DIGITAL MODELLING FOR PERFORMANCE PREDICTION
OF HYSTERESIS MOTORS

CENTRE FOR NEWFOUNDLAND STUDIES

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DIGITAL MODELLING FOR PERFORMANCE PREDICTION
OF HYSTERESIS MOTORS

by

Sidde Gowda Deve Gowda; B.E.

A Thesis submitted in partial fulfilment
of the requirements for the degree of
Master of Engineering

Faculty of Engineering and Applied Science
Memorial University of Newfoundland

December 1979

St. John's
Newfoundland, Canada
To My Parents
ABSTRACT

The performance prediction of the hysteresis motor depends largely on the success of optimized representation of the actual B-H loop of its rotor hysteresis material. Digital simulation of the typical hysteresis materials like 17% cobalt steel, 36% cobalt steel and Oerstt-70 alloys having coercivity lying between 4 and 20 kA/m and remanent flux density lying between 0.8 and 1.3 T are carried out. The simulation is based on the modified Fröhlich's approach. Reasonably close agreement is found between the simulated and those supplied by the power magnet manufacturing company.

On the basis of parallelogram approximations analytical models of the circumferential-flux hysteresis motor have been given. The motor field equations are then solved, to predict the terminal quantities, using the digital B-H loop modelling. The air-gap power of the hysteresis motor is studied as a function of coercive force, remanent flux density, saturated relative permeability and unsaturated relative permeability of the hysteresis material. A series of tests were carried out using 17% cobalt steel hysteresis rotor. The reasonably close agreement between the terminal quantities predicted from the digital simulation and those measured experimentally validates the usefulness of digital simulation of the hysteresis motor.
ACKNOWLEDGEMENTS

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GLOSSARY OF SYMBOLS

- \( r_g \): Radius at the Airgap
- \( r_h \): Radius to Centre of Hysteresis Material
- \( g \): Length of Airgap
- \( H \): Magnetic Field Intensity
- \( H_c \): Coercive Force of Hysteresis Material
- \( B \): Flux Density
- \( h' \): Radial Thickness of the Hysteresis Material
- \( N_s \): Number of Stator Turns
- \( P \): Number of Poles
- \( P_r \): Residual Flux Density in the Hysteresis Material
- \( F_{h0} \): Magnetic Potential Drop in The Hysteresis Material
- \( F \): General Symbol for Magnetic Potential
- \( F_{\theta} \): Magnetomotive Force at Angle \( \theta \)
- \( F_{0\theta} \): Magnetic Potential Drop Across \( R_0 \)
- \( F_{p\theta} \): Magnetic Potential Drop Across \( R_p \)
- \( B_c \): Peak Magnetic Flux Density
- \( \Delta F \): Incremental Change in Magnetic Potential
- \( B_h \): Radial Flux Density in Hysteresis Material
- \( H_h \): Magnetic Field Intensity in The Hysteresis Material
- \( l \): Axial Length of Rotor
- \( L_g \): Airgap Inductance
- \( E_p \): R.M.S Voltage of Non-linear Element
- \( E_g \): Airgap Voltage
- \( E_a \): Supply Voltage to The Phase 'a'
\( R_p \) : Incremental Reluctance Derived From \( R_a \) and \( R_0 \).

\( F_q \) : Maximum Force of \( F_q \).

\( \theta \) : Radian Angle.

\( \omega \) : Stator Angular Frequency.

\( \mu_{rs} \) : Saturated Relative Permeability.

\( \mu_{r0} \) : Unsaturated Relative Permeability.

\( \mu_0 \) : Permeability of Free Space.

\( m \) : Number of Phases.

\( s \) : Slip.

\( R_g \) : Airgap Reluctance.

\( R_0 \) : Unsaturated Incremental Reluctance.

\( R_a \) : Stator Resistance.

\( \phi_\theta \) : Flux at Angle \( \theta \).

\( \phi_{h\theta} \) : Flux Through the Idealized Hysteresis Element at an Angle \( \theta \).

\( L_0 \) : Inductance Dual \( R_0 \).

\( L_p \) : Inductance Dual \( R_p \).

\( R_p \) : Saturated Incremental Reluctance.

\( \phi_{p0} \) : Angular Flux Density Through Square Loop Element.

\( M_s \) : Saturation Magnetization Intensity.

\( M(a,b) \) : Critical Intensity of a Preisach-Neel's Elemental Segment (T).

\( S(a,b) \) : Distribution Function of Preisach-Neel's (T.m^2A^2).
1.1 GENERAL OUTLINE

Motor and systems designers have recently taken a long second look at the hysteresis motors' unique characteristics. Its uniform torque, low starting current, and lack of synchronising problems make this type of motor favourable to a number of modern industrial applications. Of all the unique features of a synchronous hysteresis motor, its flat speed-torque characteristics, nearly constant power factor, low noise level and simple rotor construction stand foremost.

The major starting problem which has been inherent in induction machines does not arise at all in case of hysteresis machine, as the resistance of the hysteresis rotor is ideally a direct function of the slip. The hysteresis machine can accelerate all the load that it can carry to synchronous speed irrespective of the load inertia, as it possess inherent built-in constant synchronising torque, unlike an induction machine. In comparison to induction or synchronous motors, it has almost no stability problem. The hysteresis motor is a type of synchronous motor that once was thought of limited usefulness. Development in permanent magnet materials having more hysteresis energy per unit volume like Alnico-5, Simonda 81, Orestat-70, P6-alloy, Vicaly and cobalt-steel alloys etc., have led to the production of hysteresis motors in fractional horsepower range, which has got intensive applications as drive motors in clocks, recording equipments, gyros, computer...
tape-driven and in general where constant torque, speed and quiet operation are required. Its starting and accelerating current is low in order of about 150 percent of full-load current requirement.

Apart from the above mentioned merits, it has also a number of demerits, such as low power factor, high magnetising current, and low efficiency associated with high parasitic losses in its rotor magnetic material. Therefore the use of hysteresis motors have been limited to certain special fields where efficiency might not count so much.

In recent years the development of new and improved designs of small and medium-sized brushless synchronous motors are gaining momentum [1]. In part, this is due to a changing market for synchronous a.c. drives and development in power electronics. The combination of inverter and synchronous motors has distinct advantages [2-5] over other forms of drive for applications requiring precise speed combined with smooth starting capability, constant torque and noiseless operation. For this type of applications hysteresis motors are now widely used along with others, particularly reluctance and a.c. permanent magnet types.

The foremost thing to consider seriously in the design of electrical equipment and machinery is to minimise the hysteresis and eddy current losses as they are detrimental to the efficiency and performance. However, in the case of the hysteresis machine 'hard' magnetic materials are used, which are usually not conductive to lamination. The word 'soft' and 'hard' meant low and high coercive force materials respectively.
1.2 CONSTRUCTION

A hysteresis motor has no winding on the rotor. Generally, it exhibits substantially constant torque from stand still to synchronous speed. Its special characteristics result from the hard magnetic material of which the rotor is made and simple rotor construction. The cross-section of the hysteresis machine is shown in the Fig. (2.1). Unlike other synchronous motors, the hysteresis motor has a perfectly round and symmetrical rotor. That means that the rotor has no salient poles. Hence it has no preferred position for synchronising. In its simple form, it has a conventional slotted and laminated stator with phase windings and a homogeneous cast sleeve (in the present work 17% cobalt steel is used) of permanent magnet material comprising active part of the rotor. The active sleeve is secured to the shaft over a non-magnetic support. (in the present work aluminium sleeve is used).

1.3 OPERATION

The hysteresis characteristic of its rotor magnetic material is the main cause for the development of the driving torque. Hence the name is hysteresis motor. The principle of developing its fundamental driving torque is quite simple. When an alternating voltage is impressed across the stator terminals, the alternating currents establish a rotating field in the rotor, which causes the flux density to lag behind the magnetic intensity due to the hysteresis phenomena of its magnetic material. The angle by which the flux density lags the magnetic intensity is termed as hysteresis lag angle. The phase angle between the stator magnetomotive force and the resultant airgap flux density gives rise to the driving torque. Lag angle depends only on
the area of hysteresis loop of the rotor magnet and independent of the
dependent frequency of magnetisation neglecting eddy currents.

Since the driving torque is directly proportional to the
area of the loop, the developed torque in the hysteresis motor is the
same all the way from zero to synchronous speed. At synchronous speed
the operation of the motor is accomplished exclusively by the hysteresis
torque, as the eddy current torque due to the main fundamental field is
zero. In a hysteresis motor, when the rotor is locked and the stator
field is rotating, the flux density at any point in the rotor follows
a major hysteresis loop, with a frequency equal to the stator supply
frequency. When the motor is accelerating to the synchronous speed,
the rotor field moves backward at a diminishing rate with respect to the
rotating field produced by the stator. At any one point on the rotor,
the frequency of the hysteresis loop decreases because of the decrease
of slip, until at synchronous speed, when the hysteresis cycle completely
stops. At the synchronous speed the rotor develops magnetic poles
similar to the d.c. excited synchronous motor. The magnetic potential
and flux density waves are no longer moving relative to the rotor, as
it attains its synchronism. Thus the rotor containing permanent
magnet material rotating at synchronous speed will have fundamental
waves of the flux and magnetomotive force tied to it. Each element
of the rotor ceases to operate cyclically on major B-H loop, and carries
a constant flux density.

For sinusoidal revolving m.m.f., the airgap flux density wave is
distinctly non-sinusoidal, because of the hysteresis nature of the rotor
magnetic material. This has been illustrated in the Fig. (11) for
FIG. 1.1 FLUX DISTRIBUTION AROUND HYSTERESIS ROTOR FOR SINUSOIDAL MMF WAVE.
the hysteresis machine [7]. Thus, at synchronous mode of operation the rotor does not experience any time-varying effects of harmonics present in the non-sinusoidal flux wave.

1.4 LITERATURE REVIEW

The hysteresis motor was first explored as a torque producing device by Steinmetz [9] in 1908. He was the pioneer who put a stepping stone to the world of hysteresis machine. The next major contribution was by Teare [10] in 1940, who showed a method of calculating the torque from known field configuration of magnetic flux and magnetomotive force in the hysteresis material of the rotor, on the assumption of sinusoidally revolving stator mmf. Further research was carried out by Roters [11] on the theory of development of torque from both hysteresis lag angle and from total loop energy points of view. Finally, Roters showed that parasitic losses which occur in the rotor, influenced by the local flux oscillations could be greatly reduced by designing the stator with closed slots. This opened the gate for the practical hysteresis motors in fractional horse power range.

In the analyses [10, 11] the effect of eddy currents flowing in the rotor magnetic material on the air-gap field was neglected. Because of the peculiar shape of the magnetic hysteresis loop of the rotor, there remained a major problem in predicting the equivalent magnetic or dual electric equivalent parameters of it. Hence, the foremost problem in the analysis of a hysteresis machine was the treatment of the hysteresis loop of the rotor magnetic material. The analysis of the hysteresis machine based on the actual hysteresis loop becomes almost
a prohibitive due to its property of non-linearity. Lot of research [7-14, 21-23, 25] have been undertaken to approximate the hysteresis loop so as to facilitate the formulation of equations in its analysis.

Tserra [10] was the pioneer who made such an approximation in his analysis of motor. He replaced the actual hysteresis loop by means of an inclined ellipse, which has almost the same maximum values of B and H. The elliptical representation was further extended by Roters [11] and Robertson [12] for fractional horsepower hysteresis motor.

Miyari and Kakaoka [13] extended the elliptical representation still further neglecting the space harmonics in both the stator mmf and the airgap flux density waves. They assumed the permeability of the rotor hysteresis ring to be finite and neglected the eddy currents flowing in the magnetic ring. Following Miyari and Kakaoka's, O'Kelly [14] analysed the hysteresis motor by the equivalent Kron primitive machine, in which the rotor hysteresis material was replaced by closed coils with self reactance being assumed equal to mutual reactance. However, it is claimed that the rotor reactance is better represented in parallelogram approximations developed by Copeland and Siemon [7], in the analyses of radial flux hysteresis machine. The same authors continued their research further and gave a very good account of the analysis of the circumferential-flux type rotor [8]. The flux density distribution in the machine is found [8], using a parallelogram loop approximation to the B-H characteristics of the hysteresis material. An equivalent circuit of the motor was developed.

Copeland and Siemon [7, 8] used the parallelogram method to model the hysteresis loop and predicted the rotor parameters and hence
the internally developed torque of the machine in terms of the permeabilities and hysteresis lag angle of the rotor material. Fig. (1.2) shows a parallelogram model. The parallelogram loop model was analysed further by neglecting the rotor eddy currents field effects due to space harmonics on the air-gap field at the synchronous mode of operation.

Later in 1969 Rahman, Copeland and Slemon [21] developed expressions to predict the parasitic losses in terms of the air-gap field, the stator current and rotor hysteresis material characteristics:

However the analyses were limited to synchronous mode of operation only. The general analytical models for polyphase hysteresis motor at both synchronous and subsynchronous speeds were developed [15].

Steady-state equivalent circuit models were developed using the parallelogram approximation for both synchronous and subsynchronous modes of operation. The parasitic losses associated with the rotor hysteresis material, the stator iron loss and saturation effects are best represented by suitable parameters in the general equivalent circuit model [15].

The Preisach-Neel's model has been illustrated in Appendix A.

Experiences with miniature motors gave the impressions that low efficiency and low power factor are the inherent characteristics of the hysteresis motors, making large ratings of such motors impractical. Integral horsepower hysteresis motors with improved efficiency have been built successfully [15-17]. Using "Scaling Techniques", the performances of large motors were studied [18, 19], and it was found that very encouraging results in terms of efficiency and power could be obtained for larger ratings.
FIG. 1.2 MODELLING OF HYSTERESIS LOOP BY PARALLELOGRAM
1.5 SCOPE

The scope of the present research lies in the computer simulation of B-H loop for low-coercivity permanent magnet materials that are most suitable for the rotor of a hysteresis motor, having coercivity between 0.8 and 1.3 T and to compare with the actual B-H loop available, supplied by the permanent magnet manufacturing company. Based on the computer simulation the motor field equations are solved and the terminal properties of the machine are predicted. Experiments are carried out using hysteresis rotor made of 17% cobalt steel to verify the validity of digital models.

1.6 BRIEF OUTLINE ON THE REMAINING CHAPTERS

Chapter II presents the analysis of the hysteresis machine based on Copeland and Slemion's model.

Chapter III describes the various methods of modelling B-H loop and their adoptability to study the performance of hysteresis motor.

In Chapter IV an attempt is made to simulate the B-H loop of various magnetic materials suitable for rotor of hysteresis motor.

Chapter V presents a method of representing hysteresis which includes the effect of minor loops and computer solution to find the terminal quantities of the hysteresis motor by the digital simulation method.

In Chapter VI, test results of the performance of the hysteresis motor using 17% cobalt steel rotor is given.

Conclusions of the research are presented in Chapter VII.

This concluding Chapter also contains suggestions for the work in future.
CHAPTER II

ANALYTICAL MODEL

2.1 INTRODUCTION

The aim of this chapter is to analyse the circumferential flux hysteresis machine with the help of rectangular loop approximation to the B-H characteristic of the hysteresis material. The flux density distribution is found and also the equivalent circuit is developed. Improvement over the rectangular approximation is also carried out. Two approximations are carried out for the B-H model.

2.2 THE CIRCUMFERENTIAL FLUX HYSTERESIS MACHINE

Fig. (2.1) shows a cross-section of a hysteresis machine with a circumferential-flux rotor. The stator is considered to have an m-phase 2-pole winding each phase of which has its turns sinusoidally distributed in a large number of slots. Unlike in the case of radial flux machine, the flux density after crossing the air-gap radially, must be directed circumferentially around the hysteresis material to complete its path.

The flux distribution in the hysteresis ring is shown in the Fig. (2.2), where the magnetic path is radial in the air-gap and circumferential in the hysteresis material. It is assumed that the flux distribution is uniform inside the ring and there is no flux penetration into the non-magnetic sleeve.

The turns \( N_b \) of phase 'a' are assumed to be distributed
FIG. 2.1 CROSS-SECTION OF CIRCUMFERENTIAL-FLUX HYSTERESIS MACHINE
FIG. 2.2 FLUX PATTERNS IN AIRGAP AND HYSTERESIS RING
with a conductor density of
\[ n_a = \frac{N_a}{2} \mid \sin \theta \mid \text{ conductors per radian} \quad (2.1) \]

Let the currents in phase a be expressed in the form of the following equation:
\[ i_a = \hat{i} \sin \omega t \text{ amperes} \quad (2.2) \]

and let the stator currents form a balanced polyphase set of sequence a, b, c. Consider the incremental sector of the machine shown in Fig. 2.3. Proceeding clockwise around the path shown, the magnetomotive force of the enclosed conductor is
\[ AF = -\frac{mN_6}{4} \cos (\omega t - \theta) \Delta \theta \text{ amperes} \quad (2.3) \]

Assuming that the stator iron has essentially infinite permeability, the incremental mmf of equation 2.3 is absorbed in producing the difference in air-gap flux density at \( \phi + \Delta \theta \) with respect to \( \phi \) and in magnetic field intensity \( B_{h0} \) of the hysteresis material. Thus,
\[ AF = \frac{B_{h0}}{\mu_0} \left[ B_{G}(\phi + \Delta \theta) - B_{G} \right] \Delta \theta \text{ amperes} \quad (2.4) \]

Equations 2.3 and 2.4 may be combined to give the differential equation
\[ \frac{B_{h0}}{\mu_0} \frac{dB_{h0}}{d\theta} = H_{h0} \hat{r} \hat{f} \frac{mN_6 I}{4} \cos (\omega t - \theta) \quad (2.5) \]

At this stage, it is assumed that the properties of the hysteresis material can be represented by the ideal rectangular loop as shown in Fig. (2.4). In this idealization, the flux density \( B_h \) in the hysteresis material can increase only with the field intensity \( H_h \) is equal to the coercive force \( H_c \) and can decrease only when
FIG. 2.3  INCREMENTAL SECTION OF MACHINE, SHOWING
POSITIVE DIRECTION OF FLUX DENSITY AND
FIELD INTENSITY VECTORS
FIG. 2.4 IDEALIZED MAGNETIZATION CHARACTERISTIC FOR HYSTERESIS MATERIAL
$H_B = -H_c$. Let $K$ be a measure of the stator current in per unit of the stator current required to produce coercive force:

$$K = \frac{m N_s I}{4 \pi H_c B_c} \quad \ldots \quad 2.6$$

For $K > 1$, there will be a part of the hysteresis material for which $H_{th} = H_c$. For this part, equation 2.5 can be written as

$$\frac{dB_{th}}{d\theta} = \frac{H_c B_c}{8} \left[ 1 - K \cos (\omega t - \theta) \right] \quad \ldots \quad 2.7$$

This equation has a solution of the form

$$H_{th} = \frac{H_c B_c}{8} \left[ \theta + K \sin(\omega t - \theta) + C \right] \text{webers per meter}^2 \quad 2.8$$

where $C$ is a constant.

The range of application of equation (2.8) and the value of the constant $C$ must now be determined. Since positive coercive force exists over this region, it follows that the flux density in the material encompassed is increasing with time:

$$\frac{dB_{th}}{dt} > 0 \quad \ldots \quad 2.9$$

Throughout this range, the operating point for material is on the right-hand vertical side of the characteristic of Fig. (2.4). The magnetic field in the machine rotates clockwise in Fig. (2.1) with respect to the rotor at subsynchronous rotor speeds. The flux density $B_{th}$ in the material, therefore, will reach a maximum positive value at that point where the rate of application of m.f. is reduced to the value which is just sufficient to maintain coercive force.

From equations (2.5) and (2.6), this particular condition occurs
at the following position:
\[
a = \sqrt{1 - \cos^2 \theta}
\]  
(2.10)

For the continuity of flux in the machine, the air-gap flux per unit of angle \( \theta \) must equal the rate of change of flux with \( \theta \) in the hysteresis material. Thus the following relationship is established:
\[
\frac{dB_{h\theta}}{d\theta} = r_{e}B_{g0} 
\]  
(2.11)

Since \( \frac{dB_{h\theta}}{d\theta} \geq 0 \) over the range of equation (2.8) and since \( B_{h\theta} \) is a function of \((\omega t - \theta)\), the result is
\[
\frac{dB_{h\theta}}{d\theta} \leq 0.
\]
From equation 2.11, the flux density \( B_{g0} \) is, therefore, negative. At \( \omega t - \theta = \alpha \), then \( B_{g0} \) must be zero. Equation (2.8), consequently becomes:
\[
B_{g0} = \frac{\mu_{0}r_{e}H_{c}}{g} \left[ \delta + K \sin (\omega t - \theta) - (\omega t - \alpha) - K \sin \alpha \right] 
\]  
(2.12)

The other limit of range of application for equation (2.12) occurs at \((\omega t - \theta) = \delta\)

where \( B_{g0} \) again reaches zero. This value may be determined by iterative solution of equation (2.12) equated to zero:
\[
K \sin \delta - \delta = \sqrt{K - 1 - \cos \delta + \frac{1}{K}} 
\]  
(2.13)
for values of \( K \).

Because of the symmetry of the machine, there is a similar range in which \( B_{h\theta} = -H_{c} \), and
\[
B_{g0} = \frac{\mu_{0}r_{e}H_{c}}{g} \left[ -\delta + K \sin (\omega t - \theta) + (\omega t - \alpha - \pi) + K \sin \alpha \right] 
\]  
(2.14)

Fig. (2.5) shows \( B_{g0} \) as a function of \( \theta \) at \( t = 0 \) for one specific value of \( K \). Between the two parts of the solution given by equations (2.12) and (2.14), the air-gap flux density...
FIG. 2.5  AIR GAP FLUX DENSITY AS A FUNCTION OF $\theta$ AT $t=0$ AND $K<1.862$
is equal to zero.

Equations (2.12) and (2.14) give the solution for values of \( K \) from 1.0 to the value for which \( \delta - \alpha = \pi \). Substituting into equations (2.13) and (2.10) shows that this condition obtains at \( K = K_c \) where

\[
K_c = \left( \frac{\pi}{2} + 1 \right)^{1/2} = 1.862 \quad \cdots \cdots \cdots (2.15)
\]

At this value of \( K \), \( \alpha = 57.5 \) degrees.

For operation with \( K > K_c \), the solution of equation (2.8) still applies for the half of the machine in which positive coercive force exists. The constant \( C \) must, however, be re-evaluated by setting \( B_{00} = 0 \) in equation (2.8) for \( \theta = \omega t - \alpha \) and \( \theta = \omega t - \alpha + \pi \).

Solution of two resultant equations gives

\[
\alpha = \sin^{-1} \left( \frac{\pi}{2K} \right) \quad \cdots \cdots \cdots \cdots (2.16)
\]

and

\[
C = -\omega t - \frac{\pi}{2} + \sin^{-1} \frac{\pi}{2K} \quad \cdots \cdots \cdots (2.17)
\]

The constant for the region of negative coercive force may be evaluated in a similar manner.

To facilitate determination of the flux linkage of the stator winding, let the air-gap flux density \( B_{00} \) be expressed as a Fourier series in \( \sigma \), where

\[
\sigma = \omega t - \theta \quad \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots (2.18)
\]

For the range \( 0 \leq K < 1.862 \), using equations (2.12) and (2.18) it can be shown that

\[
B_{00} = \frac{2}{\pi} \frac{\mu_s r H_c}{g} \left\{ \int_{\alpha}^{\beta} \left[ -\sigma + K \sin \sigma + \cos^{-1} \frac{11}{K^2 - 1} \right] \sin \sigma \sin \delta \pi \frac{\sin \theta}{\sin \delta} \right\} \sin \sigma
\]
\[
\begin{align*}
&= 2 \left( \frac{\mu_0 R^2 H_c}{4 \pi} \right) \left\{ \left[ \frac{2}{2} \left[ -\frac{\sqrt{K^2 - 1}}{K} - 1 \right] \times \frac{K}{1} + 2 \sin \delta + 2 \cos \delta \left( \frac{K \sin \delta}{2} - \delta a - \sqrt{K^2 - 1} \right) \right] \sin \sigma \\
&\quad + \frac{1}{2} \left[ \right. \left. -2 \sin \delta \left( \delta + \sqrt{K^2 - 1} \right) - 2 \cos \delta \cos 2\delta + \frac{K}{2} \cos 2a + 2K \right] \cos \sigma \right\} + \\
&\quad + \text{higher odd harmonic terms}
\end{align*}
\]

Neglecting the higher harmonic terms, the R.H.S. of the above equation is of the type

\[ A \cos \sigma + B \sin \sigma = J \sin \beta \]

Where

\[
A = \frac{1}{\pi} \frac{\mu_0 R^2 H_c}{g} \left[ -2 \sin \delta \left( \delta - a + \sqrt{K^2 - 1} \right) - 2 \cos \delta - \frac{K}{2} \left( \cos 2\delta - \cos 2a \right) + 2K \right]
\]

\[
B = \frac{1}{\pi} \frac{\mu_0 R^2 H_c}{g} \left[ -\frac{\sqrt{K^2 - 1}}{K} \left( \delta - a \right) + 2 \sin \delta + 2 \cos \delta \left( \frac{K \sin \delta}{2} - \delta a - \sqrt{K^2 - 1} \right) \right]
\]

Therefore

\[ J = \sqrt{A^2 + B^2} \]

\[ \beta = \tan^{-1} \left( \frac{B}{A} \right) \]
2:3 EQUIVALENT CIRCUIT OF IDEALISED CIRCUMFERENTIAL-FLUX MACHINE

The flux linkage $\lambda_a$ of phase a of the stator winding due to the air-gap flux may be determined by first finding the flux linkage of a single turn with sides $\theta$ and $\theta + \pi$,

$$\lambda_a = \int_{\theta}^{\theta+\pi} E_{g0} r g d\theta$$

$$= -2\mu_0 r_h H_{c r g} L_j \frac{\cos (\omega t - \theta - \beta)}{s} \quad \ldots \quad (2.20)$$

+ higher odd harmonic terms, using equation (2.20) to define $E_{g0}$.

The axial length of the rotor is 1. The air-gap flux linkage of the complete stator winding of phase a is then found to be, using equation (2.1):

$$\lambda_a = \int_{0}^{\pi} N_s \sin \theta \lambda_0 d\theta$$

$$= \frac{N_s}{2} \frac{H_{c r g} L_j}{s} \sin (\omega t - \beta) \text{ webers} \quad \ldots \quad (2.21)$$

This flux linkage has no time harmonic terms. Following equations (7) and (27) Ref. (7), let the inductance corresponding to the air-gap reluctance be

$$L_g = \frac{\mu_0 N_s^2 r_g L_j}{8} \text{ Henrys} \quad \ldots \quad (2.22)$$

Let the effective value of current in phase a's $I_a$

Then the effective value of the voltage induced in the stator winding of phase a's.
\[ E_p = \frac{\lambda a}{\sqrt{2}} \quad \ldots \quad (2.23) \]

Combining equations (2.21), (2.22) and (2.23) gives

\[ E_p = \omega L_{g} \left( \frac{4\pi H_{C}}{2\mu N_{s}} \right) \sqrt{\frac{Z}{2}} \text{ Volts (rms)} \quad \ldots \quad (2.24) \]

The similarity between this equation (2.24) and equation (39) of Ref. (7) demonstrates that this circumferential-flux machine can be represented by a simple electrical equivalent circuit of the form shown in Fig. (2.6).

Here the stator leakage inductance \( L_{L_s} \) and the stator resistance \( R_s \) have been added to extend the circuit to the terminals of the machine.

2.4 IMPROVED MODEL FOR B-H LOOP

The equivalent circuit for circumferential-flux machine shown in Fig. (2.6) is simpler than that developed for the radial-flux machine in Fig. (10) of Ref. (7). The reason for this is that the B-H loop of Fig. (2.4) assumes infinite unsaturated permeability and zero saturated permeability for the hysteresis material. A better model for a practical B-H loop would be that of Fig. (2.7) in which the unsaturated relative permeability \( \mu_{r0} \) is finite and the saturated relative permeability \( \mu_{rs} \) is greater than unity.

The relationship between the flux densities in the air gap and in the hysteresis material is given in equation (2.11). Differentiating the eqn. (2.11) with respect to \( \theta \) and combining the result with equation (2.5) gives...
FIG. 2.6  EQUIVALENT CIRCUIT FOR CIRCUMFERENTIAL-FLUX MACHINE, BASED ON B-H LOOP OF FIG. 2.4
where the relationship between the circumferentially directed \( B'_{h0} \) and \( H'_{h0} \) is shown in Fig. (2.7). Following a procedure similar to that demonstrated in Fig. (4) of Ref. [7], the relationship of \( B'_{h} \) and \( H'_{h} \) in Fig. (2.7) can be related to the \( B_{h}/H_{h} \) characteristic of Fig. (2.4) by the equations

\[
B'_{h} = B_{h} + \frac{\mu_{p} \mu_{0} H_{h}}{g} \quad \text{webers per meter} \quad (2.26)
\]

and

\[
B'_{h} = B_{h} + \frac{\mu_{p} \mu_{0} H_{h}}{g} \quad \text{webers per meter} \quad (2.27)
\]

where

\[
\mu_{p} = \frac{\mu \rho \omega}{\sigma} \quad (2.28)
\]

To conform with the analysis of the Ref. [7], let

\[
\phi'_{h0} = h_{0} B'_{h0} \quad \text{webers} \quad (2.29)
\]

and

\[
\phi'_{h0} = h_{0} B'_{h0} \quad \text{webers} \quad (2.30)
\]

Inserting equations (2.26), (2.27), (2.29) and (2.30) into equation (2.25) gives

\[
R_{g} \frac{d^{2}\phi'_{h0}}{d\theta^{2}} = R_{0} \phi'_{h0} + R_{p} \left( \phi'_{h0} - \phi_{h0} \right) - \frac{\omega n_{A}}{4} \quad \cos (\omega t - \theta) \quad (2.31)
\]
FIG. 2.7 IDEALIZED MAGNETIZATION CHARACTERISTIC WITH FINITE UNSATURATED PERMEABILITY AND SATURATED PERMEABILITY > 1.0
where the air-gap reluctance \( R_g \), the incremental unsaturated reluctance \( R_0 \), and the effective incremental saturated reluctance \( R_p \) of the hysteresis material per unit angle are given by

\[
R_g = \frac{g}{\mu_0 r g l} \quad \text{amperes per weber-radian} \quad (2.32)
\]

\[
R_0 = \frac{\xi_h}{\mu_0 \mu_0 h l} \quad \text{amperes per weber-radian} \quad (2.33)
\]

\[
R_p = \frac{\xi_h}{\mu_0 \mu_0 h l} \quad \text{amperes per weber-radian} \quad (2.34)
\]

Equation (2.31) is of second order and is nonlinear. This may be represented by the equivalent magnetic circuit shown in the Fig. (2.8).

For specific values of the reluctances \( R_g \), \( R_p \) and \( R_0 \) and for a given value of current \( I \), a set of solutions can be obtained; but this set of solutions is too cumbersome to evaluate and plot for ranges of values of all parameters. By representing the element \(-R_g \frac{d^2 q \phi}{d\theta^2}\) of equation (2.31) by a simple reluctance, thus making equation (2.31) a linear algebraic one, a simple but approximate solution could be developed. If the distribution of flux \( \phi \) with respect to \( \theta \) is sufficiently close to sinusoidal in form, a double differentiation with respect to \( \theta \) would cause a shift of a half period in the wave or a reversal of sign. The element then could be represented by the air-gap reluctance \( R_g \), that is

\[
-R_g \frac{d^2 q \phi}{d\theta^2} = R_g \phi \quad (2.35)
\]
FIG. 2.8 EQUIVALENT MAGNETIC CIRCUIT FOR INCREMENTAL WEDGE OF CIRCUMFERENTIAL-FUX MACHINE

\[ \frac{mN_0^2 I}{4} \cos(\omega t - \theta) \]

\[ R_g \left( \frac{d^2 \phi_{h\theta}}{d\theta^2} \right) = R_g \phi_{h\theta}' \]

\[ \frac{\phi'_{h\theta} - \phi_{h\theta}}{R_p} = \gamma_h H_{h\theta} \]

\[ R_0 \phi_{h\theta}' \]
This is a very big approximation. Then the equivalent circuit of Fig. (2.8) would contain only reluctance, a source mmf, and the ideal rectangular-loop hysteresis element. In this analysis, Thévenin's theorem is applied at the terminals of the ideal hysteresis element in Fig. (2.8) to represent the remainder of the circuit by a mmf ($F_q$) per unit angle, in series with a reluctance ($R_q$) per unit angle, where

$$F_q = \left( \frac{R_p}{R_p + R_q + R_g} \right) m N A t^2 \cos (\omega t - \theta) \text{ amperes} \ldots \ldots (2.36)$$

$$R_q = \frac{R_p (R_g + R_q)}{R_p + R_q + R_g} \text{ amperes per weber-radian} \ldots \ldots (2.37)$$

The resultant single-loop magnetic circuit is analyzed in Ref. [7] and, after Fourier analysis, the fundamental component of flux per unit angle in the ideal rectangular-loop element is given by

$$\phi_{p5} = \frac{\tau H_c}{R_q} J \cos (\omega t - \theta + \delta) \ldots \ldots \ldots (2.38)$$

where factor $J$ and angle $\delta$ for this first approximate solution are plotted as a function of ratio $K$, as defined in equation 2.36, in Fig. (2.9).

The dual electric equivalent circuit of Fig. (2.8) is shown in the Fig. (2.10), wherein the ideal hysteresis element is represented by the source voltage $E_p$. 
FIG. 2.9 - COMPARISON OF FACTOR K AND ANGLES FROM FIRST AND SECOND APPROXIMATE ANALYSIS OF FIG. 2.7 WITH THAT OBTAINED IN EXACT ANALYSIS OF LOOP OF FIG. 2.4.
FIG. 2.10  EQUIVALENT CIRCUIT OF CIRCUMFERENTIAL FLUX MACHINE
BASED ON B-H LOOP OF FIG. 2.7
where

\[ E_p = \omega L_q \left( \frac{4\pi H_c}{\sqrt{2mN_s}} \right)^{J/\Phi} \text{ volts (rms)} \ldots (2.39) \]

\[ L_q = \frac{mvN_s^2}{8R_q} \text{ Henry} \ldots \ldots \ldots \ldots (2.40) \]

substituting the value of \( R_q \) from equation (2.37) in equation (2.40),

\[ L_q = \frac{mvN_s^2 (R_p + R_0 + R_g)}{8R_p (R_0 + R_g)} \text{ Henry} \ldots \ldots (2.41) \]

defines the inductances in Fig. (2.10) are related to their corresponding reluctances in Fig. (2.8), by equation similar to (2.41).

Factors \( J \) and angle \( \beta \) for the exact first-approximation analyses should be identical when \( \mu_r = 0 \), and \( \mu_r = \infty \) in Fig. (2.7), leaving only the air-gap reluctance in Fig. (2.8). In Fig. (2.9), the approximate analysis gives values of \( J \) which are high for \( K<K_c \) and low for \( K>K_c \) while the values of \( \beta \) are low for all values of \( K \). This approximate analysis is, however, close enough to be useful. For machines in which the air-gap reluctance is not the dominant element in the magnetic circuit of Fig. (2.8), this first approximate analysis should be quite accurate. For machines in which the air-gap reluctance is the dominant quantity, the exact analysis developed at the beginning of this chapter gives the best accuracy.

2.5 SECOND METHOD OF APPROXIMATE ANALYSIS

The values of \( K, J \) and \( \beta \) for the second approximate solution are shown in the Fig. (2.9), together with those of the first
approximate solution and the exact solution for the simple B-H loop. The results of the second approximate solution are quite close to that of the exact solution for large values of $K$. With a high current in the stator giving a large value of $K$, the mmf drop in the reluctance, particularly the air gap, greatly exceeds that required for coercive force in the hysteresis material. Under this condition, the flux is expected to be nearly sinusoidally distributed in space and time, and the approximation on which the analysis is based is reasonably accurate.

From another viewpoint, the exact solution showed that, for $K>1.862$, the rotor material spends all its time on the sides of the B-H loop, making an instantaneous jump across the top and bottom of the loop at the transition angles $a$ and $a+x$. Equations of type (2.8) then apply throughout the air gap. For large values of $K$, this solution approaches a sinusoid. From equation (2.11), the circumferential flux, being differentially related to the air-gap flux, will rapidly approach a sinusoid because of the large values of $K$.

2.6 TORQUE IN THE MACHINE

The torque of the machine is equal to the power crossing the airgap per unit of angular velocity of the rotating field.

Using the equation (2.36), (2.39) and (2.41), the torque for a 2-pole $m$-phase machine is
The function \( JK \sin \beta \) is plotted in Fig. (2.11) for 

- the exact analysis of the simple B-H loop model,
- the first approximate analysis.

The expression of the torque given by the equation 

\[
T = \frac{mL_a}{Z_m N_b} \left( \frac{4r_h H_c}{Z_m N_b} \right)^2 \quad \text{JK \sin \beta \ Newton-meters} \quad (2.42)
\]

is however valid for parallelogram model neglecting the parasitic losses at synchronous mode. Detailed derivation of sub-synchronous phasor representation including both mmf and flux parasitic losses are given in Ref. [15]. Fig. (17) of Ref. [7] shows the predicted characteristic together with the measured curve of maximum torque near synchronism as a function of stator current. The rounding of the experimental curve is explained by the corresponding rounding of the B-H loop, particularly near the saturation.

In the analytical approach of developing the necessary equations to determine the motor terminal quantities such as voltage, current and power factor, the non-linear characteristic of B-H loop is simplified utilising the linear properties. The precise consideration of minor loops also becomes a difficult task. This gives rise to erroneous terminal quantities of the hysteresis motor. Also it is difficult to represent the B-H loop, maintaining all the basic qualities of it by any analytical means.

The torque produced in a hysteresis motor is proportional to the actual area of the hysteresis loop. Therefore, the best method to accommodate the the hysteresis loops taking into consideration...
FIG. 2.11 TORQUE FUNCTION $JK \sin \beta$ FOR VARIOUS METHODS OF ANALYSIS
Saturation is perhaps by modelling of B-H loop by discrete digital points. Hence the modelling of the B-H loop using digital techniques is the next best alternative.
CHAPTER III

MODELLING OF B-H LOOP

3.1. INTRODUCTION

It is a well known fact that the analysis of the hysteresis machine depends how we represent the actual B-H loop, keeping its basic properties similar to that of the original one. The analysis of a circuit having non-linear element is a difficult task. To represent the B-H loop and to find the motor dimensions, in terms of the properties of the rotor magnetic material, it is very essential to represent it in a suitable way. The analysis of the machine becomes impossible if one sticks to the actual B-H loop. Analyses of the motor behaviour have been made using ellipse and parallelogram to represent the hysteresis non-linearity. Copeland and Slenon [7, 8] use the field parallelogram approach, Robertson and Zaky [12] use the field ellipse approach, and O'Kelly [24] uses the circuit ellipse method. It is necessary to know the characteristic curves of the ring material and also to be able to represent them in some way.

Poritsky and Butler [26] consider the non-linear relationship of the B-H curve to find the mathematical interpretation. The first attempt was made to represent a certain portion of the loop by an empirical formula, e.g., \( B = B_s (1 - a (H_s H_c) \). In addition, Gillot and Abrams [27], Bullingham, Bernol [28] Zakrazewski and Pietras [29] have also suggested the static B-H loop representation.
FIG. 3.1  CHARACTERISTIC CURVE OF HYSTERESIS RING MATERIAL.
In addition, the suggestions given by Fisher and Moser [30], Davis [31], Trutt, Erdelyi and Hopkins [32], and Widger [33], are also of high importance in the static B-H loop representation.

The approximate theory for calculating the torque of the hysteresis motor was first developed by Teare [10]. In hysteresis machine, full representation of B-H loop is very essential. In the uniform rotating field, the B-H relationship for components is an inclined ellipse. Because of this, Teare assumed an elliptical model for the hysteresis loop. The equivalent ellipse is chosen so that the area and the maximum value of B are the same as those of the corresponding loop. Thus the hysteresis loop is replaced by an elliptical model in his analysis of the hysteresis motor. The elliptical representation was further extended by Roters [11], Miyairi [13], and Robertson [12] for the fractional hp. synchronous hysteresis motor. O’Kelly [14] further extended the elliptical representation of hysteresis loop, and analysed the hysteresis motor accordingly by replacing the rotor material by closed coils.

In case of elliptical representation, the B-H loop is modified to an elliptical shape. By this method of representation, the higher remanent flux density which is required for the higher starting torque is possible. Thus, in this method the area of the hysteresis loop is almost replaced by the same area as that of the B-H loop. The area of the loop, can easily be found which is proportional to the torque produced in the rotor material. Preisach-Neél’s model has been used by J. Peard and M. Polojadoff in the study of the performance of hysteresis motor. This model seems to work very well for particular type of
hysteresis material. However, it is found that the Prölich approach gives a better B-H loop for most of the hysteresis materials.

3.2 BASIC ELLIPTICAL MODEL

Considering the concept of complex permeability the B-H relation can be defined as

\[ B = \mu R e^{j\theta} \]

where \( \theta \) is known as the hysteresis angle. If the magnetising field is \( H_0 \sin \omega t \), then

\[ B = \mu_0 H_0 \sin (\omega t - \theta) \]

Eliminating the sinusoidal time function the equation of the elliptical hysteresis is:

\[ \frac{H}{(H_0 \sin \theta)^2} + \frac{B}{(\mu R H_0 \sin \theta)^2} = \frac{2BH_0 \cos \theta}{\mu R (H_0 \sin \theta)^2} = 1 \]  \tag{3.1}

The semi-major axis, semi-minor axis and the angle \( \tau \) shown in the Fig. (3.2) are approximately \( \mu R H_0 \), \( H_0 \sin \theta \) and \( \cos \theta / \mu R \) respectively. Therefore the area of the loop is approximately \( \mu R H_0^2 \sin \theta \), where \( \mu R \) is determined from the static B-H loop as follows:

\[ \mu R = \frac{B_0}{H_0} \]  \tag{3.2}

The hysteresis angle is obtained by equating the area of static B-H loop to that of the area of the ellipse

\[ \pi \mu R H_0^2 \sin \theta = \text{Area of the actual loop} \]

Taking into consideration the elliptical approximations, the complex permeability \([34, 24]\) of the rotor hysteresis material is given by

\[ \mu = \mu R e^{j\beta} \]

such that the magnetic field intensity \( H \) is written as

\[ H = E_0 \left( \frac{H}{H_0} e^{-j\omega t} \right) \]  \tag{3.3}
FIG. 3.2  ELLIPTICAL REPRESENTATION OF B-H LOOP
The corresponding magnetic flux density $B$ is given as

$$B = R_0 \left( B_0 e^{j(\omega t - \theta)} \right) \quad \ldots \ldots \quad (3.4)$$

The rotor hysteresis material's relative permeability $\mu_r$ and hysteresis lag angle $\theta$ can be easily obtained from the given hysteresis loop of the material.

### 3.3 Frölich's Model

In Fig. (3.3) the major $B-H$ loop is essentially divided into four portions and each portion is represented by Frölich's curve, with centers are at $c$ and $f$ ($H = \pm H_0, B=0$). The $abcd$ portion of the loop as shown in Fig. (3.3) is opposite in sign to that of defa. Therefore, if the upper portion of the loop can be represented by some formula, then lower portion can be found automatically. The portion $gbc$ is represented as follows,

$$B_{gbc} = \frac{H + H_{cc}}{E + F \left( H + H_{cc} \right)} \quad \ldots \ldots \quad (3.5)$$

A Frölich curve is used in order to obtain the actual curve i.e., abc,

$$B_{abc} = \frac{(H + H_{cc})}{E + F \left( H + H_{cc} \right)} + CH \quad \ldots \ldots \quad (3.6)$$

If $H < 0$, then $C = 0$.

The portion of the loop $cd$ is represented by
\[ B_{\text{cd}} = \frac{(H + H_{c})}{E - EE(H + H_{c})} \] ............ (3.7')

Similarly,
\[ B_{\text{def}} = \frac{(H - H_{cc})}{E - F(H - H_{cc})} + GH \] ............ (3.8')

and:
\[ B_{\text{ef}} = \frac{(H - H_{c})}{EE + FE(H - H_{c})} \] ............ (3.9')

In the above equations (3.5 - 3.9), the values of \( E, F, EE, FF \), are all constants, and they are obtained by plotting the reciprocals of \( B \) and \( H \), which give a straight line. The slope of the line is \( E \) or \( EE \) and the intercept of the line on the Y-axis is \( F \) or \( FF \), depending on the portion of the loop.

The equation of the straight line representing gbc portion of the curve is:
\[ \frac{1}{B} = \frac{E}{H} + F \] ............ (3.10')

Similarly for the portion ed it is
\[ \frac{1}{B'} = \frac{EE}{H} + FF \] ............ (3.11')

The \( B-H \) relationship in the material is no longer described by the \( B-H \) loop for the surface, when the amplitude of the applied magnetic field begins to decrease within the material. But it is described by one of the minor loops of Fig. (3.3).
The method by which any of these is determined, using the known data describing the major saturated hysteresis loop is described as below.

The portion of the major loop fits well with the similar portion of the minor loop by changing the centre \((H = H_c, B = 0)\). That is by changing the value of coercive force \((H_c)\) of the major \(B-H\) loop to the value of the coercive force corresponding to the minor loop.

Equations (3.6) and (3.7) are modified as follows, in order to calculate the value of flux density \((B)\) for the minor loops,

\[
B_{abc} = \frac{(H + H_c)}{E + F} + CH - (B_{r_{max}} - B_r). \quad (3.12)
\]

if \(H < 0\), \(G = 0\)

\[
B_{cd} = \frac{(H + H_c)}{EH - EF} \quad \ldots \ldots \ldots \quad (3.13)
\]

Similarly equations (3.8) and (3.9) are changed to

\[
B_{def} = \frac{(H - H_c)}{E - F} + CH + (B_{r_{max}} - B_r). \quad (3.14)
\]

if \(H > 0\), then \(G = 0\)

\[
B_{fg} = \frac{(H - H_c)}{EH + EF} \quad \ldots \ldots \ldots \quad (3.15)
\]

The \(H_c\) and \(B_{r_{max}}\) are the values of coercive force and residual magnetisation respectively of the loop from which the values of flux density are calculated for the loop nested within it.
3.4 PARALLELGRAM MODEL

Copeland and Slemon introduced the parallelogram model in the analysis of hysteresis motor. They predicted the fundamental developed torque in terms of machine dimensions and hysteresis material characteristics, namely the permeabilities and hysteresis lag angle. The parallelogram modelling has been carried out, considering the width of it is equal to twice the coercive force of the material. It is however, claimed that the rotor reactance is represented in a better way, compared to elliptical model.

In the hysteresis material the magnetic flux per unit angle $\phi_0$ is related to the magnetic potential $E_{BH}$ across the material by the idealized characteristic of Fig. (3.5). This is derived from the B-H characteristic of the rotor hysteresis material Fig. (3.4) and is linearly related to it by

$$\frac{\phi_0}{B_H} = \eta_H^1 \text{ meters}^2 \ldots \ldots \ldots (3.16)$$

and

$$\frac{E_{BH}}{B_H} = h \text{ meters} \ldots \ldots \ldots \ldots \ldots (3.17)$$

Any flux excursion on the Fig. (3.5) is governed by a straight-line relation of the $y = mx + c$, where $\frac{1}{R_q}$ is the slope if the state-point is on the left or right hand side of the loop, and $\frac{1}{R_q}$ if the state-point is within the outer boundaries of the loop. These incremental reluctances per unit angle are given by expressions
Fig. 3.4 Idealized Magnetization Characteristic for Hysteresis Material.
FIG. 3.5  FLUX PER UNIT ANGLE VS MAGNETIC POTENTIAL FOR HYSTERESIS MATERIAL IN MACHINE
\[ R_0 = \frac{h}{\mu_{kr} \mu_0 R_b} \text{ amperes per weber radian} \ldots \quad (3.18) \]

\[ R'_0 = \frac{h}{\mu_{kr} \mu_0 R_b} \text{ amperes per weber radian} \ldots \quad (3.19) \]

This characteristic may be represented as a linear reluctance per unit angle \( R_0 \) in series with a vertically sided \( \phi/F \) characteristic as shown in Fig. (3.6a) and (3.6b). The linear reluctance \( R_0 \) of Fig. (3.6a) has an effect which is equivalent to an extension of the air gap. The new loop has vertical sides, i.e., zero reluctance. Because of the subtraction of \( \frac{1}{R_0} \) everywhere from the slope of the loop, any flux excursion within the outer boundaries of the new loop will occur with a slope \( \frac{1}{R_p} \) where

\[ R_p = R_0 - R_0 \text{ amperes per weber radian} \ldots \quad (3.20) \]

The nonlinear characteristic of Fig. (3.6b) can be further simplified by representing it as a linear reluctance per unit angle \( R_p \) in parallel with a rectangular loop nonlinear element as shown in Fig. (3.6c) and (3.6d).
FIG. 3.6a RELUCTANCE PER UNIT ANGLE \( R_0 \)

FIG. 3.6b ORIGINAL CHARACTERISTIC OF FIG. 3.5 WITH SERIES RELUCTANCE \( R_0 \) SUBTRACTED
FIG. 3.6c  RELUCTANCE PER UNIT ANGLE $R_0$

FIG. 3.6d  CHARACTERISTIC OF FIG. 3.6b WITH PARALLEL RELUCTANCE $R_0$ SUBTRACTED
CHAPTER IV

DIGITAL SIMULATION OF B-H LOOP

4.1 INTRODUCTION

In the preceding chapter different ways of graphical and analytical representation of B-H loops have been introduced. Consideration of idealised B-H characteristics enables the desirable properties or 'goodness' of the rotor material of a hysteresis machine to be assessed. Type of B-H loop approximations to be made depends upon the magnetic properties of the rotor magnetic materials.

It has been learnt from the fundamental principle of hysteresis machine, that the torque produced is directly proportional to the area of the B-H Loop of the rotor magnetic material. Thus the entire performance of the machine depends primarily on the optimized representation of the actual B-H loop of the material.

A number of attempts were made by several investigators [22, 35, 36, 37] to develop a computer programme in order to simulate B-H loop of non-linear elements. In 1970, J.S. Everatt developed an algorithm to simulate the B-H loop of any non-linear element based on the modified Prülich's approach, given in the next section.

The digital simulation of hysteresis loop plays a major role, in predicting the behaviour of hysteresis motor by the use of modern digital computers available. In terms of machine dimensions,
winding data, and in terms of the properties of the rotor magnetic material.

4.2 MODIFIED FRÖLICH'S MODEL

By using the more successful mathematical representation derived from Lamont's law, which states that the permeability of the magnetic material is proportional to its degree of magnetisation,

\[ \mu = K (B_g - B) \]  \hspace{1cm} (4.1)

where K is a constant.

Since \( \mu = \frac{B}{H} \)

\[ \frac{B}{H} = K B_g - KB \]  \hspace{1cm} (4.2)

\[ B = \frac{K B_g}{1 + KH} = \frac{H}{B + c H} \]  \hspace{1cm} (4.3)

where b and c are constants, c being \( 1/B_g \).

In applying these relations to the permanent magnet demagnetisation curve it is necessary to displace the curve by the amount of coercive force \( H_c \). The equation for a demagnetisation curve then becomes:

\[ B = \frac{H + H_c}{b + c(H + H_c)} \]  \hspace{1cm} (4.4)

when \( H = 0 \), \( B = B_r \),

and \( B_r = \frac{H_c}{b + c H_c} \)  \hspace{1cm} (4.5)

\[ b = \frac{H_c}{B_r} \frac{H_c}{B_g} \]  \hspace{1cm} (4.6)

or since \( c = \frac{1}{B_g} \)
\[ B = \frac{H + H_c}{(H_c/B_r) + (H/B_m)} \]  \hspace{1cm} (4.7)

To find the value of \( B_m \) which makes \( BH \) a maximum, the above expression is multiplied by \( H \) and differentiated and equated to zero.

\[ BH = \frac{H^2 + HH_c}{b + c(H + H_c)} \]  \hspace{1cm} (4.8)

\[
\frac{d(BH)}{dH} = \frac{[b + c(H + H_c)] [(2H + H_c) - (H^2 + H_c)c]}{(b + c(H + H_c))^2} = 0 \hspace{1cm} (4.9)
\]

Solving this expression yields the following expressions for \( B_m \) and \( H_m \):

\[ B_m = \frac{(1/c)(\sqrt{1 - cB_r} - 1)}{} \]  \hspace{1cm} (4.10)

\[ H_m = \frac{H_c/cB_r}{\sqrt{1 - cB_r} - 1} \]  \hspace{1cm} (4.11)

In a similar way, the expressions for the remaining quadrants of the loop can be developed. Thus, the simulation of the complete hysteresis loop is made by analytical expressions.

Everatt developed a numerical method utilising the modified Frolich curve, which made the computer simulation of non-linear hysteresis loop possible for specific application. This technique is utilised to simulate the entire \( B-H \) loop for hysteresis motor application. The entire programming is based on the Everatt's method.

4.3 ALGORITHM

The magnetisation curve of the hysteresis material can
be represented by [35],

\[ B^* = \frac{aB_n H^*}{1 + bH^*} + \mu_0 H^* \quad \ldots \ldots \ldots \quad (4.12) \]

which is a modification of the Prölich curve. Where \( H^* > 0 \):

- \( B^* \) = Flux density
- \( B_n \) = saturation flux density
- \( H^* \) = magnetic field intensity

\( a \) and \( B_n \) are constants which are determined from the actual magnetisation curve. The constant \( b \) is under the control of the algorithm and is initially equal to \( a \). The hysteresis loop is constructed from four adjoining curve segments, is shown in Fig. (4.1). Table (4.1) contains the values of \( B_m \) and \( H_m \), which are the values of \( B \) and \( H \) at the last tip of the loop encountered. \( H_c \) is a function of \( B_m \) and \( H_{c_{max}} \). In the beginning \( H_c \) is zero and is recalculated at each tip, according to the rules:

\[ H_c = H_{c_{max}} \frac{|B_m|}{B_n} \quad \text{where} \quad |B_m| \leq B_n \quad \ldots \ldots \quad (4.13) \]

\[ H_c = H_{c_{max}} \quad \text{where} \quad |B_m| > B_n \quad \ldots \ldots \quad (4.14) \]

Table (4.1) clearly shows that, the constants \( a \) and \( b \) are equal on segments 1 and 3. But the values of \( a \) and \( b \) are recalculated at each entry to segments 2 or 4.

4.4 **Simulation Examples**

The algorithm developed for modified Prölich approach given in earlier section is utilised in this section. This method
FIG. 4.1 GROWTH OF SIMULATED B-H LOOP
<table>
<thead>
<tr>
<th>Segment</th>
<th>Conditions</th>
<th>Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Segment 1</td>
<td>$B &gt; 0$, $H$ increasing</td>
<td>$B^* = B$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$H^* = H - E_c$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$b = a$</td>
</tr>
</tbody>
</table>

| Segment 2 | $B > 0$, $H$ decreasing | $B^* = B$ |
|           |                    | $H^* = H - E_c$ |
|           |                    | $b = \frac{\|B\| - \delta_0 (\|B\| + E_c)}{\|H^*\| + E_c}$ |

| Segment 3 | $B < 0$, $H$ decreasing | $B^* = B$ |
|           |                    | $H^* = H - E_c$ |
|           |                    | $b = \frac{\|B\| + \delta_0 (\|B\| + E_c)}{\|H^*\| + E_c}$ |

| Segment 4 | $B < 0$, $H$ increasing | $B^* = B$ |
|           |                    | $H^* = H - E_c$ |
|           |                    | $b = \frac{\|B\| + E_c}{\|H^*\| + E_c}$ |

Table 4.1: Definitions of the Four Segments and Appropriate Changes of Variables.
have been tested in simulating 17% cobalt steel, 36% cobalt steel and oerstid-70 alloys.

Table (4.2) contains the pertinent magnetic properties of 17% cobalt steel, 36% cobalt steel and oerstid-70. However, it is known that $H_c$, $B_r$, $B_{sat}$, and $H_e$ of the hysteresis material vary to some extent depending upon its past history. The parameters of the respective hysteresis material given in Table (4.2) are inserted into the computer programme. The values of $H_c$ and $H_{cmax}$ are set within the reasonable value. It should be noted that the intersection on the $H$ axis increases with decreasing ratio of $B_r/H_c$.

The actual B-H loop of 17% cobalt steel, 36% cobalt steel, and oerstid-70 alloys are shown in Fig. (4.2), (4.3), and (4.4) respectively. The simulated B-H loop of 17% cobalt steel, 36% cobalt steel, and oerstid-70 are shown in Fig. (4.5), (4.6) and (4.7) respectively. Comparison between the simulated B-H loop and the actual B-H loop shows a very close agreement in all respect.

4.5 EFFECT OF HYSTERESIS PARAMETERS ON AIR-GAP POWER OF THE HYSTERESIS MACHINE

Fig. (4.8) shows the plot of air-gap power vs the unsaturated relative permeability. Fig. (4.9) shows the relation between the air-gap power and $B_r$, which indicates that there is an enormous increase in air-gap power as the $B_r$ increases. The plot shown in Fig. (4.10) clearly reflects that there is a very less change in the air-gap power, as the saturated relative permeability is varied. The
Table 4.2 Pertinent Magneto-Electric properties of Hysteresis Materials.

<table>
<thead>
<tr>
<th></th>
<th>17% Cobalt Steel</th>
<th>Oerstet-70</th>
<th>36% Cobalt Steel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_x$</td>
<td>0.95 T</td>
<td>0.85 T</td>
<td>0.90 T</td>
</tr>
<tr>
<td>$B_y$</td>
<td>1.50 T</td>
<td>1.20 T</td>
<td>1.45 T</td>
</tr>
<tr>
<td>$H_C$</td>
<td>12.80 kA/m</td>
<td>12.50 kA/m</td>
<td>19.85 kA/m</td>
</tr>
<tr>
<td>$\mu_{rs}$</td>
<td>14.60</td>
<td>14.90</td>
<td>12.00</td>
</tr>
<tr>
<td>$H_a$</td>
<td>28 kA/m</td>
<td>25 kA/m</td>
<td>34 kA/m</td>
</tr>
</tbody>
</table>
FIG. 4.2  B-H LOOP FOR 17% COBALT STEEL (ACTUAL)
FIG. 4.3  B-H LOOP FOR 36% COBALT STEEL (ACTUAL)
FIG. 4.4 B-H LOOP FOR OERSTED-70 (ACTUAL)
FIG. (4.5). B-H LOOP FOR 17% COBALT STEEL (SIMULATED).
FIG. (4.6). B-H LOOP FOR 98% COBALT STEEL (SIMULATED).
FIG. (4.7). B-H LOOP FOR OERSTIT-70 (SIMULATED).
PLOT AIR-GAP POWER VS UNSATURATED RELATIVE PERMEABILITY
KEEPING THE FOLLOWING CONSTANT

(1). REMANENT FLUX DENSITY
(2). COERCIVE FORCE
(3). SATURATED RELATIVE PERMEABILITY

FIG. (4.6). AIR-GAP POWER VS UNSATURATED RELATIVE PERMEABILITY
Plot air-gap power vs remanent flux density; keeping the following constant:
1. Unsaturated relative permeability
2. Saturated relative permeability
3. Coercive force
PLOT AIR-GAP POWER VS SATURATED RELATIVE PERMEABILITY:
KEEPING THE FOLLOWING CONSTANT
(1). UNSATURATED RELATIVE PERMEABILITY
(2). REMANENT FLUX DENSITY
(3). COERCIVE FORCE

FIG. (4.10). AIR-GAP POWER VS SATURATED RELATIVE PERMEABILITY
plot of air-gap power vs coercive force shown in Fig. (4.11), clearly indicates that there is a sudden rise in air-gap power up to certain value of the coercive force (about 13 kA/M), and decrease in power beyond this value of $H_c$. 
PLOT AIR-GAP POWER VS COERCIVE FORCE:
KEEPING THE FOLLOWING CONSTANT
(1). UNSATURATED RELATIVE PERMEABILITY
(2). SATURATED RELATIVE PERMEABILITY
(3). REMANENT FLUX DENSITY

FIG. (4.11). AIR-GAP POWER VS COERCIVE FORCE
5.1 INTRODUCTION

The equations developed [8] to study the behaviour of the hysteresis motor is the landmark in the evolution of the hysteresis motor. These equations are developed by modelling the properties of rotor material with the help of idealised parallelogram of B-H loop. However, the equations developed to predict the torque of the idealised machine was exclusive off the parasitic losses associated with the rotor magnetic material, which is due to the excursion of the minor loop caused by the eddy current effect and tooth ripple present in the air-gap flux density.

5.2 EQUATIONS OF MOTOR

The equations used are based on those developed by Copeland and Slamon [8] for circumferential flux motor. The cross section of the circumferential flux motor is as shown in the Fig. (5.1). It is important to note that the rotor core of the circumferential flux is non-magnetic. The flux crosses the airgap radially and follows the circumferential path in the hysteresis ring.

The fundamental motor field equations for an m-phase, p-pole machine are given as:

\[ \frac{dB}{dt} = \frac{1}{\mu_0} \left( \frac{H_0}{h} + \frac{m}{h} \right) \cos \left( \frac{2\pi}{p} \left( wt - \varphi \right) \right) \]
Fig. 5.1 Cross-section of circumferential flux hysteresis motor showing dimensions and elemental segments.
\[ \frac{dB_h}{d\theta} = \frac{rB}{g} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (5.2) \]

According to the notation shown in the section of the motor Fig. (5.1), the following equations are developed for nth of

\[ N = 2\pi/\Delta \theta \text{segments} \quad [22] : \]

\[ I(n) = r_h \Delta \theta \frac{h}{\mu_0 r \Delta \theta} \left[ 2B_h(n) - B_h(n - 1) - B_h(n + 1) \right] \quad (5.3) \]

\[ B_h(n) = f[H_h(n)] \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (5.4) \]

\[ B_g(n) = \frac{h}{2\pi \Delta \theta} \left[ B_h(n + 1) - B_h(n - 1) \right] \quad (5.5) \]

Equations (5.3) and (5.4) are solved for given stator mmf distribution \( I(n,T), n = 1, 2, \ldots \ldots N \). By knowing the previous state of the rotor at \( T - \Delta T \), the magnetic state of the rotor of the idealised motor at time \( T \) is calculated. Equation (5.5) then gives the air gap flux, and from this the flux linkage per phase is computed knowing the stator winding distribution. The value of the stator induced voltage is then computed, and hence the motor shaft torque, by omitting windage and friction loss is carried out.

In the present study, provision is made to insert the stator winding resistance and leakage inductance in the above equations, and the corresponding changes are made in the computer programme to obtain the stator terminal voltage.

5.3 COMPUTER MODEL OF THE HYSTERESIS LOOP

The digital technique used in modelling the hysteresis
loop is based on the piecewise linear approximations. [22]. The model
developed has got close association with the behaviour of many other
permanent magnet materials which have got the required properties, as
rotors of the hysteresis motors.

The above mentioned model is found useful in solving the full
motor equations by iteration method, which is a necessary factor
for this type of problem. The magnetic flux density is defined
by \( y(t) \) and the magnetic field strength is defined by \( x(t) \). \( T \) refers to
time.

Thus the magnetic state of the material can be defined at any
instant of time. The previous state of the material at any time is
given by \( (T-\Delta T) \). The previous state requires the three 'history' par-

The allowable values of \( x \) and \( y \) are bounded in parallelogram set by \( H_s, B_s, \alpha, \beta \), and the tail representing the saturation
Fig. (5.2a). In general, changes within this boundary follow lines
with a slope \( \alpha \) equal to that of the top of the parallelogram. Each
point \((x,y)\), however, has a horizontal recoil line associated with it,
of length equal to or less than \( 2\alpha \), bounded by the points \((b,y)\) and
\((c,y)\). For large changes in \( x \), the horizontal line is dragged along
by one of its ends. When \( x \) changes in direction, there is a horizontal
movement to the other end of the line, and it is then dragged in the
opposite direction. There are nine possible modes of operation
\((-45, 45, 90, m, \text{ integer})\).

The length of the recoil line in each mode is given in Table (5.1)
FIG. 5.2a LOOP PARAMETERS
### Table 5.1 Length of recoil line

<table>
<thead>
<tr>
<th>Mode m</th>
<th>Length of recoil line</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \rho &lt; (c' - b) &lt; 2\rho )</td>
</tr>
<tr>
<td>-2, -1, 1, 2</td>
<td>( \rho )</td>
</tr>
<tr>
<td>-3, 3</td>
<td>( 0 &lt; (c - b) &lt; \rho )</td>
</tr>
<tr>
<td>-4, 4</td>
<td>0</td>
</tr>
</tbody>
</table>
The possible H/H path is shown in the Fig. (5.2b). The 'history' parameters at each turning point of H are given in table (5.2).

<table>
<thead>
<tr>
<th>Point</th>
<th>O</th>
<th>L</th>
<th>P</th>
<th>M</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0</td>
<td>(x_3)</td>
<td>(x_2)</td>
<td>(x_4)</td>
<td>(x_1)</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>(y_3)</td>
<td>(y_2)</td>
<td>(y_4)</td>
<td>(x_1)</td>
</tr>
<tr>
<td>b</td>
<td>-(\rho)</td>
<td>(x_3 - \rho)</td>
<td>(x_4)</td>
<td>(x_1)</td>
<td></td>
</tr>
<tr>
<td>c</td>
<td>+(\rho)</td>
<td>(x_3)</td>
<td>(x_2 + 2\rho)</td>
<td>(x_4)</td>
<td>(x_1 + \rho)</td>
</tr>
<tr>
<td>m</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>
FIG. 5.2b POSSIBLE B/H PATH AS H IS VARIED
The subroutine employed in this case has to perform a long series of tests to establish new mode. To determine the new value of \( y \), \( b \) and \( c \) at each mode, a function routine is employed. However, it does not require nonlinear function storage.

In the computer programme, 360 electrical degrees are divided into 36 segments. Idealised m.m.f distributions, corresponding to 18 and 3 slots per pole is used and the equations are solved for 180 electrical degrees only. An error function is used based on the successive approximations, which is reduced to a small value. The error function is of the form:

\[
e^2(n) = \left| I(n) - F[H_h(n), B_h(n), B_h(n-1), B_h(n+1)] \right|^2.
\] (5.6)

The flow diagram for motor-equation program is shown in the Appendix E.
CHAPTER VI

RESULTS AND DISCUSSION

6.1 INTRODUCTION

In this chapter an attempt is made to study the performance characteristics of the hysteresis motor experimentally and to compare it with the computed results using the digital simulation method. The rotor hysteresis material used is made of 17% cobalt steel, supplied by the Permanent Magnet Manufacturing Company.

6.2 RING SPECIFICATION

Before going to study the performance of the hysteresis motor, it is essential to know the characteristic curve of the ring material and also to represent it in the form of $B-H$ loop.

The heat treatment and annealing of the 17% cobalt steel was carried out at the Permanent Magnet Manufacturing Company. But the company could not provide the final $B-H$ loop of the hysteresis ring material supplied. However, on testing the ring in the motor assembly, it was found that the sample ring did not give rise to the same values of $B$ and $H$ as specified for 17% cobalt steel material. However, on the basis of experimental results the expected $B-H$ loop of the sample ring was simulated and at values of $B_1 = 0.80 \, T$, $\mu_{rs} = 10.00$, $\mu_f = 100.00$ and $H_s = 9500.00 \, A/m$, the computer results and experimental results agree within the limit of tolerance.
6.3 EXPERIMENTAL SETUP AND MEASUREMENT

Details of the experimental setup are given in the Appendix B. Two digital wattmeters were used to measure the input power to the machine. The design data of the hysteresis machine is given in the Appendix C.

The hysteresis motor was loaded by means of a d.c. work machine mechanically coupled with the experimental Mawdesley's Generalised machine. The machine was slowly loaded to the point of pull-out by varying the load resistance connected to the work machine.

The total input power less the total copper loss gives the air gap power. The output of the hysteresis machine was calculated. The air-gap power less the rotor parasitic loss and friction and windage and core-loss gives the shaft output power. The parasitic loss was measured experimentally.

6.4 RESULTS

The efficiency and the full load input power factor of the hysteresis machine were calculated by knowing the output power and from the known values of terminal voltages and currents. Comparison of the experimental results and computed results show a very close agreement. Tables (6.1) and (6.2) show experimental results and computed results respectively.

The agreement between computed and measured results of Figs. (6.1) and (6.2) is reasonably good in view of the complexity
Table 6.1 Performance Results (Measured)

<table>
<thead>
<tr>
<th>Current $I_p$ (A)</th>
<th>Pull-in Voltage $V_{L-L}$</th>
<th>Power Factor $\beta$</th>
<th>Air-gap Power (Watts)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.18</td>
<td>178.5</td>
<td>0.36</td>
<td>1077.50</td>
<td>58.0</td>
</tr>
<tr>
<td>9.84</td>
<td>190.0</td>
<td>0.37</td>
<td>1107.50</td>
<td>58.3</td>
</tr>
<tr>
<td>10.28</td>
<td>198.0</td>
<td>0.34</td>
<td>1184.00</td>
<td>60.0</td>
</tr>
<tr>
<td>10.82</td>
<td>205.0</td>
<td>0.33</td>
<td>1218.20</td>
<td>59.0</td>
</tr>
<tr>
<td>10.86</td>
<td>206.0</td>
<td>0.33</td>
<td>1235.70</td>
<td>56.50</td>
</tr>
<tr>
<td>10.94</td>
<td>208.0</td>
<td>0.33</td>
<td>1230.20</td>
<td>59.0</td>
</tr>
</tbody>
</table>
Table 6.2 Performance Results (Computed)

<table>
<thead>
<tr>
<th>Current $I_p$ (A)</th>
<th>Pull-in Voltage $V_{L-I}$ (Volt)</th>
<th>Power Factor</th>
<th>Air-gap Power (Watts)</th>
<th>Efficiency (I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.18</td>
<td>190.0</td>
<td>0.41</td>
<td>998.0</td>
<td>73.0</td>
</tr>
<tr>
<td>9.84</td>
<td>199.0</td>
<td>0.39</td>
<td>1277.0</td>
<td>72.0</td>
</tr>
<tr>
<td>10.28</td>
<td>204.0</td>
<td>0.39</td>
<td>1261.0</td>
<td>71.0</td>
</tr>
<tr>
<td>10.82</td>
<td>210.0</td>
<td>0.34</td>
<td>1275.0</td>
<td>69.0</td>
</tr>
<tr>
<td>10.86</td>
<td>210.7</td>
<td>0.34</td>
<td>1270.0</td>
<td>69.5</td>
</tr>
<tr>
<td>10.94</td>
<td>212.0</td>
<td>0.34</td>
<td>1274.0</td>
<td>69.0</td>
</tr>
</tbody>
</table>
of the non-linearity of the hysteresis material and approximations made in the analysis. Fig. (6.1) shows the plot of terminal voltage vs stator phase current. Fig. (6.2) shows the plot of terminal voltage vs airgap power.
CHAPTER VII

CONCLUSIONS

Type of B-H loop approximations to be made depends upon the magnetic properties of the material. The performance of the hysteresis machine primarily depends upon the optimized representation of the actual B-H loop of the material, as it is used in predicting the terminal performances of the hysteresis motor. The digital technique based on the modified Frölich's approach is used in the simulation of the typical hysteresis materials like 17% cobalt steel, 36% cobalt steel and Cerastit-70 alloys. These materials have coercivity lying between 4 and 20 kA/m, and remanent flux density between 0.8 and 1.3 T. The program was developed (Appendix B) for PDP 1160 computer, and automatic plotting of B-H loop was carried out. It was observed that the intersection on the H-axis increases with the decrease in ratio of \( B_r/B_c \). It was found that the maximum and minimum values of \( B_c \) critically determines the simulation of the loop. Comparison of the simulated B-H loops with those supplied from the permanent magnetic company show a very close agreement in all respects.

Based on parallelogram approximation of the B-H loop, the motor field equations were developed for the circumferential-flux type [8]. These equations were solved numerically to predict the terminal values of the motor. Using current as input, the airgap flux and hence line voltages are computed with the given stator windings. A complete computer algorithm is developed and the listings are...
included in Appendix E. The main computer program was first developed for IBM 360/55 systems. It was also made adoptable for PDP 11/60 for integration with the graphic plotters.

The digital method of simulating the hysteresis loop and solving the motor equations numerically, allow the steady state behaviour of the hysteresis motor to be predicted from its known dimensions, winding data and hysteresis material's magneto-electric properties. The hysteresis ring of the test rotor was made of 17% cobalt steel alloy. The parallelogram approximation of the B-H loop is used in the numerical analysis of motor performance predictions. Test results of the terminal quantities at synchronous speed indicate good correlation between the measured and the calculated values.

Effects of pertinent hysteresis parameters like coercive force, remanent flux density, saturated relative permeability, and unsaturated relative permeability, on the airgap power of the hysteresis motor are studied.

The terminal properties of the hysteresis machine may be improved further by using the modified Prölich methods to solve the motor equations instead of the parallelogram. Developing a complete computer program in order to obtain, perhaps more accurate terminal quantities based on the modified Prölich model is one of the future works that should be carried out.
REFERENCES


APPENDIX A

PREISACH–NÉEL'S MODEL

J. Perard and M. Poloujadoff [36] used this model in the study of the 'Asynchronous performances of hysteresis motor under unbalanced conditions'. It seems to work very well for hysteresis materials like vicalloy. However, it is found that the modified Fröhlich approach gives a better B–H loop for materials like simonds–81 and garstt-70.

This model originates from a graphical representation of the hysteresis phenomenon given by Preisach [37, 38]. It is according to physical reality in low fields as shown by Néel [39] who gave a theory concerning the wall displacement in this case in 1942. Thus the name is Preisach–Néel's model.

Magnetisation curve of a Preisach–Néel's elemental segment is shown in the Fig. (A1), by considering a small sample of magnetic material as the superposition of an arbitrary large number of elemental segments. Each one is characterized by two critical values of field intensity a and b ('a > b') and has a rectangular hysteresis loop m(H) given in Fig. (A1). Each elemental segment is also represented in the (a, b)–plane by a point below the first bisector (a > b) shown in Fig. (A2). If the intensity of the field sample is H, the magnetisation intensity of an elemental segment is given as:

\[ m = M_s \text{ if } b < a < H \]  \hspace{1cm} (A1)

\[ m = -M_s \text{ if } H < b < a \]  \hspace{1cm} (A2)
FIG. A1  MAGNETIZATION CURVE OF A PREISACH–NÉEL'S ELEMENTAL SEGMENT
FIG. A2 - PREISACH - NIELSEN'S DIAGRAM
m = \pm M_a \text{ if } b \leq H < a \quad \text{(A3)}

In the last case the sign of \( m \) is dependent on the previous history of the material. Therefore, the calculation must start from a state in which the value of \( m \) is known for all elemental segments.

Let 'n' be the total number of elemental segments in the sample. Then \( d_n \), the number of elemental segments having a representative point located in the rectangle defined by the coordinates \((a, a + da, b, b + db)\). The \( M \), the average value of \( m \) elemental segment

\[
M = \frac{1}{n} \int m \, d_n
\]

Next the model is completed by a distribution function \( S(a, b) \) related to the probability, \( \left( \frac{dn}{n} \right) \) of finding an elemental segment having critical fields in the intervals \((a, a + da)\) and \((b, b + db)\). The function related to this is defined in the following way:

\[
\frac{dn}{n} = \frac{1}{M_a} S(a, b) \, dab \quad \text{...(A5)}
\]

The magnetisation of the sample is given by:

\[
M = \int_{R^+} S(a, b) \, dab - \int_{R^-} S(a, b) \, dab \quad \text{...(A5)}
\]

if \( R^+ \) and \( R^- \) are the regions of the \((a, b)\) plane, where \( m \) is equal to \(+ M_a\) and \(- M_a\) respectively. Since the materials have the same properties, if all variations of \( H \) are changed in sign, \( S(a, b) \) is symmetrical with respect to the second bisection. In other words, \( S(a, b) = S(-b, -a) \). This function can be determined from the experimental knowledge of the rising magnetisation curve and the largest hysteresis cycle of the material. To determine the numerical values of \( S(a, b) \), the method proposed by Biorti and Pescetti [40] is used.
Based on the above principle, J. Papard and M. Foulds developed a computer programme to compute the flux density \( B = \mu_0 H + M \), knowing \( B(t) \).

At the beginning, the material is non-magnetised (\( H = 0, M = 0 \)), the magnetised intensity of the elemental segment is given by the Fig. (A3a). Then

\[
\begin{align*}
m &= S(a, b) \text{ below the second bisector,} \\
m &= -S(a, b) \text{ above the second bisector,}
\end{align*}
\]

\[
\int_{t_1}^{t_2} S(a, b) \, db = \int_{t_1}^{t_2} S(a, b) \, db 
\]

\( H(0) = 0 \)  

(A6)  

(A7)

In a first stage, \( H(t) \) increases from zero up to \( H^M \). For a given value of \( H \), \( M \) has changed from \( -M_M \) into \( +M_M \). Then for the elemental segments represented by points of the triangle \( T \), Fig. (A3b). Thus:

\[
M(t) = M(0) + 2 \int_{T} S(a, b) \, db
\]

(A8)

Later \( H^M \), the final value of \( H \), when \( H = H^M \) (Fig. A3c), is called. Suppose in a second stage, \( H(t) \) decreases from \( H^M \), \( M \) decreases from \( M_M \) and is given by

\[
M(t) = M_M - 2 \int_{T} S(a, b) \, db
\]

as long as \( H > -H^M \) and \( H \) is steadily decreasing (Fig. A3d). If \( H \) reaches \( -H^M \), Fig. (A3e) shows that \( M \) reaches \( -M_M \). If \( H \) decreases further, \( M(H) \) will be again the magnetisation curve. If, however, \( H \) increases again just after having reached \( H^M \) (with \( H^M > -H_M \)), \( M \) is given by:

\[
M(t) = M^M + 2 \int_{T} S(a, b) \, db
\]

as long as \( H < H^M \) and \( H \) is decreasing, Fig. (A3f).
FIG. A3  ILLUSTRATION OF PREISACH – NÉEL’S MODEL.
APPENDIX B

DESCRIPTION OF THE Hysteresis Motor UNDER STUDY

The stator of the hysteresis machine has a polyphase distributed winding. The rotor consists of a hysteresis ring supported by an aluminium sleeve. A hysteresis motor is a smooth cylindrical rotor electrical machine.

The Mawdleys Generalised Machine is used in the experimental study. The stator of the experimental hysteresis motor is the stator of the Mawdleys Generalised Machine. The stator of the Generalised Mawdleys Machine has a conventional 4-pole a.c. winding in 48 slots. The ends of all the 48 coils are brought out to 96 terminals symmetrically arranged in four concentric circles. Thus, there is a unique flexibility of running the machine in several modes.

The arrangement of the physical relationship between the position of the coil sides and the arrangement of the terminals is shown in the adjoining photograph. Coil sides 1-48 coloured red, are in slot position shown and the ends of these coil sides are connected to terminals 1-48, coloured red (not visible in the photograph). Coil sides 1-48, coloured blue are connected to terminals 1-48, coloured blue. The position of the slot numbers 1, 2, etc. are marked on the external connection plate.

There is a provision to measure the air gap flux, through the built-in search coils, which is provided in the stator. Single search wires are provided in the tops of the stator slots 1, 7, 9, 12 and 13. The search coils of various pitches can be achieved with this
The direct measurement of the temperature rise of the stator core is made possible by means of a built-in thermo couple. The ends of the thermo couple are brought out to the left-bottom corner of the connection plate. The insulation of the stator winding has a temperature tolerance limit of 110°C as measured by the thermocouple.

The stator core temperature can also be measured with the help of a mercury thermometer, through a groove at the top of the stator core surface. The Mawdsley's Generalised Machine is coupled to a d.c. work machine. The d.c. work machine is used to load the experimental hysteresis machine.

In addition a built-in a.c. tacho-generator is provided for measuring the rotor speed, in the Generalised Machine set.

The torque measuring unit is the outstanding feature of the Generalised Machine set. The torque measuring unit facilitates accurate measurement of both steady state and transient torques. The torque to be measured is transmitted by a hollow shaft whose tapered end fits coaxially with the experimental rotor shaft, while the other end is coupled with the d.c. work machine. The measured torque is to be obtained ultimately as an electrical output from the converter unit.
APPENDIX C

DESIGN DATA OF THE EXPERIMENTAL MACHINE

STATOR SPECIFICATIONS

The stator of the experimental hysteresis machine is the Mawdsleys Generalised Machine stator having the name plate data as follows.

Normal stator volts 200/220, stator No. 1

Stator core
Core material Sheet steel 0.457 mm
Outside diameter 279.400 mm
Inside diameter 152.400 mm
Slot depth 27.080 mm
Slot width 2.540 mm
Tooth width 7.430 mm
Number of slots 48

Slots were tapered towards the air gap having 4.8 mm bottom radius and 2.8 mm tip radius.

Stator winding
Type of winding double layer lap
Number of coils 48
Number of conductors per slot 54
Number of turns per coil 27
Conductor dia 1.22 mm
<table>
<thead>
<tr>
<th>Material</th>
<th>Copper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean length of turn</td>
<td>736.600 mm</td>
</tr>
<tr>
<td>Slot skew</td>
<td>1 slot pitch</td>
</tr>
<tr>
<td>Conductor covering</td>
<td>Polyvinyl acetate</td>
</tr>
<tr>
<td>Class of insulation</td>
<td>E</td>
</tr>
<tr>
<td><strong>Rotor Specification</strong></td>
<td></td>
</tr>
<tr>
<td>Diameter of the ring</td>
<td>151.250 mm</td>
</tr>
<tr>
<td>Outer diameter of the aluminium sleeve</td>
<td>118.500 mm</td>
</tr>
<tr>
<td>Ring depth</td>
<td>16.500 mm</td>
</tr>
<tr>
<td>Ring length</td>
<td>105.000 mm</td>
</tr>
<tr>
<td>Internal diameter of aluminium sleeve</td>
<td>97.400 mm</td>
</tr>
<tr>
<td>( r_\text{h} ) of the sample</td>
<td>57.500 mm</td>
</tr>
<tr>
<td>( r_\text{g} ) of the sample</td>
<td>67.560 mm</td>
</tr>
<tr>
<td>Shell thickness of sample</td>
<td>37.500 mm</td>
</tr>
<tr>
<td>Length of each sample</td>
<td>101.200 mm</td>
</tr>
<tr>
<td>Diameter of the shaft</td>
<td>75.500 mm</td>
</tr>
<tr>
<td>Air gap</td>
<td>0.508 mm</td>
</tr>
<tr>
<td>Stator resistance per phase</td>
<td>1.100 Ohm</td>
</tr>
<tr>
<td>Stator leakage reactance per phase</td>
<td>3.700 Ohm.</td>
</tr>
</tbody>
</table>
APPENDIX D

FILE NAME: OST70.FTN

COMPUTER PROGRAMME TO SIMULATE THE B-H LOOPS OF THE HYSTERESIS MATERIALS LIKE 17% COBALT STEEL, 36% COBALT STEEL AND OERSTIT-70 (16% COBALT STEEL) ALLOYS.

SIMULATION OF OERSTIT-70 ALLOY IS GIVEN HERE AS AN EXAMPLE.

COMMON/FUN/CA, BSAT, ALPHA
DIMENSION H(128), B(128)
CALL PLOTS(2)
NEND=126
2. 0., 500., 1000., 2000., 3000.,
3. 4000., 5000., 6000., 7000., 8000.,
4. 9000., 10000., 11000., 12000., 13000.,
5. 14000., 15000., 16000., 17000., 18000.,
6. 19000., 20000., 21000., 22000., 23000.,
7. 24000., 0., 0., /.

PI = 4. * ATAN(1.)
FMU0 = 4. * PI / 1.0E-7
FMURS = 14.6
ALPHA = FMU0 * FMURS
BSAT = 1.2
B(1) = 0.
HC = 12500.
CB = CONB(0., 0., HC)
CA = 11.0E-4
M = 0.

DO 111 N = 2, NEND
B(N) = B(N-1)
CALL FROLOP(H(N-1), B(N), HC(N), M, HC, CB)
CONTINUE

D WRITE (5, 200)
200 FORMAT (' ', 'MAGNETIC FIELD STRENGTH ', ' / )
DO 5 K = 1, NEND
5 H(K) = H(K) / 1000.
D WRITE (5, 210) (H(K), K = 1, NEND)
210 FORMAT (4F12.8)
D DO 7 K = 1, NEND
7 B(K) = B(K) * 10.
D WRITE (5, 230) (B(K), K = 1, NEND)
D220 FORMAT (' ', 'FLUX DENSITY ', ' / )
230 FORMAT (4F12.5)
CALL AX1S (1.5., ' ', -1.6., 0., -30., 10.)
CALL AX1S (4.2., ' ', 1.8., 80., -1.5., 5.)
H(NEND+1) = -30.
H(NEND+2) = 10.
B(NEND+1)=1.5
B(NEND+2)=.5
CALL SYMBOL (1, 8, 14, 'FIG (4.7)'. B-H LOOP FOR
*OERSTI-70 ( SIMULATED )'. 0, 50)
CALL LINE (H, B, NEND, 1, 8, 4, 14)
CALL SPLINE (H, B, NEND, 1, 8, 32, 0)
CALL PLEXIT
100 STOP.
END
FUNCTION BXV(X, B)
.COMMON /FUN/CA, BSAT, ALPHA
BXV=(CA*BSAT*X)/(1+B*X)+(ALPHA*X)
RETURN
END
FUNCTION CONB(U, V, W)
.COMMON /FUN/CA, BSAT, ALPHA
CONB=(CA*BSAT)/(ABS(V)-ALPHA*(ABS(U)+W)-1./(ABS(U)+W)
RETURN
END
FUNCTION YFX(X, Y, S, XX)
YFX=(XX-X)*S+Y
RETURN
END
FUNCTION ERR(X, Y, B)
.COMMON /FUN/CA, BSAT, ALPHA
ERR=(CA*BSAT*X)/(1+B*X)+(ALPHA*X)
RETURN
END
SUBROUTINE FROLOP(X, Y, XX, M, HC, CB)
.COMMON /FUN/CA, BSAT, ALPHA
HCMIN=12500.
HCMAX=12800.
IF (M.EQ.-2) GO TO 90
IF (M.EQ.2) GO TO 70
IF (M.EQ.-1) GO TO 50
IF (M.EQ.1)GO TO 30
DEAl ... WITH: HOOE ...

\[ \text{IF } (X - X) \geq 11, 12, 13 \]

\[ \text{YY} = \text{YFX}(X, Y, \text{ALPHA}, XX) \]

\[ \text{IF } (\text{YY}) = 14, 15, 15 \]

\[ \text{XSTAR} = \text{XX} + \text{HC} \]

\[ \text{YSTAR} = -\text{YY} \]

\[ \text{IF } (\text{XSTAR} \leq 0) \text{ GO TO 19} \]

\[ \text{ERRM2} = \text{ERR}(\text{XSTAR}, \text{YSTAR}, CB) \]

\[ \text{IF } (\text{ERRM2}) = 17, 18, 18 \]

\[ \text{YSTAR} = \text{BXY}(\text{XSTAR}, CB) \]

\[ \text{Y} = -\text{YSTAR} \]

\[ \text{M} = -2 \]

\[ \text{RETURN} \]

\[ \text{XSTAR} = \text{XX} - \text{HC} \]

\[ \text{Y} = \text{BXY}(\text{XSTAR}, CA) \]

\[ \text{M} = 1 \]

\[ \text{RETURN} \]

\[ \text{Y} = \text{YY} \]

\[ \text{M} = 0 \]

\[ \text{RETURN} \]

\[ \text{XSTAR} = \text{XX} - \text{HC} \]

\[ \text{YSTAR} = \text{YY} \]

\[ \text{IF } (\text{XSTAR} \leq 0) \text{ GO TO 17} \]

\[ \text{ERR1} = \text{ERR}(\text{XSTAR}, \text{YSTAR}, CA) \]

\[ \text{IF } (\text{ERR1}) = 19, 17, 17 \]

\[ \text{RETURN} \]

\[ \text{YY} = \text{YFX}(X, Y, \text{ALPHA}, XX) \]

\[ \text{IF } (\text{YY}) = 20, 21, 21 \]

\[ \text{XSTAR} = \text{XX} - \text{HC} \]

\[ \text{YSTAR} = -\text{YY} \]

\[ \text{IF } (\text{XSTAR} \leq 0) \text{ GO TO 23} \]

\[ \text{ERRM1} = \text{ERR}(\text{XSTAR}, \text{YSTAR}, CA) \]

\[ \text{IF } (\text{ERRM1}) = 22, 23, 23 \]

\[ \text{XSTAR} = \text{XX} - \text{HC} \]

\[ \text{YSTAR} = \text{BXY}(\text{XSTAR}, CA) \]

\[ \text{Y} = -\text{YSTAR} \]
M = -1
RETURN

Y = YY
M = 0
RETURN

XSTAR = XX + HC
YSTAR = YY
IF (XSTAR .LE. 0.) GO TO 22
ERRM2 = ERR(XSTAR, YSTAR, CB)
IF (ERR2) 23, 26, 26

Y = BXY(XSTAR, CB)
M = 2
RETURN

C MODE M = 0 CALCULATIONS ARE COMPLETED
C MODE M = 1

IF (XX - H) 31, 12, 32
IF (ABS(Y) - BSAT) 40, 41, 41

HC = HCMAX
GO TO 42

HC = HCMIN + (HCMAX - HCMIN) * ABS(Y) / BSAT
CB = CONB(X, Y, HC)
IF (XX + HC) 33, 34, 34

XSTAR = XX - HC
YSTAR = BXY(XSTAR, CA)
Y = -YSTAR
M = 1
RETURN

XSTAR = XX + HC
Y = BXY(XSTAR, CB)
M = 2
RETURN

XSTAR = XX - HC
Y = BXY(XSTAR, CA)
M = 1
RETURN

C TRY MODE M = -1
50 IF (XX-XX) 51.12.52
51 XSTAR=XX-HC
   YSTAR=BXY(XSTAR,CA)
   Y=YSTAR
   RETURN
52 IF (ABS(Y)-BSAT) 60.81.61
51 HC=HCMAX
   GO TO 62
60 HC=HCMIN+(HCMAX-HCMIN)*ABS(Y)/BSAT
62 CB=CONBCX,Y,HC)
   IF (XX-HC) 53.54.54
53 XSTAR=XX-HC
   YSTAR=BXY(XSTAR,CB)
   Y=YSTAR
   M=2
   RETURN
54 XSTAR=XX-HC
   Y=BXY(XSTAR,CA)
   M=1
   RETURN
C  TRY WITH MODE M=2
70 IF (XX-XX) 71.12.13
71 IF (XX+HC) 73.74.74
73 XSTAR=XX-HC
   YSTAR=BXY(XSTAR,CA)
   Y=YSTAR
   M=1
   RETURN
74 XSTAR=XX+HC
   Y=BXY(XSTAR,CB)
   M=2
   RETURN
C  TRY WITH MODE =-2
90 IF (XX-XX) 11.12.91
91 IF (XX-HC) 99.94.94
83 XSTAR=XX+HC
YSTAR = BXY(XSTAR, CB)
Y = YSTAR
M = 2
RETURN
XSTAR = XX - HC
Y = BXY(XSTAR, CA)
M = 1
RETURN
END
APPENDIX E

FLOW DIAGRAM OF THE MOTOR EQUATION PROGRAM USED TO PREDICT THE TERMINAL PERFORMANCE OF HYSTERESIS MOTOR

1. input motor dimensions, hysteresis ring parameters and winding details.

2. initialise the state of the hysteresis ring and set the other initial condition.

3. input time step of stator current, angle $\phi$.

4. time $T = 0$

5. $T = T + \Delta T$

6. compute $I(a,T)$, $1 \leq n \leq 18$

7. call subroutines $FLXLIN$, $TORQUE$, $LOSS$ and $UPDATE$

8. store magnetic state of rotor at $T - \Delta T$

9. $H_h(n,T) = H_h(n,T) + \Delta H(a,T)$;

   $i \leq n \leq 18$
enter hysteresis routine to find $B_h(n,T); 1 \leq n \leq 18$

using magnetic state of the rotor at $T - \Delta T$

compute error terms in motor equations $\epsilon(n); 1 \leq n \leq 18$

$\epsilon \leq \epsilon_{max}$

no

compute $B_g(n,T); 1 \leq n \leq 18$

compute flux linkage with two stator windings and hence compute current, phase voltage, terminal voltage, phase angle

compute shaft torque, power from torque; parasitic loss, output power; efficiency

$T = T_{max}$

no

yes

STOP

adjust $\Delta H_n(n,T)$ by an amount depending on $\epsilon(n)$
APPENDIX F

C FILE NAME: HYSMOT.FTN
C
C COMPUTER PROGRAMME TO PREDICT THE TERMINAL
C PERFORMANCE OF HYSTERESIS MOTOR.
C
COMMON PI
COMMON /GAP/RO,ALPHA,BETA,TOP,BOT,EF,RIG,PX,QX,RX,SX,TX,UX,VX,WX
COMMON /CURR/FM,FIP, VHBP, VHCP, REL, VHB, VHC, ARC, MODE, MODEP
COMMON /FLUX/VOLTSP, NT, DAREA, PSI, FIG
COMMON /POWER/VH, FI, TORQ, VOL
COMMON /PARAS/GAPDIA, CONLCS, PLOSS
COMMON /WIND/WINDIN, DTHETA, PHI, FREQ, PP
DIMENSION FI(36), FM(18), FIG(36)
DIMENSION FIP(18), VHBP(18), VHCP(18), MODEP(18)
DIMENSION VHB(18), VHC(18), MODE(18), VHC36(18), VHSCAL(18)
DIMENSION NINCA(8), STACRA(8), PHIA(8)
DIMENSION NINCA(8), STACRA(8), PHIA(8)
PI=4, *ATAN(1.)
GAPDIA=(50.5/1000.
AIR GAP=.508/1000. *1.
RING=16.38/1000.
RINDIA=GAPDIA-RING
RINWID=100.10/1000.
FMUO=4, *PI/100.0E8
PP=2.
DTHETA=PI/(18, *PP)
REL=(2, *AIRGAP*RING)/(FMUO*GAPDIA*DTHETA)
CONTF=(2, *RING)/(GAPDIA*DTHETA)
ARC=RINDIA*DTHETA/2.
DAREA=RINWID*GAPDIA*DTHETA/2.
VOL=PI*RINDIA*RING*RINWID
WRITE (5,801)
801 FORMAT (/,' CIRCUMFERENTIAL FLUX MACHINE PARAMETERS',/) WRITE (5,802) GAPDIA, RINDIA
802 FORMAT (8H GAP DIA=,E10.4, 1HM, 10H RING DIA=,E10.4, 1HM)
WRITE (5,803) RING, RINWID
FORMAT (12H RING DEPTH=, E10.4, 1HM, 7H WIDTH=, E10.4, 1HM)
WRITE (5, 803) AIRGAP

804 FORMAT (9H AIR GAP=, E10.4, 1HM)
C. SETUP WINDING PARAMETERS
WIN5IN=27.*12.*1.732/(PI*PI)
SLOTS=48,
FREQ=60.
RES=1.1
X1R=3.7
WRITE (5, 852) WIN5IN, RES, X1R.

852 FORMAT (16H WINDING FACTOR=, F7.3, 5H OHMS, 25H STATOR
#PHASE RESISTANCE=, F5.3, 3H OH, 7H REACT=, F5.3)
WRITE (5, 853) SLOTS, PP, FREQ

853 FORMAT (21H TOTAL NO. OF SLOTS=, F6.3, F6.3, 11H POLE PAIRS.
#19H SUPPLY FREQUENCY=, F5.1, 5H HERZ)
FMURS=10.
D TYPE = 'FMURS'
D ACCEPT #, FMURS
FMURO=100.
D TYPE = 'FMURO'
D ACCEPT #, FMURO
RESIST=.28E-6
ALPHA=FMURS*FMURO
BETA=FMURO*FMURO
RO=400.
TOP=.8
D TYPE = 'TOP'
D ACCEPT #, TOP
RIG=9500.
D TYPE = 'RIG'
D ACCEPT #, RIG
WRITE (5, 881)

881 FORMAT (/, 'HYSTERESIS RING PARAMETERS', /)
WRITE (5, 882) FMURS, FMURO, TOP, RIG

882 FORMAT (6H MU SAT=, F5.1, 11H MU UNSAT=, F5.1,
#11H BR (REM)=, F4.2, 6H TESLA, 4H HC=, E10.3, 4H A/M)
WRITE(6,883) RESIST

883 FORMAT (18H RING RESISTIVITY=E10.2,5H OHM-M)

W=PI/10.

BOT=TOP

EF=RIG

PX=(TOP+BETA*RIG)/(BETA-ALPHA)

WX=PX

SX=(TOP+BETA*RIG)/(BETA-ALPHA)

TX=SX

RX=(TOP+BETA*(RIG-RO))/(BETA-ALPHA)

UX=RX

QX=RX-RO

C INITIALISE THE HYSTERESIS RING AND CALCULATE THE LOSS EXPRESSION FACTORS

DO 160 J=1,18

VH(J)=0.

FIC(J)=0.

VHB(J)=RO

VHC(J)=RO

MODE(J)=0

160 CONTINUE

BETA1=SQRT(.5*(-1.+SQRT(1.+CHI**4)))

ETA1=.98

A1=.43

CONL0S=COAREA*SAPDIA*FREQ*PI*BETA1*A1*HIPA1*ETA1/(2.*ALPHA)

NSTART=1

DATA NINCA/10,10,10/

D DATA STACRA/7.24,5.31,6.23/

D DATA STACRA/6.29,9.27,10.17/

D DATA STACRA/10.06,10.82,9.31/

D DATA STACRA/9.39,9.18,9.7/

D DATA STACRA/9.84,9.02,10.29/

D DATA STACRA/10.37,10.49,10.82/

D DATA STACRA/10.86,10.84,10.9/

D DATA STACRA/10.9,10.98,11.06/
DATA STACRA/8.7,8.7,8/
DATA PHIA/0.0,0.0/
NNN=3
DO 1000 III=1,NNN
NINC=NINCA(III)
STACUR=STACRA(III)
PHI=PHIA(III)
NEND=NSTART+NINC
FMP=STACUR*MINDIN**3./2.
100 C
BASIC COMPUTATION LOOP TO DETERMINE THE B AND H

C IN THE HYSTERESIS RING AT SUCCESSIVE INTERVALS
DO 65 NT=NSTART,NEND,
T=NT
DO 2 J=1,18
SEG1=J
FMC(J)=FMP-ETHETA*PP*SIN(CWT-PP*SEG1*ETHETA-PHI)
100
FIP(J)=FIC(J)
VHBP(J)=VHBC(J)
VHCP(J)=VHCC(J)
MODEP(J)=MODE(J)
CONTINUE
201 CALL UPDATE
CONTINUE
65 CONTINUE
NT=NEND
FIC(1)=(FIC(2)+FIC(18))*CONTF/2.
DO 7 J=2,17
FIC(J)=(FIC(J+1)+FIC(J-1))*CONTF/2.
CONTINUE
7
FIC(18)=-(FIC(1)+FIC(17))*CONTF/2.
CALL FLXLIN
CALL TORQUE(PP)
CALL LOSS
PSIDES=PSI*180./PI
PHIDES=PHI*180./PI
VINPH=(VOLTS+STACUR*RES*COS(PSI)+STACUR*X1R*SIN(PSI)
VOUTPH=(STACUR*X1R*COS(PSI)-STACUR*RES*SIN(PSI))
VCOMP = SQRT(VINPH**2 + VOUTPH**2)
VRMS = VCOMP / 1.414
CURRMS = STACUR / 1.414
POW1 = TORQ * FREQ * 2 * PI / PP
POW2 = 3. * VOLTS * STACUR * COS(PSI) / 2.
POW3 = 3. * RES * STACUR / STACUR / 2.
POW4 = POW1 - PLOSS
EFF = POW4 / POW2
WRITE (5, 711) NT, STACUR, PHIDEG

711 FORMAT (/12H, TIME STEP =, I3, 27H STATOR CURRENT AMPLITUDE =, 
& F8.2, 16H FLUX ANGLE =, F7.2, 3HDEG)
WRITE (5, 712) VOLTS, PSIDEG, VCOMP

712 FORMAT (/16H, AIR GAP VOLTS =, F8.2, 2H V, 16H AT PHASE ANGLE =, 
& F9.2, 14H TERM. VOLTS =, F8.2, 2H V)
WRITE (5, 713) TORQ, POW1

713 FORMAT (/8H, TORQUE =, E10.4, 4H N-M, 20H POWER FROM TORQUE =, 
& E10.4, 6H WATTS)
WRITE (5, 714) POW2, PERR

714 FORMAT (/8H, ERROR =, F7.2, 1H)
WRITE (5, 715) POW3, PLOSS

715 FORMAT (/25H, STATOR RESISTANCE LOSS =, E10.4, 6H WATTS, 
& 17H PARASITIC LOSS =, E10.4, 6H WATTS)
WRITE (5, 716) POW4, EFF

716 FORMAT (/15H, OUTPUT POWER =, E10.4, 6H WATTS, 
& 13H EFFICIENCY =, F6.1, 1H)
WRITE (5, 12)

12 FORMAT (31H, AIR GAP FLUX DENSITY IN WB/M2)
WRITE (5, 13) (FIGCJ), J = 1, 18

13 FORMAT (9F8.4/9F8.4)
WRITE (5, 14)

14 FORMAT (9H, HYST FLUX DENSITY)
WRITE (5, 15) (FIGCJ), J = 1, 18

15 FORMAT (9F8.4/9F8.4)
DO 152 J = 1, 18
VHSCAL(J) = VH(J) / 1000.

152 CONTINUE

WRITE (6, 150)

150 FORMAT (/29H HYST MAGNETIC FIELD IN KA/M:
WRITE (6, 151) (VHSCAL(J), J=1, 18)

151 FORMAT (8F8.3/8F8.3)

ISTART = NEND

1000 CONTINUE

STOP

END

C 'UPDATE' CALCULATES HYSTERESIS RING FLUX DENSITY
C AND MAGNETIC FIELD STRENGTH FOR EACH SEGMENT OF THE RING.
C CALLS ON SUBROUTINE LOOP.

SUBROUTINE UPDATE

COMMON PI

COMMON /GAP/R0, ALPHA, BETA, TOP, BOT, EF, RIS, PX, QX, RX, SX, TX, UX, VX, W
COMMON /CURR/FM, FIP, VHB, VHCP, REL, VH, VHC, ARC, MODE, MODEP

COMMON /POWER/VH, FI, TORQ, VOL

DIMENSION F1(36), FM(18), VH(36), VHC(18), MODE(18)

DIMENSION F1P(18), VHCP(18), VHCP(18), MODEP(18)

DIMENSION ERROR(18), FIDIST(18)

EPSIL = RIS / 1000.

VHINC = RIS / 10.

D TYPE * 'TEST', ARC

16 J=1

HC = 0

17 N=1

1701 IF (J.EQ.1) GO TO 1702

1702 FIDIST(J) = 2. * F1(J) - F1(J-1) - F1(J+1)

GO TO 1700

1703 FIDIST(1) = 2. * F1(1) + F1(18) - F1(2)

GO TO 1700

1700 FIDIST(18) = 2. * F1(18) - F1(17) - F1(1)

1708 ERROR(J) = CVH(J) - (FM(J) - REL * FI2(J)) / ARC

IF (ABS(ERROR(J)) - EPSIL) 18, 18, 18
18  MC=MC+1.
    IF (MC.EQ.16) GO TO 1930
1800  IF (J.EQ.18) GO TO 1920
    J=J+1
    GO TO 17
19  SG=SIGN11,:ERROR(J>)
1902  IF (N.EQ.1) GO TO 1910
    IF (SSG+SG) 1800,1800,1810
1810  VH(J)=VH(J)-SG*WHINC
    FI(J)=FIP(J)
    VHB(J)=VHB(P(J)
    VHC(J)=VHC(P(J)
    MODE(J)=MODEP(J)
    CALL LOOP (VH(J),FI(J),VHB(J),VHC(J),MODE(J))
    SSG=SG
    N=N+1
    GO TO 1781
1920  WHINC=WHINC+.66
    GO TO 16
1930  RETURN
    END
    FUNCTION XFY(X,Y,S,Y)
    XFY=(YY-Y)/S+X
    RETURN
    END
    FUNCTION YFX(X,Y,S,XX)
    YFX=(XX-X)*S+Y
    RETURN
    END
C SUBROUTINE FLXLIN FIRST CALCULATES THE FLUX LINKED
C WITH A SINGLE TURN COIL WITH SIDES AT J AND J+16.
C AND THEN ADDS THE CONTRIBUTION OF A SET OF COILS TO
C FROM THE TOTAL FLUX LINKED WITH TWO ORTHOGONAL COILS
C FROM THESE THE PEAK PHASE VOLTS AND PHASE ANGLE
C ARE FOUND
C SUBROUTINE FLXLIN.
COMMON PI
COMMON /FLUX/VOLTS, NT, DAREA, PSI, FIG
COMMON /WIND/WINDIN, DTHETA, PHI, FREQ, PP
DIMENSION FIG(36), FL(36)
DO 4 J=19,36
FIG(J) = FIG(J-18)
4 CONTINUE
DO 1 K=1,18
FL(K) = 0.
KEND=K+18
DO 2 J=K, KEND
FL(K) = DAREA*FIG(J)+FL(K)
2 CONTINUE
1 CONTINUE
FLA=0.
FLB=0.
COMMON WINDIN, DTHETA, PP
DO 3 J=1,18
SEG=J
FLA=COMMON*SIN(SEG*DTHETA, PP)*FL(J)+FLA
FLB=COMMON*COS(SEG*DTHETA, PP)*FL(J)+FLB
3 CONTINUE
VOLTS=FLA*FREQ*2.*PI*PP
VOLTSB=FLB*FREQ*2.*PI*PP
VOLTS=SQRT(VOLTS*VOLTS+VOLTSB*VOLTSB)
T=NT
WT=T*PI/10.
ANG=0.
IF (VOLTSB) 10, 15, 7
7 IF (VOLTS) 9, 9, 11
9 ANG=PI
10 ANG=ANG+PI
11 PSID=ATAN(VOLTS/VOLTSB)
12 PSI=PSID-WT+ANG+PHI
13 IF (PSI) 13, 14, 14
13 PSI=PSI+2.*PI
RETURN
15  PSID=PI-SIGN(PI/2.),VOLTA
      ANG=PI
      GO TO 12
END
SUBROUTINE TORQUE(POLPAR)
   "TORQUE" CALCULATES THE HYSTERESIS LOOP AREA
   FORMED BY THE B-H DISTRIBUTION IN THE RING
   COMMON PI
   COMMON /POWER/VH,FI,TORO,VOL.
   DIMENSION VH(36),FI(36)
   DO 4 J=19,36
      VH(J)=VH(J-18)
      FI(J)=FI(J-18)
   CONTINUE
   TORQA=POLPAR*VOL*(VH(1)-VH(36))*(FI(1)+FI(36))/(4.*PI)
   DO 1 J=2,36
      TORQ2=(VH(J)-VH(J-1))*(FI(J)+FI(J-1))/(4.*PI)
      TORQ=TORQ+POLPAR*VOL*VH(J)
   CONTINUE
   RETURN
   END
   'LOOS' ESTIMATES PARASITIC LOSS IN THE
   HYSTERESIS RING WHEN THE MOTOR IS IN
   THE REGION OF SYNCHRONISM. INCLUDES FLUX
   PARASITIC LOOS ONLY
   SUBROUTINE LOSS
   COMMON /WIND/WINDIN_DTHETA,PHI,FREQ,PP
   COMMON /PARAS/6A/R1A,CONLOS,LLOSS
   COMMON /FLUX/VOLTSP,NT,DAREA,PSI,FIG
   DIMENSION FIG(36)
   PLOSS=0.
   DO 1 J=1,36
      PLOSS=PLOSS+CONLOS*FIG(J)*FIG(J)
   CONTINUE
   RETURN
SUBROUTINE LOOP IS PARALLELOGRAM MODEL OF THE Hysteresis loop including minor, recoil, loops
SUBROUTINE LOOP (X,Y,B,C,M)

COMMON PI
COMMON GAP.RO.,ALPHA.,BETA.,TOP.,BOT.,EF.,RIG.,PX.,RX.,SX.,TX.,UX.,VX.,W
IF (M.GE.2) GO TO 41
IF (M.LE.-2) GO TO 51
AT FIRST DEALS WITH MOMPES -1,0, AND 1 ONLY
IF (X.LT.B) GO TO 36
IF (X.LT.C) GO TO 35
IF (M.NE.-1) GO TO 34
IF (X.LT.VX) GO TO 33
IF (X.LT.WX) GO TO 32
Y=YFX(G.,TOP.,ALPHA.,X)
B=X
C=X
M=4
RETURN

32  Y=YFX(RIG.,0.,BETA.,X)
C=X
B=XFY(G.,TOP.,ALPHA.,Y)
M=3
RETURN

33  Y=YFX(RIG.,0.,BETA.,X)
C=X
B=X-RO
M=1
RETURN

34  D=(Y-ALPHA.M+BETA.M*.RIG.)/(BETA.-ALPHA.)
IF (X.GT.D) GO TO 31
Y=YFX(C.,Y.,ALPHA.,X)
C=X
B=X-2.,RO
B1=XFYCEF.,0.,BETA.,Y)
IF (B.GT.B1) GO TO 348
B=B0
340 H=0
RETURN
35 RETURN
36 IF (M.NE.1) GO TO 40
37 IF (X.GT.OX) GO TO 38
38 Y=YFX(0.,BOT,ALPHA,X)
B=X
C=X
H=-4
RETURN
39 Y=YFX(EF,0.,BETA,X)
B=X
C=XYF(0.,BOT,ALPHA,Y)
H=-3
RETURN
30 Y=YFX(EF,0.,BETA,X)
B=X
C=X+RO.
H=1
RETURN
40 A=(CY-ALPHA*B+BETA*EF)/(BETA-ALPHA)
IF (X.LT.A) GO TO 37
Y=YFX(B,Y,ALPHA,X)
B=X
C=X+2.*RO
C1=XYF(RIG,0.,BETA,Y)
IF (C.LT.C1) GO TO 400
C=C1
400 M=0
RETURN
C DEALS WITH MODES 2,9 AND 4.
41 IF (X.LT.B) GO TO 46
42 Y=YFX(0.,TOP,ALPHA,X)
B=X
C=X
M=4
RETURN
43    IF (X.LT.C) RETURN
44    IF (M.NE.3) GO TO 45
45    IF (X.GT.WX) GO TO 42
        Y=YFX(CIG,0,BETA,X)
        C=X
        B=XFY(0,TOP,ALPHA,Y)
        M=3
        RETURN
46    IF (X.GT.VX) GO TO 44
47    Y=YFX(C,Y,ALPHA,X)
        C=X
        B=X-RO
        RETURN
48    IF (X.GT.WX) GO TO 42
49    IF (X.GT.UX) GO TO 50
50    IF (X.GT.SX) GO TO 49
51    IF (X.GT.QX) GO TO 48
52    IF (X.GT.PX) GO TO 47
        Y=YFX(0,BOT,ALPHA,X)
        B=X
        C=X
        M=4
        RETURN
53    Y=YFX(CEF,0,BETA,X)
        B=X
        C=XFY(0,BOT,ALPHA,Y)
        M=3
        RETURN
54    Y=YFX(CEF,0,BETA,X)
        B=X
        C=X+RO
        M=1
RETURN
40  \( Y = YFX(0..TOP, \alpha, X) \)
    \( B = X \)
    \( C = X + RO \)
    \( M = 2 \)
    RETURN
60  \( Y = YFX(0..TOP, \alpha, X) \)
    \( B = X \)
    \( C = XFY(RIG, 0..BETA, Y) \)
    \( M = 3 \)
    RETURN
C DEALS WITH MODES -2, -3 AND -4
51  IF (X.LT.B) GO TO 56
    IF (M.NE.-4) GO TO 511
    IF (X.LT.PX) GO TO 58
510 IF (X.LT.RX) GO TO 56
    IF (X.LT.TX) GO TO 54
    IF (X.LT.VX) GO TO 53
    IF (X.LT.WX) GO TO 52
    \( Y = YFX(0..TOP, \alpha, X) \)
    \( B = X \)
    \( C = X \)
    \( M = 4 \)
    RETURN
511 IF (X.GT.C) GO TO 512
    RETURN
512 GO TO 510
52  \( Y = YFX(RIG, 0..BETA, X) \)
    \( C = X \)
    \( B = XFY(0..TOP, \alpha, Y) \)
    \( M = 3 \)
    RETURN
53  \( Y = YFX(RIG, 0..BETA, X) \)
    \( C = X \)
    \( B = X - RO \)
    \( N = 1 \)
RETURN
54 \( y = yfx(0..bot., \alpha, x) \)
\( c = x \)
\( b = x - xo \)
\( m = -2 \)
RETURN
55 \( y = yfx(0..bot., \alpha, x) \)
\( c = x \)
\( b = xyf(ef, 0..beta, y) \)
\( m = -3 \)
RETURN
56 IF (M. EQ. -4) GO TO 58
IF (M. EQ. -3) GO TO 57
IF (X. GT. XX) GO TO 60
57 IF (X. GT. PX) GO TO 59
58 \( y = yfx(0..bot., \alpha, x) \)
\( b = x \)
\( c = x \)
\( m = -4 \)
RETURN
59 \( y = yfx(ef, 0..beta, x) \)
\( b = x \)
\( c = xyf(0..bot., \alpha, y) \)
\( m = -3 \)
RETURN
60 \( y = yfx(b, y, \alpha, x) \)
\( b = x \)
\( c = x + xo \)
RETURN
END