RESOURCE PORTFOLIO

FOR USE WITH COURSE 1 OF THE PROPOSED ATLANTIC CURRICULUM FOR SENIOR HIGH MATHEMATICS

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Resource Portfolio

For use with Course 1 of the proposed Atlantic curriculum for

Senior High Mathematics

by

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A project submitted to the School of Graduate Studies

in partial fulfillment of the requirements for the degree of

Master of Education

Faculty of Education

Memorial University of Newfoundland

May 1996

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Part 1

Overview of Project and Literature Review

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INTRODUCTION

There are many forces at work in our society which are profoundly changing the face of mathematics - how it is taught and how it is assimilated by students. In recent times the advent of computer technology, graphing utilities, electronic spreadsheets, numerical analysis packages, and inexpensive electronic calculators has caused a re-examining of old traditions in the mathematics classroom and has prompted a need for curricular changes which harmonize with the new technology. Mathematics educators are called upon to prepare students for the twenty-first century by teaching mathematics in a new and different way, in a way which stimulates and benefits not only college-bound students, but all other students entrusted to the school system.

This call for renewal did not come solely with the 90's - rather it was felt much earlier with the realization that the new mathematics was serving the needs of only a few and was not a mathematics for all. A majority of students simply opted out of the subject by either accepting failure as inevitable or by moving to a more basic, remedial type of mathematics. It was felt that students were not learning a form of mathematics that could help them become productive members of society. As newer and better technology was introduced which began to change the face of society and which could handle many of the tedious tasks of mathematics classrooms the call for reform was further strengthened. That call is felt today more than ever before.

The call includes the teaching of nontraditional mathematics topics, such as statistics and probability, which more adequately prepare students for the mathematical needs of modern society. It means teachers have to place less emphasis on training students in procedures and algorithms in arithmetic and algebra, as has traditionally been the case, and place more emphasis on using the new technology to create better conceptual understanding of mathematics.

This project offers support for that change within the new curriculum which is being planned for Newfoundland and Labrador schools, as per Course 1 objectives of the proposed Atlantic curriculum. A brief overview of the project is given, followed by a literature review concentrating on the call for change in mathematics, stemming back to the 1980's. The review will examine why it was felt change was necessary and the nature of the proposed changes. In particular this literature review will focus on the role of the National Council of Teachers of Mathematics (NCTM) in the movement and their leadership in effecting change.

Brief Overview of Project

The main thrust of this project is the development of a teacher resource for Course 1 (Grade 10) of the new Atlantic curriculum for mathematics, including ideas which may be applicable to the present Gr. 10 curriculum as outlined in *Math 1300 - Course Description* (Government of Nfld & Labrador, 1992). It is being completed at a time when the secondary mathematics curriculum for Newfoundland and Labrador schools is on the threshold of revision. Plans are ongoing to implement a new common curriculum in mathematics in the high schools of the four Atlantic provinces by September, 1997. Piloting is scheduled to take place in Newfoundland and Labrador schools in September, 1996. The curriculum's agenda is based strongly on the recommendations of the NCTM *Standards* (NCTM,1989).

Many high school mathematics teachers today would probably agree that mathematics textbooks at the high school level do not contain the various types of learning activities that are recommended by the National Council of Teachers of Mathematics in documents such as the NCTM *Standards* (NCTM, 1989). As a result, activity-based sessions depend upon teacher time to find appropriate materials to make mathematics more interesting and applicable. The new curriculum will more adequately reflect these standards, and this project is intended to be in harmony with that curriculum.

The two main parts of this project are:

- a literature review of the new movement in mathematics stemming back to the 1980's. The ideas in this review provide rationale for this project.

- a resource portfolio which includes (1) current topics in secondary mathematics in general - cooperative learning, graphing calculators, and spreadsheets and (2) activities and ideas, some original, some referenced, which may prove useful in implementing the objectives of Course 1 of the Atlantic curriculum.

Activities in this project will be placed on web pages of the World Wide Web to offer on-line the ideas put forth in the resource. As well links will be made to other Internet material pertinent to topics in Course 1.

A Review of the Literature

A review of the literature of the 1980's reveals that tremendous efforts have been made in several countries to develop a mathematics curriculum adequate to meet the needs of a majority of students. National and international studies and inquiries have been conducted into mathematics education to address topics such as: the relationship of mathematics to its applications, teaching mathematics to non-mathematicians, using technology in the teaching of mathematics, introducing the real world into mathematics classes, women and mathematics, and teaching mathematics to the low and high achievers. One very extensive study into the teaching of mathematics was conducted in the early 80's in Great Britain. The report on this study - *Mathematics counts. Report of the committee of enquiry into the teaching of mathematics in schools* (Great Britain, Department of Education and Science, 1982), also known as *The Cockcroft Report* is very applicable today.

The Cockcroft Report - Great Britain

The Cockcroft Report (1982) resulted from an inquiry into the teaching of mathematics in schools in Great Britain. This inquiry emerged because of concern over low attainment in mathematics in many of the country's schools. One of the fundamental statements made in this report is that "Low attainment in mathematics can occur in children whose general ability is not low" (The Cockcroft Report, 1982, p.98). Some of the factors contributing to this, according to the report, include inappropriate teaching of mathematics, lack of confidence in the subject by the students, and lack of continuity. It points out the isolation in which mathematics is often taught and the excessive preoccupation with sequences of skills which do not arise from any specific problems and which do not lead to the learning of incidental information.

> Mathematics lessons in secondary schools are very often not about anything. You collect like terms, or learn the laws of indices, with no perception of why anyone needs to do these things. . . . in mathematics the incidental information which one might expect (current exchange and interest rates, general knowledge of climate, communications and geography, the rules and scoring systems of games, social statistics) is rarely there because most

> teachers in no way see this as part of their responsibility when teaching mathematics. We believe that this points out in a very succinct way the need . . . to relate the content of the mathematics course to pupils' experience of everyday life (The Cockcroft Report, 1982, pp. 8-9).

The report indicates that mathematics teachers need to be aware of the mathematical techniques that are required for the study of other subjects and aware that the curriculum should be designed so that students will be familiar with the necessary mathematical topics before they are encountered in other areas. As well, the report stresses the need for a liaison between teachers so that those who need to use mathematics in other subject areas will do so with an approach that does not conflict with that which is used in the mathematics classroom. The report emphasizes the importance of mathematics as a means of communication, and highlights the need for students to be able to understand and use graphs, charts, and tables in non-mathematical curricular areas: "Teachers of other subjects, as well as mathematics teachers, need to be aware of the part which mathematics can play in presenting information with clarity and economy, and to encourage pupils to make use of mathematics for this purpose" (p. 9).

The Cockcroft Report (1982) outlines some major changes which should take place in the teaching of mathematics. It calls for changes in the way teachers present mathematics, for more discussion of mathematical topics between teachers and students, and between students themselves, for more practical work and more application of mathematics to everyday situations, and for more investigations and making of generalizations by the students.

With respect to the use of calculators, the report states that the advantages more than compensate for any possible disadvantages, and suggests that the use of calculators has not produced any adverse effect on computational ability. It stresses that "the availability of a calculator in no way reduces the need for mathematical understanding on the part of the person using it" (p. 111).

With respect to the low attainment of mathematics in Great Britain, Ahmed (1984), one of the members of the Committee of Inquiry that produced the Cockcroft Report, feels there are probably several reasons why ineffective systems exist and why teachers get emersed in them. The reasons are very complex, he feels, and often teachers are aware of the failure of the system they are working in. He feels that their perception of the constraints which force them to operate in a certain way, such as class size, time, class disruptions, and lack of motivation by pupils, is often misleading and mixed up. He sees the need for teachers to have collections of "situations" and investigations to offer to pupils. These should be structured so that they provide an opportunity for follow-up work for those who are competent and interested.

The National Commission on Excellence in Education - U.S.

The need for change in mathematics education in Great Britain in the 1980's was echoed by the National Commission on Excellence in Education in the United States. A report by the Commission (1983) states that "Our nation is at risk. Our once unchallenged preeminence in commerce, industry, science, and technological innovation is being overtaken by competitors throughout our world" (p. 5). The report cites several indicators to support the statements made. Those involving mathematics included evidence that many 17-year olds did not possess higher order intellectual skills, nearly 40% could not draw inferences from written material, and only one-third could solve a mathematics problem requiring several steps. It states: "Between 1975 and 1980, remedial mathematics courses in public 4-year colleges increased by 72 percent and now constitute one-quarter of all mathematics courses taught in those institutions" (p. 9). The report recommends that the mathematics students learn in high school should equip them to:

(a) understand geometric and algebraic concepts

(b) understand elementary statistics and probability

(c) apply mathematics in everyday situations

(d) estimate, approximate, measure, and test the accuracy of their calculations (p. 25).

In addition to the traditional sequence of studies available for college-bound students, a new and equally demanding mathematics curriculum was recommended for those who did not plan to continue their formal education immediately following high school. The report's recommendations are based on the beliefs that "everyone can learn, that everyone is born with an urge to learn which can be nurtured, that a solid high school education is within the reach of virtually all, and that life-long learning will equip people with the skills required for new careers and for citizenship" (p. 24).

The National Council of Teachers of Mathematics (NCTM)

The National Council of Teachers of Mathematics (NCTM) has been advocating reform in mathematics education since it became apparent that the "new mathematics" of the 60's and 70's did not provide the type of change needed. In 1980 they issued *An Agenda for Action*, a set of recommendations intended to provide a framework for the improvement of school mathematics in the 1980's. It was recommended that more mathematics study be required for all students, with a greater range of options to accommodate the diverse needs of the student population. One of the major goals was the development of problem solving skills. As well, emphasis was placed on using calculators and computers at all grade levels. This was not to imply computational skills were obsolete; rather, these skills would be complemented with more than computational facility. It was felt that calculators and computers should be used in imaginative ways for exploring, discovering, and developing mathematical concepts. At the same time, critical classroom interaction of students with peers and teachers should not be replaced by the use of these tools in isolated activities.

The NCTM *Curriculum and Evaluation Standards for School Mathematics* (1989) represents the first effort to establish national standards for school mathematics. What is significant about this document is the fact that it is the work of mathematics teachers from across the nation. The standards put forth were intended as "statements of criteria for excellence" and as "facilitators of reform" to help teachers prepare students for the 21st century.

It is recognized by NCTM that the educational goals of the past era, namely the industrial age, are not in tune with the economic needs of today's information society. The *Standards* (NCTM, 1989) recognizes new goals for education which include the following:

> *Mathematically literate workers* - Workers must be competent in technology and communication and understand the mathematics involved in various problems in today's work world.

> *Lifelong learning* - School mathematics must equip students with skills such as problem solving which will help them accommodate to changed conditions over their lifetime.

> *Opportunity for all* - Women and minority ethnic groups must study more mathematics and be more represented in science and technical careers.

> *Informed electorate* - Citizens must be able to understand issues in a technological society.

The five goals that NCTM (1989) articulates for all students are that they:

1. learn to value math
2. become confident in their ability to do math
3. become mathematical problem solvers
4. learn to communicate mathematically
5. learn to reason mathematically

Some of the goals echo those of *The Cockcroft Report* (1982). Students, for example, should be encouraged to read, write, and discuss mathematics; to develop mathematical habits of mind; to understand the role of mathematics in human affairs; and above all to develop mathematical power. NCTM (1989) feels that these goals are very important and are in contrast to the way students have been taught mathematics for decades. Traditionally, the teacher has been in complete control, discussion of mathematics topics has been limited, and students have learned to do mathematics by imitating examples placed on the chalkboard by teachers. NCTM (1989) feels that this trend should not continue in today's classrooms and that the conceptual understanding of mathematics should take precedence over the practice of algorithms. The 9 - 12 standards call for "a shift in emphasis from a curriculum dominated by memorization of isolated facts and procedures . . . to one that emphasizes conceptual understandings, multiple representations and connections, mathematical modelling, and mathematical problem solving" (NCTM, 1989, p. 125).

High school mathematics has historically served the needs of a small group of students - namely those college-bound or interested in studying pure mathematics. The *Standards* (NCTM, 1989) document advocates a mathematics program in secondary schools which will serve the needs of all students when they graduate by making mathematics mc,e applicable to their daily living. By having them explore genuine world problems; understand mathematical models, structures, and simulations; and use statistics to examine data and graphs to explore functions, students will be better equipped to understand many real life phenomena. Access to appropriate calculators and computers and appropriate use of them for investigation and exploration, the Council feels, would help in their overall understanding of concepts. The emphasis they recommend is one that shifts from "knowing math" to "doing math", with a stress on activities that grow out of problem situations.

National Research Council

At the same time that the NCTM *Standards* (1989) were being introduced, the National Research Council (1989) was responding to the evidence that "current mathematical achievement of U.S. students is nowhere near what is required to sustain our nation's leadership in a global technological society" (p. 1). It was felt that a complacent America was tolerating underachievement and minimum expectations in mathematics education and that something needed to happen to sufficiently prepare students for secondary requirements and on-the-job demands. The National Research Council felt that "Children can succeed in mathematics. If more is expected, more will be achieved" (p. 2).

Research on learning, according to the National Research Council (1989), shows that most students cannot effectively learn mathematics by merely listening and imitating. Ironically, most mathematics is taught that way. The Council suggests that mathematics becomes useful to a student only when it has been developed through personal engagement. Students need to construct proper understandings of mathematical concepts. The teaching of mathematics, the National Research Council posits, is often inappropriate for the way most students learn.

More than any other subject, mathematics screens students out of some scientific and professional careers, and, according to the National Research Council (1989) it is the "worst curricular villain in driving students to failure in school" (p. 7). The National Research Council (1989) stresses the importance of high-quality mathematics for all citizens of society and feels that mathematical literacy is essential as a foundation for democracy in a technological age: "Discussion of important health and environmental issues (acid rain, waste management, greenhouse effect) is impossible without the language of mathematics; solutions to these problems will require a public consensus built on the social fabric of literacy" (p. 8). According to the National Research Council (1989) high school students should know enough about chance to understand health and environmental risks, should know enough mathematics to understand investments, should be able to study data and draw scientific conclusions, and should be able to interpret graphs, even if they do not pursue further mathematics.

Following the publication of their report to the nation and the publication of NCTM's *Standards* (1989) document, the National Research Council (1990) proposed a framework for reform in mathematics education. They promoted a complete redesign of the content of school mathematics, based on the goals set forth by NCTM (1989). The rationale for change put forth by the National Research Council reflects earlier thoughts. Mathematics education, it feels, must change for various reasons including:

(1) *Changes in the need for mathematics* - As the economy shifts from one of an industrial base to an information base, workers will need more analytical skills, as opposed to the traditional mechanical skills. They will need to know how to integrate graphs and analyse the statistical data used by marketing companies, businesses, etc.

(2) *Changes in mathematics and how it is used* - The advent of computers, for example, has led to applications of mathematics that were once not practical.

(3) *Changes in the role of technology* - Computers and calculators are profoundly affecting how mathematics is taught and have caused a need for readjustment in the teaching approach to almost every math topic taught.

(4) *Changes in American society* - As mathematics and technology changes, the workplace demands that schools change. Most jobs of the next century will require more mathematical skills than ever before

(5) *Changes in understanding of how students learn* - Learning is a process of assimilating new information with prior knowledge and constructing meaning in an active way and thus mathematics education must reflect that.

(6) *Changes in international competitiveness* - Students in other countries have taken the lead over those in the U.S. in terms of mathematical accomplishments. Other countries differ from the U.S. in terms of math content taught and math expectations.

The National Research Council (1990) emphasizes the need for students to be able to analyse the quantitative data that is so prominent in today's society - graphs, charts, and statistics. A great deal of societal decisions are made from interpreting such data. At the same time, they feel that cultural and historical aspects of mathematics should not be ignored.

Group Learning in Mathematics

The NCTM *Curriculum and Evaluation Standards* (1989) points to the need for variety in approaches to instruction, including opportunities for discussion of mathematical concepts and problems among students, as well as project work and group assignments: "Active student participation in learning through individual and small-group explorations provides multiple opportunities for discussion, questioning, listening, and summarizing" (p. 140). Being able to communicate mathematically is viewed as one of the keys to understanding mathematics. One of the ways to do this is through group learning.

Over the years different forms of group learning have no doubt been used in mathematics classrooms for peer teaching, group projects, laboratory activities, etc. Students have been assigned to groups to work on tasks such as mathematics assignments, problem sets, or review sheets. The trend, however, has not been very widespread, for most mathematics teaching, according to the *Standards* (1989), has been in the form of the lecture approach. While some mathematics educators may have experienced high success with groups, others no doubt have had opposite experiences. Those teachers with negative group experiences would probably agree that problems such as low quality interactions among students, lack of interest by under-achievers, domination by some group members, poor attitudes from others, and cognitive barriers have made them feel very uncomfortable about groups in their classrooms and have discouraged their use. To add to this, many would probably agree that determining the progress of individuals in traditional type groups has been very difficult. Problems such as these can not only threaten teacher control but may prevent meaningful learning. It was not until the introduction of cooperative learning in recent years that the whole idea of group learning has been viewed through a different set of lenses and has taken on new meaning .

The term *cooperative learning* refers to an instructional strategy whereby students of all ability levels work together in small groups to achieve a common learning goal. It represents a more structured form of group learning and has been advocated for use in practically any subject and any grade level as an option to traditional learning and traditional-type groups. All students share responsibility for learning as they help each other achieve success. Slavin (1987) sees cooperative methods as very viable alternatives to the traditional classroom techniques in which competition for grades and other rewards by one student could reduce the chances of success of another. He cites three concepts which are central to all types of student team learning and cooperative learning - team rewards, individual accountability, and equal opportunities for success. Students of low ability contribute to the success of others by improving over their past performances. All types of achievers are challenged to do their best. Slavin (1987) has indicated that if students are rewarded for doing better than they themselves have done in the past, their motivation to achieve will be greater than it would be if their rewards were based on their performance compared with that of others. Rewards for individual improvement make success neither too difficult nor too easy for students to achieve. Allowing this improvement to be used to gain points for a team increases the motivation to learn all the more.

Slavin (1991) describes several student team learning methods which he has studied and advocated for use with cooperative groups since the 1980's. Two of these, Student Teams-Achievement Divisions (STAD) and Teams-Games-Tournament (TGT), are applicable for subjects at the secondary school level and provide a framework for group work . In STAD students are assigned to four- or five-member teams, each made up of students mixed in performance level, sex, and ethnicity. The teams study material presented by the teacher, using whatever method they wish to master the material, until all members understand what has been taught. Following practice sessions, students take quizzes on the material they have been studying. They are individually accountable for the quizzes and thus not allowed to help one another. Team scores are compiled from the individual results, with each student's improvement since the last quiz used in determining team results. Teams with the highest scores are recognized in weekly one-page class newsletters. According to Slavin (1991), the change in the classroom is dramatic. Students begin helping each other instead of resenting those who are high achievers and intimidating those who are weak. They see learning activities as being sociable and fun, and somewhat under their control.

The second procedure applicable for secondary students is the Teams-Games-Tournament, a method whereby students play academic games to show their mastery of the subject matter. Competitions take place that use tournament tables of three students, all comparable in performance. Low achievers compete with low achievers, high compete with high, etc. so that the competition is always fair. Even though teams stay together for about six weeks, the tournament table assignments change every week according to a system that maintains the equality of the competition. Fair and equal competition makes it possible for students of all levels to contribute maximum points to their teams if they try hard. As with STAD, weekly publications highlight high-scoring teams and table winners. The teaching pattern, team practice, assessment, and equal opportunities for success are similar to those for STAD. According to Slavin (1991), the use of academic games and quizzes makes this method more exciting that of STAD. He cites one case in which students stayed after school and missed their buses to attend a playoff. Teachers, he says, have reported that students normally not interested in school have shown special interest in their studies after becoming involved in TGT, even to the point of coming after class for materials to take home to study.

The two examples mentioned above, STAD and TGT, are merely two of the ways suggested to set up groups in secondary classrooms, which could be effective for mathematics learning as well as for learning in other subjects. Another effective way to set up groups, as cited in the literature (Aronson, 1978, cited in Artzt & Newman, 1990), is the Jigsaw method. Using Jigsaw (or some variation thereof) the teacher gives students a common task to handle and then assigns subsections of that task to each individual so he/she may become the expert in that section. Students with the same subsections meet in expert groups to discuss them, and return to their own teams to teach their teammates all that they have gleaned. Then students take individual quizzes, the results of which are compiled into team scores. As with STAD, the improvement score system is used in determining team scores.

A group learning technique called Learning Together (Johnson & Johnson, 1975, cited in Artzt & Newman, 1990) has been supported as an effective learning strategy in the mathematics classroom. Using this method, students meet in heterogeneous groups of four or five members and work on assignment sheets. When the group has agreed to the solutions, a single answer sheet is submitted for correction to represent the group's work. In order for group learning to have taken place, Johnson & Johnson stress that the following conditions must exist: positive interdependence, face-to-face interactions, social skills, individual accountability, and group processing.

Hilke (1990) sees cooperative learning as a method which encourages supportive relationships, good communication skills, and higher-level thinking abilities. The goals of cooperative learning, she says, are to foster academic cooperation among students, encourage positive group relationships, develop students' self-esteem, and enhance academic achievement. The peer academic support, according to Hilke, is not available in other types of learning. She feels that if teachers carefully plan the process and teach the appropriate skills to students, the method could be very effective. The social skills which must be taught include careful listening, taking turns to ensure participation by everyone, respect for other viewpoints, and assistance to those with difficulty.

As they learn to discuss the concepts, ask questions, help others, and become individually accountable, their self-esteem will undoubtedly increase. According to

Weissglass (1990), "...the small group approach changes the teacher's focus from being answer-oriented to being process-oriented. The teacher no longer collects worksheets to assess students' progress; instead, the teacher becomes an observer and facilitator of small-group interaction, paying attention to the ongoing learning process" (p. 307). Weissglass has found this to be both rewarding and frustrating, noting that when he lectures it is often easy to isolate himself from reality and ignore the fact that many students do not understand the lesson. In small groups, however, he can see what it is they don't understand and help them with it.

At the University of Idaho, Keeler & Steinhorst (1994), who applied cooperative learning to an introductory course in statistics to those students who had the poorest background and motivation, reported very favourable results. Students who completed the course in cooperative groups had higher mean final score averages than other students taught in the traditional manner. As well, the percentage of students completing the course was higher. Students showed much enthusiasm for group activity (especially working in pairs) and thought the process focused their attention on what was important. The results suggest that students become more actively engaged and learn better when placed in cooperative groups.

Artzt & Newman (1990) feel that in order for cooperative learning to be successful, group members need to learn the proper skills. The students in a group must perceive themselves as dependent on one another, have certain expectations of each other, and be concerned for all members. They must talk to one another, explain, communicate and justify ideas and when necessary engage in intellectual conflict. Each member must feel comfortable about speaking and be given the opportunity to do so. Members must develop collaborative skills that enable them to work together effectively, no matter what ability levels are present. As well, according to Artzt & Newman, students must be willing to share responsibility for one another's learning as well as be individually accountable for their own learning. Incentives and rewards are important intrinsic motivators, as are the social aspects and togetherness that students experience.

According to Slavin (1991), cooperative learning is a very thoroughly researched instructional method and reviews usually indicate a positive effect on student achievement when two essential features, group goals and individual accountability, are present. Simply asking students to work together is not enough, he claims. The group's success must depend on the individual learning of all members. His own research efforts, as well as those of others (Newman & Thompson, 1987; Davidson, 1985, cited in Slavin, 1991) show that when these elements are present, cooperative learning can be a very effective means of increasing student achievement.

In the mathematics classroom there are special opportunities for effective learning by small heterogeneous groups of the type mentioned. Different students have different approaches to solving mathematics problems. In sharing, discussing, and communicating these approaches with one another, they can develop new insights and perspectives which otherwise would not be learned. The call for reform in mathematics education (NCTM, 1989) emphasizes more communicating of ideas in an activity-based approach. Topics such as statistics and probability, data analysis, functions, geometry, and algebra can be explored in small groups using concrete, diagrammatic, or computer models. The small-group approach will allow the teacher time to interact with the students, observe their approaches to problems, and offer suggestions where possible. When mathematics is learned in a more relaxed setting, there is less possibility that students will develop anxiety towards the subject. An example of a group learning activity in mathematics which contains some cooperative learning ideas is given in the resource portfolio section (part 2) of this project.

Graphing Calculators and Mathematics

Graphing calculators have become known as very functional, practical, and relatively affordable tools for teaching and learning mathematics. The position of NCTM with respect to their use is made clear in the *Curriculum and Evaluation Standards* (1989). It states that calculators (and computers) should be available to students at all times, but should be used wisely.

> Calculators and computers for users of mathematics, like word processors for writers, are tools that simplify, but do not accomplish the work at hand. Thus, our vision of school mathematics is based on the fundamental mathematics students will need, not just on the technological training that will facilitate the use of that mathematics. ... the use of calculators does not eliminate the need for students to learn algorithms. Some proficiency with paper-and-pencil computational algorithms is important, but such knowledge should grow out of problem situations that have given rise to the need for such algorithms." (p. 8)

One of the dominant features of graphics calculators is their ability to display graphs of algebraic functions and statistics and data of real life phenomena.

Students can zoom, plot, and trace as they inspect graphs, investigate patterns and draw conclusions. All operations can be performed with great speed, allowing more time for further exploration and investigation. As stated by NCTM (1989), students learn to establish links between algebraic and graphical representations, crystallizing their understanding of functional concepts. Since several graphs can be viewed at once, comparisons can be made and intersections noted. The effect of making changes to algebraic parameters can be seen instantly in graphical form, giving students a broader picture of the concept of algebraic functions and providing more opportunity for conceptual understanding, as recommended by NCTM *Standards* (1989).

Graphing calculators can be used to solve problems that were once either unsolvable algebraically or too difficult to solve. Interesting real-world situations can be analysed and explored. Systems of linear equations can be solved graphically by zooming in on the intersection of the graphs and zeros of polynomial functions can be found by checking x-intercepts. As recommended by NCTM (1989), this is not to imply that the device should be used to do all the student's work; rather it should be used to complement student efforts and help students more fully understand algebraic processes and concepts.

Research results which support the perceived benefits of graphics calculators is relatively new. Wilson and Krapfl (1994) reviewed several reports addressing the relationship between graphics calculators used by secondary students and their understanding of functions. These reports are nearly unanimous in their claim that graphics calculators are beneficial to students, one of their chief benefits being their ability to conceptually connect alternative representations of functional ideas. One study reviewed by Wilson and Krapfl was conducted by Ruthven (1990, cited in Wilson & Krapfl, 1994) who compared the performance of 47 secondary students in England who used graphics calculators in a year of pre-calculus study with the performance of 40 students, similar in background, who did not have access to graphing technology. The study revealed that students with access to the calculators were better able to make links between graphic and algebraic representation of functions. As well, it showed that performance by female students was enhanced even more than that of males.

One of the results of a study by Rich (1990, cited in Wilson & Krapfl, 1994) was that students using graphics calculators viewed graphs more globally. That is, they understood the importance of domain, intervals where the function increases and decreases, asymptotic behaviour, and end behaviour. Another study reviewed by Wilson & Krapfl (Dick & Shaughnessy, 1988, cited in Wilson & Krapfl, 1994) showed that students not only improved in math performance but in their disposition. It was found that some students were impressed by the power of calculators, while others found them to make mathematics learning more enjoyable. As well, calculators seemed to reduce anxiety and diminish uncertainty.

The article by Wilson & Krapfl (1994) reflects NCTM's (1989) call for change in the mathematics teaching and learning environment. NCTM (1989) claims that the suitable use of graphing technology will "transform the mathematics classroom into a laboratory... where students use technology to investigate, conjecture, and verify their findings....and the teacher encourages experimentation" (p. 128). Prior to graphics calculators exploration by students was very difficult, if not impossible. According to Wilson & Krapfl (1994) their availability has the potential to change the mathematics classroom to one of less lecturing and less tedious manual graphing, to more problem solving, more discovery learning, more investigating, and more higher order questioning.

Dunham and Dick (1994) found encouraging results when reviewing experimental studies on the use of graphing calculators. One study by Harvey (1993, cited in Dunham & Dick, 1994) analysed data from a 1988-1989 extensive field test of a graphing-intensive curriculum, the Computer and Calculator in PreCalculus Project.

In comparing mean scores on a "calculus readiness" test for fifty-five schools using graphing technology with those of twenty-two control schools with traditional pre-calculus courses, Harvey found significant differences favouring the graphing-intensive curriculum. Osborne (1992, cited in Dunham & Dick, 1994) used the same data as above and found that those students receiving the calculator instruction subsequently attained calculus placement on the Calculus Readiness posttest at nearly twice the rate of those receiving traditional instruction.

Research has found some problems with the use of graphics calculators which need to be highlighted at this point. Williams (1993, cited in Wilson & Krapfl, 1994) found that graphics calculators can be confusing for some students, even after they have been given instruction and experimentation time. Some had serious misconceptions about the scaling features of calculators. Just because something is on the screen doesn't mean students see or make sense of it. Experts sometime see relationships that are missed by students who sometimes make erroneous connections or draw invalid conclusions. Another pitfall highlighted in this research is that sometimes graphs are not represented accurately. It may be difficult, if not impossible, to detect points of discontinuity. Some screens do not have enough pixels to accurately represent graphs. Another concern which Wilson & Krapfl (1994) point out is the tendency to rely too heavily on graphs. Some exact solutions can be found only by algebraic methods, (eg. solving systems of equations), whereas the graphical representation may be an approximation which can be used to verify algebraic solutions. Methods for finding *exact* solutions should not be abandoned. As well, students should not feel that all numbers are rational since they appear that way on graphical displays. Underlying mathematical principles cannot be ignored while using these powerful tools. With awareness of potential problems, educators can indeed reduce their effects and allow the device to have maximum positive impact on learning.

There are obviously many ways in which graphics calculators can be used in the high school curriculum. Several of these are given in the resource portfolio section of this project.

Spreadsheets and Mathematics

A spreadsheet is a very large rectangular array of cells which can be used to represent a mathematical environment. According to NCTM (1995) the use of computer spreadsheets in the secondary mathematics classroom is becoming more widespread as teachers realize their potential for facilitating the study of a variety of topics. Many such programs, such as Microsoft Excel and Lotus for Windows have some very powerful, yet user-friendly options, including easy formula copying, chart wizards, and graphical menus. Their very extensive capabilities facilitate representation of functions in various graphical and tabular forms. Line graphs, bar graphs, scatterplots, and 3-dimensional displays are but a few of the possible ways to represent data.

NCTM (1989) emphasizes the importance of visualization in mathematics learning, as well as the forming of conjectures, guessing, exploring, the making of generalizations and problem solving. Dugdale (1994) feels that spreadsheets provide opportunities for all of these if used appropriately in the mathematics classroom. According to Dugdale (1994), "a traditional solution answers only to specific questions, whereas a spreadsheet model can give a more global view of the problem situation, and it is more readily used as a general solution to a family of problems" (p. 45). She sees it as a natural environment for problem solving and mathematics modelling. Dugdale reports on a project involving 32 teachers of mathematics in K- 12 who explored spreadsheet problem solving methods and developed models appropriate for classroom use. The use of spreadsheets for math modelling as well as for charts and graphs was new to the participants. To familiarize teachers with the spreadsheet's capabilities, they were shown examples of their use to generate the Fibonacci sequence and extract square roots. Graphs of square roots were added to highlight features not apparent in tables. It was the graphs that drew the most attention and facilitated observation.

Some of the models that were offered to the teachers for browsing or were generated by teachers themselves included spreadsheets for solving travel problems, modelling predator-prey populations, comparing daily allowances, and solving volume/surface area problems - all applications which could be applied in a mathematics classroom.

Dugdale's (1994) project has significant implications for mathematics teachers. One interesting point is that teachers gained sufficient facility with the spreadsheet to begin designing one of their own after approximately three hours of guided activity. Teachers found that once models were set up, initial values could be altered to investigate any number of similar problems. As Dugdale points out, functions can be defined recursively and investigated dynamically.

Spreadsheets in the secondary school have a variety of applications. Students can use them to explore algorithms for solving linear equations with one unknown or for solving a system of linear equations with two unknowns. They can be used for solving word problems, analysing data, constructing graphs of data, producing lines of best fit, and generating tables to observe patterns. Some examples are given in the resource portfolio section of this project.

Concluding Remarks

The literature suggests many implications for change in the teaching and learning environment of today's rapidly changing technological society. One of the underlying themes is that teachers must no longer transmit mathematical knowledge to the minds of students - rather students must construct their own knowledge under the guidance of teachers. The traditional "chalk and talk" method which caused students to sit passively as teachers "pumped" information into their minds will not adequately prepare students for today's society. Teachers need to promote higher order thinking skills and encourage a deep understanding of concepts.

Teachers need to have on hand a variety of projects, problems, and approaches to accommodate a very diverse student population. Many different scenarios of solutions to problems could arise as different students take different approaches to solving them. The teacher must be ready for on-the-spot questions at all times, acting as a facilitator of knowledge. The teacher must be willing to step back from the role of expert and be willing to work together with the students to research answers to problems until a suitable answer is found, if one exists. The voices which echo these thoughts through the literature need to be heard and analyzed by teachers. Many voices are calling for change in the way mathematics is taught. Experts in areas such as cooperative learning and technology are presenting very viable alternatives to traditional methods of teaching and learning. These alternatives need to be considered.

Teachers need to be willing to consider the powerful technology which exists, analyze the educational possibilities of this technology, review the suggestions and ideas offered by computer enthusiasts for incorporating this technology into mathematics classes, question the consequences, and make professional judgements. Teachers more than anyone else in the field of education can feel the pulse of learning in their classrooms. Mathematics teachers know when real mathematics learning is taking place. They can distinguish between conceptual understanding and memorization of rules. They can feel the mathematical empowerment which filters into the minds of some students and they can see the mathematical struggle of others. Being open to attempt some of the current technological ideas is a low-risk proposition for them, for they will know what works best once alternatives are examined, and they will know whether or not these alternatives bring students to deeper levels of mathematical understanding. It's only when deep levels of understanding are reached that mathematical programs can be considered valid and effective.

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Resource Portfolio

For use with Course 1 of the proposed Atlantic curriculum for Senior High Mathematics

by

Mary C. Moore

A project submitted to the School of Graduate Studies
in partial fulfillment of the requirements for the degree of
Master of Education

Faculty of Education
Memorial University of Newfoundland
May 1996

Resource Portfolio

Part 2

Note to Reader: This section of the project is the actual resource, designed for use by teachers of Course 1 of new Atlantic Curriculum for Mathematics. Since it is intended to serve as a "stand-alone" document, it contains repetition of some key points in Part 1. Included are some similar introductory comments, some basic points regarding the role of the National Council of Teachers of Mathematics (NCTM), points regarding the value of group learning, and details on the use of graphing calculators and spreadsheets. These topics reflect current trends in mathematics education and thus it was felt advisable to repeat the pertinent details in the teacher resource section.

Table of Contents

Introduction

There are many forces at work in our society which are profoundly changing the face of mathematics - how it is taught and how it is assimilated by students. In recent times the advent of computer technology, graphing utilities, electronic spreadsheets, numerical analysis packages, and inexpensive electronic calculators has caused a reexamining of old traditions in the mathematics classroom and have prompted a need for curricular changes which harmonize with the new technology. Mathematics educators are called upon to prepare students for the twenty-first century by teaching math in a new and different way, in a way which stimulates and benefits not only college-bound students, but all students entrusted to the school system.

This call for renewal did not come solely with the 90's. Rather, it was felt much earlier with the realization that the new mathematics was serving the needs of only a few and was not a mathematics for all. A majority of students simply opted out of the subject by either accepting failure as inevitable or moving to a remedial mathematics. It was felt that students were not learning a form of mathematics that could help them become productive members of society. As newer and better technology was introduced which began to change the face of society and which could handle many of the tedious tasks of math classrooms the call for reform was further strengthened. That call is felt today more than ever before.

The call includes the teaching of nontraditional mathematics topics, topics such as statistics and probability, which more adequately prepare students for the mathematical needs of modern society. It means teachers have to place less emphasis on training students in procedures and algorithms in arithmetic and algebra, as has traditionally been the case, and place more emphasis on using the new technology to create a better conceptual understanding of mathematics.

This resource will attempt to offer support for change within the new mathematics curriculum which is being planned for Newfoundland and Labrador schools. It touches on the use of group learning, graphing calculators, and computer technology in the mathematics classroom and presents ideas for implementing some of the objectives of Course 1 (Grade 10) of the upcoming Atlantic mathematics curriculum. These objectives are in keeping with the National Council of Teachers of Mathematics *Curriculum and Evaluation Standards* (1989). Presently, most mathematics textbooks at the high school level do not reflect the goals of the NCTM standards and, as a result, activity-based sessions depend upon teacher time to find appropriate materials to make mathematics more interesting and applicable. The new Atlantic curriculum will no doubt more adequately reflect these standards, and this project is intended to be in harmony with that curriculum.

This resource is being written at the dawn of change, for the Atlantic curriculum in mathematics is presently under development. As a result, some of the ideas presented may not apply to the intended grade level, depending on the depth of treatment which will be given some topics In addition, ideas will not be equally applicable to all ability levels taking Course 1. The extent to which some material can be used will depend on factors such as class size, time restraints and availability of resources.

The Role of NCTM in Change

The National Council of Teachers of Mathematics has been advocating reform in mathematics education since it was apparent that the "new mathematics" of the 60's and 70's did not provide the type of change needed. In 1980 they issued an *Agenda for Action*, a set of recommendations intended to provide a framework for the improvement of school mathematics in the 1980's. It was recommended that more math be required for all students with a greater range of options to accommodate the diverse needs of the student population. One of the major goals was the development of problem solving skills. As well, emphasis was placed on using calculators and computers at all grade levels. This was not to imply computational skills were obsolete; rather, these skills would be complemented with more than computational facility.

The NCTM *Curriculum and Evaluation Standards for School Mathematics* (1989) represents the first effort to establish national standards for school mathematics. What is significant about this document is the fact that it is the work of mathematics teachers. The standards put forth by them are intended as "statements of criteria for excellence" and as "facilitators of reform" to help teachers prepare students for the 21st century.

It was recognized by NCTM that the educational goals of the past era, namely those of the industrial age, are not in tune with the economic needs of today's information society. The *Standards* (1989) recognizes new goals for education: ***mathematically literate workers*** - workers must be competent in technology and communication and understand the mathematics involved in various problems in today's work world; ***lifelong learning*** - school mathematics must equip students with skills such as problem solving which will help them accommodate to changed conditions over their lifetime; ***opportunity for all*** - women and minority ethnic groups must study more mathematics and be more represented in science and technical careers; ***an informed electorate*** - citizens must be able to understand issues in a technological society.

The five goals which NCTM (1989) articulates for all students are that they:

1. learn to value math
2. become confident in their ability to do math
3. become mathematical problem solvers
4. learn to communicate mathematically
5. learn to reason mathematically

The *Standards* (1989) advocate that students should be encouraged to read, write and discuss mathematics, develop mathematical habits of mind, understand the role of mathematics in human affairs and above all develop mathematical power. This is in contrast to the way students have been taught mathematics for decades.

Traditionally, the teacher has been in complete control, discussion of math topics has usually been limited, and students have learned to do math by imitating examples placed on the chalkboard by teachers. Since this trend has been the norm for quite some time, it naturally still exists in many classrooms today. The practice of algorithms has traditionally taken precedence over conceptual understanding and will undoubtedly remain that way until the call for change is felt by all mathematics educators. That call, as stated in the 9 - 12 standards, is for "a shift in emphasis from a curriculum dominated by memorization of isolated facts and procedures . . . to one that emphasizes conceptual understandings, multiple representations and connections, mathematical modelling, and mathematical problem solving" (NCTM, p. 125)

Mathematics has historically served the needs of those students who were college-bound or interested in studying pure mathematics. Many of these students have prospered in the traditional mathematics classroom and have entered post secondary education with sufficient mathematical backgrounds to meet the challenges ahead. Others students, however, have graduated from high school with the memory of horrendous mathematical experiences which affect them for life. For many of them math anxiety and feelings of incompetence returns everytime a topic related to math arises, to such an extent that they avoid the simplest of everyday math operations. The *Standards* (NCTM, 1989) document calls for a change in this regard and advocates a mathematics program in secondary school that will serve the needs of all students when they graduate. The document depicts the mathematics environment as one which will benefit all students by making mathematics more applicable to their daily living. By having them explore genuine world problems, understand mathematical models, structures, and simulations, use statistics to examine data and graphs to explore functions, students are better equipped to understand many real life phenomena. The emphasis has shifted from "knowing math" to "doing math," with a stress on activities that grow out of problem situations.

Group Learning in Mathematics

The NCTM *Curriculum and Evaluation Standards* (1989) points to the need for variety in approaches to instruction, including opportunities for discussion of math concepts and problems among students, as well as project work and group assignments. "Active student participation in learning through individual and small-group explorations provides multiple opportunities for discussion, questioning, listening, and summarizing" (p. 140). Being able to communicate mathematically is viewed as one of the keys to understanding mathematics. One of the ways to do this is through effective group learning.

Over the years group learning has been used in mathematics classrooms in the form of peer teaching, group projects, laboratory groups, etc. Students have, at times, been assigned to groups to work on tasks such as math assignments, problem sets, or review sheets. The trend, however, has not been widespread, for most math learning has been in the form of the lecture approach. While some math educators may have experienced high success with traditional groups, others have no doubt experienced problems with them. Problems such as low quality interactions among students, lack of interest by under-achievers, domination by some students, cognitive differences among students and poor attitudes have no doubt made teachers feel uncomfortable about using groups in the classroom. To add to this, difficulty in determining the progress of individuals in traditional type groups has been a deterrent for many. Such problems can not only threaten teacher control but prevent meaningful learning. It was not until the introduction of cooperative learning in recent years that the whole idea of group learning has been viewed through a different set of lenses and has taken on new meaning.

The term *cooperative learning* refers to an instructional strategy whereby students of all ability levels work together in small groups to achieve a common learning goal. It has been advocated for use in practically any subject and any grade level as an option to traditional-type learning. All students share responsibility for learning as they help each other achieve success. Slavin (1987) sees cooperative methods as very viable alternatives to the traditional classroom techniques in which competition for grades and other rewards by one student could reduce the chances of success of another. He cites three concepts which are central to all types of student team learning and cooperative learning - *team rewards, individual accountability, and equal opportunities for success.* Students of low ability contribute to the success of others by improving over their past performances. Slavin (1980) has indicated that if students are rewarded for doing better than they themselves have done in the past, their motivation to achieve will be greater than it would be if their rewards were based on their performance compared with that of others. Rewards for individual improvement make success neither too difficult nor too easy for students to achieve. Allowing this improvement to be used to gain points for a team increases the motivation to learn.

While a complete unit on cooperative learning in mathematics is beyond the scope of this project, some ideas are presented to illustrate how it can be used. As well, more information is included in the literature review section of this project.

Student Team Learning

Using this method the teacher presents a lesson in mathematics, eg. graphing linear inequalities, and has the students meet in small heterogeneous groups to complete a set of worksheets on the lesson. Students in each group help one another with the topic to ensure that all group members know how to accurately complete the sheets. The students are then individually evaluated on graphing inequalities and the scores they contribute to their groups (teams) are based on the degree to which they have improved over previous individual scores. Points could be awarded, for example, for every 5% improvement a student has made over previous test results. Scores of students who consistently do well could also be considered for points.

Jigsaw

Using Jigsaw to deal with the topic of graphing linear inequalities, a teacher assigns each team member a special topic to learn (or review). For example, one student per group may be asked to review graphing linear equations. That student then meets with the team members of the other teams who are reviewing the same topic. After delving into the topic with the group the student returns to his or her own team to review the topic with other group members in preparation for the upcoming topic.

A group learning activity

The following activity on maximum volume contains some cooperative learning ideas. The lesson requires students to cut small congruent squares from the corners of a larger square to determine what size of a cutout will give maximum volume when the sides are folded up. The exercise could be completed in one class period of 40 minutes. Another class period will be needed to discuss findings.

Cooperative learning ideas involved: The students are expected to use a learning technique called Learning Together which requires group members to work together on a solution. When the group agrees on the solution, one answer will be submitted for the group. Each group member is accountable for understanding the solution and one group member will be asked to present the answer in class. If random selection is perceived to cause anxiety for some students, it may be best to have students decide among themselves who presents in class. Group members will be expected to discuss the solution either in their journals or on an upcoming test. As well as individual accountability, it is important that there exist positive interdependence, face-to-face interaction, and good social skills. It may be best to form groups that are heterogeneous in a multitude of ways. The groups could consist of different ability levels, cultural backgrounds, physical makeups, etc.

Student worksheet follows.

✎ Student Sheet - Maximum Volume

Problem: You have a square piece of paper 16 cm by 16 cm or 20 cm by 20 cm (depending on your group). From the corners you will cut four small congruent squares and make a pan. Many different size pans can be made, depending on the side lengths of the cutout squares. The question arises as to what size square could be cut out so that the pan will have the greatest volume.

Using the instructions outlined below, work collectively in your assigned groups to determine a possible solution to this problem. At the end of the session *all* group members will be expected to understand the group solution. All members will be expected to discuss the solution either in their journals or on an upcoming test. One member from each group will be randomly selected to discuss findings in class.

Instructions/questions:

1. Cut from each corner of the square paper a 1 cm. by 1 cm. square and fold up the sides to make a pan. Tape the sides in place. Fill in the blanks across from the 1.0 in the chart below.

2. If your paper is 16 cm by 16 cm, your answers should be 14, 196, 1, and 196. If your paper is 20 cm by 20 cm, your answers should read 18, 324, 1,

and 324. Complete the table for the other side lengths of cut-out squares.

Note: Volume = lwh

Measure of side of cut-out (cm.)	Measure of side of base (cm.)	Area of base of pan (cm.)	Height of pan (cm.)	Volume of pan (cubic. cm.)
1.0				
2.0				
3.0				
4.0				
5.0				
6.0				
7.0				
8.0				

3. From the data collected can you determine which size of cut-out square will give the largest possible volume? Have a group member record thoughts at this point.

4. On a graph, plot the length of a side of each cutout square (x) and the resulting pan volume (y). What does the graph tell you about maximum volume? Record.

5. In the blank spaces in column 1 add other sizes of the cutout square. Have each group member choose a different number, such as 1.5, for side length and fill in the necessary information. Use calculators if necessary.

6. Explain why you think the answer you have decided upon for maximum volume is the best possible answer. Is it possible that your answer contains some degree of error? Explain.

7. Review the problem carefully for class presentation. One group member will be randomly selected to present findings and the group work will be assessed by that person's response.

Maximum Volume Problem

Sample answers to max. volume problem (a 16 by 16 square)

side sq	Side base	Area base	height	volume
1	14	196	1	196
2	12	144	2	288
3	10	100	3	300
4	8	64	4	256
5	6	36	5	180
6	4	16	6	96
7	2	4	7	28
8	0	0	8	0
1.5	13	169	1.5	253.5
2.5	11	121	2.5	302.5
3.5	9	81	3.5	283.5
4.5	7	49	4.5	220.5
2.6	10.8	116.64	2.6	303.264
2.7	10.6	112.36	2.7	303.372
2.75	10.5	110.25	2.75	303.1875
2.8	10.4	108.16	2.8	302.848
2.9	10.2	104.04	2.9	301.716

For an alternate method of solving this problem - using spreadsheets -

see Page 29. Sample answers to a 20cm by 20cm sheet are also given on Page 29.

Follow-up activity

As a follow-up assignment, have students examine the surface area of various boxes that are used everyday - detergent boxes, juice packs, cereal boxes, etc. to determine whether or not the manufacturer is getting maximum volume for the amount of surface area used by the packaging.

Graphing Calculators

Graphing calculators have become very functional, practical, and relatively affordable tools for teaching and learning mathematics. The position of the National Council of Teachers of Mathematics with respect to their use is made clear in the *Curriculum and Evaluation Standards* (1989). The Council feels that calculators (and computers) should be available to students at all times, but should not be used to replace the fundamental mathematics students will need.

One of the dominant features of graphing calculators is their ability to display graphs of algebraic functions as well as graphs of statistics and data of real life phenomena. Students can zoom, plot, and trace as they inspect graphs, investigate patterns and draw conclusions. All operations can be performed with great speed, allowing more time for exploration and investigation. Students learn to establish links between algebraic and graphical representations, crystallizing their understanding of functional concepts. Since several graphs can be viewed at once, comparisons can be made and intersections noted. The effect of making changes to algebraic parameters can be seen instantly in graphical form, giving students a broader picture of the concept of algebraic functions and providing more opportunity for conceptual understanding.

Graphing calculators can be used to solve problems that were once either unsolvable algebraically or too difficult to solve. Interesting real-world situations can be analysed and explored. Systems of linear equations can be solved graphically by zooming in on the intersection of the graphs and zeros of polynomial functions can be found by checking x-intercepts. In keeping with the *Standards* document (NCTM, 1989) this is not to imply that the device should be used to do all the student's work; rather it should be used to complement student efforts and help students more fully understand algebraic processes and concepts. Research into the use of graphics calculators is cited in the literature review section of this project.

While the reviews generally are very positive towards their use, a few problems have been highlighted in the research. These devices can be confusing for some students, even after instruction and experimentation time has been given. There are some misconceptions about the scaling features of calculators. Students do not always understand what's on the screen, for experts sometimes see relationships that are missed by students who make erroneous connections or draw invalid conclusions. Sometimes graphs are not represented accurately. It may be difficult, if not impossible, to detect points of discontinuity. Some screens do not have enough pixels to accurately represent graphs. As well, some exact solutions can be found only by algebraic methods, eg. solving systems of equations, whereas the graphical representation may be an approximation and can be used to verify algebraic solutions. Methods for finding exact solutions should not necessarily be abandoned. As well, students should not feel that all numbers are rational since they appear that way on graphical displays. Underlying mathematical principles cannot be ignored while using these powerful tools.

With an awareness of potential problems, educators can reduce negative effects and allow the device to have maximum positive impact on learning. There are obviously many ways which graphics calculators can be used in the high school curriculum.

For examples of their use see:

Graphing Activities	Page 91
The Quadratic Function	Page 110
Quadratics in Real Life	Page 119
Exploration of quadratics	Page 114
Line of Best Fit	Page 123
Matrix Applications	Page 133

Spreadsheets and Mathematics

A spreadsheet is a very large rectangular array of cells which can be used to represent a mathematical environment on a computer screen. Its use in the secondary mathematics classroom is becoming more widespread as teachers realize its potential for facilitating the study of a variety of topics. Many of today's spreadsheets, such as Microsoft Excel and Lotus for Windows have some very powerful, yet user-friendly options, including easy formula copying, chart wizards, and graphical menus. Their very extensive capabilities facilitate representation of functions in various graphical and tabular forms. Line graphs, bar graphs, scatterplots, and 3-dimensional displays are but a few of the possible ways to represent data.

NCTM (1989) emphasizes the importance of visualization in mathematics learning, and the need to conjecture, guess, explore, make generalizations, and problem solve. Spreadsheets seem to provide opportunities for all of these in the mathematics classroom. Their value as a graphing tool for all types of graphs at the secondary level is especially great.

An example of Spreadsheet Use in Mathematics

Grade 10 students could use spreadsheets to explore (or review) algorithms for solving linear equations with one unknown. The algorithm for solving a linear equation of the form ax + b = c is given below. The second diagram displays the formulas entered into the spreadsheet. In making changes to row 6 all other parts of the spreadsheet change accordingly and students can see the effect on the workings (and solution) of those changes.

	A	B	C	D	E	F	G	H
1	Solving	Linear	Equations					
2								
3	Note:	Only the	numbers	in row 6	can be	changed		
4								
5								
6	3	x	+	6		=	18	
7								
8	3	x	+	6	-6	=	18	-6
9								
10	3	x				=	12	
11								
12		x				=	4	

	A	B	C	D	E	F	G	H
1	Solving	Linear	Equations					
2								
3	Note:	Only the	numbers	in row 6	can be	changed		
4								
5								
6	3	x	+	6		=	18	
7								
8	=+A6	x	+	=+D6	=+-D6	=	=+G6	=+-D6
9								
10	=+A6	x				=	=+G6-D6	
11								
12		x				=	=+G10/A6	
13								

The first chart gives the appearance of the worksheet as the data is being entered. To enter that data, use the steps which follow:

To write the equation ...

In A6 enter 3 In B6 enter x In C6 enter ^+ In D6 enter 6

In F6 enter "= In G6 enter 18

To enter the algorithm for solving the equation ...

In A8 enter +A6 In B8 enter x In C8 enter ^+ In D8 enter + D6

In E8 enter +-D6 In F8 enter "= In G8 enter +G6 In H8 enter +-D6

etc. ... until all formulas are entered as per the second chart. The chart will resemble the first chart above as the data is being entered. To view the formulas, as per the second chart, use the formula display option. In Microsoft Excel this is accomplished by going into Tools/Options..../View and checking formulas. For clarity of viewing it may be necessary to decrease the column width when the formulas are displayed. Note: In Microsoft Excel, cells which use a + sign as their first character will show an equal sign in front of the entry when formulas are displayed (as in G8).

The spreadsheet can be saved as a file and retrieved for use when teaching students the process involved in solving linear equations. Algorithms for other mathematical procedures, such as solving systems of linear equations, can be also be entered and used in a similar manner. Students could be asked to design algorithms for other mathematical procedures that are familiar to them.

A student exploration sheet for this spreadsheet is given below. This sheet can be used by students in the computer lab when exploring the above algorithm.

For other examples of spreadsheet use see:

Maximum Volume Problem	Page 29
Height and Shoe Size	Page 37
NBA Ticket Prices	Page 84
Mixture Problem	Page 108

✎ Student Sheet - Spreadsheets

Solving Linear Equations - An Exploration

Student Name: ______________________________

1. What changes are made in the equation from row 6 to row 8?

2. What changes are made from row 8 to row 10?

3. What is the final step in finding the value of x in row 12?

4. Change the number 6 in D6 to 9. Answer each of the above questions again.

5. Keep the 9 in D6 and change the 3 in A6 to a 2. How does this change the answer? Now try changing the 18 in G6 to some other number. Explain what happens.

6. Make several other changes in row 6 and watch the results. Use what you learned in this exercise to explain how linear equations of the form ax + b = c are solved using pencil and paper.

Maximum Box Problem Revisited

Using Spreadsheets

The maximum box problem given earlier could be completed using spreadsheets such as Lotus 1-2-3 (Win) or Microsoft Excel. Students could enter formulas for each of the items and have the spreadsheet perform the calculations.

For a 20 cm. by 20 cm. sheet of paper, they could begin by first entering the numbers 1 to 9 in column A, as per the given worksheet. To enter the formulas they can use the following:

In B5 enter +20 - 2*A5	In C5 enter +B5^2
In D5 enter +A5	In E5 enter +C5*D5

They can then copy the formulas to the end of the numbers entered, generate the values, and look for patterns. They will notice that for this initial set of integers the iargest volume occurs when the side of the cutout square has a measure between 3 and 4.

The next step would be for them to try values between 3 and 4. If they use the decimal values given in the chart they will see that the maximum volume lies between 3.3 and 3.4.

They can continue the trial and error process until they are satisfied that they have found the maximum volume. They can also use the spreadsheet program to generate a graph with the x-axis representing the side length of the cutout square and the y-axis representing volume. The same process could be used for several other sheet sizes. Students should attempt to make generalizations as they generate various spreadsheets.

Exploration Spreadsheet for Maximum Volume

Using a 20 cm. by 20 cm. sheet

	A	B	C	D	E
1	Maximum	Volume	- Using a	20 x 20	sheet (cm)
2					
3	Side of	Side of	Area of	Height of	Volume of
4	Cut out sq	Base	Base	Pan	Pan
5	1	18	324	1	324
6	2	16	256	2	512
7	3	14	196	3	588
8	4	12	144	4	576
9	5	10	100	5	500
10	6	8	64	6	384
11	7	6	36	7	252
12	8	4	16	8	128
13	9	2	4	9	36
14					
15	3.1	13.8	190.44	3.1	590.364
16	3.2	13.6	184.96	3.2	591.872
17	3.3	13.4	179.56	3.3	592.548
18	3.4	13.2	174.24	3.4	592.416
19	3.5	13	169	3.5	591.5
20	3.6	12.8	163.84	3.6	589.824
21	3.7	12.6	158.76	3.7	587.412
22	3.8	12.4	153.76	3.8	584.288
23	3.9	12.2	148.84	3.9	580.476
24					
25	3.35	13.3	176.89	3.35	592.5815
26	3.37	13.26	175.8276	3.37	592.539
27	3.39	13.22	174.7684	3.39	592.4649
28					
29	3.36	13.28	176.3584	3.36	592.5642
30					
31	3.33	13.34	177.9556	3.33	592.5921
32					
33	3.34	13.32	177.4224	3.34	592.5908
34					

Atlantic Curriculum in Mathematics - Course 1

Ideas for teaching Selected Topics

Topic: Data Analysis and Discrete Math

Subtopic: Scatterplots and Correlation

As part of the proposed Atlantic curriculum students are expected to be able to explore, recognize, represent, and apply patterns and relationships both formally and informally. The activities which follow are designed to get students thinking about the appropriateness of scatterplots to represent data, to get them started on gathering and exploring data and to show them how to find equations to best represent data. The activities are as follows:

Scatterplots and Correlation

Relationship between forearm and waist size

Note to teacher: The story (NCTM, 1988) given on the student sheet which follows could be presented to students to get them thinking about ways to appropriately represent data and to introduce them to scatterplots. The lesson could take the following form:

- have students read the story silently and think about the questions which follow

- ask students to gather data from one another similar to that of the story (they could work in pairs) and record their measurements on the chalkboard

- examine the data which students have collected and ask them to discuss their thoughts on how this data could be appropriately represented

- do a rough sketch of the points on the chalkboard or overhead, discuss the relationships which exists and examine any points which do not seem to follow the general relationship

- have students complete the lesson by answering the questions on their sheets.

This lesson could be completed in one class period. Part of a second period may be needed to discuss questions assigned to students, as per the student sheet.

Note: Some students in the class may be sensitive about having personal data (eg. waist size) collected during class time. If this possibility exists, it may be better to ask students to collect the data in advance from friends or family members and bring to class when needed.

Student Sheet - Forearm and Waist size

Name:____

Read the following story.

A student, who was helping at a yard sale, watched as a customer looked through the pile of jeans for sale. The man would bend his hand back at the wrist, bend his arm at the elbow, and then wrap the waist of the pants around his forearm. The man explained that his waist was the same size as his forearm, so he never needed to try slacks on for size. The student was studying data analysis in his mathematics class, so he decided to gather some data and model the relationship between the forearm circumference and waist size.

Think: Would the method described above work for most people? What would be a good way to represent the data to see if a relationship exists between forearm and waist size?

Under the guidance of your teacher you are going to examine similar data in class. **After you examine this data**, give your thoughts on a good way to represent this sort of data and explain why the method is appropriate. Do you think it is possible to design a model to predict a person's waist size?

Scatterplots and Correlation

Height and Shoe Size

Note to teacher:

The following activity (NCTM, 1993) requires students to determine the relationship between height and shoe size by designing a scatterplot and constructing a model of the points. Data from the class could be collected *in advance of the activity* so it could be distributed to them or placed on an overhead when needed. A sample form for data collection is given at the end of the student sheet. If data is collected in advance, the student sheet could be completed in one class period, with unfinished work assigned for homework. Another class period could be used to discuss student answers and review the concept of designing models to represent data.

Note: This activity could also be very effectively completed by using a spreadsheet to handle the data.

✎ Student Sheet - Height and Shoe size

Name: ____

1. Using the data on height and shoe size, plot the pairs of points for all people on a grid.

2. Do the points seem to fall into a linear pattern? If necessary, draw an oval as narrow as possible around the points. The narrower the oval, the closer the points are to following a line.

3. Draw a line through the middle of your points. The line should also run through the middle of your oval.

4. Your line is a *geometric model* for the relationship between height and shoe size. You can use the model to estimate your shoe size. What shoe size would you estimate from the graph? _______ How does it compare with your actual shoe size? _______

5. Choose two points on your line whose coordinates are integers. They do not have to be points of your original data but should be fairly far apart. Record the coordinates . ___________________________

6. Use the two points to find the slope of the line. __________________

7. Use the slope and one of your two points to find the y-intercept (0,b) of the line. Write the y-intercept. ___________ Equation: ___________________

8. Your equation is an *algebraic model* for the relationship between height and shoe size. Use this model to calculate your predicted shoe size. Do this by substituting your height for x in the equation and rounding the shoe size to the nearest 0.5.

   What shoe size was predicted?_________ How close is the predicted size to your real shoe size? __________

9. Compare your misses with those of other students. Whose line did the best job of predicting shoe size for members of your class? Why?

10. What shoe size does your equation predict for a person who is 125 cm. tall? __________ Do you think this prediction is reasonable? Why or why not?

11. Can you give other ways to represent this data? Do you think a scatterplot is the best of the possible ways? Why or why not?

Chart - Height and Shoe Size

Height	Shoe Size	Predicted shoe size	Amount of miss

Scatterplots and Correlation

Forearm and foot lengths

Note to teacher: The following activity requires students to explore the relationship between forearm and foot lengths of fifteen students. The data for this activity is supplied to the students and arranged first in order of ascending foot length and then into three groups of five points. This data could be ignored and substituted with real data collected from students in the class. The activity requires students to first find the median points for each of the three groups and then find an equation to best represent the relationship expressed by the data. It could lead into a discussion of ways to find lines of best fit.

Time required: This exercise could be completed in one class period.

Answers to nos. 1 and 4 of student sheet

1. (22.7, 24.2) (24.7, 26.2) (27.4, 28.9)   4. $a = f + 1.5$

Note: See Page 123 for ways to find line of best fit.

✎ Student Sheet - Forearm and Foot lengths

(Hirsh, 1986)

Name: ____

Below are the lengths, in centimetres, of one foot and forearm of each of fifteen students. They have been arranged first in order of ascending foot length and then into three groups of five points.

Lower group:	foot length	21.5	22.3	22.7	23.4	23.9
	arm length	23.8	24.2	23.8	24.5	24.6
Middle group:	foot length	24.3	24.7	24.7	24.8	25.2
	arm length	25.5	25.8	26.2	27.3	27.2
Upper group:	foot length	25.6	26.5	27.4	28.9	29.5
	arm length	27.4	26.8	29.1	28.9	30.5

1. For each of the three groups, find the median of the foot lengths and the median of the arm lengths and enter them as coordinates in the spaces below. These three points are called the median points of each group.

   Lower group (________,________) Middle group (________,________)

   Upper group (________,________)

2. Plot each of the individual points and the median points for each group on a graph.

3. Use a ruler to draw the best possible straight line through the three median points.

4. Use your line to find an equation that relates forearm length (a) to foot length (f). What equation did you find? ______________________

Scatterplots and Correlation

Predicting Male Height

Note to teacher: This exercise, which takes approximately one class period, requires students to examine the relationship between the height of a male at 30 months and his height at 19 years. Data is supplied to the students in ascending order of height at 30 months and is divided into three groups. Students are required to find the best equation to express this relationship, as well as equations to find a *least* and *greatest* estimate of height at 19 years

The exercise could be used in reverse by having class members use the information gained to estimate their height at 30 months based on their present height. If any of them have records available at home they could check the accuracy of their estimates.

The answers to the student sheet are as follows:

1.	(90.3, 179.6)	(91.2, 182.0)	(94.7, 187.4)
3.	$H = 2h - 1$	4. $H = 2h - 3$	5. $H = 2h + 2.2$
6.	a. 185 cm	b. 182.2 cm	c. 188.2

✎ Student Sheet - Predicting Male Height

(Hirsh, 1986)

Name: ______________________________________

The height of a boy at the age of 2 years and 6 months (30 months) is said to be one-half of his mature height. Below are the heights, in centimetres, of 15 boys at 30 months and at 19 years.

1. Find the median points for each of the three groups.

						Median Point
Height (h) at 30 months	89.0	89.9	90.3	90.8	90.9	(________)
Height (H) at 19 years	178.0	177.1	179.6	181.8	184.0	(________)
Height (h) at 30 months	91.0	91.1	91.2	91.2	91.9	(________)
Height (H) at 19 years	180.5	182.0	183.1	180.1	185.1	(________)
Height (h) at 30 months	92.9	93.3	94.7	95.4	96.1	(________)
Height (H) at 19 years	182.0	186.3	187.4	187.9	189.4	(________)

2. Plot the points (including the median points) on a grid and then draw a straight line through the median points. Put h on the x-axis and H on the y.

3. Use the equation of the line to express the relationship between h and H.

____

4. Draw a second line, parallel to this median-fit line, so that all the points are either on this line or *above* it. Find the equation of this line.

____

5. Draw a third line, parallel to the median-fit line, so that all the points are either on this line or below it. Find the equation of this line.

____

6. A boy was 93 cm tall when he was 30 months old.

   a. Use your equation from exercise 3 to predict the height of this boy at 19 years of age. ________________

   b. Use the equation from exercise 4 to find a *least* estimate of his height at 19 years. ________________

   c. Use the equation from exercise 5 to find a *greatest* estimate of his height at 19 years. ________________

Scatterplots and Correlation

Other Suggestions to Teacher

Have students compare:

a person's height with the distance around his/her head

a person's waist and the distance around his/her neck

finger length and height

Topic: Data Analysis and Discrete Math

Subtopic: Interpreting Graphs

As part of Course 1 of the proposed Atlantic curriculum students are expected to not only draw graphs to represent certain data, but to interpret what is happening in a given graph. The following two exercises could be used to encourage discussion and comprehension of information presented in graphical form. The examples are applicable because of their real world application, as emphasized by NCTM *Standards* (1989). Both of the exercises and class discussion of them would require two - three class periods. The exercise, as per student sheets, are:

1. Matching graphs of rainfall with the appropriate city Page 48

2. Interpreting what is happening in given graphs (Kaur, 1992). Page 51

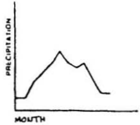
A sample story for one of the graphs in No. 2 is given on Page 53

Student Sheet - Graph Match-up

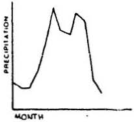
Name: ______________________________

Each graph below depicts change in precipitation for a city over a period of one year. Match each graph with one of the cities in the table given below.

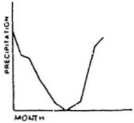
1. ______________________



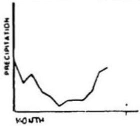
2. ______________________



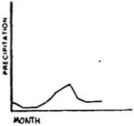
3. ______________________



4. ______________________



5. ______________________



6. ______________________

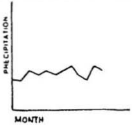


Table of fictitious cities

	Barmal	Westbur	Maribi	Rampur	Chadd	Kenmor
Jan.	7	19	11	2	6	3
Feb.	7	13	6	1	5	3
March	9	12	8	1	5	7
April	8	8	4	2	9	9
May	9	5	3	4	15	1
June	8	2	1	5	23	14
July	9	0	2	6	18	11
August	10	1	2	6	17	10
Sept	8	2	2	3	22	11
Oct.	7	8	4	2	21	8
Nov.	10	15	8	2	7	4
Dec.	9	17	9	2	4	4

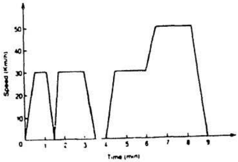
Student Activity

Interpreting Information from Graphs

Name ____

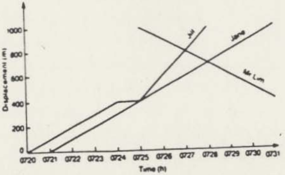
EXAMPLE 1

A man is driving from home to work. The graph below shows his speed at any time during his nine minute trip. Make up a story about his trip to explain the graph.



EXAMPLE 2

Town A and Town B are 1 km. apart. The graph shows the journey of the twin sisters (Jill and Jane) and Mr. Lim one morning. Make up a story about their trip to explain the graph.



Interpreting Information from Graphs

Note to teacher: Here is a sample story (Kaur, 1992) for Example 1. written by a student.

Mr. Test-tube, the famous biologist, was on his way to work one morning. He started his car and increased his speed to 30km/hr and drove consistently for about 1 ½ min. before stopping his car at a traffic junction. He started his car again and drove at a constant speed for 2 minutes, then he went over a bump, so he stopped his car to have a look outside. There was no damage so he started his car again and drove for 2 minutes at a constant speed of 30 km/hr before spotting an eagle in the sky. He decided to follow the eagle as part of an experiment so he accelerated to 50 km/hr and drove at that pace for about 2 minutes before the eagle came to rest in the top branches of a very tall tree. As he was driving an uncovered convertible he slowed down till he finally reached his workplace 9 min after leaving home.

Topic: Data Analysis and Discrete Math

Subtopic: Inferences from Articles

Data can appear not only in the form of tables, charts, and various types of graphs from government and corporations, but can also surface in articles from magazines and newspapers and consumer bills sent regularly to households. The Atlantic curriculum for mathematics points to the need for students to be able to draw inferences from various forms of data. The following activities are designed to help students do that. They are:

Inferences from Articles

Interpreting Data from Newspaper Articles

Note to teacher: The following activity requires students to examine life expectancy of bills of various denominations (U.S. currency). In order to do the activity students should be familiar with finding lines of fit using a graphing calculator. (Refer to Page 124 for information on how to use linear regression on a graphing calculator to find a line of best fit). if they have access to the Internet or other resources such as Statistics Canada they may be able to find similar information for Canadian bills.

Student Sheet

Inferences from a newspaper article[1]

Name: ______________________________

Consider the data that appeared in the following article in the New York Times, May 22, 1994.

In Recycling of Greenbacks, New Meaning for Old Money

For decades, the Federal Reserve has disposed of its old, worn-out currency by shredding it and burying the multi-billion-dollar confetti in landfills. But as the number of available landfills declines and concern over the environment grows, Federal Reserve banks across the country are trying to find companies that can recycle the 7,000 tons of tired money - enough to fill 1,750 dump trucks - that is shredded each year.

That amounts to 715 million bills, with a face value of nearly $10 billion, roughly 3 percent of the value of the currency in circulation. So far, the Federal Reserve's efforts have involved the use of old bills in roofing tiles, particle board, fuel pellets, stationery, packing material, and artwork.

The Federal Reserve Bank of Los Angeles, for example, is negotiating with Terra Roofing Products of Fontana, Calif. to use shredded bills in its fireproof roofing shingles. The shingles, which are made of cement and wastepaper, do not burn like wooden ones and have become popular in California as a safeguard against firestorms.

Aaron Cohen, Terra's chief executive, said the roofing company had conducted extensive studies indicating that adding a small amount of shredded money increases the strength of the shingle. At the 37 regional banks and branches of the Federal Reserve, old bills are shredded, then placed in a high-pressure processor that forms the residue into "bricks" that weigh 2.2 pounds apiece. Each brick contains 1,000 bank notes.

The following information was provided with the article re the life expectancy of bills of various denominations.

$1 bills	-	18 to 22 months	$20 bills	-	4 years
$5 bills	-	2 years	$50 bills	-	9 years
$10 bills	-	3 years	$100 bills	-	9 years

The number of bills of each denomination currently in circulation was given as follows:

Denomination		**In circulation (thousands)**	**Denom.**		**In circulation (thousands)**
$1	-	5 300 000	$100	-	1 570 000
$2	-	437 000	$500	-	295
$5	-	1 280 000	$1 000	-	170
$10	-	1 260 000	$5 000	-	355
$20	-	3 500 000	$10 000	-	345
$50	-	7 100 000			

EXERCISES

1. From the information given, estimate the total value of American currency in circulation.

2. Using a graphing calculator find the line of best fit for both sets of data. Discuss the goodness of fit (See Page 123 - Finding lines of best fit)

3. Suppose you were in charge of introducing a new bill into circulation in the United States. What denomination would you choose? Use the equations from problem 2 to determine how many bills you would print and how long you would expect them to last. What are the drawbacks to this approach?

4. Obtain data from Statistics Canada or the Internet and repeat the exercises for Canadian bills.

[1]Adapted with permission from "Media Clips", edited by Ron Lancaster. *The Mathematics Teacher*, copyright February, 1995 by the National Council of Teachers of Mathematics.

Inferences from a Newspaper Article - Answers to 1 and 2

1. Here students can do the arithmetic to find there are \$5 300 000 000 in \$1 bills, \$874 000 000 in \$2 bills, 6 400 000 000 in \$5, etc. for a total value of \$612 716 500 000.

2. In the display below, x represents the denomination and y represents the corresponding life expectancy for the line of best fit for the number of bills in circulation. (The first answers are obtained when 1.5 is used as the life expectancy for \$1 bills).

```
LinReg
y=ax+b
a=0.0809917355
b=2.239256198
r=.9071174339
```

In the next display x represents the denomination and y represents the corresponding number of bills in circulation.

```
LinReg
y=ax+b
a=-301.2408999
b=2315933.922
r=-.3953066726
```

The goodness of fit is quite good for the first set of data and very poor for the second set.

Inferences from Articles

Interpreting data from a light and power bill

Note to teacher: The following exercise is an example of how information from everyday bills can be used in math class to heighten student awareness of the mathematics which is involved in such items and to help them interpret and understand this mathematics. While the activity which follows involves a genuine light bill from a rural Newfoundland community, students may wish to use bills from their own households.

This exercise can be covered in *one class period* or assigned as an at-home activity. It is suggested that it be used as a starting point to get students involved in analysing other similar types of data. Phone bills, municipal tax bills, cable tv bills, and insurance bills could be used in a similar manner.

Student Sheet

Interpreting data from a light and power bill

Student Name: ______________________________

Below is a representation of a light and power (including heat) bill from a resident in a rural community in Newfoundland received in January for the period Dec. 15, 1995 to Jan. 15, 1996.

Meter No.	Past Due	Pres. date	Past reading	Pres. reading	Multipler	KWH used	Days
247160	DEC 7	JAN 10	67675	70767	1	3092	34

CUSTOMER NAME:

Previous Balance	Payments to DEC. 15/95	Adjustments	Forfeited Disc.	Interest	Balance Forward
$100.94	$100.94				$0.00

BASIC CUSTOMER CHARGE		$16.73
*** 3092 KWH @ 6.522 CENTS		$201.66
DISCOUNT 1.5%	$3.28 CR	
ELECTRIC SERV. CHARGES		$215.11
GOODS & SERVICES TAX (7.0%)		$15.06
DONATION TO "SHARE THE LIGHT"		$ ----------
TOTAL AMT. DUE ON OR BEFORE JAN 25/96		$ 230.17

1. Interpret the information in the bill and explain the mathematics that is involved.

2. The bill shows the previous month's balance as $100.94. Use the information from the bill to determine the number of kilowatt(kwh) hours which was used since then.

3. Establish a formula for determining basic customer charge for a month.

4. The following table represents the basic customer charge for this same household for the other months of 1995, rounded to the nearest dollar. The winter of 1995 in this community was an exceptionally harsh winter with average mean temperatures for the months of Jan - March being -13^{0} C.

Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
255	267	251	219	155	135	95	93	122	112	101	230

Assume this customer wanted to opt into a plan whereby the yearly amount would be spread evenly over twelve months. Using the information given for 1995, make a prediction on a reasonable monthly amount for the upcoming year, keeping in mind that the any outstanding balance or credit will be determined at the end of the year. What factors need to be considered in determining the monthly charge for 1996? Is it sufficient to find the mean monthly charge for 1995?

Inferences from Articles

Project: Critiquing articles from newspapers

Note to teacher: The following project is designed to stimulate student interest in examining scatterplots, bar graphs, box plots, and various other representations found in the media, as per the objectives of Course 1 of the Atlantic curriculum for high school mathematics. Students need to be able to critique representations of real life data, examine the assumptions made, check for misrepresentations of data, and suggest better ways to display information. This project will encourage them to think about graphs that appear daily and question their validity for representing information.

It is suggested that

- students be given approximately *three to four weeks* to complete this project
- some class time be used in working on the project so that students may receive guidance as they search through and critique articles.

This activity could be effectively completed using a *cooperative learning* approach if so desired. (See the literature review for more information on cooperative learning techniques, and page 15 for an example of a cooperative learning activity).

Student Project Sheet

Critiquing articles from newspapers

Name. ________________________________

For this assignment you are to collect 6 - 8 statistical articles from newspapers, magazines, etc. Look for bar graphs, scatterplots, line graphs, tables of data, and any other type of statistical information which is found in the media. Attempt to find at least three different types of representations. For each graph or statistical display answer the following questions

1. Write an analysis of the information represented.

2. Examine any assumptions which are made.

3. Is the data effectively represented? Explain. If not, suggest other ways which the data could be effectively illustrated.

4. Discuss whether or not the data is misleading in any way, gives the reader wrong impressions, or omits pertinent information.

5. Discuss whether or not information in the representation could be used to make predictions.

6. Formulate some questions which come to mind as a result of examining the data.

Topic: Data Analysis and Discrete Math

Subtopic: Designing an experiment to study a problem

Students of the new Atlantic curriculum in mathematics will be expected to gain insight into statistical methods by conducting experiments to study problems. They are encouraged to relate their experiments to other disciplines. Following are ideas for experiments which could be used to help them gain insight into conducting various types of experiments. The ideas are:

Designing an experiment to study a problem

A Statistical Research Project on Weather

Note to teacher: This activity requires students to explore relationships among weather variables in groups of three or four over a four week period. There are several other aspects of weather which may be investigated in addition to those given on their handout. For example, they could investigate:

- pressure
- wind speed
- humidity
- number of sunny days in certain areas
- places with many foggy days
- areas with high snowfall accumulations
- areas which become blocked with ice or icebergs

There are obviously many aspects of weather which can be investigated, so students should not feel restricted. They will find an abundance of weather information on the Internet if they have access.

Students who are not academically inclined may like to compare weather variables for two cities, their highs and lows, foggy and sunny days, etc. They could investigate whether or not a city's weather is fairly predictable for a particular season. They could investigate questions such as:

- Based on weather information, which city do you recommend should host the Summer Games - St. John's or Corner Brook?

✎ Student Sheet - A Statistics Project

Variables Affecting Weather

The following project is a statistical research project which is to be completed in groups of three to four. You will research various aspects of weather and explore relationships between weather variables.

Week One: Data Collection During the first week of this project make initial preparations by searching the Internet (where possible) for weather info, checking newspapers, magazine articles, library books, etc. and watching weather reports on television to gather enough information for the first group meeting so that the group can decide which aspects of weather they want to research. You will be given a class period at the end of the first week to discuss the data collected by your group and agree on a research question.

Week Two: Formulation of Hypothesis There are many variables with respect to weather that can be explored. Two or three variables will be sufficient for this project. For example, you may decide to research the relationship between temperature and precipitation. Once all data has been analyzed and a research question has been finalized, formulate a hypothesis. Each group member will be responsible for writing a paragraph (as a journal entry) on group discussions at the end of the second week.

Week 3: Further Data Collection and Analysis Construct tables and draw graphs to show the relationships between the variables you have selected. The more data your group collects, the better. The more work each member can do in advance of the meetings, (compiling tables, etc.) the more effective the group meetings will be.

Week 4: Finalize details and submit project. Group members must divide the work for the final report and include in the report the responsibilities of each member. Your final submission is to answer the following questions.

Project Questions

1. State the question which your group investigated and the initial hypothesis.
2. Describe how group members collected the data on weather - where and when it was collected and comment on its accuracy.
3. Place the data in tables so that all information will be easily readable.
4. Construct graphs wherever possible which will assist in the interpretation and presentation of the data.
5. Describe in detail the inferences you drew from the data regarding the relationships between the weather variables. Compare these findings with your initial hypothesis. Was your initial hypothesis correct? If not, suggest reasons for the change.
6. Discuss anything else which you feel is relevant to this project. This could include comments on the group process or any changes your group would make if you were to do the same project again. Did the project give you any ideas for further research on weather?
7. Describe briefly the role played by each group member.

Designing an experiment to study a problem

Investigating Short-Term Memory

Note to teacher: This activity, adapted from Bowman (1994), can be conducted in one class period. Students are asked to conduct an experiment on memory recall by compiling and reading aloud lists of words to their partners, and analysing the results. The student sheet is self-explanatory.

Expected Result: Early and late words in the list are recalled better than those in the middle, for, according to psychologists, early words find their way into long term memory whereas the words in the middle are overwritten in short term memory by those at the end.

To conclude lesson: Graph the results for the entire class on the overhead or blackboard. Let the x-axis represent the numbers 1 to 12 to indicate the position of the word in the list, and the y-axis represent the frequencies of correct responses for each word.

Other experiments could be conducted using the same idea. For example:

- Students could investigate the effect of using words that are related. A single reader may be best for the entire class.

- Students could follow the original idea, but instead of waiting 10 seconds to begin recall, have the hearer count backwards in 7's from 256 before recalling the words. The arithmetic exercise will most likely obliterate the contents of working memory.

Student Sheet

Experiment - Investigating Short Term Memory

Objective: To carry out an experiment which would investigate the operation of short term memory by studying the way a list of words is recalled.

Task: Each student is to compose a list of words, read them aloud to a partner (in turns), and ask the partner to recall as many as possible.

Protocol: In composing the words the following protocol must be observed:

- there must be 12 words
- they must be nouns so that there is less chance of sentence construction
- they must not be in alphabetical order
- there must not be any unusual words

In reading the words aloud, the following protocol must be observed:

- the reader should say the word "elephant" silently after each word so that the list is read at an even pace
- the list should be read only once and not displayed in written form
- a 10 second period should elapse before the subject starts recalling the words
- a period of 30 seconds should be given for recall

The reader is to record the results by placing 1's and 0's in the list to indicate whether the words in the list were recalled or not.

Investigating Short-Term Memory
Word Lists

To indicate recall use 1 (Yes) or 0 (No).

Words	Recall (1 or 0)
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	
10.	
11.	
12.	

Designing an experiment to study a problem

A Capture-Recapture Simulation

Note to teacher:

This activity can be conducted in *one class period.* It is a simulation of capture-recapture which is often used to estimate the size of populations, such as fish and wildlife, which are very difficult to count. All pertinent information is given on the student sheet. Paper cut-outs could be used instead of beans or macaroni to represent the fish.

A suggested follow-up exercise on salmon populations accompanies the activity.

For a related activity on dynamical systems see Page 125.

✎ Student Sheet

A sampling technique to estimate the size of a population

Name: ______________________________________

The following technique is a simulation of capture-recapture which is often used to estimate the size of populations, such as fish and wildlife, which are very difficult to count. The simulation uses macaroni (or white beans) .

Procedure: Fill about 3/4 of a bag with macaroni or beans. Leave room for shaking.

- Take out a handful and mark all of them with a colored marker. Record the number marked and put them back into the bag.

- Shake the bag to mix the colored ones throughout the others. Then take out a couple of handfuls.

- Count and record the number of marked pieces in this sample as well as the total number in the sample. Use the following proportion to make an estimate of the number of pieces in the bag.

$$\frac{\text{\# marked in sample}}{\text{sample size}} = \frac{\text{\# marked in bag}}{\text{total number } x}$$

- Take out 4 more samples and repeat the above step with each sample. You will then have a total of five estimates.

- Find the average of the five estimates to give an estimate of the total number in the bag.

You can use the following table to keep track of your data.

Sample #	# Beans in Sample	# Marked Beans	Estimate of total (Using proportion)
1			
2			
3			
4			
5			
			Average:

A Sampling Technique

Follow-up Activity - Salmon Populations

Note to teacher: This exercise is designed to encourage students to think about how fish populations, in this case salmon, can be affected in a certain area over a period of time. It is also in keeping with NCTM's (1989) idea that students should experience mathematics in other subject areas across the curriculum. Students should already be familiar with how fish populations are estimated using the capture-recapture idea.

Student Sheet - Salmon Populations

Name. ______________________________

You are conducting research into salmon populations in your area to determine whether or not recent developments and increased fishing are negatively affecting the stocks The following chart gives information on salmon counts in the area before this year's introduction of salmon This data and the answers to the questions which follow are relevant to your research

Note. Salmon were introduced into your area only two years ago, so assume there were no salmon before that time.

Type of Salmon	Salmon Added Last Year	Estimated No. of Salmon	Max. Salmon to be introduced
Coho	1530	490	900
Pink	890	445	2350
Sockeye	990	660	5000
Spring	2700	1335	2200
Total Salmon	**6110**	**2930**	**10450**

Questions to Answer:

1 What percent of each type of salmon survived after the first year?

Type	% survived
Coho	
Pink	
Sockeye	
Spring	

2. Which type of salmon appears to have the best survival rate after the first year? Which type has the poorest survival rate after the first year?

Best: ______________________ Poorest: ______________________

3. It is estimated that one tenth of the salmon died due to the change in environment. This being the case, how many of each type must have been caught by fishermen. ____________________

4. It is estimated that twice as many fish will be caught by fishermen next year. This being the case, and keeping in mind the 10% death rate due to change, how many will need to be introduced this year to account for the catches?

5. After one more year, the initial fish will begin to reproduce. If the reproduction survival rate was approximately 1.4 per fish, how would that affect the number of fish to be introduced next year?

Topic: Data Analysis and Discrete Math

Subtopic: Critiquing Graphs and Reports

Students are expected to interpret data from various representations, including articles, tables, written reports, statistical charts, etc. Following are two ideas for doing this.

1. Critiquing Weather Data - A chart on daily temperature data with questions for students. Page 80

2. A chart on NBA ticket prices with questions for students to answer. Page 83

Critiquing Graphs and Reports

Critiquing Weather Data

Note to Teacher: The objective of this lesson is to encourage students to analyse and critique everyday statistical data, and where possible make generalization and predictions. Weather data for March 1988 and 1989 are used for this exercise.

Here is an example of a generalization that students could make as they work with the chart.

- March 1989 was no hotter than 1988 (Mean difference = 10.67, standard deviation of difference = 14.89, standard error = 4.29, and t = 2.49)

The student sheet and data chart follow.

Student Sheet

Critiquing Weather Data

Name: ______________________________

Use the chart below to answer the following questions.

1. Is it possible to use the first 20 days of March to predict the temperature for the remaining 11 days of the month? Examine both years and discuss whether or not you think this is possible within 60 % accuracy.

2. Based on the temperatures given for March of each year, predict the temperature for April 1 of each year. If possible, obtain the actual temperature and compare with your predictions.

3. Analyse the table carefully and make some generalizations based on the given information. Consider comparing highs and lows of each year, as well as weather variability.

Follow-up Activity:

Over a period of one week, keep daily track of the *forecast* high and low temperatures in the local newspaper. Each day check the actual high and low temperatures to determine the accuracy of the forecast.

Daily High and Low Temperatures (Walker, 1993)

	March 1989				March 1988			
Day	High	Rank	Low	Rank	High	Rank	Low	Rank
1	46	3	43	14	66	15	49	24
2	59	10	30	2	60	5	57	27
3	71	17.5	43	14	69	18.5	59	28
4	72	19	54	19	75	27	61	29.5
5	67	15	55	21	74	26	44	17
6	58	8	38	9	86	31	52	26
7	50	4.5	31	3.5	47	1	40	7
8	53	6	37	7.5	54	2	41	10
9	56	7	43	14	60	5	42	13.5
10	45	2	33	5	66	15	42	13.5
11	44	1	37	7.5	66	15	49	7
12	50	4.5	23	1	69	18.5	48	22.5
13	59	10	31	3.5	65	13	47	21
14	62	12.5	35	6	69	18.5	45	18
15	59	10	40	10.5	71	23	42	13.5
16	62	12.5	49	17.5	55	3	43	16
17	71	17.5	40	10.5	61	8	33	1
18	80	23	55	21	73	24.5	39	4.5
19	80	23	57	23	62	10.5	51	25
20	64	14	47	16	60	5	37	3
21	69	16	42	12	70	21.5	34	2
22	76	21	49	17.5	80	28	48	22.5
23	86	25	55	21	82	29	61	29.5
24	88	27.5	65	28.5	83	30	66	31
25	88	27.5	62	25	61	8	41	10
26	87	26	65	28.5	62	10.5	46	19.5
27	90	30.5	64	27	73	24.5	41	10
28	90	30.5	68	31	61	8	46	19.5
29	89	29	67	30	63	12	40	7
30	80	23	63	25	70	21.5	39	4.5
31	73	20	58	24	69	18.5	42	13.5

Critiquing Graphs and Reports

NBA Ticket Prices[1]

Note to teacher: The chart which follows on NBA ticket prices appeared in the Toronto Star on November 5, 1993. Students are to analyse the information in the chart and answer the questions which follow. A spreadsheet is recommended for doing some of the calculations and making predictions.

[1]Adapted with permission from "Media Clips," edited by Ron Lancaster and Charlie Marion. *Mathematics Teacher*, copyright January, 1995 by the National Council of Teachers of Mathematics. All rights reserved.

Student Sheet - NBA Ticket Prices

The average ticket prices charged for the NBA games at the 27 home sites over the past four seasons (source: Team Marketing Report):

	93 - 94	92 - 93	91 - 92	90 - 91
New York	$39.66	$36.14	$30.50	$27 17
Chicago	$36.45	$32.98	$29.40	$25.88
Phoneix	$36.06	$33.39	$21.67	$20.33
LA Lakers	$32.84	$32.28	$47.11	$39.06
Boston	$31 45	$31.45	$24.80	$21.60
Orlando	$31.28	$27.08	$15.00	$15.00
San Antonio	$29.93	$25.84	$26.00	$23.11
New Jersey	$29.62	$24.67	$21.50	$19.17
Detroit	$29.41	$29.41	$27.07	$27.07
Utah	$28.61	$27.19	$22.57	$23.25
Portland	$27.81	$26.11	$24.36	$24.56
Seattle	$27.78	$26.42	$19.57	$18.00
Houston	$27.36	$25.12	$22.17	$18.56
Cleveland	$26.89	$25.13	$21.43	$18.83
Golden State	$26.74	$24.75	$22.25	$20.17
Washington	$24.83	$23.49	$19.75	$15.50
Miami	$24.62	$21.20	$18.25	$15.00
Atlanta	$24.26	$24.26	$20.00	$16.80
Sacramento	$22.89	$19.03	$19.67	$19.67
Philadelphia	$22.70	$23.06	$19.67	$19.80
LA Clippers	$22.45	$22.86	$20.40	$20.40
Charlotte	$22.43	$20.31	$22.44	$22.00
Dallas	$22.30	$22.30	$19.38	$19.38
Milwaukee	$21.59	$19.77	$18.14	$17.14
Denver	$21.14	$17.70	$17.43	$17.43
Minnesota	$20.74	$20.74	$17.56	$17.56
Indiana	$20.71	$19.38	$19.50	$15.20
NBA AVERAGE	**$27.12**	**$25.16**	**$22.52**	**$20.24**

Questions:

1. At the bottom of each column, the NBA average price is given for each season. How was this average calculated? Is this number really the average ticket price? Can you think of another way to calculate the average price?

2. Which teams increased their ticket prices by the greatest percent from 1990 - 1991 through 1993 - 1994?

3. Suppose that you are a sports writer interested in investigating whether all NBA teams have raised their prices over the last four seasons by basically the same amount. What calculations would you do to investigate this question? Use a spreadsheet to do the calculations and attempt to predict the prices for the 1994 - 1995 season.

4. If possible, obtain the prices for the 1994 - 1995 season and compare with your predictions.

Topic: Relations and Functions

Subtopic: Representing Functions in Multiple Ways

The new Atlantic curriculum for mathematics requires students to develop an in-depth understanding of functions in general and to acquire an efficiency with graphing a variety of linear and non-linear functions. Students will be expected to graph in both traditional (table of values, etc) and non-traditional ways (graphing utility).

The following activities are intended to serve as stimulation for learning about functions and graphing and as reinforcement of concepts. They are:

Representing Functions in Multiple Ways

A Function Game

To the teacher: A Description of the Game

The game Barriers (Friedlander et al, 1988) described here is intended to heighten student motivation and stimulate the process of algebra concept-formation. The student is required to analyse simultaneously a collection of functions and perform, in a game situation, a large number of matchings between a set of graphically or algebraically presented functions and a set of function-properties.

Prerequisite: The minimum prerequisite knowledge to play this game is an introduction to functions in general, and to linear functions in particular.

The Game: The game contains 9 Property (Function) cards, 6 Barrier cards, and 26 Function cards.

Getting Started: The 6 Barrier cards and 26 Function cards are shuffled and placed face-down in one pile as the bank. Each player receives 4 cards from the bank, as well as 3 Property cards randomly selected from the 9 (the remaining 3 are put aside). The Property cards are placed face up on the table.

Moving: Each player, at his/her turn, picks a card from the bank, and "plays" one of the 5 cards now in his/her possession as follows:

(a) Places a Function card beside one of his/her Property cards that correctly describes a property of that function, or (b) Bars his/her opponent with a Barrier card,

or (c) Returns a card to the bottom of the bank pile.

Which ever option is chosen, at the end of this stage the player again holds 4 cards.

Barring: A barred player cannot add a Function card to his/her Property cards (move a.) unless he/she removes the Barrier by *covering* it with a corresponding Function card. *Covering* counts as a move, but the player may also choose to leave the Barrier temporarily and opt for moves (b) or (c).

Refilling the bank: An empty bank may be refilled with the Function cards discarded in the process of removing Barriers.

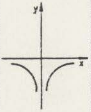
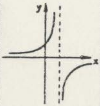
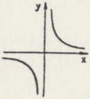
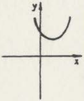
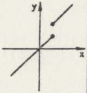
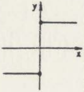
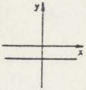
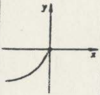
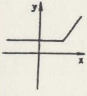
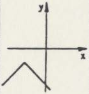
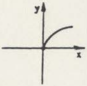
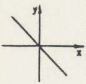
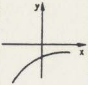
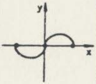
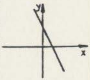
The Winner: The first player to collect three Function cards beside each of *two* (of his three) Property cards is the winner.

Note: The number and variety of problems posed to a player during one class period (2 - 3 games) would take many textbook pages to present, and a much longer period of time to be understood and answered.

The rules may seem complicated at first, but with teacher help will become clear.

The game can be played at different difficulty levels. Lower ability students can use the cards as a sorting activity - i.e., given a Property card, find all the corresponding Function cards, or vice versa, find all the Property and Barrier cards which correspond to a given Function card.

Functions Game - Cards

$g\ R \to R$ $g(x) = \begin{cases} 3 & x \le 0 \\ 4 & x \ge 0 \end{cases}$	$h\ R \to R$ $h(x) = \lvert x \rvert + 1$	$g\ x\ x \neq 0 \to R$ $g(x) = \frac{1}{x}$	$f\ R \to R$ $f(x) = -3$	$k\ R \to R$ $k(x) = x^2$	$m\ R \to R$ $m\ x \to 7$
$h\ R \to R$ $h\ x \to -x^2 - 3$	$k\ x \mid x \neq 0 \to R$ $k(x) = \begin{cases} 3 & x < 0 \\ -3 & x > 0 \end{cases}$	$w\ R \to R$ $w(x) = -3x$	$f\ R \to R$ $f(x) = 2x$	g :negatives: $\to R$ $g(x) = x^2$	
					
					
		**Function Cards**			

PROPERTY CARDS				
Increasing throughout the domain.	Zero is not in the domain	Linear function	The function has a zero value	Zero has a positive image
The domain contains all but one real number	The function has positive or zero values only	The origin is on the graph	The function has only negative values	

BARRIER CARDS		
Decreasing or constant throughout the domain. **Stop!**	The domain does not contain all the real numbers. **Stop!**	The function has no zero value. **Stop!**
The origin is not on the graph. **Stop!**	The images of all positive numbers are negative. **Stop!**	Partly increasing and partly decreasing. **Stop!**

Representing Functions in Multiple Ways

Graphing Activities[1]

Note to teacher: The first two activities in this section require students to graph several equations to form a figure The equations are either linear, quadratic, or absolute functions, with restrictions given for domains and ranges

The figures formed by the graphs are:

- Activity 1 - A heart with Cupid's arrow
- Activity 2 - A sailboat

When students have completed these activities they should feel ready to pursue the *graphing-art project* which is given. In this project they work in reverse to the two previous activities. They are expected to

- design a picture which contains graphs of lines, parabolas, and absolute values.
- formulate equations to correspond to each part of the picture.

[1]Adapted with permission from "Graphing Art" by Fan Disher. *Mathematics Teacher*, copyright February, 1995 by the National Council of Teachers of Mathematics.

Student Sheet - Graphing Activity 1

Graph each of the following equations on one set of coordinate axes. Watch the given restrictions on the domains and ranges. Tell what picture is formed.

	Equations	Restrictions on Domain/Range
1.	$y = -\frac{1}{9}(x - 6)^2 + 14$	D: [0,12]
2.	$y = -\frac{1}{9}x^2 - \frac{4}{3}x + 10$	D: [-12, 0]
3.	y = - 2x + 34	D: [12, 13]
4.	2x - y = -34	R: [-13, -12]
5.	x = 13 or x = -13	R: [5, 8]
6.	$y = \lvert(\frac{21}{13})x\rvert - 16$	D: [-13, 13]
7.	y = 1	D:[-13,-10.5] ∪ [-5,5] ∪ [10.5, 13]
8.	x = - | y - 1| + 13 or x = - |y - 1| - 13	R: [0,2]

Student Sheet - Graphing Activity 2

Graph each of the following equations on one set of coordinate axes. Watch the given restrictions on the domains and ranges Tell what picture is formed.

	Equations	Restrictions on Domain/Range
1.	x = -2	R: [-8, -1] ∪ [13, 16]
2.	y = 15 or y = 16	D: [-2,0]
3.	x = 0	D: [15, 16]
4.	y = -7/5 ∣ x + 2∣ + 13	D: [-12, 8]
5.	$x \quad 2(y - 10)^2 - 3$	R: [9, 11]
6.	$y \cdot -\frac{1}{3}x + 7$	R: [-5.08, 3]
7.	$y - 6 \quad -\frac{1}{3}(x + 3)$	D: [-6.23, 4.88]
8.	x + 3y = 10	D:[-7.19, 6.44]
9.	$y \quad \frac{1}{18}(x \cdot 2)^2 - 16$	D: [-14, 10]
10.	y = -1	D: [-12, 8]
11.	y = -8	D: [-14, 10]

Graphing-Art Project

1. Using graph paper, draw a picture containing graphs of lines, parabolas, and absolute values. The picture must contain graphs of a minimum of ten (10) different equations. Keep track of the equations which correspond to each part of the picture. At least two must be lines; at two, parabolas; and at least two, absolute-value graphs. One of the absolute-value graphs or one of the parabolas must have a horizontal line of symmetry. The remaining equations may be any of the three.

2. Domains and ranges may be determined by solving equations simultaneously or by using the graphing calculator to graph the functions and trace the curves to find points of intersection. If these points are not integers, specify them to the nearest hundredth.

3. Hand in your graph with the different parts of your graph numbered to coincide with the equations. Have your equations printed neatly in the table provided and give the restrictions on the domain and range for each. Give your name and the name of your picture on the top of the equation page.

Graphing Project - Equation Form

Student Name:____________________ Project Title: ____________________

Equations	Restrictions on Domain and Range

Topic: Relations and Functions

Subtopic: Determining the equation of a line

The following activity is intended to be an enjoyable starting point for introducing equations of lines. It may prove effective for several reasons.

- It can take place outside the classroom, in a school yard or gymnasium, giving students a break from regular routine.
- Students do not need pencils or exercises. They do the math mentally in a very relaxed setting.
- Many concepts can be introduced using this activity, including parallel and perpendicular lines and systems of equations.

A description follows.

A Teacher-led Activity

Title: The Human Coordinate System[1]

Objective: To present equations of lines by having the student bodies represent points on the Cartesian coordinate system.

Description:

This activity is intended to be an enjoyable starting point for introducing equations of lines. It can take place in the classrooom (depending on the arrangement of desks), in the gymnasium, or in the school yard. Begin by arranging the students in rows up and down and across so the lines both ways are straight. The student in the center represents the origin and each other student represents a point in the system, depending on position from the origin. Each is asked to name his or her coordinates. They are to remain seated until called upon to stand.

All students with x-coordinate equal to 1 are asked to stand. Students are told that this vertical line represents the equation $x = 1$. These students sit down and students with y-coordinate of 2 are asked to stand. They can see that this horizontal line represents the equation $y = 2$. This procedure continues until the type of equation representing an horizontal or vertical line is obvious.

Moving to equations involving x's and y's, the teacher then asks all students whose x and y coordinates are the same to stand up. They will see a straight line running through the origin. Students whose x and y add up to 4 are asked to stand ie. $x + y = 4$. These students sit down and students whose x and y have a difference of 2 are asked to stand ie. $x - y = 2$ The process continues with a variety of types of equations until the students get the idea and can make several generalizations.

Extending the idea.....

This idea can also be used to help in the understanding of solutions to ***systems of equations.*** Students with x- and y- coordinates totalling 1 can be asked to stand at the same time as students whose x- minus y- coordinate equals 1. The teacher asks if there is someone standing in both lines. Students can see that the person with coordinates (1,0) stands in both lines and thus satisfies the two equations: $x + y = 1$ and $x - y = 1$. The meaning of a solution to a system becomes obvious as more of these are 'physically' practiced.

The idea can also be extended to **parallel lines** by simultaneously representing such equations as $x + y = 1$ and $x + y = 2$, or to **perpendicular lines** by simultaneously representing the equations $x + y = 4$ and $x - y = 2$. Many other concepts and principles could no doubt be represented, including solving linear inequalities and systems of linear inequalities. The active physical involvement of the students and the visual effect involved should lead to better understanding by students.

[1]Adapted with permission from "The Human Coordinate System," by Richard Crouse. *Mathematics Teacher*, copyright February, 1991 by the National Council of Teachers of Mathematics. All rights reserved.

Topic: Relations and Functions

Subtopic: Solving Systems of Linear Equations

NCTM *Curriculum and Evaluation Standards* (1989) encourages a move away from student memorization of procedures and towards student thinking and empowerment. Alper et al (1995), stress the importance of having students reason on their own rather tnan memorize procedures.

> . . . students often forget procedures. If, on the one hand they have no way of recapturing the procedure except through direct memory, they are then lost. If, on the other hand they have confidence they can rediscover a forgotten procedure, their mathematical knowledge is much more powerful. (p. 715)

The next activity asks students to reason on their own and discover for themselves the point of intersection of two graphs. It is entitled:

This is followed by:

✎ Student Sheet - Get the Point?

Your group's goal in this activity is to discover a method for finding the point of intersection of two lines. This is to be done without guessing or using graphs, but by working with the equations of the two straight lines.

Your written report on the activity should include the following:

- Solutions to equations 1 (a - e)
- Two of the problems made up for question 2 by individual group members, with solutions
- Your group's written directions for question 3

Each group will make an oral presentation on its results for question 2.

Questions

1. For each of the following pairs of equations, find the point of intersection of their graphs by some method *other than graphing or trial and error.* When you think you have each solution, check it by graphing or by substituting the values into the equations.

a.	$y = x$	and	$3x = y + 4$
b.	$Y = 2x + 5$	and	$y = 3x - 7$
c.	$3x + 2y = 13$	and	$y = 4x + 1$
d.	$7x - 3y = 31$	and	$y - 5 = 3x$
e.	$5x + 3y = 17$	and	$2y + 1 = 3x$

2. Each person in the group should make up a pair of linear equations, and find the point of intersection. Then group members should trade problems (without giving solutions), and work on each other's problems, trying to find the point of intersection without guessing or graphing.

3. As a group, develop and write down general directions for finding the coordinates of the point of intersection of two equations for straight lines without guessing or graphing. Make your directions easy to follow.

[1]Adapted with permission from "Is this mathematics class?", by Lynne Alper, Dan Fendel, Sherry Fraser, and Diana Resek. *Mathematics Teacher*, copyright November, 1995 by the National Council of Teachers of Mathematics.

Solving Systems of Linear Equations

A real-world problem[1]

Note to teacher: NCTM *Curriculum and Evaluation Standards* (1989) emphasizes the importance of applying mathematics to common everyday problems. The following word problem is an example of how mathematics can be applied to nutrition.

Here is the solution to the problem which follows:

Define three functions representing the amounts of energy, protein, and carbohydrates.

$$A:(c,p,r) \rightarrow 3.5c + 6p + 3r = 1200$$

$$B:(c,p,r) \rightarrow 0.03c + 0.25p + 0.04r = 30$$

$$C:(c,p,r) \rightarrow 0.7c + 0.2p + 0.8r = 150$$

Enter the system of equations and use a graphing tool to solve for c, p, r to arrive at $c = 147.619$ $p = 96.8253$ $r = 34.1269$

[1]Adapted with permission from "A technology-intensive approach to algebra" by M. Kathleen Heid and Rose Mary Zbiek. *Mathematics Teacher*, copyright November, 1995 by the National Council of Teachers of Mathematics.

✎ Student Sheet - A real world problem

Solving Systems of Linear Equations

A hiker plans to carry only high-energy, lightweight food - a mixture of peanuts and raisins and some cocoa powder to make drinks from water. The nutritional value of these foods is shown in the following table:

Nutrition available from each gram of food

	Cocoa	Peanuts	Raisins
Energy in calories	3.5	6.0	3.0
Protein in grams	0.03	0.25	0.04
Carbohydrates in grams	0.7	0.2	0.8

1. On the packages for cocoa, peanuts, and raisins, the recommended single serving is 28 grams of cocoa (before water is added), 28 grams of peanuts, and 85 grams of raisins. Calculate the calorie, protein, and carbohydrate yield from a snack that consists of a single serving of each food.

2. What combination of cocoa (c), peanuts (p), and raisins (r) can yield a snack with 1200 calories, 30 grams of protein, and 150 grams of carbohydrates?

Mixture problems without using systems of equations

To the teacher: A description of the method involved

Students often become confused with mixture problems. They often cannot visualize answers or make reasonable guesses before they do the mathematics. These problems have traditionally been solved using systems of equations. Using a tug-of-war[1] approach may help students who are having difficulty visualize the solution. That is the intent of this exercise.

Problem:

> White jelly beans cost $0.25 a pound and blue jelly beans cost $0.85 a pound. How many pounds of white jelly beans must be added to 25 pounds of blue jelly beans to arrive at a mixture worth $0.45 a pound?

With an ***intuitive approach***, we would begin by reasoning that if the mixture contained the same amount of white and blue jelly beans, it would cost an amount that is right in the middle. A $0.60 difference exists between the two types. A middle price would be $0.25 + $0.30 (half the difference), or $0.55. But the problem states that the new mixture must be worth $0.45. Therefore, the mixture must contain more white jelly beans than blue. Also, it must be greater than 25 pounds.

Then we would reason that the price difference between the white jelly beans and the mixture is $.20. The price difference between the blue jelly beans and the mixture is $.40. As a ratio, the price differences compare 1:2. It makes sense that the amounts will follow this ratio. Since more white jelly beans are needed, the ratio of white to blue must be 2:1. Therefore, since the mix has 25 pounds of blue, it must have 50 pounds of white.

Using the ***tug-of-war approach***, imagine the white jelly beans having a tug-of-war game with the blue jelly beans. Both want the final price to be closer to their price.

1. Draw the "rope" (see figure below).

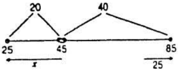
2. Label the three numbers you know: the left point on the rope is the price of the white jelly beans (25), the right point is the price of the blue jelly beans (85), and the knot will be located at the final price (45), and closer to the left side. Try to approximate where 45 would land between 25 and 85.

3 Draw arrows to indicate the pulling force in each direction. Since the white jelly beans are pulling harder, make the left-pointing arrow longer than the right-pointing arrow.

4. Label what you know - the pull to the right is 25. Label what you don't know - the pull to the left is x.

5. Calculate the gap between the knot and the left side (20) and between the knot and the right side (40). Since the gap on the left is half as big, the left side must be pulling twice as hard. Therefore, we must have 50 pounds of white jelly beans.

Students will eventually discover the rule: gap x pull = gap x pull. Multiply the gap on the left (20) by the pull to the left (the unknown). Set this quantity to the gap on the right (40) times the pull to the right (25). Solving for the unknown yields 50 pounds. See diagram next page.



This formula allows for all mixture problems to be solved, even if the gaps do not come out in a nice ratio like 1:2.

Following is a practice exercise for students which involves percents. The answer is 40 ounces.

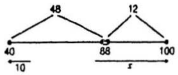
[1]Adapted with permission from "Tug of War," by William D Telford, Jr. *Mathematics Teacher*, copyright February, 1993 by the National Council of Teachers of Mathematics. All rights reserved.

Student Sheet

Tug-of-War Exercise - An acid-mixture problem

Solve the following problem using the tug-of-war diagram given below.

A solution that is 40% acid must be strengthened to become a solution that is 88% acid. If we have 10 ounces o   0% solution, how much pure acid must be added?



Acid Mixture Problem - Using Spreadsheets

Note to Teacher: The above problem can also be solved using spreadsheets, as per the diagram below.

Columns A and B contain the original amount of acid and the original amount of solution. Column C contains the amount of acid added and column D contains the total amount of acid after additions (An + Cn)

Column E gives the total amount of solution after acid is added (Bn + Cn)

Column F gives the percent of acid in the total solution - (Dn/En) * 100
As we can see we need to add 40 ounces to have an 88% solution.

	A	B	C	D	E	F
1	An	acid	mixture	problem		
2	Amounts	are in	ounces			
3						
4	original acid	original amt	acid added	acid after	total	% acid
5				addition	volume	in total
6						
7	4	10	0	4	10	
8	4	10	1	5	11	45.45455
9	4	10	2	6	12	50
10	4	10	3	7	13	53.84615
11	4	10	7	11	17	64.70588
12	4	10	10	14	20	70
13	4	10	15	19	25	76
14	4	10	20	24	30	80
15	4	10	25	29	35	82.85714
16	4	10	30	34	40	85
17	4	10	35	39	45	86.66667
18	4	10	36	40	46	86.95652
19	4	10	38	42	48	87.5
20	4	10	40	44	50	88
21	4	10	42	46	52	88.46154

Solving Linear Systems (Other)

Refer to the activity on The Human Coordinate system (Page 96) for a starter session on solving systems of equations by graphing.

Refer to the spreadsheet algorithm (Page 24) for solving linear equations. Using similar ideas a spreadsheet algorithm could be formulated for solving systems of linear equations

Topic: Relations and Functions

Subtopic: The Quadratic Function

One of the topics listed in the Atlantic curriculum for grade 10 mathematics is the introduction of the quadratic function. $f(x) = ax^2 + bx + c$ The following unit proposes to introduce the topic using graphing calculators and computer technology. Prior to these lessons students will have become familiar with the linear equation $ax + by = c$, the corresponding linear function $f(x) = mx + b$, and the related graphs. To begin this unit students will be exposed to real life examples of these functions and their graphs, will study terminology related to these functions, learn to interpret the equations prior to graphing, and examine the effects that parameter changes will have on the graphs.

This unit is divided into three main lessons:

LESSON 1 Introduction

In this lesson the teacher introduces the quadratic function, discusses some real life examples, uses the graphics calculator to generate graphs of quadratics and introduces terms related to the graph. The main goal of this lesson is to enable students to visualize graphs of quadratics and to make real world 'sense' out of them without getting bogged down in tedious calculations. The graphics calculator will play a major role in facilitating this understanding. The manual calculations will be integrated after a general understanding of the concepts is realized. If students have their own grapher, or if class sets are available, they can generate the graphs along with the teacher. This lesson can be covered in approximately one 40 minute class period.

Procedure

Begin by introducing the term "quadratic", its derivation (the Latin word "quadratus", meaning "square"), its general shape when graphed (a parabola), and ask students to suggest some real world examples. Such examples could include the path of an object as it is thrown into the air, the dive of a plane, the jump of a basketball player, the launching of a missile, the path of an object thrown from a building or parabolic curves in art and architecture.

The following real world example could be used.

> An object thrown upwards with a starting speed of "v" metres a second, from a height "H", after "t" seconds has a height given by $h = vt - 4.9t^2 + H$.

The teacher could substitute values for "v" and "H" and use an overhead graphing calculator, such as the Casio fx -7700 or TI-82 to plot the graph. The TRACE function is turned on and students are asked to identify the height of the object after various seconds. They can see that height varies with time, and that the height reaches a peak (its maximum value) before it starts getting smaller. The teacher could guide the discussion with questions such as "How long does it take for the object to hit the ground?" Students can see from the graph that this question refers to an x-intercept, the point at which h=0. Teacher asks "What x values (domain) and y values (range) are included on the graph? The graph itself, with the aid of the TRACE function reveals these values.

Vary the altitude, H, from which the object is thrown and its starting speed, v. A new graph is produced, and the discussion centres around what is happening to the graph as these variables change. As various graphs are displayed, the terms VERTEX, AXIS OF SYMMETRY, X-INTERCEPTS, MAXIMUM/MINIMUM points, and DOMAIN and RANGE can be discussed.

Students should realize that some quadratic functions represent real life occurrences but are not as visually apparent as the path of an object. For example:

> *A producer of fuel from coal estimates that the cost C dollars per barrel for a production run of x thousand dollars is given by the quadratic formula* $C = 9x^2 - 180x + 940$.

Graph the equation and use TRACE to show students how the cost (y coordinate) is associated with the number of barrels produced. With teacher led discussion, students will see from the graph the number of barrels that should be produced to keep cost to a minimum. As well, they can see how cost soars beyond that point.

Class understanding could be evaluated by presenting students with a graph and asking them to record pertinent information.

An example that could be used here is the graph of $h = -5t^2 + 6t + 3$, showing a diver's height above water "t" seconds after he leaves the diving board on a forward somersault dive. After using their own graphers or inspecting the graph from the overhead, students are asked to write down

I. his maximum height above the water, and the time it takes to reach this height
ii. the approx. height he reaches after 1 sec, 2 sec, etc.
iii. how long it takes before he enters the water

For homework, students could be asked to write a journal entry on the day's lesson.

LESSON 2 - Exploration

In this lesson students will use appropriate software in the computer lab to produce and explore several graphs of quadratics, some of which are in standard form, $y = a(x - h)^2 + k$. They will look closely at what happens to the shape and direction of the graphs when parameter changes are made and take note of vertex, axis of symmetry, x-intercepts (if any), and domain and range.

Note: If it isn't possible to use the lab, this lesson could be completed using graphics calculators (teacher directed or on an individual basis, if class sets are available).

Time Required: This lesson will require approximately *two class periods*, which gives time for communication among teachers and students, and students and students regarding the exploration session in the lab. Once all discussion is finalized students will need to spend some time learning how to change quadratics from general to standard form.

Procedure

Several quadratic equations are presented that look different from the general one presented in lesson 1, but students are advised that they are simply different forms of the general equation. For example, the quadratic equation $y = 3x^2$ is a general quadratic that can be written as $y = 3x^2 + 0x + 0$. Students are cautioned that $y = ax^2 + bx + c$ is not quadratic if "a" has a value of zero. The coefficients "b" and "c" could equal zero (either one or both), but never "a". Students are also given equations such as $y = 2(x - 3)^2$ and are asked to use graphing calculators and other pertinent information to put them in the form $y = ax^2 + bx + c$. They should already know how to use methods of simplifying learned earlier to find the general form. These methods could be reviewed at this point and used in verifying answers. The idea here is to keep students in touch with the general form they learned in lesson 1. They are informed that the look of the equation will change to simplify manipulations and graphing, but embedded in the equation is its general form. (Exercises supplied on handout)

Next, students are asked to *investigate* several quadratic equations by producing them on the computer screen using a computer software package such as Best Grapher (or another similar graphing package). They are asked to record their discoveries as they progress and are told not to be limited by the graphs given on the handout. *Exploration is encouraged..*

The handout used for the lesson can be submitted and used for evaluation. As well, the effectiveness of the lesson can be evaluated by having individual students present their findings regarding parameter changes and by sharing with the class results of further exploration.

Student Sheet - Lesson 2

Topics:

1) Using the graph of $y = a(x - h)^2 + k$ and other pertinent information to find the general form of quadratic equations.
2) Exploring the effect that parameter changes have on the general shape of the graph of quadratic functions.

Exercise 1. For the first topic consider the following for $f(x) = ax^2 + bx + c$

When $x = 0$ $\quad f(x) = c$

When $x = 1$ $\quad f(x) = a(1) + b(1) + c = a + b + c$
When $x = -1$ $\quad f(x) = a(1) + b(-1) + c = a - b + c$

Thus $\quad f(1) - f(-1) = 2b$

Rearranging the equation $f(1) = a + b + c$, we know that $a = f(1) - (b + c)$

In summary, finding f(0), f(1), and f(-1) can give you the information you need to find the general form. Keeping this in mind, use a graphing calculator or graphing software to graph each of the following and use the trace function to help you express each equation in the general form $y = ax^2 + bx + c$.

EQUATION	GENERAL FORM
1. $y = (x + 2)^2$	________________
2. $y = (x - 5)^2$	________________

3. $y = 2(x-1)^2 - 6$ ____________________

4. $y = 2(x + 3)^2$ ____________________

5. $y = -3(x - 2)^2$ ____________________

2. Using computer software (eg. Best Grapher) sketch each of the following graphs and record the pertinent information in the table. When finished each table, write the discoveries you have made.

Equation	Vertex	Axis of sym.	x-intercepts	2 pts. on graph
$y = x^2$				
$y = 2x^2$				
$y = -2x^2$				
$y = \frac{1}{2}x^2$				
$y = \frac{-1}{3}x^2$				

Discoveries: ____

____________________ ____________________________

Equation	Vertex	Axis of sym	x-intercepts	2 pts. on graph
$y = (x - 4)^2$				
$y = (x + 4)^2$				
$y = 2(x + 4)^2$				
$y = -2(x + 3)^2$				
$y = \frac{-1}{2}(x + 3)^2$				

Discoveries: ____

____

Equation	Vertex	Axis of sym	x-intercepts	2 pts. on graph
$y = x^2 + 6$				
$y = 2x^2 + 6$				
$y = x^2 - 6$				
$y = -3x^2 - 6$				
$y = (x - 3)^2 + 4$				
$y = 2(x - 3)^2 + 5$				

Discoveries: ____

____

The Quadratic Function

LESSON 3 - A real-life situation

Activity: Finding a quadratic equation for a real-life situation

There are several examples of parabolic equations in real life which can be integrated into mathematics class. One such example[1] is the flow of water from the school water fountain. Here is how this activity works.

Students first collect data from a school water fountain. They have to decide where to start with their measurements and what height to use, for their equations will depend on their initial assumptions. Wire can be used to capture the nature of the flow to model the fountain. Once the measurements are decided upon, an equation can be generated that has roots 0 and R and height h.

For example, if the roots are 0 and 9.75, the students would first write

$$y = x(9.75 - x).$$

However, the height is approximately 8.4. The above equation gives a height of 23.75 when they complete the square as shown below:

$$y \quad -x^2 + 9.75x$$

$$y \quad -1(x^2 - 9.75x + (4.875)^2) + (4.857)^2$$

$$y \quad -1(x - 23.75)^2 + 23.75$$

To arrive at an equation having the desired height, students need to change -1 to some other constant.

$$k(4.875)^2 = 8.4$$

$$k = 0.35345$$

Thus the equation: y = 0.35345x(9.75 - x)

Finding the constant can be a tedious process. However, a graphing calculator can be programmed, or a BASIC computer program written, to find the constant by entering the max/min value and the height. The programs appear below.

Program for the TI-81, TI-82, or TI-85 calculator.

```
:Disp "Input the Max or Min"
:Input M
:Disp "Input the first root"
:Input R
:Disp "Input the second root"
:Input S
:M/R*S-((R+S)/2)^2) STO Z  (Its derivation can be shown)
:Disp "The constant is"
:Disp Z
```

BASIC program the will run on Apple or IBM

```
10 PRINT "INPUT THE MAX OR MIN"
20 INPUT M
30 PRINT "INPUT THE FIRST ROOT"
40 INPUT R
50 PRINT "INPUT THE SECOND ROOT"
60 INPUT S
70 PRINT THE CONSTANT IS"
80 PRINT M/(R*S-((R+S)/2)²)
```

[1]Adapted with permission from "Finding Quadratic Equations for real-life situations" by Theodor Korithoski. *The Mathematics Teacher*, copyright Feb. 1996, by the National Council of Teachers of Mathematics.

Note: The above activity could be completed over *two 40 minute class periods*. Students will need the first period to collect the data and start setting up the equation. During the second period they will find the correct equation and learn how to use a graphing calculator or a Basic program to assist them in this or other similar situations.

Miscellaneous

The activities in this section could be used at various points throughout Course 1 of the high school Atlantic curriculum for mathematics. Some are intended to supplement on-going class activities - eg. finding line of best fit and matrix applications, others are intended as mathematical stimulation for students outside the regular curriculum - eg. constructing a Rhombicosidodecahedron, while others, e.g. Statistics Jeopardy and tessellations are intended to bring fun to learning mathematics.

The activities are:

Finding Line of Best Fit

The line of 'best' fit can be found using a couple of different procedures. The first procedure produces a reasonable line called the *median-median* line. Before attempting this method it is wise to look closely at the scatterplot to be sure the relationship between the variables is linear. The steps (NCTM, 1988) for finding the median-median line are as follows:

1. Separate the data into three groups of equai size according to the values of the horizontal coordinate.

2. Find the summary point for each group based on the median x-value and the median y-value.

3. Find the equation of the line (Line L) through the summary points of the outer groups.

4. Slide L one-third of the way to the middle summary point

    a. Find the y-coordinate of the point on L with the same x-coordinate as the middle summary point.

    b. Find the vertical distance between the middle summary point and the line by subtracting y-values.

    c. Find the coordinates of the point P one-third of the way from the line L to the middle summary point.

5. Find the equation of the line through the point P that is parallel to line L.

Finding Line of Best Fit - **Using a graphing calculator**

A line of best fit can be found using linear regression on a graphing calculator. The steps used to do this on the TI-82 are as follows:

Press STAT to display the stat edit menu.

Press "1" to edit

In the column marked L_1 enter the x-coordinates, and in the column marked L_2 , enter the y-coordinates.

Press STAT ▸ 5 to select Linear Regression LinReg(ax + b)

Press ENTER

The display gives the value for: "a" - the slope of the line
"b" - the y-intercept of the line
"r" - the correlation coefficient

Dynamical Systems[1]

To the teacher: Dynamical systems can be used in the mathematics classroom to stimulate student interest in mathematics. It is a topic which can incorporate group learning, spreadsheets, and graphing calculators, and is in tune with the recent focus on revamping the mathematics curriculum and its delivery.

A dynamical system is basically anything that changes over time. Examples include the weather, tides, populations, etc. The basic procedure used in dynamical systems is the iteration of functions. For a given function, F, and a given initial seed x_0, we iterate to produce the sequence

$$x_0, F(x_0), F(F(x_0)), F(F(F(x_0))) = F^3(x_0), \ldots.$$

This sequence is called the *orbit of* x_0, and the goal is to predict eventual behavior. Orbits of simple functions such as $F(x) = x^2$ can be easily produced by entering the value of x_0 into the calculator and repeatedly pressing the x^2 key. Much of the literature that dealing with mathematical dynamical systems begins with an analysis of the *logistic function*

$$F(x) = cx(1 - x)$$

In a very simplistic sense this function can be used to model population growth where x_0 is the initial population and the parameter *c* is a constant that reflects various conditions, such as food supply, temperature, etc. The variable *x* is restricted between 1 and 0 and represents a fraction of the limiting capacity of the population.

It is advisable to use a spreadsheet or programmable calculator to analyse orbits, for many different orbits are possible for the logistic function. After experimenting with different functions, initial seeds, and parameter values, it is advisable to use graphical analysis to get a visual sense of the mathematics involved.

Suppose we wish to look at the behavior of a particular population in which the initial population is 10 percent of the limiting capacity of the environment. We would need to look at the orbit of $x_0 = 0.1$. This can be done in the following manner:

> Starting on the x-axis at $x_0 = 0.1$, draw a vertical line segment until it hits the graph of F(x) at $(0.1, F(0.1))$. From here draw a horizontal line segment until it hits the diagonal y = x at $(F(0.1), F(0.1))$. Notice that directly below this intersection, on the x-axis, is $x_1 = F(x_0)$. Repeat this procedure starting with x_1 to get x_2, with x_2 to get x_3, etc. The orbit can be followed along the diagonal, with each step representing one iteration. These steps converge to the right-hand fixed point. The orbit of an initial seed to the right of both fixed points converges as before, indicating an attracting fixed point.

From the above steps students can conclude that when c = 1.8 and the initial population is 10 percent of the limiting capacity, then eventually the population will grow to about 44 percent of the limiting capacity and remain there. Since accurate graphing is essential to this procedure computers and graphing calculators are invaluable.

A more usable form of the logistic function to model situations such as fish stocks is:

$$F(x) = x(1 + r) - \frac{rx^2}{L}$$

where *r* is the unrestricted growth rate and L is the limiting capacity.

After discussing the above ideas (for more details on this topic see the reference below), the class should be ready to complete the project which follows. They may work alone or in pairs.

[1]Adapted with permission from "Stimulating Mathematical Interest with Dynamical Systems" by Marilyn B. Durkin. *Mathematics Teacher*, copyright March, 1996 by the National Council of Teachers of Mathematics.

Student Project

Dynamics of Population Growth

You have been hired to do a study on trout populations for the Department of Fisheries and Oceans in a remote lake in rural Newfoundland. Previous studies show that the carrying capacity of this lake is 20 000 trout and that the unrestricted growth rate is 0.7. As more people are building houses and cabins closer to this lake, the department is trying to decide whether to stock the lake before opening it to fishing. Your answers to the following questions are relevant to this study.

Answer all questions in a concise and logical manner. Use mathematical analysis, algebra, graphs, etc. to support your answers.

1. Assuming that no stocking is done and that no fishing is allowed, develop a dynamical system to model the size of the trout population.

2. Suppose that the province has just suffered a severe drought. As a result, the trout population of the lake has fallen to 50 percent of its carrying capacity. How many years will pass before the population recovers to within 1 000 trout of its limiting capacity?

3. After much debate it is decided to open the lake for fishing immediately with no stocking done. It is expected that the fishers will remove 2 500 trout each year. What will be the effect on the lake in the long run? Could the lake survive another drought of the same severity in the immediate future?

4. Suppose the area becomes really popular and the number of people going there to fish increases significantly over the next couple of years, so much so that the lake loses 4 500 trout per year. If this rate continues, how long will the trout population survive?

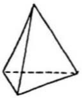
5. Decide the maximum number of trout that could be removed each year and still have the trout population survive.

6. Assume the fishing level evens out to 4 500 trout removed each year. How much stocking would be needed annually to maintain a stable population? Take minor catastrophes into account.

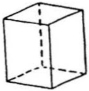
Investigating Platonic Solids[1]

The five solids shown below are the only *regular* solids which can be constructed. They are called regular because all faces, edges, and angles are congruent. These solids are called Platonic solids after the Greek philosopher Plato. They can be constructed using specific numbers of polygons as sides:

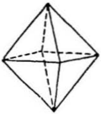
1. Tetrahedron (fire) - 4 triangles



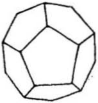
2. Hexahedron (earth) - 6 squares



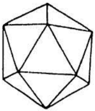
3. Octahedron (air) - 8 triangles



4. Dodecahedron (universe) - 12 pentagons



5. Icosahedron (water) - 20 triangles



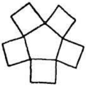
Question for students: Can you discover why only five regular solids can be made? Remember, all faces of a regular solid must be regular polygons of the same size and shape.

[1]Adapted with permission from "Polyhedra from cardboard and elastics," by John Woolaver. *Mathematics Teacher*, copyright April, 1977 by the National Council of Teachers of Mathematics.

Steps to constructing a Rhombicosidodecahedron[1]

A Rhombicosidodecahedron is a semiregular solid which has several regular polygons of different shapes and faces. It is a member of the set referred to as Archimedes solids.

1. Using elastics, attach a square to each side of a pentagon.
2. Attach triangles between the squares. This will result in a saucer-shaped assembly.
3. Attach a pentagon to a square.
4. Attach squares to the four remaining sides of the pentagon. Then attach triangles as needed to fill in the spaces between the squares.



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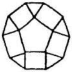

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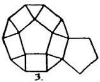

3.

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Student Sheet

Activity: Students explore the idea of going into a business to sell kits to make square patios

Take the following problem:

You are going into business for yourself to sell kits to make square patios. The components for the patios are these:

(a) Tiles that are 1 square foot in size (use square crackers to model)

(b) Frames to lattice in the tiles (use plain toothpicks)

(c) Corners to attach the frames together (gumdrops)

(d) Border stabilizers that are second, heavy-duty frames for the outside edges (colored toothpicks)

You can sell the materials alone as a kit for a do-it-yourself version or charge more for an installed patio.

The prices are as follows:

Materials	Kit	Installed Patio
Border stab.	\$1	\$2
Tiles	\$2	\$3
Corners	\$1	\$3
Frames	\$3	\$5

Before opening your business, inform potential customers about how many pieces of each component they need for square patios of various sizes and what the cost will be for the kit or the installed patio. *Be prepared to answer their questions quickly and accurately.*

Think about the mathematics you can use to prepare for the opening of your business. Record your thoughts. Your teacher will offer some suggestions.

Notes to teacher: The Square-Patio problem

The matrix application can be introduced to students and they can put their charts on a graphing calculator such as the TI-82

The following row matricies represent the number of border stabilizers, tiles, corners, and frames of patios of sides 1,2, and 3.

[A] [4 1 4 4]

[B] [8 4 9 12]

[C] [12 9 16 24]

Row Matrices

The following cost matrix represents the price of the kit alone (first column) and the installed cost (second column).

[D] [[1 2]
[2 3]
[1 3]
[3 5]]

Cost Matrix

The product of the row matrix for a certain sized patio and the cost matrix will give two costs - the cost of the kit and the cost of the install~d patio, for each number of sides.

Generalizing show students that:

$4x$, x^2, $(x + 1)^2$, and $2x(x + 1)$

are expressions for the costs of the various parts, where x represents the number of tiles on each side.

A matrix can be entered in formula form and used to calculate any size value of x.

Enter a row matrix as expressions of x .

$[1,1 = 4x,\ 1,2 = x^2,\ 1,3 = (x + 1)^2,\ 1,4 = 2x(x + 1)]$

These polynomials can also be used to generate a "fast" table of data. Enter the following in Y_1 and Y_2:

$$Y_1 = 4x(1) + X2(2) + (X + 1)2(1) + 2X(X + 1)(3)$$
$$Y_2 = 4x(2) + X2(3) + (X + 1)2(3) + 2X(X + 1)(5)$$

Set Tableset to TBLMIN = 1 and Δ TBL = 1. Go to table and see all data for sizes represented.

[1]Adapted with permission from "The Square - Patio Problem" by Martha Lowther. *The Mathematics Teacher*, copyright Jan, 1996 by the National Council of Teachers of Mathematics. All rights reserved.

Statistics Jeopardy

The following game (Mellor, 1992), based on the TV game show Jeopardy, can serve as review and reinforcement of introductory statistics for secondary students.

Materials: a transparency of the game board and small pieces of paper to conceal the answer or clue blocks.

Procedure: Divide the class into teams of equal numbers with each team occupying its row of desks. Review the basic characteristics of Jeopardy, ie. contestants are shown answers or clues in various categories and must respond with an appropriate question. (If selected questions do not lend themselves to this format, switch to a question/answer format.) Using the overhead, display the game board with the answer or clue blocks concealed and indicate that the four categories are: measures of central tendency or measures of dispersion, graphing, symbols and sampling.

To start, arbitrarily select one of the students in seat one of the rows. This student then selects a category and point value. Example - graphing for 200 points, and the answer or clue block for this item is revealed. All students in seat one of the rows then compete on this item. i.e. try to formulate an appropriate question. The seat one student who raises his/her hand first then poses the question. If the student is incorrect, the next seat one student of a competing team who raises his/her hand can try, and so on until a correct response is given. If none succeed in posing an appropriate answer, the item may be opened up to all students. The score board is then updated with the team that had the correct response obtaining the point value that was selected. Normally, the teams that make incorrect responses lose the same number of points but will not be assigned scores lower than zero. The student in seat two of the successful team on the first item then selects a category and point value and the seat two students "play off" The game continues in this manner until all the answer or clue blocks on the game board are revealed. The team with the highest score wins the game.

STATISTICS JEOPARDY - GAME BOARD

Pts.	Measure of Central Tendency or Dispersion	Graphing	Symbols	Sampling
100	Difference between the largest and smallest values of a set of values	Type of graph where the area of each rectangle is proportional to frequency	Σ	Large body of data from which a sample is selected.
200	Middle value in a set of ordered data	Type of graph obtained by connecting the midpoint of each rectangle of a histogram	[illegible]	Type of sample where each member of the population has a random chance of being selected
300	Measure of central tendency not influenced by extreme values.	Type of graph for representing parts of a whole.	S_x	Type of sample taken from a particular segment of a population.
400	For a normal distribution, the percentage of data values within one standard deviation of the mean	Finding information about points situated between the given points on a graph	μ	Type of sample which cannot be reincorporated into the populat-ion
500	Measure of central tendency which, if it exists, is a member of the data set	Finding information about points situated beyond the given points on a graph.	σ	A type of sampling where the population is divided into classes and then a part of the sample is taken from each class.

Statistics Jeopardy - Correct Responses

Points	Measure of Central Tendency or Dispersion	Graphing	Symbols	Sampling
100	What is the range?	What is a histogram?	What is the sum of?	What is a populat-ion?
200	What is the median?	What is a frequency polygon?	What is the sample mean?	What is a random sample?
300	What is the median?	What is a circle graph?	What is the sample standard deviation?	What is a clustered sample?
400	What is 68 % of 500?	What is interpolation?	What is the populat-ion mean?	What is a destructive sample?
500	What is the mode (or median)?	What is extrapolation?	What is the populat-ion stand. deviation?	What is a stratified sample?

Note:

These are only four of the many possible categories. For example, a category on 'distributions' may contain some of the following items:

100 - Name for the curve of the normal distribution (bell curve)

200 - A non-symmetrical distribution which is skewed to the right (positively skewed distribution)

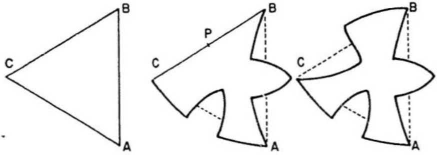
300 - The degree of flatness or 'peakedness' of a distribution (kurtosis).

Categories do not have to based on specific topics of content. For example, a category for words in statistics beginning with 's' could be used.

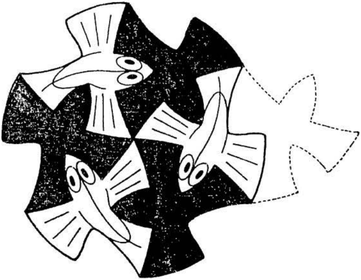
Tessellating with Hexagons

The following activity[1] is based on the work of the artist M.C. Escher (1898 -1972) who is famous for work with tessellations. By skilfully altering a basic polygon, such as a triangle or hexagon, he was able to produce intricate, artistic tessellations.

STEP 1. Start with an equilateral triangle ABC. Mark off the same curve on both sides AB and AC as shown. Mark off another curve on side BC that is symmetric about the midpoint P. Choose curves carefully so that a figure suitable for tessellating will be formed.



STEP 2: Fit together six of these figures to form a hexagonal array Take the basic figure and continue with the tessellation to fill the sheet.



[1]Adapted with permission from "Designs with tessellations," by Evan Maletsky. *Mathematics Teacher*, copyright April, 1974 by the National Council of Teachers of Mathematics. All rights reserved.

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