A COMPARATIVE STUDY OF DEDUCTIVE PROOFS IN GEOMETRY EDUCATION BETWEEN THE U.S. AND THE PEOPLE'S REPUBLIC OF CHINA

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By

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Abstract

During the last two decades, there has been a reassessment of deductive proof education due to the students' difficulties with deductive proof writing. Questions such as "what are we doing ? " and "why are we doing it ?" have been thought extensively by geometry educators and teachers.

The purpose of this study is to help geometry educators and teachers rethink deductive proof education. By comparing students' van Hiele levels, curricula, examinations, supplementary materials, and method of instruction in the U.S. and the People's Republic of China, this study will enable educators and teachers in both countries to recognize past failures in deductive proof education in their own country.

The results of the study show that the curricula of deductive proofs are not well-accepted in either the U.S. or the People's Republic of China. The inadequacies of textbooks, and the teachers' heavy reliance on them have caused many problems in the U.S. Entrance examinations and supplementary materials impose heavy demands on students' geometric thinking and cause a failure in the deductive proof education in China.

More research needs to be done on the factors influencing the students' achievement in deductive proof education. The resultant changes made by research will improve the quality of deductive proof education.

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CHAPTER I

Overview Of the Study

Introduction

Before the development of Greek culture there were no theorems or demonstrations, and deductive proofs (Szabo, 1972; Siu, 1993). The methods of deductive proofs were discovered by the Greeks through the development of geometry from a practical science into a deductive system. Thales (c 625 - 547 B. C.), born in Miletus, Greece, became the first person to prove deductively some geometric relationships, and was considered "the Father of Deductive Reasoning". He was, however, still unable to organize the propositions into any deductive system. His student, Pythagoras of Samos (c 585 - 501 B. C.), honored by the Romans as "the Wisest and Bravest of the Greeks", proved deductively the "first great theorem" -- the Pythagorean theorem. The methods of deductive proofs was developed by Plato (429 - 348 B. C.), who had studied geometry with Pythagoreans. His greatest contribution to geometry was the discovery of the method of analysis in constructing deductive proofs. He insisted on accurate definitions, clearly stated assumptions, and logical deductive proofs. Euclid (365 - 275 B. C.), a young Greek mathematician and a student at Plato's Academy, organized all the various isolated geometric discoveries and deductions of earlier generations into one single deductive system (Lightner, 1991).

Since ancient Greece, the methods of deductive proofs have been considered to be an essential characteristic of mathematics by Western Thought (Hanna & Jahnke, 1993). Meanwhile, learning to write deductive proofs has been an important objective for students learning geometry. During the sixties and early seventies the American "new math" reforms resulted in a much greater emphasis on deductive proofs in the school mathematics curriculum. Educators and mathematicians (Smith & Henderson, 1959; Swain, 1963; Lester, 1975) agreed that the study of deductive proofs should enter the school curriculum at the earliest possible time that is consistent with children's intellectual development. Lester (1975) suggested that "every effort be made to determine the most appropriate places at which to introduce students to the various aspects of proof" (p. 14). However, "exaggerated formalism, unsuccessful teaching experiences and a critical public eventually led to the demise of this reform, and in turn to a critical reassessment of mathematics education" (Hanna & Jahnke, 1993, p. 422). This reassessment has influenced the attitude and practice of deductive proof education at the secondary level. Today, the role or meaning of deductive proofs in school mathematics is being greatly debated by the mathematics educators and teachers in North America.

The methods of deductive proofs were translated into China by Matteo Ricci (1552-1610), the most prominent Jesuit missionary in China at the time of the Ming Dynasty, and Xu Guang-Qi (1562-1633), a famous mathematician in ancient China. Ricci said in his journal, "nothing pleased the Chinese as much as the volume on the Elements of Euclid. This perhaps was due to the fact that no people esteem mathematics as highly as the Chinese" (Siu, 1993, p. 345).

The style and context of deductive proof education in the People's Republic of China have significant differences from those in North America. In China, Geometry courses require students to have abilities to see relationships among several geometric figures, to make simple inferences, and to prove geometric statements deductively. They require students to possess a high degree of technical knowledge in formal deduction, logical thinking, and logical expression. Most geometry content in high schools in the U.S. is studied in grades 8 and 9 in middle schools in China. There is a unified mathematics curriculum, called the New Unified Series, to be used throughout the country. The texts are more theoretical than those in North America. Not only do the amount and depth of mathematics in Chinese texts surpass those in North American texts, but students in China do far more complicated problems than those of the students in North America.

Statement of the Problem

For the last two decades there has been many discussions and great debates on deductive proofs. Some American educators (Shaughnessy & Burger, 1985) suggested that the students' introduction to geometry should be informal without deductive proofs. Considering that few students achieve any competence in deductive proof writing, and many have negative attitudes about it (Senk 1985; Shaughnessy & Burger 1985), it may be helpful to introduce students informally to geometry.

Introducing students informally to geometry does not mean an abandonment of deductive proofs. Prevost (1985) said that today, geometric content is often emphasized on computational skills. This results in students' very weak grasp of geometric concepts at the grade ten. Cox (1985) & McDonald (1989) found that students enrolled in the informal geometry course were not as capable of producing accurate structures of geometric content as those enrolled in the formal geometry course. Although the students in the informal geometry course had above average IQ's and all successfully completed the course in plane geometry, the level of understanding exhibited by these students was at best superficial.

We must continue to teach our students how to write deductive proofs and how to read them critically in order to help them have a complete understanding of the geometry that they are studying (Reisel, 1982). Considering that there are many problems in learning and teaching deductive proofs, deductive proofs as a mathematics content area need to be rethought. We need to think extensively about our teaching---what are we doing? and why are we doing it? We also need to think about students' learning styles, their stages of intellectual development, and learning environment.

Purposes of the Study.

To rethink deductive proof education, we need to share materials and methods found to be effective in teaching and learning deductive proofs. An international comparative study on deductive proof education can create a challenge to educators to rethink the educational practice within their own country. It encourages educators to re-evaluate curricula, class sizes, teaching materials, and learning styles which are all crucial factors in any educational process (Lamon, 1971; Chang, 1984). Lee (1982) & Chang (1984) compared the higher education system of the Republic of China to that of the United States, and saw the values of the international exchange programs. This study compares deductive proof education between the U.S. and the People's Republic of China. It will be an informative challenge to investigate, compare, and summarize the deductive proof education of the two countries. It will help us recognize past failures in deductive proof education in both countries and present a long-term goal for deductive proof education.

This study will enable us to utilize the best available resources from both countries. It will provide recommendations and implications for deductive proof education in both countries. It is designed to achieve the following objectives:

(1) to investigate, compare, and summarize deductive proof education at the secondary level; and

(2) to encourage further study, discussion, and possible refinement of deductive proof education in both countries.

This study will provide answers to the following questions:

(1). Can a student in an urban school in China be assigned a van Hiele level?

(2). What percentage of students at the beginning of the eighth grade in urban schools in China are prepared for learning deductive proofs?

(3). Which level of instructional language can be used to teach geometry at the eighth grade in urban schools in China?

(4). Are there any differences between the van Hiele levels of mental development in geometry of grade eight students in urban schools in China and those of grade ten students in the United States?

Significance of the Study.

This study will make the following significant contributions to the field of deductive proof education in both countries:

(1) to help the understanding of the critical issues of deductive proof education at the secondary level in both countries;

(2) to recognize past failures in deductive proof education in both countries; and

(4) to present a long-term goal for deductive proof education.

Limitations of the Study.

This study is limited to secondary geometry education in both countries. A study related to Chinese students' van Hiele level was conducted in the urban middle schools in China at the beginning of the fall, 1994. It had the following limitations:

(1). Students sampled were enrolled in grade eight for the first time in the fall of 1994. Students repeating grade eight were eliminated from the sample;

(2). the population from which the sample was drawn consisted of all grade eight students in the chinese urban schools. An urban school was selected giving a sample size of 95 students whose students' ID numbers were even; and

(3). students were tested during the school day.

CHAPTER II

Review of the Literature

This chapter will briefly describe how to write and read deductive proofs in geometry, discuss students' difficulties with deductive proofs, and examine the debate and discussions on deductive proof education. An introduction to the van Hiele model will be made. The model will be used to explain why many students find learning deductive proofs in geometry such a difficult task. Due to the shortage of the literature in China at the present time, the review will mainly focus on the literature in the U.S.

Deductive Proof Education

Deductive Proofs

It seems clear that a mathematical proof is "a careful sequence of steps with each step following logically from an assumed or previously proved statement and from previous steps" (NCTM, 1989; Pereira-Mendoza & Quigley, 1990, p. 9). That is, a proof is an exercise in logic (Otte, 1994). This definition is especially true for a deductive proof in geometry.

Geometry, the best and simplest of all logics (Hope & Jack, 1990), is the study of the properties and characteristics of certain sets, such as lines, angles, triangles, circles, and planes. Four thousand years ago, the Egyptians were concerned with land measures and became very proficient in dealing with these practical aspects of geometry due to the annual floods of the Nile River. Two thousand years later, the Greeks moved away from this practical concept of geometry and developed it into a logical system (Weiss, 1972). This system emphasizes accurate definitions, clearly stated assumptions, and logical deductive proofs. It consists of undefined terms, definitions, postulates, and theorems.

In the formal, rigorous development of geometry, the only properties of points, lines, and planes that can be mathematically accepted are those coming from these postulates, definitions, and theorems (Weiss, 1972). A postulate is an assumption considered to be true without having been proven. A theorem is usually a statement that consists of the "if" part, called the hypothesis, or the given information, and the "then" part, called the conclusion, or the aspect to be proved. To prove a theorem in geometry, there needs to be five parts:

1. the statement of the theorem;
2. a diagram that illustrates the hypothesis;
3. a list, in term of the figure, of what is given;
4. a list, in term of the figure, of what you are to prove; and
5. a series of statements and reasons, such as given information, definitions, postulates, and theorems that have already been proved, which lead from the hypothesis to the conclusion.

Basically, there are only three kinds of mathematical proofs in geometry:

(a). Direct Proof: from a hypothesis or a "given" to prove another statement. The "proof" will take the form of a series of statements, the first one directly derivable from the hypothesis and each succeeding one derivable from its antecedent.

(b). Indirect Proof: sometimes it is difficult or even impossible to find a direct proof, but it may be easy to reason indirectly. If the theorem to be proven is: a-->b, and we can not think of a way of arguing from hypothesis or given "a" to prove the conclusion "b", it is useful to know that the contraposition and the condition are equivalent. Thus, if a-->b, then: b'-->a'. This kind of proof always suggest the possibility of taking the negation of the statement to be proved and arguing from that to see if from it the negative of the hypothesis can be obtained. If so, then we have proved the theorem. In such a proof we prove that something can not be false by showing that if it were false we would arrive at a contradiction. A indirect Proof is usually written as follows:

1. assuming temporarily that the conclusion is not true;
2. reasoning logically until you reach a contradiction of a known fact; and
3. pointing out your temporary assumption must be false, and that the conclusion must then be true.

(c). Two-Way Proofs: sometimes a theorem is stated in the form: a<--->b recognizable by such phrases as "if and only if", "necessary and sufficient condition that". The students should be taught unambiguously to recognize such phraseology and to realize that it always means that they must contrive two proofs, not just one. They must prove that: a-->b. When that is done, they must then prove that b-->a.

All of the three kinds of proofs are deductive proofs. They are deduced by using logical reasoning. Direct Proof and Indirect Proof are studied during grade ten in secondary schools in the U.S. while Direct Proof is studied by Chinese students during grade eight and Indirect Proof during grade nine. Two-Way Proofs are studied during grade ten in China. They are not studied by students in the U.S.

Problems in Deductive Proof Education

Many geometry teachers find that most high school students have a lot of difficulties with deductive proofs. They do not understand the role or meaning of an axiomatic system. Shaughnessy & Burger (1985) said in their article, "despite our best efforts to teach them, even the most capable algebra students may struggle and get through geometry by sheer willpower and memorization but with little understanding of the logical system we have been developing all year" (p. 419). There are two kinds of problems in learning deductive proofs: the technical rational problem and the psychological-emotional problem.

In writing deductive proofs, students need to possess some degree of technical knowledge in formal deduction, logical thinking, and logical expression. Unfortunately, many teachers find that in geometry courses, a number of students do not know much about writing a deductive proof. They usually will not be able to follow strictly deductive reasoning from the start. They often begin with a more or less disorder period of logical thinking in which they try to find a series of statements and reasons in order to deduce the conclusion from the hypothesis (Stone, 1971). The technical problem has become a great barrier in deductive proof education.

The psychological-emotional problem.

Another kind of problem in deductive proof education is the psychological-emotional problem. Much research indicates that many students has a dualistic view of the nature of deductive proofs. Balacheff (1988) & Chazan (1993) found that some students viewed a deductive proof in geometry as a proof only for a single case which was pictured with the associated diagram. On an item which assessed students' understanding of the generalization principle of deductive proofs, Williams (1979) & Chazan (1993) found that 20% of the students did not realize that a given deductive proof proved a relationship for all triangles, with only 31% of the students appreciating the generalization principle. In Schoenfeld's (1989) & Chazan (1993) research, students were asked to prove results deductively and then to make a construction. They first proved correctly the deductive results and then conjectured a solution to the construction problem that flatly violated the results they had just proven. Fischbein (1982) & Chazan (1993) interpreted this result as an indication that the students were not aware of the distinctions between an inductive and a deductive proof. That is, students who held this belief did not understand the generalization principle for deductive proofs (Williams, 1979; Chazan, 1993); they did not understand that the validity of the conclusion was meant to be generalizable to all figures which satisfied the givens (Chazan, 1993).

While many students do not appreciate the generalization principle of a deductive proof, some students measure a series of examples to prove geometry statements without writing a deductive proof. They believe that an evidence is a deductive proof. Martin (1989) & Chazan (1993) surveyed 101 American preservice elementary school teachers and found that 68% of the sample believed that an inductive proof was a sufficient "proof", only 6.4% felt the need for a deductive proof. These dualistic views of the nature of deductive proofs are more likely to cause failures in teaching students to read deductive proofs critically.

Debate and Discussions on Deductive Proof Education

Although geometry has been thought by many to be a powerful tool to train students to write formal proofs, student's difficulties with deductive proofs make geometry a largely debated topic in school curricula. Fey (1984) & Masingila (1993, p. 38) called geometry "the most troubled and controversial topic in school mathematics today". Jean Dieudonne (Masingila, 1993, p. 38) claimed "Euclid must go!". Stone (1971) thought that "the 'synthetic' method imposes heavy demands on the learner's 'geometrical intuition' and 'mathematical ingenuity'" (p. 91). And, "there is a diversity of views on what high school geometry is" (Pereira-Mendoza & Robbins, 1977, p. 189). Pereira-Mendoza & Robbins (1977) found that teachers tended to feel that informal skills in geometry were important while educators favoured deductive ability.

Debate and discussions on deductive proof education continue today. But there seems to be a general agreement that doing deductive proofs in geometry is the most difficult topic of school mathematics (Carpenter et al., 1980; Usiskin, 1982; Senk, 1985, 1989). About 85 percent of high school graduates in the United States have not mastered deductive proofs that underlies the structure of a standard geometry course (Senk, 1985). Many Chinese students have rejected the topic of deductive proofs, and few students can prove deductively a geometric problem on a higher school mathematics entrance examination.

We need to think extensively why many students in both countries find learning deductive proofs such a difficult task. The van Hiele model can be used to explain the students' difficulties with deductive proofs and to help us recognize past failures in deductive proof education.

The Van Hiele Model

The van Hiele model was proposed by Dutch educators, P. M. van Hiele and his late wife, Dina van Hiele-Geldof in 1957. As experienced mathematics teachers in Montessori schools, they met with the same difficulties that we all encounter in teaching deductive proofs to our students. Based on classroom observations, the van Hieles believed that "the students moved sequentially from one level of thinking to the next as their capability increased" (Gutierrez et al., 1991, p. 237), and that instruction played an important role in raising the geometric thinking level of their students (Gutierrez et al., 1991).

The van Hiele Model

The van Hiele model is a mental image model. It is composed of two main aspects. The first consists of five levels of geometric thinking, and the second is concerned with the development of intuition in students.

Five levels of geometric thinking.

The first aspect of the van Hiele model is five levels of geometric thinking. The van Hieles thought that all students entered at the ground level with the ability to name and identify the common geometric figures, but Usiskin (1982) & Pegg (1985) reported that some students did not enter at the ground level. The majority of

American studies used a 1 to 5 scale. They allocated a base level, level 0, for the students who did not have the visualization ability (Pegg, 1985).

The five levels of geometric thinking have been described by Hoffer (1981) and Shaughnessy & Burger (1985) as follows:
Level 1: Visualization. Students at this level have the ability to name and identify the common geometric figures, but they can only see a whole geometric figure, no attention is given to its components.

Level 2: Analysis. At this level, students have the ability to think of a particular geometric figure and note its properties that it must have (necessary conditions), but they do not have the ability to see how one figure relates to other figures.

Level 3: Informal deduction. At this level, students have the ability to, see relationships among several geometric figures, appreciate the role of general definitions, and make simple inferences.

Level 4: Formal deduction. At this level, students have the ability to reason formally within the context of a mathematical system, complete with undefined terms, postulates, an underlying deductive system, definitions, and theorems.

Level 5: Rigor. At this level, students have the ability to compare deductive systems based on different postulates, and to study various geometries in the absence of concrete models.

The development of intuition in students.

The second aspect of the van Hiele model concentrates on the phases of learning by which means a teacher can assist the growth of the students through the various levels of thinking. Progress from one level of thinking to the next is more dependent on instruction than on students' age or maturation (Fuys & Geddes, 1984). Van Hiele emphasized that instruction should be directed at the student's level. He noted that many failures in teaching geometry resulted from a language barrier: the teacher used the language of a higher level than that which was understood by the students (Gutierrez, et al.; 1991).

Research on the Van Hiele Model

Since the 1960's, the van Hiele model has caused enthusiastic responses. Russian educators conducted extensive research on this model, and made resultant changes in geometry curriculum in the early 1960's. In 1976, Wirszup introduced this model to American mathematics educators by giving a detailed account of the model in the light of the experiences in Russia. In the 1980's, Australian mathematics educators did some informal investigatory work about this model (Pegg, 1985). Furthermore, there has been a growing interest in research on the model in the last 10 years (Hoffer, 1983; Senk, 1985; Fuys et al., 1988; Gutierrez & Jaime, 1989; Gutierrez et al., 1991). Much of the research was related to the van Hiele levels, the students' level of thinking, and curriculum and instruction.

The van Hiele levels.

Previous studies have drawn some conclusions relevant to the van Hiele levels of development in geometry. Mayberry (1983), Fuys et al. (1985), and Pegg (1985) found that the van Hiele levels appear to be hierarchical in nature. That is, "what is viewed by the pupil as of paramount importance at one level is subsumed by a new perception at the next level" (Pegg, 1985, p. 6-7). The van Hiele levels also have a logical structure: the recognition of a figure in Level 1 is essential to the development of its properties in Level 2. These properties are required in Level 3 to form the essential prerequisites for the understanding of a mathematical system at Level 4. The properties in Level 4 are essential to compare systems based on different axioms and to study various geometries in the absence of concrete models in Level 5 (Pegg, 1985).

The Students' Levels of Thinking.

One of the important focuses of research on the van Hiele model is to determine the students' levels of geometric thinking (Gutierrez et al., 1991). Many researchers used a written test to determine a student's van Hiele level (Usiskin, 1982; Gutierrez & Jaime, 1987; Gutierrez et al., 1991). In 1982, Usiskin constructed a 25-item multiple-choice Van Hiele Geometry Test and administered it to all the students enrolled in the 10th-grade geometry course in 13 schools in the United States. In his study, Usiskin (1982) found that over two-thirds of students taking the test could be assigned a van Hiele level. He noted that when the fifth level was excluded, the 3 out of 5 criterion assigned 85% of the students to a van Hiele level and the 4 out of 5 criterion assigned 92% of them (Wilson, 1990). Based on these findings, Usiskin (1982) concluded that individual student can be assigned a van Hiele level. This conclusion has been confirmed by many other studies (Usiskin & Senk, 1990).

Instructional aspects of the van Hiele model. One important instructional aspect of the van Hiele model is that students can not be expected to understand instruction at a level higher than their level. Shaughnessy & Burger (1985) found that it was very likely that the teacher and the students were reasoning about the same concept but at different levels. This indicates that students may have vastly different geometric concepts in mind than teachers think they do when teachers are teaching a course in geometry. Davey & Holliday (1992) found that if instruction was at a higher level of mental development than the students' level, some students might reject the subject, and others may wish to please the teacher and just accept what the teacher says without any understanding.

Relationships Between High School Students' Achievement in Learning Deductive Proofs and Van Hiele Levels of Geometric Thinking

On previous parts of this chapter, it was shown that to learn deductive proofs in geometry is to understand a system of deductive reasoning, to organize results into a deductive system of axioms, major concepts and theorems, and minor results derived from these (axioms, major concepts and theorems). That is, to learn deductive proofs in geometry is to think geometry at van Hiele' level 4 (formal deductive level).

A major question needs to be answered: which van Hiele level is necessary for students to learn deductive proofs in geometry?

In 1983, Senk conducted a research that consisted of 751 students who had studied deductive proofs in geometry and fitted the van Hiele model in the CDASSG project (Usiskin, 1982). By administering "CDASSG Proof Test" to her sample, Senk (1989) found that van Hiele level accounted for 25% of the variance in deductive proof writing achievement for students who took the "Van Hiele Geometry Test" in the fall of 1982, and 34% of the variance in the spring ($p < .0001$).

Based on these findings, Senk (1989) indicated that high school students' achievement in learning deductive proofs in geometry is positively related to van the Hiele level of geometric thinking. She noted that a students who started a high school geometry course with Level 0 had little chance of learning to write deductive proofs later in the year. A students who started with Level 1 was likely to be able to do some simple deductive proofs by the end of the year, but such a student had less than one chance in three of mastering deductive proof writing. A student who started with Level 2 had at least a 50-50 chance of mastering deductive proofs by the end of the year, and a student who entered with Level 3 had an even greater chance of mastering deductive proof writing. She concluded that "although there is no individual van Hiele level that ensures future success in proof writing, Level 2 appears to be the critical entry level" (1989; p. 319).

By also administering "Test for Knowledge of Standard Content" to the same sample of students, Senk (1989) found that the students' entering geometry knowledge accounted for 37% of the variance in proof writing achievement when it was used to examine the relationships between deductive proof writing achievement and their van Hiele level in the fall; students' entering geometry knowledge and van Hiele levels accounted for about 40% of the variance in deductive proof writing scores in the fall and 60% in the spring. Therefore, "much of a student's achievement in writing geometry proofs is due to factors within the direct control of the teacher and the curriculum" (Senk, 1989; p. 319).

Research on the van Hiele model suggests some changes that seem appropriate for the existing school geometry curriculum at the secondary level. It suggests that instruction should be directed at the student's level. It also suggests that teachers should assist students to develop their geometric thinking levels in order to prepare them for learning deductive proofs. Therefore, it is necessary for us to think of the geometric thinking level of our students in order to improve the quality of deductive proof education. The next chapter will attempt to find and compare the van Hiele levels of the students in the U.S. and the People's Republic of China. The students' van Hiele level found will be used to analyze and compare deductive proof education in both countries.

CHAPTER III

The Van Hiele Levels of Students in the U.S. and the People's Republic of China

The American Students' Van Hiele Levels

In 1982, Usiskin conducted the Cognitive Development and Achievement in Secondary School Geometry (CDASSG) project to address a variety of questions relating to the van Hiele model. One of the research questions in his study was "how are entering geometry students distributed with respect to the levels in the van Hiele schemes" (Usiskin, 1982, p1). "The Van Hiele Geometry Test" was conducted to determine the distribution of the van Hiele Level of the student in the U.S.. [As the only van Hiele instrument available that can be group-administered to assign students' van Hiele levels in the United States or Canada (Usiskin & Senk, 1990), "The Van Hiele Geometry Test" has made a valuable contributior to research on van Hiele levels (Crowley, 1990)]. The test has five subtests, each of which contains five items. The items within a subtest were written to correspond directly to characteristic behaviours that students exhibit at each level, which were described by the van Hieles (Usiskin, 1982; Crowley, 1990).

In Usiskin's (1982) study, a student was assigned a van Hiele level based on the sequence of subtests mastered. Mastery of a subtest was determined by the following rules: if a student met the criterion (there were two criteria for passing a level: 3 out of 5 items correct and 4 out of 5 items correct) for passing levels from 0 to n, and failed to meet the criterion for levels n + 1 and above, then the student was assigned to the level n; if the student could not be assigned to any level, then that student was said to not be fit into a level.

By administrating "The Van Hiele Geometry Test" to 2699 students enrolled in a one-year geometry course (grade ten) in 13 schools selected from throughout the United States to provide a broad representation of community socio-economies, Usiskin (1982) found that a student in the U.S. could be assigned a van Hiele level, and over 70% were at level 0 (30.7%) or level 1 (42.7%) using 4 out of 5 criterion. The numbers and percentages of students at each of the van Hiele levels found by CDASSG are presented in Table 1. Also, the number of students who were below the visualization level, and those who did not fit the theory are included. The criterion used in Table 1 is 4 out of 5 (80%) items correct. CDASSG study showed that it was possible to classify 91.9% percent of the students into a van Hiele level or below the visualization level. The majority of students were at or below the visualization level. This result suggested that a formal language can not be used to teach geometry at the beginning of grade ten in the U.S.

Table 1

Numbers and Percentages of Students at Each van Hiele Level Using A 4 Out of 5 Criterion

Level	Number	Percentage
Nofit	191	8.1%
0	726	30.7%
1	1008	42.7%
2	338	14.3%
3	93	3.9%
4	5	0.2%
Totals	2 361	100.0%

Note: The data in table 1 are from "Van Hiele Levels And Achievement In Secondary School Geometry" CDASSG Project by Usiskin, Z., 1982. (ERIC Document Reproduction Service No. ED 220 288)

Although the CDASSG project was conducted in 1982, its results still represent the entering geometry students distributed with respect to the levels in the van Hiele schemes. Usiskin & Senk (1990) said in their article:

The test (The Van Hiele Geometry Test) has been more widely used than we would ever have imagined. Over 100 individuals have formally requested and received permission from us to duplicate it. The test has almost universally been used to determine van Hiele levels for a set of individuals (Usiskin & Senk,1990; p. 242).

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Despite the low reliability coefficients, the results of the CDASSG study are rather robust. In return for permission to duplicate the test (the Van Hiele Geometry Test), we ask to receive a copy of the results. None of the studies that have come to our attention has found performance in high school geometry significantly different from that of the students in our study (Usiskin & Senk,1990; p. 244).

The Chinese Students' Van Hiele Levels

In order to thoroughly compare deductive proof education between the U.S. and the People's Republic of China, we conducted a study focusing on the van hiele levels of the students at the beginning of grade eight in urban schools in China. The reasons for conducting the study only in urban schools are that the urban schools in China are more similar to those of the schools in the U.S. than rural ones in China, and that the great differences in the sections of educational and socioeconomic conditions between the urban and rural areas in China. For example, the educational level of rural laborers in China is the lowest of any group. The Third Population Survey of China (1982) found that for every thousand people employed in the labor force, only 0.4 persons had a college education, 53.4 had a senior high school education, 214.7 had a junior high school education, 371.6 had elementary school education, and 359.9 were illiterate or semiliterate (Wu, 1989 - 90). Moreover, most rural schools in China are smaller than the urban ones. They can not offer the breadth and depth of curricula that urban schools do. In the extensive mountainous and pastoral regions, many rural elementary schools are taught by literate peasants and do not extend beyond four of five years.

Purposes

This study was concerned with four research questions with respect to students' level of mental development in geometry at the beginning of the eighth grade in urban schools in China. These questions, along with the corresponding statistical analysis used to test the hypotheses, or describe the data collected, are given below:

Question 1.

Can a student in urban schools in China be assigned a van Hiele level?

Question 2.

What percentage of students at the beginning of the eighth grade in urban schools in China are prepared for learning deductive proofs?

Question 3.

Which level of instructional language can be used to teach geometry in grade eight in urban schools in China?

Question 4.

Are there any differences between the van Hiele levels of mental development in geometry of grade eight students in urban schools in China and those of grade ten students in the United States?

Null Hypothesis: There is no significant difference in the van Hiele levels of mental development in geometry of grade eight students in urban schools in China and those of grade ten students in the United States.

Questions 1, 2, and 3 were answered by administering a modified version of the Van Hiele Geometry Test to 95 grade eight students at FuZhou Yian Yun middle school in China on October 12, 1994. Table 2 was constructed to show the numbers and percentages of students at various van Hiele levels using 4 out of 5 criterion.

To answer question 4, the null hypothesis was tested by using the chi-square test for homogeneity of the van Hiele levels of students in urban schools in China and those of the students in the United States. A contingency table was constructed for 4 out of 5 criteria by using the van Hiele levels of students in urban schools in China and those of students in the United States. The fall result of the CDASSG project (Usiskin, 1982) was used for the latter group of students. The level of significance selected was .05 in both instances.

Method

Design of the study.

This study gathered data on 95 students in grade eight at Fuzhou Yian Yun middle school. The school is an ordinary urban middle school and generally represents urban schools in China with similar educational and socioeconomic conditions.

Instrument.

The instrument used in the study is a modified version of the Van Hiele Geometry Test (Usiskin, 1982). The criteria used in the test is 4 out of 5 items correct because it reduces the effect of guessing. The modified Van Hiele Geometry Test consists of the first 20 items on the original test; that is, the items dealing with the first four levels. The last five items on the original test were excluded since "the existence and/or testability of level 5 (Rigor) had been questioned" (Usiskin, 1982, p. 79). A copy of the modified Van Hiele Geometry Test is contained in Appendix with appropriate instructions and answer sheet.

Test administration.

The Van Hiele Geometry Test and answer sheets were sent to FuZhou Yian Yun middle school in China on October 12, 1994. The following instructions were given to the teachers:

1. they were asked to administer the test before the end of October, 1994;

2. the test was administered to all grade eight students whose students' ID numbers were even in the school;

3. the time allowed for the test was exactly 30 minutes; and

4. the answer sheets were returned immediately after the test was completed.

The Results

The numbers and percentages of students at each of the van Hiele levels are presented in Table 2. Also, the number of students who were below the visualization level, and those who did not fit the model are included. The criterion used in Table 2 is 4 out of 5 items correct at each level. This study showed that it was possible to classify 80 percent of the students into a van Hiele level or below the visualization level. 2 students or 2.1 percent were below the visualization level; 28 students or 29.5 percent were at the visualization level; 26 students or 27.4 percent were at the analysis level; 16 students or 16.8 percent were at the informal deductive level; and only 4 students or 4.2 percent were at the formal deductive level. This study showed that, excluding not fit students, there were 48.4 percent students who were ready for learning deductive proofs at the beginning of grade eight. The majority of students were at the visualization, analysis, or informal deduction level. This study suggested that an informal language can be used to teach geometry in grade eight in urban schools in China.

Table 3 is a contingency table for the 4 out of 5 criterion to test homogeneity. The chi-square value was found to be 126.96 (> 11.07, $p < .05$) which resulted in rejection of the null hypothesis of question 4.

Table 2

Numbers and Percentages of Students at Each Van Hiele Level Using A 4 Out of 5 Criterion

Level	Number	Percentage
Nofit	19	20.0%
0	2	2.1%
1	28	29.5%
2	26	27.4%
3	16	16.8%
4	4	4.2%
Totals	95	100.0%

Table 3

Contingency Table for 4 Out of 5 Criterion to Test Homogeneity

Level	Sample: China	Sample: the U.S.	Totals
Nofit	19 (20.0%)	191 (8.1%)	210
0	2 (2.1%)	726 (30.7%)	720
1	28 (29.5%)	1 008 (42.7%)	1 036
2	26 (27.4%)	338 (14.3%)	364
3	16 (16.8%)	93 (3.9%)	109
4	4 (4.2%)	5 (0.2%)	9
Totals	95	2 361	2 456

The four research questions can be answered on the basis of the above results:

1. a student in an urban school in China can be assigned a van Hiele level;

2. excluding not fit students, there are 48.4 percent of students who are ready for learning deductive proofs at the beginning of grade eight in urban schools in China;

3. an informal language can be used to teach geometry at the eighth grade in urban schools in China; and

4. there are relationships between the students' van Hiele levels

of mental development in geometry of grade eight students in urban schools in China and those of grade ten students in the United States.

Discussion

The first major conclusion indicates that a beginning grade eight student in an urban school in China can be assigned a van Hiele level. However, the percentage of students who did not fit the van Hiele model was greater than those of students in the U.S. One possible reason for the result could be the lack of training that Chinese students had in taking the multiple-Choice standard test. Stevenson (1992) indicated that "results from cross-national studies can be greatly distorted if the research procedures are not comparable in each area and if the test materials are not culturally appropriate" (p. 70). The lack of training might have some Chinese students, who were slow thinkers, spend a little valuable time to familiarize the test. Then, they had to rush the test in order to complete more items without careful thinking. This might result in mastering nonconsecutive levels.

Another possible reason for the result could be encouraging memorization and probably rote learning in urban schools in China. Items 14 and 15 on the Van Hiele Geometry Test (Usiskin, 1982) can be memorized without understanding. In this study, 12 out of 20 test takers who did not fit the van Hiele model mastered level 3 while they did not master level 2.

The second major conclusion indicates that with the aid of instruction, about half of the students (the percentage might be greater if students who did not fit the van Hiele model had been considered) at the beginning of grade eight in urban schools in China may raise their geometric thinking level to the formal deduction level before the end of grade nine. However, there is still 31.6 percent of the students (the percentage might be greater if the students who did not fit the van Hiele model had been considered) who are not ready for learning deductive proofs. Considering that Chinese teachers often present geometry in an abstract manner, these students will not likely be able to master deductive proofs by the end of grade nine.

The third major conclusion indicates that geometry instruction in grade eight in urban schools in China should be informal. After one year of study, it may be possible to use the formal language to teach deductive proofs.

The fourth major conclusion is related to the van Hiele levels of students in grade eight in urban schools in China, and those of students in grade ten in the United States. The van Hiele levels of students at the beginning of grade eight in urban schools in China are significantly higher than those of students at the beginning of grade ten in the United States. This result may be due to hard work of Chinese students. Chinese students believe that achievement depends on diligence. Stevenson (1992) indicated that "the idea that increased effort will lead to improved performance is an important factor in accounting for the willingness of Chinese and Japanese children, teachers and parents to spend so much time and effort on the children's academic work" (p. 74).

This chapter compares the van Hiele levels of the students in grade ten in the U.S. to those of the students in grade eight in the People's Republic of China. The comparison is made without considering the students' age. However, it may be reasonable because students' progress from one level of thinking to the next is more dependent on instruction than on their age or maturation (Fuys & Geddes, 1984). The next chapter will attempt to compare deductive proof education between the U.S. and the People's Republic of China by using the van Hiele model and the van Hiele levels of students in both country.

CHAPTER IV

Deductive Proof Education in the U.S. and the People's Republic of China

Deductive Proof Education in the U.S.

The precollege education system in the United States consists of two levels: elementary school and secondary school. However, the education systems are different among states. The most popular systems are:

(1) 6-3-3 systems (six years of elementary school, three years of junior high school, and three years of senior high school);

(2) 6-6 systems (six years of elementary school and six years of secondary school);

(3) 6-2-4 systems (six years of elementary school, two years of junior high school, and four years of senior high school); and

(4) 8-4 systems (eight years of elementary school and four years of secondary school)

All students are required to study mathematics in secondary schools. However, the mathematics curriculum of the secondary school varies in content. There are three levels of mathematics courses for students to choose from. The first level consists of algebra, trigonometry, plane geometry, and sometimes, calculus, statistics, and computer science. The second level consists of elementary algebra and general mathematics. The third level consists of laboratory mathematics, consumer mathematics, and business mathematics. Most college-bound students choose the first level, and students with lower mathematics ability choose the second or even third Level (Chang, 1984).

Textbooks define limits to the content of the curriculum (Brown, 1973; McKnight, et al., 1987; Westbury, 1990; Chandler & Brosnan, 1995) and provide structure for 75% to 95% of classroom instruction (Tyson and woodward, 1989; Chandler & Brosnan, 1995). Most Mathematics and science teachers rely on a single textbook for instruction (Weiss, 1987; Flanders, 1994). The U.S. has never used a unified set of mathematics textbooks. However, textbook publishers in the U.S. are careful to investigate statewide curriculum goals and the Curriculum and Evaluation Standards' guidelines in developing unified goals for mathematics instruction (Jiang & Eggleton, 1995).

Both statewide geometry curriculum and the Curriculum and Evaluation Standards (NCTM, 1991) require college-bound students to master deductive proofs in geometry in grades 9-12. For example, in the State of New Mexico, the geometry course requires college-bound students to have:

(1) knowledge of two-dimensional and three-dimensional figures and their properties;

(2) the ability to think of two-dimensional and three-dimensional figures in terms of symmetry, congruence, and similarity;

(3) the ability to use the Pythagorean Theorem and special right-triangle relationships;

(4) the ability to draw geometrical figures and use geometrical modes of thinking in solving problems; and

(5) appreciation of the role of proofs (New Mexico Commission on Higher Education & New Mexico State Dept. of Education, 1987, p.11-2).

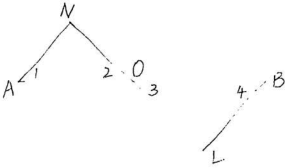
Textbook publishers also stress the importance of deductive proofs. Jurgensen & Brown (1990) thought that "teaching students to write proofs is one of the toughest jobs of a geometry teacher" (p. T55). They (1990) made the following suggestions for teaching deductive proofs:

> Objective 6: Understanding the Organization of Proofs.
> This sixth objective is to assist students to understand the organization of proofs. To read a direct proof of a theorem, students need to see the organization as containing at least these components: a statement of the theorem, a figure, a "given", a "prove", and a proof with statements and reasons listed in logical order. The key to reading the proof is developing an interrelationship among these parts, which involves looking up and down and from side to side many times. Although many theorems are presented without proof in this textbook, it is important for your students to develop an appreciation of this orderly method of reasoning. The partially proved theorems and exercises will help students

gain skill in writing proofs in two-column form. Some of the exercises call for the writing of complete proofs. Indirect proof is presented as an Application. Your better students may enjoy experimenting with this method (p. T37).

Most high school geometry textbooks presented material at or above van Hiele level 3 and had problems that often jumped from level 0 to level 3 (Geddes et al, 1982; Shaughnessy & Burger, 1985). The following problems in a textbook (Jurgensen & Brown, 1990) generally represent requirements of deductive proof writing in grade ten in the secondary school in the U.S. They require students to formally deduce the conclusions from the hypothesis, or to think at van Hiele level 4.

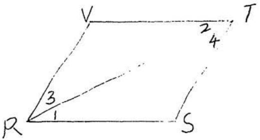
Given: $\angle 1 = \angle 2$; $\overline{AN} \parallel \overline{LB}$. Proven: $\angle 4 = \angle 3$ (p. T9).



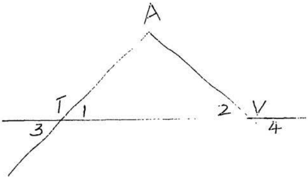
Write a complete proof in two-column form.

Given: $\overline{VT} \parallel \overline{SR}$; $\overline{VR} \parallel \overline{TS}$

Prove: $\triangle RST \cong \triangle TVR$ (p. T10).



Given: $\angle 3 = \angle 4$. Prove: AT = AV (p. T11).



The level of the language used for instruction is often above the analysis level in geometry courses. The majority of teachers used the Traditional Lecture/Demonstration method to teach their classes. Seventy eight percent of the teachers used geometric solids less than once a week. Most high school teachers reported that they taught 5 mathematics classes per day, 5 days per week in the secondary school. Thus, little attention could be given to individual needs after classes (ETS & NAED, 1991).

Considering that over 70 percent of secondary school students have only levels 0 or 1 of geometric thinking prior to taking geometry (Usiskin, 1982), and that level 2 is necessary for students to learn deductive proofs (Senk, 1989), the geometry textbooks and instruction were not well-designed for secondary school students in the U.S.

Currently, research on the van Hiele model has made resultant changes in geometry curriculum and instruction. Educators (Wirszup, 1976; Fuys, Geddes, & Tischler, 1985; Senk, 1989) have demonstrated ways to raise the van Hiele levels of students in elementary and secondary schools. Standards (NCTM, 1989) advocated educators and teachers to consider the importance of sequential learning as expressed by van Hiele model:

> "Evidence suggests that the development of geometric ideas progresses through a hierarchy of level. Students first learn to recognize whole shapes and then to analyze the relevant properties of a shape. Later they can see relationships between shapes and make simple deductions. Curriculum development and instruction must consider this hierarchy" (NCTM, 1989, p. 48; Teppo, 1991; p. 214).

The Standards (NCTM, 1989) made the following suggestions for geometry curriculum and instruction:
Standard 9, "Geometry and Spatial Sense" for grades K-4 (NCTM, 1989; Teppo, 1991), calls for students to be able to "describe,

model, draw, and classify shapes; investigate and predict the results of combining, subdividing, and changing shapes; develop spatial sense". It recommends that students learn to recognize geometric shapes by using a variety of "everyday objects and other physical materials." This learning represents students' geometric thinking at the van Hiele's visualization level (Teppo, 1991).

Standard 12, "Geometry" for grades 5-8 (p. 112), calls for students to "identify, describe, compare, and classify geometric figures; visualize and represent geometric figures..., explore transformations of geometric models; understand and apply geometric properties and relationships" (NCTM, 1989; Teppo, 1991; p. 215). These learning activities continue the development of geometric thinking begun in grades K-4. They help "definitions become meaningful, relationships among figures be understood, and students be prepared to use these ideas to develop informal arguments" (NCTM, 1989,p. 112; Teppo, 1991; p. 215). They develop students' analysis and informal deduction ability.

Within the span of grade K to grade 8, students deepen their understanding of concepts of geometry and are provided with essential preparation for the study of deductive proofs in secondary school geometry. The systematic geometry instruction before the secondary school is necessary to insure the students' later success in learning deductive proofs. "The specific language of the standards, with the inclusion of examples of activities for students, serves as an excellent blueprint for the incorporation of the van Hiele theory into American mathematics education" (Teppo, 1991; p. 215). They help to ensure that effective classroom instruction occurs.

Deductive Proof Education in the People's Republic of China

Curriculum.

The educational system in the People's Republic of China is modeled after the Russian system. Precollege education in China is organized into three levels: primary school (grades 1-6), junior middle school(grades 7-9), and senior middle school (grades 10-12). Student ages 7-12 attend primary school; ages 13-15 attend junior middle school; and ages 16-18 attend senior middle school.

A school's curriculum is determined by the Ministry of education. Textbooks are commissioned by the ministry of education and written by university faculty and committees of experienced teachers in 'key' schools, which serve as college-preparatory schools for high-achieving students. Students attend schools six days a week, for 9-10 months each year. The curriculum of the middle school is quite demanding. Both ordinary and 'key' middle schools offer fourteen courses: Chinese, Mathematics, Foreign languages (English being the most common, but also Russian, Japanese, French and German), Physics, Chemistry, Biology, History, Politics, Geography, Health Education, Physical Education, Music, Art, and labor Skills. Mathematics has an important role in primary and secondary education because it is considered to be the bases for the study of all other subjects. Students study mathematics from grade 7 to grade 12 in middle schools. There is a unified mathematics curriculum, called the New Unified Series, to be used throughout all of the country. The mathematics curriculum includes a study of algebra, plane and solid geometry, trigonometry, analytic geometry, probability and statistics, and one semester of calculus (differential and integral).

The teaching of geometry begins in the primary school. In grade three and four, teachers teach students to construct, protract, measure, and classify basic geometry figures. Most geometry content in secondary schools in the U.S. is taught in grade 8 and 9 in middle schools in China. The following is the geometric syllabus of mathematics curriculum at the secondary level in China. It has given much more emphasis to deductive proofs than those in the U.S:

Geometry in Grade 8:

1. Fundamental concepts (16 class hours)
2. Parallel and perpendicular lines (18)
3. Triangles (40)
4. Quadrilaterals (20)
5. Area of polygons and Pythagoras theorem (8)

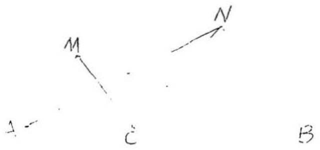
Geometry in Grade 9:

1. Similaries (36 class hours)
2. Circles (48) (Leung, 1987; p. 41).

4. Polyhedral angles and regular polyhedra (7 class hours): Polyhedral angles, properties of polyhedral angles. Regular polyhedra, transformations of polyhedra (Leung, 1987, p. 49).

Textbooks present geometry material at the van Hiele level 3 or level 4. Deductive proofs are treated at a depth that is hard to be found in any other contemporary syllabus of equivalent level. Problems of deductive proofs in geometry textbooks place emphasis on accurate definitions, clearly stated assumptions, and logical deductive proofs. They require students to deduce formally. They are far more complicated than those in the U.S. textbooks. The following questions generally represent the complicated level of deductive proofs in the geometry textbooks in middle schools in China:

Given: Referring to the diagram, Point C refers to Line AB, △ ACM and △ CBN are equilateral triangles. Prove: AN = BM (MSMG, 1993, p. 115).



Geometry in Grade 10:

Solid Geometry

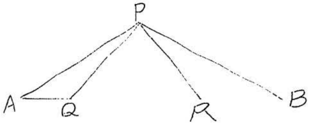
1. Lines and planes (28 class hours):

Planes, basic properties of planes, drawing plane figures. Relationship between the position of two lines, two lines perpendicular to the same line, angles with corresponding sides parallel, angles between two lines in different planes. Relationship between the positions of a line and a plane, determination and properties of a line being parallel to a plane, determination and properties of a line being perpendicular to a plane, projection of a tilted line on a plane, angle between a line and a plane, the theorem of three perpendicular lines and its converse of two planes being parallel, dihedral angles, determination and properties of two planes being perpendicular (Leung, 1987; p. 48-49).

2. Polyhedra and solids of revolution (29 class hours):

The concepts, properties, drawings and areas of prisms, pyramids and frustums of pyramids. The concepts, properties, drawings and areas of cylinders, cones and frustums of cones. The concepts, properties, drawings and areas of spheres, polar caps and their areas. The concept of volume and formulas, prismoids and their volumes, volumes of spheres and spherical segments.

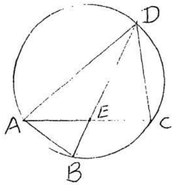
Given: Referring to the diagram, PQR is an equilateral triangle, $\angle APB = 120^{\circ}$. Prove: (1) $\triangle PAQ \sim \triangle BPR$; (2) $AQ \cdot RB = QR^{2}$. (MSMG, 1993, p. 263).



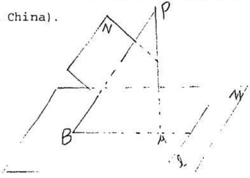
Examinations.

In the U.S., there are no entrance examinations to the elementary, junior high, or senior high schools. Nation-wide examinations do not test the students' ability of writing deductive proofs. However, the deductive proof writing is tested wildly in local- and nation-wide examinations in China. The problems of deductive proofs in entrance examinations, especially the nation-wide examinations, require a high level of logical thinking and logical expression. These examinations place much pressure on deductive proof education. The following problems generally represent the complicated level of deductive proof writing on mathematics entrance examinations in China:

(12%). Referring to the diagram, in the inscribed quadrilateral ABCD of the circle, chord AC, BD meet at E, and $AD \cdot AB = CD \cdot CB$. Prove that E is the midpoint of AC (the 1984 senior middle schools unified recruitment examination in Guangzhou; Leung, 1987).



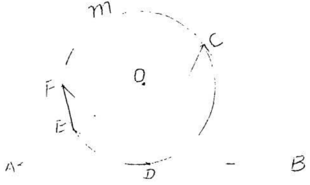
(10%). A helicopter is at the point P and A is its orthogonal projection on a horizontal plane M. An object B on plane M is seen from the helicopter (B is not the same point as A) and the straight line PB cuts at a right angle through the window plane of the aircraft, which is the plane N (see the figure). Prove that the plane N intersects plane M, and that the Line l is perpendicular to line AB (Mathematics Problems of the 1980 Higher Education Entrance Examination in China).



Supplementary materials.

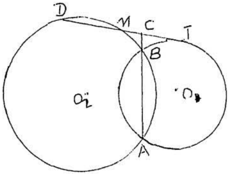
In the U.S., "...limited use was made of resources beyond the textbook for either content or teaching methods." (McKnight, et al, 1987; Jiang & Eggleton, 1995, p. 188). In contrast, Chinese teachers often demonstrate challenging supplementary problems of deductive proofs in classes. For example, in the geometry course in grade 9, about 30% class time is spent on demonstrating the challenging problems of deductive proofs from non-textbooks used to prepare the students for higher school entrance examinations. The following questions generally represent the complicated level of deductive proofs in supplementary materials:

Referring to the diagram, AB is the hypotenuse of Rt $\triangle$ ABC and is tangent to $\odot$ O, D is the point of tangency and the midpoint of AB, point C refers to $\odot$ O, $\odot$ O meets AC at E, F refers to $\overset{\frown}{EMC}$ of $\odot$O. Prove: (1) $\angle DFC = 2 \angle EFD$; (2) $BD^2 = AC \cdot DE$; and (3) $2 \sin \angle B$ $CD \cdot DF$ (Peng, 1994, p. 35).



Referring to the diagram, given $\odot O_1$ meets $\odot O_2$ at A, B. DT is tangent to $\odot O_1$, T is the point of tangency. DT meets $\odot O_2$ at D, M; and DM = MT.

Prove: CT = 2 CM (Zheng, 1993, p. 52).



These problems are so difficult that sometimes the students will not be able to solve them within a single class period. It may have the positive result that students are challenged to "apply their knowledge and experience in new and increasingly more difficult situations" (NCTM, 1989; Jiang & Eggleton, 1995, p. 189). However, the extracurriculum is not well accepted by many Chinese students who think geometry only at level 2 or 3. It impose heavy demands on the students' geometrical intuition and mathematical ingenuity. It causes a failure in deductive proof education.

Instruction.

Unlike the U.S., deductive proof education in China does not derive its pedagogy from a cohesive body of educational theory. Few teachers similar with the van Hiele model and little research has been conducted in the area of deductive proof education. The instructional practices are lacking in theoretical guidance.

The instructional level of deductive proofs is at Level 3 (informal deductive level) or Level 4 (formal deductive level). Sometimes, a whole lesson is designed to help students express themselves in a rigorous mathematical language on some definitions, postulates, and theorems, such as the theorem of three perpendicular lines. Many teachers require their students to memorize proofs of theorems in textbooks because these proofs may be tested in high school entrance mathematics examinations. Considering that most students' geometric thinking level is at level 2 or 3, the instruction is at a higher level than the students' level. The instruction can not be understood by most students.

Teaching loads in middle schools in China are lighter than those in the United States. A mathematics teacher teaches 2 classes per day. However, with every class containing 45 - 55 students, teachers give little attention to individual differences among students. They assume that all children can learn, and neither teaching style nor content is altered to suit the needs of different students.

Currently, Chinese mathematics educators are rethinking deductive proof education. They have recognized that emphasizing only on the formal, rigorous development of geometry has little educational value for both higher mathematics and real-world (Zhang, 1993). Many teachers are eager to adopt methods that teach problem solving rather than mere memorization of facts. The new national policy of compulsory education stated that the educational goal for grades 1-9 should be to increase the cultural and educational quality of the whole nation rather than to meet the students' needs of entering higher schools (Jiang & Eggleton, 1995). In response to the new national policy of compulsory education, the curriculum reform in Shanghai (1988) made the following suggestions for geometry curriculum and instruction:

In grades 1-5, geometry course should help students have the ability to name and identify the common geometric figures, to note the simple properties that it must have (necessary conditions), and to do simple geometric computation by using everyday objects and other physical materials (Zhang, 1993). This learning represents students' geometric thinking at the visualization and analysis levels.

In grades 6-7, through informal exploring of geometry by using the transformations and movements of geometric models, the geometry course should help students understand and apply geometric properties and relationships, and the role of proofs (Zhang, 1993). These learning activities represent students' geometric thinking at the informal deduction level and help students prepare for formal deduction.

In grades 8-9, geometry course should help students master formal deductions and improve their logical thinking ability. The students first learn to write simpler geometric proofs related to Triangles and Quadrilaterals, then improve their logical thinking by writing complicated proofs related to Similaries and Circles (Zhang, 1993).

China is planning to lower the theoretical depth of geometry instruction. A new unified geometry curriculum, in which the sequential learning in geometry and the goal of increasing the cultural and educational quality of the whole nation are considered, has been used to teach deductive proofs in middle schools since 1994.

In summary, the curricula of deductive proofs were not well-accepted for students in both countries. Most students in grade ten in the U.S. do not prepare for formal deductions. Many students in grade eight in China reject the topic of deductive proofs. However, the educational scenes of deductive proofs are changing greatly in both the U.S. and the People's Republic of China. In the U.S., NCTM (1989) has brought the van Hiele model closer to actual implementation (Teppo, 1991) in deductive proof education. Chinese educators have also begun to rethink deductive proof education.

CHAPTER V

Conclusions and Recommendations

Conclusions

In this study, we have attempted to compare deductive proof education between the U.S. and the People's Republic of China. Students' geometric thinking levels, curricula, instructions, examinations, and supplementary materials are compared to help educators and teachers rethink deductive proof education. This study shows that there are significant differences between the deductive proof education in the U.S. and the People's Republic of China.

In the U.S., most educators, teachers, and students think that geometry is more than deductive proofs. Teachers present geometry in a more concrete manner than teachers do in China. The instruction is still at a higher level of mental development than the students' level. The inadequacies of textbooks and teachers' heavy reliance on them have caused many problems in deductive proof education in the U.S.

In China, many teachers and students view geometry as only deductive proofs. Although the geometric thinking levels of Chinese students are higher than those of the students in the U.S, the curriculum and instruction are presented in a more abstract manner than those in the U.S. There are a variety of entrance examinations to test the students' achievement in deductive proofs writing. These examinations place much pressure on deductive proof education and drive the curriculum. They cause many students to reject the topic of deductive proofs.

The results of this study have implications for deductive proof education at the secondary level in both countries. They suggest that instruction should fit the cognitive level of the students. Deductive proof writing should build on the strong geometry foundation knowledge which students have already developed. It would appear that for the majority of students at the beginning of grade eight in urban schools in the People's Republic of China, instruction at the analysis or informal deduction level would be most appropriate. At least the plane geometry course in grade eight should be informal without deductive proofs. In the U.S., educators have suggested that more informal geometry courses at the secondary school level are needed (Cox, 1985).

Recommendations for Further Research

One of the important objectives of this study is to encourage further study, discussion, and possible refinement of deductive proof education at the secondary level in both countries. It has been suggested that there are a number of factors contributing to the students' achievement in deductive proof writing, such as the students' geometric thinking level, entry geometry knowledge (Senk, 1989), curriculum and instruction, academic motivations, et al.

The following studies are suggested to be conducted in China for further research:

1. that a study be conducted in both rural and urban middle schools to determine the students' van Hiele levels;

2. that a study be conducted in both rural and urban middle schools to determine students' achievement in deductive proof writing;

3. that a study be conducted in both rural and urban middle schools to find that relationships among students' achievement in deductive proof writing, students' entry geometry knowledge, students' van Hiele levels, teaching and learning style, instructional time spent on teaching deductive proofs, academic motivations, and students' age; and

4. that a study be conducted to find the best way to raise the van Hiele level of students in both rural and urban middle schools.

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Appendix

The Van Hiele Geometry Test*

(modified version)

Directions

Do not open this test booklet until you are told to do so.

This test contains 20 questions. It is not expected that you know everything on this test.

When you are told to begin:

1. Read each question carefully.

2. Decide upon the answer you think is correct. There is only one correct answer to each question. Cross out the letter corresponding to your answer on the answer sheet.

3. Use the space provided on the answer sheet for figuring or drawing. Do not mark on this test booklet.

4. If you want to change an answer, completely erase the first answer.

5. If you need another pencil, raise your hand.

6. You will have 30 minutes for this test. Wait until your teacher says that you may begin.

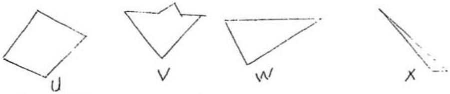
*From "Van Hiele Levels And Achievement In Secondary School Geometry" CDASSG Project by Usiskin, Z., 1982. (ERIC Document Reproduction Service No. ED 220 28).

The Van Hiele Geometry Test
(modified version)

1. Which of these are squares?

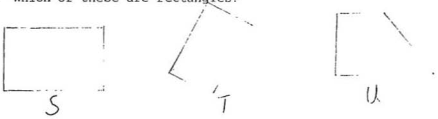
(A) K only
(B) L only
(C) M only
(D) L and M only
(E) All are squares.

2. Which of these are triangles?



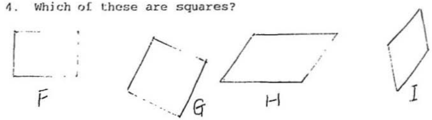
(A) None of these are triangles.
(B) V only
(C) W only
(D) W and X only
(E) V and W only

3. Which of these are rectangles?



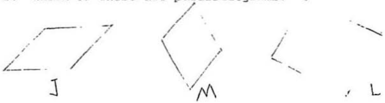
(A) S only
(B) T only
(C) S and T only
(D) S and U only
(E) All are rectangles

4. Which of these are squares?



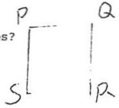
(A) None of these are squares
(B) G only
(C) F and G only
(D) G and I only
(E) all are squires

5. Which of these are parallelograms?



(A) J only
(B) L only
(C) J and M only
(D) None of these are parallelograms
(E) All are parallelograms

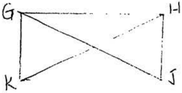
6. PQRS is a square.



Which relationship is true in all squares?

(A) PQ and RS have the same length.
(B) QS and PR are perpendicular
(C) PS and QR are perpendicular.
(D) PS and QS have the same length.
(E) Angle Q id large than angle R.

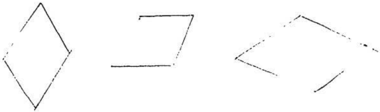
7. In a rectangle GHJK, GJ and HK are the <u>diagonals</u>.



Which of (A)-(D) is <u>not</u> true in <u>every</u> rectangle?

(A) There are four right angles.
(B) There are four sides.
(C) The diagonals have the same length.
(D) The opposite sides have the same length.
(E) All of (A)-(D) are true in every rectangle.

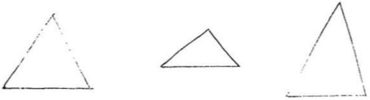
8. A <u>rhombus</u> is a 4-sided figure with all sides of the same length. Here are three examples:



Which of (A)-(D) is <u>not</u> true in every rhombus?

(A) The two diagonals have the same length.
(B) Each diagonal bisects two angles of the rhombus.
(C) The two diagonals are perpendicular.
(D) The opposite angles have the same measure.
(E) All of (A)-(D) are true in every rhombus.

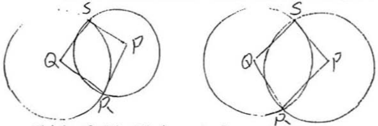
9. An isosceles triangle is a triangle with two sides of equal length. Here are three examples.



Which of (A)-(D) is true in every isosceles triangle?

(A) The three sides must have the same length.
(B) One side must have twice the length of another side.
(C) There must be at least two angles with the same measure.
(D) The three angles must have the same measure.
(E) None of (A)-(D) is true in every isosceles triangle.

10. Two circles with centers P and Q interest at R and S to form a 4-sided figure PRQS. Here are two examples.



Which of (A)-(D) is not always true?

(A) PRQS will have two pairs of sides of equal length.
(B) PRQS will have at least two angles of equal measure.
(C) The lines PQ and RS will be perpendicular.
(D) Angles P and Q will have the same measure.
(E) All of (A)-(D) are true.

11. Here are two statements.

Statement 1: Figure F is a rectangle.
Statement 2: Figure F is a triangle.

Which is correct?

(A) If 1 is true, then 2 is true.
(B) If 1 is false, then 2 is true.
(C) 1 and 2 cannot both be true.
(D) 1 and 2 cannot both be false.
(E) None of (A)-(D) is correct.

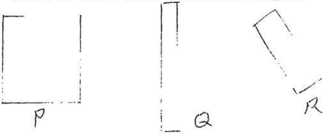
12. Here are two statements.

Statement S: $\triangle$ ABC has three sides of the same length.
Statement T: In $\triangle$ ABC, $\angle$B and $\angle$C have the same measure.

Which is correct?

(A) Statements S and T cannot both be true.
(B) If S is true, then T is true.
(C) If T is true, then S is true.
(D) If S is false, then T is false.
(E) None of (A)-(D) is correct.

13. Which of these can be called rectangles?



(A) All can
(B) Q only
(C) R only
(D) P and Q only
(E) Q and R only

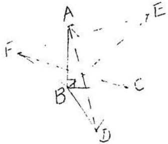
14. Which is true?

(A) All properties of rectangles are properties of all squares.
(B) All properties of squares are properties of all rectangles.
(C) All properties of rectangles are properties of all parallelograms.
(D) All properties of squares are properties of all parallelograms.
(E) None of (A) (D) is true.

15. What do all rectangles have that some parallelograms do not have?

(A) opposite sides equal
(B) diagonals equal
(C) opposite sides parallel
(D) opposite angles equal
(E) None of (A)-(D).

16. Here is a right triangle ABC, Equilateral triangles ACE, ABF, and BCD have been constructed on the sides of ABC.



From this information, one can prove that AD, BE, and CF have a point in common. What would this proof tell you?

(A) Only in this triangle drawn can we be sure that AD, BE, and CF have a point in common.
(B) In some but not all right triangles, AD, BE, and CF have a point in common.
(C) In any right triangle, AD, BE, and CF have a point in common.
(D) In any triangle, AD, BE, and CF have a point in common.
(E) In any equilateral triangle, AD, BE, and CF have a point in common.

17. Here are three properties of a figure.

Property D: It has diagonals of equal lenth.
Property S: It is a square
Property R: It is a rectangle.

Which is true?

(A) D implies S which implies R
(B) D implies R which implies S
(C) S implies R which implies D
(D) R implies D which implies S
(E) R implies S which implies D

18. Here are two statements.
I: If a figure is a rectangle, then its diagonals bisec each other.
II: If the diagonals of a figure bisec each other, the figure is a rectangle.

Which is correct?

(A) To prove I is true, it is enough to prove that II is true.
(B) To prove II is true, it is enough to prove that I is true.
(C) To prove II is true, it is enough to find one rectangle whose diagonals bisect each other.
(D) To prove II is false, it is enough to find one non-rectangle whose diagonals bisect each other.
(E) None of (A)-(D) is correct.

19. In geometry:

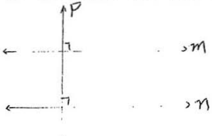
(A) Every term can be defined and every true statement can be proved true.
(B) Every term can be defined but it is necessary to assume that certain statements are true.
(C) Some term must be left undefined but every true statement can be proved true.
(D) Some term must be left undefined and it is necessary to have some statements which are assumed true.
(E) None of (A)-(D) is correct.

20. Examine these three sentences.

(1) Two lines perpendicular to the same line are parallel.
(2) A line that is perpendicular to one of two parallel lines is perpendicular to the other.
(3) If two lines are equidistant, then they are parallel.

In the figure below, it is given that lines m and p are perpendicular and lines n and p are perpendicular. Which of the above sentences could be the reason that line m is parallel to line n?

(A) (1) only
(B) (2) only
(C) (3) only
(D) Either (1) or (2)
(E) Either (2) or (3)



Answer Sheet

Name: Student ID Number:

1. A B C D E
2. A B C D E
3. A B C D E
4. A B C D E
5. A B C D E

6. A B C D E
7. A B C D E
8. A B C D E
9. A B C D E
10. A B C D E

11. A B C D E
12. A B C D E
13. A B C D E
14. A B C D E
15. A B C D E

16. A B C D E
17. A B C D E
18. A B C D E
19. A B C D E
20. A B C D E