A DESCRIPTIVE STUDY OF LEVEL THREE ADVANCED MATHEMATICS STUDENTS' CONCEPTUAL UNDERSTANDING OF THE ROOTS OF POLYNOMIAL FUNCTIONS

ANNETTE WINDSOR

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by

Annette Windsor, B.A., B.Ed.

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Abstract

The purpose of this descriptive study was to investigate level III advanced mathematics students' understanding of the roots of polynomial functions as a result of an integrated approach with the graphing calculator. It examined the students' ability to work with the symbolic, tabular, and graphic representations of polynomial functions in their quest to identify its roots. It also explored what students had to say about the graphing calculator and its features and their reaction to its inclusion in the learning process.

One class of thirty one students of Mathematics 3201 participated in the study. Each student completed a manual of thirteen activities designed specifically for the TI-82 graphing calculator to address the curricular objectives for the polynomial unit. Students were required to formulate a written definition for the root of a polynomial function, at the beginning and conclusion of the study. All were interviewed as many as five times and all wrote a final unit test and questionnaire.

After the integrated approach with the graphing calculator, students were able to formulate and articulate a coherent explanation of the root of a polynomial function. Additionally, many could also provide significant detail regarding the different aspects of a root and demonstrated a reasonable degree

of proficiency with polynomial functions expressed symbolically, graphically, and in tabular form.

Most students responded positively to the integration of the graphing calculator and learned to use it efficiently and effectively by the time the study concluded. It proved to be most popular in a supportive role to verify work and to provide insight so busy work could be kept at a minimum. It helped most students appreciate that the graph of a polynomial function was integrally connected to the algebra they had learned in past courses.

Not all students, however, were pleased with the frequent use of the calculator. Some felt that it detracted from their ability to master the algebraic procedures that were paramount in "real" mathematics.

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A special thanks to the many teachers who permitted so many of their students to miss class time in order to be interviewed. The assistance of the students in the target group is also gratefully acknowledged. Without the support of both, the study would have been impossible.

Two other special people who deserve recognition and thanks are my mother for looking after my son and my home and my husband for his encouragement and his help solving the many computer related problems that arose throughout the course of my master's program. Their support was invaluable.

Preface

For the past eleven years I have been teaching math in one senior high school in this province. For all of these years I have entertained and struggled with a question crucial to my professional growth as a teacher. What exactly is mathematics?

A difficult question indeed.

Occasionally, I dare ask my students. This year was one such occasion. Their responses surprised, didn't surprise, intrigued, pleased, and disappointed me. Some of them said that mathematics is:

-the science of numbers

-addition, subtraction, multiplication, division, algebra, geometry, calculus, functions, fractions, square roots, trigonometry, theorems, and postulates

-story problems, problem solving, and a way to develop your mind and your reasoning skills

-more than funny symbols and long numbers, it is a part of our lives

-a way to explain the world in a simplified way

-a subject we must take in school

-it is a class that I have had to take 5 or more times a week for the past thirteen years

-formulas that I'll probably never use in life

-isn't overly useful and is really only a pain to many

-what allows us to construct buildings, boats, and airplanes

-finding slopes, drawing graphs, solving for x, and proving proofs
-used in statistics and scientific research
-the basis for all science

In the course of building and refining my own sense of what mathematics is all about, I became interested in the graphing calculator and the potential it offered me to shift away from the monkey see monkey do approach so prevalent with this subject. The graphing calculator was the accessible and relatively affordable tool that offered a real alternative in the classroom. For me this instrument could provide the bridge from abstract symbolic mathematics to the development of a greater appreciation for the meaning behind the symbols. It seemed to have the potential to move students beyond the prescription/imitation modus operandi common in school mathematics to speculation, exploration, and determination. In a non-judgmental way the calculator promised to make its user more likely to explore, observe, adjust, and conclude.

It is my hope that the use of the calculator and the approach taken during the study has, in some small way, helped some of my students develop a greater sense of freedom and responsibility for their own learning.

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CHAPTER ONE

1.1 Introduction

For the past twenty years there has been a call for fundamental reform in all levels of the teaching and learning of mathematics. The response has been slow and arduous.

In general, teaching mathematics is not vastly different than that which was typical when the call was issued. Stakeholders in the educational process have not embraced the vision that has culminated in the Standards document (NCTM, 1989) with its demand that students learn more and better mathematics and that instruction they receive be radically reformed. The fact that the changes are so pervasive and presumably costly may partially explain the sluggish response.

However, technological tools such as the graphing calculator challenge and defy the status quo. In the hands of the masses, software and calculators, easily programmed and capable of symbolic manipulation, will press the issue of reform. Educators will be forced to abandon the security of traditional curricula rich in memorization and the paper and pencil wizardry of simplifying, factoring, and manipulating in favor of one that promotes skills crucial in the workplace of today and tomorrow.

Hopefully, the findings of this research will benefit other senior high mathematics teachers in their quest to understand what role the graphing calculator can play in their teaching of mathematics, how it can be beneficial to their students, how they might integrate it into their classrooms, and even why they should.

1.2 A Focus for the Study

This study is descriptive rather than experimental or comparative. It does not attempt to make any specific comparisons among the target group for this study to groups from previous years nor does it attempt to statistically quantify the findings. Rather it investigates and describes the effect of an instructional approach that uses the graphing calculator on students' conceptual understanding of polynomial functions. Specifically, it attempts to answer the questions:

1. Does a unit of instruction which includes regular and frequent use of the graphing calculator, as one element of the instructional approach, help students develop an understanding of the concept of the roots of polynomial functions?

2. Which representation of function, tabular, graphic, or algebraic do students choose to use and work with when determining the roots of a polynomial function? Why?

3. What growth or development, if any, have the students exhibited in their ability to make the link between the symbolic factors of the polynomial equation and its real roots?

4. What do students say about the integration of the graphing calculator into their learning of mathematics that would reveal their attitude towards this device?

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CHAPTER TWO

REVIEW OF THE RELATED LITERATURE

2.1 Introduction

The literature review is divided into three parts. The first presents a brief overview of the history of calculating and calculating devices, presenting it as a phenomenon not exclusive to modern day invention. The second part delves into the calculator and the place it has occupied in the mathematics classroom over the past twenty years. The final part of the review discusses some of the potential benefits to be derived from the graphing calculator and then presents some of the research findings on graphing technology in mathematics education. Since much of the literature related to the graphing calculators is activity rather than research based, the author has also incorporated an overview of the research related to the microcomputer as it pertains to the learning of mathematics.

2.2 The History of Computing

Calculating is not the brainchild of modern day society. Historically speaking, calculating is as diverse and dynamic as it is old. Since pre-historic times, calculating has been both an integral part of everyday commerce and a source of wonder and fascination for some of the greatest minds in history.

In an ongoing search for speed, efficiency, and accuracy, efforts to enumerate and compute have resulted in the development of procedures and devices that facilitated these processes. Humanity's earliest attempts at enumeration include such things as stones in a bag, notches on a stick, and tally marks in the sand or on the wall, usually indicating a one to one correspondence. Approximately five thousand years ago, these practices were rendered obsolete by the appearance of counting boards and devices like the abacus (Moursund, 1981).

The eventual emergence of number systems such as those used by the Greeks and Romans brought significant changes to the process of computation. Rather than a collection stones, notches, or tally marks a single symbol or series of symbols was used to denote quantity. The Hindu-Arabic number system was even more versatile because it introduced the powerful notion of place value which permitted the development of algorithms for addition, subtraction, multiplication, and division. Algorithms eventually developed into mathematical tables that could be used by those working in the areas of astronomy, navigation, and weaponry (Moursund, 1981). These mathematical tables provided an efficient source of answers to frequently occurring problems without the necessary drudgery of repetitive calculations.

In the 1600s Napier made a significant contribution to computing through his idea to use rods in order to perform the multiplication and division of whole numbers (Moursund, 1981).

These rods came to be called Napier's bones. Napier later developed the concept of logarithms, permitting multiplication and division of any decimals to be reduced to addition and subtraction through the use of logarithmic tables. These tables continued to be quite valuable to scientists and engineers until well into the second half of this century. However, with the advent of electronic computational devices the tables became redundant and inefficient.

From the seventeenth to the twentieth century, mechanical calculating devices were invented and sometimes built. However, it was not until the nineteenth century that they became commercially available. Due to their limited availability and high cost, they had little impact on education (Moursund, 1981). Chalk and blackboard and paper and pencil continued to be the preferred medium in educational circles.

The quest to build an efficient calculating device received its biggest impetus from a new found ability to harness electrical power and build electrical motors. With each decade in the twentieth century came improvements in circuitry until the electronic digital computer was born (Moursund, 1981). Progress continues to be ambitious and rapid. This is evidenced by the numerous improvements that have been made to the scientific calculator, the graphing calculator, and the home computer, and the lap-top to the extent that by the time a purchase is made, new technology becomes available that eclipses the capabilities of that just procured.

2.3 The Calculator

> With the decrease in the cost of the minicalculator, its accessibility to students at all levels is increasing rapidly. Mathematics teachers should recognize its potential contribution as a valuable instructional aid. In the classroom, the minicalculator should be used in imaginative ways to reinforce learning and to motivate learners as they become proficient in mathematics. (Mathematics Teacher, 1978, p.92)

This is the official position of the NCTM on the use of calculators as stated in 1974. In the twenty year period since, this position has not changed. Instead it has been reaffirmed, repeated, and refined to reflect the changes in technology that have since occurred. Further support for its integration came in 1980 in the NCTM's Agenda for Action: Recommendations in the 1980s. One of their recommendations was that mathematics programs take full advantage of the power of calculators and computers at all grade levels (p. 1). At the end of that decade NCTM released <u>Curriculum and Evaluation Standards for School Mathematics</u>. This document reiterated its former position regarding calculator use for all levels of the curriculum:

> Because technology is changing mathematics and its uses, we believe that appropriate calculators should be available to all students at all times (NCTM, 1989, p. 8)

Despite the many invitations to adopt the calculator as an integral part of the learning environment, it still has had little actual impact on the curriculum, both in what is taught and how it is taught.

A major reason for this resistance could be a fear that the calculator will be detrimental to the students' acquisition, maintenance, and facility with basic mental arithmetic and paper and pencil algorithms. The conflicting circumstances between potential benefits on one hand, and faithfulness to the old ways on the other hand, gave rise to one of the largest bodies of research in mathematics education (Hembree & Dessart, 1992).

Though some research has yielded ambiguous findings, there is considerable agreement among many as to the effects of using calculators on basic computational skills, achievement, testing, concept development, problem solving, and attitude.

In 1986 Hembree and Dessart assumed the task of integrating the findings of seventy nine research reports. They performed a meta-analysis to determine the effects of using calculators on students' achievement on tests, conceptual knowledge, computation, problem solving, and attitude. To synthesize the various findings they transformed results to a common numerical base called effect size. A positive effect size indicated a study favoring the calculator treatment. In all, 524 'effects' were measured in the seventy nine studies. These effects were then grouped according to grade level, student ability, and common aspects of performance or attitude so that the results could be tested for statistical significance (Hembree & Dessart, 1992).

In some of these seventy nine studies, experimental groups were permitted to use calculators during testing situations. Comparison groups were allowed only paper and pencil. The difference in the average scores in those studies showed a clear advantage to those who had used the calculator for instruction and testing (Hembree & Dessart, 1992). For tests with calculators, students of average and low ability showed positive effects that seemed moderate to large (Hembree & Dessart, 1992).

This result is not particularly surprising. However, it is interesting that for tests without calculators there was small but significant effects observed for average students at all grade levels. This indicated that the use of calculators during instruction advanced the students' skills with written algorithms (Hembree & Dessart, 1992).

Much of the research literature did not reach any conclusions regarding the relationship between the calculator and conceptual development. In the face of a lack of evidence to support any such claims, many concluded that, at the very least, the calculator posed no threat to the development of conceptual knowledge. However, numerous articles, which could only be classified as opinion literature, hypothesized that the calculator should be instrumental in helping students to understand concepts better (Branca, Breedlove, & King, 1992).

The relationship between the calculator and problem solving, however, is clearer than it is for conceptual knowledge. There were observable gains in the area of problem solving as a result of using the calculator. The scores of high and low ability students in problem solving showed a moderate improvement as a result of improved computation and strategy use (Hembree & Dessart, 1992). Studies on the use of calculators and problem solving summarized by Suydam found either positive effects or no significant differences when calculators were used (Szetela & Super, 1987). In their study, carried out on 290 grade seven students, Szetela and Super (1987) concluded that the problem solving group with calculators were slightly more successful that the group who had received traditional instruction.

Computational skills were also studied in order to determine what impact the calculator might have had. In general, Hembree and Dessart (1992) concluded that the calculator could apparently advance the average students' computational skills while doing no harm to the computational skills of low and high ability students. Subsequent research by Hembree and Dessart (1992) uncovered studies that found the calculator to be advantageous for computation for average ability students, as well as those of low ability. The general literature also theorized that computation would be enhanced as a result of the calculator and that errors would be the result of problem misconceptualization or calculator keystroking (Shuard, 1992).

Performance aspects such as concept development, problem solving, testing, and computation were not the only factors that were of interest to those studying the impact of calculators on mathematics education. Students' attitudes towards the calculator were also a topic of investigation. Specifically, those students who had been exposed to the calculator during their instruction displayed a better attitude towards mathematics than did those who had had no contact with the device (Hembree & Dessart, 1992). The Szetela and Super (1987) study, conducted for an entire school year on twenty-four seventh grade classes supported this finding because the calculator group scored significantly higher on the attitude-toward-problem-solving-test than did the control group.

2.4 The Graphing Calculator

2.4.1 Introduction

Computers have been available on a more or less limited basis in schools since the late sixties. In the 1980s it was widely anticipated by educators that the microcomputer would revolutionize mathematics instruction. Yet the changes predicted never materialized. Two reasons might explain this reality. First, computers were neither readily available nor accessible, for economic reasons. Secondly, and just as importantly, the role of the microcomputer had not been clearly defined or teachers properly trained in its use.

The availability and accessibility of microcomputers continues to be an issue today and considering the limited financial resources of many school systems, it is likely that it will continue to be an obstacle in the future. However, the introduction of the graphing calculator in the late 1980s has provided a real alternative to the microcomputer. These hand held devices, now widely available and relatively inexpensive, have the potential to enhance the mathematics curriculum. They are sophisticated, easy to use devices that are forcing educators to re-think the role of technology in instruction. The more recent symbolic manipulation capabilities of some graphing calculators and the ease of programming that is typical of these instruments are forcing educators to re-examine what it is about mathematics that they value and to reassess what it is they think that students should know in order to be mathematically literate.

2.4.2 Potential of the Graphing Calculator

One of the greatest benefits to be derived from the use of the graphing calculator in the senior high mathematics curriculum is the opportunity it affords students to explore, interact, and test. For example, the capacity for exploration of functional relations of all kinds is virtually unlimited. Its potential will not begin to be realized if it is used primarily for rote computation and template problem solving.

Because students regularly have to interpret the answers supplied and re-adjust their input, it is an ideal tool to broaden students' critical thinking skills (Dion, 1990). One way in which critical thinking skills are developed is through the user's control over the viewing window. This feature offers a wonderful opportunity for students to explore, to experiment, to question, and to adjust and re-adjust their thinking. For example, the default screen can produce misleading graphs unless more suitable axes and scales are defined by the user. The user learns to refine his or her skill of analyzing the domain and range of a function (Kelly, 1993). The beauty of this exercise is that students discover, through experience, the need for thoughtful use of the calculator and inadvertently learn the meaning of the old phrase garbage in, garbage out (Dion, 1990).

The graphing calculator has enormous potential in a senior high mathematics program. It can be particularly useful for: solving systems of equations and inequalities, solving absolute value problems, working with trigonometric identities, exploring polynomial relationships, investigating calculus topics, studying polar and parametric equations, and researching statistical concepts. This list is, by no means, complete.

Solving systems of linear equations and polynomial equations, in general, is a very common and useful application of the graphing calculator. The visual image displayed by the calculator reinforces the solution set obtained by the traditional paper and pencil methods and helps students see the connection between symbolic expressions and graphical representations.

The graphing calculator is ideal for solving absolute value equations (Horak, 1994) and verifying trigonometric identities (Kelly, 1993). Examining absolute value equations using the graphing calculator enables students to see relationships among an equation, its associated graph, and its derived solutions (Horak, 1994). The visual image displayed by entering $y=|3x+2|$ and $y=|x+2|$ should help students to better appreciate the solution set for $|3x+2|=|x+2|$. Similarly, the graphing calculator offers an effective way to investigate trigonometric identities. Students can determine whether the left and right hand side of the equation are indeed equal by observing the graph or graphs that result from entering equations for the left and right side individually. Screens that display the same graph for both equations entered suggest that the identity is true for all values of the variable (Kelly, 1993). Topics that are rarely seen at the secondary level can now be introduced much earlier because of the power of the graphing calculator. Graphs of polar and parametric equations are now readily accessible with the aid of these hand held computers (Demana & Waits, 1989). The fact that producing polar and parametric graphs by hand takes so long makes it an ineffective instructional strategy (Demana & Waits, 1989). However, many hand held graphing calculators now have the built-in parametric graphing utilities that automate the curve construction process (Foley, 1992). They can simultaneously plot related curves and have a user controlled trace that displays a numerical readout of the parameter value and the coordinates associated with each plotted point (Foley, 1992). Though this may not be something that one would want to explore with every mathematics class, at least the potential is there to explore and enrich the more advanced classes.

Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) recommends the integration of statistical concepts and applications into the senior mathematics curriculum. Relating equations and data, finding the line of best fit, and being able to make interpretations and predictions from data are some skills that are advocated (Rubenstein, 1992). The graphing calculator, with its vast statistical capability can make this a realistic objective. Capable of producing scatter plots, graphs and parameters for curves of best fit, and correlation coefficients, among other things, the graphing calculator is an ideal tool for developing the necessary skills for statistical analysis (Rubenstein, 1992).

An imposing, but fundamental concept that can be simplified and clarified by the graphing calculator is the concept of limit. Limits are typically studied in first year university mathematics courses and in some secondary programs. It can be an intimidating and difficult topic for many students because of the many rules associated with evaluating limits for different types of functions. The beauty of the graphing calculator is that it provides the student with the opportunity to see what the concept means in graphical and tabular terms in a dynamic way that is not always possible with a traditional chalk and talk approach. In the context of limits, the graphing calculator also helps students develop a greater appreciation for the continuous nature of number (Heid as cited in Hart, 1992).

The potential of the graphing calculator within the branch of mathematics called calculus, both differential and integral, can not be overestimated. Newton's method for approximating roots is an ideal topic to explore with the graphing calculator because of its capability, under the draw option, to add tangents lines where indicated. The calculator is particularly useful because of its speed and precision in drawing these tangent lines and its dynamic capability to zoom in on the graph to determine if the tangent lines are converging on the root of the original function. Again, because of the speed of the calculator several different types of functions can be explored in order to expose students to situations where the successive approximations may or may not converge.

Just as the concept of limit can be clarified when explored in conjunction with the calculator, so can the derivative. The derivative is the cornerstone of the entire branch of mathematics known as differential calculus and lends itself nicely to the exploration with the graphing calculator. For example, the differentiability or non-differentiability of a given function and it being locally linear can be seen dynamically when zooming in on the graph (Dick, 1992).

These are only a few topics in calculus where the calculator can be put to good use. Its graphical capabilities impart a richer meaning to calculus concepts and hopefully leave the student with a less impoverished view of functions as mere symbolic expressions to be manipulated (Dick, 1992).

The graphing calculator adds a visual dimensions to the world of mathematics and moves it beyond the strictly symbolic. The students learn from experience that mathematics is much more than pencil gymnastics.

2.5 Literature Review of Graphing Technology

In general, the research literature does indicate that students who use graphing technology are able to function at higher levels of graphical understanding (Browning as cited in Hart, 1991), are better able to relate graphs to their equations (Rich as cited in Dunham and Dick, 1994), are better able to read and interpret graphical information (Boers-van Oosterum, 1990), and have a greater understanding of the connections among graphic, numeric, and symbolic representations (Beckmann 1989; Browning 1989; Hart 1992).

Though the research literature on the use of graphing calculators is limited because of the newness of the technology, much of what is available has attempted to compare overall achievement between experimental groups for whom graphics technology is available and control groups taught in the traditional way.

A recent study out of Hawthorn, Victoria from the Swinburne Institute of Technology (Boers and Jones, 1992) looked at the effects of the graphing calculator on students' achievement and attitudes in a first year undergraduate calculus course. There were 320 subjects in this particular study, all of whom were first year students at the Institute. The subjects were divided into two lecture groups according to their declared major and their reported mathematics background. The first group (n=150) included Math/Computer Science majors and Double Computer Science majors, as well as those who had low math scores in their final year of high school and those who had some deficiency in their math background. The latter two gathered a week before the semester began to brush up their prerequisite skills and to be introduced to the graphing calculator. They were referred to as the bridging students. The second group (n=170) consisted of students majoring in Medical Biophysics & Instrumentation, Computing & Instrumentation, Computer Aided Chemistry, and Computer Aided Bio-Chemistry. The course content was the same for both groups except for the first two weeks when the first group studied financial mathematics and the second group studied vector analysis. Assessment was similar for both groups. All students in both groups were required to purchase a graphing calculator among their course materials. It was used extensively in the topics of graphing functions, equations and inequalities, and limits. Particular emphasis was placed on the capacity of the calculator to act as checking device for analytically derived knowledge (Boers & Jones, 1992).

With both groups, Boers and Jones (1992) investigated student attitudes towards the graphing calculator, the impact it had on performance, and the impact it had on the bridging students. The attitude surveys were administered to all students in April '91 and September '91 before the course began in order to assess how the changes were being anticipated. In general, response to the proposal to include the graphing calculator in their instruction was positive and any change in attitude over the five month interim was in the positive direction. Many students reported to like the fact that problems were now presented and analyzed algebraically and graphically. Negative reactions generally reflected a fear that, potentially, the calculator would erode skills. The results of the surveys showed that females, typically, rated their math ability lower than males and reported being more anxious than males. Also more females said they found the graphing calculator difficult to use and were confused when problems were analyzed algebraically and graphically. Ironically, females were less likely than males to make the statement I find myself experimenting with a problem rather than just trying to get the solution, but they disagreed, more than males, that the graphing calculator was a waste of time (Boers & Jones, 1992).

Despite the insecurity shown, females did not perform more poorly than males. In fact, the mean score for females at the end of the semester was 62.49 whereas it was 58.61 for the males. For females with a Math and Computer Science major, the graphing calculator was associated with a dramatic improvement in performance relative to that of male students in the same group. Prior to 1991, males had slightly outperformed females, but after 1991 the women outperformed the men and by a larger margin. This difference was even more dramatic among the bridging students who had chosen this major. No comparison could be drawn from the Double Computer Science majors because there were too few females on which to base the comparison.

The overall performance of the bridging students did not seem to be positively or negatively effected by the introduction of the graphing calculator. In fact, there was no change in the relative performances of bridging and normal entry students. Subsequent interviews with bridging students revealed that they felt the calculator was a useful device, but not a major factor in helping them to bridge the gap between high school math and university math.

The Ohio State C^2PC Project was an enormous study designed to incorporate graphing technology into a precalculus course. For the first two years of the project, the materials were piloted by the authors and others in five, then nine, high school classrooms. Subsequently, the course was field tested by over 2000 students in 86 high schools and 40 colleges. Interactive software or graphing calculators were an integral part of the course for the experimental groups while instruction for the comparison groups emphasized paper and pencil skills and algebraic manipulation (Dunham, 1992). Results from ANOVA showed that there were no significant differences between experimental and comparison groups on the pre-test, but that the C^2PC schools significantly outperformed comparison schools on the posttest. Similar results were found using ANCOVA with pretest scores as the covariate (Dunham, 1992).

Dunham (1992) implies a need for cautious optimism when interpreting the results of the study because of its lack of uniformity among the experimental groups. Variations occurred in the length of class time, pacing, type of graphing technology used, frequency of use, and student profiles. As well, comparison classes were even more varied in syllabi and texts. Consequently, further investigation is needed to confirm these exciting results. During the Heid study (as cited in Hart, 1992) students used computers as a primary means of computation during the first twelve weeks of a calculus course and developed paper and pencil methods for the last three weeks. MuMath, software capable of symbolic manipulation, was used by those in the experimental groups. Data was collected in a variety of ways from each of the two experimental groups of fifteen to twenty students and one comparison class of approximately one hundred. Results of the study showed that students who used the computers had a broader and deeper understanding of the course concepts and performed almost as well on a final exam of routine skills (Heid as cited in Hart, 1992). Even though the mean score of the three groups was not significantly different, the experimental groups outperformed the comparison group on 14 of the 16 conceptual questions on the exam, but did worse on the paper and pencil techniques. Further analysis revealed that the experimental students were more creative, accurate, and detailed in their responses and better able to tie ideas together (Heid as cited in Hart, 1992).

The Palmiter research (as cited in McClendon, 1992) confirmed Hart's findings. This study compared an experimental group with two control groups to investigate the use of a computer algebra system in an introductory calculus course. At the end of a five week period the experimental group was given a computational and a conceptual exam with questions extracted from the final exam of the second control group. The experimental group was allowed to use the computer software they had been using previously. The first control group wrote a similar exam during the tenth week. Also, after eleven weeks both the experimental group and the first control group wrote a common exam. The computer was not permitted. The analysis showed that the experimental group performed significantly better on the conceptual exam at week five than the control group did at week ten. As well, the experimental group scored significantly better on the computational exam in 45 minutes than the control group did in 90 minutes. Finally, though there was no significant difference on the mean score between the groups on the conceptual portion of the final exam, the experimental group performed significantly higher overall (Palmiter as cited in McClendon, 1992). Palmiter hypothesized that emphasis on the computational skills for the last five weeks and the absence of the computer during the exam interfered with the groups performance on the conceptual questions (Palmiter as cited in McClendon, 1992).

Another study that supports the contention that computers can be a valuable tool in the acquisition of conceptual knowledge is the Schrock study (Schrock as cited in Hart, 1991). Working with three sections of calculus I, two control groups and one experimental group, Schrock investigated the differences between students' understanding of calculus concepts. The experimental group used a computer algebra system while the control groups were taught in the traditional manner. The study focused on the differences exhibited in computational skills and on the ability to solve application problems. The results of a conceptual exam given after the thirteenth week indicated a significant difference between the means of the control group and the experimental group. However, the final exam showed that, though the mean score of the experimental group was higher, the difference was not significant. The analysis of the conceptual questions on the exam, however, showed a significant difference in favor of the experimental group. Similar findings were noted on an application exam that both groups wrote. Shrock (as cited in Hart, 1991) concluded that the experimental students showed no loss in computational skills and that a calculus course emphasizing the development of concepts through the use of technology rather than skills acquisition would have a positive effect on students.

Melin (as cited in Hart, 1992) reached similar conclusions after a four week experiment with two groups of calculus I students. The experimental group (n=24) used the graphing calculator during their instruction while the control group (n=24) did not. The experimental instruction occurred after the first of the two departmental exams administered to both groups.

Even though the graphing calculators were not permitted on exam two, the experimental group performed significantly better.

Further support for these results was found in the Stout study (as cited in Hart, 1991). One of the two groups involved used the graphing calculator to explore the concept of the derivative while the other was taught in the traditional manner without the benefit of graphics technology. Two five item tests were administered to both groups for which no calculators were allowed. On Test 1, where they had to draw the graph of the derivative given the graph of a function, the calculator group scored significantly higher. On the second test, drawing the function from the graph of its derivative, there was no significant difference.

Tall (as cited in Beckmann, 1988) studied the effectiveness of using computer graphics to develop understanding of the derivative. Using the software programs MAGNIFY and GRADIENT, Tall conducted a study of 112 sixteen year olds. Tall found that the 43 students in the experimental sections had fewer difficulties understanding the concept of derivative and performed significantly better than the control group on stretching the derivative, recognizing the graph of a derivative, specifying a non-differentiable function, and relating the derivative to the gradient and the gradient function. Performance of manipulative techniques was not significantly different for either group.

Ruthven (1990) and Quesada and Maxwell (1992) are two other studies that found significant differences in favor of experimental groups who used graphing calculators in a pre-calculus course. Students' responses to a survey in the Quesada and Maxwell (1992) study also indicated that the graphing calculator was perceived by students to be a helpful tool for understanding the course content.

The collective results of these and other studies are encouraging, but Dunham and Dick (1994) caution that it would be unwise to assume that simply carting a set of calculators into the classroom will have some magical effect on students and that attributing significant differences in achievement to them would be irresponsible.

Other studies have been less quantitative than those mentioned thus far. One, for example, probed teaching philosophies as manifest in use of graphing calculators in the classroom. Another explored the effects of an alternate approach to introductory calculus that highlighted the graphing calculator while a third focused on the connection between symbolic, numeric, and graphical representations. Studies of this sort did not attempt to measure achievement and were less quantitative in their methodology and analysis.

Simmt (1993) explored teachers' expressed and manifested philosophies of mathematics and mathematics education as these philosophies were articulated and worked out in the context of making decisions for utilizing the graphing calculator in their instruction. Six teachers were observed while using graphing calculators to teach lessons on the quadratic function. Simmt (1993) found that the graphing calculators were used primarily to provide graphical images so the students could observe, investigate, and generalize about the transformations performed on the quadratics. The calculators were also used to verify student work. For the most part, the calculators were not used to facilitate and/or encourage students to make conjectures and prove or refute ideas.

The ways in which the teachers used the calculators and their reasons for doing so varied among the six being studied. Pragmatically, the teachers believed that the calculator offered instructional variety and enabled them to generate more examples in a shorter period of time. One teacher felt that his students had more confidence in the accuracy of the calculator and as such it enabled them to work on their own with less need for teacher assistance. All but one of the teachers felt that the calculator motivated the students and two expressed their belief that the students had a better understanding of the concepts in the unit.

The different activities chosen by the teachers in the study, when, how, and how often the calculators were used were all indicative of the existing philosophies. The choices made by two of the teachers indicated a belief that mathematics can be done inductively and that a person can be led to the truth behind the mathematical task. Mathematics was seen by one as a human activity, by another as a process, and still another as a well structured body of pure knowledge. All considered it to be sequential and logical. One teacher made the decision not to include the calculator in further instruction because he felt that students were not doing mathematics when they were looking at graphs.

The McClendon study (1991) is another that did not take a quantitative approach. McClendon developed a graphics calculator study guide to be used in the first term of an introductory calculus course. The guide was intended to emphasize conceptual understanding and to provide calculator problem solving techniques to enhance or replace traditional paper and pencil problem solving techniques. McClendon found that most students were able to work independently on the homework assignments, used the calculators freely on all tests and homework, thought they understood calculus concepts, and felt they were better able to solve problems because they could use the calculator. Student performance on the tests were encouraging and the exit survey showed an increase in confidence and an improved attitude towards the calculator.

In the Hart study (1992) 324 students from 12 institutions were part of an experimental curriculum, using supercalculators, to emphasize the connection between symbolic, numerical, and graphical representations. The experimental students showed a greater facility with graphical and numerical representations and were better able to tie the three representations together than the traditional students. Also, individuals exhibited preferences for certain representations. How they used the calculator was tied to their management of the representations. Furthermore, the data indicated that those lacking confidence in symbolic manipulation tended to use the calculator more readily, that routine calculations done on the calculator were looked at least critically, and that confidence in the graphical information conveyed was tied to having prior information. Hart (1992) also felt that the results indicated that grades were not a good indicator of the quality of the connections among the representations and that technology appeared to affect student learning in a positive way by helping them develop richer concept images. Hart concluded that students had to be guided in the proper use of the calculator in order to receive its positive effects.

The literature, however, is not unanimous in its findings regarding the relationship between graphing technology and achievement in mathematics. Some of the literature available contends that the use of graphics technology does not have a significant impact upon student achievement. Others claim that though it may have no significant impact, neither does it negatively affect skill acquisition. Some researchers claim that studies attempting to link achievement and graphing technology offer little insight into how the technology affects student learning.

Judson (as cited in Hart, 1992), Hamm (as cited in Hart, 1992) , and Hawker (1986) all report no significant advantage for students who were exposed to the computer throughout their instruction. In general, achievement did not improve significantly for students using software capable of symbolic manipulation. Judson noted, however, that the students' ability to understand and apply the concepts was not negatively impacted by their reliance on the software to perform the algebraic calculations and that their motivation to learn the concepts was increased. Hawker commented that the computer as an adjunct to a course with no fundamental revision to the curriculum was bound to result in no advantage to the student.

In other studies, Rich (1991), Shoaf-Grubbs (1992), and Army (1992) found no difference in overall pre-calculus achievement between experimental and control groups (as cited in Dunham and Dick, 1994). Giamati (1991) found significant differences in favor of the control group.

Dunham and Dick (1994) claim that experimental studies attempting to measure and attribute gains in achievement to the presence of graphing technology offer us little insight. Truly isolating the effects of the technology on the students' achievement is extremely difficult to do. It is their belief that research now needs to focus on and investigate things other than achievement. For example, what aspects of graphing calculators bring about improved understanding, what role do multiple representations play in learning mathematics with graphing calculators, what paper and pencil skills retain their importance, can technology impede understanding, and what accounts for the successes and failures in using graphing calculators in learning mathematics (Dunham and Dick, 1994)?

3

CHAPTER THREE

SETTING, SUBJECTS, AND DESIGN & PROCEDURE

3.1 Setting

This study was carried out in September and October of 1996 with one class of level III students in St. John's, Newfoundland.

3.2 Subjects

There was one class of 31 advanced mathematics 3201 students who participated in the study, 19 females and 12 males. The students were randomly assigned by the scheduling program Thesis to either of the three sections offered at the school. One of these three sections was the target group for the study and was taught by the researcher. All students assigned to this particular class were required to secure parental or guardian consent. Provisions were made for students not wanting to participate in the study or without the necessary permission. They were permitted to move to one of the other two sections available if their schedules permitted or to absent themselves from the study, if not the class. One student decided to change program to the easier level III mathematics 3200 course; all others chose to remain. Students were also advised in their permission letters that they were free to opt out of the study, if not the class, anytime throughout the unit. None did so.

Within the target group, four students had their own graphing calculator, fifteen purchased one, and twelve borrowed from the class set available at the school.

3.3 Design & Procedure

The polynomial unit is the first of five in mathematics 3201. It comprises 20% of the year's work. Taking into consideration time out for exams during the year, this unit should have taken seven weeks to complete. It took eight.

During this time students received instruction relative to the objectives outlined in the curriculum guide established by the Provincial Department of Education. These objectives were not necessarily done in sequence nor were they done with the textbook as the primary resource.

The target group used graphing calculators, one per student, throughout their instruction. A manual of thirteen graphing calculator activities, found in Appendix A, was specifically developed for these students to use that paralleled many of the objectives of the polynomial unit. It included a series of tables, questions, and writing exercises that students were required to complete. Each student had his or her own copy of the manual.

The approach taken in the activities was primarily inductive. Calculators were available on a daily basis and were regularly supplied, on a sign out basis, for homework assignments.

The first several classes were spent getting to know each other and the mechanics of using the graphing calculator, such as keying sequences necessary for proper execution of the order of operations. Activity 1 was done collectively by the researcher and the class. Subsequent class time was spent in groups going through the activities from the manual. Written work from the manual was collected and corrected on a bi-weekly basis during the eight week period. A few weeks into the study, an optional Saturday class was held. Its purpose was to allow students to ask questions, catch up, or forge ahead.

In the interest of time, several activities from the manual were assigned for homework so that students would be better prepared to discuss their findings and to arrive at some conclusions within the framework of a group during class time. The last of the thirteen activities was completed in the large class setting with the guidance of the researcher. After the manual was completed, all students were assigned one of four appointment times to come together in a group, of no more than six, to discuss and, hopefully, resolve any concerns, problems, or questions that might have surfaced throughout the manual or the classes in general.

During the course of the eight weeks, individual interviews were held and four quizzes were administered. In the interviews, questions relative to the roots of polynomial functions were posed and discussed. Six of the thirty-one students in the class were selected to be interviewed on a weekly basis. These selections were made based on the students' history in past mathematics courses, their sex, and the aptitude they had displayed thus far. Efforts were made to select a heterogeneous group that was dependable but also represented the varying abilities present in the class. Other students were interviewed as schedules allowed and subject teachers within the school permitted.

All of the interviews were formal and semi-structured. They investigated students' understanding of the roots of polynomial functions and their ability to form links among the symbolic, graphic, and tabular representations of functions. All were audio or video taped.

In addition to the focus on the roots of polynomial functions, this study probed the attitudes of students towards learning mathematics with the graphing calculator. Two surveys were completed by all students in the target group at the beginning and conclusion of the unit. The purpose of the two surveys was to establish a portrait of the class in terms of their attitudes towards the calculator before and after the polynomial unit, to determine if there had been any changes in attitude towards its integration, and if the overall attitude was positive or negative. The final questionnaire also attempted to reveal students' reactions to the calculator and in what capacity they found it to be most beneficial.

Periodically throughout the eight weeks, the class came together with the researcher as a large group, during class time, to review some of the concepts that had arisen in the manual that needed to be explored and discussed algebraically and graphically. The last two weeks of the eight focused primarily on building algebraic skills and using the calculator to verify solutions achieved manually and to show how it could be used to minimize unnecessary algebraic calculations. Specific exercises from the course's text and from previous tests given by the researcher were selected for practice.

At the end of the eight week period, time outside of the regular school day was set aside for students to write a unit test and respond to a written questionnaire. This arrangement was necessary because approximately two hours was needed to complete both. Students agreed to write either Friday evening from 5 to 7 p.m. or Sunday morning from 10:30 a.m. to 12:30 p.m. Two versions of the test were prepared. All test papers and questionnaires were collected at the conclusion of the testing period to protect the integrity of the test and to prevent those writing on Sunday from seeing the sorts of questions that were asked Friday. Two students did not write during those times and a third version of the test was administered four days later. One of the two still did not write at that time even though she was advised of the plan two days in advance. Two additional times were scheduled for this student. She did not show up for either. She received mark of 0 for her unexcused absence. The analysis and summary relative to the unit test will be based on the thirty students who wrote. Also, one of the thirty students who wrote the unit test did not submit the questionnaire as requested. It was never submitted despite several requests. The analysis and conclusion relative to the questionnaire will be based on the twenty nine who responded.

Though the students used the graphing calculator throughout their work on the polynomial unit, the study attempted to investigate, in detail, only a few aspects of the students' experiences. Specifically, this study attempted to answer the questions:

1. Does a unit of instruction which includes regular and frequent use of the graphing calculator, as one element of the instructional approach, help students develop an understanding of the concept of the roots of polynomial functions?
2. Which representation of function, tabular, graphic, or algebraic do students choose to use and work with when determining the roots of a polynomial function? Why?
3. What growth or development, if any, have the students exhibited in their ability to make the link between the symbolic factors of the polynomial equation and its real roots?
4. What do students say about the integration of the graphing calculator into their learning of mathematics that would reveal their attitude towards this instrument?

4

CHAPTER FOUR

ANALYSIS OF DATA

4.1 Introduction

The purpose of this study was to investigate Level III Advanced Mathematics 3201 students' understanding of the roots of polynomial functions, their ability to form links among the symbolic, graphic, and tabular representations of function, to gain insight into the features and characteristics of the calculator that the students found most useful, and to track their attitudes towards this device throughout the unit as a result of the integrated approach with the graphing calculator.

This chapter presents the analysis in four parts. The first begins in 4.2 on page 37 and explores students' initial and final definitions of a root to see if there has been any progress evident. The second part, 4.3, on page 43 gives an overview of student performance on the final unit test and questionnaire relative to the focus of the study on the roots of polynomial functions. The third, on page 62 in section 4.4, presents two student vignettes while the fourth, found in 4.5 on page 87, describes the data as it pertains to students' attitudes towards the calculator as elicited in the surveys, the questionnaires, and in informal discussions.

4.2 What is a Root?

The concept of a root is not a new one for the students in Math 3201. The first unit in 3201 actually builds upon that which was introduced through quadratics in the previous course. Objective 3 of Unit Three from the Level II Advanced Mathematics 2201 course states:

Students will be expected to:

3.9.1 Interpret the x-intercepts of the graph of the equation $y=ax^2+bx+c$ as the roots of the equation $ax^2+bx+c=0$.

(Government of Newfoundland and Labrador Department of Education Division of Program Development, 1995, p. 121)

The commentary for objective 4 says that students should be constantly reminded of the graphical interpretation of the roots. Objective 4 states:

Students will be expected to:

4.2.2 Find the quadratic equation when the roots are given.

4.6.2 Use the discriminant to tell if an equation has two equal real roots.

4.6.3 Use the discriminant to tell if an equation has two unequal real roots.

4.6.4 Use the discriminant to tell if an equation has no real roots.

4.6.5 Use the discriminant to determine the number of x-intercepts of the graph of $y=ax^2+bx+c$.

(Government of Newfoundland and Labrador Department of Education Division of Program Development, 1995, p. 132)

These objectives reveal that students of Math 2201 have been introduced to the concept of a root in a significant way. Instruction, as dictated by the objectives for the course, would have included an exposure to roots that are real and imaginary, algebraic and graphical.

At the end of the second week of classes, after having done some preliminary work with the calculator and the first introductory activity from the lab manual, the students were asked to give a written response to a series of six questions. These questions, included in Appendix B, did not require mathematical solutions, but explanations as to what certain phrases and terms meant. One of these questions asked students to describe what was meant by the term root and to use a diagram to supplement their explanation if at all possible.

In general, there was considerable confusion regarding the meaning of this term. Students did not have an adequate understanding of roots despite their prior experience with the concept. Seventeen of the thirty present that day were unable to formulate any kind of coherent response. Eight of these omitted the question altogether. Others said that it was the origin of an equation, the coordinates used to plot an equation on a graph, when a number can be broken down in multiples, the basic steps needed to overcome a problem, the beginning of an equation, the beginning of a number that when multiplied upon another becomes 2x2=4 or 4x4=16. Others described roots as numbers which when multiplied would give another number, the unknown variable in an equation, or the origin of a number in its lowest form.

One of the common misinterpretations that emerged from these responses is the students' confusion between root and square root. This is not surprising given the fact that teachers regularly ask for the root of a number instead of specifying the type of root desired such as square root. However, this question was asked in the context of five other questions relative to the solving of equations in one variable.

Several of the responses described above reveal a general confusion between the term root in a mathematical context with that of its homonyms which mean source, origin, and beginning.

In addition to those responses described above, five students defined a root to be the number for x that makes the equation true. None of these five provided a diagram, but they did give an algebraic example of a quadratic that had been factored and the roots presented. However, only two of these algebraic examples correctly included zero for one side of the equation which is necessary to actually generate the roots. Some of these five responses described a root as:

The x values which you solve for in an equation.

The value of x when plugged into a given equation the answer is 0.

Another four students described a root in the same terms as those noted above, but they did not offer an example. Like the others, they also neglected to supply a diagram as requested.

The final group of four who answered the question adequately gave a graphical explanation of a root and supplied a diagram. Each of the four described it as the place where the graph crosses the axis though one did say the y-axis and two others qualified their answers by later including that x=0. None of the four offered an algebraic interpretation.

The roots of polynomial functions was a primary focus for the study. Asking for a written definition early in the semester permitted the researcher to assess students' understanding of the concept as the school year began. It also served as a basis of comparison against which to judge later work.

One way in which the researcher was able to gain some insight into the progress, or lack thereof, relative to this concept was a written definition completed by all students in the target group three weeks after the completion of the polynomial unit. Students were given the last ten minutes of a class period to formulate a response. They were not advised in advance that this would be expected of them. The statement presented to the students said:

> Explain, in as much detail as possible, what is meant by the word root in mathematics.

This statement is very similar to the way it was worded at the beginning of the unit and was deliberately general so as to allow students to bring in different aspects of roots if they so desired.

All students in the target group were present and gave a written response. None said they were unable to provide an answer as eight had done previously. All, but three, supplied a diagram of a polynomial function with x-intercepts indicated and labeled as the roots. The three that neglected to include a diagram, did describe a root in graphical terms as the place where the graph cuts, crosses, or touches the x-axis. Two students included a specific example of a quadratic function with its roots listed and labeled on a graph. All other students answered the question in general terms, with some giving more detail than others.

Somewhere in their answer, fifteen students described the root symbolically in terms of its equation by saying:

A root is the value(s) of x in an equation that make it equal to 0.

The number that makes the equation true.

This is where x will have a value and y will be 0.

It is the value of x which causes y to equal 0.

The solutions or answers to an equation.

The values of the factors when they are equal zero.

Is the solution of an equation.

Nine of those in the target group more accurately described the place where the graph cuts the x-axis as a **real** root. Eighteen students included in their explanation, a comment to the effect that roots could be imaginary as well as real. One remarked that there has to be an equal number of them, another two said that they could be identified by the presence of the letter i, twelve observed that they could not be visible on the x-axis, and eight said that they are solutions for the algebraic equation that make it true. Some of the other comments about a root were that it was synonymous with the word zero, that the number of roots was determined by the degree of the equation, and was also related to the number of turns in the graph. Three mentioned single, double, and triple roots as types of real roots and one of the three illustrated the concepts with diagrams. Three in the group also recalled that the real roots could be further classified as rational or irrational, though they did not pursue the idea by describing what either meant.

One student from the class included in his response a definition of root as it pertains to the numeric phenomenon square root. He did, however, also define root in graphical terms. One interesting comment from the group was that the if the roots could be determined, then an equation for the graph could be found.

One student revealed her difficulty with the idea that imaginary roots cannot be represented graphically. She described them as being located at the critical values of the polynomial function, or where the turns occur. The only other difficulty with the concept of root that surfaced in this activity involved identifying the roots from the graph. One student described the root to have the opposite sign to what actually appeared on the graph. For example, if the graph intersected the x-axis at 3, then, according to her, the root was actually -3. This confusion resulted, in all likelihood, because of the work done with the factors of polynomial functions wherein if the factor was (x+3), then the root that resulted from letting the factor equal zero was -3. Though there was no evidence of this confusion on the test three weeks previous, none of the questions specifically depicted a polynomial graph from which the roots would have to be listed. The closest question of this sort gave a quartic graph with two single roots and one double root that had to be used to identify two possible equations whose graph might look like the one supplied. The student mentioned above answered the question correctly on the exam.

In general, the students seemed to have acquired an adequate working definition for root. Most were mathematically correct and indicated that they understood the concept both in terms of its algebraic and graphic components. Some of the responses were quite brief and certainly could have included more detail. The vagueness of the statement and the ten minute time limit might have contributed to this result.

4.3 Student Performance

4.3.1 Unit Test

In general, results from the unit test, found in Appendix C, indicated that the majority of students had achieved most of the objectives set forth in the curriculum guide. The class average was 78 with a median of 78.5 and a mode of 86. The standard deviation was 12.3. A stem and leaf plot of the marks for the unit test can be found in Table 1 on the following page.

There were no restrictions on the use of the calculator throughout the test. One question in the second part, however, specifically requested an algebraic solution as opposed to one derived on the calculator. Students were advised that if they chose to solve one of the problems in the second part with the graphing calculator, they had to carefully explain the steps taken to arrive at the answer.

Table 1

Stem and Leaf Plot of Marks for Polynomial Unit Test

						9		
						9		
						8		
						6	9	
						5	6	7
					9	5	6	7
					9	4	6	4
					8	4	2	4
					7	1	2	2
			2	4	3	1	0	0
1	2	3	4	5	6	7	8	9

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For eight weeks students had regular access to and practice with graphing calculators while exploring polynomials. The description that follows provides some detail regarding their performance on those questions relative to the focus of the study.

The first question in the objective section of all versions of the test presented a polynomial that could not be factored using any of the standard approaches. Students were required to use the TABLE feature of the calculator to identify the factors of the polynomial. Responses offered insight into students' ability to make the link between the symbolic factors and its subsequent roots.

Twenty four of the students correctly identified all three factors for the polynomial function while six did not get the answer completely correct. Three of these six gave the roots, not the factors. Two gave two of the three correct factors and one student either misread the TABLE or made a keying error when entering the function.

The second part of that question asked for the ordered pairs used to identify the factors. Of the five that answered incorrectly, the most common error was to reverse the ordered pairs or to give only two of the three pairs required for a correct answer. The vast majority of students showed that they were able to interpret the function written symbolically and in tabular form.

The fourth question or 2b asked the reader to identify the number of imaginary roots indicated by the given graph. Sixteen students answered correctly. The major source of difficulty for this problem seemed to be that students were unable to correctly recall the relationship that exists between the number of turns and the degree of a polynomial function which would then permit them to subtract the number of real roots from the total number of roots. Some of the fourteen that answered incorrectly did not have a clear idea as to what a single, double, or triple roots looked like which in turn prevented them from subtracting the correct number of real roots from the degree of the polynomial to get the number of imaginary roots. Some also appeared to think that imaginary roots could be found at the critical points of the graph if the turn was completely above or below the x-axis. In general, many students exhibited confusion with this aspect of a root.

The sixth question on the test paper, number 4, asked the reader what change they would make to a given quadratic function so that it would have two real roots. This question attempted to test students' understanding of what a real root meant graphically. Twenty four answers were correct. Of the six remaining, the majority gave an answer that was too general. Most of the answers considered unsatisfactory made a general statement without giving specifics. For example, one said he would change the 2, without saying which two or what change he would make.

The variety of responses that students were able to suggest, made this problem quite revealing in terms of the students' grasp of the concept of a real root. The researcher expected that students would alter the constant. However, recommendations were made to change the degree of the polynomial, to change the sign on the leading coefficient, to change the linear coefficient, or to change the constant term. Students' responses showed clearly that they understood, in graphical terms, that a real root was the same as an x-intercept.

The tenth question, number 8, focused on the relationship between the discriminant of a quadratic equation and the number and type of roots that it would have. Of the ten that gave an incorrect answer, the problem seemed to be that they did not know that a positive number, a negative number, or a zero for the discriminant revealed information about the number and type of roots that the quadratic would have. A few of the ten were not specific enough, saying something like all real roots and others had the relationship among positive, negative, and zero discriminants confused. These students seemed to have a vague idea what was required but they could not recall the specifics. One, for example said two equal real roots.

Though previous questions indicated that students, in general, knew what a real root was, only two thirds of the class knew the special relationship between the discriminant of a quadratic and its roots.

In question 9, students had to supply a polynomial equation that would have a triple root at a specified number. Of the two students who gave an incorrect response, one made a mistake when putting the answer in polynomial form, which was not required, and the other had the degree correct, but the wrong factor. This question indicated that students were easily able to make the connection between the appearance of a polynomial graph and its algebraic equation.

In question number 12, students had to infer that if (x-5) was a factor, then 5 was a root. This information was in turn used to solve for k by substituting 5 into the given equation. This question, like number one, tested students' grasp of the link between symbolic factors and subsequent roots. Only three gave an incorrect response while one left it blank. Those that answered incorrectly were careless with the signs and/or calculations and consequently ended up with the wrong value for k.

The most common error for number 13 was neglecting to put an exponent of 2 on the factor that represented a double root as indicated by the given graph. Among the other typical mistakes, were reversing the signs in the factors or giving specific values to the roots labeled as -a, -b, and c. In general, however, students were able to read the graph, identify the roots, and generate the corresponding factors; further evidence of their ability to work with functions expressed symbolically and graphically.

The final question from Part A asked students to describe and explain what they knew about one of the roots of a function represented by a table of values. Fourteen of the thirty students who wrote the test did not receive full marks for this question. Six of these answered only half of the question by neglecting either to explain **why** or **what** the table revealed. One student was unable to express herself coherently and another five were apparently completely unable to interpret what the numbers in the table indicated about the roots of the function because they either left it blank or said the function would have to have an imaginary root between 2 and 3. This question indicated a fairly common problem among the students. Only slightly more than half of the class was able to correctly interpret the function expressed in tabular form and provide a complete and coherent answer.

All three versions of the test had four questions in the second part that required detailed solutions as well as a final answer. In general, the average mark for this part of the exam, as a percent, was 81.3 for the class, indicating that the students had a slightly greater degree of difficulty with the first part of the test than the second.

Twenty three students of thirty wrote version two of the final test. The first question in Part B of version two asked students to find the specific equation of a cubic function given a double root at -4, an ordered pair on the x-axis, and an additional point in the first quadrant. Fourteen students answered the question correctly. Six made minor errors such as leaving out a negative sign while finding the leading coefficient. One student performed addition instead of multiplication when determining the leading coefficient and another wrote the factors for the double root with the wrong sign despite the fact that the other factor was written properly. The solutions for two different students indicated that they were able to find all three factors for the polynomial, but had no idea that they were supposed to continue and find a specific value for the leading coefficient. All of the twenty three students who wrote this version of the test demonstrated a knowledge of the link between roots and their corresponding factors. The same number were also able to interpret information about the third root written in tabular form and use it to generate the third factor.

All versions of the test included a quartic equation written in polynomial form, for which students had to find all the real and/or imaginary roots. None of the quartics were factorable by using the standard methods. Students were expected to present an algebraic solution.

Twenty three of the students had a perfect answer or a minor mistake such as an error with signs, an incorrect use of brackets in the workings, or a mistake in the final statement of the solution that involved the absence of a negative sign that should have been included. More serious problems included an arithmetic error in the synthetic division that went undetected even though the calculator should have helped the student pick up his mistake. A couple of students did not go beyond finding the first root because it did not occur to them to check to see if it was a double root. The appearance of the graph on the calculator should have alerted them to this fact. This, in turn prevented them from finding the two remaining roots that were irrational. The final difficulty occurred because a few of the students were unable to factor the polynomial by grouping once one of the roots was identified. Only two of the thirty had no idea how to approach the problem.

In general, a large number of students were able to take the symbolic expression of the function, change it to a factored format, and generate the roots. This algebraic procedure was simplified by the graphical display offered by the calculator.

The second question that had to be done by all students was to find the point of intersection of a cubic and a linear function. Twenty five of the thirty students achieved a perfect or near perfect score on the question. Nine of these lost part of one mark because they did not provide enough detail in their solution. They neglected to describe that they had to change the window in order to find the point of intersection where one of the curves was tangent to the other. Students that received less than four out of a possible five for the question found only one point of intersection, entered an incorrect equation into the calculator, or made an error recording the quotient after using synthetic division which, in turn, resulted in an incorrect solution for the system.

This question did not specifically reveal anything about students' understanding of roots. It did, however, give some indication how students prefer to solve problems if given the opportunity to choose. There was no prescribed method for solving this problem and all but one did so by using the graphing calculator.

Two versions of the test required that students find two different quartic functions given three of its four roots. This question caused more difficulty for the students than any other. Thirteen of the students scored 3 or better out of 4. The primary source of difficulty for the other students was that they were unable to correctly apply the formulas for the sum and product of the roots to find the quadratic with the irrational roots or the imaginary roots. Two students tried to find the quartic in one step using the formulas for the sum and product of the roots and incorrectly assumed that there was no quadratic or linear term in the polynomial. Two students gave only one quartic in their solution while others forgot to give the graphs that were required. Several students lost marks for the graphs because they did not indicate the correct number of real roots. This problem would have been overcome if they had changed the window or zoomed in on the graphs in order to see more clearly that some of the roots were very close together. One student found two of the quadratics individually but did not know how to put these quadratics together to find the quartic. Three students identified the wrong factors for the irrational roots. They reversed the signs even though they had no problem creating the correct factors for the rational roots.

If the roots of a polynomial are integral or simple rationals, like ½, then students have no trouble generating the symbolic factors. However, if the roots are complex or irrational then students experience more difficulty.

The first version of the test had a final question that involved finding the value of the linear coefficient of a cubic given that it had a double root. Four of the six who wrote this test had a perfect solution. One of these took a trial and error approach until he found the value of m to be 24. The others used the formulas for the sum and product of the roots to create a system of two unknowns that they solved by substitution. Another took this same approach but made a minor error when she substituted 2r for s instead of -2r. The sixth student in this group had nothing written for this problem because he had no idea where to begin. Though he was advised he could stay for a few extra minutes to complete the problem, he elected to leave.

The final question on version three of the test asked the reader to find a cubic polynomial whose roots were two less than the roots of a given polynomial. The student who wrote this exam, correctly identified the roots of the given polynomial as -4, 2, and 2. However, when finding the polynomial whose roots were two less than these, she made an arithmetic error and said that 2 less than -4 was 6 instead of -6.

4.3.1.1 Summary of Students' Performance

In general, the majority of students seemed to know that a real root is an x-intercept, were able to identify the factors of a polynomial by looking at its table of values, and were able to modify algebraic representations of a function in order to cause its graph to change in specific ways. They were also quite successful creating an equation for functions having single, double, and triple roots at specified numbers. The majority were also able to make the connection between roots and factors. This knowledge, subsequently, permitted them to perform substitutions and solve for missing coefficients. Answers in the second part of the unit test also indicated that students used the calculator regularly for problems requiring algebraic solutions and exclusively when permitted. They used the calculator for the insight it offered into some problems so that unnecessary, busy work could be kept to a minimum and also to verify their algebraic solutions.

Problem areas in the test were most evident because of the number of students that answered incorrectly and the type of answers presented. Difficulties surfaced in the question that asked for the number of imaginary roots indicated by a graph. Almost half of the class answered incorrectly which indicated that several were unable to identify the difference between the appearance of single, double, and triple roots. Though most did subtract the number of real roots from the total number of roots, they arrived at the wrong solution because they miscalculated the number of real roots. This same question revealed some confusion about imaginary roots and whether or not they were visible on the graph of the polynomial at the critical points. Though this was not a prevalent misconception throughout the class, it might have been useful to use the TRACE and/or MAX/MIN features of the calculator more regularly to show students that imaginary roots were not represented as part of the graph as indicated by the numbers at the bottom of the screen describing the location of the cursor.

The final two questions from the first section of the test also indicated some problems. Even though the students should have recognized the graph as a quartic, one fifth of those who wrote gave a cubic polynomial for the answer by leaving out the exponent of two on the binomial that represented the double root. This error seemed to be based more on carelessness than conceptual confusion. The final question that resulted in less than a perfect score for almost half of the class was the one that required students to interpret a table of values. Lack of attention to the fact that the question was asking for two things and the inability to coherently explain their interpretation were the chief reasons that students did poorly on this question. Many only answered half of this question.

For the second part of the test, students were successful in finding the equation of a specific cubic, identifying points of intersection of two curves, solving for all real and imaginary roots for a polynomial, and finding the missing linear coefficient when told that polynomial had a double root. They were least successful with the question that required them to find two possible quartic polynomials given two rational roots and two irrational roots or two rational roots and two imaginary roots. In general, this question was done poorly by just over half of the class for two reasons. First of all, many were not able to correctly apply the formulas for the sum and product of the roots. Others made careless mistakes with the signs in the factors, did not provide the two answers that were required, or were careless with the graphs by not indicating the correct number of real roots. In general, successfully finding the symbolic form of a function declined as the roots become more complex. Students worked well with functions that were represented symbolically, graphically, or in tabular form. However, they experienced a greater degree of difficulty if the roots provided were non-integral.

4.3.2 Final Questionnaire

For approximately the last hour of the two hour period allotted for the unit test, all students completed a questionnaire and an attitude survey. One student of the thirty who wrote the unit test did not submit the questionnaire. Consequently, the analysis and conclusion will be based on the remaining twenty nine.

The first seven questions from the questionnaire posed specific mathematical problems that did not necessarily require a mathematical solution but a written explanation as to how the problem could be solved and how it could be verified.

The first problem did not require a mathematical solution, but an explanation as to how a student might check to see that the quadratic he or she had found, had the two irrational roots given in the statement of the problem.

Answers on these seven questions were categorized as 1, 2, 3, or 4. A 1 was a good answer and for the first question and would have to look something like:

> Once the quadratic equation was determined, plug the equation in y= and then check 2nd TABLE and see that y=0 when $x=1\pm\sqrt{2}$. That will show that these are real roots since they cross the x-axis.

If 2 was assigned, then it indicated that the student had the right idea but did not provide enough detail. An answer of this sort for the first question would look like:

I would graph my written solution and then go to CALC/ROOT

This particular student was assigned a 2 because she did not say what she would be looking for and how she would know if her solution was correct.

If a student was given a 3 for his or her answer, it meant that he or she had the right idea about how to do the problem but made an algebraic error during the process. An error of this sort would be something like not following the order of operations when solving an algebraic equation. Only two of the questions, numbers 3 and 5, would warrant a rating of 3 because only these two required a mathematical answer as opposed to a written explanation.

A rating of 4 was reserved for those students who left the question blank, provided an answer that was incorrect, or who were too vague. The following is one such example:

I would graph the equation and then analyze the graph.

Table 2, below, depicts the numbers of students whose answers were coded as 1, 2, 3, or 4. These codes were assigned to the first seven problems on the questionnaire.

Table 2

Numbers of Students Whose Answers Were Coded as 1, 2, 3, or 4.

Question	1	2	3	4
1	18	5	N/A	6
2	13	13	N/A	3
3	15	14	0	0
4	20	8	N/A	1
5	7	7	3	12
6	13	11	N/A	5
7	7	22	N/A	0

Four of the six students who rated a 4 on question 1 left it out altogether. Another gave an answer that was far too general and the final student had a misconception as to what an imaginary root was. This particular student interpreted $1\pm\sqrt{2}$ as imaginary rather than irrational.

Almost four fifths of the class chose the calculator as a means of verifying their work even though it could also have been done algebraically using the quadratic formula. They demonstrated that they knew how to use the calculator effectively for a problem of this sort. As many chose the tabular representation of function to verify the roots as those who chose to use the graph and the ROOT program under the CALC menu.

The second question asked students how they would know for sure if the given function had a positive real root. Thirteen students referred, in some way, to the graph crossing the x-axis, but did not say on what side. They rated a two. Answers of this sort read:

> Because it is of an odd degree and therefore must intersect the x-axis at some point.

This answer indicated that they had the right idea, but were not specific enough. Twenty-six of the twenty-nine responses showed that students did know that a real root could be seen on the graph as an x-intercept.

The only problem evident in the answers for the third problem was that all fourteen students either did not read the instructions carefully enough or they misinterpreted what was being asked. All fourteen neglected to actually state the point or points of intersection even though they did explain, correctly, how to find them.

Twenty of twenty-nine answers for number four were satisfactory. Eight others explained what they would do on the calculator but did not say exactly what they would be looking for. For example, one said:

> Enter the equation in the y= list.
> Go back to the homescreen and press VARS.
> Go the y-vars and press 1 then 1.
> Enter y(2).

Though this particular student had a clear idea as to how to proceed, he did not say what he would be looking for when he entered y(2). One student gave no response at all. The vast majority of students made the connection between a symbolic factor and its corresponding root.

The fifth question presented the greatest difficulty of all problems on the questionnaire. Those answers that rated a two did so for a couple of reasons. Most students commented only on the second part of the question saying that they would check their answer by using the CALC/ROOT option or the TABLE feature. Others described correctly how they would use the vertex form of a quadratic and the calculator to find the equation, but they did not actually find the function as required.

Three of the students whose answers rated a three for the fifth question made the same error. In the process of finding the quadratic described, they incorrectly applied the order of operations, thus arriving at the wrong value for the leading coefficient. However, one of these students went on to describe how she would verify her work while the others seemed to recognize that that their solution was incorrect, but did not know how to correct it.

Twelve answers for problem five rated a four. They did so for several reasons. There was no explanation included as requested, the question was virtually untouched, or there was serious algebraic errors in the presentation of the solution. The remaining answers categorized as fours were so general in their descriptions of how to verify the solution that they had to be considered unsatisfactory. For example, one student described checking the vertex using the MAX feature but said nothing about checking the roots. Another couple of students went astray when they used an incorrect version of the vertex form of a quadratic function.

Almost half of the class did not recognize that sketching the graph manually and using its symmetry would have helped them find the other root. There was a definite confusion among the students as to how to use the tabular information given to sketch the graph and to generate its algebraic equation.

In question 6, five students of eleven were given a 2 when they said they would substitute the root into the equation to solve for m, but they incorrectly identified the root as 2 instead of -2. However, previous problems on the questionnaire and from the test relative to this concept were answered correctly, indicating that this particular error resulted from carelessness rather than from a conceptual misunderstanding. Other students' answers that rated a 2 were not specific enough as to how they would check to see if their value of m was correct and what they would be looking for on the calculator. For example, one student said:

> Put the equation in y= and graph.
> See if that graph has the same roots by using the TABLE feature.

A better answer would have said something about looking in the table for the point (-2,0).

The final question that required a mathematical solution was number seven. Twenty two answers rated two. Seven rated one. All twenty-two students had the same problem with this question. They were not specific enough as to how they would verify their work. They neglected to say that they would check the roots **and** the y-intercept, both of which would have to be correct in order to know that they had the only correct answer.

4.4 Student Vignettes

All students in the target group were interviewed at least once, many students twice, and six students a total of four or five times each. Of these six, two were chosen, one male and the other female, as subjects for the student vignettes that follow. For the sake of convenience they will be referred to as Mark and Amy. All statements will be preceded by either an S or I to denote who is speaking, S for student and I for interviewer.

The vignettes describe the first and last interviews conducted with these students in an attempt to reveal what progress, if any, has been made with regards to the concept of the roots of polynomial functions. Each vignette is followed by a brief analysis of the interaction between the interviewer and the student.

4.4.1 Mark

Mark is a seventeen year male whose recent history, in terms of grades, is quite respectable in the high school mathematics courses. He has completed the last two courses of the high school program with a low A in each. He was willing to be interviewed and did not appear to be overly nervous.

Before Mark attempted to answer any of the established questions for the first interview, he was told that he could start anywhere after he read aloud the problems listed. Also, he was informed that there was no prescribed method that had to used for any of the questions. He was free to answer the questions algebraically, numerically, on the calculator, or by trial and error if he thought it might be appropriate.

4.4.1.1 First Interview

The questions were hand written and offered to the student.

1. Given the equation $13x-41y=22$, what are the x and y-intercepts?
2. The polynomial equation $y=x^3-4x$ has 2, 0, and -2 as its roots. True or false. Support your answer.
3. Approximate the real roots of $6x^8-19x^4+10=0$

He read them aloud and said he did not know what to do with the first one. He decided to attempt the third. After he explained that the question meant to find the x values, Mark tried to factor. Factoring was not as easy as he thought because he did not know what to do after he took the x^4 out of the first two terms.

A lengthy discussion ensued regarding what an equation in factored form would look like. Though we did arrive, inductively, at a consensus that a factored expression would have to have series of brackets and/or terms written side by side with no connecting operational symbols, Mark realized that he did not know how to proceed with the specific equation given.

I: What do you normally do when a method you've chosen is causing you difficulty?

Do something else he responded as he picked up the calculator and entered the equation in the y= list. Nothing came up on the screen when he pressed GRAPH so he zoomed out. Again nothing happened.

I: What are you going to do now?

S: See if there's anything in the table that will give me the numbers for x... probably will have to look for y=0.

I: Why would you look for the y to be 0?

S: Because when y is 0 that means that x is on the y intercept.

His use of graphing terminology was confused so we spent several minutes talking about a better way to describe some of the critical features of a graph. We tried to clarify root, x intercept, y intercept, and axis.

Mark continued using the TABLE feature but the numbers in the second column got progressively larger in the negative direction as he kept his finger on the down cursor. He briefly suggested that maybe the ZERO feature or the TRACE feature would be more helpful. After some thought, however, he said that he would need to see the graph first in order for these features to be effective. This had been the problem all along, he could not get the graph to appear on the screen because he did not know how to alter the viewing window or even that he should. His next choice was to use Tblset which allows the user to control the independent variable in the TABLE. However, because he could not see the graph he had no idea what value to enter. He was stumped.

Unable to proceed with this problem, I suggested we try number two. Laying down the calculator, he picked up his pen. He said:

S: Well this one when $y=x^3-4x$ has roots 2, 0, and -2. I can put in those roots for x and see if it equals 0 on the y side.

I: Good. Go ahead.

His work confirmed what he suggested when the first two values satisfied the equation. He had some trouble with the negative sign in the last example so we discussed the difference between -2^2 and $(-2)^2$.

I: Any idea how you might do this problem on the calculator? As he entered the function in the y= list, he described a root as where the graph crosses the y intercept.

I: You keep saying that.. where the graph crosses the y intercept.

S: where the graph crosses the x intercept

I: crosses the...

S: x axis

I: The place or point is called the x intercept but your graph crosses the x axis... say it again.

S: So a root is where the graph crosses the x axis.

The same problem that caused Mark to abandon the third problem recurred. He saw only one of the three roots on the screen. He decided to use TABLE again and was able to describe the numbers in the list as all the points on the graph. We spent several minutes discussing how to get the numbers in the TABLE that we wanted by using the Tblset feature. Eventually, he got the three values 2, 0, and -2 to show 0 for the y coordinate by setting -2 to be the initial value and the increment to be 2.

However, he still was not able to get the graph to show these three roots so he decided to try the TRACE feature. I needed to tell him to keep his finger down on the cursor button. The graph slowly panned to the left so he was able to see that the graph eventually intersected the x axis in three places. But, because the cursor did not land directly on the x axis, as indicated by the coordinates at the bottom of the screen, Mark decided to use the ZERO program under the CALC menu to find the roots. All three were confirmed.

I: What did you learn about the graph when it took you so long to find these roots?

S: I don't know...that it didn't all fit on the screen?

I: That's a good way to describe it. It didn't all fit on the same screen.

After some prompting, guidance, and exploration, Mark said that he would put the calculator back on the standard screen. After doing so, he was able to get a global image of the function. Asked what he had learned through this exercise, he was not quite able to articulate the importance of controlling the viewing window for different functions.

I: Now having done number 2, any idea how you might do number 3?

The quick response was no.

When asked again what a root was, Mark said it's the value for x. Asked for another interpretation, he said the x intercept.

I: What is the difference between question 2 and question 3?

S: That one gave you the roots.

I: Right. This one gave you the roots. This one said **you** find them. What are you going to do with number 3?

S: Put it back in y= ..and ..it probably wasn't in standard before.

Mark seemed to realize that the source of his difficulties before might have been the numbers used in the viewing window. The interview concluded with a brief explanation of how important controlling the window is when looking at the graph of a function so as to get a better idea of how it looks globally. Due to time constraints, question 1 wasn't addressed.

4.4.1.2 Fifth Interview

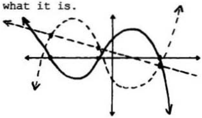
The fifth interview had four questions:

1. Tell me as much as you can (as much as possible) about the roots of polynomial functions. Be as specific as possible.

2. Is it possible for a polynomial function to have no rational roots, but still cross the x axis? Does $y=x^3+x^2-3$ have any rational roots? How can you tell?
3. There is one thing that the system, in red, has in common with the green cubic. Tell me what it is.

Red (---)

Green (▬)



4. How many roots does $y=-3x^2+5x+7$ have? How do you know? What are the roots? Name a second quadratic with the same roots.

The interview began with Mark reading aloud the first problem. He was asked to recall everything he could about roots by thinking back over the classes and labs that he had completed throughout the unit. I recorded his responses.

S: They're the parts on the graph that cut the x axis.

S: That means the y equals 0.

S: The highest degree of the polynomial, let's say 3, means there are 3 roots.

S: Um.. the shape of the graph of a parabola has two roots.

S: A straight line or a diagonal line is one root.

S: Like an s, three roots.

S: Three roots means two turns... Or however many roots there are, there is one less turn.

S: There are real roots and imaginary roots.

I: Can you tell me anything about either category or both categories?

S: Real roots are actually on the x axis.

S: And imaginary, they don't.

I: Then what are they?

S: Um.. well they're negative numbers, well not really negative numbers but …

I: How do you identify imaginary numbers? How do you know if that's an imaginary number or not?

S: Well let's say there are three roots and it cuts the x axis in one spot like a straight line, then there two other roots so they're imaginary.

I: So you're talking about something a cubic that would look something like this?

S: Yeah.

I: So I know there is one real root and two imaginary roots but what exactly are imaginary roots?

S: fake numbers sort of

I: It has to have what in it for me to say, yes, that's an imaginary number?

S: i.

This dialogue was followed with an explanation for several minutes, by the interviewer to Mark, about what an imaginary root represented. The equation $x^2+1=0$ was examined algebraically and graphically in an attempt to explain what the solutions $\pm i$ represented.

The dialogue then reverted back to looking for more details about roots.

S: Imaginary roots have to have an even degree...there can't just be one or three.. there has to be two or …

I: Do you know why there has to be even number?

S: Because of the plus and minus.

A brief explanation followed about the consequences of systematically finding the roots of a polynomial until the final quotient was a quadratic. The quadratic formula would then be applied to yield two imaginary roots, never one or three. The student's facial expression and subtle comments indicated that he seemed to be following the explanation.

After a brief recap of some of the points mentioned about roots thus far, I asked for additional ideas. Mark responded after a lengthy pause.

S: Negative b over a is the product, no sum of the roots.

I: If you remember that one, you must remember the other one.

S: c over a

I: You're the first one to remember these formulas so far. There's one other little stipulation about this one.

S: c is not always c if there's about ten numbers, then c will be the last number.

I: OK. There's something else about the degree...if the degree is even...? What about if it's odd?

S: It's negative.

At this point the student said he could not recall anything else about roots. I responded by asking if there were different types of real roots. After a considerable pause the student said he did not know. I suggested to him that he might look over activity number twelve that night. He responded by saying that he guessed that the answer I was looking for was yes.

In an attempt to draw more information from the student and to see if he was familiar with certain other aspects of real roots, I drew a diagram and asked Mark to describe the three types of real roots illustrated. He responded correctly by referring to each intercept as either a single, double, or triple root. Successful with the graph, Mark was asked if he could tell what kind of real roots might be indicated from an equation. Again his answers indicated that he was able to identify single, double, and triple roots in an equation based on the degree of the factor.

The third question of the interview asked Mark to identify what the system had in common with a cubic. He studied the diagram for eight seconds then said

S: The point of intersection is the same as the roots.

I: What part of the point of intersection?

S: The x.

I: Right. The ys are certainly not the same, are they because the ys are..

S: That would be -3, or whatever, and that would be 0.

I: So, the x coordinate here is the same as ?

S: The root.

Mark had no problems with this question so we moved onto the next. Asked how many roots the function in number 4 had, his quick response was two because its highest degree was a 2. He then had to find the roots. He immediately picked up the calculator, entered the function in the y= list, and went to the TABLE.

I: You went to the TABLE first. It wasn't helping?

S: No. It wasn't showing 0 in the y side.

I: You do know that you can get it, don't you? How can you get it?

S: Tblset, but that's too much trouble. It's just as well to go to ZERO.

He read off both answers using the ZERO feature. Though he was able to describe the roots received as real, he did not recall that they could be rational or irrational. He smiled and said that he was supposed to find that out for homework that night.

The final part of the question asked him for a second quadratic with the same roots. Using the digits that the calculator yielded and rounding them off, he formed a new quadratic, in factored form. A brief discussion followed about the possibility of finding a second quadratic using the original function in polynomial form. He needed to be prompted into saying that all terms would have to be divided or multiplied by the same value instead of just the leading term. The interview then concluded with a reminder to review activity twelve to determine the different types of real roots.

4.4.1.3 Analysis

The earlier interview with Mark revealed three things. The first was that the link between the symbolic and graphical representations of function was tenuous at best. His thinking about roots was in symbolic terms. He described finding a root algebraically when he said it means finding the x values. However, three weeks previous he had described the root as the origin of an equation, so there had been some progress in this regard. Later in the same interview he repeated this definition during the last question attempted. He said that a root was the value for x. His perception of a root in these algebraic terms was also emphasized twice more when he chose to substitute the values -2, 0, and 2 into the function to see if it yielded a zero for the y and when his first effort to approximate the real roots for the third question was to try factoring. He did not seem to know why this was appropriate, only that it was something he was used to doing.

Even though Mark had a calculator at his disposal for this interview, his first inclination was to solve the problems presented by algebraic means. However, when he did decide to use this device, or it was suggested to him, he was uncertain about some of its features. At this point in the study he had had access to a calculator, during class time, for five weeks. He had to use it to complete the tables in the manual and to answer some of the questions that followed. Despite this, he had considerable difficulty getting the graph to appear on the screen in some kind of identifiable form. He had little idea how to control the viewing window and what features he should use when.

Mark's seeming indifference to the calculator and his lack of fluidity when using it, suggest that it takes a considerable period of time for him to use it efficiently and effectively.

The third revelation that emerged from this interview concerned the student's use of the language. He had difficulty articulating what a factored expression would look like and struggled with his definition of a real root throughout the interview. This was not a surprise given the fact that the physical and pedagogical structure of most math classes encourages exchange primarily between teacher and student. There is little opportunity for language development.

The last interview conducted with Mark indicated that he had made progress in terms of his ability to connect the symbolic, graphical, and tabular representations of function. His initial comment about roots was that it was the parts of a graph that cut the x axis which also means that the y has to be 0. Later he described real roots as actually being on the x axis while imaginary roots were not. He was also able to identify single, double, and triple roots from a particular equation and could generate a second quadratic, in factored form, that would have the same roots as that which was given. Furthermore, when using the calculator he immediately accessed the TABLE feature to look for the number 0 in the y column. He was able to work with functions in symbolic form, in graphical form, and as from a table of values.

His use of the calculator was efficient and fluid. He was able to decide quickly what method to use for different questions and was able to adjust himself accordingly when a particular approach wasn't suitable. This was evident when he had to find the roots for the quadratic and chose to use TABLE, but quickly decided against it when 0 did not appear in the second column. Without hesitation he claimed that the Tblset feature was too troublesome to use and immediately computed the roots using the CALC/ZERO program.

He was slightly behind where he should have been in terms of his ability to categorize real roots as rational or irrational. He also did not have a clear understanding about imaginary roots, how they are represented, and what role they played algebraically. However, his use the language or terminology had improved, his answers were said less like questions, and his responses were less hesitant. Mark had an air of confidence about him, seemed comfortable with the questions asked, and realized that he needed more work with a few of the concepts.

4.4.2 Amy

Amy is a seventeen year old female with a friendly disposition and a relatively strong background in math in terms of grades; she finished the previous two courses with a low eighty in each. She was quite willing to be interviewed and was enjoyable to work with. She was enthusiastic and had a habit of saying oh yeah when something became apparent to her. She relaxed early into the interview and appeared to forget she was being recorded.

4.4.2.1 First Interview

Following the preliminary preamble of getting to know something about her educational background, her friends at school, and a little about her family, the interview began. After reading aloud the three prepared questions, she was told that the problems could be done in any manner and in any sequence. With a slight hesitation she choose the third. Shortly afterwards she picked up the calculator and said that she did not know how to do it on paper because the numbers were too large. We discussed the relationship between the degree of a polynomial and the number of roots. When Amy realized that the problem had eight roots, she laughingly said that she did not think she should begin with this problem after all. I convinced her to persevere so she picked up the calculator again, entered the equation in y=, and graphed the function. The appearance of the graph was jumbled together. I asked her if she could change the screen in some way so she decided to zoom out but found that it went tighter together. Zooming in again brought her back to where she started. She concluded that zooming in would not help so I encouraged her to do so once more. Though she was satisfied with the graph, she was disappointed not to see any zeros in the y column when she accessed the TABLE feature. She wasn't sure how to proceed. She asked if it was OK to try something else like TRACE and found one root to be around .904. She said:

that's the closest because it jumps

We discussed characteristics of graphs that were nearly vertical around the root and how very small changes in the x produced large changes in the y. Going back to the TABLE, Amy learned, through guided discovery over the course of several minutes, how to control the values displayed by using Tblset. Entering the initial value she found using the TRACE option, and changing the increment to smaller values to allow better approximations for the root, she arrived at a correct answer. She seemed excited by her discovery and repeated the process until a second root was found around 1.259.

S: Do we have to keep going and get eight of these?

I: How many times does it cross the x axis?

S: I think it was 4 on the graph.

I: OK. So what does that mean to you?

S: That there'd be four roots.

I: Four ___

S: Four real roots.

Reference to the morning class that day and our discussions about different types of roots, helped her to remember that there could also be imaginary roots. We discussed how one might tell from the graph of an equation like the one entered, how many real and imaginary roots there would be. Since she had already located and identified two of the real roots, it was decided not to continue with this problem.

Amy chose to try number 2 next. She picked up the pencil and said:

S: I'm just gonna try it.

Changing the f(x) to zero, she was unable to explain why.

S: But it's just how I know how to do it.

I: Why did you let that y=0?

S: Because the y=0 where the x...where the line cuts the x axis...OK, so now I understand. Now I'm gonna factor it. So x...so this one is working out. So it's true.

Her factors indicated that the roots are 2, 0, and -2 so she concluded that statement was true. However, apparently she did not understand before why the y was equal to 0.

Moving onto the third problem, Amy seemed confused.

S: I forget how to put that equation together. It's like x and y have to be on different sides.

I: Why?

S: Because that's like you say y=... because they're. I don't know how to explain. I don't know what to do here. I know like once I have the equation, to find the x intercepts you let y=0 and to find y, you let x=0. So...I don't know what to do.

I: But, haven't you just described what to do?

S: Yeah. Can you do this equation like it is without having to change it around?

I: Why don't you try it and see what happens?

Haltingly, Amy described what she was doing and eventually found the x and y intercepts. When she was finished and satisfied with her answer, I asked her if she would be able to do it on the calculator.

S: Well by the y is 41y and I don't know how to do it. Unless you change it around and get rid of 41 by dividing it on that side and then the other side by 41 too.

She continued until the equation was in y= form. Running out of time, I asked her to just explain what she would now do with the calculator.

S: Then I'd graph it just to look at it and then...then you might see by looking at the graph where it intersects.

I: What about if its not going to be an integer? What are you going to do?

S: Um. So this is a line so it only intersects once..so you could do it the other way like TRACE or one of the other methods.

I: So if you're using TRACE, where would you stop?

S: On the point where it intersects the y axis and then you could find that point where it intersects the x axis.

I: Should that give you the same answers?

S: I would say.

She wasn't convinced. The interview concluded at that point because another student was waiting.

4.4.2.2 Fifth Interview

Amy was interviewed four other times, the last one being twenty six days after the one described in the preceding section. The questions asked were the same as those noted in the fifth interview for Mark.

Slowly and confidently, Amy described some of the features that she could recall about roots. All comments were recorded:

S: Roots are the points where the graph intersects the x axis.

S: The number of roots you can tell by looking at the highest degree.

S: They are the zeroes of the factors.

I: You're telling me then how you can find a root?

S: Yeah.

S: When there's a double root it looks like a parabola.

I: So, does that mean that the whole graph is a parabola?

S: No. That at a certain point, if it's a double root, it looks like a parabola, but it could go on and cut again.

Amy then drew a diagram to illustrate what certain types of roots looked like. She drew a single root and a triple root. She had already described a double root.

S: I can't really remember anything else except how to find them.

I: OK. So, how do you find them?

S: You could factor your equation or you could use the quadratic formula.

The discussion then branched into different types of factoring that might be used to solve an equation. Amy was able to recall grouping, common factor, and difference of squares. She needed to be reminded about the trinomial method and the rational roots theorem as other possible ways to find linear factors. In the process of recalling these methods of factoring, she mentioned the calculator as a way of finding the roots of a polynomial. The TRACE, TABLE, and CALC/ROOT features were the three methods on the calculator that she listed.

When Amy was unable to recall anything else about roots, I suggested to her that she read down through the list of questions for the interview. This helped her to recall that there could be real or imaginary roots. When asked if there were different types of real roots, she responded by saying rational roots. Some prompting did elicit irrational as well. However, there was a certain degree of uncertainty about how to identify whether real roots were rational or irrational. For example, when asked if it was possible for a polynomial to have no rational roots but still cross the x axis, she said:

S: I think it would be because it's still just a number. I'm not sure but I figure it could just cut at any point.

I: We just talked about different categories of roots. There are only two types. They are either ...?

S: Real or imaginary.

I: Right. What's real?

S: Where it cuts the x axis.

I: Right. Now where it cuts is either this type or that type. Which of two types does it have to be?

S: Rational or irrational.

We then went back to re-read question number two. Amy paused for several seconds before she said that if it did not have rational roots, it could still have irrational roots.

We finished the interview by discussing the function given in the second question and whether or not it had any rational roots. Amy thought that she would look at the calculator first to answer the question. The answers, found by using the CALC/ROOT, appeared to be irrational according to her because there was no repeating pattern. I then felt it necessary to review the rational roots theorem with her to verify the fact that the function in number 2 did not have any rational roots. She needed to be reminded that a way to show, algebraically, that a function did not have any rational roots, was to show that none of the possible rational roots resulted in a remainder of 0 using synthetic division or the remainder theorem.

Once again, because of time constraints, we did not have the opportunity to complete all of the questions.

4.4.2.3 Analysis

In the first interview Amy was awkward with the mechanics of using the calculator. When the first graph she entered did not produce a satisfactory graph, she needed to be prompted to change the screen. Also, when she zoomed out and then back in again, she found herself back where she started and concluded that zooming in would not help change the appearance of the graph. I needed to suggest that she zoom in again a second time. Also, later in the interview when the TABLE feature did not produce a zero in the second column, she was unaware of the role of Tblset and immediately decided to try an alternate method. Finally, she did not attempt to use the CALC/ROOT option at any time, though it might have been more efficient for some of the problems.

Twice during the interview, Amy demonstrated a lack of understanding about the connection between the graphical and symbolic expressions of an equation. For example, when trying to prove or disprove the second statement, she picked up her pencil, changed the f(x) to 0 and started to factor. She was unable to explain why f(x) was equal to 0 and why factoring would work. She said, but it's just how I know how to do it. Her efforts to answer the questions presented to her seemed to be based more on the methods she had been taught to use in the past than on an understanding of what each question was looking for. However, when questioned, she did stop and think about it. Some connection seemed to be formed as she tried to articulate her answer. She exclaimed, OK, so now I understand.

She may have experienced an insightful moment for the question referred to above but the same tenuous link between the symbolic and graphical expressions of an equation resurfaced again. Amy recited that in order to find the x intercept, you let the y=0 and to find the y intercept, you let the x=0. However, because the linear equation was in standard form and not slope y intercept form, she did not know what to do. I reminded her that she had already described what needed to be done. She seemed to be operating more from memory and previous experience than from a graphical understanding of what she was being asked to find.

The researchers' impression that Amy had tended to develop answers based on memory rather than an understanding of the graphical dimension was justified in the final questionnaire. She said of herself:

I know what is going on rather than just memorizing things.

As noted about Mark, Amy also had problems expressing herself. However, it was not so much that her use of the language was confused, but that she seemed unable to articulate what she was thinking. Several times during the interview she said I don't know how to explain it or I don't know what to do. She was attempting to recall things that she had learned how to do in the past. Because she had not understood the graphical implications of the procedures she had learned, she had difficulty explaining what was asked in the questions. Writing solutions on paper was comfortable. Explaining them was not. However, one of the discussions did appear to help her understand why she was letting the $y = 0$ when she was solving equations.

The fifth interview indicated that Amy was fairly proficient in her ability to describe some of the important features of the roots of polynomial functions. Early in the interview Amy demonstrated that she was able to interpret roots in graphical and algebraic terms. She described roots as points on the x axis where the graph intersects the x axis and as the zeroes of the factors of the equation. In addition to finding the roots by factoring, she recalled that they could also be found on the calculator by either of the three methods TRACE, CALC/ROOT, or TABLE. Her comments about single, double, and triple roots were graphical rather than symbolic.

Some confusion emerged when Amy tried to classify different types of roots. The suggestion that she read through the question paper helped her to recall, quickly, that roots could be real or imaginary. She did not elaborate about what imaginary roots meant algebraically or graphically. The only type of real root that she was able to name, when asked, was rational. Her hesitation during this part of the interview stemmed from her lack of understanding that real roots had to be either rational or irrational. Her response to the question "Can a polynomial have no rational roots, but still cross the x axis?" was hesitant and uncertain. She slowly responded, "I think it would be because it's still just a number. I'm not sure but I figure it could just cut at any number." Due to her lack of confidence with the concept of rational and irrational roots, it seemed appropriate to conclude the interview with further discussion and written work relative to showing that the polynomial in question two did not have any rational roots but that it did have an irrational one.

4.5 Students' Attitudes Towards the Graphing Calculator

4.5.1 To Use or not to Use?

At the beginning of the polynomial unit all students completed a survey. The survey can be found in Appendix D. The intention was to reveal students' background experience with scientific and graphing calculators. It also attempted to assess initial feelings and perceptions regarding mathematics and the potential use of graphing calculators during the instruction, home practice, and testing phases of the learning experience.

Overall, the results indicated a fairly enthusiastic group who had had little exposure to the graphing calculator, experienced slight apprehension about its implementation, and felt somewhat positive about its use in class, at home, and for testing purposes. The responses to questions relative to the aforementioned were not unanimous but they were positive, albeit guardedly so. A little over two thirds of the class felt that they had a good idea as to what mathematics was all about, enjoyed the subject, and felt that the use of the calculator should not be restricted. Most of the others were uncertain of, rather than negative towards these assertions.

A second group of statements on the survey were more specific about the potential benefits of the graphing calculator. Responses to these statements, however, were not as strongly positive. Approximately half of the class felt that the calculator would enhance their understanding and help them become more successful in the polynomial unit. The same number felt that it could be used to verify their algebraic solutions and that mathematical problems could be done interchangeably on the calculator and by hand. Most of the remaining students were uncertain about the validity of such statements while only a few claimed that the calculator would not prove to be a useful alternative that would help them understand the concepts better.

The reservation towards the calculator that emerged in these questions might be partly explained by the fact that so few had had any experience with or exposure to this device.

Student attitudes were also assessed in the second part of the final questionnaire. Six questions attempted to gauge their overall reactions to the calculator, what features, if any, proved to be most useful, what effect it had on their confidence, and if they would like to continue using the calculator in the future.

During the course of the polynomial unit, the class used the graphing calculators daily. It was, however, used in a very limited capacity in relation to its total capabilities.

Asked what features helped them to learn about the graphing calculator, the students were overwhelmingly positive. Many said they appreciated the graphing utility and the opportunity it afforded them to just see the function and to observe how the function behaved and was affected by its degree. One insightful student commented that the calculator helped him to understand that a graph is just a series of points while another said that the calculator helped her understand the shapes of functions.

One of the most popular feature of the calculator among the students was the CALC menu with its many options. Nearly all students described the root option as a convenient, quick, and easy feature to use. Several remarked that it also helped them to better understand the concept of a root and that seeing the intercepts and intersections was beneficial. Four students mentioned the ZOOM capabilities, saying that it gave them a better idea about the appearance of double and triple roots. For example, one person said:

> What appears to be a double root may actually be two roots.

Other comments included:

> ZOOM provides more detail.
>
> ZOOM allowed me to see things that I may have passed over.
>
> ZOOM lets me see points of tangency.

Three students said that they liked the ability of the calculator to TRACE along the curve and one, in particular said:

> It showed me that a graph continues on and on and doesn't stop at a point.

The other feature of the TI-82 that many students found functional and versatile was TABLE.

> I can see where the roots are because the y=0.

In general, students said that TABLE was fast, easy, and helped them to learn and understand about roots.

A few of the random, but interesting comments made by the students include:

> More exploration is possible.

I get a better mental picture of how to do different things.

I can check my answers in more than one way.

The calculator helped me understand why the algebraic work made sense.

The only comments made that might be interpreted as somewhat negative were:

It didn't helped me learn but saved me time and helped me check my work.

and

I didn't like having to adjust the viewing window because it was time consuming.

At the beginning of the unit, a few students expressed a concern that they would become overly reliant on the calculator and not be able to solve problems algebraically. The ninth question of the final questionnaire addressed this concern and asked students to respond accordingly in light of their own experience.

Judging from the responses received, most fears were alleviated throughout the course of the unit. The vast majority of students were quite positive about the role the calculator had played in their learning. Seven of the twenty nine students who completed the questionnaire were positive about the calculator, but qualified their responses. Twenty one offered only positive comments and one in the target group said that it had impeded his understanding because it did not help him understand factoring.

Several students testified to the benefits of the calculator by saying that first they felt they would fall behind other classes but now felt they understood the algebra better. Students said:

> The calculator helped because without it I would not have been able to see every function that I dealt with and wouldn't have the same complete understanding of polynomial functions... it saves me a lot of busy work.
>
> It helps me understand aspects of polynomial functions like shape, size, and direction.
>
> At first I was dependent on the calculator, but now I can do most things both ways... it gave me another way to solve problems and a way to check my work easily.

Several said that the calculator enhanced their understanding of zeroes and points of intersection. They liked the way it enabled them to check their work and they felt that they were able to work things out on paper as well as before. One student in particular concurred with much of this assessment and added:

> It gives me a faster way to explore the effects of certain things on polynomials.

Four other supporters stated:

> With the calculator I can see the graph, its roots, and curves...I know if answers are correct and I don't need to second guess my work if the calculator backs it up.

> It helped the speed at which I was able to learn because I was able to do more practice in the same amount of time.

Students who qualified their answers said things like:

> It helped me understand polynomial functions, but I don't feel as confident in my own ability to solve problems.
>
> I don't think that it had much effect on whether or not I understand polynomial functions but it did come in handy when working out and checking some equations.
>
> It helped me after I was shown how to work it out on paper.

Many of the students in the target group were united in their reactions to the calculator as a valuable tool due to the fact that it provided a quick and efficient way to do many problems related to polynomials. Most of all, it enabled them to feel more secure about their own work because much of it could be verified on the calculator. Many of these reactions were confirmed in two other questions.

Asked if they would like to continue to use the calculator in later units of the 3201 course, the answer was a resounding yes. Even the student throughout the study who was most reticent about using the calculator felt she would like to continue having one available to her. She did, however, think that it should not be used as much and that it would be more beneficial after a concept was taught to use as a shortcut or as a check. One other student was not sure and reported that she was forgetting how to do things algebraically.

Again and again students remarked that the calculator was a tremendous boost to their confidence, that they were more likely to check their work, and were less likely to make silly mistakes.

The last item on the questionnaire was not one that students had to respond to. Rather it provided those interested a forum for their own concluding comments. The responses were varied.

The fact that the calculator had been a positive influence was evident by comments referring to the calculator as fast, easy, and more interesting that just algebra. Among those who responded to this question and felt nervous and confused about the experience at first, said that things balanced out in the end. Others felt that the calculator allowed them to understand rather than just memorize and that it was possible to get a better perspective of how functions look instead of just seeing it as a bunch of numbers and letters. Other interesting and exciting reactions referred to the calculator as a great tool for visualizing that allowed more interaction between classmates and one that should be beneficial for university.

One student did comment that too much time had been spent on the calculator and not enough on the course while another suggested that more examples from the text should have been done.

The last activity required for all students in the target group was to complete an attitude survey. Results from the survey, essentially, reiterated that which was said in the questionnaire. All students, except one, were more confident in their mathematics when they had the graphing calculator. The dissenter from the group did not disagree, but was uncertain. Four were unsure or did not find the calculator to be motivating or interesting and the same number felt uncertain or in agreement that the calculator had actually decreased their interest in mathematics. The vast majority said that the calculator and manual were easy to use and that the graphical and algebraic analysis helped them to understand the underlying mathematical ideas being studied. The concluding statements in the questionnaire suggest that many found the calculator to be an ideal tool for exploration that could be used to verify their work and that they would recommend to a friend. Three felt that they may be able to make suggestions for changes to the lab manual that would make it clearer for future students to use.

5

CHAPTER FIVE

SUMMARY, LIMITATIONS, IMPLICATIONS, AND RECOMMENDATIONS

In this chapter a summary of the study and discussion of the findings are presented. Implications of the study for instruction are discussed and recommendations for future research are included.

5.1 Summary

Many of today's workplaces are struggling to make the transition from a top down operational and managerial style to a team based approach that involves its employees in the decision making process. Schools have a responsibility to better prepare their clients to occupy these positions. Students need to learn how to learn, to question, to explore, to make sense of their findings, to communicate, and to work within a group structure. The graphing calculator can play a small role in helping to create an environment that encourages and promotes these types of activities.

This descriptive study was designed to investigate Level III Advanced Mathematics 3201 students' conceptual understanding of the roots of polynomial functions as a consequence of an integrated approach to the unit with the TI-82 graphing calculator. Thirty one students participated in the study. Data were collected through interviews, specific writing activities, a unit test, two surveys, and a final questionnaire.

Following the completion of the data collection phase of the study, the taped interviews were reviewed and the unit test, writing activities, and final questionnaire were analyzed to determine students' understanding of the roots of polynomial functions. Information relative to their facility with graphic, symbolic, and tabular representations of function, as exhibited by their use of the calculator, and their attitude towards this device was sought.

5.2 First Purpose of the Study

The general purpose of this study was two-fold. The first was to find answers to the four questions proposed in light of the integrated approach to the polynomial unit with graphing calculators. Each of the four questions will be addressed separately in subsections 1 through 4.

5.2.1 Question 1

> Does a unit of instruction which includes regular and frequent use of the graphing calculator, as one element of the instructional approach help students develop an understanding of the concept of the roots of polynomial functions?

An analysis of the data suggested that the approach taken in the unit, which included the graphing calculator as an integral component, did help students understand the concept of the root of a polynomial function. Since the graphing calculator was used regularly and frequently during the classroom experience, it seems reasonable to conclude that it had some impact on student learning. The initial and final definitions of a root submitted by all students in the target group and the results of the final test and questionnaire suggest that the majority of students completed the unit with an adequate understanding of the roots of polynomial functions, both graphic and algebraic.

The unit test and questionnaire indicated that the majority of students were able to interpret and apply many aspects of a root. Most students' answers included references to the graphing calculator to support their explanations. There was a general understanding that a root was the value that satisfied the equation, making it a true statement, and that it also represented the place on the x axis where the graph intersected. Most understood the difference between real and imaginary roots, were able to identify single, double, and triple real roots, and knew the relationship between the degree of a polynomial with several terms and the number of turns in its graph. The fact that imaginary roots could not be interpreted graphically caused difficulty for a few.

The mean, median, mode, and distribution of marks for the unit test and the quality of answers offered in the questionnaire attest to the fact that students finished the unit with a fairly sound understanding of a root.

The individual definitions submitted by all students at the beginning and conclusion of the study reflect the progress made, relative to the concept, for each student. Without advance notice and without help from others, all students formulated a written description of a root twice during the unit. The second definition was requested three weeks after the study concluded. This suggests that the instructional approach used enabled students to retain what they had learned. Despite the lapse in time, all were able to formulate a coherent statement that, for all but one, encompassed its graphical and symbolic qualities.

5.2.2 Question 2

> Which representation of function, tabular, graphic, or algebraic do students choose to use and work with when determining the roots of a polynomial function? Why?

In general, most students became proficient with the graphic and symbolic representation of function. Both offered a straightforward approach for students to use when finding roots. The tabular display, on the other hand, was slightly less popular because it was time consuming. It required students to read, analyze, adjust, and possibly re-adjust until a root could be determined to a certain degree of accuracy. Consequently, though many found the table convenient to use for simple functions with integral roots, it was not the preferred choice when the roots were non-integral. However, at the conclusion of the study, the vast majority of students were able to extract information relative to the roots of a polynomial whether it was written in tabular form, in symbolic form, or represented graphically.

Given a specific function, students typically decided to enter the function first, generate the graph, and then observe the table or access the CALC menu. What they elected to do after observing the graph depended on what the question was looking for. The point is that most were able to demonstrate a degree of flexibility with the calculator and adjust themselves accordingly.

As the study progressed, there was certainly a growing tendency for students to choose the calculator more frequently as an alternate way to solve a problem, answer a question, or verify a solution. Initially, in class and during interviews, students chose to attempt problems using the procedural techniques they had seen and used before, even though some solutions could have been achieved more efficiently with the calculator. As the unit progressed students were more apt to choose the calculator. This seemed to happen as students became more familiar with the mechanics and dynamics of the calculator and as their awareness of the link between the symbolic and graphic representations of function blossomed.

As the study unfolded and students became more familiar with the calculator, their preferences for different representations of functions varied.

More than two thirds of the students in the target group indicated in the final questionnaire that they liked the graphing capabilities of the calculator. Many claimed that being able to see the graphs of the equations they were working with not only helped them understand what a root was, but also helped them understand the algebra better.

The TABLE option was also a popular feature among the students. During the interview process many regularly checked the table of values first when trying to identify the roots of a function. This method was especially popular for functions that looked relatively simple and whose roots were integral. Students found roots for such functions easy to generate and identify from the table by looking for zero in the second column. However, students typically found that the CALC/ROOT option was a convenient and accurate alternative for all functions and consequently it was the more popular choice.

Despite the presence and potential of the calculator, students, in general, still wanted to know how to answer questions on paper by performing algebraic procedures. They liked the support offered by the calculator and the insight it provided for the concepts studied, but many still wanted to know how to solve problems manually. After all, that's what math had been all about for the past several years. However, by the time the study was complete, most students were able to exhibit the algebraic skills necessary and were not detracted by the calculator. For example, one student commented:

I am still able to work problems out on paper as well as before.

5.2.3 Question 3

> What growth or development, if any, have the students exhibited in their ability to make the link between the symbolic factors of the polynomial equation and its real roots?

The third question of the study addressed the skill of factoring and whether or not the approach taken with the graphing calculator helped students make the link between the symbolic factors of a polynomial and its roots.

Factoring has been and still is a problem for many students, even at this level of mathematics. This group is no exception. Regardless, being able to write a polynomial in factored form is fairly important in terms of identifying its roots.

Many of the students in the target group had some difficulty factoring polynomials, particularly if the grouping or trinomial methods needed to be applied. This difficulty escalated if two factors were additive inverses of each other or if the common factors were strictly symbolic rather than numeric. Evidence from the final test and questionnaire suggested that most of the students were able to easily identify the roots from the factors and, conversely, the factors from the roots even if they had difficulty factoring. Errors that did occur in problems that tested this concept, resulted from carelessness with signs and/or exponents. This assumption is made because the errors did not consistently occur throughout any one paper and only one student of the thirty had the concept of roots and factors reversed.

Factoring improved somewhat after students were advised that the TABLE or CALC features could be used in conjunction with the graph to pick out some of the rational roots of a polynomial. These, in turn, enabled them to identify some of its factors. Some students who claimed to have a history of difficulty with factoring were excited by this possibility.

5.2.4 Question 4

> What do students say about the integration of the graphing calculator into their learning of mathematics that would reveal their attitude towards this device?

This question focused on student attitudes towards the integration of the graphing calculator into the polynomial unit of the level three advanced mathematics course. In general, this device was well received by the students. Most were intrigued and content to be part of the study though some were a little anxious at first.

As a whole, the class was relatively positive about the inclusion of the graphing calculator into their learning of mathematics. There was, however, an undercurrent of reluctance to accept this instrument among a very small number of students.

Some comments made at the beginning of the semester, during writing activities requested, indicated a slight anxiety about falling behind other classes, becoming too reliant on the calculator, and not being able to solve problems manually. Some of these concerns subsided as the semester progressed. One student, for example, became one of the staunchest supporters of the calculator and finished the unit with a test score of 97%. His answers on the questionnaire indicated a strong grasp of the concepts studied. He stated that he found the calculator to be a tremendous help to him and that he would continue to use it .

However, a couple of students continued to feel that they were not learning real mathematics and that they would prefer to be taught how to do exercises step by step. One of these, in particular, felt this way and was more at ease in the classroom when exploration was at a minimum and procedures were demonstrated at the board. Her reaction to the integrated approach with the graphing calculator seemed to be a consequence of her vision of what mathematics was supposed to be. She also perceived the calculator to be doing the work for her which she feared would be detrimental when time came for an exam. Her responses in the final attitude survey indicated a dislike and mistrust of the calculator though, oddly enough, she said she would like to continue using it in subsequent units, though not as much. She finished the unit with a score in the high eighties.

Three other students did not seem to be overly enthused about the potential of the graphing calculator. Their reaction, however, was harder to gauge because they had put little or no effort into the activities required for the unit. One of the three was regularly absent and another never wrote the final test and questionnaire despite the fact that she was given five different opportunities to do so.

The replacement teacher who has since taken over the teaching duties of the researcher has commented that the calculator continues to be an integral tool in the classroom. Students regularly reach for the calculator during instruction and practice. This is not a surprising development considering the feedback received from all students saying they would like to continue using this device in subsequent units and comments on the questionnaire describing their preference for the calculator as a means of helping them to solve and verify written solutions.

5.3 Second Purpose of the Study

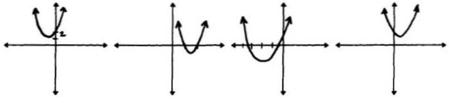
The second purpose of this study was suggested in the preface of this paper and was alluded to at the beginning of the chapter. This goal was to help students become more active in the learning process; to show them how to find answers for themselves rather waiting for them to be delivered fait accompli.

One of the questions on the final questionnaire attempted to provide some insight into student progress relative to this goal. Students were asked to investigate the role of the linear coefficient in a quadratic function and to summarize their findings.

Six of the twenty nine students did not attempt to answer the question. Of the remaining twenty three students, two proposed a possible investigation, but did not carry it out. Another two summarized what they found, but did not describe what their investigation entailed. However, two thirds of the class did develop a plan, carry it out, and present their findings.

All of the responses to this question described an inductive approach. Several quadratics that differed only in the value of the linear coefficient were entered and graphed. The resulting changes were described. One student's answer was as follows:

$y=1x^2+bx+2$ where b= 1, -3, 4, -1/2



If b is negative, the graph is on the right side of the y axis.

If b is positive the graph is on the positive side of the y axis.

The closer b is to zero, the closer the vertex is to the y intercept.

The farther b is from zero, the farther the vertex is from the y intercept.

The fact that the majority of the students were able to formulate a plan, carry it out, and summarize their findings is encouraging and exciting.

The graphing calculator played a role in the students' ability to offer an answer to this question. Over the course of the eight weeks that this study took to complete, students were involved in group situations where they had to investigate, describe, and make conclusions on a regular basis. The results of this question indicate that given the opportunity to learn how to explore, observe, and summarize, students will do just that.

5.4 Limitations

The descriptive, rather than quantitative, nature of the study and the research design used prohibits the researcher from claiming that the positive results achieved were due solely to the integration of the graphing calculator. Though it did play an integral role, the results were presumably influenced by other factors.

The increased interaction, student to student, certainly would have had some influence on the ability of students to construct and refine their own understanding of what a root was. Increased communication because of the physical and pedagogical structure of the class probably coerced some into becoming more active than they might have been. The interview process may have been beneficial for some because it provided a one-on-one opportunity to articulate and review some of the qualities of roots. The increased time spent on this unit should also have had a positive effect on the students' success with the concepts studied.

The results of this study are also not generalizeable to the larger population because it was conducted with so few students and had no control group. What it has attempted to do is to provide some insight into the progress made by a small number of students who used the graphing calculator regularly while learning about polynomial functions. Hopefully, it will give other teachers some idea how the graphing calculator might be beneficial for some students and how they might integrate it into their own instruction.

5.5 Implications

The increased use of technology in the workplace and the evolution towards employees sharing in the decision making process indicates that educators need to help students become more active and responsible for their own learning. Students need to become more adept at using technology effectively. They need help to learn how to question, observe, describe, categorize and generalize. They need to learn to trust their instincts and to have confidence in their own abilities. And they need to learn how to do all of these things collectively as well as individually. This is a very tall order but school mathematics can play a small but significant part in helping students to develop these skills. In particular, proper and regular use of the graphing calculator can help promote these behaviors.

The results of this study have two implications for instruction. First of all, if one of the aims of mathematics educators is to have students understand what they are learning in high school algebra, then they must establish, more firmly, the link between the symbolic and graphic. Despite efforts to integrate the algebraic and graphic components of the high school mathematics curriculum, students see algebra as algebra and graphing as a separate and unrelated entity. However, frequent exposure to the graphical dimension, available through the graphing calculator, over an extended period of time does help students to bridge the gap from the symbolic to the graphic. The results, however, are not immediate. Formation of the link between the two representations will not be complete in a just a few classes with the graphing calculator. It takes several classes just for students to become familiar with the mechanics of this device before any thought can be given to the significance of what the calculator is showing. However, as students be come familiar with the device, they will probably be more inclined to choose it in their search for answers.

The second implication for instruction is that not all students will be enthused about the intrusion of the graphing calculator into the learning process. Some will be frustrated, especially initially, because they have to remember a variety of procedures for using this device. It will not motivate all students and the novelty will gradually abate. Though many students will gradually develop an appreciation for it, others never will because it forces them into activities that they are not comfortable with. Students whose motto seems to be just show me what to do and when to do it and I will be satisfied will probably never appreciate the power of this device.

Hopefully, these comments will be of value for teachers in planning instruction that will result in improved success for many students.

5.6 Recommendations

The focus of this study was to determine if the graphing calculator had any effect on students' ability to understand and determine the roots of a polynomial using the different representations of functions available on the graphing calculator.

The findings of this research suggest additional areas that might also be explored with the graphing calculator. One such area involves student retention over the long term. Specifically, what effect does using a graphing calculator have on students' ability to retain and build upon concepts learned previously? Further investigation could be done to determine what effect the graphing calculator has on student learning in subsequent units of the senior high mathematics program and in what capacity students continue to use this device.

A similar study with younger students might provide educators with insight into ways that mathematics could be made more relevant and less procedural. This could reduce the repetition and overlap that characterizes our curriculum so that more and better mathematics might be available for students during the latter school years.

Further study could also be carried out with students who seem most reticent to use the graphing calculator to see if prolonged and regular use of the calculator would encourage them to see beyond the procedural. Could the graphing calculator help these students gain an appreciation for mathematics as a vehicle that promotes critical thinking instead of viewing it as a subject reserved for the classroom that has little or no value beyond the walls of the school after exams are complete and marks are awarded?

This study and others indicate that the graphing calculator has the potential to improve the quality of mathematics that is learned in school if it is used effectively on a continuing basis. It can bring about change in classroom dynamics, alter the role of both teacher and student, and serve as a catalyst for mathematics learning. Further study is necessary to answer these and other questions relative to the graphing calculator in order to convince math educators, in general, that this tool can be a valuable implement in the classroom.

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Appendix A
Graphing Calculators in the Mathematics Classroom

This manual is designed to be used with the TI-82 graphing calculator during the first unit of the Advanced Mathematics 3201 course entitled Polynomial Functions. As you progress through this unit on polynomial functions you will be incorporating the activities and exercises into your regular classroom routine, transforming your classroom experience into one of experimentation where you have the ultimate responsibility for your own learning. You must be a **doer of** math and not a **receptacle for** math. Your participation and attention will be critical while you ask yourself questions like: Why? What would happen if...? Would this be the same or different than? How can I make this change? and so on.

As a teacher and researcher I am interested in the graphing calculator as a tool in the mathematics classroom. My primary purpose during this investigation is to determine whether or not the graphing calculator can enhance your conceptual understanding of the roots of polynomial functions. Your **learning** will be directly related to your efforts to interpret, synthesize, and apply what you see on the screen. Consequently, it is extremely important for you to answer the questions posed, respond to the statements presented, and also to share your conclusions, insights, questions, and concerns with your teacher and your classmates. Complete the activities in the order in which they are presented. Each activity is followed by a blank page to use for the writing activities that I will be asking you to complete as the unit progresses. It will also be a good place to bring up concerns or difficulties that might surface. Please do not proceed on to subsequent activities if you are having any problems.

The exercises that have been included in this manual are only a sample of what the TI-82 can do. For further guidance in the use of this calculator, please ask for an owner's manual, available from me. The next two pages are a menu map for the TI-82. This map will give some idea of it's extensive capabilities. Explore and enjoy!

ACTIVITY #1 - Introducing the TI-82 Graphing Calculator!

The Keyboard

The keys are grouped by both colour and zone. The four zones are: graphing keys, editing keys, advanced function keys, and scientific keys.

Very top row of the keypad - Graphing and Table keys

Some of rows 2 and all of row 3 - Editing keys, except for LINK, LIST, and STAT.

Rows 3 and 4 - Advanced function keys each with pull down menus

Rows 5-10 - Scientific calculator keys

Most of the keys have three meanings. To access the second meanings in the light blue above each key, press the light blue 2nd key first. To access the operation or letter in white, use the ALPHA found below the 2nd key. If you are typing in a program you can use the A-LOCK by pressing 2nd ALPHA.

Display

Adjust the display contrast -press 2nd and keep your finger on the up arrow to darken screen
-press 2nd and keep your finger on the down arrow to lighten screen

When you've adjusted the contrast up to 9 (displayed in the top right hand corner) its time to get new batteries.

Calculator Settings

The calculator can be returned to factory settings by pressing 2nd and MEM. You then press 3 or cursor down to reset. However, this will delete all data and programs stored in your calculator if you continue, so be careful. You may not want this to happen.

Mode Menu

The mode menu will control the display, both graphically and numerically. Cursor down to select an alternate setting for any of the options listed. Activated settings are highlighted.

Math Keys

The third and fourth rows contain keys that allow the user to access both math and variable values. An entry can be selected by using the cursor keys or entering the number of the desired choice and then pressing ENTER.

Screens

Homescreen - When the calculator is turned on, the homescreen is displayed . This is the primary screen for the TI-82 and is where calculations are entered and results displayed. You may return to the home screen at any time by pressing 2nd QUIT (jokingly referred to as quit and go home).

Graphics Screen -This screen is used to display graphs you have entered in the y= list
Press GRAPH.

Menu Screen - This type of screen displays a menu of selections. Cursor down and highlight your choice or enter the number of your choice.

Table Screen - This screen displays a table of values for the dependent and independent variables. This screen can be altered by pressing 2nd WINDOW (Tblset) For example, in the TABLE SETUP menu, TblMin lets you decide what the value of the independent variable will be and ΔTbl allows you to decide what the increment will be.

List Screen - This screen offers as many as six lists for the user to input data.

y= Screen - The screen on which you enter equations to be graphed. In the y= list, you can de-activate a function so that its graph will not be displayed by placing the cursor on the = sign and pressing ENTER.

Functions in the y= list have to be removed by pressing CLEAR or by using the DEL key.

Calculations

To use the TI-82 as a calculator you will be using the buttons CLEAR, MATH, and 2nd, the cursor keys, as well as the bottom six rows. Brackets must be used to group expressions if you want an operation performed on the entire group. For example, $\sqrt{(12-4\times 2)}$

The $\wedge$ (called carat key) interprets any number that follows it as an exponent.

The negative in brackets at the bottom of the keypad must be used for a negative sign, **not** the subtraction symbol in the far right column.

The DEL key will delete the symbol that the cursor in on. 2nd DEL will insert a space immediately to the left of the cursor position.

The last entry keyed can be displayed again by pressing 2nd ENTER. This permits you to cursor back and edit what you typed previously, saving you keystrokes. If you press 2nd ENTER twice, it will display the second last expression entered and so on.

The vertical bar in the upper right corner indicates that the calculator is operating. You can press ON to interrupt any calculation or if the calculator appears to be hung up.

Function Evaluation

After entering a function as y_1 in the y= list, return to the home screen, press 2nd VAR 1 1 (number), and ENTER. When you key this sequence, your homescreen should look like $Y_1(3)$. The calculator will produce the answer on the right side of the screen

Graphing a Function

After a function has been entered in the y= list, you will need to view it. Do so by pressing GRAPH. Depending on the type of function that you are working with, you may want to change your viewing window. The ZOOM menu allows you to alter your window in many ways. The STANDARD option automatically sets up a coordinate plane that extends from -10 to 10 on both the x and y axes. If you do not want any of the predetermined options available in this menu, press WINDOW and enter your own selections. Experiment with the ZOOM menu to find out how the selections will alter the appearance of your graph.

If you are not satisfied with the viewing window that you have chosen, you can interrupt the calculator while it is still active by pressing ON.

ACTIVITY #2 - Using the TI-82 to Solve a Linear System

Purpose: This activity explores a few of the capabilities of the TI-82 and attempts to show two ways that a linear system may be solved, numerically and graphically.

Objective: At the conclusion of this exercise, you should be able to demonstrate how to use the following features on the graphing calculator: Y= , GRAPH , WINDOW , CALC , INTERSECT, ZOOM, ZBOX , & TABLE.

The TI-82, besides being a normal scientific calculator, can perform a variety of operations. It will accept and execute programs. It will graph or display a variety of graphs and allow the user to explore the relationship that exists between an algebraic equation and its graph. It can be used to help you solve mathematical problems and to add a visual dimension to the learning of mathematics that is usually only available with the use of computers.

You are encouraged to explore with your calculator and to ask questions of other students and your teacher. Please do not move onto subsequent activities if you are experiencing any difficulty. This manual is meant to be a record of your learning, so be sure to complete the charts and to answer the questions that follow.

ACTIVITY:

1. Press y= . Press CLEAR if there is anything entered on this screen. You can enter a maximum of 10 functions on this screen

2. On the y= screen, key in $y = 2x - 1$, ENTER , and GRAPH . Press ZOOM 6 to set up what is called a standard axes. The calculator will display the graph of the linear equation you have just entered. Press 2nd Graph to have a table of values displayed for the function. You can scroll up or down to see other ordered pairs.

3. Press WINDOW . This key allows you to see the maximum and minimum values that the calculator displays on the x and y axes, as well as the scale that is used. You may adjust these parameters if you wish by entering values on each line followed by ENTER. Experiment! Press ZOOM 6 again to set up a standard axes

4. Press y=. Key in $y = x + 1$ as y_2. Press GRAPH. You now have the graph of two lines which intersect in the first quadrant. Press 2nd GRAPH to display some of the ordered pairs on this function. Do you notice anything about the lists for y_1 and y_2? By a method of your choosing, find out **algebraically** where these lines meet. Use the small space provided below.

5. Press 2nd TRACE to access the CALC feature. A new menu will appear, numbered 1 through 7. Press 5 to access the INTERSECT program or go down with your arrow keys to 5 and then press ENTER.
6. You must now move the boxed shaped cursor with the left or right arrow keys as close as possible to the point of intersection. The text at the bottom of the screen will tell you the location of the cursor. Press ENTER three times in succession, reading the text at the bottom, until it displays the coordinates of the point of intersection.
7. Press ZOOM . There are nine features that you can access. Press 1 to choose ZBOX . You now have the capability to determine the size of the viewing screen by drawing a box around the point of intersection of the two lines. The cursor should appear as a small plus sign. It may be hiding on one of the lines so press the arrow keys a few times so you can see it. You are now going to box on the point of intersection using the arrow keys and ENTER. Press ENTER when you want to anchor the first corner of your rectangular box. Now, move the cursor left or right as far as you want and then move it up or down so as to enclose the point of intersection. When you are satisfied with the size of the box, press ENTER for the **second** time to anchor the other endpoint of the diagonal. The viewing screen will be re-drawn, showing only the part of the graph that you enclosed with the ZBOX feature. This feature allows you to get more accurate values for the coordinates if they happen to be **non-integral**.

8. Again press 2nd GRAPH to access the TABLE feature. Your screen will display three columns entitled X, Y_1 , and Y_2 . This is a table of values for both of the linear equations you just entered. The first and second columns represent the ordered pairs for the first linear equation while the first and third columns represent the ordered pairs for the second equation. Is there any ordered pairs that these two functions share? What is the significance of having the same ordered pair in both lists? Explain below.

9. Experiment with your TI-82 by entering additional equations in the y= screen. You will need to delete the existing equations by moving up or down with the arrow keys to the desired equation and pressing CLEAR . With several graphs on the screen at the same time, your screen becomes cluttered and difficult to analyze.

10. Using your TI-82 complete the table on the next page. Review your results with a friend to see that both of you have the same solution set and that you understand how to use the calculator before you progress any further.

11. You may need to adjust the viewing window so that you can see the point of intersection of the systems in the table. You decide the values you will use to define your window. When you enter the equations, fractional values must be enclosed in brackets.

System	**Slope Intercept Form**	**Algebraic Solution**	**Solution on Calculator**	**Sketch**
$2x-3y+1=0$ $x+3y-8=0$				
$-2x+2y+8=0$ $-x+2y-8=0$				
$x-3y=0$ $-2x+4y+1=0$				

Not all linear systems have **one** solution. Make up one example of a linear system that does not have one solution. Verify that your example is correct by using your calculator. Name the system and sketch its graph below. Share your answer with someone else in the class. Can there be more than one correct answer? I want you to get up and find someone else in the class who has a system that does not have one solution, but whose graphs looks different than yours. What is your friend's algebraic solution and what does it look like?

ACTIVITY #3 - The Graphs of Different Types of Functions

Purpose: This activity is designed to help you make some observations about the differences between polynomial functions and non-polynomial functions.

Objective: At the conclusion of this activity, you should be able to describe at least one way in which the two types of functions, polynomial and non-polynomial, differ according to their graphs and their algebraic equations.

Your text describes a **polynomial** function to be any equation of the form:

$$y = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots\dots + a_2 x^2 + a_1 x^1 + a_0$$

where all exponents are **non-negative** integers. The numbers a_n, a_{n-1}, ..., a_2 a_1, and a_0 are the numerical coefficients of the polynomial and represent real numbers.

Some of the equations on the next page represent polynomial functions, while others do not. Enter each equation on the calculator and sketch the graph indicated on the screen next to the equation of the function. Experiment with your viewing window to make sure that you have a global picture of the function's graph. When you have drawn all of the graphs displayed on the calculator in the spaces next to the equations, tear out the whole page, cut each out, and organize them into one of two groups, polynomial or non-polynomial, according to their similarities. Tape them on the blank page that follows. For each category that you create, **describe** directly below the equation why you have chosen to include this one. For example, you could describe what characteristics or features they have in common and what distinguishes them from other graphs not included in the category?

$$y = x^2 + 5x + 1$$

$$y = \frac{x-2}{x+3}$$

$$y = 2\log x + 3$$

$$y = \sin x$$

$$y = 2 \cdot 3^x$$

$$y = \frac{x^2+1}{x-1}$$

$$y = -x^3 + 3x - 2$$

$$y = \cos x$$

$$y = \log(x-1)$$

$$y = \frac{1}{2} \cdot 2^x$$

Polynomials

Non-Polynomials

ACTIVITY #4 - The Roots of Different Types of Functions

Purpose: To use the TRACE feature, TABLE feature, and the ZERO feature of the TI-82 to find the root or roots of a function.

Objective: At the conclusion of this activity you should be able to use the TRACE, TABLE, and ZERO(under the CALC menu) features of the TI-82 to find the root(s) of a function.

In the previous activity, page 18, there are ten functions. You must find all of the real roots for each function using each of the three methods outlined below a minimum of three times each. Complete the table on the next page.

Method One - TRACE allows you to move the cursor along any function being displayed while the text at the bottom of the screen identifies the x and y coordinates of its position. When the graph is on the screen, press TRACE and the left or right arrow keys. As you trace along the curve, the view will pan to the left or right constantly adjusting the viewing window.

Method Two - 2nd TABLE allows you to set up a table of values for each function so you can determine the root(s). You can change the increment in your table by using 2nd WINDOW and entering a different value for ΔTbl. This helps you find more accurate roots, though not necessarily exact.

Method 3 - The ZERO feature allows you to find a root. Press 2nd TABLE, 2, enclose the root on the screen to the left and the right (called the left and right bound) by pressing ENTER. Press ENTER again after the guess text appears at the bottom of the screen. The root will be displayed or a very good approximation to it.

Explain to me, in detail, what is meant by the **root** of a function. You may check with any of the books in the classroom, but I don't want their definition. I want yours. You might find a diagram helpful to supplement your explanation.

Function	Sketch	Root(s)	Method

Which method or methods did you prefer to use to find the root(s) of a function? Discuss your reasoning.

ACTIVITY #5 - Functions of the Form $f(x) = ax^2 + bx + c$

Purpose: To recognize the basic shape of the graphs of functions of the form $f(x) = ax^2 + bx + c$ and to observe what effect changing the parameters a, b, and c will have on the graphs.

Objective: At the conclusion of this activity, you should be able to:

1) Predict the shape of functions of the form $f(x) = ax^2 + bx + c$, describe how it will open and identify the parameter that causes the opening to change.

2) Point out where c is located on the graph for each function and explain algebraically why this is so.

3) Compare the x-intercept(s) and the discriminant for each equation and describe what the discriminant will indicate about the graph.

4) Describe the relationship between the algebraic solutions and the x-intercepts.

Use your graphing calculator to graph each function in the y= list. Complete the table below. A reminder that the ZERO feature is under 2nd TRACE and will allow you to find the root(s). You will need to block in each root with the left and right arrow keys.

Equation	Opens	y Inter	Value of the Discriminant	# of x-inter.	Sketch	Real Sol'ns
$f(x)=x^2-5x-5$						
$f(x)=-3x^2+7x-5$						
$f(x)=9-x^2$						
$f(x)=.5x^2+4x+8$						
$f(x)=x^2+x+1$						

What is the basic shape of all of the functions of the form $f(x)=ax^2+bx+c$?

2. For each function state whether "a" is positive or negative.

   1. __________
   2. __________
   3. __________
   4. __________
   5. __________

   Compare this with the sixth column. What do you notice?

3. **How** does the leading coefficient affect the graph of the quadratic equation? What effect does the size of $|a|$ have on size of the opening of the parabola? Discuss your observations.

4. State the value of "c" for each equation.

   1. __________
   2. __________
   3. __________
   4. __________
   5. __________

   Compare this with the third column. What do you notice? There is an algebraic reason why this is so. Discuss.

5. Circle the part of the quadratic formula known as the discriminant. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ The type of number that results from calculating the discriminant tells something important about the x-intercepts. What information does the discriminant convey about the x-intercepts of the graph? Explain in terms of the algebra of the formula why this is so.

6. Use factoring or the quadratic formula to find the solutions for each equation.

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________

Look at your table. What does this tell you?

ACTIVITY #6- Further Investigations of Functions of the Form $f(x) = ax^2 + bx + c$

Purpose: 1. To learn how to find the vertex of a parabola using the calc feature of the TI-82.
2. To find the axis symmetry for the graph of a quadratic equation.

Objective: At the conclusion of this activity, you should be able to:

1. Use the max or min feature of the CALC menu on the TI-82 to find the vertex of a quadratic function.

2. Determine the equation of the axis of symmetry of any quadratic function

3. Point out the relationship that exists between the x-coordinate of the vertex and the axis of symmetry.

4. Find the y-coordinate of the vertex if given the x-coordinate. You should be able to do this on your calculator as well as on paper.

The vertex of a parabola is either a maximum or a minimum point. Press 2nd TRACE, select max or min, block in the vertex to the left and right by using the cursor keys and ENTER. The text at the bottom of the screen will display the coordinates of the vertex. Sometimes the calculator will not give you the exact vertex, but a very good decimal approximation. For example, -5.961E-7 actually means -.0000005961 and is more or less the number zero.

Use your graphing calculator to complete the table and answer the questions that follow.

Equation	Sketch	Max/Min Point	Axis of Symmetry
$f(x)=2x^2-4x+1$			
$f(x)=9-4x^2$			
$f(x)=-2(x-5)^2+4$			
$f(x)=5x^2$			
$f(x)=1x^2-6x+9$			

1. Evaluate the expression -b/2a for each equation in the table.

   1. ____________
   2. ____________
   3. ____________
   4. ____________
   5. ____________

2. For each quadratic equation in the table, use the CALC menu, select value, enter the x-coordinate (you might have to round off) of the ordered pairs in the third column of your table. What does the calculator give you back? What do you notice?

   1. ____________
   2. ____________
   3. ____________
   4. ____________
   5. ____________

3. Based on your answers to questions 1 and 2, describe a second way to find the vertex of a quadratic function beside using the max/min feature in the calc menu.

4. How would you find the y-value of the vertex using pencil and paper instead of the calculator?

5. What is special about the vertex of the graph of the quadratic function?

Activity #7 - Polynomial functions of the Form

$$f(x)=ax^{n}+bx^{n-1}+...+k$$

Purpose: This exercise is designed to help you investigate the shape of certain polynomial functions of the form $f(x)=ax^{n}+bx^{n-1}+...+k$ where n = 2, 3, 4, and 5.

Objective: At the conclusion of this activity, you should be able to predict the shape and direction of polynomial functions of the form $f(x)=ax^{n}+bx^{n-1}+...+k$ where n = 2, 3, 4, 5. You should also be able to predict the number of turns that will occur in the graph and how many zeroes there will be.

Use you TI-82 to graph each of the functions noted on the following page. Complete the table and answer the questions that follow. You may use the TRACE and ZOOM IN, or 2nd TRACE 1 features of the calculator to find the roots or zeroes of the functions or do it algebraically by looking at the factors. Complete the following table. Pay particular attention to the relationships you see developing in columns 3, 4, and 6. You may, occasionally, have to change the viewing window in order to see more of the graph.

Equation	Each Linear Factor	Degree	a	Sketch	# of Roots	# of Turns	List the Roots
$f(x) = 7 - x^2$							
$f(x) = x^3 + 5x^2 + 6x$							
$f(x) = -3x^3 - 1x^2 + 3x + 1$							
$f(x) = (x^2 - 1)(x^2 - 3) - (x^2 - 1)$							
$f(x) = x^3 - 9x^2$							
$f(x) = (x^3 - 3x^2)(x^2 + 3x + 2)$							
$f(x) = -(x^2 - 5x + 6)(x^3 - x)$							

What do you notice about the linear factors and the roots?

Is there any relationship between the degree of a polynomial of the form $f(x) = ax^n + bx^{n-1} + ... + k$ and the number of roots that it has? If so, what is this relationship?

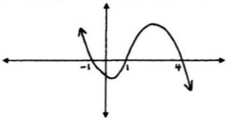
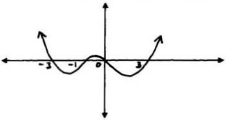
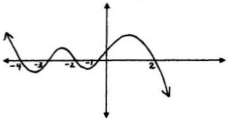
What is the relationship between the degree of a polynomial of the form $f(x) = ax^n + bx^{n-1} + ... + k$ and the number of times that it turns direction?

What is the relationship between the zeroes of the linear factors of the functions and the x-intercepts?

How many roots would you expect for a polynomial function $f(x) = ax^{10} + bx^9 + cx^8 + ... + m$?

What shape do you expect the polynomial $f(x) = ax^3 + bx^{n-1} + ... + k$ to have? Which term in the following function $f(x) = x^3 - 7x^2 + 7x + 15$ has the greatest effect on maintaining this shape? Experiment with your calculator, systematically removing one term at a time to see how each term influences the shape. Discuss.

In the table below, three graphs are provided for you. You have to see if you can reproduce two similar graphs on your calculator by entering two different equations. Remember that the equations can be entered in factored form as well as in standard form.

Sketch	Equations and their Graphs
	
	
	

ACTIVITY #8 - Polynomial Functions of the Form $f(x) = ax^n$

Purpose: In this activity you will use the TI-82 to explore how the coefficient a and the exponent n affect the shape and direction of the graph of a polynomial function strictly of the form $f(x) = ax^n$.

Objective: At the conclusion of this lab, you should be able to describe the basic shape expected for polynomial functions of the form $f(x) = ax^n$ where n $\in$ 2, 3, 4, and 5. You should also be able to describe what effect a positive "a" value will have on the graph, as opposed to a negative "a" value for each equation and describe how the polynomial graphs are similar or different.

Use your TI-82 to graph each of the following polynomial functions. Complete the chart below and answer the questions that follow. For the columns increases where and decreases where, write your answer algebraically using an inequality symbol. For example, $x \rangle -2$.

Function	a	n	Root(s)	Increases Where	Decreases Where	Sketch
$f(x) = x^2$						
$f(x) = 2x^4$						
$f(x) = -\frac{1}{2}x^2$						
$f(x) = -3x^4$						

What do the graphs of the functions with the even values of n have in common? Be specific.

How do the graphs of the functions with the even values of n differ? What do you suppose is causing these differences in the graphs? Be specific.

Function	a	n	Roots	Increases Where	Decreases Where	Sketch
$f(x) = x^3$						
$f(x) = 2x^5$						
$f(x) = \frac{1}{2}x^2$						

$f(x) = -\frac{1}{2}x^3$						
$f(x) = -2x^3$						
$f(x) = -3x^5$						

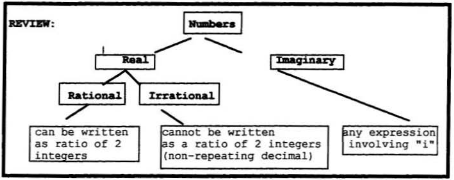
What do the graphs of the functions with the odd values of n have in common? Be specific.

If the function is of an odd degree, how do their graphs differ? You should be able to make two observations. Be specific.

ACTIVITY #9 - The Roots of Polynomials

Purpose: This activity is designed to clarify the relationship between the roots of polynomial functions, the x-intercepts, and whether the roots are rational, irrational, or imaginary.

Objective: At the conclusion of this activity you should be able to determine if a polynomial function has real roots or imaginary roots, and if they are real, whether they are rational or irrational.



a).Graph $y= x^2+x+1$

Find the roots algebraically________

b).Graph $y= x(x^2+1)$

Find the roots algebraically________

c) Graph $y=(x+3)(x^2-7)$

Find the roots algebraically________

d).Graph $y=(x^2-5)(x^2+1)$

Find the roots algebraically________

2. How many:

real roots does #1 have___imaginary roots___total # of roots___

#2 have___imaginary roots___total # of roots___

#3 have___imaginary roots___total # of roots___

#4 have___imaginary roots___total # of roots___

3. How many times does the graph intersect the x-axis?

#1______

#2______

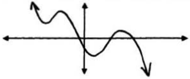
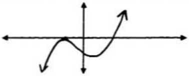
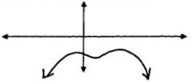
#3______

#4______

4. What do you notice about your answers to questions 2 and 3?

5. What is the relationship between the degree of each polynomial in each example and its total number of roots?

To complete the following table, in the second column list the number and type of roots for each graph.

Graph	Number and Type of Root
	
	
	

ACTIVITY #10 - Single, Double, and Triple Roots of Polynomial Functions

Purpose: To be able to predict the shape of a function if its equation has real roots that are single, double and/or triple.

Objective: At the conclusion of this activity, you should be able to describe the relationship between the degree of a factor and the behavior of the graph at the zero of that factor, specifically for factors of degrees of 1, 2 and 3.

Use your graphing calculator to complete the table.

Function	List Each Linear Factor	List the Roots	Sketch	Appearance of Graph at Each Root (passes through without flattening out, is tangent, or passes through and momentarily flattens out)
$f(x)=x^3-x^2$				
$f(x)=(x+1)^2(x-2)^3$				
$f(x)=-x^4-2x^3$				
$f(x)=x^3(x^2+3x+2)$				
$f(x)=x^3+x^2-4x-4$				

For each function in the table, complete the chart below.

Each Different Factor	Degree of Each Factor	Single/Double/Triple Root
____________	____________	____________
____________	____________	____________
____________	____________	____________
____________	____________	____________
____________	____________	____________
____________	____________	____________
____________	____________	____________
____________	____________	____________
____________	____________	____________
____________	____________	____________
____________	____________	____________
____________	____________	____________

ACTIVITY #11 - Remainder and Factor Theorems

PART A.

Purpose: To discover the relationship that exists between the value of a function for a certain number and division of a polynomial by a linear binomial.

Objective: By the end of the activity you should be able to articulate the relationship that exists between a polynomial function's value for a specific number "a" and division of that polynomial by the linear binomial (x-a).

Enter $f(x) = -3x^4 + 2x^3 - x^2 + 5x + 11$ in your y= list.

1. Divide f(x) by each linear factor listed below. Record the quotient and the remainder.

   (x-2) Quotient ______________ Remainder ____________

   (x+2) Quotient ______________ Remainder ____________

   (x-1) Quotient ______________ Remainder ____________

   (x+1) Quotient ______________ Remainder ____________

2. Go to the homescreen and enter

   2nd VAR 1 1 (the zeroes of the linear factors above) ENTER

   This should look like $y_1(2)$

   Record the input and the output.

   f( ) = ________

   f( ) = ________

   f( ) = ________

   f( ) = ________

3. What do you notice about exercise #1 and #2? Be specific.

4. If any function f(x) is divided by a linear binomial such as (x-a), what will be the remainder?

PART B.

Purpose: To investigate the relationship between a polynomial function's remainder when divided by (x-a) and possible factors of that polynomial.

Objective: When you finish this exercise you should be able to articulate the relationship that exists between certain remainders of polynomials and their factors.

1.a) Use your calculator to compute $6 \div 3$

What is the quotient? _________

What is the remainder? ________

What does this tell you about the number 3?

b) Use your calculator to calculate $377286 \div 546$

What is the quotient? _________

What is the remainder? ________

What does this tell you about the number 546?

2. Enter $f(x) = 2x^4 - 3x^3 - 5x^2 + 9x - 6$ in the y= list.

Compute $f(x) \div (x-2)$ by any method.

What is the quotient? ________________

What is the remainder? _______________

What does this tell you about the linear divisor $(x-2)$?

What is one of the zeroes of $f(x)$? _____________

Use your calculator to compute $f(2)$. ___________

3. Enter $g(x) = x^4 + 3x^3 - 35x^2 - 39x$ in the y= list. Compute $g(x) \div (x+1)$.

What is the quotient? ________________________

What is the remainder? ________________________

What does the answer to the question directly above tell you about the linear divisor $(x+1)$?

4. Turn off or CLEAR f and g. Consider the function $p(x) = (x+8)(x-3)(x-5)(x+4)(x+6)$. Enter $p(x)$ in the y= list. Sketch the graph of $p(x)$. Adjust your window so you get a global picture of the function.

Use TRACE or CALC 2 to find its roots. ______________________________

Use your calculator to evaluate $p(-8), p(3), p(5), p(-4)$, and $p(-6)$ ____

What do you notice?

What does this mean about the linear binomials listed in the original function?

5. If $f(x)$ is divided by $(x-a)$, what will the remainder be? _____________

If $f(a) = 0$, what does that tell you?

If a is one of the x-intercepts of the graph of $f(x)$, then

$f(a) =$ ______?

If a is one of the x-intercepts of the graph of $f(x)$, then

what is one of the factors of $f(x)$?

ACTIVITY #12 - Rational and Irrational Roots

Purpose: To determine whether the roots of a polynomial equation are rational or irrational.

Objective: At the conclusion of this activity, you should be able to look at the graph of a polynomial function on your calculator to determine if the roots of the function are real or imaginary. If they are real, then you should be able to determine if they are rational or irrational. You may do this in either of two ways, by using your calculator or by using the Rational Roots Theorem and synthetic division.

Use your graphing calculator to complete the table.

Equation	Total # of Roots	Sketch	Total # of Real Roots	List the x intercepts
$f(x)=2x^3-5x^2-1$				
$f(x)=3x^3-4x^2-5x+2$				
$f(x)=x^3-x^2+3$				
$f(x)=-x^4-4x+2$				

1.a) List all of the positive and negative factors of the constant term. We'll refer to these as p.

____

____

____

____

b) List all of the positive and negative factors of the leading coefficient. We'll refer to these as q.

____

____

____

____

c) Now list all of the possible p/q combinations, positive and negative, from the lists in a) and b). These are called the possible rational roots of the function.

d) Circle the numbers from the list above that are the same as the x-intercepts in the table for each function.

2.a) How **many** real roots does each function in the table have.

b) How do you know?

c) What are the real roots of all of these functions?

d) If any of the roots are exact, verify that they are listed among the possible rational roots in 1 c) for that function. State yes **or** not in list.

3. What can you conclude about the x-intercepts for each function that are not listed in 1 c) as a possible rational roots?

ACTIVITY #13 - Systems of Equations

Purpose: To determine the solution for a system of polynomial equations.

Objective: At the conclusion of this activity, you should be able to:

1. Solve a function in x in two different ways and describe what you did.
2. Use the calculator to support your algebraic solution
3. Describe the relationship between the algebraic solution set and the points of intersection of the graph of the system.

Use your graphing calculator to sketch each set of graphs on the same axis and find the points of intersection.

Equations	**Sketch**	**Coordinates of each point of intersection**
$f(x) = x^3 + 3x$ $f(x) = 2$		
$f(x) = 2x^3 + 3x^2$ $f(x) = 1$		
$f(x) = x^3 - x$ $f(x) = 3x$		

$f(x)=x^3+4x^2$ $f(x)=3x+18$		

Algebraically, solve each system in the chart. What do you notice about each solution as compared to the points of intersection of the system?

Use your graphing calculator and the Roots option under the CALC menu to solve each equation in the following table .

Equation	Sketch	Roots
$f(x) = x^3 + 3x - 2$		
$f(x) = 2x^3 + 3x^2 - 1$		
$f(x) = x^3 - x - 3x$		
$f(x) = x^3 + 4x^2 - 3x - 18$		

What do you notice about the third column of both tables?

Describe two ways to use your calculator to find the values of x that make the equation $x^2 - 3x + 1 = 2x^2 + x$ a true statement.

Appendix B

Questions

1. Explain as clearly as possible what the following terms, phrases, or directions mean.

   a) Solve the equation for x : $x^2+5x = -6$

   b) Find the solutions for $2x^3-3x^2 = 9x-10$.

   c) State the zeroes of the function $0 = x(x-7)(2+x)$.

   d) Find all the real & imaginary roots for $(x-2)=3-4/(x-3)$.

2. a) What is meant by the term root? Use a diagram to supplement your explanation.

   b) What other words are synonymous with the word root?

Appendix C

Mathematics 3201 - Test One
Unit One - Polynomials

Part A - Objectives. Place your answers on the answer sheet provided. 18 marks.

1. a) Use the **TABLE** feature on your calculator to find the factors of the function $f(x)=x^3-5x^2-8x+12$

   b) Name the **ordered pairs** that you used to identify the factors of $f(x)$.

2. a) What is the **degree** of the polynomial graphed as follows:

   b) How many imaginary roots, if any, does the polynomial function have?

3. Where is the vertex for the function $f(x)=15-2x-x^2$? Describe it as a maximum or a minimum.

4. What one change would you make to the function $f(x)=2x^2+3x+4$ so that it would have two **real** roots?

5. What is the remainder when $2x^7+3x^2+x$ is divided by $(x+1)$?

6. Find the equation for the linear function that passes through $(2,0)$ and $(-2,-16)$.

7. Make up a **quadratic** function such that the sum of its roots would be 5 and the product of its roots would be -12.

8. If the discriminant of $f(x)=ax^2+bx+c$ is)0, how many roots of each type does the function have?

9. If a cubic polynomial has a triple root at 5, what could its equation be?

10. What is the value of the polynomial for x=-3 as indicated by the synthetic division?

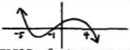
$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -8 & 5 & -1 \\ & & -3 & 9 & -3 & -6 \\ \hline & 1 & -3 & 1 & 2 & -7 \end{array}$$

11. $f(x)=8x^3+9x^2-16x-5$. Evaluate $f(7)$ by using 2nd VARS.

12. Find the value of k if you know that $(x-5)$ is a factor of

$P(x)=\frac{1}{5}x^4-kx-100$.

13. Name two different functions whose graph would look like:



14. The TABLE feature on your calculator contains the following digits for a function in y_1.

x	y_1
2	-1
3	2

What do you know about one of the roots of y_1 ? **Explain**

PART B - Please answer all of the questions that follow on the sheet provided. Be sure to support all of your answers. If are not required to solve a problem by algebraic methods, then you must clearly **explain** how you arrived at your answer. For example, if you choose to do a problem on the calculator, then itemize the steps that you followed.

1. Find the equation of the cubic function that has a double root at -4 if it also passes through (2,0) and (1,50).
   4 marks
2. Algebraically determine all of the real and/or imaginary roots for $f(x)=x^4+2x^3-2x^2-6x-3$
   5 marks
3. Find the point or points of intersection of the two functions
   $f(x)=-2x^3+x^2+3x-1$
   $f(x)=-x+2$
   5 marks
4. Find **two different** polynomial functions of **least degree** with $1+\sqrt{2}$, 0 and 5 as three of its roots. Your answers do not have to be in polynomial form. Include a sketch next to each function so I can see how the functions are similar and how they are different.
   6 marks

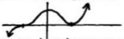
Mathematics 3201 - Test One
Unit One - Polynomials

Part A - Objectives. Place your answers on the answer sheet provided. 18 marks.

1. a) Use the **TABLE** feature on your calculator to find the factors of the function $f(x)=x^3-4x^2-25x+28$

   b) Name the **ordered pairs** that you used to identify the factors of $f(x)$.

2. a) What is the **degree** of the polynomial graphed as follows:



b) How many imaginary roots, if any, does the polynomial function have?

3. Where is the vertex for the function $f(x)=15-2x+x^2$? Describe it as a maximum or a minimum.

4. What one change would you make to the function $f(x)=-2x^2+3x-4$ so that it would have one **real** root?

5. What is the remainder when $2x^7+3x^2+x$ is divided by $(x+2)$?

6. Find the equation for the linear function that passes through $(-2,0)$ and $(2,-16)$.

7. What is the sum and product of the roots for $-4x^3+6x^2-4x-13=0$?

8. If the discriminant of $f(x)=ax^2+bx+c$ is $=0$, how many roots of each type does the function have?

9. If a **quartic** polynomial has a triple root at 0, what could its equation be?

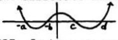
10. What is the quotient indicated by the synthetic division below?

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -8 & 5 & -1 \\ & & -3 & 9 & -3 & -6 \\ \hline & 1 & -3 & 1 & 2 & -7 \end{array}$$

11. $f(x)=8x^3+9x^2-16x-5$. Evaluate $f(-8)$ by using 2nd VARS.

12. Find the value of k if you know that 5 is a root of $P(x)=\frac{1}{5}x^4-kx-100$.

13. Name two different functions whose graph would look like:



14. The TABLE feature on your calculator contains the following digits for a function in y_1.

x	y_1
2	-1
3	2

What do you know about one of the roots of y_1 ? Explain

PART B - Please answer all of the questions that follow on the sheet provided. Be sure to support all of your answers. If are not required to solve a problem by algebraic methods, then you must clearly **explain** how you arrived at your answer. For example, if you choose to do a problem on the calculator, then itemize the steps that you followed.

1. Find the equations of **two** quartic functions that have $1+2i$ and $-\sqrt{5}$ as two of its four roots. Sketch the graph of each function next to its equation.
   4 marks
2. Algebraically determine all of the real and/or imaginary roots for $f(x)=-x^4+2x^3+15x^2-14x-56$
   5 marks

3. Find the point or points of intersection of the two functions
   $f(x)=-2x^3+x^2+3x-1$
   $f(x)=-x+2$
   5 marks
4. Find the value of the integer m so the function $f(x)=-2x^3+mx-32$ has a double root.
   6 marks

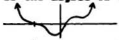
Mathematics 3201 - Test One
Unit One - Polynomials

Part A - Objectives. Place your answers on the answer sheet provided. 18 marks.

1. a) Use the **TABLE** feature on your calculator to find the factors of the function $f(x)=2x^3+3x^2-23x-12$

   b) Name the **ordered pairs** that you used to identify the factors of $f(x)$.

2. a) What is the **degree** of the polynomial graphed as follows:



   b) How many imaginary roots, if any, does the polynomial function have?

3. Where is the axis of symmetry for the function $f(x)=5-2x+x^2$?

4. What one change would you make to the function $f(x)=-2x^2+3x+4$ so that it would have two imaginary roots?

5. What is the remainder when $2x^7+31x^2-x$ is divided by $(x+3)$?

6. Find the equation for the linear function that passes through $(-2,0)$ and $(2,-16)$.

7. Create a quadratic equation whose roots would add up to 6 and would multiply to be 10.

8. If a **quartic** polynomial has a triple root at 0, what could its equation be?

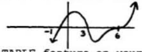
9. Does the synthetic division below indicate that -3 is a root for the given polynomial? Explain.

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & -8 & 5 & -1 \\ & & -3 & 9 & -3 & -6 \\ \hline & 1 & -3 & 1 & 2 & -7 \end{array}$$

10. $f(x) = 8x^3 + 9x^2 - 16x - 5$. Evaluate $f(-8)$ by using 2nd VARS.

11. Find the value of k if you know that 5 is a root of $P(x) = \frac{1}{5}x^4 - kx - 100$.

12. Name two different functions whose graph would look like:



13. The TABLE feature on your calculator contains the following digits for a function in y_1.

x	y_1
2	-1
3	2

What do you know about one of the roots of y_1 ? Explain

PART B - Please answer all of the questions that follow on the sheet provided. Be sure to support all of your answers. If are not required to solve a problem by algebraic methods, then you must clearly **explain** how you arrived at your answer. For example, if you choose to do a problem on the calculator, then itemize the steps that you followed.

1. Find the equations of **two** cubic functions that have $2-\sqrt{3}$ and -3 as two of its three roots. Sketch the graph of each function next to its equation.
   4 marks
2. Algebraically determine all of the real and/or imaginary roots for $f(x) = x^4 + 2x^3 - 2x^2 - 6x - 3$
   5 marks

3. Algebraically determine the point or points of intersection of the two functions $f(x) = x^3 + 4x^2$ $f(x) = 3x + 18$

   5 marks

4. Find a cubic function whose roots are two less than the roots of $f(x) = -2x^3 + 24x - 32$

   6 marks

Mathematics 3201
Polynomials and the Graphing Calculator - Questionnaire

Directions: Answer all of the following questions. Some of the questions are not looking for mathematical solutions, but your descriptions of **how** you would solve the problem or how you would check your answers using the calculator.

1. Suppose you were asked to find a quadratic equation with roots $1 \pm \sqrt{2}$. Once your written solution was complete, **how** would you use your calculator to verify your answer?

2. How do you know for sure if $P(x) = x^9 - \sqrt{5}x^4 + \sqrt{6}$ has a positive real root?

3. Find the point or points of intersection of the two functions $f(x) = x^3 - 2x$ and $P(x) = x^9 - \sqrt{5}x^4 + \sqrt{6}$. **Explain** how you solved this problem.

4. How could you use the calculator to decide if $(x-2)$ is a factor of $P(x) = x^5 - x^4 - 3x^2 - 2$?

5. Find the other root for the quadratic function whose vertex is $(1,12)$ if it also passes through $(-1,0)$. How do you know if your answer is correct?

6. Suppose you were asked to find the value of m in $f(x) = x^3 + 5x^2 + mx + 4$ if $(x+2)$ **is** one of its factors. What would you do with the calculator to determine if you had found the correct value for m?

7. How many quartic functions will have these particular roots? How many quartic functions will have these roots and also pass through $(0,-3)$?
   If you actually had to find the equation, how would you know if your answer was correct?

8. What features of the graphing calculator, if any, helped you learn about polynomial functions? Discuss.

9. At the beginning of the unit, some students expressed a legitimate concern that they would become too reliant on the calculator and would not be able to work problems out on paper. Respond to this concern in light of what we have done over the past several weeks. Do you feel that the calculator enhanced or impeded your understanding of polynomial functions? Discuss.

10. We used the calculator during this unit to investigate the role of a and c for quadratics $y=ax^2+bx+c$. How would you investigate the role of the b? Use the space below to show what you would do to investigate how b affects the graph. What conclusions can you formulate after your investigation?

11. Do you think that you would like to continue to use the graphing calculator in later units of this course if one is available to you? Why or why not? Has it had any affect on your confidence in your solutions?

12. In what capacity, if any, did you find the graphing calculator most useful to you? Discuss.

13. Are there any other comments that you would like to make about using calculators or about this unit?

Appendix D - Attitude Surveys

Student Questionnaire 1

Write any comments you wish to add after any of the questions or add them to the back of this page. Use the following code for your answers:

SA-Strongly Agree D-Disagree A-Agree

SD-Strongly Disagree NS-Not Sure

_______1. I am very familiar with **scientific** calculators and use them all the time in math class and some other classes.

_______2. I have used graphing calculators regularly in past mathematics classes.

_______3. Students should be able to use the graphing calculator any time during math class.

_______4. Students should be able to use the graphing calculator for all homework.

_______5. Students should be able to use the graphing calculator on all tests and quizzes.

_______6. Using a graphing calculator in math class will increase my understanding of the concepts being studied.

_______7. The graphing calculator is used primarily to verify that algebraic work is correct.

_______8. Any problem that can be solved on the calculator can be done using paper and pencil.

_______9. Any problem that can be solved using paper and pencil can be done on the graphing calculator.

_______10.I am a little nervous about using the graphing calculator in my math course.

_______11. I trust the answers that I get on my scientific calculator.

_______12. I am confident that the graphing calculator will help me be more successful understanding polynomials.

_______13. Most math problems can be solved in more than one way.

_______14. Many problems in math can be solved by drawing and interpreting the graph of the problem.

_______15. I have used computers and graphing software before so I think that will help me with the graphing calculator.

_______16. I enjoy math.

_______17. I have a good idea as to what mathematics is all about.

Comments:

Graphing Calculator Questionnaire 2

Read each statement and circle the letter of the descriptor that best describes your reaction.

1. The graphing calculator helps me feel more confident about my solutions.
   A. Strongly agree B. Agree C. Not Sure D. Disagree E. Strongly Disagree

2. I found the graphing calculator to be motivating and interesting.
   A. Strongly agree B. Agree C. Not Sure D. Disagree E. Strongly Disagree

3. Using the graphing calculator has decreased my interest in mathematics
   A. Strongly Agree B. Agree C. Not Sure D. Disagree E. Strongly Disagree

4. The TI-82 is easy to use.
   A. Strongly Agree B. Agree C. Not Sure D. Disagree E. Strongly Disagree

5. In general, I was able to do the lab activities without too much difficulty and answer the questions that followed.
   A. Strongly Agree B. Agree C. Not Sure D. Disagree E. Strongly Disagree

6. When a problem is analyzed both graphically and algebraically, it helps me to understand the underlying mathematical ideas.
A. Strongly Agree B. Agree C. Not Sure D. Disagree
E. Strongly Disagree

7. The graphing calculator has helped me to understand at least one mathematical idea/technique that I didn't understand before.
A. Strongly Agree B. Agree C. Not Sure D. Disagree
E. Strongly Disagree

8. Because of the graphing calculator, I find myself exploring a mathematical problem rather than just trying to get the solution.
A. Strongly Agree B. Agree C. Not Sure D. Disagree
E. Strongly Disagree

9. I like to use the graphing calculator to check my work.
A. Strongly Agree B. Agree C. Not Sure D. Disagree
E. Strongly Disagree

10. I would recommend the graphing calculator to my friends, especially since we can use it for the public exam.
A. Strongly Agree B. Agree C. Not Sure D. Disagree
E. Strongly Disagree

11. I can recommend some changes to a lab activity that would make it clearer for the students using the manual.
A. Strongly Agree B. Agree C. Not Sure D. Disagree
E. Strongly Disagree

Comments: