A DESCRIPTIVE STUDY OF GRADE FOUR AND GRADE SIX STUDENTS' UNDERSTANDING OF BAR GRAPHS

JUDITH MARY MELLOR, B.Sc., B.Ed.

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BY

©Judith Mary Mellor, B.Sc., B.Ed.

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ABSTRACT

The purpose of this descriptive study was to investigate grade four and grade six students' understanding of the information conveyed by bar graphs. In particular, the effects of various characteristics of familiarity of the graph topic, the arrangement of the data, and scale on students' ability to read, interpret and predict from bar graphs were examined.

Five elementary schools participated in the study. This resulted in a sample of 121 grade four students and 127 grade six students. Each student was administered a written test designed by the author which consisted of four bar graphs with three questions per graph. On the basis of the written responses, 35 students from grade four and 37 students from grade six were given audiotaped interviews to obtain additional information. The responses for each graph were then categorized, and the major errors were discussed in terms of the frame theory model developed by Davis (1984).

At least 15 types of errors were documented. While some of these were reading-language and computation errors, the majority were graph-based errors. Four general categories were identified namely: data arrangement, topic, scale, and the fact the information was not shown on the graph.

Overall, students at both grade levels had little difficulty reading bar graphs, more difficulty interpreting bar graphs, and had major difficulty knowing when prediction from bar graphs was possible. The frequency of reading-language, computation, and particularly scale errors was higher at the grade four level than at the grade six level.

However, errors involving pattern arrangements of the data occurred in similar frequencies for both grades and it was concluded that both grade four and grade six students have similar but flawed graph frames.

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CHAPTER I

INTRODUCTION

Today it is recognized that every adult should be able to effectively process the large amounts of information encountered in everyday life. Such information is frequently in graphical form with business, government and the news media all utilizing graphs to display information. For example, such information might consist of statistical data on consumer sales, the national budget, or unemployment. The value of the use of graphs in displaying information has been described by Weintraub (1967):

> They present concepts in a concise manner or give at a glance information which would require a great deal of descriptive writing. They often distil a wealth of information into a small amount of space. (p. 345)

Furthermore, the ease by which graphs can now be produced by computers has led to their increased use by society. This increased use implies that schools need to help students become competent in utilizing graphs to their maximum potential.

Educational authorities have recognized for some time that it is not sufficient for students just to be able to directly read information from a graph. In a position paper on basic skills in mathematics, the National Council of Supervisors of Mathematics listed reading and drawing conclusions from graphs as one of ten vital skills (NCSM, 1977). The National Council of Teachers of Mathematics in An Agenda for Action, (NCTM, 1980) called for an increased emphasis on drawing inferences and predicting from data. This increased emphasis is reflected in the recently released NCTM Standards document (1989) where graphing is included in the probability and statistics standards. The value of progressing beyond just reading information directly from graphs has been reinforced by Kirk, Eggen, and Kauchak (cited in Curcio, 1987) when they stated that the maximum potential of a graph is actualized when the reader is capable of interpreting and generalizing from the data presented.

Despite the suggestion of many educational organizations that increased attention be paid to the development of graphing skills, the results of the Fourth National Assessment of Educational Progress indicated that students at both the elementary and secondary levels have difficulty with items that require more than a literal reading of the graph (Brown et al., 1988; Kouba et al., 1988). This deficiency was also noted in the three previous National Assessments of Educational Progress (Carpenter, Coburn, Reys & Wilson, 1978; Carpenter, Kepner, Corbitt, Lindquist & Reys, 1980; Carpenter, Lindquist, Mathews & Silver, 1983; Lindquist, Carpenter, Silver and Mathews, 1983).

Significance of the Study

To date there has been little research on graphing. As Kosslyn and Pinker (1983) stated:

> Even a casual perusal of the literature immediately convinces one that there is a real need for research on charts and graphs, and that there is a real need for a systematic approach to the topic. Research on charts and graphs is, in a word, scanty. (p. 6)

The limited research available indicates that particular features of the content and presentation of graphs can contribute to students' difficulties in understanding the information displayed in a graph.

There is a need for more research to determine specific student errors in graphing and the factors which contribute to these errors. Information resulting from this research would be valuable to teachers in planning instruction and in helping students to overcome their difficulties.

Purpose of the Study

This study investigated the effects of various characteristics of graphical displays on grade four and grade six students' ability to read, interpret, and predict from bar graphs. Specifically, it attempted to answer the following questions.

Question 1: What difficulties do grade four and grade six students have in reading, interpreting and predicting from bar graphs?

Question 2: What differences exist between grade four and grade six students' ability to read, interpret, and predict from bar graphs?

Definition of Terms

The following terms were used throughout this study and are clarified here.

Bar graph. A bar graph is the "graphical representation of frequencies and magnitudes by rectangles drawn with lengths proportional to the frequencies or magnitudes concerned" (Kendall & Buckland, 1982, p. 13). The rectangles are contained within perpendicular labelled axes and each rectangle is separated from the one next to it. This study was limited to vertical bar graphs.

Read from a bar graph means to obtain facts that are explicitly stated on the graph.

Interpret from a bar graph means to generate information using the four basic mathematical operations. In this study the operations were limited to addition and subtraction.

Predict from a bar graph means to make generalizations based on the graphical representation of the data.

Theoretical Model

The theoretical model used in this study to interpret student responses is that developed by Davis (1984). This model uses the concept of a frame - a knowledge representation structure that is stored in memory - to describe how people process information. In terms of the model, processing of information from a source starts with the selection of a cue from the information which results in selection of a frame from memory. Data from the source is then mapped to the variables or slots of the frame, hence the general frame information and the information from the source are brought together. This "instantiated" frame is then used as a data base for decisions.

By examining the students' responses to a variety of graphical displays it is possible to create a description of a graphical frame. Errors can then be described and explained in terms of the inadequacies of the frame. For example, students make errors when faced with variable scales within the context of different graphical problems. By examining students' responses on questions on reading, interpreting, and predicting from graphs with different scale factors, it may be possible to determine whether the errors are the result of an incomplete frame which is incorrectly completed, an incorrect default evaluation, or a complete but incorrect frame. Such information, particularly when it results from the examination of performance at different grades, helps formulate a "picture" of the development of frames and can provide a partial basis on which to build appropriate activities to correct the errors. This "picture" can also provide a basis for the design of materials to be used to teach graphical concepts.

Limitations of the Study

Any research study has inherent limitations that restrict the generalizability of and interpretation of the results. Two specific limitations are discussed in this section.

This study was limited to the vertical bar graph as a graph form and to two grades, four and six. Consequently, the generalizability of the results to other graph forms and to other grade levels is restricted.

Although the sample for the study was not randomly selected there is no reason to believe it is not representative of grade four and six students. However, the use of interviews with a selected subset of the sample restricts the interpretation of the results. It is difficult to establish to what extent the students interviewed are representative of the sample as a whole. It is also possible that the students might have had difficulty verbalizing the actual thought processes they had used.

Summary

The increased recognition of the importance of graphing skills and the lack of studies on graphing indicate a need for more research in this area. Furthermore, the development of information processing models such as that of Davis (1984) provides a framework within which it is possible to explain how children process information in graphical form.

The purpose of this study was to investigate grade four and grade six students' understanding of the information conveyed by bar graphs. In particular, this study examined the effects of various characteristics of graphical displays on students' ability to read, interpret, and predict from such displays.

Before attempting specifically to apply frame theory to the comprehension of bar graphs, a review of the related literature on graphing is presented.

on the data in each graph, these students had to assess the validity of the two generalizations. The control group were asked to merely inspect the results of the experiment. A post test designed to measure general inspection of the graphs showed the control group scoring higher than the experimental group on all items except for one on generalizations. This meant the generalization "cues" were a detriment to overall learning rather than an aid.

Other Studies on Graphing

Many of the early studies on graphing were designed to dotermine the relative effectiveness of different methods of representing quantitative data (Washburne, 1927; Thomas, 1933; Peterson & Schramm, 1954; Culbertson & Powers 1959; Feliciano, Powers & Kearl, 1963). These studies were not always restricted to graphs and often included tables. However, all the studies included bar graphs as one of the graphic forms. The populations used in these studies were quite diverse: elementary and/or junior high students (Washburne, 1927; Thomas, 1933), high school students and women homemakers (Feliciano et al., 1963), recent high school graduates in the farming occupation (Culbertson & Powers, 1959), and male Air Force entrants (Peterson & Schramm, 1954). Many of these studies ordered the graphic forms according to difficulty. For example, Thomas (1933) found that children could read most easily pictorial graphs, followed by pie graphs, two dimensional graphs, and finally line graphs. MacDonald-Ross (1977) reviewed the more extensive of these studies, those by Washburne (1927), Culbertson & Powers (1959), and Feliciano et al. (1963). He found the conclusion of Washburne (1927) that no one graphic form was superior in all respects to other forms to be justified. Some conclusions pertaining to bar graphs from these studies were given. For example, Culbertson and Powers (1959) concluded that for the evaluation and comparison of specific quantities both horizontal and vertical bar graphs were easier to read than line graphs. They also found that horizontal bars were preferable to vertical bars in that they provided more room for labelling. Feliciano et al. (1963) found that for their general audience, horizontal bar graphs produced better test scores than long or short tables or text, and that scores improved when a horizontal bar graph accompanied by text was used.

More recently, efforts have been made to conduct studies that would assess students' competencies in graphing. While some of these studies include bar graphs, a separate detailed analysis was generally not available for this graphical form.

Wainer (1980) administered a test (table, line graph, bar graph, pie graph) to third through fifth graders to measure their "graphicacy". Three types of question were used: elementary - requiring the extraction of exact information, intermediate - requiring the detection of trends, and comprehensive - requiring the comparison of whole structures. Third graders had considerably more difficulty than fourth and fifth graders with the graphs, but there was only a slight difference in the performance of the fourth and fifth graders.

Curcio (1981, 1987) administered a test (pictograph, bar graph, circle graph, line graph) to fourth and seventh graders. The questions were designed to reflect three levels of graph comprehension: the ability to read the data, between the data, and beyond the data. The grade seven students performed better on the test than the grade four students. Graph comprehension for the grade fours was found to be related to both reading and mathematics achievement, and to prior knowledge of the topic, math content and graph form. A similar relationship was found for the grade sevens except that prior knowledge of the topic and graph form were not included. While no correlation was found between sex and graph comprehension for the grade four students, a low but significant correlation was found for the grade sevens. In a follow-up to the Curcio (1981) study, Curcio and Smith-Burke (1982) undertook a task-based interview study to examine how grade four and grade seven students process information in graphical form. They found that the students tended to be very persistent in their errors despite additional information or countersuggestions by the interviewers. One of the graphs used in this study was a bar graph showing the height of children that had been designed with the height measurements decreasing from bottom to top on the vertical axis. It was reported that "a number of fourth and seventh graders failed to note, process, and/or adjust for inconsistent information on the redesigned graph on height" (p. 20).

Padilla, McKenzie and Shaw, Jr. (1986) used the Test of Graphing in Science (TOGS) to investigate the line-graphing ability of students in grades seven through twelve. The TOGS is a multiple choice test developed by McKenzie and Padilla (1986) to measure subskills necessary for line graph construction and interpretation. They found that the grade seven and the grade eight students scored lower than the high school students. Starting with the grade nines, an increase in the mean scores (with the exception of grade eleven) was noted. The students performed best on the subskills of plotting points and determining coordinates, and most poorly on the subskills of scaling axes and using a best fit line.

Wavering (1989) used a test requiring construction of three different line graphs with students in grades six through twelve. The responses were classified into one of nine categories ranging from no attempt to make a graph to a complete graph with a statement of the relationship between the variables. The categories in between represented increasingly more successful attempts at ordering the data, scaling the axes, and recognition of a relationship between the variables. Middle school students generally gave responses in the first four categories while high school students gave responses in categories five through nine.

Overall, these studies indicated that graphing ability increased with grade level. Students were persistent in their errors and had the most difficulty answering questions requiring higher level cognitive skills; namely those that require more than a literal reading of the graph. Furthermore, these problems seem to exist for all the types of graph studied.

The only category of graphs that have been studied extensively is graphs of physical phenomena (Kerslake, 1977; Bell & Janvier, 1981; Clement, 1985; Barclay, 1985; Clement, Mokros, & Schultz, 1985; Mokros & Tinker, 1987). Two major categories of errors have been noted in both school and college populations. These are confusing slope with height, and confusing the graph of an event with a picture of the event. For example, Clement et al. (1985) gave grade seven and eight students a problem dealing with graphs of temperature vs. time of day and found that they confused the highest and lowest points on the graph with where the temperature was rising or falling most rapidly. Mokros and Tinker (1987) reported that many grade seven and grade eight students when asked to draw a speed vs. time graph for a bicycle travelling uphill, downhill, and over a level stretch, simply drew a picture of a hill. Microcomputer-based labs have been shown to be successful in reducing these errors (Barclay, 1985; Linn, Layman & Nachmias, 1987; Mokros & Tinker, 1987).

Teaching of Graphing Skills

The literature contains advice on how to teach graphing skills. For example, a detailed "how to do it" approach to interpreting and constructing bar graphs, pictographs, line graphs, and circle graphs was given by Hawkins (1980). This included steps for the construction of the graphs followed by a strategy to teach students how to interpret graphs. The teaching strategy involved guiding students from concrete, specific encounters with the data to those requiring higher cognitive ability. Guidance through this analytic process is provided by the teacher asking questions designed to induce specific thinking tasks. The teaching strategy required four levels of questions, with the highest level requiring students to summarize, conclude and generalize from the data.

A study by Kauchak, Eggen and Kirk (cited in Eggen et al., 1978) supported the idea that questions can be used to increase the amount learned from graphs. The use of structured questions to induce specific thinking tasks is also supported by the NCTM (1989) and forms the basis of a recent elementary and middle school activity book by Curcio (1989) for developing graphing skills.

Weintraub (1967) in reviewing some of the early works on graphing concluded that the skills of reading and interpreting from graphs must be taught. Furthermore, he suggested a developmental sequence for teaching the various types of graphs. He suggested starting with pictographs, then circle or pie graphs, vertical bar graphs, horizontal bar graphs, two dimensional graphs, and concluding with line graphs.

Padilla et al. (1986) wrote that graphing skills should be emphasized in both the science and math curriculum starting in the early grades. In particular, attention should be focused on graphing subskills that are known to cause difficulty. For example, students have difficulty scaling axes and using a best fit line.

Curcio (1987) suggested that students should be involved in graphing activities to build and expand the relevant schemata needed for graph comprehension. Specifically, she recommended that children should collect their own data, and should be encouraged to verbalize the relationships and patterns observed in it.

Wavering (1989) suggested the teaching of graphing begin below the sixth grade with seriation activities, one-to-one correspondence activities, and with recognition of patterns. Formal graphing skills should be taught in the later grades using data from student experiments that can be graphed using the reasoning processes that students are developing during that time.

The use of microcomputer-based labs with middle school students indicates that graphing skills are improved with their use (Barclay, 1985; Mokros & Tinker 1987; Linn et al., 1987). The results of these studies indicated a decrease in errors with graphs of physical phenomena when microcomputer-based science labs are used. However, not all researchers support the use of microcomputers to facilitate graphing. For example, Wavering (1989) stated "with the increasing use of computers to generate graphs for students, if students are not given opportunities to work their way through their own graphs, it could be that logical development and understanding of graphing may be short-circuited" (p. 379).

While the teaching suggestions are consistent with the limited research on graphing, there is a need for more extensive research to provide a broader foundation for instructional development.

Familiarity of Topic, Arrangement of Data, and Scale

Familiarity of Topic

The idea that the familiarity of the topic of a graph can affect understanding of the graph has been acknowledged for some time. The belief that familiarity with the topic improved graph comprehension resulted in attempts in some of the early studies on the relative effectiveness of various data forms to control for familiarity of topic. For example, Washburne (1927) used graphs and text on the economic history of Florence in order that the topic might be equally unfamiliar to all students. In another study Culbertson and Powers (1959) instructed students to answer from the graphs provided and not from their prior knowledge of the topic.

The idea that students' graph comprehension improves when the topic is familiar has been examined empirically by Curcio (1981, 1987) and Curcio and Smith-Burke (1982).

Curcio (1981, 1987), showed that familiarity with the graph topic does contribute to a students' ability to understand a graph. Specifically, fourth graders were found to rely more on the topic of a graph for its meaning than the seventh graders. An exploratory descriptive study by Curcio and Smith-Burke (1982) involving fourth and seventh graders provided additional information on how prior knowledge of the graph topic affects graph comprehension. Some aspects of prior knowledge of the graph topic which resulted in errors were; failure to use prior knowledge when required, use of inadequate prior knowledge, relying solely on prior knowledge and not on the graph, and being led astray by prior knowledge. For example, on a graph showing the average time of sunset for June through December, a student gave an incorrect response by using prior knowledge of the path of the sun setting. Being led astray by this prior knowledge was prompted by the appearance of the graph which reminded the student of the way the sun sets.

Studies on the understanding of graphs of physical phenomena have indicated that students frequently make the error of confusing the graph with a picture of the event. One interpretation of this error is that the students familiarity with the topic interferes with their comprehension of the graph. For example, Clement, Mokros and Schultz (1985) stated there is a "tendency to incorrectly superimpose existing knowledge about a physical phenomena upon a graphing problem" (p.1). Bell and Janvier (1981) noted the same error and referred to the situation in the graphical problem as a "distractor". They proposed that "graphs should be introduced and analyzed in graphical terms without reference to situations" (p. 41).

Overall, there is evidence that familiarity of the graph topic does affect graph comprehension.

Arrangement of Data

Predicting from data requires recognizing patterns or trends in the data. The test of graphicacy used by Wainer (1980) and the Test of Graphing in Science (TOGS) used by Padilla, McKenzie, and Shaw Jr (1986) both included items which required students to detect trends shown in the data. It was a conclusion of both studies that students had more difficulty with these items than those that required a direct reading of the graph. Specific details on the nature of these difficulties was not reported in either study.

In an earlier study, Washburne (1927) pointed out that "both the logical and visual arrangement of data have an important effect on learning" (p. 374). This conclusion was reviewed by MacDonald-Ross (1977) and found to be justified.

There is limited evidence that students have difficulties detecting patterns and trends in data, and that the arrangement of the data has an effect on the students' comprehension of the graph.

Scale

Kerslake (1977) was involved in a study of students' understanding of graphs for the project Concepts in Secondary Mathematics and Science based at Chelsea College, London. One of the questions on a graphing test given to students aged 12-15 tested the students' responses to a change in scale. Students were presented with three line graphs, two of which represented the same information but with a change in scale, and asked to select the two which represented the same information. Most of the incorrect responses were from those who chose the graphs with the greatest superficial resemblance.

Padilla et al. (1986) using the Test of Graphing in Science (TOGS) found that for line graphs students were successful on only 32% of the items on scaling of axes. Specific details on the nature of the difficulties were not reported. A study by Wavering (1989) showed that when middle school students were asked to construct line graphs from the data given they made virtually no attempt to scale on either axis of the graph. Ninth, tenth, eleventh and twelfth graders efforts ranged from partially scaling to complete scaling of the data on both axes.

Huff (1954) discussed how graphs can be used effectively to misrepresent data. One specific aspect of the graph that was discussed was scale, and he showed how changing the scale of a graph can change the visual appearance of the data to the point where its message is misconstrued.

In conclusion, this review of the literature indicates that there is very little research on graphing, particularly on bar graphs. The limited research available indicates that students have difficulties answering questions that involve more than a literal reading of the graph, but does not give details of these difficulties. There is also some evidence that familiarity of the graph topic, arrangement of the data, and scale can affect students' comprehension of a graph. The present study extends the previous research by providing a description of specific errors in comprehending bar graphs and the factors which contribute to these errors. Furthermore, it provides additional information on the effect of familiarity of topic, arrangement of the data, and scale on the understanding of the information conveyed by bar graphs.

CHAPTER III

DESIGN AND PROCEDURE

The purpose of this study was to investigate students' understanding of the information conveyed by bar graphs. This chapter describes how the research was conducted and includes a description of the population, sample, pilot study, procedure, and method of analysis.

Population and Sample

The population for this study consisted of students at the grade four and grade six level. Two local school boards provided a sample consisting of students from five schools. One school was selected for the pilot study and the other four for the main study.

The sample for the main study consisted of 121 students in grade four and 127 students in grade six. The interview component was a subset of these students: 35 from grade four and 37 from grade six.

Pilot Study

The pilot study was conducted in November 1989. This phase involved 30 students in grade four and 46 students in grade six. All students completed a written test designed by the researcher to measure students' ability to read, interpret, and predict from bar graph displays. On the basis of their performance on the written test, five students from grade four and six students from grade six were then selected for follow-up interviews which were audiotaped.

The major aims of the pilot study were:

1. To determine if the allotted time for the test items was sufficient.
2. To determine if the wording of any specific items created difficulty for the students.
3. To determine if the overall level of difficulty of the test items was appropriate for the grade levels concerned.
4. To form a preliminary error categorization system.

Following completion of the pilot study minor modifications in the format of some of the graphs were made. Some graphs were eliminated from the main study since they did not provide any additional information to that obtained from other graphs. An error categorization system was formulated and will be discussed under the main study.

Main Study

Procedure

Students at all schools were aware in advance that they would be participating in a mathematics research study. On meeting the students the researcher informed them of the purpose of the study and that it involved a written component and for some students a short interview at a later date.

Written component

Following the initial briefing, a written test was administered by the researcher. Each individual test consisted of four bar graphs with three questions per graph. Once they had received the papers students were told to examine each graph carefully and to answer as many questions as they could. They were advised to move on to the next question or graph if they had prolonged difficulty with any part of the test.

The tests were compiled in such a manner that twenty subsets of four of the twelve graphs used in the study were ordered seven different ways. This was done to minimize any schooling effect as well as any possible boredom effect.

Interviews

After the tests had been completed an initial perusal of the written responses was made and students selected for interview. The selection was made so that a variety of different responses were chosen to obtain as complete a spectrum of results as possible. The interviews were audiotaped, did not normally exceed fifteen minutes, and were conducted within three school days from the date of the test.

At the start of the interview students were told by the researcher that they would be taken through the test and asked to explain their answers. The student's written test was then placed in front of him/her. The questions were read out loud by the interviewer with the students supplying answers and explanations for the answers. If the students appeared frustrated with a question or engaged in long periods of silence they were given the option of proceeding to the next question. In some cases the students were asked additional questions to those on the paper. These questions generally pertained to the reasonableness of the answer given or graphing in general.

Analysis

Initially the type and nature of the errors were categorized according to the error categorization system developed from the pilot study. After this initial categorization the errors were analyzed within Davis's frame theory. Finally, the research questions were answered.

Question 1: What difficulties do grade four and grade six students have in reading, interpreting and predicting from bar graphs?

The overall mean success rates of each grade for the read, interpret, and predict questions were calculated. The major error types for each question category were summarized in terms of Davis's frame theory.

Question 2: What differences exist between grade four and grade six students' ability to read, interpret and predict from bar graphs?

A comparison of the mean success rates of each grade for the read, interpret, and predict questions was made. The similarities and differences in the major error types for each question category were noted.

Error Categorization System

Reading-language or Computation Errors (R/C)

Reading-language Errors

a) Student does not understand the question.

Example: When the total is required the largest frequency is given instead.

b) Student incorrectly reads values from the graph.

Example: The value 350 is read from the graph as 330.

Computation Errors

c) Student uses the wrong operation.

Example: Values from the graph are added instead of subtracted.

d) Student makes an error in performing the required operation.

Example: 500 - 300 = 100

Graph-based Errors (G)

a) Student does not attend to scale.

Example: Each horizontal space or guideline is counted as one unit. No reference is made to the scale indicated on the vertical axis.

b) Other scale errors.

Example: Each horizontal space or guideline is counted as representing 10 cm instead of 20 cm.

c) Student does not understand that it is sometimes possible to predict from graphs.

Example: An answer of "No, because it is not there (on the graph)" is given to predict questions.

d) Student does not make a prediction when one is warranted by the graph topic and the pattern in the data.

Example: A prediction for a child's allowance in 1990 (patterned allowance data for 1986-1989 was given) is not made.

e) Student forces a pattern on the data and follows it when asked a predict question.

Example: When faced with non-patterned data a "pattern" is made up. A value for an item not shown on the graph is obtained by following this pattern.

f) Student follows an existing pattern in the data when either the graph topic does not allow for prediction or the outcome would not be reasonable.

Example: By following a pattern in the data a value is obtained for the height of a ten-year old height which is greater than that shown of a nineteen-year old.

g) Student believes the absence of a pattern is the sole reason for their inability to predict from a non-patterned graph even though the graph topic does not allow for prediction.

Example: An answer of "No, because there is no pattern" is given to a question asking if it is possible to predict a value for an item not shown on the graph. The student states that a value could have been obtained had the data been patterned.

h) Student sees the graph as a picture.

Example: Each bar in the graph is seen as a picture of an item such as a tree.

i) Other graph errors involving pattern not included in a-h above.

Example: Refusal to predict a correct value for an item not shown on the graph on the basis that it would break an existing pattern in the data.

Topic Errors (T)

a) Students misuse their prior knowledge of the topic in their attempts to predict from the graph.

Example: Toronto is a large city, so any numerical values associated with the city will also be large.

b) Student believes their lack of prior knowledge of the topic results in their inability to predict from the graph.

Example: Not having visited Toronto, it is not possible to predict any values associated with the city.

Unexplained Error

Student gives an incorrect answer which does not fit any of the above categories. These errors were often unexplainable by the students themselves during their interview.

<u>Incomplete answer</u>

a) Student gives the desired yes response to the predict question but does not indicate a value.

Example: An answer of yes it is possible to predict Jane's allowance in 1990 is given, but the 1990 allowance value is not supplied.

b) Student gives the desired no response but with insufficient explanation.

Example: "No, because it is too hard" and "No, because I don't know".

CHAPTER IV

ANALYSIS OF DATA

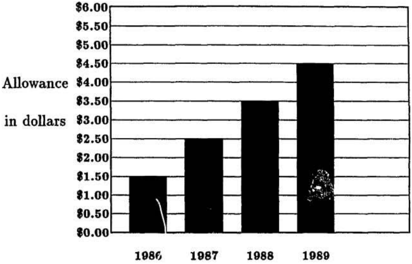
The purpose of this study was to investigate grade four and grade six students' comprehension of the information conveyed by bar graphs. This Chapter presents the analysis of the data in three parts. The first summarizes student performance on the twelve graphs using the error categorization system described in Chapter III and includes a description of the errors. The second describes the major types of errors in terms of Davis's frame theory. The final section examines the data in terms of the stated research questions.

Summary of Student Performance on Individual Graphs

To check the reliability of the researcher's coding the following procedure was undertaken. A random sample of 30 graphs (15 selected randomly from each of grade four and grade six), together with the coding scheme were given to a colleague. The colleague coded these graphs independently of the researcher and then compared the results. This resulted in an intercoder reliability of 88%.

JANE'S ALLOWANCE

The following graph shows Jane's allowance for the last four years.



1. In what year was Jane's allowance $3.50? ______

2. How much more allowance did Jane get in 1988 than in 1986? ______

3. Can you tell what Jane's allowance is in 1990?
   Circle **YES** or **NO.**

   If you circled **YES**, draw the bar for Jane's allowance in 1990 on the graph and explain your answer. ______________________________

   ____

   If you circled **NO**, explain why you circled **NO.** ______________________________

   ____

Figure 1. Graph I Jane's Allowance (patterned)

Graph I

This graph shows allowance in a patterned format (See Figure 1).

Interviews: Grade four-15 Grade six-12

Question 1. (Read)

Results:

Table 1

Graph I Literal Reading Question

Response	Grade Four (n=55)	Grade Six (n=57)
Correct Answer	53 (96%)	55 (96%)
R/C - a	1 (2%)	---
R/C - b	---	1 (2%)
No response	1 (2%)	1 (2%)

Very few students had difficulty at the literal level, with 96% of the grade four students and 96% of the grade six students obtaining the correct answer (See Table 1).

Question 2. (Interpret)

Results:

The most common reading-language error was to give either the allowance value for 1986 or 1988 (R/C-a), (16% grade four; 4% grade six) with a few students saying 1987 or both values (See Table 2). A small percentage of students made computational errors, either adding instead of subtracting (R/C-c) or making other

Table 2

Graph I Interpret Question

Response	Grade Four* (n=55)	Grade Six* (n=57)
Correct Answer	26 (47%)	48 (84%)
R/C - a	10 (18%)	3 (5%)
R/C - c	3 (5%)	---
R/C - d	2 (4%)	1 (2%)
G - a	3 (5%)	2 (4%)
G - b	7 (13%)	1 (2%)
Unexplained error	3 (5%)	1 (2%)
No response	1 (2%)	1 (2%)

* Total not 100% due to rounding.

computational errors (R/C-d).

All the graph based errors involved scale. Five percent of grade four and 4% of grade six students indicated that they had not attended to scale (G-a) but had, instead, counted the horizontal spaces or guidelines for the interval 1986-1988 saying the answer was $4.00. Thus, they counted each space as a unit of one dollar. The interviews revealed another scale error (G-b) in which students added all or some of the values on the vertical axis. Some previously unexplained larger values were then placed in this category which then accounted for 13% of the grade four errors compared to only 2% of the grade six errors.

Overall, the grade four students performed poorly on this question (47% correct) relative to the grade six students (84% correct). The poorer performance of the grade fours appears to be due to their difficulty understanding the question and their difficulty with scale.

Question 3. (Predict)

Results:

Table 3

Graph I Predict Question

Response	Grade Four (n=55)	Grade Six* (n=57)
Correct Answer	42 (76%)	46 (81%)
R/C - b	1 (2%)	---
G - c	7 (13%)	6 (11%)
G - d	---	1 (2%)
Incomplete - a	---	1 (2%)
Unexplained error	3 (5%)	2 (4%)
No response	2 (4%)	1 (2%)

* Total not 100% due to rounding.

The majority of students, 76% of grade four and 81% of grade six students, had no difficulty predicting Jane's allowance in 1990 to be $5.50 (See Table 3). Both in their written comment and in the interviews, however, it was clear that not all students had been attending to the scale of the graph. This was evidenced by comments such as "the graph goes up by two spaces", "her allowance goes up $2.00", "it went up three (lines)", and "her allowance increases fifty cents" as explanations for how they arrived at the correct answer.

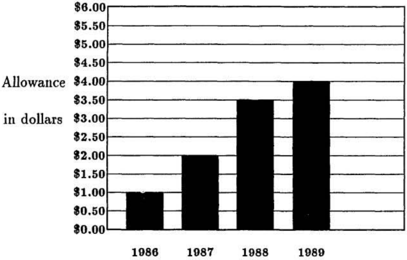
Almost all graph based errors were a response of the form "No, because 1990 is not shown on the graph" (G-c) which was given by 13% of the grade fours and 11% of the grade sixes. In interviews when students were encouraged to re-examine the graph, some still insisted a prediction was not possible or spoke of the pattern and an uncertainty of whether to follow it. For example, one student said "she could get $5.50

but I am not sure". The only other graph based error was a response by a student that "many answers were possible" (G-d).

Overall, students in both grades performed well on this question.

JANE'S ALLOWANCE

The following graph shows Jane's allowance for the last four years.



1. In what year was Jane's allowance $3.50? ______

2. How much more allowance did Jane get in 1988 than in 1986? ______

3. Can you tell what Jane's allowance is in 1990?
   Circle **YES** or **NO.**

   If you circled **YES**, draw the bar for Jane's allowance in 1990 on the graph and explain your answer. ____________________

   ____

   If you circled **NO**, explain why you circled **NO.** ____________________

   ____

Figure 2. Graph II Jane's Allowance (non-patterned)

Graph II

This graph is a non-patterned version of Graph I. The accompanying questions are the same (See Figure 2).

Interviews: Grade four-16 Grade six-20

Question 1. (Read)

Results:

Table 4

Graph II Literal Reading Question

Response	Grade Four* (n=55)	Grade Six (n=58)
Correct Answer	52 (95%)	58 (100%)
R/C - a	1 (2%)	---
R/C - b	2 (4%)	---

* Total not 100% due to rounding.

The results of this question are given in Table 4 and were similar to those in Graph I, with students performing very well.

Question 2. (Interpret)

Results:

The percentage of students in each grade who gave correct answers are given in Table 5 and was similar to that in Graph I.

As was the case in Graph I, the most common error was giving the allowance figure for either 1986 or 1988 (R/C-a). Some computational errors (R/C-d) were again

Table 5

Graph II Interpret Question

Response	Grade Four* (n=55)	Grade Six (n=58)
Correct Answer	25 (45%)	46 (79%)
R/C - a	6 (11%)	4 (7%)
R/C - d	2 (4%)	3 (5%)
G - a	14 (25%)	2 (3%)
G - b	3 (5%)	3 (5%)
Unexplained error	3 (5%)	___
No response	2 (4%)	___

* Total not 100% due to rounding.

present but the error of adding the 1988 and 1986 allowance, (R/C-c), noted in Graph I was not present.

As in Graph I, all graph based errors were scale errors. Most noticeable here was that 25% of the grade four students compared to only 5% of the grade four students on Graph I did not attend to the scale (G-a). Most of the students who did not attend to scale gave an incorrect answer of $5.00, with a few saying $4.00 or $6.00. The interviews revealed that students had obtained these answers by counting correctly or incorrectly either the horizontal spaces or guidelines on the graph for the interval 1986-1988.

Overall, the grade four students performed poorly on this question (45% correct) relative to the grade six students (79% correct). The grade fours made far more errors of the form (G-a), 25%, compared to only 3% for the grade sixes.

Question 3. (Predict)

Results:

Table 6

Graph II Predict Question

Response	Grade Four (n=55)	Grade Six* (n=58)
Correct Answer	10 (18%)	11 (19%)
R/C - a	---	2 (3%)
G - c	8 (15%)	8 (14%)
G - e	29 (53%)	28 (48%)
T - a	3 (5%)	---
Incomplete - b	1 (2%)	7 (12%)
No response	[illegible] 7%)	2 (3%)

* Total not 100% due to rounding.

Students had difficulty with this question with only 18% of grade four students and 19% of grade six students giving the correct answer (See Table 6). Those who gave the correct answer gave reasons such as "because there is no pattern", "each year her allowance goes up a different amount", "it might be anywhere". The actual success rate for this question might be higher as statements such as No, I don't know", "No, it's too hard", and "No, because you can't tell", were classified as incomplete. However, the overall performance would have still been poor.

A small percentage, 3%, of grade six students answered both yes and no. While these students were not among those interviewed their response was interpreted as an indication that they had not understood the question (R/C-a).

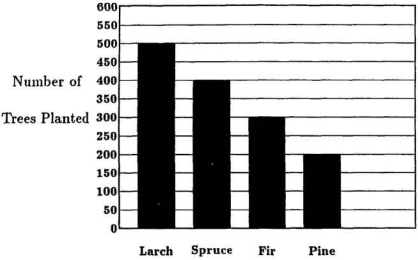
There were 15% of the grade four students and 14% of the grade six students who indicated that it was not possible to obtain an answer because 1990 was not on the graph (G-c). These figures were similar to those seen in Graph I. The most common graph-based error was to give a value for the 1990 allowance. There were 53% of the grade four students and 48% of the grade six students who were willing to give the allowance (G-e). These students thought that the data showed a pattern that should be followed. Many of them elaborated on the pattern they had found making incorrect statements such as "it goes up $0.50 a year" and "it goes up $1.50 a year". Some students used an increase of $0.50 on the basis of examining the 1988-89 increase. One of the more ingenious attempts at creating a pattern was to note that increments of one, two and three spaces between the bars had already been used so the increment to obtain the 1990 allowance was four spaces. In the interviews students often persisted with these "patterns" (some quite confidently and others rather hesitatingly), or stated that they could not explain the answer they had given. For example, one student stated " Five dollars. I'm not sure if this is right. I think the pattern went 2,3,1 and then starts at 2 again".

Although Graph I and Graph II were both allowance graphs topic errors were made only on Graph II. For a small percentage of grade four students, 5%, their prior knowledge of the topic interfered with their comprehension of the graph (T-a). One student stated he thought of a real allowance before giving an answer of $5.50. The other students gave answers of $4.50, one based on the assumption that "as she gets older her allowance does not increase as much", the other based on the assumption that Jane would be doing more chores and consequently "a little bit more work for a little bit more pay".

Overall, the performance of grade four and grade six students on this question was poor. The major error types and percentage of these errors for each grade was similar.

TREES PLANTED

The following graph shows the number of each type of tree planted in a town in England by a forestry group.



1. How many fir trees were planted? ______
2. What was the total number of trees planted? ______
3. Elm trees were also planted. Can you tell how many elm trees were planted? Circle **YES** or **NO**.

   If you circled **YES**, draw the bar for the elm trees on the graph and explain your answer. ________________________________

   ____

   If you circled **NO**, explain why you circled **NO**. ________________

   ____

Figure 3. Graph III Trees Planted (patterned)

Graph III

This graph shows a patterned arrangement of the number of four types of trees planted (See Figure 3).

Interviews: Grade four-16 Grade six-14

Question 1. (Read)

Results:

Table 7

Graph III Literal Reading Question

Response	Grade Four* (n=55)	Grade Six (n=58)
Correct Answer	52 (95%)	57 (98%)
G - a	1 (2%)	1 (2%)
G - h	2 (4%)	---

* Total not 100% due to rounding.

Very few students had difficulty at the literal level with 95% of the grade four students and 98% of the grade six students obtaining the correct answer (See Table 7). The few errors were graph based errors. An answer of 6, obtained from counting the number of horizontal spaces comprising the fir bar (G-a) was seen at both grade levels. Perceiving the bar for the number of fir trees as depicting a tree (G-h) resulting in answer of one, only occurred at the grade four level.

Question 2. (Interpret)

Results:

Table 8

Graph III Interpret Question

Response	Grade Four* (n=55)	Grade Six (n=58)
Correct Answer	28 (51%)	42 (72%)
R/C - a	12 (22%)	4 (7%)
R/C - d	6 (11%)	3 (5%)
G - a	2 (4%)	---
G - b	[illegible] (2%)	2 (3%)
G - h	2 (4%)	—
Unexplained error	1 (2%)	4 (7%)
No response	3 (5%)	3 (5%)

* Total not 100% due to rounding.

This question required students to add, unlike the interpret question for Graph I and Graph II which required students to subtract. The results are given in Table 8.

A common reading-language error was to give the largest number of trees planted, 500, (R/C-a), (22% grade four; 7% grade six). The students' lack of understanding of the word total was evident during interviews as they continuously pointed to the highest point on the graph. Computational errors also occurred (R/C-d), (11% grade four; 5% grade six).

A small percentage of students made the graph based errors of not attending to scale (G-a), other scale errors (G-b), or perceived the graph as a picture (G-h).

Overall, the grade four students performed poorly on this question (51% correct) relative to the grade six students (72% correct). The poorer performance of the grade four students appears to be due to their greater difficulty with the word total.

Question 3.

Results:

Table 9

Graph III Predict Question

Response	Grade Four (n=55)	Grade Six (n=58)
Correct Answer	2 (4%)	4 (7%)
R/C - a	1 (2%)	---
G - c	9 (16%)	10 (17%)
G - f	38 (69%)	41 (71%)
T - b	1 (2%)	---
Incomplete - b	---	2 (3%)
No response	4 (7%)	1 (2%)

Very few students at each grade level gave the correct answer (See Table 9). Most students did not realize that despite the pattern in the data, the number of elm trees could not be determined. Students who gave the correct answer gave reasons such as "well who knows but who plants them", and "there could be any amount planted", to support their answer. Included in this category was a grade six student who when explaining his answer stated that the answer could not be one of the values already on the graph.

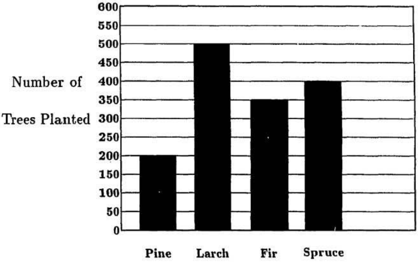
There were 16% of the grade four students and 17% of the grade six students who indicated that it was not possible to obtain an answer because elm trees were not shown on the graph (G-c). The most common graph based error was to give a value for the number of elm trees planted (G-f). There were 69% of the grade four students and 71% of the grade six students who gave such a value. Most of these students followed the pattern in the data and gave an answer of 100. A small number of students continued the 'pattern' in the other direction and gave an answer of 600, and some gave other values which they did not explain.

The only topic error noted was that of a grade four student who stated that he could not answer the question because he had never seen an elm tree (T-b).

Overall, both grades performed poorly on this question, with the major error types and the percentage of these errors for each grade being similar.

TREES PLANTED

The following graph shows the number of each type of tree planted in a town in England by a forestry group.



1. How many fir trees were planted? ______

2. What was the total number of trees planted? ______

3. Elm trees were also planted. Can you tell how many elm trees were planted? Circle **YES** or **NO**.

   If you circled **YES**, draw the bar for the elm trees on the graph and explain your answer. ____________________

   ____

   If you circled **NO**, explain why you circled **NO**. ____________________

   ____

Figure 4. Graph IV Trees Planted (non-patterned)

Graph IV

This graph is a non-patterned version of Graph III. The accompanying questions are the same (See Figure 4).

Interviews: Grade four-15 Grade six-17

Question 1. (Read)

Results:

Table 10

Graph IV Litoral Reading Question

Response	Grade Four* (n=55)	Grade Six (n=57)
Correct Answer	42 (76%)	54 (95%)
R/C - b	3 (5%)	---
G - a	4 (7%)	2 (3%)
G - h	3 (5%)	---
Unexplained error	2 (4%)	---
No response	1 (2%)	1 (2%)

* Total not 100% due to rounding.

Table 10 shows that students at the grade four level had a lower success rate than in Graph III (96% correct) with 76% obtaining the correct answer. As in Graph III, very few grade six students had any difficulty with 95% obtaining the correct answer.

The error of incorrectly reading the value from the graph (R/C-b) was made by 5% of the grade four students who gave incorrect responses of 35, 330, and 300.

The graph based errors were the same type as for Graph III, not attending to scale (G-a) and perceiving the graph as a picture (G-h).

Question 2. (Interpret)

Results:

Table 11

Graph IV Interpret Question

Response	Grade Four* (n=55)	Grade Six (n=57)
Correct Answer	22 (40%)	39 (68%)
R/C - a	19 (35%)	7 (12%)
R/C - d	4 (7%)	4 (7%)
G - a	1 (2%)	3 (5%)
G - b	2 (4%)	1 (2%)
G - h	4 (7%)	1 (2%)
Unexplained error	1 (2%)	1 (2%)
No response	2 (4%)	1 (2%)

* Total not 100% due to rounding.

Table 11 shows that students at the grade four level had a lower success rate than in Graph III (51% correct) with 40% obtaining the correct answer. The percentage of grade six students obtaining the correct answer was 68% and similar to that on Graph III (72% correct).

As was the case in Graph III, students had difficulty with the meaning of the word total (R/C-a) and also made computational errors (R/C-d).

As in Graph III students made the graph based errors of not attending to scale (G-a), other scale errors (G-b), or perceiving the graph as a picture (G-h). Students who gave an answer of 4 were categorized as perceiving the graph as a picture although those who made different errors in question 1 might have misunderstood and given the total type of trees.

Overall, the grade four students performed poorly on this question (40% correct) relative to the grade six students (68% correct). As was the case in Graph III the grade fours were hindered by their lack of understanding of the word total.

Question 3. (Predict)

Results:

Table 12

Graph IV Predict Question

Response	Grade Four* (n=55)	Grade Six* (n=57)
Correct Answer	2 (4%)	3 (5%)
R/C - a	3 (5%)	2 (4%)
G - c	13 (24%)	12 (21%)
G - e	6 (29%)	10 (18%)
G - g	8 (15%)	17 (30%)
T - a	---	1 (2%)
T - b	2 (4%)	---
Incomplete - b	5 (9%)	8 (14%)
No response	6 (11%)	4 (7%)

* Total not 100% due to rounding.

Students had difficulty with this question with only 4% of the grade four students and 5% of the grade six students obtaining the correct answer (See Table 12). The actual success for this question might have been higher because as for other graphs statements such as "No, I don't know" and "No, you can't tell" were categorized as incomplete although the overall success rate would have still been poor.

A few students at each grade level did not understand the question (R/C-a).

These students either stated they did not understand the question, or gave an inappropriate response such as making reference to the height of trees.

There were 24% of the grade four students and 21% of the grade six students who indicated it was not possible to obtain an answer because elm trees were not on the graph (G-c). The most common error was to give a value for the number of elm trees planted. There were 29% of the grade four students and 18% of the grade six students who gave such a value (G-e). Most of these students thought the data showed a pattern that should be followed. In the interviews they either persisted with incorrect statements about patterns they had found or could not explain what they had done.

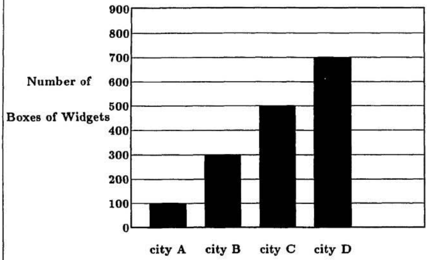
Another graph based error was the students' belief that the absence of a pattern was the sole reason for their inability to find an answer (G-g). There were 15% of the grade four students and 29% of the grade six students who made this error with those who were interviewed stating that they could have found an answer if there had been a pattern in the data.

A small percentage of students made topic errors, either being misled by the topic (T-a) or believing that their lack of knowledge of the topic was a hinderance (T-b). One student gave a value for the number of elm trees based on her understanding of the suitability of English weather for growing trees. The other students decided they could not give an value for the number of elm trees because they did not know what elm trees were.

Overall, both grades performed poorly on this question.

BOXES OF WIDGETS

The following graph shows the number of boxes of Widgets that were made in each of four cities in 1988.



1. In which city was 700 boxes of Widgets made? ____

2. How many more boxes of Widgets were made in city D than in city A? ____

3. A fifth city, city E, also made Widgets in 1988. Can you tell how many boxes of Widgets were made in city E? Circle **YES** or **NO**.

   If you circled **YES**, draw the bar for Widgets made in city E on the graph and explain your answer. ____________

   ____

   If you circled **NO**, explain why you circled **NO**. ____________

   ____

Figure 5. Graph V Boxes of Widgets by Lettered City (patterned)

Graph V

This graph shows a patterned arrangement of the number of Widgets produced by cities A, B, C, and D (See Figure 5).

Interviews: Grade four-12 Grade six-15

Question 1. (Read)

Results:

Table 13

Graph V Literal Reading Question

Response	Grade Four (n=33)	Grade Six (n=34)
Correct Answer	33 (100%)	34 (100%)

Students had no difficulty with this question, with all of the students obtaining the correct answer (See Table 13).

Question 2. (Interpret)

Results:

Table 14 shows that reading-language and computation errors were seen only at the grade four level. Nine percent of the students obtained an answer of 700 (R/C-d). Interviews showed that the students were incorrectly "counting on" from city A to city D. As seen in other graphs, students often counted the horizontal guidelines, in this case starting with the one marking the top of the bar for city A.

The only graph based error was not attending to scale (G-a). Thirty-six percent

Table 14

Graph V Interpret Question

Response	Grade Four* (n=33)	Grade Six (n=34)
Correct Answer	14 (42%)	27 (79%)
R/C - a	1 (3%)	---
R/C - d	3 (9%)	---
G - a	12 (36%)	6 (18%)
Unexplained error	1 (3%)	---
No response	2 (6%)	1 (3%)

* Total not 100% due to rounding.

of the grade four and 18% of the grade six students obtained an answer of 6 or 7 by "counting on" but not attending to scale.

Overall, the grade four students performed poorly on this question (42% correct) compared to the grade six students (79% correct). The grade fours made far more errors of not attending to scale (G-a) than the [illegible]

Question 3. (Predict)

Results:

Only a few grade six students obtained the correct answer (See Table 15). These students realized that despite the pattern in the data, the number of boxes of Widgets made in city E could not be determined.

There were 12% of the grade four students but only 2% of the grade six students who indicated it was not possible to determine the answer because city E was not shown on the graph (G-c). The most common error was to give a value for the number of Widgets made in city E (G-f). There were 72% of the grade four and 85%

Table 15

Graph V Predict Question

Response	Grade Four* (n=33)	Grade Six* (n=34)
Correct Answer	---	2 (6%)
G - c	4 (12%)	1 (2%)
G - f	24 (72%)	29 (85%)
Incomplete - b	1 (3%)	1 (3%)
No response	4 (12%)	1 (3%)

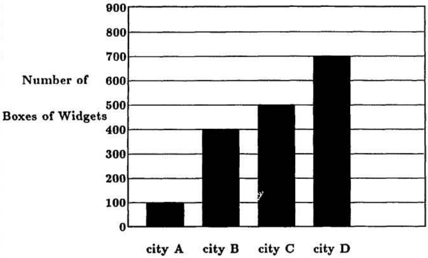
* Total not 100% due to rounding.

of the grade six students who gave such a value. Nearly all of these students followed the pattern in the data and gave an answer of 900 with a few students giving an answer of 800. The explanations elaborating on these answers often contained incorrect statements about the topic and scale of the graph, perhaps indicating the students were preoccupied with the visual appearance of the data. For example, "everyone is 2 ft tall so, it means it is 900", "each city makes $200 more", "in each city there are 3 more boxes", and "each city makes twice as much as the one before".

Overall, both grades performed extremely poorly on this question.

BOXES OF WIDGETS

The following graph shows the number of boxes of Widgets that were made in each of four cities in 1988.



1. In which city was 700 boxes of Widgets made? _____

2. How many more boxes of Widgets were made in city D than in city A? _____

3. A fifth city, city E, also made Widgets in 1988. Can you tell how many boxes of Widgets were made in city E? Circle **YES** or **NO**.

   If you circled **YES**, draw the bar for Widgets made in city E on the graph and explain your answer. _____

   If you circled **NO**, explain why you circled **NO**. _____

Figure 6. Graph VI Boxes of Widgets by Lettered City (non-patterned)

Graph VI

This graph is a non-patterned version of Graph V. The accompanying questions are the same (See Figure 6).

Interviews: Grade four-12 Grade six-10

Question 1. (Read)

Results:

Table 16

Graph VI Literal Reading Question

Response	Grade Four (n=33)	Grade Six (n=34)
Correct Answer	33 (100%)	34 (100%)

As in Graph V students had no difficulty with this question with all students obtaining the correct answer (See Table 16).

Question 2. (Interpret)

Results:

Table 17 shows that a small percentage of students made reading-language errors and gave inappropriate responses (R/C-a), or made computation errors (R/C-d) by incorrectly using the "counting on" technique.

As was the case in Graph V, the only graph based error was not attending to scale (G-a). There were 24% of the grade four students and 29% of the grade six students who made this error.

Table 17

Graph VI Interpret Question

Response	Grade Four (n=33)	Grade Six (n=34)
Correct Answer	19 (58%)	23 (58%)
R/C - a	2 (6%)	---
R/C - d	1 (3%)	1 (3%)
G - a	8 (24%)	10 (29%)
Unexplained error	1 (3%)	---
No response	2 (6%)	---

Question 3. (Predict)

Results:

Table 18

Graph VI Predict Question

Response	Grade Four* (n=33)	Grade Six* (n=34)
Correct Answer	2 (6%)	1 (3%)
R/C - a	1 (3%)	---
G - c	3 (9%)	7 (21%)
G - e	16 (48%)	20 (59%)
G - g	6 (18%)	2 (6%)
Incomplete - b	3 (9%)	---
No response	2 (6%)	4 (12%)

* Total not 100% due to rounding.

Students had difficulty with this question with only a small percentage of students giving the correct answer (See Table 18). As in other graphs, the students who stated "I don't know", and "I can't tell" had their answers categorized as incomplete.

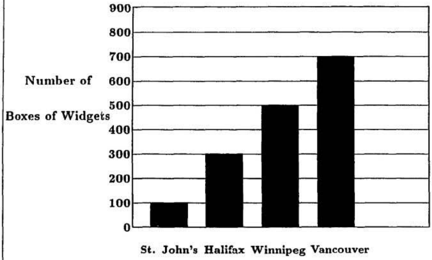
As was the case in Graph V, some students (9% grade four; 21% grade six) indicated it was not possible to determine an answer because city E was not shown on the graph (G-c). The most common error was to give a value for the number of boxes of Widgets made in city E (G-e). There were 48% of the grade four students and 59% of the grade six students who gave such a value. Most of these students thought the data showed a pattern which should be followed. All but a few grade six students gave answers of 800 or 900, ie values which continued the increasing trend. As in other non-patterned graphs most students tried to describe the patterns they had seen, while others gave no clear explanation for their answer. For example, one student explained an answer of 800 by writing "...it goes 1,3,1,2, and then it would drop down to 1 again". As in Graph V, students' explanations often contained incorrect statements about topic and scale.

Another graph based error was students' beliefs that the absence of a pattern was the sole reason for their inability to obtain an answer (G-g). There were 18% of the grade four students and 6% of the grade six students who made this error.

Overall, both grades performed extremely poorly on this question.

BOXES OF WIDGETS

The following graph shows the number of boxes of Widgets that were made in each of St. John's, Halifax, Winnipeg, and Vancouver in 1988.



1. In which city was 700 boxes of Widgets made? ______
2. How many more boxes of Widgets were made in Winnipeg than in Halifax? ______
3. Widgets were also made in Toronto in 1988. Can you tell how many boxes of Widgets were made in Toronto? Circle **YES** or **NO.**

   If you circled **YES**, draw the bar for Widgets made in Toronto on the graph and explain your answer. ______

   If you circled **NO**, explain why you circled **NO.** ______

Figure 7. Graph VII Boxes of Widgets by City Name (patterned)

Graph VII

This graph is similar to Graph V except that real place names are used for the cities (See Figure 7).

Interviews: Grade four-7 Grade six-8

Question 1. (Read)

Results:

Table 19

Graph VII Literal Reading Question

Response	Grade Four (n=33)	Grade Six (n=35)
Correct Answer	30 (91%)	34 (97%)
NG - a	3 (9%)	---
No response	---	1 (3%)

Students had little difficulty with this question with 91% of the grade four students and 97% of the grade six students obtaining the correct answer (See Table 19).

Question 2. (Interpret)

Results:

The most common error was the graph based error of not attending to scale (G-a) which was made by 30% of the grade four students and 14% of the grade six

Table 20

Graph VII Interpret Question

Response	Grade Four (n=33)	Grade Six (n=35)
Correct Answer	20 (61%)	28 (80%)
G - a	10 (30%)	5 (14%)
G - b	1 (3%)	---
Unexplained error	---	1 (3%)
No response	2 (6%)	1 (3%)

students (See Table 20). However, one student who was interviewed subtracted 300 from 500 but gave an answer of 2. The explanation was to the effect that while it was really 200, the answer in final form was 2.

Overall, the grade four students performed poorly on this question (61% correct) relative to the grade six students (80% correct). The grade four students made more errors of not attending to scale (G-a).

Question 3. (Predict)

Results:

The results of this question are shown in Table 21. As was the case in Graph V, students did not predict on the basis that Toronto was not shown on the graph (G-c), (15% grade four; 14% grade six), or followed the existing pattern when it was not reasonable to do so, (G-f), (58% grade four; 57% grade six).

A small percentage of grade four students, 6%, believed their lack of prior knowledge about the topic resulted in their inability to answer the question. One student stated he could not answer the question because he did not know what

Table 21

Graph VII Predict Question

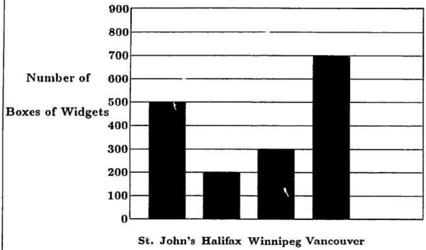
Response	Grade Four (n=33)	Grade Six* (n=35)
Correct Answer	---	2 (6%)
R/C - a	1 (3%)	1 (3%)
G - c	5 (15%)	5 (14%)
G - f	19 (58%)	20 (57%)
T - b	2 (6%)	---
Incomplete - b	2 (6%)	5 (14%)
Unexplained error	3 (9%)	---
No response	1 (3%)	2 (6%)

Widgets were, the other stated that she needed to know more about Toronto to be able to give an answer.

Overall, both grades performed extremely poorly on this question.

BOXES OF WIDGETS

The following graph shows the number of boxes of Widgets that were made in each of St. John's, Halifax, Winnipeg, and Vancouver in 1988.



1. In which city was 700 boxes of Widgets made? ______
2. How many more boxes of Widgets were made in Winnipeg than in Halifax? ______
3. Widgets were also made in Toronto in 1988. Can you tell how many boxes of Widgets were made in Toronto? Circle **YES** or **NO.**

   If you circled **YES**, draw the bar for Widgets made in Toronto on the graph and explain your answer. ______

   If you circled **NO**, explain why you circled **NO**. ______

Figure 8. Graph VIII Boxes of Widgets by City Name (non-patterned)

Graph VIII

This graph is a non-patterned version of Graph VII. The accompanying questions are the same (See Figure 8).

Interviews: Grade four-8 Grade six-9

Question 1. (Read)

Results:

Table 22

Graph VIII Literal Reading Question

Response	Grade Four (n=33)	Grade Six (n=36)
Correct Answer	30 (91%)	36 (100%)
G - a	2 (6%)	---
No response	1 (3%)	---

The results of this question are given in Table 22 and were similar to those of Graph VII, with students performing very well.

Question 2. (Interpret)

Results:

Table 23 shows that both grades performed well on this question (79% grade four; 86% grade six) having a higher success rate than in Graph VII (61% grade four; 80% grade six). There were fewer errors of not attending to scale (G-a) than in Graph VII, particularly at the grade four level, 9% compared to 30%.

Table 23

Graph VIII Interpret Question

Response	Grade Four (n=33)	Grade Six* (n=36)
Correct Answer	26 (79%)	31 (86%)
R/C - a	1 (3%)	1 (3%)
R/C - c	1 (3%)	---
G - a	3 (9%)	2 (6%)
Unexplained error	---	2 (6%)
No response	2 (6%)	---

* Total not 100% due to rounding.

Question 3. (Predict)

Results:

Table 24

Graph VIII Predict Question

Response	Grade Four* (n=33)	Grade Six* (n=36)
Correct Answer	1 (3%)	5 (14%)
R/C - a	1 (3%)	---
G - c	9 (27%)	10 (28%)
G - e	10 (30%)	11 (31%)
G - g	2 (6%)	2 (6%)
T - a	2 (6%)	1 (3%)
T - b	2 (6%)	---
Incomplete - b	4 (12%)	7 (19%)
No response	2 (6%)	---

* Total not 100% due to rounding.

Students had difficulty with this question with only 3% of the grade four students and 14% of the grade six students obtaining the correct answer (See Table 24). As discussed for other graphs, the actual success rate might have been higher due to the number of answers which were incomplete although the overall performance would have still been poor.

Students made the same type of graph based errors as in Graph VI, a similar graph without the use of real place names. However, fewer students forced a pattern on the data and then followed it (G-e). This could be due to the fact that the data in Graph VIII is not ordered in magnitude while the data in Graph VI is. It would appear that students were more likely to think they saw a pattern in data that was ordered.

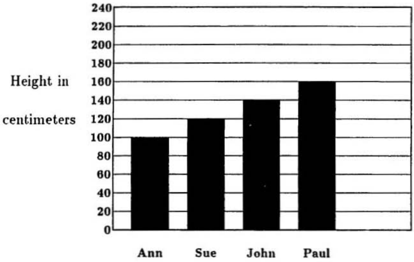
A small number of topic errors were made. A few students gave a value for the number of boxes of Widgets produced in Toronto based on the fact that Toronto is a large centre (T-a). A misunderstanding of the topic of the graph led one grade six student completely astray (T-a). Interpreting the graph as showing sales for past years, she reasoned that Toronto, being a newcomer to selling Widgets, would not sell as many as the other cities and assigned it 100 boxes.

As was the case in Graph VII, not having been to Toronto and not knowing what Widgets are, were also given as reasons for not being able to give a value for Toronto (T-b).

Overall, both grades performed poorly on this question.

HEIGHT OF THE SMITH CHILDREN

The following graph shows the height of four of the Smith children.



1. How tall is Ann? ______
2. How much shorter is Ann than John? ______
3. A fifth child in the family is called Mary. Can you tell how tall Mary is? Circ'e **YES** or **NO**.

   If you circled **YES**, draw the bar for Mary on the graph and explain your answer. ____________________

   ____

   If you circled **NO**, explain why you circled **NO**. ____________________

   ____

Figure 9. Graph IX Height of the Smith Children by Name (patterned)

Graph IX

This graph shows the height, in a patterned format, of four children by name.

Interviews: Grade four-7 Grade six-8

Question 1. (Read)

Results:

Table 25

Graph IX Literal Reading Question

Response	Grade Four (n=33)	Grade Six (n=34)
Correct Answer	31 (94%)	34 (100%)
R/C - a	1 (3%)	---
R/C - b	1 (3%)	---

Table 25 shows that very few students had difficulty at the literal level with 94% of the grade four students and 100% of the grade six students obtaining the correct answer.

Question 2. (Interpret)

Results:

All the graph based errors were scale errors (See Table 26). Twenty-one percent of the grade four students and 3% of the grade six students indicated they had not attended to scale (G-a) by giving an answer of 2. The interviews revealed other scale errors (G-b) where students had used increments other than 20 cm. One

Table 26

Graph IX Interpret Question

Response	Grade Four* (n=33)	Grade Six (n=34)
Correct Answer	14 (42%)	28 (82%)
R/C - a	1 (3%)	---
R/C - b	---	1 (3%)
G - a	7 (21%)	1 (3%'
G - b	8 (24%)	4 (12%)
No response	3 (9%)	---

* Total not 100% due to rounding.

student had obtained an answer of 300 cm by multiplying 3 (the number of horizontal guidelines from the top of the bar representing Ann's height to the one representing John's height) by an increment of 100 cm. A few previously unexplained values of 200 cm and 300 cm were then placed in this category, (G-b), which then accounted for 24% of the grade four errors and 12% of the grade six errors.

Question 3. (Predict)

Results:

Almost none of the students realized that despite the pattern in the data, Mary's height could not be determined (See Table 27).

There were 15% of the grade four students and 12% of the grade six students who indicated it was not possible to obtain an answer because Mary was not shown on the graph (G-c).

The most common error was to give a value for Mary's height (G-f). There were 73% of the grade four students and 82% of the grade six students who gave a

Table 27

Graph IX Predict Question

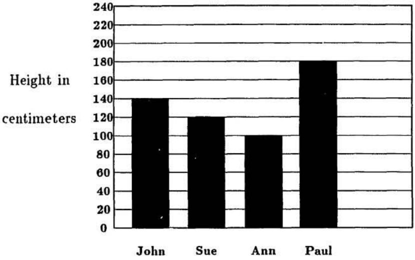
Response	Grade Four (n=33)	Grade Six (n=34)
Correct Answer	---	1 (3%)
G - c	5 (15%)	4 (12%)
G - f	24 (73%)	28 (82%)
Incomplete - b	3 (9%)	1 (3%)
No response	1 (3%)	---

value of 180 cm for Mary's height by following the pattern in the data. Students' explanations of their answers often contained incorrect statements. Some of these indicated confusion with the scale used. For example: "it goes up 10 cm", "each was 1 cm taller than the other", and "each child's height went up 100 cm". A few explanations, "Mary is older so she is taller", and "Mary is the oldest" indicate that besides the pattern shown, students prior knowledge of height increasing with age was also influencing their comprehension.

Overall, both grades performed extremely poorly on this question.

HEIGHT OF THE SMITH CHILDREN

The following graph shows the height of four of the Smith children.



1. How tall is Ann? ______
2. How much shorter is Ann than John? ______
3. A fifth child in the family is called Mary. Can you tell how tall Mary is?
   Circle **YES** or **NO.**

   If you circled **YES**, draw the bar for Mary on the graph and explain your answer. ______________________________

   ____

   If you circled **NO**, explain why you circled **NO**. ______________________________

   ____

Figure 10. Graph X Height of the Smith Children by Name (non-patterned)

Graph X

This graph is a non-patterned version of Graph IX. The accompanying questions are the same (See Figure 10).

Interviews: Grade four-8 Grade six-10

Question 1. (Read)

Results:

Table 28

Graph X Literal Reading Question

Response	Grade Four (n=33)	Grade Six (n=34)
Correct Answer	32 (97%)	33 (97%)
R/C - a	1 (3%)	---
G - a	---	1 (3%)

The results of this question are given in Table 28 and were similar to those of Graph IX, with students performing very well.

Question 2. (Interpret)

Results:

Table 29 shows that the results of this question (48% grade four correct; 76% grade six correct) were similar to those in Graph IX (42% grade four correct; 82% grade six correct). As was the case in Graph IX, all graph based errors were scale errors.

Table 29

Graph X Interpret Question

Response	Grade Four (n=33)	Grade Six (n=34)
Correct Answer	16 (48%)	26 (76%)
G - a	7 (21%)	3 (9%)
G - b	7 (21%)	3 (9%)
Unexplained error	2 (6%)	2 (6%)
No response	1 (3%)	---

Overall, the grade four students performed poorly on this question (48% correct) relative to the grade six students (76% correct). The grade fours had more difficulty with scale.

Question 3. (Predict)

Results:

Table 30

Graph X Predict Question

Response	Grade Four* (n=33)	Grade Six* (n=34)
Correct Answer	2 (6%)	2 (6%)
R/C - a	1 (3%)	3 (9%)
G - c	4 (12%)	6 (18%)
G - e	11 (33%)	16 (47%)
G - g	4 (12%)	6 (18%)
T - a	5 (15%)	1 (3%)
Incomplete - b	6 (18%)	---

* Total not 100% due to rounding.

Students had difficulty with this question with only 6% of the grade four students and 6% of the grade six students giving the correct answer (See Table 30). The actual success rate for the grade four students might have been higher because as for other graphs statements such as "No, I don"t know", and "No, I can't tell" were categorized as incomplete. However, overall performance would have still been poor.

There were a small percentage of students who either stated they did not understand the question or gave an inappropriate response (R/C-a).

There were 12% of the grade four students and 18% of the grade six students who indicated it was not possible to obtain an answer because Mary was not shown on the graph (G-c). The most common error was to give a value for Mary's height (G-e). There were 33% of the grade four students and 47% of the grade six students who gave a value. Most of these students thought that the data showed a pattern that should be followed. The most common value given by the grade six students was 160 cm. The students indicated they thought of 160 as a missing value in a pattern. For example, one student stated, "each one went up by 20 starting at 100, so I looked and 160 was not there".

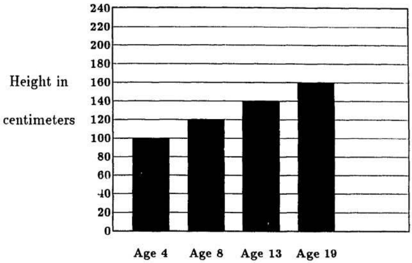
Another graph based error was students belief that the absence of a pattern was the sole reason for their inability to find an answer (G-g). There were 12% of the grade four students and 18% of the grade six students who made this error. Those students who were interviewed stated they could have found an answer if there had been a pattern in the data.

Fifteen percent of the grade four students and 3% of the grade six students were misled by the topic (T-a). One student gave a value for Mary's height based on the fact that he had five people in his family but could not elaborate on this during the interview. Another student wrote "Mary sounds the tallest" and assigned a value accordingly. The other students made reference to Mary being the smallest, a baby, and a newborn, and gave a value accordingly. It is possible that these students had a language problem with "the fifth child".

Overall, both grades performed poorly on this question.

HEIGHT OF THE SMITH CHILDREN

The following graph shows the height of four of the Smith children ages 4, 8, 13, and 19.



1. How tall is the 4 year old? ______

2. How much shorter is the 4 year old than the 19 year old? ______

3. A fifth child in the family is 10 years old. Can you tell how tall the 10 year old is? Circle **YES** or **NO**.

   If you circled **YES**, draw the bar for the 10 year old on the graph and explain your answer. ______________________________

   ____

   If you circled **NO**, explain why you circled **NO**. ______________________________

   ____

Figure 11. Graph XI Height of Smith Children by Age (patterned)

Graph XI

This graph shows the height, in a patterned format, of four children by age (See Figure 11).

Interviews: Grade four-12 Grade six-17

Question 1. (Read)

Results:

Table 31

Graph XI Literal Reading Question

Response	Grade Four (n=33)	Grade Six (n=36)
Correct Answer	33 (100%)	36 (100%)

Students had no difficulty at the literal level with all students obtaining the correct answer (See Table 31).

Question 2. (Interpret)

Results:

Students performed moderately well on this question with 73% of the grade four students and 75% of the grade six students obtaining the correct answer (See Table 32).

The reading-language error, (R/C-a), was giving the height of the nineteen-year old as the answer (3% grade four; 8% grade six).

All of the graph based errors were scale errors. Twelve percent of the grade

Table 32

Graph XI Interpret Question

Response	Grade Four (n=33)	Grade Six (n=36)
Correct Answer	24 (73%)	27 (75%)
R/C - a	1 (3%)	3 (8%)
R/C - b	1 (3%)	---
G - a	4 (12%)	4 (11%)
G - b	3 (9%)	2 (6%)

* Total not 100% due to rounding.

four students and 11% of the grade six students indicated they had not attended to scale (G-a) by giving an answer of 3 cm. The interviews revealed other scale errors, (G-b), (9% grade four; 6% grade six) similar to those that occurred in Graph IX and Graph X.

Overall, the grade four students performed almost as well as the grade six students on this question.

Question 3. (Predict)

Results:

Thirty six percent of the grade four students and 19% of the grade six students realized that the height of the ten-year old would most likely fall between that of the eight-year old and the thirteen-year old and indicated an appropriate value (See Table 33).

The reading-language errors were students who stated they did not understand the question, answered both yes and no, or gave an inappropriate response of subtracting the ages of the children (R/C-a).

Table 33

Graph XI Predict Question

Response	Grade Four* (n=33)	Grade Six (n=36)
Correct Answer	12 (36%)	7 (19%)
R/C - a	2 (6%)	1 (3%)
G - c	4 (12%)	9 (25%)
G - f	5 (15%)	7 (19%)
G - i	3 (9%)	11 (31%)
T - a	1 (3%)	---
T - b	1 (3%)	---
Unexplained error	2 (6%)	---
No response	3 (9%)	1 (3%)

* Total not 100% due to rounding.

There were 19% of the grade four students and 25% of the grade six students who indicated it was not possible to obtain an answer because the ten-year old was not on the graph (G-c). Some students (15% grade four; 19% grade six) followed the pattern in the data (G-f). All but a few of these students gave a value of 180 cm for Mary's height. When asked during the interview whether it was reasonable for the ten-year old to be taller than the nineteen-year old, one student suggested the ten-year old was on stilts in an attempt to justify his answer. Another student admitted his answer of 180 cm was unreasonable but after re-examining the graph stated he did not want to change it. One of the answers other than 180 cm was 140 cm. The student who gave this answer realized that if she continued the pattern by adding 20 cm to the height of the nineteen-year old, the answer was not reasonable. Since the pattern involved increments of 20 cm, she continued the "pattern" by subtracting 20 cm to obtain a more reasonable answer.

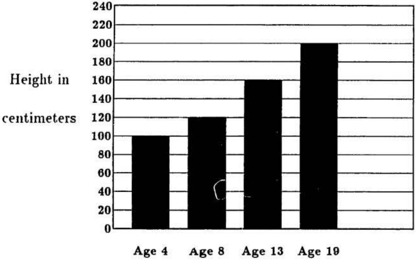
The G-i category (9% grade four; 31% grade six) consisted of other types of errors related to pattern. The most common error was the refusal by students to give a value for the height of the ten-year old because it would involve breaking the existing pattern of increments of 20 cm if a value between 120 cm and 140 cm was given. Another error was the refusal by students to give a value because the only answer they could think of, 180 cm, was unreasonable.

A few grade four students made topic errors. One student did not use any prior knowledge of height in relation to age (T-a) and stated the ten-year old could be any height. Another student stated he was unable to give a value for the height of the ten-year old child because he did not know the child personally (T-b).

Overall, both grades performed poorly on this question. However, the grade four students did better (36% correct) than the grade six students (19% correct). The grade sixes had more difficulty in dealing with the presence of a pattern in the data than the grade fours.

HEIGHT OF THE SMITH CHILDREN

The following graph shows the height of four of the Smith children ages 4, 8, 13, and 19.



1. How tall is the 4 year old? ______

2. How much shorter is the 4 year old than the 19 year old? ______

3. A fifth child in the family is 10 years old. Can you tell how tall the 10 year old is? Circle **YES** or **NO.**

   If you circled **YES**, draw the bar for the 10 year old on the graph and explain your answer. ______

   ______

   If you circled **NO**, explain why you circled **NO.** ______

   ______

Figure 12. Graph XII Height of Smith Children by Age (non-patterned)

Graph XII

This graph is a non-patterned version of Graph XI. The accompanying questions are the same (See Figure 12).

Interviews: Grade four-10 Grade six-8

Question 1. (Read)

Results:

Table 34

Graph XII Literal Reading Question

Response	Grade Four (n=33)	Grade Six (n=35)
Correct Answer	33 (100%)	34 (97%)
No response	---	1 (3%)

The results of this question are given in Table 34 and were similar to those in Graph XI, with students performing very well.

Question 2. (Interpret)

Results:

There were 48% of the grade four students and 89% of the grade six students who obtained the correct answer to this question (See Table 35). The success rate for the grade fours was much lower than that in Graph XI (72% correct) and higher for the grade sixes than that in Graph XI (75% correct).

The reading-language errors were responses of either of the four-year old's

Table 35

Graph XII Interpret Question

Response	Grade Four* (n=33)	Grade Six (n=35)
Correct Answer	16 (48%)	31 (89%)
R/C - a	3 (9%)	---
G - a	10 (30%)	1 (3%)
G - b	1 (3%)	---
Unexplained error	2 (6%)	2 (6%)
No response	1 (3%)	1 (3%)

* Total not 100% due to rounding.

height or the nineteen-year old's height (R/C-a) (9% grade four; 0% grade six). It was thought that the success rate might actually be lower than that calculated due to the fact that the height of the four-year old was the same value as the correct answer. During the interviews, however, only one student was found to be just taking the height of the four-year old.

As was the case in Graph XI, all graph based errors were scale errors. Thirty percent of the grade four students and 3% of the grade six students did not attend to scale (G-a), with a few of these students also miscounting horizontal spaces or guidelines.

Overall, the grade four students (48% correct) performed poorly on this question relative to the grade six students (89% correct). The grade four students made far more errors of not attending to scale (G-a).

Question 3. (Predict)

Results:

Table 36

Graph XII Predict Question

Response	Grade Four* (n=33)	Grade Six (n=35)
Correct Answer	15 (45%)	17 (49%)
G - c	2 (6%)	5 (14%)
G - e	5 (15%)	---
G - g	4 (12%)	5 (14%)
G - i	1 (3%)	1 (3%)
Unexplained error	4 (12%)	5 (14%)
No response	2 (6%)	2 (6%)

* Total not 100% due to rounding.

Forty-five percent of the grade four students and 49% of the grade six students realized that the height of the ten-year old would most likely fall between that of the eight-year old and that of the thirteen-year old and gave an appropriate value (See Table 36).

All of the errors were graph based. There were 6% of the grade four students and 14% of the grade six students who indicated it was not possible to obtain an answer because the ten-year old was not on the graph (G-c). The error of forcing a pattern on the data and attempting to follow it (G-e) was made only at the grade four level. These students (15%) all obtained answers of either 220 cm or 240 cm. Another graph based error was students belief that the absence of a pattern was the sole reason for their inability to find an answer (G-g) (12% grade four; 14% grade six).

A few students tried to force a pattern on the data but stopped when they saw that their answer was not reasonable in that the ten-year old would be taller than the nineteen-year old (G-i).

Overall, slightly less than half of the students were successful on this question.

Frame Analysis

Students at the grade four and grade six level had previously studied graphs, and with the exception of a few students, exhibited evidence of possessing a graphical frame. On the basis of Davis's frame theory and the data collected the following description of a graph frame has been formulated.

By the nature of a graph frame it contains general information about graphs. Operationally, when a student is faced with a graphical problem to solve, the student retrieves the frame from memory and seeks input for certain frame variables or slots in the form of specific information from the graph. Specific slots appear to exist for explicit features of the graph such as axes labels, title, scale, and data arrangement. In addition to these specific slots, there exist slots for appropriate information from other, more general frames namely; reading-language, computation, and topic.

Data is mapped correctly or incorrectly from the graph to the graph frame with slots that cannot be filled from the graph data filled by default, ie by relying on past experience. When the specific information from the graph is combined with the general information in the graph frame, the frame is said to be instantiated. This instantiated graph frame is then used as a data base for any further information processing about the graph. Figure 13 illustrates this conceptualization of a graph frame.

Errors in reading, interpreting, and predicting from graphs reveal something of the inner workings of the graph frame. An analysis of the observed errors in terms of deficiencies of the instantiated graph frame is now given.

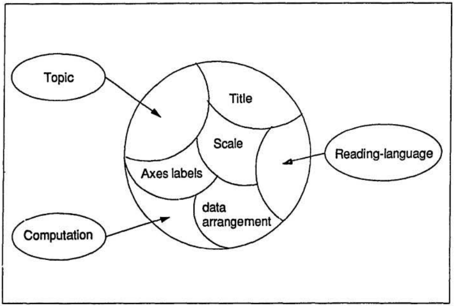


Figure 13. Conceptualization of a Graph Frame

Reading-Language or Computation Errors

Reading-language or computation errors were made by students when answering literal, interpret, and predict questions. These errors were most common for the interpret questions.

Two views of these errors are that they can be attributed to deficiencies in the general frames, per se, from which information is drawn to instantiate the graph frame or from errors made in mapping information from these general frames to the graph frame. Both possibilities result in a flawed graph frame being used to solve a graphical problem and will explain the errors discussed under this heading.

For example, in Graphs I and II, when asked how much more allowance Jane got in 1988 than in 1986, students gave answers which included either the allowance value for 1988 or 1986, both values, or the sum of the values. One student gave an answer of $3.50 "because 1988 was $3.50" and another wrote down $3.50 + $1.50 = $5.00. From the written test and the interviews it was concluded that the graph frame contained incorrect information on either the reading-language or computation required to answer the question.

In another example from Graphs III and IV, students when asked for the total number of trees planted gave the largest number planted. In the interviews students consistently pointed to the bar representing the greatest frequency and several used words such as most and biggest when explaining their answer. It was concluded that the graph frame contained incorrect information on the meaning of the word total.

It appears that while students have slots within their graphical frames for the retrieval of information from more general reading-language and computation frames, processing errors surface. Due to the nature of the study and the internalized character of a frame, it is not possible to determine whether these errors are due to deficiencies in the generalized reading-language or computation frames, or are associated with the actual mapping of information from such frames into the graph frame. The implications of these two possibilities are discussed in Chapter V.

Graph-Based Errors

There are four categories of errors that fall within this general heading, namely: data arrangement, topic, scale, and the fact that the information was not shown on the graph.

Data Arrangement

The effect of the visual arrangement of data on students' ability to predict from a graph was built into the study. Three types of arrangements were used: patterned data in order of magnitude; non-patterned data in order of magnitude; and non-patterned data not ordered in magnitude. Many errors in prediction occurred for each type of data arrangement.

The written test and interviews suggest that for many students their concept of patterning was mapped to the graph frame. Students have had experience with patterning in the primary grades and have formulated a connection between patterns and predicting missing values. An analysis of the data also suggests that this idea of pattern, whether correct or incorrect, is not used within the graph frame with the appropriate constraint that should be imposed by a consideration of the graph topic. Whether the topic information does not exist within the graph frame or the link to use it in conjunction with pattern is missing is not clear. This is illustrated in the following examples.

In Graph XI, students predicted the height of the ten-year old to be more than that of the nineteen-year old based on the pattern in the graph. When the unlikelihood of the ten-year being taller than the nineteen-year old was pointed out during the interview, one student in an attempt to protect the flawed frame suggested the possibility of the ten-year old being on stilts rather than change the answer.

For graphs with non-patterned data some students sought and found a "pattern". This occurred both in cases where the data was ordered in magnitude and where the data was not ordered in magnitude. For example, in Graph VI when faced with non-patterned data associated with the manufacture of Widgets in cities A, B, C, and D students forced a pattern and used it to predict for City E. Similar problems occurred even when the graph topic was familiar (such as allowance), but the situation did not allow for prediction. During the interviews, the students often hesitated when describing these 'patterns' but then continued on rather than admit something was incorrect. This can also be viewed as an attempt to protect the flawed frame.

Another error in non-patterned graphs where any attempt to search for a pattern made no conceptual sense, was for students to cite the absence of a pattern as the reason for their inability to predict. For example, in Graph IV (a non-patterned arrangement of the number of pine, larch, fir, and spruce trees planted) students stated they could not predict the number of elm trees planted because there was no pattern in the data. During the interviews the students said that such a prediction would have been possible had the data been patterned.

For many of the graphs where it was not possible to predict, students were asked if alternate answers to the one they obtained on the basis of a pattern (real or perceived) were possible. Some students said yes, and then realized it was therefore not possible to predict. Others said yes but then indicated they still wished to follow the pattern. A third group of students said no, the pattern had to be followed. These answers indicate there are different degrees of difficulty with pattern.

Overall, it appears the majority of students have a pattern slot in their graph frame. The analysis of the data revealed two majors deficiencies in this slot. One deficiency is the presence of incorrect information that a pattern must exist in the data. This is evidenced by the fact that many students forced "patterns" not only for data that were ordered in magnitude but also for data that were not ordered in magnitude. Another deficiency is that the slot is not linked to a topic slot (when one exists). Often students believed that the absence of a pattern, regardless of the fact the topic was unsuitable for prediction, meant they could not predict. The result of these two deficiencies was that in pairs of graphs on the same topic where prediction was not possible because of the nature of the topic, performance was often equally poor even though the organization of the data were different. For example, Graph III and Graph IV, and Graph IX and Graph X.

Topic Errors

Students' ability to use knowledge of the topic in predicting from bar graphs was another factor built into the study. Topics of differing degrees of familiarity were used. For example, allowance and height were considered to be familiar topics. The topic of trees was also familiar but the different types of trees used were not familiar to all students. While Widgets was an unfamiliar topic, two versions of this graph used city names which were familiar to the students. There is limited evidence from the graphs showing height by age that when the topic is familiar the students accommodate better. **Overall, students made the same types of errors regardless of the topic.** This is because the majority of errors were pattern related and as discussed earlier, topic information did not exist within the graph frame or, if it did, the link to use it in conjunction with pattern was missing. The remainder of this section discusses errors in cases where the topic was known to form part of the graph frame.

A small percentage of students in their written work and in interviews did indicate that topic formed part of their graph frame. However, an analysis of the data suggested that the information on topic, whether correct or incorrect was sometimes not used appropriately within the graph frame.

For example, in Graph VIII one student predicted a value for the number of boxes of Widgets produced in Toronto based on the incorrect knowledge that large centres always produced large quantities. For the same graph, another student stated he could not predict because he did not know what a Widget was.

Another example was in Graph XI, a patterned version of height by age. During the interviews students provided evidence of topic within the graph frame by indicating that the height of the ten-year old should be between that of the eight-year old and nineteen-year old. However, they then stated it was not possible to give such a value as the answer because it would break the pattern shown in the data. While the correct topic information existed it was not used appropriately in conjunction with the pattern slot of the graph frame. It appeared in one sense that pattern "dominated" over topic.

Scale

The nature of the data implied that, for some students, scale does not form part of the graph frame. An example of the subsequently flawed graph frame hindering students at the literal reading level was in Graph III where a few students indicated the number of fir trees was six, by counting the number of blocks comprising the bar representing the number of fir trees. An example at the interpret level was in Graph V where a considerable number of students (36% grade four; 18% grade six) indicated that there were either 6 or 7 more boxes of Widgets produced in City A than in City D by counting the horizontal spaces or guidelines between the bars.

Students who provided evidence that scale forms part of the graph frame when answering a reading or interpret question, often failed to use it in elaborating on their answer to a predict question. Hence explanations to the predict questions often contained inaccurate statements such as "it went up two spaces" when there was an increase of two 10 cm increments between two items. There are three possible frame explanations for this. First, the student's concept of scale is only partially formed. Second, there is no link between pattern and scale within the graph frame. A third explanation is that it is a reading-language problem, per se.

Information not shown on the graph

In answering the predict question some students indicated they could not predict on the basis that the information was not shown on the graph. For example they gave explanations such as "because it's not on the graph" and "I cannot see it up there (on the graph)". In the interviews the researcher acknowledged the information was not on the graph and asked them to re-examine the graph to see if there was any way they could use it to predict. The majority of these students either continued to say no, or if they noticed a pattern were unsure if they could use it.

It was concluded that within the graph frame there was possibly no topic or pattern slot, or if it existed the students did not use it in this situation. In either case, when the graph frame was initially retrieved from memory, input information on topic and pattern was not required and therefore not sought.

Overall, several deficiencies have been noted in students' graphical frames. A limited number of problems exist with the reading-language and computation slots of the frame. Other more serious problems exist with the pattern, topic, and scale slots of the frame. Students' information about pattern appears to be incorrectly developed or not linked to other slots of the frame, particularly topic, which often makes it difficult to verify the topic slot actually exists. The scale slot of the graph frame often does not exist or is not properly developed. Another frame deficiency is the failure to predict because the information is not on the graph. Some students tend to envisage the graph as "complete" and do not realize that it can be used to obtain additional information.

Research Questions and Results

Question 1: What difficulties do grade four and grade six students have in reading, interpreting and predicting from bar graphs?

Students had very few difficulties with the literal reading of bar graphs with mean success rate., for grade four students being 95% and for grade six students being 98%.

There were more difficulties with the interpretation questions with mean success rates for grade four students being 52% and for grade six students being 78%. Most of the errors in this category were reading-language, computation, or scale errors. In fact, the mean percentage of errors that could not be explained by these reasons was 8% for grade four and 4% for grade six and most of the 8% and 4% were either no response or unexplained errors.

The reading-language and computation errors were attributed to either errors within the respective general reading-language or computation frames from which information is mapped to the graph frame or errors in the actual mapping process. The majority of errors at the interpret level were scale errors with the frame analysis indicating that in many of these cases scale did not form part of the graph frame.

The level of performance on prediction questions was extremely low with the mean success rates for grade four students being 16% and for grade six students being 18%. Most of the student difficulties in predicting from bar graphs were attributed to correct or incorrect pattern information being used within the graph frame without the appropriate constraint or consideration of the graph topic. The students' information on pattern appears to not be linked to the topic information in the graph frame. In many cases it was not even possible to verify that topic formed part of the graph frame.

There were also some students who indicated that they did not realize a graph can sometimes be used to obtain values not shown. It was concluded that the graph frame of these students was limited, and possibly did not contain any topic or pattern information.

<u>Question 2</u>: What differences exist between grade four and grade six students' ability to read, interpret and predict from bar graphs?

There is little difference between grade four and grade six students' ability to read bar graphs with mean success rates for grade four being 95% and for grade six being 98%.

Grade four students performed poorly on interpret questions compared to the grade six students with the mean success rate for grade four being 52% and for grade six being 78%. Overall, the grade fours made more reading-language and computation errors than the grade sixes and considerably more scale errors. In fact, the mean percentage of errors that were scale errors was 25% for grade four and 12% for grade six.

Students at both grade levels performed poorly when answering predict questions with the mean success rates for grade four being 16% and for grade six being 18%. Most of these errors involved pattern and were of a similar type and frequency for both grades.

This concludes the analysis of the data pertaining to grade four and grade six students' understanding of the information conveyed in bar graphs. The following chapter, Chapter V, provides a summary of the study and the conclusions. Implications for instruction and recommendations for future research are presented.

CHAPTER V

SUMMARY, IMPLICATIONS, AND RECOMMENDATIONS

In this Chapter a summary of the study and discussion of findings are presented. Implications of the study for instruction are presented and recommendations for future research are included.

Summary

In today's highly technological society it is a necessity that every person should be able to process the large amounts of information encountered in everyday life. Such information is increasingly encountered in graphical form partly due to the advances in computer graphics which have allowed for more efficient production and better quality. To be proficient at processing information students must be able to do more than literally read a graph. They should be able to interpret graphs and to predict, where possible, from the data.

This study was designed to investigate grade four and grade six students' ability to read, interpret, and predict from bar graphs. Data were collected through administration of a written test to all students in the sample followed by a short audiotaped interview with selected students. The sample consisted of 121 students in grade four and 127 students in grade six. The interview component involved 35 students from grade four and 37 students from grade six.

Students were briefed by the researcher on the purpose of the study and that it would involve a written component with the possibility of a follow-up interview. Each student was then administered a written test consisting of four bar graphs each with three questions: a literal reading question, an interpret question, and a predict question. Students were told to examine each graph carefully and to answer as many of the questions they could within the 20-minute time restriction imposed. On the basis of errors on the written test, selected students were given audiotaped interviews to obtain more information on the nature of their errors. The interviews were approximately fifteen minutes, and were conducted within three school days from the date of the test.

Following the completion of the interview sessions, the type and nature of the errors were categorized. After this initial categorization a frame analysis was undertaken and finally the research questions were answered.

An analysis of the data suggested that the students made at least 15 types of errors in reading, interpreting, and predicting from bar graphs. While some of these were reading-language or computation errors, the majority were graph-based errors. The analysis further indicated there were four general categories of graph-based errors namely: data arrangement, topic, scale, or the fact that the information was not shown on the graph.

Reading-language and computation errors could be explained by deficiencies in the general reading-language or computation frames from which information was drawn, or errors in the process of mapping this information to the graph frame. For example, when asked how much more allowance Jane got in 1988 than in 1986, students added the allowance for 1986 to that of 1988 rather than subtracting it.

While it appears that most students have a pattern slot within their graph frames, this slot is either incorrectly developed or not correctly linked to other slots of the frame. In situations where the nature of the graph topic would suggest that searching for a pattern made no conceptual sense students used existing patterns to predict, forced patterns in order to predict, or cited the absence of a pattern for their inability to predict. For example, when faced with non-patterned data showing the number of larch, fir, pine, and spruce trees planted, students forced a pattern to determine the number of elm trees planted.

Most scale errors were attributed to the lack of a scale slot in the graph frame. Other scale errors suggested the presence of only a partially formed scale slot or the lack of a link between scale and other slots the graph frame. For example, students stated that Ann was 2 cm shorter than John, when Ann was 40 cm (two increments of 20 cm) shorter than John.

Students tended to envisage the graph in some "complete" manner and stated they could not predict because the information was not shown on the graph. For example, students could not determine Jane's allowance in 1990 (patterned allowance data for 1986-89 was given) since "1990 was not on the graph". It was concluded that the graph frames of these students was very limited with possibly no pattern or topic slots.

Overall, students in both grade four and grade six had little difficulty in reading bar graphs but had considerably more difficulty interpreting bar graphs. Knowing when prediction from bar graphs was possible appeared to be extremely difficult for nearly all students. The frequency of reading-language, computation, and particularly scale errors was higher for the grade fours than for the grade sixes. However, errors involving pattern occurred in similar frequencies for both grades. It was concluded that students at the grade four and the grade six level have similar but flawed graph frames.

Discussion

The present study has identified errors in understanding the information presented in bar graphs and analyzed these errors in terms of deficiencies of students' graphical frames. A possible explanation for how an incorrect graph frame develops is now discussed.

Davis's (1984) frame theory model indicates that frames are personal constructs created as the result of experience. This does not mean that students will have very different frames. In fact, large numbers of students have been identified as having developed very similar frames. Davis attributes this to their shared school experience and because there are underlying principles that govern information processing in general.

The results of this study suggest that a significant percentage of students possess similar but flawed graph frames. One explanation for the development of the flawed frame is students' lack of appropriate experience with graphs together with the nature of information processing. For example, it is a common characteristic of human information processing to initially overgeneralize. Davis states that it is only with experience that limitations are developed that prevent overgeneralizing. In this study students were seen to have an overgeneralized concept of the use of pattern and, consequently, they used pattern without the limitation of topic to determine values for items not shown on the graph. It is possible that students lacked the appropriate experience with graphs to develop a link between the pattern and topic slots of the graph frame. It is also possible that the graphical experiences of students had been so limited that a topic slot of the graph frame was never developed.

It can be argued the use of current elementary school texts (both in mathematics and other subjects), together with teaching time constraints, results in students receiving less instruction, and much less varied experience, than what is necessary to become proficient at processing graphical information. Most of these experiences are at the literal reading and interpret level with students seldom constructing and drawing their own graphs. Prediction is often mentioned only in situations where it is possible, hence students never assess why a prediction is possible. Furthermore, work with patterns in non-graphical situations may encourage students to follow patterns even when it is not reasonable. Such experiences would explain why the general graph information in students' graph frames that becomes part of the instantiated frame is often limited or incorrect, and that information retrieved from other frames and brought to the graph frame is incorrect.

Specific suggestions based on the results of this study on what constitutes appropriate experience with graphing are given in the next section.

Implications

The increased use of graphs by society indicates a necessity for educators to help students become adept at utilizing graphs to their maximum potential. The results of this study, therefore, have several implications for instruction.

Since difficulties in processing information contained in graphs has been found to exist, teachers must be made aware of where they exist and the factors that contribute to these difficulties. This information would be of value in planning instruction that would result in greater success for students.

The question arises of what can be done to possibly reduce the incidence of errors. This study has shown that both grade four and grade six students have difficulties with scale. Specifically, it suggests that students have either no scale slot in their graph frame or one that is only partially formed. It is possible that by exposing students to different scale factors on the same and different graphical problems that a scale slot will be properly developed. For example, a child's allowance for the last several years could be displayed using scale factors of $0.50, $0.75, and $1.00. With the use of computer graphing students could quickly see the effect of the chosen scale on the image. The results of this study also indicated that when students were not attending to scale they were often attempting to count horizontal guidelines on the graph instead. Most elementary school mathematics texts provide these guidelines to assist students in matching vertical and horizontal data entries. It is suggested that students' attention be drawn to the function of these guidelines and as students become more experienced with graphs their use be discontinued.

This study has shown that students have almost no correct ideas about predicting from the data on a bar graph, particularly when it is or is not appropriate. Despite recommendations by educational authorities for increased emphasis on this skill, the students' written work and interviews suggested that their experience with it has been very limited. The suggestions in the literature that students construct their own graphs, verbalize relationships in the graph, discuss the significance of any trends in the data, and check the appropriateness of their answers would seem to be a logical starting point for broadening this experience.

More specifically, the results of this study suggest an alarming reliance of students on the idea of pattern when asked if prediction from the data were possible.

It appeared the pattern slot of students' graph frames was incorrectly developed or not linked to other slots of the frame. Since the concept of pattern is mapped to the graph frame, teachers should first examine the materials students use for non-graph exercises involving pattern to ensure they are appropriate. Students experiences with graphs should then include exposure to patterned and non-patterned data for topics which prediction is and is not conceptually sound. In particular, the same data should be shown with different orders of presentation. For example, export data for several countries could be displayed both in order of increasing magnitude and in a non-ordered form. Students could then be asked to compare the graphs.

The task of developing skills in reading, interpreting, and predicting from graphs is not solely the responsibility of the mathematics teacher. Graphs can be found across the entire curriculum, particularly in science and social studies. Furthermore, reading, interpreting, and predicting from data are skills not confined to work with graphs. It is important to realize, therefore, that all teachers have a role to play if students are to become adept at processing information.

Recommendations for Future Research

The focus of this study was to determine the difficulties of grade four and grade six students in understanding the information conveyed by bar graphs. Only through the documentation of student errors and attempts to understand the processes underlying these errors can educators hope to improve students' graph comprehension. This requires carefully designed research.

The results of this study suggest several recommendations for future research. First, since the present study was limited to grade four and grade six, and to one graph form, similar studies should be conducted at other grade levels and with other graph forms. This would provide information on the grade level at which the idea of predicting from bar graphs is no longer "dominated" by the pattern, and whether students have the same over-reliance on pattern when predicting from other graph forms. The aim of this research would be to formulate a description of the development of students' graph frames that could be used in designing instruction.

Second, a study that provides more detailed information on the inner workings of a graph frame should be conducted. Such extended research should address specifics about graph frames that could not be determined within the present study. Factors that could be examined include: determination of the existence and nature of errors in the mapping of information to the graph frame; determining in answers to predict questions where pattern appears to "dominate" over topic whether the topic slot of the graph frame actually exists; and determination of the existence and nature of linking mechanisms between slots of the graph frame; particularly pattern and topic.

Third, a study should be conducted to determine if specific instruction on areas of difficulty with bar graphs results in a new or corrected graph frame. For example, a unit on graph skills focusing on the three levels: read, interpret, and predict, with attention to identified areas of difficulty could be developed for use at a particular grade level. If specific instruction on areas of difficulty is found to be successful it would suggest that students have modified their previous graph frame or replaced it with a new one. If it is not successful, it would suggest that the previous flawed graph frame still exists. If such instruction was found to be successful, a follow-up study investigating the effect of different strategies of instruction could be conducted. For example, a study that examines the effect of students collecting and organizing their own data on their ability to predict could be conducted.

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