TWO INSTRUCTIONAL STRATEGIES AND THE HAND-HELD CALCULATOR AS VARIABLES IN TEACHING THE SOLUTION OF VERBAL PROBLEMS IN ALGEBRA

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by

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ABSTRACT

The purpose of this study was to investigate the relative effectiveness of three procedures, Initial Variable (IV), Guess and Test (GT), and Guess and Test with Hand-Held Calculator (GTC), for translating word problems into equations and then solving the problems.

The investigator chose 78 grade-seven students from a rural high school. They were first divided equally into high- and low-ability groups on the basis of their IQ scores. Then, within each ability level, equal numbers of students were randomly assigned to one of the treatment groups. In the IV procedure, the student initially introduced a variable for the unknown. Then, he wrote an appropriate equation which he solved. In the GT procedure, the student first guessed the solution to the problem and then checked it. If the solution was incorrect, the procedure was repeated at least twice so that the student hopefully saw a pattern that suggested an appropriate equation for the problem. In the GTC procedure, the student followed the same steps as in the GT procedure but used the hand-held calculator to aid in the computation.

Ten 40-minute class periods were used for instruction in each procedure and two 40-minute class periods for administration of a posttest and retention test to each group. Each GTC student had a calculator for the two tests. On both tests students were to find an equation for each verbal problem and then solve it. This produced two different scores, an equation score and a solution score for each test. A five-point Likert-type attitude test was also administered at the beginning of the last instruction period. Instruction in all three

treatment groups was by the investigator.

The data was analysed by use of a 3 x 2 ANOVA. This analysis was performed separately for each of the posttest equation criterion, posttest solution criterion, retention test equation criterion, retention test solution criterion, and attitude test.

The investigator concluded that none of the three treatment procedures was superior to another in terms of initial learning and retention. However, students' ability, as measured by IQ, was a significant factor in determining students' problem-solving ability. None of the three treatment procedures significantly improved students' attitude toward mathematics. No significant interactions occurred with respect to achievement or attitude between the treatment groups and ability levels.

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CHAPTER 1

THE PROBLEM

Brief Overview of Problem Solving

Problem solving has been recognized both as a significant factor influencing the growth of mathematics from the time of Euclid (Kinsella, 1970) and as the principal reason for studying mathematics (National Council of Supervisors of Mathematics Position Paper on Basic Mathematics Skills, 1978). Skypek (1977) quoted a Georgia State Legislator, who spoke at an annual statewide Quality of Life Conference on the theme "Back to the Basics - the Three R's" as having said, "I think problem solving is the basic skill in mathematics" (p. 5).

Recognizing problem solving as a basic skill in mathematics today was emphasized by Henderson and Pingry (1953) when they wrote the following:

> If life were of such a constant nature that there were only a few chores to do and they were done over and over in exactly the same way, the case for knowing how to solve problems would not be so compelling. All one would have to do would be to learn how to do a few jobs at the outset. From then on he could rely on memory and habit. Fortunately-or unfortunately depending upon one's point of view-life is not simple and unchanging. Rather it is changing so rapidly that about all we can predict is that things will be different in the future. In such a world the ability to adjust and to solve one's problems is of paramount importance (p. 233).

It is widely recognized that one learns mathematics by doing mathematics, and one learns problem solving by solving problems. Henderson and Pingry (1953) emphasized this when they stated, "From what we know about learning, there is only one way students can learn to solve problems - by solving problems and studying the process" (p. 233).

Background of the Study

One method of improving the problem-solving processes is by teaching students to use heuristic precepts and strategies. The groundwork for such was laid by Polya, accredited as 'The Father of Modern Heuristic', in How to Solve It (1957) and Mathematical Discovery (1962, 1965). Information-processing theories have created interest in heuristics applied to problem-solving and evidence from research in artificial intelligence supports Polya's theories (Kilpatrick, 1969, p. 527).

The word "heuristic" comes from the Greek word heuriskein which means "to discover". In the plural, heuristics suggest a repertoire of techniques used for discovering solutions to problems (Hughes, 1976, p.v). Hughes (1976), a former student of Polya's, referred to this collection of techniques as "the heuristic tool box" (p. 88). The maxims or techniques listed by Hughes (1976) included searching for a pattern, drawing a figure, and finding a related problem. Hughes' techniques are essentially the same as those found among the list given by Polya in How to Solve It (1957). Hughes (1976) stated, "There is no end to the list of maxims; hence, we merely stop. It should be enlarged by the practitioner of heuristics who, not finding a convenient tool in his box, fashions a new one" (p. 89).

This study was the investigation of the specific problem-solving technique of guessing rather than the investigation of general heuristics In an investigation of general heuristics, the impact of various problem-solving techniques, for example, drawing a diagram, guessing, and finding a related problem would be studied. The guessing technique was given special attention by Polya in How to Solve It (1957) and Mathematics and Plausible Reasoning, Volume 2 (1954) as well as being the theme of the movie produced by the Mathematical Association of America, Let Us Teach Guessing, featuring Polya. Polya (1971) stated his maxim concisely as, "Let us teach proving by all means but let us also teach guessing" (p. 324). Polya (1971) further elaborated on his "guess-and-test" strategy as follows:

> Let me recommend here just one little practical trick. Before the students do a problem, let them guess the result, or part of the result. The boy who expresses an opinion commits himself; his prestige and self-esteem depend a little on the outcome, he is impatient to know whether his guess will turn out right or not, and so he will be actively interested in his task and in the work of the class - he will not fall asleep or misbehave.
>
> In fact, in the work of the scientist, the guess almost always precedes the proof. Thus, in letting your students guess the result, you not only motivate them to work harder, but you teach them a desirable attitude of mind. (p. 328).

With renewed emphasis in heuristics coupled with a new technological aid, the hand-held calculator, it is the investigator's belief that the "guessing strategy" in problem-solving will receive special attention.

Purpose of Study and Problem Statements

The purpose of this study was to investigate the relative effectiveness of two procedures, termed Guess and Test and Initial Variable, for translating word problems into equation form and then solving the problems. In addition, the study included an assessment of the hand-held calculator in the Guess and Test procedure.

Specifically, the study was directed by the following questions:

1. Does the Guess and Test procedure for setting up equations and then solving word problems lead to higher achievement and an improved attitude as compared to the Initial Variable procedure?
2. Does the use of the hand-held calculator with the Guess and Test procedure lead to higher achievement and an improved attitude as compared to the Guess and Test procedure without the hand-held calculator?
3. Does the student's ability level, as measured by IQ, influence achievement and/or attitude in any of the treatment groups?

Description of Treatments

Initial Variable (IV). This approach which is found in many school textbooks is the same as the step method outlined by Bassler, Beers, and Richardson (1975). They give the following six steps for directing students to solve verbal problems:

1. Read the problem carefully ...
2. Decide what question the problem asks and choose a variable to represent the unknown ...
3. Consider the other information given in the problem and how it relates to the unknown ...
4. Write an equation or equations expressing the given relationships.
5. Solve the equation or equations. [Only the problems solved by setting up one equation are considered in this study.]

6. Check the answer (p. 172).

Guess and Test (GT). The approach used here for solving verbal problems in algebra is essentially the same as that outlined by Kinsella (1970) in his discussion of some general-teaching methods for solving algebra problems.

The objective of the guess-and-test strategy for solving verbal problems in algebra was to write an equation which the students would eventually solve. The actual climax of the guessing came when the student wrote the equation, since the solving was more or less mechanical.

Specifically, the student was asked to guess the value of the unknown and then check it. He wrote the operational steps of the checking in table form. If the guess was incorrect, as it usually was, he was asked to repeat the procedure at least once more. He was then asked to look for a pattern in the table that would hopefully suggest a way of deriving an appropriate equation by replacing the guessed value in the operational steps by a variable, say x. He then solved the resulting equation and checked the answer in the verbal statement of the problem.

Guess and Test with Hand-Held Calculator (GTC). This approach is identical to the Guess and Test approach except that the calculations were performed with the aid of a hand-held calculator.

Significance of Study

Although most of Doblaev's (1969) study consisted of the analysis of thought processes in setting up equations for algebraic problems as reflected by students' reports of what they were thinking as they worked the problems, the reasons he gave for the significance of his study can be quoted here, in slightly paraphrased fashion, as being significant for this study.

First, as reported by Doblaev, the solution of problems by means of equations constitutes the most important part of secondary school algebra.

Secondly, as reported by Doblaev, it is generally accepted that setting up equations is a topic in which pupils have greater difficulty than in solving algebraic examples.

Thirdly, as reported by Doblaev, most methodologists and teachers agree that procedures for setting up equations have been less developed than other areas of algebra.

Since textbooks usually describe only the initial-variable method for setting up equations, the procedure of guess-and-test needs more exploration to determine its effectiveness. If effective, and the calculator-aided guess-and-test method is even more effective, textbook materials, especially calculator-based materials, need be developed to take full advantage of the procedures. This suggestion is reflected in Recommendation 9 of the Report of the Conference on Needed Research and Development on Hand-Held Calculators in School Mathematics (Estry and Payne, 1976):

> Curriculum materials should be developed that teach problem-solving strategies more effectively and that build pupils' confidence in their ability to solve problems. (p. 18).

Particularly, significant was the grade level of the students. Since these grade-seven students were translating verbal problems into equations for the first time, it was important to know if the heuristic-based procedure, guess-and-test, was superior to the more traditional procedure, initial-variable, in this intital stage of algebraic problem solving.

CHAPTER 2

REVIEW OF THE RELATED LITERATURE

Introduction to Calculators

Only within the last ten years has the hand-held calculator been available to students in significant numbers, but its use in that short span of time has generated considerable debate. Countless articles have been written discussing its pros and cons. Bell (1978) wrote that "the questions are not simply whether to use or not to use calculators, but how, with whom, when, in what ways, and in the service of what objectives" (p. 173). These types of questions have been addressed by the National Advisory Committee on Mathematical Education (NACOME) Report (1975), the Report of the Conferences on Needed Research and Development on Hand-Held Calculators in School Mathematics (Esty and Payne, 1976), and the report prepared for the National Science Foundation under the directorship of Suydam (1976).

The NACOME report (1975) recommended that a calculator should be available for each mathematics student no later than the end of grade eight.

Concern for the use of the calculator is reflected in Recommendation 6 of the Report of the Conference on Needed Research and Development on Hand-Held Calculators in School Mathematics. It was stated that:

> Materials should be developed to exploit the calculator as a teaching tool at every point in the curriculum to test a variety of ideas and possibilities pending emergence of calculator-integrated curriculums. (p. 18)

Recommendation 6 of Suydam's report, Electronic Hand Calculators: The implications for Pre-College Education, reflected the same concern for using calculators in schools:

> Place more emphasis on problem-solving strategies. Use practical, realistic, significant problems, and more applications. (p. 40)

Gawronski and Colblentz (1976) pointed out that when calculators are used to eliminate the drugery of calculations, more time will be available to devote to problem-solving skills. Suydam and Weaver (1977) were of the same opinion when they stated, "The focus can be on strategies and process when the calculator is used, with less emphasis on computation within the problem-solving context" (p. 42).

Research on Calculators

Szetela (1979), in reviewing the literature for his study on calculator use in trigonometry, found the effects of the calculators on mathematics learning and attitude encouraging. He found that of 40 findings in 26 studies reported in Bulletin No. 9 of the Calculator Information Center, the calculator groups performed significantly higher on paper-and-pencil tests than noncalculator groups 19 times, there were no significant differences 18 times, and in only 3 cases did the noncalculator group achieve significantly higher scores. Szetela (1979) also found that of seven findings reported on attitudes towards mathematics in calculator studies, six of the findings produced nonsignificant differences, with only one producing a significant difference in favor of calculator-based instruction. In his own study, Szetela found that the calculator subjects achieved significantly higher scores than noncalculator subjects on a quiz after two and one-half weeks of instruction, but on the final achievement test given after three and one-half weeks, the calculator subjects did not retain a statistically significant margin of superiority, although they did retain higher scores. He found no significant difference in attitude towards learning. He considered this finding on attitude to be consistent with other calculator research on attitudes towards mathematics.

Of eight studies reported in the report Electronic Hand Calculators: The Implication for Pre-College Education, (Suydam, 1976), except for one study which was a survey of the calculator use in schools, all studies reported favourable findings with the use of the calculator. One of the studies did report that the calculator groups achieved higher on concepts and computation but not as high on problem solving.

The search of the literature revealed two studies that examined the effects of the use of the electronic calculator on problem-solving achievement with one of them examining its effects on attitude. Ward (1978) found that there was no significant difference in problem-solving achievement or attitude towards mathematics between calculator and noncalculator groups. Kasnic (1977) found the same results regarding problem-solving achievement. In addition, he found no significant difference between the low-ability calculator group using the calculator on a posttest of problem-solving ability and the four high-ability groups, one of which was a calculator group using the calculator on the posttest. This led him to conclude that low-ability problem-solving students could profit from using a calculator on a test of problem-solving since it reduces the computational difficulty.

Two other studies are relevant to the extent that they focused on the impact of the calculator on the study of algebra. Quinn (1975) in a study with eighth- and ninth-grade algebra students, found that the use of a programmable calculator was not justified should justification mean superior algebra achievement. He found no improvement in attitude at the grade-eight level on any of the six attitudinal scales used, but at the grade-nine level there was an indication that the programmable calculator could be a helpful aid in maintenance and improvement of some aspects of student attitude. Cooper (1977), in an analysis of the effects of the hand-held calculator on a college algebra class, found that there was no significant difference in attitude or achievement between groups who used calculators and those who did not.

Introduction to Solving Verbal Problems in Algebra

Fawcett and Cummins (1970) described a procedure for setting up equations for word problems in algebra that is essentially the same as the Guess and Test procedure used in this study. They pointed out that the beginning experiences of students with word problems is very important, since it sets the stage for solving similar problems in the future. It is for this initial stage that they recommended the guess-and-test procedure. They contended that with practice and growth, the student will eventually be able to set up the equation immediately for the kinds of problems which he understands. In other words, after the student has practiced the guess-and-test procedure, he will then shorten his work by using the initial-variable procedure.

An opinion similar to that of Fawcett and Cummins was put forth by Nyberg (1966). He discussed what he considered a general method of solution for all verbal problems in algebra. His general method which he said is like the ancient "Rule of False Position" is essentially the guess-and-test procedure. He did not believe the method to be any better than the "traditional" (initial-variable) method, but he found that after a student had worked a few problems by the general method (guess-and-test procedure) he could then better understand the "traditional" method.

Research on Solving Verbal Problems in Algebra

The search of the literature revealed two studies that related directly to the verbal problem-solving strategies used in this study, but no studies were found that compared different strategies for solving verbal problems in algebra in which one of the strategies was calculator based. Settle (1977) compared the guess-and-test and "traditional procedures (initial-variable) for writing relevant equations to verbal problems in elementary algebra. He found that the guess-and-test approach to developing skill in equation construction for verbal problems in elementary algebra was superior to the "traditional" approach in terms of initial learning, but no such superiority existed in terms of transfer or retention. Crowe (1975), in a study similar to that of Settle, but with no retention test or transfer test, found no significant difference between the two strategies, the guess-and-test strategy and the initial-variable strategy. He also found no significant interaction between ability levels and methods.

A study comparing two strategies of instructing students to solve verbal problems and which closely parallels this study is that of Bassler, Beers, and Richardson (1975). They found that students instructed in a step method (Polya Method) of problem-solving scored higher than students taught a translation strategy (Dahmus Method) on the equation criterion, but there was no difference on the solution criterion.

Several studies that analyzed solutions to word problems have somewhat peripheral relevance to this study. In a study conducted with students who just completed grade eight, Kilpatrick (1967) found the group that used the least trial-and-error had most trouble with word problems. In a study conducted with grade-nine general students, Dalton (1974) found that subjects who used trial-and-error tended to be more effective problem solvers. These conclusions, although in agreement with each other, are incompatible with that reported by Suydam and Weaver (1977) in a discussion of research on problem solving. They reported that blind guessing and trial-and-error are considered to be the most unsuccessful strategies. Kantowski (1977), in a study of processes involved in mathematical problem solving, found evidence to indicate that as problem-solving ability developed, less trial-and-error and guessing were observed.

The results of several other studies bear some relevance to his present review. Post (1967) concluded that a special study of a structure of the problem-solving process appeared not to enhance the problem-solving ability of grade-seven students, but he did find that intelligence was a significant factor in the determination of a problem-solving ability. Bratton (1977) found that instruction in the use of Polya's heuristic approach to solve college algebra word problems did not prepare subjects to solve such problems any better than instruction in a traditional method.

CHAPTER 3

METHODOLOGY

Design

A 3 x 2 factorial design was selected for the study by using an extension of the "Experimental Group-Control Group: Randomized Subjects" design (Kerlinger, 1973, pp. 331-334).

		Treatments		
		IV	GT	GTC
Intelligence	High		Achievement or Attitude Scores	
	Low			

Figure 1. Experimental Design

Instruction and Materials

Ten 40-minute class periods were used for instruction in each method and two 40-minute class periods for administration of the posttest and retention test to each group. No homework was given. Instruction in all three groups was performed by the investigator.

Verbal problems of equivalent nature to those given in the grade-seven textbook (Fleenor, Eicholz, and O'Daffer, 1974) were used. Problems to be used in all three treatment groups were essentially the same except that in the GTC group some problems were worked involving larger numbers but using the same principles as problems involving smaller numbers.

Examples of types of problems used in the study are given as follows:

Example 1: A certain number is added to 29 and the result is 84. What is the number?

Example 2: Ten is subtracted from three times a number and the result is 31. Find the number.

Example 3: A man averaged 58 kilometres (km) per hour on a trip. If his trip was 696 km, how long did it take him to make the trip?

Example 4: (This example would be worked in the GTC group only.)

Two hundred fifty is added to seventeen times a number and the result is 2834. Find the number.

The following is an example of a problem with an illustration of the three treatment methods:

Problem: A certain number is multiplied by 3 and then 9 is subtracted from the product. The result is 6. What is the number?

Initial Variable Procedure. The student first determines the basic unknown number. He then writes the following statement: Let x (or some other variable) represent the number. Next, he translates the problem into an equation, say $3x - 9 = 6$. He then solves the equation and checks the solution in the verbal statement of the problem.

Guess and Test Procedure. The student first guesses the answer, say 6. He then checks by writing $3.6 - 9 = 18 - 9 = 9$. Since 9 is greater than 6, he knows that his guess was too much. He then guesses a second time. If he guesses, say 4, he writes $3.4 - 9 = 12 - 9 = 3$.

This will show him that his guess was too small.

Guess	Test Your Guess	Results
6	$3.6 - 9 = 9$	Too large
4	$3.4 - 9 = 3$	Too small

The working is organized in table form. He now looks for a pattern in the table that hopefully suggests a way of deriving an appropriate equation by replacing the guessed value by x and writing $3x - 9 = 6$. This step is emphasized by the teacher as the key component of the procedures. He then solves the equation and checks the solution in the verbal statement of the problem.

Guess and Test with Calculator Procedures. The student follows the same steps as in Guess and Test but uses the calculator to aid in the computation.

Instrumentation

The two achievement tests were constructed by the investigator. Since the same achievement tests were given to all three treatment groups, each test was administered to all students at the same time. Each GTC student used a calculator for the two tests.

A posttest was administered on the class day immediately following instruction. The test consisted of ten verbal problems, seven of which were equivalent to the verbal problems presented in class and three of a more complex nature than the other seven.but based on principles presented in class. Students were to find an equation for each problem and then solve it. Two different scores, an equation score and a solution score, were obtained for each problem.

Four weeks after the posttest, a retention test was administered. Students were not aware of this test until the time of its administration. The test was identical to the posttest except that the numerical values in the problems were varied.

An attitude scale was given to determine if students' attitude toward mathematics was significantly different among the IV, GT, and GTC groups. The test was administered at the beginning of the lesson on the last period of instruction. The scale, a five-point Likert type with 26 items, was developed by Suydam and Trueblood. The internal consistency reliability of the scale is approximately .95. It was selected from scales reported by Suydam (1974).

Grading

In order to obtain the equation score, each student was given a score from zero to two on each problem, calculated as follows:

0 – did not attempt to write the equation or the equation, if written, did not relate to the correct form.

1 – equation that could have been correct except for mistakes such as translating "sixteen less than a number" as 16 – x or omitting brackets in translating "twice the sum of a number and two"

2 – wrote the correct equation.

The student's equation score on each test was the sum of the equation scores for the problems.

In order to obtain the solution score on each achievement test, each student was given a score of zero to three for each problem, calculated as follows:

0 – did not attempt to solve the equation or, if attempted, showed no evidence of a solving technique.

1 - showed evidence of a solving technique for the correct equation—but arrived at an incorrect answer.

2 - arrived at the correct solution without obtaining an appropriate equation.

3 - solved the correct equation, thus producing a correct answer for the verbal problem.

The student's solution score on each test was the sum of the solution scores for the problems.

Subjects

The investigator studied the total grade-seven enrolment, consisting of 78 students in three classes, in a small urban environment.

The students were first divided equally into two ability groups, designated as High (upper half) and Low (lower half), based on their Lorge-Thorndike verbal IQ scores taken from the cumulative records. Within each ability level, equal numbers of students were assigned at random to one of the three treatment groups. The three treatment methods, Initial Variable (IV), Guess and Test (GT), and Guess and Test with Calculator (GTC) were then randomly assigned to the three treatment groups.

Randomization was achieved by using the table of random digits in Glass and Stanley (1970) and following the procedure outlined by them on p. 213.

The Lorge-Thorndike Verbal IQ test was used to divide students into ability groups, since verbal comprehension is necessary for the understanding of word problems. Also, this was the only IQ test available in the school.

Statement of Research Hypotheses

The study investigated 15 hypotheses. The three basic types of hypotheses for each of five ANOVA calculations were:

$H_0 1$: There is no significant difference with respect to achievement among the three treatment groups (IV, GT, GTC).

Statistically, $H_0 1$: $\mu_{.1} = \mu_{.2} = \mu_{.3}$

$H_0 2$: There is no significant difference with respect to achievement between the two ability levels (High, Low).

Statistically, $H_0 2$: $\mu_{1.} = \mu_{2.}$

$H_0 3$: There is no significant interaction with respect to achievement between the three treatment groups (IV, GT, GTC) and the two intelligence levels (High, Low).

Statistically, $H_0 3$: $(\mu_{ij} - \mu_{i.} - \mu_{.j} + \mu) = 0$

where $i = 1, 2$, and $j = 1, 2, 3$.

The 15 hypotheses, three for each of the posttest equation criterion, posttest solution criterion, retention test equation criterion, retention test solution criterion, and attitude test, are of the same type as $H_0 1$, $H_0 2$, and $H_0 3$ except for the attitude test, in which the dependent variable "achievement" is replaced by "attitude".

Statistical Procedure

A 3 x 2 ANOVA was used to analyze the data collected. This analysis was performed separately for each of the posttest equation criterion, the posttest solution criterion, the retention test equation criterion, the retention test solution criterion, and the attitude test. All hypotheses were tested at the .05 level of significance. If significant F-ratios ($p < .05$) occurred, the Scheffee procedure (Glass and Stanley, 1970, pp. 443-445) was used to examine pair-wise contrasts of treatment group mean scores.

CHAPTER 4

RESULTS

Analysis of Data and Findings

The data for each of the posttest, retention test, and attitude test were analyzed by a 3 x 2 analysis of variance. This analysis was performed separately for each of the posttest equation criterion, posttest solution criterion, retention test equation criterion, retention test solution criterion, and attitude test. All hypotheses were tested at the .05 level of significance.

Posttest equation criterion results. Table 1 contains the means and standard deviations of scores for the posttest equation criterion. The overall F-ratios for the data of Table 1 are summarized in Table 2. The results indicated by the overall F-ratios are as follows:

1. There was no significant difference with respect to achievement on the posttest equation criterion among the three treatment groups.
2. There was a significant difference with respect to achievement on the posttest equation criterion between the two ability levels. The high-ability students had a significantly larger overall mean score than the low-ability students.
3. There was no significant interaction with respect to achievement on the posttest equation criterion between the three treatment groups and the two ability levels.

These results indicate that there was no significant difference due to treatments in deriving equations for algebraic word problems. However, the high-ability students performed significantly better than the low-ability students. There was no significant interaction between treatments and ability levels.

Table 1

Group Size, Means, and Standard Deviations*
for Postbest Equation Criterion

	Treatment		
	IV	GT	GTC
High Ability	15.08 (2.33) n = 13	14.92 (1.68) n = 12	13.77 (1.83) n = 13
Low Ability	12.62 (2.02) n = 13	12.89 (2.20) n = 9	12.15 (1.99) n = 13
Overall	13.85 (2.48) n = 26	14.05 (2.13) n = 21	12.96 (2.05) n = 26

* Standard deviations in parentheses

Table 2

Analysis of Variance of Scores for Posttest Equation Criterion

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F_o	Probability $F \geq F_o$
Treatments	13.87	2	6.94	1.71	0.19
Ability	75.17	1	75.17	18.53*	0.00
Interaction	2.33	2	1.16	0.29	0.75
Error	271.81	67	4.06		
Total	365.67	72	5.08		

* Significant at the .05 level

Posttest solution criterion results. Table 3 contains the means and standard deviations of scores for the posttest solution criterion. The overall F-ratios for the data of Table 3 are summarized in Table 4. The results indicated by the overall F-ratios are as follows:

1. There was no significant difference with respect to achievement on the posttest solution criterion among the three treatment groups.
2. There was a significant difference with respect to achievement on the posttest solution criterion between the two ability levels. The high-ability students had a significantly larger overall mean score than the low-ability students.
3. There was no significant interaction with respect to achievement on the posttest solution criterion between the three treatment groups and the two ability levels.

These results indicate that there was no significant difference due to treatments in solving algebraic word problems. However, the high-ability students performed significantly better than low-ability students. There was no significant interaction between treatments and ability levels.

Table 3

Group Size, Means, and Standard Deviations*
for Posttest Solution Criterion

	Treatment		
	IV	GT	GTC
High Ability	19.54 (5.24) n = 13	17.58 (3.40) n = 12	17.92 (3.12) n = 13
Low Ability	14.77 (4.78) n = 13	16.33 (4.12) n = 9	15.38 (4.17) n = 13
Overall	17.15 (5.48) n = 26	17.05 (3.68) n = 21	16.65 (3.12) n = 26

*Standard deviations in parentheses

Table 4

Analysis of Variance of Scores for Posttest Solution Criterion

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F_o	Probability $F \geq F_o$
Treatments	3.32	2	1.66	0.09	0.91
Ability	160.30	1	160.30	9.01*	0.00
Interaction	37.47	2	18.73	1.05	0.36
Error	1192.45	67	17.80		
Total	1393.77	72	19.36		

* Significant at the .05 level.

Retention test equation criterion results. Table 5 contains the means and standard deviations of scores for the retention test equation criterion. The overall F-ratios for the data of Table 5 are summarized in Table 6. The results indicated by the overall F-ratios are as follows:

1. There was no significant difference with respect to achievement on the retention test equation criterion among the three treatment groups.
2. There was a significant difference with respect to achievement on the retention test equation criterion between the two ability levels. The high-ability students had a significantly larger overall mean score than the low-ability students.
3. There was no significant interaction with respect to achievement on the retention test equation criterion between the three treatment groups and the two ability levels.

These results indicate that there was no significant difference due to treatments in deriving equations for algebraic word problems. However, the high-ability students performed significantly better than the low-ability students. There was no significant interaction between treatments and ability levels.

Table 5

Group Size, Means, and Standard Deviations*
for Retention Test Equation Criterion

	Treatment		
	IV	GT	GTC
High Ability	14.83 (2.41) n = 12	14.92 (2.35) n = 12	13.25 (4.49) n = 12
Low Ability	12.46 (2.22) n = 13	13.00 (1.87) n = 9	11.08 (5.58) n = 12
Overall	13.60 (2.57) n = 25	14.10 (2.32) n = 21	12.17 (5.08) n = 24

* Standard deviations in parentheses

Table 6

Analysis of Variance of Scores for Retention Test Equation Criterion

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F_o	Probability $F \geq F_o$
Treatment	41.97	2	20.98	1.73	0.19
Ability	81.58	1	81.58	6.72*	0.01
Interaction	0.58	2	0.292	0.02	0.98
Error	776.97	64	12.14		
Total	905.37	69	13.12		

* Significant at the .05 level

Retention test solution criterion results. Table 7 contains the means and standard deviations of scores for the retention test solution criterion. The overall F-ratios for the data of Table 7 are summarized in Table 8. The results indicated by the overall F-ratios are as follows:

1. There was no significant difference with respect to achievement on the retention-test solution criterion among the three treatment groups.
2. There was a significant difference with respect to achievement on the retention test solution criterion between the two ability levels. The high-ability students had a significantly larger overall mean score than the low-ability students.
3. There was no significant interaction with respect to achievement on the retention test solution criterion between the three treatment groups and the two ability levels.

These results indicate that there was no significant difference due to treatments in solving algebraic word problems. However, the high-ability students performed significantly better than low-ability students. There was no significant interaction between treatments and ability levels.

Table 7

Group Size, Means, and Standard Deviations*
for Retention Test Solution Criterion

	Treatment		
	IV	GT	GTC
High Ability	19.58 (4.68) n = 12	20.75 (3.25) n = 12	18.67 (5.31) n = 12
Low Ability	14.69 (4.68) n = 13	17.11 (4.20) n = 9	17.33 (4.94) n = 12
Overall	17.04 (5.22) n = 25	19.19 (4.03) n = 21	18.00 (5.06) n = 24

* Standard deviations in parentheses

Table 8

Analysis of Variance of Scores for Retention Test Solution Criterion

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F_o	Probability $F \geq F_o$
Treatment	38.87	2	19.43	0.92	0.40
Ability	188.45	1	188.45	9.00*	0.00
Interaction	39.59	2	19.80	0.94	0.39
Error	1340.15	64	20.94		
Total	1620.98	69	23.49		

* Significant at the .05 level

Comparison of posttest and retention test results. Since the posttest and retention test scores were not compared statistically, a graphical comparison of the mean scores was made (see Figures 2 and 3).

For the GT group, the overall mean score for the retention test equation criterion indicates an increase over that of the posttest, whereas that for each of the IV and GTC groups indicates a decrease, more dramatically for the GTC group. This pattern of increase-decrease (see Figure 2) is manifested in the mean scores of the high- and low-ability groups except for the high-ability GT group where the mean scores are equal.

The overall mean score for the solution criterion of the retention test increased over that of the posttest for both the GT and GTC groups but decreased slightly for the IV group. This pattern of increase-decrease (see Figure 3) is manifested in the mean scores of the high- and low-ability groups except for the high-ability IV group where the mean scores are approximately equal. The increase in the mean scores for the retention test solution criterion over those for the posttest solution criterion was more noticeable for the high-ability GT group and low-ability GTC group than for the other ability groups.

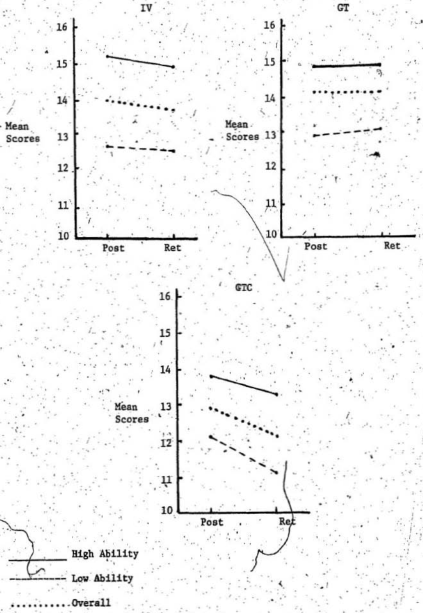

Figure 2. Graphical comparison of mean scores for equation criterion of posttest and retention test for the three treatment groups.

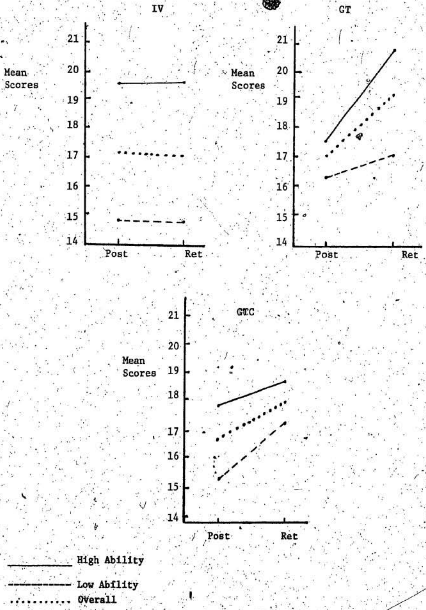

Figure 3. Graphical comparison of mean scores for solution criterion of posttest and retention test for the three treatment groups.

Attitude test results. Table 9 contains the means and standard deviations of scores for the attitude test. The overall F-ratios for the data of Table 9 are summarized in Table 10. The results indicated by the overall F-ratios are as follows:

1. There was a significant difference with respect to attitude toward mathematics among the three treatment groups.
2. There was no significant difference with respect to attitude toward mathematics between the two ability levels.
3. There was no significant interaction with respect to attitude toward mathematics between the three treatment groups and the two ability levels.

These results indicate that there was a significant difference in attitude toward mathematics due to treatments. However, there was no significant difference between high-ability students and low-ability students. There was no significant interaction between treatments and ability levels.

The investigator proceeded to test for the significant difference in attitude among the three treatment groups by utilizing the Scheffe method. The Scheffe analysis for the .05 simultaneous confidence intervals is summarized in Table 11. Since all confidence intervals included zero, no pairwise significant differences were detected.

Table 9

Group Size, Means, and Standard Deviations*
for Attitude Test

	Treatment		
	IV	GT	GTC
High Ability	92.31 (16.76) n = 13	90.83 (17.03) n = 12	73.92 (16.55) n = 13
Low Ability	84.08 (22.50) n = 13	86.67 (17.33) n = 9	78.69 (17.27) n = 13
Overall	88.19 (19.88) n = 26	89.05 (16.85) n = 21	76,31 (16.75) n, = 26

* Standard deviations in parentheses

Table 10

Analysis of Variance of Scores for Attitude Test

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F_o	Probability $F \geq F_o$
Treatments	2489.92	2	1244.96	3.81*	0.03
Ability	106.36	1	106.36	0.33	0.57
Interaction	571.12	2	285.56	0.87	0.42
Error	21898.94	67	326.85		
Total	25103.81	72	348.66		

* Significant at the .05 level.

Table 11

Simultaneous Confidence Intervals Around Differences Between Pairs of Treament Means for Attitude Test

Difference Between Means	$\sqrt{MS_w \ldots}$ *	.05 Confidence Interval
1. $\bar{X}_{.1.} - \bar{X}_{.2.} = -0.86$	13.31	(-14.71, 12.45)
2. $\bar{X}_{.1.} - \bar{X}_{.3.} = 11.88$	12.59	(-0.71, 24.47)
3. $\bar{X}_{.2.} - \bar{X}_{.3.} = 12.75$	13.31	(-0.56, 26.06)

1.* $\sqrt{MS_w\left(\frac{1}{n_1}+\frac{1}{n_2}\right)(3-1)\ {}_{.95}F(2,67)} = \sqrt{326.85\left(\frac{1}{26}+\frac{1}{21}\right)(2)\ (3.15)}$

$= 13.31$

2. $\sqrt{MS_w\left(\frac{1}{n_1}+\frac{1}{n_3}\right)(3-1)\ {}_{.95}F(2,67)} = \sqrt{326.85\left(\frac{1}{26}+\frac{1}{26}\right)(2)\ (3.15)}$

$= 12.59$

3. $\sqrt{MS_w\left(\frac{1}{n_2}+\frac{1}{n_3}\right)(3-1)\ {}_{.95}F(2,67)} = \sqrt{326.85\left(\frac{1}{26}+\frac{1}{26}\right)(2)\ (3.15)}$

$= 13.31$

Discussion of Results

Since no significant difference occurred with respect to achievement on the equation criterion of either the posttest or retention test among the three treatment groups, it appears, in this experiment, that none of the three treatments was more effective than another for teaching students to write algebraic equations for verbal problems. But from a close examination of the graphs of the overall group means for the equation criterion (see Figures 4 and 5), it is evident that the GTC group exhibited somewhat inferior performance to that of either the IV or GT group. Since the graphs of the overall group means for the attitude test (see Figure 6) reflect a similar inferiority for the GTC group, the investigator suspects that the use of the hand-held calculator may have presented difficulties for the students. Maybe more than one class period is necessary to allow for the students to become sufficiently competent in the use of the calculator especially in knowing when to use it. It seems that students relied too much on the calculator in setting up correct equations, whereas the calculator should only act as a tool to check their guesses. In fact, the only occurrence of students not attempting to set up equations was on the retention test in the GTC group where three students did not set up equations but proceeded to solve the problems without them.

The IV group performed well on the equation criterion compared to the other two groups. Possibly, students felt competent working with the formal approach because it more closely resembled previous experience. The introductory instruction, which was the same for all three groups, was oriented toward a formal approach. Because of the lack of familiarity with a more informal approach by the GT and GTC

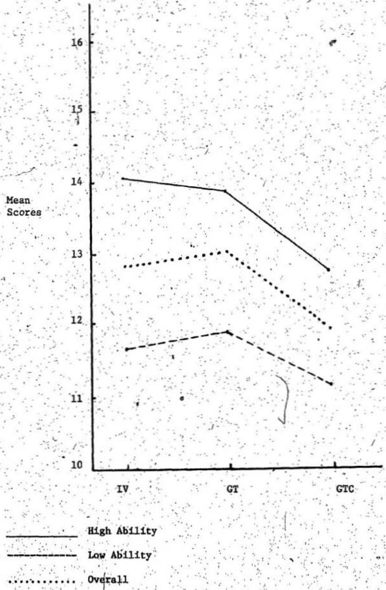

Figure 4. Graphical representation of mean scores for posttest equation criterion for the three treatment groups.

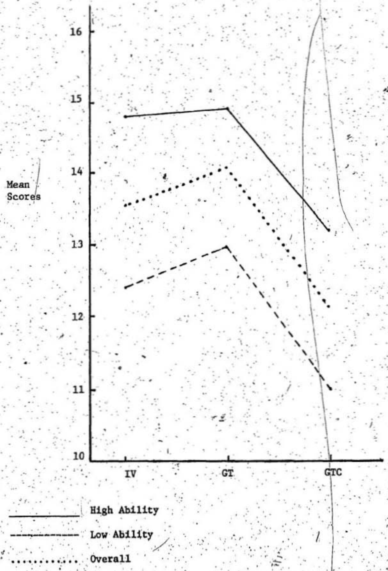

Figure 5. Graphical representation of mean scores for retention test equation criterion for the three treatment groups.

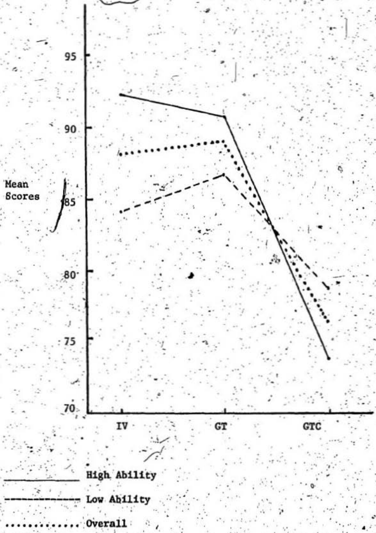

Figure 6. Graphical representation of mean scores for attitude test for the three treatment groups.

students, more time may have been necessary to acquaint the students with the informal guess-and-test procedure. With more time given to developing the procedure, performance superior to that of the IV students might have resulted.

A not unexpected finding was the significantly higher mean scores for the high-ability students than for the low-ability students for the equation criterion of each of the posttest and retention test. Since the graphs of the equation criterion for each of the posttest and retention test mean scores for both high- and low- ability students (see Figures 4 and 5) are almost parallel, essentially no interaction occurs. From Figures 4 and 5 it can be observed that, for both high- and low-ability students, the GT group performed better than either the IV group or GTC group except for the posttest where the high-ability GT group performed only slightly lower than the high-ability IV group. This probably suggests that the informal guess-and-test procedure, if developed to a greater extent than in this study, may be better than the more formal initial-variable procedure in setting up equations. This is probably especially true for low-ability students. Since the graphs for the retention test equation criterion have a parallel nature similar to the graphs for the posttest equation criterion, it appears that for retaining knowledge of deriving equations each treatment worked equally well with both ability levels.

As for the equation criterion, no significant difference occurred with respect to achievement on the solution criterion of either the posttest or retention test among the three treatment groups. However, high-ability students obtained higher mean scores than the low-ability students for the solution criterion of each of the posttest and retention test.

Although not statistically significant, interaction did occur on the posttest solution criterion between ability levels and treatments (see Figure 7). From Figure 7, it can be observed that the solution-criterion mean score for the high-ability students is higher for the IV group than for either the GT group or GTC group, whereas for the low-ability students it is lower for the IV group than for either the GT group or GTC group. As for the equation criterion, the guess-and-test procedure seems to be better for low-ability students than the initial-variable. By guessing possible solutions for a problem, the low-ability students may be better able to decide on the reasonableness of an answer than if the student had chosen an initial-variable to represent the unknown. Knowing that one guess is too large and another is too small allows the student to roughly estimate the solution. For the high-ability student the initial-variable procedure seems to be better than the guess-and-test procedure. Problems of a more complex nature may be required for high-ability students to appreciate the purpose of the guess-and-test procedure.

Although the results were not statistically significant, the graphs in Figure 8 suggest that some interaction did occur on the retention test solution criterion between ability levels and treatments. In Figure 8, the graphs of the mean scores illustrate that for the low-ability students the GTC groups attained a higher score than either the IV group or GT group, whereas for the high-ability students the GTC group attained a lower mean score than either the IV group or GT group. Again, as for the retention test equation criterion for high-ability students, the GT group performed better than either the IV group or GTC group,

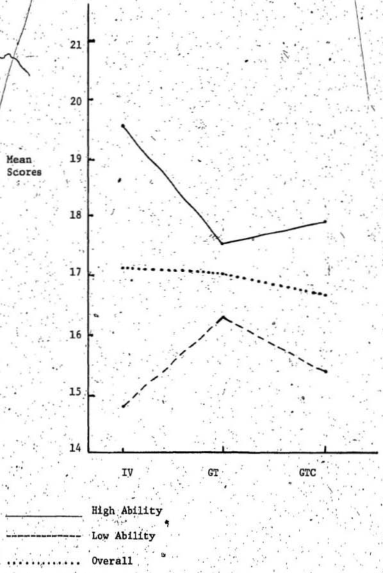

Figure 7: Graphical representation of mean scores for posttest solution criterion for the three treatment groups.

but not for the low-ability students since the GT group performed better than only the IV group. The mean score for the low-ability students being higher for the GTC group than for the other two groups seems to suggest that with increased familiarity with both the calculator and the informal guess-and-test procedure, low-ability students could become better problem solvers than those subjected to either the informal initial-variable procedure or the calculator-unaided formal guess-and-test procedure. Perhaps given problems of a more complex nature, the high-ability GTC students would have performed similarly to that of the low-ability GTC students on the retention test solution criterion. On observation of the graphs of the overall group means for the retention test solution criterion (see Figure 8), it is evident that the GT group performed better than either the IV group or GTC group. From this, it appears that the guess-and-test procedure is better for remembering solutions to verbal problems than the initial-variable procedure. If the procedure had been developed to a greater extent, it may have resulted in performance superior to that of the initial-variable procedure.

Interaction on the attitude test, although not statistically significant, occurred between ability levels and treatments. From the graph of the overall group means (see Figure 6), it can be observed that overall the GT students obtained the highest score and the GTC students, the lowest. As suggested earlier, possibly the reason for the low overall mean score for the GTC students was because of lack of familiarity with the calculator. Another interesting observation is that for the GTC group, unlike the other two treatment groups, the low-ability students attained a higher mean score than the high-ability students. This would seem to give more weight to the previous suggestion that with increased familiarity with both the calculator

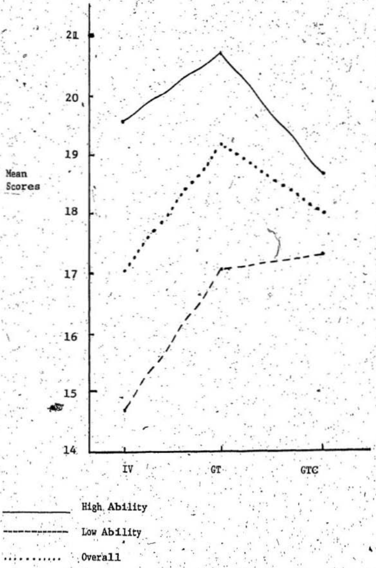

Figure 8. Graphical representation of mean scores for retention test solution criterion for the three treatment groups.

and the informal guess-and-test procedure, low-ability students might become better problem solvers than those subjected to either the informal initial-variable procedure or the formal guess-and-test procedure without the calculator.

Although one of the ten class periods was used to introduce the calculator, the investigator feels that more time was necessary, especially since the students had not previously used calculators in school. Even with one day devoted to introducing the calculator, the students nevertheless finished the work in ten class periods as was intended.

Because of the moderately high overall mean scores on the achievement tests, it appears that grade seven is not too early to begin solving algebraic word problems. It also appears that once techniques for solution are learned, they appear resistent to forgetting. This is especially true for the GT group, since it has higher overall mean scores for both the equation and solution criteria of the retention test than for the respective criteria of the posttest, and also for the GTC group which achieved a higher overall mean score for the solution criterion of the retention test than for the solution criterion of the posttest. The investigator was assured by the regular mathematics teacher that no teaching on algebraic word problems had occurred between the posttest and retention test.

The results of this study agree with that of Crowe(1975) in which he found no significant difference between the guess-and-test and initial-variable strategies for translating word problems of algebra. However, Settle (1977) found that the guess-and-test approach for translating verbal problems in elementary algebra was significantly better than the "traditional" approach (initial-variable) on the posttest but not on the retention test. This present study found no such significance but the GT group did achieve a higher overall mean score than the IV group on all test criteria except the solution criterion of the posttest where it was only slightly lower. Some possible reasons for Settle's results being different from the results of this study might be that Settle used written materials for the students as well as two instructors for teaching.

In both the GT and GTC groups only one student used a table for the guess-and-test procedure on the posttest and no student on the retention test. This seems to agree with the contention of Fawcett and Cummins (1970), that with practice and growth the student will eventually be able to set up the equation immediately for the kinds of problems which he understands.

That the GTC group wasn't significantly different than either the IV group or GT group on attitude agrees with that reported by Szetela (1979). He reported that in Bulletin No. 9 of the Calculator Information Center (1977) of the seven findings reported on attitude toward mathematics in calculator studies, six of the findings produced nonsignificant differences.

No study reported in this paper compares specifically the impact of the hand-held calculator on solving algebraic word problems. But both Quinn (1975) and Cooper (1977), who studied the impact of the calculator on the study of algebra, found no significant difference in attitude or achievement between groups who used calculators and those who did not. The results of this study yield similar conclusions.

CHAPTER 5

SUMMARY, LIMITATIONS, CONCLUSIONS, AND IMPLICATIONS

Summary

The purpose of this study was to investigate the relative effectiveness of two procedures, termed Guess and Test and Initial Variabhe, for translating word problems into equations and then solving the problems. In addition, the study included an assessment of the hand-held calculator in the Guess and Test procedure. Specifically, the study was directed by the following questions:

1. Does the Guess and-Test procedure for setting up equations and then solving word problems lead to higher achievement and an improved attitude as compared to the Initial Variable procedure?
2. Does the use of the hand-held calculator with the Guess and Test procedure lead to higher achievement and an improved attitude as compared to the Guess and Test procedure without the hand-held calculator?
3. Does the student's ability level, as measured by IQ, influence achievement and/or attitude in any of the treatment groups?

An examination of the research related to problem solving in mathematics revealed only two studies directly related to the guess-and-test and initial-variable procedures. Settle (1977) compared the guess-and-test and "traditional" (initial-variable) procedures for writing revelant equations to verbal problems in first-year algebra.

He found that the guess-and-test approach to developing skill in equation construction for verbal problems in elementary algebra was superior to the "traditonal" approach in terms of initial learning but no such superiority existed in terms of transfer or retention. The other study conducted by Crowe (1975) found no significant difference between the two strategies and no significant interaction between ability levels and methods.

To accomplish the purpose of this study, the investigator chose the total grade-seven enrolment of 78 students from a rural high school. The 78 students were first divided equally into high- and low-ability groups on the basis of their Lorge-Thorndike verbal IQ scores. Then, within each ability level, equal numbers of students were assigned at random to one of three treatment groups. The three treatment procedures Initial Varible (IV), Guess-and-Test (GT), and Guess and Test with Calculator (GTC) were then randomly assigned to the three treatment groups. In the GT procedure, the student first guessed the solution to the problem and then checked it. If the solution was incorrect, the procedure was repeated. The process was repeated at least twice, so that the student hopefully saw a pattern that suggested an appropriate equation for the problem. The solution to the problem was then found by solving the equation. In the IV procedure, the student initially introduced a variable for the unknown. Then, he wrote an appropriate equation which he solved to get the solution to the problem. In the GTC procedure, the student followed the same steps as in the GT procedure but used the hand-held calculator to aid in the computation.

Ten 40-minute class periods were used for instruction in each procedure and two 40-minute class periods for administration of a posttest and retention test to each group. Each GTC student had a calculator for the two tests. On both tests, students were to find an equation for each verbal problem and then solve it. This produced two different scores, an equation score and a solution score, for each test. A five-point Likert type of attitude test was also administered at the beginning of the last instructional period. Instruction was performed in all three treatment groups by the investigator.

The analysis of the data was performed by use of a 3 x 2 analysis of variance (ANOVA). This analysis was performed separately for each of the posttest equation criterion, posttest solution criterion, retention test equation criterion, retention test solution criterion, and attitude test. All hypotheses were tested at the .05 level of significance. On the basis of the ANOVA results, the major findings were stated and conclusions drawn.

Limitations

The study was limited by the use of the three experimental treatments (IV, GT, GTC). To make a fuller assessment of the hand-held calculator, one other experimental treatment group, Initial Variable with Hand-Held Calculator, (IVC), would be necessary. In addition to the four treatment groups, two other treatment groups, IVC and GTC both working with more complex problems than the other IV and GTC groups, would allow for an assessment of the hand-held calculator relative to the level of difficulty of problems solved.

The study was limited to grade seven from one particular high school. Although the grade level was probably an advantage since it was the first time in the students' mathematical development that they had been exposed to strategies for solving verbal problems by setting up algebraic equations, the question might be raised whether the duration of 10 days for the study was sufficient time to effectively develop the strategies.

Although no problems were encountered as to the appropriateness of the instructional materials, it must be pointed out that the materials were not subjected to a pilot study.

The relatively low reliability coefficient of .68 (but reasonable for a 10-item test) for the posttest may have affected the detection of any differences in treatment effects.

Although absenteeism was very low during both instruction and the writing of the tests, it should be noted that the 5 students absent from the posttest were all from the GT group. One of these students was from the high-ability level and the other four were from the low-ability level. These five students were not entered in the analysis of data for the attitude and retention tests. Three other students were absent from the retention test. One was from the high-ability IV group, one from the high-ability GTC group, and the other from the low-ability GTC group.

Conclusions

Subject to the inherent limitations of this study and based on the findings, the following conclusions are presented. Since there were no significant differences among the three instructional procedures for translating and solving algebraic word problems, the investigator concluded that none of the three treatment procedures, Guess-and Test, Guess-and Test with Calculator, or Initial Variable, was superior to another in terms of initial learning and retention. However, students' ability, as measured by IQ, was a significant factor in performance for all three treatments. The ANOVA results did provide evidence of a significant difference in attitude toward mathematics among the three treatment groups but an application of the Scheffé analysis detected no significant differences between groups. Therefore, the investigator concluded that none of the three treatment procedures improved students' attitude toward mathematics. No significant interactions occurred with respect to achievement or attitude between the three treatment groups and ability levels.

Implications for Classroom Teacher

The results of this study would seem to imply that both the IV and GT procedures could be employed for instruction in the solution to verbal problems in algebra. The results do not seem to indicate that the use of the hand-held calculator would improve students' problem solving ability in algebra. Possibly this was because of the short exposure time students had to the calculator or because the problems weren't sufficiently complex to take full advantage of the calculator. Possibly, a combination of the two instructional procedures along with the use of the calculator for more complex problems would be more effective for a wider range of student ability than if one procedure were used alone.

The results of this study support the assertion that intelligence is a significant factor in determining problem-solving ability. This may not be surprising to the classroom teacher, but nevertheless the implications are important. Rather than accept the assertion as an immutable fact, the classroom teacher should explore ways of narrowing the gap between high- and low-ability problem solvers. Since the low-ability GTC students obtained a higher mean score for the retention test solution criterion than either the low-ability IV students or low-ability GT students, increased exposure to the use of the calculator for low-ability students might help considerably in narrowing this gap.

Because of the moderately high overall mean scores on the achievement tests, it would seem to imply that grade seven is not too early to begin solving algebraic word problems and once techniques for solution are learned, they are reasonably resistant to forgetting.

Implications and Suggestions for Future Research

The results of this study seem to imply that, for algebraic word problems typical of most textbooks, neither of the two problem-solving strategies, guess-and-test or initial-variable, is superior to the other. Also, the guess-and-test strategy aided by the calculator appears not to be superior to the guess-and-test strategy without the calculator. It is suggested that further investigation be undertaken to explore possible uses of the calculator, especially its use in different problem-solving strategies for solution of verbal problems in algebra;- questions of the following sort, for example, could direct further exploration. What would be the effect on students' problem-solving ability for solution of algebraic word problems if students were given prolonged exposure to the calculator? Would a problem-solving strategy be more effective if students used the calculator with problems of a more realistic and/or more complex nature? Would one problem-solving strategy be more effective than another if both were aided by the calculator and the strategies were given sufficient time to be developed?

The results of this study also seem to imply that high-ability students, as measured by IQ, are better able to solve algebraic word problems than low-ability students, irregardless of whether the initial-variable, guess-and-test, or guess-and-test with calculator is used. Further investigation is suggested to determine to what extent the problem-solving ability of low-ability students can be improved by the prolonged use of the calculator.

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Appendix A

Description of Instructional Materials for IV, GT, and GTC Procedures

– Selected Parts From Teacher's Notes

Description of Instructional Materials

Ten 40-minute class periods were used for instruction in each method and two 40-minute class periods used for administration of the posttest and retention test to each groups.

The instructional materials were organized into four sections each with a specific objective as follows:

SECTION 1: OBJECTIVE: The student should be able to translate verbal phrases into mathematical language.

SECTION 2: OBJECTIVE: The student should be able to write a related equation for an equation such as 2 . X = 36, and then find its solution.

SECTION 3: OBJECTIVE: The student should be able to translate a word problem into an equation.

SECTION 4: OBJECTIVE: The student should be able to translate a word problem into an equation, then solve the equation to find the solution to the word problem.

For the GTC group, a day was allotted to introduce the calculator. The objective for the day was:

> The student should be able to use the hand-held calculator to make calculations involving the four basic operations of addition, subtraction, multiplication, and division.

More specifically, the work completed on each day was as follows:

Description of days work	Group IV	Group GT	Group GTC
Translation of verbal phrases into mathematical language.	Day 1	Day 1	Day 1
Reviewing of previous days work and translation of more complex verbal phrases than those of the first day.	Day 2	Day 2	Day 2
Introducing the calculator.	N/A	N/A	Day 3
Review of equations.	Day 3	Day 3	Day 4
Translation of word problems (mostly number problems) into equations.	Day 4	Day 4	Day 5
Translation of word problems (mostly number problems) into equations and then solving them to solve the problems.	Day 5	Day 5	Day 6
Solving problems related to motion and age.	Day 6	Day 6	Day 7
Review of work by solving 25 problems.	Days 7, 8,9,10	Days 7, 8,9,10	Days 8, 9,10

Appendix B

Introductory Instructional Materials

for IV, GT, and GTC Procedures

- Selected Parts From Teacher's Notes

IV - Days 1, 2, 3

GT - Days 1, 2, 3

GTC- Days 1, 2, 4 (Day 3 for introduction of calculator)

IV, GT, and GTC - Day 1

SECTION 1: OBJECTIVE: The student should be able to translate verbal phrases into mathematical language.

The terms, variable and open expression, are reviewed at this point.

Variable: To represent unknown numbers we use symbols, such as x, □, △, n or y. Each of these symbols is called a variable. Any letters, although x is probably the most common, can be used for a variable. The variable is used to represent any number.

Open Expression: △ represents some number and △ + 5 represents another number. We do not know which number △ + 5 refers to since we haven't assigned a value to △. For this reason △ + 5 is refered to as an open expression. If a value of 2 is given to △, then △ + 5 is 7.

A discussion of the following examples is given to show the meaning of translating word phrases into mathematical symbols.

1. The result of adding a number to 3: $3 + x$
2. The sum of a number and 4: $n + 4$
3. The result of a number decreased by 2: $y - 2$
4. The result of decreasing 18 by a number: $18 - y$
5. The product of a number and 4: □ x 4
6. Eight times some number: $8x$
7. The result of 10 divided by a number: $10 \div x$ or $\frac{10}{x}$

8. The result of a number divided by 6: $x \div 6$ or $\frac{x}{6}$

9. One-third of a number: $\frac{1}{3} \times n$

Discussion:

In translating the word phrases into mathematical language, we will follow these steps.

1. Read the word phrase carefully.
2. Choose a variable to represent the unknown in the phrase.
3. Consider other information given in the word phrase and how it relates to the unknown.
4. Write an open expression for the given relationships.

Example 1

+

The result of adding a number to 3.

x

Open expression: $3 + x$

$3 + x$ is the literal translation but $x + 3$ is also correct because of the commutative property. Give an example. If the number is 7, then $3 + 7 = 10$, and $7 + 3 = 10$.

The following word phrases are used to show that it isn't necessary to put braces under the word phrase to indicate the parts to be translated into mathematical language.

Example 1

10 added to a number

All that is necessary is to think, "Let the unknown be x (or some other symbol) then the mathematical expression is 10 + x".

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.
.

Exercise 1

Directions: Write an open expression for each word phrase.

Example: x less than 15 ___15 - x___

1. 5 added to a number.
2. A number decreased by 6.
3. One-half of a number.
4. Eight subtracted from a number.
5. Two-thirds of a number.
6. Twelve decreased by a number.
7. A number added to 8.
8. The product of 3 and some number.
9. A number is doubled.
10. A number divided by 10.
11. 12 divided by a number.
12. One-half of a number which is then decreased by 6.

Correct and discuss the examples.

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GT and IV - Day 3 (GTC - Day 4)

SECTION 2: OBJECTIVE: The student should be able to write a related equation for an equation such as $2 \cdot x = 36$, and then find its solution.

Review of equations (open number sentences):

A number sentence with = (is equal to) is called an equation. It means that the number represented by the expression on the left-hand side of the equal sign is another name for the number represented on the right-hand side. An equation can be either true or false.

Examples:

The equation $4 + 3 = 7$ is true.

The equation $3 + 2 = 6$ is false.

Some equations with variables are neither true nor false but are made true or false by replacing the variable with a number.

Example:

$n + 6 = 8$

When n is replaced by 1, the equation is false but when n is replaced by 2, the equation is true.

A number that makes an equation true is called a solution of the equation.

To solve an equation is to find its solution.

Solution of equations:

Since the solving of equations by using the Addition Principle and the Multiplication Principle is usually not introduced at the grade seven level, the solving of equations by writing related equations is reviewed.

For an equation like $4 + 3 = 7$, there are two related equations, $7 - 3 = 4$ and $7 - 4 = 3$.

Similarly, for an equation like $x + 4 = 7$, there are two related equations, $7 - 4 = x$ and $7 - x = 4$.

To solve $x + 4 = 7$, we choose the related equation $7 - 4 = x$ and get $3 = x$ (or $x = 3$). We can check the solution in the original equation $x + 4 = 7$.

Check: $3 + 4 = 7$.

For an equation like $3.4 = 12$, there are two related equations.

$$\frac{12}{4} = 3 \quad \text{and} \quad \frac{12}{3} = 4$$

Similarly, for an equation like $3 \cdot x = 12$, there are two related equations.

$$\frac{12}{3} = x \quad \text{and} \quad \frac{12}{x} = 3$$

To solve $6 \cdot n = 12$, we write the related equation

$$\frac{12}{6} = n \quad (\text{or } n = \frac{12}{6})$$

Then, $n = 2$

We can check the solution in the original equation $6 \cdot n = 12$.

Check: $6 \cdot (2) = 12$

Exercise 4

Directions: Solve the equations.

Example: $3 \cdot x = 9$

$$x = \frac{9}{3} = 3$$

1. $5 + 2 = x$
2. $5 + y = 12$
3. $t + 2 = 10$
4. $3 \cdot x = 12$
5. $7 \cdot y = 21$
6. $12 = 3 \cdot y$
7. $3 \cdot y + 2 = 20$
8. $6 + 12 = 3x + 3$
9. $\frac{1}{2} x + 4 = 20$

Appendix C

Developmental Instructional Materials

for IV Procedure

- Selected Parts From Teacher's Notes

Days 4, 5, 6

IV - Day 4

SECTION 3: OBJECTIVE: The student should be able to translate a word problem into an equation.

In this section, we will learn how to write equations for word problems. We will not solve the equations at this stage.

Main Steps for Writing Equations for Word Problems

1. Read the problem carefully.
2. Decide what question the problem asks and choose a variable to represent the unknown.
3. Consider the other information given in the problem and how it relates to the unknown.
4. Write an equation.

Example 1

Eight is multiplied by a number and the result is 104

8 x n = 104

What is the number?

Analysis

First, we read the problem carefully and then we choose a variable, say n, for the unknown which in this case is "the number" asked for in the question. Considering the information related to the unknown we translate it into an equation, as shown, getting 8 . n = 104.

We won't attempt to solve the equation yet.

A similar analysis is performed for Examples 2 and 3.

Example 2

Four times a number increased by 10 is 50. What is the number?

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.
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Example 3

Joe said, "I am thinking of a number. If you add 8 to it and then multiply the result by 2, you get 150." What was Joe's number?

.
.
.

Exercise 5

Directions: Write an equation for each word problem. Do not solve the equation.

Example

A number increased by 5 is 20. What is the number?

Equation: $x + 5 = 20$

1. Six times a number is 72. What is the number?
2. 36 is 5 more than some number. What is the number?
3. If some number is subtracted from 45, the result is 36. What is the number?
4. Five times the difference of 16 subtracted from a number is 49. What is the number?
5. If I multiply by 10, then add 8, I get 58. What is the number?
6. Two-thirds of a number increased by 10 is 8. What is the number?

7. The sum of Tom's age and his father's age is 50 years. If Tom is 13 years old, how old is his father?

8. Twice the sum of a number and 14 is 56. What is the number?

Correct and discuss the examples.

Day 5

SECTION 4: OBJECTIVE: The student should be able to translate a word problem into an equation, then solve the equation to find the solution to the word problem.

Main Steps for Solving Word Problems

1. Read the problem carefully.
2. Decide what question the problem asks and choose a variable to represent the unknown.
3. Consider the other information given in the problem and how it relates to the unknown.
4. Write an equation.
5. Solve the equation.
6. Check the answer in the original problem.

Example 1

A certain number added to 5 is equal to 9. What is the number?

$$x + 5 = 9$$

Analysis

First, we read the problem carefully and then we choose a variable for the unknown, say x. We translate part of the problem to get an open expression as x + 5, and then using the rest of the information in the problem, we write an equation.

$$x + 5 = 9$$

Solution: Let x represent the number.

$$x + 5 = 9$$

$$x = 9 - 5$$

$$x = 4$$

Check: 4 added to 5 is equal to 9.

A similar analysis is performed for Examples 2 and 3.

Example 2

Two-thirds of a number increased by 4 is 24. What is the number?

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Example 3

Tom had some marbles. After a game with Joe, Tom had 10 more than when he started. If he now has 30 marbles, how many marbles did Tom start with?

.
.
.

Exercise 6

Directions: Write an equation for each word problem, then solve the equation to obtain the solution to the problem.

Example

A certain number is added to 35 and the result is 65. What is the number?

Solution: Let x represent the number

$35 + x = 65$

$x = 65 - 35 = 30$ Ans. The number is 30.

Check: 30 added to 35 is 65 (or $30 + 35 = 65$)

1. Ten is multiplied by a number and the result is 120. What is the number?
2. From 72, a number is subtracted to obtain 36. What is the number?
3. Twelve more than a number is equal to 17. What is the number?
4. One-half of a number which is then increased by 8 is 18. What is the number?
5. If I multiply a number by 8, then add 16, I get 40. What is the number?
6. Several boys had 25 candies apiece. Another boy had 30 candies. All the boys had a total of 130 candies. How many boys had 25 candies?

Correct and discuss the examples.

Appendix D

Developmental Instructional Materials for GT Procedure

- Selected Parts From Teacher's Notes

Days 4, 5, 6

GT - Day 4

SECTION 3: OBJECTIVE: The student should be able to translate a word problem into an equation.

Examples 1-3 are used to illustrate the "guess and test" strategy.

Example 1

Eight is multiplied by a number and the result is 104. What is the number?

The following table is drawn on the blackboard and at the same time students are handed duplicated sheets of similar tables. This is to encourage students to go through the steps especially at the beginning.

Guess	Test Your Guess	Result

A student is asked to guess the number. Each student then puts the guess in a table under the column labelled "Guess" and then makes a calculation to determine if the gues is correct. The calculated number is then put in the table under the heading "Result" together with how it was obtained.

The process is continued with several other students each being asked to guess a number and after each number guessed each student in class is asked to make a calculation to determine if the guess is correct.

A table similar to the following will result.

Guess	Test Your Guess	Result
6	8 . 6 = 48	Too small
12	8 . 12 = 96	Too small
15	8 . 15 = 120	Too large

The calculations are filled in on the blackboard only after each student has completed the appropriate steps.

The students are now asked to look for a pattern in the table and then with the help of this pattern write a general equation for the word problem.

Possible equation: 8 . x = 104

Similar discussions are carried out for Examples 2 and 3.

Example 2

Four times a number increased by 10 is 50. What is the number?

Example 3

Joe said, "I am thinking of a number. If you add 8 to it and then multiply the result by 2, you get 150". What was Joe's number?

Exercise 5

Directions: Write an equation for each word problem. Do not solve the equation.

1. Six times a number is 72. What is the number?
2. 36 is 5 more than some number. What is the number?
3. If some number is subtracted from 45, the result is 36. What is the number?
4. Five times the difference of 16 subtracted from a number is 49. What is the number?

5. If I multiply a number by 10, then add 8, I get 58. What is the number?
6. Two-thirds of a number increased by 10 is 8. What is the number?
7. The sum of Tom's age and his father's age is 50 years. If Tom is 13 years old, how old is his father?
8. Twice the sum of a number and 14 is 56. What is the number?

Correct and discuss the examples.

Day 5

SECTION 4: OBJECTIVE: The student should be able to translate a word problem into an equation, then solve the equation to find the solution to the word problem.

An equation for each of the following examples is derived by the method similar to that discussed in Example 1 on Day 4. When the equation is found, each student will then solve it and check the solution in the original problem.

Example 1

A certain number added to 5 is equal to 9. What is the number?

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.
.

Example 2

Two-thirds of a number increased by 4 is 24. What is the number?

.
.
.

Example 3

Tom had some marbles. After a game with Joe, Tom had 10 more than when he started. If he now has 30 marbles, how many marbles did Tom start with?

.
.
.

Exercise 6

Directions: Write an equation for each word problem, then solve the equation to obtain the solution to the problem.

1. Ten is multiplied by a number and the result is 120. What is the number?
2. From 72, a number is subtracted to obtain 36. What is the number?
3. Twelve more than a number is equal to 17. What is the number?
4. One-half of a number which is then increased by 8 is 18. What is the number?
5. If I multiply a number by 8, then add 16, I get 40. What is the number?
6. Several boys had 25 candies apiece. Another boy had 30 candies. All the boys had a total of 130 candies. How many boys had 25 candies?

Correct and discuss the examples.

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.
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Appendix E

Developmental Instructional Materials

for GTC Procedure

– Selected Parts From Teacher's Notes

Days 3, 5, 6, 7

GTC – Day 3

SECTION 3: OBJECTIVE: The student should be able to use the hand-held calculator to make calculations involving the four basic operations of addition, subtraction, multiplication and division.

For the students to become familiar with their calculator, the following examples are discussed:

Example 1: 8 + 5 = 13

Example 2: 5 + 8 = 13

The answers to Examples 1 and 2 are the same. This illustrates that the commutative property is true for addition.

Example 3: 2568 + 3697 = 6265

Note the speed at which the answer can be found for the large numbers.

Example 4: 10 - 2 = 8

(Push 10 first, not 2)

Example 5: 2 - 10 = -8

The answer to Example 5 is not possible in the set of whole numbers. Examples 4 and 5 illustrate that the commutative property doesn't hold for subtraction.

Example 6: 9281 - 8469 = 812

The answer can be obtained quickly.

Example 7: 8 x 6 = 48

Example 8: 6 x 8 = 48

Examples 7 and 8 illustrate that the commutative property is true for multiplication.

<u>Example 9</u>: 9854 x 36 = 354, 744.

The answer can be obtained quickly.

Similar discussions are carried out for the remaining examples.

<u>Example 10</u>: $14 \div 7 = 2$ or $\frac{14}{7} = 2$

<u>Example 11</u>: $7 \div 14 = 0.5$ or $\frac{7}{14} = 0.5$

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<u>Example 12</u>: $4704 \div 84 = 56$

.
.
.

<u>Example 13</u>: $\frac{1}{2} \times 10 - 2 = 3$

.
.
.

<u>Exercise 4</u>

Directions: Compute the following with your calculator.

Example: 985 + 231 = 1216

1. 89 + 64
2. 1435 + 2891
3. 943 - 899
4. 88 - 59
5. 456 x 129
6. 98 x 942
7. $55,955 \div 589$
8. $5607 \div 63$
9. $\frac{1}{2} \times 98 + 157$
10. $\frac{1}{3}(2688 - 3)$

11. 181 + 2 x 41

12. 3 x 98 - 45

Day 5

SECTION 4: OBJECTIVE: The student should be able to translate a word problem into an equation.

Examples 1-3 are used to illustrate the "guess-and-test" strategy. (From now on calculators will be used.)

Example 1

Twelve is multiplied by a number and the result is 672. What is the number?

The following table is drawn on the blackboard and at the same time students are handed duplicated sheets of similar tables. This is to encourage students to go through the steps especially at the beginning.

Guess	Test Your Guess	Result

A student is asked to guess the number. Each student then puts the guess in a table under the column labelled "Guess" and then makes a calculation to determine if the guess is correct. The calculated number is then put in the table under the heading "Result" together with how it was obtained.

The process is continued with several other students each being asked to guess a number and after each number guessed each student in class is asked to make a calculation to determine if the guess is correct.

A table similar to the following will result:

Guess	Test Your Guess	Result
39	12 . 39 = 468	Too small
45	12 . 45 = 540	Too small
55	12 . 55 = 660	Too small

The calculations are filled in on the blackboard only after each student has completed the appropriate steps.

The students are now asked to look for a pattern in the table and then with the help of this pattern write a general equation for the word problem.

Possible equation : 12 . x = 672

Similar discussions are carried out for Examples 2 and 3.

Example 2

Twenty-five times a number increased by 42 is 1617. What is the number?

Example 3

Joe said, "I am thinking of a number. If you add 19 to it and then multiply the result by 15, you get 1560." What was Joe's number?

Exercise 6

Directions: Write an equation for each word problem. Do not solve the equation.

1. Seventeen times a number is 884. What is the number?
2. 197 is 56 more than some number. What is the number?
3. If some number is subtracted from 521, the result is 72. What is the number?
4. Fourteen times the difference of 16 subtracted from a number is 1022. What is the number?
5. If I multiply a number by 36, then add 150, I get 5190. What is the number?
6. Two-thirds of a number increased by 16 is 62. What is the number?
7. The sum of Tom's age and his father's age is 71 years. If Tom is 19 years old, how old is his father?
8. Thirty-five times the sum of a number and 14 is 2450. What is the number?

Correct and discuss the examples.

Day 6

SECTION 5: OBJECTIVE: The student should be able to translate a word problem into an equation, then solve the equation to find the solution to the word problem.

An equation for each of the following examples is derived by the method similar to that discussed in Example 1 on Day 5. When the equation is found, each student will then solve it and check the solution in the original problem.

Example 1

A certain number added to 25 is equal to 209. What is the number?

Example 2

Two-thirds of a number increased by 25 is 71. What is the number?

Example 3

Tom had some marbles. After a game with Joe, Tom had 14 more than when he started. If he now has 51 marbles, how many marbles did Tom start with?

Exercise 1

Directions: Write an equation for each word problem, then solve the equation to obtain the solution to the problem.

1. Twenty-four is multiplied by a number and the result is 1344. What is the number?

2. From 156 a number is subtracted to obtain 36. What is the number?
3. Twenty-seven more than a number is equal to 102. What is the number?
4. One-half of a number which is then increased by 56 is 84. What is the number?
5. If I multiply a number by 14, then add 46, I get 396. What is the number?
6. Several boys had 25 candies apiece. Another boy had 30 candies. All the boys had a total of 130 candies. How many boys had 25 candies?

Correct and discuss the examples.

Appendix F

Practice Materials for IV, GT, and GTC Procedures

– Selected Parts From Teacher's Notes

IV – Days 7, 8, 9, 10

GT – Days 7, 8, 9, 10

GTC – Days 8, 9, 10

IV - Days 7, 8, 9, 10

During the remaining days students will work solving problems. Students who have difficulties will be helped. They will work for about thirty minutes of each period and the remaining approximately ten minutes will be taken for correcting and discussion.

Sheet handout to students

Exercise 8

Directions: Write an equation for each word problem, then solve the equation to obtain the solution to the problem.

Example:

A number increased by 4 is 19. What is the number?

Solution: Let n represent the number.

$n + 4 = 19$

$n = 19 - 4 = 15$

Answer: The number is 15.

Check: 15 increased by 4 is $15 + 4 = 19$. (IV only)

1. A certain number is added to 36 and the result is 85. What is the number?
2. From 78, a number is subtracted to obtain 54. What is the number?
3. Seven times a number which is added to 16 is 65. What is the number?
4. When 5 is added to a certain number and the sum is multiplied by 6, the result is 120. What is the number?

5. A jet plane flew 6 hours at the same speed and travelled a distance of 5700 kilometers. How many kilometers did it travel in 1 hour?
6. Twice the sum of Tom's age and Joe's age is 100 years. If Joe's age is 20 years, how old is Tom?
7. If 25 is added to a certain number and the sum is divided by 3, the result is 25. What is the number?
8. Tom worked for two weeks and earned $85. If he earned $25 the first week, how much did he earn the second week?
9. If I multiply a certain number by 5 and subtract 12 from the product, the result is 28. What is the number?
10. Three times the number of boys in grade seven is equal to the number of girls. If the number of girls is 150, how many boys are there?
11. The Anderson family went on a trip. They travelled 200 kilometers a day for a number of days, and 100 kilometers on the last day. If they travelled 1500 kilometers on the trip, how many days did they travel 200 kilometers?
12. The result of decreasing 45 by a number is 18. What is the number?
13. One-third of a number is 42. What is the number?
14. Eight times a number which is then divided by 3 is 56. What is the number?
15. Six times a number which is then decreased by 5 is 79. What is the number?

16. A car travelling at an average speed of 90 km/hr travels for 8 hours. How far does it travel?

17. Susan bought 14 candies. If she spent 70 cents, how much did each candy cost?

18. Twice the sum of Harry's age and Tom's age is 50. If Harry is 14 years old, how old is Tom?

19. Eight years ago, Susan was twelve years old. How old is she now?

20. Sixty-five divided by a number is 5. What is the number?

21. Joan is 144 centimeters (cm) tall. She is 14 cm taller than Karen. How tall is Karen.

22. In 5 hours an airplane travels 3300 km. What is the average speed of the ariplane?

23. If a certain number is multiplied by 4 and 18 is added to the product, the result is 62. What is the number?

24. Bill and Dean together have 70 rabbits. If Dean has 28 rabbits, how many does Bill have?

25. One-quarter of a number which is then decreased by 5 is 14. What is the number?

GTC - Day 8, 9, 10

During the remaining days students will work solving problems. Students who have difficulties will be helped. They will work for about thirty minutes of each period and the remaining approximately ten minutes will be taken for correcting and discussion.

Calculators will be used to do the calculations.

Sheet Handout to students

Exercise 9

Directions: Write an equation for each word problem, then solve the equation to obtain the solution to the problem.

1. A certain number added to 136 and the result is 385. What is the number?
2. From 178 a number is subtracted to obtain 54. What is the number?
3. Seven times a number which is then added to 16 is 65. What is the number?
4. When 17 is added to a certain number and the sum is multiplied by 13, the result is 2223. What is the number?
5. A jet plane flew 6 hours at the same speed and travelled a distance of 5700 kilometers? How many kilometers did it travel in 1 hour?
6. Twice the sume of Tom's age and Joe's age is 124 years. If Joe's age is 27 years, how old is Tom?
7. If 25 is added to a certain number and the sum is divided by 3, the result is 41. What is the number?
8. Tom worked for two weeks and earned $185. If he earned $48 the first week, how much did he earn the second week?
9. Three times the number of boys in grade seven is equal to the number of girls. If the number of girls is 168, how many boys are there?
10. If I multiply a certain number by 28 and subtract 98 from the product, the result is 1722. What is the number?

11. The Anderson family went on a trip. They travelled 247 kilometers a day for a number of days, and 138 kilometers on the last day. If they travelled 2114 kilometers on the trip, how many days did they travel 247 kilometers?

12. The result of decreasing 126 by a number is 89. What is the number?

13. One-ninth of a number is 146. What is the number?

14. Eight times a number which is ten divided by 3 is 56. What is the number?

15. Six times a number which is then decreased by 27 is 507. What is the number?

16. A car travelling at an average speed of 94 km/hr travels for 18 hours. How far does it travel?

17. Susan bought 14 chocolate bars. If she spent $3.64, how much did each chocolate bar cost?

18. Twice the sum of Harry's age and Tom's age is 54. If Harry is 14 years old, how old is Tom?

19. Eight years ago, Mrs. Smith was 39 years old. How old is she now?

20. Three hundred forty-three divided by a number is 7. What is the number?

21. Joan is 144 centimeters (cm) tall. She is 14 cm taller than Karen. How tall is Karen?

22. In 5 hours an airplane travels 3335 km. What is the average speed of the ariplane?

23. If a certain number is multiplied by 4 and then 18 is added to the product, the result is 62. What is the number?

24. Bill and Dean together have 70 rabbits. If Dean has 28 rabbits, how many does Bill have?

25. One-quarter of a number which is then decreased by seventy-six is two hundred six. What is the number?

Appendix G

Posttest

Name: ______________________________

Directions: For each word problem write an equation in one variable, then solve the equation to find the solution to the word problem. Do your work on the paper provided.

1. A certain number is added to 35 and the result is 52. What is the number?
2. 8 is multiplied by a certain number and the result is 128. What is the number?
3. Judy has 78 candies. She divided them among her friends and each received 13. How many friends did Judy have?
4. From 46 a number is subtracted to obtain 29. What is the number?
5. I am thinking of a number. If I multiply it by 6 and add 14, I get 86. What is the number?
6. If 18 is added to a number and the sum is divided by 12, the result is 8. What is the number?
7. 6 times the sum of Mary's age and Tom's age is 156. If Mary's age is 12 years, how old is Tom?
8. If the result of 39 times the difference of 17 subtracted from one-half of a number is equal to 3510, what is the number?
9. Joe's father went on a trip. He travelled 350 kilometers a day for a number of days and 150 kilometers on the last day. If twice the distance that he travelled is equal to 5200 kilometers, how many days did he travel 350 kilometers.
10. If 132 is added to 7 times a certain number and the sum is divided by 24, the result if 16. What is the number?

Appendix H

Retention Test

Name: ______________________________

Directions: For each word problem write an equation in one variable, then solve the equation to find the solution to the word problem. Do your work on the paper provided.

1. A certain number is added to 41 and the result is 65. What is the number?
2. 6 is multiplied by a certain number and the result is 90. What is the number?
3. Judy has 112 candies. She divided them among her friends and each received 14. How many friends did Judy have?
4. From 84 a number is subtracted to obtain 36. What is the number?
5. I am thinking of a number. If I multiply it by 8 and add 16, I get 128. What is the number?
6. If 16 is added to a number and the sum is divided by 14, the result is 6. What is the number?
7. 5 times the sum of Mary's age and Tom's age is 150. If Mary's age is 14 years, how old is Tom?
8. If the result of 48 times the difference of 13 subtracted from one-half of a number is equal to 3120, what is the number?
9. Joe's father went on a trip. He travelled 320 kilometers a day for a number of days and 120 kilometers on the last day. If twice the distance that he travelled is equal to 5360 kilometers, how many days did he travel 320 kilometers?
10. If 124 is added to 6 times a certain number and the sum is divided by 73, the result is 4. What is the number?

Appendix I

Attitude Test

This is to find out how you feel about mathematics. You are to read each statement carefully and decide how you feel about it. Then indicate your feeling by putting a circle around one of the five possible responses.

SA - if you strongly agree

A - if you agree

U - if you are undecided

D - if you disagree

SD - if you strongly disagree

1. Mathematics often makes me feel angry.	SA	A	U	D	SD
2. I usually feel happy when doing mathematics problems.	SA	A	U	D	SD
3. I think my mind works well when doing mathematics problems.	SA	A	U	D	SD
4. When I can't figure out a problem, I feel as though I am lost in a mass of words and numbers and can't find my way out.	SA	A	U	D	SD
5. I avoid mathematics because I am not very good with numbers.	SA	A	U	D	SD
6. Mathematics is an interesting subject.	SA	A	U	D	SD
7. My mind goes blank and I am unable to think clearly when working mathematics problems.	SA	A	U	D	SD
8. I feel sure of myself when doing mathematics	SA	A	U	D	SD
9. I sometimes feel like running away from my mathematics problems.	SA	A	U	D	SD
10. When I hear the word mathematics, I have a feeling of dislike.	SA	A	U	D	SD
11. I am afraid of mathematics.	SA	A	U	D	SD
12. Mathematics is fun.	SA	A	U	D	SD
13. I like anything with numbers in it.	SA	A	U	D	SD
14. Mathematics problems often scare me.	SA	A	U	D	SD

15. I usually feel calm when doing mathematics problems.	SA	A	U	D	SD
16. I feel good toward mathematics.	SA	A	U	D	SD
17. Mathematics tests always seem difficult.	SA	A	U	D	SD
18. I think about mathematics problems outside of class and like to work them out.	SA	A	U	D	SD
19. Trying to work mathematics problems makes me nervous.	SA	A	U	D	SD
20. I have always liked mathematics.	SA	A	U	D	SD
21. I would rather do anything else than do mathematics.	SA	A	U	D	SD
22. Mathematics is easy for me.	SA	A	U	D	SD
23. I dread mathematics.	SA	A	U	D	SD
24. I feel especially capable when doing mathematics problems.	SA	A	U	D	SD
25. Mathematics class makes me look for ways of using mathematics to solve problems.	SA	A	U	D	SD
26. Time drags in a mathematics lesson.	SA	A	U	D	SD

END

15-04-82

FIN