THE DEVELOPMENT, IMPLEMINTATION, AND EVALUATION OF MATHEMATICS LABORATORIES IN AN ELEMENTARY EDUCATION MATHEMATICS METHODS COURSE

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ABSTRACT

The main purpose of this study was to develop mathematics laboratories for a mathematics education methods course, Education 2341, and to evaluate the laboratories and their effectiveness in aiding students in meeting the objectives of the course. A second purpose was to determine any changes in attitude toward four aspects of mathematics education as a result of completing Education 2341.

The procedure was carried out in five steps. First, the mathematics laboratories were developed. During the time period when these laboratories were being piloted, an Evaluative Questionnaire and interview questions were designed and tested. The mathematics laboratories were then revised by using data collected from the Evaluative Questionnaire, interviews, and the activity evaluation sheets. The activity evaluation sheets had been administered after each laboratory.

The main study was conducted during the 1980 winter semester, with the laboratories being held on a weekly basis. Each laboratory consisted of three activities each of which included two questions. These questions provided feedback on the effectiveness of the laboratories. The students were given activity evaluation sheets for each laboratory in order to

obtain data for future revisions. At the close of Education 2341 the Evaluative Questionnaire and interviews were administered. Finally, the Connelly Taxonomized Attitude Questionnaire was administered before and after the 1980 winter semester of Education 2341.

The effectiveness of the laboratories in aiding students meet the objectives was examined. The laboratories were effective in aiding students to (1) develop follow-up activities given the information that elementary school children had exhibited difficulty with a previous activity on the same topic; (2) suggest how a given activity might be altered to accomplish different objectives; (3) suggest an alternate activity to accomplish the same objective as a given activity. The laboratories were ineffective in aiding students to suggest an extension activity for an elementary school student given the information that the student had exhibited no difficulty with a previous activity on the same topic. The results in this study were inconclusive for determining the effectiveness of the laboratories in aiding students to adapt an activity in mathematics designed for an university student for use in an elementary school classroom.

The attitudes of students toward (1) mathematics; (2) individualizing mathematics instruction and the use of guided discovery techniques in mathematics; and (3) teaching mathematics were increased during the course. Attitudes toward the existence and use of logical structure in mathematics did not change significantly as most students entered the course with a positive attitude.

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CHAPTER I

STATEMENT OF THE PROBLEM

Education students at Memorial University of Newfoundland who are studying to be elementary school teachers are required to complete two mathematics and two mathematics education courses. The mathematics courses provide an understanding of the mathematical concepts involved in programs currently used in the elementary schools. The subject matter covered in one course is primarily geometry while the other mathematics course emphasizes the structure of the number systems. The first mathematics education course familiarizes the student with "methods" appropriate for teaching a major portion of the elementary school mathematics curriculum. The second mathematics education course, Education 2341, is a logical extension of the other three courses. It deals with the remaining elementary school mathematics topics, in addition to providing student teachers with the practical experiences of working with a small group of elementary school children.

Originally, Education 2341 consisted of two components, lectures and in-school sessions. The lectures covered theories in diagnostic and activity-oriented teaching and learning. The practical aspect of the course, the in-school sessions, gave the students an opportunity to apply these theories. Each student was assigned one or two children who received activity-oriented instruction based on the results of diagnostic tests of their mathematical abilities. The student teacher was responsible for developing both the diagnostic tests and the activities with which they taught the child.

Often the students experienced difficulty in applying the theories of diagnostic and activity learning given in the lectures to the practical situation of the in-school sessions. Moreover, the students were not familiar enough with the materials that were needed in their activities. An attempt was made in recent semesters to alleviate these problems. The mathematics education room was well stocked with concrete materials for use in the development of mathematical concepts and skills and it was opened to students for six hours a week. It was hoped that exposure to these materials would aid students in their preparation of the in-school activities. This method has resulted in students having only a minimal knowledge of how concrete materials can be used to illustrate mathematical concepts. Since one of the objectives of the course is the attainment of this knowledge, an alternate method of instruction was needed.

Purposes of the Study

The purposes of this study were to develop mathematics laboratories for the mathematics education methods course, Education 2341, and to evaluate the laboratories and their effectiveness in aiding students in meeting the objectives of the course. A further purpose was to determine any changes in attitude towards mathematics and mathematics teaching as a result of completing Education 2341.

Questions to be Investigated

In conjunction with the above purposes, mathematics laboratories were developed to seek the answers to the set of questions listed below. Questions 1 to 5 were asked after completion of a mathematics laboratory or a set of laboratories. Questions 6 to 9 were asked with reference to the complete course:

1. Will a student in Education 2341 be able to suggest a follow-up activity, given the information that an elementary school student had exhibited difficulty with a previous activity on the same topic?

2. Will a student in Education 2341 be able to suggest how a given activity might be altered to accomplish different objectives?

3. Will a student in Education 2341 be able to suggest an alternate activity to accomplish the same objective as a given activity?

4. Will a student in Education 2341 be able to suggest an extension activity for an elementary school student, given the information that the student had exhibited no difficulty with a previous activity on the same topic?

5. Will a student in Education 2341 be able to adapt an activity in mathematics designed for a university student, for use in an elementary school classroom?

6. Will a student's attitude toward mathematics change between the first and last weeks of enrollment in Education 2341?

7. Will a student's attitude toward the existence and use of logical structure in mathematics change between the first and last weeks of enrollment in Education 2341?

8. Will a student's attitude toward individualizing mathematics instruction and the use of guided discovery techniques in mathematics change between the first and last weeks of enrollment in Education 2341?

9. Will a student's attitude toward teaching mathematics change between the first and last weeks of enrollment in Education 2341?

Rationale for the Study

Copeland (1974) stated that:

> Children in the elementary school, however, are not ready to work at the abstract level with formal logic and proofs. They are very much a part of the physical world. Mathematics for them should be exploration and discovery--an inductive approach through the physical world with concrete objectives (p. 208).

Earlier, Flavell (1963), while discussing the work of Piaget, had taken a similar position. Hollis (1973) stated that:

> . . . while research evidence is contradictory and inconclusive it appears that children learn mathematics more readily when instruction includes manipulative materials (p. 49).

Hollis (1973), Flavell (1963), and Copeland (1974) are saying that children should be taught with a teaching strategy that uses manipulative materials. Though this is what is recommended, many elementary school teachers to date are not using these procedures. A recent national survey completed by the National Science Foundation (N.S.F.) (Fey, 1979) reported that:

> . . . information from the N.S.F. studies raises doubts about the extent to which any of these proposals for innovative pedagogy have influenced predominate instructional patterns (p. 11).

The results of the survey ". . . suggest very common use of an instructional style in which teacher explanation and questioning is followed by student seatwork on paper and pencil assignments" (p. 12). Suydam (1977) reported studies that support this observation on patterns of classroom instruction.

The importance of how the teacher teaches cannot be overemphasized. As Spodek (1972) explained, "Often it is the way mathematics is taught rather than mathematics itself that creates learning difficulties" (p. 139). If teaching procedures are causing obstacles for children learning mathematics then it is time those "old" teaching strategies are exchanged for "new" ones.

In the United Nations publication, New Trends in Mathematics Teaching, it is stated that:

> The stress then should be not so much on the new (although there is new mathematics) but on new procedures of learning involving activity, doing, understanding, discovery, intention (Fehr, 1973, p. 9).

The question is, how does one get the student teacher to use these methods. The Second International Congress on Mathematical Education (1973) recommended that:

> . . . the style of training teachers must reflect what we actually wish teachers to do in their classrooms. In this training as in actual teaching, materials must be present and the student teacher involved with them. To talk about them or to look at pictures is not sufficient (Howson, 1973, p. 54).

In recent years mathematics laboratories have been used to aid teachers in learning to use activity-oriented instruction. Student teachers need to develop elementary mathematical concepts by using concrete objects while they are devising ways of using materials with children. LeBlanc (1972) argued that:

> Another shortcoming is in the disparity of what the instructor says and what the instructor does. The pre-service teacher is advised to use concrete materials and objects when she becomes a teacher, but such advice is rarely accompanied by experience in the methods classroom with such materials (p. 40).

Leith (1972) and Fitzgerald (1971) investigated whether the active learning approach was more effective than the lecture-discussion method for preparation of elementary school teachers. Leith concluded that the students who participated in the active learning approach increased significantly in their achievement on mathematics and exhibited a more positive attitude towards mathematics. Fitzgerald's study indicated no significant change in the student's attitudes or achievement, but a very high correlation was found between prospective elementary school teachers attitude towards mathematics and their achievement in that subject. Even though the use of mathematics laboratories may result in an increase in the education student's understanding of basic mathematical concepts and improve their attitude, will it result in their using the activity approach in their own teaching?

Boonstra (1970) analyzed student teachers classroom instructional patterns after they had participated in two mathematics laboratories. Results showed that there was no significant change in the student teachers' behavior. Boonstra concluded that two laboratories were not sufficient to train teachers to use an activity-oriented approach.

Postman (1971) studied the effects of mathematics laboratory experiences on the teaching behavior of pre-service elementary school teachers. He concluded that participating in mathematics laboratories was not sufficient to cause teachers to use the laboratory approach when teaching.

Unkel (1971) conducted a study where the student teachers were taught through the use of mathematics laboratories. He investigated whether this method was an effective way to train teachers. These student teachers tutored students from grades one to six using manipulative aids. The elementary students were tested to see if achievement increased. The results showed that the students' achievement in grades one, two, three, five, and six increased significantly and the student teachers' improved significantly on their knowledge of basic mathematics concepts.

To date no studies could be found which used a combination of mathematics laboratories, lectures, and tutoring. This study attempted to link these three components and, in particular, investigated the role of the laboratories.

Plan for the Study

The following procedure was selected to obtain the data necessary to fulfill the purposes of this study. This procedure consisted of five major steps.

First, the mathematics laboratories were developed. Topics for the laboratories were based on materials that were covered in Education 2341 lectures or types of lessons required to be taught in the in-school sessions.

Secondly, the mathematics laboratories were piloted with students enrolled in Education 2341 during the Fall of 1979. During this same time period an Evaluation Questionnaire and interview questions were designed and administered to determine the reliability of the Questionnaire and to provide an opportunity to practice and refine interviewing techniques. The Evaluation Questionnaire and interviews were later used in evaluating the laboratories.

Thirdly, the mathematics laboratories were revised by using information gathered from the students, professors, and laboratory assistants who had participated in Education 2341 during the piloting stage. These data were obtained from activity evaluation sheets administered after each laboratory and from the responses to interview questions. The necessary changes were made to the instruments during this phase.

Fourthly, the experiment was conducted during the 1980 winter semester, with the laboratories being on a weekly basis for students enrolled in Education 2341. The students were given activity evaluation sheets for each laboratory in order to obtain data for future revisions. At the close of Education 2341 the Evaluation Questionnaire and interviews were administered once more.

Finally, the Connelly Taxonomized Attitude Questionnaire (1973) was administered before and after the 1980 winter semester of Education 2341.

Limitations of the Study

1. The sample for the study was not randomly selected from the population.

2. The reliability and validity of homemade instruments were not as extensively tested as in standardized instruments.

3. The subjects used in this study were not the same participants as used to determine the validity of the Connelly Taxonomized Attitude Questionnaire.

Definition of Terms

Activity learning is learning whereby the student is actively participating usually by manipulating concrete objects.

Materials are concrete objects that can be used in teaching mathematical concepts, skills, and generalizations. For example, Cuisenaire rods are employed in the place value laboratory.

Positive attitude is a rating (of 56 to 80) on the Connelly Taxonomized Attitude Questionnaire.

Neutral attitude is a rating (of 41 to 55) on the Connelly Taxonomized Attitude Questionnaire.

Negative attitude is a rating (of 16 to 40) on the Connelly Taxonomized Attitude Questionnaire.

Elementary grades are grades kindergarten to six.

Elementary school teachers are teachers who instruct in any or all grades from kindergarten to six.

Overview of the Report

This report of the development, implementation, and evaluation of mathematics laboratories of an elementary education mathematics course is organized into five chapters. A brief summary of the content of each chapter follows.

In Chapter I the problem, purposes, objectives, rationale, and plan of the study have been discussed. Also included were the limitations of the study as well as the definition of terms used in the study.

A review of the related literature and research is presented in Chapter II.

Chapter III is a discussion of the pilot stage. This includes the development of the mathematics laboratories and the instruments used in the main study.

Research methodology is found in Chapter IV. This entails the sample, procedures, method of data collection, and analysis.

The presentation and analysis of data with respect to the assessment of the study's questions is contained in Chapter V.

Chapter VI contains a discussion of the results. In addition, a summary of the study and recommendations for further research are included.

CHAPTER II

REVIEW OF THE LITERATURE

Learning through an activity approach is a learning process which has been strongly supported by theory. Two renowned contributors to theories of learning supporting this view are Piaget and Dienes. Their theories will be briefly summarized, followed by a number of research studies on the topic. Studies are reported on elementary school students' achievement and attitudes towards mathematics when learning by an activity-oriented approach. If Dienes and Piaget are correct in asserting that children best learn from a teaching method that is activity-oriented, then teachers should be using this method.

Further research is presented on how university education students' achievement and attitudes are affected by activity learning. If teachers are to use the activity-oriented approach then teachers need to be taught how to use an activity-oriented approach with students. Finally, studies concerned with the effects of the activity learning approach on student teachers' instructional style are reviewed.

Theory on Activity Learning

Activity learning is not a new concept in education. It can be traced back throughout the ages and throughout the world. The essence of this approach is captured in the Chinese proverb:

I hear and I forget
I see and I remember
I do and I understand

In recent years this philosophy of learning has once more come to the forefront, mostly due to the work of Piaget and Dienes. The theories of learning that each has developed are briefly described below.

Flavell (1963) described four stages of development in Piaget's learning theory. The first stage of development, the sensorimotor stage, begins at birth and extends to approximately 1.5 years. It is here that the child learns to differentiate himself from the world. He sees that he is an object among objects. The child obtains knowledge by manipulating these objects.

The pre-operational stage, stage two, is characterized by the child's irreversibility of thought and inability to conserve. The child is unable to ignore his own perception. When the child is requested to reverse the order of a pattern in front of him he is unable to successfully complete the task. The child may think that a collection of six objects is not equal to a second collection of six objects if one collection has more space between the objects.

Conservation and reversibility are present in the third stage, concrete operations. This stage usually occurs between the ages of seven and twelve, which makes it most pertinent to this study, since these are the ages of most elementary school children.

The concrete operations stage received this name because it is during this stage that the child needs to manipulate material objects and ideas that are real to him in order to understand a concept. The concept needs to be presented in a concrete manner since children of this stage are not yet able to think abstractly. For example, when children are introduced to addition, the manipulation of objects by them would be a concrete experience but the sentence 2 + 3 = 5 by itself would be abstract. For other children who have an understanding of the addition concept, sentences such as this might be considered concrete.

The final stage, formal operations, is characterized by the child's ability to think at an abstract level. The child is able to reason on the basis of symbolic presentation.

There are four factors which Piaget claims influence the child's development through the stages. These are maturation, social transmission, physical experiences, and equilibration.

Maturation is the maturing of the child's physiological system. "The average age at which children go through each stage can vary considerably from one social environment to another" (Piaget, 1972, p. 7). It can be argued that age is one factor that aids development but not the only factor.

Piaget emphasizes the importance of the interaction of the child with his environment, social transmission. It is necessary for the child to have an interaction of his thoughts with others. The interaction aids to refine the child's own ideas.

The third factor, physical experience, contributes to development by the reinforcement of the concepts of conservation. This development extends from the pre-operational stage to the beginning of formal operations, the time when the child is in elementary grades. The physical experience that Piaget refers to is the manipulation of physical materials. A child in the elementary grades who cannot think abstractly is able to better understand a concept if the concept has been taught by using a physical model.

Equilibration is a process of transition through the stages accomplished by assimilation and accommodation. Assimilation is the child "fitting" a new experience into the child's existing framework of thought patterns. Experiences that cannot be "fitted" into this existing framework cause the child to modify his concept to include the experiences. This is called accommodation. The child is in a state of equilibrium until he encounters a new experience. The child must either accommodate or assimilate this experience before he is able to return to a new equilibrium. The development from pre-logical to logical reasoning is a continuous restructuring of the child's theoretical framework.

Duckworth (1964) stresses that the first three factors can occur within a passive individual but for intellectual development, the fourth factor is a vital component. The individual has to be active. She stated that:

> Piaget finds it necessary to call upon the factor of the individual's own activity. An individual comes to see the world as coherent, as structured, to the extent that he acts upon the world, transforms it, and succeeds in coordinating these actions and transformations (p. 2).

For the child who thinks at the concrete level this activity with the real world cannot be abstract. It must be active with concrete situations.

The term "active" can be viewed in two ways. One is doing things in social interaction, rather than individually. The other is acting on physical objects. Piaget stresses that these activities must not be an end in themselves but must be a medium through which the child passes to obtain the final stages of formal operations.

The child must pass through the four stages of development to become an abstract thinker. Elementary school children are usually in the second or third stage. At these stages the child is able to think concretely but not abstractly. Piaget encouraged the use of concrete situations to aid the child to pass through these two stages and into formal operations.

Dienes' theory of learning is more specifically related to mathematical instruction than is Piaget's theory. Dienes' (1973) cycle for effective learning includes the following six steps: 1) construction, 2) analysis, 3) abstraction, 4) representation, 5) symbolization, and 6) formalization.

Construction is the "play" stage, where the child randomly manipulates materials in a structured system. At the second stage, analysis, restrictions are imposed on play similar to the rules of a game. These restrictions give the child structured experiences that have a structure similar to that of the desired concept.

The next step is to expose the child to several different experiences which have a structure similar to that of the desired concept. When the child is able to compare and identify the components that are similar in these different experiences the child is working at the third level, abstraction. It is here that the child is leaving the concrete stage and is able to deal with the abstract nature of the relations.

Once the child has abstracted the desired concept or principle from the different experiences he must be able to represent what he has abstracted. The child has to either invent or learn a language to express the concept or principle.

The final stage, formalization, is the process where the child forms a system of the descriptions made at the symbolization level. It is at the formalization stage that the basic rules are used to deduce other parts of the descriptions not experienced by the child.

Dienes, like Piaget, asserts that children need to manipulate materials, and this manipulation of materials should be part of a structured plan. Effective learning does not happen by chance. Children must be exposed to materials within a directed plan of action. Piaget's and Dienes' theories both support the view that children should have experiences with a variety of materials that embody the concept.

If children learn most effectively through activity teaching in the elementary grades then teachers should be using this mode of instruction. Children's

> . . . learning should be related to the factual and concrete world of objects, events, and facts, and much less to explanations such as conventionally given in mathematics (Dienes, 1970, p. 265).

The basic premise of this study is that teachers should be teaching students by using a variety of activities.

Research in Activity Learning

Research in activity learning has not yielded conclusive results. In this section several research studies on the effect of activity learning on elementary school children in mathematics are discussed. Following this, studies on the effect of activity learning on perspective elementary teachers' mathematics achievement and attitude are reported.

Kieren (1969) reviewed the research on activity learning for the period from 1964 to 1969. He found that there was no shortage of literature available on the topic but that the research was lacking in control and generalizability. The following is a summary of Kieren's review.

Sutton-Smith (1967) researched the role of play in cognitive development. A high correlation was found between play and information-seeking behavior. Williams (1967) and Biggs (1965) reported on the comparison of the traditional method of teaching to three manipulative methods. Two methods were unimodel and the third multimodel. Biggs found high IQ boys did better on understanding, motivation, and attitude when using the unimodel method rather than the traditional method. The traditional method did facilitate good computation but high anxiety. However, the multimodel group surpassed the other groups in all three areas mentioned above. Other research Kieren reviewed compared the manipulation of Cuisenaire products to traditional teaching methods. This research was inconclusive with studies showing both positive and negative results. The research was highly criticized for unrepresentative samples, Hawthorne effect, and conclusions going beyond the data.

Another project reported by Kieren was the Madison project (1967). The purpose of this project was to develop classroom experiences using manipulative aids, train teachers to use them, and to test the experiences. Interviews with students on this project showed that students in grades six to nine did not like complicated mental tasks but did enjoy physical activities. Data from these interviews prompted the Madison project to extend the physical materials and laboratories. This extension included students in grades kindergarten to nine and also prospective mathematics teachers in college courses.

The main conclusion Kieren reached was that activity learning can be used practically and effectively but cautions against its use in a wholesale manner. The following are studies conducted after 1969.

A study was conducted by Nicholas (1973) on discovery learning using materials to teach multiplication and division to third graders. The experimental group used manipulative materials and pupil discovery, whereas the control group used semi-concrete materials, abstractions, teacher exploration, and exposition. The results indicated a significant difference in favor of the concrete materials and discovery method on achievement and attitude tests.

Johnson (1971) investigated the effect of activity-oriented lessons on the achievement and attitudes of seventh grade students in mathematics. Results showed that low and middle ability students were aided in learning some mathemathics concepts by use of activity-oriented lessons. The attitudes of the students were not significantly changed. Johnson also mentioned that this difference may have been caused by other factors in the experiment such as the teacher or the topic taught.

Smith (1974) studied sixth to eighth graders using the laboratory approach with regard to its effects in their achievement and attitude. Both groups had the same instructor. The results showed no significant change in the students' achievement and attitude when taught by the laboratory approach.

Davidson (1973) studied the understanding of mathematical concepts of grade three and four children using the textbook and drill, or textbook with concrete materials. No significant differences were found between the two groups on achievement. Davidson did conclude that the geometry sections were enriched by the addition of concrete materials.

Fennema (1972) investigated the relative effectiveness of a symbolic and a concrete model in learning a selected mathematical principle. The results showed that children could learn a principle when taught by either model if tested by direct recall. If the learning was tested for transfer or for an extension of the taught principle the children who were taught by the symbolic model performed better. The students who had been taught in this study by the symbolic model had been taught with the aid of concrete materials the year before. Fennema concluded that students who have used concrete materials in their past are later able to learn by symbolic models.

Curry (1970) investigated whether concrete and semi-concrete materials would aid in teaching clock arithmetic to third graders. The tests were designed to measure computation and understanding of the principles. The concrete and semi-concrete groups both were significantly superior on these variables to those taught by the abstract approach.

DeVenney (1972) described a study conducted during 1965-68 by the School Mathematics Study Group (SMSG). Seventh graders from ten schools were the experimental group which used manipulative materials, whereas seventh graders of five different schools were taught at an abstract level by teacher exposition. At the end of grade seven, results showed that the control group had a higher mean gain on the SAT computation scale but on a SMSG scale of mathematical concepts the experimental group showed significantly greater gains. Attitudes of both groups were shown at the beginning of the study to be negative toward mathematics whereas at the conclusion of the study the attitudes of the experimental groups were shown to be highly positive while the control group attitudes remained the same.

Moody, Abell and Bausell (1971) studied the effect of activity-oriented instruction upon original learning, transfer, and retention. Ninety subjects were randomly assigned to one of four different treatments. Group A participants received multiplication instruction using manipulative materials. Group B (rote treatment) received instruction on multiplication from a textbook. Group C (rote-word problems treatment) received multiplication word problems along with the same instruction given to Group B, and Group D (control treatment) received no instruction on multiplication. Analysis of the results demonstrated that there was no significant differences in original learning, transfer, and retention of multiplication facts for the three instructional methods.

Ormody (1971) investigated the question of whether concrete and semi-concrete materials would contribute to the learning of mathematics. Three grade six classes were taught by the investigator for 45 minutes per day for eleven days. In all classes achievement was reported significantly higher for both the concrete and semi-concrete groups than the symbolic group that used no materials.

Maffei (1976) described a mathematical program in which mathematical concepts were taught using physical models. A diagnostic test at the beginning and end of the three-month period showed a 25 percent increase in the number of problems scored correctly. Discipline problems were noticeably lower. Slow students were beginning to develop a positive self-concept with respect to their ability to do mathematics.

In the studies reported previously, as in other studies found to date, experiments have now shown activity learning to result in students achieving significantly less than when taught by any other method. Activity learning has not been shown to create a negative attitude towards mathematics by students. The research has indicated that activity learning has either resulted in increased achievement and/or attitudes or has shown no significant difference between the two approaches used.

Two surveys on mathematical instruction in a laboratory setting done by Hynes, Hynes, Kysilka, and Brumbaugh (1973), and Vance and Kieren (1971) indicated that research done in the area of mathematics instruction in a laboratory setting is far from consistent and conclusive but children do appear to learn more readily when instruction includes the use of manipulative materials.

Activity-oriented instruction in the elementary school mathematics classes was strongly supported by the Cambridge Conference on School Mathematics.

The following report was made:

> A comment might be made on the role of physical equipment in the earliest grades. Whether one thinks in terms of the premathematical experiences that are embodied in the manipulation of physical materials, whether one regards these physical objects as aids to effective communication between teacher and child or whether one regards them as attraction objects that increase motivation, the conclusion is inescapable that children can study mathematics more satisfactorily when each child has abundant opportunity to manipulate suitable physical objects (Cambridge Conference, 1963, p. 35).

Studies on Prospective Teachers

If one accepts the premise that an activity approach is to be employed in the elementary grades, then the question arises as to how teachers can be trained to use this method. Dienes (1970) and LeBlanc (1972) suggest a close relationship between content of the elementary mathematics programs and the instructional processes taught elementary school teachers in mathematics. The Second International Congress on Mathematical Education supports training teachers by

> . . . the use of manipulative materials, apparatus and other concrete aids, to encourage, through early experience of working with physical models, the ability to abstract mathematical relations and patterns. Such materials provide open-ended situations which give enjoyment as well as independence of approach. But prospective teachers need to work with the materials and become familiar with their uses. As Piaget says, it is often particularly difficult for the teacher of mathematics, who, because of his profession, has a very abstract type of thought, to place himself in the concrete perspective which is not necessarily that of his young pupils. It is to help a teacher to bridge this gap that he needs to experience for himself the dawning of new understanding through handling suitable concrete materials (Howson, 1973, p. 46).

The following studies have been reviewed to determine if activity-oriented instruction can be used to teach prospective teachers to use an activity-oriented approach and if activity learning affects the student teacher's mathematics achievement and attitude.

Two investigators, Leith (1972) and Fitzgerald (1971), compared an active learning approach to a lecture discussion method for prospective elementary teachers. Leith (1972) used one control and one experimental group. The 29 subjects in the control group were taught mathematics by lectures and discussion while the 28 subjects of the experimental group were taught by a combination of lectures, student recitation, discussion, readings, laboratories, and student activities. The mathematics laboratories took place in the classroom during regular classroom hours. In addition the students in the experimental group were encouraged to attend voluntary laboratories that were open for one hour per week. The control group participants were given equal time for office hours. Data collected from a standardized mathematics attitude scale and a teacher-constructed achievement test showed that the experimental group achieved significantly higher and obtained a more positive attitude toward mathematics than the lecture-discussion control group.

Fitzgerald (1971) studied 73 prospective elementary teachers who were divided into four groups. Two randomly selected groups received different amounts of time of mathematics laboratory instruction and lectures while the remaining students comprised the two control groups, who were taught by a lecture-discussion method. One of the experimental groups had half lecture and half mathematics laboratories; the other had two-thirds lectures and one-third mathematics laboratories. Age was studied by dividing each group into those over 24 years and those 24 years and under. Analysis of the achievement on mathematics of prospective elementary school teachers showed the achievement was "neither improved nor damaged" by using mathematics laboratories. Analysis also showed that age was not a factor in achievement or attitude towards mathematics when being taught by using mathematics laboratories.

Hendrickson (1969) studied the relative effectiveness of three methods of teaching mathematics to prospective elementary school teachers. The subjects were 90 students taking a required mathematics course intended for elementary education students. The subjects were randomly selected and assigned to one of three methods. One of the four lectures of the preservice teachers was exchanged for a mathematics laboratory for the first method; for a session of pattern discovery, mathematical creativity, and problem-solving for the second method, and remained as a lecture for the third method. Both experimental groups scored higher on an achievement test than the control group. However, the differences were not significant. The attitudes of the students towards mathematics were not significantly different for any one of the three groups on the two attitude scales, the Arithmetic Attitude Scale and the Math Attitude Scale but a third scale, the Mathematics Semantic Differential, did indicate significant differences with the control group scoring highest and the mathematics laboratories lowest.

Warkentin (1975) investigated the effect of mathematics instruction using manipulative models on attitude and achievement of prospective teachers. One hundred and forty-nine students (six sections of students enrolled in a theory of arithmetic course) who comprised the experimental group received instruction by a laboratory approach while the 197 subjects (the other nine sections of the same course) in the control group received the traditional lecture approach. Results of an achievement test indicated a significant difference in the mean achievement in favor of the control group, but for attitudes the significant difference was in favor of the experimental group. Warkentin stated that the achievement test examined content of the textbook which was completed by the control group but due to time constraints the same content was not completely covered by the experimental group.

The studies reported above illustrate the effects that mathematics laboratories may have on student teachers' achievement and attitude towards mathematics. However, they do not show if the use of laboratories aids preservice teachers in learning to teach mathematics using manipulative aids. Two studies which investigated this factor were conducted by Fuson (1975) and Boonstra (1970).

Fuson (1975) studied the effects on preservice elementary school teachers of learning mathematics and the effects of teaching mathematics through the active manipulation of materials. The subjects were the 16 Master of Science in Teaching students who completed 20 sessions of two and one half hours each. The sessions included content from elementary school mathematics and a model on how to teach this mathematics to children. These classes were taught by having the subjects actively manipulate objects. Some time was spent on pedagogy and discussion. From half-hour, taped interviews not one student reported a preference for a more formal abstract course. While all students felt that manipulating materials was useful for teaching mathematics, the materials did present problems and limitations. Fuson (1975) found from examination of video-tapes of the subjects teaching pre-set concepts, that all subjects used a manipulative object to aid in teaching. The video-taping also showed that the subjects taught in a more learner-focused manner than teacher centered. Information collected from supervisors of the subjects after they had completed one-quarter of practice teaching showed that each student teaching mathematics used manipulative aids. Subjects' attitudes towards mathematics increased significantly.

Boonstra (1970) conducted a case study on the effects of two mathematics laboratory experiences upon the teaching behavior of prospective elementary school teachers. Subjects selected were those who were currently doing student teaching in grades four, five or six. After completing the mathematics laboratories the students were asked to teach an aspect of the topics covered in the laboratories to the class they were student teaching. Data were collected by tape and video recorders in the classroom. From analysis of the tapes Boonstra concluded that the two mathematics laboratories were not sufficient to effect student teachers to use manipulative materials or for the instruction to be more student-centered.

Postman (1971) investigated the effects of mathematics laboratory experiences on the teaching behavior of preservice elementary school teachers. The study had 14 preservice teachers in the experimental group and four in the control group. The students were video-taped twice, once before receiving six weeks of laboratory experience and once after. The laboratory approach was associated with a gain in the percentage of teacher comments directed to small groups. Since the study had such a limited number of subjects, a second experiment was conducted. Twenty preservice teachers were each video-taped once. In both studies the preservice teachers were given one of three mathematical topics to teach. Each of the three topics could very easily be taught with the aid of manipulative materials. Yet the tapes showed that the preservice teachers spent 90 percent of their time talking to a class and materials were not used. Postman concluded that preservice teachers receiving activity learning instruction procedures do not benefit from receiving instruction by mathematics laboratories.

Unkel (1971) studied mathematics laboratories and a guided discovery approach in the teaching and learning of mathematics by children and prospective teachers. The study investigated whether learning through activity can train prospective teachers to teach by guided discovery using a laboratory setting. The 29 prospective teachers tutored 66 students from grades one to six twice weekly for a nine-month period. Pre- and post-achievement tests given to the children showed a significant increase in achievement for grades one, two, three, five, and six.

From the above studies it is apparent that activity-oriented instruction can affect student teachers' mathematics achievement and attitude. Leith (1972) showed that students instructed by an activity approach achieved significantly higher and obtained a more positive attitude toward mathematics. Fitzgerald's (1971) results reported earlier showed no significant difference between the activity-oriented and nonactivity-oriented groups. Hendrickson (1969) found the experimental group to have a higher score on the mathematics achievement test but was not significantly different from the control group on two of three attitude scales.

Warkentin (1975) found the nonactivity-oriented group to have a score that was significantly higher in achievement but significantly lower on their attitudes towards mathematics as compared to the experimental group.

The four investigators, Fuson (1975), Boonstra (1970), Postmann (1971), and Unkel (1971) give contradictory results on the effects mathematics laboratories have on the pre-service teacher ability to learn to teach mathematics using manipulative aids.

Summary

Piaget and Dienes' theories give a theoretical rationale for using an activity-oriented approach when teaching elementary school children. Research investigating elementary children's learning of mathematics indicated that other methods of teaching did not significantly increase the elementary students' mathematical achievement or alter favorably their attitude towards the learning of mathematics when compared to teaching using the activity approach. Thus it appears plausible to teach elementary children by an activity-oriented approach. Fey (1979) reported that the National Science Foundation survey found lecturing and teacher questioning to be the principle instructional style used. LeBlanc (1972) suggested that teachers be taught by the same method as they will be using to teach their students, that is, the activity approach.

Research was reviewed on the effects of activity learning on prospective elementary teachers' mathematical achievement, their attitude towards learning mathematics, and whether the activity instructional style will make teachers teach using the activity approach. The studies reported show that teachers can learn to use activity-oriented instruction when taught by mathematics laboratories. They further indicate that the prospective elementary school teachers' mathematics achievement may improve but there still exists a question of doubt as expressed in the studies of Warkentin (1975) and Hendrickson (1969).

The same can be said for the student teacher's attitude toward mathematics and the teaching of mathematics. It appears that mathematics laboratories are a feasible method to teach prospective elementary school teachers how to use the activity-oriented approach. This study provides laboratories that are activity-oriented to obtain information regarding prospective elementary school teachers' ability to design effective activities.

CHAPTER III

PILOT STAGE

In this chapter the pilot stage of the study is described. The pilot stage was designed to achieve the following objectives: 1) to develop and pilot a set of mathematics laboratories, 2) to develop and pilot an evaluation instrument, 3) to develop and pilot an interview format.

The chapter begins with an overview of the pilot stage followed by an explanation of the development of the mathematics laboratories. The development of the instruments is then described and the modifications made to them for the main study are outlined. Finally, the modifications made to the laboratories as a result of the piloting are explained.

Overview of the Pilot Stage

In the fall of 1979 a set of mathematics laboratories were developed. Each laboratory had one of the following topics as a central theme: place value, geometry, logic, probability and statistics, calculators, practice games, and concept development. The rationale for the selection of these topics will be discussed later. Each laboratory consisted of four separate activities, each requiring 20 to 30 minutes to complete.

The sample for the initial development was the prospective elementary school teachers enrolled in Education 2341 during the fall of 1979. There were two classes, one with 16 students and the other 22. The room where the mathematics laboratories were held was open for two hours, three days per week. Students went to the laboratory when they wished, for whatever length of time was required to complete any three of the four activities.

Each activity had four general questions at the end. These questions required the student to either adapt the activity to new circumstances or to design alternate activities. They were used in the study as a means of evaluating the effectiveness of the laboratories in meeting course objectives. Students were permitted to complete the questions at any time and were required to submit them to their professor at the end of the semester. Each activity also included an activity evaluation sheet which the student was required to complete and submit to the laboratory assistants before leaving the laboratory on the day when the activity was completed.

Whereas the activity evaluation sheet was used to evaluate the mathematics activities individually, an Evaluation Questionnaire and interview questions were designed to evaluate the mathematics laboratory experience as a whole. The Evaluation Questionnaire was administered twice in a two-week interval near the end of the semester to determine its reliability.

Interviews were conducted with a sample of students to determine if additional information on the effectiveness of the laboratories could be obtained by this means. Also it provided the opportunity to develop suitable interview questions and enabled the interviewer to practice audio taping interviews.

A standard attitude scale was selected to determine the effect of the entire course on the attitude of the students enrolled in Education 2341 towards four aspects of mathematics education. This instrument is described in detail later in this report.

Development of the Mathematics Laboratories

The choice of the topics for the laboratories was influenced by the objectives of the course, Education 2341, for the fall semester of 1979. Practice and developmental activities were included as part of the in-school requirements of the course; therefore, these two topics were a logical choice. Also, units on geometry and calculators were components of the Education 2341 lectures; therefore, these two topics were included. Since geometry is a major component of elementary school mathematics, two separate laboratories were constructed with this theme. Place value is a topic that is a prerequisite to most algorithms for computing with whole number and furthermore this concept is embedded in a variety of manipulative aids. Logic and probability and statistics were topics not generally covered in Education 2341 lectures but were thought to be excellent areas for enrichment in elementary school mathematics. Most of the activities used in the logic laboratory required the use of problem-solving strategies. The philosophy that mathematics is taught to aid in problem-solving, and that logical thinking aids problem-solving were the two basic premises for including a laboratory which required an intuitive approach to logic.

Four 20-minute activities were selected for each of the eight laboratory themes. The criteria used to select activities under each theme were as follows:

1. Each activity used concrete materials that were available to elementary school teachers. Either they could be bought from a commercial firm or they could be easily collected or made by the teacher.

2. The selection of topics and materials was as diverse as possible within each laboratory and between laboratories.

3. The topics were considered mathematically significant.

4. The activities were adaptable to different grade levels and, when possible, to different topics.

The activities were prepared to interest and challenge the elementary school student teachers. It was thought that if the laboratories were at the level of elementary school students, the student teachers would find the activities trivial and would not experience the problems children have when first encountering a new concept. For example, a multiplication race game normally played in an elementary classroom would be in base 10 but in the laboratory it was played in base 5 for the prospective elementary school teachers.

Activities were chosen from various mathematics education sources, but were often modified for use in this pilot stage. The activities that were selected for use in the main study can be found in Appendix A.

Activity Questions

For each activity each student received a handout which consisted of the objectives, materials, size of group, directions, and four questions about the activity. These questions were to be submitted to the professor at the end of the semester. The questions were designed so that the education students would have to reflect on the activity and the materials used in it, and extend their knowledge to different situations. The questions were:

1. How would one adapt this activity for use in the classroom?

2. What would be a follow-up exercise if the child had difficulty with this activity?

3. What is an alternate activity to accomplish the same objective?

4. Suggest a variation of this activity to accomplish different objectives.

Instruments

Whereas answers to the above questions provided an opportunity to evaluate if Education 2341 students were meeting certain objectives of the course by completing the activities, other methods were used to evaluate the laboratories and the activities within them. These included an activity evaluation sheet for each activity, an Evaluative Questionnaire administered after all laboratories had been completed, interviews with a sample of students at the end of the study, and personal observations made by the laboratory assistants. Each of these methods is described below.

Activity Evaluation Sheet

An activity evaluation sheet was developed and used in the mathematics laboratories to provide information to improve the laboratories. An activity evaluation sheet was included with each activity. The students were asked to rate each activity according to six criteria. The criteria were:

1. The activity meets the stated objectives.
2. The directions are clear and understandable.
3. The activity provides for students at different ability levels and different needs.
4. The activity would motivate students.
5. Did <u>you</u> learn something new from the activity?
6. Did <u>you</u> enjoy the activity?

The criteria were selected to enable Education 2341 students an opportunity to evaluate the mathematics activities from their own perspective. These criteria were judged by the students on an ordinal scale from excellent (5) to bad (1) with each judgement receiving a corresponding number of points.

For each criteria in a given activity the ratings were totaled and averaged. The mean responses given for each activity during each laboratory session of the piloting stage are reported in Table 1. The mean responses ranged from 2.4 to 4.6 with 3.0 as the median. Any activity that had a rating on one of its criteria below 3.5 was examined for modifications. These mean responses were used to determine the strengths and weaknesses of an activity as perceived by Education 2341 students.

TABLE 1

Average Responses for each Activity during each Laboratory during Pilot Stage

Laboratory	Activity	1	2	3	4	5	6
1	1	4.0	3.0	3.6	3.8	3.4	3.7
	2	4.3	3.4	3.9	3.9	3.7	3.4
	3	4.2	4.2	3.4	4.2	4.1	4.4
	4	4.3	4.1	4.0	3.9	3.3	3.8
2	1	4.0	3.3	4.0	4.1	3.2	3.6
	2	3.5	3.9	3.9	3.6	2.6	3.0
	3	3.5	3.3	3.5	3.8	3.3	3.3
	4	4.3	3.3	3.8	4.0	3.9	3.6
3	1	4.3	4.0	4.0	3.6	3.1	3.6
	2	4.3	4.4	3.7	3.7	3.7	3.7
	3	4.3	4.1	4.1	4.1	3.2	4.2
	4	4.4	3.6	3.6	4.3	3.6	3.5
4	1	4.1	3.8	3.6	4.1	4.0	3.8
	2	4.2	3.7	3.6	3.8	3.5	3.6
	3	3.7	3.9	3.7	3.6	3.8	3.3
	4	3.9	4.1	3.9	4.0	4.0	4.0
5	1	3.8	3.4	3.3	3.3	3.5	3.1
	2	4.3	4.5	4.4	4.6	4.3	4.1
	3	3.8	3.6	3.8	3.6	3.3	3.0
	4	3.8	3.8	3.6	3.6	3.8	3.0
6	1	4.1	3.6	3.9	4.0	3.8	3.9
	2	4.1	3.9	3.9	3.4	3.1	2.9
	3	4.2	4.1	4.0	4.0	3.6	4.1
	4	3.8	3.9	3.6	3.9	4.0	4.0
7	1	4.1	4.2	4.2	3.9	3.7	3.6
	2	4.0	3.9	4.0	3.6	3.3	3.6
	3	4.4	3.6	3.9	4.0	3.4	3.6
	4	3.7	3.7	3.2	3.3	3.2	3.1
8	1	4.1	4.3	3.9	4.1	3.9	3.8
	2	4.2	4.1	4.0	3.8	3.5	3.6
	3	4.3	4.2	3.9	4.1	3.9	4.1

It should be noted that the first three activities for each laboratory reported in Table 1 correspond to the activities used in the main study with the fourth activity of each laboratory being discarded after the pilot study. Laboratory number 8 does not have a fourth activity listed since this activity was considered a prerequisite to the other three activities. The fourth activity was eventually incorporated into the lectures to be completed by the students before they participated in the eighth laboratory.

Space was provided on the activity evaluation sheets where general comments could be made about the activity. A typical selection of these comments and suggested improvements can be found in Table 2. The numbers of positive and negative comments were calculated for each activity and these results can be found in Table 3.

The only change made to the activity evaluation sheet between the pilot stage and the main study was the change of position of the general comments section. To encourage students to comment more on the activities, the general comment section was moved from the bottom of the evaluation sheet to the top. The final form of the activity evaluation sheet can be found in Appendix B.

The Evaluative Questionnaire

The second instrument was a questionnaire. Its main purpose was to obtain the students' reaction to the

TABLE 2

Typical Student Comments Extracted from Activity Evaluation Sheets Administered during the Pilot Stage

1. I enjoyed it because it made me think.
2. It might frustrate or discourage a slower child.
3. It showed still another way to help children learn place value.
4. The directions are unclear.
5. I was able to play a game while learning to subtract by regrouping.
6. This is boring.
7. Activity is a bit too long.
8. It's intriguing.

mathematics laboratories. Though the questionnaire had 52 items dealing with all three components of the course, only those items that directly pertained to the mathematics laboratories are reported in this study.

Each item was a statement or question to which the student was asked to respond by picking one of the four or five responses that best described his feelings on that particular topic. The responses varied in degree where (a) gave the strongest positive reaction, (b) a slightly

TABLE 3

Frequency of Positive/Negative Comments on Activities during Pilot Stage

Laboratory Activities		Total Positive Comments	Total Negative Comments	Total Number of Comments
1	1	6	6	12
	2	16	4	20
	3	25	1	26
	4	17	3	20
2	1	6	1	7
	2	10	10	20
	3	4	5	9
	4	18	9	27
3	1	7	1	8
	2	11	–	11
	3	17	1	18
	4	9	1	10
4	1	6	2	8
	2	7	2	9
	3	5	5	10
	4	10	3	13
5	1	3	–	3
	2	2	–	2
	3	2	2	4
	4	–	1	1
6	1	8	2	10
	2	3	2	5
	3	9	–	9
	4	4	2	6
7	1	2	1	3
	2	1	–	1
	3	2	2	4
	4	1	1	2
8	1	2	2	4
	2	3	–	3
	3	1	–	1

less positive reaction while (e) was the strongest negative reaction. The reverse held for negatively stated questions and statements. There were 38 positively stated statements and 14 negatively stated ones. At the end of the Evaluative Questionnaire there were six open-ended questions offering students ample opportunity to express their opinions on the different components of the course.

During the pilot stage the questionnaire was administered on two different occasions, the second occurring two weeks after the first. This was done to determine the instrument's reliability. The responses to each item were compared for each student. If a student gave the same response to an item on both administrations of the Evaluative Questionnaire a score of 1 point was given. If a student's response changed by one degree, one-half point was given. If there was a change of more than one degree in the student's responses on an item, no points were given. The sum of points for each item was totaled and averaged. An average of 75 percent was arbitrarily set as the lower limit for a reliable item. An item that received an average of 75 percent or higher was termed reliable and remained unchanged. If the average was lower than 75 percent the item was dropped from subsequent use of the test or modified to remove any ambiguity.

Another purpose of the Evaluative Questionnaire was to provide feedback on the mathematics laboratories.

The multiple-choice questions did provide additional feedback from the students but the open-ended questions did not accomplish this objective. The comments were repetitious of those given on the activity evaluation sheet completed in the laboratories. Therefore, it was decided to eliminate these questions from the Evaluative Questionnaire during subsequent administrations. A list of the questions on the Evaluative Questionnaire pertaining to the mathematics laboratories can be found in Appendix C. The entire questionnaire used in the main study is found in Appendix D.

Ten of the 14 statements pertaining to the mathematics laboratories were positively stated and the remaining four statements were negatively stated. There were 15 students who participated in the first administration of the Evaluative Questionnaire. The results of the first administration of the multiple-choice questions on the Evaluative Questionnaire pertaining to the mathematics laboratories are also found in Appendix C. The following is a discussion of these results.

The students' reactions to questions 1, 4, and 5 were favorable, indicating that the students were satisfied with the structure, organization, and staffing of the mathematics laboratories. In question 3 most students expressed the wish to have to do only two of the three activities required for each laboratory because of the one-hour time restriction. This request was taken into consideration for the main study. The decision made was to extend the laboratory period to two hours. Excluding one of the three activities would have decreased the variety of experiences which the students were then receiving. Responses from questions 2, 6, 7, 8, 11, and 12 indicated that the majority of the students perceived the mathematics laboratories as an asset in learning how to teach mathematics.

Students differed in their opinions as to whether the mathematics laboratories required too much work, as can be seen from the responses made to question 13. Responses to question 9 indicated that the objectives of the mathematics laboratories could have been more clearly stated to the students from the beginning of the semester, yet few perceived any difficulty with the laboratories as seen from their responses to question 11. The overall rating of the mathematics laboratories from students writing the first administration of the Evaluative Questionnaire was between good and very good.

Interviews

Interviews were conducted with a sample of the Education 2341 students at the end of the pilot stage. These interviews were to (1) check students' reactions to being taped by audio cassette, and (2) provide further feedback on the laboratories from the student. The interviews included six open-ended questions about the mathematics laboratories.

In the pilot stage 25 percent of the students enrolled in Education 2341 were interviewed. Each student was interviewed individually for 15 minutes. If a student wished to extend a discussion topic he was encouraged to do so. The questions were meant only to provide structure for the evaluative discussion and to insure that all the crucial areas were discussed with each student. The student responses were recorded on audiotapes. The interview questions yielded responses similar to those presented in Table 4. From the interviews it was concluded that the students had difficulty with the four questions at the end of each activity. Since many students had not completed the questions until the end of the semester, this caused difficulty in remembering the activities in enough detail to answer any question accurately. The students found the questions too general and having to answer the same four questions every time became very monotonous.

The piloting of the interview procedure aided in obtaining a standard method of interviewing and demonstrated the usefulness of the questions. A list of the interview questions pertaining to the mathematics laboratories is included in Appendix E.

Subjective Observations

The observations of the laboratory assistants were recorded while students were participating in the laboratories.

TABLE 4

Typical Selection of Student Comments Extracted from Interviews Administered during the Pilot Stage

1. The laboratories were interesting.
2. There should be three activities in one hour.
3. The laboratories made one aware of the different materials needed to teach mathematics.
4. The laboratories should be examples of activities at the elementary level.
5. The laboratories were good.
6. Mathematics in some of the laboratories was too difficult.

Notes were made on whether a particular activity needed an extra explanation due to confusion with procedures or mathematical difficulties.

These observations either supported or refuted any written comments made by the students concerning the difficulty and clarity of the activities. They also indicated whether the laboratories could be used in the future without a laboratory assistant or what role was necessary for the laboratory assistant.

Modifications of Mathematical Laboratories

Data from the activity evaluation sheets, Evaluative Questionnaire, interviews, and subjective observations were used by the two laboratory assistants and the professors teaching Education 2341 to select three of the four activities for use in the main study. The exclusion of an activity was based on lack of adaptability, duplication of materials or concepts, and too difficult or too trivial as compared to the other activities.

Minor changes, such as clarification of directions, variation of examples, or a decrease in length were made in the remaining 24 activities. One major alteration was made on the activity on quadrilaterals and their properties using geostrips. The students' reaction to that activity during the pilot stage was very negative. The activity was found to be long, boring, and tedious. The theme of the activity was considered significant enough to be included, but the activity was altered considerably from its original form.

Another major change made to the mathematics activities was the deletion of the four questions discussed earlier. It was intended that students should stop after each activity and think on how it could be adapted in the four ways suggested by the questions. It was decided that the same objective could be accomplished by asking any two of the four questions for each activity. This would shorten the question period. Furthermore, the questions were made more specific to each activity. The following example is from an activity in the probability and statistics laboratory. The example required students to answer a mathematical question that should be possible if the student had learned the concept taught in the activity and requested that the student design an activity to teach the same concept but with a different approach.

> Ms. Soffis went to Florida with blue, yellow, and white bikinis plus brown, black, and tan sandals. How many different outfits could she wear to the beach?
>
> The above question, with the aid of a doll and clothes, could be an alternate activity to accomplish the same objective as the "counting" activity. Describe a different activity that would accomplish the same objective.

The changes suggested in this chapter for the mathematics laboratories, procedures, and instruments were implemented in the main study. The purpose of the pilot stage was to develop and pilot various mathematics laboratories and instruments for the main study. The process that was necessary to complete the aims for the pilot stage has been described in this chapter.

CHAPTER IV

DESIGN OF MAIN STUDY

The design of the study is described in this chapter. Beginning with an explanation of the evaluation model, the curriculum development aspect of this study is described. This is followed by a description of the population and sample. The procedure and instruments used in the main study are then discussed. The method of evaluating the laboratories is presented and finally, a description of the methods of analysis is presented for each of the questions asked in the study.

Evaluation

> Evaluation has for too long been viewed by education practitioners as a negative, constricting, threatening activity to be tolerated only under stress (Provus, 1975, p. 29).

Evaluation should be a positive happening which aids the program, the educators, and the students. Evaluation can serve two functions: program revision (formative) and final assessment (summative). Stake (1969) stated that formative evaluation provides information for outsiders. Taba's (1969) model of curriculum development includes as a part of the process, a feedback into each step of the development. This allows for program revision. This support for formative evaluation in curriculum development was the basis for its selection as the evaluation process for this study.

The evaluation model used in this study was adapted from the models of Taba (1969) and Hartung (1961) and is illustrated in Figure 1.

Formative evaluation occurs at each step in the model. If changes are required in the curriculum then they are made and the process continues. This provides for continual improvement in the curriculum.

The objective of this study was to answer the questions stated in the first chapter. Since the aims of Education 2341 are both cognitive and affective, test situations were selected that were suitable for both domains.

Taba (1969) stated that information should be collected in a variety of ways, both by objective and subjective means. Often the standardized pencil and paper tests are not sufficient to supply all the necessary information. Taba suggested diaries, observations, questionnaires, school records, essays, and interviews as means of collecting information. In this study, data were collected by both objective and subjective means for each of the cognitive and affective domains. The affective domain was objectively tested by the attitude scale and the activity evaluation sheets and subjectively by the Evaluative

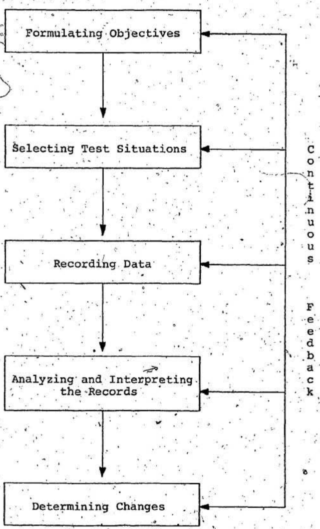

FIGURE 1. Model used for evaluating mathematics laboratories

Questionnaire and interviews. The cognitive domain was tested by the questions at the end of each activity. Information on revision of the laboratories was obtained by paper and pencil methods on the Evaluative Questionnaire, and activity evaluation sheets, besides the more subjective approach of laboratory assistant observations and student interviews.

Hartung (1961) states that when determining changes one has to include objectives, instructional methods, materials, facilities, learning activities, and techniques of evaluation. Any one, or all, of these may need minor or major alterations before going through a new cycle. For this study the objectives were modified, instructional methods and learning activities were extended, and materials were varied. These changes were a result of the mathematics laboratories being piloted in the fall semester before being used a second time during the winter in the main study.

Population and Sample

The population under consideration in this study was all prospective elementary teachers who were studying at Memorial University to obtain a bachelor's degree in primary or elementary education.

The sample used in the main study included all prospective elementary school teachers completing Education

2341 in the winter of 1980. There were four classes, with enrollments of 20, 20, 16, and 14 students, respectively. The classes were taught by three different instructors. Students enrolled in special programs in the faculty of education were not included in the sample.

Procedure

During the winter semester the mathematics laboratories were conducted by two graduate students, one of whom was the researcher. Each of the laboratories was two hours in length and contained three activities. Students worked in small groups which were formed by individual choice. At the end of each activity there were two questions related to the activity to be answered by each student. These were submitted to the laboratory assistant, along with the activity evaluation sheet, at the end of the laboratory period.

The mathematics laboratories were evaluated by means of the evaluation sheets, the questions completed in the laboratory, the Evaluative Questionnaire, and interviews. The Evaluative Questionnaire was completed during the last Education 2341 lecture, while the interviews were held several days later but before the final examination in the course. The interviews were conducted with 36 randomly selected students enrolled in Education 2341.

The students' attitude toward mathematics and the teaching of mathematics was studied by the use of an attitude scale. This scale was administered during the first 20 minutes of the first lecture of Education 2341 in the 1980 winter semester and administered again during the last Education 2341 lecture prior to answering the Evaluative Questionnaire. In Figure 2 a chronological flowchart of events which formed the procedure is presented.

Instruments Used in the Main Study

The instruments used in the main study were the activity evaluation sheets, Evaluative Questionnaire, interviews, and the Connelly Taxonomized Attitude Scale. The activity evaluation sheet described in the previous chapter was used again in the main study to collect additional suggestions for modifications to the laboratories that may not have been covered in the first set of changes. The Evaluative Questionnaire and interviews were used to obtain feedback about the mathematics laboratories. On the Evaluative Questionnaire students were required to select a response to questions about the laboratories while the interviews were used to serve as a check on these responses. The interviews allowed students an opportunity to express their views by answering open-ended questions about the mathematics laboratories.

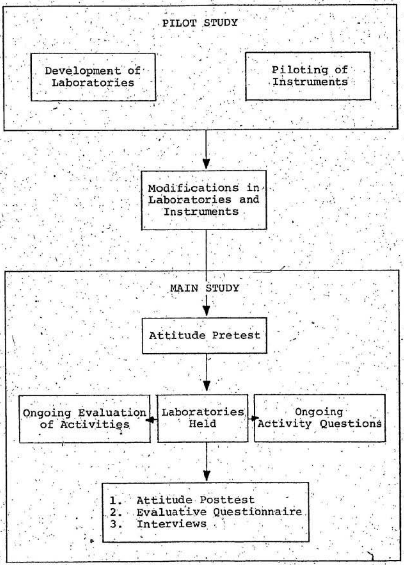

FIGURE 2. Chronological flowchart of events of the procedure.

The activity evaluation sheets, Evaluative Questionnaire, and interviews were developed and piloted in the pilot stage of the study and were described in Chapter 3. The Connelly Taxonomized Attitude Scale, a standardized instrument, is described below.

Attitude Scale

The attitudes of Education 2341 students were measured by the Connelly Taxonomized Attitude Scale (Connelly, 1973). The instrument measured four related areas of mathematics education: attitudes toward mathematics, attitudes toward the existence and use of logical structure in mathematics, attitudes toward individualizing mathematics instruction and the use of guided discovery techniques in mathematics, and attitudes toward teaching mathematics. The instrument used a five-point Likert scale. It contained 64 items: 44 positively stated items and 20 negatively stated items.

Connelly administered the instrument to a sample of 146 students in Education 325 at Kent State University. Alpha reliability coefficients of 0.91 and 0.94 were obtained for the pretest and posttest, respectively. It was assumed that this instrument was equally valid and reliable for the population investigated in the present study. A copy of this instrument is included in Appendix F.

For this study it was decided to treat the scores of the four areas separately. Also, instead of using Connelly's procedure of having one score as the dividing line between positive and negative attitudes, a buffer zone of 14 points was considered a neutral attitude. The possible scores for each area range from 16 to 80. A score from 16 to 40 was considered a negative attitude, 41 to 55 was a neutral attitude, and 56 to 80 was a positive attitude.

Evaluation of Mathematics Laboratories

This study sought answers to the questions stated in Chapter I. Since there were a variety of questions, several different modes of data collection were necessary.

As was discussed in the curriculum development section, the mathematics laboratories were included in Education 2341 during the 1980 winter semester and were evaluated in the same manner as were the laboratories in the pilot stage. Data were collected from the activity evaluation sheets, the Evaluative Questionnaire, the interview questions, and notes made by the two laboratory assistants. These data are presented in the next chapter in the same format as was previously used in the development of the mathematics laboratories section during the pilot study. That is, a table of the mean responses to questions on each activity, a sample of suggested improvements and comments on the laboratories, and the totals of positive and negative comments are presented. This information was taken from the activity evaluation sheets. The responses to the multiple choice questions on the Evaluative Questionnaire are given in tabular form, as are typical comments from the interviews. The above data were used to determine if there were any additional changes needed to be made to the laboratories.

Study Questions and Analysis

The analysis of the study questions involved an interpretation of two sets of data, the data obtained from student responses on the questions which follow each activity in the mathematics laboratories and the Connelly Taxonomized Attitude Questionnaire.

The questions after each activity were answered by students at the end of each laboratory. These answers were graded on a 1 to 5 ordinal scale. If a student named a correct material that could be used in the adaption he received one point; if the student named a material and described an activity that would be impossible to duplicate if one did not have prior knowledge to his references then he received two points; if the material was named and a vague but sequential explanation of the adapted activity was given, three points were alotted; the student who named the material and gave a detail, sequential explanation but a routine activity received four points; the student who named the material and showed evidence of a creative adapted activity obtained five points.

For the grading of these questions the two laboratory assistants discussed the grading scheme and then separately graded a student's answer. After comparing grades and a final consultation, the laboratory assistants graded two students' answers separately. The same grades were given by the two assistants for all questions.

The answers to these activity questions were used to determine answers to the first five questions of the study. These questions are discussed further below.

Study question one. Will a student in Education 2341 be able to suggest a follow-up activity, given the information that an elementary school student had exhibited difficulty with a previous activity on the same topic? Four questions from the activities were relevant to this study question. They are summarized in Table 5.

Study question two. Will a student in Education 2341 be able to suggest how a given activity might be altered to accomplish different objectives? Nine questions from the activities were relevant to this study question. They are summarized in Table 6.

TABLE 5

Activity Questions Relevant to Study Question One

Laboratory	Activity	Question Number
1	1	1
3	2	1
5	3	2
6	2	1

TABLE 6

Activity Questions Relevant to Study Question Two

Laboratory	Activity	Question Number
1	3	2
1	2	1
2	1	2
2	3	1
3	2	2
3	1	2
4	1	1
4	2	1
5	2	1

Study question three. Will a student in Education 2341 be able to suggest an alternate activity to accomplish the same objective as a given activity? Eighteen questions from the activities were relevant to this study question. They are summarized in Table 7.

Study question four. Will a student in Education 2341 be able to suggest an extension activity to an elementary school student, given the information that the student had exhibited no difficulty with a previous activity on the same topic? Two questions from the activities were relevant to this study question. They are summarized in Table 8.

Study question five. Will a student in Education 2341 be able to adapt an activity in mathematics designed for a university student, for use in an elementary school classroom? Eleven questions from the activities were relevant to this study question. They are summarized in Table 9.

Each activity question was used to provide a separate answer to the study question to which it was relevant. This meant, for example, that study question one was answered on four separate occasions, once for each activity question related to it.

Two alternative procedures for providing answers to the study questions were considered and rejected. First, one might have considered a study question to be answered

TABLE 7

Activity Questions Relevant to Study Question Three

Laboratory	Activity	Question Number
1	1	2
1	2	2
2	2	2
2	3	2
3	3	2
3	1	1
4	3	2
4	1	2
4	2	2
5	1	2
5	2	2
6	3	2
6	2	2
7	1	2
7	2	2
7	3	2
8	3	2
8	2	2

TABLE 8

Activity Questions Relevant to Study Question Four

Laboratory	Activity	Question Number
7	3	1
8	1	1

TABLE 9

Activity Questions Relevant to Study Question Five

Laboratory	Activity	Question Number
1	3	1
2	1	1
2	2	1
3	3	1
4	3	1
5	1	1
5	3	1
6	3	1
7	1	1
8	3	1
8	2	1

positively if a certain percentage of related activity questions were satisfactorily answered. This was rejected due to the low number of activity questions used for each study question. Second, the responses of each type for all activity questions relevant to a given study question could have been added and the percentage of students receiving a score of less than three determined. A criterion could then be applied to answer the study question similar to that used to answer the activity question. This method was rejected since it was considered that, although the activity questions were similar in that they were related to the same study question, they were sufficiently different among themselves that it made little sense to add the scores together.

Study questions six through nine were evaluated using the Connelly Taxonomized attitude Questionnaire. The number of students with positive, neutral, and negative attitudes were tabulated before and after completion of Education 2341. The dependent t-test was used to determine if a significant change at the 0.5 level of significance had occurred in the students' attitude between the first and last weeks of enrollment in Education 2341. The following hypotheses were tested.

1. There is no significant change in the students' attitude toward the existence and use of logical structure in mathematics between the first and last weeks of enrollment

in Education 2341.

2. There is no significant change in the students' attitude toward mathematics between the first and last weeks of enrollment in Education 2341.

3. There is no significant change in the students' attitude toward individualizing mathematics instruction and the use of guided discovery techniques in mathematics between the first and last weeks of enrollment in Education 2341.

4. There is no significant change in the students' attitude toward teaching mathematics between the first and last weeks of enrollment in Education 2341.

CHAPTER V

ANALYSIS OF DATA

In this chapter the analysis of data related to the study's questions is reported. First, the effectiveness of the laboratories is considered, thus providing answers to the first five questions of the study. The remaining four questions consider the change of student's attitudes between the first and last lectures in Education 2341. Finally, in light of the data collected on the activity evaluation sheets, Evaluative Questionnaire, interview questions, laboratory questions used to study the laboratory effectiveness, and observations made by the laboratory assistants, modifications to the laboratories are suggested.

Laboratory Effectiveness

The laboratory effectiveness was determined by examining students' responses on the laboratory questions answered after each activity. Each activity included two questions requesting students to modify the activities according to some given specifications.

Each of these activity questions provided an answer to a study question. For example, study question one was answered by examining the responses to four separate activity questions.

The scoring for the laboratory questions was as follows. The highest possible score was five which indicated that the student had accurately described a creative activity using a suitable material for adoption of the activity. If the activity was considered to be routine rather than creative, and included a suitable material, then the student received four points. An answer that was vague but described an activity which could be duplicated was valued at three points. An explanation that described an activity using suitable materials but was too vague to be followed accurately by others was awarded two points. If suitable alternate materials were described without a description of the adapted activity, one point was awarded. Students who suggested inappropriate activities and materials were given a score of zero. The procedure for establishing the reliability for the above judgements was discussed in chapter four under the section, study questions and analysis.

The results of the analysis of the first five questions of the study are presented below.

Study Question 1

Will a student in Education 2341 be able to suggest a follow-up activity given the information that an elementary school student had exhibited difficulty with a previous activity on the same topic?

The four activity questions used to evaluate study question 1 were previously listed in Table 5 in Chapter IV. The percentage of students who received each score on each question was determined and these data are reported in Table 10. There were two activity questions on which less than 20 percent of the students received a score of less than 3, thus indicating a positive answer to study question 1. These questions were labeled 3 : 2 : 1 and 6 : 2 : 1. In this notation the first numeral refers to the laboratory number, the second to the activity, and the third to the activity question. On the remaining two questions, there were 32.3 percent and 26.0 percent of the students who gave responses that received a score of less than 3. Although these percentages were modest they were beyond the stipulated criterion, thus these questions indicated a negative answer to study question 1. These results are discussed in Chapter VI.

Study Question 2

Will a student in Education 2341 be able to suggest how a given activity might be altered to accomplish different objectives?

The nine activity questions used to evaluate study question 2 were previously listed in Table 6 in Chapter IV.

TABLE 10

Percentages of Student Responses for Study Question 1

Activity Questions	N	-*	0	1	2	3	4	5
				Responses				
Lab : Act : Q No.								
1 : 1 : 1	68	-	-	13.2	19.1	26.5	35.3	5.9
3 : 2 : 1	70	-	1.4	2.9	4.3	5.7	11.4	74.3
5 : 3 : 2	63	1.6	-	7.9	17.5	41.3	22.2	9.5
6 : 2 : 1	62	-	-	1.6	4.8	21.0	41.9	30.6

*Note: Students who completed the activity but did not answer the question.

The percentage of students who received each score on each question was determined and these data are reported in Table 11. There were six activity questions on which less than 20 percent of the students received a score of less than 3, thus indicating a positive answer to study question 2. These questions were 1 : 2 : 1, 1 : 3 : 2, 3 : 2 : 2, 4 : 1 : 1, 4 : 2 : 1, and 5 : 2 : 1. On the remaining three questions, 2 : 1 : 2, 2 : 3 : 1, and 3 : 1 : 2 the percentage of students who obtained a score under 3 were 36.9 percent, 33.3 percent, and 49.1 percent. These percentages were above the 20 percent criterion; thus for these questions study question 2 was answered negatively. The results are discussed further in Chapter VI.

TABLE 11

Percentages of Student Responses for Study Question 2

Activity Questions	N	Responses -	0	1	2	3	4	5
Lab : Act : Q No.								
1 : 2 : 1	69	4.3	2.9	-	7.2	36.2	42.0	7.2
1 : 3 : 2	68	-	-	-	2.9	47.1	38.2	11.8
2 : 1 : 2	65	3.1	4.6	1.5	27.7	6.2	47.7	9.2
2 : 3 : 1	63	-	9.5	17.5	6.3	14.3	34.9	17.5
3 : 1 : 2	69	1.4	1.4	27.5	18.8	36.2	14.5	-
3 : 2 : 2	70	1.4	-	-	-	4.3	8.6	85.7
4 : 1 : 1	69	-	-	-	4.3	30.4	59.4	5.8
4 : 2 : 1	69	1.4	-	-	13.0	8.7	63.8	13.0
5 : 2 : 1	64	-	-	1.6	9.4	17.2	65.6	6.3

Study Question 3

Will a student in Education 2341 be able to suggest an alternate activity to accomplish the same objective as a given activity?

The 18 activity questions used to evaluate study question 3 were previously listed in Table 7 in Chapter IV. The percentage of students who received each score on each question was determined and these data are reported in Table 12. There were 12 activity questions on which less than 20

TABLE 12

Percentages of Student Responses for Study Question 3

Activity Questions	N	Responses –	0	1	2	3	4	5
Lab : Act : Q No.								
1 : 1 : 2	68	2.9	1.5	7.4	5.9	17.6	30.9	33.8
1 : 2 : 2	68	–	–	–	10.3	54.4	20.6	14.7
2 : 2 : 2	67	–	–	7.5	–	14.9	59.7	17.9
2 : 3 : 2	63	6.3	3.2	4.8	4.8	9.5	39.7	31.7
3 : 1 : 1	69	–	–	15.9	17.4	43.5	20.3	2.9
3 : 3 : 2	69	1.4	–	8.7	10.1	18.8	39.1	21.7
4 : 1 : 2	69	–	1.4	8.7	5.8	18.8	47.8	17.4
4 : 2 : 2	69	1.4	–	8.7	4.3	5.8	30.4	49.3
4 : 3 : 2	69	–	1.4	23.2	4.3	18.8	43.5	8.7
5 : 1 : 2	64	–	–	–	1.6	7.8	70.3	20.3
5 : 2 : 2	64	–	–	–	6.3	14.1	20.3	59.4
6 : 2 : 2	62	–	–	6.5	–	37.1	24.2	32.3
6 : 3 : 2	62	–	3.2	35.5	1.6	14.5	12.9	32.3
7 : 1 : 2	64	–	–	7.8	10.9	12.5	56.3	12.5
7 : 2 : 2	64	–	–	–	28.1	39.1	7.8	25.0
7 : 3 : 2	63	–	–	7.9	6.3	4.8	52.4	28.6
8 : 2 : 2	67	–	–	9.0	11.9	43.3	26.9	9.0
8 : 3 : 2	67	–	–	1.5	4.5	26.9	38.8	28.4

percent of the students received a score of less than 3, thus indicating a positive answer to study question 3. These questions were 1 : 1 : 2, 1 : 2 : 2, 2 : 2 : 2, 2 : 3 : 2, 4 : 1 : 2, 4 : 2 : 2, 5 : 1 : 2, 5 : 2 : 2, 6 : 2 : 2, 7 : 1 : 2, 7 : 3 : 2 and 8 : 3 : 2. On the remaining six questions, the percentage of responses that received a score of less than 3 were 33.3 percent, 20.2 percent, 28.9 percent, 40.3 percent, 28.1 percent, and 20.9 percent. Each of these were greater than the criterion of 20 percent, thus study question 3 was answered negatively for these six questions. These results are discussed in further detail in Chapter VI.

Study Question 4

Will a student in Education 2341 be able to suggest an extension activity for an elementary school student, given the information that the student had exhibited no difficulty with a previous activity on the same topic?

The two activity questions used to evaluate study question 4 were previously listed in Table 8 in Chapter IV. The percentage of students who received each score on each question was determined and these data are reported in Table 13. On the two questions, 23.8 percent and 50.7 percent of the students received a score of less than 3, indicating an unsatisfactory response to the question and a negative answer to study question 4. These results are

TABLE 13

Percentages of Student Responses for Study Question 4

Activity Questions	N	-	0	1	2	3	4	5
			Responses					
Lab : Act : Q No.								
7 : 3 : 1	63	1.6	-	12.7	9.5	30.2	17.5	28.6
8 : 1 : 1	67	-	4.5	11.9	34.3	23.9	6.0	19.4

discussed in Chapter VI.

Study Question 5

Will a student in Education 2341 be able to adapt an activity in mathematics designed for an university student, for use in an elementary school classroom?

The 11 activity questions used to evaluate study question 5 were previously listed in Table 9 in Chapter IV. The percentage of students who received each score on each question was determined and these data are reported in Table 14. There were six activity questions on which less than 20 percent of the students received a score of less than 3, thus indicating a positive answer to study question 5. These questions were 1 : 3 : 1, 4 : 3 : 1, 5 : 1 : 1, 5 : 3 : 1, 7 : 1 : 1 and 8 : 3 : 1. On the remaining five

TABLE 14

Percentages of Student Responses for Study Question 5

Activity Questions	N	Responses -	0	1	2	3	4	5
Lab : Act : Q No.								
1 : 3 : 1	68	-	-	-	2.9	35.3	61.8	-
2 : 1 : 1	65	-	-	6.2	50.8	15.4	12.3	15.4
2 : 2 : 1	67	-	-	1.5	55.2	20.9	13.4	9.0
3 : 3 : 1	69	-	-	1.4	21.7	37.7	30.4	8.7
4 : 3 : 1	69	-	-	7.2	10.1	18.8	43.5	20.3
5 : 1 : 1	64	-	-	1.6	10.9	32.8	39.1	15.6
5 : 3 : 1	63	-	-	-	17.5	46.0	34.9	1.6
6 : 3 : 1	62	-	-	-	32.3	29.0	38.7	-
7 : 1 : 1	64	-	-	-	12.5	39.1	32.8	15.6
8 : 2 : 1	67	-	-	1.5	22.4	43.3	23.9	9.0
8 : 3 : 1	67	-	-	-	1.5	55.2	41.8	1.5

questions, 57 percent, 56.7 percent, 23.1 percent, 32.3 percent and 23.9 percent of the students received a score of less than 3, indicating an unsatisfactory response to the question and a negative answer to study question 5. These results are discussed in Chapter VI.

Student Attitudes

The attitudes of the students were measured by the Connelly Taxonomized Attitude Scale. This attitude scale tested four separate areas of mathematics education. The scale was employed to test the student's attitude towards:

Attitude 1: The existence and use of logical structure in mathematics.

Attitude 2: Mathematics

Attitude 3: Individualizing mathematics instruction and the use of guided discovery techniques in mathematics.

Attitude 4: Teaching mathematics.

The attitude scale was administered twice, once on each of the first and last days of Education 2341 lectures. Students' scores were included in the analysis if the student completed the attitude scale on both occasions. There were 59 of the 70 students who met that requirement.

The highest possible score on each scale was 80, indicating the most positive attitude and the lowest possible score was 16, indicating the most negative attitude. A score from 56 to 80 was considered to be a positive attitude, 41 to 55 a neutral attitude, and 16 to 40 a negative attitude.

In Table 15 the number of students who had positive, neutral, and negative attitudes toward the four distinct areas of mathematics education for the two administrations is illustrated.

Table 15

Frequencies of Student Attitudes

	Attitude 1		Attitude 2		Attitude 3		Attitude 4	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Positive	58	55	44	51	58	59	37	45
Neutral	1	4	14	8	1	–	20	14
Negative	–	–	1	–	–	–	2	–

A dependent t-test was performed on the attitude scores to determine the significance of any attitude change between the two occasions for each of the scales. The results of this testing are shown in Table 16. The results are discussed below with respect to each of the four related hypotheses.

TABLE 16

T-test Values for the Differences in Attitudes Between Pretest and Posttest

Tested Attitudes	N	Pretest $\overline{X}$	Pretest S	Posttest $\overline{X}$	Posttest S	t values
Attitude 1	59	63.00	4.23	64.31	5.77	1.97
Attitude 2	59	60.54	7.92	63.42	7.37	4.06*
Attitude 3	59	65.56	5.53	69.00	4.63	5.38*
Attitude 4	59	56.83	6.58	58.97	6.20	3.29*

*$p < 0.01$

Hypothesis 1

There is no significant change in the students' attitude toward the existence and use of logical structure in mathematics between the first and last weeks of enrollment in Education 2341.

From Table 15 it can be seen that the number of students with a positive attitude toward the existence and use of logical structure in mathematics decreased from 58 to 55. However, the mean scores for this attitude, as reported in Table 16, increased from 63 to 64.31. This indicated that the students who obtained a more positive attitude towards the existence and logical structure in mathematics had a greater change in this direction than those students who had become more negatively inclined. The results of the dependent t-test indicated the value of t was 1.97. Since this value is not significant at the 0.05 level the null hypothesis was not rejected. The probability of obtaining the value of 1.97 is less than 0.10.

Hypothesis 2

There is no significant change in the student's attitude toward mathematics between the first and last weeks of enrollment in Education 2341.

As shown in Table 15 the number of students with a positive attitude towards mathematics increased over the semester with a corresponding decrease in the number who had negative and neutral attitudes. This shift can also be observed in the means reported in Table 16 where the means were 60.54 and 63.42 on the pre- and posttests, respectively.

The t-value of 4.06 was significant at the .01 level of significance thus indicating a significant change in student attitudes toward mathematics. Education 2341 students had a significantly more positive attitude toward mathematics after completing Education 2341 than before participating in the course. The null hypothesis was therefore rejected.

Hypothesis 3

There is no significant change in student's attitude toward individualizing mathematics instruction and the use of guided discovery techniques in mathematics between the first and last weeks of enrollment in Education 2341.

As can be observed from Table 15 little change occurred in student attitude for individualizing instruction and the use of guided discovery techniques. Yet from Table 16 it can be seen that there was an increase in the means from 65.56 to 69.00.

The t-value of 5.28 was significant at the .01 level of significance thus indicating a significant change in student attitudes toward individualizing mathematics instruction and the use of guided discovery techniques. Education 2341 students had a more positive attitude toward individualizing mathematics instruction and the use of guided discovery techniques upon completion of the course. The null hypothesis was therefore rejected.

Hypothesis 4

There is no significant change in student's attitude toward teaching mathematics between the first and last weeks of enrollment in Education 2341.

As shown in Table 15 the number of students with a positive attitude towards the teaching of mathematics increased with a corresponding decrease in the number of negative and neutral attitudes. This shift can also be observed in the means reported in Table 16. The mean score increased from 56.83 to 58.97.

The t-value of 3.29 was significant at the .01 level of significance thus indicating a significant difference in student attitudes towards teaching mathematics. Education 2341 students had a more positive attitude toward teaching mathematics upon completion of the course. The null hypothesis was therefore rejected.

Ongoing Development of the Mathematics Laboratories

Activity Evaluation Sheets

Activity evaluation sheets, administered after each activity, were designed so that Education 2341 students could evaluate the activity immediately after completion. Each activity was evaluated on six specified criteria. The criteria were:

1. The activity meets the stated objectives.
2. The directions are clear and understandable.
3. The activity provides for students at different ability levels and different needs.
4. The activity would motivate students.
5. Did you learn something new from the activity?
6. Did you enjoy the activity?

In addition, students were asked to make any comments they wished about the activities. The criteria were judged by the students on an ordinal scale from excellent (5) to bad (1).

The points on each criterion for each activity, as rated by the Education 2341 students, were then averaged and tabulated. The average ratings for each criterion for each activity are reported in Table 17.

It can be observed from Table 17 that the average ratings were above three for each of the criteria on every activity. The means ranged from 3.2 to 4.5 as compared to the range of 2.4 to 4.6 during the pilot stage (see Table 2 in Chapter III). The percentages of ratings in selected intervals are presented in Table 18 for both the pilot and main study. From the table it can be observed that in the main study 50 percent of the ratings exceeded a score of 40 whereas less than 30 percent of the ratings met this criterion during the pilot. In general, this indicated an overall higher rating of the activities during the main study.

For example, one can compare the students' evaluation of the activity "Geo-strips and quadrilaterals" to observe the value of an ongoing evaluation of the laboratories. The activity titled Geo-strips and quadrilaterals (laboratory 4, activity 3) received the following average scores on the criteria during the pilot study: 3.7, 3.9, 3.7, 3.6,

TABLE 17

Average Student Responses for Each Activity

Lab : Act	Criteria 1	2	3	4	5	6
1 : 1	4.2	4.0	3.3	4.4	3.7	4.2
1 : 2	4.5	3.7	3.8	4.2	3.5	4.1
1 : 3	4.5	4.5	3.5	4.2	4.0	4.2
2 : 1	4.3	3.8	4.0	4.2	3.7	4.1
2 : 2	4.4	3.9	3.8	3.7	4.0	3.6
2 : 3	4.0	3.8	3.8	4.0	3.5	3.6
3 : 1	4.2	3.8	3.8	3.5	3.3	3.2
3 : 2	4.3	4.2	4.1	4.1	3.7	3.9
3 : 3	4.4	4.5	4.2	4.4	3.7	4.2
4 : 1	4.4	4.3	4.2	4.3	3.9	4.2
4 : 2	4.3	4.1	4.0	4.2	3.7	4.1
4 : 3	4.3	4.2	4.1	4.2	4.1	4.2
5 : 1	4.3	4.2	3.9	4.2	3.9	4.1
5 : 2	4.2	4.1	3.7	3.9	4.0	3.7
5 : 3	4.3	4.2	4.0	4.0	3.7	3.6
6 : 1	4.1	3.7	3.8	4.1	4.0	4.1
6 : 2	4.3	4.2	3.9	3.7	3.2	3.4
6 : 3	4.4	4.4	4.1	4.2	3.9	4.3
7 : 1	4.2	4.3	3.9	4.0	3.5	3.7
7 : 2	4.3	4.3	3.9	3.9	3.7	3.7
7 : 3	4.2	4.2	3.9	3.9	3.9	3.7
8 : 1	4.4	4.5	4.0	4.1	4.0	3.8
8 : 2	4.2	4.3	3.8	3.6	3.4	3.4
8 : 3	4.5	4.5	4.0	4.4	3.9	4.3

TABLE 18

Comparison of Student's Evaluation of Activities Between the Pilot and the Main Study

	Percentage of Ratings in each Interval		
	Pilot Study		Main Study
Ratings Interval	including all activities	including only the activities used in the main study	
0.0 - 2.5	0.00	0.00	0.00
2.6 - 3.0	3.20	3.50	0.00
3.1 - 3.5	18.30	21.50	8.33
3.6 - 4.0	52.70	45.80	41.67
4.1 - 4.5	25.30	28.50	50.00
4.6 - 5.0	0.50	0.70	0.00

3.8, and 3.3 and for the main study it received: 4.3, 4.2, 4.1, 4.2, 4.1, and 4.2. Because this activity, mentioned earlier in this report, bored the students in the pilot study, major changes were made in it for the main study. The increase in its ratings indicated that these changes resulted in a more positive reaction to the activity by the students.

Another activity that was changed between the pilot and main study was "geoboards" (laboratory 5, activity 3).

Geoboards were rated 3.8, 3.6, 3.8, 3.6, 3.3, and 3.0 during the pilot study and 4.3, 4.2, 4.0, 4.0, 3.7, and 3.6 during the main study. The changes in the student evaluation for this activity were not as great as the above activity, but positive differences can be observed.

All activities that had a rating less than 3.5 on any of the criteria were re-examined to determine the probable cause. The activities in question were 1 : 1, 3 : 1, 6 : 2, and 8 : 2.

The first activity, 1 : 1, was a card game in which the skill of naming equivalent fractions was practiced. The low score for this activity on the third criterion, "the activity provides for students at different ability levels and different needs," was justified. Since this was the initial activity of the first laboratory, students may have been unaware of how simply this activity could be adapted for other topics or for other levels of students.

The other three activities, 3 : 1, 6 : 2, and 8 : 2 received low scores on the fifth criterion, "Did you learn something new from the activity," and on the sixth criterion, "did you enjoy the activity." All three of these activities contained mathematics taught at the elementary classroom level. One activity was concerned with multiplication, the other classification and seriation of numbers, and the third, multiplying decimals using blocks and arrays. Each of these topics is mathematically trivial to university students and might not have been challenging to them.

As mentioned earlier the activity evaluation sheets had space for students to make suggestions and comments about the activities. In Table 19 a selection of typical comments is presented. This selection includes both positive and negative comments. Many of the comments report the students' reaction to the activity while others were suggestions to aid in improving the laboratory. For example, it was suggested that the symmetry activity could be made simpler if the designs had been pre-cut. These suggested improvements were sometimes helpful while others were unrealistic to implement into the particular activity. Some comments were contradictory to each other.

The number of positive and negative comments made about each activity are reported in Table 20. The total number of comments in the main study was 2004 as compared to 286 comments (see Table 4) made during the piloting of the laboratories. This was a substantial difference even when one takes into consideration the differences in enrollment between the main study and the piloting of the laboratories. During the pilot, the enrollment was approximately one-half of the enrollment in Education 2341 during the main study. From Table 20 it can be observed that as the semester progressed the number of comments made on the activities tended to decrease. In the first laboratory of the main study the number of comments averaged 185 on each activity while on the eighth laboratory the average number of comments

TABLE 19

Typical Student Comments from Activity Evaluation Sheets

Positive

1. It provided enjoyment for people with different abilities.
2. It makes you concentrate and check answers.
3. Mind-expanding.
4. It gave me a greater understanding of place value.
5. It provides an opportunity for the student to do and see the activity as it really is.
6. Discovery is shown well here.
7. I loved it.
8. It is exciting for children.
9. The activity encouraged me to be very observant at all times.
10. It can be altered to accomplish a variety of objectives.

Negative

1. The rules seemed a little difficult.
2. The game had so many possibilities you didn't know what to choose.
3. Kids would get bored very quickly throwing a die 100 times.
4. Nothing but a brain-teaser.
5. I don't think graph paper is a very helpful aid to teaching multiplication.
6. Activity could be made simpler by having designs already cut.
7. I don't really like cards.
8. Time-consuming and a lot of work.
9. The slower child would feel frustrated; it is aimed at the older students with good concentration.
10. Instructions could be clearer.

TABLE 20

Frequency Table of Positive and Negative Comments from Activity Evaluation Sheets

Activity	No. of Positive Comments	No. of Negative Comments	Total No. Comments
1 : 1	120	40	160
1 : 2	142	49	191
1 : 3	144	61	205
2 : 1	73	27	100
2 : 2	67	28	95
2 : 3	66	32	98
3 : 1	56	29	85
3 : 2	80	11	91
3 : 3	118	6	124
4 : 1	61	7	68
4 : 2	48	16	64
4 : 3	75	7	82
5 : 1	61	8	69
5 : 2	38	18	56
5 : 3	43	13	56
6 : 1	51	14	65
6 : 2	34	13	47
6 : 3	56	4	60
7 : 1	33	14	47
7 : 2	27	19	46
7 : 3	33	19	52
8 : 1	42	2	44
8 : 2	27	14	41
8 : 3	54	4	58

was only 48. This held for both positive and negative comments. A possible explanation for this occurrence is that at the beginning of the semester students found it novel to be asked to evaluate the activities and thus pursued the act with enthusiasm. As the semester passed this novelty effect decreased and the number of comments declined.

Evaluative Questionnaire

Whereas the activity evaluation sheets were used to evaluate the individual activities at the time the students participated in them, the Evaluative Questionnaire was used to give an overall evaluation of the laboratories by the students after all laboratories had been completed. There were 14 multiple choice statements on the Evaluative Questionnaire for which the students had to select the reaction that most closely described their feelings on some aspect of the laboratories. A list of the questions and the number of responses for each reaction for each question on the Evaluative Questionnaire pertaining to the mathematics laboratories can be found in Appendix C.

Questions 1, 4, and 5 received responses from the students participating in the main study similar to the responses made from the students in the pilot study. That is, the students perceived the structure, organization, and staffing of the mathematics laboratories as being adequate.

Question 3 had been changed from the pilot study due to the increase from one to two hours in time available for each laboratory. From the students' responses in the main study 74 percent felt that two hours was sufficient time to complete the three activities.

Responses to questions 6, 7, 8, 11, and 12 indicated that the majority of the students perceived the mathematics laboratories as an asset towards teaching mathematics. When comparing the response from the pilot study to the responses of the main study, the main study had a higher percentage of responses in the more positive reactions. This indicated that students in the main study perceived the laboratories as a greater asset towards teaching mathematics than their counterparts in the pilot study. This finding is discussed further in Chapter VI. The high number of "b" ratings on question 2 indicated that students saw the laboratories as useful in helping them select activities for their in-school sessions; however, the students may have missed the more general usefulness of this information--to teach mathematical concepts using materials. The students' reactions may have been due to the obvious practicability of applying these laboratories to their in-school sessions. On the other hand, teaching mathematics using materials was an indirect generalization from the laboratories that students may not have been aware of at that time.

On question 13 in the pilot study students were divided in deciding if the amount of work required in Education 2341 was worth the effort. This differs for the main study where the majority of students did not think the course was too much work. In the main study, responses for question 9 also differed from those of the pilot study. The majority of the students indicated that the objectives were clearly outlined from the beginning of the course. The overall rating of the course for the main study was also higher than the rating in the pilot stage. There was a higher percentage of students in the main study who felt the mathematics laboratories were very good and even outstanding.

Interview Questions

The interviews, like the Evaluative Questionnaire, were used to evaluate the mathematics laboratories in their entirety after Education 2341 students had completed the course. A list of the interview questions pertaining to the mathematics laboratories can be found in Appendix D. Other questions asked the students during the interviews could be used to evaluate other aspects of the course. These questions, though not utilized in this study, can be found in Appendix F.

Students who were interviewed noted the following as the major items contributing to their knowledge from the mathematics laboratories.

1. Learned how to modify new activities (16 times)
2. Learned about materials (14 times)
3. Obtained new ideas (9 times)
4. Learned new mathematics (4 times)

Thirty-two of the 36 students interviewed stated that the mathematics in the laboratories was not too difficult for them. The most frequent response to the question "Why should the activities not be used directly in the school session" was that having to modify the activity provides practice for the time they will be teaching in the schools. Seven students felt the laboratories should be examples of activities that could be used directly in the in-school sessions. The remaining 29 students did not agree.

Twenty-one of the students stated that the questions at the end of the activities made you think. Other responses to this question were:

1. I would learn just as much without questions (6 times).

2. The questions should be more varied and the wording should be more specific (5 times).

3. The questions gave you practice in adapting activities (5 times).

4. Groups rushed through the questions to get out of the laboratory (4 times).

5. Questions were ambiguous (4 times).

As to decreasing the number of laboratories, 31 of the 36 students stated "No"; one laboratory per week was acceptable.

Education 2341 students responded that the mathematics laboratories helped them with respect to the in-school sessions in the following ways. The students:

1. Obtained new ideas (15 times).
2. Learned about materials (15 times).
3. Didn't use any activity from the laboratory in the in-school sessions (6 times).
4. Learned new activities (4 times).

Student reactions to the mathematics laboratories with respect to the lectures were as follows:

1. When the professor referred to a material in class, I knew what he was talking about (7 times).
2. There was no relationship between the mathematics laboratories and the lectures (7 times).
3. The mathematics laboratories and lectures ran parallel; what was talked about in the lecture would have been done in an activity in the laboratory (4 times).
4. I can't remember (4 times).

Summary

In this chapter data were presented regarding the effectiveness of the laboratories. Each of the first five questions of the study was answered several times, once for each relevant activity question. In the next chapter, the study questions are discussed further. The remaining four questions, considering the change of students' attitudes between the first and last lectures in Education 2341, have been answered in this chapter and are discussed in the following chapter. Finally, data obtained on the activity evaluation sheets, Evaluative Questionnaire, interview questions used to study the laboratory effectiveness and observations made by the laboratory assistants, provided the material on which to base suggestions for modifying the laboratories. The implications of the data collected are discussed in the next chapter.

CHAPTER VI

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

A summary of the study, conclusions that were drawn from the analysis of the data, and recommendations for the course Education 2341 and for further investigations are included in this chapter.

Summary

In the past several decades, many educators became aware of the benefit of using manipulative aids to assist children in learning mathematics. Though research supports the above practice, a recent national survey (Fey, 1979) found the principal instructional style was still teacher explanation and questioning. Recommendations to alleviate this problem suggested that pre-service elementary school teachers should be taught by an activity approach using manipulative materials. Research indicated that this was a plausible method of instructing mathematics to elementary school teachers, but the results of research reviewed on the effects of the teaching styles of elementary school teachers after being taught by an activity approach was inconclusive.

The purposes of this study were to develop mathematics laboratories for the mathematics education methods course, Education 2341, and to evaluate the laboratories and their effectiveness in aiding students to adapt, design, and extend mathematics activities for use in the elementary school classroom. Any changes in student attitude towards mathematics and teaching mathematics as a result of completing Education 2341 were also determined.

In order to fulfill the above purposes the following questions were considered:

1. Will a student in Education 2341 be able to suggest a follow-up activity, given the information that an elementary school student had exhibited difficulty with a previous activity on the same topic?

2. Will a student in Education 2341 be able to suggest how a given activity might be altered to accomplish different objectives?

3. Will a student in Education 2341 be able to suggest an alternate activity to accomplish the same objective as a given activity?

4. Will a student in Education 2341 be able to suggest an extension activity for an elementary school student, given the information that the student had exhibited no difficulty with a previous activity on the same topic?

5. Will a student in Education 2341 be able to adapt an activity in mathematics designed for an university

student for use in an elementary school classroom?

6. Will a student's attitude toward mathematics change between the first and last weeks of enrollment in Education 2341?

7. Will a student's attitude toward the existence and use of logical structure in mathematics change between the first and last weeks of enrollment in Education 2341?

8. Will a student's attitude toward individualizing mathematics instruction and the use of guided discovery techniques in mathematics change between the first and last weeks of enrollment in Education 2341?

9. Will a student's attitude toward teaching mathematics change between the first and last weeks of enrollment in Education 2341?

The sample for the initial development of the mathematics laboratories was the prospective elementary school teachers enrolled in Education 2341 during the fall of 1979. There were two classes, one containing 16 students and the other 22. The sample used in the main portion of this study was the prospective elementary teachers completing Education 2341 in the winter of 1980. There were four classes, the first contained 20 students, the second 14, the third 20, and the fourth 16.

Eight mathematics laboratories were developed, each having a central mathematical theme. Each laboratory consisted of three activities, each of which involved a different aspect of the theme. These laboratories were incorporated into Education 2341 which previously had consisted of theoretical lectures and practical sessions where the student teachers were involved with two or three elementary school children in mathematics.

Questions at the end of each activity in the mathematics laboratories were employed to answer the first five study questions stated above. The remaining four study questions were answered with the aid of the standardized Connelly Taxonomized Attitude Scale. A continuing evaluation and development of the mathematics laboratories was conducted using the following instruments:

1. Activity evaluation sheets provided a fast method of evaluating the activities and gave students an opportunity to comment on the activities.

2. An Evaluative Questionnaire provided student reactions to the set of mathematics laboratories.

3. Interviews gave an open-ended method of obtaining student reactions and suggestions for improvements on the mathematics laboratories.

4. Observations made by the laboratory assistants provided a subjective conformation to the results found from the three instruments listed above.

Each study question had several activity questions related to it. Each activity question was considered to be successfully answered if less than 20 percent of the students obtained a score under three.

A dependent t-test for means was performed on the pretest and posttest attitude scores to determine if significant changes in attitudes occurred.

Conclusions and Discussion

Effectiveness of Laboratories

The effectiveness of the laboratories was determined by examining students' responses on the activity questions. Each activity question was considered to be successfully answered if less than 20 percent of the students obtained a score under three on a five-point scale. The evaluations of these activity questions provided the answers to the first five study questions.

The student responses to each activity question provided a partial answer to one of the five study questions. In study question one, two of the four activity questions relating to this study question met the above criterion. For the remaining two questions less than 35 percent of the students obtained a score under three.

Difficulties that the students encountered with this type of question may have been a result of being in the position of student teacher rather than any inadequacy on their part. For example, lessons which student teachers were usually required to prepare were for a single session and, unlike regular classroom teachers, they were not required to supply follow-up activities. If any follow-up activities were required it was to continue with the concept being taught. Because of their inexperience, it was often difficult for a student teacher to know the kinds of problems children have with the concepts being taught.

The collective results of the four activity questions indicated that students can successfully develop follow-up activities, given the information that elementary school children had exhibited difficulty with a previous activity on the same topic.

In study question two, six of the nine activity questions relating to this study question met the accepted criterion. The other three questions had less than 50 percent of the students obtaining a score under three.

When the student responses of the nine activity questions were observed as a unit, study question two was answered in the affirmative. Often verbal explanations on how the laboratories in which the students participated could be adapted for other activities were described by the laboratory assistants. This provided the students with examples and ideas on how they might have made other adaptions of the activities in the laboratory and with their own teaching sessions. Also, the large number of times that this type of question was asked gave students more opportunity to perfect this skill.

On study question three, 12 of the 18 activity questions were answered successfully. The remaining six activity questions had less than 45 percent of the students obtaining a score under three. With those results study question three was also answered affirmatively. Often in the laboratory sessions, suggestions were given to students on alternate methods of teaching the same concept. These examples provided the stimulus for discussion between members of the group to create even more alternate ways to teach the concept. The large number of times that this question was asked was also an asset.

For study question four, will a student in Education 2341 be able to suggest an extension activity for an elementary school student, given the information that the student had exhibited no difficulty with a previous activity on the same topic, neither of the two activity questions were successfully answered by the required number of students. For one of the activity questions, approximately one-quarter of the students obtained a score of under three and for the other activity question more than one-half of the students obtained a score less than three. This study question was similar to study question one in that it required the student to suggest follow-up activities. Suggesting an activity to follow an already established activity appeared to be more difficult than suggesting an activity without regard for what is to follow. Also, the few times this question was asked was a limiting factor. With those results, the answer to study question four was negative.

Six of the 11 activity questions for study question five were successfully answered. Three of the remaining five activity questions had less than 35 percent of the students receiving a score less than three. The remaining two activity questions had over 50 percent receive under three. Overall these results were inconclusive. Further research is necessary to determine if these laboratories could effectively aid students to adapt an activity in mathematics designed for an university student for use in an elementary school classroom. The problem that students encountered was their ability to present concepts on a level that was understandable to a child. For example, one student's adaption of the calculator activity on multiplying and dividing by powers of 10 included decimal numbers when it would have been advisable to start with only whole numbers for elementary school children.

The effectiveness of the laboratories was considered to be satisfactory, given that four of the five study questions were answered in the affirmative. That is, the students, after completion of Education 2341, were able to design or modify activities according to given specifications.

Attitudes

Most Education 2341 students' attitudes were positive toward the existence and use of logical structure in mathematics but it did not change significantly between the first and last weeks of enrollment in Education 2341. That is, hypothesis 1 was not rejected. To understand, or even to be aware of a logical structure in mathematics, one needs a background in mathematics. Students in Education 2341 are required to have completed two university mathematics courses. The results of the attitude pretest indicated that after completing these courses most students had an appreciation of the logical structure in mathematics. The results of posttest indicated that this appreciation was maintained after the completion of Education 2341.

Education 2341 students' attitudes toward mathematics significantly increased between the first and last weeks of enrollment in Education 2341. That is, hypothesis 2 was rejected. Education 2341 was designed to encourage students to have a more favorable attitude toward mathematics. By the results of the attitude scale this goal was met. Students were able to partake in the practical aspect of teaching as well as learning through activities in a small group setting. This combination with direct guidance in lectures was successful in positively changing students' attitudes toward mathematics.

Education 2341 students' attitudes toward individualizing mathematics instruction and the use of guided discovery techniques in mathematics increased significantly between the first and last weeks of enrollment in Education 2341. That is, hypothesis 3 was rejected. Two possible factors contributed to students obtaining a more favorable attitude toward individualizing mathematics instruction and the use of guided discovery techniques in mathematics. The first was lectures in Education 2341 on diagnostic testing and discovery learning. The second was having each Education 2341 student instruct one to three elementary students. The Education 2341 students in the practical sessions became aware through immediate feedback from the elementary students that people learn at different rates and each has his own problem to overcome in order to learn. This "on-the-spot" training was effective in facilitating a positive change in the Education 2341 student attitudes.

Education 2341 students' attitudes toward the teaching of mathematics increased significantly between the first and last weeks of enrollment in Education 2341. That is, hypothesis 4 was rejected. Another goal of Education 2341 was to encourage students to obtain a favorable attitude toward the teaching of mathematics. This was accomplished by giving the students an opportunity to have practical teaching sessions with elementary children. These sessions were observed by the professor and a time was alotted for the discussion of problems encountered in these sessions. These stimulating, in-school sessions with professional support gave the Education 2341 student the confidence that is needed to teach mathematics.

The students' attitudes toward the four aspects of mathematics education was positive at the conclusion of Education 2341. In three of the four areas the students' attitudes had increased significantly. On the fourth aspect, the existence and use of logical structure in mathematics, the students when entering Education 2341 had already obtained a positive attitude. Thus, the aim, to give students a more favorable attitude toward mathematics and the teaching of mathematics, was successfully accomplished.

The Development of the Mathematics Laboratories

The ongoing development of the mathematics laboratories was largely dependent on the ratings and reactions of the students on the activity evaluation sheets, Evaluative Questionnaire, and interviews on the mathematics laboratories. Referring to the evaluation model used in this study and described in Chapter IV, after the data had been collected, analyzed, and interpreted the necessary changes in the laboratories were made. The revised laboratories were then re-evaluated to produce more effective laboratories.

The high average ratings on the activity evaluation sheets indicated a favorable reaction to the mathematics laboratory. Also, the increase in average responses from the piloting of the laboratories to the main study indicated that the changes made to the laboratory between these two administrations resulted in improvements to the laboratories from the Education 2341 students' point of view.

From the Evaluative Questionnaire both groups of students, from the pilot and the main studies, perceived the structure, organization, and staffing of the mathematics laboratories as being adequate. Seventy-five percent of the students in the main study agreed that two hours per week was sufficient time to complete the laboratories. This suggests that the change from one hour to two hours for each laboratory was necessary since students in the pilot felt that one hour was insufficient to complete the three activities in each laboratory. In both the pilot and the main study the majority of the students perceived the mathematics laboratories as an asset toward teaching mathematics but the main study had a higher percentage of responses in the more positive reactions. One reasonable explanation for this change is that the laboratories had been improved for the main study and thus were more effective in aiding students to design and modify the activities given in the laboratories and to teach their in-school sessions.

Another difference observed between the pilot and main study was the student's perception of the amount of work required for the course. Most students in the main study indicated that they did not feel that the amount of work required was over-demanding.

There are two explanations that can account for this result. First, the students in the pilot study had not known they would be doing laboratories in Education 2341 and perceived the laboratories as extra work while the students in the main study knew before the course that the laboratories were a requirement. Secondly, in the pilot stage the questions at the end of each activity could be submitted at the end of the semester. This meant that students were required at some time to complete this assignment. In the main study, the questions were completed after each activity and immediately passed to the laboratory assistant. In this case the questions were not perceived as an extra assignment.

More students in the main study stated that the objectives were clearly outlined from the beginning of the course. The overall rating of the course was higher for the main study than for the pilot stage.

Comparing results from the Evaluative Questionnaire for the pilot and main study indicated that the revisions made to the mathematics laboratories were successful. The responses of the students who had been interviewed also supported this statement. The responses showed that students found the laboratories to contribute to their knowledge of teaching mathematics as well as requiring them to think. From the Evaluative Questionnaire and interview questions students indicated that they felt the mathematics laboratories were a worthwhile experience.

The higher ratings obtained on the instruments for the mathematics laboratories in the main study as compared to the pilot study provided strong support for having a continuing evaluation in the development of the mathematics laboratories. The success of the mathematics laboratories in the main study was due to the changes made to the original laboratories because of the results of the evaluations.

Implications and Recommendations

From this study it is recommended that the mathematics laboratories should be continued in the course, Education 2341. As indicated in the evaluation model the laboratories should undergo ongoing revision to correct weaknesses. More specifically, greater emphasis should be placed on suggesting extension activities for an elementary school student given the information that the student had exhibited no difficulty with a previous activity on the same topic.

Activities that will aid in improving the students' attitudes toward the existence and use of logical structure in mathematics could also be included in the laboratories. In conjunction with the evaluation model of this study, the findings from the main study must be considered with respect to the objectives of the Education 2341 course. If it is considered necessary for students to be able to provide extension activities or to have a more positive attitude toward the existence and use of logical structure in mathematics, then these topics must become objectives of the course. Appropriate changes would then have to be made in Education 2341 to have students meet these objectives.

Since the laboratories were considered to be successful, it should not be necessary in the future to undergo such extensive evaluations as was done for this study. It is recommended that the student evaluation sheets and the questions at the end of each activity be continued. The student evaluation sheets require minimal time to be completed by the student and it is a fast and effective method to keep a check on students' reactions to the laboratories. The use of activity questions is useful in two ways. First, they require the student to analyze and synthesize the information in each laboratory and secondly, they provide information on the students' ability to design and modify activities under given specifications.

Possible extensions of this study are:

1. The evaluation of the three components of Education 2341, that is, the lectures, in-school sessions, and the mathematics laboratories working together as a unit.

2. The evaluation of the three prerequisite courses of Education 2341, to determine if these courses are adequate to prepare a student for Education 2341.

3. The evaluation of the effect Education 2341 has on the teaching styles of elementary school teachers after they have taught for several years.

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APPENDIX A

ACTIVITIES AND SOURCES FOR THE MATHEMATICS LABORATORIES

Lab #1 Practice
Activity #1

CRAZY FRACTIONS

Objective: (1) The student should be able to name a fraction equivalent to a given one.

(2) The student should be able to rename whole numbers as fractions.

Materials: A deck of forty-nine CARDS for the following set of fractions: halves, thirds, fourths, fifths, sixths, eighths and tenths. There are also CARDS for the whole numbers one, two, three and four.

Size of Group: 2 to 4 players.

Source: Ernest Carlisle
Games and Puzzles for Elementary and Middle School Mathematics, p. 122
N.C.T.M.
Virginia
1976

Directions:

The dealer gives each player six CARDS, turning one card face up (starting the discard pile), and placing the remaining CARDS in the center of the table face down. Play starts with the player to the left of the dealer discarding a CARD of the same suit (same denomination) or of equivalent value to the top CARD of the discard pile, or a whole number CARD. 2/2, 3/3, 6/6, etc. are considered whole number CARDS. Whole number CARDS act as wild CARDS because a player may RENAME a whole number CARD to any suit he chooses. During his turn, a player may discard or DRAW a CARD from the deck. A player does not both draw and discard unless the drawn CARD is discardable. The object of the game is for a player to get rid of his CARDS.

Twenty-five points are awarded to the player who gets rid of all his CARDS plus the total of the CARDS remaining in all other hands--one point for each fraction less than 1/2; two points for each fraction equal to or greater than 1/2; and three points for each whole number. Before play begins you should agree on the number of points (usually 100) that will constitute a game.

Lab #1 Practice
Activity #1

Questions about the Activity:

1. If during a game of "Crazy Fractions" you observed that a child was unable to identify an equivalent fraction, what would you suggest as a follow-up exercise to solve this problem? (Hint: The section on equivalent fractions in the grade 4 book of the I.S.M. series may be able to help you.)

2. Can you think of another activity which accomplishes the same objectives as "Crazy Fractions" (an example is in the 4th grade book of the I.S.M. series on p. 325)?

Lab #1 Practice
Activity #2

CONTIG

Objective: (1) The student should be able to recall addition, subtraction, multiplication and division facts.

(2) Given three numbers the student should be able to calculate a variety of answers depending on the combination of operations selected.

Materials: Playing board
Markers
Three dice
Score pad

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	44	45	48	50	54	55
60	64	66	72	75	80	90	96
100	108	120	125	144	150	180	216

Size of Group: 2 to 5 players

Source: Arithmetic Teacher
Vol. 19, 388-390
May 1972

Lab #1 Practice
Activity #2

Directions:

To begin play each player in turn rolls all three dice and determines the sum of the three numbers showing. The player with the smallest sum begins play. Play then progresses from left to right.

The first player rolls the dice. He must use one or two operations on the three numbers shown on the dice. He is then allowed to cover the resulting number on the board with a marker. When he has finished his turn, he passes the dice to the player on his right. He may not cover a number which was previously covered. Only one number may be covered each turn.

To score in CONTIG you must cover a number on the board which is adjacent vertically, horizontally or diagonally to another covered number. One point is scored for each adjacent covered number.

When a player rolls the dice and is unable to produce a number which has not already been covered he must pass the dice to the next player. If he incorrectly passes the dice, believing he has no play when, in fact, he does have a play, or if he makes an error in calculations, any of the players may call out the mistake. The first player to call attention to the error may place his marker on the proper uncovered number and receive the appropriate points. This does NOT affect the turn of the player citing the error.

A cumulative score is kept for each player. A player is eliminated from further play in a game when he fails, in three successive turns, to produce a number that can be covered. When all players have experienced three successive failures to produce a coverable number, the game ends. The player with the highest cumulative score wins.

SAMPLE PLAY (four players)

	Roll of dice	Number covered	Points scored
Player 1	2, 3, 4	9 (2 + 3 + 4 = 9)	0
Player 2	2, 4, 5	10 ([4 ÷ 2] x 5 = 10)	1
Player 3	1, 5, 6	11 ([1 x 5] + 6 = 11)	1
Player 4	3, 4, 6	2 (4 − [6 ÷ 3] = 2)	3

Lab #1 Practice
Activity #2

Questions about the Activity:

1. Last semester a student changed the whole numbers on the dice to fractions and limited the operations to multiplication and division. Can you suggest another way that this game can be modified to aid in teaching different objectives?

2. In the Math Laboratory there is a board game called "Roll the Product"; it can be used to practice some of the objectives stated for "Contig". Can you suggest another activity which could be used by a child to practice "Contig's" objectives?

Lab #1 Practice
Activity #3

RACE GAME: MARATHON RUN

Objective: The student should be able to recall the multiplication facts in base 5.

Materials:
- Gameboard
- Multiplication fact CARDS
- Multiplication fact sheet
- Hazard CARDS
- Markers
- Die

Size of Group: 2 to 4 players

<u>Directions</u>:

Each player throws the die. The player with the <u>lowest</u> score goes first and the play proceeds left to right.

The first player throws the die and draws a card. The player then states the answer to the problem on the card.

The answer is checked by using the multiplication fact sheet.

If the player is correct he gets to move the number of spaces that is on the die.

If a player lands on a hazard space, he draws a hazard CARD and follows the directions on the CARD.

The player who reaches the finsih line first is the winner.

Lab #1 Practice
Activity #3

Questions about the Activity:

1. In the grade 6 book of the I.S.M. series addition, subtraction, and multiplication in base 5 and 6 are taught. "Marathon Run" could be used for children who are taught these concepts. For those children who only study base 10, how can this game be modified for their use?

2. "Marathon Run" is a game used to practice multiplication facts; what other operations or concepts could be practiced using this game? How do you have to change the game to achieve these new objectives? (BE CREATIVE).

Lab #2 Place Value
Activity #1

GROUPING AND PLACE VALUE THROUGH GAMES

Objective: The student should be able to state that the Hindu-Arabic numeration system is a place value system which has a $b^n \ldots b^3$ b^2 b^1 b^0 configuration.

Materials: One set of multibase blocks base 5
One set of poker chips
Four abaci
Two dice
Four chip tills

Size of Group: 4 players

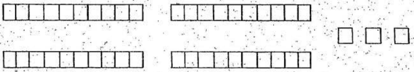
Note: This is a developmental activity.

Source: A.F. LeBlanc, D.R. Kerr Jr., M. Thompson
Numeration
Addison-Wesley Publishing Company
Don Mills
1976

Lab #2 Place Value
Activity #1

This activity is comprised of three games, one with multibase blocks, one with colored chips, and one with an abacus. These games develop three important concepts in numeration: grouping, trading and place value.

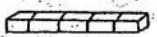
In our numeration system, grouping is done in sets of ten. Grouping is an efficient method to keep track of large numbers.

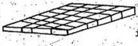


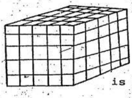
represents 43.

Game 1 uses both concepts grouping and trading.

 is a unit.

 is a long.

 is a flat.

 is a block.

Directions:

Game 1 - Tradeup to a Black, Base 5

The object of the game is to win a block. Each player rolls a die. The player with the smallest score goes first. The play proceeds from left to right. Each player rolls a die and takes as many units as the die indicates. Suppose on the first roll, player A rolls a 2 and takes two units. On the second roll, player A rolls a 4. Player A now takes four more units and has a total of six units. He now trades five units for one long. The game continues, each player trading units for longs, longs for flats, and finally five flats for a block. The first player to get a block wins.

a) When you have won a long, how many units have you won? ______________

Could you actually count them on the long? ______

b) When you have won a flat, how many units have you won? ______________

Could you actually count them on the flat? ______

c) It would be impossible to count all the units in a block, but you could trade a block for ______ flats. How many units could you count in the set of flats equivalent to a block? ______________

Lab #2 Place Value
Activity #1

Game 2 is a transitional step between the grouping concept and place value. Color chips are used to represent different numbers of objects. Thus a single chip represents a number of objects rather than a single object. The choice of color for the chips is completely arbitrary. If base 10 is used and if 1 yellow represents 1 object, and 1 blue represents 10 yellow, and 1 green represents 10 blue, and 1 red represents 10 green, then 34 would be represenfed by:

B B B
Y Y Y Y

Game 2 - <u>Trading for Yellow</u>

In this game, we will use 1 for 3 trade, i.e., Base 3.

1 yellow = 1 object
1 blue = 3 yellow
1 green = 3 blue
1 red = 3 green

Trade the chips in each of these tills for YELLOW chips.

Record the answer:

<u>Example</u>

R	G	B	Y
		•	•

=

R	G	B	Y
			••••

= 4Y

(a)

R	G	B	Y
		•	••

= ____Y

(b)

R	G	B	Y
	•	•	•

= ____Y

R	G	B	Y
	••	•	••

= ____Y

R	G	B	Y
•			•

= ____Y

Suppose you have two blue chips and four yellow chips. How many units does this represent in base 6? ____

Lab #2 Place Value
Activity #1

Game 3 focuses on place value. In place value, the position that a representation holds determines the value. A 4 may represent 4 or 40 or 400, depending on its position (example - 444). In this game an abacus is used. A bead on the ones wire represents a single object; a bead on the tens wire represents 10 and a bead on the hundreds wire represents 10 tens. The number of objects a bead represents depends on both the place value of the wire and the base being used.

Game 3 - <u>Trading on an Abacus</u>

This game is similar to game 1. Each player rolls the die and records the number on the abacus. The game is played in base seven.

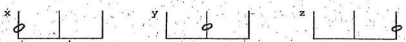
For base seven the abacus place value is:

7 x 7's	7's	1's
7^2	7^1	7^0

Suppose player A rolls 6 on his first turn. He flips 6 ones on the first wire of his abacus. On his second turn player A rolls 5. He flips 5 ones on the first wire giving him 11 ones on his first wire. He can now trade seven of these ones to 1 seven. The abacus now has 1 seven chip and 4 ones chip.

The winner is the person who trades 7 seven chips to receive the first 49 chips.

Which abacus represents the greatest value x, y, or z? ____
Assume that the base is 10; how much larger (smaller) is the number represented in x than in y? ____ y than in z? ____ x than in z? ____



Assume that the base is 8; how much larger (smaller) is the number represented in x than in y? ____ y than in z? ____ x than in z? ____

Lab #2 Place Value
Activity #1

Questions about the Activity:

1. How can these games be adapted for use in an elementary classroom?

2. How could you vary one of the games to accomplish different objectives?

Example: New objective.

Using a 1 for 10 trade the student should be able to trade a given number of chips so that he has a minimum.

In this game the child receives 37 yellow chips. He trades 10 yellow chips for 1 blue chip three times, leaving him with

R	G	B	Y
		•••	•••••••

Lab #2 Place Value
Activity #2

GROUPING AND PLACE VALUE IN BASES OTHER THAN 10

Objective: Given a numeral in base 10, the student should be able to construct the numeral in any base using a varied selection of manipulative aids.

Materials: Popsicle sticks
Rubber bands
Unifix cubes
Place value charts
Place value bins

Size of Group: 2 to 4

Source: Numeration
John F. LeBlanc
Addison Wesley Publishing Company
Don Mills
1976
Activity 5 & 11

Lab #2 Place Value
Activity #2

Directions:

Grouping aids are materials which have been or can be physically bundled together to represent a number. Place value aids indicate a number by the position of the material object.

a) Using each manipulative aid given, construct each of the following numerals in base 10.

i) 19_{10} ii) 30_{10}

b) Remember base 10 receives its name because it is bundled in groups of 10. Thus base 5 would be bundled in groups of 5; and base 7 would be bundled in groups of 7, etc.

Example: 15_{10} would be constructed in base 6 using Dienes blocks as follows:

The numeral would be written as 23_6.

Using the same manipulative aids as in (a) show the following base ten numerals in base 6. Also write the numeral in base 6.

i) 19_{10} ii) 30_{10}

DRAW a picture of popsicle sticks bundled together to represent 19_{10} in base 3. Write the numeral in base 3.

Lab #2 Place Value
Activity #2

Questions about the Activity:

1. This activity is designed to aid 2341 students in understanding place value. How could it be used in an elementary classroom?

2. What other materials could you use to teach place value? How would you use these materials in an activity teaching place value?

Lab #2 Place Value
Activity #3

PLACE VALUE GRIDS AND PATTERNS

Objective: The student should be able to state that "123.45" is a symbol that represents a set consisting of one hundred, two tens, three ones, four tenths and five hundredths.

Materials:
Grids and patterns
Crayons (three colors)
Lima beans
20 paper bags
10 rubber bands
Graph paper

Size of Group: 2 to 4

Source: Experiences in Mathematical Ideas
Unit 1, Experience 3
N.C.T.M.
1970

Lab #2 Place Value
Activity #3

Directions:

This activity has four stations.

STATION A: SINK THE SUB

Place an X in each of ten cells in your unmarked grid. Then turn over the grid that shows the path of a submarine and see if you made any hits. If you did, list the numbers that show the values of the cells where the hits were made.

__________ __________ __________

__________ __________ __________

__________ __________ __________

__________ __________ __________

Your grid is the last page of this booklet. If you ran your finger down the grid on a vertical path, how much would the number increase (decrease) between cells?

STATION B: BREAK THE CODE

Below each numeral write the letter that has that value in the grids at this station. When you are through, you will discover a secret message.

3.46 201 0.04 8.3

565.9 77.77

Lab #2 Place Value
Activity #3

0									9
90									99

Lab #2 Place Value
Activity #3

STATION C: HOW MANY BEANS?

Find the smallest number of bags of beans needed to represent each number below. Following the example given, write the number of beans in each bag and the number of bags.

Number	Bags	Number of bags
167	100, 40, 20, 4, 2, 1	6
132		
483		
461		
400		
306		
1,301		

STATION D: HOW MANY STARS?

Fill in after the number of each pattern the number of STARS it contains.

Pattern	Number of STARS
1	
2	
3	
4	
5	
6	

If the units digit in a numeral is increased by one, by how much does the number increase? ______

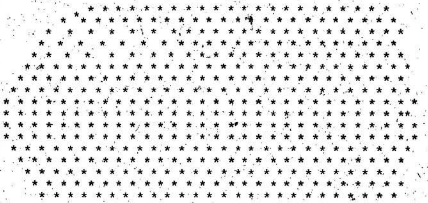
If the tens digit in a numeral is increased by one, how much does the number increase? ______

If two adjacent digits are switched, will the number represented get bigger or smaller? ______

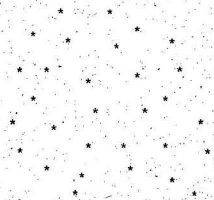
What is the largest number expressed by a three-digit numeral? ______

Lab #2 Place Value
Activity #3

A



B



Lab #2 Place Value
Activity #3

C

* * * * *
* * * * * *
* * * * * *
* * * * *
* * * *
* * *
* *

D

* * * * * * * * * *
* * * * * * * * *
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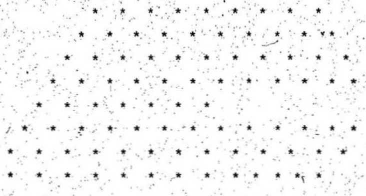
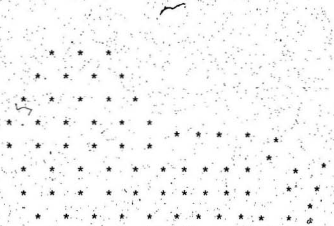
Lab #2 Place Value
Activity #3

E



F



Lab #2 Place Value
Activity #3

Questions about the Activity:

1. This activity can be used to accomplish different objectives. For example: Given decimals the student should be able to write fractions with denominators of either hundreds or tens.

   In "Break the Code" the child could be asked to represent each decimal as a fraction. The code would then be hidden in fraction notation. Can the activity be used for any other objectives?

2. On page 224 of the grade 6 book of the I.S.M. series, there is an example of another activity with the same objectives as "Break the Code". Can you suggest any other activities?

Lab #3 Development
Activity #1

MULTIPLICATION

Objective: (1) Given a set of number blocks, the student should be able to combine and trade them to obtain the smallest number of blocks.

(2) Using graph paper, a student should be able to multiply 2-digit by 2-digit numbers.

Materials: A place value playing board
A set of number blocks

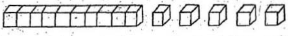
Size of Group: 2 to 4

Source: Mark A. Spibell, Carole E. Greenes & Robert E. Willuitt
Multibase Activities
Base 4
Creative Publications
Palo Alto
1974

Lab #3 Development
Activity #1

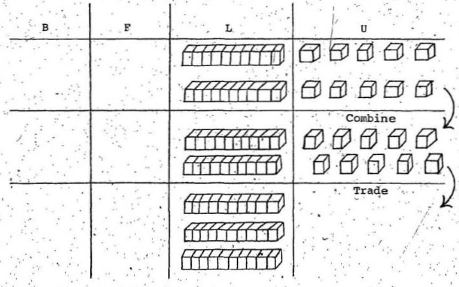
Directions:

Make two piles of wood, each like this:



Put the two piles together on your playing board. Combine the two piles and trade if possible.

Example:

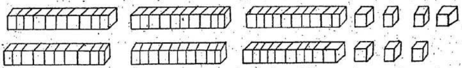


Is your answer 3 longs?

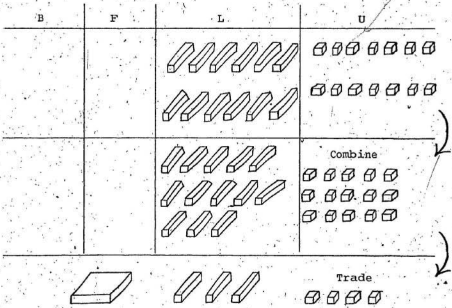
We can express this problem using numerals like this, $\begin{array}{r} 15 \\ +15 \\ \hline 30 \end{array}$ or we can call it two piles of 15, and write $\begin{array}{r} 15 \\ \times 2 \\ \hline 30 \end{array}$

Lab #3 Development
Activity #1

Make two piles of wood, each like this:



Combine the two piles on your playing board and trade if possible.



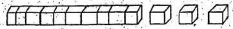
Is your answer 1 flat, 3 long, 4 units?

We can express this problem using numerals like this, 67 +67 = 134 or we can call it two piles of 67 and write 67 x2 = 134

$$\begin{array}{r} 67 \\ +67 \\ \hline 134 \end{array} \qquad \begin{array}{r} 67 \\ \times 2 \\ \hline 134 \end{array}$$

Lab #3 Development
Activity #1

Make three piles of wood, each like this:



Combine the three piles on your playing board and trade if possible.

Is your answer 3 long and 9 units?

We express this problem using numerals like this, $\begin{array}{r}13\\ \underline{x3}\\ 39\end{array}$ or like this, $\begin{array}{r}13\\ 13\\ \underline{+13}\\ 39\end{array}$

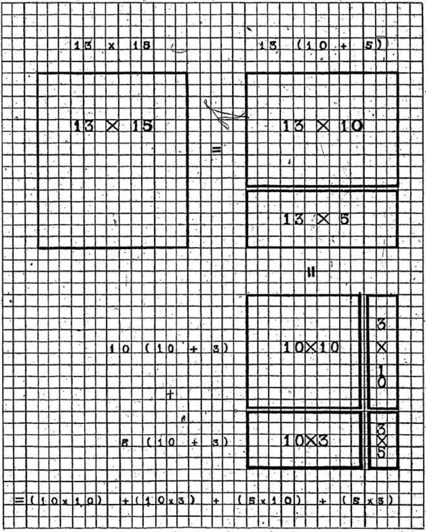
13 is easily represented with blocks, but if one wanted to model $\begin{array}{r}13\\ \underline{x3}\end{array}$ … model $\begin{array}{r}13\\ \underline{x15}\end{array}$ one would need 15 piles of

which would require many blocks and much space. Another way to illustrate 13 x 15 is to use the distributive property, namely

$$\begin{aligned}13 \times 15 &= 13 \times (10 + 5)\\ &= (13 \times 10) + (13 \times 5)\\ &= [(10 + 3) \times 10] + [(10 + 3) \times 5]\\ &= [(10 \times 10) + (3 \times 10)] + [(10 \times 5) + (3 \times 5)]\\ &= [10 \times 10) + (3 \times 10) + (10 \times 5) + (3 \times 5)\end{aligned}$$

Using a graph paper and arrays is a convenient method of modelling multiplication using the distributive property. (See next page)

Lab #3 Development
Activity #1



Lab #3 Development
Activity #1

Using the enclosed graph paper, solve

$$24 \times 17 = 24 \times (10 + 7)$$
$$= (24 \times 10) + (24 \times 7)$$
$$= [(20 + 4) \times 10] + [(20 + 4) \times 7]$$
$$= [(20 \times 10) + (4 \times 10)] + [(20 \times 7) + (4 \times 7)]$$
$$= (20 \times 10) + (4 \times 10) + (20 \times 7) + (4 \times 7)$$

Next try 17 x 19, first using graph paper and then using the distributive property.

Lab #3 Development
Activity #1

Lab #3 Development
Activity #1

Questions about the Activity:

1. What other activities can be used to teach multiplication? (Page 162 of Grade 3 book of the I.S.M. series may give you some new ideas). Can you think of any others?)

2. This activity can be modified to teach division. Give an outline of how the division algorithm can be developed in the elementary grades. (Remember the "multiplication" activity was designed for you, thus it includes abstract and concrete models).

Lab #3 Development
Activity #2

SPECIAL NUMBERS

Objective: Using cubes, the student should be able to construct rectangles to represent a number.

Materials: A set of cubes (100).

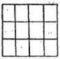
Size of Group: Two to four.

Source: L. Pereira-Mendoza
Rectangles, Trees, and Factoring
School Science and Mathematics,
Vol. 74, p. 708
December 1974

Lab #3 Development
Activity #2

Directions:

Take 12 cubes and form a rectangle. For example:

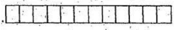


Record the number of cubes on each side of the rectangle.

Number of cubes	Side 1	Side 2
12	3	4

Can you form a different rectangle which has a different number of cubes on each side?

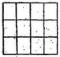
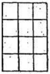
For example:



Number of cubes	Side 1	Side 2
12	3	4
12	1	12

Can you form any other different rectangles with 12 cubes? If so, how many cubes form the sides? ______

For this activity

 is the same as 

The rectangle is merely turned sideways.

Lab #3 Development
Activity #2

Follow the same procedures for 1 to 24 cubes. Remember to keep a record of all the rectangles you can.

No. of cubes	Side 1	Side 2	No. of different rectangles
1	1	1	1
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12	3	4	3
	2	6	
	1	12	
13			
14			
15			
16			
17			
18			
19			
20			
21			
22			
23			
24			

Lab #3 Development
Activity #2

Did you find that the number of different rectangles depended on the number of cubes you used?

____

Which numbers made the least number of different rectangles?

____

____

Why? ____

What concept is being developed in this activity? __________

____

Suggest a plan to complete the development of the concept of prime number.

____

Lab #3 Development
Activity #2

Questions about the Activity:

1. If a child stated that the "special" numbers were odd numbers, what follow-up activity would you give to change this error?

2. This activity can be modified so that a student would find the number of cubes needed to make a square--example 1, 4, 9, and 16 cubes all make rectangles with equal sides, i.e., square numbers. Suggest another concept that can be taught using a variation of this activity. Explain how the activity would be conducted.

Lab #3 Development
Activity #3

ADDITION

<u>Objective:</u> (1) Given a set of Cuisenaire rods, the student should be able to add two rods whose sum is equal to an orange rod.

(2) Given a set of Cuisenaire rods, the student should be able to subtract two rods if the largest given rod is orange.

<u>Materials:</u> A set of Cuisenaire rods.

<u>Size of Group:</u> Two to four.

<u>Source:</u> P. Davidson, A. Fair, G. Galton
<u>Student Activity Cards for Cuisenaire Rods.</u> Cuisenaire Company of America.
New Rochelle
1972

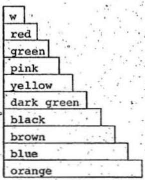
Lab #3 Development
Activity #3

Directions:

Whenever presenting new materials, one should give the learner a period of time to "play" with the materials and become familiar with them.

Part A is your "play" time.

(1) Make a staircase using one rod of each color.

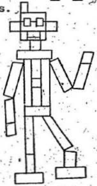


(2) Make another staircase and fit the two together so that the stairs go up one side and then down the other.

(3) Now try to fit the two staircases together in other ways.

(4) This is my dog. What does yours look like?

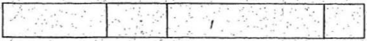
(5) This is Mr. Robot. Make him with rods.



CAN YOU MAKE MRS. ROBOT? TRY.

Lab #3 Development (cont'd.)
Activity #3

(1) This is a rod train



Using all the colored rods, make some trains.
Make a short train.
Make a long train.
Make a red train.
Make a green train.

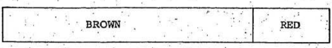
Hitch all your trains together.

Make a train of five red cars. Make a train of four purple cars. Which is longer? ________________

Lab #3 Development
Activity #3

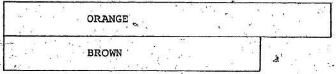
Part B. Stories about rods.

1. Put a brown rod and a red rod end to end.



Find a rod that equals brown plus red.
Brown plus red = ____________________

2. Take an orange rod. Put a brown rod beside it.



Find a rod that equals orange minus brown.
Orange minus brown = ____________________

3. Use your rods to help you complete these stories.

   1. Pink plus yellow = ____________________
   2. Blue minus pink = ____________________
   3. Blue minus yellow = ____________________
   4. Green plus red plus pink = ____________________
   5. Dark green minus green = ____________________
   6. Red plus red plus red = ____________________

4. Make up a story for your partner. Then, let him make a story for you to do.

Lab #3 Development
Activity #3

Questions about the Activity:

1. This activity can be modified to teach multiplication facts by repeated addition. Could you suggest how such an activity could be presented for an elementary class?

   OR

   Can you think of any other concepts that could be taught using this activity? If so, please explain.

2. On page 250 of the Grade 1 book of the I.S.M. series, there is an activity which could be used with concrete materials. This activity achieves one of the objectives that is in the "Addition" activity. Suggest another activity that accomplishes the "Addition" activity objectives (pennies and dimes are often used to accomplish these objectives).

Lab #4 Geometry
Activity #1

REFLECTION

Objective:

(1) Given a mira the student should be able to name the parts of a mira plus the object and image.

(2) Given a mira, object, direction and length of the object's movement the student should be able to draw the length and direction of the object's image.

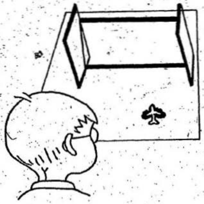
Materials: 4 discs
4 miras

Size of Group: 1 to 4

Source: Mira Math for Elementary School
Mira Math Company
Creative Publishers
Palo Alto
1973

FIRST FLIGHT

1. GROUNDED

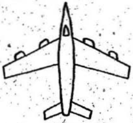


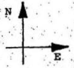
a) Look at the picture above. What do you think the boy sees in his mira?

b) To find out, place your mira in front of the airplane shown near the bottom of this page. What do you see?

c) Now slide your mira slowly up the page. Now bring it back. What happens?

d) Move the mira in other ways. What do you see?



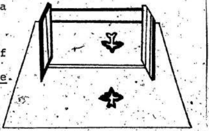


2. IMAGE AND OBJECT

a) Place your mira on page 1, as shown in this picture.

The airplane in front of the mira is called the object.

The airplane you see in the mira is called the reflection-image of the object. We usually just call it the image.



b) You can tell the image from the object by their colors.

What color is the image? What color is the object?

c) Move the mira slowly. What happens to the image? Does the object move when the mira is moved?

d) Lift the mira slowly--up and away. Watch the image. You're flying.

3. TEST PILOT

a) You have just been named test pilot of the "red demon" on page 11 Park it nose to nose with the object airplane.

b) Your first mission is:

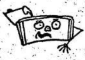
Take off backwards, stall the plane, then glide back onto the runway.

Don't crash into the other plane.

c) Your second mission is:

Take off backwards, then circle the airfield.

When your plane spins, don't worry--you're a test pilot.

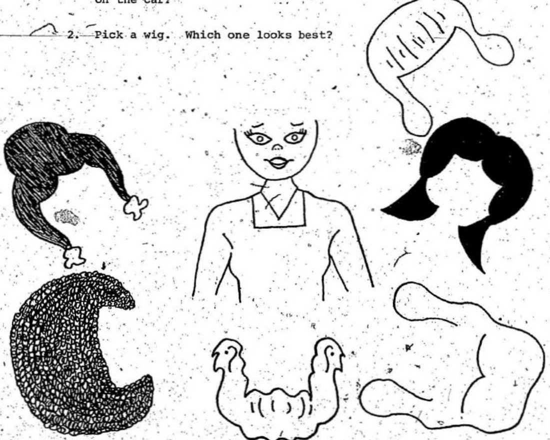


MIRA FUN

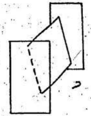
1. Use the mira to back the car into the garage. Watch the fenders.

The car is out of gas. Can you "park" the garage on the car?

2. Pick a wig. Which one looks best?

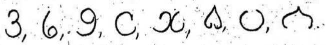


Place your mira near the figure below as shown in the picture.



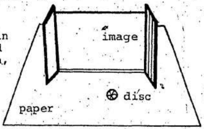
Slide the mira closer to the figure. What did you make? ______________________________

Can you make these figures?



What else can you make?

FOLLOW THE LEADER



You will need a sheet of plain paper, your mira, and a round disc (such as a checker, coin, button)

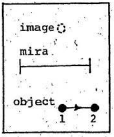
Place the disc in front of your mira, as shown.

You are now ready for Investigations 1 & 2, below.

1. a) Move the disc:
   left, right, toward the mira, away from it, in other ways.
   As you move the disc, watch the image.
   Is there a pattern in the way it moves?

   b) This time, plan where you are going to move the disc.
   Before you move it, think of how the image will move.
   Then test your answer by moving the disc.

2. In this investigation you will learn more about how the image moves.

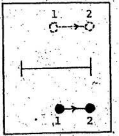


This picture is not finished. It shows a move of the object from position 1 to position 2. It does <u>not</u> show the move of the image.

Use your disc and mira to find how the image moves.

Then draw this move on the picture. Show it this way: ◌-->--◌ (1, 2)

Does your picture look like the one below?

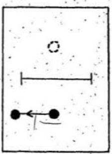


If it does, you are ready for the next page.

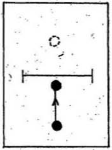
Look at each picture on the next page. An object move is shown, the corresponding image move is <u>not</u>.

Use your disc and mira to discover how the image moves. Then finish the picture.

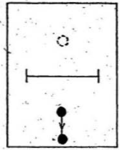
a)



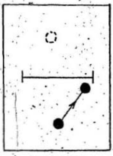
b)



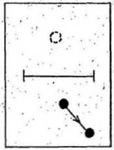
c)



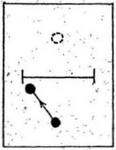
d)



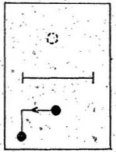
e)



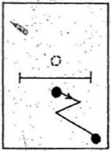
f)



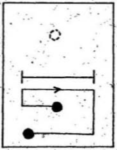
g)



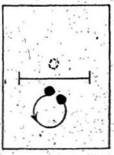
h)



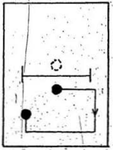
i)



j)



k)



TEST YOUR PICTURES

Use your mira to test your pictures.

Stand it on top of the mira in each picture.

Does the red image fit on the image move you have drawn?

Questions about the Activity:

1. In this activity the mira is used to aid in discovering some properties of reflection. The mira can also be used to aid in teaching symmetry. Either describe an activity using the mira to demonstrate symmetry or some other different concept.

2. There are many ways to present reflection in an activity. There is an example on page 286 of the grade 6 book of the I.S.M. series. Suggest a different activity that could be used.

Lab #4 Geometry
Activity #2

SYMMETRY

Objective: Given directions the student should be able to cut symmetrical designs.

Materials: Square pieces of paper
Scissors
Pins

Size of Group: 2 to 4

Source: Paper Folding
N.C.T.M.
Donovan A. Johnson
Washington, D.C.
1972

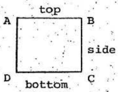
Discussion:

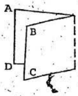
This activity provides an intuitive introduction to line and point symmetry. The relaxed situation is designed to stimulate curiosity. Emphasis is not on understanding the underlying concepts.

Directions:

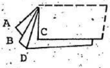
PART A: Folding Two Perpendicular Creases

Take a square piece of paper and fold one side edge into the opposite side edge.





Take your folded paper and fold the top edge down into the bottom edge.



The paper is now 1/4 the size of the initial piece. This is a necessary skill for the next three parts.

PART B: Line Symmetry

Fold two perpendicular creases, keeping the paper folded.

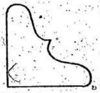
Cut any edge into a planecurve with scissors.

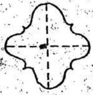
Unfold the paper.

Discuss the results with your partner. Is there anything similar about your cuttings? ____________

____

Example: TRY ONE different than this one.





PART C: Line and Point Symmetry

Fold two perpendicular creases, dividing the paper into quadrants.

Keep the paper folded.

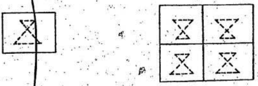
Form a design or geometric figure by pricking through the four layers of paper with a pin.

Unfold the paper. Discuss the results with your partner. How are your own designs similar and different for each quadrant? ______________________________

____

____

Example:



PART D: Symmetrical Design

Fold two perpendicular creases, dividing the paper into quadrants.

Fold once more bisecting the folded right angles.

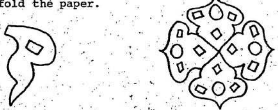
Keep the paper folded.

Trim the edge opposite the 45° angle so that all folded parts are equal.

While the paper remains folded, cut odd-shaped notches and holes. Be sure to leave parts of the edges intact.

Unfold the paper.

Ex le



What are some similarities and differences in your new des n? ____

____

____

Questions about the Activity:

1. This symmetry activity may be too complicated for elementary school children, but on Page 272 in the book of the I.S.M. series there is an example for children's use. Describe another way that the "symmetry" activity could be modified for an elementary classroom.

2. In this activity and the one mentioned in the example above, the symmetry line is given and the student must describe if the figure has two sides alike or different, i.e., symmetrical or not. On page 112 in the grade 4 book of the I.S.M. series the activity gives the figure, asks if it is symmetrical; if so, where and how many lines of symmetry does it have? Can you suggest a different way to teach symmetry?

Lab #4 Geometry
Activity #3

GEO STRIPS AND QUADRILATERALS

Objective: (1) The student should be able to state the differences and similarities among different quadrilaterals.

Materials: A set of geo strips.

Size of Group: 2 to 4

Source: Dynamic Geometry
H. Shaw D.C.P.
Invicta Plastics Limited
OADBY

Discussion:

The geo strips are in four colors - red, blue, yellow, and white. There are different lengths in each color. The strips will be referred to by letters and numbers as follows:

RED STRIPS

There are three RED strips.
The long red strip is R3.
The medium length red strip is R2.
The short red strip is R1.

R3 is __________ as long as R2.
R2 is __________ as long as R1.
R3 is __________ as long as R1.

BLUE STRIPS

There are two blue strips.
The long one is B2.
The short one is B1.

B2 is __________ as long as B1.

YELLOW STRIPS

There are two yellow strips.
The long one is Y2.
The short yellow strip is Y1.

Y2 is __________ as long as Y1.

WHITE STRIPS

There are two white strips.
The long white strip is W2.
The short one is W1.
The white strips are special lengths. W2 is longer than W1.

The strips fix together by means of brass fasteners.

Using geo strips, construct as many different quadrilaterals as you can. You should get at least six.

(Hint: To keep your construction rigid, insert a diagonal).

Lab #4 Geometry
Activity #3

Did you get seven? ______________

If you can name your newly constructed figures write them below.

A. ______________
B. ______________
C. ______________
D. ______________
E. ______________
F. ______________
G. ______________
H. ______________

Take each figure and determine if it has the following characteristics:

	Fig. A	B	C	D	E	F	G	H
1. Only 2 opposite sides parallel								
2. Opposite sides parallel								
3. Opposite sides congruent								
4. Opposite angles congruent								
5. All four sides congruent								
6. All four angles congruent								
7. All four sides and all four angles congruent								
8. Diagonals bisect each other								
9. Both diagonals are equal length								

Lab #4 Geometry
Activity #3

From the above table you can determine that rectangles have opposite sides parallel and congruent and all four angles are congruent, but it is a little more difficult to determine the converse, i.e., if all angles are congruent, is it a rectangle?

Using your constructed figures and table, try the following questions.

Correctly complete the following statements by using the <u>most descriptive</u> of the words rectangle, rhombus, or square.

1. If one angle of a rhombus has a measure of 90, the rhombus is a ___________.

2. If a parallelogram is equilateral, the parallelogram is a ______________.

3. If one angle of a parallelogram is a right angle, the parallelogram must be a ___________ but may be a ___________.

4. An equilateral and equiangular parallelogram is a ___________.

Complete each of the following with one of the words, Always, Sometimes, Never.

1. A quadrilateral is ___________ a parallelogram.
2. A square is ___________ a parallelogram.
3. A square is ___________ equilateral.
4. A rectangle is ___________ equiangular.
5. A trapezoid is ___________ a parallelogram.

Lab #4 Geometry
Activity #3

Questions about the Activity:

1. Besides describing distinguishing features of quadrilaterals, what other concept in the elementary curriculum can be clarified through the use of geo strips? Describe an activity that accomplishes this objective.

2. The "Geo Strip" activity is one approach to distinguishing geometric shapes. On page 198 of the grade 2 book of the I.S.M. series there is an example of another approach. Suggest a third approach.

Lab #5 Geometry
Activity #I

TESSELLATIONS

Objective: (1) The student should be able to tessellate a plane using a variety of polygons.

(2) The student should be able to state which regular polygons tessellate.

(3) The student should be able to state the reason why a regular polygon will or will not tessellate.

Materials: Pattern Blocks
Regular Polygons
Red Counters
Grease Pencils

Size of Group: 2 to 4

Source: J.A. LeBlanc, D.R. Kerr Jr., M. Thompson
Activities 1, 2, and 4
Addison-Wesley Publishing Company
Don Mills
1976

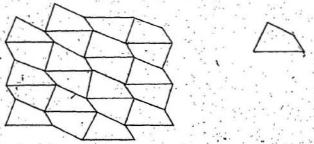
Lab #5 Geometry
Activity #1

Directions:

Using the pattern blocks and the red counters, guess which shapes can be used to tessellate the table top.

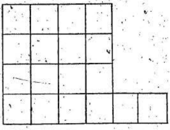
Note: Tessellate means to cover a plane without overlapping or leaving gaps.

Below is an example of a plane being tessellated by an irregular quadrilateral.



Now try the pattern blocks and counters to see which ones actually do tessellate the table top.

Lab #5 Geometry
Activity #1



This is an example of a pattern produced by tessellating a square.

Which of the regular posterboard polygons can tessellate a plane?

Regular triangle ________________
Square ________________
Regular pentagon ________________
Regular hexagon ________________
Regular octagon ________________

Why do some regular polygons tessellate and others don't? To answer this question one needs to know the angles of each regular polygon. The following is a list of hints to solve this problem.

a) The sum of the angles of a triangle is 180°.
b) Each polygon shape can be cut up into triangular shapes.
c) Each regular polygon has equal sides and equal angles.

The measure of an interior angle for a triangle is ________.

Square ________________
Regular pentagon ________________
Regular hexagon ________________
Regular octagon ________________

d) When a shape tesselates, the sum of the angles where the vertices meet is 360°.

Why do some regular polygons tessellate and others don't?

Lab #5 Geometry
Activity #1

Questions about the Activity:

1. How would you adapt this activity for use in an elementary classroom?

2. Given a combination of pattern blocks a child should be able to tessellate to make a variety of patterns or designs. This is an objective for an alternate tessellation activity. An example of these patterns can be found on the cover of the Pattern Block Books. Suggest another activity for tessellations.

Lab #5 Geometry
Activity #2

SOLIDS

Objective: Given a solid, the student should be able to state the plane shape which results from intersecting the solid with a plane.

Materials: Modeling clay
A thin wire
Geometric solids

Size of Group: 2 to 4

Source: Mathematical Experiences in Early Childhood
L.D. Nelson & W. Liedtke
Encyclopaedia Britannica Publications Ltd.
Toronto
1972

Lab #5 Geometry
Activity #2

Directions:

Using modeling clay, construct the following solids:

a) sphere
b) cylinder
c) cone
d) cube
e) tetrahedron

A cross section is made by slicing a wire through a solid. The cross section is a plane shape which results from intersecting a solid shape with a plane. For example, if you "sliced" a cube parallel to one of its outside faces, the cross section would be a square.

a "sliced" solid cube

cross section is a square

If you "sliced" one of the corners of the cube, the cross section is a triangle.

a "sliced" solid cube

cross section is a triangle

Experiment with your newly constructed solids. "Slice" them to obtain the following cross sections. State which solids produced the cross sections. It is not necessary to find all these cross sections; do as many as you can.

quadrilateral	______	octagon	______
hexagon	______	circle	______
ellipse	______	hyperbolia	______
rectangle	______	pentagon	______
square	______	parabolia	______
triangle	______	trapezoid	______

If you find two solids can produce the same cross section, add the second solid to your list.

Lab #5 Geometry
Activity #2

Questions about the Activity:

1. This activity could be modified to study Euler's Rule. If you are not familiar with this Rule look it up on page 284 of the grade 5 book of the I.S.M. series. (If you are still doubtful ask one of the lab assistants.) How else could you modify this activity to accomplish a different set of objectives?

2. This activity is just one way to approach the study of solids. On pages 106-107 in the grade 4 book of the I.S.M. series there is an example of a different approach. Suggest an activity that could be used to study solids.

Lab #5 Geometry
~~Activity~~ #3

GEOBOARDS

Objective: The student should be able to state a geometric shape which provides the greatest area for the smallest perimeter.

Materials: Geoboard
Elastic bands

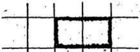
Size of Group: 2 to 4

Source: Instruction Booklet for the 'Invicta' Basic Shapes Set
Invicta Plastics Ltd.
OADBY

Lab #5 Geometry
Activity #3

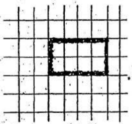
Directions:

Area on a geoboard is 1 square unit for every empty box. The distance between nails is 1 unit. The perimeter is found by counting these lengths. For example, put an elastic band as shown below.



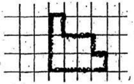
The shaded area, of the space enclosed by the elastic, has an area of 2 square units. The perimeter is 6 units long. Set up the following situations on the geoboard.

a)



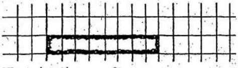
What is the area? ____
What is the perimeter? ____

b)



What is the area? ____
What is the perimeter? ____

c)



What is the area? ____
What is the perimeter? ____

Try the same activity keeping a constant area of 16 square units. Which geometric shape provides the smallest perimeter for a given area? ____________________

Lab #5 Geometry
Activity 3

Try the same activity keeping a constant area of 9 square units. Which geometric shape provides the smallest perimeter for a given area? ________________________

What would you conclude from these results?

____

____

Lab #5 Geometry
Activity #3

Questions about the Activity:

1. How could one adapt this activity for use in an elementary class?

2. Describe an activity for the child who did not understand the concept area (or perimeter). An example can be found in the follow-up activity on page 143 in the grade 2 book in the I.S.M. series.

Lab #6 Logic
Activity #1

KALAH

An Ancient Game of Mathematical Skill

Objectives: (1) The student should be able to increase his/her quantitive judgement (see note below)

(2) The student should be able to increase his development of intuitive decision making.

(3) The student should be able to count.

Materials: A Kalah playing board
Dried beans

Size of group: 2

Source: Games and Puzzles for Elementary and Middle School Mathematics
N.C.T.M.
John B. Haggerty
Virginia
1976, p. 207

Quantitive judgement is the "quality of the mind to make a specific decision based upon the array of the opposing position. This decision is entirely intuitive and is based upon a grasp of special relationships within the scope of that which is loosely defined as 'human intelligence.'"[1]

[1] Jerome S. Bruner, The Process of Education (Cambridge: Harvard University Press, 1961), p. 64.

Lab #6 Logic
Activity #1

Discussion:

This game promotes the development of pure reason in the form of quantitative judgement without a trace of chance or probability. The game is as old as civilization itself and had been continuously played in the Near and Far East for seven thousand years.

A quotation from June 14/63 Time tells of the background of the game. Carved on a vast block of rock in the ancient Syrian City of Aleppo are two facing ranks of six shallow pits with larger hollows scooped out at each end. The same design is carved on columns of the temple at Kar Nak in Egypt, and it appears in early tomb paintings in the Valley of the Nile. It is carved in the Theseum in Athens and in rock ledges along caravan routes of the ancient world. Today the same pits and hollows are to be found all over Asia and Africa, scratched in the bare earth, carved in rare woods, or ivory inlaid with gold.

The basic rules are so simple that even a young pre-school child can play the game after a short demonstration. A demonstration game is the most practical way to introduce this game to a group of pupils.

Directions:

The game is played by two players who sit on opposite sides of the playing board. Each player deposits three counters in each of the six cups on his side of the board. The object of the game is to collect as many counters as possible in the kalah, the large container, at each player's right.

The method of determining who moves first can be decided by the players. Each player, in turn, takes all the counters that are in any one of the six cups on his side of the board and distributes them one by one in each cup, going to his right. If a player has enough counters to go beyond his kalah, he distributes them in his opponent's cup, skipping the opponent's kalah. These counters now belong to the opponent.

There are two important elements that give the game its challenging strategy. If the player's last counter lands in his own kalah, he gets another turn; if his last counter lands in an empty cup on <u>his side</u> of the board, he captures all his opponent's counters in the cup opposite and puts them in his own kalah, along with his capturing piece.

Once a counter is placed in either kalah, it remains there until the round is ended. A capture ends the move, and the play goes to the opponent. A round of play is over when all six cups on one side are empty. The other player adds the remaining counters in his cups to the ones that are

Lab #6 Logic
Activity #1

in his kalah. The score is determined by who finishes with the most counters. If the winning player has collected 23 counters, his score is 5 because each player begins with 18 counters. The winning score for the game can be determined according to the situation or playing time allowed, but a score of 40 is usually the goal.

Note: Lower grades can begin learning to play by simply putting one counter in each cup. More experienced players or faster students can play with as many as six counters in each cup.

Lab #6 Logic
Activity #1

Questions about the Activity:

1. After Kalah has been demonstrated to a class, how else could you use it?

2. List other games that need a logical strategy to win. Example (1) NIM (if you are not familiar with this game and wish to learn it ask one of the lab assistants). (2) MASTERMIND.

Lab #6 Logic
Activity #2

PRE-NUMBER

Objective: The student should be able to sort, classify and order objects into a number of different categories.

Materials: Cylinders
9 dowel sticks
1 bead .
1 picture
1 pattern block

Size of Group: 2 to 6

Source: J. Piaget

Lab #6 Logic
Activity #2

Discussion:

Classifying is an important mental activity at the pre-school level. It is basic to many mathematical ideas, including logic. In a given collection one has to observe a common attribute in a number of the members of the collection. These members with the common attribute are then grouped together. The child should be able to tell why this particular group was separated from the other members of the collection. One attribute often used by children is color. All the red objects may be separated from all the silver and all the black objects.

Directions: (Classification)

Using the bead, pattern block and picture, classify each object and each set of objects in as many different classifications as possible. List the classifications in the space provided below. One example is given for each set.

Bead: contains a hole

Pattern Block: regular hexagon

Picture: floats to the ground when dropped

Bead and Pattern Block: concrete object

Bead and Picture: weighs less than 3 g.

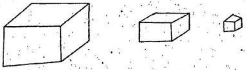
Pattern Block and Picture: color is all or part yellow

Bead, Pattern Block and Picture: used in a classification activity

~~Lab #6 Logic~~
Activity #2

Discussion:

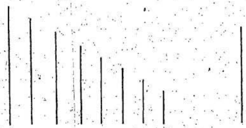
Another mathematical strategy that is developed in the child after classification is ordering or seriation. In classifying, the child had to observe one common attribute in a set of the members of the collection. In ordering, he has to find the common attribute and then order the objects in the set according to the magnitude of that characteristic in each object. For example, if the child has a set of blocks, he orders them according to the size.



Directions: (Part B)

Presented here are three activities which require a child to seriate. Even though these activities are of an elementary nature, you should work through them.

Present the child with a series of sticks, but omit one of the sticks.

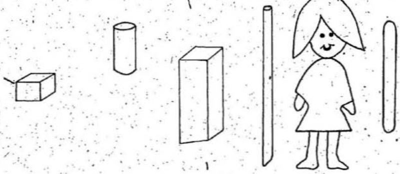


Lab #6 Logic
Activity #2

Give the omitted stick to the child, and ask him to fit it into the series.

Present the child with cylinder containers which are a variety of lengths and widths. The child is asked to arrange the set in order. After he has arranged them by one dimension, he should be asked to try and find another way to arrange them.

Present the child with a set of objects which vary in shape, size and dimensions.



The child is asked to arrange these objects in order. Due to the variety of dimensions, it is difficult for the child to pick one dimension and stay with it as a basis for ordering.

Note: The concept of cardinal number is developed through the child's understanding of classification and seriation.

Lab #6 Logic
Activity #2

Questions about the Activity:

1. If a child had difficulty seriating the dowel sticks what could be a follow-up activity that would help him with his problem? On page 25 of the kindergarten book of the I.S.M. series there is one example.

2. In this activity an object was given and the student must classify it. On pages 38 and 39 of the kindergarten book of the I.S.M. series a different kind of classification activity is demonstrated. Suggest a classification or seriation activity for a primary class.

Lab #6 Logic
Activity #3

TRAIN GAME

Objective: (1) The student should be able to sort attribute blocks by color, shape, thickness, and size.

(2) The student should be able to select a block according to its attributes.

Materials: 1 set of attribute blocks

Size of group: 4 to 8

Source: Elementary Mathematics Laboratory Experiences
Joseph R. Hooten Jr.
Michael L. Mahaffey
Charles E. Merrill Publishing Company
Columbus
1973, pp. 54-55

Lab #6 Logic
Activity #3

Directions:

Part A:

Sort the blocks by color.
How many groups do you have? ________

Sort the blocks by size.
How many groups do you have? ________

Sort the blocks by shape.
How many groups do you have? ________

Sort the blocks by thickness.
How many groups do you have? ________

The complete set should have

________ X ________ X ________ X ________ OR ________ pieces.

Part B:

Divide the attribute blocks evenly among all players. One person is arbitrarily selected to go first. The play then proceeds from left to right.

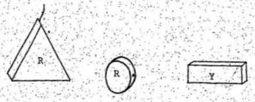
The first person places one attribute block on the table stating its color, shape, size and thickness. The next person has to place an attribute block which has two attributes the same and two attributes different from the first block. If this has been successfully completed, the third person must lay a block which has two attributes the same and two attributes different as the second block played. The game carries on so that a train is formed by the blocks on the table.

If an incorrect piece is played, it must be taken back and the play is continued with the NEXT player.

If you do not have a block to play, you miss your turn. The first person to play all his pieces is the winner, or the person with the least number of blocks left after all possible plays have been exhausted.

Lab #6 Logic
Activity #3

Example play:



Block #1: large red fat triangle

Block #2: small red fat circle

Block #3: small yellow fat rectangle

Lab #6 Logic
Activity #3

Questions about the Activity:

1. How would you adapt the train game for young children?

2. On page 197 in the grade 2 book of the I.S.M. series there is a follow-up activity which uses attribute blocks to aid children in learning to sort attribute blocks in a logical manner. Suggest another activity that uses attribute blocks to facilitate logic.
(Hint: The writings of the mathematician Zolten Dienes may be able to help in answering this question)

Lab #7 Probability
Activity #1

MARBLES

Objective: Given the total number of marbles in a container, the student should be able to state the quantity of each color in the container.

Materials: Probability containers
Marbles

Size of Group: 2 to 4

Source: Quantifying Chance
R.J. Souviney
Arithmetic Teacher
December 1977
Vol. 25, No. 3

Lab #7 Probability
Activity #1

Directions:

There are 30 marbles in container A. The marbles are a mixture of black, white, and red.

Take the container, turn it upside down, and record the color that appears on the chart below. Repeat this process 30 times, each time recording the result.

White	Black	Red

What is your guess as to the:

Number of white marbles in the container ____.
Number of black marbles in the container ____.
Number of red marbles in the container. ____.

Now repeat the experiment a second time, still using container A, and record the results in the chart below.

White	Black	Red

What is your guess this time as to the:

Number of white marbles ______.
Number of black marbles ______.
Number of red marbles ______.

Is there a discrepancy in your guess this time, compared to the first time?

Pool all the data from the two experiments to find the:

Total number of white observed ________.
Total number of black observed ________.
Total number of red observed ________.

Divide your totals by the number of experiments performed.

Lab #7 Probability
Activity #1

Now what is your guess for the:

Number of white marbles in container A ______.
Number of black marbles in container A ______.
Number of red marbles in container A ______.

Does this guess differ from any of your other guesses?

Now repeat the same experiment a third time, still using container A, and record the results on the chart below.

White	Black	Red

What is your guess this time as to the number of

White marbles ______.
Black marbles ______.
Red marbles ______.

Is there a discrepancy in your guess this time compared to guess #1 and guess #2?

Pool all the data from the three experiments to find the:

Total number of whites observed ______.
Total number of blacks observed ______.
Total number of reds observed ______.

Divide your totals by the number of experiments performed.

Now, what is your guess for the number of:

White marbles in container A ________.
Black marbles in container A ________.
Red marbles in container A ________.

Does this guess differ from any of your other guesses?

Open container A and see how many white and black marbles it contains. Were any of your guesses correct? Which guess was closest to the actual situation? Why do you think that was so?

Lab #7 Probability
Activity #1

Questions about the Activity:

1. How would you adapt this activity for a child?

2. If there had been five colors and 60 marbles, would you follow the same procedures? If not, what modifications would you suggest? Why?

Lab #7 Probability
Activity #2

DICE

Objective: Given any sum of two dice, the student should be able to state the probability of obtaining that sum by throwing the dice.

Materials: Two dice

Size of Group: 2 to 4

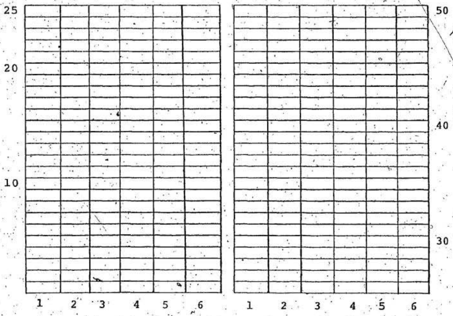
Source: Instructor Guide
Probability and Statistics Workshop

Lab #7 Probability
Activity #2

Directions:

If you were to toss a die 100 times, which number would you expect to appear most frequently?

Roll the die 100 times:
Record the number by shading one box after each toss.



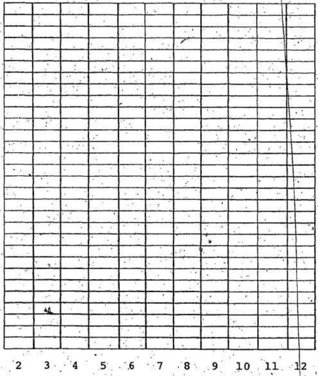
From the data, which number would appear most frequently?

____

Lab #7 Probability
Activity #2

If you tossed two dice 100 times, which sum would you expect to appear most frequently?

Roll the two dice 100 times. Record the sum by shading one box after each toss.



Lab #7 Probability
Activity #2

Take each sum and find how many ways it can be made by the throw of the dice.

Example: Sum = 4

	Die #1	Die #2	Total Sum
Toss #1 =	2	2	4
Toss #2 =	1	3	4
Toss #3 =	3	1	4

These would be written (2,2), (1,3), (3,1). The first position representing Die #1 and the second position representing Die #2.

Sum = 5

	Die #1	Die #2	Total Sum
Toss #1	4	1	5
Toss #2	1	4	5
Toss #3	2	3	5
Toss #4	3	2	5

i.e., sum 5 = (4,1), (1,4), (2,3), (3,2)

Sum 2 =
Sum 3 =
Sum 4 =
Sum 5 =
Sum 6 =
Sum 7 =
Sum 8 =
Sum 9 =
Sum 10 =
Sum 11 =
Sum 12 =

How many different ways can a pair of dice fall? ________
How many of the ways show a sum of 5? ________

Then the probability of a sum of 5, P(5) is $\frac{\quad}{36}$ or ________

Complete the table of various sums of two dice on a single toss.
Express all answers as common fractions in lowest terms.

P(2) =	P(6) =	P(10) =
P(3) =	P(7) =	P(11) =
P(4) =	P(8) =	P(12) =
P(5) =	P(9) =	P(13) =

Lab #7 Probability
Activity #2

Questions about the Activity:

1. If you tossed two octahedrons, whose faces were labeled from 1 to 8, what sum would appear most often?

   Note: An octahedron has 8 equilateral triangular faces--there is a model in the Math Lab.

2. On pages 332-333 in the grade 6 book of the I.S.M. series there is an alternate activity for determining all possible outcomes in a probability experiment. Suggest another activity to accomplish this objective for an elementary classroom.

Lab #7 Probability
Activity #3

COUNTING

<u>Objective:</u> Given a variety of objects, the student should be able to state the number of different arrangements that can be made.

<u>Materials:</u> Cuisenaire rods

<u>Size of Group:</u> 2

<u>Source:</u> J.F. LeBlanc
<u>Probability and Statistics</u>
<u>Brevard Teaching Center</u>
Florida
1975

Lab #7 Probability
Activity #3

Directions:

Take four different-coloured rods.
Make a train by lining the four rods end to end.
Record the colour sequence.

Example:

Red	White	Orange	Pink

R W O P

Now arrange the train by putting the card in a different order.

Red	Pink	Orange	White

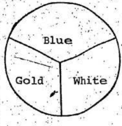
RPOW

How many different arrangements can you find for this four-car train? List them below.

If you had five different rods, how many arrangements would you find?

How many different routes can you take to get from St. John's to Florida? List them below.

Lab #7 Probability
Activity #3



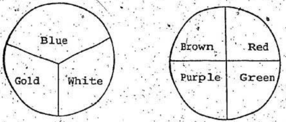
Spinner A

If you had spinner A, there are three possible outcomes:

1. Blue
2. Gold
3. White

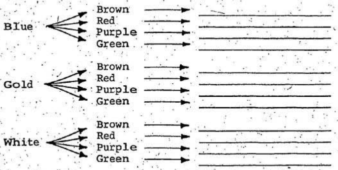
When there are two spinners, the number of possible outcomes increase.

Spinner A



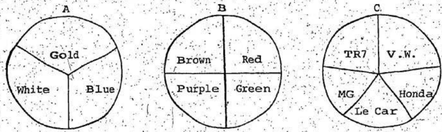
Spinner B

What are the possible outcomes?

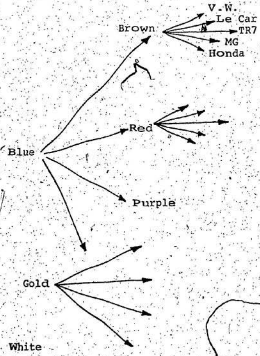


Lab #7 Probability
Activity #3

Three spinners increase the number of outcomes.



A car company used the three spinners to decide the colours of cars they would use for demonstration models. How many different cars would be used as demonstrators?



Lab #7 Probability
Activity #3

Questions about the Activity:

1. If a child could do the "counting" activity without difficulty, describe a follow-up extension activity. (Either combinations or permutations could be developed).

2. Ms. Soffis went to Florida with blue, yellow, and white bikinis plus brown, black, and tan sandals. How many different outfits could she wear to the beach?

   The above question with the aid of a doll and clothes could be an alternate activity to accomplish the same objective as the "counting" activity. Describe a different activity that would accomplish the same objective.

Lab #8 Calculator
Activity #1

CIRCLES

Objective: The student should be able to state the relationship between diameters and circumferences.

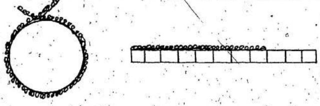
Materials: calculators
string
metric ruler
circles (A to B)

Source: Instructional Aids in Mathematics
N.C.T.M. 34th Yearbook, p. 243
Washington, D.C.
1973

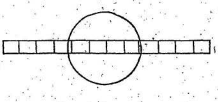
Lab #8 Calculator
Activity #1

Directions:

Place the string around the edge of circle A. Mark where the string meets. Now stretch the string along the ruler to find the distance around the circle, i.e., its circumference. Record the results on the enclosed chart.



Measure the distance across the widest section of a circle, i.e., find the length of its diameter. Record the result on the chart.



Using the calculator:

1) add the circumference to the diameter
2) subtract the diameter from the circumference
3) multiply the circumference by the diameter
4) divide the circumference by the diameter

Record the results on the enclosed chart.

Repeat the above process on Circles B, C, D, E, F, G, and H. Record all results on the enclosed chart.

Look at your recordings. Do you see any relationship between the circumference and the diameter? What is it? What number have you recreated? What is its name?

Lab # 8 Calculator
Activity # 1

Circle	Circumference	Diameter	+	-	x	÷

Lab #8 Calculator
Activity #1

Questions about the Activity:

1. Using the calculator as a teaching aid, describe a follow-up activity as an extension to the "circles" activity.

2. Describe an activity which includes the use of calculators and a concept that is not studied in today's lab.

Lab #8 Calculator
Activity #2

DECIMALS

Objective: The student should be able to state the pattern that exists when a number is multiplied or divided by a power of 10.

Materials: Calculator per participant.

Size of Group: 2 to 4

Lab #8 Calculator
Activity #2

Directions:

Multiply 384 by 10; 100; 1000; 10,000 and 100,000.

Record all your results on the chart on the next page.

Multiple 384 by .1; .01; .001; .0001; and .00001.

Record all your results.

Multiply 675.823 by .1; .01; .001; .0001; and .00001.

What similarities and differences do you perceive in your results? Does the power of 10 affect your result? How? If you were asked to multiply 93.4 by 10,000, could you do it without a calculator or pencil? Try it.

If you had repeated the experiment but had divided by powers of 10 instead of multiplying, how do you think the numbers would have changed? Can you divide 93.4 by .001 without a calculator or pencil?

If you find this question difficult, repeat the above experiment but divide everywhere it says to multiply and record your results on Chart B.

Lab #8 Calculator
Activity #2

Chart A (Multiplication)

Number	Power of 10	Result
384	1	384
384	10	

Lab #8 Calculator
Activity #2

Chart B (Division)

Number	Power of 10	Result

Lab #8 Calculator
Activity #2

Questions about the Activity:

1. How would you simplify this activity for elementary children?

2. This activity is one way decimals and calculators can be used together. Another way to use a calculator with decimals is to show that 1/10 = .1, 2/100 = .02, etc. Suggest another activity using a calculator to aid in teaching decimals.

Lab #8 Calculator
Activity #3

ESTIMATION

<u>Objective:</u> Given a number, the student should be able to estimate the square root of that number.

<u>Materials:</u> calculator
a set of 50 red discs
deck of estimation cards

<u>Size of group:</u> 2

Lab #8 Calculator
Activity #3

Directions:

The players decide between themselves who is to go first. The player who will not play first takes the calculator. The estimation cards are shuffled and placed face down on the table. Divide the red discs evenly between the two players.

The first player picks a card. He says the number written on the card out loud and then makes an estimate of its square root.

Example: If player A had picked 144, his thought process would be somewhat like the following: "Well, 10 x 10 = 100; so the square root of 144 is greater than 10. And 15 x 15 is 225, so the square root of 144 is less than 15 but greater than 10."

Suppose player A guessed 13, player A gives 1 red disc to the "bank" and player B will multiply 13 x 13 on the calculator and will tell his opponent the answer, 169.

Player A now thinks "169 is not too much bigger than 144, so maybe its 12." He pays another red disc, and player B multiplies 12 x 12 to get 144. The answer is correct, and roles of the two players switch to give player B a chance to estimate and player A uses the calculator.

Remember, every time a player wants to make a guess, he must pay 1 red disc to the bank. The player who "spends" all 25 red discs is the loser.

Lab #8 Calculator
Activity #3

Questions about the Activity:

1. How could you adapt this activity for elementary school children?

2. Describe another activity which includes calculators as an aid in teaching estimation.

APPENDIX B

ACTIVITY EVALUATION SHEET

EDUCATION 2341

Evaluation Sheet

Lab # ________________ Activity # ________________

General comments and suggested improvements:

____

____

____

Rate on a 1 to 5 scale by circling the desired response.

5 - Excellent
4 - Good
3 - Fair
2 - Poor
1 - Bad
NA - Not applicable

The activity meets the stated objectives:

NA 1 2 3 4 5

The directions are clear and understandable:

NA 1 2 3 4 5

The activity provides for students at different ability levels and different needs:

NA 1 2 3 4 5

The activity would motivate students:

NA 1 2 3 4 5

Did <u>you</u> learn something new from the activity?

NA 1 2 3 4 5

Did <u>you</u> enjoy the activity?

NA 1 2 3 4 5

APPENDIX C

A LIST OF THE QUESTIONS AND THE NUMBER OF RESPONSES FOR EACH QUESTION ON THE EVALUATIVE QUESTIONNAIRE PERTAINING TO THE MATHEMATICS LABORATORIES

Responses from Pilot Stage	Responses from Main Study	Questions
		1. Were the laboratory assistants willing to give personal help in the mathematics laboratories? I felt
15	61	a) welcome to seek personal help as often as I needed it.
–	5	b) the laboratory assistant would give personal help if asked.
–	–	c) hesitant to seek personal help.
–	–	d) the laboratory assistant was unsympathetic and uninterested in my problems.
		2. Did the mathematics laboratories prepare you to do activities in the school sessions?
–	–	a) The mathematics laboratories are too difficult to be of any use to elementary school children.
2	49	b) From the mathematics laboratories I was able to adopt activities to be used in the school session.
13	17	c) From the mathematics laboratories I got ideas on how materials can be used to teach mathematical concepts.
		3. I recommend that only two activities should be done in the mathematics laboratory since three 20-minute activities usually take more than one hour (used only in the pilot study).
4		a) strongly agree
7		b) agree
3		c) undecided
–		d) disagree
1		e) strongly disagree

Responses from Pilot Stage	Responses from Main Study	Questions
		3. There is sufficient time to complete three activities and discuss the questions in the two-hour laboratory period. (This replaced question 3 in the pilot stage).
	18	a) strongly agree
	30	b) agree
	-	c) undecided
	15	d) disagree
	3	e) strongly disagree
		4. The composition of the small groups for the mathematics laboratories should be
13	62	a) left open; as it was this semester
2	3	b) decided by the students at the beginning of the semester
-	1	c) decided by the staff at the beginning of the semester
		5. How well were the mathematics laboratories organized?
8	42	a) extremely well organized
6	22	b) adequately organized
-	1	c) less organization than would
-	-	seem desirable
1	1	d) no apparent organization
		e) too tightly organized
		6. How would you rate the value of the mathematics laboratories on clarifying the use of manipulative aids?
6	44	a) extremely valuable
7	16	b) very useful
2	6	c) useful
-	-	d) minimal use
-	-	e) useless
		7. How would you rate the value of the mathematical laboratories on clarifying particular teaching strategies?

Responses from Pilot Stage	Responses from Main Study	Questions
3	24	a) extremely valuable
4	27	b) very useful
6	9	c) useful
2	6	d) minimal use
-	-	e) useless
		8. How would you rate the mathematics laboratories' effectiveness to your investment of time and effort?
3	18	a) very high value for my effort
7	26	b) high value for my effort
3	20	c) moderate value for my effort
2	2	d) low value for my effort
-	-	e) no value for my effort
		9. How clear were the objectives and purposes of the mathematics laboratories?
4	44	a) were clearly outlined from the beginning
7	17	b) became clear as the course developed
4	5	c) became somewhat clear as the course progressed
-	-	d) were referred to only indirectly
-	-	e) were never made clear
		10. I had a great deal of difficulty with the mathematics laboratories.
-	-	a) strongly agree
1	2	b) agree
1	4	c) undecided
10	33	d) disagree
3	27	e) strongly disagree
		11. How would you rate your understanding of elementary school mathematics as a result of the mathematics laboratories?
6	29	a) I learned a lot.
7	25	b) My understanding improved.
1	8	c) A few ideas were new to me.
1	3	d) I learned very little.
-	1	e) I learned almost nothing.

Responses from Pilot Stage	Responses from Main Study	Questions
		12. How would you rate your understanding of how children learn mathematics as a result of the mathematics laboratories?
1	26	a) I learned a lot.
11	30	b) My understanding improved.
1	6	c) A few ideas were new to me.
1	4	d) I learned very little.
-	-	e) I learned almost nothing.
		13. How have you felt about the work that has been required in the mathematics laboratories?
5	5	a) There has been too much work.
4	35	b) There has been a great deal of work but it was worth the effort.
3	11	c) I notice no difference from the work required in other education courses.
3	15	d) There was not much work.
		14. What would be your overall rating for the mathematics laboratories?
-	11	a) outstanding
7	36	b) very good
5	15	c) good
3	3	d) adequate
-	1	e) poor

APPENDIX D

EVALUATIVE QUESTIONNAIRE

General Information

Name: ____________________

Number of courses completed on your degree ________

What is your major? ____________________

Math courses completed:

1150 ________ others ________

1151 ________ ________

1010 ________ ________

1011 ________ ________

Number of years of teaching experience, if any ________

Was student teaching completed

a) prior to this semester?
b) during this semester?
c) or is not yet completed?

Approximate number of hours per week spent on this course including lectures, mathematics laboratories, school sessions, preparation for activities, study, etc. ________

The Course in General

1. How well was the course organized?

   a) extremely well organized
   b) adequately well organized
   c) less organization would seem desirable
   d) no apparent organization
   e) too tightly organized

2. How well did this course contribute to your mathematical needs?

   a) made a very important contribution
   b) was valuable, but not essential
   c) was moderately helpful
   d) made a minor contribution
   e) made no contribution at all

3. Would you recommend this course to a good friend whose interests and background are similar to yours?

   a) recommend highly
   b) generally recommend
   c) recommend with reservations
   d) definitely not recommend

4. Log books are only an evaluation aid for the professors and serve no useful purpose for me.

   a) strongly agree
   b) agree
   c) undecided
   d) disagree
   e) strongly disagree

5. The course consists largely of "fun and games" activities but little is really learned that is of use in the classroom.

   a) strongly agree
   b) agree
   c) undecided
   d) disagree
   e) strongly disagree

<u>Lectures</u>

1. During lectures, I felt that questions and comments were always welcome.

   a) strongly agree
   b) agree
   c) undecided
   d) disagree
   e) strongly disagree

2. The professors seemed

   a) always prepared
   b) almost always prepared
   c) usually prepared
   d) frequently not prepared
   e) never prepared

3. The theory that is taught in this course is idealistic and cannot be used in a regular classroom.

a) strongly agree
b) agree
c) undecided
d) disagree
e) strongly disagree

4. I should be given more detailed instructions regarding how lesson plans should be written.

a) strongly agree
b) agree
c) undecided
d) disagree
e) strongly disagree

5. I would prefer more lectures on algorithms for basic operations than lectures on topics such as learning theories, diagnosis, calculators and geometry.

a) strongly agree
b) agree
c) undecided
d) disagree
e) strongly disagree

Mathematics Laboratories

1. Were the laboratory assistants willing to give personal help in the mathematics laboratory? I felt

a) welcome to seek personal help as often as I needed it.
b) the laboratory assistant would give personal help if asked.
c) hesitant to seek personal help.
d) the laboratory assistant was unsympathetic and uninterested in my problems.

2. Did the mathematics laboratories prepare you to do activities in the school session?

a) The mathematics laboratories are too difficult to be of any use to elementary school children.
b) From the mathematics laboratories I was able to adopt activities to be used in the school session.
c) From the mathematics laboratories I got ideas on how materials can be used to teach mathematical concepts.

c) undecided
d) disagree
e) strongly disagree

5. When working with one student and materials you can use activity learning since you can question and interact with the children but that kind of interaction would be impossible in a regular classroom.

a) strongly agree
b) agree
c) undecided
d) disagree
e) strongly disagree

6. The materials are good as aids in learning some concepts but the child seems to be able to do most mathematics better if he is taught by a pencil and paper method.

a) strongly agree
b) agree
c) undecided
d) disagree
e) strongly disagree

The questions in the following section will be asked with respect to the school sessions, mathematics laboratories, and the lectures. Please select the letter which best describes your reaction to each of the three sections of the course. (Key: L = Lectures, ML = Mathematics Laboratories, SS = School Sessions).

	L	ML	SS
For example:			
I really enjoyed the	e	e	e

a) strongly agree
b) agree
c) undecided
d) disagree
e) strongly disagree

	L	ML	SS
1. How well were the three sections organized?	___	___	___

a) extremely well organized
b) adequately organized
c) less organization than would seem desirable

L ML SS

d) no apparent organization
e) too tightly organized

2. How would you rate the value of the three sections on clarifying the use of manipulative aids? ___ ___ ___

a) extremely valuable
b) very useful
c) useful
d) minimal use
e) useless

3. How would you rate the value of the three sections on clarifying your view of what is involved in teaching? ___ ___ ___

a) extremely valuable
b) very useful
c) useful
d) minimal use
e) useless

4. How would you rate the three sections' effectiveness to your investment of time and effort? ___ ___ ___

a) very high value for my effort
b) high value for my effort
c) moderate value for my effort
d) low value for my effort
e) no value for my effort

5. How clear were the objectives and purposes of the three sections? ___ ___ ___

a) were clearly outlined from the beginning
b) became clear as the course developed
c) became somewhat clear as the course progressed
d) were referred to only indirectly
e) were never made clear

6. I had a great deal of difficulty with the three sections. ___ ___ ___

a) strongly agree
b) agree
c) undecided
d) disagree
e) strongly disagree

	L	ML	SS
7. How would you rate your understanding of elementary school mathematics as a result of the three sections?	___	___	___

a) I learned a lot.
b) My understanding improved.
c) A few ideas were new to me.
d) I learned very little.
e) I learned almost nothing.

8. How would you rate your understanding of how children learn mathematics as a result of the three sections?	___	___	___

a) I learned a lot.
b) My understanding improved.
c) A few ideas were new to me.
d) I learned very little.
e) I learned almost nothing.

9. How have you felt about the work that has been required in the three sections?	___	___	___

a) There has been too much work.
b) There has been a great deal of work but it was worth the effort.
c) I notice no difference from the work required in other education courses.
d) There was not much work.

10. What would be your overall rating for the three sections?	___	___	___

a) outstanding
b) very good
c) good
d) adequate
e) poor

Other

1. If I am teaching in a school that cannot afford mathematical materials then I feel adequately able to construct some materials myself.

a) strongly agree
b) agree
c) undecided
d) disagree
e) strongly disagree

2. My preparation for teaching elementary mathematics is quite adequate.
   a) strongly agree
   b) agree
   c) undecided
   d) disagree
   e) strongly disagree

Open-ended Questions

1. Do you feel prepared to lead activities in elementary mathematics?

   ____

2. Have you constructed any mathematical materials through the course? If so, describe them.

   ____

   ____

   ____

3. Do you think the course should be structured differently? If so, how?

   ____

   ____

   ____

   ____

4. Strengths of the course were:

   ____

   ____

   ____

   ____

   ____

   ____

3. There is sufficient time to complete three activities and discuss the questions in the two-hour laboratory period.

   a) strongly agree
   b) agree
   c) undecided
   d) disagree
   e) strongly disagree

4. The composition of the small groups for the mathematical laboratories should be

   a) left open; as it was this semester.
   b) decided by the students at the beginning of the semester.
   c) decided by the staff at the beginning of the semester.

School Sessions

1. The student(s) I had were very difficult to teach.

   a) strongly agree
   b) agree
   c) undecided
   d) disagree
   e) strongly disagree

2. I would prefer to have taught a whole class mathematics instead of individual students.

   a) strongly agree
   b) agree
   c) undecided
   d) disagree
   e) strongly disagree

3. Most of the time the child was unresponsive to questions.

   a) strongly agree
   b) agree
   c) undecided
   d) disagree
   e) strongly disagree

4. The lessons for the school session take so long, I don't see how I could realistically use manipulative materials when teaching regularly.

   a) strongly agree
   b) agree

5. Weaknesses of the course were:

6. I would suggest the following:

APPENDIX E

A LIST OF INTERVIEW QUESTIONS PERTAINING TO THE MATHEMATICS LABORATORIES

1. What did you learn from the mathematics laboratories?

2. Do you think the mathematics in the laboratories was too difficult?

3. Do you think the laboratories should be examples of activities that you could use directly in the school sessions? Why?

4. What did you think of the questions at the end of the activities? How should they be changed?

5. Do you think there should be a smaller number of laboratories? Which ones do you think should be left out?

6. Do you think the mathematics laboratories helped you with the in-school experience? How?

7. Do you think the mathematics laboratories helped clarify any of the ideas discussed in the lectures? How?

APPENDIX F

CONNELLY TAXONOMIZED ATTITUDE SCALE

Name: ____________________

Directions: This is not a test and will not be used in any way to produce a grade for you. Your responses will be kept confidential. The items on this instrument are statements about mathematics. For each item select the response which best describes your impression of the statement. The response choices are:

A - strongly agree
B - agree
C - no opinion
D - disagree
E - strongly disagree

____ 1. The basic properties of the various number systems are used often in the development of mathematical concepts.

____ 2. I have nothing but contempt for mathematics.

____ 3. The pupil should be guided to discover mathematical concepts and not simply told facts.

____ 4. It frightens me to think about teaching mathematics.

____ 5. Seeing the basic properties of the various number systems used in so many different contexts has helped me to realize the usefulness of the logical structure of mathematics.

____ 6. I regard mathematics as a lasting tribute to man's ignorance.

____ 7. I know that experimentation is helpful in mathematics.

____ 8. I wish I didn't have to teach mathematics.

____ 9. The great interrelationship of mathematical concepts helps to organize the teaching of mathematics.

____ 10. I feel under a great strain in a mathematics class.

____ 11. Mathematics provides many opportunities for the student to develop his own solutions to problems.

____ 12. Teaching mathematics is a waste of time.

____ 13. The logical structure of mathematics is the strongest tool I'll have in the teaching of mathematics.

____ 14. Mathematics makes me feel as though I'm lost in a jungle.

____ 15. I plan to have my pupils discover everything I want to teach them in mathematics.

____ 16. Since the content of the elementary school mathematics program is so basic, I know everything I need to know in order to teach at that level.

____ 17. Mathematics is like a pyramid with each concept being founded on those which came before it.

____ 18. Mathematics makes me feel uncomfortable.

____ 19. There are several different but acceptable ways to deal with most mathematical concepts.

____ 20. I could be happy teaching nothing but mathematics all day long.

____ 21. I find that organizing mathematical concepts around the logical structure of mathematics helps me to understand them.

____ 22. Mathematics is mainly pencil pushing.

____ 23. I intend to give my pupils many independent investigations in mathematics where they are free to do experimental work.

____ 24. Mathematics is the subject I most enjoy teaching.

____ 25. I will make full use of the logical structure of mathematics in my approach to teaching.

____ 26. The every existence of humanity depends on mathematics.

____ 27. I plan to provide extensive class time for pupils to experiment with their own mathematical ideas.

____ 28. The teaching of mathematics is at least as important as the teaching of other school subjects.

____ 29. A teacher most certainly should employ a systems approach through the logical structure of mathematics as the learning of mathematical concepts is greatly facilitated by this approach.

____ 30. Mathematics may be compared to a great tree, ever putting forth new branches.

____ 31. Mathematics is inductive and deductive, making use of both intuition and rigorous logic.

____ 32. I would like to be an elementary mathematics specialist.

____ 33. Mathematical systems, correctly perceived, provide aesthetic satisfaction a kin to that of a beautiful painting.

____ 34. Mathematics is a subject which I have enjoyed studying in school.

____ 35. In discussing mathematics, I prefer that conversation be developed on an intuitive basis rather than being concerned with careful statements.

____ 36. I really enjoy teaching mathematics.

____ 37. Mathematics is disorganized and unstructured.

____ 38. My general attitude toward mathematics is favorable.

____ 39. Students of all abilities should learn better when taught by guided discovery techniques.

____ 40. Teaching mathematics represents a challenge to me.

____ 41. Using the logical structure of mathematics for developing mathematical concepts is much better than the rote memorization of isolated facts called for in more traditional approaches.

____ 42. I feel mathematics is the greatest means for increasing the world's knowledge.

____ 43. I feel that mathematics teachers should just tell their students the facts that they want learned and give them enough practice to insure the required learning.

____ 44. As a teacher, I feel that pupils need to know mathematics because they will probably use it every day.

____ 45. I know I would be severely handicapping any people I taught if I did not show them the many uses of the basic properties of the various number systems.

____ 46. Mathematics is stimulating to me.

____ 47. Guided discovery methods of teaching mathematics just frustrate students.

____ 48. I like teaching mathematics so much that I would enjoy being an advisor to a mathematics club.

____ 49. I feel that a systems approach through the logical structure of mathematics is the only approach I would think of using in teaching mathematics.

____ 50. Working with various mathematical topics is fun.

____ 51. I'll be too busy showing pupils how to work specific examples to use guided discovery lessons in teaching mathematics.

____ 52. I now realize I'll have to learn much more mathematics to effectively teach mathematics.

____ 53. If I never hear of the "logical structure of mathematics" again, it will be too soon.

____ 54. I see nothing wrong with learning a variety of mathematical topics.

____ 55. Guided discovery methods of teaching mathematics take too much time to be of any value.

____ 56. In our technical society, providing pupils with a good mathematics background is important.

____ 57. Mathematics is a series of unrelated facts to memorize.

____ 58. I feel mathematics helps make other subjects easier to understand.

____ 59. I don't plan to employ guided discovery techniques in my teaching of mathematics as it only creates complete chaos in the classroom.

____ 60. I am happier teaching a mathematics class than teaching any other class.

____ 61. The logical structure of mathematics is apparent only to mathematicians.

____ 62. Mathematics fascinates me.

____ 63. One of the difficulties with mathematics is that there is only one correct way of treating a mathematical concept.

____ 64. I like to teach mathematics, but I prefer to teach other subjects.

APPENDIX G

A LIST OF INTERVIEW QUESTIONS NOT PERTAINING TO THE MATHEMATICAL LABORATORIES

1. What is your name?

2. Have you done or are you doing your student teaching?

3. Where did you obtain the ideas for your in-school activities?

4. Did you use reference books from the laboratories?

5. Which books did you buy for the course? Why?

6. Did you use Arithmetic Teacher?

7. Did you use the Investigating School Mathematics series?

8. Describe briefly your best activity?

9. Why do you think it is good?

10. What did you learn from the in-school experience?

11. Do you think writing up the lesson each week benefited you?

12. What did you learn from the self-evaluation of each lesson?

13. What did you learn from your interaction with the children?

14. Did you have any problems with the school sessions? Can you suggest any improvements?

15. What did you learn from the lectures?

16. Do you think there is enough theory taught in this course?

17. If you consider the school sessions as the practical part of the course how does the theory relate to the school sessions?

18. Did the theory ever affect your in-school experiences? How? Why?

19. Do you see this course as 50 percent practical and 50 percent theoretical?

20. Why do you think your professor encouraged you to use materials, give diagnostic tests each week and to build a student-teacher bond with your pupils?

21. Did you have any problems with the lectures? Can you suggest any improvements?

22. Do you think a regular 15-minute time arranged with your professor is a good idea? Why?

23. Do you have any suggestions as to how the course could be changed to make an improvement?