THE EFFECTS OF A SUPPLEMENTARY ACTIVITIES IN AN ACADEMIC GEOMETRY COURSE ON STUDENT ACHIEVEMENT AND ATTITUDES

JOSEPH CATER MURCELL

THE EFFECTS OF SUPPLEMENTARY ACTIVITIES IN AN ACADEMIC GEOMETRY COURSE ON STUDENT ACHIEVEMENT AND ATTITUDES

by

© Joseph Cater Murcell, B.Ed., B.A., B.Sc.

An Internship Report submitted in partial fulfillment
of the requirements for the degree of
Master of Education

Department of Curriculum and Instruction
Memorial University of Newfoundland

June 1982

St. John's Newfoundland

ABSTRACT

The purpose of this study was to develop a set of activities to supplement a unit of work in the ninth grade academic geometry program, and to evaluate the benefits to be derived from it. In order to do this the following four questions were considered:

(1) Does the use of a unit involving supplementary activities result in students attaining competence with geometric concepts?

(2) Does the use of a unit involving supplementary activities have any effect on student attitudes toward geometry?

(3) How effective is each of the supplementary activities in helping students achieve each objective?

(4) What are some of the problems encountered using the activity approach to geometry?

A unit of work, supplemented with 14 activities, was taught to two intact ninth grade academic classes of 28 students each. The activities were developed by the investigator or were selected from other sources.

To determine student achievement on the unit, two tests constructed by the investigator were administered. Both tests were designed to test whether the behavioural objectives for the unit had been achieved. The objectives for the unit were written at three cognitive levels: computation, comprehension, and application.

An analysis of the test results showed that many of

the students failed to achieve mastery of the geometric concepts at all three cognitive levels. However, the findings did show that the students were more successful at the lower cognitive levels than they were at the application level.

A modified form of Aiken's Scale of Attitudes Toward Mathematics was given as a pretest and post-test to determine whether the unit had any effect on student attitudes toward geometry. A dependent t-test for means was performed on the pretest-post-test attitude scores. The results showed that students made a significant positive attitudinal change toward geometry during the seven week period in which unit one was taught.

The problems encountered with the activity approach to geometry were noted. They were: unfamiliarity with the approach and unfamiliarity working with concrete materials by both teacher and students, too much time had to be spent with the slower learners, and student tendency to jump to a conclusion on limited experience.

On the basis of the findings of this study, several recommendations were made. The most obvious was that the activity approach can be used as an alternative approach to the expository approach. Other recommendations were that in future studies, the long term effects of supplementary activities be assessed, and that the subjects be randomly selected from a wide range of ability levels.

ACKNOWLEDGEMENTS

The investigator wishes to express his thanks to all those who have helped him in any way to complete this study.

He is especially thankful to his advisor, Dr. Dale Drost, for his help and expert advice and the other committee member, Dr. Lionel Mendoza, for his help and guidance. Thanks are also expressed to Dr. Frank Riggs and Mr. Graham Blundon for serving on the examining committee.

The investigator also would like to thank Mr. Joe Roberts, Program Coordinator for Mathematics and Science for the Port aux Basques Integrated School Board and Mr. Brendan Croskery, Principal of St. James Regional High, Port aux Basques for their help throughout the study.

Finally, the investigator wishes to thank his wife, Jean, for the help, patience, and understanding she has portrayed over the past six years while he has been on the graduate program.

TABLE OF CONTENTS

LIST OF TABLES

CHAPTER I

THE PROBLEM

Secondary school geometry is frequently described as "the most disliked subject" and as often as "the subject in which most students achieve poorly." Many prominent mathematics educators, such as Wilkinson (1970) and Friedman (1978), who are concerned about this problem, feel that changing the content is only a partial answer to the problem and that a change in methods of instruction is necessary as well. As a result, many of those concerned with improving student achievement and attitudes toward geometry are directing their attention toward the methodology of teaching geometry. One approach that has attracted much attention from educators and researchers has been the activity or laboratory approach.

Deans (1971) stated that "the laboratory approach exposes children to a wide range of manipulative, concrete materials, and practical activities from which they can abstract mathematical ideas" (p. 20). Kidd, Myers, and Cilley (1970) stated that "the activity approach will do far more good than the traditional approach to build enthusiasm for and confidence in mathematics, to teach students to use their own ingenuity and to relate mathematical ideas and symbols to real objects" (p. 10).

Riggs (1974) gave several reasons why activities should be used in mathematics, (i) they provide motivation,

(ii) they are good because children like them, (iii) they provide an opportunity for pupils to be creative and imaginative, (iv) they help students of different mathematical abilities, and (v) they aid in understanding. He also stated that it is as important for high school students as it is for elementary school students to manipulate objects and discover certain relationships in mathematics.

In addition to educators, some psychologists including Piaget, Dienes, and Bruner have studied the effects of activity materials on achievement of mathematical concepts as well as attitudes and have supported the use of these materials. Bruner (1966) suggested that the course for learning mathematical ideas is first through an enactive representation, then an iconic representation, and finally a symbolic representation. He stated that following this approach, the learner not only understands the abstract idea but he also has a stock of concrete images which embody the abstraction. Vigilante (1967), expressing similar views to those of Bruner, stated that children learn best when they have tangible, manipulative or visual materials to use. He maintained that "it is only after having experiences on the enactive level that an individual can come to generalize on the basis of these experiences, which provide for him the firm foundation necessary to deal with geometry on the symbolic level" (p. 455).

Piaget (1964) suggested that knowledge cannot be acquired solely from the blackboard or drill, but must be actively constructed by the child through interaction with his environment. He stated that:

> to know an object, is not simply to look at it and make a mental copy or image of it. To know an object is to act on it. To know is to modify, to transform the object, and to understand the process of this transformation, and as a consequence to understand the way the object is constructed. (Piaget, 1964, p. 171)

The theories of Bruner and Piaget are supported by Dienes. He concluded, from his many studies conducted in the classroom, that children can best learn mathematical concepts by first having the freedom to manipulate and experiment with activity materials. Dienes (1967) stated that the learner must be given every opportunity to be exposed to activities. These activities, he said, give the students (i) the opportunity to think for themselves, (ii) the opportunity to appreciate the order and pattern which is the essence of mathematics, and (iii) the needed skills.

It seems that psychologists and educators agree that one of the major pedagogical goals of mathematics instruction should be the active participation of pupils in the learning process. One way to have students actively involved is through the use of activities. The effects of a set of supplementary activities on achievement in and attitudes toward geometry were examined in this study.

Statement of the Problem

In Newfoundland, geometry is presently a part of the required mathematics curriculum for academic students in grade nine. Many students achieve poorly in geometry and as mentioned earlier, it is often said to be the most disliked subject in school. To try and improve not only the achievement of the students in geometry, but also improve their attitudes toward geometry, supplementary activities were implemented into the regular academic program. Therefore the problem under investigation in this study was to determine what effect supplementary activities had on achievement in and attitudes toward geometry.

Purpose of the Study

The purpose of this study was to develop and evaluate activities to supplement a unit of work in the ninth grade textbook Math is/Geometry (Ebos, Tuck, Drost, and Hatcher, 1981). The activities made use of concrete materials to facilitate concept formation and required the active involvement of each student.

The effects of the supplementary activities on student achievement in and attitudes toward geometry were investigated. More specifically the following questions were considered:

1. Does the use of a unit involving supplementary activities result in students attaining competence with geometric concepts?

2. Does the use of a unit involving supplementary activities have any effect on student attitudes toward geometry?

3. How effective is each of the supplementary activities in helping students achieve each objective?

4. What are some of the problems encountered using the activity approach in geometry?

Rationale for the Study

The learning of mathematics is primarily concerned with concept formation rather than the memorization of facts. The mental processes involved in the formation of concepts are much more complex than those associated with the memorization of facts. Although there is little disagreement among psychologists regarding the role of concept formation in the learning of mathematics, there are several existing theories about how best to foster concept formation. As suggested earlier, the belief that activities enhance the learning of mathematics has gained much validity from learning theories such as those suggested by Bruner (1966), Piaget (1964), and Dienes (1967). They contend that students can have a better understanding of concepts if they discover these concepts by themselves through experiences related to the physical world.

The Cambridge Conference on School Mathematics in its 1963 report, Goals for School Mathematics, made a strong case for activity materials when it stated: "the conclusion is inescapable that children can study mathematics more satisfactorily when each child has abundant opportunity to manipulate suitable physical objects" (p. 35). Thus there appears to be strong support among educators and psychologists for the position that manipulation of the activity materials should precede the requirement of abstracting an idea. In short, it appears that activity materials are essential in an instructional program for geometry.

Educators and psychologists have also emphasized the need and urgency for research into the use of activity materials at all grade levels. Begle (1969), Kieren (1969), and Bernard (1972) stated that although considerable research has been done to assess the effectiveness of using activity materials in teaching mathematics, the results of these studies suggest that the effectiveness of this approach is still an open question. Friedman (1978) suggested that "we should increase our efforts to determine those situations in which the manipulative materials strategy is most promising" (p. 80). He also stated that "we still have to travel a long way on this research road before we can definitely answer Kieren's questions: for whom, for which topics, and with what materials are manipulative and playable activities valuable" (p. 80).

At issue in this study was the effectiveness of supplementary activities on certain geometric concepts at the grade nine level.

Definition of Terms

Academic Student

A person who has an average general ability in mathematics. He is one who has a greater degree of success when concepts are presented in a manner which de-emphasizes mathematical rigor and emphasizes practical application and logical reasoning. He will attain the essence of the concept with instruction and practice (Mathematics Bulletin, 1981-82, p. 28).

Activity Materials

The manipulative materials used for instruction such as geoboards, rubber bands, activity sheets, ruler, protractor, dotted paper, and graph paper.

Activity Approach

An instructional approach where the student is given the opportunity to develop mathematical concepts through his involvement with activity materials.

Limitations of the Study

A study of this kind has a number of limitations that could influence the results. The duration of the study was seven weeks. The possibility exists that this time was insufficient to develop the concepts completely and effectively.

The tests used to measure achievement in the geometry unit were teacher constructed and were not standardized.

However, both tests were designed to test the behavioural objectives of the unit and were assumed to be valid and reliable.

Only students from two intact grade nine academic classes, at one high school, were included in the sample. Thus the sample was not necessarily representative of the larger population of academic mathematics students.

The supplementary activities selected for the experiment dealt only with certain geometric concepts at the grade nine level. Hence, generalization could not be made about the effectiveness of this method on other geometric concepts. Some concepts might be taught more effectively by some other method of instruction.

Each class was taught by the investigator. The possibility of teacher bias existed.

Another limitation of the study is that students could have been more enthusiastic about geometry than they normally would have been because of the novelty of the use of manipulative aids.

Outline of the Study

In this chapter, the purpose of the study and a rationale for the study have been discussed. A review of the related literature is presented in Chapter II. Chapter III contains a discussion of the procedures followed in conducting the study and the methods used to collect and analyze the data. The results obtained from the data analysis are described in Chapter IV. The conclusions of the study and implications and recommendations for future research are summarized in Chapter V.

CHAPTER II

REVIEW OF THE RELATED LITERATURE

In this chapter various research studies on activity learning are reviewed to determine what effect, if any, activities have on achievement in geometry and attitudes toward geometry. The studies are divided into two groups: the first group compared the activity oriented approach to some other method of instruction, and the second group dealt with the mathematics laboratory and its effects on achievement in and attitudes toward mathematics.

Studies Involving the Activity Oriented Approach

In recent years, there has been a growing interest by educators and psychologists in the way in which children learn most effectively. Psychologists such as Piaget and Dienes, among others, have suggested that learning at certain stages of the child's cognitive development proceeds from the concrete to the abstract. Proponents of the theory that learning proceeds from the concrete to the abstract have suggested that experiences with activity materials must be provided for learners in order for them to learn effectively at the abstract level. This interest in the use of activity materials in the learning of mathematics has brought an increase in the quantity and variety of instructional materials available for instructional use. Numerous research studies, some of which are reviewed below, have been conducted in an attempt to evaluate the impact of these materials on mathematical achievement and attitudes toward mathematics.

Monier (1977) investigated the effects of an activity approach to teaching geometry in certain high schools in Afghanistan. A total of eight teachers and 612 students were randomly selected to participate in the study. The activity approach consisted of 48 activities which were introduced as learning modules supplementing the regularly used approach. The regular approach consisted of lectures, use of textbooks, and recitation based on memorization only. Three intermediate tests and a comprehensive final examination were used to measure achievement. The results showed that in comparison to the regular approach, the activity approach significantly helped students to: (i) improve their performance in overall understanding of geometry, (ii) achieve higher levels in creative thinking, (iii) develop greater ability to explain geometric concepts, (iv) improve their ability in solving geometric concepts, (b) develop the ability to recall geometric concepts better, and (vi) develop greater ability in setting up complete proofs for geometric theorems.

Studies done by Bring (1971) and Johnson Jr. (1971) reported significant differences in favor of the groups using the activity materials. Bring (1971) investigated the effects of concrete activities on achievement of objectives in metric and non-metric geometry with fifth and sixth grade students. The students were divided into two groups characterized by the amount and type of concrete activities afforded the students during the experiment. Bruner's levels of understanding and Piaget's stages of intellectual growth were employed in the formulation of unit materials and overall development of the study. Two post-tests were administered one week apart to measure achievement. Analysis of the data indicated that students using concrete activities achieved higher than students deprived of the concrete activities.

Johnson Jr. (1971) investigated the effects of three treatments on the achievement of objectives related to perimeter, area, and volume. Ninety-six students in grades four, five, and six were categorized on factors of reading ability, age, and sex. They were assigned randomly to one of three treatments. The three "semiprogrammed" treatments included either a "maximum", a "moderate", or a "minimum" amount of concrete materials in the instruction. Two post-tests were given two weeks apart. Johnson reported that a high degree of concreteness yielded higher achievement of objectives in these topics of each of the post-tests.

The results of these three studies indicated that the use of activity materials can be effective in the teaching of geometric concepts. However, other studies were reviewed which revealed that student manipulation of activity materials is not always more effective than other instructional methods on achievement in geometry.

Prigge (1974) investigated the effects of three instructional settings on the learning of selected geometric concepts. One hundred sixty-nine third grade students were randomly assigned to the three treatments. The experimental period was 10 days and students studied instructional materials prepared with an individualized, programmed format. Treatment W groups studied the geometric concepts using paper and pencil activities. Treatment M groups studied the identical concepts using activities that made use of manipulative aids such as the geoboard and georuler, and also paperfolding. Treatment S studied the same concepts using activities with geometric solids such as cubes and prisms. Data from one post-test and two retention tests were analyzed. Significant differences were found favoring the S treatment. The evidence indicated that children of lower ability were better able to learn the selected concepts when activities involving solids were included in the presentation.

Wong (1979) was concerned with the effects of handling manipulative devices on the learning of selected concepts in geometry by 13 year olds. The experiment was designed to determine who should handle the concrete materials; the teacher, the pupils, no one, or both, in the learning of geometric concepts. Four units of instruction were designed to be used in the four treatment modes. The 120 students who participated in the study were not randomly assigned. All pupils were taught by the same teacher, spent the same amount of time on each unit and took the same test. The results showed no significant difference between the four groups in their mean performance on the achievement test and post-test.

Norman (1977) designed a study to determine whether a significant difference existed between the effects of two methods of instruction, namely activity oriented and traditional textbook. A unit of work on geometry of the circle was taught to two groups of 12 students randomly assigned from a general grade ten class. One group was taught using the textbook; the other group used an instructional package devised by the investigator. A criterion referenced test constructed by the investigator was given as a pretest and a post-test to measure achievement. Attitudes were measured by issuing a pretest and post-test attitudes questionnaire. Analysis of the data indicated no significant gain in achievement between the groups after the treatments were employed. However, a significant gain in attitudes toward geometry was found to exist in favor of the activity oriented group.

Research studies that dealt specifically with geometry at or near the grade nine level using the activity oriented approach were rare. The group of studies which will be discussed next has some relevance to this study, although the mathematics is not geometry. The studies reviewed used concrete materials to help evaluate the effectiveness of the activity oriented approach on mathematical achievement and attitudes toward mathematics. Also the students involved were at or near the grade nine level.

London (1978) compared the effectiveness of an activity oriented mode of instruction to a conventional textbook mode. Two teachers volunteered to participate and four of their eighth grade classes were selected for the study. Each teacher taught two classes, one by the activity approach and one by the conventional approach. The materials used by the activity classes were various manipulation devices and sets of physical objects accompanied by written instructions. The two other classes, the control group, used only the textbook. An achievement test constructed by the investigator was given as a pretest and a post-test. A pretest and a post-test of attitudes toward mathematics using the semantic differential was also given. Analysis of the data indicated no significant gain in achievement in and attitudes toward mathematics for the activity group.

Two other studies were reviewed for the effects of concrete materials on the learning of mathematical concepts, and no significant difference was found between the activity approach and some conventional approach. Wilkinson (1971) investigated the effectiveness of using supplementary materials to teach a variety of mathematical concepts to eighth grade students. Six classes consisting of 136 students participated in the study for 46 days. Two classes consisted of students who were achieving at grade level or above, two classes below grade level and two classes at all three levels. The findings showed no significant gain in attitudes toward mathematics by using supplementary materials. They also indicated that supplementary materials were not effective when used for teaching students who were achieving below grade level.

The purpose of a study by Bobelstein (1978) was to examine the effects of supplementary materials and activities on achievement in and attitudes toward mathematics in a first year algebra class. Two hundred eighty-three students from 14 first year algebra classes were involved in the study. A pretest on arithmetic achievement and an attitude inventory were administered to determine whether equivalent groups existed. The attitude inventory was administered four times during the year and an algebra achievement test was given at the end of the study. Results showed that using supplementary activities and materials did not seem to have any positive effect on attitudes as the attitudes of both groups declined during the year. There was also no significant difference in achievement in algebra between the groups.

More positive results with the use of concrete materials were found by Purser (1973), who conducted a study to determine if certain activities using manipulative aids were associated with student gain in achievement and retention scores in mathematics at the seventh grade level. Three hundred thirty-nine students of high, low, and medium ability served as the sample for the study (169 experimental and 170 control). Activity packages were developed by the investigator to be used in the study. Results showed a significant difference in achievement and retention favouring the group using the activity materials.

Results supporting Purser's conclusions were found by Scheer (1977) who conducted a study to investigate what effect supplementary activities would have on mathematical achievement, attitudes toward mathematics, and self-concept in the classroom. Forty-two students, ranging in age from eight to 15 years, were selected for the study. Two groups were set up, 21 in each group, matched by age, grade, and sex. To measure achievement, a pretest was given at the start of the study and two post-tests were given, one at the end of the study and another six weeks after termination. The Revised Mathematics Attitude Scale was used to determine any change in attitudes toward mathematics. Results showed that the children who participated in the supplementary activities showed greater gain in mathematical achievement at the termination of their session and six weeks after than did children who did not participate in the activities. There was also a significant difference in attitudes toward mathematics favoring the treatment group.

Although the majority of the studies reviewed concluded that the use of activities by school children had a beneficial effect upon their achievement in or their attitudes toward mathematics, others reviewed showed conflicting results. It also appeared from the research reviewed that activities can be effective with students of medium or low ability, especially when the subject area is geometry. The objective of this study was to examine whether supplementary activities have any effect on student achievement in geometry and attitudes toward geometry of grade nine academic students.

Studies Involving the Mathematics Laboratory

The studies reviewed in this section dealt with the effects of the mathematics laboratory approach on achievement in mathematics and attitudes toward mathematics. Research studies that dealt specifically with geometry at or near the grade nine level using the laboratory approach were rare. However, since the studies reviewed on the laboratory approach made use of the concrete materials, their results were relevant to this study.

Davidson and Walter (1972) pointed out that students involved with the mathematics laboratory are actively involved in "doing mathematics" at the concrete level. The students are provided with an opportunity to manipulate objects, to think about what they have done, to discuss and write about their feelings, and to build necessary skills.

The results of the first studies reviewed favored the mathematics laboratory approach over other methods of mathematics instruction. Whipple (1972) compared the laboratory method of teaching geometry with an individualized instructional approach. Four classes of eighth grade students were chosen for the study which lasted for 14 days. The two experimental groups were taught by the laboratory approach with emphasis placed on the use of manipulative models. The two control groups used individual instructional units. Whipple reported that the students who used the laboratory approach with manipulation materials scored higher than students using the individualized instructional units. He also found that students exposed to the laboratory approach showed a significant increase in attitudes toward mathematics when compared to students not using the laboratory approach.

Similar results were found by Wasylyk (1970, cited in Vance & Kieren 1971) who organized a mathematics laboratory to teach measurement to low ability grade nine students. The students first worked in small groups using concrete materials. This activity was followed by class discussions, problem solving sessions, and project sessions, each with a specific purpose. Analysis of the data indicated that the achievement of students in the laboratory group was significantly higher than that of those in the control group taught the same topics using a conventional method. It was also found that the laboratory group exhibited significantly higher attitudes toward mathematics.

Despite the fact that the three previous studies have reported improved mathematical achievement resulting from mathematics laboratories, a larger number of studies found no such advantage. Wilkinson (1970) conducted an experiment to investigate whether the laboratory method was more effective for teaching geometry to sixth graders than the conventional method. There were two experimental treatments: one which consisted of laboratory units which contained worksheets and manipulative materials, and a second where cassette tapes which contained a verbatim recording of all directions and questions on the laboratory worksheets were used. The control group was taught by teacher and textbook. The results indicated that the students in the experimental group did as well on the geometry achievement test as students in the group taught by their teacher. Wilkinson also reported that the laboratory approach did not significantly affect pupils' attitudes toward mathematics but that the method appeared to be more effective in geometry with students of middle and low ability.

In a year long study, Johnson (1971) sought to determine the effectiveness of using activity oriented lessons in seventh grade mathematics. Students were randomly assigned to one of six classes. Two classes used only the textbook, two classes used instructional materials other than the textbooks, and the final two classes used the textbook augmented by enrichment activities. The three units of work taught to all groups were number theory, geometry, and measurement, and rational numbers. Achievement tests were given at the end of each unit of work. A pretest and a post-test of attitudes toward mathematics using the semantic differential was also given. Johnson concluded that the performance of students taught exclusively by the activity approach was inferior to that of students receiving textbook based or activity enriched instruction. The results showed no significant difference in attitudes toward mathematics between the two groups. However, there was some evidence that the laboratory lessons in the study of measurement and geometry were effective for low and middle ability students.

Similarly, Corwin (1978) investigated the effects of laboratory experiments and manipulative aids on achievement and attitudes of students in high school geometry. Eight teachers and 334 students (165 control and 169 experimental) were selected for the study. Each teacher taught two classes, one with activities that were keyed to selected sections of the textbook while in the other class the same topics were taught in a lecture discussion format. Corwin concluded that using laboratory activities in a geometry classroom did not improve or hinder student's achievement in and attitudes toward mathematics. However, the students in the experimental group felt that the experiments and the manipulative aids did help them visualize and understand the geometry concepts.

Kujawa (1976) conducted a study with grades four, five, and six students to determine whether supplementary mathematics laboratories made a difference in student's mathematical achievement and/or attitudes toward mathematics. The 46 students selected for the experimental group attended a mathematics laboratory, in subgroups of seven or eight, three days a week, 40 minutes a day, for a period of 15 weeks. The mathematics laboratory was in addition to the regular mathematics class. The laboratory was divided into three formats: structured activities, semi-structured activities, and free choice activities. Some of the activities utilized were games, programmed materials, diagnostic instruments, and puzzles. The experimenter did the teaching, grading, and reporting. He concluded that no conclusive evidence was found to support the supplementary mathematics laboratory as an instructional approach.

The findings of a study by Smith (1973) were similar to those found by Wilkinson (1970), Johnson (1971), and Corwin (1978) on the laboratory approach. He investigated the extent to which mathematics laboratory experiences enabled middle school students to gain in achievement in mathematics and to develop more positive attitudes toward mathematics. Eighty-two students from grades six, seven, and eight were chosen for the study. The experimental group used activities designed to correlate with the objectives and lessons of the regular mathematics class. The control group was taught the same objectives in a regular classroom setting by means of a conventional method. Both groups were taught by the same teacher. The results showed no significant difference in achievement scores between the two groups. It was also found that mathematics laboratory instruction did not significantly affect attitudes toward mathematics.

Vance (1969, cited in Vance & Kieren 1971) investigated a laboratory program to see what effect activities and games selected to supplement and to enrich the regular course had on attitudes toward mathematics and achievement in mathematics. Two groups of combined grade seven and eight students were used for the study over a three month period. The experimental group, working in groups of two, worked on activity lessons using concrete materials. The control group was taught the same lesson by their teacher in a regular classroom setting. It was concluded that students did learn new mathematical ideas in the laboratory setting, but overall did not achieve as well as the students in the regular classroom. Although attitudes toward mathematics among the groups were not significantly different, student reaction was more favorable to the laboratory setting than to the classroom setting.

From the results of the studies reviewed on the laboratory approach, it appears that students can learn mathematical ideas from a laboratory setting. Some studies indicated that with students of low ability, concrete representation and laboratory approaches appear most effective. Vance and Kieren (1971) concluded that the mathematics laboratory promotes better attitudes toward mathematics. However, from the studies reviewed there is only limited evidence to support this conclusion. At best it can be said that pupils can learn from such instructional approaches (Vance & Kieren, 1971) which may be used to meet individual needs (Brousseau, 1973).

Summary

Research conducted for the purpose of investigating the effects of activity materials on achievement in mathematics have provided conflicting results. Several studies reviewed concluded that the activity approach produced superior results while other studies concluded that the activity approach was no more effective than any other method of instruction. There was, however, considerable support for the use of the activity materials with certain groups of students. Prigge (1974), Wasylyk (1970), Wilkinson (1970), and Johnson (1971) indicated that students of below average mathematical ability benefit more from the activity approach than do students of high mathematical ability.

There was very little research found which supported the idea that students involved with activity materials have more positive attitudes toward mathematics. Most studies reviewed showed no change in the attitudes of students toward mathematics whether taught by the activity approach or some conventional approach.

From reviewing the literature, it was concluded that the effects which supplementary activities will have on student achievement in geometry and attitudes toward geometry cannot be predicted with any certainty. For that reason, the purpose of this study was to investigate what effects, if any, supplementary activities will have on achievement in geometry and attitudes toward geometry of grade nine academic students.

CHAPTER III

DESIGN AND PROCEDURE

In this chapter the manner in which the investigation was conducted is described. Included is a description of the sample used in the study, the instructional materials, procedure, evaluation instruments, and methods employed to analyze the data.

Description of Sample

The study was conducted at a regional high school in a middle sized rural Newfoundland community. The school offered a tri-level program in mathematics at the grade nine level; the advanced, academic, and practical courses. Students were streamed for each level on the basis of their mathematical achievement and the recommendations of the grade eight mathematics teacher. The school had five classes of grade nine students; one advanced, three academic, and one practical.

The subjects for this study were two intact classes of grade nine academic geometry students. Each class consisted of 28 students. The students were of average ability. The classes were not randomly selected but were those that were taught by the investigator.

Instructional Materials

For this study, the instructional unit was unit one, "Working with Geometry" selected from the grade nine textbook Math is/Geometry (Ebos et al, 1981).

Behavioural objectives were written and categorized at three cognitive levels: computation, comprehension, and application, as discussed later in the section on evaluation instruments. The categorization was done by the investigator. The objectives are listed in Appendix A.

The unit was supplemented with 14 activities developed by the investigator or selected from other sources. These activities are found in Appendix B. Activities 2, 3, and 9, with some changes, were adopted from Krulik (1972). Activity 8 was taken from Olson (1975). Activity 11 was adopted from Rudisill and Wall (1979) and activities 12 and 13 were developed by Jencks and Peck (1974). The remaining activities were developed by the investigator. The supplementary activities involved the use of concrete materials such as the square geoboard and the circular geoboard, and methods such as paper cutting and paperfolding.

The activity sheets used contained a clear statement of each objective to be investigated, a listing of the materials needed, and the procedures to follow to carry out the activities.

A teacher's manual, found in Appendix C, was also prepared. In the manual, the purpose of each activity was stated and the places in the curriculum where the activity or activities were to be implemented were explained. The manual also informed the teacher of the prerequisite concepts for each activity.

Evaluation Instruments

In this section a description of the instruments used during the study to collect the data is presented.

Achievement Tests

The two achievement tests, developed by the investigator, are found in Appendix D. The first test was developed to determine whether the objectives listed for the first three topics were met by students. The second test was developed to determine whether the objectives for the last three topics were met.

The tests were designed to evaluate the objectives of the unit which were written at three cognitive levels: computation, comprehension, and application. According to Wilson (1971), test items to evaluate objectives at the computation level place emphasis on the recall of basic facts and terminology or the manipulation of problem elements according to the rules students have learned. For comprehension, test items require either recall of concepts and generalizations, or transformation of problem elements from one mode to another. To evaluate objectives at the application level test items are constructed so that the students have to recall relevant knowledge, select an appropriate operation, and then perform that operation.

The objectives are general in the sense that they cover the entire unit. For example objective 2 which states that "the student, through the use of inductive reasoning, should be able to determine properties of geometric figures" does not name a specific geometrical figure but all figures mentioned in the unit.

In Table 1 classification of each objective as computation, comprehension, or application, and the matching test items to evaluate that objective are presented. To determine reliability of the classification of test items, each test item was classified as computation, comprehension, or application by two other mathematics educators. One of these was a program coordinator for mathematics in a nearby district, while the other was the principal of the school where the study was conducted. For Test I they differed on six items while for Test II they differed on four items. They disagreed mostly on items that were either computation or comprehension. This might have been due to the fact that the dividing line between levels is sometimes fine and computation level items are sometimes incorporated within the comprehension level.

In Table 2, a summary of how each test item related to an activity is given. Some of the activities had a matching test item(s) of each type: computation, comprehension, or application, while the other activities had

Table 1

Classification of Objectives and Matching Test Items

Test I

Cognitive Level	Objective*	Matching Test Item(s)
Computation	1 (a)	2, 18
	1 (c)	1, 3, 5, 6, 7, 11, 17, 20, 23
Comprehension	1 (b)	27
	3	4, 8, 9, 12, 13
Application	2	24
	6	14, 15, 25
	7	10, 16, 19, 21, 22, 26, 28

Test II

Cognitive Level	Objective*	Matching Test Item(s)
Computation	1 (a)	4, 12, 13, 14
	1 (c)	3, 9, 16, 18 (ii), 18 (iii)
Comprehension	1 (b)	23
	3	2, 5, 6, 7, 11, 18 (iv)
	4	19
Application	2	20
	5	21, 22
	6	1, 17, 18 (i)
	7	8, 10, 15

* See Appendix A for objectives

Table 2

Activities, Matching Test Items and Cognitive Level

Activity	Test	Matching Test Item(s)		
		Computation	Comprehension	Application
1		1, 7, 23	-	10
2		-	12	14, 26
3		-	-	21, 25
4	I	-	8, 13	24
5		3	9	24
6		11, 18, 20	27(b) (c)	-
7		5	-	16, 19, 28
8		-	19	21
9		3, 4, 13	23 (2)	22
10		16	5	15
11	II	12, 18(ii) (iii)	6, 23(i)	-
12		-	7	1, 8, 17
13		-	11, 23 (3)	18(i)
14		9	-	10, 20

matching test item(s) at one or two of the cognitive levels. These activities were such that it was very difficult to get test items at all three cognitive levels. They were activities 1, 2, 3, 4, 6, 8, 11, 13, and 14.

The items on the tests were scored by giving one point if the item was answered correctly and zero points if not answered correctly. To obtain an estimate of the reliability of the test scores the Kuder-Richardson Formula 20 was used. The reliability coefficient for each achievement test was 0.65.

Student Attitude Questionnaire

The attitude pretest and post-test were parallel forms of a 24 item questionnaire designed by Aiken (1974) and modified by the investigator to correspond to geometry. A copy of the questionnaire is included in Appendix E.

Prior to completing the questionnaire, the students were informed that it was not a test and were asked to express their honest feelings in their response to the questions. The students responded to each question by checking one of the five letters (a, b, c, d, e) which were coded as: strongly agree, agree, no opinion, disagree, and strongly disagree, respectively. Twelve of the items on the questionnaire were positively stated while the other twelve items were negatively stated.

The scale used for scoring students response to each item is found in Appendix E. The most positive attitudes toward geometry was indicated by a score of 96 and the most negative attitudes toward geometry was indicated by a score of zero. A score of 48 was considered neutral. The reliability coefficient for the questionnaire was 0.81.

Procedure

The unit, including the activities designed to supplement the unit, was taught to two grade nine academic classes by the investigator over a period of seven weeks. The students received nine 40 minute sessions of instruction per six day cycle. The time spent on each activity is outlined in Table 3.

In addition to the periods allocated for the activities, two periods were used to administer achievement tests. The first test was given at the completion of the first three topics. The second test was given at the completion of the unit. The remaining periods were spent on the textbook material. In summary, 50 class periods were used for the completion of the unit. The attitude questionnaire for students was given as a pretest two days prior to the start of the unit and as a post-test two days after the completion of the unit.

The students involved used their own textbook accompanied by a booklet of activities. For each supplementary lesson concrete materials were provided for the students.

Table 3

Periods Allotted to Activities

Activity	Periods
1	2
2	1
3	1
4	1
5	2
6	½
7	3
8	1
9	2
10	2
11	2
12	1½
13	1
14	1

The supplementary activities were implemented into the lesson to help develop a concept or to discover a relationship. The purpose of each supplementary activity and the procedure to follow to do the activity were explained to the students. While the students were doing each activity, the investigator acted as a guide giving assistance when requested or needed but encouraging the students to complete the activities and hence to develop concepts through their own efforts. The investigator ensured that paperfolding, paper cutting, and the construction of geometric figures on the geoboard were done satisfactorily and with the necessary precision. After each lesson, appropriate exercises from the textbook were assigned to the students to do in class or at home for homework.

During the teaching of the unit the problems encountered with the activities were recorded by the investigator.

Pilot Study

A pilot study was conducted during the first month of school with the grade nine academic class that was not participating in the study. The purpose of the pilot study was mainly to examine the wording of the instructions, to ensure that students understood and followed the instructions, and to determine which activities were suitable and where they actually fitted into the regular curriculum.

The supplementary activities were integrated into Unit one by the investigator. After the completion of the unit, each activity was evaluated by the investigator using the results of the achievement tests administered. Also considered were student comments on the clarity of the objectives, directions, and presentation.

On the basis of this evaluation no supplementary activities were omitted or added, but some alterations were made to the instructions of activities 1, 4, 5, 7, 9, and 10. It seemed that many students, particularly those new to the approach, needed fairly specific instructions that indicated how they were to proceed.

Also a pilot study was done on the two achievement tests. The purpose of this pilot study was to examine (i) the time to do the tests, (ii) the wording of the test items to see if students knew what was being asked, and (iii) to examine the suitability of the items.

On the basis of this pilot study it was felt that not all of the objectives were adequately tested by both tests. Also, Test I did not require an entire period to complete. In Test I objectives 1 and 7 were not adequately tested so three items were deleted and five items added that dealt specifically with these objectives. For Test II objective 5 was not adequately tested so two items were deleted and two more added. No changes were made in the wording of any of the test items.

One problem that was encountered by the investigator at the beginning of the pilot study was that many of the students were unfamiliar with working with activities. Many of the students became quite impatient and wanted to be told what the conclusion would be when they could not arrive at it immediately. However, after much encouragement that they should develop concepts through their own efforts, this attitude changed. Also the students were not used to working with concrete materials. Much individual help was needed when these materials were in use.

Analysis of Data

To answer question one; "Does the use of a unit involving supplementary activities result in students attaining competence with geometric concepts?", two achievement tests were administered to each student. Each item on the tests were designed to test whether the objectives of the unit were met. A behavioural objective was considered met if the corresponding item(s) on the achievement test was correctly answered by an average of 75 percent of the students who completed the exam.

Since the objectives of the unit were written at three cognitive levels: computation, comprehension, and application, all the test items for each level were grouped together to determine whether the 75 percent performance level was achieved for each level of objectives.

In order to answer question two; "Does the use of a unit involving supplementary activities have any effect on student attitudes toward geometry?", the modified form of the Aiken's Scale of Attitudes Toward Mathematics described earlier was administered as a pretest and a post-test to each student. The following null hypothesis was tested: "There is no significant change in attitudes toward mathematics (geometry) between the beginning and end of the instructional unit." A dependent t - test for means was performed to determine any significant difference between the scores.

To answer question three; "How effective is each of the supplementary activities in helping students achieve each objective?", each activity was designed around a specific objective. Some items on the achievement tests were matched with this objective. The activity was considered effective in helping students achieve that objective if the corresponding items on the achievement test were answered correctly by an average of 75 percent of the students who completed the exam.

To answer question four; "What are some of the problems encountered using the activity approach to geometry?", any problems encountered with the unit and the activities were noted. These problems were discussed in the analysis of the results in Chapter IV.

CHAPTER IV

ANALYSIS OF DATA

In this chapter the analysis of the data collected during the investigation is reported. The results related to each of the four questions asked in Chapter I are stated.

Student Achievement

To answer question one; "Does the use of a unit involving supplementary activities result in students attaining competence with geometric concepts?", the results of the two achievement tests were analyzed. In Table 4 the data for Test I is reported while the data for Test II is given in Table 5.

For most of the objectives, as indicated in Tables 4 and 5, more than one related test item was constructed to test whether that objective had been met. To obtain the information on the number of students answering test items related to each objective correctly, the number of correct responses for each item was tabulated and the average taken. In some instances this average was artifically low or high depending on how large a range there was between the highest number of correct responses and the lowest number of correct responses. Objective 3, for example, had 11 matching test items. The number of correct responses for each item ranged from 30 to 54 with the greatest number of correct responses lying above 40. Since two of the items related to that

Table 4

Performance on the Objectives at each Cognitive Level on Test I

Cognitive Level	Objective	Number of Test Items	Mean Number of Correct Responses per Objective	Mean Percentage of Correct Responses per Objective
Computation	1(a)	2	42*	75
	1(c)	9	45	80
Comprehension	1(b)	1	49	88
	3	5	40	72
Application	2	1	36	65
	6	3	42	75
	7	7	40	72

* Maximum is 56

Table 5

Performance on the Objectives at each Cognitive Level on Test II

Cognitive Level	Objective	Number of Test Items	Mean Number of Correct Responses per Objective	Mean Percentage of Correct Responses per Objective
Computation	1(a)	4	35*	62
	1(c)	5	43	76
Comprehension	1(b)	1	42	75
	3	6	43	77
	4	1	30	54
Application	2	1	45	80
	5	3	36	65
	6	2	38	68
	7	3	31	55

* Maximum is 56

objective were answered poorly this caused the average number of students answering test items related to that objective to be low. This also occurred with objectives 1(c), 5, and 7.

Question one was answered by comparing the mean percentage of students answering test item(s) related to each objective correctly with the expected outcome of 75 percent. In each of Tables 4 and 5 the mean percentage of students answering test item(s) related to each objective is given. The objective, the number of test items related to each objective, and the cognitive level at which the objective is written are also reported.

The data given in Table 4 for Test 1 show that all the objectives written at the computation level were met. At the comprehension level the 75 percent criterion was met for objective 1(b) but not for objective 3. The 75 percent criterion was achieved for obejctive 6 at the application level but not for objectives 2 and 7.

The data given in Table 5 for Test II show that the 75 percent criterion was met for objective 1(c) at the computation level but not for objective 1(a). At the comprehension level, objectives 1(b) and 3 were considered met while objective 4 was not. The 75 percent criterion was achieved for objective 2 at the application level but not for objectives 5, 6, and 7.

When the data of both tests were combined, as reported in Table 6, only objective 1(c) at the computation level and

Table 6

Performance on the Objectives at each

Cognitive Level on Test I and II Combined

Cognitive Level	Objective	Number of Test Items	Mean Number of Correct Responses per Objective	Mean Percentage of Correct Responses per Objective
Computation	1(a)	6	39*	69
	1(c)	14	44	78
	1(b)	2	46	81
Comprehension	3	11	42	75
	4	1	30	54
Application	2	2	41	73
	5	3	36	65
	6	5	40	72
	7	10	36	65

* Maximum is 56

objectives 1(b) and 3 at the comprehension level met the 75 percent criterion.

In order to compare student achievement at the three cognitive levels: computation, comprehension, and application, the mean percentage of students answering test items belonging to each level correctly for each of the two achievement tests is reported in Table 7.

From the data given in Table 7 the criterion of an average of 75 percent of the students answering the test items at each level correctly was met only at the computation and comprehension levels for Test I. However, from the results test items at the computation and comprehension levels were answered by the greatest number of students and items at the application level were answered by the least number of students.

In summary, a definite yes or no answer cannot be given. In some of the cases the criteria is met while in others it is not. Therefore, further research is needed before a definite answer to this question can be given.

Student Attitudes

To answer question two; "Does the use of a unit involving supplementary activities have any effect on student attitudes toward geometry?", the results of the student attitude questionnaire were analyzed. Students' scores were included only if the student completed the questionnaire on both occasions.

Table 7

Mean Percentage of Students Answering Test Items at each Cognitive Level Correctly

	Cognitive Level		
	Computation	Comprehension	Application
Test I	78	80	71
Test II	69	68	67
Total	74	74	69

A dependent t-test for means was performed on the set of difference scores from these questionnaires. In Table 8 the mean and standard deviation for each of the pretest and

Table 8

Data for Student Attitude Questionnaire

Test	N	$\bar{X}$	S_X	t-value
Pretest	56	55.41	12.79	2.49
Post-test	56	57.73	11.72	

$p < 0.05$

post-test scores, and the t-value are reported.

A t-value of 2.49 indicated a significant gain in student attitudes toward geometry at the 0.05 level of significance during the seven week period in which unit one was taught. Therefore the null hypothesis "there is no significant change in attitudes toward mathematics (geometry) between the beginning and end of the instructional unit" was rejected.

Supplementary Activities

To answer question three; "How effective is each of the supplementary activities in helping students achieve each objective?", an analysis was done on the results of the test items that were matched to each activity. In Table 9 the data for activities 1 - 7 is presented while the data for activities 8 - 14 is given in Table 10. Some of the activities had at least one matching test item written at each of the computation, comprehension, and application levels, while others had matching test item(s) at one or two of the cognitive levels. Each activity was considered effective if the corresponding test items were answered correctly by an average of 75 percent of the students. Activities 1, 2, 3, 6, 7, 10, 11, 13, and 14 met the 75 percent criterion.

The 75 percent criterion was also achieved for test items at different cognitive levels for some of the activities.

Table 9

Performance on Items on Test I Related to Activities 1 - 7

Activity	Cognitive Level of Test Item(s)	Number of Test Items	Mean Percentage of Correct Responses per Cognitive Level	Mean Overall Percentage of Correct Responses
1	Computation	3	82	78
	Application	1	74	
2	Comprehension	1	91	82
	Application	2	73	
3	Application	2	84	84
4	Comprehension	2	65	65
	Application	1	65	
5	Computation	1	54	
	Comprehension	1	77	64
	Application	2	61	
6	Computation	3	90	89
	Comprehension	2	88	
7	Computation	1	84	79
	Application	3	73	

Table 10

Performance on Items on Test II Related to Activities 8 – 14

Activity	Cognitive Level of Test Item(s)	Number of Test Items	Mean Percentage of Correct Responses per Cognitive Level	Mean Overall Percentage of Correct Responses
8	Comprehension	1	54	59
	Application	1	63	
9	Computation	3	51	
	Comprehension	1	75	64
	Application	1	66	
10	Computation	1	96	
	Comprehension	1	82	83
	Application	1	71	
11	Computation	3	77	82
	Comprehension	2	86	
12	Comprehension	1	59	60
	Application	3	60	

Table 10 (Continued)

Activity	Cognitive Level of Test Items(s)	Number of Test Items	Mean Percentage of Correct Responses per Cognitive Level	Mean Overall Percentage of Correct Responses
13	Comprehension	2	79	82
	Application	1	84	
14	Computation	1	93	75
	Application	2	58	

The computation items for activities 1, 2, 6, 7, 10, 11, and 14 met the 75 percent criterion. The comprehension items for activities 2, 5, 6, 9, 10, 11, and 13 also met the 75 percent expected outcome, while this criterion was only met on activities 3 and 14 for the application items.

The data, as presented in Tables 9 and 10, indicated that activities appear to be most effective in helping students achieve objectives at the lower cognitive levels than at the application level.

Problems with the Activity Approach to Geometry

To answer question four; "What are some of the problems encountered using the activity approach to geometry?", the problems noted by the investigator during the study were analyzed.

One problem noted by the investigator was that students were unfamiliar with this type of approach and generally had a feeling of frustration when they were not able to come up with a conclusion immediately. Therefore some time was used to encourage students to discover and generalize on their own.

Another problem noted was that students were not familiar working with concrete materials, such as the geoboard, and paperfolding. Therefore considerable time had to be spent ensuring that students used the materials correctly.

Another problem encountered by the investigator was that more time than expected had to be spent with the slower students. As a result, many of the other students were deprived of immediate help, creating a situation where students' restlessness, and distracting noise sometimes existed in the classroom.

The investigator also noted that the students tended to jump to conclusions, to generalize on the basis of limited experience. The students had to be reminded that inductive reasoning does not prove that the conclusion is true, and that the more examples are used, the surer we can be of the conjecture.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The purpose of this study was to develop a set of activities to supplement a unit of work in the ninth grade academic geometry program, and to evaluate the benefits derived from it. In order to do this the following four questions were considered:

(1) Does the use of a unit involving supplementary activities result in students attaining competence with geometric concepts?

(2) Does the use of a unit involving supplementary activities have any effect on student attitudes toward geometry?

(3) How effective is each of the supplementary activities in helping students achieve each objective?

(4) What are some of the problems encountered using the activity approach to geometry?

In order to gather the required data, two intact ninth grade academic classes of 28 students each were selected and the treatment was employed. Fourteen activities were developed by the investigator or were selected from other sources to supplement the unit. To determine student achievement on the unit, two tests, constructed by the investigator, were administered. Both tests were designed to test whether the behavioural objectives for the unit had been achieved. A behavioural objective was considered met if the corresponding item(s) on the achievement test were correctly answered by an average of 75 percent of the students who completed the exam. Since the objectives for the unit were written at three cognitive levels; computation, comprehension, and application, the test items for each level were grouped together to determine whether the items for each level were answered correctly by an average of 75 percent of the students.

The tests were also designed so that most of the activities had matching test items written at three different cognitive levels; computation, comprehension, and application. Each activity was considered effective in helping students achieve that objective if the corresponding items on the achievement test were answered correctly by an average of 75 percent of the students.

A modified form of Aiken's Scale of Attitudes Toward Mathematics was given as a pretest and post-test to determine whether a unit involving supplementary activities had any effect on student attitudes toward geometry. A dependent t-test for means was performed on the set of difference scores.

The problems encountered by the investigator with the activity approach to geometry during the course of the study were recorded.

Conclusions

The results of this study were consistent with those by Vance (1969), Norman (1977), and Johnson (1971). They showed that the activity approach can be an effective method of instruction. The conclusions in this study are made relative to the four questions asked.

An analysis of the test results showed that many of the students failed to achieve mastery of all the geometric concepts at the three cognitive levels. This low level of attainment by some students could be attributed to several factors. One such factor was the unfamiliarity with the instructional approach. Many of the students were only familiar with the expository method of teaching and therefore had to make adjustments to the activity method of teaching. These students, learning to make discoveries on their own for the first time, quite often became frustrated when they were not able to come up with a conclusion immediately. This undoubtedly caused the students to miss some of the concepts and not fully understand others.

Another contributing factor was the tendency of students to jump to conclusions on the basis of limited experiences. Many of the students failed to understand that the more examples that are used, the more certain they can be of their conclusions. These results must also be interpreted in light of the limitation of the short time duration of the study. Probably students did not spend enough time on each activity to develop a full understanding of the concepts.

Also it was noted by the investigator that many of the students lacked some of the necessary prerequisite skills. This could undoubtedly be due to the omissions, or non emphasis, of the geometry content in previous grades in favour of developing other mathematical skills. The practice of omitting topics at the elementary level deprives the student of the proper prerequisite skills.

The findings did, however, show that students were more successful in answering items at the computation and comprehension levels than they were at the application level. This could be attributed to the fact that there might not have been enough time allotted to the objectives at the application level. Another factor could have been that students only seemed to be familiar with answering questions that required the recall of basic facts or concepts. It seemed that up to this point the students had done very little in terms of answering questions that required the recall of relevant knowledge, select an appropriate operation, and then perform that operation.

The data collected from the attitude questionnaire showed that students made a significant positive attitudinal change toward geometry. This result must be interpreted in the light of the limitation that the manipulation of the geoboard, paperfolding, and paper cutting could have been a novelty, hence could produce a significant attitude change over a short period of time. Also the positive attitudinal change towards geometry could have been brought about by the teaching approach used. Beberman (1958) stated that the key to maintaining interest in mathematics is that students be given ample opportunities to discover generalization by themselves.

The achievement test data indicated that many of the supplementary activities were effective in helping students achieve the related objectives. The activities appeared to be most effective, however, in helping students achieve mastery of the content at the lower cognitive levels. This result must be interpreted in the light of the limitation of the short time duration of the study. Perhaps more time is needed to teach objectives at the application level. It is apparent that the relative merits of economy of time and increased learning must be weighed carefully when formulating instructional decisions regarding supplementary activities.

Recommendations

On the basis of the findings of this study, several recommendations can be made. The most obvious is that the activity approach can be used as an alternative approach to the expository approach in developing certain geometric concepts. It is evident from the findings that supplementary activities can be effective in helping some students achieve mastery of the content at the lower cognitive levels. This seems to imply that work in geometry should not be restricted to just paper and pencil activities, but should include activities where students become involved with concrete materials. Since most textbooks do not provide adequately for this, teachers must provide such materials themselves.

When determining the effects of supplementary activities in an academic geometry course on student achievement and attitudes, the time period of the study should be considered. Thus a second recommendation is that a more extensive study on a larger scale be conducted to assess long term effects of supplementary activities on student achievement and attitudes.

Since this study was carried out on students of average ability, a third recommendation is that further research on ability levels is needed to clarify the effects of supplementary activities on achievement in and attitudes toward geometry of students of different abilities.

The subjects for this study were not randomly selected, but were two intact classes of grade nine academic students, and no control group was used. A fourth recommendation is that in future studies a control group be used and the students be randomly selected. This would provide more detailed information on achievement in and attitudes toward geometry.

The investigator was the only teacher involved in the study. A fifth recommendation is that in future studies other teachers be involved to determine how they feel toward using supplementary activities.

REFERENCES

Aiken, L.R. Attitudes toward mathematics and science in Iranian middle schools. School, Science and Mathematics, 1979, 79, 229 - 234.

Beberman, M. An emerging program of secondary school mathematics, Cambridge: Harvard University Press, 1958.

Begle, E.G. The role of research in the improvement of mathematics education. Education Studies in Mathematics, 1969, 2, 232 - 244.

Bernard, R.P. Historical development of the laboratory approach. Dissertation Abstract International, 1973, 33, 5028A.

Bobelstein, R.H. Supplementary materials and activities, their effort on attitudes and achievement in first year algebra. Dissertation Abstract International, 1979, 39, 6603A.

Bring, C.R. Effects of varying concrete activities on the achievement of objectives in metric and non-metric geometry by students of grades five and six. Dissertation Abstract International, 1972, 32, 3375A.

Brousseau, A.R. Mathematics laboratories: should we or should we not? School Science and Mathematics, 1973, 73, 99 - 105.

Bruner, J.S. Towards a theory of instruction, Cambridge: Harvard University Press, 1966.

Corwin, V.A. A comparison of learning geometry with or without laboratory activities using manipulative aids and paperfolding techniques. Dissertation Abstract International, 1978, 38, 6584A - 6585A.

Davidson, P.S., & Walter, M.I. The laboratory approach. In W.C. Lowry (ed). The Slow Learner in Mathematics, Thirty-fifth Yearbook of the National Council of Teachers of Mathematics, Washington, 1972, 221 - 281.

Davis, R.B. The Madison Project of Syracuse University. The Mathematics Teacher, 1960, 53, 571 - 575.

Deans, E. The laboratory approach to elementary mathematics. Today's Education, 1971, 60, 20 - 22.

Dienes, Z.P. Some basic processes involved in mathematics learning. In J.M. Scandura (ed). Research in Mathematics Education, Washington: NCTM, 1967, 21 - 34.

Ebos, F., Tuck, B., Hatcher, G., & Drost, D. Math is/Geometry, Ontario: Nelson of Canada Ltd., 1981.

Friedman, M. The manipulate materials strategy: the latest pied piper? Journal for Research in Mathematics Education, 1978, 19, 78 - 80.

Goals for School Mathematics, report of the Cambridge conference on School mathematics, Boston: Houghton Mifflin Co., 1963.

Jenchs, S.M., & Peck, D.M. 'Thought starters' for the circular geoboard. The Mathematics Teacher, 1974, 67, 228 - 233.

Johnson, R.E. The effects of activity oriented lessons on achievement and attitudes of seventh grade students in mathematics. Dissertation Abstract International, 1971, 32, 305A.

Johnson, R.L. Effects of varying concrete activities in perimeter, area, and volume by students of grade four, five, and six. Dissertation Abstract International, 1971, 31, 4624A.

Jungst, D.G. Elementary Mathematics: Laboratory Manual, Boston: Allyn and Bacon, Inc. 1975.

Kidd, K., Myers, S., & Cilley, D. The laboratory approach to mathematics, Chicago: Science Research Associates, Inc., 1970.

Kieren, T.E. Activity learning, Review of Education Research, 1969, 39, 509 - 522.

Krulik, S. A mathematics laboratory handbook for secondary schools, Toronto: W.B. Saunders Co., 1972.

Kujawa, E., Jr. The effects of a supplementary mathematics laboratory on the mathematical achievement and attitudes toward mathematics of students in the intermediate grades: four, five, and six. Dissertation Abstract International, 1976, 37, 3378A.

London, E. A comparative study of the achievement of urban eighth grade mathematics students using an activity oriented mode of instruction and a conventional textbook mode. Dissertation Abstract International, 1978, 39, 2113A.

Mathematics Bulletin, Grade 7-11, Division of Instruction, Department of Education, Province of Newfoundland, 1981-82.

Monier, M.I. Some effects of an activity approach to teaching geometry in the high schools in Afghanistan. Dissertation Abstract International, 1977, 38, 2630A - 2631A.

Norman, F. Activity oriented instruction versus traditional textbook method in tenth grade geometry: a comparative study. St. John's, Newfoundland, Memorial University of Newfoundland, 1977.

Olson, A.T. Mathematics through paperfolding. Washington, D.C. NCTM, 1975.

Piaget, J. Cognitive development in children: development and learning. Journal of Research in Science and Teaching, 1964, 2, 176 - 186.

Prigge, G.R. The differential effects of the use of manipulative aids on the learning of geometric concepts by elementary school children. Dissertation Abstract International, 1974, 35, 3307A.

Purser, J.D. The relation of manipulative activities, achievement and retention in a seventh grade mathematics class: an exploratory study. Dissertation Abstract International, 1973, 31, 3255A - 3256A.

Riggs, F. Activities in mathematics. Teaching Mathematics, 1974, 1, 3 - 4.

Rudisill, M., & Wall, C. Circular geoboard tasks. In L.E. Voclko (ed). Manipulative activities and games in the mathematics classroom, National Education Association Publication, The Curriculum Series, 1979.

Scheer, J.K. Effects of supplemental diagnostic/prescriptive teaching on mathematical achievement, attitude, and self concept. Dissertation Abstract International, 1977, 38, 681A.

Smith, E.B.D. The effects of laboratory instruction upon achievement in and attitudes toward mathematics on middle school students. Dissertation Abstract International, 1974, 34, 3715A.

Vance, J.H., & Kieren, T.E. Laboratory settings in mathematics: what does research say to teachers. The Arithmetic Teacher, 1971, 18, 585 - 589.

Vigilante, N.J. Geometry for primary children: considerations. The Arithmetic Teacher, 1967, 14, 453 - 459.

Whipple, R.M. A statistical comparison of the effectiveness of teaching metric geometry by the laboratory and individual instruction approaches. Dissertation Abstract International, 1972, 33, 2699A.

Wilkinson, G.G. The effects of supplementary materials upon academic achievement in and attitudes toward mathematics among eighth grade students. Dissertation Abstract International, 1971, 32, 1994A.

Wilkinson, J.D. A laboratory method to teach geometry in selected sixth grade mathematics classes. Dissertation Abstract International, 1970, 31, 4637A.

Wilson, J.W. Evaluation of learning in secondary school mathematics. In Bloom, B.S., Hastings, J.T., & Madaus, G.F. (ed). Handbook on formative and summative evaluation of student learning, New York: McGraw Hill, Inc. 1971, 643 - 696.

Wong, M.L. The effects of handling manipulative devices on the learning of selected concepts in geometry - a study with thirteen year olds in Malaysia. Dissertation Abstract International, 1979, 39, 5384A - 5385A.

Appendix A

Behavioural Objectives

Behavioural Objectives

1. For each term in the vocabulary list, the student should be able to:
   (a) give the definition.
   (b) construct an example.
   (c) recognize an example.
2. The student, through the use of inductive reasoning, should be able to determine properties of geometric figures.
3. The student should be able to:
   (a) recall.
   (b) illustrate the properties (listed in the property list) of geometric figures.
4. Using compass and straight edge, the student should be able to perform the following constructions.
   (a) To copy a given angle.
   (b) To bisect an angle.
   (c) To construct a line perpendicular to a given line at a point on the line.
   (d) To construct a perpendicular to a line from a point not on the line.
   (e) To construct the perpendicular bisector of a line segment.
   (f) To construct a line parallel to a given line through a point not on the line.
5. The student, using his knowledge of constructions, should

be able to construct geometric figures.

6. Given the measure of various angles in a diagram, the student will have to use his knowledge of certain geometric properties to calculate the measure of various unknown angles in the diagram.
7. The student, using his knowledge of properties of geometric figures, should be able to solve problems related to these figures.

<u>Vocabulary List</u>

line segment
ray
angle
acute angle
obtuse angle
straight angle
right angle
reflex angle
opposite rays
adjacent angles
supplementary angles
complementary angles
vertically opposite angles
congruent angles
congruent segments
parallel lines
perpendicular lines
corresponding angles
alternate angles
polygon
regular polygon
triangle
quadrilateral
pentagon
hexagon

heptagon

octagon

nonagon

decagon

isosceles triangle

equilateral triangle

scalene triangle

acute angled triangle

right angled triangle

obtuse angled triangle

equianglar triangle

parallelogram

square

rhombus

rectangle

trapezoid

altitude of a triangle

median of a triangle

bisector

orthocenter

circumcentre

incentre

centroid

circle

circumference

radius

arc

diameter

semicircle

chord

sector

segment of a circle

inscribed angle

central angle

tangent

inscribed quadrilateral

diagonal

Property List

1. If two lines intersect, vertically opposite angles have equal measures.
2. If two sides of a triangle are congruent, then the angles opposite these sides are congruent.
3. If two angles of a triangle are congruent, then the sides opposite these angles are congruent.
4. The three angles of an equilateral triangle are congruent.
5. The measure of an exterior angle of a triangle is equal to the sum of the measures of the two opposite interior angles.
6. The sum of the measures of the interior angles of a triangle is 180°.
7. The sum of the measures of the interior angles of a quadrilateral is 360°.
8. If two parallel lines are cut by a transversal, the corresponding angles are congruent.
9. If two parallel lines are cut by a transversal, the alternate angles are congruent.
10. If two parallel lines are cut by a transversal, the interior angles on the same side of the transversal are supplementary.
11. In a given polygon, the number of diagonals that can be drawn from any one vertex is three less than the number of sides.

12. In a given polygon, the number of triangles formed by constructing all the diagonals from any one vertex is two less than the number of sides.
13. The sum of the measures of the interior angles of any polygon is equal to the number of triangles formed by constructing all the diagonals from any one vertex times 180°.
14. In a parallelogram, any two opposite angles have equal measures.
15. In a parallelogram, any two opposite sides are both congruent and parallel.
16. In a parallelogram, any two consecutive angles are supplementary.
17. In a rectangle, the diagonals have equal measures.
18. In a square, the diagonals have equal measures.
19. In a rhombus, any two opposite angles have equal measures.
20. In a parallelogram, the diagonals bisect each other.
21. In a rhombus, the diagonals are right bisectors of each other.
22. The altitudes of a triangle meet in one point called the orthocenter.
23. The bisectors of the angles of any triangle meet in one point called the incentre.
24. The right bisectors of the sides of any triangle meet in one point called the circumcentre.
25. The medians of a triangle meet in one point called the

centroid.

26. In an equilateral triangle the right bisector of a side of the triangle passes through a vertex.

27. In an isosceles triangle, the bisector and the altitude of the vertical angle coincide.

28. The perpendicular drawn from the center of a circle to a chord divides the chord into two congruent segments.

29. The measure of a central angle is equal to the measure of its intercepted arc.

30. The measure of an inscribed angle is equal to half the measure of its intercepted arc.

31. The measure of the inscribed angle is equal to half the measure of the central angle intercepted by the same arc.

32. Inscribed angles intercepted by the same arc have the same measures.

33. Opposite angles of an inscribed quadrilateral are supplementary.

34. When a line is tangent to a circle, it is perpendicular to the radius drawn to the point of contact.

35. Tangents drawn to a circle at each end of a diameter are parallel.

36. The right bisector of a chord contains the center of the circle.

37. Congruent chords are equidistant from the center of the circle.

Appendix B

Supplementary Activities

Activity #1

Performance Objective

The student should be able to:

(i) construct the following geometric figures - line segments and angles.

(ii) construct angles that are acute, right, obtuse, and straight.

(iii) construct geometric figures such as line segments and angles that are congruent.

Materials

Geoboard

Rubber Bands

Procedure

1. Line segments. On a geoboard a line segment is represented by looping a rubber band around two nails. Make a line segment on your geoboard.
2. Repeating the procedure in exercise 1, make the:
   (i) longest line segment possible.
   (ii) next longest line segment possible.
   (iii) shortest line segment possible.
3. Angles. On a geoboard an angle is represented by showing a pair of line segments which share exactly one point - a point which is an endpoint of each segment.
   (a) Show an angle whose sides touch as many nails

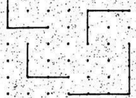
as possible.

(b) Can you show another angle whose sides touch the same number of nails as the one in exercise 1?

4. On the geoboard, show an angle with the smallest opening possible. While leaving one of the rubber bands around the same nails, keep moving the other rubber band so that different angles are formed. What do you notice about the angles?

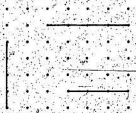
5. Since angles of different sizes can be made, show on the geoboard:

(a) an angle that looks like the corner of a book.



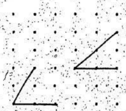
This kind of angle is called a right angle. On the geoboard, show different models of right angles.

(b) an angle that looks like the edge of a book.



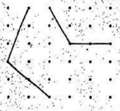
This kind of angle is called a <u>straight angle</u>. On the geoboard, show different models of straight angles.

(c) an angle that is smaller than a right angle.



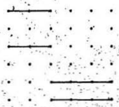
This kind of angle is called an <u>acute angle</u>. On the geoboard, show different models of acute angles.

(d) an angle that is smaller than a straight angle but larger than a right angle.



This kind of angle is called an <u>obtuse angle</u>. On the geoboard, show different models of obtuse angles.

6. On the geoboard, form a model of two congruent line segments as <u>shown</u> in the diagram.



What do you notice about congruent line segments?

7. (a) Show other models of two or more congruent line segments.

(b) Show a model of two noncongruent line segments.

8. (a) On the geoboard, form a model of two congruent angles as shown in the diagram.

What do you notice about congruent angles?

(b) Show other models of two angles that are congruent.

(c) Show a model of two angles that are not congruent.

Activity #2

General Objective

The student, through the use of inductive reasoning, should be able to determine properties of geometric figures.

Performance Objective

The student should be able to find the sum of the measures of the interior angles of a triangle.

Materials

Construction Paper

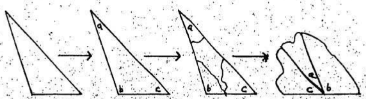
Scissors

Ruler

Pencil

Procedure

1. Draw a triangle on the construction paper and cut it out. Label each angle of the triangle. Cut the triangle into three pieces so that each piece has one angle from the original triangle. Place the three original angles so that their vertices are a common point and the sides of the middle piece touch a side of each of the outside pieces with no overlapping. Pictorially, the procedure has been as illustrated.



What is true of the final figure which has been constructed? How many degrees are in the angle formed by the outside edges of the two outside pieces?

2. Cut out another triangle of different size from the construction paper. Repeat the procedure you followed in exercise 1 using this triangle. What can you tell about the sum of the three angles of this triangle?

3. Repeat the procedure with another triangle. What can you tell about the sum of the three angles of this triangle? Does it seem that this result would be true for all triangles?

4. What conclusion can you make about the sum of the measures of the interior angles of a triangle?

Activity #3

General Objective

The student, through the use of inductive reasoning, should be able to determine the properties of geometric figures.

Performance Objective

The student should be able to determine the sum of the measures of the interior angles of a quadrilateral.

Materials

Construction Paper

Scissors

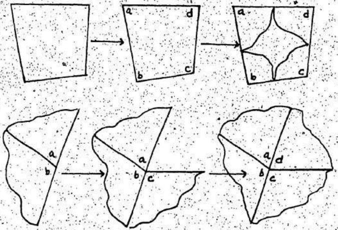
Ruler

Pencil

Procedure

1. Draw a quadrilateral on the construction paper and cut it out. Label each angle of the quadrilateral. (For this discussion use angles a, b, c, and d.) Cut the quadrilateral into four pieces so that each piece has one angle from the original quadrilateral. Place angle "a" on a surface. Put angle "b" besides "a" so that they have a common vertex and their sides are touching. Now place angle "c" with angles "a" and "b" so that all three have a common vertex and also place it so that a side of angle "c" touches a side of either angle "a" or angle "b". Angle "d"

should fit in the remaining space so that it has the common vertex that the other three angles have and the sides of angle "d" touch the sides of two of the angles with no overlapping. Pictorially, the procedure has been as shown in the diagram.



What is the measure of the angle formed by the outside edges of the first and last pieces?

2. Draw two more quadrilaterals of different sizes and shapes and repeat the procedure described previously. What can you tell about the sum of the four interior angles of each quadrilateral?

3. Write a probable conclusion about the sum of the measures of the interior angles of a quadrilateral.

Activity #4

General Objective

The student, through the use of inductive reasoning, should be able to determine the properties of geometric figures.

Performance Objective

The student should be able to determine some properties of angles and triangles.

Materials

Paper

Ruler

Pencil

Scissors

Procedure

1. (a) On the paper, draw two lines AB and CD intersecting at O to form vertically opposite angles. Fold and crease the paper through vertex O, placing BO on CO. Do AO and DO coincide? What do you notice about ∠AOC and ∠BOD?

(b) On the paper, draw again the two lines AB and CD intersecting at O. This time, fold and crease the paper through vertex O, placing CO on AO. Do BO and DO coincide? What do you notice about the angles?

(c) Using the above results write a probable conclusion about vertically opposite angles.

2. (a) Draw an isosceles triangle ABC with AB = AC. Fold and crease the paper so that AB lies on AC. What do you notice about ∠B and ∠C?

(b) Draw another isosceles triangle. Fold the paper so that the two congruent sides are placed together. What do you notice about the two angles?

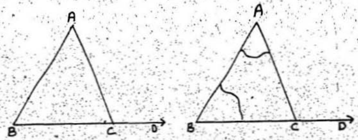
(c) Using the above results write a probable conclusion.

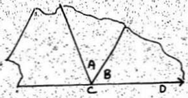
3. (a) Draw an equilateral triangle ABC. First, fold and crease the paper so that AB lies on AC. What do you notice about ∠B and ∠C? Secondly, open the paper and refold and crease so that BC lies on BA. What do you notice about ∠C and ∠A? Finally, open and refold so that CB lies on CA. What do you notice about ∠A and ∠B? What can you conclude about ∠B, ∠A and ∠C?

(b) Draw another equilateral triangle and repeat the procedure described in part (a).

(c) Using the above results write a probable conclusion.

4. (a) On the paper, draw triangle ABC with exterior angle ACD as shown.





Cut part of triangle ABC out so that you can cut angles A and B off as shown. Then place ∠A and ∠B on ∠ACD as shown with their vertices at C. What do you notice about ∠A, ∠B, and ∠ACD?

(b) Draw another triangle with an exterior angle and repeat the procedure described in part (a).

(c) Using the above results write a probable conclusion?

Activity #5

General Objective

The student, through the use of inductive reasoning, should be able to determine the properties of geometric figures.

Performance Objective

The student should be able to determine the properties of pairs of angles when two lines are parallel.

Materials

Paper

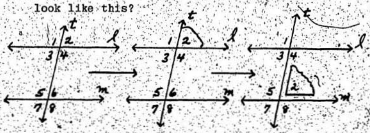
Pencil

Ruler

Scissors

Procedure

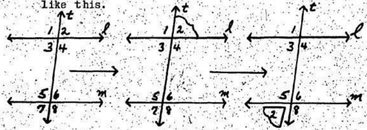
1. (a) Draw two parallel lines ℓ and m cut by transversal t on the paper. Label the angles using the numbers from 1 to 8. Then pick a pair of corresponding angles and cut one of the angles out and place it on top of the other angle. What do you notice? Pictorially, it would look like this?



(b) Using the same diagram, pick another pair of corresponding angles and repeat the procedure as described in part (a). What do you notice?

(c) Write a probable conclusion about corresponding angles.

2. (a) Draw another pair of parallel lines ℓ and m cut by transversal t. Pick a pair of alternating angles and cut one of the angles out and place it on top of the other angle. What do you notice? Pictorially, it would look like this.

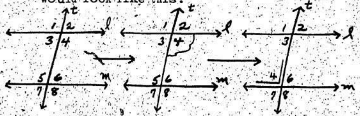


(b) Using the same diagram, pick another pair of alternating angles and repeat the procedure as described in part (a). What do you notice?

(c) Write a probable conclusion about alternating angles.

3. (a) Draw another pair of parallel lines ℓ and m cut by transversal t. Pick a pair of interior angles on the same side of the transversal and cut one of the angles out and place it by the side of the other angle. What do you

notice about the two angles? Pictorially, it would look like this.



(b) Using the same diagram, pick out another pair of interior angles on the same side of the transversal and then repeat the procedure as described in part (a). What do you notice about the two angles?

(c) Write a probable conclusion about the interior angles on the same side of the transversal.

Activity #6

Performance Objective

The student should be able to construct the following geometric figures: triangles, quadrilaterals, and other polygons.

Materials

Geoboard

Rubber Bands

Procedure

On the geoboard, polygons are represented by stretching a rubber band (or rubber bands) around three or more nails to enclose a region on the board.

1. Using a geoboard:

   (a) form a model of a polygon by looping a rubber band around three nails which are not on the same line segment. This figure is called a <u>triangle</u>. Form other models of triangles on the geoboard. How many sides does a triangle have?

   (b) form a model of a polygon by looping a rubber band around four nails which are not on the same line segment. This figure is called a <u>quadrilateral</u>. Form other models of quadrilaterals on the geoboard. How many sides does a quadrilateral have?

(c) form a model of a polygon by looping a rubber band around five nails which are not on the same line segment. This figure is called a pentagon. Form other models of pentagons on the geoboard. How many sides does a pentagon have?

(d) form a model of a polygon by looping a rubber band around six nails which are not on the same line segment. This figure is called a hexagon. Form other models of hexagons on the geoboard. How many sides does a hexagon have?

(e) form a model of a polygon by looping a rubber band around seven nails which are not on the same line segment. This figure is called a heptagon. Form other models of heptagons on the geoboard. How many sides does a heptagon have?

(f) form a model of a polygon by looping a rubber band around eight nails which are not on the same line segment. This figure is called an octagon. Form other models of octagons on the geoboard. How many sides does an octagon have?

Activity #7

General Objective

The student, through the use of inductive reasoning, should be able to determine the properties of geometric figures.

Performance Objective

Given a polygon, the student should be able to determine:

(i) the relationship between the number of sides of a polygon and the number of diagonals that can be drawn from any one vertex.

(ii) the sum of the measures of the interior angles of a polygon.

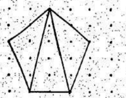
Materials

Geoboard

Rubber Bands

Procedure

1. Use the geoboard to form a model of a pentagon and the diagonals of the pentagon from any one vertex as shown.



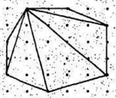
How many diagonals can be drawn from any one vertex?

2. Using the geoboard, form a model for each of the following polygons, and then determine the number of diagonals that have a given vertex as an endpoint.

Name of polygon	Number of sides	Number of diagonals with given vertex as an end-point
triangle	3	0
quadrilateral	4	
hexagon		
heptagon		
octagon		

3. From the pattern you have discovered in exercise 2, write a formula to express the relationship between the number of sides of a polygon and the number of diagonals drawn from any one vertex.

4. Using the geoboard, form a model of a heptagon and the triangles that can be formed by the diagonals drawn from any one vertex as shown.



(i) How many triangles are formed by drawing all the diagonals from one vertex?

(ii) What is the sum of the measures of the interior angles of a heptagon?

5. Using the geoboard, form a model for each of the following polygons. First determine the number of triangles formed by the diagonals drawn from any one vertex, and then the sum of the measures of the interior angles of the polygon.

Name of polygon	Number of sides	Number of triangles formed by drawing all diagonals from one vertex	Sum of the measures of the interior angles
triangle	3		180
quadrilateral	4	2	2 . 180 = 360
pentagon	5		
hexagon			
octagon			

6. From the pattern discovered in exercise 5, write a formula to express the relationship between the number of triangles formed by drawing all the diagonals from one vertex and the sides of a polygon.

7. From the pattern discovered in exercise 5, write a formula for finding the sum of the measures of the interior angles of a polygon.

Activity #8

Performance Objective

The student should be able to:

(i) bisect an angle.

(ii) construct a line perpendicular to a given line.

(iii) construct a perpendicular to a line at a point on the line.

(iv) construct the perpendicular to a line from a point not on the line.

(v) construct the perpendicular bisector of a line segment.

Materials

Paper or Waxed Paper

Pencil

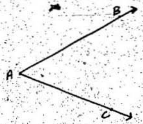
Ruler

Procedure

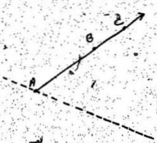
Some of the constructions done by compass and straight edge can be done by paper folding. You will need some paper and a set of instructions to guide you in these paper folding activities. New paper should be used for each activity.

(i) To bisect an angle. Construct a model of an angle on a piece of paper as shown in Step 1. Fold the paper so that the two sides of the angle coincide, then make a sharp crease along the fold as shown in Step 2. Unfold the paper as shown in Step 3;

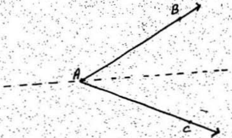
the crease is the bisector of angle BAC.



Step 1



Step 2



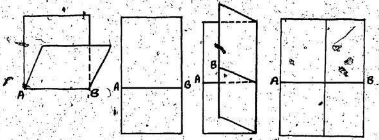
Step 3

(ii) A line perpendicular to a given line.

(a) Fold the paper to make a given line AB.

(b) Open the paper to its original form.

(c) Refold the paper so that part of the given line can be made to fold over onto itself. Then the perpendicular can be constructed.



(iii) The perpendicular to a line at a point on the line.

(a) Fold the paper to make a given line AB.

(b) Open the paper to its original form and place a point P on the line AB.

(c) Refold the paper so that part of the given line AB is folded over onto itself and that the crease passes through the given point P.

(iv) Following similar directions as described in exercise 3, fold the paper to construct a perpendicular to a given line from a point not on the line.

(v) The perpendicular bisector of a given line segment.

(a) Fold the paper to make a line segment AB.

(b) Open the paper to its original form.

(c) Refold the paper so that the endpoints of the given line segment AB are coincident. Why is the crease CD the perpendicular bisector of AB? Is this point equally distant from A and B?

Activity #9

General Objective

The student, through the use of constructions, should be able to construct geometric figures.

Performance Objective

The student should be able to determine the lines containing the altitudes, the perpendicular bisectors of the sides, the medians, and the angle bisectors of a triangle.

Materials

Paper for paper folding

Pencil

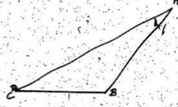
Ruler

Procedure

1. (a) On a piece of paper draw an acute triangle ABC. By letting points A and B coincide, we can draw along the resulting crease to obtain the perpendicular bisector of segment AB. Obtain all three perpendicular bisectors, one for each side of the triangle. Do the perpendicular bisectors of the sides of triangle ABC have a point in common? Is the point inside or outside the original triangle?

   (b) Repeat part (a) using a model of a right triangle.

(c) Trace triangle ABC as shown. Repeat part (a) using the traced triangle ABC.



(d) State the generalization suggested in parts (a), (b), and (c).

2. (a) On a piece of paper draw an acute triangle PQR. Fold the three altitudes. An altitude from a vertex can be formed by folding the opposite side upon itself such that the crease passes through the given vertex. The crease is the altitude. Obtain all three altitudes, one from each vertex of the triangle. Do these three lines meet in a single point? Is the point inside or outside the original triangle?

(b) This time, draw a right angled triangle on the paper and paper fold the three altitudes of this triangle. Do these three altitudes have a point in common? Where?

(c) Draw an obtuse triangle on your paper and paper fold the three altitudes of this triangle. Do these three altitudes have a point in common? Is the point outside or inside the original triangle?

(d) State the generalization suggested in parts (a), (b), and (c).

3. (a) On a piece of paper draw a right angled triangle XYZ. Fold the three angle bisectors. An angle bisector can be formed by folding one side of the angle on top of the other side and creasing. The crease through the vertex is the angle bisector. Obtain all three angle bisectors, one for each angle of the triangle. Do the three angle bisectors of triangle XYZ have a point in common?

(b) Repeat part (a) using an acute triangle XYZ.

(c) State the generalization suggested in parts (a) and (b).

4. (a) On a piece of paper draw an obtuse triangle DEF. Fold the three medians. To fold a median to a given side first bisect the side. This can be done by folding it upon itself such that the endpoints coincide. Then fold a crease through the midpoint and the opposite vertex. This crease is a median. Obtain all three medians of the triangle. Do the medians of triangle DEF have a point in common?

(b) Repeat part (a) using an acute triangle DEF.

(c) State the generalization suggested in parts (a) and (b).

Activity #10

General Objective

The student, through the use of inductive reasoning, should be able to determine the properties of geometric figures.

Performance Objective

The student should be able to determine the properties of a parallelogram.

Materials

Paper

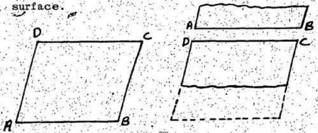
Scissors

Pencil

Ruler

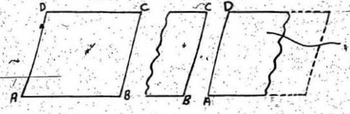
Procedure

1. (a) On the paper, construct a parallelogram. Label each angle A, B, C, and D. Cut the parallelogram out and then cut the parallelogram into two pieces as shown and place on a surface.



What do you notice about the segments AB and CD?

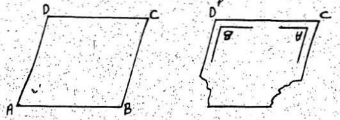
(b) On the paper, construct the same parallelogram. Label each angle A, B, C, and D. Cut the parallelogram out and then cut the parallelogram into two pieces as shown and place on a surface.



What do you notice about the segments AD and BC?

(c) Write a probable conclusion about the opposite sides of a parallelogram.

2. (a) On the paper, construct another parallelogram. Label each angle A, B, C, and D. Cut the parallelogram out and then cut ∠A and ∠B off as shown and place on a surface.

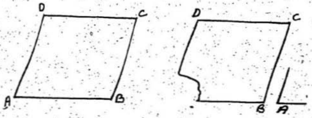


What do you notice about ∠ A and ∠C? ∠ B and ∠D?

(b) Draw another parallelogram and repeat the procedure as described in exercise 2 (a). What do you notice about the opposite angles?

(c) Write a probable conclusion about the opposite angles of a parallelogram.

3. (a) On the paper, construct another parallelogram and label each angle A, B, C, and D. Cut the parallelogram out and then cut off ∠A as shown and place on a surface.



What do you notice about ∠ A and ∠ B?

(b) Draw another parallelogram and repeat as described in exercise 3 (a). What do you notice about the two consecutive angles?

(c) Write a probable conclusion about any two consecutive angles of a parallelogram.

Activity #11

Performance Objective

Given a circle, the student should be able to illustrate the basic parts of the circle.

Materials

- Rubber Bands
- String
- Protractor
- Ruler
- Circular Geoboard

Procedure

A circle is a plane closed curve. It separates the plane into three subsets: the circumference, the interior of the circle and the exterior of the circle (Figure 1).

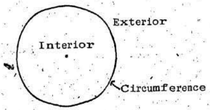


Figure 1

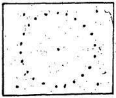
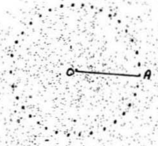


Figure 2

The geoboard you will be using is called a circular geoboard, and it has a nail in the interior of the circle called the center.

1.



(a) Connect the center of the circle with nail A on the circumference of the circle. This line segment is called the radius (r). Use your ruler to measure the radius OA.

(b) Connect the center of the circle with any other nail on the circumference of the circle. Measure the radius.

(c) What can you conclude about the distance between the center of the circle and any point on the circumference?

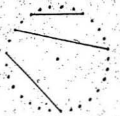
2. We can connect a rubber band to two nails on the circle so that it passes through the center of the circle. This is called a diameter (d) of the circle. Connect nails to make four diameters in different directions. Measure the lengths of these diameters. What do you notice about the lengths of the diameters?

3. Use your ruler to measure the length of one of the diameters. Compare your results with the length of

a radius in exercise 1. What do you notice about the length of the diameter and the length of the radius?

4. Use a rubber band to connect any two nails on the circumference of the circle. This line segment is called a chord. A chord begins on the circumference of the circle and ends on the circumference of the circle.

(a) Make some chords on the geoboards as shown.

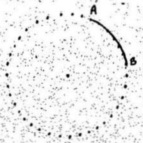


(b) Look at the shortest chord you can make. How many of these chords can you make?

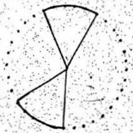
(c) What is the name of the longest chord you can make?

(d) How many of these can you make?

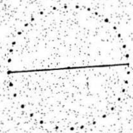
5. A chord cuts off a portion of the circumference of the circle called an arc. Put your finger on point A. Trace along the circumference of the circle until you come to point B. This is called arc AB. Form some arcs on your geoboard.



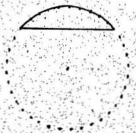
6. A sector is a figure made by two radii and the arcs between their endpoints. Form some sectors on your geoboard.



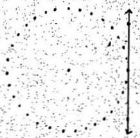
7. Divide the circle into two equal sectors. These sectors are called semicircles. What chord divides the circle into two equal parts?



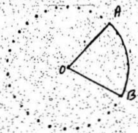
8. A <u>segment</u> is a chord and the arc between its end-points. Make some segments on your geoboard.



9. Connect two adjacent exterior nails. Notice that the line formed barely touches one nail on the boundary of the circle. This line is said to be tangent to the circle and is, therefore, called a <u>tangent line</u>. Practice making some tangent lines.

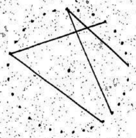


10. Stretch a rubber band forming arc AB to the center of the circle as shown.



The angle formed is said to be subtended by arc AB. The angle is also called a central angle. A central angle is an angle formed by two radii of a circle. What two radii form the central angle subtended by arc AB? Make some central angles on your geoboard.

11. Make any two chords on your geoboard so that their point of intersection is a point on the circumference of the circle.



These angles are called inscribed angles. Make some inscribed angles on your geoboard.

Activity #12

General Objective

The student, through the use of inductive reasoning, should be able to discover the properties of geometric figures.

Performance Objective

The student should be able to state the relationship between the measure of an inscribed angle and a central angle which intercepts the same arc.

Materials

Rubber Bands

String

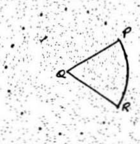
Protractor

Ruler

Circular Geoboard

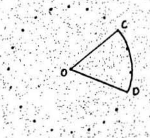
Procedure

1. Use the geoboard to set up the central angle as shown in the diagram.



Arc PR and central angle PQR are measured in degrees. Use a protractor to measure angle PQR. The number of degrees in angle PQR will be the number of degrees in arc PR.

2. On the geoboard, make the central angle subtended by arc CD.

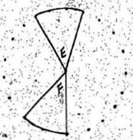


(a) What is the degree measure of angle COD?

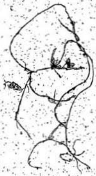
(b) What is the degree measure of arc CD?

3. (a) Make the smallest central angle you can form on your geoboard. What is the degree measure of the angle?

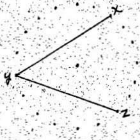
(b) Each central angle formed by connecting two nails that are next to each other with the center equals 15°.



What do you think a central angle like angle E will measure? What is the degree measure of angle F? Write a formula that will give us the number of degrees in our central angle for this circle.

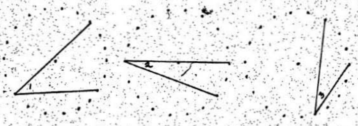


4. Use the geoboard to set up an inscribed angle as shown in the diagram.



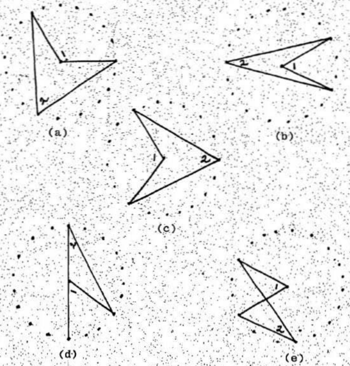
Arc XZ and inscribed angle XYZ are measured in degrees. Use a protractor to measure angle XYZ. What do you notice about the degree measure of the inscribed angle and the intercepted arc?

5. What can you say about the degree measure of angle 1, angle 2, and angle 3 in the following diagrams?



6. Form each diagram as shown on the geoboard. Find the degree measure of angle 1 and angle 2 for each diagram. Record results in this chart.

	a	b	c	d	e
angle 1					
angle 2					



(a) (b) (c) (d) (e)

Compare the degree measure of the inscribed angles and the central angles.

7. Using your results from exercise 6, write a conclusion about the inscribed angle and its corresponding central angle.

Activity #13

General Objective

The student, through the use of inductive reasoning, should be able to determine the properties of geometric figures.

Performance Objective

Given an angle inscribed in a semicircle, the student should be able to write a generalization about the measure of all angles inscribed in a semicircle.

Materials

Rubber Bands

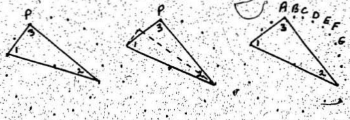
String

Circular Geoboard

Ruler

Procedure

Set up the following figures on your geoboard.



(a) (b) (c)

(i) In figure (a) find the degree measure of angle 1. How many degrees are there in each of the other angles of the triangle?

(ii) Start with the same figure used in (a) but move P to a new nail (figure b). What happens to the degree measures of angles 1, 2 and 3?

(iii) Now move P to points A, B, C, D, E, F, and G. Use the following chart to record the results.

	A	B	C	D	E	F	G
angle 1							
angle 2							
angle 3							

What happens to angles 1, 2, and 3? Did you notice anything unique about the measure of angle 3?

(iv) Write a generalization about the measure of the angle inscribed in a semicircle.

Activity #14

General Objective

The student, through the use of inductive reasoning, should be able to determine the properties of geometric figures.

Performance Objective

Given an inscribed quadrilateral, the student should be able to state the relationship between its opposite angles.

Materials

Rubber Band

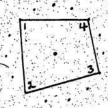
String

Circular Geoboard

Ruler

Procedure

1. Using the geoboard, set up a quadrilateral by connecting any four nails on the circumference of the circle similar to the one shown below.



Record the degree measure of each of the four interior angles.

$\angle 1 =$ $\angle 4 =$

$\angle 2 =$ $\angle 1 + \angle 3 =$

$\angle 3 =$ $\angle 2 + \angle 4 =$

2. Repeat exercise 1 three times, and each time use a different quadrilateral which has its vertices on the circumference of the circle. Record your results.

angles	quadrilateral 1	quadrilateral 2	quadrilateral 3
$\angle 1$			
$\angle 3$			
sum			
$\angle 2$			
$\angle 4$			
sum			

3. Write a general conclusion of what you have discovered about the angles of a quadrilateral inscribed in a circle.

Appendix C

Teacher's Manual

Activity #1

Instructions for Teachers

The goal of this activity is to utilize the square geoboard to introduce students to some concepts in geometry such as: line segments and angles. Before this activity is introduced to the students, time should be spend discussing the terms: point, line, plane, and space. Care must be taken to ensure that students know the purpose of the activity and the procedure to follow to carry out the activity. Also, have the students make a drawing on their books of the models they have formed on the geoboards for future reference.

This activity is to be used in the first section "Vocabulary in Geometry" when students are doing questions 1, 3, 4, 11, and 12 on pages 3 and 4. This activity serves three purposes:

(i) students can form models as indicated in each exercise by placing rubber bands around certain nails, thus gaining experience in what line segments and angles look like. While students are forming models on the geoboard, have them define each concept, thereby answering questions 1 and 4.

(ii) Before students are taught to measure the size of an angle with a protractor, they should have experience in making models of angles and noticing that they can be of different sizes. This can be done by placing a long rubber band on the geoboard to form an angle; keep one side fixed, move the free side of the band to form the various angles which you want to depict. Once the students recognize the different sizes of angles that can be formed, you can introduce the question dealing with specific kinds of angles. When right angles and straight angles are understood, models of acute and obtuse angles can be made and identified.

(iii) So that students can become familiar with the term congruent, show them that figures that are congruent fit exactly on top of each other.

This activity, along with the five questions mentioned previously, will take two periods.

Answers

3. (b) yes
4. The size of the angle keeps changing.

Activity #2

Instructions for Teachers

Some geometric facts can be verified with certain procedures used on physical materials. The sum of the measures of the angles of a triangle equaling 180° can be demonstrated by cutting the triangle into three pieces, each of which contains one of the vertices of the triangle. If the angles of these three vertices are placed so that they are adjacent, the exterior edges of the three will form a straight angle, the measure of which is 180°.

Care must be taken to ensure that students understand the purpose of the activity and the procedure they must follow to carry out the activity. Notice that in this activity students are asked to state a generalization. Each student should be encouraged to state this generalization as early as possible, thus enhancing his ability to formulate hypotheses.

This activity can be used at the beginning of section 2 "Properties of Angles and Triangles" on page 5 when the teacher is introducing inductive reasoning. It can be used as an example of inductive reasoning. At the same time it will help verify the fact that the sum of the measures of the interior angles of a triangle is 180°.

Answers

1. It forms a straight angle: 180°
2. 180°
3. 180°; yes
4. The sum of the measures of the interior angles of a triangle is 180°.

Activity #3

Instructions for Teachers

The sum of the measures of the angles of a quadrilateral equaling 360° can be demonstrated by cutting the quadrilateral into four pieces, such that each would contain one of the quadrilateral's vertices. If the angles of these four vertices are placed together with no overlapping, a complete turn or rotation is made, the measure of which is 360°. Care must be taken to ensure that students understand the purpose of the activity and the procedure to follow to carry out the activity. Encourage students to make their own generalization.

This activity is to be used consecutively with activity 2 when the teacher is introducing inductive reasoning. This activity, along with activity 2, will take up part of a lesson.

Answers

1. 360°
2. 360°
3. The sum of the measures of the angles of a quadrilateral is 360°.

Activity #4

Instructions for Teachers

This activity will show students that some properties of geometric figures (angles and triangles) can be verified by use of paper folding and paper cutting. This can be a useful activity to motivate the students. Waxed paper should be used over the standard type paper because it forms very visible creases. New paper should be used for each activity. Before you introduce the activity to the students, review the topic on isosceles and equilateral triangles.

This activity is to be introduced as a follow up activity to questions 3, 4, 6, and 7 on pages 6 and 7 to help verify the properties that students have discovered by means of a ruler and protractor. This activity will take up part of one lesson.

Answers

1. (a) Yes, they have the same measure.
   (b) Yes; they have the same measure.
   (c) If two lines intersect, the vertically opposite angles have equal measure.
2. (a) They have equal measures.
   (b) They have equal measures.
   (c) If two sides of a triangle have equal measures, then the angles opposite these sides are equal.
3. (a) They have the same measures; they have the same measures; they have the same measures; all three angles have the same measures.
   (c) If all sides of a triangle are congruent, then the angles of the triangle are congruent.
4. (a) The sum of the measures of $\angle A$ and $\angle B$ is equal to the measure of $\angle ACD$.
   (c) The measure of an exterior angle of a triangle is equal to the sum of the measures of the two

opposite and interior angles.

Activity #5

Instructions for Teachers

This activity shows students that some properties of parallel lines can be discovered by paper cutting. Before this activity is introduced, students should be able to recognize corresponding angles, alternate angles, and supplementary angles. Care should be taken that students understand the purpose of the activity and the procedure to follow to carry out the activity.

This activity will take the place of questions 3, 4, and 5 on page 11. These three questions can be used as a follow up to the activity so that the students will not conclude that consideration of two cases is sufficient to reach a conclusion.

Answers

1. (a) They have the same measure.
   (b) They have the same measure.
   (c) If two parallel lines are cut by a transversal, the corresponding angles are congruent.
2. (a) They have the same measure.
   (b) They have the same measure.
   (c) If two parallel lines are cut by a transversal, the alternate angles are congruent.

3. (a) They are supplementary.
   (b) They are supplementary.
   (c) If two parallel lines are cut by a transversal, the interior angles on the same side of the transversal are supplementary.

Activity #6

Instructions for Teachers

The goal of this activity is to introduce students to the different types of polygons. Utilize the geoboard for this activity.

Before beginning this activity, it might prove useful to review the notion of open and closed figures, and what a polygon is.

Have students, as they make each type of polygon on the geoboard, draw a similar one on their own book for future reference. Also have students define each type of polygon that is indicated. This activity can be readily used at the beginning of the section on "Names of Polygons" (page 12). This activity can be very useful in introducing students to the different types of polygons.

Answers

1. (a) 3
   (b) 4
   (c) 5

(d) 6

(e) 7

(f) 8

Activity #7

Instructions for Teachers

The goal of this activity is to utilize the geoboard to help students develop a formula to find the sum of the measures of the interior angles of any polygon. Before using this activity, students should know that the sum of the measures of the interior angles of a triangle is 180°. Have students, as they form each model on the geoboard as described in the activity, draw it on their own paper for future reference. You may have to carefully guide the students through this activity so that they can arrive at the correct generalization. This activity can be readily used to develop a formula for finding the sum of the measures of the interior angles of a polygon instead of using questions 6 and 7 on page 13. You can use question 8 on page 15 as a practice exercise.

Answers

1. 2

2.

Name of polygon	Number of sides	Number of diagonals with given vertex as an endpoint
triangle	3	0
quadrilateral	4	1
hexagon	6	3
heptagon	7	4
octagon	8	5

3. Number of diagonals = n - 3, where n represents the number of sides of the polygon.

4. (i) 5

(ii) 5 x 180 = 900

5.

Name of polygon	Number of sides	Number of triangles	Sum of the measures
triangle	3	1	1 . 180 = 180
quadrilateral	4	2	2 . 180 = 360
pentagon	5	3	3 . 180 = 540
hexagon	6	4	4 . 180 = 720
octagon	8	6	5 . 180 = 1080

6. Number of triangles = n - 2, where n represents the number of sides of the polygon.

7. Sum of the interior angles of a polygon = (n - 2) . 180

Activity #8

Instructions for Teachers

Paper folding can be a very useful activity for motivating students. This activity provides a variety of paper folding activities which can be useful for basic compass and straight edge constructions. Waxed paper should be used because it forms very visible creases. New paper should be used for each activity.

The activity can be used as a follow up to the basic constructions on pages 15 - 17. This will show the students that some of the geometric constructions done with a compass and a straight edge can also be done by paper folding.

Activity #9

Instructions for Teachers

Paper folding activities are not necessarily limited to just the basic constructions. They can also be useful in helping students discover properties of triangles. Waxed paper should be used because it forms very visible creases. New paper should be used for each activity. Before using this activity, students should be able to recognize and define the following terms: altitude, median, perpendicular (right) bisector, and an angle bisector of a triangle. This activity can be used as a follow up to help the students verify the properties of triangles which they had discovered when they were using compass and straight edge in the exercises on pages 18 and 19.

Answers

1. (a) Yes, inside the original triangle.
   (b) Yes, meet on the hypotenuse.
   (c) Yes, outside the original triangle.
   (d) The perpendicular (right) bisectors of the sides of any triangle meet at a point.
2. (a) Yes, inside the original triangle.
   (b) Yes, vertex point of the right angle.
   (c) Yes, outside the original triangle.
   (d) The altitudes of a triangle meet at one point.
3. (a) Yes, inside the original triangle.
   (b) Yes, inside the original triangle.
   (c) Yes, inside the original triangle.
   (d) The angle bisectors of a triangle meet at a point.
4. (a) Yes.
   (c) The medians of a triangle meet at a point.

Activity #10

Instructions for Teachers

This activity shows the student that some properties about parallelograms can be discovered by paper cutting. Before using this activity students should be able to recognize and define the following terms: congruent angles, congruent segments, parallel lines, and supplementary angles. Care should be taken to ensure that students understand the purpose of the activity and the procedure to follow to carry out the activity. This activity can be used along with questions 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, and 25 on page 22 to discover properties of quadrilaterals.

Answers

1. (a) They are congruent and parallel.
   (b) They are congruent and parallel.
   (c) In a parallelogram, opposite sides are both congruent and parallel.
2. (a) They are congruent; they are congruent.
   (b) They are congruent.
   (c) In a parallelogram, opposite angles are congruent.
3. (a) They are supplementary.
   (b) They are supplementary.
   (c) In a parallelogram, any two consecutive angles of a parallelogram are supplementary.

Activity #11

Instructions for Teachers

The goal of this activity is to introduce students to the basic parts of a circle. Utilize the circular geoboard for this activity.

Care must be taken that students understand the purpose of the activity and the procedure they must follow to carry it out.

This activity can be used at the beginning of the section "Relations with Circles" on page 24 to introduce students to what a circle is and to illustrate the basic parts of the circle. Have students, as they make models of certain parts of the circle on the geoboard, draw the same diagram on their book for future reference. Also while you are doing this activity, have students write a definition for each term.

Answers

1. (a) Answers will vary.
   (b) Answers will vary.
   (c) The distance from the centre of the circle to any point on the circumference is the same.
2. They have equal measures.
3. The length of the diameter is twice the length of the radius of the circle.
4. (b) Answers will vary.
   (c) Diameter.
   (d) Answers will vary.
7. Diameter.
10. OA and OB.

Activity #12

Instructions for Teachers

The main idea presented in this activity is that the measure of an inscribed angle is half the measure of the central angle which subtends the same arc. Before using this activity, students should review the definitions of such terms as chord, arc, central angle, and inscribed angle, and must understand that a central angle has its vertex at the center of the circle and an inscribed angle has its vertex on the circle. Work through the activity at a pace slow enough to allow as many students as possible to follow the reasoning. This activity can be used instead of questions 7 and 8 on page 25 in helping students discover that "the measure of an inscribed angle is half the measure of the central angle which subtends the same arc". When the activity is completed, questions 7 and 8 can be assigned as a follow up and, if necessary, put them on the board for further discussion. This lesson should take approximately one period.

Answers

2. (a) $\angle COD = 75°$

   (b) arc CD = 75°

3. (a) 15°

   (b) $\angle E = 45°$; $\angle F = 75°$; the degree measure of the central angle is equal to the degree measure of the arc.

4. $\angle XYZ = 52\frac{1}{2}$; arc XZ = 105; the degree measure of the inscribed angle is one half the degree measure of its intercepted arc.
5. $\angle 1 = 37\frac{1}{2}$; $\angle 2 = 45$; $\angle 3 = 30$.
6.

angle 1	105	60	105	60	45
angle 2	$52\frac{1}{2}$	30	$52\frac{1}{2}$	30	$22\frac{1}{2}$

The inscribed angle is half the central angle.
7. The measure of an inscribed angle is half the measure of the central angle which subtends the same arc.

Activity #13

Instructions for Teachers

The main idea presented in this activity is that the measure of an angle inscribed in a semicircle is always 90°. Before using this activity, students should review such terms as: arc, chord, and inscribed angle. The following property should also be reviewed, "the measure of an inscribed angle is half the measure of its intercepted arc".

Have the students work through the activity at a pace slow enough so that most of them will be able to follow the reasoning. This activity can be used, instead of questions 11 and 12 on page 25, to introduce the property that the measure of an angle inscribed in a semicircle is always 90°.

These two questions can be used as a follow up later in the lesson to verify the conclusion reached in the activity.

Answers

(i) $\angle 1 = 75$; $\angle 2 = 15$; $\angle 3 = 90$.

(ii) $\angle 1 = 60$; $\angle 2 = 30$; $\angle 3 = 90$.

$\angle 1$ is getting smaller and $\angle 2$ is getting larger, but $\angle 3$ remains the same.

(iii)

angle 1	60	$52\frac{1}{2}$	45	$37\frac{1}{2}$	30	$22\frac{1}{2}$	15
angle 2	30	$37\frac{1}{2}$	45	$52\frac{1}{2}$	60	$67\frac{1}{2}$	75
angle 3	90	90	90	90	90	90	90

$\angle 1$ and $\angle 2$ change but $\angle 3$ remains the same.

(iv) The measure of an angle inscribed in a semicircle is always 90°.

Activity #14

Instructions for Teachers

The main idea presented in this activity is that the opposite angles of an inscribed quadrilateral are supplementary.

Before using this activity, students should be able to define and recognize inscribed angles and supplementary angles. Also the property "the measure of an inscribed angle is half the measure of its intercepted arc" should be reviewed.

Have students work through the activity at a pace slow enough to allow them to follow the reasoning. Care must be shown that students understand the purpose of the activity. This activity can be used instead of questions 13 and 14 on page 25 to introduce the property that the opposite angles of an inscribed quadrilateral are supplementary. After the activity has been completed these questions can be assigned to verify the conclusion that the students have discovered.

Answers

1. $\angle 1 = 75$ $\angle 4 = 60$

   $\angle 2 = 120$ $\angle 1 + \angle 3 = 180$

   $\angle 3 = 105$ $\angle 2 + \angle 4 = 180$

2. Answers will vary.
3. The opposite angles of an inscribed quadrilateral are supplementary.

Appendix D

Achievement Tests

Achievement Test #1 Academic Geometry Grade IX

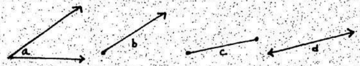
Name: ____________________

Class: ____________________

Section A

From the four items given after each statement, select the one which is the most appropriate and place the number of that item in the bracket provided at the right.

1. Which of the following figures shown below represents a segment? .... ( )

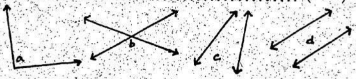


1. a 2. b 3. c 4. d

2. A set of points in a line consisting of one endpoint and all the points in one direction is called a(n) .... ( )

   1. ray 3. line
   2. segment 4. angle

3. Which of these figures represents two parallel lines? .... ( )



1. a 2. b 3. c 4. d

4. Suppose A and B are two acute angles P° and Q° respectively. $\angle A$ and $\angle B$ are complementary angles if and only if .... (   )

1. $P + Q < 90$ 3. $P + Q = 90$
2. $P + Q = 180$ 4. $P + Q \angle 180$

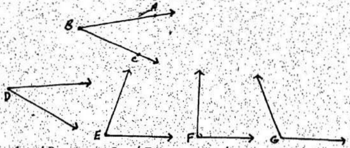
5. In which figure below is the dotted line a diagonal? .... (   )

1. a 2. b 3. c 4. d

6. Given triangle PQR. If $\angle P = 70°$, $\angle Q = 80°$, and $\angle R = 30°$, what name is given to triangle PQR? .... (   )

1. equilateral 3. isosceles
2. scalene 4. right angled

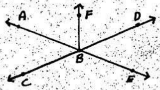
7. Which of the following angles below is congruent to $\angle ABC$? .... (   )



1. $\angle D$ 2. $\angle E$ 3. $\angle F$ 4. $\angle G$

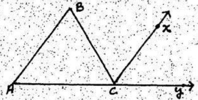
8. In the diagram below, which angle is congruent to ∠ABC? .... (   )

1. ∠ABF
2. ∠ABD
3. ∠DBE
4. ∠FBD



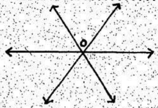
9. In the diagram, the lines AB and CX are parallel and A, C, and Y lie on the same line. Name an angle at C congruent to B?...(   )

1. ∠BCY
2. ∠BCX
3. ∠ACY
4. ∠XCY

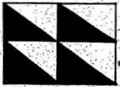


10. If six rays are drawn from a point O to form six congruent angles, what is the size of each angle? .... (   )

1. 60
2. 120
3. 30
4. 80



11. The shaded shapes in the figure are called .... (   )

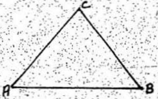


1. circles
2. squares
3. rectangles
4. triangles

12. The sum of the angles of a triangle is ... (   )
1. between 90 and 180
2. 360
3. 180
4. between 180 and 360

13. An exterior angle of a triangle equals the .... (   )
1. sum of the interior and opposite angles
2. sum of the interior angles
3. sum of the exterior angles
4. difference of the interior and opposite angles

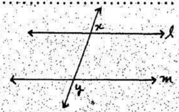
14. If angles A and B are the same size and if angle A measures 75°, what is the measure of angle C? .... (   )



1. 45
2. 30
3. 60
4. 75

15. In the diagram below line 1 is parallel to line m. If the measure of angle X equals 60°, what is the measure of angle Y? .... ( )

1. 60
2. 30
3. 80
4. 120

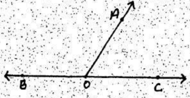


16. The number of triangles that can be formed in a decagon when all the diagonals containing any one vertex are drawn is ...... ( )

1. 7
2. 6
3. 8
4. 9

17. In the drawing below, if BOC is a straight line then $\angle$ AOB and $\angle$ AOC can be described as .... ( )

1. equal
2. adjacent and complementary
3. complementary and vertically opposite
4. adjacent and supplementary

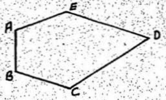


18. A polygon has .... ( )

1. exactly 3 sides
2. three or more sides
3. exactly 5 sides
4. more than 3 sides

19. What is the sum of the measures of the interior angles of polygon ABCDE shown below? .... ( )

1. 180
2. 360
3. 540
4. 450



20. Which of the figures below is an octagon?..( )

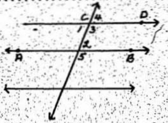
1. a 2. b 3. c 4. d

21. Polygon ABCD is a quadrilateral. If the sum of the measures of three of its angles is 280°, what is the measure of the fourth angle? .... ( )

1. 40 3. 100
2. 60 4. 80

22. A road builder wants to widen a road so that the new edge CD goes through point C and is also parallel to the old edge AB. Which of these choices guarantees AB and CD are parallel? .... (   )

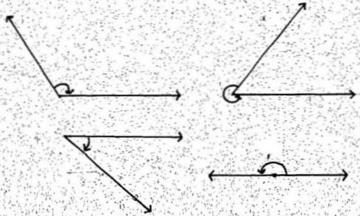
1. $\angle 1 \cong \angle 3$
2. $\angle 4 \cong \angle 2$
3. $\angle 1 \cong \angle 4$
4. $\angle 2 \cong \angle 5$



Section B

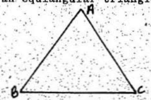
Answer each question in the space provided.

23. Using a protractor measure the following angles, and then identify what type of angle is illustrated.



24. An equiangular triangle is one in which all three angles have equal measures.

(a) Draw an equiangular triangle ABC.



Measure AB, BC and AC. What do you notice?

(b) Draw another triangle and measure the sides. What do you notice?

(c) Using the above results, write a probable conclusion.

25. Find the missing measures in the following diagram.

26. In triangle ABC, ∠B is 1° more than 3 times ∠A and ∠C is 3° more than 4 times ∠A. Find the angles.

27. Draw a neat diagram to illustrate the following.

(a) vertically opposite angles

(b) obtuse scalene triangle

(c) hexagon

28. If three of the interior angles in a pentagon have a measure of 100° each, and each of the other two angles contains X°, what is the value of X?

Achievement Test #2 Academic Geometry Grade IX

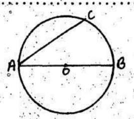
Name: ______________________

Class: ______________________

Section A

From the four items given after each statement, select the one which is the most appropriate and place the number of that item in the bracket at the right.

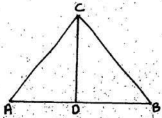
1. In the figure shown below AB is a diameter of a circle O and arc AC = 100°. The measure of $\angle$ CAB is .... (   )
   1. 40
   2. 50
   3. 80
   4. 100



2. The diagonals of a rectangle .... (   )
   1. bisect the angles of the rectangle
   2. are parallel
   3. are equal in length
   4. none of these

3. Using the figure shown below, in triangle ABC, $\overline{CD} \perp \overline{AB}$. Which of the following must be true? .... (   )

1. $\overline{CD}$ bisects $\overline{AB}$
2. $\overline{CD}$ is an altitude
3. $\overline{CD}$ is a median
4. $\overline{CD}$ bisects $\angle ACB$



4. A segment joining one vertex of a triangle and the midpoint of the opposite side is called .... ( )

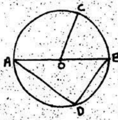
   1. median 3. right bisector
   2. angle bisector 4. altitude

5. The following condition "opposite angles are congruent" does not satisfy one of the quadrilaterals given below .... ( )

   1. parallelogram 3. rhombus
   2. rectangle 4. trapezoid

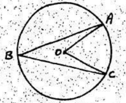
6. Which of the line segments in the figure below is a diameter? .... ( )

   1. $\overline{OC}$
   2. $\overline{BD}$
   3. $\overline{AB}$
   4. $\overline{AD}$



7. In the diagram below, the inscribed angle ABC and the central angle AOC are subtended by arc AC. The measure of the inscribed angle ABC equals .... ( )

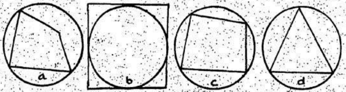
1. the central angle AOC
2. twice the central angle AOC
3. one-third the central angle AOC
4. one-half the central angle AOC



8. If the measure of a central angle is 50°, then the inscribed angle intercepting the same arc measures .... (   )

1. 100 3. 60
2. 50 4. 25

9. Which of the following diagrams fit the definition of an inscribed quadrilateral?.. (   )

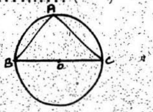


1. a 2. b 3. c 4. d

10. In an inscribed quadrilateral ABCD, $\angle A = 80°$. The measure of $\angle C$ is .... (   )

1. 50 3. 10
2. 100 4. 80

11. In the diagram, whenever BC is a diameter, which of the following statements is always true? .... (   )



1. ∠ A is always a right angle
2. ∠ C is always 45°
3. ∠ B is always a right angle
4. ∠ C is always a right angle

12. The part of a circle bounded by an arc and two radii is called a .... (   )

1. segment 3. diameter
2. chord 4. sector

13. Two altitudes of a triangle intersect at the vertex of the triangle. The triangle is .... (   )

1. isosceles 3. right
2. equilateral 4. obtuse

14. A diagonal of a rectangle divides a rectangle into two .... (   )

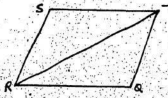
1. obtuse triangle
2. right angled triangle
3. acute triangle
4. equiangular triangle

15. In a parallelogram ABCD, if ∠ A = 75° what is the measure of ∠B? .... (   )

1. 105 3. 115
2. 75 4. 100

16. In the diagram below, which segment seems to be congruent to $\overline{ST}$? .... ( )

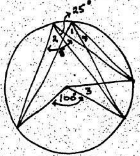
1. $\overline{TQ}$
2. $\overline{QR}$
3. $\overline{RS}$
4. $\overline{RT}$



Section B

Read each question carefully before answering it.

17. In order, find the measures of angles 1, 2, 3, 4, and 5.



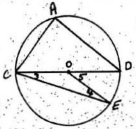
18. In the figure, $\overline{CD}$ is a diameter.

(a) Since $\overline{CD}$ is a diameter, what is the measure of $\angle A$?

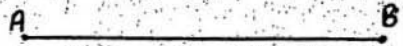
(b) What kind of angle is $\angle DOE$?

(c) What kind of angle is $\angle DCE$?

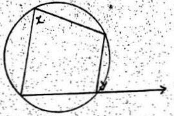
(d) What do you notice about $\angle 3$ and $\angle 4$?



19. Copy this line AB. Using pencil, ruler, and compasses only, draw a line perpendicular to AB at point B. Do not rub out your construction lines.



20. (a) Draw an inscribed quadrilateral on your paper as shown. Using the protractor measure ∠x and ∠y. What do you notice?



(b) Draw another inscribed quadrilateral similar to the one shown in part (a). Using the protractor measure the exterior angle and its opposite interior angle. What do you notice?

(c) Write a probable conclusion about an exterior angle and its opposite interior angle in an inscribed quadrilateral.

21. Draw a segment AB. With center A and radius AB, draw an arc; and with center B and radius AB, draw another arc to cut the former arc at C. Explain why the measure of angle BAC is 60°.

22. Construct triangle ABC so that $\angle C = 90°$ and CB = 3 cm. The hypotenuse is 6 cm. Find the incentre.

23. Draw diagrams to show that you understand the meaning of these terms.

    (a) tangent to a circle

    (b) median of a triangle

    (c) an angle in a segment of a circle

Appendix E

Student Questionnaire

Appendix E

Student Questionnaire

Name: ____________________

Class: ____________________

Directions

This is not a test and will not be used in any way to produce a grade for you. The items on this instrument are statements about geometry. For each item select a response which best describes your impression of the statement and place your response in the space provided at the left. The response choices are:

A – Strongly Agree

B – Agree

C – No Opinion

D – Disagree

E – Strongly Disagree

______ 1. Geometry is not a very interesting subject.

______ 2. I want to develop my geometrical skills and study this subject more.

______ 3. Geometry is a very worthwhile and necessary subject.

______ 4. Geometry makes me feel nervous and uncomfortable.

______ 5. I have usually enjoyed studying geometry in school.

______ 6. I don't want to take any more geometry than I have to.

______ 7. Other subjects are more important to people than geometry.

______ 8. I am very calm when studying geometry.

______ 9. I have seldom likedstudying geometry.

______ 10. I am interested in acquiring further knowledge of geometry.

______ 11. Geometry helps to develop the mind and teaches a person to think.

______ 12. Geometry makes me feel uneasy and confused.

______ 13. Geometry is enjoyable and stimulating to me.

______ 14. I am not willing to take more than the required amount of geometry.

______ 15. Geometry is not especially important in everyday life.

______ 16. Trying to understand geometry doesn't make me anxious.

______ 17. Geometry is dull and boring.

______ 18. I plan to take as much geometry as I can during my education.

______ 19. Geometry has contributed greatly to the advancement of civilization.

______ 20. Geometry is one of my most dreaded subjects.

______ 21. I like trying to solve new problems in geometry.

______ 22. I am not motivated to work very hard on geometry.

______ 23. Geometry is not one of the most important subjects for people to study.

_____ 24. I don't get upset when trying to do geometry lessons.

Directions for Scoring

The mathematics attitude scale can be scored on four subscale variables: enjoyment in mathematics (items 1, 5, 9, 13, 17, and 21); motivation in mathematics (items 2, 6, 10, 14, 18, and 22); importance of mathematics (items 3, 7, 11, 15, 19, and 23); and fear of mathematics (items 4, 8, 12, 16, 20, and 24). Items 1, 4, 6, 7, 9, 12, 14, 15, 17, 20, 22, and 23 are scored according to the following key: E = 4, D = 3, C = 2, B = 1 and A = 0. Items 2, 3, 5, 8, 10, 11, 13, 16, 18, 19, 21, and 24 are scored according to the following key: E = 0, D = 1, C = 2, B = 3 and A = 4. The four subscale scores can also be combined to yield a total score.