"THE DEVELOPMENT OF A SYSTEMATIC APPROACH TO THE TEACHING OF MATHEMATICS BASED ON BLOOM'S MODEL FOR MASTERY LEARNING"

CENTRE FOR NEWFOUNDLAND STUDIES

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THE DEVELOPMENT OF A SYSTEMATIC APPROACH TO THE TEACHING
OF MATHEMATICS BASED ON ELCOM'S MODEL FOR MASTERY LEARNING

by


An Internship submitted in partial fulfillment
of the requirements for the degree of
Master of Education

Department of Education
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St. John's
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ABSTRACT

This paper attempts to present a description of a curriculum development project in an eighth grade Mathematics class, developed and implemented to fulfill the requirements for the degree of Master of Education in curriculum studies at Memorial University.

In the introduction, the writer attempts to justify the need for change in many areas of education. He discusses some basic conditions that may contribute to the retardation of student achievement such as individual differences, the feeling of inadequacy experienced through failure, the types of instruction used, and the practice of continuous promotion. It is suggested that, although many advances have been made in education during the last decade, many changes are still necessary in the field of instruction. The writer hypothesizes that the introduction of mastery learning as an approach for teaching would increase the achievement of our students as well as permit a greater emphasis on individualized instruction.

The concept of mastery learning refers to an approach of instruction that provides for complete mastery of the subject matter. It is characterized by (1) an analysis of the subject matter (2) an emphasis on feedback/correction procedures (3) a development of specific corrective measures, and (4) an evaluation of student progress. The model of mastery learning was conceptualized by J.B. Carroll in 1963 and transformed into an effective strategy for instruction by B.S. Bloom in 1968. It is based on the premise that all students can master the prescribed tasks if the time supplied is sufficient. The paper includes the variables that determine the rate of learning. They are: aptitudes, ability to understand instruction, quality of instruction, perseverance,
In Chapter 4, the report describes the implementation of this strategy in a Grade 8 Mathematics class in a Junior High School. The implementation followed the basic steps for development as proposed by Bloom and Carroll. These were: (1) Prerequisite planning (2) Adoption strategies (3) Organization of materials (4) Instructional procedures, and (5) Summative evaluation. Besides a student profile, the chapter includes a detailed account of the procedures followed to correct the diagnosed weaknesses, the progress made in student achievement, and the changes observed in student attitudes.

The final chapter summarizes the basic strengths and weaknesses of the mastery learning approach. The writer concludes that the major strengths of the approach are the advantages gained through superior planning, the feedback and correction procedures which transform group based instruction into individualized instruction, the extent it caters to individual differences, and the apparent degree of success it acclaims. The major objections to the approach are related to the administrative problems that it generates and the difficulty experienced in teaching for objectives beyond the knowledge level.

It is hoped that this paper will assist other teachers who are so inspired to try this or similar projects in their classrooms.

C.G.W.
# CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>A. Focus on change</td>
<td>1</td>
</tr>
<tr>
<td>B. Premise for change</td>
<td>2</td>
</tr>
<tr>
<td>C. Recent changes in Newfoundland education</td>
<td>10</td>
</tr>
<tr>
<td>D. A suggested innovation</td>
<td>11</td>
</tr>
<tr>
<td>II. REVIEW OF LITERATURE</td>
<td>13</td>
</tr>
<tr>
<td>A. Concept of mastery learning</td>
<td>13</td>
</tr>
<tr>
<td>B. History of mastery learning</td>
<td>15</td>
</tr>
<tr>
<td>C. Variables of learning</td>
<td>17</td>
</tr>
<tr>
<td>D. Steps of development</td>
<td>24</td>
</tr>
<tr>
<td>III. IMPLEMENTATION PROCEDURES</td>
<td>27</td>
</tr>
<tr>
<td>A. Prerequisite planning</td>
<td>27</td>
</tr>
<tr>
<td>B. Adoption strategies</td>
<td>31</td>
</tr>
<tr>
<td>C. Organization of materials</td>
<td>33</td>
</tr>
<tr>
<td>D. Instructional procedures</td>
<td>38</td>
</tr>
<tr>
<td>IV. TREATMENT, EVALUATION AND RESULTS</td>
<td>39</td>
</tr>
<tr>
<td>A. Student profiles</td>
<td>39</td>
</tr>
<tr>
<td>B. Experimental treatment</td>
<td>40</td>
</tr>
<tr>
<td>C. Evaluation</td>
<td>48</td>
</tr>
<tr>
<td>D. Discussion</td>
<td>52</td>
</tr>
</tbody>
</table>
V. SUMMARY, LIMITATIONS AND CONCLUSIONS ............... 55
   A. Achievement ........................................ 55
   B. Attitude changes ..................................... 56
   C. Strengths of mastery learning ...................... 57
   D. Limitations of mastery learning .................... 59
   E. Summary .............................................. 62

BIBLIOGRAPHY ............................................... 64

APPENDIX ...................................................... 67
LIST OF FIGURES

Figure 1. Diagram of experimental treatment ........ 49

Figure 2. Grades assigned to the mastery class at midterm ......................................... 52

Figure 3. Grades assigned to the mastery class in June .................................................. 53

Figure 4. Grades assigned in 1976 vs grades assigned in 1975 ........................................ 53
CHAPTER 1
INTRODUCTION

A. Focus on change

Education is approaching a critical period in its history. It is approaching a period when society is requiring educational institutions, more than ever before, to account for the quality of their education. However, accountability has not been easy.

Society has been spending tremendous amounts of money on education during the last decade and is now expecting quality education in return. This injection of large amounts of money has provided new schools and facilities, new advances in instructional procedures as well as more and better educated teachers. Despite this, education is receiving a fair amount of criticism.

Criticism is evident from two main sources. Internally, educators have expressed the opinion that schools' most important objective is developing those attributes of students which will enable them to live effectively in a complex society. Block (1971) believes education has progressed very little towards this goal despite the advances made in the last decade. He said,

Despite great advances in knowledge about student learning and the investment of tremendous amounts of time, effort, and money, our schools still have not moved very far toward the goal of increased learning for all students. Present policies and practices continue to reproduce the same normal achievement distribution in the learning of classroom after classroom of students that are produced in the learning of the students' parents and perhaps grandparents. Thus the schools continue to provide successful and rewarding learning experiences for only one-third of our learners (p.2).

The second source of criticism is society. The lay public, as well as scholars in the various disciplines, are considering the
investment made in education and are doubting whether the progress made in education has kept pace with changes made in other areas of society. Indeed, society has a valid reason for doubting if education is providing success for only one-third of its students as is suggested by Block.

However, not all people are criticizing the efforts of education. Many are attempting to defend the practices of the schools. Taba (1962) said,

"Since World War II, the public schools have grown too rapidly to develop unassailable programs. They have been too plagued with growing enrollments, mass attendance, and a shortage of teachers, buildings, and finances to do an adequate job of curriculum development (p.1)."

and consequently levied the blame for such criticism on the social changes. She said,

"Today, as in the 1890's, the ferment in education is caused by the transforming effects of technology and science on society, with criticism focusing on the failure of the schools to solve the problems created by that transformation (p.1)."

Despite these many criticisms there is still faith in education. Society knows that its hope for the future can be found in the quality of education offered to its citizens. Therefore, if society does not observe the schools and their programs as changing to meet its needs, then it should foster as much change as necessary.

B. Premise for change

Change is imminent. Trump (1967) suggests that for added quality
schools need to focus on change. The writer believes that the learning experience of the students will improve if change is made in some of the present conditions of the educational system. All of these conditions are worthy of discussion. However, in this paper, the writer will consider only four of them. These are:

1. Individual differences
2. Student failure
3. Types of instruction
4. Promotion policies

1. Individual differences

In educational circles it has been a well accepted fact that all students are different. They differ in their home circumstances, attitudes, interests, and their ability to learn. These differences have caused a wide variety of opinions among educators as to their extent and the approaches to be taken to teach students who need special treatment because of these differences. However, opinions have changed drastically over the past decade due to the amount of knowledge acquired about students and how they differ. Previously, according to Bloom (1972), teachers accepted the idea that only some students had the ability to learn to a mastery level. He said,

Throughout the world, the proportion of students expected to fail in school varied from 5 percent to as high as 75 percent. We differed also in the causes we invoked to explain school learning differences - genetics, motivation, socioeconomic status, language facility, docility, etc. But, we assumed that most of the causes of success or failure in school learning lay outside the school's or the teacher's responsibility (p.4).

If teachers adhere to this theory, then it is likely they provide learning experiences for the group rather than for individuals.
This places the onus on the students to utilize the opportunities provided by this group instruction. Then, if they don't succeed, it is easy for teachers to defend themselves by using the normal curve as a method of grading. It is hoped that this practice is not as prevalent as in the past. Many teachers are beginning to reconsider this practice and to accept Bloom's theory of student differences. He said,

More recently, we have come to understand that under appropriate learning conditions, students differ in the rate at which they can learn—not in the level to which they can achieve or in their basic capacity to learn. Fundamental research on these ideas is still in process. Studies in which these ideas have been applied to actual school subjects reveal that as many as 90 percent of the students can learn these school subjects up to the same standard that only the top 10 percent of students have been learning under usual conditions. As this research proceeds, special conditions have been discovered under which both the level of learning and the rate of learning become much the same from student to student. That is, there is growing evidence that much of what we have termed individual differences in school learning is the effect of particular school conditions rather than of basic differences in the capabilities of our students (p. 5).

Once teachers accept the theory that individual differences may be largely due to the effect of school conditions, they will accept the responsibility to do more to equalize opportunity within their teaching. It will also require them to study other methods and implement these methods found helpful in other areas.

However, teachers should be cautioned against innovating for change sake. Not all innovations have been as helpful as expected, but most have attempted to provide suitable activities and experiences to help some students toward further achievement. One example of such an innovation was programmed instruction. It developed as a result of advances in technology and provided assistance to many.
students, but did not materialize as a teaching method to the extent educators predicted it would (Morgan, 1973). However, many features of programmed instruction were retained. These included: setting up clearly defined objectives, careful sequencing of materials, constant evaluation and feedback. These features, as well as others, should characterize any innovation that attempts to assist in coping with student differences.

2. Student failure?

During the past decade school officials have been concerned with the increase in the number of students who experience emotional difficulties of some degree and eventually drop out of school. They have attempted to relate this drop out problem to the success or failure experienced by those students. Bloom (1971) summarized strong evidence to support such a relationship. He said,

There is considerable evidence that repeated success in school over a number of years increases the probability of the student’s gaining a positive view of himself and high self-esteem. Similarly, there is evidence that repeated failure or low performance in school increases the probability of the student’s developing a negative view of himself and a lowered self-esteem. While these relationships between school marks and self-concept are relatively clear, much additional research is needed to establish the casual and interactive links between school achievement and self-view over a number of years. ... repeated success in school over a number of years (especially at the primary school level) appears to increase the likelihood that an individual can withstand stress and anxiety more effectively than individuals who have a history of repeated failure or low marks in school. To put it bluntly, repeated success in coping with the academic demands of the school appears to confer upon a high proportion of such students a type of immunization against emotional illness (p. 6).

Success or failure, although determined by the assignment of grades and judgements made by teachers as to the capability of the students, may be more closely related to how the students actually
view themselves. After an accumulation of 'passing' or 'failing' grades the students make their own judgements as to their performance. They tend to view themselves as having a high, moderate or low degree of capability. If the view is negative, they develop a negative attitude towards themselves and the learning situation, or if the results are positive, then they will enter the next learning situation with more vigor and determination to complete it successfully. In summary, the students' attitudes may be consequences of their feeling of adequacy or inadequacy of their own self-concept.

The school environment should provide individuals with evidence of their adequacy to enable them to surmount crises and periods of stress. This can be accomplished by having students experience as much success as is realistically possible. Therefore, any innovations in school learning should provide such experiences.

3. Types of instruction

The instructional approach presently used in Newfoundland schools, especially in the high schools, appears to be group based rather than student centered. Bloom (1971), in a recent lecture, comments on group-based instruction. He said,

In group based learning situations there is a great deal of pressure on the student to learn at the rate of the group. That is, there is every effort to maintain a set schedule of work and the student is expected to 'keep up with the class'. Undoubtedly, this group pressure for conformity to the set schedule of learning activities makes it easier for the fast students who may have little difficulty in keeping within the pace of learning set by the teacher but makes it difficult for the slower learner who have more problems of maintaining the learning pace set by the teacher of the class (p.9).
It may be easy to understand why many teachers prefer this form of instruction for it helps them cope with the large numbers of students found in their classes. In those large classes teachers find it difficult to group students and provide adequate instructional time and materials for these groups.

As Bloom suggests, in the group-based instruction there is a tendency for the teacher to set the pace to coincide with the average of the class, leaving the slower learners to 'catch up' on their own or lose interest and experience yet another failure. This type of instruction discriminates against the students of the lower and upper levels of ability. The students at the upper level of ability may be more likely to find ways of advancing their knowledge through individual study and private reading. The teachers' concern, therefore, should be with the slow learners for they are the ones who have most to lose from group-based instruction.

The introduction of 'streaming' in larger schools provided the opportunity for homogeneous grouping and, hence, permitted a wider diversification of teaching methods. Instruction, under this condition, can be directed to a group possessing the same level of ability. A change in method, however, may not be sufficient unless it is accompanied by a sufficient amount of time for mastery of the concepts and skills being taught.

4. Promotion policies

It is not an uncommon occurrence today to hear people criticizing a lack of knowledge and skills by our graduates. Laymen expect all students to be proficient in the basics of mathematics and language
upon completion of Grade 11. If they find that this is not the case, they criticize the schools for their lack of teaching. Only recently, a manager, friend remarked, "What is being done in our schools? I cannot hire a student to work in my office without having to check his work for errors in language and computation." While this may not be the case generally, it certainly appears to be a trend of which educators have to be aware.

If some students are not proficient in the use of English and mathematics at the completion of Grade 11, then it could partly be attributed to the promotion policies of our schools. In 1971, the public examinations in Newfoundland were dropped for Grades 9 and 10 in favour of school examinations. This move paved the way for teachers to adopt their own policies regarding student promotion, permitting all schools to determine its standards for these grades in relation to the students' ability. Under these conditions, it is reasonable to assume that a small proportion of our students reach and graduate from Grade XII without having mastered all the necessary skills.

Another factor that has contributed to this dilemma is that of continuous promotion, where students are progressed from grade to grade regardless of their achievement. This trend is necessitated by a lack of building spaces and the belief that any student should be promoted with his peer group. It appears that this 'social promotion' in the Primary and Elementary schools has caused a 'bottleneck' effect in the highschools. More and more students are entering highschool only to find they are ill-prepared for the work required of them there. Both teachers and students are frustrated. We need continuous promotion but not without continuous progress.
It is, therefore, the duty of the teachers to assure a higher level of achievement by the students before they are promoted.

C. Recent changes in Newfoundland education

Since confederation in Newfoundland, the view that changes are necessary in education has been made urgent by educators and laymen alike. This view has resulted in significant strides having been made to discard some of the hindrances to progressive education in this province. The Government began after confederation to make such changes, but it was not until the recommendations by a Royal Commission on Education in 1964 were implemented that significant advances were made.

The Royal Commission headed by Dr. P.J. Warren (1967) summarized some of the basic changes that have taken place to improve education at the school level. Warren said,

The professionalization of teaching has been reflected in the classrooms. New approaches to instruction have been introduced .... In a growing number of schools, there is now no rigid division of the day into lesson periods, the children are not made to sit quietly in rows of desks, the teaching is much less authoritarian than it was, physical punishment is seldom used, and less emphasis is placed on written examinations. Students are permitted to progress more at their own rate, to work in small groups or individually, and to learn from a wide variety of resources (p.6).

Further changes were evident at the department level to grant the schools more autonomy in setting and grading their own examinations. Warren (1973) discussed these changes when he said,

In June 1970, the Department of Education announced that, beginning in 1971, public examinations would be dropped in Grades IX and X, for a trial period of five years. The major purpose of this move was to encourage schools to take the initiative in improving the quality of instruction and in establishing their own evaluative criteria for promotion
purposes .... Later regulations stated that as of June 1972, high schools approved by the Minister would be permit-
ted to assign 50% of the final mark for the Grade XI exam-
ination certificate (p.7).

Warren viewed many of the changes made in schools with great
optimism and suggested that the reorganization had helped students
considerably. It is undeniable that it has facilitated learning to
a large extent as it was advocated to do. The positive results of
such changes may be seen in the increasing numbers of students who
are continuing in secondary education, as well as a similar
increase in teaching facilities, buildings and resources. It would
be difficult to suggest however, that all changes were for the best.
In the opinion of the writer, the reorganization has contributed
largely to an overcrowding of students in our classrooms. Conse-
quently, direct progress of student learning has been hindered
rather than aided by such a change. This condition requires that
future changes concentrate on the instructional aspect of teaching.

D. A suggested innovation

The introduction of this paper has so far attempted to outline
some of the weaknesses in the educational system of this province
and has suggested some steps that have already been taken to overcome
these weaknesses. It has concluded that organizational changes are not
sufficient, that more emphasis should be placed on the instructional
aspect of teaching in the various classrooms. What is needed is an
approach whereby more students are taught to the level of mastery, there-
by allowing a higher proportion of students to acquire greater levels
of achievement. This approach should exhibit a more systematic form
of development, similar to that proposed by Smith and Nagel (1972) and by Davies (1974). Smith and Nagel said,

The systems approach views the entire educational program as a system of closely interrelated parts. It is an orchestrated learning pattern with all parts harmoniously integrated into the whole: the school, the teacher, the students, the media, and the materials. Such an approach integrates the older, more familiar methods and tools of instruction with the newer ones -- the computer, television, programmed instruction, and simulations to name a few (p.86).

Whereas, this description of the systems approach outlines the components of work in the instructional process, other descriptions focus on the step-by-step procedures for designing such an effective instructional system. Davies comments on a learning systems design as follows:

This methodology consists of systematic procedures for planning, designing, carrying out, and evaluating the total process of learning and teaching. It is directed at achieving specific objectives and is based on research in human learning and communication. Applying this methodology will produce a learning system which arranges human and non-human resources in an efficient manner to bring about effective student learning (p.302).

The writer is a proponent of this type of design, and believes in the importance of the four procedures: planning, designing, implementing and evaluating in developing any educational program. Consequently, it has resulted in a study of the mastery learning approach as proposed by Carroll (1963), Block (1971) and Bloom (1971).

It is the thesis of the writer that a mastery learning approach will assist in the solution of the problems discussed in this paper. Students would be taught as individuals and advanced at a rate that is compatible with their abilities. The type of instruction used is student centered rather than group based and promotion is
based on the mastery of the objectives decided on at the beginning of the year. Finally, the development of this approach would exemplify the systems approach outlined above.

The remainder of this paper will be devoted to exploring the potential possibilities of utilizing this learning for mastery approach in teaching Mathematics that may, in fact, allow most students to achieve to high academic levels. The theory of mastery learning will be discussed and the effects of such an implementation in a Newfoundland school will be examined.
CHAPTER II
REVIEW OF LITERATURE

A. Concept of mastery learning

An understanding of any innovation requires a knowledge of the literature written on that innovation. The writer considers three questions relevant to mastery learning. What does the concept 'mastery learning' mean? How did the concept develop? What are the steps for developing a mastery learning model?

Block (1971) concluded that mastery learning was a powerful new approach to student learning which can provide almost all students with the successful and rewarding learning experiences now allowed to only a few. It proposes that all or almost all students can master what they are taught (p.3).

Thus, mastery learning refers to an approach of instruction that provides for the complete mastery of the subject matter. It does not specify the time for mastery, since the major variable of this approach is the amount of time students are allowed.

Wentling (1973) said,

Mastery learning, which incorporates the requirement and expectation that all students will meet a predetermined criterion or mastery level before progressing on to the next unit, is credited with being one of the major developments in instructional technology (p.50).

Wentling's comment on mastery learning summarizes the main characteristics of this approach. It suggests that students progress to one level, only after a complete mastery of the previous level has been accomplished. Complete mastery will depend on the amount of time allowed the students, since some students may need as much as five times the time required by others.
The concept of mastery learning refers to a model of instruction which has the following distinctive features:

1. A clearly defined sequence of skills and concepts structured on the basis of a 'task analysis' process which is generally associated with the learning theory of Gagne.

2. An evaluation system which specifies 'mastery' criteria for each skill or concept in the sequence.

3. A pacing system which insures that the learner has achieved the mastery criteria for lower level prerequisite skills and concepts in the hierarchy prior to further attempts at learning related higher level concepts and skills.

Many models of mastery learning have been developed in the past few years, but most are based on the Carroll model. One such model was developed by Mueller in 1973. In this model, Mueller said,

The mastery instructional model is as follows, complete with rationale: Research indicates that most students can learn almost anything, given enough time and proper prerequisite learning. Therefore, if we arrange all instructional tasks into their proper learning sequence, almost all students will be able to accomplish each and every task, eventually. If each student is taught until he masters a particular learning unit, he can then be sent on to the next task or goal, ad infinitum. Students are not compared with other students in amount learned, and no student is ever at the bottom of a distribution, because there is no distribution of test scores (p.5).

In this model, students are not competing against other students in the class. They have no fear of not completing the work, since they work at a rate at which they are capable until they have mastered the tasks prescribed. Students progress at different rates depending on their ability, interests and aptitudes. It is apparent from this...
approach that not all students would complete all the levels prescribed. Reporting of progress in the mastery approach is different. Teachers should not report any progress until complete mastery of a task is achieved.

Variations of this mastery concept would include the Winnetka Plan developed in 1922 (Block, 1971), the general programmed-learning approach championed by Skinner, and it can be found today as the basis for various commercial and non-commercial programs designed to facilitate the individualization of instruction.

B. History of mastery learning

The idea of learning for mastery is quite old. As early as the 1920's there were attempts made to produce mastery in student learning. At that time two projects were initiated. One was the Winnetka Plan of Carleton Washburne in 1922, the other was an approach developed by Professor Henry C. Morrison in 1926 at the University of Chicago Laboratory School (Block, 1971).

These projects had several common features. First, mastery was defined in terms of the objectives each student was expected to achieve. Second, the instruction was arranged into several, well-structured learning tasks. Third, the plans expected students to attain mastery before proceeding to the next task. Fourth, the feedback on the adequacy of students' learning was achieved by progress tests administered during and at the completion of the task performance.

Both of these approaches shared many of the major features
of programmed learning. However, the inadequacies recognized in
programmed instruction, were not as apparent in the mastery
learning approach. It did not need expensive equipment and
programs, it helped more students master more work in less time,
and because it did not designate a specific amount of time for
completion, it provided for individualized instruction.

A model for mastery learning was officially proposed in
the 60's (Carroll, 1963); including the new factor of time
allowed for students to finish the activities. Block (1971),
while discussing the model, said,

...The model proposed that the quality of the student's
instruction and his ability to understand it interacted
to extend the time he needed for task mastery beyond that
normally required by his aptitude for the task. If both
the quality of his instruction and his ability to under-
stand it were high, then he would require little or no
additional learning time. However, if they were both low,
then he would require much additional time (p.6).

It was Benjamin Bloom in 1968 who transformed this model
into an effective tool for mastery learning. Bloom's working
model emphasized the ample feedback/correction procedures to
ensure that each student's unit instruction was of optimal
quality. When correction was determined necessary by feedback
devices, Bloom's model suggested supplementary materials to be
applied to help the student overcome his unit learning problems
before the group instruction continued.

Bloom's conception of mastery learning was adapted and
developed for use in Seoul National University by B.M. Chung and H.G. Kim
(Morgan, 1973), both of whom have written about their experiences
in the use of this concept. Many other successful strategies have
been easily and inexpensively implemented at all levels of education since Bloom's model first appeared. Mastery approaches have been tried in all subjects ranging from Arithmetic to Philosophy and Physics with success experienced in each attempt.

G. Variables of learning

The concept of mastery learning suggests that it is, under appropriate conditions, the rate at which students learn that differs and not the level to which they achieve nor their basic capacity to learn. Further, there is growing evidence that individual differences in learning are most likely due to the effects of school conditions upon individuals rather than basic differences in student capabilities.

The rate of learning for each student will depend upon several variables. These variables, as suggested by both Bloom (1968) and Carroll (1963) are:

1. Time
2. Aptitude
3. Ability to understand instruction
4. Quality of instruction
5. Perseverance

1. Time

Many studies done in the past ten years, including those done by Block (1970) and Glaser (1968), have found that students can master the material offered to them if enough time is available for them to spend on the task. Some students master a task quickly while others do not. The mistake made by educators is to advance those that do not at the same time as the others. The schools are guilty of doing this very frequently.
This is partly a consequence of the large amount of material they are required to teach, partly because of the very great variation that exists in the amounts of time that children need for learning and the extent of grouping that may result. In all fairness, teachers do attempt to assist the 'slow' students 'catch up' by offering them special assistance in instruction and further practice. However, it is inevitable that some students advance without having mastered the previous task, because they have not been given sufficient time.

What then, is meant by the time needed for mastery? How does the time required depend upon other variables of learning? Time, according to Carroll (1963), does not mean "elapsed time" but the actual amount of time during which the person is oriented to the learning task and actively engaged in learning. When time for mastery is discussed in this paper, it means the time during which the student is paying attention and is trying to learn.

For Carroll, the time spent on learning is the key to mastery. He agrees that the time required is likely to be affected by the student's aptitudes, verbal ability, the quality of instruction received in class, and the quality of the help received outside of class.

2. Aptitude

Carroll (1963) defines aptitude as the amount of time required by the learner to achieve mastery of a given learning task. Implicit in this definition is the assumption that, given enough time, almost
all students can master a learning task to an A-level of competence. In other words, most students reach the same criterion level but at different rates. Further evidence to support this claim has been presented by Glaser (1968) and Atkinson (1968) who both found in their studies that most students eventually reached mastery on given learning tasks, but they did so at varying rates (Bloom, 1971).

Bloom and his Chicago group, after conducting a study of aptitude distribution in relation to student performance, concede that there are differences between students at the extreme ends of an aptitude continuum and the majority of the student population. At the top of the distribution are 1 - 5% of the students who have a special talent for a subject and learn that subject with ease. At the opposite end of the distribution are another 1 - 2% who have special disabilities for some learning task such that they may never be able to learn to a mastery level.

This difference in student aptitude from subject to subject was tested by Carroll (1967). He, as well as Kim (1968), found that the level of achievement in any subject depended upon, to some extent, the aptitude displayed by students towards that subject. Carroll further concluded that aptitudes are predictive of both the rate at which and the level to which students will learn.

3. Ability to understand instruction

Another cause for a student's non-achievement is related to his inability to understand the instruction given. Bloom (1968) defines the ability of the learner to understand exactly what it is he is to learn and the procedures that are necessary for achievement. The student's inability to understand the instruction given may stem
from the fact that most instruction is verbal. Yates (1957) did studies on the procedures used by teachers to place students in the secondary school. He found that there was a definite relationship between a student's verbal ability and success in the secondary school, and that the student's ability to understand instruction would determine the level of achievement. Behr (1967) emphasized the fact that since students are likely to perform better if the material is presented in a mode which emphasizes their abilities, the instruction should be varied to coincide with any special abilities the students may have. If a student has a low verbal ability, instruction would not be as effective if it were highly verbal. Block (1971) said,

"Regardless of the approach used to increase a student's ability to understand instruction, the evidence is clear that the use of only a single mode of instruction hampers the learning of students who are weak in the aptitudes required to learn in that mode." (p.93)

There is no doubt that the ability to understand instruction depends upon the quality of instruction given, and indirectly, achievement depends on both.

4. Quality of instruction

The fourth variable is the quality of instruction given to the student. Carroll (1963) defines the quality of instruction as the degree to which the presentation, the explanation and the sequential order of the elements within the learning task is at the appropriate level for an individual given his present stage of learning. The quality is determined by the clarity and appropriateness of the materials, the amount of participation in
and practice of the learning by each student, and the amount and types of reinforcement given to each learner. The teacher prepares the materials such that the student can learn it as rapidly and efficiently as possible. Carroll (1963) says,

This means, first, that the learner must be told, in words that he can understand, what he is to learn and how he is to learn it. It means that the learner must be put into adequate sensory contact with the material to be learned (for example, one must insure that the learner will adequately see or hear the materials of instruction). It also means that the various aspects of the learning task must be presented in such an order and with such detail that, as far as possible, every step of the learning is adequately prepared for by a previous step. It may also mean that the instruction must be adapted for the special needs and characteristics of the learner, including his stage of learning. All these things may be summarized in what we call quality of instruction (p. 726).

The quality of instruction may be affected by such things as the teacher, the textbooks, workbooks, files and teaching machine programs (Carroll, 1963). If the quality of these things is less than optimal, more time will be needed to teach the learning tasks.

Much of the research done in mastery learning has been done to determine what effect the quality of instruction has upon achievement. Airasian (1967) found that the quality of instruction affects both the student's learning rate and achievement level. In his experiments, he improved instruction and found a great improvement in the achievement level. Anthony (1967) found that the use of frequent and varied reinforcements is also important in the production of optimal quality of instruction needed for greater learning. Cronbach (1969)
found such a high correlation between this variable and achievement that he expressed an urgency for curriculum makers and teachers to develop approaches to instruction which will best serve the needs of different groups of learners. It is apparent that most students can master what we have to teach them, and it is the task of instruction to find the means which will enable our students to master the subject under consideration.

5. Perseverance.

The students who have demonstrated a low achievement appear to lack the persistence required to complete tasks assigned them. This perseverance, translated in terms of time, is the time the learner is willing to spend on learning. A long perseverance includes a willingness to withstand discomfort, fatigue, strain and minor illness. It includes a willingness to face immediate failure in view of a successful termination of the task.

Many students are not willing to persevere for the amount of time required for mastery. They may regard the task as too difficult, or unnecessary to learn. They may start to learn and later become distracted or bored, or even lose confidence in their ability to learn. The extent of their perseverance may be dependent upon a series of other factors including attitudes towards school, economic-socio conditions, self image and inadequate knowledge of their capabilities. It is difficult to measure the amount of perseverance of students but the most direct evidence would come from observations of the amount of time the students actively engage in learning.
Certainly, a mastery model supposes that students who cannot persever to a reasonable level cannot be capable of mastering the tasks required of them.

The perseverance of students in their work was studied by Seashore (1942), Weiner (1965) and Carroll (1967). Weiner indicated that students high in achievement motivation persisted longer if their trials were failures than if they were successes. However, those having a low achievement motivation persisted longer if their trials were successes. It emphasizes the importance of success and failure as an influence on persistence.

Seashore's studies, carried out many years ago, expressed the importance of external and internal reinforcement on the persistence of students. Poor quality of instruction may decrease the perseverance for students of high and low intelligence. This observation was made by Carroll (1967) in his book on school learning.

The writer suggests that, differentiated from these, there may exist a sixth variable which determines the students' rate of achievement. This is the motivational aspect of learning. Although related to the others, the writer believes this variable may have causes related to the home. The students, in their relationships at home, appear to develop attitudes which contribute to their learning adequacies or inadequacies.

In conclusion, the rate of a student's learning may depend on any combination of these six variables. Underachievement results whenever perseverance is less than some 'reasonable' level.
whenever the quality of instruction is poor, whenever the time allowed is less than adequate for learning. If learning is to be successfully accomplished, then the educators must attempt to modify the factors to any extent possible. Carroll (1963) supports this hypothesis, for he says,

"We have a feeling about the relative amenability of different factors in achievement to manipulation or treatment: "Aptitude" is regarded as relatively resistant to change, whereas it is the hope of the psychologist that he can readily intervene to modify "perseverance," "quality of instruction," or "opportunity for learning." To some extent, this feeling is justified not only by logic but also by research findings—by the research on the apparent constancy of the IQ, on the effect of various instructional variables, etc. On the other hand, if aptitude is largely a matter of prior learnings, it may be more modifiable than we think, ... (p.731).

D. Steps of development

Most approaches read by the writer emphasized several basic steps which were necessary for development. These basic steps were adhered to in the planning of the proposal for implementation in the next chapter. However, to complete the literature written on this topic, the writer believes it necessary at this time to outline briefly these basic steps.

1. The first step is the selection of the subject for which a mastery strategy would be developed (Block, 1971). The subject chosen should be easily subdivided into simple task units, which could be considered as steps in the overall progress of the students. Block suggests also that another important criterion for selection would be the subject having the minimal
prerequisite learning. This means it would be advantageous to select a subject which is new to the student's curriculum, thus eliminating the variable of student's prior knowledge.

2. Then, the subject material must be divided into many instructional units which are introduced into a sequential fashion. The sequence is important because the mastery of each unit depends upon mastery of the previous unit.

3. To recognize when students have attained mastery, the objectives of instruction must be specified in terms of what the student is expected to learn. The objectives should coincide with those established in the previous non-mastery teaching or based on new teaching materials. The objectives establish a standard which becomes the sole criterion for judging student performance.

4. The fourth step, according to Block (1971), is to translate the objectives into specific evaluation procedures, whereby the students' performance can be determined. These procedures take the form of evaluation instruments. The instruments are used for the formative and summative evaluation. They are diagnostic tests that are administered at the completion of each unit to provide feedback on the adequacy of the students' learning. To determine if mastery has been achieved a summative evaluation is required before the students advance to the next level.

5. The instruction activities are designed to help students towards immediate mastery. However, they may need to be supplemented with appropriate learning correctives so that they
can eventually master the unit of work. Block (1971) outlined several of these corrective techniques. Among these are re-teaching, tutoring, use of alternative learning materials, audio-visual methods and small group sessions.

6. The final step is the administration of the tests to determine mastery for the purpose of advancing students to the next level, or to provide information necessary to individualize instruction within a mastery approach. Basically, formative evaluation seeks to identify learning weaknesses prior to the completion of instruction on a course segment. Summative evaluation is 'final' and grades assigned on their basis are likely to follow the student throughout his scholastic career. These occur infrequently and cover relatively large blocks of instructional materials.

In this chapter, the writer has attempted to include a fairly detailed account of the literature read and the research previously performed on mastery learning. In the next chapter, it is the intent of the writer to outline a proposal for the implementation of this approach in line with the suggestions recorded in this chapter.
CHAPTER III

IMPLEMENTATION PROCEDURES

It is the intention of the writer, in this chapter, to discuss the implementation of the mastery learning approach in an attempt to develop an instructional design in the teaching of mathematics. The design, when completed, should demonstrate the general application of the systems approach as described in the introduction of this paper. A model (Merrill, 1971), outlining this approach, was foremost in the mind of the writer as this design was being developed.

The implementation of this mastery learning project will be discussed under the following headings:

A. Prerequisite planning
B. Adoption strategies
C. Organization of materials
D. Instructional procedures

A. Prerequisite planning

The implementation of any approach to teaching requires an extensive amount of preplanning before instruction begins. This was especially true of the mastery learning project. The preplanning included a decision as to the school, grade and subject to be used for the innovation. The school chosen was the one which the innovator knew best, or at least had been teaching in for several years. The school was the Botwood Junior High school, in which he had spent a total of nine years working with students of the same age level, in the area of Mathematics. The subject of Mathematics was also a significant choice for the writer, since there had been
a gradual inclination towards this approach by the writer over the past three years.

The school is a typical Newfoundland junior high school, in that it serves approximately 250 students in Grades 8 and 9. The teaching staff comprises a membership of twelve teachers who teach practically full-time, while the principal teaches a little more than half-time. The school building was originally built for approximately 85 students, but by the extension of several portable classrooms it accommodates 250 students in rather crowded conditions. The school is located in a community of about 5000 people many of whom are lower-income and government assisted families.

The curriculum includes the core subjects of English, Science, Mathematics, Social Studies, French and Religion with other subjects recently introduced including Health, Science and Current Events. An integral part of the curriculum may be described as the "third curriculum" and includes sports and clubs such as the Glee Club, Chess Club, French Club, as well as committees to arrange for other activities during the year. By Grade 9, the students are streamed according to their achievement during the previous year's work. This means four classes in each grade grouped homogeneously with programs which differ significantly from high achievers to low achievers.

The general philosophy of assigning grades and promotion in this school has changed drastically during the past five years. Previously, many students were required to repeat the grade. Whereas, students are now promoted almost 'en masse' to the high school. The rare few who are not promoted are given the option of attending a summer school
session with extra help given in the subject failed. These students are promoted to the low achievement class in Grade 10. This situation, together with the general disinterest of many students, has contributed to a very low achievement level. Many students go through the act of working when they are aware that even if they do not master the required work they will be given a 'pass' and 'pushed on' to the next grade. However, out of those being promoted to the next grade, many fail because of inadequate background in the prerequisite work. Perhaps, the answer lies in completing only the concepts that students can master and continuing with the remainder of the program the following year. Under these conditions, most students could and should receive 'A' grades. The implementation of mastery learning strategies would allow many students to achieve at higher levels and eliminate the high failure rate and negative learning experiences. Such an implementation would also remove the need to divide students into academic and general classes.

The emphasis in this school has been in language, both oral and written, and mathematics with a trend in this area to a more basic and practical approach in its teaching. With this in mind, the writer selected Arithmetic in an average Grade 8 class to attempt the mastery approach. If successful, the approach would be implemented with the same class next year in Algebra. The writer is of the opinion that once success has been established in one class, the mastery approach could be extended to a complete grade and even to the entire school.

Arithmetic was chosen because of its extension possibilities.
into Algebra and because it is the most convenient subject for arranging sequentially. Arithmetic consists of a number of well defined units whose learning is cumulative in that the learning of any unit is probably dependent upon the learning of all prior units. Bloch (1971) attempted to explain the success of mastery learning strategies in sequentially learned subjects when he said,

The learning of any sequentially arranged subject depends upon the learning of each of its units. If at each stage in the sequence the student learns the material upon which the next unit builds, then his learning throughout the sequence is likely to be adequate (p. 56).

Bloom (1971) also suggested this type of subject for the development of a mastery learning approach. He said that a successful mastery approach could be developed in a subject that was "closed", that is, one which is composed of a finite set of ideas, and whose content has changed little over the past years. The writer concurs with Bloom when he said, "Early courses in basic required subjects (e.g. Arithmetic, English, Reading, Mathematics, and Science) are good candidates (for mastery learning techniques)" (p. 67).

The choice of subject and grade was also influenced by the fact that students introduced to mastery learning techniques at this stage would benefit from a curriculum which progressed in this manner from grade to grade as it was implemented in a higher grade level each year. Another reason for the writer's choice of Grade 8 was that students were entering junior highschool and had worked under a strategy which permitted them to progress only after mastering the previous topics - a strategy which is very similar to the mastery learning approach. Finally, by utilizing mastery learning techniques
at a relatively early grade level, the students' future educational achievement could, in all likelihood, be expected to improve.

B. Adoption strategies

Having made preliminary decisions as to the grade and subject for implementation, it was then necessary to devise a strategy for the adoption of such an approach. This strategy required the innovator to establish a working relationship with the principal, teachers, parents and, not the least, the students.

The innovator, in the search for information on mastery learning, selected several articles to use in approaching other co-workers of the school. The writer included the following: Benjamin S. Bloom (1971), "Mastery Learning and Its Implications for Curriculum Development", James H. Block (1971), Mastery Learning: Theory and Practice, and John B. Carroll (1963), "A Model of School Learning". These articles were used by the innovator to extend any explanation required on mastery learning.

At a convenient time, the concept was discussed with the principal, who had believed in such an approach to teaching for some time. The complete plan for implementation was explained at this interview and all questions answered as completely and carefully as possible. An assurance of positive results and hard work was emphasized throughout the interview. The innovator was to leave no doubt as to the possible benefits to be derived from this change.

Once the idea of such an approach was accepted by the principal,
the theory of such an approach was explained to the students themselves. Since the students were the ones who would be directly affected by any change, the innovator explained it to them very carefully, outlining the actual commitment by them and the benefits to be derived from such an approach. The students were very receptive to such a change, which encouraged the innovator.

Most teachers are generally independent of other staff members in their teaching approach, so consequently, approval by the staff was not necessary. However, the writer was of the opinion that for better relations and to assure more assistance, the innovator should present his theory to the staff members. It was assumed that some would accept the idea and offer assistance, while others would not. From those who were willing, the innovator asked for suggestions on alternative strategies to particular problems and help to maintain confidence in the students. To assist those members who were not convinced, further information was supplied. Copies of the three articles selected were displayed in a conspicuous place in the staff room and all members encouraged to read them. The innovator made it clear that he was prepared to present and discuss the articles on mastery learning during a staff meeting, if necessary.

Next, it was decided to inform the students' parents of what the innovation meant to their children. A letter was sent to the parents explaining the main themes of mastery learning and the benefits that would be derived from such a learning approach. Care was taken, however, to prevent the attitude that the innovation was just an experiment or something new to try out for a while. Rather,
it was emphasized that, if successful, the approach would be extended to other classes. It was also explained that with the additional time allowed students on each topic, they would probably achieve at higher academic levels.

C. Organization of materials

After the preliminary planning had been completed, the innovator then began the selection and arrangement of the actual material to be taught. The selection of material is made by the Department of Education in conjunction with the Provincial Mathematics Committee. However, it is the duty of a teacher to comply with the Department of Education, on one hand, but to consider the educational needs of the students on the other. It remained only for the teacher to take the program, whether prescribed or changed from that prescribed, and perform a task analysis on the program.

Davies (1971) emphasized the task analysis as an important step in developing an instructional design. He said,

One of the very first steps in developing an educational or training programme is to analyze the nature of the actual task involved. Some tasks, of course, are purely academic or intellectual in nature, others are primarily concerned with physical skills. However, regardless of the nature of the task, it is necessary to determine both the ingredients and the characteristics of the topic or job that the student has to learn (p. 36).

As Davies pointed out, the task analysis included (1) a listing of these tasks to be learned (2) a description of these tasks in the form of objectives (3) identification of an acceptable performance, and (4) establishment of a means to determine when
mastery is completed. The writer included a fourth step which needed to be finalized before instruction began - the development of a reporting system.

First, a task analysis began with an hierarchical ordering of the units and the tasks within each unit. This hierarchical ordering was necessary because such ordering corresponds to students' mental development as they pass from the easy to the more difficult stages of learning. By ordering the tasks the students were permitted to progress from familiar material into new and unfamiliar material. This ordering in mathematics is especially important for the concepts and principles are built on previously learned concepts.

The innovator considered the units prescribed by the Department of Education, deleted those units which were deemed unnecessary and included topics which were necessary to bring the students prerequisite learning up to the required level. It was decided to postpone some topics until later in the year to provide for a sequential arrangement of units. Later, when students had developed some knowledge of the mastery approach, the topics in Geometry, Measurement, and Probability were taught concurrently with other units on the number systems, but on different days. The innovator planned to be flexible and make changes as would be deemed necessary and advantageous to the students.

As the year progressed, it became evident that the program planned at the outset would not be completed even by the most capable students. This was due to a number of factors, but mainly
because of the capabilities of the students and the insertion into the program of the new unit on the Metric System. Consequently, it was necessary to revise the initial program to suit the students' capabilities and school scheduling. Two chapters were finally deleted to be done the following year. They included work on Exponents and Integers, which are closely related to the Grade 9 program and could be done the following year.

This analysis required a great deal of time and effort, and seemed to involve going into much unnecessary detail. However, the writer believed the analysis to be essential if intelligent decisions were to be made of the learning needs of the students and the necessary teaching strategies.

Second, a task analysis included the description of those tasks in the form of objectives. The formulation of objectives has long been neglected in the instructional process. As Bloom (1968) pointed out, to develop mastery in each student's learning, teachers should be able to recognize when students have attained it. This recognition is only possible if the tasks to be learned are described in terms of well written objectives. The objectives actually define what mastery is expected.

The writer realizes that a mere writing of objectives would not assist learning - it is the way that these objectives are used that will do this. The innovator did, at the beginning of the year, construct a list of general objectives designed from the determined content. These objectives were included with others from other content areas to be presented to the school board. It was then necessary to prepare
a list of specific objectives for each unit. During the process of instruction, the innovator displayed these objectives on duplicated sheets as well as on the bulletin board. Students required an explanation and a constant reminder of these goals, for mastery can only be achieved if they are aware of exactly what must be mastered.

The third step for task analysis was well stated by Block (1971) when he said,

"The crucial step, however, is the translation of these objectives of instruction into specific summative evaluation procedures whereby the evidence required to judge and grade each student's learning at a subject's completion can be gathered." (pg. 67)

As Block has suggested, the setting of mastery goals required some predetermined standard of performance and the establishing of evaluation procedures. The performance standard was absolute. It indicated the specific proportion of items tested a student would exhibit before he could be judged to have mastered the subject (Block, 1971). It was set prior to summative evaluation and served as the sole criterion against which each student's performance was judged.

The writer did not find from the literature any hard and fast rules for setting this standard but used the A-level of 80% as the mastery level. This was a familiar standard used in the school and therefore necessitated very little explanation to parents and students. The innovator was not concerned about the fact that many of the students would be recorded as A's, for this was the main aim of this approach. The writer realized that regardless of the instruction given and the time allowed, some students could not reach the A-level. Consequently, other levels were established. B and C - grades were set for those who would not reach mastery, while a F-grade signified
unsatisfactory progress. It should be pointed out that once the standard was set all students were graded relative to that standard, and not relative to other students' performance.

The means of determining mastery was established. These generally were paper and pencil tests which were constructed to test the objectives of the unit for which they were designed. The writer did not believe these to be ideal instruments to use, however, he believed that the construction of new testing instruments at that time would be both tedious and costly. The textbook includes at the end of each unit a complete mastery test which is especially designed to test the objectives for that unit. When these were used, it was necessary to construct parallel tests to avoid any concern about students repeating the same test. At the beginning of the year, the teacher arranged for two periods in this class, when only half the class would be present. These two periods were used to give more individual assistance, as well as to do some oral testing of some students. This oral testing was one of the criteria for the assignment of grades and the regrouping of students when it was felt necessary. The use of oral testing is itself an innovation for the teacher since he has never used this as a criterion for grading before.

Fourth, a new approach in teaching necessitates a new way of reporting the students' progress to parents. It was coincidental that this year our school was in the process of recommending a new reporting system. The writer was given the opportunity to work with other members of the staff to develop a new report card. This report card included a cover page with a principal's message, an attendance report,
as well as loose sheets for each subject teacher to report for that subject. Each teacher completed a prepared form for that subject and included it for the parent's perusal. The report for mathematics included a letter grade, space for a list of concepts completed and comments by the teachers on achievement and time needed for achievement of these concepts. This coincided with the requirements for a mastery learning approach.

D. **Instructional procedures**

The fourth stage in the implementation of a mastery learning approach was that of planning the instructional strategies. These strategies, although an integral part of the treatment stage had to be carefully planned before implementation was possible. These strategies will be discussed in Chapter IV under the heading 'Experimental treatment'.
CHAPTER IV
TREATMENT, EVALUATION AND RESULTS

The successful implementation and completion of any mastery learning strategy was considered by the writer to be dependent upon the selection of materials and techniques to be used in instruction. For this reason, the writer developed an instructional model to exhibit the general components of such an approach. This model is displayed on page 41. Before examining this model, it may be necessary to include a brief description of the students participating in this project. Finally, this chapter includes an evaluation of the progress experienced during and at the completion of the project.

A. Student profiles

The original intention of the teacher and the administration of the school was to limit the class size to 27 students, but it was soon evident that this was not feasible. To do this, it meant that other classes would be overcrowded, necessitating the addition of five new members to the class. Two of these transferred from schools in Ontario and New Brunswick, two others were transferred from other classes after the first evaluation period, and one requested to be placed in this class later in the year. This gave the project a sample of 32 students. Although an increase of only five students, this added an increased burden on group work and the correcting of completed work by the teacher.

The sample (class) consisted of 15 boys and 17 girls with an
age differential of five years, ranging from 12 years to 17 years. This age differential contributed to a wide diversity of maturity, interests, personal needs, environmental factors and ability, all of which are contributing factors to achievement. Their I.Q. ratings ranged from 67 to 98 on the Otis testing scale. However, more significant was the mean I.Q. for the class. The mean I.Q. was 80, indicating a class of below-average intelligence. Other information was obtained from the cumulative records completed by their previous teachers. Such comments as 'lack of interest' and 'lack of parental help and support' aided the teacher in dealing with some of the problems experienced with these students later in the year.

B. Experimental treatment

The most significant difference in the introduction of mastery learning and the traditional approach was the strategies used in instruction. The classroom operation of mastery learning followed the stages described by Block (1971). These stages are as follows: (1) Pretesting, (2) Grouping, (3) Instruction, (4) Formative testing, (5) Application of learning correctives, and (6) Summative evaluation. Many of these are basically the steps to be followed under a traditional approach with the main difference being the continuous evaluation and regrouping of the students. The traditional approach would apply some correctives to the group as a whole without making provision for those having mastered the objectives to move to the next task.

A pretest was administered to ascertain the students' present
Fig. 1. Diagram of Experimental Treatment

- PRETEST for PLACEMENT
- OBJECTIVES for the UNIT
- TOPIC A
  - TEACHER DIRECTED INSTRUCTION
  - TOPIC B
  - TOPIC C
- LAST TOPIC
- UNIT 1
- UNIT 2
- UNIT 3
- MASTERY TEST A
- ALTERNATE LEARNING STRATEGIES
  - Audio Visual Aids, practice sheets, private tutoring, homework assignments, repeat instruction, etc.
- MASTERY TEST B
- UNIT 1... Summative Testing

...etc.
working level and the extent to which they had obtained their pre-
requisite skills. The administration of the test was kept to a
minimum in time and included questions on the four basic units
completed in Grade 7. The test indicated that most students in
the class had completed the units prescribed for Grade 7, but
had not developed the requisite skills in any great depth. The
decision to start all members of the class at the beginning of the
course was made for this reason as well as for administrative
expediency related to the fact that this was the first attempt at
such a project.

The students were then given a list of objectives on the first
unit. These objectives were explained and displayed on bristol-
board for the duration of the unit. The objectives were written in
behavioural fashion as much as possible and became the direction
for instruction and student studies.

The teacher presented the lesson to the whole group on the task
prescribed, using the methods and in the mode the teacher believed
to be most appropriate for the attainment of the objectives. Students
were encouraged to put forth appropriate learning effort throughout
the first instructional period rather than at the time of small
group activity. After completion of the lessons and practice on this-
topic, the first stage of the formative evaluation began.

Block (1971) described the formative evaluation as an integral
part of the teaching-learning process. He said,

The instruments (used in formative evaluation) are brief, so
that they do not take up inordinate amounts of instructional
time. They are also diagnostic. Each instrument tests those
skills students must learn from a given instructional unit
if they are to master the major desired skills. A formative
instrument administered at the close of a unit, therefore, provides an in-depth picture of what skills each student has or has not learned. Consequently, it suggests in what ways his original instruction must be supplemented if he is to complete his learning before proceeding to a new instructional unit (p.69).

In response to the suggestions made by Block in his article, the teacher, at the end of instruction, administered short tests to determine mastery. The tests were not graded. Rather, they were designated as 'satisfactory mastery' or 'more work needed'. The mastery marking gave those who received it positive evidence of their academic achievement. It is believed that this evidence reinforced their approach to the learning, suggested their study habits were satisfactory, and generated positive interest in and attitudes towards the learning (Block, 1971). Other students saw their mistakes and understood why they needed more work in this area. They were encouraged to do more work so that they, too, could attain a 'mastery' marking next time.

The writer (teacher) believed the mastery learning approach to be most effective at this point in the learning process. The students received immediate feedback indicating their difficulties as diagnosed by the mastery tests. The extent of the difficulties were recorded so that the teacher could treat the students individually, in small groups, or during further class instruction. The teacher's duty became one of regrouping according to the information gained, and then translating this information into specific instructional procedures whereby the teacher could correct these learning difficulties.

The techniques or strategies applied have been summarized under the heading 'Alternate Learning Strategies' in Fig. 1 on page 41. These
strategies varied from topic to topic depending on the weaknesses observed. For some students, the teacher only repeated instruction in the area of the weaknesses and, by using prepared work sheet materials, gave further practice before they attempted mastery test B. For still others the problem seemed to be more severe. It appeared that their difficulties were caused by a lack of prerequisite learning. These students showed a lack of interest and understanding in their work assignment, and therefore, required a different treatment from the regular class instruction. For them, it was necessary to use a variety of corrective measures. However, readers should realize that those measures discussed below constitute a repertoire of techniques used by the teacher during the year, and that only one or two may be used at any one time.

The application of small group instruction was probably the best way to assist students to overcome their difficulties, since the teacher was freed from the class as a whole to spend time with those who had most difficulties. In small groups, the teacher gave individual instruction to problems that had been diagnosed. To perform this type of instruction, the writer grouped students with not more than six or eight in a group, depending on the learning difficulties of the students. Also arranged for the class, was one period for girls and one for boys where individualized instruction was foremost in the mind of the teacher. The ideal, when referring to small group instruction, would be one teacher with one student, which is called tutoring. Since such instruction was difficult to arrange, it was done only in specially arranged sessions, and then
infrequently. The teacher did, however, on several occasions require students to remain after school for a few minutes while special and additional instruction was given. Although beneficial in some respects, the writer was not impressed with the enthusiasm shown by some students in this regard. As a follow-up to these small group sessions, the correctives were extended to the home by the use of assigned work.

For some students, the original approaches to teaching were not sufficient, so it was necessary to use alternative learning materials. These materials took the form of workbooks and programmed instruction, other textbooks, the use of audio-visual methods, and games and puzzles.

The writer of this paper has been attempting over the past years to rewrite the chapters of Arithmetic in simplified language and including many more exercises to assist the students. These were given to the students as supplementary material to be done independently of the teacher. Although related to programmed instruction, these were not programmed to any great extent. The main purpose of these materials was to provide the drill and the specific problem solving practice the students needed for learning. Other textbooks dealing with the same units of work were readily available, so were used to expand the activities the teacher could afford the students.

The use of audio-visual materials further extended individualization of instruction. Some students found it difficult to learn from traditional approaches, but readily responded to visual methods of instruction. The teacher used filmstrips, and visual illustrations
to assist those students. Many of these aids were obtainable from the
School Board office, which supplied these filastrips, motion pictures,
and many other services to the school. However, emphasis was placed
on low-cost resources that could be used with existing audio-visual
and duplicating equipment. Consequently, the teacher often confined
selection to those materials that could be obtained easily or those
that could be inexpensively made by the teacher.

Another corrective which the teacher considered worthy of using
was an ‘exercise pool’. The teacher developed on file cards an ex-
ample pool of exercises to be used by the students to increase their
efficiency in each unit of work and as review materials. They coinci-
dided with the specified objectives of the unit and was continually
modified and expanded to include the important aspects of the unit.
They were organized to make their use by teachers both easy and
efficient. Once the pool was considered adequate, they were used as
an evaluation technique as well as for regular practice.

The writer recognized these approaches as only corrective
measures to be used when the situations require it. They did not
replace the actual instructional period as the students began each
task of the unit.

The final stage in the instructional approach was the adminis-
tration of the summative evaluation tests. The primary purpose of
this evaluation was to grade students according to their achievement
of the course objectives. These were administered infrequently, at
the end of each unit of work. The grades obtained from these tests
were affixed to the students’ report cards and cumulative records.
Unlike the formative evaluation which was done during the instructional
period, this evaluation was considered to be final. The tests used for this evaluation were achievement tests constructed to coincide with the objectives specified at the outset of the program. The final evaluation accounted for 50% of the total evaluation and served as the chief criteria for the final placement of students.

The placement of students from the mastery class, although not considered to be a step in the mastery plan, required much deliberation on the part of the teacher. The fact that some students had not completed all the topics posed some problems late in the year, but these problems were rectified to some extent during the late stages by intensive work using traditional approaches. The writer was unable to find, in the literature on mastery learning, any information on placing students, but considered this a necessary step in the completion of such an approach to teaching. The teacher found it advantageous to develop a philosophy for the advancing of students before such a move was made. To the extent possible, students were placed in accordance with the amount of work covered successfully, the extent of their achievement throughout the year, their level of maturity and their success in other subjects.

The writer offered the following suggestions pertaining to the placement of students from the mastery class. Those students who had mastered all topics prescribed for the course should be classified as A-students and placed in the A-class of Grade 9. This class, after spending a minimal amount of time on review, would begin the study of Algebra and Geometry. Those students who constantly showed an
inability to reach the mastery level should be placed in the basic course for Grade 9. The remainder of the students should be classified as B or C and, thus, placed in the B or C classes. In these classes a longer period should be devoted to the review and completion of basic concepts before continuing into Algebra. It was also suggested that a summer school, which has been organized and operated by the School Board for the past five years, be utilized for those students who had difficulty in completing the course. The summer school could be considered an extension of the regular semester for students to complete the unfinished topics. Most of these suggestions were considered acceptable by the administration and staff and implemented at the end of the year.

The writer realized that the success of this innovation required a large degree of flexibility. Consequently, the writer was prepared to divert from the original plan as much as was deemed necessary, but it was found that only slight differences were necessary to implement the mastery learning approach developed in this project.

C. Evaluation

The success or failure of any project can be assessed by observation of three major determinants. These are: (1) Achievement
(2) Student opinion, and (3) Teacher perception.

The teacher considered the evaluation of student achievement to be a continuous process, so reflected this in the progress made from topic to topic. However, twice during the year the school requirements made possible a summative evaluation of these students.

During the month of January, an examination was administered to test the students on work completed to-date. The results of the grades assigned in the mastery class are recorded in Fig. 2.

![Graph](image)

Figure 2: Grades assigned to the mastery class at midterm.

The bar graph shows a definite skew effect towards the higher grade levels. Indeed, over 70% of the students received an A-grade or a very high B-grade. These results may not be significant since the examination only covered those tasks already completed by those students, and amounted to only a part of the year's work.

At the end of the school year a similar examination was given culminating the complete year's work. The results of this examination are compiled in Fig. 3 on page 50.
Figure 3. Grades assigned to the mastery class in June.

Again the graph indicates a definite trend towards the higher grades. The graph shows approximately 60% of the students having received an A-grade or a very high B-grade.

The final graph is a broken line graph which is used to compare the grades assigned to the mastery class with those received last year under traditional teaching strategies.

Figure 4. Grades assigned in 1976 vs grades assigned in 1975
The teacher made use of one period during the final week to determine the students' attitudes towards this new approach. During this period, the teacher explained that he wanted the students' opinions on their likes and dislikes of the Mathematics class this year. Without identifying themselves, they were urged to answer two questions on a sheet of paper provided. These questions were:

1. What did you like about the Mathematics class this year?
2. What did you not like about the class?

As expected, most students had definite ideas as to the worth or shortcomings of the mastery approach. They expressed these ideas very adequately in the replies they gave to the questions. Since these reflect the opinions of the writer to a great extent, it was decided to include them. The following lists have been compiled from the students' answer papers with slight changes in wording to correct errors in language.

To the question, "What did you like about the Mathematics class this year?", the students replied:

1. You were allowed to check with others doing the same work.
2. It helped us gain confidence in our work.
3. It helped us get better grades.
4. The teacher had more time to spend with some groups.
5. We worked harder to get into a certain group.
6. I worked better with a group of friends.
7. The whole group asks for help from the teacher, not one person.
8. We had an opportunity to improve grades by repeating similar tests.
9. If you were slow, you had a chance to catch up.
10. The teacher learned where you made mistakes.
11. It is easier to learn when you go over it a second time.
12. We didn’t rush through the work as we did in other years.
13. We could get help from the group, rather than the teacher. You might understand your friends better than the teacher.
14. The work got done faster and time passed more quickly.
15. The teacher made sure that everyone knew how to do it.
To the question, "What things did you not like about the class?", the students replied:

1. I disliked the comment 'more work necessary'. It meant failing.
2. I couldn't work with others in the class.
3. There was more talking done in the class.
4. It was a lot more of hard work.
5. Those people in some groups sort of kept to themselves.
6. I disliked some people moving ahead of me.
7. The teacher interrupted when he was working with other groups.
8. I didn't like having to get 80% to be considered satisfactory.
9. I couldn't get help from the teacher sometimes because he was working with another group.
10. The people in some groups couldn't catch up with the fast group.
11. You would have to show people in your group.
12. When you were in some groups you felt 'dumb' or 'put down'.

The conclusions made by the teacher related to the success or failure of the project, and other observations perceived by the teacher will be summarized in the next section of the paper on page 55 below or in the general conclusions of Chapter 5.

D. Discussion

The teacher reaction to this approach was very positive since it encouraged a large degree of involvement on the part of the teacher. The involvement began very early in the year when the teacher was responsible for ordering the tasks, writing the objectives, and preparing instructional materials. With the experience gained from this involvement, the teacher acquired, as well, a motivation to become more creative and attempt other design projects.

While using the traditional approach, the teacher discovered that much time usually elapsed before any factual evidence of student weaknesses was received. However, under the mastery learning approach, this was not possible. A constant feedback on student achievement was evident.
This information, besides informing the teacher of the action to be taken, reinforced the learning and assured the students that their mode of learning and their approach to study was adequate.

The best indicator of the success of a project is the achievement accomplished by the students. This is evident in this project. The number of A and B grades received by the mastery students, as indicated in Figure 3 on page 50, is considered by the teacher to be significant. Never before had the students received better grades in mathematics than they did in other subjects and, consequently, their attitudes towards mathematics began to change. This success appears to be more significant to the teacher when it is considered that all students in the experimental group wrote the same final exam as did other Grade 8 students in the school, and that little time was provided for review. Although the mastery approach permits students to complete varying amounts of the program, this was not feasible for those students. Some would be working with different teachers in different schools next year, so would be placed at an unfair disadvantage if the program was not completed. Consequently, the teacher used the last six weeks to finish the program for all students.

The success of the project is also evident when one considers the proportion of those students attaining mastery as compared with those from the previous year. However, the teacher recognizes the difficulty of stating valid conclusions from the comparison made in Figure 4 on page 50, because of the many variables that contribute to student achievement but were not controlled during the project. Even though there were different teachers, different programs and different levels of maturity, the fact remains that they did receive
better grades this year under the mastery learning approach.

A very obvious change in student attitudes was observed in their attitudes towards testing. The students were not accustomed to writing formal tests in the Elementary School and, therefore, had great apprehension towards them in the Junior High School. Once they realized that the tests to be given were directly related to the objectives, the dread they had experienced gradually subsided. They soon learned that the tests were easy to anticipate by considering the objectives they had received for each unit.

From their comments listed in this chapter, the students concluded that working in groups is advantageous. The students enjoyed working in groups because it gave them an opportunity to work and discuss their difficulties together before approaching the teacher for help. This approach usually permitted some to help weaker students in the group and released the teacher to work with others individually.

In conclusion, the writer believes that as a result of this project, the students in the class will progress to Grade IX with added confidence and anticipation. If this happens, then the project was worthwhile.
CHAPTER V

SUMMARY, LIMITATIONS, AND CONCLUSIONS

An evaluation of such a project requires the formulation and reporting of some general conclusions related to the changes observed in student achievement and both student and teacher attitudes. The conclusions, as outlined in this chapter, will describe as well the basic strengths and limitations of the approach based on the writer's perception of the project. This description could assist readers in making other attempts at this or similar projects.

A. Achievement

The writer concludes that there was a definite improvement in students' achievement under the mastery learning conditions. As it already has been reported, the mastery students under the present project achieved considerably higher than they did under traditional methods. This could be attributed to the many outstanding features of the mastery approach. These include: grouping, feedback and correction, individualized instruction, and the continuous success of those students. Whatever the reason, most students did appear to improve their grades this year. Whether or not the difference was sufficient enough to be considered significant is questionable, because no statistical comparison was made. Even though most students did better under the mastery approach, several fell quite short of the standard set for mastery.

The success of students to attain the objectives defined at the beginning of the course is dependent upon the skills previously learned. Therefore, lack of success in some students may be attributed to their incomplete mastery of the prerequisites. It may be only
speculation, but the writer is of the opinion that more effective learning would result if the mastery learning approach was applied over the complete hierarchy of skills.

B. Attitude changes

The mastery learning approach used in this study appeared to produce significant changes in the attitudes of some students, while no change for the better was evident in others. The better students were interested in the completion of each topic and got tremendous satisfaction out of achieving at an A-level. They received the confidence that appeared to be lacking as they began the program and with this confidence they were able to accomplish much more work.

However, the slower students found no satisfaction from being told that they needed more work to complete the topic. Indeed, they soon associated the fact that they didn't move to the next task with that of failure. Students in this category generally fell further behind. They did not, as expected, work harder to master the topics, but instead, became lazy and despondent. It was necessary to encourage them or revert to tactics used in the traditional approach of giving them a failing grade and moving them ahead with their classmates.

The concept of grouping did not take on the significance for the slower students as it did for the others. There was a tendency for them to waste time and aim only at the least possible amount of work, rather than try to complete as much work as others in the class. It was found that to maximize their efforts, it was necessary to place a 'pacer' in the group. The pacer would be a person who was probably slow but thorough in his/her work.
The teacher's (writer's) attitude towards the approach was very favourable. The writer preferred this type of instruction as compared with traditional methods, although this method was much more time consuming and demanding than any other method used. In spite of the demands placed on the teacher, a great deal of satisfaction has accrued from this curriculum development project.

C. Strengths of mastery learning

The mastery approach has many strengths that could be utilized in any teaching situation. The writer will summarize these as follows:

1. There is every reason to believe that this approach would yield an increase in the grades assigned to most students. The increase was definitely evident in the number of A-grades assigned as compared to the results of the previous year under traditional methods.

2. The use of feedback/correction procedures in this approach transforms group-based instruction into individualized instruction. The feedback received from formative testing reinforces the learning of the better students and diagnoses effectively any weaknesses of the others before much time elapses. It gives direction to teachers as to the corrective measures to be taken immediately, and not at the end of the school semester as under traditional approaches. The diagnosis should be accompanied by a very specific prescription if the students are to rectify their errors of learning.

3. Successful attempts at completing the prescribed work provide students a basis for the development of intrinsic motivation for
learning. Those students who accomplish mastery are motivated to do as well in the next task while the other students, who do not reach mastery on the first attempt, do so on successive attempts. This success enables them to enter the next learning task with a better grasp of the preceding learning tasks in the series. This success increases their confidence and interest in the task, and prevents the frustration that is usually associated with failure.

4. Mastery learning includes the elements of good instruction.
It incorporates the principles found in programmed instruction but does not rely on the rigid nature of this method of teaching. It is socially acceptable to students since there is a constant interpersonal relationship with teachers as well as with other students.

5. The extent that it caters to individual differences has to be a major strength of the approach. Individuals are taught, to some extent, independent of others. Special instructional materials are constructed to fit the strengths and weaknesses of the individual. The student is graded against his achievement of the objectives and is not compared with others in the class. This approach is a definite attempt to individualize instruction.

6. Finally, the approach adheres to the principles of development advocated in the introduction of this paper. These include: preplanning, specification of objectives, group instruction, formative evaluation, individualized instruction, and final evaluation. Therefore, the approach is based on some fundamental principles of learning and teaching.
D. Limitations of mastery learning

For any innovator, the advantages of such an innovation appear to outweigh the disadvantages. This is apparent for two reasons: Firstly, the innovator is dedicated to the innovation selected and, secondly, the readings found while researching the topic are usually in favour of the innovation. Therefore, it is logical to assume that this report would include the favourable aspects of mastery learning. However, the evaluation of any project should include any limitations observed by the writer during the project. In this section, the writer will cite a number of limitations observed in his experience with this mastery learning project.

1. In mastery learning, it is necessary to identify specific performance objectives and appropriate instruction and time for mastery of these objectives, but it is difficult to prepare these objectives beyond the knowledge and application level. The very concept of 'mastery' denotes the acquisition of knowledge. Consequently, the apparatus used to determine mastery does rely on the knowledge acquired. It does not measure the ability of students to solve problems, only the skills acquired in the solving of problems. It does not permit social development objectives like 'learning leadership skills' or 'understanding of interpersonal relationships'. These can best be taught by the use of field trips and other social group activities for which mastery learning does not provide. There is also a tendency for the objectives to be set for the slowest members of the class. Such classes are of little challenge (and often a waste of time) for even the average ability class members let alone the fastest learners. This would defeat the whole purpose of mastery learning.
2. The most obvious drawback of mastery learning is that this approach is difficult to apply in schools where organization is based on grades and time periods to complete these grades. The idea of mastery suggests a continuous advancement from topic to topic as each one is mastered. Not all students can be expected to achieve mastery of all topics at the same time. Therefore, the decision to promote them to the next grade becomes a difficult one. The writer, in his proposal, suggested several alternatives that would be considered in promoting the student but recognizes that some may not be feasible because of the grade structure and the student's achievement in other subject areas. For the mastery model to be effective, students must be allowed to pass to subsequent grades at any time during the school year.

3. A third drawback of the mastery model is that it is very costly to operate. Since students progress individually rather than as a class, ideally, they should be instructed individually. As the number of groups gets larger there should be provisions made for tutoring or the development of a very elaborate series of self-instructional materials. This requires a tremendous expenditure in teacher time and school money. The teacher should be given a smaller teaching load, allowing time to prepare such materials and monitor student progress. However, presently the mastery programs in education do not permit this. Further, the cost in time and money required to develop tools for measurement of mastery would be in excess of what the writer considers feasible. Even if the money were available, it would be difficult to build in the objectivity needed for evaluating mastery test items.
4. Another drawback of the mastery approach is the difficulty of reporting. The present practice of most schools is to report at equal intervals of time during the school year. This is impractical under a successful mastery approach. Students are completing the topics at various stages of the school year and their progress should only be reported after mastery is reached. The use of the grade 'incomplete' would have to be utilized on evaluation and reporting would have to occur at different times for different students. This practice would require further explanation to parents, since they have become accustomed to receiving reports at the same time.

5. One of the basic principles which characterizes mastery learning is that students progress at their own rate without any references to other students in the class. This is not necessarily good for the students. They will and do live in a culture where competition is a respected value. Thus, it is difficult (and perhaps unwise) to restrict people from comparing themselves with others. Certainly, students should compare themselves to aid them in deciding future education, jobs and other options in life. Further, it is evident that students still see differentiation although in another form. They become aware of students ahead of them rather than those getting better grades.

6. Finally, many educators are not convinced of the very concept of mastery learning. They argue that, for some areas, there are several degrees of mastery, whereas, in others nothing less than complete mastery is satisfactory. The mastery approach assumes that
everything must be learned by all students, and that all topics must be learned to mastery. These assumptions are not necessarily true in all cases. As prerequisites, some topics are not necessary for the next, but this is contrary to the mastery approach. Consequently, it is reasonable to assume that only some subjects lend themselves to the mastery approach.

E. Summary

In summary, the mastery learning approach, although having many limitations when implementation is attempted in the present school system, may outline a basic strategy that is more successful than other strategies for these reasons:

1. It can be implemented to some degree by individual teachers who have limited resources, if they are prepared to expend the necessary time.

2. The teacher is more involved than in the traditional approach. The teacher structures the content, resources and methods to best satisfy the set objectives. The teacher must endeavour to improve the method of record keeping that was used in traditional approaches.

3. It outlines a clear approach for instruction that diagnoses students' weaknesses, gives remedial treatment and evaluates the results of such treatment.

4. It exemplifies better than other methods the systems approach to development.

5. It uses techniques that could be used in any method of instruction with some variation.
6. Most of all, it improves the achievement of students to some degree.

It is hoped that this description of the approach attempted will stimulate other teachers into similar projects in an attempt to fulfill our responsibility to have students attain their potential and thereby find satisfaction from their future life.


APPENDIX

The writer, while preparing this design paper, considered as essential the following levels of development:

A. Student analysis
B. Task analysis
C. Behavioural objectives
D. Media - selection of materials
E. Teaching strategies
F. Mastery test
G. Formative evaluation

In this section of the design paper, the writer selected samples of the materials prepared and used by the teacher. The selections are taken from the Unit 'Formulas, Areas and Volumes'.
A. Student analysis

The planning of such a program involved an analysis of the students' interests, capabilities and needs. The teacher attempted to identify several attributes and characteristics of the students by doing a preliminary study of the following:

A. Personal characteristics
   1. I.Q. rating
   2. Environmental factors
   3. Personal factors
   4. Physical conditions

B. Academic capabilities

The academic capabilities of the students were ascertained by investigating their past achievements as well as by the administration of a pre-test. The pre-test tested their knowledge of the units done previously in the Elementary School.
B. Task analysis

Topic 1: Relationships of polygons and circles.
(a) Develop the concept of perimeter.
(b) Develop the concepts of polygon, triangles, quadrilaterals, square, rectangle and pentagon.
(c) Calculate the perimeters of polygons.
(d) Develop the concepts of circle, radius, chord, diameter and circumference.
(e) Establish the basic relationships of circles.

Topic 2: Finding areas.
(a) Find examples of area in everyday activities.
(b) Develop the concept of area.
(c) Introduce the units to be used.
(d) Calculate area of rectangles.

Topic 3: Study of formulas.
(a) Introduce the meaning of formulas.
(b) Find some uses for formulas.
(c) Name the components of a formula.
(d) Examine several formulas.

Topic 4: Finding areas using formulas.
(a) Calculate the area of squares.
(b) Develop the concepts of height and base.
(c) Calculate the area of parallelograms.
(d) Develop the formula for area of triangles.
(e) Calculate the area of triangles.
(f) Develop the concept of trapezoid.
(g) Calculate the area of trapezoids.

Topic 5: Finding the Volume by using formulas.
(a) Develop the concepts of solids, cylinders, cubes, pyramids and cones.
(b) Calculate the volume of each of the solids.
C. Behavioural objectives

Topic 1:
General Objective: Students should understand and apply the basic relationships found in polygons and circles.

Behavioural Objectives:
At the end of the chapter, Formulas, Areas and Volumes,
1. The student will write a sentence to explain 'perimeter'.
2. The student will state, on the answer page, the rule for finding the perimeter of polygons.
3. Given a number of polygons with the measurements of their respective sides, the student will calculate their perimeter.
4. From four diagrams drawn on paper, the student will identify those representing polygons.
5. The student will write the name of five out of six polygons whose diagrams are shown on flash cards.
6. The student will identify the four parts of a circle that will be shown on a diagram.
7. Given the radius of a circle, the student will calculate the diameter.
8. Given the diameter of a circle, the student will calculate the radius.
9. The student will state the relationship which exists between the diameter and the circumference of a circle.
10. Given the diameter of any circle, by using the formula \( C = \pi d \), the student will calculate the circumference of a circle.

Topic 2:
General Objective: Students should understand the meaning of area and be able to calculate areas of polygons.

Behavioural Objectives:
At the end of the chapter,
1. The student will list at least four situations which require the calculation of areas.
2. The student will state the meaning of the area of a polygon.
3. The student will count the units contained in each polygon presented, to find its area.
4. The student will name three units used in finding areas.
5. The student will state the number of centimeters in 1 metre, the number of metres in 1 kilometre, etc.

6. Given the dimensions of at least two rectangles, the student will calculate the area of these rectangles.

7. Given the dimensions of a polygon that can be divided into rectangles, the student will calculate the area of this polygon.

Topic 3:

General Objective: Students should understand what formulas are and the use made of them.

Behavioural Objectives:

At the end of the chapter,

1. The student will write a sentence to explain what a formula is.

2. The student will write formulas to represent known relationships about polygons, which are written for him.

3. Given any formula, the student will name the three component parts.

4. Given the diameter of a circle, the student will use the formula \( C = \pi d \) to find the circumference.

5. Given the dimensions of a rectangle, the student will use the formula \( A = l \times w \) to find the area.

Topic 4:

General Objective: Students should be able to use formulas to find the areas of polygons.

Behavioural Objectives:

At the end of the chapter,

1. Given the measurement of a side of a square and the formula \( A = s \times s \), the student will calculate the area of a square.

2. Given a diagram of a parallelogram, the student will name the base and height.

3. Given a diagram of a triangle, the student should name the base and the height.
4. From a list of diagrams, the student will select and name the circles, triangles, parallelograms, and trapezoids.

5. Using mathematical symbols, the student will write the formula for finding the area of each of the following:
   (a) Parallelogram  (c) Trapezoid
   (b) Triangle  (d) Circle

6. Given the dimensions of each of these polygons in No. 5, and by using the formulas, the student will calculate their areas.

Topic 5:

General Objective: Students should be able to use formulas to find volumes of three-dimensional solids.

Behavioural Objectives:

At the end of the chapter,

1. The student will identify and name on paper the following solids presented by the teacher: cylinders, cones, cubes, pyramids, and spheres.

2. The student will list at least three things for each from the environment, which have the shape of the solids listed above.

3. The student will state the meaning of 'volume of a solid'.

4. Using the mathematical symbols, the student will write formulas for finding the areas of the following solids:
   (a) cylinders  (b) cones  (c) cubes  (d) pyramids  (e) spheres

5. Given the dimensions of each solid given in No. 4, the student will calculate the volume of these solids.
D. Media - selection of materials

The selection of media was made after consideration of the following criteria:

1. The accessibility or ease in using the materials.
2. The purchase or preparation costs.
3. The time required to locate or prepare each item.
4. The ease with which they could be used by the teacher and the students.
5. The type of materials best suited to the teaching strategies.

The following materials were used in this chapter:

1. Blackboard and chalk.
2. Paper and pencils.
3. Pictures.
4. Mathematical models.
5. Mathematical sets.
6. Flash cards.
7. Prepared diagrams.
12. Filmstrips (when available).
E. Teaching strategies

The teaching strategies outlined here are those first used in introducing the topic and are not to be confused with the alternate teaching strategies applied after the first mastery test. These are described in detail in the main part of the text.

The following principles were incorporated into the lessons which follow:

1. Attempt to establish a need for the learning of each concept.
2. Present the lesson in a sequential fashion such that successive lessons result from the previous one.
3. Use examples and situations from student activities to stimulate his thinking and motivation.
4. Lead students to discover the various concepts by the use of questioning and class discussion.
5. Make use of teaching aids to best acquire the desired terminal behaviour of the students.
6. Use discussion accompanied by demonstration and student activity.
7. Limit the amount of practice given during the lesson itself, but assign sufficient practice for reinforcement after the lesson has been taught.

Lesson 1: The concept of perimeter.

Step 1: Outline the objectives for the lesson.

Step 2: To establish a need for learning this particular concept the teacher would begin a discussion related to the students' interests and activities. A discussion of baseball would result in such questions as 1. How far away is home plate from the mound? 2. How far must you hit a ball to drive it over the left field fence? 3. How many sides has the ball field?

A statement like 'Did you know that the ball field has a...
perimeter of approximately 2000 ft.? Would certainly have the student wondering, first, what is meant by perimeter and second, how do you measure it.

Step 3: Leading from question 3, have a student draw the shape of the tall field on the board. At this time a picture of a similar shaped object would be shown to the class, or a model which possesses this shape. The teacher tells the students the shape on the blackboard is a PENTAGON.

After drawing other shapes on the blackboard, the teacher asks the class how they are alike. The class will eventually supply the answers:

1. They are made up of segments.
2. They are on the same plane.
3. They are all closed figures.

The teacher explains that because of these things they all belong to the POLYGON family. The teacher then asks students to draw other members of the family and label the drawings with their appropriate names. The students are asked to make a bulletin board display of the family.

Step 4: Students are then required to find the distance around the polygons. After they see that you must add up the length of all the sides, you tell them this distance is called the perimeter.

Step 5: To reinforce this concept, have the students use a tape to measure the perimeter of the desk, the classroom, the school and report back to the next class.

Step 6: Have students write the meaning of perimeter on their answer books and state the rule for finding the perimeter.

Step 7: Further reinforcement is possible by assigning written exercises prepared in advance.
Lesson 2: The parts of a circle.

Step 1: Begin this lesson with a discussion on some topic which might interest the students. Use the topic 'large trees', for this is related to the shape of circles. Show pictures of very large trees, or bring a cut section from a large tree for comparison purposes. Tell the students of the Douglas Fir in British Columbia which has a road passing through its base. Then use questions like:
1. How wide are most roads?
2. How big around do you think this tree is?
3. Would the distance around be more than or less than the width of the road?

Step 2: Make a statement like - Did you know that the world's largest living thing is a tree? This tree is the General Sherman tree in California, whose circumference is 102 ft., and diameter is 92 ft.

Step 3: Before students continue they have to be familiar with this new shape and the terms related to it. The circle and its various parts would be explained and named by the use of transparencies and the overhead projector. Use these overlays to show the parts separately and then together for purposes of comparison. The following concepts are to be introduced: circle, diameter, radius and chord.

Step 4: The teacher would ask questions as follows: (1) Can we find the distance around such a circle? (2) Can we call this distance the 'perimeter' of a circle? After eliciting the answer 'yes', we would indicate that this is a special polygon (one with an infinite number of sides) so we call its perimeter the 'circumference'. No doubt the idea of a special polygon having an infinite number of sides will stimulate further discussion.

Step 5: By the further use of overlays, students would be helped to see the relationship between the length of a radius and the diameter in a circle.

Step 6: Have the students write the relationship in a full sentence on their answer books.

Step 7: Make an assignment to the class to further reinforce the ideas learned.
Lesson 3: The relationship between the radius and diameter of a circle.

Step 1: The teacher would have asked the students on the preceding day to bring to class a wheel, a water bucket, pie plate, and a tin can. The teacher first asks the class the reason why these are selected.
   1. They all represent circles.
   2. They are of various sizes.

Ask the students to wrap a string around the circles. They are helped to see that the length of the string represents the circumference of the circles. This would be shown by teacher using a model. If they have acquired the concept of 'diameter', they will be able to measure it by measuring the length of the longest chord. The teacher would demonstrate this by the use of a model.

Step 2: The teacher asks the students to find the circumference and diameter of each object and summarize their findings in a table that they had drawn on their page previously.

<table>
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<th>diameter</th>
<th>circumference</th>
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</tbody>
</table>

Step 3: The teacher asks the class if there is an apparent relationship between the two. Some students will see that the circumference is a little more than three times the diameter. Then the teacher tries to be a little more specific. He asks the class to divide the length of the diameter into the length of the circumference. The students will be surprised to find that the answer in each case is approximately 3.1.

Step 4: After the students have discovered this fact the teacher begins to summarize the discovery. The summary would be as follows: The circumference is approximately 3.1 times the diameter for any circle. This number constant is represented by the Greek letter \( \pi \) which has been approximated to 3.14159... For calculation purposes, we may use the value 3.14.

Step 5: The teacher may add further historical notes to the teaching of this concept.

Step 6: The teacher reinforces the ideas by a class assignment.
F. Mastery test - Topic 1

The student will answer the following questions on the pages provided. Think carefully and write your answers legibly.

Topic 1: Relationships of Polygons and circles.
1. Write a sentence to explain the term 'perimeter'.
2. State the rule for finding the perimeter of any polygon.
3. Calculate the perimeter of each of the following polygons:
   
   ![Diagram of polygons]

4. Identify each of the following as a polygon or not.
   
   ![Diagram of shapes]

5. The teacher will flash six cards at the end of the test to show polygons. Write the name of each polygon on your answer page.

6. In the following diagram, name the parts of the circle that are indicated with letters.

   ![Diagram of circle with letters]

7. If the radius of a circle is 3.5 cm., calculate the diameter.
8. If the diameter of a circle is 24 cm., calculate the radius.
9. State the relationship which exists between the diameter and the circumference of a circle.
10. If the diameter of a circle is 15 cm., use the formula $C = \pi d$ to find its circumference.
G. Formative evaluation

An evaluation of the students' progress was made during and at the end of the teaching period. Constant observation and checking of the assigned work was necessary to determine progress and further grouping. At the end of each topic a test was administered to determine whether mastery had been achieved. The tests were not graded. Rather, they were designated as 'satisfactory mastery' or 'more work necessary'. The performance level of 80% was established at the outset of this project and used throughout to determine in which of the two groups to place the students. The mastery test used in the first topic of the unit is enclosed on page 78. Those students who received a 'satisfactory mastery' rating progressed to the next topic while those who received a 'more work necessary' rating were given special treatment as described in Chapter IV on page 40.