A COMPARATIVE STUDY OF AN ENACTIVE AND AN ICONIC APPROACH TO THE TEACHING OF PERIMETER AND AREA TO SEVENTH GRADE STUDENTS

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by

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ABSTRACT

The purpose of the study was to compare the effects on achievement and retention of an enactive and an iconic approach to the teaching of perimeter and area concepts to seventh grade students. Four intact classes of heterogeneously grouped students were selected for the study. Treatments were randomly assigned to the classes. The students were stratified into low, middle, and high ability groups by means of the Canadian Tests of Basic Skills. In the enactive treatment two classes used concrete materials in the form of 5 x 5 geoboards while the two classes in the iconic treatment used a semi-concrete material in the form of 5 x 5 dot paper on which they drew the same figures that the enactive group made on the geoboards.

Two parallel sets of lesson plans, a posttest, and a retention test were constructed by the researcher. A pilot study was conducted prior to implementation of the main study. The posttest was administered immediately after the instructional period which lasted about 20 teaching days, and the retention test was administered five weeks later. Data were collected from 103 students and a two-way analysis of variance was used to analyze the scores on each test. Scheffé tests were used to investigate differences among the three ability levels.

Six null hypotheses were tested at the 0.05 level of significance, and all were rejected. The results were as follows: There was a significant difference in mean scores on both the posttest and retention test in favour of the iconic treatment. There was a significant difference in mean scores among the three ability levels on both the posttest and retention test. There was a significant interaction between treatment and ability with respect to scores on both the posttest and retention test.

Based on the results of the study, it was recommended that further research be conducted using a larger sample and dealing with a different topic, at a lower grade level, or with students who have had previous experience with manipulative materials.

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THE PROBLEM

Need for the Study

In recent years there has been a noticeable trend in mathematics instruction towards increasing the active participation of students in the learning process. For the most part, activity methods have been employed with elementary school students. The junior high school years, however, have long been regarded as a time when student interest in school begins to decline and unfavourable attitudes toward learning start to develop. The wider use of manipulative materials, often within a mathematics laboratory approach, is an attempt to provide an alternative to the traditional textbook-chalkboard type of instruction.

Educators have varying conceptions of activity-based instruction. Suydam and Higgins (1977) pointed out that the common element found in these different conceptions is student involvement in the process of learning mathematics. This involvement is not only intellectual; the student is actively involved in doing or seeing something done.

To many educators the term "activity learning" is synonymous with "mathematics laboratory". Kulm (1976) described the mathematics laboratory approach as a kind of teaching and learning strategy that involves the student in discovering something for himself. Conclusions drawn by the student are then based on his own activity and observations rather than on information given to him directly by the teacher (Weiss, 1978). Jolly (1978) described the laboratory approach to teaching simply as a method of planning and organizing mathematical experiences in such a way as to get more active student involvement in mathematics classes and make more use of physical materials in the classroom.

The use of manipulative materials is not new in the teaching of mathematics. As Schussheim (1978) pointed out, many of the experiments in mathematics education that took place in the late 1960s and early 1970s called for the use of manipulative materials or the establishment of mathematics laboratories. Weaver (1971) wrote:

> Experience, activity, interest and so on, are rapidly becoming shibboleths for elementary school mathematics education, often enshrined within a nebulous math-lab approach to instruction. (p. 263)

He wrote further:

> The crucial factor associated with experience and activity is appropriateness. . . . But strong leadership is needed to suggest promising activities and experiences that are appropriate for the attainment of particular mathematical goals or objectives within thoughtfully planned, systematic programs of instruction. (p. 264)

Kieren (1971) noted that while the use of manipulative activities was currently in vogue, research was needed into the role and effects of manipulatives in teaching mathematics. Suydam and Higgins (1977), in a review of research on the use of manipulatives, noted that because of an intuitive belief in the importance of using manipulative materials, some mathematics educators see little need for further research on the use of such materials.

Suydam and Higgins repudiated this view by stating:

> There are too many studies where the use of manipulative materials is "only as good as" regular instruction to believe that we know all that is needed about the use of materials. Our understanding of the details of effective use is shockingly scant. There is an obvious need for new research efforts on the use of materials in activity learning in elementary school mathematics. (p. 107)

This statement is significant in view of the fact that the National Council of Teachers of Mathematics (NCTM), in An Agenda for Action: Recommendations for School Mathematics of the 1980s, recommended that:

> Teachers should use diverse instructional strategies, materials, and resources, such as the provision of situations that provide discovery and inquiry as well as basic drill; the use of manipulatives, where suited, to illustrate or develop a concept or skill. (p. 12)

Bruner (1966) proposed that a theory of instruction should specify the following: experiences that predispose the individual toward learning; ways in which a body of knowledge should be structured so that it can be grasped by the learner; and effective sequences for instruction. Bruner distinguished three systems of processing information by which individuals construct models of the world: through action, imagery, and language. He used the terms enactive, iconic, and symbolic to identify these three modes of representation. Bruner stated that any idea, problem, or body of knowledge could be presented to students in these three modes. The task of teaching a subject to a child at any particular age is one of representing the structure of that subject in terms of the child's way of viewing things.

Bruner (1964) suggested that learning mathematics begins with instrumental activity, a kind of definition of things by doing them. Such operations become represented and summarized in the form of images. Finally, with the help of symbolic notation, the learner comes to grasp the abstract properties underlying particular concepts.

Purpose of the Study

The study was designed to collect and analyze data from seventh grade students participating in two modes of learning, one utilizing concrete and the other semi-concrete materials. The treatment in which concrete materials were used was called the enactive mode. The treatment in which semi-concrete materials were used was called the iconic mode. The concrete materials consisted of geoboards and rubber bands while the semi-concrete materials consisted of dot paper (geopaper) on which figures could be drawn. The mathematical content used in the study included the topics of perimeter and area of polygons.

Answers to the following questions were sought. To what extent were students able to learn and retain mathematical concepts after using concrete and semi-concrete materials? To what extent were the same objectives of the concrete approach achieved by students using the semi-concrete approach? Was there any interaction between treatment and ability with respect to achievement when the students were stratified into low, middle, and high ability groups?

General Hypotheses

The effects of the two treatments were tested under the following general hypotheses. Each of these hypotheses was tested for a posttest and a retention test. They are stated more specifically in Chapter III.

1. There is no significant difference between the two treatment groups in their performance on a mathematics achievement test.

2. There is no significant difference between the three ability groups in their performance on a mathematics achievement test.

3. There is no significant interaction between treatment and ability with respect to student performance on a mathematics achievement test.

Definition of Terms

The following is a brief description of terms used in the study or encountered in the review of the literature.

Enactive: An enactive representation is characterized by a set of actions for arriving at a certain concept; the student manipulates objects directly.

Iconic: An iconic representation is characterized by a set of images or graphics that stand for the concept; the student deals with pictures rather than concrete objects.

Symbolic: A symbolic representation is characterized by a set of symbolic or logical propositions that present the concept in an abstract manner; the student deals with mathematical symbols.

Concrete: In the context of this study, concrete and enactive are synonymous. Each student was provided with a manipulative device, a geoboard, which served as a model to which they related the principles and problems introduced to them.

Semi-concrete: In the context of this study, the semi-concrete approach and the iconic mode are synonymous. Each student drew diagrams on dot paper which then served as a visual model for the principles and problems presented to them.

Achievement: Achievement refers to the degree of understanding of instructional content as measured by a student's score on an achievement posttest administered immediately after the study.

Retention: Retention refers to the degree of recall of the principles in the instructional content as measured by a student's score on a retention test administered five weeks after the study was completed.

Activity learning: Activity learning refers to school learning situations in which the student develops mathematical concepts through active participation. The process might involve the manipulation of physical objects, measuring, drawing, counting, comparing, seeking patterns, and recording data.

Manipulative materials: These are objects or things that the pupil is able to feel, touch, handle, and move. They are intended to provide an embodiment of the mathematical principles or ideas being explored.

Mathematics laboratory approach: The mathematics laboratory approach is a kind of teaching and learning strategy which places students in an activity oriented situation which involves them in discovering something for themselves.

High ability student: In the context of this study this refers to a student who has scored in the top one-third of the sample on the mathematics section of the Canadian Test of Basic Skills.

Middle ability student: This refers to a student who has scored in the middle one-third of the sample on the mathematics section of the Canadian Test of Basic Skills.

Low ability student: This refers to a student who has scored in the bottom one-third of the sample on the mathematics section of the Canadian Test of Basic Skills.

Scope and Limitations

The topic chosen for this study was one that is common to many junior high mathematics programs; perimeter and area of polygons, and, in particular, the development of area formulas for three- and four-sided polygons.

For the enactive mode, the students used 5 x 5 geoboards and rubber bands as manipulative materials. For the iconic mode, the students used 5 x 5 dot paper on which they drew the same diagrams that the enactive group made on their geoboards. Two parallel sets of lesson plans, which allowed the teacher a realistic and active role in the presentation of the materials, were written for both treatments.

In each of the treatment modes the students were stratified into high, middle, and low ability groups. Comparisons were made on scores on achievement measures and retention of achievement.

The study had several limitations. First, it was not possible to randomly assign students to treatment groups. Instead, intact classes were used, with random assignment of treatments to classes. Second, the same teachers did not teach both treatment groups. These two limitations were partly controlled in the following manner: Only schools with a single grade seven class were used in the assignment of treatments to classes, thereby making the intact classes selected as heterogeneous as possible. Four classes were used; two for each treatment. The directions in the teacher's lesson plans were very specific and parallel in both treatments, thereby ensuring some control of the teacher effect. The teachers were directed to scrupulously follow these directions.

A third limitation was due to the relatively short duration of the study, namely, one month. The results cannot be generalized to an extended period of time such as an entire school year.

Chapter II

REVIEW OF RELATED LITERATURE

The literature review in this chapter is organized into several sections. First, a brief history of activity learning is outlined. Next, a rationale for the use of manipulative materials is provided, followed by a rationale for the use of manipulatives in teaching perimeter and area. In the next section, Bruner's learning theory, which is of particular relevance to the study, is discussed. This is followed by a review of studies which focus on Bruner's modes of representation: enactive, iconic, and symbolic. Finally, a review of research studies that involved instructional approaches similar to the ones employed in this study is provided. Research studies dealing with immediate achievement are considered first; then, studies dealing with both immediate achievement and retention are reviewed. Since this study was concerned with the teaching of a unit of geometry at the seventh grade level, only those studies dealing with geometry, and involving sixth, seventh, or eighth grade students are reported. Some dealt specifically with perimeter and area while others dealt with related topics in geometry.

History of Activity Learning

Current writers generally attribute the forces behind the use of manipulative materials to the work of Piaget (1953, 1970), Bruner (1960, 1966), Dienes (1971), and others. However, early influences can be traced back to the nineteenth century.

Leeb-Lundberg (1970), in an article comparing the Frobelian kindergarten with an activity oriented elementary class of today, pointed out that both contain many similar concrete materials. As early as 1826, Froebel had formulated a theory about the teaching of elementary school geometry which started with three-dimensional bodies constructed by the children themselves.

Austin (1927) proposed that concrete materials be used in teaching high school geometry. He wrote that "pupils should be permitted to observe the laws of geometry operating in concrete form before they are required to do logical thinking (p. 287)."

In 1954, the NCTM published a yearbook entitled Emerging Practices of Mathematics Education (Clark, 1954). In a section entitled "Laboratory Teaching in Mathematics" is found the following statement:

> Laboratory techniques have long been used in public schools in such areas as science, dramatics, home economics, and shop. Teachers have long been urged to use laboratory techniques in the teaching of mathematics. Enough teachers are doing that, so that we may well consider laboratory teaching

> as one of the emerging practices in teaching mathematics. (p. 101)

Laboratory techniques were described as an approach to teaching and learning mathematics which provided opportunities for students to abstract mathematical ideas from their own experiences. Students were expected to be actively engaged in the doing of mathematics; they were not to be passive observers in the learning process.

During the 1960s and early 1970s the use of manipulative materials in the classroom received strong support from many quarters. The Cambridge Conference writers expressed the opinion that in order for each child to learn mathematics satisfactorily there must be abundant opportunity to manipulate suitable physical objects (Adler, 1966). Two popular teacher training projects, the Madison Project in the United States, described by Davis (1964), and the Nuffield Project in England, described by Matthews (1968), both advocated the use of manipulatives in the classroom. Many articles appeared in education journals discussing the advantages of activity learning. The entire December 1971 issue of the Arithmetic Teacher was devoted to the topic of mathematic laboratories.

Toward the middle 1970s, as Suydam and Higgins (1977) and Friedman (1978) have noted, the interest in manipulative materials declined. The mathematics laboratory in particular has received much less attention in the literature since that time than it had previously. In fact, it would appear that the earlier emphasis given in the literature to the use of manipulative materials did not produce a proportionate emphasis in the classroom. The 1975 report of the National Advisory Committee on Mathematical Education (NACOME) noted that despite the strong effort to base much elementary and junior high school instruction on laboratory or activity-centred models utilizing many forms of manipulative materials, it was not clear that manipulative materials were widely used at all. For instance, 35 percent of the elementary school teachers in the NCTM survey in the report had never used the mathematics laboratory and 10 percent had never used manipulative materials at all (NACOME, 1975).

The most recent survey conducted by the NCTM to determine the beliefs about objectives and priorities for school mathematics for the 1980s was through a project entitled Priorities in School Mathematics (PRISM). Forty-eight percent of the respondents favoured increasing the emphasis on mathematics laboratories in the 1980s, while 34 percent opted for not changing the amount of emphasis. Support for introducing basic ideas through laboratory investigations or experiments with materials varied with topic, but was supported by most samples. Of lay samples surveyed, 93 percent supported the use of physical materials and models while support from professional samples varied from 62 to 83 percent (NCTM, 1981).

Among the eight recommendations of the NCTM's An Agenda for Action: Recommendations for School Mathematics of the 1980s, the use of diverse instructional strategies, materials, and resources was suggested in recommendation 4. Considered in relation to the findings of the PRISM report, it is worth noting that while some support was given to increasing the emphasis on mathematics laboratories and the introduction of ideas through laboratory investigations, support for using manipulative materials, small group instruction, and out of class activities to teach mathematics throughout the 1980s was stronger.

Rationale for the Use of Manipulative Materials

The basic philosophy underlying the use of manipulative materials is that students learn better by doing and applying. Sims and Oliver (1950) contended that one reason for so much difficulty in learning mathematics was the verbalistic nature of the usual teaching style and the amount of symbolism used.

Kline (1970), in a paper entitled "Logic vs. Pedagogy", made a strong plea for an approach to mathematics which is more intuitive and less rigorous. He wrote:

> It is the contention of this paper that understanding is achieved intuitively and that the logical presentation is at best a subordinate and supplementary aid to learning and at worst

> a decided obstacle. Intuition should fly the student to the conclusion, make a landing, and then perhaps call upon plodding logic to show the overland route to the same goal. If this contention is correct, then the intuitive approach should be the primary one in introducing new subject matter at all levels. (p. 266)

Grossnickle (1954) stated the rationale for the use of manipulative material in this way:

> If a child is able to make discoveries and generalizations in quantitative situations by the use of symbols he should not use manipulative materials. On the other hand, if he cannot deal understandably with quantitative situations by use of symbols, he should use objective materials to discover relationships among quantities. The pupil should be encouraged at all times to operate at the highest level of abstraction at which he understands the work. (p. 134)

Such statements have been typical of a collection of voices asking for a more intuitive and less deductive approach to the teaching of mathematics. The activity approach to learning and the use of physical materials is an attempt to provide a method to help students analyze and think abstractly.

Psychological investigations into the ways children learn, conducted by Piaget, Bruner, Dienes and others, have led to the development of the following principles which, according to Reys (1971), form the basic foundation underlying the rationale for using manipulative materials in learning mathematics:

> 1. Learning is based on experience.
> 2. Sensory learning is the foundation of all experience and thus the heart of learning.

3. Learning is a growth process and developmental in nature.
4. Learning is characterized by developmental stages.
5. Concept learning is the essence of learning mathematics.
6. Learning is enhanced by motivation.
7. Learning proceeds from the concrete to the abstract.
8. Learning requires active participation by the learner.
9. Formulation of mathematical abstractions is a long process.

In addition, Barson (1971), Cathcart (1971), and Vance (1971) included the following among the aims of activity learning: Activity learning can cultivate favourable attitudes toward mathematics. It can encourage and develop creative problem solving. It can allow for individual differences in the manner and speed at which children learn. Activity learning can enable children to see the origin of mathematical ideas and to model the way mathematicians behave, while promoting discussion of mathematics among students. Finally, activity learning can be used to enrich and vary instruction and also to review and reinforce basic concepts.

However, Friedman (1978) warned that on the basis of the evidence to date "an instructional strategy that gives preeminence to the use of manipulative materials is unwarranted (p. 79)." He stated that the research community had an obligation to inform the public of "this latest Pied Piper." Proponents of activity learning, however, stress the importance of careful selection of manipulative materials. Reys (1971) wrote:

> The mere use of manipulative materials does not ensure that they are being used properly. Manipulative materials must be used at the right time and in the right way if they are to be effective. Failure to select appropriate manipulative materials and failure to use them properly can destroy their effectiveness. (p. 555)

Reys further advised that teachers not make excessive use of manipulative materials, but use them only when they represent an integral part of the instructional program and when the program could not be achieved better without the materials.

In summarizing the theoretical arguments for the use of manipulative materials in mathematics learning, Kieren (1971) stated that materials have a fundamental position in the sequence of learning activities, can provide an information seeking, non-authoritarian environment, and can contribute a readiness foundation for later ideas. At the same time, Kieren remarked that research was needed before the broad question could be answered: "For whom, for which topics, and with what materials are manipulative and play-like activities valuable? (p. 232)"

Rationale for Using Manipulative Materials to Teach the Concepts of Perimeter and Area

Jamaski (1978) indicated the need to evaluate students' understanding of area in different ways. He argued that many students memorize and apply formulas without any insight into what area is all about. Woodward (1982) observed that one particular seventh grade student of high ability did not understand the concept of area and could not conceptually distinguish between area and perimeter. That particular student's previous experience with these concepts was abstract in nature, having been given formulas and asked to calculate perimeter and area. Woodward suggested that the use of physical materials and activities which included the counting of unit squares would have allowed the student to develop his own area formulas and thus gain a proper understanding of the concept of area.

From the recent results of the National Assessment of Educational Programs (NAEP), reported by Hirstein (1981), several common misconceptions about area measurement among 13 year olds were evident. Only 51 percent of the students could correctly find the area of a rectangle when presented a picture of a rectangle without unit squares on the figure. In fact, 23 percent gave the perimeter of the rectangle rather than the area. Only four percent could give the area of a right triangle given the lengths of the three sides and without unit squares on the figure. Only 12 percent correctly found the area of a square when a picture and a length of one side was given. Many students, instead, found the perimeter.

Hirstein (1981), in an analysis of the NAEP findings, concluded that the difficulties shown by the students seemed to result from misconceptions about area and confusion between the concepts of perimeter and area rather than from computational weakness. Hirstein remarked that it would appear from the results that the suggested methods for introducing area using manipulative materials have not been widely used. He emphasized that the use of materials, such as graph paper and geoboards to count unit squares and generalize relationships, to illustrate the concepts of area cannot stop in grade school. Such activities with area should begin during the early school years and continue throughout the mathematics program.

Bruner's Theory of Learning

While the learning theories of Piaget, Bruner, and Dienes have provided a theoretical framework supporting the use of manipulative materials in the classroom, the learning theory of Bruner is of particular relevance to this study. Since the terms, enactive and iconic, were borrowed from Bruner, his theory is discussed in some detail.

It should be noted, however, that all three learning theories have some common elements. Piaget (1953, 1970) and Dienes (1971) emphasize that concrete and semi-concrete materials should be embodied in mathematical instruction before students are required to think abstractly. Both also emphasize that most children under 12 years of age are not able to think abstractly.

Bruner (1966) considers stages of development in much the same light as Piaget. He maintains that the three modes of representation; enactive, iconic, and symbolic, are acquired by an individual in stages, but unlike Piaget, fixes no age intervals at which this acquisition takes place. He does assert that the stages of development do form a fixed sequence. A very young child knows about objects through the uses to which they are put. Later he begins to know things through mental images he forms of them and can reproduce things by drawing. When he reaches the final stage, he develops the ability to know things through symbolic means, such as language or mathematical symbols. Beyond this stage a person has the ability to use more than one mode in solving problems. In fact, it is the interplay between modes that is the rule rather than the exception. This interplay between the three modes and the development of intellectual powers is reflected to a large extent in how one learns mathematics. Bruner conjectures that when a student encounters a concept for the first time, it would be best to present instruction in the natural sequence of development: enactive, iconic, and symbolic modes. Bruner (1966) stated:

> If it is true that the usual course of intellectual development moves from enactive through iconic to symbolic representations of the world, it is likely that an optimum sequence will progress in the same direction. (p. 49)

However, he went on to state that actions, pictures, and symbols vary in their difficulty and utility for people of different ages and different backgrounds. He stated further:

> There is no unique sequence for all learners, and the optimum in any particular case will depend upon a variety of factors, including past learning, stage of development, nature of the material, and individual differences. (p. 49)

The purpose of the present study was to compare the effects of enactive versus iconic modes in relation to achievement and retention and not with the effects of the sequencing of the three modes. However, at the end of the study the students were expected to operate in the symbolic mode as they would after completing a unit on perimeter and area at the seventh grade level. In view of the fact that Bruner stated, there was "no unique sequence for all learners," the enactive-symbolic sequence and the iconic-symbolic sequence were compared to some extent.

Bruner's learning theory lends much support to the use of concrete and semi-concrete materials in the classroom.

Grasping the structure of a subject is understanding the subject, according to Bruner, in a way that permits other things to be related to it meaningfully. Concrete and semi-concrete embodiments of mathematical principles and ideas should help the student see these relationships.

Studies Focusing on Bruner's Modes of Representation

A review of the literature was undertaken to locate studies dealing specifically with comparisons between enactive and iconic modes of representation. Few studies were found at any grade level that referred specifically to these two modes; even fewer were found that used Bruner's terms in the title to a study. Therefore, the studies cited in this section, unlike the following section, are not limited to those involving geometric concepts or to the grade six-to-eight levels. Each study reported made some reference to Bruner's modes of representation as a theoretical basis for the study.

Scott and Neufeld (1976) conducted a study which focused upon the question of whether concrete manipulative materials contributed more to children's mathematical concept formation than do pictorial representation. Three second grade classes in each of three elementary schools were involved in the study. Twenty carefully controlled lessons on beginning multiplication were prepared for three modes of instruction, namely, manipulative, pictorial, and abstract. The abstract mode served as a comparative control. A test, prepared by the investigator, was administered before and after the instructional period. The test scores for the manipulative and pictorial groups were compared using analysis of covariance, with IQ and pretest scores as covariates. The researchers concluded that there was no significant difference between pictorial and manipulative modes of their instruction in their ability to affect children's concept formation in beginning multiplication.

Fennema (1972) conducted a study to compare the effects on the learning of a mathematical principle of two modes of presentation: one involving a meaningful symbolic model and the other a meaningful concrete model. Second grade students were taught a previously unlearned mathematical principle, namely, multiplication defined as the union of equivalent disjoint sets. One recall test and three transfer tests were given as posttests. It was found that groups that were exposed to the symbolic model did somewhat better in overall learning of the principle and in direct recall. No significant differences were found on the test of concrete transfer. The group that had learned with symbolic models performed better in the two tests of symbolic transfer. In one test the difference in mean scores was significant. While there were no significant differences in the overall learning of the principle, the children who were exposed to the symbolic model were able to transfer this learning to untaught symbolic examples of the principle better than the children who had learned with the concrete model. The subjects in this study had participated during the previous year in a mathematical program which emphasized the manipulation of concrete objects. Fennema concluded that since they had pre-symbolic experiences, they were able to learn with symbols, and the use of the symbolic model with its greater generalizability was more effective.

Abkemeir and Bull (1976) conducted a study to determine whether figural or symbolic modes in programmed materials were more effective in teaching a three class session in beginning algebra. One hundred sixty beginning algebra students were randomly assigned to a figural group where they were presented with arrow diagrams and function machines, or to a symbolic group where they were presented with symbols, formulas, and sets of ordered pairs. A posttest was given at the end of the three sessions, and a retention test was given a week later. Each test presented some items in figural, symbolic, and "neutral modes". There were no significant differences between learning test or retention test performance of 87 figural subjects or the 73 symbolic subjects.

McIntyre and Reed (1976) conducted a study to test whether specific types of visual devices, based on Bruner's modes of representation, might have a significant effect on the learning of concepts of electrostatics by elementary school children. The enactive devices were felt-cloth models whose transformations could be perceived without recourse to imagery or language. The iconic visual devices were pictures of the enactive devices which required the use of imagery for transformation. The symbolic devices were pictures in which the enactive figures were replaced by alphabetic characters which required interpretation as well as imagery for their use. Six classes of fourth grade students were randomly selected for the study and randomly assigned to the three treatments. From an analysis of variance of the data, it was concluded that there were no significant differences among the three treatments with respect to learning of the concepts of electrostatics.

Rogers (1977) conducted a study in which play was used to teach mathematics to children ranging from ages seven to ten. The content chosen for the study included factors, multiples, common factors, prime numbers, and factoring. In the "factor" game, for example, the teacher and students were given numbers. If the teacher's number was 12, she would catch 2, 3, 4, 6 and put them in "jail". After several rounds of the game the principle of factors was discovered by the child. Prime numbers and the other principles were taught in the same manner. Two teachers were involved in the study. For one teacher a control group was taught the same mathematical content by formal means.

Thus, the control group was tested in the manner in which they were taught while the play group was formally tested on content which they had only experienced in play. It was concluded that the experimental group did slightly better than the control group. The difference was not statistically significant. For the other teacher, instead of a control group, the formal test was translated into "play way" form. Since all the students had taken the formal test, each student acted as his own control. Mean play way test scores were found to be substantially higher than formal test scores. When the three small groups were combined into one group the difference was significant at the 0.01 level.

One point illustrated in this experiment was that not only could instruction be presented to children in different modes, but that knowledge could be demonstrated by the students in various ways. In one case, a student could not state the pairs of factors of 12 but could recite the multiplication tables by rote. However, in the jailbreak game, that student applied the principle enactively with no difficulty in releasing fellow prisoners.

Smith, Szabo, and Trueblood (1980) conducted a three-week study to test the effectiveness of three methods of teacher presentation on manipulative measurement skills using linear measurement tasks. Sixty-six first and second grade students were randomly assigned to three treatment groups described as abstract, graphic, and manipulative.

Verbal instruction was used in the abstract mode, pictures and charts in the graphic mode, and concrete materials in the manipulative mode. It was found that on a posttest measuring manipulative output, the group that received manipulative instructional input scored significantly higher than the group receiving graphic instructional input. On a posttest requiring manipulative output, the group that received manipulative instructional input did not score significantly higher on posttests of linear measurement than the group that received abstract instructional input.

Bruner (1964) conducted a study with four eight-year-old children during which they were closely observed during 24 hours of mathematics instruction over a six-week period. They were given instruction in factoring, the distributive and commutative properties of addition and multiplication, and finally, in quadratic equations. The children had a series of graded problem cards which they could go through at their own pace. Mathematical ideas were first presented through concrete instructions using building blocks, balance beans, and other materials. From these constructions the students were encouraged to form perceptual images of mathematical ideas. Then the child was encouraged to develop a notation for describing the concepts.

Bruner reported that the children not only understood the abstractions they had learned, but also had a store of concrete images that served to exemplify the abstractions. When the children searched for a way to deal with new problems, the task was usually carried out, not simply by abstract means, but also by matching up images. The tentative conclusion reached was that a good stock of visual images was needed for embodying abstraction when learning mathematics.

Through the years, the idea has been propounded that the learning of mathematics progresses through three levels: concrete, pictorial, and symbolic. Bruner (1966) discussed these three stages using new labels: enactive, iconic, and symbolic. From a review of the research dealing with these modes of representation, the results with regard to the effectiveness of one mode over the other appear inconclusive. In one study, there was no significant difference between enactive and iconic. In another, there was no significant difference between iconic and symbolic. One favoured symbolic over enactive while another favoured enactive over symbolic. One study favoured enactive over iconic. Bruner (1966) noted that each of the three modes has a unique way of representing events, but all are capable of partial translation, one into the other.

Research Studies Involving Manipulative Materials in Teaching Geometry in Grades Six to Eight

As was stated previously, only those studies dealing with the use of manipulative materials in teaching geometry at the sixth, seventh, or eighth grade levels are reported. It should be noted that in some of the studies there was no distinction between activity oriented instruction and the mathematics laboratory. What one researcher labels a mathematics laboratory approach, another often labels activity oriented instruction, or instruction involving the use of manipulative materials. For the most part, the studies are reported in terms of the label used by the particular researcher. The most relevant studies, that is, those related to perimeter and area, are reported before those dealing with other geometric concepts. First, studies dealing with immediate achievement are reported, then, studies dealing with both immediate achievement and retention.

Immediate Achievement

Smith (1974), in a two week study of underachieving seventh graders, investigated the effects on achievement of three approaches involving manipulative materials. Each approach was concerned with developing and applying formulas for finding areas of quadrilaterals and triangles. The three approaches were classified as expository, multimodal, and unimodal. The expository group used a lecture-discussion-drill approach without the use of manipulatives. The multimodal approach employed a multiple concrete embodiment, while the unimodal approach used a single concrete embodiment through a numeric approach. A pretest and post-test were prepared by the investigator to assess achievement of students' ability to (1) state area formulas; (2) apply area formulas using integers; and (3) apply area formulas using rational numbers.

Smith reported the following results. The expository approach was found to be significantly superior to both the unimodal and multimodal approach for developing the area formulas of a right triangle. The expository approach was significantly better than the multimodal approach for developing the area formula of a parallelogram. The expository approach was superior to the multimodal approach for teaching students to apply, with integers, the area formula of a right triangle and superior to the unimodal approach for teaching application of the area formula of a parallelogram using rational numbers. No other significant differences were found among 16 comparisons.

Jolly (1978) conducted a study to investigate the effects of the use of a laboratory approach to teach selected concepts of perimeter, area, and volume to seventh grade students. Intact classes were used, and all students had been segregated by sex and ability at the beginning of the school year. During 13 treatment days two experimental groups of "average" students studied the geometric concepts using manipulative aids such as geoboards, dot paper, paper figures, rectangular solids, and mimeograph materials. The two control groups of average students studied the same concepts in a lecture-discussion approach with the only aid being the chalkboard. It was concluded that there was no significant difference between the achievement scores of students in the experimental group and those of students in the control group.

R. E. Johnson (1971), in a year long study, investigated the effects of activity oriented lessons on the achievement and attitudes of seventh graders in mathematics. Six classes were taught by two teachers. The study involved three modes: textbook mode, activity mode, and enriched mode, where activities from the second treatment supplemented the text. The areas of mathematics presented were number theory, rational numbers, geometry, and measurement. For the units taught on number theory and rational numbers, the performance of students taught solely by the activity approach was poorer than that of students taught by a textbook-based or activity-enriched approach. In the unit on geometry and measurement, however, there was some evidence that the activity lessons were more effective for low and middle ability students.

Wilkinson (1971) devised a set of activities on metric geometry for presentation to sixth grade students over a period of 20 consecutive school days. One experimental treatment used laboratory units containing worksheets and manipulative materials while the second experimental treatment used individual cassette tapes which contained verbatim the directions and questions on the worksheets. In the third treatment, the students were taught by a traditional teacher-textbook method. Analysis of covariance was used to treat the data, covariates being pretest scores in geometry achievement and non-verbal intelligence. No significant differences in geometry achievement were found among the three treatments. Wilkinson expressed the opinion that brighter students may have had their thinking slowed when forced to use physical materials.

Wong (1979) conducted a study to determine the effects of the handling of manipulative devices on the learning of selected concepts in geometry among 13 year olds. The experiment was designed to see who should handle the concrete materials: the teacher, the pupils, both, or neither. Several geometric concepts were studied in these four treatments, including quadrilaterals and diagonal planes. No significant differences were found at the 0.05 level among the four treatments on an achievement test. On a subtest of non-transfer items of the achievement test, the scores of students in the treatment where both the teacher and the pupils handled the manipulative devices and of those in the treatment where only the teacher handled the manipulative devices were significantly higher than the scores of the students in the treatment where only the pupils handled the manipulative devices. Wong reported that while the overall results were inconclusive, a consistent pattern in the ordering of the mean scores suggested that if manipulative devices are to be used in teaching mathematics, they should be handled by both the teacher and the pupils and not by pupils alone.

Anderson (1958) conducted an eight week study to determine the effects of using a kit of 16 visual-tactual stimuli on the achievement of eighth graders in learning a unit on area, volume, and the Pythagorean relation. Nine classes in the experimental group made use of manipulative materials while nine classes in the control group used no manipulative aids. No significant differences were found at the 0.05 level, but Anderson reported that the scores in the experimental group were moderately higher than those in the control group. The results were inconclusive as to whether low ability students profited more than high ability students from the use of visual-tactual aids.

Small (1966) conducted a study to investigate the effects of activity instruction on sixth grade low achievers. Concepts involving linear measure and place value were formulated in each of three modes: concrete, semi-concrete where the student could refer to a drawing, and abstract where no manipulative or pictorial aids were involved. Small reported that, in general, the students functioned more efficiently when the concepts were expressed in the concrete and semi-concrete modes. Furthermore, he reported that their performance depended in part on the mathematical areas being tested and that the ability to operate with different instructional modes seemed to be an individual problem which required identification for each student.

Whipple (1972) carried out a 14 day study to investigate the effectiveness of teaching metric geometry to eighth graders by the laboratory and by an individualized instruction approach. Two experimental classes made use of manipulative materials while two control classes used individualized instruction units. On a posttest for achievement the experimental group scored three points higher than the control group. The experimental group showed about a five point superiority in computing areas and volume using actual objects. Neither of the two differences was reported to be significant.

Immediate Achievement and Retention

R. L. Johnson (1971), in a study that included sixth-grade students, investigated the effects of three approaches to the teaching of a unit on perimeter, area, and volume. The three treatments were designated maximum, moderate, and minimum. The maximum approach consisted of a semi-programmed text together with physical models for each student. The moderate approach included the same semi-programmed text but no physical models. In the minimum approach the students were deprived of the use of drawings and illustrations which were a part of the semi-programmed text. Ninety-six students from grades four, five, and six were categorized on factors of sex, reading ability (2 levels), and age (2 levels). A posttest was given at the end of the unit and a retention test two weeks later. Johnson reported that the treatment effect produced significant differences between test means at the 0.01 level of significance on the posttest and the retention test. The drop in means from the posttest to the retention test was less than one point for the maximum treatment group, nearly two points for the moderate treatment group, and three points for the minimum treatment group. The overall results, Johnson reported, gave evidence that a high degree of concreteness yielded higher mean scores on both achievement and retention measures.

Zirkle (1981) conducted a study to investigate the effects of manipulative materials on achievement and retention of the concept of area measurement by sixth and seventh grade students. One hypothesis tested was that the use of manipulatives by the student or the use of manipulatives by the teacher would result in greater student achievement and retention than would the use of pictorial aids by the teacher. Another hypothesis tested was that the use of manipulative or pictorial aids would result in greater retention than would the rote learning of formulas by the students. Three sixth grade classes and six classes of grade seven students were involved in the study. The area measurement test was given immediately after the treatment as an achievement test and then again six weeks later as a retention test. The SRA Mathematics Achievement Test was given as a pretest. For the sixth graders there were significant differences on both the achievement test and the retention test favouring the group taught by pictorial aids. For the seventh graders there were no significant differences on either the achievement test or the retention test accounted for by treatment. For the seventh graders there was a significant difference on the retention test favouring the group using manipulative and pictorial aids over the group taught area measurement by rote memorization of formulas.

Bring (1972) studied the effects of varying concrete activities on the achievement of fifth and sixth grade students studying a unit in geometry. Four classes of fifth and sixth grade students, divided into two treatment groups, were given a two week semi-programmed unit on volume, congruence, symmetry, and isometrics of an equilateral triangle and a square. One treatment group was given a supplement of concrete materials and activities; the other treatment did not have this supplement and, where possible, pictures were replaced by verbal descriptions. A pretest was given before the treatments were begun. A posttest was given at the end of the unit and a retention test one week later.

No significant difference in achievement was found on the posttest scores. On the retention test, there was a significant difference favouring the concrete treatment.

Cohen and Walsh (1980) conducted a study to determine the effects of an individualized and a traditional mode of instruction on learning and retention of a unit in geometry at the junior high level. Students in three seventh and two eighth grade classes were randomly assigned to the two treatment groups. The individualized group employed SRA Computapes with each taped lesson accompanied by worksheets that provided visual reinforcement of the mathematics on the tape. In the traditional group, the lessons were primarily expository in format and consisted of oral and written drill and required assignments from the textbooks. The experiment ran for five 45 minute periods per week for six weeks. On a posttest administered at the conclusion of the experiment, and a retention test administered three weeks later, there were no significant differences in achievement or retention due to mode of instruction. There was no significant interaction between ability and mode of instruction.

Vance (1969) conducted a study to investigate the effects of implementing a mathematics laboratory at the grade seven and eight levels. Students were randomly assigned to one of three groups: mathematics laboratory group, class discovery group, and the control group.

Students in the laboratory group worked in pairs from written instructions, rotating through a set of ten activity lessons based on concrete materials. In the discovery group the laboratory activities were presented to whole classes of students by their teachers who demonstrated the activities using concrete materials. Both the laboratory group and the discovery group spent one out of every four classes in the experimental settings. The control group continued with its regular instruction for all the allotted class time.

It was found that the use of 25 percent of class time in mathematics for informal exploration of new mathematical ideas did not adversely affect achievement in the regular program over a three month period. In addition, from tests of achievement, retention, transfer, and divergent thinking, it was concluded that students learned new mathematical ideas in the laboratory setting, although their test scores were slightly lower than those of the students in the class discovery situation. Both laboratory and class discovery groups scored higher than students in the regular program on the same four measures. There were no significant differences among the three groups at either grade level on an achievement test based on work covered in the regular mathematics program during the study.

Richards (1971) compared the effectiveness of a verbal and a verbal-manipulative program of instruction in teaching the accurate reading of a ruler to sixth grade students. Two experimental and one control group were selected in each of two schools. One experimental group used a verbal program of instruction and the other used a verbal-manipulative program. Analysis of variance was used in conjunction with t-tests to analyze the data from a post-test and a retention test. No significant differences were found between either the posttest or the retention test scores of the two experimental groups.

Purser (1973) carried out a study to determine if certain manipulative activities using measuring instruments were associated with gains in achievement and retention scores in mathematics at the seventh grade level. A series of four units of learning packages was developed comprising two areas of mathematics: fractions and decimals; and two areas of measuring: using a ruler and using a micrometer. The learning packages for the experimental group consisted of manipulative materials while those of the control group consisted of paper and pencil type problems. Fifteen grade seven classes were each divided into three ability groups: low, medium, and high. Students of all ability levels in the experimental group scored significantly higher on a posttest and retention test than students of the corresponding ability levels in the control group.

Summary

There were three purposes to the review of the literature. One was to provide a background to research on the use of manipulative materials by outlining the history of activity learning and giving a rationale for the use of manipulative materials in teaching mathematics. Another was to provide a review of studies which focused on modes of presentation. The third purpose was to provide a summary of the research which has been done at the grade six, seven, and eight levels on the use of manipulatives in teaching topics in geometry. The search of the literature showed a shortage of relevant studies on geometry at the seventh and eighth grade levels, particularly perimeter and area studies.

From the findings of the review of empirical research it was concluded that, for immediate achievement, seven of the 15 studies favoured the use of manipulative materials while nine showed no significant difference for treatment effects. Only one study found an expository approach to be more effective than the use of manipulatives. In one study it was suggested that pictorial aids used by the teacher are more effective than manipulative materials used either by the teacher or by the students. Of the seven studies in which retention was examined, the use of manipulatives was favoured in four. It should be noted that while in nine studies there were no significant differences for treatment on immediate achievement, in two of the nine the use of manipulatives was favoured for low ability students and in two other studies the use of manipulatives had positive effects on retention measures.

Although the results of the studies varied, it was concluded from the review that the use of manipulative and pictorial aids was as effective, and sometimes more effective, than the non use of materials in teaching geometry at the junior high level.

Chapter III

METHODS AND PROCEDURE

In this chapter a description of the methods and procedure used in carrying out the study is presented. First, the population and sample are described. This is followed by a description of the instruments used, the treatment groups, the qualifications of the teachers, the instructional content, and instructional procedure used in the study. A description of the research design and the hypotheses tested is provided. Next, the methods used to analyze the data are given. Finally, the pilot study is described and the outcomes reported.

Sample

The population consisted of students enrolled in grade seven under the Placentia-St. Mary's Roman Catholic School Board. The subjects for the study were 109 seventh grade students in four classes in the Placentia district. Five schools in that area each contained a single grade seven class. Two other schools each contained two grade seven classes, but in each of the schools, the students were streamed into an academic and a general class. Since randomization of students to treatments was not possible, only those five schools with a single grade seven class were selected to participate in the study, thereby providing intact classes that were assumed to be heterogeneous. These schools were located in five different towns within the district. A pilot study was carried out in one of the schools. The remaining four schools participated in the main study. Two schools contained grades K to 8, while the other two schools contained grades 7 to 11. The four intact classes were randomly assigned to the two treatments in the study.

Materials and Instruments

Two sets of eight lesson plans, one set for each of the two treatments, were used in the presentation of the instructional content of the study. These lesson plans were prepared by the investigator. A set of behavioural objectives was written for each lesson, and the content validity of the lessons verified by two mathematics educators and four graduate students in mathematics education. The two sets of lesson plans were parallel in oral and written instructions and written format. The primary difference was that in one set the students were instructed to make geometric figures on the geoboard while in the other set they were instructed to draw the same figures on dot paper. For most lessons the only difference in written format was the substitution of the word "geopaper" in the iconic treatment for the word "geoboard" of the enactive treatment. The lesson plans for the enactive treatment are included in Appendix A.

An achievement posttest consisting of 25 items was constructed by the investigator. The content validity was ensured by including items for each of the behavioural objectives. In addition, the items were judged valid by a panel of mathematics educators. The reliability of the test was found to be 0.87, using the Kuder-Richardson formula 20 on the pilot study data. The posttest was administered immediately after the instructional period. A parallel form of the posttest was written and administered five weeks later as a retention test.

Each test was designed to measure four levels of the student's mastery of the subject matter in the lesson plans, namely, recall, algorithm, comprehension, and application. These levels were based on the taxonomy of learning of Wilson (1971). Space was provided on the test paper for the students to work out solutions to the items. Correct items were scored 1 while incorrect items were scored 0. The achievement posttest and retention test are included in Appendix B.

The Canadian Test of Basic Skills (CTBS) was used to classify the students into three ability groups: high, middle, and low. The exact criterion used was the CTBS score obtained by the student on the 48-item Mathematics Concept Test and the 32-item Mathematics Problem Solving Test, the reliability coefficients of which are 0.86 and 0.79 respectively (King, 1975). Students who scored in the top one-third of the sample were classified as high ability, those in the middle one-third as middle ability, and those in the bottom one-third as low ability. The Canadian Test of Basic Skills has been used throughout the province of Newfoundland as a measure of academic ability and a predictor of success at particular grade levels.

Treatment Groups

The two treatments used in this study were labelled as the enactive mode and the iconic mode. In the enactive treatment, the students used 5 x 5 geoboards and rubber bands to assist them in completing exercises in the unit. The geoboard was considered to be a concrete aid. In the iconic treatment, the students used 5 x 5 dot paper, described as a semi-concrete material, on which they drew the geometric figures that were made on the geoboard by the students in the enactive treatment. In the lesson plans, the 5 x 5 dot paper was referred to as geopaper. Each student was provided with his or her own geoboard or geopaper by the teacher.

Instructional Content

In both treatment groups the students proceeded through the same unit of work on the concepts of perimeter and area. The instructional content was designed to lead the students to the discovery of the formulas for the perimeter of a rectangle and square as well as the area formulas for the rectangle, square, triangle, parallelogram, and trapezoid. Applications of these concepts were given to the students by using student lesson sheets.

From discussions with the teachers involved in the study, it was determined that the students had little or no previous contact with the subject matter in any depth. Geometry appears to take a secondary position to other course material in the elementary grades. Roberts (1979), in a study on the teaching of geometry in the elementary grades, reported that far too little time was spent in teaching geometry. The main reason given by teachers was that there was not enough time. Prior to this study, however, the four teachers involved had covered a chapter in the grade seven text introducing the students to elementary geometric concepts such as point, line segment, line, plane, angle, parallel lines, and polygons.

Teacher Qualifications

The four teachers involved were all experienced teachers with several years experience in teaching mathematics at the junior high level. Three teachers each had two university credits in mathematics and two credits in mathematics education while one teacher in the iconic treatment had one university credit in mathematics. Two were male and two were female. All had a Newfoundland teaching certificate of grade V or VI.

Instructional Procedure

Mimeographed copies of the lesson plans were distributed to each of the teachers. Each lesson contained a set of behavioural objectives, an instructional session as part of the teacher's presentation, student lesson sheets, and answers to all exercises in the lesson plans. The total instructional period was about 20 teaching days.

The procedure for presenting each lesson was as follows. First, the teacher stated the purpose and the behavioural objectives of the lesson. The teacher then used a guided discovery technique to teach the students. The students were asked to "think along", to engage in class discussion, and demonstrate their solutions to the rest of the class. The teachers during both treatments used an overhead projector, with a transparent geoboard in the enactive treatment and a transparent sheet of dot paper in the iconic treatment. Though guided by specific questions, the students were free to make their own observations and to express in their own way any conclusions or relationships drawn from their observations. By following specific directions in the teacher lesson plans, the teacher played an active role in giving directions, encouraging discussion, and getting feedback from the students.

After the instructional session by the teacher, the student lesson sheets were distributed to the students. These extended the discovery aspect of the lesson as students answered questions, recorded data, and drew conclusions which led to the generalization of the perimeter and area formulas. Each lesson contained practice exercises, as well as more challenging ones called "Think" exercises. In addition, there were assigned exercises from their regular textbook, School Mathematics 1 (Fleenor et al., 1974). This was the only use that was made of the student text during the study. Here, the role of the teacher was also an active one. The teacher was directed, in the lesson plans, to walk around the classroom encouraging the students to experiment in making or drawing the figures, and giving assistance when students were having difficulty or losing interest.

The students were permitted to take the geoboards and geopaper home to finish exercises on the lesson sheets. However, the exercises from the student textbooks were usually given as homework assignments. Since the schools were located in different towns, there was no exchange of materials between the students in the two treatments. All homework assignments were checked in a class discussion the following school day.

The teachers were instructed to adhere strictly to the lesson plans in the study. The researcher met with each teacher before the study began and was in contact with each teacher during the study to ensure that the lessons were presented in the manner required. The achievement posttest was administered immediately after the instructional period, and the retention test was administered five weeks later.

Research Design

In this study, it was not possible to randomly assign students to treatment groups. Instead, four intact classes were randomly assigned to the two treatments: enactive and iconic. The students in each treatment were classified into three ability levels: high, middle, and low, based on their CTBS scores. After the instructional period, student achievement was measured by means of a posttest, and again five weeks later by means of a retention test. The design was a 2 x 3 factorial design in which treatment and ability formed the independent variables. The dependent variables were the students' achievement scores obtained on the posttest and retention test. The design is illustrated in figure 1.

		TREATMENTS	
		Enactive	Iconic
ABILITY	High		
	Middle		
	Low		

Figure 1. The factors and levels of the design

Hypotheses

Answers to several questions were sought in the study: To what extent were students able to learn and retain mathematical concepts after using concrete and semi-concrete materials? To what extent were the same objectives of the concrete approach achieved by students using the semi-concrete approach? Was there any interaction between treatment and ability with respect to achievement when the students were classified into low, middle, and high ability groups?

To answer these questions, the following six null hypotheses were tested by using a two-way analysis of variance.

1. There is no significant difference in mean scores between the two treatment groups on a posttest for achievement.

2. There is no significant difference in mean scores among the three ability groups on a posttest for achievement.

3. There is no significant interaction between treatment and ability with respect to scores on a posttest for achievement.

4. There is no significant difference in mean scores between the two treatment groups on a retention test for achievement.

5. There is no significant difference in mean scores among the three ability groups on a retention test for achievement.

6. There is no significant interaction between treatment and ability with respect to scores on a retention test for achievement.

Analysis of Data

All six null hypotheses were tested at the 0.05 level of significance. The main effects due to treatment as well as interaction of treatment with ability were displayed graphically. The Scheffé method was used to investigate the difference between pairs of group means for the three ability levels where the ability factor was significant and interpretable.

Pilot Study

A pilot study was conducted with an eighth grade class at the school where the investigator was teaching. The pilot study was carried out early in the 1981 fall semester. None of the students had had any previous contact with the concepts contained in the unit designed for the study, neither as part of their mathematics program in grade seven, nor previously in the grade eight program. The pilot teacher had previously taught mathematics at this level for several years. The investigator taught one of the lessons for one day when the pilot teacher was absent. The pilot teacher reported to the investigator on the progress of the students after each of the eight lessons in the unit.

The purposes of the pilot study were:

1. to determine the time required to complete each lesson;

2. to determine any difficulties with the written and oral instructions, with the exercises in the student lesson sheets, and with the concrete materials used;

3. to clarify the role of the teacher within the unit;

4. to obtain a reliability coefficient for the achievement posttest as well as the time allotment needed for completion of the test.

From the pilot study several observations were made. It was determined that for most of the lessons two 40-minute class periods were required. Checking solutions to exercises given as homework assignments sometimes required a third class period. The written and oral instructions were found to be clear and required no major rewording. The exercises in the student lesson sheets were found to be of sufficient variety to provide a challenge to all the students without being judged too difficult.

It was determined that instruction would be more effective if each student had his or her own geoboard or dot paper and worked individually, rather than with another student. However, this did not mean that class discussion was discouraged.

The teacher's role was confirmed to be an active rather than a passive one. Instructions to the teacher, which had been explicitly written into the lesson plans, were shown to be adequate in allowing the teacher to play an active role. These procedures were explained more fully under Instructional Procedure earlier in this chapter. The need for the teacher to sometimes intervene and show the solutions to some of the exercises was noted. For example, if an exercise required five particular shapes with a particular area measurement, then, if the students were having difficulty discovering the fifth one, it was better for the teacher to directly guide the students toward the solution, in the interest of time allotment and class control.

A 25-item posttest administered at the end of the pilot study was found to have a reliability measure of 0.87, as measured by the Kuder-Richardson formula 20. No student required more than 40 minutes to complete the test.

In summary, no major revisions were required in the lesson plans for the study. It should be noted that these lesson plans had been prepared by the investigator in consultation with two mathematics educators and four mathematics teachers during the 1981 summer session at Memorial University of Newfoundland and had been subjected to several revisions.

Chapter IV

RESULTS AND ANALYSIS

In this study, four intact classes of seventh grade students were randomly assigned to the two treatment groups, namely, enactive and iconic. Students were previously classified as being of low, middle, or high ability on the basis of their CTBS scores. The independent variables were treatment and ability. The dependent variables were the scores that the students received on an achievement posttest and a retention test. In this chapter the data collected in the study and the results of testing the null hypotheses are reported.

Sample

The data from 103 students were considered in the analysis of the results. A total of 109 students began the study, but data from six of these were not used because of absences during one of the testing periods.

In Table 1, the number of students in each ability level and the range of CTBS scores for each of the levels is reported. To classify the students into the different ability levels, a frequency distribution of the scores was made. Students who scored in the top one-third of the distribution were classified as being of high ability, those who scored in the middle one-third as being of middle ability, and those in the bottom one-third as being of low ability.

Table 1

Number of Students in each Ability Level

Ability Level	Range of CTBS Scores	Number of Students
Low	20-30	36
Middle	31-37	36
High	38-61	31

In Table 2 the number of students in each cell of the factorial design for the study is given. The number of students in the high ability level in the enactive treatment was low in comparison to the same level in the iconic treatment.

Table 2

Number of Students in each Cell

	Treatment		
Ability	Enactive	Iconic	Total
Low	14	22	36
Middle	18	18	36
High	8	23	31
Total	40	63	103

The data for individual students obtained from the Canadian Test of Basic Skills, the posttest, and retention test are reported in Appendix C.

Analysis of Data

The scores from the posttest and retention test were analyzed using a two-way analysis of variance. The Statistical Package for the Social Sciences (SPSS) subprogram ANOVA provides an analysis of variance for factorial designs with unequal and disproportionate cell frequencies, as this study required. The classic experimental procedure which was employed in the analysis of variance is described in the SPSS manual (Nie, et al., 1975). The Scheffé analysis for multiple comparison of means was used to investigate significant differences between the three ability levels where the ability factor was significant and interpretable. All hypotheses in the study were tested at the 0.05 level of significance.

Analysis of Achievement Posttest Scores

In Table 3, a summary of the results of the analysis of variance for the achievement posttest is presented. Both the treatment and ability factors were found to be significant.

Table 3

Analysis of Variance for Posttest

Source of Variation	Sum of Squares	DF	Mean Square	F	Significance of F
Main Effects	1076.64	3	358.88	36.72	0.00
Treatment	551.67	1	551.67	56.45	0.00
Ability	372.83	2	186.41	19.08	0.00
Treatment by Ability	59.38	2	29.69	3.04	0.05
Within Cells	947.93	97	9.77		
Total	2083.95	102	20.43		

However, since the treatment-by-ability interaction was also found to be significant, the significance of the main effects had to be interpreted with caution. The means for each cell of the design on the achievement posttest are reported in Table 4.

Table 4

Means of Posttest Achievement Scores

Ability	Treatment		Total for Ability
	Enactive	Iconic	
Low	14.29*	18.73	17.00
Middle	14.72	21.39	18.06
High	20.38	23.04	22.35
Total for Treatment	15.70	21.06	18.98

*maximum = 25

A graph of these mean scores is presented in Figure 2. The profiles drawn in the graph show that at each ability level the means for the iconic group were greater than the means for the enactive group. As seen by the positive slopes, the significant difference in treatment is clearly in favour of the iconic treatment for each ability level.

For the ability factor, the profiles suggest that the means for the high ability level were greater than the means for the low ability level. A Scheffé analysis verified that this difference was significant at the 0.05 level. Since the mean of the middle ability group was close to the mean of the low ability group in the enactive treatment and close to the mean of the high ability group in the iconic treatment, it could not be concluded that the means of the middle ability level were significantly different from the means of the low or high ability levels. A Scheffé analysis showed that, at the 0.05 level, there was no significant difference between the low and middle ability levels, but that there was a significant difference between the middle and high ability levels. A summary of the results of the Scheffé analysis is given in Table 5.

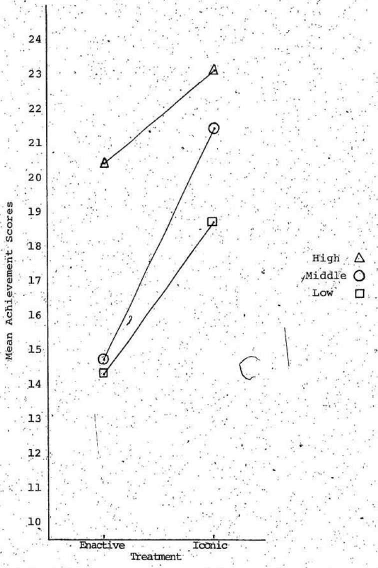

Figure 2. The mean scores on the achievement posttest

Table 5

Differences Between the Ability Levels on the Achievement Posttest, Scheffé Analysis

Contrast	Difference Between Means	Critical Difference
low-middle	-1.06	$\sqrt{(I-1)\,{}_{.95}F_{I-1,\,N-IJ}}$
middle-high	-4.29*	= 2.49
low-high	-5.35*	(I = 3, J = 2, N = 103)

*$\underline{p} < 0.05$

As shown in Figure 2, the interaction between treatment and ability was ordinal. While the high ability students scored higher on the achievement posttest than the middle ability students, and the middle ability students scored higher than the low ability students in both the enactive and iconic treatments, the superiority was not the same for both treatments. The middle ability group showed a greater superiority to the low ability group in the iconic treatment than in the enactive treatment. At the same time, the high ability group was not as superior to the middle ability group in the iconic treatment as it was in the enactive treatment. Since the middle ability group did relatively better in the iconic treatment, it could be said that the iconic treatment was relatively more effective with middle ability students. In fact, the middle ability group scored higher in the iconic treatment than did the high ability group in the enactive treatment.

Hypotheses Related to the Achievement Posttest

Hypothesis one: There is no significant difference in mean scores between the two treatment groups on a posttest for achievement.

Hypothesis one was rejected since the iconic group scored significantly higher than the enactive group on the achievement posttest.

Hypothesis two: There is no significant difference in mean scores among the three ability groups on a posttest for achievement.

Hypothesis two was rejected since a Scheffé analysis revealed a significant difference in the mean scores between the low and high ability levels, and between the middle and high ability levels. However, there was no significant difference in the mean scores between the low and middle ability levels.

Hypothesis three: There is no significant interaction between treatment and ability with respect to scores on a posttest for achievement.

Hypothesis three was rejected since the interaction between treatment and ability was found to be significant at the 0.05 level in the two-way analysis of variance. From a graphical display of the posttest means, it was concluded that the interaction was ordinal, with the middle ability group scoring relatively better in the iconic treatment.

Analysis of the Retention Test Scores

The results of the analysis of variance for the retention test scores are reported in Table 6. These results paralleled the results for the posttest. There was a significant difference between treatments and between ability levels, and there was a significant interaction between treatment and ability. Since interaction was, again, found to be significant, the significance of the main effects had to be interpreted with caution.

Table 6

Analysis of Variance for Retention Test

Source of Variation	Sum of Squares	DF	Mean Square	F	Significance of F
Main Effects	1675.61	3	558.54	40.14	0.00
Treatment	568.80	1	568.80	40.88	0.00
Ability	900.83	2	450.42	32.37	0.00
Treatment by Ability	97.07	2	48.53	3.49	0.03
Within Cells	1349.72	97	13.92	25.48	0.00
Total	3122.40	102	30.61		

The means of the retention test are reported in Table 7.

Table 7

Means of Retention Test Scores

Ability	Treatment: Enactive	Treatment: Iconic	Total for Ability
Low	10.43*	13.73	12.44
Middle	11.28	18.78	15.03
High	18.00	21.35	20.48
Total for Treatment	12.32	17.95	15.77

*maximum = 25

A graph of these mean scores is presented in Figure 3. The profiles in the graph are similar to the profiles in the graph of the posttest means. At each ability level the mean for the iconic group was greater than the corresponding mean for the enactive group. The significant difference in treatment was, again, clearly in favour of the iconic group.

By observing the means and the graph, the means for the high ability group appeared to be significantly greater than the means for the low ability group. A Scheffé analysis verified that the difference was significant at the 0.05 level. As with the posttest results, due to the different type of performance by the middle ability group, it could not be concluded that the means of the middle ability level were significantly different from the means of the low or high ability levels by observation alone.

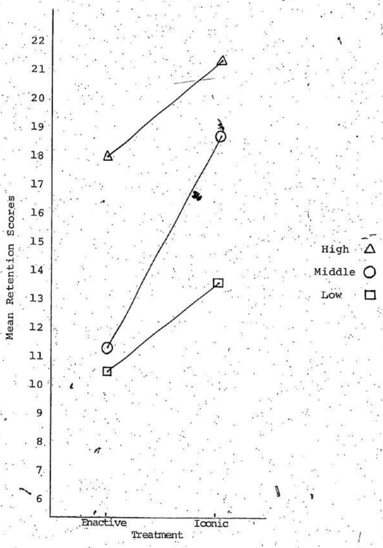

Figure 3. The mean scores on the retention test.

However, a Scheffé analysis confirmed that there was a significant difference between the low and middle ability levels and between the middle and high ability levels. A summary of the Scheffé procedure for testing the differences between the ability levels for the retention test is given in Table 8.

Table 8

Differences Between the Ability Levels on the Retention Test, Scheffé Analysis

Contrast	Difference Between Means	Critical Difference
low-middle	-2.59*	2.49
middle-high	-5.45*	
low-high	-8.04*	

*$p < 0.05$

The interaction of the mean scores on the retention test was ordinal, as was the case for the posttest means. Likewise, for the retention test, it could be said that the middle ability students did relatively better in the iconic treatment. Again, the middle ability group also scored higher in the iconic treatment than did the high ability group in the enactive treatment.

Hypotheses Related to the Retention Test

Hypothesis four: There is no significant difference in mean scores between the two treatment groups on a retention test for achievement.

Hypothesis four was rejected since the iconic group scored significantly higher than the enactive group on the retention test.

Hypothesis five: There is no significant difference in mean scores among the three ability groups on a retention test for achievement.

Hypothesis five was rejected since a Scheffé analysis revealed a significant difference in mean scores between all possible pairs of ability levels, that is, between low and middle, middle and high, and low and high ability groups. Interaction between treatment and ability was ordinal.

Hypothesis six: There is no significant interaction between treatment and ability with respect to scores on a retention test for achievement.

Hypothesis six was rejected since interaction between treatment and ability was found to be significant beyond the 0.05 level in the two-way analysis of variance. A graphical display of the retention test means showed that the interaction was ordinal, with the middle ability group scoring relatively better in the iconic treatment.

Summary

All six null hypotheses, which were tested at the 0.05 level in a two-way analysis of variance, were rejected. A significant difference was found on both the posttest and retention test for the treatment effect in favour of the iconic group. For the ability factor, a significant difference was found between low and high ability levels and between middle and high ability levels on the posttest. On the retention test, a significant difference was found between all three pairs of ability levels. It should be noted that the difference in mean scores between the low and middle ability students in the enactive treatment was small on both the posttest and retention test. A significant interaction between treatment and ability was found on both the posttest and retention test. For both tests, the interaction was ordinal. The iconic treatment was relatively more effective with the middle ability level.

Chapter V

SUMMARY, DISCUSSION, AND RECOMMENDATIONS

In this chapter a summary of the study and a discussion of the results are presented. Then, recommendations for further research and implications for classroom teachers are suggested.

Summary

The study was designed to investigate the relative effectiveness of enactive and iconic presentations in the teaching of selected concepts of perimeter and area to seventh grade students. The students were classified as being of low, middle, or high ability, according to their scores on the Canadian Test of Basic Skills which was administered one month prior to the beginning of the study.

A 2 x 3 factorial design was used in the study, with four intact classes being randomly assigned to the enactive and iconic treatments. All classes studied the same concepts for five weeks using activities prepared by the researcher. The enactive group used 5 x 5 geoboards in their activities while the iconic group used 5 x 5 dot paper. A posttest was administered immediately after the instructional period was completed, and a retention test was administered five weeks later. A two-way analysis of variance was used to analyze the scores on each test. Scheffé tests were used to investigate differences among the three ability levels.

The six null hypotheses were tested at the 0.05 level, and all were rejected. The findings, summarized for the posttest and retention test together, were as follows:

1. There was a significant difference between treatments in the mean scores on both the posttest and the retention test in favour of the iconic treatment.

2. There was a significant difference in mean scores between the three ability levels on both the posttest and the retention test. On the posttest, the significant difference was between the low and high ability students and between the middle and high ability students. There was no significant difference between the low and middle ability students. On the retention test, there was a significant difference between all three pairs of ability levels; that is, between students of low and middle ability, between those of low and high ability, and between those of middle and high ability.

3. There was a significant interaction between treatment and ability with respect to the scores on both the posttest and the retention test. The pattern of the results for both tests was very similar. In both cases, the interaction was ordinal.

Discussion

One of the main purposes of the study was to determine if seventh grade students could benefit from the use of concrete and semi-concrete materials in learning mathematics and to determine the relative effectiveness of the two approaches. It was found that seventh grade students showed greater achievement and retention of the particular concepts in the study when these concepts were presented to them in an iconic mode than when presented in an enactive mode. While these findings are not inconsistent with those reported in some prior research studies (R. E. Johnson, 1971; Zirkle, 1981), the performance of the students in the enactive treatment was lower than what was expected; and the magnitude of the difference in favour of the iconic was not expected. A possible explanation is that students at the seventh grade level are at the stage of mental development where they are more receptive to an iconic presentation than to an enactive presentation, or at least for the topic of perimeter and area used in this study.

A different explanation might be that topics in plane geometry naturally lend themselves to drawing diagrams and that students are more familiar with paper and pencil to draw diagrams than with geoboards and rubber bands for the construction of geometric figures. Drawing diagrams may have provided the iconic group with a more permanent mental image of the geometric figures. It is possible that a third treatment in which students used geoboards together with dot paper might have produced even higher success. This possibility could be investigated in future research.

A second purpose of the study was to determine if there was a significant interaction between treatment and ability. The findings from both the posttest and retention test indicated that there was a significant interaction between treatment and ability. The interaction for both tests was ordinal. The iconic treatment was shown to be more effective than the enactive treatment for all three ability levels. On both tests, the low ability students in the iconic treatment scored higher than the middle ability students in the enactive treatment, and the middle ability group in the iconic treatment scored higher than the high ability students in the enactive treatment. Overall, it could be said that the greatest effect of treatment was with the middle ability level. On both tests, the middle ability students scored much closer to the high ability students in the iconic treatment than in the enactive treatment. In the enactive treatment, the middle ability students only scored slightly higher than the low ability students.

It was expected that the use of concrete materials in the enactive treatment would result in higher achievement for the low ability students than was shown in the study.

Similarly, it was not expected that the middle ability students would score as low in the enactive treatment as was shown. Therefore, these findings have to be viewed with caution, because to draw the conclusion that the use of concrete materials in teaching is harmful to low and middle ability students would not be warranted, in view of a large body of research showing the benefits of manipulative materials in classroom instruction. However, some concrete materials may be confusing to low ability students and a distraction to high or middle ability students, as was suggested by Wilkinson (1971). Possibly, for particular mathematical concepts, either a concrete or a semi-concrete presentation is better, or perhaps a combination of concrete and semi-concrete might be more appropriate than either method used separately.

While the iconic treatment was shown to be superior to the enactive treatment for each of the three ability levels, the close performance of the middle ability students and the high ability students in the iconic treatment on both the posttest and retention test was unexpected. In fact, the middle ability students scored higher in the iconic treatment than did the high ability students in the enactive treatment. Again, it would appear that for the average or middle ability student, the semi-concrete materials were more appropriate than the concrete materials for the particular concepts in this study. For the high ability students in the enactive mode, the concrete materials may have been a form of distraction to them rather than presenting any particular difficulty.

In the period between the administration of the posttest and the retention test, the students in the sample studied units on decimals and percents and did not work with the geometry concepts. As would be expected, there was a decrease in scores for both groups from the posttest to the retention test administered five weeks later. The greatest decrease in percentage points was for the low ability students in the enactive treatment, a drop of 27 percent. The next largest decrease was for the low ability students in the iconic treatment, a drop of 26 percent. This would not be totally unexpected since the time between administration of the tests was five weeks. The decrease for the middle ability students in the enactive treatment, 23 percent, was relatively large, but not surprising since their performance in the enactive treatment was only marginally better than the low ability students. As would be expected, the high ability students showed the smallest decrease. For the high ability students the decrease was smallest in the iconic treatment: 7 percent compared with 11 percent in the enactive treatment. However, the decrease for the high ability students in the enactive treatment was only one-half a percentage point less than the middle-ability students in the iconic treatment. This was not surprising since the middle ability students in the iconic treatment actually scored higher on both tests than the high ability students in the enactive treatment.

The results of this study seem to support the use of the Canadian Test of Basic Skills in classifying students into ability levels for the purpose, at least, of teaching mathematics since there was a significant difference between two pairs of ability levels on the posttest and among the three pairs of ability levels on the retention test. If significant interaction between teaching methods and ability does exist, then having a means of classifying students into ability levels would be very useful.

Recommendations and Implications

The following recommendations are based on the results of the study and on the observations made by the researcher.

1. It is recommended that further research be conducted using a larger sample and dealing with a different topic in geometry or a topic from another area of mathematics.

2. It is recommended that a similar study be conducted at a lower grade level to investigate whether there is a particular grade level at which one mode of presentation is superior to the other for the same concepts in geometry.

3. It is recommended that a similar study be conducted with students who have had previous experience

with manipulative materials.

4. It is recommended that a similar study of longer duration be conducted since one of the limitations in this study was its relatively short duration, namely, one month.

Based on the results of the study, the following implications for classroom teachers are suggested:

It is recommended that the decision to use concrete or semi-concrete materials be considered carefully. Semi-concrete materials might be more effective than concrete materials for some mathematical concepts and with students at a particular ability level. For the content dealt with in this study, dot paper proved to be more effective. Such materials are inexpensive and readily accessible to classroom teachers.

For certain topics, the use of manipulative materials should be considered as an alternative to the usual expository approach to teaching mathematics. From discussions with the teachers involved in the study, it was concluded that both teachers and students were positive in their attitudes toward the use of both concrete and semi-concrete materials.

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APPENDICES

APPENDIX A

LESSON PLANS FOR THE ENACTIVE TREATMENT

LESSON 1

THE GEOBOARD

Purpose

The overall purpose is to introduce students to the geoboard by having them make geometric figures on the geoboard. Since these figures were included in Unit C, Module 1, of the grade seven text, the lesson can serve as a review of some of that material. Questions related to the figures can allow for class discussion and feedback from the students.

Behavioral Objectives

The student will be able to make the following geometric figures on the geoboard:

- line segments
- angles
- triangles
- 4-sided polygons
- n-sided polygons

Introduction

Explain that a geoboard is a board on which nails or pegs are arranged in an orderly fashion to represent sets of points. Geoboards come in a variety of shapes which you can demonstrate by showing some of the different models. Students have a 5 x 5 geoboard with pegs in a square arrangement. Let them count the pegs.

Line segments

Explain the following aspects of the geoboard by placing the above on the transparent geoboard.

1. A line segment is represented by a rubber band stretched between two pegs.

2. To measure the length of a segment a unit is chosen. In this case the distance between two adjacent pegs in a horizontal or vertical direction is 1 unit.

3. Distance is not measured diagonally since the distance between two consecutively placed pegs is not 1 unit. Students can verify this with a ruler.

4. Parallel lines (horizontal and vertical pairs).

Exercise: Line segments

Direct students to:

(a) make a horizontal, vertical, and diagonal line segment on their geoboards.

(b) make a line segment 1 unit in length.

(c) make a line segment 3 units in length.

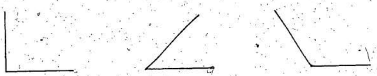
(d) make the longest horizontal line segment possible.

Place each of the above exercises on the transparent geoboard after the students have placed each on their boards.

Angles

Explain that an angle is represented on the geoboard by showing two line segments which share a common end point. Instruct the students to make the following angles:

1. right 2. acute 3. obtuse



As the students make each, review the definitions in a discussion. Illustrate each on the transparent board after allowing students to make theirs.

Triangles

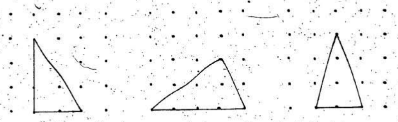
Explain that on a geoboard a triangle is formed by stretching one rubber band around three pegs which are not in the same line segment.

Tell students to make the following triangles on their geoboards, obtaining the definitions in a class discussion.

right triangle -- a triangle with one right angle

scalene triangle -- a triangle with no sides congruent

isosceles triangle -- a triangle with two sides congruent



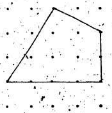
Allow a different student to demonstrate one each on the transparent geoboard.

Ask students to try to make an equilateral triangle. Explain that it is not possible on their geoboards because the distance between two consecutive diagonally placed pegs is not the same as that between two consecutive horizontal or vertical pegs (as was explained earlier).

Quadrilaterals

Explain that on a geoboard a quadrilateral is formed by stretching a rubber band around four pegs, no three of which are on the same line segment.

Illustrate with this example.



Instruct the students to make some more quadrilaterals on their geoboards.

Allow the students some time to experiment by making some shapes of their own on the geoboard while you distribute the student lesson sheets to the class. These pages allow the student to follow written instructions similar to that which will be found in all the lessons.

Move around the class, checking the students' progress, and initiating class discussion. In this way, each type of quadrilateral shown should be defined. These are the quadrilaterals for which area formulas will be found in later lessons. Allow a student to demonstrate each of the figures made, either by having it placed on the transparent geoboard or holding it up for the class to see.

The following definitions would be appropriate. State these after students have verbalized their own in the class discussion.

parallelogram (exercise 1) -- A quadrilateral whose opposite sides are parallel.

Point out that opposite sides are also congruent.

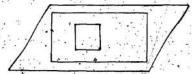
trapezoid (exercise 2) -- A quadrilateral with one pair of parallel sides.

rectangle (exercise 3) -- A parallelogram with all right angles.

square (exercise 4) -- A rectangle with all sides congruent.

Equivalent definitions on the part of students would be acceptable. For example, square--four-sided figure with all sides equal and all angles right angles.

Make sure that students realize that all squares are rectangles, and all rectangles are parallelograms.



STUDENT LESSON SHEETS 1

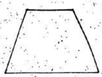
Exercises

(1) Make a quadrilateral with the same shape as shown here. On your figure what is the length of the longest side that can be measured on the geoboard? ____________

How would you describe the figure? ________________

Is this quadrilateral a square? ___ rectangle? ____ parallelogram? ___

Make two more such quadrilaterals on your geoboard.

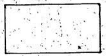


(2) Make a quadrilateral with the same shape as shown here. How does it differ from the quadrilateral in the previous exercise? ___________

____

Is the figure a rectangle? _____ trapezoid? _____ parallelogram? ____

Make some more quadrilaterals of this type on your geoboard.



(3) Make a quadrilateral with the same shape as shown here.

Are all sides of the figure of the same length? __________

What is the length of each side?

____

Is this quadrilateral best described as a parallelogram? ______

rectangle? ______ square? ______

Make two more quadrilaterals of this type on your geoboard.

(4) Make a quadrilateral with the same shape as shown here.

What is the length of each side of your figure? __________

Is your quadrilateral a

parallelogram? ______

rectangle? ______ square? ______

Make the biggest figure of this type on your geoboard. What is the length of each side? __________

Make the smallest such figure on your geoboard. What is the length of each side? __________

On your geoboard make several closed figures with more than 4 sides.

Make a 14-sided figure.

What name is given to any closed figure that has sides made up of line segments? ______________

LESSON 2

PERIMETER

Purpose

To develop the concept of perimeter.

Behavioral Objectives

The student will be able to:

1. verbalize and write the definition of perimeter as the measure of the distance around a polygon.

2. find the perimeter of n-sided polygons when the measure of each side is given.

3. express measurements in the following metric units: metre, centimetre, millimetre, kilometre.

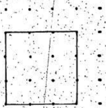
4. write the formula for finding the perimeter of a rectangle as $P = 2 \cdot \ell + 2 \cdot w$ or $P = 2 \cdot (\ell + w)$.

5. find the perimeter of a rectangle using the above formula.

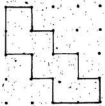
Introduction

Introduce the lesson by stating that it will consist mainly of making polygons on the geoboard and making measurements on them.

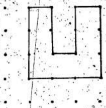
Make figure A on the transparent geoboard. Instruct students to copy it on their geoboards and find the total measure of the distance around it. After getting some verbal answers show the class, on the transparent board, how to find the measure.



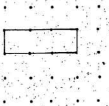
A



B



C



D

Do exactly the same for figures B, C, and D.

Then, pose the question: What name is given to the measurement of the distance around a polygon? ____________

Discuss briefly the following:

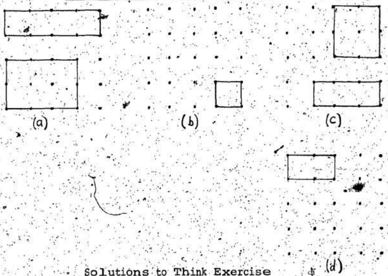
1. Perimeter can be found for any plane geometric figure.

2. In the real world, the standard unit of length in the metric system is the metre (m), other common units being the centimeter (cm), the millimetre (mm), and the kilometre (km).

3. Two examples in the immediate environment--perimeter of the classroom and perimeter of the chalkboard.

Next, distribute the student lesson sheets to the class. Move around the room to check student progress. Allow them to demonstrate figures to the class either on the transparent geoboard or by holding them up for the class to see them. The solutions to the Exercises (a, b, c) should be placed on the transparent geoboard.

Solutions to Practice Exercise



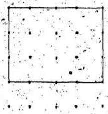
Solutions to Think Exercise

(a) 

$P = 4 \cdot s$

(b) $P = a + b + c$

(c) $P = 24$ units

(d) 

$P = 2 \cdot 4 + 2 \cdot 3$

$= 8 + 6$

$= 14$ m

(e) Width = 3 units

$P = 2 \cdot \ell + 2 \cdot w$

$20 = 14 + 2 \cdot w$

$20 - 14 = 2 \cdot w$

$6 = 2 \cdot w$

$3 = w$

Could explain the solution this way along with a simple example on the transparent geoboard.

STUDENT LESSON SHEETS 2

Define perimeter: ______________________________

____

1. On your geoboard make the largest rectangle that is not a square.
   What is its length? ______
   width? ______
   perimeter? ______
2. On your geoboard make a rectangle with length 4 units and width 2 units. Find its perimeter (P).
   P = ______
3. Make two other rectangles, find their perimeters, and record the length and width of each.
   Rectangle 1: length ____ Rectangle 2: length ____
   width ____ width ____
   perimeter ____ perimeter ____
4. If the measure of the length of a rectangular region is "ℓ" units and the width is "w" units, write a formula for finding the perimeter (P).
   P = or P =
5. Using the formula, find the perimeter of a rectangle which has length 3 units and width 4 units.
   Check your solution by making the rectangle on your geoboard and finding its perimeter.

6. Make one more rectangle on your geoboard and find its perimeter by using the formula.

P =

Exercises

Make the following figures on your geoboard. Record your solutions by sketching each figure and marking its dimensions.

(a) two different rectangles each with a perimeter of 10 units.

(b) a square with a perimeter of 4 units.

(c) a square and a rectangle each with a perimeter of 8 units.

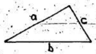
(d) a rectangle with a perimeter of 6 units and width 1 unit. What is its length? ____________

Think

(a) Since a square is a special type of rectangle, write a formula for finding the perimeter of a square with a side "s" units in length.

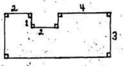
P =

(b) If a, b, and c are the lengths of a sides of a scalene triangle write a formula for finding the perimeter.



P =

(c) Find the perimeter of the following polygon:



(d) A rectangular shaped room measures 4 metres by 3 metres. Make a model of the shape of the room on your geoboard and find its perimeter. Use the perimeter formula.

(e) If a rectangle with a perimeter of 20 units has a length of 7 units, what is the measure of its width?

Textbook Exercises

C-40 Number 4

C-41 Numbers 2 (A-D), 6

LESSON 3

INTRODUCTION TO AREA

Purpose

To develop the concept of area by counting unit squares.

Behavioral Objectives

The student will be able to:

1. state that the unit for measuring area is the square unit.

2. write and verbalize the definition of area of a polygon as the number of square units contained in its interior.

3. find the area of a polygon by counting the number of unit squares contained in its interior.

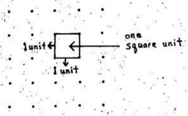
4. state the common metric units, and their symbols, for measuring area--square metre (m^2), square centimetre (cm^2), square millimetre (mm^2), square kilometre (km^2).

Introduction

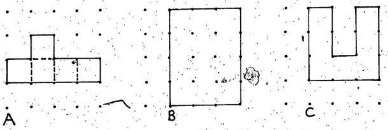
Introduce the lesson by stating that it deals with measurement, but of a different type than in the previous lesson, namely, the measurement of the interior of a figure.

Make a 4 x 3 rectangle on the transparent geoboard and state that it is possible to measure the space in the interior of such a figure if a small square region is chosen as the unit of measurement.

Instruct the students to make the smallest possible square on their geoboards, after which you make the same on the transparent board.



Explain that this small square will be used as a standard unit to measure the interior of polygons on the geoboard. Its sides which are 1 unit in measure enclose a space called 1 square unit.



Make a copy of figure A on the transparent geoboard.

Tell the students to copy it on their geoboards and get a measure of its interior by counting the number of unit squares inside. If necessary, partition the figure with rubber bands as shown by the broken lines.

After getting feedback about figure A do the same as above for figures B and C.

At this point, ask the students to state the name given to the measure of the number of square units contained in the interior of a plane figure or polygon. ______________

Briefly discuss the following:

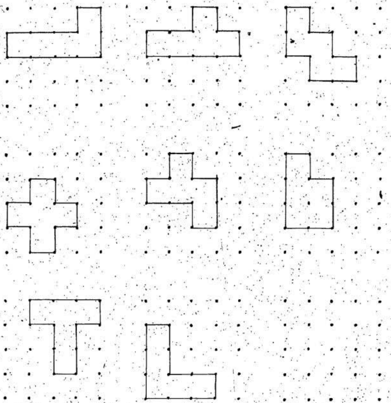
1. Area can be found for any plane region. The geoboard being used here limits it to polygons.

2. The common metric units for measuring area, and their symbols, are the square metre (m^2), square centimetre (cm^2), square millimetre (mm^2), and square kilometre (km^2). Mention that the symbols, example, m^2, will be discussed in a later lesson. For now, it is important to know that the unit for measuring area is the square unit as opposed to the linear unit for perimeter. A square centimetre, for example, would be a small square having sides 1 centimeter in length. Its area, being 1 square centimetre, would be written as 1 cm^2.

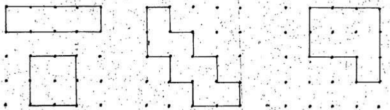
Next, distribute the student lesson sheets to the class. Move around the room to check student progress. Exercises (a-e) should not pose much difficulty. Check the solutions to these by placing them on the transparent geoboard or allowing students to place them on the transparent geoboard, or hold the figures up for the class to see.

Solutions to Practice Exercise

(a)

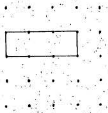


(b)

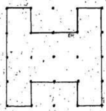


(c) (c)

There are other solutions.



(d)



(e)

<u>Solutions to Think Exercise</u>

(a) i. 16 square units

ii. 7 square units

(b) It will take 600 squares of the size shown to cover the surface of the sheet.

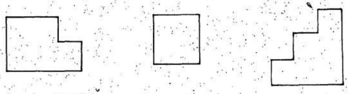
(c) 1 kilometre.

STUDENT LESSON SHEETS 3

What kind of unit is used to measure area? ____________

Write a definition for area. ____________________

On your geoboard make a polygon that has the same shape as each of those below, and find the area of each.



Exercises

Make the following on your geoboard. Record the shapes by sketching each on plain paper and marking the dimensions.

(a) eight polygons with different shapes, each having an area of 5 square units.

(b) a rectangle and a square, each having an area of 4 square units.

(c) two polygons with different shapes, each having an area of 7 square units.

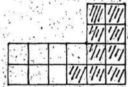
(d) a rectangle with an area of 3 square units and a perimeter of 8 units.

(e) the largest polygon with the shape of the letter "H".

Find its area. ____________

Think

(a) The following is a model of a room with the area divided into square metres. A section is covered with carpet, as shown by the shaded region.

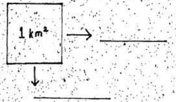


i. What is the total area of the room? ____________

ii. What is the area of the floor that remains to be covered? ____________

(b) If this region ⟶ ☐ is one square centimetre, what is meant by the following statement:
The area of this sheet is 600 cm^2. ____________

(c) If the following figure is a model of a square kilometre, 1 km^2, then what is the length of each side of the square?



LESSON 4

AREA (Continued)

Purpose

1. To extend the concept of the area of polygons by counting unit squares and parts of unit squares.

2. To show how a diagonal divides the area of a rectangle.

Behavioral Objectives

The student will be able to:

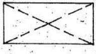
1. define a diagonal as a segment joining two (nonconsecutive) vertices of a polygon.

2. state that a diagonal cuts a rectangle in half with the two sections formed being triangles.

3. find the area of polygons by counting unit squares and parts of unit squares.

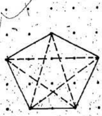
4. find the area of polygons using the principle that a diagonal cuts the area of a rectangle in half.

Begin the lesson with a short discussion in which you review the definition of a diagonal (behavioral objective #1). Make a rectangle on the transparent geoboard.



Ask the students to copy it on their geoboards and place all possible diagonals on their figure.

After checking the answer, place the following pentagon on the transparent geoboard, and have the students copy it. Ask them to place all possible diagonals on it.



Allow a student to demonstrate by holding up the geoboard for the class to see.

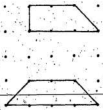
Next, make a unit square on the transparent geoboard. 

Ask the students to copy it on their board, and place one diagonal on it.

Ask: How does the diagonal cut the unit square? <u>("in half" is the response that is needed)</u>.

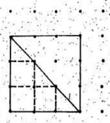
Ask: What is the area of the triangular sections on each side of the diagonal? (½ <u>square unit</u>)

Make the following polygons on the transparent geoboard.



Ask the students to copy each and find the area by counting.

Next, make a square with side 3 units, and on it make a diagonal. Ask the students to copy it and find the area of the triangular section on one side of the diagonal by counting. Suggest that they partition the section as shown below.

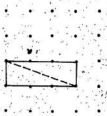


Ask them to compare the area of the triangular section to the area of the square.

(They should see that the area is ½ the area of the square.)

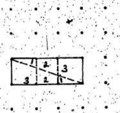
Instruct the students to make a square with side 2 units and to do the same as they did for the previous square. Ask the same question.

Next, make a 3 by 1 rectangle on the transparent geoboard. Ask the students to copy it, and on it make a diagonal.



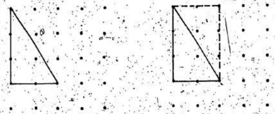
In this case, the students cannot count ½ square units in finding the area of a triangular section. Instruct them to find the area of a triangular section on one side of the diagonal. Then ask for a verbal explanation. They should see that the diagonal cuts the area of the rectangle in half.

Some students may see it more clearly if the rectangle is partitioned, as shown below, and the matching sections of each triangle pointed out.



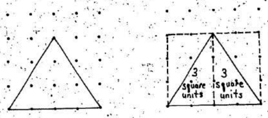
If the two triangles have the same area, then the area of each must be ½ the area of the rectangle.

Next make the following triangle on the transparent geoboard. Ask the students to copy it and find its area.



Allow the students time to find the area, and then ask for a verbal explanation. Show the class that the area of the triangle is ½ that of a rectangle that can be made on the triangle.

Next, make the following triangle on the transparent geoboard.

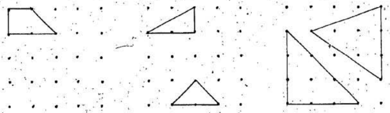


Ask students to copy it and find its area. If necessary, suggest that they construct a rectangle on each half of the triangle as shown by the broken lines above.

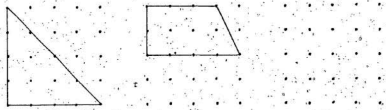
This exercise may require careful discussion. Each small rectangle is 6 square units. Since a diagonal divides a rectangle in half, each triangular section has an area of 3 square units. Therefore, the original triangle has an area of 6 square units (3 + 3).

Next, distribute the student lesson sheets to the class. Move around the room encouraging the students to experiment with the exercises on their own. Allow them time to work on the exercises before showing them the solutions on the transparent geoboard or having the students show their solutions.

Solutions for Practice Exercises

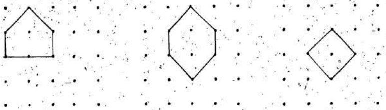


(a) (b) (c)

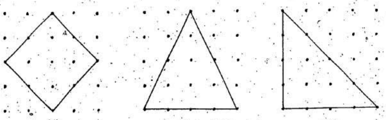


(d) (e)

Solutions for Think Exercise



(a) (b) (c)



(d) (e) (f) (f)

STUDENT LESSON SHEET 4

Define diagonal. ______________________________

____

On your geoboard make a rectangle 4 units long and 1 unit wide. On it make a diagonal and find the area of a triangular section on one side of it.

Explain how you found the area. ______________________

____

Exercises

Make the following on your geoboard. Make a sketch of each on blank paper.

(a) a 4-sided polygon with an area of $1\frac{1}{2}$ square units.

(b) two triangles with different shapes, each with an area of 1 square unit.

(c) a triangle with an area of $4\frac{1}{2}$ square units.

(d) a right triangle with perpendicular sides 4 units in measure.

(e) a 4-sided polygon with an area of 7 square units.

Think

Make the following on your geoboard. Make a sketch of each on blank paper.

(a) a 5-sided polygon with an area of 3 square units.

(b) a 6-sided polygon with an area of 4 square units.

(c) a square with an area of 2 square units.

(d) a square with an area of 8 square units.

(e) the largest possible isosceles triangle with a base of 4 units. Find its area. ____________

(f) the largest possible triangle. What is its area? ____________

Textbook Exercises

C-45 Number 1 (A-C)

Number 2 (A-J)

LESSON 5

AREA OF A RECTANGLE

Purpose

To develop the formula for the area of a rectangle.

Behavioral Objectives

The student will be able to:

1. define the height of a rectangle as the length of a side which is perpendicular to a second side called the base.

2. use the terms length and width in place of base and height when finding the area of a rectangle.

3. state the formula for the area of a rectangle as $A = b \cdot h$ and $A = \ell \cdot w$.

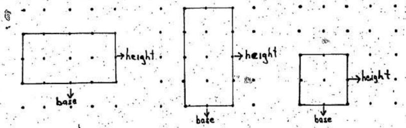
4. compute the area of a rectangle given the measure of its sides.

5. write the common metric symbols for area--m^2, cm^2, mm^2, km^2.

Introduction

Introduce the lesson by asking the students to recall the definition of a rectangle--a parallelogram with four right angles, or an equivalent description.

Make the following rectangles, one at a time, on the transparent geoboard, and ask the students to copy each on their boards.



In a discussion, define the height of a rectangle (stated in behavioral objective #1). Point out that in a rectangle, base and height are commonly called length and width. To be consistent with the formula for area of a parallelogram, the terms base and height will be used in first stating the formula.

Instruct the students to get the measure of the base and height for each of the above rectangles.

Check the answers by getting verbal responses, and then distribute the student lesson sheets to the class. By working through these sheets, the students should discover the formula for the area of a rectangle. If, after completing the table, some students do not see the relationship, $A = b \cdot h$, suggest that they study the table carefully. The hint that the multiplication operation is used could be given, if necessary.

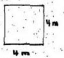
Move around the class and check student progress. Encourage them to attempt all the exercises and make use of the formulas that have been derived so far.

Allow the students to check the solutions to all the practice exercises by letting the students place the solutions on the transparent geoboard.

Use the explanation to <u>Think</u> exercise (a) to explain the symbols for the metric units for area, square metre, as m^2, for example.

Having done exponents, the students should realize that $S \cdot S$ can be written as S^2 just as $3 \cdot 3$ can be written as 3^2.

Take a square with a side of 4m as an example.

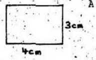


Explain as follows:

$$A = 4m \times 4m$$
$$= 4 \cdot 4 \cdot m \cdot m$$
$$= 16m^2$$

$m \cdot m = m^2$ i.e. M (exponent 2)

Take, as another example, a rectangle 4 cm by 3 cm

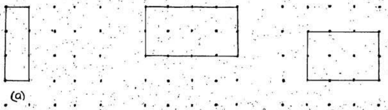


Explain as follows:

$$A = \ell \cdot w$$
$$= 4\text{ cm} \cdot 3\text{ cm}$$
$$= 4 \cdot 3 \cdot \text{cm} \cdot \text{cm} \quad \text{since } \text{cm} \cdot \text{cm} = \text{cm}^2$$
$$= 12\text{ cm}^2$$

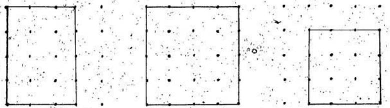
Explain that in finding area we measure plane surfaces on two dimensional surfaces. Therefore, the unit is symbolized with exponent 2. With perimeter, we only measure length--one dimensional, symbolized by exponent 1. 10 cm could be written 10 cm^1.

Solutions for Practice Exercise



(a) (b) (c)

$A = b \cdot h = 1 \cdot 3 =$
3 square units



(d) (e) (f)

$A = b \cdot h = 3 \cdot 4 =$
12 square units

Solutions for Think Exercise

(a) $A = s \cdot s$ or $A = s^2$

(b) (24,1), (12,2), (8,3), (6,4)

(c) 36 metres

(d) 22 square units

(e) 18 Area of wall $120 \times 60 = 7200\ cm^2$

Area of one tile $20 \times 20 = 400\ cm^2$

$$400\overline{)7200}^{\;18}$$

STUDENT LESSON SHEET 5

Make four different rectangles on your geoboard and complete the table below for each rectangle you make. Also, sketch the shapes of the rectangles on blank paper and mark the dimensions on each.

Rectangle	Number of unit squares in bottom row (BASE)	Number of rows of unit squares (HEIGHT)	Total number of unit squares	AREA
A				
B				
C				
D				

Do you see a pattern in the table that indicates a special relationship among the measures of the base, height, and area that would enable you to easily find the area of a rectangle without counting unit squares?

What is that relationship? ______________________

If we let A = area, b = base, and h = height, write a formula for finding the area of a rectangle.

A = __________

If the base of a rectangle is commonly called the length (ℓ) and the height called the width (w), write an equivalent formula for the area of a rectangle.

A = ________________

Exercises

Make the following on your geoboard, sketch each on blank paper, and mark the dimensions:

a) rectangle with an area of 3 square units and a height of 3 units.

What is the measure of its base? ________

Check the area by the formula.

$A = b \cdot h$ = ____ x ____ = ____

b) rectangle with an area of 8 square units. What is the measure of its length? ________

width? ________

c) a rectangle with an area of 6 square units. What is its perimeter?

P = __________

d) a rectangle with a perimeter of 14 units and a base of measure 3 units.

What is its height? ________

Find the area using A = b x h ____________

e) a rectangle with length of measure 4 units and area of 16 square units.

What is its width? ______

What special type of rectangle is it? ______

What is its perimeter? ______

f) a rectangle with the largest possible area and having a perimeter of 12 units.

What is the measure of the sides of the rectangle?

______

What is the area of the figure? ______

Think

a) If a square is a special type of rectangle with sides of measure "s" units write two equivalent formulas for the area of a square.

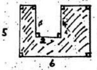
A = ______ or A = ______

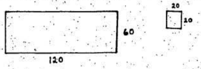
b) If a rectangle has an area of 24 square units, what are some possible measures of its sides?

______

c) If the area of a square room is 81 square metres, what is its perimeter? ______

d) Find the area of the shaded regions.





e) A section of wall measures 120 cm long and 60 cm wide. A certain brand of square tiles measures 20 cm on each side. Find out how many tiles would be needed to cover the section of wall.

Textbook Exercises

C - 47 Number 1 (A)

Number 2 (A)

S - 25 (Set 42) Number 1 (A,D)

Number 2 (A-C)

LESSON 6

AREA OF A PARALLELOGRAM

Purpose

To develop the formula for the area of a parallelogram by showing the relationship of the area of the parallelogram to the area of a rectangle with the same measure for the base and height.

Behavioral Objectives

The student will be able to:

1. define the height of a parallelogram as the perpendicular distance from one side to the opposite side which is called the base.

2. locate the height and base in a parallelogram.

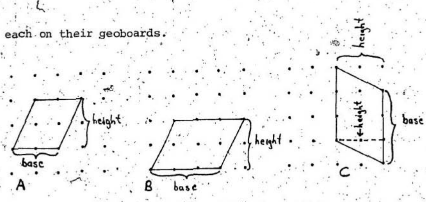
3. write and verbalize the formula for the area of a parallelogram as $A = b \cdot h$.

4. find the area of a parallelogram, given the measures of the base and height.

Introduction

Introduce the lesson by asking students to recall the definition of a parallelogram (a quadrilateral that has opposite sides parallel).

Place the following parallelograms, one at a time on the transparent geoboard, and ask the students to copy each on their geoboards.



After placing parallelogram A on the transparent geoboard, pose the questions: How tall is the parallelogram? ________ How wide is the parallelogram along the bottom? ________

Ask if they agree that it is 2 units tall and 2 units wide along the bottom, that is, if the height is 2 units and the base is 2 units. Point out the base and height as indicated in figure A. Emphasize that the height is not the measure of a side of the parallelogram.

After the students have copied parallelogram B, ask them to get the measure of the base and height. Then, ask for a possible definition for the height of a parallelogram. In a short discussion define the height as stated in behavioral objective #1.

Place parallelogram C on the transparent geoboard, and ask the students to copy it. Again, tell them to find the measures of the base and height. If they have difficulty locating the base, suggest that they can rotate the geoboard until the base is at the bottom of the parallelogram. For this figure, show that the height can also be seen within the parallelogram as a perpendicular segment from the vertex to the base.

Next, proceed to develop the formula for the area of a parallelogram.

Place parallelogram A on the transparent geoboard and ask the students to copy it on their boards.

A B C

Instruct them to find the area of the parallelogram by counting unit squares. Tell them to record the area and also the measures of the base and height.

Next, on the transparent geoboard, transform the parallelogram into a rectanble by the following steps. Instruct the students to do the same on their boards.

1. Place a different coloured rubber band around the triangular section at the left of the parallelogram as shown in figure B.

2. Move the triangular section from the left side over to the right side of the parallelogram as shown in

figure C. In the same motion, move in the rubber band representing the left side of the parallelogram so that it becomes a side of the rectangle.

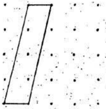
Instruct the students to find and record the area of the rectangle, as well as the measures of the base and height. Compare the area of the original parallelogram to the area of the rectangle that was formed ___(same)___. Compare the base and height of the original parallelogram to the base and height of the rectangle. ___(same)___ Point out that in the transformation from parallelogram to rectangle, the shape of the figure was changed but that the area of the figure was not changed.

Review the above procedure starting with original parallelogram A.

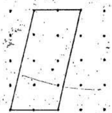
Next, distribute the student lesson sheets to the class. Making rectangles from the parallelograms should give the students a better grasp of the concept of area. Some students may deduce the area formula in computing and recording the base, height, and area of the parallelogram.

Move around the class checking student progress. Suggest that they make use of any formulas learned so far. Place the solutions to all geoboard exercises on the transparent geoboard, or let the students place them on it.

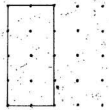
Solutions for Practice Exercise



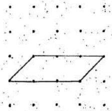
(a)



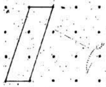
(b)



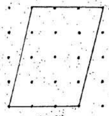
(c)



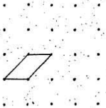
(d)



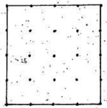
(d)



(e)



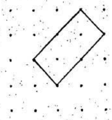
(f)



(g)

Solutions for Think Exercise

(a)



(b) 112 m^2

(c) 6 cm

(d) 12 square units

Area large parallelogram	$5 \times 3 = 15$
Area small parallelogram	$3 \times 1 = 3$
Area unshaded region	$15 - 3 = 12$ square units

STUDENT LESSON SHEETS 6

Make three parallelograms, one at a time, on your geoboard, and complete the table below for each. Change each parallelogram into a rectangle as you did before.

Parallelogram	Measure of base	Measure of height	Area Parallelogram	Measure of base Rectangle	Measure of height Rectangle	Area Rectangle
A						
B						
C						

Do you see a special relationship among the measures of the base, height, and area of a parallelogram that would allow you to find the area if given the measures of the base and height?

What is that relationship? ____________________

Write a formula for finding the area of a parallelogram.

A = ____________________

Complete the following statement: The area of a parallelogram is ____________ the area of a rectangle which has the same measure of base and height as that parallelogram.

Exercises

Make the following on your geoboard. Record the shape of each on blank paper.

(a) a parallelogram with area 4 square units, base 1 unit, and which is not a rectangle.
What is its height? ________

(b) a parallelogram with base 2 units and height 4 units.
What is its area? ________

(c) a parallelogram with the same area as (b) but having a different shape.

(d) a parallelogram with an area of 3 square units and which is not a rectangle.
What is the measure of its base? ______ height? ______

(e) the largest possible parallelogram that is not a rectangle. What is its area? ______ base? ________
height? ________

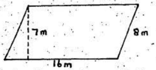
(f) the smallest possible parallelogram that is not a rectangle. What is its base? ______ height? ________
area? ________

(g) the largest possible type of parallelogram. What type is it? ________ What is its area? ________

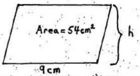
Think

(a) On your geoboard make a parallelogram with an area of 4 square units and which has one pair of parallel sides twice as long as the other pair.
What type of parallelogram is it? ________

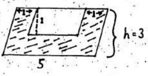
(b) Find the area of this parallelogram region.



(c) What is the measure of the height of this parallelogram?



(d) Find the area of the shaded region of this parallelogram shaped figure. The unshaded region is also parallelogram in shape.



Textbook Exercises

C-47 Number 3 (A)

S-25 (Set 42) Number 3(A)

LESSON 7

AREA OF A TRIANGLE

Purpose

To develop the formula for the area of a triangle.

Behavioral Objectives

The student will be able to:

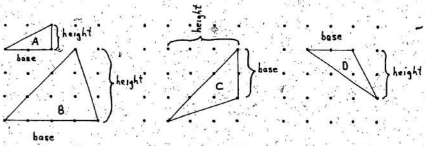
1. define the height of a triangle as the perpendicular distance from a vertex to the opposite side or the extension of the side and the base as being the width of this side.

2. locate the base and height in a triangle.

3. write and verbalize the formula for the area of a triangle as $A = \frac{1}{2}b \cdot h$.

4. find the area of a triangle given the measures of its base and height.

Make the following triangles, one at a time, on the transparent geoboard and ask the students to copy each on their geoboards.



State that the height of triangle A is 1 unit and the base is 2 units. Ask the students if they agree. Point out the height and base of the triangle.

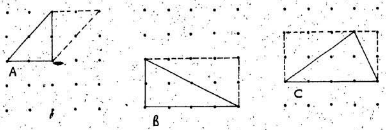
After the students have copied triangle B ask them to find the measures of the base and height. Get some verbal answers, and then ask for a possible definition of the height of a triangle. Point out, as for the parallelogram, that the height is not the side of this triangle.

After the students have copied triangle C ask them to find the measures of the base and height. If they have difficulty in locating the base, suggest that they rotate the geoboard until the base is on the bottom. After finding the measure of the base and height in that position, ask the students to rotate the board until the figure is in its original position and to find the measures with the triangle in that position. Point out the base and height and define them as stated in behavioral objective #1.

Place triangle D on the transparent geoboard. Ask the students to copy it and find the measures of the base and height.

Next, distribute the student lesson sheets to the class, and proceed to develop the formula for the area of a triangle.

Place triangle A on the transparent geoboard,



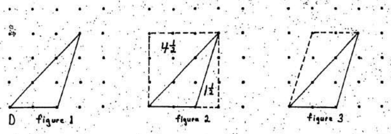
After the students have copied it, ask them to find and record in the table, the base, height, and the area by counting. Using a different coloured rubber band, make a parallelogram on the triangle as shown by the broken lines. After the students have done the same, ask them to find and record the base, height, and area of the parallelogram. They should use the formula $A = b \cdot h$ rather than count squares.

Next, place triangle B on the transparent geoboard, and proceed as for triangle A.

Next, place triangle C on the transparent geoboard and proceed as before.

Again, ask the students to record the same measures as for the first two triangles.

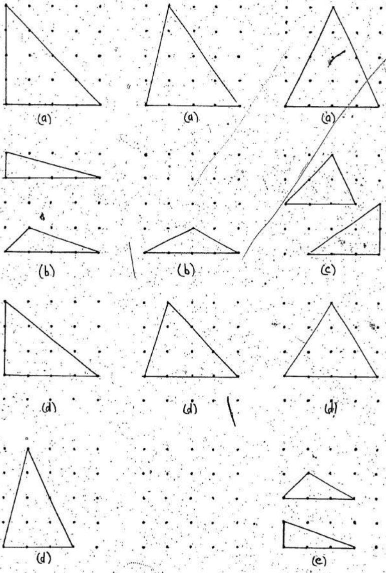
Next, place triangle D on the transparent geoboard and ask the students to copy it on their geoboards.



To find the area of this triangle by counting, it is necessary to make a rectangle on the triangle as shown by the broken lines above (figure 2). Ask the students to make such a rectangle and record its area. (9) Next, ask them to find the areas of the two right triangles on both sides of the original triangle. If necessary, remind the students to use the fact that a diagonal cuts the area of a rectangle in half. Area of the large right triangle = (4½). Area of the small right triangle = (1½). Therefore, the area of the original triangle is (3) square units, found by subtracting the sum of the areas of the two right triangles from the area of the rectangle (9 - [4½ + 1½]). Ask the students to record the area of the triangle in the table. Then, instruct them to remove the rectangle and make a parallelogram on the original triangle, as shown above (figure 3). Ask them to complete the table for triangle D.

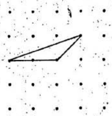
Review the above steps for triangle D. Then, instruct the students to complete the table for triangles E and F described below it and answer the questions that follow. A discussion of these questions should enable the students to grasp the formula for the area of a triangle. Place the solutions to all the geoboard exercises on the transparent geoboard for the students to see.

Solutions for Practice Exercises



<u>Solutions for Think Exercises</u>

(a)



(b) i $A = \frac{1}{2}b \cdot h = \frac{1}{2} \cdot 6 \cdot 4 = 12$ sq. units

ii $A = \frac{1}{2}b \cdot h = \frac{1}{2} \cdot 8 \cdot 5 = 20$ sq. units

iii Not enough information

(c) Area $= \frac{1}{2}\, b \cdot h = \frac{1}{2}\, 1 \cdot 4 = 2$ sq. units

Each new triangle will have area 2 sq. units because the measure of the base and height remains the same for each triangle.

(d) $A = \frac{1}{2}\, b \cdot h$

$= \frac{1}{2}\, 20 \cdot 18$

$= 180\ m^2$

(e) $A = \frac{1}{2}\, b \cdot h$

$400 = \frac{1}{2}\, 25 \cdot h$

$h = 32$

Students may reason in different ways to get the answer to (e). Some may divide 400 by 25 and then double the result. Others may use trial and error.

STUDENT LESSON SHEETS 7

Triangle	Measure of Base	Measure of Height	Area	Parallelogram	Measure of Base	Measure of Height	Area
A				on A			
B				on B			
C				on C			
D				on D			
E				on E			
F				on F			

triangle E -- an isosceles triangle with base 2 units and height 2 units

triangle F -- scalene with base 1 unit and height 2 units. To find the area of the triangle make a rectangle around the triangle as was done for triangle C.

Study the above table and answer the questions below.

Does a triangle and the parallelogram constructed on it have the same measure of base and height? ________

How does the area of a triangle compare with the area of a parallelogram constructed on it? ______________

Recall the formula for the area of a parallelogram.

A = ______________________

Do you see a special relationship among the measures of the base, height, and area of a triangle that would enable you to find the area when given the measures of the base and height?

What is that relationship? ______________________

____

Write a formula for finding the area of a triangle.

A = ______________

Exercises

On your geoboard make the following:

(a) a triangle with base 4 units and height 4 units. What is its area? ____________

(b) three different triangles with base 4 units and height 1 unit.

What is the area of each? ____________

Explain why. ______________

(c) a triangle with base 3 units and area 3 square units. What is its height? ____________

(d) four different triangles, each with an area of 6 square units.

What is an easy method of finding the base and height without experimenting on the geoboard first? ____________

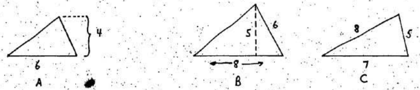
____

(e) A triangle has area of $1\frac{1}{2}$ square units and base 3 units. What is the area of the parallelogram that could be built on the triangle? ______________

Think

(a) On your geoboard make an obtuse triangle with an area of 1 square unit.

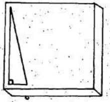
(b) Find the area of the following triangular regions:



Which figure contains too much information? ____________

Which contains too little information? ____________

(c) On your geoboard, make a right triangle with base 1 unit and height 4 units as shown. Find its area. ________



Move point P to the right, one nail at a time, making a new triangle each time. Find the area of each triangle. How do the areas change? ____________

Explain why. ____________

____

____

(d) A triangular region has a base of 20 metres and a height of 18 metres. Find its area. ____________

(e) A triangular region with a base of 25 cm has an area of 400 square centimetres. What is the measure of its height? ____________

Textbook Exercises

C - 49	Number 1 (A)
	2 (A)
	4
	5 (A, C)
S - 25 (Set 43)	Number 1 (A, D, E, H)
	Number 2 (A-D)

LESSON 8

AREA OF A TRAPEZOID

Purpose

To develop the formula for the area of a trapezoid.

Behavioral Objectives

The student will be able to:

1. define the height as the perpendicular distance from one parallel side to the other parallel side, which is considered to be the base.

2. find the measures of the base and height.

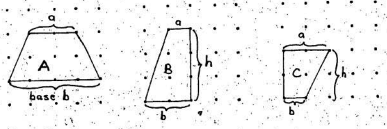
3. write and verbalize the formula for the area of a trapezoid as $A = \frac{1}{2}(a+b)\cdot h$

4. find the area of a trapezoid given the measures of the height and the two parallel sides.

Since the formula for the area of a trapezoid is more complex than the formulas encountered so far, it will be developed by a more guided discovery approach. With the 5 x 5 geoboard and the approach used, it is possible to develop the formula only for trapezoids that have parallel sides of 1 and 2 units.

Begin by reviewing the definition of a trapezoid (a quadrilateral with two parallel sides). Place three quadrilaterals that are not trapezoids on the transparent geoboard, and for each ask the question: Is this a trapezoid?

Next, place trapezoid A on the transparent geoboard. Ask the students to copy it on their boards, and find the measures of the base and height.



The student will probably have little difficulty finding the base and height at this stage. In any case, point out that either of the two parallel sides could be considered the base. Define the height of a trapezoid as stated in behavioral objective #1.

Point out that the measure of the side parallel to the base is required in the formula for the area of a trapezoid. It is necessary to name it by means of a letter. State that the side parallel to the base will be known as "a", and that, as before, the base is "b" and the height is "h".

Place trapezoid B on the transparent geoboard. After the students have copied it on their boards, ask them to find the measures of "a", "b" and "h". After checking the measures, make trapezoid C on the transparent geoboard. Ask students to copy it on their boards and, again, find the measures of "a", "b" and "h". For this figure, ask them to find the area by counting.

Next, ask the students if they can see a method for developing a formula to find the area of a trapezoid. If there is no response, give some hints, one at a time:

1. Build a figure on the trapezoid.

2. The method is similar to that used for the area formula for a triangle.

After some discussion, state that the method involves making a parallelogram on the trapezoid and comparing the areas.

Make trapezoid D on the transparent geoboard.

Ask the students to copy it on their geoboards and, in a table, record the measures of "a", "b", "h", and the area.

Trapezoid	a	b	h	area
D				

Next, using a different coloured rubber band, form a parallelogram on the trapezoid as shown above. Point out that this is equivalent to a rotation or turning of the original trapezoid. Tell the students to rotate their boards to see this.

Next, ask the students to find the measures of the base, height, and the area of the parallelogram and record the measures in a table.

Parallelogram	base	height	area
on D			

Compare the area of the original trapezoid to the area of the parallelogram ____(½)____.

Ask: How much longer is the measure of the base of the parallelogram compared with the base of the original trapezoid?

Record: longer by ____(1)____ units.

Ask: Is there a side in the original trapezoid with this measure? ____(side "a")____

Point out, on the transparent geoboard, that if we picture the original trapezoid being rotated, then the measure of "a" was added to the base "b", forming the base of the parallelogram which now has measure "a + b". Tell the students to write a + b above "base" in the above table.

Ask: Does the height of the trapezoid change? ____(No)____

Ask the students to recall, in words, the formula for a parallelogram. <u>(base x height)</u>

Next, ask the students to write a formula for the area of the <u>particular parallelogram</u> that was made on the trapezoid?

<u>$A = (a+b)\cdot h$</u>

Explain the formula by pointing out that the base of this particular parallelogram has measure "a+b".

Ask the students to look at the areas that they have recorded for the trapezoid and the parallelogram.

After pointing out that the area of the trapezoid was found to be ½ the area of the parallelogram, ask the students to write a formula for the area of a trapezoid.

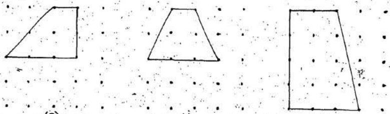
$A = \frac{1}{2}(a+b)\cdot h$

Discuss and review the above formula and the procedure that was used to develop it. Summarize by stating that any trapezoid can be considered to be ½ of a parallelogram that can be formed on the trapezoid. Since we can write the formula for that particular trapezoid as $(a+b)\cdot h$, then we can write the formula for the trapezoid as $\frac{1}{2}(a+b)\cdot h$.

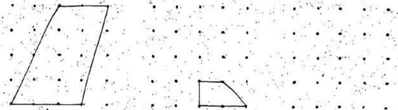
Next, distribute the student lesson sheets to the class. After they have completed the table let them answer the questions that follow. Then check the answers in a short discussion of the questions.

Place the solutions to all geoboard exercises on the transparent geoboard or allow the students to hold up their geoboards so that the solutions can be seen.

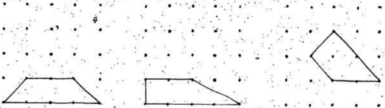
Solutions for Practice Exercises



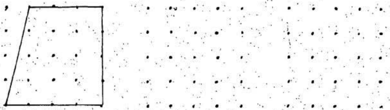
(a) (a) (b)



(b) (c)



(d) (d) (d)



(e)

Solutions for Think Exercise

(a) $A = \frac{1}{2}(a+b)\cdot h$
$= \frac{1}{2}(12+18)\cdot 7$
$= 105\ m^2$

$A = \frac{1}{2}(a+b)\cdot h$
$= \frac{1}{2}(200+120)\cdot 50$
$= 8{,}000\ cm^2$

(b) $a = 33 - 8 = 25$ mm
$h = 22$ mm
$b = 33$ mm

$A = \frac{1}{2}(a+b)\cdot h$
$= \frac{1}{2}(25+33)\cdot 22$
$= 638\ mm^2$

(c) $A = \frac{1}{2}(a+b)\cdot h$
$6 = \frac{1}{2}(a+3)\cdot 3$
$a = 1$

(Students will probably use different reasoning to solve (c))

(d) i $A = \frac{1}{2}(a+b)\cdot h$
$= \frac{1}{2}(12+8)\cdot 6$
$= \frac{1}{2}(20)\cdot 6$
$= 60\ m^2$

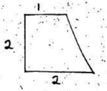
ii $A = \frac{1}{2}(a+b)\cdot h$
$= \frac{1}{2}(14+10)\cdot 7$
$= \frac{1}{2}(24)\cdot 7$
$= 84\ mm^2$

STUDENT LESSON SHEETS 8

Make the following trapezoids on your geoboard and record the measures of "a", "b", "h", and the area in the table. Then make a parallelogram on the trapezoids and complete the table for the parallelogram.

A a trapezoid having base 2 units, height 3 units, and the side parallel to the base 1 unit in measure.

B a trapezoid with the following shape and measures.



Trapezoid	Measure of a	Measure of b	Measure of h	Area	Parallelogram	Measure of base	Measure of height	Area
A					on A			
B								

How does the area of the trapezoid compare with the area of a parallelogram formed on the trapezoid? ________

____

How many pairs of parallel sides are there in a trapezoid? ________

What letter is used to represent the measure of the side parallel to the base of a trapezoid? ________

If a parallelogram were formed on a trapezoid that has parallel sides of measures "a" and "b", then what would be the measure of the base of that parallelogram?

____

What would be the formula for the area of a parallelogram formed on the trapezoid? ________

Write a formula for the area of the trapezoid.

____

Exercise

Make the following on your geoboard:

(a) a trapezoid with an area of 4 square units. What is the measure of a? ______ b? ______ h? ______

(b) two different trapezoids each having a = 2 units, b = 3 units, and h = 4 units. What is the area of each? ________

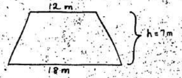
(c) the smallest possible trapezoid.
What is its area? ________

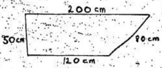
(d) three different trapezoids each having an area of 3 square units.

(e) the largest possible trapezoid.
What is its area? ________
What is the difference in length between the two parallel sides? ________

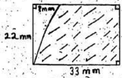
Think Exercise

(a) Find the area of the following trapezoid shaped regions.





(b) Find the area of the shaded region in the figure below.



(c) Find the measure of "a" if

b = 3 km

h = 3 km

area = 6 km^2

(d) Find the area of the following trapezoids given these measures:

i. a = 12 cm
b = 8 cm
h = 6 m

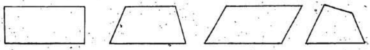
ii. a = 14 mm
b = 10 mm
h = 7 mm

APPENDIX B

ACHIEVEMENT POSTTEST AND RETENTION TEST

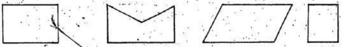
POSTTEST

1. Which of the following polygons is a rectangle?



A B C D

2. Which of the following is not a quadrilateral?



A B C D

3. Which of the following is the correct formula for finding the perimeter of a rectangle?

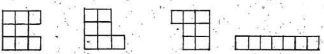
   A. $P = \ell + w$ B. $P = \ell \cdot w$

   C. $P = 2\cdot\ell + 2\cdot w$ D. $P = 2\cdot\ell$ x $2\cdot w$

4. The correct unit among the following for measuring area is

   A. cm B. cm^2 C. cm^3, D. none of these

5. Which region has the greatest area if each small square is a unit square?



A B C D

6. To say that the area of a square room is 9 square metres means that

A. the distance around the room is 9 m

B. the length of the room is 9 m

C. the length and width of the room are both 9 m

D. the room can be covered with 9 squares each having an area of 1 square metre

7. How many diagonals can be drawn in a rectangle?

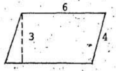
A. 0 B. 1 C. 2 D. 3

8. Which of the following is the correct formula for finding the area of a rectangle?

A. $A = \ell + w$ B. $A = \ell \cdot w$

C. $A = 2 \cdot \ell + 2 \cdot w$ D. $A = 2 \cdot \ell \times 2 \cdot w$

9. What is the height of the following parallelogram?



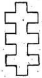
A. 6 B. 4 C. 3 D. 7

10. Mr. Smith wishes to fence in his backyard with a rope. To do this he would find

A. perimeter B. area

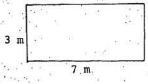
C. a diagonal D. none of these

11. Find the perimeter of the following figure if the measure of each straight segment is 1 unit.

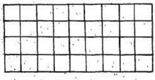


____

12. Mr. Jones has a rectangular shaped yard with dimensions as shown. How much rope would be needed to fence off the yard?



13. On the following grid, shade in a region that has an area of $7\frac{1}{2}$ square units.



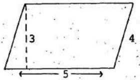
14. What is the area of a rectangle that has length 8 cm and width 3 cm.

15. A rectangular shaped room measures 5 m long and 4 m wide. If carpeting costs $14 per square metre, find the cost of carpeting the room.

16. If the area of a square shaped room is 64 cm^2, what is its perimeter?

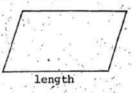
17. Write the formula for finding the area of a parallelogram.

18. Find the area of the following parallelogram.



____

19. Mr. Jones wishes to change the shape of his parallelogram shaped patio without changing the area or the length of the patio. Tell, in your own words, how he can do this.



____

____

____

____

20. Write the formula for finding the area of a triangle.

____

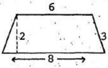
21. Find the area of a triangle that has base 8 cm and height 4 cm.

____

22. A rectangular shaped sign has an area of 48 cm^2 and a length of 12 cm. What is its width?

____

23. What are the values of a, b, and h in the following trapezoid?

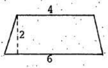


a = __________

b = __________

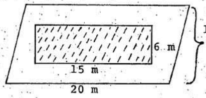
c = __________

24. Find the area of the following trapezoid.



____

25. Find the area of the unshaded region in the following diagram which has a shaded rectangle within a parallelogram.



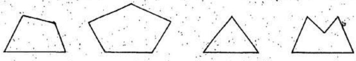
____

RETENTION TEST

1. How many of the following figures are rectangles?

A. 0 B. 1 C. 2 D. 3

2. Which of the following is a quadrilateral?



A B C D

3. Which of the following is the correct formula for finding the perimeter of a rectangle?

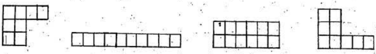
A. $P = 2\cdot\ell \times 2\cdot w$ B. $P = \ell\cdot w$

C. $P = \ell + w$ D. $P = 2\cdot\ell + 2\cdot w$

4. The correct unit among the following for measuring area is

A. m B. m^3 C. m^2 D. none of these

5. Which of the following has the smallest area if each small square is a unit square?



A B C D

6. To say that the area of a piece of cardboard is 15 square centimetres means that

   A. the cardboard can be covered with 15 squares each having an area of 1 square centimetre

   B. the length of the cardboard must be 15 cm

   C. the distance around the cardboard must be 15 cm

   D. the length and width of the cardboard are both 15 cm

7. How many diagonals can be drawn in a parallelogram?

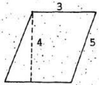
   A. 0 B. 4 C. 1 D. 2

8. Which is the correct formula for finding the area of a rectangle?

   A. $A = \ell + w$ B. $A = 2 \cdot \ell + 2 \cdot w$

   C. $A = 2 \cdot \ell \times 2 \cdot w$ D. $A = \ell \cdot w$

9. What is the height of the following parallelogram?

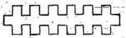


______

10. Mr. Jones wishes to put a railing around his swimming pool. To do this, he would need to find

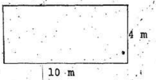
    A. area B. a diagonal C. perimeter D. none of these

11. Find the perimeter of the following figure if the measure of each straight segment is 1 unit.



______

12. Mr. Smith has a rectangular shaped yard with dimensions as shown. How much rope would be needed to fence off the yard?



____

13. On the following grid, shade in a triangle that has an area of 4½ square units.

14. What is the area of a rectangle that has a length of 12 m and a width of 9 m?

____

15. A rectangular shaped lawn measures 20 m long and 12 m wide. If large sods cost $2 per square metre, find the cost of sodding the lawn.

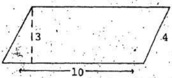
____

16. If the area of a square shaped room is 36 m^2, what is its perimeter?

____

17. What is the formula for finding the area of a parallelogram?

18. Find the area of the following parallelogram.



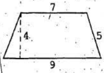
19. Mr. Smith wishes to change the shape of his rectangular shaped patio without changing the area or the length of the patio. Tell, in your own words, how he can do this.

20. Write the formula for finding the area of a triangle.

21. Find the area of a triangle that has base 12 cm and height 6 cm.

22. A rectangular shaped wall has an area of 72 m^2 and a width of 6 m. What is its length?

23. What are the values of a, b, and h in the following trapezoid?

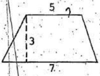


a = ________

b = ________

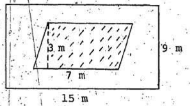
h = ________

24. Find the area of the following trapezoid.



____

25. Find the area of the unshaded region in the following diagram which has a shaded parallelogram within a rectangle.



APPENDIX C

RAW SCORES FOR THE CANADIAN TESTS OF BASIC SKILLS, THE POSTTEST, AND THE RETENTION TEST

Enactive Treatment

Ability	Student	CTBS	Posttest	Retention test
	1	20	14	9
	2	20	12	11
	3	24	11	4
	4	26	16	10
	5	26	17	9
	6	26	10	10
Low	7	28	18	9
	8	29	20	10
	9	30	18	15
	10	21	11	8
	11	26	6	9
	12	26	12	9
	13	26	16	16
	14	29	19	17
	15	32	20	11
	16	33	10	10
	17	33	19	21
	18	34	11	6
	19	34	15	7
	20	36	16	8
	21	36	17	12
Middle	22	37	16	8
	23	31	15	14
	24	31	8	5
	25	33	17	15
	26	33	17	19
	27	34	16	11
	28	34	10	9
	29	34	16	13

Ability	Student	CTBS	Posttest	Retention test
	30	35	14	9
Middle	31	36	16	14
	32	37	12	11
	33	40	21	17
	34	47	20	14
	35	41	21	17
	36	44	11	8
High	37	45	22	21
	38	47	21	23
	39	48	24	23
	40	50	23	21

Iconic Treatment

Ability	Student	CTBS	Posttest	Retention test
	1	20	23	13
	2	22	10	7
	3	22	13	10
	4	23	22	19
	5	23	15	10
	6	23	21	18
	7	23	19	15
	8	24	17	11
	9	24	20	11
	10	25	23	21
Low	11	26	19	20
	12	27	15	10
	13	27	19	14
	14	27	16	10
	15	28	20	11
	16	29	16	11
	17	30	22	9
	18	30	21	19
	19	21	20	16
	20	23	19	20
	21	23	24	10
	22	26	18	17
	23	31	15	12
	24	31	25	23
	25	32	24	13
Middle	26	32	22	22
	27	33	22	18
	28	33	21	17
	29	33	19	15

Ability	Student	CTBS	Posttest	Retention test
	30	34	21	23
	31	34	24	20
	32	34	24	21
	33	37	22	19
	34	37	21	19
Middle	35	32	19	18
	36	32	16	18
	37	33	23	18
	38	33	24	19
	39	34	20	20
	40	35	23	23
	41	38	20	18
	42	38	23	23
	43	40	21	14
	44	41	24	24
	45	42	21	20
	46	49	24	22
	47	51	23	22
	48	61	25	24
	49	38	23	22
High	50	38	23	21
	51	39	23	19
	52	41	24	19
	53	42	23	24
	54	43	22	22
	55	44	25	24
	56	44	21	21
	57	45	25	23
	58	46	20	17
	59	49	24	22
	60	49	24	23

Ability	Student	CTBS	Posttest	Retention test
	61	51	24	21
High	62	53	23	22
	63	57	25	24