The Principle Of Excluded Middle Then And Now: Aristotle And *Principia Mathematica*

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There is a very long tradition from the fourth century B.C. to the nineteenth century, in which the logic of Aristotle was studied, commented on, criticized at times, though never dethroned, the logic which dominated western thought until the twentieth century. It is this we call "Aristotelian Logic". That logic regarded as a science is appropriately defined by Kant in these words: "Logic is ... a science *a priori* of the necessary laws of thinking, not, however, in respect of particular objects but all objects in general: it is a science, therefore, of the right use of the understanding and of reason as such, not subjectively, i.e. not according to empirical (psychological) principles of how the understanding thinks, but objectively, i.e. according to *a priori* principles of how it ought to think."[1]

In the development of Aristotelian logic, these necessary laws of thought are derived by reference to thought itself as it expresses being and non-being. There are three very general principles[2] commonly called the Laws (or Principles) of Thought which specify what it is to think of something scientifically: the Principle of Identity, which requires that the object must be thought as having an immutable nature (A is A); the Principle of Contradiction, where it cannot be thought as at once having a certain character and not having that character (A cannot be B and not B); and the Principle of Excluded Middle, where it either has that character or property or does not have it (A is either B or not B). The fuller implications of these three principles will be treated below, where their logic is contrasted with the significance of these principles for modern truth-functional logic.

[1] . Immanuel Kant, *Logic* (trans. Robert Hartman and Wolfgang Schwarz), Indianapolis, 1974, 18.

[2] .A fourth principle, the Principle of Sufficient Reason, is added by speculative philosophers such as Leibniz, Hegel, Heidegger. But we shall remain silent about it.

Modern logic was given its classical formulation in *Principia Mathematica*.[3] In that work the Principle of Identity (p>p) appears as Th.2.08, the Law of Contradiction -(p.-p) as Th. 3.24, the Law of Excluded Middle (pv-p) as Th. 2.11.[4] Although these are distinct theorems, the three principles of Aristotelian Logic are clearly only interdefinitions in Classical logic, for

(p>p) = (pv-p) = -(p.-p).

In short, they collapse into one another. As theorems moreover they themselves become elements of proofs of subsequent theorems.

Aristotle gives these principles distinct interpretations.[5] Of the Principle of Contradiction he says, "A principle which one must have if he is to *understand* anything is not an hypothesis; and that which one must know if he is to know anything must be in his possession for every occasion. Clearly, then, such a principle is the most certain of all; and what this principle is we proceed to state. It is: `The same thing cannot at the same time both belong and not belong to the same object and in the same respect.'" The Principle of Excluded Middle is not derived from the Principle of Contradiction; it follows rather from the definition of what is truth and falsehood: "What is more, there cannot be anything between two contradictories, but of any one subject, one thing must either be asserted or denied. This is clear if we first define what is truth and what is falsehood. A falsity is a statement of that which is that it is not, or of that which is not that it is; and a truth is a statement of that which is that it is, or of that which is not that it is not. Hence, he who states of anything that it is, or that it is not, will either speak truly or speak falsely. But of what is neither being nor nonbeing it is not said that it is or that it is not."[6] How do these two principles differ? One way of expressing the difference, Aristotle's way, is this: the Principle of Contradiction assures that not everything is true; the Principle of Excluded Middle that not everything is false. In his words:

> The statement of Heraclitus, that everything is and is not, seems to make everything true, but that of Anaxagoras, that an intermediate exists between two contradictories, makes everything false; for when things are

[3] .Alfred North Whitehead and Bertrand Russell, Cambridge, 1910. In this century the logic of *Principia Mathematica* [henceforth PM] has so succeeded that it is now called "Classical logic", and so it shall be called here.

[4] .Russell calls these theorems by these names. He states that the Principle of Identity given there is not the same as the `law of identity', which is inferred from the Principle later in the work. See *Principia Mathematica to *56*, Cambridge, 1967, pp. 99, 101, 111.

[5] But he does not name them Contradiction, Identity, Excluded Middle. That is a later addition. 6. *Metaphy*. IV, 1005b15-20, trans. Hippocrates G. Apostle, Bloomington, Ind., 1966, 58.

[6] *Ibid.,* 1011b23-29.

> blended, the blend is neither good nor not-good, so that it is not possible to say anything truly.[7]

The Principle of Identity is not derived from either of the others, but from Aristotle's reflections on the unity and being of substance: "To ask why something is itself is to inquire into nothing, for the fact or the existence of something must be clear. Thus, *the fact that something is itself*, this is the one answer, and the one cause in all cases, as, for example, in the questions `Why is a man a man?' and `Why is the musical musical?', unless one were to answer that each thing is indivisible from itself, since to be one for each thing is to be indivisible from itself. But this [that a thing is itself] is common to all things and a short answer for all of them."[8] The Principle of Identity is not as it might appear the abstract reiteration of a term as subject and predicate. In its propositional form, as Hegel observes, there is promise of a distinction between subject and predicate, as well as identity.[9] A true appreciation of this principle is found in Leibniz, who holds that all truth is identity, but not an empty repetition stripped of difference. The Principle of Identity is rather the assertion of the unity of what is different.

These three principles in the long tradition of Aristotelian logic are distinct from one another, serving as regulative principles of the understanding rather than as rules of inference or elements of proofs. In these two respects they stand opposed to their interpretation in Classical logic. There is another fundamental difference between the two interpretations: Aristotelian logic holds these principles as self-evident.[10] Classical logic regards them as tautological elements of a formal system which itself is at most a theory of ratiocination. This requires elaboration for the sake of the subsequent argument.

The formal system of PM regarded as a `theory' of logic[11] is not to be judged true or false, but more or less adequate to that which it is devised to order and systematize. The elements of the formal system are given `interpretations' which, ideally, make the axioms turn out true in domains of discourse to which the formal system is applied. But as with theorems in general, the proof of the pudding is in the eating, in the deductions made from the theory and their agreement with experience, in the case of PM in the theorems

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7 *Ibid*., 1012a25-29.

8 *Ibid*. VII, 1041a15-20

9 *Encyclopedia Logic, n.*115.

10 Aristotle states emphatically, for example, that though the Principle of Contradiction is the most certain of all, it cannot be demonstrated. But the position of one who says that it is possible for the same thing to be and not to be [to deny then the Principle of Contradiction] can be refuted "if only our opponent says something." Aristotle produces, in *Metaphy*. IV, ch.4 (1006a29 - 1009a5) at least seven "demonstrations by refutation", as he calls them, to show that he who denies the Principle of Contradiction must assume it to effect his denial.

11 This position was early expressed by Ernst Nagel in "Logic without Ontology", in *Naturalism and the Human Spirit*, ed. J.H.Krikorian, New York, 1944.

deducible from the axioms and their applicability to logical interests. The three principles in question as formulated in the Formal System PM are simply valid-within-the-system. We shall see that the Principle of Excluded Middle as formulated in Th. 2.11 of PM, (pv-p), presents difficulties when interpreted as a logical principle with absolute validity.

A. Difficulties With The Principle Of Excluded Middle Of Classical Logic, (Pv-P)

There are deviations from Classical logic for various reasons, some relatively innocuous, others so fundamental they amount, in Quine's expression, to "changing the subject."[12] Among the innocuous `deviant logics' are many-valued logics, where propositions are not simply divided into `true' and `false', but, say, `true', `false' and `possible'. Such logics obviously deny the Principle of Excluded Middle, (pv-p), of Classical logic, but still use the two-valued logic as their paradigm. In Intuitionist logic there occurs a fundamental deviation.

Let us recall the history of the birth of Classical logic, and the reasons for the Intuitionist deviation from it. With the production of non-Euclidean geometries in the nineteenth century, there was a perceived crisis in the foundations of mathematics. The consistency of rival geometries showed that axioms and postulates were not self-evident truths, but were themselves to be judged within axiom systems for consistency, completeness, independence, but not for truth. Mathematics then required a more rigorous foundation than simple conformity to our intuitions of counting and measuring. Moreover, there was great interest in the counter-intuitional results of Cantor concerning the comparison and ordering of infinite collections,[13] the hierarchy from those that are countable (as the natural numbers) to those that are uncountable (the real numbers). In his development of abstract set theory, Cantor produced the ascending series of transfinite cardinals, results which were as exciting as they were mind-bending. But just as his work was winning general acceptance, contradictions and paradoxes began to appear: the Burali-Forti paradox in 1897, Russell's paradox (concerning the set of all sets that are not members of themselves), and Cantor's own paradox in 1899. There was pressing need, therefore, to secure mathematics on a firm foundation.

Mathematicians of the latter part of the century set about to secure these foundations in two ways: (1) to derive mathematics from a logical system itself more fundamental than mathematics [Frege in his *Begriffsschrift* and *Grundgesetze*, Russell in PM, the `logicists'

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[12] W.O.Quine, *Philosophy of Logic*, Cambridge MA, 1970, 81: "Here, evidently, is the deviant logician's predicament: when he tries to deny the doctrine he only changes the subject."

[13] Galileo and others anticipated the idea, first formulated by Dedekind in 1888, that an infinite set is one that can be put in 1-1 correspondence with a proper subset of itself. Cf. W. and M. Kneale, *The Development of Logic*, Oxford, 1966, 440. Stephen Kleene, *Mathematical Logic*, NewYork, 1967, 176-7, adds: "In 1638, Galileo noted the `paradox' that the squares of the positive integers can be place in 1-1 correspondence with all the positive integers, contrary to the axiom of Euclid that the whole is greater than any of its parts."

in short]; (2) to formulate mathematics as a formal axiomatic system, and prove the system consistent, that is, free from contradiction by finitist methods[14] [Hilbert and his followers, the `formalists' in short]. Both programs failed, the logicist program because of the discovery of the `antinomies' resulting from self-referentiality[15], the formalist program because of Godel's incompleteness proofs.[16]

`Intuitionism' rose out of the ashes of the destructive effect of the `antinomies', and found new impetus in the results of Godel. Brouwer, its early twentieth century leader, maintained that the need for a Fregean logical foundation or rigorous axiomatization only appeared because mathematics had extended itself beyond its limits. In a paper "The untrustworthiness of the principles of logic" (1908), he criticized the unexamined use of the laws of Classical logic, in particular the Law of Excluded Middle, (p v -p). According it absolute validity in cases extending to all natural numbers, for example, leads to results unacceptable to Intuitionists. For consider, say, the proposition p: "There is an uninterrupted run of 1000 nines in the decimal expansion of B." Such a proposition is virtually undecidable, for though one might very improbably find such a run and hence affirm the proposition as true, there is no way to prove the proposition false, the infinite sequence of digits not being exhaustible. The intuitionist rejects the application of (pv-p) where infinite sequences or an infinite set is in question. Thus, for him the principle does not have universal validity.

Of course the intuitionist rejection of the principle (pv-p) cannot be done without discarding other elements of the logic of PM, (p>p), -(p.-p) [given their equivalence to (pv-p)], and also one half of the law of double negation, (p=--p).[17] Rejecting (pv-p),

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[14] That is, procedures that do not involve the conception of the completed infinite.

[15] There are, of courses, the various devices (such as Russell's `theory of logical types') for avoiding the production of the known antinomies without forsaking general set theory . But these are stop-gap measures. As Quine [*The Ways of Paradox*, Cambridge, Mass., 1976, 16] aptly observes: "We cannot simply withhold each antinomy-producing membership condition and assume classes corresponding to the rest. The trouble is that there are membership conditions corresponding to each of which, by itself, we can innocuously assume a class, and yet these classes together can yield a contradiction. ... I remarked earlier that the discovery of antinomy is a crisis in the evolution of thought. In general set theory the crisis began sixty years ago [written in 1961] and is not yet over." Nor is it over today.

[16] What Godel proved in 1931 was that no deductive system, whatever its axioms, is able theoretically to prove all the truths of elementary number theory. He did this by constructing a sentence in elementary number theory which is true if and only if it is not a theorem of number theory, the complete analogue, in layman's terms, of "I am unprovable." As Stephen Kleene explains, "Godel's sentence `I am unprovable' is not paradoxical. We escape paradox because (whatever Hilbert may have hoped) there is no a priori reason why every true sentence must be provable." ["The Work of Kurt Godel", *The Journal of Symbolic Logic*, 41 (1976), in S.G. Shanker ed, *Godel's Theorem in Focus*, London, 1988, 54.] Godel showed that any number-theoretic formal system, consisting of a logic to which is added number-theoretic axioms, is incomplete if it is consistent. The logical part, the formal system PM, has been proven complete. Thus, the number-theoretic axioms must ever be incomplete, and with this result Hilbert's program is demolished.

[17] Intuitionists must reject --p>p (hence p=--p), for if p=--p, then --p>p immediately reduces to p>p (i.e. to pv-p).

intuitionist logic must also reject *reductio ad absurdum*, which relies in one of its steps on (pv-p).[18] The logic that remains lacks the simplicity, convenience and familiarity of the logic of PM, and the arsenal of tools familiar to the mathematician (*reductio ad absurdum* and mathematical induction among them) is considerably reduced. The system that is produced can be construed as a fragment of the logic of PM, having PM as its only complete enlargement.[19] Obscured perhaps in the logic which is left is the intuitionist's demand for constructive proofs for mathematical objects -- he is intolerant of arguments which purport to produce mathematical objects simply by showing the falsity of the assumption of the non-existence of such objects. For most mathematicians the price the intuitionist would exact is too high, even as they grant the cogency of his reasons , and themselves recognize a constructive proof as sounder than that which has been demonstrated non-constructively.

Although Brouwer's `intuitionism' antedates Godel's incompleteness theorems, the position was strengthened by Godel's results which demonstrated that there are formally undecidable propositions of PM and related systems. In Godel's own words, "...it can be proven rigorously that in *every* consistent formal system that contains a certain amount of finitary number theory there exist undecidable arithmetic propositions and that, moreover, the consistency of any such system cannot be proved in the system."[20] Quine, notwithstanding his complete loyalty to Classical logic, is moved to say, "The excess of admitted questions over possible answers seems especially regrettable when the questions are mathematical and the answers mathematically impossible."[21]

If there is dispute about the propriety or justification of a logical principle such as pv-p, how might one settle the dispute? Intuitionists reject the principle in question, whereas present-day mathematicians, who tend to describe themselves as Platonists or realists,[22] accept the principle, though sometimes with reservations:

> There are also differences of viewpoint concerning the lengths to which one may be prepared to carry one's Platonism - if, indeed one claims to be a Platonist. ...When all the ramifications of set theory are considered, one comes across sets which are so wildly enormous and nebulously constructed that even a fairly determined Platonist such as myself may

[18] A *reductio ad absurdum* proof of, say, t runs this way: assume -t. Derive a contradi ction q.-q. Then -t > (q.-q), which is absurd. Thus, because tv-t, then t.

[19] As A. Tarski proved. Cf. Kneale and Kneale, 574. But to construe it as a `fragment of PM' is to misconstrue it, for intuitionist logic intends its results to be radically different from PM.

[20] Kurt Godel, "On Formally Undecidable Propositions of *Principia Mathematica* and Related Systems I" (1931) in .Shanker, 40-1.

[21] *Philosophy of Logic*, 87.

[22] `Realists' because they hold that mathematical conjectures are true or false prior to and independent of the proofs whereby they are established; thus they cannot go so far as intuitionists in rejecting some form of `either p or not p'.

> begin to have doubts about their existence ... There may come a stage at which the sets have such convoluted and conceptually dubious definitions that the question of the truth or falsity of mathematical statements concerning them may begin to take on a somewhat `matter-of-opinion' quality rather than a `God-given' one.[23]

Mathematicians of various stripes have difficulties, it would seem, with the principle pv-p. If they will not go so far as the intuitionists in a radical solution to these difficulties, they must nonetheless find them annoying and, as Quine might put it, `regrettable'.

B. The Principle Of Excluded Middle In Aristotelian Logic

There is confusion about Aristotle's understanding of the Principle of Excluded Middle originating, we shall argue, in an inadequacy of Classical logic -- it is a blunt instrument -- to express the Aristotelian position. The chief criticism of the Aristotelian account and confusion about it centre on Aristotle's reservations about the principle applied to future contingents in *De Interpretatione*, ch.9. An analysis of the argument there and in ch. 7 of that work will show Aristotle's account is clear and unambiguous, and at the same time reveal what is inadequate if the argument is approached under the paradigm of Classical logic.

Aristotle begins *De.Interp.*, ch. 9, with a statement of what is called the Principle of Bivalence,[24] although here limited to what is or what has taken place: "With regard to things present or past propositions whether positive or negative are true of necessity or false." [18a28-9] As shown in Aristotle's account of `opposition', affirmative/negative universal propositions are opposed as contradictories to negative/affirmative particular propositions, and there one contradictory must be true and the other false.[25] But two universal propositions, one affirmative and the other negative are opposed as contraries; it is impossible that both propositions are true, though both might be false.[26] Aristotle is at pains here to distinguish two sorts of negations: any universal proposition can be the

[23] Roger Penrose, *The Emperor's New Mind*, Oxford, 1989, 113.

[24] Jan Lukasiewicz, *Aristotle's Syllogistic*, Oxford, 1951, 82: "...the so-called principle of bivalence which state that every proposition is either true or false, i.e. that it has one and only one of two possible truth values: truth and falsity." He continues: "This principle must not be mixed up with the law of excluded middle, according to which of two contradictory propositions one must be true."

[25] As "Every man is white" and "Not every man is white" ["Some man is not white."]

[26] As "Every man is white" and "No man is white", ch.7, 17b16-22. Subcontrariety is not an `opposition' in Aristotle's sense: "The particular affirmative and particular negative do not have opposition properly speaking, because opposition is concerned with the same subject." St. Thomas Aquinas and Cajetan, *Comm. On Interpretation*, trans. Jean T. Oesterle, Milwaukee, 1962, 90. The subjects of particular propositions having the same subject term are indeterminate for any singular thing, thus may or may not intersect.

negation of another as its *contrary* (All S is P; All S is nonP), and here they can be false at the same time; or as its *contradictory*(All S is P; Some S is nonP), and here one must be true and the other false. Therefore, "Of two opposites it is not the case always that one must be true and the other false." In Classical logic there is only the one form of opposition, propositional negation. This marks a fundamental difference between the two logics. The Classical logician, blind to contrariety, interprets what he reads in Aristotle solely (and therefore inadequately) as propositional negation. [27]

"Socrates is not white", an example of a singular proposition, is the proper negation of "Socrates is white", and here too, as with contradictory propositions, one must be true and the other false because such propositions regarding things present (or past) are determinate. But not so with opposed singular propositions about future contingent matters. It is this case which is the subject matter of Ch. 9, and the question is whether in singular propositions about future contingencies, propositions such as "There will be a sea battle tomorrow", "There won't be a sea battle tomorrow", it is necessary that one of the opposites be true and the other false.

The problem is set in a metaphysical context of contingency and necessity. Does the analysis of truth and falsity in propositions and being and non-being in things actual and past imply a fatalistic necessity of being and non-being in future things? If what has been said should suggest that *per impossible* all events come about of necessity, then we must subject the Principle of Excluded Middle to further scrutiny. The Philosopher's gaze will be directed toward singular propositions concerning that which may or may not come to pass in the future. But why singular propositions, such as `Socrates will be executed tomorrow', and not universal propositions, `All living things will die' for example? Singular propositions pertaining to what will or will not be can, some of them at least, be said to be contingent, whereas universal propositions as universal have their predicates necessarily in their subjects, are "big with the future" as one might say, as death is there given in the being of the living thing.[28] Why again only propositions about the future? What is different about that which is past or present? There is nothing of contingency in what has been, nor in what is actual. The actual as actual is beyond the contingency of what may or may not be - it is realized possibility.

Aristotle states the dimensions of the problem in this way:

> For if every affirmation or negation is true or false [Principle of Excluded Middle] it is necessary for everything either to be the case or not to be the case. For if one person says that something will be and another denies this

[27] See Fred Sommers, *The Logic of Natural Language*. Oxford, 1982, viii, Chaps.13 and 14, and Appendix B. Sommers' book was largely inspired by the difference between these two forms of opposition and the one form in Classical logic.

[28] Aquinas says, "...those things that take place contingently pertain exclusively to singulars, whereas those that per se belong or are repugnant are attributed to singulars according to the notions of their universals." *Comm. on Interp.*, 104. Thus `Socrates will die' is determinately true, having nothing of contingency in it even though of the future.

> same thing, it is clearly necessary for one of them to be saying what is true -- if every affirmative is true or false [Principle of Bivalence]; for both will not be the case together under such circumstances. ... it follows that nothing is or is happening, or will be or will not be, by chance or as chance has it, but everything of necessity and not as chance has it (since either he who says or he who denies is saying what is true.)[29]

The argument of Chap. 9 shows that what follows from the assumption that one of opposites must be true and the other false is untenable because impossible. Then (at 19a23) Aristotle proceeds most directly to express the distinctions required to state the truth of the matter.

There are different grades of contingency that are threatened if opposed propositions about future singular things must be one true and one false, and a different analysis is appropriate to each of them. As long as something will be in the future, it will be there in one way or another in its cause, determinately in some cases and therefore necessarily (`Socrates will die'), as an inclination in other cases but such that the cause could be impeded (`Socrates will be executed tomorrow'), lastly as a potency purely (`The cat will catch the mouse.') [30] For our purposes the second and third cases yield different and perhaps unexpected results.

When two opposed propositions speak of future contingencies, as in Aristotle's example "There will be a sea-battle tomorrow", "There won't be a sea-battle tomorrow", even if there is a strong inclination toward the former -- the ships on both sides assembled, the conflict between the warring parties extreme, the weather propitious, Aristotle insists that we must reject the conclusion that either he who says there will be such a battle speaks truly or he who says there won't be speaks truly. There is nothing determinate to make one or other of those pronouncements true.[31] But if neither proposition is true, then this proposition is also not true (given that it says one of the disjuncts is true):

> (1) `Either the sea-battle will take place tomorrow or the sea-battle won't..[32]

Because it is impossible for the same thing to be and not to be at the same time, what is true is necessarily true *when* it is true, but not before it is true.[33] But, adds

[29] 18a34- b8, Aristotle's *Categories and De Interpretatione*, trans. J.LAckrill, Oxford, 1963, 50.

[30] Aquinas, *Comm.on Interp*.. 107.

[31] Cf. 18a35 - b8 for the *reductio ad impossibile* of the position.

[32] A proper symbolization of this proposition would not be the disjunction (pv-p) of Classical logic, but `a is P or a is not-P'. See Sommers, 308-9.

[33] "What is, necessarily is, when it is; and what is not, necessarily is not, when it is not." 19a23, Ackrill, 52.

Aristotle, "it is not possible to say neither is true; that is, to say that a thing neither will take place nor will not take place." (18b17) Thus, this proposition is not acceptable:

> (2) `The sea-battle neither will take place nor will not take place.'

That proposition also illicitly says something determinate -- it asserts that both disjuncts are false.

Is there a proper assertion falling under the Principle of Excluded Middle in this case? Clearly (pv-p) won't do, for it asserts the truth of one of the disjuncts and would thus fall under the same objections as (1). One could say, with Aristotle:

> Clearly then it is not necessary that of every affirmative and opposite negation one should be true and the other false. For what holds for things that are does not hold for things that are not but may possibly be or not be; with these it is as we have said. (19a39) [34]

and that is as explicit a denial of the Principle of Excluded Middle, at least as given in Classical logic, as one could find. Furthermore, it should be clear that Aristotle is here denying the truth or falsity of

> (3) The sea-battle will take place tomorrow.
>
> (4) The sea-battle won't take place tomorrow.

Thus, in the argument of Chap. 9, it must be said that propositions are not always true or false, a denial of the universal applicability of the Principle of Bivalence.

It is therefore not a little strange to read Kneale's analysis of Chap. 9: he says that what Aristotle is apparently doing is questioning the Principle of Bivalence while accepting the Principle of Excluded Middle. Interpolating Aristotle's words, Kneale concludes:

> For while he asserts that `everything must either be or not be, or about to be or not be [19a27-30], he also says `It is not necessary that of every affirmation and denial of opposed statements one should be true and the other false. For in the case of that which exists potentially but not actually the rule which applies to that which exists actually does not hold good.'[19a39-b4]. This appears to mean that the disjunction of a statement

[34] There are other assertions of some apparent form of Excluded Middle, at 19a30 for example: "I mean, for example, it is necessary for there to be or not to be a sea-battle tomorrow; but it is not necessary for the sea battle to take place tomorrow, nor for one not to take place..." Ackrill, 53. Sommers, 308-9, suggests what he calls a categorial principle, `A sea-battle will-or-won't take place tomorrow." This has the merit of expressing the potentiality of the situation today appropriately, and at the same time one is not led to the conclusion rejected most forcefully by Aristotle that one or other disjunct is actually true.

> and its negation can be true without either the original statement of its negation being true. In other words, Aristotle is trying to assert the Law of Excluded Middle while denying the Principle of Bivalence. We have already seen this is a mistake..[35]

The Principle of Excluded Middle that Kneale thinks Aristotle is asserting is the notorious (pv-p) of Classical logic. But Aristotle is questioning both Bivalence and Excluded Middle as the argument above has shown, though neither in the form (pv-p), to which Kneale reduces both in his argument. In trying to understand these passages Kneale is an unwitting slave to an inappropriate principle . His "solution" (that Aristotle should recognize that the sentence `There will be a sea-battle tomorrow' expresses the same proposition as the sentence `There is a sea-battle' uttered tomorrow) shows that he interprets each from the simplistic principle (pv-p) of Classical logic, a principle completely inadequate to the discussion, and missing Aristotle's difficulties entirely. Kneale's so-called "solution" is Aristotle's problem: if today a proposition about tomorrow's contingency is true, how can this be without falling into the untoward consequences Aristotle has put before us. Storrs McCall offers a `proof' similar to Kneale's which begins: "What Aristotle seems not to have noticed is that the two doctrines, that `p' is true if and only if p, and that some propositions are neither true nor false are incompatible."[36] But Tarski's criterion of truth, " p is true if and only if p", differs radically from Aristotle's "to say of what is that it is not, or of what is not that it is, is false; while to say of what is that it is, or of what is not that it is not, is true." [*Metaphy.*1110b26-28] Tarski's criterion is sentential (p is a sentence), `Platonist' in intention (sentences are true and false apart from any "saying" of them), and properly applied as Tarski himself shows in formal, not natural language. Aristotle's definition is appropriate to the categorical proposition (Saying of all or some S that is P that it is P), with safeguards against the radical Platonist/realist implications of Tarski's definition, and properly applied in natural language.

Peter Geach gives a completely bogus reading of Aristotle's statement "It is not possible to say neither is true; that is to say, that a thing neither will take place nor will not take place," (the passage analyzed above). In a series of steps acceptable only to Classical logic, he reduces `Not: neither F nor not-F' to `Not:not:either F or not-F', then by Double Negation to `Either F or not-F' -- the Classical Law of Excluded Middle. He writes:

> People have tried to maintain (sometimes appealing to three-valued logic) that of a pair of contradictory predictions relating to a future continency neither need be true. (Sometimes they say that neither need be *determinately* true; but this qualification, though it may make their doctrine easier to swallow, is quite devoid of sense.) Oddly enough, they claim as precedent the famous chapter ix of Aristotle's *De Interpretatione*.

[35] Kneale and Kneale, 48, underlining mine.

[36] McCall in "A Non-Classical Theory of Truth", *American Philosophical Quarterly*, Vol. 7 (1979), 83-86.

> In fact, Aristotle expressly rejects the idea of such a breakdown of our Law (*op.cit*. 18b,18-20). Moreover, he supplies a strong argument against the idea. What is now true to say that a thing will be, it will be true to say that it is or has been; so, if it is now true to say of Jones that he is neither going to be hanged tomorrow nor not, then tomorrow it will be true to say of him that he neither has been hanged nor has not; and this sort of result, Aristotle says, is absurd (*op.cit*. 18b,22-25)[37]

As the argument above shows, Aristotle in that passage stands against asserting any indeterminate proposition as true. Far from "expressly rejecting the breakdown of our Law", he is not accepting a Principle of Excluded Middle in any of its guises when it purports to determine the indeterminate. One could hardly have read Aristotle in a more bizarre manner.[38]

To complete our analysis of Chap.9, there is still the case to be considered where a future contingent is in its cause as a bare potency. Aristotle gives this example: "Thus, this coat may be cut in two halves; yet it may not be cut in two halves. It may wear out before it is cut." If it should happen that the coat is destroyed this day by fire, then the two propositions `This coat will be cut in half' and `This coat will not be cut in half' are both false, there being no coat. As Aristotle explains in *Categories* x

> The statement that `Socrates is ill' is the contrary of `Socrates is well'. Yet we cannot maintain even here that one statement must always be true and the other must always be false. For, if Socrates really exists, one is true and the other false. But if Socrates does not exist, both the one and the other are false.[39]

Where the subject of a proposition is vacuous, then it and its negation are related as contraries, there being only the opposition of the predicate, not contradictories, and both propositions are false. This would obtain for all propositions with vacuous `definite descriptions', for example the hackneyed `The present king of France is bald.' Sommers would extend this analysis also to propositions where predication is a category mistake.

The reservations Aristotle has and distinctions he makes about his own Principle of Excluded Middle (and Bivalence) are grounded in his metaphysics. We have grown accustomed to another logic and narrower metaphysics; and so we must stretch our understanding to be equal to these profound arguments. For Aristotle logical propositions are related to truth and falsity as things to being and non-being. When things are

[37] "The Law of Excluded Middle" in Peter Geach, *Logic Matters*, Berkeley, 1972, 81.

[38] Sommers, 310, says, "Not one of the distinctions needed for an understanding of chapter 9 is available to the interpreter who comes to it with a Fregean organon."

[39] 13b15-18, trans. Harold P. Cook, London (Loeb), 1962, 95.

indeterminate either because they are not actual or not completely given in their causes, propositions about them must be similarly indefinite. This is ever the case in things that neither always are (necessary being) nor always are not (impossibilities), but sometimes are and sometimes are not. There are, no doubt, eternal truths in the temporal world, but not all truths are eternal. And so it is not necessary that of every affirmation and its negation, one must be true and the other false.

C. Concluding Reflections On Interpreting Philosophy

Classical logic has been described as a "blunt instrument" in dissecting Aristotle's argument: the subtleties required are just not available to it. There is something likewise less than satisfactory about the application of Classical logic to mathematics, a certain uniformity and bluntness there too which the Intuitionist rejects. There are other cases not mentioned in this paper, where the generic character of Classical logic, its homogenous `one-size-fits-all' approach becomes intolerable. Heisenberg's `uncertainty principle', fundamental to the structure of quantum mechanics, cannot be accommodated to Classical logic; nor, it should be added, could Descartes, Leibniz, Kant, Hegel, to mention only some from the history of modern philosophy.

Classical logic, with its Principles of Double Negation and Excluded Middle, is not without its metaphysics, changing, developing, evolving, from its earliest statements in Frege and Russell, expressed in Wittgenstein's *Tractatus*, through the analytical philosophies that give priority to language, to more recent analysts who give priority to the structure of thought expressed in language. What do these philosophies which take Classical logic for granted have in common, and what, if anything, do those other philosophical positions share in their common opposition to the prevailing logic? Except for obvious differences which grow out of matters we have addressed, these questions are beyond the scope of this paper.[40]

It is clear that those who subscribe to the Classical position on Bivalence and Excluded Middle, do express some form of `realism' [Dummett's characterization]: there is for them a "world" which is what it is whether we know it or not; and true propositions are simply those that express what is the case with the world. The world might be completely unknown, many of its "truths" unknowable in principle, as the mathematical `realists' (or `Platonists' as they prefer to call themselves) would grant; nonetheless "truths" about the "world" would subsist, "absolute, external and eternal."[41] How have we come to this again as a viable philosophical position, after the whole history of modern philosophy which

[40] These questions are addressed in Michael Dummett's book, *The Logical Basis of Metaphysics*, Cambridge MA, 1990. He calls the two groups `realists' and `anti-realists'. `Realists', he says, share a common doctrine, which cannot be said of the `anti-realists'. What unites them in their opposition to the `realists' is simply their rejection of the Principle of Bivalence.

[41] In the words of Penrose about mathematics, *op.cit.*, 113.

stands opposed to it? From the Cartesian *cogito*, to the subjective idealism of Berkeley and Hume, to the `Copernican Revolution' of Kant, we have long known the `egocentric predicament' of attempting to assert the purported existence of a world apart from a known world.

To trace the history of this (largely) twentieth century `realism' would be long, and out of place here. Whatever its origin, in Frege's rejection of what he calls "psychologism" or elsewhere, this much can be said, that the logic which he invented was so radical and so remarkably compelling and productive that it effected its own revolution. Frege succeeded in reducing all of classical mathematics to a single formal system. But that would have had only esoteric interest. His revolution in logic lies in his doctrine that singular sentences are atoms, and more complex propositions are simply truth-functions of these atoms. Once a mechanical calculus of truth-functions was provided, then Classical logic became an extraordinarily powerful technique for manipulating these atoms, ordering them in ways that serve the prevailing interest in the logic of contingencies, the concatenation of `facts' for practical and theoretical purposes. After Russell made it popular, Classical logic applied its calculus to ever widening fields, to switching circuits (in 1930), to scientific discourse (from the late 20's), to the computer (the Turing machine, its theoretical model, in 1937), to every form of academic interest claiming to be science, and to every sort of technological interest. In its wonderful simplicity, power and decidability it is said to have advanced in a very short time beyond everything achieved in logic in its previous 2000 year history. In this case, it is the logic which carries the metaphysics with it: the `realism' of our times is consequent on the unquestioned success and authority of Classical logic.

If anything else is to be gleaned from the matters analyzed here, perhaps it is this, that Classical logic is a particularly inept instrument to analyze those philosophies which stand opposed to the `realism' it demands. Yet we see such analyses everywhere in philosophical literature, presented as though they were objective assessments of positions they can at best dogmatically oppose.

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