SIMULATION STUDY OF GENERALIZED LINEAR MIXED EFFECTS MODELS ON FISHERY DATA

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Simulation study of generalized linear mixed effects models on fishery data

by

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Abstract

The generalized linear model (GLM) is a class of versatile models suitable for for several types of dependent variables. GLMs are commonly used to model maturity data. Generalized linear mixed models (GLMM) are a useful extension of the GLM with the addition of random effects. GLMMs have previously been used to improve the estimates of the maturities and provide better predictions of maturities in the near future. Dowden (2007) used GLMMs to model a Atlantic cod maturity data set. His research found that GLMMs improved maturity estimates and forecast accuracy over the GLM commonly used. The results also revealed potential year effects in the cod data. This may be due to actual year effects or some other source such as sampling error. In general it is unknown whether year effects are present in a data set. In this practicum we first provide an overview of Dowden's results. Then we conduct a simulation study to investigate which GLMM provides the most accurate estimates of the simulated maturities and parameters under a range of simulation factors including the presence of year effects. The two GLMMs used to model the simulated data are an autoregressive (AR) mixed model and a AR mixed effects model with random year effects (AR YE). In this research we find the AR YE model appears to be more appropriate than the AR model when the presence of year effects are unknown. The AR YE model's estimates are similar or better than the AR model's and it also tends to be either as efficient or more efficient depending on the presence or size of the year effects.

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Chapter 1

Introduction

Fisheries management is concerned with making regulatory decisions that ensures sustainable production from the fish stock while promoting the economic and social well-being of fishermen and industries. They make decisions on how much to develop the fishery, the limits on the locations and times where fishing will take place as well as the size and number of fish that are allowed to be caught. The success of the fishery depends on the health of the stock and thus management must take this into account along with the effect that their decisions will have on the stock (Gulland, 1983).

Stock assessment uses information on the past and current status (size, age-structure, etc) of a fish stock in the construction of quantitative statistical and mathematical models to make the best predictions possible about the alternative management choices. This provides information to help make reasoned policies for the present and, as more data becomes available, help refine or modify these decisions. For instance, stock assessment can be used to calculate that by avoiding catching fish below a certain size a fish stock suffering from growth overfishing can recover while increasing the total catch weight (Gulland, 1983). Different management approaches may achieve the same biological yield and these choices often involve a trade off between the average yield and the variability of yield. Stock assessment is used to provide estimates of the nature of this trade off (Hilborn et. al. 1992).

Fisheries go through stages of development and the role of stock assessment changes with each stage. Early in the development of a fishery, usually not much is known about the fish stock such as the size of the stock and the natural life span of the fish. The role of stock assessment here is to provide rough initial estimates on the distribution, size and productivity of the stock as well as help develop monitoring requirements to provide better assessments in later stages of the development of the fishery. As development proceeds, the data updates and provides feedback on the population parameters management uses in the decision process, and through regular assessments it could possibly provide early warning of overfishing. When the fishery is fully developed, it can be used to search for better policy options by providing a framework of calculations for fish growth, movement, mortality, and vulnerability to fishing. It may also be used to determine if the current catch and effort statistics are likely to give a misleading picture of the trends and health of the stock and if a more systematic sampling program would be worth while. At this stage, the stock sometimes has been over fished and would need to go through a period of rehabilitation. Management may then decide to use tough policies which reduce the amount of fish being caught. Here stock assessment is used to estimate how long the rebuilding would take under different amounts of reduction. Another way management may decide to rebuild the stock is through technologies such as fish hatcheries. Stock assessment is then used to help measure if the process is working or if it is having deleterious side effects on the stock or on other fish stocks. (Hilborn et. al. 1992)

A common measure of the productive potential of a fish stock is spawning stock biomass (SSB). It is the total weight of the mature component of a stock, calculated as the product of biomass-at-age and proportion mature-at-age (i.e. maturities) summed over all ages in the stock. It is important to have good estimates of maturities to produce good estimates of SSB's. The age at maturity is an important life history characteristic in stock assessment; it may be an indicator of stress. A lowering in the age at maturity is sometimes associated with a reduction in the population size or higher mortality rates. Thus good estimates of maturity are important for successful management of the stock as changes in maturation affects the productivity of the stock and amount of fish that can be harvested without affecting the standing stock size for future years.

Data on North Atlantic cod in Northwest Atlantic Fisheries Organization (NAFO) subdivision 3Ps were collected during annual research vessel trawl surveys conducted by Fisheries and Oceans Canada (DFO). The data were collected using a length stratified sampling scheme (Doubleday 1981). During 1960 to 2005 a total of 25,810 cod were collected, of which 13,535 where female. The ages of the fish were also determined by counting annual growth increments in small ear bones called otoliths. Due to sex-specific differences in maturation, males and females were treated separately and for the purposes of this practicum only females were used in the analysis. These data were summarized into the annual proportion mature and total number examined at ages 1 to 14.

A cohort model is used to model the 3Ps cod maturity data. A cohort is a group of individuals with the same birth year. Generalized linear models (GLMs), and in particular logistic regression models, are commonly used to model the relationship between the proportion mature and age in a cohort. A problem with this approach is that data are accumulated annually for recent cohorts, and when this data is included in the estimation of the cohort model then the resulting maturities may change considerably. If the changes are large then this can lead to retrospective variation in SSB estimates and this can undermine the credibility of advice. For example, it may happen that in 2008 the estimated proportion mature at age 5 in 2008 could be quite different than the proportion mature at age 5 in 2008 estimated in 2009 because of the addition of new data collected in 2009. This would lead to a difference in the estimate of SSB for 2008 made in estimation year 2008 compared to the estimate of SSB for 2008 made in estimation year 2009. This can create much havoc in the advisory process. It is highly desirable to use cohort models that provide accurate and precise estimates that do not produce large retrospective differences in maturity estimates - and consequently SSB estimates.

In retrospective analysis the data after a specified year are dropped from the model and maturities are forecasted. Then the data from the first year dropped are added back into the model and the maturities are forecasted again. This is done repeatedly until the full data set is used in the model. Retrospective variance is the change in the forecasted maturities as new data is added to the model. We show that generalized linear mixed models (GLMM; see Section 1.1) can reduce the retrospective variance and improve short-term forecasts of maturities compared to the commonly used GLM logistic regression models. Short-term (e.g. 3 years) forecasts of maturities are routinely provided in stock assessments to use when forecasting stock status in response to proposed fishery management actions.

GLMMs have been previously used to improve estimates of the maturities (i.e. proportion mature at age) and provide predictions of maturities in the near future. In these models, the proportion mature is considered to be an increasing function of age, which is the regression covariate. The functional relationship is modeled separately for each cohort. Dowden (2007) studied maturity models that include fixed effects components where the parameters were unknown constants to be estimated, and random effects components where parameters were treated as random variables. Dowden investigated the use of GLMMs to improve maturity estimates and forecasted maturities. He found that GLMMs provided better inferences than GLMs for modeling maturities in Atlantic cod stocks. GLMMs fit the data better for the majority of observations and improved maturity estimates and forecast accuracy. An overview of Dowden's results is provided in Chapters 2 and 3. While forecast accuracies were improved with the use of GLMMs, retrospective variation can still be large. I will conduct a simulation study using two GLMMs with the best results when applied to the 3Ps data. I will investigate which GLMM has the most accurate estimates of the simulated maturities and parameters under a range of simulation factors.

1.1 The Generalized Linear Mixed Model

Two types of models are briefly reviewed in this section. The first is the generalized linear model (GLM). The second is the generalized linear mixed model (GLMM), which is an extension of GLM.

A GLM consists of three components (Dobson, 1990). The first component is a set of independent random response variables, denoted as $\mathbf{Y} = (Y_1, \ldots, Y_n)$. Each Y_i is assumed to depend on a single parameter η_i and share the same distribution from the exponential family. A distribution belongs to the exponential family if it can be written in the form

$$f(y|\eta) = exp\left[a(y)\,b(\eta) + c(\eta) + d(y)\right] \tag{1.1}$$

where Y depends on a single parameter η. Many distributions such as the Normal, Poisson and Binomial belong to the exponential family. The distribution (??) is in canonical form if $a(y) = y$. Therefore the distribution of Y_i can be written as

$$f(y_i|\eta_i) = exp\left[y_i b_i(\eta_i) + c_i(\eta_i) + d_i(y_i)\right] \tag{1.2}$$

The second component of a GLM is the linear predictor

$$\eta = \mathbf{X}\beta. \tag{1.3}$$

where β is a $p \times 1$ vector of unknown parameters and $\mathbf{X}$ is the matrix of the explanatory variables. $\mathbf{X} = [\mathbf{x}_1', \ldots, \mathbf{x}_n']'$ is a $n \times p$ matrix. This linear predictor, η, is equal to a monotone, differentiable link function of the expected value of Y_i,

$$g(\mu_i) = \mathbf{x}_i'\beta \tag{1.4}$$

which is the third component of the GLM, where $E(Y_i) = g^{-1}(\mathbf{x}_i'\beta) = \mu_i$ and $g^{-1}(\cdot)$ is the inverse of the link function $g(\cdot)$.

GLMMs are widely used in ecological applications and in other areas. This model extends GLM by adding random effects to the linear predictor (e.g. Bolker et.al., 2008). GLMMs are useful for accommodating overdispersion in count data based on binomial, negative binomial or Poisson distributions, and accounting for the dependence among the response variables which is inherent in longitudinal data. A GLMM consists of a response variable $\mathbf{Y} = (Y_1, \ldots, Y_n)$ with explanatory variables associated with the fixed and random effects, vectors $\mathbf{x}_i$ and $\mathbf{z}_i$ respectively. Let $\mathbf{X} = [\mathbf{x}_1', \ldots, \mathbf{x}_n']'$, and $\mathbf{Z} = [\mathbf{z}_1', \ldots, \mathbf{z}_n']'$. In this model the linear predictor becomes

$$\eta = \mathbf{X}\beta + \mathbf{Z}\delta \tag{1.5}$$

where δ is an $q \times 1$ vector of random effects parameters and $\mathbf{Z}$ is an $n \times q$ matrix of explanatory variables. $\mathbf{X}$ and β are defined as in (??). η is equal to a differentiable monotonic link function $g(\cdot)$ such that

$$g(\mu) = \mathbf{X}\beta + \mathbf{Z}\delta. \tag{1.6}$$

Given δ, the conditional mean is given by

$$E(\mathbf{Y}|\delta) = g^{-1}(\mathbf{X}\beta + \mathbf{Z}\delta) \tag{1.7}$$

where $g^{-1}(\cdot)$ is the inverse link function. The random effects δ are assumed to have a multivariate normal distribution with mean 0 and covariance matrix $\mathbf{D} = \mathbf{D}(\theta)$, where θ is an unknown vector of variance parameters (Breslow et.al., 1993). The data vector $\mathbf{y}$ can be written as

$$\mathbf{y} = \mu + \mathbf{e} \tag{1.8}$$

where $\mathbf{e}$ is a vector of unobserved errors. Given μ, $\mathbf{e}$ has mean $E(\mathbf{e}|\mu) = 0$ and covariance

$$cov\,(\mathbf{e}|\mu) = \mathbf{R}_{\mu}^{1/2}\mathbf{R}\mathbf{R}_{\mu}^{1/2} \tag{1.9}$$

where $\mathbf{R}_{\mu}^{1/2}$ is a diagonal matrix containing evaluations at μ of a known variance function for the GLMM and $\mathbf{R}$ is a variance-covariance matrix of unknowns (Wolfinger and O'Connell, 1993).

1.1.1 Estimation Methods for the Generalized Linear Mixed Models

GLMM parameters β and θ can be estimated using the ML approach, in which the values of these parameters are chosen to maximize the likelihood. The likelihood function $L(\beta, \theta|\mathbf{y})$ is the same as the joint probability density function $f(\mathbf{y}|\beta, \theta)$ but with a shift of emphasis to the parameters β and θ with the response variables $\mathbf{y}$ fixed. The likelihood is based on the marginal distribution of Y,

$$L(\beta, \theta | y_1, \ldots, y_n) = \int \left[\prod_i f_{Y_i|\delta}(y_i|\delta) \right] f_\delta(\delta)\, d\delta \tag{1.10}$$

where the vector $\delta \sim f_\delta(\delta)$ with $E(\delta) = 0$ and $var(\delta) = \mathbf{D}$. For the models in this practicum, $f_\delta(\delta)$ is normal with mean 0 and variance $\mathbf{D}$. The likelihood must be integrated over all possible values of the random effects. The ML estimating equations (i.e. score equations) come from taking the log of equation (**??**) and differentiating the log likelihood with respect to β and θ. The MLEs are the zero-root of the score function.

The ML estimators of the variance components do not take into account the loss in degrees of freedom from estimating the fixed effects. The restricted maximum likelihood (REML) method is a modification of ML that takes the degrees of freedom for the fixed effects into account when estimating the variance components. β is not involved in the REML estimates, as a result the variance components estimates are unaltered by a change in the value of β with $\mathbf{X}$ unchanged. The REML estimators are also less sensitive to outliers in the data than MLEs (McCulloch et al., 2008). For linear mixed models, the variance components are estimated from linear combinations of the data, $\mathbf{K}'\mathbf{y}$, that do not involve β. $\mathbf{K}'$ is chosen to have as many linearly independent rows as possible satisfying $\mathbf{K}'\mathbf{X} = 0$ and then maximum likelihood is based on $\mathbf{K}'\mathbf{y}$ (McCulloch et. al., 2008). For nonlinear mixed models, REML estimates are more complex. The normality assumption on the random effects for linear mixed models yields a closed-form expression for the marginal likelihood, while the same assumption for a nonlinear mixed model leads to a computationally intensive likelihood involving multi-dimensional integration. Lindstrom and Bates (1990) have proposed a linear mixed-effects (LME) approximation to REML for nonlinear mixed models. Noh and Lee (2008) have modified the REML procedure of linear mixed models to obtain hierarchical-likelihood estimators for nonlinear mixed models. When GLMMs have large numbers of random effects, the calculations may be slow or infeasible. Other methods such as the penalized quasi-likelihood, Laplace approximation and Gauss-Hermite quadrature have been developed to solve this problem by approximating the likelihood (Bolker et.al., 2008).

Penalized quasi-likelihood (PQL) is one of the simplest and most widely used methods to estimate the parameters of a GLMM. PQL uses a quasi-likelihood rather than a true likelihood. PQL starts with a first order Taylor's approximation of the mean function about the current estimate of β and the prediction of the random effects δ. We can write $\mathbf{Y} = \mu + \varepsilon$ where $\varepsilon = \mathbf{Y} - \mu$. A first-order Taylor's approximation of $\mu = \mu(\eta) = \mu\{\eta(\beta, \delta)\}$ about the current estimate of β and prediction of δ (denoted as $\tilde{\beta}$ and $\tilde{\delta}$) is, using the chain rule,

$$\mu = \tilde{\mu} + Diag\left\{ \left.\frac{\partial \mu(\eta)}{\partial \eta}\right|_{\beta=\tilde{\beta},\delta=\tilde{\delta}} \right\} \left\{ \left.\frac{\partial \eta(\beta,\delta)}{\partial \beta'}\right|_{\beta=\tilde{\beta},\delta=\tilde{\delta}} \left(\beta - \tilde{\beta}\right) + \left.\frac{\partial \eta(\beta,\delta)}{\partial \delta'}\right|_{\beta=\tilde{\beta},\delta=\tilde{\delta}} \left(\delta - \tilde{\delta}\right) \right\}.$$

Recall that $\eta = \mathbf{X}\beta + \mathbf{Z}\delta$ so

$$\frac{\partial \eta(\beta,\delta)}{\partial \beta'} = \mathbf{X} \text{ and } \frac{\partial \eta(\beta,\delta)}{\partial \delta'} = \mathbf{Z}.$$

Note that

$$Diag\left\{ \frac{\partial \mu(\eta)}{\partial \eta} \right\} = Var(\mathbf{Y}|\delta) = \mathbf{V}.$$

Let $\tilde{V}$ denote the evaluation of V at $(\tilde{\beta}, \tilde{\delta})$. The Taylor's approximation of μ can be written as

$$\mu = \tilde{\mu} + \tilde{\mathbf{V}}\mathbf{X}\left(\beta - \tilde{\beta}\right) + \tilde{\mathbf{V}}\mathbf{Z}\left(\delta - \tilde{\delta}\right),$$

and the approximation of the mean function leads to

$$\mathbf{Y} = \tilde{\mu} + \tilde{\mathbf{V}}\mathbf{X}\left(\beta - \tilde{\beta}\right) + \tilde{\mathbf{V}}\mathbf{Z}\left(\delta - \tilde{\delta}\right) + \varepsilon.$$

Subtracting $\tilde{\mu}$ from both sides of this equation, and then multiplying by $\tilde{\mathbf{V}}^{-1}$ yields

$$\tilde{\mathbf{V}}^{-1}\left(\mathbf{Y} - \tilde{\mu}\right) = \mathbf{X}\left(\beta - \tilde{\beta}\right) + \mathbf{Z}\left(\delta - \tilde{\delta}\right) + \tilde{\mathbf{V}}^{-1}\varepsilon;$$

hence

$$\tilde{\mathbf{V}}^{-1}\left(\mathbf{Y} - \tilde{\mu}\right) + \mathbf{X}\tilde{\beta} + \mathbf{Z}\tilde{\delta} = \mathbf{X}\beta + \mathbf{Z}\delta + \tilde{\mathbf{V}}^{-1}\varepsilon.$$

Define the adjusted dependent variable (e.g. McCullagh and Nelder, 1989) as

$$\tilde{\mathbf{Y}} = \tilde{\mathbf{V}}^{-1}\left(\mathbf{Y} - \tilde{\mu}\right) + \mathbf{X}\tilde{\beta} + \mathbf{Z}\tilde{\delta}$$

and the standardized residual as $\varepsilon_s = \tilde{\mathbf{V}}^{-1/2}\varepsilon$. The Taylor's approximation can be written in a standard linear mixed-model form,

$$\tilde{\mathbf{Y}} = \mathbf{X}\beta + \mathbf{Z}\delta + \tilde{\mathbf{V}}^{-1/2}\varepsilon_s$$

where $E\left(\tilde{\mathbf{Y}}\right) = \mathbf{X}\beta$ and, ignoring the variability in $\tilde{\mathbf{V}}$, $Cov\left(\tilde{\mathbf{Y}}\right) = \mathbf{ZDZ}' + \tilde{\mathbf{V}}^{-1}$.

PQL involves a double iterative process in which a linear mixed model is used to estimate the mean parameters β and the random effect variance parameters θ, and then the random effects are predicted usually as best linear unbiased estimators based on the current estimates of β and $\mathbf{D}$. These current estimates are then used to update $\tilde{\mathbf{V}}$ and $\tilde{\mathbf{Y}}$ and the procedure is iterated until convergence. The procedure is doubly iterative because estimation of the linear mixed-model variance parameters involves an iterative procedure. REML estimation for θ may also be used in the linear mixed-model stage. PQL may yield biased estimates if the standard deviations of the random effects are large. Using the Laplace approximation reduces the bias (Bolker et.al., 2008).

The Laplace approximation is a method used to approximate the integral in equation (**??**), and the approximation is very accurate for normally distributed random effects (McCulloch et. al., 2008). Another method to approximate the likelihood is to use the Gauss-Hermite quadrature (GHQ). For some smooth functions, GHQ can approximate the integrals for the likelihood as a weighted sum. The GHQ approximation is more accurate than the Laplace, but it is slower. As the number of random effects increases, the speed of GHQ decreases rapidly, making it infeasible for analysis with more than two or three random factors (Bolker et.al., 2008). Also if the function whose integral is to be approximated is not smooth or if it is not properly centered, the approximation may be poor (McCulloch et. al., 2008).

The pseudo-likelihood (PL)/ restricted pseudo-likelihood (REPL) approaches are iterative procedures similar to PQL and involve estimating the fixed and random effects, β and δ, from linear mixed-model equations, and estimating the unknown parameters in $\mathbf{D}$ and $\mathbf{R}$ using either ML or REML in turn until convergence. The PL/REPL estimation procedure is based on a Gaussian approximation and Taylor's theorem while PQL is based on a quasilikelihood. PL/REPL starts with an initial estimate of μ, $\hat{\mu}$ which is used to compute

$$\mathbf{v} = g(\hat{\mu}) + (\mathbf{y} - \hat{\mu})g'(\hat{\mu}). \tag{1.11}$$

The ML or REML estimation procedure is then used to fit a weighted linear mixed model with response variable $\mathbf{v}$, fixed and random effects model matrices $\mathbf{X}$ and $\mathbf{Z}$, and diagonal weight matrix $\hat{\mathbf{W}} = \mathbf{R}_{\hat{\mu}}^{-1}[g'(\hat{\mu})]^{-2}$. This yields estimates of $\hat{\mathbf{D}}$ and $\hat{\mathbf{R}}$

which are then compared with the old estimates. If the difference is not sufficiently small, the mixed model equations

$$\mathbf{H}\begin{bmatrix}\hat{\alpha}\\ \hat{\beta}\end{bmatrix} = \begin{bmatrix}\mathbf{X}'\hat{\mathbf{W}}^{1/2}\hat{\mathbf{R}}^{-1}\hat{\mathbf{W}}^{1/2}\mathbf{v}\\ \mathbf{Z}'\hat{\mathbf{W}}^{1/2}\hat{\mathbf{R}}^{-1}\hat{\mathbf{W}}^{1/2}\mathbf{v}\end{bmatrix} \tag{1.12}$$

where

$$\mathbf{H} = \begin{bmatrix}\mathbf{X}'\hat{\mathbf{W}}^{1/2}\hat{\mathbf{R}}^{-1}\hat{\mathbf{W}}^{1/2}\mathbf{X} & \mathbf{X}'\hat{\mathbf{W}}^{1/2}\hat{\mathbf{R}}^{-1}\hat{\mathbf{W}}^{1/2}\mathbf{Z}\\ \mathbf{Z}'\hat{\mathbf{W}}^{1/2}\hat{\mathbf{R}}^{-1}\hat{\mathbf{W}}^{1/2}\mathbf{X} & \mathbf{Z}'\hat{\mathbf{W}}^{1/2}\hat{\mathbf{R}}^{-1}\hat{\mathbf{W}}^{1/2}\mathbf{Z}+\hat{\mathbf{D}}^{-1}\end{bmatrix} \tag{1.13}$$

are solved for $\hat{\alpha}$ and $\hat{\beta}$. The estimate of $\hat{\mu}$ is updated by substituting $\hat{\alpha}$ and $\hat{\beta}$ in the expression $\hat{\mu} = g^{-1}(\mathbf{X}\hat{\alpha} + \mathbf{Z}\hat{\beta})$. These steps are repeated until convergence (Wolfinger and O'Connell, 1993).

The expectation-maximization (EM) algorithm is another way to calculate ML/REML estimates. It is an iterative algorithm which alternates between two steps, calculating the conditional expected values, and maximizing simplified likelihoods. The likelihood is simplified by the invention of "missing" data which is combined with the observed data to form what is called the complete data. For GLMMs the random effects are assumed to be the missing data. The expectation of the log likelihood of the complete data with respect to conditional distribution of δ given $\mathbf{y}$ is calculated. This expectation is then maximized with respect to the parameters. The log likelihood is recalculated with the new parameter estimates and maximized again until convergence. There are various simulated EM algorithms that can be used to approximate the conditional expected values such as the Monte Carlo and the stochastic approximation algorithms (McCulloch, 2008).

1.2 Statistical Software Packages

In this practicum, two procedures produced by the SAS Institute are used for estimating parameters in GLMs and GLMMs, PROC GENMOD and PROC GLIMMIX. The GENMOD procedure fits GLMs to the data by maximum likelihood estimation of the parameter vector β. There is generally no closed form solution for the maximum likelihood estimates. The GENMOD procedure estimates the parameters through an iterative process. The GLIMMIX procedure fits GLMMs based on linearizations, using a Taylor series expansion to approximate the GLMM as a linear mixed model. By default the estimation method for models containing random effects is the restricted pseudo-likelihood method (REPL) with an expansion around the current estimate of the best linear unbiased predictors of the random effects. This is the estimation method we used to fit GLMMs.

1.3 Outline of the Practicum

In Chapter 2 the fixed effects and mixed effects cohort models are applied to the maturity data. Their fit will be examined by how well the estimated proportions mature compare to the observed and how accurately each model forecasts future maturities. I show that simple cohort models explain much of the variability in maturity data, but that there are additional patterns in the data that are common across adjacent cohorts. These are referred to as year effects. In Chapter 3 a year effect is added to the mixed effects model and applied to the data. The fit of this model is examined and compared to the previous models in Chapter 2. A simulation study is carried out in Chapter 4. The mixed effects model with and without year effects will be applied to simulated data with and without the presence of year effects. These models will be compared in terms of how accurately they estimate the proportions mature, and their accuracy in estimating the model parameters. Chapter 5 summarizes the conclusions.

Chapter 2

The Fixed Effects and Mixed Effects Models

2.1 Introduction

Fisheries management considers changes in SSB in their stock projections, which requires maturities to be forecasted for the next several years. Maturity, as a function of age within a cohort, tends to change smoothly over time. For 3Ps cod, Dowden (2007) investigated two types of models, fixed effects and mixed effects models, to improve estimates of maturities, especially for unfinished cohorts, and to improve forcasted maturities. Unfinished cohorts are recent cohorts where the cod have not reached the age at which all are mature. For analysis of the 3Ps cod data, a cohort is unfinished if it has not reached age 14. Figure ?? displays the concept of finished and unfinished cohorts. Each cell represent the data collected in a given year for a given age group. Each diagonal set of cells represents the data collected by year and age for a given cohort. The white cells represent unfinished cohorts. The data collected for these cohorts cover the younger ages and do not reach the maximum age by the last year of data collection. The grey cells represent finished cohorts. The data collected for these cohorts cover the older ages and reach the maximum age by the last year. In this chapter we evaluate two of the models Dowden (2007) used, a fixed effects (FE) logistic regression model, and a autoregressive mixed effects (AR) model with no over-dispersion. The models are applied to the 3Ps female cod data for ages up to 14. This dataset contained 13329 observations from years 1960 to 2005. Table ?? summarizes the proportion mature and the numbers sampled by age.

To evaluate the performance of these models we look at how the estimated maturities change over time, how they compare with the observed proportions, and how accurately the models forecasted maturities. Retrospective analysis is used to determine how accurately each model forecasted maturities. In retrospective analysis, the data after a specified year, called the retro year, is removed and the maturities are predicted three years ahead. For example, if the retro year was 2000, only the data up to year 2000 is used to forecast maturities for 2001, 2002 and 2003. These predicted maturities are then compared with the estimated maturities obtained when all the data are used for estimation.

For the fixed effects approach, the predicted maturities for new cohorts in the forecast period were computed by averaging the three closest cohorts. This averaging procedure was also used for unfinished cohorts which have insufficient data to estimate maturities. For the mixed effects approach, the correlation structure was used to predict maturities. The retrospective performance of the models was examined for each year since 1995. The retrospective metric

$$\rho = \sum_{y=1995}^{Y} |\hat{p}_{a,y+3,y} - \hat{p}_{a,y+3,Y}| \tag{2.1}$$

was used to measure the prediction error at age a for each model. Here $\hat{p}_{a,y+3,y}$ is the predicted proportion mature at age a in year $y+3$ obtained using data up to retrospective year $y \leq Y$ and Y is the last year in the full data set. Substantial differences between predicted and estimated maturities cause problems when estimating SSB. These differences can be a source of retrospective error in SSB, such that in annual assessments the SSB in some past year is revised as new data are added to the assessment. Large retrospective errors can undermine the credibility of assessment advice.

2.2 Fixed Effects (FE) Model

The fixed effects approach refers to a generalized linear model (GLM). More specifically, it is a logistic regression model with

$$p_c(a) = \frac{exp(\beta_{0c} + \beta_{1c}a)}{1 + exp(\beta_{0c} + \beta_{1c}a)} \tag{2.2}$$

where $p_c(a)$ is the probability that a fish in cohort c is mature at age a. The parameters β_{0c} and β_{1c} are unknown parameters that are estimated separately for each cohort c. For some cohorts, the data are insufficient to estimate the parameters as illustrated by Figure ??. These are some of the very young and the very old cohorts. Cohort B covers only a small range of young ages and cohort D covers only a small range of old ages. Both cohorts do not cover enough of the maturity ogive to reliably estimate the maturity curves. For the 3Ps cod dataset, after the 2000 cohort, only young cod were observed, covering only the lower portion of the maturity ogive. Similarly, before the 1954 cohort, only the old cod were observed, covering only the upper portion of the maturity ogive. These cohorts do not cover enough of the range of the ogive to estimate the parameters, therefore parameters were estimated only for the 1954-2000 cohorts. Probabilities for the other cohorts were obtained by averaging over the adjacent three years.

Define A_{50} to be the age of 50% maturity, $p_c(A_{50}) = 0.5$. From equation (??), A_{50} can be calculated as $A_{50} = \frac{-\beta_{0c}}{\beta_{1c}}$ and can be estimated as

$$\hat{A}_{50} = \frac{-\hat{\beta}_{0c}}{\hat{\beta}_{1c}}, \tag{2.3}$$

where $\hat{\beta}_{0c}$ and $\hat{\beta}_{1c}$ are the estimates of the logistic regression parameters.

The maturity range (MR) is defined as the difference between the age at 75% maturity and the age at 25% maturity. MR is calculated by $MR = A_{75} - A_{25} = \frac{-log(9)}{\beta_{1c}}$ and is estimated as

$$\widehat{MR} = \hat{A}_{75} - \hat{A}_{25} = \frac{-log(9)}{\hat{\beta}_{1c}}. \tag{2.4}$$

2.2.1 FE Results

The intercept β_{0c} and slope β_{1c} varied greatly across cohorts with little trend (Figure ??), especially in the 1963 cohort where β_{0c} decreased to -135.2090 and β_{1c} increased to 22.5732. $\hat{A}_{50}$ had a downward trend and had much less variation. $\widehat{MR}$ also varied wildly across cohorts with little trend. $\widehat{MR}$ had a large decrease for the 1963 cohort, and a large increase in the 1980 cohort, indicating a large difference between the rate at which these cohorts matured and their adjacent cohorts.

The estimated proportions mature for ages 4 to 8 increased over time (Figure ??), especially after 1990. Some cohorts did not have enough data for the fixed effects model to estimate the proportion mature. For these years, the average of the three adjacent cohorts were used as estimates. These averages are the flat lines in the beginning and end of the plot.

The retrospective analysis in Figure ?? shows large differences between the forecasted and subsequently estimated proportions mature. The variability between forecasted and estimated maturities is greatest in ages 4 to 6, with the largest retrospective errors occurring in ages 5 and 6. The older ages have less variability because the proportions mature at these ages are close to one since 1995.

Most of the estimated proportions mature are close to the observed for all cohorts (Figure ??). Within each cohort as age increases the proportions mature should also increase. Most of the observed proportions mature fall on a smooth ogive curve for each cohort in Figure ??. For some cohorts there are observations which are followed by a decrease in proportion mature in the next year. For example for the 1999 cohort the observed proportion mature at age 5 is 0.745 and decreases to 0.502 at age 6. These observation may indicate potential year effects and cause problems in estimating maturities especially for unfinished cohorts. Data are collected annually for unfinished cohorts and the maturity estimates are updated. A year effect in the data may cause the proportions mature to be overestimated. With the addition of new data the estimated ogive will shift away from the observation with the year effect fitting the observed maturities better. Observed proportions mature that fall away from the estimated maturity ogive will result in large residuals and retrospective errors.

The χ^2 residuals showed greater variability for some years (Figure ??). Five years have residual less than -5 which were truncated and displayed as solid circles. The arrows indicate six years where the 95% confidence intervals did not cover zero. Confidence intervals for the average annual χ^2 residuals are calculated as

$$\bar{r}_y \pm 1.96\sqrt{\sigma^2_{r_y}/n_{r_y}} \tag{2.5}$$

where $\bar{r}_y$ is the mean of the χ^2 residuals in year y, $\sigma^2_{r_y}$ is the variance of the χ^2 residuals and n_{r_y} is the number of residuals. Although most of the confidence intervals covered zero there are years where most of the residuals are positive or most are negative, which suggests there may be some real year effects in the data.

2.3 Autoregressive Mixed Effects (AR) Model

New data is collected each year for unfinished cohorts, and the parameters estimated by the fixed effects model can change substantially from year to year with the addition of data. The mixed effects model was used to help reduce this problem by treating some parameters as random effects. This random component is autocorrelated to account for how maturities tend to change smoothly over time. The parameters β_{0c} and β_{1c} appear to be autocorrelated over time. This model is

$$p_c(a) = \frac{exp\{(\beta_0 + \delta_{0c}) + (\beta_1 + \delta_{1c}) \times a\}}{1 + exp\{(\beta_0 + \delta_{0c}) + (\beta_1 + \delta_{1c}) \times a\}} \tag{2.6}$$

where δ_{0c} and δ_{1c} are autocorrelated random cohort effects and β_0 and β_1 are fixed effects common to all cohorts. δ_{0c} and δ_{1c} are assumed to be random variables from a normal distribution with mean zero and autocorrelated over time; that is, $\delta_{0c} \sim N(0, \sigma^2_{\delta_0})$, $\delta_{1c} \sim N(0, \sigma^2_{\delta_1})$, $Corr(\delta_{0j}, \delta_{0k}) = \gamma_0^{|j-k|}$ and $Corr(\delta_{1j}, \delta_{1k}) = \gamma_1^{|j-k|}$. These are AR(1) correlations with γ_0 and γ_1 respectively.

To predict the logistic regression model's random slopes and intercepts for each cohort, β_0 and β_1 are added to δ_{0c} and δ_{1c},

$$\hat{\beta}_{0c} = \hat{\beta}_0 + \hat{\delta}_{0c}, \tag{2.7}$$

$$\hat{\beta}_{1c} = \hat{\beta}_1 + \hat{\delta}_{1c}. \tag{2.8}$$

These $\hat{\beta}_{0c}$ and $\hat{\beta}_{1c}$ are used to predict A_{50} and MR using equations (??) and (??)

respectively.

2.3.1 AR Results

The $\hat{\beta}_{0c}$, $\hat{\beta}_{1c}$ and $\widehat{MR}$ do not vary as much across cohorts (Figure ??) and were much smoother than the FE model. $\hat{\beta}_{1c}$ increased over time while $\widehat{MR}$ decreased over time, especially from the 1980 to 1990 cohorts. The $\hat{A}_{50}$ was similar to that of the fixed effects model.

The proportion mature for ages 4 to 8 increased over time (Figure ??). The estimates were smoother than the FE model but the basic trends were similar. The variation in the retrospective results was less than the fixed effects model (Figure ??) and the size of retrospective errors, ρ, were smaller for all ages. The retrospective results also varied more smoothly over the years. However, there are still large retrospective errors for some years in the younger ages.

The proportion mature are relatively close to the observed and similar to the FE model's estimates (Figure ??). The χ^2 residuals showed similar variability to that of the FE model (Figure ??). Six years have residuals beyond ± 5. These are truncated and displayed as solid circles. The 95% confidence intervals did not cover zero for two years. Many years have residuals with mostly the same sign indicating some potential year effects.

2.4 Discussion

The AR model is a better choice than the FE for modeling maturation rates in Atlantic cod stocks. While both models had a similar estimated proportions mature which are close to the observed, the AR estimates are smoother. The AR model also had smaller retrospective errors than the FE model suggesting improved forecast accuracies. Although there was an improvement in the retrospective results, the AR model still had large retrospective errors for some years. The χ^2 residuals from both the FE and AR models also suggested some potential year effects. Dowden added year effects to the AR model to further improve maturity estimates and forecasted maturities. This model and its results are presented in the next chapter.

Table 2.1: Summary of the 3Ps female cod data by age

	Proportion mature			Numbers sampled		
Age	min	max	mean	min	max	total
1	0.000	0.000	0.000	1	15	142
2	0.000	0.000	0.000	2	97	1131
3	0.000	0.000	0.000	9	183	1845
4	0.000	0.235	0.036	14	136	2000
5	0.000	0.848	0.242	13	109	1897
6	0.053	1.000	0.543	4	146	1727
7	0.333	1.000	0.808	1	98	1508
8	0.667	1.000	0.938	1	84	1064
9	0.797	1.000	0.979	1	66	773
10	0.846	1.000	0.995	1	41	509
11	1.000	1.000	1.000	1	28	324
12	0.667	1.000	0.989	1	26	214
13	1.000	1.000	1.000	1	14	123
14	1.000	1.000	1.000	1	11	72

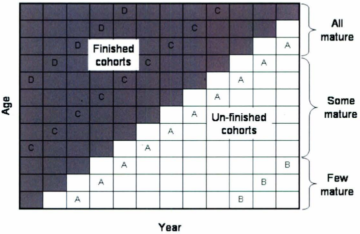

Figure 2.1: Conceptual diagram of a time series of maturity at age data. Grey colored cells represent finished cohorts, and white cells represent unfinished cohorts. Typically there are few to no fish mature at younger ages and all fish are mature at older ages. There is insufficient data to reliably estimate the maturity curves for cohorts B and D, but there should be sufficient data to estimate the maturity curves for cohorts A and C.

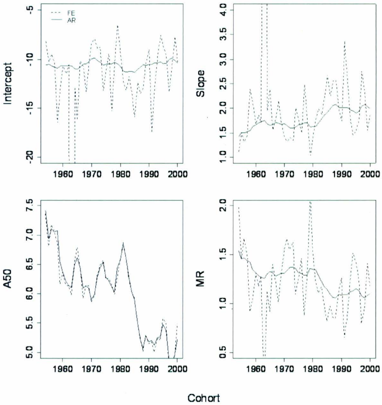

Figure 2.2: Estimates for 3Ps cod with the fixed effects model (FE) and autoregressive mixed effects model (AR). Top left panel: intercepts (β_{0c}). Top right panel: slopes (β_{1c}). Bottom left panel: $\hat{A}_{50}$. Bottom right panel: $\widehat{MR}$

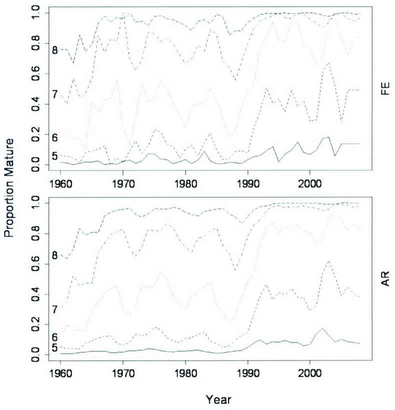

Figure 2.3: 3Ps cod proportions mature at ages 4-8 vs year. Ages 5-8 are listed on the left side of each line. Top panel: Fixed effects model (FE). Bottom panel: autoregressive mixed effects model (AR).

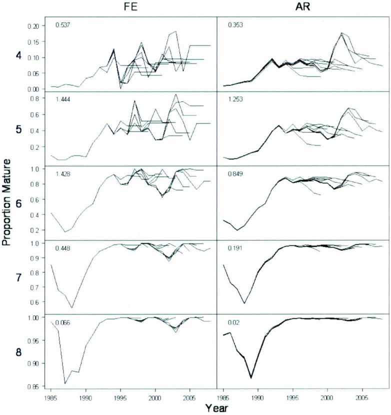

Figure 2.4: Retrospective analysis for 3Ps cod, ages 4-8 (listed in the left margin). The retrospective metric ρ is shown in the top left corner of the panels. Column 1: Fixed effects model (FE). Column 2: autoregressive mixed effects model (AR).

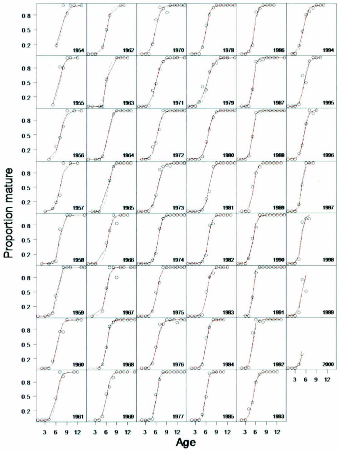

Figure 2.5: 3Ps cod proportions mature at age estimated from the fixed effects model (FE; solid black line) and the autoregressive mixed effects model (AR; dashed red line). Observations are plotted as circles (○).

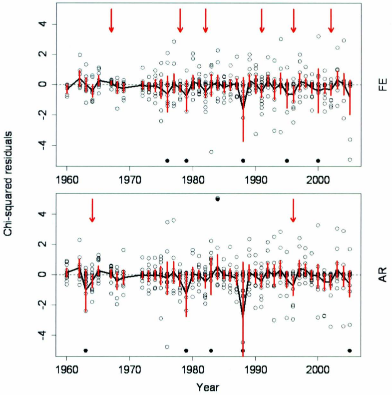

Figure 2.6: χ^2 residuals ($\circ$) for the 3Ps cod maturity data. Solid symbols are truncated ($<$ or $>$ $\pm$ 5). Red vertical lines are 95% confidence intervals (CI's). The arrows indicate where the CI's did not cover zero. The solid black line connects the average χ^2 residuals. Top panel: Fixed effects model (FE). Bottom panel: autoregressive mixed effects model (AR).

Chapter 3

Autoregressive Mixed Model with Year Effects

3.1 Introduction

The third approach Dowden (2007) used was a mixed effects model with cohort and year effects (YE). These year effects may be caused by environmental conditions such as food and temperature which can vary year to year, leading to more or less individuals making the decision to become mature. Sampling may also contribute to year effects. Maturation rates can vary across a population. If the full range of the population is not covered by the sampling or if there is an annual variation in the distribution of samples across the population range, then the calculated proportion mature may not be representative of the population. The year effects model will be described in the next section and its performance will be evaluated and compared to the models in the previous chapter.

3.2 AR with Year Effects (AR YE) Model

Define $p_{cy}(a)$ as the probability that a fish from cohort c is mature at age a in year y. The autoregressive mixed model with year effects (AR YE) is

$$p_{cy}(a) = \frac{exp\{(\beta_0 + \delta_{0c} + \eta_{0y}) + (\beta_1 + \delta_{1c}) \times a\}}{1 + exp\{(\beta_0 + \delta_{0c} + \eta_{0y}) + (\beta_1 + \delta_{1c}) \times a\}}. \tag{3.1}$$

In this model the random slope effects for age δ_{1c}, are autocorrelated similar to equation (??). The random intercept effects are composed of two effects, an autocorrelated cohort effect δ_{0c}, and a simple uncorrelated year effect η_{0y}. The η_{0y}'s are i.i.d $N(0, \sigma_\eta^2)$. A year effect is assumed to be the same for all cohorts in that year.

The $\hat{A}_{50}$'s and the $\widehat{MR}$'s do not directly include $\hat{\eta}_{0y}$'s. For the AR YE model, the maturity ogive for a cohort is no longer a logistic linear function of age, because each age has a year effect. For illustration purposes we assumed the year effects were due to sampling and not reflective of changes in the population. Therefore, the year effects were treated like nuisance parameters and $\hat{A}_{50}$'s and $\widehat{MR}$'s were based on only $\hat{\beta}_{0c}$ and $\hat{\beta}_{1c}$, which are calculated as in equations (??) and (??) respectively.

For estimating the proportions mature it is straight-forward to include the year effects. If the year effects are real in the population then the estimated proportions mature should include the $\hat{\eta}_{0y}$'s. If the year effects are not real and are instead sampling errors, then the $\hat{\eta}_{0y}$'s should be treated as a nuisance parameter and excluded in estimating the maturities. For the analysis of the estimated maturities over time and the retrospective analysis, the year effects will be treated as both a predictive parameter (YE+) or as a nuisance parameter (YE-). However, additional information will be required to resolve whether the year effects are "real" or just due to sampling deficiencies. This is beyond the scope of this practicum.

3.2.1 AR YE Results

The $\hat{\beta}_{0c}$, $\hat{\beta}_{1c}$ and $\widehat{MR}$ do not vary much across cohorts and are much smoother than the AR model (Figure ??). $\hat{\beta_{1c}}$ increased over time while $\widehat{MR}$ decreased over time, especially from the 1980 to 1990 cohorts and had a similar shape to the AR model. The $\hat{A}_{50}$ was also smoother than the FE and AR models.

The year effects are treated in two ways, as a nuisance parameter (AR YE-) where $\hat{\eta}_{0y}$ is not used in computing the proportion mature estimates, and as a predictive parameter (AR YE+) where $\hat{\eta}_{0y}$ is included. The proportion mature for age 4 to 8 increased over time for both AR YE- and AR YE+ (Figure ??). The AR YE+ model estimates are rougher than the AR model but smoother than the FE model. The AR YE- model estimates are the smoothest of all the models.

The retrospective results (Figure ??) for the AR YE+ model varied less than both the FE and and the AR models. Its retrospective errors, ρ, are less than the FE model for all ages and slightly smaller than the AR model for ages 5 to 7. The retrospective results for the AR YE- model varied more smoothly than all the other models. The AR YE- model's retrospective errors are also the smallest of all the models for all ages. However substantial retrospective errors are still present for some years.

Most of the AR YE estimated proportions mature are close to the observed (Figure ??). The AR YE maturity estimates are similar to the AR estimates, however for some cohorts the AR YE estimates appear a little closer to the observed. For example the 1954, 1957 and 1958 cohorts. The confidence intervals for the χ^2 residuals from the AR YE model all covered zero (Figure ??). Also there are no years where most of the residuals share the same sign as in both the FE and AR models.

3.3 Discussion

Mixed effects models are the more appropriate choice for modeling maturities in Atlantic cod stocks, with the best model being the AR YE model. Using the AR YE model on the 3Ps cod data improved the maturity estimates, for two reasons. While the AR YE model maturity estimates are similar to the AR estimates, for some cohorts the estimates are closer to the observed maturities than the AR and FE models. The AR YE- model also has the smoothest maturity estimates over time. Using the AR YE model also improved forecast accuracies. The AR YE- model has the best retrospective results with the smallest retrospective errors.

The 3Ps cod data appeared to have potential year effects. From the observed maturities in Figure ??, some observations for a given age had higher proportions mature then the next age. Also from the χ^2 residuals in Figure ??, most of the residuals for some years share the same sign. This may be due to actual year effects or some other source such as sampling error. In general it is unknown whether there are year effects present in a data set. If no year effects are present in the data then the AR model would be the most appropriate model to fit the data while if year effects are present then the AR YE model would be most appropriate. In Chapter 4 a simulation study is conducted to see how the estimates are affected by using the AR YE model to fit data with no year effects present as compared to the AR model estimates. Also the AR model will be used to fit data with year effects and compared to the AR YE model estimates.

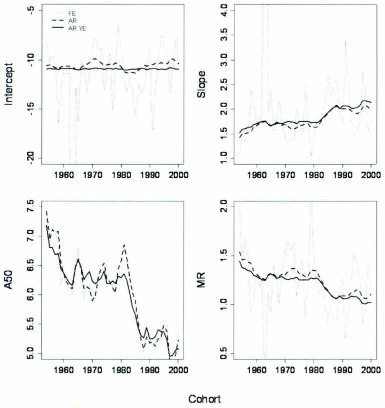

Figure 3.1: Estimates for 3Ps cod with the fixed effects model (FE), autoregressive mixed effects model (AR) and the AR model with year effects (AR YE). Top left panel: intercepts (β_{0c}). Top right panel: slopes (β_{1c}). Bottom left panel: $\hat{A}_{50}$. Bottom right panel: $\widehat{MR}$

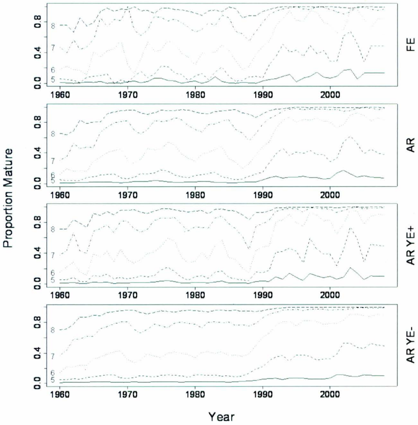

Figure 3.2: 3Ps cod proportions mature at ages 4-8 vs year. Ages 5-8 are listed on the left side of each line. Top panel: Fixed effects model (FE). Second panel: Autoregressive mixed effects model (AR). Third panel: AR with year effects as predictive parameters (AR YE+) Bottom panel: AR with year effects as nuisance parameters (AR YE-).

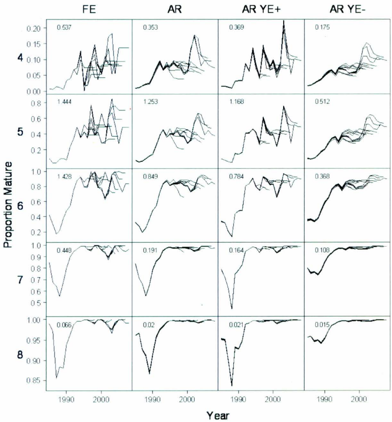

Figure 3.3: Retrospective analysis for 3Ps cod, ages 4-8 (listed in the left margin). The retrospective metric ρ is shown in the top left corner of the panels. Column 1: Fixed effects model (FE). Column 2: Autoregressive mixed effects model (AR). Column 3: AR with year effects as predictive parameters (AR YE+). Column 4: AR with year effects as nuisance parameters (AR YE-).

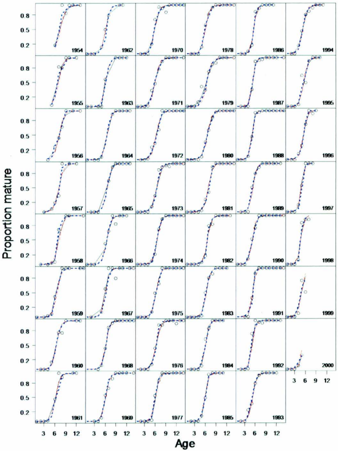

Figure 3.4: 3Ps cod proportions mature at age estimated from the fixed effects model (FE; solid black line), the autoregressive mixed effects model (AR; dashed red line), and the autoregressive mixed effects model with year effects (AR YE; dotted blue line). Observations are plotted as circles (○).

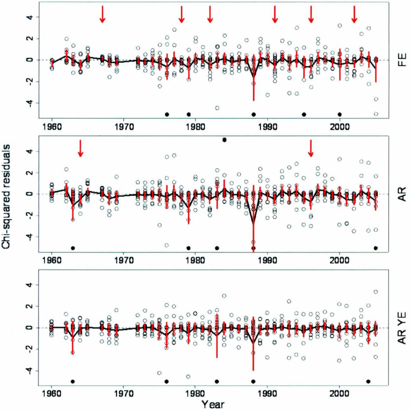

Figure 3.5: χ^2 residuals ($\circ$) for the 3Ps cod maturity data. Solid symbols are truncated ($<$ or $>$ $\pm$ 5). Red vertical lines are 95% confidence intervals (CI's). The arrows indicate where the CI's did not cover zero. The solid black line connects the average χ^2 residuals. Top panel: Fixed effects model (FE). Middel panel: autoregressive mixed effects model (AR). Bottom panel: AR with year effects (AR YE).

Chapter 4

Simulation study

4.1 Introduction

In this chapter I examine the accuracy of the estimators of the AR and AR YE models using simulated data. Simulated data sets were generated both with and without the presence of year effects. To generate data without year effects the AR model parameter estimates from the 3Ps data were used. Similarly, data with year effects were generated using the AR YE parameter estimates. Using the 3Ps estimates makes the simulated data sets resemble real Atlantic cod data. I then base the values of the fixed and random cohort slopes and intercepts on these parameter estimates, which are then used to generate the simulated data sets as described below. A set of simulation factors are also used in generating the data sets. These simulation factors are used to vary the generated data sets in terms of the range of ages the fish are maturing and in the difference in the proportion mature between cohorts, which are also described below.

The steepness of the maturity ogive was varied for each simulated data set. Changing the steepness of the maturity ogive changes the range of ages where the fish in a given cohort are actively becoming mature. This is known as the active range. At the very young ages all the fish are immature, and at the very old ages all the fish are already mature. The middle ages, where the fish are actively making the decision to become mature, are of interest. The maturity range (MR) can be used to measure how broad or narrow the active range is. Recall that MR is the difference in ages at which 75% and 25% of the cohort are mature. The fixed intercept and slope used in generating simulated data are based on a MR simulation factor, where β_0 and β_1 are calculated as

$$\begin{aligned} \beta_1 &= log(9)/MR \\ \beta_0 &= -A_{50}/\beta_1 \end{aligned} \tag{4.1}$$

I varied the active range of the simulated data by basing β_0 and β_1 on three different levels of MR. Based on the estimated $\widehat{MR}$ of the 3Ps cod data, a low level $MR = 1.0$, a medium level $MR = 1.3$, and a high level $MR = 1.6$ were chosen. A low value of MR indicates a narrow active range. Once the cohort starts to mature, it matures quickly. A higher MR indicates a broader active range and the cohort maturing more slowly. The β_1's for generating the simulated data sets were calculated using these MR values and equation (**??**). The middle of the range of A_{50}'s for the 3Ps cod data was approximately 6.25. This value was used in equation (**??**) to set the value of β_0 to generate the simulated maturity data.

The contrast over cohorts was also varied for each simulated data set. The contrast over cohorts refers to the difference in maturities between cohorts. For example when there is little contrast over cohorts, the proportions mature at each age are similar for all cohorts. When there is a large contrast between cohorts, there are large differences in proportions mature at each age. I varied contrast over cohorts by using a cohort effect (CE) simulation factor. Basing the random intercept and slope on the 3Ps data estimates and on CE, δ_{0c} and δ_{1c} are calculated as

$$\begin{aligned}
\delta_{0c} &= CE \times \hat{\delta}_{0c} \\
\delta_{1c} &= CE \times \hat{\delta}_{1c}
\end{aligned} \tag{4.2}$$

where $\hat{\delta}_{0c}$ and $\hat{\delta}_{1c}$ are the 3Ps cod data estimates and CE is the cohort effect. I used the AR estimates of $\hat{\delta}_{0c}$ and $\hat{\delta}_{1c}$ to generate data without year effects and the AR YE estimates to generate data with year effects. Three levels of cohort effect were chosen, $CE = 0.5$, $CE = 1.0$ and $CE = 1.5$. The low level of CE decreases the difference between cohorts by 50%, the medium level leaves the contrast over cohorts the same as in the 3Ps data, and the high level increases the contrast over cohorts by 50%. Using these levels and the AR or AR YE random effects estimates, the δ_{0c}'s and δ_{1c}'s for the simulated data are calculated using equation (??). Note that the random effects were fixed when generating the simulated data.

When year effects are present in the simulated data, the size of the year effects were also varied by using a year effect (YE) simulation factor. Using YE and the 3Ps random year effects estimates, the random year effects used to generated the simulate data are calculated as

$$\eta_{0y} = YE \times \hat{\eta}_{0y} \tag{4.3}$$

where $\hat{\eta}_{0y}$ is the AR YE model's estimated random year effects and YE is the year effect simulation factor. Three levels of YE are used, a low (YE=0.5), medium (YE=1) and high level (YE=2). The low level of YE decreases the size of the year effects by 50%, the medium level leaves the year effects the same as the 3Ps data estimates and the high level doubles the year effects.

To generate data without year effects, the $3 \times 3 = 9$ combinations of MR and CE were first used to set values for β_0, β_1, and δ_{0c}, δ_{1c} using equations (??) and (??) and the 3ps AR model estimates. These new fixed and random effects were then used in the AR model (equation ??) to calculate new proportions mature. Maturity data were simulated by randomly generating the number of mature fish a thousand times from the binomial distribution, using the new proportions mature and the real total number of fish caught in each age and year from the 3Ps data (i.e. the n's). As a result a thousand simulated data sets with no year effects present were generated for each combination of MR and CE.

Similarly, to generate data with year effects, the $3 \times 3 \times 3 = 27$ combinations of MR, CE and YE were used to set values for β_0, β_1, δ_{0c}, δ_{1c} and η_{0y} using equations (??), (??) and (??) and the 3ps AR YE model estimates. These were then used in the AR YE model (equation ??) to calculate proportions mature. The number of mature fish were then randomly generated a thousand times from the binomial distribution using the new proportions mature and the real total number of fish caught. As a result a thousand simulated data sets with year effects were generated for each combination of MR, CE and YE. In the following sections both the AR and the AR YE models are used to fit the simulated data sets. The original estimates of the models used to generate the data are considered to be the true population values and are compared to the simulated values and the results are presented below.

4.2 The Values of A_{50} and MR Used to Generate Simulated Datasets

Introducing the MR factor and the cohort effect (CE) affects the estimated $\hat{A}_{50}$ and $\widehat{MR}$. This section examines how changing the MR and CE levels affected these estimates. Recall that A_{50} is the age at which 50% of the cohort is mature. Figure

?? shows the A_{50}'s calculated as in section (2.3) using the fixed and random effects generated by equations (??) and (??). Each panel shows the cohort-specific A_{50}'s from a different combination of the three MR and CE levels. It shows that when the levels of both the MR factor and CE are low there is little difference in A_{50} over all the cohorts. When the levels of MR and CE are both high, there is a large downward trend in A_{50}. Also when the level of CE increases given a level of MR, the annual variation in A_{50} increases. When MR increases for a given level of CE, the annual variation in A_{50} also increases, however the amount of variation depends on the level of CE. At the low level of CE, when MR increases there is only a slight increase in the variation of the A_{50}'s, while at the high level of CE, the increase of variation is much larger. Most of the change in annual variation of A_{50} is due to changes in CE.

Figure ?? shows the values of MR over cohorts for each combination of simulation factors. Here I refer to the MR simulation factor as $\overline{MR}$. It shows that at the low levels of both $\overline{MR}$ and CE there is little change in MR over cohorts. At the high levels of $\overline{MR}$ and CE the MR for each cohort is larger and there is a large downward trend. Figure ?? also shows that as CE increases the annual variation in MR increases at a given level of $\overline{MR}$. As the level of $\overline{MR}$ increases given a level of CE, the overall MR for each cohort increases and there is also more variation.

4.3 AR model Results

The results were simplified by determining which factors are important in modeling maturities. To achieve this, I first calculated the absolute bias,

$$|b_{ay}| = |\bar{p}_{ay} - p_{ay}| \tag{4.4}$$

where $\bar{p}_{ay} = \frac{1}{K}\sum_{i=1}^{K}\hat{p}_{ay(i)}$ is the average proportion mature at age a in year y, $|b_{ay}|$ is the absolute bias at age a in year y, K is the number of iterations, $\hat{p}_{ay(i)}$ is the estimated proportion mature at age a in year y for iteration i and p_{ay} is the simulated population value of the proportion mature at age a in year y. The absolute bias was then fitted with a GLM using factors CE, MR, YE, age and method. Method refers to the model used to fit the data, either AR or AR YE. In order to determine which factors were significant a ANOVA table was computed based on the GLM of the absolute bias and the results are displayed in Table ??. The ANOVA table shows that YE, method and age are important factors with significant interactions between method and YE, and between CE and age. Since MR is not an important factor nor does it have any significant interactions with other factors, MR is dropped to simplify the results. Also since CE only has a significant interaction, only the low and high levels of CE are used to simplify the results. Detailed results for all the simulation factors are presented in the Appendix.

Table ?? shows the absolute bias averaged over MR and years for each method, age, CE and YE factor. For the AR model, absolute bias is small when there are no year effects present (YE = 0) or when YE is small. It increases with the level of YE for each age and CE factor. As age increases the absolute bias increases between ages 5 and 6, then decreases between ages 6 and 8. Also there is little change in the absolute bias between levels of CE.

In Figures ?? to ??, some summaries of the estimated proportions mature are compared with simulation population values for ages 5 to 8. To simplify the results based on the significant factors in Table ??, the figures shown here are for all the four levels of YE, the low and high levels of CE (CE=0.5, CE=1.5) and the medium level of MR (MR=1.3). The figures for all levels of the simulation factors are displayed in the Appendix. In the first column of these figures, the AR model was used to estimate the proportions mature. As a measure of the model's performance for each set of simulation factors, the bias and the root mean squared error (RMSE) are calculated as

$$\begin{aligned} b_y(a) &= \sum_{i=1}^{K}(\hat{p}_{ay}(i) - p_{ay})/K \\ RMSE_y(a) &= \sqrt{\sum_{i=1}^{K}(\hat{p}_{ay}(i) - p_{ay})^2/K} \end{aligned} \tag{4.5}$$

where $b_y(a)$ is the average bias in year y for age a, $RMSE_y(a)$ is the average RMSE in year y for age a, K is the number of iterations, $\hat{p}_{ay}(i)$ is the estimated proportion mature at age a in year y for iteration i and p_{ay} is the simulated population value of the proportion mature at age a in year y. The average absolute values of $b_y(a)$ over years and the average $RMSE_y(a)$ over years is displayed in Figures **??** to **??**, and calculated as

$$\begin{aligned} \bar{b}(a) &= \frac{1}{n_Y}\sum_{y=y_1}^{Y}|b_y(a)| \\ \overline{RMSE}(a) &= \frac{1}{n_Y}\sum_{y=y_1}^{Y} RMSE_y(a) \end{aligned} \tag{4.6}$$

where $\bar{b}(a)$ is the average absolute bias for age a, $\overline{RMSE}(a)$ is the average RMSE for age a, n_Y is the total number of years, and y_1 and Y are the first and last years respectively.

The AR model performs well in estimating proportions mature close to the true proportions when there are no year effects present (Figures **??** and **??**). When YE=0 the average absolute bias and average RMSE are small and close to zero for all ages.

The red dashed lines are the 95% and 5% quantiles of the estimated proportions mature. These quantiles are fairly close to the simulated proportions mature for most years, with the simulated proportions mature falling between them for all ages and simulation factors. When year effects are present (Figures **??** - **??**), the mean estimated proportions mature are smoother than the simulated proportions. As a result the mean estimates are not close to the simulated population values for some years when large jumps occur. As the year effects increases the average absolute bias and average RMSE also increase for all ages. When the year effects are low (YE=0.5, Figures **??** - **??**), the simulated proportions mature for most years fall between the 95% and 5% quantiles and the mean estimates remain close. As the year effects get larger (Figures **??** - **??**), the simulated proportions fall outside the quantiles for more years and the mean estimates become less accurate. Changing the levels of CE changed the range of proportions mature for ages 5 to 8. Increasing CE given a level of YE and MR increased the range of proportions mature for each age, although the accuracy of the model remains similar with little difference in the average absolute bias and average RMSE between the low level of CE (CE=0.5) and the high level of CE (CE=1.5).

Figures **??** to **??** show histograms of the estimated fixed intercept (β_0) and fixed slope (β_1) for each level of YE. These figures show combinations of simulation factors CE (CE=0.5, CE=1.5) and MR (MR=1.3). The figures for all combinations of the simulation factors are presented in the Appendix. In the first column are the AR model estimates. The red vertical lines indicate the value of the true parameters used to generate the data. These values were calculated using equation (**??**). To measure the accuracy of the estimates, the bias and RMSE are calculated for these parameters as

$$
\begin{aligned}
bias &= \frac{1}{K}\sum_{i=1}^{K}\left(\hat{\theta}_i - \theta\right) \\
RMSE &= \sqrt{\frac{1}{K}\sum_{i=1}^{K}\left(\hat{\theta}_i - \theta\right)^2}
\end{aligned}
\tag{4.7}
$$

where $\hat{\theta}_i$ is the parameter estimate for iteration i, θ is the value of the true parameter and K is the number of converged iterations. These values are shown at the top of each figure panel. Table **??** shows the percentage of iterations that converged when fit with the AR model.

Figures **??** and **??** show the histograms for the fixed intercept (β_0). These figures show that when there are no year effects present (YE=0) the AR model estimates are fairly close to the true fixed intercept with a small bias close to 0. When year effects are present the AR model tends to overestimate the intercept, with both the bias and RMSE increasing with YE. The bias and RMSE are similar for both levels of CE.

Figures **??** and **??** show the histograms for the fixed slope (β_1). These figures show that when year effects are not present, the AR model estimates are close to the true fixed slope with the bias and RMSE close to 0. When year effects are present the model tends to underestimate the fixed slope, with the bias and RMSE increasing with YE. There is little difference between the low and high levels of CE when year effects are low or not present. Here the high level of CE has slightly higher biases and RMSEs. When YE=1.0 and YE=2.0, the difference between the low and high levels of CE is greater, with CE=1.5 having higher biases and RMSEs.

Figures **??** to **??** compares the estimated and true effects for the random intercept (δ_{0c}) and random slope (δ_{1c}) over cohorts. The first column shows the AR model estimates. As measures of accuracy the bias and RMSE for each cohort is calculated as

$$
\begin{aligned}
b_c &= \sum_{i=1}^{K} (\hat{\theta}_c(i) - \theta_c)/K \\
RMSE_c &= \sqrt{\sum_{i=1}^{K} (\hat{\theta}_c(i) - \theta_c)^2/K}
\end{aligned}
\tag{4.8}
$$

where b_c and $RMSE_c$ are the average bias and RMSE in cohort c respectively, K is the number of iterations, $\hat{\theta}_c(i)$ is the estimated random effect in cohort c for iteration i and θ_c is the simulation value of the random effect in cohort c. The average absolute bias and RMSE are displayed in the bottom right corner of the figures and are calculated as

$$
\begin{aligned}
|\bar{b}| &= \sum_{c=c_1}^{C} (|b_c|)/n_c \\
RMSE &= \sum_{c=c_1}^{C} (RMSE_c)/n_c
\end{aligned}
\tag{4.9}
$$

where $|\bar{b}|$ and RMSE are the average absolute bias and average RMSE over cohorts, c_1 and C are the first and last cohorts respectively and n_c is the total number of cohorts.

Figures **??** and **??** compares the estimated random intercepts (δ_{0c}) to the population values over cohorts. The AR estimates are smoother than the population values when no year effects are present. The estimates are closest to the population values when YE=0.5 and have the smallest average absolute bias and RMSE. As YE increases to 1.0 and 2.0, the estimates vary more widely and the absolute bias and RMSE increases. The red dashed lines are the 95% and 5% quantiles of the estimated random intercept. The simulated random intercept falls between these lines for most cohorts except when YE=2.0. There is little difference between the low and high level of CE with the average absolute biases and RMSEs being similar given YE.

Figures ?? and ?? compares the estimated random slopes (δ_{1c}) to the population values over cohorts. The AR estimates are close to the population values for most cohorts except when YE is high. The average absolute biases and RMSEs are similar when YE=0 to YE=1.0. When YE=2.0, the AR estimates vary more widely than the population values and the average absolute bias and RMSE increases. The absolute biases and RMSEs tend to be lower when CE is high. As YE increases up to 1.0, the absolute biases and RMSEs increase slightly when CE is low and decrease slightly when CE is high. The population random slope falls between the 95% and 5% quantiles for most cohorts except when YE=2.0. When CE is low and YE is 0.5 to 1.0, the estimated random slopes for the first and last few cohorts are not as close as the other cohorts and the population values fall outside the 95% and 5% quantiles.

4.4 AR YE model Results

Table ?? shows the absolute bias averaged over MR and years for each method, age, CE and YE factor. This table shows that the average absolute bias is very similar for both the AR and AR YE method when no year effects are present. Here the AR methods's absolute bias tends to be slightly smaller. When year effects are present, the AR YE method's absolute bias is much smaller than the AR method's. As YE increases, there is little change in the AR YE method's absolute bias while the AR method's absolute bias increases substantially.

The efficiency of the two models was also compared. To do this first the RMSE for each model is calculated as in (??). Using the RMSEs for both models the relative efficiency is calculated as

$$RE_y(a) = \frac{RMSE_y(a)(AR)}{RMSE_y(a)(AR\ YE)} \tag{4.10}$$

where $RE_y(a)$ is the relative efficiency for age a in year y and $RMSE_y(a)(AR)$ and $RMSE_y(a)(AR\ YE)$ are the RMSEs for age a in year y for the AR and AR YE models respectively. When $RE_y(a)$ is greater than 1, the AR model's RMSE is greater than the AR YE model's hence the AR YE model is more efficient for age a in year y. When $RE_y(a)$ is less than 1, the AR model is more efficient for age a in year y. When $RE_y(a)$ equals 1, the models have the same efficiency. Using the relative efficiency, a GLM was fitted with factors CE, MR, YE and age. Based on the results a ANOVA table was computed (Table **??**). The ANOVA table shows that YE and age are significant factors and interactions between YE and age, and CE and YE are also significant. Since MR is not a significant factor and has no significant interactions, it can be dropped from further analyses of the simulation results.

Table **??** shows the relative efficiency average over MR and years. This table shows that the AR YE model is more efficient than the AR model when year effects are present and almost just as efficient when no year effects are present. It shows that as the YE increases the average relative efficiency also increases, meaning that the AR YE model becomes more efficient than the AR model. When YE=0 and YE=0.5 the models are very similar, however the AR model is slightly more efficient when YE=0 and the AR YE model is slightly more efficient when YE=0.5. Relative efficiency depends on age when year effects are present. Relative efficiency tends to decrease from age 5 to 6, then increase from age 6 to 8. The relative efficiency also depends on CE when YE=2.0. When year effects are not present or small, there is little difference between the levels of CE. However when YE=2.0 the relative efficiency for a given age decreases as the level of CE increases.

Table **??** shows the percentage of iterations that converged when fit with the AR

YE model. The AR YE model tends to converge less often than the AR model, especially when YE, CE and MR factors are all high.

The AR YE model's mean estimated proportions mature are close to the population values for all combinations of YE (Figures ?? - ??). When no year effects are present the AR YE model estimates are very similar to the AR model's with similar average absolute biases and RMSEs close to 0 (Figures ?? and ??). When year effects are present, the AR YE model fits the simulated proportions mature better than the AR model. When YE is low the AR YE model fits slightly better than the AR model, with slightly smaller average absolute bias and RMSE. As YE increases the fit of the AR model worsens while the fit of the AR YE model tends to stays the same. The simulated population proportion mature falls between the 95% and 5% quantiles for most years. There is little difference in the fit of the AR YE model between the low and high levels of CE. The average absolute bias and RMSE remains similar for all levels of YE.

The AR YE model tends to overestimate the fixed intercept (β_0, Figures ?? and ??). When YE=0 the AR YE model estimates are similar to the AR model's and fairly close to the true parameter value with a bias close to 0. As YE increased the bias and RMSE increased slightly however, the AR YE estimates are closer to the true fixed intercept than the AR model's, especially when YE=2.0. The low level of CE has slightly smaller biases and RMSEs than the high level for all levels of YE.

The AR YE model tends to underestimate the fixed slope (β_1, Figures ?? and ??). As with the fixed intercept, when YE=0 the AR YE model estimates are similar to the AR model's. For all levels of YE, the bias and RMSE of the AR YE model remain small and close to zero. When year effects are present, the AR YE model estimates are closer to the true fixed slope than the AR model's, especially when year effects are high. There is little difference between the levels of CE, however the bias and RMSE tend to be smaller when CE=0.5.

Figures ?? and ?? shows that when year effects are not present or low (YE=0.5) the AR YE model random intercept (δ_{0c}) estimates are similar to the AR model's estimates and have similar absolute biases and RMSEs. As year effects increase, the AR YE model's random intercept estimates remain close to the simulated random intercepts with little change in the absolute bias and RMSEs. When YE is greater than or equal to 1, the AR YE model random intercept estimates are closer to the simulated than the AR model's and have smaller absolute biases and RMSEs, especially when YE=2. There is little difference in the fit of the AR YE model between the levels of CE given a level of YE. However the bias and RMSEs are slightly smaller when CE is high. For all levels of YE, the simulated random intercepts fall between the 95% and 5% quantiles for most cohorts.

The AR YE model random slope (δ_{1c}) estimates are similar to the AR model estimates and have similar absolute biases and RMSEs except when year effects are high (Figures ?? and ??). When YE=2 the AR YE model estimates are closer to the simulated random slope than the AR model and have a smaller absolute bias and RMSE. As YE increases, the AR YE model's absolute bias and RMSE tends to increase slightly when CE is low, and decrease slightly when CE is high. The AR YE model estimates remain close to the simulated random slopes for all levels of YE. However when CE is high the estimates are closer and the simulated random slopes fall between the 95% and 5% quantiles for more cohorts. When CE is low and year effects are present, the AR YE model estimates for the first and last few cohorts tend not to be as close to the simulated random slope as the other cohorts, with the simulated values falling outside the 95% and 5% quantiles.

Figure ?? compares the mean estimated year effects (η_{0y}) to the simulated population value for each combination of CE and YE levels when MR=1.3. The average absolute bias and RMSE displayed in the bottom right corner of each panel are calculated as in equations (??) and (??) where θ now represents random year effects and c represents the year. When no year effects are present, the AR YE model random year effects estimates are small and close to the simulated value, 0. Both the absolute bias and RMSE are small and close to zero. When year effects are present, the estimated year effects are close to the simulated with the simulated year effects falling between the 95% and 5% quantiles for most years, however the absolute bias and RMSE are higher than when YE=0. The absolute bias and RMSE tends to decrease when YE=1 and increase when YE=2. There is little difference between the levels of CE, with the absolute bias and RMSE remaining similar for all levels.

4.5 Discussion

When fitting data without year effects, the AR model would be more appropriate than the AR YE model. However, comparing the fit of the AR YE model on simulated data without year effects with the AR model, there is very little difference between them. The absolute biases averaged over the MR factor and years are very similar for both models and are all close to zero. Both models also have very similar efficiency since the relative efficiencies averaged over the MR factor and years are just under 1. The AR YE model's estimated maturities as well as it's fixed and random effects are very close to the AR model's, with very similar biases and RMSEs. The year effects estimated by the AR YE model are close to zero and not significant.

When fitting data with year effects, the AR YE model would definitely be more appropriate than the AR model. Comparing the fit of the AR and AR YE models on simulated data with year effects shows that the AR YE model tends to fit better than the AR model, especially when the year effects are large. The AR YE model's absolute biases, averaged over the MR factor and years, are smaller than the AR model's. The AR YE model is also more efficient since the relative efficiencies averaged over the MR factor and years are all over 1. The AR YE model's mean estimated maturities are closer to the simulated population values with smaller absolute biases and RMSEs than the AR model for all levels of YE, however the estimates are only slightly better when YE is low. The AR YE model also tends to estimate the fixed and random effects better than the AR model. When year effects are low, the AR YE model fixed and random effects estimates are similar to the AR model's with slightly smaller biases and RMSEs. As the year effects increase, the AR YE model's estimates are closer to the simulated effects than the AR model's.

Another difference between the models is the number of iterations that converged. The AR model converged more often than the AR YE model. For example when YE, CE and MR levels are all high the AR model converged for 98.2% of the iterations where as the AR YE model converged for 73.4%.

Table 4.1: ANOVA table based on a GLM of the absolute bias with factors CE, MR, YE, age and method.

Factor	Df	Deviance	Residual Df	Residual Deviance
Null			14111	24.4576
Method	1	2.1959	14110	22.2617
CE	2	0.0109	14108	22.2508
MR	2	0.0783	14106	22.1725
YE	3	2.3326	14103	19.8399
Age	3	0.4471	14100	19.3928
Method $\times$ CE	2	0.0108	14098	19.3820
Method $\times$ MR	2	0.0727	14096	19.3093
Method $\times$ YE	3	2.1328	14093	17.1765
Method $\times$ Age	3	0.0667	14090	17.1099
CE $\times$ MR	4	0.0102	14086	17.0997
CE $\times$ YE	6	0.0054	14080	17.0942
CE $\times$ Age	6	0.1079	14074	16.9863
MR $\times$ YE	6	0.0455	14068	16.9408
MR $\times$ Age	6	0.0333	14062	16.9075
YE $\times$ Age	9	0.0710	14053	16.8364

Table 4.2: Absolute bias averaged over MR and years.

		YE							
		0		0.5		1.0		2.0	
CE	Age	AR	AR YE	AR	AR YE	AR	AR YE	AR	AR YE
0.5	5	0.00732	0.00736	0.0201	0.0109	0.0392	0.0103	0.0713	0.01020
	6	0.01570	0.01560	0.0380	0.0240	0.0627	0.0215	0.1050	0.02040
	7	0.01320	0.01320	0.0283	0.0176	0.0500	0.0158	0.0949	0.01490
	8	0.00562	0.00569	0.0122	0.0073	0.0240	0.0069	0.0536	0.00718
1.0	5	0.01050	0.01050	0.0232	0.01460	0.0434	0.01400	0.0779	0.01420
	6	0.01390	0.01390	0.0327	0.02210	0.0544	0.02040	0.0968	0.01830
	7	0.01280	0.01280	0.0251	0.01680	0.0443	0.01600	0.0817	0.01460
	8	0.00746	0.00754	0.0153	0.00991	0.0287	0.00938	0.0574	0.00878
1.5	5	0.01170	0.01160	0.0246	0.0170	0.0436	0.0161	0.0807	0.01600
	6	0.01040	0.01050	0.0260	0.0179	0.0434	0.0168	0.0810	0.01490
	7	0.01090	0.01090	0.0215	0.0149	0.0383	0.0145	0.0693	0.01370
	8	0.00755	0.00754	0.0174	0.0114	0.0318	0.0105	0.0586	0.00899

Table 4.3: Percentage of 1000 iterations that converged when the AR model was used to fit the simulated data. The simulation factors used to generate the simulated data are shown in the top row (MR), the first column (YE) and second column (CE).

		MR		
YE	CE	1.0	1.3	1.6
0.0	0.5	100%	100%	100%
	1.0	100%	99.6%	99.7%
	1.5	98.7%	97.0%	97.1%
	0.5	100%	100%	100%
0.5	1.0	100%	99.2%	98.7%
	1.5	99.5%	97.3%	98.9%
	0.5	100%	100%	100%
1.0	1.0	100%	98.8%	99.1%
	1.5	99.3%	95.8%	96.0%
	0.5	100%	99.9%	99.8%
2.0	1.0	97.2%	98.2%	97.4%
	1.5	98.6%	97.7%	98.2%

Table 4.4: ANOVA table based on a GLM of relative efficiency ($AR_{RMSE}/ARYE_{RMSE}$) with factors CE, MR, YE and age.

Factor	Df	Deviance	Residual Df	Residual Deviance
Null			7055	71101
CE	2	57	7053	71044
MR	2	17	7051	71027
YE	3	7822	7048	63205
Age	3	417	7045	62787
CE × MR	4	1	7041	62786
CE × YE	6	54	7035	62732
CE × Age	6	49	7029	62684
MR × YE	6	9	7023	62674
MR × Age	6	19	7017	62656
YE × Age	9	854	7008	61801

Table 4.5: Relative efficiency averaged over MR and years.

		YE			
CE	Age	0	0.5	1.0	2.0
0.5	5	0.983	1.17	1.82	3.62
	6	0.985	1.10	1.51	2.69
	7	0.987	1.14	1.72	3.58
	8	0.990	1.22	2.07	5.73
1.0	5	0.988	1.12	1.67	3.12
	6	0.989	1.09	1.47	2.68
	7	0.990	1.10	1.61	3.59
	8	0.991	1.14	1.79	4.98
1.5	5	0.989	1.09	1.55	2.88
	6	0.991	1.08	1.45	2.80
	7	0.992	1.07	1.50	3.48
	8	0.991	1.09	1.59	4.23

Table 4.6: Percentage of 1000 iterations that converged when the AR YE model was used to fit the simulated data. The simulation factors used to generate the simulated data are shown in the top row (MR), the first column (YE) and second column (CE).

		MR		
YE	CE	1.0	1.3	1.6
0.0	0.5	100%	100%	100%
	1.0	99.8%	99.5%	97.5%
	1.5	97.5%	90.2%	92.4%
0.5	0.5	100%	99.9%	99.9%
	1.0	100%	99.3%	98.1%
	1.5	96.0%	94.6%	95.3%
1.0	0.5	100%	99.9%	100%
	1.0	99.5%	97.9%	95.9%
	1.5	96.8%	92.6%	94.3%
2.0	0.5	100%	100%	100%
	1.0	96.5%	96.4%	81.5%
	1.5	98.8%	87.2%	73.4%

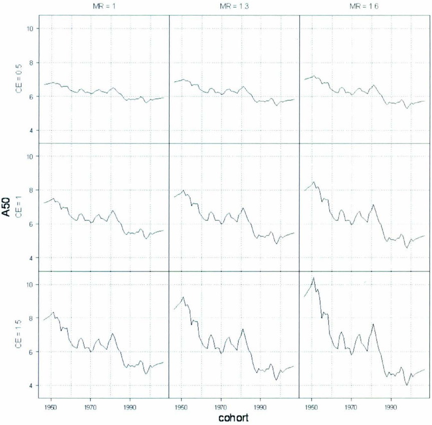

Figure 4.1: A_{50}'s used to generate simulated data. The simulation factors used to generate the data are shown in the left margin (CE) and on the top ($\overline{MR}$).

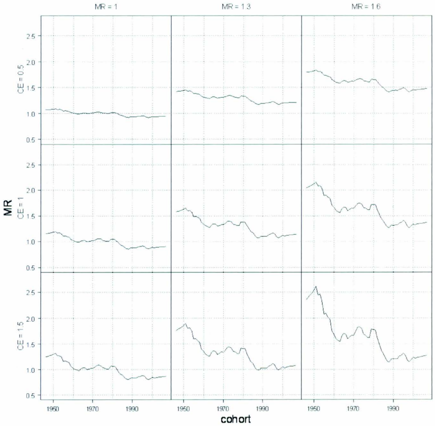

Figure 4.2: MR's used to generate simulated data. The simulation factors used to generate the data are shown in the left margin (CE) and on the top ($\overline{MR}$).

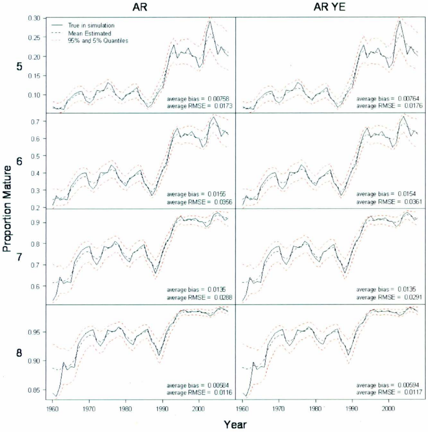

Figure 4.3: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model ($YE = 0$). Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with no year effects generated using simulation factors $CE = 0.5$ and $MR = 1.3$.

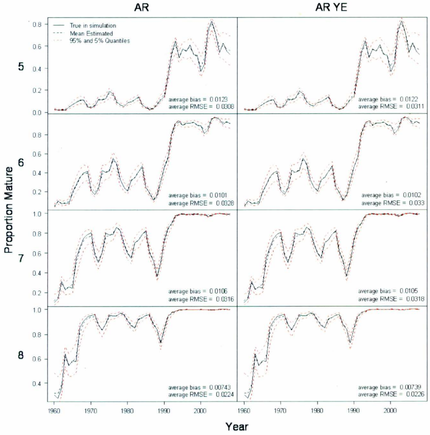

Figure 4.4: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model ($YE = 0$). Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with no year effects generated using simulation factors $CE = 1.5$ and $MR = 1.3$.

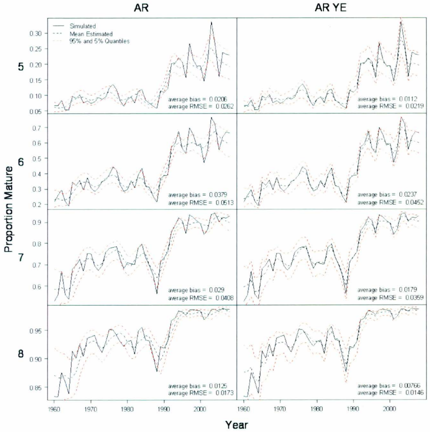

Figure 4.5: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 0.5$, $CE = 0.5$ and $MR = 1.3$.

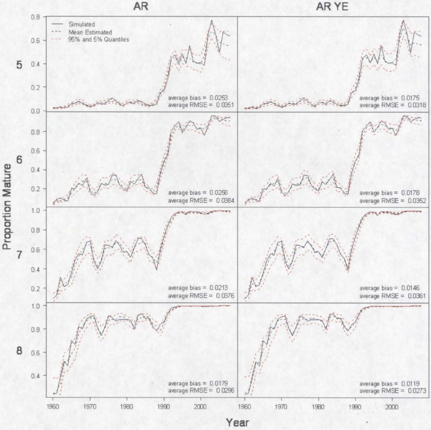

Figure 4.6: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 0.5$, $CE = 1.5$ and $MR = 1.3$.

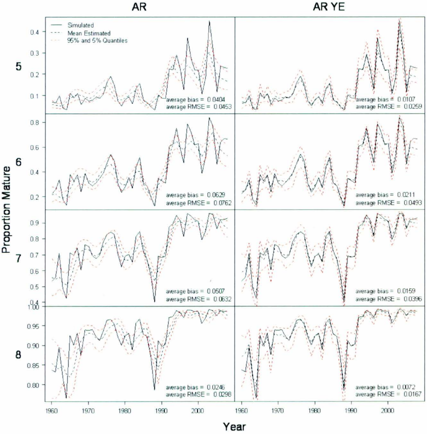

Figure 4.7: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 1.0$, $CE = 0.5$ and $MR = 1.3$.

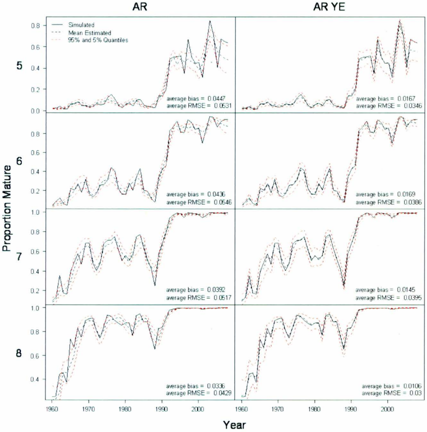

Figure 4.8: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 1.0$, $CE = 1.5$ and $MR = 1.3$.

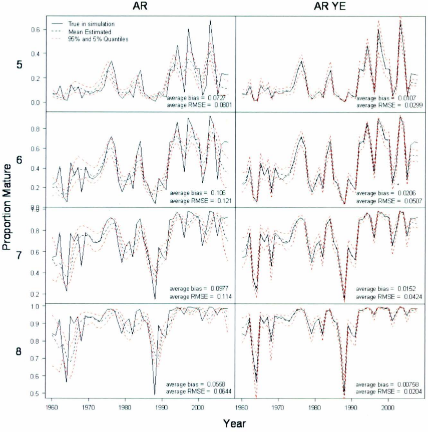

Figure 4.9: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 2.0$, $CE = 0.5$ and $MR = 1.3$.

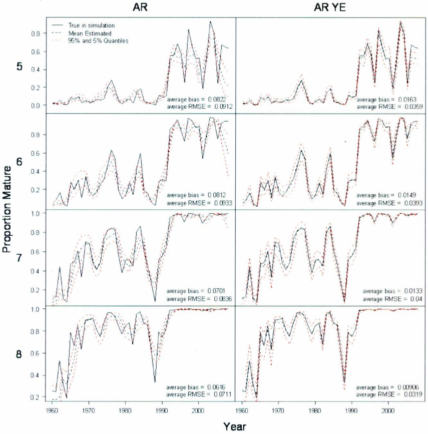

Figure 4.10: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 2.0$, $CE = 1.5$ and $MR = 1.3$.

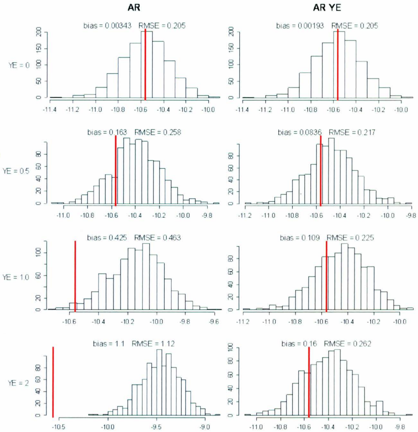

Figure 4.11: Histograms of the estimated fixed intercept. Simulated data were generated using simulation factors CE=0.5 and MR=1.3. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

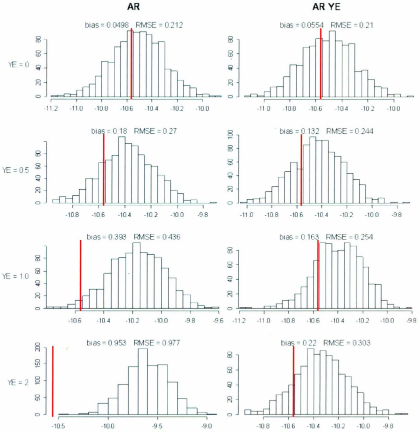

Figure 4.12: Histograms of the estimated fixed intercept. Simulated data were generated using simulation factors CE=1.5 and MR=1.3. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

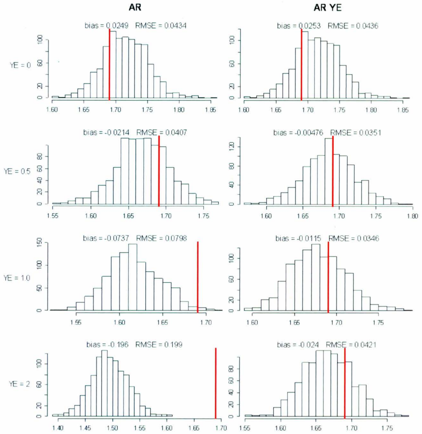

Figure 4.13: Histograms of the estimated fixed slope. Simulated data were generated using simulation factors CE=0.5 and MR=1.3. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

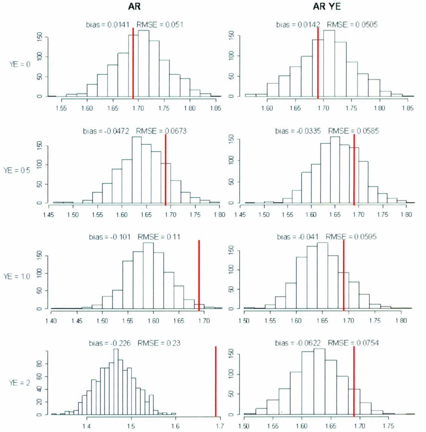

Figure 4.14: Histograms of the estimated fixed slope. Simulated data were generated using simulation factors CE=1.5 and MR=1.3. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

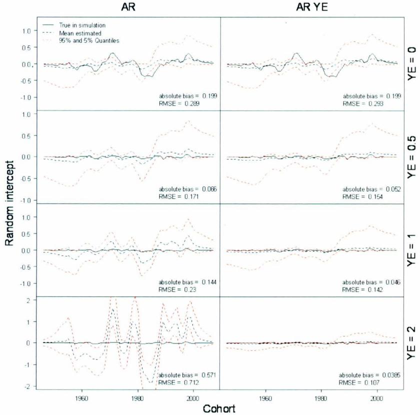

Figure 4.15: Simulated and mean estimated random intercept vs cohorts. Simulated data were generated using simulation factors CE=0.5 and MR=1.3. The YE factor used is listed in the right margin. Column 1: Random intercept estimated using the autoregressive (AR) mixed effects model. Column 2: Random intercept estimated using the AR model with year effects (AR YE).

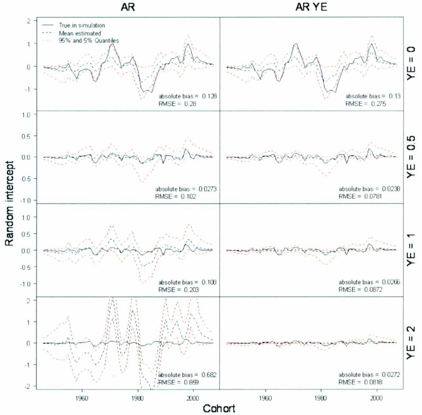

Figure 4.16: Simulated and mean estimated random intercept vs cohorts. Simulated data were generated using simulation factors CE=1.5 and MR=1.3. The YE factor used is listed in the right margin. Column 1: Random intercept estimated using the autoregressive (AR) mixed effects model. Column 2: Random intercept estimated using the AR model with year effects (AR YE).

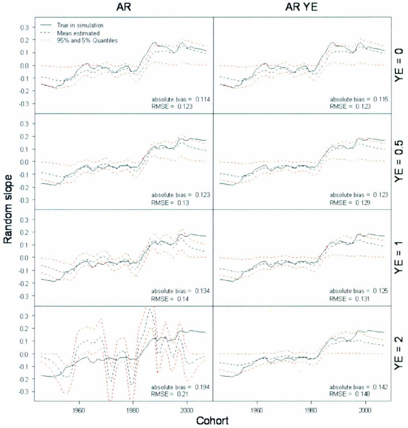

Figure 4.17: Simulated and mean estimated random slope vs cohorts. Simulated data were generated using simulation factors CE=0.5 and MR=1.3. The YE factor used is listed in the right margin. Column 1: Random slope estimated using the autoregressive (AR) mixed effects model. Column 2: Random slope estimated using the AR model with year effects (AR YE).

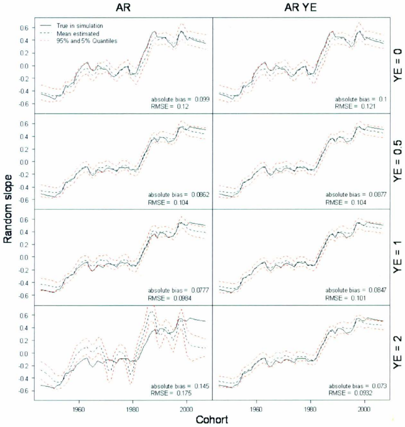

Figure 4.18: Simulated and mean estimated random slope vs cohorts. Simulated data were generated using simulation factors CE=1.5 and MR=1.3. The YE factor used is listed in the right margin. Column 1: Random slope estimated using the autoregressive (AR) mixed effects model. Column 2: Random slope estimated using the AR model with year effects (AR YE).

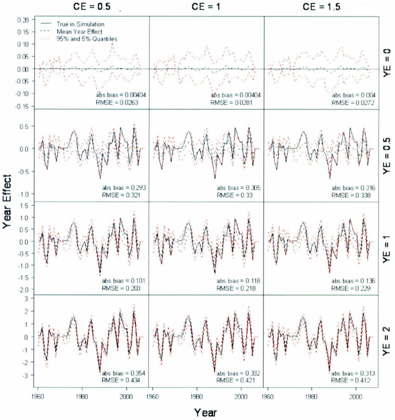

Figure 4.19: Simulated and mean estimated year effects vs years. Simulated data were generated using simulation factor MR=1.3. The CE and YE factors used are listed in the top and right margins respectively.

Chapter 5

Conclusion

To investigate if the use of mixed models could improve maturity estimates and forecast accuracies in Atlantic cod stocks, Dowden (2007) applied three models to the 3Ps cod maturity data. A generalized linear model called the fixed effects (FE) model, and two generalized linear mixed models- an autoregressive (AR) mixed effects model and a AR mixed effects model with random year effects (AR YE). Dowden's results showed that the mixed effects models are more appropriate choice for modeling maturities in Atlantic cod stocks with the AR YE model improving maturity estimates and forecast accuracies the most. All the models had similar estimated proportions mature, which are close to the actual proportions, however the AR YE model estimated proportions changed more smoothly than the other models when the year effects were treated as a nuisance parameter. The estimated $\hat{\beta}_{0c}$, $\hat{\beta}_{1c}$, $\widehat{MR}$ and $\hat{A}_{50}$ where also smoother over time for the AR YE model. The retrospective analysis showed that all the models have problems with accuracy in forecasting maturities for some years and have some large retrospective errors. However the AR YE model where the year effects are treated as a nuisance parameter had the best retrospective results, with the proportions mature varying less than the other models and with smaller retrospective errors.

Based on the observed maturities and the χ^2 residuals of the 3Ps cod data, there appeared to be some potential year effects in the data. Investigating the source of these potential year effects are beyond the scope of this practicum. Choosing the most appropriate mixed model to fit the cod maturity data depends on whether or not these potential year effects are real. The AR model is more appropriate when no year effects are present in the data, while the AR YE model is more appropriate if year effects are present. To investigate whether the AR or the AR YE model is more appropriate for modeling maturities when it is unknown if year effects are present in the data, a simulation study was conducted. Maturity datasets were generated using the 3Ps cod data parameter estimates of the AR and AR YE models and using three simulation factors, a cohort effect (CE) factor, maturity range (MR) factor and a year effect (YE) factor. Using three different levels of these simulation factors provides a range of contrast between cohorts, active ranges and the sizes of the year effects. Both the AR and AR YE models were applied to the simulated data.

From the results of the simulation study, the AR YE model appeared to be more appropriate than the AR model to fit data when the presences of year effects are unknown. Based on the ANOVA tables of the absolute bias and relative efficiency, MR was not a significant factor and was dropped. Comparing the absolute bias of the models averaged over MR and years showed that for a given age and level of CE, the bias for both models were very similar and close to 0 when no year effects were present. When year effects are present, the AR YE model's absolute bias is smaller than the AR model's for each given age and level of CE and YE. As the year effects increased, the absolute bias of the AR model increased while the AR YE model's bias had little change.

The relative efficiency also averaged over MR and years showed that the AR YE model is either as efficient or more efficient than the AR model depending on the year effects. When year effects are not present, there is very little difference in the efficiency of the models, with the AR model only very slightly more efficient then the AR YE model. With the presence of year effects, the AR YE model becomes more efficient than the AR model for all ages and levels of CE. As the year effects increases, the AR YE model becomes even more efficient than the AR model. This shows there is little risk in using the AR YE model. The performance of the AR YE model on Atlantic cod maturity data is very similar to the AR model when no year effects are present, and more accurate and efficient when they are present.

The AR YE model maturity estimates are similar or more accurate than the AR model's depending on the level of year effects. When there were no year effects present in the simulated data, the results of the AR YE model were very similar to the AR model results. With year effects in the data, the AR YE model fit better than the AR model. When the year effects were small or not present, the AR model's estimated maturities are close to the simulated population values. As the year effects increased, the AR model's estimated maturities became less accurate with the average absolute bias and the RMSE increasing as the year effects increased. The AR YE model's estimated maturities remained close to the simulated population values for all levels of year effects with only slight increases in the RMSE as the year effects increased.

Comparing the AR YE model's estimated year effects with the simulated values showed that the estimates are most accurate when year effects are low. When no year effects were present, the estimated year effects were close to zero with very small average absolute bias and RMSE. When year effects were present, the estimated year effects were closest to the simulated population values for the medium level of year effects (YE=1). As the year effects increased the estimates became less accurate with the average absolute bias and RMSE increasing.

The fixed and random effects estimates of the AR and AR YE model were also compared at each level of YE. Histograms of the fixed intercept and fixed slope estimates showed that when no year effects were present, the estimates of the AR and AR YE model were very similar and close to the simulated values. Both models tend to overestimate the fixed intercept and underestimate the fixed slope when year effects were present. The AR YE model estimates for both fixed effects were closer to the simulated values than the AR model estimates. As the year effects increased, the accuracy of the AR model estimates worsened with both the bias and RMSE increasing. There was little change in the accuracy of the AR YE estimates, with the bias and RMSE increasing slightly.

Comparing the random effects estimates of both models with the simulated values showed that when no year effects were present or year effects were low, the random intercept and random slope estimates of the AR and AR YE model were very similar. When year effects are present, the AR YE model estimates are closer to the simulated values than the AR estimates. As the year effects increased, the accuracy of the AR model estimates worsened with the average absolute bias and RMSE increasing with YE. The accuracy of the AR YE model estimates changed very little as YE increased, with the average absolute bias and RMSE of the random intercept estimates decreasing slightly and the random slope estimates absolute bias and RMSE increasing slightly.

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Appendix A

Figures from the Simulation Study

A.1 Simulated and mean estimated proportion mature vs year figures

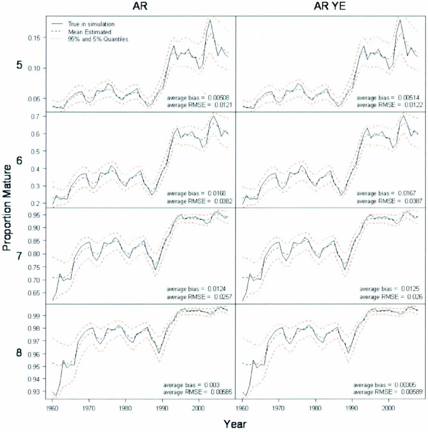

Figure A.1: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with no year effects generated using simulation factors $CE = 0.5$ and $MR = 1.0$.

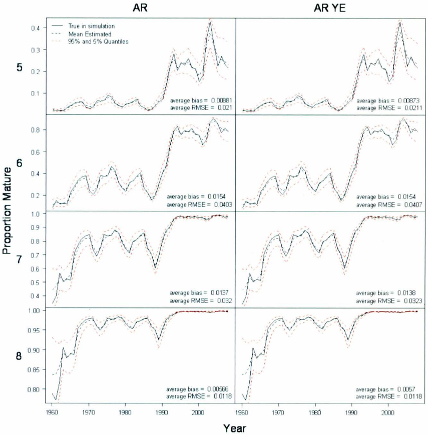

Figure A.2: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with no year effects generated using simulation factors $CE = 1.0$ and $MR = 1.0$.

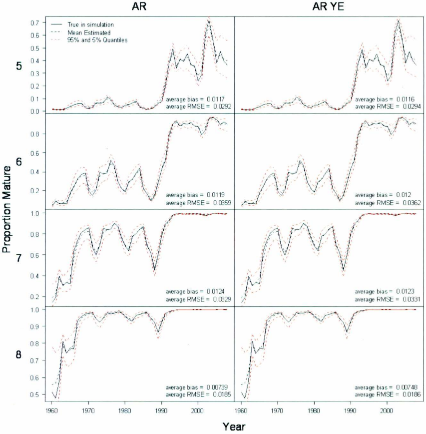

Figure A.3: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with no year effects generated using simulation factors $CE = 1.5$ and $MR = 1.0$.

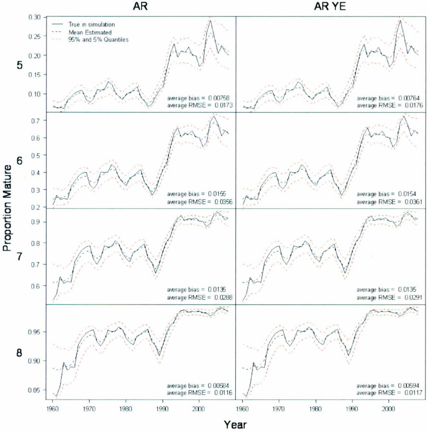

Figure A.4: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with no year effects generated using simulation factors $CE = 0.5$ and $MR = 1.3$.

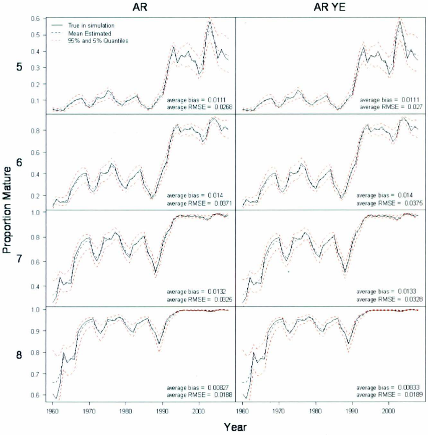

Figure A.5: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with no year effects generated using simulation factors $CE = 1.0$ and $MR = 1.3$.

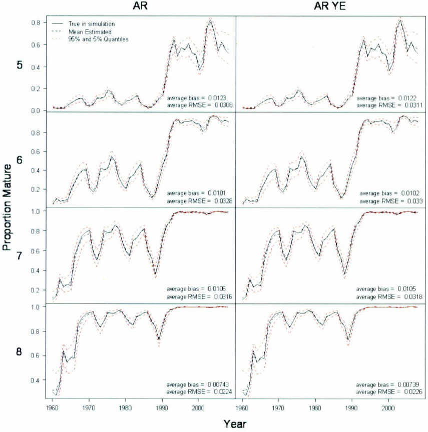

Figure A.6: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with no year effects generated using simulation factors $CE = 1.5$ and $MR = 1.3$.

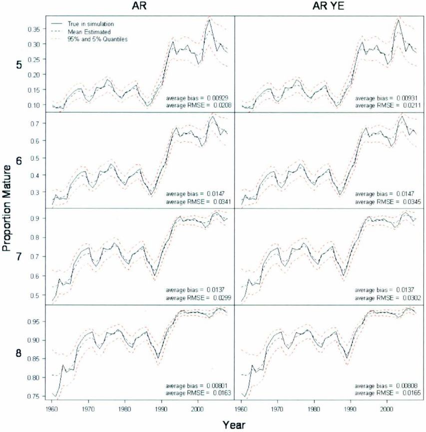

Figure A.7: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with no year effects generated using simulation factors $CE = 0.5$ and $MR = 1.6$.

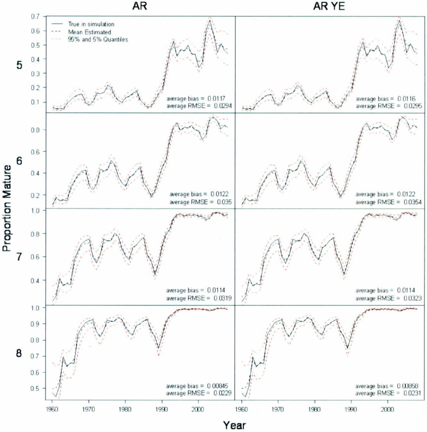

Figure A.8: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with no year effects generated using simulation factors $CE = 1.0$ and $MR = 1.6$.

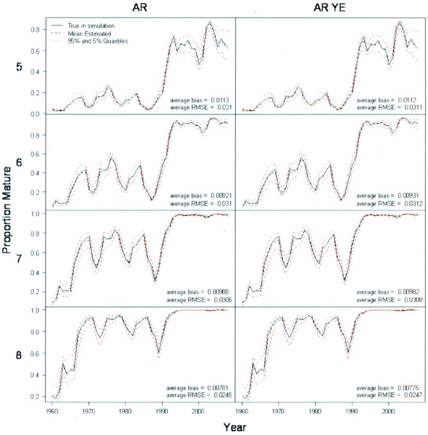

Figure A.9: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with no year effects generated using simulation factors $CE = 1.5$ and $MR = 1.6$.

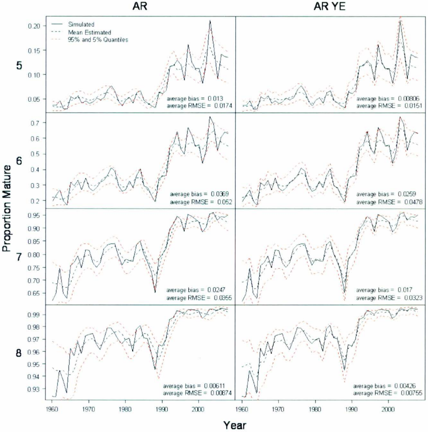

Figure A.10: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 0.5$, $CE = 0.5$ and $MR = 1.0$.

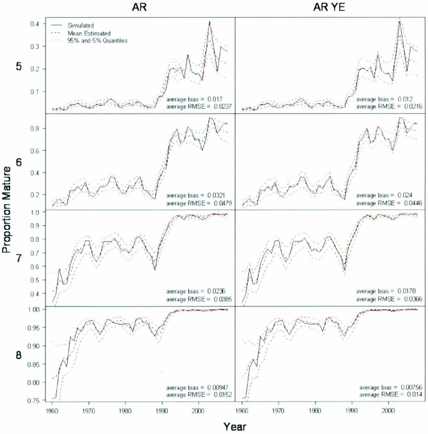

Figure A.11: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 0.5$, $CE = 1.0$ and $MR = 1.0$.

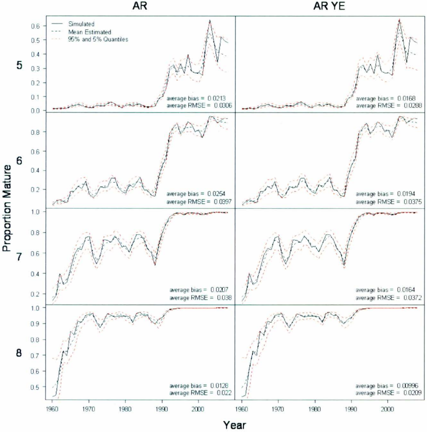

Figure A.12: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 0.5$, $CE = 1.5$ and $MR = 1.0$.

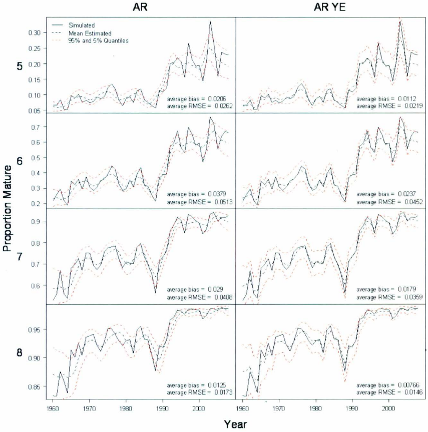

Figure A.13: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 0.5$, $CE = 0.5$ and $MR = 1.3$.

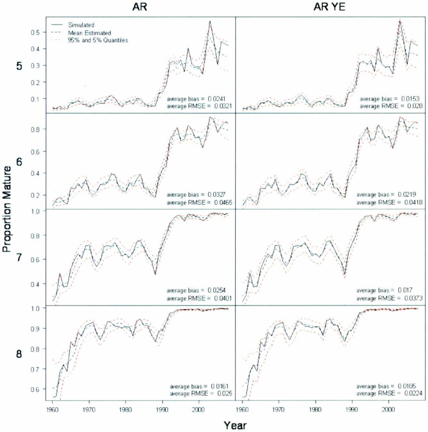

Figure A.14: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 0.5$, $CE = 1.0$ and $MR = 1.3$.

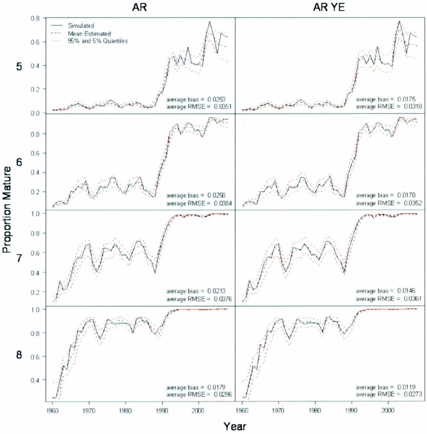

Figure A.15: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 0.5$, $CE = 1.5$ and $MR = 1.3$.

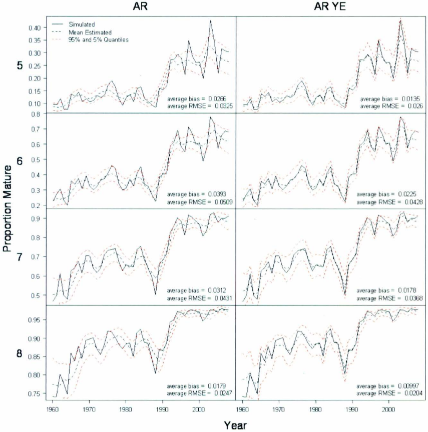

Figure A.16: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 0.5$, $CE = 0.5$ and $MR = 1.6$.

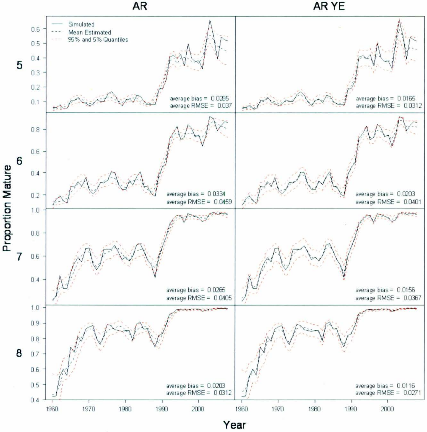

Figure A.17: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 0.5$, $CE = 1.0$ and $MR = 1.6$.

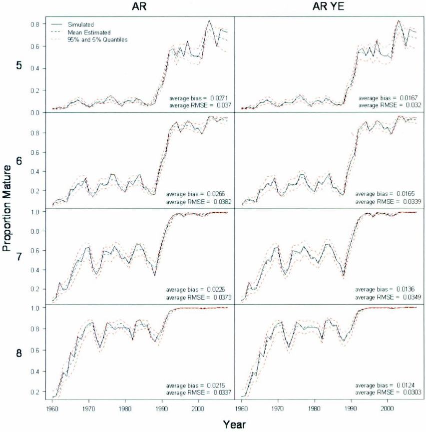

Figure A.18: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 0.5$, $CE = 1.5$ and $MR = 1.6$.

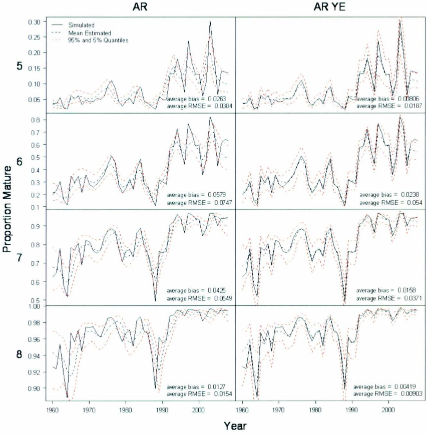

Figure A.19: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 1.0$, $CE = 0.5$ and $MR = 1.0$.

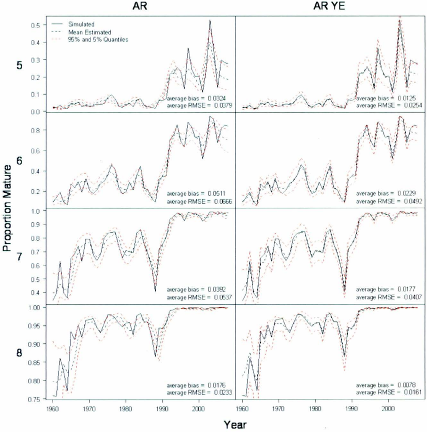

Figure A.20: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 1.0$, $CE = 1.0$ and $MR = 1.0$.

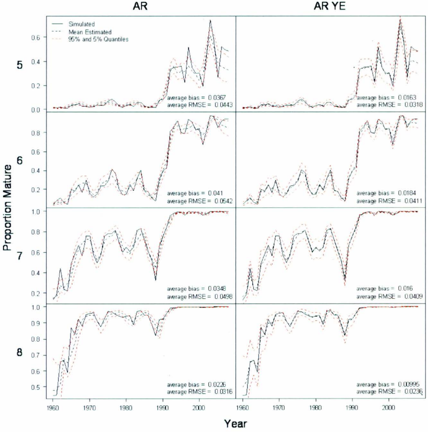

Figure A.21: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 1.0$, $CE = 1.5$ and $MR = 1.0$.

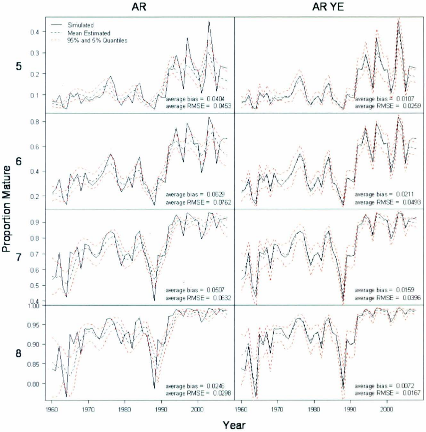

Figure A.22: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 1.0$, $CE = 0.5$ and $MR = 1.3$.

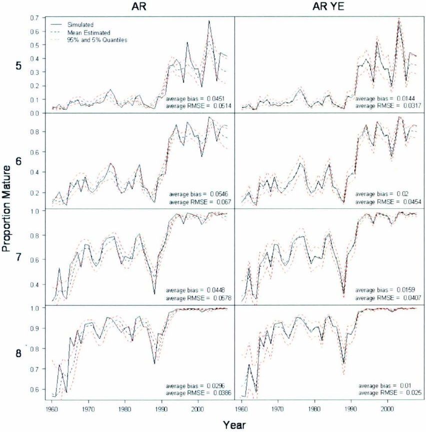

Figure A.23: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 1.0$, $CE = 1.0$ and $MR = 1.3$.

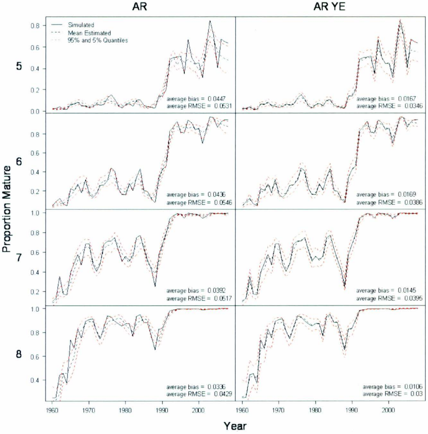

Figure A.24: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 1.0$, $CE = 1.5$ and $MR = 1.3$.

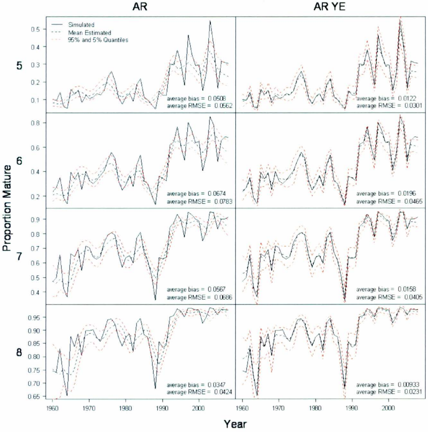

Figure A.25: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 1.0$, $CE = 0.5$ and $MR = 1.6$.

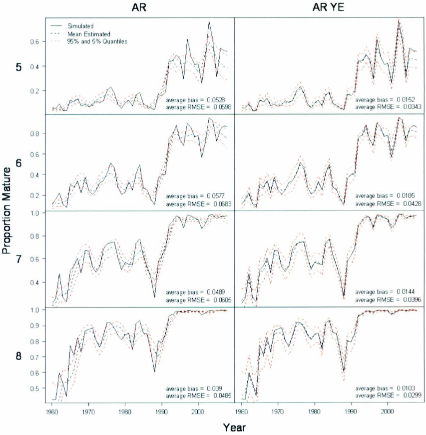

Figure A.26: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 1.0$, $CE = 1.0$ and $MR = 1.6$.

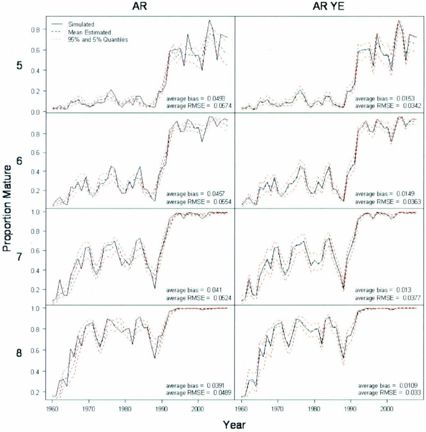

Figure A.27: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 1.0$, $CE = 1.5$ and $MR = 1.6$.

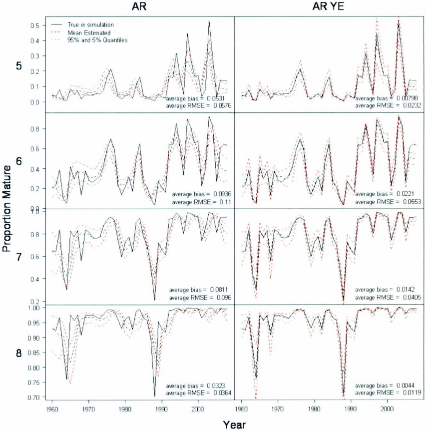

Figure A.28: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 2.0$, $CE = 0.5$ and $MR = 1.0$.

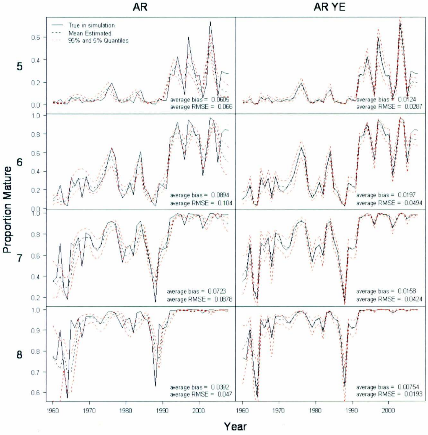

Figure A.29: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 2.0$, $CE = 1.0$ and $MR = 1.0$.

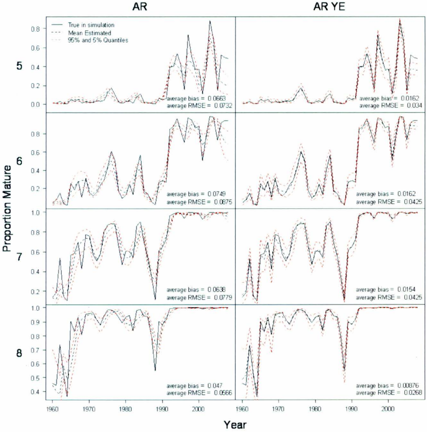

Figure A.30: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 2.0$, $CE = 1.5$ and $MR = 1.0$.

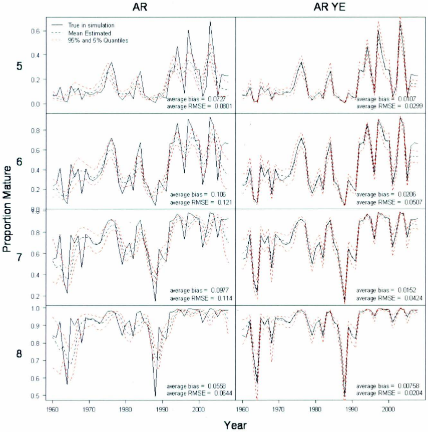

Figure A.31: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 2.0$, $CE = 0.5$ and $MR = 1.3$.

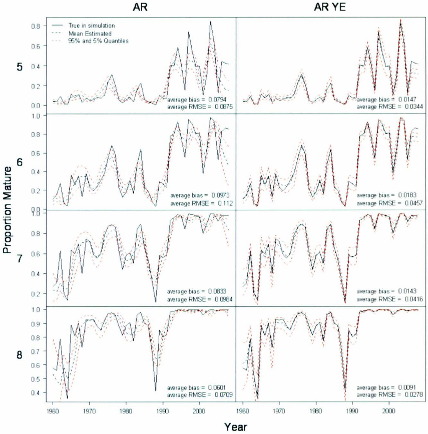

Figure A.32: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 2.0$, $CE = 1.0$ and $MR = 1.3$.

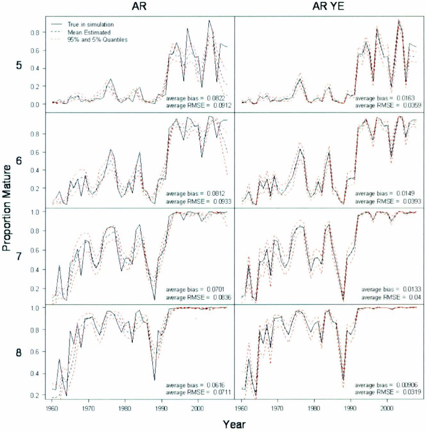

Figure A.33: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 2.0$, $CE = 1.5$ and $MR = 1.3$.

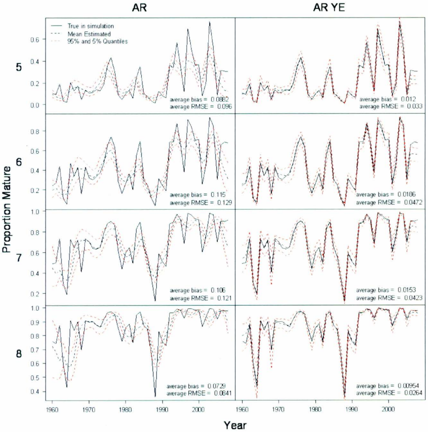

Figure A.34: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 2.0$, $CE = 0.5$ and $MR = 1.6$.

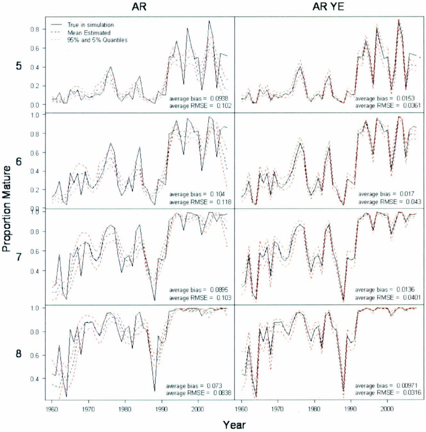

Figure A.35: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 2.0$, $CE = 1.0$ and $MR = 1.6$.

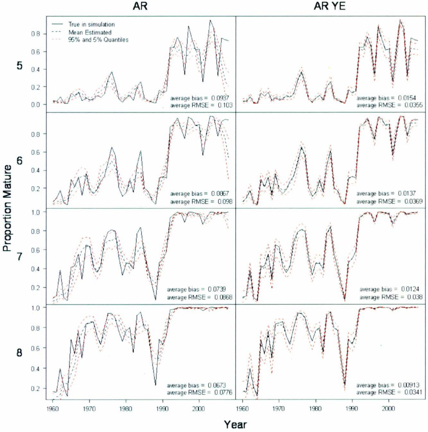

Figure A.36: Simulated and mean estimated proportion mature vs year. Column 1: Proportions mature estimated using the autoregressive (AR) mixed effects model. Column 2: Proportions mature estimated using the AR model with year effects (AR YE). Simulated data with year effects generated using simulation factors $YE = 2.0$, $CE = 1.5$ and $MR = 1.6$.

A.2 Histograms of the estimated fixed intercept

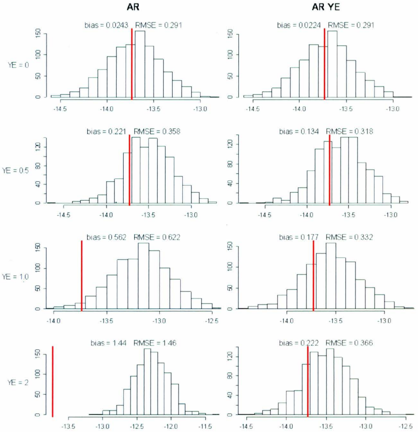

Figure A.37: Histograms of the estimated fixed intercept. Simulated data were generated using simulation factors CE=0.5 and MR=1.0. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

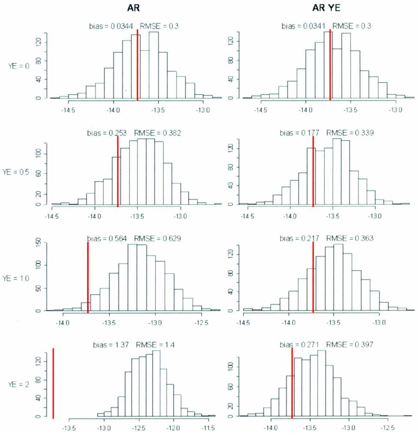

Figure A.38: Histograms of the estimated fixed intercept. Simulated data were generated using simulation factors CE=1.0 and MR=1.0. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

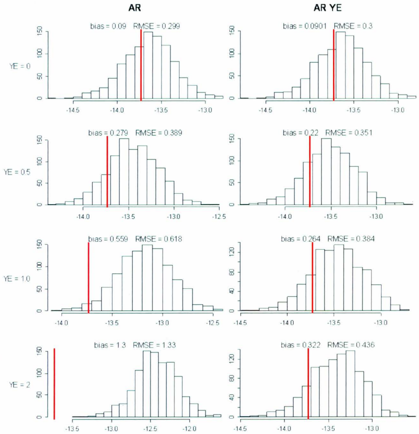

Figure A.39: Histograms of the estimated fixed intercept. Simulated data were generated using simulation factors CE=1.5 and MR=1.0. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

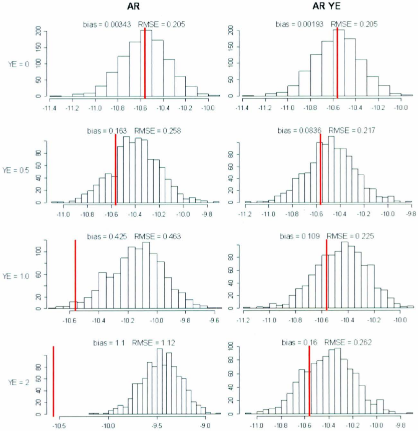

Figure A.40: Histograms of the estimated fixed intercept. Simulated data were generated using simulation factors CE=0.5 and MR=1.3. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

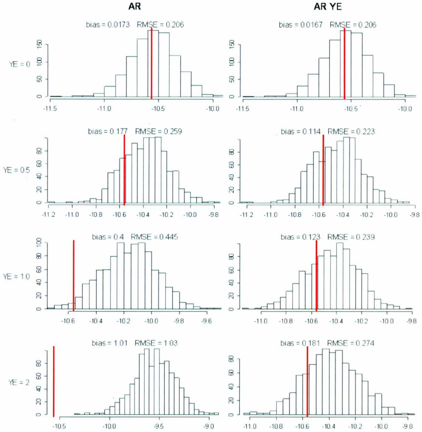

Figure A.41: Histograms of the estimated fixed intercept. Simulated data were generated using simulation factors CE=1.0 and MR=1.3. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

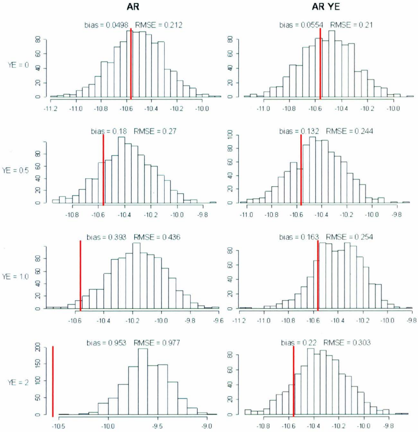

Figure A.42: Histograms of the estimated fixed intercept. Simulated data were generated using simulation factors CE=1.5 and MR=1.3. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

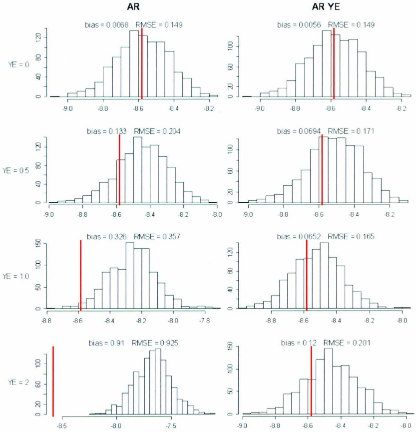

Figure A.43: Histograms of the estimated fixed intercept. Simulated data were generated using simulation factors CE=0.5 and MR=1.6. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

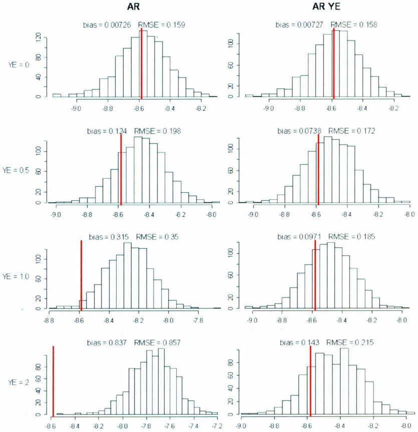

Figure A.44: Histograms of the estimated fixed intercept. Simulated data were generated using simulation factors CE=1.0 and MR=1.6. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

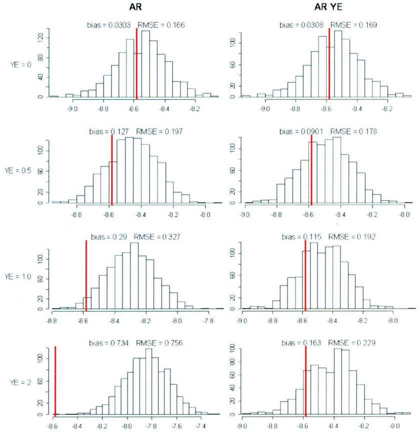

Figure A.45: Histograms of the estimated fixed intercept. Simulated data were generated using simulation factors CE=1.5 and MR=1.6. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

A.3 Histograms of the estimated fixed slope

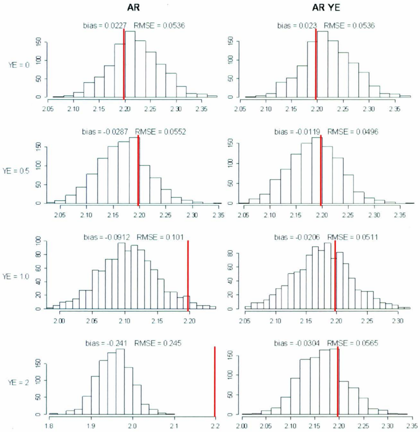

Figure A.46: Histograms of the estimated fixed slope. Simulated data were generated using simulation factors CE=0.5 and MR=1.0. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

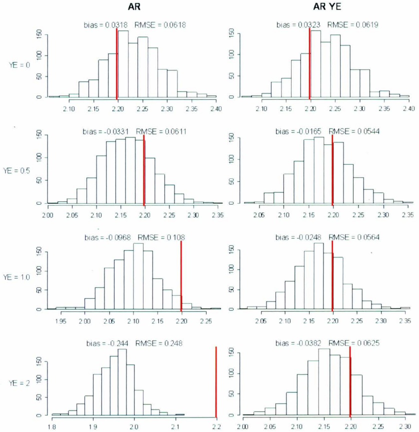

Figure A.47: Histograms of the estimated fixed slope. Simulated data were generated using simulation factors CE=1.0 and MR=1.0. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

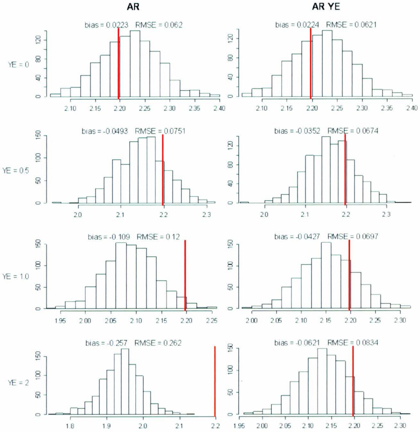

Figure A.48: Histograms of the estimated fixed slope. Simulated data were generated using simulation factors CE=1.5 and MR=1.0. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

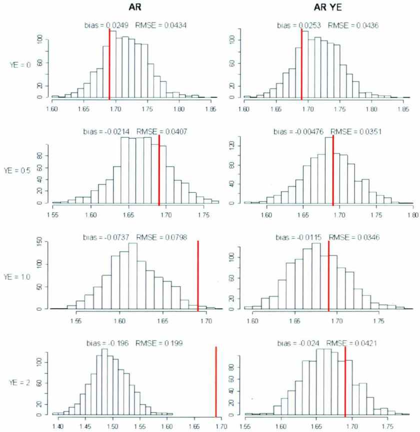

Figure A.49: Histograms of the estimated fixed slope. Simulated data were generated using simulation factors CE=0.5 and MR=1.3. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

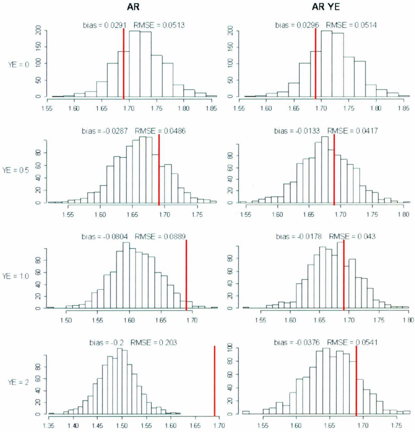

Figure A.50: Histograms of the estimated fixed slope. Simulated data were generated using simulation factors CE=1.0 and MR=1.3. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

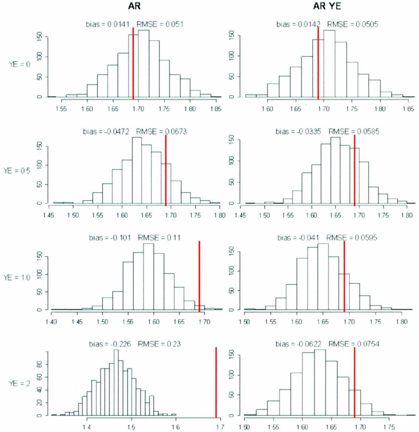

Figure A.51: Histograms of the estimated fixed slope. Simulated data were generated using simulation factors CE=1.5 and MR=1.3. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

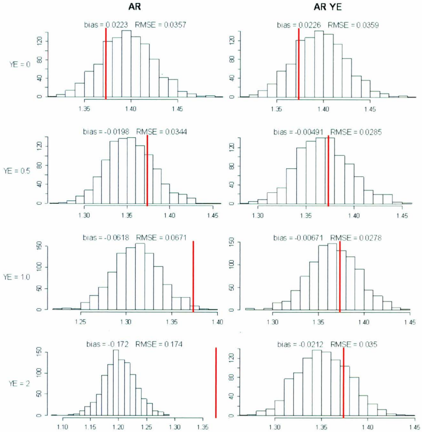

Figure A.52: Histograms of the estimated fixed slope. Simulated data were generated using simulation factors CE=0.5 and MR=1.6. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

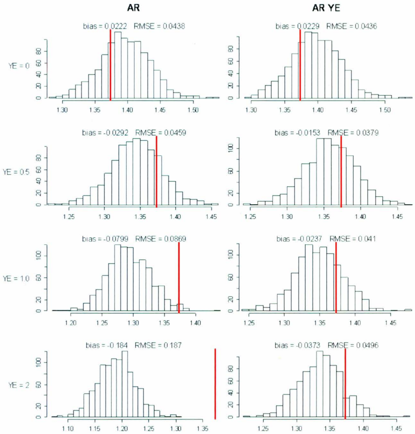

Figure A.53: Histograms of the estimated fixed slope. Simulated data were generated using simulation factors CE=1.0 and MR=1.6. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

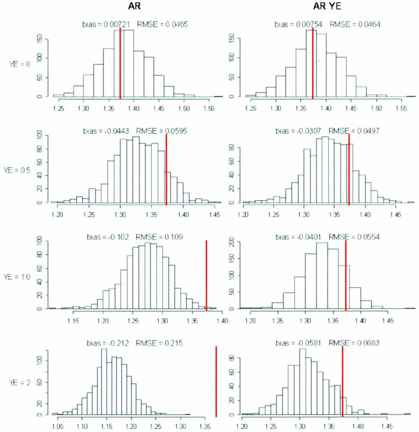

Figure A.54: Histograms of the estimated fixed slope. Simulated data were generated using simulation factors CE=1.5 and MR=1.6. The YE factor used is listed in the right margin. Vertical lines indicate the value of the true parameter used to generate the data. First column: AR model estimates. Second column: AR YE model estimates.

A.4 Simulated and mean estimated random intercept vs cohorts figures

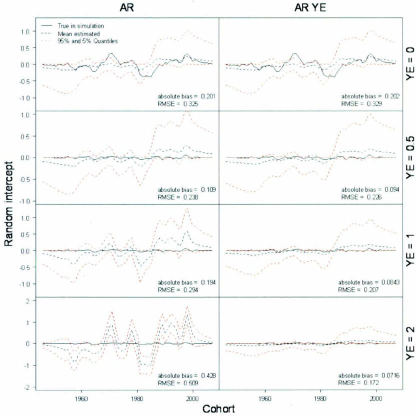

Figure A.55: Simulated and mean estimated random intercept vs cohorts. Simulated data were generated using simulation factors CE=0.5 and MR=1.0. The YE factor used is listed in the right margin. Column 1: Random intercept estimated using the autoregressive (AR) mixed effects model. Column 2: Random intercept estimated using the AR model with year effects (AR YE).

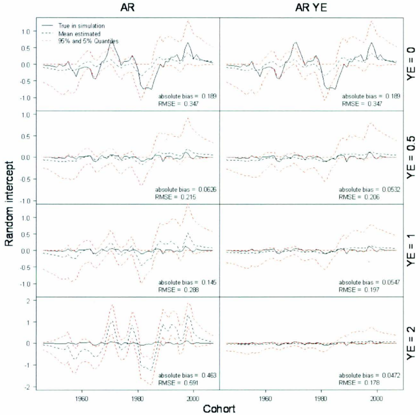

Figure A.56: Simulated and mean estimated random intercept vs cohorts. Simulated data were generated using simulation factors CE=1.0 and MR=1.0. The YE factor used is listed in the right margin. Column 1: Random intercept estimated using the autoregressive (AR) mixed effects model. Column 2: Random intercept estimated using the AR model with year effects (AR YE).

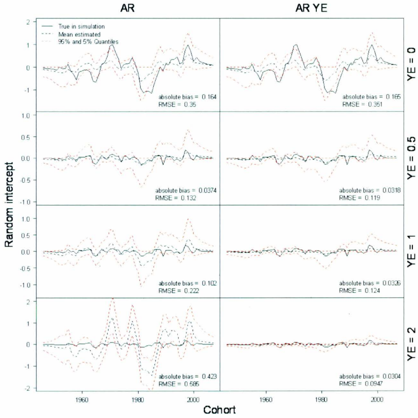

Figure A.57: Simulated and mean estimated random intercept vs cohorts. Simulated data were generated using simulation factors CE=1.5 and MR=1.0. The YE factor used is listed in the right margin. Column 1: Random intercept estimated using the autoregressive (AR) mixed effects model. Column 2: Random intercept estimated using the AR model with year effects (AR YE).

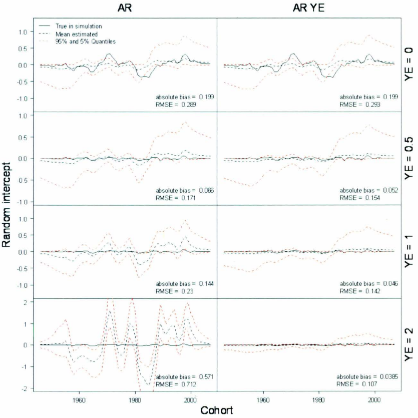

Figure A.58: Simulated and mean estimated random intercept vs cohorts. Simulated data were generated using simulation factors CE=0.5 and MR=1.3. The YE factor used is listed in the right margin. Column 1: Random intercept estimated using the autoregressive (AR) mixed effects model. Column 2: Random intercept estimated using the AR model with year effects (AR YE).

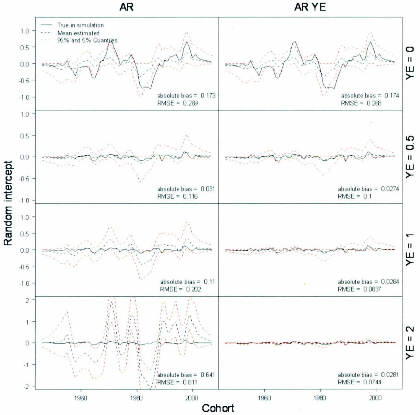

Figure A.59: Simulated and mean estimated random intercept vs cohorts. Simulated data were generated using simulation factors CE=1.0 and MR=1.3. The YE factor used is listed in the right margin. Column 1: Random intercept estimated using the autoregressive (AR) mixed effects model. Column 2: Random intercept estimated using the AR model with year effects (AR YE).

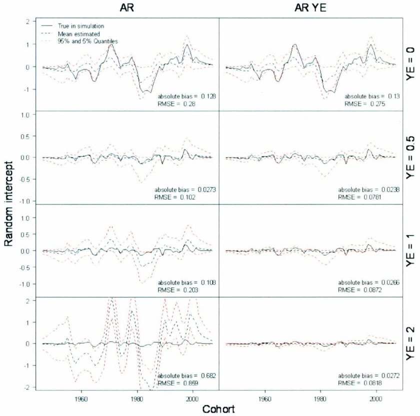

Figure A.60: Simulated and mean estimated random intercept vs cohorts. Simulated data were generated using simulation factors CE=1.5 and MR=1.3. The YE factor used is listed in the right margin. Column 1: Random intercept estimated using the autoregressive (AR) mixed effects model. Column 2: Random intercept estimated using the AR model with year effects (AR YE).

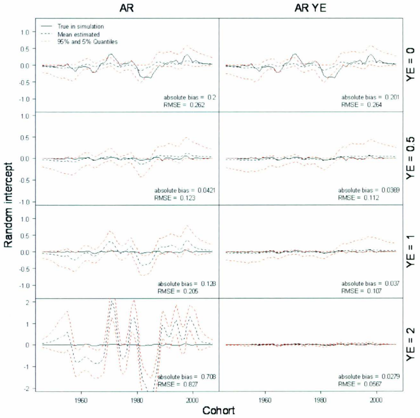

Figure A.61: Simulated and mean estimated random intercept vs cohorts. Simulated data were generated using simulation factors CE=0.5 and MR=1.6. The YE factor used is listed in the right margin. Column 1: Random intercept estimated using the autoregressive (AR) mixed effects model. Column 2: Random intercept estimated using the AR model with year effects (AR YE).

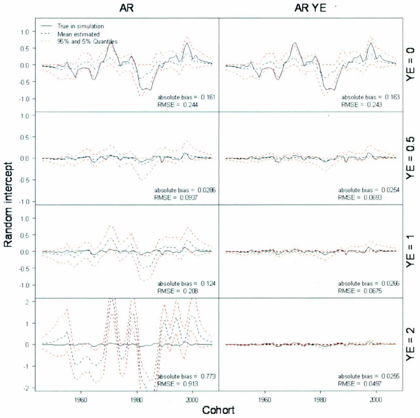

Figure A.62: Simulated and mean estimated random intercept vs cohorts. Simulated data were generated using simulation factors CE=1.0 and MR=1.6. The YE factor is listed in the right margin. Column 1: Random intercept estimated using the autoregressive (AR) mixed effects model. Column 2: Random intercept estimated using the AR model with year effects (AR YE).

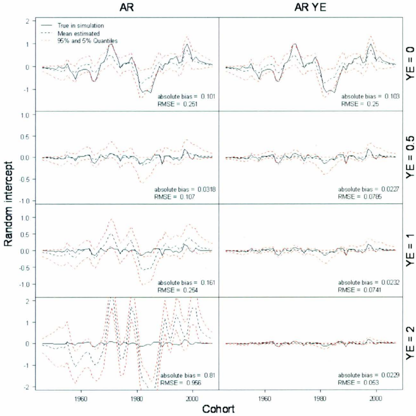

Figure A.63: Simulated and mean estimated random intercept vs cohorts. Simulated data were generated using simulation factors CE=1.5 and MR=1.6. The YE factor used is listed in the right margin. Column 1: Random intercept estimated using the autoregressive (AR) mixed effects model. Column 2: Random intercept estimated using the AR model with year effects (AR YE).

A.5 Simulated and mean estimated random slope vs cohorts figures

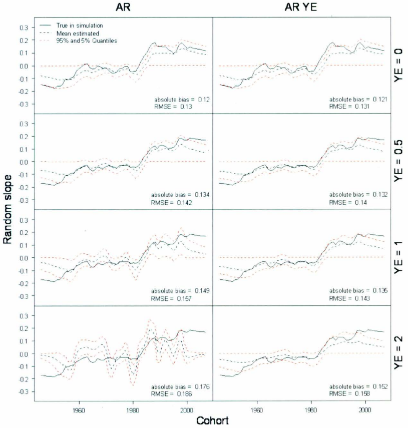

Figure A.64: Simulated and mean estimated random slope vs cohorts. Simulated data were generated using simulation factors CE=0.5 and MR=1.0. The YE factor used is listed in the right margin. Column 1: Random slope estimated using the autoregressive (AR) mixed effects model. Column 2: Random slope estimated using the AR model with year effects (AR YE).

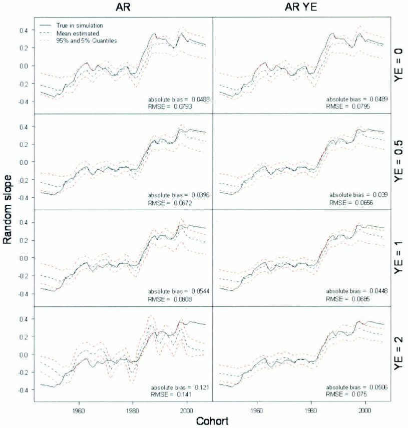

Figure A.65: Simulated and mean estimated random slope vs cohorts. Simulated data were generated using simulation factors CE=1.0 and MR=1.0. The YE factor used is listed in the right margin. Column 1: Random slope estimated using the autoregressive (AR) mixed effects model. Column 2: Random slope estimated using the AR model with year effects (AR YE).

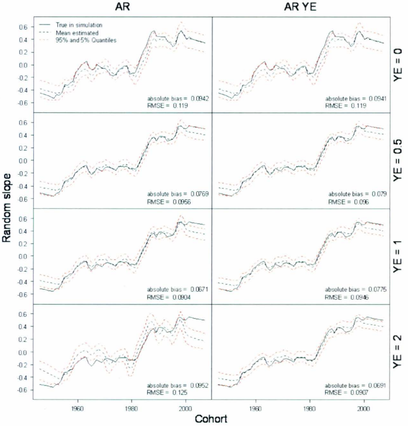

Figure A.66: Simulated and mean estimated random slope vs cohorts. Simulated data were generated using simulation factors CE=1.5 and MR=1.0. The YE factor used is listed in the right margin. Column 1: Random slope estimated using the autoregressive (AR) mixed effects model. Column 2: Random slope estimated using the AR model with year effects (AR YE).

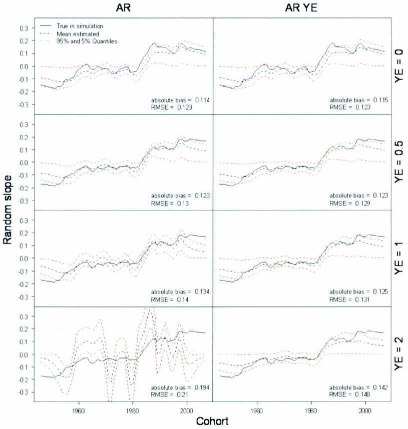

Figure A.67: Simulated and mean estimated random slope vs cohorts. Simulated data were generated using simulation factors CE=0.5 and MR=1.3. The YE factor used is listed in the right margin. Column 1: Random slope estimated using the autoregressive (AR) mixed effects model. Column 2: Random slope estimated using the AR model with year effects (AR YE).

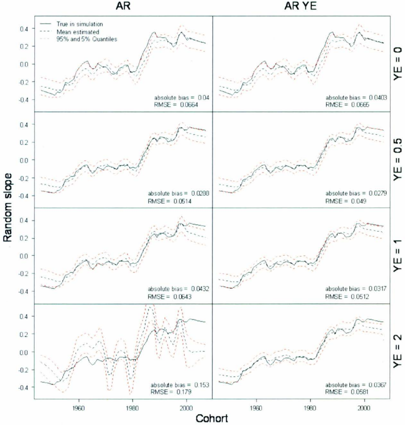

Figure A.68: Simulated and mean estimated random slope vs cohorts. Simulated data were generated using simulation factors CE=1.0 and MR=1.3. The YE factor used is listed in the right margin. Column 1: Random slope estimated using the autoregressive (AR) mixed effects model. Column 2: Random slope estimated using the AR model with year effects (AR YE).

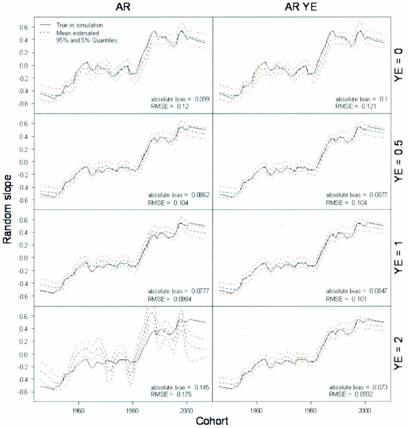

Figure A.69: Simulated and mean estimated random slope vs cohorts. Simulated data were generated using simulation factors CE=1.5 and MR=1.3. The YE factor used is listed in the right margin. Column 1: Random slope estimated using the autoregressive (AR) mixed effects model. Column 2: Random slope estimated using the AR model with year effects (AR YE).

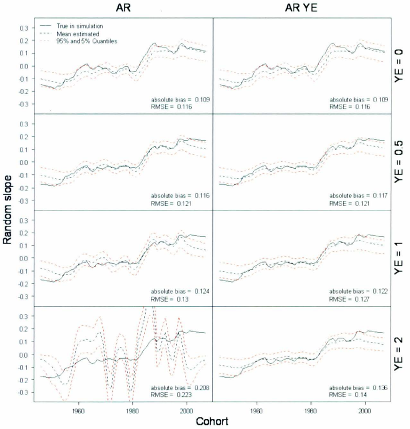

Figure A.70: Simulated and mean estimated random slope vs cohorts. Simulated data were generated using simulation factors CE=0.5 and MR=1.6. The YE factor used is listed in the right margin. Column 1: Random slope estimated using the autoregressive (AR) mixed effects model. Column 2: Random slope estimated using the AR model with year effects (AR YE).

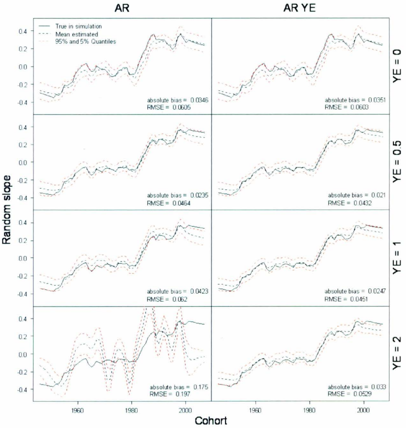

Figure A.71: Simulated and mean estimated random slope vs cohorts. Simulated data were generated using simulation factors CE=1.0 and MR=1.6. The YE factor used is listed in the right margin. Column 1: Random slope estimated using the autoregressive (AR) mixed effects model. Column 2: Random slope estimated using the AR model with year effects (AR YE).

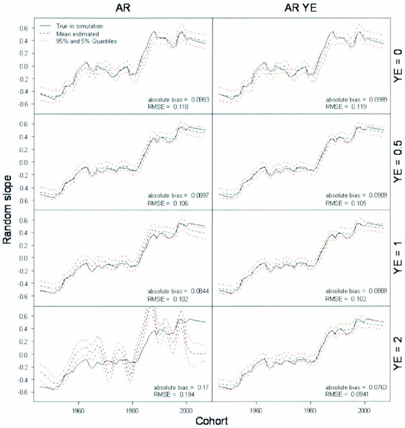

Figure A.72: Simulated and mean estimated random slope vs cohorts. Simulated data were generated using simulation factors CE=1.5 and MR=1.6. The YE factor used is listed in the right margin. Column 1: Random slope estimated using the autoregressive (AR) mixed effects model. Column 2: Random slope estimated using the AR model with year effects (AR YE).

A.6 Simulated and mean estimated year effects vs years figures

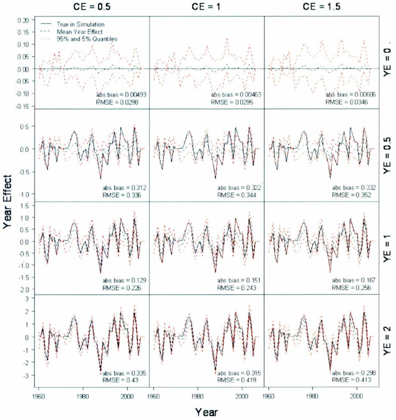

Figure A.73: Simulated and mean estimated year effects vs years. Simulated data were generated using simulation factor MR=1.0. The CE and YE factors used are listed in the top and right margins respectively.

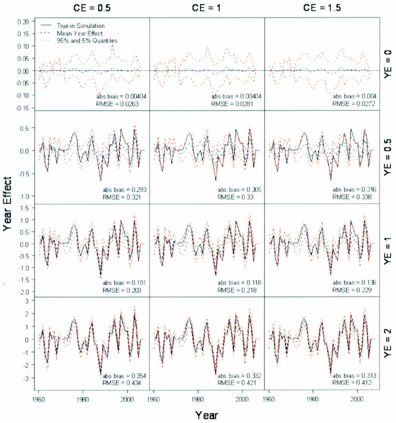

Figure A.74: Simulated and mean estimated year effects vs years. Simulated data were generated using simulation factor MR=1.3. The CE and YE factors used are listed in the top and right margins respectively.

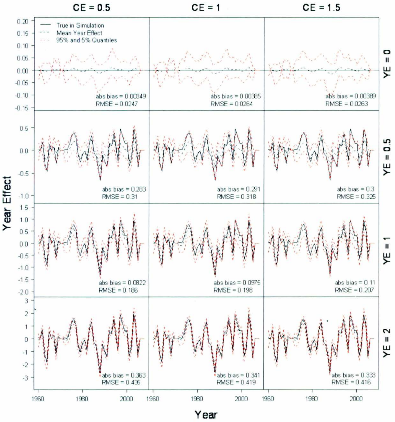

Figure A.75: Simulated and mean estimated year effects vs years. Simulated data were generated using simulation factor MR=1.6. The CE and YE factors used are listed in the top and right margins respectively.