

A GRADE 9 COORDINATE GEOMETRY UNIT:  
BRIDGING BASIC SKILLS AND THE APEF  
CURRICULUM

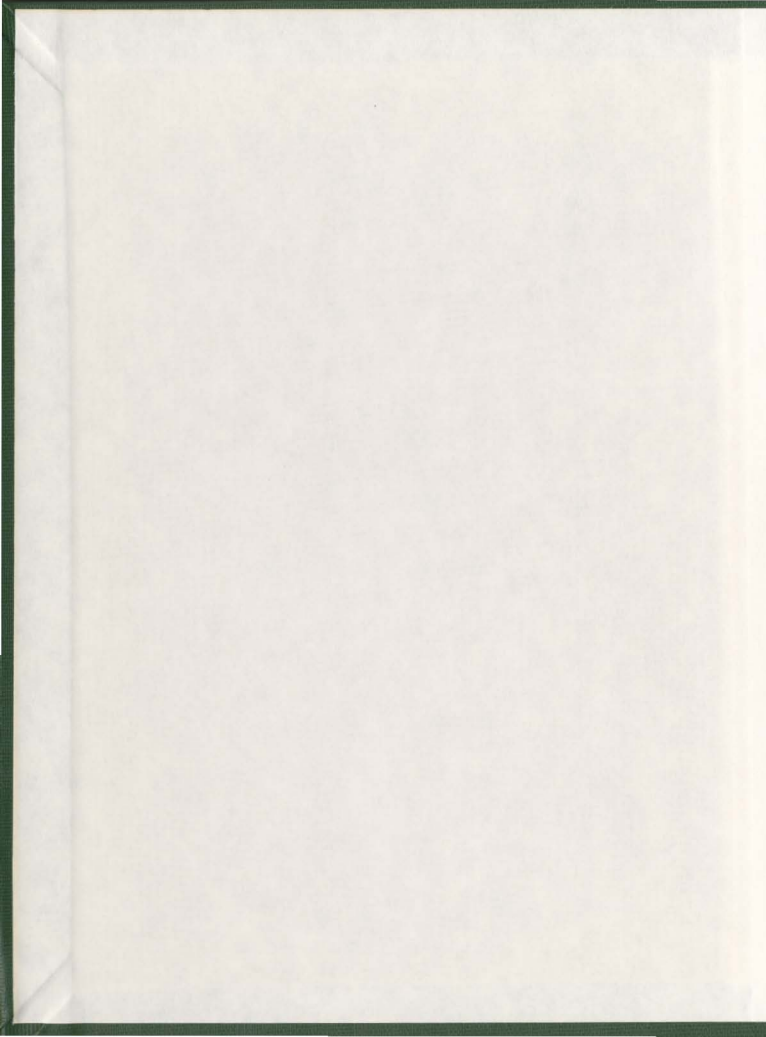
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**A GRADE 9 COORDINATE GEOMETRY UNIT:  
BRIDGING BASIC SKILLS AND THE APEF  
CURRICULUM**

by

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**A project submitted to the school of Graduate Studies  
in partial fulfilment of the  
requirements for the degree  
Master of Education**

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## **Abstract**

The changing philosophy towards how students learn mathematics has rekindled the confrontation between investigative and traditional pedagogical camps. Labelled the math wars, both groups are blaming the other for the poor mastery of basic mathematical skills as indicated by student assessment scores. Societal, educational researcher and student views of the basic skills issues are presented. Acknowledging that each student may learn in a different way and at a different rate, the performance and mastery type students are examined metacognitively. A number of sample pedagogical practices that reflect the philosophy of this project have been chosen for presentation. A coordinate geometry project, moulded with ideas of mathematics learning as advocated by the Atlantic Provinces Education Foundation (APEF), is presented which reflects the conviction that both constructivist and traditional ideologies of mathematics should be a balanced part of student learning.

### **Acknowledgments**

I wish to acknowledge the support and encouragement of my family and friends during this educational journey. To Isabelle and Gordon, my parents, who with pride for their children. believed education is always a worthy goal; to Debbie, my wife, for her love, patience and shoulder; to Mary for her motivation and editing; to Carol for her encouragement and advice: and Dr. John Grant McLoughlin for helping tie it all together. Thank you.

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## **PREAMBLE**

A topical issue in mathematics is North American students' lacklustre performance on traditional standardized tests. The blame appears to be centred around the belief that students have not mastered fundamental or basic math skills (Anthony, 1999; Burns, 1998; Carnine, Dixon, & Jones, 1994; Colvin, 1999; Cornwall, 1999; Friesen, Clifford, & Jardine, 1999; Raimi & Braden, 1998; Silver, 1997; Wittman, Marcinkiewicz, & Hamodey-Douglas, 1998). So called "math wars" have sprung up in districts that have begun to introduce newer mathematical curriculum reflecting the constructivist view (Burns, 1998; Colvin, 1999). Traditionalists are appalled by the low student achievement on assessments such as the Third International Mathematics and Science Study (TIMSS) (Silver, 1997). They have used the results of these and other similar assessments to question whether a brief, investigative exposure of children to the many math concepts they are expected to master is enough to facilitate learning (Anthony, 1999; Cornwall, 1999; Carnine, et al., 1994). Proponents of the investigative math approach have argued that current assessment techniques do not reflect how the student learns constructively, and they argue that the traditional approach to mathematics teaching and learning has not been successful in the past (Battista, 1999; Bowman, 1998; Devlin, 1999; O'Brien, 1999; Pita, 2000). The implementation of the Atlantic Provinces Education Foundation (1999) (APEF) curriculum at the local level represents a shift towards this investigative approach. Student preparedness in terms of basic skills continues to be an ongoing concern with parents and educators. This project attempts to address the pending transitional issue through a local junior high context.

## **LITERATURE REVIEW**

This project is intended to offer a bridge to intermediate students in instructional (and learning) style by presenting the Grade 9 mathematics unit, Coordinate Geometry, in a more investigative manner than the current curriculum in place now suggests. Yet it does not propose an absolute investigative approach as practice is an important method in achieving mastery of mathematical concepts (Bachelis, 1999; Carnine, et al., 1994; Cornwall, 1999; Raimi & Braden 1998; Silver, 1997; Wittman, et al.,1998). The philosophy adopted for this unit is a middle of the road approach, a balance between the investigative and traditionalist camps.

The literature review begins with details of a telephone conversation with Dr. Herb Gaskill, Head of the Mathematics and Statistics Department at Memorial University of Newfoundland, thus providing an introduction to local concerns. Before proceeding further, it is important to adopt a working definition of basic mathematical skills. The definition is accompanied by consideration of the skills issue as seen through the eyes of society, educational researchers, and the students themselves. The focus then shifts to a brief examination of student math anxiety followed by a metacognitive discussion of performance and mastery oriented students, and sample teaching practices that reflect the philosophy of this project.

### **Local Concerns**

Dr. Herb Gaskill, Head of the Mathematics and Statistics Department at Memorial University of Newfoundland stated in a telephone interview on March 9, 2001, that approximately

35% of first year test takers have failed the Mathematics Skills Inventory (MSI) exam over the past 3 years of its administration. Hennebury (1999) summarizes the MSI and its function as a placement test designed to gauge basic skills and predict which students may experience difficulty in first year mathematics courses. The MSI comprises 100 multiple choice items consisting of 20 sections of 5 topically related questions each, ranging from fractions to algebra to trigonometry. Note that the MSI is required to be written and passed prior to students registering for their first finite math or precalculus course.

As provided by Dr. Gaskill, the following table is a summary of the MSI marks attained by approximately 1,100 students in Academic or Advanced high school math courses for the academic years 1998 - 1999 and 1999 - 2000.

Table 1. Summary of Mean High School and MSI Marks

	1998-1999		1999-2000	
	High School Mark	MSI Mark	High School Mark	MSI Mark
Advanced Math 3201	73.13	66.16	70.14	66.86
Academic Math 3200	74.34	51.20	72.98	53.05
Academic Math 3203	64.18	44.27	67.65	47.42

Anthony (1999) quotes Student Union VP Academic, Keith Dunne, "These numbers [in the above table] are almost the same as those last year – on a test designed to gauge basic skills, it is appalling that so many students are not performing up to par. Clearly the K - 12 system is failing students" (p.1). Jen Anthony, VP External expressed, "Memorial has taken a proactive

approach here in attempting to ensure that students are not entering courses for which they are unprepared. The fault lies solely at the feet of the Department of Education. There must be a returned emphasis to basic skills - things like fractions, grammar and comprehension are being ignored for the sake of learning how to surf the web" (p.1). Although additional research may reveal contradictory views on this matter, the glaring fact that students have demonstrated poor basic skills is of great concern.

### **Defining Basic Skills**

The Basic Skills Agency of London, England defines basic skills as "the ability to read, write and speak in English and use mathematics at a level necessary to function and progress at work and in society in general" (1997, p. 4). According to Usnick (1991), when elementary teachers refer to student mastery of basic skills they really mean speed and accuracy, with an emphasis on speed (for general computational work). Schwartz (1999) believes that "Basics are those fundamental building blocks on which later learning often rests" (p.2). In isolation, the basics may not be the most interesting mathematics, but they are essential for the student to problem solve. Friesen, Clifford, and Jardine (1999) define basics for any discipline as the "smallest, most clearly testable bits and pieces" (p.8).

There is some ambiguity regarding the definition of basic skills. Some researchers use the terms 'basics' and 'basic skills' interchangeably. The definition is contextual, in that it depends upon the level at which the student is being taught. For example, consider what is expected of a student who is entering calculus versus a student in Grade 9 math. For the purpose of this paper,

basic skills will refer to background knowledge and/or skills (such as comprehension) and procedural tasks (such as cross multiplying) required for a student to master a particular math concept.

### **Differing Perspectives of Mathematical Skills Issues**

Central to the topical issue of basic skills are the perspectives of society, educational researchers and students. The positions on the basic skills problem vary considerably within and between these groups. These views are presented here.

#### **Societal Views**

Burns (1998) states that there are essentially two opposing views to the teaching of mathematics. One group of parents and educators favour paper and pencil exercises over investigative hands-on activities. Another group of parents and educators believe that this skill-and-drill approach hinders true understanding and instead prefer a model of math instruction that promotes problem solving. This contentious issue is currently being emotionally debated in areas where an investigative type of math curriculum has replaced the traditional math. The investigative math utilizes real world problems that literally drive the curriculum. The students draw on all math disciplines in trying to solve them. But, more and more parents who are opposing this approach, are upset as they believe their children are not learning basic mathematical skills effectively. On the other hand, proponents of the problem solving orientation contend most rote learning undermines understanding.

Colvin (1999) reports on the direction of math reform in Escondido, California where

parents, *not* educators, decided in January 1999 that the math curriculum which emphasised problem solving with real-world situations was inadequate for their children. They wanted a return to a traditional math program. The proof, they felt, was shown in the poor reporting by the students on a *traditional* standardized test administered by the school district. "Taught traditionally, many students failed to learn math well enough to use it as a tool in their daily lives. The proposed solution to that problem – having students ponder real-world dilemmas – didn't necessarily teach them much math" (p. 26).

Battista (1999) reports that opponents of math reform clamour for a return to the basics "as they cite isolated examples of alleged failures of mathematics reform [while] they ignore the countless failures of traditional curriculum" (p. 425). The quality of math education is threatened as educational decision makers, ignorant of how students learn mathematics, adopt policies that violate or go against professional recommendations of mathematics reform. The critical or basic skills for the modern student are not pencil and paper computation but "problem solving, reasoning, justifying ideas, making sense of complex situations and learning new ideas independently" (p. 428).

O'Brien (1999) argues that we cannot return to the basics for improvement because we have been there all along and they have failed us badly. He writes that the 'back to the basics' bandwagon ironically is appealing to the very parent who they themselves were not very good at math or even disliked math because of the 'parrot math' approach they experienced. O'Brien questions what is wrong with allowing children to find meaning in math. Battista (1999) points out that there is little stigma today associated with anyone admitting that they are poor in math.

### **Educational Researchers' Views**

A 1996 survey conducted by the Basic Skills Agency (1997) assessed the basic skills of 15,579 first year, full-time English college students upon entrance to 18 English colleges. Of these, 4,470 students were monitored throughout the year. On average, one-third were identified as needing basic skills support. Those who received the support were three times as likely to complete their courses than those who did not. Three-quarters of those who received basic skills support completed their first year of studies whereas only just over half of the students who required, but did not receive any specific support, finished their first year of studies.

Bowman (1998) reports that the National Council of Teachers of Mathematics (NCTM) has criticized the traditional curricula as being too computational. Rote learning, the council judges, has failed to promote reasoning, insight, and problem solving. It contends that math learned by youngsters is nonsensical and thus the students tend to question their own ability to understand and learn. Many professionals in math, science and technology note that in today's Information Technology (I.T.) environment new knowledge is quickly generated and disseminated. There is an increasing demand placed on students to possess a broader range of skills and knowledge. Bowman however, notes that there is a backlash against active and interactive learning. The negative reaction stems from a number of observations: a lack of adequate assessment techniques for this type of learning; the use of standardized tests as a means of determining a child's learning (many schools and parents advocate methods more likely to raise a student's score); the decreased ability of students to perform traditional computational skills; and finally, the lack of inservicing for teachers.

Carmine, Dixon, and Jones (1994) observe the apparent failure of discovery learning with

lower achievers and question why it is being touted again as a pedagogical practice for low achievers. Silver (1997) reports that there is a lack of algebraic competence among American high school graduates as demonstrated by the increasing number of remedial math courses in the colleges. Cornwall (1999) identifies three of Canada's mathematical problems: a glaring shortage of specialist Math teachers in Canada; the NCTM approach of learning math through manipulative and calculator use from kindergarten to grade four; and a spiral curriculum. She does briefly describe two "success" stories of schools which used a non-spiral curriculum and whose teachers effectively taught math by adopting an approach that concentrated on basic number facts and problem solving exercises.

Devlin (1999) recounts evidence showing that "many people are unable to apply in real-world contexts the arithmetic procedures that they learned in the classroom" (p.3). Cossey (1999) believes that despite the publicised California Mathematics Standards, schools continue to abandon the attempt to produce critical thinkers in favour of moulding students adept at symbolic and numerical manipulations. Countries whose students scored well on the TIMSS seem to have one thing in common, namely, a focused curriculum. Using a curriculum packed with numerous concepts does not permit time for 'user-friendly' methods of teaching. "Students won't have time to reflect on their work or to engage seriously in solving novel problems involving these ideas" (Cossey, 1999, p. 442). Carnine, et al. (1994) also question the number of mathematical concepts or topics to which students are exposed as it exceeds the number of concepts taught in the Asian countries.

In order to address all listed objectives in curriculum guides and therefore complete the curriculum, many teachers resort to direct instruction. That is, there simply is not enough time for



constructivist learning. However, as indicated by Henigin (2000) and Pita (2000), direct instruction does not bode well for North American students when they are compared to their international peers. Based upon the TIMSS-R's index of teachers' emphases on mathematical reasoning and problem-solving (EMRPS), Pita (2000) notes that "Emphasizing reasoning and problem-solving was related to performance, with students at the high and medium levels [EMRPS] having higher average achievement than those at the low level, both internationally and for most countries." In fact, the top five ranked countries for both 'Average Mathematics Achievement of Eighth-grade Students, by Nation' and EMRPS were identical in placement Henigin (2000) and Pita (2000).

Schoen, Fey, Hirsch, and Coxford (1999) also believe that when faced with uncertainty generated by the ongoing "math wars," teachers will retreat to the traditional, more comfortable style of teaching – direct instruction (p. 444). They believe that reform in mathematics is being initiated for a variety of reasons. First, the development of the calculator and computer has enabled the student to spend less time on pencil and paper operations. Focus can now change regarding procedural skills in algebra and arithmetic. Second, examining how mathematics is used in the workplace has revealed that topics in statistics, probability, and areas of discrete mathematics require more in-depth treatment than provided by the current traditional curriculum. Third, unfavourable experiences with the traditional, abstract method of instruction helps promote the belief that students learn better when key ideas are developed from students working with concrete, real life problems. The math is more meaningful and the acquired skills could be thus transferred to other fields of endeavour. Fourth, reports from the business sector indicate the mathematical traits desired in graduating students: general mathematical understanding and skills,

well developed ideas for examining and analysing problems, and the ability to communicate ideas for the solution of those problems.

Colvin (1999) notes at the time of his publication that Interactive Math was used in more than 100 schools across the US and students of this mathematical approach to math did no worse [nor better] than traditionally taught students on college entrance exams. This can be viewed as a 'feather in the cap' for advocates of Interactive Math as that program devotes relatively little time to algebra and geometry, both of which are dominant topics on the American SAT. Yet, whichever way Americans have been taught over the last 100 years, Colvin (1999) muses that most have wound up with "little usable mathematics knowledge" as indicated in the TIMSS and other assessment results (p.28). North American math curriculum, in general, has been described similarly to California's as "a mile wide and an inch deep" in its delivery of lightly explored, numerous concepts (Cossey, 1999). Although Canada has fared better on the TIMSS and the TIMSS-R than the United States, the most successful countries on these international assessments tend to teach fewer concepts, thus, permitting their students time to develop a deeper understanding of what is taught (Cossey, 1999).

Raimi and Braden (1998) in reporting on math standards in the United States, state that "One can no more use mathematical 'concepts' without a grounding in fact and experience, and indeed memorization and drill, than one can play a Beethoven sonata without exercise in scales and arpeggios" (p. 34). It is important for students to not only know facts and possess skills but to be able to use them. Bachelis (1998) reports the preliminary results of a survey of freshman students at Wayne State University who had graduated either from a high school's Core Plus program (a math curriculum that emphasizes critical thinking over traditional memorization) or

from a nearby high school that promoted traditional math teaching. Bachelis' (1999) early analysis clearly favoured traditional math over reform-type math as 69% of the Core Plus survey respondents were forced to take remedial classes when they entered college as a result of not learning enough basic algebra for the college math courses. Based on his study, Bachelis reflects that if the intent of reform math was to reach more students and make math more meaningful, then the cure is worse than the disease.

### **Students' Views**

Wittman, Marcinkiewicz, and Hamodey-Douglas (1998) believe that students' frustration and negative attitudes towards math come from the nonmastery of fundamental mathematical skills and the failure of the schools to promote this mastery. Donato (1989), in a study of fifth grade students, discovered that a number of the students disliked math because they did not understand it and they became apprehensive with the announcement of assignments or tests. The parents of those students reinforced their children's beliefs by saying the children were never good at math. They added that even when the parents themselves were in school they had also experienced difficulty. As a result, the parents had low expectations and stated that there was "nothing that anyone could do to help their child learn and use math effectively" (p. 17).

Stacey and MacGregor (1997) report that students' initial difficulty with algebra can be traced to a limited understanding of number and operations and a difficulty expressing in writing what they do understand. Philippou and Christou (1998) recorded responses from a questionnaire given to 162 first year prospective primary teachers attending the University of Cyprus in 1992. The questions focussed on reasons for disliking mathematics. The responses were "I was afraid of it" (29%), "because of poor teaching" (27%), and "lack of teacher enthusiasm" (25%) (p.198).

Ruffell, Mason, and Allen (1998) queried twenty graduate students who had completed one year of studies in education on their attitudes toward mathematics. When asked to list some positive and negative experiences of mathematics, 58% of the positive responses were related to 'the self' such as "something clicked", whereas 38% of the negative experiences were teacher related, such as "he confused us with wrong answers." The reported positive experiences were mostly individual, whereas the negative were often collective.

### **Math Anxiety**

Wittman et al., (1998) state that "Students whose mathematical problem-solving performance is hindered by the continual need to review facts and procedures will learn to view mathematics as an anxiety provoking experience" (p. 8). Mathematics anxiety, as reported by Levine (1995), is often viewed as debilitating and is linked to poor performance in both academic settings and daily life. Levine states that it is females in middle and secondary schools who report higher rates of anxiety than males, yet they often score better on tests prior to secondary school. At that point the trend in performance switches. The author believes society (media and popular culture) contributes to this trend by supporting males in choices made for mathematics-oriented careers such as engineering. Donato (1989) wrote in her study of fifth grade children that the apprehension problem was largely from the students who had weak math skills, especially in problem solving.

Miller, Mercer, and Dillon (1992) contend that failure at math problems leads to math anxiety. They feel that math instruction should focus on helping students understand math concepts by using a sequence of teaching by concrete methods (with the use of manipulative

objects), then by progressing to semiconcrete (using, for example, pictorial representation), and finally to abstract teaching, where the sole focus is numbers. Though their research stems from focussing on students with learning problems, the sequence could be applied to all students. The authors also believe it is important to use various manipulatives when instructing at the concrete level as it assists student with generalization. Miller et al. (1992) outline four steps appropriate for the three levels of instruction. First, give an advance organizer. The advance organizer is used to connect the previous lesson to the current one. It provides a rationale for learning the skill/concept. Second, demonstrate the skill and have students model the process. Third, provide guided practice with feedback. Fourth, provide independent practice. The authors believe that "Rote memorization of math facts does not teach students to understand mathematical concepts. Teachers are realizing that their instruction must include more than ongoing drill and practice . . . to ensure continued success and progress in mathematics, students should be taught conceptual understanding prior to memorization of the facts" (p.108). Student automaticity of basic skills is necessary to form a strong basis for the continuing development of problem solving skills.

### **Metacognitive Approaches to Styles of Teaching**

In this part of the literature review, an examination of mastery and performance oriented students is detailed before a discussion of two components of metacognition (being aware of how one learns). The section concludes with a selection of ideas that have influenced the nature of this project.

#### **Mastery and Performance Students**

Ames (1992) distinguishes between mastery and performance goal oriented students in

discussing student achievement and motivational processes. How students interpret their success in mastering skills and concepts differs for these students. Central to the mastery goal concept is that effort, not ability, is directly responsible for success whereas the main element to the performance goal philosophy is that ability, not effort, is directly responsible for success.

Mastery oriented students focus attention on the intrinsic value of learning, through which students are oriented toward developing new skills and attempting to understand what it is they are doing. They take pride and satisfaction from successful effort and generally spend increasing amounts of time on learning new tasks. They employ self-regulatory strategies to facilitate their learning.

Performance oriented students place more emphasis on one's self-worth. Public recognition is important. Hence learning is viewed as a way to achieve a goal of enhancing one's self-worth. The use of effort to a performance goal oriented person can be both good and bad. If great effort does not bring success then one's self-concept of ability is threatened. Thus, those individuals tend to avoid challenging tasks as they perceive any failure as a reflection on their own ability.

By being aware of the contrasting perceptions of reason for success (failure), the teacher consciously attempts to avoid presenting learning activities that promote one process over the other. The trend in the last few years in North American math education has been to move away from the old practice of 'drill and instill' toward an approach that encourages problem solving or investigative hands-on activities. However, the difficulty, as Ames (1992) points out, is that evaluation is usually oriented toward performance goals through *traditional* standardized testing. Students are getting mixed signals when taught in one fashion and tested in another. This factor of non-complementary teaching and testing can adversely affect test performance. The author

believes that how the teacher structures the classroom will affect how students approach and engage in learning.

### **Metacognition**

Palincsar and Brown (1987) discuss two components of metacognition (being aware of how one learns). The first is the knowledge one has about his/her own cognitive processes and the second is the actual regulation of the analytical activity. The authors believe the hallmarks of metacognitive instruction are the acquisition and control of learning strategies. A metacognitive approach to mathematical teaching includes three items: developing the students awareness of what the problem is asking; teaching the student appropriate strategies for solving the problem; and teaching the student to monitor the use of the strategies employed. Specifically, in the section of their paper involving improving students' math performance when using algorithms, they focused on two instructional studies. In the first study, DeCorte and Verschaffel (as cited in Palincsar and Brown, 1987), examined problem solving activities of first and second grade students concentrating on the thinking process rather than the actual mechanical operation of the algorithm to promote mastery. The experimental students were given practice verifying their answers and focussed instruction on processes that led to successful solutions. The researchers discovered that experimental students made fewer algorithmic errors than the control group in posttesting immediately after and one month after the two week intervention period. DeCorte and Verschaffel employed the strategy of having the student think aloud as a problem was solved. They discovered the experimental group made spontaneous use of the conceptual knowledge they had acquired in the intervention period. The second study by Lloyd, Saltzman, and Kauffman (as cited in Palincsar and Brown, 1987) involved counting for six numbers (ie. counting by 2s, 3s, 4s, 5s, 7s and 10s). The researchers discovered that for students identified as learning disabled,

preskill training through modeling and drill was not very effective. Strategy and cue-training, explicitly taught, were found to be necessary.

Gourgey (1998) describes integrating metacognition with mathematics teaching, thereby encouraging students to monitor (and improve) their progress as they develop foundations for problem solving. By planning, monitoring, and evaluating one's own performance, "Metacognition enables one to use knowledge strategically to perform most efficiently" (p. 82). For example, consider problem solvers. Being aware of how and what one learns is a considerable difference between novice and expert problem solvers. Novices will usually select one strategy and pursue it without evaluating what they are doing. Time is lost on wild goose chases. The experts, however, will attempt to understand the problem fully and possibly use various approaches as they, with a higher degree of confidence, evaluate the efficiency of one approach over another.

### **Pedagogical Practices**

There are various methods and approaches that teachers and researchers have used to best determine students' areas of mathematical understanding and/or difficulty. Such examples are highlighted whose practice are incorporated into the development of the subsequent sections on coordinate geometry.

Wilcox and Zielinski (1997) suggest that students should be forced to explain their reasoning. If time and opportunity are permitted for this exploration, the students could very well develop a richer understanding of the concept or task. Montague (1995) believes that learning mathematics is now generally accepted to be a constructivist process. The trend is to replace deductive teaching and rote learning with environments which foster students actively participating in their own learning.



Some may argue that students may be proficient in carrying out procedural tasks with high degrees of competence but this does not necessarily translate to a good understanding of the conceptual mathematics involved in the procedure, especially for tasks that are not too complex and easily memorized. Teachers have known 'good' students to struggle with questions they felt the student should have easily tackled. The frustration stemmed from not being able to employ previously learned skills to what students perceived to be new types of questions. The students had previously demonstrated that they were efficient with mathematical procedure, but now were unsure where to apply it in new situations. This problem relates to the lack of emphasis on real world mathematics. How well is the student able to make connections from what is done or practiced in class to an actual or new situation involving the same procedure? The connection could be reinforced by presenting the student with interesting or varied real life situations for the investigations versus repetition of procedural tasks.

Turner, Styles, and Daggs (1997) believe the intermediate school years are crucial for promoting a student's present and future interest in mathematics. They cite four strategies to consider when involving them in mathematics. First, make math instruction more challenging and less rote, thus creating an environment which encourages students' thinking. Second, select activities that could support students' independent thinking and autonomy. These activities would permit students to explore the concept being examined so that they can become the classroom expert. Third, the activities employed should be of a collaborative nature as students can and do learn from one another. Finally, relate the mathematics to the students' lives.

Friesen et al. (1999) believe that the philosophy and the growing practice is to combine instruction of the basic skills, which are traditionally taught independently, and the concept being focused upon. The shift is towards producing critical thinkers. By employing different strategies in

our teaching we first appeal to more students as each respond differently to different styles and. secondly, the student may become better aware of the skills they already possess and it may lead to increased confidence.

### **Summary of the Literature**

The emotional debate in North America concerning mathematical curricula has been initiated by undesirable test scores produced from the local to the international level. Blame by both parents and educators has been placed on generally one of two pedagogical approaches used for teaching mathematics. Both of these methods, the traditional and the investigative, have their supporters and opponents. A series of views from society, educational researchers and students are presented on the issue that students today are lacking basic mathematical skills. Researchers believe the nonmastery of basic skills leads to frustration, anxiety, and resulting negative attitudes that develop towards the subject. This vicious cycle could be broken by making math more meaningful for the student through teaching that recognizes that there are two distinctly motivated individuals, (performance and mastery oriented). Tasks and activities that are varied and personally challenging, with well defined goals and contain social components can encourage students to put forward effort, and actually become involved in their own learning. Requiring students to explain their reasoning, incorporating basic number facts in instruction, and using real-life problem solving enhance student learning.

As indicated by data gathered by Pita (2000), students of teachers who had placed emphasis on mathematical reasoning and problem solving generally did much better on assessments similar to the TIMSS and TIMSS-R. Mathematical reasoning is encouraged more

through the investigative mathematical approach than the traditional 'drill and skill'. But, to obtain the level of understanding expected of students, there is strong opinion and evidence that the child must possess basic mathematical skills. As stated previously, this project's definition of basic mathematical skills refers to specific background knowledge/skills (such as comprehension) and procedural tasks (such as cross multiplying) required by the student to master a particular math concept. Lacking such, the student is disadvantaged from the outset.

### **CONTEXT FOR THE PROJECT**

The direction of mathematics teaching in the province of Newfoundland is moving towards an investigative approach. Embracing the Atlantic Provinces Education Foundation (APEF) philosophy of mathematics learning, new math courses reflecting the APEF rationale, were introduced in Grade 1 and Level I (Grade 10) at the start of the 2000 - 2001 school year. Changes at the intermediate level will commence with the introduction of a new Grade 7 curriculum in Fall 2001. The current Grade 9 mathematics program is not slated for revision until September 2003. Therefore students in Grade 9 will continue to be taught (for the most part) in the traditional manner until the newer math is brought on stream (Atlantic Provinces Education Foundation, 1999).

As stated earlier, this project was designed to offer a small bridge to the students in instructional (and learning) style by presenting the Grade 9 unit, Coordinate Geometry, in a more investigative manner. As students will be exposed to that approach to their learning upon entry into highschool, I felt it was worthwhile to develop this unit to formally introduce the students to the manner through which they will be learning mathematics, beginning next school year. This

project was designed to teach the curriculum objectives of a Grade 9 *Coordinate Geometry* unit. The objectives are stated in the Intermediate Mathematics Curriculum Guide – the program designed by the Newfoundland and Labrador Department of Education and Training in 1995 and currently being taught in the province for the school year 2000-2001. This project was implemented in a rural setting with a Junior High Grade 9 class of 28 students of mixed ability.

The Coordinate Geometry unit has a time allotment of 10% of the Grade 9 mathematics program, which encompasses approximately 17 classes (including assessment). This project provides lessons for each intended outcome as articulated in the Intermediate Mathematics Curriculum Guide.

The unit was chosen to be taught as a guided, investigative approach to help students achieve the stated curriculum objectives. An investigative perspective can help bring topics vividly to life as the student is encouraged to learn through a ‘hands on’ approach. It can: help tap into natural curiosity and, within the confines of a lesson, allow students to explore the chosen subject matter; facilitate conceptual learning for both the performance and mastery oriented students as they are encouraged to make conjectures; and address math anxiety as students begin to assume ownership of their learning. The project was devised with ideas about mathematics learning as advocated by the APEF, “mathematics learning is an active and constructive process .... learners are individuals who bring a wide range of prior knowledge and experiences, and who learn via various styles” (Atlantic Provinces Education Foundation, 1999, p. 2).

Constructivist theory states that in learning, individuals build or construct their own knowledge. They build upon existing knowledge by creating connections to new concepts and, in a way, create their own understanding of the world around them. For the purposes of this project, a genuine constructivist approach would be an unrealistic method to employ given the time

constraints and overabundance of concepts and mastery levels the students are expected to obtain. On the other hand, the traditional explanation followed by 'drill and skill' approach has not been overly successful either. Therefore the project offers a middle of the road approach: investigative and exploratory (teacher directed), but coupled with an emphasis on strengthening and maintaining basic skills and concepts through practice.

The remaining part of the project is divided into two distinct sections: a Teacher's Learning Outcome unit composed of 7 learning outcomes/objectives (as prescribed by the provincial Department of Education and Training) and an accompanying Student Worksheets unit. The sections are intended to work together. The objectives on each sheet are identified in the top right hand corner. All outcomes and corresponding worksheets can readily be grouped together. It is intended that other teachers can readily avail of this resource. The expectation is that several other Grade 9 math teachers in the Avalon West School Board will employ this material in the next two years preceding the curriculum change.

In relevant sections, collaborative work for small and large groups is encouraged to help facilitate conceptual learning of the stated objectives. The questions in each section are by no means an exhaustive list. It is possible that they could be used as a single resource to help achieve the intended outcomes. Some students require more (or less) practice, or perhaps even a different approach to their learning than that offered by these questions. In addition, reference has been made to the resource texts suggested by the Intermediate Mathematics Curriculum Guide. Readers, if they wish, may reorder the units by collating a Teacher's Learning Outcome objective with the appropriate Student Worksheet. Please note that outcomes 3 and 4 were grouped together in the same objective section to facilitate a smoother flow of content.

Students will:	Assumed Background Knowledge
<p><i>to calculate the distance between two points on a coordinate plane using the Pythagorean Theorem.</i></p>	<ul style="list-style-type: none"> <li>Pythagorean Theorem</li> <li>simplifying radicals</li> </ul>

A: Individual or small group activity

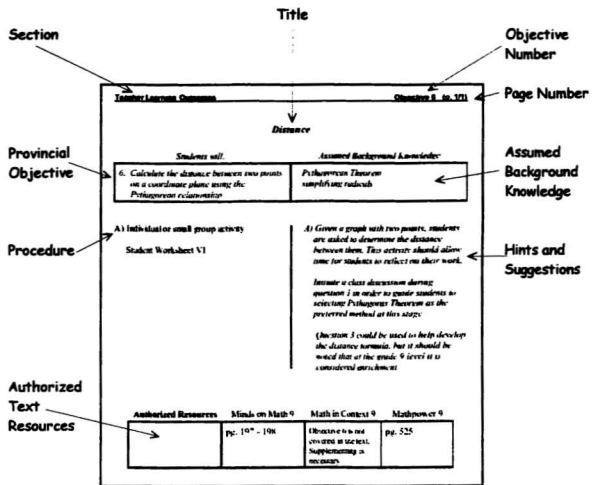
Student Worksheet: All

A: Given a graph with two points, students are asked to determine the distance between them. This activity should allow for a rich discussion of the Pythagorean Theorem. A class discussion, either initiated or asked for by students, is needed in order to guide students in seeing the connection between the distance formula and the Pythagorean Theorem.

In the second problem, students are asked to determine the distance between two points on a coordinate plane. This activity should allow for a rich discussion of the Pythagorean Theorem.

Authorized Resources	Standards: Math 1	Math 1: Content	Math 1: Overview
	<p><b>Grade 9</b></p>	<p>Math 1: Content</p>	<p>Math 1: Overview</p>

# Layout of a Teacher's Learning Outcomes Page



The Provincial Objectives are matched with the Student Worksheets as follows:

- |                    |   |                  |
|--------------------|---|------------------|
| Objective 1        | → | Worksheet I      |
| Objective 2        | → | Worksheet II     |
| Objectives 3 and 4 | → | Worksheet III/IV |
| Objective 5        | → | Worksheet V      |
| Objective 6        | → | Worksheet VI     |
| Objective 7        | → | Worksheet VII    |

Objectives 6 and 7 may be introduced at any point in the unit but it is recommended that objectives 1 - 5 are done in order of presentation.

## **Teacher's Learning Outcome**

### ***Objective 1***



**Methods of Representing Relations**

<i>Students will:</i>	<i>Assumed Background Knowledge</i>
<i>1. Represent relations using various methods:</i> <ul style="list-style-type: none"><li><i>• arrow diagrams</i></li><li><i>• table of values</i></li><li><i>• mapping relation</i></li><li><i>• equation</i></li><li><i>• a set of ordered pairs</i></li></ul>	<ul style="list-style-type: none"><li><i>• ratios</i></li><li><i>• substitution</i></li><li><i>• plotting coordinate points</i></li><li><i>• order of operations with integers</i></li></ul>

**A) Function Game**

*A) One whole class approach is to start with a game. The experience may help the student retain the intended concepts a little easier. There are a number of variations of the Guess My Rule or Functions Game, but essentially the teacher or student thinks of an equation (which they keep to themselves). Each student then take turns giving a number to the teacher who replies with an answer based on the equation. (The input (x) and the output (y) can be recorded as a table of values, mapping diagrams or as sets of ordered pairs to represent relations.) This is continued until a student is able to correctly give the rule/equation.*

**B) Student Worksheet I**

*B) To strengthen the students' understanding of the number of ways of representing relations, worksheets are used.*

Authorized Resources	Minds on Math 9	Math in Context 9	Mathpower 9
	pg. 460 - 465, 469 - 472	pg. 370 - 373	pg. 500 - 503 507 - 509

## **Teacher's Learning Outcome**

### ***Objective 2***

*Solving Systems of Equations*

<i>Students will:</i>	<i>Assumed Background Knowledge</i>
2. Solve a system of linear equations of the form $y = mx + b$ using a graph generated from a table of values.	<ul style="list-style-type: none"><li>• interpretation of linear graphs</li><li>• substitution</li><li>• plotting coordinate points using table of values</li></ul>

## A) Activity

A) Use a tiled floor labelled A - D ... rows and 1 - 4 ... columns. Two students are selected to act as chess men where their movements will be dictated by one of two equations. The equations should be earlier determined by the teacher to be a system of intersecting equations. On the board will be placed the two equations with uncompleted table of values underneath each (the x-elements should be given.) As each ordered pair is determined, the corresponding student moves to the proper square. Most students should be quick to point out that the identical ordered pairs are determined for each chess man, especially when both students attempt to occupy the same square.

A variation on this game would be to again divide the class into two teams. Each chessman will move as their coordinate is correctly determined, the common point being previously given to the class. The first to arrive at the intersecting point through the series of correct coordinates, wins!

A graphing calculator would certainly be

B) Discussion of the concept of *point of intersection*.

C) Student Worksheet II

*beneficial in facilitating conceptual learning for this objective.*

*B) Elicit examples from the students like the game of Battleships, overhead/profit margins and so on to stress the significance or importance of a common point between different entities.*

Authorized Resources	Minds on Math 9	Math in Context 9	Mathpower 9
	pg. 478 - 479	pg. 382 - 385	pg. 518 - 519

## **Teacher's Learning Outcome**

### ***Objectives 3 and 4***

**What is Slope?**

<i>Students will:</i>	<i>Assumed Background Knowledge</i>
3. <i>Determine the slope of a line using the relationship: <math>\text{slope} = \frac{\text{rise}}{\text{run}}</math></i> 4. <i>Link visual characteristics of slope with its numerical value.</i>	<ul style="list-style-type: none"><li>• <i>simplify fractions</i></li><li>• <i>distinguish between x and y elements</i></li><li>• <i>perform operations of addition and subtraction of integers</i></li><li>• <i>evaluate using absolute value with positive and negative slopes</i></li></ul>

A) Discuss the concept of slope

*A) Steepness, inclination, slant, tilt, even "rate of change" are some similar concepts.*

*Elicit examples from the students (for example skiing, tobogganing, stairs)*

B) Investigation

*B) Place students in small groups of 2 – 4.*

*Materials*

- 4 screws
- 2 straws
- 2 balloons
- adhesive tape
- a metre stick for each group
- a level for each group

*About 3 m apart, set two screws into one of the class room walls, one 0.5 m above the floor, the other 1 m. String a line (eg. cat gut) from each screw and insert each through a straw. Attach a balloon to each straw in a way that will allow the balloon to be inflated. Tie each line to either one of two anchors set into the floor 5 m away. At the anchors on the floor, equally inflate each balloon. At the same time release the balloons to shoot up the lines.*

Guide students to measure slope without calculating angles.

*The investigation actually does not begin not with the balloons going up the lines but when they are deflated and begin to slide back down the lines.*

*After the straws/balloons stopped, discuss*

C) Demonstrate why horizontal lines have no slope and it is undefined for vertical lines.

D) Distinguish between positive and negative slopes.

*the factors that affect steepness. In this case, why one straw/balloon came down quicker than the other. The intent is to collectively agree that rise and run (or similar terms) affect steepness slope.*

*Each group is given a metre stick and level, and should measure the plumb height (rise) at any point along an assigned line and from the base of that measurement, the distance to the screw on the floor (run).*

*Each group is to calculate the slope from their data. Collect and display all the information from each group on the chalk board for comparison and discussion.*

C) *Show that the straw/balloon will not move on a horizontal line. That the straw can't "grip" the line when assembled vertically.*

D) *Additional reinforcement could be done with a graphing calculator. This is a quick, visual method for demonstrating positive/negative slopes.*

*Using graph paper, (volcano model) show two lines that have the same steepness and same length but ask if they really the same lines.*



*If not, how can they have the same slope if they are going in opposite directions?*



E) Student Worksheet III/IV.

*Is there a way we could distinguish between lines that have the same slopes but are going in different directions?*

*E) Remind students to be consistent in how they calculate rise and run and to use the points in question 3.*

Authorized Resources	Minds on Math 9	Math in Context 9	Mathpower 9
	pg. 143 - 145, 456	pg. 134 - 138	pg. 520 - 522

## **Teacher's Learning Outcome**

### ***Objective 5***

### Graphing

<i>Students will:</i>	<i>Assumed Background Knowledge</i>
5. Graph a linear equation using the slope and y-intercept.	• plot coordinate points

- A) Explore the concepts of slope and y-intercept and their effect on a graphed line.

Student Worksheet III/IV

- A) Using the suggestion in the Intermediate Curriculum Guide (p. 140), students will given sets of equations with the same slopes or y-intercepts. Asked to graph, they are to determine how parts of the linear equation affect the line.

*As in grade 8, the students can use a table of values to graph the lines in questions 1 and 2. Suggest to the students that they should choose values for the x-variable that will provide an integer answer if multiplied by a fractional slope.*

*The students can be given the sheets to work on individually or in small groups. As a group, they can each take an equation to work at and then compare their conjectures before reporting them to the class as a whole. Thus, all become actively involved in their own learning.*

*When asked to graph without tables of values, if time permits the students may 'discover' how  $m$  and  $b$  interact or be shown how to use the concepts effectively.*

*At this point, the students are formally introduced to the equation  $y = mx + b$ .*

*In questions 3, 4, and 5, the students should be asked to state  $m$  and  $b$ .*

Authorized Resources	Minds on Math 9	Math in Context 9	Mathpower 9
	pg. 473 - 477	Objective 5 is not covered in the text. Supplementing is necessary.	Objective 5 is not covered in the text. Supplementing is necessary.

## **Teacher's Learning Outcome**

### ***Objective 6***

***Distance***

<i>Students will:</i>	<i>Assumed Background Knowledge</i>
6. Calculate the distance between two points on a coordinate plane using the Pythagorean relationship.	<ul style="list-style-type: none"><li>• Pythagorean Theorem</li><li>• simplifying radicals</li></ul>

A) Individual or small group activity

Student Worksheet VI

A) Given a graph with two points, students are asked to determine the distance between them. This activity should allow time for students to reflect on their work.

Initiate a class discussion during question 1 in order to guide students to selecting Pythagoras Theorem as the preferred method at this stage.

Question 3 could be used to help develop the distance formula, but it should be noted that at the grade 9 level it is considered enrichment.

<b>Authorized Resources</b>	<b>Minds on Math 9</b>	<b>Math in Context 9</b>	<b>Mathpower 9</b>
	pg. 197 - 198	Objective 6 is not covered in the text. Supplementing is necessary.	pg. 525

## **Teacher's Learning Outcome**

### ***Objective 7***

*Distance*

<i>Students will:</i>	<i>Assumed Background Knowledge</i>
7. Determine the midpoint of a segment on the coordinate plane	<ul style="list-style-type: none"><li>• plotting coordinate points</li><li>• perform operations of addition and subtraction of integers</li><li>• interpretation of number lines</li></ul>

A) Student Worksheet VII

A) After discussing the concept of average and midpoint, the worksheet could be used in a group by the individual student.

Authorized Resources	Minds on Math 9	Math in Context 9	Mathpower 9
	pg. 142	pg. 139 - 141	pg. 525



# Student Work Sheets

## **Student Worksheet I**

**Methods of Representing Relations*****Guess My Rule***

1. I have a rule (equation) in mind. For each number ( $x$ ) that you give me, I'll put it in the rule and give you an answer ( $y$ ). Write down each number that is given to me and its corresponding answer in the tables below. Try and guess the rule that I am thinking of!

Game 1

$x$	$y$
-----	-----

Game 2

$x$	$y$
-----	-----

Game 3

$x$	$y$
-----	-----

Equation \_\_\_\_\_

Equation \_\_\_\_\_

Equation \_\_\_\_\_

A set of *ordered pairs* is one method to record what values make the equation true or solves the equation.

2. Consider the equation  $y = -x + 4$ . Circle all of the following ordered pairs that are solutions for the equation.  
A) (1, 3)   B) (2, 2)   C) (3, 2)   D) (6, -2)   E) (7, -4)
3. Consider the equation  $y = (x - 2)/2$ . Circle all of the following ordered pairs that are solutions for the equation.  
A) (0, 1)   B) (2, 0)   C) (-2, -2)   D) (-6, -3)   E) (-3, -3)

4. For each of the following, complete the table of values, write them as ordered pairs, and then write an equation.

a) The product of two numbers is 24.

$x$	$y$
1	
	12
3	
	6
5	

ordered pairs: \_\_\_\_\_

equation: \_\_\_\_\_

b) The sum of two numbers is -10.

$x$	$y$
-2	
	-9
0	
	-12
2	

ordered pairs: \_\_\_\_\_

equation: \_\_\_\_\_

5. Consider the linear equation  $y = 3$ .

a) Complete the following table

$x$	-2	-1	0	1	2
$y$					

Each  $x$ -element  
maps to the  $y$ -  
element 3.

-2   -1   0   1   2



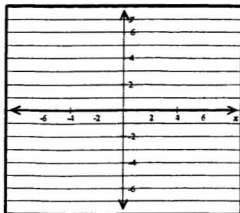
b) Rewrite the data as ordered pairs.

\_\_\_\_\_

c) Describe the pattern that you see.

\_\_\_\_\_  
\_\_\_\_\_

d) Plot the results.



e) What type of line is created when all the  $x$ -values are different but the  $y$ -value does not change? \_\_\_\_\_

6. Consider the linear equation  $x = 4$ .

a) Complete the following table.

$x$					
$y$	-2	-1	0	1	2

Each  $y$ -element maps to the  $x$ -element 4.

-2 -1 0 1 2



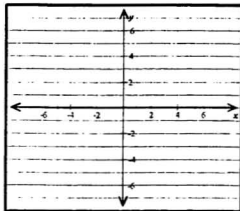
b) Rewrite the data as ordered pairs.

\_\_\_\_\_

c) Describe the pattern that you see.

\_\_\_\_\_

e) Plot the results.



- f) What type of line is created when all the  $x$ -values are the same but the  $y$ -values differ? \_\_\_\_\_

7. The posted speed limit on the highway is 100 km/h.

- a) Construct a table of values for time ( $t$ ) and distance ( $d$ ) for  $t = 0, 1, 2, 3, 4$ .

$t$	$d$

- b) Rewrite the data as ordered pairs.

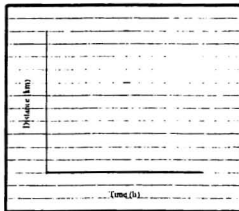
\_\_\_\_\_  
\_\_\_\_\_

- c) Describe the pattern that you see.

\_\_\_\_\_  
\_\_\_\_\_

- d) Plot the results with time on the  $x$ -axis.

- e) Keeping in mind that the speed is a constant 100 km/h, write an equation to find the distance  $d$  for each hour.  
 $d =$  \_\_\_\_\_



8. Consider a  $5 \times 5$  checkerboard.



- a) Determine the number of different sized squares in a  $5 \times 5$  checkerboard and record your results in the table on the right.

eg.  is a  $2 \times 2$  square

Size of Square	Number of Squares
$1 \times 1$	
$2 \times 2$	
$3 \times 3$	
$4 \times 4$	
$5 \times 5$	

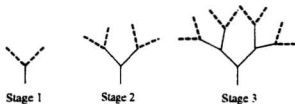
- b) Describe the pattern that relates the size of the square to the number of squares.

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9. In a similar fashion as a Fractal Tree, each additional segment produces two new breaks/branches as the stages develop.



- a) Complete the table to the right.

Stage	Number of New Branches
1	2
2	
3	
4	
5	

- b) Express the data as ordered pairs.

---

- c) Describe the relationship developing between the stages and the number of new branches being produced.

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- d) Let  $s$  = the stage. Try and write an equation to find the number of new branches ( $b$ ) for each stage. (Hint! It is not a linear relationship!)

$b =$  \_\_\_\_\_



## **Student Worksheet II**

**Solving Systems of Equations**

1. Jerry has an agreement with his Aunt to shovel out her driveway after each snowfall for \$10. Carol's Aunt paid her \$40 up front and will pay her \$5 for each time she shovels out her driveway.

a) Complete each table of values below for calculating how much each could earn.

Jerry										
Number of Snowfalls	1	2	3	4	5	6	7	8	9	10
Earnings to Date										

Carol										
Number of Snowfalls	1	2	3	4	5	6	7	8	9	10
Earnings to Date										

- b) Who makes the most money after 5 snowfalls? \_\_\_\_\_  
After 10 snowfalls? \_\_\_\_\_

c) Graph both sets of data on the graph to the right.

d) At what point do the graphs intersect?  
What does this point mean?

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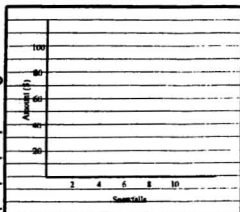
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2. A faulty satellite has a geostationary orbit (remains in a fixed position with respect to the land beneath it) at 12 km above the space station. An astronaut jets off from the station at a constant rate of 3 km/h towards the stationary satellite.

a) Complete each table of values on the next page for determining the distance from the space station of both the satellite and the astronaut.

Satellite						
Hour						
Distance (from the Space Station)						

Astronaut						
Hour						
Distance (from the Space Station)						

b) Graph both sets of data on the graph to the right.

c) At what point do the graphs intersect?  
What does this point mean?

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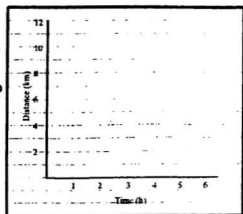
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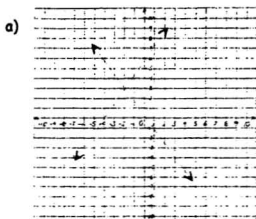
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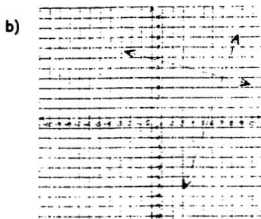
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3. Determine the point of intersection from each graph.



point of intersection: a) \_\_\_\_\_



b) \_\_\_\_\_

4. There appears to be no point of intersection in the graph on the right. Explain why.

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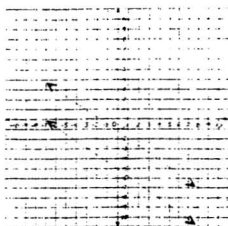
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5. In this question you are asked to examine pairs of intersecting linear equations. How could you determine their point of intersection?
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- 

On this paper, find the point of intersection for each system.

(Hint: Use  $\{x \mid x = -2, -1, 0, 1, 2, x \in \mathbb{I}\}$ )

a)  $y = 3 - 6x$  and  $y = 2x + 3$

b)  $y = 3x + 9$  and  $y = -5x - 7$

c)  $y = x + 4$  and  $y = -8 + 7x$

d)  $y = -13 - 5x$  and  $y = -6 + 2x$

e)  $y = -4x + 3$  and  $y = x - 7$

f)  $y = -2x - 6$  and  $y = -5x - 9$

g)  $y = -11 + 6x$  and  $y = -11 - 3x$

h)  $y = 7x - 5$  and  $y = -4x + 6$

6. List all the methods you have used so far for determining the point of intersection of two lines.

## **Student Worksheet III / IV**

*What is Slope?*

1. What factors affect steepness?

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2. Record the data from the experiment in the table on the right.

3. As a hill becomes steeper, does its slope become greater or smaller? Why?

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4. Stairs have a "comfortable measurement" so that they are neither too steep nor too shallow. Research the measurement.

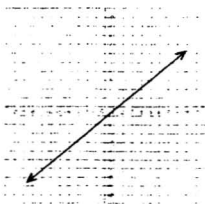
5. Given a graph of a straight line that has no numbered axis, explain how you could tell whether its slope was negative or positive.

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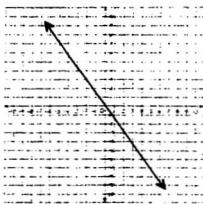
6. For the line, select one part of the line and determine its slope. Repeat this on another section of each line.

a)



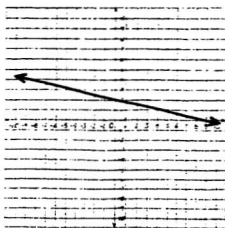
Slope: i) \_\_\_\_\_ ii) \_\_\_\_\_

b)



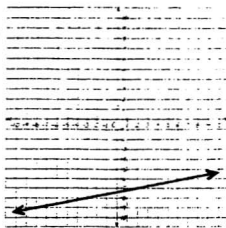
Slope: i) \_\_\_\_\_ ii) \_\_\_\_\_

c)



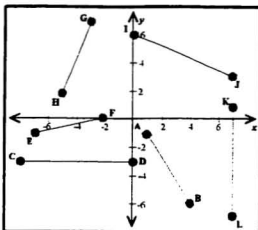
Slope: i) \_\_\_\_\_ ii) \_\_\_\_\_

d)



Slope: i) \_\_\_\_\_ ii) \_\_\_\_\_

7. Determine the slope of each line segment.



AB = \_\_\_\_\_

CD = \_\_\_\_\_

EF = \_\_\_\_\_

GH = \_\_\_\_\_

IJ = \_\_\_\_\_

KL = \_\_\_\_\_

8. Determine the slope through the following points.

a) C(1, 4) and D(3, 5)

b) S(-10, 15) and T(20, -10)



c)  $H(0, -8)$  and  $I(9, 4)$ d)  $M(2, 14)$  and  $N(2, -12)$ 

9. State whether the line connecting the points has a slope that is either positive, negative, zero or undefined.

a)  $Y(18, 3)$  and  $Z(-2, 3)$ 

\_\_\_\_\_

b)  $A(-25, -4)$  and  $B(-30, 14)$ 

\_\_\_\_\_

c)  $R(12, 0)$  and  $S(-4, 6)$ 

\_\_\_\_\_

d)  $P(-9, 14)$  and  $Q(-9, 3)$ 

\_\_\_\_\_

10. Match the slopes to the given lines.

i. \_\_\_\_  $-\frac{1}{8}$ 

ii. \_\_\_\_ undefined

iii. \_\_\_\_ 1

iv. \_\_\_\_  $\frac{1}{8}$ 

11. a) What is meant by a ramp having a slope of  $\frac{2}{3}$ ? a slope of  $\frac{1}{3}$ ?  
Which is steeper than the other and why?

\_\_\_\_\_  
\_\_\_\_\_

b) Which is steeper  $\frac{3}{5}$  or  $-2$ ? Discuss.

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c) Which line is steeper,  $y = 3$  or  $x = 3$ ? Discuss.

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## **Student Worksheet V**

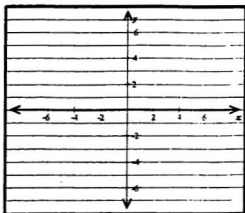
## Graphing

1. Using tables of values, graph each equation on the same axis.

a)  $y = 3x - 4$

b)  $y = \frac{1}{2}x - 4$

c)  $y = -\frac{2}{3}x - 4$



- d) What does each equation have in common?
- 

- e) What do the graphed lines have in common?
- 

- f) Write a conjecture about these linear equations and their graphs.
- 

- g) Think of a name for the constant term in the equations above that reflects what it does when graphed. Afterwards, as a class, we will decide on a common name.
- 

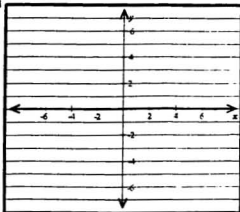
- h) Determine the slope for each line. Compare it to the linear equation. What do you notice?
-

2. Using tables of values, graph each equation on the same axis.

a)  $y = -\frac{1}{3}x$

b)  $y = -\frac{1}{3}x + 4$

c)  $y = -\frac{1}{3}x - 3$



- d) What does each equation have in common?
- 
- e) What do the graphed lines have in common?
- 
- f) Write a conjecture about these linear equations and their graphs.
- 
- g) Determine the slope for each line. Compare it to the linear equation. What do you notice?
- 
- h) Think of a name of the x-coefficient in the equations that reflects what it represents when graphed. Afterwards, as a class, we will decide on a common name.
-

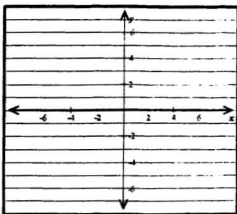
3. Examine the following two equations:

$$y = 2x + 4$$

$$y = 2x - 7$$

- a) What do both equations have in common?
- \_\_\_\_\_

- b) Predict how the lines will be graphed.
- \_\_\_\_\_



- c) Graph the lines **without** using a table of values.

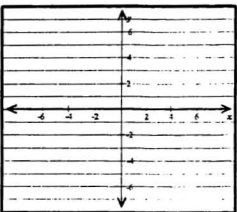
4. Examine the following two equations:

$$y = -4x - 3$$

$$y = 5x - 3$$

- a) What do both equations have in common?
- \_\_\_\_\_

- b) Predict how the lines will be graphed.
- \_\_\_\_\_



- c) Graph the lines **without** using a table of values.

5. Summarize how you may graph a linear equation by determining two of its points.
- \_\_\_\_\_
- \_\_\_\_\_

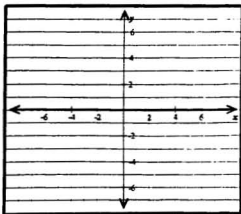
5. Graph the following equations without using a table of values.

a)  $y = \frac{4}{3}x - 5$

b)  $y = -\frac{5}{2}x + 1$

c)  $y = -\frac{2}{3}x$

d)  $y = 7$



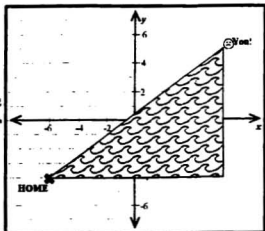
## **Student Worksheet VI**



# Distance

1. Examine the diagram to the right.

You're late again! There are two ways for you to get home around either side of the pond. (Obviously) a long way and a shorter one.



- a) How could you determine the longer distance around the pond? Record it.

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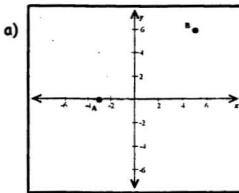
- b) How could you find the shorter distance? Think of as many different ways as you can. Record the distance.

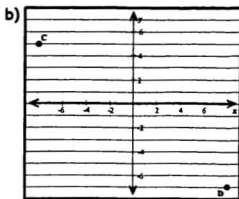
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- c) What is the difference between the two distances? \_\_\_\_\_

2. Determine the distance between the points in the graphs below. Show your workings to the right of each graph.





3. Given just two coordinate points (eg.  $A(-3, 5)$  and  $B(2, 6)$ ), how could you determine the distance between them?

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*Without using any graph paper, can you think of a method that could give you lengths of the legs of the triangle? (Hint, compare the lengths of the triangle's legs and the coordinate points)*

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4. Determine the shortest distance between the following points:

a)  $H(0, 0)$  and  $I(-4, -6)$

b)  $N(9, -3)$  and  $O(2, 8)$

c)  $F(-7, 0)$  and  $G(7, 0)$

d)  $T(-2, -7)$  and  $U(-11, 4)$

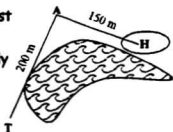
e)  $Q(12, 6)$  and  $R(8, -3)$

f)  $S(0, -10)$  and  $H(10, 0)$

g)  $A(-3, 5)$  and  $B(2, 6)$ h)  $D(14, 1)$  and  $E(9, -5)$ i)  $Y(6, 2)$  and  $Z(-1, 9)$ j)  $J(-8, -11)$  and  $K(0, 0)$ 

5. This golf hole has what is called a "dog leg" where most people hit around the pond for a distance of about 350 m. If you were to try and tee off from  $T$  directly to the hole  $H$ , how far must you hit the ball?

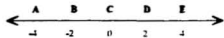
Note:  $\angle TAH$  is a right angle.



## **Student Worksheet VII**

## Midpoint

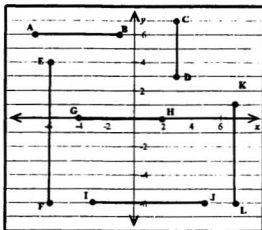
1. Referring to the number line on the right, what is the midpoint of segment AC? \_\_\_\_\_



2. What is the value corresponding to point A? \_\_\_\_\_
3. What is the value corresponding to point C? \_\_\_\_\_
4. What is the midpoint of segment BE? \_\_\_\_\_
5. What is the value corresponding to point B? \_\_\_\_\_
6. What is the value corresponding to point E? \_\_\_\_\_

7. Examine the values of each segment and its midpoint that you found. Write a conjecture describing the relationship between the coordinates of the endpoints of a segment and the coordinate of the midpoint it has.
- \_\_\_\_\_
- \_\_\_\_\_

8. Write the coordinates of each point and the midpoint for each segment.



- a) A \_\_\_\_\_ B \_\_\_\_\_ midpoint \_\_\_\_\_
- b) C \_\_\_\_\_ D \_\_\_\_\_ midpoint \_\_\_\_\_
- c) E \_\_\_\_\_ F \_\_\_\_\_ midpoint \_\_\_\_\_
- d) G \_\_\_\_\_ H \_\_\_\_\_ midpoint \_\_\_\_\_
- e) I \_\_\_\_\_ J \_\_\_\_\_ midpoint \_\_\_\_\_
- f) K \_\_\_\_\_ L \_\_\_\_\_ midpoint \_\_\_\_\_

9. Determine the coordinates of the midpoint from the following endpoints of a segment.

a)  $C(1, 4)$  and  $D(3, 14)$

b)  $V(-10, 15)$  and  $W(20, -10)$

c)  $H(0, -8)$  and  $I(10, 4)$

d)  $M(2, 14)$  and  $N(2, -12)$

e)  $Y(18, 3)$  and  $Z(-2, 3)$

f)  $A(-25, -2)$  and  $B(-31, 14)$

g)  $R(12, -7)$  and  $S(-4, 11)$

h)  $J(-9, 5)$  and  $K(-2, 23)$

i)  $E(9, 0)$  and  $F(-9, 0)$

j)  $T(-11, 7)$  and  $U(6, -4)$

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