The Association Between Spatial and Mathematical Skills: The Role of The Mental Number Line (MNL)

By

Felix Ayesu

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Abstract

Background

Decades of research on numerical cognition have demonstrated that individuals with strong spatial skills tend to be high math achievers. Furthermore, recent studies have indicated that spatial ability is malleable, implying that interventions designed to enhance mathematical learning can be modelled through spatial skill-related tasks. However, the precise mechanism underlying the association between spatial and mathematical skills has yet to be fully understood. An emerging spatial representation theory, tested among children, proposes that the mental number line (MNL), a fundamental cognitive representation underlying the spatial representation of numbers, could be a potential mechanism for the strong link between spatial and mathematical skills. This dissertation investigated the validity of the spatial representation theory among adults by conducting three independent studies that examined various aspects of the theory.

Study 1 explored the relationship between spatial skills and mathematics in adults and the mediating role of the MNL. Studies 2 and 3 posited that, given the spatial representation theory, would spatial anxiety predict mathematical skills, and if so, would such a relationship be mediated by the MNL?

Method

Ninety-two (92) undergraduate students completed assessments of their mental rotation, spatial visualization, Bounded Number Line (BNL) and Unbounded Number Line estimation tasks (UNL), general cognitive ability, and four math areas in study 1. One hundred and twenty-two (122) undergraduate students were tested on their spatial skills, spatial and math anxieties, BNL, and four mathematical domains in study 2. Finally, in study 3, two hundred and twenty-one (221) adults were tested on their spatial skills, spatial and math anxieties, UNL and BNL, and four mathematical domains.

Results

The result indicated that after controlling for general cognitive ability, MNL mediated the relationship between spatial and mathematical skills in adults. However, in contrast to findings among children, the UNL estimation appeared to play a more significant role in this relationship. The result also revealed that spatial anxiety significantly predicted mathematical skills and that the UNL mediated this relationship, even after controlling for general and math anxieties. Overall, the results of the dissertation supported the spatial representation theory, but with some modification. The theoretical and educational implications of these findings are further discussed in Chapter 5.

Keywords: Mental number line, spatial skills, mathematical skills, spatial anxiety

General Summary

Researchers have investigated for many years how our ability to understand numbers is connected to spatial skills. So far, the evidence suggests that people who perform well in spatial tasks, such as mental rotation or visualizing objects in space, also do well in mathematics. More recent studies have shown that mathematic skills can be improved by improving spatial abilities. However, why these two skills are interlinked remains to be seen. One idea that has gained attention in studies involving children is the spatial representation theory, which suggests that our ability to represent numbers and place them on a number line plays a role in mediating our math and spatial skills. This ability to map numbers in space along an imaginary line is supported by a mental representation called the mental number line, which resembles a physical linear number line. To investigate this theory in adults, we conducted three separate studies. In the first study, we examined how spatial skills, mental number line, and math abilities are related in adults. In the second and third studies, we explored whether anxiety about spatial tasks affects math skills. We also checked whether the mental number line plays a role in this relationship. In Study 1, 92 college students took tests on spatial skills, math abilities, and number line estimation. In Study 2, 122 college students were tested for their spatial skills, anxiety about spatial tasks and math, and math abilities. Study 3 involved 221 adults, and their spatial skills, spatial and math anxieties, mental number line, and math abilities were assessed. The results confirmed that the mental number line connected spatial and math skills in adults. However, unlike what we see in children, a different tool for assessing the mental number line, called the Unbounded Number Line mediated the relationship between spatial skills and mathematics in adults. We also found for the first time that feeling anxious about spatial tasks can predict math abilities, and that the Unbounded Number Line appears to be involved in this connection.

Co-Authorship Statement

I hereby declare that this thesis incorporates material that is the result of joint research as follows:

Chapter 1 of this dissertation includes the outcome of the manuscript, which has the following authors: Dr. Darcy Hallett and Nicole Eddy. Dr. Hallett provided monumental feedback that shaped that paper in many forms. He contributed to the data analysis, proofreading, and editing of the manuscript. Nicole Eddy assisted with Data collection for study 1.

Chapters 2 and 3 of the dissertation include the outcome of the two under the supervision of Dr. Darcy Hallett. Dr. Hallett wrote most of the initial javascript code to create the bounded and unbounded number line estimation tasks used in the online surveys of these studies. He provided important feedback that shaped these studies into their present shape. He also assisted in data analysis for these two studies and edited the thesis during it writing stage.

In all cases, key ideas, primary contributions, study designs, data analysis, interpretation, and writing were performed by me.

Dedication

In loving memory of my late mother, Ms. Margaret Djan Ayesu, whose belief in my potential and unwavering encouragement have been a guiding light throughout my academic journey. Although you are no longer with us, your unwavering belief in my potential continues to be a driving force in my life. Your memory lives on in the dedication and determination you instilled in me, and this thesis is dedicated to your memory. To my beloved wife, Mrs. Krista Lynn Ayesu, your staunch support, and understanding have contributed immensely to this academic endeavour. Your love and encouragement have made the journey worthwhile, and I dedicate this work to you with all my heart. I extend my heartfelt gratitude to my family, who have stood by me with love and encouragement. Your collective support has been a source of strength and inspiration. This thesis stands as a testament to the remarkable individuals who have touched my life, and I dedicate it to each of you with deep appreciation.

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CHAPTER ONE: GENERAL INTRODUCTION

Introduction

In recent years, growing reliance on technology has highlighted the significance of Science, Technology, Engineering, and Mathematics (STEM) in modern societies and economies. Consequently, many countries have prioritized the improvement of education in these areas. Mathematical skills, which are essential components of other STEM fields, have received particular attention as policymakers and education stakeholders are urged to take steps to enhance mathematical education (Organisation for Economic Co-operation and Development [OECD] report, 2019). Indeed, recent research indicates that a lack of mathematical proficiency can adversely affect not only individuals, but society as a whole. For instance, low math literacy is associated with difficulty securing well-paying jobs, high school dropout, earning a low income, experiencing high unemployment, and managing finances ineffectively (OECD, 2019). De Bruin and Slovic (2021) investigated the correlation between the ability to use numbers (numeracy skills), and financial well-being in 141 countries. They found that low numeracy skills were associated with inadequate financial well-being across various cultural backgrounds.

Understanding the neurocognitive basis of numerical skills is a critical first step toward enhancing mathematical skill development. Ultimately, researchers and educators can create and implement suitable curricula and interventions to facilitate mathematics learning, by understanding the neurological and cognitive foundations of mathematics development. Over the years, cognitive, behavioural, and neuroimaging research has consistently demonstrated a strong link between spatial and mathematical skills, suggesting that thorough comprehension of spatial skills could facilitate mathematics learning. Wai et al. (2009) discovered that spatial skills influence mathematical learning and are a major factor in STEM fields. A data analysis of 400,000 individuals revealed that high scores in spatial skills in grades 9-12 predicted higher levels of education, careers in STEM fields, and higher earnings. Moreover, brain imaging studies have found a consistent overlap in the brain regions involved in processing spatial and mathematical skill-related tasks. Overlapping brain regions have typically been reported in the frontal gyrus, intraparietal sulci, and posterior and anterior parietal cortices (Cattaneo et al., 2009; Dumontheil & Klingberg, 2012; Hubbard et al., 2003; Zorzi et al., 2002; Zago et al., 2008). For example, the intraparietal sulcus and Angus gyri, located in the parietal region of the brain, are involved in numeracy-related activities. Transcranial Magnetic Stimulation studies further support this idea, demonstrating heightened activity in these areas during numerical processing (Cattaneo et al., 2009). In a longitudinal study, Dumontheil and Klingberg (2012) found that not only is the left intraparietal sulcus involved in visual working memory activities, but that this connection also predicted arithmetic performance two years later, suggesting that strong activities in this area could lead to better mathematical skill development.

Behavioral studies have also demonstrated a robust and consistent positive relationship between spatial and mathematical skills, where individuals with strong spatial skills tend to be good mathematical problem solvers (Attit et al., 2020; Frick, 2018; Mix et al., 2014; Xie et al., 2019). Despite the generally significant correlation between spatial and mathematical skills, Xie et al. (2019) concluded from a meta-analysis that the relationship between spatial and mathematical skills is not linear, indicating that not all spatial skill tasks are equally related to all mathematical domains. Moreover, the authors found that age moderated the association between spatial and mathematical skills, and a robust association was found among children and adolescents. In contrast, no significant relationship was found for adults. Consequently, it is essential to further investigate spatial and mathematical skill interconnections to understand which specific spatial skills are related to different mathematical domains and how those associations change with age.

Indeed, recent evidence shows that spatial skill training can improve spatial skill performance (Mix et al., 2014) and that the improvement in spatial skills appears to transfer to mathematical skill improvements (Hawes et al., 2022). This evidence suggests that mathematical instruction can be modelled through spatially related tasks to enhance mathematics learning. However, spatial and mathematical abilities are considered multidimensional (Hawes et al., 2022), and it remains unclear how different spatial dimensions relate to specific mathematical domains. Moreover, as reported in a recent meta-analysis by Hawes et al. (2022), there is a lack of understanding of the mechanisms underlying the significant relationships between spatial and mathematical skills, hindering educators' and learners' capacity to take advantage of these associations. Thus, understanding the connection between spatial and mathematical skills, especially regarding the consistency of such relationships, delineating how specific spatial skills relate to specific mathematical domains, and understanding the mechanism underlying the relationship, would enable spatial interventions to be tailored to appropriate mathematical domains to foster mathematical learning.

The goals of this dissertation are twofold. First, given the lack of studies exploring spatial and mathematical skill relationships in adults, this dissertation focuses on examining how spatial and mathematical skills are related among adults. Unlike previous studies, this dissertation employs a more comprehensive approach involving multiple spatial dimensions and assesses how each dimension is related to various advanced mathematical skills. Second, this dissertation explores the potential cognitive mechanisms that interconnect spatial and mathematical skills. In summary, this dissertation adds to our understanding of the cognitive processes linking spatial and mathematical skills, bolstering the theoretical basis for exploring how spatial instruction can enhance mathematical learning. In this chapter, I present a review of spatial skills, how they relate to math, and suggested theoretical explanations for the association between spatial and mathematical skills.

Domains of Spatial Skill

Spatial skills have been extensively researched as a cognitive concept essential for various facets of life, including problem-solving, decision-making, and navigation. This facilitates our capacity to mentally manipulate and understand spatial relationships in real-life experiences and imagined spaces (Atit et al., 2022). Researchers from various disciplines, including psychology, neuroscience, and education, have been highly interested in studying spatial skills to understand the underlying mechanisms and contributing elements. Despite considerable research in this area, the categorization of spatial skills remains inconclusive as different studies have operationally defined spatial skills based on what they purport to measure. Although the exact number of dimensions has yet to be determined, contemporary agreements are related to the multidimensionality of spatial skills. Several spatial skill domains have been identified using various statistical tools and theoretical considerations (Buckley et al., 2018; Hegarty et al., 2006; Schneider & McGrew, 2012; Shipley & Newcombe, 2015).

One way of classifying spatial abilities is through factor analysis, a statistical method used to determine the weightings of various cognitive skills (Buckley et al., 2018; Schneider & McGrew, 2012). The Cattell-Horn-Carroll (CHC) theory provides a hierarchical framework for understanding these skills, with general intelligence at the highest level (Schneider & McGrew, 2012). The CHC theory posits that general intelligence is a central coordinator for more specific cognitive skills, such as visual processing, fluid reasoning, psychomotor speed, quantitative knowledge, and auditory processing speed. Visual-spatial skill factors include mental rotation, spatial visualization, spatial orientation, spatial relations, and spatial perception, positioned in the middle of the hierarchy. Thus, spatial skills are closely related to general human intelligence but are separate from other cognitive skills, such as reading, writing, and fluid reasoning. Lohman (1988) conducted a meta-analysis identifying three fundamental components of spatial skills: spatial visualization, spatial orientation, and spatial relations. Spatial visualization involves mentally visualizing moving and changing objects without physical presence. Conversely, spatial orientation involves perceiving and understanding objects from various perspectives. Spatial relations involve comprehending and interpreting spatial connections between objects or components in a given setting, including proximity, distance, direction, size, shape, and orientation. These skills allow individuals to recognize, interpret, and visualize the spatial arrangement in two-dimensional and three-dimensional settings, enabling them to navigate and interact effectively with their physical surroundings. Hegarty et al. (2006) discovered that spatial skills could be classified into small- and large-scale categories. Small-scale skills are evaluated using paper-and-pencil tests such as mental rotation and spatial visualization. In contrast, large-scale skills are assessed through tasks that require participants to draw upon their real experiences in real-world environments, such as map reading.

One of the most cited classifications of spatial skill is provided by Newcombe and Shipley (2015), who proposed a comprehensive classification of spatial skills described along two main dimensions: intrinsic-extrinsic, and dynamic- static. The intrinsic-extrinsic dimension centers on the nature of spatial information, differentiating between information grounded in the characteristics of objects and their arrangement and information tied to an object's location and relation to a reference frame. The dynamic-static dimension pertains to whether spatial information is transformed through actual movement or mental simulation. Dynamic properties concern the spatial aspects of objects and their connection to a reference frame after undergoing transformation, whereas the static properties remain unchanged. Recent studies have supported this theoretical framework (see Newcombe, 2018 for reviews). Based on Shipley and Newcombe's (2015) categorization, spatial skills can be divided into four distinct domains: intrinsic-static, intrinsic-dynamic, extrinsic-static, and extrinsic-dynamic. Intrinsic-static involves the processing of objects without altering their form, as observed in tasks such as embedded figures and picture completion. Intrinsic-dynamic refers to the manipulation of objects or shapes through physical or mental transformations, as demonstrated by tasks, such as mental rotation, block design, and paper folding. Extrinsic-static spatial skills are centered on processing connections among objects or shapes without transformation, as is evident in tasks such as interpreting water levels or static map reading. Conversely, extrinsic-dynamic spatial skills encompass understanding the relationships among objects or shapes involving transformations, exemplified by tasks such as perspective-taking, spatial navigation, and dynamic map reading.

However, some studies questioned the validity of distinguishing between dynamic and static spatial tasks. For instance, Mix et al. (2018) conducted research on spatial tasks involving both dynamic-static and intrinsic-extrinsic dimensions among kindergarteners, third graders, and sixth graders. Through factor analysis, it was discovered that a two-factor model representing extrinsic and intrinsic categories better accounted for the data from kindergarteners and third graders compared to sixth graders, who displayed only one converging factor. According to Mix et al. (2018), the dimensionality of spatial ability can be represented by whether internal or external information is considered when completing spatial tasks. Therefore, while existing literature does not definitively categorize spatial skills, evidence suggests that they consist of multiple dimensions. This dissertation aligns with Newcombe and Shipley's (2015) categorization of spatial skills by utilizing corresponding measures, as in recent investigations exploring the relationship between mathematical abilities and spatial skills. Specifically, spatial visualization and mental rotation, categorized as intrinsic-dynamic spatial abilities, are highly correlated with STEM education, particularly mathematical skills (Attit et al., 2021; Buckley et al., 2018; Frick, 2019).

How are Spatial and Mathematical Skills Related?

One of the well-researched areas of numerical cognition is the relationship between spatial and mathematical skills. Researchers have consistently found evidence that people who perform well in spatial skills also tend to perform better in mathematical skills (Atit et al., 2021; Frick, 2019). Evidence supports an association between spatial and mathematical skills across study designs and measurement techniques. For example, in a recent meta-analysis, Atit et al. (2021) found a moderate but consistent correlation between spatial and mathematical skill. Similarly, a meta-analysis by Xie et al. (2019) found that spatial and mathematical skills were related across longitudinal and cross-sectional studies. Moreover, the authors found that age moderated the relationship between spatial and mathematical skills, where spatial and mathematical skills were significantly related to children and adolescents but not adults. Xie et al. (2019) observed that the non-significant association between spatial and mathematical skills in adults could be due to the limitations of their study, particularly the small sample size of the effect sizes, which resulted from the limited number of studies conducted on adults.

Specifically, the authors noted that a limited number of studies have examined the relationship between spatial and mathematical skills in adults, leading to the inclusion of a small number of effect sizes in their meta-analysis. Although they found no significant association between spatial and mathematical skills among adults, some studies found a significant relationship. Wei et al. (2011) examined undergraduate students' cognitive correlates of their mathematical skills. They found that mental rotation, figure analysis, and visual-spatial working memory were significantly correlated with advanced mathematical skills, even after controlling for general cognitive processes, number sense, and language. In contrast, Haciomeoglu (2015) tested the relationship between spatial orientation, spatial visualization, algebra, and geometry. They found that whereas spatial visualization correlated with algebra and geometry, spatial orientation did not correlate with these math areas. Overall, there is a lack of evidence regarding adults' spatial and mathematical skills, and the existing evidence does not provide a conclusive account of how spatial skills relate to mathematical skills.

Although most studies have focused on the link between spatial and mathematical skills during the early stages of learning and development, understanding this relationship in adults may also enhance how mathematical learning can benefit from spatial skills during the later stages of development. Recent research suggests that spatial training can enhance spatial skills (Mix et al., 2018). As demonstrated in a recent meta-analysis by Hawes et al. (2022), spatial training that resulted in improved spatial skills also improved mathematics performance, with the effect being more profound in studies involving older participants than younger participants. Indeed, for adults with weak mathematical skills, this evidence implies that it is always possible to catch up with their mathematical skills as they can receive spatial training to enhance their mathematical learning. To improve mathematical skills through spatial skill training, it is essential to first understand how specific spatial capacities are connected to specific mathematical domains, and to explore the underlying mechanisms that may explain these associations. This dissertation investigates the relationships between different spatial skill dimensions and various mathematical domains and the potential mechanisms that may be responsible for these connections.

Why are Spatial and Mathematical Skills Related?

Evidence from neuroimaging studies suggests that similar brain areas are involved in numerical and spatial processing (Dumontheil & Klingberg, 2012; Bisiach & Luzatti, 1978; Hubbard et al., 2003; Zacks, 2008; Zorzi et al., 2002; Zorzi et al., 2002). The shared neural mechanism theory is reinforced by research on individuals with parietal lobe damage who exhibit deficits in spatial and numerical abilities. For instance, individuals suffering from Gerstman syndrome demonstrate symptoms such as difficulties in performing mathematical calculations or comprehending mathematical concepts (acalculia) as well as an impaired ability to recognize and name individual fingers (finger agnosia) (Gerstman, 1940). Moreover, hemispatial neglect patients who exhibit impairments in attending to specific spatial areas due to damage to the parietal lobe demonstrate deficits in their ability to determine the midpoints of objects (Bisiach & Luzatti, 1978; Zorzi et al., 2002). Functional MRI studies in monkeys and humans have identified the intraparietal sulcus as a specific site for potential interactions between numerical and spatial dimensions (Hawes et al., 2019; Hubbard et al., 2003; Kadosh et al., 2008). In addition, higher-level spatial skills, such as mental rotation, also connect to these parietal regions, as indicated by studies of the neural correlates of mental rotation (Zacks, 2008).

Within the realm of cognition, a substantial body of research suggests that numbers are spatially organized, called the spatial representation of numbers theory (Dehaene et al., 1992, 1993, 2009; Gunderson etbal., 2012; Tam et al., 2019; Hawes & Ansari, 2020; LeFevre et al., 2013; Shaki et al., 2009; Toomarian & Hubbard, 2018; Pato, Fisher, Nuerk & Cress, 2016). According to this theory, the foundation of mathematical skill development lies in the ability to make sense of numbers through magnitude representation. More than three decades of studies on magnitude representation have revealed that people tend to automatically associate relatively smaller numbers to the left side of space, and relatively larger numbers to the right side of space, called the Spatial Numerical Association Response Codes [SNARC] (Dehaene et al., 1992, 1993; Pato, Fisher, Nuerk & Cress, 2016; Shaki et al., 2009; Toomarian & Hubbard, 2018). Spatial Numerical Associations have been studied using magnitude judgement tasks where participants are asked to decide which of two numbers is smaller or larger or parity judgement tasks that requires participants to determine whether a number is even or odd. Results from these studies have revealed that participants appear to have faster response rate and commit less error when they responded with to larger numbers on the right and responded to smaller numbers with the left hand (Dehaene et al., 1992, 1993, 2009; Shaki et al., 2009; Toomarian & Hubbard, 2018). These spatial biases have been interpreted as evidence of the existence of an internal Mental Number Line (MNL) (Dehaene et al., 1992, 1993, 2009; Shaki et al., 2009; Toomarian & Hubbard, 2018). The MNL is a cognitive representation that humans use to understand and compare numerical magnitudes. It’s like an imaginary ruler in our minds that helps us organize numbers in a linear fashion. Imagine a line stretching from left to right. On this line, smaller numbers are positioned to the left, and larger numbers are to the right. For example, if you think of the numbers 1, 2, 3, and so on, you will naturally arranges them from left to right. When we encounter numbers, our brains automatically map them onto this mental number line. For instance, if you see the number 7, your mind places it somewhere along the line, closer to the larger numbers. The MNL influences our mathematical abilities. It is linked to concepts like counting, addition, subtraction, and estimation. When we perform mental calculations, we mentally move along this line to compare magnitudes and determine relationships between numbers. Interestingly, the orientation of the MNL can vary across cultures (Shaki et al., 2009). In Western cultures, it typically follows a left-to-right orientation (like reading a book). However, some cultures (e.g., Arabic speakers) may have a right-to-left orientation due to their writing systems (Shaki et al., 2009). Dyscalculic individuals often struggle with magnitude representation (Cirino et al., 2015; Friso-van den Bos et al., 2015). Their MNL may be less accurate or less well-developed as they find it challenging to mentally order numbers, estimate quantities, or compare magnitudes (Cirino et al., 2015; Friso-van den Bos et al., 2015). The MNL relies on spatial skills.

The SNARC effect reflects the automatic or unconscious processing of numbers in space; however, other spatial processing of numbers requires effortful or conscious processing of numbers in space (Hawes & Ansari, 2020). Effortful or conscious spatial processing of numbers is typically assessed in studies using number line estimation tasks (Hawes & Ansari, 2020). In the number line estimation, participants are typically shown a number line flanked by numbers at the start and end points and are asked to determine the position of some given numbers called the Bounded Number Line (BNL) (more on this task will be provided in the next section). Although both automatic and effortful spatial processing of numbers is underlined by the internal mental number line framework (Hawes & Ansari, 2020) and, therefore, should be expected to be related to mathematical learning, the relationship between the automatic spatial processing of numbers and mathematics proficiency appears to be inconsistent. For example, Viarougue et al. (2014) reported a negative correlation between the SNARC effect and 2D mental rotation tasks, suggesting that strong spatial skills facilitate operational performance in the number line. However, the relationship between the SNARC effect and numerical skills remains inconclusive, with some studies finding no significant relationship between the SNARC effect and numeracy skills (Cipora & Nuerk, 2013). In contrast, other studies reported a significant but negative relationship between the SNARC effect and numeracy skills (Hoffman et al., 2014). Nevertheless, individuals with high mathematical anxiety appear to exhibit stronger SNARC effects (Nunez et al., 2021).

The number line estimation, mirroring the MNL, which is reflective of the effortful spatial representation of numbers, seems to show a significant relationship with mathematical proficiencies, such as arithmetic and counting (Schneider et al., 2018). Indeed, a number of studies have suggested that training individuals in number line estimation leads to improvement in number line estimation and that this improvement is transferred to numerical reasoning (Fisher et al., 2011; Link et al., 2013), suggesting that improved performance on number line estimation could prime a more refined internal mental number line (Hawes & Ansari, 2020; Fisher et al., 2011). Taken together, while the effortful representation of numbers in space is predictive of mathematics, the automatic representation of numbers does not seem to be conclusively predictive of numerical abilities. The question is to what extent does possessing strong spatial ability influence mathematics learning? Recent studies have sought to examine the interconnection between high-level spatial abilities, such as mental rotation, spatial visualization, spatial scaling, and mathematics. Based on data from children, these studies concluded that spatial abilities influence the effective development of the number line (theoretically similar to the mental number line), leading to improvements in mathematics (Gunderson et al., 2012; Tam et al., 2019; Lefevre et al., 2013).

In a longitudinal study conducted by Gunderson et al. (2012), it was established that the ability of children to mentally transform two-dimensional figures at age five predicted their scores in symbolic calculations at eight years of age. Moreover, their findings revealed that knowledge of number lines (using BNL) assessed at age six significantly mediated the relationship between spatial skills and performance in symbolic calculations. Based on these results, they concluded that spatial skills enhance magnitude representation, subsequently leading to improved mathematical ability. In addition, Tam et al. (2019) examined the role of number lines in the relationship between spatial and mathematical skills. The authors found that participants with higher spatial skills scored higher on the number line tasks and that this relationship was mediated by numerical representation. However, although Frick (2019) found a significant relationship between spatial skills and arithmetic, they failed to find a significant mediating role for the number line measured in the relationship between spatial and mathematical performance in children. Overall, these studies provide a unified framework for how spatial skills interconnect with mathematics through the number line (see figure 1).

Although the spatial representation theory reviewed above appears to identify the number line as the mechanism interconnecting spatial skills and mathematics, the data supporting the theory so far have come from children and to the best of my knowledge, no study has attempted to examine the extent to which the association between high-level spatial ability and mathematics are mediated by the number line among adults. It is important to note that the use of the term mental number line (MNL) does not suggest that the MNL is not the same thing as a number line task, but rather a conceptual framework for understanding the mechanism underlying the spatial numerical associations. Another issue that relates to how the number line has been assessed in the current literature supporting the spatial representation theory. In all studies so far, the BNL has been employed to represent the number line. As previously mentioned, because the mental number line is thought to underly the number line estimation (Hawes & Ansar, 2020), enhanced performance on the number line should theoretically be interpreted as a refinement of the underlying mental number line's development. However, the reliance on performance patterns on the BNL as a reflection of the development of the mental number line has recently been challenged by recent studies that have argued that the BNL may elicit proportion knowledge rather than exclusively tapping into the underlying MNL. In the section, below, I present a brief overview of the recent controversy surrounding the use of the BNL as a measure of the MNL.

Assessing the mental number line: Bounded or Unbounded number line?

Number line estimation is an effortful and intentional way of processing numbers in space (Hawes & Ansari, 2020). Theoretically, the mental number line is thought to underlie this effortful spatial representation of numbers, with good performance on the number line indicating a strong spatial and number connection (i.e., enhanced mental number line). Typically, number line estimation involves the use of a BNL task in which participants are presented with a horizontal line representing a bounded interval of numbers (e.g., from 0 to 1000). For example, a BNL task may start with 0 and end with 1000. Participants are informed that the line corresponds to this specific numeric range and that their task is to estimate and mark the appropriate positions on the line for the various numbers presented to them. For instance, during a trial, participants might be shown the number "25" and asked to identify the position of this number on the line between 0 and 1000.

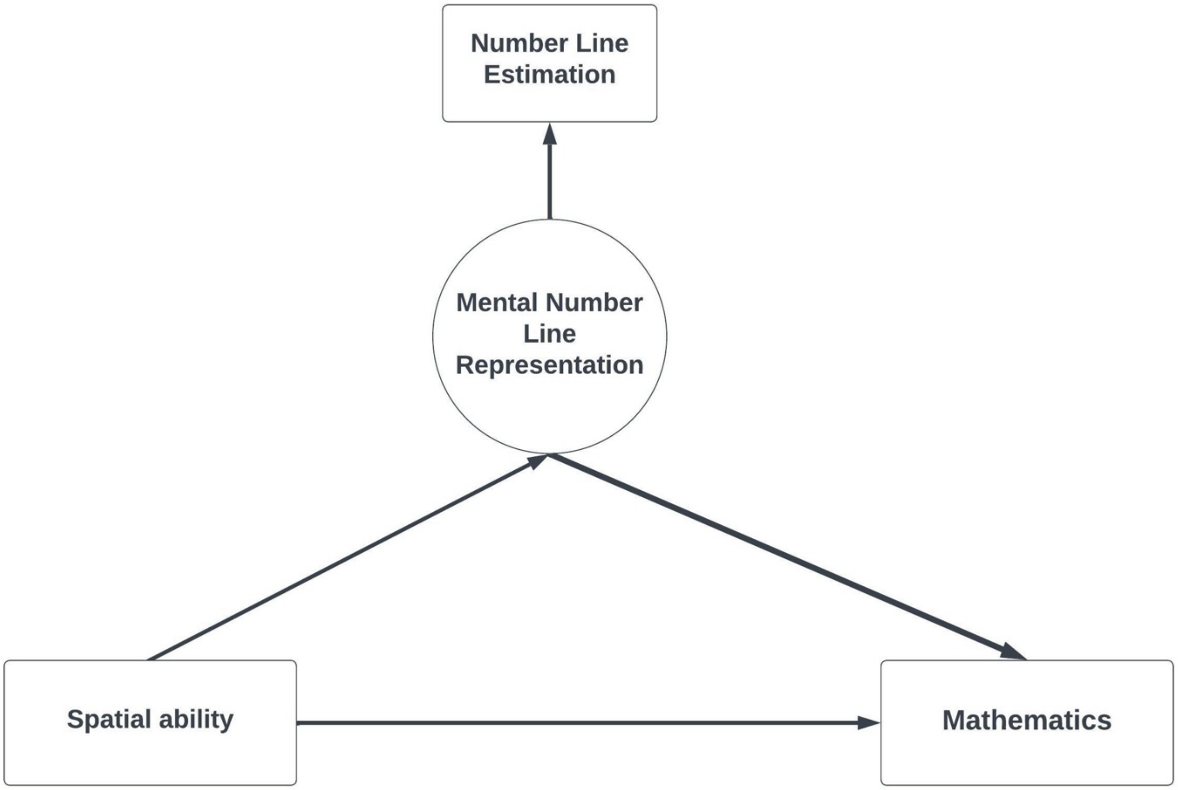
In a series of studies, Siegler and colleagues have revealed a developmental trajectory for these BNL tasks, with notable changes observed from childhood to adulthood. Young children exhibit limited accuracy in their estimations within bounded numerical intervals, whereas adults demonstrate improved precision and understanding (Siegler & Booth, 2004; Booth & Siegler, 2006; Opfer & Siegler, 2007). Siegler and Booth (2004) showed that children often display compressed estimation patterns. They observed that children tended to overestimate the position of smaller numbers and underestimate larger numbers, resulting in a compressed pattern when placing numbers on a number line. For instance, when given numbers such as 10, 20, 40, 50, 60, 70, 80, and 90, and asked to position them on a number line starting from 0 and ending at 100, children may place 10 at position 20, 20 at position 40, and so on. This resulted in little space on the number line to place the remaining numbers and the numbers were not evenly spaced. This error pattern was described as a logarithmic pattern. As children's cognitive skills mature, their performance on BNL estimation tasks improves and becomes more accurate. Booth and Siegler (2006) found that older children exhibit greater accuracy and a reduced compressed pattern as they become more proficient in evenly distributing their estimations across the interval. Adults' performance on the BNL is described as linear, as they evenly space numbers on BNL estimation tasks. Booth and Siegler (2006) characterized the pattern of BNL development as log-to-linear. Indeed, a recent meta-analysis involving 263 effect sizes found that performance on the BNL moderately correlated with mathematical skills and that the correlation was moderated by age, where the correlation seems to increase with age (Schneider et al., 2018). One interpretation based of Siegler and colleagues' theory, children perform poorly on the number line estimation tasks because of a limitation on their underlying mental number line. As children gain more experience, their mental number line becomes more enhanced, leading to improvement in their number line estimation performance.

However, recent studies have challenged the log-to-linear theory, contending that the BNL may require participants to rely on their knowledge of proportion instead of directly accessing the internal number line mechanism in completing those measures (Barth & Paladino, 2011; Cohen & Blanc-Goldhammer, 2011). It is important to note that proportion knowledge maybe relative magnitude (Jirout and Newcombe, 2018), and maybe reliant to some extent on the number line representation, but does not capture the entirely of the mental number line that underlies absolute magnitude representation. Participants completing the BNL tasks often rely on the line's boundaries and midpoints to determine number positions (Barth & Paladino, 2011). This reliance was theorized to lead to a more accurate performance at the line's endpoints and middle, with variations in accuracy linked to the numerical distances between whole numbers at these points. Interestingly, Cohen and Blanc-Goldhammer (2011)'s data analysis favoured a Cyclic Power Model (fitted by a cyclic function), where on the BNL, participants committed less error at the middle and endpoints of the number line, suggesting that BNL might induce proportion understanding instead of an absolute magnitude representation. Although proportion judgement is also a magnitude representation, this differs from the intuitive magnitude representation believed to underlie general numerical skills development. When we rely on the intuitive magnitude representation for completing a number line task, we automatically map any given number on our internal number line without relying on the boundaries of the number line tasks to determine the position of a given number on the number line. For example, when participants are asked to find the location of "50" on a bounded number line task, they activate their intuitive magnitude representation by mapping "50" onto their mental number line, which allows them to determine where to place the number. Thus, a pure magnitude measure allows direct access to our internal magnitude representation underlined by the mental number line organized as a mental number line.

Cohen and Blanc-Goldhammer (2011) devised a new version of the number line estimation task called the unbounded number line estimation task (UNL). This version features a labelled start, unspecified endpoint, and unit measure as reference points for positioning specified numbers. Recent research supports the efficacy of the UNL task as it minimizes the use of proportion-based strategies seen in traditional bounded number line tasks. Reinert et al. (2015) used eye tracking to examine adult eye fixation patterns when completing BNL and UNL task estimation tasks. Based on the Cyclic Power Model, the authors examined whether participants showed increased eye fixation at the starting, middle, and endpoints of the BNL and UNL tasks. The results revealed that for BNL tasks, participants appeared to show increased eye fixation at these points when placing numbers on the line. In contrast, the participants showed decreased fixation on these points on the number line in the UNL task. The authors concluded that BNL elicited proportionate strategies. Georges and Schiltz (2021) explored elementary school children's performance on UNL and BNL tasks, revealing a potentially divergent cognitive construct underlying these tasks. Their findings suggested a stronger reliance on proportion judgment in BNL tasks. There is disagreement regarding whether magnitude representation is best assessed using BNL or UNL, with current evidence supporting UNL as a more appropriate measure of pure magnitude.

Figure 1

A Diagrammatic Illustration of the Spatial Representation Framework



Note. The figure shows the spatial representation of the framework. Spatial skills predict mathematical skills and this relationship is mediated by a number line knowledge. The mental number line is the underlying mechanism for magnitude representation.

In contrast to BNL, no study has investigated how UNL tasks relate to spatial and mathematical skills in the context of mediation. Given the controversy surrounding the use of the BNL as a measure of our intuitive magnitude representation and the suggestion that the UNL may be a purer measure of magnitude (Barth et al., 2016; Cohen & Blanc-Goldhammer, 2011; Reinert et al., 2019; Reinert et al., 2015; Reinert et al., 2021; Siegler & Booth, 2004; Siegler & Booth, 2007; Siegler & Opfer, 2003), it is essential to determine how the UNL relates to spatial and mathematical skills.

The Present Dissertation

The above review shows that, although the relationship between spatial and mathematical skills seems well established in children and adolescents, evidence from adult studies is limited and inconclusive. However, mathematical skills are not limited to childhood and adolescence. Adults can benefit from the association between spatial and mathematical skills. Consequently, this dissertation investigated the relationship between spatial and mathematical skills among adults. Building on previous evidence from children (Gunderson et al., 2012; Tam et al., 2019), suggesting that the spatial representation account of spatial and mathematical skills' associations exists through a mediational analysis, the current dissertation aims to extend spatial representation theory among adults. However, considering the new evidence supporting UNL tasks as a better measure of magnitude representation, it is essential to examine the roles of these two number line estimation tasks in the relationship between spatial and mathematical skills. Employing these two tasks and comparing their relationships with spatial and mathematical skills have extended our understanding of the interplay between these variables in adults. This approach was particularly valuable, because it explored whether the same patterns found in children could be replicated in adults. Three separate studies were conducted in total.

Study One aimed to extend spatial representation theory to an adult sample. This study was the first to examine the role of number line representation measured by BNL and UNL estimations in mediating the relationship between spatial and mathematical skills among adults. Thus, Study One included both traditional BNL tasks and a recently developed UNL version of the task.

Study Two examined another facet of the spatial representation account and its theoretical implications. It was argued that if the spatial representation account holds true, it would be expected that factors that may impact spatial reasoning would also influence mathematical skills. One factor that has been shown to be negatively related to spatial ability is spatial anxiety (Attit et al., 2021; Lyons et al., 2018), where individuals with high spatial anxiety have low spatial skill task scores (Attit & Rocha, 2021). Mathematics anxiety is related to mathematics (Ramirez et al., 2013). In addition, recent studies have found that spatial and mathematics anxiety are correlated (Ferguson et al., 2015). Although many studies have examined mathematical anxiety and its relationship with mathematics, very few have explored how spatial anxiety relates to spatial skills, and no study has attempted to investigate how these affective aspects of spatial skills relate to mathematical skills. Therefore, Study Two was approached from an exploratory perspective. In addition to testing how spatial anxiety relates to mathematical skills, this study included a measure of BNL to examine its association with spatial anxiety and mathematical skills. By incorporating these variables, this study sheds light on the intricate connections among spatial anxiety, MNL representation, and mathematical skills in adults.

Study Three aimed to further explore the findings of Studies One and Two and to establish a more comprehensive understanding of the spatial representation account. This study examines how the affective aspects of spatial and mathematical skills are related across domains. Thus, we examined whether spatial anxiety was related to mathematical skills. In addition, this study investigated how the affective aspects of spatial and mathematical skills are related to the magnitude representation measured by bounded and unbounded number lines.

By examining the relationships between these variables, this dissertation sheds light on the role of MNL in the relationship between spatial and mathematical skills. This research contributes to a better understanding of the cognitive mechanisms underlying the relationship between spatial and mathematical skills and informs future research on potential interventions for enhancing mathematical learning. Furthermore, using scores of both BNL and UNL estimation tasks to infer the role of the underlying MNL representation and provides theoretical insights into the validity of these tasks in assessing the internal representation of number magnitudes. Ultimately, this study contributes to the field's understanding of how spatial skills and MNL influence mathematical skills, offering implications for educational practices and interventions that support mathematical learning.

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CHAPTER 2: THE ASSOCIATION BETWEEN SPATIAL AND MATHEMATICAL SKILLS IN ADULTS: THE ROLE OF MENTAL NUMBER LINE.

Felix Ayesu, Darcy Hallett, and Nicole Eddy
Research Center for the Development of Mathematical Cognition,
Department of Psychology,
Memorial University of Newfoundland

Author Note

Correspondence regarding this article should be addressed to Felix Ayesu, Psychology Department, 232 Elizabeth Ave, St. John's, NL, A1B 3X9, Canada. Email: fayesu@mun.ca.
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Abstract

Spatial and mathematical skills exhibit a consistent association. However, the underlying mechanism connecting them remains unclear. Recent studies of children have proposed that the Mental Number Line (MNL), a fundamental cognitive representation involving the spatial representation of numbers, is a potential explanation for the strong link between spatial and mathematical skills. We tested this hypothesis using adults, while also testing whether the best measure of MNL understanding is a bounded number line (where both ends of a number line are labelled), or an unbounded number line (where only the zero point and a small interval from that point are labelled). Ninety-two (92) undergraduate students were assessed for their knowledge of four mathematical domains (fractions, algebra, arithmetic, and geometry) as well as their knowledge of mental rotation, spatial visualization, and MNL representation, using both bounded and unbounded number line tasks. Structural Equation Modelling revealed that spatial skill significantly predicted algebra, fractions, and arithmetic performance but not geometry. Furthermore, the unbounded number line estimation task was the only mediator that significantly explained the relationship between spatial skills and these mathematic areas, suggesting that the unbounded number line may be a better indicator of magnitude representation and, therefore, MNL in adults.

Keywords: spatial skills, mathematical cognition, mental number line, unbounded number line, bounded number line.

Educational Relevance Statement

In this study, we provided evidence that individuals with strong spatial skills also tend to excel in mathematical performance. This connection is influenced by proficiency in number line estimation, specifically a more recent version of the task known as the Unbounded Number Line. These findings provide significant theoretical insights that could influence future research, and suggests the possibility that utilizing spatial ability tasks to improve mental number line ability can positively affect the development of mathematical skills.

Introduction

Decades of research on numerical cognition across diverse age groups and using various methodologies and designs seem to conclude that people with strong spatial abilities also tend to be high mathematical achievers (Hawes et al., 2022; Xie et al., 2019; Atit et al., 2022). The robust association between spatial skills and mathematics suggests that if the mechanism underlying spatial and mathematical associations is identified, it could serve as an avenue for improving mathematical learning (Hawes et al., 2022; Xie et al., 2019; Atit et al., 2022). Indeed, recent studies have demonstrated that spatial skills are malleable (Cheng & Mix, 2014; Hawes et al., 2022; Mix & Battista, 2018), as spatial skills training transfer to mathematics (Hawes et al., 2022). However, the current literature does not provide a unified framework for understanding the mechanism underlying the spatial-mathematical associations.

Neuroimaging studies provide evidence suggesting the involvement of similar brain areas in numerical and spatial processing (Dumontheil & Klingberg, 2012; Bisiach & Luzatti, 1978; Hubbard et al., 2003; Zacks, 2008; Zorzi et al., 2002; Zorzi et al., 2002). This shared neural mechanism theory gains support from research on individuals with parietal lobe damage, revealing deficits in both spatial and numerical abilities. For instance, those with Gerstman syndrome struggle with mathematical calculations (acalculia) and recognizing individual fingers (finger agnosia) (Gerstman, 1940). Additionally, hemispatial neglect patients, facing challenges in attending to specific spatial areas due to parietal lobe damage, exhibit difficulties in determining the midpoints of objects (Bisiach & Luzatti, 1978; Zorzi et al., 2002). Functional MRI studies in monkeys and humans highlight the intraparietal sulcus as a potential site for interactions between numerical and spatial dimensions (Hawes et al., 2019; Hubbard et al., 2003; Kadosh et al., 2008). Higher-level spatial skills like mental rotation also correlate with these parietal regions, as evidenced by studies on the neural correlates of mental rotation (Zacks, 2008).

In the domain of cognition, extensive research supports the spatial representation of numbers theory (Dehaene et al., 1992, 1993, 2009; Gunderson et al., 2012; Tam et al., 2019; Hawes & Ansari, 2020; LeFevre et al., 2013; Shaki et al., 2009; Toomarian & Hubbard, 2018; Pato, Fisher, Nuerk & Cress, 2016). This theory posits that the foundation of mathematical skill development lies in the ability to comprehend numbers through magnitude representation. Over three decades of studies on magnitude representation have revealed an automatic association of relatively smaller numbers to the left side of space and relatively larger numbers to the right side, termed Spatial Numerical Association Response Codes (SNARC) (Dehaene et al., 1992, 1993; Pato, Fisher, Nuerk & Cress, 2016; Shaki et al., 2009; Toomarian & Hubbard, 2018). Spatial Numerical Associations observed in magnitude judgment tasks and parity judgment tasks indicate faster responses and fewer errors when participants associate larger numbers with the right side and smaller numbers with the left hand (Dehaene et al., 1992, 1993, 2009; Shaki et al., 2009; Toomarian & Hubbard, 2018). These biases are interpreted as evidence of a metaphorical internal number line (MNL). The MNL is a cognitive representation that humans use to understand and compare numerical magnitudes. It’s like an imaginary ruler in our minds that helps us organize numbers in a linear fashion. Imagine a line stretching from left to right. On this line, smaller numbers are positioned to the left, and larger numbers are to the right. For example, if you think of the numbers 1, 2, 3, and so on, you’ll naturally arrange them from left to right. When we encounter numbers, our brains automatically map them onto this mental number line. For instance, if you see the number 7, your mind places it somewhere along the line, closer to the larger numbers. The MNL influences our mathematical abilities. It’s linked to concepts like counting, addition, subtraction, and estimation. When we perform mental calculations, we mentally move along this line to compare magnitudes and determine relationships between numbers. Interestingly, the orientation of the MNL can vary across cultures. In Western cultures, it typically follows a left-to-right orientation (like reading a book). However, some cultures (e.g., Arabic speakers) may have a right-to-left orientation due to their writing systems (Shaki et al., 2009). Dyscalculic individuals often struggle with magnitude representation (Cirino et al., 2015; Friso-van den Bos et al., 2015). Their MNL may be less accurate or less well-developed as they find it challenging to mentally order numbers, estimate quantities, or compare magnitudes (Cirino et al., 2015; Friso-van den Bos et al., 2015). The MNL relies on spatial skills (Dehaene et al., 1992, 1993, 2009; Shaki et al., 2009; Toomarian & Hubbard, 2018; Gunderson et al., 2012). While the SNARC effect occur automatically other spatial processing of numbers demands intentional effort (Hawes & Ansari, 2020).

Effortful spatial processing of numbers is typically evaluated using number-line estimation tasks, which similar to the underlying MNL (Hawes & Ansari, 2020). In these tasks, participants viewed a number line with flanking numbers at the start and end points, aiming to determine the position of given numbers, referred to as the Bounded Number Line (BNL). Despite the shared framework of an internal mental number line underlying both automatic and effortful spatial processing of numbers (Hawes & Ansari, 2020), their relationship with mathematical learning appears inconsistent. Studies have reported varying associations between the SNARC effect and numeracy skills, with some indicating a negative correlation with 2D mental rotation tasks (Viarougue et al., 2014) and others suggesting a significant but negative relationship with numeracy skills (Hoffman et al., 2014). Individuals with high mathematical anxiety seem to exhibit stronger SNARC effects (Nunez et al., 2021). On the other hand, number line estimation, reflective of the effortful spatial representation of numbers, shows a significant relationship with mathematical proficiencies such as arithmetic and counting (Schneider et al., 2018). Training in number line estimation leads to improved numerical reasoning, implying that enhanced performance on the number line can refine the internal mental number line (Hawes & Ansari, 2020; Fisher et al., 2011). It is important to note that the term 'mental number line' (MNL) does not imply actual knowledge but serves as a conceptual framework for understanding the mechanism underlying spatial numerical associations.

To what extent does possessing high-level spatial skills lead to improvements in math performance? In other words, what mechanism connects spatial abilities to the spatial numerical associations that could confer high spatial ability as an advantage for numerical reasoning and mathematics in general? Recent studies have explored the interconnection between high-level spatial abilities, such as mental rotation, spatial visualization, spatial scaling, and mathematics, suggesting that spatial abilities influence the effective development of the number line, leading to improvements in mathematical skills (Gunderson et al., 2012; Tam et al., 2019; Lefevre et al., 2013). In a longitudinal study, Gunderson et al. (2012) examined the relationship between mental rotation, linear number line estimation, and mathematical achievement in children. Study One found that seven-year-old's mental rotation scores assessed during the first three months of the school year significantly predicted their BNL scores six months later. They also found that mathematical achievement at the beginning of the school year was a significant predictor of children's BNL estimation knowledge six months later. In Study Two, Gunderson et al. (2012) found that spatial skills measured by mental transformation tasks at age 5 predicted symbolic calculation scores at age 8. Notably, linear number line knowledge assessed at age six significantly mediated the relationship between spatial skills and symbolic calculations. Based on these results, Gunderson et al. (2012) conclude that spatial skills improve linear number line knowledge, leading to better mathematical skill performance.

This evidence was confirmed among Chinese second graders by Tam et al. (2019), who found that number line estimation tasks significantly mediated the relationship between spatial skills and calculation, as well as spatial skills and performance on mathematical word problems. In contrast, number line estimation was not a significant mediator in the relationship between spatial skills and the arithmetic fact retrieval task. Other studies reported findings that contradict the results of Tam et al. (2019). For example, Frick (2019) failed to find a significant mediating role for MNL, measured by performance on the BNL, in the relationship between spatial and mathematical skills. Despite spatial representation theory pointing to the number line as the mechanism connecting spatial skills and mathematics, existing data supporting this theory primarily stem from studies on children. To date, no study has examined the extent to which the association between high-level spatial ability and mathematics is mediated by number lines among adults.

Assessing the magnitude representation: BNL or UNL estimation

Number line estimation is an intentional process of spatially processing numbers (Hawes & Ansari, 2020). The mental number line theoretically underlies this effortful spatial representation with good performance, indicating a robust spatial and numerical connection (i.e., enhanced mental number line). Traditionally, number line estimation involves a Bounded Number Line (BNL) task, where participants are presented with a horizontal line representing a bounded interval of numbers (e.g., 0 to 1000). For instance, a BNL task may start at 0 and end at 1000, requiring participants to estimate and mark the appropriate positions for various numbers within this range (Siegler & Booth, 2008). Research indicates that children commonly exhibit errors in estimation, frequently overestimating or underestimating positions, with a tendency toward overestimation for smaller numbers (Siegler & Booth, 2008). For instance, when placing numbers such as 5, 20, 45, 67, and 85 on a number line bounded by 0 and 100, children might place 5 at a location corresponding to 20, and 20 at a location corresponding to 45. This overestimation of smaller numbers leaves less space for larger numbers, causing the positions of larger numbers to be compressed, consequently leading to an underestimation of relatively larger values. The children's performance becomes more accurate within the same numerical range as they age. However, they persist in making similar errors when presented with more complex tasks involving different boundaries, such as 1–1000. Siegler et al. (2008) described this error pattern as a logarithmic representation in children. By contrast, adults tend to exhibit a linear pattern in the estimation of BNL (Siegler & Booth, 2008).

The log-to-linear results have been interpreted as a manifestation of the developmental trajectory of the internal number line underlying magnitude representation development, where children show logarithmic patterns as they are still developing their magnitude representation. The linear performance pattern in adults occurs because their internal number line is fully developed, leading to reduced errors (Siegler et al., 2008; Opfer & Siegler, 2007). However, others have argued that the developmental changes on the traditional BNL may not be a consequence of representational changes in magnitude but tasks specific measurement skills (Cohen & Snecka, 2014). Moreover, it has been suggested that the performance patterns of BNL may stem from the use of proportion knowledge rather than exclusively relying on pure magnitude knowledge (Barth & Paladino, 2011; Cohen & Blanc-Goldhammer, 2011; Georges & Shiltz, 2021; Reinert et al., 2019). Among seven-year-old children, Barth and Paladino (2011) found that children's data on BNL tasks were a better fit by a proportion judgment model relative to a logarithmic model, suggesting that children's performance was more linear than that proposed by Siegler et al. (2008). They found that children appeared to make a more accurate estimate of the BNL task if the number to be placed on the number line was close to the numerical distance at the midpoint, the starting point, and the endpoint of the number line. Thus, the labeled endpoints of the BNL task disclose the value of the whole. Consequently, children rely on these positions and compare them with the numbers to be placed on the number line before making their judgment.

Although proportional skill is undoubtedly helpful in mathematical understanding (Fitzpatrick & Hallett, 2019), if the BNL estimation task reflects the understanding of proportion more than absolute magnitude representation, it may not be the best measure of intuitive magnitude representation, and hence the internal number line. Instead of the BNL estimation task, Cohen and Blanc-Goldhammer (2011) proposed a version of the number line estimation task known as the UNL estimation task, which they claimed was more reflective of pure magnitude representation. The UNL estimation task has a labeled starting point, but the endpoint was not marked. However, a unit point was marked on this task, which served as a reference point for participants to gauge the location of the specified numbers. Cohen and Blanc-Goldhammer (2011) contended that the UNL estimation task is a more effective tool for accessing magnitude representations, as it limits the use of proportion-based strategies. Cohen and Blanc-Goldhammer (2011) found that both BNL and UNL estimation tasks revealed a similar numerical bias described as an accelerating power function. However, the BNL estimation task involved a proportion estimation strategy, leading to proportion-related variance.

In contrast, the UNL estimation task showed a pattern of scalar variance consistent with estimating integers. Based on this result, the authors concluded that the UNL estimation task was a more accurate measure of the numerical bias of integers than the BNL version.

Reinert et al. (2015) conducted an eye-tracking study to investigate the eye-fixation patterns of adults while performing BNL and UNL estimation tasks. Using the Cyclic Power Model as a reference (i.e., a curvilinear model where accuracy decreases from the starting point and then increases again as it approaches the midpoint and then repeats this pattern between the midpoint and the endpoint), they examined whether participants exhibited increased eye fixation at the starting point, midpoint, and endpoint of the number line when completing the BNL and UNL estimation tasks. They found that participants displayed increased eye fixation at these points when placing numbers on the line during BNL estimation tasks. Conversely, participants exhibited reduced fixation on these points on the number line during the UNL estimation tasks, concluding that BNL estimation may elicit proportion estimation strategies. In another study, Reinert et al. (2019) aimed to understand the nature of magnitude knowledge in tasks involving UNL and BNL estimations by comparing them with non-symbolic numerosity estimation tasks. Non-symbolic estimation tasks involve comparing dots and indicating quantities using Arabic numbers (verbal) or symbolic numbers (e.g., 1, 2, and 3). Participants tended to underestimate in the perception task (dot comparison) and overestimate in the production task (produce dots based on symbolic numbers), known as a bidirectional mapping process. The researchers hypothesized that any task claiming to access magnitude knowledge should have error patterns similar to non-symbolic estimation. They found that UNL estimation tasks displayed similar overestimation and underestimation patterns as non-symbolic numerosity estimation tasks. In contrast, these patterns were reversed in the BNL estimation task. This result further suggests that UNL estimation aligns more closely with pure magnitude judgments than the BNL task.

In summary, previous research has primarily focused on assessing magnitude estimation using BNL estimation tasks, revealing a log-to-linear pattern, with children displaying logarithmic error patterns and adults showing a linear pattern, implying developmental changes in magnitude representation. Recent studies (Barth & Paladino, 2011; Cohen & Blanc-Goldhammer, 2011; Reinert et al., 2015; 2019) have challenged this log-to-linear theory, suggesting that BNL estimation tasks may rely more on proportion knowledge than pure magnitude knowledge. Instead, the UNL estimation task, featuring a labeled starting point but an unspecified endpoint, was proposed as a better measure of magnitude representation, and therefore, the underlying MNL representation.

Present Study

The above reviews have highlighted the potential of the UNL as a measure of the internal MNL and, hence, magnitude representation skills. Moreover, the identified mechanism underlying the relationship between spatial and mathematical abilities is number line knowledge (Gunderson et al., 2012; Tam et al., 2019). However, the existing evidence is constrained and needs more exploration across diverse age groups, particularly in the context of adults. Although studies have consistently revealed the relationship between spatial and mathematical skills in children and adolescents, the clarity of this association diminishes in adults (Attit et al., 2022; Hawes et al., 2022; Xie et al., 2019). Furthermore, the impact of spatial training on mathematical skills appears to be influenced by age, suggesting an age-dependent effect (Hawes et al., 2022). Therefore, extending the investigation beyond children to encompass adults is crucial, as findings may diverge from those observed in younger populations.

The present study aimed to examine the correlation between spatial and mathematical skills in adults, evaluating multiple advanced-level mathematical domains and various spatial domains, outcomes that have yielded mixed results in prior research (Haciomeroglu, 2015; Thompson et al., 2013). Additionally, past studies attributing the mediation of the spatial and mathematical skills relationship to the number line have exclusively employed scores on the Bounded Number Line (BNL) task (Gunderson et al., 2012; Tam et al., 2019). However, considering recent indications that the Unbounded Number Line (UNL) task might tap into more pristine magnitude representational skills (Reinert & Moeller, 2021), exploring the UNL task's relationship with spatial and mathematical skills is pertinent. The current study delved into assessing both BNL and UNL tasks to elucidate their respective roles in shaping the connection between spatial and mathematical skills. The following research questions were investigated.

1. How do different spatial and advanced mathematical domains relate in adults?
2. Does mental number line, measured by number line estimation mediate the relationship between spatial and advanced mathematical skill domains?
3. What is the role of BNL and the UNL in the relationship between spatial and advanced mathematical skills?

Method

Participants

Ninety-two (92) undergraduate students from a mid-sized university in Atlantic Canada participated in this study (28 males, 64 females; *Mean Age* = 21.00, *SD* = 1.79). Participants were compensated with course credit or $20 for their participation. As a result of the COVID-19 lockdown, two participants did not complete all measures and their data were removed from further analysis. Thus, the data from only 90 participants were included in the final analysis. This study was reviewed and approved by the Research Ethics Board at the university in question.

Measures

Spatial skill tasks:

Spatial skills were assessed using two tasks: mental rotation and spatial visualization. Within the categorization of spatial skills provided by Newcombe and Shipley (2015), mental rotation falls under intrinsic-dynamic spatial skills. On the other hand, spatial visualization can be classified as an intrinsic static spatial skill. These areas of spatial skill have previously been found to be related to basic mathematical calculations and algebraic arithmetic (Frick, 2019; Haciomeroglu, 2015; Thompson et al., 2013).

Mental Rotation Test (MRT)

The revised Mental Rotation Test (Vandenberg & Kluse, 1978, and modified by Peters et al., 1995) was used to assess mental rotation. This 24-item measure presented five figures per item. Each item comprised a separate figure on the left and four figures on the right. All four figures on the right-hand side differ in terms of orientation. Two of the four figures on the right are the same as the first figure on the left side but have been rotated around the vertical axis of the left figure. Participants were required to identify the two forms among the four figures on the right, which were the same as those on the left. One point was awarded for each pair of correctly identified figures, and 0 points for incorrect pairs. The maximum and minimum scores are 24 and 0, respectively. Higher scores indicate higher mental rotation skills. The internal consistency of this measure is high (Cronbach's alpha = 0.85). See Appendix A.

Spatial Visualization: Surface Development Test (SDT)

Spatial visualization was assessed using a Surface Development Test (SDT). This measure, developed by Ekstrom et al. (1976), contained 12 items, each consisting of a pair of net and solid shapes. Participants were required to match the edges of the net shape to those of the solid shape when the shape was folded along the dotted lines of the net figure. To complete the test, participants had to visualize the open and closed forms of the shapes and match the numbers on the net to the letters on the solid shapes. One point was awarded for a correct match and zero points for an incorrect match. To correct for guessing, for each participant, a fraction of their incorrect answers was subtracted from their correct answers (total correct answers - 1/5 of the total incorrect answers). High scores indicate better performance, and low scores indicate poor performance. The internal consistency (Cronbach's alpha) was 0.89. See Appendix B.

Mathematical Skills: Math Placement Test (MPT)

The mathematical skills of the participants were evaluated using a paper-and-pencil test taken from the Mathematical Placement Test (MPT). The MPT is a diagnostic test of basic mathematics skills administered by the Department of Mathematics and Statistics at the university where this study was conducted. Mathematics educators designed it to assess competence levels in varied core areas of mathematics covering up to at least 12th grade. A total of 45 items were selected from the MPT question pool. These items were subsequently grouped into four mathematical domains: fractions, algebra, geometry, and arithmetic. To ensure that the items were correctly grouped into their respective mathematical domains, two of the co-authors of this study and a graduate student were asked to independently classify the items into fractions, algebra, geometry, and arithmetic. The inter-rater agreement was 100%, indicating that each rater classified items in the same mathematical domain. Sixteen (16) items were classified as fractions, 10 as algebra, 14 as arithmetic, and 5 as geometry questions.

These questions were multiple choices that required participants to solve the problem and then select the correct answer from four options (a-d). This means that participants required conceptual and procedural knowledge in each mathematical domain. For the arithmetic items, participants were asked to perform arithmetic computations (e.g., simplify 10 + 325 + 14 + 61). Similarly, for the fraction items, participants were required to perform arithmetic computations on multiple fractions, including decimals (e.g., to simplify and reduce to the lowest term:1/3(1/2(6))). Algebra requires finding variables in algebraic equations (for example, if $5v + 3 = 2v – 3$, then $v =?$). For the geometry items, the participants were shown various model figures comprising triangles, rectangles, and circles. They were then asked to determine the areas and perimeters of specified parts of the figures. In addition, participants were not allowed to use a calculator but used their own computational and mathematical knowledge. Thus, these items assessed the comprehensive proficiency of each domain. The percentage of correct responses was determined for each domain, where a higher percentage represented higher scores.

Cronbach's alpha was calculated for each of the four subscales. Arithmetic ($\alpha = .68$), fractions ($\alpha = .78$), and algebra ($\alpha = .63$) demonstrated reasonable to good reliability. However, the geometric measure demonstrated poor reliability ($\alpha = .24$). We decided to retain the geometric measure given the high inter-rater agreement that the component items were geometry items, but we will regard the results with this measure with caution.

Magnitude representation: bounded and unbounded number line tasks

The participants completed both BNL and UNL assessments. The BNL task was based on Link et al.'s (2014) study, with lines ranging from 0-115 (see Figure 2; Panel A). A range of 115 was chosen to make it more difficult and to help create variability among the adult participants, while also keeping the numbers in the same range as those that were used for the UNL task. For this assessment, participants were asked to place 20 numbers on an individual number line, with each line measuring 23 cm in length. The lines were staggered to prevent participants from using their previous answers as reference points. Participants were not permitted to use physical means of measurement to place the numbers on the lines (e.g., a ruler or finger spacing). Each participant was asked two training questions but did not receive any feedback on the accuracy of their responses. The UNL task was adopted from Reinert et al. (2015). Participants were required to place a given number on a number line for this task, with a starting point and no endpoint, but a unit measurement that served as a guide (see Figure 2; Panel B). There were 20 empty lines, with each line measuring 23 cm in length. Each page consisted of three items, except for the final page, which contained two. The starting points of the lines were staggered to ensure that the participants did not use the line lengths of other items as a lead. The unit guide or anchor was consistently 1.1 cm long, varying from quantified representations of 2 to 12. Participants were required to place numbers ranging from 0 to 115 on the line, and two training questions were provided for each participant. Similar to the BNL task, participants did not receive any feedback regarding the accuracy of their responses to the training questions. In addition, they were not permitted to use physical means of measurement to place numbers on the lines (e.g., ruler or finger spacing).

For both the BNL and UNL tasks, participants had three minutes to complete the task and were scored using the percent absolute error (PAE) principle [PAE= (estimated number – target)/scale x 100%, Booth & Siegler, 2008] for each item. For example, placing 81 in place of 90 yielded a PAE of 6.96% [((81-90)/115) × 100]. The mean PAE was computed for all 20 items, and the mean of these items was calculated to create an overall performance score. Higher scores on these tasks indicated more errors and poor performance.

Figure 2

Samples of Number Line Estimation Task

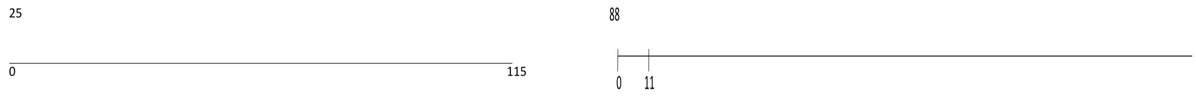


Fig. 2; Panel A. An example of the BNL task

Fig. 2; Panel B. Example of UNL task

General Cognitive Skills: Nonsense Syllogism Test

To control for general cognitive skills, we adopted the Nonsense Syllogism Test developed by Ekstrom et al. (1976) as a proxy measure. Participants were required to complete formal syllogisms using nonsensical content for this task. Two preamble statements and conclusions were made for each item. Participants were asked to identify whether the conclusions deduced from the two preamble statements were logical. There were 30 items on this task divided into two parts, each allotted four minutes. High scores are indicative of high general cognitive skills in this task.

Procedure

Participants were tested individually in a quiet laboratory setting. The researcher provided instructions and answered questions before the participants began each task. The entire testing process took approximately two (2) hours and was divided into two sets in two 1-hour sessions held on separate days. The Surface Development Test, Mental Rotation, and two number line tasks were in Set 1, whereas the logical reasoning and Math Placement Test were in Set 2. Half of the participants completed Set 1 in their first session and Set 2 in their second session, whereas the other half completed Set 2 in their first session and Set 1 in their second session. All measurements were performed using a pencil and paper.

Analysis

Data analysis was performed using R Studio (version 2023.06.1+524). The MVN package (version 5.9) was used to assess the multivariate normality (Korkmaz et al., 2014). The lavaan package (version 0.6-5; Rosseel, 2012) was used for SEM and mediation analyses. Using the Mahalanobis multivariate test, we first conducted a multivariate outlier test on the data of the 90 participants. Five (5) outliers were detected and were excluded from the analysis (Mah < 16.919, $p < 0.05$). Therefore, only the data from 85 participants were included in the final analysis. A multivariate Mardia normality test indicated nonsignificant skewness (170.939, $p < 0.360$), but significant negative kurtosis (-2.023, $p = 0.042$). Consequently, model fit indices with robust standardized error estimates were used, as this adjusted the standard errors for non-normality correction (Satorra & Bentler, 1988). Structural Equation Models (SEM) were used to evaluate the relationships between spatial skills, mathematical competencies, and MNL.

Results

Table 1 presents the results of the descriptive statistics. Based on PAE scores, comparing performance on BNL and UNL tasks, performance on the BNL task was better than on the UNL task (*Mean difference* = 4.28, $t_{168} = 10.0$, $p < 0.001$, Cohen's $d = 1.54$). As shown in Table 2, all mathematical domains had moderate to high significant correlations. In addition, mental rotation and spatial visualization were significantly correlated. Logical reasoning was found to have a significant correlation with arithmetic and mental rotation and was also significantly correlated with BNL (*p*= 0.05), but not with the remaining measures. The results also revealed that BNL scores were significantly correlated with the fraction, geometry, mental rotation, and spatial visualization, but did not significantly correlate with any of the remaining measures. At the same time, the BNL scores had a low error level and were very positively skewed, suggesting the task may have had a floor effect. The UNL was also significantly correlated with fraction, algebra, and arithmetic, but did not correlate significantly with geometry. In addition, UNL was significantly correlated with mental rotation and spatial visualization.

Table 1

Descriptive statistics (N = 85)

		M	*SD*	Min	Max	Skewness	Std. error
1.	Fraction	64.6	23.3	12.5	100.0	-0.2	0.3
2.	Arithmetic	74.0	18.3	21.4	100.0	-0.6	0.3
3.	Algebra	71.5	22.4	10.0	100.0	-0.6	0.3
4.	Geometry	54.4	24.4	0.0	100.0	-0.2	0.3
5.	Mental rotation	13.5	5.0	3.0	23.0	-0.1	0.3
6.	Spatial visualization	35.7	11.8	12.2	56.7	-0.1	0.3
7.	BNL	5.4	2.1	2.4	14.6	1.6	0.3
8.	UNL	9.7	3.4	2.0	21.4	0.4	0.3
9.	Logical Reasoning	56.2	12.7	20	80.0	-0.2	0.3

Note. BNL is a Bounded Number Line task, UNL is Unbounded Number Line task; and is reported as the mean Percent Absolute Error (PAE). All mathematical domains and logical reasoning tests were scored as percentage correct. Spatial visualization included a correction factor to control chance guessing, and mental rotation was the total correct response out of the 24 items.

Table 2

Correlations among the variables (N= 85)

	1	2	3	4	5	6	7	8
1. Fraction								
2. Arithmetic	0.70***							
3. Algebra	0.59***	0.52***						
4. Geometry	0.38***	0.27*	0.21					
5. Spatial Visualization	0.24*	0.24*	0.31**	0.04				
6. Mental Rotation	0.21	0.20	0.27*	0.12	0.65***			
7. BNL	-0.22*	-0.19	-0.16	-0.17	-0.28*	-0.31**		
8. UNL	-0.35***	-0.32**	-0.47***	-0.13	-0.44***	-0.29**	-0.31*	
9. Logical Reasoning	0.14	0.23*	0.09	-0.09	0.14	0.26*	-0.21	-0.01

Note. $p < 0.05$ *; $p < 0.01$ **; $p < 0.001$ ***

Structural Equation Models (SEM)

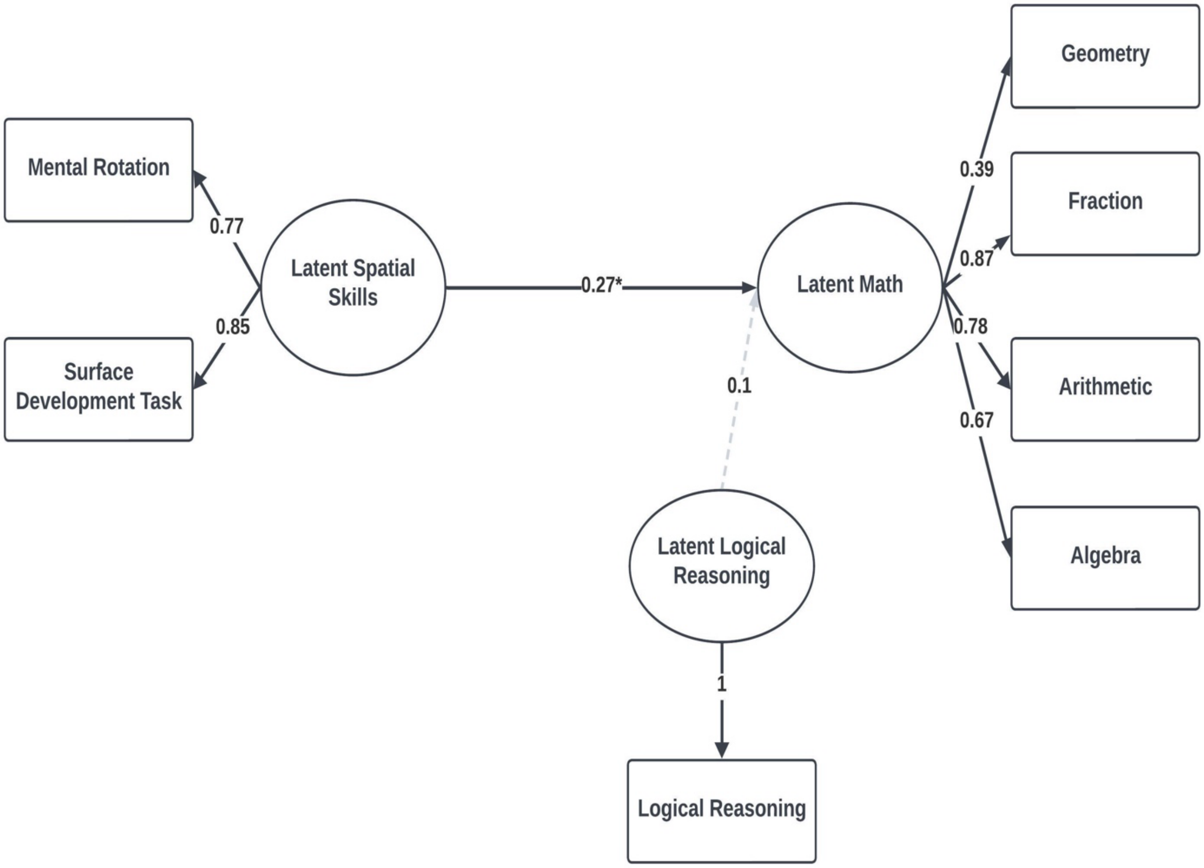
We used SEM to examine various research questions. SEM is a statistical tool used to model complex relationships, typically at the latent level. According to Hu and Bentler (1999), good model fit selection criteria include a nonsignificant chi-squared ($\chi 2$) test, Comparative Fit Index (CFI) > 0.95, Standardized Root Mean Residual (SRMR) < 0.08, and Root Mean Square Error Approximation (RMSEA) < 0.06. Aron et al. (2013) suggested that RMSEA < 0.1 and a 90% confidence interval are considered a good fit. As stated previously, we used robust fit indices for all models as those corrected for slight non-normality in our data (Santorra & Bentler, 1994; Savalei, 2019). We selected the best model in all the models tested by considering modification indices and what made theoretical sense.

How are Spatial Skills, MNL, and Mathematical Skills Related in Adults?

The first research question examined whether spatial skills were related to advanced mathematical skills at the latent level in adults. We examined this in Model 1. First, we created a Latent Spatial Skill variable that consisted of mental rotation and spatial visualization, a Latent Mathematical Skills variable that consisted of all mathematical domains, and a Latent Logical Reasoning variable from logical reason scores. Mathematical skills were then predicted from spatial skills, while controlling for logical reasoning. Figure 1 shows the path diagram. The results from Model 1 show that the robust fit indices were good: $\chi 2(12, n = 85) = 17.899$, $p = 0.119$, scaling correction factor for Satorra-Bentler correction = 1.016, CFI = 0.963, RMSEA = 0.076, and SRMR = 0.059. This result confirms that spatial skills predict mathematical skills, even when general cognitive skills are controlled.

Figure 3

Model 1. Path Diagram Showing the Relationship Between Spatial and Mathematical Skills



Note. The diagram above shows the relationship between spatial and Mathematical skills, while controlling for logical reasoning. Continuous dash lines indicate a nonsignificant path, while continuous thick lines show significant paths. $p < 0.01$ = **; $p < 0.001$ = ***.

Model 1 confirmed that spatial skills are related to advanced mathematical skills. Therefore, we introduced the mediator in Model 2. We created Latent BNL and UNL skills from BNL and UNL scores, respectively. These variables were then entered into the model specification as mediators in the relationship between latent spatial and math skills. By including both MNL tasks in the model, we could determine the indirect effect of each BNL while controlling for the effect of UNL and vice versa. All model fit indices were excellent in this model: $\chi 2(12, n = 85) = 17.899$, $p = 0.119$, scaling correction factor for Satorra-Bentler correction = 1.016, CFI = 0.963, RMSEA = 0.076, and SRMR = 0.059. Once the mediating variables were introduced, the direct effects of spatial and mathematical skills were no longer significant ($b = 0.101$, $p = 0.494$). In addition, the indirect effect of spatial skills on mathematical skills through BNL skills was not nonsignificant ($b = 0.031$, $p = 0.564$). By contrast, the indirect effect of spatial skills on mathematical skills through UNL was significant, suggesting that only UNL skills significantly mediated this relationship. Figure 2 presents the path diagram and relationships for Model 2.

Figure 4

Model 2. Path Diagram Showing the Relationship Between Spatial and Mathematical Skills and magnitude representation

Note. The diagram above shows the relationship between spatial and Mathematical skills, BNL and UNL, while controlling for logical reasoning. Continuous dash lines indicate a nonsignificant path, while continuous thick lines show significant paths. $p < 0.01$ = **; $p < 0.001$ = ***.

How is spatial skill related to specific mathematical skill domains, and what is the role of the BNL and UNL tasks in the relationship?

In Model 3, we broke down the relationship between spatial skills, magnitude representations, and mathematical skills into distinct paths by separately including the mathematic domain measures as outcomes rather than having an overall latent math skills factor. Specifically, this model isolates spatial and mathematical skills relations and the mediation of magnitude representations (UNL and BNL) along various mathematical dimensions. Given the multidimensionality of both spatial and mathematical skills, it is essential to identify the relationship between spatial skills and specific mathematical areas, as this would inform any potential spatial training interventions tailored to a specific mathematical domain for a more effective result. Therefore, in Model 3, spatial skill was entered as a predictor for arithmetic, fractional, and algebra. As shown in Table 2, none of the correlations were significant for geometry. Subsequently, geometry was not included in this model. Logical reasoning was used as the control variable in this model.

The results indicated that all fit indices were good: $\chi 2(8, n = 85) = 12.687$, $p = 0.123$, scaling correction factor for Satorra-Bentler correction = 1.041, CFI = 0.974, RMSEA = 0.085, and SRMR = 0.042. Figure 3 shows the path diagram of Model 3. As shown in Table 3, the indirect paths from spatial skills through UNL skills were significant for fraction, algebra, and arithmetic. However, the indirect path from spatial skill to all mathematical dimensions through BNL was nonsignificant. Moreover, the direct paths between spatial skills and all mathematical skills were not nonsignificant: fraction [$b = 0.03$, $p = 0.842$], algebra [$b = 0.209$, $p = 0.150$], and arithmetic [$b = 0.106$, $p = 0.396$]. After controlling for general cognitive skills, the results indicate that UNL skills fully mediated the relationships between spatial skills and fraction, arithmetic, and algebra.

Table 3

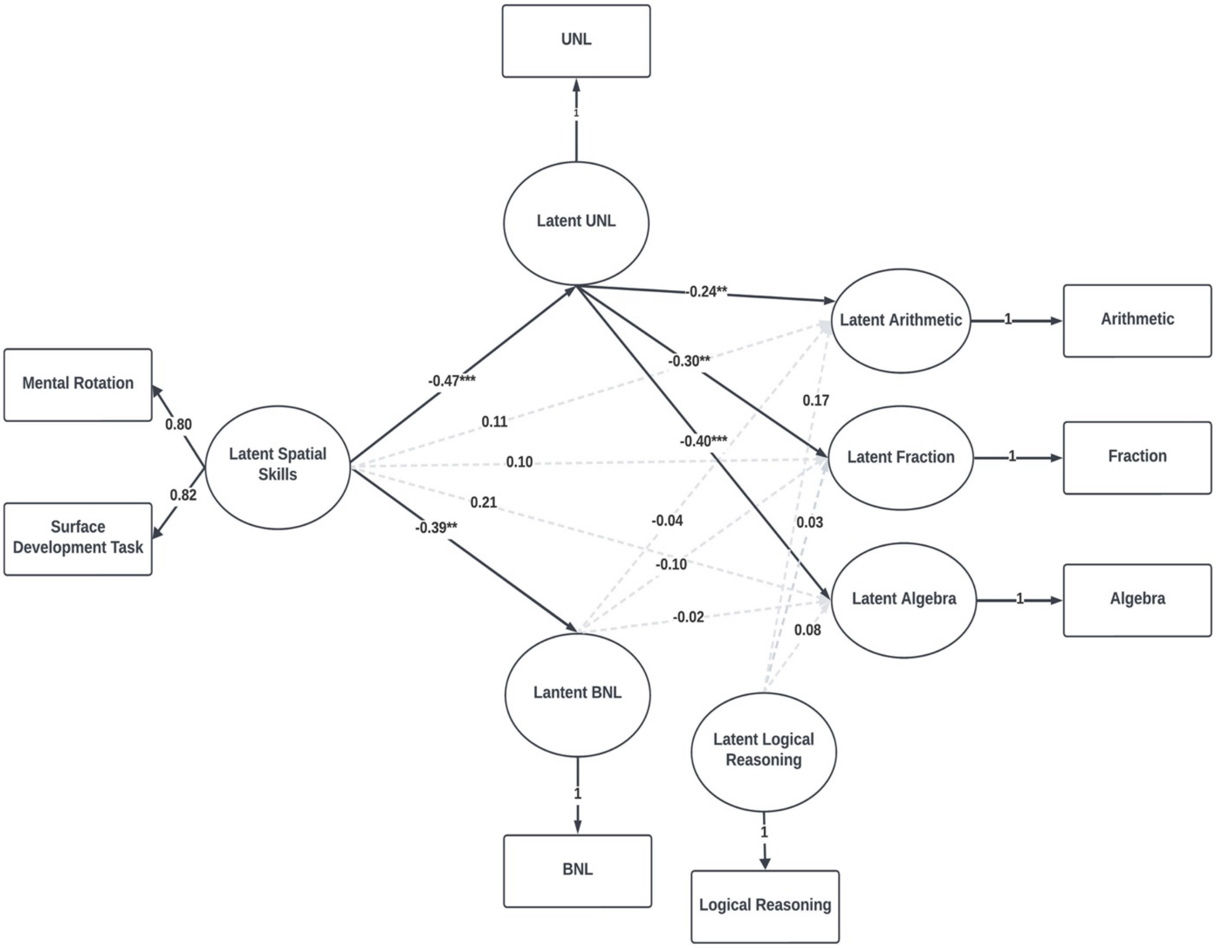
Indirect Effects of UNL and BNL and Spatial Skills on the Different Mathematical Skills

Mediators	Mathematical Domains	Indirect effects		
		ab	*p*	95% CI
UNL	Fraction	0.143	**0.019**	[0.077, 0.408]
	Algebra	0.187	**0.002**	[0.067, 0.432]
	Arithmetic	0.116	**0.036**	[0.017, 0.353]
BNL	Fraction	0.039	0.454	[-0.073, 0.079]
	Algebra	0.014	0.684	[-0,091, 0.060]
	Arithmetic	0.009	0.796	[-0.151, 0.035]

Note. Bold-faced *p's* are significant paths. The table shows the indirect effect of the BNL and UNL in the relationship between spatial skills and the different mathematical domains.

Figure 5

Model 3: Mediation Model showing Paths from UNL and BNL knowledge to Mathematical skills



Note. The diagram above shows various mediation paths connecting spatial skills to mathematical domains through BNL and UNL knowledge. Continued thick lines indicate significant paths, whereas faint dashed lines indicate nonsignificant paths.

Discussion

We examined the mediating role of the number line in the spatial–mathematical associations, building on recent evidence in children (Gunderson et al., 2012; Tam et al., 2019). Unlike previous studies, we incorporated BNL and UNL versions of the number line task to assess the magnitude representation. First, we verified the expected relationship between spatial skills and mathematics in an adult sample. The results from our first model further strengthen the widely held claim that spatial skills predict mathematical skills in an adult sample even after controlling for general cognitive skills. Interestingly, we found that spatial skills did not significantly predict geometry, even though at first glance, geometry is the mathematical area that is most visibly spatial. It is important to note that the geometry task consisted of only five items and the measure had poor internal consistency, which could have contributed to the lack of results for this measure. Nevertheless, this finding suggests that the relationship between spatial skills and mathematical understanding is not driven by competence in mathematical domains that are inherently spatial. Instead, this relation may work through a mechanism similar to that proposed by spatial representation theory, where spatial skills are tied to magnitude representation underlined by the MNL, which subsequently leads to improved mathematical performance. This finding is consistent with previous research that failed to find a significant relationship between spatial skills and geometry in children (Kyatta & Lehto, 2008).

Our study also revealed that a distinct measure of magnitude representation mediated the relationship between spatial and mathematical skills compared to previous studies with children. Notably, our results did not support previous findings that the BNL mediates the existing links between spatial skills and mathematics (Gunderson et al., 2012; Tam et al., 2019). Instead, the UNL explained the relationships between algebra and spatial skills, fractions and spatial skills, and arithmetic and spatial skills. Additionally, we found that BNL was not correlated with any mathematical skill except for fractions in our adult sample. This finding is significant as it is the first study to present evidence suggesting that BNL does not strongly correlate with mathematical skills in adults. Previous research has indicated that the relationship between BNL and mathematics in children weakens with age, and increasing evidence suggests that BNL and UNL knowledge may involve different cognitive mechanisms (Georges & Schiltz, 2021; Reinert, 2019). However, it is also possible that this task was too easy for adults, resulting in a floor effect that restricted variability that may have correlated with the math performance measure, although there was still enough variability to correlate with the spatial tasks.

Another reason the bounded number line did not mediate the spatial skills-math performance relation is that the spatial representation theory could be viewed as a developmental phenomenon. From this perspective, it may not be essential for the BNL task to mediate the relationship between spatial and mathematical skills in the adult sample. The relationship between spatial and magnitude representation measured by number line may be crucial during the early years of learning, with this connection weakening over time. However, spatial skills remain beneficial in the early stages of learning. This argument explains why the BNL estimation measure may no longer mediate the association between spatial and mathematics in adults. Nevertheless, this argument does not account for the observed relationship between mathematical performance and UNL performance later in adulthood, suggesting that magnitude representation still plays a role in explaining the spatial skills-mathematical performance relationship, even in adulthood. This raises questions about what BNL and UNL tasks measure and how they change with age.

Given the nature of the mathematical tasks in this study, it is plausible that proportion reasoning (potentially measured by BNL knowledge in adults) is not relevant to solving these problems. Our findings align with recent evidence suggesting that the UNL may be a more comprehensive measure of magnitude representation (Cohen & Blanc-Goldhammer, 2011; Jung et al., 2020; Link et al., 2014; Reinert et al., 2019). If the UNL task is a purer measure of magnitude knowledge, spatial representation theory predicts it will play a more significant role than the BNL task as a mechanism connecting spatial and mathematical skills. Furthermore, it may be a superior measure of magnitude knowledge once magnitude representation becomes linear. In early ages, a BNL may be the best way to differentiate between those with a linear and logarithmic number line representation. Once participants understand the number line to be linear, the UNL may be the better measure of the precision of that understanding and, therefore, contribute to explaining the relationship between spatial skills and mathematical performance.

Limitations

The results of this study have limited generalizability for several reasons. First, the participants in this study were university students enrolled in at least one psychology course during the data collection period. These students are typically not engaged in active or formal mathematical learning. Consequently, the results may not reflect typical students who engage in mathematical learning. Second, this study used a cross-sectional design, which may not have provided the best approach for investigating the relationship between spatial skills and mathematical learning over time. A longitudinal design would be a more effective approach, as it would allow researchers to determine when the role of the BNL diminishes, and the role of the UNL becomes more prominent. Finally, this study is correlational, meaning it can only identify relationships between variables and cannot establish causality. Therefore, future studies should adopt an experimental design that can help control for the effects of other cognitive constructs, such as working memory, which is highly related to spatial and mathematical skills. Additionally, future studies should include a larger sample size, especially for the structural equation model, to further validate the correlational relationships found in this study.

Conclusion

In summary, this study further supports the spatial representation theory, specifically in explaining why spatial skills are related to mathematical performance. The results suggest that the UNL may be a more accurate measure of magnitude knowledge than the BNL for adults. These findings raise questions about how magnitude representation changes with age and how different measures capture these changes. These findings open exciting avenues for future research. If the relationship between spatial skills and mathematics remains mediated by magnitude representation underlined by the MNL, and indicated by performance on the number line even in adulthood, there is potential to enhance mathematical learning in adults through training using the UNL.

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CHAPTER 3: RELATIONSHIP BETWEEN SPATIAL ANXIETY AND MATHEMATICAL SKILLS

Abstract

There is a strong connection between spatial and mathematical skills. The spatial representation theory proposes that spatial and mathematical skills and a Mental Number Line (MNL) are intricately linked and play a crucial role in developing numerical skills. According to this theory, strong spatial skills enhance MNL representation, leading to improved mathematical learning. A notable implication of this spatial representation framework is that spatial processing difficulties can negatively affect mathematical learning. This study explored the association among spatial anxiety, mathematical anxiety, mathematical proficiency, and MNL representation, measured by linear number line estimation performance. We analyzed data from 48 undergraduate students who completed evaluations measuring spatial anxiety, mathematical anxiety, number line estimation, and mathematical proficiency. These results suggest a significant correlation between spatial anxiety and mathematical anxiety. Additionally, mathematical anxiety significantly correlated with performance in algebra and geometry but not in arithmetic or fractions. However, we did not find evidence of a cross-domain relationship between spatial anxiety and mathematics or between spatial anxiety and number line estimation.

Keywords: Spatial anxiety, spatial ability, mathematical skills, mathematical anxiety

Introduction

Whether spatial skills and mathematics are related is no longer in doubt, yet there is a lack of a unified framework explaining the mechanism interconnecting these two abilities. An emergent theory posits that higher-level spatial abilities are interconnected to mathematics through magnitude representation accessed with a number line in children (Lefevre et al., 2013; Tam et al., 2019; Gunderson et al., 2012). The mechanism underlying magnitude representation is a so-called mental number line similar to an actual linear number line. Thus, a strong performance on a linear number line is taken as a strong magnitude representation resulting from an enhanced internal number line representation (Dehaene, 1992; Lefevre et al., 2013; Tam et al., 2019; Gunderson et al., 2012). Although this spatial representation theory still needs more research to verify its practical implications, it provides direct evidence suggesting that having a strong spatial ability could lead to more effective development of the so-called mental number line, which subsequently leads to improved number representation (Lefevre et al., 2013; Tam et al., 2019; Gunderson et al., 2012). It is important to note that within the realm of the spatial representation theory, the mental number line [MNL] is a cognitive construct, and the behavioural manifestation of this construct has been inferred from performance on an actual number line estimation. If the spatial representation theory is true, then it would be expected that factors impairing spatial abilities could eventually negatively affect numerical processing, as weak spatial skills could lead to an ineffective mental number line, leading to low numeracy skills. An alternative way of testing the spatial representation theory is to examine how impairment in spatial ability influences mathematics. Thus, this theory raises the question of how deficits in spatial skills influence mathematical learning and vice versa.

Spatial anxiety, an apprehension in performing spatial activities, is negatively related to spatial ability (Lyons et al., 2018). Specifically, high spatial anxiety is related to low spatial ability and vice versa (Lyons et al., 2018). Examining the cross-domain effect between spatial anxiety and mathematics will facilitate and provide a more unified framework for understanding the mechanism underlying the spatial and mathematical skills associations. This study examined the relationship between spatial anxiety and mathematics, building on the spatial representation theory. In the sections below, I provide an overview of spatial anxiety, how it relates to spatial ability, and why it might be related to mathematics. First, we examine how mathematics performance is related to its affective component, mathematical anxiety.

Relationship between Mathematical Anxiety and Mathematical Performance

In mathematical learning, decades of research suggest that some individuals have apprehensions when solving mathematical problems, called mathematical anxiety. Studies investigating the relationship between mathematical anxiety and performance have reported a strong negative correlation, where higher levels of mathematical anxiety are associated with poorer performance on mathematical tests (Ashcraft & Krause, 2007; Douglas et al., 2018; Ganley et al., 2020; Kukowski et al., 2016; Necka et al., 2015). For example, Ramirez et al. (2013) found that higher levels of mathematical anxiety were associated with poorer mathematical achievement and lower mathematical course-taking intention among Latino students. The negative relationship between mathematical anxiety and mathematical skills is especially pronounced in specific populations. For example, Ma and Xu (2004) found that female college students who reported higher levels of mathematical anxiety had lower mathematical achievement scores than their male counterparts, indicating that the effect of mathematical anxiety may be more profound among women than men. Other studies have found that mathematical anxiety has a greater significant impact on students with lower levels of mathematical skills (Ashcraft & Moore, 2009) and on individuals who are more anxious in general (Wang et al., 2015).

Necka et al. (2015) examined the relationship between mathematical anxiety and mathematical performance while considering the role of self-mathematical overlap, which refers to the extent to which individuals identify themselves as good at math. The researchers found that stronger self-mathematical math anxiety was not related to math performance. Additionally, researchers have found that the negative relationship between mathematical anxiety and mathematical performance is more robust for those with weaker self-mathematical overlap, suggesting that an individual's identification with math may influence the relationship between mathematical anxiety and performance. Ma and Xu's (2004) longitudinal panel analysis investigated the causal order of mathematical anxiety and achievement among high school students in China. Their results indicated that mathematical anxiety and mathematical achievement have a reciprocal relationship, with each affecting the other over time. In a meta-analysis, Barroso et al. (2020) examined the relationship between mathematical anxiety and mathematical achievement by analyzing 747 effect sizes from studies conducted between 1992 and 2018. The results showed a small-to-moderate, negative, statistically significant correlation between mathematical anxiety and achievement. The authors identified several moderators of this relationship, including grade level, mathematical skill level, and the type of mathematical anxiety scale used.

These studies suggest that mathematical anxiety negatively affects mathematical performance, with factors such as working memory, mathematical self-concept, and self-mathematical overlap playing a role in this relationship.

Why is Mathematical Anxiety Related to Mathematical Performance?

A cognitive deficit model, similar to the spatial representation model, is typically used to explain the relationship between mathematical anxiety and performance. On the one hand, this model suggests that problems with basic number skills, such as counting, number recognition, and symbolic number comparison, or with fundamental cognitive skills, such as spatial processing, can result in performance difficulties that may affect children's emotional responses to learning mathematics and lead to the development of mathematical anxiety (Maloney, 2016; Maloney & Beilock, 2012). Ma and Xu (2004) provided evidence to support this model in a longitudinal cross-lagged study of American high school students. They found that low mathematical achievement predicted later mathematical anxiety, indicating that poor mathematical achievement could increase mathematical anxiety. Furthermore, studies conducted with young children indicate that mathematical anxiety can develop as early as the first or second grade, providing evidence for a strong association between fundamental number skills, likely MNL development, and mathematical anxiety (Krinzinger et al., 2009; Vukovic et al., 2013; Wu et al., 2012).

However, some researchers have suggested that high mathematical anxiety may impede the efficiency and effectiveness of MNL, leading to poor mathematical performance. Ashcraft et al. (1994) investigated the effects of mathematical anxiety on working memory, a cognitive process essential for completing mathematical tasks. The results showed that high levels of mathematical anxiety were associated with reduced working memory capacity, which in turn led to poorer performance in mathematical tasks. In a follow-up study, Ashcraft et al. (2000) examined whether mathematical anxiety affected the skills required to perform mental calculations. The results showed that individuals with high mathematical anxiety levels had slower and less accurate mental calculations than those with low mathematical anxiety levels. Ashcraft et al. (2001) explored the relationship between mathematical anxiety and performance among undergraduate students. They found that mathematical anxiety was negatively correlated with mathematical performance and that this relationship was mediated by working memory capacity. Ashcraft et al. (2007a) investigated the relationship between mathematical anxiety and automaticity when performing basic arithmetic operations. They found that individuals with high mathematical anxiety levels were less automatic at performing basic arithmetic operations than those with low mathematical anxiety levels. Similarly, Ashcraft et al. (2007b) examined whether mathematical anxiety affects the skills to estimate numerical quantities, and the skills believed to be underlined by the internal mental number line. The results showed that individuals with high mathematical anxiety levels were less accurate at estimating numerical quantities than those with low mathematical anxiety levels. Based on these studies, we propose that high levels of mathematical anxiety may weaken the underlying MNL, leading to poor performance in mathematical skills.

Nunez-Pena et al. (2019) investigated the relationship between mathematical anxiety and MNL knowledge in undergraduate students. This study found that individuals with high mathematical anxiety tended to perform poorly on more complex number line estimation tasks. In contrast, those with low levels of mathematical anxiety performed better on the same task. It is important to note that a limitation of the study is that it did not control for overall mathematical skills; as such, the extent to which participants' mathematical skills influenced the observed relationship between number line estimation knowledge and mathematical anxiety is unclear. Other studies have investigated how mathematical anxiety relates to MNL representations in non-number line estimation tasks. Georges et al. (2016) investigated how mathematical anxiety relates to the spatial-numerical association code (SNARC) effect, a phenomenon used to describe the existence of an MNL. The authors found that greater mathematical anxiety was associated with a stronger SNARC effect. Combining this result with other studies which found that a stronger SNARC effect was related to low math performance (e.g., Hoffman et al., 2014), it could be concluded that low-level number cognition, depicted by low number-space associations, maybe a potential risk factor for mathematical anxiety.

Taken together, there appears to be some support for the spatial representation theory when examined from the perspective of how mathematical anxiety relates to mathematical skills. Mathematical anxiety can negatively impact the development of the MNL, which underlies mathematical skills, leading to poor mathematical performance. In addition, a deficient MNL, possibly resulting from low spatial skills, may lead to poor mathematical performance, which can lead to high mathematical anxiety. It is also possible that the association between mathematical anxiety and MNL is due to the shared variance between mathematical anxiety and spatial anxiety, which drives this relationship.

What is Spatial Anxiety, and How is it Related to Spatial Skills?

According to Alvarez-Vergas, Lyons, and colleagues (2020), spatial anxiety is a domain-specific disorder, evidenced by apprehension in completing spatially relevant tasks. Spatial anxiety appears to originate from deficits in brain functions. For example, neuroimaging studies have found reduced activity in Bilateral Vestibulopathy (BVP) brain regions in individuals with high levels of spatial anxiety (Kremmyda et al., 2016). Similar to spatial skills, recent studies have claimed that spatial anxiety is a multidimensional deficit. Individuals may have different apprehensions depending on their exposure to specific spatial situations. Using factor analysis, Lyons et al. (2018) identified three sub-dimensions of spatial anxiety. The first is mental manipulation anxiety, which pertains to apprehensions about processing spatial objects that require mental rotation. The second is navigation anxiety, which pertains to apprehensions about navigating the environment. The third is mental imagery anxiety, which pertains to apprehensions regarding imagining different objects and their spatial locations. High levels of spatial anxiety are negatively related to spatial skill (Ramirez et al., 2012; Kremmyda et al., 2016). For example, among 7-year-olds, Ramirez et al. (2012) found that individuals who scored high on a spatial anxiety scale were likely to perform poorly on mental rotation tasks. Furthermore, researchers have reported that individual differences in mental rotation and spatial anxiety interact with working memory capacity. Individuals with a high working memory capacity tended to perform worse on mental rotation tasks when they had higher spatial anxiety scores.

In another study, Attit and Rocha (2021) found that individuals with high levels of spatial anxiety were likely to perform poorly on spatial skill tasks and more likely to avoid spatial situations. Specifically, the authors investigated the relationship between K-12 teachers' spatial skills, spatial anxiety, and the likelihood of including spatially relevant tasks in classroom activities. They found that the level of mental manipulation anxiety significantly predicted the level of performance on spatial skills, whereas mental imagery anxiety did not. Additionally, they found that teachers' spatial skills were predictive of their level of incorporation of spatially relevant activities in the classroom; highly spatially anxious teachers were less likely to include spatial reasoning activities in their teaching and vice versa. Additionally, among adults, Ferguson et al. (2015) reported that individuals with a poor sense of direction were likely to have high anxiety about navigating their environment. However, it is currently not understood whether avoiding spatial reasoning tasks necessarily leads to poor performance on spatial skill tasks or vice versa. Moreover, spatial anxiety has been found to explain gender differences in adults' mental rotation performance (Alvarez-Vergas et al., 2020). Few studies have examined the effect of spatial anxiety on spatial skills. However, the results reported thus far indicate that high spatial anxiety is related to low spatial skills, whereas low spatial anxiety is also related to high spatial skills.

Relationship between Mathematical Anxiety, Spatial Anxiety, and Spatial skills

Spatial skill influences the development of MNL based on the spatial representation theory, which then helps in effectively developing numerical skills (Gunderson et al., 2012). Moreover, according to the spatial deficit theory of mathematical anxiety, low spatial skills could lead to inefficient MNL, which can lead to low mathematical performance and, consequently, high mathematical anxiety (Ferguson et al., 2015; Georges et al., 2016). Taken together, these theories suggest that spatial skills and mathematical anxiety are interrelated. Indeed, few studies have examined whether performance patterns in spatial skills can be explained by mathematical anxiety, and the results so far have shown a significant negative correlation. For example, Ferguson et al. (2015) assessed undergraduate students' mathematical anxiety and spatial skills, measured by a self-reported sense of direction. The authors found that spatial skill (sense of direction) significantly predicted mathematical anxiety and that this effect persisted even after controlling for age and general anxiety.

Mathematical anxiety has also been associated with spatial anxiety. In a follow-up study by Ferguson et al. (2015), participants were assessed for spatial anxiety, their sense of direction and mathematical anxiety. After controlling for sense of direction, sex, and general anxiety, spatial anxiety remained a significant predictor of mathematical anxiety. These findings suggest that individuals with high mathematical anxiety levels may have high spatial anxiety and poor spatial skills. Maloney et al. (2012) and Douglas and LeFevre (2018) confirmed that individuals with high mathematical anxiety also had a worse sense of direction, lower mental rotation skills, and stronger connections between numbers and space. Overall, there seems to be evidence that mathematical anxiety is related to spatial anxiety and skills.

Present Study

The review above demonstrates that domain-specific anxiety is a valuable predictor of skills in the respective domains. Thus, mathematical anxiety was negatively related to mathematical performance. Similarly, spatial anxiety was negatively correlated with spatial skill. However, there is a cross-domain effect of mathematical anxiety on spatial skills, indicated by a positive correlation between mathematical anxiety and spatial skills. Based on the spatial representation framework, the next logical step is to examine the cross-domain relationship between spatial anxiety and mathematical skills. This investigation is crucial in light of the ongoing campaign among education and psychology researchers to revise mathematics curricula to include spatially relevant activities (Gunderson et al., 2012; Tam et al., 2019; Mix et al., 2016), as incorporating spatial reasoning into mathematics learning may enhance the effective grasp of mathematical concepts and improve performance. However, suppose that spatial anxiety is negatively related to mathematical skills. In that case, exposure to spatial activities in the classroom may lead to unintended consequences, such as a decline in mathematics performance for individuals with apprehension towards spatial activities. Understanding the relationship pattern between spatial anxiety and mathematical skills is crucial to avoid such a consequence. For example, even if spatial anxiety is negatively related to mathematical skills, spatial skills can still be inculpated in the classroom as part of instruction, but simultaneously with an intervention for those with high spatial anxiety.

The current study was exploratory and aimed to address this gap in the literature by investigating the interrelationships between spatial anxiety and mathematical skills. Consistent with spatial representation theory, we expected spatial anxiety to be related to mathematical skills. Given the multidimensionality of spatial anxiety, we included a spatial anxiety measure consisting of multiple subscales that measured different spatial anxieties. By doing so, we were able to assess whether different spatial anxieties relate differently to different mathematical skills. In addition, a measure of general anxiety was included, as previous research has suggested that controlling for spatial and general anxiety reduces the relationship between mathematical anxiety and spatial skills (Ferguson et al., 2015). Additionally, this study examined whether spatial anxiety was related to MNL. If MNL is the interconnecting mechanism in the spatial and mathematical relationship, then anxieties affecting spatial and mathematical skills would be related to MNL.

Method

Participants

A total (122) undergraduate psychology students were recruited through the Psychology Research Experience Program at a mid-sized university in Atlantic Canada for the first data-collection session. Only 69 students completed the second data-collection session. Although no demographic data were collected in the first session, we compared the 69 students who completed both sessions with the 53 students who only completed the first session on the first-session variables and found that they did not differ in any of the measures except general anxiety (the student who did not complete the second session had higher general anxiety scores, $t_{(120)}$ =

2.72, *p* = .008, Cohen's *d* = 0.50). Some participants were compensated with course credits, while others received $20 compensation if they completed all sessions. The Research Ethics Board at the university in question approved this study.

Measures

Spatial Anxiety Scale (SAS)

Participants completed a self-reported spatial anxiety questionnaire adopted from Lyons et al. (2018). The spatial anxiety scale consists of 24 items that assess how anxious the participants feel when performing spatial tasks. The spatial anxiety questionnaire assesses anxiety in three spatial areas: spatial navigation (e.g., asked to follow directions to a location across town without the use of a map), spatial imagery (e.g., given a test in which you were allowed to look at and memorize a picture for a few minutes, and then given a new, similar picture and asked to point out any differences between the two pictures), and spatial mental manipulation (for example, asked to imagine and mentally rotate a 3-dimensional figure. Each subscale consists of eight items, with participant responses ranging from one ("not at all") to 5 ("very much"). The mean score for each subscale was computed, with high scores indicating high anxiety in the respective domains. See Appendix 3.

Abbreviated Mathematical Anxiety Scale (AMAS)

The AMAS consists of 24 questions asking participants to rate their level of anxiety when performing math-related tasks. Participants were given the option to select choices ranging from "not at all' (scored 1) to 'very much' (score 5). The mean anxiety scores were computed, with high scores representing high anxiety levels.

The Generalized Anxiety Disorder Scale (GAD-7)

General anxiety was assessed using the GAD-7 adopted from Spitzer et al. (2006). On the GAD-7, participants rated their level of anxiety on a 4-point scale consisting of seven items. The choices on the GAD-7 range from "not at all" (score of 1 to "nearly every day" (score of 5). High scores are indicative of high anxiety levels.

Mathematical Placement Test

Mathematical skills were assessed using a paper-and-pencil task selected from the Mathematical Placement Test. The MPT is a diagnostic test of basic mathematics skills administered by the Department of Mathematics and Statistics at the university where this study was conducted. Mathematics educators designed the test to assess competence levels in specific core areas of mathematics covering up to at least 12th grade, including arithmetic of integers, rational numbers, algebraic expressions, trigonometry, geometry, and fractions. We selected 45 items comprising four mathematical proficiencies: fractions, algebra, geometry, and arithmetic. Namely, there were five geometry items, 14 arithmetic items, 16 fraction items, and 10 algebra items. The second author and a graduate student at the university rated these items in four mathematical domains with an inter-rater reliability of 100%. The measurement was divided into three parts to ensure all fields were completed or attempted before the 40 min expired. Part 1 consisted of algebra and geometry; participants were allowed 15 minutes to complete it. Part 2 consisted of a fraction that lasted for a maximum of 15 minutes, and the arithmetic in part 3 took 10 minutes to complete. The participants were not permitted to use a calculator while completing these tasks.

Number Line Estimation Tasks: BNL

A number line estimation task was included to explore the possibility of a relationship between MNL and spatial and mathematical anxiety. This task required participants to slide a bar on a screen to determine the location of specified numbers. The number line knowledge was bounded by a starting number of zero and an endpoint of 10000. The numbers to be determined were adopted from Nunez et al. (2019) with slight modifications. Overall, there were 22 items on the number line estimation tasks, which were completed in four minutes. The participant's scores were calculated using the Percentage Absolute Error (PAE) ([Estimated number – To be estimated numbers]/value of the line) × 100. High PAE scores indicate high errors and, hence, low performance.

Procedure

Data collection was conducted using the Qualtrics software. The assessments were divided into two sessions. Participants completed the Spatial Anxiety Scale, Abbreviated Mathematical Anxiety Scale, GAD-7, and Number Line Estimation Task in the first session. The first session lasted 30 min. Session two consisted of mathematics achievement questionnaires and was allotted 60 minutes. Once the participants signed up for the study, they were redirected from the SONA website to Qualtrics, where they had access to it. They had to read the consent form and indicate their agreement before completing the questionnaire. After the participants finished each session, they were redirected to SONA, where the system automatically granted course credits. The paid participants were given a chit to receive money from the department office. The measures were presented in random order within each session. For example, in session one, some participants received the GAD-7 first, whereas others received it as the last measure. The same was done for session two, where some participants may have received algebra first, but others would have it last. All the participants completed the first session before the second session.

Analysis

Data analysis was performed using R (version 4.1.2) and the Jamovi software (version 2.2.5.0). We used pairwise deletion for these analyses, which means that any analyses that used only data from Session 1 had a larger sample size than those that used data from Session 2. Nevertheless, prior to the final analysis, several data-screening procedures were performed. Some participants completed the questionnaires too quickly, which may have affected the data accuracy and fidelity. For instance, even though the estimated completion time for the session was approximately 30 min, certain participants finished it in less than a minute. To address this issue, we excluded seven participants who completed session one in less than 300 seconds, leaving a total sample size of 115. Two of the seven people who were eliminated had Session 2 data, which means their Session Two data were also eliminated. From these 67 participants, we eliminated 18 who completed session two in less than 900 seconds and one who scored less than chance. This resulted in a final sample size of 115 people with session 1 data and 48 with both session 1 and session 2 data. Given the small sample size and the lack of correlations between the spatial anxiety measures and any of the mathematics measures, we did not conduct structural equations models to examine their relationships further.

Results

The descriptive statistics for all measures are presented in Table 4. Correlation analysis was performed to examine the relationships between the different measures, and the results are presented in Table 5. All mathematics measures exhibited significant correlations with each other. Also, the findings revealed significant correlations between mathematical anxiety and all spatial anxiety domains. Moreover, mathematical anxiety was positively correlated with algebra and arithmetic but showed no significant correlation with fractions or geometry. However, we did not find a significant correlation between spatial anxiety and any of the mathematical domains. In addition, the number line estimation task significantly correlated with fraction, arithmetic, and algebra, but not geometry.

An inadequate statistical power could explain the lack of a significant correlation between spatial anxiety and mathematical performance. To investigate this possibility, we performed a power analysis to assess whether the non-significant correlations could be attributed to insufficient power. Using our sample size of 48 participants and the observed correlations presented in Table 5, we conducted a power analysis specifically for the negative correlations between spatial anxiety and mathematical skills. The results revealed extremely low power (approximately 0.114) and low values (< 0.1) in some instances. To gain further insight, we conducted a priori power analysis to achieve a power of 0.8, which allowed us to detect a medium effect size in such relationships. The analysis indicated that a sample size of at least 85 was necessary to achieve a medium effect size. These findings suggest that the limited sample size of our study may have contributed to the lack of significant correlations between spatial anxiety and mathematical skills.

Table 4

Descriptive statistics

	Mean	*S.D*	*Min*	*Max*	*Skewness*	*Std. error*
1. Fraction	61.5	12.0	37.5	81.3	-0.1	0.3
2. Arithmetic	76.3	12.8	42.9	100.0	-0.8	0.3
3. Algebra	48.3	19.2	0.0	70.0	-0.6	0.3
4. Geometry	47.5	26.0	0.0	100.0	-0.3	0.3
5. General anxiety	2.5	0.9	1.0	4.0	0.1	0.2
6. Mathematical anxiety	2.9	0.8	1.0	4.9	-0.1	0.2
7. Spatial Imagery Anxiety	2.5	0.9	1.0	5.0	0.7	0.2
8. Mental Manipulation anxiety	3.0	1.1	1.0	5.0	-0.0	0.2
9. Spatial navigation anxiety	3.0	1.0	1.0	5.0	0.0	0.2
10. Number line estimation	5.5	5.0	1.2	34.0	3.8	0.2

Note. The number line estimation tasks were measured using the PAE, and all mathematical measures were presented as percentages. *N* = 115 for session 1 measures (Mathematical anxiety, Spatial Imagery Anxiety, Mental Manipulation Spatial navigation anxiety, Number line estimation). *N* = 48 for all session 2 measures (fraction, arithmetic, algebra, and geometry).

Table 5

Pairwise Correlations among the Variables in the Study

		1	2	3	4	5	6	7	8	9
1	Algebra	-								
2	Arithmetic	0.44**								
3	Fraction	0.60***	0.36*							
4	Geometry	0.34*	0.24	0.29*						
5	General Anxiety	-0.28	-0.24	-0.04	-0.07					
6	Mathematical Anxiety	-0.47***	-0.33*	-0.27	-0.19	0.60***				
7	Spatial Imagery Anxiety	0.05	-0.10	0.09	0.26	0.302**	0.35***			
8	Manipulation Anxiety	-0.11	-0.09	-0.07	0.05	0.36***	0.57***	0.53***		
9	Spatial Navigation Anxiety	0.10	0.08	-0.13	0.12	0.40***	0.40***	0.53***	0.56***	
10	Number Line Estimation	-0.48***	-0.52***	-0.40**	-0.05	0.19*	0.22*	0.20*	0.16	0.03

Note. $p < 0.001$ = ***; $p < 0.01$ = **; $p < 0.05$ = *. $N = 115$ for correlation among all anxiety measures and number line estimation measure, $N = 48$ for correlations involving all math measures sessions

Discussion

The current study examined the spatial representation theory, which proposes a connection between spatial skills, mathematical learning, and the MNL representation. According to this theory, proficient spatial skills facilitate the development of efficient internal number line representations, thereby enhancing numerical skills. The current study took an alternative approach to examine spatial-mathematical associations by investigating whether an affective aspect of these skills has a cross-domain relationship with one another. Specifically, we examined how mathematical and spatial anxiety relates to mathematical and spatial skills within and across domains. If spatial representation theory accurately predicts that spatial skills support mathematical learning through MNL representation, then one would expect that deficits affecting spatial anxiety would also affect mathematical skills, as the underlying MNL would be affected. Similarly, deficits affecting mathematical skills should negatively affect spatial skills as the underlying MNL is impacted.

Given that spatial anxiety is a multidimensional deficit, we included three subscales of spatial anxiety and examined their correlations with the four mathematical areas. However, we also included mathematical anxiety, which represents an affective aspect of math, because spatial anxiety is related to spatial skills. In doing so, we were able to assess whether the emotional factors that affect mathematical anxiety are related to the emotional factors of spatial skills. Consistent with Ferguson et al.'s (2015) findings, our results demonstrated a significant correlation between all subscales of spatial anxiety and mathematical anxiety, indicating that individuals with higher apprehension about spatial situations were more likely to experience anxiety when engaging in math-related tasks. This suggests a potentially shared cognitive and emotional component in spatial and mathematical processing.

Nevertheless, although spatial anxiety measures were related to mathematical anxiety as expected, the data did not support the predicted relationship between spatial anxiety and mathematical skills. None of the three subscales of spatial anxiety was significantly related to any of the four categories of mathematical skills. The sample sizes underlying these correlations were fairly small, so it could be that the relationships would be statistically significant if the design had more power. However, these correlations are both positive and negative, and mostly close to zero. The strongest correlation in this group was in the opposite direction to what would be predicted (i.e., those who scored higher on spatial imagery anxiety tended to do better on the geometry question). The most parsimonious interpretation of these results is that there was no relationship between spatial anxiety and mathematical skills.

Other correlations involving mathematical skills measures, which had a smaller sample size, were consistent with expectations, but not overly so. The mathematical skills subscales were positively correlated with each other, except geometry, which was not significantly related to arithmetic. Our data revealed a significant association between mathematical anxiety and algebra as well as arithmetic, but not with fractions or geometry, suggesting that some areas of mathematics may elicit higher mathematical anxiety levels than others. Although the reasons behind the specific relationship between mathematical anxiety, algebra, and arithmetic remain unclear, one possible explanation could be the nature of the problems in these domains. In algebra, variables are introduced to represent unknown numbers, potentially increasing the level of complexity. This complexity could contribute to increased anxiety among individuals with mathematical anxiety when facing problems in these areas, thereby negatively affecting their performance. In addition, it is also plausible that highly anxious individuals would find these mathematical areas even more difficult, leading to poor performance. The lack of a significant correlation between mathematical anxiety and fractions or geometry suggests that mathematical anxiety may not directly affect performance in these specific areas. However, it needs to be clarified why these areas are any less complex than arithmetic. Ultimately, this may be an issue with a lack of power due to the low attrition rate, resulting in a small sample size included in the analysis.

The number line estimation was significantly correlated with algebra and fractions but not with arithmetic or geometry. Moreover, number line estimation was significantly correlated with mathematical anxiety and spatial imagery anxiety but not with mental manipulation anxiety or spatial navigation anxiety. The lack of a significant correlation between spatial anxiety and mathematical skills could be attributed to inadequate statistical power. A power analysis assessed whether the non-significant correlations were due to insufficient statistical power. The analysis revealed extremely low power values, indicating that a large sample size is required to detect meaningful relationships. These findings suggest that the limited sample size may have contributed to the lack of a significant correlation between spatial anxiety and mathematical skills.

In summary, this study found a significant correlation between mathematical anxiety and various types of spatial anxiety, indicating a shared emotional component between spatial and mathematical processing. Mathematical anxiety is positively associated with algebra and arithmetic skills, but not fractions or geometry. The number line estimation is related to algebra and fractions as well as mathematical and spatial imagery anxiety. The study did not find significant correlations between spatial anxiety and mathematical skills but found positive correlations between spatial anxiety scales and specific mathematical areas. These mixed relationships may be due to variations in the relationship between spatial anxiety and spatial skills. The lack of significant correlations may also be attributed to insufficient statistical power resulting from the small sample size used or, as detailed below, some methodological issues in the data that question the fidelity of the data.

Conclusion and Future Directions

In conclusion, this study suggests a complex relationship among spatial anxiety, mathematical anxiety, and mathematical skills. Although there was a significant correlation between mathematical anxiety and various spatial anxieties, the association between spatial anxiety and mathematical skills was not statistically significant. This finding challenges the assumption of a straightforward negative relationship between spatial anxiety and mathematical performance. This study also highlights the importance of considering the emotional factors of mathematical and spatial anxiety to understand the cognitive processes underlying mathematical performance.

However, the limitations of this study, such as the small sample size and inadequate statistical power, warrant caution when interpreting the results, and further research is required to elucidate the nature of these relationships. Therefore, future studies should address these limitations. First, as this study was conducted during the early part of the COVID pandemic, it did not utilize best practices in online studies, such as screening participants and using attention check items. The utilization of these practices would increase the fidelity of the data. Second, this study used a bounded number line measure to predict spatial representation theory, even though Study 1 of this thesis suggested that an UNL might be a better indicator of MNL in an adult population. Future studies should include a measure of the unbounded number line. Third, no measures of spatial skill were included in this study. It would further validate our theoretical predictions if we could demonstrate that spatial anxiety is related to spatial skills as well as mathematical skills. Finally, as previously mentioned, the sample size should be increased. The subsequent study in this thesis addressed these issues to test this hypothesis.

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CHAPTER 4: THE RELATIONSHIP BETWEEN SPATIAL ANXIETY AND MATHEMATICS: THE ROLES OF BOUNDED AND UNBOUNDED NUMBER LINE TASKS.

Abstract

Spatial and mathematical skills are highly related; however, the mechanisms of their interconnection remain unclear. Recent studies have proposed that Mental Number Line (MNL) representation, a mechanism underlying the spatial representation of numbers, is the mediating factor in the spatial and mathematical associations. One way to examine this hypothesis is to investigate how a deficit in spatial skills relates to mathematical performance and vice versa. In this study, we examined how spatial anxiety, an affective component affecting efficient spatial skills, relates to mathematical performance. Furthermore, we examined whether MNL mediates the relationship between spatial anxiety and mathematical skills. We sampled 191 adults and tested them on different subscales of spatial anxiety, spatial skills, mathematical skills, and bounded and unbounded number line estimations. Using structural equation models, we found that spatial anxiety significantly predicted mathematical performance even after controlling for general anxiety. Notably, the relationship between spatial and mathematical anxiety was mediated by an unbounded number line estimation. This result supports spatial representation theory, suggesting that high spatial anxiety could lead to poor spatial skills, which could lead to inefficient MNL representation and, consequently, relate to mathematical performance.

Keywords: Spatial skills, spatial anxiety, mental number line, mathematical anxiety

Introduction

Extensive research spanning several decades has consistently shown a positive association between spatial skills and mathematical achievement (Alloway & Passolunghi, 2011; Casey et al., 2015; Uttal et al., 2013; Chen & Zhou, 2011; Frick, 2018; Gunderson et al., 2012; Kyatta & Lehto, 2008; Mix et al., 2018; Wai et al., 2009). Recent studies have also sought to uncover ways of leveraging spatial-mathematical connections to enhance mathematical learning. However, to fully harness the benefits of this association, it is crucial to understand its underlying neurocognitive dimensions. Understanding the underlying neurocognitive mechanisms of spatial and mathematical associations will enable the development of tailored spatial interventions to enhance mathematical learning, based on individual needs. The spatial representation theory offers a plausible explanation for the connection between spatial skills and mathematics (Gunderson et al., 2012; Tam et al., 2019). According to this theory, the Mental Number Line (MNL) is the mechanism underlying the relationship between spatial and mathematical skills. Thus, strong spatial skills facilitate the efficient development of the MNL, leading to enhanced numerical processing (Gunderson et al., 2012).

Support for spatial representation theory mostly comes from behavioural studies that have used number line estimations as a mediating model, with mathematics skill scores as the outcome and spatial skill as the predictor (Georges et al., 2017; Gunderson et al., 2012; Tam et al., 2019). These studies have shown that number line estimation significantly mediates the relationship between mathematical and spatial skills. However, a point of contention in the spatial representation model revolves around the measurement of the MNL representation. Most studies investigating the mechanisms underlying spatial and mathematical skills using the spatial representation approach have employed the Bounded Number Line (BNL) estimation task

(Gunderson et al., 2012; Xie et al., 2019). In BNL estimation tasks, participants are required to indicate the position of a given number on an empty number line with labelled starting points and endpoints. For example, participants were asked to place 20 on a number line starting from 0 and ending at 100.

Earlier studies have observed developmental changes in the BNL estimation tasks, with younger children's performance described as logarithmic and adults displaying linear representation (Siegler & Booth, 2006). This developmental change is interpreted as a representational shift of magnitude knowledge development, and hence the MNL. However, emerging evidence indicates that the performance patterns observed in BNL estimation tasks may not accurately reflect the magnitude representation underlined by the MNL but may be influenced by the strategy used when completing the BNL (Cohen & Blanc-Goldhammer, 2011; Jung et al., 2020; Link et al., 2014; Reinert et al., 2019). These studies proposed that participants may rely on proportion judgments rather than purely on magnitude knowledge when completing BNL estimation tasks. For instance, a cyclic power model has been found to align better with children's data on these tasks, showing that participants' errors decrease when the target number is closer to specific points, such as the startpoint, midpoint, and endpoint of the number line. This suggests that participants may rely on these points, suggesting a proportion judgment with a particular sensitivity to the halfway point (Cohen & Blanc-Goldhammer, 2011; Jung et al., 2020; Link et al., 2014). Additionally, eye-tracking studies have revealed increased fixation by participants around the starting, middle, and endpoints of the BNL (Reinert et al., 2019). To address these limitations, a new version of the number line estimation task, the Unbounded Number Line (UNL) estimation task, has been proposed as a better measure of magnitude representation and MNL knowledge. In this task, participants indicated the position of specified numbers on an empty number line with no labelled endpoint. For example, one may ask participants to indicate the position of 20 on an empty number line that starts at 0, but the endpoint is not labelled. Instead, the scale of the line is defined by a labelled interval (e.g., the number line has a mark indicating where five would be). An increasing body of research argues that the UNL estimation task may provide a more accurate representation of the MNL. Furthermore, evidence suggests that bounded and UNL estimation tasks may involve different cognitive mechanisms, emphasizing the need to examine their roles independently in the relationship between spatial and mathematical skills (Reinart et al., 2019).

The first study of this dissertation aimed to investigate spatial representation theory, examining the role of the MNL as a possible mechanism underlying the spatial-mathematics relations. This study involved a sample of adults and measured various spatial skills, such as mental rotation and spatial visualization, along with four mathematical areas: arithmetic, fractions, algebra, and geometry. Furthermore, two number line estimation tasks were included, encompassing both the bounded and unbounded scenarios. Using structural equation models with multiple mediations, the findings indicate that spatial skills significantly predict mathematical aptitude. Notably, only the UNL estimation task, not the BNL task, emerged as significant independent mediators in the relationship between spatial and mathematical skills. The results from Study One confirmed spatial representation theory, implying a connection between spatial skills and mathematical performance in adults. Moreover, the findings suggest that MNL could be the underlying factor, with UNL estimation tasks serving as more accurate measures than BNL.

To gain a deeper understanding of the spatial representation theory, a more comprehensive approach would be to examine how other factors, such as deficiencies in either spatial or mathematical skills, can affect the development of MNL knowledge. From the perspective of spatial representation theory, MNL serves as a domain-specific cognitive resource that develops from the shared resources between spatial and mathematical skills. Therefore, if this proposition is true, one would expect that impairments in spatial skills could hinder the efficiency of MNL and affect mathematical skills. Spatial anxiety is an affective factor of spatial skills defined as apprehension related to spatial situations, which has been shown to be related to spatial skills (Lyons et al., 2018). Prior studies that investigated spatial anxiety among adults relied on a spatial anxiety measure that was recently described as being dominated by self-reported measures of individuals' apprehension in navigation situations (Lyons et al., 2018). A better measure of spatial anxiety should include items that assess apprehension in broader spatial situations.

Spatial anxiety has also been considered a multidimensional deficit encompassing various dimensions related to specific spatial situations (Lyons et al., 2018). This characterization of spatial anxiety entails that the anxiety experienced while navigating differs from that experienced while mentally rotating objects. Although the exact number of spatial anxieties remains unclear, a recent factor analysis revealed three subscales of spatial anxiety: Mental Manipulation Anxiety, Navigation Spatial Anxiety, and Mental Imagery Anxiety (Lyons et al., 2018). Mental Manipulation Spatial Anxiety involves feeling apprehensive while performing tasks that require the rotation of 2D or 3D objects, such as "imagine and mentally rotate a 3-dimensional figure." Navigation Spatial Anxiety arises from performing tasks that involve external environmental navigation, such as "trying to find your way around an unfamiliar city." Mental Imagery Spatial Anxiety stems from apprehensions related to tasks involving understanding spatial relations among objects without transforming them mentally, such as "describing a person's face you have only met once."

There is limited research on the relationship between spatial anxiety and spatial skill. However, existing research suggests that high levels of spatial anxiety are associated with lower levels of spatial skills and vice-versa (Kremmyda et al., 2016; Lawton, 1994; Ramirez et al., 2012). For example, Ramirez et al. (2012) found that 7-year-olds with high scores on a spatial anxiety scale performed poorly on mental rotation tasks. Additionally, individual differences in mental rotation and spatial anxiety interact with working memory capacity, with high spatial anxiety and high working memory capacity associated with poorer performance on mental rotation tasks (Ramirez et al., 2012). Attit and Rocha (2021) found that individuals who reported high spatial anxiety also demonstrated a greater tendency to avoid spatial situations and exhibited poorer performance on spatial skill tasks. Furthermore, the authors found a significant association between mental manipulation anxiety and performance on spatial skills among K-12 teachers, whereas mental imagery anxiety did not exhibit such an association. Additionally, Ferguson et al. (2015) discovered that a poor sense of direction positively correlates with heightened navigation anxiety. Alvarez-Vergas et al. (2020) found that spatial anxiety is an explanatory factor for gender differences in mental rotation performance, with higher levels of spatial anxiety linked to lower performance.

Examining the relationship between different spatial anxieties and skills is crucial, especially when considering new measures with multiple spatial anxiety subscales. Lyons et al. (2018) developed and validated multidimensional spatial anxiety subscales by investigating their connections with their respective spatial skill subdimensions. For example, they discovered that mental manipulation anxiety predicted mental rotation skills, whereas navigation anxiety predicted map-reading skills. However, no significant relationship was found between imagery anxiety and the embedded figures task. The authors explained that this might be because the imagery anxiety items do not capture the same cognitive processes underlying the embedded figures task. They suggested that using a different measure to represent spatial imagery accurately could yield a significant result. Thus, it appears that the connection between spatial anxiety and spatial skills is not linear and that different spatial anxieties are related to different spatial skills. To understand this complexity, it is necessary to examine how different subscales of the spatial anxiety scale relate to different spatial skill dimensions. In this study, we measured two spatial skill dimensions and assessed their connection to different spatial anxiety domains.

The reviews mentioned above suggest that, although mixed evidence exists regarding the relationship between spatial anxiety and spatial skills, there is evidence of a significant relationship between spatial anxiety and spatial skills in general. If high spatial anxiety is related to poor spatial skills, might this relationship also apply to mathematical skills, given that spatial and mathematical skills are strongly related? The spatial representation theory might offer the possibility of testing this relationship. According to the spatial representation framework, high spatial anxiety is expected to render spatial skills inefficient, leading to poor number line representation and mathematical performance. It is important to note that the reason for the significant relationship between spatial anxiety and spatial skills varied. For example, as reported in previous studies, individuals with high spatial anxiety tend to avoid spatial situations (Gunderson et al., 2013). However, individuals with high spatial anxiety may instead (or also) have poor spatial skills (Krymmyda et al., 2016). In either case, it would be expected that high spatial anxiety would be related to poor spatial skills, which in turn would be related to the reduced efficacy of the MNL and subsequently lead to poor mathematical skills. Thus, not only should spatial anxiety be related to spatial skills and MNL, but it should also be related to mathematical skills.

In the exploratory study outlined in Chapter 3, we examined the relationship between spatial anxiety, the MNL, and mathematical performance. Our investigations revealed no significant correlations between spatial anxiety and MNL or between spatial anxiety and any of the measured mathematical domains. This lack of correlation between spatial anxiety and MNL was not unexpected, given that the BNL task was used. As previously mentioned, the BNL task has been demonstrated to be more reflective of proportion knowledge than magnitude knowledge that underlies the MNL representation. Additionally, despite the nonsignificant correlations discovered between spatial anxieties and mathematical skills, a power analysis indicated that the study lacked the statistical power to detect any potential effects, given the small sample size of 48 participants.

The Present Study

This study explored the relationship between spatial anxiety, MNL, and mathematical skills in a large group of adults. To address the controversy surrounding using BNL scores to represent the MNL, we included an UNL estimation task. We examined how both the BNL and UNL estimation tasks relate to spatial anxiety. Although our findings from Study One support spatial representation theory, as indicated by a significant mediation role for UNL performance, which is theoretically a representation of the MNL, in the relationship between spatial skills and mathematical skills, this evidence is novel. Additionally, one of the interesting findings from Study One was that spatial skill did not predict performance on geometry but predicted performance on all other measures. This finding implies that spatial skills have different relationships with mathematical skills. Therefore, it is essential to test the relationship between spatial and mathematical skills across multiple mathematical domains to delineate the mathematical dimensions related to spatial skills. Determining this would enable appropriate spatial training that is tailored to mathematical learning. Based on the lack of correlation between spatial skills and geometry in Studies One and Two (and the low reliability of the measure), we did not include geometry as a mathematical domain in this study. In the current study, we instead included percentage knowledge as an additional mathematical domain to test whether percentage is a mathematical area predicted by spatial skills.

In summary, the current study aimed to replicate the results of Study One with a much larger sample size, explore the relationship between the various subscales of spatial anxiety and different spatial skill dimensions beyond those identified by Lyons et al. (2018), and test whether spatial anxiety interferes with the relationship between spatial skills and mathematical performance. Therefore, the following research questions were examined.

1. Are spatial anxieties related to the measures of spatial skills used in this study?
2. In these data, does MNL mediate the relationship between spatial and mathematical skills (i.e., does it replicate our results from Study 1)?
3. Are spatial anxieties related to mathematical skills?
4. Does MNL mediate the relationship between spatial anxiety and mathematics, if any such relationship exists?

Method

Participants

A total of 221 participants were initially recruited through Prolific for the study. Data from three participants were excluded because of technical computer errors, and two more participants withdrew their consent. Another 23 people were eliminated because they failed one of the attention checks, and three participants were eliminated because they completed both sessions in less than 30 min, which was one-quarter of the allotted time and too fast to trust that the measures were completed conscientiously. This resulted in a final sample of 190 participants. Among these participants, there were 95 women (*Mean Age* = 27.1 years) and 95 men (*Mean Age* = 27.8 years). The educational backgrounds of the participants varied as follows:17.4% had graduate-level education; 28.4% had a high school diploma or A-level qualification; 5.3% had secondary school education (e.g., GED/GCSE); 10.0% had technical/community college education; and 38.4% had completed an undergraduate degree. Regarding ethnicity, 63.2% of the participants were white, 21.6% were black, 2.1% were Asian, 8.9% were mixed-race, and 4.2% belonged to other ethnic groups.

Procedure

All the measures were completed online and hosted by Qualtrics. Participants who signed up from Prolific also had access to the study's link and had to read and agree to a consent form before completing the various tasks. The tasks were conducted in two separate sessions. The first session consisted of a spatial anxiety scale, mathematical anxiety rating, and spatial skill measurements. The second session consisted of mathematical skills and general anxiety measurement. Overall, the participants needed approximately two hours to complete all measures. The presentation of the tasks was randomized within sessions using the Qualtrics Automatic Randomization Function. Participants received US$18.00 compensation for their time participating in the study. The researcher audited participant submissions before the payment was issued to ensure they passed attention checks and met all educational requirements. This study was reviewed and approved by the Interdisciplinary Committee on Ethics in Human Research (ICEHR) at the Memorial University of Newfoundland and the Labrador.

Measures

Spatial Anxiety Scale (SAS)

The spatial anxiety scale was adopted from Lyons et al. (2018) and consists of 24 items that assess how anxious participants feel when performing spatial tasks. Eight (8) of the items assessed spatial navigation anxiety (e.g., asked to follow directions to a location across town without the use of a map), eight (8) assessed spatial imagery anxiety (e.g., given a test in which participants were allowed to look at and memorize a picture for a few minutes, and then given a new, similar picture and asked to point out any differences between the two pictures), and eight (8) assessed spatial mental manipulation anxiety (e.g., asked to imagine and mentally rotate a 3-dimensional figure). The choices ranged from 'not at all' (0) to 'very much' (4). High scores are indicative of high spatial anxiety. Each subscale consisted of eight items. The internal consistency (Cronbach's alpha) of these subscales was high [Mental Manipulation Anxiety, alpha = 0.85], [Spatial Navigation Anxiety, alpha = 0.89] and [Spatial Imagery Anxiety, α = 0.87].

Abbreviated Mathematical Anxiety Scale (AMAS)

The AMAS consists of nine questions asking participants to rate their level of anxiety when performing math-related tasks (Hopko et al., 2003). Participants were given the option to select choices ranging from 'not at all' (scored 0) to' very much' (scored 4). High scores are indicative of high anxiety levels. The internal consistency of this measure is high (α = 0.95).

Generalized Anxiety Disorder Scale (GAD-7)

The Generalized Anxiety Disorder Scale (GAD-7), adopted from Spitzer et al. (2006), was used to assess general anxiety levels. On the GAD-7, participants rated their level of anxiety on a 4-point scale consisting of seven items. Choices on the GAD-7 range from "not at all" (score of 0 to "nearly every day" (score of 4). Higher scores indicate higher anxiety levels. The internal consistency of this measure is high (α = 0.92).

BNL Estimation Task

The BNL task consisted of 20 lines, starting at 0 and ending at 115. Participants were asked to click on the screen to indicate the positions of the numbers on each number line. Each line was placed on a separate screen within a display margin of 800 × 200 mm to prevent participants from using their previous answers as reference points. Participants were required to place numbers ranging from 0 to 115 on the line, and two training questions were provided for each participant. The participants were given 4 min to complete the task.

UNL Estimation Task

In the UNL task, we adopted the task used by Reinert et al. (2015) with a few modifications. Participants were asked to slide a bar on a number line with a starting point and unit measurement as a reference. They consisted of 20 lines, each 650 mm long and 5 mm high. Each line was placed on a separate screen within a display margin of 800 × 200 mm to prevent participants from using their previous answers as reference points. The unit measurement or anchor was consistently 10 mm in length. Participants were required to place numbers ranging from 0 to 115 on the line, and two training questions were provided for each participant. The participants were given four minutes to complete the task.

For both BNL and UNL estimation tasks, the mean percent absolute error (PAE) [PAE= (estimated number –target)/scale, Booth & Siegler, 2008] was computed.

Mental Rotation Task

The Revised Mental Rotation Test (Vanderberg & Kluse, 1978; modified by Peters et al., 1995) was used to assess mental rotation. This 24-item measure presented five figures per item.

Each item comprised a separate figure on the left and four figures on the right. All four figures on the right-hand side differ in terms of orientation. Two out of the four figures on the right were the same as the first figure on the left side but had been rotated around the vertical axis of the left figure. Participants were required to identify the two forms among the four figures on the right, which were the same as those on the left. One point was awarded for each pair of correctly identified figures, and 0 for incorrect pairs. The maximum and minimum scores are 24 and 0, respectively. High scores are indicative of higher mental rotation skills. The task was divided into two parts to ensure that participants could attempt all the items. Participants were given four minutes to complete each part and required eight minutes to complete all parts. The internal consistency of this measure is high (α = 0.89).

Spatial visualization Task

Spatial visualization, the ability to mentally transform 3-dimensional objects into 2-dimensional ones and vice versa, was evaluated using the Surface Development Test (SDT). This test is a subset of a collection of cognitive tests developed by Ekstrom et al. (1976). This measure contained 12 items, with each item consisting of a pair of net and solid shapes. The participants were required to pair the edges of the net shape with those of the solid shape when folded along the dotted lines of the net figure. To complete this test, participants were required to visualize the open and closed forms of the shapes and identify the number on the net corresponding to the letter on the robust figure. One point was awarded for correct matching, and 0 points were given for an incorrect match. High scores represented superior spatial visualization, and low scores indicated poor spatial visualization skills. The internal consistency of this measure is high (α = 0.90). The test is divided into two parts. Each part was completed within 6 minutes and required a total of 12 minutes.

Mathematical Placement Test (MPT)

Mathematical skills were assessed using a paper-and-pencil task selected from the Mathematical Placement Test (MPT). The MPT is a diagnostic test for basic mathematics skills administered by the Department of Mathematics and Statistics at the university where this study was conducted. Mathematics educators designed the test to assess competence levels in specific core areas of mathematics covering up to at least 12th grade, including the arithmetic of integers, rational numbers, algebraic expressions, trigonometry, geometry, percentage, and fractions. Forty-five (45) items were selected comprising four mathematical proficiencies: fractions, algebra, percentage, and arithmetic. There were 14 arithmetic, 16 fractions, 5 percentage, and 10 algebraic items. Participants were not permitted to use a calculator while completing these tasks. The participants were expected to complete the measurement within 45 min. To ensure that participants had the opportunity and time to attempt questions from each domain, each domain was presented as a block of questions and randomized across blocks. Cronbach's alpha was calculated for each of the four subscales. Arithmetic (alpha = 0.6), fraction (alpha = 0.6), and algebra (alpha = 0.5) demonstrated reasonable to good reliability and geometry (alpha = 0.6).

Analysis

Data cleaning and transformations were conducted on Snowflake cloud platform using structured query language (SQL). The data were further analyzed in R version 1.2.5019 using the lavaan package (version 0.6-5; Rosseel, 2012). First, we conducted a multivariate outlier test on the data of the 190 participants using the Mahalanobis test. Thirteen (13) outliers were detected and excluded from the remaining analysis (Mah < 22.262, $p < 0.05$), leaving 177 participants. In addition, sixteen (16) participants had missing data for at least one of the measures and were excluded from further analysis. Therefore, the data from 161 participants were included in the final analysis. Structural Equation Models were used to evaluate the relationship between spatial skills and mathematical competencies. Although the multivariate skewness was not nonsignificant, there was some univariate significant skewness (see Table); therefore, SEM models with robust, standardized error estimates were used. According to Satorra and Bentler (1998), these fit indices adjust the standard errors for nonnormality.

Results

The correlational analysis in Table 7 reveals several significant associations between variables of interest. All mathematical measures exhibited significant positive correlations, indicating a strong interrelation between different mathematical skills. As predicted, the three subscales of spatial anxiety were negatively related to the two spatial skills measures and positively related to mathematical and general anxiety. However, the relationship between spatial anxiety and mathematical achievement varied. Mental Manipulation Spatial Anxiety was (negatively) related to percentage and arithmetic scores but not fraction and algebra scores. Navigation Spatial Anxiety was negatively related to all mathematical measures except algebra. However, Imagery Spatial Anxiety was negatively related to arithmetic measures only.

The UNL task, measured as the degree of error, demonstrated significant positive correlations with Mental Manipulation Spatial Anxiety, suggesting that higher levels of spatial anxiety in this area were associated with poor performance in this task. However, UNL was not significantly correlated with navigation or imagery spatial anxiety. Simultaneously, the UNL task showed negative correlations with the measures of spatial skill and all mathematical measures. This suggests that people who make more errors in the UNL have weak mathematical performance and worse spatial skills. In contrast, the BNL task did not correlate significantly with any mathematical measure, suggesting a weaker association between this task and overall mathematical skills. In addition, the BNL task did not correlate significantly with any spatial skill or spatial anxiety subscale scores. Overall, the patterns of correlations demonstrated the expected relationships among spatial anxiety, spatial skills, and mathematical skills, while also suggesting that UNL knowledge was more correlated with these measures than BNL knowledge.

Similarly, mathematical anxiety demonstrated significant negative correlations with all mathematical measures, indicating that higher levels of mathematical anxiety are associated with poorer performance on mathematical tasks. Mathematical anxiety also showed a significant positive correlation with all spatial anxieties, indicating a link between mathematical and spatial anxiety. Furthermore, mathematical anxiety was negatively correlated with all measures of spatial skills, suggesting that higher levels of mathematical anxiety are associated with lower levels of spatial skills. Finally, mathematical anxiety positively correlated with the BNL and UNL tasks.

Table 6

Description statistics (N=161)

		Mean	*S.D*	*Min*	*Max*	*Skewness*	*Std. error*
1.	Mental Rotation	11.6	5.7	1	21	-0.1	0.2
2.	Surface Development Task	20.8	8.52	5	37	0.2	0.2
3.	UNL	7.15	3.1	2.1	20.7	1.6	0.2
4.	BNL	29.5	2.1	22.1	35.5	0.0	0.2
5.	Mathematical Anxiety	1.6	0.8	0.1	3.5	0.2	0.2
6.	General Anxiety	1.1	0.8	0.0	3.0	0.5	0.2
7.	I - Anxiety	1.6	0.9	0.0	3.7	0.4	0.2
8.	M - Anxiety	1.9	1.0	0.0	4.0	0.1	0.2
9.	N - Anxiety	1.8	0.9	0.1	4.0	0.1	0.2
10.	Percentage	67.6	18.2	0.0	80.0	-1.5	0.2
11.	Algebra	38.0	22.9	0.0	80.0	-0.3	0.2
12.	Fraction	55.9	14.5	13.3	80.0	-0.6	0.2
13	Arithmetic	69.1	14.9	21.4	92.9	-0.5	0.2

Note. All mathematics measures were in percentages, UNL were in PAE

Table 7

Correlation Among the Various Variables (N= 161)

		1	2	3	4	5.	6.	7.	8.	9.	10.	11	12
1.	Mental Rotation												
2.	Surface Development Task	0.70***											
3.	UNL	-0.30***	-0.32***										
4.	BNL	0.03	-0.03	-0.20*									
5.	Mathematical Anxiety	-0.16*	-0.17**	0.11*	-0.07								
6.	General Anxiety	-0.02	-0.03	0.01	-0.01	O.47***							
7.	I - Anxiety	-0.16*	-0.21**	0.01	0.02	0.46***	0.25**						
8.	M - Anxiety	-0.36***	-0.41***	0.18*	0.01	0.54***	0.23**	0.65***					
9.	N - Anxiety	0.19*	-0.17*	0.01	0.04	0.58***	0.31***	0.65**	0.67***				
10	Percentage	0.34***	-0.39***	-0.36***	0.09	-0.24**	-0.08	-0.08	-0.20**	-0.20*			
11.	Algebra	0.34***	0.34***	-0.27**	0.02	-0.21**	-0.02	-0.02	-0.12	-0.01	0.40***		
12	Fraction	0.27***	0.21***	-0.25***	0.10	-0.30***	-0.08	-0.11	-0.14	-0.29***	0.49***	0.43***	
13	Arithmetic	0.32***	0.34***	-0.15	-0.15	-0.41***	-0.05	-0.19*	-0.13*	-0.23***	0.38***	0.50***	0.39***

Note. $p < 0.001$ = ***; $p <0.01$ = **; $p < 0.05$ = *. I – anxiety is Spatial Imagery Anxiety, M – anxiety is Mental Manipulation Anxiety, and N–anxiety is Navigation Anxiety.

Table 8

Partial Correlations (N = 161)

		1	2	3	4	5.	6.	7.	8.	9.	10.	11
1.	Mental Rotation											
2.	Surface Development Task	0.70***										
3.	UNL	-0.28***	-0.31***									
4.	BNL	0.03	-0.03	-0.20*								
5.	Mathematical Anxiety	-0.15	-0.21**	0.12	-0.07							
6.	I - Anxiety	-0.15	-0.22**	0.01	0.03	0.40***						
7.	M - Anxiety	-0.37***	-0.42***	0.18*	0.01	0.50***	0.63***					
8.	N - Anxiety	-0.18*	-0.19*	0.07	0.03	0.52***	0.63***	0.64***				
9.	Percentage	0.16*	-0.39***	-0.36***	0.09	-0.23**	-0.06	-0.19*	-0.18*			
10	Algebra	0.23**	0.34***	-0.27***	0.02	-0.24***	-0.02	-0.12*	-0.01	0.41***		
11.	Fraction	0.20**	0.34**	-0.25**	0.10	-0.29***	-0.09	-0.13	-0.26***	0.35***	0.54***	
12	Arithmetic	0.26***	0.33***	-0.21**	-0.02	-0.45***	-0.15*	-0.22**	-0.23**	0.16*	0.53***	0.52***

Note. $p < 0.001$ = ***; $p < 0.01$ = **; $p < 0.05$ = *. I – anxiety is Spatial Imagery Anxiety, M – anxiety is Mental Manipulation Anxiety, and N–anxiety is Navigation Anxiety. General anxiety was used as the control variable.

Relationships Between Spatial Anxiety and Spatial Skills

The first research question investigated the association between spatial anxiety and spatial skill. The correlation table (Table 7) reveals significant correlations between all measures of spatial anxiety and spatial skills. However, to ensure that this relationship was not solely influenced by general anxiety levels, we conducted a partial correlation analysis while controlling for general anxiety. The results are presented in Table 8. After controlling for general anxiety, imagery anxiety significantly correlated with surface development task, but not mental rotation. Mental manipulation anxiety significantly correlated with mental rotation as well as surface development task. Navigation anxiety also correlated significantly with surface development task ans well as mental rotation. These findings, generally indicate that individuals with higher spatial anxiety exhibit poorer mental rotation and spatial visualization skills. Notably, our results confirm and extend those of Lyons et al. (2018), demonstrating a cross-subdimensional relationship between different spatial anxieties and their corresponding spatial skill components and other spatial components.

Structural Equation Models (SEM)

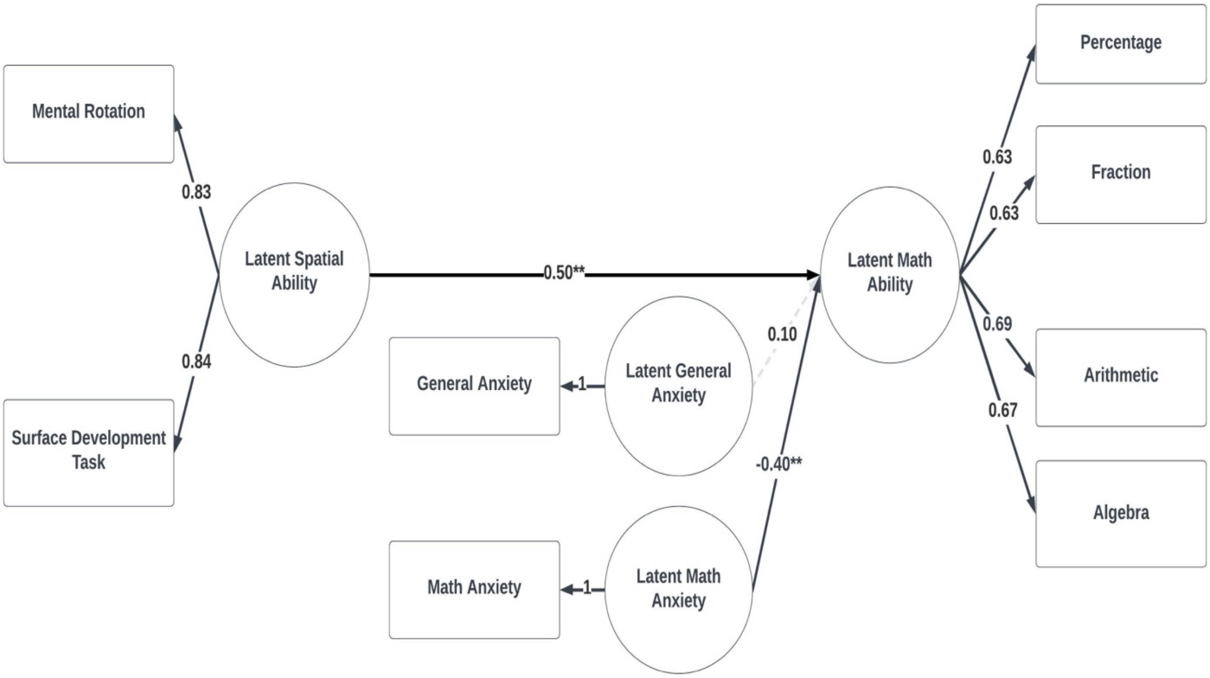
As shown in Table 7, the results of the correlations reflect the relationships between spatial anxieties and spatial skills, spatial anxieties and mathematical skills, and the BNL and UNL. We conducted a series of structural equation modelling (SEM) analyses to assess the predictive relationships between variables of interest. SEM is a statistical technique useful for modelling complex relationships, particularly those involving latent variables, and can account for possible measurement errors. Satorra-Bentler robust fit indices, which correct for slight non-normality in the data, were used in all models (Satorra & Bentler, 1994; Savalei, 2019). Generally, the criteria for good model fit selection, as suggested by Hu and Bentler (1999), include a nonsignificant chi-square ($\chi 2$) test, Comparative Fit Index (CFI) greater than 0.95, Standardized Root Mean Residual (SRMR) less than 0.08, and Root Mean Square Error Approximation (RMSEA) less than 0.06. Aron et al. (2013) also recommended that an RMSEA of less than 0.1 and a 90% confidence interval is considered a good fit. It is important to note that these criteria are mere suggestions, and, as such, the decision to retain a model also depends on the hypothesis and theoretical considerations. We adopted a nested model to determine some of these models, where we tested the models without any controls and then added the control and mediating variables, where necessary.

Replicating Study 1: Does MNL mediate the relationship between spatial and mathematical skills in these data?

The second research question sought to ascertain whether the observed pattern of association between spatial skills, mathematics, and MNL found in Study One remained consistent in the present dataset. Consistent with Study One, we investigated this relationship at both the general (Model 1a and Model 1b) and specific levels (Model 2a and Model 2b) of mathematical performance. This approach allowed us to confirm the general connection between spatial skills and mathematics at the latent level while also examining how spatial skills relate to distinct mathematical areas. To achieve this, we created a latent spatial skill variable by combining mental rotation and spatial visualization tasks (surface development task). Similarly, for Models 1a and 1b, the latent mathematical skills variable was created by combining arithmetic, algebra, fractions, and percentages. The mediating variable is represented by UNL scores. We omitted BNL in this analysis because it did not significantly correlate with any of the predictor variables. In addition, general and mathematical anxieties were controlled in the model specification for all models. Model 1a tested the relationship between spatial and mathematical skills without the mediating variable (MNL). In this model, general and mathematical anxieties were regressed as controls for mathematical skills, and spatial skills were then included. See Figure 6 for the path diagram. The results from Model 1a demonstrated excellent fit indices: $\chi 2(16, n = 161) = 25.239$, $p = 0.065$, scaling correction factor for Satorra-Bentler correction = 1.043, CFI = 0.973, robust RMSEA = 0.061, SRMR = 0.042, and TLI = 0.952. As shown in Figure 6, spatial skills still significantly predicted mathematical skill performance, after controlling for general and mathematical anxiety.

Figure 6

Model 1a. Path Diagram Showing the Relationship Between Spatial and Mathematical Skills

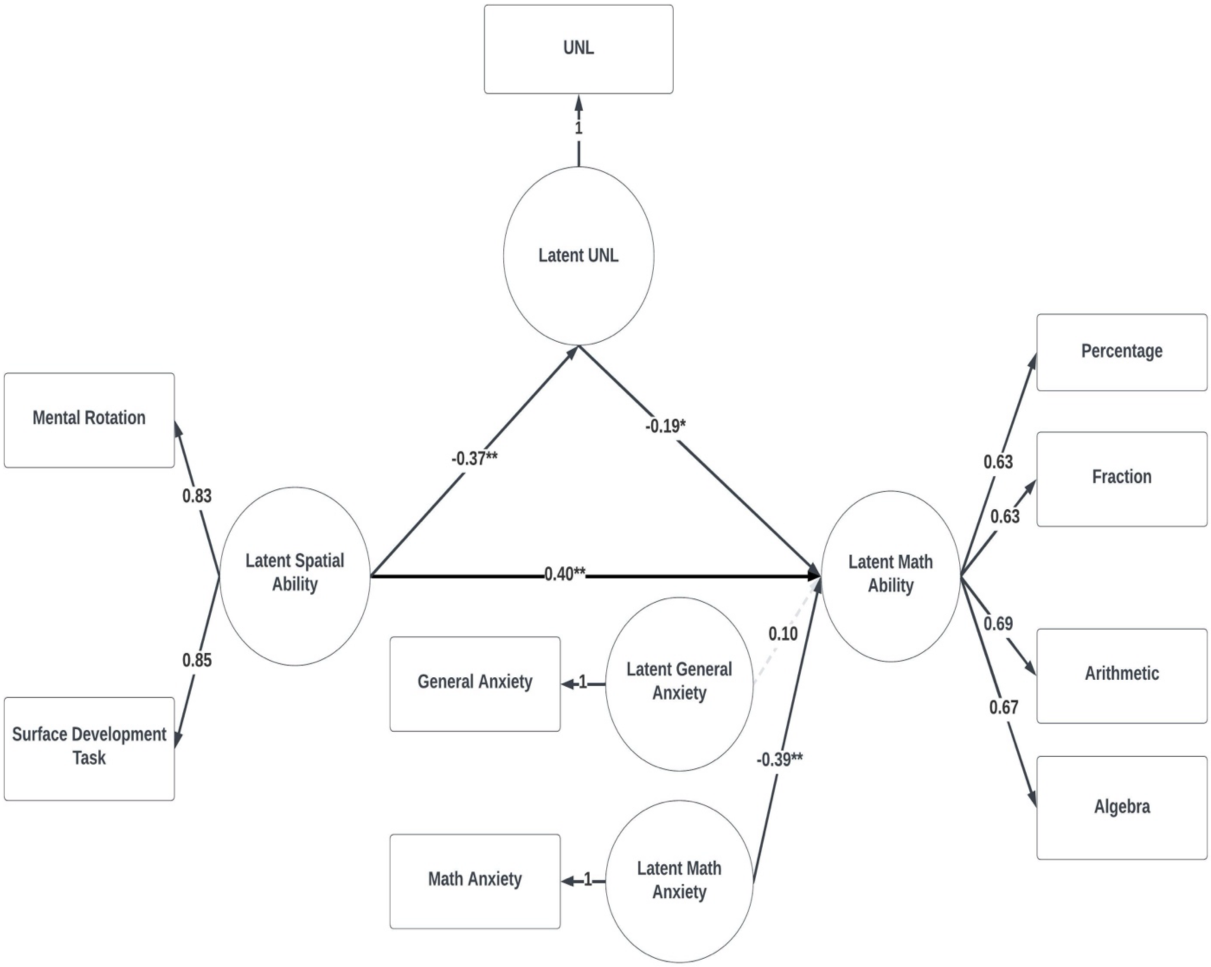


Note. The diagram above shows the relationship between spatial and mathematical skills. Oval shapes represent latent variables, and rectangular shapes represent observed variables. Continuous thick lines illustrate a significant path, and faint continuous dash lines represent a nonsignificant path. $p < 0.001$ = ***.

Given the results of Model 1a, MNL was introduced as a mediator in Model 1b. A Model diagram is shown in Figure 7. The results from Model 1b demonstrated good fit indices: $\chi 2(22, n = 161) = 32.588$, $p = 0.068$, scaling correction factor for Satorra-Bentler correction = 1.055, CFI = 0.971, robust RMSEA = 0.056, SRMR = 0.045, and TLI = 0.952. Mediation analysis showed that the direct path between spatial and mathematical skills was significant, even after controlling for general and mathematical anxieties, and the effect of MNL was partialled out ($b = 0.435$, $p < 0.001$). Similarly, the indirect effect of MNL on the path relationship between spatial and mathematical skills was significant ($b= 0.073$, $p = 0.031$). This finding indicates that MNL partially mediates the relationship between spatial and mathematical skills.

Figure 7

Model 1b. Path Diagram Showing the Relationship Between Spatial and Mathematical Skills, and MNL

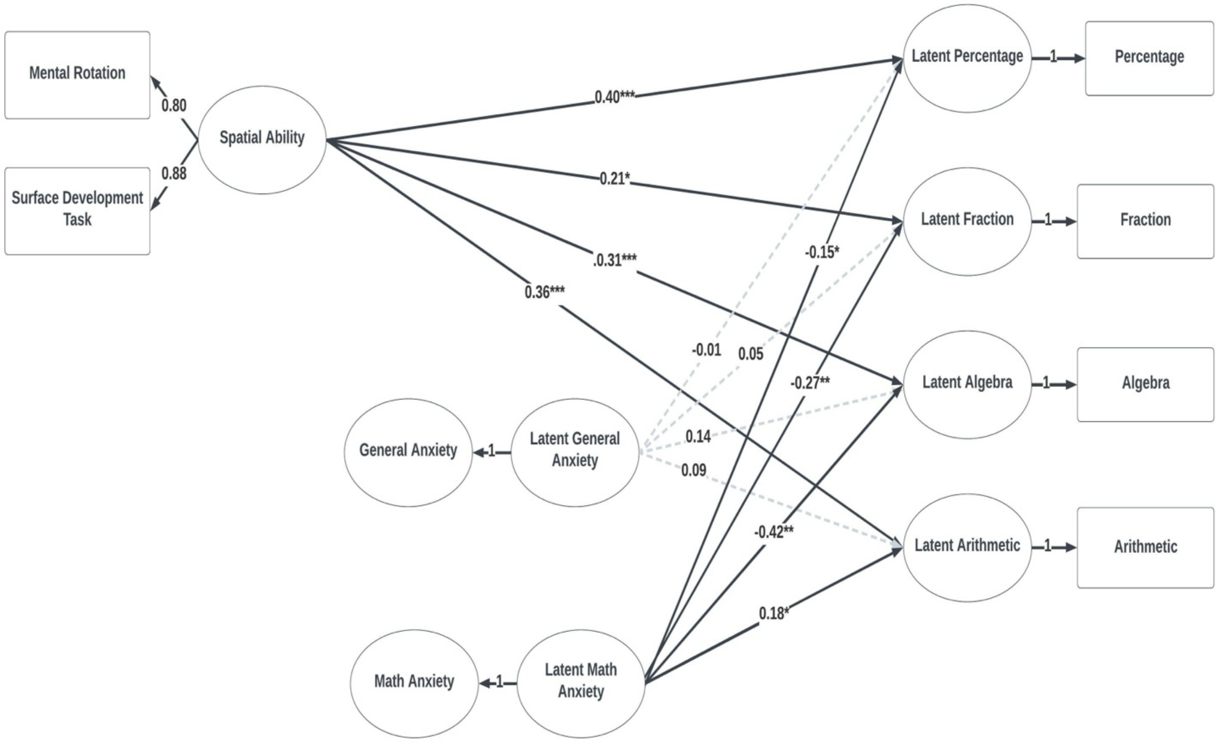


Note. The diagram above shows the relationship between spatial and mathematical skills and MNL. Oval shapes represent latent variables, and rectangular shapes represent observed variables. Continuous thick lines illustrate a significant path, and faint continuous dash lines represent a nonsignificant path. $p < 0.001$ = ***.

In Models 2a and 2b, we sought to break up the relationship between spatial and mathematical skills as well as the mediation analysis of component parts. Thus, we sought to examine how spatial skills relate to different mathematical skill domains, and investigate whether MNL mediates all relationships. To achieve this, Model 1b was modified so that each mathematical domain formed a separate latent variable, and these latent variables were entered as dependent variables. Thus, latent scores of fractions, algebra, percentage, and arithmetic were predicted from spatial skills, while controlling for general and mathematical anxieties. A path diagram is shown in Figure 8. The results indicated excellent fit indices: $\chi 2(5, n = 161) = 4.041$, $p = 0.544$, scaling correction factor for Satorra-Bentler correction = 0.972, CFI = 1.00, robust RMSEA = 0.00, SRMR = 0.012, and TLI = 1.015. As shown in Figure 8, spatial skills significantly predicted performance in terms of fraction, algebra, arithmetic, and percentage after controlling for general and mathematical anxiety.

Figure 8

Model 2a. Path Diagram Showing the Relationship Between Spatial and Mathematical skills, and MNL



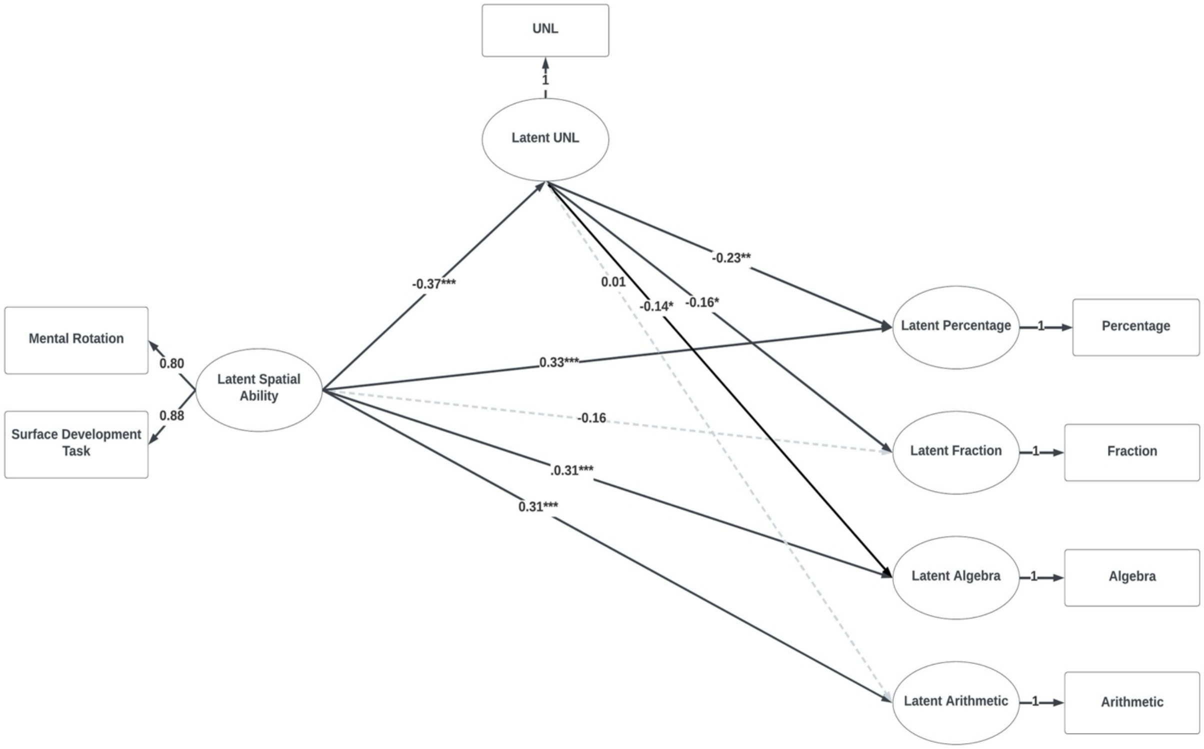
Note. The diagram above shows the relationship between various spatial and mathematical skills and MNL. $p < 0.05$ = ***; $p < 0.01$ = **; $p < 0.001$ = ***. Oval shapes represent latent variables, and rectangular shapes represent observed variables. Continuous dash lines indicate nonsignificant paths and thick lines indicate significant paths. Note that general and math anxieties were controlled in this model.

MNL skills were then introduced in Model 2b as mediators in all path specifications. As shown in Figure 9, the results demonstrated that all robust fit indices were excellent: $\chi2(8, n = 161) = 4.431, p = 0.816$, scaling correction factor for Satorra-Bentler correction = 0.964, CFI = 1.00, robust RMSEA = 0.00, SRMR = 0.013, and TLI = 1.041. See Figure 9 for the path diagram. Overall, the findings demonstrate the expected mediation. MNL significantly, but partially, mediated the relationship between spatial skill and percentage (indirect effect: $b = 0.083, p = 0.023$; direct effect: $b = 0.326, p <0.001$), fraction (indirect effect: $b = 0.060, p = 0.033$; direct effect: $b = 0.157, p = 0.058$), and algebra (indirect effect: $b = 0.051, p = 0.052$; direct effect: $b = 0.314, p < 0.001$). As the direct effect for fractions was no longer significant, we could make a case in which this effect was fully mediated, but the *p*-value of this direct effect bordered on significance; therefore, it is more parsimonious to think of this as a partial mediation. However, the mediation effect was not significant for arithmetic (indirect effect: $b = -0.005, p = 0.860$; direct effect: $b = 0.351, p < 0.001$).

In summary, the findings of Models 2a and 2b provide valuable insight into the predictive role of spatial skills in specific mathematical areas. These results support the hypothesis that individuals with higher spatial skills excel in various mathematical tasks, particularly algebraic, arithmetic, and percentage knowledge tasks. The significant but partial mediation effects found for MNL in these relationships suggest that it plays a crucial role in explaining the relationship between spatial skills and various mathematical domains, and confirms the spatial representation account in these data. The absence of mediation effects of MNL on the relationship between spatial skills and arithmetic implies the existence of other underlying mechanisms that influence this relationship. In addition, the fact that MNL did not mediate the arithmetic and spatial skills relationship suggests that spatial representation theory may not account for some relationships between spatial and mathematical skills when the affective aspects of these skills are considered.

Figure 9

Model 2b. Path Diagram Showing the Relationship Between Spatial and Mathematical skills, and MNL



Note. The diagram above shows the relationship between various spatial and mathematical skills and MNL. $p < 0.05$ = ***; $p < 0.01$ = **; $p < 0.001$ = ***. Note that this model builds on Model 2a and controls for general and math anxieties. For ease of reading, this diagram did not include the paths for math and general anxiety, but the model specification included them, and they showed the same pattern as they did in Model 2a. Oval shapes represent latent variables, and rectangular shapes represent observed variables. Continuous dash lines indicate nonsignificant paths, and thick lines indicate significant paths.

How is Spatial Anxiety Related to Mathematical Skills?

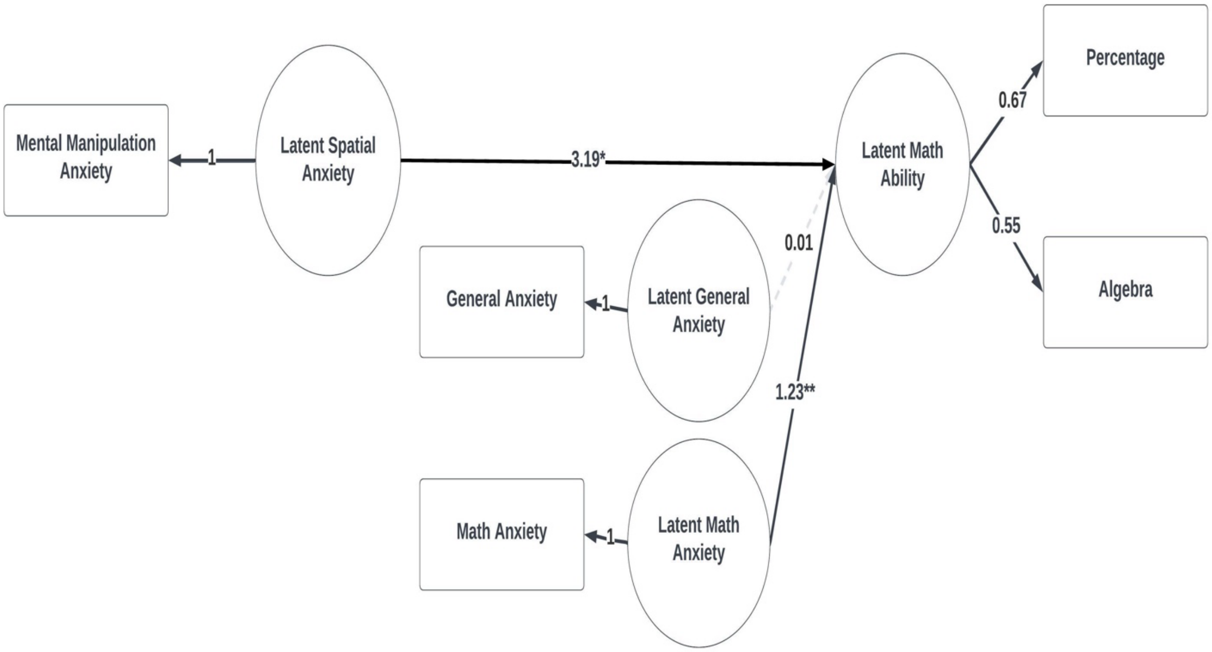
Our third research question investigated whether spatial anxiety and mathematics are related. We examined how and why spatial anxiety and mathematical skills may be related to spatial representation. The results from the correlation table (Table 6) illustrate significant relationships between various mathematical skills and spatial anxiety. However, when examined within the framework of the spatial representation account, the mechanism underlying this relationship was predicted to be MNL. Thus, high spatial anxiety is expected to lead to inefficient MNL, which would result in poor mathematical skills. Based on this premise, our variables of interest in the models were spatial anxiety, MNL measures, and mathematical skills. As shown in Table 6, there were differences in the correlations between the MNL and spatial anxiety measures, as well as mathematical anxiety measures. Consequently, except for the control variables, only measures that showed significant correlations with one another were included in our model specification level. Two mathematical domains met the criteria (Percentage and Algebra) and were included in all the models reported here.

Several nested SEM models are used to examine this relationship at the latent level. We created three latent variables in all models: Latent Math Skills, Latent Spatial Anxiety, and Latent MNL. Algebra and percentage were the observed variables for Latent Math Skills, UNL was the observed variable for MNL, and mental manipulation and Spatial Anxiety were the observed variables for spatial anxiety. To examine the relationship between spatial anxiety and mathematical skills, latent mathematical skills were entered as the dependent variables. General and mathematical anxiety, as well as spatial anxiety, were entered as predictors. We begin the model selection process by running hierarchical models and then examining any modification indices that are theoretically viable. We provide a model path diagram displaying the standardized estimated coefficients (b) for each model.

Model 3 explored the relationship between spatial anxiety and mathematical skills while controlling for general and mathematical anxiety. The path diagram is shown in Figure 10. The robust model fit indicators for this model were as follows: $\chi 2(3, n = 161) = 3.258$, $p = 0.353$, scaling correction factor for the Satorra-Bentler correction = 1.059, CFI = 0.998, RMSEA = 0.023, SRMR = 0.017, and TLI = 0.991. The regression analysis results indicated that spatial anxiety significantly predicted mathematical skills, after controlling for general and mathematical anxiety.

Figure 10

Model 3. Path Diagram Showing the Relationship Between Spatial Anxiety and Mathematical skills



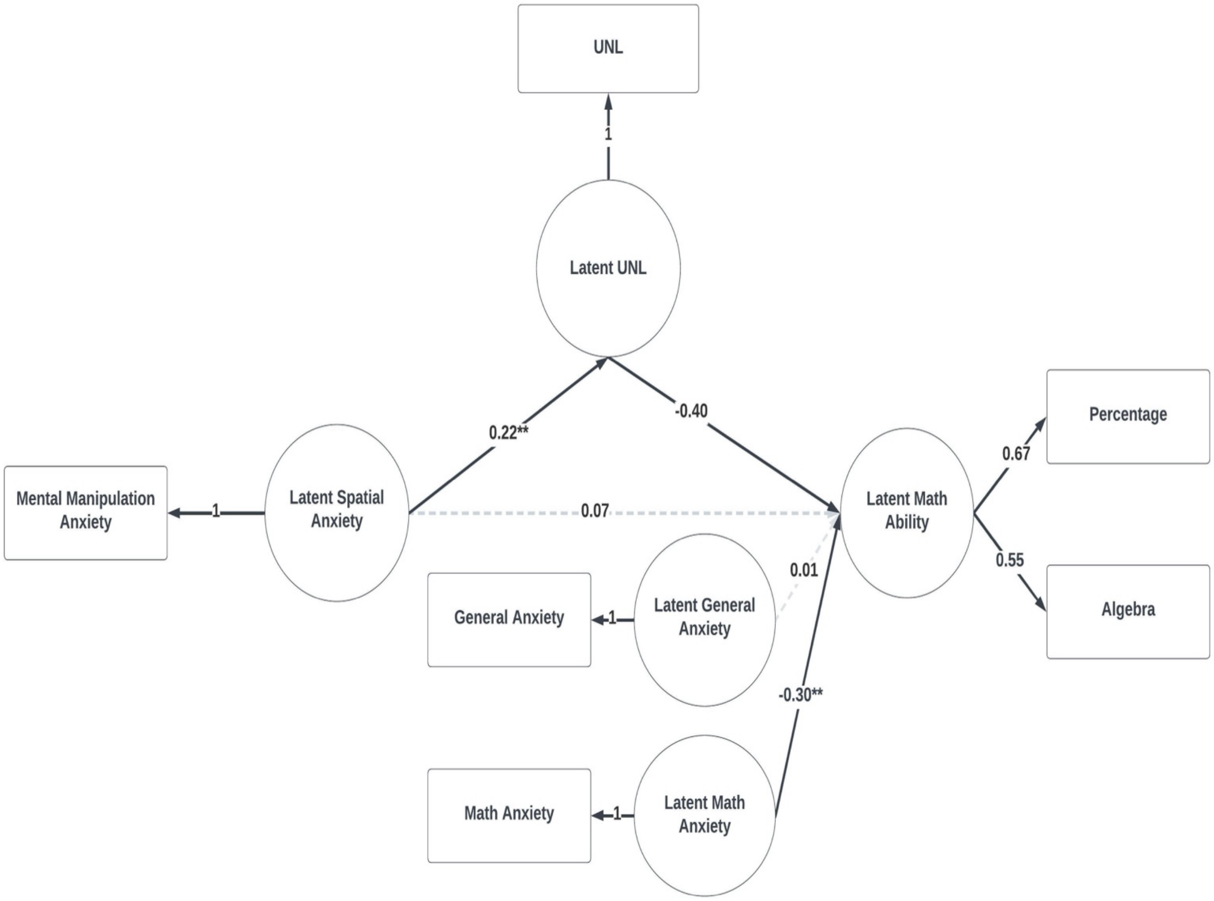
Note. The diagram above shows spatial anxiety significantly predicted mathematical skills after controlling for general and mathematical anxiety. $p < 0.05$ = ***; $p < 0.01$ = **; $p < 0.001$ = ***. Oval shapes represent latent variables, and rectangular shapes represent observed variables. Continuous dash lines indicate nonsignificant paths, and think lines indicate significant paths. Spatial anxiety significantly predicted mathematical skills.

Does MNL Mediate the Relationship Between Spatial Anxieties and Mathematical Skills?

Given the evidence from Model 3, which indicates that spatial anxiety is related to mathematical skills, the third research question examined whether MNL mediates this relationship. In Model 4, MNL was included in the regression model; therefore, mathematical skills were predicted by general, mathematical, MNL, and spatial anxiety. The regression results showed that spatial anxiety was no longer directly predictive of mathematical skills after controlling for general and mathematical anxiety. The results demonstrated that all robust fit indices were excellent: $\chi2(5, n = 161) = 5.765$, $p = 0.330$, scaling correction factor for Satorra-Bentler correction = 0.945, CFI = 0.995, robust RMSEA = 0.029, SRMR = 0.029, and TLI = 0.986. As shown in Figure 11, for the path diagram, the direct effect of MNL on the relationship between spatial and mathematical skills was not nonsignificant ($b = -0.071$, $p = 0.474$). In contrast, the indirect effect of MNL on spatial anxiety and mathematical skills was significant ($b = -0.89$, $p = 0.043$). Based on this result, it can be concluded that spatial anxiety predicts mathematical skills and MNL fully mediates this relationship.

Figure 11

Model 4. Path Diagram Showing the Relationship Between Spatial Anxiety and Mathematical skills

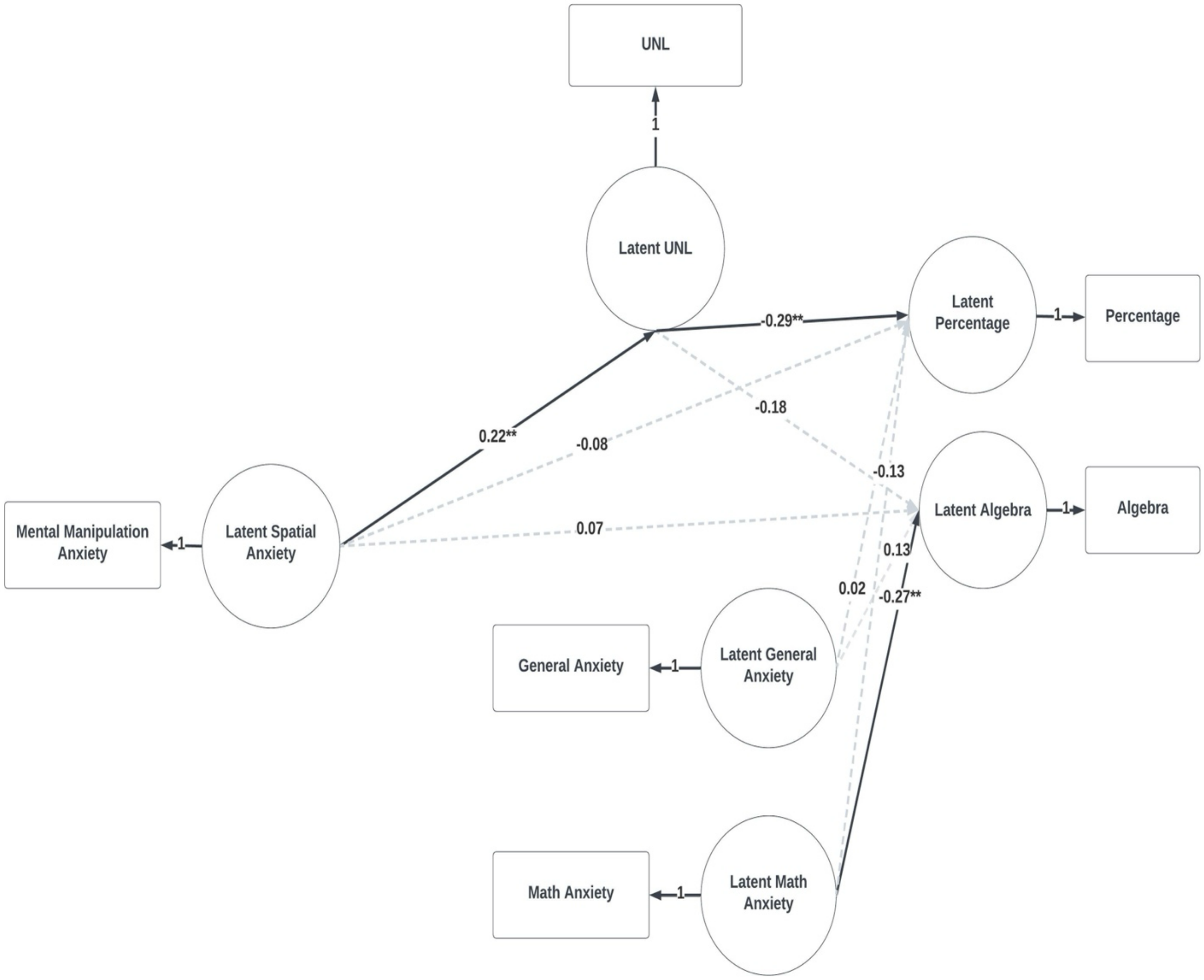


Note. The diagram above shows that Spatial anxiety significantly predicted mathematical skills. $p < 0.05$ = ***; $p < 0.01$ = **; $p < 0.001$ = ***. Oval shapes represent latent variables, and rectangular shapes represent observed variables. Continuous dash lines indicate nonsignificant paths, and think lines indicate significant paths.

Although the results of Model 4 indicated that spatial anxiety is related to mathematical skills and that MNL mediates this relationship, Model 5 examined this relationship at a more specific level. Thus, in Model 5, we separately predicted algebra and percentage scores by spatial anxiety while controlling for general and mathematical anxieties in each regression specification. Figure 12 shows the path diagram. The results from Model 5 indicated that all robust fit indices were excellent: $\chi2(2, n = 161) = 2.462$, $p = 0.292$, scaling correction factor for Satorra-Bentler correction = 0.810, CFI = 0.998, robust RMSEA = 0.033, SRMR = 0.029, and TLI = 0.982. The results showed that MNL fully mediated the relationship between spatial anxiety and percentage (indirect effect [$b = -0.065$, $p = 0.033$] and direct effect [$b = -0.078$, $p < 0.241$]). However, MNL did not significantly mediate the relationship between spatial anxiety and algebra (indirect effect [$b = -0.040$, $p = 0.093$] or direct effect [$b = 0.008$, $p < 0.931$]). This suggests that either the relation between spatial anxiety and mathematical skills is mediated by MNL only for specific mathematical skills (e.g., percentages), or that the effect of the mediation for other mathematical skills is small enough that it did not reach the threshold of statistical significance in this study.

Figure 12

Model 5. Path Diagram Showing the Relationship Between Spatial Anxiety and Mathematical skills



Note. The diagram above shows the relationship among spatial anxiety, algebra and fractions. General anxiety and math anxiety are control variables. $p < 0.05$ = ***; $p < 0.01$ = **; $p < 0.001$ = ***. Oval shapes represent latent variables, and rectangular shapes represent observed variables. Continuous dash lines indicate nonsignificant paths, and think lines indicate significant paths.

Discussion

This study investigated the relationship between spatial anxiety, spatial skills, and mathematical performance. Our findings provide important insight into these relationships and their implications. The first research question explored the association between spatial anxiety and spatial skill. The results showed that higher levels of spatial anxiety were significantly associated with poorer performance on mental rotation and surface development tasks, even after controlling for general anxiety. This suggests that spatial anxiety plays a role in predicting individual performance on these spatial tasks. Lyons et al. (2018) demonstrated that individuals with high spatial anxiety tend to exhibit poorer spatial task performance, particularly those involving mental rotation. Our results extend their findings by suggesting that spatial anxiety is not confined to specific spatial domains and may have a pervasive impact on diverse spatial situations. However, the precise directionality of these relationships remains to be determined as the current results are correlational. For example, it is plausible that individuals with deficient spatial processing skills experience heightened apprehension when confronted with spatial tasks. However, it is also conceivable that impairments in spatial anxiety may lead to disruption of the cognitive mechanisms underlying spatial representation, subsequently impeding spatial processing. Future longitudinal studies are necessary to better understand the relationship between spatial and mathematical anxiety and MNL knowledge. Nevertheless, our results have educational implications, especially in the current wave of inculcating spatial reasoning tasks and activities in classrooms.

For our second research question, we tested whether spatial skills predict mathematical performance in adults and then, as in Study One, whether the MNL mediates that relationship. Spatial skills predicted mathematical performance in our structural equation models, both when we treated math performance as a single latent variable (see Model 4) and for each math domain (see Model 5). Additionally, MNL, represented by the UNL task, mediated the relationship between spatial skills and mathematical performance (see Models 1b and 2b). Although these results replicate those of Study 1, there were some differences. First, when broken down into separate mathematical domains, there was no significant mediation with arithmetic, while UNL mediated the relationship between spatial skills and arithmetic in Study One. Second, almost all these mediations are partial mediations, as the direct path from spatial skills to mathematics was still significant even after the mediating variable was included in the model. In Study One, these relationships were fully mediated. This may be because Study One also had BNL tasks in the model, which took enough variance away from the direct effect to make it nonsignificant. It is also possible that the larger sample size in this study gave us more power to detect a direct effect. Either way, we can be confident that this pattern of mediation exists, even if we are not sure if it is partial or full mediation.

Our next research question investigated whether spatial anxiety is related to mathematical performance. Indeed, as demonstrated by the analysis, spatial anxiety relates not only to spatial skills but also to mathematical skills. Specifically, the results revealed that spatial anxiety significantly predicted performance in overall mathematical skills, even after controlling for general and mathematical anxiety. Moreover, the findings indicated that the effect of spatial anxiety on mathematical performance may vary across mathematical domains. The mediation analysis results indicated that MNL significantly mediated the relationship between spatial anxiety and mathematical performance (see Model 4), suggesting that spatial anxiety could lead to low MNL, which, in turn, would lead to low mathematical performance. Furthermore, the evidence suggests that spatial anxiety may have a detrimental impact on the development and utilization of the spatial skills necessary for successful mathematical problem-solving.

One interesting finding was the significant association between spatial anxiety, the UNL, and mathematical scores. The data suggest that the relationship between spatial anxiety and mathematics is mediated by MNL, as measured by UNL. It remains unclear whether weak MNL skills, as indicated by poor performance on UNL estimation, contribute to the development of domain-specific anxieties in spatial and mathematical skills, or whether heightened levels of apprehension in processing spatial tasks contribute to weaker MNL skills. These findings are consistent with previous research conducted by Gunderson et al. (2012), supporting the idea that robust spatial skills contribute to efficient MNL, ultimately enhancing mathematical representation. Within the framework of spatial representation, we propose that high spatial anxiety is linked to low spatial skills, leading to compromised MNL that subsequently results in lower mathematical performance. Alternatively, people with high spatial anxiety may avoid tasks involving spatial processing. In this case, high spatial anxiety may prevent individuals from practicing spatial-related tasks, thereby depriving themselves of the stimulations that could facilitate their spatial skills, and, subsequently, their MNL and mathematical skills. Nevertheless, these results further support recent claims from emerging studies, indicating that BNL and UNL knowledge have distinct developmental trajectories and that the UNL may serve as a purer measure of magnitude knowledge (Reinert et al., 2015; 2019; Cohen et al., 2020; Cohen & Blanc-Goldhammer, 2011).

Another interesting finding of the current study is that BNL did not correlate significantly with any of the mathematical measures. While the relationship between BNL and various mathematical competencies has been extensively explored in previous research, little attention has been paid to understanding how scores on the UNL relate to mathematical skills. Most studies investigating this relationship have focused on children and have primarily assessed basic arithmetic computations. For example, Jung et al. (2020) discovered a significant correlation between BNL and performance in basic arithmetic operations such as subtraction, addition, and multiplication. Conversely, they found no significant correlation between UNL scores and these same measures. Interestingly, the authors found that children employed different strategies when solving the BNL and UNL. Specifically, they claimed that the proportion strategy fitted the BNL, while the magnitude estimation-based strategy was used in the UNL. These findings suggest that proportional strategies drive the association between BNL and basic mathematical skills. Thus, BNL assessment relies on proportion estimation, which may predict basic arithmetic skills but may not necessarily reflect distinct cognitive skills assessed by BNL (Link et al., 2014).

In other research, one of the common phenomena used to indicate the existence of the MNL is the spatial–numerical association response code (SNARC) effect (Dehaene et al., 1993). The SNARC effect is an observation that, during magnitude comparison tasks, people tend to respond faster to smaller numbers on their left side and to larger numbers on the right side. Interestingly, previous studies failed to find any significant association between the SNARC effect and mathematical skills in children (Gibson et al., 2016; Schneider et al., 2009) and adults (Bonato et al., 2007; Cipora et al., 2013; Fisher et al., 2014; Toomaran et al., 2019). Similarly, Jung et al. (2020) reported that UNL scores did not correlate with arithmetic computations in children. Thus, although the SNARC effect and UNL appear to be effective representations of the MNL, paradoxically, the SNARC effect is not related to mathematical skills, just as UNL scores are non-predictive of mathematical competencies in children, as reported by these past studies. The lack of a significant correlation between UNL and mathematical skills has been interpreted as an indication that pure magnitude representation does not relate to mathematical skills.

The evidence from our study contradicts these findings by previous studies discussed above (Bonato et al., 2007; Cipora et al., 2013; Fisher et al., 2014; Toomaran et al., 2019; Gibson et al., 2016; Schneider et al., 2009). Specifically, our findings reveal that a significant predictive relationship exists between UNL and mathematical performance in adults. In addition, the areas of mathematics included in the current study have never been explored in any existing studies. We argue that specific mathematical skills, particularly those requiring higher-level processing and involving complex conceptual and procedural knowledge, such as algebra, fractions, and arithmetic, as assessed in our study, are associated with magnitude knowledge assessed by the UNL. This assertion is supported by a recent review by Cipora et al. (2020) that underscores the importance of considering factors such as task difficulty and age when investigating the relationship between UNL and BNL knowledge.

Conclusion

The relationship between spatial and mathematical skills is strong and rooted in a shared cognitive representation called MNL knowledge. Our study supports the spatial representation hypothesis but with some modifications. Specifically, we found for the first time that differences in mathematical performance could be explained by differences in spatial anxiety, even after controlling for mathematical and general anxiety. Furthermore, spatial skills and number line knowledge mediated the relationship between spatial anxiety and mathematical skills. Based on these results, we can propose that, among adults, spatial anxiety is related to poor spatial skills, and these poor skills could result in inefficient MNL skills, ultimately leading to lower performance in mathematical tasks.

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CHAPTER 5: GENERAL DISCUSSION

Introduction

This dissertation aimed to investigate the relationship between spatial and mathematical skills. Although previous research suggests that these skills are predictive of each other (Casey et al., 2015; Chen & Zhou, 2014; Frick, 2018; Mix et al., 2014; Research Council [NRC], 2006; Uttal et al., 2013; Xie et al., 2019) and recommends incorporating spatially relevant tasks in mathematical education (NRC, 2006), the underlying mechanism of this relationship remains unclear. The spatial representation theory (Gunderson et al., 2012; Tam et al., 2019) attempts to explain the link between spatial and mathematical skills, suggesting that they have a shared cognitive representation as an internal mental number line (MNL) on which numbers are mapped onto space. Accordingly, spatial skills contribute to efficient MNL representation, which supports effective numerical skill development. Three separate studies were conducted to examine this theoretical framework further. Study One investigated whether spatial representation was evident in an adult sample using a different tool for measuring the MNL. Study Two examined the relationship between spatial anxiety, spatial skills, and mathematical skills. Study Three aimed to extend Study Two by examining whether MNL mediates the relationship among spatial anxiety and mathematical and spatial skills. Although the findings from each study have been presented in their respective chapters, this chapter aims to synthesize the results of all three studies and provide a comprehensive summary.

The Unbounded Number Line, not the Bounded Number Line, Mediates the Association between Spatial and Mathematical Skills

Gunderson et al. (2012) and Tam et al. (2019) examined the mediating role of MNL among children and concluded that it was indeed a significant mediator in the relationship between spatial and mathematical skills. The results of this thesis confirm, for the first time, that the mediational role of MNL in the association of spatial and mathematical skills also exists in adults. However, unlike children, the results of Study One indicate that the unbounded number line (UNL) was the mediating variable in the relationship between spatial and mathematical skills in adults. This result is consistent with recent evidence suggesting that the UNL may be a more accurate measure of the MNL than the bounded number line (BNL), which is the mechanism underlying spatial and mathematical interconnections (Reinert et al., 2015; 2019). However, despite increasing evidence indicating that UNL is more reflective of pure magnitude representation and the underlying MNL representation, this evidence is weakened by findings that UNL measures did not predict mathematical performance in previous studies in children (Georges & Schiltz, 2021; Jung et al., 2020; Schneider et al., 2018). Some researchers have explained that the non-significant relationship between scores on the UNL and mathematical skills coincides with other evidence suggesting that nonsymbolic numerical representation tasks did not significantly predict mathematical performance (Wei et al., 2012). Nonsymbolic tasks usually involve comparing large numbers of dots in order to identify which group of dots has more dots. Like the UNL, these tasks are proposed to underlie the representation of magnitude (Dehaene et al., 1993; Gobel et al., 2011; Reinert et al., 2019), but not necessarily the MNL. However, the findings in this thesis suggest that UNL is predictive of mathematical skills, at least in adults.

Siegler and Lortie-Fortie (2014) proposed an integrative theory of magnitude development, positing that MNL is a dynamic structure that continually increases in accuracy with age and experience rather than a fixed representation of magnitudes. They further suggested that distinct aspects of MNL underlie different mathematical skills. During the early stages of MNL's representation, it is relatively imprecise and constrains the range of the magnitude representation capacity of young children. In adolescence and adulthood, the MNL becomes more precise, leading to an improvement of magnitude representation. Indeed, as Hawes et al. (2022) demonstrated, the effect of spatial training on mathematics learning was moderated by age, with an increase in age (from 3-20) corresponding to an increase in the effect of transfer of spatial training on mathematical skills. The BNL and UNL estimation tasks may assess different aspects of MNL at different stages of development. This study is the first to examine how UNL performance is related to a range of high-level mathematical skills while also coincidentally looking at these skills in adults. These findings suggest that UNL performance may be either more central in predicting higher-level mathematical skills or more representative of the MNL representation in adults than children. This result implies that BNL fails to reflect MNL once it has reached a more precise level of development. At the same time, the UNL continues to tap into the MNL 's representation, even during later stages of development, leading to a significant relationship between the UNL and higher-level mathematical skills. Therefore, while BNL estimation tasks may be limited to the range of the MNL representation that they can access, UNL estimation tasks may have an unlimited range of the MNL representation and remain relevant through adulthood.

Spatial Anxiety Predicts Mathematical Performance

The findings of the third study reveal an interesting and significant association between high spatial apprehension and low mathematical performance. Although this is a correlational relationship and does not imply causation, this observation suggests that spatial anxiety may have a detrimental effect on students' mathematical skills over and above its association with general and math anxiety. Importantly, this study identified MNL as an underlying mechanism connecting spatial skills and mathematical performance in individuals with spatial anxiety. This finding further supports the spatial representation account, which proposes that strong spatial skills contribute to enhanced MNL, ultimately leading to improved mathematical performance. MNL is a cognitive representation of numerical magnitude and is believed to be crucial in mathematical reasoning and problem-solving. The results of this study imply that individuals with high spatial anxiety may have compromised spatial skills, leading to compromised MNL and subsequently resulting in lower mathematical skills. Furthermore, this suggests that spatial anxiety may hinder the development and utilization of spatial skills necessary for successful mathematical problem-solving.

The theoretical proposal drawn from these findings is that spatial skills are closely tied to MNL, which influences mathematical performance. By extension, individuals with high spatial anxiety may face challenges in developing strong spatial skills, leading to compromised MNL and, consequently, lower mathematical skills. This conclusion aligns with the spatial representation hypothesis and highlights the importance of spatial skills in supporting mathematical reasoning and performance.

Comprehensive Remodeled Spatial Representation Framework

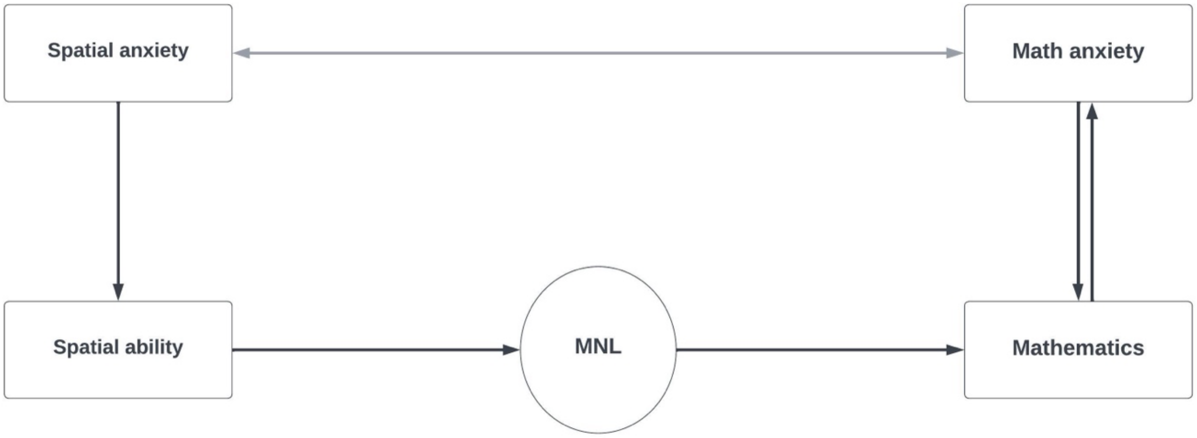
The results from all three studies in this dissertation provide support for the spatial representation theory. Gunderson et al. (2012) proposed that spatial and mathematical skills are related because they are connected by the same cognitive mechanism, the internal MNL. The findings from Study One showed that MNL, measured with the UNL estimation task, significantly mediated the relationship between spatial skills and mathematical proficiencies assessed. Studies Two and Three found that the affective aspects of spatial and mathematical skills in the form of spatial and mathematical anxieties were related to spatial and mathematical skills.

These pieces of evidence further strengthen spatial representation theory in two ways. First, the results extend spatial representation theory to an adult sample. This dissertation provides evidence that the underlying mechanism of spatial and mathematical skill associations may be MNL. To the best of our knowledge, this is the first study to replicate the mediating role of MNL on the relationship between spatial and mathematical skills. Second, this dissertation strengthened the spatial representation theory by demonstrating that spatial anxiety relates to mathematical skills. Although the literature has shown that mathematical anxiety is related to mathematical and spatial skills, suggesting that it could be a shared mechanism underlying spatial and mathematical skills, no study has examined whether spatial anxiety is related to mathematical skills (Alvarez-Vergas et al., 2020; Ferguson et al., 2015; Lyons et al., 2018; Maloney, 2011; Maloney & Beilock, 2012; Maloney et al., 2011; Zhang et al., 2019).

Through this dissertation, we have gained new insights into the role of spatial anxiety in the relationship between spatial and mathematical skills. Study Three examined the direct link between spatial anxiety, MNL, mathematical proficiency, and spatial skills. Our results showed that spatial anxiety predicts MNL and mathematical proficiency and that MNL significantly mediates the relationship between spatial anxiety and math, providing evidence for spatial representation theory. Although our study showed associations between spatial anxiety, spatial skills, and mathematical skills, the exact nature of these relationships remains unclear. It is plausible that spatial anxiety could result in deficiencies in spatial skills, which could hinder effective MNL development, resulting in lower mathematical skill development. Alternatively, it is possible that the inability to develop an effective MNL could contribute to spatial anxiety, decreased spatial skills, and, subsequently, reduced mathematical performance. Thus, the directionality of the relationships between spatial anxiety, spatial skills, and mathematical skills requires further research to determine the causal pathways involved. In light of these findings, we propose remodelling spatial representation theory to integrate spatial and mathematical anxiety related to MNL and the spatial mathematical associations (see Figure 13). This would provide a more comprehensive understanding of the complex relationships between these variables.

Figure 13

A comprehensive model of the spatial representation framework.



Note. The diagram shows how spatial skills, spatial anxiety, mathematical anxiety and mathematical skills are interrelated with the MNL.

Theoretical and Educational Implications

Previous studies have primarily focused on the connection between spatial and mathematical skills and MNL measured by BNL in children (e.g., Gunderson et al., 2012; Tam et al., 2019), but prior to this dissertation, no study had attempted to test the applicability of the spatial representation theory in adults. While understanding how spatial skills can enhance mathematical learning in the early stages is essential, it is also necessary to investigate how this relationship persists in advanced stages of learning. Recent studies have indicated that individuals with mathematical learning disabilities tend to have poor performance in spatial skill tasks (Ferguson et al., 2015) and demonstrate decreased activity in the intraparietal regions of the brain (McCaskey et al., 2020), an area also implicated in spatial processing (Catteneo et al., 2009; Hawes et al., 2019; Hubbard et al., 2003). Hence, it is critical to explore the cognitive mechanisms underlying spatial-mathematical associations in the early stages and during adulthood, as these disabilities can persist throughout life.

The results of these studies suggest that MNL training using an UNL can enhance mathematical learning in adults. Moreover, the findings shed light on the link between the UNL and math. Unlike previous studies that mainly focused on arithmetic assessment in children (e.g., Gunderson et al., 2012; Tam et al., 2019), our study is the first to establish a link between UNL scores and various mathematical domains beyond basic arithmetic. If future studies replicate this result in longitudinally designed and well-controlled studies, it will strengthen the argument behind using spatial interventions to enhance mathematics learning, especially with interventions that align with the UNL. For example, our data suggest that MNL measured by the UNL may be the mechanism connecting spatial skills to mathematics. Therefore, the next theoretical step would be to examine whether this result is stable across development and whether training people in the UNL could specifically lead to improvements in mathematics.

Moreover, while the existing evidence supports previous findings that indicate a strong association between spatial skills and mathematical skills (Hawes et al., 2022; Uttal et al., 2013) and suggests that mathematical instruction could incorporate spatial reasoning tasks to improve performance, one should acknowledge that spatial anxiety could also affect not only spatial skills but also mathematical performance. Suppose further research confirms the suggestions of the results presented here. In that case, additional interventions will need to be provided for individuals with high spatial anxiety so that any spatial intervention provided in the classroom does not negatively impact their mathematic learning.

Limitations and Future Directions

The generalizability of this study's findings is limited for several reasons. First, the sample of participants in Studies One and Two was restricted to university students enrolled in at least one psychology course during the data collection period. In Study Three, only adults with at least a high school education were recruited. Therefore, the results of this dissertation may not be representative of the general population of students engaged in mathematical learning. Furthermore, these participants may not represent a broader population of students with diverse backgrounds and experience in mathematics learning. Second, longitudinal designs will be more effective at capturing the developmental trajectories of these skills across various age groups, including children and adolescents. This could allow researchers to explore when the role of the BNL in capturing the underlying magnitude representation and, therefore, the MNL diminishes, and the role of the UNL becomes more significant. Furthermore, the sample sizes in Studies One and Two were relatively small, particularly for the structural equation model, which may limit the statistical power and generalizability of the findings. Although Study Three had a relatively larger sample size, future studies should include more extensive and more diverse samples to improve the robustness of their results and to better understand the complex relationships between spatial skills and mathematical learning.

To better understand the complex relationships between spatial skills and mathematical learning and provide more comprehensive insights into educational practices, future studies should strive to use longitudinal designs with larger and more diverse samples and employ experimental methods to establish causal relationships between variables. This would help researchers capture the developmental trajectories of these skills across diverse age groups and provide more robust evidence for educational interventions.

Conclusion

This dissertation explored the underlying mechanisms contributing to the well-established connections between spatial abilities and mathematical skills. Guided by the spatial representation theory, the results offered additional confirmation for the MNL hypothesis, positing that the mental representation of magnitude knowledge mediates the links between high-level spatial skills and mathematical proficiency. Additionally, the data indicated a correlation between elevated spatial anxiety and lower achievement in mathematics. Hence, educators incorporating spatial reasoning tasks into their teaching should be mindful of individuals with heightened apprehension towards spatial activities. Currently, the effective support for individuals with high spatial anxiety in educational settings remains unclear. Nevertheless, it is imperative to identify individuals with substantial spatial anxiety and implement interventions aimed at reducing their apprehension in the classroom. This proactive approach has the potential to assist individuals in enhancing their utilization of spatial task processing, ultimately leading to improvements in mathematical performance.

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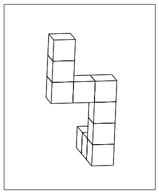
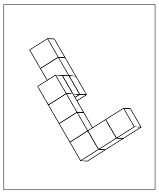
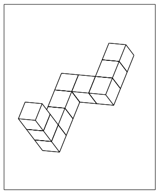
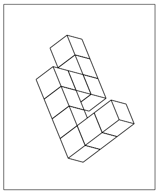
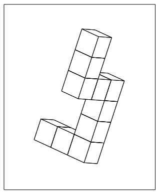
Appendix A

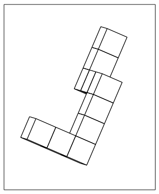
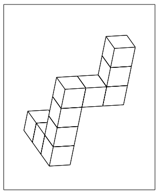
MENTAL ROTATIONS TEST (MRT-A)

This test is composed of the figures provided by Shepard and Metzler (1978), and is, essentially, an Autocad-redrawn version of the Vandenberg & Kuse MRT test.

Please look at these five figures

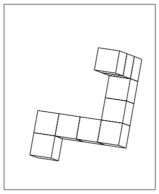
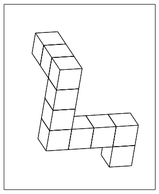
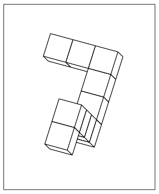
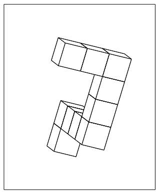
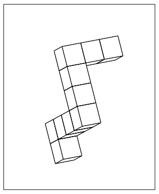
Note that these are all pictures of the same object which is shown from different angles. Try to imagine moving the object (or yourself with respect to the object), as you look from one drawing to the next.

Here are two drawings of a new figure that is different from the one shown in the first 5 drawings. Satisfy yourself that these two drawings show an object that is different and cannot be "rotated" to be identical with the object shown in the first five drawings.

Now look at
this object:
1.

Two of these four drawings show the same object.
Can you find those two? Put a big X across them.

If you marked the first and third drawings, you made the correct choice.

Appendix B

SPATIAL ANXIETY 10

Spatial Anxiety Scale

Table 3 – Spatial Anxiety Scale

Subscale	Item
M	Asked to imagine the 3-dimensional structure of a complex molecule using only a 2-dimensional picture for reference
M	Asked to determine how a series of pulleys will interact given only a 2-dimensional diagram
M	Asked to imagine and mentally rotate a 3-dimensional figure
M	Asked to imagine a 3-dimensional structure of the human brain from a 2-dimensional image
M	Asked to imagine the motion of a mechanical system given a static picture of the system
M	Imagining on a test what a 3-dimensional landscape model would look like from a different point of view
M	Asked to imagine the 3-dimensional shape created by rotating a complex 2-dimensional plane on an exam
M	Using a 3-dimensional model of an airport to complete a homework assignment
N	Finding your way to an appointment in an area of a city or town with which you are not familiar
N	Finding your way back to your hotel after becoming lost in a new city
N	Asked to follow directions to a location across town without the use of a map
N	Finding your way back to a familiar area after realizing you have made a wrong turn and become lost while driving
N	Trying to get somewhere you have never been to before in the middle of an unfamiliar city
N	Trying a new route that you think will be a shortcut without the benefit of a map
N	Asked to do the navigational planning for a long car trip
N	Memorizing routes and landmarks on a map for an upcoming exam
I	Asked to recall the shade and pattern of a person's tie you met for the first time the previous evening
I	Asked to give a detailed description of a person's face whom you've only met once
I	Asked to recall the exact details of a relative's face whom you have not seen in several years
I	Asked to recreate your favorite artist's signature from memory
I	Describing in detail the cover of a book to a bookseller because you've forgotten both the title and author of the book
I	Tested on your ability to create a drawing or painting that reproduces the details of a photograph as precisely as possible
I	Asked to imagine and describe the appearance of a radio announcer or someone you've never actually seen
I	Given a test in which you were allowed to look at and memorize a picture for a few minutes, and then given a new, similar picture and asked to point out any differences between the two pictures

Note. Table 3 gives the complete final Spatial Anxiety Scale broken into its three subscales: Mental Manipulation (M), Navigation (N), and Imagery (I). Instructions: "The items in the questionnaire below refer to situations and experiences that may cause tension, apprehension, or anxiety. For each item, mark the response that describes *how much you would be made to feel anxious by it*. Work quickly, but be sure to think about each item." Response options: 'not at all', 'a little', 'a fair amount', 'much', 'very much'. Scoring: 0 (not at all) to 4 (very much); sum scores across the 8 items for each subscale.

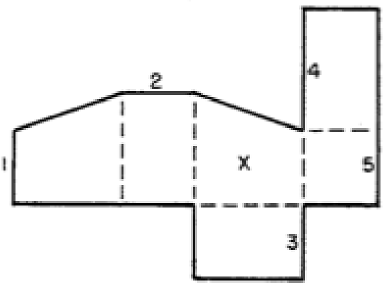
Appendix C

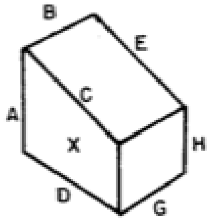
Name ______________________________

SURFACE DEVELOPMENT TEST — VZ-3

In this test you are to try to imagine or visualize how a piece of paper can be folded to form some kind of object. Look at the two drawings below. The drawing on the left is of a piece of paper which can be folded on the dotted lines to form the object drawn at the right. You are to imagine the folding and are to figure out which of the lettered edges on the object are the same as the numbered edges on the piece of paper at the left. Write the letters of the answers in the numbered spaces at the far right.

Now try the practice problem below. Numbers 1 and 4 are already correctly marked for you.





1: H
2:
3:
4: C
5:

NOTE: The side of the flat piece marked with the X will always be the same as the side of the object marked with the X. Therefore, the paper must always be folded so that the X will be on the outside of the object.

In the above problem, if the side with edge 1 is folded around to form the back of the object, then edge 1 will be the same as edge H. If the side with edge 5 is folded back, then the side with edge 4 may be folded down so that edge 4 is the same as edge C. The other answers are as follows: 2 is B; 3 is G; and 5 is H. Notice that two of the answers can be the same.

Your score on this test will be the number of correct letters minus a fraction of the number of incorrect letters. Therefore, it will not be to your advantage to guess unless you are able to eliminate one or more of the answer choices as wrong.

You will have 6 minutes for each of the two parts of this test. Each part has 2 pages. When you have finished Part 1 (pages 2 and 3), STOP. Please do not go on to Part 2 until you are asked to do so.

DO NOT TURN THIS PAGE UNTIL ASKED TO DO SO.