THE MATHEMATICAL NEEDS OF HIGH SCHOOL STUDENTS AS PERCEIVED BY MATHEMATICS INSTRUCTORS IN POST-SECONDARY INSTITUTIONS IN NEWFOUNDLAND

RANDELL MERCER

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by

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ABSTRACT

The purposes of this study were: (1) to establish the rank ordering of a set of general objectives for secondary school mathematics by trade school mathematics instructors and university mathematics instructors; (2) to determine the relative importance of each objective for the mathematics program of Newfoundland High Schools, as perceived by each group; and (3) to analyze and compare these perceptions in an effort to determine any trend in the way these objectives are perceived by each group relative to the other.

The instrument used for collecting the data was derived from a survey and an analysis of literature relating to the needs and abilities of high school mathematics students. The objectives used in the instrument, which were formulated in consultation with a group of mathematics educators, represented nine different content areas and two behavioral levels. The final form of the instrument consisted of all the possible distinct combinations, in pairs, of 18 objectives (153 pairs).

Twenty instructors from the faculty of the Department of Mathematics at Memorial University of Newfoundland, and a similar number from the mathematics staffs of the various trade schools throughout the province of Newfoundland were selected randomly. These individuals were to complete the instrument by selecting from each pair the objective which was considered more important to the secondary school

mathematics program.

The results of the data collected were analyzed using several procedures. It was found that there were areas of agreement as well as disagreement in the rankings of the objectives. On the basis of the findings of the study the following conclusions were drawn:

1. Trade school mathematics instructors indicated that the objectives dealing with applications and measurement were of the highest relative importance, while the university mathematics instructors indicated that the objectives dealing with algebra were of the highest relative importance. Both groups indicated that objectives dealing with probability and statistics were of the least relative importance.

2. There was no significant difference attached to the importance of the cognitive level of the objectives by either group; that is, both the trade school and university mathematics instructors indicated that there was no significant difference in relative importance between the objectives of high cognitive behavior and those of low cognitive behavior.

3. There was a significant interaction effect between group membership and the content area of the objectives. The trade school mathematics instructors indicated that the objectives for all content areas differed in relative importance, with the exception of those for logic and relations, geometry and graphs, and algebra and number systems. The university mathematics instructors indicated

significant differences in relative importance of the objectives for all content areas, except of those objectives for measurement, geometry, graphs and applications. Furthermore, the university mathematics instructors, when compared with the trade school mathematics instructors, attached more relative importance to the objectives dealing with geometry, graphs, algebra, relations and functions, probability and statistics, and logic. However, in the case of objectives dealing with applications and measurement the trade school mathematics instructors indicated a higher degree of relative importance than did the university mathematics instructors.

4. There was a significant inconsistency in the rankings of university mathematics instructors.

The study concluded with several implications of the results and suggestions for further research.

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THE PROBLEM

With the post-war developments in mathematics education, from the UICSM reforms (1951) through to the realizations resulting from the launching of Sputnik (1957), a stage was set for quite a dramatic change in mathematics curricula. From the 1950's through the 1960's we have witnessed the modification, modernization, and improvement of the mathematics programs in our schools. In recent months, however, the post-secondary institutions of this province have expressed considerable concern about the mathematical preparation of students who enter these institutions. It appears that there is some controversy relative to the question "What is expected of high school mathematics?" In reference to this question, two implications appear evident: (1) all students should be given the opportunity to obtain a basic set of mathematical competencies; however, (2) the mathematical preparation of students on post-secondary programs will vary.

The basic mathematical needs of every citizen in a society depend on various factors within that society. In a recent article by Edwards, et al. (1972) the authors suggest that

> Technology, the educational level of society, and occupational requirements

> are examples of factors that influence the basic mathematical competencies and skills needed by enlightened citizens (p. 677).

Although curricula can be reviewed and altered to provide each person with the basic mathematical competencies considered essential by society, this may not be sufficient. The complexity of our technological society has become such that the use of mathematics has expanded beyond its use by individuals in their private lives to become a tool which is used by increasingly more and more people in the operation of the society in which they live. Edwards, et al. (1972) suggest that

> There are ~~three~~ basic ways to view mathematics:
> 1. Mathematics as a tool for effective citizenship and personal living
> 2. Mathematics as a tool for the functioning of the technological world
> 3. Mathematics as a system in its own right (p. 672).

This being the case, then the response to the question "What is expected of high school mathematics?" would certainly be influenced by the orientation of the institution involved towards the basic ways of viewing mathematics.

There appear to be factions of people involved with mathematics teaching who feel that the reforms of the 1960's have provided the necessary content, and that sufficient mathematics can be taken from it. On the other hand, there seems to be the feeling that the mathematics being taught in our high schools is lacking, due to its dissociation from application and practice. The proponents of this line of thought seem to be in favor with a program more oriented towards the development of mathematics skills. Such views are the elements of a controversy which would have to be considered when assigning direction to a curriculum for secondary school mathematics.

Statement of the Problem

The purposes of this investigation were (1) to establish a rank ordering of a set of general objectives for secondary school mathematics by concerned groups of post-secondary mathematics instructors; (2) to determine the relative importance of each objective for the mathematics program of Newfoundland High Schools, as perceived by each group; and (3) to analyze and compare these perceptions in an effort to determine any trend in the way these objectives are perceived by each group relative to the other.

Answers were also sought for the following questions:

(i) What rankings result from applying a method of paired comparisons to 18 objectives for secondary school mathematics with trade school mathematics instructors making the choices?

(ii) What rankings result from applying a method of paired comparisons to 18 objectives for secondary school mathematics with university mathematics instructors making the choices?

(iii) How are these rankings correlated?

(iv) Do trade school mathematics instructors and university mathematics instructors agree on the cognitive level of the objectives for secondary school mathematics as to importance?

(v) Do trade school mathematics instructors and university mathematics instructors agree on the content area of the objectives for secondary school mathematics as to importance?

Need for the Study

Curriculum development has been known to have evidenced failure, in many cases, due to unsystematic or haphazard approaches. In many cases, attempts at program change may not even be classified as curriculum development. What appears to have happened is that programs have been borrowed and imposed on a particular educational setting on the merits of their success or popularity elsewhere. However, one of the basic steps in any systematic approach to curriculum development is to establish a set of aims or guidelines based on the students' needs. Oakes (1965) makes the following suggestion:

> It is recommended that as new mathematics programs are developed, primary consideration be given to the formulation of a brief list of objectives; ..... (p. 278).

The Report of the Secondary School Curriculum Committee of the NCTM (1959) points out that in any undertaking of curriculum development in mathematics, one must be guided "under the control and direction of a carefully constructed body of objectives of educational endeavor (p. 395)".

Johnson and Rising (1967) suggest that there is no clear consensus concerning the question of the product of mathematics instruction. Furthermore, this problem finds its roots in the lack of clear sets of goals for teaching mathematics.

In view of these suggestions, an effort to determine the mathematical needs of secondary school students seems in order. Subsequently, it is hoped that such a study might provide some input into future curriculum development in secondary school mathematics in this province.

Limitations of the Study

The present study does not attempt to provide sufficient evidence for making a curriculum decision. It merely attempts to provide and interpret some information relative to making such a decision.

No claim is made that the list of objectives for secondary school mathematics is exhaustive. The study will be confined to the content areas used in the NLSMA studies and the broad behavioral levels as noted on page 32.

The samples of mathematics instructors in post-secondary institutions are unbiased to the extent that they were selected randomly from the total lists of trade school and university mathematics instructors in post-secondary institutions of Newfoundland. However, no attempt is made to extrapolate the data to represent mathematics instructors outside of these particular groups.

Definition of Terms

1. Trade school mathematics instructors:
   - mathematics instructors involved in programs, at the trade and technical levels, being offered at the vocational schools throughout the province of Newfoundland, including the College of Trades and Technology, the College of Fisheries and the district vocational schools, during the academic year 1974-75.

2. University mathematics instructors:
   - the faculty members of the Mathematics Department at Memorial University of Newfoundland, who were engaged in the teaching of mathematics courses during the academic year 1974-75.

3. List of objectives:
   - a list of 18 statements of general expected outcomes of a secondary school mathematics program, which incorporate aspects of content and behavior.

4. Objectives of low cognitive behavior:
   - objectives which relate to such behavioral abilities as to know, manipulate, compute, and translate.

5. Objectives of high cognitive behavior:
   - objectives which relate to such behavioral abilities as to interpret, analyze, abstract, discover, transfer, and synthetize.

6. Content areas:
   - areas of mathematical content for secondary school mathematics classified as follows: (1) systems of numbers, (2) measurement, (3) geometry, (4) graphs, (5) algebraic expressions and sentences, and their solutions, (6) relations and functions, (7) probability and statistics, (8) logic, and (9) applications.

Overview of the Report

In this chapter the writer has attempted to provide an outline of the problem to be studied and a justification of the rationale for the study. Chapter II will be devoted to a brief review of the related literature. Chapter III contains a detailed description of the instrument used and the data collection procedures. The results of the data analysis are contained in Chapter IV. The final chapter includes a summary of the study, conclusions related to findings, some implications of the results, and suggestions for further research.

CHAPTER II

REVIEW OF THE LITERATURE

The purpose of this chapter is to discuss briefly the functions of objectives for school mathematics and to indicate certain factors which influence the composition of such objectives. In addition, an investigation into the nature of objectives for school mathematics, from an historical perspective, is summarized. This includes an examination of major reports concerning the nature of the content of mathematics courses, and the effects of these reports on the development of mathematics programs to follow.

Objectives as an Educational Issue

The literature, related to the idea of program development, quite frequently supports the notion that program development is most likely to succeed if subjected to a predetermined set of general objectives or guidelines. However, the terminology 'objectives' lends itself to several interpretations.

It is the contention of Harding (1968) that the term 'objectives' for secondary school mathematics lends itself to three interpretations, which can be referred to as three levels of specificity. One level may be classified as broad major goals. Examples of these would be such statements as: to provide understanding of the interaction between mathematics and reality; to understand that the question "Why?" is important to ask, and that in mathematics, an answer is not always supplied by merely giving a detailed proof (Buck, 1965); or to develop the talents of the individual to the greatest possible extent. These broad generalities are limited in their use as guides to curriculum development since they lend themselves to much controversy on philosophical grounds.

At the other extreme level are objectives expressed in terms of specific behaviors to be carried out by the students. These point out explicitly the specific level of observable performance expected of the student upon completion of a learning sequence (Montague and Butts, 1968). These objectives are useful in planning the sequence of a course of study, and are useful in evaluating the products of learning.

Between these two extremes of educational objectives there is a category of objectives characterized by an intermediate level of generality. It is the objectives of this level of generality that provide guidelines for program development without fragmenting the discipline into dissipated bits of knowledge or without setting the stage for a philosophical controversy. Harding (1968) points out that

> Such objectives provide an organizational framework within which to identify the behavioral characteristics desired. Some

> attention seems to be required at this intermediate level of generality to the objectives of secondary school mathematics. A general framework must be established within which to set specific goals of instruction (p. 6).

The idea of stating a set of general objectives, as an initial step in any attempt at curriculum development, is an issue whose existence can be traced well back into this century. Reeve (1924) stated that

> ...... a clear statement of the general and specific objectives of every phase of school work is the first step toward the achievement of worthwhile results (p. 192).

Franklin Bobbit (Oakes, 1965) suggested that

> ...... the major task of curriculum making is the discovery of the goals in a general way and the planning of the general outlines of the routes (p. 5).

In his examination of the literature on objectives for secondary school mathematics from 1920-1940, Oakes (1965) concluded that the starting point of mathematics curriculum development is a development of a list of objectives.

Johnson and Rising (1967) have suggested that, in addition to the problem of deciding how and by whom the content should be determined, a major unresolved problem in mathematics is the lack of clear goals for mathematics teaching.

Taba (1962) stated that the task of formulating educational objectives is twofold. First, there is the task of determining a set of goals (or aims) for the program. The second task is to define or determine the context in which to achieve these aims and the specific levels of attainment that are required. Furthermore, she points out that the establishment of a rational and operational basis of objectives results only to the extent that the specific objectives are consistently related to the aims of the program.

To relate this line of reasoning to general objectives and their relevance to mathematics education, Greenberg (1974) reiterated

> Yet there can be no meaningful application of behavioral objectives without prior decisions regarding the goals of mathematics education and, more narrowly, what Allendoefer, (1971) calls the 'general objectives', that is, what topics to teach (p. 640).

Presently, there is very little agreement as to what the general objectives of mathematics education are. During June 1973, the National Science Foundation held a conference of nearly fifty top mathematics educators at Snowmass, Colorado (Greenberg, 1974). The central issue at this conference was the need for a national study group to deal with the formulation of the general objectives of mathematics education necessary for the general education of 'everyman'. For such a gathering to concern itself with the need for the formulation of a list of objectives for mathematics education, would seem to warrant that some consideration be given to the contention made by <u>The Report of the Secondary School Curriculum Committee</u> of the National Council of Teachers of Mathematics (NCTM, 1959) that

> Many efforts to formulate such a list of objectives produce unsatisfactory results because the nature of the task is not clearly

> understood. Some persons view the task primarily as one of securing a list of subject matter concepts and abilities. ......Other persons view the task as being primarily one of indicating certain types of desirable behavior. .....
> While each of these points of view can lead to identification of essential elements of a valid set of objectives, it can do no more than provide an incomplete guide to effective instruction. The task of setting up a truly significant list of objectives involves not only the specification of behavioral elements but also the specification of subject-matter concepts and abilities and the establishment of relations between the two sets of elements (p. 395).

In reference to an article on needed research concerning mathematics curriculum (Romberg & DeVault, 1967-68), F.J. Crosswhite emphasized again that the present curriculum suffers from a lack of a discernible set of goals. Furthermore, survey techniques should be applied to adult populations (which conceivably, would include post-secondary teachers) in an effort to provide a platform for establishing general objectives relevant to the students' future mathematical needs.

To summarize the underlying rationale for the statement of a list of objectives as a preliminary step in program development, Allendoerfer (1971) quite appropriately stated:

> It is a general principle of rational behavior that no one should start an activity in any field of human endeavor until he has thought through just what he wishes to accomplish. Indeed, some of the great follies of our time have been perpetuated by those who act just for the sake of action, with no thought

> of their objectives. Thus, in every aspect of teaching and curriculum development, we should begin by stating our general objectives (p. 168).

Factors Influencing the Formulation of Objectives

There is also the question 'what factors influence the nature of the content of these general objectives?'. Mueller (1967) suggested that a common feature of recent courses in mathematics is that they present content in a way which is pleasing to the mathematician and not necessarily appropriate for the majority of students.

Weiss (1967) urged that the planners of mathematics curriculum should relate to the needs of industry and citizenship. However, he noted that the problem of determining who makes the decisions concerning the content of the mathematics curriculum is a difficult one. Greenberg (1974) took somewhat of a democratic view in suggesting that the direction of curriculum development must be determined by the goals and values of society as a whole, and not by the content specialists and experts in learning theory. Along the same note, Marshall Stone (1961) suggested that the kinds of mathematics available through school curriculum may, out of necessity, be the dictates of modern technology.

However, if high school mathematics is to be considered as a link in a person's total educational sequence, one must consider Ferguson's (1970) conjecture that

> ...... the high school must meet the boundary conditions placed upon it by the various colleges and industries that receive the products of the high school (p. 383).

To be more explicit there is a need to examine the mathematical expectations, as perceived by the personnel in the post-secondary institutions, of those students that feed into them from the high schools.

This appears to be a reoccurrence of the line of thought put forth by Carpenter (1949). He pointed out that we not only need to analyze the mathematical needs of the college preparatory student but also the mathematical needs of those students who enter industry and the commercial fields. He stated:

> ...... another serious difficulty, which prevents any real reorganization in mathematics curriculum from taking place, is our failure to recognize worthwhile objectives of mathematics instruction other than satisfying a college entrance requirement or as a prerequisite for more mathematics (p. 43).

In summary, one could conclude that the need for a statement of objectives is an important step in curriculum development. An examination of expected outcomes, and the variety and variability of such outcomes enable educators to develop a broad conception of the objectives of mathematics education to the extent that these objectives do not become static, and are accountable for the needs of all students.

An Historical Review of the Nature of Objectives for School Mathematics

In a study concerned with objectives for mathematics education in the United States for the period 1920-1960, Oakes (1965) suggested that the aims of mathematics education changed over time, and that such change may be related to the cultural environment or societal conditions of the time. He further implied that at various times throughout this century, various professional and governmental groups, concerned with mathematics education, undertook the effort to formulate reports for the purpose of mathematics educational reform to meet the demands of the time. Following the publication of such reports, there was a period when educators responded to and even expanded on the objectives that were set forth.

Objectives for mathematics in secondary school education -- 1920 to 1940

The first of such reports, The Reorganization of Mathematics in Secondary Education, was published by the National Committee on Mathematical Requirements in 1923 (Harding, 1968). The aims of mathematics education, as put forth in this report, were considered in reference to three categories: (1) practical aims, (2) disciplinary aims, and (3) cultural aims.

Practical aims were oriented towards the knowledge and understanding of the content matter. These included such things as (1) a command of the fundamental processes and laws of arithmetic and algebra, (2) interpreting graphic representation, and (3) a familarity with geometric forms.

Disciplinary aims were related mainly to one's utilization of his thought processes. Such aims were as follows: (1) the ability to recognize any relevant information and to disregard irrelevant information, (2) the acquisition of mental habits and attitudes, and (3) the acquisition of the idea of relationship or dependence (the function concept).

Cultural aims were more abstract and mainly concerned with the development of appreciations and ideals, such as (1) appreciation of the beauty in geometric forms, (2) ideals of perfection as in precision of statement and thought, and (3) appreciation of the power of mathematics (Oakes, 1966).

This report became the basis for discussions concerning mathematics objectives until about 1940 -- articles were written in efforts to formulate objectives of these types or to specify, in a more detailed manner, the nature of these aims.

Langley (1930) suggested four reasons for teaching geometry: (1) logical thinking, (2) knowledge of geometric facts and relations, (3) acquisition of measurement formulae and methods, and (4) cultivation of space perception. These objectives were more oriented toward the disciplinary aims.

Breslich (1932) expressed the consensus that the development of the various characteristics of functional thinking may be set up as an objective to be attained. There appeared to be, at this time in the development of mathematics education, much emphasis expressed in favor of developing the power of reasoning or method of thought. Mossman (1938) replaced the goal of teaching mathematics as a tool, and instead related the notion of the role of mathematics in the development of one's power of thought.

Hassler and Smith (1930) emphasized two objectives in teaching algebra: (1) the ability to think and (2) the appreciation of the role of mathematics in the development of civilization. They also stressed, as an overriding objective in mathematics, the concept of function.

During the period from 1920-1940, there appeared to be agreement, in the writings of specialists in mathematics education, concerning the following objectives: (1) the ability to reason, (2) increased development of computational skills, and (3) understanding through application (Oakes, 1965). Oakes also pointed out that there was increasing emphasis on the importance of the concept of function as a unifying notion in mathematics.

Although there appeared to be some commonality in the objectives put forth by mathematics educators during this period, there is some evidence to indicate certain discrepencies between the perceptions of teachers and those of the educators. Fawcett (1938) reported that although the objectives expressed by the educators emphasized the ability to understand and reason, the actual classroom situation resorted to rote memorization. Shibli (1932) reported similar evidence in that his study showed that teachers placed little emphasis on the process of deductive thinking, an objective emphasized by the educators.

To summarize briefly the change that had occurred, in reference to the nature of objectives of school mathematics up to the 1940's, would be to say that there was a move from the theory of mental discipline, as put forth by Thorndike, to teaching for transfer (Betz, 1949). Thorndike's notion of mental discipline, which was prominent at the turn of this century, basically implied that mere exposure to the subject matter automatically resulted in increased development of one's mental powers, a notion which underlay the teaching of Latin even into the 1960's. The National Committee of 1923 began to break away from this idea and began movements aimed at improving the teaching of mathematics through emphasizing the development of one's power to reason. This frame of reference was maintained in the development of objectives for school mathematics during the 1920's and 1930's (Breslich, 1949), and led to the notion of teaching for transfer which was recognized as a central objective of education by the 1940's (Betz, 1949).

Objectives for mathematics in secondary school education -- 1940 to 1955

In 1940, two reports on mathematics education were published which appeared to have a definite impact on the development of objectives for school mathematics in the years to follow. The first of these reports was published by the Joint Commission of The Mathematical Association of America and The National Council of Teachers of Mathematics. This report (NCTM, 1940) favored the idea that the development of educational objectives was an important first step in program development or improvement. The opinion was expressed that such objectives were of two distinct types: (1) objectives were factual; for example, those objectives concerned mainly with facts, skills, organized knowledge, accurate concepts, etc.; and (2) objectives were psychological, in the sense that they related to the student's individual mode of behavior; for example, those objectives concerned with work habits, attitudes, interests, modes of thought, appreciations, etc. Furthermore, this Joint Commission was of the opinion that the objectives of a general program for secondary school mathematics should encompass the following content areas: (1) number and computation; (2) geometric form and space perception; (3) graphic representation; (4) elementary analysis; (5) logical thinking; (6) relational thinking, and (7) symbolic representation and thinking. With reference to both the factual and psychological needs, the Joint Commission constructed a set of objectives to encompass the whole of the content areas mentioned above. These objectives included the following: (1) the ability to think clearly, (2) the ability to use information, concepts, and general principles, (3) the ability to use fundamental skills, (4) the development of desirable attitudes, and (5) the development of interests and appreciations. They also warned against the over-emphasis on the function concept because there are important and interesting parts of mathematics which do not relate to it.

Oakes (1965) points out that during the same year that the Joint Commission Report was published, the Progressive Education Association also published a report. However, this report stressed the fulfillment of the students' needs through mathematics rather than being oriented toward the subject matter itself. The major objectives of this report were: (1) to meet the needs of the individual, (2) to foster the development of democracy as a way of life, and (3) to achieve the development of personality in a manner consistent with democratic living. This group was of essentially the same opinion concerning the function concept as the Joint Commission. However, it was not primarily concerned with the mathematics per se, but rather with the utilization of those mathematical applications which would prove fruitful in the development of the individual. Consequently, program development was oriented towards those concepts involved in problem solving such as formulating the problem, understanding approximation, understanding concepts basic to operations, etc.

The reports of 1940 apparently had set the stage for a struggle between the child-centered and subject-centered enthusiasts. However, efforts were being made to balance these two ideas in the construction of texts. To comply with this notion, Butler and Wren (1951) suggested that the objectives of secondary mathematics include the following:

1. proficiency in fundamental skills
2. comprehension of basic facts
3. appreciation of significant objectives
4. development of desirable attitudes
5. efficiency in making sound applications
6. confidence in making intelligent and independent interpretations (p. 16).

With the advent of post-war reports on secondary education, the concept of tracking became a serious issue and resulted in certain implications in reference to objectives for school mathematics. Nevertheless, the underlying motivation for the tracking concept, that is, the varying needs and abilities of the students, was by no means a new idea. Reeve (1924) called for a "re-organization to meet the varying needs of pupils (p. 452)".

The reports of the Commission on Post-War Plans appeared to realize that curriculum development could no longer continue on the assumption that our schools were dealing with an homogeneous population. They rationalized that the needs of students varied amongst individuals, and that these differences required different school mathematics programs. So, basically they attempted to endorse the tracking concept. Furthermore, these reports indicated a reaction to the concept of specialization (Oakes, 1965), and consequently designed, as one of its tracks, a general mathematics program (Butler & Wren, 1970) which aimed

> ...to develop the abilities, attitudes, understanding and behavior patterns which should be the common experience of all educable men and women (p. 30).

The basic mathematics for this general mathematics program, according to these authors, were to include

> ...some fundamental knowledge of the nature of proof; the basic concepts of the structure of our number system, algebraic and geometric structures; the nature of measurement; the concepts of relation and function; and basic statistical measures (p. 31).

The notion of mathematics in general education, which is evident in the 1923 report of the National Committee and the Joint Commission report of 1940, and more overtly supported by the Commission on Post-War Plans, emphasized a greater variety of topics and more immediate applications. Furthermore, the difference between the traditional course and the general mathematics course was not so much in the basic subject matter as in the point of view and method of treatment, as expressed by Butler and Wren (1960).

> The appropriate development of relevant basic mathematical skills, concepts, and principles will be the first responsibility of teachers regardless of track. The differentiation will come in the interpretation, supplementation, and enrichment of this development. For one track this aspect of the program will be oriented in the context of the user of mathematics; in the other it will assume the point of view of a forward look toward more advanced work in mathematics (p. 47).

Oakes (1965) summarizes the nature of and attitude toward objectives during the 1940-55 era in the following manner:

> In the period following the two reports of 1940, specialists in mathematics education attempted to rationalize the opposing demands of the child-centered and subject-centered advocates. Recommendations for double-track programs, special courses, and attention to the needs of both extremes of the ability scale reflected the desire to give more attention to the needs of the students. On the other hand, the emphasis placed upon the acquisition of skills and basic facts indicated recognition of the necessity of developing competence in the specific areas. In agreement with the point of view of the fifteenth yearbook there was a tendency to attempt to express objectives in terms of expected behavioral changes. The influence to the National Committee Report is strongly evidenced by constant reference to the practical, disciplinary, and cultural objectives. Still present, also, was the emphasis upon the training in thinking to be gotten from the mathematics courses with special emphasis upon geometry. Possibly in response to the increasing influence of those who advocated emphasis upon generalization of ideas and concepts rather than upon the isolation of identical elements, the lists of stated objectives were becoming more abbreviated and there was a greater concern with the whole problem of stated objectives (p. 95-96).

Objectives for mathematics in secondary school education -- post 1955

The big issue that was prominent during the post 1955 years of school mathematics reform dealt with the understanding of concepts and structure. The first effort at curriculum development of this nature was undertaken at the University of Illinois, and is commonly referred to as the UICSM project. In the development of its program, the group concerned, carried out the process in the light of three premises: (1) a consistent rather than a disjointed exposition of high school mathematics leads to a better understanding of the subject matter; (2) high school students have a profound interest in mathematics; and (3) manipulative skills are necessary for the purpose of concept development. This last notion is analogous to the present controversial issue that an exposure to a rich source of concepts appears to be more beneficial than an exposure to manipulative tasks, even to the point where the former is inclusive of the latter (UICSM Project Staff, 1957).

The above line of thought is also evident in the report published by the Commission on Mathematics in 1959. This report emphasized the point that, although the manipulative skills have an important place in mathematics programs, it takes second place to understanding the underlying ideas and concepts. Furthermore, with the advent of this report a new attitude towards objectives seemed to be emphasized -'objectives were no longer used to sell the cause of mathematics, and consequently, became increasingly concerned with the development of mathematical ability. The Report of the Commission on Mathematics (1959), suggested a program of the following nature:

> College preparatory math should include topics selected from algebra, geometry (demonstrative and co-ordinate), and trigonometry - all broadly interpreted. The point of view should be in harmony with contemporary mathematical thought; emphasis should be placed upon basic concepts and skills, and upon the principles of deductive reasoning regardless of the branch of mathematics from which the topic is chosen. In every case, the standard of substance and content should be commensurate with that of the course outlined in Chapter 4. Courses designed for other purposes (e.g. consumer math., business math., shop mathematics) are not acceptable (p. 60-61).

The objectives of the course described in this report were listed as the following:

1. Strong preparation, both in concepts and skills, for college mathematics at the level of calculus and analytic geometry.
2. Understanding of the nature and role of deductive reasoning - in algebra, as well as in goemetry.
3. Appreciation of mathematical structure ('patterns') - for example, properties of natural, rational, real, and complex numbers.
4. Judicious use of unifying ideas - sets, variables, functions, and relations.
5. Treatment of inequalities along with equations.
6. Incorporation with plane geometry of some co-ordinate geometry, and essentials of solid geometry and space preception.
7. Introduction in grade 11 of fundamental trigonometry - centered on co-ordinates, vectors, and complex numbers.

8. Emphasis in grade 12 on elementary functions (polynomial, exponential, circular).
9. Recommendation of additional alternative units for grade 12: either introductory probability with statistical applications, or an introduction to modern algebra.

Following the repercussions caused by the struggle for world technological supremacy of the late 1950's, various groups were organized to implement reform in the school mathematics programs. One group, the SMSG (School Mathematics Study Group), in taking on such a role, pointed out that because of the increasing inability to predict, even to the scope of the very near future, there was a need to teach mathematics to enable the student to learn more mathematics later. In light of this philosophy, the major objectives for mathematics by this group, which is reminiscent of the Commission on Mathematics Report, were (1) to understand the nature of mathematics and (2) to understand the processes of mathematics (Harding, 1968).

During the summer of 1963 a group of mathematicians and mathematics users met to propose a school mathematics program for the future. The result of this meeting was the Report of the Cambridge Conference on School Mathematics, which presented the following proposals: (1) a downward compression of content; (2) elimination of drill; (3) early treatment of probability and statistics, and notions of calculus; and (4) the development of one's analytic powers.

Again the emphasis on understanding comes through distinctively.

One of the more prominent aspects of mathematics education in the post-1955 era has been the attention given to structure (Rosskopf, 1957), and to the knowledge and understanding of major mathematical concepts, a point which is emphasized in the twenty-fourth yearbook of the National Council of Teachers of Mathematics. Furthermore, Oakes (1965) maintains that developments, in school mathematics reform of this more recent era, reflect the suggestions put forth by the Commission on Mathematics in reference to objectives for program development.

Summary

Objectives for school mathematics have been a feature of mathematics reform throughout this century. However, such reforms and the objectives for school mathematics associated with them have been related to the societal conditions of that particular era. Consequently, the emphasis in school mathematics has shifted from the development of one's power to reason (1920 to 1940 era) to the appropriate development of basic mathematical skills and competencies (1940 to 1955 era) to the understanding of the structure and major concepts of mathematics (post-1955 era).

The need for objectives for secondary school mathematics appears to be affirmative. It is the opinion of many educators (Reeve, 1924; Taba, 1962; Oakes, 1965; Allendoerfer, 1971; Greenberg, 1972) that the statement of objectives for secondary school mathematics should be the initial step in program development. More narrowly, Harding (1968) indicated that objectives of an intermediate level of generality are more beneficial in attempting to establish a framework for mathematics instruction. Furthermore, it has been suggested (Carpenter, 1949; Ferguson, 1970) that one must examine the various boundary requirements of the post-secondary institutions as a prerequisite to program development. It is the purpose of this investigation to explore this idea by examining the mathematical outcomes of secondary education as perceived by personnel who are involved in the teaching of mathematics in various post-secondary institutions.

CHAPTER III

DESIGN OF THE STUDY

This study was proposed to answer questions related to the perceptions of post-secondary mathematics instructors regarding objectives for the secondary school mathematics program. In order to answer such questions an instrument was constructed which consisted of a list of objectives for secondary school mathematics. The instrument was used to obtain information from selected trade school and university mathematics instructors in various post-secondary institutions throughout the province of Newfoundland.

This chapter gives a description of how the list of objectives was formulated, how the samples were selected, and how the instruments were administered.

Choosing the Objectives

The list of objectives used in this study and the framework for its construction were the results of a survey and analysis of the literature pertinent to the needs and abilities of secondary school students in mathematics. Particular reference was made to writings by such prominent groups as the NCTM and the SMSG, to such studies as those conducted by NLSMA and the International Study of Achievement in Mathematics, and to a doctoral dissertation by D.E. Bolivar.

The objectives were not direct reprints from one source, but rather were paraphrases of statements from different sources. Furthermore, Krathwohl (1965) suggested that there are at least three levels of detail for expressing objectives which he classified as follows: (1) the most general level, (2) the intermediate level, and (3) the most specific level. Of these three levels of detail he indicated that the intermediate level of specificity provides better results when seeking agreement on a curriculum. Consequently, this level for expressing objectives was sought in this study.

Several problems were encountered in the process of constructing the initial list of objectives. It was attempted to make the list comprehensive although the nature of the study restricted its practical length. Furthermore, an effort was made to preserve meanings and avoid, as much as possible, ambiguity at the intermediate level of specificity of the objectives. Nevertheless, whether a list of objectives of this nature is comprehensive is always subject to controversy since different levels of meanings are attached to the statement of an objective by different individuals, or even by a single individual, in different contexts. In consideration of these factors the objectives were formulated on the basis of the following dimensions: (1) the content area of the objective, and (2) the behavioral level of the objective.

<u>The content area of the objective</u>. There was an attempt to make the list comprehensive in reference to content area. This attempt was based on the agreement, found in the NLSMA studies, that there are a number of basic content areas with which students should be familiar upon completion of school mathematics programs (Romberg & Wilson, 1968). These content areas were classified as follows: (1) systems of numbers, (2) measurement, (3) geometry, (4) coordinate systems and graphs, (5) algebraic sentences and their solutions, (6) algebraic expressions, (7) relations and functions, (8) probability and statistics, (9) logic, and (10) applications. Such a classification of mathematical content by a group which has, since 1962, been primarily concerned with mathematics achievement in our schools warrants its appropriateness as a guide for the development of a list of objectives for school mathematics.

For purposes of this study the ten content areas listed above were reduced to nine by combining the areas of algebraic sentences and their solutions, and algebraic expressions. Such a move resulted from the use of objectives, formulated from the analysis of the literature, which were inclusive of both of these content areas.

<u>The behavioral level of the objective</u>. In addition to the classification of the objectives by content area, they were also classified into two broad categories of behavior. These were defined as (1) low cognitive behavior,

which included the abilities to know, manipulate, compute and translate, and (2) high cognitive behavior, which included the abilities to interpret, analyze, transfer and synthetize.

A problem encountered, relating to the behavioral level of the objectives, involved the mutual exclusion of the objectives at the intermediate level of specificity. For example, an objective requiring the high behavioral ability to analyze might be inclusive of the low behavioral abilities to know and to compute. Consequently, the use of some low behavioral abilities were unavoidably implied in the objectives requiring high behavioral abilities.

The original survey of the literature, pertinent to the aims or objectives of school mathematics, resulted in a list of 35 objectives. After examination of the list by a group of mathematics educators at Memorial University, it was suggested that the list be refined with reference to two criteria:

(1) the objectives should be explicit in the sense that the reader will be limited in his interpretation of them. Although such an effort would prove to be virtually impossible since the wording of such statements is subject to different interpretations by different people, an attempt was made to limit the degree of interpretability as much as possible.

(2) the objectives should be distinct with reference to content area and behavioral level.

The latter criterion did not imply that the behavior was to be a specific performance on the part of the student. Rather, due to the generality of each item, the attainment of such an objective (or aim) would require the performance of a number of specific behaviors. Furthermore, the nature of the study was not to explore specific performances, but rather to provide a scheme whereby one could categorize the items involved, with reference to cognitive requirements. Hopefully, this would then provide a basis upon which to examine any differences that might arise in the types of objectives preferred by each group.

The initial list of 35 objectives was subjected to careful study in an effort to eliminate any repetitions and ambiguities that existed and to make combinations where sufficient overlap warranted it. This resulted in a list of objectives organized as follows: two objectives representative of each content area, one of which was classified as low cognitive while the other was high cognitive. Such a scheme yielded a total list of 18 objectives.

The list of 18 objectives was resubmitted to the committee for validation. They were asked to examine the objectives on the basis of the previously stated criteria. After this examination of the objectives, the committee came together as a group to pool their comments, and make the necessary suggestions. The committee felt that there was no change required in the nature of the objectives, and the changes that were suggested related to grammatical composition which might disguise the true meaning of the statement.

It should be reiterated that this list of objectives for school mathematics was the result of what the literature implied to be representative of appropriate aims to be attained upon completion of school mathematics programs. There was no claim that this list of objectives is by any means exhaustive. The only claim made was that this list is representative of such sources as mentioned above, whose credentials and opinions warrant little dispute.

The final list of objectives is presented below:

System of Numbers

1. To acquire the basic computational skills related to the real number system and the subsets thereof, including various algorithms associated with these numbers.
2. To be able to achieve economy in computations by making use of one's understanding of the structure and operations of the real number system.

Measurement

3. To develop a facility for measurement, with respect to determining length, area, volume, etc., and to the terminology

and relations of various measurement systems.

4. To develop an understanding of the nature of measurement, relative to the notions of precision, accuracy, and estimation, and their effects in interpreting the meaning of a solution to a problem.

Geometry

5. To be able to apply the properties of geometric figures, such as similarity, congruency, the Pythagorean theorem, etc. in the solution of a problem.

6. To develop an understanding of the structure of geometry, which includes the basic assumptions upon which geometry is built and how geometric facts and relations can be generated from these assumptions.

Graphs

7. To be able to take a set of data, tabulate it, and present it in meaningful graphical form.

8. To be able to analyze and interpret data, as presented in graphs and tables, and to draw inferences relevant to the solution of the problem under consideration.

Algebraic Expressions and Sentences, and their Solutions

9. To develop elementary skills in algebraic manipulations, including the solution of inequalities and linear, quadratic, simultaneous, polynomial, logarithmic and exponential sentences, and the use of algebraic algorithms.

10. To be able to analyze and select the appropriate algebraic processes in problem solving.

Relations and Functions

11. To be able to represent the relationship between two sets of numbers by using co-ordinate graphs, tables, algebraic or trigonometric sentences.

12. To be able to recognize the concept of function as a relevant and unifying notion throughout the mathematical knowledge that one has acquired.

Probability and Statistics

13. To develop the ability to apply basic concepts and principles of probability and statistics.

14. To develop the ability to interpret statistical data for the purpose of

making inferences or drawing conclusions.

Logic

15. To acquire the ability to follow proofs by comprehending the sequence of the premises and conclusions involved.

16. To be able to carry through a consistent argument to a valid conclusion.

Applications

17. To acquire a familiarity with the applications of mathematics to the fields of the physical sciences, industry and technology, and consumerism.

18. To be able to select from his mathematical knowledge the necessary mathematics which can be applied to a specific real life situation.

The Instruments

Each of the 18 objectives was paired with each of the other objectives to produce a total of 153 possible distinct pairs. Each of these pairs was assigned a code number and was printed on a 3" x 8" strip of paper, which varied in color depending on the group to which it was assigned. These two procedures were attempts to reduce time consumption in the transfer of information from each instrument to the recording sheets.

The 153 pairs of objectives were arranged according to a set of procedures derived from suggestions made by Torgenson (1958) in an effort to counterbalance

> changes in performance due to fatigue or practice effects, or for judgement based in part on factors other than the relative magnitudes of the discriminal processes (p. 167).

These procedures included the following: (1) each objective appeared first in one-half the pairs of which it was a member; (2) five sets of the 153 objectives were arranged randomly using a table of random numbers; (3) these random sets of objectives were assigned to one-half the members of each group so that only two members of each group received a set of objectives arranged in the same order; and (4) the order of presentation of the pairs of objectives was reversed for the other half of the respondents. Each set of objectives was then stapled at one end, forming a booklet.

Population and Samples

This study involved the mathematics instructors who were teaching in various post-secondary institutions in the province of Newfoundland during the academic year 1974-75.

Group I was classified as university mathematics instructors. The total list of 39 mathematics instructors was obtained from the Mathematics Department of Memorial University. The names on this list were arranged in alphabetical order, and a random sample of 20 subjects were chosen from a table of random numbers using a procedure described by Glass and Stanley (1970).

Group II was classified as trade school mathematics instructors. Again, the total list of 39 trade school mathematics instructors was obtained from the principals of the district vocational schools in cooperation with the Vocational Education Division of the Provincial Department of Education, the registrar of the College of Trades and Technology, and the Academic Department Head at the College of Fisheries. A sample of 20 subjects was selected from this population, using the same techniques as was used with Group I.

The Administration of the Instruments

In consideration of the small size of the samples (20 subjects each), and an estimation of 40% to 60% return of mail surveys (Kerlinger, 1964), it was decided to administer the instruments personally. The administration of the instruments was accompanied by a set of instructions (see Appendix B). The booklets of the pairs of objectives were delivered personally to the individuals involved, from April 18, 1975 to May 16, 1975. The task of each respondent was to select from each pair of objectives the more relevant or important one for high school mathematics by indicating X in the space provided to the left of that objective. A choice had to be made for each pair. The respondents were given a minimum of one day to complete the booklet, after which time the instruments were collected personally.

Analysis

Analysis of the group frequencies and of the individual frequencies for each objective was carried out on the responses given in each instrument. Based on the two sets of group frequencies, a correlation coefficient (r) was determined to examine the relationship between the responses of both groups. However, a correlation of the frequencies for each objective by each subject of the overall samples was also carried out. The 37 x 37 correlational matrix, resulting from this procedure, enabled not only a comparison of the rankings by the two groups, but also a comparison of the within-group rankings of each group. To be specific, Quadrant 1 (TXT) of Figure 1 (relative to correlational matrix) provided information concerning the correlation of rankings within the group of trade school mathematics instructors, while Quadrant 4 (UXU) provided a similar type of information with regards to the university mathematics instructors. Quadrant 2 (TXU) and Quadrant 3 (UXT) provided correlations between the two groups. The analytic procedure used compared the two sets of within-group correlations (r_w) with each other as well as with the between-group correlations (r_b) by using random samples of 'r' from the respective quadrants of Figure 1.

	1 2 3 … 20	21 22 … 37
1 2 3 … 20	Rankings by Trade School Mathematics Instructors correlated with each other (TXT)	Rankings by Trade School Mathematics Instructors correlated with the rankings by University Mathematics Instructors (TXU)
21 22 … 37	Rankings by University Mathematics Instructors correlated with the rankings by Trade School Mathematics Instructors (UXT)	Rankings by University Mathematics Instructors correlated with each other (UXU)

Figure 1. Model of the Correlational Matrix for Individual Rankings of the Objectives (Pearson Product-Moment Correlation Coefficients)

N.B. Variables 1 to 20 represent the individuals in the group of Trade School Mathematics Instructors. Variables 21 to 37 represent the individuals in the group of University Mathematics Instructors. Each variable has 18 observations associated with it...one for each objective. (TXT), (TXU), (UXT), and (UXU) symbolize the text in the respective quadrants.

COGNITIVE LEVEL

SUBJECTS	High (H)	Low (L)	Group Means
(U)	$\bar{X}_{hu}$	$\bar{X}_{lu}$	$\bar{X}_{.u}$
(T)	$\bar{X}_{ht}$	$\bar{X}_{lt}$	$\bar{X}_{.t}$
Cognitive level means	$\bar{X}_{h.}$	$\bar{X}_{l.}$	$\bar{X}_{..}$

CONTENT AREA

SUBJECTS	Number Systems	Measurement	Geometry	Graphs	Algebra	Relations & Functions	Probability & Statistics	Logic	Applications	Group Means
(U)	$\bar{X}_{nu}$	$\bar{X}_{mu}$	$\bar{X}_{gu}$	$\bar{X}_{gru}$	$\bar{X}_{au}$	$\bar{X}_{fu}$	$\bar{X}_{pu}$	$\bar{X}_{lu}$	$\bar{X}_{apu}$	$\bar{X}_{.u}$
(T)	$\bar{X}_{nt}$	$\bar{X}_{mt}$	$\bar{X}_{gt}$	$\bar{X}_{grt}$	$\bar{X}_{at}$	$\bar{X}_{ft}$	$\bar{X}_{pt}$	$\bar{X}_{lt}$	$\bar{X}_{apt}$	$\bar{X}_{.t}$
Content area means	$\bar{X}_{n.}$	$\bar{X}_{m.}$	$\bar{X}_{g.}$	$\bar{X}_{gr.}$	$\bar{X}_{a.}$	$\bar{X}_{f.}$	$\bar{X}_{p.}$	$\bar{X}_{l.}$	$\bar{X}_{ap.}$	$\bar{X}_{..}$

Figure 2. Models of Two-Way Analysis of Variance

A two-way analysis of variance was also applied to the data as represented in Figure 2. From this procedure F-ratios were determined, which related to the interaction effects presented in Question 4 concerning group membership and the content area of the objectives, and in Question 5 concerning group membership and the behavioral level of the objectives.

Summary

In this chapter the writer has provided a description of the structure of the instrument used in the study, the procedures employed in the data collecting process, and a brief overview of the analytic techniques to be applied to the data. Chapter IV will be concerned primarily with the reporting of the analysis of the data.

CHAPTER IV

ANALYSIS OF THE DATA

In this chapter an analysis of the data, collected through the use of the instrument described in Chapter III, is presented. The analysis of the data, relevant to each group under study, was directed in a fashion which would be responsive to the five questions proposed in Chapter I. As a brief overview of the analytic procedure, it is noted that, in response to Questions 1 and 2, which are concerned with the ranking of the objectives by both groups, the rankings were established from frequencies derived from the instruments. Question 3, which is concerned with an examination of the correlations of the rankings, was approached on the basis of correlational analysis coupled with analysis of variance. Responses to Questions 4 and 5, relating to the interaction effects involving group membership and the behavioral level of the objectives, and group membership and the content area of the objectives respectively, were sought entirely on the results produced by analysis of variance supported by a method of individual cell comparison of means (Winer, 1971).

The Responses

The collection of the data was terminated on May 16, 1975. The return for the group of trade school mathematics instructors was 100%. However, in the case of the university mathematics instructors 85% or 17 out of a possible 20 respondents returned the instrument in a completed form. Two of the three university mathematics instructors, who did not complete the instrument, forwarded a brief explanation of their refusal to cooperate, while the other informed me of her reasons verbally.

Treatment of Responses

The responses of each individual were tabulated on a frequency sheet, which provided a frequency for the number of times each objective was selected, by an individual, over each other objective. These individual frequencies were then transferred to group frequency tables. Tables 1 and 2 assigned to each objective a score resulting from the number of times each objective was judged more important than each of the other objectives. Based on the group frequency for each objective, the objectives were ranked for each of the two groups (Table 3). With these basic data the writer proceeded to investigate each of the proposed questions. The results of the testing, related to each question, are reported in the order in which the questions were stated in Chapter I.

Results Relating to Questions 1 and 2

Question 1. What are the rankings of the 18 objectives for secondary school mathematics resulting from

Table 1

Results of Trade School Mathematics Instructors' Rating of Importance of Objectives

Objectives	Individuals 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
1	15	8	8	11	4	8	5	2	14	10	12	11	9	7	13	13	16	15	4	17	202
2	11	11	8	12	0	5	6	11	15	14	15	13	8	13	13	13	10	13	9	11	211
3	14	14	12	16	14	14	5	17	17	13	14	16	14	11	5	6	17	15	15	9	258
4	16	11	15	15	14	16	1	10	16	14	13	14	17	10	2	11	8	10	16	16	245
5	9	14	7	15	11	6	5	13	11	11	6	6	5	5	7	8	5	12	12	13	181
6	7	3	6	7	11	4	2	4	7	14	2	6	2	1	3	4	4	7	7	3	104
7	2	10	2	10	8	7	9	8	6	6	3	3	9	3	2	7	3	8	3	11	120
8	6	8	9	9	6	10	7	12	9	5	4	1	10	6	12	11	5	10	5	6	151
9	10	9	13	5	8	11	8	7	8	8	12	9	12	12	14	16	15	6	4	15	202
10	10	13	12	9	14	11	15	10	5	14	9	12	11	15	11	9	13	7	12	5	217
11	6	11	3	8	5	4	13	9	3	4	5	3	11	7	8	9	9	5	9	9	141
12	3	5	8	2	4	4	12	6	0	8	8	8	4	5	0	6	11	1	1	4	100
13	0	0	4	1	2	0	9	3	4	1	3	0	0	0	4	0	5	3	1	9	49
14	1	3	6	3	4	8	6	6	9	2	3	3	2	4	10	6	3	7	3	6	95
15	5	3	4	1	5	5	5	1	2	4	6	8	7	12	6	12	0	0	10	3	99
16	8	6	3	4	13	14	17	6	2	0	8	10	2	10	10	12	7	2	12	3	149
17	15	11	17	14	16	13	14	14	12	15	14	13	14	16	16	7	9	17	14	14	265
18	15	13	16	11	14	13	14	14	13	10	16	17	16	16	17	3	13	15	16	9	271

Table 2

Results of University Mathematics Instructors' Rating of Importance of Objectives

Objectives	Individuals 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Total
1	13	15	17	11	16	16	16	16	12	11	12	11	5	4	16	8	16	--	--	--	215
2	4	11	16	7	16	1	12	12	8	2	7	13	4	3	13	11	10	--	--	--	150
3	8	7	13	4	4	9	15	13	7	12	12	7	7	9	13	8	4	--	--	--	152
4	10	3	3	7	6	2	4	6	5	14	9	9	8	7	8	6	6	--	--	--	113
5	16	7	14	3	8	2	6	10	7	5	10	6	7	2	14	10	9	--	--	--	136
6	16	10	9	3	7	9	1	11	4	15	2	2	12	7	10	8	11	--	--	--	137
7	5	8	2	12	2	6	11	6	14	6	7	15	4	12	6	1	0	--	--	--	117
8	7	7	4	10	4	14	12	6	11	11	10	17	6	13	4	6	5	--	--	--	147
9	15	17	15	14	16	10	17	16	2	9	7	14	13	10	17	12	16	--	--	--	220
10	13	14	12	16	14	10	3	16	17	9	15	8	16	8	14	11	15	--	--	--	211
11	8	8	5	10	10	8	8	7	12	0	7	11	9	13	8	7	8	--	--	--	139
12	0	0	7	17	13	9	0	1	14	3	0	1	3	8	2	10	11	--	--	--	99
13	7	1	1	5	7	7	2	2	0	6	3	11	4	8	1	6	4	--	--	--	75
14	5	5	0	3	7	10	11	1	1	12	2	16	3	13	0	12	1	--	--	--	102
15	4	11	10	6	10	10	5	11	9	2	14	3	14	5	5	6	12	--	--	--	137
16	7	14	8	7	12	16	7	8	14	15	17	4	17	10	5	8	15	--	--	--	184
17	10	8	6	8	0	6	12	9	5	4	5	1	13	10	9	12	3	--	--	--	121
18	5	7	11	10	1	8	11	2	11	17	14	4	8	11	8	11	7	--	--	--	146

Table 3

Ranking of Objectives for Each Group

Objective	Mathematics Instructors	
	Trade School	University
1	7.5	2
2	6	6
3	3	5
4	4	15
5	9	12
6	14	10.5
7	13	14
8	10	7
9	7.5	1
10	5	3
11	12	9
12	15	17
13	18	18
14	17	16
15	16	10.5
16	11	4
17	2	13
18	1	8

the responses of trade school mathematics instructors using a paired comparison procedure?

Question 2. What are the rankings of the 18 objectives for secondary school mathematics resulting from the responses of university mathematics instructors using a paired comparison procedure?

Results. Table 3 (page 49) provides the rankings of the objectives for each group of mathematics instructors involved in the study. To gain some insight into any distinctive features of agreement or disagreement in the rankings of both groups of instructors Table 4 provides a comparison of objective ranks. The degree of agreement between the two groups can be illustrated by comparing the proximity of the ranks associated with a particular objective which was rated by each group.

Upon inspection of Table 4, it was observed that the exact commonality of rank for an objective by both groups occurred in only two instances. Objective 2 (number systems-high cognitive) was ranked number 6 for both groups while Objective 13 (probability and statistics-low cognitive) received a rank of 18 in both cases. Furthermore, in 39% of the cases the same objective, in both rankings, differed by two or fewer ranks. On the other hand, however, it was found that in about 28% of the cases the same objective, rated by both groups, differed by six or more ranks. To be specific, based on the ratings by both groups, Objective 2 (number systems-high cognitive), 3 (measurement-low

Table 4

Group Comparison of Objective Ranks

Rank	Objectives Rated by Trade School Mathematics Instructors	Rank	Objectives Rated by University Mathematics Instructors
1	18(applications-high)*	1	9(algebra-low)
2	17(applications-low)	2	1(number systems-low)
3	3(measurement-low)	3	10(algebra-high)
4	4(measurtment-high)	4	16(logic-high)
5	10(algebra-high)	5	3(measurement-low)
6	2(number systems-high)	6	2(number systems-high)
7.5	1(number systems-low)	7	8(graphs-high)
7.5	9(algebra-low)	8	18(applications-high)
9	5(geometry-low)	9	11(relations-low)
10	8(graphs-high)	10.5	15(logic-low)
11	16(logic-high)	10.5	6(geometry-high)
12	11(relations-low)	12	5(geometry-low)
13	7(graphs-low)	13	17(applications-low)
14	6(geometry-high)	14	7(graphs-low)
15	12(relations-high)	15	4(measurement-high)
16	15(logic-low)	16	14(probability-high)
17	14(probability-high)	17	12(relations-high)
18	13(probability-low)	18	13(probability-low)

Note -- contained in the parenthesis following the objective number we find (content area of the objective-behavioral level of the objective). *Example -- objective 18, which involves the content area of applications at the high cognitive level of behavior, is ranked number 1 by the Trade School group.

cognitive), 7 (graphs-low cognitive), 10 (algebra-high cognitive), 12 (relations and functions-high cognitive), 13 (probability and statistics-low cognitive) and 14 (probability and statistics-high cognitive) differed by two or fewer ranks. Of these, Objectives 3 (measurement-low cognitive), 7 (graphs-low cognitive) and 12 (relations and functions-high cognitive) were ranked higher by the trade school mathematics instructors, while Objectives 10 (algebra-high cognitive) and 14 (probability and statistics-high cognitive) were ranked higher by the university mathematics instructors. Again based on the ratings by both groups Objectives 4 (measurement-high cognitive), 9 (algebra-low cognitive), 16 (logic-high cognitive), 17 (applications-low cognitive) and 18 (application-high cognitive) differed by six or more ranks. Of these, Objectives 4 (measurement-high cognitive), 17 (applications-low cognitive) and 18 (applications-high cognitive) were ranked higher by the trade school mathematics instructors, while Objectives 9 (algebra-low cognitive) and 16 (logic-high cognitive) were ranked higher by the university mathematics instructors.

In comparing the rankings for both groups, it was readily observed that in what is considered the ranks of most importance, for example ranks 1 through 5, there were two common objectives - Objective 3 (measurement-low cognitive) and Objective 10 (algebra-high cognitive. However, in the case of the trade school mathematics instructors, it was observed that the objectives dealing with applications seemed to be considered the most important objectives. Whereas, it appeared that the university mathematics instructors suggested that objectives dealing with algebra were the most important of those considered.

At the other extreme end of the ranking scale, for example, ranks 14 through 18, it was found that a degree of commonality between the rankings of both groups was again evident, with objectives dealing with relations and probability and statistics occurring in three of the lowest five ranks for both groups. Furthermore, it can be observed that both groups considered objectives dealing with probability and statistics to be of least importance.

Results Relating to Question 3

Question 3. How are the rankings by the trade school mathematics instructors correlated with the rankings by the university mathematics instructors?

Results. The correlation coefficient between the group scores for each objective (as shown in Tables 1 and 2) was 0.44. This indicated a positive correlation between the rankings of both groups which was significantly different from zero correlation, using a one-tailed t-test. However, relative to analysis of group data, questions arise concerning the consistency of responses by the individuals whom the group analysis represents. The analysis used in this study to investigate such internal factors utilized a correlational procedure. The set of frequencies corresponding to each objective for each respondent was subjected to a correlational analysis using the Pearson Product-Moment Correlation (DEST 02) computer program which was prepared by the Division of Educational Research Services of the University of Alberta. This procedure provided a correlational matrix concerning the amount of consistency (or spread) in the scores associated with each objective over the group of 37 mathematics instructors involved in the study. A model of this matrix and a diagramatic breakdown into components for further analysis is provided in Figure 1 (page 42).

The correlational matrix was considered to have three defineable components, identifiable in its four quadrants. Quadrant 1 entries represented correlation coefficients which provided an indication of the amount of consistency (or spread) among the individual rankings associated with the objectives within the group of trade school mathematics instructors. In other words, we have a set of within group correlation coefficients for this particular group of mathematics instructors. Similarly, Quadrant 4 provided a set of within group correlation coefficients for the group of university mathematics instructors. The third component in the matrix was identified in the second and third quadrants as representing the amount of consistency (or spread) between the individual rankings by the two groups involved in the study; that is, a set of between group correlation coefficients.

From each of the above components of the correlational matrix, a random sample of 30 correlation coefficients was selected for the purpose of comparing the correlations of the three components in order to establish any differences that may have existed. However, the sampling distribution of correlation coefficients is not normal, and consequently, when testing the null hypothesis of a population correlation being other than a zero correlation, one must make use of the Fisher Z transformation of the correlation coefficient (r) (Edwards, 1963). This transformation normalizes the distribution of the correlation coefficients. Using this approach, one can then apply a one-way analysis of variance to the three sets of correlation coefficients in order to determine the existence of significant differences.

The means and standard deviations of the Fisher Z's corresponding to the three random samples of correlation coefficients are presented in Table 5.

When a one-way analysis of variance was applied to these data, as shown in Table 6, it was found that a significant difference ($p<.05$) existed among the three sets of scores; that is, among the correlation coefficients for the rankings of objectives within the group of university mathematics instructors, the correlation coefficients for the rankings of objectives within the group of trade school mathematics instructors, and the correlation coefficients for the rankings of objectives between the trade school and

Table 5

Mean Scores and Standard Deviations of Fisher Z's associated with the Correlation Coefficients (r)

	Within Trade School Group (n=30)		Within University Group (n=30)		Between Trade School and University Group (n=30)	
	$\overline{X}$	SD	$\overline{X}$	SD	$\overline{X}$	SD
Z	.4463	.2661	.2610	.3223	.1060	.2052
r	.418		.255		.106	

Note -- r represents the correlation coefficient associated with Z.

Table 6

Summary of Analysis of Variance involving the Components of the Correlational Matrix (TXT, UXU, TXU)

Source of Variation	Sums of Squares	Degrees of Freedom	Mean Squares	F-ratio	Probability
Between groups	1.7407	2	.8704	12.0467*	<.001
Within groups	6.2856	87	.0722		

*reject at .05 level of significance

Note -- TXT = correlation coefficients for the within trade school mathematics instructors group

UXU = correlation coefficients for the within university mathematics instructors group

TXU = correlation coefficients for the between trade school and university mathematics instructors group university instructors. Further analysis, using the Scheffe' comparison of means, as shown in Table 7, found that the correlation coefficients for the within university mathematics instructors group did not differ significantly ($p>.05$) from those for the between trade school and university mathematics instructors group. However, the difference between the correlation coefficients for the within university mathematics instructors group and those for the within trade school mathematics instructors group, and the difference between the correlation coefficients for the within trade school mathematics instructors group and those for the between trade school and university mathematics instructors group were significant ($p<.05$).

Although the correlation coefficient (Table 5, page 56) corresponding to the mean Fisher Z of each group was somewhat different from zero in a positive direction, a t-test of the hypothesis of zero correlation (Edwards, 1963) showed that the mean correlation coefficient for the within trade school mathematics instructors group was significantly different ($p<.05$) from zero correlation, while the mean correlation coefficients for the other two components were not significantly different ($p>.05$) from zero correlation.

The test of homogeneity of the 30 values of r, in each case, was used to test the hypothesis that the 30 correlation coefficients (r) were homogeneous; that is, in each case, the 30 values of r were all estimates of the same population value (Edwards, 1963). This test required the

Table 7

Summary of the Scheffe' Method of Multiple Comparisons of the Means of the Components of the Correlational Matrix

(N=90)

Group	Sample Mean	n
TXT	$\bar{X}_1=.4463$	30
UXU	$\bar{X}_2=.2610$	30
TXU	$\bar{X}_3=.1061$	30

Contrast	ψ	σ_ψ^2	ψ/σ_ψ
$A=\mu_1-\mu_2$	.1853	.0048	2.6670*
$B=\mu_1-\mu_3$	.3402	.0048	4.9019*
$C=\mu_2-\mu_3$	.1549	.0048	2.2319

Note -- *reject at .05 level of significance

ψ = difference between means

$\sigma_\psi^2 = MS_w(\frac{1}{n_1} + \frac{1}{n_2})$

μ_i = mean of population i

finding of

$$\chi^2=\sum_{i=1}^{k} (n_i-3)(Z_i)^2 - \frac{[\sum_{i=1}^{k} (n_i-3)(Z_i)]^2}{\sum_{i=1}^{k} (n_i-3)}$$

with (k-1) degrees of freedom where k is the number values of r(k=30). It was found that the χ^2 value for the within university mathematics instructors group was a significant value ($p<.05$). The χ^2 values for the other two components were nonsignificant values; that is, one could conclude that the r values were homogeneous for these two particular components. In other words, it was found that the correlation coefficients associated with the rankings within the group of university mathematics instructors were not significantly homogeneous. However, the correlation coefficients associated with the rankings within the group of trade school mathematics instructors and those associated with the rankings between the trade school and university mathematics instructors were significantly homogeneous.

Results Relating to Question 4

Question 4. Do trade school mathematics instructors and university mathematics instructors agree on the cognitive level of the objectives for secondary school mathematics as to importance?

Results. Question 4 is concerned primarily with the interaction effect between the group to which the respondent belongs and the behavioral level of the objective.

The mean scores and standard deviations of the scores of the trade school mathematics instructors and the university mathematics instructors on the objectives of both low and high behavioral levels are shown in Table 8. A two-way analysis of variance, which was applied to these data, yielded the results shown in Table 9. Upon inspection of the means in Table 8, and the F ratio in Table 9, it was found that there was no significant interaction effect ($p > .05$) between group membership and behavioral level of the objectives. In other words, there was no significant difference ($p > .05$) in relative importance attached to the behavioral level of the objectives by either group.

Results Relating to Question 5

Question 5. Do trade school mathematics instructors and university mathematics instructors agree on the content area of the objectives for secondary school mathematics as to importance?

Results. Question 5 deals primarily with the interaction effect between the group membership of the respondents and the content area of the objectives.

The mean scores and standard deviations, relevant to the interaction effect expressed in Question 5 concerning group membership interacting with the content area of the objectives, are listed in Table 10. A two-way analysis of variance was applied to these data, the results of which

Table 8

Mean Scores and Standard Deviations for Both Groups of Instructors on the High and Low Behavioral Levels of the Objectives

	High Cognitive Objectives		Low Cognitive Objectives	
	Mean	SD	Mean	SD
Trade School Mathematics Instructors	77.15	5.204	75.85	5.204
University Mathematics Instructors	77.29	9.279	75.71	9.279

Table 9

Summary of Analysis of Variance Involving the Behavior Level of the Objectives and the Different Groups of Instructors

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F-ratio	Probability
Group	0	1	0	0	>.25
Behavior level	2.07	1	2.07	.7041	>.25
Interaction	0.0232	1	0.0232	.0079	>.25
Within group	3784.16	70	2.94		

Table 10

Mean Scores and Standard Deviations for Both Groups of Instructors

on the Various Content Areas of the Objectives

	Number Systems		Measurement		Geometry		Graphs		Algebra		Relations & Functions		Probability & Statistics		Logic		Applications	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Trade School Mathematics Instructors	20.65	7.05	25.15	7.39	14.25	5.64	13.55	4.88	20.95	4.38	12.05	4.89	7.20	4.44	12.40	7.21	26.80	6.11
University Mathematics Instructors	21.47	7.65	15.59	4.66	16.06	7.08	15.53	8.00	25.35	5.24	14.00	7.19	10.41	7.84	18.88	7.42	15.71	6.12

are shown in Table 11. The results showed that there was a significant interaction effect ($p<.05$) between group membership and the content area of the objectives.

Where significant differences of the mean scores were noted, individual cell comparisons were made to determine at what levels of the factors involved such interaction effects existed. For this purpose, the analysis between all possible pairs of means, in a logical grouping of means, was carried out using the Newman-Keuls procedure (Winer, 1971). This procedure, for each of the multiple cell comparisons, is outlined in Tables 12, 13, and 14.

Inspection of Table 12 shows that for the trade school mathematics instructors there was a significant difference ($p<.05$) between the means of all content areas; with the exceptions of non-significant differences between the means for logic and relations, the means for geometry and graphs, and the means for algebra and number systems. In the case of the university mathematics instructors (Table 13) non-significant differences ($p>.05$) existed between the means for measurement, geometry, graphs and applications, while for all other individual cell mean comparisons significant differences were found.

In Table 14 there are nine different individual cell mean comparisons, one for each content area. In each case the comparison involved the mean score for the trade school mathematics instructors and the mean score for the university mathematics instructors in a particular content area. An

Table 11

Summary of Analysis of Variance Involving the Content Area of the Objectives and the Different Groups of Instructors

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Squares	F-ratio	Probability
Group	0	1	0	0	>.25
Content area	365.06	8	44.51	20.14*	<.001
Interaction	148.86	8	18.61	8.42*	<.001
Within group	12774.75	315	2.21		

*reject at .05 level of significance

Table 12

Individual Cell Comparisons - Content Area Means for Trade School Mathematics Instructors

Ordered Means:	7.20	12.05	12.40	13.55	14.25	20.65	20.95	25.15	26.80	r	Critical Value
(i) 7.20	--	4.85*	5.20*	6.35*	7.05*	13.45*	13.75*	17.95*	19.60*	9	1.49
12.05		--	0.35	1.50*	2.20*	8.60*	8.90*	13.10*	14.75*	8	1.45
12.40			--	1.15*	1.85*	8.25*	8.55*	12.75*	14.40*	7	1.41
13.55				--	0.70	7.10*	7.40	11.60*	13.25*	6	1.37
14.25					--	6.40*	6.70*	10.90*	12.55*	5	1.31
20.65						--	0.30	4.50*	6.15*	4	1.23
20.95							--	4.20*	5.85*	3	1.12
25.15								--	1.65*	2	0.94
26.80									--		

(ii) $S_{\overline{AB}} = \sqrt{MS_{w,cell}/n_h} = \sqrt{2.21/18.38} = 0.34$

(iii) r:	2	3	4	5	6	7	8	9
$q_{.95}(r,315)$:	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.39
Critical Value = $S_{\overline{AB}}\, q_{.95}(r,315)$:	.94	1.12	1.23	1.31	1.37	1.41	1.45	1.49

N.B. $S_{\overline{AB}}$ = standard error of cell mean

n_h = harmonic mean of cell frequencies $\left(= \frac{pq}{\Sigma\Sigma(1/n_{ij})}\right)$

r = the number of steps two means are apart in an ordered sequence

* = reject at .05 level of significance

Table 13

Individual Cell Comparisons - Content Area Means for University Mathematics Instructors

Ordered Means:	10.41	14.00	15.53	15.59	15.71	16.06	18.88	21.47	25.35	r	Critical Value
(i) 10.41	--	3.59*	5.12*	5.18*	5.30*	5.65*	8.47*	11.06*	14.94*	9	1.49
14.00		--	1.53*	1.59*	1.71*	2.06*	4.88*	7.47*	11.35*	8	1.45
15.53			--	0.06	0.18	0.53	3.35*	5.94*	9.82*	7	1.41
15.59				--	0.12	0.47	3.29*	5.88*	9.76*	6	1.37
15.71					--	0.35	3.17*	5.76*	9.64*	5	1.31
16.06						--	2.82*	5.41*	9.29*	4	1.23
18.88							--	2.59*	6.42*	3	1.12
21.47								--	3.88*	2	.94
25.35									--		

(ii) $S_{\overline{AB}} = \sqrt{MS_{w.cell}/n_h} = \sqrt{2.21/18.38} = 0.34$

(iii)

r:	2	3	4	5	6	7	8	9
$q_{.95}(r, 315)$:	2.77	3.31	3.63	3.86	4.03	4.17	4.29	4.39
Critical Value = $S_{\overline{AB}}\, q_{.95}(r, 315)$:	.94	1.12	1.23	1.31	1.37	1.41	1.45	1.49

*reject at 0.05 level of significance

Table 14

Individual Cell Comparisons - Group Means for Each Content Area

	Number Systems		Measurement		Geometry	
Ordered Means	20.65	21.47	15.59	25.15	14.25	16.06
Differences	0.82		9.56*		1.81*	

	Graphs		Algebra		Relations & Functions	
Ordered Means	13.55	15.53	20.95	25.35	12.05	14.00
Differences	1.98*		4.40*		1.95*	

	Probability & Statistics		Logic		Applications	
Ordered Means	7.20	10.41	12.40	18.88	15.71	26.80
Differences	3.21*		6.48*		11.09*	

$r = 2$

$q_{.95}(r, 315) = 2.77$

Critical Value $= S_{\overline{AB}}\, q_{.95}(r, 315) = 0.94$

*reject at 0.05 level of significance

analysis of these data showed that there was a significant difference ($p<.05$) in all cases except in the content area of number systems. Furthermore, upon inspection of Table 10, it can be seen that, with the exceptions of applications and measurement, in all of the other content areas (geometry, graphs, algebra, relations and functions, probability and statistics, and logic) the university mathematics instructors had the higher means.

Summary of the Findings

This chapter contains various analyses of the data collected in this study, and the results relating to the questions which were associated with the major purposes of the study outlined earlier.

The purposes of this study were to establish rank orderings of a set of objectives for secondary school mathematics by trade school and university mathematics instructors, and to determine the perceptions by these two groups as to the importance of the objectives in the secondary school mathematics program; and from there to analyze and compare these perceptions relative to each group. It was found that there was a degree of agreement between the two groups in the rankings of the objectives in the sense that Objectives 2 (number systems-high cognitive), 3 (measurement-low cognitive), 7 (graphs-low cognitive), 10 (algebra-high cognitive), 12 (relations and functions-high cognitive), 13 (probability and statistics-low cognitive), and 14 (probability and statistics-high cognitive) differed by two or fewer ranks when the rankings for each of these objectives, by both the trade school and university mathematics instructors, were compared. However, a degree of disagreement also existed in the rankings of the objectives in the sense that Objectives 4 (measurement-high cognitive), 9 (algebra-low cognitive), 16 (logic-high cognivite), 17 (applications-low cognitive) and 18 (applications-high cognitive) differed by six or more ranks when the rankings for each of these objectives, by both the trade school and university mathematics instructors, were compared. It was also found that both groups rated the objectives dealing with probability and statistics as ranking least in relative importance. The group of trade school mathematics instructors rated objectivesdealing with applications and measurement as highest in relative importance while the group of university mathematics instructors rated objectives dealing with algebra as highest in relative importance.

Homogeneous correlation coefficients were found among the correlations of the rankings within the group of trade school mathematics instructors, while the correlations of the rankings within the group of university mathematics instructors were not homogeneous. This inconsistency of ranking within the university group was supported by a t-test of the hypothesis that the correlations are not significantly different from zero correlation, which showed that the mean correlation for the within university mathematics instructors group was not significantly different from zero correlation. This, however, was not the case for the within-trade-school mathematics instructors group.

The null hypothesis of no interaction effect between group membership and cognitive level of the objectives was shown to be non-significant. However, there was a significant interaction effect between group membership and content area of the objectives. There was no significant difference between the means of logic and relations and functions, between the means for geometry and graphs, and between the means for algebra and number systems as ranked by the trade school group. For the university group there were non-significant differences among the means for measurement, geometry, graphs, and applications. In both cases, significant differences were found between all other content areas by each group. Furthermore, with the exception of number systems, the means of each other content area differed significantly between the two groups.

Implications arising from these findings will be discussed in the next chapter.

CHAPTER V

SUMMARY, CONCLUSIONS, IMPLICATIONS AND RECOMMENDATIONS

This chapter provides an overview, in retrospect, of the problem under investigation, the instrumentation involved in the collection of the data, the population about whom the study is concerned, and the analysis applied to the data. Conclusions arising out of the findings of the study are also presented. Furthermore, the writer provides some discussion relating to possible implications of the results and some suggestions for further research.

Summary of the Investigation

The present study was designed primarily to examine the perceptions of two groups of post-secondary mathematics instructors (trade school mathematics instructors and university mathematics instructors) relative to a set of general objectives for secondary school mathematics. Furthermore, attempts were made to analyze and compare these perceptions in an effort to determine any trends in the ways by which these groups perceived the objectives, relative to each other.

Questions explored. Relating to the primary purpose of the study the following questions were specified and explored:

(i) What rankings result from applying a method of paired comparisons to 18 objectives for secondary school mathematics with trade school mathematics instructors making the choices?

(ii) What rankings result from applying a method of paired comparisons to 18 objectives for secondary school mathematics with university mathematics instructors making the choices?

(iii) How are these rankings correlated?

(iv) Do trade school mathematics instructors and university mathematics instructors agree on the cognitive level of the objectives for secondary school mathematics as to importance?

(v) Do trade school mathematics instructors and university mathematics instructors agree on the content area of the objectives for secondary school mathematics as to importance?

The instruments. In order to gather the necessary data, an appropriate instrument was devised. Following a survey of the literature and projects pertinent to the study of the needs and abilities of secondary school mathematics students an initial list of 35 objectives of an intermediate level of specificity was produced. This initial list of objectives was edited and revised in consultation with a panel of mathematics educators to produce a final list of 18 objectives. These objectives, in their final form, represented nine different content areas of mathematics, each of which was, in turn represented by one objective classified as an objective of high cognitive behavior, and another objective classified as one of low cognitive behavior. Each of the objectives was then paired with each other objective to produce a possible 153 distinct pairs of objectives. The instrument, in its final form, consisted of 153 pairs of objectives, for each of which an individual respondent was asked to make a choice as to which objective in each pair he considered more important to secondary school mathematics.

Samples. The respondents were selected randomly from the faculty of the Mathematics Department of Memorial University of Newfoundland and, the mathematics staffs of the various trade schools throughout the province of Newfoundland. Replies were obtained from 20 trade school mathematics instructors and 17 university mathematics instructors.

Analysis. The collected data were subjected to several analytic procedures in response to the questions under investigation. Appropriate correlational analysis, analysis of variance, and individual cell comparisons measures were used to evaluate the data collected by the instrument.

Conclusions

Comparison of the rankings of objectives by both groups does not imply total disagreement between the trade school mathematics instructors and the university mathematics instructors as to the relative importance of the objectives for secondary school mathematics. In 39% of the cases the same objective, rated by both groups, differed by two or fewer ranks. These included objectives dealing with systems of numbers, algebraic expressions and sentences, and their solutions, relations and functions, and probability and statistics at a high cognitive level of behavior, and objectives dealing with measurement, graphs, and probability and statistics at a low cognitive level of behavior. Nevertheless, in 28% of the cases the same objective, rated by both groups, differed by six or more ranks, thus indicating a degree of disagreement between the two groups. These included objectives dealing with measurement, logic, and applications at a high cognitive level of behavior, and objectives dealing with algebraic expressions and sentences, and their solutions, and applications at a low cognitive level of behavior.

In the case of what would be considered the important ranks, for example ranks 1 through 5, it was found that Objective 10 (algebra-high cognitive) and Objective 3 (measurement-low cognitive) were common to both rankings. Of the objectives assigned to the lower ranks or ranks of least importance, for example ranks 14 through 18, it was again found that a degree of agreement existed. In this case, Objective 12 (relations and functions-high cognitive), Objective 14 (probability and statistics-high cognitive) and Objective 13 (probability and statistics-low cognitive) were common to both rankings.

Based on the rankings (Table IV, page 51) it appeared that the trade school mathematics instructors attached more importance to the objectives which had straight forward and rather direct implications involved in trade-oriented programs, for example, those objectives dealing with applications and measurement. This same group also tended to attach relatively little importance to objectives dealing with structure and assumptions in mathematics, for example, those objectives dealing with the function concept and the structure of geometry, as well as to those objectives dealing with notions of probability and statistics. As a group, the university mathematics instructors rated objectives dealing with algebraic expressions and sentences, and their solutions high in relative importance. However, as was the case with the trade school group, the university group rated objectives dealing with probability and statistics, and the function concept very low in relative importance. But in the interpretation of the results relating to group data, consideration must be given to the question of the internal consistency of the rankings. This question is given some attention in the following discussion.

In the analysis dealing with the question of how the rankings of both groups were correlated, it was found that a significant correlation existed between the rankings, associated with the objectives, by both groups. However, a further analysis which tested the significance of the correlations between the scores associated with the objectives by individual respondents (refer to Figure 1, page 42) showed that the mean correlation coefficient associated with the rankings for the objectives by the individual trade school mathematics instructors differed significantly from the corresponding mean correlation coefficient associated with the rankings by the individual university mathematics instructors. Furthermore, t-tests of the hypotheses of zero correlation showed that of the mean correlation coefficients for the rankings within the trade school group and within the university group only the mean correlation coefficient associated with the individual trade school mathematics instructors differed significantly from zero correlation. This would seem to imply that the rankings within the university group were not correlated with each other. In addition, it was found that the correlation coefficients associated with the individual university mathematics instructors were not significantly homogeneous; that is, there was a significant degree of variability in the correlational values of the rankings within the university group. Based on these findings it can be argued that there was variation among the university mathematics instructors as to the relative order of importance of the objectives. However, this was not the case for the rankings by the trade school group. Consequently, any interpretation of the results relating to the university mathematics instructors, as a group, must be subjected to the finding that there was a significant inconsistency between individual university mathematics instructors in responding to the relative importance of the objectives.

The null hypothesis of no [illegible] the group to which the respondent belongs and the behavioral level of the objectives is non-significant. This would suggest that the behavioral level of the objectives was not considered a differentiating factor among the objectives, by either group of instructors.

However, the null hypothesis of no interaction effect between the group to which the respondent belongs and the content area of the objectives is significant. It was found that trade school mathematics instructors indicated no significant differences in importance between the objectives for geometry and graphs, between the objectives for algebra and number systems, and between the objectives for logic and relations and functions. For the objectives of all other content areas there were significant differences in importance between them. The university mathematics instructors indicated no significant differences in importance between the objectives for measurement, geometry, graphs, and applications, while the level of relative importance between the objectives for all other content areas differed significantly. Furthermore, it was found that there was no significant difference in the relative importance placed on the objectives dealing with number systems by both groups. However, in all other content areas, with the exception of the objectives for applications and measurement, the university mathematics instructors attached significantly greater importance on the objectives than did the trade school mathematics instructors.

Relative to the interpretation of the results concerning the interaction effects involving the group of university mathematics instructors, it should be remembered that the test for homogeneity of the correlation coefficients associated with the rankings of the objectives by the individual university mathematics instructors showed that the rankings of the objectives were significantly non-homogeneous. This would imply that any conclusions drawn, relative to the group data for the university mathematics instructors, would have to be done so in view of the inconsistency within this group with respect to the rankings of the objectives.

In summary the conclusions drawn from the study could be enumerated as follows:

1. There was a degree of agreement between the rankings of the objectives for secondary school mathematics by the group of trade school mathematics instructors and the group of university mathematics instructors. However, the trade school mathematics instructors group indicated that objectives dealing with applications and measurement were

the most important, in that order, while the university mathematics instructors group indicated the objectives dealing with algebra to be the most important. However, both groups indicated that the objectives dealing with probability and statistics were of least importance relative to the other objectives. Both groups also attached relatively little importance to the objective dealing with the concept of function as a unifying notion in mathematics.

2. There was a significant consistency in the rankings by the trade school mathematics instructors, whereas this was not the case with the university mathematics instructors.

3. There was no significant difference attached to the importance of the cognitive level of the objectives by either group.

4. The trade school mathematics instructors group did not indicate any difference in the relative importance of the objectives for geometry over those for graphs, of the objectives for algebra over those for number systems, and of the objectives for logic over those for relations and functions. The university mathematics instructors group did not indicate any difference in relative importance among the objectives for measurement, geometry, graphs, and applications.

5. Both groups astached the same relative importance to the objectives for number systems. However, the university mathematics instructors attached a greater degree of import-

ance to the objectives dealing with geometry, graphs, algebraic expressions and sentences; and their solutions, relations and functions, probability and statistics, and logic than did the group of trade school mathematics instructors. On the other hand, the trade school mathematics instructors placed greater importance on objectives dealing with applications and measurement than did the group of university mathematics instructors.

6. The interpretation of these conclusions should be made relative to Conclusion 2.

Implications and Recommendations

The findings of the study do not indicate total disagreement between the group of trade school mathematics instructors and the group of university mathematics instructors concerning the relative importance of the objectives for secondary school mathematics. However, the high rankings of importance attached to the objectives for applications and measurement by the group of trade school mathematics instructors as opposed to that of the group of university mathematics instructors is not surprising since these are two practical aspects of mathematics in trade-oriented courses. On the other hand, both groups appear to agree on the relative non-importance of the objectives for probability and statistics. One can conjecture that this may relate to the relatively little coverage presently given, in secondary school mathematics courses, to this particular content area.

Another finding which certainly has implications regarding the interpretation of the results of this study relates to the question of the consistency of the rankings associated with the individuals in both groups. The non-homogeneity of the correlation coefficients of the rankings for the university mathematics instructors suggests that the question of agreement between the two groups must be answered in light of the question of agreement within the group of university mathematics instructors. It is plausable to suggest that the source of disagreement within the university group of mathematics instructors relates to the background of the instructors. This group is composed of individuals who specialize or have interests in particular contents areas. Whereas the individuals in the group of trade school mathematics instructors, for the most part, have a general background in mathematics and, furthermore, have a somewhat common goal oriented towards the practicalities of mathematics. This would provide a basis for a greater degree of consistency in opinion regarding the importance of objectives for secondary school mathematics.

Although it was suggested that similarity or dissimilarity among the backgrounds of the individuals involved in this study may have affected the consistency of response within each group, one might also point out that the consistency of response may be related to the nature of the course of study for which the individual instructor is responsible. For instance, in the trade schools we find mathematics courses of two basic types--courses for technology programs and courses for trade programs. The level of mathematics in these programs are distinctively different. However, the number of trade school mathematics instructors who are totally involved in the technology programs are very few. Due to the overlap of instructors in both the technology programs and the trade programs, and the fact that most of the trade school mathematics instructors are involved in the trade programs, it is not surprising that there was a degree of consistency in their responses concerning secondary school mathematics. Within the university group we again find two subgroups--junior division instructors and senior division instructors. In this case, we have junior division instructors who are concerned primarily with the teaching of mathematics to students who are fresh out of the secondary school environment. On the other hand, it is the senior division instructors who receive students who have passed through a stage of orientation and learning maturity, so to speak. It would not be unreasonable, then, to suggest that these two groups would differ in their perceptions of the mathematical needs of high school students. Therefore, it is recommended that a similar study be carried out using subgroups of those groups used in this study. These would include junior and senior division instructors at the university and instructors involved in technology and trade programs at the trade schools.

It was found that there was no significant difference as to the relative importance of low cognitive objectives and high-cognitive objectives by both groups. This would seem to have some bearing on the controversy which suggests that the trade schools courses are more concerned with low level or skill-oriented objectives as opposed to somewhat higher cognitive level objectives for university mathematics courses. There appears to be a consensus of opinion that the trade schools primarily require the students to be well equipped in the operationally-oriented aspects of mathematics, thus indicating the acquisition of a low level of cognitive behavior. The view is also prevalent that the university mathematics departments are more concerned about the structural and conceptual frameworks in mathematics; a notion which would suggest the acquisition of a high level of cognitive behavior. Although this would appear to be the case, the findings of this study seem to indicate otherwise. But again this must be interpreted in light of the question of the within group consistency mentioned earlier. Nevertheless, it is suggested that an examination of secondary school mathematics programs, with respect to more precise and well defined behavioral levels of the objectives, be undertaken. It is, therefore recommended that a study similar to the present study which involves behavioral objectives of various cognitive levels, analogous to Bloom's taxonomy, be carried out.

When considering the mathematical needs of students upon completion of secondary school mathematics programs some consideration should be given to those students who do not pursue further education on a post-secondary level.

A final recommendation for further research is to investigate the mathematical needs of secondary school graduates entering businesses or other professions which require minimal mathematical knowledge.

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APPENDIX A

LETTER OF INTENT TO RESPONDENTS

March 27, 1975

Dear Mr.

I am a graduate student in the Department of Curriculum and Instruction at Memorial University. My area of specialization is Mathematics Education at the high school level. I am about to undertake a study in an attempt to determine the mathematical needs of high school students, as perceived by mathematics instructors at the post-secondary levels.

The results of a random sampling technique provided a sample of post-secondary mathematics instructors in which your name has occurred. On this basis, I am seeking your cooperation in the collection of the required data.

It is not my intention to impose any rigid time requirements on you, and I feel that any unnecessary inconveniences on your part should be avoided. However, I am sure that you can appreciate that one must establish somewhat of a flexible scheduling of time for collecting the required information. What I propose is as follows: I will deliver the instrument on 1975 and leave it with you overnight. I will return on the following day to collect it. I would further point out that the completion of the required information is estimated to take about three-quarters (3/4) of an hour of your time.

The purpose of this letter is to inform you, in advance, of my intentions. If, however, the above time creates any difficulty, I would appreciate it if you would inform me accordingly, so that we can make the necessary alternate arrangements.

Thank you,

Randell Mercer

c.c. Principal

APPENDIX B

INSTRUMENT AND ACCOMPANYING INSTRUCTIONS

Instructions to University Mathematics Instructors

Enclosed you will find a booklet containing 153 strips of paper. On each strip there are two possible objectives for a high school mathematics program. Please place an X in the blank opposite the objective which you consider more relevant to such a program. Assume that a choice must be made in each case, even though you may have no strong personal preference in some cases. I will pick up the booklet after a 24 hour period.

Your opinions are sought because, as a professional mathematician, you have a view of mathematics in depth which is needed in curriculum and construction. An analysis of the responses will be performed in an effort to determine a set of guidelines for high school mathematics curriculum development. A summary of the results will be made available to you, at your request, upon completion of this study.

Thank you,

Randell Mercer

Instructions to Trade School Mathematics Instructors

Enclosed you will find a booklet containing 153 strips of paper. On each strip there are two possible objectives for a high school mathematics program. Please place an X in the blank opposite the objective which you consider more relevant to such a program. Assume that a choice must be made in each case, even though you may have no strong personal preference in some cases. I will pick up the booklet after a 24 hour period.

Your opinions are sought because, as a teacher of post-secondary mathematics, you are aware of the mathematical needs of students entering your institutions from the high schools, and consequently you can provide an input which is needed in curriculum construction. An analysis of the responses will be performed in an effort to determine a set of guidelines for high school mathematics curriculum development. A summary of the results will be made available to you, at your request, upon completion of this study.

Thank you,

Randell Mercer

The list of objectives:

1. To acquire the basic computational skills related to the real number system and the subsets thereof, including various algorithms associated with these numbers.
2. To be able to achieve economy in computations by making use of one's understanding of the structure and operations of the real number system.
3. To develop a facility for measurement, with respect to determining length, area, volume, etc., and to the terminology and relations of various measurement systems.
4. To develop an understanding of the nature of measurement, relative to the notions of precision, accuracy, and estimation, and their effects in interpreting the meaning of a solution to a problem.
5. To be able to apply the properties of geometric figures, such as similarity, congruency, the Pythagorean theorem, etc. in the solution of a problem.
6. To develop an understanding of the structure of geometry, which includes

the basic assumptions upon which geometry is built and how geometric facts and relations can be generated from these assumptions.

7. To be able to take a set of data, tabulate it, and present it in meaningful graphical form.
8. To be able to analyze and interpret data, as presented in graphs and tables, and to draw inferences relevant to the solution of the problem under consideration.
9. To develop elementary skills in algebraic manipulations, including the solution of inequalities and linear, quadratic, simultaneous, polynomial, logarithmic and exponential sentences, and the use of algebraic algorithms.
10. To be able to analyze and select the appropriate algebraic processes in problem solving.
11. To be able to represent the relationship between two sets of numbers by using co-ordinate graphs, tables, algebraic or trigonometric sentences.
12. To be able to recognize the concept of

function as a relevant and unifying notion throughout the mathematical knowledge that one has acquired.

13. To develop the ability to apply basic concepts and principles of probability and statistics.

14. To develop the ability to interpret statistical data for the purpose of making inferences or drawing conclusions.

15. To acquire the ability to follow proofs by comprehending the sequence of the premises and conclusions involved.

16. To be able to carry through a consistent argument to a valid conclusion.

17. To acquire a familiarity with the applications of mathematics to the fields of the physical sciences, industry and technology, and consumerism.

18. To be able to select from his mathematical knowledge the necessary mathematics which can be applied to a specific real life situation.

Each of the 18 objectives were combined in pairs to give a total of 153 possible distinct pairs. Each respondent was required to indidate which objective in each of the 153 pairs that he considered more important for secondary school mathematics. An example of a pair of objectives, found in the instrument, is given below. The X to the left of the objective indicates the selection of the objective considered more important in this particular pair.

Example:

___ To acquire the basic computational skills related to the real number system and the subsets thereof, including various algorithms associated with these numbers.

X To be able to achieve economy in computations by making use of one's understanding of the structure and operations of the real number system.