IMPROVING THE GPR REFLECTION METHOD FOR ESTIMATING SOIL
MOISTURE AND DETECTION OF CAPILLARY FRINGE AND WATER TABLE
IN A BOREAL AGRICULTURAL FIELD

by

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ABSTRACT

Improving the GPR reflection method for estimating soil moisture and detection of capillary fringe and water table in a boreal agricultural field

by

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The objective of this thesis was to monitor the soil moisture (SM) and water table depth (WTD) in an agricultural field using ground-penetrating radar (GPR). First, SM was estimated using hyperbola-fitting method (27-50 cm depth range) and compared with vertically installed 30 cm long Time Domain Reflectometry (TDR) probe data. TDR-measured and GPR-estimated SM were not significantly different, and the root mean square error (RMSE) was 0.03 m$^3$ m$^{-3}$. Second, the depth of the capillary fringe ($D_{CF}$) was estimated distinguishing the reflections from the top of the capillary fringe in a GPR radargram. A site-specific strong linear relationship ($R^2 = 0.9778$) of $D_{CF}$ and measured-WTD was developed. RMSE between GPR-based WTD and actual WTD was 0.194 m. Proposed average capillary height for the particular site throughout the growing season (0.741 m) agrees with the existing literature and would be beneficial for the agricultural water management in the region.

Keywords: agricultural, boreal, capillary fringe, GPR, hyperbola fitting, soil moisture, water table depth
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<tr>
<td>BRF</td>
<td>Boreal Ecosystem Research Facility</td>
</tr>
<tr>
<td>BH</td>
<td>Borehole</td>
</tr>
<tr>
<td>c</td>
<td>Electromagnetic wave velocity in free space/ speed of light</td>
</tr>
<tr>
<td>CMP</td>
<td>Common mid-point</td>
</tr>
<tr>
<td>CO</td>
<td>Common offset</td>
</tr>
<tr>
<td>D&lt;sub&gt;CF&lt;/sub&gt;</td>
<td>Depth to the capillary fringe</td>
</tr>
<tr>
<td>EM</td>
<td>Electromagnetic</td>
</tr>
<tr>
<td>GovNL</td>
<td>Government of Newfoundland and Labrador</td>
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<tr>
<td>GPR</td>
<td>Ground Penetrating Radar</td>
</tr>
<tr>
<td>max</td>
<td>Maximum</td>
</tr>
<tr>
<td>min</td>
<td>Minimum</td>
</tr>
<tr>
<td>MMSA</td>
<td>Multiresolution Monogenic Signal Analysis</td>
</tr>
<tr>
<td>MOP</td>
<td>Multi offset profile</td>
</tr>
<tr>
<td>n</td>
<td>Number of samples</td>
</tr>
<tr>
<td>n.d</td>
<td>No date</td>
</tr>
<tr>
<td>NL</td>
<td>Newfoundland</td>
</tr>
<tr>
<td>PBRS</td>
<td>Pynn’s Brook Research Station</td>
</tr>
<tr>
<td>r</td>
<td>Correlation coefficient</td>
</tr>
<tr>
<td>R</td>
<td>Normal incidence reflection coefficient</td>
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<tr>
<td>RMSE</td>
<td>Root Mean Square Error</td>
</tr>
<tr>
<td>Rx</td>
<td>Receiver antenna</td>
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<tr>
<td>SM</td>
<td>Soil moisture</td>
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<tr>
<td>Abbreviation</td>
<td>Description</td>
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<tr>
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<tr>
<td>TDR</td>
<td>Time Domain Reflectometry</td>
</tr>
<tr>
<td>TRIME</td>
<td>Time-domain Reflectometry with Intelligent Micro Elements</td>
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<tr>
<td>$t_{rw}$</td>
<td>Reflected wave travel time</td>
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<tr>
<td>TWTT</td>
<td>Two-way travel time of GPR waves</td>
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<tr>
<td>Tx</td>
<td>Transmitter antenna</td>
</tr>
<tr>
<td>$v$</td>
<td>Average radar velocity</td>
</tr>
<tr>
<td>$v_{rw}$</td>
<td>Reflected wave velocity</td>
</tr>
<tr>
<td>$v_t$</td>
<td>Total volume of soil</td>
</tr>
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<td>Volume of water</td>
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<td>Water table depth</td>
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<td>Zero offset profile</td>
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<td>$\alpha$</td>
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<td>$\varepsilon$</td>
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<td>$\varepsilon_r$</td>
<td>Relative permittivity (dielectric constant)</td>
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<td>$\theta_m$</td>
<td>Gravimetric soil moisture content</td>
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<td>$\theta_v$</td>
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<td>$\sigma$</td>
<td>Electrical conductivity</td>
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CHAPTER 1

Introduction and Overview
1.1 Introduction

Maintaining the ideal soil moisture (SM) in the root zone (unsaturated zone) at each growing phase, and understanding the behavior and contamination possibilities of groundwater (saturated zone) are the main components of sustainable water management in agriculture. Soil moisture is the temporary water storage at the unsaturated condition in between atmosphere and groundwater reservoir; the key factor that helps the growing plants by allowing to uptake their nutrients from the soil. Water table demarcates the boundary between saturated-unsaturated soils.

SM data may also be useful for crop yield forecasting and early warning of droughts, organic matter decomposition, heat transfer at the land-atmosphere interface and, to manage insect and disease control (Agrios, 1997; Champagne et al., 2012; Dari et al., 2018; Engman, 1991; Ghorbani et al., 2008; Koster, 2010; Schwingshackl et al., 2017). Additionally, SM also helps to improve agricultural practices without environmental degradation (Engman, 1991) and regulates the distribution of precipitation between runoff and infiltration (Burns, 1974; Li et al., 2017). The SM is neither homogeneously dispersed nor static over time, because of the heterogeneity of soil properties, topography, land cover, and the non-uniformity of rainfall and evapotranspiration (Engman, 1991). A large number of data are needed to measure SM in a larger area. Gravimetric SM ($\theta_m$) of a particular soil sample can be derived from mass loss by oven drying a moist sample, which is considered as the direct and the standard method. Destructive, non-repeatable, and time and labor consuming circumstances of the gravimetric method directed researchers toward indirect methods of SM determination (Huisman et al., 2003; Lambot et al., 2004). The indirect method could be either large-scale or point-scale measurements. Remote sensing techniques
such as satellites are being used for large-scale SM detection, but has low resolution and restricted to the soil surface (Errico et al., 2000). Recognized indirect methods for SM measurements are; neutron probes (Chanasyk & Naeth, 1996; Kodikara et al., 2013) electromagnetic probes (time domain reflectometry/ TDR) (Brandyk et al., 2016; Galagedara et al., 2003b), capacitance sensors (Zotarelli et al., 2011), electrical resistance probes (Williams, 1980), heat pulse sensors (Price, 1982), fiber optic sensors (Ciocca et al., 2012), cosmic-ray sensors (Sigouin et al., 2016) and gamma-ray scanners (Baldoncini et al., 2019; Carroll, 1981). Electromagnetic probes among all of them have become dominant and standard during the last few decades, but low sampling volume and limiting to point measurements are the main disadvantages (Fisher et al., 1992; Galagedara et al., 2003a; Greaves et al., 1996; Van Overmeeren et al., 1997; Weiler et al., 1998).

A reliable indirect method which is repeatable and rapid to obtain high-resolution SM data at large-scales would be worthy than conventional point measurements (Galagedara et al., 2005; Huisman et al., 2003; Takeshita et al., 2004). Ground penetrating radar (GPR) operates in between 10 to 1200 MHz electromagnetic frequency bandwidth and is primarily reliable for determining SM at a sample volume of 1 m$^3$ or more (Davis & Annan, 1989; Galagedara et al., 2003a; Grote et al., 2003; Hubbard et al., 2002). GPR is a non-destructive, portable and time-effective method for large-scale applications (Alumbaugh et al., 2002; Bikowski et al., 2010; Doolittle et al., 2006; Du & Rummel, 1996; Galagedara et al., 2003a, 2003b, 2005; Gao et al., 2012; Grote et al., 2002; Huisman et al., 2001, 2003; Jacob & Hermance, 2004; Jadoon et al., 2010; Minet et al., 2010; Parkin et al., 2000; Rucker & Ferre, 2004; Stoffregen et al., 2002; Wijewardana & Galagedara, 2010).
In addition, GPR is capable of non-intrusively detecting the water table and subsurface anomalies such as cavities and sub-surface flow paths (Doolittle et al., 2006; Gish et al., 2002; Paz et al., 2017; Shih et al., 1986). Irrigation is needed in agriculture if the soil moisture in the unsaturated zone and/or the water table depth (WTD) is not appropriate for the plant root system to absorb water and nutrients. In contrast, a drainage system must be used to control the excess amount of surface/groundwater in agricultural fields. However, intensive agricultural activities contaminate soil and water resources (Heinse & Link, 2013; Nimmo, 2009). When the contaminants get into the groundwater, it is not only harmful to human use but also difficult to control spreading into other areas. Therefore, studying groundwater dynamics is also important with SM measurements. The accuracy of WTD measurements over a large area can be improved by increasing the number of monitoring wells, but it is costly and time-consuming.

When using GPR, the accuracy of WTD estimations depends on the average radar velocity (v) from the surface to the water table (Doolittle et al., 2006; Johnson, 1992; Kowalczyk et al., 2018). The user should define v during data collection and processing within a range of 0.060 m/ns (wet) to 0.150 m/ns (dry) in common soils (Davis & Annan, 1989). Additionally, in a GPR radargram, a prominent and early reflection can come from the capillary fringe or the transition zone, but not from the actual water table (Daniels et al., 2004; Endres et al., 2000; Klenk, 2014; Pyke et al., 2008). These limitations affect the precision of GPR based WTD estimations. However, the importance of knowing radar wave velocity v of subsurface materials, influence of SM on the v, presence of capillary fringe, and tracing water table as a zone on the GPR radargram rather than a sharp boundary are well documented (Annan et al., 1991; Bano, 2006; Bevan et al., 2003; Endres et al., 2000; Loeffler & Bano, 2004; Nakashima et al.,
Number of studies related to water table monitoring using GPR have also been reported in agricultural practices and/or groundwater pollution studies during the last two decades (Conant et al., 2004; Corbeanu et al., 2002; Gish et al., 2002; Lambot et al., 2008; Lunt et al., 2005; Mahmoudzadeh et al., 2010; Talley et al., 2005; Tsoflias & Becker, 2008). The GPR method has also been used in water table studies in other applications such as studying preferential flows in mining (Grandjean & Gourry, 1996), groundwater dependent ecosystems (Molina-Sánchez et al., 2015; Zurek et al., 2015), and civil/structural engineering works (Slowik, 2013).

1.1.1 Rationale of the study

Long-lasting winter permits only one crop season in Newfoundland and Labrador (NL), Canada (Kavanagh, 2014). High soil acidity, and low soil organic matter, cation exchange capacity, nutrients retention and water holding capacity, and poor soil structure with sandy soils are the other major barriers for agricultural production in the region (Department of Fisheries and Land Resources, n.d.). A recent hydrological modelling in western Newfoundland soils showed lower saturation, quicker hydrological responses, and higher infiltration and percolation rates in podzolic soils when compared with most other agricultural soils (Altdorff et al., 2017). Low intensity of farming due to these circumstances has cut down farmers’ profits and trends towards agriculture.

Studies have been carried out and ongoing for introducing winter crops (Kavanagh, 2014), alternative fruits and vegetables, forage mixtures for yield and
quality (Haverstock, 2014), different soil amendments, and integrated pest management (Madore, 2014). Currently, the Government of Newfoundland and Labrador has engaged in long-term multidisciplinary research projects in collaboration with Memorial University to improve agricultural practices and production in NL. However, the impact of spatio-temporal variations of SM and fluctuation of WTD during the growing period is not well understood in podzolic soils in NL. Without this knowledge, irrigation and drainage practices cannot be improved for increasing the agricultural and water productivity. In addition, groundwater fluctuation and underground flow paths are not well studied at the regional scale to achieve better WTD management and reduce groundwater contamination threats under different agricultural management practices. The present study was conducted to fill this knowledge gap.

1.1.2 Objectives

Therefore, the main goal of this study was to monitor the spatio-temporal variation of SM and WTD using the GPR technique throughout the growing season. Following main objectives were considered in order to accomplish the main goal. These two objectives were achieved by two separate research studies as discussed in the next two chapters. Specific objectives with respect to each study are given in the relevant chapter.

1. To establish a methodology to estimate the spatio-temporal variation of SM in the agricultural root zone using GPR hyperbola analysis.

2. To distinguish the capillary fringe reflection in a GPR profile for precise WTD estimation.
1.1.3 Thesis organization

This thesis encompasses four chapters, including two chapters as research papers.

Chapter One: starts with an overview, rationale of the study and overall objectives. It also provides the theoretical background of GPR for studying SM and WTD.

Chapter Two: establishes a methodology to estimate SM in the crop root zone. This also includes a comparison of the sample area of TDR- and GPR-based SM determination.

Chapter Three: focuses on precise WTD estimation. This chapter shows the capability of WTD estimation using GPR as well as the challenges of distinguishing the water table reflection from the capillary fringe reflection. The proposed method provides a good augment of precise WTD estimation using GPR.

Chapter Four: includes the general discussion, conclusions, and further directions.

1.2 Theoretical Background

1.2.1 Basic principles of GPR

Estimation of SM and WTD from GPR are based on the transmission and reflection of an electromagnetic wave that is transmitted to the studied medium (Chanzy et al., 1996). The transmitter antenna (Tx) generates a short pulse of radar wave which
propagates away in a broad beam (Daniels et al., 2004). Electrical properties of the subsurface cause part of the transmitted signal to be reflected, refracted and attenuated. The reflected signal is detected by the receiver antenna (Rx) (Davis & Annan, 1989). As indicated by Figure 1.1, several radar waves may reach the Rx, and wave propagation geometry may vary according to the survey method (Du & Rummel, 1994; Huisman et al., 2003; Paz et al., 2017). The direct ground wave directly propagates from the Tx to the Rx through the ground. The airwave directly propagates through the air between the Tx and Rx. The reflected or refracted waves characterize energy returned directly at a boundary where different electrical properties are encountered.

![Figure 1.1: Electromagnetic wave propagation paths with respect to different GPR survey methods (after Paz et al., 2017).](image)

1.2.2 SM estimation from the GPR velocity

SM can be expressed as volumetric water content, $\theta_v$, ($\theta_v = \text{soil bulk density} \times \theta_m$) that can be defined as the ratio of the volume of water, $v_w$, to the total soil volume, $v_t$ as given in Equation 1.1 (Swiss Federal Institute of Technology, 2017).
\[ \theta_v = \frac{v_w}{v_t} \]  \hspace{1cm} (1.1)

The relative permittivity (\( \varepsilon_r \)) also known as the dielectric constant of a material is the ratio between its absolute permittivity (\( \varepsilon \)) and the permittivity of a vacuum (\( \varepsilon_0 \)) as given in Equation 1.2. Permittivity is a material property that affects the Coulomb force between two point charges in a material (IEEE Standard Board, 1998).

\[ \varepsilon_r = \frac{\varepsilon}{\varepsilon_0} \]  \hspace{1cm} (1.2)

Radar signal velocity (\( v \)) of a non-magnetic and low-loss geological material is related to \( \varepsilon_r \) and the electromagnetic wave propagation velocity in free space (\( c = 0.3 \text{ m/ns} \)) as given in Equation 1.3 (Davis & Annan, 1989; Neal, 2004; Schmelzbach et al., 2012).

\[ v = \frac{c}{\sqrt{\varepsilon_r}} \]  \hspace{1cm} (1.3)

Several empirical and mixing models have been developed to relate SM to \( \varepsilon_r \) (Jones & Or, 2002). The relationship between \( \varepsilon_r \) and \( \theta_v \) (Eq. 1.4) was suggested by Topp et al. (1980) is widely being used in vadose zone hydrology.

\[ \theta_v = -5.3 \times 10^{-2} + 2.92 \times 10^{-2}\varepsilon_r - 5.5 \times 10^{-4}\varepsilon_r^2 + 4.3 \times 10^{-6}\varepsilon_r^3 \]  \hspace{1cm} (1.4)

Root mean square error (RMSE) is a measure of the differences between the predicted values by a model or an estimator and the observed values and can be calculated by Equation 1.5.

To evaluate the accuracy of GPR-based \( \theta_v \) estimations by
comparing with gravimetric SM as the standard method or with TDR as the commonly used indirect method, RMSE is generally used.

\[ \text{RMSE} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (p_i - o_i)^2} \]  

(1.5)

where \( p_i \) is the predicted value, \( o_i \) is the observed value and \( N \) is the sample size.

1.2.3 Water table determination from the GPR

The amount of pore spaces filled with water generally increases with the depth of a soil profile and reaches the zone of saturation (phreatic zone). The upper surface of the zone of saturation is called water table and, it reflects more than 40% of GPR wave energy (Kowalczyk \textit{et al.}, 2018) in coarse-grained soils. Accordingly, water table can give continuous, mostly flat reflections with high amplitude in GPR radargrams (Davis & Annan, 1989; Greaves \textit{et al.}, 1996; Van Overmeeren, 1998) and has become an essential method in groundwater studies (Beres & Haeni, 1991; Tsolias \textit{et al.}, 2001).

The real part of the dielectric constant (\( \varepsilon_r \)) and the electrical conductivity (\( \sigma \)) control the propagation of EM waves. Attenuation coefficient (\( \alpha \)) can be derived using Equation 1.6 (Davis & Annan, 1989; Neal, 2004; Algeo \textit{et al.}, 2016).

\[ \alpha \approx \frac{1}{2c\varepsilon_0} \frac{\sigma}{\sqrt{\varepsilon_r}} \]  

(1.6)

Additionally, Bentley & Trenholm (2002) have described that capillary fringe reflection has the opposite to that of airwave. Doolittle \textit{et al.} (2006) reported that oscillations in the reflected radar pulse due to water table result in a series of bands to represent the water table in a radar profile.
The normal incidence reflection coefficient (R) is (Davis & Annan, 1989),

\[
R = \frac{\sqrt{\varepsilon_{r_1}} - \sqrt{\varepsilon_{r_2}}}{\sqrt{\varepsilon_{r_1}} + \sqrt{\varepsilon_{r_2}}}
\]  \hspace{1cm} (1.7)

where the dielectric permittivity above and below the water table reflection are \(\varepsilon_{r_1}\) and \(\varepsilon_{r_2}\), respectively.

### 1.2.4 Assumptions and limitations

It is assumed in GPR analysis that propagation and reflection of electromagnetic waves occur instead of induction in a material. Because GPR operating frequency bandwidth exceeds the material’s transition frequency and displacement currents are dominant in that frequency spectrum (Davis & Annan, 1989). However, this assumption is not acceptable for the materials with high magnetic permeability.

GPR wave propagation is three-dimensional. However, most GPR applications assume that wave propagation in two-dimensions to simplify the analysis. Under natural soil conditions where anisotropic soil matrix is present, this assumption would not be fully accurate.

Empirical derivations in most GPR studies consider dielectric and lossless or low-loss materials. GPR would not be used as a reflection imaging technique if this assumption is not applicable.

### 1.3 Potential impacts of the study

At the global scale, the world is moving to unpredictable climate changes. Expected extreme weather events may cause food security challenges due to land and water degradation. Local food production is the best option to achieve food security in
the region. Natural barriers of inherent soil properties on agriculture in NL have already created a lack of crop production in the province. Therefore, more challenges may happen in the future.

Maintaining the required SM in the root zone results in optimum crop yields. A profitable agricultural industry may invest in novel technologies, research, and developments to increase the local crop production. In contrast, intensive agriculture may cause groundwater pollution. Implementing the best possible water management with a better understanding of groundwater fluctuation of the region will reduce these threats. As a result, sustainable use of land and water resources would be ensured for future generations.

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CO-AUTHORSHIP STATEMENT

The research objectives were conceived and defined under the supervision of Dr. Lakshman Galagedara (principle supervisor), who also provided the funding for the research. Dr. Mumtaz Cheema (co-supervisor) and Dr. Adrian Unc (committee member) were also collaborators of the same research project on “Hydrogeophysical Characterization of Agricultural Fields in Western Newfoundland using Integrated GPR-EMI”, which was led by Dr. Galagedara.

I, Chameera Illawathure as the thesis author, was responsible for data collection, analysis and data interpretation. I personally wrote the first draft of my thesis including the two main chapters (2 & 3) in manuscript format that make up the core of my thesis. Dr. Galagedara provided research plans and guidance for the entire fieldwork. He also guided the data processing and analysis, and comprehensively reviewed and revised my thesis. Dr. Cheema and Dr. Unc reviewed the entire thesis and steered me in the right direction of how to analyze my raw data and the possible output of my data presentation.
CHAPTER 2

Soil moisture estimation from GPR hyperbola fitting in the
agricultural root zone
2.1 Introduction

More than 60% of the root biomass of most temperate crops exists within the upper 30 cm of the soil profile (Fan et al., 2016). Soil moisture (SM) in this upper soil layer is a key factor that determines plant growth through water and nutrient uptake from the soil (Seneviratne et al., 2010). SM also plays a major role in the hydrological cycle as it regulates the distribution of precipitation between storm runoff and water storage (Burns, 1974). Knowledge of the SM in shallow root zones is also useful to optimize sowing depth. SM shows both spatial and temporal variations. Therefore, it is challenging to measure SM with respect to time and space at the field scale, especially in the shallow root zone.

Ground Penetrating Radar (GPR) is capable for large-scale, non-destructive estimation of SM (Davis & Annan, 1989; Galagedara et al., 2003; Huisman et al., 2003; Lambot et al., 2004; Minet et al., 2012; Tran et al., 2014). The direct groundwave method for determining SM using GPR is well established but has limitations to be applicable for the root zone (Galagedara et al., 2005b). Some of the limitations are difficulties of distinguishing the airwave from groundwave and, shallow penetrating depths (Galagedara et al., 2005a; Galagedara et al., 2005b). Further, manual picking of the leading edge of the groundwave is associated with errors, and even automatic picking cannot be performed efficiently (Galagedara et al., 2003). The lack of a practical approach to determine SM in the shallow agricultural root zone impedes non-destructive, portable and time-effective advantages of GPR.

The two-way travel time of GPR reflected wave could be used to estimate the SM of a large area in natural field conditions (Lunt et al., 2005). Reflection from a subsurface point reflector (i.e., a drainage pipe or a piece of rock) can trace out a hyperbola
in a GPR radargram. The depth and the shape of the object, and the matrix (Maas & Schmalzl, 2013) influence the shape of the hyperbola. SM can be estimated through the reflected wave velocity and corresponding permittivity of the soil matrix if the depth of reflection is well known (Davis & Annan, 1989; Huisman et al., 2003; Topp et al., 1980). The estimated wave velocity roughly represents an average value of the soil layer from the ground surface to the reflector. Information about the soil volume of which the hyperbola analysis could optimally describe propagation velocity has not been well documented. The estimated moisture from a hyperbola analysis can be validated using a commonly used indirect method such as Time Domain Reflectometry (TDR) (Galagedara et al., 2005; Huisman et al., 2001). Gravimetric sampling, which is the direct method, is not feasible due to the comparatively large sample volume of GPR (Dobriyal et al., 2012) and due to its destructive nature, which hinders sample repeatability.

Full-wave inverse modeling of the GPR data has demonstrated its benefits in terms of accuracy and automatization for SM mapping (Lambot et al., 2004; Lambot & André, 2014; Minet et al., 2012; Tran et al., 2014). The method relies on a 3D electromagnetic model accounting for the radar-antenna-medium system and its inversion to retrieve the medium permittivity and correlated moisture. This approach requires a specific calibration of the radar instrument. The method has not been made available yet, but is expected to be released soon as an online tool on www.gprsensing.com (S. Lambot & A. De Coster, Université catholique de Louvain, Belgium).

On the other hand, TDR is a commonly used indirect method to measure SM, but still has a comparatively smaller sample volume than GPR (Topp et al., 2008).
Therefore, an adequate number of TDR measurements are needed to represent the GPR sample volume. The knowledge of the TDR sample volume that covers the GPR sample volume is essential for GPR data validation. Ferré et al. (1998) described the TDR sample volume as a cylindrical shape that contributes to the overall SM measurement. This cylindrical soil volume has a sample radius as a function of the TDR probe separation distance and height equivalent to the length of the TDR probe (Ferré et al., 1998). The soil outside the sample volume has no significant input to the total TDR probe response. Introducing TRIME (Time-domain Reflectometry with Intelligent Micro Elements) technology, current TDR probes can measure larger sample volumes than conventional TDR probes (IMKO Micromodultechnik, Germany).

Similar to TDR, the volume of the porous medium that contributes to the GPR wave velocity estimation varies with the configuration of each GPR system. The GPR characterization scale is also influenced by the soil properties themselves. In addition, the GPR sample volume and the resolution of its moisture estimation mainly differs with different survey parameters and different center frequencies (Galagedara et al., 2005). Medium to high frequency (e.g., 250, 500 and 1000 MHz) GPR instruments are sold as transducers in which antennas and electronics are assembled to optimize the performance (Sensors & Software Inc., Canada). They are relatively small and therefore ideal for use throughout the growing season even with the presence of crops. Employing the appropriate GPR frequency with a feasible survey type for data acquisition increases the accuracy of the results as well as the efficiency of the method.

The traditional GPR hyperbola method for SM estimation is applicable only if point reflectors are present in the soil being studied and if clear hyperbolas are visible in the radargram. However, finding shallow hyperbolas in a radargram is a major
limitation of this method for use as a practical tool for rapid SM estimation in the root zone. Some recent research has focused on increasing the efficiency of the hyperbola fitting method by minimizing the human error associated with identifying and fitting hyperbolas. Techniques such as MMSA (Multiresolution Monogenic Signal Analysis) are useful to detect hyperbolas accurately (Qiao et al., 2015). Real-time detection and fitting of hyperbolas have also been recently investigated (Dou et al., 2017). An automated hyperbola detection algorithm has been calibrated to determine the existence of a hyperbola by the ‘ambiguity zone for the human brain,’ which is a relatively fast method compared to other methods (Mertens et al., 2016). Instead of using the traditional hyperbola fitting method in commercially available software, a modified algorithm such as MMSA would increase the accuracy and efficiency of the proposed GPR method. As noted in the recent review of current GPR-based SM measurement methods by Liu et al. (2017), the automated recognition of hyperbolic signals should not only improve the efficiency of the method, but it eliminates the need to know the depth of the reflector. To avoid the dependability of the hyperbola fitting on different analysts (Sham & Lai, 2016), we followed a systematic guideline for GPR data processing. We hypothesized that the same accuracy for SM estimation in the upper 30 cm of the soil profile of an experimental plot could be achieved by analyzing hyperbolas regardless of the depth from 27-50 cm.

Motivated by the recent advancements of the GPR hyperbola analysis and challenges encountered in the field, we aimed at evaluating a straightforward GPR method to estimate SM in the shallow agricultural root zone. TDR data were used as the reference to evaluate GPR data with special consideration given to sample geometry of both methods, which has not yet been well documented. For that purpose, a
systematic TDR data collection was introduced to coincide with the GPR sample area. Consequently, the objectives of this study were to use hyperbola fitting method to; (i) determine the soil volume that hyperbolic reflections describe in terms of radar wave velocity and (ii) evaluate a practical GPR-based approach to estimate SM within the upper 30 cm of the soil profile.

2.2 Theoretical Background

The GPR wave propagation velocity \( (v_{rw}) \) for a monostatic antenna can be estimated by the depth to the known reflector method (ASTM D6432-11; Daniels et al., 2005). Limitations are: depth is not known in most cases, and the two-way travel time \( (t_{rw}) \) of radar wave only at the apex of the hyperbola is used for the calculation. Velocity sounding method can be used to determine the radar velocity in a multiple antenna-offset survey (Dix, 1955).

2.2.1 Traditional hyperbola fitting method

![Diagram of traditional hyperbola fitting method](image)

Figure 2.1: Arrow-headed lines represent the wave propagation paths assuming straight-ray propagation.
Equation 2.1 determines the average $v_{rw}$ from the ground surface to a sub-surface point reflector for $t_{rw}$ with respect to horizontal distance from the reflector to the antenna, $x$ (Fig. 2.1) (Huisman et al., 2003).

$$v_{rw} = \frac{\sqrt{(x - 0.5a)^2 + d^2} + \sqrt{(x + 0.5a)^2 + d^2}}{t_{rw,x}}$$  \hspace{1cm} (2.1)$$

Where $a$ is the antenna separation and $d$ is the depth to the top of the reflector.

### 2.2.2 Modified hyperbola detection algorithms

Assuming that (i) the electromagnetic properties of the soil above the buried reflector do not vary, (ii) wave propagation follows straight ray paths, (iii) the antenna and antenna-medium coupling do not affect observed propagation times, (iv) the target reduces to a point and (iv) the antenna is at the soil surface, the two-way travel times measured over a point reflector in a monostatic configuration of GPR antennas follow a hyperbolic function (e.g., Mertens et al., 2016). Equation 2.2 which is a general shape of a hyperbola can be derived using the Pythagorean theorem. Hyperbola parameters appearing in Equation 2.2 are illustrated in Figure 2.2.

$$\frac{t^2}{t_0^2} - \frac{|y - y_0|^2}{\frac{t_0^2 v_{rw}^2}{4}} = 1$$  \hspace{1cm} (2.2)$$

Where $v_{rw}$ is the reflected wave velocity in the soil, $a$ and $b$ in Figure 2.2 are $t_0$ and $(t_0, v_{rw}/2)$, respectively. Therefore, $v_{rw}$ can be estimated for a known $b$ and position of the apex of the hyperbola.
Equation 2.2 can be modified accounting for the radius of the reflector for a common-offset method (CO) with a significant antenna separation (Sham & Lai, 2016). Still, most of the computational algorithms currently used are based on the assumption that the reflector is a point. Otherwise, the calculation would be more complicated. It is, however, worth noting that neglecting the radius or shape of the target is expected to result in errors in the permittivity estimates (Sham & Lai, 2016).

2.3 Method Development

2.3.1 Site description and experimental setup

A field study was conducted in a flat experimental plot of 15 m × 4 m (49.073 N, 57.561 W) at Pynn’s Brook Research Station, Pasadena, NL, Canada (Fig. 3). Eroded and dissected lacustrine, glacial veneer and organic deposits are dominant with a shallow unconfined sandy aquifer (Newfoundland & Labrador Geological Survey, 2014). Shallow soil at the adjacent cornfield is gravelly loamy sand (sand 82.0 ± 3.4%, silt 11.6 ± 2.4% and clay 6.4 ± 1.2%) (Badewa et al., 2018). Soil textural analysis (n=8) at the experimental plot revealed that the shallow soil layer up to a depth of 50 cm is loamy sand which is the same as the 0-15 cm soil profile as described by Sadatcharam
Laboratory tests for determining basic soil properties were carried out at Boreal Ecosystem Research Facility (BERF) of Grenfell Campus-Memorial University of Newfoundland, Corner Brook, NL, Canada.

### 2.3.2 Data acquisition

Data collections were carried out in 2017 (September 22, October 3, 20, 24) and 2018 (June 01 and 29). Both CO and Common mid-point (CMP) methods (Davis and Annan, 1989; Huisman et al., 2003) were used for GPR data acquisition. Twelve CO surveys were carried out using 250, 500 and 1000 MHz center-frequency transducers of the PulseEKKO® Pro GPR system (Sensors and Software Inc., Canada) after burying eight different types of point reflectors ranging from 27-70 cm depth below the surface. Buried materials were; three hollow metals, one aluminum can and four plastic bottles filled with three different concentrated salt solutions and one with tap water. Sizes of the reflectors were from a maximum of 4.5 l (30 cm × 15 cm × 10 cm) and to a minimum of 2 l plastic water bottle. These reflectors were buried along two straight lines 2 m apart (Fig. 2.3). GPR grid surveys were performed at 0.5 m spacing resulting in nine grid lines in total and, each grid line was 15 m in length. Two survey lines of each grid survey were done exactly along the traces of the buried objects (Fig. 2.4a).

CMP surveys using 500 MHz transducers were also carried out at each buried location (October 20 and 24, 2017). Thirteen systematic SM measurements were collected using vertically installed 30 cm long TRIME-PICO64 TDR probes (IMKO Micromodultechnik, Germany) at the center and 10, 20 and 30 cm radii from top of each reflector together to coincide with the CMP surveys (Fig. 2.4b). Four TDR measurements at each radius were collected at 0°, 90°, 180° and 270° with respect to
the GPR survey direction and one TDR measurement was collected exactly on top of
the reflector (Fig. 2.4b). Figure 2.5 illustrates the vertical cross-sectional sample area
of systematic TDR data collection over a buried reflector.

![Diagram of experimental setup](image)

Figure 2.3: Plan view of the experimental setup; R₁-R₈ are buried reflectors (not to scale).

Before winter started, buried reflector locations were marked using plastic
pegs, and the level of the ground surface was marked in each peg. In 2018, two field
campaigns were performed after the spring thaw. In each day, two GPR lines along the
buried reflectors were carried out using all three frequencies with systematic TDR data
collection as before. Then buried pits were opened up carefully to verify the depth and
positions of the reflectors.
Figure 2.4: (a) A GPR CO survey over the buried objects using 1000 MHz center-frequency transducer fixed to an odometer. (b) A systematic TDR data collection at a buried location; measuring tapes cross at top of the buried object where the center TDR measurement was performed. Dashed arrow line indicates the GPR survey direction.

Figure 2.5: Vertical cross section of systematic TDR sampling area over a buried reflector. Sample area of the center TDR probe is shown by the shaded area. Distances represent TDR locations from the center along the transect (see also Fig. 4b).
2.3.3 TDR probe calibration

Twenty-one (21) TDR measurements were collected by vertically installed 30 cm long TDR probes along one GPR transect at each buried object. Just after each TDR measurement, soil samples (2.5 cm in diameter) were collected from 0-30 cm depth using a soil auger (ASTM D1452) from the same location \((n=6)\). Soil bulk density was measured using undisturbed samples collected at the middle and two ends of the experimental plot (ASTM D7263-09). Gravimetric soil moistures were determined by the oven drying method, 105°C for 24h, and then converted to volumetric SM \((\theta_v)\) using the average bulk density of the soil.

2.3.4 GPR data processing

GPR data processing was done using EKKO Project V3 R1 software (Sensors and Software Inc.). SM at each buried location was estimated through the calculation of the relative permittivity \((\varepsilon_r)\) (Topp et al., 1980) using \(v_{tw}\) obtained from GPR. The data processing methodology is shown in Figure 2.6. The procedure started by checking the major deflections of GPR traces in a particular radargram. The flow chart shown in Figure 2.6 is for a positive first major deflection. If it was negative, re-picking was done with the negative transition, and the negative edges were picked by automatic time picking. When the deflection was not clear and difficult to make a decision, both transitions and relevant auto picking criteria were used. Each trace was shifted to align first-breaks in a line (Fig. 2.7).
Figure 2.6: Systematic guidelines for GPR data processing (steps illustrated by dashed lines were not followed for this analysis)
Figure 2.7: Positive edges of both first-break and hyperbolic reflection are picked by automatic time picking after a successful re-pick processing.

If the first break is a positive deflection, all other later reflection events were picked at the positive edge to obtain the actual reflected wave travel time. After this re-picking procedure, each data file was exported, and the time-zero picking was checked by automatic time picking of the positive edge of the airwave. Time zero method described by Galagedara et al. (2003) was used for time-zero calibration and, to minimize the error of time-zero drift. Locations of the apex of the hyperbola by automatic picking were used to minimize the subjective error of manual hyperbola fitting.

### 2.3.5 Testing the hypothesis

A 30 cm long TRIME-PICO64 TDR probe has a sample radius of 5 cm in the plane perpendicular to the probe length (IMKO Micromodultechnik, Germany). We considered a 2-dimensional projection of TDR and GPR sample volumes for the comparison of $\theta$, estimations from both methods. Therefore, seven TDR measurements, coincided with the GPR sample area for a buried location, were collected along the
GPR transect (Fig. 2.4b and Fig. 2.5). The total TDR sample area for a buried location that is projected to the GPR section is 70 cm × 30 cm = 2100 cm² (Fig. 2.5).

A GPR frequency of 500 MHz was used for CMP analysis. First, we compared GPR sample areas of ~30 cm deep hyperbolic reflections from CO and CMP methods. TDR data points along the survey line were only considered for the comparison. We used RMSE to compare GPR and TDR measurements. Only three hyperbolas at 27, 31 and 34 depths (to represent around 30 cm depth) were selected for the comparison in three survey days. Sample area for \( \theta_v \) estimations from CO and CMP methods are compared in Section 2.4.3. In the second step, we compared the CO survey method of 250 and 500 MHz for \( \theta_v \) estimations over ~30 cm deep hyperbolic reflections statistically using RMSE value and Mann-Whitney test (Hammer et al., 2001; Hettmansperger & Sheather, 1986).

To check the spatial representativeness of using GPR, we compared GPR estimated \( \theta_v \) with different criteria used to average the TDR data. The TDR averaging criteria used were: (i) TDR1: all TDR measurements (30 cm radius) collected at a buried location \((n=13)\); (ii) TDR2: up to 20 cm radius including the center \((n=9)\); (iii) TDR3: only 10 cm radius \((n=4)\); (iv) TDR4: only at 20 cm \((n=4)\); (v) TDR5: only at 30 cm \((n=4)\); (vi) TDR6: across the transect \((n=7)\); and (vii) TDR7: along the transect \((n=7)\).

Finally, to check the hypothesis, we obtained the \( \theta_v \) estimations by hyperbola fitting regardless of the depth of the hyperbola (27 – 50 cm depth range). For the comparison of GPR-estimated and TDR-measured \( \theta_v \), we used above-mentioned statistical methods.
2.4 Results and Discussion

2.4.1 Probe calibration results

Average soil bulk density of 0-30 cm soil profile was 1.39 (±0.09) g cm\(^{-3}\) \((n=6)\). TDR measured \(\theta_v\) in m\(^3\) m\(^{-3}\) \((\text{min}=0.1034, \text{max}=0.1798, \text{median}=0.1298; n=21)\) and gravimetrically measured \(\theta_v\) in m\(^3\) m\(^{-3}\) \((\text{min}=0.0907, \text{max}=0.1618, \text{median}=0.1239; n=21)\) were within a RMSE of 0.0161 m\(^3\) m\(^{-3}\) (see also Appendix 2.1). Average bulk density of the soil \((n=28)\) and the RMSE of \(\theta_v\) estimation \((n=10)\) between TDR and gravimetric methods of a shallow soil were 1.31 g cm\(^{-3}\) and 0.018 m\(^3\) m\(^{-3}\), respectively, in a previous study conducted at the same site (Badewa et al., 2018). Table 2.1 compares different TDR criteria used to average \(\theta_v\) measurements at a buried location.

All the averaging criteria agree with the overall average (TDR1) having medium to high positive correlations with TDR1 and TDR2 showing the highest correlation. TDR5 has the lowest correlation with the overall average. TDR2 also shows a high correlation with other criteria except for TDR5.

Table 2.1: Pearson correlation coefficients \((p\text{-values within brackets})\) of TDR measurements with different criteria used to average the TDR data

<table>
<thead>
<tr>
<th></th>
<th>TDR1 ((n = 13))</th>
<th>TDR2 ((n = 9))</th>
<th>TDR3 ((n = 4))</th>
<th>TDR4 ((n = 4))</th>
<th>TDR5 ((n = 4))</th>
<th>TDR6 ((n = 7))</th>
</tr>
</thead>
<tbody>
<tr>
<td>TDR2</td>
<td>0.940 ((n = 9)) (0.000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TDR3</td>
<td></td>
<td>0.837 ((n = 4)) (0.000)</td>
<td>0.940 ((n = 4)) (0.000)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TDR4</td>
<td></td>
<td></td>
<td>0.845 ((n = 4)) (0.000)</td>
<td>0.863 ((n = 4)) (0.000)</td>
<td>0.650 ((n = 4)) (0.016)</td>
<td></td>
</tr>
<tr>
<td>TDR5</td>
<td></td>
<td></td>
<td></td>
<td>0.714 ((n = 4)) (0.006)</td>
<td>0.432 ((n = 4)) (0.140)</td>
<td>0.293 ((n = 4)) (0.332)</td>
</tr>
<tr>
<td>TDR6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.907 ((n = 7)) (0.000)</td>
<td>0.851 ((n = 7)) (0.000)</td>
</tr>
<tr>
<td>TDR7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.936 ((n = 7)) (0.000)</td>
</tr>
</tbody>
</table>
2.4.2 Error estimation of the hyperbola fitting method

Time related to apexes (Fig. 2.8) of the hyperbolas \((n=59)\) in all three frequencies for automatic picking (median=7.156 ns) and hyperbola fitting (median=7.410 ns) were not significantly different \((p-value=0.6167)\) at the 95% significance level of the Mann-Whitney test. Radar wave velocities obtained by the hyperbola method are close to the theoretical velocities calculated by Equation 2.1 (Appendix 2.2). The RMSE between the actual and the hyperbola method for depth and position of buried objects are within 0.05-0.06 m for all frequencies (Table 2.2; see also Appendices 2.2-2.4). The data processing procedure presented in Section 2.3.4 is associated with these RMSE levels.

Figure 2.8: A hyperbola traced out in a GPR radargram (left); GPR traces exported to automated time picking software and positive edges of the airwave and reflected wave are picked (middle); magnified view (right). Arrows are aligned exactly on top of the reflector and two-way travel time corresponded to that trace (trace number 696 in this example) was substituted into Equation 1 after time-zero correction for the velocity calculation.
Table 2.2: The root mean square error (RMSE) between hyperbola fitting and automatic time picking (for theoretical calculations) for details of the apex of the hyperbola

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>N</th>
<th>Depth (m)</th>
<th>Position (m)</th>
<th>Velocity (m/ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>11</td>
<td>0.05</td>
<td>0.06</td>
<td>0.02</td>
</tr>
<tr>
<td>500</td>
<td>33</td>
<td>0.05</td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>250</td>
<td>19</td>
<td>0.06</td>
<td>0.06</td>
<td>0.01</td>
</tr>
</tbody>
</table>

The RMSE of hyperbola-derived velocity was 0.01-0.02 m/ns when compared with the theoretical velocity. A difference of 0.02 m/ns in radar velocity is equivalent to about 0.01 m$^3$m$^{-3}$ difference in $\theta_v$ in wet soils (within the range of 0.067-0.098 m/ns). In dry soils (within the range of 0.099-0.150 m/ns), a difference of 0.02 m/ns in radar velocity could make a maximum of 0.06 m$^3$m$^{-3}$ difference in $\theta_v$. However, the theoretical velocity is based on a measurement of exactly on top of the buried reflector. Hyperbola derived velocity is a measure of an average number of data points and represents an average value within the GPR sample area. Therefore, RMSE of 0.02 m/ns was accepted to proceed to $\theta_v$ estimation (Section 2.4.4).
2.4.3 GPR sample area of $\theta_v$ estimations with the hyperbola method

Figure 2.9: Cross section of GPR and TDR sample areas over a 30 cm deep reflector with systematic data collection; comparison of CO (above) and CMP (below). Note: transmitted wave in blue and reflected wave in red. T= Transmitter, R=Receiver.

Figure 2.9 compares sample areas of the TDR with CMP and CO methods for a 500 MHz survey in a default mode. Percentages of TDR sample area covered by the two GPR survey methods are listed in Table 2.3. The GPR sample area for $\theta_v$ estimation depends on the depth of the hyperbola used for $v_{rw}$ estimation (Fig. 2.10). Table 2.4 presents the percentage of 30 cm long TDR probes’ sample area covered by 30 cm and 70 cm hyperbola depths (70 cm depth was selected to show the full coverage of TDR sample area by GPR sample area).
Figure 2.10: Vertical cross-section of GPR and TDR sample areas over 30 cm deep (dotted triangle) and 70 cm deep (outside large triangle) reflectors with TDR sample area (dashed rectangle).

Table 2.3: Percentage of TDR sample area covered by two GPR survey methods (500 MHz)

<table>
<thead>
<tr>
<th>Sample area (cm²)</th>
<th>TDR sample area covered by GPR</th>
<th>Unrelated area out of total GPR sample area</th>
</tr>
</thead>
<tbody>
<tr>
<td>CMP 1200.0</td>
<td>49.8 %</td>
<td>12.9 %</td>
</tr>
<tr>
<td>CO 1837.5</td>
<td>71.4 %</td>
<td>18.4 %</td>
</tr>
</tbody>
</table>

Table 2.4: Percentage of 30 cm TDR sample area covered by two hyperbola depths

<table>
<thead>
<tr>
<th>Depth of hyperbola (cm)</th>
<th>TDR sample area covered by GPR</th>
<th>Unrelated area out of total GPR sample area</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>71.4 %</td>
<td>18.4 %</td>
</tr>
<tr>
<td>70</td>
<td>100 %</td>
<td>18.4 %</td>
</tr>
</tbody>
</table>

* Total TDR sample area (70 cm × 30 cm) = 2100 cm²
2.4.4 Hyperbola fitting using CMP and CO methods

The software for analyzing GPR data allows fitting half of the hyperbola traced out in a CMP survey output to estimate the average wave velocity to a selected reflector. The correct reflection event should be picked by careful observation. We considered 500 MHz surveys for CMP data analysis (Fig. 2.11c). Table 2.5 presents the RMSE of TDR measured $\theta_v$ vs. $\theta_v$ estimated by using CO and CMP hyperbola fitting methods for 500 MHz. Three hyperbola depths (<34 cm) selected for the comparison imply that CMP-hyperbola estimated $\theta_v$ is less accurate (RMSE = 0.05 m$^3$ m$^{-3}$) than CO-hyperbola (Fig. 2.11 a & b) estimated $\theta_v$ (RMSE = 0.02 m$^3$ m$^{-3}$) when compared to TDR measured $\theta_v$.

Table 2.5: Root mean square error (m$^3$ m$^{-3}$) of GPR-estimated and 30 cm TDR-measured $\theta_v$ for CO and CMP methods of 500 MHz GPR

<table>
<thead>
<tr>
<th>GPR Method</th>
<th>Sample size*</th>
<th>Average TDR $\theta_v$ along the transect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>10 cm (n=3)</td>
</tr>
<tr>
<td>CMP - hyperbola fitting</td>
<td>6</td>
<td>0.054</td>
</tr>
<tr>
<td>CO – hyperbola fitting</td>
<td>6</td>
<td>0.018</td>
</tr>
</tbody>
</table>

* Number of buried locations
Figure 2.11: Hyperbolas traced out from buried reflectors 1-3 (a) and 5-8 (b) in CO and all reflectors in CMP (c) surveys using 500 MHz on October 24, 2017.
2.4.5 Evaluation of a practical approach to estimate $\theta_v$ in the root zone

TRIME-PICO64 TDR probes provide reliable $\theta_v$ estimations at point-scale with an RMSE of 0.016 m$^3$ m$^{-3}$ when compared to the gravimetric method. In addition, the TDR measurements are consistent as Badewa et al. (2018) reported similar accuracy (RMSE = 0.018 m$^3$ m$^{-3}$) during the previous growing season at the same location. This accuracy has allowed TRIME-PICO64 TDR probes (especially in sandy soils) to be used as a common used indirect method of $\theta_v$ estimation (IMKO Micromodultechnik, 2017). Many researchers have argued the low representativeness of the point-scale measurements of TDR (Hignett et al., 2008). Therefore, apart from the advantage of the increased sample volume of TRIME-PICO technology (IMKO Micromodultechnik, Germany), a systematic TDR data collection was introduced in this study to be compatible with the large sample volume of the GPR hyperbola analysis method. However, the high spatial variability of the field water content, high gravel content and the artificial compaction due to human and machinery activities within very shallow depths can have a negative impact of the $\theta_v$ estimation using the proposed GPR method (Badewa et al., 2018).

We compared the sample areas of TDR and GPR considering a 1 m longitudinal section along the GPR transect. In the CO method, the default mode of 500 MHz records a GPR trace every 2 cm, so that there will be 51 data points for a 1 m long transect. If the antenna step size is 2 cm, CMP data collection will have 21 traces for a 1 m length. The sample area of a 500 MHz GPR-hyperbola for $\theta_v$ estimation over a point reflector at 30 cm depth is higher in the CO method (1837.5 cm$^2$) than the CMP method (1200.0 cm$^2$). Consequently, the CO method produced more accurate $\theta_v$ estimation (RMSE = 0.02 m$^3$ m$^{-3}$) than the CMP method (RMSE = 0.05 m$^3$ m$^{-3}$) with
500 MHz (Table 2.5). It should also be noted that the unrelated area considered for the \( \theta_v \) estimation is higher in CO than CMP, which could potentially increase the error in CO method under high spatial variability of \( \theta_v \). When the minimum antenna separation was used, 500 MHz had 11.25 cm (antenna separation/2) distance from the center to either transmitter or receiver. Since two antennas in a CMP survey were moving apart from the center, this initial cone-shaped soil volume of 11.25 cm radius and 30 cm height would not be accounted for in the velocity estimation (Fig. 2.8). Though the accuracy of estimated \( \theta_v \) by CMP-hyperbola would increase by increasing the survey distance, the maximum distance could be limited depending on the strength of the signal received by the receiver (Greaves et al., 1996). Therefore, despite the practicability of the CO-hyperbola fitting method, it also provides a reliable \( \theta_v \) estimation (RMSE = 0.02 m\(^3\) m\(^{-3}\)) along a survey line when the appropriate frequency and the survey parameters are used (Galagedara et al., 2003).

Selection of the best GPR frequency for estimating field \( \theta_v \) has always been a trade-off. Higher frequencies penetrate only shallow depths and give a higher resolution while low frequencies penetrate deeper but give a lower resolution. Our preliminary data showed that it would not be possible to use 1000 MHz transducers in such a field survey. Because of the low weight and smaller footprint (0.08 m × 0.25 m approx.) of 1000 MHz transducers, it would not guarantee the proper ground coupling, and they would result in noisy data due to the inherent surface roughness. In such cases, 250 MHz would be a better alternative. However, the resolution aspect plays a major role in the accuracy of \( \theta_v \) estimation. In a default mode 250 MHz (antenna separation = 38 cm) has 5 cm sampling interval whereas 500 MHz (antenna separation = 22.5 cm) has 2 cm sampling interval. For instance, 1 m length of a CO GPR transect (50 cm
horizontal distance to both sides of a buried reflector) will have 21 and 51 GPR traces for 250 MHz and 500 MHz, respectively. All the TDR averaging criteria agreed that 500 MHz had a lower error (max RMSE = 0.03 m$^3$ m$^{-3}$) than 250 MHz (RMSE = 0.06 m$^3$ m$^{-3}$) for hyperbola depths < 34 cm (Table 2.6). In most cases, agricultural fields have low spacing between crop rows, which do not allow operating long/larger instrumentation in-between them. Approximate transducer footprints of 250 MHz and 500 MHz are 0.30 m × 0.70 m and 0.15 m × 0.40 m, respectively.

Table 2.6: Statistical comparison of 30 cm long TDR average $\theta_v$ and GPR-hyperbola estimated $\theta_v$

<table>
<thead>
<tr>
<th>GPR frequency (MHz)</th>
<th>n</th>
<th>Test$^a$</th>
<th>TDR average</th>
<th>TDR1</th>
<th>TDR2</th>
<th>TDR3</th>
<th>TDR4</th>
<th>TDR5</th>
<th>TDR6</th>
<th>TDR7</th>
</tr>
</thead>
<tbody>
<tr>
<td>250</td>
<td>6</td>
<td></td>
<td></td>
<td>0.056</td>
<td>0.056</td>
<td>0.057</td>
<td>0.055</td>
<td>0.057</td>
<td>0.058</td>
<td>0.056</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(b)</td>
<td></td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
<td>0.081</td>
<td>0.005</td>
<td>0.005</td>
<td>0.005</td>
</tr>
<tr>
<td>500$^\dagger$</td>
<td>9</td>
<td></td>
<td></td>
<td>0.023</td>
<td>0.024</td>
<td>0.026</td>
<td>0.022</td>
<td>0.023</td>
<td>0.023</td>
<td>0.024</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(b)</td>
<td></td>
<td>0.427</td>
<td>0.348</td>
<td>0.653</td>
<td>0.236</td>
<td>0.391</td>
<td>0.270</td>
<td>0.178</td>
</tr>
<tr>
<td>500$^\ddagger$</td>
<td>9</td>
<td></td>
<td></td>
<td>0.036</td>
<td>0.041</td>
<td>0.045</td>
<td>0.037</td>
<td>0.027</td>
<td>0.033</td>
<td>0.039</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(b)</td>
<td></td>
<td>0.185</td>
<td>0.057</td>
<td>0.185</td>
<td>0.133</td>
<td>0.426</td>
<td>0.157</td>
<td>0.157</td>
</tr>
<tr>
<td>500$^\S$</td>
<td>18</td>
<td></td>
<td></td>
<td>0.030</td>
<td>0.033</td>
<td>0.037</td>
<td>0.031</td>
<td>0.025</td>
<td>0.029</td>
<td>0.032</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(b)</td>
<td></td>
<td>0.117</td>
<td>0.079</td>
<td>0.159</td>
<td>0.069</td>
<td>0.200</td>
<td>0.103</td>
<td>0.079</td>
</tr>
</tbody>
</table>

As per the limitations mentioned in Section 2.4.4, we evaluated a practical approach of 500 MHz to estimate $\theta_v$ within the agricultural root zone (30 cm). Eighteen hyperbolas ranging from 27-50 cm depth that collected from three field campaigns were used for the comparison. Statistical comparison of GPR-estimated $\theta_v$ and TDR-

---

$^a$ (a) Root mean square error (RMSE) / (m$^3$ m$^{-3}$), (b) Mann-Whitney test, p-value
$^\dagger$ hyperbola depths = 27, 31, 34 cm
$^\ddagger$ hyperbola depths = 40, 45, 50 cm
$^\S$ hyperbola depth range = 27-50 cm
measured $\theta_v$ is summarized in Table 2.6. The \textit{p-values} for the Mann-Whitney test are less than the significance level ($\alpha = 0.05$) in all cases except TDR4 for 250 MHz. Therefore, the data support the hypothesis that there is a difference between the population medians. Meanwhile, the \textit{p-values} for Mann-Whitney test for all cases tested are higher than the significance level ($\alpha = 0.05$) for 500 MHz. Therefore, the data support the hypothesis that there is no difference between the medians of GPR and TDR measured $\theta_v$.

Reflections from the shallow hyperbolas do not represent the complete TDR sample area (Fig. 2.10). If the hyperbola depths were more than 70 cm, 100% of the 30 cm TDR sample area would be covered by GPR (Table 2.4). However, additional unrelated sample area would be added to the GPR sample area. Unrelated sample area within 0-30 cm out of the total GPR sample area was the same whether hyperbola depth was 30 or up to 70 cm. Nevertheless, additional larger sample area was added for GPR $\theta_v$ estimation from deeper hyperbola. Even taking advantage of the fully covered TDR sample area, additional sample area from 30-50 cm layer would significantly affect the total GPR response. This sampling area coverage resulted in a lowered RMSE (max 0.04 m$^3$ m$^{-3}$) for deeper hyperbolas.

Further, this RMSE level was higher than the RMSE error obtained by Lunt \textit{et al.} (2005) for $\theta_v$ estimations from a GPR reflection study. They used a calibrated neutron probe to compare the $\theta_v$ estimations of GPR and acquired RMSE of 0.018 m$^3$ m$^{-3}$. However, they conducted the survey using 100 MHz antennas and used detailed borehole information for their analysis, which would not always be practical in an agricultural field. It should be noted that the sources of errors associated with the proposed method are not only due to the experimental setup but also due to the
assumptions based on straight-ray wave propagation. Estimating the $\varepsilon_r$ using a mixing model such as Topp’s equation (Topp et al., 1980) may also readily result in 1-2% errors in terms of absolute moisture (Mukhlisin & Saputra, 2013).

The need for prior knowledge of the shallow soil profile might be a constraint of this proposed method (Grote et al., 2002). Also, this method might not be applicable if texturally different soil layers exist within the shallow depths. As mentioned by Lunt et al. (2005), transient reflector geometry and inconsistency of GPR reflection amplitude under wet conditions are also challenges for the accuracy of this method. The proposed approach can be used to estimate $\theta_i$ in the root zone with a priori knowledge such as geological strata, the presence of shallow soil horizons, and the thickness of $A_P$ horizon in the shallow soil profile. CO surveys carried out in a silage-corn field adjacent to the experiment plot using the same 500 MHz transducers in the same configuration showed both well- or ill-shaped shallow hyperbolas in a radargram without burying reflectors. Integrated or segregated application of modern hyperbola fitting algorithms (Dou et al., 2017; Mertens et al., 2016; Qiao et al., 2015) would increase the efficiency and the accuracy (avoiding the subjective error of hyperbola fitting) of this proposed method. The capability of this approach to be used in the spatial and temporal variation of $\theta_i$ in managed agricultural fields for efficient water management will be our next research question.

### 2.5 Conclusion

We proposed a noninvasive practical approach to estimate SM rapidly in shallow soils with relatively large samples volumes. GPR data were evaluated using a systematic TDR data collection with special reference to sampling geometry of both
methods. According to our understanding, this approach has not yet been discussed in the literature. Besides, the subjective error of hyperbola fitting was minimized by following a data processing guideline. Our field experiment revealed that SM measurement using the proposed hyperbola fitting method agreed with 30 cm long TDR probe data with an RMSE of 0.02 m$^3$ m$^{-3}$ for shallow (< 34 cm), and an RMSE of 0.04 m$^3$ m$^{-3}$ for deeper hyperbola analysis. We calculated the representative sample areas of GPR CO and CMP methods with respect to TDR sample area. Reflections over a 30 cm deep point reflector along a 1 m GPR survey length of CO and CMP methods were compared. The CO method covered larger (71.4 %) TDR sample area (30 cm × 70 cm) than the CMP method (49.8 %).

Further, we showed that deeper hyperbolic reflections cover a larger sample area for $\theta_v$ estimation than shallower hyperbolic reflections when comparing with the TDR method. Our analysis showed that the CO method (RMSE = 0.02 m$^3$ m$^{-3}$) was more accurate than the CMP method (RMSE = 0.05 m$^3$ m$^{-3}$) when the velocity was estimated using shallow hyperbolas. It was also revealed that 1000 MHz transducers were not successful due to poor ground coupling in irregular surfaces of crop fields. In a comparison of the CO method, 500 MHz (RMSE = 0.02 m$^3$ m$^{-3}$) gave better SM estimations than 250 MHz (RMSE = 0.06 m$^3$ m$^{-3}$) for shallow hyperbolas, mainly due to the higher resolution and higher compatibility with the TDR sample area. The error of 500 MHz SM estimations, when compared to 30 cm long TDR probe data, were considerably different for two depth ranges (27-34 cm or 40-50 cm) of hyperbolas tested. These results do not support our hypothesis that the same accuracy for $\theta_v$ estimation in 30 cm deep shallow soil profile of the experimental plot could be achieved by fitting the hyperbolas regardless of the depth from 27-50 cm. However, the estimated
\( \theta \), using the proposed GPR hyperbola fitting (27-50 cm) is still within an RMSE of 0.03 m\(^3\) m\(^{-3}\) and is not statistically different from the TDR measurements in this study. Therefore, evaluating the GPR estimated \( \theta \), with a systematic TDR probes arrangement along the GPR transect is recommended. Probe arrangement should be compatible with the GPR sample geometry (i.e., long TDR probes at the top of the reflector and shallow probes towards both sides of the transect).

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Sadatcharam, K. 2019. Assessing potential applications of multi-coil and multi-frequency electromagnetic induction sensors for agricultural soils in Western Newfoundland (Unpublished master’s thesis). Memorial University of Newfoundland, St. John’s, NL, Canada


CHAPTER 3

Distinguish the capillary fringe reflection in a ground penetrating radar profile for precise water table depth estimation
3.1 Introduction

Knowledge of the water table depth (WTD) is essential for many environmental prospects including water management in agriculture. WTD, where hydraulic pressure equals atmospheric pressure, demarcates the saturated-unsaturated soil boundary. Naturally, a transition zone called capillary fringe presents on top of the water table, which is nearly saturated but having negative water pressure (de Marsily, 1986). Capillary fringe in the vadose zone provides space, water and nutrients for plants and soil organisms (Selker et al., 1999; Tindall et al., 1999). Both WTD and the depth to the top of the capillary fringe (#D_{CF}#) are subjected to seasonal fluctuations resulting in diverse water management practices in agricultural lands throughout the growing season.

WTD can be measured through a borehole (BH), but generally unfeasible at large field-scales. High-resolution subsurface images of ground penetrating radar (GPR) can be used to measure shallow WTD especially in coarse-grained soils (Doolittle et al., 2006; Paz et al., 2017; Shih et al., 1986). In GPR, a transmitter antenna (Tx) transmits high frequency (10 to 1200 MHz) short pulses of electromagnetic (EM) energy into the subsurface and a receiver antenna (Rx) captures the transmitted energy (Davis & Annan, 1989). Transmitted EM energy (from now on referred to as GPR wave) can be reflected, refracted or attenuated (Annan, 2005; Daniels et al., 2004; Strobach et al., 2010). GPR wave propagation through the subsurface is highly sensitive to soil moisture (SM) (Agliata et al., 2018; Algeo et al., 2016; Annan, 2005; Huisman et al., 2003). Water table reflects more than 40% of GPR wave energy in coarse-grained soils (Kowalczyk et al., 2018). Accordingly, the water table can give continuous,
mostly flat reflections with high amplitude in GPR radargrams (Greaves et al., 1996; Gueting et al., 2015; Khalil et al., 2010; Mahmoudzadeh et al., 2010 & 2012; Seger & Nashait, 2011; Van Overmeeren, 1998). Based on this advantage, GPR has become an essential method in groundwater studies (Beres & Haeni, 1991; Tsoflias et al., 2001). Early researches, for example, Johnson (1992), Livari & Doolittle (1994), van Overmeeren (1994 & 1998), reported the capability of GPR method to detect water table.

GPR field techniques for water table studies have been developed over the decades. Annan et al. (1991) suggested that it would be essential to have a sharp boundary between saturated and unsaturated zones in a GPR profile for precise WTD estimation. Longer wavelengths (i.e., lower frequencies) are recommended even though the resolution of the radar profile decreases with increasing wavelength (Annan et al., 1991). Loeffler & Bano (2004) have also found that GPR frequencies higher than 900 MHz do not identify the top of the saturated zone due to the effect from the capillary fringe and the transition zone. Therefore, most of the studies have been carried out using GPR frequencies lower than 250 MHz (Bano, 2006). Nakashima et al. (2001) and Takeshita et al. (2004) used common-midpoint (CMP) data acquisition of GPR to explain the multiple reflections from the water table. Besides, Nakashima et al. (2001) concluded that CMP data allow estimating the variation of relative permittivity ($\varepsilon_r$) and therefore calculating the radar wave velocity ($v$) along the soil profile. However, common-offset (CO) survey method with 100 MHz has been mostly used for the water table studies (Paz et al., 2017). GPR techniques were employed during pumping tests to measure the temporal fluctuation of WTD (Bevan et al., 2003; Endres et al., 2000; Tsoflias et al., 2001). Further, GPR has been used for various groundwater studies.
(Corbeanu et al., 2002; Conant et al., 2004; Doetsch et al., 2012; Gish et al., 2002; Lambot et al., 2008; Lunt et al., 2005; McClymont et al., 2012; Oliver & Woodroffe, 2016; Schmelzbach et al., 2011 & 2012; Słowik, 2014; Talley et al., 2005; Tsoflias & Becker, 2008; Yang et al., 2013).

However, shallow water table associated with closely spaced soil horizons is challenging to interpret only with the aid of GPR outputs (van Overmeeren, 1998). With the advances in sophisticated sensor technology, real-time WTD and SM data would help to improve the GPR outputs (Doolittle et al., 2006). Still, a site-specific GPR data validation is needed to distinguish the accurate water table reflection. Therefore, the objective of this study was to calibrate and validate a site-specific relationship between GPR-estimated $D_{CF}$ and measured-WTD data in an agricultural field.

### 3.2 GPR theory

There are three characteristics to consider when interpreting WTD from a GPR profile (Fig. 3.1). First, saturation decreases from bottom to top of the capillary fringe (within the transition zone) subsequently increasing the radar wave velocity (Daniels et al., 2005). The thickness of this zone depends on the amount, size (diameter) and interconnectivity of soil pores (Bear, 1972; Doolittle, 2006). Second, the oscillations of the reflected radar pulse due to the transition zone result in a series of bands to represent the water table in a radar profile (Doolittle et al., 2006). Top of the capillary fringe gives an early and robust reflection than the actual water table (Bentley & Trenholm, 2002; Igel et al., 2016). The wetting front also has the same reflection as the water table (Rejiba et al., 2012). Third, two-way travel time (TWTT) correspondence to the maximum absolute amplitude of the airwave ($t_{air}$) is opposite from that of a reflection
event \((t_{\text{reflect}})\) (Bentley & Trenholm, 2002) (Fig. 3.1). In addition to the above characteristics, the maximum absolute amplitude occurs at the second half of the respective GPR wavelet (Booth et al., 2010).

Figure 3.1: Increment of capillary rise with decreasing pore size resulting a comparatively more substantial difference between the WTD measured in a borehole, and saturation boundary reflection observed using a GPR profile in clay-rich soils than that of sand-rich soils (modified from Paz et al., 2017)

Equation 3.1 gives radar signal velocity \((v)\) of a non-magnetic and low-loss geological material.

\[
v = \frac{c}{\sqrt{\varepsilon_r}} \quad (3.1)
\]

Where \(\varepsilon_r\) is the relative permittivity and \(c\) is the electromagnetic wave propagation velocity in free space (= 0.3 m/ns) (Davis & Annan, 1989; Neal, 2004; Schmelzbach et al., 2012).

In addition, depth to a known reflector method (Daniels et al., 2004) can be used to calculate the GPR reflection wave velocity \((v_{rw})\) of a monostatic antenna using Equation 3.2.
\[
v_{rw} = \frac{2D}{t}
\]  
(3.2)

Where \( t \) is the TWTT of the reflected GPR wave from a reflection boundary and \( D \) is the depth to the boundary (\textit{ASTM D6432-11}).

\( \varepsilon_r \) of the soil just below the ground surface can be calculated using GPR direct groundwave method using Equation 3.3 (Huisman et al., 2003).

\[
\varepsilon_r = \left[ \frac{c (t_{GW} - t_{AW}) + x}{x} \right]^2
\]  
(3.3)

Where \( t_{GW} \) and \( t_{AW} \) are direct groundwave and airwave arrival time, respectively, from Tx to Rx antenna and \( x \) is the antenna separation.

\( \varepsilon_r \) can be derived from SM measured as volumetric water content, \( \theta_v \) using an empirical model. Topp \textit{et al}. (1980) suggested the first and well-known model (Mukhlisin & Saputra, 2013) as given in Equation 3.4.

\[
\varepsilon_r = 3.03 + 9.3\theta_v + 146.0\theta_v - 76.7\theta_v
\]  
(3.4)

3.3 Materials and Methods

3.3.1 Study area

The experimental site was located at a silage cornfield and a grass field in Pynn’s Brook Research Station (PBRS), Pasadena, NL, Canada. The area is gently sloping with a 2-5\% slope, and depth to the bedrock is >1 m from the surface (Kirby, 1988). Details of the observed shallow soil profile are given in Table 3.1. The top soil \((\varepsilon_{r1}, t_1)\) is an organic soil layer with gravels. Immediately below the top layer is the Ap horizon \((\varepsilon_{r2},\)
t_2) and be classified as loamy sand (sand = 82.0 ± 3.4%; silt = 11.6 ± 2.4%; clay = 6.4 ± 1.2%) (Badewa et al., 2018). The average bulk density and porosity of the loamy sand layer (n=28) is 1.31 g cm^{-3} (±0.07) and 51% (±0.03), respectively (Badewa et al., 2018).

A well-sorted sandy soil layer was observed from 0.35-3.47 m depth by hand auguring. The average capillary height was approximated as 0.70 m for unsaturated sandy layer (ε_t_3, t_3) according to Liu et al. (2014). The average WTD measured in all GPR survey days (n=16) was 2.55 m. Therefore, the average t_3 was considered as 1.50 m (2.55-0.70-0.35). The average precipitation is 1113 mm per year with 410 mm falling as snow, and annual mean temperature is 4 °C as recorded at the nearest weather station, Deer Lake, NL for last 30 years (https://weather.gc.ca).

<table>
<thead>
<tr>
<th>Soil layer</th>
<th>Depth range (m)</th>
<th>Layer thickness (m)</th>
<th>Relative permittivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top soil</td>
<td>0 - 0.05</td>
<td>t_1 = 0.05</td>
<td>ε_r_1</td>
</tr>
<tr>
<td>Loamy sand</td>
<td>0.05 - 0.35</td>
<td>t_2 = 0.30</td>
<td>ε_r_2</td>
</tr>
<tr>
<td>Sand (unsaturated)</td>
<td>0.35 - top of the capillary fringe</td>
<td>t_3 = WTD_{w}-0.7-0.35</td>
<td>ε_r_3</td>
</tr>
</tbody>
</table>

### 3.3.2 Site preparation and data collection

Following materials and instruments were used for data acquisition, processing and interpretation of this study.

PulseEKKO® Pro GPR system (Sensors and Software Inc., Canada) with 100 and 250 MHz center frequencies

Decagon Em50 data logger, and water level, electrical conductivity-, temperature- and SM-probes (METER group Inc., USA)

EKKO Project V3 R1 and IcePicker V3 R7 GPR data processing Software (Sensors and Software Inc., Canada)
The silage cornfield was relatively flat and 43 m long. The main GPR survey line was marked between the cornfield and the grassland using wooden pegs (Fig. 3.2 left). A shallow groundwater monitoring BH (3.47 m deep) was constructed at 19 m position of the GPR survey line. The perpendicular distance between BH and the survey line was 0.5 m (Fig. 3.2 right). A water level-, electrical conductivity- and temperature-sensor, connected to a data logger (Em50 – Meter Group Inc., USA), was installed at the bottom of the BH. The water level sensor measures the height of the water column in the BH. Three SM probes (probe length = 5 cm each) were installed horizontally at 0.1 m, 0.2 m and 0.3 m depths and connected to the same data logger (Fig. 3.3 left). An additional temperature-sensor was installed together with the SM probe at 0.2 m depth. GPR surveys were carried in between the locations of SM probes and the BH (Fig. 3.3 right). The probes were oriented perpendicular to the GPR survey direction.

Background GPR surveys were carried out; (i) before construction of the BH, (ii) after the construction of the BH, but before installation of the water level sensor, and (iii) after installation of the water level sensor. Sixteen 250 MHz GPR CO surveys (43 m in length, antenna separation = 0.38 m, sampling interval = 0.05 m) were performed in 2017 and 2018. Three 100 MHz GPR CO surveys (~30 m in length, antenna separation = 1.0 m, sampling interval = 0.25 m) were also conducted under wet, median and dry conditions in 2018 along the same GPR line.
Figure 3.2: Photograph captured during a 100 MHz survey at the starting point of the GPR survey line (left). Plan view of the borehole (BH) location with GPR survey line and the location of soil moisture (SM) probes (right). A-A1 cross section is illustrated in Figure 3.3. The intersection of the A-A1 section and GPR survey line is marked as “x.”

Figure 3.3: A-A1 vertical cross-sectional view of the soil profile and the details of the borehole (BH) (Left). The sampling areas of soil moisture (SM) probes (Right). “x” marks the position of the GPR survey line.
Three basic GPR data processing steps were applied using EKKO Project V3 R1 Software (Sensors and Software Inc., Canada) as listed below.

- Edit the first-break
- Apply dewow and SEC2 gain
- Background subtraction – full filter length

After completing above basic processing, GPR files were exported to IcePicker V3 R7 software (Sensors and Software Inc., Canada) for automatic time picking.

3.3.3 Defining the average GPR velocity

The average $\varepsilon_r$ from the surface to the top of the capillary fringe ($\bar{\varepsilon}_r$) is needed to define an average $v_{nw}$ when using Equation 3.1. Three soil layers ($\varepsilon_r1$, $\varepsilon_r2,$ and $\varepsilon_r3$) were considered to get $\bar{\varepsilon}_r$. First, the $\varepsilon_r$ of 0-0.05 m soil depth ($\varepsilon_{r1}$) was calculated using the GPR direct groundwave (Eq. 3.3). Twelve GPR traces near the SM probes were considered for groundwave analysis (Fig. 3.4).

SM data logging interval was 60 min. Therefore, one daily mean SM datum had 24 replicated measurements. Each SM probe has a cylindrical sampling volume (radius ~ 0.05 m, volume = 0.715 L) that covers 0.05 m soil heights both above and below the probe (Fig. 3.3 right) (https://www.metergroup.com; Sakaki et al., 2008). Daily mean SM at three depths was converted to daily mean $\varepsilon_r$ value using Equation 3.4. An average $\varepsilon_r$ for the soil layer between 0.05-0.35 m ($\varepsilon_{r2}$) was obtained from those $\varepsilon_r$ values at three SM probe depths.

$\varepsilon_r$ of the soil layer from 0.35 m depth to the top of the capillary fringe ($\varepsilon_{r3}$) was assumed based on the literature and onsite weather data (Table 3.2 and Appendix 3.1).
Figure 3.4: GPR wave paths related to twelve GPR traces at the intersection of the GPR survey line and the A-A\textsubscript{1} plane. A 250 MHz CO survey with 0.38 m antenna separation and 0.05 m sampling interval was considered. Tx\textsubscript{1} and Tx\textsubscript{12} = transmitter positions, Rx\textsubscript{1} and Rx\textsubscript{12} = receiver positions, corresponding to first and twelfth trace. $x = \text{intersection of A-A}_1$ plane (refer to Fig. 3.2). Note that GPR wave paths assumed to be straight.
Table 3.2: Relative permittivity ($\varepsilon_r$) values for the assumed moisture condition of the sand layer below 0.35 m depth and above the top of the capillary fringe

<table>
<thead>
<tr>
<th>Assumed moisture condition</th>
<th>$\varepsilon_r$</th>
<th>Soil moisture (SM) % (Topp et al., 1980)</th>
<th>Reference for $\varepsilon_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry</td>
<td>3-5</td>
<td>3-8</td>
<td>Davis &amp; Annan, 1989</td>
</tr>
<tr>
<td>Mostly dry</td>
<td>6-7</td>
<td>10-13</td>
<td>Cihlar &amp; Ulaby, 1974</td>
</tr>
<tr>
<td>Slightly dry</td>
<td>8-9</td>
<td>15-17</td>
<td>Martinez, 2001</td>
</tr>
<tr>
<td>Median</td>
<td>10-11</td>
<td>19-21</td>
<td>Cihlar &amp; Ulaby, 1974</td>
</tr>
<tr>
<td>Slightly wet</td>
<td>12-15</td>
<td>23-28</td>
<td>Charlton, 2008; Reynolds, 1997</td>
</tr>
<tr>
<td>Mostly wet</td>
<td>16-18</td>
<td>29-32</td>
<td>Martinez, 2001</td>
</tr>
<tr>
<td>Saturated</td>
<td>20-30</td>
<td>~40</td>
<td>Davis &amp; Annan, 1989</td>
</tr>
</tbody>
</table>

Then the weighted average of $\varepsilon_{r1}$, $\varepsilon_{r2}$, and $\varepsilon_{r3}$ was considered as the $\varepsilon_{\bar{r}}$. For this purpose, the percentage sample area of each layer was calculated with respect to total GPR sample area (Fig. 3.4 and Table 3.3).

Table 3.3: Percentage sample area of each soil layer out of total GPR sample area on the B-B1 plane related to twelve GPR traces collected top of the SM probes

<table>
<thead>
<tr>
<th>Soil layer</th>
<th>Polygon (refer to Fig. 3.4)</th>
<th>Area (m²)</th>
<th>Percentage out of total GPR sample area (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top soil</td>
<td>pqsr</td>
<td>0.0462</td>
<td>3.4</td>
</tr>
<tr>
<td>Loamy sand</td>
<td>rsut</td>
<td>0.2667</td>
<td>19.5</td>
</tr>
<tr>
<td>Sand (unsaturated)</td>
<td>tuvw</td>
<td>1.0561</td>
<td>77.1</td>
</tr>
<tr>
<td>Total</td>
<td>pqwv</td>
<td>1.3690</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Let $w$ to be the weight and $x$ to be the data number, then weighted average ($\bar{x}$) equals to;

$$\bar{x} = \frac{\sum_{i=1}^{n} w_i \times x_i}{\sum_{i=1}^{n} w_i}$$  (3.5)
\[ \varepsilon_r = \frac{(3.4\% \times \varepsilon_{r1}) + (19.5\% \times \varepsilon_{r2}) + (77.1\% \times \varepsilon_{r3})}{100\%} \]

\[ \bar{\varepsilon}_r = \frac{(3.4 \times 23.0) + (19.5 \times 10.5) + (77.1 \times 10.5)}{100} \]

\[ \bar{\varepsilon}_r = 10.9 \]

Table 3.4: Calculation of the average reflected wave velocity \( (v_{rw}) \) through the estimation of the average relative permittivity \( (\bar{\varepsilon}_r) \) from surface down to the top of the capillary fringe

<table>
<thead>
<tr>
<th>Date</th>
<th>WTD (m)</th>
<th>( t_3 ) (m)</th>
<th>( \varepsilon_{r1} )</th>
<th>( \varepsilon_{r2} )</th>
<th>( \varepsilon_{r3} )</th>
<th>( \bar{\varepsilon}_r )</th>
<th>( v_{rw} ) (m/ns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6/23/2017</td>
<td>2.47</td>
<td>1.42</td>
<td>23.0</td>
<td>10.5</td>
<td>10.5</td>
<td>10.9</td>
<td>0.091</td>
</tr>
<tr>
<td>7/6/2017</td>
<td>2.55</td>
<td>1.50</td>
<td>4.0</td>
<td>12.2</td>
<td>6.0</td>
<td>7.1</td>
<td>0.112</td>
</tr>
<tr>
<td>7/28/2017</td>
<td>2.74</td>
<td>1.69</td>
<td>13.2</td>
<td>7.5</td>
<td>6.0</td>
<td>6.5</td>
<td>0.117</td>
</tr>
<tr>
<td>8/18/2017</td>
<td>2.85</td>
<td>1.80</td>
<td>10.9</td>
<td>7.8</td>
<td>6.0</td>
<td>6.5</td>
<td>0.117</td>
</tr>
<tr>
<td>8/29/2017</td>
<td>2.90</td>
<td>1.85</td>
<td>19.5</td>
<td>8.1</td>
<td>4.0</td>
<td>5.3</td>
<td>0.130</td>
</tr>
<tr>
<td>9/15/2017</td>
<td>2.91</td>
<td>1.86</td>
<td>14.0</td>
<td>11.2</td>
<td>4.0</td>
<td>5.7</td>
<td>0.125</td>
</tr>
<tr>
<td>10/3/2017</td>
<td>2.77</td>
<td>1.72</td>
<td>6.2</td>
<td>11.6</td>
<td>7.0</td>
<td>7.9</td>
<td>0.107</td>
</tr>
<tr>
<td>11/7/2017</td>
<td>2.63</td>
<td>1.58</td>
<td>13.1</td>
<td>13.7</td>
<td>10.5</td>
<td>11.2</td>
<td>0.090</td>
</tr>
<tr>
<td>6/1/2018</td>
<td>2.24</td>
<td>1.19</td>
<td>15.2</td>
<td>12.2</td>
<td>13.5</td>
<td>13.3</td>
<td>0.082</td>
</tr>
<tr>
<td>6/20/2018</td>
<td>2.33</td>
<td>1.28</td>
<td>14.0</td>
<td>12.5</td>
<td>8.0</td>
<td>9.1</td>
<td>0.100</td>
</tr>
<tr>
<td>6/29/2018</td>
<td>2.31</td>
<td>1.26</td>
<td>19.5</td>
<td>12.1</td>
<td>7.0</td>
<td>8.4</td>
<td>0.103</td>
</tr>
<tr>
<td>7/20/2018</td>
<td>2.54</td>
<td>1.49</td>
<td>20.0</td>
<td>8.6</td>
<td>6.0</td>
<td>7.0</td>
<td>0.113</td>
</tr>
<tr>
<td>8/9/2018</td>
<td>2.61</td>
<td>1.56</td>
<td>6.5</td>
<td>12.8</td>
<td>4.0</td>
<td>5.8</td>
<td>0.125</td>
</tr>
<tr>
<td>9/7/2018</td>
<td>2.75</td>
<td>1.70</td>
<td>4.0</td>
<td>12.8</td>
<td>4.0</td>
<td>5.4</td>
<td>0.129</td>
</tr>
<tr>
<td>10/2/2018</td>
<td>2.56</td>
<td>1.51</td>
<td>8.0</td>
<td>12.4</td>
<td>10.5</td>
<td>10.8</td>
<td>0.091</td>
</tr>
<tr>
<td>10/31/2018</td>
<td>1.86</td>
<td>0.81</td>
<td>23.3</td>
<td>13.1</td>
<td>17.0</td>
<td>16.5</td>
<td>0.074</td>
</tr>
</tbody>
</table>

3.3.4 Estimating the DCF from GPR

GPR traces that used for \( v_{rw} \) calculation were considered for the DCF estimation as well. Accordingly, the mean TWTT to the capillary fringe reflection (mean \( t_{CF} \)) was
obtained from twelve GPR traces. The subjective error was high when picking the leading edge of the wavelet (Galagedara et al., 2003). Therefore, TWTT related to the absolute maximum amplitude of the airwave (t_{air}) and the reflection event (t_{reflect}) were picked. This procedure was similar to the direct groundwave analysis by Grote et al. (2003). The correct t_{CF} was determined by using Equation 3.6.

\[ t_{CF} = t_{reflect} - t_{air} \] (3.6)

D_{CF} was then estimated from the mean t_{CF} and the calculated v_{rw} using Equation 3.2. D_{CF} of eight GPR surveys in 2017 were calibrated using a linear regression model. WTD measured by the water level sensor (WTD_m) at the same time of GPR survey was used for the calibration. Next, the WTD was predicted for eight survey days in 2018 using the calibration equation obtained. The predicted WTD (WTD_p) and WTD_m were compared using a 1:1 plot and root mean square error (RMSE). In a second step, WTD_m and D_{CF} for all 16-survey days were plotted in a linear regression plot to examine an average capillary height. Slope and the intercept of the regression line as well as the prediction line were compared statistically with that of the 1:1 line.

3.4 Results and Discussion

3.4.1 Site-specific relationship for WTD_m vs. D_{CF}

D_{CF}, which derived using Equation 3.2 for all 16 GPR survey days are given in Table 3.5. The WTD_m fluctuation related to GPR survey days (n=16) was from 1.85 m to 2.91 m. For the entire period of the study covering growing seasons in 2017 and 2018 (496 days), the WTD_m varied between 1.58 – 2.95 m. The shallowest WTD_m (1.58 m) was observed in the spring of 2018 (April 30, 2018), and the deepest WTD_m (2.95 m) was found in the summer of 2017 (Sept 10-12 2017). Throughout the studied period,
the average WTD$_m$ was 2.48 m with an annual average of 2.69 m in 2017 and 2.34 m in 2018 during the growing season (from May to the end of October). These data imply that 2017 had a relatively dry growing season was also confirmed with the onsite weather data collected (Appendix 3.1).

Table 3.5: GPR estimated depth to the top of the capillary fringe (D$_{CF}$) which is derived from the mean two-way travel time (TWTT) to the capillary fringe reflection (t$_{CF}$) using Equation 3.2 for all GPR surveys. Standard error (SE) of mean, and minimum (Min), median and maximum (Max) of t$_{CF}$ time picks are also given.

<table>
<thead>
<tr>
<th>Date</th>
<th>Mean t$_{CF}$ (ns)</th>
<th>SE of mean</th>
<th>t$_{CF}$ (ns) Min</th>
<th>Median</th>
<th>Max</th>
<th>D$_{CF}$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6/23/2017</td>
<td>35.07 ±1.70</td>
<td>0.43</td>
<td>32.65</td>
<td>35.08</td>
<td>38.04</td>
<td>1.59</td>
</tr>
<tr>
<td>7/6/2017</td>
<td>29.21 ±1.93</td>
<td>0.58</td>
<td>27.97</td>
<td>28.51</td>
<td>34.80</td>
<td>1.62</td>
</tr>
<tr>
<td>7/28/2017</td>
<td>32.40 ±5.39</td>
<td>1.44</td>
<td>27.71</td>
<td>28.82</td>
<td>39.78</td>
<td>1.90</td>
</tr>
<tr>
<td>8/18/2017</td>
<td>36.43 ±2.87</td>
<td>0.70</td>
<td>32.11</td>
<td>37.81</td>
<td>39.67</td>
<td>2.13</td>
</tr>
<tr>
<td>8/29/2017</td>
<td>33.59 ±2.90</td>
<td>0.70</td>
<td>30.21</td>
<td>32.29</td>
<td>39.16</td>
<td>2.17</td>
</tr>
<tr>
<td>9/15/2017</td>
<td>34.31 ±3.86</td>
<td>1.11</td>
<td>27.70</td>
<td>35.23</td>
<td>38.83</td>
<td>2.14</td>
</tr>
<tr>
<td>10/3/2017</td>
<td>36.84 ±3.69</td>
<td>1.02</td>
<td>32.80</td>
<td>36.26</td>
<td>42.15</td>
<td>1.96</td>
</tr>
<tr>
<td>11/7/2017</td>
<td>40.23 ±4.53</td>
<td>1.26</td>
<td>33.09</td>
<td>39.45</td>
<td>44.36</td>
<td>1.81</td>
</tr>
<tr>
<td>6/1/2018</td>
<td>38.08 ±2.06</td>
<td>0.47</td>
<td>35.64</td>
<td>39.35</td>
<td>40.59</td>
<td>1.56</td>
</tr>
<tr>
<td>6/20/2018</td>
<td>33.35 ±3.82</td>
<td>0.83</td>
<td>30.18</td>
<td>31.38</td>
<td>40.54</td>
<td>1.67</td>
</tr>
<tr>
<td>6/29/2018</td>
<td>31.18 ±1.35</td>
<td>0.33</td>
<td>30.07</td>
<td>30.73</td>
<td>34.40</td>
<td>1.60</td>
</tr>
<tr>
<td>7/20/2018</td>
<td>32.89 ±3.33</td>
<td>0.77</td>
<td>28.07</td>
<td>35.20</td>
<td>37.56</td>
<td>1.85</td>
</tr>
<tr>
<td>8/9/2018</td>
<td>30.25 ±2.01</td>
<td>0.44</td>
<td>28.15</td>
<td>28.80</td>
<td>32.69</td>
<td>1.89</td>
</tr>
<tr>
<td>9/7/2018</td>
<td>30.05 ±2.67</td>
<td>0.67</td>
<td>27.78</td>
<td>29.29</td>
<td>36.47</td>
<td>1.96</td>
</tr>
<tr>
<td>10/2/2018</td>
<td>36.93 ±2.65</td>
<td>0.61</td>
<td>34.33</td>
<td>35.42</td>
<td>41.31</td>
<td>1.68</td>
</tr>
<tr>
<td>10/31/2018</td>
<td>31.64 ±2.78</td>
<td>0.61</td>
<td>27.93</td>
<td>31.27</td>
<td>35.56</td>
<td>1.18</td>
</tr>
</tbody>
</table>

The WTD$_m$ at the same time of GPR survey, and corresponding D$_{CF}$ are plotted in Figure 3.5 (left). Linear regression of the WTD$_m$ vs. D$_{CF}$ for GPR surveys in 2017 implies that there is a strong linear regression with an $R^2$ of 0.9778 (WTD$_m$ = 0.6956 D$_{CF}$ + 1.3884) between these two parameters.
Figure 3.5: Linear regression plot of measured water table depth (WTD$_m$) vs. GPR estimated depth to the capillary fringe (D$_{CF}$) for 2017 data (n=8) (left). The 1:1 plot of the predicted water table depth (WTD$_p$) vs. WTD$_m$ in 2018 (n=8) (right).

\[
\text{WTD}_m = 0.6956 \times \text{D}_{CF} + 1.3884 \\
R^2 = 0.9778
\]

Figure 3.6: Temporal variability of the measured water table depth (WTD$_m$) and the estimated depth to the capillary fringe using GPR (D$_{CF}$) for both years and, the predicted water table depth (WTD$_p$) for 2018.

WTD$_p$ for 2018 based on the D$_{CF}$ and the regression model vs. WTD$_m$ in 2018 were plotted in a 1:1 plot (Fig. 3.5 right). The slope of the prediction line (1.5967) and
the slope of 1:1 line (1.000) are significantly different at \( \alpha=0.05 \) (p-value= 0.004, df= 12, \( t \) critical= 2.179 < \( t=3.536 \)) (Appendix 3.2). The error of WTD predication is high during the wet survey days and overestimated from the 1:1 line (Fig. 3.5 right). This behaviour could be due to the fact that the capillary fringe would not fluctuate uniformly with the WTD fluctuation. As Bentley & Trenholm (2002) stated, the capillary height is higher when the WTD is increasing (during discharging) while it is lower when the WTD is decreasing (during recharging). This feature could not be captured by a regression equation (Fig. 3.6). Therefore, the regression model might not be suitable when there is a sudden decrease in WTD like during heavy or long-lasting rain events. The rain event at the end of the growing season of 2018 was unexpectedly high (Appendix 3.1); consequently, resulted the maximum error of WTD prediction on Oct 2018 (Fig. 3.6). However, it is worth noting that the RMSE of \( WTD_m \) vs. \( WTD_p \) (0.194 m) is possibly acceptable for the scale of application in most agricultural practices.

In general, the \( D_{CF} \) cannot be measured directly under field conditions (Salim, 2016). The proposed method provides a non-invasive approach to estimate \( D_{CF} \), which is more beneficial in agricultural fields especially during the growing season. The advantage of the proposed method is that both WTD and \( D_{CF} \) could be estimated in real time. The results would have been improved if a broader range of measured data were available under different SM conditions.

As seen in Figure 3.7, \( WTD_m \) vs. \( D_{CF} \) for all survey days have a linear relationship (\( WTD_m = 1.0123 \times D_{CF} + 0.741 \)) with an \( R^2 \) of 0.911. The slopes of the regression line (1.012) and the slope of 1:1 line (1.000) are not significantly different at \( \alpha=0.05 \) (p-value= 0.885, df= 28, \( t \) critical= 2.048 > \( t=0.146 \)) (Appendix 3.2).
Therefore, the intercept of the regression line (0.741 m, n=16) can be considered as the average capillary height within these two growing seasons in 2017 and 2018. This analysis allows defining an average capillary height for the study site throughout the growing season. The average value agrees with the value of ~0.70 m capillary height for the same soil conditions described by Liu et al. (2014).

![Figure 3.7: Comparison of the measured water table depth (WTD\textsubscript{m}) and the estimated depth to the capillary fringe (D\textsubscript{CF}) based on GPR data for all 16 GPR surveys. 1:1 line indicates the measured and GPR based water table depth.](image)

3.4.2 Challenges of the proposed method

In GPR, the Rx only records different amplitudes of the receiving signals with respect to time. The GPR interpreter observes the radar events in a radargram and obtains relevant TWTTs. Without knowing the \( v_{rw} \), it is impossible to derive D\textsubscript{CF} (e.g., using Eq. 3.2). Under these circumstances, there are two challenges to estimating the D\textsubscript{CF} in a GPR radargram. First, picking the TWTT of the capillary fringe reflection correctly. Second, knowing or properly assuming the average \( v_{rw} \) from the surface down to the top of the capillary fringe reflection.
Procedures available in the literature were combined to overcome the challenge of picking the correct TWTT. The standard error of the mean tCF was >1 other than 4 cases in 2017: 1.44 (July 28), 1.26 (Nov 7), 1.11 (Sept 15) and 1.02 (Oct 3). Standard deviation (SD) of the mean tCF was above 3 ns for those four survey days. The error associated with these four surveys is mostly due to a high signal-noise ratio in the data acquisition. The lowest SDs of the mean tCF were 1.70 and 1.35 for June 23, 2017 and June 29, 2018, respectively. The inconsistency of GPR reflection amplitude under wet conditions as stated by Lunt et al. (2005) could be a reason for the error of time picking.

The challenge of defining \( v_{rw} \) is determining the average \( \varepsilon_r \) of the material above the capillary fringe. \( \varepsilon_r \) controls the \( v_{rw} \) and the reflection coefficients at interfaces. Common \( v_{rw} \) values suggested from the literature may not be accurate for heterogeneous soil profiles. In addition, seasonal fluctuation of WTD and capillary height can remarkably change the average \( \varepsilon_r \). However, \( v_{rw} \) can be measured by using multi-offset GPR survey methods such as CMP and WARR (wide-angle reflection and refraction). Nevertheless, it is time- and labor-consuming to carry out multi-offset surveys in every field campaign (Huisman et al., 2003; Paz et al., 2017). In the present study, three different \( \varepsilon_r \) values were used for different soil layers. A weighted average of \( \varepsilon_r \) was calculated based on the GPR sampling geometry in order to minimize the error of assuming an average \( \varepsilon_r \).

The average capillary height considered (0.70 m) based on the study of Liu et al. (2014) to estimate the \( \varepsilon_{c3} \) is closer to the average capillary height obtained from this study (0.741 m). It should mention that taking an average capillary height would be reasonable under static conditions, but not always suitable if the seasonal fluctuation of WTD is high during the time of interest.
Figure 3.8: Two-way travel time (TWTT) picks of the maximum amplitude of the reflection ($t_{\text{reflect}}$), after time-correction ($t_{\text{CF}}$), and the mean $t_{\text{CF}}$. GPR-estimated (250 MHz) depth to the capillary fringe ($D_{\text{CF}}$) and the measured water table depth ($WTD_{\text{m}}$) at the borehole (BH) are presented under dry soil moisture condition on Aug 29 (above), under median soil moisture condition on Oct 03 (middle), and under wet soil moisture condition on June 23 (below), in the growing season of 2017.
Figure 3.9: Comparison of 100 MHz (left) and 250 MHz (right) GPR radargrams. Two-way travel time (TWTT) picks after time-correction (t_CF), and the mean t_CF are shown. Measured water table depth WTD_m at the borehole (BH) are presented under wet soil moisture condition on June 01 (above), under median soil moisture condition on July 20 (middle), and under dry soil moisture condition on Aug 09 (below), in the growing season of 2018.
The GPR velocity is independent of the frequency (for frequencies above 100 MHz) and dependent only on the dielectric permittivity and the magnetic permeability (Reppert et al., 2000). Therefore, the same velocity derived from 250 MHz could be used to analyze 100 MHz data on the same day. GPR profiles from 250 MHz give high-resolution than 100 MHz. Thus, multiple reflections near the water table can be clearly observed (Fig. 3.8). However, those reflections give an undulated boundary. In contrast, 100 MHz give low-resolution images with a relatively flat and clear boundary for the water table zone and with less multiple reflections. The water table reflections picked in a 100 MHz radargram are closer to WTD$_m$ than that of 250 MHz (Fig. 3.9). Figure 3.9 clearly shows that the error between the GPR water table and the actual water table (WTD$_m$) is low under dry conditions and the error increases when deceasing WTD. Both frequencies perform well under dry SM conditions than wet SM conditions because of low signal attenuation under dry SM conditions (Daniels, 2004). Present results imply that 250 MHz is suitable to examine shallow D$_{CF}$ whereas 100 MHz is suitable to examine deeper WTD.

3.5 Conclusion

The shallow water table in a sandy aquifer, with the presence of shallow soil horizons, was difficult to interpret only with GPR data. Real-time WTD data were used to aid the validation of GPR based WTD data. $\varepsilon_r$ of shallow soil was determined using continuously measured SM data. GPR sampling geometry was also considered to improve the radar velocity assumptions. A site-specific strong linear relationship ($R^2 = 0.9778$) between D$_{CF}$ and WTD$_m$ was developed ($WTD_m = 0.6956 D_{CF} + 1.3884$) using eight GPR surveys throughout the growing season of 2017. The regression model was
validated using eight monthly GPR surveys in the same location in 2018. Low RMSE (0.194 m) of predicted- and measured- WTD implies that the proposed non-invasive method would be beneficial for precise WTD and $D_{CF}$ estimations in agricultural fields.

A regression model was developed using all GPR data collected for two growing seasons using the same approach. As a result, an average capillary height of 0.741 m was suggested for the particular site throughout the growing season. The suggested capillary height not only has a strong linear relationship ($R^2=0.911$, $n=16$) but also comparable with the existing literature. The developed model is recommended to modify using the data collected for many growing seasons covering a wider variability of water table. Then the knowledge of the local variation of WTD and $D_{CF}$ throughout the growing season would be transferred to end users of the agricultural sector in the region.

**References**


Igel, J., Stadler, S., Günther, T. 2016. High-resolution investigation of the capillary transition zone and its influence on GPR signatures. 16th International Conference on Ground Penetrating Radar (GPR), Hong Kong, 2016, pp. 1-5, doi:10.1109/ICGPR.2016.7572603


McClymont, A.F., Hayashi, M., Bentley, L.R., Liard, J. 2012. Locating and characterizing groundwater storage areas within an alpine watershed using time-lapse gravity, GPR and seismic refraction methods. Hydrological Processes, 26, pp. 1792–1804


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CHAPTER 4

Summary
4.1 Summary and Future Work

Both drought and flooded or waterlogged farmlands are challenges for agricultural producers in Canada. Long winters, as well as the high soil moisture (SM) levels due to heavy spring rains or rapid snowmelt, can make saturated soil conditions. As a result, agricultural fields become unreachable or difficult to work delaying start of the growing season. A late growing season reduce the crop growth and development and eventually yield of the crops. Proper seed germination and seedling establishment cannot be guaranteed since wet soils are slow to warm up. During the early growing stage, if the SM conditions are still not favorable, the plant root system may be stunted. Further, oxygen demand in the root zone goes high under waterlogging conditions, which may cause death or low productivity of plants having underdeveloped root systems. During the later growing season with dry SM conditions, the root systems should be deep enough to optimize the plant nutrient and water uptake from the soil. Therefore, the SM condition in the root zone, behavior of the water table and capillary zone throughout the growing season are important parameters for water and nutrient management in agricultural fields. The present study proposed a non-invasive method to measure SM and water table depth (WTD) in the agricultural fields using ground-penetrating radar (GPR).

The receiver antenna (Rx) of GPR detects the radar signals that reflected from the interfaces having different relative permittivity ($\varepsilon_r$). GPR records the radar wave travel time from the Tx back to the Rx with respect to the amplitude of the reflected wave. This phenomenon provides a basis to estimate the depth of the reflection interface. However, the reflected wave velocity ($v_{rw}$) from the surface down to the reflection interface should be known. On the other hand, if the depth to the reflection is
known, the $v_{rw}$ can be estimated. Major advantages of the GPR method are non-invasive and time-effective data collection ability in large-scale. Therefore, the GPR technology is suitable to use in the agricultural fields.

The first study obtained $v_{rw}$ from hyperbola fitting in GPR to estimate the SM in the agricultural root zone. Most published studies focused on groundwave velocity to estimate the SM. Groundwave propagates from the Tx to the Rx through the immediately below the ground surface, so that the SM estimation from the groundwave method is applicable for very shallow depths. Since this study focused on the agricultural root zone, the depth of investigation should be at least 30 cm from the surface. Hyperbola fitting method is also challenged by finding shallow hyperbolas in a radargram.

Therefore, this study hypothesized that the same accuracy for SM estimation in 30 cm deep soil profile of the experimental plot could be achieved by fitting the hyperbolas in the depth range from 27 to 50 cm. A systematic TDR data collection was introduced in this study, which could reduce the bias of different sampling volumes of GPR and time domain reflectometry (TDR). Thirteen TDR samples (vertically installed 30 cm long) per one hyperbola location at a maximum radius of 30 cm expected to be a good representation for GPR sample volume and GPR estimated SM data evaluation. In addition, a comparison of the sample areas of the systematic TDR collection and GPR in a 2D plan provides useful information to select the appropriate GPR survey type. The subjective error associated with the hyperbola fitting was reduced by following predefined data processing guidelines. The proposed method would be more efficient when it combines with the latest hyperbola-fitting algorithms (e.g., Dou et al.,
2017; Lambot & André, 2014; Mertens et al., 2016; Minet et al., 2012; Qiao et al., 2015; Tran et al., 2014).

The second study estimates the WTD throughout the growing season. There are some limitations in GPR, sometimes can be advantageous. Limitation one is centimeters-scale accuracy of WTD is difficult due to the interference from the capillary fringe in GPR data profile. The advantage is distinguishing the water table- and capillary fringe-reflections allows estimation of both WTD and the depth to the capillary fringe ($D_{CF}$) simultaneously. WTD and $D_{CF}$ are important parameters to ensure the ideal water availability to the crop root system or identification of potential water logging conditions at each growing phase of a crop.

Limitation two is that the direct groundwave velocity is only valid for very shallow depths as mentioned earlier. The advantage is $\varepsilon_r$ of a shallow soil layer can be derived from the direct groundwave velocity (Huisman et al., 2003; Galagedara et al., 2003; Galagedara et al., 2005). The shallow soil layer is the most sensitive to the SM variation (i.e., for $\varepsilon_r$ variation) which can influence the $v_{rw}$ of shallow depths.

Limitation three is that a GPR radargram needs an accurate $v_{rw}$ from the surface down to the water table/capillary fringe for precise depth estimations. GPR multiple-offset surveys are capable of estimating $v_{rw}$, but they are time and labor consuming. Therefore, $v_{rw}$ was assumed based on $\varepsilon_r$ estimations. However, for deep reflections or rough estimations of $\varepsilon_r$, the straight ray path assumption would be valid. The above assumption can add a significant error for the shallow reflections. This limitation was addressed by considering a weighted average of $\varepsilon_r$ from different soil layers. $\varepsilon_r$ of a deep soil layer was assumed based on the literature. $\varepsilon_r$ of a middle soil layer was calculated
based on measured SM at different depths. $\varepsilon_r$ of the topsoil was determined using the GPR direct-groundwave method.

One GPR center frequency should have been performed well in both studies to monitor SM and WTD simultaneously. However, there is a trade-off between the resolution of the GPR image and the signal penetrating depth. Consequently, 500 MHz high-resolution data acquisition was compatible with SM estimations at the root zone whereas the penetration depth ($< 2.5$ m under particular conditions) of 500 MHz radar frequency was not sufficient to capture the WTD depths at the study site. However, this thesis focused on three main objectives. Two of them were achieved by the study described in Chapter 2, and the other objective was achieved by the study described in Chapter 3.

<table>
<thead>
<tr>
<th>Objective</th>
<th>Conclusion(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 To examine the soil volume that hyperbolic reflections describe in terms of radar wave velocity</td>
<td>Reflections over a 30 cm deep point reflector along 1 m GPR survey length were compared. The CO method covered larger (71.4 %) TDR sample area (30 cm × 70 cm) than the CMP method (49.8 %).</td>
</tr>
<tr>
<td>2 To evaluate a practical GPR-based approach to estimate $\theta_v$ within the upper 30 cm of the soil profile</td>
<td>SM measurement using the proposed hyperbola fitting method agreed with 30 cm long TDR probe data with an RMSE of 0.02 m$^3$ m$^{-3}$ for shallow ($&lt; 34$ cm), and an RMSE of 0.04 m$^3$ m$^{-3}$ for deeper hyperbola analysis.</td>
</tr>
<tr>
<td>3 To calibrate and validate a site-specific relationship between GPR-estimated $D_{CF}$ and measured-WTD (WTD$_m$)</td>
<td>A site-specific relationship was developed $WTD_m = 0.6956 \ D_{CF} + 1.3884$ ($R^2 = 0.9778$, n=8). RMSE = 0.194 m for predicted- and measured- WTD.</td>
</tr>
</tbody>
</table>

RMSE = 0.19 m$^2$.
An average capillary height (0.741 m) throughout the growing season is suggested for the particular site which is comparable with the literature.

**Future Works**

The tested systematic TDR probes arrangement could be modified. It is recommended having more TDR sampling locations along the GPR survey line. Probe arrangement should be compatible with the GPR sample geometry (i.e., long TDR probes at the top of the reflector and shallow probes towards both sides of the survey line).

During the data analysis, it was observed in GPR radargrams, that the groundwater reflections near the borehole were free from the capillary fringe, and represented the correct WTD. Thus, the observed height of capillary fringe can be used to correct the GPR measured WTD. However, this observation could not be evaluated with the present study. Therefore, it is recommended to continue the study related to Chapter 3 with an additional two observation boreholes along the GPR survey line.

The second study revealed a methodology to estimate both D<sub>CF</sub> and WTD within a same GPR profile while the first study discussed the uncertainty/accuracy of estimating SM within the root zone using the hyperbola reflection method. A future study should focus on developing and testing a methodology to measure shallow SM-using the direct groundwave, SM within the root zone-using hyperbola fitting, together with D<sub>CF</sub> and WTD simultaneously in one GPR profile. If this can be achieved, GPR can be used to derive and map hydrological properties within the vadose zone over larger areas in achieving sustainable agricultural water management.
APPENDICES
APPENDIX 2.1: One to one (1:1) plot of volumetric water content ($\theta_v$) measured by TDR and gravimetric sampling.
APPENDIX 2.2: One to one (1:1) plot of GPR measured radar wave velocity ($v_{rw}$) (m/ns) by hyperbola fitting and theoretical calculation using two-way wave travel time ($t_{rw}$).
APPENDIX 2.3: One to one (1:1) plot of GPR estimated position (m) vs. actual position (m) of buried location along the GPR transect
APPENDIX 2.4: One to one (1:1) plot of GPR estimated depth (m) vs. actual depth (m) of buried reflector
Mann-Whitney Test and CI: GPR estimated $\theta_v$, Overall TDR (Depth 27-50 cm)

<table>
<thead>
<tr>
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<th>N</th>
<th>Median</th>
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</thead>
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<tr>
<td>VWC-GPR</td>
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<td>0.16476</td>
</tr>
<tr>
<td>Overall</td>
<td>18</td>
<td>0.15115</td>
</tr>
</tbody>
</table>

Point estimate for $\eta_1 - \eta_2$ is 0.01010
95.2 Percent CI for $\eta_1 - \eta_2$ is (-0.00156, 0.02442)
$W = 383.0$
Test of $\eta_1 = \eta_2$ vs $\eta_1 \neq \eta_2$ is significant at 0.1173
The test is significant at 0.1171 (adjusted for ties)

Mann-Whitney Test and CI: GPR estimated $\theta_v$, Overall TDR (Depth 27-34 cm)

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</thead>
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<td>VWC-GPR S</td>
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<td>0.15776</td>
</tr>
<tr>
<td>overall S</td>
<td>9</td>
<td>0.15187</td>
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</tbody>
</table>

Point estimate for $\eta_1 - \eta_2$ is 0.00934
95.8 Percent CI for $\eta_1 - \eta_2$ is (-0.00820, 0.02818)
$W = 95.0$
Test of $\eta_1 = \eta_2$ vs $\eta_1 \neq \eta_2$ is significant at 0.4268
The test is significant at 0.4265 (adjusted for ties)

Mann-Whitney Test and CI: GPR estimated $\theta_v$, Overall TDR (Depth 40-50 cm)

<table>
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<tr>
<td>overall D</td>
<td>9</td>
<td>0.15043</td>
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</table>

Point estimate for $\eta_1 - \eta_2$ is 0.01433
95.8 Percent CI for $\eta_1 - \eta_2$ is (-0.00700, 0.04467)
$W = 101.0$
Test of $\eta_1 = \eta_2$ vs $\eta_1 \neq \eta_2$ is significant at 0.1853
The test is significant at 0.1846 (adjusted for ties)
APPENDIX 2.6: Hyperbola analysis datasheet for 500 MHz

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<tr>
<th>Date</th>
<th>F</th>
<th>Reflector #</th>
<th>Depth (m)</th>
<th>Velocity (m/ns)</th>
<th>$\epsilon_0$</th>
<th>GPR $\theta_0$</th>
<th>Average of TDR measured $\theta_0$ (m$^3$/m$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Overall</td>
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<tr>
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<td></td>
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<td>0.085</td>
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<td>0.2337</td>
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APPENDIX 3.1: Graphs of measured data

Temporal variation of water table depth (WTD) and soil moisture (SM) at three depths measured near the borehole (above), daily rainfall and daily average WTD fluctuation (below) throughout the study period.
Time lag for water table response to cumulative P-E (daily mean precipitation minus daily mean evapotranspiration); one day time lag (above), 8-days time lag (middle), and 16-days time lag (below).
Comparing the slopes of regression line and prediction line with 1:1 line

To test whether the slopes for two independent populations are equal, following null and alternative hypotheses were tested:

\( H_0: \beta_1 = \beta_2 \) i.e. \( \beta_1 - \beta_2 = 0 \)

\( H_1: \beta_1 \neq \beta_2 \) i.e. \( \beta_1 - \beta_2 \neq 0 \)

The test statistic is

\[
t = \frac{b_1 - b_2}{\sqrt{\frac{s_{b1}^2}{n_1} + \frac{s_{b2}^2}{n_2}}} \sim T(n_1 + n_2 - 4)
\]

\( n = \) sample size; \( b_1 \) and \( b_2 \) are slopes

\[
s_b = \frac{s_{y|x}}{s_x \sqrt{n - 1}}
\]

\( s_{y|x} \) = standard error of predicted \( y \) for each \( x \) in the regression

\( s_x = \) standard deviation

If the null hypothesis is true then

\[
\beta_1 - \beta_2 \sim N(0, s_{b1-b2})
\]

Where

\[
s_{b1-b2} = \sqrt{s_{b1}^2 + s_{b2}^2}
\]
Comparison of regression line and 1:1 line

<table>
<thead>
<tr>
<th></th>
<th>Regression</th>
<th>1:1 Line</th>
<th>$s_{b1-b2}$</th>
<th>0.085</th>
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<td>16</td>
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<tr>
<td>b</td>
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<td>df</td>
<td>28</td>
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<td>0.000</td>
<td>$\alpha$</td>
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<tr>
<td>$s_{x}$</td>
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<td>p-value</td>
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Since $t < t_{\text{critical}}$ and $p\text{-value}> \alpha$, two slopes are not significantly different at $\alpha=0.05$.

Comparison of prediction line and 1:1 line

<table>
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<th></th>
<th>Regression</th>
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Since $t > t_{\text{critical}}$ and $p\text{-value}< \alpha$, two slopes are significantly different at $\alpha=0.05$. 