Multi-Objective Optimization Application in Power Systems

by

Yasamin Rahnamafard, B.Eng.

A thesis submitted to the School of Graduate Studies in partial fulfillment of the requirements
for the degree of Master of Engineering

Faculty of Engineering and Applied Science

Memorial University of Newfoundland

14 March 2019

St. John’s Newfoundland and Labrador Canada
Abstract

For hundreds of years, electricity supply industries have been in the hands of monopoly utilities, until engineers conceived the management of this industry as a challenging multi-objective optimization problem. Multiple objectives need to interact in divergent or competing interests to win the competition and to deliver the electrical energy and keep the lights on. The most highlighted objectives are the economic dispatch of power for satisfying consumers with lower power bill costs, the minimum loss in transmission lines, the environmental emission reduction, and the reliability and stability of the voltage and generated power. The consideration of mentioned goals push for a tighter coordination of planning and operation scheduling in a power system and raises interest in using multi-objective optimization methods in power system applications.

The Optimal Power Flow (OPF) is used to determine an optimal operating condition for power systems while considering the limitations and system constraints such as: generator capability, line capacity, bus voltages, and power flow balances. Among many optimization methods, the Teaching-Learning Based Optimization (TLBO) algorithm has gained wide acceptance in different areas of science and engineering. In this study, the Teaching-Learning-Based Optimization algorithm is used to optimize the Optimal Power Flow problem considering total fuel cost of generation, emission, voltage deviation and, active power transmission losses in single and multi-objective cases. The method has been applied on the IEEE 30-bus and IEEE 57-bus test systems for several OPF objectives including minimization of fuel cost, emission, voltage deviation, and power losses, and the results reflect the effectiveness of the method.
Acknowledgments

I would like to express my gratitude to my supervisor Dr. Benjamin Jeyasurya for his kind advice, guidance and constant support during my research.

Special appreciation is given to the Natural Sciences and Engineering Research Council of Canada (NSERC), the School of Graduate Studies and Memorial University of Newfoundland for providing financial support throughout my master’s program.

I would also like to thank Dr. Wayne Raman-Nair, Dr. Siu O’Young, Dr. Faisal Khan for the graduate courses they taught which helped me to fulfill my course requirements for this master’s program.

I would like to dedicate my thesis to my beloved parents Hassan Rahnamafard and Zahra Seyed Hossein. Words cannot express my feelings for my lovely sisters Baharak and Roshank Rahnamafard and my supportive brother, Hossein Rahnamafard. Thank you for your unconditional love and for believing in me, thank you for encouraging me in all areas of my life and for inspiring me to follow my dreams.
# Table of Contents

1. **Introduction** .................................................................................................................. 13
   1.1 Research Objectives ........................................................................................................ 14
   1.2 Thesis Organization .......................................................................................................... 15

2. **Review of Optimization Techniques** ............................................................................. 16
   2.0 Introduction ..................................................................................................................... 16
   2.1 Optimization ................................................................................................................... 17
      2.1.1 The Standard Form of the Optimization Problem .................................................. 17
   2.2 An Overview of Classifications of Optimization Techniques ....................................... 20
      2.2.1 Continuous Optimization versus Discrete Optimization ....................................... 20
      2.2.2 Unconstrained Optimization versus Constrained Optimization .............................. 21
      2.2.3 Single Objective or Multi Objectives Functions ...................................................... 21
      2.2.4 Deterministic Optimization versus Stochastic Optimization ................................. 21
      2.2.5 Choice of Method ..................................................................................................... 23
   2.3 Local Optimization Algorithms ...................................................................................... 24
      2.3.1 Newton’s Method ...................................................................................................... 26
      2.3.2 Unconstrained Optimization .................................................................................... 28
      2.3.3 Constrained Optimization ....................................................................................... 29
         2.3.3.1 Penalty Method ................................................................................................. 30
         2.3.3.2 Barrier Method ................................................................................................. 30
         2.3.3.3 Sequential Unconstrained Minimization Techniques (SUMT) ......................... 31
         2.3.3.4 Lagrange Multiplier .......................................................................................... 32
      2.3.4 Non-Gradient Based Methods ................................................................................... 32
   2.4 Global Optimization Algorithms ...................................................................................... 33
      2.4.1 Classification and History ......................................................................................... 34
   2.5 Teaching-Learning-Based Optimization (TLBO) algorithm ........................................ 37
      2.5.1 TLBO Overview ....................................................................................................... 37
      2.5.2 TLBO Algorithm ..................................................................................................... 39
      2.5.3 Case Study of a Constrained Optimization Problem with TLBO Algorithm .......... 42
   2.6 More Case Studies of Optimization Problems .................................................................. 46
3.3.3 The OPF Constraints ........................................................................................................ 86  
  3.3.3.1 Equality Constraints .................................................................................................. 86  
  3.3.3.2 Inequality Constraints .............................................................................................. 87  
  3.3.4 OPF Solution Methodologies ...................................................................................... 88  
  3.3.5 Case Study of Minimizing Cost for 7-Bus Power System ........................................ 90  
  3.3.6 Case Study of Minimizing Cost for 37-Bus Power System ........................................ 92  
  3.4 Security-Constrained Optimal Power Flows (SCOPF) ................................................ 95  
  3.4.1 Case Study of SCOPF .................................................................................................. 96  
  3.5 Conclusions .................................................................................................................... 100  
  
 4. Multi-Objective Optimization .............................................................................................. 101  
  4.0 Introduction .................................................................................................................... 101  
  4.1 Single Objective Optimization ....................................................................................... 101  
  
  4.1.1 Single Objective Optimization Formulation ............................................................... 101  
  
  4.1.2 Case Study of Single Objective Optimization ............................................................ 103  
  4.2 Multi-Objective Optimization ......................................................................................... 104  
  4.3 Non-Dominated and Dominated Points, Pareto Solution ............................................ 105  
  4.4 Classical Multi-Objective Optimization Methods .......................................................... 109  
  4.5 Weighted Sum Method .................................................................................................... 111  
  4.7 Case Studies of Multi-Objective Optimization ............................................................... 112  
  
  4.7.1 Case Study 1 ............................................................................................................... 112  
  
  4.7.2 Case Study 2 ............................................................................................................... 114  
  4.8 Conclusions .................................................................................................................... 115  
  
 5. Applying Multi-Objective Optimization Techniques for Solving Optimal Power Flow .......................................................... 116  
  5.0 Introduction .................................................................................................................... 116  
  5.1 Multi-Objective Optimization Problem Formulation .................................................... 116  
  5.2 Case Studies ................................................................................................................... 120  
  
  5.2.1 IEEE 30-Bus Power System ....................................................................................... 121  
  
    Case 1: Minimization of Fuel Cost .................................................................................... 123  
    Case 2: Minimization of Emission .................................................................................... 124  
  

List of Figures

Figure 2.1: Graphical Representation of Nonlinear Programming Problem in x-Space [5]..... 18
Figure 2.2: Global and Local Optimal of a Two-Dimensional Function [5] .......................... 19
Figure 2.3: Optimization Taxonomy [6] .................................................................................. 22
Figure 2.4: Classification Schematic of Optimization Techniques For Engineering Systems [8] .......................... 23
Figure 2.5: Gradient-based Optimization [9] ......................................................................... 25
Figure 2.6: Newton’s Method Uses Curvature Information to Take a More Direct Route [10] 27
Figure 2.7: One Dimensional Multi-Model Function [9] .......................................................... 33
Figure 2.8: The Taxonomy of Global Optimization Algorithms [16] ........................................ 34
Figure 2.9: Metaheuristic Classification [19] .......................................................................... 36
Figure 2.10: Model for the distribution of marks obtained by learners taught by two different teachers [20] .......................................................................................................................................................... 38
Figure 2.11: Model for the distribution of marks obtained for a group of learners [20] ....... 38
Figure 2.12: Flowchart of TLBO [20] [22] ............................................................................... 41
Figure 2.13: Constrained function of constrained nonlinear optimization problem case study 47
Figure 3.1: A typical bus in a network [1] .................................................................................. 60
Figure 3.2: Economic Dispatch, Considering All Generators in One Shared Bus [1].......... 61
Figure 3.3: One line diagram for 7-Bus Power System [28] .................................................... 78
Figure 3.4: Economic Dispatch for a 37-bus Power System [29] ............................................. 79
Figure 3.5: OPF solution methodologies [34] ....................................................................... 89
Figure 3.6: One Line Diagram of Optimal Power Flow for 7-Bus Power System with OPF [28] ................................................................................................................................................................................. 90
Figure 3.7: One Line Diagram of 37-Bus Power System .......................................................... 92
Figure 3.8: One Line Diagram of 37-Bus Power System with No Control on Area [29] ...... 97
Figure 3.9: One Line Diagram of 37-Bus Power System OPF with OPF Control on Area [29] 97
Figure 3.10: SCOPF of 37-Bus Power System [29] .............................................................. 98
Figure 4.1: Local and Global Optimum Points [4] ................................................................. 102
Figure 4.2: Non-dominated and dominated points [15] ......................................................... 106
Figure 4.3: Pareto Optimal Solution [13] ............................................................................................................. 107
Figure 4.4: Objective Function Plot, Trade-Off Region is Between the Green Lines [12] ........ 108
Figure 4.5: Pareto Optimal Front plot [12] ............................................................................................................. 108
Figure 4.6: Classification of Multi-Objective Optimization Methods and the Role of Human in
the Optimization Processes [43] ................................................................................................................................. 110
Figure 4.7: Classification of Goal-Oriented Multi-Objective Optimization Methods and
Algorithms [20] ......................................................................................................................................................... 111
Figure 4.8: Optimization Toolbox Setting .............................................................................................................. 113
Figure 4.9: Pareto Front of Case Study 1 .................................................................................................................. 113
Figure 4.10: Pareto Front of Case Study 2 .................................................................................................................. 114
Figure 5.1: Convergence of Case 1 for IEEE 30-Bus System ...................................................................................... 123
Figure 5.2: Convergence of Case 2 for IEEE30-Bus System ....................................................................................... 124
Figure 5.3: Convergence of Case 3 for IEEE 30-Bus System ...................................................................................... 125
Figure 5.4: Convergence of Case 4 for IEEE 30-Bus System ...................................................................................... 126
Figure 5.5: Convergence of Case 5 for IEEE 30-Bus System ...................................................................................... 127
Figure 5.6: Convergence of Case 6 for IEEE30-Bus System ....................................................................................... 128
Figure 5.7: Convergence of Case 7 for IEEE30-Bus System ....................................................................................... 129
Figure 5.8: Convergence of Case 8 for IEEE30-Bus System ....................................................................................... 130
Figure 5.9: Convergence of Case 9 for IEEE 30-Bus System ...................................................................................... 132
Figure 5.10: Simulation Results of Optimum Solution for IEEE-30 Bus System Single-
objective Cases ...................................................................................................................................................... 134
Figure 5.11: Simulation Results of Optimum Solution for IEEE-30 Bus System Multi-objective
Cases ...................................................................................................................................................................... 136
Figure 5.12: Convergence of Case 10 for IEEE 57-Bus System ................................................................................... 138
Figure 5.13: Convergence of Case 11 for IEEE 57-Bus System ................................................................................... 139
Figure 5.14: Convergence of Case 12 for IEEE 57-Bus System ................................................................................... 140
Figure 5.15: Convergence of Case 13 for IEEE 57-Bus System ................................................................................... 141
Figure 5.16: Convergence of Case 14 for IEEE 57-Bus System ................................................................................... 142
Figure 5.17: Convergence of Case 15 for IEEE 57-Bus System ................................................................................... 143
Figure 5.18: Convergence of Case 16 for IEEE 57-Bus System ................................................................................... 144
Figure 5.19: Fitness Function Convergence of Case 17 for IEEE 57-Bus System .............................................. 145
Figure 5.20: Fitness Function Convergence of Case 18 for IEEE 57-Bus System .......... 146
Figure 5.21: Simulation Results of Optimum Solution for IEEE 57-Bus System Single-objective Cases .............................................................. 148
Figure 5.22: Simulation Results of Optimum Solution for IEEE-57 Bus System Multi-objective Cases ............................................................... 150
List of Tables

Table 2.1: Constrained and Unconstrained Optimization Classification ........................................... 28
Table 2.2: Algorithm (TLBO pseudocode) [22] .................................................................................. 40
Table 2.3: Initial population .................................................................................................................. 43
Table 2.4: New values of the variables, objective function and the penalties (teacher phase) .................. 43
Table 2.5: Updated values of the variables, objective function and the penalties based on fitness comparison (teacher phase) .................................................................................................................. 44
Table 2.6: New values of the variables, objective function and the penalties (learner phase) .................. 44
Table 2.7: Updated values of the variables, objective function and the penalties based on fitness comparison ................................................................................................................................................. 44
Table 2.8: Lagrange multiplier method, constrained optimization for the case study ......................... 48
Table 2.9: Newton-Raphson method and iterative solution, Lambda= 0.4 ............................................ 50
Table 2.10: Newton-Raphson method and iterative solution, Lambda= -2 .......................................... 50
Table 2.11: MATLAB Optimization Toolbox Commands [25] .............................................................. 53
Table 2.12: Results for Multivariable Minimization Problem Using MATLAB Function fmincon .......... 56
Table 3.1: Economic Dispatch Solution Neglecting Losses and Including Generator Limits [27] ....... 67
Table 3.2: B-Coefficients for 7-Bus Power System ................................................................................. 77
Table 3.3: Economic Dispatch Case Study for 7-Bus System ................................................................. 78
Table 3.4: Economic Dispatch Case Study for 37-Bus System ............................................................... 80
Table 3.5: Optimal Power Flow Case Study for 7-Bus System ............................................................... 91
Table 3.6: OPF of 7-Bus Power System, Minimizing Generation Cost .................................................. 91
Table 3.7: Optimal Power Flow Case Study for 37-Bus System ............................................................. 93
Table 3.8: OPF of 37-Bus Power System, Minimizing Generation Cost .................................................. 94
Table 3.9: Contingency Analysis and Violation List ............................................................................. 98
Table 3.10: Comparing Generators Power Output in 37-Bus Power System .......................................... 99
Table 3.11: SCOPF of 37-Bus Power System, Minimizing Generation Cost .......................................... 99
Table 5.1: Summary of case studies ....................................................................................................... 120
Table 5.2: The Main Characteristics of The IEEE 30-Bus Test System [31]......................... 121
Table 5.3: Cost and Emission Coefficient of Generators for IEEE 30-bus System [31]........ 122
Table 5.4: Simulation Results of Optimum Solution for IEEE-30 Bus System- Single Objective Cases ........................................................................................................................................................................ 133
Table 5.5: Simulation Results of Optimum Solution for IEEE-30 Bus System, Multi-Objective Cases ........................................................................................................................................................................................................................................................................................................ 135
Table 5.6: The Main Characteristics of the IEEE 57-Bus Test System [31]...................... 137
Table 5.7: Cost and Emission Coefficient of Generators for IEEE 57-Bus System [31]....... 137
Table 5.8: Simulation Results of Optimum Solution for IEEE 57-Bus System ............... 147
Table 5.9: Simulation Results of Optimum Solution for IEEE-57 Bus System, Multi-objective Cases ........................................................................................................................................................................................................................................................................................................ 149
Table 5.10: Comparison of the Simulation Results for Minimizing Fuel Cost with Different AI Methods........................................................................................................................................................................................................................................ 151
Table 5.11: Comparison of the Simulation Results in Multi-Objective Optimization of Fuel Cost and Voltage Deviation, Case 5, IEEE30-Bus System................................................................. 152
Chapter 1

1. Introduction

Electrical energy is considered to be the most popular form of energy globally, because it can be transported at high efficiency and at reasonable prices [1]. For about hundreds of years, the electricity supply industries where in the hands of monopoly utilities, until engineers conceived the management of this industry as a challenging optimization problem. By increasing the size, complexity, and scope of these optimization problems, professional and powerful computer programs were designed and deployed to refine the most efficient algorithms for the planning and the operation of power systems. By introducing competition to the electricity supply industry, a single organization is no longer in charge. Multiple objectives need to interact in divergent or competing interests to win the competition and to deliver the electrical energy and keep the light on [2]. The most highlighted objectives are the economic dispatch of power for satisfying consumers with lower power bill costs, the reliability and stability of generated power, the minimum loss in transmission lines, and the security constraints in a power system.

The power transmitted through transformers and transmission lines is restricted to the given limits, which arise due to thermal, voltage, or stability considerations. The Optimal Power Flow (OPF) is used to determine an optimal operating condition for power systems while considering the limitations and system constraints such as generator capability, line capacity, bus voltages, and power flow balances. Increasing the power quality and reliability, using the available reserves are other possible target goals in solving an OPF problem. In this study, the OPF is formulated as a multi-objective problem of minimizing fuel cost, emission, transmission losses, voltage deviation, etcetera.
Security Constrained Optimal Power Flow (SCOPF) is defined as an effective strategy to enforce the reliability criteria for the secure operating of the system. The achieved operating point ensures that if any possible contingency happens, the post-contingency states remain secure [3].

The consideration of mentioned goals push for the tighter coordination of planning and operation scheduling in a power system and raises interest in using multi-objective optimization methods in power system applications.

Among the many optimization methods, the Teaching-Learning-Based Optimization (TLBO) algorithm has gained wide acceptance among optimization researchers [4]. In this study, the TLBO algorithm is used to optimize the OPF problem in several single and multi-objective cases. The method has been applied on the IEEE 30-bus and IEEE 57-bus test systems for several OPF objectives such as minimization of fuel cost, emission, voltage deviation, and power losses.

1.1 Research Objectives

The focus of this research is to present the application of multi-objective optimization methods in power system problems. Different objectives of the OPF problem have been introduced with constraints and limitations.

The TLBO method is used to optimize different OPF objectives, in a single or multi-objective optimization problem, that reflects the performance of power systems. Furthermore, the TLBO algorithm has been applied to the optimization problem as an effective method.

The principal goals of this research are summarized as follows:

- To study applications of optimization techniques in power systems.
- To study the constrained optimization methods and applications for Economic Dispatch (ED) and Optimal Power Flow (OPF).
- To research and explore the multi-objective optimization methodologies in power system operations.
- To study security constraints in power system problems.
To study, analyze, and visualize the operation and planning of a power system using MATLAB, as a powerful computer programming software, and the PowerWorld simulator.

To study the effectiveness of the Teaching-Learning-Based Optimization (TLBO) algorithm in the Optimal Power Flow problem for several single and multi-objective cases.

1.2 Thesis Organization

This thesis consists of 6 chapters. Chapter 2 provides a review of optimization techniques. A comprehensive model of optimization problems and the classification of methods are discussed. Local and global algorithms are introduced. In addition, the application of MATLAB optimization toolbox is briefly introduced to solve several case studies and illustrates the effectiveness of algorithms for general optimization problems.

Chapter 3 deals with theories and principles behind three major types of problems commonly referred to in power systems literature as power flow, Economic Dispatch (ED) and Optimal Power Flow (OPF). Different formulations of ED are applied to achieve the least-cost generation dispatch, considering and neglecting transmission loss and generation limits. The main principles of the optimal power flow problem and Security Constraint Optimal Power Flow (SCOPF) are proposed. For each type of problem, case studies and examples are provided to illustrate the concepts. MATLAB and Power World simulator are used to analyze and simulate the 7-bus and 37-bus power systems.

Chapter 4 presents the concepts and classification of single and multi-objective optimization problems. The Teaching-Learning-Base Optimization (TLBO) algorithm is introduced and the method is implemented on a constraint optimization problem.

Chapter 5 is concerned with optimizing 4 different objectives of the OPF problem: fuel cost, environmental emission, voltage deviation, and transmission losses. The constrained OPF problem is formulated and optimized in different scenarios, as a single objective or multi-objective optimization problem, and the results are compared.

Chapter 6 highlights the key contribution of the research presented in this thesis along with suggestions for future works.
Chapter 2

2. Review of Optimization Techniques

2.0 Introduction

“For since the fabric of the universe is most perfect, and is the work of a most wise Creator, nothing whatsoever takes place in the universe in which some form of maximum or minimum does not appear.” Leonhard Euler

Many researchers have shown that humankind has the instinctive desire to make the most beneficial decisions and choose the optimal solution between possible options, for the purpose of heightening the quality of life. From the early ages of civilization until now, there have been ongoing struggles toward developing the core principles of optimization techniques to make the best choices among any inferior situation.

There have been extensive efforts toward this aim by dedicated scientists, engineers, mathematicians, and philosophers, which have resulted in the development of methods, tools, and intelligent softwares we use today. It is imperative to discuss the major achievements in this area.

This chapter draws the focus to major optimization techniques, and briefly explains the mathematical theory and foundations behind them.

---

1 Leonhard Euler, Introduction to De Curvis Elasticis, Additamentum I to his Methodus Inveniendi Lineas Curvas Maximi Minimive Proprietate Gaudentes 1744; translated on page 10-11, "Leonhard Euler's Elastic Curves", Old father et al 1933.
2.1 Optimization

Optimization is the process of maximizing or minimizing the desired objective function while satisfying the problem constraints. Scientists have shown that an abundant number of processes and structures in nature follow an optimum system status. For example, one of nature’s optimization processes is genetic mutation for survival. As well, the crystalline structure of metal sand alloys is defined by unit cells that are formed by atoms in the minimum energy positions. The sphere of a liquid droplet in zero gravity is the geometric form of the minimum surface area for a given volume. The architecture of a honeycomb is one of the most compact packaging structures. Many experts and managers use optimum solutions as a key in their organizations and businesses to strive for excellence. This attitude helps them to achieve consumer satisfaction by identifying optimum solutions from different feasible scenarios. A production factory would benefit hugely from small savings in a mass-produced part. By minimizing the weight of a vehicle, fuel consumption can be affected significantly and this would increase the efficiency. A company needs to consider the optimum number of labor resources for a maximized profit. An automated production line could minimize production time per item by optimizing the process cycle and could ultimately maximize efficiency and benefit [1].

The challenge of increasing the efficiency of known optimization techniques has been an ongoing struggle over the decades. Today, applications of optimization are found in all aspects of our lives.

2.1.1 The Standard Form of the Optimization Problem

It is important to present the standard form of optimization problems before reviewing the optimization methods. Optimizing a scenario in an engineering problem can be defined as minimizing or maximizing a function with respect to inequality and equality constraints. Considering real cases, the optimization model can be formed by objectives and constraint functions, which may be linear or nonlinear, convex or non-convex, explicit or implicit functions.
The standard form for a single-objective, non-linear, constrained optimization problem is provided in Equation 2.1.

\[
\begin{align*}
\text{Minimize} & \quad f(x) & (2.1) \\
\text{Subject to} & \quad g_i(x) \leq 0 & i = 1, \ldots, m \\
& \quad h_j(x) = 0 & j = 1, \ldots, \ell \\
& \quad x_i^L \leq x \leq x_i^U
\end{align*}
\]

where \( x = (x_1, x_2, \ldots, x_n)^T \) is a column vector of \( n \) real-valued design variables, which are modified to obtain the optimal solution. In Equation 2.1 \( f(x) \) represents the objective (or goal) function, \( g_i(x) \) is an inequality constraint and \( h_j(x) \) stands for an equality constraint function. The searchable design region is determined by the upper and lower bounds, \( x_i^L \) and \( x_i^U \), of the design variables, referred to as the side constraints [2].

Figure 2.1 represents a nonlinear, constraint optimization problem for \( n=2 \) design variables, where constant curves for objective function contours are drawn. The optimum is expressed by the highest contour that passes through the feasible region. In this example, \( \Omega = \{ x : g(x) \leq 0, h(x) = 0, x_i^L \leq x_i \leq x_i^U \} \). \( \Omega \), a subset of \( R^n \), is called the feasible region. Vectors \( x_i^L \), \( x_i^U \) represent the explicit lower and upper bounds on the design variables, respectively. The optimization problem is presented in Equation 2.2.

\[
\begin{align*}
\text{Minimize} & \quad f(x) & (2.2) \\
\text{Subject to} & \quad x \in \Omega
\end{align*}
\]

Figure 2.1: Graphical Representation of Nonlinear Programming Problem in x-Space [5]
Figure 2.2 represents the global and local optima of a two-dimensional function. Ashok D.B [5] defines the mathematical definition of global and local optima in his book according to the following definitions [5].

Local Maximum: By definition, a (local) maximum \( x^* \in \mathbb{X} \) of an objective function \( f : \mathbb{X} \to \mathbb{R}^n \) is an input element with \( f(x^*) \geq f(x) \) for all \( x \) neighboring \( x^* \). If \( \mathbb{X} \subseteq \mathbb{R}^n \), \( \forall x^* \exists \varepsilon > 0, f(x^*) \geq f(x), \forall x \in \mathbb{X}, |x - x^*| < \varepsilon \).

Local Minimum: A (local) minimum \( x^* \in \mathbb{X} \) of an objective function \( f : \mathbb{X} \to \mathbb{R}^n \) is an input element with \( f(x^*) \leq f(x) \) for all \( x \) neighboring \( x^* \). If \( \mathbb{X} \subseteq \mathbb{R}^n \), \( \forall x^* \exists \varepsilon > 0 : f(x^*) \leq f(x), \forall x \in \mathbb{X}, |x - x^*| < \varepsilon \).

Local Optimum: A local optimum \( x^* \in \mathbb{X} \) of an objective function. If \( f : \mathbb{X} \to \mathbb{R}^n \) is either a local maximum or a local minimum.

Global Maximum: A global maximum \( x^* \in \mathbb{X} \) of an objective function. If \( f : \mathbb{X} \to \mathbb{R}^n \) is an input element where \( \forall x \in \mathbb{X}, f(x^*) \geq f(x) \).

Global Minimum: A global minimum \( x^* \in \mathbb{X} \) of an objective function If \( f : \mathbb{X} \to \mathbb{R}^n \) is an input element where \( \forall x \in \mathbb{X}, f(x^*) \leq f(x) \).

Figure 2.2: Global and Local Optimal of a Two-Dimensional Function [5]

Global algorithms are highly efficient for solving problems with integer and/or discrete variables. In a discrete or integer optimization problem, some or all of the design variables are
restricted to be an integer or discrete value. Generally, implicit functions appear in this class of problems.

The side constraints in an optimization problem are commonly implemented directly on the algorithm rather than considering equality or inequality constraints. By definition, solving a constrained optimization problem is the process of optimizing an objective function with respect to one or more equality and/or inequality constraints, with or without side constraints. An equality constraint can either be violated or satisfied, while an inequality constraint can be violated, active, or satisfied. An active inequality constraint is one for which \( g_j(x) = 0 \). An unconstrained optimization problem may have side constraints, while it will not be affected by any equality or inequality constraints [5].

2.2 An Overview of Classifications of Optimization Techniques

Optimization techniques are used to find the best solution to the problem presented in Equation 2.1. The approach is to form the optimum objective function by choosing the best combination of design variables in the searchable region while satisfying all the equality, inequality and side constraints. Referring to local or relative optima for some cases, more than one optimum may exist.

An important step in optimizing a problem is classifying the optimization model. This section presents an overview of the optimization model classification for the various optimization problem types [6].

2.2.1 Continuous Optimization versus Discrete Optimization

In an optimization process, the appropriate algorithm can be identified based on the nature of the design variables, which can be either continuous or discrete values. In discrete optimization problems, the variables are only allowed to take on values from a set of discrete elements (a subset of integers). In a continuous optimization problem, any real value can be implemented in the model.

According to the property of smoothness in a continuous function, it is feasible to derive information about the neighborhoods of a point, based on the provided values of the objective
function and constraints in that specified point. Many discrete optimization problems make use of generating continuous domain subsets to simplify the computation process.

2.2.2 Unconstrained Optimization versus Constrained Optimization

Optimization problems can be categorized based on the existence of constraints in the problem structure. An unconstrained optimization problem does not include any active/violated constraints in its model. In a constrained optimization problem, the objective function is supposed to be optimized with respect to one or more equality and/or inequality constraints. Also, in some applications, a constrained problem is reformulated by using a penalty factor in the objective function and an unconstrained optimization problem will arise in this case.

2.2.3 Single Objective or Multi Objectives Functions

The number of objective functions in an optimization problem can define another category of optimization methods. Single-objective optimization corresponds to the minimum or maximum value of a single objective function. The research of multi-objective optimization methods received impetus in many practical engineering fields, where the problem needs to achieve a compromise between two or more conflicting objective functions and finds the best set of values for each one of them.

2.2.4 Deterministic Optimization versus Stochastic Optimization

Deterministic optimization methods use accurate known data, while in a stochastic optimization process, the set of information is not actual (as a result of measurement error or representing uncertain data about a future process).

The deterministic solutions “become computationally impracticable for problems of realistic size, either because the model grows too large, or because the solution procedures are too lengthy, or both, and heuristics provide the only viable scheduling techniques for large projects” [7]. In uncertain optimization methods, the uncertainty is incorporated into the mathematical formula of the model.

Stochastic optimization models make use of probabilistic distribution functions through the feasible region of the problem (these functions are either available or can be estimated). The
The main goal is to approach a strategy that is feasible for all possible data and optimizes the future efficiency of the model. Some references [4] have presented another classification of optimization techniques based on the nature of the equations involved in the objective function. The most famous methods in this category include linear/ nonlinear optimization, geometric optimization, and quadratic optimization.

There are many options for classifying the most common approaches in optimization techniques. In the following section, a brief overview of a broad classification, as either local or global algorithms, is provided. Figures 2.3 and 2.4 represent two different taxonomies of optimization techniques [6], [8].

![Optimization Taxonomy](image)

Figure 2.3: Optimization Taxonomy [6]
2.2.5 Choice of Method

Several factors should be considered in deciding which particular method to solve a given optimization problem. Some of these factors are [4]:

1. The type of problem to be solved (general nonlinear programming problem, geometric programming problem, etc.)
2. The availability of a ready-made computer program
3. The calendar time required for the development of a program
4. The necessity of derivatives of the functions \( f \) and \( g_j, j = 1, 2, \ldots, m \)

5. The available knowledge about the efficiency of the method

6. The accuracy of the solution desired

7. The programming language and quality of coding desired

8. The robustness and dependability of the method in finding the true optimum solution

9. The generality of the program for solving other problems

10. The ease with which the program can be used and its output interpreted [4].

2. 3 Local Optimization Algorithms

Local optimization techniques are heuristic methods that are widely used in a variety of engineering problems. The optimization procedure in this classification consists of searching in the feasible criterion among candidate solutions by making local changes in each outcome solution. This strategy is continued until a set of solutions deemed to be optimal is found or a time boundary is met.

Most local optimization techniques make use of gradient information to find the optimal solution. The popularity of these methods is due to their high efficiency, as they can solve problems with a large number of variables while demanding little problem-specific parameter tuning. However, heuristic methods and local search algorithms do not guarantee the best optimum solution but attempt to yield a local optimum in the feasible region. They also have difficulties in solving discrete optimization problems.

Gradient-based algorithms follow a two-step process to reach the optimum solution in most cases. This process can be explained using Figure 2.5 of a blindfolded boy on a hill. The boy is supposed to reach the highest point on the hill (maximizing the objective function) while staying inside the fences (satisfying the constraints and staying in feasible criterion). The design variables are the \( x \) and \( y \) coordinates of the boy.
Based on the gradient optimization technique, the blindfolded boy can use a two-step process in order to reach the top of the hill (a single step in the x-direction and another step in the y-direction), while trying to estimate his next step direction based on information gained from previous steps. His walk in this direction will continue until no further progress can be made, which may include reaching a fence. In this position, he is able to choose a new direction by taking another two steps inside the fences (as a feasible region). Then he can continue the process until he reaches the peak of the hill. This two-step iterative method of searching for the optimum solution can be mathematically summarized according to Equation 2.3.

\[ x^q = x^{q-1} + \alpha^* S^q \]  

(2.3)

**S:** Searching direction  
**\( \alpha^* \):** Optimum step size

In this example, the first step is to use gradient information, which determines the searching direction ‘\( S \)’ to move. The second step provides the optimum step size, \( \alpha^* \), which represents the number of iterations that should be followed in this direction until no progress can be yielded.

Termination of local optimization techniques can be based on different choices, such as time boundaries or the number of iterations. Different search directions can be applied to the optimization problem. A usable direction that could enhance the objective function is desired for an unconstrained optimization problem or a constraint optimization problem without active/violated constraints.

On the other hand, the required search direction for a constraint optimization scenario with violated constraints is desired to overcome the constraint bounds and is called a feasible direction. If the problem includes active constraints and no violated constraint, the search
direction method should be both usable and feasible, which simply means that it should not violate any constraint bounds and it should improve the objective function at the same time. Considering the one-dimensional search, the best step size could be found through many different methods, including the golden section search, the Fibonacci search, and many variations of polynomial approximations. Then the desired technique can be combined with a specific gradient-based algorithm to perform the required one-dimensional search.

Karush-Kuhn-Tucker (KKT) [5] conditions can be used to determine whether a constrained local optimum has been found by the presented gradient information. Karush-Kuhn-Tucker conditions provide the necessary conditions for a local optimum and can be summarized as:

1. The optimum design point $x^*$ must be feasible.
2. At the optimum design point, the gradient of the Lagrangian must vanish according to Equation 2.4,

$$
\nabla f(x^*) + \sum_{j=1}^{m} \lambda_j \nabla g_j(x^*) + \sum_{k=1}^{p} \lambda_{m+k} \nabla h_k(x^*) = 0 \tag{2.4}
$$

where the Lagrange multipliers, $\lambda_j \geq 0$ and $\lambda_{m+k}$, are unrestricted in sign.

3. For each inequality constraint $\lambda_j \nabla g_j(X) = 0$, where $j = 1,...,m$.

Although the Karush-Kuhn-Tucker conditions are effective in determining the local optimum, their satisfaction cannot guarantee that the global optimum has been achieved [5].

2.3.1 Newton’s Method

Newton’s algorithm is one of the classical gradient-based optimization solutions. It has been classified as an iterative unconstrained algorithm that is derived from a second-order Taylor series expansion of the objective function around the initial design point $x^0$ according to Equation 2.5.

$$
f(x) \approx f(x^0) + \nabla f(x^0)^T(x - x^0) + \frac{1}{2}(x - x^0)^T H(x^0)(x - x^0) \tag{2.5}
$$

where $H(x^0)$ is the Hessian matrix, and contains the second-order gradient information (partial derivatives) of the objective function in a square matrix. Differentiating Equation 2.5 with respect to $x$ and setting the result equal to zero according to the Karush-Kuhn-Tucker conditions, the updated formula in Equation 2.6 for the current design point will result.

$$
x = x^0 - H(x^0)^{-1} \nabla f(x^0) \tag{2.6}
$$
And for higher dimensions, the above iterative scheme can be generalized in Equation 2.7.

\[ x_{n+1} = x_n - \gamma H(x_n)^{-1} \nabla f(x_n) \] (2.7)

Newton’s method is modified to include a small step size \( \gamma \in (0, 1) \).

A presented classical form of the Newton algorithm in equation 2.6 makes use of a fixed step size of \( \gamma = 1 \) and provides the search direction (descent direction) of \( -H(x^0)^{-1} \nabla f(x^0) \).

The method follows a quadratic rate of convergence, requiring a single step to reach the optimum point for any positive definite quadratic function. In a practical approach, Newton’s method is adjusted to include a one-dimensional search that enhances both the efficiency and robustness of the method.

The method has a highly desirable quadratic convergence rate; meanwhile, the calculation process associated with finding the second-order gradient information in the Hessian matrix would be highly sophisticated. As a result, most gradient-based methods make use of first-order gradient information to simplify the method.

Figure 2.6 represents a comparison of gradient descent route (green) and Newton’s method route (red) for minimizing a function (with small step sizes) where Newton’s method uses curvature information to take a more direct (shorter) route.

![Figure 2.6: Newton’s Method Uses Curvature Information to Take a More Direct Route](image)
Table 2.1 provides a summary of the constrained and unconstrained optimization classification based on direct/indirect search methods.

### Table 2.1: Constrained and Unconstrained Optimization Classification

<table>
<thead>
<tr>
<th>Direct Search Methods</th>
<th>Nonlinear - Unconstrained Optimization Techniques</th>
<th>Nonlinear - Constrained Optimization Techniques</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Random Search Methods</td>
<td>Random Search Methods</td>
</tr>
<tr>
<td></td>
<td>- Random Jumping Method</td>
<td>Complex Method</td>
</tr>
<tr>
<td></td>
<td>- Random Walk Method</td>
<td>Sequential Linear Programming (Sequential</td>
</tr>
<tr>
<td></td>
<td>- Random Walk Method with Direction Exploitation</td>
<td>Unconstrained Minimization Techniques – SUMT)</td>
</tr>
<tr>
<td></td>
<td>Grid Search Method</td>
<td>Sequential Quadratic Programming</td>
</tr>
<tr>
<td></td>
<td>Univariate Method</td>
<td>Zoutendijk’s Method of Feasible Directions</td>
</tr>
<tr>
<td></td>
<td>Pattern Directions</td>
<td>Rosen’s Gradient Projection Method</td>
</tr>
<tr>
<td></td>
<td>Powell’s Method</td>
<td>Generalized Reduced Gradient Method</td>
</tr>
<tr>
<td></td>
<td>Simplex Method</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Indirect Search Method</th>
<th>Nonlinear - Unconstrained Optimization Techniques</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Gradient of a Function</td>
</tr>
<tr>
<td></td>
<td>Steepest Descent (Cauchy) Method</td>
</tr>
<tr>
<td></td>
<td>Conjugate Gradient (Fletcher-Reeves) Method</td>
</tr>
<tr>
<td></td>
<td>Newton’s Method</td>
</tr>
<tr>
<td></td>
<td>Marquardt Method</td>
</tr>
<tr>
<td></td>
<td>Quasi-Newton Methods</td>
</tr>
<tr>
<td></td>
<td>Davidon–Fletcher–Powell Method</td>
</tr>
<tr>
<td></td>
<td>Broyden–Fletcher–Goldfarb–Shanno Method</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Indirect Search Method</th>
<th>Nonlinear - Constrained Optimization Techniques</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Penalty Function Method</td>
</tr>
<tr>
<td></td>
<td>- Interior Penalty Function Method</td>
</tr>
<tr>
<td></td>
<td>- Exterior Penalty Function Method</td>
</tr>
<tr>
<td></td>
<td>Extended Interior Penalty Function Methods</td>
</tr>
<tr>
<td></td>
<td>Linear Extended Penalty Function Method</td>
</tr>
<tr>
<td></td>
<td>Quadratic Extended Penalty Function Method</td>
</tr>
<tr>
<td></td>
<td>Penalty Function Method for Parametric Constraints</td>
</tr>
<tr>
<td></td>
<td>Augmented Lagrange Multiplier Method</td>
</tr>
</tbody>
</table>

### 2.3.2 Unconstrained Optimization

There are two popular iterative methods involved in unconstrained classification of optimization techniques: the conjugate gradient method (also referred to as the Fletcher-Reeves) and the Broyden- Fletcher-Goldfarb-Shanno (BFGS) method [5] [4].

The Fletcher-Reeves algorithm makes use of the conjugate search direction for finding the optimum solution of the objective function, using the information of the last iteration. The method produces the exact solution after a finite number of iterations and can be thought of as a direct method. The method efficiently minimizes a quadratic function in, at most, \( n \) steps (where...
n is the size of the system matrix), while the computing process requires only small memory space.

The BFGS method is the most efficient variable metric method that could reach the desired search direction by gaining the information from the previous n iterations. The method makes an approximation to the inverse of the Hessian matrix, \( H(x_0)^{-1} \) in equation ... , and uses specified updates of information by gradient evaluations. The BFGS method belongs to the classification of Quasi-Newton methods.

Unconstrained problems may arise when an appropriate penalty factor is implied in a constrained problem or the constraints are eliminated. The general form of an unconstrained problem is shown in Equation 2.8.

\[
\text{Minimize } f(x_1, x_2, ..., x_n) \\
\text{Where } x = [x_1, x_2, ..., x_n]^T \text{is a column vector of } n \text{ real-valued design variables.}
\] (2.8)

2.3.3 Constrained Optimization

One of the most useful approaches for solving a constrained optimization problem is to convert a multi-dimensional constrained problem into a series of the equivalent one-dimensional unconstrained problem by using penalty and barrier methods.

Penalty methods are used to search for a feasible point that could satisfy all the constraints simultaneously. These techniques implement a penalty factor to the objective function, considering the effect of any constraint violations. Barrier methods are typically used to prevent the infeasible region from growing while we already have a feasible point relative to inequality constraints. In summary, the penalty method will drive the solution into a feasible region and the barrier method will keep the answer in the infeasible region (preventing from leaving the feasible region) [11].
2.3.3.1 Penalty Method

Considering the primary objective function as shown in Equation 2.9:

\[ \text{Minimize } f(x) \quad (2.9) \]

Subject to

\[ g_i(x) \leq 0, \quad i = 1, \ldots, p \]
\[ h_j(x) = 0, \quad j = 1, \ldots, m \]

The penalized objective function can be formulated as Equation 2.10.

\[ \text{Minimize } f(x) \quad (2.10) \]

where

\[ F(x, \rho, \beta) = f(x) + P(x, \rho, \beta) \]
\[ P(x) = \sum_{j=1}^{m} \rho_j h_j^2(x) + \sum_{i=1}^{p} \beta_i g_i^2(x) \]

Then, the penalty parameters \( \rho_j \) and \( \beta_i \) are given by Equation 2.11.

\[ h_j = \begin{cases} 0 & \text{if } h_j(x) = 0 \\ \rho_j & \text{if } h_j(x) \neq 0 \end{cases} \quad (2.11) \]
\[ \beta_i = \begin{cases} 0 & \text{if } g_i(x) \leq 0 \\ \beta_i & \text{if } g_i(x) > 0 \end{cases} \]

2.3.3.2 Barrier Method

The goal of the barrier method is to prevent point \( x \) from ever approaching the boundary of the feasible region. The method introduces a barrier function \( B(g(x)) \geq 0 \), which is differentiable and approaches \( \infty \), as \( g_i(x) \to 0^- \) and requires a start point that over satisfies the constraint.

For example:

\[ - \sum_{j=1}^{m} \ln(-g_i(x)) b(x) = - \sum_{j=1}^{m} \frac{1}{g_i(x)} \]

Barrier function method is shown by Equation 2.12.

\[ B(\epsilon^{(k)}) = \min \{ f(x) + \epsilon^{(k)} b(x) \} \Rightarrow x^{(k)} \quad (2.12) \]
s.t. \( g(x) < 0 \)

Where the parameters sequence \( \{\epsilon^{(k)}\}_{k \in \mathbb{N}} \) satisfies the condition

\[ 0 < \epsilon^{(k+1)} < \epsilon^{(k)} \text{ and } \epsilon^{(k)} \to 0. \]

### 2.3.3.3 Sequential Unconstrained Minimization Techniques (SUMT)

The Sequential Unconstrained Minimization Techniques (SUMT) method, presented by Fiacco and McCormick [12], is used to convert the constrained problem to the equivalent unconstrained optimization problem. SUMT implements the penalty method in order to find a suitable initial point, as well as taking advantage of the barrier method to make sure that the point will over satisfy the inequality constraints.

The steps of the SUMT search for the optimal solution are shown in the following sequence.

1. Choose the following parameters
   - R Initial value for \( r \)
   - \( \theta \) Control rate of reduction \( r \)
   - \( \epsilon \) Satisfactory error tolerance
   - \( k \) = Iteration number (set to be initially 0)

2. Search for an initial point \( x_0 \) that could over satisfy the inequality constraints and boundary conditions of an objective function while being close enough to satisfy the equality constraints as well.

3. Form the barrier function \( B(x, r) \), which consists of a specific term for each of the constraints and bounds. Combine the original form of an unconstrained objective function with this function to create the new objective equation. Minimize \( F(x) = f(x) + B(x, r) \).

4. Find the optimal point \( x_{k+1} \) for the function \( F(x) \) using the starting point \( x_k \) (using unconstrained Non-Linear Programming(NLP) methods, such as gradient base methods).

5. Consider the stopping conditions, if \( |x_{k+1} - x_k| \leq \epsilon \), then \( x_{k+1} \) is the optimal solution and the cycle is over. Otherwise, reduce the value of \( r: r \leftarrow r \theta \), increase the value of \( k: k \leftarrow k + 1 \) and start from step 2 [12] [13].
2.3.3.4 Lagrange Multiplier

The Lagrange multiplier method can find the solution to optimization problems with equality constraints (Equation 2.13).

\[
\text{Minimize } f(x) \quad (2.13)
\]

\[
\text{Subject to: } h_i(x) = 0, \quad j = 1, \ldots, m < n
\]

The Lagrange function can be formed as Equation 2.14.

\[
L(x, \lambda) = f(x) + \sum_{j=1}^{m} \lambda_j h_j(x) = f(x) + \lambda^T h(x) \quad (2.14)
\]

\[
\frac{\partial L}{\partial x_i}(x^*, \lambda^*) = 0, \quad i = 1, \ldots, n
\]

\[
\frac{\partial L}{\partial \lambda_j}(x^*, \lambda^*) = 0, \quad j = 1, \ldots, m
\]

Where \( \lambda \) represents \( m \times 1 \) vector of Lagrange multipliers, one for each constraint, \( x^* \) is the minimum solution, and \( \lambda^* \) is a set of associated Lagrange multipliers.

In case of a quadratic objective function and linear constraint, partial derivative could be found according to Equation 2.15.

\[
L(x, \lambda) = \frac{1}{2} x^T A x + b^T x + c + \lambda^T (C x - d) \quad (2.15)
\]

\[
\frac{\partial L}{\partial x} = A x + b + C^T \lambda = 0
\]

\[
\frac{\partial L}{\partial \lambda} = C x - d = 0
\]

Results are presented in Equation 2.16 by the linear system, which can be solved directly.

\[
\begin{bmatrix}
A & C^T \\
C & 0
\end{bmatrix}
\begin{bmatrix}
x^* \\
\lambda^*
\end{bmatrix} =
\begin{bmatrix}
-b^* \\
d
\end{bmatrix} \quad (2.16)
\]

2.3.4 Non-Gradient Based Methods

Gradient–free optimization methods are among the commonly applied numerical techniques used to find the local optimum of an objective function in a multidimensional space. Powell’s conjugate direction method and the Nelder-Mead simplex algorithm are well-known approaches.
in this classification. Both methods are capable of finding the local minimum in a non-linear, unconstrained optimization problem [14].

The Nelder-Mead simplex method performs a multi-dimensional search by heuristic algorithms. The method is efficient and the computational process is compact and robust, while it does not require the function to be derivate or smooth. The process simply makes use of a simplex (a triangle in ‘n’ dimensions) and generates a sequence of triangles in different shapes, for which the function values at the vertices get smaller and smaller [11].

2.4 Global Optimization Algorithms

“Global optimization is a branch of applied mathematics and numerical analysis that deals with the global optimization of a function or a set of functions according to some criteria” [15]. Typically, these techniques are proposed to return the best possible set of input values for which, the objective function generates the best optimal solution (among multiple optima), considering the constraints of the model.

In Figure 2.7, the minima at $x \approx -1$ represent the concept of local optimal (local minimum) in respect to the objective function $f(x)$, while $x=0$ represents a globally optimal solution (global minimum) for this function. Depending on the adopted method, all three points could be reached by local optimization methods.

![Figure 2.7: One Dimensional Multi-Model Function [9]](image)
2.4.1 Classification and History

Thomas Weise [16] presented a rough taxonomy of global optimization methods in his book, named ‘Global Optimization Algorithms- Theory and Application’ presented, shown in Figure 2.8. Global optimization techniques are mainly classified as either deterministic or probabilistic algorithms.

Figure 2.8: The Taxonomy of Global Optimization Algorithms [16]
A deterministic algorithm is a mathematical function that generally defines a unique set of solution to the specific set of input values in its domain. The deterministic method makes use of an explicit relation between the set of solution candidates and their fitness by exploring the search space. On the other hand, if the relation is very complicated or the search space has a large dimension, deterministic methods will no longer be efficient and probabilistic algorithms will be used to make the optimization feasible.

Probabilistic methods were refined 50 years ago; the most famous branch of this family are known to be Monte Carlo approaches. These techniques guarantee to improve the solutions in a short time, while the results might not be the global answer [16]. Heuristics algorithms are mathematical functions used in deterministic algorithms for global optimization to define the processing order of candidate solutions, while probabilistic methods may only rely on parameters that have been chosen by heuristic algorithms [16]. A metaheuristic method is a higher-level procedure, proposed to generate an efficient solution to an optimization problem. These algorithms, which are often nature-inspired, can be applied to solve complex optimization problems with uncertain, stochastic, dynamic information. For example, evolutionary algorithms are inspired by the behavior of biological evolution, and candidate solutions are considered as individuals competing in a virtual environment [17] [18]. Figure 2.9 presents different methods in each classification of metaheuristic groups which are the most popular metaheuristic methods between global optimization techniques.
Figure 2.9: Metaheuristic Classification [19]
2.5 Teaching-Learning-Based Optimization (TLBO) algorithm

2.5.1 TLBO Overview

Teaching-Learning-Based Optimization (TLBO) algorithm is a popular method among metaheuristic techniques with relatively competitive performances. The TLBO algorithm is a nature-inspired method proposed by Rao and colleagues [20]. This method is also a population-based algorithm which uses a population of solutions to proceed to the optimal solution. The population is considered as a group or class of learners. The most attracting point about the algorithm is that it does not need any algorithm-specific parameter to be tuned and only common controlling parameters like population size and the number of generations is required for its working. The method has gained wide acceptance for its simplicity and the ability to find the global or near-global optimum solutions in less number of function evaluations rather than other metaheuristic algorithms [21].

TLBO is inspired by the process of knowledge in the classroom environment. The method consists of two steps: ‘Teacher phase’ and ‘Learner Phase’. ‘Teacher Phase’ means acquiring knowledge from a teacher, and ‘Learner Phase’ means learning by interacting with classmates. The subjects offered to the learners are considered to be different design variables for the objective function of the optimization problem. The learners’ result is analogous to the ‘fitness’ of the optimization problem [20] [22]. The teacher is considered as the best optimal solution to the problem in the first step. TLBO focus on the influence of the teacher on the learners and the influence of learners on their classmates. The learners’ results are typically represented by the student’s mean grade. The teacher attempts to empower the students by increasing their knowledge and improving their grades. This will consequently result in a better class average based on his or her capabilities and teaching skills. On the other hand, despite the great efforts made by the teacher for increasing student’s grades, the quality of students sitting in the class cannot be ignored as an important factor that affects their grades. Quality of students is assessed through the mean value of the class population. Moreover, the teacher is responsible to make efforts to bring the students into a higher level of knowledge. A new teacher with better qualifications and skills would be required to teach the classroom at this higher level [20] [22].
Some notes have reported remarkable results about TLBO outperforming ES, PSO, ABC, DE, and GEM on a number of benchmarks and constrained optimization problems where TLBO reaches better or equal optimal solutions much faster than the other methods, as reported in [22] [23].

Figure 2.10: Model for the distribution of marks obtained by learners taught by two different teachers [20]

Figure 2.10 shows the distribution of marks achieved by the learners in two different classes [20]. Assuming that there are two different teachers for each class, T1 and T2, teaching the same subject to the same merit level learners in the classes. Curve 1 and Curve 2 represent the obtained marks, by learners taught by teacher T1 and T2 respectively, while both distributions are assumed to be normal. M1 and M2 represent the mean factors, as the main difference between both groups’ results. As it can be seen in Figure 2.10, Curve 2 represents better marks obtained by students taught by teacher T2. Clearly, the comparison indicates that a better instructor conducts a better mean for the results of the learners.

The learners also improve their level of knowledge by interacting with other learners, which will increase their results.

Figure 2.11: Model for the distribution of marks obtained for a group of learners [20]
Figure 2.11 illustrates a normal distribution model for the marks obtained by the learner for a class with Curve-A having mean MA. By considering the teacher as the most knowledgeable person in the group, the best learner in the society will be imitated as a teacher (TA in Figure 2.11). The teacher TA will try to improve the mean level of the class according to his or her teaching capabilities. Teacher TA will put maximum efforts to proceed the mean MA toward the new mean, MB, which is close to her or his level of knowledge depending on his or her capabilities; but the learners will gain the knowledge not only according to the quality of the teacher but also depending on the merit level of the students present in the class, which is judged from the mean value of the society. The quality of the students can improve to the stage MB by the efforts of teacher TA, where the learner requires a superior teacher with greater knowledge than themselves. Consequently, the algorithm will be followed by the new Curve-B with new teacher TB.

2.5.2 TLBO Algorithm

Table 2.2 presents the TLBO algorithm. As a population-based algorithm, the initial phase in TLBO starts with randomly generating a population of candidate solutions in the search space. The problem search space consists of \( n \) dimensions, and each dimension should satisfy its upper and lower boundaries. The process is divided into two parts namely: the ‘Teacher Phase’ and the ‘Learner Phase’ [20] [24].

- **Teacher Phase**
  In this phase, the teacher tries to increase the knowledge of learners and increase the mean result of the class in the subject taught by him or her depending on his or her capability [20]. This procedure is exposed in the algorithm between lines (11–14) in Table 2.2. [22]

- **Learner Phase**
  In this phase, the students improve their level of knowledge by interacting with other learners. A learner can improve his or her knowledge if the other randomly selected learner has more knowledge than him or her. The procedure for the learner phase is given in lines (27-28) in Table 2.2 [22].
Table 2.2: Algorithm (TLBO pseudocode) [22]

1. \( n \): dimension of the problem
2. \( m \): population size
3. MAXITER: maximum number of iterations
4. Initialization()
5. while ITER < MAXITER
6.   Elite ← SelectBest(P,Elite)
7.   for \( i = 1 : m \)
8.     \( T_i = \text{round}(1 + \text{rand}) \)
9.     \( X_{\text{mean}} ← \text{mean}(X_i) \)
10. \( X_{\text{teacher}} ← \text{best}(X_i) \)
11. \( X_{\text{new},i} = X_i + \text{rand} \cdot (X_{\text{teacher}} - (T_i \cdot X_{\text{mean}})) \)
12. if \( f(X_{\text{new},i}) < f(X_i) \)
13.   \( X_i ← X_{\text{new},i} \)
14. end if
15. \( j ← \text{randi}(m) \)
16. if \( j \neq i \)
17.   if \( f(X_i) < f(X_j) \)
18.     \( X_{\text{new},j} = X_i + \text{rand} \cdot (X_i - X_j) \)
19.   else
20.     \( X_{\text{new},j} = X_i + \text{rand} \cdot (X_j - X_i) \)
21.   end if
22. end if
23. if \( f(X_{\text{new},i}) < f(X_i) \)
24. \( X_i ← X_{\text{new},i} \)
25. end if
26. end for
27. P ← ReplaceWorstWithElite(P,Elite)
28. P ← RemoveDuplicateIndividuals(P)
29. ITER ← ITER + 1
30. end while

Figure 2.12 presents the flowchart of Teaching-Learning-Based Optimization (TLBO) algorithm.
Initialize no. of students (population), no. of subjects (design variables) and termination criterion

Calculate the mean of each design variable

Identify the best solution and the values of variables

Modify the other values of variables based on best solution

\[ \text{Difference Mean}_{\mu i} = c_i (X_{\text{Mean} i} - T_j M_j) \]

\[ X'_{ji} = X_{ji} + \text{Difference Mean}_{\mu i} \]

Is solution corresponding to \( X'_{ji} \) better than the solution corresponding to \( X_{ji} \)?

No
Keep the previous

Yes
Replace the previous

Select two solutions \( X_{\text{rand} p j} \) and \( X_{\text{rand} q j} \), randomly

Is solution \( X_{\text{rand} p j} \) better than the solution \( X_{\text{rand} q j} \)?

No

Yes

\[ X''_{jp j} = X_{jp j} + r_i (X_{\text{rand} q j} - X_{\text{rand} p j}) \]

\[ X''_{jq j} = X_{jq j} + r_i (X_{\text{rand} p j} - X_{\text{rand} q j}) \]

Is solution corresponding to \( X''_{jp j} \) better than that of \( X''_{jq j} \)?

No
Reject

Yes
Accept

Keep the previous

Replace the previous

Is the termination criterion satisfied?

Yes
Report the solution

No
2.5.3 Case Study of a Constrained Optimization Problem with TLBO Algorithm

In this case study, a constrained benchmark function of Himmelblau is solved with the TLBO algorithm adopted from [21]. The objective function needs to be minimized by values of $x_1$ and $x_2$ considering the given constraints.

\[
\text{Minimize } f(x) = (x_1^2 + x_2 - 11)^2 + (x_1 + x_2^2 - 7)^2
\]

\[
\text{Subject to:}
\]
\[
g_1(x) = 26 - (x_1 - 5)^2 - x_2^2 \geq 0,
\]
\[
g_2(x) = 20 - 4x_1 - x_2 \geq 0,
\]
\[
-5 \leq x_1, x_2 \leq 5
\]

The known solution to this benchmark function is $0$ for $x_1 = 3$ and $x_2 = 2$ and $g_1(x) = 18$ and $g_2(x) = 16$ according to [21]. The population size is assumed to be five (numbers of learners). There are two design variables, $x_1$ and $x_2$, and one iteration as the termination criterion.

Table 2.3 presents the initial population which is randomly generated within the ranges of variables and provides the corresponding values for the objective function. The mean values of $x_1$ and $x_2$ are calculated. The lowest value of $f(x)$ is considered as the best learner in this minimization problem (it is supposed as an equivalent to the teacher). The penalty factors will be assigned to the objective function in case of any violation to the given constraints upon to the designer/decision maker (in this case the penalty $p_1$ for violation of $g_1(x)$ is considered as $10 \times (g_1(x))^2$ and the penalty $p_2$ for violation of $g_2(x)$ is considered as $10 \times (g_2(x))^2$). The fitness function is called $f'(x)$ and is formulated according to Equation 2.17 as a minimization problem considering the penalty factors. The fitness function, $f'(x)$, is called the pseudo-objective function.

\[
f'(x) = f(x) + 10 \times (g_1(x))^2 + 10 \times (g_2(x))^2 \tag{2.17}
\]
In this step, the teacher tries to improve the mean result of the learners. The $\text{difference}_{\text{mean}}$ values for $x_1$ and $x_2$ are calculated by assuming the random numbers of $r_1 = 0.25$ and $r_2 = 0.43$ for $x_1$ and $x_2$ respectively and $T_f = 1$, shown by Equations (2.18) and (2.19):

$$\text{difference}_{\text{mean}}(x_1) = 0.25 \times (3.22 - 2.093) = 0.2817 \quad (2.18)$$

$$\text{difference}_{\text{mean}}(x_2) = 0.43 \times (0.403 - 1.697) = -0.55642 \quad (2.19)$$

The values under the $x_1$ and $x_2$ columns in Table 2.3 are added by the calculated $\text{difference}_{\text{mean}}$ values. Table 2.4 presents the new results for $x_1$ and $x_2$, penalties and the values of the objective function and pseudo-objective function.

Completing the teacher phase of the TLBO algorithm, the results of $f'(x)$ in Table 2.3 and 2.4 are compared and the best values are placed in Table 2.5.
Table 2.5: Updated values of the variables, objective function and the penalties based on fitness comparison (teacher phase)

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$f(x)$</th>
<th>$g_1(x)$</th>
<th>$p_1$</th>
<th>$g_2(x)$</th>
<th>$p_2$</th>
<th>$f'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.22</td>
<td>0.403</td>
<td>13.13922</td>
<td>22.66919</td>
<td>0</td>
<td>6.717</td>
<td>0</td>
<td>13.13922</td>
</tr>
<tr>
<td>0.47275</td>
<td>1.73258</td>
<td>94.22118</td>
<td>2.50217</td>
<td>0</td>
<td>16.37642</td>
<td>0</td>
<td>94.22118</td>
</tr>
<tr>
<td>3.46375</td>
<td>-0.22142</td>
<td>12.76312</td>
<td>23.5909</td>
<td>0</td>
<td>6.36642</td>
<td>0</td>
<td>12.76312</td>
</tr>
<tr>
<td>1.94175</td>
<td>4.03658</td>
<td>136.437</td>
<td>0.35312</td>
<td>0</td>
<td>8.19642</td>
<td>0</td>
<td>136.437</td>
</tr>
<tr>
<td>2.49575</td>
<td>0.31058</td>
<td>39.32602</td>
<td>19.63227</td>
<td>0</td>
<td>9.70642</td>
<td>0</td>
<td>39.32602</td>
</tr>
</tbody>
</table>

The learner phase considers the interaction between students to improve their knowledge. In this example, the interaction is assumed to have a random manner. Table 2.6 shows the new values of $x_1$ and $x_2$ in the first step of learner phase while the random numbers are considered to be $r_1 = 0.47$ for $x_1$ and $r_2 = 0.33$ for $x_2$.

Table 2.6: New values of the variables, objective function and the penalties (learner phase)

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$f(x)$</th>
<th>$g_1(x)$</th>
<th>$p_1$</th>
<th>$g_2(x)$</th>
<th>$p_2$</th>
<th>$f'(x)$</th>
<th>Interaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5112</td>
<td>-0.03576</td>
<td>92.96006</td>
<td>25.75979</td>
<td>1.99096</td>
<td>0</td>
<td>92.96006</td>
<td>1 &amp; 2</td>
<td></td>
</tr>
<tr>
<td>1.42356</td>
<td>1.26332</td>
<td>75.2906</td>
<td>11.61309</td>
<td>13.04244</td>
<td>0</td>
<td>75.2906</td>
<td>2 &amp; 5</td>
<td></td>
</tr>
<tr>
<td>3.57831</td>
<td>-0.42747</td>
<td>12.38652</td>
<td>23.79606</td>
<td>6.11423</td>
<td>0</td>
<td>12.38625</td>
<td>3 &amp; 1</td>
<td></td>
</tr>
<tr>
<td>2.20213</td>
<td>2.807</td>
<td>20.67471</td>
<td>10.29267</td>
<td>8.38448</td>
<td>0</td>
<td>20.67471</td>
<td>4 &amp; 5</td>
<td></td>
</tr>
<tr>
<td>2.75613</td>
<td>-0.919</td>
<td>30.24144</td>
<td>20.12048</td>
<td>9.89448</td>
<td>0</td>
<td>30.24144</td>
<td>5 &amp; 4</td>
<td></td>
</tr>
</tbody>
</table>

The results of $f'(x)$ in Tables 2.5 and 2.6 are compared and the best values are placed in Table 2.7 completing the learner phase.

Table 2.7: Updated values of the variables, objective function and the penalties based on fitness comparison

<table>
<thead>
<tr>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$f(x)$</th>
<th>$g_1(x)$</th>
<th>$p_1$</th>
<th>$g_2(x)$</th>
<th>$p_2$</th>
<th>$f'(x)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.22</td>
<td>0.403</td>
<td>13.13922</td>
<td>22.66919</td>
<td>0</td>
<td>6.717</td>
<td>0</td>
<td>13.13922</td>
</tr>
<tr>
<td>1.42356</td>
<td>1.26322</td>
<td>75.2906</td>
<td>11.61309</td>
<td>0</td>
<td>13.04244</td>
<td>0</td>
<td>75.2906</td>
</tr>
<tr>
<td>3.57831</td>
<td>-0.42747</td>
<td>12.38652</td>
<td>23.79606</td>
<td>6.11423</td>
<td>0</td>
<td>12.38625</td>
<td>3 &amp; 1</td>
</tr>
<tr>
<td>2.20213</td>
<td>2.807</td>
<td>20.67471</td>
<td>10.29267</td>
<td>8.38448</td>
<td>0</td>
<td>20.67471</td>
<td>4 &amp; 5</td>
</tr>
<tr>
<td>2.75613</td>
<td>-0.919</td>
<td>30.24144</td>
<td>20.12048</td>
<td>9.89448</td>
<td>0</td>
<td>30.24144</td>
<td>5 &amp; 4</td>
</tr>
</tbody>
</table>

As can be seen from the tables, the value of $f'(x)$ is minimized from 13.13922 at the beginning of iteration to 12.38652 at the end of the first iteration. The TLBO algorithm is able to reach the known results of this benchmark (0 for $x_1 = 3$ and $x_2 = 2$ and $g_1(x) = 18$ and $g_2(x) = 16$ according to [21]) by increasing the number of iterations. Computer programs such as
MATLAB, can be used to execute the TLBO algorithm for solving the unconstrained and constrained optimization problems.
2.6 More Case Studies of Optimization Problems

2.6.1 Case Study of Unconstrained Nonlinear Optimization Problem

The goal of this case study is to minimize the presented unconstrained function, by setting the partial derivatives equal to zero and solving the function by parameters value.

\[ f(x_1, x_2, ..., x_n) = x_1^2 + 2x_2^2 + 3x_3^2 + x_1x_2 + x_2x_3 - 8x_1 - 16x_2 - 32x_3 + 110 \]

Equating the first derivatives to zero, result in

\[ \frac{\partial f}{\partial x_1} = 2x_1 + x_2 - 8 = 0 \]
\[ \frac{\partial f}{\partial x_2} = x_1 + 4x_2 + x_3 - 16 = 0 \]
\[ \frac{\partial f}{\partial x_3} = x_2 + 6x_3 - 32 = 0 \]

Using MATLAB and \( X = A\backslash B \) command, the optimal solution for the function is resulted,

\[ (\hat{x}_1 \quad \hat{x}_2 \quad \hat{x}_3) = (3 \quad 2 \quad 5) \]

where the function value is:

\[ f(3 \quad 2 \quad 5) = 2 \]

To see if this point is the minimum, the Hessian matrix is formed.

\[ H(\hat{x}) = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 6 \end{bmatrix} \]

Using MATLAB function eig(H), the eigenvalues are found to be 1.55, 4.0 and 6.45 which are all positive. Thus the Hessian matrix is a positive definite matrix and \((3 \quad 2 \quad 5)\) is a minimum point.
2.6.2 Case Study of Constrained Nonlinear Optimization Problem

2.6.2.1 Solution by Lagrange Multiplier Method

In this case study, the main propose is to apply Lagrange multiplier method for solving constrained parameter optimization to determine the minimum distance from the origin of the ‘xy’ plane to a circle described by:

\[ (x - 8)^2 + (y - 6)^2 = 25 \]

The minimum distance is obtained by minimization of the distance square, given by:

Objective Function: \( f(x, y) = x^2 + y^2 \)

The MATLAB plot command is used to plot the circle shown in Figure 2.13 [1].

![Figure 2.13: Constrained function of constrained nonlinear optimization problem case study](image)

Forming the Lagrange function, we obtain:

\[ \ell = x^2 + y^2 + \lambda [(x - 8)^2 + (y - 6)^2 - 25] \]

The necessary condition for extrema are:

\[ \frac{\partial \ell}{\partial x} = 2x + \lambda (2x - 16) = 0 \]
\[ 2x(\lambda + 1) = 16\lambda \]

\[ \frac{\partial \ell}{\partial y} = 2y + \lambda(2y - 12) = 0 \]

\[ 2y(\lambda + 1) = 12\lambda \]

\[ \frac{\partial \ell}{\partial \lambda} = (x - 8)^2 + (y - 6)^2 - 25 = 0 \]

Eliminating \( \lambda \) from the first two equation results in:

\[ y = \frac{3}{4}x \]

Substituting in third equation yields:

\[ \frac{25}{16}x^2 - 25x + 75 = 0 \]

The solutions of above quadratic equations are \( x = 12 \) and \( x = 4 \). Thus, the corresponding extrema are at points (4,3) with \( \lambda = 1 \), and (12, 9) with \( \lambda = -3 \). It is clear that the minimum distance is at point (4,3) and the maximum is at point (12, 9). From this example, clearly, the minimum distance is 5, located at (4,3). To distinguish this points, the second derivatives are obtained and the Hessian matrix evaluated at these points are formed. The matrix with positive eigenvalues is a positive definite matrix and the parameters corresponds to the minimum point.

| Table 2.8: Lagrange multiplier method, constrained optimization for the case study |
|-----------------|-----|-----|-----|
| \( \lambda \)  | X   | Y   | f (x) |
| Minimum Distance | 1   | 4   | 3   | 25  |
| Maximum Distance | -3  | 12  | 9   | 225 |
2.6.2.2 Solution by Newton-Raphson Method

In many optimization problems, the direct solution is not possible and the equation needs to be solved with iterative methods. A significantly superior method applicable for solving presented continues function in case study 2, is the Newton-Raphson method.

From the first two equations, x and y are found. These are

\[
x = \frac{8\lambda}{\lambda + 1}
\]

\[
y = \frac{6\lambda}{\lambda + 1}
\]

Substituting in third equation results in

\[
f(\lambda) = \frac{100\lambda^2}{(\lambda + 1)^2} - \frac{200\lambda}{\lambda + 1} + 75 = 0
\]

This is a non-linear equation in terms of \( \lambda \) and can be solved by the Newton-Raphson method.

\[
\Delta \lambda^{(k)} = -\frac{\Delta f(\lambda)^{(k)}}{\left( \frac{df}{d\lambda} \right)^{(k)}}
\]

\[
\lambda^{(k+1)} = \lambda^{(k)} + \Delta \lambda^{(k)}
\]

\[
\frac{df(\lambda)}{d\lambda} = \frac{200\lambda}{(\lambda + 1)^3} - \frac{200}{(\lambda + 1)^2} = \frac{-200}{(\lambda + 1)^3}
\]

The following commands show the procedure for the solution of the given case study, solved by the Newton-Raphson method and an initial estimate of \( \lambda = 0.4 \).

Table 2.9 and Table 2.10 summarize the results of this method. After five iterations the solution converges to \( \lambda = 1.0, x = 4 \), and \( y = 3 \), corresponding to the minimum length. If the program is run with the initial estimate of \( -2 \), the solution converges to \( \lambda = -3.0, x = 12 \), and \( y = 9 \), which corresponds with maximum length [1].
Table 2.9: Newton-Raphson method and iterative solution, Lambda= 0.4

<table>
<thead>
<tr>
<th>Iter</th>
<th>Df</th>
<th>J</th>
<th>DLambda</th>
<th>Lambda</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0000</td>
<td>26.0204</td>
<td>-72.8863</td>
<td>0.3570</td>
<td>0.4000</td>
<td>2.2857</td>
<td>1.7143</td>
</tr>
<tr>
<td>2.0000</td>
<td>7.3934</td>
<td>-36.8735</td>
<td>0.2005</td>
<td>0.7570</td>
<td>3.4468</td>
<td>2.5851</td>
</tr>
<tr>
<td>3.0000</td>
<td>1.0972</td>
<td>-26.6637</td>
<td>0.0411</td>
<td>0.9575</td>
<td>3.9132</td>
<td>2.9349</td>
</tr>
<tr>
<td>4.0000</td>
<td>0.0337</td>
<td>-25.0505</td>
<td>0.0013</td>
<td>0.9987</td>
<td>3.9973</td>
<td>2.9980</td>
</tr>
<tr>
<td>5.0000</td>
<td>0.0000</td>
<td>-25.0001</td>
<td>0.0000</td>
<td>1.0000</td>
<td>4.0000</td>
<td>3.0000</td>
</tr>
</tbody>
</table>

Table 2.10: Newton-Raphson method and iterative solution, Lambda= -2

<table>
<thead>
<tr>
<th>Iter</th>
<th>Df</th>
<th>J</th>
<th>DLambda</th>
<th>Lambda</th>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0000</td>
<td>75.0000</td>
<td>200.0000</td>
<td>-0.3750</td>
<td>-2.0000</td>
<td>16.0000</td>
<td>12.0000</td>
</tr>
<tr>
<td>2.0000</td>
<td>27.8926</td>
<td>76.9346</td>
<td>-0.3625</td>
<td>-2.3750</td>
<td>13.8182</td>
<td>10.3636</td>
</tr>
<tr>
<td>3.0000</td>
<td>8.1227</td>
<td>38.1258</td>
<td>-0.2131</td>
<td>-2.7375</td>
<td>12.6042</td>
<td>9.4531</td>
</tr>
<tr>
<td>4.0000</td>
<td>1.2823</td>
<td>26.9480</td>
<td>-0.0476</td>
<td>-2.9506</td>
<td>12.1013</td>
<td>9.0760</td>
</tr>
<tr>
<td>5.0000</td>
<td>0.0454</td>
<td>25.0682</td>
<td>-0.0018</td>
<td>-2.9982</td>
<td>12.0036</td>
<td>9.0027</td>
</tr>
<tr>
<td>6.0000</td>
<td>0.0001</td>
<td>25.0001</td>
<td>-0.0000</td>
<td>-3.0000</td>
<td>12.0000</td>
<td>9.0000</td>
</tr>
</tbody>
</table>

The following commands show the MATLAB code for this case study, Newton-Raphson method [1].

```matlab
% Iterative solution Using Newton method
iter = 0; % Iteration counter
Df = 10; % Error in Df is set to a high value
Lambda = input('Enter estimated value of Lambda = ');
fprintf('
')
disp(['     Iter      Df         J       DLambda   Lambda   x      y'])
while abs(Df) >= 0.0001 % Test for convergence
    iter = iter + 1; % No. of iterations
    x = 8*Lambda/(Lambda + 1); %
```
\[
y = \frac{6 \cdot \Lambda}{\Lambda + 1};
\]
\[
Df = (x - 8)^2 + (y - 6)^2 - 25; \quad \% \text{ Residual}
\]
\[
J = -\frac{200}{(\Lambda + 1)^3}; \quad \% \text{ Derivative}
\]
\[
\text{Delambda} = -\frac{Df}{J}; \quad \% \text{ Change in variable}
\]
\[
disp([\text{iter}, Df, J, \text{Delambda}, \Lambda, x, y])
\]
\[
\Lambda = \Lambda + \text{Delambda}; \quad \% \text{ Successive solution}
\]

End

2.6.3 MATLAB Optimization Toolbox

The Optimization Toolbox includes programs or m-files that can be used to solve different types of optimization problems. Many publications provide information on the optimization toolbox, including algorithms and examples for different programs [25].

The use of any program or m-file in the optimization toolbox requires the following steps:

- Selecting the appropriate program or m-file to solve the specific problem at hand.
- Formulation of the optimization problem in the format expected by MATLAB. In general, this involves stating the objective function in a specific form such as a “minimization” type and the constraints in a specific form such as “less than or equal to zero” type.
- The distinction between linear and nonlinear constraints.
- Identification of lower and upper bounds on design variables.
- Setting/changing the parameters of the optimization algorithm (based on the available options) [4], [25].

Each program or m-file in MATLAB can be implemented in several ways. For illustration, the help commands and the response for the program fmincon are shown below. The function fmincon can be used in 12 different ways as indicated below (by the help command). The differences depend on the available data in the mathematical model of the problem and the information required from the solution to the problem. In using the different function calls, any data missing in the mathematical model of the optimization problem needs to be indicated using a null vector as [ ]. Note that the response is edited for brevity [4].

```
>> help fmincon

FMINCON finds a constrained minimum of a function of several variables.

FMINCON attempts to solve problems of the form:

\[
\text{min } F(X) \quad \text{subject to:}
\]
```
A*X  <= B, Aeq*X  = Beq  (linear constraints)
C(X)  <=  0, Ceq(X) = 0  (nonlinear constraints)
LB  <=  X  <=  UB  (bounds)

X=FMINCON (FUN,X0,A,B)  X=FMINCON (FUN,X0,A,B,Aeq,Beq)  X=FMINCON
(FUN,X0,A,B,Aeq,Beq,LB,UB)
X=FMINCON (FUN,X0,A,B,Aeq,Beq,LB,UB,NONLCON)
X=FMINCON (FUN,X0,A,B,Aeq,Beq,LB,UB,NONLCON,OPTIONS)
X=FMINCON (FUN,X0,A,B,Aeq,Beq,LB,UB,NONLCON,OPTIONS,...
P1,P2,...) [X,FVAL] = FMINCON (FUN,X0,...)
[X,FVAL,EXITFLAG] = FMINCON (FUN,X0,...)
[X,FVAL,EXITFLAG,OUTPUT]=FMINCON (FUN,X0,...)
[X,FVAL,EXITFLAG,OUTPUT,LAMBDA] =FMINCON (FUN,X0,...)
[X,FVAL,EXITFLAG,OUTPUT,LAMBDA,GRAD]=FMINCON (FUN,X0,...)
[X,FVAL,EXITFLAG,OUTPUT,LAMBDA,GRAD,HESSIAN]=FMINCON
(FUN,X0,...).
Table 2.11 shows some of the MATLAB Optimization Toolbox commands [25].

Table 2.11: MATLAB Optimization Toolbox Commands [25]

<table>
<thead>
<tr>
<th>Type of optimization problem</th>
<th>Standard form for solution by MATLAB</th>
<th>Name of MATLAB program or function to solve the problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Function of one variable or scalar minimization</td>
<td>Find $x$ to minimize $f(x)$ with $x_1 &lt; x &lt; x_2$</td>
<td>fminbnd</td>
</tr>
<tr>
<td>Unconstrained minimization of function of several variables</td>
<td>Find $x$ to minimize $f(x)$</td>
<td>fminunc or fminsearch</td>
</tr>
<tr>
<td>Linear programming problem</td>
<td>Find $x$ to minimize $f^T x$ subject to $[A]x \leq b, [A_{eq}]x = b_{eq}, l \leq x \leq u$</td>
<td>linprog</td>
</tr>
<tr>
<td>Quadratic programming problem</td>
<td>Find $x$ to minimize $\frac{1}{2} x^T [H]x + f^T x$ subject to $[A]x \leq b, [A_{eq}]x = b_{eq}, l \leq x \leq u$</td>
<td>quadprog</td>
</tr>
<tr>
<td>Minimization of function of several variables subject to constraints</td>
<td>Find $x$ to minimize $f(x)$ subject to $c(x) \leq 0, c_{eq} = 0$</td>
<td>fmincon</td>
</tr>
<tr>
<td>Goal attainment problem</td>
<td>Find $x$ and $\gamma$ to minimize $\gamma$ such that $F'(x) - w\gamma \leq \text{goal}$, $c(x) \leq 0, c_{eq} = 0$</td>
<td>fgoalattain</td>
</tr>
<tr>
<td>Minimax problem</td>
<td>Minimize $\max_{x} { F_i(x) }$ such that $c(x) \leq 0, c_{eq} = 0$</td>
<td>fminimax</td>
</tr>
<tr>
<td>Binary integer programming problem</td>
<td>Find $x$ to minimize $f^T x$ subject to $[A]x \leq b, [A_{eq}]x = b_{eq}$, each component of $x$ is binary</td>
<td>bintprog</td>
</tr>
</tbody>
</table>
2.6.3.1 Case Study Using MATLAB Optimization Toolbox: Unconstrained Optimization with Objfun

Find the minimum of the Rosenbrock’s parabolic valley function, given by:

\[ f(x_1, x_2) = 100 \left( x_2 - x_1^2 \right)^2 + (1 - x_1)^2 \]

starting from the initial point: \( X_1 = \{-1.2, 1.0\} \), \( X^* = \{1\} \)

\[ f_1 = 24.0 \quad f^* = 0.0 \]

Solution

Step 1: Writing an M-file objfun.m for the objective function.

```matlab
function f = objfun(x)
    f = 100 * (x(2) - x(1) * x(1))^2 + (1 - x(1))^2;
```

Step 2: Invoking the unconstrained optimization program in a new MATLAB file.

```matlab
clear all
warning off
x0 = [-1.2, 1.0]; % Starting guess
f = objfun(x0)
options = optimset('LargeScale', 'off');
[x, fval] = fminunc (@objfun, x0, options)
```

The values of function value at the starting point is \( f = 24.2000 \). Optimization completed because the size of the gradient is less than the default value of the optimality tolerance. The final result is obtained for \( x = [1.0000 \quad 1.0000] \) where the minimum of the Rosenbrock’s parabolic valley function is \( 2.8358e-11 \).
Case Study of Multivariable Minimization Problems Using MATLAB Function f-mincon

The solution of multivariable minimization problems, with inequality and equality constraints, using the MATLAB function f-mincon is illustrated in this section [4].

Find the solution of presented function starting from the initial point $X_1 = \{0.1 \ 0.1 \ 3.0\}^T$

$$\text{Minimize } f(x) = x_1^3 - 6x_1^2 + 11x_1 + x_3$$

Subject to

$$x_1^2 + x_2^2 - x_3^2 \leq 0$$
$$4 - x_1^2 - x_2^2 - x_3^2 \leq 0$$
$$x_3 - 5 \leq 0$$
$$-x_i \leq 0, i = 1, 2, 3$$

Solution

Step 1: Write an M-file objfun.m for the objective function.

```matlab
function f = objfun(x)
    f = x(1)^3 - 6*x(1)^2 + 11*x(1) + x(3);
```

Step 2: Write an M-file constraints.m for the constraints.

```matlab
function [c, ceq] = constraints(x)
% Nonlinear inequality constraints
    c = [x(1)^2 + x(2)^2 - x(3)^2; 4 - x(1)^2 - x(2)^2 - x(3)^2; x(3) - 5; -x(1); -x(2); -x(3)];
% Nonlinear equality constraints
    ceq = [];
```

Step 3: Invoke constrained optimization program (write this in new MATLAB file).

```matlab
clc
clear all
warning off
x0 = [.1, .1, 3.0];
% Starting guess
f = objfun(x0)
[c, ceq] = constraints(x0)
options = optimset('LargeScale', 'off');
```
[x, fval] = fmincon (@objfun, x0, [], [], [], [], [], [], @constraints, options)
[c, ceq] = constraints (x) % Check the constraint values at x

The results for function value and constraints at the starting point is presented in Table 2.12. Local minimum found that satisfies the constraints. According to solver stopping criteria details, the optimization completed because the objective function is non-decreasing in feasible directions. Constraints are satisfied within the default value of the constraint tolerance.

Table 2.12 summarizes the results achieved with MATLAB optimization toolbox using the fmincon function.

Table 2.12: Results for Multivariable Minimization Problem Using MATLAB Function f-mincon

<table>
<thead>
<tr>
<th>Variables</th>
<th>Initial Point</th>
<th>Optimum Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_1$</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>$x_2$</td>
<td>0.1</td>
<td>1.4142</td>
</tr>
<tr>
<td>$x_3$</td>
<td>3</td>
<td>1.4142</td>
</tr>
<tr>
<td>$x_1^2 + x_2^2 - x_3^2 \leq 0$</td>
<td>-8.98</td>
<td>0</td>
</tr>
<tr>
<td>$4 - x_1^2 - x_2^2 - x_3^2 \leq 0$</td>
<td>-5.02</td>
<td>0</td>
</tr>
<tr>
<td>$x_3 - 5 \leq 0$</td>
<td>-2</td>
<td>-3.5858</td>
</tr>
<tr>
<td>Constraints</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-x_1 \leq 0$</td>
<td>-0.1</td>
<td>0</td>
</tr>
<tr>
<td>$-x_2 \leq 0$</td>
<td>-0.1</td>
<td>-1.4142</td>
</tr>
<tr>
<td>$-x_3 \leq 0$</td>
<td>-3</td>
<td>-1.4142</td>
</tr>
<tr>
<td>$f(x)$</td>
<td>Minimize $f(x) = x_1^3 - 6x_1^2 + 11x_1 + x_3$</td>
<td>4.041</td>
</tr>
</tbody>
</table>
2.7 Conclusions

This chapter has provided an overview of optimization techniques typically used in engineering optimization applications. Optimization methods are being applied in a wide spectrum of industries and have reached a degree of maturity with rapid advances in computational techniques. In recent years, large-scale engineering systems, with multi-objective functions and a set of constraints, can be optimized with both accuracy and efficiency.

In this chapter, the mathematical principles and foundations behind the optimization algorithms were discussed. The methods were classified as either local or global algorithm, while both constrained and unconstrained optimization problems were considered. The knowledge behind available algorithms would help the decision-maker choose the appropriate method, based on the optimization case at hand. Three numerical case studies with constraint and unconstrained objective function have been discussed. In the next chapter, the application of these studies in power systems will be explained explicitly.
Chapter 3

3. Optimal Power Flow

3.0 Introduction

The Optimal Power Flow (OPF) problem has a long research history spanning more than 45 years. A comprehensive model and formulation of the economic dispatch problem including voltage and operating constraints were introduced and it was later named the optimal power flow problem.

The main focus of OPF studies is based on the minimization of operational cost while considering the system constraints. There have been some efforts to point out other objective functions of the OPF problem, such as minimization of active and reactive power losses, variation in deregulated electricity markets, voltage stability indexes and minimization of emission index in recent years [26].

The main objective of this chapter is to provide a brief review of the Economic Dispatch (ED) problem and introduce the lambda iteration method through a few case studies. The load flow problem is presented and a comprehensive model of the OPF problem with different states has been represented. The concept of Security Constraint Optimal Power Flow (SCOPF) in a power system has been discussed and simulated on two standard case studies.

3.1 Power Flow

Power flow studies are the backbone of power system analysis and design. They are necessary for planning and designing the future expansion of power systems as well as determining the best operation of existing systems. In addition, power flow analysis is required for many analyses, such as transient stability and contingency studies. The principal information obtained from a power flow study is the magnitude and phase angle of the voltage at each bus and the real and reactive power flow in each line. In any interconnected power system of N buses,
the power injections at the buses are given by a set of $2N$ nonlinear simultaneous equations presented in order by Equations 3.1 and 3.2 [1] [27].

$$P_G^k - P_D^k = V_k \sum_{i=1}^{N} [V_j G_j^k \cos(\delta^k - \delta^j) + B_j^k \sin(\delta^k - \delta^j)] \quad (3.1)$$

$$Q_G^k - Q_D^k = V_k \sum_{i=1}^{N} [V_j G_j^k \sin(\delta^k - \delta^j) - B_j^k \cos(\delta^k - \delta^j)] \quad (3.2)$$

Where $k = 1, 2, ..., N$

- $V_k$ is the voltage magnitude at bus $k$
- $\delta^k$ is the voltage angle at bus $k$
- $P_G^k$ is the active power generation at bus $k$
- $P_D^k$ is the active power demand at bus $k$
- $Q_G^k$ is the reactive power generation at bus $k$
- $Q_D^k$ is reactive power demand at bus $k$
- $G_j^k$ is the real part of $(k,j)$ element of the bus admittance matrix
- $B_j^k$ is the imaginary part of $(k,j)$ element of the bus admittance matrix

Generally, the goal of power flow studies is to determine the specification of three different types of buses in the network. At each bus, two of four quantities $\delta_i$, $|V_i|$, $P_i$, and $Q_i$ are specified, and the remaining two are calculated.

### 3.1.1 Load Buses

A load bus or PQ bus is a non-generator bus where the total amount of active and reactive power generation is zero, usually the real power and reactive power of the load are known (according to previous records, load forecast or measurements). The unknown quantities are $\delta_i$, $|V_i|$, which need to be calculated. This is usually a generator bus.
3.1.2 Voltage Controlled Buses

At voltage controlled buses, the voltage magnitude is kept at a constant and is generally called the generator bus or PV bus. Both of the quantities, $|V_l|$ and $P_g$, are specified and the $\delta_l$ and $Q_l$ are the two unknown quantities that must be determined by power flow analysis.

3.1.3 Slack Bus

At the slack bus, the voltage angle $\delta_k$ is set to zero as a reference for the angles of all other bus voltages in the network. Figure 3.1 shows a bus schematic. The voltage magnitude of the slack bus $|V_k|$ is also specified. This bus has the ability to either absorb or supply active or reactive power. The other two unknown quantities, $P_G^k$ and $Q_G^k$, should be calculated by power flow analysis [1].

![Diagram of a typical bus in a network](image)
3.2 Economic Dispatch (ED)

The main objective of the Economic Dispatch (ED) problem is to minimize the total fuel cost of generation. The problem is subject to meet a set of equality constraints with respect to load and generation balances. The upper and lower limits on the output of generating units would provide a set of inequality constraints which needs to be satisfied by the Economic Dispatch solution.

3.2.1 Economic Dispatch Neglecting Losses and No Generator Limits

The simplest scenario for solving the Economic Dispatch problem neglects transmission losses. In this case, the problem does not consider system configuration and line impedance [1] [27]. This model assumes that the system consists of only one bus where all the generators and loads are connected. Figure 3.2 shows the shared bus in the simplest case of Economic Dispatch. The Economic Dispatch is the simplest model used when the transmission distance is very short and the load density is very high.

Since transmission lines are neglected, the total demand, $P_D$, is the sum of all generation. A cost function, $C_i$, is assumed to be known for each plant such that the objective function (total production cost) is minimum and subject to the constraint as defined by the Equations 3.3 and 3.4 [1].

Minimize $C_t = \sum_{i=1}^{n_g} C_i = \sum_{i=1}^{n_g} (\alpha_i + \beta_i P_i + \gamma_i P_i^2)$  \hspace{1cm} (3.3)
Subject to: \[ \sum_{i=1}^{n_g} P_i = P_D \] (3.4)

\( C_t = \text{total fuel cost} \)

\( P_D = \text{total demand} \)

\( P_i = \text{generation of } i\text{th plant} \)

\( n_g = \text{total number of dispatchable generating plants} \).

A typical approach is to augment the constraints into an objective function by using the Lagrange multipliers according to Equation 3.5.

\[ \mathcal{L} = C_t + \lambda (P_D - \sum_{i=1}^{n_g} P_i) \] (3.5)

The minimum of this unconstrained function is found where the derivative of the function to its variables is zero (Equation 3.6).

\[ \frac{\partial \mathcal{L}}{\partial P_i} = 0 \implies \frac{\partial C_t}{\partial P_i} + \lambda(0 - 1) = 0 \quad \implies \quad \frac{\partial C_t}{\partial P_i} = \frac{dC_i}{dP_i} = \lambda \] (3.6)

Therefore all units have the same incremental operating cost, denoted here by \( \lambda \), in order to minimize the total operating cost, \( C_t \).

The coordination equation can be expressed as Equation 3.7.

\[ P_i = \frac{\lambda - \beta_i}{2\gamma_i} \] (3.7)
A simple example of the economic dispatch problem neglecting losses and generation limits is presented below. The example is cited from the Saadat textbook [1].

The fuel cost functions for three thermal plants are given by:

\[ C_1 = 500 + 5.3 P_1 + 0.004 P_1^2 \,\$/hr \]
\[ C_2 = 400 + 5.5 P_2 + 0.006 P_2^2 \,\$/hr \]
\[ C_3 = 200 + 5.8 P_3 + 0.009 P_3^2 \,\$/hr \]

Where \( P_1, P_2, \) and \( P_3 \) are in Megawatts (MW). The total load demand is 800 MW. Neglecting line losses and generator limits, the economic dispatch results and the total cost in \$/hr can be found by using either analytical or iterative methods [1]. The analytical solution uses the derivative of cost functions using the Equation 3.6, in order to find the same incremental operating cost according to the following equation:

\[
\frac{\partial C_1}{\partial P_1} = 5.3 + 0.008 P_1 = \lambda \\
\frac{\partial C_2}{\partial P_2} = 5.5 + 0.012 P_2 = \lambda \\
\frac{\partial C_3}{\partial P_3} = 5.8 + 0.018 P_3 = \lambda \\
\lambda = \frac{\partial C_1}{\partial P_1} = \frac{\partial C_2}{\partial P_2} = \frac{\partial C_3}{\partial P_3}
\]

\[ P_1 + P_2 + P_3 = 800 \,\text{MW} \]

Converting the above equations to matrix form:

\[
\begin{bmatrix}
1 & 1 & 1 & 0 \\
0.008 & 0 & 0 & -1 \\
0 & 0.012 & 0 & -1 \\
0 & 0 & 0.018 & -1
\end{bmatrix}
\begin{bmatrix}
P_1 \\
P_2 \\
P_3 \\
\lambda
\end{bmatrix}
= 
\begin{bmatrix}
800 \\
-5.3 \\
-5.5 \\
-5.8
\end{bmatrix}
\]

Then the dispatch generation of each unit and the incremental fuel cost are:

\[ P_1 = 400 \,\text{MW} \]
\[ P_2 = 250 \,\text{MW} \]
\[ P_3 = 150 \,\text{MW} \]
\[ \lambda = 8.5 \,\$/MWh \]
The total fuel cost is:

\[ C_T = C_1 + C_2 + C_3 \]

\[ C_T = 500 + 5.3 \times (400) + 0.004 \times (400)^2 + 400 + 5.5 \times (250) + 0.006 \times (250)^2 + 200 + 5.8 \times (150) + 0.009 \times (150)^2 = 6,682.5 \text{ $/h} \]

The iterative solution relies on the gradient method and uses an initial value for the coordination equation.

In this case, the initial value is assumed to be \( \lambda^{(1)} = 6.0 \text{ $/MWh} \). To form the coordination equations using Equation 3.5, we will have:

\[ P_1^{(1)} = \frac{\lambda - \beta_1}{2\gamma_1} = \frac{6.0 - 5.3}{2 \times 0.004} = 87.5000 \text{ MW} \]

\[ P_2^{(1)} = \frac{\lambda - \beta_2}{2\gamma_2} = \frac{6.0 - 5.5}{2 \times 0.006} = 41.6667 \text{ MW} \]

\[ P_3^{(1)} = \frac{\lambda - \beta_3}{2\gamma_3} = \frac{6.0 - 5.8}{2 \times 0.009} = 11.1111 \text{ MW} \]

Since \( P_D = 800 \text{ MW} \) the error \( \Delta P \) is:

\[ \Delta P^{(1)} = 800 - (87.5 + 41.6667 + 11.1111) = 659.7222 \text{ MW} \]

\[ \Delta \lambda^{(1)} = \frac{1}{2 \times 0.004} + \frac{1}{2 \times 0.006} + \frac{1}{2 \times 0.009} = \frac{659.7222}{263.8888} = 2.5 \text{ $/MWh} \]

\[ \lambda^{(2)} = 6.0 + 2.5 = 8.5 \text{ $/MWh} \]

\[ P_1^{(2)} = \frac{\lambda - \beta_1}{2\gamma_1} = \frac{8.5 - 5.3}{2 \times 0.004} = 400 \text{ MW} \]

\[ P_2^{(2)} = \frac{\lambda - \beta_2}{2\gamma_2} = \frac{8.5 - 5.5}{2 \times 0.006} = 250 \text{ MW} \]

\[ P_3^{(2)} = \frac{\lambda - \beta_3}{2\gamma_3} = \frac{8.5 - 5.8}{2 \times 0.009} = 150 \text{ MW} \]

\[ \Delta P^{(2)} = 800 - (400 + 250 + 150) = 0 \text{ MW} \]
Since $\Delta P^{(2)} = 0$, the equality constraint is met in two iteration. Therefore, the economic dispatch results are:

\[
\begin{align*}
    P_1 &= 400 \text{ MW} \\
    P_2 &= 250 \text{ MW} \\
    P_3 &= 150 \text{ MW}
\end{align*}
\]

The total fuel cost is:

\[
C_t = C_1 + C_2 + C_3 = 500 + 5.3(400) + 0.004(400)^2 + 400 + 5.5(250) + 0.006(250)^2 \\
+ 200 + 5.8(150) + 0.009(150)^2 = 6,682.5 \text{ $/hr}$
\]

3.2.2 Economic Dispatch Neglecting Losses and Including Generator Limits

Considering the generation limits of a power plant, the power output of any generator should not exceed its rating, nor should it be below the necessary rating for stable boiler operations. Thus, the generations are restricted to lie within given minimum and maximum limits. The problem is finding the real power generation for each generator such that the total fuel cost function is minimum, that it is subject to the equality constraint given by Equation 3.4 and the inequality constraints given by Equation 3.8 [1] [27].

\[
P_{l\,(\text{min})} \leq P_l \leq P_{l\,(\text{max})} \quad i = 1, ..., n_g \quad (3.8)
\]

Where $P_{l\,(\text{min})}$ is the minimum generating limit for plant $i$ and $P_{l\,(\text{max})}$ is the maximum generating limit for plant $i$.

The numerical solution for an estimated $\lambda$, can be found from the coordination equation which is presented by Equation 3.7, and iteration is continued until Equation 3.4 is satisfied. As soon as any plant reaches its maximum or minimum power generation limits, the plant becomes kept at the limit. In effect, the plant output becomes a constant, and only the unviolated plants must operate at an equal incremental cost [1].

A simple example of the economic dispatch problem neglecting losses and including generation limits is presented below. This example is cited from Glover’s textbook [27].
An area of an interconnected power system has two fossil-fuel units operating on economic dispatch. The variable operating costs of these units are given by:

\[ C_1 = 10P_1 + 8 \times 10^{-3}P_1^2 \quad \text{$/hr} \quad 100 \leq P_{G_1} \leq 600 \text{ MW} \]
\[ C_2 = 8P_2 + 9 \times 10^{-3}P_2^2 \quad \text{$/hr} \quad 400 \leq P_{G_1} \leq 1000 \text{ MW} \]

Where \( P_1 \) and \( P_2 \) are in Mega Watts (MW).

The goal is to determine the power output and the incremental operating cost of each unit such that the results could minimize the total operating cost function \( (C_T) \), while the total load demand \( (P_D) \) varies between 500MW to 1500 MW. The generating unit inequality constraints will be considered and transmission losses can be neglected [27].

At light loads, unit 2 operates at its lower limit of 400 MW, where its incremental operating cost is \( \frac{dc_2}{dp_2} = 15.2 \) $/MWh. In this case, the additional load comes from unit1 until \( \frac{dc_1}{dp_1} = 15.2 \) $/MWh.

\[ \frac{dC_1}{dP_1} = 10 + 16 \times 10^{-3}P_1 = 15.2 \text{ $/MWh} \]

\[ P_1 = 325 \text{ MW} \]

At this point the total power is : \( P_T = 325 + 400 = 725 \text{ MW} \)

For \( P_T \) less than 725 MW, where \( P_1 \) is less than 325 MW, the incremental operating fuel cost of the area is determined by unit 1 alone.

At heavy loads, unit 1 operates at its upper limit of 600 MW, where its incremental operating cost is \( \frac{dc_1}{dp_1} = 19.6 \) $/MWh. Additional load comes from unit 2 for all values of \( \frac{dc_2}{dp_2} \) greater than 19.6 $/MWh. At \( \frac{dc_2}{dp_2} = 19.6 \) $/MWh:

\[ \frac{dC_2}{dP_2} = 8 + 18 \times 10^{-3}P_2 = 19.60 \text{ $/MWh} \]

\[ P_2 = 644 \text{ MW} \]

At this point the total power is : \( P_T = 600 + 644 = 1244 \text{ MW} \)
For $P_T$ greater than 1244 MW, where $P_2$ is greater than 644 MW, the incremental operating fuel cost of the area is determined only by unit 2.

For $725$ MW $< P_T < 1244$ MW, neither of the units has reached a limit value and the economic dispatch solution is the same as values given in Table 3.1 [27]. Table 3.1 summarizes the results of the economic dispatch problem which considers the generator limits while neglecting the transmission losses in this example [27].

<table>
<thead>
<tr>
<th>$P_T$</th>
<th>$P_{G1}$</th>
<th>$P_{G2}$</th>
<th>$\lambda = \frac{dC}{dP}$</th>
<th>$C_T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MW</td>
<td>MW</td>
<td>MW</td>
<td>$$/\text{MWhr}$</td>
<td>$$/\text{hr}$</td>
</tr>
<tr>
<td>500</td>
<td>100</td>
<td>400</td>
<td>11.60</td>
<td>5720</td>
</tr>
<tr>
<td>600</td>
<td>200</td>
<td>400</td>
<td>13.20</td>
<td>6960</td>
</tr>
<tr>
<td>700</td>
<td>300</td>
<td>400</td>
<td>14.80</td>
<td>8360</td>
</tr>
<tr>
<td>725</td>
<td>325</td>
<td>400</td>
<td>15.20</td>
<td>8735</td>
</tr>
<tr>
<td>800</td>
<td>365</td>
<td>435</td>
<td>15.84</td>
<td>9899</td>
</tr>
<tr>
<td>900</td>
<td>418</td>
<td>482</td>
<td>16.68</td>
<td>11,525</td>
</tr>
<tr>
<td>1000</td>
<td>471</td>
<td>529</td>
<td>17.53</td>
<td>13,235</td>
</tr>
<tr>
<td>1100</td>
<td>524</td>
<td>576</td>
<td>18.38</td>
<td>15,030</td>
</tr>
<tr>
<td>1200</td>
<td>576</td>
<td>624</td>
<td>19.22</td>
<td>16,910</td>
</tr>
<tr>
<td>1244</td>
<td>600</td>
<td>644</td>
<td>19.60</td>
<td>17,765</td>
</tr>
<tr>
<td>1300</td>
<td>600</td>
<td>700</td>
<td>20.60</td>
<td>18,890</td>
</tr>
<tr>
<td>1400</td>
<td>600</td>
<td>800</td>
<td>22.40</td>
<td>21,040</td>
</tr>
<tr>
<td>1500</td>
<td>600</td>
<td>900</td>
<td>24.20</td>
<td>23,370</td>
</tr>
</tbody>
</table>

3.2.3 Economic Dispatch Including Generator Limits and Losses

When transmission distances are very small and load density is very high, transmission losses may be neglected and the optimal dispatch of generation is achieved with all plants operating at the equal incremental production cost. However, in a large interconnected network where power is transmitted over long distances with low load density areas, transmission losses are a major factor and they affect the optimum dispatch of generation. One common practice for including the transmission line losses is to express the total transmission loss as a quadratic function of the generator power outputs [1] [27].
The basic quadratic form is the same as Equation 3.3 while considering 3.8 and 3.9:

\[
C_t = \sum_{i=1}^{n_g} C_i = \sum_{i=1}^{n} \alpha_i + \beta_i P_i + \gamma_i P_i^2 \quad (3.3)
\]

\[
P_{i_{(\min)}} \leq P_i \leq P_{i_{(\max)}} \quad i = 1, ..., n_g \quad (3.8)
\]

\[
\sum_{i=1}^{n_g} P_i = P_D + P_L \quad (3.9)
\]

Where \(P_{i_{(\min)}}\) and \(P_{i_{(\max)}}\) are the minimum and maximum generating limits for plant \(i\) respectively. The common approach is to augment the constraints into the objective function by using the Lagrange multipliers which is shown by Equation 3.10:

\[
\mathcal{L} = C_t + \lambda \left( P_D + P_L - \sum_{i=1}^{n_g} P_i \right) + \sum_{i=1}^{n_g} \mu_i(\max)(P_i - P_{i_{(\max)}}) + \sum_{i=1}^{n_g} \mu_i(\min)(P_i - P_{i_{(\min)}}) \quad (3.10)
\]

The minimum of this unconstrained function is found where the derivative of the function to its variables are zero according to Equation 3.11 [1].

\[
\begin{cases}
\frac{\partial \mathcal{L}}{\partial P_i} = 0 \\
\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \\
\frac{\partial \mathcal{L}}{\partial \mu_i(\max)} = P_i - P_{i_{(\max)}} = 0 \quad (3.11) \\
\frac{\partial \mathcal{L}}{\partial \mu_i(\min)} = P_i - P_{i_{(\min)}} = 0 \\
\frac{\partial C_t}{\partial P_i} + \lambda \left( 0 - \frac{\partial P_L}{\partial P_i} - 1 \right) = 0
\end{cases}
\]

Also,

\[
C_t = C_1 + C_2 + \cdots + C_{n_g} \quad (3.12)
\]

\[
\frac{\partial C_t}{\partial P_i} = \frac{\partial C_i}{\partial P_i} \quad (3.13)
\]

\[
\frac{\partial C_i}{\partial P_i} + \lambda \frac{\partial P_L}{\partial P_i} = \lambda \quad i = 1, ..., n_g \quad (3.14)
\]
The term $\frac{\partial P_L}{\partial P_i}$ in Equation 3.14 is known as the incremental transmission loss. From the substitution of equation $\frac{\partial C}{\partial \lambda} = 0$ in Equation 3.10, as well as consideration of Equation 3.9, Equation 3.14 will be derived. This equation can be simplified in the form of Equation 3.15 [1].

$$\frac{\partial C_i}{\partial P_i} \left( \frac{1}{1 - \frac{\partial P_L}{\partial P_i}} \right) = \lambda \quad i = 1, \ldots, n_g \quad (3.15)$$

or

$$L_i \frac{\partial C_i}{\partial P_i} = \lambda \quad i = 1, \ldots, n_g$$

Where $L_i$, is known as a penalty factor of plant $i$ and is given by Equation 3.16:

$$L_i = \frac{1}{1 - \frac{\partial P_L}{\partial P_i}} \quad (3.16)$$

An example of the economic dispatch problem that considers generation limits and transmission losses is presented below. In this example, the generators’ cost functions, the transmission loss model and the incremental fuel cost for the area are provided by the given information.

An area of an interconnected power system has two fossil-fuel units operating on economic dispatch. The variable operating costs of these units are given by [27]:

$$C_1 = 10P_1 + 8 \times 10^{-3}P_1^2 \quad \$/hr \quad 100 \leq P_{G1} \leq 600 \text{ MW}$$
$$C_2 = 8P_2 + 9 \times 10^{-3}P_2^2 \quad \$/hr \quad 400 \leq P_{G2} \leq 1000 \text{ MW}$$

and the transmission loss model is given by:

$$P_L = 1.5 \times 10^{-4}P_1^2 + 2 \times 10^{-5}P_1P_2 + 3 \times 10^{-5}P_2^2$$

Where $P_1$ and $P_2$ are in megawatts. The goal of this example is to determine the power output of each unit, the total transmission losses, the total load demand, and the total operating cost, $C_T$, when the area $\lambda = 16.00 \$/MWh. Substituting the following model in Equation 3.15:

$$\frac{\partial C_1}{\partial P_1} \left( \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} \right) = \frac{10 + 16 \times 10^{-3}P_1}{1 - (3 \times 10^{-4}P_1 + 2 \times 10^{-5}P_2)} = 16.00$$
\[
\frac{\partial C_2}{\partial P_2} \left( \frac{1}{1 - \frac{\partial P_L}{\partial P_2}} \right) = \frac{8 + 18 \times 10^{-3} P_2}{1 - (6 \times 10^{-5} P_2 + 2 \times 10^{-5} P_1)} = 16.00
\]

Rearranging the two equations:

\[
20.8 \times 10^{-3} P_1 + 32 \times 10^{-5} P_2 = 6.00 \\
32 \times 10^{-5} P_1 + 18.96 \times 10^{-3} P_2 = 8.00
\]

The results will be: \(P_1 = 282 \, MW\), \(P_2 = 417 \, MW\)

Substitution of the values in the problem loss model:

\[
P_L = 1.5 \times 10^{-4} P_1^2 + 2 \times 10^{-5} P_1 P_2 + 3 \times 10^{-5} P_2^2 = 19.5 \, MW
\]

Total load demand according to Equation 3.7 will be:

\[
P_D = \sum_{i=1}^{n_g} P_i - P_L = P_1 + P_2 - P_L = 282 + 417 - 19.5 = 679.5 \, MW
\]

Total operating cost is:

\[
C_T = C_1 + C_2 = 10(282) + 8 \times 10^{-3}(282)^2 + 8(817) + 9 \times 10^{-3}(417)^2 = 8357 \$/h
\]

It is worth mentioning that this example has been solved for the case which the exact value of \(\lambda\) is provided by the given information. The solution should be modified with the same approach as explained in the previous example, where the \(\lambda\) is not known but the total demanded load is given by the information. As explained earlier, in this case, the generators’ inequality constraints will be considered to find the common \(\lambda\). If one or more units reach their limit values, then these units are held at their limits and the remaining units operate at equal incremental fuel cost \(\lambda\). The
common $\lambda$ for the units that are not at their limits will be used as the incremental fuel cost of the area. Knowing the $\lambda$, the power output of each unit will be determined and consequently, the total transmission losses will be calculated according to the given loss model.
3.2.4 Derivation of Loss Formula

The system loss can be calculated according to loss coefficient or B-coefficient methods, developed by Kron and adopted by Kirchmayer [1]. This section explains the algorithm of B-coefficients and includes a review of a case study.

Assuming the total transmission losses over all buses is:

\[ P_L + Q_L = \sum_{i=1}^{n} V_i I_i^* = \sum_{i=1}^{n} \sum_{j=1}^{n} I_i Z_{ij} I_j^* \]  \hspace{1cm} (3.17)

\[ Z_{ij} = R_{ij} + jX_{ij} \]  \hspace{1cm} (3.18)

Where \( P_L \) and \( Q_L \) are the real and reactive power loss of the system.

\( Z_{ij} \) is the impedance matrix

\( I_i \) and \( I_j \) are the injected bus currents.

\( R_{ij} \) is the real part of the impedance matrix

\( Z_{ij} \) is the imaginary part of the bus impedance matrix

The bus impedance matrix is symmetrical, so the \( Z_{ij} = Z_{ji} \) and the real power loss equation can be formed as:

\[ P_L = \sum_{i=1}^{n} \sum_{j=1}^{n} I_i R_{ij} I_j^* = I_{bus}^T R_{bus} I_{bus}^* \]  \hspace{1cm} (3.19)

Considering individual bus currents of each load buses vary as a constant complex fraction of the total load current:

\[ I_{lk} = l_k I_D \]  \hspace{1cm} (3.20)

Where

\( l_k \), is the complex fraction,

\( I_{lk} \), is individual bus currents,

\( I_D \), is the total current

Assuming bus 1 to be the reference bus (slack bus), the voltage at bus 1 can be defined by:
\[ V_1 = \sum_{i=1}^{n_g} Z_{1i} I_{gi} + \sum_{i=1}^{n_d} Z_{1k} I_{Lk} \]  
(3.21)

Where:

\( n_g \), is the number of generator buses

\( n_d \), is the number of load buses

Substituting Equation 3.20 into Equation 3.21 results in:

\[ V_1 = \sum_{i=1}^{n_g} Z_{1i} I_{gi} + I_D T \]  
(3.22)

Where:

\[ T = \sum_{k=1}^{n_d} l_k Z_{1k} \]  
(3.23)

Assuming \( I_0 \) is the current flowing out of bus 1, with all other current set to be zero, then \( V_1 \) is:

\[ V_1 = -Z_{11} I_0 \]  
(3.24)

By substituting Equation 3.24 in Equation 3.22 and Equation 3.20, the load currents become:

\[ I_{LK} = \rho_k \sum_{i=1}^{n_g} Z_{1i} I_{gi} + \rho_k Z_{11} I_0 \]  
(3.25)

Where:

\[ \rho_k = -\frac{l_k}{T} \]  
(3.26)

Reforming the generator currents with the above relations in matrix form results in:

\[
\begin{bmatrix}
I_{g1} \\
I_{g1} \\
\vdots \\
I_{L1} \\
I_{L2} \\
I_{Ln_d}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & \cdots & 0 & 0 \\
0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 1 & 0 \\
\rho_1 Z_{11} & \rho_1 Z_{12} & \cdots & \rho_1 Z_{1ng} & \rho_1 Z_{11} \\
\rho_2 Z_{11} & \rho_2 Z_{12} & \cdots & \rho_2 Z_{1ng} & \rho_2 Z_{11} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\rho_k Z_{11} & \rho_k Z_{12} & \cdots & \rho_k Z_{1ng} & \rho_k Z_{11}
\end{bmatrix}\begin{bmatrix}
I_{g1} \\
I_{g1} \\
\vdots \\
I_{L1} \\
I_{L2} \\
I_{Ln_d}
\end{bmatrix} =
\begin{bmatrix}
I_{gnLg} \\
\vdots \\
I_0
\end{bmatrix}
\]  
(3.27)

In short form:

\[ I_{bus} = CI_{new} \]  
(3.28)
Substituting in Equation 3.19, Equation 3.29 forms as:

\[ P_L = I_{\text{new}}^T C^T R_{\text{bus}} C^* I_{\text{bus}}^* \]  

(3.29)

The generator current can be formed as:

\[ I_{gl} = \frac{P_{gi} - Q_{gi}}{V_i^*} = \psi_i P_{gi} \]  

(3.30)

Where:

\[ \psi_i = \frac{1 - j \frac{Q_{gi}}{P_{gi}}}{V_i^*} \]  

(3.31)

Adding the current \( I_0 \) to the column vector current \( I_{gi} \) in Equation 3.30 results in:

\[
\begin{bmatrix}
I_{g1}^1 \\
I_{g2}^2 \\
\vdots \\
I_{gn_g}^{ng} \\
I_0^g
\end{bmatrix} = 
\begin{bmatrix}
\psi_1 & 0 & \ldots & 0 & 0 \\
0 & \psi_1 & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \ldots & \psi_1 & 0 \\
0 & 0 & \ldots & 0 & I_0^g
\end{bmatrix} 
\begin{bmatrix}
P_{g1} \\
P_{g2} \\
\vdots \\
P_{gn_g} \\
1
\end{bmatrix}
\]  

(3.32)

or in short form

\[ I_{\text{new}} = \psi P_{G1} \]  

(3.33)

Where

\[
P_{G1} = 
\begin{bmatrix}
P_{G1} \\
P_{G2} \\
\vdots \\
P_{G_{ng}} \\
1
\end{bmatrix}
\]  

(3.34)

Substituting Equation 3.33 in Equation 3.29 forms in:

\[ P_L = P_{G1}^T \psi^T C^T R_{\text{bus}} C^* \psi^* P_{G1}^* \]  

(3.35)

The resultant matrix from equation 3.34 is in complex form so the real power loss can be derived as:

\[ P_L = P_{G1}^T R[H] P_{G1}^* \]  

(3.36)
Where:

\[ H = \psi^T C^T R_{bus} C^* \psi^* \]  

(3.37)

H is also known as Hermitian matrix and the real part of H is found by the form

\[ R[H] = \frac{H + H^*}{2} \]  

(3.38)

The above matrix can be formed as follows:

\[
R[H] = \begin{bmatrix}
B_{11} & B_{12} & \cdots & B_{1ng} & B_{01}/2 \\
B_{21} & B_{22} & \cdots & B_{2ng} & B_{02}/2 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
B_{ng1} & B_{ng2} & \cdots & B_{ngng} & B_{0ng}/2 \\
B_{01}/2 & B_{02}/2 & \cdots & B_{0ng}/2 & B_{00}
\end{bmatrix}
\]  

(3.39)

The power loss can be calculated according to Equation 3.40

\[
P_L = \begin{bmatrix} P_{g1} & P_{g2} & \cdots & P_{gn_g} \end{bmatrix} \begin{bmatrix}
B_{11} & B_{12} & \cdots & B_{1ng} \\
B_{21} & B_{22} & \cdots & B_{2ng} \\
\vdots & \vdots & \ddots & \vdots \\
B_{ng1} & B_{ng2} & \cdots & B_{ngng}
\end{bmatrix} \begin{bmatrix} P_{g1} \\
P_{g2} \\
\vdots \\
P_{gn_g}
\end{bmatrix} + \begin{bmatrix} P_{g1} & P_{g2} & \cdots & P_{gn_g} \end{bmatrix} \begin{bmatrix}
\frac{B_{01}}{2} \\
\frac{B_{02}}{2} \\
\vdots \\
\frac{B_{0ng}}{2}
\end{bmatrix} + B_{00}
\]  

(3.40)
3.2.5 Economic Dispatch for 7-Bus Power System

The main objective of the economic dispatch for the 7-bus power system [28] is minimizing the total generators fuel costs while considering the generators limits. Figure 3.3 shows the single line diagram of the 7-bus power system. The system contains 5 generators, 5 loads, 7 buses and 11 transmission lines. The total load demanded by this system is 760MW and 130MVA. The reference bus is located at bus 7. For the base case, the total generation cost is 16935.93 $/hr and the total transmission loss is 7.94 MW.

Using the cost coefficients of the 5 generators in this 7-bus system, the total generation operating cost function is expressed by Equation 3.41 [28]:

$$C_i(P_{gi}) = (A_i + B_iP_{gi} + C_i(P_{gi})^2) * f_c \tag{3.41}$$

Where

- $f_c$: fuel cost ($/Mwh$)
- $A_i$: fuel cost dependent value (Mbtu/hr)
- $B_i, C_i$: coefficient of the cost model

$$C_1(P_{g1}) = \left(373.5 + 7.62P_1 + 0.0013P_1^2 \right) * 2.04$$
$$C_2(P_{g2}) = \left(403.61 + 7.519P_2 + 0.0014P_2^2 \right) * 2.061$$
$$C_4(P_{g4}) = \left(253.24 + 7.836P_4 + 0.0013P_4^2 \right) * 2.093$$
$$C_6(P_{g6}) = \left(388.93 + 7.573P_6 + 0.0013P_6^2 \right) * 2.139$$
$$C_7(P_{g7}) = \left(194.28 + 7.771P_7 + 0.0019P_7^2 \right) * 2.574$$

$$C_T = \left(373.5 + 7.62P_1 + 0.0013P_1^2 \right) * 2.04 + \left(403.61 + 7.519P_2 + 0.0014P_2^2 \right) * 2.061$$
$$+ \left(253.24 + 7.836P_4 + 0.0013P_4^2 \right) * 2.093$$
$$+ \left(388.93 + 7.573P_6 + 0.0013P_6^2 \right) * 2.139$$
$$+ \left(194.28 + 7.771P_7 + 0.0019P_7^2 \right) * 2.574 = 16459.78 \text{ ($/hr$)}$$
The transmission loss is calculated with B-coefficients according to Equation 3.40 and has formed the equality constraints. The power generation limits of each generator are considered as the inequality constraints. Table 3.2 provides B-coefficients for 7-bus power system solved by MATLAB.

Table 3.2: B-Coefficients for 7-Bus Power System

<table>
<thead>
<tr>
<th>B</th>
<th>0.0212</th>
<th>0.0072</th>
<th>0.0035</th>
<th>0.0002</th>
<th>-0.004</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0072</td>
<td>0.0087</td>
<td>0.0018</td>
<td>0.0011</td>
<td>-0.0037</td>
</tr>
<tr>
<td></td>
<td>0.0035</td>
<td>0.0018</td>
<td>0.0167</td>
<td>-0.0048</td>
<td>-0.0072</td>
</tr>
<tr>
<td></td>
<td>0.0002</td>
<td>0.0011</td>
<td>-0.0048</td>
<td>0.0094</td>
<td>-0.0022</td>
</tr>
<tr>
<td></td>
<td>-0.004</td>
<td>-0.0037</td>
<td>-0.0072</td>
<td>-0.0022</td>
<td>0.0103</td>
</tr>
<tr>
<td>B0</td>
<td>0</td>
<td>0.0004</td>
<td>0.0013</td>
<td>-0.0009</td>
<td>-0.0012</td>
</tr>
<tr>
<td>B00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.22E-04</td>
</tr>
</tbody>
</table>

According to equation 3-15 the total power loss can be calculated:

\[
P_L = \begin{bmatrix} P_{g1} & P_{g2} & \cdots & P_{gn} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1n} \\ B_{21} & B_{22} & \cdots & B_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n1} & B_{n2} & \cdots & B_{nn} \end{bmatrix} \begin{bmatrix} P_{g1} \\ P_{g2} \\ \vdots \\ P_{gn} \end{bmatrix} + \begin{bmatrix} B_{01}/2 \\ B_{02}/2 \\ \vdots \\ B_{0n}/2 \end{bmatrix}\]

\[
\begin{bmatrix} P_{g1} & P_{g2} & \cdots & P_{gn} \end{bmatrix} \begin{bmatrix} B_{01}/2 \\ B_{02}/2 \\ \vdots \\ B_{0n}/2 \end{bmatrix} = 26.11 MW
\]
Figure 3.3: One line diagram for 7-Bus Power System [28]

Table 3.3 illustrates the results of the running economic dispatch solution with the MATLAB Optimization Toolbox and using the f-mincon function for achieving the minimum fuel cost of generation for the 7-bus power system. The total fuel cost is 16459.78 $/hr with loss of 26.11 MW. Economic dispatch provides the system with the minimum generation fuel cost and the generation outputs are changed.

The results show that the active power output of generators has been changed and the total fuel cost has reached its minimum value.

Table 3.3: Economic Dispatch Case Study for 7-Bus System

<table>
<thead>
<tr>
<th>Generation (MW)</th>
<th>Number</th>
<th>Name</th>
<th>Base case (MW)</th>
<th>Economic Dispatch by MATLAB (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bus-1</td>
<td></td>
<td>101.85</td>
<td>100</td>
</tr>
<tr>
<td>2</td>
<td>Bus-2</td>
<td></td>
<td>189.26</td>
<td>345.09</td>
</tr>
<tr>
<td>4</td>
<td>Bus-4</td>
<td></td>
<td>95.03</td>
<td>102.2</td>
</tr>
<tr>
<td>6</td>
<td>Bus-6</td>
<td></td>
<td>200.33</td>
<td>238.82</td>
</tr>
<tr>
<td>7</td>
<td>Bus-7</td>
<td></td>
<td>200.65</td>
<td>0</td>
</tr>
<tr>
<td>Total Generation (MW)</td>
<td></td>
<td></td>
<td>786.11</td>
<td>786.11</td>
</tr>
<tr>
<td>Total Load (MW)</td>
<td></td>
<td></td>
<td>760</td>
<td>760</td>
</tr>
<tr>
<td>Total Loss (MW)</td>
<td></td>
<td></td>
<td>7.94</td>
<td>26.11</td>
</tr>
<tr>
<td>Total Hourly Cost ($/hr)</td>
<td></td>
<td></td>
<td>16935.93</td>
<td>16459.78</td>
</tr>
</tbody>
</table>
3.2.6 Economic Dispatch for 37-Bus Power System

The goal of the economic dispatch power system for the 37-bus, 9 generator power system [27] is to minimize total generator fuel costs while meeting the generators real power limits and the power flow equations. Figure 3.4 shows the 37-bus power system solved by the PowerWorld Simulator [28]. The total load demanded by this system is 813.70 MW and 280.7 MVAR.

![Economic Dispatch for a 37-bus Power System](image)

Table 3.3 summarizes the results of running economic dispatch solution in order to achieve the minimum fuel cost of generation for the 37-bus powers system. The results show that the active power output of generators has been changed and the total fuel cost has reached its minimum value. The software considers the transmission losses in simulation results. Economic Dispatch solution defines the most efficient approach with respect to total generation costs while it might increase the total losses or ignore some of the constraints in the system.
Table 3.4: Economic Dispatch Case Study for 37-Bus System

<table>
<thead>
<tr>
<th>Bus Number</th>
<th>Bus Name</th>
<th>Base case (MW)</th>
<th>Economic Dispatch by PowerWorld(MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>WEBER69</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>48</td>
<td>BOB69</td>
<td>16</td>
<td>0</td>
</tr>
<tr>
<td>44</td>
<td>LAUF69</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>50</td>
<td>ROGER69</td>
<td>42.09</td>
<td>42.09</td>
</tr>
<tr>
<td>54</td>
<td>BLT69</td>
<td>106.78</td>
<td>106.08</td>
</tr>
<tr>
<td>53</td>
<td>BLT138</td>
<td>140</td>
<td>140</td>
</tr>
<tr>
<td>28</td>
<td>JO345</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>28</td>
<td>JO345</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>31</td>
<td>SLACK345</td>
<td>190.94</td>
<td>217.68</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Total Generation (MW)</th>
<th>825.8</th>
<th>825.8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total Load (MW)</td>
<td>813.8</td>
<td>813.8</td>
</tr>
<tr>
<td></td>
<td>Total Loss (MW)</td>
<td>10.79</td>
<td>11.59</td>
</tr>
<tr>
<td></td>
<td>Total Hourly Cost ($/hr)</td>
<td>15848.33</td>
<td>15555</td>
</tr>
</tbody>
</table>
3.3 Optimal Power Flow

Economic dispatch ignores the limits imposed by the devices in the transmission system. This is a serious shortcoming in power generation scheduling. The power transmitted through transformers and transmission lines is restricted to lie within given limits, which arise due to thermal, voltage, or stability considerations. The combination of economic dispatch, with either the full ac power flow or a dc power flow, and enforcing the system constraints, is an efficient solution for optimizing the power generation. The result is known as the Optimal Power Flow (OPF). There are several methods for solving the OPF problem [27].

3.3.1 OPF Formulation

The mathematical formulation of the optimal power flow problem is stated as a nonlinearly constrained optimization problem. The problem is subject to satisfaction of nonlinear equality constraints and nonlinear inequality constraints. The objective function is expressed by Equation 3.42 [3] [30].

\[
\begin{align*}
\text{Minimize} & \quad F(x, u) \\
\text{Subject to:} & \quad g(x, u) = 0 \\
& \quad h(x, u) \leq 0
\end{align*}
\]  

(3.42)

where \( u \) is the vector of control or independent variables, \( x \) is the vector of state or dependent variables. \( f(x, u) \) is the set of objective functions of OPF, \( g(x, u) \) is the set of inequality constraints, \( h(x, u) \) is the set of equality constraints.

The OPF problem has many control variables that need to be specified and subjected to a variety of equality and inequality constraints. These constraints include limits on all control variables, power flow equations, generation/load balance, branch flow limits (MW, MVAR, MVA), bus voltage limits, active/reactive reserve limits, generator MVAR limits, corridor (transmission interface) limits.
The OPF state variables, also called \( x \), are defined by Equations 3.43 and the control variables, \( u \), are defined by the Equation 3.44 [30] [31].

\[
x^T = \left[ P_{SG}, V_{L_1}, ..., V_{L_{NL}}, Q_{G_1}, ..., Q_{G_{NG}}, S_{l_1}, ..., S_{nl} \right]
\] (3.43)

\[
u^T = \left[ P_{G_2}, P_{G_{NG}}, V_{G_1}, ..., V_{G_{NG}}, Q_{C_1}, ..., Q_{C_{NC}}, T_1, ..., T_{NT} \right]
\] (3.44)

Where

- \( P_G \): Active power generated at the PV buses
- \( V_G \): Voltage magnitude at PV buses
- \( Q_C \): Reactive power supplied by all shunt reactors
- \( T_{NT} \): Transformer load tap changer magnitude
- \( P_{SG} \): Active power of the slack bus
- \( V_L \): Voltage magnitude at PQ buses (load buses)
- \( Q_G \): Reactive power of all generator units
- \( S_l \): Transmission Line loading (Load flow)
- \( NL \): Number of load buses
- \( NG \): Number of generator buses (PV buses)
- \( NC \): Number of shunt VAR compensators
- \( NT \): Number of transformers
- \( nl \): Number of transmission lines

### 3.3.2 The Objective Functions

Six objective functions have been considered for optimizing the OPF problem. These objective functions vary from fuel cost generation, active and/or reactive power transmission loss, reactive power reserve margin, security margin index, and environmental emission. Some of these objective functions are conflicting in nature, which makes the OPF problem complicated [30]. A multi-objective concept will be discussed in chapter five to solve the OPF problem.
3.3.2.1 Minimization of Fuel Cost of Generation (FC)

The main objective function of the optimal power flow problem is minimizing the generation fuel cost of thermal units. The generation fuel cost objective function can be expressed by a quadratic function according to Equation 3.45.

\[
\text{Minimize } TFC = \sum_{k=1}^{N_G} FC_k(P_{Gk}) \tag{3.45}
\]

\[
FC_k(P_{Gk}) = a_k P_{Gk}^2 + b_k P_{Gk} + c_k
\]

Where

- \( FC \): Total fuel cost generation of the power system
- \( FC_k(P_{Gk}) \): Fuel cost function of the \( k^{th} \) generator
- \( N_G \): Number of generators including the slack generator
- \( a_k \): Quadratic cost coefficient of the \( k^{th} \) generator
- \( b_k \): Linear cost coefficient of the \( k^{th} \) generator
- \( c_k \): Basic cost coefficient of the \( k^{th} \) generator
- \( P_{Gk} \): Real power output of the \( k^{th} \) generator.

3.3.2.2 Minimization of Active Power Transmission Loss

Active power transmission loss is one of the objective functions of the OPF problem. The problem goal is to minimize the total power loss in transmission lines and transformers in the network, represented by Equation 3.46 [30].

\[
\text{Minimize } P_L = \sum_{k=1}^{N_G} P_{Gk} - \sum_{k=1}^{N_G} P_{Dk} \tag{3.46}
\]

Where

- \( P_L \): Total \( RL^2 \) loss in the transmission lines and transformers of the network
- \( P_{Gk} \): Active power generation at \( k^{th} \) bus
- \( P_{Dk} \): Active power demand at \( k^{th} \) bus
3.3.2.3 Minimization of Reactive Power Transmission Loss

Voltage stability at load buses is particularly associated with the availability of reactive power. Based on this, minimizing reactive power losses is another important objective in optimal power flow analysis. The reactive power loss can be expressed as the difference between total generated reactive power and the total demanded reactive power in the network. Reactive power losses in a network can be minimized by the optimal scheduling of generated reactive power, transformer tap settings, and other compensating devices. In practice, the $Q_{loss}$ is not necessarily positive. Equation 3.47 represents the formulation of reactive power transmission loss [30].

\[
\text{Minimize } Q_{loss} = \sum_{k=1}^{N_G} Q_{Gk} - \sum_{k=1}^{N_G} Q_{Dk}, \quad K = 1, \ldots, N_b \quad (3.47)
\]

Where

$Q_{loss}$: Total reactive power transmission Loss in the power system

$Q_{Gk}$: Total reactive power generation at $k^{th}$ bus

$Q_{Dk}$: Total reactive power demand at $k^{th}$ bus

3.3.2.4 Maximization of Reactive Power Reserve Margin

Optimizing the reactive power reserves is one of the most important objectives in optimal power flow analyses. The amount of reactive power reserves can be assumed as a measure of the degree of voltage stability in a power network. With this perspective, maximizing the reactive power reserve margins and seeking to distribute reserves among the generators, are proposed in OPF studies. The objective can be achieved by minimizing the function presented by Equation 3.48.

\[
\text{Minimize } F = \sum_{k=1}^{N_G} \left[ \frac{Q_k^2}{Q_{k max}} \right] \quad (3.48)
\]

Where

$Q_k$ : Reactive power of $k^{th}$ generator

$Q_{k max}$ : Maximum reactive power of $k^{th}$ generator

84
3.3.2.5 Minimization of Emission Index (EI)

The Emission or Environmental Index (EI), is an index from the viewpoint of environmental conservation. The atmospheric pollutants such as nitrogen oxides (NOx) and sulfur oxides (SOx) caused by fossil-fueled thermal units can be modeled separately. However, the OPF problem seeks to minimize the total (Ton/h) emission $E(P_G)$ of these pollutants, which can be stated by the following equations. As indicated, the amount of emissions is given as a function of the generator active power output, which is the sum of quadratic and exponential functions according to Equation 3.49 [30].

$$\text{Minimize } FE = \sum_{k=1}^{N_G} E_k(P_{Gk}) \quad (3.49)$$

$$E_k(P_{Gk}) = \sum_{k=1}^{N_G} \alpha_k P_{Gk}^2 + \beta_k P_{Gk} + \gamma_k + \omega_i e^{(\mu_k P_{Gk})}$$

$FE$: Total emission of the power system

$E_k(P_{Gk})$: Emission function of $k^{th}$ generator

$\alpha_k, \beta_k, \gamma_k, \omega_i$ and $\mu_k$: Coefficients of the $k^{th}$ generator

3.3.2.6 Maximization of Security Margin Index (SMI)

The last objective function is the Security Margin Index (SMI). The OPF problem seeks to operate all the transmission lines connected in a network to their maximum capability according to Equation 3.50 [30].

$$\text{Minimize } SMI = \sum_{k=1}^{N_G} (S_{li}^{\text{max}} - S_{li}) \quad (3.50)$$
3.3.2.7 Minimization of Voltage Deviation

Voltage deviation is a measure of voltage quality in the power network. The index of deviation is also important from the security aspect. The indicator is formulated as a cumulative deviation of voltages for all load buses (PQ buses) in the network from a nominal value of unity. It can be defined mathematically as presented by Equation 3.51 [31], [32].

\[
VD = \left( \sum_{p=1}^{NL} |V_{LP} - 1| \right)
\]

\(V_{LP}\): voltage at load buses

3.3.3 The OPF Constraints

The optimal power flow problem has many control variables that are subjected to meet a variety of equality and inequality constraints. These control variables include generator bus voltages, active power in all generator units, switchable shunt reactors, and transformer tap positions. The OPF constraints are categorized into equality and inequality constraints.

3.3.3.1 Equality Constraints

The power flow equations require the net injection of the active and reactive power at each bus to be zero. These equations form the equality constraints of the OPF problem, reflecting the physics of the power system and its limitations and are presented by Equations 3.52 and 3.53.

Active Power Constraints:

\[
P^k_G - P^k_D = V_k \sum_{j=1}^{N} \left[ V_j \left[ G_j^k \cos(\delta^k - \delta^j) + B_j^k \sin(\delta^k - \delta^j) \right] \right]
\]

(3.52)

Reactive Power Constraints:

\[
Q^k_G - Q^k_D = V_k \sum_{j=1}^{N} \left[ V_j \left[ G_j^k \sin(\delta^k - \delta^j) - B_j^k \cos(\delta^k - \delta^j) \right] \right]
\]

(3.53)
Where \( k = 1, 2, \ldots, N \)

\( V_k \): Voltage magnitude at bus \( k^{th} \)

\( \delta^k \): Voltage angle at bus \( k^{th} \)

\( P_G^k \): Active power generation at bus \( k^{th} \)

\( P_D^k \): Active power demand at bus \( k^{th} \)

\( Q_G^k \): Reactive power generation at bus \( k^{th} \)

\( Q_D^k \): Reactive power demand at bus \( k^{th} \)

\( G_j^k \): Real part of \((k, j)\) element of the bus admittance matrix

\( B_j^k \): Imaginary part of \((k, j)\) element of the bus admittance matrix

### 3.3.3.2 Inequality Constraints

Inequality constraints are a set of constraints that represent the systems operational and security limits and boundaries.

Active and reactive power generation constraints for all units should be considered according to inequalities presented by Equation 3.54.

\[
P_{Gk}^{Min} \leq P_{Gk} \leq P_{Gk}^{Max} \quad (3.54)
\]

\[
Q_{Gk}^{Min} \leq Q_{Gk} \leq Q_{Gk}^{Max}
\]

Where,

\( P_{Gk} \): Active power generated by \( k^{th} \) generator

\( Q_{Gk} \): Reactive power generated by \( k^{th} \) generator

\( P_{Gk}^{Min} \): Minimum active power generated by \( k^{th} \) generator

\( P_{Gk}^{Max} \): Maximum active power generated by \( k^{th} \) generator

\( Q_{Gk}^{Min} \): Minimum reactive power generated by \( k^{th} \) generator

\( Q_{Gk}^{Max} \): Maximum reactive power generated by \( k^{th} \) generator

\( P_{Gi}^{Max} \): Maximum MW generation by \( i^{th} \) generator

\( P_{Gi}^{Min} \): Minimum MW generation by \( i^{th} \) generator
Bus voltage magnitude constraints are presented by Equation 3.55.

\[ V_{k_{\text{Min}}} \leq V_k \leq V_{k_{\text{Max}}} \]  
(3.55)

Where

\( V_k \): Voltage at \( k^{th} \) bus
\( V_{k_{\text{Min}}} \): Minimum acceptable voltage at \( k^{th} \) bus
\( V_{k_{\text{Max}}} \): Maximum acceptable voltage at \( k^{th} \) bus

The reactive power source capacity constraint is provided by Equation 3.56.

\[ Q_{ck_{\text{Min}}} \leq Q_{ck} \leq Q_{ck_{\text{Max}}} \]  
(3.56)

The transformer tap position constraints are represented by Equation 3.57.

\[ T_{k_{\text{Min}}} \leq T_k \leq T_{k_{\text{Max}}} \]  
(3.57)

The line thermal limit constraints for all transmission lines are according to Equation 3.58.

\[ |S_i| \leq S_{i_{\text{max}}} \]  
(3.58)

**3.3.4 OPF Solution Methodologies**

A first comprehensive survey regarding optimal power dispatch was given by H.H. Happ [33]. The solution methodologies of OPF can be broadly classified into two groups; conventional (classical) methods and intelligent methods.

Traditionally, conventional methods are effectively used to solve the OPF problem. Classical methods are based on mathematical programming techniques and can meet the requirements of objective functions, type of application, and nature of constraints. Mathematical formulations have to be simplified to find the solutions because of the extremely limited capability to solve real-world large-scale power system problems. Classical methods are weak in handling qualitative constraints. They have poor convergence, they may get stuck at local optimum, they can find only a single optimized solution in a single simulation run, they become too slow if the
number of variables is large and they are computationally expensive for the solution of a large system [26].

To overcome the shortcomings of conventional methods, intelligent methods based on Artificial Intelligence (AI) have been developed in the recent past. The major advantages of the AI methods are their relative versatility in handling various qualitative constraints, as well as their ability to find multiple optimal solutions in a single simulation run. Therefore, they are quite suitable for solving multi-objective optimization problems. In most cases, they can converge to the global optimum solution at a fast rate. The other advantages of intelligent techniques are their ability to learn and their ability to appropriately solve non-linear modeling problems. On the other hand, some drawbacks, include large dimensionality and the choice of training methodology [26]. Figure 3.5 summarizes two classifications of OPF solution methodologies.

![Figure 3.5: OPF solution methodologies](image-url)

Figure 3.5: OPF solution methodologies [34]
3.3.5 Case Study of Minimizing Cost for 7-Bus Power System

The goal of OPF for the 7-bus power system case is to minimize generator fuel cost function while satisfying power flow equations, bus voltage magnitudes, the slack generator active power limits and specified branch flow limits (MVA).

Figure 3.6 shows the single line diagram of the 7-bus power system and OPF results by PowerWorld [28]. The total load is 760MW and 130 MVAR. For the base case, the total fuel cost is 16939$/hr and transmission loss is 7.94MW.

![Figure 3.6: One Line Diagram of Optimal Power Flow for 7-Bus Power System with OPF [28]](image)

The 7-bus power system contains 12 unknown variables including 4 controllable variables and 8 dependent variables. The controllable variables are four generators active power output. The dependent variables are the two bus voltage magnitudes and six phase angles. MATLAB Optimization Toolbox and the f-mincon function is applied to achieve the minimum generation cost of this power system.

Table 3.5 summarizes the results of the optimal power flow solution for the 7-bus power system without considering the security constraints. Power and loss are in MW, and the cost is in $/hr.
The total hourly cost results for the 7-bus case is achieved 16978.74 $/hr, using MATLAB Optimization Toolbox, and the f-mincon function. The results of running the case in PowerWorld Simulator are presented.

Table 3.5: Optimal Power Flow Case Study for 7-Bus System

<table>
<thead>
<tr>
<th>Number</th>
<th>Name</th>
<th>Base case (MW)</th>
<th>OPF by PowerWorld (MW)</th>
<th>OPF by MATLAB (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bus-1</td>
<td>101.85</td>
<td>127.54</td>
<td>127.48</td>
</tr>
<tr>
<td>2</td>
<td>Bus-2</td>
<td>170.08</td>
<td>187.19</td>
<td>187.1</td>
</tr>
<tr>
<td>4</td>
<td>Bus-4</td>
<td>95.03</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>6</td>
<td>Bus-6</td>
<td>200.33</td>
<td>200.17</td>
<td>200.17</td>
</tr>
<tr>
<td>7</td>
<td>Bus-7</td>
<td>200.65</td>
<td>200.49</td>
<td>205.5</td>
</tr>
<tr>
<td></td>
<td>Total Generation (MW)</td>
<td>767.94</td>
<td>765.39</td>
<td>770.25</td>
</tr>
<tr>
<td></td>
<td>Total Load (MW)</td>
<td>760</td>
<td>760</td>
<td>760</td>
</tr>
<tr>
<td></td>
<td>Total Loss (MW)</td>
<td>7.94</td>
<td>5.24</td>
<td>10.25</td>
</tr>
<tr>
<td></td>
<td>Total Hourly Cost ($/hr)</td>
<td>16935.93</td>
<td>16890</td>
<td>16978.74</td>
</tr>
</tbody>
</table>

Table 3.6 compares the results of minimizing the total fuel cost function with economic dispatch and OPF methods for the 7-bus power system. Economic dispatch has the lowest cost and the base case has the highest cost. The table also illustrates that the cost of the OPF is less than the base case but it is more expensive than the results of economic dispatch. While considering the transmission lines limits, OPF methods are preferred and will prevent the overloading of the line capacity.

Table 3.6: OPF of 7-Bus Power System, Minimizing Generation Cost

<table>
<thead>
<tr>
<th>Generation (MW)</th>
<th>Bus Name</th>
<th>Base Case (MW)</th>
<th>Economic Dispatch (MW)</th>
<th>OPF(MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus-1</td>
<td>101.85</td>
<td>100</td>
<td>127.48</td>
<td></td>
</tr>
<tr>
<td>Bus-2</td>
<td>170.08</td>
<td>345.09</td>
<td>187.1</td>
<td></td>
</tr>
<tr>
<td>Bus-4</td>
<td>95.03</td>
<td>102.2</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Bus-6</td>
<td>200.33</td>
<td>238.82</td>
<td>200.17</td>
<td></td>
</tr>
<tr>
<td>Bus-7</td>
<td>200.65</td>
<td>0</td>
<td>205.5</td>
<td></td>
</tr>
<tr>
<td>Total Hourly Cost ($/hr)</td>
<td>16935.93($/hr)</td>
<td>16459.78($/hr)</td>
<td>16978.74($/hr)</td>
<td></td>
</tr>
</tbody>
</table>
3.3.6 Case Study of Minimizing Cost for 37-Bus Power System

The goal of the OPF for the 37-bus, 9 generator power system [27] [28] is to minimize total generator fuel costs while satisfying all the power flow constraints. The power plant includes 9 generators, 26 loads, 37 buses and 43 branches. Bus 31 is the slack bus. Figure 3.7 shows the single line diagram of the 37-bus power system solved by the PowerWorld Simulator [27]. The total load is 813.64 MW and 280.7 MVAR. The base case total fuel cost is $15,848.33/hr and the transmission loss is 10.79 MW.

Figure 3.7: One Line Diagram of 37-Bus Power System
Table 3.7 summarizes the results of running optimal power flow by the PowerWorld Simulator in order to minimize the fuel cost of generation for 37-bus powers system.

### Table 3.7: Optimal Power Flow Case Study for 37-Bus System

<table>
<thead>
<tr>
<th>Generation (MW)</th>
<th>Bus Number</th>
<th>Bus Name</th>
<th>Base Case (MW)</th>
<th>Optimal Power Flow by PowerWorld (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>WEBER69</td>
<td></td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>48</td>
<td>BOB69</td>
<td></td>
<td>16</td>
<td>150</td>
</tr>
<tr>
<td>44</td>
<td>LAUF69</td>
<td></td>
<td>20</td>
<td>150</td>
</tr>
<tr>
<td>50</td>
<td>ROGER69</td>
<td></td>
<td>42.09</td>
<td>205.96</td>
</tr>
<tr>
<td>54</td>
<td>BLT69</td>
<td></td>
<td>106.08</td>
<td>31.31</td>
</tr>
<tr>
<td>53</td>
<td>BLT138</td>
<td></td>
<td>140</td>
<td>0</td>
</tr>
<tr>
<td>28</td>
<td>JO345</td>
<td></td>
<td>150</td>
<td>42.09</td>
</tr>
<tr>
<td>28</td>
<td>JO345</td>
<td></td>
<td>150</td>
<td>140</td>
</tr>
<tr>
<td>31</td>
<td>SLACK345</td>
<td></td>
<td>190.94</td>
<td>106.08</td>
</tr>
<tr>
<td><strong>Total Generation (MW)</strong></td>
<td></td>
<td></td>
<td><strong>825.1</strong></td>
<td><strong>825.4</strong></td>
</tr>
<tr>
<td><strong>Total Load (MW)</strong></td>
<td></td>
<td></td>
<td><strong>813.7</strong></td>
<td><strong>813.6</strong></td>
</tr>
<tr>
<td><strong>Total Loss (MW)</strong></td>
<td></td>
<td></td>
<td><strong>10.79</strong></td>
<td><strong>11.14</strong></td>
</tr>
<tr>
<td><strong>Total Hourly Cost ($/hr)</strong></td>
<td></td>
<td></td>
<td><strong>15848.33</strong></td>
<td><strong>15624</strong></td>
</tr>
</tbody>
</table>

Table 3.8 compares the results of minimizing the total fuel cost function with economic dispatch and OPF methods for the 37-bus power system. Economic dispatch reaches the lowest cost (15555 $/hr) and the base case has the highest cost (15848.33 $/hr). The table also illustrates that the cost of the OPF is less than the base case but is more expensive than the results of the ED (15624 $/hr).

The total generation cost in OPF will increase as a result of considering limits and constraints at each bus and transmission line.
Table 3.8: OPF of 37-Bus Power System, Minimizing Generation Cost

<table>
<thead>
<tr>
<th>Bus Number</th>
<th>Bus Name</th>
<th>Base Case (MW)</th>
<th>ED (MW)</th>
<th>OPF (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>WEBER69</td>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>48</td>
<td>BOB69</td>
<td>16</td>
<td>0</td>
<td>150</td>
</tr>
<tr>
<td>44</td>
<td>LAUF69</td>
<td>20</td>
<td>20</td>
<td>150</td>
</tr>
<tr>
<td>50</td>
<td>ROGER69</td>
<td>42.09</td>
<td>42.09</td>
<td>205.96</td>
</tr>
<tr>
<td>54</td>
<td>BLT69</td>
<td>106.08</td>
<td>106.08</td>
<td>31.31</td>
</tr>
<tr>
<td>53</td>
<td>BLT138</td>
<td>140</td>
<td>140</td>
<td>0</td>
</tr>
<tr>
<td>28</td>
<td>JO345</td>
<td>150</td>
<td>150</td>
<td>42.09</td>
</tr>
<tr>
<td>28</td>
<td>JO345</td>
<td>150</td>
<td>150</td>
<td>140</td>
</tr>
<tr>
<td>31</td>
<td>SLACK345</td>
<td>190.94</td>
<td>217.68</td>
<td>106.08</td>
</tr>
<tr>
<td>Total Hourly Cost ($/hr)</td>
<td>15848.33($/hr)</td>
<td>15555 ($/hr)</td>
<td>15624 ($/hr)</td>
<td></td>
</tr>
</tbody>
</table>
3.4 Security-Constrained Optimal Power Flows (SCOPF)

A common objective of OPF problem is minimizing the total generation cost. Although it ensures the economic operation of power systems, it does not include contingency constraints and the solution might be insecure. Security constrained optimal power flow (SCOPF) is defined as an operating strategy to enforce (N-1) reliability criteria for the secure operating point. Any single contingency should not induce any further constraint violations and the design should avoid cascading failure of the network [35].

The inclusion of contingencies requires some pre-contingent and post-contingent control actions. Pre-contingent control actions normally consider line overloads and limitation, while post-contingent control actions are applied for reactive power control and generator outage re-dispatch [29].

Optimal power flow formulations can be modified to ensure that there are no contingency violations in any post-contingency states. The Security Constrained OPF (SCOPF) takes the form of Equation 3.59, denoted by C, with the given set of contingencies in the OPF problem [36] [35].

\[
\text{Minimize } C(x,u) \tag{3.59}
\]

Subject to satisfaction of nonlinear equality constraints:

\[
\text{Subject to: } h(x,u) = 0
\]

and nonlinear inequality constraints:

\[
g(x,u) \leq 0
\]

\[
\forall m \in C: \\
g^m(x,u) \leq 0 \\
h^m(x,u) = 0,
\]

where \(g^m\)and \(h^m\) represent the network constraints under contingency \(m\), and are referred to as the security (or contingency) constraints. For example, if contingency \(m\) considers a particular line being out of service, then the network model in constraints \(g^m\) would not contain that line [35].
A security assessment can be applied as a static security assessment or dynamic security assessment [37]. The study presented in this thesis will consider static security. The SCOPF starts the search for optimal operating point by running OPF with N constraints, then the contingency analysis is run to recognize the potential contingency cases. If there is no constraint violation in the network, the OPF will provide the optimal solution for SCOPF. If a security violation is induced by outages, the complete security constraint should be applied. The OPF and each of the contingency power flows should be re-executed until the OPF can satisfy all contingency constraints. The new optimal operating point ensures that the solution will be feasible for all contingencies under consideration. This will clearly show the concept of an inherent tradeoff between optimality and reliability in the network. Frequently adjusting the generation schedules and voltages set points will result in a less economic but more reliable operating solution [36] [35]. Many techniques are used to analyze the contingencies and SCOPF. PowerWorld Simulator has the contingency analysis function which provides the user with the contingency violation information.

3.4.1 Case Study of SCOPF

The power system with 37-bus has been considered for analyzing the SCOPF, adopted from [29]. The system has been slightly modified, mostly with some transmission lines enhancements and transformers limits to better improve its SCOPF performance. The goal for the SCOPF is to minimize generator fuel costs while considering the system security constraints and limitation. In this case, the SCOPF has focused on a single transmission line outage only. The one-line diagram of the system is shown in Figure 3.8, while there is no area control enforced to the system. Some selected information, such as total load, total losses, current MW generation by each generator, and its linear cost value are presented by a table in one line diagram. Considering this model as the base case, the total fuel cost is 25918$/hr and the transmission losses are 40.98 MW. The total load is 1421MW and 478.1 MVar. Total generation is 1462.3MW and 576 MVar.
Figure 3.8: One Line Diagram of 37-Bus Power System with No Control on Area [29]

Figure 3.9 shows the same system while the area control option has been set to apply optimal power flow control in PowerWorld Simulator. The total load is approximately 1421 MW and the cost is $29752/h. The total loss is 22.92 (Because of tolerances in the solution, the total load, losses, and costs might change slightly by repeating a solution with the same load scalar).

Figure 3.9: One Line Diagram of 37-Bus Power System OPF with OPF Control on Area [29]
The formulation of SCOPF can be defined according to the set of Equations 3.38. While the objective function of the study will remain the same as OPF solution, the optimization problem will consider the necessary constraints with respect to an outage of a single transmission line. Figure 3.10 shows the system while the full security constrained OPF method is performed.

![Figure 3.10: SCOPF of 37-Bus Power System [29]](image)

Table 3.9 provides the results regarding contingency analysis and a number of violations that have been happened in each method.

<table>
<thead>
<tr>
<th>Control Option</th>
<th>Number of Violations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case, with No Control Option</td>
<td>363</td>
</tr>
<tr>
<td>Optimal Power Flow Control</td>
<td>19</td>
</tr>
<tr>
<td>Security Control Optimal Power Flow</td>
<td>7</td>
</tr>
</tbody>
</table>

The SCOPF solution is the best possible compromise between finding an optimal solution and enforcing the constraints. It is found that in some situations it is not possible to enforce all the constraints. Otherwise, it would not be possible to find a feasible solution for the system [38].
Table 3.10 compares the output power of generators in 37-bus case study while performing the OPF and SCOPF solution.

Table 3.10: Comparing Generators Power Output in 37-Bus Power System

<table>
<thead>
<tr>
<th>Number of Bus</th>
<th>Name of Bus</th>
<th>Base Case (MW)</th>
<th>OPF (MW)</th>
<th>SCOPF (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>RUDDER69</td>
<td>0</td>
<td>40.46</td>
<td>30.04</td>
</tr>
<tr>
<td>16</td>
<td>CENTURY69</td>
<td>23.23</td>
<td>26.96</td>
<td>100</td>
</tr>
<tr>
<td>20</td>
<td>FISH69</td>
<td>76.14</td>
<td>80.24</td>
<td>110</td>
</tr>
<tr>
<td>28</td>
<td>AGGIE345</td>
<td>400</td>
<td>400</td>
<td>359.87</td>
</tr>
<tr>
<td>31</td>
<td>SLACK345</td>
<td>726.61</td>
<td>399.79</td>
<td>374.1</td>
</tr>
<tr>
<td>37</td>
<td>SPIRIT69</td>
<td>0</td>
<td>140</td>
<td>96.57</td>
</tr>
<tr>
<td>44</td>
<td>RELLIS69</td>
<td>58.73</td>
<td>60</td>
<td>60</td>
</tr>
<tr>
<td>48</td>
<td>WEB69</td>
<td>0</td>
<td>30.34</td>
<td>60</td>
</tr>
<tr>
<td>53</td>
<td>KYLE138</td>
<td>177.53</td>
<td>186.84</td>
<td>210.95</td>
</tr>
<tr>
<td>54</td>
<td>KYLE69</td>
<td>0</td>
<td>80</td>
<td>38.98</td>
</tr>
</tbody>
</table>

The results of considering the outage of a single transmission line, are given in Table 3.11. Power is in MW, Voltage is in per unit and Cost is in $/hr. The total hourly cost for SCOPF is 30184 $/hr with the loss of 40.88 MW. The table illustrates that the output generation of each unit and total fuel cost are changed during the outage to ensure the optimal and secure operation of the system [37]. Evidently, the cost of SCOPF (30184 $/hr) is higher than the cost of normal OPF (29752 $/hr) which is justifiable by the significant drop in the number of system violation from 19 violations while using OPF to 7 violations while performing SCOPF. Hence the higher cost is required when the security constraints are included. All the results have been provided by PowerWorld Simulator.

Table 3.11: SCOPF of 37-Bus Power System, Minimizing Generation Cost

<table>
<thead>
<tr>
<th>Area Summary Totals</th>
<th>Base Case</th>
<th>OPF</th>
<th>SCOPF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MW</td>
<td>Mvar</td>
<td>MW</td>
</tr>
<tr>
<td>Load</td>
<td>1420.8</td>
<td>478.1</td>
<td>1421.1</td>
</tr>
<tr>
<td>Generation</td>
<td>1462.3</td>
<td>566.3</td>
<td>1444.6</td>
</tr>
<tr>
<td>Losses</td>
<td>40.88</td>
<td>235.91</td>
<td>22.92</td>
</tr>
<tr>
<td>Hourly Cost($/hr)</td>
<td>25916 ($/hr)</td>
<td>29752($/hr)</td>
<td>30184 ($/hr)</td>
</tr>
</tbody>
</table>
3.5 Conclusions

This chapter has discussed the fundamentals of Economic Dispatch (ED), Optimal Power Flow (OPF), and Security Constraint Optimal Power Flow (SCOPF) and the solution to solve them. The goal of the economic dispatch problem is to minimize the total fuel cost of generation while satisfying power system limitations and constraints. The problem is subject to meet a set of equality constraints with respect to load generation balance, while as a serious shortcoming, it ignores the limits imposed by the devices in the transmission system.

The OPF, as an efficient solution to power generation scheduling, combines the economic dispatch with either the full ac power flow or a dc power flow, while enforces the system constraints. The Security Constraint Optimal Power Flow (SCOPF) includes contingency constraints analysis and ensures the secure operation of the system while having the same objective of OPF with respect to minimizing the total generation costs of the power system. It is important to highlight the fact that the optimal generation schedule with security constraints considerations will require a higher cost in comparison to other methods.

Different case studies with respect to each concept (ED, OPF and SCOPF) are presented and simulated using MATLAB and PowerWorld Simulator to explain the solution methodologies. Case studies include standard IEEE 7-bus and IEEE 37-bus power systems.
4. Multi-Objective Optimization

4.0 Introduction
The primary intention of this chapter is to provide an introduction to the general concepts and principles of multi-objective optimization. Then, classifications for multi-objective optimization algorithms are presented.

Single-objective optimization corresponds to the minimum or maximum value of a single objective function. Multi-objective optimization methods receive impetus in many practical engineering fields, where the problem needs to achieve a compromise in the presence of trade-offs between two or more conflicting objective functions and find the best set of values for each one of them.

The first part of this chapter explains the theoretical concept of the single objective optimization problem and the formulation. Then, a comprehensive model of multi-objective optimization methods is presented. The final section deals with practical insights using two case studies.

4.1 Single Objective Optimization

Single-objective optimization corresponds to the minimum or maximum value of a single objective function. The problem could be formed by either an objective function or constraint functions, with linear or nonlinear, convex or non-convex, explicit or implicit characteristics.

4.1.1 Single Objective Optimization Formulation

By definition, a function of one variable $f(x)$ is said to have a relative or local minimum at $x = x^*$ if $f(x) \leq f(x^* + h)$ for all sufficiently small positive and negative values of $h$. Also, a point $x^*$ is called a relative or local maximum if $f(x) \geq f(x^* + h)$ for all values of $h$ sufficiently close to zero [4].
A function $f(x)$ is said to have a global or absolute minimum at $x^*$ if $f(x^*) \leq f(x)$ for all $x$ in the domain over which $f(x)$ is defined. Similarly, a point $x^*$ will be a global maximum of $f(x)$ if $f(x^*) \geq f(x)$ for all $x$ in the domain. Figure 4.1 shows the difference between the local and global optimum points.

A single objective optimization problem can be formulated as finding the variable $x = x^*$ in the interval $[a, b]$ such that $x^*$ minimizes the objective function $f(x)$.

The standard form for a single-objective, non-linear, constrained optimization problem is provided in Equation 4.1 [4].

\[
\begin{align*}
\text{Minimize} & \quad f(x) \\
\text{Subject to} & \quad g_i(x) \leq 0 \quad i = 1, \ldots, m \\
& \quad h_j(x) = 0 \quad j = 1, \ldots, \ell \\
& \quad x_L \leq x \leq x_U
\end{align*}
\]

where $x = (x_1, x_2, \ldots, x_n)^T$ is a column vector of $n$ real-valued design variables corresponding the optimal solution. In Equation 4.1, $f(x)$ represents the objective function, $g_i(x)$ an inequality constraint and $h_j(x)$ an equality constraint function. The searchable design region is defined by the upper and lower bounds (or side constraints), $x_L$ and $x_U$, of the design variables. Figure 4.1 shows the local and global optimum points for function $f(x)$. 

\[A_1, A_2, A_3: \text{Relative Maxima}\]
\[A_2: \text{Global Maxima}\]
\[B_1, B_2: \text{Relative Minima}\]
\[B_1: \text{Global Minima}\]

Figure 4.1: Local and Global Optimum Points [4]
4.1.2 Case Study of Single Objective Optimization

Determine the maximum and minimum values of the function

\[ f(x) = 12x^5 - 45x^4 + 40x^3 + 5. \]

Solution:
Since

\[ f'(x) = 60(x^4 - 3x^3 + 2x^2) = 60x^2(x - 1)(x - 2), \]

\[ f'(x) = 0 \]

at \( x = 0, x = 1, \text{ and } x = 2. \)

The second derivative is

\[ f''(x) = 60(4x^3 - 9x^2 + 4x) \]

At \( x = 1, f''(x) = -60 \) and hence \( x = 1 \) is a relative maximum. Therefore,

\[ f_{\text{max}} = f(x = 1) = 12 \]

At \( x = 2, f''(x) = 240, \) and hence \( x = 2 \) is a relative minimum. Therefore,

\[ f_{\text{min}} = f(x = 2) = -11 \]

At \( x = 0, f''(x) = 0 \) and hence we must investigate the next derivative:

\[ f'''(x) = 60(12x^2 - 18x + 4) = 240 \quad \text{at} \quad x = 0 \]

Since \( f'''(x) \neq 0 \) at \( x = 0, x = 0 \) is neither a maximum nor a minimum, and it is an inflection point [4].
4.2 Multi-Objective Optimization

The Multi-Objective Optimization Problem (MOP) involves mathematical approaches to minimize or maximize several objective functions simultaneously, considering a set of constraints. It can be used in a theoretical or practical scenario, where the decision maker seeks an optimal set of answers considering tradeoffs between conflicting objectives. There are many applications associated with the implementation of multi-optimization techniques, including planning, programming, operation, and engineering. Consequently, these applications become a benefit to economics, business and many other fields of science, all of which try to achieve a compromise between objectives, in order to find the optimal solution.

Multi-optimization problems are characterized by having no unique ideal solution that can link competing objectives while meeting various constraints at the same time. Defining the priorities of the decision maker can lead to the best mathematical equality set of solutions. As a conceptual example of a multi-objective optimization problem, assume that we need to buy a flight ticket. The objectives in this decision would be the price of the ticket, a specific budget, the duration of the flight, the airline’s service qualities, the time limitations, baggage allowance and regulations, flight insurance, etc. It is clear that there is no close link between the competing objectives in this problem, which can lead to a unique set of answers highly dependent on the decision maker’s priorities.

For solving a multi-objective optimization problem, deciding the type of solution, which can be either local or global, is the initial step. This step will be generally followed by choosing a suitable mathematical method for the intended purpose. Then, objective functions and system constraints should be defined explicitly. By choosing the appropriate method, results would be found; if the results were unsatisfactory, options or start points will be changed. This cycle will continue until the best answer is achieved.

Many advanced software, such as MATLAB toolboxes, accelerate the pace of engineering and science. These technologies have been introduced to make mathematical computing more precise for this kind of application.
The general form of multi-objective optimization is formulated as finding the set of values for
decision vector $x$ according to Equation 4.2, such that:

\[
\begin{align*}
\text{Minimizes/Maximizes} & \quad f_m(X), \quad m = 1, 2, \ldots, M \\
\text{Subject to} & \quad g_j(X) \geq 0, \quad j = 1, 2, \ldots, J \\
& \quad h_k(X) = 0, \quad k = 1, 2, \ldots, K \\
& \quad x_{j_{\text{Lower Band}}} \leq x_i \leq x_{j_{\text{Upper Band}}}, \quad i = 1, 2, \ldots, N
\end{align*}
\]

Equation 4.2 defined $f_m(X)$ as the objective function set (also called objective criteria, payoff functions, cost functions, or value functions); $M$ is the number of objectives, $X$ is a vector of
design variables (also called decision variables), where $N$ is the number of independent variables
$x_n$; $g_j(X)$ and $h_k(X)$ are inequality and equality constraints respectively; $x_{j_{\text{Lower Band}}}$ and $x_{j_{\text{Upper Band}}}$
are the restricting boundary conditions for decision variables.

4.3 Non-Dominated and Dominated Points, Pareto Solution

When comparing the solution of a single objective optimization problem, the objective
function values will determine the superiority of one solution over another, while in a multi-
objective optimization problem this concept would be explained by the definition of dominance.
A vector of objective functions, $F(x^*) \in \mathbb{Z}$, is non-dominated if there does not exist another
vector, $F(x) \in \mathbb{Z}$, such that $F(x) < F(x^*)$ with at least on $F_i(x) < F_i(x^*)$ Otherwise, $F(x^*)$ is
dominated. In other words, the solution $X$ dominates $Y$, if $X$ has the same or better solution for
all the objectives (equally no worse than $Y$ in all objectives); and also the solution presented by
$X$ is strictly better than $Y$ in at least one objective function. In this case, it could be defined that
$X$ dominates $Y$ or equally $Y$ is dominated by $X$ [13]. Figure 4.2 shows the non-dominated and
dominated points for function $f_1$ and $f_2$ [13].

105
• $x_1$ dominates $x_4$, $x_4$ is dominated by $x_1$.
• $x_3$ dominates $x_5$, or $x_5$ is dominated by $x_3$.
• $x_3$ and $x_2$ neither of the solutions dominates ($x_3$ and $x_2$ are not dominated with respect to each other).

Within the entire search space in a given set of solutions, the non-dominated solution set is a set of all the solutions that are not dominated by any member of the solution set. The non-dominated set of the entire feasible decision space is called the Pareto-optimal set. The boundary defined by the set of all point mapped from the Pareto optimal set is called the Pareto optimal front [13].

Figure 4.3 shows the Pareto-optimal solution. Many of the multi-objective optimization methods convert multiple objectives into a single-objective optimization function by a user-defined procedure. This is known as a scalarized problem. Achieving an appropriate function of scalarization would guarantee the solutions obtained by the Pareto optimality technique [13], [3].
The following example shows a Pareto front plot for a multi-objective function using \textit{fgoalattain}, adopted from MATLAB tutorial [39].

Minimize: \[ [f_1(x) ; f_2(x)] \]

\textit{where} \[ f_1(x) = \sqrt{1 + x^2} \]
\[ f_2(x) = 4 + 2\sqrt{1 + (x - 1)^2} \]

The two objectives in our equations are shifted and scaled versions of the convex function \(\sqrt{1 + x^2}\).

In this case, it can be seen by inspection that the minimum of \(f_1(x)\) is 1, and the minimum of \(f_2(x)\) is 6. Figure 4.4 plots the objective functions when the variable \(x\) varies between -0.5 and 1.5, the two objectives have their minima at \(x=0\) and \(x=1\). Figure 4.5 represents the set of solution for Pareto optimal front.
Figure 4.4: Objective Function Plot, Trade-Off Region is Between the Green Lines [12]

Figure 4.5: Pareto Optimal Front plot [12]
4.4 Classical Multi-Objective Optimization Methods

Classical multi-objective optimization methods have been refined for 40 years; there have been many algorithms suggested by researchers in this area, considering the classification of different methods. Cohon [40] presented the two following types of classification for Multi-Objective Optimization algorithms:

- Generating methods
- Preference-based methods

In generating methods, the decision-maker should choose one solution through the generated set of non-dominated solutions. In this case, there is no priority affecting knowledge with the final decision. While considering the preference-based methods, optimization is based on some known preference for each one of the objectives. Hwang and Masud [41] and later Miettinen [42] completed and improved the above classification according to the following classes:

- No-preference methods
- Posteriori methods
- Priori methods
- Interactive methods

In no-preference methods, a heuristic algorithm is used to generate a single optimal solution, while the objectives are not being affected by priority or importance. In posterior methods, a set of Pareto-optimal solutions would be generated iteratively using the preference knowledge of each objective. Some information about the algorithm parameter is necessary for classical methods to ensure generating Pareto optimal solutions [41][42]. Mathematical programming and evolutionary algorithms are the two main subcategories of posterity classification.

In priori methods, sufficient preference information is expressed before the solution process. Famous applications associated with this method include the utility function method, lexicographic method, and goal programming.

Interactive methods of multi-objective optimization problems are iterative based algorithms that involve the continuous interaction of the decision maker to find the most favorable solution at each iteration in order to get Pareto optimal solution set of interest [42]. Figure 4.6 represents a classification of multi-objective optimization methods and the role of the human in the
optimization process. Figure 4.7 provides the classification of goal-oriented multi-objective optimization methods and algorithms.

Figure 4.6: Classification of Multi-Objective Optimization Methods and the Role of Human in the Optimization Processes [43]
4.5 Weighted Sum Method

Weighted Sum (WS) is one of the most common classical approaches used for defining the Pareto optimal set. This method scalarizes a set of objectives into a single objective by adding each objective pre-multiplier by a user-defined weight factor. The weight of each factor would be defined by the objective relative importance in the problem [13].

\[
\text{Minimizing } \quad f(x) = \sum_{m=1}^{M} w_m * f_m(x), \quad m = 1, 2, ..., M \quad (4.3)
\]

Subject to:

\[
\begin{align*}
& g_j(X) \leq 0 & j &= 1, 2, ..., J \\
& h_k(X) = 0 & k &= 1, 2, ..., K \\
& x_i^{(L)} \leq x_i \leq x_i^{(U)} & i &= 1, 2, ..., N
\end{align*}
\]
In Equation (4.3), \( w_m \) is defined as the weight of the \( m^{th} \) objective function, \( g_j (X) \) and \( h_k (X) \) are inequality and equality constraints. \( x_j^{(L)}_{\text{Lower Band}} \) and \( x_j^{(U)}_{\text{Upper Band}} \) are the restricting boundary conditions for decision variables.

4.7 Case Studies of Multi-Objective Optimization

Two case studies of multi-objective optimization problems with general propose are discussed in this chapter which are adopted from MathWorks online website. The first case has two objective function and two variables, without any constraint. The second case study is a problem which consists of two objective functions, three variables, and two constraints.

4.7.1 Case Study 1

This case has a two-objective fitness function \( f(x) \), where \( x \) is also two-dimensional: objective functions to be minimized are:

\[
\begin{align*}
  f_1 &= 4x_1^6 - 2x_1^2 + 4x_1x_2 + 4x_2^6 - x_1^2x_2^2 \\
  f_2 &= x_2^6 - 8x_1^2x_2^2 + x_1^6 + 2x_1x_2
\end{align*}
\]

The following function file is created before proceeding, and is stored it as mymulti1.m in MATLAB® path.

```matlab
function f = mymulti1(x)
    f(1) = 4*x(1)^6 - 2*x(1)^2 + 4*x(1)*x(2) + 4*x(2)^6 - (x(1)^2)*(x(2)^2)
    f(2) = x(2)^6 - 8*(x(1)^2)*(x(2)^2) + x(1)^6 + 2*x(1)*x(2)
```

Figure 4.8 represent the MATLAB optimization toolbox setting with respect to this case study. By specifying the population size of the case for 60 for two variables, and setting the lower bound between [-5 -5] and upper bound for [5 5], the Pareto plot has been generated and represented in Figure 4.9.
Figure 4.8: Optimization Toolbox Setting

Figure 4.9: Pareto Front of Case Study 1
4.7.2 Case Study 2

In this case, there are two constraints that should be considered in two objective functions, which have to be minimized are with three variables.

\[
\begin{align*}
    f_1 &= x_1^4 + x_2^4 + x_3^4 \\
    f_2 &= x_1 x_2 (\cos x_3)^2 + x_1 x_3 (\cos x_2)^2 + x_2 x_3 (\cos x_1)^2
\end{align*}
\]

Subject to:

\[
\begin{align*}
    2x_1 + x_2 + x_3 &= 10^5 \\
    -100 &\leq x_1, x_2, x_3 \leq 100
\end{align*}
\]

By using the Weighted Sum (WS) technique, the objectives can be linearly combined as a simple objective function. Using the MATLAB optimization toolbox, by specifying the function tolerance by \(1e^{-8}\), and setting the lower bound between \([-100 -100 -100]\) and upper bound for \([100 100 100]\), the Pareto plot has been generated and presented in Figure 4.1.

Figure 4.10: Pareto Front of Case Study 2
4.8 Conclusions
This chapter has introduced the general concepts and principles of the single objective and multi-objective optimization problems. The Pareto optimal solution has been introduced as a set of acceptable trade-off optimal solutions (non-inferior solutions). Different classifications for multi-objective optimization algorithms are presented. Weighted Sum (WS) method as one of the most common classical approaches used for defining the Pareto optimal set. Additionally, the MATLAB optimization toolbox is used for solving two numerical case studies presented in this chapter.
Chapter 5

5. Applying Multi-Objective Optimization Techniques for Solving Optimal Power Flow

5.0 Introduction

The primary propose of this chapter is to solve the Optimal Power Flow (OPF) problem as a multi-objective optimization problem while considering the system constraints. Teaching-Learning Base Optimization (TLBO) algorithm is used to optimize the OPF problem in several single and multi-objective cases. The method has been applied to 18 different cases on the IEEE 30-bus and IEEE 57-bus test systems for several OPF objectives such as minimization of fuel cost, emission, voltage deviation, and power losses. Simulation results and comparisons highlight the effectiveness of the method for solving multi-objective OPF problems and relating the results to the performance of the power system.

5.1 Multi-Objective Optimization Problem Formulation

Four objectives of the optimal power flow problem are discussed in this section. The proposed objective functions are fuel cost generation, active power transmission losses, voltage deviation, and environmental emission. It is assumed that all the generating units are thermal with a cubic cost model for the fuel expressed as a cubic function of the output of the generating units. Some of these objective functions are conflicting in nature, which makes the OPF problem complicated. A multi-objective concept will be applied to solve the OPF problem [30]. The mathematical formulation of the Optimal Power Flow problem can be stated as a nonlinear constrained optimization problem. The problem is subject to the satisfaction of nonlinear equality constraints and nonlinear inequality constraints.
The function is expressed by Equation 5.1 for the problem with two objective functions.

\[
\text{Minimize} \{ F_1(x, u), F_2(x, u), \ldots, \text{and } F_n(x, u) \} \quad (5.1)
\]

Subject to: 
\[
\begin{align*}
g(x, u) &= 0 \\
h(x, u) &\leq 0
\end{align*}
\]

The OPF problem has many control variables that need to be both specified and subjected to a variety of equality and inequality constraints. In the above equation, \( F \) is the objective function to be minimized. \( x \) is the vector of dependent variables (state vector), and \( u \) is the vector of independent variables (control variables) presented in Equations 5.2:

\[
\begin{align*}
u^T &= [P_{G2}, \ldots, P_{NG}, V_{G1}, \ldots, V_{NG}, Q_{C1}, \ldots, Q_{NC}, T_1, \ldots, T_{NT}] \\
x^T &= [P_{SG}, V_{L1}, \ldots, V_{LNl}, Q_{G1}, \ldots, Q_{NG}, S_l, \ldots, S_{lnl}]
\end{align*}
\]

Where

- \( P_G \): Active power generated at the PV buses
- \( V_G \): Voltage magnitude at PV buses
- \( Q_C \): Reactive power supplied by all shunt reactors
- \( T_{NT} \): Transformer load tap changer magnitude
- \( P_{SG} \): Active power of the slack bus
- \( V_L \): Voltage magnitude at PQ buses (load buses)
- \( Q_G \): Reactive power of all generator units
- \( S_l \): Transmission Line loading (Load flow)
- \( NL \): Number of load buses
- \( NG \): Number of generator buses (PV buses)
- \( NC \): Number of shunt VAR compensators
- \( NT \): Number of transformers
- \( nl \): Number of transmission lines
The network constraints include limits on all control variables, power flow equations, generation/load balances, branch flow limits (MW, MVAR, MVA), bus voltage limits, active/reactive reserve limits, generator MVAR limits, corridor (transmission interface) limits. Equations 5.3 shows the equality constraints of the power system. Inequality constraints for active power constraints and reactive power constraints are provided by Equations 5.4 to 5.8.

\[ P_G^k - P_D^k = V_k \sum_{j=1}^{N} \left[ V_j \left[ G_{j}^k \cos(\delta^k - \delta^j) + B_{j}^k \sin(\delta^k - \delta^j) \right] \right] \]  \hspace{1cm} (5.3)

\[ Q_G^k - Q_D^k = V_k \sum_{j=1}^{N} \left[ V_j \left[ G_{j}^k \sin(\delta^k - \delta^j) - B_{j}^k \cos(\delta^k - \delta^j) \right] \right] \]  \hspace{1cm} (5.3)

Inequality Constraints:

\[ P_{G_k}^{Min} \leq P_{G_k} \leq P_{G_k}^{Max} \]  \hspace{1cm} (5.4)

\[ Q_{G_k}^{Min} \leq Q_{G_k} \leq Q_{G_k}^{Max} \]  \hspace{1cm} (5.5)

\[ V_{k}^{Min} \leq V_{k} \leq V_{k}^{Max} \]  \hspace{1cm} (5.6)

\[ Q_{c_k}^{Min} \leq Q_{c_k} \leq Q_{c_k}^{Max} \]  \hspace{1cm} (5.7)

\[ T_{k}^{Min} \leq T_{k} \leq T_{k}^{Max} \]  \hspace{1cm} (5.8)

Where:

\( k = 1, 2, ..., N \)

\( V_k \) : Voltage at \( k^{th} \) bus

\( \delta^k \) : Voltage angle at bus \( k^{th} \)

\( P_G^k \) : Active power generation at bus \( k^{th} \)

\( P_D^k \) : Active power demand at bus \( k^{th} \)

\( Q_G^k \) : Reactive power generation at bus \( k^{th} \)

\( Q_D^k \) : Reactive power demand at bus \( k^{th} \)

\( G_{j}^k \) : Real part of \((k,j)\) element of the bus admittance matrix

\( B_{j}^k \) : Imaginary part of \((k,j)\) element of the bus admittance matrix

\( P_{G_k} \) : Active power generated by \( k^{th} \) generator

\( Q_{G_k} \) : Reactive power generated by \( k^{th} \) generator

\( Q_{c_k} \) : The reactive power source capacity constraint

\( T_k \) : The transformer tap position constraints
It worth to mention that the control variables are self-constrained. The system generating constraints, transformer constraints, shunt VAR compensator constraints and, security constraints will satisfy the upper and lower boundaries of variables with aforementioned inequalities in solution.

The inequality constraints of state (dependent) variables can be incorporated into the objective function as a quadratic penalty term. It contains the load bus voltage magnitude, the active power of slack bus, the reactive power of generation and line loading. In these terms, a penalty factor is multiplied by with the square of the disregarded value, is added to the objective function and any unfeasible solution is declined [44]. Equation 5.9 expresses the mathematical formulation of the penalty function:

\[
Penalty = \lambda_P (P_G - P_{G_i}^{lim})^2 + \lambda_V \sum_{i=1}^{N_L} (V_L - V_{L_i}^{lim})^2 + \lambda_Q \sum_{i=1}^{N_G} (Q_G - Q_{G_i}^{lim})^2 + \lambda_S \sum_{i=1}^{N_L} (S_{li} - S_{li}^{lim})^2
\]

(5.9)

Where \(\lambda_P, \lambda_V, \lambda_Q\) and \(\lambda_S\) are the penalty factors and \(x^{lim}\) is the violated limit value of the state(dependent) variable \(x\), Equation 5.10 expresses the limit value as follows:

\[
x^{lim} = \begin{cases} 
  x^{max}, & x > x^{max} \\
  x^{min}, & x < x^{min} 
\end{cases}
\]

(5.10)
5.2 Case Studies

Several case studies with single and multi-objectives have been performed to illustrate the effect of considering multi-objectives optimization in solving the OPF problem.

Table 5.1: Summary of case studies

<table>
<thead>
<tr>
<th>Case no</th>
<th>IEEE 30-bus system</th>
<th>IEEE 57-bus system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Case 2</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Case 3</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Case 4</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Case 5</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Case 6</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Case 7</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Case 8</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Case 9</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Case 10</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Case 11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case 14</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Case 15</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Case 16</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>Case 17</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Case 18</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>
5.2.1 IEEE 30-Bus Power System

The IEEE 30-bus system is the proposed system in which the line data and the bus data are given in [31]. Different cases are considered to evaluated and investigate the basic and complex objective functions.

The system active and reactive power demands are 283.4 MW and 126.2 MW, respectively. In addition, the algorithms are implemented to solve all cases under the same conditions, i.e. system data, constraints, control variables limits. Table 5.2 provides a summary of the system specification. Bus 1 is the swing bus or slack bus. In the power flow study, the rule of the slack bus is to balance active and reactive power in the system, while satisfying power balances mentioned by Equations 5.4 and 5.5.

The first four case studies minimize the single objective functions of OPF including fuel cost, emission, transmission losses, and voltage deviation. The remaining cases are for multi-objective optimizations which are converted to single objectives with weight factors as in many previous studies [31].

Table 5.2: The Main Characteristics of The IEEE 30-Bus Test System [31]

<table>
<thead>
<tr>
<th>Items</th>
<th>IEEE 30-bus system</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Quantity</td>
</tr>
<tr>
<td>Buses</td>
<td>30</td>
</tr>
<tr>
<td>Branches</td>
<td>41</td>
</tr>
<tr>
<td>Generators</td>
<td>6</td>
</tr>
<tr>
<td>Shunt VAR compensation</td>
<td>9</td>
</tr>
<tr>
<td>Transformer with tab changer</td>
<td>4</td>
</tr>
<tr>
<td>Control Variables</td>
<td>24</td>
</tr>
<tr>
<td>Connected load</td>
<td></td>
</tr>
<tr>
<td>Load bus voltage range allowed</td>
<td>24</td>
</tr>
</tbody>
</table>

Table 5.3 presents the cost and emission coefficient of generators for IEEE 30-bus system.
Table 5.3: Cost and Emission Coefficient of Generators for IEEE 30-bus System [31]

<table>
<thead>
<tr>
<th>Generator</th>
<th>Bus</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>α</th>
<th>β</th>
<th>γ</th>
<th>ω</th>
<th>μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0.00375</td>
<td>18</td>
<td>0.037</td>
<td>4.091</td>
<td>-5.554</td>
<td>6.49</td>
<td>0.0002</td>
<td>2.857</td>
</tr>
<tr>
<td>G2</td>
<td>2</td>
<td>0</td>
<td>1.75</td>
<td>0.0175</td>
<td>16</td>
<td>0.038</td>
<td>2.543</td>
<td>-6.047</td>
<td>5.638</td>
<td>0.0005</td>
<td>3.333</td>
</tr>
<tr>
<td>G3</td>
<td>5</td>
<td>0</td>
<td>1</td>
<td>0.0625</td>
<td>14</td>
<td>0.04</td>
<td>4.258</td>
<td>-5.094</td>
<td>4.586</td>
<td>0.00001</td>
<td>8</td>
</tr>
<tr>
<td>G4</td>
<td>8</td>
<td>0</td>
<td>3.25</td>
<td>0.00834</td>
<td>12</td>
<td>0.045</td>
<td>5.326</td>
<td>-3.55</td>
<td>3.38</td>
<td>0.002</td>
<td>2</td>
</tr>
<tr>
<td>G5</td>
<td>11</td>
<td>0</td>
<td>3</td>
<td>0.025</td>
<td>13</td>
<td>0.042</td>
<td>4.258</td>
<td>-5.094</td>
<td>4.586</td>
<td>0.00001</td>
<td>8</td>
</tr>
<tr>
<td>G6</td>
<td>13</td>
<td>0</td>
<td>3</td>
<td>0.025</td>
<td>13.5</td>
<td>0.041</td>
<td>6.131</td>
<td>-5.555</td>
<td>5.151</td>
<td>0.0001</td>
<td>6.667</td>
</tr>
</tbody>
</table>

Case studies one to four provides a comparison for optimizing the single objective function of OPF problem including fuel cost function, emission, the active power losses in transmission lines, and voltage deviation separately.
Case 1: Minimization of Fuel Cost

The main objective function of the optimal power flow problem is minimizing the generation Fuel Cost (FC) of all thermal units, presented by Equation 5.11. Table 5.4 summarizes the results of applying TLBO optimization technique with MATLAB on this case study.

\[
FC = \sum_{i=1}^{NG} a_i P_{G_i}^2 + b_i P_{G_i} + c_i \tag{5.11}
\]

Where

\(FC\) : Total fuel cost generation of the power system
\(NG\) : Number of generators including the slack generator
\(a_i\) : Quadratic cost coefficient of the \(k^{th}\) generator
\(b_i\) : Linear cost coefficient of the \(k^{th}\) generator
\(c_i\) : basic cost coefficient of the \(k^{th}\) generator
\(P_{G_i}\) : Real power output of the \(k^{th}\) generator.

Figure 5.1 shows the minimization of the fuel cost function with respect to the number of evaluation. The optimization process leads to the fuel cost of 800.7832 $/hr while satisfying all system constraints.

Figure 5.1: Convergence of Case 1 for IEEE 30-Bus System
Case 2: Minimization of Emission

The Emission or Environmental Index (EI), is an index from the viewpoint of environmental conservation. The atmospheric pollutants can be modeled separately. However, the OPF problem seeks to minimize the total (Ton/h) emission $E(P_G)$ of pollutants such as nitrogen oxides (NOx) and sulfur oxides (SOx) caused by fossil-fueled thermal units, which can be stated by the following equations. As indicated, the amount of emissions are given as a function of the generator active power output, which is the sum of quadratic and exponential functions according to Equation 5.12 [30].

$$F(x, u) = \text{Emission} = \sum_{k=1}^{NG} (\alpha_i + \beta_i P_{G_i} + \gamma_i P_{G_i}^2) \times 0.01 + \omega_i e^{(\mu_i P_{G_i})} \quad (5.12)$$

Where $\alpha_i$, $\beta_i$, $\gamma_k$, $\omega_i$ and $\mu_i$ are coefficients, provided in Table 5.3 for the IEEE 30-bus system. The results are presented in Table 5.4.

![Figure 5.2: Convergence of Case 2 for IEEE30-Bus System](image)

Figure 5.2 presents the minimization of emission function with TLBO algorithm where the optimization procedure provides the best output of 0.2049 ton/hr.
Case 3: Minimization of Voltage Deviation

Voltage deviation is a measure of voltage quality in the power network. The index of deviation is also important from the security aspect. The indicator is formulated as a cumulative deviation of voltages for all load buses (PQ buses) in the network from a nominal value of unity. It can be defined mathematically as presented by Equation 5.13 [31], [32]. Figure 5.3 illustrates the convergence of function to the optimal solution of 0.1181 p.u after 100 times evaluations by the TLBO method by MATLAB. Table 5.4 provided more details about the results of implementing the algorithm on this case study.

\[
VD = \left( \sum_{P=1}^{NL} |V_{LP} - 1| \right)
\]  

(5.13)

![Figure 5.3: Convergence of Case 3 for IEEE 30-Bus System](image-url)
Case 4: Minimization of Active Power Transmission Loss

The objective function to minimize the active power transmission loss is defined by Equation 5.14:

\[ f(x, u) = P_{Loss} = \sum_{q=1}^{m} G_{q(ij)} \left[V_i^2 + V_j^2 - 2V_iV_j - 2V_iV_j\cos(\delta_{ij})\right] \]  (5.14)

Where, \( \delta_{ij} = \delta_i - \delta_j \), is the difference in voltage angle between bus \( i \) and bus \( j \), \( G_{q(ij)} \) is the transfer conductance of branch \( q \) connecting the buses \( i \) and \( j \). Figure 5.4 shows the optimization procedure result in the real power loss of 3.1804 MW in transmission lines. Table 5.4 provides the details on minimization results.

![Convergence of Case 4 for IEEE 30-Bus System](image)

Figure 5.4: Convergence of Case 4 for IEEE 30-Bus System
Case 5: Two Objectives: Minimization of Fuel Cost and Emission

The goal of the objective function is to minimize both fuel cost and emission simultaneously. The converted single objective function will follow Equation 5.15. The weight factor $\lambda_E$ is assigned the value of 1000 according to [44]. Figure 5.5 shows the convergence of the fitness function. For clarity, the convergence of each individual objective attaining the best fitness value is included in the fitness diagram for all multi-objective case studies. Table 5.5 presents the optimized solution for this multi-objective optimization problem. The best fuel cost value arrives at an increase in cost for Case 1, with the final value being 836.7029 $/hr and the ultimate value for emission is 0.2423 ton/hr after 100 evaluations.

$$f(x, u) = \left( \sum_{i=1}^{NG} \left( a_i + b_iP_{G_i} + c_iP_{G_i}^2 \right) \right) + \lambda_E \times Emission + Penalty$$

(5.15)

$$Emission = \sum_{k=1}^{NG} (\alpha_i + \beta_iP_{G_i} + \gamma_iP_{G_i}^2) \times 0.01 + \omega_i e^{(\mu_iP_{G_i})}$$

Figure 5.5: Convergence of Case 5 for IEEE 30-Bus System
Case 6: Two Objectives: Minimization of Fuel Cost and Voltage Deviation

The combined objective function of the fuel cost and the voltage deviation in the network is expressed by Equation 5.16:

\[
f(x, u) = \left( \sum_{l=1}^{N_G} a_l + b_l P_{G_l} + c_l P_{G_l}^2 \right) + \lambda_{VD} \times VD + \text{Penalty} \quad (5.16)
\]

\[
VD = \left( \sum_{P=1}^{NL} |V_{LP} - 1| \right)
\]

where the weight factor \( \lambda_{VD} \) is assigned a value of 100 as presented in [32] and [31]. Table 5.5 summarizes the results of implementing multi-objective optimization using TLBO algorithm using MATLAB and Figure 5.6 presents the convergence of fitness function in this case study.

![Figure 5.6: Convergence of Case 6 for IEEE30-Bus System](image-url)
Case 7: Two Objectives: Minimization of Fuel Cost and Real Power Loss

The multi-objective case of minimizing two objectives of fuel cost and real power loss is converted to a single objective function by considering a multiplier as the weight factor to one of the objectives, expressed by Equation 5.17:

\[ f(x, u) = \left( \sum_{i=1}^{NG} a_i + b_i P_{G_i} + c_i P_{G_i}^2 \right) + \lambda_P \times P_{Loss} + Penalty \quad (5.17) \]

\[ P_{Loss} = \sum_{q=1}^{m} G_{q(ij)} \left[ V_i^2 + V_j^2 - 2V_iV_j - 2V_iV_j \cos(\delta_{ij}) \right] \]

Where the value of \( \lambda_P \) is chosen as 40 according to [32], and \( P_{Loss} \) is the real power loss in the network, \( \delta_{ij} = \delta_i - \delta_j \), is the difference in voltage angle between bus \( i \) and bus \( j \), and \( G_{q(ij)} \) is the transfer conductance of branch \( q \) connecting the buses \( i \) and \( j \). Figure 5.7 shows the convergence of fitness function for the multi-objective optimization of OPF where the fuel cost reaches 836.7029 $/hr and real power loss. Table 5.5 summarizes the results of implementing the TLBO algorithm by MATLAB on this case study [31].

Figure 5.7: Convergence of Case 7 for IEEE30-Bus System
Case 8: Three Objectives: Minimization of Fuel Cost, Emission, Losses

The multi-objective case of minimizing fuel cost, emission, and real power loss is converted to a single objective function by multiplying a weight factor to the objectives according to Equation 5.18.

\[ f(x,u) = \left( \sum_{i=1}^{NG} a_i + b_i P_{G_l} + c_i P_{G_l}^2 \right) + \lambda_E \times \text{Emission} + \lambda_P \times P_{\text{Loss}} + \text{Penalty} \]  

(5.18)

Where emission and real power loss is expressed according to Equations 5.12 and 5.14 respectively. The weight factors are selected to balance between the objectives as 19 and 22, for emission and losses, respectively [32]. Figure 5.8 illustrates the convergence of fitness function and Table 5.5 shows the optimized values with respect to this multi-objective optimization case. The fitness function convergence revealed the best values of the function to be 830.6375 $/hr, 0.2529$ ton/hr, 5.6193 MW for fuel cost, emission and losses respectively.

Figure 5.8: Convergence of Case 8 for IEEE30-Bus System
Case 9: Four Objectives: Fuel Cost, Emission, Voltage Deviations and Losses

The aim of this case is to minimize four-conflicted objectives of fuel cost, emission, voltage deviations and losses in the network simultaneously. The objective function is defined by Equation 5.19:

\[
f(x, u) = \left( \sum_{i=1}^{NG} a_i + b_i P_{Gi} + c_i P_{Gi}^2 \right) + \lambda_E \times \text{Emission} + \lambda_{VD} \times \text{VD} + \lambda_P \times P_{Loss} \\
+ \text{Penalty}
\]

Substituting related equations, the function will be expressed according to Equation 5.20.

\[
f(x, u) = \left( \sum_{i=1}^{NG} a_i + b_i P_{Gi} + c_i P_{Gi}^2 \right) + \lambda_E \times \left( \sum_{k=1}^{NG} (a_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2) \times 0.01 + \omega_i e^{(\mu P_{Gi})} \right) \\
+ \lambda_{VD} \times \left( \sum_{P=1}^{NL} |V_{lp} - 1| \right) + \lambda_P \\
\times \left( \sum_{q=1}^{m} G_{q(ij)} [V_i^2 + V_j^2 - 2V_i V_j - 2V_i V_j \cos(\delta_{ij})] \right) + \text{Penalty}
\]

Where, \( \lambda_E \), \( \lambda_{VD} \), and \( \lambda_P \) are the weighting factors, which are selected to balance between the objectives as 19, 21 and 22, respectively [32] [31]. Table 5.5 summarizes the results of implementing the TLBO optimization algorithm by using MATLAB on this case study. Figure 5.9 illustrates the convergence of fitness function, while all of the four objectives are considered in the multi-objective OPF optimization procedure.
Figure 5.9: Convergence of Case 9 for IEEE 30-Bus System
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Min</th>
<th>Max</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>PG2 (MW)</td>
<td>20</td>
<td>80</td>
<td>48.324</td>
<td>67.861</td>
<td>59.5179</td>
<td>79.755</td>
</tr>
<tr>
<td>PG5 (MW)</td>
<td>15</td>
<td>50</td>
<td>21.6011</td>
<td>49.9999</td>
<td>50</td>
<td>49.9936</td>
</tr>
<tr>
<td>PG8 (MW)</td>
<td>10</td>
<td>35</td>
<td>21.7212</td>
<td>34.9953</td>
<td>20.3846</td>
<td>34.9985</td>
</tr>
<tr>
<td>PG11 (MW)</td>
<td>10</td>
<td>30</td>
<td>12.135</td>
<td>29.9999</td>
<td>10.9346</td>
<td>29.9999</td>
</tr>
<tr>
<td>VG1 (p.u)</td>
<td>0.95</td>
<td>1.1</td>
<td>1.0757</td>
<td>0.9865</td>
<td>1.033</td>
<td>1.0595</td>
</tr>
<tr>
<td>VG2 (p.u)</td>
<td>0.95</td>
<td>1.1</td>
<td>1.0541</td>
<td>0.9781</td>
<td>1.0199</td>
<td>1.0545</td>
</tr>
<tr>
<td>VG5 (p.u.)</td>
<td>0.95</td>
<td>1.1</td>
<td>1.0244</td>
<td>0.9573</td>
<td>1.0206</td>
<td>1.0365</td>
</tr>
<tr>
<td>VG8 (p.u.)</td>
<td>0.95</td>
<td>1.1</td>
<td>1.0317</td>
<td>0.9664</td>
<td>1.0094</td>
<td>1.0444</td>
</tr>
<tr>
<td>VG11 (p.u.)</td>
<td>0.95</td>
<td>1.1</td>
<td>1.0991</td>
<td>1.0847</td>
<td>1</td>
<td>1.0676</td>
</tr>
<tr>
<td>VG13 (p.u.)</td>
<td>0.95</td>
<td>1.1</td>
<td>1.0758</td>
<td>1.0837</td>
<td>1.0022</td>
<td>1.0691</td>
</tr>
<tr>
<td>T11 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>1.0402</td>
<td>0.9407</td>
<td>0.992</td>
<td>0.9743</td>
</tr>
<tr>
<td>T12 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9347</td>
<td>0.9</td>
<td>0.9019</td>
<td>1.0204</td>
</tr>
<tr>
<td>T15 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>1.0237</td>
<td>0.9599</td>
<td>0.9591</td>
<td>1.0338</td>
</tr>
<tr>
<td>T36 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9663</td>
<td>0.9242</td>
<td>0.9716</td>
<td>0.996</td>
</tr>
<tr>
<td>Qc10 (p.u.)</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Qc12 (p.u.)</td>
<td>0</td>
<td>5</td>
<td>3</td>
<td>0</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Qc15 (p.u.)</td>
<td>0</td>
<td>5</td>
<td>4</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Qc17 (p.u.)</td>
<td>0</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>Qc20 (p.u.)</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>Qc21 (p.u.)</td>
<td>0</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Qc23 (p.u.)</td>
<td>0</td>
<td>5</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>Qc24 (p.u.)</td>
<td>0</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Qc29 (p.u.)</td>
<td>0</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Fuel Cost ($/h)</td>
<td>800.7832</td>
<td>945.8329</td>
<td>870.632</td>
<td>967.2393</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emission (ton/h)</td>
<td>0.3648</td>
<td>0.2049</td>
<td>0.2644</td>
<td>0.2072</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Voltage Deviation (p.u.)</td>
<td>0.9073</td>
<td>0.7326</td>
<td>0.1181</td>
<td>0.6348</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Active Power Loss (MW)</td>
<td>9.046</td>
<td>3.6296</td>
<td>6.5758</td>
<td>3.1804</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.10: Simulation Results of Optimum Solution for IEEE-30 Bus System Single-objective Cases
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Min</th>
<th>Max</th>
<th>Case 5</th>
<th>Case 7</th>
<th>Case 6</th>
<th>Case 8</th>
<th>Case 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>PG2 (MW)</td>
<td>20</td>
<td>80</td>
<td>58.6876</td>
<td>50.6644</td>
<td>55.6226</td>
<td>52.0053</td>
<td>50.452</td>
</tr>
<tr>
<td>PG5 (MW)</td>
<td>15</td>
<td>50</td>
<td>28.0745</td>
<td>21.0879</td>
<td>38.1331</td>
<td>31.9342</td>
<td>31.9004</td>
</tr>
<tr>
<td>PG8 (MW)</td>
<td>10</td>
<td>35</td>
<td>34.9809</td>
<td>22.4297</td>
<td>34.9925</td>
<td>34.9939</td>
<td>34.9986</td>
</tr>
<tr>
<td>PG11 (MW)</td>
<td>10</td>
<td>30</td>
<td>27.3498</td>
<td>13.3915</td>
<td>29.9965</td>
<td>25.7703</td>
<td>29.5358</td>
</tr>
<tr>
<td>VG1 (p.u)</td>
<td>0.95</td>
<td>1.1</td>
<td>1.068</td>
<td>1.0397</td>
<td>1.0689</td>
<td>1.0718</td>
<td>1.0534</td>
</tr>
<tr>
<td>VG2 (p.u)</td>
<td>0.95</td>
<td>1.1</td>
<td>1.0545</td>
<td>1.0223</td>
<td>1.0564</td>
<td>1.0555</td>
<td>1.0368</td>
</tr>
<tr>
<td>VG5 (p.u)</td>
<td>0.95</td>
<td>1.1</td>
<td>1.0295</td>
<td>1.0188</td>
<td>1.0347</td>
<td>1.0306</td>
<td>1.0108</td>
</tr>
<tr>
<td>VG8 (p.u)</td>
<td>0.95</td>
<td>1.1</td>
<td>1.0397</td>
<td>1.0049</td>
<td>1.042</td>
<td>1.041</td>
<td>1.02</td>
</tr>
<tr>
<td>VG11 (p.u)</td>
<td>0.95</td>
<td>1.1</td>
<td>1.0532</td>
<td>1.0032</td>
<td>1.0652</td>
<td>1.0876</td>
<td>1.0999</td>
</tr>
<tr>
<td>VG13 (p.u)</td>
<td>0.95</td>
<td>1.1</td>
<td>1.0566</td>
<td>1.0157</td>
<td>1.0679</td>
<td>1.0744</td>
<td>1.0064</td>
</tr>
<tr>
<td>T11 (p.u)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9933</td>
<td>1.0118</td>
<td>0.9898</td>
<td>0.9985</td>
<td>1.0145</td>
</tr>
<tr>
<td>T12 (p.u)</td>
<td>0.9</td>
<td>1.1</td>
<td>1.0367</td>
<td>0.9007</td>
<td>0.9802</td>
<td>0.9818</td>
<td>1.0249</td>
</tr>
<tr>
<td>T15 (p.u)</td>
<td>0.9</td>
<td>1.1</td>
<td>1.0392</td>
<td>0.9911</td>
<td>1.0117</td>
<td>1.0155</td>
<td>0.9991</td>
</tr>
<tr>
<td>T36 (p.u)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9836</td>
<td>0.9588</td>
<td>0.9739</td>
<td>0.9705</td>
<td>0.9752</td>
</tr>
<tr>
<td>Qc10 (p.u)</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Qc12 (p.u)</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Qc15 (p.u)</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Qc17 (p.u)</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Qc20 (p.u)</td>
<td>0</td>
<td>5</td>
<td>2</td>
<td>5</td>
<td>5</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Qc21 (p.u)</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Qc23 (p.u)</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Qc24 (p.u)</td>
<td>0</td>
<td>5</td>
<td>5</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Qc29 (p.u)</td>
<td>0</td>
<td>5</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Fuel Cost ($/h)</td>
<td>836.7029</td>
<td>804.0932</td>
<td>859.5791</td>
<td>830.6375</td>
<td>835.9996</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emission (ton/h)</td>
<td>0.2423</td>
<td>0.3559</td>
<td>0.2286</td>
<td>0.2529</td>
<td>0.2482</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Voltage Deviation (p.u.)</td>
<td>0.5471</td>
<td>0.1181</td>
<td>0.8726</td>
<td>0.7155</td>
<td>0.2349</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Active Power Loss (MW)</td>
<td>5.4242</td>
<td>9.772</td>
<td>4.5615</td>
<td>5.6193</td>
<td>5.6224</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.11: Simulation Results of Optimum Solution for IEEE-30 Bus System Multi-objective Cases
5.2.2 IEEE 57-Bus System

The IEEE 57-bus system data [31] is considered to perform a total of 8 case studies for the system. Table 5.6 provides a summary of the system specification. The system active and reactive power demands are 1250.8 MW and 336.4 MVAR. The IEEE 57-bus test system constitutes of 7 generators, 80 transmission line, 17 transformers, and 3 capacitor banks as given in [31]. Upper and lower boundaries of voltage buses and transformer tap settings are between 0.9 & 1.1 p.u. Table 5.7 presents the cost and emission coefficient of generators for IEEE 57-bus system.

Table 5.6: The Main Characteristics of the IEEE 57-Bus Test System [31]

<table>
<thead>
<tr>
<th>Items</th>
<th>IEEE 57-bus system</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buses</td>
<td>57</td>
<td>Ref [31]</td>
</tr>
<tr>
<td>Branches</td>
<td>80</td>
<td>Ref [31]</td>
</tr>
<tr>
<td>Generators</td>
<td>7</td>
<td>Buses:1(Slack), 2, 3, 6, 8, 9 and 12</td>
</tr>
<tr>
<td>Shunt VAR compensation</td>
<td>3</td>
<td>Buses: 18, 25 and 53</td>
</tr>
<tr>
<td>Transformer with tab changer</td>
<td>17</td>
<td>Branches: 19, 20, 31, 35, 36, 37, 41, 46, 54, 58, 59, 65, 66, 71, 73, 76, and 80</td>
</tr>
<tr>
<td>Control Variables</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>Connected load</td>
<td></td>
<td>1250.8 MW, 336.4 MVAR</td>
</tr>
<tr>
<td>Load bus voltage range allowed</td>
<td>50</td>
<td>[0.94- 1.06] p.u.</td>
</tr>
</tbody>
</table>

Table 5.7: Cost and Emission Coefficient of Generators for IEEE 57-Bus System [31]

<table>
<thead>
<tr>
<th>Generator</th>
<th>Bus</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>α</th>
<th>β</th>
<th>γ</th>
<th>ω</th>
<th>μ</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>1</td>
<td>0</td>
<td>20</td>
<td>0.00375</td>
<td>18</td>
<td>0.037</td>
<td>4.091</td>
<td>-5.554</td>
<td>6.49</td>
<td>0.0002</td>
<td>0.286</td>
</tr>
<tr>
<td>G2</td>
<td>2</td>
<td>0</td>
<td>40</td>
<td>0.0175</td>
<td>16</td>
<td>0.038</td>
<td>2.543</td>
<td>-6.047</td>
<td>5.638</td>
<td>0.0005</td>
<td>0.333</td>
</tr>
<tr>
<td>G3</td>
<td>5</td>
<td>0</td>
<td>20</td>
<td>0.0625</td>
<td>13.5</td>
<td>0.041</td>
<td>6.131</td>
<td>-5.555</td>
<td>5.151</td>
<td>0.00001</td>
<td>0.667</td>
</tr>
<tr>
<td>G4</td>
<td>8</td>
<td>0</td>
<td>40</td>
<td>0.00834</td>
<td>18</td>
<td>0.037</td>
<td>3.491</td>
<td>-5.754</td>
<td>6.39</td>
<td>0.0003</td>
<td>0.266</td>
</tr>
<tr>
<td>G5</td>
<td>11</td>
<td>0</td>
<td>20</td>
<td>0.025</td>
<td>14</td>
<td>0.04</td>
<td>4.258</td>
<td>-5.094</td>
<td>4.586</td>
<td>0.00001</td>
<td>0.8</td>
</tr>
<tr>
<td>G6</td>
<td>13</td>
<td>0</td>
<td>40</td>
<td>0.025</td>
<td>15</td>
<td>0.039</td>
<td>2.754</td>
<td>-5.847</td>
<td>5.238</td>
<td>0.0004</td>
<td>0.288</td>
</tr>
<tr>
<td>G7</td>
<td>12</td>
<td>0</td>
<td>20</td>
<td>0.0322581</td>
<td>12</td>
<td>0.045</td>
<td>5.326</td>
<td>-3.555</td>
<td>3.38</td>
<td>0.002</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Case 10: Minimization of Fuel Cost

The basic objective function of this case is to optimize the fuel cost function given by Equation 5.11. The results are provided in Table 5.8, as considered in Case 1. Figure 5.12 shows the minimization of the fuel cost function with respect to the number of evaluations. The optimization process leads to a fuel cost of 41815 $/hr while satisfying all system constraints.

Figure 5.12: Convergence of Case 10 for IEEE 57-Bus System
Case 11: Minimization of Emission

Similar to Case 2, the focus of this case study is on minimizing the total emissions in the OPF optimization problem, defined by Equation 5.12. The results of the optimization procedure by the TLBO algorithm are presented in Table 5.8. In addition, Figure 5.13 shows the minimization of emission function where the optimization procedure provides the best output of 1.0902 ton/hr.

![Figure 5.13: Convergence of Case 11 for IEEE 57-Bus System](image-url)
Case 12: Minimization of Voltage Deviation

This case discusses the same objectives as Case 4. The objective function is given by Equation 5.13. Table 5.8 provides information related to this case. Figure 5.14 illustrates the convergence of function to the optimal solution of 0.7173 p.u after 100 times evaluations by using the TLBO method with MATLAB. Table 5.8 provided more details about the results of implementing this algorithm in this case study.

![Figure 5.14: Convergence of Case 12 for IEEE 57-Bus System](image)

Figure 5.14: Convergence of Case 12 for IEEE 57-Bus System
Case 13: Minimization of Active Power Loss

The aim of this case is to minimize the active power loss in transmission lines, as defined by Equation 5.14. Table 5.7 provides the coefficient and Table 5.8 shows the optimization results. Figure 5.13 shows the optimization procedure results in the real power loss of 13.3375 MW in transmission lines. Table 5.8 provides the details on minimization results.

Figure 5.15: Convergence of Case 13 for IEEE 57-Bus System
Case 14: Two Objectives: Minimization of Fuel Cost and Emission

The goal here is to minimize the fuel cost and to improve the emission function of the IEEE 57-test system at the same time. Thus, the objective function is the same as Case 5, given by Equation 5.15 where $\lambda_E$ is chosen as 1000. Figure 5.16 shows the convergence of fitness function and Table 5.9 presents the optimized solution of this multi-objective optimization problem. For clarity, the convergence of only one objective attaining the best fitness value is included in the fitness diagram for all multi-objective case studies. The best fuel cost value arrives at an increase in cost than Case 10, with final value being 41978 $/hr and for emission is 1.5153 ton/hr after 100 evaluations.

![Convergence of Case 14 for IEEE 57-Bus System](image)

Figure 5.16: Convergence of Case 14 for IEEE 57-Bus System
Case 15: Two Objectives: Minimization of Fuel Cost and Voltage Deviation

As in Case 6, this case is a multi-objective optimization problem. The main purposes of the problem concern minimization of the fuel cost and voltage profile enhancement, as given by Equation 5.16, where $\lambda_{VD}$ is chosen as 100. Table 5.9 summarizes the results of implementing multi-objective optimization using the TLBO algorithm with MATLAB and Figure 5.17 presents the convergence of fitness function in this case study.

![Figure 5.17: Convergence of Case 15 for IEEE 57-Bus System](image-url)
Case 16: Two Objectives: Minimization of Fuel Cost and Real Power Loss

Similar to Case 7, the goal focuses on the minimization of total fuel cost while monitoring the real power loss. The multi-objective problem is converted to a single function by multiplying weight factors and is used to minimize both objective functions simultaneously according to Equation 5.17. Figure 5.18 shows the convergence of fitness function for a multi-objective optimization of OPF where the fuel cost reaches 41857 $/hr and real power losses are 15.9728 MW. Table 5.9 summarizes the results of implementing the TLBO algorithm with MATLAB in this case study.

![Figure 5.18: Convergence of Case 16 for IEEE 57-Bus System](image-url)
Case 17: Three Objectives: Minimization of Fuel Cost, Emission, Losses

This case is similar to Case 8. It presents the compromise solution over the optimization of three objective functions consisting of total fuel cost, emission and real power losses at the same time. The problem formulation is given by Equation 5.18.

Figure 5.19 illustrates the convergence of the fitness function and Table 5.9 shows the optimized values with respect to this multi-objective optimization case. The weight factors are selected to balance between the objectives as 19 and 22 for emission and losses, respectively [32]. The fitness function convergence revealed the best values of the function to be 41900 $/hr, 2.0562 ton/hr, 16.1162MW for fuel cost, emission and losses respectively.

Figure 5.19: Fitness Function Convergence of Case 17 for IEEE 57-Bus System
Case 18: Four Objectives: Fuel Cost, Emission, Voltage Deviations and Losses

As explained in Case 9, the multi-objective case for minimizing fuel cost, emission, and real power losses is converted to single objective function by multiplying a weight factor to the objectives as expressed by Equations 5.19 and 5.20. The weight factors are selected to balance between the objectives as 19, 21 and 22, for emission, voltage deviation, and losses, respectively [32]. Table 5.9 summarizes the results of implementing the TLBO optimization algorithm by using MATLAB in this case study. Figure 5.20 illustrates the convergence of fitness function, while all of the four objectives are considered in the multi-objective optimization procedure.

Figure 5.20: Fitness Function Convergence of Case 18 for IEEE 57-Bus System
Table 5.8: Simulation Results of Optimum Solution for IEEE 57-Bus System

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Min</th>
<th>Max</th>
<th>Case 10</th>
<th>Case 11</th>
<th>Case 13</th>
<th>Case 14</th>
</tr>
</thead>
<tbody>
<tr>
<td>PG2 (MW)</td>
<td>30</td>
<td>100</td>
<td>98.1603</td>
<td>99.445</td>
<td>42.3358</td>
<td>10.5523</td>
</tr>
<tr>
<td>PG3 (MW)</td>
<td>40</td>
<td>140</td>
<td>49.1818</td>
<td>140</td>
<td>117.2235</td>
<td>82.5886</td>
</tr>
<tr>
<td>PG6 (MW)</td>
<td>30</td>
<td>100</td>
<td>69.8870</td>
<td>100</td>
<td>47.2586</td>
<td>98.2577</td>
</tr>
<tr>
<td>PG8 (MW)</td>
<td>100</td>
<td>550</td>
<td>456.7591</td>
<td>302.6981</td>
<td>281.0804</td>
<td>339.6432</td>
</tr>
<tr>
<td>PG9 (MW)</td>
<td>30</td>
<td>100</td>
<td>87.0398</td>
<td>100</td>
<td>59.5569</td>
<td>99.4843</td>
</tr>
<tr>
<td>PG12 (MW)</td>
<td>100</td>
<td>410</td>
<td>364.9578</td>
<td>293.3467</td>
<td>302.397</td>
<td>409.1274</td>
</tr>
<tr>
<td>V1 (p.u.)</td>
<td>0.95</td>
<td>1.1</td>
<td>0.9852</td>
<td>1.0327</td>
<td>1.0232</td>
<td>0.9665</td>
</tr>
<tr>
<td>V2 (p.u.)</td>
<td>0.95</td>
<td>1.1</td>
<td>0.9796</td>
<td>1.0212</td>
<td>1.0081</td>
<td>0.9574</td>
</tr>
<tr>
<td>V3 (p.u.)</td>
<td>0.95</td>
<td>1.1</td>
<td>0.9641</td>
<td>1.0073</td>
<td>1.0123</td>
<td>0.9574</td>
</tr>
<tr>
<td>V6 (p.u.)</td>
<td>0.95</td>
<td>1.1</td>
<td>0.964</td>
<td>0.9887</td>
<td>1.0072</td>
<td>0.9621</td>
</tr>
<tr>
<td>V8 (p.u.)</td>
<td>0.95</td>
<td>1.1</td>
<td>0.9775</td>
<td>0.981</td>
<td>1.027</td>
<td>0.9606</td>
</tr>
<tr>
<td>V9 (p.u.)</td>
<td>0.95</td>
<td>1.1</td>
<td>0.9514</td>
<td>0.9649</td>
<td>1.0051</td>
<td>0.9511</td>
</tr>
<tr>
<td>V12 (p.u.)</td>
<td>0.95</td>
<td>1.1</td>
<td>0.9633</td>
<td>0.9865</td>
<td>1.0219</td>
<td>0.988</td>
</tr>
<tr>
<td>Qc18 (p.u.)</td>
<td>0</td>
<td>20</td>
<td>8.48</td>
<td>14.07</td>
<td>7.14</td>
<td>1.6</td>
</tr>
<tr>
<td>Qc25 (p.u.)</td>
<td>0</td>
<td>20</td>
<td>22.99</td>
<td>25.46</td>
<td>12.02</td>
<td>19.41</td>
</tr>
<tr>
<td>Qc53 (p.u.)</td>
<td>0</td>
<td>20</td>
<td>7.77</td>
<td>0.07</td>
<td>12.05</td>
<td>5.76</td>
</tr>
<tr>
<td>T19 (p.u.)</td>
<td>0</td>
<td>20</td>
<td>0.9225</td>
<td>1.0895</td>
<td>1.1</td>
<td>0.9504</td>
</tr>
<tr>
<td>T20 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9025</td>
<td>1.0976</td>
<td>0.9191</td>
<td>0.9002</td>
</tr>
<tr>
<td>T31 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>1.0323</td>
<td>0.9005</td>
<td>1.0151</td>
<td>0.9293</td>
</tr>
<tr>
<td>T35 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>1.0288</td>
<td>1.0344</td>
<td>1.0003</td>
<td>0.9597</td>
</tr>
<tr>
<td>T36 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9251</td>
<td>0.9906</td>
<td>1.0395</td>
<td>1.0342</td>
</tr>
<tr>
<td>T37 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9638</td>
<td>0.9443</td>
<td>1.0243</td>
<td>0.9896</td>
</tr>
<tr>
<td>T41 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9431</td>
<td>0.9472</td>
<td>0.9604</td>
<td>0.9006</td>
</tr>
<tr>
<td>T46 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9737</td>
<td>0.9797</td>
<td>0.9183</td>
<td>0.9836</td>
</tr>
<tr>
<td>T54 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9106</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9002</td>
</tr>
<tr>
<td>T58 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9195</td>
<td>0.9255</td>
<td>0.9475</td>
<td>0.9009</td>
</tr>
<tr>
<td>T59 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9152</td>
<td>0.9226</td>
<td>0.9871</td>
<td>0.9094</td>
</tr>
<tr>
<td>T65 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9246</td>
<td>0.9809</td>
<td>0.9887</td>
<td>0.9</td>
</tr>
<tr>
<td>T66 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9088</td>
<td>0.9299</td>
<td>0.9006</td>
<td>0.9</td>
</tr>
<tr>
<td>T71 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9372</td>
<td>0.9299</td>
<td>0.9302</td>
<td>0.9003</td>
</tr>
<tr>
<td>T73 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9985</td>
<td>1.0069</td>
<td>1.0002</td>
<td>0.9002</td>
</tr>
<tr>
<td>T76 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9026</td>
<td>1.0224</td>
<td>0.9787</td>
<td>0.9841</td>
</tr>
<tr>
<td>T80 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.956</td>
<td>0.9439</td>
<td>1.0072</td>
<td>0.9</td>
</tr>
<tr>
<td>Fuel Cost ($/h)</td>
<td>41815</td>
<td>45701</td>
<td>50870</td>
<td>43223</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Emission (ton/h)</td>
<td>1.892</td>
<td>1.0902</td>
<td>1.6807</td>
<td>1.5065</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Voltage Deviation (p.u.)</td>
<td>1.5192</td>
<td>1.2582</td>
<td>0.7173</td>
<td>1.4022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Active Power Loss (MW)</td>
<td>17.9115</td>
<td>19.816</td>
<td>25.922</td>
<td>13.3375</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.21: Simulation Results of Optimum Solution for IEEE 57-Bus System Single-objective Cases
Table 5.9: Simulation Results of Optimum Solution for IEEE-57 Bus System, Multi-objective Cases

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Min</th>
<th>Max</th>
<th>Case 14</th>
<th>Case 15</th>
<th>Case 16</th>
<th>Case 17</th>
<th>Case 18</th>
</tr>
</thead>
<tbody>
<tr>
<td>PG2 (MW)</td>
<td>30</td>
<td>100</td>
<td>99.3452</td>
<td>97.6138</td>
<td>81.731</td>
<td>83.8276</td>
<td>66.1226</td>
</tr>
<tr>
<td>PG3 (MW)</td>
<td>40</td>
<td>140</td>
<td>51.6008</td>
<td>43.9625</td>
<td>46.9343</td>
<td>49.4303</td>
<td>96.5192</td>
</tr>
<tr>
<td>PG6 (MW)</td>
<td>30</td>
<td>100</td>
<td>88.7318</td>
<td>59.4725</td>
<td>64.3148</td>
<td>43.9851</td>
<td>56.9695</td>
</tr>
<tr>
<td>PG8 (MW)</td>
<td>100</td>
<td>550</td>
<td>419.6951</td>
<td>464.0246</td>
<td>442.9978</td>
<td>458.6195</td>
<td>455.0092</td>
</tr>
<tr>
<td>PG9 (MW)</td>
<td>30</td>
<td>100</td>
<td>97.8437</td>
<td>81.7688</td>
<td>86.7345</td>
<td>78.7001</td>
<td>62.1057</td>
</tr>
<tr>
<td>PG12 (MW)</td>
<td>100</td>
<td>410</td>
<td>363.5859</td>
<td>374.0378</td>
<td>399.729</td>
<td>410</td>
<td>357.1104</td>
</tr>
<tr>
<td>V1 (p.u.)</td>
<td>0.95</td>
<td>1.1</td>
<td>0.95</td>
<td>0.9869</td>
<td>0.9648</td>
<td>0.9737</td>
<td>0.9992</td>
</tr>
<tr>
<td>V2 (p.u.)</td>
<td>0.95</td>
<td>1.1</td>
<td>0.9423</td>
<td>0.9803</td>
<td>0.9598</td>
<td>0.9684</td>
<td>0.9891</td>
</tr>
<tr>
<td>V3 (p.u.)</td>
<td>0.95</td>
<td>1.1</td>
<td>0.9405</td>
<td>0.966</td>
<td>0.9509</td>
<td>0.9579</td>
<td>1.0001</td>
</tr>
<tr>
<td>V6 (p.u.)</td>
<td>0.95</td>
<td>1.1</td>
<td>0.9566</td>
<td>0.9668</td>
<td>0.9508</td>
<td>0.9684</td>
<td>1.0051</td>
</tr>
<tr>
<td>V8 (p.u.)</td>
<td>0.95</td>
<td>1.1</td>
<td>0.9781</td>
<td>0.9878</td>
<td>0.9695</td>
<td>0.9802</td>
<td>1.0191</td>
</tr>
<tr>
<td>V9 (p.u.)</td>
<td>0.95</td>
<td>1.1</td>
<td>0.9548</td>
<td>0.9639</td>
<td>0.9496</td>
<td>0.9528</td>
<td>0.9881</td>
</tr>
<tr>
<td>V12 (p.u.)</td>
<td>0.95</td>
<td>1.1</td>
<td>0.974</td>
<td>0.9837</td>
<td>0.9744</td>
<td>0.966</td>
<td>0.9878</td>
</tr>
<tr>
<td>Qc18 (p.u.)</td>
<td>0</td>
<td>20</td>
<td>14.29</td>
<td>6.9</td>
<td>7.46</td>
<td>6.49</td>
<td>5.39</td>
</tr>
<tr>
<td>Qc25 (p.u.)</td>
<td>0</td>
<td>20</td>
<td>13.5</td>
<td>12.55</td>
<td>19.23</td>
<td>12</td>
<td>24.57</td>
</tr>
<tr>
<td>Qc53 (p.u.)</td>
<td>0</td>
<td>20</td>
<td>19.6</td>
<td>8.06</td>
<td>10.65</td>
<td>9.24</td>
<td>6.45</td>
</tr>
<tr>
<td>T19 (p.u.)</td>
<td>0</td>
<td>20</td>
<td>0.9002</td>
<td>0.9001</td>
<td>0.9</td>
<td>0.905</td>
<td>0.9724</td>
</tr>
<tr>
<td>T20 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9072</td>
<td>0.9982</td>
<td>0.9</td>
<td>0.9</td>
<td>1.0532</td>
</tr>
<tr>
<td>T31 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.97</td>
<td>0.968</td>
<td>0.9871</td>
<td>0.923</td>
<td>0.9526</td>
</tr>
<tr>
<td>T35 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9</td>
<td>1.0152</td>
<td>1.1</td>
<td>0.9076</td>
<td>1.0274</td>
</tr>
<tr>
<td>T36 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>1.0288</td>
<td>0.901</td>
<td>0.9012</td>
<td>0.9</td>
<td>0.907</td>
</tr>
<tr>
<td>T37 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>1.0054</td>
<td>0.997</td>
<td>0.9879</td>
<td>0.9098</td>
<td>0.9341</td>
</tr>
<tr>
<td>T41 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9013</td>
<td>0.9145</td>
<td>0.9</td>
<td>0.9066</td>
<td>0.9311</td>
</tr>
<tr>
<td>T46 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9489</td>
<td>0.9451</td>
<td>0.9188</td>
<td>0.941</td>
<td>0.9941</td>
</tr>
<tr>
<td>T54 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9</td>
<td>0.9452</td>
<td>0.9055</td>
<td>0.9013</td>
<td>0.9998</td>
</tr>
<tr>
<td>T58 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9024</td>
<td>0.9001</td>
<td>0.9126</td>
<td>0.9</td>
<td>0.9121</td>
</tr>
<tr>
<td>T59 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9008</td>
<td>0.913</td>
<td>0.9254</td>
<td>0.9151</td>
<td>0.9712</td>
</tr>
<tr>
<td>T65 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9063</td>
<td>0.9299</td>
<td>0.9074</td>
<td>0.9</td>
<td>0.982</td>
</tr>
<tr>
<td>T66 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9002</td>
<td>0.901</td>
<td>0.9</td>
<td>0.9</td>
<td>0.9629</td>
</tr>
<tr>
<td>T71 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9003</td>
<td>0.9041</td>
<td>0.9031</td>
<td>0.9126</td>
<td>0.9063</td>
</tr>
<tr>
<td>T73 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>1.0267</td>
<td>0.931</td>
<td>0.9857</td>
<td>0.9957</td>
<td>0.9807</td>
</tr>
<tr>
<td>T76 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9048</td>
<td>0.9382</td>
<td>0.9405</td>
<td>0.9</td>
<td>0.9008</td>
</tr>
<tr>
<td>T80 (p.u.)</td>
<td>0.9</td>
<td>1.1</td>
<td>0.9</td>
<td>0.9366</td>
<td>0.9013</td>
<td>0.9</td>
<td>0.999</td>
</tr>
</tbody>
</table>

Fuel Cost ($/h)      | 41875| 41852| 41857| 41900| 42583|
Emission (ton/h)      | 1.5821| 1.9748| 1.8674| 2.0562| 1.879|
Voltage Deviation (p.u.) | 1.5599| 1.426| 1.5042| 1.78| 1.5491|
Active Power Loss (MW)| 17.1518| 18.5264| 15.9728| 16.1162| 17.3545|
Active Power Loss (p.u.) | 17.1518| 18.5264| 15.9728| 16.1162| 17.3545|
Figure 5.22: Simulation Results of Optimum Solution for IEEE-57 Bus System Multi-objective Cases
5.3 Comparative Study

The results reported in this thesis are compared to some other Artificial Intelligent (AI) methods under the same control variable limits, constraints, and system data for two case studies. It appears from Tables 5.10 and 5.11 that the proposed method outperforms many techniques used to solve different OPF problems. The results are either better or comparable to those obtained by other techniques. This shows the efficiency of the TLBO method for finding the optimum solution.

<table>
<thead>
<tr>
<th>Method</th>
<th>Cost ($/hr)</th>
<th>Method Description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>GSA</td>
<td>798.67</td>
<td>Gravitational Search Algorithm</td>
<td>[45]</td>
</tr>
<tr>
<td>DSA</td>
<td>799.09</td>
<td>Differential Search Algorithm</td>
<td>[46]</td>
</tr>
<tr>
<td>SA</td>
<td>799.45</td>
<td>Simulated Annealing</td>
<td>[47]</td>
</tr>
<tr>
<td>DE</td>
<td>799.28</td>
<td>Differential Evolution</td>
<td>[48]</td>
</tr>
<tr>
<td>TLBO</td>
<td>800.48</td>
<td>Teaching-Learning-Based Optimization</td>
<td></td>
</tr>
<tr>
<td>FPSO</td>
<td>800.72</td>
<td>Fuzzy Particle Swarm Optimization</td>
<td>[49]</td>
</tr>
<tr>
<td>IGA</td>
<td>800.80</td>
<td>Improved Genetic Algorithm</td>
<td>[50]</td>
</tr>
<tr>
<td>PSO</td>
<td>800.96</td>
<td>Particle Swarm Optimization</td>
<td>[49]</td>
</tr>
<tr>
<td>GAF</td>
<td>801.21</td>
<td>Fuzzy Genetic Algorithm</td>
<td>[49]</td>
</tr>
<tr>
<td>ICA</td>
<td>801.84</td>
<td>Imperialistic Competitive Algorithm</td>
<td>[51]</td>
</tr>
<tr>
<td>EGA</td>
<td>801.06</td>
<td>Enhanced Genetic Algorithm</td>
<td>[52]</td>
</tr>
<tr>
<td>TS</td>
<td>802.29</td>
<td>Tabu Search</td>
<td>[53]</td>
</tr>
<tr>
<td>MDE</td>
<td>802.37</td>
<td>Modified Differential Evolutionary Algorithm</td>
<td>[54]</td>
</tr>
<tr>
<td>IEP</td>
<td>802.46</td>
<td>Improved Evolutionary Programming</td>
<td>[55]</td>
</tr>
<tr>
<td>EP</td>
<td>802.62</td>
<td>Evolutionary Programming</td>
<td>[56]</td>
</tr>
<tr>
<td>RGA</td>
<td>804.02</td>
<td>Refined Genetic Algorithm</td>
<td>[57]</td>
</tr>
<tr>
<td>GM</td>
<td>804.85</td>
<td>Gradient Method</td>
<td>[58]</td>
</tr>
<tr>
<td>GA</td>
<td>805.94</td>
<td>Genetic Algorithm</td>
<td>[57]</td>
</tr>
</tbody>
</table>
Table 5.11: Comparison of the Simulation Results in Multi-Objective Optimization of Fuel Cost and Voltage Deviation, Case 5, IEEE30-Bus System

<table>
<thead>
<tr>
<th>Method</th>
<th>Cost ($/hr)</th>
<th>Voltage Deviation(p.u.)</th>
<th>Objective Function</th>
<th>Method Description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>TLBO</td>
<td>803.8268</td>
<td>0.09750</td>
<td>813.5786</td>
<td>Teaching-Learning-Based Optimization</td>
<td></td>
</tr>
<tr>
<td>DSA</td>
<td>803.8274</td>
<td>0.09770</td>
<td>813.5961</td>
<td>Differential Search Algorithm</td>
<td>[46]</td>
</tr>
<tr>
<td>GSA</td>
<td>804.3148</td>
<td>0.09330</td>
<td>813.6417</td>
<td>Gravitational Search Algorithm</td>
<td>[45]</td>
</tr>
<tr>
<td>BBO</td>
<td>804.9982</td>
<td>0.10400</td>
<td>815.1982</td>
<td>Biogeography-Base Optimization</td>
<td>[59]</td>
</tr>
<tr>
<td>PSO</td>
<td>806.3800</td>
<td>0.08910</td>
<td>815.2900</td>
<td>Particle Swarm Optimization</td>
<td>[49]</td>
</tr>
<tr>
<td>EM</td>
<td>804.2600</td>
<td>0.12700</td>
<td>816.9600</td>
<td>Electromagnetism-Like Mechanism</td>
<td>[60]</td>
</tr>
<tr>
<td>DE</td>
<td>805.2619</td>
<td>0.13670</td>
<td>818.8319</td>
<td>Differential Evolution</td>
<td>[48]</td>
</tr>
</tbody>
</table>
5.4 Conclusions

This chapter has presented the application of multi-objective optimization in solving the Optimal Power Flow (OPF) problem. The objective functions are: fuel cost function, emission, voltage deviation, and transmission losses. The Teaching-Learning-Based Optimization (TLBO) algorithm has been successfully and effectively implemented to the IEEE 30-bus and IEEE 57-bus power system for single and multi-objective cases to reflect the performance of the power system. Fitness function gradually reduces in all the minimization cases. It is worthwhile to mention that unlike the other optimization algorithm, TLBO does not require any internal parameter to tune.

Evidently, the OPF fitness function gradually reduces in all optimization cases, despite some irregularities occurring during the process of convergence due to the non-linear relationship of the objective functions with control parameters of the network. A comparative study is provided to highlight the efficiency of the algorithm in comparison with other artificial intelligence methods.
Chapter 6

6. Conclusion and Future Work

The primary purpose of this thesis is to present the general concepts and classifications of multi-objective optimization methods and draw the focus to its applications on power systems. The optimal power flow problem is a non-linear, constrained optimization problem with several objective functions. Multi-objective optimization has been implemented to optimize the OPF problem for different objective functions considering fuel cost, emission, voltage deviation, and power losses.

The Teaching-Learning-Based Optimization (TLBO) algorithm is a popular method among metaheuristic techniques with relatively competitive performances. It is inspired by passing the knowledge from teacher to learners or by the interaction between the learners themselves. TLBO is employed to optimize the OPF problem by using MATLAB for single and multi-objective OPF case studies. Simulation results and comparisons highlight the effectiveness of the method for solving multi-objective OPF problems and reflect the performance of the power system. This thesis has considered four key problems for power systems: economy, environmental emission, voltage deviation, and transmission losses. The outcomes of the presented research reflect the effectiveness of approaches and potential benefits for power system planning and operation.
6.1 Summary of the Research and Contribution of Thesis

- Application of multi-objective optimization in solving the optimal power flow problem has been presented.
- The Teaching-Learning-Based Optimization (TLBO) algorithm has been successfully and effectively implemented to standard case studies for single and multi-objective cases.
- Unlike other optimization algorithms, TLBO does not require any internal parameter to tune.
- The OPF fitness function for all presented case studies gradually reduces using TLBO method, despite some irregularities in the process of convergence, which arises due to the nonlinear relationship between the objective functions and control parameters of the network.
- The efficiency of TLBO method is shown with a comparative study with other intelligent methods for a single objective and a multi-objective case study in this thesis.
- The ability of TLBO to provide the global optimum solutions in comparatively less number of function evaluations can be studied in OPF for future works.

6.2 Recommendations of Future Work

Some of the potential research areas for improving the performance of the algorithm in the future is presented in this section.

- The concepts of learning through tutorials and self-motivated leaning can be studied in future research.
- Using the adaptive teaching factor for TLBO is a new discussion for improving the performance of the algorithm. The adaptive teaching factor allows the students to acquire partial knowledge from the instructor where as in the basic TLBO the students could learn either everything or nothing from the presented topic [61].
- Number of teachers on the fitness value of the objective function could be increased to effect the efficiency of the algorithm.
- TLBO with a variable-population scheme can be studies for optimization problems.
• Experimentation of TLBO on very large-scale systems is another interesting area for future studies in order to show the capability and efficiency of the system for handling a system with large number of variables and constraints.
• Using an adaptive penalty factor for system constraints in solving multi-objective optimal
References


