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Coefficients Calculation for the EPP FPV R/C Aircraft

TECHNICAL REPORT

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Summary

In aviation, the moments of inertia estimate how quickly an aircraft rotates. There are three moments of inertia, each along an aircraft axis (e.g. X-Y-Z). The aerodynamic coefficients characterize the forces and moments acting on an aircraft during flight. They are non-dimensional and facilitate the aerodynamic forces and moments calculation [1].

This technical report includes the calculations carried out to obtain the mass moment of inertia matrix and the aerodynamic coefficients that will serve as a base for the EPP FPV aircraft configuration file in JSBSim. However, these parameters can be used for the development of any EPP FPV computer model under any flight simulator.

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1. The EPP FPV

The EPP FPV vehicle is an Expanded Polypropylene (EPP) foam UAV manufactured and sold by HobbyKing.com [2]. It is primarily designed for First Person View (FPV) camera flight. The expanded polypropylene foam makes it light (perfect for gliding) and robust during landings and crashes, making it perfect for beginners. The thrust is provided by a propeller driven by a DC electric motor. The propulsion system is located in the rear of the fuselage to allow for electronic equipment and batteries installation in the front of the aircraft.

Figure 1. The EPP FPV [2]

2. Inertia Coefficients

The moment of inertia is a rotational inertial measurement or the tendency of a body to resist the changes on its rotational movement. Although the moment of inertia must be constant for a rigid body, it is always around a particular axis and can hold different values for other axes. The moment of inertia also depends on the mass distribution with respect to the rotational axis. The moment of inertia of a continuously distributed mass is:

$$I = \int r^2 dm \tag{1}$$

Where *dm* is the differential mass of the body and r^2 is the minimum distance to the rotational axis. Assuming that the even distribution of the mass is unknown, the mass can be substituted by the density times the volume. Then, the inertia based on the volume is calculated by:

$$I = \int r^2 \rho dV \quad (2)$$

Where ρ is the volumetric density expressed by mass/volume and dV is the differential volume.

If the body is composed of a series of geometric elements, the moment of inertia is calculated by (1st) splitting each of those elements, (2nd) calculating their moment of inertia, and (3rd) adjusting the moment of inertia by knowing the distance between the element's Centre of Mass (CM) and the system's CM. The EPP FPV aircraft is formed approximately by the following series of geometries in the corresponding projection plane:

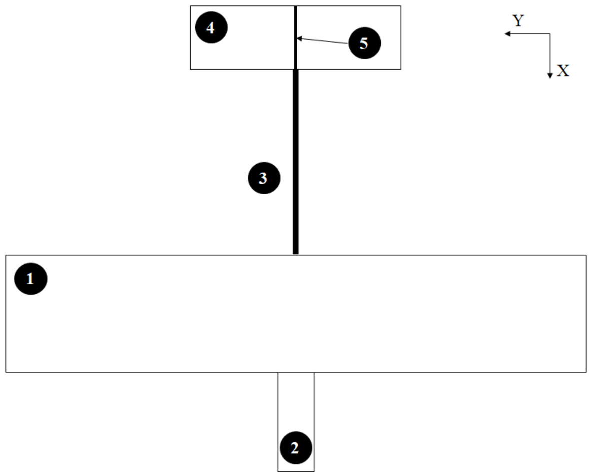

Figure 2. Top view of the UAV

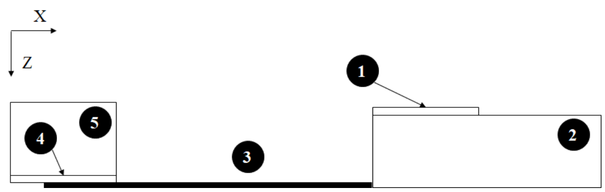

Figure 3. Side view of the UAV

Table 1. List of simple geometries in the EPP FPV

Element #	Fuselage/aerosurface name
1	Wings rectangle
2	Fuselage rectangle
3	Tube of the fuselage rectangle
4	Elevator stabilizer rectangle
5	Vertical fin rectangle

Additionally, the Steiner's Theorem (commonly known as the Parallel Axis Theorem) establishes that the moment of inertia, with respect to any axis parallel to an axis that crosses the CM, is equal to the moment of inertia of the axis that crosses the CM plus the product of the mass by the square of the distance between both axes:

$$I_{axis} = I_{axis}^{(CM)} + Mh^2 \quad (3)$$

Where I_{axis} is the moment of inertia of the parallel axis, $I_{axis}^{(CM)}$ is the moment of inertia of the axis that crosses the CM and is parallel to the first one, *M* is the mass, and *h* is the distance between both axes.

2.1. Moment of Inertia around X Axis (I_X)

Due to the aircraft symmetry along the X axis, each CM of the geometric figures belongs to that X axis (Figure 2). This means that the moments of inertia of each of the geometries around their CM are the same moments on the X axis. Therefore, the moment of inertia around the X axis is the sum of the moments of inertia generated by all the elements in the aircraft:

Table 2. Moment of inertia around X axis

Element #	Fuselage/aerosurface name	$I_X^{(CM)}$ (kg m^2)
1	Wings rectangle	0.048811653
2	Fuselage rectangle	0.000667668
3	Tube of the fuselage rectangle	7.16102E-06
4	Elevator stabilizer rectangle	0.000854486
5	Vertical fin rectangle	1.38669E-07
		$I_{XX} = \sum I_X^{(CM)} = 0.050341107$

2.2. Moment of Inertia around Y Axis (I_Y)

The EPP FPV UAV is not symmetric around the Y axis and, therefore, the moment of inertia is calculated by following the parallel axis theorem in two steps: (1st) calculating the moment of inertia around the Y axis of the corresponding element and (2nd) applying the theorem of the parallel axes (equation 3).

Based on the geometry of the aircraft in Figure 3, the moments of inertia around their own CM and the adjusted moment of inertia around the aircraft CM are:

Table 3. Moment of inertia around Y axis

Element #	Fuselage/aerosurface name	$I_Y^{(CM)}$ (kg m^2)	I_Y (kg m^2)
1	Wings rectangle	0.0007292	0.000883194
2	Fuselage rectangle	0.0080239	0.009035895
3	Tube of the fuselage rectangle	0.0059711	0.00700636
4	Elevator stabilizer rectangle	5.148E-05	7.02006E-05
5	Vertical fin rectangle	1.123E-05	1.71092E-05
			$I_{YY} = \sum I_Y = 0.017012758$

2.3. Moment of Inertia around Z Axis (I_Z)

The aircraft view for the calculation of the moment of inertia around the Z axis is the projection on the YZ plane. The measurements needed for the calculation of I_Z can be extracted from the other two projections in Figure 2 and Figure 3. Since the aircraft is not symmetric in the Z axis, the Steiner's Theorem is required (equation 3). Similarly, as calculated for I_{YY}, I_{ZZ} is obtained in two steps: (1st) calculate $I_Z^{(CM)}$ for each of the elements in the aircraft and (2nd) adjust I_Z for each element by using the parallel axis theorem:

Table 4. Moment of inertia around Z axis

Element #	Fuselage/aerosurface name	$I_Z^{(CM)}$ (kg m^2)	I_Z (kg m^2)
1	Wings rectangle	0.049540815	0.050101808
2	Fuselage rectangle	0.008691551	0.023759408
3	Tube of the fuselage rectangle	0.005971099	0.029063534
4	Elevator stabilizer rectangle	0.000905967	0.021889738
5	Vertical fin rectangle	1.13709E-05	0.004589648
			$I_{ZZ} = \sum I_Z = 0.129404136$

2.4. Product moment of inertia

By definition, the product moment of inertia of a system with respect to two planes, is the sum of the products of all the masses of the system times the distance to each of the planes. Assuming that the product moment of inertia is between the axis X and Y, then I_{XY} (or I_{YX}) is:

$$I_{XY} = I_{YX} = \sum_{i=1}^{n} m_i\, x_i\, y_i \qquad (4)$$

Where m_i is the mass of each element in the system and x_i and y_i are the distance to the planes X and Y respectively.

Similarly, $I_{XZ} = I_{ZX}$ and $I_{YZ} = I_{ZY}$ are:

$$I_{XZ} = I_{ZX} = \sum_{i=1}^{n} m_i\, x_i\, z_i \qquad (5)$$

$$I_{YZ} = I_{ZY} = \sum_{i=1}^{n} m_i\, y_i\, z_i \qquad (6)$$

Based on equation 4, I_{XY} is 0 since the aircraft is symmetric in that plane and the aircraft has its opposite in the other side of the axis, making the total sum equal to 0. The aircraft is also symmetric in the YZ plane and, therefore, $I_{YZ} = I_{ZY} = 0$.

However, there is no symmetry on the XZ plane as it is observed from Figure 3. The product of inertia for this case is calculated by using equation 5, and knowing the distance between each of the CMs and the corresponding X and Z axes:

Table 5. Product moment of inertia in XZ

Element #	Fuselage/aerosurface name	I_{XZ} (kg m^2)
1	Wings rectangle	-0.000321188
2	Fuselage rectangle	0.003813103
3	Tube of the fuselage rectangle	0.004833909
4	Elevator stabilizer rectangle	0.000622633
5	Vertical fin rectangle	0.000162956
		$I_{XZ} = \sum I_{XZ} = 0.009111413$

Finally, the matrix moment of inertia is built with all the moments of inertia around their own axis in the diagonal and the products of inertia in the off-diagonal elements:

$$\begin{bmatrix} I_{XX} & I_{XY} & I_{XZ} \\ I_{YX} & I_{YY} & I_{YZ} \\ I_{ZX} & I_{ZY} & I_{ZZ} \end{bmatrix} = \begin{bmatrix} 0.0503411 & 0 & 0.00911141 \\ 0 & 0.017012758 & 0 \\ 0.00911141 & 0 & 0.12940414 \end{bmatrix} \qquad (7)$$

3. Aerodynamic coefficients

Contrary to the moments of inertia, which were calculated from physical elements on the aircraft, the aerodynamic coefficients require wind tunnel experiments or similar. A common practice is to use the aerodynamic coefficients from known aircraft that have similar structure and response.

The mini SGS-126 (Figure 4) is a glider similar in shape to the EPP FPV. The aerodynamic responses of both are similar and, therefore, the aerodynamic coefficients of the mini SGS-126 were used as a basis for the calculation of the EPP FPV parameters.

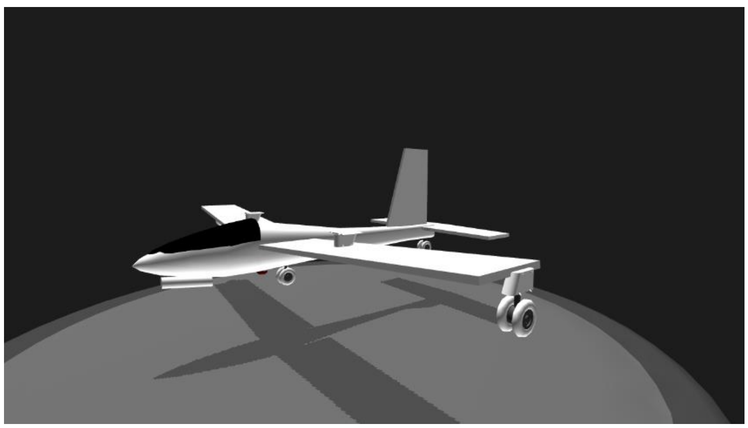

Figure 4. Mini SGS-126 Glider [3]

Considering the original aerodynamic coefficients from the mini SGS-126 [4] and the validation procedure initially proposed in [5], the following are the EPP FPV aerodynamic coefficients:

Table 6. EPP FPV drag force aerodynamic coefficients

		DRAG					
C_{D0}	Drag at zero lift			0.007			
C_D^{α}	Drag due to α	α	-0.0175	0.0	…	1.3963	1.5708
		C_D^{α}	0.01	0.015	…	1.5	1.46
$C_D^{\delta^e}$	Drag due to elevator deflection	Elevator	-1.0		0.0		1.0
		$C_D^{\delta^e}$	0.114		0.0		0.114

Table 7. EPP FPV side force aerodynamic coefficients

SIDE		
C_Y^{β}	Side force due to β	-0.83
$C_Y^{\delta a}$	Side force due to aileron deflection	-0.0456
$C_Y^{\delta r}$	Side force due to rudder deflection	0.1880

Table 8. EPP FPV lift force aerodynamic coefficients

LIFT							
C_L^{α}	Drag due to α	α	-0.1571	-0.1369	…	1.369	1.5708
		C_D^{α}	0.0	0.06	…	0.26	0.03
$C_L^{\delta e}$	Drag due to elevator deflection	-0.3420					

Table 9. EPP FPV roll moment aerodynamic coefficients

ROLL		
C_l^{β}	Roll moment due to β	-0.0313
C_l^{p}	Roll moment due to roll rate	-0.4700
C_l^{r}	Roll moment due to yaw rate	0.1500
$C_l^{\delta a}$	Roll moment due to aileron deflection	0.2500
$C_l^{\delta r}$	Roll moment due to rudder deflection	-0.0046

Table 10. EPP FPV pitch moment aerodynamic coefficients

PITCH		
C_{m0}	Pitch moment at zero lift	0.102
C_m^{α}	Pitch moment due to α	-1.573
C_m^{q}	Pitch moment due to pitch rate	-9.0000
$C_m^{\dot{\alpha}}$	Pitch moment due to α rate	-5.2000
$C_m^{\delta e}$	Pitch moment due to elevator deflection	-1.2610

Table 11. EPP FPV yaw moment aerodynamic coefficients

	YAW	
C_n^{β}	Yaw moment due to β	0.0170
C_n^{p}	Yaw moment due to roll rate	-0.1800
C_n^{r}	Yaw moment due to yaw rate	-0.0250
$C_n^{\delta a}$	Yaw moment due to aileron deflection	0.0115
$C_n^{\delta r}$	Yaw moment due to rudder deflection	-0.0370

References

[1] M.V. Cook, *Flight Dynamics Principle*, 2nd ed. 2007.

[2] HobbyKing.com, “EPP-FPV Instructions.” [Online]. Available: https://hobbyking.com/media/file/163971972X102667X57.pdf. [Accessed: 21-Apr-2017].

[3] SimplePlanes, “SGS-126 N5702S Mini Glider.” [Online]. Available: https://www.simpleplanes.com/a/kpSAvF/SGS-126-N5702S-Mini-Glider. [Accessed: 17-Dec-2018].

[4] “JSBSim Flight Dynamics Model - Code aircraft/minisgs.” [Online]. Available: https://sourceforge.net/p/jsbsim/code/ci/master/tree/aircraft/minisgs/. [Accessed: 17-Dec-2018].

[5] O. Cereceda, L. Rolland, and S. O’Young, “Validation discussion of an Unmanned Aerial Vehicle (UAV) using JSBSim Flight Dynamics Model compared to MATLAB/Simulink AeroSim Blockset,” in *2016 IEEE International Conference on Systems, Man, and Cybernetics (SMC)*, 2016, pp. 3989–3994.