

**COPULA-BASED MODELS FOR RISK ANALYSIS OF PROCESS SYSTEMS  
WITH DEPENDENCIES**

by

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## Abstract

With the increasing integration of heat and mass and the complexity of process systems, process variables are becoming strongly interdependent. Ignoring these dependencies in process safety modelling is unreasonable. The present work addresses this dependency challenge. It proposes two simple yet robust risk models for process safety analysis.

The first model is the copula-based bow-tie (CBBT) model, which revises the traditional bow-tie (BT) model by considering dependencies among the causes and failures of safety barriers. Copulas are used to simulate hypothetical dependent joint probability densities. The proposed model, along with classical BT analysis, is examined under a case study of the risk analysis of a typical distillation column. Comparing the results from both approaches in terms of the estimated probability of a potential hexane release scenario, it is shown that the dependencies of process units' malfunctions can increase the likelihood of accident scenarios to a significant extent. Further, to explore the mechanisms behind the impact of such dependencies, the effect of dependencies on the two most basic logic gates is also analyzed.

The next model developed is the copula-based Bayesian network (CBBN), which integrates linear dependence modelled by a Bayesian network (BN) and non-linear dependence by copulas. It provides more reliable estimation of accident probability when applied to real cases. Sensitivity analysis identifies the factors that play important roles in causing an accident. A diagnostic analysis is also performed to find the most probable explanation for the occurred event. Results match the accident investigation report and thus prove the effectiveness of the proposed model.

*Key words:* Risk assessment; Bow-tie; Bayesian network; Dependence; Copula; Process safety; Accident model

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## **List of Abbreviations**

BN	Bayesian network
BPCS	Basic process control systems
BT	Bow-tie
CBBN	Copula-based Bayesian network
CBBT	Copula-based Bow-tie
CE	Conditioning event
CPT	Conditional probability tables
ET	Event tree
ETA	Event tree analysis
FMEA	Failure mode and effect analysis
FT	Fault tree
FTA	Fault tree analysis
HAZOP	Hazard and operability study
IE	Initiating event
MCS	Minimum cut set
OE	Outcome event
QRA	Quantitative risk analysis
SB	Safety barrier
SIF	Safety instrumented functions
SIS	Safety instrumented systems
SF	Safety function
TE	Top event

## **Co-authorship Statement**

For all the work presented in this thesis, I am the principal author. In the design stage, my supervisor identified the research gap to be filled, which helped me to write the research proposal. I reviewed the literature and developed two revised methodologies to overcome the limitations of the currently widely used risk analysis methodologies. I applied these methodologies to practical studies, obtained simulation data and then analyzed the results. In this procedure, Dr. Faisal Khan helped by offering suggestions towards the selection of specific research aspects, such as recommending me to perform sensitivity analysis and probability updating. He contributed to reviewing and approving the discussions of results as well. I prepared the draft of the manuscript and revised it based on the feedback from Drs. Faisal Khan and Syed Imtiaz.

## **Chapter 1. Introduction and Overview**

Complex process operations involving large inventories of hazardous materials have serious safety concerns. The loss of material in such facilities may lead to low-probability but high-consequence events (Pasman, 2015), such as significant economic loss, environmental damage or multiple fatalities or injuries. These concerns are quantified in terms of financial and personnel risk. Past major accidents, for example, Bhopal (1984), Piper Alpha (1988) and Buncefield (2005), have led to the establishment of process safety management regulations. While process safety management is effective, its full potential has not yet been reached. Also, as the complexity of operations is on the rise, accident causation is becoming more complex and harder to estimate and predict (Vaughen and Kletz, 2012). This situation underscores the need for better estimation of these accident scenarios, their likelihood, quantitative risk and subsequently better safety management practices, and many qualitative and quantitative analysis methods have been developed to meet this need.

### **1.1 Quantitative Risk Analysis**

In the past, qualitative analysis was widely used for the risk assessment of hazardous substances. However, one of its obvious drawbacks is its vagueness in terminology, such as the description “a high degree of protection” (Buncefield Major Investigation Board, 2008). On the other hand, Quantitative Risk Analysis (QRA) is easy to perform and is now widely applied because the computational burden has been lessened thanks to technological progress.

QRA was first used in nuclear plants. In the 1970s, the probabilistic risk assessment for the nuclear sector was developed by the United States Nuclear Regulatory Commission. It was only at a later stage that QRA was applied to chemical process safety management. In 2012, Seveso, the European industrial safety regulatory agency, issued its third generation of safety regulations (Seveso III directive) (EU, 2012), which apply to more than 10,000 industrial establishments, many of which are chemical plants (European Commission - Environment Directorate, 2015). As a widely-used approach, QRA has been adopted to facilitate the implementation of Seveso regulations (Pasman and Reniers, 2014).

The latest trend in the development in QRA has been towards dynamic risk analysis (Villa et al., 2016). Dynamic QRA makes use of newly available information on the process system such as accident precursors or alarm databases to continuously update the risk level. The steps involved in dynamic QRA are shown in Fig. 1.1. From this comprehensive perspective, dynamic QRA is considered a robust tool for hazard and risk quantification of a process facility.

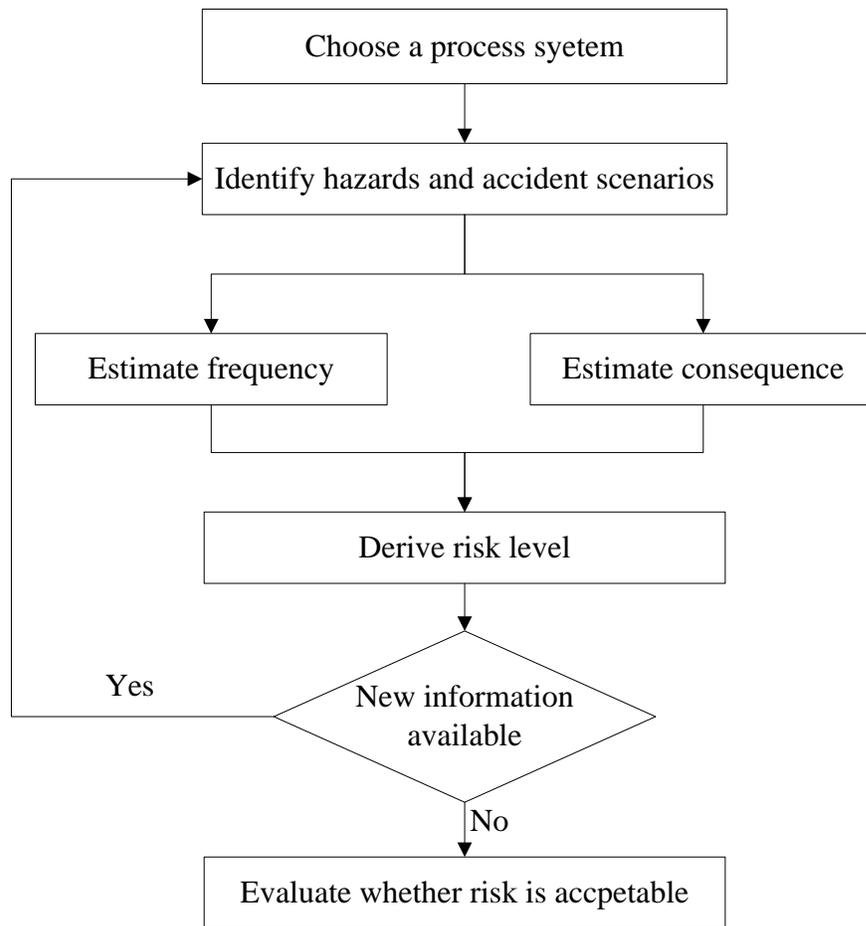


Figure 1.1 QRA steps adapted from Hashemi (2016).

Hazard and accident scenario identification is the very first step in the QRA technique, which involves a thorough review of potential abnormal situations, such as high temperature, overpressure, leakage, etc. These abnormal situations can escalate to accidents if related protection and mitigation systems fail. According to the CCPS (2001), the protection layers for process plants can be classified as belonging to the following categories: the process design itself, basic process control systems (BPCS), alarms and operator actions, safety instrumented functions (SIF), and relief valves along with an automatic shutdown system in cases of emergencies. In general, not all these safety and

control systems are applied. The number of incorporated safety systems depends on the risk acceptance criteria required by the regulating authorities.

As is reported by CCPS (2003), there are many methods available for the hazard identification of a process system: Hazard and Operability Study (HAZOP), Failure Mode and Effect Analysis (FMEA), safety checklists, etc. To limit the focus to severe hazards or credible scenarios, one may employ the maximum credible accident scenario analysis approach proposed by Khan and Abbasi (2002).

The next risk analysis steps refer to the estimation of frequencies of identified accidents and their potential consequences. This estimation can be carried out by means of probabilistic and engineering models (Crowl and Louvar, 2011).

Frequency estimation calls for the collection of failure rates or probabilities of failure in demand data. Such generic data are usually based on expert judgement and process empirical knowledge and can be collected from databases such as OREDA (2002), TNO (2005a), HSE (2009), etc. If available, plant-specific data from historical records is the best source to be integrated into the calculations. Even though such probabilistic estimation cannot fully reflect reality, it still offers meaningful and detailed predictions of potential risks.

Consequence estimation involves the determination of possible effects in terms of health loss, property loss and environmental damage resulting from undesired scenarios. There are many mathematical and empirical models available for the estimation of consequences. Interested readers may refer to Crowl and Louvar (2011) and Assael and Kakosimos (2010) for an exhaustive description of source models, fires, explosions and toxic gas dispersion

calculations. In addition, Yang et al. (2018) used computational fluid dynamics to simulate fire in a floating liquefied natural gas facility. As an alternative, Hashemi et al. (2014) developed loss functions for the overall consequence assessment of process deviations modelling five major loss categories: quality, production, asset, human health and environmental losses.

The risk level is established once the estimation results of frequency and consequence are determined. If new information on the behavior of the process system becomes available, new hazards may be identified, and the present risk level should be revised by estimating the frequency and consequence again. This updated risk profile is then compared with the acceptability criteria to confirm if it meets the requirements.

## **1.2 Specific QRA approaches**

While the previous section contributes to the overview of QRA, the current section introduces the most common approaches to performing QRA, especially for hazard identification and frequency estimation procedures.

Fault tree analysis (FTA) is a typical graphical QRA tool. When performing FTA, the top event, usually the release of hazardous materials from a container, is identified first. Next, all the possible intermediate and basic events such as the occurrence of abnormal conditions and the subsequently unfortunate failures of protection systems are found by conducting a causal analysis. The top event probability can then be obtained from the logistics shown in the developed fault tree.

Similar to FTA, event tree analysis (ETA) is also an easily-adopted risk assessment method.

ETA consists of many branches, which start from an unwanted event, normally known as the top event, and end with different outcomes. The outcomes will differ based on the performance of safety barriers that are supposed to reduce the effects of the top event.

Combining FTA and ETA will lead to the bow-tie (BT) diagram, which is considered a comprehensive QRA technique, since it presents both the causes and the consequences of a top event. Some recent adoptions of BT in chemical process safety analysis can be found in Aqlan and Mustafa Ali (2014) and Lu et al. (2015).

Among the most recently used QRA techniques is Bayesian network (BN). BN is defined as a directed acyclic graph based on Bayes' theorem (Mittnik and Starobinskaya, 2010). One of the features of BN is its capabilities in updating prior beliefs when new information becomes available. In the field of chemical process application, the accident precursor data collected throughout the lifecycle of a plant can be used to dynamically adapt the failure probabilities of the safety barriers. Based on this, a real-time risk monitoring platform is built, which is very useful in supervising the fast-changing operation conditions of a plant.

### **1.3 Dependency in risk assessment of process systems**

When conducting traditional process safety and risk analysis, it is often assumed that there is no dependency in the causations. Nevertheless, such an assumption is no longer convincing due to process integration. Taking a complex chemical plant as an example, the components within the same system, e.g., a temperature safety instrumented system, or across systems work under similar circumstances and thus are subject to similar temperature, pressure and stress. This leads to correlated failure probabilities of these

components. The simultaneous occurrence of several failures caused by inherent dependency can result in major accidents or even catastrophes. To prevent these, investigation of the potential correlation and dependency among process variables is necessary. Unfortunately, research on dependency is very limited in the process safety literature.

BN is one of the few tools available to integrate the consideration of dependency into the process of risk assessment. In BN, joint densities are defined using conditional probability tables (CPTs). A typical application of BN for modelling dependency can be found in an interesting study by Khakzad et al. (2013), where the failure probability of an alarm system was assumed to depend on whether the ignition barrier works or not. Similar work has also been presented in Ale et al. (2014) and Paskan and Rogers (2013). Even though the use of CPTs to represent simple dependency among variables is straightforward, the BN model is unable to construct complex, non-linear dependence (Mohseni Ahooyi et al., 2014).

Alternatively, as reported in Hashemi et al. (2015a), the correlation coefficient is the most widely applied tool to measure complex dependency, with the linear correlation parameter or Pearson correlation parameter used for capturing linear relationships and rank correlation coefficients for non-linear relationships. However, this single number fails to reflect more complicated dependencies (Schirmacher and Schirmacher, 2008).

To address this shortcoming, copula functions are introduced, which provide a framework for the construction of dependent multivariate distributions. Using copulas provides increased flexibility, as the variables can come from any marginal family (Nelson, 2006). It is notable that by using copulas, the estimation of marginal distributions can be separate

from the estimation of dependence structures.

The use of copula is not foreign in areas such as financial risk management; the risk assessment of nuclear plants, see Yi and Bier (1998) for instance; and transportation research. However, it was not until the last decade that risk practitioners began to notice the potential prevailing function of copula for process safety analysis. Meel and Seider (2006) performed a state-of-the-art dynamic failure assessment of an exothermic CSTR. An event tree was developed, and copula functions were used to model the dependency among the performances of the safety barriers. Pariyani et al. (2012) focused on the effect of dependence on the failure probabilities of the safety, quality and operability systems with the help of two types of copula families: the Gaussian copula and the Cuadras & Auges copula.

More recent work on the assessment of correlated process variables can be found in Oktem et al. (2013), Hashemi et al. (2015b), Yu et al. (2015) and Song et al. (2016). It is worth mentioning that in Hashemi et al. (2015b), copulas were employed to construct a multivariate loss function for the modelling of operation loss in a hypothetical de-ethanizer column. However, the research focus was on the overall risk estimation while considering the dependence between operational risk and business risk.

#### **1.4 Research scope and objective**

The scope of the thesis covers the estimation of accidents' likelihood while considering dependencies in risk analysis. The research also studies the mechanisms behind such effects of dependencies. The developed models are especially applicable to complex

process systems.

From previous subsections of the overview on the QRA technique and its popular forms and applications in process safety analysis, it can be concluded that the accurate modelling of correlation in risk assessment remains an unresolved challenge. Therefore, the overall objective of current research is the application of copula functions to fill this gap. Copula functions are incorporated in existing QRA techniques to build two novel risk assessment models:

- i) Copula-based bow-tie model (CBBT)
- ii) Copula-based Bayesian network (CBBN)

The first objective of this research is the development of the copula-based bow-tie model (CBBT), which considers dependencies in initiating events as well as safety systems. As is observed, previous published works about the application of copulas focused on dependence in event trees. As a result, only AND dependence has been studied due to the inherent attributes of an event tree. To overcome this limitation, the combination of fault tree and event tree incorporated in a bow-tie model with copulas, namely CBBT, is proposed in the present research.

With the growing popularity of the use of topological network-based approaches such as Bayesian network in risk assessment, the possibility of integrating them with copulas is becoming a subject of growing interest for researchers. This leads to the second objective of this thesis: the development of a Copula-based Bayesian network (CBBN).

## **1.5 Novelty and contributions**

This thesis presents useful methodologies which are innovative and scientifically viable to be applied to industry. It contributes to both research academia and industrial implementation.

The proposed CBBT model enables research on the effects of dependency among causation factors on not only the AND logic but the OR logic as well. In the developed revised bow-tie model for a hexane distillation unit, for instance, some correlated initiating events are under an AND gate, while others are under an OR gate. The other advantage of incorporating both FT and ET is that the root causes of an accident scenario can be fully analyzed.

The second work on the CBBN model successfully preserves the features of both BN and copula, with the former capturing conditional dependencies, while the latter modelling non-linear dependencies, among network nodes.

Even though copula is a confirmed robust tool for modelling dependency and correlation, it has not yet been universally applied in process industries, partly because of its abstract and overcomplicated appearance as presented in textbooks. To make copula easy to access, another important contribution of this work is the exploration of a simple and understandable way to use copula such that it can be added to current risk analysis tools without significant efforts or technical difficulties.

## **1.6 Thesis structure**

This thesis is written in a manuscript format, which includes two peer-reviewed journal

articles. The outlines of the following chapters are summarized as follows.

Chapter 2 presents a manuscript published in the Journal of Loss Prevention in the Process Industries. It proposes a revised bow-tie model that considers dependency with the help of copulas. To highlight the effect of dependence, the methodology is first applied to two studies on two common logic gates (AND gates & OR gates). It is then followed by a case study on the frequency estimation of the consequences resulting from a potential accident scenario of hexane release from a typical distillation column. The simulated consequence probabilities from both revised and traditional models are compared. Finally, a detailed discussion and explanation of the results is given.

Chapter 3 contains a manuscript submitted in revised form to Process Safety and Environmental Protection. It provides a novel copula-based Bayesian network model. A step-by-step description of how to construct it is presented with a demonstrative example. To validate the robustness of the proposed risk analysis model, a real-life catastrophe that happened in the U.S. is re-examined. A sensitivity analysis for this case is also conducted, identifying the most important factors. Further, to take advantage of Bayesian network, backward probability updating is performed to find the dominant causes of this accident.

Chapter 4 summarizes the conclusions of the present research. Directions for future work are also suggested.

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## Chapter 2. Risk assessment of process system considering dependencies<sup>1</sup>

### Abstract

Risk assessment is conducted in process systems to identify potential accident scenarios and estimate their likelihood and associated consequences. The bow-tie (BT) technique is most frequently used to conduct the risk assessment. It is a simple, comprehensive and straightforward technique; however, it considers independence among the causation factors (initiating events) of an accident scenario and the safety barriers in place to minimize the impact of the accident scenario. This is a serious limitation and can lead to erroneous results. This paper presents a simple yet robust approach to revise the Bow-tie technique considering interdependence. It employs copula functions to model the joint probability distributions of causations in the BT model of the accident scenario. This paper also analyzes the impact of dependence on two common logic gates used to represent the potential accident scenario. The probability of a potential accident scenario in a hexane distillation unit using both the traditional BT technique and the revised approach is compared. Results confirm that the revised approach is reliable and robust.

*Key words:* Risk assessment; Bow-tie model; Dependence; Copula function, operational risk

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<sup>1</sup> C.Guo et al. Journal of Loss Prevention in the Process Industries 55 (2018) 204-212.

## 2.1 Introduction

In chemical process industries, it is very likely for accident scenarios to occur. If safety and protection systems fail to function, these scenarios will likely escalate into catastrophic events. Therefore, it is essential to analyze the risks of existing process systems to increase awareness of accident probabilities and their possible consequences.

To identify hazards and prevent accidents, quantitative risk assessment (QRA) is one of the most widely adopted approaches (Khan et al., 2002, Khan and Haddara, 2004). The bow-tie model (BT) is a popular and traditional QRA method that contributes to risk identification and safety maintenance in process systems. However, BT is often used with the assumption that there is no dependence among the causes. While this simplifies the risk analysis process, it also decreases the accuracy of the risk estimation, since there may be interactions among causes or safety systems.

As the interrelationships among causations are drawing more attention, there are some studies assessing the correlated random variables that lead to abnormal conditions in process facilities (Hashemi et al., 2015, Yu et al., 2015). There have also been some tools to incorporate dependencies in risk assessment. For example, Bayesian Network (BN) analysis defines a joint density by means of conditional probability distributions. Khakzad et al. (2013) mapped the BT into the BN, where the dependence of safety barriers on the top event is captured. However, BN analysis has the disadvantage of not being able to construct non-linear dependence structure (Mohseni Ahooyi et al., 2014).

To overcome the limitations of these risk analysis methods, Yi and Bier (1998) devised a model that uses copula theory (Nelsen, 2006) to capture the dependence between failure

probabilities of safety barriers in a nuclear plant. Initially, the application of copulas was popular in financial analysis (Durante, F. and Sempi, C., 2015). Recently, copulas are starting to be employed in the field of risk assessment of process systems (Pariyani et al., 2012, Oktem et al., 2013). The major strength of using copulas is that the process of estimating marginal distributions is separate from the dependence structure estimation. This indicates that the margins of correlated variables can even come from different families.

In Yi and Bier's model, copula functions were applied to study the dependence in event tree analysis. Meel and Seider (2006) then built four Bayesian models to conduct dynamic failure assessment by applying this approach to an exothermic chemical reactor. Elidan (2010) proposed the Copula Bayesian Network (CBN), which was a combination of BN and copula functions. The CBN offered a framework for capturing cause-effect relationships among correlated variables with complicated dependence. Hashemi et al. (2016) developed a methodology for mapping the BN into the CBN model and the CBN structure learning that involves the selection of local copulas and associated parameters.

The objective of the present work is to develop a robust risk assessment method that considers dependence among causations factors and safety barriers. The dependence assumption is based on the nature that the components within the same system (i.e. temperature safety instrumented system etc.) or across systems of a chemical plant work under similar circumstances and thus are subject to similar temperature, pressure or stress. This leads to correlated failure probabilities of such components. The work considers dependence in both the event tree and the fault tree parts of the bow-tie. To highlight the

importance of considering dependence in risk analysis, the present study also compares the results of the consequence probabilities from the proposed methodology with the results from a conventional BT model where the dependence effect is ignored.

The remainder of this paper is organized as follows. In Section 2.2, the proposed updated risk assessment methodology with two illustrative examples is provided. This proposed methodology is then applied to a case study involving a distillation unit in Section 2.3. Section 2.4 briefly discusses the effect of dependence by analyzing the results, followed by some conclusions as presented in Section 2.5.

## **2.2 The proposed risk assessment methodology**

The proposed methodology considers dependence among the causations for risk assessment. This methodology is the revised version of the bow-tie technique. The details of this methodology are presented in Figure 2.1. To better illustrate the methodology (shown in Figure 2.1), two simple examples are presented here. These examples study the effects of dependence of common logic gates (AND/OR).

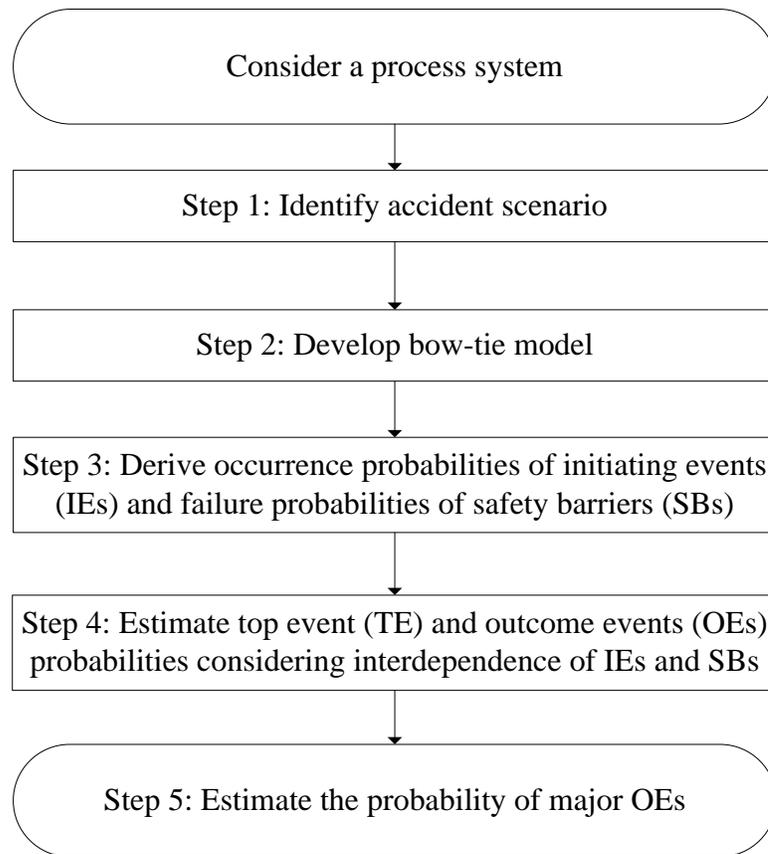


Figure 2.1 Methodology for risk assessment considering dependence.

### 2.2.1 **Step 1: Identify accident scenario**

Once a process system is selected, the probable accident scenario is developed. Subsequently, the causes of this accident scenario or top event (TE), which are called initiating events (IEs) in bow-tie analysis, are identified. The accident scenario is then further analyzed based on the failure or success of safety barriers (SBs), leading to the possible consequences or outcome events (OEs).

In the examples, a range of IEs (A, B, C and D) and two SBs (SB1 and SB2), the respective TEs and OEs are identified. OE1 refers to safe condition, where both SB1 and SB2 function

despite TE occurs. If SB1 functions but SB2 fails, a near miss outcome event is viewed to occur denoted by OE2. An incident (OE3) will occur once SB1 fails however SB2 fortunately works. Lastly, the worst OE is an accident (OE4), when neither SB1 nor SB2 succeeds in mitigating the outcome of TE.

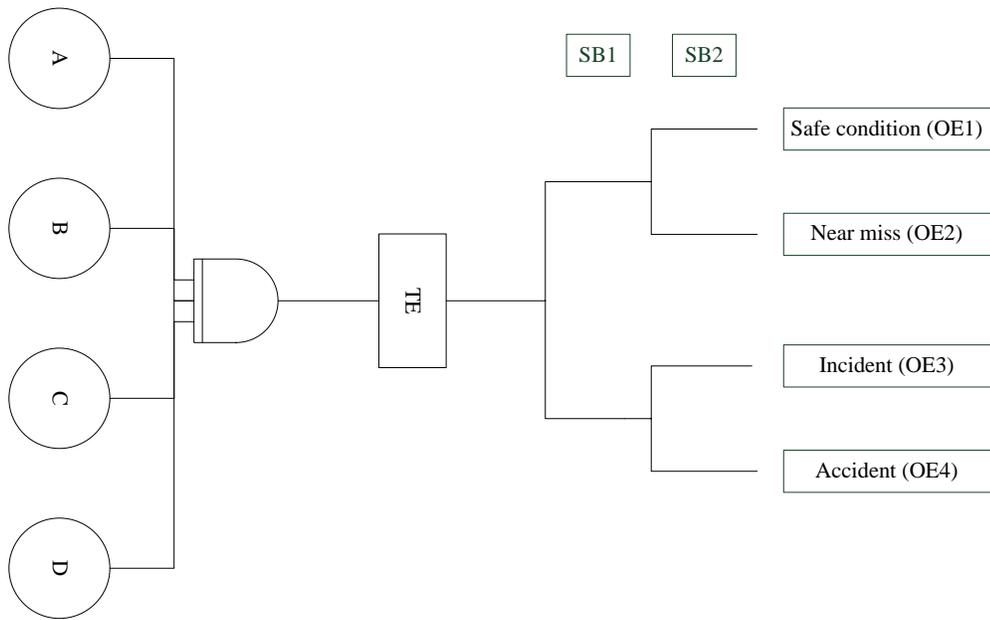
### 2.2.2 Step 2: Develop bow-tie model

The fault tree (FT) and event tree (ET) are developed based on the causality and SBs identified in the accident scenario. The bow-tie model is then created to combine FT and ET.

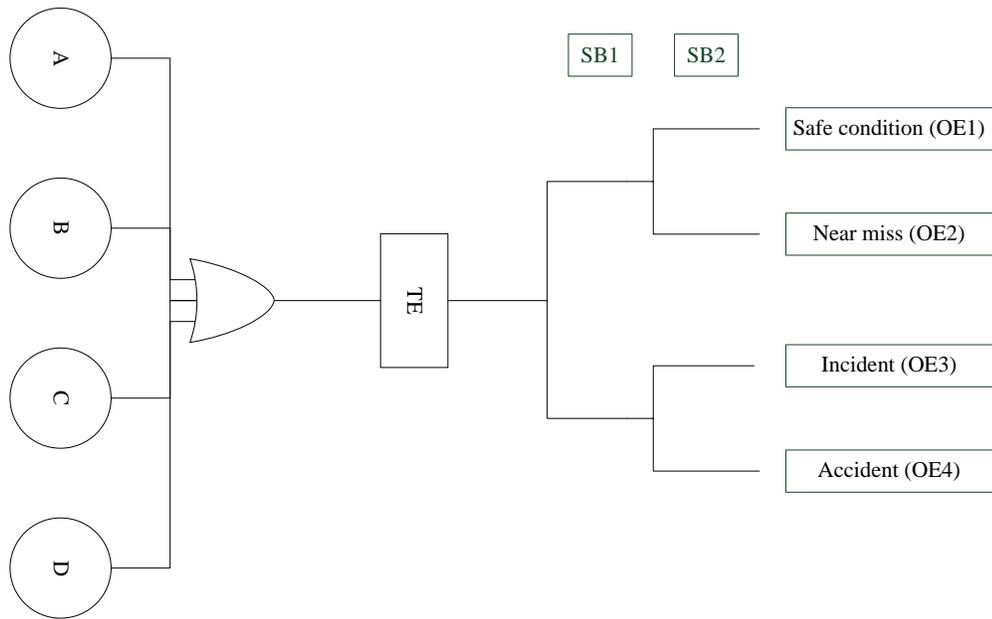
In the simulation example, for the AND gate, three cases are simulated. In the first case, IE A and IE B are connected by an AND gate, or  $A \cap B$ , as expressed mathematically. The TE is believed to occur only if A and B occur simultaneously. There is also an AND gate connecting A, B, and C ( $A \cap B \cap C$ ) in the second case, and A, B, C, and D ( $A \cap B \cap C \cap D$ ) in the third case.

Using the OR gate, three cases are also simulated. In the first case, IE C and IE D are connected by an OR gate, or  $C \cup D$  as a mathematical expression. The TE will occur if either C or D occurs or C and D occur at the same time. There is also an OR gate connecting B, C, and D ( $B \cup C \cup D$ ) in the second case, and A, B, C, and D ( $A \cup B \cup C \cup D$ ) in the third case.

Figure 2.2 shows the bow-tie models of both an AND gate example and an OR gate example in the case of 4 IEs.



(a)



(b)

Figure 2.2 Bow-tie models of the example in the case of 4 IEs (A, B, C, and D) and two logical operators: (a) AND gate; (b) OR gate.

### 2.2.3 Step 3: Derive occurrence probabilities of IEs and failure probabilities of SBs

In a classical model, discrete values for probabilities are used to estimate the occurrence probabilities of OEs. In contrast, it is assumed that IEs probabilities and failure probabilities of SBs follow the Beta distribution, with selected parameters  $a$  and  $b$  in the proposed model. The failure probability distribution function is given as:

$$f(x) \propto x^{a-1} (1-x)^{b-1} \quad (2.1)$$

The mean value is  $a/(a+b)$  and the variance is  $ab/[(a+b)^2(a+b+1)]$ . The parameters are selected such that the mean is equal to the discrete value of the IE probability or the failure

probability of the SB. Tables 2.1 and 2.2 present the parameters of the Beta distribution for each IE and each SB used in the examples, respectively. The adoption of these numbers is not rigorous but for sample calculations in the examples only.

Table 2.1 Probability distributions for the IEs.

Initiating event	Occurrence probability			
	Discrete value	Distribution	Distribution parameter	
			a	b
A	0.1	Beta	1	9
B	0.2	Beta	2	8
C	0.3	Beta	3	7
D	0.4	Beta	4	6

Safety barrier	Failure probability			
	Discrete value	Distribution	Distribution parameter	
			a	b
SB1	0.05	Beta	1	19
SB2	0.15	Beta	3	17

Table 2.2 Probability distributions for the SBs.

#### 2.2.4 Comparison study: Estimate TE and OEs probabilities considering independence of IEs and SBs

Before moving to the proposed algorithm for probability estimation that incorporates

interdependence, the traditional bow-tie method is first used for comparison purpose. It is considered that the occurrence probabilities of IEs and the failure probabilities of SBs are independent. Then the discrete occurrence probability of an OE is estimated as the discrete probability of the TE multiplied by the discrete probabilities of failure or success of various SBs along the corresponding branch. The probability of the TE is calculated as the union of minimal cut sets.

For example, the discrete probabilities of TE and OE3 in Figure 2.2-a are as follows.

$$\Pr(TE)=\Pr(A).\Pr(B).\Pr(C).\Pr(D) \quad (2.2)$$

$$\Pr(OE3)=\Pr(TE).\Pr(SB1).\Pr(\overline{SB2}) \quad (2.3)$$

where  $\Pr(A)$  and  $\Pr(B)$  are the discrete probabilities of IE A and of IE B, and  $\Pr(SB1)$  and  $\Pr(\overline{SB2})$  refer to the discrete failure and non-failure probability of safety barriers  $SB1$  and  $SB2$ , respectively. Other OEs probabilities are obtained similarly.

#### 2.2.5 **Step 4: Estimate TE and OEs probabilities considering interdependence of IEs and SBs**

##### *Algorithm for probability estimation by Monte Carlo simulations*

To capture the correlation among IEs and SBs, copula functions are used. A copula is a multivariate probability distribution, where each random variable has a uniform marginal distribution on the unit interval [0, 1]. Because of the possibility for dependence among variables, a copula can be used to construct a new multivariate distribution for dependent variables.

There are many kinds of multi-dimensional copulas. In this work, the Gaussian copula,

which is one of the most common copulas, is used. It is a simple yet flexible elliptical copula. A correlation matrix consisting of corresponding correlation parameters ( $\rho$ ) is then designed according to the interactions among IEs and SBs.

Subsequently, Monte Carlo integration is conducted to simulate the probabilities. In each trial, correlated random numbers with uniform distribution between 0 and 1 are first generated and compared with the random numbers that follow specific Beta distributions of corresponding IEs. If the uniform random number is smaller or equal to the random number of the IE, the IE will occur. The next step is the analysis of the intermediate event. If there is an AND gate connecting the IEs, the relative intermediate event will only occur when all the corresponding IEs occur. In the case of an OR gate, the intermediate event will occur when any corresponding IE occurs. By applying this analysis of the AND gate as well as the OR gate to the following intermediate events in the bow-tie model, whether the TE will occur or not in this trial can be finally confirmed.

The right side of the bow-tie model, which is the ET, is then analyzed. Similar to the simulation of IEs, correlated random numbers are generated from the copula function that is applied to SBs. The results for which SBs fail in this trial can be derived by comparing these numbers with the random numbers that represent failure probabilities of respective SBs. These results determine the branch of the ET that points to the particular OE. This simulation is conducted for a million trials. The mean occurrence probabilities of the TE along with all the OEs are obtained.

For the sake of simplicity, the correlation parameters of any two IEs in the examples are assumed to be identical, starting from 0.2 to 1. One of the correlation matrices used in the

case of  $A \cap B \cap C$  is shown in Table 2.3.

Monte Carlo simulations with one million trials are conducted for the two examples. The correlation parameters used and the resulting mean occurrence probabilities of the TEs for both independent and interdependent cases are presented in Figures 3.3 and 3.4, where  $\rho$  being 0 signifies that the IEs are completely independent therefore the probabilities are calculated by use of the method discussed in Section 2.2.4, where  $\rho = 1$  signifies that the IEs are deterministically related, while other correlation parameters that fall between 0 and 1 signify that the IEs are partly dependent.

Table 2.3 One of the correlation matrices for the case  $A \cap B \cap C$ .

	pA	pB	pC
pA	1	0.8	0.8
pB	0.8	1	0.8
pC	0.8	0.8	1

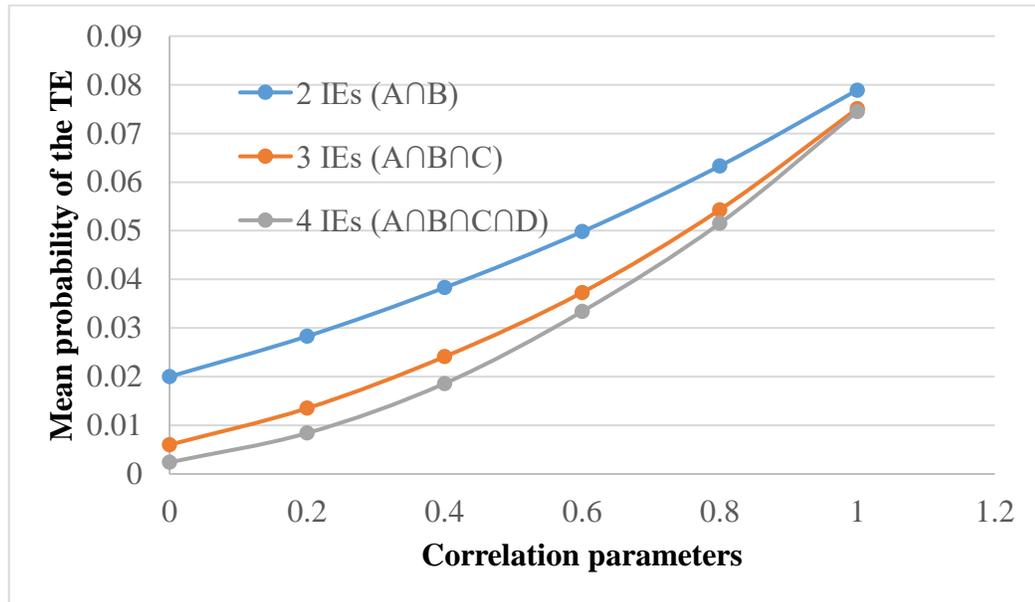
*The effect of interdependence on the probability of TE for AND gate example*

Figure 2.3 demonstrates that the mean probabilities of the TEs increase significantly as  $\rho$  rises in the AND gate example. This is due to the AND gate logic. The positive correlation among IEs improves the system reliability to some extent because if one IE does not occur, the others are less likely to occur. Nevertheless, the improvement is not significant since only one of them needs to not occur to avoid the occurrence of the TE. Conversely, the positive correlation significantly decreases the system reliability: if one IE occurs, then

others are more likely to occur; this can lead to an increased TE probability.

In addition, it is worth mentioning that the TE probability increases more dramatically when more IEs are correlated. For instance, the mean probability of the TE, given that  $\rho$  equals 1, is 3.98 times as large as when  $\rho$  equals 0 in the case of 2 IEs ( $A \cap B$ ). In contrast, the increase is 28.80 times in the case of 4 IEs ( $A \cap B \cap C \cap D$ ).

It is interesting to note that the probability of the TE tends to approach the minimal IE probability, which is 0.1 in all three cases, when the dependence becomes stronger. This is because the minimal probability becomes dominant in the case of dependence for the AND gate. If the IE with the minimal probability occurs, other IEs also tend to occur, which will cause the TE to occur. It is also notable that the TE probability is closest to 0.1 when  $\rho$  equals 1 in the case of 2 IEs ( $A \cap B$ ). The TE probabilities are farther from 0.1 when  $\rho$  equals 1 in the cases of 3 IEs ( $A \cap B \cap C$ ) and 4 IEs ( $A \cap B \cap C \cap D$ ), but the differences from the case of 2 IEs ( $A \cap B$ ) are not significant.



$\rho$	0	0.2	0.4	0.6	0.8	1
Mean probability of the TE in the case of 2 IEs ( $A \cap B$ )	0.0200	0.0283	0.0383	0.0498	0.0633	0.0789
Mean probability of the TE in the case of 3 IEs ( $A \cap B \cap C$ )	0.0060	0.0135	0.0241	0.0373	0.0543	0.0751
Mean probability of the TE in the case of 4 IEs ( $A \cap B \cap C \cap D$ )	0.0024	0.0084	0.0186	0.0334	0.0515	0.0745

Figure 2.3 The effect of interdependence among IEs on the probability of TE for AND gate example; data is also presented for analysis.

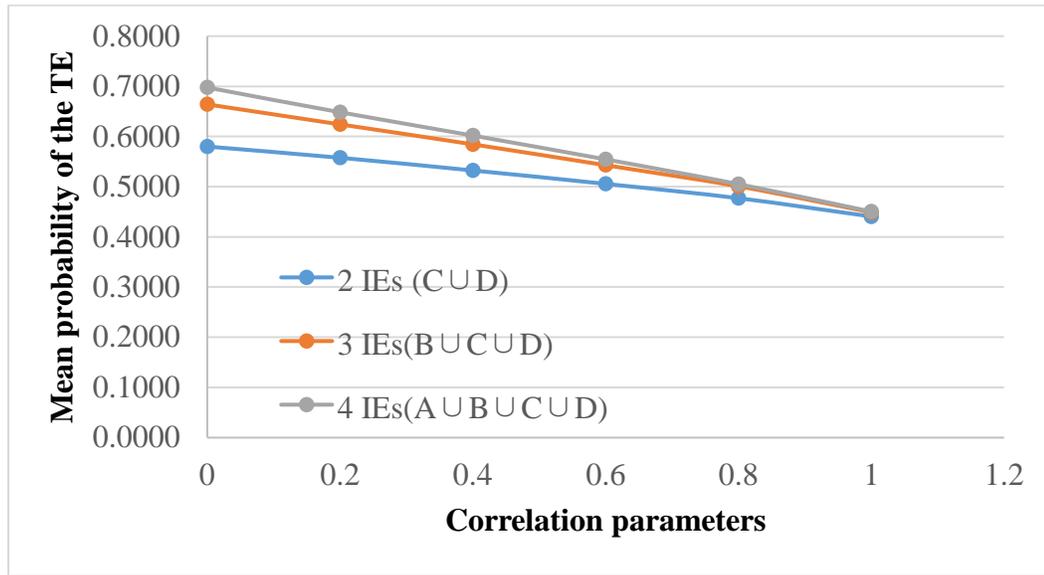
*The effect of interdependence on the probability of TE for OR gate example*

Figure 2.4 shows that the mean probabilities of the TE decrease steadily as  $\rho$  rises for the

OR gate example, which is due to the OR gate logic. The positive correlation among IEs decreases the system reliability slightly because if one IE occurs, the others are more likely to occur. However, the decrease is not significant, as the occurrence of any IE leads to the TE. Conversely, the positive correlation greatly improves the reliability: if one IE does not occur, then others are less likely to occur, leading to a decreased probability of TE.

Also worth mentioning is that the TE probability decreases more substantially when more IEs are correlated. For instance, the mean probability of the TE when  $\rho$  equals 1 is 0.76 times as large as when  $\rho$  equals 0 in the case of 2 IEs (CUD). By contrast, the ratio is only 0.65 times as large in the case of 4 IEs (AUBUCUD).

Contrary to the results in the AND gate example, the TE probability is closer to the maximal IE probability (0.4) in all three cases when there is stronger dependence. This is expected, as the maximal probability is dominant in the case of dependence for the OR gate. The occurrence of the IE with the maximal probability can lead to the occurrence of the TE. It is also notable that the TE probability is closest to 0.4 when  $\rho$  equals 1 in the case of 2 IEs (CUD). The TE probabilities are farther from 0.4 when  $\rho$  equals 1 in the cases of 3 IEs (BUCUD) and 4 IEs (AUBUCUD), but the differences from the case of 2 IEs (CUD) are not significant.



$\rho$	0	0.2	0.4	0.6	0.8	1
Mean probability of the TE in the case of 2 IEs (CUD)	0.5800	0.5576	0.5323	0.5056	0.4770	0.4407
Mean probability of the TE in the case of 3 IEs (BUCUD)	0.6640	0.6241	0.5844	0.5428	0.5007	0.4489
Mean probability of the TE in the case of 4 IEs (AUBUCUD)	0.6976	0.6483	0.6018	0.5544	0.5051	0.4504

Figure 2.4 The effect of interdependence among IEs on the probability of TE for OR gate example; data is also presented for analysis.

*The effect of interdependence on the probability of OEs*

To further study the effect of dependence of IEs and SBs on the probability of OEs, the  $\rho$  between SB1 and SB2 is considered to be 0.8. The case of  $A \cap B \cap C$ , with the correlation

parameters being 0.8, is used to perform the simulation.

Table 2.4 presents the results of the occurrence probabilities of the TE and the OEs for both independent and interdependent analyses. Results show that the outcome events' probabilities increase drastically when dependence of causations is considered, compared to the independent case. It is clear that the TE probability increases, causing all the OEs to occur. The occurrence probability of an accident increases the most substantially, approximately 42 time, and the accident probability even exceeds the incident probability in the interdependent case.

Table 2.4 Occurrence probabilities of the TE and the OEs in the case study.

Symbol	Event	Independent case (Discrete value)	Interdependent case (Mean value)	$\frac{\text{Pr}(\text{Interdependence})}{\text{Pr}(\text{Independence})}$
TE	Top event	6.00E-03	5.42E-02	9.03
OE1	Safe condition	4.85E-03	4.52E-02	9.33
OE2	Near miss	8.55E-04	6.30E-03	7.37
OE3	Incident	2.55E-04	8.33E-04	3.27
OE4	Accident	4.50E-05	1.90E-03	42.22
MOE	Major OEs	3.00E-04	2.73E-03	9.11

## 2.2.6 Step 5: Estimate the probability of major OEs

Major outcome events are defined as those consequences that cause severe loss, including fatalities or significant financial loss. In this case, incident (OE3) and accident (OE4) are considered to be major OEs. The probability of major OEs is estimated by combining the probability of OE3 and OE4. Results are presented in Table 2.4. It is clear that the occurrence probability of major OEs in the interdependent case is much larger than the probability in the independent case.

### **2.3 Application of the proposed methodology**

To test and verify the proposed methodology, a detailed case study is conducted. The methodology is applied to an accident scenario in a hexane distillation unit, adopted from a study by Markowski and Kotynia (2011). The installation is presented in Figure 2.5.

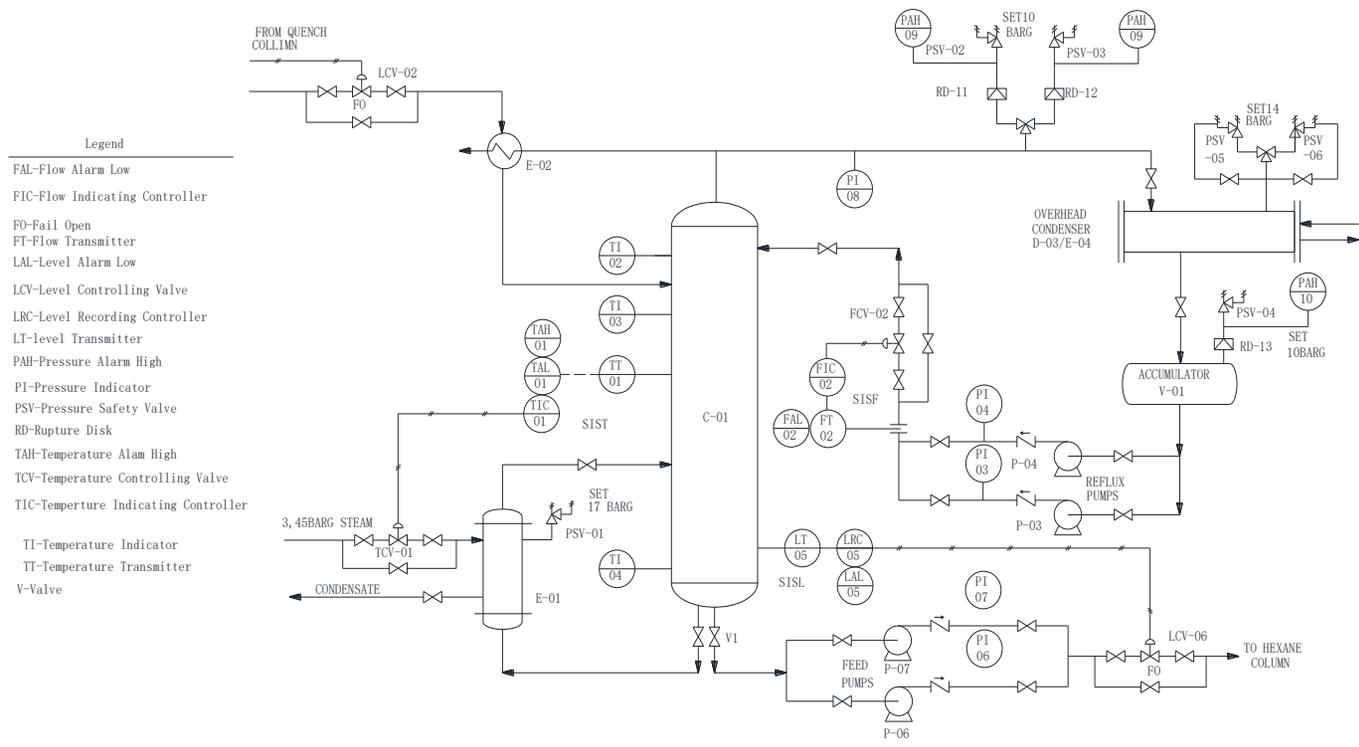


Figure 2.5 Hexane distillation column adapted from Markowski and Kotynia (2011).

### 2.3.1 Steps 1-2: Identify accident scenarios and then develop the bow-tie model

The Hazard and Operability study (HAZOP) is used to identify accident scenarios. For the sake of simplification, only the catastrophic hexane release scenario is studied (Markowski and Kotynia, 2011). Safety and protection systems of the distillation unit comprise three safety layers, as shown in Table 2.5. The bow-tie model is developed for the accident scenario and is shown in Figure 2.6, in which OE2 and OE4 are viewed as major OEs.

Table 2.5 Safety and protection systems.

Safety layer	Measure
Layer I—prevention systems	Good engineering practice (GEP) Basic Process Control Systems (BPCS) with indication and alarm in central room: BPCS <sub>PAH</sub> , BPCS <sub>TAH</sub> , BPCS <sub>TAL</sub> , BPCS <sub>LAL</sub> , BPCS <sub>FAL</sub> , BPCS <sub>TI</sub> , BPCS <sub>PI</sub>
Layer II—protection systems	Safety instrumented systems (SIS): SIS <sub>T</sub> (TT, TIC, TCV), SIS <sub>L</sub> (LT, LRC, LCV), SIS <sub>F</sub> (FT, FIC, FCV), PSV, RD
Layer III—mitigation systems	Automatic deluge system (Ads) Fire brigade (Fb)

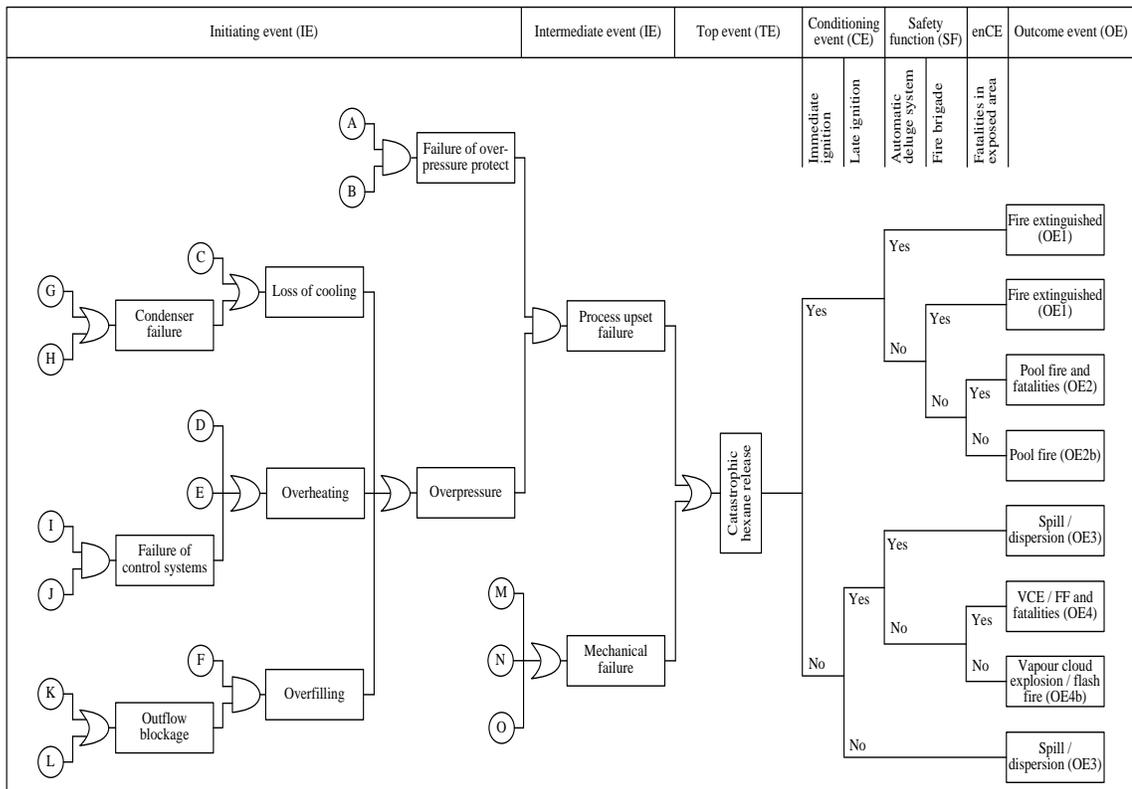


Figure 2.6 Bow-tie accident scenario model for Hexane distillation example similar to one reported in (Markowski and Kotynia, 2011).

### 2.3.2 Step 3: Derive occurrence probabilities of IEs and failure probabilities of SBs

The probability numbers for the conditioning events (CEs) as well as the safety functions (SFs) and the IEs are presented in Tables 2.6 and 2.7 respectively. It is notable that Markowski and Kotynia (2011) used fuzzy probability numbers for the IEs and standard probability numbers for the CEs, based on look-up tables developed in the LOPA book (CCPS, 2001) and experts' knowledge and experience. The failure probabilities for the SFs are assumed based on experts' judgement because they are not given in Markowski and

Kotynia (2011). In addition, all the probabilities are considered to follow Beta distribution with corresponding parameters.

Table 2.6 The probabilities of the CEs and the failure probabilities of the SFs.

Conditioning event and Safety function	Symbol	Probability			
		Discrete value	Distribution	Distribution parameters	
				a	b
Immediate ignition	II	0.1	Beta	1.00	9.00
Late ignition	LI	0.5	Beta	5.00	5.00
Failure of automatic deluge system	$\overline{Ads}$	0.04	Beta	2.00	48.00
Failure of fire brigade	$\overline{Fb}$	0.2	Beta	2.00	8.00
Fatalities in affected area	Fa	0.1	Beta	1.00	9.00

Table 2.7 Components of the IEs and their probabilities.

Initiating event	Symbol	Failure probability			
		Discrete value	Distribution	Distribution parameters	
				a	b
Failure of PI-08	A	0.1	Beta	1.00	9.00
Failure of PSV-02	B	0.01	Beta	1.00	99.00

Loss of water	C	0.1	Beta	1.00	9.00
Failure of TT-01	D	0.01	Beta	1.00	99.00
Failure of TCV-01	E	0.1	Beta	1.00	9.00
Failure of LCV-02	F	0.1	Beta	1.00	9.00
Condenser rupture	G	0.01	Beta	1.00	99.00
Fouling	H	0.001	Beta	1.00	999.00
Failure of TIC-01	I	0.01	Beta	1.00	99.00
Failure of TAH-01	J	0.1	Beta	1.00	9.00
Failure of V1	K	0.1	Beta	1.00	9.00
Failure of P-06	L	0.1	Beta	1.00	9.00
Corrosion	M	0.01	Beta	1.00	99.00
Material defect	N	0.01	Beta	1.00	99.00
Human error	O	0.01	Beta	1.00	99.00

### 2.3.3 Comparison study: Estimate TE and OEs probabilities considering independence of IEs, CEs and SFs

For comparison, it is first assumed that the occurrence probabilities of the IEs are independent of each other. Similarly, the probabilities of CEs are also considered to be independent of the performance of the SFs. This assumption of independence is adopted in Markowski and Kotynia (2011). To simplify the calculation, the IEs, CEs and SFs are

designated discrete probability numbers.

To derive the discrete probabilities of the TE and the OEs, one can adopt the method discussed in Section 2.2.4. In this case, for example, the probabilities of TE and OE2 are calculated as shown in the equations below:

$$\begin{aligned} \Pr(TE) = & \Pr(A)\Pr(B)\Pr(C) + \Pr(A)\Pr(B)\Pr(G) + \Pr(A)\Pr(B)\Pr(H) + \Pr(A)\Pr(B)\Pr(D) + \Pr(A)\Pr(B) \\ & )\Pr(E) + \Pr(A)\Pr(B)\Pr(I)\Pr(J) + \Pr(A)\Pr(B)\Pr(F)\Pr(K) + \Pr(A)\Pr(B)\Pr(F)\Pr(L) + \Pr(M) + \Pr(N) \\ & + \Pr(O) \end{aligned} \quad (2.4)$$

$$\Pr(OE2) = \Pr(TE)\Pr(II)\Pr(\overline{Ads})\Pr(\overline{Fb})\Pr(Fa) \quad (2.5)$$

where  $\Pr(A)$ ,  $\Pr(B)$ , ...,  $\Pr(Fa)$  stand for the respective discrete probabilities in Tables 2.6 and 2.7. All other OEs probabilities are obtained similarly. The discrete probability values of the TE and the OEs are summarized in Table 2.10.

#### 2.3.4 Step 4: Estimate TE and OEs probabilities considering interdependence of IEs, CEs and SFs

To demonstrate the advantage of the proposed methodology, the dependence among the IEs, CEs and SFs is considered in this case study. As Table 2.8 shows, B, E, F, and K are assumed to be correlated because they are all concerned with the failure of valves. However, the  $\rho$  between the failure of the temperature controlling valve (E) and the failure of the level controlling valve (F) is assumed to be 0.8. The  $\rho$  between the failure of the pressure safety valve (B) and the failure of valve 1 (K) is also considered to be 0.8. The  $\rho$  between B and E, B and F, E and K, or F and K is considered to be 0.6. In addition, it is assumed that the failure of the pressure indicator (A) and the failure of the pressure safety valve (B)

are correlated, with  $\rho$  being 0.8, because they compose the overpressure protection system. The  $\rho$  between A and E, F, or K is considered to be 0.6. These correlation parameters are presented in Table 2.8.

A Gaussian copula with the correlation matrix shown in Table 2.9 is applied to the CEs and SFs. In general, there is a more significantly positive correlation of the CEs and SFs with their nearer neighbors. For instance, the  $\rho$  between the performance of fire brigade and fatalities is considered to be 0.8 while that between immediate ignition and fatalities is only 0.5. This indicates that the failure of the fire brigade has a larger impact on fatalities. Probabilistic simulation with 1,000,000 iterations is done with these Gaussian copulas and the results of the mean probabilities of the TE and the OEs are summarized in Table 2.10.

Table 2.8 Correlation parameters among IEs.

	pA	pB	pE	pF	pK
pA	1	0.8	0.6	0.6	0.6
pB	0.8	1	0.6	0.6	0.8
pE	0.6	0.6	1	0.8	0.6
pF	0.6	0.6	0.8	1	0.6
pK	0.6	0.8	0.6	0.6	1

Table 2.9 Correlation parameters among CEs and SFs.

	pII	pLI	p $\overline{Ads}$	p $\overline{Fb}$	pFa
pII	1	0.8	0.6	0.6	0.5

pLI	0.8	1	0.8	0.5	0.6
p $\overline{\text{Ads}}$	0.6	0.8	1	0.8	0.7
p $\overline{\text{Fb}}$	0.6	0.5	0.8	1	0.8
pFa	0.5	0.6	0.7	0.8	1

Table 2.10 Result summary of occurrence probabilities of FOP, the TE and OEs.

Symbol	Event	Independent case (Discrete value)	Interdependent case (Mean value)	$\frac{\text{Pr}(\text{Interdependence})}{\text{Pr}(\text{Independence})}$
FOP	Failure of overpressure protection	1.00E-04	6.90E-03	6.9
TE	Hexane release	3.00E-02	3.37E-02	1.12
OE1	Fire extinguished	2.98E-03	2.99E-03	1.00
OE2	Pool fire and fatalities	2.40E-06	2.87E-04	119.58
OE2b	Pool fire	2.16E-05	2.24E-04	10.37
OE3	Spill/dispersion	2.65E-02	3.04E-02	1.15
OE4	VCE/FF and fatalities	5.40E-05	3.46E-04	6.41
OE4b	VCE/FF	4.86E-04	4.58E-04	0.94

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MOE	Major	OEs	5.64E-05	6.33E-04	11.22
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where fatalities occur

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### 2.3.5 **Step 5: Estimate the probability of major outcome events**

The probability of major outcome events is estimated by combining the probabilities of OEs where fatalities occur (OE2 and OE4). These probability results for both independent and interdependent cases are presented in Table 2.10.

## 2.4 **Discussion**

### 2.4.1 **The effect of interdependence on the probability of the top event**

Table 2.10 shows that the probability of failure of overpressure protection (FOP) increases significantly when the correlation model is applied. The probability of FOP in the interdependent case is 7.9 times larger compared to the independent case. In contrast, there is only a slight increase in the probability of the top event (hexane release). The reasons for such changes are described below.

FOP is an AND gate connecting two initiating events: A (failure of PI-08) and B (failure of PSV-02), meaning that the overpressure will only fail when both PI-08 and PSV-02 fail to function. Thus, the probability of FOP is believed to approach the probability of B, which equals 0.01 in the interdependent case. The minimal cut sets of the top event obtained from the bow-tie model is as follows.

$$\sum MCS_{TE} = ABC + ABG + ABH + ABD + ABE + ABIJ + ABFK + ABFL + M + N + O \quad (2.6)$$

It is obvious that the probabilities of  $ABC$ ,  $ABG\dots$ ,  $ABFL$  will increase when A, B, E (failure of TCV-01), F (failure of LCV-02) and K (failure of V1) are positively dependent, as defined by the correlation parameters in Table 2.8. This increase will finally result in the rise of the top event probability. However, the effect of dependence on the probability of the top event is insignificant since the combined probability of M (corrosion), N (material defect) and O (human error) is dominant in this case. Even though the increase of the top event probability is relatively small in this specific case, it is still important to pay attention to possible dependent causes of abnormal conditions in risk assessment, because the effect of dependence under other circumstances may be substantial.

#### 2.4.2 **The effect of interdependence on the probability of the outcome events**

It is clear from Table 2.10 that when considering the dependence among the initiating events, conditioning events and safety functions, the occurrence probabilities of certain outcome events change dramatically. The increase of the top event probability, as discussed previously, is the reason why the probabilities of most outcome events increase despite that of OE4b. It is notable that the probabilities of the major consequences (OE2 and OE4) increase sharply (119.58 times and 10.37 times as large as in the independent case respectively). This can be explained as the followings. Because the conditioning events and safety functions are correlated, the branch that follows the sequence of immediate ignition, failure of the automatic deluge system, failure of the fire brigade and fatalities, is most likely to occur in this event tree. Furthermore, it is clear from Eq. 2.5 that the OE2

probability tends to approach the top event probability multiplied by the minimal occurrence probability among conditioning events and safety functions. Therefore, the probability of OE2 increases most substantially. Similarly, the probabilities of OE2b and OE4 both increase greatly, but the increase is not as large as OE2. Conversely, the probability of OE4b decreases.

Table 2.10 also shows that the effect of dependence on outcome events probabilities is more prominent than that of a top event probability. This indicates that the correlated safety barriers have significant impacts on the occurrence probabilities of outcome events. If feasible, it is suggested that independent barriers be added into safety systems.

## **2.5 Conclusions**

This work has revised the bow-tie analysis by integrating the dependence among causation factors. The revised bow-tie model, when run in probabilistic mode using Monte Carlo simulations, provides more reliable and robust results. This is established revisiting a past case study and comparing the results. The considerable difference between the respective results reveals that the effect of dependence is significant and thus should be considered when assessing risks of a process system.

The revised methodology is a simple adoption of the copula function to represent the dependency. This revised methodology serves as a useful and easy to adopt tool to analyze risk in a process system. It has the ability to model a complex engineering system where dependencies are inherent. Another important feature of the revision is its ability to capture and represent dependencies among the safety barriers and most importantly, to represent a

common failure model. This work could further be improved by transforming the bow-tie into a network-based approach such as a Bayesian network or the Petri net along with dependence modeling, using the copula function.

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## Chapter 3. Copula-based Bayesian network model for process system risk assessment<sup>2</sup>

### Abstract

Risk assessment is an essential exercise for process systems from early conceptual design to operation and subsequently during decommissioning. Risk assessment methods have evolved over the past two decades from index-based methods to detailed quantitative methods. The Bayesian network (BN) is a recently developed technique used for risk assessment that utilizes updating, adapting and discrete-time-based analysis properties. Although the BN is a powerful technique, it continues to face the challenge of modelling non-linear complex correlations of process components. This paper proposes a copula-based Bayesian network model that assists in overcoming the challenge of non-linear relationships. In addition to defining conditional probabilities, the copulas are also used to describe the joint probability densities of the network nodes in the BN. Application of the proposed model is demonstrated using a process accident case study. The results reveal that the proposed model is effective in estimating more reliable accident probabilities. A sensitivity analysis is also conducted to identify important factors that need to be monitored to prevent accident occurrence. Though the focus of the present study is on process systems, the proposed model is applicable to most engineering systems.

*Key words:* Risk assessment; Bayesian network; Dependence; Copula; Process safety; Accident model

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<sup>2</sup> C.Guo et al. Submitted in revised form to Process Safety and Environmental Protection.

### 3.1 Introduction

Process industries deal with hazardous substances in large quantities. The release of these materials can result in severe consequences including the loss of life, environmental damage, and financial losses. As the nature of process operation is becoming more complex due to process integration and digitalization, process safety management is becoming a key concern, and risk assessment is an important step in process safety management. Many approaches are available to conduct risk analyses of process systems. Among them, Layer of Protection Analysis (LOPA) is a comprehensive yet easy-to-use risk assessment technique. LOPA is a semi-quantitative approach which considers three layers of safety and protection systems. The first layer is prevention systems, including Basic Process Control Systems (BPCS) with indicators and alarms; the second layer consists of Safety Instrumented Systems (SIS), while the third layer refers to mitigation systems such as the deluge system (CCPS, 2001). CCPS (2001) has proposed a range of occurrence (or failure on demand) probabilities of these systems. These probabilities are derived from plant data and expert judgement.

Although LOPA is a recommended approach and widely used, it has some inherent limitations. For instance, the causal analysis of an accident is too simple, and the predicted probability is vague and often unrealistic.

As an alternative, the Quantitative Risk Analysis (QRA) approach is a detailed logical reasoning-based method used to make more realistic probabilistic estimations of accident scenarios (CCPS, 2003). The QRA approach is built upon fault tree analysis (FTA) and event tree analysis (ETA). FTA identifies the probable initiating events i.e. failures of BPCS

and SIS that cause the top event. ETA presents all the possible outcomes resulting from a top event.

The bow-tie (BT) technique combines FTA and ETA and has been proven to be a robust risk assessment tool. De Dianous and Fievez (2006) used BT in the ARAMIS project to demonstrate risk control. Recently, BT has commonly been used together with other techniques. Lu et al. (2015), for example, proposed a risk evaluation method that combines both a risk matrix and bow-tie for natural gas pipelines. Aqlan and Mustafa Ali (2014) assessed the risk of a chemical plant by integrating lean manufacturing principles and fuzzy BT.

The Bayesian network (BN) is an emerging graphical tool used for the risk analysis of chemical process systems. In contrast to the static nature of BT, BN makes use of the accident precursor data recorded during the lifecycle of a chemical plant to conduct probability adapting. Another feature of BN is that it can consider dependent failures, which BT is unable to do. BN is dependent on the linear relationships among correlated variables defined by the means of Conditional Probability Tables (CPTs). Such dependence construction is widely discussed and used by many researchers (Ale et al., 2014, Islam et al., 2018, Khakzad et al., 2013, Paskan and Rogers, 2013). The traditional BN is unable to model complex interrelationships, such as non-linear dependence among correlated variables (Mohseni Ahooyi et al., 2014).

To address this challenge, the use of copulas is introduced to process safety analysis. Meel and Seider (2006) provided a failure assessment of an exothermic CSTR (Continuous Stirred Tank Reactor) with copulas representing dependent failure probabilities of safety

systems. Pariyani et al. (2012) studied two different copula families to model the safety, quality, and operability systems (SQOSs) interactions. As a powerful model for constructing the dependence among continuous variables, copula applications have been frequently implemented in a wide range of fields such as near-miss analysis (Oktem et al., 2013), risk monitoring in managed pressure drilling (Hashemi et al., 2016) and disruption lengths modelling (Zilko et al., 2016). Most recently, Guo et al. (2018) proposed a revised bow-tie model that incorporates copula functions. Nevertheless, the integration of copulas to BN for process modelling and risk assessment has yet to be considered.

This study presents an integration of copulas with the Bayesian network to represent non-linear dependencies. Multiple copula functions are explored to identify the most appropriate functions that define variables' dependencies. The integrated model has the strengths and flexibility of both BN and copulas. It can be applied to accident analysis in engineering fields where there are potential dependencies among the causes

Section 3.2 of this paper presents detailed steps to build a copula-based BN model with an illustrative example. In Section 3.3, the proposed model and the conventional BN are applied to a real-life reboiler rupture case study. Results from both approaches are compared and discussed in Section 3.4. Section 3.5 presents a sensitivity analysis to identify the most influential causation factors, while Section 3.6 is devoted to a diagnostic analysis of the case study. The main highlights of the current work are summarized in Section 3.7.

### **3.2 The proposed copula-based Bayesian network model**

To define the non-linear and complex dependencies of a process system, the copula-based Bayesian network (CBBN) model is proposed. This model enhances the traditional Bayesian network (BN) by equipping it with copula functions. Figure 3.1 shows the steps to develop a CBBN. An example is also provided to accommodate the detailed description of each step in the following subsections.

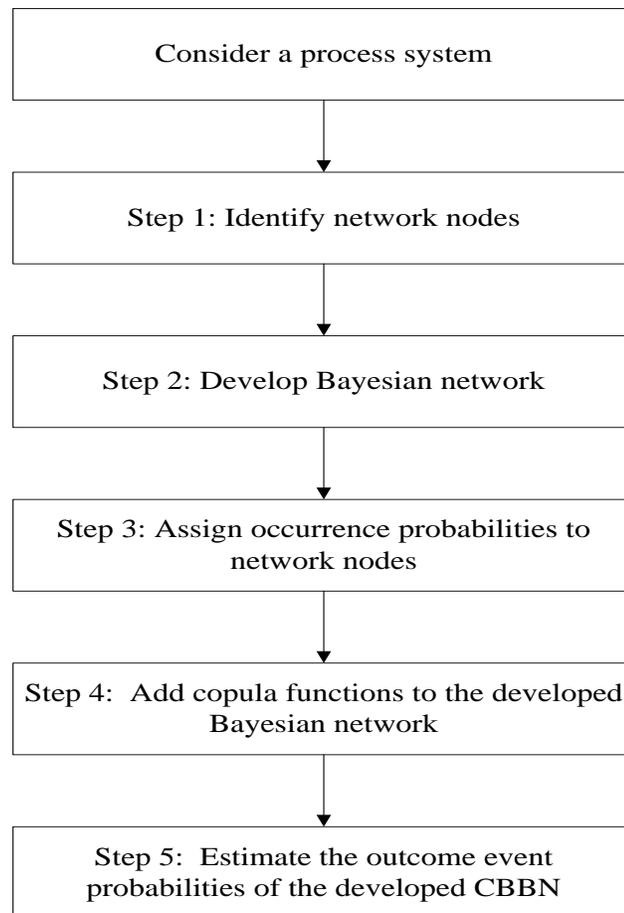


Figure 3.1 Steps for developing a CBBN.

### 3.2.1 Step 1: Identify network nodes

At first, the outcome events (OEs) of interest for the focused process system are identified. Then, the causal analysis is conducted to identify the potential root causes. These root causes and OEs are represented by network nodes in the proposed model. Each cause node has two states, occurrence and non-occurrence.

In the examples, the OEs and their causes (A, B, C) are identified. Table 3.1 presents the eight states of this OE node with the specific state combination of A, B and C.

Table 3.1 Possible outcome events based on the state combination of nodes A, B and C.

A	B	C	Outcome event
Yes	Yes	Yes	OE1
Yes	Yes	No	OE2
Yes	No	Yes	OE3
Yes	No	No	OE4
No	Yes	Yes	OE5
No	Yes	No	OE6
No	No	Yes	OE7
No	No	No	OE8

### 3.2.2 Step 2: Develop Bayesian network

Once the network nodes are identified, a Bayesian network (BN) is developed to connect the causes and the OE node. In a BN, the causal arcs between cause nodes and the OE node mean that the state of the OE node is determined by the occurrence or non-occurrence of the causes. In contrast, the arc drawn from one cause node to the other indicates that the occurrence probability of the latter cause is affected by whether the former cause occurs or not.

The BN model for the example is shown in Figure 3.2, in which A is named the root node because there are only arcs starting from it. The OE node is a leaf node with arcs merely pointing to itself, while B and C are both intermediate nodes, as there are arcs from and to them.

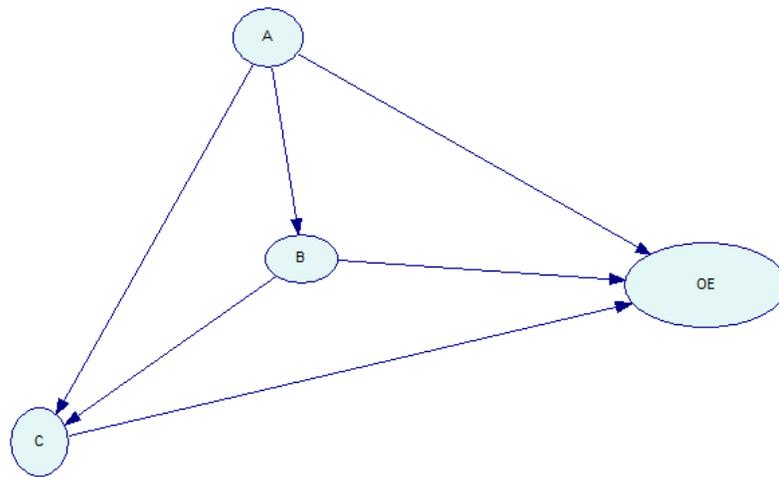


Figure 3.2 BN model for the example.

### 3.2.3 Step 3: Assign occurrence probabilities to network nodes

Occurrence probability numbers are first assigned to the respective root nodes.

Subsequently, conditional probabilities denoting conditional dependencies are assigned to the intermediate nodes. These probability numbers are estimated according to the causal relationships between the correlated nodes.

Table 3.2 presents the assumed occurrence probabilities of the network nodes in the example. For instance, the probability of  $C|A,B$  is assigned to be 0.3, meaning that the occurrence probability of C when A and B have already occurred is 0.3.

Table 3.2 Occurrence probabilities of the network nodes in the example.

Network node	Probability
A	0.3
$B A$	0.7
$B \bar{A}$	0.4
$C A,B$	0.3
$C \bar{A},B$	0.4
$C A,\bar{B}$	0.5
$C \bar{A},\bar{B}$	0.8

#### 3.2.4 Step 4: Add copula functions to the developed Bayesian network

This is the key step in building the copula-based Bayesian network (CBBN), in which copulas are employed to describe the complex dependencies among cause nodes. Copulas are useful functions that provide an easy way to create distributions modelling dependent

variables (Shemyakin and Kniazev, 2017). From the various types of multi-dimensional copulas, the multivariate normal copula, also called the Gaussian copula, has been chosen for the present study. The Gaussian copula has flexibility in modeling both positive and negative correlations (Pariyani et al., 2012). The degree of correlations is then represented by pairwise correlation coefficients between nodes, all of which compose a correlation matrix. In this way, a CBBN model is built to model both the linear and non-linear dependence within a system.

Table 3.3 presents the correlation matrix used in this example. Overall, the correlation between A and B is assumed to be positive while that between B and C is assumed to be negative. As a result, the correlation between A and C should be negative. In addition, the strength of correlations of these three pairs is assumed to be equal.

Table 3.3 Correlation parameters for the example.

	P(A)	P(B)	P(C)
P(A)	1	0.6	-0.6
P(B)	0.6	1	-0.6
P(C)	-0.6	-0.6	1

### 3.2.5 Step 5: Estimate the outcome event probabilities of the developed CBBN

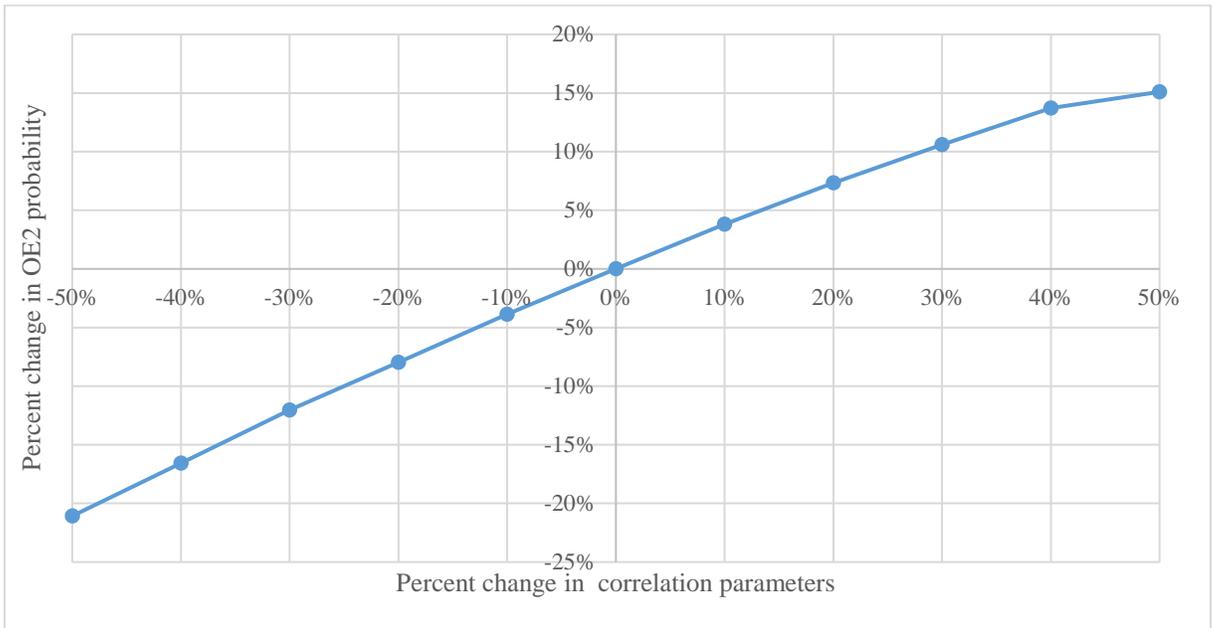
One million iterations of the Monte Carlo simulation are performed to estimate the mean occurrence probabilities of the outcome events in the developed CBBN. The simulation algorithm used here is similar to that used in Guo et al. (2018). Such simulation does bring

some computational difficulties and burdens which require programming. However, the algorithm has been realized efficiently in Matlab®.

Table 3.4 provides the outcome probabilities of the CBBN example. Moreover, to study the effect of the dependence degree on probability estimation, a sensitivity analysis for OE2 is also conducted and presented in Figure 3.3.

Table 3.4 Occurrence probabilities of the OEs for the example in BN and CBBN.

Outcome event	BN model (Deterministic value)	CBBN model (Mean value)	$\frac{P(\text{CBBN})}{P(\text{BN})}$
OE1	0.063	0.017	0.27
OE2	0.147	0.260	1.77
OE3	0.045	0.011	0.25
OE4	0.045	0.011	0.25
OE5	0.112	0.051	0.45
OE6	0.168	0.141	0.84
OE7	0.336	0.480	1.43
OE8	0.084	0.028	0.34



Correlation parameter change	-50%	-40%	-30%	-20%	-10%	0%	10%	20%	30%	40%	50%
OE2 probability	0.205	0.217	0.229	0.239	0.250	0.260	0.270	0.279	0.287	0.295	0.299
OE2 probability change	-21%	-17%	-12%	-8%	-4%	0%	4%	7%	11%	14%	15%

Figure 3.3 Variation of OE2 probability as dependence strength changes. (Data also included)

### 3.2.6 Comparison: Estimate the outcome event probabilities of the developed BN

To distinguish the proposed CBBN model from the traditional BN model, the deterministic probabilities of the outcome events in the BN are also estimated.

Taking OE7, for example, without considering copulas, the deterministic probability is simply as follows.

$$P(OE7)=P(\bar{A})P(\bar{B}|\bar{A})P(C|\bar{A},\bar{B})=[1-P(A)][1-P(B|\bar{A})]P(C|\bar{A},\bar{B})=(1-0.3)(1-0.4)0.8=0.336 \quad (3.1)$$

Similarly, other outcome event probabilities have been derived and summarized in Table 3.4.

### 3.2.7 Discussion of the results for the example

Table 3.4 shows that the probabilities of OE2 and OE 7 in CBBN are larger than those in BN. Conversely, other outcome event probabilities in CBBN are smaller than those in BN. These are caused by the effect of the copula. As Table 3.3 shows, nodes A and B are positively correlated, nodes A and C are negatively correlated, and nodes B and C are negatively correlated. Thus, when A occurs, B also tends to occur. Subsequently, C tends to not occur, leading to the increased probability of OE2. In contrast, when neither A nor B occurs, C is more likely to occur, which increases OE7 probability.

As can be seen in Figure 3.3, OE2 probability rises steadily as the dependence grows stronger. When there is a small increase or decrease in correlation parameters ( $\rho$ ), the absolute change in OE2 probability is almost identical. For instance, a +20% or -20%

change in correlation parameters only results in about a +7% or -8% change, respectively, in estimated OE2 probability, which are very close. Nevertheless, as the percentage change of correlation parameters becomes larger, the probability of OE2 decreases faster than it increases. Figure 3.3 shows that OE2 probability falls by 21%, compared to its increase of only 15% when the deviation in strength of dependence is 50%. Specifically, a -50% fall in correlation parameters (i.e.,  $|\rho|=0.3$ ) decreases OE2 probability to 0.205, approaching its value calculated in the case of the BN model (0.147). In contrast, a +50% rise in correlation parameters (i.e.,  $|\rho|=0.9$ ) increases OE2 probability to 0.299, near the probability of node A (0.3). This phenomenon was illustrated in Guo et al. (2018), which argued that for AND logic, the probability of an outcome tends to approach the minimal probabilities of its causation factors when there is high dependence among them. As a result, the maximal increased OE2 probability can only reach 0.3, showing that the limit of increase in the probability of OE2 is smaller than that of decrease.

To conclude this sensitivity analysis, the percent change in OE2 probability is symmetrical when the percent change in correlation parameters is relatively small but then becomes asymmetrical when the extent of change in correlation parameters is larger.

### **3.3 Application of the copula-based Bayesian network**

To validate the proposed CBBN model for modeling complex dependencies in risk analysis, a practical case report issued by the U.S. Chemical Safety and Hazard Identification Board is examined. As can be found in the incident report, the June 13, 2013 reboiler rupture, explosion and fire at the Williams Geismar Olefins Plant caused 2 fatalities and 167 injuries

(CSB, 2016). The ruptured reboiler used to be part of the propylene fractionator shown in Figure 3.4. It is notable that there are two reboilers: Reboiler A and Reboiler B, and both operated continuously in the original design. When the reboilers were fouled and needed cleaning, the process had to be shut down. In 2001, block valves (gate and ball valves) were installed on the reboiler piping. In this way, the process could continue to work with only one reboiler operating when the other fouled reboiler required maintenance. This fouled reboiler was then cleaned and set on standby mode (CSB, 2016).

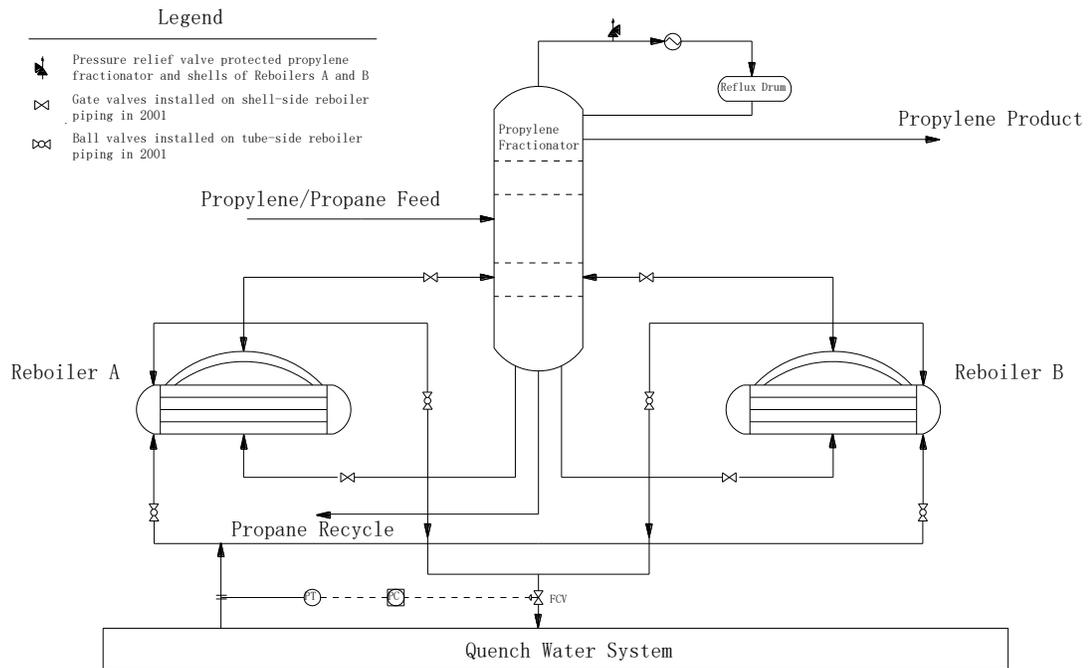


Figure 3.4 Propylene fractionator column (CSB,2016).

### 3.3.1 Steps 1-2: Identify network nodes and develop Bayesian network

Through detailed causal analysis of this incident, the root, intermediate and leaf nodes are identified. Linking all the nodes, the Bayesian network for this accident scenario is developed. Figure 3.5 shows the BN, where propane mixture releases and boiling liquid expanding vapor explosion (BLEVE) is the top event. As the performances of the safety barriers vary, this top event may result in various outcomes of different severities. These outcome events are presented in Table 3.5, in which OE3 and OE6 are considered to be major events since fatalities will occur. It should be clarified that the consideration of the safety barriers and the classification of the outcome events are partly based on (Markowski and Kotynia, 2011; Khakzad et al., 2013).

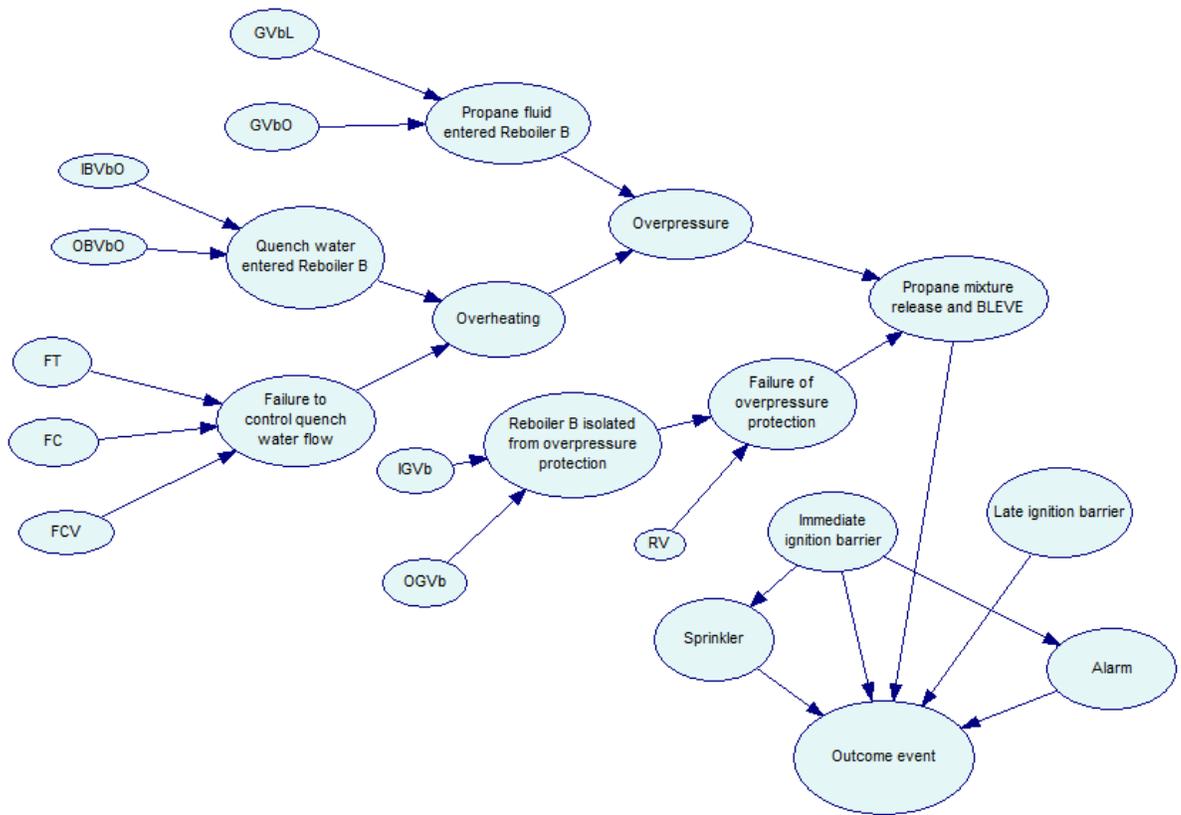


Figure 3.5 Bayesian network for propane release from Reboiler B.

Table 3.5 Outcome event nodes depending on the performance of safety nodes.

Immediate ignition barrier	Late ignition barrier	Sprinkler	Alarm	Outcome event
work	work	-	-	Dispersion (OE1)
work	fail	work	-	Dispersion (OE1)
work	fail	fail	work	Vapor cloud explosion (VCE)/Flash fire (FF) (OE2)
work	fail	fail	fail	VCE/FF and fatalities (OE3)

fail	-	work	-	Fire extinguished (OE4)
fail	-	fail	work	Fireball (OE5)
fail	-	fail	fail	Fireball and fatalities (OE6)

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### 3.3.2 Step 3: Determine occurrence probabilities of network nodes

To conduct an in-depth investigation and simulation of this real incident, occurrence probabilities of the cause nodes are assumed to follow time-dependent exponential distribution. According to the case study report (CSB, 2016), the propylene fractionator with block valves had operated for 12 years and Reboiler B had remained on standby for a period of 16 months when the incident occurred. Taking into account both the operation time and the failure frequencies collected from CCPS (2014), the failure probabilities of these causes are calculated and illustrated in Table 3.6.

For example, the reliability of the flow transmitter after 12 years of service, denoted by  $R(FT)$ , equals  $e^{-0.01/yr \times 12yrs} = 0.887$ . Therefore, the failure probability  $P(FT)$  equals  $[1 - R(FT)] = 0.113$ . Similarly, the probability that the gate valves were mistakenly opened during the 16 months (approximately 1.33 yrs) while Reboiler B was on standby is  $1 - e^{-0.01/yr \times 1.33yrs} = 0.125$ .

Table 3.6 Occurrence probabilities of the cause nodes.

Network node	Symbol	Frequency (/yr)	Probability
Gate valves leaked	GVbL	0.001	0.012

Failure of quench water flow transmitter	FT	0.01	0.113
Failure of quench water flow controller	FC	0.01	0.113
Failure of quench water flow controlling valve	FCV	0.1	0.699
Failure of inlet gate valve due to plugging, choking, structural flaw	IGVb	0.1	0.699
Failure of outlet gate valve due to plugging, choking, structural flaw	OGVb	0.1	0.699
Failure of relief valve	RV	0.01	0.113
Gate valves mistakenly opened	GVbO	0.1	0.125
Inlet ball valve mistakenly opened	IBVbO	0.1	0.125
Outlet ball valve mistakenly opened	OBVbO	0.1	0.125
Propane fluid entered Reboiler B	-	-	OR gate
Quench water entered Reboiler B	-	-	AND gate
Failure to control quench water flow	-	-	OR gate
Overheating	-	-	AND gate
Reboiler B isolated from overpressure protection	-	-	AND gate

Failure of overpressure protection	-	-	OR gate
Overpressure	-	-	AND gate
Propane mixture release and BLEVE	-	-	AND gate

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Table 3.7 Safety nodes and their probabilities (CCPS (2001); OREDA (2002)).

Network node	Symbol	Probability
Failure of immediate ignition barrier	IIB	0.2
Failure of late ignition barrier	LIB	0.5
Failure of sprinkler	SP	0.150, 0.04
Failure of alarm	AL	0.225, 0.13

While determining the failure probabilities of the safety nodes, conditional dependencies are considered. It is shown in Table 3.7 that there are two probability numbers for either sprinkler or alarm. This means that failure probabilities of the sprinkler and alarm depend on the performance of immediate and late ignition barriers (IIB and LIB). In the case of IIB works but LIB fails or in other words late ignition occurs, the failure probabilities of the sprinkler and alarm are higher (0.15 and 0.225, respectively). In the other case of IIB fails, i.e. released propane ignites immediately, the failure probabilities are lower, being 0.04 and 0.13.

### 3.3.3 Step 4: Integrate copula functions to the developed Bayesian network

After analyzing the possible interrelationships of the network nodes, reasonable correlation matrices are designed and shown in Tables 3.8-3.11. As Table 3.8 shows, the correlation parameter ( $\rho$ ) between the causes that let quench water enter Reboiler B is assumed as 0.8, which means there is very significant dependence. This is because it is likely that both the inlet ball valve (IBVbO) and the outlet ball valve (OBVbO) of Reboiler B were mistakenly opened. For a similar reason, the failures of the inlet gate valve (IGVb) and the outlet gate valve (OGVb) probability occur simultaneously, justifying the assumption that the  $\rho$  between these two should be 0.7, indicating significant dependence, as shown in Table 3.10. Table 3.9 shows that the failures of the flow transmitter (FT), flow controller (FC) and flow controlling valve (FCV) are moderately dependent (i.e.  $\rho=0.6$ ) since these components make up the quench water control system and therefore work under the same environment. Table 3.11 presents the dependence strengths among the safety nodes represented by pairwise correlation parameters. On average, it is assumed that the dependence between a safety node and its closer neighboring node is more significant due to potentially stronger interactions.

The Gaussian copulas with these correlation parameters are then added to the existing BN, building the CBBN, where both linear and non-linear dependence of the network nodes are considered.

Table 3.8 Correlation parameters between the causes of quench water entering Reboiler B.

	P(IBVbO)	P(OBVbO)
P(IBVbO)	1	0.8
P(OBVbO)	0.8	1

Table 3.9. Correlation parameters within quench water flow control system.

	P(FT)	P(FC)	P(FV)
P(FT)	1	0.6	0.6
P(FC)	0.6	1	0.6
P(FV)	0.6	0.6	1

Table 3.10 Correlation parameters between the causes of Reboiler B isolated from overpressure protection.

	P(IGVb)	P(OGVb)
P(IGVb)	1	0.7
P(OGVb)	0.7	1

Table 3.11 Correlation parameters among safety nodes.

	P(IIB)	P(LIB)	P(SP)	P(AL)
P(IIB)	1	0.8	0.7	0.6
P(LIB)	0.8	1	0.8	0.7
P(SP)	0.7	0.8	1	0.8
P(AL)	0.6	0.7	0.8	1

### 3.3.4 Step 5: Estimate the top event and outcome event probabilities of the developed CBBN

Probabilistic simulations of 1 million trials employing the algorithm described in Section 2.5 are performed to estimate the mean probabilities of the top and outcome events. Results are presented in Table 3.12.

Table 3.12 Result summary of occurrence probabilities of the top event and outcome events in both BN and CBBN.

Symbol	Event	BN model (Discrete value)	CBBN model (Mean value)	$\frac{P(\text{CBBN})}{P(\text{BN})}$
TE	Propane mixture release and BLEVE	8.83E-04	4.50E-03	5.10
OE1	Dispersion	6.53E-04	3.36E-03	5.15
OE2	VCE/FF	4.11E-05	7.23E-05	1.76
OE3	VCE/FF and fatalities	1.19E-05	1.64E-04	13.82
OE4	Fire extinguished	1.70E-04	7.54E-04	4.44
OE5	Fireball	6.14E-06	2.43E-05	3.96
OE6	Fireball and fatalities	9.18E-07	1.21E-04	132.08

### 3.3.5 Comparison: Estimate the top event and outcome event probabilities of the developed BN

As a comparison, the developed Bayesian network model, which only captures linear dependence, is also studied. The probabilities of the top event and OE6 are calculated as shown below:

$$P(TE)=(GVbL+GVbO-GVbL\times GVbO)IBVbO\times OBVbO(FT+FC+FCV-FT\times FC-FC\times FCV-FT\times FCV+FT\times FC\times FCV)(IGVb\times OGVb+RV-IGVb\times OGVb\times RV) \quad (3.2)$$

$$P(OE6)=P(TE)P(IIB)P(SP|IIB)P(AL|IIB) \quad (3.3)$$

Where  $GVbL$ ,  $GVbO$ , ...,  $P(AL|IIB)$  represent the respective probabilities in Tables 3.6 and 3.7. Similarly, the probabilities of other outcome events are derived and presented in Table 3.12.

## 3.4 Discussion

### 3.4.1 The top event probability in CBBN and BN

Table 3.12 shows that the probability of the top event in CBBN is significantly larger than that in BN, which can be explained by the effect of non-linear dependencies of the root nodes as defined by Tables 3.8-3.10. Root nodes  $IBVbO$  and  $OBVbO$  are positively correlated under the AND gate and so are the nodes  $IGVb$  and  $OGVb$ . This correlation leads to the increased probabilities of the respective intermediate nodes, which tend to approach 0.125 and 0.699, almost 8 and 1.43 times as large as the BN case. This increase

finally contributes to the increased top event probability. There is an OR gate connecting the dependent root nodes FT, FC and FV, causing the decreased probability of the intermediate node Failure to control quench water flow. However, this probability only drops from 0.763 to nearly 0.699. Such an increase for the AND gate as well as a decrease for the OR gate in intermediate node probabilities are explained in (Guo et al., 2018). As Eq. (3.2) shows, compared to the significant increased probabilities resulting from the dependence within AND gates, this decrease does not have a large effect on the top event probability. As a result, the probability of propane release and BLEVE is 5.10 times as large in CBBN as in BN.

### **3.4.2 The outcome event probabilities in CBBN and BN**

Table 3.12 shows that the probabilities of all the outcome events are also obviously greater in CBBN than in BN. In the BN for this scenario, only the conditional dependence of the performances of the sprinkler and alarm on the performance of the immediate ignition barrier are defined by Table 3.7. While in CBBN, non-linear dependence is also incorporated by correlation parameters as presented in Tables 3.8-3.11. The increased top event probability as explained in Section 4.1 accounts for the increase of all the outcome event probabilities. In particular, the probabilities of OE3 and OE6, where fatalities occur, increase most sharply. This is because the positive non-linear dependence among the safety nodes increases the occurrence probability that most or all safety barriers fail at the same time, which results in OE3 or OE6. Quantitatively speaking, similar to the demonstration

in Section 3.2.7, the occurrence probability of OE3 and OE6 in CBBN gets close to the failure probability of the sprinkler under late ignition and under immediate ignition, respectively. Therefore, as is shown in Table 3.12, the probabilistic differences in terms of the ratio between CBBN and BN for OE3 and OE6 are 13.82 and 132.08, respectively.

### **3.5 Sensitivity analysis**

Figures 3.6 and 3.7 show the tornado diagrams of the sensitivity analyses for OE6 in the developed BN and CBBN, respectively. The horizontal axis shows the absolute change in the posterior probability of OE6 when the probability of each initiating event or safety barrier changes by 20%.

As can be seen from Figure 3.6, Failure of immediate ignition barrier (IIB), Outlet ball valve mistakenly opened (OBVbO), Inlet ball valve mistakenly opened (IBVbO), Alarm failure given that immediate ignition barrier fails (AL|IIB) and Failure of sprinkler given that immediate ignition barrier fails (SP|IIB) are the most and equally sensitive causes for OE6 in the developed BN. As Eq. (3.3) shows, a 20% change in the probability of any one from these 5 nodes results in exactly a 20% change in OE6 probability.

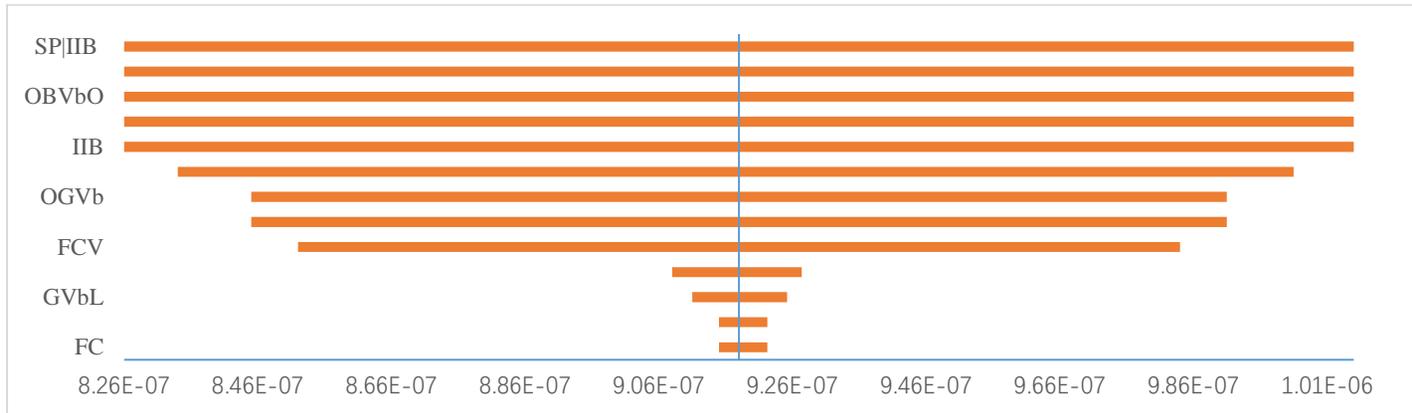


Figure 3.6 Sensitivity analysis for OE6 in BN.

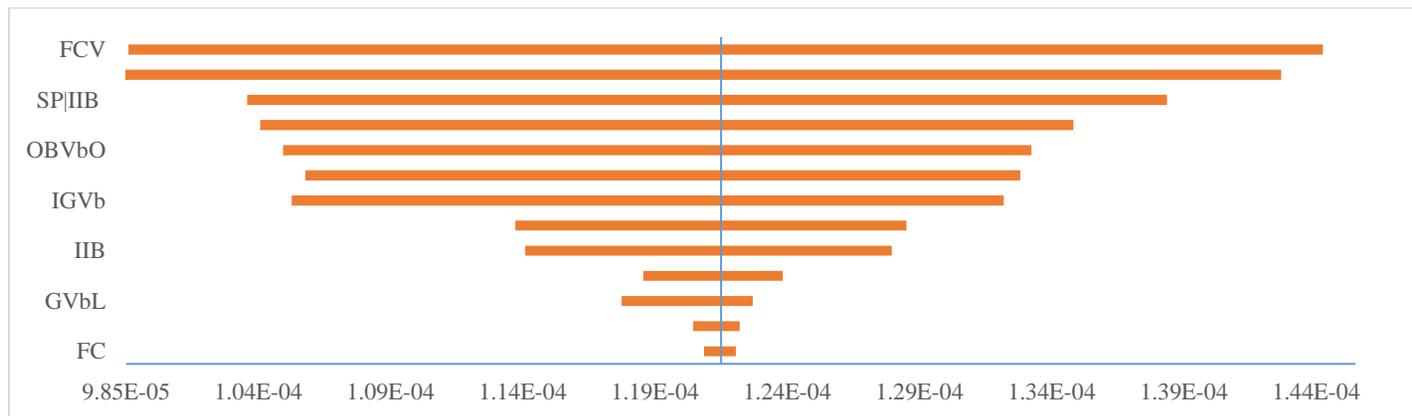


Figure 3.7 Sensitivity analysis for OE6 in CBBN.

In contrast, Figure 3.7 shows that Failure of quench water flow controlling valve (FCV) is the most sensitive cause for OE6 in the developed CBBN. This is due to the effect of dependence on the OR gate: the probability of the intermediate node tends to approach the maximal root node probability when there is dependence among root nodes. In this case, FCV accounts for the largest failure probability leading to the intermediate node Failure to control quench water flow. Consequently, FCV is dominant in determining the probability of this intermediate node and thus significantly affects the top event and then OE6 probability. Gate valves mistakenly opened (GVbO) is shown to be the second most sensitive parameter because its probability is much larger than the Gate valves leaked (GVbL) probability.

Figure 3.7 also shows that OE6 is thirdly sensitive to SP|IIB. The reason is concerned with the AND logic of an event tree, which means that OE6 probability depends more on the safety nodes with smaller probabilities. Therefore, the impact of AL|IIB and IIB becomes less significant in comparison with the case of BN. In addition, it is clear that IBVbO and OBVbO both rank as the 4<sup>th</sup> sensitive parameters. This is because of the effect of dependence on the AND gate: the intermediate node probability will get closer to the minimal root node probability when the dependence is considered. Returning to this example, the initial probability of Quench water entering Reboiler B is close to 0.699, the probability value of both IBVbO and OBVbO. When the probability value of IBVbO decreases by 20%, for instance, the intermediate node probability will get closer to this decreased probability number of IBVbO, resulting in a sharp decrease in OE6 probability. In contrast, the probability of OE6 does not change as considerably as the former analysis

for decrease when there is a 20% increase in IBVbO probability, because the intermediate node probability still tends to approach 0.699, which is the probability of OBVbO as the minimal probability.

### **3.6 Probability updating**

Probability updating is usually performed to find the most probable causes of a specific outcome event (Abimbola et al., 2015). Abimbola et al. (2015) and Khakzad et al. (2013) have conducted exhaustive updating of node probabilities in a Bayesian network based on Bayes theorem. In this paper, the updating analysis is performed by GeNIe 2.1 (<https://www.bayesfusion.com/genie/>).

According to the accident report (CSB, 2016), a rupture in Reboiler B caused BLEVE and a large amount of propane mixture release, which ignited, resulting in a fireball and two fatalities. Therefore, the state of node Outcome event is instantiated to OE6. Figure 3.8 shows the posterior probabilities of the other nodes based on this evidence. The most probable causes of OE6 are determined to be as follows. The gate valves were mistakenly opened, letting propane fluid enter Reboiler B. Quench water entered Reboiler B by mistakenly opened ball valves and the failure of the quench water flow controlling system due to the failure of the flow controlling valve, introducing heat to Reboiler B. The propane mixture was continuously heated, leading to overpressure. Unfortunately, the gate valves failed, isolating Reboiler B from the overpressure protection system. Reboiler B finally ruptured, causing BLEVE and then a fireball, killing two workers nearby, because of the failure of the immediate ignition barrier, sprinkler and alarm. This diagnostic analysis

matches CSB (2016).

Specifically, the updated occurrence probabilities of root nodes are presented in Table 3.13 by conducting backward propagation. It is shown that the main contributing factors are Gate valves leaked and Gate valves mistakenly opened, whose posterior probabilities are more than 7 times as much as their prior probabilities.

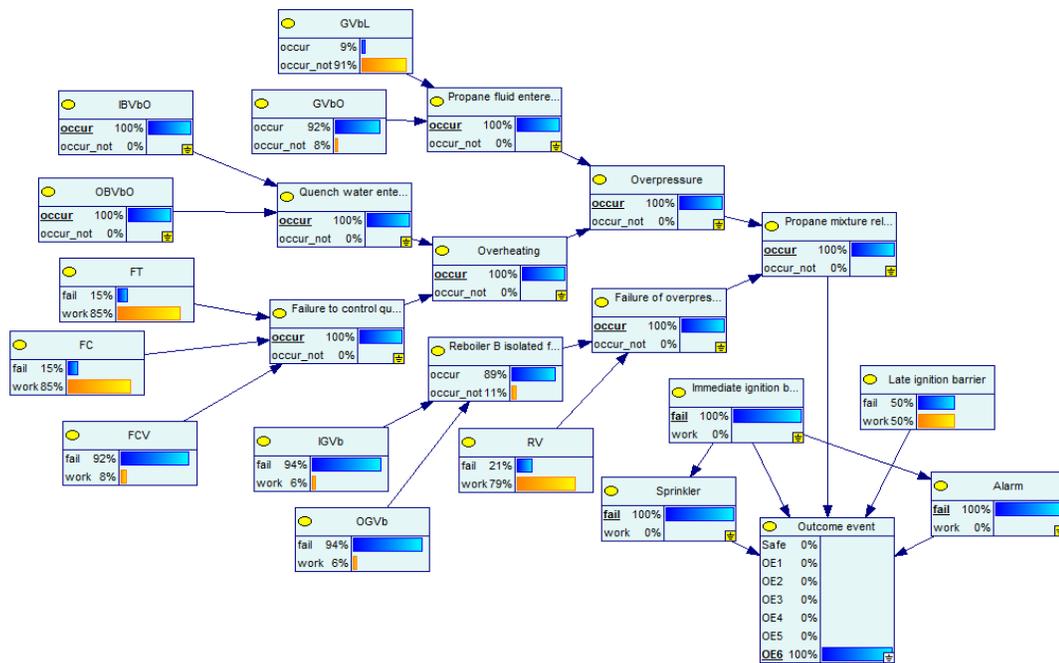


Figure 3.8 Diagnostic analysis of OE6.

Table 3.13 Updated probabilities of the nodes for OE6.

Network node	Symbol	Prior probability (Pi)	Posterior Probability (Pp)	Ratio (Pp/Pi)
Gate valves leaked	GVbL	0.012	0.089	7.46

Failure of quench water flow transmitter	FT	0.113	0.148	1.31
Failure of quench water flow controller	FC	0.113	0.148	1.31
Failure of quench water flow controlling valve	FCV	0.699	0.916	1.31
Failure of inlet gate valve due to plugging, choking, structural flaw	IGVb	0.699	0.938	1.34
Failure of outlet gate valve due to plugging, choking, structural flaw	OGVb	0.699	0.938	1.34
Failure of relief valve	RV	0.113	0.207	1.83
Gate valves mistakenly opened	GVbO	0.125	0.923	7.39

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### 3.7 Conclusions

The proposed copula-based Bayesian network model is a robust risk assessment model that preserves the strength of BN and copulas in modelling conditional non-linear dependencies. BN is able to model the cause-effect relationships between components and copulas extend conditional dependence to stochastic dependence of higher-level complexity. Moreover, the application of BN makes it possible to make probability updating and dynamic probability estimations. Such a novel combination successfully overcomes the limitations of using either the traditional BN model alone or copulas with other quantitative risk analysis approaches.

The proposed model is tested on a real case study. The results of the proposed model are compared with the results of a traditional BN. It is observed that the non-linear dependence modeled by copulas yields significant increases in outcome probabilities, which are closer to reality. This highlights the significance of dependence among causes on the occurrence of undesired events. Moreover, this case study proves that the CBBN model is innovative and scientifically viable to be implemented to industry.

The proposed revised model illustrates the use of copulas in a very simple and easy to implement way. It captures the inherently complex dependencies of process variables, e.g., common failure modes. Sensitivity analysis presents the crucial factors that affect the accident scenario. A diagnostic analysis is also performed, showing the most likely the causes of the BLEVE and propane release. Results confirm the effectiveness of this model. Results confirm the advantage of this model against other similar approaches.

The proposed model needs further tests to ensure its wider applicability. This work can also be improved by considering advanced algorithms for data processing and estimating correlation parameters.

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## Chapter 4. Summary

Quantitative risk analysis (QRA) currently plays an important role in risk assessment and safety management throughout the life cycle of process installations. To lower risks in the earliest stage, QRA can be used in risk-based design, which focuses on inherent safety. However, QRA is more often performed in the installation phase, which is after the completion of the equipment layout and safety measures. In the Introduction and Overview section of this thesis, a review of several popular QRA techniques and their strengths and limitations is presented. As is explained, most existing QRA techniques, such as bow-tie, do not take dependent failures into account. Although the Bayesian network incorporates linear dependency into the risk analysis process, it cannot model non-linear dependency in complex process systems.

To meet the need for the risk estimation of systems with complex dependencies, the thesis proposes revisions of two traditional QRA methods by integrating copula functions. One revised model is copula-based bow-tie (CBBT), which fits integrated systems where dependency exists. The other is copula-based Bayesian network (CBBN), which is considered more generally applicable than CBBT, since it captures both mutual and stochastic dependencies by combining BN and copula functions.

### 4.1 Conclusions

Both models have been applied to practical cases that occur in chemical installation sites. The causes and outcomes of accident scenarios are first identified. Monte Carlo simulations

are then employed while running these two copula-based models to count the mean occurrence times of all the possible outcomes. These simulation results are compared with the calculated deterministic probabilities from traditional bow-tie or Bayesian network analysis. As is observed, the probabilities of severe outcome events, where all the safety barriers fail to function, are considerably larger in copula-based models. This observation shows the great influence of dependence among safety barriers on the occurrence of accidents. It is also shown that the ignorance of potential dependency might result in an underestimated risk. To reduce the risk caused by dependence effects, more independent safety barriers are recommended to be integrated into process systems, if possible.

The proposed models demonstrate the use of copula in a simple and straightforward way. The stochastic and non-linear dependencies among process variables, such as common failure modes, are represented by means of copulas. Hence, these two copula-based models can be employed as useful approaches when performing the risk assessment of complex process systems with inherent dependencies. The specific conclusions for each model are presented separately in the following subsections.

#### **4.1.1 Development of copula-based bow-tie model**

By integrating the stochastic dependencies among causes and bow-tie analysis, a copula-based bow-tie model (CBBT) is developed. This revised model is first tested to study the effect of dependence among initiating events on AND gates & OR gates. It has been proven that positive dependence will increase the probability of an AND gate while decreasing the probability of an OR gate. It also shows that as dependence is more significant, an AND

gate probability value becomes closer to the minimal initiating event probability. In contrast, the probability value of an OR gate tends to approach the maximal probability value of all the initiating events. These interesting findings highlight the importance of monitoring potential dependent initiating events that may cause abnormal conditions so as to prevent top events.

#### **4.1.2 Development of copula-based Bayesian network model**

This thesis proposes a copula-based Bayesian network model, which is a powerful tool for modeling cause-effect relationships and conditional and stochastic dependencies. This model is applied to a real-life case study about a disaster resulting from a reboiler rupture. Causal analysis is performed and presented in the form of a Bayesian network. Simulation results indicate that some dependent failures should be blamed for causing the BLEVE and propane release, which match what occurred in reality. Sensitivity analysis identifies the safety systems that need more inspection and maintenance.

#### **4.2 Future work**

The two models may be examined by a broader scope of contexts beyond chemical processing industries to increase their applicability. Also, one of the limitations of the present research is that when determining failure probabilities, expert opinions rather than historical records are used. If sufficient on-site failure data become available, the proposed methodologies will be more useful. Whether or not there are dependencies should first be

analyzed. Subsequently, if dependencies do exist, advanced algorithms for processing data to estimate correlation parameters and model such dependencies should be further explored. Last, the incorporation of accident precursors to allow dynamic updates of the estimated probabilities in these copula-based models is an interesting subject for future research.