THEORETICAL AND NUMERICAL INVESTIGATION OF THE RHEOLOGY OF HEAVY CRUDE OIL

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ABSTRACT

The rheological study of heavy crude oil is important in the field of petroleum engineering. The rheological properties of heavy oil (e.g., shear stress, shear rate, viscosity, etc.) depend on several factors including temperature, pressure, surface tension, diluent type and diluent composition, pH, shear stress and thermal histories, memory, and shear conditions during the analysis. The investigation of the rheology of heavy crude oil flow is a critical issue for both upstream and downstream operations. The objective of this study is to perform an investigation on the rheological properties of heavy crude oil to show the effect of shear rate, temperature, and pressure on the viscosity and the shear stress. The aim of this work was to broaden current knowledge of the rheological behavior and flow characterization of heavy crude oil. This paper takes a new look at the shear stress-strain relationship by considering the memory effect along with temperature effect on the shear rate.

It is considered that the viscosity of the heavy crude oil is a function of pressure, temperature, and shear rate. As the heavy crude oil is considered as a Bingham fluid, Bingham model is employed here for the analysis. The experimental data from previous studies are used to complete the analysis. To develop the model, a modified Darcy's law that employs the effect of memory on the Bingham model is considered. The effect of temperature has been incorporated by the Arrhenius equation for the development of a new model to study the heavy crude oil rheological behaviors. The relationship between shear stress and viscosity has been shown at different fractional derivative order and time. The validation and the simulation of the model are performed by using the experimental and the field data from the literature. The numerical simulation of this model is conducted by using the MATLAB simulation software.

From the sensitivity analysis, it is found that the temperature has the highest impact on the viscosity over the pressure and the shear rate. On the other hand, the pressure shows a strong effect on the shear stress-shear rate relationship over the temperature. In the model analysis, it is found that the fluid memory affects in the Bingham model due to nonlinear behavior of heavy crude oil. The shear stress increases with decreasing viscosity at different fractional derivative order and time. The change in shear stress is high at large fractional derivative. The range of fractional derivative order is from 0.2 to 0.8. When fractional derivative order, $\alpha_1 = 0.8$, it shows a big change in the

viscosity and shear stress relationship compared to $\alpha_1 = 0.2$. The shear stress also increases with viscosity as the time passes. Generally, it happens during the heavy crude oil production. The proposed model is validated with experimental data that shows a good match at $\alpha_1 = 0.3$. The simulation results show that the trends are the same for the viscosity-shear stress relationship when the memory effect is considered. In this context, we try to compare the effects of pressure, temperature, and shear rate on the viscosity and the shear stress-shear rate relationship, and to develop a model by considering temperature and memory effects for heavy crude oil. The study of heavy oil rheology has a great effect on transportation and processing standpoints. The different rheological measurements through quantitative experimental simulation of shear and thermal effects can significantly affect the pipeline design. We believe that we found an innovative solution to model the heavy crude oil rheology.

DEDICATION

To my dearest parents, my beloved husband, and daughter

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CO-AUTHORSHIP STATEMENT

I, Fatema Akter Happy, hold the primary author status for all chapters in the thesis. However, each manuscript is co-authored by my supervisor and other co-researchers whose contributions have facilitated the development of this work as described below.

- **Fatema Akter Happy, Arifur Rahman, Syed Imtiaz, M. Enamul Hossain. "A Critical Review on Engineering Approach in the Context of Memory Concept for Fluid Flow through Porous Media", submitted to the Journal of Porous Media (Responded to the Reviewer Comments)**

Statement: I, Fatema Akter Happy, the primary author and carried out the research and development of the mathematical models of heavy crude oil flowing through porous media. I drafted the manuscript and incorporated the comments of the co-authors in the final manuscript. Co-authors helped me to select the appropriate model for the modeling of heavy crude oil

- **Fatema Akter Happy, Syed Imtiaz, M. Enamul Hossain. "A Comparative Study of the Effects of Temperature, Pressure, and Shear Rate with Viscosity in the Shear Stress-strain Relationship", to be submitted to the journal.**

Statement: I, Fatema Akter Happy, the primary author and carried out the research and development of the mathematical models of heavy crude oil flowing through porous media. I drafted the manuscript and incorporated the comments of the co-authors in the final manuscript. Co-authors helped me to conceive the data and to do the computational works.

- **Fatema Akter Happy, Syed Imtiaz, M. Enamul Hossain. "A Modified Memory-Based Bingham Model for Heavy Crude Oil", to be submitted to the journal.**

Statement: I, Fatema Akter Happy, the primary author and carried out the research and development of the mathematical models of heavy crude oil flowing through porous media. I drafted the manuscript and incorporated the comments of the co-authors in the final manuscript. Co-authors helped me to conceive the data and to do the computational works.

TABLE OF CONTENTS

Contents

LIST OF TABLES

LIST OF FIGURES

CHAPTER ONE

Introduction

1.1 Introduction

Crude oil has high importance in the petroleum industry. The rheological behavior of crude oil depends on a wide range of conditions including pressure, temperature, surface tension, etc. They have severe effects on the rheological properties of the crude oil. Many coils have low asphaltene content. They exhibit Newtonian fluid and single-phase behavior with low viscosity. Some crude oils do not follow the conventional Newtonian structure due to their high viscous behavior. They can only flow after applying a minimum pressure gradient, e.g., threshold pressure gradient (TPG). These types of fluids are called non-Newtonian fluids with yield property. The heavy crude oil and waxy crude oil shows non-Newtonian characteristics with yield property (Chang *et al*., 1998) and thixotropy property (Zhao and Wang, 2006).

The rheological behavior of heavy crude oil plays a vital role for the flow characterization through porous media and the rheology of heavy crude oil is heavily related to the oil recovery and well productivity (Chen and Lu, 1990; Prada and Civan, 1999; Henaut, 2003). Heavy crude oil is defined by high density and high viscosity. It differs from the rheology of light crude oil due to its viscosity and density. The flow of heavy crude oil is characterized as a non-linear seepage process at reservoir condition. The non-linear seepage process makes the situation more difficult for the flow of heavy crude oil. For the development of heavy crude oil, oil must flow from the reservoir to the well and crude oil must lift from well to the surface continuously (Chen and Lu, 1990; Alshmakhy and Maini, 2011). Therefore, it has become an important task to study the rheological behavior of heavy crude oil in the wellbore and the reservoir. There are numerous numerical and experimental studies based on the characteristics of heavy crude oil flow through porous media. Generally, three different ways are used to characterize the flow and the rheology of heavy crude oil, e.g., the composition of heavy crude oil, the rheological property of heavy crude oil, and thixotropic property. The rheology means the study of flow and the deformation of fluids after applying a certain amount of stress. Generally, shear stress means the amount of force per unit area and the shear rate means the deformation rate. For example, if two plates are moving, the velocity gradient between two plates divided by the distance is called the shear rate. The fluid will deform continuously until the applied force is not removed. Wu *et al*. (1992) investigated that the heavy crude oil does not follow Newtonian fluid's behavior. It follows Bingham fluids behavior. Bingham fluids only deform after the applying pressure gradient reaches the minimum pressure gradient. So, for the development of the model, it is assumed that heavy crude oil acts like a Bingham fluid and follows Bingham equation. Heavy crude oil does not also obey the classical Darcy's law, and it can be approximated by a Bingham fluid (Mirzadjanzade *et al.*, 1971). In this case, a modified Darcy's law is used in the theoretical development process that considers the incorporation of the threshold pressure gradient. It is also important to consider the effect of temperature, pressure, surface tension, memory, etc. to define the rheology of the heavy crude oil. The viscosity of heavy crude oil also decreases with increasing temperature and increases with increasing pressure. When heavy crude oil flows in porous media, there will be solid-liquid or liquid-liquid interactions. These phenomena will cause chemical dissolution of the medium and enlarging pores, swelling and flocculation, pore plugging and precipitation reactions, and transport of particles is obstructing the pores to change the porosity, permeability, viscosity and so on. This is called the memory effect. The change in viscosity affects the crude oil element under shear force and changes the crude oil behavior. To account the effect of memory n the shear stress and strain rate relationship, a fractional derivative is introduced in the strain rate, and a modified Darcy's law from Caputo (1998) are used in the Bingham equation. Though the effect of memory is less, it should be considered to characterize the rheology of heavy crude oil for porous media.

1.2 Objectives

The objectives of this thesis are to provide the theoretical and numerical description of the rheology of heavy crude oil. This study presents a better understanding of the major parameters affecting the heavy crude oil rheology. It has been shown that the shear rate, temperature, and pressure influence the crude oil rheology. A good agreement has been established with the literature that the temperature is a dominant factor over the shear rate and pressure on the viscosity. The effects of pressure and temperature are also incorporated in the Bingham model. A new mathematical model that considers the effect of temperature including memory effect is proposed. A detailed review of the memory effect has also been offered in this thesis. Numerical investigations of the proposed model show that the temperature and memory have considerable effects on the shear stress-strain relationship. We propose these effects in the modified Bingham equations for heavy crude oil in porous media, as there are very limited theoretical and numerical studies for the rheology of the heavy crude oil. The validation is done by comparing with other models available in the literature.

1.3 Thesis Outline

This rest of the thesis is ordered as follows: Chapter 2 presents the literature review on the non-Newtonian fluids as the crude oil as well as heavy crude oil is considered as a non-Newtonian fluid; available shear stress-strain models and viscosity models for heavy crude oil; heavy crude oil as a Bingham fluid; introduction of fractional derivative in the Bingham model, and the knowledge gaps in the research are provided. A detailed review of memory formalism has been included in chapter 3. Chapter 4 provides an investigation of shear rate, temperature, and pressure effects on the Bingham model and the viscosity. A new approach that considers the incorporation of the fractional derivative for showing the memory effect in the Bingham equations is presented. Finally, chapter 5 presents the conclusion of the thesis that highlights the overall contribution of the thesis and some guidelines are recommended for the future study.

CHAPTER TWO
Literature Review

In this chapter, the existing non-Newtonian fluid flow models for porous media and the available models for crude oil are reviewed. The rheological behavior of heavy crude oil is also discussed in this paper. The main objective of this study is to review the rheological model emphasizing the Bingham model. This study shows the flow of heavy crude oil as a Bingham fluid that considers the memory effect. A brief discussion about the importance of fractional derivative for crude oil in porous media is also concluded

2.1 Non-Newtonian Fluid

The rheology of the fluid and the morphology of the medium affects the fluid flow through porous media. There are numerous applications of porous media that deals with the Newtonian fluid. A Newtonian fluid obeys Newton's laws of viscosity, which is the shear stress is proportional to the shear rate. The mathematical expression of this law is

$$\tau = \mu\dot{\gamma} \tag{2.1}$$

Where τ is the shear stress, $\dot{\gamma}$ is the shear rate, and μ is the dynamic viscosity which defines the ratio of the shear stress and the shear rate. The dynamic viscosity does not depend on the shear rate; however, sometimes it is affected by pressure and temperature (Skelland, 1967; Chhabra and Richardson, 1999). A non-Newtonian fluid is a fluid which does not follow the characteristics of Newtonian fluid. The non-Newtonian fluid depends on the shear rate or the history of shear rate. This history of shear rate is also known as memory. The flow of the non-Newtonian fluid is affected by pressure, temperature, surface tension, and fluid memory (Hossain *et al*., 2007). Some examples of the non-Newtonian fluids are, blood, toothpaste, salt solutions, waxy crude oil, heavy crude oil, custard, ketchup, starch suspensions, paint, shampoo, and drilling mud etc. (Nguyen and Nguyen, 2012). The Non-Newtonian fluids are commonly classified into three categories: Time-dependent fluids, viscoelastic fluids, and time-independent fluids (Sochi, 2011). There are many existing models that can describe the rheological characteristics of non-Newtonian fluid flow behavior. Table 2.1 shows the examples of the non-Newtonian models which are most commonly used for different types of fluids.

Table 2.1: Examples of non-Newtonian models

Model	Rheological Behavior	Model Equations
Power-Law	Shear-thinning or pseudo-plastic fluids	$\mu = C\dot{\gamma}^{n-1}$ Or $\tau = C\,\dot{\gamma}^{n}$
Bingham Plastic	Viscoplastic fluid	$\tau = \tau_0 + \mu_p\dot{\gamma}$
Carreau	Shear-thinning or pseudo-plastic fluids	$\frac{\mu-\mu_\infty}{\mu_0-\mu_\infty} = [1+(\lambda\gamma)^2]^{\frac{n-1}{2}}$
Herschel-Bulkley	Viscoplastic fluid	$\tau = \tau_0 + C\,\dot{\gamma}^{n}$
Cross	Shear-thinning or pseudo-plastic fluids	$\frac{\mu-\mu_\infty}{\mu_0-\mu_\infty} = \frac{1}{[1+k(\dot{\gamma})^n]}$ Where, n and k are the curve fitting parameter μ_0 and μ_∞ are the limiting values of apparent viscosity at low and high share rates.
Ellis	Shear-thinning or pseudo-plastic fluids	$\mu = \frac{\mu_0}{1+\left(\frac{\tau}{\tau_{1/2}}\right)^{\alpha-1}}$ μ_0 is the zero-shear viscosity and the remaining two constants α (> 1) and $\tau_{1/2}$ are adjustable parameters.
Casson	Viscoplastic fluid	$\tau^{0.5} = \tau_0^{0.5} + (\dot{\gamma}\mu_a)^{0.5}$
Godfrey	Time-dependent fluids	$\mu(t) = \mu_i - \Delta\mu'\left(1 - e^{-t/\lambda'}\right) - \Delta\mu''\left(1 - e^{-t/\lambda''}\right)$

Stretched Exponential	Time-dependent fluids	$\mu(t) = \mu_i + (\mu_{inf} - \mu_i)\left(1 - e^{-\left(t/\lambda_s\right)^c}\right)$
Maxwell	Viscoelastic Model	$\tau + \lambda_1 \frac{\partial\tau}{\partial t} = \mu_0\dot{\gamma}$
Jeffreys	Viscoelastic Model	$\tau + \lambda_1 \frac{\partial\tau}{\partial t} = \mu_0\left(\gamma + \lambda_2 \frac{\partial\gamma}{\partial t}\right)$

2.2 Rheological Characteristics of Non-Newtonian Fluid

There are many rheological models for the flow of non-Newtonian fluids. In the next section, the available rheological models for the non-Newtonian fluid are discussed. There are three types of non-Newtonian fluids, such as time-independent fluids, time-dependent fluids, and visco-elastic fluids. In the following sections, we will discuss time-independent and time-dependent behavior only.

2.2.1 Time Independent Fluids

Time-independent fluid flow behavior can be shown by the following constitutive equation,

$$\tau_{yx} = f(\dot{\gamma}_{yx}) \tag{2.2}$$

Or by the inverse relation,

$$\dot{\gamma}_{yx} = f(\tau_{yx}) \tag{2.3}$$

These equations indicate that the value of τ_{yx} at any point of the sheared fluid can be determined by the present value of shear rate, $\dot{\gamma}_{yx}$ or vice versa. This type of fluid has no memory effect in their history (Chhabra and Richardson, 1999).

2.2.1.1 Shear-Thinning Fluids

One of the most common types of fluids is the shear thinning fluids or pseudo-plastic fluids in the engineering fields. It is characterized by an apparent viscosity, which decreases by increasing the shear rate or vice versa. The viscosity is not dependent at very high or low shear rates because at that time fluids exhibit Newtonian behavior, e.g., shear stress, and shear rate are proportional to each other and the plot of shear rate and shear stress become a straight line. Polymer solution and melts usually exhibits shear thinning behavior at very high shear rates.

2.2.1.1.1 Power Law or Ostwald de Waele Equation

A log-log plot of shear stress and shear rate over a short range of shear rate can be approached by a straight line for a shear thinning fluid. The relationship can be expressed by the following equation,

$$\tau = C\,\dot{\gamma}^{n} \tag{2.4}$$

This equation can also be represented regarding apparent viscosity

$$\mu = C\dot{\gamma}^{n-1} \tag{2.5}$$

Shear thinning fluid is characterized by the value of power-law index, n. The value of n is greater than 0 and smaller than 1 for shear thinning fluid. Depending on the molecular weight and concentration, n varies from 0.3 to 0.7 for polymer melts and solution. Here, C is the consistency index which is used to measure the consistency of the substance. These two parameters, C, and n, are mainly curve fitting parameters. Table 2.2 shows the classification of different types of fluids base on power law index.

Table 2.2: The classification of different types of fluids base on the power law index

Value of n	Types of Fluid
n<1	Shear thinning fluid
n=1	Newtonian fluid
n>1	Shear thickening fluid

2.2.1.1.2 The Cross-Viscosity Equation

Cross (1965) developed a four-parameter model which has also been used for many applications. This relationship is known as cross viscosity equation which is presented as follows,

$$\frac{\mu-\mu_\infty}{\mu_0-\mu_\infty} = \frac{1}{[1+k(\gamma)^n]} \tag{2.6}$$

In the above equation, μ_0 and μ_∞ are the apparent viscosity at low and high shear rates and n and k are the two curve fitting parameters. Table 2.3 classifies different types of fluids based on curve fitting parameter and viscosity value.

Table 2.3: Classification of the different types of fluids based on curve fitting parameter and viscosity value.

Parameters	Values	Type of Fluid or Model
k	0	Newtonian fluid
$\mu \ll \mu_0$	-	Power law model
$\mu \gg \mu_\infty$	-	Power law model

In cross viscosity equation, it is suggested that the value of n is constant. This constant value (2/3) is sufficient for the approximation of viscosity data in many areas. Later, Barnes *et al.* (1989) proposed that the value of n should be considered a fitting parameter instead of considering constant for the improvement of viscosity data.

2.2.1.1.3 The Ellis Fluid Model

At low shear rates, the deviation from the power-law model is very noteworthy. In that case, Ellis fluid model is used instead of using a power law model. The model equation of this model is given regarding apparent viscosity as follows,

$$\mu = \frac{\mu_0}{1+\left(\frac{\tau}{\tau_{1/2}}\right)^{\alpha-1}} \tag{2.7}$$

In the above equation, μ_0 is the viscosity at zero shear rate, $\tau_{\frac{1}{2}}$ and α are the fitting parameters. $\tau_{\frac{1}{2}}$ is the shear rate at which the apparent viscosity reduces to its half of zero shear value. When $\tau_{\frac{1}{2}}$ approaches to infinity, this equation indicates Newtonian fluid behavior. The value of α exhibits the degree of shear thinning behavior, e.g., the lower the value of α, the lower shear thinning behavior.

2.2.1.2 Visco-plastic Fluid Behavior

Fluid that has visco-plastic behavior can only flow or deform when the threshold stress or yield stress exceeds the normal shear stress. If yield stress does not exceed the applied shear stress, fluid will behave like an elastic solid. When the value of yield stress exceeds the applied shear stress, it behaves like a Newtonian fluid. Bingham plastic fluid and Herschel-Bulkley fluid model are the examples of visco-plastic fluid behavior. A detailed review of these models will be presented in section 2.3.

2.2.1.3 Shear-Thickening or Dilatant Behavior

This type of fluid behaves like a pseudo-plastic fluid system. Fluids that apparent viscosity increase with increasing shear rate and have no yield stress are called shear thickening fluid. Shear thickening fluid behavior is observed in concentrated suspensions ad solutions. Some examples of shear thickening fluid are corn flour in water, kaolin, TiO_2, etc. However, this type of fluid has very little interest among the researchers. Only Some researchers (Barnes, 1989; Goddard and Bashir,1990; Boersma *et al*., 1990) worked on shear thickening fluid behavior.

2.2.2 Time-Dependent

When the apparent viscosity is not only the function of the applied stress but also the function of time, e.g., previous kinematic history), it is known as time dependent fluids. It is difficult to express this type of fluid behavior by the simple mathematical equation. Some example of time-dependent fluids is cement paste, red mud suspensions, bentonite in water, coal-in-water suspensions, cream, hand lotion, and waxy crude oil. Time-dependent fluids can be divided into two groups depending on the response of material after applying shear stress over the time. One is thixotropy, and another is rheopexy. The thixotropic fluids are sheared at a constant rate. The apparent viscosity decreases with the time at which shearing occurs. Conversely, rheopectic fluids mean negative thixotropic where apparent viscosity increases over time.

However, in this study, Pure viscous fluids flow is the main concern as heavy crude oil is very high viscous fluid. For pure viscous fluid, the viscosity is the function of shear rate, and it is observed that no elastic behavior is present (Carreau *et al*., 1997).

2.3 Available Models for the Characterization of Crude Oil Rheology

Crude oil is a naturally occurring product which is composed of hydrocarbons and others organic compounds. It can be found in light oils to extra heavy oil state. Refined crude oils are classified as kerosene, Gasoline, and Asphaltene. Generally, crude oils are classified depending on its viscosity, density, or API gravity based on hydrocarbon composition. Table 2.4 shows the UNITAR classification of crude oils based on their physical properties.

Table 2.4: UNITAR classification of crude oils at 15.6 ^{0}C

	Viscosity mPa.s	Density kg/m³	API Gravity
Conventional oil	$<10^2$	< 934	$>20^0$
Heavy oil	$10^2 – 10^4$	934 – 1000	$20^0 – 10^0$
Bitumen	$>10^4$	>1000	$<10^0$

Heavy crude oil refers to the crude oil which has high density and high viscosity. It is very common to use API gravity scale for ranking the crude oil. The following equation is used to define the API gravity.

$$Gravity = 141.5/Density - 131.5[°API] \qquad \textbf{(2.8)}$$

Generally, the range of heavy crude oil is between 10-20^0 API or lower (Boduszynski *et al*., 1998).

Light crude oil refers to the liquid petroleum which has low density and low viscosity. Light crude oil can flow freely at low temperature due to its low viscosity. The API gravity of light crude oil is above 31.1^0. Crude oil with API gravity between 21.5^0 and 31.1^0 is known as the medium crude oil (http://www.essay.uk.com/free-essays/finance/crude-oil.php).

Extra heavy crude oil refers to crude oil that has less than 10 API degree. The in-situ viscosity is greater than 10,000 Cp. The extra heavy crude oil does not flow at reservoir condition (Powers, 2014)

The rheology of heavy crude oil has been explained by many authors. Some authors considered heavy crude oil as a Bingham fluid (Malkin and Khadzhiev, 2016; Liu *et al*., 2012; Chen *et al*., 2005; Evdokimov *et al*., 2001), where some experiment studies showed that power law model fitted best. Some authors also proposed Herschel-Buckley model and the Casson model. However, the Bingham model, Herschel-Buckley model, and Casson model are typically for Viscoplastic fluids. Table 2.5 shows different types of Viscoplastic models in different fields.

Table 2.5: Different visco-plastic models for different cases.

Authors	Type of Study	Model Type	Remarks
Gheorghitza (1964)	Theoretical	Bingham plastic	Discusses the motions that vary slowly in porous media for water reservoir by incorporating the hysteresis phenomena
Churaev and Yashchenko (1966)	Experimental	Bingham plastic	Sand pack filters are used to study the human sols filtration and to show the variation in filtration rate pressure gradient.
Kozicki *et al*. (1967)	Analytical	Bingham plastic	The pressure gradient flow rate relationship is derived by using Blake-Kozeny equation.
Park *et al.* (1975)	Analytical and experimental	Herschel-Bulkley and Power law	Pressure drop flow rate data are used for aqueous solutions flowing through packed beds to develop a new viscosity equation. Herschel-Buckley and power law model shows good results between experimental and predicted values. When n=1 =, the HB model is reduced to the Bingham model.
Singh *et al.* (1975)	Analytical and experimental	Bingham plastic	Capillary model is used to derive the model by using pressure gradient flow rate data for fixed and fluidized beds

Masuyama *et al.* (1983)	Experimental	Kaolin slurries (Bingham plastic)	Empirical correlations are used to show the relationship of pressure gradient flow rate doe packed beds.
AI-Fariss *et al*. (1983, 1987, 1989	Analytical and experimental	Waxy crude oils (Herschel-Bulkley)	Model represents the pressure gradient flow rate relationship for packed beds flowing through porous media.
Pascal (1981, 1983a, 1983b, 1984, 1985)	Analytical	Bingham plastic and Herschel-Bulkley	A modified Darcy's law is developed for the steady state and unsteady state flow in porous media
Kuang and Kozicki (1989)	Analytical	Bingham plastic	One-dimensional two-phase flow of Bingham fluids through porous media is studied. It has been explained that the fractional flow depends on saturation, injection velocity, and coordinate in space.
Wu *et al.* (1992)	Analytical	Bingham plastic	Flow behavior of slightly compressible Bingham fluids in porous media. It is considered that heavy crude oil is a Bingham fluid.
Vinay *et al*. (2007, 2009)	Analytical	Bingham plastic	Compressive pressure diffusion and viscous dumping for waxy crude oils by using Three dimensionless numbers (Reynolds number, compressibility number, and Bingham number)

2.3.1 Bingham Model

Heavy oil is a Bingham type of fluid, and it will not flow below the yield stress. When the shear stress is higher than yield stress, it would present the flowing characteristics of the Newtonian fluid

(Dong *et al*., 2012). The viscosity is a parameter that makes the major difference between Bingham fluid and heavy oil. The viscosity of heavy oil depends on temperature, and so it can be considered as the heavy oil is a temperature-sensitive Bingham fluid. Numerous researchers (Thomas *et al*., 1967; Pruess and Zhang, 1990; and Song *et al*., 2001) studied the threshold pressure gradient (TPG) of heavy crude oil in porous media. Some studies also showed that the heavy crude oil is a thixotropic fluid having the shear shinning property (Govier and Fogurasi 1972; Wang *et al*. 2006; Christos 2004; Rojas *et al*. 2008). Wang *et al*. (2006) investigated the flowing characteristics and rheology of Zaoyuan heavy oil. Their results show that the Zaoyuan heavy oil has a TPG and a thixotropic property in porous media. The shear stress of this oil decreases with the shear time.

Theoretical and experimental studies show that some fluids have the characteristics of Bingham type non-Newtonian behavior in porous media (Barenblatt *et. al*., 1968 and Bear, 2013, Wu *et al*., 1992; Pruess and Zhang, 1990; Ikoku and Ramey, 1978; Odeh and Yang, 1979; Savins, 1969, Van Poollen *et al*., 1969; Gogerty, 1967). In this type of flow, fluid only moves when the applied pressure gradient is greater than the minimum pressure gradient, i.e., the threshold pressure gradient (Wu *et al*., 1992). Table 2.6 shows some rheological models explained in the literature for crude oil and drilling mud as both show the same kind of rheological behavior.

Table 2.6: Different types of Rheological models for Crude Oil and Drilling Mud

Author	Study area	Model	Remarks
Wang *et al.* (2005)	Heavy crude oils	Bingham and HB model	One dimensional radial flow of heavy crude oil through porous media having yielding properties and weak thixotropic behavior.
Chen *et al.* (2005)	Heavy crude oils	Bingham model	Capillary equation is used for the modeling of fluid flow through porous media, and a pore network model is used to show the yield stress.
Vinay *et al.* (2005)	Waxy Crude Oil	Bingham model	Lagrange multipliers technique are used to develop the model, and Finite volume method is used to discretize the equation. Three conditions are considered, e.g., an isothermal condition, a non-isothermal condition considering viscosity

			changes with temperature and constant yield stress, and an isothermal condition, a non-isothermal condition considering yield stress changes with temperature and constant viscosity.
Vinay *et al*. (2006)	Waxy Crude Oil	Bingham model	The effect of compressibility on the flow pattern for isothermal transient flow for a weakly compressible Viscoplastic fluid.
Frigaard *et al*. (2007)	Waxy Crude Oil	Bingham model	Study of the displacement of weakly compressible waxy crude oil that has the significant effect of compressibility over the timescale.
Reed and Pilehvari (1993)	Drilling Mud	Power-law model	Effective diameter is introduced through annuli.
Hemphill *et al*. (1993)	Drilling Mud	Herschel–Bulkley model	H-B model can predict yield point more accurately over Bingham model and Power law model.
Maglione *et al*. (1996)	Drilling Mud	Herschel–Bulkley model	The effect of pressure and temperature are analyzed with viscosity, velocity profile, and pressure profile.
Escudier *et al*. (2002)	Drilling Mud	Cross model	Flow data and laminar annulus flow data for shear thinning fluid shows good agreement for both experimental case and predicted study.
Ozbayoglu and Omurlu (2007)	Drilling Mud	Herschel–Bulkley model	A yield power law model is proposed to calculate the frictional pressure loss for ten different mud sample.

Sorgun and Ozbayoglu (2011)	Drilling Mud	Power-law model	A Eulerian-Eulerian computational fluid dynamics (CFD) model is developed to estimate frictional pressure loss and velocity profile in concentric and eccentric annuli.
Meriem-Benziane *et al*. (2012)	Light Crude Oil	Herschel–Bulkley and Casson model	An investigation on the light crude oil rheology and its emulsions to enhance the flowability through the pipeline.
Banerjee *et al*. (2015)	Heavy Crude Oil	Power-law model	Use of surfactant to enhance the flowability of heavy crude oil.
Sami *et al*. (2017)	Heavy Crude Oil	Power-law model	An experimental study on the heavy crude oil rheology.

Bingham model is the most acceptable model for heavy crude oil due to its conciseness and satisfied accuracy (Jingwen *et al*., 2013).

2.3.2 Power Law Model

Power law model has been discussed briefly in the previous section. In this section, we will discuss non-Newtonian fluid as well as heavy crude oil characterization that considers the power law model. Yu and Dae Han (1973) considered the power law model equation for the study of the horizontally stratified flow of molten polymers. Numerous studies considered power law model for the flow characterization of polymer solutions (Bishop and Deshpande,1986; Chhabra *et al*., 1983; Xu et al., 2007, 2009; Jia *et al*., 2011; Picchi *et al*., 2015; Picchi and Poesio, 2016a, 2016b). Kumar et al. (2006) did an investigation on Indian heavy crude oil using the novel surfactant, Brij 30. They showed the effect of additives on the crude oil rheology, such as yield stress, viscosity, elastic modulus, loss modulus, complex viscosity, phase angle, and thixotropy. The authors considered the power law model, Bingham model, and Casson model for the rheological studies where the viscosity decreases with increasing the shear rate and the shear stress increases by increasing shear rate. They found that the power law model best fits for both experimental values and predicted values over the Bingham and Casson model. Hasan et al. (2010) discussed the viscosity reduction of the heavy crude oil through pipeline transportation. They showed the effect of temperature and the effect of light crude oil addition in the heavy crude oil and got the best output for power law model. Naiya *et al.* (2014) presented a study on the effect of surfactant on the synthetic crude oil rheology. They showed the effect of temperature, shear rate, and the addition of mineral oil on the viscosity. The authors also considered the power law model, Bingham model, and Casson model for the rheological studies where they found the viscosity decreases with increasing the shear rate and the shear stress increases by increasing shear rate. Their experimental studies showed that the power law model is the best model to describe the experimental data of waxy crude oil.

2.3.3 Herschel-Buckley Model

Herschel-Buckley model is used to describe the thixotropic behavior of shear thinning fluid with yield stress at the steady state condition, e.g., no thixotropic effects (Livescu, 2012). However, Herzhaft *et al*. (2002, 2006) suggested that the time-dependent yield stress shear thinning behavior would not be useful to characterize the complex viscosity behavior if the shear rate is very low and exhibits transient flow regime. Several authors considered Herschel-Buckley model for drilling mud (Founargiotakis *et al*., 2008; Kelessidis *et al.*, 2006; Hamed and Belhadri, 2009; Majidi *et al.* 2008, 2010). Cui *et al.* (2013) found that the flow behavior of heavy crude oil through porous media is not linear. They considered both the Bingham model and Herschel-Buckley model for the theoretical development of the model and developed two models. The authors found that heavy crude oil does not flow if the pressure gradient is smaller than threshold pressure gradient. Pressure gradient must be greater than the threshold pressure gradient. When n=1, the Herschel-Buckley model is reduced to the Bingham model. They suggested the Bingham model over H-B model for heavy crude oil and non-Newtonian fluids.

2.3.4 Casson Model

Casson (1959) developed a model called the Casson model. A Casson fluid has shear thinning properties with an infinite viscosity at zero shear rate. Likewise, the Bingham model, if the applied shear stress is greater than yield stress, fluid will flow. When the yield stress is greater than the shear stress. Fluid will behave like solid (Pramanik, 2014). Some examples of Casson fluids are jelly, tomato sauce, concentrated juice, soup, honey, etc. (Sheikh *et al*., 2017). There are numerous studies on Casson fluid flow and heat transfer in the literature (Mukhopadhyay, 2013a, 2013b;

Mukhopadhyay *et al*., 2013a, 2013b; Rao *et al*., 2013; Crane, 1970; Grubka and Bobba, 1985; Chen and Char, 1988; Ali, 1994; Kelessidis and Maglione, 2006; Sharma, 2012; Rashidi *et al*., 2012; Soid *et al*., 2012; Nadeem *et al*., 2013; Ramachandra Prasad *et al*., 2013; Qasim *et al*., 2013; Khan *et al*., 2015; Freidoonimehr *et al*., 2015; Ramesh and Devakar, 2015; Mohamed *et al*., 2015). Dash *et al*. (2000) performed a study on Casson fluid and found that the rate of dispersion is yield stress dependent. An extension of Dash *et al*. (2000) work had been made by Nagarani et al. (2004). They considered the effect of wall absorption in their model. Hussanan *et al.* (2016) developed a model of Casson fluid past a stretching surface in the presence of viscous dissipation and solved analytically for velocity and temperature. Khan *et al.* (2016) introduced fractional derivative in The Casson model for the flow of Casson fluid due to an infinite plate. However, there is a limited study on heavy crude oil that considers the Casson model for the flow through porous media or the pipeline transportation. Meriem-Benziane *et al*. (2012) studied the rheology of light crude oil and its emulsions. Their experiment results and simulation of yield stress of the Herschel-Buckley model and Casson model show good agreement between the predicted values and experimental values. Banerjee *et al*. (2015) performed an experimental study to improve the flowability of Indiana heavy crude oil by using the natural surfactant. They considered power law, Bingham, and Casson model for their investigation. The authors used Sodium the extract of *Sapindus mukorossi* (soapnut) plant as a surfactant to improve the rheological properties of crude oil, such as yield stress, viscosity, thixotropic properties, pour point, and interfacial tension. They found that the yield stress is reduced by 98%, viscosity 80%, thixotropic area by 94.64%, and interfacial tension by 97%. However, their experimental results show that the power-law model best fits crude oil flow behavior over the Bingham and Casson model. Sami *et al.* (2017) investigated experimentally the rheology and flow characterization of heavy crude oil by considering the power law, Bingham, and Casson models. They used the dead oil sample as heavy crude oil and found that the viscosity is decreased by 15.6% when the temperature increases from 30^0C to 60^0C. The authors found that the heavy crude oil exhibits shear thinning behavior and best fits with power-law model over Bingham and Casson model.

2.4 Heavy Crude Oil Flow Characterization

Heavy crude oils have higher Asphaltene content and high viscosity compared to light crude oil. It also contains high heavy metals, nitrogen, and sulfur content (Powers, 2014). The demand for crude oil increased on an average of 1.76% per year over the world from 1994 to 2006. This percentage was higher at 3.4% during 2003-2004 (Ghannam *et al*., 2012). The developing countries like China and India are demanding more crude oil due to the dynamic economic growth that is 1.8% global growth in 2009 (IEA, 2010). Many international organizations reported that about 80% of the world's energy demand would depend on the crude oil and natural gas (IEA, 2010). According to International Energy Agency (IEA), the heavy crude oil reserve is about 50% of the total recoverable oil reserves. However, due to the heavy viscous nature and complex rheology, the demand for heavy crude oil and extra heavy crude oil has been lowering. Because it is very costly to produce, transport, and refine the heavy crude oil. Several authors studied for the development of improving flowability of the heavy crude oil (Ghannam *et al*., 2012). The viscosity of heavy crude oil could be as high as 10^5 mPa.s at 25^0C. However, it should not exceed 400 mPa.s for the transportation of crude oils through the pipeline (Schumacher, 1980; Fruman and Briant, 1983; Nunez *et al*., 1996). In the crude oil, the carbon content is in the range of 83-87%, hydrogen content from 10-14%, and some other compounds like sulfur, nitrogen, oxygen, nickel, and vanadium may be found in the crude oil (Sjoblom et al., 2002). Heavy crude oil also contains wax and Asphaltene (Johnsen and Ronningsen, 2003) The production and transportation of heavy crude oil are sometimes very difficult due to the non-Newtonian flow behavior in the reservoir and the rheology of heavy crude oil is heavily related to oil recovery and well productivity.

2.5 Rheology of Heavy Crude Oil

The rheological behavior of heavy crude oil is very vital for the flow characterization through porous media, and the rheology of heavy crude oil is heavily related to oil recovery and well productivity (Chen and Lu, 1990; Prada and Civan, 1999; Henaut, 2003). Heavy crude oil is defined by high density and high viscosity. It differs from the rheology of light crude oil due to its viscosity and density. The flow of heavy crude oil is characterized as a non-linear seepage process at reservoir condition. The non-linear seepage process makes the situation more difficult for the flow of heavy crude oil. To the development of heavy crude oil, the oil must flow from the reservoir to the well and crude oil must lift from well to the surface continuously (Chen and Lu, 1990; Alshmakhy and Maini, 2011). Therefore, it has become an important task to study the rheological behavior of heavy crude oil in the wellbore and the reservoir. There are numerous numerical and experimental studies based on the characteristics of heavy crude oil flow through porous media.

Three different ways have been discussed to characterize the flow and the rheology of heavy crude oil, e.g., the composition of heavy crude oil, the rheological property of heavy crude oil, and thixotropic property.

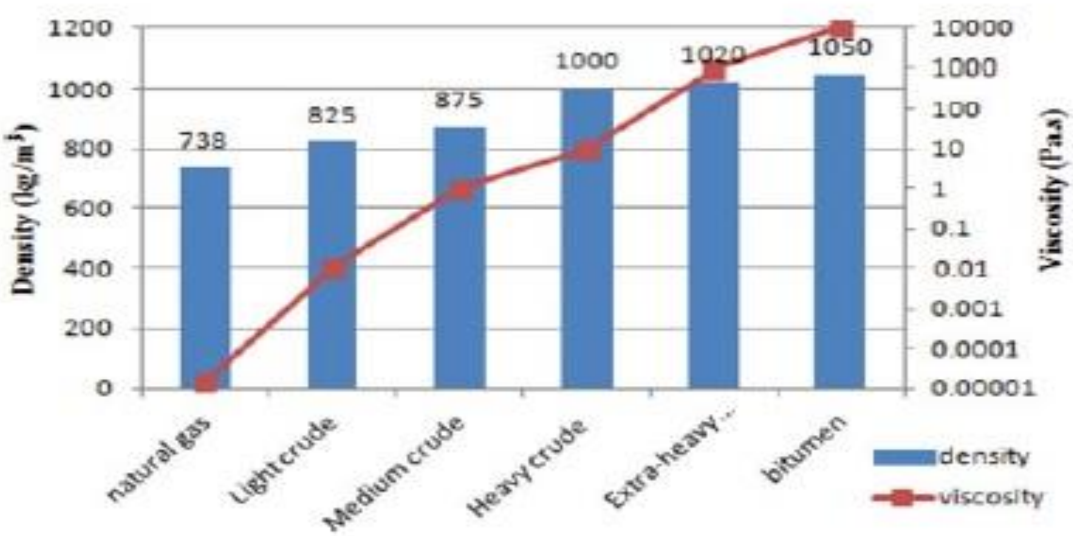

Figure 2.1: Crude Oil classification based on viscosity and density (Taken from Sami et al., 2017)

2.5.1 Composition of Heavy Crude Oil

The compositional analysis of heavy crude oil is an important tool to characterize the flow of heavy crude oil. It contains a large amount of asphaltene and resins that is the main reason for having the high viscosity (Steinborn and Flock, 1983; Schramm and Kwak, 1988; Argillier *et al*., 2002; Henaut *et al*., 20003; and Ovalles *et al*.,2011). Argillier *et al*. (2002) and Henaut *et al*. (2003) studied the effect of asphaltene and resin content on the viscosity of the oil. Their study investigated that the particles between asphaltenes might overlap if the concentration of the asphaltenes and resins reached a critical level. This is the main reason for increasing the oil viscosity as heavy crude oil shows elastic behavior due to structural change in the asphaltenes. In many cases, the viscosity increases at a high level in the absence of resins. Schramm and Kwak (1988) and Ovalles *et al*. (2011) showed that the viscosity would reduce about 2 to 3 orders of magnitude by removing asphaltenes from heavy crude oils. Additionally, some additives can be used for the reduction of oil viscosity (Ovalles *et al*., 2011).

2.5.2 Rheological Properties of Heavy Crude Oil

The rheological properties, e.g., viscosity, density, shear stress, shear rate, etc., depending on the reservoir temperature. There is a close relationship between the rheological property of heavy oil and the reservoir temperature. Heavy crude oil has a viscoelastic property while the temperature is low and behaves as a Newtonian fluid while temperature increases (Wang et al. 2006; Chen et al. 2005). Many researchers studied the Bingham model is used to describe the flowing process of heavy oil in porous media (Wu et al. 1992; Thomas et al. 1967; Pruess and Zhang 1990). For Bingham fluid, heavy oil would not flow when stress is below the yield, and it would present the flowing characteristics of Newtonian fluid above the yield stress. The major difference between Bingham fluid and the heavy oil is that the viscosity of the heavy oil is dependent on temperature, and heavy oil is a temperature-sensitive Bingham fluid. Thomas et al. (1967), Pruess and Zhang (1990) and Song et al. (2001) studied the threshold pressure gradient (TPG) of heavy crude oil in porous media. Figure 2.2 shows the general relationship for Bingham fluid regarding shear stress and shear rate. The yield shear rate is 10 MPa after which the fluid will start to flow. As shear stress increases, the rate of deformation, e.g., shear rate.

2.5.3 Thixotropic Fluid as a Shear Thinning Property

There are numerous studies which considered the heavy crude oil as a shear thinning fluid that has thixotropic properties (Govier and Fogurasi 1972; Christos 2004; Wang *et al.* 2006; Rojas *et al.* 2008). Wang *et al.* (2006) performed an investigation to characterize the flow and rheology of the Zaoyuan heavy crude oil. Their investigation shows that a startup pressure was required to initiate the flow of crude oil and the fluid flow obeys the Herschel-Bulkley rheological model. Their experiments find out that viscoelasticity behavior decreases with increasing temperature. It is also showed that the shear stress increases with increasing the shear rate and the viscosity decreases with increasing the shear rate at the different temperature. Finally, it is found that the Zaoyuan heavy crude oil has thixotropic properties and a thermal pressure gradient property (TPG). Figure 2.3 shows the flowchart describing the static viscoelastic experiments for the Zaoyuan heavy crude oil (Wang et al., 2006). The system contains a digital recorder, a pressure sensor, a switch, three containers (two for heavy oil and one for water), and a mechanic pump. Three containers, switch, and the pressure sensor is placed into an iso-thermal oven. To apply the pressure, the mechanical pump is used. Among the three containers, the first container is for water, and the other two containers are for heavy crude oil.

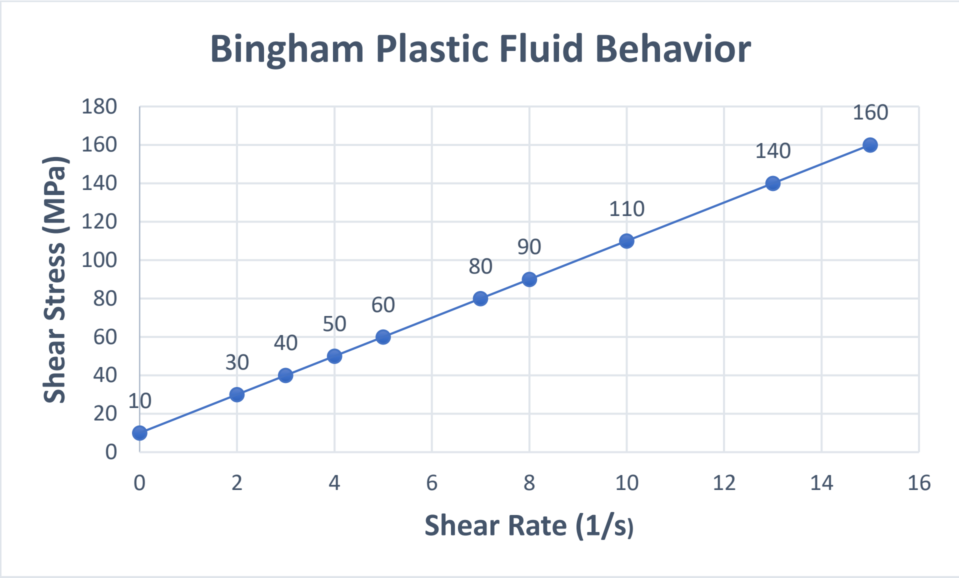

Figure 2.2: Shear stress and shear rate relationship for Bingham plastic behavior

The purpose of installing the switch is to isolate the third container (heavy oil) from the other two containers. The pressure sensor measures the pressure in the third container and transfers pressure signals to record into the digital recorder. The digital recorder calculates the pressure concerning time. Before starting the experiment, they performed the whole experiment using water to ensure no water leakage. The experiment was carried out by filling the second and third container with heavy crude oil and the first container with water. They set the iso-thermal oven temperature at the experimental temperature and delayed for approximately 10 hours to settle the heavy crude oil at the required temperature. After that, they increased the third container's pressure by the pump. The pressure increment value is a few hundred psia. Then they switched off the switch to isolate the third container and to record the pressure of the heavy oil. The pressure decreases quickly at first and then the pressure changes slowly. After a certain time, the change of pressure is not noticeable.

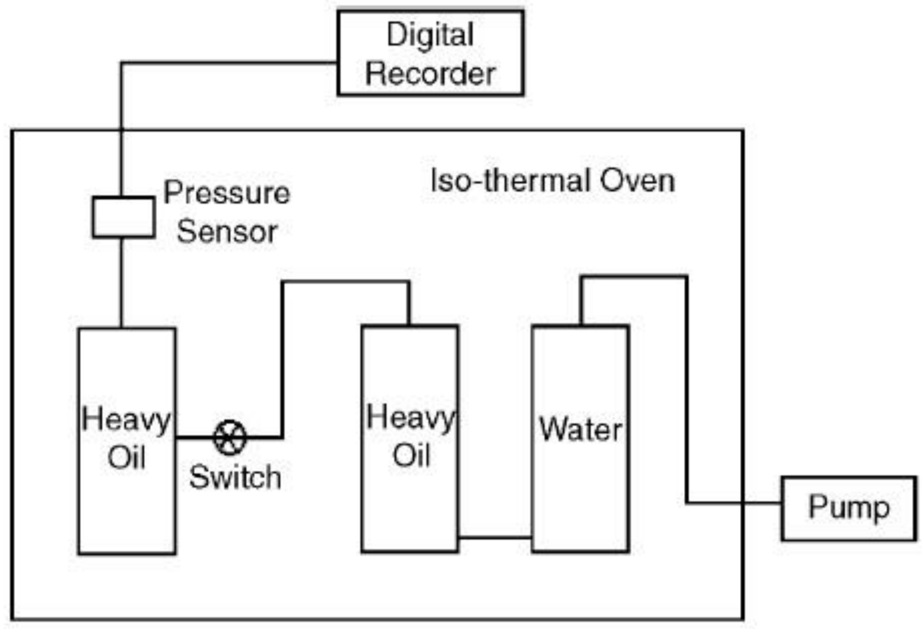

Figure 2.3: Flowchart of static viscoelastic experiments (Taken from Wang *et al*., 2006)

Rojas *et al.* (2008) suggested a model by incorporating the Arrhenius equation, power law model, and liquid crystal theory. Their proposed model deals with shear thinning properties and shows the effect of frequency and temperature on the effective complex viscosity.

2.6 Use of Fractional Derivative for showing memory effect for non-Newtonian fluid flow and heavy crude oil flow

The non-Newtonian fluid exhibits anomalous flow behavior than Newtonian fluid. There are numerous theoretical investigations which simulate and analyze the energy and mass transfer in the non-Newtonian fluids (Luikov *et al*., 1969; Li *et al*., 2011). Several experimental studies have shown that non-Newtonian fluids like muddy clay, blood, oils, polymeric solutions, paints display non-linear relationship between shear stress and velocity gradient (Tapadia and Wang, 2006; Pimenta and Campos, 2012). The Power law model, Bingham model, Herschel–Bulkley model, and Casson model have been using to characterize the flow behavior of the non-Newtonian fluid in many fields (Matsuhisa and Bird, 1965). However, there are some drawbacks to these models. Such as insufficient information about the unified constitutive equations that can present for all non-Newtonian fluids, unable to use some model parameters from one system to another system for same fluid (Huilgol and Kefayati, 2016). This work presents a comprehensive study on the non-Newtonian flow behavior for heavy crude oil to show the temperature, pressure, and memory effect.

Non-Newtonian fluids with complex behaviors have different components that may exhibits memory effects for their dynamic nature with space and time (Sun *et al*., 2016). Rock and fluid properties have a great impact on the fluid rheology and flow characterization in porous media in the petroleum reservoir. Darcy's law is the most acceptable equations to explain the flow behavior in the porous media. However, Darcy's law can't represent the true reservoir as well as fluid behavior. Therefore, the memory effect means the change in rock and fluid properties (porosity, permeability, viscosity) with time. Caputo (1998) proposed a modified Darcy's law for the characterization of natural reservoir phenomenon. In the modified Darcy's law, the flux rate is not constant with time due to complex rock and fluid interactions, whereas in the classical Darcy's law, the flux rate is constant with time. These complex rock and fluid interaction are known as anomalous diffusion that is introduced by incorporating fractional derivative in the diffusivity equation. Several studies have been discussed anomalous diffusion (Havlin and Ben-Avraham, 2002; Fomin *et al.*, 2011; Razminia *et al*., 2015a, 2015b; Zhao *et al*., 2015). There are some other studies that consider memory effect in their model (Hu and Cushman, 1991; Nibbi, 1994; Caputo, 2002; Hossain *et al.*, 2008a, Hassan *et al*., 2015a, 2015b; Obembe *et al.*, 2017). Happy *et al*. (2017) presented a detailed review of the memory effect and the solution techniques that involves with memory term, e.g., fractional derivative. The authors provided a comparison between the conventional simulation approach and the novel approach, e.g., the engineering approach in the reservoir simulation. They explained that the engineering approach could be a good tool for the formulation and solution of governing equation that is related to the memory effect. However, this memory effect can be incorporated to describe the fluid rheology, e.g., shear stress and shear rate behavior. Hossain et al. (2007) proposed a modified stress-strain model that considers the effect of temperature, surface tension, pressure, and the memory effect. They incorporated surface tension effect by introducing Marangoni number and the memory effect by introducing modified Darcy's law (Caputo, 1998). The authors assumed that the crude oil does not follow Newton's law of viscosity. The main drawback of this model is that they considered non-Newtonian fluid flow. However, they used Newton's law of viscosity. Most of the studies show that non-Newtonian fluids are the mixture of different components with different size. Some of the components are the polymer, oil, solid particles, water, and other long chain particle molecules (Evans *et al.*, 2015).

These components having dynamic and kinematic behavior can show long-term memory effects and non-local phenomena at spatially. An integer order velocity equation is used to represent the newtons constitutive equation. This equation does not consider non-local properties and memory effects as well. So, it is necessary to consider these effects for the modeling of non-Newtonian fluid. There are numerous studies that consider non-local properties and memory effects. These studies use the power law model and some other shear rate dependent dynamic viscosity model. However, it is very difficult to obtain the analytical solution of the shear rate dependent dynamic model in real field applications as it also produces non-linear equations. In some cases, this shear rate dependent model creates confusion in applications, as it has several expressions for non-Newtonian fluids.

Some recent studies (Ochoa-Tapia and Valdes-Parada, 2007; Shan *et al*., 2009; Yin *et al*., 2012; Ionescu, 2017) investigate that fractional calculus can be a good tool for the characterization of the non-Newtonian fluid as fractional calculus can consider the long-term memory effects and spatial non-local effects. Like, the time-dependent fractional derivative constitutive equations are used to describe the history dependency on fluid dynamics. Yin *et al*. (2012) proposed the time fractional derivative equation for different types of time dependent non-Newtonian fluids. They used this equation for the analysis of dynamic process for muddy clay. Shan *et al*. (2009) also used time fractional derivative equations for the seepage flow in dual-porosity media. Song and Jiang (1998) and Ionescu (2017) developed time-dependent fractional derivative models for showing the rheological behavior of Sesbania gel and xanthan gum and blood viscosity respectively. However, these fractional derivative models are based on rheology viewpoint. Most of the authors ignored the spatial non-local dependency on velocity gradient. There is very limited research about space fractional derivative that can capture the memory effects for non-Newtonian fluids. Ochoa-Tapia, and Valdes-Parada (2007) derived Darcy's law by using fractional derivative in Newton's law of viscosity. They investigated shear stress phenomena of non-homogeneous porous media by applying spatial averaging method. They considered gravitational effects in their proposed model and solved the model using Taylor series. Chen *et al.* (2015) developed a time-space fractional derivative model using time-space fractional Navier Stokes equation to show the effects of model parameters for Maxwell fluids boundary layer problems on the unsteady stretching surface. Sun *et al*. (2016) investigate theoretically this spatial non-local effect by considering fractional derivative in the shear rate. They proposed fractional derivative constitutive equation (FACE) to address the non-local effects of velocity by introducing fractional derivative velocity gradient for non-Newtonian fluids. They also proposed velocity profile, fractional Reynold's number, the frictional head loss for non-Newtonian fluids for pipe flow. They addressed the non-locality of non-Newtonian fluid flow and the relationship between the particles with different sizes. They considered Bingham model and power-law model for their theoretical investigation and validation. Table 2.7 shows the different types of fluids classification

Table 2.7: Classification of fluids with different yield stress and fractional order base on FACE model (Sun *et al*., 2016)

Type of fluid	Yield Stress	Fractional derivative order (α)
Newtonian fluid	$\tau_0 = 0$	$\alpha = 1$
Elastomer	$\tau_0 = 0$	$\alpha = 0$
Dilatant fluid	$\tau_0 = 0$	$0 < \alpha < 1$
Pseudo plastic fluid	$\tau_0 = 0$	$2 > \alpha > 1$
Bingham fluid (I)	$\tau_0 = C\ (C > 0)$	$\alpha = 1$
Bingham fluid (II)	$\tau_0 = C\ (C > 0)$	$0 < \alpha < 1$

Sun *et al*. (2016) reclassified the non-Newtonian fluids based on the fractional derivative parameter (Figure 2.4). Figure 2.4 shows that non-Newtonian fluids can be characterized based on the fractional derivative order. When α=1, it means the flowing fluid is Newtonian and α=0 means that the fluid is elastomer and ideal where viscosity $\mu = 0$.

The summary of the previous section is given as follows:

a. The memory effect can be incorporated in the shear stress-strain relationship by considering fractional derivative in the modified Darcy's law to show the variation in fluid properties with time.

b. The memory effect can be shown by incorporating fractional derivative in the shear rate to show the non-local effects.

In this thesis, these two criteria will be mathematically studied and the numerical simulation is also done by MATLAB.

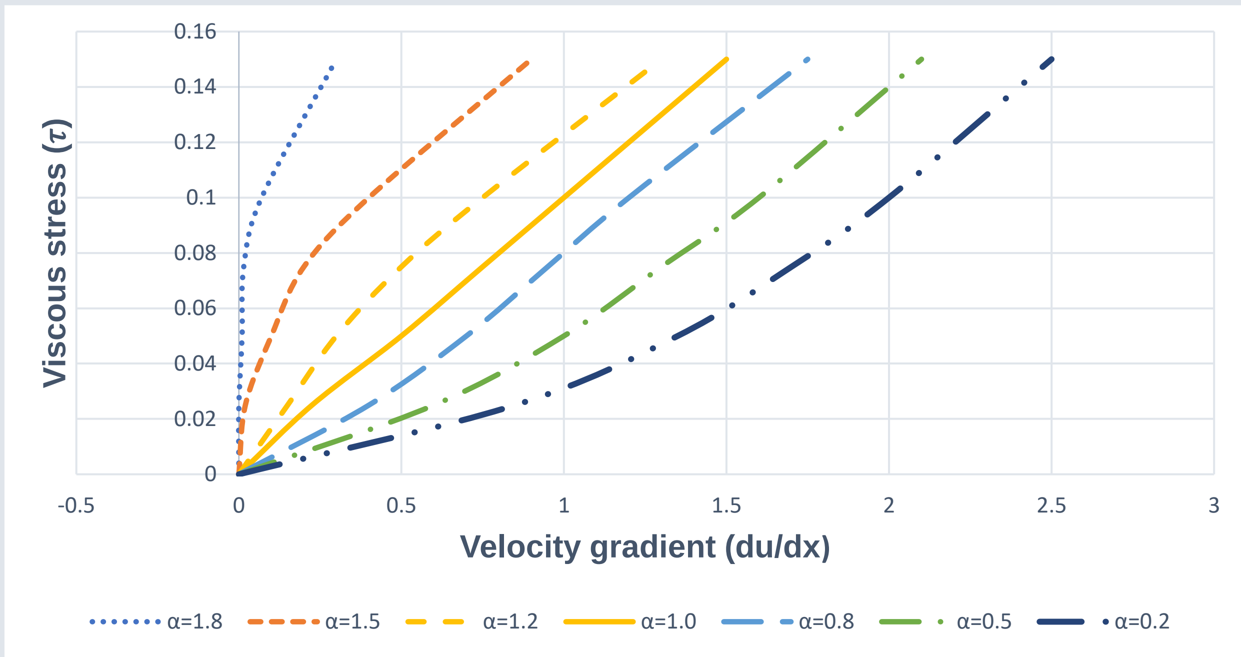

Figure 2.4: The schematic diagram of the relationship between viscous shear stress and velocity gradient at different fractional derivative orders (Sun *et al.*, 2016)

Conclusion

As heavy crude oil has complex rheological properties, it is necessary to develop a general model for heavy crude oil, especially for porous media. The above discussion has been made on the available rheological models for non-Newtonian fluids and crude oil (heavy or light). Bingham model is considered as the best model that fits with the rheology of heavy crude oil. This model also is explained in the literature. To consider the natural phenomena, memory effect is considered in the Bingham model. A brief review of memory and how it can be incorporated in the Bingham model by incorporating fractional derivative has also been discussed in the literature. In this thesis, we do a comparative study to show the effect of shear rate, temperature, and pressure on the Bingham model. A mathematical model for heavy crude oil in porous media is proposed and validated with experimental data by modifying Bingham model. In this model, we consider the effect of memory along with the effect of temperature.

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CHAPTER THREE

A Critical Review on Engineering Approach in the Context of Memory Concept for Fluid Flow through Porous Media

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Abstract

In the conventional reservoir simulation approach, after formulation of the mathematical model usually we get a set of partial differential equations (PDEs) in space and time. Besides, the incorporation of the reservoir heterogeneity makes the entire modeling process more difficult and the model equations become highly non-linear. Solution of such equations numerically requires many approximations which adds error and affects the fidelity of the solution. In order to avoid such approximations researchers have been working on different modelling approaches. In this paper, we review one of such approaches, namely, the engineering approach. Engineering approach bypasses PDE formulation in the conventional approach and leads to integro-differential equations which are easier to solve. In heterogeneous reservoir, there is always a continuous change in rock and fluid properties due to changes in pressure and temperature. Dealing with reservoir heterogeneity and time varying properties leads to nonlinearity in the system equations and is difficult to deal using conventional approaches. In this study, we critically analyzed and presented its strengths in tackling the present challenges of reservoir simulation. A comparative study is also offered with the conventional approach in reservoir simulation. Observation through this study shows that such modeling approach is more straightforward, accurate, and transparent than the conventional approach. The outcome of this study will enhance the understanding of the need for revisiting the conventional reservoir simulation approach and offer a better option to handle the current industry challenges in the area.

Keywords: Fluid flow through porous media, engineering approach, memory function, finite volume method, mathematical modeling.

Nomenclature

A_x = cross-sectional area of rock to x-direction or the flow of the fluid, ft^2 [m^2]

B = fluid formation volume factor, RB/STB or RB/scf [m^3/stb m^3]

g = gravitational acceleration, m/s^2

c_f = $c_o + c_w$ = total fluid compressibility of the formation

c_w = water compressibility of the formation, $1/pa$

c_t = $c_f + c_s$ = total compressibility of the formation

c_s = rock compressibility of the formation

K_x = absolute permeability of rock along the direction of flow

K_γ = Pseudo-Permeability. mD-$day^{1-\gamma}$

$m_{v_i}^n$ = mass of fluid contained in a unit volume or rock at block *i* at time step n, *kg*

$m_{v_i}^{n+1}$ = mass of fluid contained in a unit volume or rock at block *i* at time step n+1, *kg*

$m_i\rfloor_{x_{i-1/2}}$ = mass of fluid entering the reservoir volume element at the boundary, $x_{i-1/2}$, *kg*

$m_o\rfloor_{x_{i+1/2}}$ = mass of fluid leaving the reservoir volume at boundary, $x_{i+1/2}$, *kg*

m_{s_i} = the mass of fluid entering or leaving the reservoir volume, *i* element externally through wells, *kg*

$\dot{m}_x$ = mass flux at point *x*, *kg*

m_{a_i} = the mass of excess fluid stored in or depleted from the reservoir volume element over a time interval, *kg*

p = pressure at any time, *t*, *pa*

p_b = the bubble point pressure of the system, N/m^2

p_{ref} = pressure at a reference point at any time, *t*, *pa*

p_{i-1} = pressure of grid block i-1, psia [kPa]

p_i = pressure of grid block i, psia [kPa]

P_i^{n+1} = pressure of grid block i at a reference time t^{n+1}, psia [kPa]

P_i^n = pressure of grid block i at time at psia [kPa]

q_{sc} = well volumetric rate at the standard conditions, with temperature and can be positive

STB/D or scf/D [std m^3/d]

q = fluid flow rate, $std\ m^3/d$

q_{m_i} = mass rate enters through the well at block i, kg/s

t = time, s

t^n = time at step n, *day*

t^{n+1} = time at step n+1, *day*

u_x = velocity normal to the flow direction *x*, m/s

V_b = block bulk volume, m^3

V_{b_i} = block bulk volume at block *i*, m^3

w_x = mass rate of fluid, kg/s

$w_x\rfloor_{x_{i-1/2}}$ = mass rate of fluid at $x_{i-1/2}$, kg/s

$w_x\rfloor_{x_{i+1/2}}$ = mass rate of fluid at $x_{i+1/2}$, kg/s

Z = elevation from datum, with positive values downward, *m*

Z_{ref} = the datum reference point, *m*

$\left.\frac{\partial^2\Phi}{\partial\xi\partial x}\right|_{i-1}$ = potentials derivative with respect to time of block *i* – 1

$\left.\frac{\partial^2\Phi}{\partial\xi\partial x}\right|_{i}$ = potentials derivative with respect to time of block *i*

Abbreviations

PDEs =partial differential equations

REV = representative elemental volume

1D = one-dimensional system

2D = two-dimensional system

3D = three-dimensional system

LB = lattice Boltzmann

MBE = material balance equation

Greek Symbols

α = fractional order of differentiation, dimensionless

α_c = volume conversion factor = 5.614583 or customary units or 1 for SPE preferred SI units

β_c = the transmissibility conversion factor

$D_t^{1-\gamma}$ = Grunwald-Letnikov operator

γ = Anomalous diffusion exponent, dimensionless

Δt = time step, day

ρ = density, kg/m^3

μ_{ob} = oil viscosity at the bubble point pressure, pas

γ = fluid gravity, kg/m^3

γ_c = gravity conversion factor

η = ratio of the pseudo permeability of the medium with memory to fluid viscosity, $m^3 s^{1+\alpha}/kg$

ξ = a dummy variable for time, i.e. real part in the plane of the integral, *s*

Φ = potential, pa

Φ_{ref} = potential at a reference point, pa

ρ_{sc} = fluid density at the standard condition, $kg/\ m^3$

$\eta_x\rfloor_{x_{i-1/2}}$ = between block i – 1and *i* that are separated by a distance $\Delta x_{i-1/2}$

Γ = Euler gamma function

3.1 Introduction

Reservoir simulation plays an important role in the petroleum reservoir system to understand the composite chemical, physical, and fluid flow behavior (Ewing, 1985), to forecast the accurate and imminent performance of the reservoir, and to optimize the petroleum recovery under different operating conditions. It involves the application of geological modeling and the reservoir characterization. Geological modeling captures the geological description of the petroleum reservoir and the reservoir characterization includes the calculations of rock and fluid properties, and pressure distribution as a function of space and time (Settari and Mourits, 1998; Gutierrex and Lewis, 1998; Gutierrez *et al.*, 2001; Fanchi, 2005). Reservoir simulation is used to evaluate and reduce the risk related to the development of oil recovery. The behavior of the heterogeneous reservoir and complex rock properties (Hristov, 2013, Sprouse, 2010; Hossain and Abou-Khamsin, 2011, 2012; Khabarah and Yortsos, 1997) and the difficulty associated with the oil recovery are the major risk factors.

There are four steps involved in developing a mathematical model to analyze the reservoir behavior. Figure 3.1 depicts the interrelated steps related to reservoir simulation. First, a physical system describes the physical phenomena of nature by incorporating laws of physics. Second, the physical phenomena are converted into a mathematical form which results in a set of nonlinear PDEs. A small segment is taken into consideration in the conventional approach and mass, energy, and momentum balance equations are applied to that small segment. These steps result in a set of partial differential equations. Third, a numerical model is formulated and analyzed according to the model, usually finite difference scheme is used for numerical formulation (Mustafiz, 2008). Fourth, a computer algorithm is developed based on the numerical formulation to solve non-linear algebraic equations. Each of these four-intermediate steps is interrelated to each other in the entire modeling process.

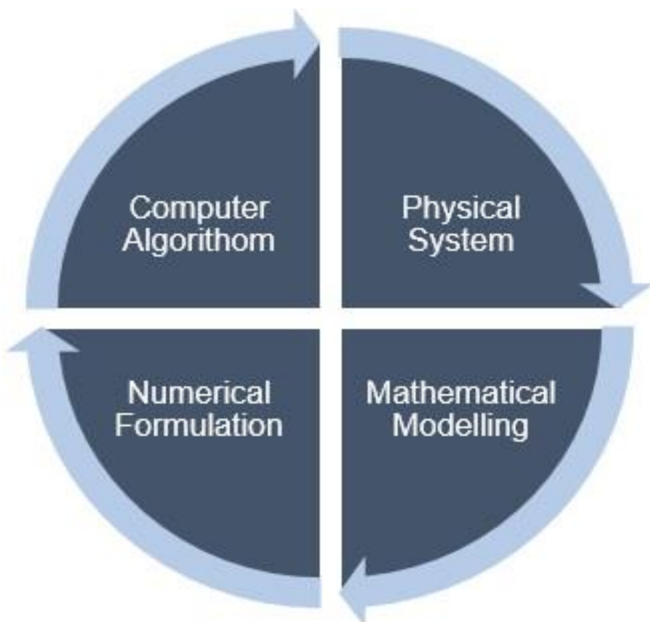

Figure 3.1: Reservoir Simulation Steps

After getting the tangible numerical results for the entire model through the computer programmer, the model output is compared with the physical system. If the outcomes do not match to an acceptable degree, adjustments are made to the previous steps to improve the model prediction. This general procedure of modeling reservoir is referred to as conventional method in this study.

There have been numerous studies in different field based on conventional approach (Swinkels and Drenth, 2000; O'Sullivan *et al.*, 2001; Ozgen, 2006; Yu and Sepehrnoori, 2014; Abbaszadeh *et al*., 2016; Alenezi and Mohaghegh, 2017; Garvey *et al*., 2017, Voskov *et al.*, 2017).

Modeling of complex and natural phenomena of fluid flow through porous media using conventional approach is a complex task for engineers and researchers. Several authors (Collins, 1976; Peachman, 1978; Ewing, 1983, 1997; Civan, 2011; Obembe *et al*., 2016; Hassan and Hossain, 2016) reviewed the models about fluid flow through porous media in detail. These methods have some limitations in the conventional approach such as: (i) this approach does not allow to put source term in the control volume, (ii) it is difficult to describe the physical system for the complex and heterogeneous reservoir. Therefore, researchers are heavily involved in finding a new reservoir simulation technique to overcome these assumptions. Another limitation of the conventional approach is an approximation in the linearization step such as the approximation of the non-linear terms with space and time, e.g., transmissibilities, production and injection, and coefficients of unknowns in the accumulation terms. These assumptions and approximations may lead to erroneous results and inaccuracy regarding reservoir properties, geometry and boundary conditions, analytical solutions and their limitations due to the complexity of natural phenomena. Also in conventional reservoir simulation, it is assumed that rock and fluid properties are invariant with time. If these assumptions are removed, the newly developed models will be nearer to the physical system and may support to enhance the prophetic capacity. However, it is difficult to relax these assumptions in conventional modeling approach as it leads to very complex nonlinear PDEs with fractional order derivatives which are difficult to solve both analytically and numerically.

Recently a new modeling approach, namely Engineering Approach has been proposed which bypasses PDEs in the modeling steps and thus are better suited to deal with the complexities of the reservoir. In most of the cases, it is not possible to solve equations analytically for real problems due to inadequate information of solid/liquid interactions (e.g., particularly under unusual thermal and mechanical restraints), and the heterogeneity and irregular shape of the reservoir system (Hossain *et al.*, 2009e). Following Abou-Kassem *et al*. (2006), the novel approach called the engineering approach is introduced that avoids limitations. It shows how to bypass the conventional formulation of model equations.

Figure 3.2 shows the steps involved in the engineering approach. First, reservoir process is discretized into grid blocks and grid points. This discretization usually removes space variable effects. The material balance equation is written in an algebraic form along with Darcy’s law. Furthermore, the formation volume factor is also written in an algebraic form in a general reservoir block over time integral $t^n \leq t \leq t^{n+1}$. The last step provides an approximation of the time integrals to develop finite difference equations or non-linear algebraic equations. Finally, these equations are solved by various numerical techniques and validated for developing a new simulator. Abou-Kassem and Osman (2008) presented an engineering approach where a block-centered grid represents the boundary conditions. They suggested first-order approximation rather than second order ones because it requires an additional equation for each reservoir boundary of a boundary grid block. They also showed that point-distributed grids and the block-centered grids are quite similar in rectangular and radial-cylindrical coordinates when one deals with constant pressure boundaries.

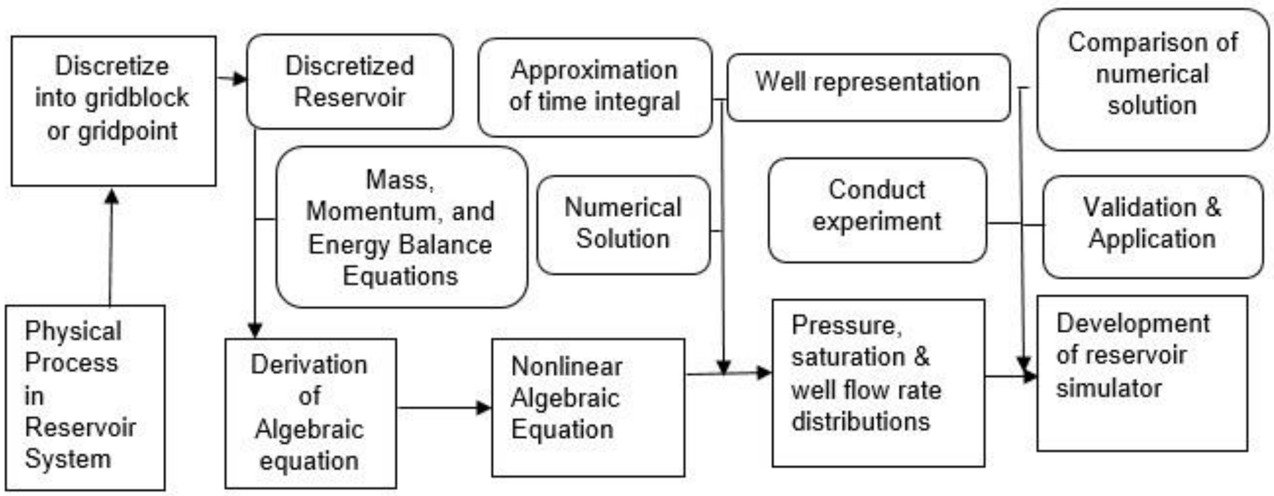

Figure 3.2: Major steps for developing reservoir simulator based on Engineering approach (Modified from Hossain, 2012)

One of the advantages of the engineering approach is that it can deal with rock and fluid memory for porous media especially when multiphase flow occurs. When multiphase flow occurs, there will be solid-liquid or liquid-liquid interactions. These phenomena will cause chemical dissolution of the medium and enlarging pores, swelling and flocculation, pore plugging and precipitation reactions, and transport of particles is obstructing the pores to change the porosity, permeability, viscosity and so on. This is called memory effect. The engineering approach can easily deal with this type of non-linear phenomena. The conventional approach can only deal with classical Darcy's law which cannot represent the complex reservoir and multiphase flow.

The objectives of this paper are to present a detailed analysis of the engineering approach and the existing conventional approach. This paper also shows the advantages of using engineering approach in the petroleum field. The study also presents a brief review of the memory concept in fluid flow through porous media, which is one of the advantages of the engineering approach. Finally, a critical analysis and comparative study of the conventional approach and the engineering approach are offered in this article.

The paper is organized as follows: in Section 2.2, we review the conventional approach and the engineering approach, in Section 2.3, we discuss the advantages of the engineering approach, and in Section 2.4, key challenges in reservoir simulation are discussed.

3.2 Reservoir Simulation Approach

In the following sections, we review both conventional approach and engineering approach and make a comparison between these two methods.

3.2.1 Conventional Approach

Odeh (1982) presented an overview of mathematical modeling where he showed the functional relationships among the variables of flow equations which results in non-linear equations. He discussed three forces, i.e., viscous forces, gravitational forces and capillary forces (Panday, 1989) for the distribution of fluids in the reservoir. Darcy's law is used to consider the effect of viscous, gravity, and capillary forces (Hilfer and Øren, 1996) on the fluid flow. The forces and mechanism for fluid flow, the spatial variations of rock and fluid properties, and the relative permeability can also be represented in the conventional approach. Mahmoudi *et al.* (2017) used a reservoir simulator to show the effect of relative permeability data using pore-scale modeling with statistical method. Liu *et al.* (2015) developed a new Lattice Boltzmann model for pore-scale modeling as the conventional computational fluid dynamics cannot deal with a multi- component multiphase type system. Another method called volume average method was first introduced by Whitaker (1967, 1968, and 1999) to transform the porous medium from microscopic to macroscopic. There are three steps in volume averaging method. First, a set of non-closed macroscopic equations are obtained by the integration of local governing equations for a representative elementary volume (REV). Second, source term is determined and the macroscopic equations are closed. In the last step, the closure problems are written and effective coefficients are determined. There are some other models like heuristic method and mixed method. These methods are also used to describe the heat transfer phenomena at local thermal equilibrium and non-local thermal equilibrium

(D'Hueppe, 2011). Minkowycz *et al.* (1999) found that the change of local thermal equilibrium depends on the rate of heat input and the sparrow number. Cai *et al.* (2016) discussed new and advanced theoretical and numerical approaches for enhanced oil recovery. Coal bed methane, shale gas, tight gas, and gas hydrate reservoirs are supplemented to the advanced theoretical and numerical approaches such as microscopic/mesoscopic modeling and fractal analysis. These approaches are used to increase the recovery. In reservoir simulation, Fractal based approach is used to characterize the flow of porous media in complex reservoir.

However, in the reservoir simulation, a reservoir modeling goes through the following major steps: formulation, discretization, well representation, linearization, solution, and validation (Abou-Kassem *et al.*, 2006). During the formulation/solution step, PDEs are resolved either analytically or numerically. Analytical solutions provide well flow rate, fluid saturations and reservoir pressure as a continuous function of time and space. On the other hand, the numerical solution presents the measures of fluid saturations and reservoir pressures are found only at discrete points of the reservoir and at discrete times. An analytical solution is usually difficult to obtain due to the highly non-linear behavior of PDEs. We use numerical solutions in the conventional method to solve the PDEs. Figure 3.3 shows the major steps involved in developing a reservoir simulator using conventional approach.

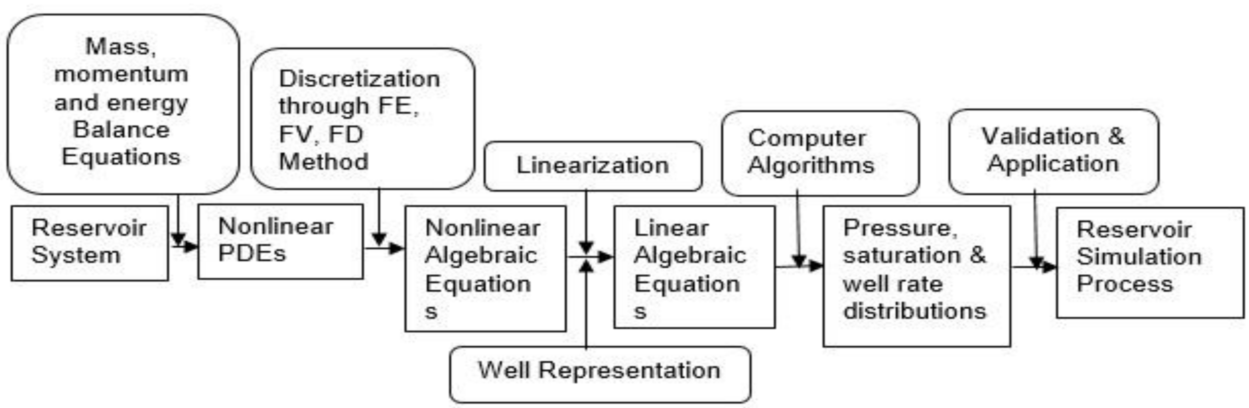

Figure 3.3: Major steps involved in developing reservoir simulator (Modified from Hossain *et al.* 2010)

To obtain model equations in an algebraic form, three consecutive steps are involved in the conventional approach: (i) derivation of PDEs describing fluid flow through porous media, (ii) discretization of the reservoir into grid block or grid points, and (iii) discretization of the resulting

PDE in space and time (Abou-Kassem, 2008). Step one contains some basic assumptions including homogeneous and isotropic formation, time-invariant rock and fluid properties, etc. These assumptions are presented in mathematical terms by imposing constraints such as constant physical properties for the control volume in the reservoir. Ertekin *et al.* (2001) used Newtonian approximations to render these control volume equations into a set of coupled, non-linear PDEs and boundary conditions that depicts fluid flow through porous media. In discretization step

PDEs are discretized, producing a set of non-linear algebraic equations. Several numerical schemes are used to discretize the resulting PDEs in space and time. Broszet (1997) carried out finite element solution for the fluids of the memory integral type. Other authors (Morgan *et al.*, 1984) solved two-phase flow problem using finite element method. Taylor series expansion is responsible for the discretization of finite difference method. The inherent assumption in Taylor series expansion is that the expansion series is valid for small increments. PDEs are written for a given time interval and a given point in space to attain nonlinear algebraic equation. The choice of the time interval (old time level, current time level, or the intermediate time level) leads to the explicit, implicit, or Crank-Nicolson formulation method. Linearization (Figure 3.3) refers to the linearization of the non-linear algebraic equation. Non-linear terms, e.g., transmissibility and coefficient of unknowns in the accumulation are approximated in this step. The nonlinear algebraic equations can be linearized and resolved by linear equation solvers. The solution contains well flow rates along with fluid saturation and pressure distributions in the reservoir. The last step to develop a simulator is validation (Abou-Kassem *et al*., 2006). In this step, model outputs are compared with real-life conditions.

3.2.2 Engineering Approach

The first two steps of the conventional approach involve rigorous PDEs and their discretization. It is possible to bypass the conventional formulation step in the engineering approach. The engineering approach uses the same finite difference equations to get non-linear algebraic equations without going through PDE's and produces integro-differential equations. There are various standard ways to derive the discretized equations for the PDE's and the non-linear algebraic equations, e.g., finite difference method, finite element method, and finite control volume method. The engineering approach is one of the derivatives of control volume approach.

3.3.2.1 Control Volume Method

In control volume method, a computational domain is broken up into smaller control volumes or domains; each control volume must satisfy the conservation of mass, momentum, and energy laws, and then satisfy these laws globally. In finite difference method, we use the values at each of the nodes. From figure 3.4(a), we can see that blue circles are indicating nodes, so, we are calculating the temperature, pressure and other fluid and rock properties in each point or nodes. In the finite volume method, instead of doing it at each node, we use the entire section (Figure 3.4(b)). Then we calculate the flux across the entire section. We use the value of one edge for the next edge. One of the advantages of the finite volume approach is if we lose any properties or values from one side, we will gain from the other side easily.

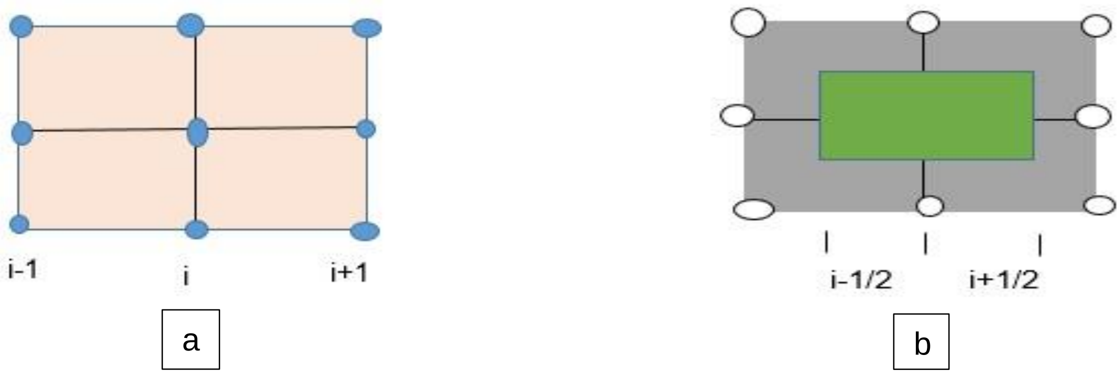

Figure 3.4: (a) Finite Difference Method, (b) Finite Volume Method

Table 3.1 shows the fundamental difference between finite difference method and finite volume method. In finite difference method, the governing equations are satisfied on individual nodes within the computational domain. However, these equations do not satisfy conservation law over the computational domain as well as globally (Mazumder, 2015). In finite volume method, these equations are satisfied with the computational domain, and this is the main strength of this approach. In both methods, error depends on the size of mesh. Extensive discussion contrasting finite volume and finite difference method can be found in (Jameson *et al.*, 1981, Vinokur, 1989; Mattiussi, 1997; Eymard *et al*., 2000; Shu, 2003). The best way is to practice governing equation with the finite volume method because it confirms the conservation of all properties in each control volume or cell and it can be in any number of grid points (Islam *et al*., 2016).

Table 3.1: A comparison between finite difference method and finite volume method

Features	**Finite Difference Method**	**Finite Volume Method**
Definition	Finite difference method is the most direct approach in which governing PDE's such as conservation of mass, energy, and momentum equation, is satisfied on individual nodes within the computational domain or reservoir.	In finite volume methods, these governing PDE's are satisfied with finite sized control volume known as cells.
Fundamental Concept	1. It is based on the differential formulation of governing equations. 2. The governing equations are satisfied only locally, not globally.	1. It is based on the integral formulation of the governing equations over any group of the control volume. (Islam *et al.*, 2016). 2. The governing equations are satisfied both locally and globally
Advantages	1. It is very hard to solve if there are two boundary conditions coexist within two blocks.	1. One can easily solve the PDE's, where two boundary conditions coexist.

Disadvantages	1. The imbalance in the flux can be reduced. However, it will never fully go away. It also depends on the mesh sizes	1. It can solve the real-life, physical problems because this approach can satisfy the conservation law. 2. The imbalance in the flux is always zero; it does not depend on the mesh sizes.

The shape and size of the cell are independent. This method is more realistic in the reservoir simulation. The engineering approach follows finite volume methods; it is also known as control volume approach.

3.2.2.2 Reservoir Discretization in Engineering Approach

In the engineering approach, fluid flow equations are presented in an algebraic form without going through the derivation of partial differential equations (Abou-Kassem *et al.,* 2006). Figure 3.5 shows reservoir discretization in the x-direction, which is involved to block centered grid and point centered grid. The blocks are related to each other, where block k and its neighboring blocks k+1, k+2, k+3, and k+4, and the distances between the points representing the blocks (Δx).

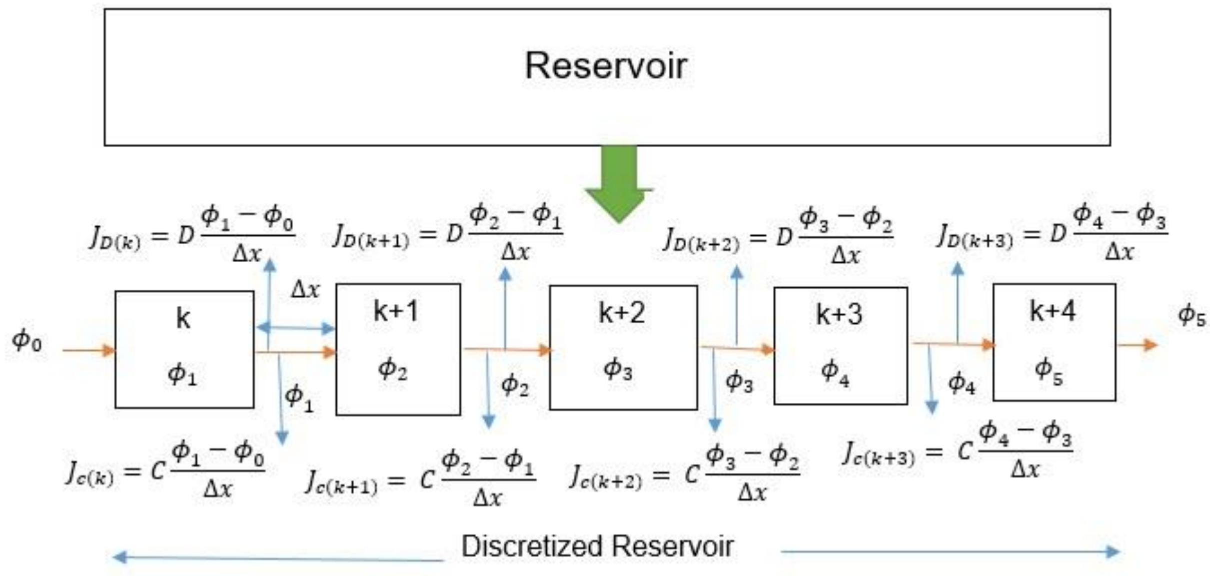

Figure 3.5: Relationships between block k and neighboring blocks in 1D flow

Here D denotes diffusion and C denotes convection. The diffusive mass flow is presented by J_D , and the convective mass flow is presented by J_c. $\phi_1, \phi_2, \phi_3, \phi_4$, and ϕ_5 represents the composition and fluid properties for k, k+1, k+2, k+3, and k+4 blocks. Abou-Kassem *et al.* (2006) also presented the engineering approach, where they showed the relationship between the studied block and neighboring blocks in the 1D flow. This approach follows three major steps: i) discretization of the reservoir into grid blocks (grid points), ii) application of fundamental engineering concept mass balance, the equation of state, and Darcy's law to derive the algebraic flow equations, and iii) extend those algebraic flow equations by approximation of the time integrals. A critical review of those basic principles and the derivation of the algebraic equations for 1D and single-phase fluid flow through porous media is given in (Duval *et al.*, 2004).

In the engineering approach, block ordering is also very important for the identification of the blocks in the reservoir. This block identification reduces the matrix computation load to get the solutions of the linear equations. There are different types of block ordering, such as zebra ordering, natural ordering, diagonal ordering, cyclic ordering, etc. The inactive blocks in the reservoir are not considered for blocks ordering, and active blocks will count (Abou-Kassem *et al.*, 2006). Natural ordering is the easiest block ordering as it is easy to program and solve. Figure 3.6 shows that engineering notation for block identification using natural ordering in a 2D reservoir consisting of a 3 × 2 matrix, where the reservoir is discretized into three blocks in the x- direction and two blocks in the y-direction.

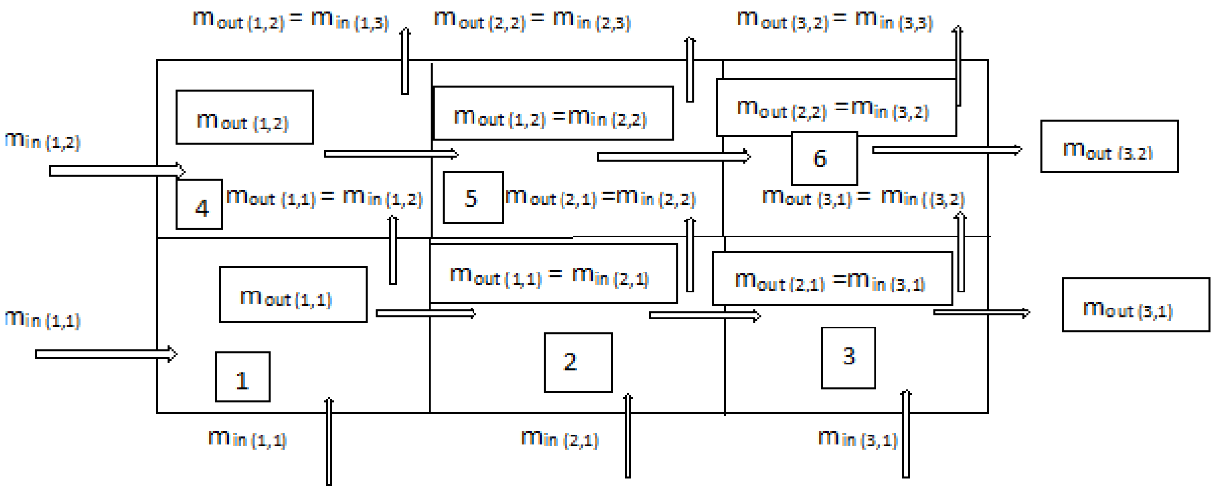

Figure 3.6: Control volume for 2D flow using engineering notation of natural ordering

3.2.2.3 Forms of Fundamental Equations in Engineering Approach

Below we show the forms of the major equations following engineering approach. As it is known that the mass conservation equation, the equation of state, and the constitutive equations are the fundamentals of the basic engineering concept. The mass conservation means that the total mass of the fluid entering in a volume element minus the mass of fluid leaving a volume element of the reservoir in the block i like figure 3.7, equals to the net increase of the mass of the fluid in the volume element of the reservoir.

$$m_i - m_o + m_s = m_a \tag{3.1}$$

Figure 3.7 shows Gridblock block *i* (or Gridpoint *i*) and its neighboring blocks (Gridblock *i* - 1 and *i* + 1) in the *x*-direction. Abou-Kassem (2007) derives how fluid enters block *i*, coming from block *i* - 1 across its $x_{i-1/2}$ face at a mass rate of $w_x\rfloor_{x_{i-1/2}}$ and leave to block *i* + 1 across its $x_{i+1/2}$ face at a mass rate of $w_x\rfloor_{x_{i+1/2}}$ at any time. The fluid mass flow rate is q_{m_i}. The fluid mass controlled in a unit volume of rock in block *i* is m_{v_i}. The fluid is flowing from left side to right side as the injection well is in the left position and the producing well in the right position. If the injection well is in the right position, the fluid must flow from the right side to the left side as the injected fluid has the considerable effects on the flow direction and velocity profile. Some terms such as $w_x\rfloor_{x_{i-1/2}}$, $w_x\rfloor_{x_{i+1/2}}$, and q_{m_i} are not the function of space. They are functions of time only as the reservoir is already discretized. The equation of state refers to the density of fluid as a function of pressure and temperature. The following equation can be written for single phase fluid.

$$B = \rho_{sc} / \rho \quad (3.2)$$

where ρ_{sc} is the density of the fluid at standard condition and ρ is the density of the of the fluid at reservoir condition.

Here, formation volume factor, $B = f(\rho)$, and density, $\rho = f(p)$

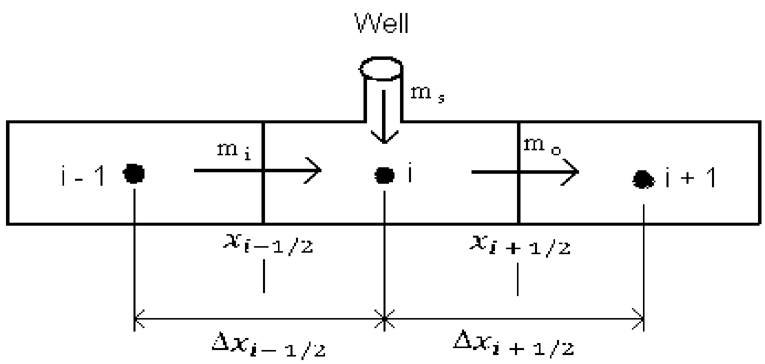

Figure 3.7. Block i as a reservoir volume element in 1D flow (redrawn from Abou-Kassem, 2007)

Mass Conservation

The following equations are involved for the derivation of mass balance equation (Abou-Kassem, 2007):

$$m_i\rfloor_{x_{i-1/2}} - m_o\rfloor_{x_{i+1/2}} + m_{s_i} = m_{a_i} \quad (3.3)$$

Where,

$$m_i\rfloor_{x_{i-1/2}} = \int_{t^n}^{t^{n+1}} w_x\rfloor_{x_{i-1/2}} dt \;; \quad (3.4)$$

$$m_o\rfloor_{x_{i+1/2}} = \int_{t^n}^{t^{n+1}} w_x\rfloor_{x_{i+1/2}} dt \;; \quad (3.5)$$

$$m_{s_i} = \int_{t^n}^{t^{n+1}} q_{m_i} dt \;; \tag{3.6}$$

$$m_{a_i} = \Delta_t (V_b\, m_v)_i = V_{b_i}\left(m_{v_i}^{n+1} - m_{v_i}^{n}\right) \tag{3.7}$$

The material balance equation for block *i* is written over a time step $\Delta t = t^{n+1} - t^n$. The relation between mass rate and mass flux

$$w_x = \dot{m}_x\, A_x \tag{3.8}$$

by incorporating volumetric velocity and fluid density mass flux ($\dot{m}_x$) is given by

$$\dot{m}_x = \alpha_c\, \rho\, u_x \tag{3.9}$$

Fluid mass per unit volume or rock (m_v) by incorporating fluid density and porosity

$m_v = \phi\, \rho$. Terms like $w_x\rfloor_{x_{i-1/2}}$, $w_x\rfloor_{x_{i+1/2}}$, and q_{m_i} are functions of time only because space is not a variable for an already discretized reservoir.

Therefore, the resulting equation is

$$\int_{t^n}^{t^{n+1}} (\alpha_c\, \rho\, u_x\, A_x)\rfloor_{x_{i-1/2}} dt - \int_{t^n}^{t^{n+1}} (\alpha_c\, \rho\, u_x\, A_x)\rfloor_{x_{i+1/2}} dt + \int_{t^n}^{t^{n+1}} (\alpha_c\, \rho\, q)_i\, dt = V_{b_i}[(\phi\, \rho)_i^{n+1} - (\phi\, \rho)_i^{n}] \tag{3.10}$$

Constitutive Equations

Darcy's law is used to describe the fluid flow through porous media which defines the rate of fluid movement into or out the reservoir. Although Forchheimer equation is also used to model the non-Darcy flow of porous media (Forchheimer, 1914; Irmay, 1958; Huang and Ayoub, 2008). Here, Darcy's law and the generalized Darcy's law are considered to describe the phenomena. Table 3.2 review the classical and modified Darcy's law for fluid flow through porous media. Abou-Kassem (2007) presented Darcy's law without considering memory concept which represents the fluid volumetric velocity (flow rate per unit cross-sectional area) from block *i* – 1 to block *i* ($u_x\rfloor_{x_{i-1/2}}$) and from block i to block *i* + 1 at any time instant *t*. Darcy's law has some assumptions, such as the fluid is homogenous, single-phase and Newtonian, no chemical reaction takes place between the fluid and the porous medium, flow condition is laminar, permeability is independent of pressure, temperature and the flowing fluid, and so on (Islam *et al.*, 2016). Caputo (2000) modified Darcy's law by considering that permeability is dependent on pressure and flowing fluid and is not constant with time. The variation of porosity, permeability, and other rock and fluid properties with time is known as memory. Hossain *et al.* (2009b) incorporated modified Darcy's law with engineering approach for reservoir modeling. Obembe *et al.* (2017) introduced a new modified memory-based constitutive equation by introducing Grunwald-Letkinov (MacDonald *et al.*, 2015; Scherer *et al.*, 2011) fractional derivative order.

Table 3.3 shows the model equations using engineering approach which results in the non-linear algebraic equation (Abou-Kassem, 2007 and Hossain *et al.*, 2009b). Hossain *et al.* (2009b) proposed a model using the engineering approach with memory concept representing the general form of diffusivity equation with memory which leads to the non-linear equation. The model equation with memory concept was derived bypassing Taylor's approximation. It consists of three parts, i.e., a derivative of a pressure derivative with memory, pressure derivative with time, η, compressibility and porosity multiplication. These terms make the equation nonlinear.

3.2.3 Contrast between Conventional Approach and Engineering Approach

In the conventional approach, flow equations are derived in the form of PDEs. In the engineering approach, the reservoir is discretized at first by several groups of grid blocks represented by grid points. This discretization process is done by replacing PDEs with the help of algebraic equations, and then the algebraic equations are solved.

The engineering approach is considered as the most vital feature due to the impulsion of engineering thinking to obtain algebraic equations (Zatzman, 2012). One does not need to go through the rigorous derivation of PDEs that are used in the conventional approach. The reservoir is discretized by a group of grid blocks or grid points, and rock properties are relegated to those grid block or grid points. This reservoir discretization removes the space variable. There is no need to discretize the differential equations, or to discretize the boundary conditions. Table 3.4 shows the fundamental differences between the conventional approach and the engineering approach (Abou-Kassem *et al.*, 2006, Abou-Kassem, 2008; Mustafiz *et al.*, 2008b). In the conventional approach, the derivatives are important when the function changes at a rate, and it is difficult to get the solution of that function. In the Newton approximation method, continuous functions are obtained using straight lines for infinitely small segments over time. This infinitely small domain function does not exist. The integration is performed if a domain which is non-existent is to be prolonged to finite, this domain makes the whole procedure sketchy (Hossain and Islam, 2010b).

Abou-Kassem (2008) showed how one could bypass the governing equations directly to Taylor series expansion by the engineering approach instead of using the conventional approach. The conventional approach also involves some assumptions, such as the model is linearized to get solutions (Hossain, 2015). The engineering approach does not involve the linearization step. It can solve the governing equations easily without going through the derivation of PDEs and linearization step.

3.3 Advantages of Engineering Approach

3.3.1 Elimination of assumptions

Engineering approach eliminates the assumptions in the governing equations. For example, in conventional approach it is often assumed that rock and fluid properties are invariant with space in the momentum balance equation (Hossain *et al.*, 2008a). In practice, this is not appropriate because rock and fluid properties are changing with time and space. In the engineering approach, the reservoir is discretized into finite-sized grid blocks or grid points, and the momentum balance equations are imposed for individual grid block having different rock and fluid properties varying with space in each block. This discretization ensures that the rock and fluid properties are not invariant with space. Another example is, Darcy's law considers homogeneous media, constant rock, and fluid properties (Hossain *et al.*, 2008a). In these cases, the current approach is to generalize Darcy's law by combining the variable rock and fluid properties during the mathematical development of fluid flow models. The engineering approach allows this modification during the process of modeling. In this approach, it is considered that rock and fluid properties are not invariant. In another word, the engineering approach eliminates the assumptions of classical Darcy's law.

3.3.2 Avoiding PDE's and formulation step

One of the most important features of the engineering approach is that it does not lead to PDE's, which is already discussed in the previous sections. Analytical solutions of nonlinear PDE's is often non-achievable and numerical solution is also very time consuming, and the solution procedures for getting the algebraic equations are more complicated for the highly non-linear system.

Table 3.2: The classical and generalized Darcy's law for fluid flow through porous media

Authors	Constitutive Equations	Equations	Permeability	Viscosity	Fractional Derivative Order	Diffusion	Memory Effect	Simulation Approach
Abou-Kassem (2007)	Darcy's law	Eq. 46, 47	Constant	Constant	0	Normal	No	Engineering Approach
Hossain *et al.* (2009b)	Modified Darcy's law	Eq. 18, 19	Variable	Constant	α	Anomalous	Yes	Engineering Approach
Awotunde *et al.* (2016)	Modified Darcy's law	Eq. 1	Variable	Constant	α	Anomalous	Yes	Conventional Approach
Obembe *et al.* (2017)	Modified Darcy's law	Eq. 4	Variable	Constant	$1-\gamma$	Anomalous	Yes	Conventional Approach

Table 3.3: The model equations in engineering approach

Authors	Model Equations	Remarks	Memory Effect
Abou-Kassem (2007)	$\int_{t^n}^{t^{n+1}} \left[T_{x_{i-\frac{1}{2}}}(p_{i-1} - p_i)\right] dt +$ $\int_{t^n}^{t^{n+1}} \left[T_{x_{i+\frac{1}{2}}}(p_{i-1} - p_i)\right] dt +$ $\int_{t^n}^{t^{n+1}} q_{sc_i} dt = \frac{V_{bi}}{\alpha_c} (\frac{\phi}{B})'_i [P_i^{n+1} - P_i^n]$	This equation describes in terms of pressure of grid block i	No
Hossain *et al.* (2009b)	$\int_{t^n}^{t^{n+1}} T_{x_{i-1/2}} \left[\int_0^t (t-\xi)^{-\alpha} \left\{\frac{\partial}{\partial \xi}\left[\left(\frac{\partial p}{\partial x}\right)_{i-1} - \left(\frac{\partial p}{\partial x}\right)_i\right] - \frac{\partial}{\partial \xi}(\gamma_{i-1/2})\left[\left(\frac{\partial Z}{\partial x}\right)_{i-1} - \left(\frac{\partial Z}{\partial x}\right)_i\right]\right\} \partial \xi\right] dt + \int_{t^n}^{t^{n+1}} T_{x_{i+1/2}} \left[\int_0^t (t-\xi)^{-\alpha} \left\{\frac{\partial}{\partial \xi}\left[\left(\frac{\partial p}{\partial x}\right)_{i+1} - \left(\frac{\partial p}{\partial x}\right)_i\right] - \frac{\partial}{\partial \xi}(\gamma_{i+1/2})\left[\left(\frac{\partial Z}{\partial x}\right)_{i+1} - \left(\frac{\partial Z}{\partial x}\right)_i\right]\right\} \partial \xi\right] dt + \int_{t^n}^{t^{n+1}} q_{sc_i} dt = \frac{V_{b_i}}{\alpha_c}\left(\frac{\phi}{B}\right)_i [p_i^{n+1} - p_i^n]$	The general form of diffusivity equation with memory using the Engineering Approach which leads to the non-linear equation. It was considered that the fluid memory is applicable for an axial flow of any single-phase fluid in porous media.	Yes

Table 3.4: Fundamental differences between the conventional approach and the engineering approach

Property	Conventional Approach	Engineering Approach
Fundamental Concept	Model equations are in differential form.	Model equations are in integral form.
Model Equations	$\frac{\partial}{\partial x}\left(\beta_c \frac{K_x}{\gamma B}\frac{\partial p}{\partial x}\right) + \frac{q_{sc}}{V_b}) = \frac{1}{\alpha_c}\frac{\partial}{\partial t}(\frac{\phi}{B})$ This PDE equation represents that flow equation for single-phase flow in 1D rectangular coordinate.	$\int_{t^n}^{t^{n+1}}\left[T_{x_{i-\frac{1}{2}}}(p_{i-1}-p_i)\right]dt + \int_{t^n}^{t^{n+1}}\left[T_{x_{i+\frac{1}{2}}}(p_{i-1}-p_i)\right]dt + \int_{t^n}^{t^{n+1}} q_{sc_i}dt = \frac{V_{bi}}{\alpha_c}(\frac{\phi}{B})'_i[P_i^{n+1}-P_i^n]$ This equation describes in terms of pressure of grid block i.
Formulation	Formulation step makes some basic assumptions that are presented in mathematical terms. These mathematical terms applied to the control volume. Later, partial differential equations are derived through formulation step to obtain non-linear	It does not involve formulation step. Natural reservoir process is discretized into a set of grid block or grid points. Algebraic equations are derived using mass conservation, the equation of state and constitutive equations.

Mathematical Concept		partial differential equations that define fluid flow through porous media.	Non-linear algebraic equations are obtained through these governing equations.
	Discretization	Non-linear PDEs equations are discretized by finite difference form.	The integral method is used to discretize the algebraic equations and to convert algebraic equations into non-linear algebraic equations.
	Linearization	Non-linear algebraic equations can be solved by linearization step because linear solver equations cannot do this task. This task is performed by approximating non-linear terms. (transmissibility, production and	It does not require any linearization step. At some point of the solution scheme, it may use this step. It involves approximation of time integral in the derivation of the algebraic equation.

		injection, and coefficients of unknowns in the accumulation terms) in both space and time.	Bocks are defined with dimensions in case of discretized reservoir and permeability, transmissibility is constant and independent of space and time.
Strength and Weakness		This approach involves the rigorous derivation of PDEs which is one of the weakest points of this approach. The variation of source or sink term with time within the time step is not considered.	This approach derives algebraic flow equations without going through PDEs and discretization. It uses fictitious well to represent boundary conditions which are very close to the physical meaning of flow equations.

Engineering approach deals more effectively with highly non-linear equations by avoiding the derivation of PDEs in the formulation step (equation 3.3 through to equation 3.10). The mass conservative equations are presented in an integral form. Table 3.4 discusses the formulation step for both the conventional approach and the engineering approach. This approach is also suitable for dealing experimentally validated mathematical models. There are numerical studies to solve PDE's in reservoir simulation. Saad (1989) and Gupta (1990) studied the treatment of non-linear terms in space and time. Some authors established advanced iterative methods such as nested Factorization, Orthomin, Block Iterative) for solving linear algebraic equations (Appleyard, 1983; Vinsome, 1976; Behie and Vinsome, 1982).

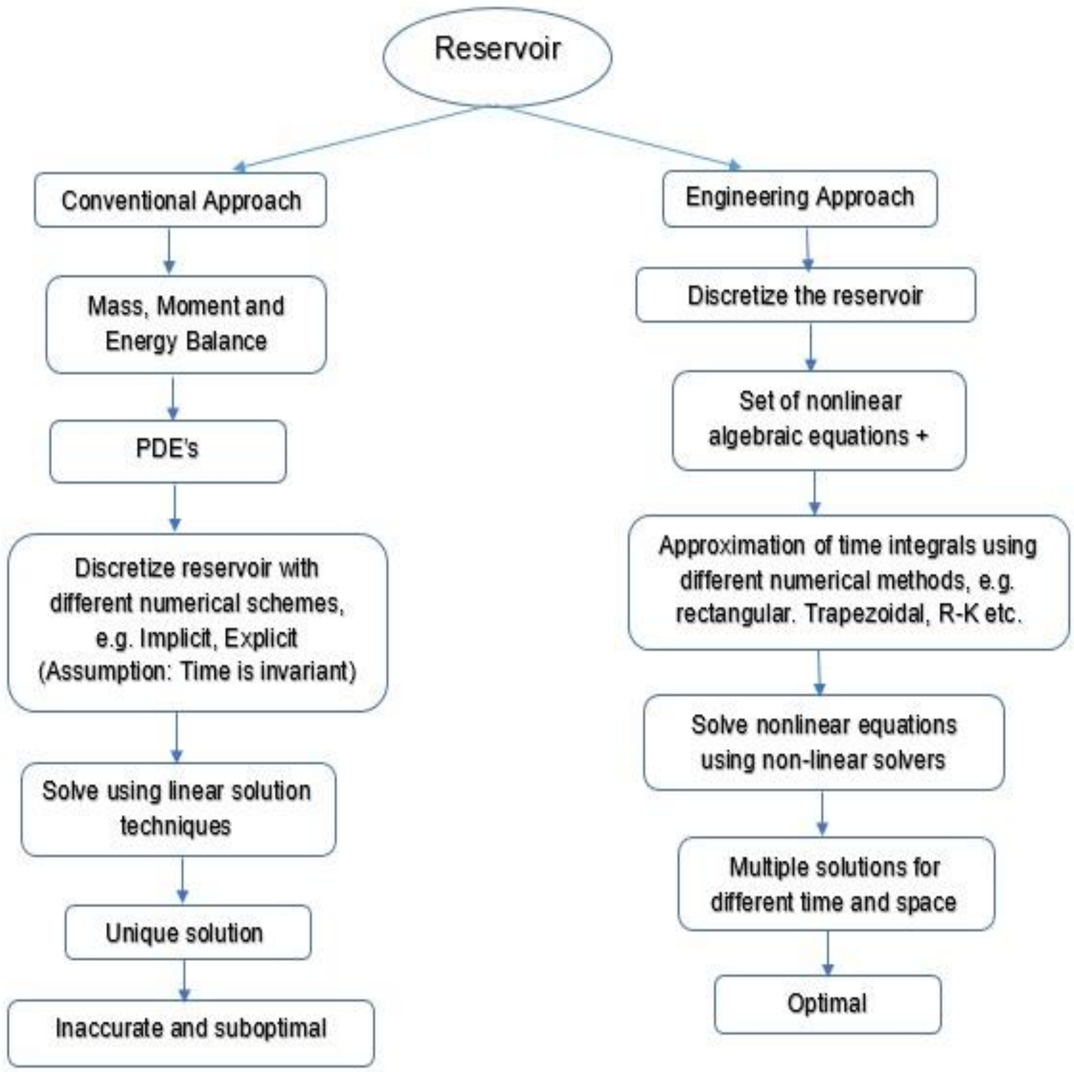

Figure 3.8: Solution steps involved in conventional approach and engineering approach

3.3.3 Inclusion of Memory concept in reservoir modeling

It is discussed earlier of this paper that the rock and fluid properties have a great effect on fluid flow through porous media in the petroleum reservoir. Darcy's law describes this phenomenon well, particularly when fluid velocity is low. In case of high velocity, this law may not be appropriate. Many authors reported that Darcy's law represents the flow through pores of a porous medium as Stokes flow over a Reprehensive elementary volume (REV) (Auriault, 1987; Bear, 1988; Rubinstein and Torquato, 1989; Ene, 1990). Many investigators (Whitaker, 1986a, 1986b; Teng and Zhao, 2000) have extended Darcy's law. Teng and Zhao (2000) derived momentum equation over a REV for a Newtonian fluid. They neglected unsteady and convection terms because these terms were negligible compared to viscous term. They showed that the generalized conditions were close to fluid flow in natural porous media. Recently, some researchers (Hu and Cushman, 1991; Nibbi, 1994; Caputo, 2000; Caputo and Kolari, 2001; Caputo, 2002; Caputo, 2014; Hossain *et al.*, 2008a, Hassan *et al*., 2015a, 2015b; Obembe *et al.*, 2017) showed that the developed theory and their models do not completely satisfy the true natural phenomena and introduced memory to their model. In table 3.2, the non-linear algebraic equations are rigorous, where Abou-Kassem (2007) considered the classical Darcy's law and Hossain et al. (2009b) considered the modified Darcy's law with the context of memory concept.

Rock and fluid properties change due to mineral precipitation, chemical reaction, temperature variations, etc. These changes lead to flow path squeezing, changing pore size, enlarging pores, and so on (Kabala and Sposito, 1991). Also, the pressure decreases due to production with time, rock and fluid properties may also vary with time. These variations of rock and fluid properties are known as memory. The rock and fluid memory are incorporated in Darcy's law to derive the diffusivity equation. In Darcy's law, the flux rate is constant with time, however, in modern Darcy's law, due to the complex rock and fluid interaction, the changes in flux rate with time have been shown by fractional diffusion equations. These complex rock-fluid interaction and behavior are known as anomalous diffusion. There have been numerous studies where fractional diffusion equations have been used to model anomalous diffusion. (Bagley and Torvik, 2000; Havlin and Ben-Avraham, 2002; Zaslavsky, 2002; AndradeLenzi *et al*., 2005, Deseri and Zingles, 2015; Fomin *et al.*, 2011; Schneider and Wyss, 1989; Razminia *et al*., 2014; Razminia *et al*., 2015a, 2015b; Zhao *et al*., 2015). Hassan *et al.* (2015a) did a comprehensive study on memory concept to model the variable rock and fluid properties over time. They used the fractional derivative to model the memory concept. They carried out a comprehensive study on different parameters, such as composite pseudo-permeability, fluid velocity, and viscosity of memory-based diffusivity equations, to capture the respective effects on pressure response of the reservoir. Rammay *et al.* (2016a) also studied a model with variable stress and pore-pressure dependent porosity and permeability over time. They established two mathematical models with space and time using Darcy's diffusivity and memory-based diffusivity equations. Obembe (2016) developed four numerical models for hot fluid injection process in a porous medium with memory concept. He considered the alteration of porosity, permeability, and dispersion for local thermal equilibrium and nonlocal thermal equilibrium. He also considered that the heat loss would occur via forced convection.

There are different types of fluid flow models with memory concept, which have the different field of applications. Table 3.5 presents the various types of fluid flow model including memory effect through porous media which have been described in the literature. Many authors studied memory concept in different fields. Zhang (2003) defined memory as a function of time and space, which depends on past events. The author developed a continuum model of second order with viscosity to derive the relationship between viscous effects in continuum flow dynamics which act like driver memory in car-following.

Several authors introduced the memory concept in their respective models of thermodynamics, heat conduction, neural glial networks and invasion percolation (Carrilo *et al.*, 2014; Carrillo, 2015; Coleman and Dill, 1973; Hristov, 2013, Sprouse, 2010; Hossain and Abou-Khamsin, 2011, 2012; Khabarah and Yortsos, 1997). Ewing *et al.* (1999) studied various finite volume element schemes for parabolic integro-differential equations. They considered their model for reactive flows or material with memory effects. Kolomietz *et al.* (2005) studied memory effects on dispersions of the kinetic energy of nuclear fission. They showed that relaxation time increases when memory effect is observed due to additional elastic forces. Kolomeitz (2014) also observed memory effects on the nuclear dynamics. He observed that memory effects depend on relaxation time. Kenkre and Sevila (2007) investigated time-dependent coefficients and memory functions of diffusivity equations. They showed when these two functions are no longer equivalent to each other; they are the spatially nonlocal generalization of others.

3.4 Challenges in Reservoir Simulation

It is still difficult to incorporate a dynamic rock model with a fluid flow model during the depletion process with a reasonable degree of accuracy using current reservoir simulators (Islam *et al.*, 2016). Most of the existing reservoir simulators do not consider the rock deformation and stress changes with the reservoir pressure during the production and do not take into consideration of the effect of change of temperature during thermal recoveries. The compaction of reservoir rock occurs due to pore size reduction, which may lead to inaccuracies of ultimate recoveries, reduction in permeability and flow rates, and collapsing to the ground and well casings damage. Branets *et al.* (2009) have extensively studied the challenges and technologies in reservoir modeling.

Table 3.5: Different types of memory-based models and their key information

Authors	Field of Application	Key Information	Remarks
Mifflin and Schowalter (1986)	Three-dimensional flows of the memory integral type of fluid	1. In case of unbounded flow regime, a residual technique is suitable, because finite element or finite difference solution would not be able to handle this type of flow regime. 2. A co-rotational Jeffreys fluid flows as a sphere in a shearing field. 3. Separation of the integral part having memory effect into velocity gradients and consideration of the torque-free laminar flow	1. The situation presented in work does not imply the true natural phenomena. 2. The model only shows how fluid viscosity changes with the stress tensor regarding time.
Ciarletta and Scarpetta (1989)	Flow of viscous fluids	1. The relationship between free energy and viscous fluids when memory is considered. 2. To see the present value of stress, a symmetric velocity gradient at every previous moment is introduced.	1. Neglect the non-linear convective term and linearized the model. 2. The theorem is a unique advantageous hypothesis.
Eringen (1991)	Flow of micro polar fluid	1. A nonlocal theory of memory-dependent micro polar fluids with orientation effects shows that the	1. The external characteristic length becomes small enough to contrast with

		internal structure of the fluid could be changed by the effect of memory.	the average radius of the gyration in infinitesimal scale
Broszeit (1997)	Flow of memory integral type fluid	1. Assume flow kinematics is known in the single integral constitutive law. 2. The relative deformation history at a particular position can be calculated for a maximum two loops for the numerical simulation of flow through the sing-screw extruder. 3. Finite element method is used for the discretization of conservation and mass equations and constitutive equations.	1. For the simulation of the flow of memory-integral type fluids, the use of finite element method is not useful, if the ratio of the size of the recirculating flow region to the size of the flow domain is low.
Daugan *et al.* (2002a, 2002b, 2004)	Flow of shear thinning fluid	1. The non-Brownian particles in a non-Newtonian fluid tend to settle along with the center line strongly for shear thinning fluid at low Reynolds number. 2. The particle position may change with time.	1. This study does not allow to perform for high Reynolds number.
Caputo (2000, and1998)	Flux in Porous Media	1. Modified the classical Darcy's law by introducing memory.	1. Caputo's models are only applicable when local phenomena are considered

		2. Permeability varies with time-dependent pressure gradient and flow. 3. Density varies with pressure by introducing a rheology in the fluid.	
Caputo and Plastino (2003)	Biological Process	1. Modifying Darcy’s law regarding the diffusion process of fluid. 2. Presented Darcy’s law regarding space fractional derivatives of the pressure. 3. The aim of using space fractional derivatives is to present the effects of the medium, which are previously affected by the fluid.	1. The authors proposed model is valid when considering local phenomena and the local variation is presented here regarding time fractional order derivatives of the pressure. 2. However, it may generate a problematic situation when a large variety of diffusion processes are taken into consideration in the biological system.
Caputo and Plastino (2004)	Flux in Porous Media	1. Modified Darcy’s constitutive equation with memory formalism. 2. Density varies with fluid pressure 3. Considered two cases, e.g., Case A and Case B.	1. The flux decays with time more rapidly and delays the effect of the pressure on the boundary with the inclusion of memory in the constitutive equation.

		4. In case of A, the green function of the pressure was computed, and closed-form formulae were found for boundary conditions, and in case of B, a skin effect was found for the flux where frequency increases with decreasing layer thickness	
Chen *et al.* (2005)	Flow of Bingham plastic fluid	1. The mobilization and subsequent flow in a porous medium of fluid have been studied with a yield stress using single-capillary expressions.	
Iaffaldano *et al.* (2005)	Diffusion Process using Sand layers' data	1.The sand layers' permeability decreases with time due to diffusion of water in sand 2.The flux rate variations are consistent with the compaction of sand as well as reduction of porosity	
Lu and Hanyga (2005)	Flow in heterogeneous porous media	1. A heterogeneous porous model is developed to simulate the wave field using Biot's Theory and the Johnson-Koplik-Dashen dynamic permeability model. 2. Memory variable satisfies first-order relaxation differential equations cannot replace the Johnson-	

		Koplik-Dashen dynamic permeability model and the viscous relaxation model	
Sullivan *et al.* (2006)	Flow of power-law fluid	1. The model describes the flow of the power-law fluid through complex porous media. 2. Lattice Boltzmann (LB) simulation techniques in three-dimensional geometry are used for the first time. 3. The shear thinning fluid characteristics are predicted by a strain-dependent relaxation parameter in both 2D and 3D media.	1. This model does not show how to calculate the shear rate directly for appropriate particle distribution functions.
Caputo and Cametti (2008)	Highly heterogeneous structures such as biological cell membranes	1. Due to the complexity of the indigenous structure, permeability is not independent of the local structure of the medium. 2. Modification of Fick's diffusion equation with memory is used to calculate the diffusion of solute concentration	1. The proposed model is applicable for highly heterogeneous structures such as biological cell membranes.

Hossain and Islam (2006b)	Fractured formation	1. Navier Stokes equation in the fracture and Darcy's law in the porous media are used for modeling 2. A formation with a fracture can behave like an open channel flow	1. Compressibility is constant with respect to pressure.
Hossain *et al.* (2007)	Stress-strain model	1. Applicable to non-Newtonian fluids, especially crude oil. 2.Temperature variations, surface tension, pressure variations, and fluid memory are incorporated in the developed mathematical model	1. The memory effect has been shown with space, which results in non-linear behavior in the stress-strain relationship
Hossain and Islam (2009c)	Fractured formation	1.Introduced a comprehensive material balance equation (MBE) with the inclusion of time dependent porosity and permeability 2. The fluid and formation compressibility as any functions of pressure 3. The improvement in the recovery factor was as much as 5%.	1.The inclusion of memory effects of fluids regarding a continuous time function in rock stress-strain relationships results in a highly non-linear MBE

Giuseppe *et al.* (2010)	Flux in Porous Media	1. Local variation of permeability affects the flow path of porous media. 2. The modification Darcy's equation is done for the pressure gradient-flux and the pressure density variations	1. Constant pressure on the boundary is considered during the laboratory experiments.
Rammay *et al.* (2016a, 2016b)	Various rock and fluid properties model	1. The models show the variation of stress and pore-pressure dependent porosity and permeability over time and the variations of PVT properties with memory. 2. Darcy's diffusivity and memory based diffusivity equations are used for the model development.	
Hossain (2016)	Variation of porosity and permeability with pressure response	1. The modification of the conventional momentum balance equations is performed. 2. The modification is done by a derivative of fractional distributed order as a memory formalism 3. Fractional order represents time-dependent diffusivity; distributed order was used to model the variation of porosity and permeability with pressure response	

The physical effect of these geo-mechanical aspects of reservoir behavior is neither insignificant nor negligible. In case of high velocity or high gas flow rates and highly fractured reservoir, the use of Darcy's law to describe the hydrocarbon flow behavior may mislead the accuracy. Sometimes, it is not possible to describe the true characteristic of the reservoir.

3.5 Additional Research Guidelines

As previously discussed that memory plays an important role as the rock and fluid properties vary and behave chaotically over time. The existing simulator and nonlinear solver cannot truly represent fluid rheological behavior. Only few researchers (Hu and Cushman, 1991; Nibbi, 1994; Caputo, 2000; Caputo and Kolari, 2001; Caputo, 2002; Hossain *et al.*, 2006b, Hossain *et al*., 2007; Hossain *et al*., 2008a; Hossain *et al*., 2008b; Hossain *et al.*, 2009b, 2009c) modified their model to capture continuous alteration of the rock and fluid properties by invoking time dependence. Model equations are needed to be solved in 2D and 3D with considering fractures and thermal stress considering compositional method. Incorporating memory into the diffusivity equations produces highly nonlinear equation. Recently, several mathematical and numerical methods have been presented which are capable of solving these highly non-linear equations and produces multiple solutions of non-linear algebraic equations (Mousavizadegan *et al.*, 2007, 2008; Mustafiz *et al.,* 2008a; Mustafiz *et al.,* 2008b; Islam *et al*., 2009). In addition, memory concept can be invoked into engineering approach, and the results from the engineering approach will be unique as well as simple without any spurious assumptions. Researchers need to develop such a model that can describe the actual phenomenon of fluid flow through porous media excluding inherent assumptions and solution strategy to efficiently solve the new model.

Besides the engineering approach, another advance in reservoir simulation is artificial intelligence and data mining. Mohaghegh (2011) introduced two new methods with artificial intelligence and data mining to enhance the recovery of fluid flow through porous media. One is surrogate reservoir models, and another one is top-down, intelligent reservoir models. The surrogate reservoir model is a copy of real and complex reservoir model including ten to hundreds of wells and thousands of grid blocks and points. Top-down, intelligent modeling can detect optimum spots by tracking hydrocarbon depletion and checking remaining reserves with the function of time. These models are also very important to characterize the complex reservoir for fluid flow through porous media.

They can handle dynamic and non-linear problems. However, these types of methods require large data set and often rely on numerical and analytical models to generate data. This is an interesting new development in reservoir simulation, however, in this review, we did not explore this direction.

Conclusion

In this study, the engineering approach and the conventional approach are analyzed and contrasted. A critical review and brief discussions are made in addition to advantages of the engineering approach and limitation of the conventional approach. Since memory-based diffusivity equations incorporate continuous alteration of rock and fluid properties with time, the results from memory-based models will be more realistic in the reservoir simulation along with engineering approach. The possible future challenges in reservoir simulation toward developing a model are also recognized. This research will help to better understand the importance of the engineering approach and the contribution of memory in fluid flow through porous media in reservoir simulation.

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CHAPTER FOUR

An Investigation of the Effects of Temperature, Pressure, and Shear Rate on the Rheological Properties of Heavy Crude Oil

Abstract

The objective of this study is to perform a study effect of different parameters on heavy crude oil rheology by using the Bingham model. As heavy crude oil is a non-Newtonian fluid, it does not follow Newton's laws of viscosity and the general Darcy's law. To account the non-Newtonian behavior, the Bingham constitutive equation is considered. The effects of shear rate, temperature, and pressure on viscosity are investigated. The general Bingham model is compared with experimental data and it shows a good match. Later, the temperature, shear rate, and pressure variations are analyzed by considering viscosity as a function of these parameters. The effect of pressure and temperature are also shown in the Bingham model. The experimental data used for this study are for heavy crude oil. As the general Bingham model fits with experimental data, it means that we can use Bingham model for the heavy crude oil. From the comparative analysis, it is found that the temperature has the utmost impact on the viscosity as well as the heavy crude oil rheology over the shear rate and the pressure. Experimental data are well matched with field data when viscosity is considered as a function of temperature. Furthermore, the viscosity decreases with increasing shear rate and increases with increasing pressure. Finally, the flow characterization and heavy crude oil rheology depend on shear rate, temperature, and pressure. The pressure has strong effect in the shear stress-strain relationship over the temperature and the temperature has the greatest impact on the viscosity. This study is the first step to model heavy crude oil rheology. In the next work, we will incorporate this knowledge for the modeling of the heavy crude oil rheology and flow characterization. Investigation though this study will help to make a better decision for crude oil rheology modeling through porous media and pipeline transportation.

Keywords: Bingham model, heavy crude oil, pressure, temperature, apparent viscosity, shear stress.

4.1 Introduction

Crude oil plays an important role among all the source of energy as it is considered the most actively traded commodities that gives a significant amount of energy supply all over the world. Global demand for crude oil has increased from 60 million per day to 84 million per day over the past two decades (Hasan *et al*., 2010). Currently, about ninety percentage of the petroleum production from the reservoirs are the conventional crude oil. This means about thirty percentage crude oil is from the unconventional reservoirs (Sami *et al*., 2017). A large portion of unproduced petroleum resources contains heavy crude oil and bitumen, and about fifty-five percentage of recoverable crude oil reserves are extra heavy crude oil and bitumen (Thakur and Rajput, 2011). However, the demand of heavy crude oil is decreasing in this 20^{th} century due to its complex rheological behavior, high cost, and technological difficulty to produce from reservoir, transport though pipeline, and refine compared to light crude oil (Sami *et al.,* 2017).

Heavy crude oils produced from different wells have different rheological properties. Production and transportation of heavy crude oil is challenging due to its non-Newtonian flow behavior. The viscosity and density of heavy crude oil are very high compared to light crude oil. Viscosity is a property that controls the crude oil flow through porous media. It also has a great influence on the flow through pipelines. Crude oil mobility is related to crude oil's viscosity as the mobility is the ratio of the permeability to viscosity. As heavy crude oil has high viscosity and low permeability, its mobility will be very low than the light crude oil. It also makes the low flow rate and recovery from the reservoir. Therefore, it has become a vital task to alternate the rheological properties of heavy crude oil which makes the crude oil flowing ability improved. It is also possible to obtain effective oil production from the reservoir. Sami *et al*. (2017) provided a density range (875 to 1000 Kg/m^3) and a viscosity range (1 to 10 Pa-s) for heavy crude oil. Some authors considered the viscosity range from 0.01 Pa-s to 10 Pa-s. Kumar *et al*. (2016) the viscosity of heavy crude oil can be reached maximum 10^5 mPa-s at 25^0C. When the density of crude oil exceeds 1000 Kg/m^3, it is considered as extra heavy crude oil, e.g., Bitumen (Hoshyargar and Ashrafizadeh, 2013).

When the shear force is applied to the crude oil, it will start to deform. This deformation will continue until the shear force is applied. The crude oil viscosity specifies the relationship between shear stress and the rate of deformation. Generally, the viscous stress helps to initiate, steady, and unify the flow and the extreme inertia force makes obstruction in the steady and unified flow and creates chaotic turbulent behavior (Hossain *et al*., 2007). It is well known that the Newtonian fluid obeys Newton's law of viscosity, whereas, the non-Newtonian fluid follows others non-Newtonian models. Heavy crude oil shows Bingham type non-Newtonian fluid behavior in porous media. In these types of fluids, a minimum pressure gradient, e.g., threshold pressure gradient is required to start the flow. The heavy crude oil flow through porous media does not follow the classical Darcy's law and a corrected Darcy's law is used for heavy crude oil followed by Dong *et al*. (2013) and Wu (2015) is incorporated and may be considered as Bingham fluid (Mirzadjanzade *et al*., 1971). Heavy crude oil shows a finite yield stress (Sochi, 2007; Sochi and Blunt, 2008; Sochi, 2010) at zero shear rate like Bingham fluid (Wu *et al*., 1992). The crude oil will only flow when the shear stress is greater than the yield stress and will behave like Newtonian fluid. However, heavy crude oil rheology is affected by several parameters. These parameters are temperature, pressure, shear rate, surface tension, pH, wax precipitation, and some other factors. Block (1951) did a critical review based on the empirical formula that describes the viscosity-pressure-temperature relationship. Although numerous researchers provided knowledge about the viscosity of heavy crude at the different temperature, pressure, and shear rate (Svrcek and Mehrotra, 1982; Mehrotra and Svrcek, 1985; Schramm and Kwak, 1988; Barrufet and Setiadarma, 2003; Yazdani and Maini, 2010; Guan *et al*., 2013). Most researchers have not considered the effect of these parameters in the shear stress-shear rate relationship. Ukwouma and Adimodi (1999) performed an experimental study in the viscosity if Nigerian oil sand bitumen. They considered the range of temperature from 50^0C to 110^0C and the shear rate range from 60-320 1/s. The authors found that despite of temperature dependency of viscosity, it also depends on shearing effects. As the temperature effects increases, the shearing effects decrease and behave as Newtonian at high temperature. They concluded that the apparent viscosity of bitumen depends on the shear rate. Mortazavi-Manesh and Shaw (2016) did an experimental study to show the effect of pressure on the Maya crude oil rheology. They found that the viscosity is a function of temperature. The authors stated that the effect of pressure is negligible compared to the temperature.

The objective of this study to provide a clear idea of the role of shear rate, temperature, and pressure on the heavy crude oil rheology. These factors are introduced regarding the shear stress. It is observed through this study that apart from the high-temperature dependency effect, the heavy crude oil rheology also depends on the shear rate and pressure.

4.2 Model Analysis

In this section, the characterization of the rheological behavior of heavy crude oil is done by using shear stress-stain relationship. For heavy crude oil, Bingham model is used to represent the shear stress and shear rate relationship.

$$\tau = \tau_0 + \dot{\gamma}\mu_a \tag{4.1}$$

For Newtonian fluid, the apparent viscosity is constant. For non-Newtonian fluid, the apparent viscosity is function of pressure, temperature, shear rate, and some other factors. First, we will consider, apparent viscosity as a function of shear rate.

$$\mu_a = f(\dot{\gamma}) \tag{4.2}$$

From literature (Taborda *et al*., 2017, Wang *et al*., 2005), it is found that the viscosity decreases by increasing shear rate. So, we can write the following relationship for viscosity and strain rate.

$$\mu_a = \mu_o e^{-B\dot{\gamma}} \tag{4.3}$$

Putting the value of μ_a from equation 4.3 into equation 4.1, we get,

$$\tau = \tau_0 + \dot{\gamma}(\mu_o e^{-B\dot{\gamma}}) \tag{4.4}$$

Heavy crude oil is a temperature dependent non-Newtonian fluid and its viscosity depends on temperature (Xin *et al.,* 2017). If we consider that the viscosity is function of temperature.

$$\mu_a = f(T) \tag{4.5}$$

The viscosity decreases with temperature and shows exponential relationship for crude oil. So, it can be written as follows,

$$\mu_a = \mu_o e^{-CT} \tag{4.6}$$

Substituting the temperature dependent viscosity value in the equation 4.1, we get,

$$\tau = \tau_0 + \dot{\gamma}(\mu_o e^{-CT}) \tag{4.7}$$

The viscosity of heavy crude oil is also a function of pressure and Mortazavi-Manesh and Shaw (2016). There is an exponential relationship between the viscosity and the shear rate. It can be expressed by the following relationships.

$$\mu_a = f(P) \tag{4.8}$$

When the pressure increases, the viscosity also increases. So, we can write as the following equation,

$$\mu_a = \mu_o e^{DP} \tag{4.9}$$

Replacing the pressure dependent viscosity value in the equation 4.1, we get,

$$\tau = \tau_0 + \dot{\gamma}(\mu_o e^{DP}) \tag{4.10}$$

Figure 4.1 shows the steps which is related to the comparative model analysis discussed through equation 4.1 to equation 4.10.

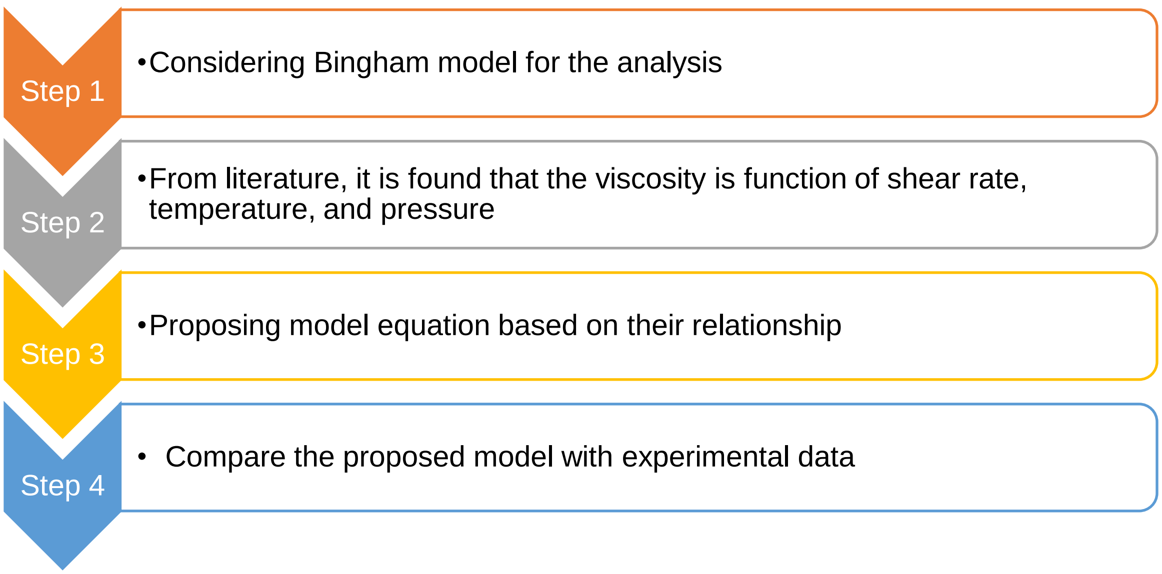

Figure 4.1: Steps related in the theoretical model analysis

4.3 Numerical Results

The data used in this study are taken from Wang *et al*. (2005), Xin *et al*. (2017), and Mortazavi-Manesh and Shaw (2016). To get the value of μ_o and B, C, and D , we use the rheological data from literature. When the viscosity is function of shear rate, the viscosity changes from 0.133301 Pas-s to 0.10756 Pas-s for the change of shear rate from 9.63313 (1/s) to 200.238 (1/s) (Taborda *et al*., 2017). For the temperature ranging from 303 K to 343.5 K (Xin *et al*., 2017), the variation in viscosity is from 7.82E-01 Pas-s to 1.44E-01 Pas-s for this condition. When the viscosity is function of pressure, the viscosity varies from 3.0 Pas-s to 4.8 Pas-s for the change of pressure from 1.42E+04 Pa to 8.76E+05 Pa (Mortazavi-Manesh and Shaw, 2016). The numerical simulation is done by using MATLAB. Table 4.1 summarizes the value of μ_o for each condition (shear rate, temperature. and pressure) and the constants that are obtained by matching the rheological data from the literature.

Table 4.1: The constant parameters when the viscosity is the function of shear rate, temperature, and pressure

Parameters	μ_o, Pas-s	B	C	D
Shear Rate	0.1322	0.001079	-	-
Temperature	5.393e5	-	0.04441	-
Pressure	3.003	-	-	4.961e-07

4.3.1 Effect of Shear Rate, Temperature, and Pressure on Viscosity

As we are considering Bingham model for the analysis, we do the comparative analysis between the Bingham model and the experimental data (Xin *et al*., 2017) (figure 4.2). The apparent viscosity is considered 783 mPa.s. It shows that the Bingham model fits well with experimental data.

After getting the parameters for the fitted exponential model, we solve the equation 4.3, 4.6, and 4.9 for different conditions. Figure 4.3 shows viscosity-shear rate relationship between the proposed equation 4.3 and the experimental data for heavy crude oil. The apparent viscosity is decreasing for increasing shear rate. It is seen from figure 4.3 that the viscosity decreases rapidly when the shear rate is small. At large shear rate, the viscosity decreases gradually. It confirms that heavy crude oil also shows shear thinning behavior. The proposed model matches the experimental data (Xin *et al*., 2017). Following the estimation of the viscosity model for

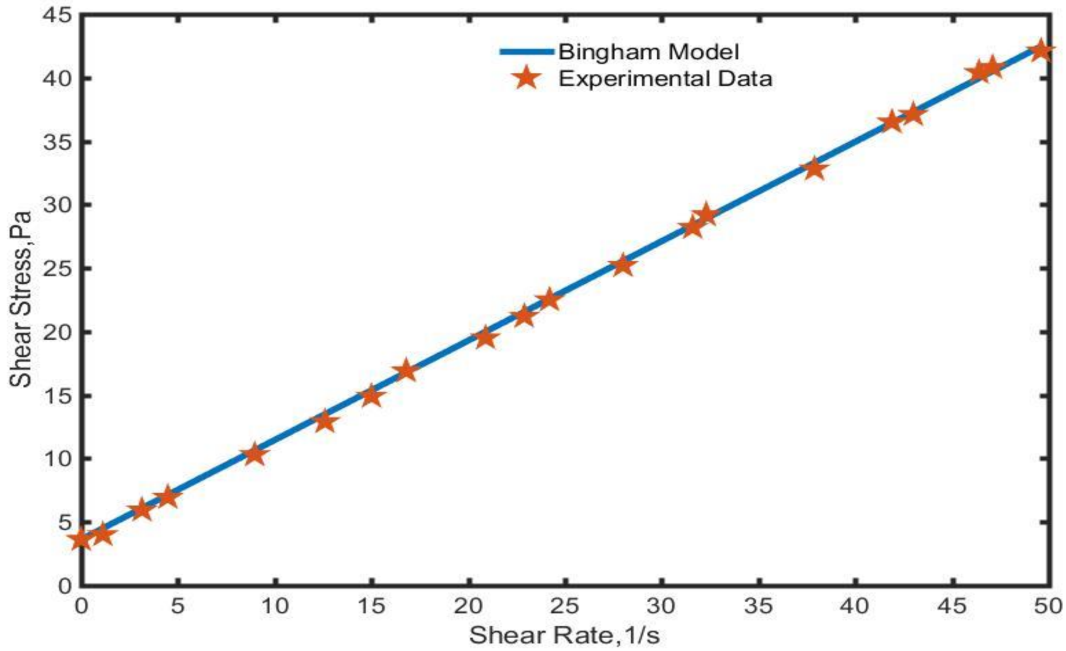

Figure 4.2: Shear stress-shear rate relationship between Bingham model and experimental data

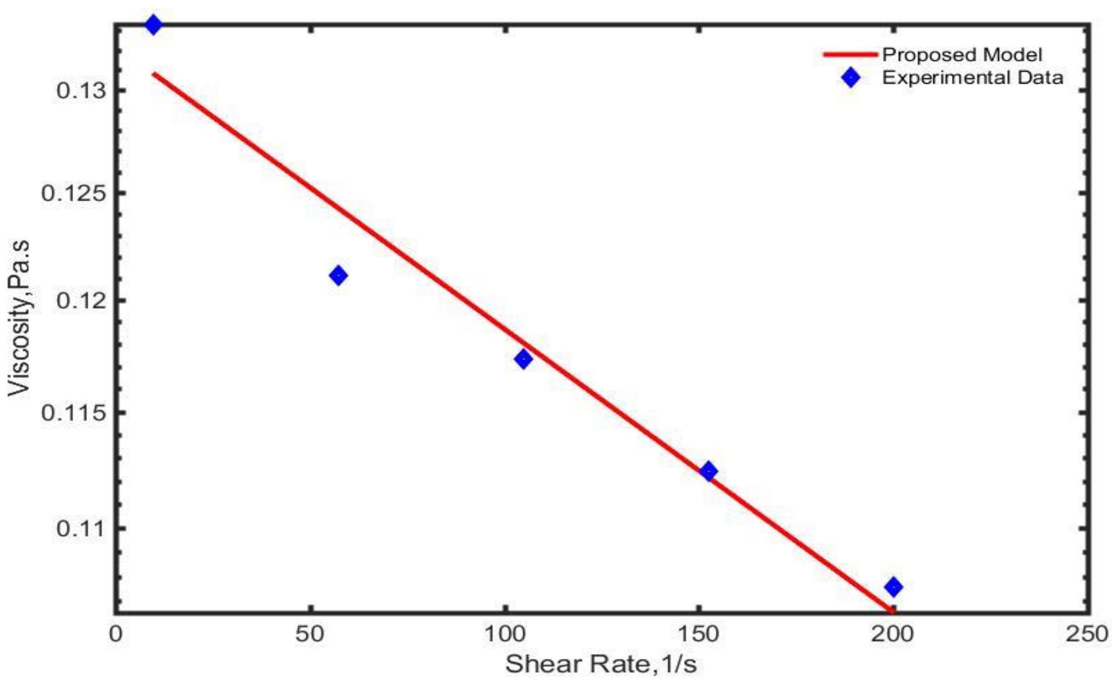

Figure 4.3: Viscosity-shear rate relationship between proposed model and experimental data

different parameters, the shear rate-viscosity dependency is incorporated in the Bingham model for heavy crude.

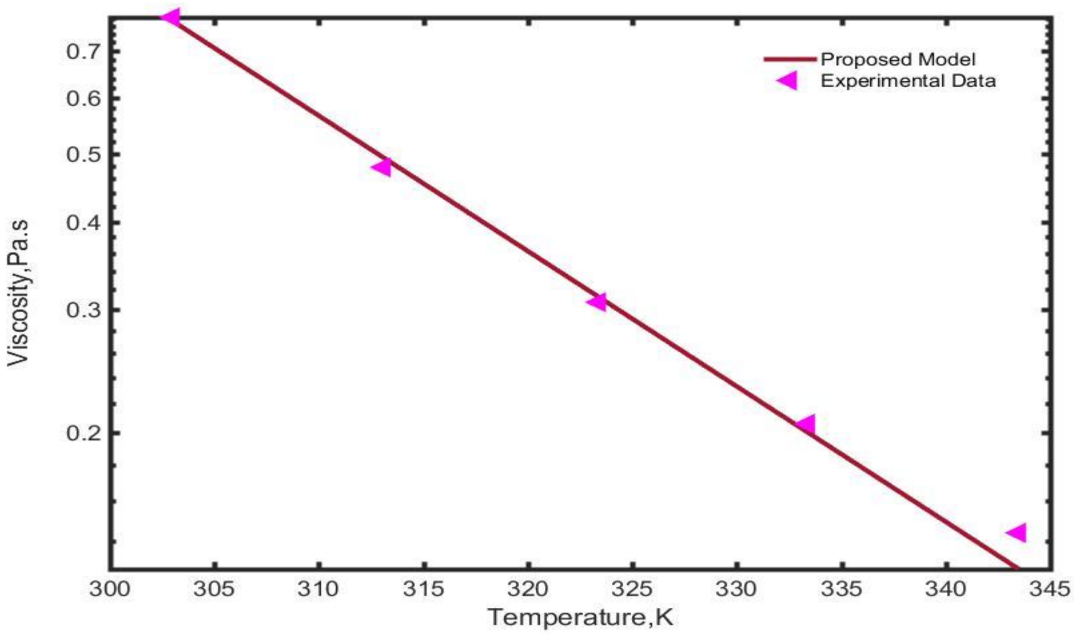

Figure 4.4: Viscosity-temperature relationship between proposed model and experimental data

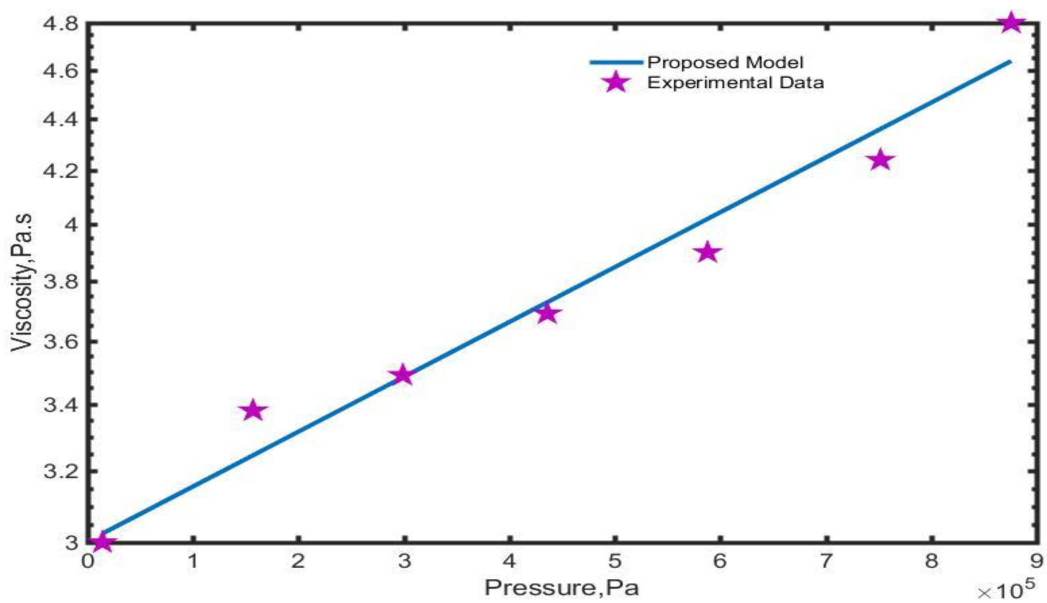

Figure 4.5: Viscosity-pressure relationship between proposed model and experimental data

Figure 4.4 shows the viscosity-temperature relationship between proposed model and experimental data. Heavy crude oil behaves as a non-Newtonian fluid at low temperature.

Viscosity decreases with the increase of temperature. The apparent viscosity decreases slowly with increasing temperature especially at large temperature. When the temperature is small, the viscosity of heavy crude oil decreases quickly due to increasing temperature. It is seen from figure 4.4 that the proposed model ($\mu_a = f(T)$) matches with the experimental data. As the trend is same and there is a good match in the analysis, the findings of this study explain that the temperature has a strong effect on the viscosity. Figure 4.5 shows the viscosity-pressure relationship between proposed model and the experimental data. It is observed that the viscosity increases with increasing pressure. From thermodynamic point of view, this relationship between viscosity and pressure is true as the pressure and the temperature has an inverse relationship and the viscosity decreases with increasing temperature.

4.3.2 Effect of Temperature and Pressure on Shear Stress

In this section, we will incorporate the effect of temperature and pressure variation on shear stress and compare the shear stress variation due to temperature and pressure effect. We use equation 4.7 and 4.10 for the analysis. When the temperature, T is $303\ K$, the rate of change of shear stress values is high (figure 4.6). At $T = 323\ K$, the rate of change of shear stress is low compared to $T = 303\ K$. The rate further decreases as the temperature increases from $323\ K$ to $343\ K$ (figure 4.6). This signifies that at high temperature the heavy crude oil will flow with low shear stress due to a reduction in viscosity. The proposed model has only been simulated however, it is not validated due to lack of experimental data.

Figure 4.7 shows the shear stress variation at different pressure. An increment of pressure value from 1.42e4 Pa to 8.76e5 Pa results in increase in shear stress from 151 Pa to 240 Pa. At low pressure and low shear rate, shear stress increases slowly. As the shear stress increases and the pressure also increases, there is a rapid increase in shear stress value. Pressure effect was not considered in determining the rheological behavior of the crude in the experiment. So, it is obvious that the proposed model will not match with the rheological data taken from the experiment. From the figures, it is observed that the pressure has more effect on the shear stress-shear rate relationship compared to the temperature. As the heavy crude oil is a high viscous fluid, additional pressure is required to apply to rise the flow rate. It can be a good decision to incorporate both the temperature effect and the pressure effect for the movement of heavy crude oil flow through porous media.

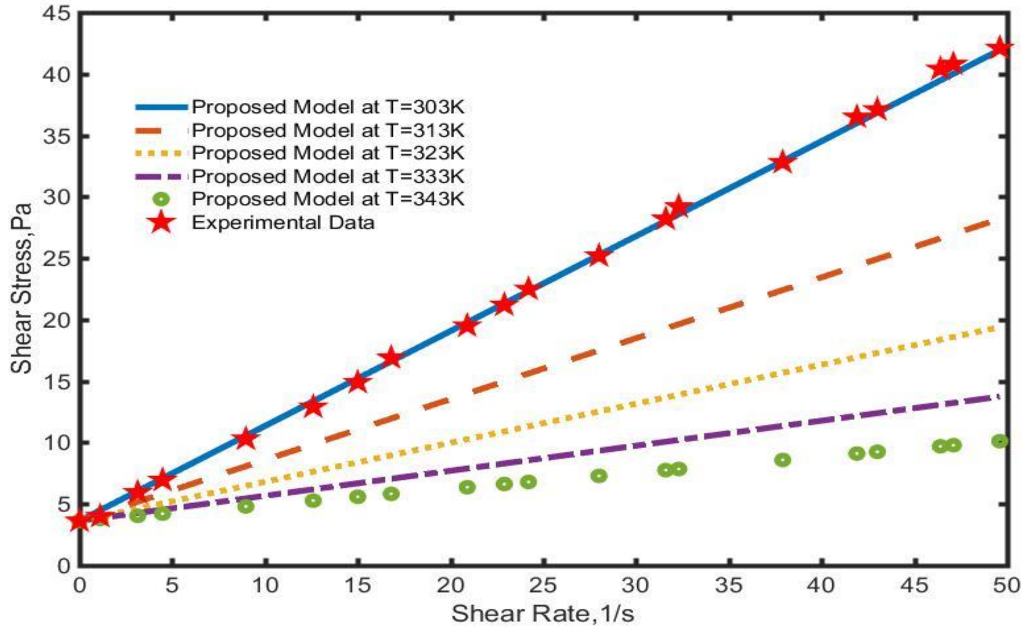

Figure 4.6: Shear stress-shear rate relationship at different temperature (K)

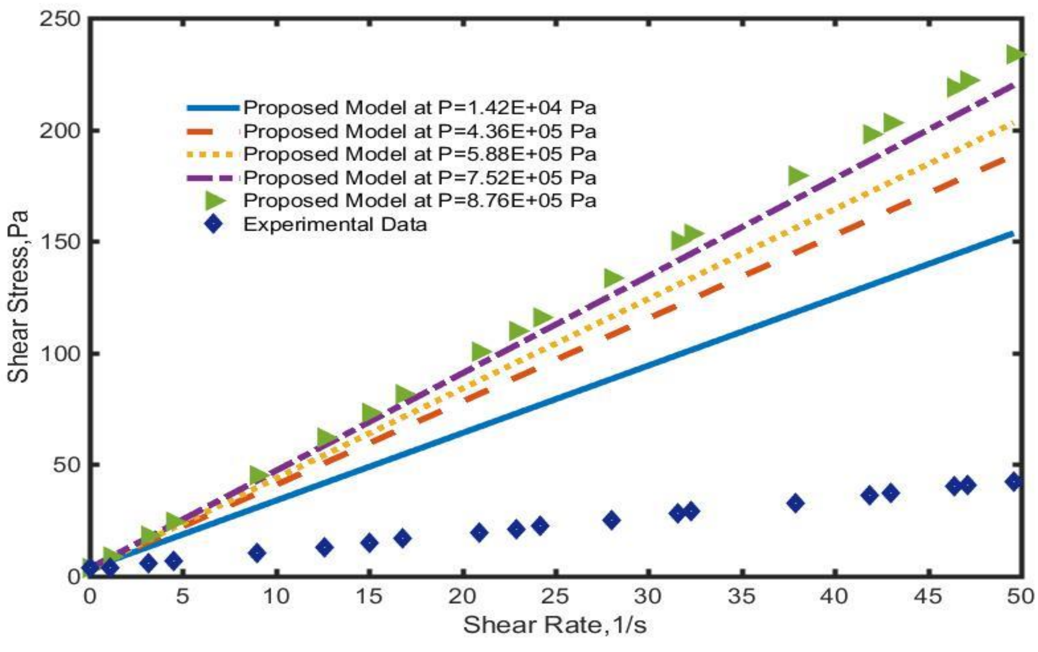

Figure 4.7: Shear stress-shear rate relationship at different pressure (Pa)

Conclusions

In this chapter a sensitivity analysis by the combination of the effect of shear rate, temperature, and pressure for the characterization of heavy crude oil. Heavy crude oil is considered as a Bingham fluid with non-Newtonian flow properties and characteristics that shows the effect of shear rate, temperature, and pressure on the viscosity in the shear stress-shear rate relationship. All parameters obtained are explained and follow an established trend as Bingham model. The change in shear stress value decreases with increasing temperature at constant pressure. As an increment in the temperature means the reduction in heavy crude oil viscosity, this requires less shear stress for the flow through porous media. If the viscosity is a function of pressure, the variation is shear stress value is high compared to temperature and the incorporation of pressure effect along with temperature would be a great idea to consider both effects in the Bingham model. The results of this research aid to increase the understanding the heavy crude oil rheology phenomenologically and open a new panorama to modeling the phenomena that involve the use of Bingham model for the heavy crude oil.

Acknowledgments

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Nomenclature

τ	Shear stress, Pa
τ_0	Yield stress, Pa
$\dot{\gamma}$	Shear rate, s^{-1}
μ_o	Oil viscosity, Pas-s
μ_a	Apparent viscosity, Pas-s
P	Pressure, Pa

$B, C, and\ D$ Constant

Reference

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CHAPTER FIVE

A Modified Memory-Based Stress-Strain Model for Heavy Crude Oil

Abstract

Heavy oil is a non-Newtonian fluid which has a strong nonlinearity in on the stress-strain relationship. To account for the non-Newtonian behavior, the Bingham model is considered in this study. This study incorporates the effect of temperature and memory in the derivation of the mathematical model. A modified shear stress-strain relationship has been proposed to capture the fluid memory effects in the Bingham model. This is done by incorporating modified Darcy's law introduced by Caputo. Three cases are introduced in the modified stress-strain model. Case 1 is studied in this paper, where, we will show the memory effect in the Bingham model. The effect of temperature is shown by Arrhenius equation. The time dependency is shown by introducing modified Darcy's law. As time increases, shear stress increases, and the rate of deformation increases. This relationship varies with the value of fractional order, α. The fractional derivative part is solved using finite difference method. Viscosity plays an important role in the rheology of heavy crude oil. Heavy oil viscosity will not change until the shear stress reaches yield stress. After reaching the required amount of shear stress, viscosity will change with shear rate. The relationship shows the non-linear behavior of heavy crude rheology. The modified shear stress-strain relationship can be used in the selection of foam and surfactant in enhanced oil recovery, pipeline transportation of heavy crude oil, well test analysis, and can be used for broader rheological study.

Key Words: heavy crude oil, Bingham model, shear stress, shear rate, viscosity

5.1 Introduction

Non-Newtonian fluids in porous media have a wide range of applications, petroleum engineering, soil remediation, filtration of the polymer solution, removal of liquid pollutants (Sochi and Blunt, 2008). There are numerous theoretical, analytical, and experimental studies on non-Newtonian fluids for single phase porous media in the past few decades (Wu and Pruess, 1998). However, there is very limited research that focuses the theoretical and numerical modeling of non-Newtonian fluids. The rheological properties of petroleum reservoir fluids play a vital role to optimize the production and pipeline transportation (Alomair *et al*., 2014). Heavy crude oil, foam, visco elastic surfactants, drilling mud, polymer solutions, waxy crude oil are some examples of non-Newtonian fluids for porous media in the petroleum field. The flow characteristics and the bulk rheology of the heavy crude oils through porous media have gained great attention due to its importance in enhanced oil recovery. In most of the cases, three important parameters, e.g., shear stress, shear rate, and viscosity are the significant parameters to characterize the fluid rheology. The impacts of these parameters especially the viscosity has a significant influence on the oil production. As viscosity decreases, the mobility of crude oil increases. This is done by lowering the viscosity. There is a number of ways to reduce the fluid viscosity. Martínez-Palou *et al.* (2011) provided a detailed review of the transportation of heavy crude oil by lowering viscosity.

Jossi *et al*. (1962) proposed a correlation where viscosity is the function of pressure, temperature, and chemical species properties. They considered the density range from 0.1 to 0.3 for their developed correlation. The authors validated the model for only non-polar substances. Some authors proposed non-local flow theories to incorporate the memory effect (Hu and Cushman, 1994). They used Darcy's law for saturated flow by applying statistical physics principle with some limited conditions. Starov and Zhdanov (2001) developed a model showing the relationship between the viscosity and the resistance coefficient (1/permeability). They used Brinkman's equation to determine the effective properties (viscosity, permeability, porosity, etc.) for fluid flow through porous media. The authors also developed a relation between the viscosity and the porosity. Abel *et al.* (2002) did a comprehensive study for a visco-elastic fluid in a porous medium over a non-isothermal stretching sheet. They considered several cases for determining the permeability, viscosity, and visco-elastic parameters. The authors found that the fluid viscosity is a function of temperature for the fluid flow through porous media. They also added that the effect of permeability parameters with the skin friction. Brenner (2005) did a review of Newton's law of viscosity for compressible fluids (liquid and gas). He used velocity gradient in place of strain rate and found a relation between the shear stress and the density. The author also showed the relationship between the deviatoric stress tensor in the Navier Stokes equation. Non-Newtonian fluids show the change in viscosity by applying the shear stress. For thixotropic fluids (pseudo-plastic fluids), the viscosity decreases by applying shear stress over time. On the contrary, for rheotropic fluids (dilatants fluids) the viscosity increases due to shear stress over time. Feys *et al.* (2009) explained the rheology behavior of fresh concrete by using two established method for shear thickening fluid behavior. They showed the effect of volume fraction in the shear stress and viscosity relationship. The authors found that the viscosity increases by increasing shear stress for shear thickening fluids, whereas, the fluid viscosity decreases with respect to the increment of the shear stress. However, the change in viscosity with shear stress is very small at low volume fractions. Barnes and Walters (1985) considered Herschel-Bulkley model for their experiment and showed that the apparent viscosity declines with increasing shear stress. Moller *et al*. (2009) experimentally proved that the existence of yield stress by showing the clear line between the solid state and the flowing state. They did their experiment for the 0.2% Carbopol, the hair gel, the foam, and the emulsion. For each case, they found a reduction of viscosity with increasing shear stress at the different time. Roberts *et al.* (2001) also showed that the shear thinning fluids viscosity decreases with increasing shear stress. They considered Bingham, Casson, and Herschel -Bulkley model to show the effects of viscosity with shear stress and shear rate.

In case of heavy crude, a minimum shear force has to be applied before the fluid starts flowing, e.g., minimum shear stress (yield stress). The yield stress concept was first introduced by Bingham and Green (1919) for Viscoplastic fluids. The importance of yield stress in porous media has been discussed by many researchers (Chen *et al*., 2005). Yield stress is a time-dependent property. The fluids that show yield stress behavior can be modeled by Bingham model (Rashaida, 2005). The flow characterization of heavy crude can be modelled by assuming heavy crude oil as a Bingham fluid (Dong *et al*., 2013). The relationship between shear stress and shear rate defines the types of fluids (light or heavy crude oil) and their flow behavior from viscosity point of view. However, it is also very important to model the structure of formation media because of the complex natural phenomena like solid and liquid interactions during thermal constraint, complex pore geometry, and rheology in porous media, etc. in the reservoir. For this reason, some authors developed a modified macroscopic flux law, e.g., modified Darcy's law, to model the flow of non-Newtonian fluids.

Hossain *et al.* (2007) used a modified Darcy's law that considers the fluid memory effect in the stress-strain relationship for light crude oil. The fluid memory means the rock and fluid properties changes with time. With time the rock and fluid properties change due to mineral precipitation, chemical reaction, temperature variations, etc. These changes lead to flow path squeezing, changing pore size, enlarging pores, and so on (Kabala and Sposito, 1991). Also, the pressure decreases due to production with time, rock and fluid properties may also vary with time. These variations of rock and fluid properties are known as memory. The rock and fluid memory are incorporated in Darcy's law to derive the diffusivity equation. In modified Darcy's law, the changes in flux rate with time have been shown by fractional diffusion equations. These complex rock-fluid interaction and behavior are known as anomalous diffusion. There have been several studies where fractional diffusion equations have been used to model anomalous diffusion. (Bagley and Torvik, 2000; Havlin and Ben-Avraham, 2002; Zaslavsky, 2002; Deseri and Zingles, 2015; Fomin *et al.*, 2011; Schneider and Wyss, 1989; Razminia *et al*., 2014; Razminia *et al*., 2015a, 2015b; Zhao *et al*., 2015). Hassan *et al.* (2015a) did a comprehensive study on memory concept to model the variable rock and fluid properties over time. They used the fractional derivative to model the memory concept. They carried out a comprehensive study on different parameters, such as composite pseudo-permeability, fluid velocity, and viscosity of memory-based diffusivity equations, to capture the respective effects on pressure response of the reservoir. Happy *et al.* (2017) and Rahman *et al.* (2017) reviewed all memory-based models, the advantages of using memory concept in porous media, and enhanced oil recovery. Most researchers incorporated memory concept for food, polymer solutions, etc. in the Bingham model. They introduced memory effect in the strain rate in terms of time. However, in this paper, the time memory effect has been modelled by introducing modified Darcy's law instead of strain rate, e.g., velocity gradient. The results show that the model follows the general trend of shear stress-strain relationship. As fluid memory is incorporated in the Bingham constitutive equations, it is expected that this model can be a good tool for the characterization of heavy crude oil in enhanced oil recovery.

5.2 Model Development

For porous media, the control volume is represented by the capillary model. Consider a capillary model that considers pores of rocks. These pores of rocks are assumed as a series of parallel capillary. Heavy crude oil is flowing in the capillary in the x-direction. Figure 5.1 and 5.2 shows the flow of heavy crude oil in a capillary.

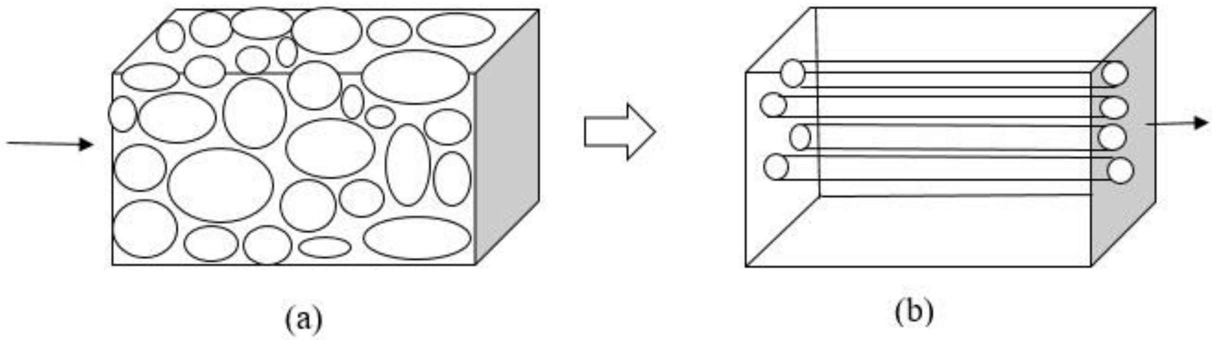

Figure 5.1: (a) a small section of porous media, (b) fluid flowing through multiple parallel capillaries (Redrawn from Al-Doury and M.M.I (2010)

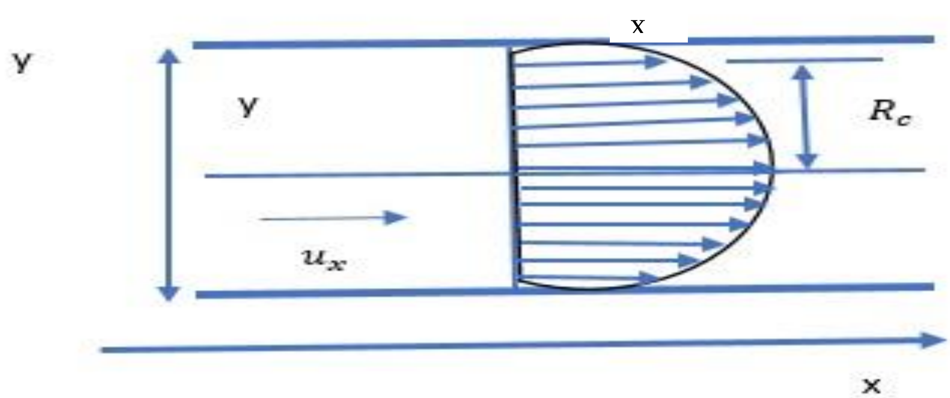

Figure 5.2: Flow of Heavy crude oil through one Capillary (Redrawn from Cui et al., 2013)

For heavy crude oil flow in capillary, a Bingham model (Eq. 1) is considered. The crude oil will not deform until the shear stress reaches the yield stress. The excess shear stress will drive to deform later. This creates two types of regions consisting of plug region ($\tau > \tau_0$) and shear region ($\tau < \tau_0$) (Rashaida, 2005).

5.2.1 Model Assumptions

For the development of the model the following assumptions were made.

1. The heavy crude is a single phase non- Newtonian fluid
2. The porous media can be represented by capillary model
3. Bingham model is time dependent

4. Modified Darcy's law is applicable for heavy crude oil
5. Reservoir rock is incompressible, and fluid is slightly compressible
6. Arrhenius equation is used to show temperature effect

5.2.2 Theoretical Development

For heavy crude oil, Bingham model is used to represent the shear stress and shear rate relationship (Zhang *et al*., 2015).

$$\tau_y = \tau_0 + \dot{\gamma}\mu_a \quad (5.1)$$

Shear rate or rate of deformation can be written as $\dot{\gamma} = \frac{du_x}{dy}$

So,

Eq. (5.1) can be written as:

$$\tau_y = \tau_0 + \frac{du_x}{dy}\mu_a \quad (5.2)$$

At temperature T, this Eq. can be written as

$$\tau_T = \tau_0 + \frac{du_x}{dy}\mu_a \quad (5.3)$$

5.2.2.1 Effect of Temperature

The molecules of liquids are much closer than gases. In the capillary tube, liquids interchange molecules between their adjacent layers. The cohesive forces for liquid hold the molecules more tightly as a rigid body. This force is the main reason for changing the viscosity. When the temperature increases, the cohesive force becomes weaker and the molecular interchange between the liquids increases. When the cohesive force decreases, shear stress also decreases. Thus, temperature plays an important role in the stress-strain relationship. Many authors showed the effect of temperature on viscosity (Avramov, 2005; Recondo *et al*., 2005; Zuritz *et al*., 2005; Haminiuk *et al*., 2006; Gan *et al*., 2006; Ghanavati *et al*., 2013). As the apparent viscosity decreases with increasing temperature (Hossain *et al*., 2009; Amani and Al-Jubouri, 2012), to account the effect of temperature we can use Arrhenius law,

$$\mu_a = \mu_0 e^{\frac{E}{RT}} \quad (5.4)$$

Leads to

$$\tau_T = \tau_0 + \left(\frac{du_x}{dy}\mu_0 e^{\frac{E}{RT}}\right) \quad (5.5)$$

5.2.2.2 Effect of Memory

Heavy crude oil has been widely modeled as a Bingham fluid due to its acceptance with the valid engineering accuracy. However, in the Bingham model, the thixotropy behavior of fluids is not considered and the viscosity is not a function of time. Cui *et al*. (2013) proposed a new viscosity equation for Bingham and Herschel-Bulkley fluid by considering the time dependency in the Al-Fariss and Pinder (1987) viscosity equation. They explained that there is a close relationship between these two types of models and the thixotropic behavior of non-Newtonian fluids. They explained that the time dependency to the viscosity is unavoidable for Bingham fluids. The results will be inaccurate without the consideration of time dependency. The authors added a time dependent term in their model. The other approach is the introduction of fractional calculus in Bingham model (Caputo and Mainardi, 1971; Bagley and Torvik, 1983). Most of researchers considered memory effect by introducing fractional calculus in the strain rate with time. Some researchers introduced space memory effect in the strain rate (Sun *et al*., 2016). In this study, to account the effect of memory, we introduce fractional derivative on the strain rate.

$$\tau_T = \tau_0 + \left(\mu_0 e^{\frac{E}{RT}}\right)\left(\frac{d^\alpha u_x}{dy^\alpha}\right) \tag{5.6}$$

Where, $\frac{d^\alpha u_x}{dy^\alpha} = f(y,t)$ and $\tau_T = g(y,t,T)$

In Darcy's law, there are some inherent assumptions (Happy *et al.,* 2017). To avoid those assumptions some researchers proposed a modified Darcy's law to account the memory effect (Hossain *et al*., 2007; Caputo, 1997; Caputo, 2000). Here, it is considered that the viscosity of Bingham fluid is dependent on time that means fluid is showing thixotropic behavior.

$$u_x = -\eta {D_t}^{\alpha_1} \frac{dp}{dx} \tag{5.7}$$

Where,

$$\eta = \frac{k_{\alpha_1}}{\mu}(t)^{\alpha_1}$$

Where, ${D_t}^{\alpha_1}u(t)$ denotes the Caputo fractional derivative of order α_1 defined by (Caputo, 1997; Awotunde *et al*., 2016)

$${D_t}^{\alpha_1}u(t) = \frac{1}{\Gamma(1-\alpha_1)}\int_0^t (t-\xi)^{-\alpha_1} u(\xi)\, d\xi \tag{5.8}$$

for positive values of α_1 and time *t* over integration step *d* τ.

$\Gamma(.)$ Standard Gamma Function

Putting the value of u_x into Eq. (5.6)

$$\tau_T = \tau_0 + \left(\mu_0 e^{\frac{E}{RT}}\right) \times \left[\frac{d^\alpha}{dy^\alpha}\left(\frac{k_{\alpha_1}}{\mu}(t)^{\alpha_1}\frac{d^{\alpha_1}}{dt^{\alpha_1}}\left(\frac{dp}{dx}\right)\right)\right] \tag{5.9}$$

Equation 9 is the modified Bingham equation that considers the shear stress is temperature, time, and space dependent. Here, the viscosity depends on temperature and time. The reason for choosing time dependency in the modified Bingham model has been discussed above. The main difference between thixotropy behavior in Bingham model and this model equation is that the shear rate is time dependent for thixotropic fluids (Yin *et al*., 2012) and in this model, the shear rate is replaced by velocity gradient. However, three cases are considered to simplify this model (Case 1, case 2, and case 3). The physical meaning and the simplifying assumptions are described below for each case.

Case 1: When, $\alpha = 1$ and $0 < \alpha_1 < 1$, Eq. 5.9 reduces to

$$\tau_T = \tau_0 + \left(\mu_0 e^{\frac{E}{RT}}\right) \times \left[\frac{d}{dy}\left(\frac{k_{\alpha_1}}{\mu}(t)^{\alpha_1}\frac{d^{\alpha_1}}{dt^{\alpha_1}}\left(\frac{dp}{dx}\right)\right)\right] \tag{5.10}$$

Here, the shear stress is only temperature and time dependent. The fluid viscosity is also temperature and time dependent. The spatial non-local phenomenon is not considered in this case and the flow rate is time dependent. As it is considered that heavy crude oil is a strongly temperature dependent fluid and it shows memory effects at the reservoir and pipeline transportation. The viscosity of heavy crude oil depends on space and time. By considering these facts, we first assume that heavy crude viscosity is time and temperature dependent (equation 5.10).

Case 2: When, $\alpha_1 = 0$ and $0 < \alpha < 1$, Eq. 5.9 reduces to

$$\tau_T = \tau_0 + \left\{ \left(\mu_0 e^{\frac{E}{RT}}\right) \times \left[\frac{d^\alpha}{dy^\alpha}\left(\frac{k_0}{\mu}\frac{dp}{dx}\right)\right] \right\}, \tag{5.11}$$

Here, the fluid viscosity is not time dependent which is one of the assumptions of general Bingham model. However, the spatial non-local phenomena are considered in this case and the Shear stress is temperature and space dependent but time independent.

Case 3: When, $0 < \alpha_1 < 1$ and $0 < \alpha < 1$,

From Eq. 5.9, we get,

$$\tau_T = \tau_0 + \left(\mu_0 e^{\frac{E}{RT}}\right) \times \left[\frac{d^\alpha}{dy^\alpha}\left(\frac{k_{\alpha_1}}{\mu}(t)^{\alpha_1}\frac{d^{\alpha_1}}{dt^{\alpha_1}}\left(\frac{dp}{dx}\right)\right)\right] \tag{5.12}$$

This case is basically the general equation of the proposed model. The range of fractional derivative orders are $0 < \alpha_1 < 1$ and $0 < \alpha < 1$. However, due to its complex and highly non-linear behavior, the solution of Eq. 5.12 is extremely difficult.

In the current study, we will focus on Case 1.

5.3 Numerical Scheme

Finite difference method is used to discretize equation 5.10. We can rewrite Equation 5.10,

From Eq. 11, we can get the results of shear stress and shear rate based on the model.

$$\tau_T = \tau_0 + \left(\mu_0 e^{\frac{E}{RT}}\right) \times \left[\frac{d}{dy}\left(\frac{k_{\alpha_1}}{\mu}(t)^{\alpha_1}\frac{d^{\alpha_1}}{dt^{\alpha_1}}\left(\frac{dp}{dx}\right)\right)\right] \tag{5.13}$$

$$=\tau_0 + \left(\mu_0 e^{\frac{E}{RT}}\right) \times \left[\frac{d}{dy}\left(\frac{k_{\alpha_1}}{\mu}(t)^{\alpha_1}\frac{1}{\Gamma(2-\alpha_1)}\frac{1}{(\Delta t)^{\alpha_1}}\sum_{J=1}^{n}[J^{1-\alpha_1} - (J-1)^{1-\alpha_1}]\left[\left(\frac{\partial p}{\partial x}\right)_i^{n-J+1} - \left(\frac{\partial p}{\partial x}\right)_i^{n-J}\right]\right)\right] \tag{5.14}$$

$$=\tau_0 + \left(\mu_0 e^{\frac{E}{RT}}\right) \times \frac{k_{\alpha_1}}{\mu}(t)^{\alpha_1}\left[\frac{d}{dy}\left(\frac{1}{\Gamma(2-\alpha_1)}\frac{1}{(\Delta t)^{\alpha_1}}\sum_{J=1}^{n}[J^{1-\alpha_1} - (J-1)^{1-\alpha_1}]\left[\frac{p_{i+1}{}^{n-J+1} - p_i{}^{n-J+1}}{\Delta x} - \frac{p_{i+1}{}^{n-J} - p_i{}^{n-J}}{\Delta x}\right]\right)\right] \tag{5.15}$$

$$=\tau_0 + \left(\mu_0 e^{\frac{E}{RT}}\right) \times \frac{k_{\alpha_1}}{\mu}(t)^{\alpha_1}\left[\frac{d}{dy}\left(\frac{1}{\Delta x\Gamma(2-\alpha_1)}\frac{1}{(\Delta t)^{\alpha_1}}\sum_{J=1}^{n}[J^{1-\alpha_1} - (J-1)^{1-\alpha_1}]\ [p_{i+1}{}^{n-J+1} - p_i{}^{n-J+1} - p_{i+1}{}^{n-J} + p_i{}^{n-J}]\right)\right] \tag{5.16}$$

$$=\tau_0 + \left(\mu_0 e^{\frac{E}{RT}}\right) \times \frac{k_{\alpha_1}}{\mu}(t)^{\alpha_1}\ \frac{d}{dy}\left\{\frac{1}{\Delta x\Gamma(2-\alpha_1)}\frac{1}{(\Delta t)^{\alpha_1}}\sum_{J=1}^{n}[J^{1-\alpha_1} - (J-1)^{1-\alpha_1}]\ [p_{i+1}{}^{n-J+1} - p_i{}^{n-J+1} - p_{i+1}{}^{n-J} + p_i{}^{n-J}]\right\} \tag{5.17}$$

$$=\tau_0 + \left(\mu_0 e^{\frac{E}{RT}}\right) \times \frac{k_{\alpha_1}}{\mu}(t)^{\alpha_1}\ [(\frac{1}{\Delta x\ \Gamma(2-\alpha_1)\ (\Delta t)^{\alpha_1}})\ (\sum_{J=1}^{n}[\ J^{1-\alpha_1} -\ (J-1)^{1-\alpha_1}]\ )]\ \{[P_{i+1,j+1}^{n-J+1} - P_{i+1,j+1}^{n-J} -\ P_{i,j+1}^{n-J+1} + P_{i,j+1}^{n-J}\ ] -\ [P_{i+1,j}^{n-J+1} - P_{i+1,j}^{n-J} -\ P_{i,j}^{n-J+1} + P_{i,j}^{n-J}\ ]\} \tag{5.18}$$

So, the discretization of equation 11 can be finally written as follows

$$\tau_T = \tau_0 + \left(\mu_0 e^{\frac{E}{RT}}\right) \times\ \frac{k_{\alpha_1}}{\mu}(t)^{\alpha_1}\ [(\frac{1}{\Delta x\ \Gamma(2-\alpha_1)\ (\Delta t)^{\alpha_1}})\ (\sum_{J=1}^{n}[\ J^{1-\alpha_1} -\ (J-1)^{1-\alpha_1}]\ )]\ \{[P_{i+1,j+1}^{n-J+1} - P_{i+1,j+1}^{n-J} -\ P_{i,j+1}^{n-J+1} + P_{i,j+1}^{n-J}\ ] -\ [P_{i+1,j}^{n-J+1} - P_{i+1,j}^{n-J} -\ P_{i,j}^{n-J+1} + P_{i,j}^{n-J}\ ]\} \tag{5.19}$$

5.4 Simulation Results: Effect of Memory on the Viscosity and Shear Stress Relationship

In this section, we present the results of simulations to test heavy crude oil rheology on the model described in Section 5.2. The model equations were implemented in MATLAB using the function provided in (Caputo, 2000) as a template. The developed numerical scheme uses finite difference method for discretization and the Caputo's fractional derivative definition for solving modified Bingham model problem and shows the relationships among those rheological properties.

The reservoir length, width, and height are considered 5000 m, 100 m, and 50 m. The permeability and porosity is 30 mD and 30%. It is considered that the reservoir is completely sealed, and the heavy crude oil is producing at a constant rate at an initial pressure, 4000 psi ($P_i = 27579028$ Pa). The API gravity of heavy crude oil is 15 with an initial viscosity, $\mu_0 = 1.78\text{e-}3$ Pas-s. The initial oil production rate is $q_i = 8.4 \times 10^{-9} m^3/s$ and the initial oil velocity is $u_i = 1.217 \times 10^{-5} m^2/s$. The values of rheological parameters used for simulation are taken from Hossain *et al.* (2009) and Zaman (2017) that are given in Table 5.1.

Table 5.1: Input parameters for the simulation study

Parameters	Value
τ_0	139.4,150,184.9, and 244 Pa
k_{α_1}	30e-15 m^2
μ	1.78e-3 Pas-s to 10 Pas-s
E	50.52 KJ/mol
R	8.314 J/mol-K
T	25.5^{0}C or 298.5K
ΔT	298.5-298= 0.5 K
t	4e4 s

In this study, we showed the change in viscosity on the shear stress for different α_1 and time for heavy crude oil. For figure 5.3 and 5.4, we use the temperature 303 K from the previous chapter in this study. As we found that this value fits with experimental data for the proposed Bingham model.

Figure 5.3 shows the viscosity is decreasing with increasing shear stress for different yield stress at fractional derivative order 0.2. The value of yield stress is taken from Zhang *et al.* (2017) for heavy crude oil. The viscosity range is considered from 1.78 mPa. s to 10,000 mPa.s for heavy crude oil with API gravity ranging from 10-20 (Alomair *et al.*, 2014). The temperature range is from 20^0C to 160^0C. As heavy crude oil is highly viscous non-Newtonian fluid, it will not flow under small shear stress. After reaching minimum shear stress, it will start to flow. From figure 5.3, it is seen that the heavy crude oil will be allowed to flow at different yield stress depending on their rheological behavior. The API gravity of heavy crude oil changes at different temperature. At that point the high viscous fluid will show higher yield stress value.

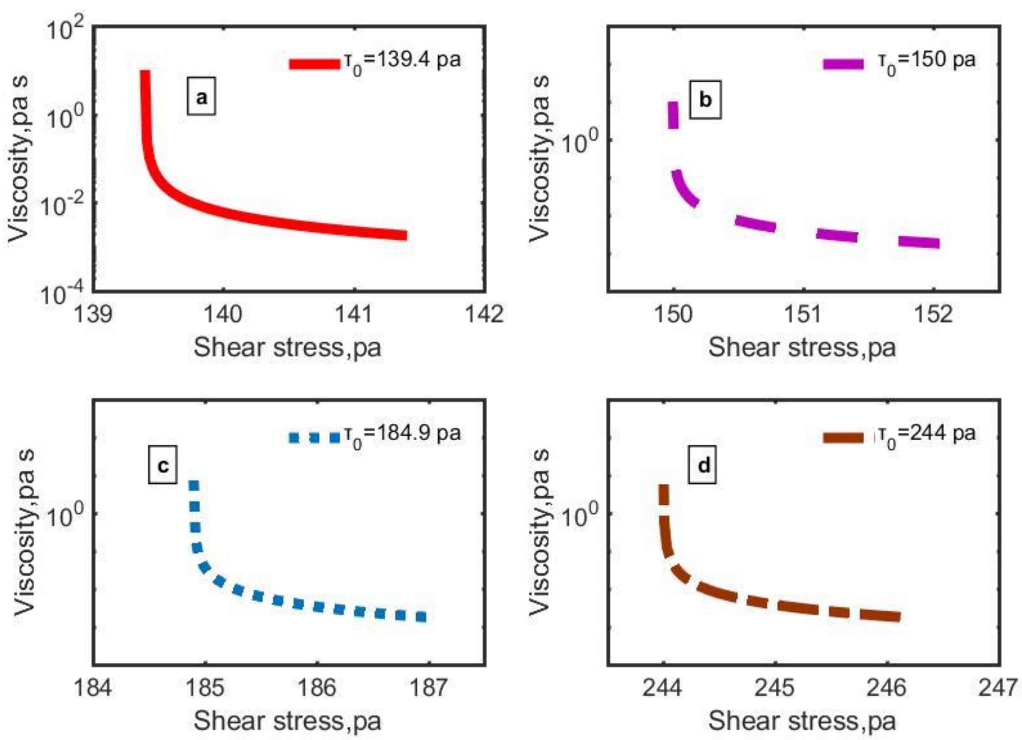

Figure 5.3: The relationship between the viscosity and the shear stress (when $\tau_0 = 139.4\ pa, 150\ pa,\ 184.9\ pa,$ and $244\ pa$) at the fractional derivative order, $\alpha_1 = 0.3$.

The change in shear stress with viscosity and fractional derivative order indicates the effect of memory. As α_1 increases, the chaotic behavior in the reservoir also increases. These increments result in increased shear stress value. Figure 5.4 shows the change on shear stress with viscosity at different fractional derivative order (0.2, 0.4, 0.6, and 0.8). As the value of $\alpha_1$is increasing, the shear stress value is also increasing. These imply that the flow of heavy crude oil has memory effect while it is flowing through porous media. It also means that at low fractional order derivative value, fluid will start to deform or flow quickly than at high fractional order derivative value. It also means that the viscosity will decrease more rapidly for the lower value of α_1. This happens due to the chaotic behavior of fluid memory in the reservoir. The curve trend is same for each case.

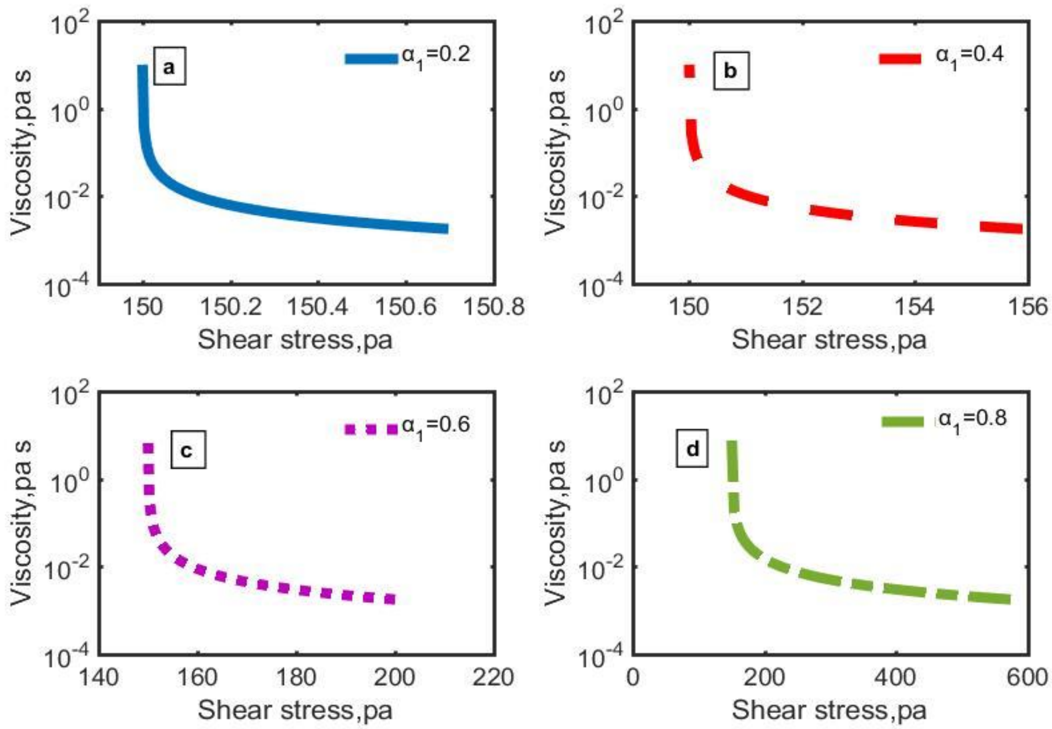

Figure 5.4: The relationship between the viscosity and the shear stress (when $\tau_0 = 150\ pa$) at the fractional derivative order, $\alpha_1 = 0.2, 0.4, 0.6,$ and 0.8.

Figure 5.5 shows the variation in shear stress as a function of viscosity at different time t (20, 40, 80, and 100 months) for heavy crude oil. The value of the fractional derivative order is considered 0.3 for this calculation. This value was experimentally validated by Zaman (2017) to get the pressure data for the individual grid in the reservoir. It is observed from figure 5.5 that the change of viscosity with time remains constant up to the value of shear stress is 153 Pa. After that the rate of change of viscosity is more rapid for shorter time periods than the longer time periods.

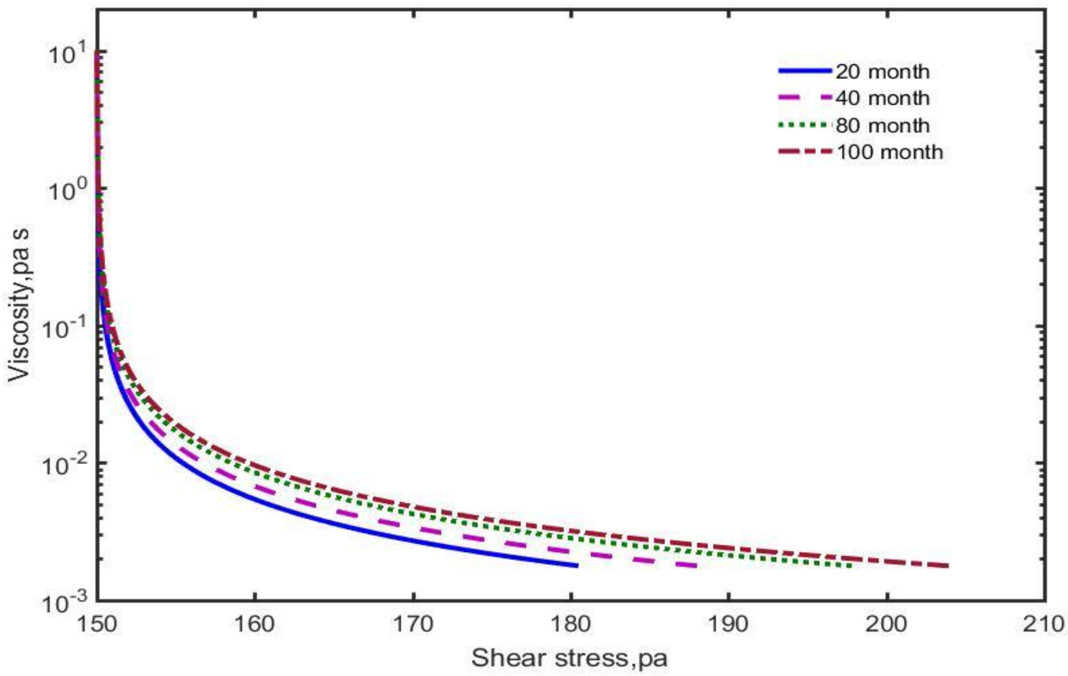

Figure 5.5: Shear stress variation as a function of the viscosity of heavy crude oil at different time, t=20, 40, 80, and 100 months (when $\alpha_1 = 0.3$ and $\tau_0 = 150\ Pa$)

Figure 5.6 shows the change in flux with respect to the time. The rapid change in flux in the first few hours indicates unsteady state condition exists. After a certain time, flux does not change significantly with time. This means the reservoir reaches the steady state conditions. We considered 580 blocks for the reservoir.

The value of $\alpha_1 = 0.3$ was validated with field data by Iaffaldano *et al.* (2015) and numerically by Tareq (2017) to capture the memory effect. For this reason, we have considered $\alpha_1 = 0.3$, and other field pressure data and experimental pressure data in each block from Iaffaldano *et al.* (2015) and Zaman (2017) for comparison.

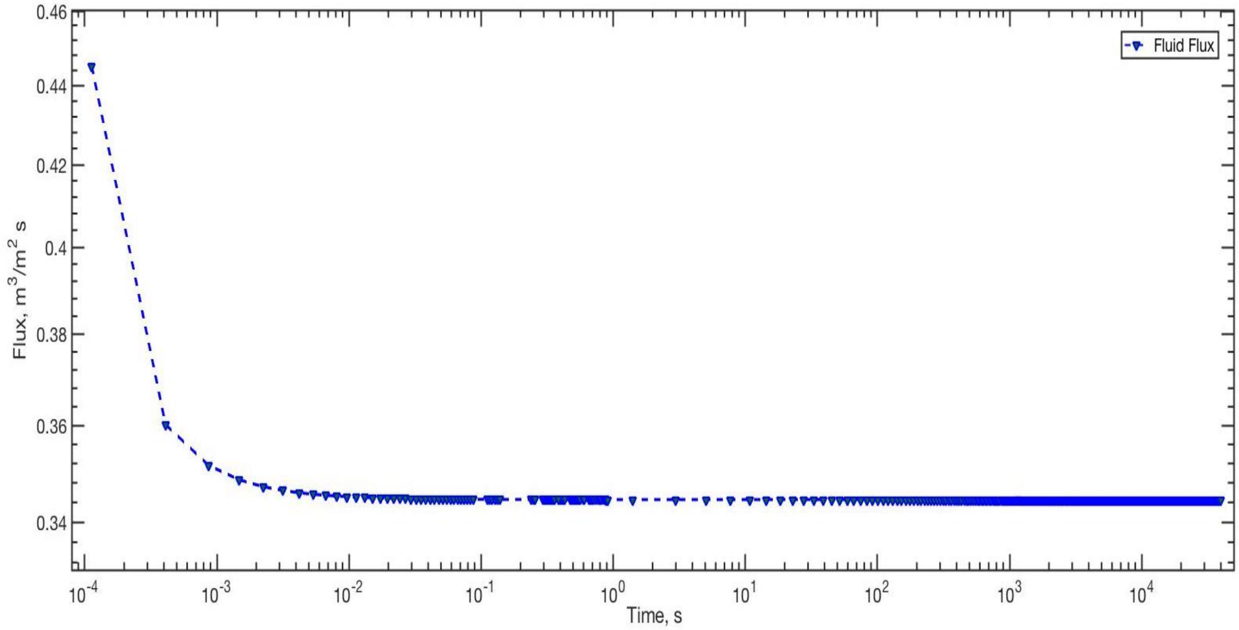

Figure 5.6: Variation of flux with time for $\alpha_1 = 0.3$

5.5 Validation and simulation of the proposed model

The validation is done with experimental data from the literature (Kaur and Jaafar, 2014) for heavy crude oil. All parameters are considered at the same condition as the experimental data. For both cases, it is showed that by applying the shear stress, the heavy crude oil's viscosity decreases (figure 5.7). We can see from figure 5.7 that if the viscosity of heavy crude oil decreases, at first few seconds, there will be no change in the shear stress. This is because, the heavy crude will only flow after reaching the minimum yield stress. After a certain time, the viscosity will decrease exponentially with shear rate. From the figure 5.7, it is concluded that the heavy crude oil will flow when the yield stress developed, and the viscosity reduces. In the experimental data, it is seen that the change shear stress with viscosity is very low. It happens as the experimental procedure was carried out for a short time period (10-15 min). The yield stress value started from 50 Pa, because it was not possible to get less yield stress value as the rheometer would not allow to lower the value. If it were obtained a less yield stress value, it would show a good presentation in the viscosity and shear stress relationship. In our proposed model, the yield stress value is obtained at 20 Pa. After getting the required shear stress, oil will start to flow and behaves like as a shear thinning fluid. The fluid memory effect shows that the value of fractional order derivative and reservoir life influences the yield stress value.

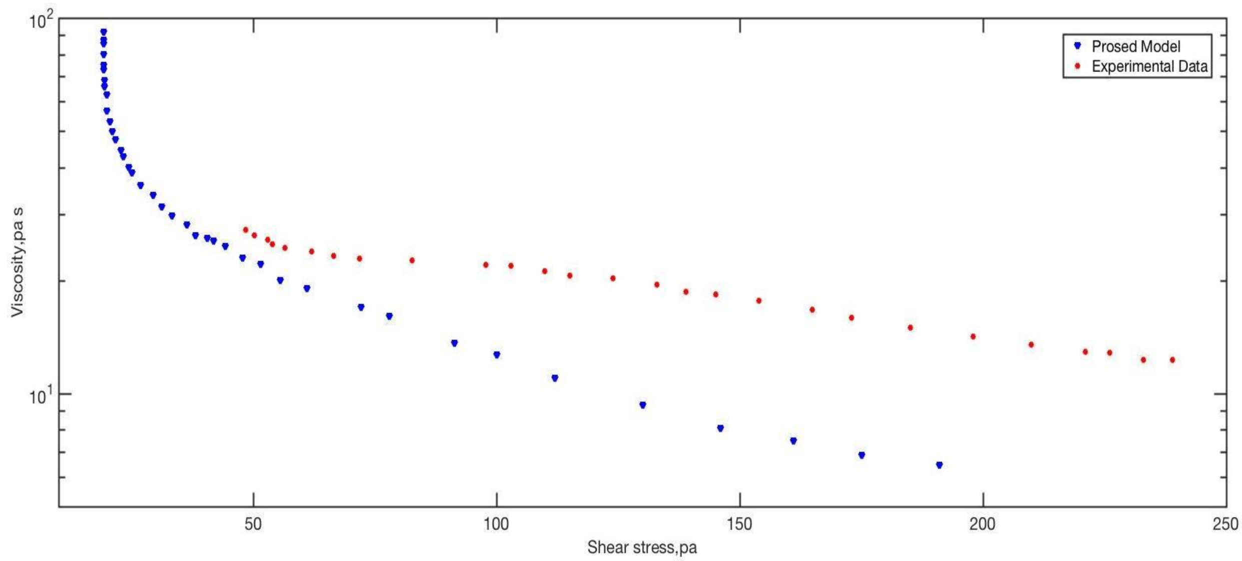

Figure 5.7: Validation between the proposed model and the experimental data from the literature

From figure 5.8 shows the decreasing trend of viscosity with shear stress, when the simulation is done for both field and experimental conditions. The field and experimental conditions described in Iaffaldano *et al*. (2006) and Zaman (2017) respectively were used to simulate the fractional derivative part of equation 5.19. Here, it is considered that the fractional derivative order, α_1, value is 0.3. This value is proposed by Zaman (2017) for field and experimental condition. The main difference is the decreasing trend for both conditions. In field condition it follows the exponential trend but in experimental condition it is straight line. This can happen due to the error in data recording or other factors that cannot represent the true field conditions. From the observation, it is seen that the newly developed model matches both field case and experimentally. In figure 5.8, the yield stress value is considered 150 Pa and the other parameters are same as E, R and ΔT.

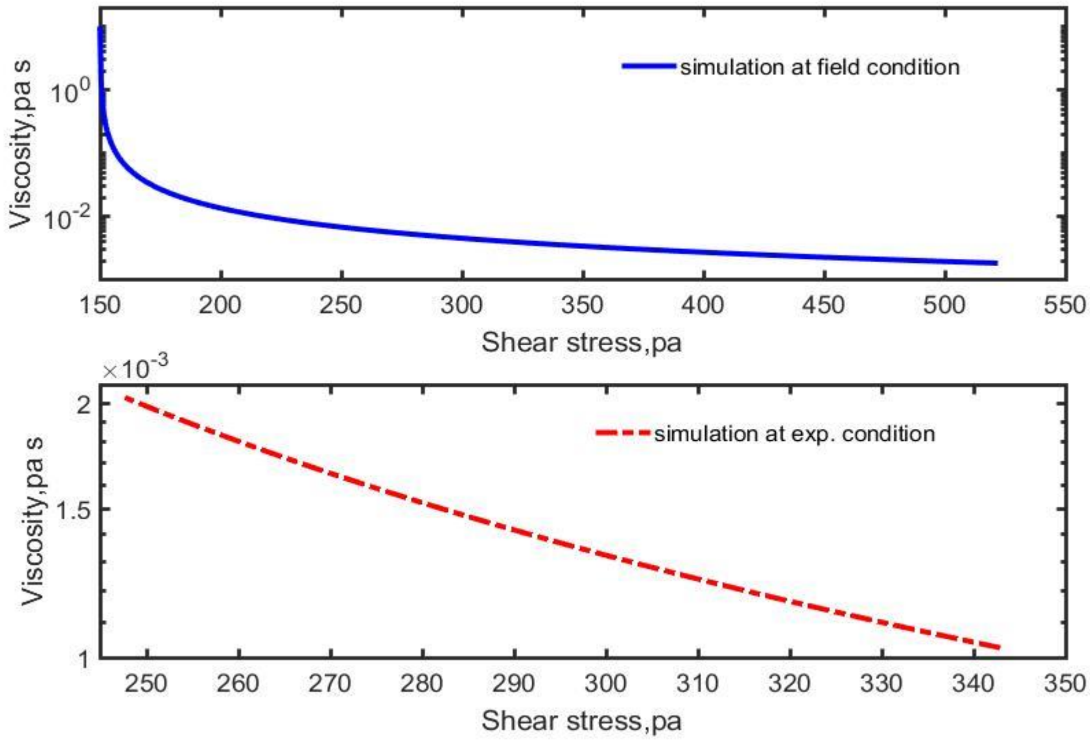

Figure 5.8: Variation of the shear stress with viscosity change at $\alpha_1 = 0.3$, (a) simulation using field data as inputs, (b) simulation using an experimental data as inputs

Conclusion

The mathematical model introduces the effect of temperature and the fluid memory effect. The fluid memory effect is showed by incorporating fractional order derivative and pseudo-permeability with time. The model proves that there is a strong relationship between the stress-strain relation with fluid memory. The increase in shear stress value with increasing fractional derivative order and time indicates the memory effect on the stress-strain relationship for heavy crude oil. This study reveals that how the Bingham fluid model can be fitted for heavy crude oil by invoking memory mechanism. This tool can help to capture the reservoir heterogeneity and inelasticity. The results match with the existing shear stress and viscosity data.

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Nomenclature

τ	Shear stress, Pa
τ_0	Yield stress, Pa
$\dot{\gamma}$ or $\frac{du_x}{dy}$	Shear rate, s^{-1}
μ_p	Bingham fluid viscosity, Pas-s
E	Activation energy for viscous flow, KJ/mol
R	Universal gas constant, KJ/mole- K
T	Temperature, K
u_x	Velocity in the x-direction, m^2/s
μ_0	Initial viscosity, Pas-s
$\frac{\partial p}{\partial x}$	Pressure gradient, Pa/m
λ	Threshold pressure gradient, Pa/m
M	Mobility, m^2/Pas-s
α, α_1	Fractional derivative order
y	Distance in the y-axis, m
${D_t}^{\alpha_1}$	Caputo derivative operator
k_{α_1}	Pseudo-permeability, $m^{2^{\alpha_1}}$
μ	Oil viscosity, Pas-s
η	Ratio of the pseudo permeability of the medium with memory to fluid viscosity, $m^3 s^{1+\alpha_1}/kg$
Γ	Euler gamma function

t	Time, s
ξ	A dummy variable for time i.e. real part in the plane of the integral
n	Old time level
$n+1$	New time level
p_{i+1}	Pressure of grid block i+1, psia, Pa
p_i	Pressure of grid block i, psia, Pa
i	Control volume centroid counter
Δx	Size of grid block in x direction, m

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CHAPTER SIX

Conclusions and Recommendations

6.1 Conclusion

This study has investigated the rheological behavior of heavy crude oil by considering Bingham model. The evidence from this study implies that the heavy crude oil exhibits non-Newtonian flow characteristics. In this study, it is considered that viscosity as a function of shear rate, temperature, and pressure. We have obtained satisfactory results that shows the temperature has significant importance on the viscosity of the heavy crude oil over shear rate and pressure. Later, the effect of temperature and the pressure is combined in the Bingham model by considering that the viscosity is function of these parameters. We have found that the pressure and the temperature have noteworthy effects in the shear stress-shear rate relationship. We have also introduced an innovative way to model the heavy crude oil rheology by using Bingham model. We have managed to incorporate the temperature variation and another new concept that is almost neglected in the fluid rheology called memory effect. Fractional derivative is introduced to consider this effect. This effect increases the non-linearity in the model equations. These model equations are solved by using Caputo's fractional derivative approach, and MATLAB programing is performed to simulate the model. Our study provides a springboard for a new way to define heavy crude rheological behavior and flow characterization. Our investigations into this area are still in progress to confirm our hypothesis more accurately. The current study is only limited by considering that shear stress is temperature and memory dependent. However, it can be considered the non-local spatial effects in the strain rate. This study is the first step towards enhancing our understanding of the temperature and time-dependent crude properties characterization by introducing fractional derivative in the stress-strain Bingham model. We hope our research will be helpful in solving the difficulty of heavy crude oil rheology and flow characterization through porous media.

6.2 Future Guidelines

To further our research, we intend to consider non-local spatial effect in the shear stress-shear rate relationship. Future studies, which take non-local spatial effect in the shear rate into account, will need to be performed. Besides, the effect of pressure can also be considered in the modified Bingham model. Future experimental investigations are needed to estimate the viscosity variation with shear stress and the shear stress variation with time for heavy crude oil. Future studies should deal with memory effects for the characterization of heavy crude oil. Our results are encouraging and should be validated by a large time scale.

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