Experimental Investigation of Oil-Water Separation Using Gravity and Electrolysis Separation

Master's Thesis

In Mechanical Engineering

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Abstract

This study investigates experimentally the performance of oil-water separation using two methods; gravity separation and electrolysis separation. Conventional oil-water gravity separation tested against numerical and categorical factors are based on a statistical method called the analysis of variance. Therefore, a factorial design experiment is conducted where each factor has a high and low level respectively as follows: water flow rate 4 gal/min and 1 gal/min, oil flow rate 4 gal/min and 1 gal/min, temperature 35 °C and 25 °C, number of compartments (2) and (1), and type of separator inlet deflector plate and elbow. These factors are combined in one correlation which describes the process. The developed correlation is able to account 98% of data variability with a 97% success of data prediction which indicates that the correlation describes the operation perfectly. The electrolysis cell separation is conducted using one factor at the time method. It is found that increasing temperature, voltage, and (NaCl) salt increases the performance of oil-water separation, while increasing the pH or the volumetric oil to water ratio acts adversely.

Acknowledgments

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Nomenclature

Abbreviatio	Description	Unit
n		
A_P	Plate Area	m^2
A_C	Mixture cross section Area	m^2
B_{χ}	Reaction Force	N
C_D	Drag Coefficient	Dimensionless
d	Diameter	m
F_B	Body Force	N
F_d	Drag Force	N
F_g	Gravity Force	N
F Value	Probability Factor	Dimensionless
G	Gas Specific Gravity	Dimensionless
g	Gravity Constant	$\frac{m}{s^2}$
GOR	Gas to Oil Ratio	Dimensionless
h	Distance	m
IQR	Interquartile range	Depends on sample Unit
L	Length	m
\dot{m}_w	Water flow rate	$\frac{kg}{s}$
P	Pressure	Ра
P Value	Probabilty Factor	Depends on sample Unit
Q_1	First quartile	Depends on sample Unit
Q_3	Third Quartile	Depends on sample Unit
Q_{gas}	Gas Flow Rate	m^3
		S
Q_{oil}	Oil Flow Rate	$\frac{m^3}{m^3}$
		S

R^2	Residual Error	Dimensionless
R^2_{adj}	Adjusted Residual Error	Dimensionless
Re_D	Reynolds Number	Dimensionless
R^2_{pred}	Predected Residual Error	Dimensionless
SS	Standard Deviation	Depends on sample Unit
SS_E	Sum of sequares deviation within Each group	Depends on sample Unit
SS_T	Sum of Sequares deviation within grand Mean	Depends on sample Unit
SS_x	Sum of Squares	Depends on sample Unit
t	Time	Second
T	Temperature of Gas	F
и	Velocity component in x direction	$\frac{m}{s}$
V	Velocity Vector	$\frac{m}{s}$
V_h	Horizontal Velocity	$\frac{m}{s}$
V_v	Terminal Vlocity	$\frac{m}{s}$
v	Velocity component in y direction	$\frac{m}{s}$
A	Volume	m^3
\forall_g	Volume of Gas	Ft^3
\forall_s	Maximum Efficient Capacity	Ft^3
W	Velocity component in z direction	$\frac{m}{s}$
\bar{X}	Observations Average	Depends on sample Unit
Z	Normal Distribution Transformer	Depends on sample Unit
μ	Viscosity	Pa.s
μ_d	Median	Depends on sample Unit
ρ	Density	$\frac{kg}{m^3}$

$ ho_g$	Gas Density	$\frac{kg}{m^3}$
ρ_l	Liquid Density	$\frac{kg}{m^3}$
$ ho_o$	Oil Density	$\frac{kg}{m^3}$
$ ho_w$	Water Density	$\frac{kg}{m^3}$
σ_{ij}	Stress acting on ij plane in i direction	$\frac{N}{m^2}$

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Chapter 1 Introduction

1.1 Background

The cost of emulsion in oil production is very expensive, because of the industrial process required to meet crude oil specifications for transportation, storage and marketing. An increase of one part per million of water reduces the price between 0.85 to 1.3 \$ per barrel. Basically, Emulsion is a scattering of water droplets in oil by forming a rigid interfacial films and prevent droplets from coalescing. The stability of this film depends on numerous of factors such as oil density, temperature, drop size, and pH. Currently, there are different methods to tackle crude oil emulsion such as chemical demulsification, pH adjustment, filtration, heat treatment. However, this thesis only focuses on gravity settling method and electrical demulsification.

Gravitational separation is not only used in oil industry, but it is also used in several industries such as petroleum, food, pulp and paper, manufacturing. It is one of the conventional methods of separating minerals, especially in the petroleum industry where water, oil and gas separators were among the first equipment to be used in the industry. The technique relies on the differences in the gravities of distinct minerals. This method gained a good reputation in the mining circles, because of the low capital cost as well as low operating cost. Over past two and a half decades, gravity separation equipment have evolved making

the technique the preferred mining method. The internal design of the oil separator is important. In fact, the main objectives of gravity separator internals are to changing the flow momentum and provide enough space and time for settling and coalescing. So far, there is no study which consider all the parameters which affect these objective in one mathematical correlation. Therefore, in this study kinematic, physical, and qualitative properties will be experimented in order to investigate the factors relationship among each other and how they contribute to the main objective of separation.

Moreover, the evaluation of emulsion stability was found affected by applying an electrical field. The use of electrical field as a method to evaluate emulsion stability, has been developed by Kilpatrick et al. (2001). In their experiment a sample of emulsion injected between parallel electrode plates where a direct current voltage is applied and increased gradually. In response to voltage increasing the water droplets tend to split from the emulsion. This study investigates more factors temperature, salt NaCl, pH, and oil to water ratio besides to the electrical field.

1.2 Research Objectives

The focus of this research is to test the performance of oil-water separation against different factors. Two experiments are conducted for this reason. In the first experiment, the separation depends on gravity forces; the water content left over in the oil is monitored throughout the entire experiment. Also, the data

gained is analyzed statistically. In the second experiment, an electrical field is used as the method to separate oil from water and is tested versus numerous factors. The amount of oil recovered in each experiment is the main focus. However, the data gained from this experiment are not statistically analyzed because they have been one factor at the time.

1.3 Thesis Organization

This thesis is organized in six chapters. The first chapter outlines the work. Chapter two is a basic overview of the gravity separator. It reviews gravity separator history, the types and workings of gravity separators as well as control systems and separating problems encountered in the field. Finally, it describes the safety devices attached to the separator to avoid accidents. The third chapter provides a theoretical background of the gravity separator, electrochemical reactions, and the statistical technique used later. It provides a theoretical background for modeling and design of the separation process, by starting with the main equation of fluid mechanics. Also, it briefly explains how the electrochemical reaction behaves. The last part of this chapter describes in detail the factorial design and the analysis of variance statistical technique. Chapter four covers everything related to the gravity separator experiment starting from the setup to model verification. Chapter five discusses details related to the electrolysis cell experiment and presents all related results. The last chapter six is a conclusion of the work and provides suggestions for future study.

Chapter 2 Gravity Separator

2.1 Historical Background

In early times, oil companies dug sumps on well-sites as reservoirs for production. Thus, the water, mud or other debris mixed with the oil settles to the bottom while the oil could be skimmed off from the surface. All the gas produced was simply released to the air. However, there were problems in this separation process, the surface water ran into the sumps; as well as dirt and other debris, creating some concerns.

By 1861, the sump separation system was replaced by the use of large wooden tanks which minimized the loss of oil [18]. These tanks were connected to the wellhead where the gas and oil could be separated in the tank, the oil drawn off in another tank and the gas vented to the atmosphere. By 1863, bolted-iron tanks were used to ship oil to market, were used later on oil leases and almost replaced the wooden tanks [18]. However, these tanks were large and difficult to transport. Moreover, they were open at the top, which allowed rain and debris into tanks.

As well as the need for better separation, it was recognized that the safety at well sites would improve if the gas were captured or burnt rather than released to the air. Also, a gas can be used as fuel for the drilling engines. Therefore, tanks became used only for storage and not separation. The progress of the separator

is given by J. H. A. Bone's 1865 description of early oil field problems [1]. The first separator used in the oil field was in 1865. Simply, it was a barrel placed on top of the oil-lease tank. The petroleum flowed in through one side on the top of the barrel and the oil outlet was placed at the bottom of the barrel. The gas entering the flow into the barrel had its own outlet on top of the barrel opposite to the inlet. The next step in oil separator improvement was the installation of the separator outside the oil-lease. It was known as the Iron Gas Tank. The Oil Well Supply Company Catalog in 1884 described one separator called Ashton Separator. It contained the basic components of today's separators [1]. It had a gas-liquid inlet, gas outlet, liquid outlet, floater and manual control valve for controlling, and a pressure relief valve. These principles were also applied by the Bougher Patent in 1895 to the separator mounted on top of storage tanks [1]. They were provided with a heat coil for better separation.

The demand of longer transmission lines contributed to the improvement of separators. In 1904 the Oil Well Supply Company Catalog listed a high pressure oil-gas separator. The size of barrel separator increased, was removed from the top of the storage tank and placed on the ground. Its test pressure reached 150 psi. This separator had no internal devices, but had a unique oil level controller which increased oil recovery and made the separator a prominent device.

In 1906 A. S. Cooper recognized the problem of entrained gas and developed a separator which could handle this problem [1]. His separator provided a

tangential inlet to spread the effluent to a bigger area to release more gas. The separator had a deep-coned bottom for sand removal.

With the construction of gas plants, the demand for high pressure equipment increased. Consequentially, the importance and awareness of the gravity separator increased and companies specializing in design and making separators began to appear. One of the popular companies in this field produced the Trumble Separator, it became famous in 1915 [1]. It gave the first real emphasis to oil scrubbing with its top feeding inlet. Also, the Smith company manufactured many separators when the value of gas started to be recognized. Those separators were designed for handling higher pressure and the inlet stream was slightly baffled.

During the 1920s the crude oil prices increased rapidly and better separation became extremely important. A. M. Ballard introduced the dual valve control separator in 1920, in which, if the liquid increased to an undesirable level, the gas outlet valve gradually closed to build up pressure in the separator to push oil down [1]. In order to avoid backflow in this separator, the inlet line was submerged in the liquid. Thus, the gas spread was completely neglected. This problem was solved in 1921 by D. G. Lorrain [1]. In this design, the dual valve feature was provided and the inlet was equipped with a check valve to avoid the flowback problem. In 1928, the J. P. Walker patent focused on the oil-scrubbing feature [1]. Also, the separator provided with spiraling downward and upward

flow for oil and gas respectively. It was designed to increase the gas velocity, and a centrifugal force gathering the oil particles in the liquid section. Moreover, M. F. Waters in 1929 designed a separator which emphasized better liquid separation in the uprising gas section [1].

these designers paid more attention to the liquid section of the separator. During the 1930s high pressure operations increased. Thus, stages separation were used. All the free gas was removed in stages from high to low pressure.

Unfortunately, during the next half century, most of the advancements of surface equipment have been attributed to companies rather than to people. Ken Arnold, a surface-equipment designer and president of a facilities-engineering concern reported: "Surface production equipment designers and inventors simply got overlooked by oil and gas historians [18]. The evolution surface facilities has been, until very recently, the design and fabrication of equipment to quickly supply need, not a formal development of technology. And, in most cases, companies, not people, got credit" [18].

This chapter reviews the history of gravity separators with focus on currently used separators and the challenges being experienced in the oil industry.

2.2 Introduction

Petroleum wells produce oil and gas and most often water. It is always required to split these three fluids from each other for marketing or safe

environmental disposal. Thus, separation of oil and gas is an important process in the petroleum industry. It does not only depend on the selected technology but also on the characteristics of the fluids, such as drop size, concentration, waxing and fouling tendency [8]. The technique relies on the difference in the gravities of the distinct fluids. Generally, there are three main principles applied in the separation technology to achieve an efficient physical separation, momentum, gravity, and coalescing [28]. Any kind of separator may apply one or more of these concepts. Momentum force is used for initial bulk separation between the liquid and gas phases by changing the direction of the effluent. Gravitational force is used by minimizing the velocity which allows the heaviest fluid to settle to the bottom while the lightest one settles to the top of the given space. However, very tiny droplets cannot be separated by gravity. Thus, these small droplets can be coalesced to form larger droplets which can be separated by gravitational force. For this study, only gravity separators will be discussed in greater detail in the following sections.

2.3 Gravity Separators

Gravity separators are used to handle multiphase flow in one vessel and separate the effluent to the main components. As gravity separators basically depend on gravitational force, their efficiency increases by reducing gas velocity. They are often classified based on their configuration as vertical and horizontal, or based on their functions as three phase separator and two phase separator;

where the former type splits the effluent to gas, oil, and water while the latter splits the flow to liquid and gas streams. Also, they might be classified by their operation pressure as low pressure (10 to 180 psi), medium pressure (230 to 700 psi), and high pressure (975 to 1500 psi) [28].

2.3.1 Horizontal Separator

A typical configuration for a horizontal separator is shown in figure (2-1). This is most commonly used when the amount of liquids is relatively high or the amount of dissolved gas is big. It can be operated in two or three phases. Effluent enters the separator and strikes the inlet diverter which causes a sudden change in the momentum; this makes the lighter fluid, which is gas, separate and flow to the top of the heavy fluid, which is liquid. After that, the gravity force will work on splitting the oil from the water, the oil will be dumped through the oil outlet valve and the water through the water outlet valve.

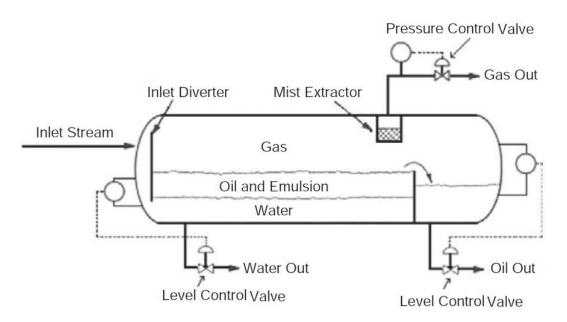


Figure (2-1) Horizontal Separator [22]

This type of separator has a bigger gas-liquid contact surface area compared to similarly vertical separator. This allows the gas bubbles stuck in the liquid to be liberated as well as this assisting the free passage of collapsed foam. Then, the mist extractor coalesces the tiny drops flowing with the gas to form bigger drops which then fall to the bottom due to gravity. However, horizontal separators have some disadvantages; they need a bigger space and become harder to use when sand, mud, wax, or paraffin are produced. Also, horizontal separators normally have less liquid surge capacity.

2.3.2 Vertical Separators

A typical configuration for a vertical separator is shown in figure (2-2). It is usually used to handle a relatively large liquid slug without carry-over into the gas

outlet or to serve a relatively high gas to liquid ratio. Moreover, it needs a smaller space compared to the horizontal separator. The effluent enters the separator through the side; the inlet diverter does the bulk separation by causing a sudden change in the momentum, as in the horizontal separator. The gas flows upward through a mist extractor to coalesce the liquid drops and leaves the separator as dry gas via the gas outlet valve. A downcomer guides the liquid through the oilgas interface so as not to disturb the oil skimming action taking place.

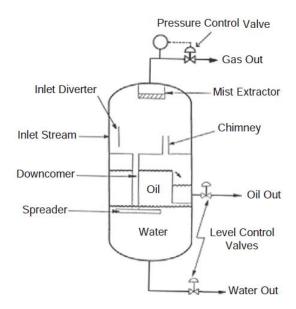


Figure (2-2) Vertical Separator [22]

The chimney works as a pressure equalizer between the lower and upper section of the gas. The gravitational force separates the oil from the water and each of them is damped through a specific valve. The disadvantages of the vertical separator is that it is more expensive and requires a large dimeter to handle the same gas handled by a horizontal separator.

2.3.3 Spherical Separators

A typical configuration for a vertical separator shown in figure (2-3). In the 1960s spherical separators were popular due to their low price and low volume occupied, especially in high pressure applications, which means less steel is required for a given pressure.

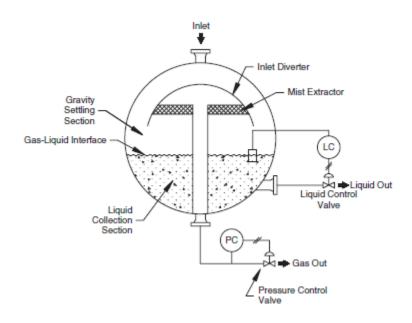


Figure (2-3) Spherical Separator [29]

The well stream enters the top of the separator, the liquid spreads thinly over the hemispherical inlet diverter and accumulates in the lower part of the vessel. Gas proceeds along the same initial path between the hemispherical baffle and the separator shell. At the lower edge of the inlet diverter, the gas passes into the chamber of the separator and rises through the mist extractor. Gas leaves the separator through the upturned liquid and activates a float or level controller to operate the oil valve on the dump line of the separator. The use of spherical

separators have been almost completely discontinued by the industry because they are not economical for large gas capacities and their surge capacities are very limited [29]. Moreover, they cannot handle foaming and their controllers are very sensitive.

2.4 Vessel Internals

The gravity separator evolved with time as a result of increasing production and to minimize costs. Economic factors played a crucial role in the modification that introduced today's gravity separators [29]. One of these modification was the vessel internals. As mentioned in previous sections, the separator does not only do a bulk separation to gas, oil, and water but also enables the tiny drops to separate. That is why there are many internal configurations, which exist to increase the efficiency of the separator. In this section some of them will be explained in more detail.

2.4.1 Inlet diverters

Inlet diverters provide primary separation between the gas and the liquid. Droplets of 300 to 500 microns in diameter fall into the liquid settling section in this stage. There are three main types of inlet diverters; baffle plates, centrifugal diverters, and elbows. The baffle plate can be a spherical dish or flat plate, as shown in figure (2-4). These shapes will make a sudden change in the direction and velocity of the flow and thus split the gas and the liquid.

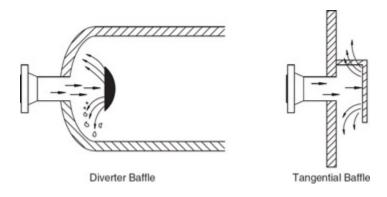


Figure (2-4) Baffle Diverter Plates [29]

Liquid has more energy than gas at the same velocity because it has higher density. This means, the liquid will not be able to change direction or velocity easily. This will make the liquid strike the diverter and fall to the bottom while the gas tends to flow around the diverter. The design of the baffles depends on the structural supports required to resist the impact force of the flow. The centrifugal inlet diverters separate the flow using centrifugal force instead of impact force, as shown in figure (2-5). They usually have inlet nozzles to create fluid velocity about (6 m/s) around a chimney with a diameter of less than two-thirds of the diameter of the vessel.

Centrifugal diverters can efficiently separate liquids and minimize the possibility of foaming and emulsifying problems, but they are very sensitive to the flow rate, which makes them not recommended for a production operation where the flow is expected to be unsteady.

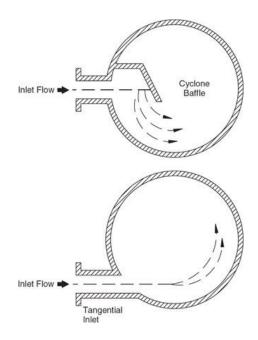


Figure (2-5) Centrifugal Inlet Diverters [29]

Moreover, half open pipes which are a modified version of 90° elbows, are also used as inlet devices, as shown in figure (2-6). They can be used for both horizontal and vertical separators. Basically, they are a piece of pipe with a length of up to three times the inlet diameter, welded to the 90° elbow.

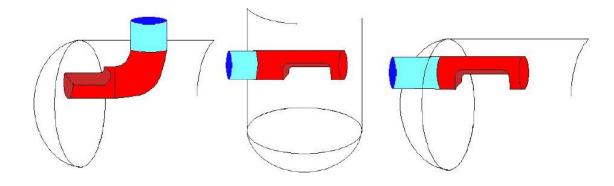


Figure (2-6) Half Open Pipe Inlet [25]

In a horizontal vessel's top entry, the last section of the half open pipe should be closed at the end and horizontally pointing opposite to the flow direction. It should be closed at the end and facing to the bottom in the case of vertical vessel's, or horizontal vessel's side entry. These inlet devices have a simple design to ensure that the total length of the vessel is effectively used as the gas can leave the vessel unless it travels through the entire vessel's length. However, it might send gas with liquid to the bottom, which can be difficult to liberate when there is emulsion.

2.4.2 Wave Breakers

Wave breakers are perforated baffles or plates, as shown in figure (2-7), which placed normal to the flow direction. Theses baffles limit the wave propagation caused by incoming fluid, especially in a horizontal separator. The wave's action inside the separator must be eliminated in order to control the liquid level and maintain a steady flow.

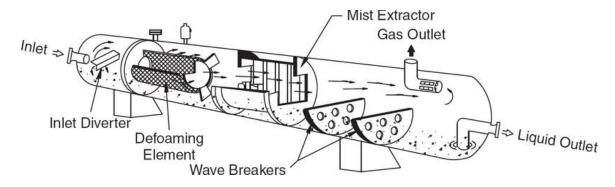


Figure (2-7) Horizontal Separator fitted with Wave Breakers [29]

2.4.3 Defoaming Plates

When gas bubbles are liberated from the liquid, foaming may occur at the interface, which can adversely affect the separator's performance. Sometimes, chemicals are added at the inlet to stabilize foaming. However, a more effective solution is to force the foam to pass through a series of inclined parallel plates or tubes, as shown in figure (2-8). This provides additional surface area which breaks up the foam and it collapses into the liquid.

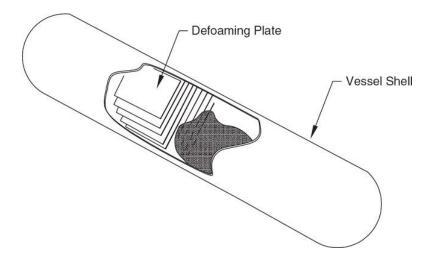


Figure (2-8) Defoaming Plates [29]

2.4.4 Vortex Breaker

Liquid outlets may form vortices or whirlpools while liquids are leaving the separator. This may suck gas down into the liquid outlets. Thus, the separator is

equipped with a vortex breaker, as shown in figure (2-9) which prevents a vortex from developing.

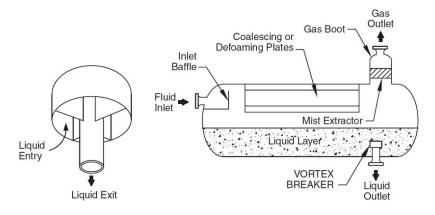


Figure (2-9) Vortex Breaker [29]

2.4.5 Mist Extractor

Mist extractors are used to remove the small liquid drops from the gas stream before it is discharged from the separator. Liquid carry-over with gas does not only depend on vessel's internal parts, but also on the liquid droplet size. Basically, the vessel internals work on increasing the droplet size and gravity takes care of the rest. There are many types of mist extractors. However, all of them are based on the balance between the gravitational and drag forces acting on the droplets. Figure (2-10) shows the most commonly used, called a Knitted-wire-mesh mist extractor. This type can remove drops of sizes above (3 to 10 µm) and is usually specified by a certain thickness (75 to 175 mm) and mesh density (160 to 190 kg/m³) and installed right before the gas outlet.



Figure (2-10) Wire-Mesh Mist Extractor [29]

Another type of mist extractor called the vane-type is widely used, especially for higher velocities. It is installed upstream of the previous type, as it removes bigger drops with diameter sizes 50 to 150 microns and make the flow uniform to be easily handled later. As illustrated in figure (2-11), as the gas flows between the plates, which are arranged between (5 to 75mm) with (150 to 300 cm) depth in the flow direction, the droplets coalesce and fall.

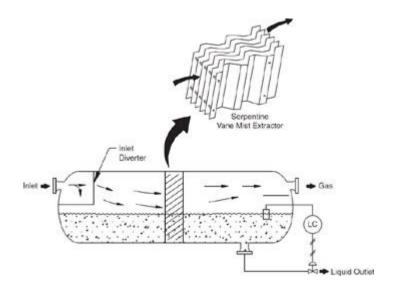


Figure (2-11) Horizontal Separator fitted with Vane-Type Mist Extractor [29]

2.5 Separator Controllers

It is important to maintain steady operating conditions inside the separator by keeping a balance between the inflow and the outflow. Otherwise, undesirable consequences such as effects on speed and the retention time of the fluid occur. Moreover, this can lead to a wide ranging harm to people, environment, and equipment. Thus, the pressure and the liquid level are controlled using automatic control valves ACVs which are operated as discussed below.

2.5.1 Pressure Controller

In order to maintain constant pressure inside the separator vessel, the inflow pressure should be very close to the outflow pressure. The inflow pressure is determined by the separator's upstream conditions while the outflow pressure can be adjusted by using gas pressure controller-proportional action, as shown in figure (2-12).

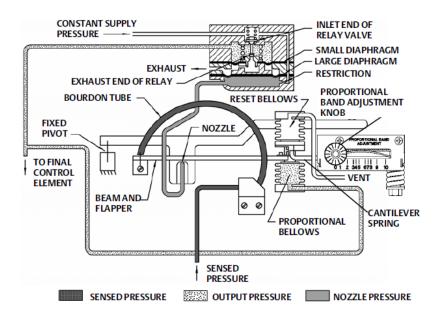


Figure (2-12) Pressure Controller [12]

When the sensed pressure from the separator outflow decreases, the Bourdon tube moves the flapper toward the nozzle. Due to the continues pressure supply, the nozzle pressure starts to build up until the diaphragm is lifted up. Thus, the inlet end of relay valve will open which increases the output pressure. Because the separator is equipped with a normally open automatic control valve as the final control element in the gas outflow for safety purposes, which will be discussed later, increasing the output pressure will throttle the gas line thereby increasing the separator pressure. At the same time, part of the output pressure flows to proportional bellows, which causes the flapper to move away from the nozzle, decreasing the nozzle pressure and pushing back the diaphragms to their original position. Finally, the flapper will stay still at an balanced distance from the nozzle and steady state operation will be achieved. However, it will be very

difficult to achieve a steady state if the system either reacts quickly or slowly to a change of the separator pressure because the automatic control valve will travel from fully open to fully close in a few seconds. Therefore, the control system is equipped with a proportional band adjusted to make the system react to the sensed pressure within an acceptable range rather than at specific point.

2.5.2 Level Controller

The level interfaces should be kept constant in order to ensure steady state operation. The gas oil interface level depends on the gas oil ratio. However, the oil water interface depends on the water cut percentage in the case of the three phase separator. As the oil outflow line is equipped with a normally closed automatic control valve as a final control element for safety, the Oil Level-Proportional Action shown in figure (2-13) works by applying more pressure if the level increases and vice versa in case of decreasing the level.

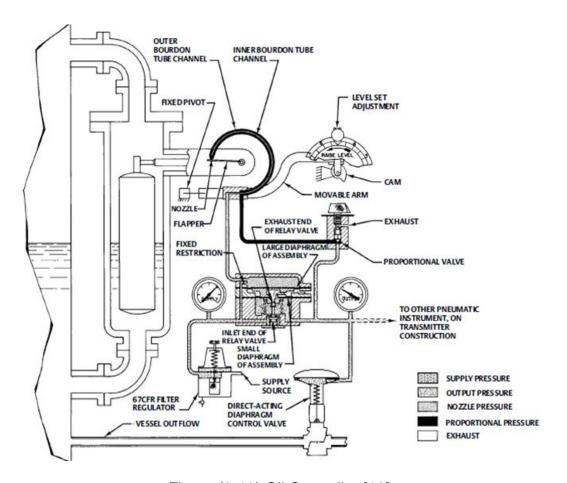


Figure (2-13) Oil Controller [12]

When the level of the oil increases, the buoyant force of the oil lifts the plunger. Consequently, the flapper moves toward the nozzle and closes the gap between them. This makes the nozzle pressure which passes through the inner Bourdon tube channel, the pressure builds up due to the continuous air supply. Then the diaphragm will be pushed down and the relay valve will leave its seat. Thus, the output pressure goes up, forcing the automatic control valve to open and let more oil flows; thereby, the oil level will decrease. At the same time, part of the output pressure will be sent to proportional valve and then to the outer Bourdon tube channel. This will make the Bourdon tube move away from the flapper and an

equilibrium will be achieved. However, as mentioned earlier, in the pressure controller, it is very difficult to maintain the level if the controlling system responds quickly to the level changes. Therefore, a proportional band valve is attached to the output pressure line which vents some of the pressure to slow down the response.

2.6 Inlet Flow Patterns

The importance of the feed pipe separator has only recently been quantified. Figure (2-14) shows the classifications of the flow patterns encountered in most gravity separators. Basically, the classification of these types depends on the relative amount of gas and liquids in the feed pipe. A flow pattern map developed in 1974 by Mandhane is shown in figure (2.14) below. While the horizontal axis parameter represents the superficial gas velocity, the vertical axis represents the superficial liquid velocity. The superficial velocity is also known as the actual velocity and defined as the volumetric flow rate of the matter divided by the total cross-section area.

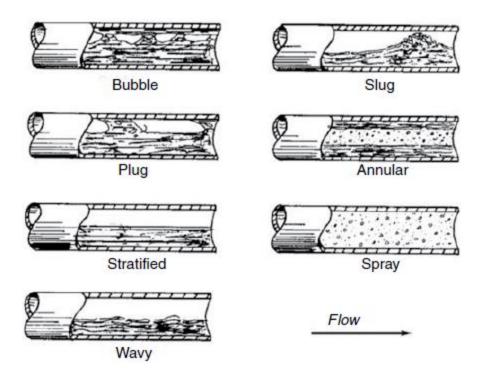


Figure (2-14) Flow Patterns [3]

It should be taken into account in separator design or selection that the amount of liquid droplets entrained into the gas will typically increase rapidly as the transition to annular flow approaches. For decreasing the entrained liquids into the gas and for a smooth steady state operation in the separator, a stratified or wave flow pattern is desired. In general, it can be said that decreasing superficial gas velocity by increasing the inlet feed pipe diameter provides a good solution to prevent entrainments into the gas.

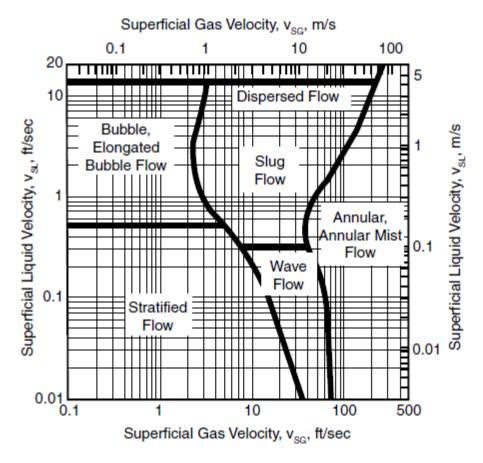


Figure (2-15) Flow Patterns [3]

2.7 Separator Operating Problems

Most of the major problems encountered in the operation of oil and gas separators are caused by foam, paraffin, sand and emulsions.

2.7.1 Foamy Crude

The major cause of foam is impurities in crude oil such as carbon dioxide and completion fluids other than water. Crude oil's tendency to create foam is determined by Lab tests [8]. Foaming in the separating vessel causes three main issues: Firstly, the mechanical control of the liquid level is aggravated because it

must deal with three phases instead of two. Secondly, the vessel has a large volume to weight ratio and occupies a big space, which may not allow a sufficient amount of oil and gas velocities. Thirdly, in some cases it becomes impossible to remove separated gas or degassed oil without entraining some of the foamy material in either the oil or gas outlets. Vessel internal design can break the foam by providing enough separation time or a sufficient coalescing surface. Also, as the occurrence of foam reduces by increasing the temperature crude oil, a heat exchanger used upstream of the separator is used to prevent foam creation.

2.7.2 Paraffin

Separator operation can be adversely affected by an accumulation of paraffin. Mist extractors in particular are likely to malfunction and be plugged by the accumulation of paraffin. The use of plate type or centrifugal separator should be considered when paraffin causes potential problem. Also, a solvent or another type of internal cleaners can be used. However, the temperature of liquid should be kept at a level to avoid paraffin in the first place.

2.7.3 Sand

Sand can be very troublesome in separators. These difficulties are evidenced mainly by a cut-out of the valve trim, damaging metering devices, plugging separator internals and accumulating in the bottom of the equipment. A design that will promote good separation which has a sufficient number of traps

for sand accumulation may be difficult to attain. The experience of a separator manufacturer is invaluable in providing a workable solution to the sand problem.

2.7.4 Emulsion

Emulsion can cause problems in the operation of three phase separators. If stable emulsion formed between the oil and water phases upstream of the separator or in the vessel, separation of these phases is very difficult. When emulsion tendencies are present, the settling time required to achieve separation of oil and water becomes much longer than normally required. In this case, it is necessary to remove the water and oil phases from this vessel and direct them to another separator for further processing. However, it is possible to overcome this problem by heating up the flow before it enters the separator, as the foaming problem.

2.8 Separator Safety Devices

The separator works at high pressure with flammable fluids. Thus, safety regulations must be considered while dealing with this equipment to prevent human injuries or environmental damage. Firstly, the separator must be equipped with a check valve at the inlet. This valve allows the fluids to flow into the separator and keeps them inside the separator vessel in case of a pressure drop occur upstream from the separator due to any failure. Also, it must be equipped with a safety relief valve to keep the separator pressure at less than the allowed working pressure. Moreover, if the supply of constant pressure to the

controllers fails, the normally open automatic control valve of the gas line prevents over-pressurizing the separator vessel, while the normally closed automatic control valve of the oil line prevents oil from damping at a high rate, which may cause an environmental issue. Finally, the separator must be grounded to avoid an electrostatic charge. Also, the annular mist flow region is often the worst and should be avoided if possible.

Chapter 3 Theoretical Background

3.1 Fluid Mechanics Governing Equations

The separation processes inside the gravity separator vessel are discussed in this chapter. The concepts of mass conservation and linear momentum are applied to understand how gas, oil and water split from each other. Generally, the basic laws of fluid mechanics can be formulated in terms of a finite system known as the Eulerian approach or using an infinitesimal system known as Lagrangian approach. The Eulerian approach concerns in the gross behavior by focusing on the properties of a flow at a given point in space as a function of time, while the Lagrangian approach involves the detailed behavior of the flow, tracking the fluid particles point by point. Therefore, the Eulerian approach uses integral equations while the Lagrangian approach is expressed using differential equations. A brief demonstration of these equations will be shown later. Full detail can be found in fluid mechanics text books [13].

3.1.1 Conservation of Mass

The balance of mass for a fixed control volume can be described as follow [14]:

Rate at which mass accumulates with in the volume = Rate at which mass enters the volume – Rate at which mass leaves the volume

The mass of fluid inside the control volume is $\iiint \rho d \forall$. Thus, the rate of mass change inside a fixed control volume in the space is $\iiint \frac{\partial \rho}{\partial t} d \forall$. The rate at which mass enters the control volume is $\iint \rho V \cdot n \, dS$, where $(\rho V \cdot n \, dS)$ is the mass rate of flow out of the area (dS) and $(V \cdot n)$ is the normal component of the velocity to the surface, which is positive if the mass flows out of the control volume and negative if the mass enters the control volume. Therefore, the conservation of mass described above can be written as [14]:

$$\iiint \frac{\partial \rho}{\partial t} d\forall + \iint \rho V \cdot n \, dS = 0 \tag{3-1}$$

The surface integral can be converted to a volume integral by using the divergence theorem which states that [10]:

$$\iint W. n \, dS = \iiint \nabla \cdot W d \forall \tag{3-2}$$

where W is an arbitrary vector.

In order to obtain the differential form of mass conservation equation, applying the divergence theorem on the integral form of mass conservation equation yields:

$$\iiint \left[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) \right] dV = 0$$

Since this is a general case for any arbitrary matter and the integral must vanish, this leads to the differential form of the mass conservation equation which is:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho V = 0 \tag{3-3}$$

Also, by expanding this equation and rearranging the terms, an alternative expression can be written as:

$$\frac{D\rho}{Dt} + \rho \nabla \cdot V = 0 \tag{3-4}$$

3.1.2 Conservation of Linear Momentum

Newton's law of momentum can be described as follows [14]:

Rate at which momentum accumulates within the volume + Rate at which momentum enters the volume - Rate at which momentum leaves the volume = net force acting on the volume.

The momentum net rate which accumulates within the control volume is $\frac{d}{dt}\iiint\rho V\ d\forall$. The momentum net rate which enters the control volume through the surface is $\iint\rho V\ V.n\ dS$. The net forces acting on the control volume are caused by stresses which are acting on the surface, and gravity which is acting on the fluid body. These can be presented respectively as $\iint\sigma_{ij}.n\ dS$, and $\iiint\rho g\ d\forall$. The force acting on the surface has two component, a normal force component which is known as a pressure force and acts toward the control volume, or

tangential force component which is created due to viscous forces and known as shear forces and acts along the plane area of the control volume. The gravity force is the force which a mass gains due to the gravity attraction. Plugging all these terms into Newton's law of momentum, yields the integral form of the momentum equation as follow [14]:

$$\frac{d}{dt} \iiint \rho V \ d\forall + \iint \rho V \ V \cdot n \ dS = \iint \sigma_{ij} \cdot n \ dS + \iiint \rho g \ d\forall$$
 (3-5)

By applying the divergence theory to the left side of equation (3-5):

$$\frac{d}{dt} \iiint \rho V \ d \forall + \iint \rho V \ V \cdot n \ dS = \iiint \frac{\partial (\rho V)}{\partial t} d \forall + \iiint \nabla \cdot (\rho V V) d \forall$$

Expanding the left side yields:

$$\frac{d}{dt} \iiint \rho V d \forall + \iint \rho V V \cdot n dS = \iiint V \left[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho V) \right] + \rho \left[\frac{\partial V}{\partial t} + V \cdot \nabla V \right] d \forall$$

The first bracket on the left hand side of the last equation vanishes, as it represents the continuity equation (3-3). Therefore, the net rate of momentum change can be represented by the following equation:

$$\frac{d}{dt} \iiint \rho V d \forall + \iint \rho V V \cdot n dS = \iiint \rho \frac{DV}{Dt} d \forall$$
 (3-6)

Similarly, applying the divergence theory to the right side of the equation (3-5) yields:

$$\iint \sigma_{ij}. n \, dS + \iiint \rho g \, d\forall = \iiint (\nabla . \, \sigma_{ij} + \rho g) \, d\forall$$
 (3-7)

Combining equations (3-6), and (3-7) yields what is known as Cauchy's equation [6]:

$$\rho \frac{DV}{Dt} = \nabla \cdot \sigma_{ij} + \rho g \tag{3-8}$$

Thus, Cauchy's equation components of the Cartesian coordinates are:

$$\rho \frac{Du}{Dt} = \rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{yx}}{\partial y} + \frac{\sigma_{zx}}{\partial z}$$
(3-9)

$$\rho \frac{Dv}{Dt} = \rho g_y + \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\sigma_{zy}}{\partial z}$$
(3-10)

$$\rho \frac{Dw}{Dt} = \rho g_z + \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} + \frac{\sigma_{zz}}{\partial z}$$
(3-11)

Note that fluid mechanics problems cannot be solved by using Cauchy's equation alone if the stresses are not expressed by primary unknowns. First, separate between the pressure stresses and viscous stresses. Also, the shear stresses need to be expressed. For this study, the fluid will be Newtonian, continuum, laminar, and incompressible. Thus, the stress tensor of the moving fluid will be in the following form [31]:

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} = \begin{pmatrix} -P_x & 0 & 0 \\ 0 & -P_y & 0 \\ 0 & 0 & -P_z \end{pmatrix} +$$

$$\mu \begin{pmatrix} 2\frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} & \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} & 2\frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \\ \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} & \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} & 2\frac{\partial w}{\partial z} \end{pmatrix}$$

$$(3-12)$$

Considering the x component of Cauchy's equation, combining equation (3-12) into equation (3-9) yields:

$$\rho \frac{Du}{Dt} = -\frac{\partial P}{\partial x} + \rho g_x + \mu \left(2 \frac{\partial^2 u}{\partial x^2} + \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \frac{\partial}{\partial z} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right) \right)$$
(3-13)

Recall that:

$$\frac{\partial}{\partial y} \left(\frac{\partial v}{\partial x} \right) = \frac{\partial}{\partial x} \left(\frac{\partial v}{\partial y} \right) \quad and \quad \frac{\partial}{\partial z} \left(\frac{\partial w}{\partial x} \right) = \frac{\partial}{\partial x} \left(\frac{\partial w}{\partial z} \right)$$

Rearranging equation (3-13) yields:

$$\rho \frac{Du}{Dt} = -\frac{\partial P}{\partial x} + \rho g_x + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \mu \frac{\partial}{\partial x} \left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right]$$
(3-14)

The square bracket term vanishes to satisfy the continuity equation. So, the x component of the momentum equation can be written as:

$$\rho \frac{Du}{Dt} = -\frac{\partial P}{\partial x} + \rho g_x + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)$$
(3-15)

Similarly, the y and z components can be found:

$$\rho \frac{Dv}{Dt} = -\frac{\partial P}{\partial y} + \rho g_y + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right)$$
(3-16)

$$\rho \frac{Dw}{Dt} = -\frac{\partial P}{\partial z} + \rho g_z + \mu \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right)$$
(3-17)

Expressing these three components in one vector form yields:

$$\rho \frac{DV}{Dt} = -\nabla P + \rho g + \mu \, \nabla^2 V \tag{3-18}$$

This equation is known as Navier-Stokes equation.

3.2 Separation Processes Modeling

The governor equations have been introduced briefly in the previous section; the main principles of separation technology, which are momentum, gravity, and coalescence will be modeled in this section.

3.2.1 Momentum Change

The effluent strikes the inlet diverter, as shown in figure (3-1). The gas and liquid enter the separator from the cross section area (A_i), which represents the

flow line, at the same horizontal velocity (u_i) , normally strikes the plate of area (A_P) and then flow along the plate.

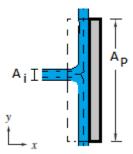


Figure (3-1) Effluent Strikes the Inlet Diverter

Obviously, using the Eulerian approach to analyze such a problem is easier. The dot lines represent the control volume under investigation which has been chosen. So that the left and right side areas are equal to the plate area. The free body diagram in (x) direction is shown below in figure (3-2). The reaction force of the plate is denoted by (B_x) ; it is assumed to be in the positive direction. The separator pressure (P_S) acts on the left side of the control volume.

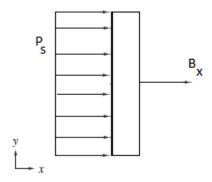


Figure (3-2) Effluent free body diagram [13]

Since there are no body forces in the horizontal direction, the incompressible steady state momentum equation can be simplified and the reaction force will be [13]:

$$B_{x} = \rho A_{P} u_{i}^{2} - P_{S} A_{P} \tag{3-19}$$

The reaction force (B_x) is compounded from the summation of liquid and gas reaction forces. Significantly, the reaction force due to liquid is much larger than that caused by gas as a result of densities difference. This makes the liquid and gas reflect differently. Considering the vertical direction, the liquid is heavier than the gas, so it will fall to the bottom of the vessel. This behavior proves that the change of momentum caused by the deflector plate causes a separation between the liquid phase and the gas phase.

3.2.2 Gravity Separation

The previous process ended with the liquids settled at the bottom of the separator vessel. These liquids are normally oil and water. As the oil droplets are lighter than water droplets, they will rise at a vertical velocity (V_v) which is used to size the separator. At this velocity there will be a forces balance between the drag force (Fd), the bouncy force (F_B), and the gravity force (F_g). As shown in figure (3-3), which is described by the following equation:

$$F_d + F_B = F_a \tag{3-20}$$

Each of those forces can be expresses as following [6]:

$$F_d = \frac{1}{2}\rho_o C_D \pi V_v^2 (\frac{d}{2})^2 \qquad F_B = \frac{4}{3}\pi \rho_o g(\frac{d}{2})^3 \qquad F_g = \frac{4}{3}\pi \rho_w g(\frac{d}{2})^3$$

$$F_B = \frac{4}{3}\pi\rho_o g(\frac{d}{2})^3$$

$$F_g = \frac{4}{3}\pi \rho_w g(\frac{d}{2})^3$$

where:

C_D: Drag coefficient

 ρ_o : Oil density

d: Oil droplet diameter

g: Gravity constant

 ρ_w : Water density



Figure (3-3) Droplet Balance

Defining the forces in equation (3-20), yields the terminal velocity equation:

$$V_v = \sqrt{\frac{4 gd}{3 C_D} \left(\frac{\rho_w - \rho_o}{\rho_o}\right)}$$

However, the terminal velocity is too slow and the Reynolds number (Re_D) is very low. Thus, creeping flow can be assumed and the drag coefficient can be found as [31]:

$$C_D = \frac{24}{Re_D}$$

where:

$$Re_D = \frac{\rho_o V_v d}{\mu}$$

 μ : viscosity of the continuous phase

Defining the terminal velocity equation, yields the Oil-Water settling velocity:

$$V_v = \frac{1}{18} g d^2 (\frac{\rho_w - \rho_o}{\mu})$$
 (3-21)

3.2.3 Coalescing Separation

When the effluent strikes the inlet diverter and bulk separation between liquids and gas takes place, some tiny drops of liquid will be carried over with the gas. These drops are caught later by the means of drag force. A pack of coalescing plates as shown in figure in (3-4), is used for this purpose.



Figure (3-4) Coalescing Plates Pack [17]

The coalescing plates are a set of sheets arranged a few millimeters apart. The main purpose is to create a film liquid by means of drag to let the gas flows free

of liquid drops. This process can be modeled as a stratified flow as shown in Figure (3-5) below:

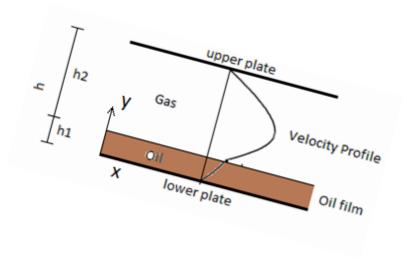


Figure (3-5) Stratified Flow

In order to model this process certain assumptions should be stated first:

- Laminar flow and Newtonian fluid
- Steady flow
- Velocity profile is function in y only
- Incompressible flow
- The developing length is very small

Adding these assumption to the momentum equation (3-15), the governing equations of the gas and oil regions respectively will be as the follows:

$$\frac{\partial^2 u_{gas}}{\partial y^2} = 2\alpha_{gas} \tag{3-22}$$

$$\frac{\partial^2 \mathbf{u}_{\text{oil}}}{\partial \mathbf{y}^2} = 2\alpha_{\text{oil}} \tag{3-23}$$

where:

$$\alpha_{\rm gas} = \frac{1}{2\mu_{\rm gas}} \left[\left(\frac{\partial P}{\partial x} \right)_{\rm gas} - \rho g_{\rm x} \right] \tag{3-24}$$

$$\alpha_{\text{oil}} = \frac{1}{2\mu_{\text{oil}}} \left[\left(\frac{\partial P}{\partial x} \right)_{\text{oil}} - \rho g_{x} \right]$$
 (3-25)

The general solution of equations (3-22) and (3-23) can be written as:

$$u_{gas} = \alpha_{gas} y^2 + Ay + B$$
; $h1 \le y \le h$ (3-26)

$$u_{oil} = \alpha_{oil} y^2 + Cy + D; \ 0 \le y \le h1$$
 (3-27)

Where the first derivative of the general solution can be found as:

$$\frac{\partial u_{gas}}{\partial y} = 2y\alpha_{gas} + A; \quad h1 \le y \le h$$
 (3-28)

$$\frac{\partial u_{oil}}{\partial y} = 2y\alpha_{oil} + C; \quad 0 \le y \le h1$$
 (3-29)

Now, boundary conditions should be applied to evaluate the four constants A, B, C, and D.

Firstly, the oil velocity at the lower plate equals zero for the no-slip condition, yielding:

$$D = 0$$
 (3-30)

Secondly, the oil and gas shear stresses are equal to each other at the interface.

This leads to:

$$\mu_{oil} \frac{\partial u_{oil}}{\partial y}_{y=h1} = \mu_{gas} \frac{\partial u_{gas}}{\partial y}_{y=h1}$$
 (3-31)

Adding equations (3-31) to (3-28) then combining with (3-29) yields:

$$C = \frac{2\mu_{gas}}{\mu_{oil}}\alpha_{gas}h1 + \frac{\mu_{gas}}{\mu_{oil}}A - 2\alpha_{oil}h1 \tag{3-32}$$

Thirdly, the gas velocity at the upper plate equals zero for the no-slip condition. As in equation (3-26) leads to:

$$B = -\alpha_{gas}h^2 - Ah \tag{3-33}$$

Finally, the oil and gas velocities of equations (3-26) and (3-27) are equal to each other at the interface. This yields:

$$\alpha_{gas}h1^2 + Ah1 + B = \alpha_{oil}h1^2 + Ch1$$
 (3-34)

Applying equations (3-32) and (3-33) into equation (3-34), then the constant A can be expressed in the two following forms:

$$A = \alpha_{gas} h \frac{\left[\frac{h_{1}^{2}}{h^{2}} \left(1 - \frac{2\mu_{gas}}{\mu_{oil}} + \frac{\alpha_{oil}}{\alpha_{gas}}\right) - 1\right]}{\frac{h_{1}}{h} \left(\frac{\mu_{gas}}{\mu_{oil}} - 1\right) + 1}$$
(3-35)

$$A = \alpha_{oil} h_1 \frac{1 + \frac{\alpha_{gas}}{\alpha_{oil}} (1 - \frac{2\mu_{gas}}{\mu_{oil}} - \frac{h^2}{h_1^2})}{\frac{\mu_{gas}}{\mu_{oil}} - 1 + \frac{h}{h_1}}$$
(3-36)

The velocity profile of the gas region can be found by applying from (3-33) and (3-35) into (3-26) while the velocity profile of the oil region can be found by applying from (3-30), (3-32) and (3-36) into (3-27):

$$u_{gas} = \alpha_{gas}h^{2} \left[\left(\left(\frac{y}{h} \right)^{2} - 1 \right) \right) + \frac{\left[\frac{h1^{2}}{h^{2}} \left(1 - 2\mu + \frac{1}{\alpha} \right) - 1 \right] \left(\frac{y}{h} - 1 \right)}{\frac{h1}{h}(\mu - 1) + 1} \right]$$
(3-37)

$$u_{oil} = \alpha_{oil}h1^{2} \left[\left(\frac{y}{h1} \right)^{2} + \left[2\mu\alpha + \frac{\mu + \alpha\mu \left(1 - 2\mu - \frac{h^{2}}{h1^{2}} \right)}{\mu - 1 + \frac{h}{h1}} - 2 \right] \frac{y}{h1}$$
 (3-38)

where:

μ: gas to oil dynamic viscosity ratio

 α : The ratio between α_{gas} and α_{oil} .

From (3-37) and (3-38) the gas and oil flow rate per unit depth can be calculated using the following relations:

$$Q_{oil} = \int_0^{h1} u_{oil} dy \tag{3-39}$$

$$Q_{gas} = \int_{h1}^{h} u_{oil} dy$$
 (3-40)

This will lead to:

$$Q_{oil} = \frac{\alpha_{oil}h1^3}{6} \left[2 + 6\mu\alpha + \frac{3\mu + 3\alpha\mu\left(1 - 2\mu - \frac{h^2}{h1^2}\right)}{\mu - 1 + \frac{h}{h1}} - 6\right]$$
(3-41)

$$Q_{gas} = \frac{\alpha_{gas}h^{3}}{6} \left[\left(1 - \frac{h1}{h} \right)^{2} \left[-4 - \frac{2h1}{h} + \frac{3h}{h1} \left[\frac{\left[\frac{h1^{2}}{h^{2}} \left(1 - 2\mu + \frac{1}{\alpha} \right) - 1 \right]}{\frac{h1}{h}(\mu - 1) + 1} \right] \right]$$
(3-42)

3.3 Separation Mathematical Modeling

In previous sections the separation processes have been modeled. This section focuses on the physical geometry of the separator vessel. Figure (3-6) below shows the gross flow in and out of the three-phase separator.

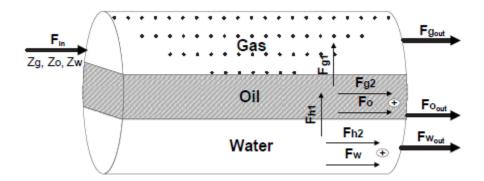


Figure (3-6) Three-Phase Components Gross Separation [26]

The molar flow (F_{in}) , which consists of Z_g , Z_o , Z_w molar fractions of gas, oil, and water respectively, enters the separator. As shown, the hydrocarbon splits into two parts. The first one is (F_{h1}) which separates due to the gravity forces, while the other (F_{h2}) stays with water component (F_w) and together form the stream flow (F_{wout}) . This flow will be subjected to a later separation process or treated before disposal based on the percentage of oil. Also, there will some gas dissolved in the oil. Some of this gas will be liberated (F_{g1}) and flow out through the gas outlet. The oil discharge (F_{Oout}) is a combination of oil (F_o) and dissolved gas (F_{g2}) .

The separator design should be able to handle the normal operating condition which leads to complete separation of oil. This can be satisfied by providing the longest path of oil drops, as shown in figure (3-7) below. In order to size the separator that way the design parameters Ac, h, , and ψ should be considered as will be shown below.

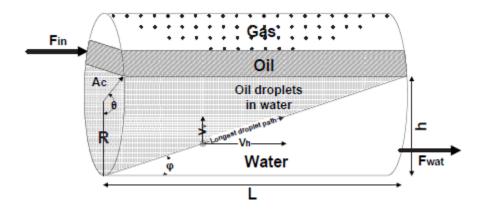


Figure (3-7) Oil Separation under Normal operation [26]

The vertical velocity component (V_v) of the oil drops are already expressed by equation (3-21) while the horizontal component is determined in the steady state by dividing the separator length (L) by the retention time (t).

$$V_h = \frac{L}{t} \tag{3-43}$$

where the retention time is defined as the time needed for the oil particle to move from the inlet to the oil outlet. It can be found from the following equation:

$$t = \frac{\rho_w \forall_w}{\dot{m}_w} \tag{3-44}$$

where:

 (\forall_w) : is the volume of the water

 (\dot{m}_w) : is the water flow rate

Now, the angle (ψ) which provide the longest oil droplet path can be expressed as:

$$\psi = \tan^{-1} \frac{V_v}{V_h} \tag{3-45}$$

The interface level of water and oil (h) can be determined as follows:

$$h = R(1 - \cos \theta) \tag{3-46}$$

where (R) is the cross section radius.

Also, the area (Ac) Can be determined as:

$$A_C = \frac{\forall_w}{L} \tag{3-47}$$

The cross section area (Ac) can also be expressed as function of the angle (θ) as below:

$$A_C = R^2 \theta - 0.5R \sin 2\theta \tag{3-48}$$

3.4 Techniques Developed by Operating Companies

This section shows techniques used by operating companies to model two phase separators.

3.4.1 Canadian Design

The Canadian design of two phases separator is based on four separation volumes which are added to determine the total separator volume.

- Minimum Shutdown Volume is defined by the low level shutdown being placed (12") from the bottom of the vessel.
- Holding Time Volume is calculated by multiplying the average liquid rate by a specific time, which is usually ten minutes. This volume is bounded to the previous volume.
- Stabilized Slug Volume is the additional volume if necessary to maintain normal operating condition.
- Gas Volume is calculated from the following equation [8]:

$$\forall_g = 0.157 \sqrt{\frac{\rho_l - \rho_g}{\rho_g}} \tag{3-49}$$

where:

 \forall_q : Gas Volume in cubic feet

 ρ_l : Liquid density in pound mass per cubic feet

 ρ_q : Gas density in pound mass per cubic feet

3.4.2 Middle East Design

One company in the Middle East designed its own separator in 1947 [8]. By collecting data from over one hundred of their gas and oil plants, they could size the separator within a ten per cent error of the required volume of the separator. They developed an empirical equation based on the concept of three volumes, a lower volume for the oil, the gas volume at the top, and foam volume in the middle. A minimum oil volume is required to prevent foam being sucked into the oil outlet. In the upper part of the separator enough space should be available for the gas without causing oil entrainment. Most of the separator volume is occupied by the foam when it is operating at maximum capacity. However, the volume needed to break the foam depends on the properties of the crude oil. The following formula is used in this technique [8]:

$$\forall_{s} = \frac{6.5d^{2} - 4d}{3.1 \times 10^{-3} \times \frac{GOR}{n} \sqrt{\frac{TG}{P} + 0.552D\sqrt[3]{\frac{d}{L}}(\frac{105}{L - 15})}}$$
(3-50)

Where:

 \forall_s : Maximum efficient Capacity in cubic feet

L: Separator Length in feet

D: Separator Diameter in feet

T: Temperature of the Gas in Fahrenheit

P: Separator pressure in psi

G: gas specific gravity

R: gas to oil volume ratio

n: Factor for number of gas outlets. This can be 1, 1.8, or 2.5 for one gas outlet, two gas outlets, or three gas outlets respectively.

Coupling with the above empirical formula, the following procedure is considered:

- The calculation should be based on the required rate and increased by 10%.
- A margin between 3 inches and 6 inches should be added to the vessel diameter to handle fluctuations.

The dimension determined previously, should always be compared with an existing installation that works in a similar condition.

3.5 Electrochemical Reactions

The relation between chemistry and electricity dates back to 1793 when Alessandro Volta produced electricity using a voltaic cell. Today, its main application is producing dry batteries used in cars, cell phones etc. Later, in 1800; Nicholson and Carlisle were the first to decompose the water by applying an electrical source. This has different applications such as hydrogen production, and oil water separation the aim of this study. These two inventions are similar to some extent and different in the sense that the former produces electricity, while the latter needs electricity to occur. The following examples gives more details regarding these two types of electrochemical reactions.

In a voltaic cell electricity is produced due to a chemical reaction. As shown in figure (3-8) below, electrons naturally move from metallic zinc metal when connected to the metallic cupper via an electrical wire. Due to this process the zinc oxidizes because of losing electrons and is called anode, while the cupper is reduced as it gains electrons and is called cathode.

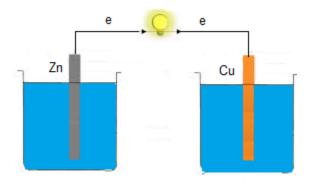


Figure (3-8) Voltaic Cell

Electrolysis is a process where electricity is used to make a chemical change occur which would not happen otherwise. Basically, in this reactions the neutron compound will be broken into its main elements. As shown in figure (3-9) below, a battery is used to pull the electrons from the oxygen and push them to the hydrogen. The anode is the bar connected to the positive side of the battery, while the cathode is the one connected to the negative side.

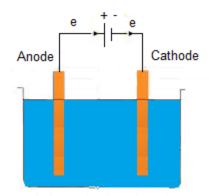


Figure (3-9) Electrolytic Cell

3.6 Design of Experiment

It is important to observe the system while it is in operation to understand how the process works. Conducting an experiment by changing factors and observing the outputs shows the relationship between the cause and effect. The statistical analysis of results obtained depends on how the experiment is performed. Factorial design of a two level experiment will be employed for this study and will be statistically analyzed using a technique called the Analysis of Variance (ANOVA) for short. The next sections will provide an over view of these concepts.

3.7 Factorial Design of Two Levels

To perform the factorial design experiment, each factor is set to two levels. high and low, which are usually coded as (+1) and (-1) respectively. Because the factors might be quantitative, such as the amount of flow rate or qualitative, such as present or absent identity. Then, all the possible combination of these levels are run. Therefore, this design is denoted as (2k) which represents the number of runs, where (k) is the number of factors. Each run is called a treatment, while together they form an experiment. The effect of a factor is defined as the change in response produced by the change in level of the factor. As well as, the ability to combine the numerical and categorical factors in one correlation, the difference between factorial design and one factor at the time "OFAT" is illustrated in Figure (3-10) where, for example, each of factors A and B has two levels coded (-1) and (+1) for low and high levels respectively. For OFAT, the effect of factor "A" is measured by the difference between the results at a certain level of "A" with the two combinations of factor "B" such that (A+1B-1- A-1B-1)), and similarly the effect of factor "B" is measured so that (A-1B-1- A-1B+1)).

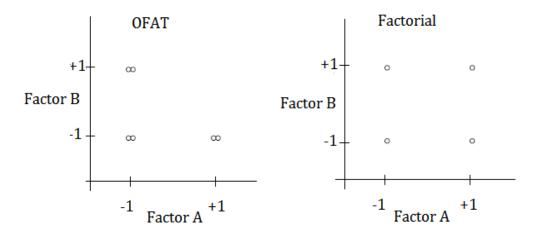


Figure (3-10) One Factor at a Time vs Factorial Design Experiments

Because of the possibility of experimental error, it is desirable to take two observations, which raises the treatment numbers to six, and estimate the effect by the average of the two results. Using factorial design, by one other combination (A+1B+1), the effect of factor "A" can be measured by taking the average of the effect of factor "A" at each level of factor "B" and similarly the effect of factor "B" can be measured. This means that only four treatments are needed to get precise result. This is just for two factors. In other words, factorial design significantly decreases the number of treatments needed.

3.8 Two to the Power of K Factorial ANOVA

This section provides a brief view of the statistical concepts without describing details, which can be done using software or even manually, as will presented numerically in a later chapter. Generally, the factorial ANOVA deals with at least

two independent factors, where the two levels of each factor are paired with the two levels of the other factors. The main goal is to finish with a correlation that describes the phenomena with acceptable accuracy. This can be achieved by following the following analysis procedure.

3.8.1 Estimate Factor Effects

The first step is to investigate the factor and its interaction effects. This provides information regarding which factors and interactions are important and in which direction they are contributing to the result. This concept can be explained by explanatory data, which can be any quantity, as shown in figure (3-11), which shows made up data, which could be of any quantity. The first line shows how the results change with factor A at the low level of factor B, while the second line shows the behavior of results with changing factor A at the high level of factor B.

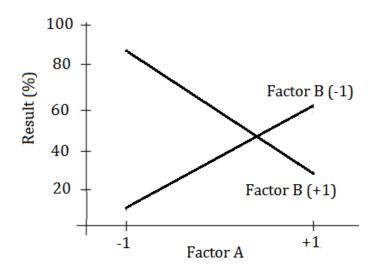


Figure (3-11) Two Factors effect

This figure demonstrates main factor effects and interaction. It can be clearly seen that there is a change in the results, even though the level of factor B is fixed. This means that the data obtained at level (-1) of factor A is different from that at level (+1), shown that, factor A contributes to the result. Similarly, it can be observed that, at the same level of factor A, there is a difference in the results. Therefore, factor B is also an important factor. Also, the two lines of the figure above are not parallel. Thus, it can be said that, factor A has failed to produce the same effect at a different level of B. Signifies, there is an interaction between the factors which should be considered.

3.8.2 Form Initial Model

In forming an initial model for the (2^k) factorial design experiment, usually a full model is used, which includes all factors and interaction effects. It has the following form [23]:

$$y = \mu + \sum_{i=1}^{k} \beta_{i} x_{i} + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \beta_{ij} x_{i} x_{j} + \sum_{i=1}^{k-2} \sum_{j=i+1}^{k-1} \sum_{z=i+1}^{k} \beta_{ijz} x_{i} x_{j} x_{z} + \cdots$$

$$+ \sum_{i=1}^{1} \sum_{j=i+1}^{2} \sum_{z=i+1}^{3} \dots \sum_{n=k}^{k} \beta_{ijz \dots k} x_{i} x_{j} x_{z} \dots x_{n+\epsilon}$$

$$(3-51)$$

where:

μ: mean

y: observation of the phenomena

x_s: the coded level of the factors 1,2,3,....k.

Bi: regression model, which is half of the main factor effect

β_{ii}: regression model, which is half of the two factors interaction effect

β_{ijz}: regression model, which is half of the three factors interaction effect

βijz..k: regression model, which is half of the (k) factors interaction effect

 ϵ : Residual; which is the difference between the predicted and observed value

Note that, for most engineering applications the effects of three or more factor interaction are not important unless descriptive factors involved.

3.8.3 Perform Statistical Testing "ANOVA"

The analysis of variance (ANOVA) is a statistical method used to compare the means of two or more groups to identify which variables contribute significantly to the result. Basically, it starts by stating a hypothetical assumption that the factor has nothing to do with the result, then checks the probability of the hypothesis's validity. There are many types of ANOVA test. However, the Factorial ANOVA is the only one discussed here as it will be used to analyze the obtained data in a later chapter. In order to rely on the results of this technique, the following assumptions should be met:

Data distributed normally

It is important to look at the data graphically before starting the analysis to make sure it does not violate an assumption required to ascertain analysis. The data obtained could have skew, kurtosis or follow other than normal distribution. One of most famous methods to check the normality assumption is to construct the normal distribution curve.

Independence of error

This means that the errors of runs are independent of one another. Basically, this relies on how the experiment is conducted. There are three requirements which need to take place to say there is an error independency. First, the runs should be done randomly. Secondly, the variable should be at same initial condition for each run. If for example, the pressure is a factor and it happen to be the same for

two following runs. In the second run the whole procedure of obtaining that amount of pressure should be repeated. Finally, runs should be blocked once there is a source of disturbance, for example, if some of the runs will be conducted using a different device or person. In this case a systematical block design is needed to eliminate the source of disturbance.

Absence of outlier data

An outlier is an extreme point which stands out from the rest of the distribution. A useful tool used to specify whether the data is an outlier or not is the box plot. It is a box which excludes all the outlier data. Basically, it considers five values. This depends on the minimum and maximum value of results, the median, first and third quartile.

3.8.4 Refine the Model

At this point, the insignificant variables should be removed and the previous steps again repeated.

3.8.5 Residual

The difference between the actual and computed value of a result is called a residual. Once the experimental model is developed, it is important to know how well this model works. Thus, it is crucial to be sure of the model's validity. This error should be as small as possible. This can be verified graphically by using a scatter plot or numerically by computing the residual error (R²) which

specifies the percentage of the overall variability explained by the model. It ranges between zero and one, a higher (R^2) means better results. However, the potential problem of (R^2) is that it increase with the number of factors, even if they are insignificant. Therefore, it is more useful to also look at the adjusted residual error (R^2_{adj}) as it takes into account the size of the factors, as will be shown in later chapters.

Chapter 4 Gravity Separator Experiment

4.1 Gravity Separator Experimental Setup

The working fluids mixed in the experiment are water and mineral oil "WO 15". The density of the oil at 15 °C is 859 kg/m³, while the viscosity at 40 °C is 15 cSt. The flow diagram of the experiment is shown in figure (4-1). It consists of different items connected with the (1") pipe system and fittings. These items have varied functions as follows:

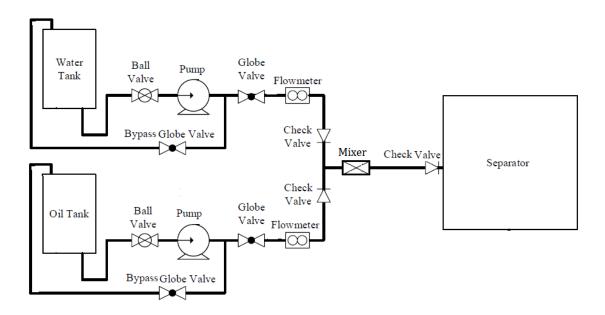


Figure (4-1) System Configuration

4.1.1 Tanks

The system contains two tanks, as shown in figure (4-2), One stores the water, while the other one is for oil. Each of them has a 877 Liter capacity. For the sake of safety, each tank is equipped with a ball vale. Its main function is to

isolate the tank in case of a leaking downstream from the tank and it allows maintenance to take place.



Figure (4-2) Storage Tank

Also, each tank is suited with a Full-Coverage Drum Heater as shown in Figure (4-3). The range of the heater is between (10 °C) and (232 °C) which can be adjusted via a temperature controller.



Figure (4-3) Heater

4.1.2 Pumps

The system is equipped with two pumps of as shown in figure (4-4). The pumps are used to move the oil and water toward the separator.



Figure (4-4) Pump

Also, the pump outlet is connected to a bypass globe valve to prevent the system from over pressurizing.

4.1.3 Flow Meters

Both the water and oil lines are equipped with flow meters with computer display as shown in figure (4-5). The range of the flow meter is (6 gal/min).



Figure (4-5) Flow Meter

A global valve is installed upstream from each of the flow meters to control the flow, while a check valve is installed downstream to prevent flow back.

4.1.3 Mixer

The streams of water and oil need to be well mixed after they leave the metering devices. Thus, a mixture shown in figure (4-6) is used to make sure good mixing is achieved by guiding the flow into specific paths as shown in figure (4-7)



Figure (4-6) Mixer



Figure (4-7) Flow after Mixer

4.1.4 Separator

The separator diagram is shown in figure (4-8) below. Its width is 35cm. It can be operated with one compartment by removing the removal plate.

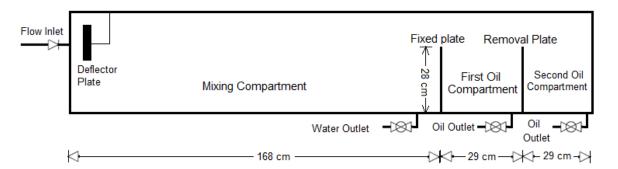


Figure (4-8) Separator diagram

The oil and water split in the separator by means of gravity, where the lighter fluid, which is oil, floats at the top and jumps the fixed plate. Figure (4-9) shows a real picture of the separator.



Figure (4-9) Separator

Also, the inlet of the separator might be an elbow or deflector plate as, shown in figure (4-10).



Figure (4-10) Separator inlet shapes

4.2 Gravity Separator Experiment

The main result investigated in the experiment is the water cut, which is the percentage of water content remaining after the separation process; less water cut means less pollution and more profit. This result is based on five factors, A, B, C, D and E, which are water flow rate, oil flow rate, number of oil compartments, temperature, and type of inlet, elbow or deflector plate, respectively as in table (4-1) below:

Table (4-1) Experiment Variables

Factor	Symbol	High	High Value	Low Value	Low Value
	·	value	Code		Code
Water Flow Rate	А	4 gal/min	+1	1 gal/min	-1
Oil Flow rate	В	4 gal/min	+1	1 gal/min	-1
No. of Compartments	С	2	+1	1	-1
Temperature	D	25 °C	+1	35 °C	-1
Type of Inlet	Ш	Deflector Plate	+1	Elbow	-1

These factors are investigated at two values or categories and coded to be combined in one correlation. The numerical factors are coded as (+1) for the higher value and (-1) for the lower value, while the descriptive factors are coded randomly to (+1) and (-1). All the combinations of the factors and the results are shown in table (4-2):

Table (4-2) Experiment Runs

Table (4-2) Experiment Runs								
Run	A:Water Flow rate	B:Oil Flow Rate	C:Number of Compartments	D:Temperature	E:Inlet Type	Water Cut		
Unit	gal/min	gal/min		°C		%		
1	-1	-1	-1	-1	-1	3.00		
2	1	-1	-1	-1	-1	7.00		
3	-1	1	-1	-1	-1	1.00		
4	1	1	-1	-1	-1	4.00		
5	-1	-1	1	-1	-1	2.50		
6	1	-1	1	-1	-1	6.50		
7	-1	1	1	-1	-1	0.75		
8	1	1	1	-1	-1	3.50		
9	-1	-1	-1	1	-1	2.80		
10	1	-1	-1	1	-1	6.75		
11	-1	1	-1	1	-1	0.75		
12	1	1	-1	1	-1	3.80		
13	-1	-1	1	1	-1	2.25		
14	1	-1	1	1	-1	6.00		
15	-1	1	1	1	-1	0.00		
16	1	1	1	1	-1	3.40		
17	-1	-1	-1	-1	1	2.80		
18	1	-1	-1	-1	1	6.85		
19	-1	1	-1	-1	1	1.50		
20	1	1	-1	-1	1	4.50		
21	-1	-1	1	-1	1	2.25		
22	1	-1	1	-1	1	6.00		
23	-1	1	1	-1	1	0.00		
24	1	1	1	-1	1	3.70		
25	-1	-1	-1	1	1	3.00		
26	1	-1	-1	1	1	7.00		
27	-1	1	-1	1	1	0.95		
28	1	1	-1	1	1	3.60		
29	-1	-1	1	1	1	2.50		
30	1	-1	1	1	1	5.00		
31	-1	1	1	1	1	0.00		
32	1	1	1	1	1	3.90		

4.3 Estimate the effects

The effect is defined as the change in result respecting to the factor. This can be calculated from table (3.2). Firstly, multiply the factor column by the results column and compute the average of the summation of each level separately. Then, the effect will be the difference between the average of the results at the higher level and the lower level. Similarly, the interaction effects can be calculated by adding the interaction column, which is a result of the factor levels product. For example, in order to measure the interaction of factors AB, add a new column from the product of the factor A column and factor B column and repeat the same procedure. This generates table (4-3) below, which will also be used to construct the Pareto plot.

Table (4-3) Effects and Interactions

Model	Effects	Absolute value of the	Cumulative	% of the	
		Effect		Cumulative	
A	3.466	3.466	3.466	37.109	
В	-2.303	2.303	5.769	61.767	
С	-0.691	0.691	6.460	69.165	
AB	-0.284	0.284	6.744	72.206	
D	-0.259	0.259	7.003	74.979	
ABC	0.253	0.253	7.256	77.687	
ABCE	0.216	0.216	7.472	80.000	
BCDE	0.178	0.178	7.650	81.906	
CE	-0.166	0.166	7.816	83.683	
BCD	0.166	0.166	7.982	85.460	
ABCD	0.159	0.159	8.141	87.163	
ABE	0.153	0.153	8.294	88.801	
BE	0.147	0.147	8.441	90.375	
ABD	0.134	0.134	8.575	91.809	
ADE	-0.116	0.116	8.691	93.051	
ВС	0.084	0.084	8.775	93.951	
CDE	0.078	0.078	8.853	94.786	
ACDE	-0.066	0.066	8.919	95.493	
AD	-0.066	0.066	8.985	96.199	
BD	-0.059	0.059	9.044	96.831	
ABCDE	0.059	0.059	9.103	97.463	
DE	0.053	0.053	9.156	98.030	
BDE	-0.047	0.047	9.203	98.533	
BCE	0.034	0.034	9.237	98.897	
Е	-0.028	0.028	9.265	99.197	
AE	-0.022	0.022	9.287	99.433	
ACD	-0.016	0.016	9.303	99.604	
ACE	0.016	0.016	9.319	99.775	
CD	-0.009	0.009	9.328	99.872	
ABDE	0.009	0.009	9.337	99.968	
AC	0.003	0.003	9.340	100.000	

Moreover, it is important to recognize which effects may contribute significantly to the results. This can be displayed better by constructing a Pareto plot, which is a chart showing the distribution of data in descending order of frequency with a cumulative line on the secondary axis as a percentage of the total, as shown in figure (4-11).

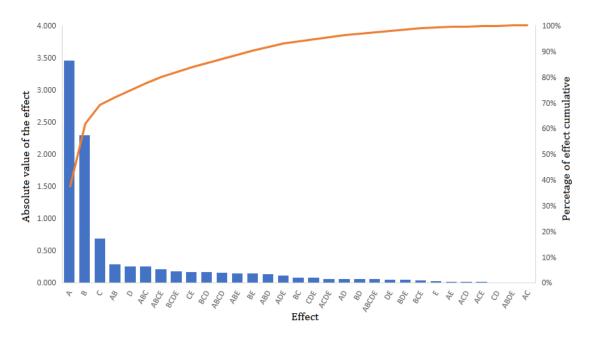


Figure (4-11) Pareto Chart

This chart provides a good indication of which effects are important. Clearly, effects A, B, C, AB, D, and ABC contribute more than the rest of the effects to the results. However, a statistical analysis (ANOVA) is still needed.

4.4 ANOVA Test

The analysis of variance explained in this section is refined with the important model effects only. It is based on the following hypothesis with a (95%) level of confidence, i.e. level of significant is (α =0.50):

Null Hypothesis: There is no difference in means.

Alternative Hypothesis: there is at least one different mean.

In order to decide which one is true, the following sum of squares should be calculated [23]:

$$SS_x = \frac{(n2^{k-1} \times effect_x)^2}{n2^k} \tag{4-1}$$

where:

SS_x: Sum of squares deviation around the mean of main effect or interaction (x)

n: Number of replicas which is one for this study

K: Number of levels which is two for this study

$$SS_T = \sum_{n=1}^{N} (x_n - \bar{x})^2$$
 (4-2)

where:

SS_T: sum of squared deviation from the grand mean

x: observation value

 \bar{x} : observations average

N: number of observations

$$SS_E = SS_T - \sum SS_x \tag{4-3}$$

where:

SS_E: The sum of the squared deviation of each group.

Before calculating the mean squares (MS), the sum of squares degree of freedom (df) should be specified based on the number of squared deviations. SS_T is the sum of (N) observed squared deviations around one point, which is the grand mean. This is the degree of freedom is (N-1) where that point is lost. In the case of effects, there are always two groups, which leads to two degrees of freedom in total, one for the high level group and one for the low level group. However, once the sum of squares is calculated within the mean of the groups, this costs one degree of freedom. Logically, the leftovers of (N) degrees of freedom belong to the SSE. After that, the mean squares (MS) can be calculated by dividing the sum of the squares by the degrees of freedom [23].

$$Mean Sequares = \frac{Sum \ of \ Square}{Degree \ of \ Freedom} \tag{4-4}$$

Now, the (F) statistical value can be obtained by dividing the mean squares of the groups by the mean square of the error as in the following equation [17]:

$$F = \frac{Mean \, Square}{Mean \, squares \, of \, Error} \tag{4-5}$$

Provided with the preceding information, many different types of software can calculate the probability value (P). For this study, Design Expert is used and generates the following:

Table (4-4) ANOVA TEST Table

Source	Sum of Squares	df	Mean Square	F value	P value	Decision
Model	144.09	8	18.01	195.53	< 0.0001	significant
A Water Flow rate	96.08	1	96.08	1043.08	< 0.0001	significant
B Oil Flow Rate	42.44	1	42.44	460.67	< 0.0001	significant
C Number of Compartments	3.82	1	3.82	41.42	< 0.0001	significant
D Temperature	0.54	1	0.54	5.84	0.024	significant
AB	0.65	1	0.65	7.02	0.0143	significant
AC	7.81E-05	1	7.81E-05	8.48E-04	0.977	insignificant
ВС	0.057	1	0.057	0.62	0.4397	insignificant
ABC	0.51	1	0.51	5.56	0.0272	significant
Error	2.12	23	0.092			
Cor Total	146.21	31				_

The summary of the ANOVA test in the above table shows that the factors A, B, C, D and the interactions AB, and ABC are significant model terms where the Probability "P" value is less than (0.05). The interactions BC, and AC are

insignificant but must be considered in the analysis for the hierarchy correction.

4.5 Statistical Analysis Assumption

The data gained from the experiment will be analyzed statistically to contract the information provided in table (4-2) into a simple correlation. This

correlation will be checked and validated by following the next steps:

4.5.1 Normality Check

The distribution of data should be classified for a correct analysis. Firstly,

both the median (μ_d) and the standard deviation (SS) should be calculated. For

the median, the observations should be rearranged in ascending order; then the

median is located. In the case of an even set of observations, it will be the

average of the two middle data. The standard deviation can be calculated using

the following equation [17]:

$$SS = \sqrt{\frac{\sum (X - \overline{X})^2}{N - 1}} \tag{4-6}$$

where:

X: observation

91

 \bar{X} : average of the observation

N: number of observations

Note that all statistics probability tables use what are called standard probability tables. Therefore, a (z) transform to data is used to linearly transform the distribution from a certain median and standard deviation to a standard distribution of a zero median and unity standard deviation, based on the fact which that the relation amongs data will not be affected by addition or subtraction, multiplication or division. Thus, this can be done by simply subtracting the median from each observation. This will reduce the median for the new set of data to zero. Moreover, with dividing all the data using a constant means dividing the standard deviation by that number. Thus, by dividing the new set of data by the earlier computed standard deviation, it will be completely transformed, as described by the following equation:

$$Z = \frac{X - \mu_d}{SS} \tag{4-7}$$

Now, the mathematical function of normal distribution is defined as [16]:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{z^2}{2}}$$
 (4-8)

Finally, the mathematical function of distribution can be drawn versus (Z) as shown in figure (4-12) which illustrates that the data are normally distributed.

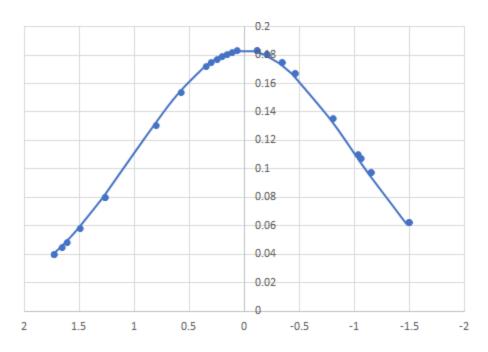


Figure (4-12) Standard Normal Distribution of Data

4.5.2 Error independence

The experiment runs are conducted in random order as shown in figure (4-13).

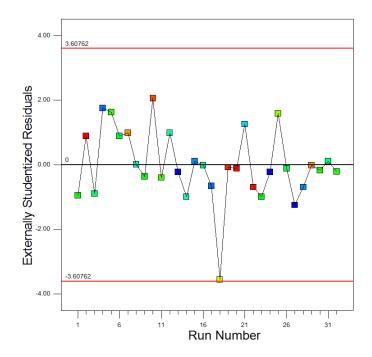


Figure (4-13) Runs vs Residual

The figure shows that the residual is well distributed and there is no obvious pattern, which means there is variance equality and error independence among runs.

4.5.3 Data Outlier

All the observations obtained in table (4.2) should satisfy the following inequality to confirm that there is no outlier data [16]:

$$Q_1 - 1.5 IQR \le x \le Q_3 + 1.5 IQR \tag{4-9}$$

where:

Q₁ and Q₃: First and third quartiles respectively

IQR: Interquartile range

x: Observation

As is known, the median (Q_2) falls in the middle of the observation. Similarly, the first quartile falls at the middle of the first half of the data, while the third quartile falls at the middle of the second half of the data. By arranging the data from smallest to highest it is found that the median $(Q_2=3.2)$, the first quartile $(Q_1=1.875)$, and the third quartile $(Q_3=4.75)$. The interquartile range is the difference between the upper and the lowest quartiles (IQR=2.875). Substitution as in the preceding inequality, yields:

$$-2.4375 \le x \le 9.0625$$

Therefore, the data gained fall in the acceptable range and there is no outlier data.

4.6 Results

It was shown in the previous sections that the water flow rate, oil flow rate, number of compartments, and temperature are significant factors, while the inlet type did not contribute significantly to the results. Also, it was shown that, there is a two factors interaction effect between the water flow rate and the oil flow rate. Moreover, the three factors interaction of water flow rate, oil flow rate, and number of compartments also showed a significant effect. In this chapter, these statements will be displayed graphically. Finally, a correlation describes the process that will be developed and verified.

4.6.1 Effects

Fig (4-14) below shows the effect of water flow "factor A".

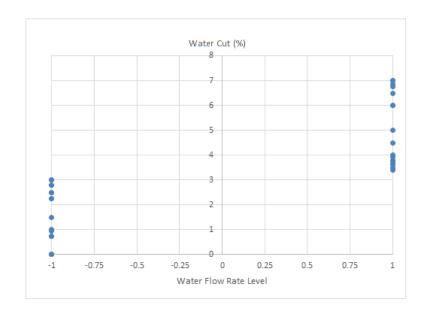


Figure (4-14) Water Flow levels Vs Water Cut

It can be seen that the water content increases with an increase in the water flow rate. Increasing the water flow rate requires more retention time for oil to separate which leads to higher water containment at the end of the process.

However, the oil flow rate "factor B" acts differently, as shown in figure (4-15) below. Increasing the oil flow rate decreases the water cut. Increasing the volume of the oil creates a chance to have more oil than water at the out let, even though the retention time is shorter.

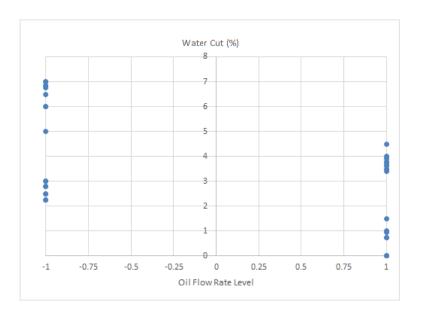


Figure (4-15) Oil flow levels Vs Water Cut

From the preceding argument, it can be concluded that there is an interaction effect between water flow rate and oil flow rate as shown in figure (4-16) below. This interaction exists because the water to oil flow rate ratio is also important. When the water to oil flow ratio is the same, then a lower flow rate of oil provides better results.

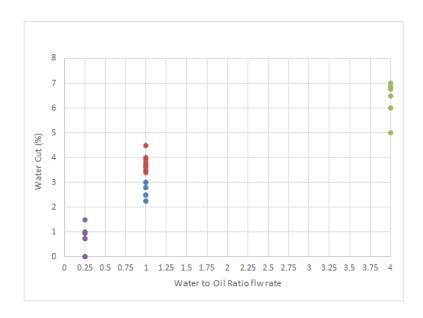


Figure (4-16) Water to Oil Ratio Vs Water Cut

Increasing the volume of the separator increases the retention time. Figure (4-17) below shows that the average results for one separator compartment show more water content than the average results of two separator compartments.

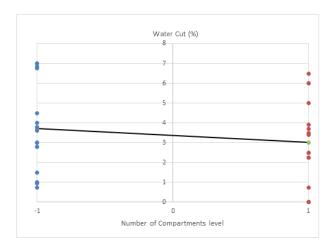


Figure (4-17) Number of Compartment levels Vs Water Cut

Also, increasing the flow temperature improves the separation, as shown in figure (4-18). This results from decreasing both viscosity and density. However,

here the effect is not very obvious, because the temperature increased slightly due to the experimental setup.

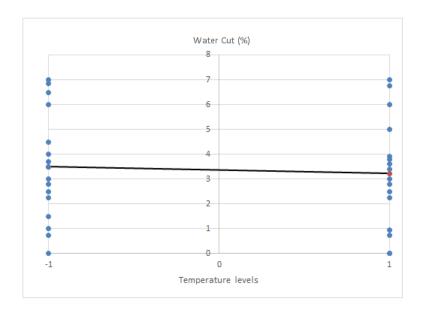


Figure (4-18) Temperature levels Vs Water Cut

4.6.2 Correlation

The linear relationship between the coded variables is presented by the following correlation:

$$Water Cut (\%)$$

$$= 3.36 + 1.73A - 1.15B - 0.35C - 0.13D - 0.14AB$$

$$+ 0.042BC + 0.13ABC$$
(4-10)

The residual error (R²) for this correlation is calculated as following [23]:

$$R^{2} = \frac{Sum \ of \ sequares \ of \ the \ model}{Total \ Sum \ of \ sequares} \tag{4-11}$$

Applying this equation, the residual error will be (0.9855). However, as stated earlier, it is more precise to calculate the adjusted residual error (R²_{adj}) as follows [23]:

$$R^{2}_{adj} = \frac{Mean \, Sequares \, of \, the \, model}{Total \, Mean \, Sequares} \tag{4-12}$$

It is found to be (0.9805). This means that the developed correlation explains (98.05%) of the variability. Moreover, the correlation ability of a prediction measured by the Press statistics (R²_{Pred}). Higher (R²_{Pred}) means better prediction and It is calculated by the following equation [23]:

$$R_{Pred}^{2} = 1 - \frac{Total\ sum\ of\ sequares\ of\ predicted\ data}{Total\ Sum\ of\ sequares\ of\ measured\ data}$$
 (4-13)

Applying the preceding equation, it is found that; the correlation is likely to predict the data with (97.19%) accuracy.

However, the model should be also subjected to conditions not used in creating the model to make sure it predicts data perfectly. The table below shows six results of such conditions followed by graphical illustrations.

Table (4-5) Validation Runs

Water	Oil flow	No. of	Temperature	Calculated	Measured
Flow Rate	rate	compartments	remperature	Result	Result
gal/min	gal/min		С	%	%
2.50	2.50	1.00	25	3.85	4.80
3.50	3.50	2.00	25	3.45	3.80
3.65	2.85	1.00	25	4.90	5.35
2.50	3.50	2.00	25	2.65	3.00
3.65	2.50	1.00	25	3.00	3.75
2.50	3.50	1.00	25	3.50	4.25

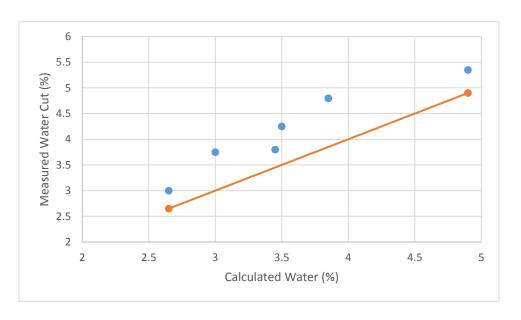


Figure (4-19) Validation Runs

The straight line shown in figure (4-19) is the calculated results based on the developed equation, while the scatter points are the measured results. The vertical distance between the scatter points and the line is the residual or error. On the first hand the data do not deviate much from the calculated value, which

means the correlation is good. But all the residual are positive for some unknown reasons.

Chapter 5 Experiments of Applying an Electrical Field

5.1 Experimental Setup

The objective of this experiment is to investigate the contribution of applying an electrical field for oil recovery. This is conducted using five different variables, temperature, voltage, salt NaCl, hydrogen ion concentration "pH", and oil to water ratio. The experimental setup of this experiment is quite simple compared to the previous experiment. Figure (5-1) shows the actual setup. It consists of a source of DC current. Its positive supply is connected to a graphite rod "Anode" with a red wire, and its negative charge connected to a steel sheet "Cathode" with a black wire.

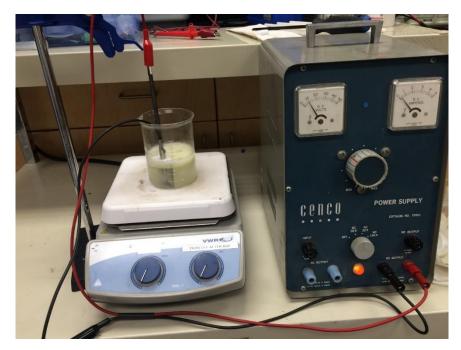


Figure (5-1) Electrolysis Cell

Both the anode and the cathode are immersed in a solution of (200 ml) of water and vegetable oil. A mixer with a heater is used to get good mixture and control the temperature. Approximately (3 ml) of detergent is added to make sure of a

good mixing, as shown in figure (5-2) below. Note that some amount of salt "NaCl" should be added to the solution in order to start the reaction.

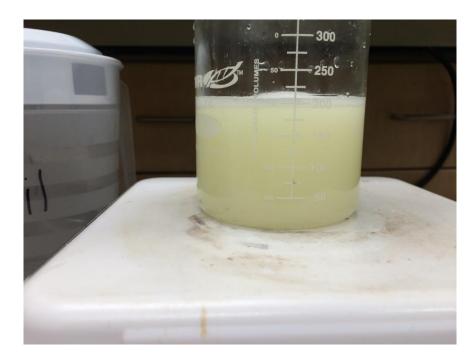


Figure (5-2) Oil and water mixture

5.2 Experiment Results

As was mentioned in the previous section, oil recovery was tested versus five different factors temperature, voltage, salt amount NaCl, hydrogen concentration pH, and oil to water ratio. In all of the experiments oil was not separated alone. Instead, a layer of foam was created as shown in figure (5-3).

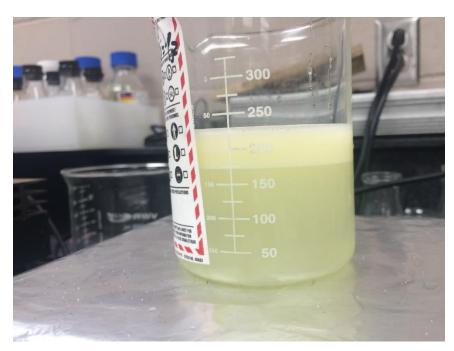


Figure (5-3) Foam Layer after Separation

The created foam is removed to the other container, then an antifoam 204 is used to break up this foam and measure how much oil has been separated, as shown in figure (5-4). In this section, the sensitivity of the results is exhibited for each of those factors.



Figure (5-4) Separation after adding foam

5.2.1 Oil recovery versus Temperature

In this experiment the percentage of oil recovery is tested against five points of temperature, as shown in figure (5-5). The other parameters are:

- NaCl (2g)
- Water (160 ml)
- Oil (40 ml)
- Volts (4 V)
- pH (7.32)

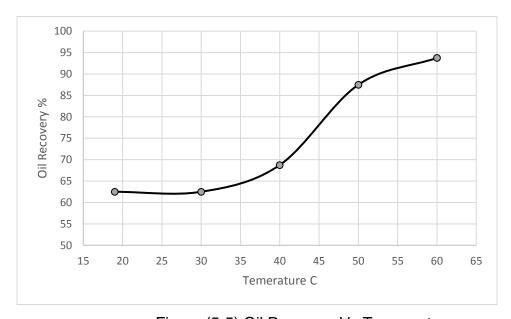


Figure (5-5) Oil Recovery Vs Temperature

It can be seen that increasing the temperature increases the efficiency of the separation, because this contributes to increasing the viscosity of the fluids. However, it seems that if the temperature is increased more, it might reach a peak point after which there are no benefits anymore from increasing the temperature.

5.2.2 Oil recovery versus Voltage

In this experiment the percentage of oil recovery is tested at five different points of voltage, as shown in figure (5-6). The other parameters are:

- NaCl (2g)
- Water (160 ml)
- Oil (40 ml)
- Temperature (21 °C)
- pH (7.37)

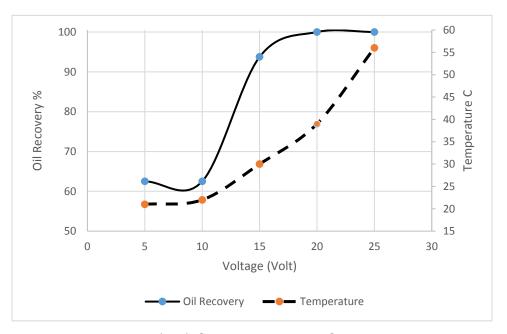


Figure (5-6) Oil Recovery Vs DC Voltage

As shown above, oil separation increases by increasing the voltage. From (15 v) and more, two things are noticed. Firstly, most of the water will evaporate. Secondly, there is a rapid increase in the temperature of the solution, as shown by the dashed curve above.

5.2.3 Oil recovery versus NaCl

In this experiment the percentage of oil recovery is tested with five different amounts of NaCl, as shown in figure (5-7). The other initial parameters are:

- Volt (5 V)
- Water (160 ml)
- Oil (40 ml)
- Temperature (22 °C)
- pH (7.34)

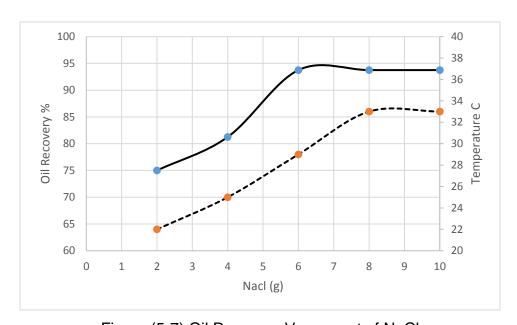


Figure (5-7) Oil Recovery Vs amount of NaCl

The oil separation efficiency increases rapidly with the increase in the amount of NaCl, as it increases the electrical conductivity. This continues to certain level and then it maintains the same performance. Also, there is a slow increase in the temperature noticed during the process, as illustrated by the dashed curve.

5.2.4 Oil recovery versus pH

In this experiment the percentage of oil recovery is tested at five different levels of pH, as shown in figure (5-8). This is controlled by adding sulfuric acid (H₂SO₄). The other initial parameters are maintained as follows:

- NaCl (2g)
- Water (160 ml)
- Oil (40 ml)
- Volt (5 V)
- Temperature (21 °C)

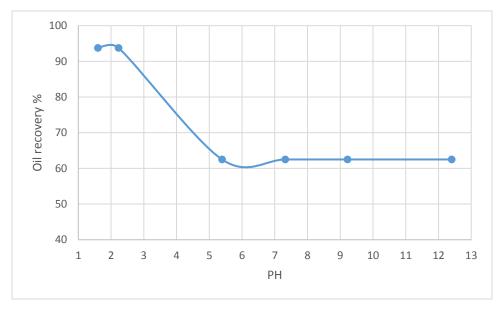


Figure (5-8) Oil Recovery Vs pH

It can be clearly seen from the figure above that separation improved when the solution is acidic.

5.2.5 Oil recovery versus volumetric oil to water ratio

In this experiment the percentage of oil recovery is tested using five different amounts of volumetric oil to water ratio, as shown in figure (5-9). The other initial parameters are maintained as follows:

- NaCl (2g)
- Total solution (200 ml)
- Volt (4 V)
- pH (7.34)
- Temperature (23°C)

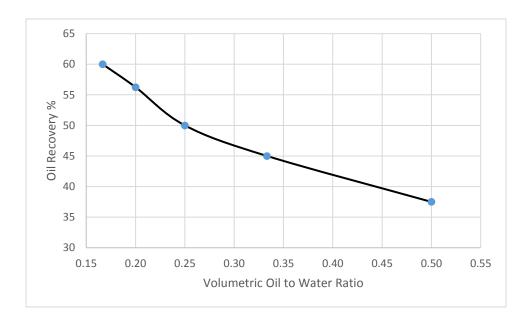


Figure (5-9) Oil Recovery Vs Volumetric oil to water ratio

Figure (5-9) shows that the efficiency of oil separation decreases almost linearly with increasing the oil content in the mixture.

Chapter 6 Conclusion

6.1 Research Findings

This study investigated the separation of oil from a water-oil mixture using two different scenarios. For each, an was experiment conducted and explained extensively in the previous chapters. These two experiments had the same purpose, to separate oil from water, but they are different in the sense that the former one, which is the gravity separator test, dealt with large quantity of the mixture flowing in pipes and then it slowed down in a bigger space to give the gravity the chance to separate the oil. In the later electrolysis cell experiment, the solution was at rest and of a much smaller quantity compared to that in the gravity test. The separation occurred due to breaking the bonds of the water. Both findings are summarized briefly in this section.

6.1.1 Gravity Separator Test

In this experiment a statistical approach called Analysis of variance "ANOVA" was employed to interpret the data, as shown in detail in chapter four. The results showed that the water flow rate, oil flow rate, number of compartments and temperature contributed to the results in such a way that all of them decreased the water cut percentage, which is the water remain percentage after the separation process, except for the water flow rate, which acts differently. Moreover, interactions between the water flow rate and oil flow rate were found to contributes adversely to the water cut. Also, other factors interaction had a positive contribution to the water cut, including the oil flow rate with the number of

compartments, and the three factor interactions of water flow rate, oil flow rate, and the number of compartments. The shape of the inlet was found to have no contribution to the results at all. All of this information is summarized in one simple coded equation (4-10). This model was tested against real measurements and showed an error between (8%) and (20%).

Water Cut (%)
=
$$3.36 + 1.73A - 1.15B - 0.35C - 0.13D - 0.14AB$$
 (4-10)

where the coded factors can be (-1) for the low value or (+1) for the higher value as shown in chapter four table (4-1).

+ 0.042BC + 0.13ABC

Table (6-1) Experiment Variables

Factor	Symbol	High	High Value	Low Value	Low Value
		value	Code		Code
Water Flow	Α	4 gal/min	+1	1 gal/min	-1
Rate					
Oil Flow rate	В	4 gal/min	+1	1 gal/min	-1
No. of	С	2	+1	1	-1
Compartments					
Temperature	D	25 °C	+1	35 °C	-1
Type of Inlet	E	Deflector	+1	Elbow	-1
		Plate			

6.1.2 Experiment of Applying an Electrical Field

The electrolysis cell technique used for the second experiment is shown in chapter five. This experiment was conducted by investigating one factor at a time. Therefore, the interaction between the factors cannot be found and no expression is developed to describe the entire process, but each factor was tested at many points compared with the gravity separator experiment. From this experiment it was found that increasing the solution temperature, the voltage, or the amount of salt increased the oil recovery, which is the percentage of oil gained after the separation with respect to the original oil content in the solution. Also, it was found that the percentage of oil recovery increases with an increase in the pH of the solution, while it decreases by increasing the volumetric oil to water ratio.

6.2 Future Work

This study can be developed and used as a background for future work using both experiments. Some experiments were not conducted due to some limitations such as the quantity of oil available and range of flow meters.

For the gravity separation experiment, the range of levels can be widely expanded, especially the oil flow rate, the water flow rate, and the temperature. Also, different set-ups of runs with many levels of each factor can be conducted to check the curvature, which cannot be checked by using just two levels in a

very small range. Also, the separator length can be added as a factor and is expected to be a very important factor.

The one factor at a time experiment conducted for the electrolysis cell experiment did not investigate the factors interactions and did not combine them in one equation. The results of this study can be used to design a statistical experiment with different levels of each factor to find a formula collecting all the variables together to give a full picture of the event.

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