Shear Behaviour of Steel Fibre-Reinforced High Strength Lightweight Concrete Beams

Without Web Reinforcement

By

Fahad Alzahrani

Submitted to

The School of Graduate Studies

in partial fulfillment of the requirement for the degree of

Master of Engineering

Faculty of Engineering and Applied Science

Memorial University of Newfoundland

May 2018

St. John's Newfoundland Canada

Dedication

This thesis is dedicated to my beloved mother and my lovely wife for having sacrificed a lot for me.

Abstract

The main objective of this investigation was to determine the influence of adding two different shapes with different lengths of steel fibres on the shear behaviour of lightweight and normal weight concrete beams with normal and high concrete grades.

Thirty-six prisms of (100 mm wide, 100 mm deep, and 400 mm long) and seventy two cylindrical samples of (100 mm diameter × 200 mm high) were cast and tested to determine the concrete mechanical properties for specimens. These samples were tested in order to discover the role of steel fibres on enhancing concrete properties in general. The modulus of rupture, flexural toughness, toughness, compressive strength and splitting tensile strength were inspected based on the small-scaled material samples.

In the structural experiment, a group of twelve large-scaled reinforced concrete beams without shear reinforcement were primarily analyzed, designed and tested in the structures lab at Memorial University of Newfoundland (MUN). These specimens were built to study the load-deflection curves, shear and flexural behaviour, concrete and steel strains and the ultimate load resistance. Simply supported beams with dimensions of (200 mm wide, 400 deep, and 2900 mm long) were structurally tested, analyzed and discussed. in order to investigate the previous responses. Three factors were proposed in this experiment. The first factor was the type of the aggregates and the second parameter taken into consideration was the concrete compressive strength that divided the beams into two groups of high and normal strengths. Thirdly, two different lengths of steel fibres with different end-shapes were considered as the third variable in order to evaluate the effects of the length of the steel fibres on the shear behaviour. All beams contained 1.46% of longitudinal tension reinforcement ratio. Besides this, a fixed concrete cross

section was suggested for all beams. Testing specimens were setup on a specified constant shear span-to-depth ratio of 3. According to a recommendation by ACI, a fixed volume fraction of 0.75% of steel fibres was added to SFRC beams. The specimens with long fibres resisted higher shear stresses and were more ductile than the ones reinforced with shorter fibres. Overall, the presence of both short and long steel fibres improved beams shear resistance by a range varied from 35% to 72% compared to reference RC beams. However, shear strength of beams with long steel fibres enhanced more by an average amount of 10% in contrast with short SFs beams.

Acknowledgement

After thanking the generous God for giving me the patience and power to successfully accomplish my research, I would like to sincerely appreciate my supervisor, Dr. Amgad Hussein, for giving me valuable advice and sharing with me his vast knowledge during the entire period of the study. His untiring guidance and outstanding supervision have enabled me to complete this dissertation successfully.

I would also like to extend my gratitude to Mr. Shawn and Mr. Jason for helping me pour my samples and test them.

In addition, I would like to give my best regards to all the faculty members of Civil Engineering Department at the University. I also greatly appreciate their assistance given to me at various points of time during the study.

Finally, I offer my deepest appreciation to my friends, Majed Sager, Abdullah Ali and Hadi Alyami for their steady support and assistance during my research.

Table of Contents

List of Tables

List of Figures

List of Symbols

A_s	Reinforcement area, mm^2
A_c	Beam concrete cross section, mm^2
a	Shear span, mm
a/d	Shear span-to-depth ratio
b_w	Web width of beam, mm
d	Effective depth "distance from extreme compression fibre to centroid of tension reinforcement", mm
D_f	Diameter of the fibre, mm
E_c	Modulus of elasticity of concrete, MPa
E_f	Modulus of elasticity of longitudinal FRP reinforcement. MPa
f	Stress at any strain ε
f'_c	Concrete compressive strength, MPa
f_r	Modulus of rupture, MPa
f_{sp}	Splitting tensile strength, MPa
f_y	Yield strength of FRP reinforcement, MPa
F	Fibre factor, $F = V\,(L_f/D_f)\,\beta$
I	Cross sectional moment of inertia, mm^4
L	Span length, mm
L_f	Length of the fibre, mm
L_f/D_f	Aspect ratio of fibre
M_n	Nominal moment calculated based on equivalent stress block, kN.m
M_{cr}	Cracking moment calculated at first crack, kN.m

P_{cr}	Applied load indicated by the testing machine at first crack, kN
P_u	Ultimate applied load indicated by the testing machine at peak, kN
P_f	Applied load indicated by the testing machine at failure, kN
V_{cr}	Applied load indicated by the testing machine at first shear crack, kN
V_f	Fibre volume fraction
v_n	Nominal shear resistance calculated based on empirical equations, MPa
α_1	Ratio of average stress in rectangular compression block to the specified f'_c
β_1	Ratio of depth of rectangular compression block to depth of the neutral axis
β	Factor for fibre shape and concrete type
λ	Factor to account for concrete density based on types of aggregates
μ	Ductility ratio
ρ_s	Longitudinal reinforcement ratio, $\rho_s = A_s / bd$
ε	Strain at a given stress
ε_{cu}	The maximum compressive strain in concrete at failure = 0.0035
τ	Average interfacial bond stress of fibre matrix = 4.15 MPa
ACI	American Concrete Institute
CSA	Canadian Standard Association
EC2	Eurocode 2
SFRC	Steel Fibre Reinforced Concrete
LVDT	Linear Variable Differential Transformer
BSF	Brittle Shear Failure
DSF	Ductile Shear Failure
FF	Flexural Failure

Chapter One

Introduction

1.1 General

Quite often, numerous varieties of materials have been embedded in concrete mixtures in order to meet the exact requirements of reinforced concrete beams. Historically, straws and horsehair were commonly used for this purpose. Indeed, inclusion of these materials in reinforced concrete beams constituted the technology concerning the addition of fibres into the concrete mixes in the earlier times. As time passed by, the use of straws and horsehair in reinforced concrete beams gave way to the use of stronger materials such as steel. Initially, that is in the early years of the 20th century, steel was introduced into the reinforced concrete beams in the form of pins and spikes. Although the primary use of adding steel reinforcing fractional segments such as spikes and pins was reported in the first decade of the 20th century, the actual usage of manufactured steel fibres began only in 1963 when researchers included 64 mm long and 1 mm thick steel fibres in their experiments in order to understand the concrete behaviour. Over the years, this theory has been somewhat developed by virtue of the revolution in the civil engineering industries that lead to the production of several categories of fibres of different strengths, shapes and materials. These fibres differed from each other in their lengths, diameters and strengths. In the last decade of the 20th century, adding steel fibres into the reinforced concrete components effectively enhanced the ductility and shear resistance of the concrete beams. Several studies have been carried out in the recent times in this field to verify how a certain amount of steel fibres could enhance the behaviour of concrete structural members. Most reinforced concrete specimens tested by researchers have demonstrated that the addition of steel fibres enabled the concrete to resist higher shear stresses and changed its brittle mode of failure into a ductile form. Standard specifications and recommendations of steel fibres for fibres -reinforced concrete have been reported by ACI.

Even though steel fibres can be replaced by glass and synthetic fibres as a means to prevent the occurrence of corrosion in the reinforced concrete member, many studies have shown that steel fibres were more efficient than their synthetic counterparts in terms of the beam capacity and in particular against the shear stresses. Some studies claim that the manufacture and production of stiff hooked-end steel fibres with high tensile strengths might be the reason behind the positive effects of steel fibres in the recent times as compared to the past when only straight fibres with limited strengths were available. Appendix A includes the digital photographs of the specimens at failure.

1.2 Nature of the Problem

Since the use of steel fibres into reinforcing concrete beams has started, a minimum amount of steel fibres of 0.75% of concrete volume has been recommended by ACI. However, there is a lack of supportive research that suggest the most efficient and suitable types and length of steel fibres either with normal weight or lightweight concrete. Due to the fact that this field of study may still be an issue of concern in terms of studying the shear behaviour of steel fibre-reinforced concrete beams either with normal or high strength. Although some predictive models have been proposed by previous researchers in this area, they are still quite limited for special cases that might not fit all other types of beams with different shape and length steel fibres, higher compressive strength, different types of aggregates and different shear span to depth ratio.

1.3 Objectives and Scope of Research

The scope of this research is carrying out real large-scaled steel fibre-reinforced concrete beams in order to consider the size effect, to prove the importance of lengths and end-shape of steel fibres. The main objective of this investigation is to analyze and compare the performance of both steel fibre-reinforced beams either normal weight concrete or lightweight concrete, without the use of the conventional stirrups, against shear stresses. Steel fibres have been used in this experimental research in order to prove their efficiency against using the conventional web reinforcement. Although most codes recommend using minimum usual shear reinforcement, this research aims to satisfy all the requirements.

As was observed, most reviewed studies till date have been published on normal weight fibre reinforced concrete beams, whereas only a few investigations have been done on the lightweight fibre reinforced concrete beams without transverse reinforcement. Therefore, this research attempts to cover and link both types of materials with the same properties and at the same circumstances.

1.3.1 Experimental Plan

The major aspect covered in this study, with the aim of achieving the objectives mentioned previously, is the determination of the positive role of adding two different sizes of steel fibres into both the normal and high strength reinforced concrete beams (RCB), either normal weight or lightweight reinforced concrete beams, on the shear behaviour. Testing twelve specimens, fully controlled and entirely monitored under the four-point loading test i.e. “simply supported beams subjected to two concentrated symmetrical and vertical downward forces”, will help to generate sufficiently satisfactory evidence. Length of steel fibres, type of aggregates, and concrete compressive strength are the three investigation parameters. Many concrete trial mixes will be designed and tested after 28 days to select the proper mixtures for the experiments. A 5000 psi (35 MPa) is the target average concrete compressive strength at 28 days for normal strength concrete reinforced beams, whereas a 10000 psi (70 MPa) average concrete compressive strength is the target for the high strength reinforced concrete beams. On the other hand, the steel fibres volume fraction (V_f), and the shear span to depth ratio (a/d) will remain fixed at certain values for all samples. 0.75% of steel fibres of concrete volume are used while all specimens tested at a shear span-to-depth ratio of 3.

1.4 Thesis Organization

This thesis comprises of six chapters. The first chapter namely, Introduction, states the main topics and objectives of the research and touches upon the experimental plan and analytical program. The following aspects will be detailed in the subsequent chapters:

I. The general shear behaviour and the ductility of reinforced concrete beams without transverse reinforcement subjected to axial forces due to the addition of steel fibres.

II. The effects of the concrete compressive strength on the shear capacity.

III. The difference between adding either short or long steel fibres into the members.

Chapter 2 of the thesis namely, Literature Review, mentions and reviews the current codes and standards associated with using steel fibres and lightweight aggregates in structural members. Certain published papers concerning the shear behaviour of steel fibre reinforced normal weight and lightweight concrete beams have been reviewed in this chapter. This review has been done in order to consider the proposed mix designs, discussions and recommendations of these studies. Chapter 3 namely, Experimental Program, describes the material properties, geometry and specifications of specimens and explains in detail the setup of this experiment. The fourth chapter namely, Experimental Findings, puts forth a comparison between six steel fibre reinforced normal weight concrete beams and six steel fibre reinforced lightweight concrete beams in terms of propagation of cracks, shear resistance, and ductility. In addition, the load-deflection curve is obtained and examined in this chapter. Chapter 5 discusses and analyzes the experimental results and observations versus the theoretical and predictive values. Finally, Chapter 6 concludes the thesis by discussing the advantages of replacing conventional stirrups in steel fibre reinforced Concrete beams and steel fibre reinforced lightweight concrete beams with steel fibres. The topics of this research are presented in Figure 1.1.

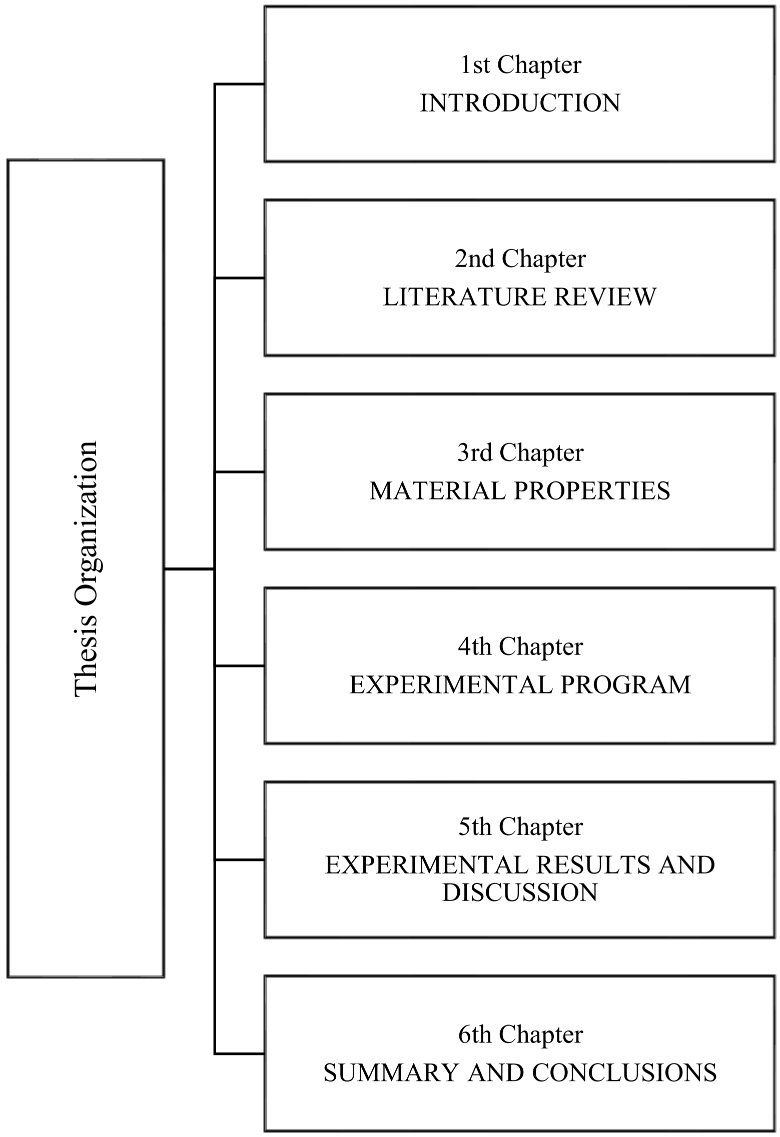


Figure 1.1: Thesis organization.

Chapter Two

Literature Review

2.1 General

Reinforced concrete beams without web reinforcement can relatively resist compressive stresses but barely resist tensile stresses and tend to suddenly fail in brittle mode. This weakness against tensile stresses can be overwhelmed using either stirrups or steel fibres. The inclusion of a sufficient amount of steel fibres with an adequate aspect ratio noticeably generally enhances the shear capacity of RC beams and may strongly replace the need for stirrups. The presence of steel fibres mainly transforms the brittle failure into a ductile form. In fact, most SFRC beams tend to fail in flexure. According to Chalioris (2013), Figure 2.2 below shows a typical prediction modes of failure and load-deflection responses of SFRC members subjected to four-point loading. The energy absorption, stiffness and ductility are also improved in SFRC beams. This is attributed to the additional strengths and energies required to fracture the concrete and pull out the fibres. This increases the post-cracking carrying loads and this feature extends the post-peak-descending curve in the load-deflection relationship in SFRC beams. However, the RC beams without shear reinforcement tend to show insignificant post-cracking ductility. Steel fibres generally improve the mechanical properties of concrete. To be more specific, concrete beams with steel fibres elastically deform especially under tension. Micro cracks initially start and macro cracks are then localized after the elastic response until a fracture occurs at low strains relatively. In most studies, the pre-cracking strength is not increased, while the most significant improvement due to using steel fibres is in the post-cracking stage. This can be mainly recognized across calculating the energy absorption and widths of cracks. The improvement of these characteristics mentioned previously is mainly attributed to the increase in the concrete tensile strength due to the presence of steel fibres. Steel fibres randomly distributed in the concrete improve residual strength, because they transfer tension stresses by bridging across sections with cracks. Steel fibres consistently demonstrate desirable reinforcement materials. Randomly-distributed and short, discrete steel fibres are shown to improve the tensile properties of the resultant composite; “especially the post-cracking behaviour”, Ning et al. (2015). Steel fibres consistently control crack propagation as mentioned before as well as increase toughness, stiffness, ultimate flexural strength while on the other hand reducing the deflection problems, Ning et al. (2015). Experiments that add knowledge to the topic of SFRC and cracking behaviour are necessary to improve structural safety. More researches in the field of steel fibre reinforced lightweight and normal weight concrete beams will be cited in the following sections.

An upgrade of the structural serviceability limit state is gained when sufficient amount and proper type of steel fibres are used. The serviceability limit state is a definition of the stage where the member is still able to structurally sustain and show strength and integrity to keep its desired role upon its planned service life span. The control of cracking is the only part to commonly assure the enhancement of the serviceability. Due to the presence of steel fibres, SFRC beams are not expected to exhibit large crack widths, Ning et al. (2015).

Double-hooked end long steel fibres have been structurally developed recently. This type of fibres has higher aspect ratio than the single hooked short fibres. Therefore, the longer the steel fibres with higher aspect ratio, the greater tensile strength and high modulus of elasticity shall be achieved. Therefore, better structural properties such as an enhancment of ductility, toughness, stiffness and post-cracking capacity are expected.

2.2 Shear Resistance Mechanism

After the formation of diagonal shear cracks in members without web reinforcement, Joint ASCE/ACI Committee 445 (1998) stated that the shear stresses in these beams without stirrups are resisted by a combination of five mechanisms shown in Figure 2.1. The resultant of the contribution from these five mechanisms is termed as the concrete shear resistance, V_c

(1) Shear resistance of un-cracked concrete compression zone, V_{cz}

(2) Vertical component, (V_{ay}), of the interface shear, (V_a) (aggregate interlock)

(3) Dowel force of longitudinal reinforcement, V_d

(4) Arching action, and

(5) Residual tensile stress across the cracks, f_t

2.3 Database Investigation on Steel Fibre-Reinforced Concrete Beams

A database of 147 FRC beams and 45 reference beams without fibres were presented by Parra-Montesinos (2006). Out of this database, 102 specimens with deformed steel fibres (hooked or crimped) were considered slender beams ($a/d \geq 2.5$). These beams consisted of a range of effective beam depth between 180 mm and 570 mm, a range of shear span to depth ratio between 1.0 and 6.0, a range of concrete compressive strength between 17.8 MPa and 103 MPa, and a range of fibre volume fraction varied from 0.25% to 2.0%. Furthermore, the flexural reinforcement ratio ranged between

0.37% and 4.58% while the aspect ratio of fibres varied from 50 to 100 and the tensile strength of these fibres ranged between 1000 MPa to 1240 MPa. The researcher presented the effect of steel fibres on the shear strength of beams. The author recommended that the required minimum shear reinforcement (stirrups or hoops) could be replaced by adding 0.75 % of deformed steel fibers in NWC beams. Therefore, 0.75% of steel fibres are selected in this research in order to check the efficiency of this recommended amount of steel fibres with LWC beams.

A comprehensive survey of the available literature on the shear behaviour of fibre reinforced concrete was presented by Cuenca (2015). The researcher presented an in-depth analysis of the influence of fibre reinforcement on shear behaviour of SFRC beams. The author summarized the effects of different parameters examined by previous researchers. As a result of this comprehensive study, Cuenca (2015) concluded that using a fibre volume fraction of 0.75% can be used as a substitution for minimum shear reinforcement for concrete beams.

2.4 High Strength Normal Weight Steel Fibre-Reinforced Concrete Beams

An experiment of 18 concrete beams were investigated by Ashour et al. (1992) . All specimens were simply supported beams subjected to a four-point load. The authors classified their beams into three series based on three factors (a/d, V_f, and ρ) in different levels. These beams generally consisted of a range of shear span to depth ratio between 1.0 and 6.0, a range of concrete compressive strength between 92 MPa and 101 MPa, and a range of fibre volume fraction varied from 0.5% to 1.0%. Furthermore, the flexural reinforcement ratio ranged between 0.37% and 4.58% while the aspect ratio of fibres was

75. The dimensions of all the specimens were constant and valued at 125 mm × 250 mm (d = 215 mm). The span lengths of beams were 930 mm, 1360 mm, 2220 mm, and 3080 mm. As a result of this study, the authors concluded that the mode of failure could be transformed to be more ductile particularly with larger *a/d*. Besides, the stiffness and shear capacity of beams increase when steel fibres are used. Increasing the level of fibre volume fraction from 0% to 1.5% enhance the shear strength by about 97% and 32% for short and slender beams, respectively.

Imam et al. (1994) studied sixteen high strength concrete beams with and without steel fibres in order to design two empirical expressions for the prediction of shear strength based on the addition of steel fibre. The dimensions of all the specimens were constant and valued at 200 mm × 350 mm. The span length of beams were and 3600 mm. All specimens were singly reinforced without stirrups. The author classified the beams into four groups based on three factors (a/d, V_f, and ρ) in different levels. These beams generally consisted of a range of shear span to the depth ratio between 1.75 and 4.5, concrete compressive strength between 108.5 MPa and 112 MPa, and fibre volume fraction varied from 0% to 0.75%. Furthermore, the flexural reinforcement ratio ranged between 1.87% and 3.08% while the aspect ratio of fibres used was 75 and the yield strength was 2000 MPa. As a result of this study, the author concluded that adding steel fibers increased the ultimate shear strength, and stiffness, while it decreased deflection of the beam. Moreover, use of fibres transformed the failure mode into a more ductile one. As the value of the span to depth ration decreases the shear strength is influenced by the anchorage condition and thus are not viable for support of the subject.

The research of Holschemacher et al. (2010) aimed to check how the properties of concrete beams with steel reinforcing depend on the fibres to prevent similar failures that are typically seen in concrete. Eighteen beams (150 mm × 150 mm × 700 mm) were cast with three different fibre contents, all with less than 1% of volume. Two different reinforcing bars (2 ϕ 6 mm and 2 ϕ 12 mm) and three types of fibres were selected: two straight fibre types with end-hooked having different ultimate tensile (1100 MPa) and one corrugated fibre type (2000 MPa). The fibres had an aspect ratio of 50. The concrete compressive strengths ranged between 67 MPa and 115 MPa. The maximum aggregate size of the concrete used was 32 mm, which was shown to have caused a reduction in the effectiveness of fibres within the failed cross sections. It was also observed that there is a dependence of the post-cracking load on the fibre content. Specimens with a fibre content of 60 kg/m^3 with longitudinal reinforcement failed in compression only. The authors concluded that for all selected fibre contents; a more ductile behaviour and greater load capacity in the post-cracking stage were achieved. Due to the fact that this study investigated such small-scale specimens to be structurally evaluated, a further research on the use of normal-strength fibre with end hooks in full-scale beams was recommended.

Jongvivatsakul et al. (2013) presented an examination of the shear carried by the fibers in fiber reinforced concrete, or FRC beams. Four beams out of eight specimens consisted of steel fibres (V_f = 0.5% or 1%) with different lengths (30 and 60 mm). The dimensions of the beams were according to the Japan Concrete Institute standards, and was valued at 100 × 100 × 400 mm and the shear span to effective depth ratio was 2.8, while the large-scaled specimens made with stirrups were 150 × 300 × 1800 mm at the same shear span-to-depth ratio. The specimens contained 2.7% of tension reinforcement bars and they had compressive strengths ranged between 46 and 62 MPa. Although the effective depth was quite satisfactory in order to be comparable to the current experiments, the researchers revealed lower ductility than the predicted observations due to using 0.5% of either short or long steel fibers. This might be related to testing these specimens under 2.8 shear span-to-depth ratio with small constant moment zone. However, one of the common observations with the current experiment was that the specimen with long steel fibers showed greater capacity than that with shorter ones. These beams consist of multiple types, and combinations, of fibers. A proposal was also presented to predict the shear capacity of the fibers. This was accomplished by performing tests on eight fiber reinforced concrete beams. The tests performed were four-point bending tests. Five types of fibers were used for the purpose of testing and verifying the tests. These fibers included 30 mm steel, 60 mm steel, polypropylene, polyvinyl alcohol, and polyethylene terephthalate fibers. Apart from the fibers themselves, hybrid fibers were also used for testing. There were three combinations, which were tested for hybrid fibers. Crack surface displacement and tension softening curves were also used for the investigation of the stress transferred through the diagonal crack. It was observed that the material type and the different combinations of fibers had a significant impact on the stress. It was also observed that the increase in the energy of fracture caused the angle of crack and stress to increase, while the length of the diagonal crack decreased with the increase in the energy of the fracture. The stress and the area of the surface of the crack were used for examining the shear in fibers. The calculated results suggested that the test results were satisfactory. One more task was completed by the researchers was the formulation of a relation for the estimation of shear. This equation was a function of the energy of fracture, stirrup reinforcement ratio, and the effective depth. The projection of the shear resistance of fiber reinforced concrete beams was also made possible through use of another derived relation. This equation also proved to be in good agreement with the experimental results for fiber reinforced concrete beams involving different combinations of fibers. The equation was also verified for results of multiple FRC beams.

An experimental study of 12 normal weight concrete beams (loaded in three-point bending) was conducted by Shoaib et al. (2014), on the size effect in shear for steel fibre-reinforced concrete beams without web reinforcement. The overall heights of the beams were 308, 600, and 1000 mm, while the shear span to effective depth ratio was set at 3. A width of 300 mm was used for all specimens. The effective depth was set between 258 mm and 920 mm. The reinforcement ratio ranged between 1.9 and 4.0%. SFRC specimens contained 1% of hooked-end steel fibres with an aspect ratio of 55 and tensile yield strength of 1100 MPA. The specimens had compressive strengths of 23, 41, and 80 MPa. The study was focused on examining the shear and flexural behaviours, response to ductility, and fracture. The key parameters used in the test were the concrete compressive strength, fibre content, and percentage of longitudinal reinforcement; shear span to effective depth ratio, and the number of transverse stirrups. End-hooked steel fibres were used while the volume fraction was varied between specific ranges. Trial outcomes exhibited that the integration of steel fibres upgraded the flexural and shear capacities along with other parameters. The integration of a minimum of 0.5% of fibre content in the concrete members while using stirrups altered the mode of fracture from brittle to ductile, while a minimum content of fibre of 1.0% was needed to accomplish the ductile response of the concrete members without the use of stirrups. Simple relations were also obtained by means of a curve fitting technique on the accessible trial data to forecast the shear capacities of medium to large flexural members with changing the fibre content. The tests conducted in the study included load-displacement response, ultimate load carrying capacity, displacement ductility response, crack propagation and failure mechanism, state of strain in tension reinforcement bars, and predictions were made using analytical techniques based on these results. The authors concluded that the shear capacity of SFRC beams was greater than the capacity of the corresponding reinforced reference concrete members. However, the normalized shear stress at failure decreased as the member depth increased. This was attributed to a size effect in shear occurs in SFRC beams without web reinforcement. The authors finally proposed modifications to the ACI 318-11 provisions for shear in SFRC beams.

Biolzi and Cattaneo (2015) studied the shear-flexure of 36 reinforced concrete beams with and without stirrups under a four-point bending test in order to determine the efficiency of the steel fibres on shear and flexure behaviours. The cross-section for beams were constant 150 mm $\times$300 mm while the spans were 240 mm, 290 mm, and 340 mm. All beams were classified based on the concrete compressive strength (normal versus high strength up to 100 MPa), shear span-to-depth ratio (1.5, 2.5, and 3.5), and the presence of web reinforcement (none, stirrups, steel fibres). Furthermore, all beams contained $2\phi 16$ mm longitudinal reinforcement and fibre volume fraction of 1%. Besides, the aspect ratio of fibres used ranged between 48 and 79 while the yield strengths varied between 1250 MPa and 2300 MPa. Beams without fibres or stirrups were initially tested in order to determine the brittleness of the beams. For concrete beams, it was observed that for short beams (*a/d* = 1.5) the moment increased due to the arch action. It was concluded that the inclusion of 1% of steel fibres by volume into high-performance concrete beams increased the shear, bending strength and ductility such that the beams failed due to the yield of the longitudinal reinforcement. Thus, the fibres tend to be more efficient than stirrups.

Gomes et al. (2017) evaluated the performance of 24 beams (140 mm × 260 mm × 2200 mm) under shear test. The flexural reinforcement was $4\phi 20$ mm for all specimens. Different doses of steel fibres were 0, 0.5, 1 and 2% by volume. The steel fibres used were called Dramix 5D 65/60 BG fibres, with non-deformable hook and ultra-high tensile strength. In order to assess the shear strength of the beams without stirrups, the 24 beams were subjected to a point load with a constant shear span to depth ratio of 2.7. The values of concrete compressive strength were up 70 MPa. The authors concluded that the shear strength significantly increased as the dosage of Dramix 5D 65/60 BG fibres was increased. The ductility was also enhanced in FRC beams. The average experimental shear strength was found to be considerably higher than the resistance estimates as provided by design codes. It was concluded from the study that building codes (RILEM, EHE, and Model Code 2010) are too conservative for the specimens and the materials based on the present study.

At the outset, the study by Kim et al. (2017) determined the impact that steel fibres have on the least shear reinforcement of high-strength concrete beams. A total number of 8 simply supported beams (260 mm × 400 mm × 4020 mm), subjected to transverse loading, were classified based on the presence and absence of fibres and stirrups. The specimens consisted of two reference reinforced concrete beams, two beams with steel fibres, two members were reinforced with stirrups, and two specimens had a web reinforcement of steel fibres and stirrups. The concrete compressive strength ranged from 21 MPa (normal strength) to 63 MPa (high strength), while the shear span-to-depth ratio was 4. Furthermore, all beams contained 1.72% of the longitudinal reinforcement and fibre volume fraction of 0.75%. Besides, the aspect ratio of fibres used was 60 while the yield strength was 1336 MPa. In fact, the study only tested one beam out of the two beams contained steel fibres (without stirrups) with high concrete compressive strength. Therefore, it is worth stating that quite a considerable number of specimens have to be used in order to establish with adequate certainty if the required test parameters do offer such a trend as to be used as a benchmark in other applications. The study utilized a length between loads (400 mm) made the beam to exhibit properties of the beam in short distance particularly for constant moment zone. The authors concluded that the fibre volume fraction of 0.75% significantly improved the shear capacity of high-strength concrete beams. They stated that the tension zone depth increased as the contribution of the steel fibers increased due to the change of concrete strength from 40 MPa to 60 MPa.

2.5 High Strength Lightweight Steel Fibre-Reinforced Concrete Beams

Tang et al. (2009) investigated the shear behaviour of 24 simply supported lightweight concrete beams without web reinforcement. These beams were subjected to four-point bending test. The dimensions of the beams used in the experiment included a beam width of 120 mm, a span length of 1500 mm, an overall height of 200 mm. The key variables of the study included compressive strengths of 23, 43, and 53 MPa and a shear span to effective depth ratio of 1.5, 2.0, 2.5, and 3.0. All beams had a reinforcement ratio of 1.33%. The shear behaviour of these beams was compared to the normal weight concrete beams (NWC). The results for the shear strengths of all the beams were recorded and later carefully examined against each other for the comparison between LWC and NWC beams. It was observed that normal weight concrete beams had their cracks going through the aggregates, while they really should have been going around them. It was also found that the load versus displacement curves of LWC beams were flatter as compared to the curves of NWC beams while keeping the strength same and at similar span to depth ratios. This may be explained through the consideration of the rigidity of beams. It was observed that there was not a significant difference in the crack shear strength of LWC and NWC beams. These computations were also in agreement with calculations of the ACI codes. The researchers concluded that the shear failure paths of LWC beams were found to be smoother as compared to the shear failure paths of the NWC beams, and that the slopes of the rising portion of the load versus displacement curves were flatter for LWC beams as compared to the curves of NWC beams with same strength and span to depth ratios.

Kang et al. (2011) presented a study that examined the shear behaviour of 12 steel fibre-reinforced lightweight concrete beams without stirrups. All specimens had a fixed cross-sectional size (125 mm × 250 mm) with different overall span lengths of 1600, 2000, and 2400 mm. The key variables that were tested in the study included the shear span to effective depth ratios of 2, 3, and 4, the volume fractions of steel fibres were 0, 0.5, and 0.75 percent, and the type of concrete including lightweight and normal weight concrete. The compressive strength of specimens ranged between 39.6 and 57.2 MPa. These beams were examined by subjecting them to four-point loads in order to examine the influence of steel fibres on the shear capacity of lightweight beams. Out of all the beams, some were lightweight concrete beams reinforced with steel fibres, while others were normal weight concrete beams reinforced with steel fibres. It was found, with the help of the results of the study, that the addition of steel fibres in a volume fraction of 0.75 percent enhanced the shear strength of the concrete beams by 30 percent and an enhancement in ductility of 5.3 or higher was also observed. The trial outcomes also suggested that the shear span to effective depth ratio influenced the shear capacity negatively. A few models to predict the shear capacity of steel fibre reinforced concrete beams were also assessed by the researchers. The already existing data from literature, along with the data from their own study, were used for this purpose. Studying the effect of variation of the different parameters, the comparison between the behaviour of the lightweight reinforced concrete beams and normal weight reinforced concrete beams led the researchers to build a relation for the prediction of the shear strength of steel fibre reinforced concrete beams, which was also found to be projecting close to accurate values for the shear strength of such beams.

2.6 Size Effect in Steel Fibre-Reinforced Concrete Beams (SFCB)

Minelli et al. (2013) conducted a research, which involved the examination of the influence of the size of nine reinforced concrete beams (loaded in three-point bending) on shear behaviour. No conventional shear reinforcement was used in any of the beams. The height of the beams, used in the experiment, lied within the range of 500 and 1500 mm. A shear span to effective depth ratio of 3 was used. The depths of the beams used were 500 mm, 1000 mm, and 1500 mm, while a same width of 250 mm was used for all beams. The compressive strength ranged between 33.1 and 38.7 MPa. The specimens contained flexural reinforcement bars of 8ϕ. 14, 8ϕ20, and 8ϕ24. SFRC beams contained different amounts of hooked-end steel fibres with an aspect ratio of 63 and tensile yield strength of 1100 MPA. A special consideration was given to the investigation of the size effect in concrete beams. It was concluded from the experiment results and observations that the size effect was substantially limited. It was found through the experiment that fibre reinforcement of concrete assisted in developing a single shear crack fracture. Since concrete is brittle, the ductilitys absent in the failure mechanism of the concrete beams without web reinforcement. However, this may be reformed through the integration of steel fibres to the beams. Since steel fibres are known to be ductile, the integration of steel fibres to the concrete beams also allows the concrete beams to become somewhat ductile as compared to the brittle nature of the reference concrete beams. The author finally stated that a relative low fibre volume fraction could significantly enhance the load-carrying capacity and ductility.

Shoaib et al. (2015) presented another study showing the shear behaviour of six lightweight concrete members reinforced with steel fibres without the use of stirrups. The overall heights of the beams ranged from 308 mm to 1000 mm. The overall depth of the specimens was below 300 mm. A shear span to effective depth ratio of 3.0 was used for all specimens. The compressive strength ranged between 22 and 31 MPa. The reinforcement ratio ranged between 1.9 and 4.0%. SFRC specimens contained 1% of hooked-end steel fibres with an aspect ratio of 55 and tensile yield strength of 1100 MPA. The researchers conducted their study in association with their previous research on normal weight SFRC beams in 2014. The researchers also noticed that the overall depth of the specimens that were reported, in the studies that were available, never went beyond

300 mm. This limitation prevented the prior researches and studies in the validation of the size effect in lightweight concrete beams. Therefore, the researchers conducted an experiment on multiple large sized lightweight steel fibres reinforced concrete beams without the use of stirrups. The overall height of the lightweight SFRC beams was within a specified range, while the shear span to effective depth ratio was also set at a specified value. The specimens were reinforced with a specified volume fraction of hooked-end steel fibres with varying ratios of reinforcement. The researchers observed during the experiment and from the results of the experiment that the shear strength decreased with an increase in the depth of the members. The authors concluded that the normalized shear stress at failure drops with a rise in the effective depth of the beam. This indicates the role of the size effect in shear for SFR lightweight concrete members without stirrups. The shear strengths predicted according to ACI 318-14 and CSA A23.3-14 codes were less than the observed experimental shear capacities. For lightweight SFRC beams, predictions of shear strength by the FIB MC2010 models were more accurate than predictions values by ACI318-14 and CSA A23.3-14 codes. This could be attributed to the direct consideration of the presence of steel fibres in predicting the shear strength for SFRC beams by FIB MC2010 models.

2.7 Analytical Prediction Models

A study was performed by Zhang et al. (2016) for the investigation and prediction of the shear strength of steel fibre reinforced concrete beams without the use of any stirrups. Since the beams used for testing were taken from different previous studies, the shear span to effective depth ratio varied from 1.0 to 6.0. Similarly, the values of beam width, beam span length, and beam depth also varied in a wide range and were not included in the study. A theoretical approach was suggested in this study, which was based on the modified compression field theory. The modified compression field theory could be used to predict the shear strength behaviour of SFRC beams without the use of stirrups. The tensile stress-strain constitutive equations were considered to be determining the tensile influence of the steel fibres in the overall tensile behaviour of the beams, and the constitutive equations were also developed to be accounting for the distribution of the steel fibres in the SFRC beams. A hundred-and-thirty-nine shear failure tests were used to verify the proposed theoretical model in this study. The type of beams that were used for the purpose of verifying the theoretical model included steel fibre reinforced concrete beams and reinforced concrete beams without stirrups. The study also examined and explained the contribution of fibre volume, concrete strength, longitudinal steel ratio, and shear span to effective depth ratio to the shear strength, which was theoretically predicted using the proposed model. The researchers concluded from the results of the study that the theoretical model, which was proposed in this study, was predicting the estimation of shear strength to a good estimate, and concluded that the model could be used to predict the shear strength of such beams accurately. Although this claim of the prediction of the shear strength of a particular type of beams is a bit overdoing, the predictions of the model were in good agreement with the selected beams from the literature.

Jain and Singh (2016) conducted another study for the examination of the capacity of deformed steel fibres in applications involving a shear force reinforcement requirement, which was minimum. The dimensions of the specimen were 150 mm × 300 mm and a span length of 1770 mm, while the simply supported span was 1470 mm. A shear span to effective depth ratio of 3.5 was used. This study was found to be stimulating, as it fixated on something, which was quite different as compared to all other studies in the same scope. An assessment of the ability of crimped and hooked-end steel fibres to be used as minimum shear reinforcement in RC beams prepared with two different grades of concrete was completed. To accomplish this, the control samples were made from the beams, which were believed to be satisfactory. The fibre-reinforced beams also showed fluctuating degrees of multiple cracking at ultimate loads. The shear strength of the FRC beams was found to be more than a low value endorsed in the literature. The grade of concrete was found to be of little importance in this regard. A comparison of the strength of the two types of deformed fibres revealed that the beams reinforced with the hooked-end fibres were found to have up to 38% higher shear strength than the crimped fibres. A simple model for shear strength was also suggested for the calculation of the behaviour of fibre reinforced concrete. The proposed model was tested along with seven other shear strength models. The seven models were selected from the literature. The proposed model predicted fairly good values. However, a model proposed by other researchers from the selected literature was found to be projecting a better approximation.

Imam et al. (1995) presented an analytical model for predicting the shear strength of reinforced high-strength concrete beams. The dimensions of all the specimens were constant and valued at 200 mm × 350 mm. All beams had span length of 3600 mm. All specimens were singly reinforced without stirrups. The author classified the beams into four groups based on three factors (a/d, V_f, and ρ) in different levels. These beams generally consisted of a range of shear span to the depth ratio between 1.75 and 4.5, concrete compressive strength between 108.5 MPa and 112 MPa, fibre volume fraction varied from 0% to 0.75%. Furthermore, the flexural reinforcement ratio ranged between 1.87% and 3.08% while the aspect ratio of fibres used was 75 and the yield strength was 2000 MPa. It was determined that the addition of steel fibres to high strength concrete beams increased the ultimate shear strength and transformed the behaviour from brittle failure into a ductile mode. This is influenced due to the effect of the relative beam size, along with the maximum aggregate size. By adding up to 1.0% by volume by weight of steel fibres to high strength concrete beams, the ultimate strength can be increased by 113%. While using the steel fibres as reinforcement, longitudinal bars do not have a considerable effect on the failure mode is thus dependent primarily on the stability of the flexural-shear crack. The model within the study indicates that with a reduction of beam strength the effect of the addition of steel fibres can increase the relative flexural capacity, which reduces the suddenness of failure. It was then suggested that the optimum fibre volume percentage increases as the steel ratio increases for an effective depth of 300 mm.

Imam et al. (1997) investigated the role of the fibres in increasing the beam strengths full flexural capacity in HSC beams. The shear resistance tends to increase the ultimate strength in order to reach the nominal flexural capacity. It was observed that for up to 140 MPa compressive strength, the effective depth could impact the reduction of the relative moment. At a critical shear span-to-depth ratio for a given beam, that shear effect can be obtained analytically such that the failure mode of a beam can be easily predicted. The empirical equations however, are valid only for values of shear span to depth ratios larger than 1.

Imam (2000) concluded that the average gain of the ultimate shear strength due to the addition of steel fibres varies from 14% to 141% depending on the span to depth ratio. Within the four modes of failure, the cracks on the reinforced concrete beams are closer in those with steel fibres than that in beams without fibres. This shows that the steel fibres redistribute the stresses beyond cracking. The conclusion of this work is that with an increase in the shear span to depth ratio, the shear problem of HSC beams can be related to the stability of the flexural-shear crack and the flexural failure.

Noghabai (2000) tested beams of high-strength concrete (90 MPa) for shear and bending where various types of fibres, up to 1% by volume, were used. It was determined that some beams performed better than others and this heavily was influenced by beam depth. For beams with an effective depth smaller than 300 mm the steel fibres did not increase the capacity, but over 500 mm the fibres did not increase the toughness of the beam enough. The volume of fibres used in the beam was highly dependable on the effective depth. For beams that were lower than 250 mm, 1% fibres by volumes caused compressive failure due to too many substitutes, while with depths over 700 mm the 1% by volume was not enough to increase the strength of the beam. This study focused on using a non-linear truss model in order to predict the carrying capacity of the beam loads. The author validated the ability of the model to predict the strength of the beams up to 700 mm effective depth, where after that capacity became harder to isolate.

Padmarajaiagh and Ramaswamy (2001) tested 13 beams (105 mm × 240 mm × 2200 mm) with a concrete compressive strength up to 65 MPa. The shear span to depth ratio used ranged from 1.97 to 2.35 for prestressed and fibre-reinforced beams. The experiment composed of using two different types of steel, 2-15M and 2-20M, while the fibre volume fractions of mild steel fibres (similar to hooked end with asepect ratio of 80) were 0, 0.5, 1.0 and 1.5%. An aggregate size of 12.5 mm was used. Analytical model using this equation was developed in order to predict the shear strength of prestressed high-strength concrete beams containing steel fibres. From this model, it was determined that beams that had fibres located only within the shear span over the full cross-section had load deformation responses compared to beams that had fibres the full length of the beam. It was concluded that the aggregate size influences the shear strength of the concrete significantly and that the empirical relation to their proposed equation was a better fit than that of other researchers. Fibers altered the failure mode from a brittle shear to a ductile flexure. Furthermore and without compromising the overall structural performance, this study stated that an equivalent amount of fibers could replace stirrups as minimum shear reinforcement. The authors concluded that nominal minimum shear reinforcement might be provided with fibers for safety.

Kwak et al. (2002) presented a study for the investigation of the effect of steel fibre reinforcement on the shear behaviour of twelve concrete beams (125 mm $\times$ 250 mm) with different spans of 1548, 1972, and 2396 mm. All beams were loaded in four-point bending test setup. The span-to-depth ratios were used as 2, 3, and 4. Three levels of fibre volume fractions were 0, 0.5, and 0.75% with an aspect ratio of 63 and a yield strength of 1079 MPa. All specimens had a flexural reinforcement of $2\phi 16$ (1.5% steel ratio). The results of the experiments showed that the shear strength was enhanced with the enhancement of the volume fractions of steel fibres. It was also observed the shear strength increased with a decrease in shear span to effective depth ratio, while the shear strength also increased when the compressive strength of concrete was increased. An enhancement of fibre content in the concrete exhibited an alteration of the mode of failure from shear to flexure. The examination of the models proved that the model proposed by the authors, along with one other model, predicted the shear strength of the beams with most accuracy. Therefore, the study may be considered a success because it produced a model for the accurate projection of shear resistance values. The study also proved to be successful in checking and validating the accuracy of the models, for estimating the shear behaviour of concrete beams, found in literature; and one of the models was also found to be predicting accurate values along with the model presented by the researchers themselves.

Test results of nine partially prestressed concrete T-beams, both with and without steel fibres, are presented in the paper published by Thomas and Ramaswamy (2006). The specimens had a flange width of 375 mm, a flange height of 100 mm, a web width of 150 mm and an overall height of 350 mm. In this analysis, three different strengths of concrete were used, up to 85 MPa, with a shear span to depth ratio of 2.65. An amount of 1.5% of steel fibres with an aspect ratio of 55 was used. The test used a control beam without fibres for each concrete type, coupled with two steel fibre reinforced beams with different fibre configurations. It was shown that for fibre reinforcement in web portioned beams improved the shear-resisting capacity. This was used in order to present a model to predict the test results of reinforced concrete beams over partial and full depths. It was found that the presence of shear resisting capacity increased by 11 to 20% with the addition of steel reinforced fibres. It is recommended that reinforcement of stirrups from the web to the flange be used for the reduction of the effects of cold joints.

An investigation of the shear behaviour of high strength concrete beams was also conducted by Tahenni et al. (2016). They also studied the effect of the addition of steel fibres to such beams. All the beams used for testing were of the same exact dimensions of 100 × 150 mm, and a span length of 900 mm, while the shear span to effective depth ratio of the beams was 2.2. The main testing parameters were set to be the volume fraction of steel fibres of 0, 0.5, 1.0, 2.0, and 3.0%; and the aspect ratios valued at 65 and 80. All specimens consisted of 1.16% longitudinal reinforcement ratio and they arched a high compressive strength up to 60 MPa. In fact, the specified dimensions used in their study do not show a realistic indication due ignoring the size effect by choosing such a small effective depth less than 300 mm. Nevertheless, the results showed that the long steel fibres enhance the structural behaviour more than the short fibres in terms of resistance and ductility. The overall result was the enhancement of the ductility in high strength concrete beams through the addition of steel fibres. It was also observed that the addition of a sufficient quantity of steel fibres to high strength concrete beams also mitigated the effect of cracking to a high degree. The ratio of shear span to effective depth holds a great value in the determination of the shear strength of fibre reinforced concrete beams. It is the most important factor that affects the shear behaviour of reinforced concrete beams. Other parameters that also affect the shear behaviour of RCC beams include compressive strength, the ratio of longitudinal reinforcement, the amount of aggregate, and the presence of transverse stirrups. All of these parameters pointed in the direction of the steel fibres contributing to the shear strength of reinforced concrete beams. This impact of steel fibres was examined both qualitatively and quantitatively, through the examination of twenty-four high strength reinforced concrete beams with steel fibres with and without the addition of stirrups. These beams were tested through four-point bending tests. The integration of stirrups in the reinforced concrete beams had little to no effect on their shear strength. The fibre-reinforced beams exhibited an increase in shear strength, and the width of the cracks formation was very narrow. A model was also proposed for the simulation of the effect of steel fibres on the shear strength of FRC beams, and the model was found to be much more effective as compared to other models found in literature. The authors concluded that the steel fibres did not affect the concrete compressive strength of HSC and the ascending portion of stess-strain curve also was not modified by the presence of steel fibres. The addition of steel fibres increased the ductility of HSC beams as the tensile strength of HSC increased by about 39%

Swamy et al. (1993) presented a study for the influence of steel fibres on the shear capacity of nine lightweight concrete I-beams with a span 3000 mm long. The beams with a compression flange of 295 mm × 52 mm, and a tension flange of 115 mm × 87 mm. The flexural reinforcing ratios were 1.6, 2.8 and 4.3%. The shear span to effective depth ratios was 2.0, 3.4, and 4.9. The range of fibre volume fraction was between 0 and 1%. Crimped steel fibres (100 aspect ratio) with an ultimate strength of 1560 MPa were used. A maximum size of 14 mm of coarse lightweight aggregates was used. The compressive strengths ranged between 40 and 45 MPa (obtained from testing 100 mm cubes). The authors researched the effectiveness of steel fibres in the enhancement of the shear capacity of the lightweight concrete beams when the steel fibres were used as the sole reinforcement materials. It was observed that the addition of steel fibres caused a few more positive effects on the behaviour of the concrete beams. These effects included a controlling of the dowel and shear cracking, a reduction in the spalling of the cover, assisting in the preservation of the ductility and the overall integrity of the concrete member. Another positive aspect of the addition of 1% of steel fibres was that the ultimate shear strength of the beams increased by about 60 to 210 percent, while the ductility increased by 20 to 150% compared to the equivalent beams with 0% of fibres. The authors also stated that the LWC beam deformations were reduced due to the addition of 1% steel fibres. Furthermore, the first cracking loads of flexure and shear, number of cracks with smaller width were greater in SFRC beams than in reference concrete counterparts. The presence of 1% of fibres also stopped the expected significant tensile splitting between the concrete and longitudinal reinforcing bars in shear-tension failure particularly. Moreover, the addition of steel fibres mitigated the fast propagation of cracks experienced in reference concrete beams. The researchers also proposed a truss model for projecting the ultimate shear resistance of the reinforced concrete beams. They also concluded that fibres played as shear reinforcement by transforming the shear failures into flexural failures for beams (containing 1.6 or 2.8% of tension reinforcing bars) tested at large shear span-to-depth ratios. Although steel fibres enhanced the cracking patterns, deformations and beam capacities, shear failure did occur prior to flexure. Upon inspection of the model, it was found that the model also predicted the values of shear resistance, which were acceptable for both normal weight and lightweight concrete beams.

2.8 American Concrete Institute (ACI)

The building code requirements for structural concrete and detailed commentaries against these codes are found in the document presented by the American Concrete

Institute (ACI 318-14). The first clause in the code for minimum shear reinforcement, i.e. 9.6.3.1, states that a minimum area of shear reinforcement will be delivered in all conditions except for the construction of the beam with steel fibre reinforced normal weight concrete, which is in conformance with the clauses 26.4.1.5.1(a), 26.4.2.2(d), and 26.12.5.1(a) while the compressive strength of concrete was less than or equal to 40 MPa. Clause 26.4.1.5.1(a) is concerned with the compliance requirements in steel fibre reinforcement, and states that the steel fibre reinforcement employed for shear resistance needs to satisfy conditions. The first condition is that the steel fibres must be deformed, and they also must conform to the standard set by the American Society for Testing and Materials. The second condition stated that the steel fibres must possess a length to diameter ratio of no less than 50, and no more than 100. Clause 26.4.2.2(d) was concerned with the design information of the concrete mixture requirements. The clause stated that the steel fibre reinforced concrete employed for shear resistance must comply with two conditions. The first condition was the conformance to a specific standard set by the American Society for Testing and Materials, while the second condition was the constraint of the mass of deformed steel fibres being more than a specified weight of concrete. Clause 26.12.5.1(a) is concerned with the compliance requirements for the acceptance of steel fibre reinforced concrete. The clause stated that the concrete used for shear resistance needs to comply with three requirements. The first requirement was the acceptance criteria for the compressive strength of standard cured specimens. The second requirement was that the residual strength after flexural testing according to a specific standard set by the American Society for Testing and Materials at a specified deflection at mid-span should be greater than a specific percentage of the first peak strength measured in a flexural test, and a specific percentage of a specific ratio of concrete compressive strength. The third requirement was that the residual strength after the flexural test according to a specific standard set by the American Society for Testing and Materials at a specified deflection at mid span was greater than a specified percentage of the first peak strength measured in a flexural test, and a specified percentage of a specific ratio of concrete compressive strength. It was inferred that deformed steel fibres might be used in place of stirrups. Finally, The ACI 318-14 Code partially allows using a fibre volume fraction of 0.75% of the volume of concrete composition (crimped or hooked fibers) for a compressive strength less than 40 MPa and beam depth less than 600 mm in order to replace the minimum shear reinforcement.

2.9 Concluding Remarks

It was observed in all publications, that the size effect of the beams or sections was not given adequate coverage. The beam dimensions used in this study were 200 mm in width, 400 mm in height, and 3200 mm in length. While a shear span to effective depth ratio of 3 was used. As compared to the beam dimensions used in this study, the beam dimensions used in most previous studies were relatively smaller. A large number of SFR normal concrete beams are available in the literature with structural beam height equals or less than 250 mm. However, the size effect was considered in this study, which was neglected in most of the previous research. Furthermore, the size effect was investigated in both lightweight and normal weight concrete beams, providing an opportunity to conduct research in the areas where it was never done before. In fact, very limited data for the use of steel fibres as shear reinforcement in lightweight concrete beams with high strength. And only one beam in the literature contained double-hooked end steel fibres with 60 mm long. See Table 2.1 that shows cases where $A_{v,min}$ is not required if $0.5\phi V_c < V_u \leq \phi Vc$.

From an in-depth study of the literature available regarding the effect of the use of steel fibres as reinforcements in concrete beams, it may be said that the use of steel fibres has a definite positive impact on the performance of concrete beams. First of all, the addition of steel fibres provides ductility to the otherwise brittle concrete beams, or members. Secondly, an increase in the shear strength, and flexural strength, of the concrete beams was also observed in several of the studies. Overall, the impact of steel fibres on the performance of concrete beams may be considered as a positive one, and the use of steel fibres for the reinforcement of concrete members may be recommended in view of all the advantages that they have to offer, while subsiding the very few conditional disadvantages that they might offer.

Table 2.1: Proportions of the used high strength mixtures [Adopted from ACI 318M-14].

Beam type	**Conditions**
Shallow depth	$h \leq 250$ mm
Constructed with steel fiber-reinforced normal weight concrete conforming to 26.4.1.5.1(a), 26.4.2.2(d), and 26.12.5.1(a) and with $f_c' \leq 40$ MPa	$h \leq 600$ mm and $V_u \leq \varphi 0.17 \ f_c' b_w d$

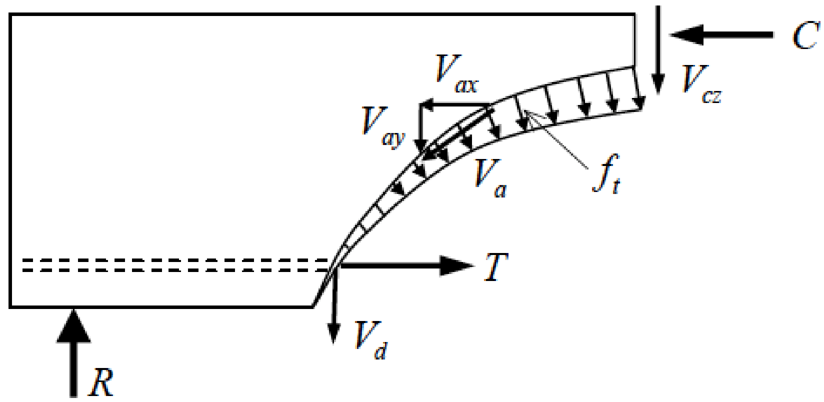


Figure 2.1: Typical responses of SFC elements under flexural–shear loading

[Adopted from Alam Md. S. (2010)].

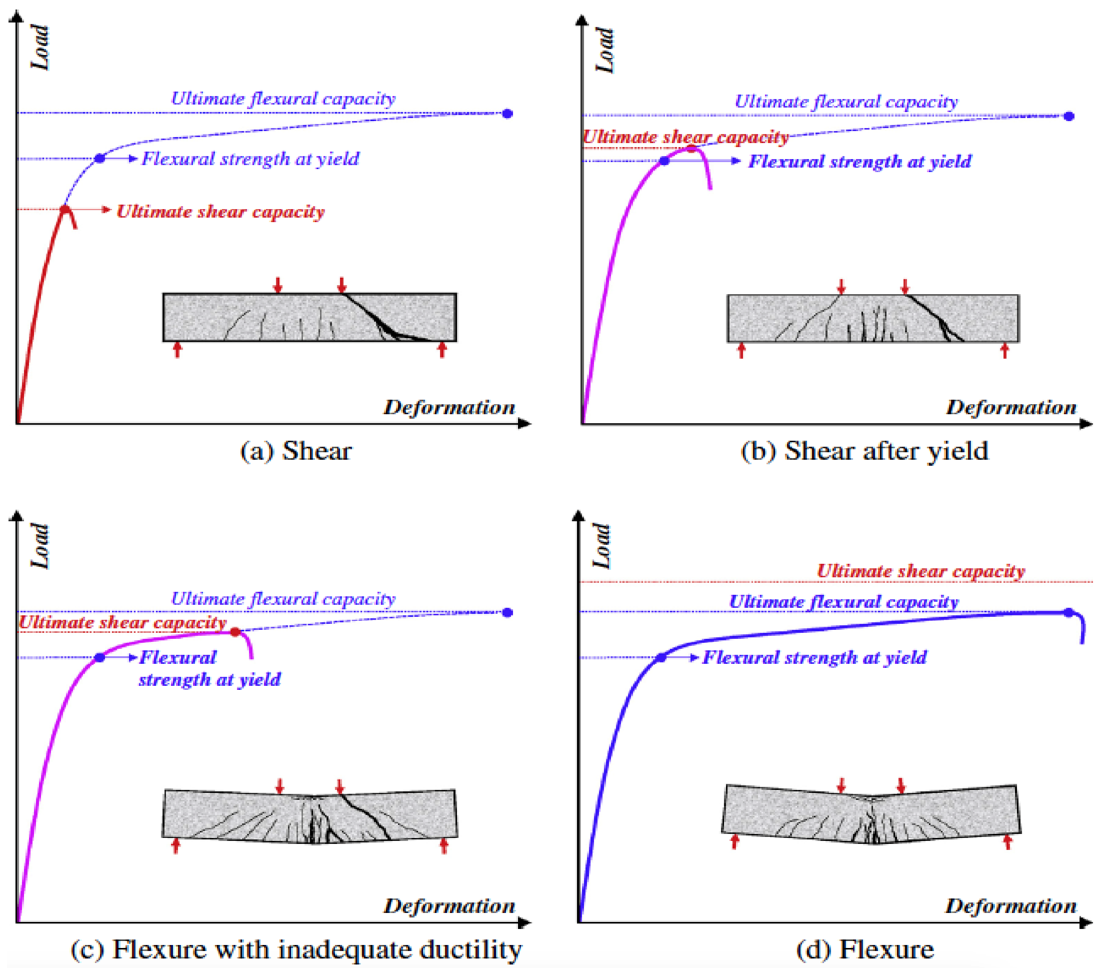


Figure 2.2: Internal forces in a cracked beam without stirrups [Adopted from Chalioris (2013)].

Chapter Three

Material Properties

3.1 Introduction

This chapter shows types of materials used in this investigation. Four different mixtures were designed based on type of aggregates, targeted concrete compressive strength and the type of steel fibres added to the mix. All these mixtures are presented in Table 3.1 and 3.2. The specifications of steel fibres are also listed in Table 3.1 according to the manufacturer. Material samples were tested and hardened material mechanical properties are listed in Table 3.3. Thirty-six prisms of (100 mm wide, 100 mm deep, 400 mm long) and seventy-two cylindrical samples of (100 mm diameter × 200 mm high) were cast and tested to determine the concrete mechanical properties for specimens. These samples were tested in order to discover the role of steel fibres in enhancing concrete properties in general. According to ASTM standards, the modulus of rupture, flexural toughness, toughness, compressive strength and splitting tensile strength were carried out from testing the small-scaled material samples.

Specimens have been leled and identified according to a simple manner that consider the first letter of the specimen ID based on the type aggregates either “N” for normal weight or “L” for lightweight, while the second letter stands for the compressive strength of the beam. That is, “N” represents the normal compressive strength while “H” letter represents the high compressive strength. Furthermore, the “B” letter stands for the word beam. More specifically and due to using a fixed amount of 0.75% of steel fibres, the number at the end of each specimen's ID explains the length of used steel fibres while beams without a number at the end have zero content of steel fibres.

3.2 Materials

3.2.1 Concrete

Four types of concrete were produced based on the type of aggregates and concrete compressive strength and types of steel fibres. Normal weight and lightweight concrete were designed to target 35 MPa and 70 MPa to represent normal and high strength specimens, respectively. Ordinary Portland general use cement was blended with all kinds of aggregates. A substance called superplasticizer was added to those beams with steel fibres in order to improve the workability of the mix. The mix with 0.75% fibres of short steel fibres showed better workability than mixtures with long fibres while pouring the beams. All of these materials were mixed and poured in the concrete laboratory at Memorial University of Newfoundland.

3.2.2 Steel Reinforcing Bars

Deformed steel reinforcing bars of 15M and 20M were used against the flexural moment. Besides, 10M deformed steel double-legged stirrups were placed to clamp the longitudinal bars within their detailed spacing. The 35M bars were cut and perpendicularly placed as spacers on the bottom reinforcing bars in order to hang the second reinforcing row and separate it from the first one. Testing both 20M and 15M steel reinforcing bars showed yield strengths of 440 MPa and 420 MPa, respectively. Therefore, average yield strength was taken as 430 MPa for both reinforcing bars.

3.2.3 Steel Fibres

Steel fibres with two different lengths and two different ends were brought to this research. Four specimens were cast with 35 mm long single hooked-end steel fibres, whereas four other specimens were poured with 60 mm long double hooked-end steel fibres.

Short single-hooked and long double-hooked steel fibres demonstrate different pull-out behaviours of the two used steel fibres. Long steel fibres showed greater tensile strength and this might be attributed to the length and the end-shape. Besides this, stress vs. strain curves showed that the tensile strength of long double-hooked fibres are higher than the single-hooked fibres with 35 mm long.

According to the manufacturer, testing the 3D (35 mm long and 0.55 mm in diameter) and 5D (60 mm long and 0.90 mm in diameter) steel fibres had tensile strengths of 1345 MPa and 2300 MPa and aspect ratios of 64 and 67, while the modulus of the elasticity values were 185000 MPa and 210000 MPa, respectively. Figure 3.1 illustrates the shapes of steel fibres used in this experiment.

3.3 Type of Concrete Mixtures

Four types of concrete mixtures were produced based on the database of a chain of trials conducted in this research. These types of mixtures were classified based on the type of aggregates and the type of steel fibres. The type of aggregates and length of steel fibres played a main role in the mixtures.

3.3.1 Normal Strength of Lightweight and Normal Weight Mixture

Six mixtures of the twelve specimens in this experiment were designed to conform to the specified normal strength concrete of 35 MPa. Half of these six beams were poured using 10 mm normal aggregate size with (0.50 water to cement ratio and 1.20 course to fine aggregate ratio). The other half of the beams contained the same aggregate size and quantities mentioned previously, but with lightweight aggregates and about 8% more retarders. Table 3.1 lists the quantities in kilograms per cubic meter for normal strength concrete of both lightweight and normal weight aggregates.

Some plain and fibre reinforced concrete trial mixtures, either with (0.40 or 0.45 water to cement ratios), showed high concrete compressive strength behaviour with less workability. Therefore, the main objective of using such a high water to cement ratio was to increase the workability and maintain the targeted range of specified normal strength. Although some studies claim that using high water to cement ratio might affect the durability of the concrete beam, this choice was fixed after executing many concrete trial mixtures. Four out of six normal strength beams contained a fixed amount of 0.75% of steel fibres, either short or long ones.

3.3.2 High Strength of Lightweight and Normal Weight Mixture

The remaining six specimens in this research were targeted to correspond to high strength concrete of 70 MPa. A maximum size of 10 mm diameter of both lightweight and normal weight aggregates were used in this group of specimens. A water to cement ratio of 0.35 and a coarse to fine aggregate ratio of 1.50 were chosen after many trial mixture runs in order to reach the specified high compressive strength. An additional amount, about 8%, of retarders was added to the lightweight concrete mixtures. Table 3.2 lists the quantities in kilograms per cubic meter for high strength concrete of both lightweight and normal weight aggregates.

Workability was negatively affected by the selected water to cement ratio of 0.35. Therefore, high rate water reducers “superplasticizer” liquid was used in the mixture to increase its viscosity by breaking the bonds among the components and enhancing the hydration between cement and water. Water to cement ratios of 0.30 and 0.28 were accomplished in a series of concrete trial batches to reach the specified high strength concrete. However, bad workability was observed at the end of those concrete mixture trials. Some researchers have stated that when less water to cement ratio is added to the concrete mixture, less hydration is achieved. A water to cement ratio of 0.35 was precisely chosen after conducting many concrete trial mixtures. Four specimens out of six high strength ones were reinforced with a fixed amount of 0.75% of short and long steel fibres.

3.4 Properties of Fresh Concrete

In order to assess the workability and consistency of unhardened concrete before casting, concrete slump cone test was carried out in this research. This test has always been recommended by ASTM to evaluate the workability of fresh concrete in its plastic state with aggregate sizes less than 37.5 mm. The cone of a slump test had a fairly smooth internal surface with a top diameter of 100 mm and a bottom diameter of 200 mm with a height of 300 mm. This cone was gradually tamped with three equivalent layers of fresh concrete. Each one-third was tamped and compacted 25 times by a metal roundedhead rod with 12 mm diameter. Strength of plastic fresh concrete against segregation was inspected by personal experience and it was obviously noticeable to the naked eye. This test was repeated with each single batch before pouring the whole amount of fresh concrete in the formwork of the specimens. All batches with a slump within the range from 19 mm to 22 mm were approved, whereas the other rejected batches were associated with either lesser or higher than specified slump. Steel fibres content did not play a negative role in concrete workability in this study since an amount recommended by ACI was used. Low levels of slumps mostly occurred in concrete batches with steel fibres. Although two different types of aggregates were used in this research, the workability was not markedly affected owing to the use of the same aggregate size. Since different water contents were used, normal strength concrete batches seemed more workable than those batches with high strength concrete before the use of the superplastisizers admixture. Higher concrete slumps were observed in plain concrete batches without steel fibres. Although short steel fibres seemed more workable than long fibres during pouring, the effect of types and length of steel fibres on workability was almost negligible due to the use of almost similar aspect ratios (L_f / D_f).

3.5 Properties of Hardened Concrete

3.5.1 Compressive Strength

A group of small-scaled cylindrical concrete samples of 100 mm diameter and a depth of 200 mm were prepared and cured for each specimen according to ASTM and were capped and tested at the same time as the testing of the large-scaled specimens. Concrete compressive and tensile strengths were mainly obtained by testing concrete cylinders. Some studies previously stated that steel fibres slightly enhance hardened concrete properties and ductility. Therefore, concrete properties were positively affected by the addition of steel fibres as has been mentioned in the chapter of literature review.

Concrete grade of reference lightweight beams was declined by about 2% to 5%. However, it could be observed that the presence of steel fibres upgraded lightweight concrete grade between 2% to 4%. Kang et al. (2011) reported that an increment of the compressive strength of SFRLC by 20% for beams containing $V_f = 0.75\%$. This might be an evidence of the higher efficiency of steel fibres with lightweight concrete compared to its efficiency in normal weight concrete, Kang et al. (2011). Although concrete grade of LHB60 went down by about 15% compared to the NHB60, this might be attributed to over rodding the cylindrical samples. In general, the slight upgrade of concrete grade might be more obvious in normal weight concrete with steel fibres. This could be highly expected and attributed to the lower rigidity of lightweight aggregate compared to the normal aggregates. Therefore, lightweight coarse aggregates might crush earlier than the normal weight aggregates under the same compressive loads. Guneyisi (2015) has observed that for lightweight aggregate reinforced concrete, the addition of steel fibres does not have significant influence on compressive strength.

In general, the concrete grade slightly improved by the addition of either short or long steel fibres. More specifically, short fibres improved concrete grade of lightweight concrete beams while the normal weight concrete compressive strength enhanced by the presence of longer fibres more. Overall, the slight upgrade of concrete grade might be more obvious in normal weight concrete with steel fibres. This could be highly expected and attributed to the lower rigidity of lightweight aggregate compared to the normal aggregates. Therefore, lightweight coarse aggregates might crush earlier than the normal weight aggregates under the same compressive loads. Guneyisi (2015) has detected that for lightweight aggregate reinforced concrete, the addition of steel fibres does not have significant influence on compressive strength. According to Kang et al. (2011), the observed compressive strength of SFRLC increased by about 20% for V_f = 0.75%. It was noticeable that the concrete compressive strength does not play a main role on either increasing or decreasing the stiffness of NWC beams. However, LHB specimen showed higher stiffness by an average amount of 23% compared to LNB member. This might be attributed to the fact that the paste of high concrete grade could usually be stiffer than the coarse aggregate itself. As a result, an average rate of 80% enhancement in the energy absorption was achieved when the concrete strength was increased from 35 MPa to 70 MPa, approximately.

3.5.2 Splitting Tensile Strength

According to ASTM standards, splitting tensile strength of hardened concrete were measured by applying a load along the cylindrical sample with a rate of 1333 N/s up to failure. It was considerably observed that the tensile strength of concrete increased with the presence of the steel fibres. The steel fibres formed more ductile cleavage in concrete samples. It was observed that the short fibres tended to be more efficient than long fibres in increasing the compressive strength while the long fibres were more effective than short ones in enhancing the tensile strength. See prisms in Figure 3.2 (a) and (b).

3.5.3 Modulus of Rupture

Concrete hardened prisms were cured and monitored under the same conditions as the cylindrical samples and they were both tested in the same day the beams were examined. Simple small-scaled beams (100 mm × 100 mm × 400 mm) with third-point loading setup were tested in order to measure the modulus of rupture according to ASTM standards. Table 3.3 shows the reinforcement ratio for flexures, fibre volume fraction, shear span-to-depth ratio, average values of concrete compressive, tensile strengths, and modulus of rupture in given units.

Table 3.1: Proportions of the normal strength mixtures.

Material	Normal strength normal weight properties		Normal strength lightweight properties	
	SG	(kg/m^3)	SG	(kg/m^3)
Cement (GU)	3.15	350	3.15	350
Coarse aggregates	2.60	985	-	-
Expanded slate 1/2" LWA (SSD)	-	-	1.53	580
Fine aggregates	2.60	821	2.60	821
Water	1.00	175	1.00	175
Steel fibres (0.75%)	7.85	59	7.85	59
Retarders	-	600 (ml/m^3)	-	650 (ml/m^3)

GU = Type GU Canadian Portland cement, similar to type 1 ASTM C150 cement (ASTM, 2012b).

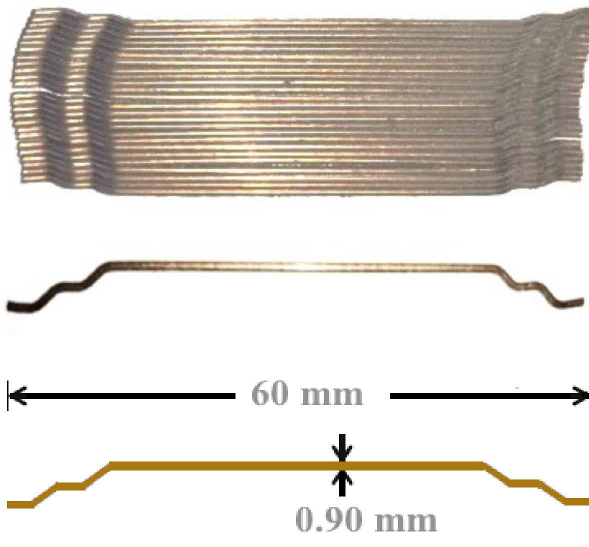
Table 3.2: Proportions of the used high strength mixtures.

Material	Normal strength normal weight properties		Normal strength lightweight properties	
	SG	(kg/m^3)	SG	(kg/m^3)
Cement (GU)	3.15	530	3.15	530
Silica fume	2.20	42	2.20	42
Coarse aggregates	2.60	946	-	-
Expanded slate 1/2" LWA (SSD)	-	-	1.53	556
Fine aggregates	2.60	630	2.60	630
Water	1.00	200	1.00	200
Steel fibres (0.75%)	7.85	59	7.85	59
Retarders	-	600 (ml/m^3)	-	650 (ml/m^3)
Superplastisizers	-	3200 (ml/m^3)	-	3500 (ml/m^3)

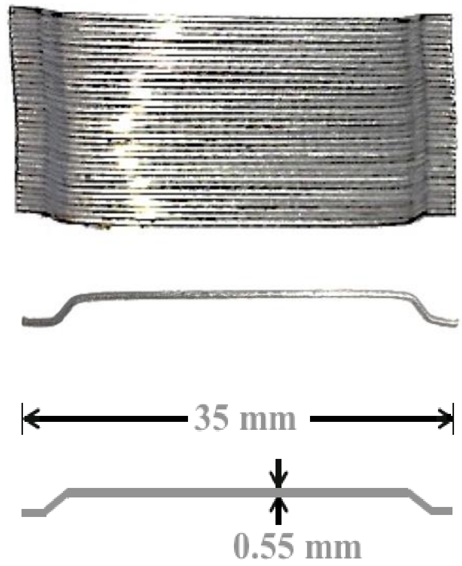
Table 3.3: General specimens details and material properties.

Specimen ID	Fibre volume V_f (%)	Fibre length L_f (mm)	a/d	Concrete compressive strength $f_c^{'}$ (MPa)	Splitting tensile strength f_{sp} (MPa)	Modulus of rupture f_r (MPa)
NNB	0.00	-	3	36.4	3.3	5.6
LNB	0.00	-	3	35.7	3.2	5.3
NHB	0.00	-	3	66.7	3.4	5.9
LHB	0.00	-	3	61.6	3.4	5.9
NNB35	0.75	35	3	35.5	4.4	6.8
LNB35	0.75	35	3	38.9	5.5	7.7
NHB35	0.75	35	3	63.2	4.5	7.1
LHB35	0.75	35	3	64.9	5.5	7.9
NNB60	0.75	60	3	37.3	7.5	10.1
LNB60	0.75	60	3	36.3	7.1	9.6
NHB60	0.75	60	3	73.0	7.7	10.9
LHB60	0.75	60	3	62.1	7.5	10.5

In all specimens, ρ_s = 1.46%.

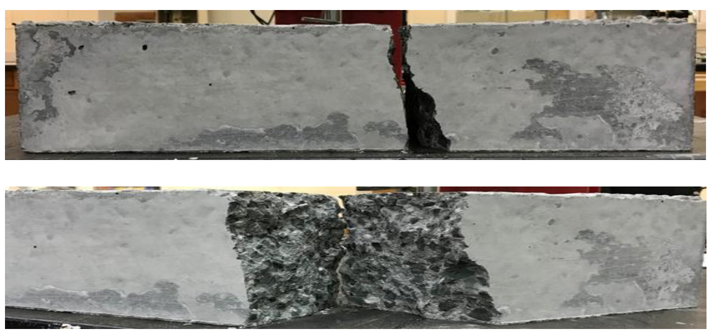


(a) Double-hooked end long fibres.



(b) Single-hooked end short fibres.

Figure 3.1: Steel fibres used in the study.



(a) Concrete without steel fibres.

(b) The bridging action of steel fibre in prisms.

Figure 3.2: Prisms at failure.

Chapter Four

Experimental Program

4.1 Introduction

The experimental program is detailed in this chapter. The experimental specimens, formwork fabrication, proportioning, mixing, casting and curing are described in the following sections of this chapter. Figures (4.3, 4.4, 4.5, and 4.6) show a typical formwork, reinforcement fabrication, casting and curing of a specimen, respectively. Twelve beams were cast in the concrete laboratory while testing them was in the structures lab at Memorial University of Newfoundland. All were cast and cured under the same environmental conditions. All beams were tested after a period of time ranged between 28 and 30 days from the casting date. For every single specimen, small-scaled samples were cured as the large-scaled beams were under curing. Small and large-scale specimens were tested, monitored and observations were recorded in the same testing day.

4.2 Test Specimens

Twelve beams (200 mm width, 400 mm height, and 3200 mm long) were cast, instrumented and tested, six were made with normal weight aggregate and the other six had lightweight aggregate. The beams were divided into three groups based on the presence and characteristics of fibres. The first group contained 0% steel fibres as reference beams, while the second and third groups had 0.75% steel fibres in two different lengths; 35 mm and 60 mm, respectively. Within each group, lightweight and normal weight aggregates were used to produce normal and high strength beams. Each specimen had a flexural reinforcement ratio of 1.46%. The design of the beams was in order to determine the influence of the length of steel fibre, types of aggregates, and concrete compressive strength on shear behaviour of reinforced concrete beams at a shear span-to-depth ratio of 3.

4.3 Formwork and Fabrication

Plywood sheets with a thickness of 25 mm was cut and assembled to host concrete mixtures in order to form this investigation's specimens. The plywood sheet had to be extended by an additional plywood sheet in order to cover the full span of the beam. The sides of the formwork were cut according to the overall height of each beam. The bottom side of the formwork had to be cut according to the width of the beam. A sufficient number of clamps and bracings were used to maintain the specified dimensions and geometry of the beams. The first layer of 20M bars was placed on concrete spacer to maintain uniform concrete clear cover while the second layer of 15M bars was placed on a small piece of 35M bars as a spacer. 3-10M stirrups were placed on each side in order to hold longitudinal bars together in their locations and to prevent anchorage failure.

4.4 Proportioning and Mixing

A portable electrical-powered drum concrete mixer with an unmixed capacity of 0.150 m^3 was used to produce the whole quantity of the required materials for all specimens in this study. A quantity of 0.256 m^3 of concrete was required for each specimen. Although two batches of 0.150 m^3 could have satisfied the amount of concrete required per beam, four batches had to be produced in order to maintain the efficiency rate of the mixer. That is, the low rate of mixer revolution of about 30 rpm played a major role in choosing a small mixing quantity. Therefore, a specific short period of time had to be set and monitored for each batch to maintain the consistency of mixture and to construct homogenous concrete in order to preserve uniform compressive concrete strengths. All concrete construction materials were proportioned by using scoops and loaded in separate buckets. Each bucket filled with a concrete portion had to be manually lifted and unloaded into the mixer just before blending them together. Coarse aggregates had to be first mixed with steel fibres in order to disperse and disband them uniformly before adding the remaining binder proportions. Volumes of chemical admixtures such as retarders and superplastisizers were measured by using a glass-measuring jug and then added to the bucket with water.

4.5 Pouring and Curing

Pouring stage started with the casting and curing of normal and high strength reference concrete specimens with normal weight aggregates. Buckets and trays were directly filled and used to transport and pour concrete mixtures into the beams' formworks. Concrete mixtures were uniformly cast, distributed, compacted and tapped layer by layer using an electrical-powered vibrator with a diameter head of 35 mm to avoid undesirable concrete honeycombs and segregations especially in the presence of steel fibres and low water content. Finishing steel trowels were used to polish and level the top surfaces of the beams to make them smooth for gluing concrete strain and crack concrete gages. After casting, plastic rolls were placed to cover the specimens and the sampling cylinders for 28 days for curing purpose in order to maintain the water content.

The other four normal weight concrete beams were cast right after testing and analyzing the reference normal weight concrete beams. Same casting procedures were repeated with the entire remaining lightweight concrete beams except that wet burlap rolls were used for curing normal weight concrete beams. All specimens were demolded after 7 days in their formworks and kept for curing for about a month.

4.6 Test Setup

The specimens were tested in the structures laboratory at Memorial University of Newfoundland. Two vertical columns of W310 × 107 sections form the setup frame. The columns are braced with two C310 × 45 sections on both sides. 15 mm thick plates are used to stiffen the bottoms side of the columns. These columns are supported on two 20 mm thick plates to avoid any possible bending. Besides this, bolts with 40 mm diameter are used to mount the columns on 1000 mm thick reinforced structural floor. Due to the fact that as the front column will experience tensile force, the bottom side of this column is braced with two 152 mm × 152 mm angles in order to distribute the loads and to simplify the use of four additional bolts. The actuator is supported with two C460 × 86 sections joined to the columns. In order to avoid the warping of the flange, two plates of 15 mm thick are used to stiffen both channels. For producing more stiffness to these channels, they are attached to each other using horizontal plates on both top and bottom sides.

4.7 Instrumentations

Three Linear Variable Differential Transformers (LVDTs) were placed at three different locations underneath the specimen. These LVDTs were located in order to measure the deflections of the beam at the centre of the beam and at the projection of concentrated loads. These LVDTs were also used to check the symmetry of the loading on the beam. The mid-span displacement measurements from the built-in LVDT of the hydraulic actuator were also recorded as a ram deflection in millimeters.

Strain gauges with 10 mm long were used for all specimens. The resistance of the strain gauges was 120 Ω with a gauge factor of 2.07 ± 0.5%. Six electrical resistance strain gauges were instrumented during the test on each specimen as shown in Figure 4.1. Four internal electrical strain gauges were placed on the reinforcement at four different locations in order to determine steel strains. Two out of these strain gages were located at mid-span of longitudinal bars while the remaining two gages were placed at east and west mid-shear spans. To be more specific, one gauge was placed at the center of each shear span. On the other hand, two external strain gauges were placed at the top mid-span of the beam. These strain gauges were placed on compression surface of the beam at mid-span in order to measure concrete strains. The strain gauges were glued on the outer bars in order to avoid hitting them while vibrating the concrete during poring. Strain gauges were covered by protective sealant and then shielded with a shrink tube waxed in order to avoid reaching the water to them during pouring. The concrete surface was grinded by an electrical grinder and then a very thin layer of epoxy paste was placed right in the middle on the top surface of the beam. Two concrete strain gages were placed on top mid-span of each specimen.

4.8 Test Procedures

A load increment of about 9 kN was gradually applied and maintained for all specimens until the observation of the first flexural cracks. Then, the first cracks were marked and measured using a microscope with 50 divisions of 0.02 mm width per division. The loading increment was almost doubled after marking the first cracks. Concrete cracks were traced, marked and measured as the loads increased. For safety purposes, marking and measuring cracks process was stopped at approximately 75% of the predicted load capacity of the specimen. This was assigned in order to ensure that the crack widths were measured at the serviceability limit state, which is 40% of the calculated ultimate resistance. Subsequently, the test was stopped and recorded data was saved.

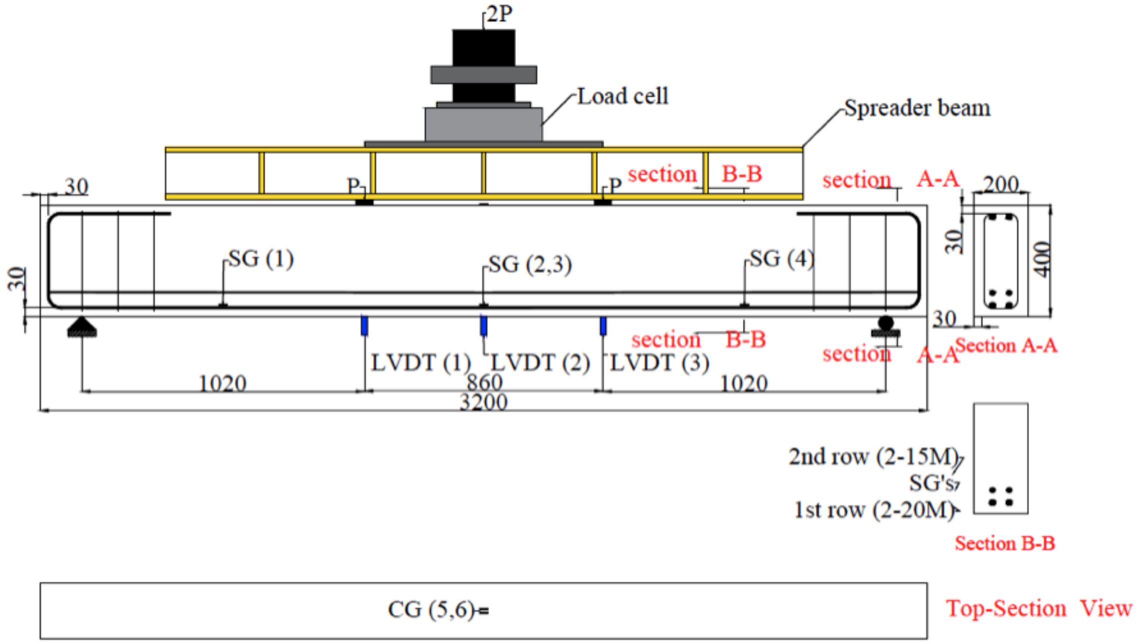


Figure 4.1: Typical detailed specimen and setup.

Figure 4.2: Specimens during loading.

Figure 4.3. Preparation of formwork.

Figure 4.4. Fabrication of steel reinforcement.

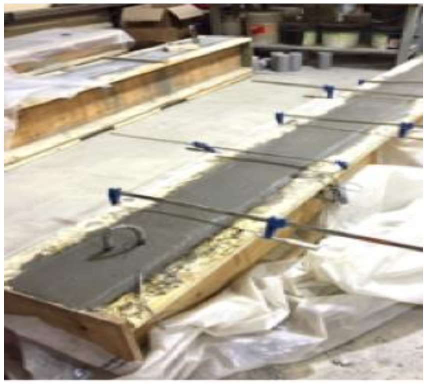


Figure 4.5. Pouring a specimen.

Figure 4.6. Curing after casting.

Chapter Five

Experimental Results and Discussions

5.1 Introduction

This chapter presents the test results of the beams. The details of the beams are mentioned in chapter three of the thesis. Twelve beams were cast, instrumented and tested, six were made with normal weight aggregate and the other six had lightweight aggregate. The beams were divided into three groups based on the presence and characteristics of fibres. The first group contained 0% steel fibres as reference beams, while the second and third groups had 0.75% steel fibres in two different lengths; 35 mm and 60 mm, respectively. Within each group, lightweight and normal weight aggregates were used to produce normal and high strength beams. Each specimen had a flexural reinforcement ratio of 1.46%.

The results are presented in this chapter in terms of load-deflection responses, mode of failure, crack patterns, concrete and steel strains.

Due to the large amount of experimental data, only a few representative results are used in the presentation.

5.2 Load-Deflection Behaviour

The deflections were measured using LVDT's as detailed previously in section 4.7. Figure 5.1 shows a typical load versus deflection curves obtained from the three LVDTs. The deflections from the LVDTs located underneath the load points were almost identical. This indicated that the load on the beams was symmetric. The reaction on each support is equal to one-half the actuator load. Hence, the shear force is equal to one-half of the total applied load by the actuator. Thus, this load, in kN, is used in the discussion and analysis of test results.

The load-deflection curves for the specimens failed in flexure could be idealized into three stages. At early loads, the beam is un-cracked and behaves in a linear elastic manner. After the formation of the first crack, and as the load is increased, several hairline cracks start to appear and propagate. This stage ends when the load-deflection curve starts to change slope. The slope of the curve is the un-cracked stiffness of the beam. Stage 2 starts at the end of stage 1 and continuous up to the yield point of the beam. Straight lines could represent both stages. With further increase in the load, the curve horizontally extends and goes into plastic state till failure, which represents stage 3.

Figure 5.2 shows the load versus deflection curves for three typical modes of failure: brittle shear, ductile shear and flexure shear.

All the reference specimens, without steel fibres, failed in brittle shear. A sudden drop in the load occurred when the beams reached their shear capacity.

It should be noticed in Figure 5.6 that NHB60 specimen was poured and tested twice in order to check and confirm the accuracy of the testing setup and procedures. As a result and as expected, NHB60-2 beam showed almost similar behaviour and results as NHB60 (the original beam used in calculation and discussion).

5.2.1 Effect of Types of Aggregates

The beams in Group 2 contained 35 mm long fibres. The lightweight aggregate beams showed lower capacity with higher deformation, in terms of ultimate displacement, compared to the normal weight specimens. For example, beams LNB35 and NNB35 had a capacity of 146.1 kN and 155.7 kN, respectively and deflection values of 38 mm and 17 mm, respectively, at failure. A similar trend was also observed for high strength concrete beams LHB35 and NHB35.

The specimens NNB35 and NHB35 with normal weight aggregates failed in ductile shear mode and hence they did not develop adequate ductility.

The beams in Group 3 with double-hooked end 60 mm fibres showed different trends than those specimens in Group 2. The normal weight beams in this group showed more ductility and higher capacity than the lightweight concrete specimens. For example, LNB60 and NNB60 showed a capacity of 164.1 kN and 165.5 kN, respectively and deformation values of 32 mm and 42 mm, respectively, at failure. Furthermore, LHB60 and NHB60 specimens with high strength concrete also showed the same behavior.

5.2.2 Effect of Length of Fibres

In general, the double-hooked end long fibres showed an enhancement in the load carrying capacity and deflection at ultimate load for beams made with different type of aggregates and concrete strengths.

This is evident by comparing the load and deflection of the corresponding beams LNB35 vs. LNB60, NNB35 vs. NNB60, LHB35 vs. LHB60, and NHB35 vs. NHB60. See Figures 5.4 and 5.5.

Steel fibres of 60 mm long enhanced deflection at peak for all members by an average of 45% more compared to deflections of beams with short fibres. The deflections at failure for all beams with long steel fibres ranged between (7% and 61%) except LNB60 deflection that dropped by 23% in contrast with the NNB60 member displacement. This might be attributed to weak interlock resistance across lightweight concrete aggregates. The higher number of short steel fibres might mitigate the interlock weakness in normal strength lightweight concrete beams. On the other hand, lower number of long steel fibres in normal concrete grade with lightweight aggregate may arise lead to serious interlock failure. Besides all of these values mentioned previously, long steel fibre thus developed the deflection at the serviceability limit state (service load is taken as 40% of ultimate shear force) by about 3% and 13% in both normal weight and lightweight concrete beams, respectively. Consequently, load capacities of beams containing long steel fibres were greater than beams reinforced with short steel fibres by an average amount of 10%. This might be attributed to the higher bond due to the longer length, which leads to a higher aspect ratio and double-hooked end form of those steel fibres. This was a good agreement with Tahenni et al. (2016) who showed in one of their specimen with steel fibres of 60 mm long failed at a greater load than the ultimate load of the beam with short fibres.

5.2.3 Effect of Concrete Compressive Strength

Overall, the concrete compressive strength had a noticeable effect on the capacity and deflection of SFRC beams. The HSC beams in Group 2, with 35 mm fibres, showed higher resistance and deformations compared to the normal strength specimens. For example, LNB35 and LHB35 had a capacity of 146.1 kN and 169.4 kN, respectively, and deflection values of 38 mm and 50 mm, respectively, at failure. Similar behavior was noticed when the capacity and deflection values of NHB35 are compared to NNB35 values. Furthermore, similar trend was found for high strength specimens with double-hooked end long fibres. For example, LNB60 and LHB60 showed a resistance of 164.1 kN and 178.9 kN, respectively, and deflection values of 32 mm and 55 mm, respectively, at failure.

As a result concrete strength increased the load capacity for reference concrete beams by an average rate of 70%, higher concrete strength achieves, greater load capacity gains regardless of the type of aggregates. Besides, both the higher strength normal weight and lightweight reference concrete beams showed an improvement rate of 38%, approximately, in term of deflection at a given peak load. Ultimate load resistance increased by an approximate rate of 16% for all types of concrete beams with steel fibres. NHB60 beam deflection increased by about 10% compared to NNB60 sample, while NHB35 dramatically showed an increase by 58% in its deflection in contrary with NNB35 specimen. Furthermore, in lightweight beams with either length of steel fibres was used, deflection developed by about 20% in average compared to the same beams with lower concrete grade. Thus, it was obviously detected that the concrete compressive strength highly improved the load capacity and increased deflections of specimens.

5.3 Load-Strain Behaviour

Typical curves of the subjected load versus reinforcement steel and concrete strains for the two beams that failed in ductile shear and the beams failed in flexure are shown in Figure 5.7, 5.8, 5.9, and 5.10. Linearity and elasticity of both concrete and steel strains were expected to appear as the specimens were under loading until first cracks were occurred. However, steel strains developed faster than concrete strains and this might be attributed to the high tensile stresses in the extreme extension fibre against the small compressive stresses on the extreme compression fibre. To be more specific, steel strains were inconsiderable at that first linear stage. As the load was gradually increased and after beams were cracked, mid-span strains of reinforcing rebar rapidly increased as the cracks were opened and expanded within the constant moment zone while strains at mid-shear span were slightly lower. This particularly happened in reference concrete beams when cracks initially developed rapidly within the constant moment zone while there was almost no considerable cracks within shear span zones. On the other hand in beams with steel fibres, strains either in the east or west mid-shear span showed similar trends as the mid-span in beams without steel fibres. Although it was expected that the mid-shear span strains would increase faster than the mid-span strain as what happened in reference concrete beams, they kept developing approximately similar values and this might be attributed to the addition of steel fibres that led to a uniform propagation of cracks and symmetric flexural mode of failure. Loads went down as the cracks grew and highly opened in a short period of time and this was an obvious indication for beams failed in shear.

In general, concrete strains showed fluctuated behaviour from the beginning of the test up to collapsing of beams. To be more specific, the maximum concrete strains at failure are show in Table 5.3. The beams that failed in shear before reaching the flexural capacity have shown lower concrete strains at failure while maximum concrete strains of those specimens failed in flexure have exceeded 0.0035.

5.4 Crack Patterns

The propagation of cracks was monitored and marked after every load step for all beams. This was carried out to classify flexural cracks from the diagonal shear cracks in order to define the general mode of failure and crack patterns of the beams. As expected, vertical hairlines cracks initiated from the extreme tension fibre upward towards the neutral axis within the constant moment. At the end of each load step, a small horizontal dash was drawn at the end of each single crack as the load was written right beside this dash in kips. As the applied load was increased, the vertical flexural cracks propagated. Flexural cracks also appeared within the shear span on each side in all specimens. Within the shear spans as the load was increased, the vertical cracks started to propagate in an inclined manner towards the loading points. The reason behind this cracking pattern was the increase of shear stresses. The start of the inclined cracks was considered to be the first shear-cracking load. Some shear cracks near the supports formed a wide diagonal crack. These diagonal shear cracks kept developing in a short period of time to suddenly and fail the beam in shear. Although some beams have developed diagonal cracks, those beams have not failed in shear due to their higher shear capacity that were produced by the addition of steel fibres. Beams failed in shear mode of failure showed wider diagonal shear cracks than those diagonal cracks have been developed in beams failed in flexure mode of failure. However, these diagonal cracks were limited and controlled by the addition of steel fibres until the concrete was crushed at the top of the beam within the constant moment zone. Due to the addition of steel fibres into concrete beams, an increased number of cracks with smaller widths developed, which is in a good agreement with Swamy et al. (1993). Tang et al. (2009) observed that a brittle mode of failure tends to occur to LWC beams without web reinforcement; for beam without shear reinforcement or steel fibres, the higher the concrete compressive strength, the greater the brittleness develops at failure. This phenomena should be noted obviously in LHB as the beam extremely fractured within the diagonal shear crack.

According to ASCE-ACI Committee 426 (1973), the inclined flexural cracks develop to eventually form a diagonal tension crack in beams with shear span-to-depth ratio between 2.5 and 6.0. This was obviously observed for beams failed in shear. The values shown in crack patterns Figures 5.11, 5.12, and 5.13 at each crack tip represent the actuator load in kilo-pounds (kips). This actuator load was considered twice the value of the load at each loading point. Table 5.1 shows the experimental crack width and spacing at service loads and failure.

In general, the slope of diagonal cracks was higher in NWC specimens than in LWC beams regardless of the presence of steel fibres. Even though the slope of the inclined crack increased as the long steel fibres were used in the normal weight concrete beams either with high or with normal concrete compressive strength, there was no significant trend of the slope of the inclined crack on the lightweight specimens group. The higher inclination of diagonal cracks in beams with long steel fibres might be attributed to the double-hooked end shape and the length itself. The anchorage resistance is expected to increase with the use of such double-hooked long steel fibres with high aspect ratio. On the other side, the normal weight beams failed in flexure showed greater inclination of diagonal cracks while lightweight beams failed in flexure showed almost similar slope of inclined cracks. This might be due to the weakness of the lightweight aggregates interlock compared to the interlock across the normal weight aggregates. In other word, the inclined cracks in normal weight beams could not cross through the aggregates which would have led the crack to take longer and curved path to continue upwards. Tang et al. (2009) reported that the cracks would usually rather penetrate through the coarse aggregate particles than around them in LWC beams. “The reason is the interlocking of the cement paste onto the rough surface pores of the LWA and thus improving bond strength between aggregate surfaces and cement paste”. On the other hand and in the lightweight beams, inclined cracks do not face boundaries to pass through the lightweight aggregates and cut them apart which would have made their inclined path softer upward without curvature. The shear cracks of LWC beams are smoother than those of NWC beams, Tang et al. (2009).

5.5 Failure Modes

Three case scenarios of failure of specimens conducted in this research were abbreviated and listed in Table 5.3. BSF abbreviation stands for brittle shear failure while DSF symbolized beams failed in ductile shear failure. Those beams failed in ductile shear-tension failure showed some flexure resistance with inadequate ductility. Furthermore, FF code stands for ductile flexural-compression failure for all specimens failed in flexure before even reaching their shear capacity. All modes of failure showed a good agreement with Chalioris (2013). Photographs of specimen at failure are shown in Appendix A. Beams generally showed regular types of failure as expected except NNB35 and NHB35 specimens, which failed in ductile shear-tension failure. As shown in Figure 5.12, the first scenario of failure was pure diagonal shear-tension failure that is obviously attributed to the absence of shear reinforcement. A brittle manner of failure occurred for all reference concrete beams (NNB, NHB, LNB, and LHB) regardless of types of aggregates and concrete compressive strength. Tang et al. (2009) stated that the lightweight concrete beams without web reinforcement tend to fail in shear. On the other hand and as presented in Figures 5.12, and 5.13, the flexure scenario of failure had to happen due to presence of steel fibres in NNB60, NHB60, LNB35, LHB35, LHB35, and LHB60 specimens. This was attributed to the enhancement of the shear capacity for those members with steel fibres. Although two specimens of normal weight concrete beams contained 0.75% of single-hooked 35 mm long steel fibres (NNB35 and NHB35), they failed in ductile shear-tension mode of failure. As discussed earlier, this ductile sudden failure may be attributed to the weak bond between such short steel fibres of 35 mm long with single-hooked end and NWC regardless of the concrete compressive strength. In other words, the only way for those short fibres to break the bond in such NWC with high toughness was to pullout of the matrix. In contrast, the short steel fibres have successfully prevented the shear failure in LWC beams. More specifically, LNB35 and LHB35 members fully resisted shear stresses to eventually fail in flexure. This behaviour might be attributed to the low toughness of LW aggregates that would rather allow the short steel fibres to break through the lightweight aggregates than pulling out from the mortar. This might be clearer when long steel fibres of 60 mm long have successfully increased the shear resistance of normal weight concrete beams irrespective of concrete compressive strength. For example, NNB60 and NHB60 specimens showed ductile flexure-compression failure. It could be claimed that the bond between concrete and steel fibres was sufficient to sustain and resist tensile stresses until beams failed in flexure. This higher bond might be attributed to both length of steel fibres and double-hooked end shape of the used long steel fibres, which were distinctly superior. In beams with 1.55% of tension steel, fibres were able to replace shear reinforcement to produce flexural failures, Swamy et al. (1993). This indicated reasonable agreements with this investigation when tension reinforcement was used 1.46% where all lightweight beams with steel fibres failed in flexure. Biolzi and Cattaneo (2016) reported that crushing of the compressive zone in a ductile manner was the failure mode of HSC beams with steel fibres, which proved that the longitudinal reinforcement has yielded. As mentioned previously, all specimens were designed and poured without web reinforcement. Therefore, shear mode of failure was strongly predicted for beams without steel fibres. And as it was expected before testing reference beams, all of those beams have been poured without steel fibres have shown a sudden failure with pure diagonal shear cracks. On the other hand, beams with steel fibres were expected to show either fully or partially resistance against shear stresses. To be more specific, all specimens fully resisted shear stresses except those beams in normal weight with short steel fibres of 35 mm long, which failed in shear. This was in a reasonable agreement with Tahenni et al. (2016) that reported two of their high strength specimens with 0.5% and 1.0% of short steel fibres of 35 mm long showed shear mode of failure while all of their other specimens with 1% of long steel fibres of 60 mm long were failed in flexure. This might be attributed to unexpected pullout action that occurred twice in two specimens of normal weight concrete beams irrespective of concrete compressive strength. Therefore, it might be said that the pullout of short steel fibres can possibly be earlier to fail normal weight concrete beams in shear before steel fibres can even show their efficiency against tensile stresses. This showed a good agreement with Padmarajaiah, D. K., and Ramaswamy, A. (2001)

who stated that in their paper. Either non fibrous beams, and partially prestressed beams with mild steel fibres “similar to hooked-end” of 40 mm long over the full length but having no shear stirrups failed in a pure diagonal shear failure in partially prestressed beams Padmarajaiah, D. K., and Ramaswamy, A. (2001). This could be related to the high stiffness of normal aggregates which would show lower interaction with short steel fibres compared to the lightweight aggregates that have lower stiffness and they can be easily involved with steel fibres. This pullout action was not an issue when using short steel fibres with lightweight aggregates neither when using long steel fibres regardless of types of aggregates and concrete compressive strength. Even though lightweight concrete beam without steel fibres and due to its lower stiffness were slightly more ductile than normal weight concrete specimen, this was not an obvious trend in steel fibrous concrete beam either types of aggregates.

5.5.1 Effect of Types of Aggregates

Regardless the type of aggregates, all reference concrete beams failed in a brittle shear mode of failure due to the absence of web reinforcement and that was natural and expected. The significant behaviour was that all lightweight concrete beams with steel fibres failed in flexure while two normal weight beams with short steel fibres regardless of concrete grade showed diagonal ductile shear modes of failure. This might be attributed to the good bond interaction expected between lightweight aggregates and steel fibres.

5.5.2 Effect of Length of Fibres

All concrete beams reinforced with double-hooked end steel fibres of 60 mm long failed in flexure regardless of concrete grate and type of aggregates. Although the beams capacities and deflections were improved, short steel fibres of 35 mm long failed normal weight concrete specimens with either normal or high concrete grade in shear.

5.5.3 Effect of Concrete Compressive Strength

Regardless type of aggregates, all reference concrete beams failed in brittle shear due to the absence of web reinforcement and that was natural and expected. The significant behaviour was that all lightweight concrete beams with steel fibres failed in flexure while two normal weight beams with short steel fibres regardless of concrete grade showed diagonal shear modes of failure. This might be attributed to the good bond interaction expected between lightweight aggregates and steel fibres. Tang et al. (2009) mentioned that the cracks would usually rather penetrate through the coarse aggregate particles than around them in LWC beams. "The reason is the interlocking of the cement paste onto the rough surface pores of the LWA and thus improving bond strength between aggregate surfaces and cement paste". This was in a reasonable agreement with Tahenni et al. (2016) that reported two of their high strength specimens with 0.5% and 1.0% of short steel fibres of 35 mm long showed shear mode of failure while all of their other specimens with 1% of long steel fibres of 60 mm long were failed in flexure.

All concrete beams reinforced with double-hooked end steel fibres of 60 mm long failed in flexure regardless of concrete grate and type of aggregates. Although the beams capacities and deflections were improved, short steel fibres of 35 mm long failed normal weight concrete specimens with either normal or high concrete grade in shear. This was in a reasonable agreement with Tahenni et al. (2016) that reported two of their high strength specimens with 0.5% and 1.0% of short steel fibres with 35 mm long showed shear mode of failure while all of their other specimens with 1% of steel fibres of 60 mm long were failed in flexure.

All concrete beams reinforced with double-hooked end steel fibres of 60 mm long failed in flexure regardless of concrete grate and type of aggregates. Although capacities and deflections of beams with high concrete grade were noticeably improved, short steel fibres of 35 mm long failed the normal weight concrete specimens in shear irrespective of concrete grades.

5.6 Capacity of the Specimens

Based on load-deflection plots, some beams reached their peaks with different deflections. Therefore, the highest point in term of the ultimate load capacity with the longer deformation in term of the deflection can be taken as the failure shear load, V_{Exp}, with correspondence to the concrete strength contribution V_c, according to the codes. This means, the experimental shear force is considered to be the maximum applied load for beams failed in shear, while it is unknown (greater than the listed values) for beams failed in flexure. For example, NNB and NNB35 have suddenly failed in shear with maximum applied loads of 71.7 kN and 156.9, respectively. It should be noted that all applied loads are half the actual subjected actuator load. Another example of beams failed in flexure, LNB35 and LNB60 have taken longer time after cracking to eventually fail in flexure at maximum applied load of 149.1 kN and 165.1 kN, respectively. These loads were not supposed to be beams' shear capacity because those beams were failed in a flexural behaviour, which means they were still ductile to carry higher shear loads than 149.1 kN and 165.1 kN. Therefore, all beams failed in a flexural scenario did not show the actual shear strength and they could have carried more shear stresses. Tahenni et al. (2016) stated that the shear strength is increased by 47% for a quantity of fibres of 0.5% and had exceeded the enhancement achieved with the conventional web reinforcement when higher quantities of fibres were used. The shear capacity in this investigation was enhanced in beams containing steel fibres; the enhancement due to the addition of double-hooked long steel fibres with a quantity of 0.75% by concrete volume was 47% on average, while the improvement attributed to the addition of single-hooked short steel fibres with a quantity of 0.75% was 46%. To be more specific, in contrast with reference beams, the shear capacities of NNB35, LNB35, NNB60 and LNB60 specimens were enhanced by 33% on average. Furthermore, the enhancement of shear capacities of NHB35, LHB35, NHB60 and LHB60 beams ranged from 58% to 60% compared to the reference beams. This revealed a good agreement with Kim et al. (2017) who reported that the shear strength of both normal- and high-strength concrete beams with 0.75% of steel fibres considerably increased more than 50%. Although steel fibres were effective in all specimens, short and long steel fibres were more efficient in normal concrete grade with either type of aggregates.

The normalized shear and flexural strengths were calculated for all specimens in order to detect the enhancement of the steel fibres on beams' capacities. Moreover, the effect of types of aggregates, and concrete compressive strength on both shear and flexural capacity of reinforced concrete beams without stirrups. The contribution of the addition of steel fibre to either shear or flexural capacity can be calculated by comparing those beams with fibres to the reference beams. NWC beams were also compared to the LWC beams in term of affecting the beams' loading capacities. The contribution of concrete grade to the capacity was also assessed in this chapter. In general, the type of aggregates did not show a constant upgrade or downgrade of normalized strengths (Use equations (5.1) and (5.2) for calculations). In general, normalized shear and flexure strength of NNB35, NNB60, LNB35, and LNB60 improved by a range of variation between (51% and 60%) while normalized shear and flexure strengths of all high concrete grade beams with steel fibres irrespective of type of aggregates showed an increase varied from (32% to 41%). This reveals that the efficiency of steel fibres is stronger in normal grade concrete beams.

$$v_u = \frac{V_{Exp}}{\sqrt{f_c^{'}}\, bd} \tag{5.1}$$

$$M_u = \frac{M_{Exp}}{\sqrt{f_c^{'}}\, bd^2} \tag{5.2}$$

5.6.1 Effect of Types of Aggregates

The normalized flexure strength was approximately constant for reference concrete beams. NNB and NHB specimens gave a good correlation with LNB and LHB members in terms of normalized shear and flexure strengths. On the other hand, normalized shear and flexure strengths of normal strength lightweight concrete beams with 35 mm long steel fibres dropped by 11% to 18%, respectively, compared to same specimens with normal weight aggregates. This drop did not show when the concrete grade was increased. More specifically, NHB35 beam agreed with LHB35 in terms of both normalized strengths either shear or flexure. The normalized flexure strength of

NNB60 matched the value obtained from LNB60. However, LNB60 showed lower normalized shear strength by about 3% than the strength produced by NNB60. Furthermore, there was unexpected trend that might be attributed to the presence of long double-hooked end steel fibres in beams with high concrete grade. The normalized shear strength of LHB60 improved by 4% while the normalized flexural strength enhanced by 18% compared to the experimental values of NHB60. Therefore as previously mentioned in the beginning of this section, it should be noted that the type of aggregates might not directly affect the normalized strengths of beams. The length of steel fibres and concrete grade factors, which will be discussed later in this chapter, might affectively lead the type of aggregates to show unpredicted behaviour.

5.6.2 Effect of Length of Fibres

In general, length of steel fibres noticeably upgraded normalized strengths. That is, normalized shear and flexure strengths mainly increased with the presence of 60 mm long steel fibres compared to those specimens with short steel fibres. Generally, normalized flexure strength of NNB60, LNBB60, and LHB60 specimens increased by 5%, 6%, and 8%, respectively, while NHB60 member remained without enhancement in terms of normalized flexural capacity compared to NHB35 beam. On the other hand, LNB60 specimen showed no significant change in terms of normalized shear capacity while all beams with steel fibres of 60 mm long improved their shear strength by about 7%, approximately. LNB60 was the only specimen that was in a good correlation with LNB35 without showing any improvement or drawbacks in term of normalized shear strength.

5.6.3 Effect of Concrete Compressive Strength

Although normalized shear strength of reference concrete beams improved by about 22% when concrete grade increased no matter what type of aggregates used, the normalized shear and flexural strengths at failure for LWC and NWC beams generally decreased with an increase in the concrete grade.. However, the shocking observation was that the normalized shear decreased by an average rate of about 16% when steel fibres were used regardless of their lengths, or even the type of aggregates. This can be obviously predicted from equations (5.1) and (5.2) where the concrete compressive strength, $f_c^{'}$, is the denominator which directly affects the value of either shear or flexure strengths. The higher concrete grades were applied, the lower shear and flexure strength were obtained. Biolzi and Cattaneo (2016) reported substantial reduction in normalized shear strength of high strength concrete with respect to normal strength concrete; this drawback is mitigated in fibre-reinforced materials.

5.7 Post-Shear Cracking Capacity

Post shear crack capacity was obtained by extracting the first shear crack load from the ultimate capacity only for those beams failed in shear. Beams with high concrete compressive strength such as NHB and LHB specimens were dramatically enhanced by about 71% in terms of the post-shear capacity in contrast with NNB and LNB beams. Thus, concrete grade certainly has a significant role to upgrade the loading capacity after shear cracks take place. The post-shear cracking capacity of NNB35 and NHB35 beams improved by 82% and 43%, respectively, compared to NNB and NHB members, correspondingly. Both NNB35 and NHB35 specimens failed in ductile shear failure.

Furthermore, short steel fibres showed better performance in improving post shear-capacity in NWC with high strength concrete than in NWC with normal concrete grade. For example, the presence of short steel fibres showed greater post-shear capacity in NHB35 specimen than in NNB35 by 14%. It should be noted that all beams failed in flexure were enhanced in terms of post-shear capacity. However, post-shear capacities of NNB60 and NHB60 as well as all LWC beams were unknown due to failing in flexure before even reaching the ultimate shear strength. Even though NNB60 and NHB60 beams failed in flexure, the addition of both short and long steel fibres were more efficient with lightweight aggregates. Due to the observed enhancement by 20% of the concrete compressive strength of SFRLC with 0.75% steel fibres, Kang et al. (2011) stated that the efficiency of steel fibres is higher in lightweight concrete than in normal weight concrete. Tensile strength and extensibility are noticeably observed not only at first crack, but also at ultimate crack. As a result of this, the ductility of concrete is improved even at post cracking (Chanh, 2017). Randomly-distributed and short, discrete steel fibres are shown to improve several tensile properties of the resultant composite; “especially the post-cracking behavior,” Ning et al. (2015). “This influence of steel fibres was more noticeable after cracking,” Swamy et al. (1993). Overall, there was no significant influence related to the length of used steel fibres on post-shear capacity. Holschemacher et al. (2010) who stated that, in materials point of view, using steel fibres with high-strength concrete exposed higher load capacities in the post-cracking range, compared to normal strength memebers.

5.8 Ductility

The ductility ratio of beams is determined by dividing the deflection at failure on the deflection at yield stage, $\mu = \frac{\partial_f}{\partial_y}$. All RC beams showed a sudden brittle failure have zero ratios of ductility. Although NNB35 and NHB35 specimens failed in shear, they showed some ductility. However, they were smaller than the ductility ratios of those beams failed in flexure. Thus, all beams failed in flexure showed the greatest ductility ratios. In general, the presence of both short and long steel fibres approximately showed similar ductility ratios of LWC beams with either concrete grade. To be more specific, the ductility ratio of LNB35, LNB60, LHB35, and LHB60 beams were 3.5, 3.0, 4.9, and 4.9, respectively. However, double-hooked steel fibres with 60 mm long were more efficient than single-hooked fibres with 35 mm long in NWC. For instance, the ductility of NHB60 and NNB60 improved by about (60% on average) compared to the comparable specimens with 35 mm long of steel fibres. In fact, NHB60 showed the highest ductility ratio of 5.6. This might be strongly attributed to the double-hooked end shape of the long steel fibres with 60 mm long especially in normal weight aggregates with the highest interlock resistance. On the other hand in terms of the ductility, the effectiveness of 35 mm long single-hooked steel fibres was more obvious in LWC beams than in NWC members. In fact, this was clear in LNB35 beam, which showed an improvement in its ductility by about 14% compared LNB60 specimen. Although the addition of short steel fibres was more effective in LWC beam with high strength, this slight increment in the ductility of LHB35 by about 2% compared to LHB60 member could be negligible. It should be noted that the ductility ratios were grater with the presence of steel fibres in LWC with high concrete grade in particular. For example, the ductility of LHB35 and LHB60 were 4.9 and 4.8, respectively. This showed a good agreement with Holschemacher et al. (2010) who stated that, in materials point of view, using steel fibres with high-strength concrete revealed better ductile behaviour, compared to normal strength memebers. And Tahenni et al. (2016) concluded their investigation with that the addition of fibres significantly improved the ductility of HSC beams. An optimal value greater than 3 of ductility ratio is highly adequate for beams structurally subjected to large displacements in seismically active regions particularly. A ductility ratio of 4.0 was achieved in HSC beams with an adequate and effective amount of steel fibres. Such ductile beams are desired especially in zones with seismic movements, Tahenni et al. (2016). Overall, single-hooked short steel fibres with NWC beams showed the lowest ductility ratio of (1.7 on average), while the same type of steel fibre showed a great average of ductility ratio with LWC beams. This means that the short steel fibres might be a good choice for LWC beams, but not with NWC memebers in term of the ductility response. Furthermore, although long double-hooked steel fibres showed higher ductility ratio by 21% with NWC beams compared to its contribution with LWC specimens, it is still a good choice for both types of aggregates. Actually, steel fibres with higher aspect ratio showed greater achieved ductility ratio of greater than 3 with both types of aggregates irrespective of concrete grade. The enhancement in the ductility of all beams with steel fibres is correlated with the improvement of these beams loading capacity. The higher flexural capacity the beams can resist, the higher the ductility can be achieved. Biolzi and Cattaneo (2016) reported that enhancement of the ductility due to inclusion of steel fibres to HSC beams.

5.9 Stiffness

The stiffness of both un-cracked section stage (1) and cracked section stage (2) generally increased as the steel fibres were used. To be more specific, the long steel fibres mostly increased the stiffness by more than 50% in the first stage and by more than 30% in the second stage. The highest ratio was with NNB60 beam when the stiffness showed an enhancement by 91% compared to the reference beam NNB. Besides, LNB60 showed the lowest enhancement due to the addition of long steel fibres by about 26% in contrast with the other specimens contained long steel fibres. The enhancement of the stiffness due to the addition of short steel fibres also ranged from 5% to 31%. For example, the lowest improvement rate happened with NNB35 beam compared to NNB member while the greatest was in LNB35 in contrast with LNB specimen. The long fibres generally seemed more efficient in enhancing the stiffness in either stage. The higher loading capacity produced when long steel fibres were used, the bigger slope could be generated. Therefore, stiffness of long fibrous reinforced beams usually showed greater stiffness. On the other hand, the type of aggregates plays an important role of the stiffness of beams. For example, stiffness of LNB member was lower than the stiffness of NNB specimen by about 42% on average in both stages. Although stiffness of LHB beam was lower than NHB member in stage (1) and higher than NHB specimen in stage (2) by 4% in both cases, this minor influence could be negligible. Thus, LW aggregates mostly reduced stiffness of normal strength beams. Although concrete compressive strength was not a significant factor affecting stiffness, high concrete strength may mitigate the negative affect of presence of lightweight aggregates on stiffness. This might be attributed to the fact that the paste of high concrete grade could usually be stiffer than the coarse aggregate itself. Although short and long steel fibres showed almost similar stiffness in LWC beams with normal concrete grade, long steel fibres incredibly seemed to be more efficient than short fibres in term of stiffness enhancement in all types of specimens. This could be attributed to greater bond between long fibres and concrete due to the higher aspect ratio and doubled-hooked end shape in this type of fibres. Steel fibres regularly increase toughness, Ning et al. (2015). Stiffness of HSC beams enhanced with the presence of steel fibres, Biolzi and Cattaneo (2016).

5.10 Energy Absorption

The area under the load-deflection curve from the origin point up to failure point represents the flexural energy of a beam. That is, this beam needs that amount of energy to completely collapse. The absolute value was taken for each segment in order to avoid the negative values caused by the noise during testing the beams. The energy absorption of specimens in this research is listed in Table 5.3. It could be obviously noticeable that the addition of steel fibres generally improved the energy absorption. In contrast with the correspondent reference beams (NNB and LNB), the energy absorption of NNB35 and LNB35 specimens increased by about 80% on average. On the other hand, the energy absorption was improved by the addition of long steel fibres in both NNB60 and LNB60 beams by about 87% on average. Although the addition of either type of steel fibres similarly improved the energy absorption of NHB35, LHB35, NHB60, and LHB60 beams by an average amount of 67%, the presence of double-hooked long steel fibres was more efficient than the presence of single-hooked short fibres by about 10%. Both short and long steel fibres showed higher energy absorption in normal strength concrete beams than in high strength concrete beams regardless types of aggregates. Consequently, long steel fibres can be highly recommended for NWC and LWC beams with both normal and high strength concrete while short steel fibres can be a second recommended choice for beams with NWC beams with high strength and LWC beams irrespective of concrete grade. Due to the lower rigidity of lightweight aggregates compared to the normal weight aggregates, the ability of the beams to absorb higher energy prior to failure could be noticeable more in NWC beams. In other word, LWC beams showed lower energy absorption than NWC beams with the same concrete grade by about 41% on average. This could attribute to the weakness of LW aggregates interlock that leads to a lower capacity can be carried and shorter deformation can be reached by LWC beams. Hence, type of aggregates could negatively affect the energy absorption of reference LWC beams. There was a great enhancement in the energy absorption caused by increase of the concrete compressive strength. Ordinary high strength beams showed higher ability to absorb higher energy up to failure than RC beams with normal concrete grade regardless type of aggregates. As a result, an average rate of 80% enhancement in the energy absorption was achieved when the concrete strength was increased from 35 MPa to 70 MPa, approximately. Steel fibres consistently increase toughness, Ning et al. (2015).

5.11 Experimental Results vs. Codes and Proposed Equations

5.11.1 Introduction

Cracking moments are of interest because they mark the first moment of failure. The emergence of the first crack demonstrates visible structural weakness, and is a meaningful predictor of the longevity and future behaviour of the beam, as it relates to the beams strengths. The theoretical cracking moments were estimated according to the transformed section analysis at a linear elastic stage while the experimental cracking moments were calculated by multiplying the applied first cracking shear load by the specified shear span of the beam. The theoretical prediction of first cracking moments and the experimental results for first cracking moments of each of the related specimens are listed in the next section.

The maximum value on Y-Axis of load-deflection curve was taken as the failure shear load, V_{Exp}, with correspondence to the concrete strength contribution V_c, according to the codes. On the other hand, the values of shear strengths were theoretically calculated based on ACI and CSA codes as well as prediction model equations by some investigators. Those researchers proposed their equation in terms of different parameters related to the steel fibres. The experimental shear capacity were compared, discussed and plotted against the predicted results in the following section.

The ultimate flexural moment at failure takes place where the concrete reaching its maximum compressive strain value, ε_{cu} = 0.0035, when the top concrete compression zone away from shear spans is crushed. Using CSA equivalent stress block concept, the nominal moment capacities, M_n, were calculated while the experimental ultimate moment resistance values were obtained by multiplying the ultimate shear force by the specified shear span. The theoretical values of nominal moments and the experimental results for first ultimate moments of each of the related specimens are listed in the next section.

5.11.2 Experimental Cracking Moments vs. Codes Results

5.11.2.1 Introduction

In structural design, the cracking moment (M_{cr}) of concrete beams is a vital measure. It is defined as the turning force that when exceeded leads to the cracking of a concrete beam. Although some researchers claim that the steel fibres do not directly contribute to resist while the cracks were not open yet, the first cracking load slightly increased as the compressive strength increased with respect to the presence of the steel fibres were used in normal weight beams especially. On the other hand, the first cracking load showed fluctuated values in lightweight concrete beams with either types of compressive strength. That is, the cracking moment is more likely not to be significantly affected by addition of steel fibres. Therefore, the length of steel fibres would not be an effective factor. Furthermore, types of aggregates could have a slight effect of the first cracking load. In other words, lightweight beams developed slightly earlier flexural cracks than normal weight specimens with the same material properties.

Table 5.3 presents the experimental M_{cr}^{Exp} cracking moment for 12 beams determined from the experiments conducted. Table 5.5 also shows three theoretical (M_{cr}^{Code}) calculated based on the ACI, CSA, and EC2 codes. The values were calculated using the formulae below:

For ACI 318-14,

$$M_{cr}^{ACI} = f_r \frac{I_g}{y_t} \tag{5.3}$$

Where,

$f_r \equiv 0.62\,\lambda\,\sqrt{f_c^{'}}$.

For CSA A23.3-14,

$$M_{cr}^{CSA} = f_r \frac{I_g}{y_t} \tag{5.4}$$

Where,

$f_r \equiv 0.6\,\lambda\,\sqrt{f_c^{'}}$

y_t is the distance from the centroid axis of the gross section to the extreme tension fibre.

I_g is the second moment of inertia of the gross section (the steel bars are neglected).

The difference from ACI and CSA is the introduction of λ into the CSA equation, but λ can be taken 1 for normal weight aggregates and 0.85 for lightweight concrete according to CSA A23.3-14

For EC2 (BSI, 2005),

$$M_{cr}^{EC2} = f_{ctm} \frac{I_u}{(h - x_u)} \tag{5.5}$$

Where,

$f_{ctm} = 0.3\, f_{ck}^{0.67}$

x_u is the distance from the neutral axis of the section to the extreme top fibre

I_u is the second moment of area of the section

h is the height of the cross section of the beam

5.11.2.2 Normal strength reinforced concrete beams

For normal strength reinforced concrete beam**s,** it was observed that the experimental cracking moment (M_{cr}^{Exp}) has not been significantly affected by the use of fibres. For example, NNB and LNB specimens had cracking moments of 18.1 kN.m and 22.7 kN.m, respectively, while the cracking moments of all specims with 0.75% of fibres varied from 20.4 kN.m to 25.0 kN.m. That is, the presence of steel fibres does not play a significant role in enhancing cracking moment. In addition, examining the ratio $M_{cr}^{Exp} / M_{cr}^{Code}$ it is clear that the codes with the exception of EC2 generally overestimated the calculated moment. For the first specimen NNB (normal strength reinforced concrete beam (0% fibres) the experimental cracking moment is equal to 18.1 kN.m. The theoretical values calculated using the codes ACI, CSA overestimate the value at 19.9 kN.m, 19.3 kN.m while EC2 underestimates the value at 17.8 kN.m.

Examining the results from the second specimen NNB35 (normal strength reinforced concrete beam (0.75% fibres-35 mm long), the experimental cracking moment is equal to 25.0 kN.m. The theoretical values calculated using the codes ACI, CSA, and EC2 all underestimate this value. Lastly, for NNB60 (normal strength reinforced concrete beam (0.75% fibres-60 mm long), the experimental cracking moment is equal to 22.7 kN.m. The theoretical values calculated using the ACI code overestimates the value while CSA and EC2 underestimate it.

5.11.2.3 High strength reinforced concrete beams

For the high strength reinforced concrete beams, M_{cr}^{Exp} increased as the percentage of fibre increased. For example, the first beam with 0% fibres had M_{cr}^{Exp} equal to 25.0

kN.m while for the second beam with 0.75% fibres it is equal to 29.5. This represents an 18 % increase in cracking moment. In addition, examining the ratio $M_{cr}^{Exp} / M_{cr}^{Code}$ it is clear that the codes with the exception of EC2 generally overestimated the calculated moment. From the results of NHB (high strength reinforced concrete beam (0% fibres), the experimental cracking moment is 25.0 kN.m. The theoretical values calculated using the codes ACI, CSA, and EC2 all overestimate the experimental value.

For the specimen NHB35 (high strength reinforced concrete beam (0.75% fibres-35 mm long) ACI, CSA, and EC2 codes underestimate it. Lastly, for NHB60 (normal strength reinforced concrete beam (0.75% fibres-60 long), the experimental cracking moment is 29.5 kN.m. The theoretical values calculated using the ACI, CSA and EC2 underestimate this value.

5.11.2.4 Normal strength lightweight reinforced concrete beam

For the normal strength lightweight reinforced concrete beams, the results of the experiment showed that the cracking moment, M_{cr}^{Exp}, values were not significantly affected by the use of steel fibres. The cracking moment of reference beams without steel fibres ranged from 18.1 kN.m to 25.0 kN.m. 0% while SFRC beams with 0.75% fibres had cracking moments equal to 22.7 kN.m and 20.4 kN.m respectively. This represents a 10% decrease. The results obtained for the specimen LNB (normal strength lightweight reinforced concrete beam (0% fibres) show that the experimental cracking moment is 22.7 kN.m. In contrast, the theoretical values calculated using the ACI, CSA, and EC2 codes all underestimated this value.

For the sample LNB35 (normal strength lightweight reinforced concrete beam (0.75% fibres-35 mm long), the experimental cracking moment is 20.4 kN.m. Similarly, the theoretical values calculated utilizing the ACI, CSA, and EC2 codes all underestimated this value. Lastly, the specimen LNB60 (normal strength lightweight reinforced concrete beam (0.75% fibres-60 mm long), the experimental cracking moment is 20.4 kN.m. Similar results were noted where the codes ACI, CSA, and EC2 all underestimated this figure.

5.11.2.5 High strength lightweight reinforced concrete beam

The high strength lightweight reinforced concrete specimens show that M_{cr}^{Exp} increased as the percentage of fibre increased. For example, the first beam with 0% fibres had M_{cr}^{Exp} equal to 22.7 kN.m while for the second beam with 0.75% fibres it is equal to 25.0. This represents a 10 % increase in cracking moment. In addition, examining the ratio $M_{cr}^{Exp} / M_{cr}^{Code}$ it is clear that all the codes generally overestimated the calculated moment.

For the LHB35 specimen (high strength lightweight reinforced concrete beam (0.75% fibres-35 mm long) the cracking moment is 22.7 kN.m. For this specimen, the theoretical values calculated based on ACI, and CSA codes underestimate these values while EC2 code overestimates it. Lastly, for the final sample LHB60 (high strength lightweight reinforced concrete beam (0.75% fibres-60 mm long) the experimental cracking moment is 25.0 kN.m. The values calculated using ACI, and CSA codes underestimate this figure while EC2 overestimate it.

5.11.2.6 Conclusion and comparison to previous research

In general, the comparison of the experimental cracking moments to the theoretical figures revealed that generally ACI, CSA and EC2 codes generally underestimate its value. This is evidenced by the fact that the average ratios $M_{cr}^{Exp} / M_{cr}^{Code}$ for all codes ACI, CSA, and EC2 are 1.10, 1.13, and 1.09, respectively. Furthermore, ACI, CSA, and EC2 had $M_{cr}^{Exp} / M_{cr}^{Code}$ ratios ranging from 0.91 to 1.35, 0.94 to 1.39 and 0.87 to 1.43, respectively. . The coefficient of variance ratio of EC2 is greater than corresponding coefficient ratio of ACI and CSA codes. Therefore, ACI and CSA predictions are more reliable predicting the nominal shear strength of SFRC beams. The comparison above also revealed that EC2 mostly underestimated the cracking moment with normal concrete grade and overestimated them with high concrete grade.

Lastly, the results also showed that for all specimens with the exception of normal strength lightweight reinforced concrete beams showed an increase in cracking moments with presence of fibres. The main reason for the increase might be attributed to the improvement in the specimen's tensile strength. Tensile strength of HSC beams improved due to the presence of steel fibres of higher aspect ratios, Tahenni et al. (2016). Furthermore, the first flexural cracking moment increases with the addition of SFs.

5.11.3 Experimental Shear Strength vs. Codes and Predicted Equations Results

Thus, nominal shear strength is the shear strength exhibited by a member (or even its cross-section) as per the provisions as well as assumptions of the particular method of strength design before any strength-reduction (f) factor is applied.

As well, the NHB beam category revealed higher values of nominal shear force (115.6, 178.9, and 198.3 kN, respectively) when compared to their corresponding beam categories, whereas the LNB beam categories exhibited the smallest nominal shear force values compared to other corresponding categories. Another factor worth considering is the nature and type of the aggregates. According to a study by Gebreyohannes and Muzeyin (2016) revealed that a structure's shear capacity rises with increase of the size of aggregate. As well, they found that, for lightweight aggregate concrete, shear capacity is identical to capacity of mortar. (Gebreyohannes and Muzeyin, 2016). Therefore, this indeed explains the observation made concerning NHB and LHB beam categories.

It is clear that all nominal shear force values estimated based on CSA code are smaller than the experimental ones. As well, the clear trend observed in the experimental values below does not conspicuously appear in the case of CSA values. One key reason could be in the manner in which the two formulas are deployed. That is, the CSA code may not necessarily take into account the effective length of the beam. This will hence depict a lack of increasing trend as in the experimental values across the beam categories. However, one key factor to consider in the differences between the two values is the fibre reinforcement. That is, beams with 0.75% fibres exhibited significantly higher nominal shear force values as they have a higher flexural capacity brought about in them by the fibres.

Table 5.6 lists the experimental shear capacity, V_{Exp} in kilo Newton against the ACI, CSA and different predicted shear design equations, V_{Pred}. Bar charts in Figures 5.17 represents the ratio of experimental results to the codes (ACI and CSA) calculations, while Figure 5.19 shows the ratio of the specimens values to the proposed estimations by Li et al. (1992), Ashour et al. (1992), Khuntia et al. (1999), Sharma (1986), Shin et al. (1994), Kwak et al. (2002), Greenough and Nahdi (2008). Ordinary reference RC beams and SFRC members are shown in order to clarify the differences. The ratios based on theoretical values calculated by predicted equations showed an approximate uniform consistency while the rates based on the codes calculations a great gap. This is due to the fact that the codes neglect the effect of steel fibres in their equations whereas the predicted equations by investigators were specifically designed for SFRC beams. It can be noticeable from 5.6 that the average ratios of the experimental shear strengths to the theoretical code values are conservative for all beams. All ratios are highly conservative for SFRC beams in particular for the reason mentioned previously about ignoring the effect of the presence of steel fibres. ACI and CSA codes slightly underestimated the nominal shear strength for all beams except NNB sample that showed lower experimental shear strength than ACI result for the same beam. ACI and CSA did not consider steel fibres in beams in predicting shear strength. Therefore, experimental shear resitance values of beams with steel fibres were noticeably greater than the codes predictions. This is definitely attributed to the higher flexural capacity gained by the presence of steel fibres in those beams. Those samples in fact failed in flexure without even knowing how much shear stresses they could resist. That is, the actual shear strength of beams failed in flexure is highly greater than codes prediction. On the other hand, the underestimation predictions by codes for reference RC beams are purposely reduced by codes for safety reasons in order to keep the designed beams in the safe side.

ACI and CSA codes tend to underestimate the value of the nominal shear strength in all beams with and without steel fibres. This underestimation is mainly attributed to the fact that these codes do not take into account the probable existence of micro-cracks within concrete, which is usually introduced by shrinkage way before commencement of loading. Such micro-cracks normally develop and become visible macro-cracks upon loading. Also, beams with 0% fibre reinforcement mostly exhibited quite low nominal shear capacity values compared to theoretical ones. This is because such beams have very low flexural capacity hence they were even failing before even reaching their shear capacity. High strength concrete beams showed an increase in their shear capacities by 41% on average compared to normal concrete grade beams at the same type of aggregates. Although the maximum shear strength of LHB sample dropped by 3%, the type of aggregate was not that significant factor to be aware of in terms of beams shear strength and the low interlock resistance of LWC was mitigated by the presence of steel fibres. Although the presence of both short and long steel fibres improved beams shear resistance by a range varied from 35% to 72% compared to reference RC beams, shear strength of beams with long steel fibres enhanced by an average amount of 10% in contrast with short SFs beams. The greater bond produced between longer double-hooked fibres with high aspect ratio and concrete matrix might be the reason behind the enhancement of beams shear capacity.

Although ACI and CSA seem highly conservative, they showed better predictions in ordinary RC beams than their predictions in beams with steel fibres. However, design equations proposed by researchers mentioned previously revealed fairly comparable values to the real experimental results. The coefficient of variance ratios of Li et al. (1992) and Sharma (1986) are greater than corresponding coefficient ratio of Ashour et al. (1992), Khuntia et al. (1999), Greenough and Nahdi (2008) and Kwak et al. (2002) models. Therefore, Ashour et al. (1992), Khuntia et al. (1999), Greenough and Nahdi (2008) and Kwak et al. (2002) equations are the most reliably applicable in order to predict the nominal shear strength of SFRC beams. Table 5.7 then, shows that experimental v_n, or nominal shear force, when compared to theoretical v_n, is significant. This values demonists that the maximum shear forces that each beam can resist, just prior to failure, is shear, and occurs regardless of the presence of steel fibre. This indicates that the beams with steel fibre have been significantly improved, as it relates to the performance against shear forces, and so has greater resistance to failure, allowing them to fail in flexure instead of in shear mode. This research exposed a good agreement with Gomes et al. (2017) who reported that steel fibres of 60 mm long has a considerable enhancement on high strength concrete beams in term of the experimental shear strengths and these values were considerably higher than the predicted values by design codes (RILEM, EHE, and *fib*).

5.11.4 Experimental Ultimate Moments vs. Codes Results

At the outset, it is worth recalling that nominal strength in a reinforced concrete beam refers to such a beam's capacity to resist influences of applied load(s). Moment redistribution is influenced by ductility of inelastic areas in a beam, loading patterns, and the beam's geometry. In reinforced concrete (RC) beams, moment redistribution is a key application of the ductility of members in the design procedure. This section takes a look at various reinforced concrete beams (NNB, NNB35, NNB60; NHB, NHB35, NHB60; LNB, LNB35, LNB60; and LHB, LHB35, LHB60) in terms of their nominal moments and then compares them to theoretical values calculated by the codes (ACI and CSA). It should be noted that the theoretical moments were calculated according to equation (5.6) for the design assumptions of singly reinforced beams containing steel fibres (Henager and Doherty 1976). See Figure 5.21.

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) + \sigma_t b (h - e) \left(\frac{h}{2} + \frac{e}{2} - \frac{a}{2} \right) \tag{5.6}$$

Where,

e = [ϵ_t (fibres) + 0.003] c/0.003

σ_t = 1.12 $\ell / d_f \rho_f F_{be}$ (psi)

σ_t = 0.00772 $\ell / d_f \rho_f F_{be}$ (MPa)

ℓ = fibre length

d_f = fibre diameter

ρ_f = percent by volume of steel fibres

F_{be} = bond efficiency of the fre which varies from 1.0 to 1.2 depending upon fibre characteristics

e = distance from extreme cpression fibre to top of tensile stress block of fibrous concrete (Figure 5.21)

ϵ_x = tensile strain in steel at theoretical moment strength of beam, for bars = f_y / E_s; for fibres = σ_t / E_x based on fibre stress developed at pull-out (dynamic bond stress of 333 psi) (Figure 5.21)

ϵ_c = compressive strain in concrete

f'_c = compressive strength of ncrete

f_y = yield strength of reinforcing bar

A_s = area of tension reinforcment

C = compressive force

h = total depth of beam

σ_t = tensile stress in fibrous concrete

E_s = modulus of elasticity

T_{fc} = tensile force of fibrous concrete = $\sigma_t \; b\,(h - e)$

T_{rb} = tensile force of bar reinforcment = $A_s f_y$

A keen analysis of the nominal moment values reveals quite a number of observations. First, the actual nominal moment (kN.m) increases with presence of steel fibres. That is, the 0% fibre beams across all the categories have the least nominal moment compared to both 0.75% fibre 35 mm- and 0.75% fibre 60 mm-long beams. For instance, consider the NNB beam categories. The NNB has 73.1 kN.m, NNB35 160.0 kN.m and NNB60 175.2 kN.m. Clearly, it can be stated hear that, as stated in the introductory section, fibre reinforcement increases flexural capacity of beams hence the higher values (Andermatt and Lubell, 2013).

Also, the NHB beam category revealed higher values of nominal moments (117.9, 182.5, and 202.3 kN.m, respectively) when compared to their corresponding beam categories. This is attributed to the nature of steel fibre reinforcement done on the beams, which have direct influence on the tensile strength of a beam.

One trend and/or observation is quite clear; both ACI and CSA values are greater only in 0% fibres beam categories. Nominal moments of ordinary RC beams seemed overestimated due to the fact that those beams failed in shear before even reaching their flexural capacity. Although NNB35 and NHB35 beams failed in shear, they showed greater experimental nominal moments values than the correspondent values by codes due to the slight deformability produced by the addition of steel fibres. However, in all remaining cases, that is, in all the 0.75% fibre beams across all the beam categories, the experimental nominal moment values are greater than both the ACI and CSA values. This implies that they both underestimate the nominal moment for SF reinforced beams. The reason for this is usually the concrete's underestimated modulus of rapture particularly as applied in the CSA design code (Hamrat et al. (2016). Besides, the addition of steel fibres played as minimum shear reinforcements, which increased the flexural capacity of SFs beams.

However, the standard codes underestimate the value of the nominal moments in the other areas. This underestimation is mainly attributed to the fact that these codes do not take into account the probable existence of micro-cracks within concrete, which is usually introduced by shrinkage way before commencement of loading. Such micro-cracks normally develop and become visible macro-cracks upon loading. Also, all the beams with 0% fibre reinforcement exhibited significantly low nominal moment values compared to theoretical ones. This is because such beams have very low flexural capacity hence they were even failing before even reaching their shear capacity. High strength concrete beams showed an increase in their load capacities by 41% on average compared to normal concrete grade beams at the same type of aggregates. Although the maximum moment capacity of LHB sample dropped by 3%, the type of aggregate was not that significant factor to be aware of in terms of beams flexural moments. Although the presence of both short and long steel fibres improved beams flexural capacity by a range varied from 35% to 72% compared to reference RC beams, flexural moment capacity of beams with long steel fibres enhanced by an average amount of 10% in contrast with short SFs beams. The greater bond produced between longer double-hooked fibres with high aspect ratio and concrete matrix might be the reason behind the enhancement of beams flexural moment capacity. Steel fibres reliably rise ultimate flexural strength Ning et al. (2015). An alternative to the shear reinforcement is the use of SFRC. More specifically, the NNB specimen showed a lower nominal moment, because it did not have steel fibres in it, and so cracked more readily. This contrasted significantly with NNB 35 and NNB60, which showed improvement in stability, and so a later nominal moment, which was attributed to the addition of steal fibres to the beam specimens. More specifically, NNB60 was demonstrated to have the highest flexural nominal moment, which can be contributed to the fact that it had the longest individual steel fibres with double-hooked end. As a result, more concrete was between these fibres, with in the core structure of the beam, which provided better results.

NHB performed better than NNB, because of the high strength reinforcement. As such, high strength reinforced concrete can be deemed more suitable to beam construction than normal strength concrete. As with NNB, the test results demonstrate that the use of 0.75% fibre, in both NHB35 and NHB60 increased the performance of the beam, with the best performing beam in the class, with a nominal moment of 202.3 kN.m, NHB60, with 60 mm long fibres. It is also significant to note that NHB60 yielded the greatest performance, of all the specimens.

LNB was the poorest performing specimen with a low nominal moment of just 68.7 kN.m. As with the other classes of concrete, LWC specimens showed a substantial improvement upon the addition of 0.75% fibres. In fact, the performance of the concrete, in terms of nominal moment, more than double as a result of the addition of steel fibre, indicating that the fibre is more important than the strength of the concrete, overall, in determining the flexural nominal moment of the specimen, and the overall performance of the beam.

Finally, LHB specimens were tested, with significantly higher nominal moments than LNB. More critical, however, is the observation that LHB samples performed categorically better, in terms of nominal moment and beam strength, than the NNB samples. Overall, LHB60 was the second greatest performing specimen within the testing, further demonstrating the superiority of concrete reinforced by 0.75% steel fibres, with a greater length.

It is also significant to consider the experimental data as it relates to the theoretical data, generated by both the American And Canadian research institutes. The American and Canadian data, which is approximately similar, shows much higher expectations for the performance of the concrete that is not reinforced with steel fibres. More specifically, the ordinary RC specimens include NNB, NHB, LNB, and LHB all of which were predicted to perform at greater than 140 kN.m, as it relates to nominal moment. However, both NNB and LNB performed at only half of the theoretical level, approximately. In contrast, the expected level of performance, for NHB and LHB was still overestimated, when comparing the experimental results, and theoretical estimates.

Similarly, however, the estimates for the nominal moments of regarding the steel reinforced concrete were consistently underestimated. Theoretical nominal moments for NNB60 were 150.1 and 148.9 kN.m, but experimental testing found that the nominal moment actually occurred at 175.2 kN.m. In the same fashion, NHB60, the highest performing concrete specimen had theoretical nominal moments of 158.9 and 157.6 kN.m, but in experimental testing, had a significantly higher nominal moment at 202.3 kN.m. This pattern was also visible in the performance of both the 35 and 60 reinforced steels, and in both lighter weight concretes as well as the normal strength varieties.

Together the overestimation of the nominal moments of the concrete without steel fibres, together with their underestimation of the theoretical performance of the beams containing steel fibres, indicates that the steel is far more important to the overall performance of the beam than theoretically expected, or projected. As such, it can be determined that the steel, and the length of the steel fibres included in the concrete, directly results in the meaningful lengthening of the nominal moments for the beam.

More specifically, as is demonstrated in Table 5.8 below, the experimental values of M_n are substantially lower than the theoretical values calculated by the codes, in specimen NNB, NHB, LNB, and LHB. The reason for the disparity between experimental and theoretical findings, in this case, is because those beams have failed in shear, before reaching their flexural moment of capacity. As a result, there was a sudden failure that occured as a result of the sense of conventional shear reinforcement, which is accomplished through the addition of steel fibres, as is demonstrated in the other specimens that failed as a result of flexure.

The steel fibres transform the behaviour of concrete. Concrete's tension capacity is improved diagonally which results in the increase of shear capacity (Cohen, 2012).

5.11.5 Experimental Crack Widths vs. Codes Results

Figures 5.22 to 5.24 show the load versus crack width for all specimens. At the outset, application of serviceability limit states for reinforced concrete beams is normally done in order to ensure both their structural integrity and functionality under service loading circumstances Alam et al. (2010). This serviceability state is defined by such key parameters as deflection checks, control of crack spacing and width, and stress limitations in the material. As per the observation of Piyasena et al. (2004), key variables affecting crack width and spacing include: effective depth, concrete cover, beam's effective width per bar, and the diameter of the bar. Other factors are stresses in reinforcement and reinforcement distribution. Based on this overview, the two phenomena [cracking width and spacing] are analyzed in the 12 concrete reinforced specimens – NNB, NNB35, NNB60; NHB, NHB35, NHB60; LNB, LNB35, LNB60; and LHB, LHB35, LHB60.

A keen analysis of the experimental results revels a number of observations among the beam types for both measured phenomena. First, beams with 35 mm long fibres exhibited the smallest crack width across the categories (when inter-category comparison is done), except in the case of LHB beams where both 35 and 65 mm-long fibres showed the same crack width. Nonetheless, the 0% fibres beams (NNB, NHB, LNB, and LHB) across all categories exhibited the largest crack widths. Residual stresses beyond the cracking stage are dominated and cracks are noticeably confined when steel fibres are added. Steel fibres maintain the interlock strength across the diagonal cracks by stitching cracked segments altogether. Besides this, the crack width in SFRC is no longer reach 0.3 mm neither at serviceability stage nor failure, Tahenni et al. (2016). In general, number of cracks increased in SFRC beams. This could be attributed due to the improvement of concrete tensile strength produced by the presence of steel fibres. The higher tensile strength of concrete is produced, the greater deformability of SFRC beams can be achieved. Therefore, the presence of steel fibres reduces crack widths and decreases spacing among them. Another key observation is that the beam structures exhibit a decreasing crack width as one moves from NNB (0.20 mm) down to LHB (0.18 mm). This could as well be substantiated based on the fact that the nature of beams with respect to steel fibres determines crack widths. So, in high strength beams, the inclusion of more and/or high strength steel fibres (SFs) helps reduce crack widths.

On the other hand, all 60 mm-long concrete beams exhibited the shortest crack spacing across all categories. It can be noted and hence concluded at this juncture that the deflection capacity of the beam structure tends to reduce with length Alam et al. (2010). On the contrary, there seems to be no clear trend among the beam categories as one move from NNB down to LHB. Also, it can be seen clearly that the 0% fibre reinforced structures have comparably higher crack spacing across the categories, a trend that is partly attributed to their lower deflection capacity as well.

Finally, it can be observed that there are more cracks in beams with 60 mm-long fibres across the categories than those specimens with shorter fibres of 35 mm long. In engineering perspectives, reinforcement availability tends to hinder development of fracture process regions. As a result, more cracks have to develop so as to release strain energy that is stored in the structure.

The crack spacing reduces at the ultimate limit state compared to serviceability limit state. This is further explained with the number of cracks that result in the structures. This trend can be attributed to the fact that SFs presence causes beams to undergo greater deformations just before failure which leads to more number of cracks, which are wider as well. For instance, number of cracks increased in beam LHB60 (from 8 to 15). Comparison of the 'number of cracks' columns at serviceability and at failure reveals a tremendous increase in the number of cracks at failure. Normally, as cracks become numerous, the spacing reduces. Table 5.10 shows a comparison of the experimental crack width and the calculated ACI-Gergely and Lutz and ACI-Frosch counterparts of the specimens at serviceability limit state. First, it can be seen that all experimental values were below the critical crack width value set by ACI 318 (0.33 mm). At a glance, the ACI-Gergely and Lutz values are superior to the rest in all categories. Additionally, the experimental values happen to be greater than the ACI-Frosch ones for NNB beams except the NNB60 one. However, as per the NHB as well as LHB beams, all ACI-Frosch values exceed the experimenral ones. A slight comparison of the values is in the LNB beams.

Another varying observation is in the trend of the experimental cracking width values versus the calculated ACI-Gergely and Lutz and Frosch ones. The former values are seen to reduce with length, as opposed to both ACI-Gergely and Lutz and ACI-Frosch ones which increase with length. For instance, both ACI and ACI-Frosch values increase from NNB to NNB60 beams (0.21 to 1.16; 0.10 to 0.53, respectively) whereas the experimental values decrease across the same beam category (0.20 to 0.11).

One reason is that when strain values become larger, the observed difference between calculated and measured values becomes higher as well. This can be seen in the summarized results where, at steel strain of 0.0002, measured value was 0.20, ACI 0.21, and ACI-Frosch 0.10; whereas at 0.0011 strain, the values were 0.10, 1.16, and 0.53

respectively. In fact, the theoretical values are computed with equations that consider tension reinforcement stress as a main parameter and do not consider the presence, size and shape of steel fibres (or all the dimensions of the beam) that influences the general fracture mechanism. In order to provide a cross-comparison of the three values, Table 5.10 provides an overview. At a glance, it can be seen that the theoretical values by ACI-Gergely and Lutz mostly overestimated the crack width of the beams. Failure to include the effects of SFs in its ACI's model attributes to this high underestimation. As a result from experimental values, LNB beam without steel fibres showed wider crack widths by about 15% compared to NNB specimen. This might be attributed to the lower interlock strength of LW aggregates that encourages less number of cracks with longer spacing and thicker widths. The lower number of carcks could be due to the lower beams capacity. The addition of either type of fibres reduced widths of cracks by about (112% on average) in LWC and NWC beams with normal concrete grade. Even though long and short steel fibres both seemed effective in controlling crack widths in LWC and NWC beams in high strength, short fibres seemed more effective in those beams with high concrete grade especially in LHB35. Thus, the addition of 35 mm long of fibres decreased crack width of NHB35 and LHB35 beams by 61% and 44%, respectively, in contrast with NHB and LHB specimens. Crack widths of NHB60 and LHB60 members were reduced by an average amount of 36% compared to NHB and LHB. Although 60 mm long steel fibres still effective to prevent the expansion of crack width, it could be noted that the short steel fibres may be more significant in term of reducing crack width. Crack widths of all beams containing long steel fibres were wider than crack widths all beams with short steel fibres by 33%. This could be attributed to the expected higher dispersion of short steel fibres across the crack compared to the low number of long fibres that might resist the opening of cracks. Steel fibres consistently control crack propagation, Ning et al. (2015). Cracking is a complex phenomenon, in two concrete beams reinforced with SF30 and SF60, notched beam tests caused a crack from the notched edge to the point of loading (Jongvivatsakul et al. 2013). The SFRC softening behaviour was tested, and two zones were considered of significance; the first zone is at the point of the crack where the crack happens suddenly and then the stress is reduced; the second zone is where displacement increases as the stress slowly decreases of the zone (Jongvivatsakul et al. 2013). SF30 measured the highest stress, as expected. On the other hand, SF60 exhibited resistance the highest stress across the crack $u \geq 0.11$ mm. The phenomenon was explained earlier as the fibre bridging the gap produced by the crack. The reason is because the "fibre increased" allowing the bonding to increase and give improved resistance from the SF30, Jongvivatsakul et al. (2013). Biolzi and Cattaneo (2015) stated that the prediction crack width and the crack spacing according to (*fib* Model Code for Concrete Structures 2010, MC2010) were reasonable for reference concrete but unreliable for fibre-reinforced concrete. The code seemed to neglect some effective parameters (i.e., tensile strength). This investigation agreed with this statement by Biolzi and Cattaneo (2015) in the low reliability of codes predictions for crack widths of SFs beams.

Table 5.1: Experimental loads and deflection.

Beam ID	At first flexural crack		At first shear crack		At peak		At failure		Post shear crack capacity	Ductility
	Load P_{cr} (kN)	Deflection δ_{cr} (mm)	Load V_{cr} (kN)	Deflection δ_v (mm)	Load P_u (kN)	Deflection δ_u (mm)	Load P_f (kN)	Deflection δ_f (mm)	(kN)	
NNB	17.8	0.6	53.4	3.5	71.7	5.0	70.1	6.0	14.3	-
LNB	22.2	1.2	48.9	3.8	67.4	6.0	67.1	6.6	18.4	-
NHB	24.5	0.7	62.3	3.7	115.6	7.8	114.9	8.3	53.4	-
LHB	22.2	1.6	53.4	3.1	111.8	9.4	110.3	9.7	58.4	-
NNB35	24.5	1.1	75.6	4.6	156.9	16.1	155.7	16.9	81.2	1.7
LNB35	20.0	2.3	57.8	3.8	149.1	27.3	146.1	38.3	> 91.2	3.5
NHB35	28.9	0.8	84.5	4.4	178.9	25.5	178.7	25.7	94.4	2.2
LHB35	22.2	2.4	66.7	3.2	175.7	31.6	169.4	50.3	> 108.9	4.9
NNB60	22.2	0.6	80.1	4.3	171.8	30.0	165.5	42.4	> 91.7	3.9
LNB60	20.0	2.2	71.2	4.7	165.1	30.0	164.1	31.7	> 93.9	3.0
NHB60	28.9	0.8	93.4	4.6	198.3	32.9	192.7	61.8	> 104.9	5.6
LHB60	24.5	2.3	75.6	4.2	190.5	37.1	178.9	54.7	> 114.9	4.8

Table 5.2: Cracking results.

Beam ID	At service load, P_s †						At failure, P_f			
	Maximum measured crack width W_{max} (mm)	Average measured crack width W_{avg} (mm)	Min crack spacing (mm)	Max crack spacing (mm)	Average crack spacing (mm)	Number of cracks	Min crack spacing (mm)	Max crack spacing (mm)	Average crack spacing (mm)	Number of cracks
NNB	0.31	0.20	145	189	172	5	95	189	122	7
LNB	0.33	0.21	154	254	215	4	115	167	143	6
NHB	0.25	0.16	124	160	172	5	70	175	122	7
LHB	0.28	0.19	110	248	172	5	66	165	86	10
NNB35	0.15	0.10	131	189	143	6	83	164	122	7
LNB35	0.16	0.11	105	187	172	5	44	105	66	13
NHB35	0.14	0.09	116	207	143	6	53	207	107	8
LHB35	0.14	0.09	87	197	122	7	50	197	78	11
NNB60	0.14	0.10	47	152	107	8	47	152	86	10
LNB60	0.16	0.11	92	171	107	8	38	133	71	12
NHB60	0.17	0.11	107	138	143	6	82	138	95	9
LHB60	0.16	0.11	68	146	107	8	25	89	57	15

P_s † Service Load = 0.4 P_u.

Table 5.3: Experimental moments, strains, stiffness and absorption energy values.

Beam ID	Mode of failure	M_{cr}^{Exp} (kN.m)	M_{n}^{Exp} (kN.m)	V_{n}^{Exp} (kN)	Steel strain at service load	Max. concrete strain ε_c	Stiffness 10^3 (N/mm)		Energy absorption at peak 10^6 (kN.mm)	Energy absorption at failure 10^6 (kN.mm)
							State 1	State 2		
NNB	BSF	18.1	73.1**	71.7	0.0002	0.0015	21	10	0.3	0.3
LNB	BSF	22.7	68.7**	67.4	0.0004	0.0016	15	6	0.3	0.3
NHB	BSF	25.0	117.9**	115.6	0.0005	0.0011	21	8	2.0	2.0
LHB	BSF	22.7	114.0**	111.8	0.0009	0.0017	20	8	1.0	1.0
NNB35	DSF	25.0	160.0**	156.9	0.0006	0.0022	25	10	2.0	2.0
LNB35	FF	20.4	152.0	149.1***	0.0010	0.0037	19	9	3.0	5.0
NHB35	DSF	29.5	182.5**	178.9	0.0009	0.0020	29	10	4.0	4.0
LHB35	FF	22.7	179.2	175.7***	0.0010	0.0039	22	10	5.0	8.0
NNB60	FF	22.7	175.2	171.8***	0.0011	0.0038	40	11	6.0	8.0
LNB60	FF	20.4	168.4	165.1***	0.0012	0.0034	30	10	4.0	5.8
NHB60	FF	29.5	206.3	202.3***	0.0014	0.0040	38	16	6.0	13.0
LHB60	FF	25.0	194.3	190.5***	0.0016	0.0039	38	14	6.0	10.0

M_n^{Exp}** Nominal flexural moment was smaller than it should be due to a sudden shear failure before reaching the flexural capacity.

v_n^{Exp}*** Nominal shear force was smaller than it should be due to a flexural collapse prior to shear failure.

Table 5.4: Cracking moments analysis.

Beam ID	M_{cr}^{Exp} (kN.m)	M_{cr}^{ACI} (kN.m)	M_{cr}^{CSA} (kN.m)	M_{cr}^{EC2} (kN.m)	$M_{cr}^{Exp} / M_{cr}^{ACI}$	$M_{cr}^{Exp} / M_{cr}^{CSA}$	$M_{cr}^{Exp} / M_{cr}^{EC2}$
NNB	18.1	19.9	19.3	17.8	0.91	0.94	1.02
LNB	22.7	16.8	16.3	17.6	1.35	1.39	1.29
NHB	25.0	27.0	26.1	26.7	0.92	0.96	0.93
LHB	22.7	22.1	21.3	25.3	1.03	1.07	0.90
NNB35	25.0	19.7	19.1	17.5	1.27	1.31	1.43
LNB35	20.4	17.5	17.0	18.6	1.17	1.20	1.10
NHB35	29.5	26.3	25.4	25.7	1.12	1.16	1.15
LHB35	22.7	22.6	21.9	26.2	1.00	1.04	0.87
NNB60	22.7	20.2	19.5	18.1	1.12	1.16	1.25
LNB60	20.4	16.9	16.4	17.8	1.21	1.24	1.15
NHB60	29.5	28.3	27.3	28.4	1.04	1.08	1.04
LHB60	25.0	22.1	21.4	25.4	1.13	1.17	0.98
				Mean	1.11	1.14	1.09
				STDEV	0.13	0.14	0.17
				COV	0.12	0.12	0.16

Table 5.5: Proposed shear strength models for FRC beams without stirrups.

Investigator	Shear strength models, MPa
Sharma (1986)	$v_n = k\, f_t \left(\frac{d}{a}\right)^{0.25}$ Recommended by ACI Committee 544.1R-96 k = 1 if f_t is obtained by direct tension test; k = 2/3 if f_t is obtained by indirect tension test; k = 4/9 if f_t is obtained using modulus of rupture; or $f_t = 0.79\sqrt{f_c^{'}},\ f_c^{'}$ in MPa.
Ashour et al. (1992)	$v_n = \left(0.7\sqrt{f_c^{'}} + 7F\right)\frac{d}{a} + 17.2\rho\frac{d}{a},\ f_c^{'}$ in MPa.
Li et al. (1992)	For FRC, d in m. $v_n = 1.25 + 4.68\left[\left(f_r\ f_{sp}\right)^{3/4}\left(\rho\frac{d}{a}\right)^{1/3} d^{-1/3}\right]$ for $a/d > 2.5$ $v_n = 9.16\left[\left(f_r\right)^{2/3}\left(\rho\right)^{1/3}\left(\frac{d}{a}\right)\right]$ for $a/d < 2.5$
Khuntia et al. (1999)	$v_n = \left(0.167 + 0.25F\right)\sqrt{f_c^{'}},\ f_c^{'}$ in MPa. $v_n = \left(0.418\frac{d}{a} + 0.25F\right)\sqrt{f_c^{'}}$, for $a/d < 2.5$, $f_c^{'}$ in MPa.
Kwak et al. (2002)	$v_n = 2.1e\, f_{sp}^{0.7}\left(\rho\frac{d}{a}\right)^{0.22} + 0.8\left(0.41\tau F\right)^{0.97}$ e = 1 for a/d > 3.5 e = 3.5d/a for other case
Greenough and Nehdi (2008)	$v_n = 0.35\left(1 + \sqrt{\frac{400}{d}}\right) f_c^{'0.18}\left(\left(1+F\right)\rho\frac{d}{a}\right)^{0.4} + 0.90\eta_o\,\tau F$

Table 5.6: Shear strength analysis.

Beam ID	V_n^{Exp} (kN)	V_n^{ACI} (kN)	V_n^{CSA} (kN)	V_n^{Exp} / V_n^{ACI}	V_n^{Exp} / V_n^{CSA}
NNB	71.7	69.5	66.3	1.03	1.08
LNB	67.4	58.6	55.8	1.15	1.21
NHB	115.6	94.2	89.7	1.23	1.29
LHB	111.8	76.9	73.3	1.45	1.53
NNB35	156.9	68.7	58.6	2.28	2.68
NHB35	178.9	91.7	78.2	1.95	2.29
			Mean	1.52	1.68
			STDEV	0.50	0.65
			COV	0.33	0.39

Table 5.7: Ratios of the experimental shear strengths to prediction values.

Beam ID	Sharma	Ashour et al.	Li et al.	Khuntia et al.	Shin et al.	Kwak et al.	Greenough and Nehdi
NNB	0.66	0.71	0.47	1.05	0.98	0.60	0.70
LNB	0.62	0.67	0.45	1.00	0.94	0.58	0.66
NHB	0.78	0.86	0.73	1.25	1.56	0.96	1.02
LHB	0.79	0.86	0.71	1.26	1.49	0.92	1.00
NNB35	1.46	0.90	0.85	1.36	1.18	0.83	1.02
NHB35	1.24	0.87	0.95	1.16	1.33	0.94	1.09
Mean	0.93	0.81	0.69	1.18	1.25	0.80	0.92
STDEV	0.34	0.09	0.20	0.14	0.26	0.17	0.19
COV	0.37	0.12	0.29	0.12	0.21	0.21	0.20

Table 5.8: Nominal moments analysis.

Beam ID	M_n^{Exp} (kN.m)	M_n^{ACI} (kN.m)	M_n^{CSA} (kN.m)	M_n^{Exp} / M_n^{Code}	
				M_n^{Exp} / M_n^{ACI}	M_n^{Exp} / M_n^{CSA}
NNB	73.1***	145.8	144.6	0.47	0.48
LNB	68.7***	145.4	144.3	0.47	0.48
NHB	117.9***	153.2	152.0	0.77	0.78
LHB	114.0***	152.4	151.3	0.75	0.75
NNB35	160.0***	149.1	147.9	1.07	1.08
LNB35	152.0	150.7	149.5	1.01	1.02
NHB35	182.5***	157.2	156.0	1.16	1.17
LHB35	179.2	157.5	156.2	1.14	1.15
NNB60	175.2	150.1	148.9	1.17	1.18
LNB60	168.4	149.6	148.4	1.13	1.13
NHB60	206.3	158.9	157.6	1.27	1.28
LHB60	194.3	157.3	156.0	1.24	1.25
			Mean	0.97	0.98
			STDEV	0.28	0.29
			COV	0.29	0.29

M_n^{Exp}** Nominal flexural moment was smaller than it should be due to a sudden shear failure before reaching the flexural capacity.

Table 5.9: Crack spacing values at serviceability limit state.

At service loads							
Beam ID	Service load P_s † (kN)	Stress in longitudinal bars f_s (MPa)	Steel strain ε_s	Min crack spacing (mm)	Max crack spacing (mm)	Average crack spacing (mm)	Number of cracks
NNB	28.7	40.0	0.0002	145	189	172	5
LNB	27.0	80.0	0.0004	154	254	215	4
NHB	46.3	100.0	0.0005	124	160	172	5
LHB	44.7	180.0	0.0009	110	248	172	5
NNB35	62.7	120.0	0.0006	131	189	143	6
LNB35	59.6	200.0	0.0010	105	187	172	5
NHB35	71.6	180.0	0.0009	116	207	143	6
LHB35	70.3	220.0	0.0011	87	197	122	7
NNB60	68.7	220.0	0.0011	47	152	107	8
LNB60	66.0	240.0	0.0012	92	171	107	8
NHB60	79.3	280.0	0.0014	107	138	143	6
LHB60	76.2	320.0	0.0016	68	146	107	8

P_s † Service Load = 0.4 P_u.

Table 5.10: Crack widths analysis.

Beam ID	At service loads					
	Max measured crack width W_{max} (mm)	Average measured crack width W_{avg} (mm)	Max crack width - ACI W_{max} Gergely and Lutz (1973) (mm)	Max crack width - ACI W_{max} Frosch (1999) (mm)	W_{Exp}/W_{ACI}	$W_{Exp}/W_{ACI\text{-}Frosch}$
NNB	0.20	0.20	0.21	0.10	0.95	2.08
LNB	0.23	0.21	0.42	0.19	0.50	1.09
NHB	0.18	0.18	0.52	0.24	0.34	0.75
LHB	0.18	0.18	0.94	0.43	0.19	0.42
NNB35	0.09	0.09	0.63	0.29	0.14	0.31
LNB35	0.11	0.11	1.05	0.48	0.11	0.23
NHB35	0.07	0.07	0.94	0.43	0.07	0.16
LHB35	0.10	0.09	1.15	0.53	0.08	0.17
NNB60	0.11	0.10	1.16	0.53	0.09	0.19
LNB60	0.12	0.11	1.26	0.58	0.09	0.19
NHB60	0.11	0.11	1.47	0.67	0.08	0.16
LHB60	0.12	0.11	1.68	0.77	0.07	0.14
				Mean	0.23	0.49
				STDEV	0.26	0.57
				COV	0.86	0.86

Table 5.11: Crack spacing values at failure.

At failure						
Beam ID	Load P_f (kN)	Max. concrete ε_c	Min crack spacing (mm)	Max crack spacing (mm)	Average crack spacing (mm)	Number of cracks
NNB	70.1	0.0015	95	189	122	7
LNB	67.1	0.0016	115	167	143	6
NHB	114.9	0.0011	70	175	122	7
LHB	110.3	0.0017	66	165	86	10
NNB35	155.7	0.0022	83	164	122	7
LNB35	146.1	0.0037	44	105	66	13
NHB35	178.7	0.0020	53	207	107	8
LHB35	169.4	0.0039	50	197	78	11
NNB60	165.5	0.0038	47	152	86	10
LNB60	164.1	0.0038	38	132	71	12
NHB60	192.7	0.0036	82	138	95	9
LHB60	178.9	0.0039	25	89	57	15

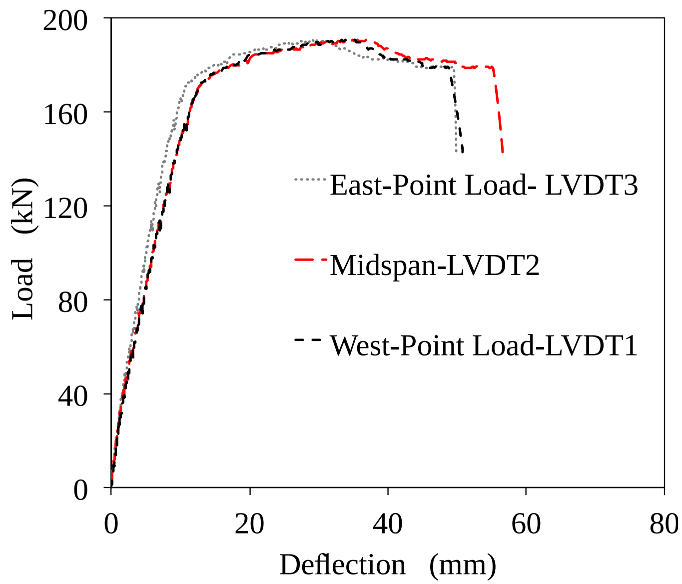


Figure 5.1: Typical load-deflection curves for beams failed in flexure (LHB60).

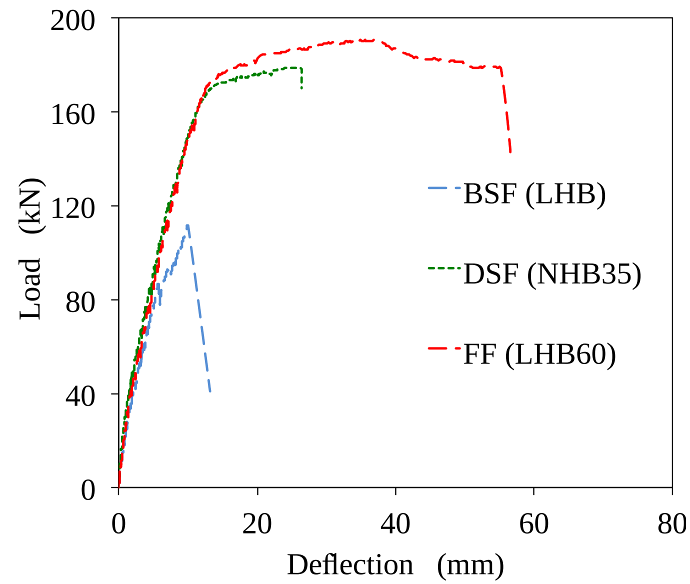


Figure 5.2: General modes of failure.

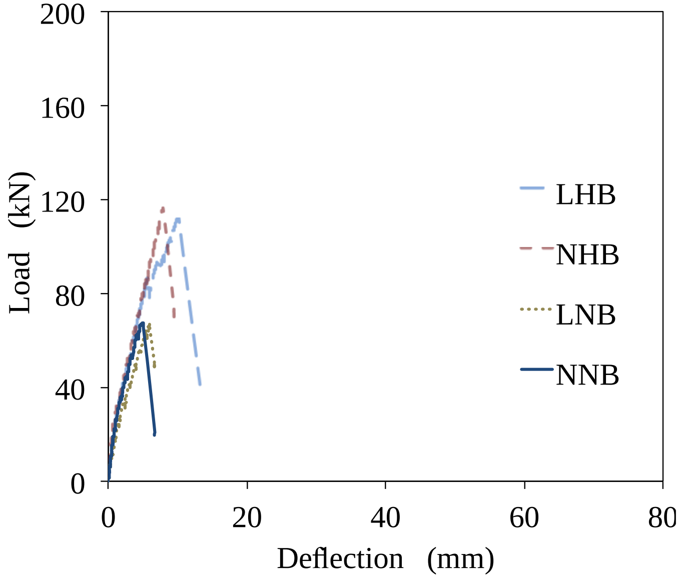


Figure 5.3: Load-deflection curves (Group 1).

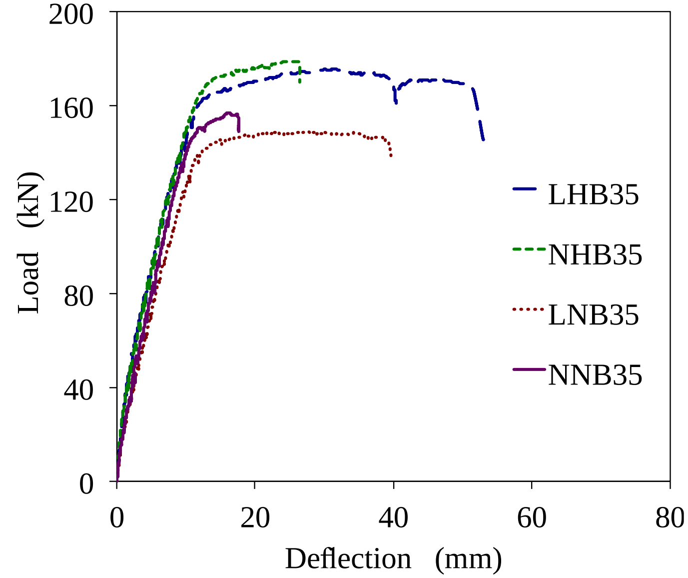


Figure 5.4: Load-deflection curves (Group 2).

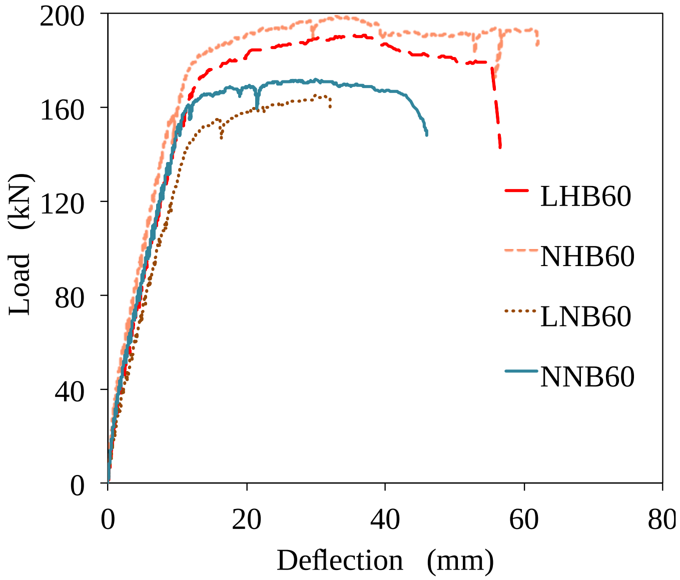


Figure 5.5: Load-deflection curves (Group 3).

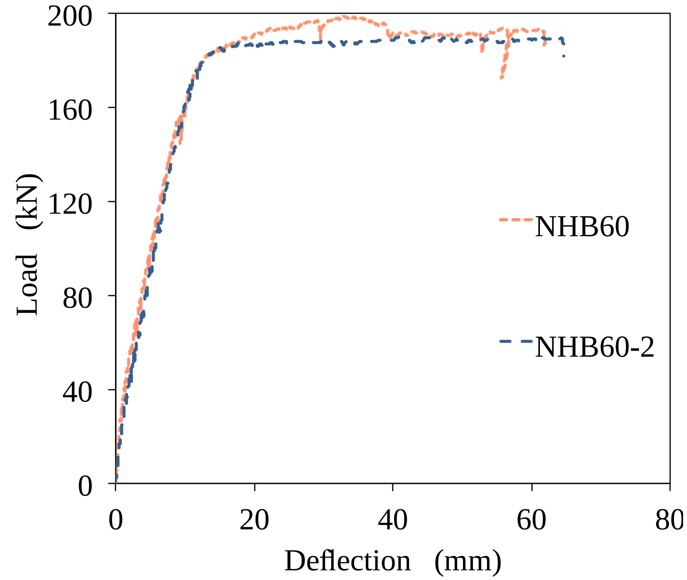


Figure 5.6: Load-deflection curves for the repeated specimen.

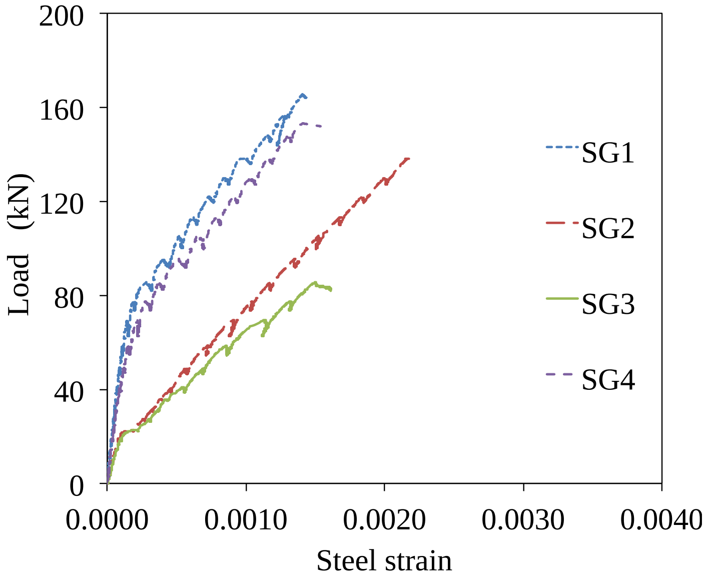


Figure 5.7: Typical load-steel strain curves (NHB60).

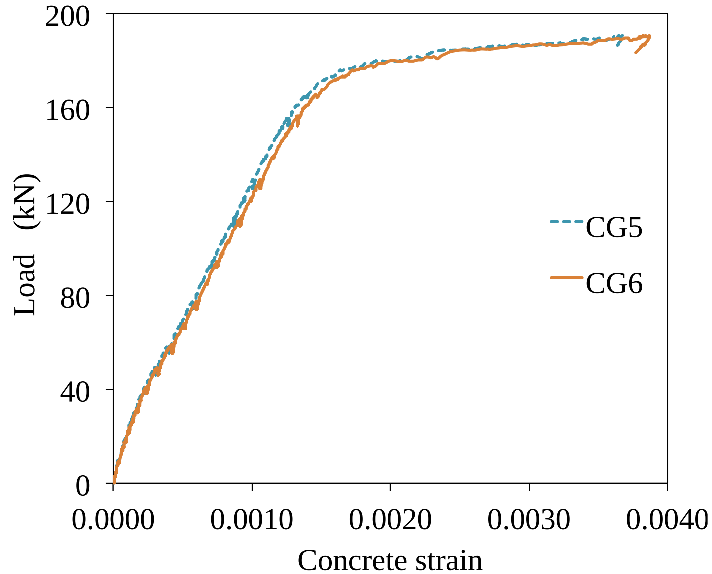


Figure 5.8: Load-concrete strain curves (LHB60).

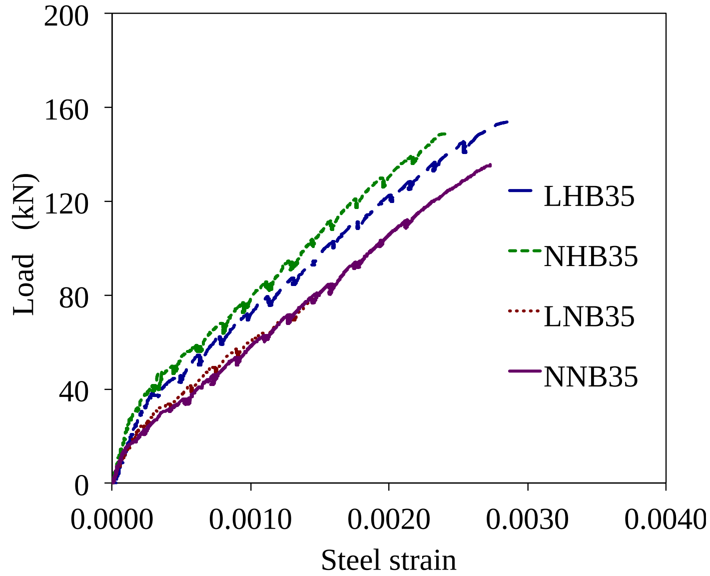


Figure 5.9: Typical load-steel strain curves (Group 2).

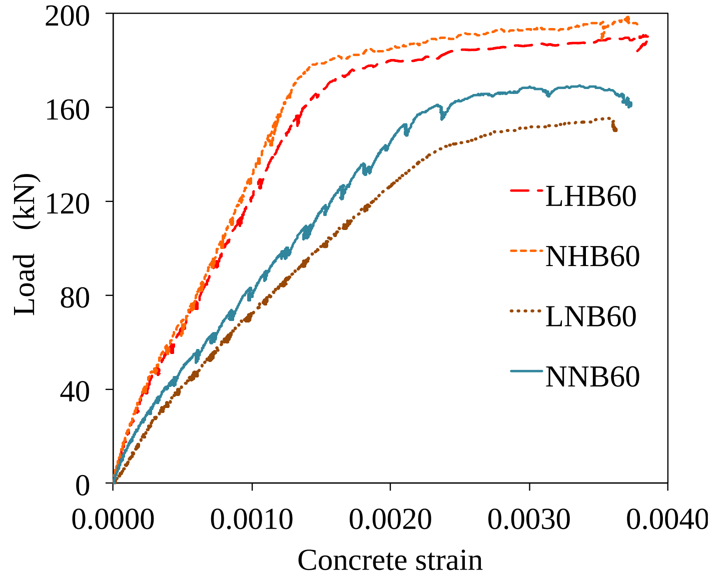


Figure 5.10: Load-concrete strain curves (Group 3).

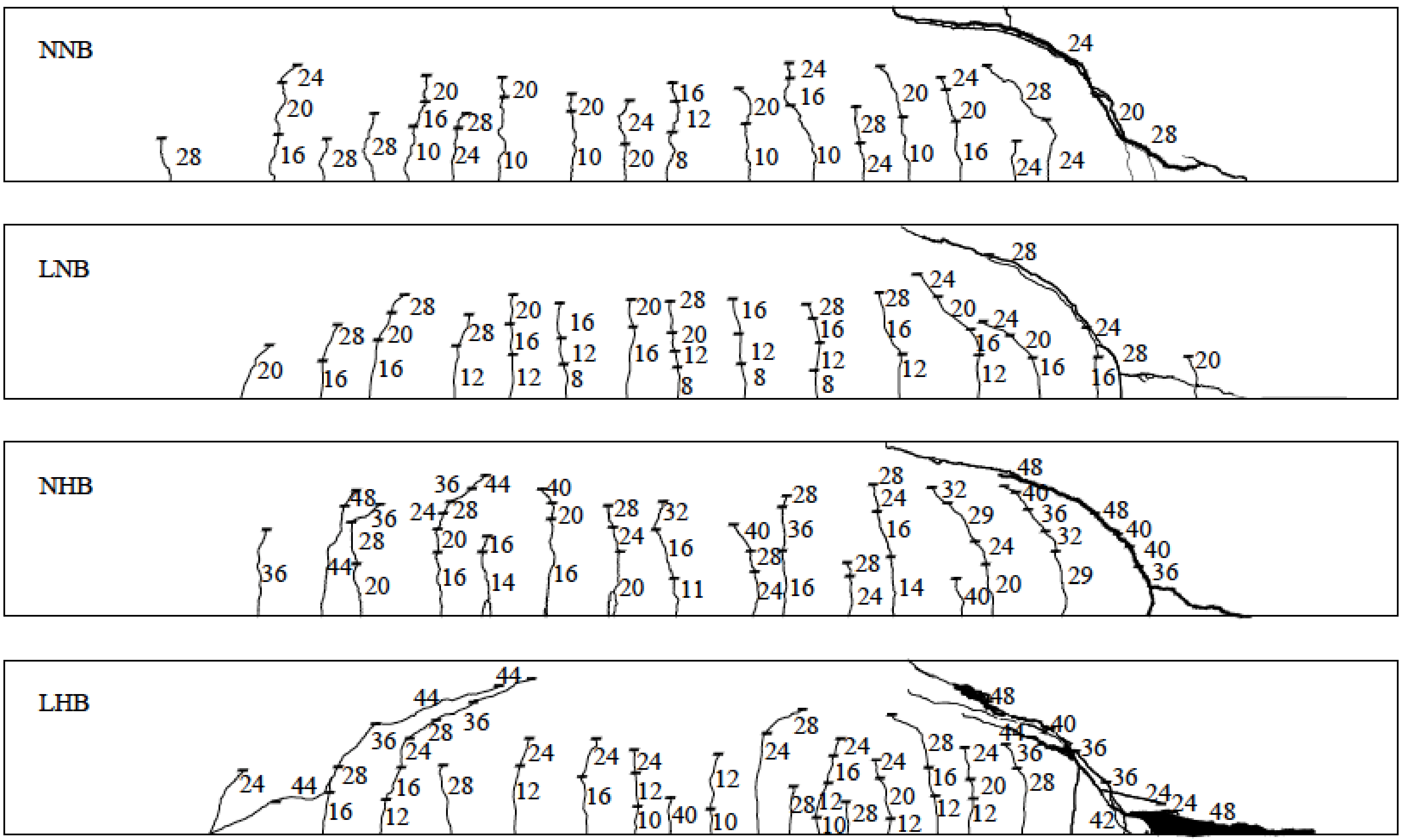


Figure 5.11: Crack patterns at failure of reference concrete beams (Group 1).

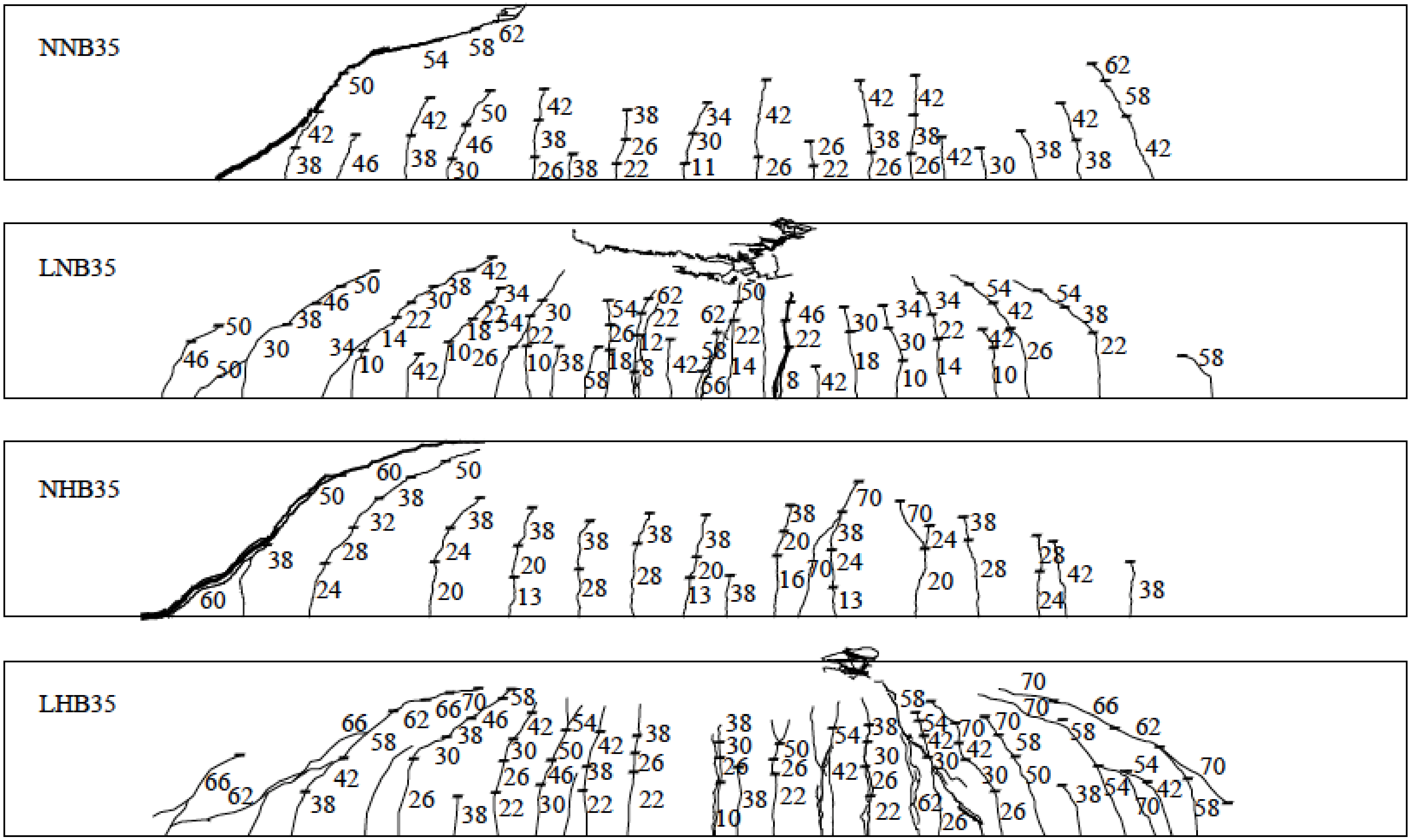


Figure 5.12: Crack patterns at failure of SFRC beams with short fibres (Group 2).

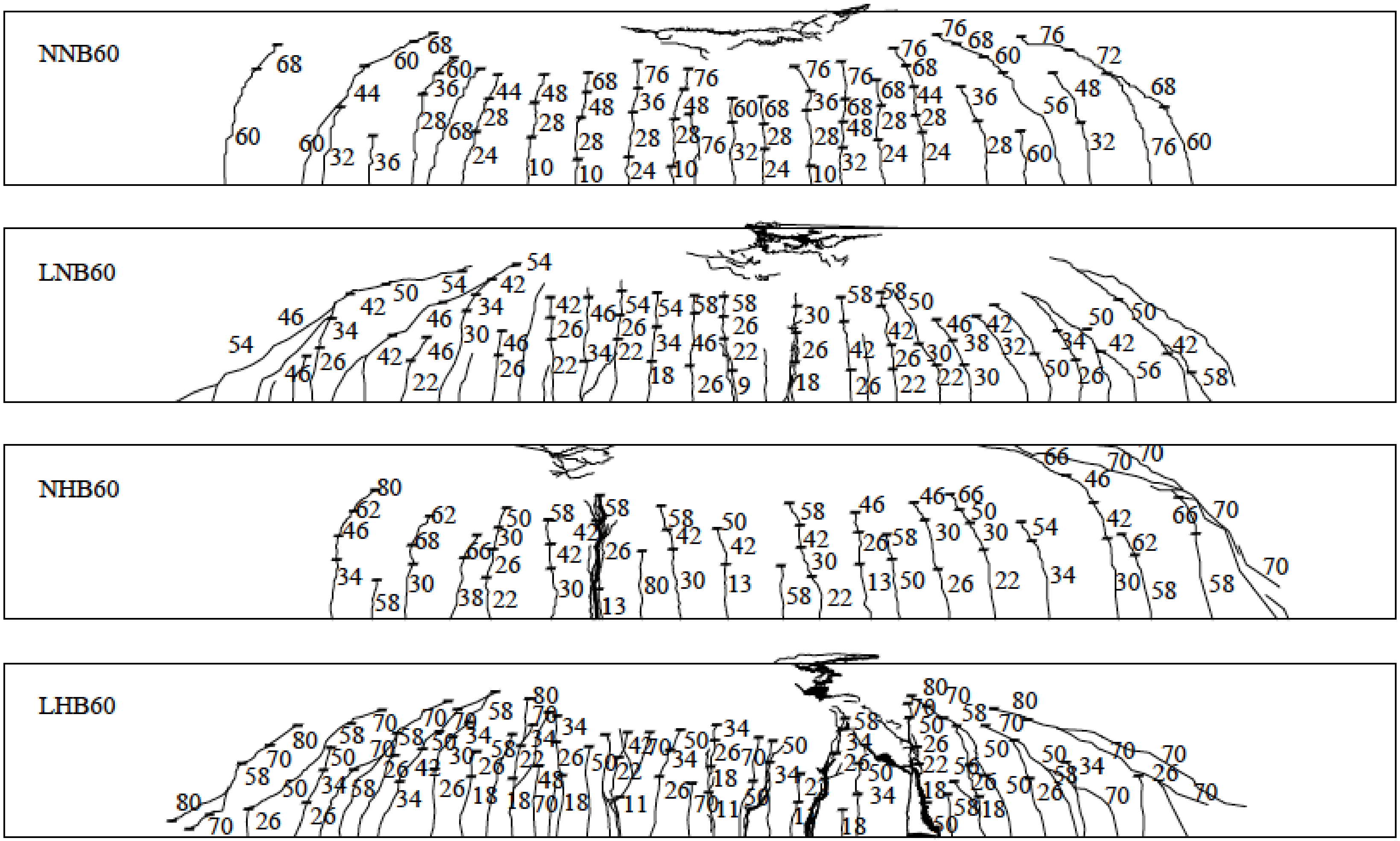
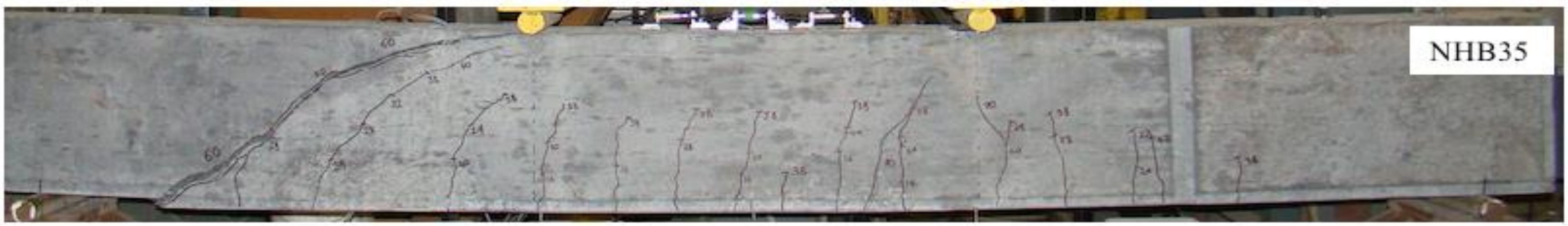


Figure 5.13: Crack patterns at failure of SFRC beams with long fibres (Group 3).

(a) Brittle shear-tension mode of failure.



(b) Ductile shear-tension mode of failure.

(c) Flexural mode of failure.

Figure 5.14: Modes of failure.

Figure 5.15: Repeated specimens (NHB60 vs NHB60-2).

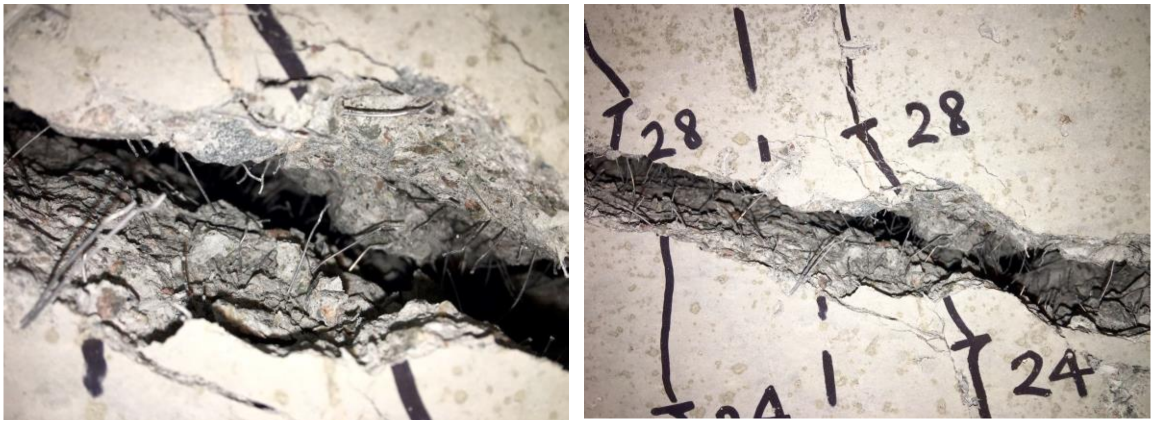


Figure 5.16: Pullout action of short steel fibres at failure.

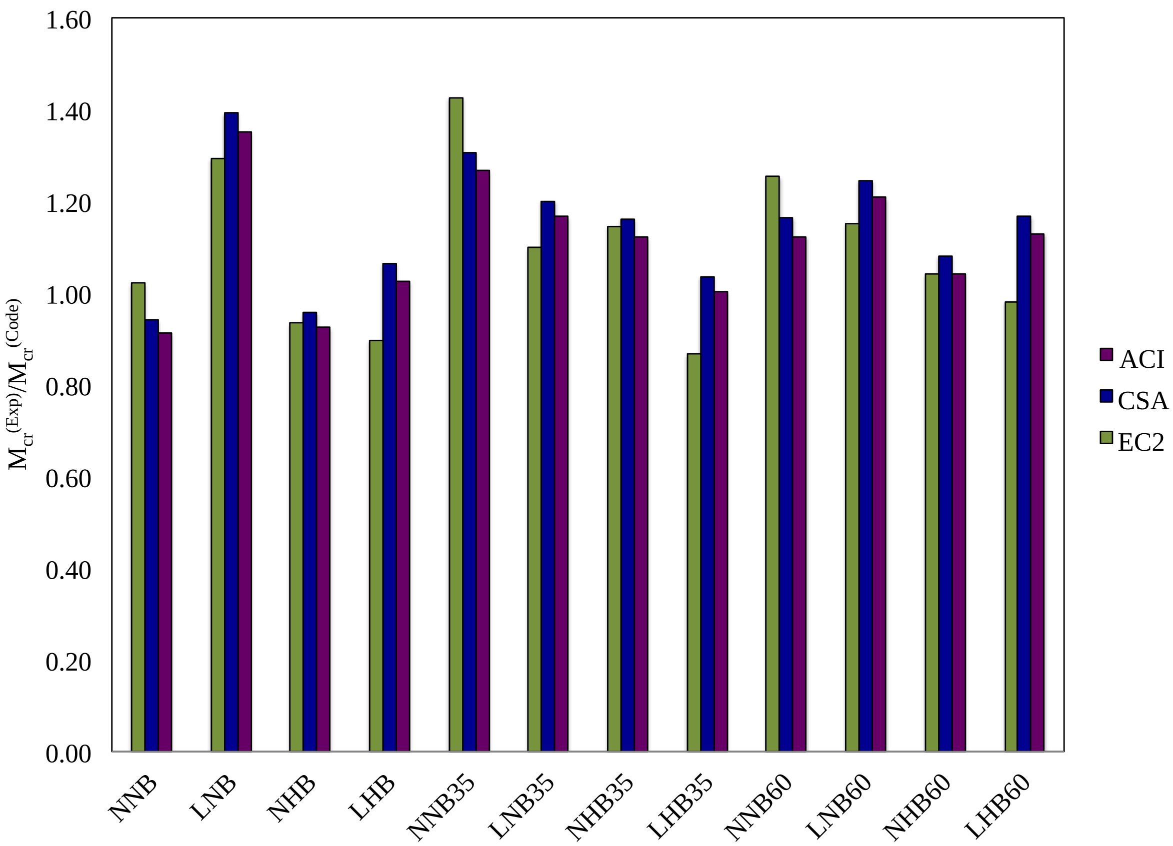


Figure 5.17: Experimental cracking moment vs. codes predicted values.

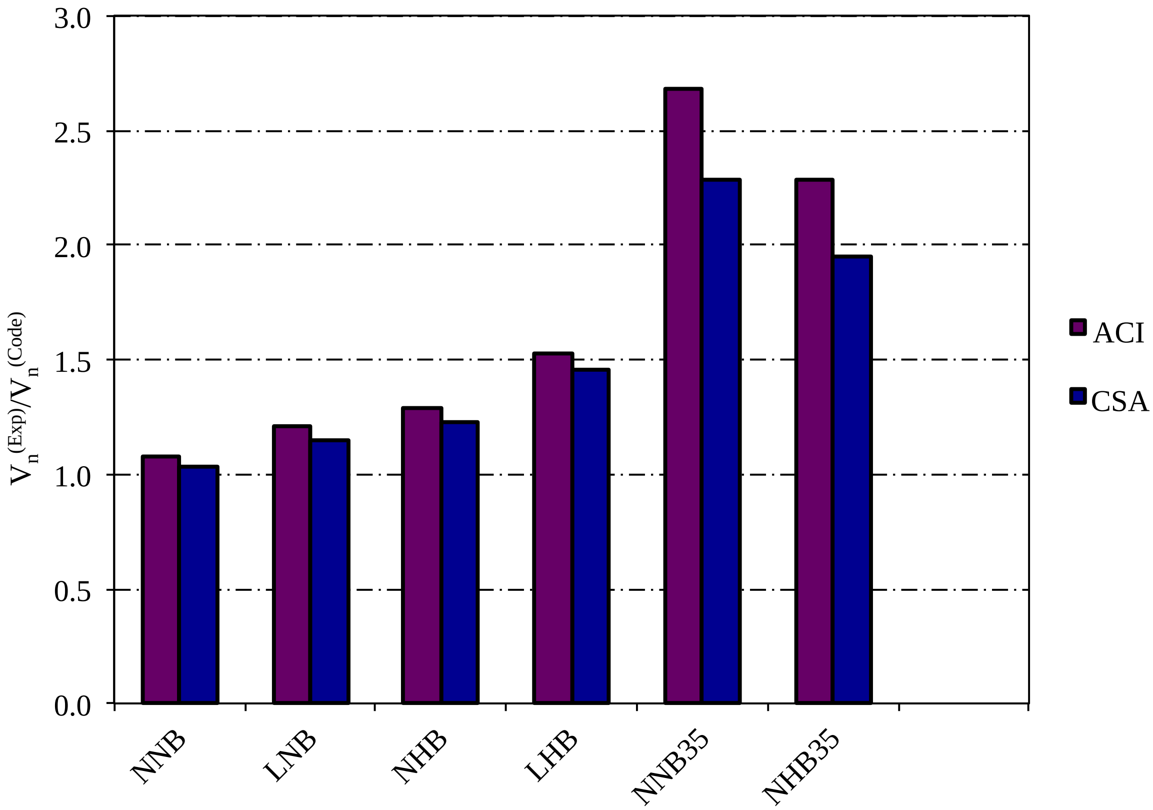


Figure 5.18: Experimental shear strengths vs. models predicted values.

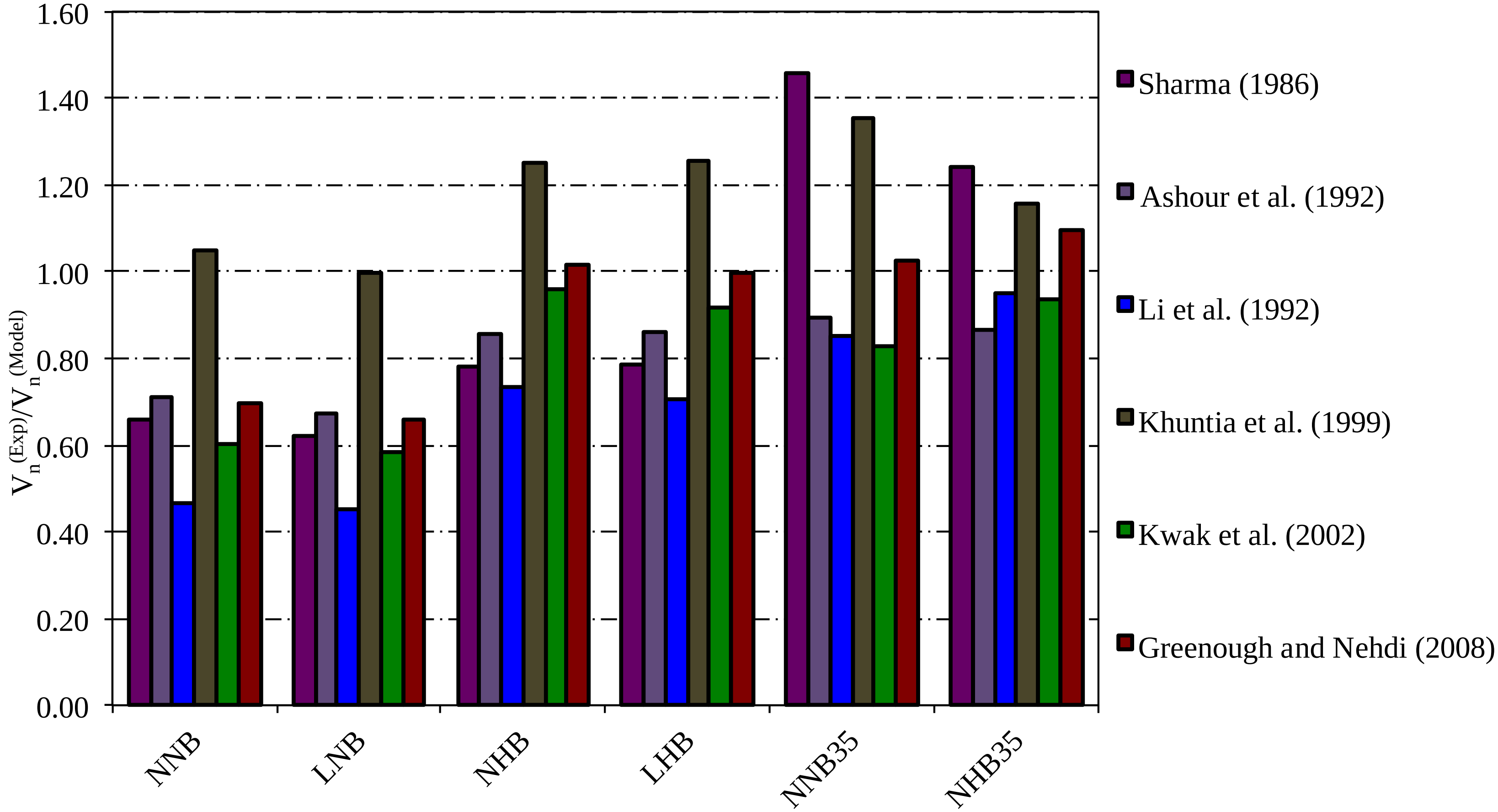


Figure 5.19: Experimental shear strengths vs. codes values.

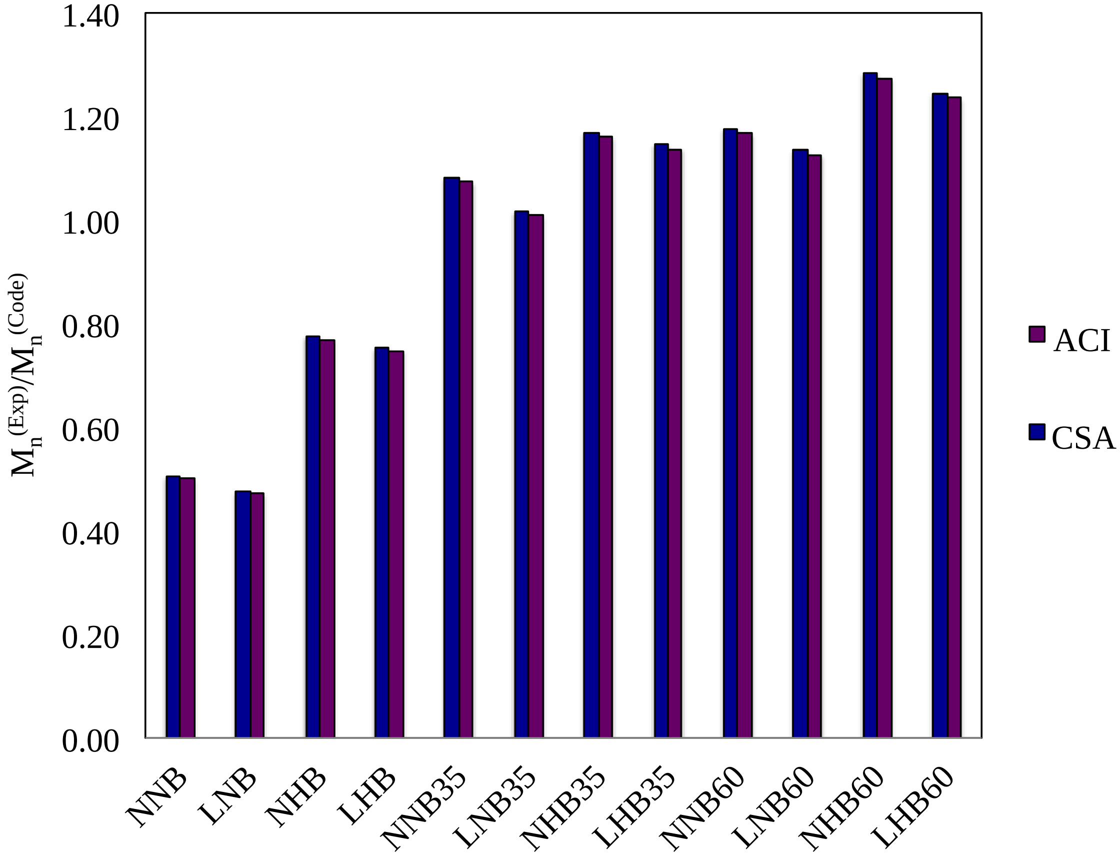


Figure 5.20: Experimental nominal moments vs. codes predicted values.

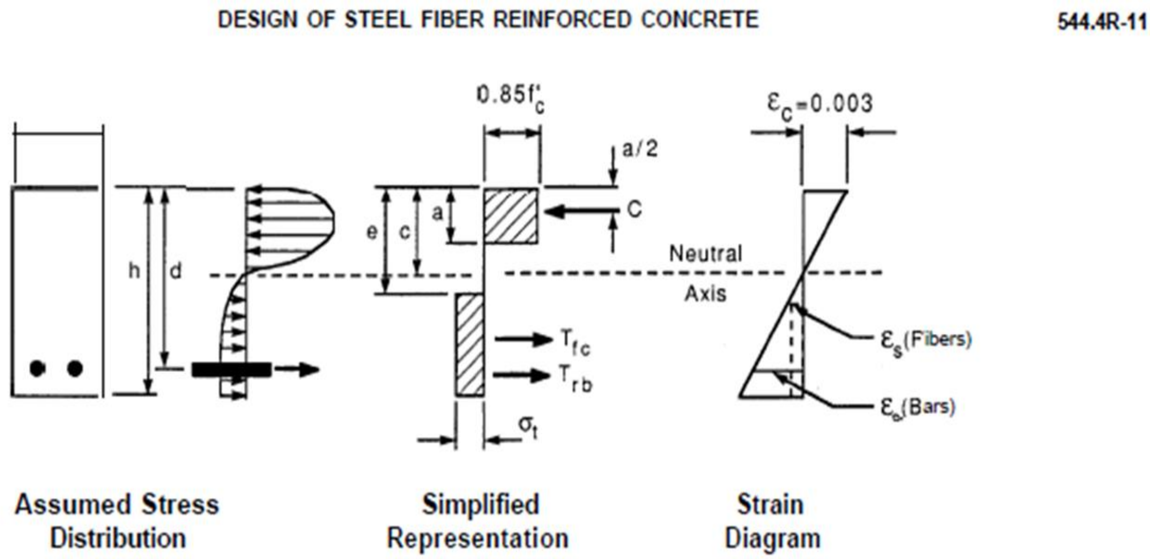


Figure 5.21: Design assumptions for analysis of singly reinforced concrete beams containing steel fibres [Adopted from Henager and Doherty 1976].

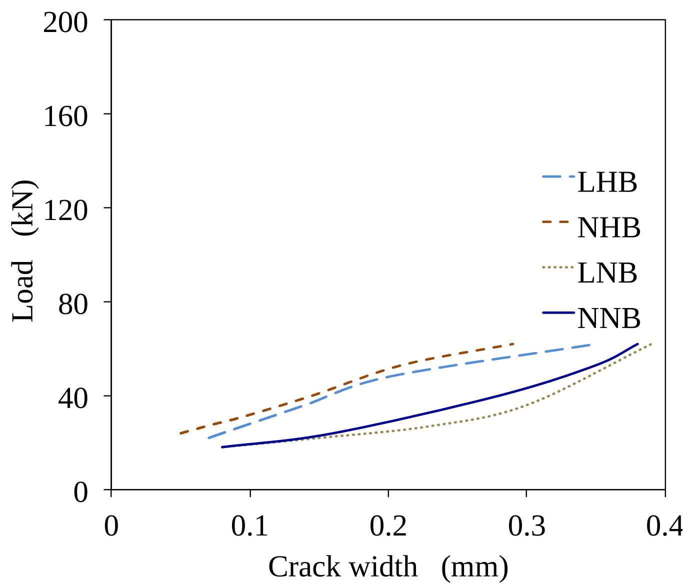


Figure 5.22: Load versus crack width of reference specimens (Group 1).

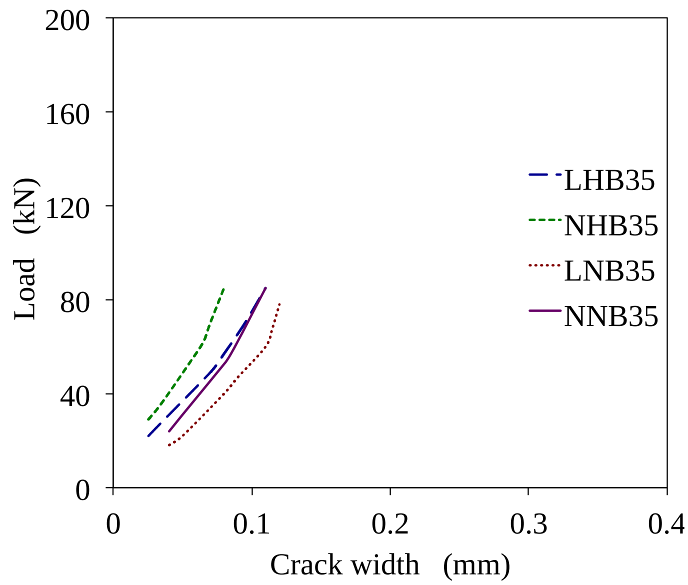


Figure 5.23: Load versus crack width at serviceability of SFRC beams with short fibres (Group 2).

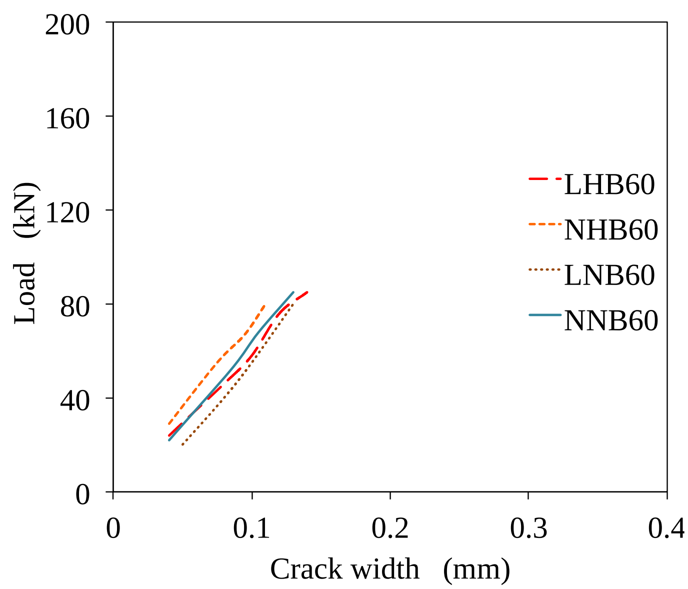


Figure 5.24: Load versus crack width at serviceability of SFRC beams with long fibres (Group 3).

Chapter Six

Summary and Conclusions

6.1 General

Steel fibre reinforced lightweight concrete can be a great composite material, which its advantages may outweigh its disadvantages. This construction material can be applied either in offshore or onshore structural projects. This comprehensive research provided preliminary evidence of the enhancement on shear behaviour of steel fibres with lightweight concrete beams with satisfactory effective depths. Most published studies in the field of steel fibres reinforced concrete beams have been respectfully reviewed and criticized in term of testing smaller specimens than those tested in this thesis.

Six steel fibre reinforced concrete beams of normal weight and six lightweights were conducted. The study is also done on normal strength and high strength beams in each category. The steel fibres used were 0% and 0.75% in each category. The lengths of the fibres used were 35 mm and 60 mm in each category. The longitudinal reinforcement ratio in all the beams is kept at 1.46%. The effect of types of aggregates, length of steel fibres, and concrete compressive strength were studied and results were presented with regard to the shear and flexure strengths, load-deflection responses, mode of failure, stiffness, energy absorption, and ductility. Shear and flexural crack widths and cracking patterns of the beams were also presented. Critical fibre content of the beams was also discussed. The possibility of replacement of minimum shear reinforcement for lightweight SFRC, given in ACI-318 Code, with steel fibres is discussed. The most efficient length of fibres for this purpose was presented.

Results of the experimental research exhibited that the addition of steel fibres enhanced the shear behaviour of both LWC and NWC beams. The presence of steel fibres usually exchanged the brittle behaviour of beams without web reinforcement to more ductile performance. Although some beams with short steel fibres failed in shear, they continued resisting higher capacity and producing better ductility. Steel fibres mitigated the brittleness property in those two SFs beams failed in shear. Moreover, steel fibres restrained the weakness of LWC interlock by keeping cracked segments stitched apart. Overall, long steel fibres were more efficient than short steel fibres in terms of load-deflection responses, stiffness, energy absorption, ductility, flexural resistance, and shear capacity. Short steel fibres were more effective than long steel fibres in controlling crack widths. The following conclusions and recommendations can be drawn from this experimental investigation:

1. All types of steel fibres transformed the brittle mode of failure to more ductile collapse. However, ductile steel fibrous beams should not always fail in flexural mode of failure especially with short steel fibres. Long steel fibres (double-hooked end with 60 mm long) always replaced the shear mode of failure by a flexural manner. Flexural mode of failure governed all specimens with long double hooked steel fibres.
2. Under four point bending test with (a/d = 3 and $\rho = 1.46\%$), crack patterns tended to quickly penetrate through LW aggregates with lower slopes compared to the inclination of cracks of NWC beams. This behaviour was attributed to the lower interlock resistance generated of LW aggregates that permit smoother straight

cracks through it upward to the load points. Big brittle failure significantly occurred in reference LWC beam with high concrete grade. This was attributed to the lower bond expected between LW aggregates with such stiffer mortar in high concrete grade.

3. Under four point bending test with (a/d = 3 and $\rho = 1.46\%$), both normal and high strength NWC beams with 0.75% of single-hooked-35 mm long steel fibres tended to fail in ductile shear mode of failure. This was attributed to the insufficient length and the end-shape of fibres that led to a lack of the bond between concrete matrix and steel fibres. Short steel fibres are used; less amount of concrete is expected to surround these fibres. Short steel fibres on the other hand were more significant in resisting shear stresses in LWC beams. This was attributed to the higher expected fusion between lightweight aggregates and the mortar that mitigated the lower interlock of lightweight aggregates and improved the bond between LWC and streel fibres with short length.
4. SFs beams experienced much more tension stiffening contribution than those beams without SFs subjected to four point bending test with (a/d = 3 and $\rho = 1.46\%$). This enhancement was attributed to the transformation in the mechanisms of crack development to more stabilized crack patterns.
5. Long steel fibres (60 mm-double-hooked end) tended to efficiently perform better than the short fibres (35 mm-single-hooked end) in strengthening capacities and deformation of beams subjected to four point bending test with (a/d = 3 and $\rho = 1.46\%$). This was attributed to the fact that more concrete was between these individual long fibres, within the core structure of the beam, which provided

higher resistance and displacement results. Although the presence of both short and long steel fibres improved beams flexural capacity by a range varied from 35% to 72% compared to reference RC beams, flexural moment capacity of beams with long steel fibres higher enhancement by an average amount of 10% in contrast with short SFs beams. Normal weight concrete specimens with steel fibres were slightly more ductile than steel fibre reinforced lightweight concrete members. This was attributed to the greater bond produced by the higher interlock resistance of normal weight aggregates.

6. Both short and long steel fibres maintained a maximum crack width at serviceability limit state of less than 0.18 mm. However, short steel fibres seemed better than long fibres in term of reducing and controlling crack widths. SFs with 35 mm long reduced crack width by about 55% on average while crack widths of those beams with long steel fibres were reduced only by 45% approximately. This was attributed to the larger o of shrt steel fibres can be within to resist the expansion of the crack, while long steel fibres would be in less number to resist enlargement of crack thicknesses. Adding 0.75% of long or short steel fibres reduced spacing. Therefore, number of cracks was increased in all beams with fibres either short or long ones.
7. In beams subjected to four point bending test with (a/d = 3 and $\rho = 1.46\%$), single-hooked end with 35 mm long can highly replace shear reinforcement in LWC beams with effective depth larger than 300 mm. However, this type of steel fibres is not recommended to replace stirrups in NWC beams. This was attributed to that the short steel fibres are expected to have insufficient overlap and the poor

end-shape that led to a lack of the bond between concrete matrix and steel fibres. Short steel fibres are used; less amount of concrete is expected to surround these fibres. Short steel fibres on the other hand were significant in resisting shear stresses in LWC beams. This was attributed to the higher expected fusion between lightweight aggregates and the mortar that mitigated the lower interlock of lightweight aggregates and improved the bond between LWC and streel fibres with short length. On the other hand, double-hooked end with 60 mm long can highly replace stirrups in both NWC and LWC beams.

8. LWC beams were expected to show low interlock capacity for beams with effective depth larger than 300 mm and subjected to four point bending test with (a/d = 3 and $\rho = 1.46\%$), yet the negative property was banned by the addition of steel fibres to those beams.
9. The transformation from normal concrete grade to a higher strength increased the load capacity of beams by 41%. Lightweight aggregates slightly dropped this load capacity for beam with the same concrete grades.
10. Although the slight drop by 3% of maximum shears strength of reference ordinary LWC with high strength sample was attributed to the lack of fusion between the very stiff mortar and lightweight aggregates with less stiffness. Even though steel fibre reinforced lightweight concrete beams generally showed lower shear resistance than normal weight concrete ones with steel fibres, the type of aggregate was not that significant factor to be aware of in term or beams shear strength and the low interlock resistance of LWC was mitigated by the presence of steel fibres.

11. Normal strength beams with steel fibres irrespective of type of aggregates showed less shear capacity compared to high strength specimens with the same properties. Although the presence of both short and long steel fibres improved beams shear resistance by a range varied from 35% to 72% compared to reference RC beams, shear strength of beams with long steel fibres enhanced by an average amount of 10% in contrast with short SFs beams. The greater bond produced between longer double-hooked fibres was attributed to the higher aspect ratio generated by the longer length and concrete matrix. Regarding length of steel fibres, ACI 318-14 code puts the requirements as: the ratio of length and diameter should be more than 50 and less than 100. The length of the fibres satisfying this requirement are the most efficient to replace minimum shear reinforcement of flexural members.
12. The workability of lightweight concrete would be negatively affected by the addition of minimum recommended amount of steel fibres before adding the sufficient quantity of workability admixtures.
13. Although some specimens showed that the onset of first flexural and shear cracks occurred earlier in members with long double-hooked steel fibres than in beams with short fibres, these specimens with long double hooked steel fibres showed higher capacity and greater deflection at failure. This was attributed to the larger number of short steel fibres exist within the section to resist the opening of first cracks, while long steel fibres would be in less quantity to resist initial cracks.
14. Adding the recommended minimum amount of 0.75% of either length of steel fibres to lightweight concrete members can efficiently replace stirrups and change the mode of failure from brittle to ductile form just like in normal weight beam

except using short fibres is not highly recommended in NWC beams to avoid the possibility of ductile shear failures.

6.2 Discussion on critical fibre content

The quantity of fibre (percentage by volume of concrete) to be used in a beam, so that it will serve the intended purpose in the most efficient manner is called “Critical Fibre Content.” If the quantity of fibre is less than the critical value, then the resistance to shear and response to ductility failure will not be effective. On the other hand, if the quantity of fibre is more than the critical value, naturally it will not be economical. Also, quantity more than critical value will not increase flexural strength. Crack control and ductility may be improved. Workability also has a role to play here. Too much of fibre quantity adversely affects workability. Critical Fibre Quantity thus depends on many factors, Cohen (2012). Parra (2006) made an extensive study of Test Results of SFRC Beams available in literature. This parametric study revealed that if the percentage of steel fibres in SFRC beams is equal to or greater than 0.5% (by volume), then the shear strength (failure shear stress) in MPa is greater than ($0.17\sqrt{f_c'}$) where f_c' is the compressive strength of concrete in MPa. This value ($0.17\sqrt{f_c'}$) is the shear strength to be provided by the minimum shear reinforcement in flexural members as per ACI 318-14 code. However, the percentage of steel fibres recommended replacing the minimum shear reinforcement as per ACI-318 code is 0.75% (the increased value is due to application of factor of safety). With more than or equal to 0.75% steel fibres by volume of concrete, the shear strength of concrete beam will not be less than ($0.3\sqrt{f_c'}$) MPa, according to Parra (2006). ACI has permitted SFRC with 0.75% (by volume) fibres in concrete as a substitute for minimum shear reinforcement requirement of flexural members subject to satisfying certain conditions regarding loading, dimensions and material. The increased shear capacity for Vf = 0.75% demonstrated that the provision of ACI 318 M-14 on using steel fibre as a minimum shear reinforcement can be further extended to SFR lightweight concrete beams with fc' up to 70 MPa.

6.3 Potential Upcoming Studies

Future studies may consider the following suggestions:

1. Combinations between an adequate amount of long and short steel fibres or placing stirrups with steel fibres are recommended especially in normal weight concrete beams.

2. Further research may focus on using long double-hooked versus single-hooked steel fibres in normal weight and lightweight concrete applications.

3. Further research may focus on studying the shear behaviour of bigger-sized SFRC beams either normal weight or lightweight concrete.

4. Further research may focus on pouring three quarters deep from the bottom tip of the beam with SFRC while the remaining quarter may only be poured with ordinary reference concrete.

References

ACI Committee 318, (2014) “Building Code Requirements for Structural Concrete (ACI318-14) and Commentary.” American Concrete Institute, Farmington Hills, Michigan, USA, 473pp.

ASTM C1609/C1609M-05, 2005, “Standard Test Method for Flexural Performance of Fiber-Reinforced Concrete (Using Beam with Third-Point Loading),” ASTM International, West Conshohocken, PA, 8 pp.

ASTM C39/C39M-05, 2005, “Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens,” ASTM International, West Conshohocken, PA, 7 pp.

Alam, Y. S., Lenormand, T., Loukili, A., and Regoin, J. P. (2010). Measuring crack width and spacing in reinforced concrete members. In B. H. Oh, Fracture Mechanics of Concrete and Concrete Structures - Recent Advances in Fracture Mechanics of Concrete (pp. 377- 382). Seoul: Korea Concrete Institute.

Andermatt, M. F., and Lubell, A. S. (2013). Behaviour of Concrete Deep Beams Reinforced with Internal Fibre-Reinforced Polymer — Experimental Study. ACI Structural Journal , 47 (110), 585–594.

Aoude, H., Belghiti, M., Cook, W. D., and Mitchell, D. (2002). Shear Strength of Steel Fibre-Reinforced Concrete Beams without Stirrups. ACI Structural Journal, 99(4). doi:10.14359/12122

Ashour, S. A., Hasanain, G. S., and Wafa, F. F. (1992). Shear Behaviour of High-Strength Fibre Reinforced Concrete Beams. ACI Structural Journal, 89(2), 176–184. https://doi.org/10.14359/2946

ASTM C39/C39M-04a (2004) "Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens, "ASTM International, West Conshohocken, Pa.

Biolzi, L., and Cattaneo, S. (2017). Response of Steel Fibre Reinforced High Strength Concrete Beams: Experiments and Code Predictions. Cement and Concrete Composites, 77, 1-13. doi:10.1016/j.cemconcomp.2016.12.002

Campione, G. (2008). Simplified Flexural Response of Steel Fibre-Reinforced Concrete Beams. Journal of Materials in Civil Engineering, 20(4), 283. https://doi.org/10.1061/(ASCE)0899-1561(2008)20:4(283)

Chalioris, C. E. (2013). Analytical Approach for the Evaluation of Minimum Fibre Factor Required for Steel Fibrous Concrete Beams Under Combined Shear and Flexure. Construction and Building Materials, 43, 317–336. https://doi.org/10.1016/j.conbuildmat.2013.02.039

Chanh Nguyen Van “Steel Fibre Reinforced Concrete.” Retrieved from < http://www.refwin.com/Final_ed/UploadFile/201182015280896.pdf >. Accessed on 29th March 2017.

Cuenca, E. (2015). On Shear behaviour of Structural Elements Made of Steel Fibre Reinforced Concrete. Universitat Politecnica de Valencia (UPV).

Cohen Michael (2012). “Structural Behaviour of Self Consolidating Steel Fibre Reinforced Concrete Beams.” M.S. Thesis, Department of Civil Engineering, University of Ottawa, Canada.

Dinh, H. H., Parra-Montesinos, G. J., and Wight, J. K. (2011). Shear Strength Model for Steel Fibre Reinforced Concrete Beams without Stirrup Reinforcement. Journal of Structural Engineering, 137(10), 1039–1051.

Ding, Y., You, Z., and Jalali, S. (2011). The Composite Effect of Steel Fibres and Stirrups on the Shear Behaviour of Beams Using Self-Consolidating Concrete. Engineering Structures, 33(1), 107–117. https://doi.org/10.1016/j.engstruct.2010.09.023

Gebreyohannes, E., and Muzeyin, S. (2016). Effect of Aggregate Size and Type on Shear Capacity of Normal Strength Reinforced Concrete Beams. AAU, School of Civil and Environment Engineering. AAU.

Gomes, J., Sousa, C., Pimentel, M., Proença, A. M., and Neves, A. S. (2017). Shear Resistance of Concrete Reinforced With Ultra-High Strength Steel Fibres. Ciência and Tecnologia dos Materiais, 29(1). doi:10.1016/j.ctmat.2016.04.004

Guneyisi Erhan, Gesoglu Mehmet, Ozturan Turan, Ipek Suleyman (2015). “Fracture Behaviour and Mechanical Properties of Concrete With Artificial Lightweight Aggregate and Steel Fibre.” Construction and Building Materials 84 pp 156-168. Elsevier Ltd.

Hamrat, M., Boulekbache, B., Chemrouk, M., and Amziane, S. (2016). Flexural Cracking Behaviour of Normal Strength, High Strength and High Strength Fibre Concrete Beams, Using Digital Image Correlation technique. Construction and Building Materials , 678–692.

Holschemacher, K., Mueller, T., and Ribakov, Y. (2010). Effect of Steel Fibres on Mechanical Properties of High-Strength Concrete. Materials and Design (1980-2015), 31(5), 2604-2615. doi:10.1016/j.matdes.2009.11.025

Imam, M., Vandewalle, L., and Mortelmans, F. (1994). Shear Capacity of Steel Fibre High-Strength Concrete Beams. Special Publication. 149, 227 – 242.

Imam, M., Vandewalle, L., and Mortelmans, F. (1995). Shear – Moment Analysis of Reinforced High Strength Concrete Beams Containing Steel Fibres. Canadian Journal of Civil Engineering, 22(3), 462-470. doi:10.1139/l95-054

Imam, M., Vandewalle, L., Moretelmans, F., and Van Gemert, D. (1997). Shear Capacity and Flexural Ductility of Reinforced High- and Normal-Strength Concrete Beams. Engineering Structures, 19(9), 738-747. doi:10.5353/th_b3121444

Imam M., Vandewalle, L. (2000). Role of Fibres in Controlling Failure Modes of High-Stregth Concrete Beams. Repair, Rehabilitation, and Maintenance of Concrete Structures. 193. 503 – 518.

Jain, K., and Singh, B. (2016). Deformed Steel Fibres as Minimum Shear Reinforcement - An investigation. Structures, 7, 126–137. https://doi.org/10.1016/j.istruc.2016.06.003

Jongvivatsakul, P., Matsumoto, K., and Niwa, J. (2013). Shear Capacity of Fibre Reinforced Concrete Beams With Various Types and Combinations of Fibres. Journal of JSCE, 1, 228–241.

Kang, T. H. K., Kim, W., Kwak, Y.-K., and Hong, S.-G. (2011). Shear Testing of Steel Fibre-Reinforced Lightweight Concrete Beams without Web Reinforcement. ACI Structural Journal, 108(5), 553–561.

Kim, C.-G., Lee, H., Park, H.-G., Hong, G.-H., and Kang, S.-M. (2017). Effect of Steel Fibres on Minimum Shear Reinforcement of High-Strength Concrete Beams. ACI Structural Journal, 114(5), 1109-1119.

Kim, K. S., Lee, D. H., Hwang, J. H., and Kuchma, D. A. (2012). Shear Behaviour Model for Steel Fibre-Reinforced Concrete Members Without Transverse Reinforcements. Composites Part B: Engineering, 43(5), 2324–2334. https://doi.org/10.1016/j.compositesb.2011.11.064

Kwak, Y., Eberhard, M. O., Kim, W., and Kim, J. (2002). Shear Strength of Steel Fibre-Reinforced Concrete Beams without Stirrups. ACI Structural Journal, 99(4), 530–538.

Lee, S. J., and Won, J. P. (2014). Flexural Behaviour of Precast Reinforced Concrete Composite Members Reinforced With Structural Nano-Synthetic and Steel Fibres. Composite Structures, 118(1), 571–579. https://doi.org/10.1016/j.compstruct.2014.07.042

Minelli, F., Conforti, A., Cuenca, E., and Plizzari, G. (2013). Are Steel Fibres Able to Mitigate or Eliminate Size Effect in Shear? Materials and Structures/Materiaux et Constructions, 1–15. https://doi.org/10.1617/s11527-013-0072-y

Md. Shah Alam (2010). “Influence of Different Parameters on Shear Strength of FRP Reinforced Concrete Beams.” PhD. Thesis, Department of Civil Engineering, Memorial University of Newfoundland, Canada.

Nasvik Joe (2012). “How To Use Steel Fibres in Concrete.” Retrieved from http://www.concreteconstruction.net/how-to/materials/how-to-use-steel-fibres-in-concrete_o

Nguyen-Minh, L., and Rovňák, M. (2011). New Formula for the Estimation of Shear Resistance of Fibre Reinforced Beams. Canadian Journal of Civil Engineering, 38(1), 23–35. https://doi.org/10.1139/L10-107

Noghabai, K. (2000). Beams of Fibrous Concrete in Shear and Bending: Experiment and Model. Journal of Structural Engineering, 126(2), 243-251. doi:10.1061/(asce)0733-9445(2000)126:2(243)

Parra-Montesinos, G. J. (2006). Shear Strength of Beams With Deformed Steel Fibres. Concrete International, 28(11), 57–66.

Padmarajaiah, S. K., and Ramaswamy, A. (2001). Behaviour of Fibre-Reinforced Prestressed and Reinforced High-Strength Concrete Beams Subjected to Shear. ACI Structural Journal, 98(5). doi:10.14359/10629

Piyasena, R., Loo, Y.-C., & Fragomeni, S. (2004). Factors Influencing Spacing and Width of Cracks in Reinforced Concrete; New Prediction Formulae. Advances in Structural Engineering , 7 (1), 49-60.

Raj, L., & Rao, A. G. (2015). Shear Strength of RC Deep Beam Panels – A Review. International Journal of Research in Engineering and Technology, 03(16), 89-103. doi:10.15623/ijret.2014.0328015

Shoaib, A., Lubell, A. S., and Bindiganavile, V. S. (2014). Size Effect in Shear for Steel Fibre-Reinforced Concrete Members without Stirrups. ACI Structural Journal, (111), 1081–1090. https://doi.org/10.14359/51686813

Shoaib, A., Lubell, A. S., and Bindiganavile, V. S. (2015). Shear Response of Lightweight Steel Fibre Reinforced Concrete Members Without Stirrups. Materials and Structures, 48(10), 3141–3157. https://doi.org/10.1617/s11527-014-0387-3

Singh, B., and Jain, K. (2014). Appraisal of Steel Fibres as Minimum Shear Reinforcement in Concrete Beams. ACI Structural Journal, 111(5), 1191–1202. https://doi.org/10.14359/51686969

Singh, H. (2014). Flexural Modeling of Steel Fibre-Reinforced Concrete Members : Analytical Investigations. ASCE Practice Periodical on Structural Design and Construction, 1–10. https://doi.org/10.1061/(ASCE)SC.1943-5576.0000244.

Swamy, R. N., Jones, R., and Chiam, A. T. P. (1993). Influence of Steel Fibres on the Shear Resistance of Lightweight Concrete I- beams. ACI Structural Journal, 90(1), 103–114.

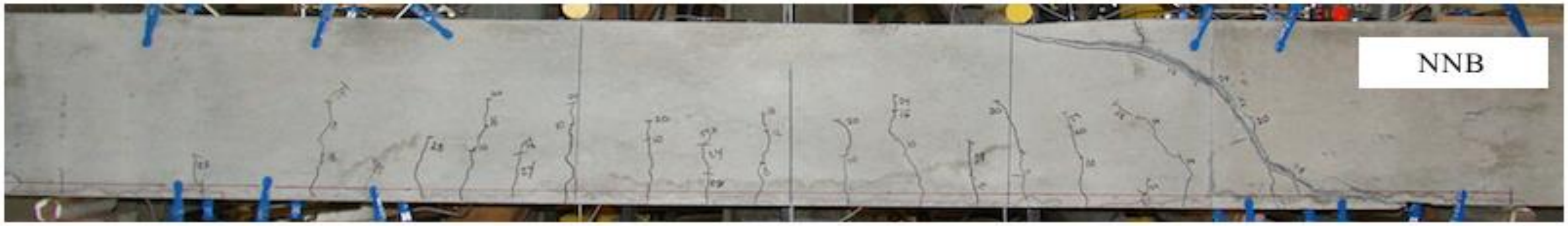
Tahenni, T., Chemrouk, M., and Lecompte, T. (2016). Effect of Steel Fibres on the Shear Behaviour of High Strength Concrete Beams. Construction and Building Materials, 105, 14–28. https://doi.org/10.1016/j.conbuildmat.2015.12.010

Tang, C.-W., Yen, T., and Chen, H.-J. (2009). Shear Behaviour of Reinforced Concrete Beams Made With Sedimentary Lightweight Aggregate without Shear Reinforcement. Journal of Materials in Civil Engineering, 21(12), 730–739.

Thomas, J., and Ramaswamy, A. (2006). Shear Strength of Prestressed Concrete T-Beams with Steel Fibres Over Partial/Full Depth. ACI Structural Journal, 103(3). doi:10.14359/15321

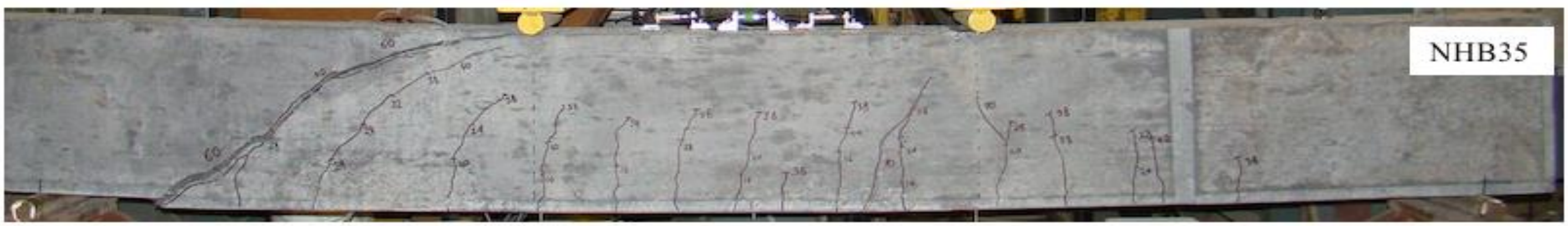
Zhang, F., Ding, Y., Xu, J., Zhang, Y., Zhu, W., and Shi, Y. (2016). Shear Strength Prediction for Steel Fibre Reinforced Concrete Beams Without Stirrups. Engineering Structures, 127, 101–116. https://doi.org/10.1016/j.engstruct.2016.08.012

Appendix




NNB35

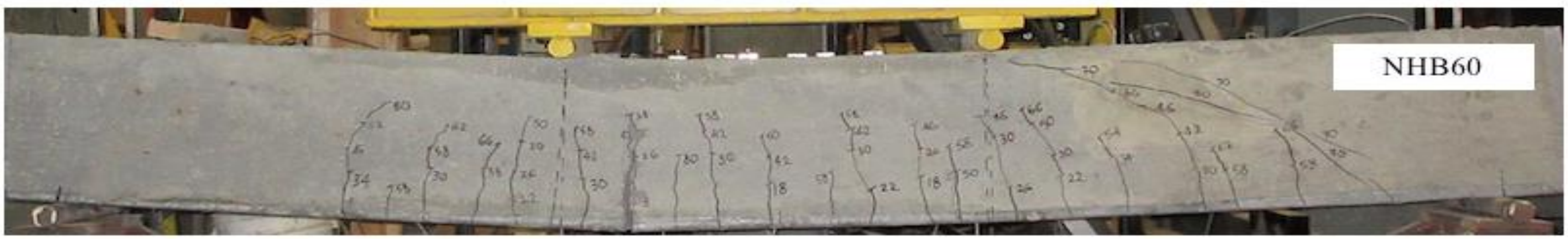

LNB35


NHB35

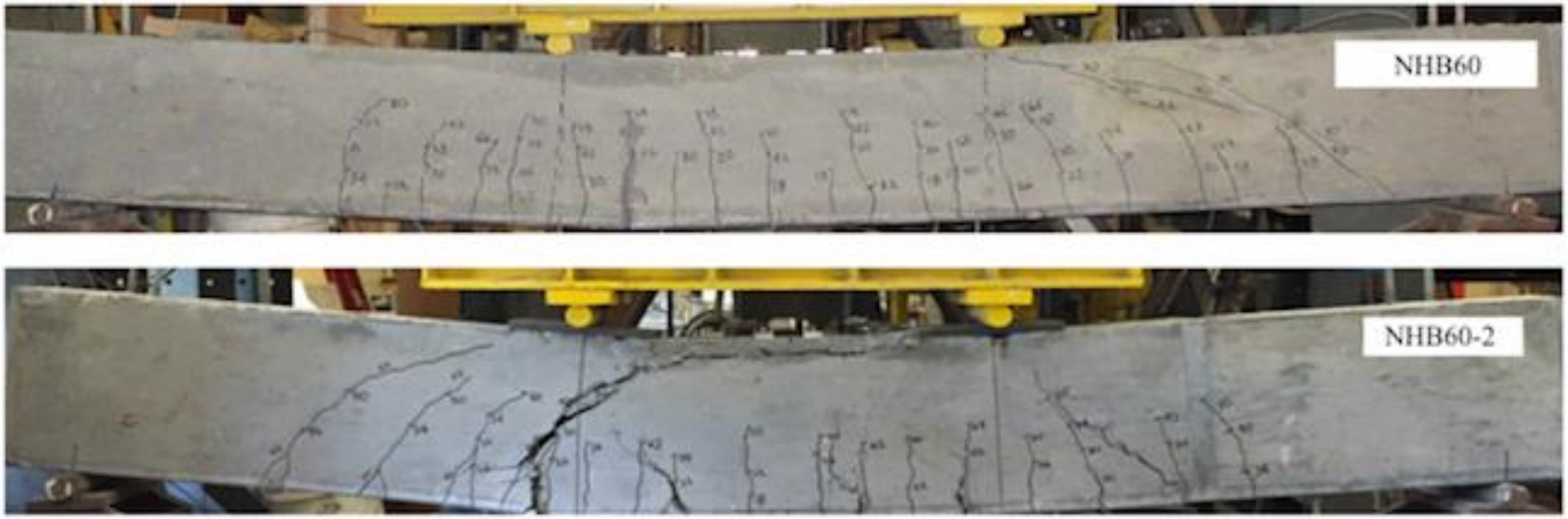

LHB35


NNB60


LNB60


NHB60


LHB60


NHB60
NHB60-2