DEVELOPMENT OF MEMORY-BASED MODELS FOR RESERVOIR FLUID CHARACTERIZATION

by

Md. Shad Rahman

A Thesis

submitted to the School of Graduate Studies

in partial fulfilment of the requirements for the degree of

Master of Engineering

Faculty of Engineering and Applied Science

Memorial University of Newfoundland

May 2018

St. John’s Newfoundland
Abstract

The petroleum industry play an important role in supplying required energy all over the world. Effective methods are required to stimulate the process. Petroleum fluids are the mixture of complex hydrocarbons. Several techniques being used to predict reserve estimation, recovery, production, enhanced oil recovery, etc. Despite of modern engineering advancement, still, there are some drawbacks, such as, conventional models, linearized rock-fluid properties models, inaccurate risk assessment, and inappropriate descriptions of thermal effects. In this research, new mathematical models for petroleum fluids (non-Newtonian) regarding various degree of complexities will be developed. The most significant component will be the continuous time function introduced to the rheology. Previous attempts are addressed in this modeling, and those models were limited for some specific cases and fluids. The current proposal will develop a comprehensive model that can be applied to different reservoir fluids irrespective to fluid origin. In addition, the proposed models will also be adjusted for a complex mixture of reservoir fluids. The model equations will be solved numerically and validated using field data and data gathered from experimental tasks available in the literature. The proposed models will be developed focusing light crude oil for reservoir conditions. The role of various factors, such as crude oil density, viscosity, compressibility, surface tension, ambient temperature, and temperature will be included in the predictive models. Model equations will be solved with non-linear solvers, as outlined earlier. This will generate a range of solutions, rather than a line of unique solutions. This analysis will increase an accuracy of the predictive tool and will enable one to assess the uncertainty with greater confidence.
To my parents, my brother, and my uncle (Late Mosharraf Hossain Khan).
Acknowledgements

I express my sincere gratitude and thanks to my supervisor, Dr. M. Enamul Hossain for his encouragement and endless guidance throughout the period of graduate study. His creativity, extremely scholarly research-oriented and professional attitude, and his continuous support contributed remarkably towards the successful completion of this thesis. He has always been there for me in good times and in bad, and without him my graduate studies would have been far more challenging. I take this big opportunity to wish him the best in his future endeavors. I would like to thank my co supervisor Dr. Salim Ahmed for his support and guidance towards my thesis completion. He always supported me and guided me on exact path to achieve my goal and complete my research task. I would like to thank all the group members of my research group for their help and support as a family. I thank Mohammad Islam Miah for his support and help to accrue more knowledge about petroleum engineering. I thank Mamun Ur Rashid to help me with canvas and creating taxonomy. I would like to thank Pulok Kanti Deb for his help and support as a senior throughout my graduate life. I thank Tareq Uz Zaman for helping me in discretization methods and with MATLAB. I would like to thank Thomas Hickey, Rasel A Slutan, Al-Amin Shuvo, Munzarin Morshed, and Dipika Deb Dipa Purkayastha for their continuous help and support throughout my studies. They have been always there for me in good and bad times. I thank Moya Crocker, Colleen Mahoney, Vanessa Coish, and Tina Dwyer for creating a friendly and enabling atmosphere at the University. I express my greatest gratitude to my family members for being a constant source of inspiration, love and affection. I would like to thank Tanzia Alam Amy for her support and inspiration in every moment of my study.

Finally, I would like to thank Research & Development Corporation of Newfoundland and Labrador (RDC), funding no. 210992; and Statoil Canada Ltd., funding no. 211162 for providing financial support to accomplish this research.
Co-Authorship Statement

I, Md Shad Rahman, hold primary author status for all the Chapters in this thesis. However, each manuscript is co-authored by my supervisor, co-supervisor, and my research mates Tareq Uz Zaman, Pulok Kanti Deb, Mohammad Islam Miah helped me to conceiving the idea and selection of appropriate techniques. Specially, Tareq Uz Zaman, Pulok Kanti Deb, Mohammad Islam Miah help to make my work easy and efficient. And my supervisor and co-supervisor always stood beside me, contributed, and facilitated to develop this work.
# Table of Contents

<table>
<thead>
<tr>
<th>Abstract</th>
<th>ii</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acknowledgements</td>
<td>v</td>
</tr>
<tr>
<td>Co-Author Statement</td>
<td>v</td>
</tr>
<tr>
<td>Table of Content</td>
<td>vi</td>
</tr>
<tr>
<td>List of Tables</td>
<td>x</td>
</tr>
<tr>
<td>List of Figures</td>
<td>xi</td>
</tr>
<tr>
<td><strong>Chapter 1</strong></td>
<td></td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Knowledge Gap</td>
<td>2</td>
</tr>
<tr>
<td>1.3 Objectives</td>
<td>3</td>
</tr>
<tr>
<td>1.4 Structure of Thesis</td>
<td>4</td>
</tr>
<tr>
<td>1.5 References</td>
<td>4</td>
</tr>
<tr>
<td><strong>Chapter 2</strong></td>
<td></td>
</tr>
<tr>
<td>A Critical Review on Reservoir Fluid Properties with the Aid of Memory</td>
<td>6</td>
</tr>
<tr>
<td>2.1 Abstract</td>
<td>7</td>
</tr>
<tr>
<td>2.2 Introduction</td>
<td>7</td>
</tr>
<tr>
<td>2.2.1 Background of Research</td>
<td>9</td>
</tr>
<tr>
<td>2.2.2 Fractional Derivative</td>
<td>10</td>
</tr>
<tr>
<td>2.2.3 Memory</td>
<td>10</td>
</tr>
<tr>
<td>2.2.4 Reservoir Rheology</td>
<td>11</td>
</tr>
<tr>
<td>2.2.5 Reservoir Fluid Properties</td>
<td>12</td>
</tr>
<tr>
<td>2.3 A Critical Analysis of the Literature</td>
<td>13</td>
</tr>
<tr>
<td>2.3.1 Rheology</td>
<td>13</td>
</tr>
<tr>
<td>2.3.1.1 Newtonian Fluid</td>
<td>16</td>
</tr>
<tr>
<td>2.3.1.2 Non-Newtonian Fluid</td>
<td>18</td>
</tr>
<tr>
<td>Section</td>
<td>Subsection</td>
</tr>
<tr>
<td>---------</td>
<td>------------</td>
</tr>
<tr>
<td>2.3.1.2.1</td>
<td>Time-Independent non-Newtonian Fluid</td>
</tr>
<tr>
<td>2.3.1.2.2</td>
<td>Time-Dependent non-Newtonian Fluid</td>
</tr>
<tr>
<td>2.3.1.2.3</td>
<td>Viscoelastic non-Newtonian Fluid</td>
</tr>
<tr>
<td>2.3.2</td>
<td>Fluid Properties</td>
</tr>
<tr>
<td>2.3.2.1</td>
<td>Oil Density</td>
</tr>
<tr>
<td>2.3.2.2</td>
<td>Oil Viscosity</td>
</tr>
<tr>
<td>2.3.2.3</td>
<td>Isothermal Compressibility Coefficient for Oil</td>
</tr>
<tr>
<td>2.3.2.4</td>
<td>Solution Gas-Oil Ratio</td>
</tr>
<tr>
<td>2.3.2.5</td>
<td>Bubble Point Pressure</td>
</tr>
<tr>
<td>2.3.2.6</td>
<td>Oil Formation Volume Factor</td>
</tr>
<tr>
<td>2.3.2.7</td>
<td>PVT Analysis</td>
</tr>
<tr>
<td>2.3.3</td>
<td>Memory Mechanism</td>
</tr>
<tr>
<td>2.3.3.1</td>
<td>Memory in Science and Engineering</td>
</tr>
<tr>
<td>2.3.3.2</td>
<td>Memory in Porous Media</td>
</tr>
<tr>
<td>2.4</td>
<td>Knowledge Gap and Future Research Direction</td>
</tr>
<tr>
<td>2.5</td>
<td>Conclusions</td>
</tr>
<tr>
<td>2.6</td>
<td>Nomenclature</td>
</tr>
<tr>
<td>2.7</td>
<td>References</td>
</tr>
<tr>
<td>2.8</td>
<td>Appendices A</td>
</tr>
</tbody>
</table>

**Chapter 3**

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Abstract</td>
<td>92</td>
</tr>
<tr>
<td>3.2</td>
<td>Introduction</td>
<td>92</td>
</tr>
<tr>
<td>3.3</td>
<td>Mathematical Model Development</td>
<td>98</td>
</tr>
<tr>
<td>Chapter 5</td>
<td>A Memory-based Fluid Density and Effective Viscosity Model for Reservoir Characterization</td>
<td>159</td>
</tr>
<tr>
<td>-----------</td>
<td>----------------------------------------------------------------------------------------</td>
<td>-----</td>
</tr>
<tr>
<td>5.1</td>
<td>Abstract</td>
<td>160</td>
</tr>
<tr>
<td>5.2</td>
<td>Introduction</td>
<td>160</td>
</tr>
<tr>
<td>5.3</td>
<td>Mathematical Model Development</td>
<td>164</td>
</tr>
<tr>
<td>5.4</td>
<td>Numerical Analysis of the Model</td>
<td>166</td>
</tr>
<tr>
<td>5.5</td>
<td>Result and Discussions</td>
<td>170</td>
</tr>
<tr>
<td>5.5.1</td>
<td>Dependency of fluid density on effective viscosity</td>
<td>170</td>
</tr>
<tr>
<td>5.5.2</td>
<td>Dependency of the Blake number on effective viscosity</td>
<td>172</td>
</tr>
<tr>
<td>5.5.3</td>
<td>Dependency of fluid flux (memory effect) on fluid flow in porous media</td>
<td>173</td>
</tr>
<tr>
<td>5.5.4</td>
<td>Dependency of apparent shear rate on effective viscosity</td>
<td>174</td>
</tr>
<tr>
<td>5.5.5</td>
<td>Comparison of proposed Density-Viscosity model with Carreau-Yasuda Model</td>
<td>176</td>
</tr>
<tr>
<td>5.6</td>
<td>Conclusions</td>
<td>176</td>
</tr>
<tr>
<td>5.7</td>
<td>Nomenclature</td>
<td>177</td>
</tr>
<tr>
<td>5.8</td>
<td>References</td>
<td>179</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Chapter 6</th>
<th>Conclusions</th>
<th>185</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1</td>
<td>Summary</td>
<td>185</td>
</tr>
<tr>
<td>6.2</td>
<td>Future work guideline</td>
<td>186</td>
</tr>
</tbody>
</table>
List of Tables

Table 2.1: Types and features of Non-Newtonian fluid ........................................ 66
Table 2.2: Considered parameters for different viscosity correlations .................... 66
Table 2.3: PVT correlations data for GOR, P_b, B_o considering ARE, AARE and STD.. 70
Table 2.4: PVT correlations data for P_b considering GOR, API, γ_g and T ............... 71
Table 2.5: PVT correlations data for B_o considering GOR, API, and γ_g .................. 74
Table 2.6: Use of Memory in various fields of science and engineering ................... 76
Table 2.7: Memory mechanism in porous media.................................................... 81
Table 3.1: Sample Reservoir Data ........................................................................... 102
Table 3.2: Experimental Data ................................................................................ 103
Table 4.1: Sample Reservoir Data ........................................................................... 137
Table 4.2: Experimental Data ................................................................................ 138
Table 5.1: Sample Reservoir Data ........................................................................... 167
Table 5.2: Experimental Data ................................................................................ 167
List of Figures

Figure 2.1: Reservoir Fluid Properties.................................................................................................. 12
Figure 2.2: Rheological study cycle.................................................................................................... 13
Figure 2.3: Applications of Rheology in various field ................................................................. 14
Figure 2.4: Classification of Rheology Fluids.................................................................................. 15
Figure 2.5: Behavior of Newtonian and non-Newtonian Fluids.................................................... 16
Figure 2.6: Shear rate vs. shear stress relationship of Newtonian fluids .................................... 17
Figure 2.7: Stress components analysis in 3-D fluid flow Reservoir Fluid Properties..... 18
Figure 2.8: A Typical plot of time independent non-Newtonian fluids...................................... 19
Figure 2.9: A typical plot for time-dependent fluid........................................................................ 21
Figure 2.10: Rock-Fluid physical characteristics in porous media.............................................. 23
Figure 3.1a: Typical Fluid Flux in Porous Media........................................................................... 94
Figure 3.1b: 3-D tangential forces of a fluid element................................................................. 101
Figure 3.2: Alignment of grid point in space................................................................................ 104
Figure 3.3: Fluid shear stress variation as a function of density for different $\alpha$ values.. 106
Figure 3.4: At field condition fluid shear stress as a function of density ......................... 107
Figure 3.5: At exp. condition fluid shear stress as a function of density .......................... 107
Figure 3.6: Fluid shear stress variation as a function of density for different months at $\alpha$= 0.3................................................................................................................................. 108
Figure 3.7: Fluid shear stress variation as a function of Blake number for different $\alpha$ values ............................................................................................................................................. 109
Figure 3.8: Fluid flux variation with time for $\alpha$= 0.3................................................................. 110
Figure 3.9: Fluid shear stress variation as a function of time for different $\alpha$ values ..... 111
Figure 3.10: At field condition fluid shear stress variation as a function of time.......... 111
Figure 3.11: At exp. condition fluid shear stress variation as a function of time ......... 112
Figure 3.12: Fluid shear stress variation as a function of strain rate for different $\alpha$ values ................................................................................................................................................. 113
Figure 3.13: At field condition fluid shear stress variation as a function of strain rate.. 113
Figure 3.14: At exp. condition fluid shear stress variation as a function of strain rate.. 114
Figure 3.15: Fluid shear stress variation as a function of strain rate for different months at
$\alpha = 0.3$ .......................................................................................................................... 114
Figure 3.16: Comparison of proposed stress-strain model with Hossain et al., (2007a)
model.................................................................................................................................. 115
Figure 3.17: Comparison of proposed stress-strain model with field results and Hossain et
al. (2007a) model. .................................................................................................................. 116
Figure 3.18: Comparison of proposed stress-strain model with exp. results and Hossain et
al. (2007a) model. .................................................................................................................. 116
Figure 4.1: Typical Fluid Flow in Porous Media................................................................. 129
Figure 4.2: 3-D tangential forces of a fluid element............................................................. 134
Figure 4.3: Alignment of grid point in space........................................................................ 139
Figure 4.4: Fluid viscosity variation as a function of shear stress for different $\alpha$ values 141
Figure 4.5: At field condition shear stress as a function of viscosity ......................... 141
Figure 4.6: At exp. condition shear stress as a function of viscosity ......................... 142
Figure 4.7: Flux change with time for $\alpha = 0.3$ .............................................................. 143
Figure 4.8: Shear stress variation as a function of time for different $\alpha$ values............ 144
Figure 4.9: At field condition shear stress variation as a function of time .................. 144
Figure 4.10: At exp. condition shear stress variation as a function of time ............... 145
Figure 4.11: Shear stress variation as a function of shear rate for different $\alpha$ values .... 146
Figure 4.12: At field condition shear stress variation as a function of shear rate .......... 146
Figure 4.13: At exp. condition shear stress variation as a function of shear rate ........... 147
Figure 4.14: Comparison of modified stress-strain model with Hossain et al. (2007a)
model.................................................................................................................................. 148
Figure 4.15: Comparison of proposed stress-strain model with field results and Hossain et
al. (2007a) model. .................................................................................................................. 149
Figure 4.16: Comparison of proposed stress-strain model with exp. results and Hossain et
al. (2007a) model. .................................................................................................................. 149
Figure 5.1: Total Energy Supply....................................................................................... 161
Figure 5.2: Sample reservoir grid point in space ................................................................. 169
Figure 5.3: Effective viscosity variation as a function of density for different $\alpha$ values 171
Figure 5.4: Effective viscosity variation as a function of density for different $\alpha$ values 171
Figure 5.5: Effective viscosity variation as a function of density for different months . 172
Figure 5.6: Effective viscosity variation as a function of Blake number for different $\alpha$
values ......................................................................................................................................... 173
Figure 5.7: Fluid flux variation as a function of time .............................................................. 174
Figure 5.8: Effective viscosity variation with shear rate for different $\alpha$ values.............. 175
Figure 5.9: Effective viscosity variation with apparent shear rate for field and exp.
condition .................................................................................................................................. 175
Figure 5.10: Comparison of proposed model with Carreau-Yasuda model ................. 176
Chapter 1

Introduction

1.1 Introduction

In this universe, every element has distinct uniqueness which can come out with the help of proper research and experimental investigation. Sometimes it become very tough to show the exact scenario with the help of proper experiments because of some limitations. This picture is more acute in petroleum industry and apparently it is difficult and expensive to conduct the experiment. In those conditions, mathematical modeling, analytical solution, numerical simulation, and scaled up ratios are the approximation approach to get relevant results on the path of research. However, the numerical solution helps to simulate the oil and gas reserve and generate new techniques to maximize the overall petroleum production. In petroleum industry, modeling and simulation are the two terms that used very frequently which are used to measure overall reserve and increase the production of any field. The mathematical modeling has a great impact on petroleum industries and several new techniques are generating every day to maximize the production.

The fluid flow through porous media is one of the most important issue for petroleum engineering. In reservoir engineering, fluid properties, rheological properties, and their relationship with formation are very important. But it is tough to model all these parameters together because of complex fluid and rheological behavior. Formation rock and fluid properties play very significant role in terms of petroleum production. The proper modeling and estimation of those rock and fluid parameters will surely help to maximize the production. However, reservoir rheological parameters are mainly focused on elastic and viscous properties of rock and fluids. For fluids, it can be measured by generating a shearing force on the fluid element surface and calculate shear stress. Usually, reservoir fluids are non-Newtonian by characteristics. In oil production process, most of the fluids
are non-Newtonian fluid and represents thixotropic behavior. Therefore, formation rheology properties consideration is very important for any production. The “time” is very important dimension for reservoir simulation. Time represents the complex behavior of reservoir and is used in developing any simulation model. To predict the pathway of any formation, the previous history is an essential factor which ultimately represents the dependency of time.

The idea of continuous rock and fluid alteration with time can be developed through model using the idea “memory’. In this thesis, the “memory” term is defined as “the properties of formation rock and fluid that help to identify the variations in rock properties (i.e., porosity, permeability) and fluid properties (density, viscosity, compressibility, ,surface tension, saturation) with respect to time and space” Several researchers are trying to incorporate the memory mechanism in porous media and presented the dependency of memory in fluid flow through porous media (Caputo, 1999, 2000; Arenzona et al., 2003, Chen et al., 2005; Hossain, 2008; Caputo and Carcione , 2013, Obembe et al., 2017). Hossian, 2008 showed the effect of fluid memory in continuous rock and fluid alteration and proposed memory based fluid models focusing petroleum study.

1.2 Knowledge Gap

Memory is a function of all possible rock and fluid properties of a given fluid and its formation over the span of time. Fluid memory is a revolutionary addition in fluid flow models still this feature is neglected in most of the research. In recent times, several researchers are trying to show the effect of fluid memory in porous media and petroleum production (Caputo, 1999, Zhang, 2003, Chen et al., 2005, Hossain 2008, Histrov, 2014, 2017. Obembe et al., 2017). The unusual behavior of non-Newtonian fluid arises the complexity in the flow medium which is due to fluid memory. With the help of fluid memory, the various fluid flow phenomena such as heterogeneity, anisotropy can be described.
Fluid viscosity plays an important role at the time of fluid flow in porous media. Viscosity helps to define fluid types during fluid flow in the formation. Like thixotropic fluid or shear thinning fluid to identify at the time of applied shear force. Thixotropic fluid is the time-dependent non-Newtonian fluid and with applied shear force viscosity decrease with time at constant shear rate. If viscosity decreases with increasing shear rate then that fluid in known as shear-thinning or pseudo-plastic fluids. Those fluids are available in nature as simple fluid or complex mixture. These fluids have a great impact on industry specially petroleum industry.

In the petroleum industry, mostly empirical correlations are used to measure fluid properties. In those correlation, the real picture of fluid properties of porous media is unable to capture. There are few models are available where fluid is considered as Newtonian and fluid memory didn’t observed. However, in nature most of the fluid are non-Newtonian (Perazzo et al., 2003, Arratia et al., 2005, Hossain, 2008) and mostly accepted Newtonian fluid water also exhibits viscous flow (Li et al., 2007). Therefore, recently researchers trying to develop fluid flow model considering non-Newtonian fluid also incorporated memory mechanism in those models (Hossain, 2008; Caputo and Carcione, 2013, Histrov, 2014, 2017a, 2017b; Obembe et al., 2017). But still there is no comprehensive fluid flow model in porous media which can represent fluid properties and memory together along with all advanced computational techniques.

1.3 Objectives

The specific objectives for this research are:

- To study fluid properties for light crude oil and incorporate memory mechanism;
- To develop comprehensive fluid model for reservoir fluid characterization;
- To develop memory-based stress-strain model focusing all fluid properties;
- To develop memory-based effective viscosity-density model with numerical validation;
- To solve the model equations numerically using field data and experimental data;
- Compare the models with existing models;
1.4 Structure of Thesis

The total research work is divided into six chapters. Those chapters contain: Chapter 1 describes the concept of the research work and identifies the knowledge gap. This chapter also provides the research objectives and the organization of the thesis. Chapter 2 provides an extended review of literature on rheology, fluid properties, memory mechanism, and fluid memory in porous media. Chapter 3 and Chapter 4 present the development of stress-strain model with memory mechanism in a comprehensive way and comparison with established model. Chapter 5 represents the development of memory-based fluid effective viscosity-density model and comparison with existing model. Finally, Chapter 6 concludes this thesis by highlighting the research contribution and few recommendations are proposed for future work pathway.

1.5 Reference


Hossain, M.E., (2008), An experimental and numerical investigation of memory-based complex rheology and rock/fluid interactions (Vol. 69, No. 11).


Chapter 2

A Critical Review on Reservoir Fluid Properties with the Aid of Memory

Preface

This paper has been submitted to the Journal of Non-Newtonian Fluid Mechanics and it is currently under review. The lead author performed the necessary literature review on fluid properties and fluid memory. The co-authors Pulok Kanti Deb helped in editing, and adding some new information and Dr. M. Enamul Hossain helped in identifying the gap in research, supervising the research, and editing the manuscript.
2.1 Abstract

Reservoir rock and fluid properties vary during any pressure difference or thermal changes in the reservoir formation. It is important to consider the rock properties such as permeability, porosity, etc. and fluid properties such as density, viscosity, PVT properties etc. as a function of time and space. Memory defines as the effect of previous events on the present, and future period of process and developments. The continuous change of rock and fluid properties can be characterized using memory mechanism. It is also significant to consider the rock, and the fluid properties as a function of time, space, and the inclusion of memory mechanism in science, and engineering study. In this paper, a detailed review of the existing rheological study, and fluid properties are presented in science and engineering. This study will provide an inclusive information on the present status of memory-based fluid flow modeling, effect of memory on rock and fluid properties during reservoir characterization. This review also offers a state-of-the-art literature survey on different fluid rheological behaviors. A literature survey provides a complete picture of memory-based models used in various fields (e.g., physics, chemistry, biology, economics, petroleum, etc.) of science and engineering. Though the role of fluid memory in porous media is significant, it is overlooked in almost all fluid models (e.g., fluid flow models, energy balance, thermal heat recovery, etc.). This paper introduces reservoir rheology and fluid properties, and demonstrates their relation to the existing mathematical models with memory mechanisms. It also focuses on the hypothetical barriers of the time function. This study will help in describing the exact picture of fluid characteristics in porous media considering memory for any reservoir scenario (e.g., reservoir simulation, thermal recovery, enhanced oil recovery, etc.).

2.2 Introduction

In the petroleum industry, the impact of reservoir rock and fluid (e.g., oil, water, and gas) properties over the span of time and space are significant. The proper identification of reservoir rock and fluid properties can significantly reduce the risk and uncertainties associated with petroleum operations. Fluid memory is one of the essential concepts that
aid in the understanding and predicting the future behavior based on its past events. The exact strength of the concept of fluid memory is that it can be used to recall all the previous incidents of the fluid properties and predict how it will act in the future. Many researchers employ the concept of fluid memory in reservoir rheology, reservoir simulation, dynamic material balance, scaling criteria for oil-water displacement, thermal recovery, and enhanced oil recovery processes of petroleum engineering. To make future predictions about fluid properties, researchers often use Newtonian flow equations, and some non-Newtonian fluid models are also considered. However, some non-Newtonian models are not yet validated or well-established.

Memory-based (MB) models are significant in fluid flow through porous media. However, the application of MB models in field applications proves very challenging. For example, developing a model for transient flow is difficult as it crosses a two-layer medium and the boundary between the two rocks can equally be important (Garra et al., 2015). Although, it is necessary to select either initial or boundary conditions. Sometimes the actual boundary layer is not considered in mathematical model (Garra et al., 2015). The sudden change of structure can cause disturbances such as fluid particles migration or displacement (e.g., oil, water or gas) (Liu and Civan, 1996; Civan, 1998; and Merlani et al., 2011). To obtain the time-dependent scenario, one can take into account that pressure \((P)\) and temperature \((T)\) change over time. Caputo (2000) established a modified Darcy’s law for reservoir fluid fluxes in porous media to show the memory mechanism. This researcher used memory mechanism to represent the time function with a fractional derivative for a stable and homogeneous condition. Recently, fractional derivative models have been used to represent unstable flows in heterogeneous porous media (Fellah et al., 2006, Hossain et al., 2007; 2008; 2009a; 2012, and Obembe et al., 2017).

Several researchers have used memory mechanism in the past few decades (Caputo, 1999; 2000; 2003; Hossain et al., 2006; 2007; 2009; 2011, and Obembe et al., 2016; 2017). However, some challenges are applying MB models in porous media. This article outlines a brief discussion of memory models and their applications in porous media, as well as
some of the challenges and future guideline for MB models for reservoir fluid properties (e.g., viscosity, density, surface tension, etc.).

2.2.1 Background of Research

In this universe, every single element or particle of any object has a relationship with its past data or history. This process happens because the flow of nature and all its particles are continuous. Not a single element can be confined or isolated from its surroundings at any condition. For any object or particle, its property, nature, and behavior can be determined from its origin and the way it traveled over the span of time. Fluid Memory is one of the most important phenomena used to describe any fluid property. However, it is overlooked by most researchers. The real scenario of continuous time function (i.e. memory) is easily determined in any fluid flow through porous media. The concept of “memory” is usually used in the branch of material science and other branches of science and engineering. For any mathematical modeling, the time function or memory of both rock formations and fluids are considered.

Hossain and other researchers (Hossain et al., 2007; 2008; 2011; and 2012) showed that most flow models did not consider the continuous alteration of reservoir rock and fluid properties. A new mathematical model based on the memory mechanism has been proposed by Hossain et al. (2007; 2008). In these research, researchers considered reservoir rheology and fluid properties over time. While these models highlight all aspects of the phenomenon, some scenarios are still not adequately addressed. The researchers considered the viscosity of the Newtonian fluid but failed to consider any relation with density or other fluid properties. These models were successfully solved numerically, however their practical validation has not been established yet. Therefore, a comprehensive fluid flow model is required to show the exact scenario of reservoir fluid properties in relation to time alteration.
2.2.2 Fractional Derivative

The fractional derivative (FD) is a modern technique used to solve ordinary differential equations (ODE), partial differential equations (PDE), and the integro-differential equations problem (Oldham and Spainer, 1974; Miller and Ross, 1993; Samko et al., 1993; Carpinteri et al., 1997; Podlubny, 1999; Hilfer, 2000; Scalas et al., 2000; Nigmatullin and Le Mehaute, 2005; Magin, 2006; Sabatier et al., 2007; and Baleanu et al., 2009). This technique is used to solve mathematical problems in various topics and problems in science and engineering such as viscoelasticity, control volume theory, heat conduction problems, thermal diffusivity, electrical and mechanical problems, chaos and fractals, etc. (Zaslavsky, 2002; West et al., 2003; Chen et al., 2004; Zaslavsky, 2005; Sabatier et al., 2007; Jesus and Machado, 2008). FD is used less in field applications because complexity arises and it is also time consuming. But its accuracy is better than conventional methods (Miller and Ross, 1993; Chen et al., 2004; Baleanu et al., 2009; Baleanu and Nigmatullin, 2010). On the contrary, there is another technique known as the conventional or classical method. In the mid 1800’s, this approach was first presented by Darcy for use in the porous medium. According to Darcy’s law, fluid flux is proportional to velocity, pressure gradient, and conductivity coefficient and it depends on the physical characteristics of the fluid medium (Scheidegger, 1960; Bear, 1972). The conventional equation explains both single phases and multiple phases flow through porous media (Muskat, 1946; Ertekin et al., 2001; Nield and Bejan, 2006; Civan, 2011). The classical Darcy model is applicable for steady-state, homogeneous, and isotropic conditions (Chan and Banerjee, 1981; Kaviany, 1995). Over the last few decades, several researchers have proposed different additions to Darcy’s flow equation by introducing the action of slip, inertia, etc. (Hossain et al., 2011; Hossain and Abu-khamsin, 2011a; Hossain and Abu-khamsin, 2012).

2.2.3 Memory

Memory can be defined as a continuous time function, which is captured through formation rock (porosity, permeability) and fluid properties (viscosity, pressure dependent fluid properties). If the memory term is incorporated in any fluid model (like stress-strain, fluid
flow, etc.), it increases the nonlinearity of the equation. Mathematically, it is represented as the form of fractional derivative. In general, the memory recalls past states and predicts future states. In the last few decades, many researchers have been using this memory concept in several disciplines such as in electromagnetic studies (Jacquelin, 1984), human biology (Cesarone, 2002; Cesarone et al., 2005), economy (Caputo and Kolari, 2001; Caputo, 2002), porous media (Caputo and Mainardi, 1971; Caputo and Plastino, 1998, 2004; Caputo, 1999, 2000, 2001; Hossain et al., 2007), rheological properties of solids (Le Mehaute and Crepy, 1983; Bagley and Torvik, 1986; De Espindola et al., 2005; Adolfsson et al., 2005) and petroleum engineering (Hossain and Islam, 2006; Hossain et al., 2007, 2008, 2009a, 2009b) to obtain more efficient results. Fluid memory adds a new way to show the time alteration in flow models for the petroleum industry (Brinkman, 1949; Bear, 1975; Sposito, 1980; Whitaker, 1986). Reservoir solid particles can be deposited along pore throats or walls to block the flow pathway and thus pore size will affect the alteration of rock and fluid properties (Hossain et al., 2009c). In the oil and gas industry, this is not a new technique, and lots of examples are found in the literature to model rheological properties, fluid flow through porous media, heat diffusion, and so on (Caputo, 1967; Kornig and Muller, 1989; Broszeit, 1997; Hilfer et al., 2000; Zaslavsky, 2002; Zhang, 2003; Metzler and Klafter, 2004; Hossain and Abu-Khamsin, 2011b; Al-Mutairi et al., 2013; Obembe et al., 2017).

2.2.4 Reservoir Fluid Rheology

Rheology is the details study of flow and deformation of materials (i.e., semi solid and liquid) and of how the flow is affected by stresses and strains with time (Bourne, 2002; Dhiman, 2012). It is used in calculating flow velocity profiles, fluid viscosity, frictional pressure losses, etc. Rheology plays a significant role in the petroleum industry, especially in reservoir characterization, drilling operations, and petroleum productions. Petroleum fluids are the mixtures of hydrocarbon, starting from a very simple gas such as methane, to complex asphalactic molecules with thousands of molecular weights (Ronningsen, 2012). The rheological characteristics of those fluids can be Newtonian, highly non-Newtonian, viscoelastic, or solid (Ronningsen, 1993a; Ronningsen, 1993b; Pedersen and Ronningsen,
In the petroleum industry, the proper measurement of rheological properties is challenging. It may cause economic losses, and in extreme conditions, it could end up in the abandonment of the production (Bagley, 1986; Drley and Gray, 1988; and Ronningsen, 2012).

### 2.2.5 Reservoir Fluid Properties

To obtain an overall scenario and an understanding of the behavior of reservoir performances, it is essential to analyze the hydrocarbon fluid properties (i.e., viscosity, density, solution gas-oil ratio, bubble point pressure, etc.) (Bateman, 2015). In figure 2.1, the most frequently used reservoir properties are shown in the diagram. The distribution of reservoir fluid phases depends on the reservoir temperature, composition, pressure, a difference of geological trap, depth, reservoir heterogeneity, and fluid migration path. Here, the fluid flows are controlled by various forces (i.e., gravity, capillary pressure, thermal and molecular diffusion, thermal convection, pressure gradients, etc.). Most researchers assumed that the reservoir fluid is static during model development, though the fluids can be in a dynamic condition in terms of geological time. Moreover, gravity works as driving force in distributing fluids. Laboratory based PVT analyses and mathematical models are used to determine those fluid properties (Dake, 1998; McCain, 2002).

**Figure 2.1:** Reservoir Fluid Properties (Dake, 1998; McCain, 2002)
2.3 A Critical Analysis of the Literature

In this review, the characteristics and behavior of fluid rheology, fluid properties, and the memory mechanisms in porous media are highlighted. It is also shown how memory techniques can be incorporated in fluid models and discussed the benefits of memory. Finally, a guideline is presented on how to develop a comprehensive fluid model by incorporating fluid rheological properties such as viscosity, density, and memory.

2.3.1 Rheology

The term rheology was invented by Bingham in the 1920’s and was inspired by a Greek quotation “Panta rei” meaning everything flows (Bingham, 1916; 1922). To show a relation between mathematics and rheology, there is equation called rheological equation of state or the constitutive equation. Rheology usually analyses mechanical properties, which also include the physical properties of solids, semisolids, and liquids, by describing the strain and flow characteristics as well as their behavior. In Figure 2.2, it is demonstrated that how rheology illustrates fluid flow relationship between the properties, structure, and processing of the materials. All three elements related to each other and without any of this element it is hard to represent the actual rheological scenario of any fluid. The linkage between every element and rheology is also shown in the following figure.

![Figure 2.2: Rheological study cycle (modified from Sochi, 2010)](image)
Rheology helps to describe the mechanical behavior of any materials as a function of stress (i.e., shear rate), strain, temperature, and pressure. In Figure 2.3, it is illustrated that how rheology is important in different branches of science and engineering. It has a great impact on physical and chemical fluid study. It is also discussed in continuum mechanics of fluids and several branches of engineering, especially in petroleum engineering during primary, secondary, and tertiary recovery operations (Drley and Gray, 1988; Ronningsen, 1993a; 1993b; and 2012; Zhao and Machel, 2012). Here, viscoelasticity helps in injectivity and increases the pressure gradients for general Darcy’s flow in any reservoir. Newtonian and non-Newtonian fluids exhibit unusual characteristics in transient flow and pressure gradient conditions for any oil and gas reservoir (Ronningsen, 1992; Ronningsen et al., 1997; Paso et al., 2009; and Wilton, 2015).

\[ \text{Figure 2.3: Applications of Rheology in various field (modified from Wilton, 2015)} \]

The rheological study is essential for dilute, polymer and concentrates colloidal systems. It helps to describe fluid flow behavior with time and space. It also describes exactly the time dependent and independent behaviors of any fluids. In rheology, the fluid is categorized into two divisions: Newtonian and Non-Newtonian. A complete picture of the fluid categories is given in Figure 2.4. In Fig 2.4, fluid is divided into two main groups,
Newtonian and non-Newtonian fluids are divided into three subgroups. Those groups are viscoelastic, time dependent, and time independent. Time dependent and time independent fluids are subcategorized according to the fluid's behavior. Figure 2.5 represents the behavior of Newtonian and non-Newtonian fluids using flow resistance as a function of shear rate. For Newtonian fluids, it will be a straight line intersecting the viscosity axis and not going through the origin. Pseudo plastic fluids and Dilatant fluids also go through the same point as Newtonian fluid. However, those shapes are concave downwards and upwards accordingly from a straight line. Bingham fluids also illustrate as concave upwards but do not follow the same point as Newtonian fluids.

Figure 2.4: Classification based on Fluids Rheology (modified from Zhao and Machel, 2012; Hamed, 2016)
2.3.1.1 Newtonian fluid

Newtonian fluids follow the Newton’s law of viscosity. Here, the Newtonian viscosity is independent of shear rate ($\gamma$) or shear stress ($\tau$) (Chhabra and Richardson, 2008) and depends on the fluid flow rate as well as the temperature and pressure. Figure 2.6, The flow curve for Newtonian fluids (water, air, gas and light crude oil) is a straight line which goes through its origin, and the slope of that line is constant ($\mu$). If the shear stress becomes double, then the shear rate will also become double, and vice versa (Chhabra and Richardson, 2008; Sochi, 2010; Ronningsen, 2012; Wilton, 2015). The rheological equation for a Newtonian fluid is given as:

$$\tau = \mu \cdot \gamma$$

(1)

Here, the viscosity is constant at fixed temperatures and pressures. In any reservoir, hydrocarbons with less than five carbon particles have an essential role in oil production. The gas that produce in a reservoir is a part of the reservoir fluids and show a vital impact on fluid viscosity and productivity of those reservoirs. Thus, gas viscosity is also considered (Ronningsen, 1993a; 1993b; and 2012).

**Figure 2.5:** Behavior of Newtonian and non-Newtonian Fluids (modified from Sochi, 2010; Wilton, 2015)
Figure 2.6: Shear rate vs. shear stress relationship of Newtonian fluids (Azar and Samuel, 2007)

When fluid flows through any porous media there are stress components that are presented in Figure 2.7. Stress components are in three direction and flow is presented in the x-direction. In Figure 2.7, Newtonian fluid observed in shearing motion, stress components in all directions are equally zero.

\[ \tau_{xx} = \tau_{yy} = \tau_{zz} \]  

(2)

To define complete explanation of a Newtonian fluid, it needs to show a constant viscosity as well as fulfills the condition of equation (2), or it maintains the Navier-Stokes equations. Though Boger fluids show constant shear viscosity but do not satisfy equation (2). So, it is defined as non-Newtonian fluids (Boger, 1976; Prilutski et al., 1983)
2.3.1.2 Non-Newtonian Fluid

Non-Newtonian fluid is defined as “shear stress versus shear rate flow curve of any fluid becomes nonlinear or does not go through the origin”. Non-Newtonian fluid can be categorized into three major groups i.e., (i) time-independent, (ii) time-dependent, and (iii) viscoelastic (Ronningsen, 1993a; 1993b; Pedersen and Ronningsen, 2000; Chhabra and Richardson, 2008). In Table 2.1, non-Newtonian fluids are categorized into three major groups. For every group, shear stress and shear rate relation, types of fluids and examples are discussed in Table-2.1.

2.3.1.2.1 Time-independent non-Newtonian fluid

The behavior of time independent fluids can be presented by a constitutive equation in simple shear form:

\[ \tau = f_i \dot{\gamma}^* \]  

(3)
The above equation represents that for any specific point, the value of $\gamma^*$ can be determined from the value of $\tau$ or vice versa. Time-independent fluids can also be classified into major three groups as: (i) shear-thinning or pseudo-plastic; (ii) visco-plastic; (iii) shear-thickening or dilatant (Chhabra and Richardson, 2008; Wilton, 2015). Time independent behavior of any non-Newtonian fluid illustrated graphical along with Newtonian fluid in Figure 8. For Newtonian fluids line goes through the origin and other time independent fluids such as pseudo plastic fluids, dilatant fluids go through the origin but do not represent as straight lines. Those fluids shape as parabolic curves. In Figure 2.8, few time independent fluids such as Bingham plastic, yield pseudo plastic does not go through origin and vary from straight line to parabolic shape.

![Figure 2.8: A Typical plot of time independent non-Newtonian fluids (modified from Sochi, 2010)](image)

Shear-thinning or pseudo plastic fluids are the most common types of time-independent fluids where viscosity decreases with increasing shear rate. At very low or high shear rates, shear-thinning fluids act as a Newtonian fluid. Sometimes graphical presentation of shear...
stress to shear rate becomes a straight line and goes through the origin (Chhabra and Richardson, 2008; and Sochi, 2010).

Visco-plastic fluid behavior is identified by the presence of a yield stress ($\tau_o$) (Chhabra and Richardson, 2008; Sochi, 2010; and Wilton, 2015). A fluid that presents linear flow curve for $|\tau_x| > |\tau_o|$ is called a Bingham plastic fluid or constant plastic viscosity. For visco-plastic fluids viscosity decreases with increasing shear rate. When shear stress is very low, the viscosity decreases immediately before the fluid starts to flow (Barnes, 1985; Astarita, 1990; Schurz, 1990; and Evans, 1992). The common examples of visco-plastic fluid are suspensions, emulsions, foodstuffs, blood and drilling muds, etc. (Barnes, 1999; Uhlherr et al., 2005).

Shear thickening or dilatant fluids are similar to pseudo-plastic fluids which do not show yield stress but viscosity increases with increasing shear rate. At low shear rates, the producing stresses are continuously small. However, it gives more friction and high shear stresses at a high shear rate. The different characteristics of this fluid drives researches to review on this fluid (Barnes et al., 1987; Barnes, 1989; Boersma et al., 1990; Goddard and Bashir, 1990). The common examples of dilatant fluids are china clay, titanium dioxide, corn flour in water, dispersions of polyvinyl chloride in dioctyl phthalate etc. (Metzner and Whitlock, 1958; Griskey et al., 1985; Boersma et al., 1990).

2.3.1.2.2 Time-dependent non-Newtonian fluid

Viscosities of fluids also depend on the time when it is subjected to the shear force (Chhabra and Richardson, 2008). If fluids are placed at shear stress for a long time, their actual viscosity becomes so much. Those fluids are: mud suspension, crude oils, foods, different water suspension, cement paste etc. Time-dependent fluid behavior can be divided into two groups, i.e., (i) Thixotropy, and (ii) Rheopexy or negative thixotropy (Chhabra and Richardson, 2008; Sochi, 2010; and Wilton, 2015).

With a constant shear rate, if the shear stress reduces with time it is known as thixotropic fluid. For thixotropic fluid, shear rate increases steadily at a constant rate from zero to a
maximum value and then reduces to zero at the same rate (Chhabra and Richardson, 2008). A hysteresis loop of shear rate is illustrated in Figure 2.9 where shear stress is increasing with shear rate up to certain point (i.e., maximum stress) and then decrease with the same rate and come to origin (Ronningsen, 1993a; 1993b; Chhabra and Richardson, 2008; Sochi, 2010; Wilton, 2015). The time-dependent behavior becomes stronger with the larger enclosed area. No hysteresis process is studied for time-independent fluid as the area of the hysteresis is zero for such fluid.

If shear stress increases with time, that fluid is known as rheopexy or negative thixotropic fluid. Rheopectic fluid is more compact compared to a thixotropic fluid as shown in Figure 2.9 where shear stress is increasing with shear rate up to certain point (i.e., maximum stress) like thixotropic fluid but more widely and then decrease with the same rate and come to origin (Ronningsen, 1993a; 1993b; Chhabra and Richardson, 2008; Sochi, 2010; Wilton, 2015). Several researchers proved the rheopectic behavior of fluids in their studies with different fluids (Freundlich and Juliushburger, 1935; Steg and Katz, 1965; Pradipasena and Rha, 1977; Keller and Keller, 1990; Tanner and Walters, 1998; Tanner, 2000; Chhabra and Richardson, 2008; Sochi, 2010; Wilton, 2015).

![Fig. 2.9: A typical plot for time-dependent fluid (modified from Chhabra and Richardson, 2008)](image)
2.3.1.2.3 Viscoelastic non-Newtonian fluid

The general law of elasticity describes that the stress applied to an object is directly proportional to the strain. In the case of tension condition, Hooke’s law applies and the proportionality coefficient is defined as Young’s modulus, G:

\[ \tau = -G \frac{dx}{dy} = -G \gamma \]  \hspace{1cm} (4)

From equation (4) shear stress is proportional to shear rate for a Newtonian fluid. Some fluids show elastic and viscous behavior with conditions. If the time-dependent behavior is ignored, then that fluid will be viscoelastic (Chhabra and Richardson, 2008; Sochi, 2010). Researchers illustrate the significances of viscoelastic fluids in different fields (Schowalter, 1978; Bird et al., 1987; Boger and Walters, 1992; Carreau et al., 1997; Larson, 1998; Tanner and Walters, 1998; Morrison, 2001; Sochi, 2010).

2.3.2 Fluid Properties

Fluid property (i.e., density, viscosity, compressibility etc.) plays a significant role in the petroleum industry. Rock and fluid properties interrelated to each other and an essential part of petroleum reservoir. Figure 2.10 depicts the rock and fluid formation characteristics for a porous medium. Rock matrix and pore fluids are shown differently and pointed the dry rock and saturated rock in fluid porous media. The conditions are shown which affect the transformation from dry to saturated rock.

2.3.2.1 Oil Density

For oil, the range for density in field varies from 30 lb/ft\(^3\) for light oil to 60 lb/ft\(^3\) for heavy crude oil where gas solubility is not considered (Ahmed, 2007; Ahmed, 2010). Usually oil density is calculated in the laboratory. If laboratory analysis is not available, empirical correlations can be used to get density at reservoir temperature and pressure. The oil density correlations can be divided into two groups based on the available data: (i) correlations use
pressure and temperature at reservoir condition to determine oil density, and (ii) correlations use PVT data, like gas-oil ration, oil gravity etc. (Ahmed, 2007).

**Figure 2.10:** Rock-Fluid physical characteristics in porous media (modified from Dunne, 2013)

Several empirical correlations are available for measuring oil densities. For Egyptian crude oil, Hanafy *et al.* (1997) presented that the correlation established by Ahmed (1988) is the most appropriate for calculating under-saturated reservoir oil density. For dead and gas-saturated reservoirs Katz (1942) and Standing’s (1981) correlations give the best estimation for oil densities (El-hoshoudy *et al.*, 2013). Very few researchers show correlations for dead oil density. In general, correlations are developed based on field data, experiments, and PVT analysis. From Katz’s chart, Standing (1981) developed a mathematical equation by incorporating (dead oil) density. Later, Ahmed (1988) published a correlation to assume the dead oil density where stock-tank oil molecular weight was not mentioned. El-hoshoudy *et al.* (2013) developed a new correlation for dead oil density considering reservoir pressure, temperature, GOR, API gravity and suspension pressure.

Standing (1981) developed a correlation for saturated oil viscosity, where gas specific gravity was used for stock tank and separator. Hanafy *et al.* (1997) established a correlation for density where he calculated density at reservoir temperature and $P_b$. El-hoshoudy *et al.*
(2013) developed a new correlation for saturated oil viscosity considering reservoir pressure, temperature, GOR, API gravity, $P_b$, and suspension pressure. Vasquez and Beggs (1980) published density correlations for the under-saturated reservoir condition. Ahmed (1988) established a correlation for oil density at the under saturated conditions. In oil industry, density is mostly determined by correlation or PVT data. However, no mathematical model is developed for density or show any relation of density with fluid other properties and fluid memory. There is a scope to develop fluid density model and capture memory effect.

2.3.2.2 Oil Viscosity

Viscosity is an important physical property for porous media. Fluid viscosity is defined as the internal resistance of the fluid with respect to external pressure to flow. Crude oil viscosity is a compact function of the reservoir temperature, reservoir pressure, API gravity, GOR, and other compositions (Ahmed, 2007; Ahmed, 2010). Researchers developed several correlations based on oil field data and laboratory analysis. Lots of researchers work on this topic to get accurate viscosity for reservoir crude oil (Ahmed 1988; El-hoshousy et al., 2013). The crude oil viscosity correlations can be sub-divided into two groups based on data of oil mixture as: (i) correlations based on crude oil composition, and (ii) correlations based on PVT data, such as API gravity, gas-oil ratio (Ahmed, 2007; El-hoshousy et al., 2013).

Oil viscosity can be measured in a laboratory while considering some conditions. Researchers used several correlations to measure crude oil viscosity however some correlations are specific for fixed region. Those correlations are still unable to capture the overall scenario of oil viscosity because of variation in crude oil composition and nature (Abdolhossein, et al., 2013). Considering reservoir pressure, crude oils viscosity correlations can be divided into three categories: (i) dead oil viscosity, (ii) saturated oil viscosity, and (iii) under-saturated oil viscosity (Ahmed, 2007; Ahmed, 2010; El-hoshousy et al., 2013; Abdolhossein, et al., 2013).
The dead oil viscosity is the most complicated to measure with correlations. Several researchers worked on dead oil viscosity and proposed some dead oil viscosity correlations (Beal, 1946; Beggs and Robinson, 1975; Glaso, 1980; Kaye, 1985; Al-Khafaji et al., 1987; Petrosky, 1990; Egbogah and Ng, 1990; Labedi, 1992; Kartoatmodjo and Schmidt, 1994; Bennison, 1998; Elsharkawy and Alikhan, 1999; Hossain et al., 2005; Naseri et al., 2005; Alomair et al., 2011; El-hoshoudy et al., 2013). Table 2.2 shows the overview of the data set used in the above-mentioned correlations. Here, the researchers developed dead oil viscosity correlation as a function of temperature and API gravity. Those correlations did not consider GOR, reservoir pressure, bubble point pressure, saturation pressure etc. Few researchers did not mention critical temperature, molar mass weight, boiling point, etc. in PVT reports and used equation of state to develop dead oil viscosity correlations (Teja and Rice, 1981; Johnson et al., 1987; Svrcek and Mehrotra, 1988; Johnson and Svreck, 1991; Mehrotra, 1991). Those models require several computations however, the accuracy is still not acceptable.

Several correlations have been established for saturated oil viscosity. If dissolved gas is present in oil, the viscosity also decreases and it is lower than the dead oil viscosity (Bergman and Sutton, 2007). Other researchers established two separate correlations for calculating crude oil viscosity at $P_b$ and below $P_b$ (Khan et al., 1987 and Labedi, 1992). Some researchers assumed saturated oil viscosity is a function of dead oil viscosity and GOR. Others proposed it as a function of dead oil viscosity and saturation pressure (Abdolhossein et al., 2013). Many other researchers established viscosity correlations for saturated oil (Chew and Connally, 1959; Beggs and Robinson, 1975; Al-Khafaji et al., 1987; Khan et al., 1987; Petrosky, 1990; Labedi, 1992; Kartoatmodjo and Schmidt, 1994; Elsharkawy and Alikhan, 1999; Hossain et al., 2005; Naseri et al., 2005; Bergman and Sutton, 2007). Table 2.2 also represents the data used to establish above mentioned correlations. In those correlations, GOR and saturation pressure data were used and ignored temperature, API gravity, reservoir pressure and bubble point pressure. Unfortunately, these correlations are developed for specific regional oil not for universal usage.
GOR is constant in the under-saturated region, so viscosity only depends on pressure (Abdolhossein et al., 2013). Several viscosity models are established for calculating viscosity in such under-saturated conditions (Beegs and Robinson, 1975; Vaequez and Beggs, 1980; Khan et al., 1987; Petrosky, 1990; Sutton and Fasrshad, 1990; Abdul-Majeed et al., 1990; Labedi, 1992; Orbey and Sandler, 1993; Kartoatmodjo and Schmidt, 1994; Almehaideb, 1997; Elsharkawy and Alikhan, 1999; Dindoruk and Christman, 2001; Abdul-Majeed et al., 1990; Labedi, 1992; Orbey and Sandler, 1993; Kartoatmodjo and Schmidt, 1994; Almehaideb, 1997; Elsharkawy and Alikhan, 1999; Dindoruk and Christman, 2001; Abdul-Majeed et al., 1990; Labedi, 1992; Orbey and Sandler, 1993; Kartoatmodjo and Schmidt, 1994; Almehaideb, 1997; Elsharkawy and Alikhan, 1999; Dindoruk and Christman, 2001; Abdul-Majeed et al., 1990; Labedi, 1992; Orbey and Sandler, 1993; Kartoatmodjo and Schmidt, 1994; Almehaideb, 1997; Elsharkawy and Alikhan, 1999). A few of these correlations present viscosity as a function of reservoir pressure. Other researchers included API gravity and dead oil viscosity (Labedi, 1992; Orbey and Sandler, 1993; Elsharkawy and Alikhan, 1999). Table 2 illustrates the datasets used in aforementioned viscosity correlations. Most of the researcher considered reservoir pressure and bubble point pressure for those correlations and overlooked saturated pressure, API gravity, temperature etc.

Mathematical model equations are used to determine viscosity in absence of sufficient laboratory data. There are few models for calculating viscosity of fluids. A few petroleum experts proposed some viscosity equations for crude oil. Some researchers also tried to incorporate memory mechanism in oil viscosity models; such as Slattery (1967), Mifflin and Schowalter (1986), Li et al., (2001), Zhang (2003), Iaffaldano et al. (2006), Hossain et al. (2007; 2008; 2009).

In petroleum engineering, viscosity is mostly determined by correlation or PVT data. However, a few mathematical models developed for viscosity and show some relationships of viscosity with other fluid properties i.e., stress-strain, surface tension and fluid memory. However, researchers failed to show the actual fluid behavior in those models. There is a scope to develop a comprehensive fluid viscosity model and capture the memory effect.

2.3.2.3 Isothermal Compressibility Coefficient for Oil

Isothermal compressibility coefficient can be defined as the change rate of per unit volume with respect to pressure. All the parameters including temperature is constant except pressure (Trube, 1957; Ahmed, 2007; Ahmed, 2010). Mathematically it can be denoted as below:
\[ c = \frac{1}{v} \left( \frac{\Delta V}{\Delta p} \right)_T \]  

(5)

This equation is used in solving problems related to transient flow. Under-saturated and saturated isothermal compressibility coefficients should be measured properly to get the accuracy for PVT analysis for any crude oil. The isothermal compressibility coefficient of the oil phase, \( c_{co} \), can be divided into two groups based on reservoir pressure such as: (i) Reservoir pressure greater than or equal to \( P_b \) termed as under-saturated isothermal compressibility coefficient, and (ii) Reservoir pressure below than \( P_b \) termed as saturated isothermal compressibility coefficient (Ahmed, 2007; Ahmed, 2010).

### 2.3.2.4 Solution Gas-Oil Ratio

When gas starts to dissolve in crude oil at a certain temperature and pressure, this is defined as Solution gas-oil ratio, \( R_s \). (Hanafy et al., 1997, Ahmed, 2007; Ahmed, 2010). The GOR is mainly a function of pressure, temperature, and API gravity (Hanafy et al., 1997, Ahmed, 2007; Ahmed, 2010). GOR increases with pressure until it reaches the saturation pressure. At \( P_b \) all the existing gases are dissolved in the oil. As such, the GOR value become maximum. There are lots of empirical correlations for GOR available in the literature in the form of PVT analysis. However, there is no comprehensive model for GOR with other fluid properties and fluid memory. Therefore, a scope is available for future researchers to develop fluid model showing GOR and other fluid properties to capture fluid memory.

### 2.3.2.5 Bubble Point Pressure

The maximum pressure at which a gas bubble starts to generate from the oil is known as the bubble point pressure, \( P_b \). \( P_b \) is mostly function of GOR, gas gravity, API gravity and temperature \( T \) (Ahmed, 2007). It can be presented mathematically as below:

\[ P_b = f \left( R_s, API, \gamma_g, T \right) \]  

(6)

In the absence of experimental \( P_b \), it is important for the petroleum engineer to make an estimation to measure \( P_b \). Over the last few decades, various graphical and mathematical
correlations have been presented for predicting $P_b$. There are lots of empirical correlations for $P_b$, which are available in the literature as PVT analysis.

### 2.3.2.6 Oil Formation Volume Factor

The ratio of the volume of oil (and the gas in solution) at the prevailing reservoir temperature and pressure to the volume of oil at standard conditions is defined oil formation volume factor (OFVF), $B_o$ (Hanafy et al., 1997, Ahmed, 2007; Ahmed, 2010). Mostly, $B_o$ is greater than or equal to unity. OFVF can be expressed mathematically as:

$$B_o = \frac{(V_o)_{pT}}{(V_o)_{sc}}$$

(7)

OFVF is an important feature for any reservoir. It is measured experimentally or uses empirical correlations to calculate. There are lots of empirical correlations for OFVF available in the literature. In most of the empirical correlations, the generalized form of OFVF is as below:

$$B_o = f (R_s, \gamma_g, \gamma_o, T)$$

(8)

### 2.3.2.7 PVT Analysis

The physical properties of reservoir fluid are one of the vital factors for any reservoir production life (Azad et al., 2014). PVT properties of crude oil such as $P_b$, OFVF, and GOR are calculated by two major approaches; equations of state and empirical correlations (Ahmed, 2007; 2010, Azad et al., 2014). Over the last 70 years, many studies have been done in this field and several correlations have been developed for calculating PVT properties. Correlations developed by Katz (1942), Standing (1947) and Lasater (1958) are the most known correlations. In recent times, several researchers are trying to use computational methods like Artificial Neural Network to measure PVT properties more accurately (Gharbi et al., 1999; Bello et al., 2008; Omole et al., 2009; Mansour et al., 2013; Shokrollahi et al., 2015). In Table 2.3, correlations measuring fluid properties such as GOR, $P_b$, OFVF are shown by considering AARE/ARE and STD. Most of the researchers did not calculate GOR in their correlations and those correlations are focused on specific
geological region. Petrosky and Farshad (1993) established correlations for GOR, $P_b$, OFVF with minimum error. Other researchers also developed correlations for GOR, $P_b$, OFVF but errors in results were much more. A few researchers also used Standing (1947); Laster (1958); Glaso (1980); Al-Marhoun (1992), and Mahmood and Al-Marhoun (1996) correlations as the standard and add some new conditions. In Table 2.4, $P_b$ correlations are presented considering GOR, API gravity, temperature and density. The datasets used in those correlations are based on specific region. Only few researchers used worldwide data but with less accuracy. Finally, in Table 2.5, OFVF correlations are presented considering GOR, API gravity, temperature and density. The datasets used in these correlations are based on certain crude oil sample and some researchers used worldwide data but still failed to capture overall scenario. In recent time, many researchers of petroleum industry are trying new techniques such as artificial neural networks to develop new models for PVT analysis. Fluid memory can play a significant role to develop more accurate PVT correlations which can be accepted worldwide.

2.3.3 Memory Mechanism

Irrespective of the research area, a variety of studies are predicting what will happen in the future. With advanced technology, future-forecasting is becoming easier day by day. Scientists are now predicting the future for different fields with more accuracy. Memory mechanism is one of the effective ways to predict future event.

2.3.3.1 Memory in Science and Engineering

Memory mechanism is used in various fields such as physics, chemistry, biology, economics, soil science, business, and different engineering disciplines including petroleum engineering. In Table 2.6 illustrates displays how memory term is used irrespective to fields of science and engineering. Here, the use of memory mechanism in various models are shown. In those models, memory is used in different ways with various conditions. Table 6 also presents the parameters that are used to develop the model, goal of that research, and field applications.
Cisotti (1911) derived an equation for energy density and introduced memory term for dielectric irreversible medium. After that Graffi (1936) developed a generalized Maxwell’s equations introducing “Memory” for the electromagnetism field where he addressed Faraday’s law, Ampere law, and Maxwell equations and considered a term $\beta$ for the history dependence factor for the electromagnetic field. Later Jacquelin (1984) also showed memory formalism for the electro-mechanism field and used fractional derivatives to show the properties of energy storage in any electrical networks. Recently Hamza et al. (2015) developed a modified mathematical model of Maxwell’s equations using Fractional calculus and urged that this model is different from the other fractional electromagnetic model in terms of fractional calculus. Researchers showed the numerical calculations of displacement, stress-strain, temperature, and induced electric/magnetic fields for special cases.

Atkinson and Shiffrin (1968) described memory as human memory and divided the memory into three stages, the sensor register, short-term store, and long-term store. In recent times, Bernacchia et al. (2011) found the group of simultaneously captured neurons is not correlated. Authors suggested a distributed, flexible neural structure for both reward valuation and memory, depending on their findings. Caputo and Mainardi (1971) developed the model for fluid dissipation based on memory and validated their model with experimental dissipation curves of different materials. Later Caputo and Plastino (1998) introduced the space fractional derivatives of pressure into Darcy’s law for the diffusion processes of fluids. Before that, authors defined memory as time-dependent fractional derivative for the diffusion of fluids in the rock.

Bruce (1983) developed a way to predict future geological trends through the study of past events and established three major paths for geologic predictions such as: climate change, element migration and geotectonic. After that Caputo (1999) investigated geological areas (i.e., geothermal) where fluids may affect pore sizes and proposed a modified Darcy’s law by introducing memory formalism to capture the effect on permeability with changing time. Furthermore, Christensen et al. (2004) described a new computational approach to minimize the operation time of any thermal simulation and proposed a dynamic gridding
technique along with fine scale orientation across the thermal front with the help of memory mechanism. In recent times, Mlodinow and Brun (2014) proposed a compact thermal relation with memory mechanism. Researchers represented that memory can record the events and interact with the system so that it can make a correlation with thermodynamic changes, i.e., entropy change.

Yamshchikov et al. (1994) studied memory for geological rock and showed several ways (i.e., emission memory effect, thermal emission memory effect, ultrasonic memory effect, electrical memory effect etc.) to carry out memory effect. But, only memory effects on stress-strain problems were highlighted. Later Xuefu et al. (1995) used memory term in rock mechanics and showed that rock has long-term history memory, behavior-reproducing memory and stress memory. Authors concluded that rocks can remember all the stresses they underwent in the past. Recently Hagemann and Stacke (2015) reviewed soil moisture–atmosphere feedback in several regions of the globe and found that for short variations of the regional climate, memory plays a significant role. Caputo and Kolari (2001) proposed an analytical model that represents the Fisher (1930) equation of tax version. For stock prices and inflation rates, they introduced memory mechanism and presented both short-run and long-run response to any increase or inflammation. Authors showed financial economics can be solved more efficiently with memory function.

Caputo (2000) reported an improvement in the representation of the flux and fluid pressure gradient during fluid transport in porous media. Author showed that these changes can be determined when periodic or constant pressure is applied to the boundary plane along with the time and space. Again Caputo (2003) presented fluid memory with the time, and space and showed this is more flexible way to represent local phenomena. Researcher advised to assign first order space derivative at the boundary layer with constant boundary pressure and consider initial medium pressure is zero. Then Zavala-Sanchez et al. (2009) defined memory term for fluid transport phenomena. They introduced a term called system “remembers” and described memory effects on effective solute spreading and mixing due to source size and positions. Recently Caputo and Carcione (2013) considered the 1-D model for water reservoir and showed memory mechanism with modified Fourier law.
Authors represented time-varying diffusivity for sediments of different grain sizes. Recently, Hristov (2014) presented diffusion model with the integral balance method and described memory term by weakly singular (power-law) kernels also showed the fading memory term by Volterra integrals, Riemann-Liouville derivative.

Cesarone et al. (2005) presented an addition to the Fick’s second law and introduced memory mechanism with Fick’s law based on fractional derivatives, also considered the indirect diffusion process across two different membranous of a biological membrane. Later Caputo and Cametti (2009) developed a fluid diffusion model based on Fick’s second equation using memory formalism and compared their results with some experiments of drug diffusion through human skin conducted by some other researchers. Space dependent diffusion constant was also proved in their study.

Hossain and Islam (2011) developed a new scaling technique for oil-water displacement and used modified Darcy law to develop the model equation. Authors also proposed a branch of scaling criteria that allows the various relationships between fluid pressure, capillary, saturation and fluid velocities incorporating the fluid memory. Later Hossain and Abu-Khamsin (2011a) developed new dimensionless numbers with memory mechanism. Those numbers will give an idea for convective heat transfer between rock and fluids with continuous alteration. Authors used the modified energy balance equation to establish the heat transfer coefficient for rock and fluid. Their established numbers would help to identify the rheological behavior of any rock and fluid system. After that, Hossain and Abu-Khamsin (2012) developed a new model based on the modified energy balance equation and introduced new dimensionless numbers for the thermal recovery process to show various heat transport mechanisms.

Wang and Li (2011) discussed a concept of “memory-dependent derivative”, which described an integral form of a common derivative of a kernel function of slipping interval. Their way of describing memory was more efficient than fractional derivatives. Authors also defined the memory in a way which is easy to understand the physical meaning and added expressive force to understand the memory dependent differential equation. Du et
al. (2013) observed that there are two stages of memory. One is the starting stage, the other is working stage. The actual relation between the fresh stage and the working stage is needed to be considered to get the accurate index of memory. Kolomietz (2014) used kinetic theory to describe the nuclear Fermi liquids and investigated the distortions that lead to scattering of particles on the Fermi-surface and relaxation of collective motion with memory (Landau, 1959). Author also observed that memory effects depended on the relaxation time and concluded that the memory formalism would give a time irreversible viscosity and time-dependent conservative force.

2.3.3.2 Memory in Porous Media

The idea of fractional derivatives in constitutive equations is not a recent theme. Fluid memory is described in lots of scientific, physical and engineering applications (Moroni and Cushman, 2001). There are plenty of examples of fluid memory in solid rheological properties, thermo-elastic, electromagnetic, heat diffusion, rock and fluid properties and other fields of research (Metzler et al., 1999; Barkai et al., 2000; Chow, 2005). This review will give the idea of fluid memory in any porous media. Table 2.7 gives a summarized view on how researchers already used memory concept in fluid flow through porous media. Here, memory models for porous media are presented sequentially. The assumptions are stated for each model with the mathematical representation. The application field and limitations of those models are also showed in the table.

Slattery (1967) used Buckingham-Pi theorem to represent viscoelastic fluid characteristics and observed memory mechanism in normal stress. The author only considered the permeability term in his study. However, that model is failed to show the overall scenario of fluid properties with the help of memory. After that Nibbi (1994) proposed a new model for viscous fluid to show the relation with free energies and used memory mechanism. Researcher presented the free energies for viscoelastic fluid but didn’t mention anything about fluid media and memory features.

Mifflin and Schowalter (1986) observed memory formalism for open or an enclosed system with three-dimensional steady state fluid flow. Researchers considered torque free laminar
flow and showed the time function as a gradient of velocity, but still that formalism is failed to show the real scenario of the time function, as their model only showed a stress-velocity relationship with time. Later, Eringen (1991) proposed a fluid model that incorporated memory concept for the micro polar property. Researcher showed fluid velocity with a relation to channel gap and observed that all fluids have their own formation and micro-scale properties and used memory function only with fluid velocity and stress. Soon after Zhang (2003) proposed the traffic flow model for micro and macroscopic fluid flow and proposed a viscosity model with second order derivatives to show the scenario of traffic flow. This model presented viscosity which is related to the driver memory. The traffic system of a road is the main base of this model. Chen et al. (2005) proposed the memory mechanism for fluid flow through porous media relating to stress and introduced the invasion percolation with memory (IPM) method to address dynamic viscous friction. Authors neglected viscous fluid flow and followed an open path for their solutions which didn’t disturb pressure distribution. However, this model failed to show the actual scenario of fluid memory. Considering the dimensionless number, Gatti and Vuk (2006) proposed a linear model considering periodic boundary for viscoelastic fluid in a 2-D flow region also considered the following assumptions: that fluid is incompressible; that the dimensionless number (Reynolds) is unity and; that the flow is isotropic homogeneous. Authors assumed fluid density, pressure and velocity are independent with respect to time to show the effect of fluid memory. However, those assumptions are only considered in conventional fluid models. With new direction, Hossain et al. (2007) proposed a mathematical model for fluid flow through porous media to show the stress-strain relationship and added temperature difference, surface tension, pressure difference and rock-fluid memory to make it a comprehensive model. The authors concluded that the actual stress-strain behavior can be presented strongly with the function of time, space, and fluid memory.

Caputo (1999) proposed a model for modified Darcy’s law by introducing fractional derivative and presented the local permeability alteration in any porous media. However, this assumed modification is only applicable when local phenomena are considered. Author
also derived the pressure distribution of fluid in a half-space with different boundary conditions. After a while, Li et al. (2001) studied non-Newtonian fluid properties and pointed out the reciprocal and cluster behavior of stress and time function but failed to show the relation of fluid media and time. Later Caputo and Plastino (2004) derived a modified constitutive relation for porous media to describe the diffusion of fluid more effectively and modified Darcy’s law by adding the fractional derivative of pressure with space. Authors presented rigorous derivation techniques of some conventional problems and illustrated in closed form and concluded that the time-memory is suitable for local phenomena and the space memory captures the differences in space. With new observations, Iaffaldano et al. (2006) observed permeability reduction during water diffusion in sand layers through experimental investigation. Researchers drew a conclusion that permeability decreased as result of matrix rearrangement and compaction, which drives to a reduction of porosity as well. This phenomenon was studied and presented by Elias and Hajash (1992) in the past also proposed a modified fluid diffusion model incorporating fluid memory mechanism through any porous media. Then Hossain and Islam (2006) represented a review of memory models and their applications considering fluid flow through porous media. They did a brief overview of how various researchers used the fluid memory mechanism for different fluid properties like viscosity, density, compressibility, free energies, diffusion etc. Then, Hossain et al. (2008) proposed a modified version of the classical diffusivity equation considering fluid memory for both rock and fluid. They derived this model introducing Caputo fractional derivative in classical Darcy law. They suggested an explicit finite difference method to solve the complex (non-linear) integro-differential equation. Recently, Rahman et al. (2016) represented a review of memory models and their applications considering fluid flow through porous media and illustrated a brief overview of how various researchers used the fluid memory mechanism for different fluid properties like viscosity, density, compressibility, free energies, diffusion etc. Authors also discussed the limitations and gave research guidelines for future researchers.
Arenzon et al. (2003) proposed a model considering thermal variation and gravity that described of hugely dense particles. Authors observed low and high densely fluid phase and showed irreversible and quasi-reversible cycles with the help of memory formalism. However, authors only discussed fluid density to show the actual scenario of the time function but didn’t show anything about fluid memory and media. Later, Sprouse (2010) developed a model with the numerical solution which was based on the short memory approach to solve the fractional diffusional heat equation using an explicit finite difference method. Researcher defined the short memory approach as prior incidents where minimum time can be neglected. With an addition, Carillo et al. (2014) developed a model and derived analytical solutions for the integro-differential equations explaining a solid heat conductor with memory and proved that their solutions were unique. Authors described that the addition of memory effects provided an alternative way to address nonlinearities in problems where the linear model approach cannot be applied. The authors considered two different models to understand the role of memory and concluded that the temperature gradient history was related to heat flux in heat diffusion problems.

Shin et al. (2003) studied inertia influenced components and found non-equilibrium characteristics in those. Researchers observed that those incidents occurred near the turbulent layer boundary. Their model is only applicable for the homogeneous formation which is not enough to show the actual behavior of fluid properties, media and time alteration. Later, Zia and Brady (2013) observed Brownian particle motion in complex fluid and described the material characteristics of any equilibrium condition. Author observed a theoretical and dynamic simulation of the transient behavior of a colloidal dispersion with respect to time but failed to show actual fluid media and memory in their study.

Hossain and Islam (2009a) studied cumulative oil production and showed time alteration. Authors included stress-strain formulation for both porous rock and fluid to generate a modified material balance equation (MBE) and claimed that the developed MBE can also be used for fractured formations with dynamic options. Authors showed an increase of 5%
oil recovery was calculated from the proposed MBE over the conventional MBE. However, researchers failed to provide solution techniques that can overcome complexity.

Hossain et al. (2009b, 2009c) derived a model to present the complex rheological behavior of fluid with memory. This phenomenon combined the bulk rheology and shear rate of fluid in porous media. Authors proposed some dimensionless number for reservoir rock and fluid properties such as porosity, permeability, heat capacities, densities, viscosities etc. Di Guiseppe et al. (2010) investigated the changes of fluid and rock properties under changing pressures and observed the changes of pore grains during fluid transport in porous media. Later Rasoulzadeh et al. (2014) showed three scales fractured porous media with memory. This model can be used to calculate flow around a production well in any oil reservoir. In recent times, Obembe et al. (2017b) presented a modified memory-based mathematical model showing fluid flow in porous media. Authors derived the model using the Grünwald–Letnikov (G–L) definition of the Riemann–Liouville (R–L) time fractional operator along with the generalized Darcy’s equation. Their proposed model is suitable for both fractal geometry and highly heterogeneous media and introduced G-L interpretation for time fractional derivative in the numerical modeling process.

Hristov (2013) established a new technique based on the overall penetration depth and used Jeffrey’s kernel theorem to get a solution for heat conduction equation with the help of fading memory. Author added a damping function to overcome the unreal behavior of the conventional equations and introduced a Volterra-type integral for the heat conduction problem. This integral had the damping function which was proposed by Cattaneo (1958). Though the proposed method follows the classical Fourier’s law, heat flux and fractional derivative related to its history, but it failed to present the overall scenario of fluid memory. Recently, Obembe et al. (2017c) developed a diffusion model with variable-order derivative (VOD). They presented time dependent diffusion behavior that was observed in heterogeneous fractured porous media. They used finite difference approximation based on control volume to handle VOD and numerically models are validated. However, they overlooked the space memory term in their model.


2.4 Knowledge Gap and Future Research Direction

There is a certain knowledge gap in fluid flow through porous media. Density and viscosity are two fluid properties which play an important part in characterizing the properties of fluid. Both properties have significant roles to maintain the fluid flow through any solid or semi-solid medium (i.e., rock). The time dependent (i.e., thixotropic fluid) and time-independent (i.e., shear-thinning fluid) fluid behavior is very important to identify fluid types under shear conditions. Some non-Newtonian pseudo plastic fluids show thixotropic characteristics with changing time and show a change in viscosity with time. This characteristic shows in very simple fluids as well as in complex liquid mixtures such as foams, micelles, slurries, pastes, gels, polymer solutions, and granular flows (Danko’ et al., 2006). In most cases, the relationship between density and viscosity is not presented well though they have a strong relation. Hossain et al. (2007; 2008) tried to figure viscosity along with shear rate but authors didn’t show any relationship with fluid density. Fluid memory is not appropriately used in fluid characterization. Their proposed models didn’t explain the solution technique to avoid the complexity. Di Guiseppe et al. (2010) showed the variations of fluid and rock properties under pressure difference and observed several changes of pore grains during fluid flow in porous media. For 1-D water reservoir, Caputo and Carcione (2013) developed a model considering fluid memory with modified Fourier law and illustrated time dependent diffusivity for sediments of different grain sizes. Rasoulzadeh et al. (2014) proposed a model for three scales fractured porous media with memory which can measure flow around a production well. Rahman et al. (2016) reviewed the fluid models and their applications based on fluid memory and showed the limitations of those models. Therefore, it is important to come up with a comprehensive fluid model to show the real picture of fluid media and memory.

Reservoir fluid properties play a vital role in petroleum production. Fluid properties are one of the most integrated and important part of reservoir engineering. However, to date, there are few mathematical models for fluid flow through porous media that can present the actual picture of rock and fluid properties. Hossain et. al (Hossain et al., 2007; 2008) established models to capture rock and fluid alteration with time. However, there are some
drawbacks. A comprehensive viscosity and density model can be developed considering time-dependent non-Newtonian fluid, more than one dimension and mostly focusing fluid memory and media. To develop a comprehensive model, compressibility, pH, gravity etc. of the reservoir should be considered. Therefore, there is a scope of future research to come up with a comprehensive fluid (e.g., viscosity-density) model incorporating memory concept.

2.5 Conclusions

This review rebuilds the significance of the applications of memory concept in the context of science and petroleum engineering. It summarizes the state-of-the-art review on the subject area. This study establishes the memory mechanism in petroleum reservoir characterization which is very essential for maximizing the operational activity and enhance the productions. The reservoir rock, and fluid properties of porous media could provide better understanding of the memory applications and effects in reservoir characterization. Though most of the fluid properties are supposed to be consistent with time, the researcher needs to capture the real changes of those properties along with time and space. The exact model and comprehensive solution of that model are also necessary for characterizing the reservoir properly. Memory can be an excellent concept for specifying the fluid properties. Application of this concept may lead to develop the comprehensive model by eliminating inherent assumptions and solving all complex nature of solution steps of the model equation. Finally, memory-based fluid flow model will help to generate a more efficient, and robust model for reservoir characterization. Thus, this approach will lead to capture the actual rock-fluid interactions and more consistent pressure, and temperature distribution in the porous media for both conventional and unconventional complex reservoir systems.

2.6 Nomenclature

List of symbols

\[ A_{xz} \] cross sectional area of rock perpendicular to the flow of heat \([m^2]\)
$a$  Corey coefficient of the oil relative permeability curve

ARE  Average relative error

AARE  Average absolute relative error

$B_o$  Oil formation volume factor [rm$^3$/sm$^3$]

$C_o$  Specific heat capacity of oil [J/kg K$^{-1}$]

$C_w$  Specific heat capacity of water [J/kg K$^{-1}$]

$C_r$  Specific heat capacity of rock [J/kg K$^{-1}$]

$c_F$  Non-dimensional form-drag constant

$c_i$  Total compressibility in porous medium [1/Pa]

$D$  Thermal diffusivity [m$^2$/s]

$dt$  Time step [s]

dt/dx  Temperature gradient along direction of heat transfer [K/m]

$E$  activation energy for viscous flow of 30 API gravity oils [KJ/mol]

FD  Fractional derivative

GOR  Gas-oil ratio [SCF/STB]

$G$  The body force term due to gravity [N]

$g$  Acceleration due to gravitation force [N]

$h$  Reservoir thickness [m]

$k$  Reservoir permeability [m$^2$]

MB  Memory Based

Ma  Marangoni number

$m$  Mass [Kg]

$m$  Temperature/viscosity parameter

OFVF  Oil formation volume factor [rm$^3$/sm$^3$]

PVT  Pressure Volume Temperature

$P$  Pressure of condensate [Pa]

$P_r$  Reservoir pressure [Pa]

$P_s$  Pressure of the system [Pa]

$P_b$  Bubble point pressure [Pa]

$q$  Oil flow (drainage) rate [m$^3$/s]

R  universal gas constant [kJ/mol-K]

$r$  Radial distance of reservoir in equation (1) [m]
\( S_o \) Oil saturation [fraction]  
\( S_g \) Gas saturation [fraction]  
\( S_w \) Water saturation [fraction]  
\( T \) Reservoir temperature [K]  
\( T^* \) Temperature gradient [K/m]  
\( T^*_D \) Dimensionless temperature distribution [dimensionless]  
\( t \) Time [s]  
\( t_D \) Dimensionless time [dimensionless]  
\( t_{cD} \) Dimensionless critical time [dimensionless]  
\( U_x \) Velocity of the advancing front of steam chamber (m/s)  
\( \bar{u} \) Velocity vector [m/s]  
\( V_P \) Pore Volume [m^3]  
\( V_S \) Solid Volume [m^3]  
\( V_{oD} \) Volume of displaced oil produce [fraction]  
\( V_{pD} \) Initial pore void filled with steam as water [fraction]  
\( v_s \) Linear velocity of steam front [m/s]  
\( W_k \) A weighting function for the numerical integration  
\( l-D \) One dimensional  
\( 2-D \) Two dimensional

Greek Letters

\( \nabla P \) Pressure gradient [Pa/m]  
\( \Delta S_o \) Change in oil saturation before/after steam front passage [fraction]  
\( \Delta x \) Size of grid block in x direction  
\( \nabla \phi \) Fluid potential gradient [N]  
\( \xi \) A dummy variable for time i.e., real part in the plane of the integral [s]  
\( \xi' \) Distance measured ahead of the front into the coded zone [m]  
\( d\xi \) Dummy time step [s]
\( \phi \)  
Porosity of fluid media [fraction]

\( \sigma \)  
Surface tension [mN/m]

\( \mu \)  
Fluid dynamic viscosity at any temperature \([Pa-s]\)

\( \mu_o \)  
Oil (dynamic) viscosity \([Pa-s]\)

\( \mu_w \)  
Water (dynamic) viscosity \([Pa-s]\)

\( \mu_{od} \)  
Dead oil viscosity \([Pa-s]\)

\( \mu_{s} \)  
Saturated oil viscosity \([Pa-s]\)

\( \mu_u \)  
Under saturated oil viscosity \([Pa-s]\)

\( \rho \)  
Density \([kg/m^3]\)

\( \rho_c \)  
Condensate density \([kg/m^3]\)

\( \rho_f \)  
Fluid density \([kg/m^3]\)

\( \rho_o \)  
Oil density \([kg/m^3]\)

\( \rho_r \)  
Dry rock density \([kg/m^3]\)

\( \rho_w \)  
Dry rock density \([kg/m^3]\)

\( \tau \)  
Shear Stress \([Pa]\)

\( \tau_T \)  
Shear stress at temperature \( T, \, 0^\circ K \)

\( \gamma \)  
Shear rate \([m/s/m]\)

\( \eta \)  
Ratio of the pseudo-permeability of the medium with memory to fluid viscosity \([m^3 s^{1+\alpha}/kg]\)

\( \alpha \)  
Fractional order of differentiation (related to the time and space), dimensionless

\( \alpha_1, \alpha_2 \)  
Derived variable for dimensionless thickness

\( \alpha_c \)  
Simplified condensate (water) or convective diffusivity

\( \alpha_{cr} \)  
Volumetric conversion factor

\( \beta_c \)  
Transmissibility conversion factor

\( \gamma \)  
Fractional order derivative

\( \Gamma \)  
Euler gamma function

\( \xi \)  
Normal distance to the advancing front of the steam chamber \([m]\)

\( \theta \)  
Inclination of the draining surface from the horizontal plane \([angle]\)
Subscripts

\(b\) Bubble point
\(e\) Effective
\(f\) Fluid
\(g\) Gas
\(o\) Oil
\(r\) Rock (matrix)
\(T\) Temperature
\(w\) Water

2.7 References


Beal, C., (1946), The viscosity of air, water, natural gas, crude oil and Its associated gases at oil field temperatures and pressures. Trans. AIME 1946.


Bruce, R.D., (1983), The past is the key to the future, Geochimica et Cosmochimica Acta, Volume 47, Issue 8, Pages 1341-1354, ISSN 0016-7037, 1983.


Caputo, M., (1999), Diffusion of fluids in porous media with memory. Geothermics. 28, 113–130, 1999


Darcy, H., (1856), Les fontaines publiques de la ville de Dijon: exposition et application ... (Google eBook), Victor Dalmont, 1856.


Dunne, J., (2013), Geologic controls on seismic amplitudes, Recoder, Sep 2013, Volume 38, issue 07


Kartoatmodjo, T., and Schmidt, Z., (1991), New correlations for crude oil physical properties, SPE Paer 23556.


Muskat, M., (1946), The flow of homogeneous fluids through porous media, 1946.


2.8 Appendix: A

Table 2.1: Types and features of Non-Newtonian fluid

<table>
<thead>
<tr>
<th>Shear rate and Shear-stress Relation</th>
<th>Type of Fluid</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of shear can be determined from shear stress at the same point and time.</td>
<td>Time independent/ Purely viscous/ Inelastic/ Generalized Newtonian fluids</td>
<td>Paint, Wet sand, Toothpaste etc.</td>
</tr>
<tr>
<td>Time of shearing and kinematic history is considered in relation between shear stress and shear rate.</td>
<td>Time-dependent fluids</td>
<td>Drilling muds, Hair gels, Printer inks etc.</td>
</tr>
<tr>
<td>To show partial elastic recovery after deformation, need to know the behavior of ideal fluids and elastic solids.</td>
<td>Viscoelastic fluids</td>
<td>Hand wash, Body cream etc.</td>
</tr>
</tbody>
</table>

Table 2.2: Considered parameters for different viscosity correlations

<table>
<thead>
<tr>
<th>Author</th>
<th>Samples origin</th>
<th>T (°F)</th>
<th>API (°API)</th>
<th>GOR (scf/STB)</th>
<th>Psw (psia)</th>
<th>P (psia)</th>
<th>Pb (psia)</th>
<th>µnd (cp)</th>
<th>µs (cp)</th>
<th>µo (cp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beal (1946)</td>
<td>U.S.A</td>
<td>98-250</td>
<td>10-52</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.86-1550</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Study</td>
<td>Dead Oil Viscosity</td>
<td>-</td>
<td>70-295</td>
<td>16-58</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>-</td>
<td>N/A</td>
</tr>
<tr>
<td>-------------------------------------------</td>
<td>--------------------</td>
<td>---</td>
<td>--------</td>
<td>-------</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>---</td>
<td>-----</td>
</tr>
<tr>
<td>Beggs and Robinson (1975)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glaso (1980)</td>
<td>North Sea</td>
<td>50-300</td>
<td>20-48</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kaye (1985)</td>
<td>Offshore California</td>
<td>143-282</td>
<td>7-41</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Al-Khafaji et al. (1987)</td>
<td></td>
<td>60-300</td>
<td>15-51</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Petrosky (1990)</td>
<td>Gulf of Mexico</td>
<td>114-288</td>
<td>25-46</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Egboagah and Ng (1990)</td>
<td></td>
<td>59-176</td>
<td>5-58</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labedi (1992)</td>
<td>Libya</td>
<td>100-306</td>
<td>32-48</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elsharkawy and Alikhan (1999)</td>
<td>Middle East</td>
<td>100-300</td>
<td>20-48</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Egbogah and Ng (1990)</td>
<td></td>
<td>59-176</td>
<td>5-58</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labedi (1992)</td>
<td>Libya</td>
<td>100-306</td>
<td>32-48</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elsharkawy and Alikhan (1999)</td>
<td>Middle East</td>
<td>100-300</td>
<td>20-48</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Study</th>
<th>Dead Oil Viscosity</th>
<th>-</th>
<th>70-295</th>
<th>16-58</th>
<th>N/A</th>
<th>N/A</th>
<th>N/A</th>
<th>N/A</th>
<th>-</th>
<th>N/A</th>
<th>N/A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beggs and Robinson (1975)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glaso (1980)</td>
<td>North Sea</td>
<td>50-300</td>
<td>20-48</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kaye (1985)</td>
<td>Offshore California</td>
<td>143-282</td>
<td>7-41</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Al-Khafaji et al. (1987)</td>
<td></td>
<td>60-300</td>
<td>15-51</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Petrosky (1990)</td>
<td>Gulf of Mexico</td>
<td>114-288</td>
<td>25-46</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Egboagah and Ng (1990)</td>
<td></td>
<td>59-176</td>
<td>5-58</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labedi (1992)</td>
<td>Libya</td>
<td>100-306</td>
<td>32-48</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elsharkawy and Alikhan (1999)</td>
<td>Middle East</td>
<td>100-300</td>
<td>20-48</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Study</td>
<td>Region</td>
<td>Saturated Oil Viscosity</td>
<td>Temperature</td>
<td>Saturated Oil Viscosity</td>
<td>Temperature</td>
<td>Saturated Oil Viscosity</td>
<td>Temperature</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------------------</td>
<td>-----------------</td>
<td>-------------------------</td>
<td>-------------</td>
<td>-------------------------</td>
<td>-------------</td>
<td>-------------------------</td>
<td>-------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Naseri et al. (2005)</td>
<td>Iran</td>
<td>105-298</td>
<td>17-44</td>
<td>8.75-54</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alomair (2011)</td>
<td>Kuwait</td>
<td>68-320</td>
<td>10-20</td>
<td>1.78-11360</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sararpardeh et al. (2013)</td>
<td>Iran</td>
<td>50-290</td>
<td>17-44</td>
<td>0.39-70</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chew and Conally (1959)</td>
<td>U.S.A</td>
<td>51-3544</td>
<td>132-5645</td>
<td>0.370-50</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beggs and Robinson (1975)</td>
<td>-</td>
<td>20-2070</td>
<td>132-5265</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Al-Khafaji et al. (1987)</td>
<td>-</td>
<td>0-2100</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Khan et al. (1987)</td>
<td>Saudi Arabia</td>
<td>24-1901</td>
<td>107-4315</td>
<td>0.130-77.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Petrosky (1990)</td>
<td>Gulf of Mexico</td>
<td>21-1855</td>
<td>1574-9552</td>
<td>0.210-7.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labedi (1992)</td>
<td>Libya</td>
<td>13-3533</td>
<td>60-6358</td>
<td>0.115-3.72</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kartoatmodji and Schmidt (1994)</td>
<td>World wide</td>
<td>2.3-572</td>
<td>15-6054</td>
<td>0.100-6.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elsharkawy and Alikhan (1999)</td>
<td>Middle East</td>
<td>10-3600</td>
<td>100-33700</td>
<td>0.050-21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Study</td>
<td>Location</td>
<td>Under-Saturated Oil Viscosity</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------------------</td>
<td>-------------------</td>
<td>------------------------------</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hossain et al. (2005)</td>
<td>Worldwide</td>
<td>19-493</td>
<td>121-6272</td>
<td>3600-360</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Naseri et al. (2005)</td>
<td>Iran</td>
<td>255-4116</td>
<td>420-5900</td>
<td>0.110-18.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bergman and Sutton (2007)</td>
<td>Worldwide</td>
<td>6-6525</td>
<td>66-10300</td>
<td>0.210-4277</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sarapardeh et al. (2013)</td>
<td>Iran</td>
<td>126-3261</td>
<td>365-5702</td>
<td>0.580-37.18</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beal (1946)</td>
<td>U.S.A</td>
<td>-</td>
<td>-</td>
<td>0.16-315</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vazquez and Beggs (1980)</td>
<td>Worldwide</td>
<td>126-9500</td>
<td>-</td>
<td>0.117-148</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Khan et al. (1987)</td>
<td>Saudi Arabia</td>
<td>-</td>
<td>107-4794</td>
<td>0.13-71</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Petrosky (1990)</td>
<td>Gulf of Mexico</td>
<td>N/A</td>
<td>N/A</td>
<td>0.22-4.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labedi (1992)</td>
<td>Libya</td>
<td>-</td>
<td>60-6358</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Orbey (1993)</td>
<td>-</td>
<td>740-14504</td>
<td>-</td>
<td>0.225-7.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kartootmodji and Schmidt (1994)</td>
<td>Worldwide</td>
<td>25-6015</td>
<td>4775</td>
<td>0.168-517</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Authors</td>
<td>Samples Origin</td>
<td>Data sets</td>
<td>GOR ARE/AARE</td>
<td>GOR STD</td>
<td>P&lt;sub&gt;b&lt;/sub&gt; ARE/AARE</td>
<td>P&lt;sub&gt;b&lt;/sub&gt; STD</td>
<td>B&lt;sub&gt;o&lt;/sub&gt; ARE/AARE</td>
<td>B&lt;sub&gt;o&lt;/sub&gt; STD</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------------------</td>
<td>---------------------------------</td>
<td>-----------</td>
<td>--------------</td>
<td>------------</td>
<td>------------------------</td>
<td>----------------</td>
<td>-----------------------</td>
<td>----------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elsharkawy and Alikhan (1999)</td>
<td>Middle East</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hossain et al. (2005)</td>
<td>Worldwide</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sarapardeh et al. (2013)</td>
<td>Iran</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standing (1947)</td>
<td>California (North America)</td>
<td>105</td>
<td>N/A</td>
<td>N/A</td>
<td>4.8% AARE</td>
<td>N/A</td>
<td>1.17% AARE</td>
<td>N/A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lasater (1958)</td>
<td>Canada, US (North America) and South America</td>
<td>137</td>
<td></td>
<td></td>
<td>3.8% ARE</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glaso (1980)</td>
<td>North Sea</td>
<td>45</td>
<td></td>
<td></td>
<td>1.28% ARE</td>
<td>6.98%</td>
<td>-0.43 % ARE</td>
<td>2.18%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Al-Marhoun (1988)</td>
<td>Middle East</td>
<td>69</td>
<td></td>
<td></td>
<td>3.66% AARE</td>
<td>4.536%</td>
<td>0.88% AARE</td>
<td>1.18%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Labedi (1990)</td>
<td>Libya, Nigeria, Angola (Africa)</td>
<td>128</td>
<td></td>
<td></td>
<td>1.24% ARE</td>
<td>17.07%</td>
<td>N/A</td>
<td>N/A</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2.3: PVT correlations data for GOR, P<sub>b</sub>, B<sub>o</sub> considering ARE, AARE and STD
<table>
<thead>
<tr>
<th>Authors</th>
<th>Location</th>
<th>N/A</th>
<th>N/A</th>
<th>N/A</th>
<th>0.57% AARE</th>
<th>0.68%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al-Marhoun (1992)</td>
<td>Worldwide (Middle East and North America)</td>
<td>700</td>
<td>N/A</td>
<td>N/A</td>
<td>0.57% AARE</td>
<td>0.68%</td>
</tr>
<tr>
<td>Casey and Cronquist (1992)</td>
<td>US Gulf Coast (North America)</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Petrosky and Farshad (1993)</td>
<td>Gulf of Mexico (North America)</td>
<td>81</td>
<td>3.8% AARE</td>
<td>2.88%</td>
<td>3.28% AARE</td>
<td>2.56%</td>
</tr>
<tr>
<td>Kartoatmojo and Schmidt (1994)</td>
<td>Indonesia, Middle East, North America and South America</td>
<td>740</td>
<td>23.2% AARE</td>
<td>83 scf/STB</td>
<td>20.2% AARE</td>
<td>171.3 psia</td>
</tr>
<tr>
<td>Farshad et al. (1996)</td>
<td>Colombia (South America)</td>
<td>98</td>
<td>-7.9% ARE</td>
<td>22.7%</td>
<td>-3.49% ARE</td>
<td>14.61%</td>
</tr>
<tr>
<td>Almehaideb (1997)</td>
<td>UAE (Middle East)</td>
<td>62</td>
<td>N/A</td>
<td>N/A</td>
<td>5% AARE</td>
<td>6.56%</td>
</tr>
<tr>
<td>Elsharkawy and Alikhan (1997)</td>
<td>Kuwait (Middle East)</td>
<td>175</td>
<td>7.87% AARE</td>
<td>10.73%</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Velarde et al. (1999)</td>
<td>Lab samples (Worldwide)</td>
<td>2097</td>
<td>4.73% AARE</td>
<td>18.2 scf/STB</td>
<td>11.7% AARE</td>
<td>263 psia</td>
</tr>
<tr>
<td>Al-Shammasi (1999)</td>
<td>Worldwide</td>
<td>1243</td>
<td>N/A</td>
<td>N/A</td>
<td>17.9% AARE</td>
<td>17.16%</td>
</tr>
<tr>
<td>Valko and McCain (2003)</td>
<td>Lab sample (Worldwide)</td>
<td>1745</td>
<td>5.2% AARE</td>
<td>N/A</td>
<td>10.9% AARE</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 2.4: PVT correlations data for $P_b$ considering GOR, $^0$API, $y_g$ and $T$
<table>
<thead>
<tr>
<th>Authors</th>
<th>Samples Origin</th>
<th>Data sets</th>
<th>Sample Data Ranges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Rs (scf/STB)</td>
<td>API (°API)</td>
</tr>
<tr>
<td>Standing (1947)</td>
<td>California (North America)</td>
<td>105</td>
<td>20-1245</td>
</tr>
<tr>
<td>Laster (1958)</td>
<td>Canada, US (North America) and South America</td>
<td>137</td>
<td>3-2905</td>
</tr>
<tr>
<td>Vasquez and Beggs (1980)</td>
<td>Worldwide (Lab samples)</td>
<td>6000</td>
<td>0-2199</td>
</tr>
<tr>
<td>Glaso (1980)</td>
<td>North Sea</td>
<td>45</td>
<td>90-2637</td>
</tr>
<tr>
<td>Al-Marhoun (1988)</td>
<td>Middle East</td>
<td>69</td>
<td>26-1602</td>
</tr>
<tr>
<td>Asgarpour et al. (1989)</td>
<td>North Sea</td>
<td>310</td>
<td>84-1680</td>
</tr>
<tr>
<td>Rollins et al. (1990)</td>
<td>Worldwide</td>
<td>541</td>
<td>4-220</td>
</tr>
<tr>
<td>Kartoatmodjo and Schmidt (1991)</td>
<td>Worldwide</td>
<td>5393</td>
<td>0-2897</td>
</tr>
<tr>
<td>Dokla and Osman (1992)</td>
<td>UAE (Middle East)</td>
<td>51</td>
<td>181-2266</td>
</tr>
<tr>
<td>Macary and El Batanoney (1992)</td>
<td>Gulf of Suez (Middle East)</td>
<td>90</td>
<td>200-1200</td>
</tr>
<tr>
<td>Petrosky and Farshad (1993)</td>
<td>Gulf of Mexico, Texas and</td>
<td>81</td>
<td>217-1406</td>
</tr>
<tr>
<td>Study Authors</td>
<td>Region</td>
<td>Minimum</td>
<td>Maximum</td>
</tr>
<tr>
<td>--------------------------</td>
<td>--------------------------------------</td>
<td>---------</td>
<td>---------</td>
</tr>
<tr>
<td>Omar and Todd (1993)</td>
<td>Louisiana (North America)</td>
<td>142-1440</td>
<td>26.6-53.2</td>
</tr>
<tr>
<td>De Ghetto et al. (1994)</td>
<td>De Ghetto et al. (1994)</td>
<td>8.61-3298</td>
<td>6-56.8</td>
</tr>
<tr>
<td>Farshad et al. (1996)</td>
<td>Farshad et al. (1996)</td>
<td>6-1645</td>
<td>18-44.9</td>
</tr>
<tr>
<td>Hanafy et al. (1997)</td>
<td>Hanafy et al. (1997)</td>
<td>7-4272</td>
<td>17.8-48.4</td>
</tr>
<tr>
<td>Velarde et al. (1999)</td>
<td>Velarde et al. (1999)</td>
<td>10-1870</td>
<td>12-55</td>
</tr>
<tr>
<td>Al-Shammasi (1999)</td>
<td>Al-Shammasi (1999)</td>
<td>6-3298</td>
<td>6-63.7</td>
</tr>
<tr>
<td>Mehran et al. (2006)</td>
<td>Mehran et al. (2006)</td>
<td>83-3539</td>
<td>18.8-48.92</td>
</tr>
<tr>
<td>Authors</td>
<td>Samples Origin</td>
<td>Data sets</td>
<td>Sample Data Ranges</td>
</tr>
<tr>
<td>-------------------------</td>
<td>----------------------</td>
<td>-----------</td>
<td>--------------------</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$R_s$ (scf/STB)</td>
<td>API ($^0$API)</td>
</tr>
<tr>
<td>Hemmati and Kharrat (2007)</td>
<td>Iran (Middle East)</td>
<td>287</td>
<td>125-2189</td>
</tr>
<tr>
<td>Moradi et al. (2010)</td>
<td>Worldwide</td>
<td>1811</td>
<td>8.861-3267</td>
</tr>
<tr>
<td>Mansour et al. (2013)</td>
<td>Egypt (Middle East)</td>
<td>43</td>
<td>45.2-1662.1</td>
</tr>
<tr>
<td>Katz (1942)</td>
<td>US (North America)</td>
<td>117</td>
<td>9.3-1313</td>
</tr>
<tr>
<td>Standing (1947)</td>
<td>California (North America)</td>
<td>105</td>
<td>20-1245</td>
</tr>
<tr>
<td>Knopp and Ramsey (1960)</td>
<td>Venezuela (South America)</td>
<td>159</td>
<td>200-3500</td>
</tr>
<tr>
<td>Vasquez and Beggs (1980)</td>
<td>Worldwide (Lab samples)</td>
<td>6000</td>
<td>0-2199</td>
</tr>
<tr>
<td>Glaso (1980)</td>
<td>North Sea</td>
<td>45</td>
<td>90-2637</td>
</tr>
</tbody>
</table>

Table 2.5: PVT correlations data for $B_o$ considering GOR, $^0$API, and $\gamma_g$
<table>
<thead>
<tr>
<th>Authors (Year)</th>
<th>Location</th>
<th>Min/Max</th>
<th>Min/Max</th>
<th>Min/Max</th>
<th>Min/Max</th>
<th>Min/Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al-Marhoun (1988)</td>
<td>Middle East</td>
<td>69</td>
<td>26-1602</td>
<td>19.4-44.6</td>
<td>0.75-1.367</td>
<td>74-240</td>
</tr>
<tr>
<td>Abdul Majed and Salman (1988)</td>
<td>UAE (Middle East)</td>
<td>420</td>
<td>0-1664</td>
<td>9.5-59.5</td>
<td>0.51-1.35</td>
<td>75-290</td>
</tr>
<tr>
<td>Kartoatmodjo and Schmidt (1991)</td>
<td>Worldwide</td>
<td>5393</td>
<td>0-2897</td>
<td>14.4-59</td>
<td>0.482-1.166</td>
<td>75-320</td>
</tr>
<tr>
<td>Dokla and Osman (1992)</td>
<td>UAE (Middle East)</td>
<td>51</td>
<td>181-2266</td>
<td>28.2-40.3</td>
<td>0.798-1.29</td>
<td>190-275</td>
</tr>
<tr>
<td>Al-Marhoun (1992)</td>
<td>Worldwide</td>
<td>11728</td>
<td>0-3265</td>
<td>9.5-55.9</td>
<td>0.575-2.52</td>
<td>75-300</td>
</tr>
<tr>
<td>Macary and El Batanoney (1992)</td>
<td>Gulf of Suez (Middle East)</td>
<td>90</td>
<td>200-1200</td>
<td>25-40</td>
<td>0.7-1.0</td>
<td>130-290</td>
</tr>
<tr>
<td>Omar and Todd (1993)</td>
<td>Malaysia (Asia)</td>
<td>93</td>
<td>142-1440</td>
<td>26.6-53.2</td>
<td>0.612-1.32</td>
<td>125-280</td>
</tr>
<tr>
<td>Petrosky and Farshad (1993)</td>
<td>Gulf of Mexico, Texas and Louisiana (North America)</td>
<td>81</td>
<td>217-1406</td>
<td>16.3-45</td>
<td>0.58-0.85</td>
<td>114-288</td>
</tr>
<tr>
<td>Farshad et al. (1996)</td>
<td>Colombia (South America)</td>
<td>98</td>
<td>6-1645</td>
<td>18-44.9</td>
<td>0.66-1.73</td>
<td>95-260</td>
</tr>
<tr>
<td>Almehaideb (1997)</td>
<td>UAE (Middle East)</td>
<td>62</td>
<td>128-3871</td>
<td>30.9-48.6</td>
<td>0.75-1.12</td>
<td>190-306</td>
</tr>
<tr>
<td>Al-Shammasi (1999)</td>
<td>Worldwide</td>
<td>1243</td>
<td>6-3298</td>
<td>6-63.7</td>
<td>0.51-1.44</td>
<td>74-341</td>
</tr>
<tr>
<td>Dindoruk and Christman (2001)</td>
<td>Gulf of Mexico (North America)</td>
<td>104</td>
<td>133-3050</td>
<td>14.7-40</td>
<td>0.601-1.027</td>
<td>117-276</td>
</tr>
<tr>
<td>Authors</td>
<td>Considered Parameters</td>
<td>Goal of the Research</td>
<td>Applied Field</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>--------------------------</td>
<td>------------------------------------------------------------</td>
<td>------------------------------------------------------------</td>
<td>-----------------------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cisotti (1911)</td>
<td>Density, Dielectric constant, Memory</td>
<td>Electro mechanism and energy study with memory</td>
<td>Energy and its application</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Graffi (1936)</td>
<td>Maxwell’s equation, Faraday’s law, Ampere law, History dependence</td>
<td>Electromagnetic study considering history</td>
<td>Electromagnetic analysis</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Atkinson and Shiffrin (1968)</td>
<td>Memory, Sensory store, Short term store, Long term store</td>
<td>To overview human memory work</td>
<td>Human memory</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Caputo and Mainardi (1971)</td>
<td>Dissipation, Solid material, Memory</td>
<td>Geophysical material study with memory mechanism</td>
<td>Geophysics and Solid</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bruce (1983)</td>
<td>Geological trend, Climate change, Element migration, Geotectonic</td>
<td>Geological and geochemical study regarding past events</td>
<td>Geological predictions</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 2.6:** Use of Memory in various fields of science and engineering
<table>
<thead>
<tr>
<th>Author(s) and Year</th>
<th>Subject Areas</th>
<th>Summary</th>
<th>Subject Areas</th>
<th>Summary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yamshchikov et al. (1994)</td>
<td>Rock memory, Space-time dynamics, Stress-strain state</td>
<td>Rock properties and rock geology with memory effect</td>
<td>Rock properties</td>
<td></td>
</tr>
<tr>
<td>Xuefu et al. (1995)</td>
<td>Memory, Long term history, Rock memory, Stress</td>
<td>Immediate stress memory of rock with the help of previous step</td>
<td>Rock mechanics</td>
<td></td>
</tr>
<tr>
<td>Caputo and Plastino (1998)</td>
<td>Darcy’s law, Fractional derivative, Diffusion, Pressure, Memory</td>
<td>Diffusion of fluid in rock with time derivative</td>
<td>Fluid diffusion</td>
<td></td>
</tr>
<tr>
<td>Caputo (1999)</td>
<td>Darcy’s law, Memory, Fractional order derivative, Permeability</td>
<td>Geothermal study and decrease of permeability with time</td>
<td>Geothermal fluids</td>
<td></td>
</tr>
<tr>
<td>Caputo (2000)</td>
<td>Pressure gradient, Porous media, Fluid flux, Memory</td>
<td>Study periodic change of fluid flux and pressure gradient with memory</td>
<td>Fluid transport</td>
<td></td>
</tr>
<tr>
<td>Caputo and Kolari (2001)</td>
<td>Fisher equation, Stock price, Inflation rate, Fractional calculus, Memory, Financial economy</td>
<td>Economic change for financial studies with variation of time</td>
<td>Economic analysis</td>
<td></td>
</tr>
<tr>
<td>Caputo (2003)</td>
<td>Space memory, Fluid memory, Local memory, Pressure,</td>
<td>Study the difference between space and fluid memory with fractional derivatives</td>
<td>Fluid behavior</td>
<td></td>
</tr>
<tr>
<td>Study</td>
<td>Content</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>---------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zhang (2003)</td>
<td>Traffic model, Memory, Space memory, Fractional derivative, Fluid flow depends on both forward and previous time step</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Caputo and Plastino (2004)</td>
<td>Porous media, Diffusion, Pressure, Boundary layer, Skin effect, Study fluid diffusion and effect of pressure towards boundary with change of time</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Christensen et al. (2004)</td>
<td>Computation, Thermal simulation, Dynamic grid, Memory, Thermal analysis and simulation with memory effect</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cesarone et al. (2005)</td>
<td>Fick’s law, Memory, Two membrane, Fractional order, Fluid diffusion process study along two membrane with memory</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Iaffaldano et al. (2006)</td>
<td>Darcy’s law, Fick’s equation, Traditional diffusion, Fractional calculus, Memory, Study fluid flow through porous media with a fluid memory and medium memory</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hossain et al. (2007)</td>
<td>Stress-strain, Fluid memory, Space memory, Chaotic behavior, Stress-strain relationship with the help of fluid and space memory and its nonlinearity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hossain et al. (2008)</td>
<td>Porous media, Continuity equation, Momentum balance equation, Crude oil, Study formation and fluid properties with space memory and time variation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Authors</td>
<td>Topics</td>
<td>Analysis</td>
<td>Field</td>
<td></td>
</tr>
<tr>
<td>-------------------</td>
<td>------------------------------------------------------------------------</td>
<td>--------------------------------------------------------------------------</td>
<td>------------------------------</td>
<td></td>
</tr>
<tr>
<td>Hossain et al. (2009)</td>
<td>Complex rheology, Memory, Shear rate</td>
<td>Study complex rheological behavior with time</td>
<td>Rheology analysis</td>
<td></td>
</tr>
<tr>
<td>Zavala-Sanchez et al. (2009)</td>
<td>Memory, transport coefficient, Medium, Dispersion coefficient</td>
<td>Transport phenomena with memory mechanism</td>
<td>Transport analysis</td>
<td></td>
</tr>
<tr>
<td>Caputo and Cametti (2009)</td>
<td>Fick’s equation, Memory, Fractional derivative, Space memory</td>
<td>Fluid diffusion considering fluid and space memory using fractional derivative</td>
<td>Fluid diffusion</td>
<td></td>
</tr>
<tr>
<td>Di Guiseppe et al. (2010)</td>
<td>Memory, Diffusivity equation, Porous media, Chemical change</td>
<td>Study rock and fluid properties with changing time and space in porous media</td>
<td>Rock and fluid properties</td>
<td></td>
</tr>
<tr>
<td>Baleanu et al. (2010)</td>
<td>Newton’s law, fractional derivatives, Memory</td>
<td>Study Newton’s law of motion with memory to make it applicable of engineering and science</td>
<td>Motion analysis</td>
<td></td>
</tr>
<tr>
<td>Bernacchia et al. (2011)</td>
<td>Neuron, Memory</td>
<td>Study neural system with time and memory</td>
<td>Neuron behavior</td>
<td></td>
</tr>
<tr>
<td>Hossain and Islam (2011)</td>
<td>Scaling technique, Darcy’s law, Memory</td>
<td>Scaling method for oil-water displacement using fluid memory</td>
<td>Reservoir scaling</td>
<td></td>
</tr>
<tr>
<td>Wang and Li (2011)</td>
<td>Kernel function, Slipping interval, Fractional derivative, Memory</td>
<td>Mathematical study to understand memory mechanism</td>
<td>Mathematical analysis</td>
<td></td>
</tr>
<tr>
<td>Authors</td>
<td>Contributions</td>
<td>Field</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------------</td>
<td>-------------------------------------------------------------------------------</td>
<td>---------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hossain and Abu- khamsin (2011)</td>
<td>Dimensionless number, Heat transfer, Rock and fluid properties, Memory, Rheological behavior</td>
<td>Reservoir characterization with fluid memory</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hossain and Abu- khamsin (2012)</td>
<td>Energy balance equation, Dimensionless number, Heat transport, Porous media, Memory</td>
<td>Study reservoir rock and fluid properties with memory mechanism</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Du et al. (2013)</td>
<td>Fractional derivative, Memory, Stage of memory</td>
<td>Study the change of a property after change of gradient of that property within a time interval</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Caputo and Carcione (2013)</td>
<td>One-dimensional, Water reservoir, Fourier law, Memory, Fractional order</td>
<td>Study water reservoir and sediment diffusivity with memory mechanism</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rasoulzadeh et al. (2014)</td>
<td>Fractured formation, Porous media, Memory</td>
<td>Fluid flow in any fractured formation with memory formulation</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mlodinow and Brun (2014)</td>
<td>Psychological, Thermal relation, Memory</td>
<td>Study thermal changes with respect to time and psychological memory arrow</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hristov (2014)</td>
<td>Memory, Fading memory, Volterra integrals, Riemann-Liouville derivative</td>
<td>Fluid diffusion for short period with memory mechanism</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Authors</td>
<td>Assumptions</td>
<td>Memory models</td>
<td>Application field</td>
<td>Limitations</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>-----------------------------------------------------------------------------</td>
<td>-------------------------------------------------------------------------------</td>
<td>-----------------------</td>
<td>-------------</td>
</tr>
<tr>
<td>Kolomietz (2014)</td>
<td>Fermi liquid, Collective motion, Memory</td>
<td>Study kinetic energy of nuclear fermi fluid with memory mechanism</td>
<td>Nuclear energy</td>
<td></td>
</tr>
<tr>
<td>Hamza et al. (2015)</td>
<td>Maxwell’s equation, Fractional calculus, Memory</td>
<td>Study electro mechanism of thermoelastic materials with memory</td>
<td>Electromagnetic analysis</td>
<td></td>
</tr>
<tr>
<td>Hagemann and Stacke (2015)</td>
<td>Soil moisture, memory, Regional climate, Max Planck model</td>
<td>Study soil moisture and climate changes with memory function</td>
<td>Soil hydrology analysis</td>
<td></td>
</tr>
<tr>
<td>Obembe et al. (2017a)</td>
<td>Porous media, Grünwald–Letnikov definition, Riemann–Liouville operator, Darcy’s law, Memory</td>
<td>Fluid flow model study with fractional operator; continuous time function</td>
<td>Fluid flow</td>
<td></td>
</tr>
<tr>
<td>Obembe et al. (2017c)</td>
<td>Variable order derivative, Porous media Finite difference approximation, Memory</td>
<td>Study fluid diffusion with variable order derivative consider memory mechanism</td>
<td>Fluid diffusion</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2.7:** Memory mechanism in porous media
<table>
<thead>
<tr>
<th>Source:</th>
<th>Description</th>
<th>Equation</th>
<th>Additional Remarks</th>
</tr>
</thead>
</table>
Porous media (Isotropic).  
Inertia effects are neglected. | \[ K = f (L, \bar{v}, t, \mu_0, P, \bar{\tau}, \alpha) \] | Viscoelastic fluid characteristics at thermal condition.  
Only considered permeability  
Other parameters didn’t consider.  
Considered regional thermodynamic state |
Fluid particles are homogeneous.  
Particles shape are spherical. | \[ \tau = \int \frac{2\eta_0}{\lambda_1} \left( 1 - \frac{\lambda_2}{\lambda_1} \right) e^{\frac{(t-t')}{\lambda_1}} + \lambda_2 \delta(t) \] \times \Gamma(t, t') dt' | 3-D state fluid flow  
Turbulent flow was not considered.  
Heterogeneous condition didn’t mention.  
Torque was ignored. |
Incompressible and homogeneous fluid flow.  
Physical space is smooth and | \[ T(x, t) = -p(x, t)I + \int_0^\infty 2\mu(\tau)d\bar{\rho}(x, t - \tau) dt, (x, t) \in \Omega_T \equiv \Omega \times (0, T) \] | Incompressible fluid flow  
Non-linear term was neglected.  
Heterogeneous condition didn’t mention. |
<table>
<thead>
<tr>
<th>Domain is bounded.</th>
<th>Solid material is used.</th>
<th>+</th>
<th>Fluid properties in micro scale</th>
<th>Nonlocal effect is neglected.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eringen (1991)</td>
<td>No heat conduction for fluid.</td>
<td></td>
<td></td>
<td>Heterogeneity was ignored.</td>
</tr>
<tr>
<td></td>
<td>Molecules are homogeneous</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Molecules shape are spherical.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$t_{kl} = -\pi \delta_{kl} + T_{kl}, T_{kl}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 2 \int_{-\infty}^{t} d\tau \int_{\sigma}^{1} \frac{1}{d\nu'} \sum_{klmn} (s = t) \frac{\delta C'<em>{mn}}{\delta\tau}, -\frac{\delta C</em>{mn}}{\delta\tau}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= 2d_{ij}(\tau) \frac{\delta x_i(\tau)\delta x_j(\tau)}{\delta x_m(\tau)\delta x_m(\tau)} 2 \sum (s = t)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$= (\lambda_0 + \lambda_1)\delta_{kl}\delta_{mn} + (\mu_0 + \mu_1)\times(\delta_{kl}\delta_{lm} + \delta_{kn}\delta_{lm})$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nibbi (1994)</td>
<td>Viscous fluid.</td>
<td></td>
<td>Free space energy in viscous fluid</td>
<td>The function of relaxation must be satisfied;</td>
</tr>
<tr>
<td></td>
<td>Incompressible and homogeneous flow</td>
<td></td>
<td></td>
<td>$\mu \in L^1(0, +\infty)$,</td>
</tr>
<tr>
<td></td>
<td>Linear and isotropic flow</td>
<td></td>
<td></td>
<td>$\mu \in L^1(0, +\infty) \cap L^2(0, +\infty)$</td>
</tr>
<tr>
<td></td>
<td>$T(t) = -p(t)I + \int_{-\infty}^{+\infty} 2\mu(s)\overline{D}(s)ds, \overline{D}$</td>
<td></td>
<td></td>
<td>Heterogeneous flow didn't mention.</td>
</tr>
</tbody>
</table>

83
| Viscous fluid.  
| Incompressible and homogeneous fluid flow.  
| Linear and isotropic porous medium.  
| Permeability declines with time in geothermal areas.  |
| \[ q = - \frac{\eta \rho_v \left( \frac{\delta^\alpha}{\delta y} \right) \left( \frac{\delta p}{\delta y} \right) \delta^\alpha p(y, t)}{\delta t} \]
| \[
= \frac{1}{\Gamma(1 - \alpha)} \int_0^t \left( t - u \right)^{-\alpha} \left( \delta p(y) \right), \text{where } 0 \leq \alpha \leq 1
\]
| Fluid flow in porous media  
| Non-Newtonian fluid properties and behavior  
| Heterogeneity was ignored.  
| The porosity was neglected.  
| Medium complexity was neglected.  |
| Li et al. (2001) | Bubble shape is spherical.  
| Bubble is homogeneous.  
| Stresses and composition are homogeneous.  |
| \[
\frac{d \tau_m}{dt} = -\alpha \tau_m + \beta \gamma_B
\]
| Non-Newtonian fluid properties and behavior  
| Heterogeneity was not considered.  
| Formation varieties was ignored.  
<p>| Fluid media was not defined.  |</p>
<table>
<thead>
<tr>
<th>Shin et al. (2003)</th>
<th>Porous medium is homogeneous.</th>
<th>$v^+ t = v^+ t_{eq} + \Delta v^+ t$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Particle motion with drag force.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mean shear rates variation is unrelated.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Gaussian fluctuating velocities of particles</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Turbulent particle is independent of mean shearing</td>
<td></td>
</tr>
<tr>
<td></td>
<td>The shear rate of the flow is independent.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$v^+ t_{eq} = -\frac{24}{R_{ep} C_D} \tau_p \frac{d}{dy} \left( \zeta_{yy} - D_{yy} \frac{d \sigma_y}{dy} \right) \frac{1}{u^*}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$v^+ t = -\frac{24}{R_{ep} C_D} \tau_p \frac{d}{dy} \left( \tau \frac{d \zeta_{yy}}{dy} \right) \frac{1}{u^*}$</td>
<td></td>
</tr>
</tbody>
</table>

Non-equilibrium characteristics for inertia influenced components of fluid.

- Formation
  - Heterogeneity was ignored.
- 2) Shear rate was not mentioned as function.
<table>
<thead>
<tr>
<th>Zhang (2003)</th>
<th>Media is isotropic.</th>
<th>$\mu(\rho) = 2\beta\tau_{\xi}\epsilon^2(\rho) = 2\beta\tau(\rho V'(\rho)\mathcal{G})^2$</th>
<th>Fluid flow through traffic model system</th>
<th>➢ Didn’t mentioned the shortcomings of Taylor function.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Considered Taylor expansion.</td>
<td>$v_t + (v + c(\rho))v_x = \mu(\rho)v_{xx}$</td>
<td></td>
<td>➢ 2) Function linearization was not specified.</td>
</tr>
<tr>
<td></td>
<td>Basis is traffic road model</td>
<td>$G^\ast$ (Monotonic, Generic function) is assumed as linear function.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Caputo and Plastino (2004)</td>
<td>Uncoupled the equations of diffusion from those of elasticity.</td>
<td>$(a + b\delta_{y1}^2) p = \left(\alpha + \beta\delta_{y2}^2\right) m(x, t) - m_0$; $(\gamma + \epsilon\delta_{y1}) q = c - d\delta_{y2}^2 \nabla p$;</td>
<td>Fluid diffusion in porous media</td>
<td>➢ The elastic reaction of the matrix was neglected.</td>
</tr>
<tr>
<td></td>
<td>Isotropic porous media.</td>
<td></td>
<td></td>
<td>➢ Neglected inertia effects.</td>
</tr>
<tr>
<td></td>
<td>All empirical parameters can</td>
<td></td>
<td></td>
<td>➢ 3) Only considered Green function.</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| | \( \gamma q(x, t) = - \left[ c + d \frac{\delta y}{\delta t} \right] \nabla p(x, t); \) 
| | \( a p(x, t) = \alpha p(x, t); \) 
| | \( \nabla \cdot q(x, t) + \frac{\delta \rho(x, t)}{\delta t} = 0; \) | Permeability reduction during fluid (water) diffusion | Ø Only considered permeability. Ø Porosity was neglected. Ø Heterogeneity was ignored. |
| Hossain et al. (2007) | Heterogeneous and isentropic formation. Fluid memory considered for viscosity, density, diffusivity and compressibility. | 
| | \( \tau_T = \frac{k^2 \Delta p A_{xz} \Gamma(1 - \alpha)}{\mu_o \eta \rho_o \varphi y c} \int_0^t (t - \zeta)^{-a} \left( \frac{\delta^2 p}{\delta \zeta^2} \right) d\zeta \) 
| | \( \times \left[ \left( \frac{\delta \sigma}{\delta T} \right) \frac{\Delta T}{\alpha_o M_a} \right] \times e \left( \frac{\sigma}{RT} \right) \right] d \sigma_x d \gamma | Stress-strain relationship for fluid flow | Ø Non-Newtonian fluid was not considered. Ø Relative permeability and fluid interface was ignored. |
- Media properties are also considered.
- Incorporate temperature and pressure effect.

| Hossain et al. (2009) | Heterogeneous and isentropic formation
- Depend on space, time, pressure and dummy variable
- Presented for one dimension
- Considered polymer fluid; Newtonian. |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \gamma_{pm} = \frac{\alpha_{SF}}{\sqrt{k\rho}} \frac{\eta}{(1-\zeta)} \int_0^t (t-\zeta)^{-\alpha} \delta^2 p \frac{\delta \zeta}{\delta \zeta \delta x} d\zeta ]</td>
<td></td>
</tr>
<tr>
<td>Modified fluid diffusivity in porous media</td>
<td></td>
</tr>
</tbody>
</table>

Modified fluid diffusivity in porous media

- 2-D or 3-D was not considered.
- Didn’t take non-Newtonian fluid.
- Only considered polymer,
<table>
<thead>
<tr>
<th>Sprouse</th>
<th>Viscous fluid.</th>
<th>( \frac{dg}{dt} = \alpha D^{1-\gamma} \nabla^2 g - \beta g; )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Incompressible</td>
<td>Where ( D^{-\gamma} f(x) = \frac{1}{\Gamma(\gamma)} \int_0^x \frac{f(t)}{(x-t)^{1-\gamma}} dt. )</td>
</tr>
<tr>
<td></td>
<td>and homogeneous</td>
<td>Describe heat flow equations</td>
</tr>
<tr>
<td></td>
<td>fluid flow.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Linear and</td>
<td></td>
</tr>
<tr>
<td></td>
<td>isotropic porous</td>
<td></td>
</tr>
<tr>
<td></td>
<td>medium.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fractional</td>
<td></td>
</tr>
<tr>
<td></td>
<td>derivative</td>
<td></td>
</tr>
<tr>
<td></td>
<td>defined based</td>
<td></td>
</tr>
<tr>
<td></td>
<td>on GL definition.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Zia and Brady</th>
<th>Heterogeneous</th>
<th>( \eta^{\text{micro}}(t; Pe) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(2013)</td>
<td>and isentropic</td>
<td>( \frac{\eta^{\text{micro}}(t; Pe)}{\eta} = -\frac{3}{4\pi} Pe^{-1} \left( 1 \right. )</td>
</tr>
<tr>
<td></td>
<td>formation.</td>
<td>( + \left( \frac{a^2}{b} \right) \varphi_b u. \int n g(r,t; Pe) d\Omega )</td>
</tr>
<tr>
<td></td>
<td>Assumed steady</td>
<td>Brownian motion in complex fluid formation</td>
</tr>
<tr>
<td></td>
<td>state.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Considered the</td>
<td></td>
</tr>
<tr>
<td></td>
<td>microstructural</td>
<td></td>
</tr>
<tr>
<td></td>
<td>perturbation.</td>
<td></td>
</tr>
</tbody>
</table>

- Heterogeneity was ignored.
- Didn’t considered compressible condition.
- Complex and isentropic porous media was not mentioned.
- Nothing mentioned for unsteady flow.
- Only considered short period.
- Group of particles were ignored.
| Obembe et al. (2017b) | ✷ Single phase flow.  
lsx  Slightly compressible fluid in reservoir rock.  
lsx  Consider Darcy’s generalized equation. | \[
\frac{\delta}{\delta x} \left[ g_x(p) D^{1-V} \left( \frac{\delta p}{\delta x} \right) \right] \Delta x + q_{sc} = V_b \frac{\delta}{\delta t} \left( \phi \frac{\partial}{\partial x} \right)
\]
Where, 
\[
g_x(p) = \frac{\eta A_x}{B_0}
\] | Modified fluid flow model in porous media  
lsx  Nothing mentioned for multi-phase flow.  
lsx  2) Ignored incompressible flow. |
| Obembe et al. (2017c) | ✷ Water saturated porous medium.  
lsx  Consider Darcy’s generalized equation  
lsx  Neglected subsequent step for calibration. | \[
\frac{\delta}{\delta x} \left[ \rho_w \eta D^{1-V} \left( \frac{\delta p}{\delta x} \right) \right] = (\rho_w \phi c_t) \frac{\delta p}{\delta t} - (\rho_w \phi \beta_t) \frac{\delta T}{\delta t}
\]
Where, 
\[
c_t = c_\phi + c_w \quad \text{and} \quad \beta_t = \beta_\phi + \beta_w
\] | Fluid diffusion model with VOD  
lsx  Only water was considered as fluid medium.  
lsx  2) Didn’t mentioned 2-D and 3-D flow. |
Chapter 3

A Mathematical Model of Stress-Strain Behavior for Reservoir Fluids to Capture the Memory Effect

Preface

This paper to be submitted to a Journal. The lead author performed the necessary literature review on fluid properties and fluid memory. The co-authors Tareq Uz Zaman helped in mathematical techniques, showed some coding techniques, Dr. Salim Ahmed reviewed the manuscript and Dr. M. Enamul Hossain helped in identifying the gap in research, supervising the research, and editing the manuscript.
3.1 Abstract:
Reservoir rock and fluid properties vary because of pressure depletion or thermal variation in any reservoir structure. Rock and fluid properties such as porosity, permeability, compressibility, fluid saturation, density, viscosity etc., play a significant role in continuous alteration of complex reservoir structure. Fluid memory is a continuous function of formation space and time which is used in various sections of porous media to forecast future events based on past events. The pressure gradient of any reservoir can be illustrated as a mathematical function of formation space and time with the help of fluid memory. Though fluid memory is one of the important fluid features in porous media usually it is neglected in the fluid models. This article introduces a new approach for stress-strain behaviour where viscous stresses and density are incorporated. The developed mathematical model shows the effect of density, porosity, pseudo permeability with viscosity consideration, pressure difference, temperature and the importance of fluid memory on the reservoir fluid stress-strain relationship. Space and time are considered to show the effect of fluid memory. The computation represents that memory effect causes complex and nonlinear characteristics for the stress-strain relationship. The objective of this study to develop a comprehensive memory based stress-strain model incorporating viscosity, density and validate the model numerically for light crude oil. The results show that fluid density has impact on shear stress. Results also show that memory mechanism effects the shear stress behaviour of a formation fluid considering porosity, density, flow regime, time and rate of strain and, for $\alpha=0.3$ both field and experimental condition shows a good match with proposed model. This mathematical model can be used for problems related to reservoir simulation, well test analysis, reservoir rheology, and enhanced oil recovery.

3.2 Introduction:
In the petroleum industry, fluid viscosity plays an important role. Enhanced oil recovery is inversely proportional to fluid viscosity as the oil flow increased by reducing the oil viscosity. The structure of fluid will modify continuously as it cannot resist the shear force and deform irrespective to the amount of shear forces. Fluid viscosity helps to show the
relationship between shear stress and strain rate. Usually, the viscous force of a fluid helps to shape and steady the flow, while inertia force of fluid influences interrupts the steady flow and show the turbulent behavior. This behavior can be identified by Blake number (modified Reynold’s number) for porous media.

A fluid that shows a simple relationship with viscous force to the strain rate is defined as the Newtonian fluid. Viscosity is used as a coefficient of proportionality for Newton’s law of viscosity which strongly relates with medium temperature and pressure. However, the shear force has no control over fluid viscosity. On the other hand, a fluid which is independent to viscosity is known as non-Newtonian fluid. For a non-Newtonian fluid, the shear force plays a part to change viscosity. Very few researchers describe this phenomenon throughout literature. Usually, irregularity arises in porous media because of complex behavior of non-Newtonian fluids. Mostly, Newton’s fluid viscosity models ignore the effects of porosity, density, fluid memory and considered Newton’s viscosity equations for the predictions (Chen et al., 2005; Gatti and Vuk, 2006, Hossain et al., 2007a, 2008a, 2008b and 2009a).

Pore structure effects oil flow in porous media. Pressure and temperature are the most important parameters of a reservoir (Hossain et al., 2007b, 2008a). The strain rate for reservoir fluid is affected by the fluid velocity, pressure difference, and reservoir temperature. Reservoir temperature and crude oil compositions are also responsible to diverge from Newtonian to non-Newtonian behavior. Hydrocarbons and nonmetallic substances such as sulfur, oxygen, nitrogen, etc., are the main components of crude oil. It is mainly subdivided into two groups based on API gravity i.e., light crude oil, and heavy crude oil. Crude oil has several subgroups such as paraffin, aromatics, resins, asphaltenes, naphthenes, etc., which may change due to change of temperature, and pressure in the reservoir. In Figure 3.1a, a typical fluid flux distribution in porous media is presented considering direction, time, and temperature.
Fluid flow in porous media is effected by formation rock and fluid properties. Usually, those properties are presented as a function of formation temperature and pressure. Formation rock works as a fluid flow medium in porous media and because of the pressure difference or thermal variation, fluid properties change (Hossain et al., 2007a, 2008a). The literature illustrates that current mathematical models are based on temperature and pressure correlated with rock and fluid properties of porous media (Arenzon et al., 2003; Shin et al., 2003, Chen et al., 2005; Hossain et al., 2007a, 2008a; Caputo and Carcione, 2013; Obembe et al., 2017).

Density is an important fluid property that controls the fluid flow in porous media. Reservoir fluid compressibility, relative permeability, viscosity, and strain rate are substantially very related to fluid density and cannot be ignored during reservoir fluid characterization. In the petroleum industry, most of the cases of fluid density is calculated from PVT data, and such data is mostly based on laboratory experiments. A few researchers have used empirical correlation to measure fluid density for a reservoir and those correlations were used for a specific geographical region (Standing, 1981; Ahmed, 1988; Hanafy et al., 1997; El-hoshousy et al., 2013). Fluid density has a significant role in the stress-strain relationship but until now it has been ignored by most researchers. There is no certain model in available literature where fluid density, viscosity, shear stress, and the rate
of strain is related. This paper will give a comprehensive idea to develop a mathematical relationship for stress-strain using porosity, permeability, fluid density, viscosity, and time. To represent the exact scenario, the stress-strain relationship should account for the effect of time, space and fluid memory. The classical reservoir simulation based on the formation permeability along with the flow of the fluid but it does not maintain the flow path. To overcome these shortcomings, the fluid memory should play an important role by monitoring flow in both conditions. The fluid memory has a unique feature as the working technique of fluid memory varies with different fluid media conditions. Fluid is a function of all possible fluid properties with space and time. Some fluids (i.e., incompressible, viscous fluids) sometimes show different behaviors which can be described more easily with fluid memory. This phenomenon is very limited in the literature.

Fluid memory is used to represent the actual picture of reservoir rock and fluid properties with time alteration in porous media. Very few researchers incorporate fluid memory in their models to show the exact scenario of time alteration in porous formation. Fluid flow in porous media is different from pipeline flow because of its complex structure. In the literature, researchers have shown the rock-fluid properties of porous media with the help of memory (Slattery, 1967; Eringen, 1991; Nibbi, 1994; Caputo, 1999; Li et al., 2001; Zhang, 2003; Chen et al., 2005; Iaffaldano et al., 2006; Hossain et al., 2007a, 2008a, 2008b and 2009a; Di Guiseppe et al., 2010; Raghavan and Chen, 2013; Rasoulzadeh et al., 2014; Rahman et al., 2016; Obembe et al., 2017).

Jossi et al. (1962) developed a viscosity correlation that considers fluid temperature, pressure, and chemical properties. Authors validated the correlation only for nonpolar fluid and the fluid density was reduced from 3.0 to 0.1. Porter and Johnson (1962) compared and evaluated fluid viscosity by two of the mostly used shear techniques: jet viscometer, and concentric cylinder viscometer. Authors found that concentric cylinder viscosity loss is less than the jet viscometer process and concluded that it might be due to the capillary effect, and kinetic energy correlations. Churchill and Churchill (1975) developed a new correlation for effective viscosity of pseudo-plastic and dilatant fluids considering shear
This correlation can also be illustrated for dynamic viscosity as a function of frequency oscillation. Vetter (1979) used Weertman’s temperature method to calculate stresses and viscosity in the asthenosphere of up to 400 km in depth by relating viscosity and ratio of temperature at the melting point. The author also considered the two creep laws and the creep rate was 1*10^3 of the stress value. Memory is not well used in current theories but some authors have developed flow theories (non-local) for classical Darcy’s law using general principles of statistical physics (Hu and Cushman, 1994). Suoqi (1997) developed a new approach to calculating fluid viscosity considering the fluid density, a balance of entropy production, and the entropy flow in an open system. The author has shown the proportional constant as a simple function of temperature and the model is compared with the Hildebrand fluidity model, and the modified Enskog theory (MET). Wernera et al. (1998) developed a mathematical model to calculate the petroleum fluid viscosity as a function of reservoir temperature and pressure. Authors considered the Kanti et al. (1989) model for the reservoir temperature, and the pressure and Grunberg and Nissan (1949) approach for fluid composition. This model is useful for big compositional rage such as heavy range of asphaltenes. Starov and Zhdanov (2001) used Brinkman’s equation to correlate viscosity of the fluid with resistance coefficient also showed fluid viscosity relationship with porosity for porous media. Boundary layer flow and heat transfer of a viscoelastic fluid in porous media for non-isothermal has been observed by Abel et al. (2002). Authors reviewed the effect of permeability, fluid viscosity, and the viscoelastic parameter for different conditions. Brenner (2005) studied Newton’s law of viscosity, and the Navier–Stokes equation and added a stress tensor parameter in the Navier–Stokes equation for compressible fluids. The author used the volume velocity (volume flux density) parameter in Newton’s law of viscosity to show the relationship between stress tensors and fluid density. Luo and Gu (2007) studied the viscosity for heavy crude oil and showed how viscosity is effected at a different temperature in presence of asphaltene. Authors used theoretical and experimental approaches to measure heavy oil viscosity at various temperatures. Islam and Carlson (2012) studied viscosity models for the geologic sequestration of CO2 at certain temperature and pressure. Authors considered water, brine,
and typical sea water and showed the effect of CO2 more acutely. Pal (2015) developed a low-shear fluid viscosity model for concentrated suspensions and considered core-shell particles thickness, and permeability. The author evaluated the model based on three different types (porous particles, solid core–hairy shell particles, and hard spheres) of particles. MacDonald and Miadonye (2017) reviewed viscosity correlations and developed a new simplistic, semi-empirical equation different than current empirical models for the viscosity of Tangleflags and Athabasca bitumen. These researchers illustrated that the proposed equation gave a low percentage of errors for viscosity measurement considering the temperature and the pressure.

Slattery (1967) used the Buckingham-Pi theorem to present viscoelastic fluid characteristics and observed memory mechanism in normal stress. The author only considered the permeability term in his study. Ciarletta et al. (1989) developed a viscosity equation using the memory mechanism for an incompressible fluid. Authors used fading memory to recall the past motions. Nibbi (1994) developed a model for fluid viscosity considering the memory mechanism as well. The Author also considered the quasi-static condition and showed a relationship between free energy, and fluid viscosity. Caputo (1999) proposed a mathematical model to modify Darcy’s law by introducing fractional derivative and presented the local permeability alteration in any porous media. However, this assumed modification is only applicable when local phenomena are considered. Li et al. (2001) did a stress analysis of air bubble for non-Newtonian fluids. Authors found two reasons for stress formations: (i) by the space of bubbles, and (ii) bubble response because of the fluid’s memory. Chen et al. (2005) proposed a model relating stress and the invasion percolation with the memory (IPM) method for porous media. Authors also showed a relation between stress and dynamic viscous friction. Hossain et al. (2007) developed a memory-based mathematical model to show the relationship between shear stress and the rate of strain in porous media. To represent the model as a comprehensive one authors also addressed the temperature difference, surface tension, pressure difference and fluid memory. Hossain et al. (2009b) derived a memory-based mathematical model to present the complex rheological behavior of fluid and proposed some dimensionless numbers for
rock and fluid properties such as porosity, permeability, heat capacities, densities, viscosities. Di Guiseppe et al. (2010) reviewed the changes of fluid and rock properties under changing pressures and observed the changes of pore grains during fluid transport in porous media. Hristov (2014) proposed the diffusion model with the integral balance method and described memory term by weakly singular power-law. Recently, Rahman et al. (2016) made a critical review of memory-based models for porous media and discussed assumptions and limitations of those models. Authors gave an overall guideline to develop a comprehensive memory-based fluid model for porous media.

Formation porosity, permeability, hydraulic diameter, fluid density, viscosity, reservoir temperature, and pressure need to be considered to develop a comprehensive memory-based stress-strain model. Fluid density is considered by Blake Number (B) which is known as the modified Reynold’s number for porous media. The fluid memory is observed with a principal variable, the order of differentiation (\( \alpha \)), and the pseudo-permeability as a ratio of permeability to fluid viscosity (\( \mu \)). The effect of the fluid memory can be determined with the variation of (\( \alpha \)). The results are illustrated in a graphical form to represent the effect of fluid density, viscosity, and fluid memory on the stress-strain relationship. The results show a nonlinear trend with time and this nonlinearity arises because the pressure is depended on fluid velocity.

3.3 Mathematical Model Development:
In the x-direction of a porous media, if a tangential force (F) is applying on the top surface of a fluid element (shown in Figure 3.1b), then the fluid element will deform. The deformation of the fluid element is because of shear forces that apply tangentially to a surface. Usually, Newton’s law of viscosity is used to represent the time-dependent fluid. It can be written as:

\[
\tau = -\mu \frac{du_x}{dy}
\]  

(1)
If the formation temperature (T) is considered in the x-direction and the rate of shear along the x axis showed as $\frac{d\mu_x}{dy} = \gamma$, then Newton’s law of viscosity in Eq. (1) can be written as below (Hossain et al., 2007a):

$$\tau = -\mu_T \gamma$$

(2)

In any fluid, there is some molecular transportation between the close by layers. This phenomenon is very acute in the gaseous medium than the liquid medium. However, interchange of liquid molecules between nearby layers is less than gases because of a cohesive force which keeps the molecules in certain place much more strongly. Cohesion shows a significant role in the liquid viscosity. If the temperature of liquid increases, cohesive bonding decreases, and the molecular transportation increases. Shear stress decreases with decreasing cohesive forces and increases with increasing molecular interchange. In consequence of this complex behavior of shear stress, several researchers have shown the importance of temperature on fluid viscosity and have developed different models based on experimental and field studies (Recondo et al., 2006; Hossain et al., 2007a, 2008a). In this model, the Arrhenius model is used to represent the relationship between temperature and fluid viscosity (Avramov, 2005; Haminiuk et al., 2006; Gan et al., 2006, Hossain et al., 2007a, 2008a). Fluid viscosity changes as a result of high pressure. If the pressure increases, the liquid molecules need more energy for their relative movement. With increasing viscosity, the stress-strain relationship is affected. Hossain et al. (2007a, 2008a) addresses this complex phenomenon though previous researchers have ignored this effect.

$$\mu_T = \mu_o e^{\left(\frac{E}{RT}\right)}$$

(3)

In Eq. (2) $\mu_o$ is the dynamic viscosity at reference temperature. Using the value of $\mu_T$ in Eq. (2) form Eq. (3) can be shown as:

$$\tau = -\mu_o e^{\left(\frac{E}{RT}\right)} \gamma$$

(4)
In porous media, two types of pores or void spaces are available (effective pores, and isolated pores). Isolated void or pore space cannot affect the flow of the porous medium (Dullien, 1992). Blake (1922) first introduced a dimensionless number for porous media to show the effect of inertia and viscous force which was also known as modified Reynold’s number. In the Blake number, fluid density, fluid velocity, hydraulic diameter for porous media, fluid viscosity and the void fraction is used to present both effects. Pressure drops in a porous medium is caused by continuous viscous and inertial losses. Several researchers have theoretically shown that the inertia (kinetic energy) loss depends on fractional void volume (Burke and Plummer, 1928; Ergun and Orning, 1949; Ergun, 1952). Ergun (1952) observed the modified Reynold’s number (Blake number) and provided the laminar and turbulent flow ranges for packed bed porous media. The system should be isolated to obtain more accurate values. In this study, the initial temperature has been considered as 298°K. In general, the formation temperature is considered as constant throughout the reservoir for a particular reservoir section at a certain depth (Yanowitch, 1967, Hossain et al., 2011, Hossain and Abu-Khamsin, 2012). Therefore, the viscosity in the Blake number is assumed constant (i.e., no change of viscosity due to temperature effect). As a result, the ratio of inertia and viscous force can be expressed by the Blake number as follows:

\[ B = \frac{u_x \rho D_h}{\mu_o (1-\epsilon)} \]  

(5)

Porosity (\(\phi\)) or void fraction (\(\epsilon\)) defined as the ratio of pore volume to bulk volume i.e., mathematically it can be written as \(\phi = \epsilon = V_p/V_b\). Thus Eq. (5) can be written as:

\[ B = \frac{u_x \rho D_h}{\mu_o (1-\phi)} \]  

(6)

\[ \mu_o = \frac{u \rho D_h}{B (1-\phi)} \]  

(7)

Using the value of \(\mu_o\) in Eq. (4) from Eq. (8) can be shown as:

\[ \tau = -\frac{u_x \rho D_h}{B (1-\phi)} e\left(\frac{E}{kT}\right) \gamma \]  

(8)
To address the fluid memory with stress relation, the fluid (mass) flux in porous media can be showed by the following equation (Caputo, 1999). As the flow is considered in x-direction, the equation can be written as:

\[ q_x = -\eta \rho_o \left[ \frac{\partial}{\partial t} \left( \frac{\partial \rho}{\partial x} \right) \right] \]  \hspace{1cm} (9)

In Eq. (9) \( q_x = \frac{q_x^l \rho_o}{h} \) and \( 0 \leq \alpha < 1 \) so Eq. (9) can be written as:

\[ u_x = -\eta \left[ \frac{\partial}{\partial t} \left( \frac{\partial \rho}{\partial x} \right) \right] \]  \hspace{1cm} (10)

Here, \( u_x \) is the fluid velocity in the reservoir. Putting the value of \( u_x \) into Eq. (8) becomes as:

\[ \tau = -\frac{\rho \, P_h}{B \, (1-\phi)} \left\{ -\eta \left[ \frac{\partial}{\partial x^u} \left( \frac{\partial \rho}{\partial x^u} \right) \right] \right\} e^{\left( \frac{E}{RT} \right)} \gamma \]  \hspace{1cm} (11)
Where, \( \eta \) is the pseudo permeability of the porous media. \( \eta \) can be showed with the below equation (Hossain 2008a; Hossain et al., 2008b) as follow:

\[
\eta = \frac{K}{\mu} (t)^\alpha
\]  

(12)

In Eq. (12), \( K \) = permeability, \( \mu \) = fluid viscosity, \( t \) = time and \( \alpha \) = order of differentiation.

Putting the value of \( \eta \) in Eq. (12) from Eq. (13) it becomes as:

\[
\tau = \frac{\rho D_h}{B (1-\phi)} \left[ \frac{K}{\mu} (t)^\alpha \left( \frac{\partial \rho}{\partial t} \right) \right] e^{\left( \frac{E}{RT} \right)} \gamma
\]  

(13)

The above model represents the effects of shear stress on the reservoir formation and fluid properties in 1-D (x-direction) and it can be expressed by a more general condition of 3-D fluid flow in an anisotropic heterogeneous formation. Though inelasticity of matrix, formation heterogeneity, and anisotropy sometimes fails to show several phenomena, fluid memory a capture all the phenomenology. The 1st part of the Eq. (13) are the effects of formation porosity, fluid density, and hydraulic diameter of porous media, the 2nd part is the effect of pseudo-permeability (i.e., permeability, viscosity ratio with time) and pressure gradient along the axis and both together represent the effect of fluid memory, the 3rd part is the effect of isothermal temperature condition, and the 4th part is the effects of strain rate according to shear stress also known as velocity gradient in y-direction.

3.4 Model Analysis:

The results of the comprehensive stress-rate of strain model can be obtained by solving Eq. (13) which is shown above. In this paper, we focused on the stress-strain relation, fluid density, viscosity, and fluid memory. We consider a sample reservoir from the production wellbore (Hossain, 2008a) and experimental data (Iaffaldano et al., 2006) for numerical calculation. The reservoir is isolated and oil is producing at a constant rate. The fluid is
assumed to be an API 32.8 gravity crude oil at 298°K temperature. All computations are carried out by MATLAB programming codes.

**Table 1:** Sample Reservoir Data (Hossain 2008a)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir length, $l$</td>
<td>5000 m</td>
</tr>
<tr>
<td>Reservoir width, $w$</td>
<td>100 m</td>
</tr>
<tr>
<td>Reservoir height, $h$</td>
<td>50 m</td>
</tr>
<tr>
<td>Porosity, $\phi$</td>
<td>30%</td>
</tr>
<tr>
<td>Permeability, $k$</td>
<td>$50 \text{ md} = 50 \times 10^{-15} \text{ m}^2$</td>
</tr>
<tr>
<td>Initial reservoir pressure, $p_i$</td>
<td>27579028 Pa (4000 psia)</td>
</tr>
<tr>
<td>Compressibility, $c$</td>
<td>$1.2473 \times 10^{-9} \text{ 1/pa}$</td>
</tr>
<tr>
<td>Initial viscosity, $\mu_0$</td>
<td>$87.4 \times 10^{-3} \text{ Pa-s}$</td>
</tr>
<tr>
<td>Initial flow rate, $q_i$</td>
<td>$8.4 \times 10^{-9} \text{ m}^3/\text{sec}$</td>
</tr>
<tr>
<td>Initial fluid velocity, $u_i$</td>
<td>$1.217 \times 10^{-5} \text{ m/sec}$</td>
</tr>
<tr>
<td>Fractional order of differentiation, $\alpha$</td>
<td>0.2-0.8</td>
</tr>
<tr>
<td>Number of grid in space, $N_i$</td>
<td>580</td>
</tr>
</tbody>
</table>

**Table 2:** Experimental Data (Iaffaldano et al., 2006)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of cylinder, $l$</td>
<td>11.6 cm</td>
</tr>
<tr>
<td>Inner diameter, $D_i$</td>
<td>10.1 cm</td>
</tr>
<tr>
<td>Volume of the cylinder, $V_c = \pi r^2 h$</td>
<td>929.374 cm$^3$</td>
</tr>
<tr>
<td>Permeability, $k$</td>
<td>26 Darcy</td>
</tr>
<tr>
<td>Viscosity, $\mu$</td>
<td>1.0266 cp</td>
</tr>
<tr>
<td>Sand density, $\rho_s$</td>
<td>2.4 g/cm$^3$</td>
</tr>
<tr>
<td>Mass of sand in cell, $M_s$</td>
<td>1550 gm</td>
</tr>
<tr>
<td>Volume of sand, $V_s$</td>
<td>645.83 cm$^3$</td>
</tr>
<tr>
<td>Porosity, $\phi$</td>
<td>0.3050913841</td>
</tr>
<tr>
<td>Fluid density, $\rho_f$</td>
<td>0.998408 g/cm$^3$</td>
</tr>
</tbody>
</table>
Compressibility, $c_t$ & $2.05743 \times 10^{-4}$ atm$^{-1}$
\hline
dp/dx & 0.01765982953 atm/cm
\hline
$\Delta p$ & 0.2048540225 atm
\hline
Number of grid in space, $N_t$ & 580
\hline

In Eq. (13), $D_h$ is used in the first part as hydraulic diameter. We consider the reservoir shape is rectangular and fully filled with fluid. If the reservoir length, $l = a$ and the width, $w = b$, then $D_h$ becomes:

$$D_h = \frac{4a b}{2(a+b)}$$

$$D_h = \frac{2a b}{(a+b)}$$

In Eq. (13) 2$^{nd}$ part is considered to show the effect of fluid memory. Here, $\left\{ \frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial P}{\partial x} \right) \right\}$ is the change of pressure with space and time. In addition, $\frac{K}{\mu}(t)^\alpha$ is used to show the effect of pseudo-permeability with time continue alteration. $\alpha$ value is showing the variation of the order of differentiation. In this paper, we calculate this $\left[ \frac{K}{\mu}(t)^\alpha \left\{ \frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial P}{\partial x} \right) \right\} \right]$ term as fluid flux ($m^3/m^2.s$). Authors used Iaffaldano et al. (2006) experimentally validated data which is simulated, and validated numerically by Zaman (2017). Different authors showed the variation of $\alpha$ value to capture the memory effect (Iaffaldano, 2006, Hossain 2008a, Histrov 2014, Obembe et al., 2017). Recently, Zaman (2017) shows that the best choice for $\alpha$ value is 0.3 which can give a good agreement for fluid memory. The results of this study are compared with the well-established Hossain et al., (2007a, 2009a) stress-strain model.

To solve the fluid memory term (partial differential equation) in Eq. (13), it is necessary to consider space and time for pressure calculation. The finite difference method is used to solve the fluid memory term $\left\{ \frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial P}{\partial x} \right) \right\}$ of Eq. (13). The discretized form of fluid memory term from Eq. (13) as follows:
By employing Eq. (19), we calculated the flux value of the fluid in porous media. For reservoir, we used 580 grids in space and for the experimental condition, we used 580 grids in space. To calculate fluid flux, all values of previous time steps were considered. In Figure 3.2, the alignment of grid point in space is shown.

![Figure 3.2: Alignment of grid point in space](image)

3.5 Results and Discussion:

3.5.1 Dependency of fluid density on shear stress

Fluid density has an important impact on viscosity, shear stress development and overall fluid flow. Though fluid density plays an important role in fluid flow, still, it is ignored in most of the research. Very few literatures are available where relationship with fluid density, shear stress and viscosity has shown for non-Newtonian fluids (i.e, semi-solids, liquids, polymer, body fluids) (Klijn et al., 1979; Cherry and Kwon, 1990; Bowles and Frimpong, 1991; Druschel and O’Rourke, 1991; Hammond and Hammond, 2001; Tuna and Altun, 2012; Dalton and Daivis, 2013; Chai and Saito, 2016; Zhao et al., 2016). In the petroleum industry, density is an important fluid property which play a significant role in fluid flow through porous media in various conditions. In this article, the relationship
between fluid density and shear stress is presented. Shear stress increases with the increasing fluid density and consequently viscosity decreases. Therefore, with the increasing fluid density shear development increase which result as a decrease of fluid flow. Figure 3.3a to 3.3d shows the variation of fluid shear stress with the change of density of crude oil for different values of $\alpha$ in semi-log plot. And the trend is showing that with the increase of $\alpha$ value (0.2-0.8) shear stress increases with also represents the effect of fluid memory. For different $\alpha$ values shear stress development is different and showing almost same trend and shape. In Figure 3.3a to 3.3d, though stress development is different but values are very close to each other and with increasing $\alpha$ value stress development decreases.

**Figure 3.3:** Fluid shear stress variation as a function of density for different $\alpha$ values

In this study, field data (Hossain, 2008a) and experimental data (Iaffaldano et al., 2006) are used to validate the proposed model equation. Figure 3.4 shows the relationship between shear stress and fluid density in field condition. Shear stress is calculated with field data and model equation. The results are showing good agreement and almost match with each other. For calculation, we considered constant flow regime, reservoir depth, and value of $\alpha = 0.3$ to observe the stress development for different density. However, Figure
3.4 shows the same trend as Figure 3.3. In Figure 3.5, stress is measured with experimental data and calculated with the proposed model equation. The proposed model equation shows a good agreement with experimental results. In Figure 3.5, for the same range of fluid density stress development is almost similar for both cases. The graph shows the same trend as Figure 3.3 and 3.4 where stress development increases with increasing fluid density.

**Figure 3.4:** At field condition fluid shear stress as a function of density

**Figure 3.5:** At exp. condition fluid shear stress as a function of density
In this article, $\alpha = 0.3$ is considered to show the effect of memory term in the model equation (Zaman, 2017). Figure 3.6 shows the same trend as Figure 3.3 where stress development increases with the change of fluid density. For different $\alpha$ values stress development is different and with time the change becomes very less. In Figure 3.6, stress development over the months (20-100 months) is not very much, still it maintains the same trend as Figure 3.3. For 20 months stress development is minimum and for 100 months its maximum when $\alpha = 0.3$ with a very small change.

![Graph showing fluid shear stress variation as a function of density for different months at $\alpha = 0.3$.](image)

**Figure 3.6:** Fluid shear stress variation as a function of density for different months at $\alpha = 0.3$

### 3.5.2 Dependency of the Blake number on fluid shear stress

Inertia and viscous force has a great effect on fluid flow through porous media. It controls the type of flow in porous media. For pipeline flow, researchers considered Reynolds number and in this study Blake number is used to show the effect of inertia and viscous force in porous media. In this paper, Blake number is calculated for certain depth at constant initial viscosity to show the effect of fluid density. Form the model equation, shear stress is inversely proportional to Blake number with all other parameter is constant. Figure 3.7a to 3.7d shows that stress with decreasing with the increasing Blake number in semi-log plot which reflects as the flow regime moves toward turbulent region stress decreases.
For different $\alpha$ values (0.2-0.8) stress development is not same though the change not too much and for $\alpha=0.8$ stress development is maximum with decreasing trend with increasing Blake number. For $\alpha=0.2$ and $\alpha=0.4$ the shape is not exactly straight line and displays a curvature shape. This scenario reflects the effect of fluid memory and non-linear behavior of the model.

![Figure 3.7: Fluid shear stress variation as a function of Blake number for different $\alpha$ values](image)

### 3.5.3 Dependency of fluid flux (memory effect) on proposed model

Fluid memory is one of the important phenomena that describes the actual scenario of fluid flow in porous media. In the literature, very few studied are available to show the effect of fluid memory in experimental cases (Iaffaldano et al., 2006, Caputo and Carcione, 2013). In this paper, experimental data (Iaffaldano et al., 2006) and sample reservoir data (Hossain, 2008a) is used and simulated to get the pressure distribution for the grids. Usually, pressure data for grids is not available is reservoir data or in experimental data. To calculate fluid flux, we considered $\alpha=0.3$ (Zaman, 2017) for both the conditions. Fluid flux ($m^3/m^2s$) is different from flow velocity (m/s) in case of porous media. In Figure 3.8, fluid flux is plotted with time and showed that flux values are almost linear when flow reaches the steady condition though for transient condition flux values fluctuated with time. The experiment (Iaffaldano et al., 2006) was run for 11 hours and for almost 6 hours the
flow was in transient condition for that reason flux values are fluctuated and after reaching the steady state condition it’s almost a linear and constant with time.

![Figure 3.8: Fluid flux variation with time for \( \alpha = 0.3 \)](image)

3.5.4 Dependence of flow time on shear stress

Time is one of the key factors when we consider fluid flow in porous media. Rock and fluid properties of any porous media change with time. As rock and fluid properties are related with all fluid flow phenomena in porous media (Caputo, 1999; Hossain, 2008a, Histrov, 2014). With time fluid flux is changing for certain \( \alpha \) value (\( \alpha = 0.3 \)) shown in Figure 3.8. In the literature, researchers have shown that how shear stress is changing with the time of fluid flow and depending on the characteristics of non-Newtonian fluid (Barnes, 1997; Chang et al., 1998; Pierre et al., 2004; Fingas and Fieldhouse, 2009; Hasan et al., 2010; Ghannam et al., 2012; Benziane et al., 2012; Dimitriou and Mckinley, 2014; Petrus and Azuraien, 2014, Kaur and Jaafar, 2014; Bao et al., 2016). Figure 3.9a to 3.9d shows that for different \( \alpha \) values stress development is different with time in semi-log plot when other parameters are considered same. With time fluid viscosity decreases because of pressure change (fluid flux) as temperature is considered in isothermal consideration. As fluid viscosity decreases with time then shear stress increases. For \( \alpha = 0.2 \) stress development is minimum and for \( \alpha = 0.8 \) stress development is maximum. For all \( \alpha \) values initial stress
with time is almost same but stress increase exponentially with time (Dimitriou and McKinley, 2014; Petrus and Azura, 2014, Kaur and Jaafar, 2014; Bao et al., 2016).

Figure 3.10 and Figure 3.11 shows the stress variation with time for both field and experiment condition. The field data plot shows a good match with proposed model results. The field stress results are showing same trend as model results but little higher stress development then model with time. For experiment condition, trend is also same and experiment results and model results show good agreement. Figure 3.10 and Figure 3.11 shows the same trend as Figure 3.9 when $\alpha = 0.3$ (Zaman, 2017) considered for both field and experimental condition.

![Figure 3.9: Fluid shear stress variation as a function of time for different $\alpha$ values](image_url)
Figure 3.10: At field condition fluid shear stress variation as a function of time

Figure 3.11: At exp. condition fluid shear stress variation as a function of time

3.5.5 Dependency of strain rate on shear stress

Fluid shear stress has a proportional relationship with rate of shear. Usually, stress increases with the change of strain and viscosity decreases (Barnes, 1997; Chang et al., 1998; Pierre et al., 2004; Hossain, 2008a; Ghannam et al., 2012; Benziane et al., 2012; Dimitriou and Mckinley, 2014; Kaur and Jaafar, 2014; Bao et al., 2016). In the proposed model equation, stress is proportional to strain rate where porosity, fluid density, hydraulic diameter, flow regime and fluid memory is considered. Figure 3.12a to 3.12d shows the shear stress development with the strain rate for different $\alpha$ values in log-log plot. For different $\alpha$ values stress development is not same and it increases with the increase of $\alpha$ values. As fluid flux is changing with the change of $\alpha$ values when other parameters remain same. Therefore, stress development increasing with strain rate and shows the effect of fluid memory. For $\alpha=0.8$ stress development is maximum with the change of strain rate.

Figure 3.13 shows the same trend as Figure 3.12 for field condition where parameters are same for both conditions. The results show good agreement for filed results with model results. Stress development is little bit higher for filed results from the model condition. In Figure 3.14, the model results are compared with experiment condition and results show good agreement and also shows the same trend as Figure 3.12 and Figure 3.13.
In this article, $\alpha=0.3$ is considered to show the effect of strain rate on shear stress. Figure 3.15 shows the same trend as Figure 12 and 13 for different month when $\alpha=0.3$. The shear stress development increase with time (month). For 20 months shear stress development is minimum and for 100 months its maximum when other parameters remain same and $\alpha=0.3$.

**Figure 3.12**: Fluid shear stress variation as a function of strain rate for different $\alpha$ values

**Figure 3.13**: At field condition fluid shear stress variation as a function of strain rate
Figure 3.14: At exp. condition fluid shear stress variation as a function of strain rate

![Graph showing fluid shear stress variation as a function of strain rate](image1)

Figure 3.15: Fluid shear stress variation as a function of strain rate for different months at $\alpha = 0.3$

![Graph showing fluid shear stress variation for different months](image2)

3.5.6 Comparison of proposed Stress-Strain model with Hossain et al., (2007a) Model

Figure 3.16 illustrates the variation of shear stress versus rate of strain of the proposed model (Eq. 13) for different $\alpha$ to compare with Hossain et al., (2007a) model in the log-log plot. The data used in both models are considered same (Hossain, 2008a). The trend and shape of the curves for both models are almost same and show the variation of shear stress value with different $\alpha$ values for both cases. In the proposed model, values of shear stress
development for $\alpha = 0.2$ is almost similar to Hossain et al., (2007a) model where shear stress has very little variation and less stress development form Hossian et al., (2007a) model. For $\alpha = 0.3$ shear stress development is initially less but with the change of strain rate it increases and stress development become higher than Hossian et al., (2007a). Figure 3.17 shows the comparison between proposed model with fields results and Hossain et al. (2007a) mode and the proposed model show similar trend, and closer to field condition than Hossain et al., (2007a) model. Same experimental data is used for all the conditions where proposed model show better outcomes than Hossain et al., (2007a) model. Stress development in filed condition is higher than proposed model and Hossain et al. (2007a) model. Figure 3.18 also shows the comparison between proposed model with experiment results and Hossain et al. (2007a) mode and the proposed model show a good match with experimental condition than Hossain et al., (2007a) model. This model considered the effect of porosity, fluid density, hydraulic diameter, viscous force, inertia force, flow regime, temperature and fluid memory. And fluid memory term is solved with finite difference method. The proposed model is showing consistent of shear stress value with increasing strain rate for both field and experiment study. In Hossain et al., (2007a) stress-strain model porosity, fluid density and memory effect is not accurately considered. Therefore, the proposed stress-strain model is more applicable to show the effect of fluid memory in the rheological study of fluid flow through porous media.
Figure 3.16: Comparison of proposed stress-strain model with Hossain et al., (2007a) model.

Figure 3.17: Comparison of proposed stress-strain model with field results and Hossain et al. (2007a) model.
3.6. Conclusions:

In this study, a comprehensive memory-based stress-strain model is proposed to characterize reservoir fluids, showed the rheological phenomenon of reservoir fluids, and the overall effect of fluid media and memory in characterization process. The analysis and numerical solution of the model shows that several fluid parameters affect the stress-strain behavior of reservoir fluid. Especially fluid density, pseudo-permeability, pressure gradient, and fluid memory have a great impact on the stress-strain behavior of reservoir fluids. Memory mechanism creates a discontinuous behavior in stress-strain relationship with the variation of fraction order of differentiation. The proposed model is validated with the available experimental and reservoir data from the literature and compared with the established stress-strain model, and showed that proposed model is more effective to get the memory effect. The proposed stress-strain model can be used in wide range of reservoir fluids characterization, and rheological study of fluid properties, media, and memory are considered to develop the model and the results show good agreement with the existing model and existing data.

7. Nomenclature:

\[ a \] Reservoir length, \( m \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>Reservoir width, m</td>
</tr>
<tr>
<td>B</td>
<td>Blake number</td>
</tr>
<tr>
<td>$D_h$</td>
<td>Hydraulic diameter, m</td>
</tr>
<tr>
<td>E</td>
<td>At 32.8 API gravity activation energy of crude oils, KJ/mol</td>
</tr>
<tr>
<td>k</td>
<td>Permeability of the reservoir, mD</td>
</tr>
<tr>
<td>p</td>
<td>Reservoir pressure, N/m$^2$</td>
</tr>
<tr>
<td>$P_i$</td>
<td>Initial reservoir pressure, N/m$^2$</td>
</tr>
<tr>
<td>$ΔP$</td>
<td>$P_T-P_o$ = Pressure difference, N/m$^2$</td>
</tr>
<tr>
<td>$p(x,t)$</td>
<td>Fluid pressure, Pa</td>
</tr>
<tr>
<td>$q_x$</td>
<td>Volumetric flow rate in x-direction, kg/m$^2$-s</td>
</tr>
<tr>
<td>R</td>
<td>Universal constant, kJ/mol-K</td>
</tr>
<tr>
<td>t</td>
<td>Time, sec</td>
</tr>
<tr>
<td>$ΔT$</td>
<td>$T_T-T_o$ = Temperature difference, °K</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature, °K</td>
</tr>
<tr>
<td>$u_x$</td>
<td>Reservoir fluid velocity in x-direction, m/s</td>
</tr>
<tr>
<td>$V_p$</td>
<td>Pore volume, m$^3$</td>
</tr>
<tr>
<td>$V_b$</td>
<td>Bulk volume, m$^3$</td>
</tr>
<tr>
<td>$y$</td>
<td>Distance from the boundary plane, m</td>
</tr>
<tr>
<td>$α$</td>
<td>Fractional order of differentiation</td>
</tr>
<tr>
<td>$η$</td>
<td>Ratio of the pseudo-permeability to fluid viscosity, m$^3$s$^{1+α}/$kg</td>
</tr>
<tr>
<td>$ε$</td>
<td>Void fraction</td>
</tr>
<tr>
<td>$ϕ$</td>
<td>Porosity of fluid media</td>
</tr>
<tr>
<td>$ρ_o$</td>
<td>Density of the fluid at reference temperature $T_o$, kg/m$^3$</td>
</tr>
<tr>
<td>$ρ$</td>
<td>Density of the fluid, kg/m$^3$</td>
</tr>
<tr>
<td>$γ$</td>
<td>At y-direction velocity gradient, 1/s</td>
</tr>
<tr>
<td>$τ$</td>
<td>Shear stress, Pa</td>
</tr>
<tr>
<td>$τ_T$</td>
<td>Shear stress at reference temperature $T_o$, Pa</td>
</tr>
<tr>
<td>$μ$</td>
<td>Dynamic viscosity, Pa-s</td>
</tr>
<tr>
<td>$μ_o$</td>
<td>Dynamic viscosity at reference temperature $T_o$, Pa-s</td>
</tr>
</tbody>
</table>
1-D One dimensional
3-D Three dimensional

3.8 References:


De Sterck, H., and Ullrich, P., (2009), Introduction to Computational PDEs, Course notes for Amath, 442.


Hossain, M.E., (2008a), An experimental and numerical investigation of memory-based complex rheology and rock/fluid interactions (Vol. 69, No. 11).


Chapter 4

A Modified Stress-Strain Model for Reservoir Fluids with Memory Mechanism

Preface

This paper to be submitted to a Journal. The lead author performed the necessary literature review on fluid properties and fluid memory. The co-authors Tareq Uz Zaman helped in mathematical techniques, showed some coding techniques Dr. Salim Ahmed reviewed the manuscript and Dr. M. Enamul Hossain helped in identifying the gap in research, supervising the research, and editing the manuscript.
4.1 Abstract

In fluid flow through porous media, the reservoir rock and fluid properties are the two most important features and have a substantial impact on fluid flow. Fluid memory is an important feature that represents the time-dependent behavior of rock and fluid. Fluid memory also illustrates the formation history and how fluid will flow in the future. Though fluid memory plays a significant role in reservoir formation, still very few researchers have considered fluid memory in fluid flow models. This article represents a modified fluid stress-strain model for porous media and shows viscous forces which also includes other rock and fluid properties. The proposed mathematical memory model illustrates the responses of formation permeability, fluid viscosity, surface tension, fluid velocity, reservoir pressure variations, and the effect of memory mechanism on the stress-strain behavior. The memory mechanism is incorporated with the stress-strain relationship and uses fraction order $\alpha$ to show the variation of time and space. The fractional order ($\alpha$) represents the effect of fluid memory for any fluid flow in porous media. Light crude oil
from a sample reservoir is considered to show the overall effect of fluid memory as a function of time and space. Reservoir pressure gradient also presented as a mathematical function of space and time to show the actual response of fluid memory. The proposed model equation is solved numerically and compared with established model, using field and experimental data available in the literature. The results show that surface tension and fluid memory has impact on shear stress. The fluid memory causes a nonlinear trend on shear stress with the increase of shear rate for different $\alpha$ values. For $\alpha=0.3$ both field and experimental condition shows a good match for proposed modified model considering memory effect and other parameters. The modified stress-strain model can be used in reservoir rheological analysis, fluid flow analysis, reservoir simulation, and EOR (enhanced oil recovery) process.

4.2 Introduction

Modern civilization drives smoothly with the help of energy and the petroleum industry is the main source of energy in recent times. Several new techniques (e.g. rheological study, well testing, enhanced oil recovery, etc.) are applying to increase the oil and gas production in the petroleum industry. Still, those techniques (i.e., mathematical models) have few shortcomings in maximizing the production (Abou-Kassem et al., 2006). Several researchers (Caputo, 1999, 2000; Zhang, 2003; Islam, 2006; Hossain and Islam, 2006; Hossain et al., 2007a, 2008b, 2009a; Hossain, 2008a; Caputo and Carcione, 2013; Al-Mutairi et al., 2013; Hristov, 2014; Rahman et al., 2016; Obembe et al., 2017a, 2017b) are trying to incorporate a memory mechanism to reduce the shortcomings and enhance the overall production.

In porous media, fluid flow through the pore spaces in the formation and those spaces are not always the same in structure. A typical sketch of fluid flow through porous media is shown in Figure 4.1. Newtonian fluids represent a simple and linear relation among stress (viscous) and strain rate. Fluid viscosity presents as proportionality coefficient for the Newton’s law and depends on mostly fluid temperature and pressure. Further, for non-Newtonian fluids viscosity is not well described and changes with applied forces and strain
rates. Usually, researchers ignore non-Newtonian fluid at the time of developing models, also researchers ignore surface tension of viscous fluids and fluid memory mechanism.

**Figure 4.1:** Typical Fluid Flow in Porous Media (modified from Miah et al., 2017)

Fluid viscosity has a great impact on oil production. The oil production of a reservoir is related to oil mobility which increases by decreasing the oil viscosity. If shear forces are applied to any fluid, the fluid will deform and start to move irrespective to the amount of shear forces. Fluid viscosity is one of the key factors to show the relation between the shear stress and strain rate. The fluid inertia forces interrupt the stabilize flow behavior and lead towards turbulent behavior.

Surface tension plays an important part in fluid flow through porous media. Several researchers have addressed the surface tension phenomenon for porous media in different ways (e.g. Marangoni effect, Capillary effect etc.). Ramakrishnan and Wasan (1986) have shown a theoretical and experimental study on drainage, and imbibition of relative permeability for wetting and non-wetting phases. Authors measured capillary pressure and used capillary numbers to represent the permeability equations. Vizika et al. (1994) reviewed the viscosity ratio (k) of forced imbibition process in porous media and proposed a theoretical model using new simulation and experimental studies. Authors showed the impact of both small and large values of capillary number (C_a). Lyford et al. (1998a, 1998b) studied oil recovery processes and used aliphatic alcohol (C_nH_{2n+1} OH) as a surfactant substance in porous media. Authors also discussed the Marangoni effect and used the
Marangoni number to show the surface gradient in porous media. Hossain et al. (2007a) developed a mathematical fluid memory model to show the stress-strain relationship in porous media. Researchers used Marangoni number ($M_a$) to show the surface gradient for porous media and introduced fluid memory in the stress-strain relationship. Cense and Berg (2009) reviewed the effect of viscous, and capillary forces for multiphase flow in porous media. Authors presented that capillary number shows a relation between viscous, and capillary forces and values of capillary number ranges $10^{-6}$-$10^{-4}$ showing the effect of capillary behavior in a multi-phase fluid flow. Ferer et al. (2011) developed a 2-D pore level model showing the response of immiscible drainage, viscous force, and capillary force. Authors increased the capillary number to get stable viscosity and the model was validated with several experiments. Datta et al. (2014) reviewed the non-wetting fluid in a complex 3-D porous medium. Authors used capillary number ($C_a$) to represent this behavior and showed a relationship with viscous and capillary forces. Guo et al. (2017) studied the capillary number theory for chemical flooding in enhanced oil recovery process. Authors discussed the capillary number range for a heterogeneous reservoir in non-Darcy condition and presented new techniques to obtain more oil recovery with better mobility.

Porter and Johnson (1962) compared fluid viscosity with two mostly used shear techniques (i.e. jet viscometer and concentric cylinder viscometer. Authors addressed that concentric cylinder viscosity loss is less than jet viscometer process and concluded that it might be due to the capillary effect and kinetic energy correlations. Churchill and Churchill (1975) developed a new correlation of effective viscosity for pseudo-plastic and dilatant fluids considering as a function of shear stress, this correlation also can be illustrated for dynamic viscosity as a function of frequency oscillation. Vetter (1979) used Weertman’s (1977) temperature method to calculate stresses and viscosity in the asthenosphere up to 400 km depth by relating viscosity and ratio of temperature at the melting point. The author also considered two creep laws and creep rate was $1*10^3$ of the stress value. Wernera et al. (1998) developed a mathematical model to calculate the petroleum fluid viscosity as a function of reservoir temperature ($T_r$) and pressure ($P_r$). Authors considered Kanti et al., (1989) model for reservoir temperature and pressure, and Grunberg and Nissan (1949)
approach for fluid composition. This model is useful for large compositional range such as heavy range of asphaltenes. Luo and Gu (2007) studied the viscosity for heavy crude oil and were able to show how viscosity is effected at a different temperature in the presence of asphaltene. Authors used theoretical and experimental approaches to measure heavy oil viscosity at various temperatures. Islam and Carlson (2012) studied viscosity models for the geologic sequestration of CO2 at certain temperatures and pressures. Authors considered water, brine, and typical sea water and showed the effect of CO2 more acutely. MacDonald and Miadonye (2017) reviewed viscosity correlations and developed a new simplistic, semi-empirical equation different than current empirical models for the viscosity of Tangleflags and Athabasca bitumen. Researchers illustrated that the proposed equation gave a low percentage of errors for viscosity measurement considering temperature and pressure.

Nibbi (1994) developed a model for fluid viscosity considering memory mechanism. The author also considered the quasi-static condition and was able to show a relationship between free energy and fluid viscosity. Caputo (1999) proposed a mathematical model to modify Darcy’s law by introducing a fractional derivative and presented the local permeability alteration in any porous media. However, this assumed modification is only applicable when local phenomena are considered. Chen et al. (2005) proposed a model relating stress and the invasion percolation with memory (IPM) method for porous media. Authors also shown the relation between stress and dynamic viscous friction. Hossain et al. (2007a) developed a memory-based mathematical model to show the relationship between stress and strain for porous media. Authors also addressed temperature difference, surface tension, pressure difference, and fluid memory to represent the model as a comprehensive one. Hossain et al. (2009b) derived a memory-based mathematical model to present the complex rheological behavior of fluid and proposed some dimensionless numbers for rock and fluid properties such as porosity, permeability, heat capacities, densities, and viscosities. Di Guiseppe et al. (2010) reviewed the changes of fluid and rock properties under changing pressures and observed the changes of pore grains during fluid transport in porous media. Hristov (2014) proposed a diffusion model with the integral
balance method and described the memory term by weakly singular power-law. Recently, Rahman et al. (2016) made a critical review on memory-based models for porous media and discussed assumptions and limitations of those models. Authors gave an overall guideline to develop a comprehensive memory-based fluid model for porous media.

Formation structure, permeability, surface tension, capillary effect, viscosity, reservoir temperature, and pressure need to be considered to develop a comprehensive model for stress-strain behavior and which can capture memory effect. For surface tension, Capillary Number \( (C_a) \) is considered to show the effect of viscous and capillary forces for porous media. The fluid memory is observed from the pressure gradient, differential order \( (\alpha) \), and the pseudo-permeability as a ratio of permeability to fluid viscosity \( (\eta) \). The actual response of the fluid memory can be determined with the variation of \( (\alpha) \). The result outcomes are illustrated in graphical form to represent the response of surface tension, fluid viscosity, and fluid memory on the stress-strain relationship. The results show a nonlinear trend with time and this nonlinearity arises pressure the fluid memory shows a nonlinear behavior with time and this nonlinearity arises because the pressure is depended on fluid velocity.

4.3 Mathematical Model Development

In the x-direction, an external shear force \( (F_x) \) is applying on the top layer of a fluid element (shown in Figure 4.2) and the fluid element will deform. The deformation of the fluid element is showing the effect of shear forces that have been applied tangentially to a surface. Usually, Newton’s law of viscosity is used to show the shear stress and rate of strain relationship for time-dependent fluids (Hossain et al., 2007a, 2008a). Mathematically it can be written as:

\[
\tau = -\mu \frac{du_x}{dy}
\]  

(1)

Temperature \( (T) \) is considered in the x-direction for the formation, and shear rate can be presented as \( \gamma \). Then Eq. (1) can be written as below:

\[
\tau = -\mu_T \gamma
\]  

(2)
Cohesion effect plays an important role in the liquid viscosity. If the temperature of fluid increases, cohesive bonding between fluid molecules decreases, and the overall molecular transportation increases. Shear stress decreases with decreasing cohesive forces and increases with increasing molecular interchange. In consequence of this complex behavior of shear stress, several researchers showed the importance of temperature on fluid viscosity and developed different models based on experimental and field studies (Recondo et al., 2006; Hossain et al., 2007a, 2008a). In this model, the Arrhenius model is used to represent the relationship between temperature and fluid viscosity (Avramov, 2005; Haminiuk et al., 2006; Gan et al., 2006, Hossain et al., 2007a; Hossain, 2008a).

\[
\mu_T = \mu_o e^{\left(\frac{E}{RT}\right)}
\]  

(3)

In Eq. (3) \(\mu_o\) is the dynamic viscosity at reference temperature. Using the value of \(\mu_T\) in equation (2) form equation (3) shows as:

\[
\tau = -\mu_o e^{\left(\frac{E}{RT}\right)} \gamma
\]  

(4)
Fluid particles and molecules may transfer to nearby layers at the time of fluid flow through porous media. This phenomenon is acuter in the gaseous medium than in liquid medium as gas molecules have fewer forces comparing liquids. Cohesive force helps to keep the molecules of fluid in a certain place more effectively. Cohesion has a prominent impact on the reservoir fluid viscosity. With the increasing fluid temperature, molecular bonding decreases and inter-molecular transportation increases. The cohesive force also has a big impact on shear stress. Shear stress reduces with decreasing cohesive forces and increases with increasing inter-molecular transfer. Researchers presented the importance of formation temperature on fluid viscosity and established several models based on field data and experimental studies (Frisch et al., 1940; Sanyal et al., 1974; Al-Besharah et al., 1989; Recondo et al., 2006; Hossain et al., 2007a, 2007b, 2008a; Wu et al., 2014; Nmegbu, 2014, Akankpo and Essien, 2015, Wu and Massoudi, 2016). Fluid viscosity also changes because

![Figure 4.2: 3-D tangential forces of a fluid element (modified from Chhabra and Richardson, 2008)](image)
of high pressure. If the pressure increases, the fluid elements need more energy for their relative movement in porous media. Hossain et al. (2007a, 2008b, 2009a, 2009b) addressed this complex phenomenon though previous researchers have ignored this effect.

Surface tension plays a significant role in fluid flow through porous media. The capillary effect for fluid layers and porous media are assumed to show the actual scenario of surface tension. The capillary effect is a process that moderate the interaction between contacting surfaces of a fluid and a solid. This influences the fluid surface from a linear condition and helps the fluid to rise or fall in a narrow space. Surface tension of a fluid can be introduced by media temperature, gravitation force, fluid density, and surface alteration through the layer (Dullien, 1992; D’Aubeterre et al., 2005). In porous media, Capillary number repents the effect of viscous force to surface tension which acting on the across of two immiscible fluids. A Capillary number is a dimensionless number which does not depend on the units of a system. In this study, the initial temperature has been considered as 298°K. In general, the formation temperature is considered as constant throughout the reservoir for a particular reservoir section at a certain depth (Yanowitch, 1967, Hossain et al., 2011, Hossain and Abu-Khamsin, 2012). Therefore, the viscosity in the Capillary number is assumed constant (i.e., no change of viscosity due to temperature effect). As a result, the ratio of viscous force and surface tension can be expressed by the Capillary number as follows:

\[ C_a = \frac{\mu_o u_x}{\sigma} \]  
\[ \mu_o = \frac{\sigma C_a}{u_x} \]  

Putting the value of Eq. (6) on Eq. (4):

\[ \tau = -\frac{\sigma C_a}{u_x} e^{\left(\frac{E_{RT}}{RT}\right)} \gamma \]  

To address the fluid memory with stress relation, the fluid flux in porous media can be showed by the following equation (Caputo, 1999). As the flow is considered in x-direction, the equation can be written as:
\[ q_x = -\eta \rho_0 \left[ \frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial p}{\partial x} \right) \right] \]  

(8)

In Eq. (8) \( q_x = \frac{d^\alpha \rho_0}{dt^\alpha} \) and \( 0 \leq \alpha < 1 \) so Eq. (8) can be written as:

\[ u_x = -\eta \left[ \frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial p}{\partial x} \right) \right] \]  

(9)

Putting the value of Eq. (9) on Eq. (7)

\[ \tau = -\frac{\sigma C_a}{\eta} \left[ \frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial p}{\partial x} \right) \right] e^{\left( \frac{E}{RT} \right) \gamma} \]  

(10)

\[ \tau = -\sigma C_a \frac{1}{\eta} \left[ \frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial p}{\partial x} \right) \right] e^{\left( \frac{E}{RT} \right) \gamma} \]  

(11)

Where, \( \eta \) is the pseudo permeability of the porous media. \( \eta \) can be showed with the below equation (Hoosain, 2008a; Hossain et al., 2008b) as follow:

\[ \eta = \frac{K}{\mu} (t)^\alpha \]  

(12)

In Eq. (12), \( K = \) permeability, \( \mu = \) fluid viscosity, \( t = \) time and \( \alpha = \) order of differentiation.

Putting the value of \( \eta \) in Eq. (11) from Eq. (12) it becomes as:

\[ \tau = \sigma C_a \frac{1}{\left[ \frac{K}{\mu} (t)^\alpha \right]} \left[ \frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial p}{\partial x} \right) \right] e^{\left( \frac{E}{RT} \right) \gamma} \]  

(13)

The above Eq. (13) illustrates the effect of shear stress and fluid properties in a reservoir formation in 1-D (x-direction) and it can be expressed in a more general condition in 3-D fluid flow for an anisotropic heterogeneous formation. Though inelasticity of matrix, formation heterogeneity, and anisotropy sometimes failed to show any phenomena, fluid memory could capture all the phenomenology. The 1st part of the Eq. (13) shows the effects
of surface tension and capillary effect of porous media. The 2\textsuperscript{nd} part is the effect of pseudo-
permeability (i.e., permeability, viscosity ratio with time) and pressure gradient along the
axis and both together represent the effect of fluid memory. The 3\textsuperscript{rd} part is the effect of
isothermal temperature condition in the formation for certain depth. Finally, the 4\textsuperscript{th} part is
the effects of strain rate according to shear stress also known as velocity gradient in y-
direction.

4.4 Numerical Analysis of The Model

The results of the proposed stress-strain model can be obtained by solving Eq. (13) which
is shown above. In this paper, we focused on the stress-strain relation, fluid surface tension,
viscosity, and fluid memory. We consider a sample reservoir from the production wellbore
(Hossain 2008a) and experimental data (Iaffaldano et al., 2006) for numerical calculation.
The reservoir is isolated and oil is producing at a constant rate. The fluid is assumed to be
an API 32.8 gravity crude oil at 298\textdegree{}K temperature. All computations are carried out by
MATLAB programming codes.

<table>
<thead>
<tr>
<th>Table 1: Sample Reservoir Data (Hossain 2008a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir length, $l$</td>
</tr>
<tr>
<td>Reservoir width, $w$</td>
</tr>
<tr>
<td>Reservoir height, $h$</td>
</tr>
<tr>
<td>Porosity, $\phi$</td>
</tr>
<tr>
<td>Permeability, $k$</td>
</tr>
<tr>
<td>Initial reservoir pressure, $p_i$</td>
</tr>
<tr>
<td>Compressibility, $c$</td>
</tr>
<tr>
<td>Initial viscosity, $\mu_o$</td>
</tr>
<tr>
<td>Surface tension</td>
</tr>
<tr>
<td>Initial flow rate, $q_i$</td>
</tr>
<tr>
<td>Initial fluid velocity, $u_i$</td>
</tr>
<tr>
<td>Fractional order of differentiation, $\alpha$</td>
</tr>
<tr>
<td>Number of grid in space, $N_t$</td>
</tr>
<tr>
<td>-------------------------------</td>
</tr>
</tbody>
</table>

**Table 4.2: Experimental Data (Iaffaldano et al., 2006)**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of cylinder, $l$</td>
<td>11.6 cm</td>
</tr>
<tr>
<td>Inner diameter, $D_i$</td>
<td>10.1 cm</td>
</tr>
<tr>
<td>Volume of the cylinder, $V_c = \pi r^2 h$</td>
<td>929.374 cm$^3$</td>
</tr>
<tr>
<td>Permeability, $k$</td>
<td>26 Darcy</td>
</tr>
<tr>
<td>Viscosity, $\mu_0$</td>
<td>1.0266 cp</td>
</tr>
<tr>
<td>Surface tension</td>
<td></td>
</tr>
<tr>
<td>Sand density, $\rho_s$</td>
<td>2.4 g/cm$^3$</td>
</tr>
<tr>
<td>Mass of sand in cell, $M_s$</td>
<td>1550 gm</td>
</tr>
<tr>
<td>Volume of sand, $V_s$</td>
<td>645.83 cm$^3$</td>
</tr>
<tr>
<td>Porosity, $\phi$</td>
<td>0.3050913841</td>
</tr>
<tr>
<td>Fluid density, $\rho_f$</td>
<td>0.998408 g/cm$^3$</td>
</tr>
<tr>
<td>Compressibility, $c_t$</td>
<td>2.05743* 10^{-4} atm$^{-1}$</td>
</tr>
<tr>
<td>$\frac{dp}{dx}$</td>
<td>0.01765982953 atm/cm</td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>0.2048540225 atm</td>
</tr>
<tr>
<td>Number of grid in space, $N_t$</td>
<td>580</td>
</tr>
</tbody>
</table>

In Eq. (13) 2nd part is considered to show the effect of fluid memory. Here, $\left\{ \frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial P}{\partial x} \right) \right\}$ is the change of pressure with space and time. In addition, $\frac{K}{\mu} (t)^{\alpha}$ is used to show the effect of pseudo-permeability with time continue alteration. $\alpha$ value is showing the variation of the order of differentiation. In this paper, we calculate this $\left[ \frac{K}{\mu} (t)^{\alpha} \left\{ \frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial P}{\partial x} \right) \right\} \right]$ term as fluid flux ($m^3/m^2 s$). Authors used Iaffaldano et al. (2006) experimentally validated data which is simulated, and validated numerically by Zaman (2017). Different authors showed the variation of $\alpha$ value to capture the memory effect (Iaffaldano, 2006, Hossain 2008a, Histrov 2014, Obembe et al., 2017). Recently, Zaman (2017) shows that the best choice for $\alpha$ value
is 0.3 which can give a good agreement for fluid memory. The results of this study are compared with the well-established Hossain et al., (2007a, 2009a) stress-strain model.

To solve the fluid memory term (partial differential equation) in Eq. (13), it is necessary to consider space and time for pressure calculation. The finite difference method is used to solve the fluid memory term \( \left\{ \frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial P}{\partial x} \right) \right\} \) of Eq. (13). The discretized form of fluid memory term from Eq. (13) as follows:

\[
\frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial P}{\partial x} \right)_{i}^{n} = \frac{1}{\Gamma(2-\alpha)} \frac{1}{(\Delta t)^\alpha} \sum_{l=1}^{n} \left[ (l+1)^{1-\alpha} - (l - 1)^{1-\alpha} \right] \left[ \left( \frac{\partial P}{\partial x} \right)_{i}^{n-l+1} - \left( \frac{\partial P}{\partial x} \right)_{i}^{n-l} \right] \]  

(14)

\[
= \frac{1}{\Gamma(2-\alpha)(\Delta t)^\alpha} \sum_{l=1}^{n} \left[ (l+1)^{1-\alpha} - (l - 1)^{1-\alpha} \right] \left[ \frac{p_{i+1}^{n-l+1} - p_{i}^{n-l+1}}{\Delta x} - \frac{p_{i+1}^{n-l} - p_{i}^{n-l}}{\Delta x} \right] \]  

(15)

\[
= \frac{1}{\Delta x \Gamma(2-\alpha)(\Delta t)^\alpha} \sum_{l=1}^{n} \left[ (l+1)^{1-\alpha} - (l - 1)^{1-\alpha} \right] \left[ p_{i+1}^{n-l+1} - p_{i}^{n-l+1} - p_{i+1}^{n-l} + p_{i}^{n-l} \right] \]  

(16)

\[
= \frac{1}{\Delta x \Gamma(2-\alpha)(\Delta t)^\alpha} \sum_{l=1}^{n} \left[ (l+1)^{1-\alpha} - (l - 1)^{1-\alpha} \right] \left[ p_{i+1}^{n-l+1} - p_{i}^{n-l+1} - p_{i+1}^{n-l} + p_{i}^{n-l} \right] \]  

(17)

Using Eq. (17) we calculated the flux value of the fluid in porous media. For reservoir, we used 580 grids in space and for experiment condition, we used 580 grids in space. To calculate fluid flux, all the values of previous time steps were considered. In Figure 4.3, the alignment of grid point in space is shown.
4.5 Results and Discussion

4.5.1 Dependency of fluid viscosity on shear stress

In Newton’s law of viscosity, fluid viscosity used as a proportional constant to show the relation between shear stress and rate of strain. In stress-strain plot viscosity presented as the slope of the curve which can be positive or negative. For any fluids, viscosity decreases with increasing shear stress. Usually, fluid viscosity shows concave up or down, decreasing shape with increasing shear stress considering positive or negative fluid velocity gradient (Branes, 1997; Vlachopoulos and Strutt, 2003; Mendes, 2010; Dalton et al., 2013; Kaur and Jaafar, 2014; Ebrahimi et al., 2015; Wen et al., 2016; Helmy, 2016, Chatzigiannakis et al., 2016). Figure 4.4 shows the change of fluid viscosity with the change of shear stress considering positive velocity gradient along the y-axis for different α values in a log-log plot. Curves shape are concave downward and fluid viscosity is decreasing with increasing shear stress for all α (0.2-0.8) values. Viscosity is decreasing with increasing shear stress initially linearly and after certain time is shows a concave shape which reflects the effect of other fluid parameters and memory. Figure 4.4a to 4.4d shows the same trend with
increasing shear stress viscosity is decreasing and for $\alpha=0.8$ shear stress development is minimum considering all other parameters are same.

Figure 4.5 shows the change of shear stress for field condition in log-log plot and field results and proposed model plot trend is almost same. Proposed model results good match with the field results and for field case stress development is little bit higher than proposed model. Plots are showing similar trend and shape as Figure 4.

Figure 4.6 shows the change of shear stress for experiment condition in log-log plot, and experimental results and proposed model trend is almost same. Proposed model results show good match with the experimental condition. For model case curve show more curvature than experiment results and stress development is also little bit higher for proposed model than experimental results. Plots are showing similar trend and shape as Figure 4.4 and Figure 4.5.

**Figure 4.4:** Fluid viscosity variation as a function of shear stress for different $\alpha$ values
4.5.2 Dependency of fluid flux (memory effect) on proposed model

Fluid memory is one of the important parameter that describes the scenario of fluid flow in porous media. In the literature, few studies are available to show the effect of fluid memory for experimental cases (Iaffaldano et al., 2006, Caputo and Carcione, 2013). In this paper, Iaffaldano et al. 2006 experimental data and Hossain, 2008a sample reservoir data is used to simulate the pressure data for the grids. To calculate fluid flux, we considered $\alpha = 0.3$ (Zaman, 2017) for both the conditions. Fluid flux ($\text{m}^3/\text{m}^2 \text{s}$) is different from flow velocity ($\text{m/s}$) in case of porous media. In Figure 4.7, fluid flux is plotted with time and showed that
flux values are almost linear when flow reaches the steady condition though for transient condition flux values fluctuated with time. The experiment (Iaffaldano et al., 2006) was run for 11 hours and for almost 6 hours the flow was in transient condition for that reason flux values are fluctuated and after reaching the steady state condition it’s almost a linear and constant with time.

Figure 4.7: Flux change with time for α = 0.3

4.5.3 Dependency of fluid flow time on shear stress

Time is one of the key factor when we consider fluid flow in porous media. Rock and fluid properties of any porous media change with time. As rock and fluid properties are related with all fluid flow phenomena in porous media (Caputo, 1999; Hossain, 2008a, Histrov, 2014). With time fluid flux is changing for certain α value (α = 0.3) shown in Figure 7. In the literature, researchers have shown that how shear stress is changing with the time of fluid flow and depending on the characteristics of non-Newtonian fluid (Barnes, 1997; Chang et al., 1998; Pierre et al., 2004; Hasan et al., 2010; Lei and Xian, 2010; Ghannam et al., 2012; Benziane et al., 2012; Dimitriou and Mckinley, 2014; Petrus and Azuraien, 2014, Bao et al., 2016). Figure 4.8a to 4.8d shows that for different α values stress development is different with time in semi-log plot when other parameters are considered same. With time fluid viscosity decrease because of pressure change (fluid flux) as temperature is considered in isothermal consideration. In Figure 4.8a to 4.8d fluid shear
stress is decreasing with time. For different $\alpha$ values stress development is different for
certain conditions. At final time, stress development is almost same for all the $\alpha$ values but
initially stress value with time is different and stress decreases exponentially with time
(Dimitriou and Mckinley, 2014; Petrus and Azuraian, 2014, Kaur and Jaafar, 2014; Bao et
al., 2016).

Figure 4.9 and Figure 4.10 shows the stress variation with time for both field and
experiment condition. The field data plot shows a good match with proposed model results.
The field stress results are showing same trend as model results. Initially stress was almost
same for both the cases but after certain time stress value for field condition is little lower
then model condition with time. For experiment condition, trend is also same for
experiment results and model results and show good agreement. Stress values for
experiment case is lower than model results though initially stress values are almost same
for both the cases. Figure 4.9 and Figure 4.10 shows the same trend as Figure 4.8 when $\alpha=
0.3$ (Zaman, 2017) considered for both field and experimental condition.

![Shear stress variation as a function of time for different $\alpha$ values](image)

**Figure 4.8:** Shear stress variation as a function of time for different $\alpha$ values
4.5.4 Dependency of shear rate on shear stress

Shear stress has a proportional relationship with rate of shear and shear stress increases with the increase of shear rate where fluid viscosity represents as a proportional factor. (Barnes, 1997; Chang et al., 1998; Pierre et al., 2004; Hossain, 2008a; Ghannam et al., 2012; Benziane et al., 2012; Dimitriou and Mckinley, 2014; Kaur and Jaafar, 2014; Bao et al., 2016). But sometimes for variation of fluids and conditions this relation not remain the same and instead of linear plot curvature trend observed. In the proposed model equation, stress is proportional to rate of strain where surface tension, capillary effect and fluid memory is considered. Figure 4.11a to 4.11d shows the shear stress development with the
strain rate for different $\alpha$ values in log-log plot. For different $\alpha$ values stress development is not same and it decreases with the increasing $\alpha$ values. As fluid flux is changing with the change of $\alpha$ value when other parameters remain same. In Figure 4.11a to 4.11d, initially stress is increasing linearly with increasing shear rate but after certain time it start to decrease with curvature shape which represent the actual effect of fluid properties and memory that considered in the proposed model equation. Therefore, stress development increasing with rate of shear and shows the effect of fluid memory. For $\alpha= 0.2$ stress development is maximum and for $\alpha= 0.8$ it’s minimum with the change of shear rate.

Figure 4.12 shows the same trend as Figure 4.11 for field condition where parameters are same for both conditions. The results show good agreement for filed results with model results. Stress development is little bit higher for filed results from the model condition and model results show the curvature trend as Figure 4.11. In Figure 4.13, the model results are compared with experiment condition and results show good agreement also shows the same trend as Figure 4.11 and Figure 4.12. The stress development in experimental condition is little less than model results though trend and shape is almost same for both the cases.

**Figure 4.11:** Shear stress variation as a function of shear rate for different $\alpha$ values
Figure 4.12: At field condition shear stress variation as a function of shear rate

Figure 4.13: At exp. condition shear stress variation as a function of shear rate

4.5.5 Comparison of modified Stress-Strain model with Hossain et al., (2007a) Model

Figure 4.14 illustrates the variation of shear stress verses rate of shear of the proposed model (Eq. 13) for different $\alpha$ to compare with Hossain et al., (2007a) model in the log-log plot. The trend and shape for both models are almost same and show the variation of shear stress value with different $\alpha$ values for both cases. In the proposed model, values of shear stress development for $\alpha = 0.2$ is mostly match with Hossain et al., (2007a) model though stress development is less. For $\alpha = 0.3$ stress development for proposed model equation is
less than $\alpha = 0.2$ also from Hossain et al., (2007a) model though trend is mostly the same. With the increase of $\alpha$ value stress development is decreasing as a result flow will increase for the proposed model equation where surface tension, pseudo-permeability, pressure gradient is considered. The stress development also decreased with increased $\alpha$ values for the modified stress-strain model. Figure 4.15 shows the comparison between proposed model with fields results and Hossain et al., (2007a) mode and the proposed model show similar trend, and closer to field condition than Hossain et al., (2007a) model. Stress development for same conditions is less than filed results and Hossain et al., (2007a) model results. Figure 4.16 also shows the comparison between proposed model with experiment results and Hossain et al., (2007a) mode and the proposed model show a good match with experimental condition than Hossain et al., (2007a) model. In Figure 4.16, trend is almost same for three plots stress development with increasing shear rate is less for experimental results which is very close to the proposed model results.

This model equation considered the effect of surface tension, capillary force, viscous force, temperature, and fluid memory. In this article, fluid memory term is solved with finite difference method. The modified model is showing consistent of shear stress value with increasing strain rate for both field and experiment study. Surface tension, capillary effect and memory mechanism was not considered properly in Hossain et al., (2007a) model. Therefore, the proposed modified stress-strain model is more applicable to show the effect of fluid memory in the rheological study of fluid flow through porous media.
**Figure 4.14:** Comparison of modified stress-strain model with Hossain *et al.* (2007a) model.

**Figure 4.15:** Comparison of proposed stress-strain model with field results and Hossain *et al.* (2007a) model.
4.6 Conclusions

In this article, a modified memory-based stress-strain model is proposed to show the continuous effect of fluid memory in the formation. This model will help to characterize reservoir fluids, show the rheological behavior of reservoir fluids, and the overall effect of fluid media and memory in characterization process. Several fluid properties are considered to develop this modified stress-strain model which can capture actual scenario. The model equation is solved numerically using field and experimental data. Surface tension, capillary effect, pseudo-permeability, pressure gradient, and fluid memory have a good amount of impact in the stress-strain behavior of formation fluids. With the effect of fluid memory, the stress development in the formation reduced with time which helps to get better fluid flow in the formation. The model validation is done with the available experimental and reservoir data from the literature, compared with the established stress-strain model. In this paper, $\alpha=0.3$ is considered to show more effective scenario of fluid memory based on literature. Fluid media and memory are considered to develop this model and the results show a good match with the existing model and data. This modified stress-strain model can be used in various formations for fluid characterization, and rheological study more effectively.

Figure 4.16: Comparison of proposed stress-strain model with exp. results and Hossain et al. (2007a) model.
4.7 Nomenclature

\( C_a \)  Capillary number
\( E \)  At 32.8 API gravity activation energy of crude oils, KJ/mol
\( EOR \)  Enhanced Oil Recovery
\( k \)  Reservoir permeability, \( mD \)
\( M_a \)  Marangoni number
\( p \)  Reservoir pressure, \( N/m^2 \)
\( P_i \)  Initial reservoir pressure, \( N/m^2 \)
\( \Delta P \)  \( P_T - P_o \) = Pressure difference, \( N/m^2 \)
\( p(x, t) \)  Reservoir fluid pressure, \( Pa \)
\( q_x \)  Volumetric flow rate in x-direction, \( kg/m^2-s \)
\( t \)  Time, \( sec \)
\( T \)  Temperature, \( ^\circ K \)
\( u_x \)  Fluid velocity in x-direction, \( m/s \)
\( y \)  Distance from the boundary layer, \( m \)
\( \alpha \)  Fractional order of differentiation
\( \eta \)  Pseudo-permeability of fluid, \( m^3s^{1+\alpha}/kg \)
\( \phi \)  Porosity of fluid media
\( \rho_o \)  Density of the fluid at initial temperature \( T_o \), \( kg/m^3 \)
\( \sigma \)  Surface tension, \( N/m \)
\( \gamma \)  Velocity gradient at y direction, \( 1/s \)
\( \tau \)  Shear stress of fluid, \( Pa \)
\( \tau_f \)  Shear stress at reference temperature \( T_o \), \( Pa \)
\( \mu \)  Dynamic viscosity of fluid, \( Pa-s \)
\( \mu_o \)  Dynamic viscosity at reference temperature \( T_o \), \( Pa-s \)
\( 1-D \)  One dimensional
\( 3-D \)  Three dimensional
4.8 References


155


M.M. Helmy, Rheological properties of sweet lupine to be used as extrusion meat additives, International Journal of Nutrition and Food Science, V 5, 7-13, (2016).
Chapter 5

A Memory-based Fluid Density and Effective Viscosity Model for Reservoir Characterization

Preface

This paper to be submitted to a Journal. The lead author performed the necessary literature review on fluid properties and fluid memory. The co-authors Tareq Uz Zaman helped in mathematical techniques, showed some coding techniques Dr. Salim Ahmed reviewed the
manuscript and Dr. M. Enamul Hossain helped in identifying the gap in research, supervising the research, and editing the manuscript.

5.1 Abstract

Rock and fluid properties are very important features for any reservoir characterization. Those properties are changing continuously because of pressure and thermal change in porous media. Rock properties such as porosity, permeability, compressibility, etc., as well as fluid properties such as density, viscosity, surface tension, and other PVT properties play a vital role in the continuous change of complex reservoir structure. Memory captures the effect of previous events and presents development for current and future events. Fluid memory is a function of space and time which is used in porous media to forecast future outcomes from the past. The pressure gradient of a reservoir can be presented with the help of fluid memory as a continuous function of time and space. Fluid memory is one of the key fluid features but it is overlooked in the fluid models. Therefore, it is necessary to
consider continuous time function for characterizing the fluid of a reservoir. This alteration of fluid properties (e.g., density and viscosity) can be characterized by memory concept. The objective of this study is to develop a memory-based density and effective viscosity model for crude oil considering continuous time function and solving the model equation numerically. In this paper, the modified model is developed based on fluid memory that shows a relationship between density-effective viscosity and pressure difference over time. The proposed model shows a good range for both shear rate and effective viscosity for zero and infinity shear region, and shows the effect of fluid memory on effective viscosity calculation.

5.2 Introduction

From the beginning of time, humans have consumed energy for several reasons. A world without energy cannot be imagined because every step of human life depends on various energy sources. The petroleum industry is one of the most important sources of energy all over the world. In Figure 5.1, the total supply of energy is shown between 2016-2017 and almost 51.50% (Oil and Natural Gas) energy is consumed by the petroleum industry. Several new techniques have been applied to upgrade the oil and gas production from a reservoir. The enhanced oil recovery (EOR) technique is one of the keys to increasing the oil and gas production (Hossain et al., 2008b, 2009a). In a reservoir, shear thinning fluids sometimes act as time-dependent thixotropic fluids. In porous media, fluid viscosity can be measured as an overall or “up-scaled” way known as apparent viscosity. For polymeric solutions (i.e., crude oil, chemical mixture, etc.) the apparent viscosity is a function of flow rate and the flow rate is also correlated with the shear rate. In porous media, the fluid flow is correlated with the fluid memory and presence of a mineral or other particle movement may reduce the response of the fluid. This decrease can create a restriction of the crude oil flow through the porous media. This paper illustrates reservoir fluid properties of porous media and proposes a relation between fluid density, viscosity, and fluid memory.
Li et al. (2001) completed a stress analysis of air bubbles for non-Newtonian fluids and found that fluid viscosity increases at a constant rate of $1 \times 10^3$ to $1 \times 10^4$. These authors have concluded that water has both solid and viscous properties. In nanometer scale, water could be used as a lubricant (Mauk, 2007). However, fluid behaves differently under molecular constraints and slow flow rate is a usual phenomenon. This kind of flow exists in nature such as the blood circulation in mammalian lungs, water films in eyes, and in artificial cases such as microchip fabrication, oil recovery, etc. (Perazzo and Gratton, 2003; Hossain, 2008a; Hossain et al., 2007, 2008b, 2009a). Generally, non-Newtonian fluid flow is considered to explain much of natural phenomena (Perazzo and Gratton, 2003; Arratia et al., 2005; Hossain, 2008a). Water that exhibits viscous flow is mostly considered Newtonian fluid, but now it is starting to be considered water as the non-Newtonian fluid. With time and research, it is clearer that nature fluids are more revealed to the non-Newtonian fluid. Though several models have already been developed considering non-Newtonian fluid, still, it is a necessity to come up with a comprehensive model which can be used for a large range of viscosity relation with fluid memory.
To present the true picture, the fluid density-viscosity relationship should consider the fluid memory in the model. The classical reservoir simulation based on the formation permeability along with the flow of the fluid but never maintain the exact flow path. Fluid memory plays an important role to overcome this problem by monitoring the flow. The fluid memory displays a unique feature as its working technique changes with time and different media conditions. The fluid memory is defined as a continuous function of time, and space considering all the fluid properties. Some fluids (i.e., incompressible and viscous fluids) show few different behaviors that can be described easily by fluid memory. The literature is very limited to show the actual picture of fluid memory phenomenon in a comprehensive way.

Fluid memory is used in porous media to show the continuous change of rock and fluid properties with time and space. Several researchers have incorporated fluid memory in their models to show the exact scenario of time alteration in porous media. The fluid flow scenario of porous media is a little bit different from pipeline flow because of its complex rheological structure. In the literature, several researchers have shown the rock-fluid properties of porous media with the help of fluid memory (Slattery, 1967; Eringen, 1991; Nibbi, 1994; Caputo, 1999; Zhang, 2003; Chen et al., 2005; Hossain et al., 2007, 2008b 2009a and 2009b; Hossain, 2008a, Di Guiseppe et al., 2010; Raghavan and Chen, 2013; Rahman et al., 2016; Obembe et al., 2017).

Kondic et al. (1996) reviewed Hele-Shaw cell using Darcy’s law to describe non-Newtonian fluid viscosity which depends on the shear-rate and pressure gradient. However, authors derivation does not agree with active shear-thinning which is only correlated to the slip layers of the flow. Miranda (2004) studied non-Newtonian fluid at tension condition to show the effect of shear stress. To develop the shear-rate model, the researcher used modified Darcy’s law and concluded that, for shear thinning fluid the adhesion force decreased with less separation. Afanasieiev et al., (2007) used the power-law, the Ellis, and Carreau model to review the drag conditions at different inclination angle for shear-thinning fluids. These researchers considered steady state condition and several rheological
parameters. However, various examples of fluids (i.e., polymeric, and suspension) represent non-linear relationship for stress-strain. The fluids (i.e., Newtonian or non-Newtonian) can be distinguished based on the rate of shear. At the very low shear rate, polymeric fluid behaves like a Newtonian fluid but at a higher shear rate, the fluid starts to behave as a non-Newtonian fluid. Usually, the power-law model is used at a higher shear rate and the Ellis model is used at lower shear rate (Afanasiev et al., 2007, Hossain, 2008a).

Frank and Li, (2005) studied Newtonian and non-Newtonian fluids and discussed some unusual behaviors of non-Newtonian fluids such as the reverse movement behind a bubble. Authors concluded that shear stress relaxation and fluid memory are the causes for that movement. Huang and Lin (2007) suggested that the memory effect and exponential alignment of length decreased with increasing thermal noise. Few models have been developed to include the fluid memory for thixotropic fluids (Lissant, 1974; Parker, 1992). Usually, fluids used in oil field applications are complex fluids (i.e., polymeric solutions) which can be shear-thinning or thixotropic fluid. In the literature, the available mathematical models (i.e., Power law, Carreau, or Cross models etc.) state the fluid rheology by defining fluid viscosity and apparent shear rate from the Darcy’s flow velocity (Cannella et al., 1989; Sorbie and Huang, 1991; Escudier et al., 2001; Lopez and Blunt, 2004). From experimental studies, researchers have drawn the conclusion that the shape of the apparent viscosity curve and bulk shear rate is similar in most cases. In the available literature, most of the experiments have been performed by Xanthan biopolymers (Chauveteau and Kohler, 1974; Chauveteau, 1982; Sani et al., 2001; Taskiroglou, 2004; Lopez, 2004a).

Several constitutive equations have been established to capture the actual rheological behavior of shear-thinning fluids (Chauveteau, 1982). Escudier et al. (2001) developed a rheological model for non-Newtonian fluids running several experiments on Xanthan gum. Lopez (2004) illustrated Carreau-Yasuda, Cross and Truncated Power-law models and presented similar outcome considering effective viscosity of shear thinning fluids for all the models. Hossain (2008a) tried to develop models for shear stress, shear rate, and
viscosity for complex polymeric fluid (e.g., crude oil) considering continuous function of time, space, and other fluid properties. In this study, the Carreau-Yasuda model is considered to develop fluid density, and effective viscosity model and incorporate the fluid memory for fluid rheology with time alteration (Carreau, 1972; Yasuda et al., 1981; Sorbie, 1989; Escudier et al., 2001; Lopez, 2004; and Hossain 2008a).

5.3 Mathematical Model Development

Many mathematical equations have been developed to capture the actual fluid rheological characteristics of shear-thinning fluids in the past. The Carreau-Yasuda, Cross, and, Power-law models give almost the same results when they represent the fluid rheology (e.g., effective viscosity) of shear thinning fluids (Escudier et al., 2001; Lopez, 2004). Therefore, the Carreau-Yasuda effective viscosity model is considered in this study to develop fluid effective viscosity and density relation as well as capture the memory effect for fluid rheology. The Carreau-Yasuda model can be written as (Carreau, 1972; Yasuda et al., 1981; Sorbie, 1989; Barnes, 1997; Escudier et al., 2001; Lopez, 2004; Hossain, 2008a and Hossain et al., 2009a):

\[
\mu_{eff} = \mu_\infty + \frac{\mu_0 - \mu_\infty}{[1 + (\lambda \gamma)^a]^\frac{n-1}{a}}
\]  

(1)

Blake (1922) proposed a dimensionless number for porous media to show the effect of inertia, and viscous force which was also known as the modified Reynold’s number. In Blake number, fluid density, fluid velocity, hydraulic diameter for porous media, fluid initial viscosity and the void fraction is used to show both effects. Several researchers used the modified Reynold’s number (Blake number) and provided the laminar and turbulent flow ranges for packed bed porous media (Burke and Plummer, 1928; Ergun and Orning, 1949; Ergun, 1952). The Blake number can be expressed as follows:

\[
B = \frac{u_x \rho D_h}{\mu_0 (1-\epsilon)}
\]

(2)
Porosity (ϕ) or void fraction (ε) defined as the ratio of pore volume to bulk volume can be written as
\[ \phi = \varepsilon = V_p / V_b. \]
Thus Eq. (2) can be written as:
\[
B = \frac{u_x \rho D_h}{\mu_o (1 - \phi)}
\] (3)
\[
\mu_o = \frac{u_x \rho D_h}{B (1 - \phi)}
\] (4)

Using the value of \( \mu_o \) in Eq. (1) from Eq. (4) can be written as:
\[
\mu_{eff} = \mu_\infty + \frac{u_x \rho D_h}{B (1 - \phi)} \frac{\mu_\infty}{[1 + (\lambda \gamma)^\alpha]^\frac{n-1}{\alpha}}
\] (5)

Caputo (1999; 2000) modified Darcy’s law by introducing the fluid memory which represents the effect of reduction of the formation permeability with time. If the fluid flow is considered in the x-direction, the mass flow rate equation can be written as (Caputo, 1999; 2000; Hossain et al., 2007; 2008b; 2009a; 2009b, Hossain, 2008a):
\[
q_x = -\eta \rho_o \left[ \frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial p}{\partial x} \right) \right]
\] (6)

In Eq. (6) \( q_x = \frac{\rho^\prime \rho_o}{A} \) and \( 0 \leq \alpha < 1 \) so Eq. (6) can be written as:
\[
u_x = -\eta \left[ \frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial p}{\partial x} \right) \right]
\] (7)

Here, \( \nu \) is the reservoir fluid velocity in porous media. Putting the value of \( \nu \) in to Eq. (5) becomes:
\[
\mu_{eff} = \mu_\infty + \frac{(-1) \rho D_h}{B (1 - \phi)} \eta \left[ \frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial p}{\partial x} \right) \right] - \frac{\mu_\infty}{[1 + (\lambda \gamma)^\alpha]^\frac{n-1}{\alpha}}
\] (8)

Where, \( \eta \) is the pseudo permeability of the porous media. \( \eta \) can be shown with the below equation (Hossain et al., 2008a, 2008b) as follows:
\[
\eta = \frac{K}{\mu} (t)^\alpha
\] (9)
In Eq. (9), \( K \) = permeability, \( \mu \) = fluid viscosity, \( t \) = time and \( \alpha \) = order of differentiation. Putting the value of \( \eta \) in Eq. (8) from Eq. (9) it becomes:

\[
\mu_{eff} = \mu_\infty + \frac{(-1)^\beta \rho \frac{dP_h}{d\tau} \left[ \mu \left( \frac{d\rho}{d\tau} \right)^\alpha \left( \frac{d\tau}{d\rho} \right)^{\beta-\alpha} \right]^{\alpha}}{\left[ 1 + (\alpha \gamma)^{\beta-\alpha} \right]} - \mu_\infty \tag{10}
\]

The above developed mathematical model represents the effect of reservoir formation, porosity, permeability, fluid density, viscosity, strain rate, and fluid memory. A comprehensive relation between density, effective viscosity, and the apparent shear rate is also shown in the model.

### 5.4 Numerical Analysis of The Model

The results of the comprehensive stress-rate of strain model can be obtained by solving Eq. (10) which is shown above. In this paper, we focused on fluid density-viscosity relation fluid memory. We consider a sample reservoir from the production wellbore (Hossain, 2008a) and experimental data (Iaffaldano et al., 2006) for numerical calculation. The reservoir is isolated and oil is producing at a constant rate. The fluid is assumed to be an API 32.8 gravity crude oil at 298oK temperature. To solve the proposed viscosity-density model presented in Eq. (10), fluid viscosities at low (\( \mu_o \)) and high shear rate (\( \mu_\infty \)) are considered as 13.2 Pa-s, and 0.00212 Pa-s respectively and the power-law index, (n), factor (a), and the time constant, (\( \lambda \)) are taken as 0.689, 0.75 and 60.7 s, respectively for numerical computation (Chauveteau and Kohler, 1974; Sorbie, 1989; Barnes, 1997; Escudier et al., 2001; Lopez, 2004a, 2004b; Hossain, 2008a; Hossain et al., 2009a). All computations are carried out by MATLAB. All computations are carried out by MATLAB.

**Table 5.1:** Sample Reservoir Data (Hossain, 2008a)

<table>
<thead>
<tr>
<th>Reservoir length, ( l )</th>
<th>5000 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir width, ( w )</td>
<td>100 m</td>
</tr>
<tr>
<td>Reservoir height, ( h )</td>
<td>50 m</td>
</tr>
<tr>
<td>Porosity, ( \phi )</td>
<td>30%</td>
</tr>
</tbody>
</table>
### Table 5.2: Experimental Data (Iaffaldano et al., 2006)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permeability, $k$</td>
<td>$30 \text{ md} = 30 \times 10^{-15} \text{ m}^2$</td>
</tr>
<tr>
<td>Initial reservoir pressure, $p_i$</td>
<td>27579028 pa (4000 psia)</td>
</tr>
<tr>
<td>Compressibility, $c$</td>
<td>$1.2473 \times 10^{-9} \text{ 1/pa}$</td>
</tr>
<tr>
<td>Initial viscosity, $\mu_o$</td>
<td>13.2 Pa-s</td>
</tr>
<tr>
<td>Initial flow rate, $q_i$</td>
<td>$8.4 \times 10^{-9} \text{ m}^3/\text{sec}$</td>
</tr>
<tr>
<td>Initial fluid velocity, $u_i$</td>
<td>$1.217 \times 10^{-3} \text{ m/sec}$</td>
</tr>
<tr>
<td>Fractional order of differentiation, $\alpha$</td>
<td>0.2-0.8</td>
</tr>
<tr>
<td>Number of grid in space, $N_t$</td>
<td>580</td>
</tr>
<tr>
<td>Length of cylinder, $l$</td>
<td>11.6 cm</td>
</tr>
<tr>
<td>Inner diameter, $D_i$</td>
<td>10.1 cm</td>
</tr>
<tr>
<td>Volume of the cylinder, $V_c = \pi r^2 h$</td>
<td>929.374 cm$^3$</td>
</tr>
<tr>
<td>Permeability, $k$</td>
<td>26 Darcy</td>
</tr>
<tr>
<td>Viscosity, $\mu$</td>
<td>1.0266 cp</td>
</tr>
<tr>
<td>Sand density, $\rho_s$</td>
<td>2.4 gcm$^{-3}$</td>
</tr>
<tr>
<td>Mass of sand in cell, $M_s$</td>
<td>1550 gm</td>
</tr>
<tr>
<td>Volume of sand, $V_s$</td>
<td>645.83 cm$^3$</td>
</tr>
<tr>
<td>Porosity, $\phi$</td>
<td>0.3050913841</td>
</tr>
<tr>
<td>Fluid density, $\rho_f$</td>
<td>0.998408 gcm$^{-3}$</td>
</tr>
<tr>
<td>Compressibility, $c_t$</td>
<td>$2.05743 \times 10^{-4} \text{ atm}^{-1}$</td>
</tr>
<tr>
<td>$\frac{dp}{dx}$</td>
<td>0.01765982953 atm/cm</td>
</tr>
<tr>
<td>$\Delta p$</td>
<td>0.2048540225 atm</td>
</tr>
<tr>
<td>Number of grid in space, $N_t$</td>
<td>580</td>
</tr>
</tbody>
</table>
In Eq. (10), $D_h$ is used in the first part as hydraulic diameter. We consider the reservoir shape is rectangular and fully filled with fluid. If reservoir length, $l = a$ and width, $w = b$, so $D_h$ becomes as:

$$D_h = \frac{4a \cdot b}{2(a+b)}$$

(11)

$$D_h = \frac{2a \cdot b}{(a+b)} ; a >> b$$

(12)

In Eq. (10) $\left[ K(t)^\alpha \left\{ \frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial P}{\partial x} \right) \right\} \right]$ is considered to show the effect of fluid memory. Here, $\left\{ \frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial P}{\partial x} \right) \right\}$ is the change of pressure with space and time. In addition, $K(t)^\alpha$ is used to show the effect of rock and fluid properties with time alteration. $\alpha$ value is showing the variation of the order of differentiation. In this paper, we calculate this term as fluid flux ($m^3/m^2 \cdot s$). We used Iaffaldano et al. (2006) experimentally validated data which is simulated, and validated numerically by Zaman (2017). Different authors showed the variation of $\alpha$ value to capture the memory effect (Iaffaldano, 2006, Hossain, 2008a, Histrov 2014, Obembe et al., 2017). Recently, Zaman (2017) shows that the best choice for $\alpha$ value is 0.3 which can give a good agreement for fluid memory. The results of this study are compared with the well-established Carreau-Yasuda model (Carreau, 1972; Yasuda et al., 1981).

To solve the fluid memory term (partial differential part) in Eq. (10), it is necessary to consider space and time for pressure calculation. The finite difference method is used to solve the fluid memory term $\left\{ \frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial P}{\partial x} \right) \right\}$ of Eq. (10). The discretized form of fluid memory term from Eq. (10) as follows:

$$\left. \frac{\partial^\alpha}{\partial t^\alpha} \left( \frac{\partial P}{\partial x} \right) \right|_t^n$$

$$= \frac{1}{\Gamma(2-\alpha)} \frac{1}{(\Delta t)^\alpha} \sum_{j=1}^{n} [J^{1-\alpha} - (J-1)^{1-\alpha}] \left[ \left( \frac{\partial P}{\partial x} \right)_t^{n-j+1} - \left( \frac{\partial P}{\partial x} \right)_t^{n-j} \right]$$

(13)
\[
= \frac{1}{\Gamma(2-\alpha)(\Delta t)^\alpha} \sum_{j=1}^{n} \left[ \frac{1}{J-1 - (J-1)^{1-\alpha}} \left( \frac{p_{i+1}^{n+1} - p_{i}^{n+1}}{\Delta x} - \frac{p_{i+1}^{n} - p_{i}^{n}}{\Delta x} \right) \right] \quad (14)
\]
\[
= \frac{1}{\Delta x \Gamma(2-\alpha)(\Delta t)^\alpha} \sum_{j=1}^{n} \left[ \frac{1}{J-1 - (J-1)^{1-\alpha}} \left( p_{i+1}^{n+1} - p_{i}^{n+1} - p_{i+1}^{n} + p_{i}^{n} \right) \right] \quad (15)
\]
\[
= \frac{1}{\Delta x \Gamma(2-\alpha)(\Delta t)^\alpha} \sum_{j=1}^{n} \left[ \frac{1}{J-1 - (J-1)^{1-\alpha}} \left( p_{i+1}^{n+1} - p_{i}^{n+1} - p_{i+1}^{n} + p_{i}^{n} \right) \right] \quad (16)
\]
Eq. (16) can be solved using initial and boundary conditions.

Using Eq. (16) we calculated the flux value of the fluid in porous media. For reservoir, we used 580 grids in space and for experiment condition, we used 580 grids in space. To calculate fluid flux, all the values of previous time steps were considered. In Figure 5.2, the alignment of grid point in space is shown.

**Figure 5.2:** Sample reservoir grid point in space (modified from De Sterck and Ullrich, 2009)

### 5.5 Results and Discussion

#### 5.5.1 Dependency of fluid density on effective viscosity

Viscosity and density are two most important fluid properties and play significant role in fluid flow through porous media. Fluid viscosity and density related closely with formation
temperature and pressure. In this study, formation temperature is considered isothermal but pressure is changing with space and time. Formation pressure is changing as a result fluid density and viscosity is also changing. Several researchers have shown the relation between pressure, fluid viscosity and density for various non-Newtonian fluids (Chou and kokini, 1987; Morris et al., 1995; Yeo and Kiran, 2000; Togrul and Arslan, 2003; Liu et al., 2006; Chan Eu, 2007; Tamara, 2008; Grem et al., 2013; Abdeen and Mohammad, 2014; Diogo et al., 2014, 2015, Abdali et al., 2016). In the literature, few resources are available focusing petroleum fluids. In this article, fluid effective viscosity represents as a function of density consider flow regime in porous media and memory effect. Figure 5.3 represents the relation between fluid density and effective viscosity in semi-log plot. In Figure 5.3a to 5.3d, different $\alpha$ values (0.2-0.8) are considered to show the effect of fluid memory in density-effective viscosity relation. Figure 5.3a shows almost a linear relation between density and effective viscosity for $\alpha=0.2$, still, plot shows a little bit curvature shape which is the effect of fluid memory. In Figure 5.3, 5.3b to 5.3d shows the same trend as figure 3a and effective viscosity increases with $\alpha$ values. For $\alpha=0.2$ gives the minimum and for $\alpha=0.8$ gives the maximum effective viscosity for a certain fluid density.

Figure 5.4 illustrates relation between fluid density and effective viscosity for field and experimental condition, and consider $\alpha=0.3$. Figure 5.4a and 5.4b both shows the same trend and shape as Figure 5.3. For field and experimental condition results show good match as the data was scaled-up. Field results show more better range of effective viscosity then experimental data considering $\alpha=0.3$ (Zaman, 2017). The proposed model shows good range and agreement for both field and experimental data.
Figure 5.3: Effective viscosity variation as a function of density for different $\alpha$ values

Figure 5.4: At field and exp. condition effective viscosity varies with density for $\alpha=0.3$

Figure 5.5 shows the change of fluid effective viscosity as a function of density for different month at $\alpha=0.3$. Figure 5.5a to 5.5d represents the same trend and shape as Figure 5.3 and Figure 4. Figure 5.5 shows the actual scenario of memory and shows the nonlinear trend of the curve very clearly. This nonlinear behavior arises because of continuous rock and fluid alteration. With the change of months, the change of effective viscosity is very less. The results show that the initial effective viscosity is around (0.06~0.08) pa and the maximum effective viscosity is around (0.12~0.14) pa.
5.5.2 Dependency of the Blake number on effective viscosity

Forces (i.e., inertia force, viscous force) have an impact on fluid flow through porous media. Those forces moderate the flow regime type and behavior of flow in porous media. Usually Reynold’s number is used in pipeline flow to determine the flow regime. In this article, Blake number (modified Reynold’s number) is used to show the effect of inertia and viscous force in porous media. Blake number is calculated for certain depth at constant initial viscosity to show the effect of fluid density. Form the model equation, effective viscosity is inversely proportional to Blake number with all other parameter is constant. Figure 5.6a to 5.6d shows that effective viscosity is decreasing with the increasing Blake number in log-log plot. For different $\alpha$ values (0.2-0.8) effective viscosity is not same though the change not too much and for $\alpha=0.8$ initial effective viscosity is maximum and for $\alpha=0.2$ its minimum. For $\alpha=0.2$ plot shows a curvature shape towards end which reflects the memory mechanism.
5.5.3 Dependency of fluid flux (memory effect) on fluid flow in porous media

In this paper, experimental data (Iaffaldano et al., 2006) and sample reservoir data (Hossain, 2008a) is used in the simulation to get the pressure data for the grids. Generally, pressure distribution for grids is not available in reservoir or experimental data. To calculate fluid flux, we considered $\alpha = 0.3$ (Zaman, 2017) for both conditions. Fluid flux $(\text{m}^3/\text{m}^2 \text{ s})$ is different from flow velocity (m/s) in case of porous media. In Figure 5.7, fluid flux is plotted with time and showed that flux values are almost linear when flow reaches the steady condition though for transient condition flux values fluctuated with time. The experiment (Iaffaldano et al., 2006) was run for 11 hours and for almost 6 hours the flow was in transient condition for that reason flux values are fluctuated and after reaching the steady state condition it’s almost a linear and constant with time.
**5.5.4 Dependency of apparent shear rate on effective viscosity**

Figure 5.8a to 5.8d shows the variation of effective viscosity with the rate of shear in log-log plotting for different α (0.2-0.8) values. The trend and shape of those plots are almost same except the data variations towards upper and lower level. Those figures trend and shape are almost as it is in the Carreau-Yasuda model (Carreau, 1972; Chauveteau and Kohler, 1974; Yasuda et al., 1981; Sorbie, 1989; Lopez, 2004b, Arratia, 2005; Boyd and Buick, 2007; Hossain et al., 2009a, Khan and Hashim, 2015; Whitty et al., 2016, Khechiba et al., 2017). The variation of data range for proposed model equation has varied from Carreau-Yasuda model because of rheological variations and scaled up. Though the shape and trend are almost same though data range is different for different values of α (0.2-0.8). With the increase of α values, the data ranges change for both shear rate and effective viscosity. From those figures, the effective viscosity-apparent shear rate curve trend, shape, and region are dependent on fluid memory and the data range are more dominant at higher (α= 0.8) fluid memory value. As α value increases, effective viscosity decreases with the increasing apparent shear rate.
Figure 5.8: Effective viscosity variation with shear rate for different $\alpha$ values

Figure 5.9 shows the relation between effective viscosity and apparent shear rate for $\alpha=0.3$. For field and exp. condition we considered $\alpha=0.3$ to show the effect of effective viscosity and apparent shear rate. In Figure 5.9, field results show good match with experimental results with up-scaling. Most of the points overlap each other and give a good range of effective viscosity value for zero and infinity shear zone at both condition. Figure 5.9 gives same trend and shape as Figure 5.8 for $\alpha=0.3$.

Figure 5.9: Effective viscosity variation with apparent shear rate for field and exp. condition
5.5.5 Comparison of proposed model with Carreau-Yasuda model

Figure 5.10 illustrates the variation of effective viscosity as a function of apparent shear rate in log-log plot for proposed model of different $\alpha$ values with Carreau-Yasuda model. The shape and trend of the curves for proposed model and Carreau-Yasuda model is almost same and most of the data are matched with each other except the scale of data variation. The proposed model showed the effect of fluid memory and give more information about formation. The proposed model also gives a large range of data in zero and infinity shear condition. Carreau-Yasuda model only applicable for power-law region whether as the proposed model is applicable for wide range of data between zero to infinity shear rate. Therefore, the proposed model is more applicable to characterize both rheological and fluid properties in porous media.

![Comparison of proposed model with Carreau-Yasuda model](image)

**Figure 5.10:** Comparison of proposed model with Carreau-Yasuda model

5.6 Conclusions

In this article, a memory-based density-effective viscosity model is proposed to characterize the rheological behavior for light crude oils. This model is solved numerically considering sample field data as well as available experimental data, and compared with the currently used conventional model (i.e., Carreau-Yasuda Model). The proposed model is very effective for representing the physical phenomena of any formation also able to
capture a large span of information, covering reservoir fluids that would not be traceable with existing models. In reservoir formation, because of shear stress, the velocity gradient (i.e., shear rate) in y-direction has a non-linear variation, which proves the linear relationship of conventional models are not acceptable. Fluid memory can represent the real picture of the above-mentioned effect. In this paper, we focused on the dependency of the effective viscosity on formation porosity, density, hydraulic diameter, and flow velocity which is related to the effect of fluid memory. The results give a good range of effective viscosity values for zero shear, power-law region, and infinity shear region and show the effect of fluid memory. This study concludes that fluid memory has a great impact on fluid flow and rheological behavior of shear-thinning fluid in porous media considering fluid and formation properties.

5.7 Nomenclature

\( a \) Reservoir length, \( m \)

\( b \) Reservoir width, \( m \)

\( B \) Blake number

\( D_h \) Hydraulic diameter, \( m \)

\( k \) Permeability of the reservoir, \( mD \)

\( L \) Length of the core, \( m \)

\( n \) Power-law exponent for Carreau–Yasuda model

\( p \) Reservoir pressure, \( N/m^2 \)

\( P_i \) Initial reservoir pressure, \( N/m^2 \)

\( \Delta P \) \( P_T - P_o \) = Pressure difference, \( N/m^2 \)

\( p (x, t) \) Fluid pressure, \( Pa \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_x$</td>
<td>Volumetric flow rate in x-direction, $kg/m^2\cdot s$</td>
</tr>
<tr>
<td>$R$</td>
<td>Universal constant, $kJ/mol\cdot K$</td>
</tr>
<tr>
<td>$t$</td>
<td>Time, $sec$</td>
</tr>
<tr>
<td>$u_x$</td>
<td>Reservoir fluid velocity in x-direction, $m/s$</td>
</tr>
<tr>
<td>$V_p$</td>
<td>Pore volume, $m^3$</td>
</tr>
<tr>
<td>$V_b$</td>
<td>Bulk volume, $m^3$</td>
</tr>
<tr>
<td>$y$</td>
<td>Distance from the boundary plan, $m$</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Fractional order of differentiation</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Ratio of the pseudo-permeability to fluid viscosity, $m^3 s^{1+\alpha}/kg$</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Void fraction</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Porosity of fluid media</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Time constant in Carreau–Yasuda model, $s$</td>
</tr>
<tr>
<td>$\rho_o$</td>
<td>Density of the fluid at reference temperature $T_o$, $kg/m^3$</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density of the fluid, $kg/m^3$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Velocity gradient at y-direction, $1/s$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Dynamic viscosity, $Pa\cdot s$</td>
</tr>
<tr>
<td>$\mu_o$</td>
<td>Dynamic viscosity at low shear rate, $Pa\cdot s$</td>
</tr>
<tr>
<td>$\mu_\infty$</td>
<td>Dynamic viscosity at high shear rate, $Pa\cdot s$</td>
</tr>
<tr>
<td>1-D</td>
<td>One dimensional</td>
</tr>
</tbody>
</table>
5.8 References


Chapter 6

Conclusion

6.1 Summary

In this thesis, all chapters have its own uniqueness and at the same time those chapters are linked to each other in the consequences of memory mechanism. The proposed models are based on memory mechanism which will open a new dimension for petroleum industry. The proposed models will help in reservoir fluid characterization considering continuous rock and fluid alteration.

Chapter 2 reviews the fractional derivatives, fluid memory, reservoir rheology, and reservoir fluid properties. This review of the literature illustrates the reservoir rheological study in petroleum engineering and shows a complete picture of reservoir fluid properties. In this literature review the definition, use, and importance of memory is shown for engineering and science as well as for porous media.

In Chapter 3, a memory-based stress-strain model is proposed to show the memory effect in porous media. This model considered formation porosity, permeability, fluid density, hydraulic diameter, modified Reynold’s number (Blake number), temperature, pseudo-permeability, pressure gradient and shear rate which makes the model as a comprehensive one. The fluid memory term has been discretized and solved the model numerically for both field and experiment conditions. The proposed model is also compared with established memory-based model and captured the trend and variations.

In Chapter 4, a mathematical model is proposed to capture memory effect for stress strain relationship where surface tension, capillary effect, pseudo-permeability, temperature, pressure gradient, and shear rate in y-direction is also considered. The proposed model is solved numerically for field, and experiment data and compared with established model where proposed model showed a good trend with the existing model.
In Chapter 5, a comprehensive model is proposed to show the relationship between fluid viscosity and density considering fluid memory effect. This model is a modified version of Carreau-Yasuda model where memory mechanism is addressed to show the effect of fluid memory in continuous rock and fluid alteration. The proposed model has solved numerically for field and experiment condition and results showed a good agreement with the existing model.

In this thesis, the mathematical models are proposed to show the effect of memory on reservoir fluid properties. The proposed models will help to characterize reservoir fluid properties and capture the effect of memory as well. The proposed stress-strain models are also considered modified Reynold’s number (Blake number) and Capillary number to show the effect of fluid density and surface tension gradients on reservoir fluids interface consequently. The results also showed how memory effect influenced the linear stress-strain relationship and presented turbulent behavior with non-linear trend. The proposed memory-based viscosity-density model captured the memory effect for viscosity-density relationship where modified Reynold’s number (Blake Number) is also mentioned. And results show that memory has a great impact on fluid viscosity-density relationship.

6.2 Future Work Guideline

- The proposed memory-based stress-strain models are solved numerically with finite difference method and numerical study is done based on some sample reservoir data and experiment data for light crude oil (Chapter 3 and Chapter 4). Therefore, the model analysis need to be done with various data form different field around the world and experimental study can be performed to check performance of the model more accurately.

- In proposed viscosity-density model, Blake number is considered instead of Reynold’s number for porous media and memory term is solved with finite difference method (Chapter 5). The numerical analysis is observed for small range of field and experimental data. Therefore, it is needed to analysis for wide range of data from different fields around the world and perform some experiment study.