DEVELOPMENT OF MEMORY-BASED MODELS FOR RESERVOIR FLUID CHARACTERIZATION

by

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Abstract

The petroleum industry play an important role in supplying required energy all over the world. Effective methods are required to stimulate the process. Petroleum fluids are the mixture of complex hydrocarbons. Several techniques being used to predict reserve estimation, recovery, production, enhanced oil recovery, etc. Despite of modern engineering advancement, still, there are some drawbacks, such as, conventional models, linearized rock-fluid properties models, inaccurate risk assessment, and inappropriate descriptions of thermal effects. In this research, new mathematical models for petroleum fluids (non-Newtonian) regarding various degree of complexities will be developed. The most significant component will be the continuous time function introduced to the rheology. Previous attempts are addressed in this modeling, and those models were limited for some specific cases and fluids. The current proposal will develop a comprehensive model that can be applied to different reservoir fluids irrespective to fluid origin. In addition, the proposed models will also be adjusted for a complex mixture of reservoir fluids. The model equations will be solved numerically and validated using field data and data gathered from experimental tasks available in the literature. The proposed models will be developed focusing light crude oil for reservoir conditions. The role of various factors, such as crude oil density, viscosity, compressibility, surface tension, ambient temperature, and temperature will be included in the predictive models. Model equations will be solved with non-linear solvers, as outlined earlier. This will generate a range of solutions, rather than a line of unique solutions. This analysis will increase an accuracy of the predictive tool and will enable one to assess the uncertainty with greater confidence.

To my parents, my brother, and my uncle (Late Mosharraf Hossain Khan).

Acknowledgements

I express my sincere gratitude and thanks to my supervisor, Dr. M. Enamul Hossain for his encouragement and endless guidance throughout the period of graduate study. His creativity, extremely scholarly research-oriented and professional attitude, and his continuous support contributed remarkably towards the successful completion of this thesis. He has always been there for me in good times and in bad, and without him my graduate studies would have been far more challenging. I take this big opportunity to wish him the best in his future endeavors. I would like to thank my co supervisor Dr. Salim Ahmed for his support and guidance towards my thesis completion. He always supported me and guided me on exact path to achieve my goal and complete my research task. I would like to thank all the group members of my research group for their help and support as a family. I thank Mohammad Islam Miah for his support and help to accrue more knowledge about petroleum engineering. I thank Mamun Ur Rashid to help me with canvas and creating taxonomy. I would like to thank Pulok Kanti Deb for his help and support as a senior throughout my graduate life. I thank Tareq Uz Zaman for helping me in discretization methods and with MATLAB. I would like to thank Thomas Hickey, Rasel A Slutan, Al-Amin Shuvo, Munzarin Morshed, and Dipika Deb Dipa Purkayastha for their continuous help and support throughout my studies. They have been always there for me in good and bad times. I thank Moya Crocker, Colleen Mahoney, Vanessa Coish, and Tina Dwyer for creating a friendly and enabling atmosphere at the University. I express my greatest gratitude to my family members for being a constant source of inspiration, love and affection. I would like to thank Tanzia Alam Amy for her support and inspiration in every moment of my study.

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Co-Authorship Statement

I, Md Shad Rahman, hold primary author status for all the Chapters in this thesis. However, each manuscript is co-authored by my supervisor, co-supervisor, and my research mates Tareq Uz Zaman, Pulok Kanti Deb, Mohammad Islam Miah helped me to conceiving the idea and selection of appropriate techniques. Specially, Tareq Uz Zaman, Pulok Kanti Deb, Mohammad Islam Miah help to make my work easy and efficient. And my supervisor and co-supervisor always stood beside me, contributed, and facilitated to develop this work.

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Chapter 1

Introduction

1.1 Introduction

In this universe, every element has distinct uniqueness which can come out with the help of proper research and experimental investigation. Sometimes it become very tough to show the exact scenario with the help of proper experiments because of some limitations. This picture is more acute in petroleum industry and apparently it is difficult and expensive to conduct the experiment. In those conditions, mathematical modeling, analytical solution, numerical simulation, and scaled up ratios are the approximation approach to get relevant results on the path of research. However, the numerical solution helps to simulate the oil and gas reserve and generate new techniques to maximize the overall petroleum production. In petroleum industry, modeling and simulation are the two terms that used very frequently which are used to measure overall reserve and increase the production of any field. The mathematical modeling has a great impact on petroleum industries and several new techniques are generating every day to maximize the production.

The fluid flow through porous media is one of the most important issue for petroleum engineering. In reservoir engineering, fluid properties, rheological properties, and their relationship with formation are very important. But it is tough to model all these parameters together because of complex fluid and rheological behavior. Formation rock and fluid properties play very significant role in terms of petroleum production. The proper modeling and estimation of those rock and fluid parameters will surely help to maximize the production. However, reservoir rheological parameters are mainly focused on elastic and viscous properties of rock and fluids. For fluids, it can be measured by generating a shearing force on the fluid element surface and calculate shear stress. Usually, reservoir fluids are non- Newtonian by characteristics. In oil production process, most of the fluids are non-Newtonian fluid and represents thixotropic behavior. Therefore, formation rheology properties consideration is very important for any production.

The “time” is very important dimension for reservoir simulation. Time represents the complex behavior of reservoir and is used in developing any simulation model. To predict the pathway of any formation, the previous history is an essential factor which ultimately represents the dependency of time.

The idea of continuous rock and fluid alteration with time can be developed through model using the idea “memory’. In this thesis, the “memory” term is defined as “the properties of formation rock and fluid that help to identify the variations in rock properties (i.e., porosity, permeability) and fluid properties (density, viscosity, compressibility, ,surface tension, saturation) with respect to time and space” Several researchers are trying to incorporate the memory mechanism in porous media and presented the dependency of memory in fluid flow through porous media (Caputo, 1999, 2000; Arenzona *et al.,* 2003, Chen *et al.,* 2005; Hossain, 2008; Caputo and Carcione , 2013, Obembe *et al.,* 2017). Hossian, 2008 showed the effect of fluid memory in continuous rock and fluid alteration and proposed memory based fluid models focusing petroleum study.

1.2 Knowledge Gap

Memory is a function of all possible rock and fluid properties of a given fluid and its formation over the span of time. Fluid memory is a revolutionary addition in fluid flow models still this feature is neglected in most of the research. In recent times, several researchers are trying to show the effect of fluid memory in porous media and petroleum production (Caputo, 1999, Zhang, 2003, Chen *et al.,* 2005, Hossain 2008, Histrov, 2014, 2017. Obembe *et al.,* 2017). The unusual behavior of non-Newtonian fluid arises the complexity in the flow medium which is due to fluid memory. With the help of fluid memory, the various fluid flow phenomena such as heterogeneity, anisotropy can be described.

Fluid viscosity plays an important role at the time of fluid flow in porous media. Viscosity helps to define fluid types during fluid flow in the formation. Like thixotropic fluid or shear thinning fluid to identify at the time of applied shear force. Thixotropic fluid is the time - dependent non-Newtonian fluid and with applied shear force viscosity decrease with time at constant shear rate. If viscosity decreases with increasing shear rate then that fluid in known as shear-thinning or pseudo-plastic fluids. Those fluids are available in nature as simple fluid or complex mixture. These fluids have a great impact on industry specially petroleum industry.

In the petroleum industry, mostly empirical correlations are used to measure fluid properties. In those correlation, the real picture of fluid properties of porous media is unable to capture. There are few models are available where fluid is considered as Newtonian and fluid memory didn't observed. However, in nature most of the fluid are non-Newtonian (Perazzo *et al.,* 2003, Arratia *et al.,* 2005, Hossain, 2008) and mostly accepted Newtonian fluid water also exhibits viscous flow (Li *et al.,* 2007), Therefore, recently researchers trying to develop fluid flow model considering non-Newtonian fluid also incorporated memory mechanism in those models (Hossain, 2008; Caputo and Carcione, 2013, Histrov, 2014, 2017a, 2017b; Obembe *et al.,* 2017). But still there is no comprehensive fluid flow model in porous media which can represent fluid properties and memory together along with all advanced computational techniques.

1.3 Objectives

The specific objectives for this research are:

- To study fluid properties for light crude oil and incorporate memory mechanism;
- To develop comprehensive fluid model for reservoir fluid characterization;
- To develop memory-based stress-strain model focusing all fluid properties;
- To develop memory-based effective viscosity-density model with numerical validation;
- To solve the model equations numerically using field data and experimental data;
- Compare the models with existing models;

1.4 Structure of Thesis

The total research work is divided into six chapters. Those chapters contain: Chapter 1 describes the concept of the research work and identifies the knowledge gap. This chapter also provides the research objectives and the organization of the thesis. Chapter 2 provides an extended review of literature on rheology, fluid properties, memory mechanism, and fluid memory in porous media. Chapter 3 and Chapter 4 presents the development of stress-strain model with memory mechanism in a comprehensive way and comparison with established model. Chapter 5 represents the development of memory-based fluid effective viscosity-density model and comparison with existing model. Finally, Chapter 6 concludes this thesis by highlighting the research contribution and few recommendations are proposed for future work pathway.

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Chapter 2

A Critical Review on Reservoir Fluid Properties with the Aid of Memory

Preface

This paper has been submitted to the Journal of Non-Newtonian Fluid Mechanics and it is currently under review. The lead author performed the necessary literature review on fluid properties and fluid memory. The co-authors Pulok Kanti Deb helped in editing, and adding some new information and Dr. M. Enamul Hossain helped in identifying the gap in research, supervising the research, and editing the manuscript.

2.1 Abstract

Reservoir rock and fluid properties vary during any pressure difference or thermal changes in the reservoir formation. It is important to consider the rock properties such as permeability, porosity, etc. and fluid properties such as density, viscosity, PVT properties etc. as a function of time and space. Memory defines as the effect of previous events on the present, and future period of process and developments. The continuous change of rock and fluid properties can be characterized using memory mechanism. It is also significant to consider the rock, and the fluid properties as a function of time, space, and the inclusion of memory mechanism in science, and engineering study. In this paper, a detailed review of the existing rheological study, and fluid properties are presented in science and engineering. This study will provide an inclusive information on the present status of memory-based fluid flow modeling, effect of memory on rock and fluid properties during reservoir characterization. This review also offers a state-of-the-art literature survey on different fluid rheological behaviors. A literature survey provides a complete picture of memory-based models used in various fields (e.g., physics, chemistry, biology, economics, petroleum, etc.) of science and engineering. Though the role of fluid memory in porous media is significant, it is overlooked in almost all fluid models (e.g., fluid flow models, energy balance, thermal heat recovery, etc.). This paper introduces reservoir rheology and fluid properties, and demonstrates their relation to the existing mathematical models with memory mechanisms. It also focuses on the hypothetical barriers of the time function. This study will help in describing the exact picture of fluid characteristics in porous media considering memory for any reservoir scenario (e.g., reservoir simulation, thermal recovery, enhanced oil recovery, etc.).

2.2 Introduction

In the petroleum industry, the impact of reservoir rock and fluid (e.g., oil, water, and gas) properties over the span of time and space are significant. The proper identification of reservoir rock and fluid properties can significantly reduce the risk and uncertainties associated with petroleum operations. Fluid memory is one of the essential concepts that aid in the understanding and predicting the future behavior based on its past events. The exact strength of the concept of fluid memory is that it can be used to recall all the previous incidents of the fluid properties and predict how it will act in the future. Many researchers employ the concept of fluid memory in reservoir rheology, reservoir simulation, dynamic material balance, scaling criteria for oil-water displacement, thermal recovery, and enhanced oil recovery processes of petroleum engineering. To make future predictions about fluid properties, researchers often use Newtonian flow equations, and some non-Newtonian fluid models are also considered. However, some non-Newtonian models are not yet validated or well-established.

Memory-based (MB) models are significant in fluid flow through porous media. However, the application of MB models in field applications proves very challenging. For example, developing a model for transient flow is difficult as it crosses a two-layer medium and the boundary between the two rocks can equally be important (Garra *et al.,* 2015). Although, it is necessary to select either initial or boundary conditions. Sometimes the actual boundary layer is not considered in mathematical model (Garra *et al.,* 2015). The sudden change of structure can cause disturbances such as fluid particles migration or displacement (e.g., oil, water or gas) (Liu and Civan, 1996; Civan, 1998; and Merlani *et al.,* 2011). To obtain the time-dependent scenario, one can take into account that pressure (P) and temperature (T) change over time. Caputo (2000) established a modified Darcy's law for reservoir fluid fluxes in porous media to show the memory mechanism. This researcher used memory mechanism to represent the time function with a fractional derivative for a stable and homogeneous condition. Recently, fractional derivative models have been used to represent unstable flows in heterogeneous porous media (Fellah *et al.,* 2006, Hossain *et al.,* 2007; 2008; 2009a; 2012, and Obembe *et al.,* 2017).

Several researchers have used memory mechanism in the past few decades (Caputo, 1999; 2000; 2003; Hossain et al., 2006; 2007; 2009; 2011, and Obembe *et al.,* 2016; 2017). However, some challenges are applying MB models in porous media. This article outlines a brief discussion of memory models and their applications in porous media, as well as some of the challenges and future guideline for MB models for reservoir fluid properties (e.g., viscosity, density, surface tension, etc.).

2.2.1 Background of Research

In this universe, every single element or particle of any object has a relationship with its past data or history. This process happens because the flow of nature and all its particles are continuous. Not a single element can be confined or isolated from its surroundings at any condition. For any object or particle, its property, nature, and behavior can be determined from its origin and the way it traveled over the span of time. Fluid Memory is one of the most important phenomena used to describe any fluid property. However, it is overlooked by most researchers. The real scenario of continuous time function (i.e. memory) is easily determined in any fluid flow through porous media. The concept of "memory" is usually used in the branch of material science and other branches of science and engineering. For any mathematical modeling, the time function or memory of both rock formations and fluids are considered.

Hossain and other researchers (Hossain *et al.,* 2007; 2008; 2011; and 2012) showed that most flow models did not consider the continuous alteration of reservoir rock and fluid properties. A new mathematical model based on the memory mechanism has been proposed by Hossain *et al.* (2007; 2008). In these research, researchers considered reservoir rheology and fluid properties over time. While these models highlight all aspects of the phenomenon, some scenarios are still not adequately addressed. The researchers considered the viscosity of the Newtonian fluid but failed to consider any relation with density or other fluid properties. These models were successfully solved numerically, however their practical validation has not been established yet. Therefore, a comprehensive fluid flow model is required to show the exact scenario of reservoir fluid properties in relation to time alteration.

2.2.2 Fractional Derivative

The fractional derivative (FD) is a modern technique used to solve ordinary differential equations (ODE), partial differential equations (PDE), and the integro-differential equations problem (Oldham and Spainer, 1974; Miller and Ross, 1993; Samko *et al.,* 1993; Carpinteri *et al.,* 1997; Podlubny, 1999; Hilfer, 2000; Scalas *et al.,* 2000; Nigmatullin and Le Mehaute, 2005; Magin, 2006; Sabatier *et al.,* 2007; and Baleanu *et al.,* 2009). This technique is used to solve mathematical problems in various topics and problems in science and engineering such as viscoelasticity, control volume theory, heat conduction problems, thermal diffusivity, electrical and mechanical problems, chaos and fractals, etc. (Zaslavsky, 2002; West *et al.*, 2003; Chen *et al.,* 2004; Zaslavsky, 2005; Sabatier *et al.,* 2007; Jesus and Machado, 2008). FD is used less in field applications because complexity arises and it is also time consuming. But its accuracy is better than conventional methods (Miller and Ross, 1993; Chen *et al.,* 2004; Baleanu *et al.,* 2009; Baleanu and Nigmatullin, 2010). On the contrary, there is another technique known as the conventional or classical method. In the mid 1800's, this approach was first presented by Darcy for use in the porous medium. According to Darcy's law, fluid flux is proportional to velocity, pressure gradient, and conductivity coefficient and it depends on the physical characteristics of the fluid medium (Scheidegger, 1960; Bear, 1972). The conventional equation explains both single phases and multiple phases flow through porous media (Muskat, 1946; Ertekin *et al.,* 2001; Nield and Bejan, 2006; Civan, 2011). The classical Darcy model is applicable for steady-state, homogeneous, and isotropic conditions (Chan and Banerjee, 1981; Kaviany, 1995). Over the last few decades, several researchers have proposed different additions to Darcy's flow equation by introducing the action of slip, inertia, etc. (Hossain *et al.,* 2011; Hossain and Abu-khamsin, 2011a; Hossain and Abu-khamsin, 2012).

2.2.3 Memory

Memory can be defined as a continuous time function, which is captured through formation rock (porosity, permeability) and fluid properties (viscosity, pressure dependent fluid properties). If the memory term is incorporated in any fluid model (like stress-strain, fluid flow, etc.), it increases the nonlinearity of the equation. Mathematically, it is represented as the form of fractional derivative. In general, the memory recalls past states and predicts future states. In the last few decades, many researchers have been using this memory concept in several disciplines such as in electromagnetic studies (Jacquelin, 1984), human biology (Cesarone, 2002; Cesarone *et al.,* 2005), economy (Caputo and Kolari, 2001; Caputo, 2002), porous media (Caputo and Mainardi, 1971; Caputo and Plastino, 1998, 2004; Caputo, 1999, 2000, 2001; Hossain *et al.,* 2007), rheological properties of solids (Le Mehaute and Crepy, 1983; Bagley and Torvik, 1986; De Espindola *et al.,* 2005; Adolfsson *et al.,* 2005) and petroleum engineering (Hossain and Islam, 2006; Hossain *et al.,* 2007, 2008. 2009a, 2009b) to obtain more efficient results. Fluid memory adds a new way to show the time alteration in flow models for the petroleum industry (Brinkman, 1949; Bear, 1975; Sposito, 1980; Whitaker, 1986). Reservoir solid particles can be deposited along pore throats or walls to block the flow pathway and thus pore size will affect the alteration of rock and fluid properties (Hossain *et al.,* 2009c). In the oil and gas industry, this is not a new technique, and lots of examples are found in the literature to model rheological properties, fluid flow through porous media, heat diffusion, and so on (Caputo, 1967; Kornig and Muller, 1989; Broszeit, 1997; Hilfer et *al*., 2000; Zaslavsky, 2002; Zhang, 2003; Metzler and Klafter, 2004; Hossain and Abu-Khamsin, 2011b; Al-Mutairi et *al*., 2013; Obembe et *al*., 2017).

2.2.4 Reservoir Fluid Rheology

Rheology is the details study of flow and deformation of materials (i.e., semi solid and liquid) and of how the flow is affected by stresses and strains with time (Bourne, 2002; Dhiman, 2012). It is used in calculating flow velocity profiles, fluid viscosity, frictional pressure losses, etc. Rheology plays a significant role in the petroleum industry, especially in reservoir characterization, drilling operations, and petroleum productions. Petroleum fluids are the mixtures of hydrocarbon, starting from a very simple gas such as methane, to complex asphaltic molecules with thousands of molecular weights (Ronningsen, 2012). The rheological characteristics of those fluids can be Newtonian, highly non-Newtonian, viscoelastic, or solid (Ronningsen, 1993a; Ronningsen, 1993b; Pedersen and Ronningsen, 2000; and Ronningsen, 2012). In the petroleum industry, the proper measurement of rheological properties is challenging. It may cause economic losses, and in extreme conditions, it could end up in the abandonment of the production (Bagley, 1986; Drley and Gray, 1988; and Ronningsen, 2012).

2.2.5 Reservoir Fluid Properties

To obtain an overall scenario and an understanding of the behavior of reservoir performances, it is essential to analyze the hydrocarbon fluid properties (i.e., viscosity, density, solution gas-oil ratio, bubble point pressure, etc.) (Bateman, 2015). In figure 2.1, the most frequently used reservoir properties are shown in the diagram. The distribution of reservoir fluid phases depends on the reservoir temperature, composition, pressure, a difference of geological trap, depth, reservoir heterogeneity, and fluid migration path. Here, the fluid flows are controlled by various forces (i.e., gravity, capillary pressure, thermal and molecular diffusion, thermal convection, pressure gradients, etc.). Most researchers assumed that the reservoir fluid is static during model development, though the fluids can be in a dynamic condition in terms of geological time. Moreover, gravity works as driving force in distributing fluids. Laboratory based PVT analyses and mathematical models are used to determine those fluid properties (Dake, 1998; McCain, 2002).

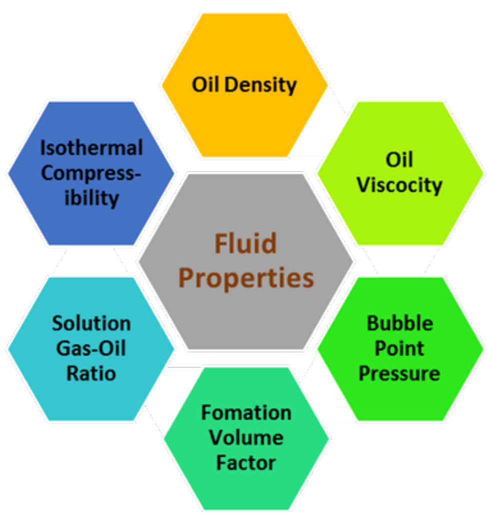

Figure 2.1: Reservoir Fluid Properties (Dake, 1998; McCain, 2002)

2.3 A Critical Analysis of the Literature

In this review, the characteristics and behavior of fluid rheology, fluid properties, and the memory mechanisms in porous media are highlighted. It is also shown how memory techniques can be incorporated in fluid models and discussed the benefits of memory. Finally, a guideline is presented on how to develop a comprehensive fluid model by incorporating fluid rheological properties such as viscosity, density, and memory.

2.3.1 Rheology

The term rheology was invented by Bingham in the 1920's and was inspired by a Greek quotation "Panta rei" meaning everything flows (Bingham, 1916; 1922). To show a relation between mathematics and rheology, there is equation called rheological equation of state or the constitutive equation. Rheology usually analyses mechanical properties, which also include the physical properties of solids, semisolids, and liquids, by describing the strain and flow characteristics as well as their behavior. In Figure 2.2, it is demonstrated that how rheology illustrates fluid flow relationship between the properties, structure, and processing of the materials. All three elements related to each other and without any of this element it is hard to represent the actual rheological scenario of any fluid. The linkage between every element and rheology is also shown in the following figure.

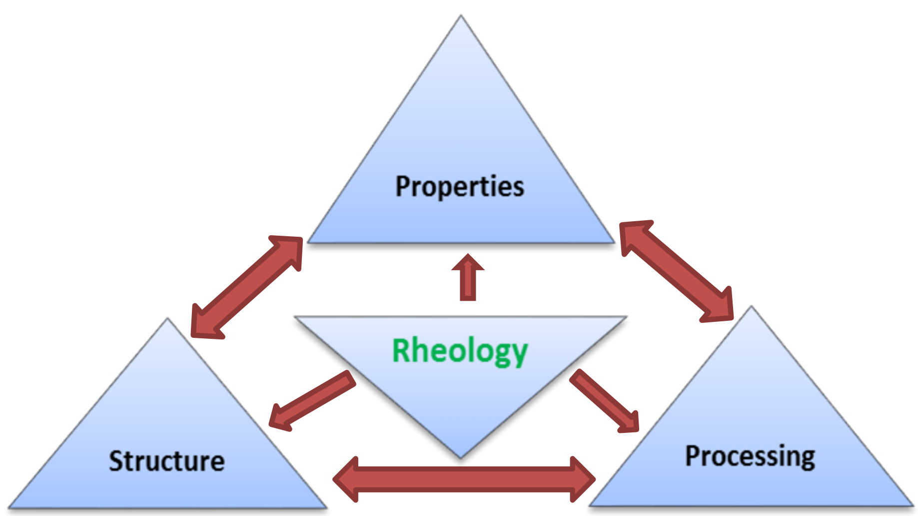

Figure 2.2: Rheological study cycle (modified from Sochi, 2010)

Rheology helps to describe the mechanical behavior of any materials as a function of stress (i.e., shear rate), strain, temperature, and pressure. In Figure 2.3, it is illustrated that how rheology is important in different branches of science and engineering. It has a great impact on physical and chemical fluid study. It is also discussed in continuum mechanics of fluids and several branches of engineering, especially in petroleum engineering during primary, secondary, and tertiary recovery operations (Drley and Gray, 1988; Ronningsen, 1993a; 1993b; and 2012; Zhao and Machel, 2012). Here, viscoelasticity helps in injectivity and increases the pressure gradients for general Darcy's flow in any reservoir. Newtonian and non-Newtonian fluids exhibit unusual characteristics in transient flow and pressure gradient conditions for any oil and gas reservoir (Ronningsen, 1992; Ronningsen *et al.,* 1997; Paso *et al.,* 2009; and Wilton, 2015).

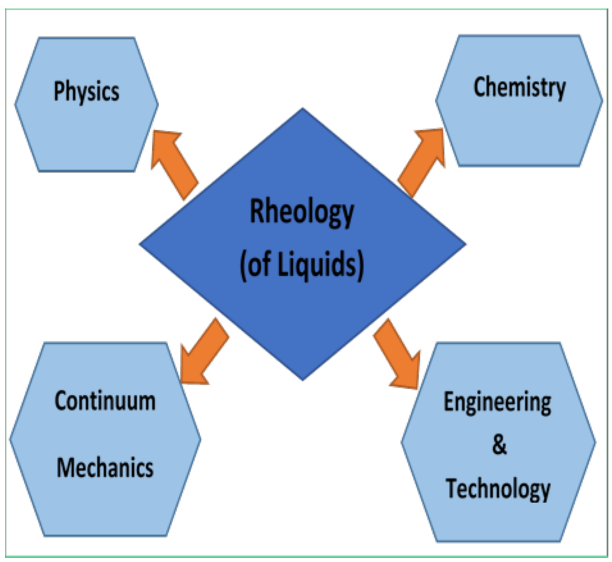

Figure 2.3: Applications of Rheology in various field (modified from Wilton, 2015)

The rheological study is essential for dilute, polymer and concentrates colloidal systems. It helps to describe fluid flow behavior with time and space. It also describes exactly the time dependent and independent behaviors of any fluids. In rheology, the fluid is categorized into two divisions: Newtonian and Non-Newtonian. A complete picture of the fluid categories is given in Figure 2.4. In Fig 2.4, fluid is divided into two main groups, Newtonian and non-Newtonian, and non-Newtonian fluids are divided in three sub groups. Those groups are viscoelastic, times dependent and time independent. Time dependent and time independent fluids are sub categorized again according to the fluids behavior. Figure 2.5 represents the behavior of Newtonian and non-Newtonian fluids using flow resistance as a function of shear rate. For Newtonian fluids, it will be a straight line intersecting the viscosity axis and not going through origin. Pseudo plastic fluids and Dilatant fluids also go through same point as Newtonian fluid. However, those shapes are concave downwards and upwards accordingly from straight line. Bingham fluids also illustrates as concave upwards but do not follow the same point as Newtonian fluids.

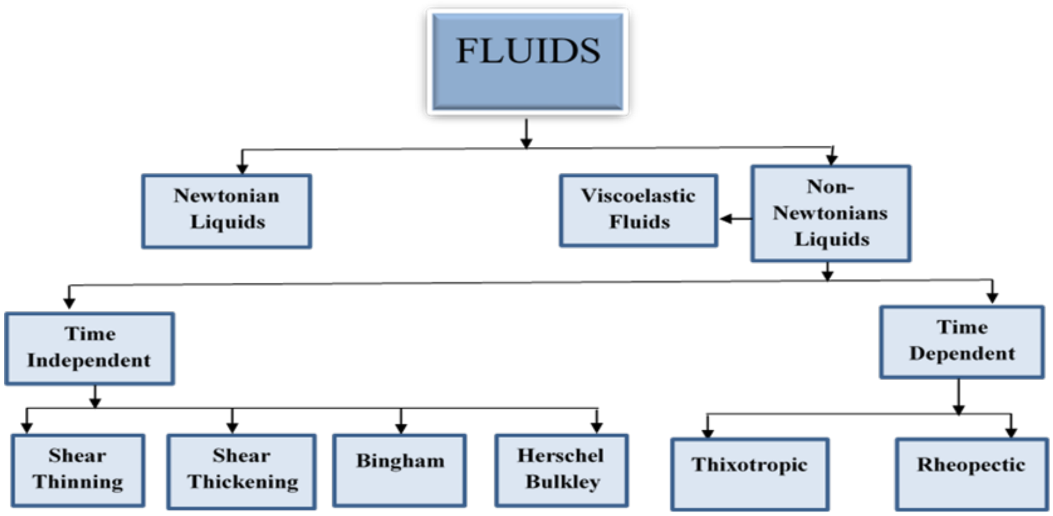

Figure 2.4: Classification based on Fluids Rheology (modified from Zhao and Machel, 2012; Hamed, 2016)

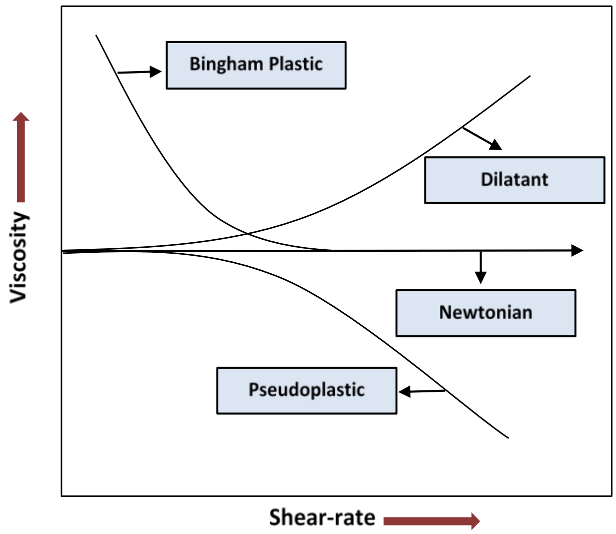

Figure 2.5: Behavior of Newtonian and non-Newtonian Fluids (modified from Sochi, 2010; Wilton, 2015)

2.3.1.1 Newtonian fluid

Newtonian fluids follow the Newton's law of viscosity. Here, the Newtonian viscosity is independent of shear rate (γ) or shear stress (τ) (Chhabra and Richardson, 2008) and depends on the fluid flow rate as well as the temperature and pressure. Figure 2.6, The flow curve for Newtonian fluids (water, air, gas and light crude oil) is a straight line which goes through its origin, and the slope of that line is constant (μ). If the shear stress becomes double, then the shear rate will also become double, and vice versa (Chhabra and Richardson, 2008; Sochi, 2010; Ronningsen, 2012; Wilton, 2015). The rheological equation for a Newtonian fluid is given as:

$$\tau = \mu . \gamma \tag{1}$$

Here, the viscosityis constant at fixed temperatures and pressures. In any reservoir, hydrocarbons with less than five carbon particles have an essential role in oil production. The gas that produce in a reservoir is a part of the reservoir fluids and show a vital impact on fluid viscosity and productivity of those reservoirs. Thus, gas viscosity is also considered (Ronningsen, 1993a; 1993b; and 2012).

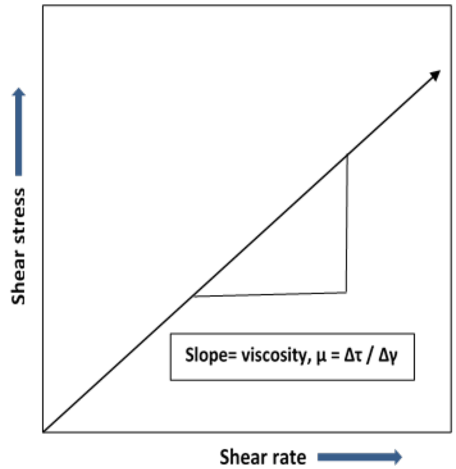

Figure 2.6: Shear rate vs. shear stress relationship of Newtonian fluids (Azar and Samuel, 2007)

When fluid flows through any porous media there are stress components that are presented in Figure 2.7. Stress components are in three direction and flow is presented in the x-direction. In Figure 2.7, Newtonian fluid observed in shearing motion, stress components in all directions are equally zero.

$$\tau_{xx} = \tau_{yy} = \tau_{zz} \tag{2}$$

To define complete explanation of a Newtonian fluid, it needs to show a constant viscosity as well as fulfills the condition of equation (2), or it maintains the Navier-Stokes equations. Though Boger fluids show constant shear viscosity but do not satisfy equation (2). So, it is defined as non-Newtonian fluids (Boger, 1976; Prilutski *et al.,* 1983)

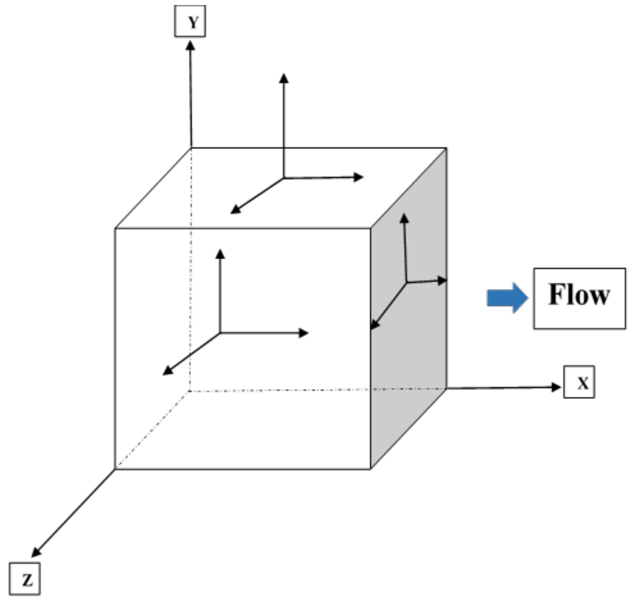

Figure 2.7: Stress components analysis in 3-D fluid flow (modified from Chhabra and Richardson, 2008)

2.3.1.2 Non-Newtonian Fluid

Non-Newtonian fluid is defined as "shear stress versus shear rate flow curve of any fluid becomes nonlinear or does not go through the origin". Non-Newtonian fluid can be categorized into three major groups i.e., (i) time-independent, (ii) time-dependent, and (iii) viscoelastic (Ronningsen, 1993a; 1993b; Pedersen and Ronningsen, 2000; Chhabra and Richardson, 2008). In Table 2.1, non-Newtonian fluids are categorized into three major groups. For every group, shear stress and shear rate relation, types of fluids and examples are discussed in Table-2.1.

2.3.1.2.1 Time-independent non-Newtonian fluid

The behavior of time independent fluids can be presented by a constitutive equation in simple shear form:

$$\tau = f_1 \gamma^* \tag{3}$$

The above equation represents that for any specific point, the value of γ^* can be determined from the value of $\boldsymbol{\tau}$ or vice versa. Time-independent fluids can also be classified into major three groups as: (i) shear-thinning or pseudo-plastic; (ii) visco-plastic; (iii) shear-thickening or dilatant (Chhabra and Richardson, 2008; Wilton, 2015). Time independent behavior of any non-Newtonian fluid illustrated graphical along with Newtonian fluid in Figure 8. For Newtonian fluids line goes through the origin and other time independent fluids such as pseudo plastic fluids, dilatant fluids go through the origin but do not represent as straight lines. Those fluids shape as parabolic curves. In Figure 2.8, few time independent fluids such as Bingham plastic, yield pseudo plastic does not go through origin and vary from straight line to parabolic shape.

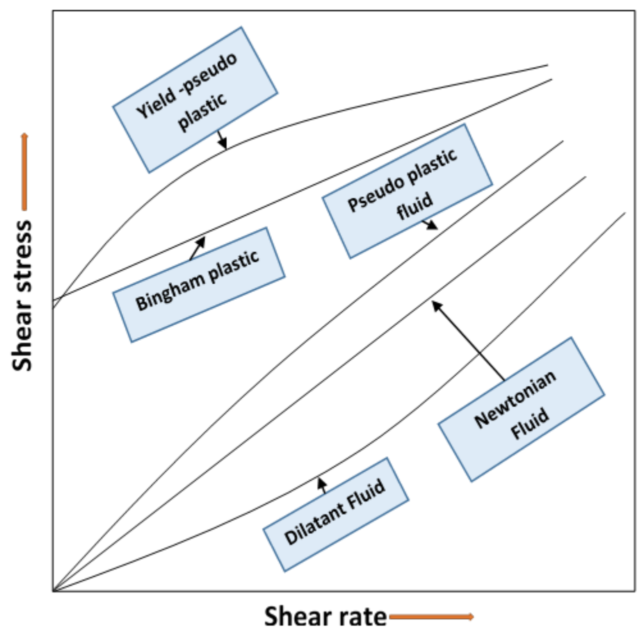

Figure 2.8: A Typical plot of time independent non-Newtonian fluids (modified from Sochi, 2010)

Shear-thinning or pseudo plastic fluids are the most common types of time-independent fluids where viscosity decreases with increasing shear rate. At very low or high shear rates, shear-thinning fluids act as a Newtonian fluid. Sometimes graphical presentation of shear stress to shear rate becomes a straight line and goes through the origin (Chhabra and Richardson, 2008; and Sochi, 2010).

Visco-plastic fluid behavior is identified by the presence of a yield stress (τ_o) (Chhabra and Richardson, 2008; Sochi, 2010; and Wilton, 2015). A fluid that presents linear flow curve for $| \tau_{yx} | > | \tau_o |$ is called a Bingham plastic fluid or constant plastic viscosity. For visco-plastic fluids viscosity decreases with increasing shear rate. When shear stress is very low, the viscosity decreases immediately before the fluid starts to flow (Barnes, 1985; Astarita, 1990; Schurz, 1990; and Evans, 1992). The common examples of visco-plastic fluid are suspensions, emulsions, foodstuffs, blood and drilling muds, etc. (Barnes, 1999; Uhlherr *et al.,* 2005).

Shear thickening or dilatant fluids are similar to pseudo-plastic fluids which do not show yield stress but viscosity increases with increasing shear rate. At low shear rates, the producing stresses are continuously small. However, it gives more friction and high shear stresses at a high shear rate. The different characteristics of this fluid drives researches to review on this fluid (Barnes *et al.,* 1987; Barnes, 1989; Boersma *et al.,* 1990; Goddard and Bashir, 1990). The common examples of dilatant fluids are china clay, titanium dioxide, corn flour in water, dispersions of polyvinyl chloride in dioctyl phthalate etc. (Metzner and Whitlock, 1958; Griskey *et al.,* 1985; Boersma *et al.,* 1990).

2.3.1.2.2 Time-dependent non-Newtonian fluid

Viscosities of fluids also depend on the time when it is subjected to the shear force (Chhabra and Richardson, 2008). If fluids are placed at shear stress for a long time, their actual viscosity becomes so much. Those fluids are: mud suspension, crude oils, foods, different water suspension, cement paste etc. Time-dependent fluid behavior can be divided into two groups, i.e., (i) Thixotropy, and (ii) Rheopexy or negative thixotropy (Chhabra and Richardson, 2008; Sochi, 2010; and Wilton, 2015).

With a constant shear rate, if the shear stress reduces with time it is known as thixotropic fluid. For thixotropic fluid, shear rate increases steadily at a constant rate from zero to a maximum value and then reduces to zero at the same rate (Chhabra and Richardson, 2008). A hysteresis loop of shear rate is illustrated in Figure 2.9 where shear stress is increasing with shear rate up to certain point (i.e., maximum stress) and then decrease with the same rate and come to origin (Ronningsen, 1993a; 1993b; Chhabra and Richardson, 2008; Sochi, 2010; Wilton, 2015). The time-dependent behavior becomes stronger with the larger enclosed area. No hysteresis process is studied for time-independent fluid as the area of the hysteresis is zero for such fluid.

If shear stress increases with time, that fluid is known as rheopexy or negative thixotropic fluid. Rheopectic fluid is more compact compared to a thixotropic fluid as shown in Figure 2.9 where shear stress is increasing with shear rate up to certain point (i.e., maximum stress) like thixotropic fluid but more widely and then decrease with the same rate and come to origin (Ronningsen, 1993a; 1993b; Chhabra and Richardson, 2008; Sochi, 2010; Wilton, 2015). Several researchers proved the rheopectic behavior of fluids in their studies with different fluids (Freundlich and Juliushburger, 1935; Steg and Katz, 1965; Pradipasena and Rha, 1977; Keller and Keller, 1990; Tanner and Walters, 1998; Tanner, 2000; Chhabra and Richardson, 2008; Sochi, 2010; Wilton, 2015).

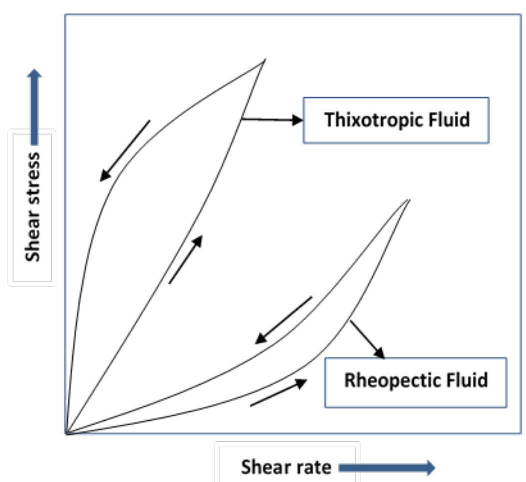

Fig. 2.9: A typical plot for time-dependent fluid (modified from Chhabra and Richardson, 2008)

2.3.1.2.3 Viscoelastic non-Newtonian fluid

The general law of elasticity describes that the stress applied to an object is directly proportional to the strain. In the case of tension condition, Hooke's law applies and the proportionality coefficient is defined as Young's modulus, G:

$$\tau = -G\frac{dx}{dy} = -G\gamma \tag{4}$$

From equation (4) shear stress is proportional to shear rate for a Newtonian fluid. Some fluids show elastic and viscous behavior with conditions. If the time-dependent behavior is ignored, then that fluid will be viscoelastic (Chhabra and Richardson, 2008; Sochi, 2010). Researchers illustrate the significances of viscoelastic fluids in different fields (Schowalter, 1978; Bird et al., 1987; Boger and Walters, 1992; Carreau et al., 1997; Larson, 1998; Tanner and Walters, 1998; Morrison, 2001; Sochi, 2010).

2.3.2 Fluid Properties

Fluid property (i.e., density, viscosity, compressibility etc.) plays a significant role in the petroleum industry. Rock and fluid properties interrelated to each other and an essential part of petroleum reservoir. Figure 2.10 depicts the rock and fluid formation characteristics for a porous medium. Rock matrix and pore fluids are shown differently and pointed the dry rock and saturated rock in fluid porous media. The conditions are shown which affect the transformation from dry to saturated rock.

2.3.2.1 Oil Density

For oil, the range for density in field varies from 30 lb/ft^3 for light oil to 60 lb/ft^3 for heavy crude oil where gas solubility is not considered (Ahmed, 2007; Ahmed, 2010). Usually oil density is calculated in the laboratory. If laboratory analysis is not available, empirical correlations can be used to get density at reservoir temperature and pressure. The oil density correlations can be divided into two groups based on the available data: (i) correlations use pressure and temperature at reservoir condition to determine oil density, and (ii) correlations use PVT data, like gas-oil ration, oil gravity etc. (Ahmed, 2007).

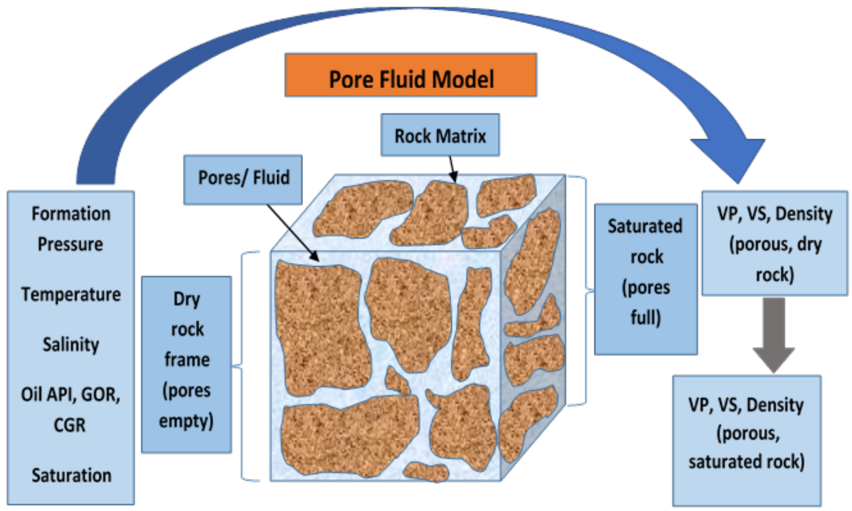

Figure 2.10: Rock-Fluid physical characteristics in porous media (modified from Dunne, 2013)

Several empirical correlations are available for measuring oil densities. For Egyptian crude oil, Hanafy *et al.* (1997) presented that the correlation established by Ahmed (1988) is the most appropriate for calculating under-saturated reservoir oil density. For dead and gas-saturated reservoirs Katz (1942) and Standing's (1981) correlations give the best estimation for oil densities (El-hoshoudy *et al.,* 2013). Very few researchers show correlations for dead oil density. In general, correlations are developed based on field data, experiments, and PVT analysis. From Katz's chart, Standing (1981) developed a mathematical equation by incorporating (dead oil) density. Later, Ahmed (1988) published a correlation to assume the dead oil density where stock-tank oil molecular weight was not mentioned. El-hoshoudy *et al.* (2013) developed a new correlation for dead oil density considering reservoir pressure, temperature, GOR, API gravity and suspension pressure.

Standing (1981) developed a correlation for saturated oil viscosity, where gas specific gravity was used for stock tank and separator. Hanafy *et al.* (1997) established a correlation for density where he calculated density at reservoir temperature and P_b. El-hoshoudy *et al.*

(2013) developed a new correlation for saturated oil viscosity considering reservoir pressure, temperature, GOR, API gravity, P_b and suspension pressure. Vasquez and Beggs (1980) published density correlations for the under-saturated reservoir condition. Ahmed (1988) established a correlation for oil density at the under saturated conditions. In oil industry, density is mostly determined by correlation or PVT data. However, no mathematical model is developed for density or show any relation of density with fluid other properties and fluid memory. There is a scope to develop fluid density model and capture memory effect.

2.3.2.2 Oil Viscosity

Viscosity is an important physical property for porous media. Fluid viscosity is defined as the internal resistance of the fluid with respect to external pressure to flow. Crude oil viscosity is a compact function of the reservoir temperature, reservoir pressure, API gravity, GOR, and other compositions (Ahmed, 2007; Ahmed, 2010). Researchers developed several correlations based on oil field data and laboratory analysis. Lots of researchers work on this topic to get accurate viscosity for reservoir crude oil (Ahmed 1988; El-hoshoudy *et al.,* 2013). The crude oil viscosity correlations can be sub-divided into two groups based on data of oil mixture as: (i) correlations based on crude oil composition, and (ii) correlations based on PVT data, such as API gravity, gas-oil ratio (Ahmed, 2007; El-hoshoudy *et al.,* 2013).

Oil viscosity can be measured in a laboratory while considering some conditions. Researchers used several correlations to measure crude oil viscosity however some correlations are specific for fixed region. Those correlations are still unable to capture the overall scenario of oil viscosity because of variation in crude oil composition and nature (Abdolhossein, *et al.,* 2013). Considering reservoir pressure, crude oils viscosity correlations can be divided into three categories: (i) dead oil viscosity, (ii) saturated oil viscosity, and (iii) under-saturated oil viscosity (Ahmed, 2007; Ahmed, 2010; El-hoshoudy *et al.,* 2013; Abdolhossein, *et al.,* 2013).

The dead oil viscosity is the most complicated to measure with correlations. Several researchers worked on dead oil viscosity and proposed some dead oil viscosity correlations (Beal, 1946; Beggs and Robinson, 1975; Glaso, 1980; Kaye, 1985; Al-Khafaji *et al.,* 1987; Petrosky, 1990; Egbogah and Ng, 1990; Labedi, 1992; Kartoatmodjo and Schmidt, 1994; Bennison, 1998; Elsharkawy and Alikhan, 1999; Hossain *et al.,* 2005; Naseri *et al.,* 2005; Alomair *et al.,* 2011; El-hoshoudy *et al.,* 2013). Table 2.2 shows the overview of the data set used in the above-mentioned correlations. Here, the researchers developed dead oil viscosity correlation as a function of temperature and API gravity. Those correlations did not consider GOR, reservoir pressure, bubble point pressure, saturation pressure etc. Few researchers did not mention critical temperature, molar mass weight, boiling point, etc. in PVT reports and used equation of state to develop dead oil viscosity correlations (Teja and Rice, 1981; Johnson *et al.,* 1987; Svrcek and Mehrotra, 1988; Johnson and Svreck, 1991; Mehrotra, 1991). Those models require several computations however, the accuracy is still not acceptable.

Several correlations have been established for saturated oil viscosity. If dissolved gas is present in oil, the viscosity also decreases and it is lower than the dead oil viscosity (Bergman and Sutton, 2007). Other researchers established two separate correlations for calculating crude oil viscosity at P_b and below P_b (Khan *et al.,* 1987 and Labedi, 1992). Some researchers assumed saturated oil viscosity is a function of dead oil viscosity and GOR. Others proposed it as a function of dead oil viscosity and saturation pressure (Abdolhossein *et al.,* 2013). Many other researchers established viscosity correlations for saturated oil (Chew and Connally, 1959; Beggs and Robinson, 1975; Al-Khafaji *et al.,* 1987; Khan *et al.,* 1987; Petrosky, 1990; Labedi, 1992; Kartoatmodjo and Schmidt, 1994; Elsharkawy and Alikhan, 1999; Hossain *et al.,* 2005; Naseri *et al.,* 2005; Bergman and Sutton, 2007). Table 2.2 also represents the data used to establish above mentioned correlations. In those correlations, GOR and saturation pressure data were used and ignored temperature, API gravity, reservoir pressure and bubble point pressure. Unfortunately, these correlations are developed for specific regional oil not for universal usage.

GOR is constant in the under-saturated region, so viscosity only depends on pressure (Abdolhossein *et al.,* 2013). Several viscosity models are established for calculating viscosity in such under-saturated conditions (Beegs and Robinson, 1975; Vaequez and Beggs, 1980; Khan *et al.,* 1987; Petrosky, 1990; Sutton and Fasrshad, 1990; Abdul-Majeed *et al.,* 1990; Labedi, 1992; Orbey and Sandler, 1993; Kartoatmodjo and Schmidt, 1994; Almehaideb, 1997; Elsharkawy and Alikhan, 1999; Dindoruk and Christman, 2001; Hossain *et al.,* 2005; Sutton and Bergman, 2006). A few of these correlations present viscosity as a function of reservoir pressure. Other researchers included API gravity and dead oil viscosity (Labedi, 1992; Orbey and Sandler, 1993; Elsharkawy and Alikhan, 1999). Table 2 illustrates the datasets used in aforementioned viscosity correlations. Most of the researcher considered reservoir pressure and bubble point pressure for those correlations and overlooked saturated pressure, API gravity, temperature etc.

Mathematical model equations are used to determine viscosity in absence of sufficient laboratory data. There are few models for calculating viscosity of fluids. A few petroleum experts proposed some viscosity equations for crude oil. Some researchers also tried to incorporate memory mechanism in oil viscosity models; such as Slattery (1967), Mifflin and Schowalter (1986), Li *et al.,* (2001), Zhang (2003), Iaffaldano *et al.* (2006), Hossain *et al.* (2007; 2008; 2009).

In petroleum engineering, viscosity is mostly determined by correlation or PVT data. However, a few mathematical models developed for viscosity and show some relationships of viscosity with other fluid properties i.e., stress-strain, surface tension and fluid memory. However, researchers failed to show the actual fluid behavior in those models. There is a scope to develop a comprehensive fluid viscosity model and capture the memory effect.

2.3.2.3 Isothermal Compressibility Coefficient for Oil

Isothermal compressibility coefficient can be defined as the change rate of per unit volume with respect to pressure. All the parameters including temperature is constant except pressure (Trube, 1957; Ahmed, 2007; Ahmed, 2010). Mathematically it can be denoted as below:

$$c = -\frac{1}{V}(\frac{\delta V}{\delta p})_T \quad (5)$$

This equation is used in solving problems related to transient flow. Under-saturated and saturated isothermal compressibility coefficients should be measured properly to get the accuracy for PVT analysis for any crude oil. The isothermal compressibility coefficient of the oil phase, c_o, can be divided into two groups based on reservoir pressure such as: (i) Reservoir pressure greater than or equal to P_b termed as under-saturated isothermal compressibility coefficient, and (ii) Reservoir pressure below than P_b termed as saturated isothermal compressibility coefficient (Ahmed, 2007; Ahmed, 2010).

2.3.2.4 Solution Gas-Oil Ratio

When gas starts to dissolve in crude oil at a certain temperature and pressure, this is defined as Solution gas-oil ratio, R_s. (Hanafy et *al*., 1997, Ahmed, 2007; Ahmed, 2010). The GOR is mainly a function of pressure, temperature, and API gravity (Hanafy et *al*., 1997, Ahmed, 2007; Ahmed, 2010). GOR increases with pressure until it reaches the saturation pressure. At P_b all the existing gases are dissolved in the oil. As such, the GOR value become maximum. There are lots of empirical correlations for GOR available in the literature in the form of PVT analysis. However, there is no comprehensive model for GOR with other fluid properties and fluid memory. Therefore, a scope is available for future researchers to develop fluid model showing GOR and other fluid properties to capture fluid memory.

2.3.2.5 Bubble Point Pressure

The maximum pressure at which a gas bubble starts to generate from the oil is known as the bubble point pressure, P_b. P_b is mostly function of GOR, gas gravity, API gravity and temperature (T) (Ahmed, 2007). It can be presented mathematically as below:

$$P_b = f\,(R_s, API, \gamma_g, T) \quad (6)$$

In the absence of experimental P_b, it is important for the petroleum engineer to make an estimation to measure P_b. Over the last few decades, various graphical and mathematical correlations have been presented for predicting P_b. There are lots of empirical correlations for P_b, which are available in the literature as PVT analysis.

2.3.2.6 Oil Formation Volume Factor

The ratio of the volume of oil (and the gas in solution) at the prevailing reservoir temperature and pressure to the volume of oil at standard conditions is defined oil formation volume factor (OFVF), B_o (Hanafy et *al*., 1997, Ahmed, 2007; Ahmed, 2010). Mostly, B_o is greater than or equal to unity. OFVF can be expressed mathematically as:

$$B_o = \frac{(V_0)_{p,T}}{(V_0)_{sc}} \tag{7}$$

OFVF is an important feature for any reservoir. It is measured experimentally or uses empirical correlations to calculate. There are lots of empirical correlations for OFVF available in the literature. In most of the empirical correlations, the generalized form of OFVF is as below:

$$B_o \ = \ f\ (R_s, \gamma_g, \gamma_o, T) \tag{8}$$

2.3.2.7 PVT Analysis

The physical properties of reservoir fluid are one of the vital factors for any reservoir production life (Azad *et al.,* 2014). PVT properties of crude oil such as P_b, OFVF, and GOR are calculated by two major approaches; equations of state and empirical correlations (Ahmed, 2007; 2010, Azad *et al.,* 2014). Over the last 70 years, many studies have been done in this field and several correlations have been developed for calculating PVT properties. Correlations developed by Katz (1942), Standing (1947) and Lasater (1958) are the most known correlations. In recent times, several researchers are trying to use computational methods like Artificial Neural Network to measure PVT properties more accurately (Gharbi *et al.,* 1999; Bello *et al.,* 2008; Omole *et al.,* 2009; Mansour et al., 2013; Shokrollahi *et al.,* 2015). In Table 2.3, correlations measuring fluid properties such as GOR, P_b, OFVF are shown by considering AARE/ARE and STD. Most of the researchers did not calculate GOR in their correlations and those correlations are focused on specific geological region. Petrosky and Farshad (1993) established correlations for GOR, P_b, OFVF with minimum error. Other researchers also developed correlations for GOR, P_b, OFVF but errors in results were much more. A few researchers also used Standing (1947); Laster (1958); Glaso (1980); Al-Marhoun (1992), and Mahmood and Al-Marhoun (1996) correlations as the standard and add some new conditions. In Table 2.4, P_b correlations are presented considering GOR, API gravity, temperature and density. The datasets used in those correlations are based on specific region. Only few researchers used worldwide data but with less accuracy. Finally, in Table 2.5, OFVF correlations are presented considering GOR, API gravity, temperature and density. The datasets used in these correlations are based on certain crude oil sample and some researchers used worldwide data but still failed to capture overall scenario. In recent time, many researchers of petroleum industry are trying new techniques such as artificial neural networks to develop new models for PVT analysis. Fluid memory can play a significant role to develop more accurate PVT correlations which can be accepted worldwide.

2.3.3 Memory Mechanism

Irrespective of the research area, a variety of studies are predicting what will happen in the future. With advanced technology, future-forecasting is becoming easier day by day. Scientists are now predicting the future for different fields with more accuracy. Memory mechanism is one of the effective ways to predict future event.

2.3.3.1 Memory in Science and Engineering

Memory mechanism is used in various fields such as physics, chemistry, biology, economics, soil science, business, and different engineering disciplines including petroleum engineering. In Table 2.6 illustrates displays how memory term is used irrespective to fields of science and engineering. Here, the use of memory mechanism in various models are shown. In those models, memory is used in different ways with various conditions. Table 6 also presents the parameters that are used to develop the model, goal of that research, and field applications.

Cisotti (1911) derived an equation for energy density and introduced memory term for dielectric irreversible medium. After that Graffi (1936) developed a generalized Maxwell's equations introducing "Memory" for the electromagnetism field where he addressed Faraday's law, Ampere law, and Maxwell equations and considered a term β for the history dependence factor for the electromagnetic field. Later Jacquelin (1984) also showed memory formalism for the electro-mechanism field and used fractional derivatives to show the properties of energy storage in any electrical networks. Recently Hamza *et al.* (2015) developed a modified mathematical model of Maxwell's equations using Fractional calculus and urged that this model is different from the other fractional electromagnetic model in terms of fractional calculus. Researchers showed the numerical calculations of displacement, stress-strain, temperature, and induced electric/magnetic fields for special cases.

Atkinson and Shiffrin (1968) described memory as human memory and divided the memory into three stages, the sensor register, short-term store, and long-term store. In recent times, Bernacchia *et al.* (2011) found the group of simultaneously captured neurons is not correlated. Authors suggested a distributed, flexible neural structure for both reward valuation and memory, depending on their findings. Caputo and Mainardi (1971) developed the model for fluid dissipation based on memory and validated their model with experimental dissipation curves of different materials. Later Caputo and Plastino (1998) introduced the space fractional derivatives of pressure into Darcy's law for the diffusion processes of fluids. Before that, authors defined memory as time-dependent fractional derivative for the diffusion of fluids in the rock.

Bruce (1983) developed a way to predict future geological trends through the study of past events and established three major paths for geologic predictions such as: climate change, element migration and geotectonic. After that Caputo (1999) investigated geological areas (i.e., geothermal) where fluids may affect pore sizes and proposed a modified Darcy's law by introducing memory formalism to capture the effect on permeability with changing time. Furthermore, Christensen *et al.* (2004) described a new computational approach to minimize the operation time of any thermal simulation and proposed a dynamic gridding technique along with fine scale orientation across the thermal front with the help of memory mechanism. In recent times, Mlodinow and Brun (2014) proposed a compact thermal relation with memory mechanism. Researchers represented that memory can record the events and interact with the system so that it can make a correlation with thermodynamic changes, i.e., entropy change.

Yamshchikov *et al.* (1994) studied memory for geological rock and showed several ways (i.e., emission memory effect, thermal emission memory effect, ultrasonic memory effect, electrical memory effect etc.) to carry out memory effect. But, only memory effects on stress-strain problems were highlighted. Later Xuefu *et al.* (1995) used memory term in rock mechanics and showed that rock has long-term history memory, behavior-reproducing memory and stress memory. Authors concluded that rocks can remember all the stresses they underwent in the past. Recently Hagemann and Stacke (2015) reviewed soil moisture–atmosphere feedback in several regions of the globe and found that for short variations of the regional climate, memory plays a significant role. Caputo and Kolari (2001) proposed an analytical model that represents the Fisher (1930) equation of tax version. For stock prices and inflation rates, they introduced memory mechanism and presented both short-run and long-run response to any increase or inflammation. Authors showed financial economics can be solved more efficiently with memory function.

Caputo (2000) reported an improvement in the representation of the flux and fluid pressure gradient during fluid transport in porous media. Author showed that these changes can be determined when periodic or constant pressure is applied to the boundary plane along with the time and space. Again Caputo (2003) presented fluid memory with the time, and space and showed this is more flexible way to represent local phenomena. Researcher advised to assign first order space derivative at the boundary layer with constant boundary pressure and consider initial medium pressure is zero. Then Zavala-Sanchez *et al.* (2009) defined memory term for fluid transport phenomena. They introduced a term called system "remembers" and described memory effects on effective solute spreading and mixing due to source size and positions. Recently Caputo and Carcione (2013) considered the 1-D model for water reservoir and showed memory mechanism with modified Fourier law.

Authors represented time-varying diffusivity for sediments of different grain sizes. Recently, Hristov (2014) presented diffusion model with the integral balance method and described memory term by weakly singular (power-law) kernels also showed the fading memory term by Volterra integrals, Riemann-Liouville derivative.

Cesarone *et al.* (2005) presented an addition to the Fick's second law and introduced memory mechanism with Fick's law based on fractional derivatives, also considered the indirect diffusion process across two different membranous of a biological membrane. Later Caputo and Cametti (2009) developed a fluid diffusion model based on Fick's second equation using memory formalism and compared their results with some experiments of drug diffusion through human skin conducted by some other researchers. Space dependent diffusion constant was also proved in their study.

Hossain and Islam (2011) developed a new scaling technique for oil-water displacement and used modified Darcy law to develop the model equation. Authors also proposed a branch of scaling criteria that allows the various relationships between fluid pressure, capillary, saturation and fluid velocities incorporating the fluid memory. Later Hossain and Abu-Khamsin (2011a) developed new dimensionless numbers with memory mechanism. Those numbers will give an idea for convective heat transfer between rock and fluids with continuous alteration. Authors used the modified energy balance equation to establish the heat transfer coefficient for rock and fluid. Their established numbers would help to identify the rheological behavior of any rock and fluid system. After that, Hossain and Abu-Khamsin (2012) developed a new model based on the modified energy balance equation and introduced new dimensionless numbers for the thermal recovery process to show various heat transport mechanisms.

Wang and Li (2011) discussed a concept of "memory-dependent derivative", which described an integral form of a common derivative of a kernel function of slipping interval. Their way of describing memory was more efficient than fractional derivatives. Authors also defined the memory in a way which is easy to understand the physical meaning and added expressive force to understand the memory dependent differential equation. Du *et* *al.* (2013) observed that there are two stages of memory. One is the starting stage, the other is working stage. The actual relation between the fresh stage and the working stage is needed to be considered to get the accurate index of memory. Kolomietz (2014) used kinetic theory to describe the nuclear Fermi liquids and investigated the distortions that lead to scattering of particles on the Fermi-surface and relaxation of collective motion with memory (Landau, 1959). Author also observed that memory effects depended on the relaxation time and concluded that the memory formalism would give a time irreversible viscosity and time-dependent conservative force.

2.3.3.2 Memory in Porous Media

The idea of fractional derivatives in constitutive equations is not a recent theme. Fluid memory is described in lots of scientific, physical and engineering applications (Moroni and Cushman, 2001). There are plenty of examples of fluid memory in solid rheological properties, thermo-elastic, electromagnetic, heat diffusion, rock and fluid properties and other fields of research (Metzler *et al.,* 1999; Barkai *et al.,* 2000; Chow, 2005). This review will give the idea of fluid memory in any porous media. Table 2.7 gives a summarized view on how researchers already used memory concept in fluid flow through porous media. Here, memory models for porous media are presented sequentially. The assumptions are stated for each model with the mathematical representation. The application field and limitations of those models are also showed in the table.

Slattery (1967) used Buckingham-Pi theorem to represent viscoelastic fluid characteristics and observed memory mechanism in normal stress. The author only considered the permeability term in his study. However, that model is failed to show the overall scenario of fluid properties with the help of memory. After that Nibbi (1994) proposed a new model for viscous fluid to show the relation with free energies and used memory mechanism. Researcher presented the free energies for viscoelastic fluid but didn't mention anything about fluid media and memory features.

Mifflin and Schowalter (1986) observed memory formalism for open or an enclosed system with three-dimensional steady state fluid flow. Researchers considered torque free laminar flow and showed the time function as a gradient of velocity, but still that formalism is failed to show the real scenario of the time function, as their model only showed a stress-velocity relationship with time. Later, Eringen (1991) proposed a fluid model that incorporated memory concept for the micro polar property. Researcher showed fluid velocity with a relation to channel gap and observed that all fluids have their own formation and micro-scale properties and used memory function only with fluid velocity and stress. Soon after Zhang (2003) proposed the traffic flow model for micro and macroscopic fluid flow and proposed a viscosity model with second order derivatives to show the scenario of traffic flow. This model presented viscosity which is related to the driver memory. The traffic system of a road is the main base of this model. Chen *et al.* (2005) proposed the memory mechanism for fluid flow through porous media relating to stress and introduced the invasion percolation with memory (IPM) method to address dynamic viscous friction. Authors neglected viscous fluid flow and followed an open path for their solutions which didn't disturb pressure distribution. However, this model failed to show the actual scenario of fluid memory. Considering the dimensionless number, Gatti and Vuk (2006) proposed a linear model considering periodic boundary for viscoelastic fluid in a 2-D flow region also considered the following assumptions: that fluid is incompressible; that the dimensionless number (Reynolds) is unity and; that the flow is isotropic homogeneous. Authors assumed fluid density, pressure and velocity are independent with respect to time to show the effect of fluid memory. However, those assumptions are only considered in conventional fluid models. With new direction, Hossain *et al.* (2007) proposed a mathematical model for fluid flow through porous media to show the stress-strain relationship and added temperature difference, surface tension, pressure difference and rock-fluid memory to make it a comprehensive model. The authors concluded that the actual stress-strain behavior can be presented strongly with the function of time, space, and fluid memory.

Caputo (1999) proposed a model for modified Darcy's law by introducing fractional derivative and presented the local permeability alteration in any porous media. However, this assumed modification is only applicable when local phenomena are considered. Author also derived the pressure distribution of fluid in a half-space with different boundary conditions. After a while, Li *et al.* (2001) studied non-Newtonian fluid properties and pointed out the reciprocal and cluster behavior of stress and time function but failed to show the relation of fluid media and time. Later Caputo and Plastino (2004) derived a modified constitutive relation for porous media to describe the diffusion of fluid more effectively and modified Darcy's law by adding the fractional derivative of pressure with space. Authors presented rigorous derivation techniques of some conventional problems and illustrated in closed form and concluded that the time-memory is suitable for local phenomena and the space memory captures the differences in space. With new observations, Iaffaldano *et al.* (2006) observed permeability reduction during water diffusion in sand layers through experimental investigation. Researchers drew a conclusion that permeability decreased as result of matrix rearrangement and compaction, which drives to a reduction of porosity as well. This phenomenon was studied and presented by Elias and Hajash (1992) in the past also proposed a modified fluid diffusion model incorporating fluid memory mechanism through any porous media. Then Hossain and Islam (2006) represented a review of memory models and their applications considering fluid flow through porous media. They did a brief overview of how various researchers used the fluid memory mechanism for different fluid properties like viscosity, density, compressibility, free energies, diffusion etc. Then, Hossain *et al.* (2008) proposed a modified version of the classical diffusivity equation considering fluid memory for both rock and fluid. They derived this model introducing Caputo fractional derivative in classical Darcy law. They suggested an explicit finite difference method to solve the complex (non-linear) integro-differential equation. Recently, Rahman *et al.* (2016) represented a review of memory models and their applications considering fluid flow through porous media and illustrated a brief overview of how various researchers used the fluid memory mechanism for different fluid properties like viscosity, density, compressibility, free energies, diffusion etc. Authors also discussed the limitations and gave research guidelines for future researchers.

Arenzon *et al.* (2003) proposed a model considering thermal variation and gravity that described of hugely dense particles. Authors observed low and high densely fluid phase and showed irreversible and quasi-reversible cycles with the help of memory formalism. However, authors only discussed fluid density to show the actual scenario of the time function but didn't show anything about fluid memory and media. Later, Sprouse (2010) developed a model with the numerical solution which was based on the short memory approach to solve the fractional diffusional heat equation using an explicit finite difference method. Researcher defined the short memory approach as prior incidents where minimum time can be neglected. With an addition, Carillo *et al.* (2014) developed a model and derived analytical solutions for the integro-differential equations explaining a solid heat conductor with memory and proved that their solutions were unique. Authors described that the addition of memory effects provided an alternative way to address nonlinearities in problems where the linear model approach cannot be applied. The authors considered two different models to understand the role of memory and concluded that the temperature gradient history was related to heat flux in heat diffusion problems.

Shin *et al.* (2003) studied inertia influenced components and found non-equilibrium characteristics in those. Researchers observed that those incidents occurred near the turbulent layer boundary. Their model is only applicable for the homogeneous formation which is not enough to show the actual behavior of fluid properties, media and time alteration. Later, Zia and Brady (2013) observed Brownian particle motion in complex fluid and described the material characteristics of any equilibrium condition. Author observed a theoretical and dynamic simulation of the transient behavior of a colloidal dispersion with respect to time but failed to show actual fluid media and memory in their study.

Hossain and Islam (2009a) studied cumulative oil production and showed time alteration. Authors included stress-strain formulation for both porous rock and fluid to generate a modified material balance equation (MBE) and claimed that the developed MBE can also be used for fractured formations with dynamic options. Authors showed an increase of 5%

oil recovery was calculated from the proposed MBE over the conventional MBE. However, researchers failed to provide solution techniques that can overcome complexity.

Hossain *et al.* (2009b, 2009c) derived a model to present the complex rheological behavior of fluid with memory. This phenomenon combined the bulk rheology and shear rate of fluid in porous media. Authors proposed some dimensionless number for reservoir rock and fluid properties such as porosity, permeability, heat capacities, densities, viscosities etc. Di Guiseppe *et al.* (2010) investigated the changes of fluid and rock properties under changing pressures and observed the changes of pore grains during fluid transport in porous media. Later Rasoulzadeh *et al.* (2014) showed three scales fractured porous media with memory. This model can be used to calculate flow around a production well in any oil reservoir. In recent times, Obembe *et al.* (2017b) presented a modified memory-based mathematical model showing fluid flow in porous media. Authors derived the model using the Grünwald–Letnikov (G–L) definition of the Riemann–Liouville (R–L) time fractional operator along with the generalized Darcy’s equation. Their proposed model is suitable for both fractal geometry and highly heterogeneous media and introduced G-L interpretation for time fractional derivative in the numerical modeling process.

Hristov (2013) established a new technique based on the overall penetration depth and used Jeffrey's kernel theorem to a get a solution for heat conduction equation with the help of fading memory. Author added a damping function to overcome the unreal behavior of the conventional equations and introduced a Volterra-type integral for the heat conduction problem. This integral had the damping function which was proposed by Cattaneo (1958). Though the proposed method follows the classical Fourier's law, heat flux and fractional derivative related to its history, but it failed to present the overall scenario of fluid memory. Recently, Obembe *et al.* (2017c) developed a diffusion model with variable-order derivative (VOD). They presented time dependent diffusion behavior that was observed in heterogeneous fractured porous media. They used finite difference approximation based on control volume to handle VOD and numerically models are validated. However, they overlooked the space memory term in their model.

2.4 Knowledge Gap and Future Research Direction

There is a certain knowledge gap in fluid flow through porous media. Density and viscosity are two fluid properties which play an important part in characterizing the properties of fluid. Both properties have significant roles to maintain the fluid flow through any solid or semi-solid medium (i.e., rock). The time dependent (i.e., thixotropic fluid) and time-independent (i.e., shear-thinning fluid) fluid behavior is very important to identify fluid types under shear conditions. Some non-Newtonian pseudo plastic fluids show thixotropic characteristics with changing time and show a change in viscosity with time. This characteristic shows in very simple fluids as well as in complex liquid mixtures such as foams, micelles, slurries, pastes, gels, polymer solutions, and granular flows (Danko' et *al*., 2006). In most cases, the relationship between density and viscosity is not presented well though they have a strong relation. Hossain *et al.* (2007; 2008) tried to figure viscosity along with shear rate but authors didn't show any relationship with fluid density. Fluid memory is not appropriately used in fluid characterization. Their proposed models didn't explain the solution technique to avoid the complexity. Di Guiseppe *et al.* (2010) showed the variations of fluid and rock properties under pressure difference and observed several changes of pore grains during fluid flow in porous media. For 1-D water reservoir, Caputo and Carcione (2013) developed a model considering fluid memory with modified Fourier law and illustrated time dependent diffusivity for sediments of different grain sizes. Rasoulzadeh *et al.* (2014) proposed a model for three scales fractured porous media with memory which can measure flow around a production well. Rahman *et al.* (2016) reviewed the fluid models and their applications based on fluid memory and showed the limitations of those models. Therefore, it is important to come up with a comprehensive fluid model to show the real picture of fluid media and memory.

Reservoir fluid properties play a vital role in petroleum production. Fluid properties are one of the most integrated and important part of reservoir engineering. However, to date, there are few mathematical models for fluid flow through porous media that can present the actual picture of rock and fluid properties. Hossain et. al (Hossain et *al.,* 2007; 2008) established models to capture rock and fluid alteration with time. However, there are some drawbacks. A comprehensive viscosity and density model can be developed considering time-dependent non-Newtonian fluid, more than one dimension and mostly focusing fluid memory and media. To develop a comprehensive model, compressibility, pH, gravity etc. of the reservoir should be considered. Therefore, there is a scope of future research to come up with a comprehensive fluid (e.g., viscosity-density) model incorporating memory concept.

2.5 Conclusions

This review rebuilds the significance of the applications of memory concept in the context of science and petroleum engineering. It summarizes the state-of-the art review on the subject area. This study establishes the memory mechanism in petroleum reservoir characterization which is very essential for maximizing the operational activity and enhance the productions. The reservoir rock, and fluid properties of porous media could provide better understanding of the memory applications and effects in reservoir characterization. Though most of the fluid properties are supposed to be consistent with time, the researcher needs to capture the real changes of those properties along with time and space. The exact model and comprehensive solution of that model are also necessary for characterizing the reservoir properly. Memory can be an excellent concept for specifying the fluid properties. Application of this concept may lead to develop the comprehensive model by eliminating inherent assumptions and solving all complex nature of solution steps of the model equation. Finally, memory-based fluid flow model will help to generate a more efficient, and robust model for reservoir characterization. Thus, this approach will lead to capture the actual rock-fluid interactions and more consistent pressure, and temperature distribution in the porous media for both conventional and unconventional complex reservoir systems.

2.6 Nomenclature

List of symbols

A_{xz}	cross sectional area of rock perpendicular to the flow of heat [m^2]

a	Corey coefficient of the oil relative permeability curve
ARE	Average relative error
AARE	Average absolute relative error
B_o	Oil formation volume factor [rm^3/sm^3]
C_o	Specific heat capacity of oil [$JKg^{-1}K^{-1}$]
C_w	Specific heat capacity of water [$JKg^{-1}K^{-1}$]
C_r	Specific heat capacity of rock [$JKg^{-1}K^{-1}$]
c_F	Non-dimensional form-drag constant
c_t	Total compressibility in porous medium [1/Pa]
D	Thermal diffusivity [m^2/s]
dt	Time step [s]
dt/dx	Temperature gradient along direction of heat transfer [K/m]
E	activation energy for viscous flow of 30 API gravity oils [KJ/mol]
FD	Fractional derivative
GOR	Gas-oil ratio [SCF/STB]
G	The body force term due to gravity [N]
g	Acceleration due to gravitation force [N]
h	Reservoir thickness [m]
k	Reservoir permeability [m^2]
MB	Memory Based
Ma	Marangoni number
m	Mass [Kg]
m	Temperature/viscosity parameter
OFVF	Oil formation volume factor [rm^3/sm^3]
PVT	Pressure Volume Temperature
P	Pressure of condensate [Pa]
P_r	Reservoir pressure [Pa]
P_s	Pressure of the system [Pa]
P_b	Bubble point pressure [Pa]
q	Oil flow (drainage) rate [m^3/s]
R	universal gas constant [kJ/ mol-K]
r	Radial distance of reservoir in equation (1) [m]

S_o	Oil saturation [fraction]
S_g	Gas saturation [fraction]
S_w	Water saturation [fraction]
T	Reservoir temperature [K]
T^*	Temperature gradient [K/m]
T_D^*	Dimensionless temperature distribution [dimensionless]
t	Time [s]
t_D	Dimensionless time [dimensionless]
t_{cD}	Dimensionless critical time [dimensionless]
U_x	Velocity of the advancing front of steam chamber (m/s)
$\vec{u}$	Velocity vector [m/s]
VP	Pore Volume [m^3]
VS	Solid Volume [m^3]
V_{oD}	Volume of displaced oil produce [fraction]
V_{pD}	Initial pore void filled with steam as water [fraction]
v_s	Linear velocity of steam front [m/s]
W_k	A weighting function for the numerical integration
1-D	One dimensional
2-D	Two dimensional

Greek Letters

∇P	Pressure gradient [Pa/m]
ΔS_o	Change in oil saturation before/after steam front passage [fraction]
Δx	Size of grid block in x direction
$\nabla \emptyset$	Fluid potendial gradient [N]
ξ	A dummy variable for time i.e., real part in the plane of the integral [s]
ξ'	Distance measured ahead of the front into the coder zone [m]
$d\xi$	Dummy time step [s]

ϕ	Porosity of fluid media [fraction]
σ	Surface tension [mN/m]
μ	Fluid dynamic viscosity at any temperature [*Pa-s*]
μ_o	Oil (dynamic) viscosity [*Pa-s*]
μ_w	Water (dynamic) viscosity [*Pa-s*]
μ_{od}	Dead oil viscosity [*Pa-s*]
μ_s	Saturated oil viscosity [*Pa-s*]
μ_o	Under saturated oil viscosity [*Pa-s*]
ρ	Density [kg/m^3]
ρ_c	Condensate density [kg/m^3]
ρ_f	Fluid density [kg/m^3]
ρ_o	Oil density [kg/m^3]
ρ_r	Dry rock density [kg/m^3]
ρ_w	Dry rock density [kg/m^3]
τ	Shear Stress [Pa]
τ_T	Shear stress at temperature T, ^{0}K
γ^*	Shear rate [m/s/m]
η	Ratio of the pseudo-permeability of the medium with memory to fluid viscosity [$m^3s^{1+\alpha}/kg$]
α	Fractional order of differentiation (related to the time and space), dimensionless
α_1, α_2	Derived variable for dimensionless thickness
α_c	Simplified condensate (water) or convective diffusivity
$\alpha_{c\prime}$	Volumetric conversion factor
β_c	transmissibility conversion factor
γ	Fractional order derivative
Γ	Euler gamma function
ξ	Normal distance to the advancing front of the steam chamber [m]
θ	Inclination of the draining surface from the horizontal plane [angle]

Subscripts

b	Bubble point
e	Effective
f	Fluid
g	Gas
o	Oil
r	Rock (matrix)
T	Temperature
w	Water

2.7 References

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2.8 Appendix: A

Table 2.1: Types and features of Non-Newtonian fluid

Shear rate and Shear-stress Relation	Type of Fluid	Example
Rate of shear can be determined from shear stress at the same point and time.	Time independent/ Purely viscous/ Inelastic/ Generalized Newtonian fluids	Paint, Wet sand, Toothpaste etc.
Time of shearing and kinematic history is considered in relation between shear stress and shear rate.	Time-dependent fluids	Drilling muds, Hair gels, Printer inks etc.
To show partial elastic recovery after deformation, need to know the behavior of ideal fluids and elastic solids.	Viscoelastic fluids	Hand wash, Body cream etc.

Table 2.2: Considered parameters for different viscosity correlations

	Author	Samples origin	T (^0F)	API (^0API)	GOR (scf/STB)	P_{sw} (psia)	P (psia)	P_b (psia)	μ_{od} (cp)	μ_s (cp)	μ_o (cp)
	Beal (1946)	U.S.A	98-250	10-52					0.86-1550		

Dead Oil Viscosity	Beggs and Robinson (1975)	-	70-295	16-58	**N/A**	**N/A**	**N/A**	**N/A**	-	N/A	N/A
	Glaso (1980)	North Sea	50-300	20-48					0.60-39		
	Kaye (1985)	Offshore California	143-282	7-41					-		
	Al-Khafaji *et al.* (1987)	-	60-300	15-51					-		
	Petrosky (1990)	Gulf of Mexico	114-288	25-46					0.72-10.25		
	Egbogah and Ng (1990)	-	59-176	5-58					-		
	Labedi (1992)	Libya	100-306	32-48					0.66-4.79		
	Kartoatmodji and Schmidt (1994)	World wide	80-320	14-59					0.50-586		
	Bennison (1998)	North Sea	39-300	11-20					6.40-8396		
	Elsharkawy and Alikhan (1999)	Middle East	100-300	20-48					0.60-33.7		
	Hossain *et al.* (2005)	World wide	32-215	7-22					12-451		

	Naseri *et al.* (2005)	Iran	105-298	17-44					0.75-54		
	Alomair (2011)	Kuwait	68-320	10-20					1.78-11360		
	Sarapardeh *et al.* (2013)	Iran	50-290	17-44					0.39-70		
	Chew and Conally (1959)	U.S.A			51-3544	132-5645				0.370-50	
	Beggs and Robinson (1975)	-			20-2070	132-5265				-	
	Al-Khafaji *et al.* (1987)	-			0-2100	-				-	
	Khan *et al.* (1987)	Saudi Arabia			24-1901	107-4315				0.130-77.4	
	Petrosky (1990)	Gulf of Mexico			21-1855	1574-9552				0.210-7.4	
	Labedi (1992)	Libya			13-3533	60-6358				0.115-3.72	
	Kartoatmodji and Schmidt (1994)	World wide			2.3-572	15-6054				0.100-6.3	
Saturated Oil Viscosity	Elsharkawy and Alikhan (1999)	Middle East	**N/A**	**N/A**	10-3600	100-33700	**N/A**	**N/A**	**N/A**	0.050-21	**N/A**

	Hossain *et al.* (2005)	World wide			19-493	121-6272				3600-360	
	Naseri *et al.* (2005)	Iran			255-4116	420-5900				0.110-18.15	
	Bergman and Sutton (2007)	World wide			6-6525	66-10300				0.210-4277	
	Sarapardeh *et al.* (2013)	Iran			126-3261	365-5702				0.580-37.18	
Under-Saturated Oil Viscosity	Beal (1946)	U.S.A	**N/A**	**N/A**	**N/A**	**N/A**	-	-	**N/A**	**N/A**	0.16-315
	Vazquez and Beggs (1980)	World wide					126-9500	-			0.117-148
	Khan *et al.* (1987)	Saudi Arabia					-	107-4794			0.13-71
	Petrosky (1990)	Gulf of Mexico					1600-10250	1574-9552			0.22-4.1
	Labedi (1992)	Libya					-	60-6358			-
	Orbey (1993)	-					740-14504	-			0.225-7.3
	Kartoatmodji and Schmidt (1994)	World wide					25-6015	25-4775			0.168-517

	Elsharkawy and Alikhan (1999)	Middle East					1287-10000	-			0.2-5.7
	Hossain *et al.* (2005)	World wide					300-3400	121-6272			3-517
	Sarapardeh *et al.* (2013)	Iran					730-5116	0.177-18.15			0.177-31

Table 2.3: PVT correlations data for GOR, P_b, B_o considering ARE, AARE and STD

Authors	Samples Origin	Data sets	GOR ARE/AARE	GOR STD	P_b ARE/AARE	P_b STD	B_o ARE/AARE	B_o STD
Standing (1947)	California (North America)	105	N/A	N/A	4.8% AARE	N/A	1.17% AARE	N/A
Lasater (1958)	Canada, US (North America) and South America	137			3.8% ARE	N/A	N/A	N/A
Glaso (1980)	North Sea	45			1.28% ARE	6.98%	-0.43 % ARE	2.18%
Al-Marhoun (1988)	Middle East	69			3.66% AARE	4.536%	0.88% AARE	1.18%
Labedi (1990)	Libya, Nigeria, Angola (Africa)	128			1.24% ARE	17.07%	N/A	N/A

Al-Marhoun (1992)	Worldwide (Middle East and North America)	700			N/A	N/A	0.57% AARE	0.68%
Casey and Cronquist (1992)	US Gulf Coast (North America)	N/A			N/A	N/A	N/A	N/A
Petrosky and Farshad (1993)	Gulf of Mexico (North America)	81	3.8% AARE	2.88%	3.28% AARE	2.56%	0.64% AARE	0.58%
Kartoatmodjoand Schmidt (1994)	Indonesia, Middle East, North America and South America	740	23.2% AARE	83 scf/ STB	20.2% AARE	171.3 psia	2.02% AARE	0.043 bbl/ STB
Farshad *et al.* (1996)	Colombia (South America)	98	-7.9% ARE	22.7%	-3.49% ARE	14.61%	0.0003% ARE	0.034%
Almehaideb (1997)	UAE (Middle East)	62	N/A	N/A	5% AARE	6.56%	1.35% AARE	1.7%
Elsharkawy and Alikhan (1997)	Kuwait (Middle East)	175	7.87% AARE	10.73%	N/A	N/A	1.43% AARE	1.96%
Velarde *et al.* (1999)	Lab samples (Worldwide)	2097	4.73% AARE	18.2 scf/ STB	11.7% AARE	263 psia	1.74%	0.0014 bbl/ STB
Al-Shammasi (1999)	Worldwide	1243	N/A	N/A	17.9% AARE	17.16%	1.81% AARE	2.27%
Valko and McCain (2003)	Lab sample (Worldwide)	1745	5.2% AARE	N/A	10.9% AARE	N/A	N/A	N/A

Table 2.4: PVT correlations data for P_b considering GOR, ^{0}API, γ_g and T

Authors	Samples Origin	Data sets	Sample Data Ranges				
			R_s (scf/STB)	API (^{0}API)	γ_g (air=1)	T (^{0}F)	P_b (psia)
Standing (1947)	California (North America)	105	20-1245	16.5-63.8	0.59-0.95	100-258	130-7000
Laster (1958)	Canada, US (North America) and South America	137	3-2905	17.9-51.1	0.574-1.223	82-272	48-5780
Vasquez and Beggs (1980)	Worldwide (Lab samples)	6000	0-2199	15.3-59.3	0.65-1.28	75-294	15-6055
Glaso (1980)	North Sea	45	90-2637	90-2637	0.65-1.276	80-280	165-7142
Al-Marhoun (1988)	Middle East	69	26-1602	19.4-44.6	0.75-1.367	74-240	130-3573
Asgarpour *et al.* (1989)	North Sea	310	84-1680	N/A	N/A	N/A	435-4060
Rollins *et al.* (1990)	Worldwide	541	4-220	14-53	0.579-1.124	60-150	890-4540
Kartoatmodjo and Schmidt (1991)	Worldwide	5393	0-2897	14.4-59	0.482-1.166	75-320	24.7-4746
Dokla and Osman (1992)	UAE (Middle East)	51	181-2266	28.2-40.3	0.798-1.29	190-275	590-4640
Macary and El Batanoney (1992)	Gulf of Suez (Middle East)	90	200-1200	25-40	0.7-1.0	130-290	1200-4600
Petrosky and Farshad (1993)	Gulf of Mexico, Texas and	81	217-1406	16.3-45	0.58-0.85	114-288	1574-6528

	Louisiana (North America)						
Omar and Todd (1993)	Malaysia (Asia)	93	142-1440	26.6-53.2	0.612-1.32	125-280	790-3851
De Ghetto *et al.* (1994)	Africa, Persian Gulf and North Sea	3700	8.61-3298	6-56.8	0.624-1.798	59-194	107-6613
Elsharkawy *et al.* (1995)	Kuwait (Middle East)	44	34-1400	20-45	0.663-1.064	130-250	317-4375
Farshad *et al.* (1996)	Colombia (South America)	98	6-1645	18-44.9	0.66-1.73	95-260	32-4138
Almehaideb (1997)	UAE (Middle East)	62	128-3871	30.9-48.6	0.75-1.12	190-306	501-4822
Hanafy *et al.* (1997)	Egypt (Middle East)	324	7-4272	17.8-48.4	0.623-1.627	107-327	36-5003
Velarde *et al.* (1999)	Lab samples (Worldwide)	2097	10-1870	12-55	0.556-1.367	74-327	70-6700
Al-Shammasi (1999)	Worldwide	1243	6-3298	6-63.7	0.51-1.44	74-341	31.7-7127
Dindoruk and Christman (2001)	Gulf of Mexico (North America)	104	133-3050	14.7-40	0.601-1.027	117-276	926-6204
Mehran *et al.* (2006)	Iran (Middle East)	387	83-3539	18.8-48.92	0.532-1.415	77.5-306	348-5355
El Banbi *et al.* (2006)	Middle East	13	1991-8280	34.1-58.5	0.642-1.332	186-312	4527-11475

Hemmati and Kharrat (2007)	Iran (Middle East)	287	125-2189	18.8-48.34	0.523-1.415	77.5-290	348-5156
Ikiensikimama and Ogboja (2009)	Niger Delta (Africa)	250	19-2948	14.78-53.23	0.564-1.294	122-264	67-6560
Moradi *et al.* (2010)	Wroldwide	1811	8.861-3267	6-56.8	0.52-1.278	67-342	36-7127
Mansour *et al.* (2013)	Egypt (Middle East)	43	45.2-1662.1	17.5-47.4	0.6777-1.2720	120-290	3500-3500

Table 2.5: PVT correlations data for B_o considering GOR, ^{0}API, and γ_g

Authors	**Samples Origin**	**Data sets**	**Sample Data Ranges**				
			R_s (scf/STB)	**API (^{0}API)**	**γ_g (air=1)**	**T (^{0}F)**	**B_o (bbl/STB)**
Katz (1942)	US (North America)	117	9.3-1313	21.8-63.7	0.575-	58-212	N/A
Standing (1947)	California (North America)	105	20-1245	16.5-63.8	0.59-0.95	100-258	1.024-2.150
Knopp and Ramsey (1960)	Venezuela (South America)	159	200-3500	15-47.5	0.605-1.67	142-307	N/A
Vasquez and Beggs (1980)	Worldwide (Lab samples)	6000	0-2199	15.3-59.3	0.65-1.28	75-294	1.028-2.226
Glaso (1980)	North Sea	45	90-2637	90-2637	0.65-1.276	80-280	1.032-2.588

Al-Marhoun (1988)	Middle East	69	26-1602	19.4-44.6	0.75-1.367	74-240	1.032-1.997
Abdul Majed and Salman (1988)	UAE (Middle East)	420	0-1664	9.5-59.5	0.51-1.35	75-290	1.028-2.042
Kartoatmodjo and Schmidt (1991)	Worldwide	5393	0-2897	14.4-59	0.482-1.166	75-320	1.007-2.144
Dokla and Osman (1992)	UAE (Middle East)	51	181-2266	28.2-40.3	0.798-1.29	190-275	1.216-2.493
Al-Marhoun (1992)	Worldwide	11728	0-3265	9.5-55.9	0.575-2.52	75-300	1.010-2.960
Macary and El Batanoney (1992)	Gulf of Suez (Middle East)	90	200-1200	25-40	0.7-1.0	130-290	1.20-2.0
Omar and Todd (1993)	Malaysia (Asia)	93	142-1440	26.6-53.2	0.612-1.32	125-280	1.085-1.954
Petrosky and Farshad (1993)	Gulf of Mexico, Texas and Louisiana (North America)	81	217-1406	16.3-45	0.58-0.85	114-288	1.118-1.623
Farshad *et al.* (1996)	Colombia (South America)	98	6-1645	18-44.9	0.66-1.73	95-260	1.06-2.064
Almehaideb (1997)	UAE (Middle East)	62	128-3871	30.9-48.6	0.75-1.12	190-306	1.142-3.562
Al-Shammasi (1999)	Worldwide	1243	6-3298	6-63.7	0.51-1.44	74-341	1.02-2.196
Dindoruk and Christman (2001)	Gulf of Mexico (North America)	104	133-3050	14.7-40	0.601-1.027	117-276	1.084-2.898

El Banbi *et al.* (2006)	Middle East	13	1991-8280	34.1-58.5	0.642-1.332	186-312	N/A
Hemmati and Kharrat (2007)	Iran (Middle East)	287	125-2189	18.8-48.34	0.523-1.415	77.5-290	1.091-2.54
El Mabrouk *et al.* (2010)	Libya (Africa)	476	10-1256	25.3-45.4	0.73-1.451	100-277	1.064-1.795

Table 2.6: Use of Memory in various fields of science and engineering

Authors	Considered Parameters	Goal of the Research	Applied Field
Cisotti (1911)	Density, Dielectric constant, Memory	Electro mechanism and energy study with memory	Energy and its application
Graffi (1936)	Maxwell's equation, Faraday's law, Ampere law, History dependence	Electromagnetic study considering history	Electromagnetic analysis
Atkinson and Shiffrin (1968)	Memory, Sensory store, Short term store, Long term store	To overview human memory work	Human memory
Caputo and Mainardi (1971)	Dissipation, Solid material, Memory	Geophysical material study with memory mechanism	Geophysics and Solid
Bruce (1983)	Geological trend, Climate change, Element migration, Geotectonic	Geological and geochemical study regarding past events	Geological predictions

Jacquelin (1984)	Electro-mechanism, fractional derivatives, Electric network, energy storage	Electro-mechanism energy study with fractional derivatives	Electric network and Energy storage
Yamshchikov *et al.* (1994)	Rock memory, Space-time dynamics, Stress-strain state	Rock properties and rock geology with memory effect	Rock properties
Xuefu *et al.* (1995)	Memory, Long term history, Rock memory, Stress	Immediate stress memory of rock with the help of previous step	Rock mechanics
Caputo and Plastino (1998)	Darcy's law, Fractional derivative, Diffusion, Pressure, Memory	Diffusion of fluid in rock with time derivative	Fluid diffusion
Caputo (1999)	Darcy's law, Memory, Fractional order derivative, Permeability	Geothermal study and decrease of permeability with time	Geothermal fluids
Caputo (2000)	Pressure gradient, Porous media, Fluid flux, Memory	Study periodic change of fluid flux and pressure gradient with memory	Fluid transport
Caputo and Kolari (2001)	Fisher equation, Stock price, Inflation rate, Fractional calculus, Memory, Financial economy	Economic change for financial studies with variation of time	Economic analysis
Caputo (2003)	Space memory, Fluid memory, Local memory, Pressure,	Study the difference between space and fluid memory with fractional derivatives	Fluid behavior

	Boundary, Fractional derivative		
Zhang (2003)	Traffic model, Memory, Space memory	Fluid flow depends on both forward and previous time step	Fluid flow
Caputo and Plastino (2004)	Porous media, Diffusion, Pressure, Boundary layer, Skin effect	Study fluid diffusion and effect of pressure towards boundary wit change of time	Fluid diffusion
Christensen *et al.* (2004)	Computation, Thermal simulation, Dynamic grid, Memory	Thermal analysis and simulation with memory effect	Thermal analysis
Cesarone *et al.* (2005)	Fick's law, Memory, Two membrane, Fractional order	Fluid diffusion process study along two membrane with memory	Fluid diffusion in biology
Iaffaldano *et al.* (2006)	Darcy's law, Fick's equation, Traditional diffusion, Fractional calculus, Memory	Study fluid flow through porous media with a fluid memory and medium memory	Fluid diffusion
Hossain *et al.* (2007)	Stress-strain, Fluid memory, Space memory, Chaotic behavior	Stress-strain relationship with the help of fluid and space memory and its nonlinearity	Reservoir rheology
Hossain *et al.* (2008)	Porous media, Continuity equation, Momentum balance equation, Crude oil	Study formation and fluid properties with space memory and time variation	Fluid flow

Hossain *et al.* (2009)	Complex rheology, Memory, Shear rate	Study complex rheological behavior with time	Rheology analysis
Zavala-Sanchez *et al.* (2009)	Memory, transport coefficient, Medium, Dispersion coefficient	Transport phenomena with memory mechanism	Transport analysis
Caputo and Cametti (2009)	Fick's equation, Memory, Fractional derivative, Space memory	Fluid diffusion considering fluid and space memory using fractional derivative	Fluid diffusion
Di Guiseppe *et al.* (2010)	Memory, Diffusivity equation, Porous media, Chemical change	Study rock and fluid properties with changing time and space in porous media	Rock and fluid properties
Baleanu *et al.* (2010)	Newton's law, fractional derivatives, Memory	Study Newton's law of motion with memory to make it applicable of engineering and science	Motion analysis
Bernacchia *et al.* (2011)	Neuron, Memory	Study neural system with time and memory	Neuron behavior
Hossain and Islam (2011)	Scaling technique, Darcy's law, Memory	Scaling method for oil-water displacement using fluid memory	Reservoir scaling
Wang and Li (2011)	Kernel function, Slipping interval, Fractional derivative, Memory	Mathematical study to understand memory mechanism	Mathematical analysis

Hossain and Abu-khamsin (2011)	Dimensionless number, Heat transfer, Rock and fluid properties, Memory, Rheological behavior	Reservoir characterization with fluid memory	Reservoir characterization
Hossain and Abu-khamsin (2012)	Energy balance equation, Dimensionless number, Heat transport, Porous media, Memory	Study reservoir rock and fluid properties with memory mechanism	Reservoir characterization
Du *et al.* (2013)	Fractional derivative, Memory, Stage of memory	Study the change of a property after change of gradient of that property within a time interval	Mechanics
Caputo and Carcione (2013)	One-dimensional, Water reservoir, Fourier law, Memory, Fractional order	Study water reservoir and sediment diffusivity with memory mechanism	Water reservoir
Rasoulzadeh *et al.* (2014)	Fractured formation, Porous media, Memory	Fluid flow in any fractured formation with memory formulation	Flow analysis
Mlodinow and Brun (2014)	Psychological, Thermal relation, Memory	Study thermal changes with respect to time and psychological memory arrow	Thermodynamics
Hristov (2014)	Memory, Fading memory, Volterra integrals, Riemann-Liouville derivative	Fluid diffusion for short period with memory mechanism	Fluid Diffusion

Kolomietz (2014)	Fermi liquid, Collective motion, Memory	Study kinetic energy of nuclear fermi fluid with memory mechanism	Nuclear energy
Hamza *et al.* (2015)	Maxwell's equation, Fractional calculus, Memory	Study electro mechanism of thermo-elastic materials with memory	Electromagnetic analysis
Hagemann and Stacke (2015)	Soil moisture, memory, Regional climate, Max Planck model	Study soil moisture and climate changes with memory function	Soil hydrology analysis
Obembe *et al.* (2017a)	Porous media, Grünwald–Letnikov definition, Riemann–Liouville operator, Darcy's law, Memory	Fluid flow model study with fractional operator; continuous time function	Fluid flow
Obembe *et al.* (2017c)	Variable order derivative, Porous media Finite difference approximation, Memory	Study fluid diffusion with variable order derivative consider memory mechanism	Fluid diffusion

Table 2.7: Memory mechanism in porous media

Authors	Assumptions	Memory models	Application field	Limitations

Slattery (1967)	❖ Incompressible fluid flow with steady state. ❖ Porous media (Isotropic). ❖ Inertia effects are neglected.	$\overline{K} = f(L, \overline{v}, t, \mu_o, P, \overline{\tau}, \alpha)$	Viscoelastic fluid Characteristics at thermal condition.	➢ Only considered permeability ➢ Other parameters didn't consider. ➢ Considered regional thermodynamic state
Mifflin and Schowalter (1986)	❖ Incompressible fluid flow with steady state. ❖ Fluid particles are homogeneous. ❖ Particles shape are spherical.	$\tau = \int \frac{2\eta_o}{\lambda_1}\left[\left(1-\frac{\lambda_2}{\lambda_1}\right)e^{-\frac{(t-t')}{\lambda_1}} + \lambda_2\delta(t)\right]$ $\times\Gamma(t,t')dt'$	3-D state fluid flow	➢ Turbulent flow was not considered. ➢ Heterogeneous condition didn't mention. ➢ Torque was ignored.
Ciarletta and Scarpetta (1989)	❖ Viscous fluid. ❖ Incompressible and homogeneous fluid flow. ❖ Physical space is smooth and	$\overline{T}(x,t) = -p(x,t)I + \int_R^\infty 2\,\mu(\tau)\overline{D}(x,t-\tau)d\tau, (x,t) \in \Omega_T \equiv \Omega\times(0.T)$	Incompressible fluid flow	➢ Non-linear term was neglected. ➢ Heterogeneous condition didn't mention.

	domain is bounded. ❖ Solid material is used.			
Eringen (1991)	❖ No heat conduction for fluid. ❖ Molecules are homogeneous ❖ Molecules shape are spherical.	$t_{kl} = -\pi\delta_{kl} + T_{kl},\ T_{kl}$ $= 2\int_{-\infty}^{t} d\tau \int_{\vartheta-\sigma}^{1} dv' \sum_{klmn}^{1} (s = t)\frac{\delta C'_{mn}}{\delta\tau}, \frac{\delta C_{mn}}{\delta\tau}$ $= 2d_{ij}(\tau)\frac{\delta x_i(\tau)\delta x_j(\tau)}{\delta x_m(t)\delta x_m(t)}\ 2\sum(s = t)$ $= (\lambda_0 + \lambda_1 j)\delta_{kl}\delta_{mn} + (\mu_0 + \mu_1)\times(\delta_{kl}\delta_{lm}$ $+ \delta_{kn}\delta_{lm}$	Fluid properties in micro scale	➢ Nonlocal effect is neglected. ➢ Heterogeneity was ignored.
Nibbi (1994)	❖ Viscous fluid. ❖ Incompressible and homogeneous flow ❖ Linear and isotropic flow	$\bar{T}(t) = -p(t)I + \int_{R}^{+\infty} 2\,\mu(s)\overline{D^t}(s)ds,$	Free space energy in viscous fluid	➢ The function of relaxation must be satisfied; ➢ $\mu \in L^1(0, +\infty)$, ➢ $\mu \in L^1(0, +\infty) \cap L^2(0, +\infty)$ ➢ Heterogeneous flow didn't mention.

Caputo (1999)	❖ Viscous fluid. ❖ Incompressible and homogeneous fluid flow. ❖ Linear and isotropic porous medium. ❖ Permeability declines with time in geothermal areas.	$q = -\frac{\eta \rho_o \left(\frac{\delta^\alpha}{\delta t^\alpha}\right)\left(\frac{\delta p}{\delta y}\right) \delta^\alpha p(y,t)}{\delta t^\alpha} = [\frac{1}{\Gamma(1-\alpha)}] \int_0^t (t - u)^{-\alpha} [\delta p(y), where\ 0 \leq \alpha \leq 1$	Fluid flow in porous media	➢ Heterogeneity was ignored. ➢ The porosity was neglected. ➢ Medium complexity was neglected.
Li *et al.* (2001)	❖ Bubble shape is spherical. ❖ Bubble is homogeneous. ❖ Stresses and composition are homogeneous.	$\frac{d\tau_m}{dt} = -\alpha\tau_m + \beta\gamma_B$	Non-Newtonian fluid properties and behavior	➢ Heterogeneity was not considered. ➢ Formation varieties was ignored. ➢ Fluid media was not defined.

	❖ Frequency is constant for the formation.			
Shin *et al.* (2003)	❖ Porous medium is homogeneous. ❖ Particle motion with drag force. ❖ Mean shear rates variation is unrelated. ❖ Gaussian fluctuating velocities of particles ❖ Turbulent particle is independent of mean shearing ❖ The shear rate of the flow is independent.	$v^{+}{}_{t} = v^{+}{}_{t,eq} + \Delta v^{+}{}_{t}$ $v^{+}{}_{t,eq} = -\frac{24}{R_{ep}}\frac{1}{C_D}\tau_p\frac{d}{dy}\left(\zeta_{yy} - D_{yy}\frac{d\bar{v}_y}{dy}\right)\frac{1}{u^*}$ $v^{+}{}_{t} = -\frac{24}{R_{ep}}\frac{1}{C_D}\tau_p\frac{d}{dy}\left(\tau_\beta\frac{d\zeta_{yy}}{dy}\right)\frac{1}{u^*}$	Non-equilibrium characteristics for inertia influenced components of fluid.	➢ Formation Heterogeneity was ignored. ➢ 2) Shear rate was not mentioned as function.

Zhang (2003)	❖ Media is isotropic. ❖ Considered Taylor expansion. ❖ Basis is traffic road model ❖ G_* (Monotonic, Generic function) is assumed as linear function.	$\mu(\rho) = 2\beta\tau_\varepsilon c^2(\rho) = 2\beta\tau(\rho V'(\rho)_*)^2$ $v_t + \{v + c(\rho)\}v_x = \mu(\rho)v_{xx}$	Fluid flow through traffic model system	➢ Didn't mentioned the shortcomings of Taylor function. ➢ 2) Function linearization was not specified.
Caputo and Plastino (2004)	❖ Uncoupled the equations of diffusion from those of elasticity. ❖ Isotropic porous media. ❖ All empirical parameters can	$\left(a + b^{\frac{\delta y2}{\delta ty2}}\right)p = \left(\alpha + \beta^{\frac{\delta y2}{\delta ty2}}\right)\{m(x,t) - m_o\};$ $\left(\gamma + \varepsilon^{\frac{\delta y1}{\delta ty1}}\right)q = \left(c - d^{\frac{\delta y2}{\delta ty2}}\right)\nabla p;$	Fluid diffusion in porous media	➢ The elastic reaction of the matrix was neglected. ➢ Neglected inertia effects. ➢ 3) Only considered Green function.

	be obtained from experiments.			
Iaffaldano *et al.* (2006)	❖ Incompressible fluid. ❖ Homogeneous fluid flow. ❖ Linear and isotropic porous medium.	$\gamma q(x,t) = -\left[c + d\frac{\delta^{\gamma}}{\delta t^{\gamma}}\right] \nabla p(x,t);$ $ap(x,t) = \alpha\rho(x,t);$ $\nabla . q(x,t) + \frac{\delta\rho(x,t)}{\delta t} = 0;$	Permeability reduction during fluid (water) diffusion	➢ Only considered permeability. ➢ Porosity was neglected. ➢ Heterogeneity was ignored.
Hossain *et al.* (2007)	❖ Heterogeneous and isentropic formation. ❖ Fluid memory considered for viscosity, density, diffusivity and compressibility.	$\tau_T = \frac{k^2 \Delta p A_{xz} \Gamma(1-\alpha)}{\mu_o \eta \rho_o \varphi y c \int_0^t (t-\zeta)^{-\alpha} \left(\frac{\delta^2 p}{\delta \zeta^2}\right) d\zeta} \times \left[\left(\frac{\delta\sigma}{\delta T}\frac{\Delta T}{\alpha_D M_a}\right) \times e^{\left(\frac{E}{RT}\right)}\right] \frac{du_x}{dy}$ Let, $I = \int_0^t (t-\zeta)^{-\alpha} \left(\frac{\delta^2 p}{\delta \zeta^2}\right) d\zeta$ $\tau_T = \frac{k^2 \Delta p A_{xz} \Gamma(1-\alpha)}{\mu_o \eta \rho_o \varphi y c I} \times \left[\left(\frac{\delta\sigma}{\delta T}\frac{\Delta T}{\alpha_D M_a}\right) \times e^{\left(\frac{E}{RT}\right)}\right] \frac{du_x}{dy}$	Stress-strain relationship for fluid flow	➢ Non-Newtonian fluid was not considered. ➢ Relative permeability and fluid interface was ignored.

	❖ Media properties are also considered. ❖ Incorporate temperature and pressure effect.			
Hossain *et al.* (2009)	❖ Heterogeneous and isentropic formation ❖ Depend on space, time, pressure and dummy variable ❖ Presented for one dimension ❖ Considered polymer fluid; Newtonian.	$\gamma_{pm} = \frac{\alpha_{SF}}{\sqrt{k\varphi}} \frac{\eta}{(1-)} \int_0^t (t-\zeta)^{-\alpha} \frac{\delta^2 p}{\delta\zeta\delta x} d\zeta$ $\mu_{eff} = \mu_\infty + \frac{\mu_0 - \mu_\infty}{\left[1 + \left(\frac{\alpha_{SF}}{\sqrt{k\varphi}} \frac{\lambda\eta}{(1-)} \int_0^t (t-\zeta)^{-\alpha} \frac{\delta^2 p}{\delta\zeta\delta x} d\zeta\right)^a\right]^{\frac{n}{a}}}$	Modified fluid diffusivity in porous media	➢ 2-D or 3-D was not considered. ➢ Didn't take non-Newtonian fluid. ➢ Only considered polymer,

Sprouse	❖ Viscous fluid. ❖ Incompressible and homogeneous fluid flow. ❖ Linear and isotropic porous medium. ❖ Fractional derivative defined based on GL definition.	$\frac{dg}{dt} = \alpha D^{1-\gamma}\nabla^2 g - \beta g;$ Where $D^{-\gamma}f(x) = \frac{1}{\Gamma(\gamma)}\int_0^x \frac{f(t)}{(x-t)^{1-\gamma}}dt.$	Describe heat flow equations	➢ Heterogeneity was ignored. ➢ Didn't considered compressible condition. ➢ Complex and isentropic porous media was not mentioned.
Zia and Brady (2013)	❖ Heterogeneous and isentropic formation. ❖ Assumed steady state. ❖ Considered the microstructural perturbation.	$\frac{\eta^{micro}(t;Pe)}{\eta} = -\frac{3}{4\pi}Pe^{-1}\left(1 + \frac{a}{b}\right)^2 \varphi_b u. \int ng(r,t;Pe))d\Omega$	Brownian motion in complex fluid formation	➢ Nothing mentioned for unsteady flow. ➢ Only considered short period. ➢ Group of particles were ignored.

Obembe *et al.* (2017b)	❖ Single phase flow. ❖ Slightly compressible fluid in reservoir rock. ❖ Consider Darcy's generalized equation.	$\frac{\delta}{\delta x}\left[g_x(p)D_t^{1-\gamma}\left(\frac{\delta p}{\delta x}\right)\right]\Delta x + q_{sc} = V_b\frac{\delta}{\delta t}\left(\frac{\phi}{B_o}\right)$ Where, $g_x(p) = \frac{\eta A_x}{B_o}$	Modified fluid flow model in porous media	➢ Nothing mentioned for multi-phase flow. ➢ 2) Ignored incompressible flow.
Obembe *et al.* (2017c)	❖ Water saturated porous medium. ❖ Consider Darcy's generalized equation ❖ Neglected subsequent step for calibration.	$\frac{\delta}{\delta x}\left[\rho_w\eta D_t^{1-\gamma}\left(\frac{\delta p}{\delta x}\right)\right] = (\rho_w\phi c_t)\frac{\delta p}{\delta t} - (\rho_w\phi\beta_t)\frac{\delta T}{\delta t}$ Where, $c_t = c_\phi + c_w$ and $\beta_t = \beta_\phi + \beta_w$	Fluid diffusion model with VOD	➢ Only water was considered as fluid medium. ➢ 2) Didn't mentioned 2-D and 3-D flow.

Chapter 3

A Mathematical Model of Stress-Strain Behavior for Reservoir Fluids to Capture the Memory Effect

Preface

This paper to be submitted to a Journal. The lead author performed the necessary literature review on fluid properties and fluid memory. The co-authors Tareq Uz Zaman helped in mathematical techniques, showed some coding techniques, Dr. Salim Ahmed reviewed the manuscript and Dr. M. Enamul Hossain helped in identifying the gap in research, supervising the research, and editing the manuscript.

3.1 Abstract:

Reservoir rock and fluid properties vary because of pressure depletion or thermal variation in any reservoir structure. Rock and fluid properties such as porosity, permeability, compressibility, fluid saturation, density, viscosity etc., play a significant role in continuous alteration of complex reservoir structure. Fluid memory is a continuous function of formation space and time which is used in various sections of porous media to forecast future events based on past events. The pressure gradient of any reservoir can be illustrated as a mathematical function of formation space and time with the help of fluid memory. Though fluid memory is one of the important fluid features in porous media usually it is neglected in the fluid models. This article introduces a new approach for stress-strain behaviour where viscous stresses and density are incorporated. The developed mathematical model shows the effect of density, porosity, pseudo permeability with viscosity consideration, pressure difference, temperature and the importance of fluid memory on the reservoir fluid stress-strain relationship. Space and time are considered to show the effect of fluid memory. The computation represents that memory effect causes complex and nonlinear characteristics for the stress-strain relationship. The objective of this study to develop a comprehensive memory based stress-strain model incorporating viscosity, density and validate the model numerically for light crude oil. The results show that fluid density has impact on shear stress. Results also show that memory mechanism effects the shear stress behaviour of a formation fluid considering porosity, density, flow regime, time and rate of strain and, for $\alpha = 0.3$ both field and experimental condition shows a good match with proposed model. This mathematical model can be used for problems related to reservoir simulation, well test analysis, reservoir rheology, and enhanced oil recovery.

3.2 Introduction:

In the petroleum industry, fluid viscosity plays an important role. Enhanced oil recovery is inversely proportional to fluid viscosity as the oil flow increased by reducing the oil viscosity. The structure of fluid will modify continuously as it cannot resist the shear force and deform irrespective to the amount of shear forces. Fluid viscosity helps to show the relationship between shear stress and strain rate. Usually, the viscous force of a fluid help to shape and steady the flow, while inertia force of fluid influences interrupts the steady flow and show the turbulent behavior. This behavior can be identified by Blake number (modified Reynold's number) for porous media.

A fluid that shows a simple relationship with viscous force to the strain rate is defined as the Newtonian fluid. Viscosity is used as a coefficient of proportionality for Newton's law of viscosity which strongly relates with medium temperature and pressure. However, the shear force has no control over fluid viscosity. On the other hand, a fluid which is independent to viscosity is known as non-Newtonian fluid. For a non-Newtonian fluid, the shear force plays a part to change viscosity. Very few researchers describe this phenomenon throughout literature. Usually, irregularity arises in porous media because of complex behavior of non-Newtonian fluids. Mostly, Newton's fluid viscosity models ignore the effects of porosity, density, fluid memory and considered Newton's viscosity equations for the predictions (Chen *et al.,* 2005; Gatti and Vuk, 2006, Hossain *et al.,* 2007a, 2008a, 2008b and 2009a).

Pore structure effects oil flow in porous media. Pressure and temperature are the most important parameters of a reservoir (Hossain *et al.,* 2007b, 2008a). The strain rate for reservoir fluid is affected by the fluid velocity, pressure difference, and reservoir temperature. Reservoir temperature and crude oil compositions are also responsible to diverge from Newtonian to non-Newtonian behavior. Hydrocarbons and nonmetallic substances such as sulfur, oxygen, nitrogen, etc., are the main components of crude oil. It is mainly subdivided into two groups based on API gravity i.e., light crude oil, and heavy crude oil. Crude oil has several subgroups such as paraffin, aromatics, resins, asphaltenes, naphthenes, etc., which may change due to change of temperature, and pressure in the reservoir. In Figure 3.1a, a typical fluid flux distribution in porous media is presented considering direction, time, and temperature.

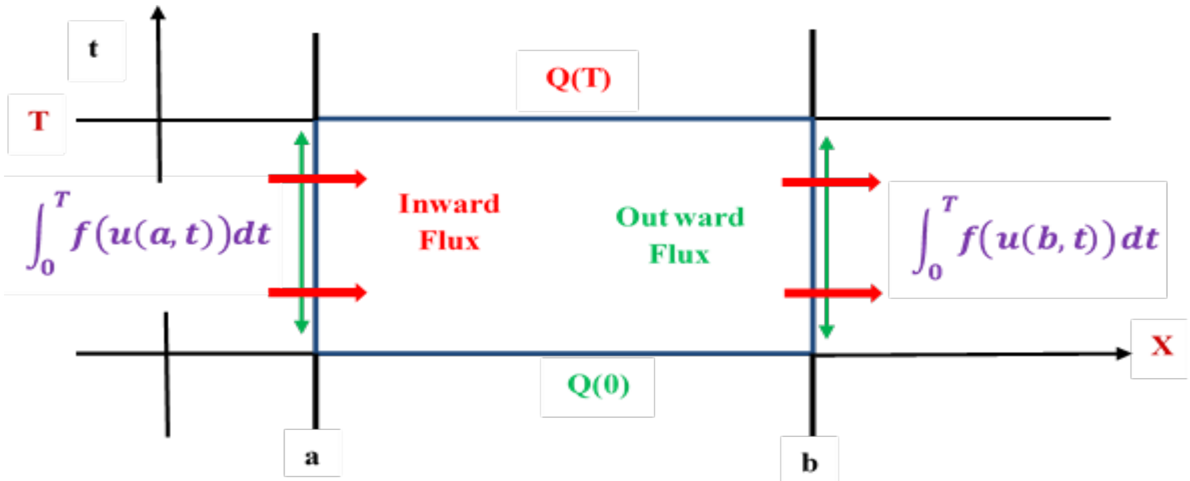

Figure 3.1a: Typical Fluid Flux in Porous Media (modified from De Sterck and Ullrich, 2009)

Fluid flow in porous media is effected by formation rock and fluid properties. Usually, those properties are presented as a function of formation temperature and pressure. Formation rock works as a fluid flow medium in porous media and because of the pressure difference or thermal variation, fluid properties change (Hossain *et al.,* 2007a, 2008a). The literature illustrates that current mathematical models are based on temperature and pressure correlated with rock and fluid properties of porous media (Arenzon *et al.,* 2003; Shin *et al.,* 2003, Chen *et al.,* 2005; Hossain *et al.,* 2007a, 2008a; Caputo and Carcione, 2013; Obembe *et al.,* 2017).

Density is an important fluid property that controls the fluid flow in porous media. Reservoir fluid compressibility, relative permeability, viscosity, and strain rate are substantially very related to fluid density and cannot be ignored during reservoir fluid characterization. In the petroleum industry, most of the cases of fluid density is calculated from PVT data, and such data is mostly based on laboratory experiments. A few researchers have used empirical correlation to measure fluid density for a reservoir and those correlations were used for a specific geographical region (Standing, 1981; Ahmed, 1988; Hanafy et al., 1997; El-hoshoudy et al., 2013). Fluid density has a significant role in the stress-strain relationship but until now it has been ignored by most researchers. There is no certain model in available literature where fluid density, viscosity, shear stress, and the rate of strain is related. This paper will give a comprehensive idea to develop a mathematical relationship for stress-strain using porosity, permeability, fluid density, viscosity, and time.

To represent the exact scenario, the stress-strain relationship should account for the effect of time, space and fluid memory. The classical reservoir simulation based on the formation permeability along with the flow of the fluid but it does not maintain the flow path. To overcome these shortcomings, the fluid memory should play an important role by monitoring flow in both conditions. The fluid memory has a unique feature as the working technique of fluid memory varies with different fluid media conditions. Fluid is a function of all possible fluid properties with space and time. Some fluids (i.e., incompressible, viscous fluids) sometimes show different behaviors which can be described more easily with fluid memory. This phenomenon is very limited in the literature.

Fluid memory is used to represent the actual picture of reservoir rock and fluid properties with time alteration in porous media. Very few researchers incorporate fluid memory in their models to show the exact scenario of time alteration in porous formation. Fluid flow in porous media is different from pipeline flow because of its complex structure. In the literature, researchers have shown the rock-fluid properties of porous media with the help of memory (Slattery, 1967; Eringen, 1991; Nibbi, 1994; Caputo, 1999; Li *et al.,* 2001; Zhang, 2003; Chen *et al.,* 2005; Iaffaldano *et al.,* 2006; Hossain *et al.,* 2007a, 2008a, 2008b and 2009a; Di Guiseppe *et al.,* 2010; Raghavan and Chen, 2013; Rasoulzadeh *et al.,* 2014; Rahman *et al.,* 2016; Obembe *et al.,* 2017).

Jossi et al. (1962) developed a viscosity correlation that considers fluid temperature, pressure, and chemical properties. Authors validated the correlation only for nonpolar fluid and the fluid density was reduced from 3.0 to 0.1. Porter and Johnson (1962) compared and evaluated fluid viscosity by two of the mostly used shear techniques: jet viscometer, and concentric cylinder viscometer. Authors found that concentric cylinder viscosity loss is less than the jet viscometer process and concluded that it might be due to the capillary effect, and kinetic energy correlations. Churchill and Churchill (1975) developed a new correlation for effective viscosity of pseudo-plastic and dilatant fluids considering shear stress. This correlation can also be illustrated for dynamic viscosity as a function of frequency oscillation. Vetter (1979) used Weertman's temperature method to calculate stresses and viscosity in the asthenosphere of up to 400 km in depth by relating viscosity and ratio of temperature at the melting point. The author also considered the two creep laws and the creep rate was 1*103 of the stress value. Memory is not well used in current theories but some authors have developed flow theories (non-local) for classical Darcy's law using general principles of statistical physics (Hu and Cushman, 1994). Suoqi (1997) developed a new approach to calculating fluid viscosity considering the fluid density, a balance of entropy production, and the entropy flow in an open system. The author has shown the proportional constant as a simple function of temperature and the model is compared with the Hildebrand fluidity model, and the modified Enskog theory (MET). Wernera et al. (1998) developed a mathematical model to calculate the petroleum fluid viscosity as a function of reservoir temperature and pressure. Authors considered the Kanti et al. (1989) model for the reservoir temperature, and the pressure and Grunberg and Nissan (1949) approach for fluid composition. This model is useful for big compositional rage such as heavy range of asphaltenes. Starov and Zhdanov (2001) used Brinkman's equation to correlate viscosity of the fluid with resistance coefficient also showed fluid viscosity relationship with porosity for porous media. Boundary layer flow and heat transfer of a viscoelastic fluid in porous media for non-isothermal has been observed by Abel et al. (2002). Authors reviewed the effect of permeability, fluid viscosity, and the viscoelastic parameter for different conditions. Brenner (2005) studied Newton's law of viscosity, and the Navier–Stokes equation and added a stress tensor parameter in the Navier–Stokes equation for compressible fluids. The author used the volume velocity (volume flux density) parameter in Newton's law of viscosity to show the relationship between stress tensors and fluid density. Luo and Gu (2007) studied the viscosity for heavy crude oil and showed how viscosity is effected at a different temperature in presence of asphaltene. Authors used theoretical and experimental approaches to measure heavy oil viscosity at various temperatures. Islam and Carlson (2012) studied viscosity models for the geologic sequestration of CO2 at certain temperature and pressure. Authors considered water, brine, and typical sea water and showed the effect of CO_2 more acutely. Pal (2015) developed a low-shear fluid viscosity model for concentrated suspensions and considered core-shell particles thickness, and permeability. The author evaluated the model based on three different types (porous particles, solid core–hairy shell particles, and hard spheres) of particles. MacDonald and Miadonye (2017) reviewed viscosity correlations and developed a new simplistic, semi-empirical equation different than current empirical models for the viscosity of Tangleflags and Athabasca bitumen. These researchers illustrated that the proposed equation gave a low percentage of errors for viscosity measurement considering the temperature and the pressure.

Slattery (1967) used the Buckingham-Pi theorem to present viscoelastic fluid characteristics and observed memory mechanism in normal stress. The author only considered the permeability term in his study. Ciarletta et al. (1989) developed a viscosity equation using the memory mechanism for an incompressible fluid. Authors used fading memory to recall the past motions. Nibbi (1994) developed a model for fluid viscosity considering the memory mechanism as well. The Author also considered the quasi-static condition and showed a relationship between free energy, and fluid viscosity. Caputo (1999) proposed a mathematical model to modify Darcy's law by introducing fractional derivative and presented the local permeability alteration in any porous media. However, this assumed modification is only applicable when local phenomena are considered. Li et al. (2001) did a stress analysis of air bubble for non-Newtonian fluids. Authors found two reasons for stress formations: (i) by the space of bubbles, and (ii) bubble response because of the fluid's memory. Chen et al. (2005) proposed a model relating stress and the invasion percolation with the memory (IPM) method for porous media. Authors also showed a relation between stress and dynamic viscous friction. Hossain et al. (2007) developed a memory-based mathematical model to show the relationship between shear stress and the rate of strain in porous media. To represent the model as a comprehensive one authors also addressed the temperature difference, surface tension, pressure difference and fluid memory. Hossain et al. (2009b) derived a memory-based mathematical model to present the complex rheological behavior of fluid and proposed some dimensionless numbers for rock and fluid properties such as porosity, permeability, heat capacities, densities, viscosities. Di Guiseppe et al. (2010) reviewed the changes of fluid and rock properties under changing pressures and observed the changes of pore grains during fluid transport in porous media. Hristov (2014) proposed the diffusion model with the integral balance method and described memory term by weakly singular power-law. Recently, Rahman et al. (2016) made a critical review of memory-based models for porous media and discussed assumptions and limitations of those models. Authors gave an overall guideline to develop a comprehensive memory-based fluid model for porous media.

Formation porosity, permeability, hydraulic diameter, fluid density, viscosity, reservoir temperature, and pressure need to be considered to develop a comprehensive memory-based stress-strain model. Fluid density is considered by Blake Number (B) which is known as the modified Reynold's number for porous media. The fluid memory is observed with a principal variable, the order of differentiation (α), and the pseudo-permeability as a ratio of permeability to fluid viscosity (μ). The effect of the fluid memory can be determined with the variation of (α). The results are illustrated in a graphical form to represent the effect of fluid density, viscosity, and fluid memory on the stress-strain relationship. The results show a nonlinear trend with time and this nonlinearity arises pressure, the fluid memory shows a nonlinear behavior with time, and this nonlinearity arises because the pressure is depended on fluid velocity.

3.3 Mathematical Model Development:

In the x-direction of a porous media, if a tangential force (F) is applying on the top surface of a fluid element (shown in Figure 3.1b), then the fluid element will deform. The deformation of the fluid element is because of shear forces that apply tangentially to a surface. Usually, Newton's law of viscosity is used to represent the time-dependent fluid. It can be written as:

$$\tau = -\mu \frac{du_x}{dy} \tag{1}$$

If the formation temperature (T) is considered in the x-direction and the rate of shear along the x axis showed as $\frac{du_x}{dy} = \gamma$, then Newton's law of viscosity in Eq. (1) can be written as below (Hossain *et al.,* 2007a):

$$\tau = -\mu_T \, \gamma \tag{2}$$

In any fluid, there is some molecular transportation between the close by layers. This phenomenon is very acute in the gaseous medium than the liquid medium. However, interchange of liquid molecules between nearby layers is less than gases because of a cohesive force which keeps the molecules in certain place much more strongly. Cohesion shows a significant role in the liquid viscosity. If the temperature of liquid increases, cohesive bonding decreases, and the molecular transportation increases. Shear stress decreases with decreasing cohesive forces and increases with increasing molecular interchange. In consequence of this complex behavior of shear stress, several researchers have shown the importance of temperature on fluid viscosity and have developed different models based on experimental and field studies (Recondo *et al.,* 2006; Hossain *et al.,* 2007a, 2008a). In this model, the Arrhenius model is used to represent the relationship between temperature and fluid viscosity (Avramov, 2005; Haminiuk *et al.,* 2006; Gan *et al.,* 2006, Hossain *et al.,* 2007a, 2008a). Fluid viscosity changes as a result of high pressure. If the pressure increases, the liquid molecules need more energy for their relative movement. With increasing viscosity, the stress-strain relationship is affected. Hossain et al. (2007a, 2008a) addresses this complex phenomenon though previous researchers have ignored this effect.

$$\mu_T = \mu_o e^{\left(\frac{E}{RT}\right)} \tag{3}$$

In Eq. (2) μ_o is the dynamic viscosity at reference temperature. Using the value of μ_T in Eq. (2) form Eq. (3) can be shown as:

$$\tau = -\mu_o \, e^{\left(\frac{E}{RT}\right)} \gamma \tag{4}$$

In porous media, two types of pores or void spaces are available (effective pores, and isolated pores). Isolated void or pore space cannot affect the flow of the porous medium (Dullien, 1992). Blake (1922) first introduced a dimensionless number for porous media to show the effect of inertia and viscous force which was also known as modified Reynold's number. In the Blake number, fluid density, fluid velocity, hydraulic diameter for porous media, fluid viscosity and the void fraction is used to present both effects. Pressure drops in a porous medium is caused by continuous viscous and inertial losses. Several researchers have theoretically shown that the inertia (kinetic energy) loss depends on fractional void volume (Burke and Plummer, 1928; Ergun and Orning, 1949; Ergun, 1952). Ergun (1952) observed the modified Reynold's number (Blake number) and provided the laminar and turbulent flow ranges for packed bed porous media. The system should be isolated to obtain more accurate values. In this study, the initial temperature has been considered as 298°K. In general, the formation temperature is considered as constant throughout the reservoir for a particular reservoir section at a certain depth (Yanowitch, 1967, Hossain et al., 2011, Hossain and Abu-Khamsin, 2012). Therefore, the viscosity in the Blake number is assumed constant (i.e., no change of viscosity due to temperature effect). As a result, the ratio of inertia and viscous force can be expressed by the Blake number as follows:

$$B = \frac{u_x \, \rho \, D_h}{\mu_o \, (1-\varepsilon)} \tag{5}$$

Porosity (ϕ) or void fraction (ε) defined as the ratio of pore volume to bulk volume i.e., mathematically it can be written as $\phi = \varepsilon = V_p/V_b$. Thus Eq. (5) can be written as:

$$B = \frac{u_x \, \rho \, D_h}{\mu_o \, (1-\phi)} \tag{6}$$

$$\mu_o = \frac{u \, \rho \, D_h}{B \, (1-\phi)} \tag{7}$$

Using the value of μ_o in Eq. (4) from Eq. (8) can be shown as:

$$\tau = -\frac{u_x \, \rho \, D_h}{B \, (1-\phi)} \, e^{\left(\frac{E}{RT}\right)} \, \gamma \tag{8}$$

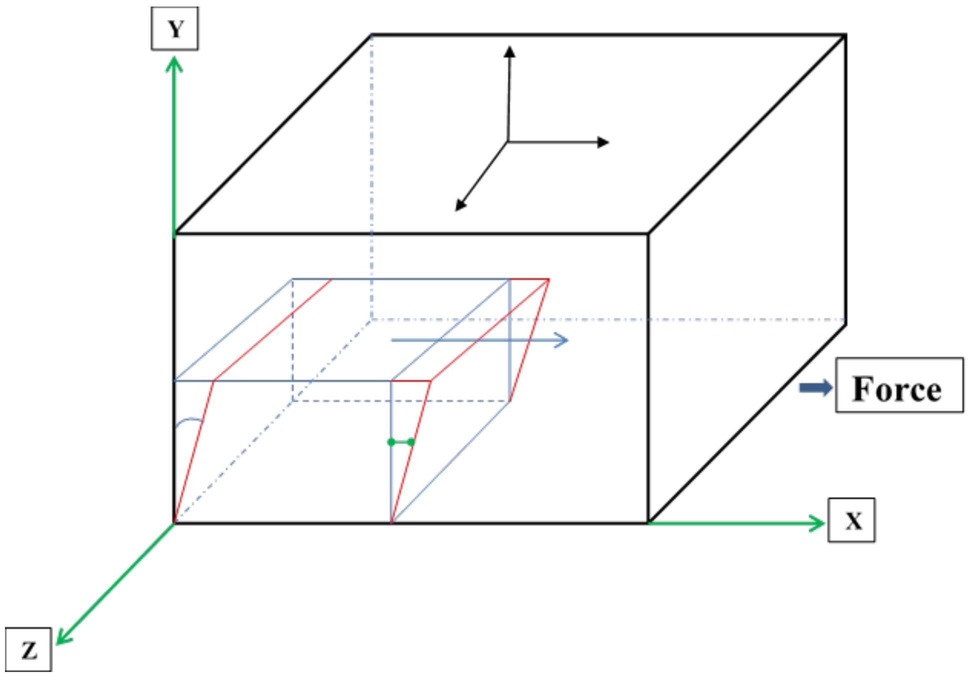

Figure 3.1b: 3-D tangential forces of a fluid element (modified from Chhabra and Richardson, 2008

To address the fluid memory with stress relation, the fluid (mass) flux in porous media can be showed by the following equation (Caputo, 1999). As the flow is considered in x-direction, the equation can be written as:

$$q_x = -\eta\, \rho_o \left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\right] \tag{9}$$

In Eq. (9) $q_x = \frac{q'_x\, \rho_o}{A}$ and $0 \leq \alpha < 1$ so Eq. (9) can be written as:

$$u_x = -\eta \left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\right] \tag{10}$$

Here, u_x is the fluid velocity in the reservoir. Putting the value of u_x into Eq. (8) becomes as:

$$\tau = -\frac{\rho\, D_h}{B\,(1-\phi)} \left\{-\eta \left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\right]\right\} e^{\left(\frac{E}{RT}\right)}\, \gamma \tag{11}$$

Where, η is the pseudo permeability of the porous media. η can be showed with the below equation (Hossain 2008a; Hossain *et al.,* 2008b) as follow:

$$\eta = \frac{K}{\mu}(t)^{\alpha} \tag{12}$$

In Eq. (12), K = permeability, μ = fluid viscosity, t = time and α= order of differentiation. Putting the value of η in Eq. (12) from Eq. (13) it becomes as:

$$\tau = \underbrace{\frac{\rho\, D_h}{B\,(1-\phi)}}_{\textbf{1st Part}} \underbrace{\left[\frac{K}{\mu}(t)^{\alpha}\left\{\frac{\partial^{\alpha}}{\partial t^{\alpha}}\left(\frac{\partial P}{\partial x}\right)\right\}\right]}_{\textbf{2nd Part}} \underbrace{e^{\left(\frac{E}{RT}\right)}}_{\textbf{3rd Part}} \underbrace{\gamma}_{\textbf{4th Part}} \tag{13}$$

The above model represents the effects of shear stress on the reservoir formation and fluid properties in 1-D (x-direction) and it can be expressed by a more general condition of 3-D fluid flow in an anisotropic heterogeneous formation. Though inelasticity of matrix, formation heterogeneity, and anisotropy sometimes fails to show several phenomena, fluid memory a capture all the phenomenology. The 1st part of the Eq. (13) are the effects of formation porosity, fluid density, and hydraulic diameter of porous media, the 2nd part is the effect of pseudo-permeability (i.e., permeability, viscosity ratio with time) and pressure gradient along the axis and both together represent the effect of fluid memory, the 3rd part is the effect of isothermal temperature condition, and the 4th part is the effects of strain rate according to shear stress also known as velocity gradient in y-direction.

3.4 Model Analysis:

The results of the comprehensive stress-rate of strain model can be obtained by solving Eq. (13) which is shown above. In this paper, we focused on the stress-strain relation, fluid density, viscosity, and fluid memory. We consider a sample reservoir from the production wellbore (Hossain, 2008a) and experimental data (Iaffaldano *et al.,* 2006) for numerical calculation. The reservoir is isolated and oil is producing at a constant rate. The fluid is assumed to be an API 32.8 gravity crude oil at 298^{o}K temperature. All computations are carried out by MATLAB programming codes.

Table 1: Sample Reservoir Data (Hossain 2008a)

Reservoir length, l	5000 m
Reservoir width, w	100 m
Reservoir height, h	50 m
Porosity, ϕ	30%
Permeability, k	50 md = 50* 10^{-15} m^2
Initial reservoir pressure, p_i	27579028 pa (4000 psia)
Compressibility, c	1.2473×10^{-9} 1/pa
Initial viscosity, μ_o	87.4×10^{-3} Pa-s
Initial flow rate, q_i	8.4×10^{-9} m^3/sec
Initial fluid velocity, u_i	1.217×10^{-5} m/sec
Fractional order of differentiation, α	0.2-0.8
Number of grid in space, N_t	580

Table 2: Experimental Data (Iaffaldano *et al.,* 2006)

Length of cylinder, l	11.6 *cm*
Inner diameter, D_i	10.1 *cm*
Volume of the cylinder, $V_c = \pi r^2 h$	929.374 cm^3
Permeability, k	26 *Darcy*
Viscosity, μ	1.0266 *cp*
Sand density, ρ_s	2.4 gcm^{-3}
Mass of sand in cell, M_s	1550 *gm*
Volume of sand, V_s	645.83 cm^3
Porosity, ϕ	0.3050913841
Fluid density, ρ_f	0.998408 gcm^{-3}

Compressibility, c_t	2.05743* 10^{-4} atm^{-1}
dp/dx	0.01765982953 *atm/cm*
Δp	0.2048540225 *atm*
Number of grid in space, N_t	580

In Eq. (13), D_h is used in the first part as hydraulic diameter. We consider the reservoir shape is rectangular and fully filled with fluid. If the reservoir length, l = a and the width, $w = b$, then D_h becomes:

$$D_h = \frac{4\ a\ b}{2\ (a+b)} \tag{14}$$

$$D_h = \frac{2\ a\ b}{(a+b)} \tag{15}$$

In Eq. (13) 2nd part is considered to show the effect of fluid memory. Here, $\left\{\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\right\}$ is the change of pressure with space and time. In addition, $\frac{K}{\mu}(t)^\alpha$ is used to show the effect of pseudo-permeability with time continue alteration. α value is showing the variation of the order of differentiation. In this paper, we calculate this $\left[\frac{K}{\mu}(t)^\alpha\left\{\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\right\}\right]$ term as fluid flux ($m^3/m^2\ s$). Authors used Iaffaldano *et al.* (2006) experimentally validated data which is simulated, and validated numerically by Zaman (2017). Different authors showed the variation of α value to capture the memory effect (Iaffaldano, 2006, Hossain 2008a, Histrov 2014, Obembe et al., 2017). Recently, Zaman (2017) shows that the best choice for α value is 0.3 which can give a good agreement for fluid memory. The results of this study are compared with the well-established Hossain *et al.,* (2007a, 2009a) stress-strain model.

To solve the fluid memory term (partial differential equation) in Eq. (13), it is necessary to consider space and time for pressure calculation. The finite difference method is used to solve the fluid memory term $\left\{\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\right\}$ of Eq. (13). The discretized form of fluid memory term from Eq. (13) as follows:

$$\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\bigg|_i^n$$

$$= \frac{1}{\Gamma(2-\alpha)}\,\frac{1}{(\Delta t)^\alpha}\sum_{J=1}^{n}\left[J^{1-\alpha} - (J-1)^{1-\alpha}\right]\left[\left(\frac{\partial P}{\partial x}\right)_i^{n-J+1} - \left(\frac{\partial P}{\partial x}\right)_i^{n-J}\right] \tag{16}$$

$$= \frac{1}{\Gamma(2-\alpha)\,(\Delta t)^\alpha}\sum_{J=1}^{n}\left[J^{1-\alpha} - (J-1)^{1-\alpha}\right]\left[\frac{P_{i+1}^{n-J+1} - P_i^{n-J+1}}{\Delta x} - \frac{P_{i+1}^{n-J} - P_i^{n-J}}{\Delta x}\right] \tag{17}$$

$$= \frac{1}{\Delta x\,\Gamma(2-\alpha)\,(\Delta t)^\alpha}\sum_{J=1}^{n}\left[J^{1-\alpha} - (J-1)^{1-\alpha}\right]\left[P_{i+1}^{n-J+1} - P_i^{n-J+1} - P_{i+1}^{n-J} + P_i^{n-J}\right] \tag{18}$$

$$= \frac{1}{\Delta x\,\Gamma(2-\alpha)\,(\Delta t)^\alpha}\sum_{J=1}^{n}\left[J^{1-\alpha} - (J-1)^{1-\alpha}\right]\left[P_{i+1}^{n-J+1} - P_{i+1}^{n-J} - P_i^{n-J+1} + P_i^{n-J}\right] \tag{19}$$

By employing Eq. (19), we calculated the flux value of the fluid in porous media. For reservoir, we used 580 grids in space and for the experimental condition, we used 580 grids in space. To calculate fluid flux, all values of previous time steps were considered. In Figure 3.2, the alignment of grid point in space is shown.

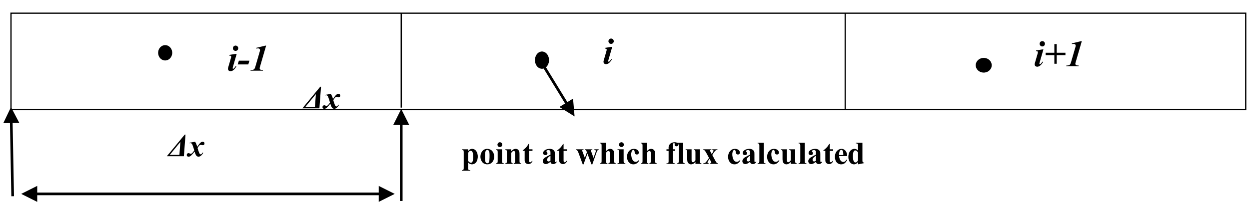

Figure 3.2: Alignment of grid point in space

3.5 Results and Discussion:

3.5.1 Dependency of fluid density on shear stress

Fluid density has an important impact on viscosity, shear stress development and overall fluid flow. Though fluid density plays an important role in fluid flow, still, it is ignored in most of the research. Very few literatures are available where relationship with fluid density, shear stress and viscosity has shown for non-Newtonian fluids (i.e, semi-solids, liquids, polymer, body fluids) (Klijin *et al.,* 1979; Cherry and Kwon, 1990; Bowles and Frimpong, 1991; Druschel and O'Rourke, 1991; Hammond and Hammond, 2001; Tuna and Altun, 2012; Dalton and Daivis, 2013; Chai and Saito, 2016; Zhao *et al.,* 2016). In the petroleum industry, density is an important fluid property which play a significant role in fluid flow through porous media in various conditions. In this article, the relationship between fluid density and shear stress is presented. Shear stress increases with the increasing fluid density and consequently viscosity decreases. Therefore, with the increasing fluid density shear development increase which result as a decrease of fluid flow. Figure 3.3a to 3.3d shows the variation of fluid shear stress with the change of density of crude oil for different values of α in semi-log plot. And the trend is showing that with the increase of α value (0.2-0.8) shear stress increases with also represents the effect of fluid memory. For different α values shear stress development is different and showing almost same trend and shape. In Figure 3.3a to 3.3d, though stress development is different but values are very close to each other and with increasing α value stress development decreases.

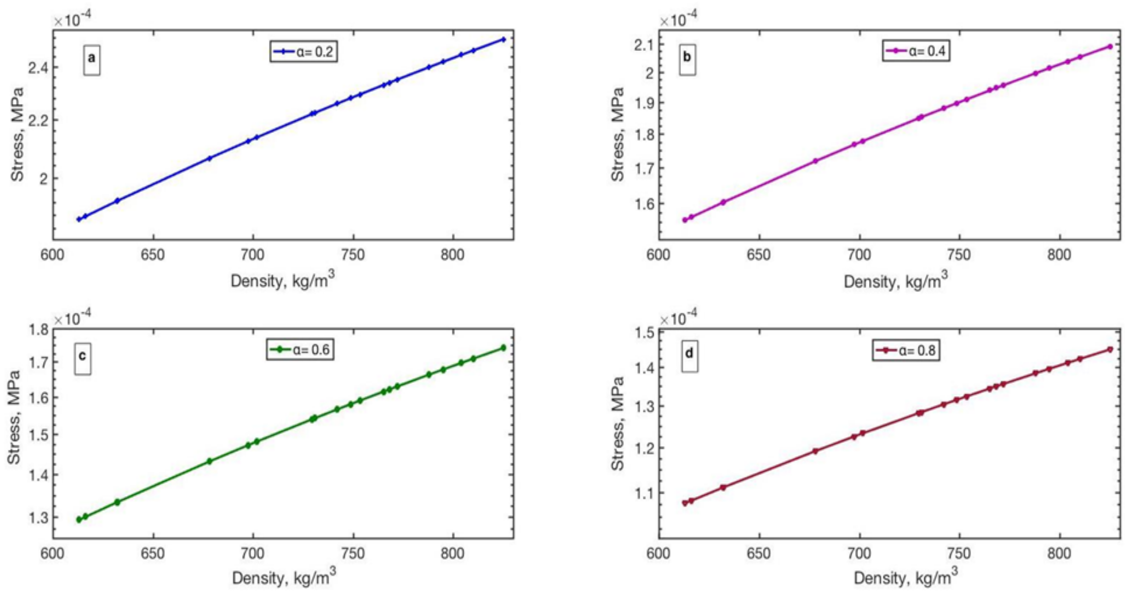

Figure 3.3: Fluid shear stress variation as a function of density for different α values

In this study, field data (Hossain, 2008a) and experimental data (Iaffaldano *et al.,* 2006) are used to validate the proposed model equation. Figure 3.4 shows the relationship between shear stress and fluid density in field condition. Shear stress is calculated with field data and model equation. The results are showing good agreement and almost match with each other. For calculation, we considered constant flow regime, reservoir depth, and value of α= 0.3 to observe the stress development for different density. However, Figure

3.4 shows the same trend as Figure 3.3. In Figure 3.5, stress is measured with experimental data and calculated with the proposed model equation. The proposed model equation shows a good agreement with experimental results. In Figure 3.5, for the same range of fluid density stress development is almost similar for both cases. The graph shows the same trend as Figure 3.3 and 3.4 where stress development increases with increasing fluid density.

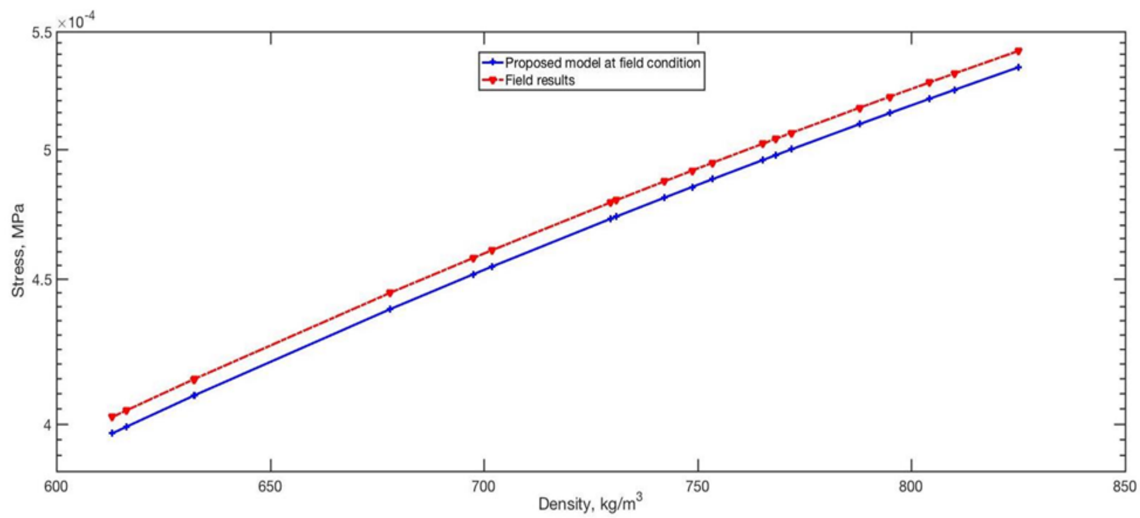

Figure 3.4: At field condition fluid shear stress as a function of density

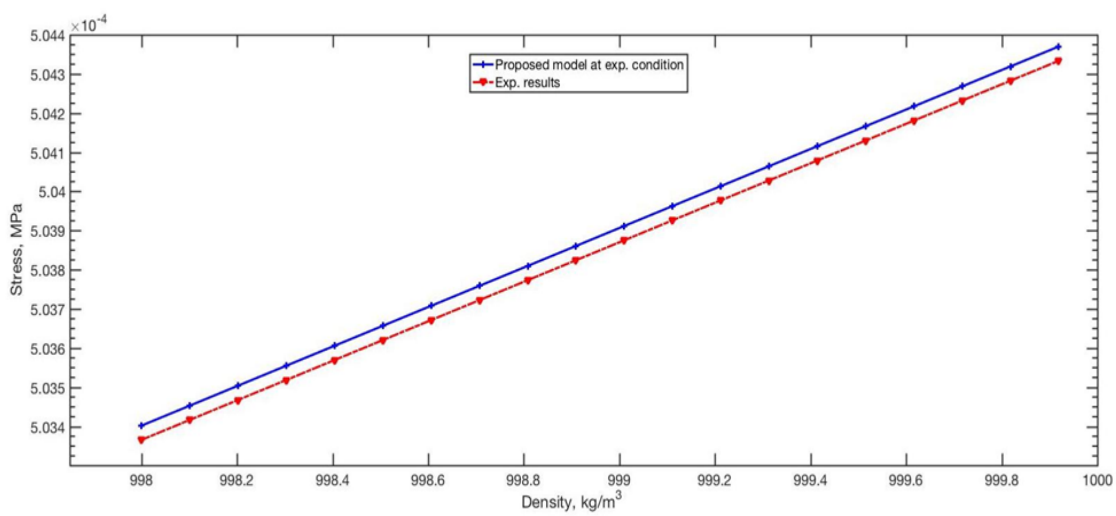

Figure 3.5: At exp. condition fluid shear stress as a function of density

In this article, α= 0.3 is considered to show the effect of memory term in the model equation (Zaman, 2017). Figure 3.6 shows the same trend as Figure 3.3 where stress development increases with the change of fluid density. For different α values stress development is different and with time the change becomes very less. In Figure 3.6, stress development over the months (20-100 months) is not very much, still it maintains the same trend as Figure 3.3. For 20 months stress development is minimum and for 100 months its maximum when α= 0.3 with a very small change.

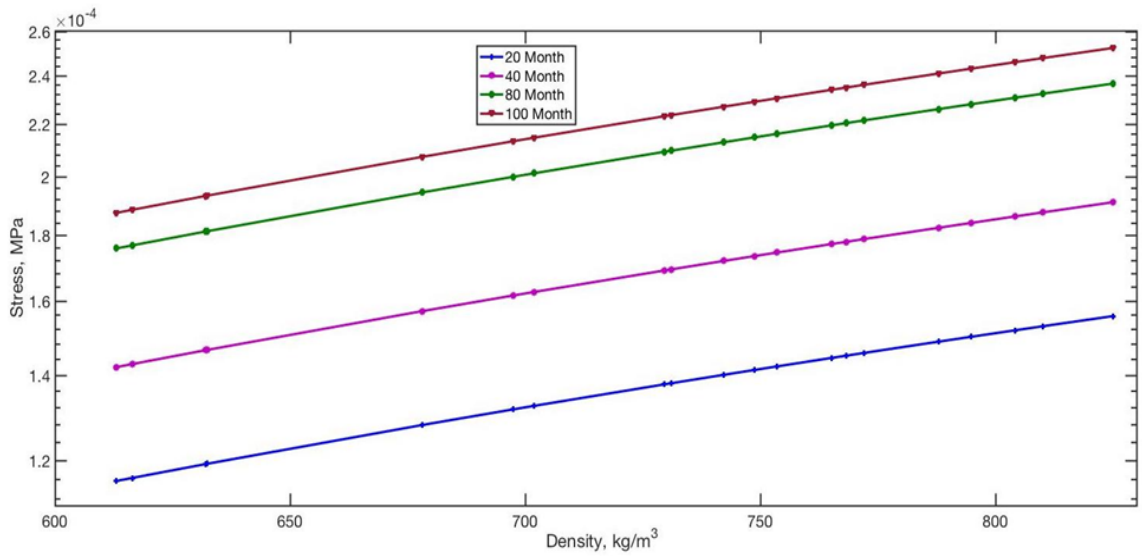

Figure 3.6: Fluid shear stress variation as a function of density for different months at α= 0.3

3.5.2 Dependency of the Blake number on fluid shear stress

Inertia and viscous force has a great effect on fluid flow through porous media. It controls the type of flow in porous media. For pipeline flow, researchers considered Reynolds number and in this study Blake number is used to show the effect of inertia and viscous force in porous media. In this paper, Blake number is calculated for certain depth at constant initial viscosity to show the effect of fluid density. Form the model equation, shear stress is inversely proportional to Blake number with all other parameter is constant. Figure 3.7a to 3.7d shows that stress with decreasing with the increasing Blake number in semi-log plot which reflects as the flow regime moves toward turbulent region stress decreases.

For different α values (0.2-0.8) stress development is not same though the change not too much and for α= 0.8 stress development is maximum with decreasing trend with increasing Blake number. For α= 0.2 and α= 0.4 the shape is not exactly straight line and displays a curvature shape. This scenario reflects the effect of fluid memory and non-linear behavior of the model.

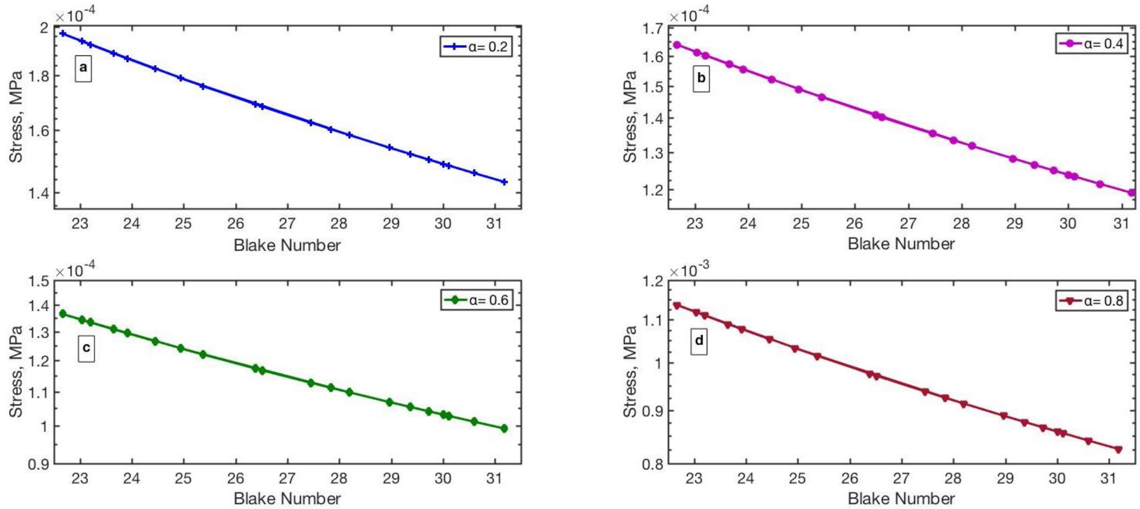

Figure 3.7: Fluid shear stress variation as a function of Blake number for different α values

3.5.3 Dependency of fluid flux (memory effect) on proposed model

Fluid memory is one of the important phenomena that describes the actual scenario of fluid flow in porous media. In the literature, very few studied are available to show the effect of fluid memory in experimental cases (Iaffaldano *et al.,* 2006, Caputo and Carcione, 2013). In this paper, experimental data (Iaffaldano *et al.,* 2006) and sample reservoir data (Hossain, 2008a) is used and simulated to get the pressure distribution for the grids. Usually, pressure data for grids is not available is reservoir data or in experimental data. To calculate fluid flux, we considered α= 0.3 (Zaman, 2017) for both the conditions. Fluid flux (m^3/m^2 s) is different from flow velocity (m/s) in case of porous media. In Figure 3.8, fluid flux is plotted with time and showed that flux values are almost linear when flow reaches the steady condition though for transient condition flux values fluctuated with time. The experiment (Iaffaldano *et al.,* 2006) was run for 11 hours and for almost 6 hours the flow was in transient condition for that reason flux values are fluctuated and after reaching the steady state condition it's almost a linear and constant with time.

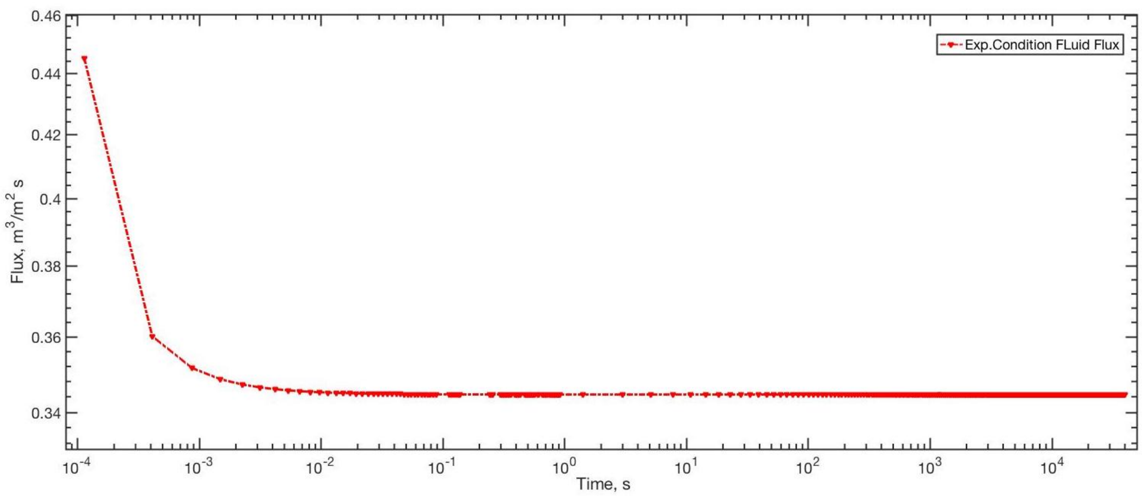

Figure 3.8: Fluid flux variation with time for α= 0.3

3.5.4 Dependence of flow time on shear stress

Time is one of the key factor when we consider fluid flow in porous media. Rock and fluid properties of any porous media change with time. As rock and fluid properties are related with all fluid flow phenomena in porous media (Caputo, 1999; Hossain, 2008a, Histrov, 2014). With time fluid flux is changing for certain α value (α= 0.3) shown in Figure 3.8. In the literature, researchers have shown that how shear stress is changing with the time of fluid flow and depending on the characteristics of non-Newtonian fluid (Barnes, 1997; Chang *et al.,* 1998; Pierre *et al.,* 2004; Fingas and Fieldhouse, 2009; Hasan *et al.,* 2010; Ghannam *et al.,* 2012; Benziane *et al.,* 2012; Dimitriou and Mckinley, 2014; Petrus and Azuraien, 2014, Kaur and Jaafar, 2014; Bao *et al.,* 2016). Figure 3.9a to 3.9d shows that for different α values stress development is different with time in semi-log plot when other parameters are considered same. With time fluid viscosity decrease because of pressure change (fluid flux) as temperature is considered in isothermal consideration. As fluid viscosity decreases with time then shear stress increases. For α= 0.2 stress development is minimum and for α= 0.8 stress development is maximum. For all α values initial stress with time is almost same but stress increase exponentially with time (Dimitriou and Mckinley, 2014; Petrus and Azuraien, 2014, Kaur and Jaafar, 2014; Bao *et al.,* 2016).

Figure 3.10 and Figure 3.11 shows the stress variation with time for both field and experiment condition. The field data plot shows a good match with proposed model results. The field stress results are showing same trend as model results but little higher stress development then model with time. For experiment condition, trend is also same and experiment results and model results show good agreement. Figure 3.10 and Figure 3.11 shows the same trend as Figure 3.9 when α= 0.3 (Zaman, 2017) considered for both field and experimental condition.

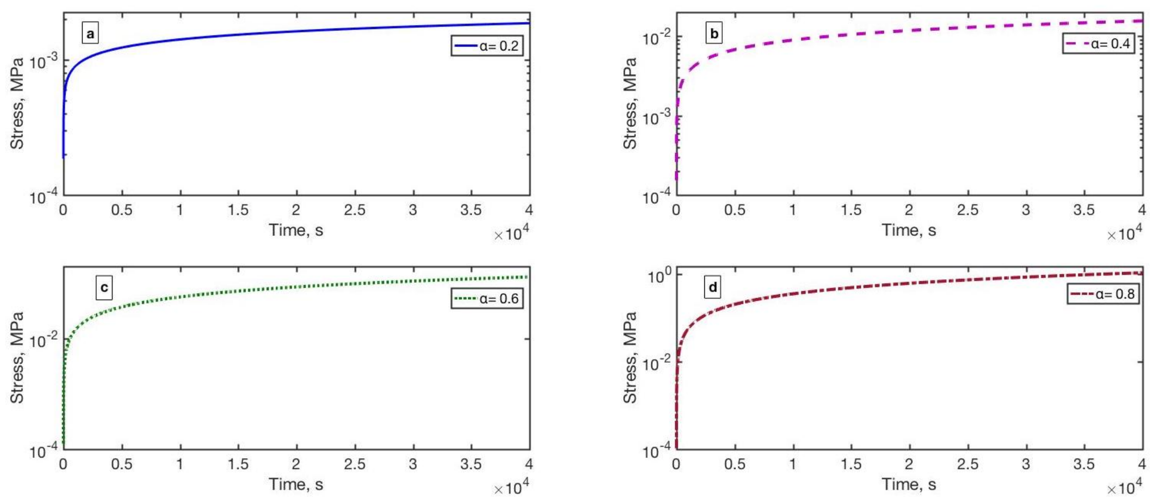

Figure 3.9: Fluid shear stress variation as a function of time for different α values

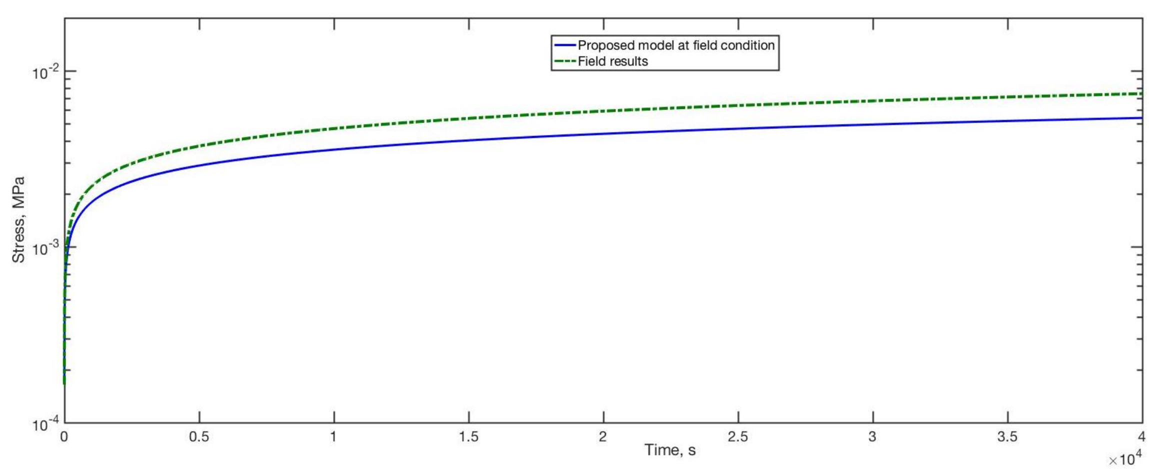

Figure 3.10: At field condition fluid shear stress variation as a function of time

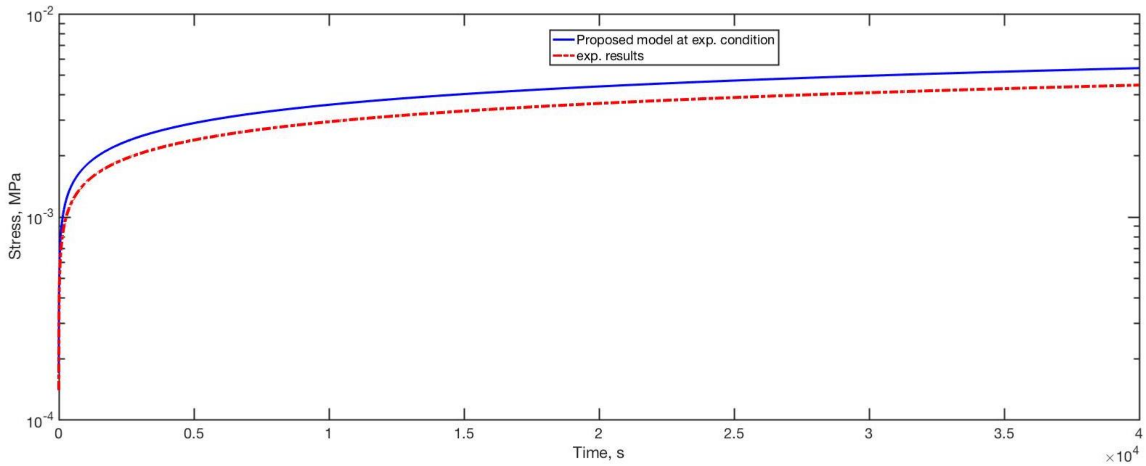

Figure 3.11: At exp. condition fluid shear stress variation as a function of time

3.5.5 Dependency of strain rate on shear stress

Fluid shear stress has a proportional relationship with rate of shear. Usually, stress increases with the change of strain and viscosity decreases (Barnes, 1997; Chang *et al.,* 1998; Pierre *et al.,* 2004; Hossain, 2008a; Ghannam *et al.,* 2012; Benziane *et al.,* 2012; Dimitriou and Mckinley, 2014; Kaur and Jaafar, 2014; Bao *et al.,* 2016). In the proposed model equation, stress is proportional to strain rate where porosity, fluid density, hydraulic diameter, flow regime and fluid memory is considered. Figure 3.12a to 3.12d shows the shear stress development with the strain rate for different α values in log-log plot. For different α values stress development is not same and it increases with the increase of α values. As fluid flux is changing with the change of α values when other parameters remain same. Therefore, stress development increasing with strain rate and shows the effect of fluid memory. For α= 0.8 stress development is maximum with the change of strain rate.

Figure 3.13 shows the same trend as Figure 3.12 for field condition where parameters are same for both conditions. The results show good agreement for filed results with model results. Stress development is little bit higher for filed results from the model condition. In Figure 3.14, the model results are compared with experiment condition and results show good agreement and also shows the same trend as Figure 3.12 and Figure 3.13.

In this article, α= 0.3 is considered to show the effect of strain rate on shear stress. Figure 3.15 shows the same trend as Figure 12 and 13 for different month when α= 0.3. The shear stress development increase with time (month). For 20 months shear stress development is minimum and for 100 months its maximum when other parameters remain same and α= 0.3.

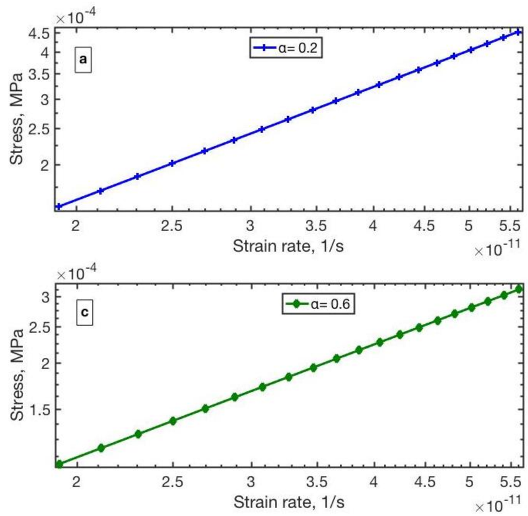

Figure 3.12: Fluid shear stress variation as a function of strain rate for different α values

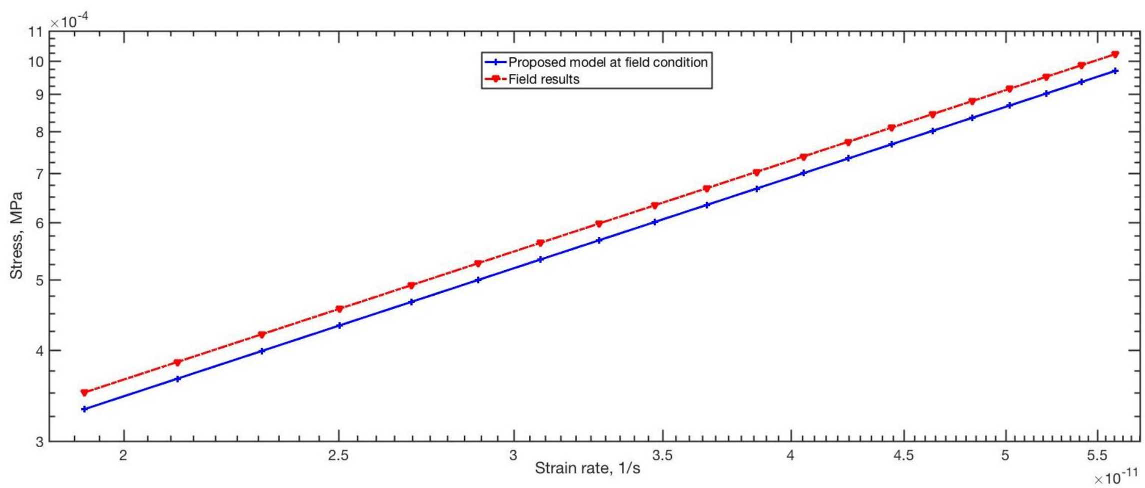

Figure 3.13: At field condition fluid shear stress variation as a function of strain rate

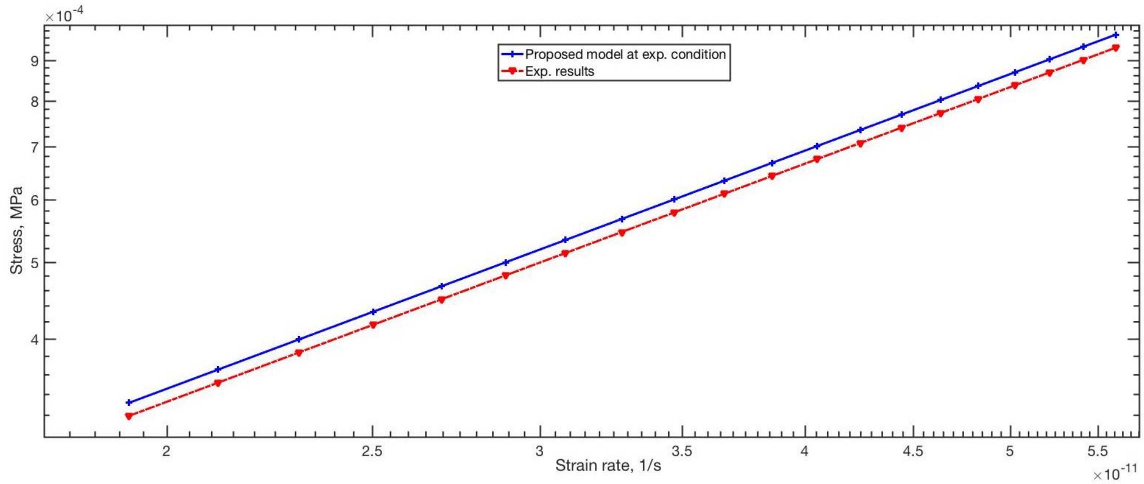

Figure 3.14: At exp. condition fluid shear stress variation as a function of strain rate

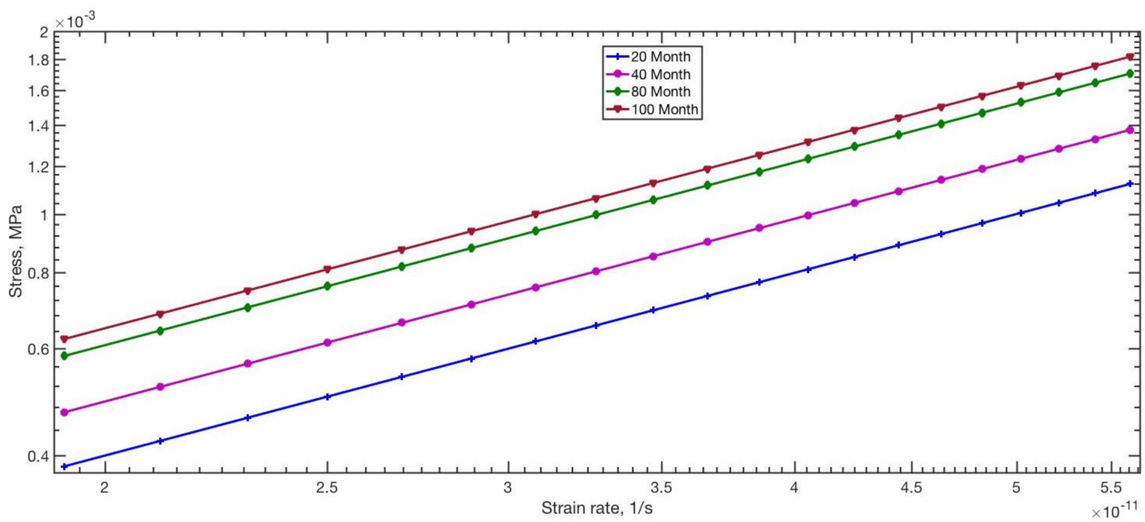

Figure 3.15: Fluid shear stress variation as a function of strain rate for different months at α= 0.3

3.5.6 Comparison of proposed Stress-Strain model with Hossain *et al.,* (2007a) Model

Figure 3.16 illustrates the variation of shear stress versus rate of strain of the proposed model (Eq. 13) for different α to compare with Hossain *et al.,* (2007a) model in the log-log plot. The data used in both models are considered same (Hossain, 2008a). The trend and shape of the curves for both models are almost same and show the variation of shear stress value with different α values for both cases. In the proposed model, values of shear stress development for α =0.2 is almost similar to Hossian *et al.,* (2007a) model where shear stress has very little variation and less stress development form Hossian *et al.,* (2007a) model. For α =0.3 shear stress development is initially less but with the change of strain rate it increases and stress development become higher than Hossian *et al.,* (2007a). Figure 3.17 shows the comparison between proposed model with fields results and Hossain et al. (2007a) mode and the proposed model show similar trend, and closer to field condition than Hossain *et al.,* (2007a) model. Same experimental data is used for all the conditions where proposed model show better outcomes than Hossain *et al.,* (2007a) model. Stress development in filed condition is higher than proposed model and Hossain et al. (2007a) model. Figure 3.18 also shows the comparison between proposed model with experiment results and Hossain et al. (2007a) mode and the proposed model show a good match with experimental condition than Hossain *et al.,* (2007a) model. This model considered the effect of porosity, fluid density, hydraulic diameter, viscous force, inertia force, flow regime, temperature and fluid memory. And fluid memory term is solved with finite difference method. The proposed model is showing consistent of shear stress value with increasing strain rate for both field and experiment study. In Hossain *et al.,* (2007a) stress-strain model porosity, fluid density and memory effect is not accurately considered. Therefore, the proposed stress-strain model is more applicable to show the effect of fluid memory in the rheological study of fluid flow through porous media

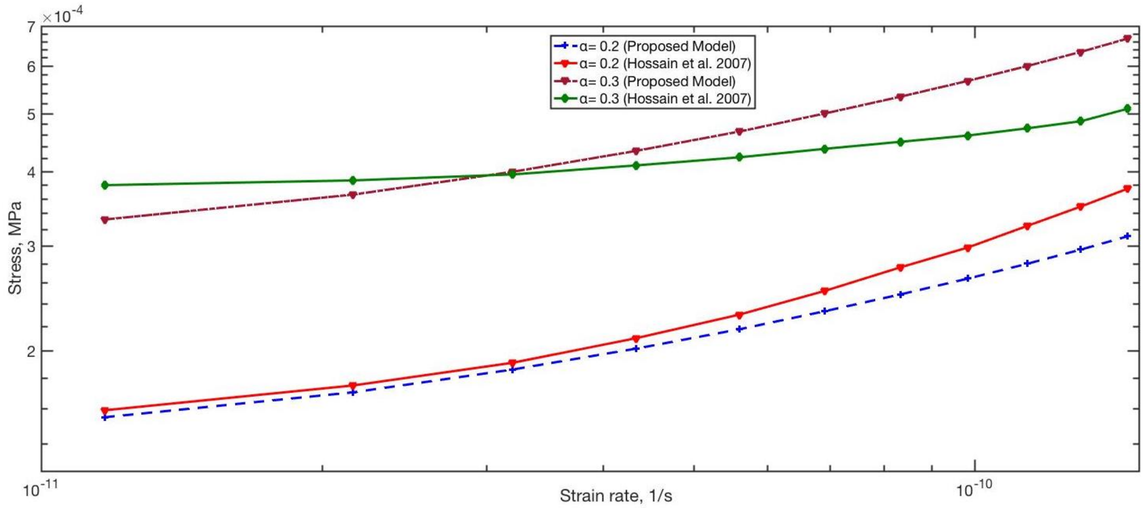

Figure 3.16: Comparison of proposed stress-strain model with Hossain *et al.,* (2007a) model.

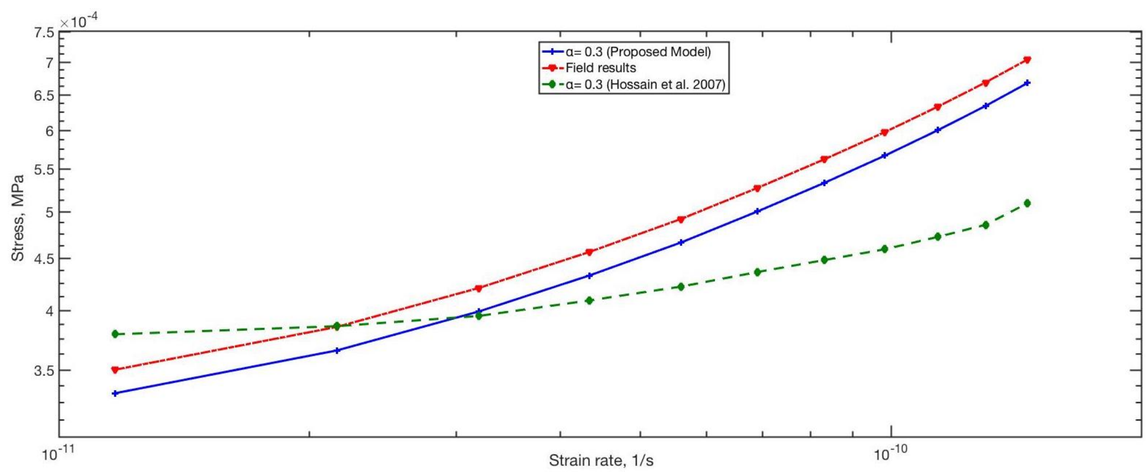

Figure 3.17: Comparison of proposed stress-strain model with field results and Hossain et al. (2007a) model.

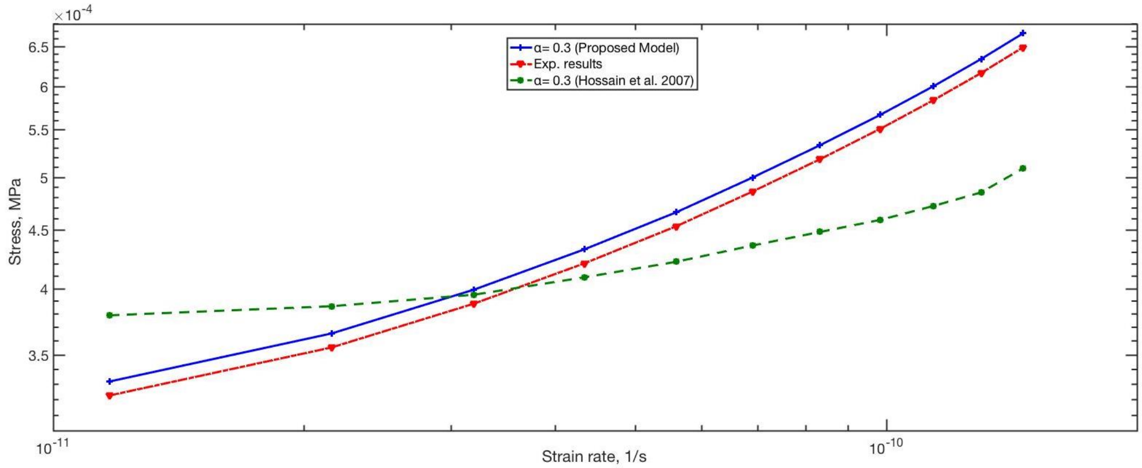

Figure 3.18: Comparison of proposed stress-strain model with exp. results and Hossain et al. (2007a) model.

3.6. Conclusions:

In this study, a comprehensive memory-based stress-strain model is proposed to characterize reservoir fluids, showed the rheological phenomenon of reservoir fluids, and the overall effect of fluid media and memory in characterization process. The analysis and numerical solution of the model shows that several fluid parameters affect the stress-strain behavior of reservoir fluid. Especially fluid density, pseudo-permeability, pressure gradient, and fluid memory have a great impact on the stress-strain behavior of reservoir fluids. Memory mechanism creates a discontinuous behavior in stress-strain relationship with the variation of fraction order of differentiation. The proposed model is validated with the available experimental and reservoir data from the literature and compared with the established stress-strain model, and showed that proposed model is more effective to get the memory effect. The proposed stress-strain model can be used in wide range of reservoir fluids characterization, and rheological study of fluid properties, media, and memory are considered to develop the model and the results show good agreement with the existing model and existing data.

7. Nomenclature:

a	Reservoir length, m

b	Reservoir width, *m*
B	Blake number
D_h	Hydraulic diameter, *m*
E	At 32.8 API gravity activation energy of crude oils, *KJ/mol*
k	Permeability of the reservoir, *mD*
p	Reservoir pressure, N/m^2
P_i	Initial reservoir pressure, N/m^2
ΔP	P_T-P_o = Pressure difference, N/m^2
$p\ (x, t)$	Fluid pressure, *Pa*
q_x	Volumetric flow rate in x-direction, kg/m^2-s
R	Universal constant, *kJ/ mol-K*
t	Time, *sec*
$\Delta T= T_T-T_o$	Temperature difference, °*K*
T	Temperature, °*K*
u_x	Reservoir fluid velocity in x-direction, *m/s*
V_p	Pore volume, m^3
V_b	Bulk volume, m^3
y	Distance from the boundary plan, *m*
α	Fractional order of differentiation
η	Ratio of the pseudo-permeability to fluid viscosity, $m^3s^{1+\alpha}/kg$
ε	Void fraction
ϕ	Porosity of fluid media
ρ_o	Density of the fluid at reference temperature T_o, kg/m^3
ρ	Density of the fluid, kg/m^3
γ	At y-direction velocity gradient, 1/*s*
τ	Shear stress, *Pa*
τ_T	Shear stress at reference temperature T_o, *Pa*
μ	Dynamic viscosity, *Pa-s*
μ_o	Dynamic viscosity at reference temperature T_o, *Pa-s*

1-D	One dimensional
3-D	Three dimensional

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Chapter 4

A Modified Stress-Strain Model for Reservoir Fluids with Memory Mechanism

Preface

This paper to be submitted to a Journal. The lead author performed the necessary literature review on fluid properties and fluid memory. The co-authors Tareq Uz Zaman helped in mathematical techniques, showed some coding techniques Dr. Salim Ahmed reviewed the manuscript and Dr. M. Enamul Hossain helped in identifying the gap in research, supervising the research, and editing the manuscript.

4.1 Abstract

In fluid flow through porous media, the reservoir rock and fluid properties are the two most important features and have a substantial impact on fluid flow. Fluid memory is an important feature that represents the time-dependent behavior of rock and fluid. Fluid memory also illustrates the formation history and how fluid will flow in the future. Though fluid memory plays a significant role in reservoir formation, still very few researchers have considered fluid memory in fluid flow models. This article represents a modified fluid stress-strain model for porous media and shows viscous forces which also includes other rock and fluid properties. The proposed mathematical memory model illustrates the responses of formation permeability, fluid viscosity, surface tension, fluid velocity, reservoir pressure variations, and the effect of memory mechanism on the stress-strain behavior. The memory mechanism is incorporated with the stress-strain relationship and uses fraction order α to show the variation of time and space. The fractional order (α) represents the effect of fluid memory for any fluid flow in porous media. Light crude oil

from a sample reservoir is considered to show the overall effect of fluid memory as a function of time and space. Reservoir pressure gradient also presented as a mathematical function of space and time to show the actual response of fluid memory. The proposed model equation is solved numerically and compared with established model, using field and experimental data available in the literature. The results show that surface tension and fluid memory has impact on shear stress. The fluid memory causes a nonlinear trend on shear stress with the increase of shear rate for different α values. For α= 0.3 both field and experimental condition shows a good match for proposed modified model considering memory effect and other parameters. The modified stress-strain model can be used in reservoir rheological analysis, fluid flow analysis, reservoir simulation, and EOR (enhanced oil recovery) process.

4.2 Introduction

Modern civilization drives smoothly with the help of energy and the petroleum industry is the main source of energy in recent times. Several new techniques (e.g. rheological study, well testing, enhanced oil recovery, etc.) are applying to increase the oil and gas production in the petroleum industry. Still, those techniques (i.e., mathematical models) have few shortcomings in maximizing the production (Abou-Kassem *et al.,* 2006). Several researchers (Caputo, 1999, 2000; Zhang, 2003; Islam, 2006; Hossain and Islam, 2006; Hossain *et al.,* 2007a, 2008b, 2009a; Hossain, 2008a; Caputo and Carcione, 2013; Al-Mutairi *et al.,* 2013; Hristov, 2014; Rahman *et al.,* 2016; Obembe *et al.,* 2017a, 2017b) are trying to incorporate a memory mechanism to reduce the shortcomings and enhance the overall production.

In porous media, fluid flow through the pore spaces in the formation and those spaces are not always the same in structure. A typical sketch of fluid flow through porous media is shown in Figure 4.1. Newtonian fluids represent a simple and linear relation among stress (viscous) and strain rate. Fluid viscosity presents as proportionality coefficient for the Newton’s law and depends on mostly fluid temperature and pressure. Further, for non-Newtonian fluids viscosity is not well described and changes with applied forces and strain rates. Usually, researchers ignore non-Newtonian fluid at the time of developing models, also researchers ignore surface tension of viscous fluids and fluid memory mechanism.

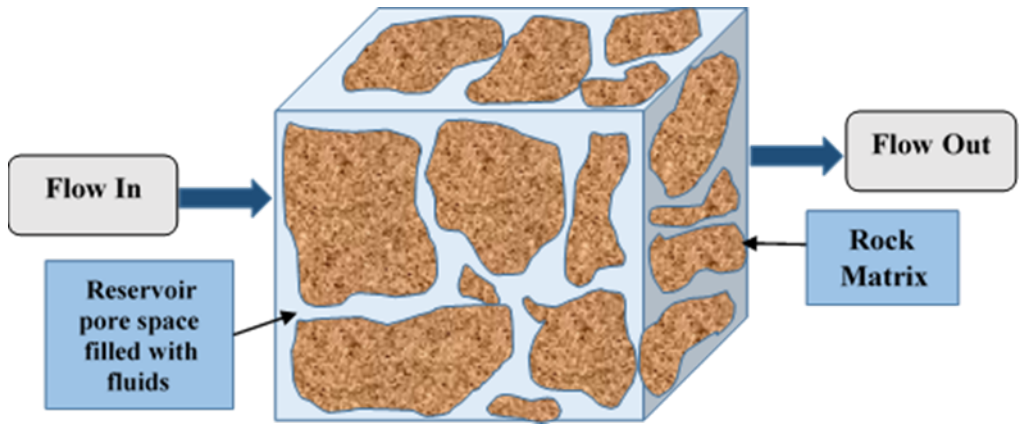


Figure 4.1: Typical Fluid Flow in Porous Media (modified from Miah *et al.,* 2017)

Fluid viscosity has a great impact on oil production. The oil production of a reservoir is related to oil mobility which increases by decreasing the oil viscosity. If shear forces are applied to any fluid, the fluid will deform and start to move irrespective to the amount of shear forces. Fluid viscosity is one of the key factors to show the relation between the shear stress and strain rate. The fluid inertia forces interrupt the stabilize flow behavior and lead towards turbulent behavior.

Surface tension plays an important part in fluid flow through porous media. Several researchers have addressed the surface tension phenomenon for porous media in different ways (e.g. Marangoni effect, Capillary effect etc.). Ramakrishnan and Wasan (1986) have shown a theoretical and experimental study on drainage, and imbibition of relative permeability for wetting and non-wetting phases. Authors measured capillary pressure and used capillary numbers to represent the permeability equations. Vizika *et al.* (1994) reviewed the viscosity ratio (k) of forced imbibition process in porous media and proposed a theoretical model using new simulation and experimental studies. Authors showed the impact of both small and large values of capillary number (C_a). Lyford *et al.* (1998a, 1998b) studied oil recovery processes and used aliphatic alcohol (C_nH_{2n+1} OH) as a surfactant substance in porous media. Authors also discussed the Marangoni effect and used the

Marangoni number to show the surface gradient in porous media. Hossain *et al.* (2007a) developed a mathematical fluid memory model to show the stress-strain relationship in porous media. Researchers used Marangoni number (M_a) to show the surface gradient for porous media and introduced fluid memory in the stress-strain relationship. Cense and Berg (2009) reviewed the effect of viscous, and capillary forces for multiphase flow in porous media. Authors presented that capillary number shows a relation between viscous, and capillary forces and values of capillary number ranges 10^{-6}-10^{-4} showing the effect of capillary behavior in a multi-phase fluid flow. Ferer *et al.* (2011) developed a 2-D pore level model showing the response of immiscible drainage, viscous force, and capillary force. Authors increased the capillary number to get stable viscosity and the model was validated with several experiments. Datta *et al.* (2014) reviewed the non-wetting fluid in a complex 3-D porous medium. Authors used capillary number (C_a) to represent this behavior and showed a relationship with viscous and capillary forces. Guo *et al.* (2017) studied the capillary number theory for chemical flooding in enhanced oil recovery process. Authors discussed the capillary number range for a heterogeneous reservoir in non-Darcy condition and presented new techniques to obtain more oil recovery with better mobility.

Porter and Johnson (1962) compared fluid viscosity with two mostly used shear techniques (i.e. jet viscometer and concentric cylinder viscometer. Authors addressed that concentric cylinder viscosity loss is less than jet viscometer process and concluded that it might be due to the capillary effect and kinetic energy correlations. Churchill and Churchill (1975) developed a new correlation of effective viscosity for pseudo-plastic and dilatant fluids considering as a function of shear stress, this correlation also can be illustrated for dynamic viscosity as a function of frequency oscillation. Vetter (1979) used Weertman's (1977) temperature method to calculate stresses and viscosity in the asthenosphere up to 400 km depth by relating viscosity and ratio of temperature at the melting point. The author also considered two creep laws and creep rate was 1*103 of the stress value. Wernera *et al.* (1998) developed a mathematical model to calculate the petroleum fluid viscosity as a function of reservoir temperature (T_r) and pressure (P_r). Authors considered Kanti *et al.,* (1989) model for reservoir temperature and pressure, and Grunberg and Nissan (1949) approach for fluid composition. This model is useful for large compositional rage such as heavy range of asphaltenes. Luo and Gu (2007) studied the viscosity for heavy crude oil and were able to show how viscosity is effected at a different temperature in the presence of asphaltene. Authors used theoretical and experimental approaches to measure heavy oil viscosity at various temperatures. Islam and Carlson (2012) studied viscosity models for the geologic sequestration of CO2 at certain temperatures and pressures. Authors considered water, brine, and typical sea water and showed the effect of CO2 more acutely. MacDonald and Miadonye (2017) reviewed viscosity correlations and developed a new simplistic, semi-empirical equation different than current empirical models for the viscosity of Tangleflags and Athabasca bitumen. Researchers illustrated that the proposed equation gave a low percentage of errors for viscosity measurement considering temperature and pressure.

Nibbi (1994) developed a model for fluid viscosity considering memory mechanism. The author also considered the quasi-static condition and was able to show a relationship between free energy and fluid viscosity. Caputo (1999) proposed a mathematical model to modify Darcy's law by introducing a fractional derivative and presented the local permeability alteration in any porous media. However, this assumed modification is only applicable when local phenomena are considered. Chen *et al.* (2005) proposed a model relating stress and the invasion percolation with memory (IPM) method for porous media. Authors also shown the relation between stress and dynamic viscous friction. Hossain *et al.* (2007a) developed a memory-based mathematical model to show the relationship between stress and strain for porous media. Authors also addressed temperature difference, surface tension, pressure difference, and fluid memory to represent the model as a comprehensive one. Hossain *et al.* (2009b) derived a memory-based mathematical model to present the complex rheological behavior of fluid and proposed some dimensionless numbers for rock and fluid properties such as porosity, permeability, heat capacities, densities, and viscosities. Di Guiseppe *et al.* (2010) reviewed the changes of fluid and rock properties under changing pressures and observed the changes of pore grains during fluid transport in porous media. Hristov (2014) proposed a diffusion model with the integral balance method and described the memory term by weakly singular power-law. Recently, Rahman *et al.* (2016) made a critical review on memory-based models for porous media and discussed assumptions and limitations of those models. Authors gave an overall guideline to develop a comprehensive memory-based fluid model for porous media.

Formation structure, permeability, surface tension, capillary effect, viscosity, reservoir temperature, and pressure need to be considered to develop a comprehensive model for stress-strain behavior and which can capture memory effect. For surface tension, Capillary Number (C_a) is considered to show the effect of viscous and capillary forces for porous media. The fluid memory is observed from the pressure gradient, differential order (α), and the pseudo-permeability as a ratio of permeability to fluid viscosity (η). The actual response of the fluid memory can be determined with the variation of (α). The result outcomes are illustrated in graphical form to represent the response of surface tension, fluid viscosity, and fluid memory on the stress-strain relationship. The results show a nonlinear trend with time and this nonlinearity arises pressure the fluid memory shows a nonlinear behavior with time and this nonlinearity arises because the pressure is depended on fluid velocity.

4.3 Mathematical Model Development

In the x-direction, an external shear force (F_x) is applying on the top layer of a fluid element (shown in Figure 4.2) and the fluid element will deform. The deformation of the fluid element is showing the effect of shear forces that have been applied tangentially to a surface. Usually, Newton's law of viscosity is used to show the shear stress and rate of strain relationship for time-dependent fluids (Hossain *et al.,* 2007a, 2008a). Mathematically it can be written as:

$$\tau = -\mu \frac{du_x}{dy} \tag{1}$$

Temperature (T) is considered in the x-direction for the formation, and shear rate can be presented as γ. Then Eq. (1) can be written as below:

$$\tau = -\mu_T \, \gamma \tag{2}$$

Cohesion effect plays an important role in the liquid viscosity. If the temperature of fluid increases, cohesive bonding between fluid molecules decreases, and the overall molecular transportation increases. Shear stress decreases with decreasing cohesive forces and increases with increasing molecular interchange. In consequence of this complex behavior of shear stress, several researchers showed the importance of temperature on fluid viscosity and developed different models based on experimental and field studies (Recondo *et al.,* 2006; Hossain et al., 2007a, 2008a). In this model, the Arrhenius model is used to represent the relationship between temperature and fluid viscosity (Avramov, 2005; Haminiuk *et al.,* 2006; Gan *et al.,* 2006, Hossain *et al.,* 2007a; Hossain, 2008a).

$$\mu_T = \mu_o e^{\left(\frac{E}{RT}\right)} \tag{3}$$

In Eq. (3) μ_o is the dynamic viscosity at reference temperature. Using the value of μ_T in equation (2) form equation (3) shows as:

$$\tau = -\mu_o\, e^{\left(\frac{E}{RT}\right)} \gamma \tag{4}$$

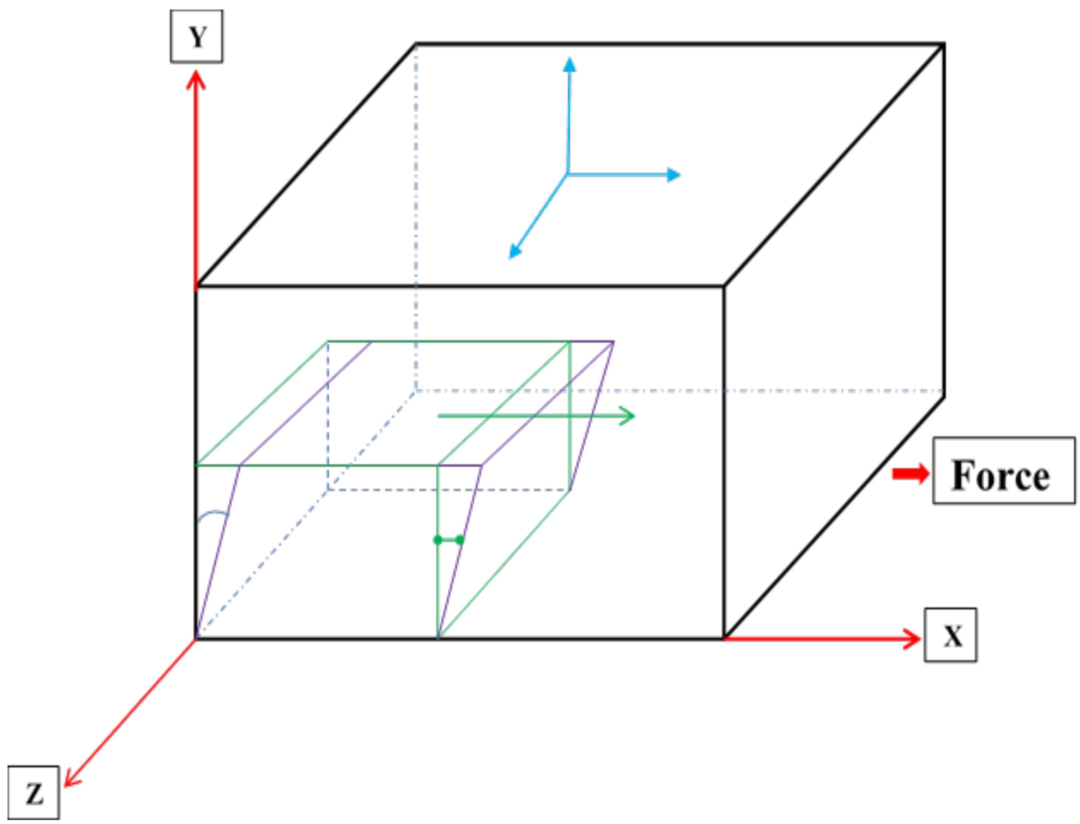


Figure 4.2: 3-D tangential forces of a fluid element (modified from Chhabra and Richardson, 2008)

Fluid particles and molecules may transfer to nearby layers at the time of fluid flow through porous media. This phenomenon is acuter in the gaseous medium than in liquid medium as gas molecules have fewer forces comparing liquids. Cohesive force helps to keep the molecules of fluid in a certain place more effectively. Cohesion has a prominent impact on the reservoir fluid viscosity. With the increasing fluid temperature, molecular bonding decreases and inter-molecular transportation increases. The cohesive force also has a big impact on shear stress. Shear stress reduces with decreasing cohesive forces and increases with increasing inter-molecular transfer. Researchers presented the importance of formation temperature on fluid viscosity and established several models based on field data and experimental studies (Frisch *et al.,* 1940; Sanyal *et al.,* 1974; Al-Besharah *et al.,* 1989; Recondo *et al.,* 2006; Hossain *et al.,* 2007a, 2007b, 2008a; Wu *et al.,* 2014; Nmegbu, 2014, Akankpo and Essien, 2015, Wu and Massoudi, 2016). Fluid viscosity also changes because of high pressure. If the pressure increases, the fluid elements need more energy for their relative movement in porous media. Hossain *et al.* (2007a, 2008b, 2009a, 2009b) addressed this complex phenomenon though previous researchers have ignored this effect.

Surface tension plays a significant role in fluid flow through porous media. The capillary effect for fluid layers and porous media are assumed to show the actual scenario of surface tension. The capillary effect is a process that moderate the interaction between contacting surfaces of a fluid and a solid. This influences the fluid surface from a linear condition and helps the fluid to rise or fall in a narrow space. Surface tension of a fluid can be introduced by media temperature, gravitation force, fluid density, and surface alteration through the layer (Dullien, 1992; D'Aubeterre *et al.,* 2005). In porous media, Capillary number repents the effect of viscous force to surface tension which acting on the across of two immiscible fluids. A Capillary number is a dimensionless number which does not depend on the units of a system. In this study, the initial temperature has been considered as 298°K. In general, the formation temperature is considered as constant throughout the reservoir for a particular reservoir section at a certain depth (Yanowitch, 1967, Hossain et *al.,* 2011, Hossain and Abu-Khamsin, 2012). Therefore, the viscosity in the Capillary number is assumed constant (i.e., no change of viscosity due to temperature effect). As a result, the ratio of viscous force and surface tension can be expressed by the Capillary number as follows:

$$C_a = \frac{\mu_o\, u_x}{\sigma} \tag{5}$$

$$\mu_o = \frac{\sigma\, C_a}{u_x} \tag{6}$$

Putting the value of Eq. (6) on Eq. (4):

$$\tau = -\frac{\sigma\, C_a}{u_x}\, e^{\left(\frac{E}{RT}\right)}\, \gamma \tag{7}$$

To address the fluid memory with stress relation, the fluid flux in porous media can be showed by the following equation (Caputo, 1999). As the flow is considered in x-direction, the equation can be written as:

$$q_x = -\eta\, \rho_o \left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\right] \tag{8}$$

In Eq. (8) $q_x = \frac{q_x' \rho_o}{A}$ and $0 \le \alpha < 1$ so Eq. (8) can be written as:

$$u_x = -\eta \left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\right] \tag{9}$$

Putting the value of Eq. (9) on Eq. (7)

$$\tau = -\frac{\sigma\, C_a}{-\eta\left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\right]}\, e^{\left(\frac{E}{RT}\right)}\gamma \tag{10}$$

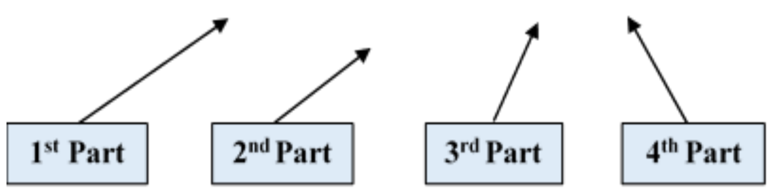
$$\tau = -\sigma\, C_a \,\frac{1}{-\eta\left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\right]}\, e^{\left(\frac{E}{RT}\right)}\, \gamma \tag{11}$$

Where, η is the pseudo permeability of the porous media. η can be showed with the below equation (Hoosain, 2008a; Hossain *et al.,* 2008b) as follow:

$$\eta = \frac{K}{\mu}\,(t)^\alpha \tag{12}$$

In Eq. (12), *K* = permeability, μ = fluid viscosity, t = time and α= order of differentiation. Putting the value of η in Eq. (11) from Eq. (12) it becomes as:

$$\tau = \sigma\, C_a \,\frac{1}{\left[\frac{K}{\mu}\,(t)^\alpha\left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\right]\right]}\, e^{\left(\frac{E}{RT}\right)}\, \gamma \tag{13}$$



The above Eq. (13) illustrates the effect of shear stress and fluid properties in a reservoir formation in 1-D (x-direction) and it can be expressed in a more general condition in 3-D fluid flow for an anisotropic heterogeneous formation. Though inelasticity of matrix, formation heterogeneity, and anisotropy sometimes failed to show any phenomena, fluid memory could capture all the phenomenology. The 1st part of the Eq. (13) shows the effects of surface tension and capillary effect of porous media. The 2nd part is the effect of pseudo-permeability (i.e., permeability, viscosity ratio with time) and pressure gradient along the axis and both together represent the effect of fluid memory. The 3rd part is the effect of isothermal temperature condition in the formation for certain depth. Finally, the 4th part is the effects of strain rate according to shear stress also known as velocity gradient in y-direction.

4.4 Numerical Analysis of The Model

The results of the proposed stress-strain model can be obtained by solving Eq. (13) which is shown above. In this paper, we focused on the stress-strain relation, fluid surface tension, viscosity, and fluid memory. We consider a sample reservoir from the production wellbore (Hossain 2008a) and experimental data (Iaffaldano *et al.,* 2006) for numerical calculation. The reservoir is isolated and oil is producing at a constant rate. The fluid is assumed to be an API 32.8 gravity crude oil at 298^{o}K temperature. All computations are carried out by MATLAB programming codes.

Table 1: Sample Reservoir Data (Hossain 2008a)

Reservoir length, l	5000 m
Reservoir width, w	100 m
Reservoir height, h	50 m
Porosity, ϕ	30%
Permeability, k	30 md = 30* 10^{-15} m^2
Initial reservoir pressure, p_i	27579028 pa (4000 $psia$)
Compressibility, c	1.2473×10^{-9} $1/pa$
Initial viscosity, μ_o	8.623×10^{-3} Pa-s
Surface tension	0.03032 N/m
Initial flow rate, q_i	8.4×10^{-9} m^3/sec
Initial fluid velocity, u_i	1.217×10^{-5} m/sec
Fractional order of differentiation, α	0.2-0.8

Number of grid in space, N_t	580

Table 4.2: Experimental Data (Iaffaldano *et al.,* 2006)

Length of cylinder, l	11.6 *cm*
Inner diameter, D_i	10.1 *cm*
Volume of the cylinder, $V_c = \pi r^2 h$	929.374 cm^3
Permeability, k	26 *Darcy*
Viscosity, μ_o	1.0266 *cp*
Surface tension	0.0726 *N/m*
Sand density, ρ_s	2.4 gcm^{-3}
Mass of sand in cell, M_s	1550 *gm*
Volume of sand, V_s	645.83 cm^3
Porosity, ϕ	0.3050913841
Fluid density, ρ_f	0.998408 gcm^{-3}
Compressibility, c_t	2.05743* 10^{-4} atm^{-1}
dp/dx	0.01765982953 *atm/cm*
Δp	0.2048540225 *atm*
Number of grid in space, N_t	580

In Eq. (13) 2nd part is considered to show the effect of fluid memory. Here, $\left\{\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\right\}$ is the change of pressure with space and time. In addition, $\frac{K}{\mu}(t)^\alpha$ is used to show the effect of pseudo-permeability with time continue alteration. α value is showing the variation of the order of differentiation. In this paper, we calculate this $\left[\frac{K}{\mu}(t)^\alpha\left\{\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\right\}\right]$ term as fluid flux ($m^3/m^2\,s$). Authors used Iaffaldano *et al.* (2006) experimentally validated data which is simulated, and validated numerically by Zaman (2017). Different authors showed the variation of α value to capture the memory effect (Iaffaldano, 2006, Hossain 2008a, Histrov 2014, Obembe *et al.,* 2017). Recently, Zaman (2017) shows that the best choice for α value is 0.3 which can give a good agreement for fluid memory. The results of this study are compared with the well-established Hossain *et al.,* (2007a, 2009a) stress-strain model.

To solve the fluid memory term (partial differential equation) in Eq. (13), it is necessary to consider space and time for pressure calculation. The finite difference method is used to solve the fluid memory term $\left\{\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\right\}$ of Eq. (13). The discretized form of fluid memory term from Eq. (13) as follows:

$$\left.\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\right|_i^n$$

$$= \frac{1}{\Gamma(2-\alpha)}\,\frac{1}{(\Delta t)^\alpha}\sum_{I=1}^{n}\left[\,I^{1-\alpha} - (I-1)^{1-\alpha}\right]\left[\left(\frac{\partial P}{\partial x}\right)_i^{n-I+1} - \left(\frac{\partial P}{\partial x}\right)_i^{n-I}\right] \tag{14}$$

$$= \frac{1}{\Gamma(2-\alpha)\,(\Delta t)^\alpha}\sum_{I=1}^{n}\left[\,I^{1-\alpha} - (I-1)^{1-\alpha}\right]\left[\frac{P_{i+1}^{n-I+1} - P_i^{n-I+1}}{\Delta x} - \frac{P_{i+1}^{n-I} - P_i^{n-I}}{\Delta x}\right] \tag{15}$$

$$= \frac{1}{\Delta x\,\Gamma(2-\alpha)\,(\Delta t)^\alpha}\sum_{I=1}^{n}\left[\,I^{1-\alpha} - (I-1)^{1-\alpha}\right]\left[P_{i+1}^{n-I+1} - P_i^{n-I+1} - P_{i+1}^{n-I} + P_i^{n-I}\right] \tag{16}$$

$$= \frac{1}{\Delta x\,\Gamma(2-\alpha)\,(\Delta t)^\alpha}\sum_{I=1}^{n}\left[\,I^{1-\alpha} - (I-1)^{1-\alpha}\right]\left[P_{i+1}^{n-I+1} - P_{i+1}^{n-I} - P_i^{n-I+1} + P_i^{n-I}\right] \tag{17}$$

Using Eq. (17) we calculated the flux value of the fluid in porous media. For reservoir, we used 580 grids in space and for experiment condition, we used 580 grids in space. To calculate fluid flux, all the values of previous time steps were considered. In Figure 4.3, the alignment of grid point in space is shown.

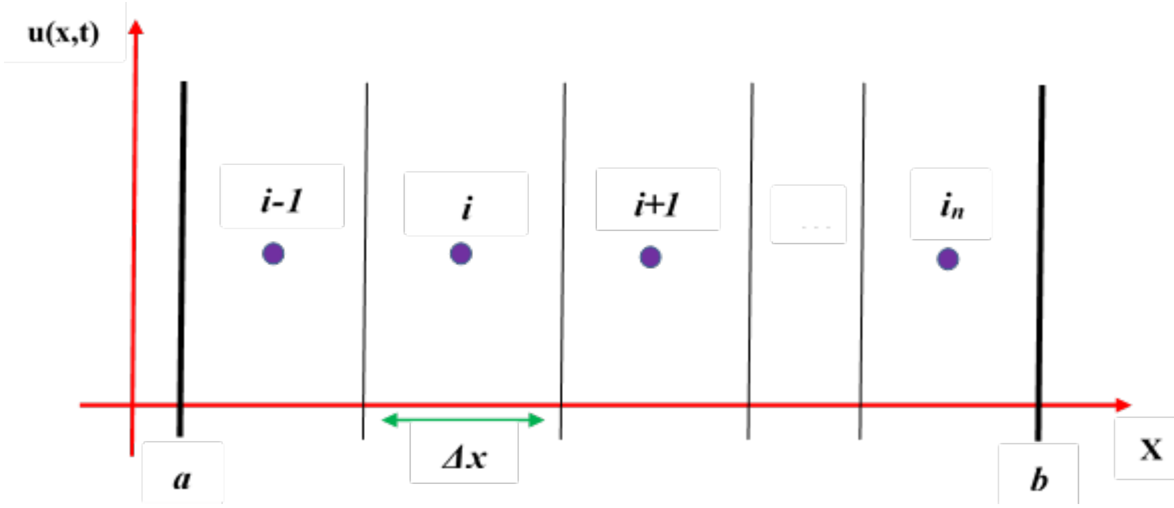


Figure 4.3: Sample reservoir grid point in space (modified from De Sterck and Ullrich, 2009)

4.5 Results and Discussion

4.5.1 Dependency of fluid viscosity on shear stress

In Newton's law of viscosity, fluid viscosity used as a proportional constant to show the relation between shear stress and rate of strain. In stress-strain plot viscosity presented as the slope of the curve which can be positive or negative. For any fluids, viscosity decreases with increasing shear stress. Usually, fluid viscosity shows concave up or down, decreasing shape with increasing shear stress considering positive or negative fluid velocity gradient (Branes, 1997; Vlachopoulos and Strutt, 2003; Mendes, 2010; Dalton *et al.,* 2013; Kaur and Jaafar, 2014; Ebrahimi *et al.,* 2015; Wen *et al.,* 2016; Helmy, 2016, Chatzigiannakis *et al.,* 2016). Figure 4.4 shows the change of fluid viscosity with the change of shear stress considering positive velocity gradient along the y-axis for different α values in a log-log plot. Curves shape are concave downward and fluid viscosity is decreasing with increasing shear stress for all α (0.2-0.8) values. Viscosity is decreasing with increasing shear stress initially linearly and after certain time is shows a concave shape which reflects the effect of other fluid parameters and memory. Figure 4.4a to 4.4d shows the same trend with increasing shear stress viscosity is decreasing and for α= 0.8 shear stress development is minimum considering all other parameters are same.

Figure 4.5 shows the change of shear stress for field condition in log-log plot and field results and proposed model plot trend is almost same. Proposed model results good match with the field results and for field case stress development is little bit higher than proposed model. Plots are showing similar trend and shape as Figure 4.

Figure 4.6 shows the change of shear stress for experiment condition in log-log plot, and experimental results and proposed model trend is almost same. Proposed model results show good match with the experimental condition. For model case curve show more curvature than experiment results and stress development is also little bit higher for proposed model than experimental results. Plots are showing similar trend and shape as Figure 4.4 and Figure 4.5.

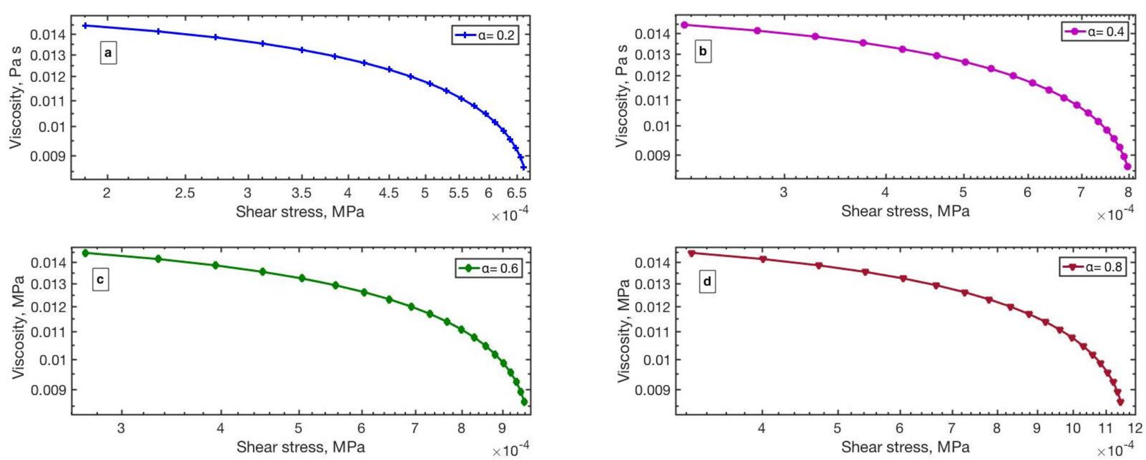


Figure 4.4: Fluid viscosity variation as a function of shear stress for different α values

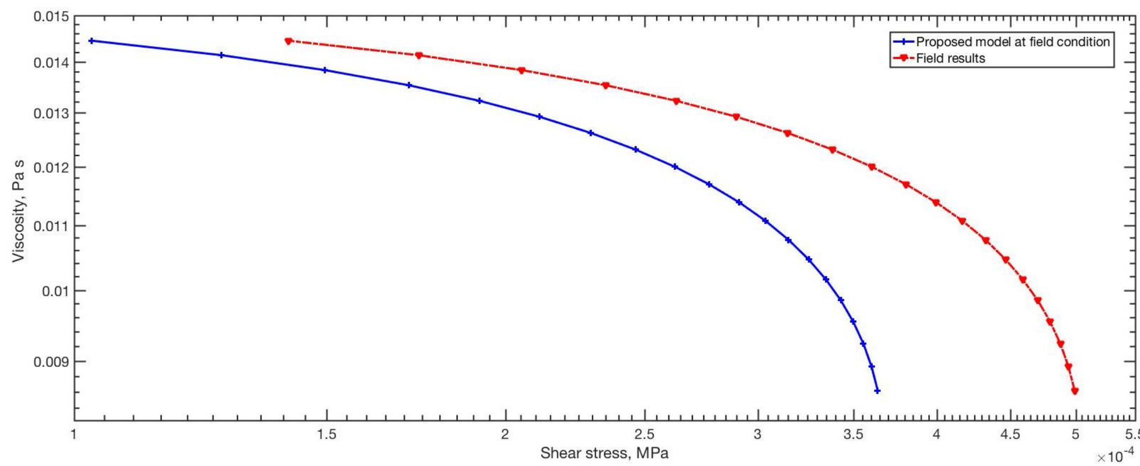


Figure 4.5: At field condition shear stress as a function of viscosity

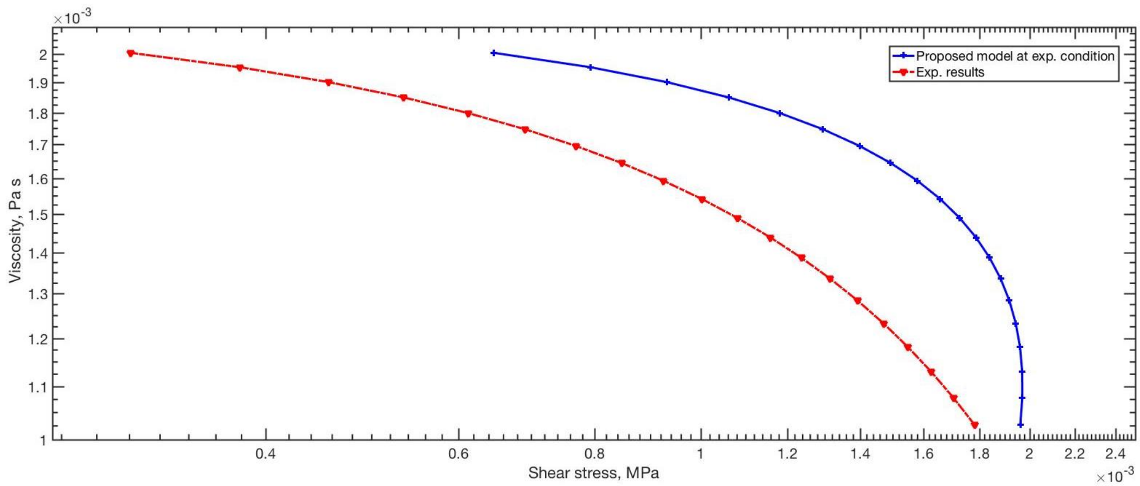


Figure 4.6: At exp. condition shear stress as a function of viscosity

4.5.2 Dependency of fluid flux (memory effect) on proposed model

Fluid memory is one of the important parameter that describes the scenario of fluid flow in porous media. In the literature, few studies are available to show the effect of fluid memory for experimental cases (Iaffaldano et al., 2006, Caputo and Carcione, 2013). In this paper, Iaffaldano et al. 2006 experimental data and Hossain, 2008a sample reservoir data is used to simulate the pressure data for the grids. To calculate fluid flux, we considered α= 0.3 (Zaman, 2017) for both the conditions. Fluid flux (m3/m2 s) is different from flow velocity (m/s) in case of porous media. In Figure 4.7, fluid flux is plotted with time and showed that flux values are almost linear when flow reaches the steady condition though for transient condition flux values fluctuated with time. The experiment (Iaffaldano et al., 2006) was run for 11 hours and for almost 6 hours the flow was in transient condition for that reason flux values are fluctuated and after reaching the steady state condition it's almost a linear and constant with time.

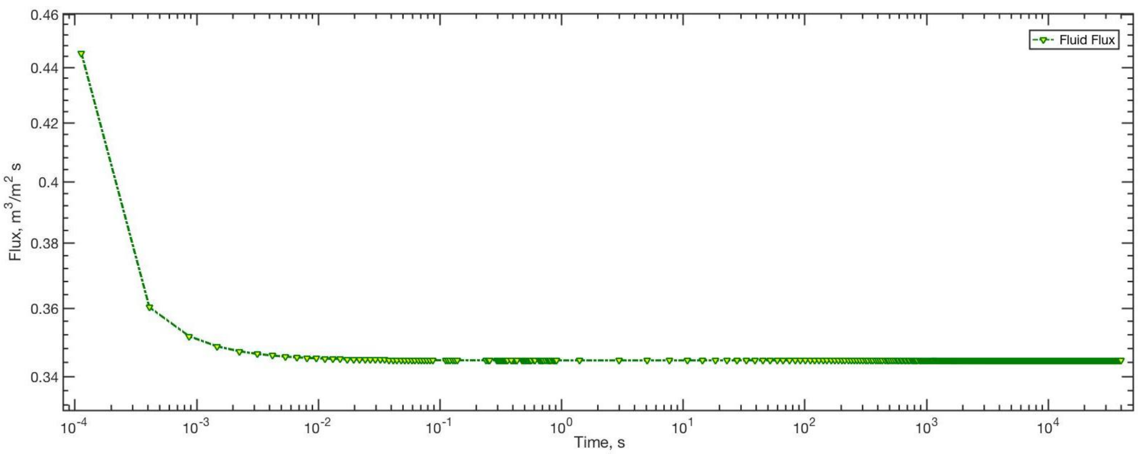


Figure 4.7: Flux change with time for α= 0.3

4.5.3 Dependency of fluid flow time on shear stress

Time is one of the key factor when we consider fluid flow in porous media. Rock and fluid properties of any porous media change with time. As rock and fluid properties are related with all fluid flow phenomena in porous media (Caputo, 1999; Hossain, 2008a, Histrov, 2014). With time fluid flux is changing for certain α value (α= 0.3) shown in Figure 7. In the literature, researchers have shown that how shear stress is changing with the time of fluid flow and depending on the characteristics of non-Newtonian fluid (Barnes, 1997; Chang *et al.,* 1998; Pierre *et al.,* 2004; Hasan *et al.,* 2010; Lei and Xian, 2010; Ghannam *et al.,* 2012; Benziane *et al.,* 2012; Dimitriou and Mckinley, 2014; Petrus and Azuraien, 2014, Bao *et al.,* 2016). Figure 4.8a to 4.8d shows that for different α values stress development is different with time in semi-log plot when other parameters are considered same. With time fluid viscosity decrease because of pressure change (fluid flux) as temperature is considered in isothermal consideration. In Figure 4.8a to 4.8d fluid shear stress is decreasing with time. For different α values stress development is different for certain conditions. At final time, stress development is almost same for all the α values but initially stress value with time is different and stress decreases exponentially with time (Dimitriou and Mckinley, 2014; Petrus and Azuraien, 2014, Kaur and Jaafar, 2014; Bao *et al.,* 2016).

Figure 4.9 and Figure 4.10 shows the stress variation with time for both field and experiment condition. The field data plot shows a good match with proposed model results. The field stress results are showing same trend as model results. Initially stress was almost same for both the cases but after certain time stress value for field condition is little lower then model condition with time. For experiment condition, trend is also same for experiment results and model results and show good agreement. Stress values for experiment case is lower than model results though initially stress values are almost same for both the cases. Figure 4.9 and Figure 4.10 shows the same trend as Figure 4.8 when α= 0.3 (Zaman, 2017) considered for both field and experimental condition.

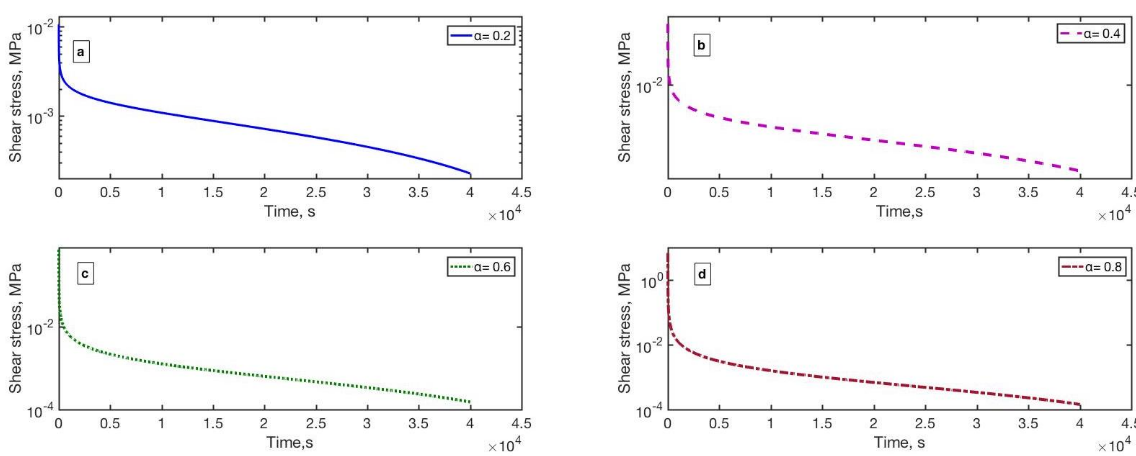


Figure 4.8: Shear stress variation as a function of time for different α values

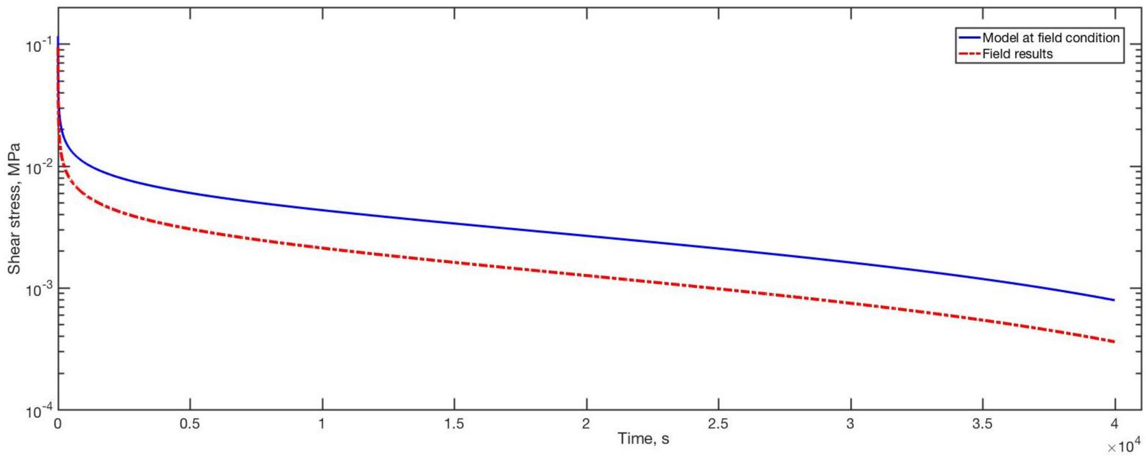


Figure 4.9: At field condition shear stress variation as a function of time

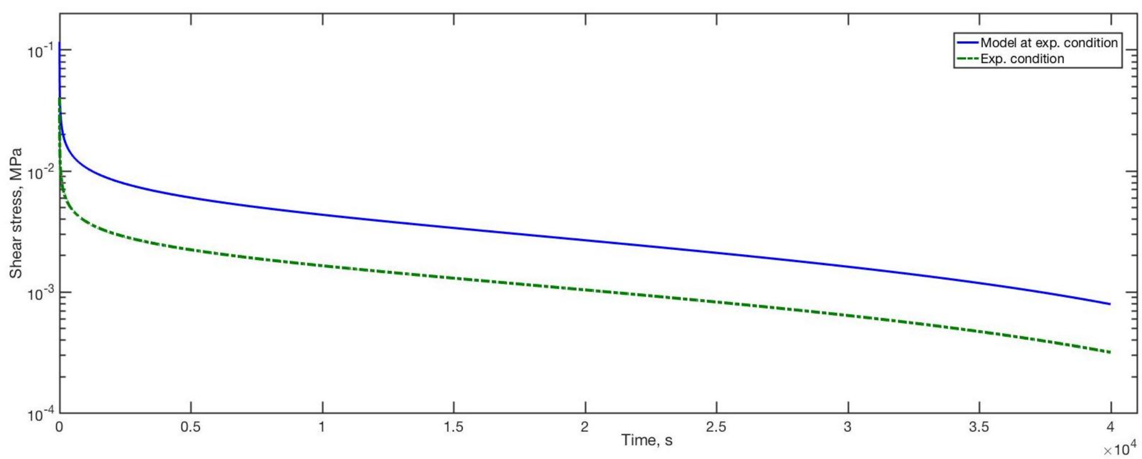


Figure 4.10: At exp. condition shear stress variation as a function of time

4.5.4 Dependency of shear rate on shear stress

Shear stress has a proportional relationship with rate of shear and shear stress increases with the increase of shear rate where fluid viscosity represents as a proportional factor. (Barnes, 1997; Chang et al., 1998; Pierre et al., 2004; Hossain, 2008a; Ghannam et al., 2012; Benziane et al., 2012; Dimitriou and Mckinley, 2014; Kaur and Jaafar, 2014; Bao et al., 2016). But sometimes for variation of fluids and conditions this relation not remain the same and instead of linear plot curvature trend observed. In the proposed model equation, stress is proportional to rate of strain where surface tension, capillary effect and fluid memory is considered. Figure 4.11a to 4.11d shows the shear stress development with the strain rate for different α values in log-log plot. For different α values stress development is not same and it decreases with the increasing α values. As fluid flux is changing with the change of α value when other parameters remain same. In Figure 4.11a to 4.11d, initially stress is increasing linearly with increasing shear rate but after certain time it start to decrease with curvature shape which represent the actual effect of fluid properties and memory that considered in the proposed model equation. Therefore, stress development increasing with rate of shear and shows the effect of fluid memory. For α= 0.2 stress development is maximum and for α= 0.8 it's minimum with the change of shear rate.

Figure 4.12 shows the same trend as Figure 4.11 for field condition where parameters are same for both conditions. The results show good agreement for filed results with model results. Stress development is little bit higher for filed results from the model condition and model results show the curvature trend as Figure 4.11. In Figure 4.13, the model results are compared with experiment condition and results show good agreement also shows the same trend as Figure 4.11 and Figure 4.12. The stress development in experimental condition is little less than model results though trend and shape is almost same for both the cases.

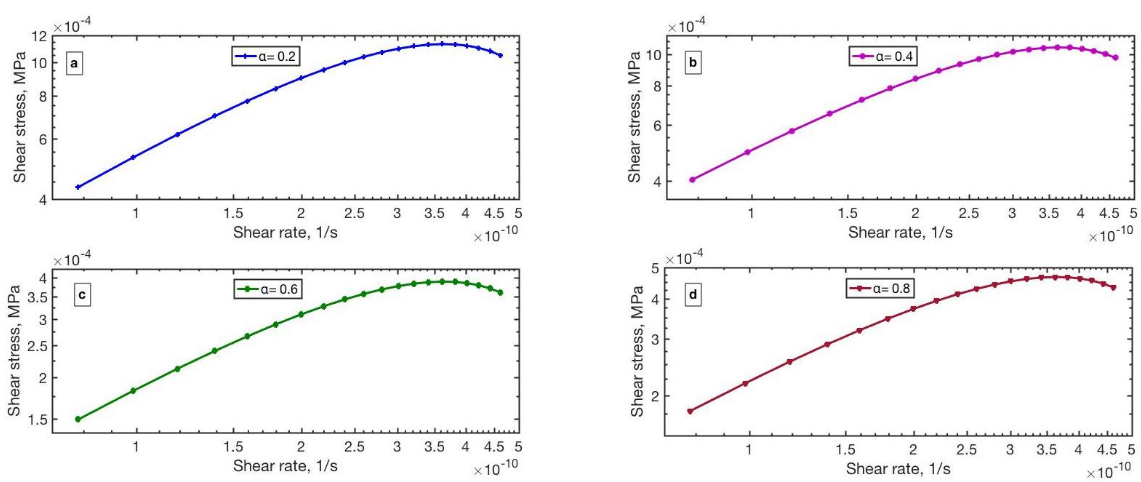


Figure 4.11: Shear stress variation as a function of shear rate for different α values

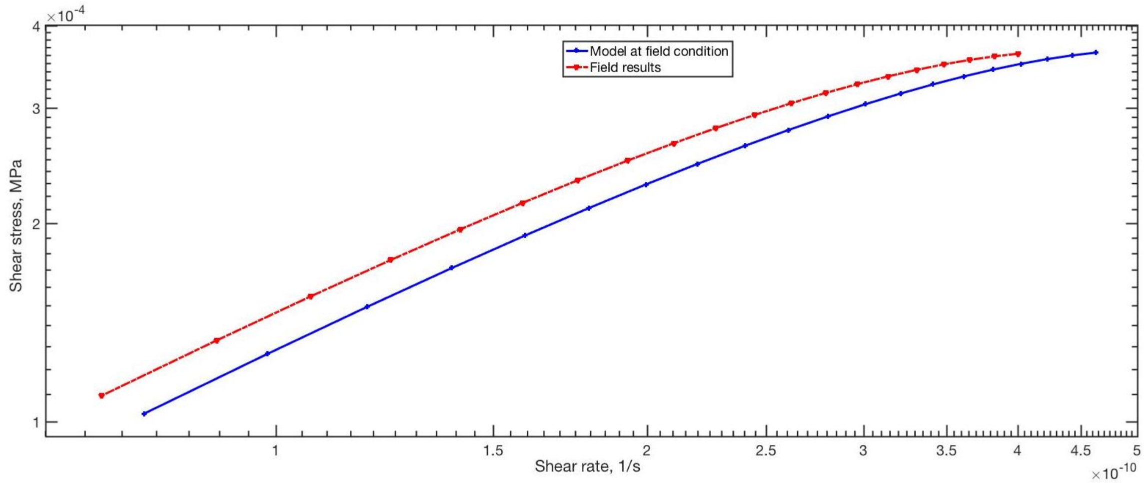


Figure 4.12: At field condition shear stress variation as a function of shear rate

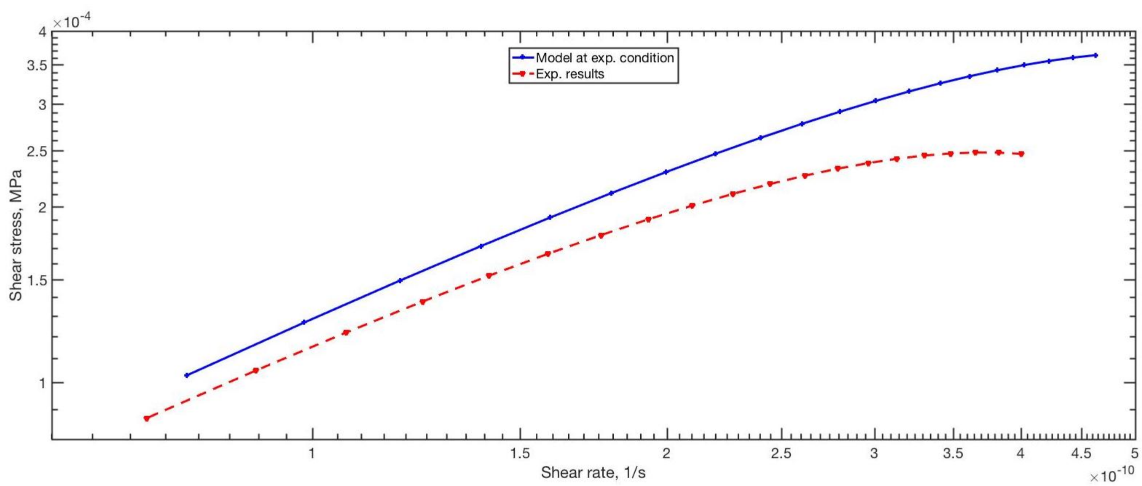


Figure 4.13: At exp. condition shear stress variation as a function of shear rate

4.5.5 Comparison of modified Stress-Strain model with Hossain *et al.,* (2007a) Model

Figure 4.14 illustrates the variation of shear stress verses rate of shear of the proposed model (Eq. 13) for different α to compare with Hossain *et al.,* (2007a) model in the log-log plot. The trend and shape for both models are almost same and show the variation of shear stress value with different α values for both cases. In the proposed model, values of shear stress development for α = 0.2 is mostly match with Hossain *et al.,* (2007a) model though stress development is less. For α = 0.3 stress development for proposed model equation is less than α =0.2 also from Hossain et al., (2007a) model though trend is mostly the same. With the increase of α value stress development is decreasing as a result flow will increase for the proposed model equation where surface tension, pseudo-permeability, pressure gradient is considered. The stress development also decreased with increased α values for the modified stress-strain model. Figure 4.15 shows the comparison between proposed model with fields results and Hossain *et al.,* (2007a) mode and the proposed model show similar trend, and closer to field condition than Hossain *et al.,* (2007a) model. Stress development for same conditions is less than filed results and Hossain *et al.,* (2007a) model results. Figure 4.16 also shows the comparison between proposed model with experiment results and Hossain *et al.,* (2007a) mode and the proposed model show a good match with experimental condition than Hossain *et al.,* (2007a) model. In Figure 4.16, trend is almost same for three plots stress development with increasing shear rate is less for experimental results which is very close to the proposed model results.

This model equation considered the effect of surface tension, capillary force, viscous force, temperature, and fluid memory. In this article, fluid memory term is solved with finite difference method. The modified model is showing consistent of shear stress value with increasing strain rate for both field and experiment study. Surface tension, capillary effect and memory mechanism was not considered properly in Hossain *et al.,* (2007a) model. Therefore, the proposed modified stress-strain model is more applicable to show the effect of fluid memory in the rheological study of fluid flow through porous media.

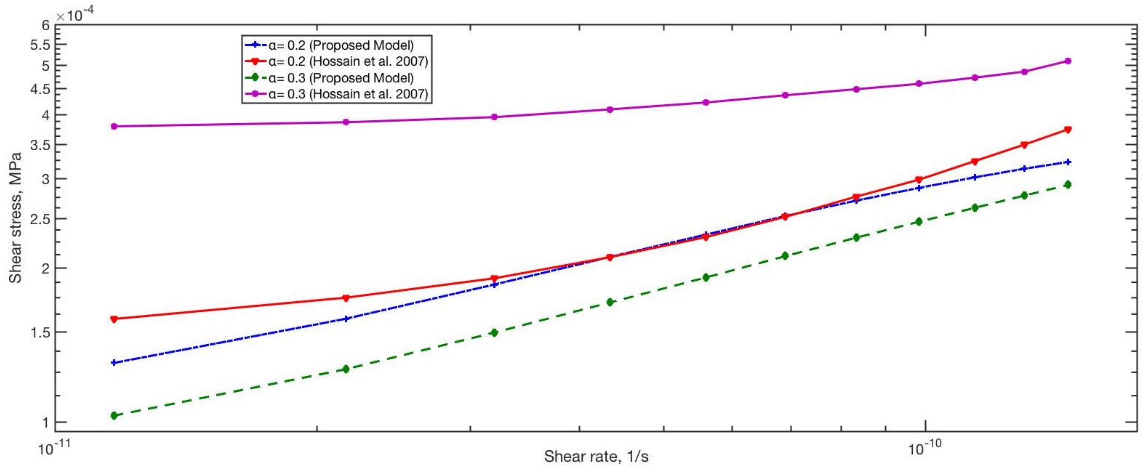


Figure 4.14: Comparison of modified stress-strain model with Hossain *et al.* (2007a) model.

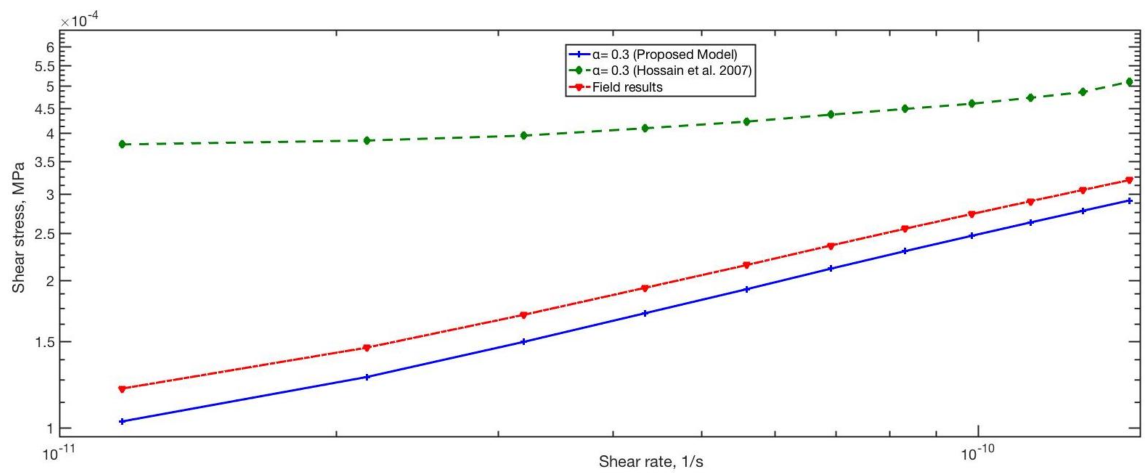


Figure 4.15: Comparison of proposed stress-strain model with field results and Hossain et al. (2007a) model.

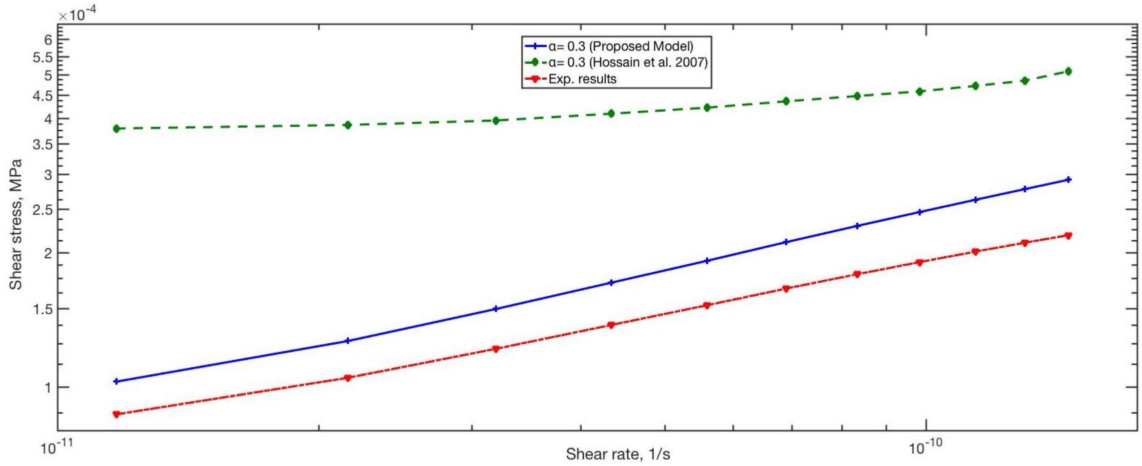


Figure 4.16: Comparison of proposed stress-strain model with exp. results and Hossain et al. (2007a) model.

4.6 Conclusions

In this article, a modified memory-based stress-strain model is proposed to show the continuous effect of fluid memory in the formation. This model will help to characterize reservoir fluids, show the rheological behavior of reservoir fluids, and the overall effect of fluid media and memory in characterization process. Several fluid properties are considered to develop this modified stress-stain model which can capture actual scenario. The model equation is solved numerically using field and experimental data. Surface tension, capillary effect, pseudo-permeability, pressure gradient, and fluid memory have a good amount of impact in the stress-strain behavior of formation fluids. With the effect of fluid memory, the stress development in the formation reduced with time which helps to get better fluid flow in the formation. The model validation is done with the available experimental and reservoir data from the literature, compared with the established stress-strain model. In this paper, α= 0.3 is considered to show more effective scenario of fluid memory based on literature. Fluid media and memory are considered to develop this model and the results show a good match with the existing model and data. This modified stress-strain model can be used in various formations for fluid characterization, and rheological study more effectively.

4.7 Nomenclature

C_a	Capillary number
E	At 32.8 API gravity activation energy of crude oils, *KJ/mol*
EOR	Enhanced Oil Recovery
k	Reservoir permeability, *mD*
M_a	Marangoni number
p	Reservoir pressure, N/m^2
P_i	Initial reservoir pressure, N/m^2
ΔP	P_T-P_o = Pressure difference, N/m^2
$p\ (x, t)$	Reservoir fluid pressure, *Pa*
q_x	Volumetric flow rate in x-direction, kg/m^2-s
t	Time, *sec*
T	Temperature, °*K*
u_x	Fluid velocity in x-direction, *m/s*
y	Distance from the boundary layer, *m*
α	Fractional order of differentiation
η	Pseudo-permeability of fluid, $m^3s^{1+\alpha}/kg$
ϕ	Porosity of fluid media
ρ_o	Density of the fluid at initial temperature T_o, kg/m^3
σ	Surface tension, *N/m*
γ	Velocity gradient at y direction, 1/*s*
τ	Shear stress of fluid, *Pa*
τ_T	Shear stress at reference temperature T_o, *Pa*
μ	Dynamic viscosity of fluid, *Pa-s*
μ_o	Dynamic viscosity at reference temperature T_o, *Pa-s*
1-D	One dimensional
3-D	Three dimensional

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Chapter 5

A Memory-based Fluid Density and Effective Viscosity Model for Reservoir Characterization

Preface

This paper to be submitted to a Journal. The lead author performed the necessary literature review on fluid properties and fluid memory. The co-authors Tareq Uz Zaman helped in mathematical techniques, showed some coding techniques Dr. Salim Ahmed reviewed the manuscript and Dr. M. Enamul Hossain helped in identifying the gap in research, supervising the research, and editing the manuscript.

5.1 Abstract

Rock and fluid properties are very important features for any reservoir characterization. Those properties of are changing continuously because of pressure and thermal change in porous media. Rock properties such as porosity, permeability, compressibility, etc., as well as fluid properties such as density, viscosity, surface tension, and other PVT properties play a vital role in the continuous change of complex reservoir structure. Memory capture the effect of previous events and presents development for current and future events. Fluid memory is a function of space and time which is used in porous media to forecast future outcomes from the past. The pressure gradient of a reservoir can be presented with the help of fluid memory as a continuous function of time and space. Fluid memory is one of the key fluid features but it is overlooked in the fluid models. Therefore, it is necessary to

consider continuous time function for characterizing the fluid of a reservoir. This alteration of fluid properties (e.g., density and viscosity) can be characterized by memory concept. The objective of this study is to develop a memory-based density and effective viscosity model for crude oil considering continuous time function and solving the model equation numerically. In this paper, the modified model is developed based on fluid memory that shows a relationship between density-effective viscosity and pressure difference over time. The proposed model shows a good range for both shear rate and effective viscosity for zero and infinity shear region, and shows the effect of fluid memory on effective viscosity calculation.

5.2 Introduction

From the beginning of time, humans have consumed energy for several reasons. A world without energy cannot be imagined because every step of human life depends on various energy sources. The petroleum industry is one of the most important sources of energy all over the world. In Figure 5.1, the total supply of energy is shown between 2016-2017 and almost 51.50% (Oil and Natural Gas) energy is consumed by the petroleum industry. Several new techniques have been applied to upgrade the oil and gas production from a reservoir. The enhanced oil recovery (EOR) technique is one of the keys to increasing the oil and gas production (Hossain *et al.,* 2008b, 2009a). In a reservoir, shear thinning fluids sometimes act as time-dependent thixotropic fluids. In porous media, fluid viscosity can be measured as an overall or "up-scaled" way known as apparent viscosity. For polymeric solutions (i.e., crude oil, chemical mixture, etc.) the apparent viscosity is a function of flow rate and the flow rate is also correlated with the shear rate. In porous media, the fluid flow is correlated with the fluid memory and presence of a mineral or other particle movement may reduce the response of the fluid. This decrease can create a restriction of the crude oil flow through the porous media. This paper illustrates reservoir fluid properties of porous media and proposes a relation between fluid density, viscosity, and fluid memory.

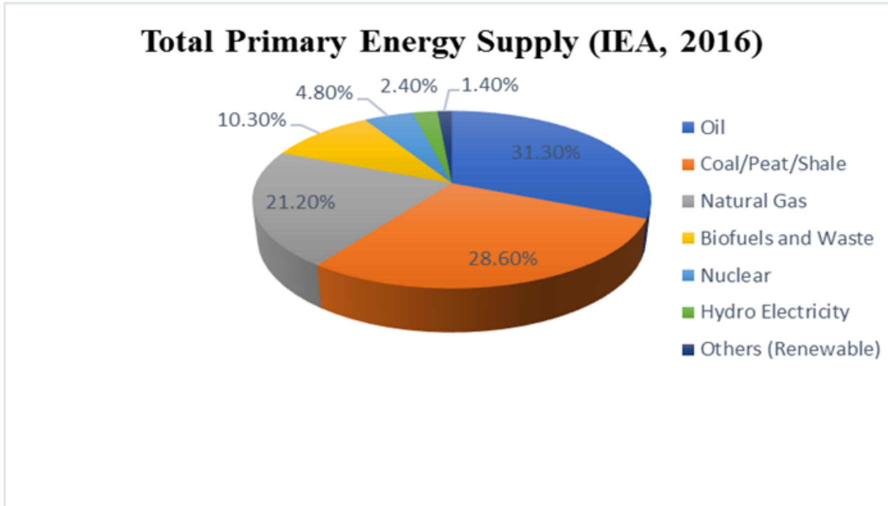

Figure 5.1: Total Energy Supply (modified from IEA - Key world energy statistics, 2016)

Li et al. (2001) completed a stress analysis of air bubbles for non-Newtonian fluids and found that fluid viscosity increases at a constant rate of 1*103 to 1*104. These authors have concluded that water has both solid, and viscous properties. In nanometer scale, water could be used as a lubricant (Mauk, 2007). However, fluid behaves differently under molecular constraints and slow flow rate is a usual phenomenon. This kind of flow exists in nature such as the blood circulation in mammalian lungs, water films in eyes, and in artificial cases such as microchip fabrication, oil recovery, etc. (Perazzo and Gratton, 2003; Hossain, 2008a; Hossain et al., 2007, 2008b, 2009a). Generally, non-Newtonian fluid flow is considered to explain much of natural phenomena (Perazzo and Gratton, 2003; Arratia et al., 2005; Hossain, 2008a). Water that exhibits viscous flow is mostly considered Newtonian fluid, but now it is starting to be considered water as the non-Newtonian fluid. With time and research, it is clearer that nature fluids are more revealed to the non-Newtonian fluid. Though several models have already been developed considering non-Newtonian fluid, still, it is a necessity to come up with a comprehensive model which can be used for a large range of viscosity relation with fluid memory.

To present the true picture, the fluid density-viscosity relationship should consider the fluid memory in the model. The classical reservoir simulation based on the formation permeability along with the flow of the fluid but never maintain the exact flow path. Fluid memory plays an important role to overcome this problem by monitoring the flow. The fluid memory displays a unique feature as its working technique changes with time and different media conditions. The fluid memory is defined as a continuous function of time, and space considering all the fluid properties. Some fluids (i.e., incompressible and viscous fluids) show few different behaviors that can be described easily by fluid memory. The literature is very limited to show the actual picture of fluid memory phenomenon in a comprehensive way.

Fluid memory is used in porous media to show the continuous change of rock and fluid properties with time and space. Several researchers have incorporated fluid memory in their models to show the exact scenario of time alteration in porous media. The fluid flow scenario of porous media is a little bit different from pipeline flow because of its complex rheological structure. In the literature, several researchers have shown the rock-fluid properties of porous media with the help of fluid memory (Slattery, 1967; Eringen, 1991; Nibbi, 1994; Caputo, 1999; Zhang, 2003; Chen *et al.,* 2005; Hossain *et al.,* 2007, 2008b 2009a and 2009b; Hossain, 2008a, Di Guiseppe *et al.,* 2010; Raghavan and Chen, 2013; Rahman *et al.,* 2016; Obembe *et al.,* 2017).

Kondic et al. (1996) reviewed Hele-Shaw cell using Darcy's law to describe non-Newtonian fluid viscosity which depends on the shear-rate and pressure gradient. However, authors derivation does not agree with active shear-thinning which is only correlated to the slip layers of the flow. Miranda (2004) studied non-Newtonian fluid at tension condition to show the effect of shear stress. To develop the shear-rate model, the researcher used modified Darcy's law and concluded that, for shear thinning fluid the adhesion force decreased with less separation. Afanasiev et al., (2007) used the power-law, the Ellis, and Carreau model to review the drag conditions at different inclination angle for shear-thinning fluids. These researchers considered steady state condition and several rheological parameters. However, various examples of fluids (i.e., polymeric, and suspension) represent non-linear relationship for stress-strain. The fluids (i.e., Newtonian or non-Newtonian) can be distinguished based on the rate of shear. At the very low shear rate, polymeric fluid behaves like a Newtonian fluid but at a higher shear rate, the fluid starts to behave as a non-Newtonian fluid. Usually, the power-law model is used at a higher shear rate and the Ellis model is used at lower shear rate (Afanasiev et al., 2007, Hossain, 2008a).

Frank and Li, (2005) studied Newtonian and non-Newtonian fluids and discussed some unusual behaviors of non-Newtonian fluids such as the reverse movement behind a bubble. Authors concluded that shear stress relaxation and fluid memory are the causes for that movement. Huang and Lin (2007) suggested that the memory effect and exponential alignment of length decreased with increasing thermal noise. Few models have been developed to include the fluid memory for thixotropic fluids (Lissant, 1974; Parker, 1992). Usually, fluids used in oil field applications are complex fluids (i.e., polymeric solutions) which can be shear-thinning or thixotropic fluid. In the literature, the available mathematical models (i.g., Power law, Carreau, or Cross models etc.) state the fluid rheology by defining fluid viscosity and apparent shear rate from the Darcy's flow velocity (Cannella *et al.,* 1989; Sorbie and Huang, 1991; Escudier *et al.,* 2001; Lopez and Blunt, 2004). From experimental studies, researchers have drawn the conclusion that the shape of the apparent viscosity curve and bulk shear rate is similar in most cases. In the available literature, most of the experiments have been performed by Xanthan biopolymers (Chauveteau and Kohler, 1974; Chauveteau, 1982; Sani *et al.,* 2001; Taskiroglou, 2004; Lopez, 2004a).

Several constitutive equations have been established to capture the actual rheological behavior of shear-thinning fluids (Chauveteau, 1982). Escudier et al. (2001) developed a rheological model for non-Newtonian fluids running several experiments on Xanthan gum. Lopez (2004) illustrated Carreau-Yasuda, Cross and Truncated Power-law models and presented similar outcome considering effective viscosity of shear thinning fluids for all the models. Hossain (2008a) tried to develop models for shear stress, shear rate, and viscosity for complex polymeric fluid (e.g., crude oil) considering continuous function of time, space, and other fluid properties. In this study, the Carreau-Yasuda model is considered to develop fluid density, and effective viscosity model and incorporate the fluid memory for fluid rheology with time alteration (Carreau, 1972; Yasuda *et al.,* 1981; Sorbie, 1989; Escudier *et al.,* 2001; Lopez, 2004; and Hossain 2008a).

5.3 Mathematical Model Development

Many mathematical equations have been developed to capture the actual fluid rheological characteristics of shear-thinning fluids in the past. The Carreau-Yasuda, Cross, and, Power-law models give almost the same results when they represent the fluid rheology (e.g., effective viscosity) of shear thinning fluids (Escudier *et al.,* 2001; Lopez, 2004). Therefore, the Carreau-Yasuda effective viscosity model is considered in this study to develop fluid effective viscosity and density relation as well as capture the memory effect for fluid rheology. The Carreau-Yasuda model can be written as (Carreau, 1972; Yasuda *et al.,* 1981; Sorbie, 1989; Barnes, 1997; Escudier et al., 2001; Lopez, 2004; Hossain, 2008a and Hossain et al., 2009a):

$$\mu_{eff} = \mu_\infty + \frac{\mu_o - \mu_\infty}{[1+(\lambda\,\gamma)^a]^{\frac{n-1}{a}}} \tag{1}$$

Blake (1922) proposed a dimensionless number for porous media to show the effect of inertia, and viscous force which was also known as the modified Reynold's number. In Blake number, fluid density, fluid velocity, hydraulic diameter for porous media, fluid initial viscosity and the void fraction is used to show both effects. Several researchers used the modified Reynold's number (Blake number) and provided the laminar and turbulent flow ranges for packed bed porous media (Burke and Plummer, 1928; Ergun and Orning, 1949; Ergun, 1952). The Blake number can be expressed as follows:

$$\mathrm{B} = \frac{u_x\,\rho\,D_h}{\mu_o\,(1-\varepsilon)} \tag{2}$$

Porosity (ϕ) or void fraction (ε) defined as the ratio of pore volume to bulk volume can be written as $\phi = \varepsilon = V_p/V_b$. Thus Eq. (2) can be written as:

$$B = \frac{u_x \rho D_h}{\mu_o (1-\phi)} \tag{3}$$

$$\mu_o = \frac{u_x \rho D_h}{B(1-\phi)} \tag{4}$$

Using the value of μ_o in Eq. (1) from Eq. (4) can be written as:

$$\mu_{eff} = \mu_\infty + \frac{\frac{u_x \rho D_h}{B(1-\phi)} - \mu_\infty}{[1+(\lambda \gamma)^a]^{\frac{n-1}{a}}} \tag{5}$$

Caputo (1999; 2000) modified Darcy's law by introducing the fluid memory which represents the effect of reduction of the formation permeability with time. If the fluid flow is considered in the x-direction, the mass flow rate equation can be written as (Caputo, 1999; 2000; Hossain *et al.,* 2007; 2008b; 2009a; 2009b, Hossain, 2008a):

$$q_x = -\eta \rho_o \left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\right] \tag{6}$$

In Eq. (6) $q_x = \frac{q_x' \rho_o}{A}$ and $0 \le \alpha < 1$ so Eq. (6) can be written as:

$$u_x = -\eta \left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\right] \tag{7}$$

Here, u is the reservoir fluid velocity in porous media. Putting the value of u in to Eq. (5) becomes:

$$\mu_{eff} = \mu_\infty + \frac{(-1)\frac{\rho D_h}{B(1-\phi)}\eta\left[\frac{\partial^\alpha}{\partial t^\alpha}\left(\frac{\partial P}{\partial x}\right)\right] - \mu_\infty}{[1+(\lambda \gamma)^a]^{\frac{n-1}{a}}} \tag{8}$$

Where, η is the pseudo permeability of the porous media. η can be shown with the below equation (Hossain *et al.,* 2008a, 2008b) as follows:

$$\eta = \frac{K}{\mu}(t)^\alpha \tag{9}$$

In Eq. (9), K = permeability, μ = fluid viscosity, t = time and α= order of differentiation. Putting the value of η in Eq. (8) from Eq. (9) it becomes:

$$\mu_{eff} = \mu_{\infty} + \frac{(-1)\ \frac{\rho\, D_h}{B\,(1-\phi)}\left[\frac{K}{\mu}\,(t)^{\alpha}\left\{\frac{\partial^{\alpha}}{\partial t^{\alpha}}\left(\frac{\partial P}{\partial x}\right)\right\}\right] - \mu_{\infty}}{[1+(\lambda\,\gamma)^{a}]^{\frac{n-1}{a}}} \tag{10}$$

The above developed mathematical model represents the effect of reservoir formation, porosity, permeability, fluid density, viscosity, strain rate, and fluid memory. A comprehensive relation between density, effective viscosity, and the apparent shear rate is also shown in the model.

5.4 Numerical Analysis of The Model

The results of the comprehensive stress-rate of strain model can be obtained by solving Eq. (10) which is shown above. In this paper, we focused on fluid density-viscosity relation fluid memory. We consider a sample reservoir from the production wellbore (Hossain, 2008a) and experimental data (Iaffaldano et al., 2006) for numerical calculation. The reservoir is isolated and oil is producing at a constant rate. The fluid is assumed to be an API 32.8 gravity crude oil at 298oK temperature. To solve the proposed viscosity-density model presented in Eq. (10), fluid viscosities at low (μ_o) and high shear rate (μ_∞) are considered as 13.2 Pa-s, and 0.00212 Pa-s respectively and the power-law index, (n), factor (a), and the time constant, (λ) are taken as 0.689, 0.75 and 60.7 s, respectively for numerical computation (Chauveteau and Kohler, 1974; Sorbie, 1989; Barnes, 1997; Escudier *et al.,* 2001; Lopez, 2004a, 2004b; Hossain, 2008a; Hossain *et al.,* 2009a). All computations are carried out by MATLAB. All computations are carried out by MATLAB.

Table 5.1: Sample Reservoir Data (Hossain, 2008a)

Reservoir length, *l*	5000 m
Reservoir width, *w*	100 m
Reservoir height, *h*	50 m
Porosity, ϕ	30%

Permeability, k	30 md = 30* 10^{-15} m^2
Initial reservoir pressure, p_i	27579028 pa (4000 psia)
Compressibility, c	1.2473 × 10^{-9} 1/pa
Initial viscosity, μ_o	13.2 Pa-s
Initial flow rate, q_i	8.4 ×10^{-9} m^3/sec
Initial fluid velocity, u_i	1.217 ×10^{-5} m/sec
Fractional order of differentiation, α	0.2-0.8
Number of grid in space, N_t	580

Table 5.2: Experimental Data (Iaffaldano *et al.,* 2006)

Length of cylinder, l	11.6 *cm*
Inner diameter, D_i	10.1 *cm*
Volume of the cylinder, V_c= $\pi r^2 h$	929.374 cm^3
Permeability, k	26 *Darcy*
Viscosity, μ	1.0266 *cp*
Sand density, ρ_s	2.4 gcm^{-3}
Mass of sand in cell, M_s	1550 *gm*
Volume of sand, V_s	645.83 cm^3
Porosity, ϕ	0.3050913841
Fluid density, ρ_f	0.998408 gcm^{-3}
Compressibility, c_t	2.05743* 10^{-4} atm^{-1}
dp/dx	0.01765982953 *atm/cm*
Δp	0.2048540225 *atm*
Number of grid in space, N_t	580

In Eq. (10), D_h is used in the first part as hydraulic diameter. We consider the reservoir shape is rectangular and fully filled with fluid. If reservoir length, *l* = a and width, *w* = *b*, so D_h becomes as:

$$D_h = \frac{4\,a\,b}{2\,(a+b)} \tag{11}$$

$$D_h = \frac{2\,a\,b}{(a+b)}\text{ ; a >> b} \tag{12}$$

In Eq. (10) $\left[\frac{K}{\mu}(t)^{\alpha}\left\{\frac{\partial^{\alpha}}{\partial t^{\alpha}}\left(\frac{\partial P}{\partial x}\right)\right\}\right]$ is considered to show the effect of fluid memory. Here, $\left\{\frac{\partial^{\alpha}}{\partial t^{\alpha}}\left(\frac{\partial P}{\partial x}\right)\right\}$ is the change of pressure with space and time. In addition, $\frac{K}{\mu}(t)^{\alpha}$ is used to show the effect of rock and fluid properties with time alteration. α value is showing the variation of the order of differentiation. In this paper, we calculate this $\left[\frac{K}{\mu}(t)^{\alpha}\left\{\frac{\partial^{\alpha}}{\partial t^{\alpha}}\left(\frac{\partial P}{\partial x}\right)\right\}\right]$ term as fluid flux ($m^3/m^2\ s$). We used Iaffaldano *et al.* (2006) experimentally validated data which is simulated, and validated numerically by Zaman (2017). Different authors showed the variation of α value to capture the memory effect (Iaffaldano, 2006, Hossain, 2008a, Histrov 2014, Obembe et al., 2017). Recently, Zaman (2017) shows that the best choice for α value is 0.3 which can give a good agreement for fluid memory. The results of this study are compared with the well-established Carreau-Yasuda model (Carreau, 1972; Yasuda *et al.,* 1981).

To solve the fluid memory term (partial differential part) in Eq. (10), it is necessary to consider space and time for pressure calculation. The finite difference method is used to solve the fluid memory term $\left\{\frac{\partial^{\alpha}}{\partial t^{\alpha}}\left(\frac{\partial P}{\partial x}\right)\right\}$ of Eq. (10). The discretized form of fluid memory term from Eq. (10) as follows:

$$\left.\frac{\partial^{\alpha}}{\partial t^{\alpha}}\left(\frac{\partial P}{\partial x}\right)\right|_i^n$$

$$= \frac{1}{\Gamma(2-\alpha)}\,\frac{1}{(\Delta t)^{\alpha}}\sum_{J=1}^{n}\left[J^{1-\alpha} - (J-1)^{1-\alpha}\right]\left[\left(\frac{\partial P}{\partial x}\right)_i^{n-J+1} - \left(\frac{\partial P}{\partial x}\right)_i^{n-J}\right] \tag{13}$$

$$= \frac{1}{\Gamma(2-\alpha)\,(\Delta t)^{\alpha}} \sum_{J=1}^{n} [J^{1-\alpha} - (J-1)^{1-\alpha}]\,[\frac{P_{i+1}^{n-J+1} - P_{i}^{n-J+1}}{\Delta x} - \frac{P_{i+1}^{n-J} - P_{i}^{n-J}}{\Delta x}] \tag{14}$$

$$= \frac{1}{\Delta x\,\Gamma(2-\alpha)\,(\Delta t)^{\alpha}} \sum_{J=1}^{n} [J^{1-\alpha} - (J-1)^{1-\alpha}]\,[P_{i+1}^{n-J+1} - P_{i}^{n-J+1} - P_{i+1}^{n-J} + P_{i}^{n-J}] \tag{15}$$

$$= \frac{1}{\Delta x\,\Gamma(2-\alpha)\,(\Delta t)^{\alpha}} \sum_{J=1}^{n} [J^{1-\alpha} - (J-1)^{1-\alpha}]\,[P_{i+1}^{n-J+1} - P_{i+1}^{n-J} - P_{i}^{n-J+1} + P_{i}^{n-J}] \tag{16}$$

Eq. (16) can be solved using initial and boundary conditions.

Using Eq. (16) we calculated the flux value of the fluid in porous media. For reservoir, we used 580 grids in space and for experiment condition, we used 580 grids in space. To calculate fluid flux, all the values of previous time steps were considered. In Figure 5.2, the alignment of grid point in space is shown.

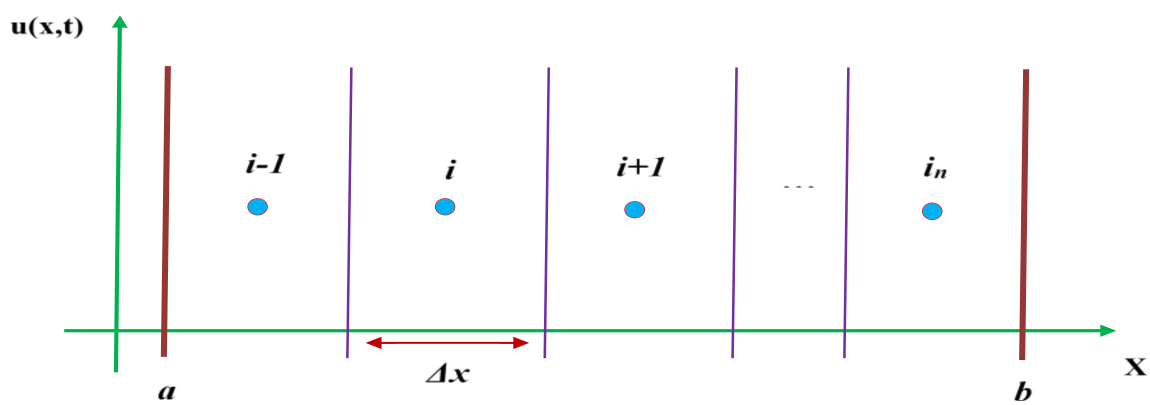

Figure 5.2: Sample reservoir grid point in space (modified from De Sterck and Ullrich, 2009)

5.5 Results and Discussion

5.5.1 Dependency of fluid density on effective viscosity

Viscosity and density are two most important fluid properties and play significant role in fluid flow through porous media. Fluid viscosity and density related closely with formation temperature and pressure. In this study, formation temperature is considered isothermal but pressure is changing with space and time. Formation pressure is changing as a result fluid density and viscosity is also changing. Several researchers have shown the relation between pressure, fluid viscosity and density for various non-Newtonian fluids (Chou and kokini, 1987; Morris *et al.,* 1995; Yeo and Kiran, 2000; Togrul and Arslan, 2003; Liu *et al.,* 2006; Chan Eu, 2007; Tamara, 2008; Grem *et al.,* 2013; Abdeen and Mohammad, 2014; Diogo *et al.,* 2014, 2015, Abdali *et al.,* 2016). In the literature, few resources are available focusing petroleum fluids. In this article, fluid effective viscosity represents as a function of density consider flow regime in porous media and memory effect. Figure 5.3 represents the relation between fluid density and effective viscosity in semi-log plot. In Figure 5.3a to 5.3d, different α values (0.2-0.8) are considered to show the effect of fluid memory in density-effective viscosity relation. Figure 5.3a shows almost a linear relation between density and effective viscosity for α= 0.2, still, plot shows a little bit curvature shape which is the effect of fluid memory. In Figure 5.3, 5.3b to 5.3d shows the same trend as figure 3a and effective viscosity increases with α values. For α= 0.2 gives the minimum and for α= 0.8 gives the maximum effective viscosity for a certain fluid density.

Figure 5.4 illustrates relation between fluid density and effective viscosity for field and experimental condition, and consider α= 0.3. Figure 5.4a and 5.4b both shows the same trend and shape as Figure 5.3. For field and experimental condition results show good match as the data was scaled-up. Field results show more better range of effective viscosity then experimental data considering α=0.3 (Zaman, 2017). The proposed model shows good range and agreement for both field and experimental data.

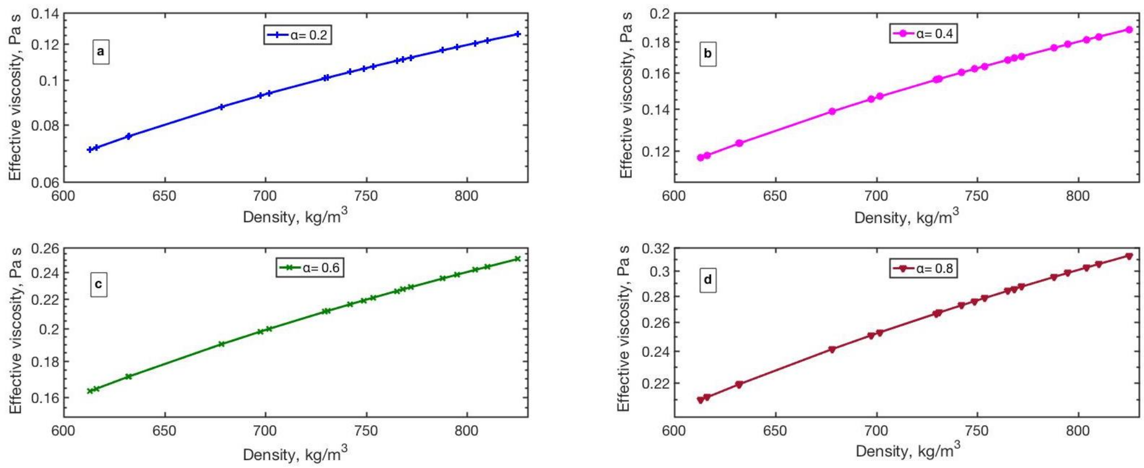

Figure 5.3: Effective viscosity variation as a function of density for different α values

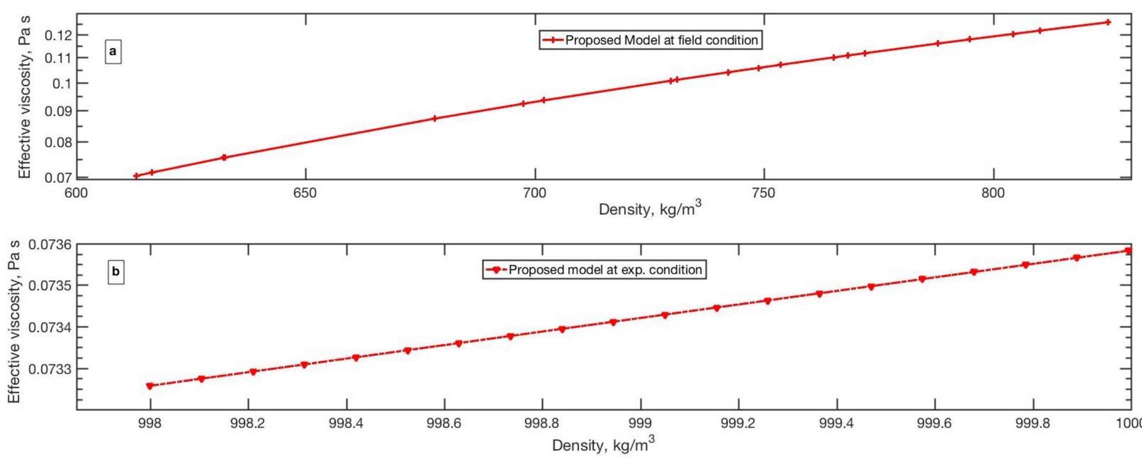

Figure 5.4: At field and exp. condition effective viscosity varies with density for α= 0.3

Figure 5.5 shows the change of fluid effective viscosity as a function of density for different month at α= 0.3. Figure 5.5a to 5.5d represents the same trend and shape as Figure 5.3 and Figure 4. Figure 5.5 shows the actual scenario of memory and shows the nonlinear trend of the curve very clearly. This nonlinear behavior arises because of continuous rock and fluid alteration. With the change of months, the change of effective viscosity is very less. The results show that the initial effective viscosity is around (0.06~0.08) pa and the maximum effective viscosity is around (0.12~0.14) pa.

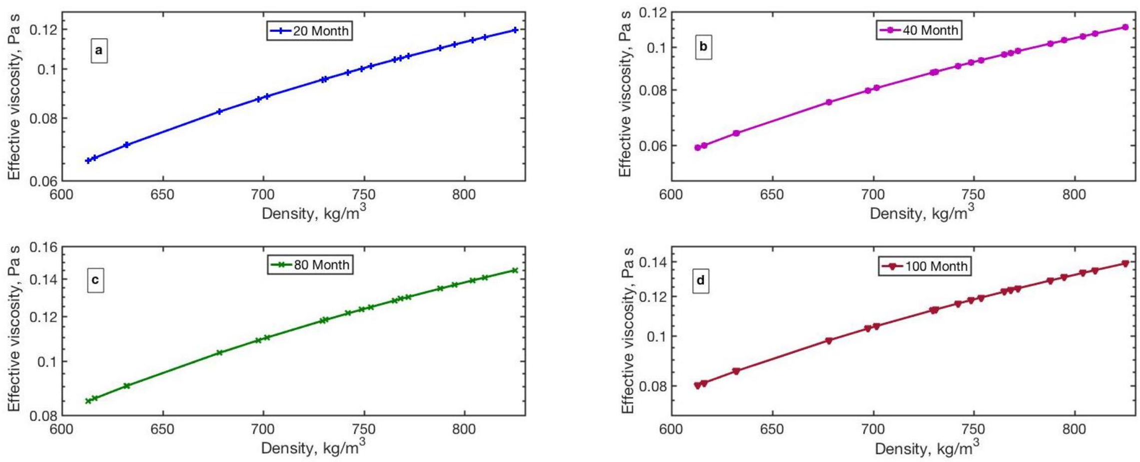

Figure 5.5: Effective viscosity variation as a function of density for different months

5.5.2 Dependency of the Blake number on effective viscosity

Forces (i.e., inertia force, viscous force) have an impact on fluid flow through porous media. Those forces moderate the flow regime type and behavior of flow in porous media. Usually Reynold's number is used in pipeline flow to determine the flow regime. In this article, Blake number (modified Reynold's number) is used to show the effect of inertia and viscous force in porous media. Blake number is calculated for certain depth at constant initial viscosity to show the effect of fluid density. Form the model equation, effective viscosity is inversely proportional to Blake number with all other parameter is constant. Figure 5.6a to 5.6d shows that effective viscosity is decreasing with the increasing Blake number in log-log plot. For different α values (0.2-0.8) effective viscosity is not same though the change not too much and for α= 0.8 initial effective viscosity is maximum and for α= 0.2 its minimum. For α= 0.2 plot shows a curvature shape towards end which reflects the memory mechanism.

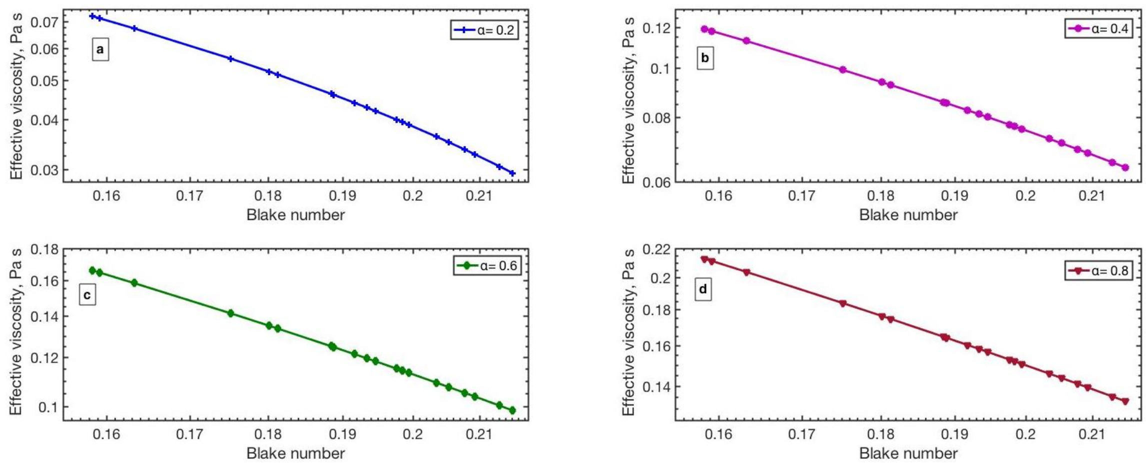

Figure 5.6: Effective viscosity variation as a function of Blake number for different α values

5.5.3 Dependency of fluid flux (memory effect) on fluid flow in porous media

In this paper, experimental data (Iaffaldano *et al.,* 2006) and sample reservoir data (Hossain, 2008a) is used in the simulation to get the pressure data for the grids. Generally, pressure distribution for grids is not available in reservoir or experimental data. To calculate fluid flux, we considered α= 0.3 (Zaman, 2017) for both conditions. Fluid flux (m^3/m^2 s) is different from flow velocity (m/s) in case of porous media. In Figure 5.7, fluid flux is plotted with time and showed that flux values are almost linear when flow reaches the steady condition though for transient condition flux values fluctuated with time. The experiment (Iaffaldano *et al.,* 2006) was run for 11 hours and for almost 6 hours the flow was in transient condition for that reason flux values are fluctuated and after reaching the steady state condition it’s almost a linear and constant with time.

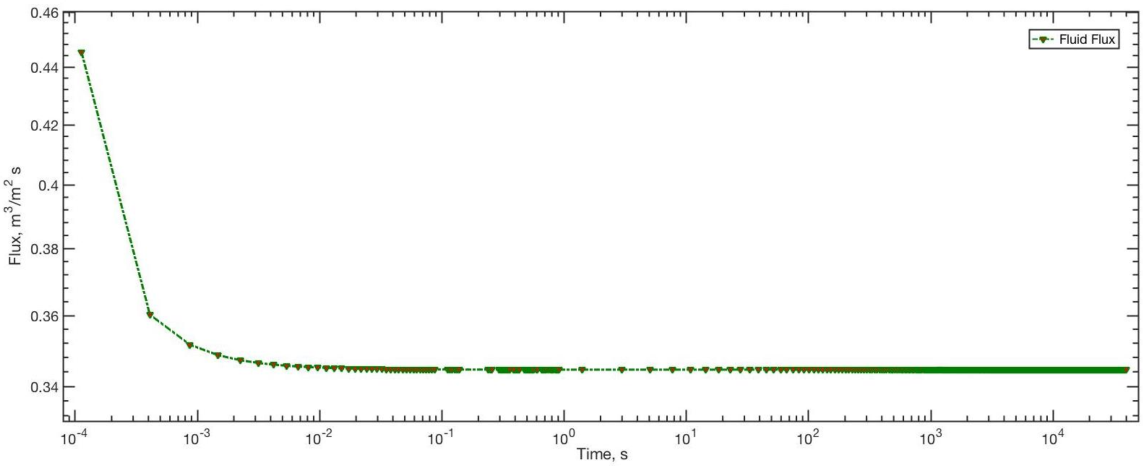

Figure 5.7: Fluid flux variation as a function of time

5.5.4 Dependency of apparent shear rate on effective viscosity

Figure 5.8a to 5.8d shows the variation of effective viscosity with the rate of shear in log-log plotting for different α (0.2-0.8) values. The trend and shape of those plots are almost same except the data variations towards upper and lower level. Those figures trend and shape are almost as it is in the Carreau-Yasuda model (Carreau, 1972; Chauveteau and kohler, 1974; Yasuda *et al.,* 1981; Sorbie, 1989; Lopez, 2004b, Arratia, 2005; Boyd and Buick, 2007; Hossain *et al.,* 2009a, Khan and Hashim, 2015; Whitty *et al.,* 2016, Khechiba *et al.,* 2017). The variation of data range for proposed model equation has varied from Carreau-Yasuda model because of rheological variations and scaled up. Though the shape and trend are almost same though data range is different for different values of α (0.2-0.8). With the increase of α values, the data ranges change for both shear rate and effective viscosity. From those figures, the effective viscosity-apparent shear rate curve trend, shape, and region are dependent on fluid memory and the data range are more dominant at higher (α= 0.8) fluid memory value. As α value increases, effective viscosity decreases with the increasing apparent shear rate.

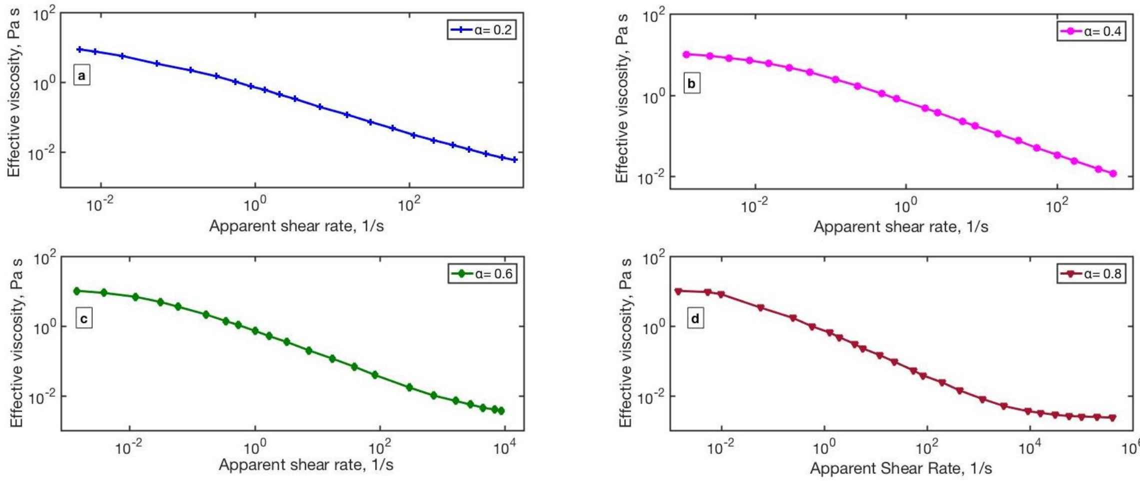

Figure 5.8: Effective viscosity variation with shear rate for different α values

Figure 5.9 shows the relation between effective viscosity and apparent shear rate for α= 0.3. For field and exp. condition we considered α= 0.3 to show the effect of effective viscosity and apparent shear rate. In Figure 5.9, field results show good match with experimental results with up-scaling. Most of the points overlap each other and give a good range of effective viscosity value for zero and infinity shear zone at both condition. Figure 5.9 gives same trend and shape as Figure 5.8 for α= 0.3.

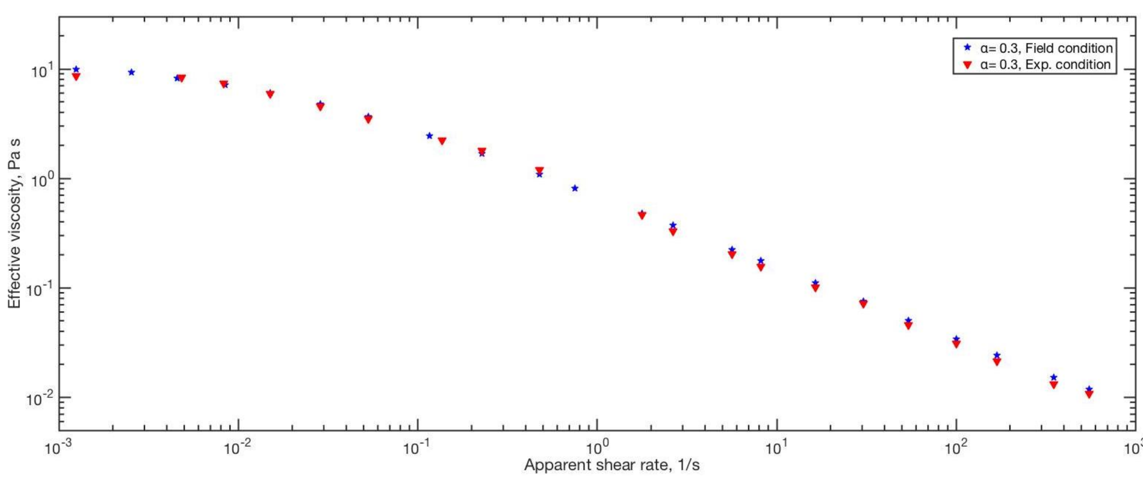

Figure 5.9: Effective viscosity variation with apparent shear rate for field and exp. condition

5.5.5 Comparison of proposed model with Carreau-Yasuda model

Figure 5.10 illustrates the variation of effective viscosity as a function of apparent shear rate in log-log plot for proposed model of different α values with Carreau-Yasuda model. The Shape and trend of the curves for proposed model and Carreau-Yasuda model is almost same and most of the data are matched with each other except the scale of data variation. The proposed model showed the effect of fluid memory and give more information about formation. The proposed model also gives a large range of data in zero and infinity shear condition. Carreau-Yasuda model only applicable for power-law region whether as the proposed model is applicable for wide range of data between zero to infinity shear rate. Therefore, the proposed model is more applicable to characterize both rheological and fluid properties in porous media.

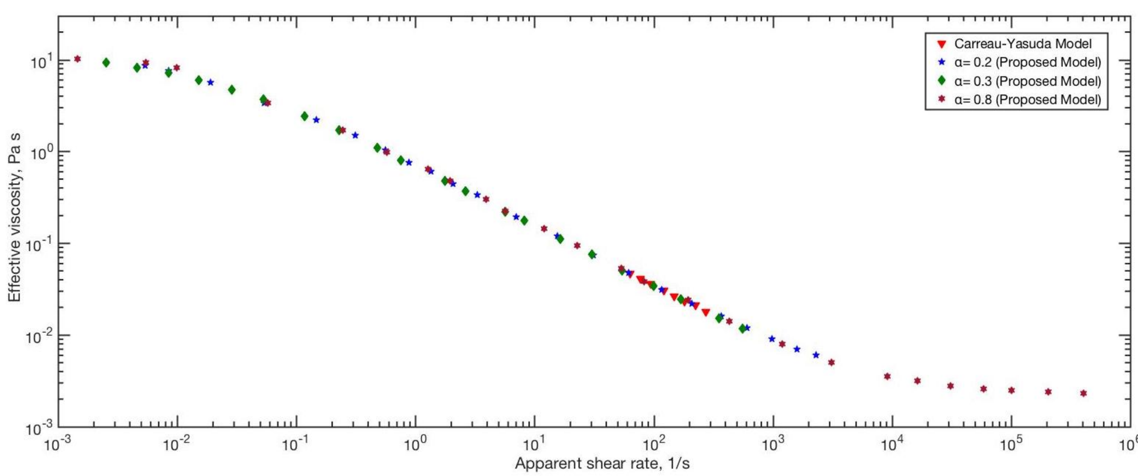

Figure 5.10: Comparison of proposed model with Carreau-Yasuda model

5.6 Conclusions

In this article, a memory-based density-effective viscosity model is proposed to characterize the rheological behavior for light crude oils. This model is solved numerically considering sample field data as well as available experimental data, and compared with the currently used conventional model (i.e., Carreau-Yasuda Model). The proposed model is very effective for representing the physical phenomena of any formation also able to capture a large span of information, covering reservoir fluids that would not be traceable with existing models. In reservoir formation, because of shear stress, the velocity gradient (i.e., shear rate) in y-direction has a non-linear variation, which proves the linear relationship of conventional models are not acceptable. Fluid memory can represent the real picture of the above-mentioned effect. In this paper, we focused on the dependency of the effective viscosity on formation porosity, density, hydraulic diameter, and flow velocity which is related to the effect of fluid memory. The results give a good range of effective viscosity values for zero shear, power-law region, and infinity shear region and show the effect of fluid memory. This study concludes that fluid memory has a great impact on fluid flow and rheological behavior of shear-thinning fluid in porous media considering fluid and formation properties.

5.7 Nomenclature

a	Reservoir length, *m*
b	Reservoir width, *m*
B	Blake number
D_h	Hydraulic diameter, *m*
k	Permeability of the reservoir, *mD*
L	Length of the core, *m*
n	Power-law exponent for Carreau–Yasuda model
p	Reservoir pressure, N/m^2
P_i	Initial reservoir pressure, N/m^2
ΔP	P_T-P_o = Pressure difference, N/m^2
$p\ (x, t)$	Fluid pressure, *Pa*

q_x	Volumetric flow rate in x-direction, kg/m^2-s
R	Universal constant, *kJ/ mol-K*
t	Time, *sec*
u_x	Reservoir fluid velocity in x-direction, *m/s*
V_p	Pore volume, m^3
V_b	Bulk volume, m^3
y	Distance from the boundary plan, *m*
α	Fractional order of differentiation
η	Ratio of the pseudo-permeability to fluid viscosity, $m^3s^{1+\alpha}/kg$
ε	Void fraction
ϕ	Porosity of fluid media
λ	Time constant in Carreau–Yasuda model, s
ρ_o	Density of the fluid at reference temperature T_o, kg/m^3
ρ	Density of the fluid, kg/m^3
γ	Velocity gradient at y-direction, 1/*s*
μ	Dynamic viscosity, *Pa-s*
μ_o	Dynamic viscosity at low shear rate, *Pa-s*
μ_∞	Dynamic viscosity at high shear rate, *Pa-s*
1-D	One dimensional

3-D Three dimensional

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Chapter 6

Conclusion

6.1 Summary

In this thesis, all chapters have its own uniqueness and at the same time those chapters are linked to each other in the consequences of memory mechanism. The proposed models are based on memory mechanism which will open a new dimension for petroleum industry. The proposed models will help in reservoir fluid characterization considering continuous rock and fluid alteration.

Chapter 2 reviews the fractional derivatives, fluid memory, reservoir rheology, and reservoir fluid properties. This review of the literature illustrates the reservoir rheological study in petroleum engineering and shows a complete picture of reservoir fluid properties. In this literature review the definition, use, and importance of memory is shown for engineering and science as well as for porous media.

In Chapter 3, a memory-based stress-strain model is proposed to show the memory effect in porous media. This model considered formation porosity, permeability, fluid density, hydraulic diameter, modified Reynold's number (Blake number), temperature, pseudo-permeability, pressure gradient and shear rate which makes the model as a comprehensive one. The fluid memory term has been discretized and solved the model numerically for both field and experiment conditions. The proposed model is also compared with established memory-based model and captured the trend and variations.

In Chapter 4, a mathematical model is proposed to capture memory effect for stress strain relationship where surface tension, capillary effect, pseudo-permeability, temperature, pressure gradient, and shear rate in y-direction is also considered. The proposed model is solved numerically for field, and experiment data and compared with established model where proposed model showed a good trend with the existing model.

In Chapter 5, a comprehensive model is proposed to show the relationship between fluid viscosity and density considering fluid memory effect. This model is a modified version of Carreau-Yasuda model where memory mechanism is addressed to show the effect of fluid memory in continuous rock and fluid alteration. The proposed model has solved numerically for field and experiment condition and results showed a good agreement with the existing model.

In this thesis, the mathematical models are proposed to show the effect of memory on reservoir fluid properties. The proposed models will help to characterize reservoir fluid properties and capture the effect of memory as well. The proposed stress-strain models are also considered modified Reynold's number (Blake number) and Capillary number to show the effect of fluid density and surface tension gradients on reservoir fluids interface consequently. The results also showed how memory effect influenced the linear stress-strain relationship and presented turbulent behavior with non-linear trend. The proposed memory-based viscosity-density model captured the memory effect for viscosity-density relationship where modified Reynold's number (Blake Number) is also mentioned. And results show that memory has a great impact on fluid viscosity-density relationship.

6.2 Future Work Guideline

- The proposed memory-based stress-strain models are solved numerically with finite difference method and numerical study is done based on some sample reservoir data and experiment data for light crude oil (Chapter 3 and Chapter 4). Therefore, the model analysis need to be done with various data form different field around the world and experimental study can be performed to check performance of the model more accurately.
- In proposed viscosity-density model, Blake number is considered instead of Reynold's number for porous media and memory term is solved with finite difference method (Chapter 5). The numerical analysis is observed for small range of field and experimental data. Therefore, it is needed to analysis for wide range of data from different fields around the world and perform some experiment study.