SHIP RESISTANCE IN ICE FLOE COVERED WATERS

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by

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THIS DISSERTATION IS DEDICATED TO

THE MEMORY OF MY PARENTS

FOUAD and BOTHIANA

ABSTRACT

The developmemt of new analytical and numerical methods for the calculation of ship resistance in broken, pack ice, or ice floe covered waters, as well as full scale and model testing, has been addressed in this dissertation.

In this study three methods for the calculation of ship resistance are presented. The first analytical formulation of ship resistance in ice fragments (The Micro Model) is based on the calculation of energy loss due to an impact between an advancing ship and an ice floe. Based on The Kinetic Theory of Gases (molecule collision and pressure) the number of possible ice floes colliding with the ship, in a given time, was estimated and consequently total ship energy loss and ship resistance was determined. The second approach for the calculation of ship resistance due to ice floes (The Macro Model) was formulated and developed based on the analysis of water drag of displaced ice floes. The Macro Model is based on the prediction of the total forces acting on the ship as a result of the displacement of a large number of ice floes out of the ship's path.

Both analytical models have been formulated to account for both the ship characteristics and ice cover properties. Numerical calculations of ship resistance for existing ship

hull forms have been performed and presented, in this study, using the newly developed models.

A numerical calculation method based on the distinct element method has been introduced in this dissertation as a new approach for the calculation of ship resistance in ice floe or pack ice.

Besides the analytical and the numerical approaches developed, a number of model test experiments were conducted in simulated broken ice cover to measure ship resistance increase due to ice fragments.

A Full-Scale field measurement program was beyond the scope of this study. However, results from existing full scale tests, available in the literature, have been collected, analysed and presented in this study.

Results from the new analytical and numerical models, model test experiments, and full scale field measurements, have been presented in this study to verify the validity of the different approaches.

This study has developed new methods or tools for calculating the ship resistance in broken ice. This study has shown that it is possible to predict with reasonable accuracy, the magnitude of ship resistance in a given ice floe or pack ice cover.

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NOMENCLATURE

A = Area.

A_f = Frontal projected area of ice floes.

A_s = Surface area of ice floes.

B = Ship beam.

C = Ice concentration.

C_b = Ship block coefficient.

C_s = Skin friction coefficient.

C_f = Form drag coefficient.

C_f' = Collision frequancy.

D = Ship displacement.

e = Coefficient of restitution.

E = Energy

F = Force in general.

F_n = Normal force.

F_t = Tangential force.

f_n = Normal force per unit length.

F_r = Froude number.

G = Acceleration of gravity.

h = Thickness.

h_w = Ice floe draft.

I = Mass moment of inertia.

k = index.

K = Coefficient.

K_c = Ice floes compacting coefficient.

l = Ice floe length.

L = Ship length.

LBP = Ship length between perpendiculars.

L_e = Length of ship entrance.

LWL = Ship waterline length.

m = Mass of ice floe.

M = Ship mass.

MFP = Mean Free Path.

n = Number of ice floes.

N_c = Number of ice floe collisions.

N_t = Tangential impulse.

N_n = Normal impulse.

N = Increase in the number of impacts.

$P(t)$ = Impulse

P = Power

Q = Distance from point of impact and maximum beam.

R_t = Ship total resistance

R_w = Open water ship resistance

R_i = Ship resistance due to ice floes.

r = Position vector

s = Distance

ds = Element of length

t = Time

U = Velocity.

v, V = Velocities before impact.

v', V' = Velocities after impact.

V_n = Normal velocity

w = Travel distance

w', w'' = Velocity and acceleration

x, y, z = Coordinates

α = Bow angle

β = Reflected angle with y-axis

δ_i = Ice floe boundary layer thickness

ζ, η, ξ = Coordinates

θ = Incident angle.

θ_1 = Reflected angle.

λ = Scaling factor

μ = Coefficient of friction

σ_i = Density of ice.

σ_w = Density of sea water

Ω = Angular velocity

CHAPTER 1

INTRODUCTION

1.1 SHIP IN ICE

Ships are often required to navigate northern seas, oceans, lakes and rivers in winter months. These ships are faced with the problem of sailing through one form or another of ice cover, floating on the surface of the water. Ice may be free floating, or anchored to the sea floor or to the shore line, either in sheet or non-consolidated form.

Type and extent of ice features depend on location, time of year, and on weather and environmental conditions of wind, current and waves.

Ice has been classified by scientists and engineers based on a number of criterion, such as by type (fresh water or sea ice) and by ice growth stage (new ice, first year ice, multi-year ice). Detailed ice glossary and classification are shown in Appendix I. However, for ship operation in ice covered waters, ice has been classified by Naval Architects into four principal types :

Continous ice cover or sheet ice.

Broken ice pieces or ice floes.

Ice ridges

Icebergs and bergy bits.

Ships advancing through continous or sheet ice experience high resistance, and therefore, require high power or a high power to displacement ratio to maintain reasonable advance speeds. This is due to the fact, that a ship has to break the ice sheet into small ice pieces which have then to be removed or displaced from a ship's path.

When available ship power is not adequate enough for the ship to break the ice sheet, at reasonable speeds; or when the ship is faced with ice thicknesses beyond its capabilities to break in a continous mode, and in case of encounter with ice ridges or ice rubble fields, ships resort to ramming. Ramming is conducted through backing of the ship to an adequate distance then speeding up and impacting the ship's bow against the ice edge, hence discharging its kinetic energy into potential, breaking and crushing energy and as a result the ice fails and the ship advances into the ice slowly through repeated cycles of impacts or rams.

In broken-, or floe-, ice infested waters, ships advance in the ice cover by pushing the ice pieces away from their path. Breaking of small to moderate ice floes seldom occurs as the ship advances through the ice cover by removing the ice blocks from its path. This process of ice floe movement and displacement results in an increase of ship resistance or a decrease of ship speed.

The resistance of a ship in broken ice is generally higher than open water resistance (could be as high as two or more times, depending on ice condition). However, this resistance is usually smaller than ice sheet breaking resistanc. Prediction of the increase of ship resistance in broken ice covered waters, due to ice floes, is the subject of this study.

In the case of ship navigation in waters infested with icebergs or bergy bits, the emphasis is placed on the early detection and the avoidance of icebergs. Ships often voluntarily slow down in these waters so that detection is more accurate and avoidance maneuvers may be performed successfully.

For any ship to operate successfully in ice infested waters it must perform her designated mission efficiently, with reasonably acceptable speeds and without damage. This dictates two basic ship requirements :

- Ship is structurally strong enough to withstand any possible ice damage.
- Ship is powerful enough to maintain a reasonably

acceptable speed in ice.

In order to meet these two requirements the ship has to operate under speed- and ice-limiting conditions. These limitations have to be met in order for the ship to operate safely. Figure 1.1 shows a presentation of limiting condition curves for ship operation, where the hatched area represents the limiting envelope where the ship can operate safely and economically in broken or pack ice.

In this study the ship has been treated as if it has unlimited strength e.g. no structure stresses or damage will occur while sailing in broken ice covered waters. The emphasis of this study is limited therefore, to the analysis of ship resistance and speed relations in ice floe fields.

Most research and development work in the area of ship performance in ice has been devoted to the problem of ship navigation in continuous or sheet ice. However broken ice coverage is typical of large areas in the Arctic and Subarctic regions yet very little attention was given to broken ice navigation problem.

1.2 BROKEN OR FLOE ICE

Broken or floe ice cover is commonly occurs in cold

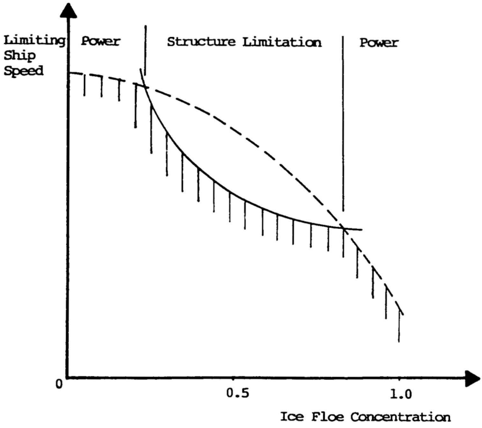

Fig.1.1 Limiting Conditions for Ship Operation in Ice

oceans, rivers or lakes by means of a number of mechanisms :

- Ice floes are formed during initial freeze up under storm conditions.
- Broken ice is produced by wind, waves, currents and thermal stresses.
- Broken ice is produced by icebreakers and ship traffic.

Ships navigating cold waters often sail through waters covered with floe or broken ice. In fact, broken ice is faced by ships more frequently than either ice sheets or ridges. Levine et al. (1974) estimated that ships in the Great Lakes sail through broken ice 75% to 90% of the time, in the winter.

Broken ice covered waters extend over large areas of the Arctic and Subarctic regions. The severity and extent of the ice coverage depends on the location, enviromental driving forces, and the time of the year.

Ice conditions have been monitored, recorded and documented by a number of nations for many years. Information on ice condition and extent are collected using ship observations, aerial photography and satellite imagery. A good example of international cooperation in ice observation in the North Atlantic is the creation and operation of the

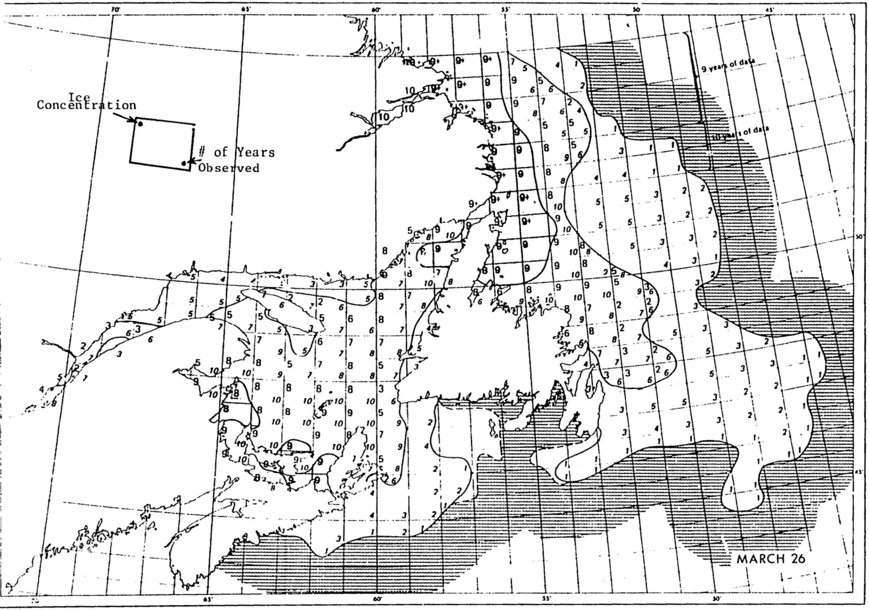

Fig.1.2 Ice Condtions Off Canada's East Coast (After Markham, 198)

International Ice Patrol. Figure 1.2 (explained in detail by Markham, 1980) shows the extent and the condition of ice coverage off Canada's East Coast during a two week period ending March 26. This ice record is based on the average ice conditions observed during the years 1963 to 1973 by the Canadian Department of Environment.

Broken ice cover can be classified on the basis of a number of parameters :

Ice floe concentration.
Size of broken ice pieces.
Type of ice.
Age of ice.
Size distribution of ice floes.
Shape of ice pieces.
Thickness of ice.

The concentration of ice is defined as the total area of ice in a given water surface area divided by this area, or area of ice per unit surface area of ocean. This ratio is expressed in a point system from 1 to 10 where 1 is equivalent to 1/10 or 10 percent ice coverage, and 10 is equivalent to 10/10 or 100 percent ice coverage. As shown in Fig.1.3, from Canadian Coast Guard (1977), pack ice coverage has also been

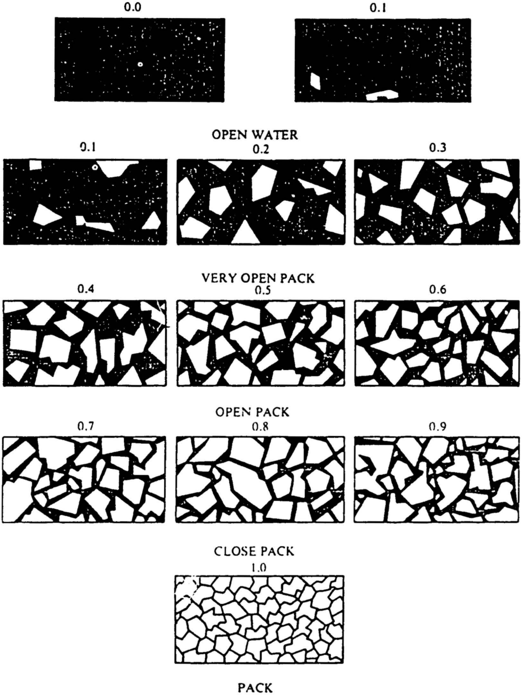

FIG. 1.3 Schematic for Ice Concentration (After CCG, 1977)

classified as, open water, very open pack, open pack, close pack and pack.

The size of ice floes can be classified into (Canadian Coast Guard, 1977):

- Giant floes which are greater than 10 km in diameter.
- Vast floes which are between 2 and 10 km in diameter.
- Big floes which are between 500 and 2000 m in diameter.
- Medium floes which are between 100 and 500 m in diameter.
- Small floes which are between 20 and 100 m in diameter
- Ice cake which is between 2 and 20 m in diameter.
- Brash ice which is less than 2 m in diameter.

Ice floe coverage is usually a mixture of different sizes of ice floes which have an identifiable size trend.

The size of ice fragments depend on many factors such as ice type and thickness, and environmental forces causing the breaking of ice, and possibly on ship navigation activity in the area. A study was conducted by Tuovinen (1979) to measure the size distribution in three different broken ice channels in the Baltic Sea. The measurements of the ice floe size were done with the aide of photographs of the channels.

The study concluded that the size distribuion of ice block follows a log-normal probability distribution function.

Ice type in broken ice covered waters, may be either first year or multi year ice, with differences in mechanical and physical properties. Also, depending on the area concerned ice may be fresh water ice, sea ice or a combination having a salinity somewhere between the two ice types.

The shape of ice fragments is perceived to be random; however, aerial photography of large Arctic areas has shown more triangular and rectangular shapes. Cusps generated by icebreakers are usually of crescent shape which are close to triangular shape.

Newly formed ice floes tend to have a circular shape, and ice fragments often lose their sharp edges and become more circular in shape under the influence of severe environmental conditions.

The thickness of broken fragments of ice depends on the original ice sheet thickness. It also depends on the extent of melting due to rain, sun and water temperature.

Ships advancing through broken ice covered waters experience an increase in resistance, or a decrease in their advancing speed, due to the extra work or energy required to remove the ice from their path.

1.3 THE PROBLEM OF SHIP IN BROKEN ICE

Most research and development attention has been devoted to the problem of breaking continuous or sheet ice. Even in the last decade efforts have been focussed on the problem of ship resistance and powering in continous ice. Little work has been done in the area of ship resistance in broken ice.

The problem of ship performance in ice floe cover is of great importance to both ship designers and operators. This problem became even more important with the tremendous increase in cold water navigation due to :

- Offshore oil and gas exploration and development.
- Ocean resource exploration.
- Mineral exploration and development in Northern regions.
- Fisheries resource utilization.
- Sovereignty and security of the economic and national zones
- Northern settlements.

Large portions of navigation areas covered with the above activities are areas infested with broken or floe ice. These activities do not always require the services of large icebreakers. In fact, the type of vessel which may experience ice floe navigation could be :

Liquid, bulk and general cargo ships.
Ferries and passanger vessels.
Offshore fishing vessels.
Fisheries patrol vessels.
Security and coast guard vessels.
Naval vessels.
Offshore oil and gas supply and service vessels.

1.4 IMPORTANCE OF SOLVING THIS PROBLEM

For any type of vessel intended for Arctic or Subarctic winter operation, it is essential to solve the problem of ship resistance prediction in different ice conditions. Vessels that are being designed for operation in ice floe infested waters, must be able to sail, with reasonable acceptable speeds, in these waters. It is therefore, important to have the knowledge and the tools to evaluate the increases of ship resistance, or the reductions in ship speed, under these floe ice conditions. This knowledge is important in the early design stages of new vessels as it dictates the powering and the propeller design requirements. This same information for existing vessels will allow for the prediction of the magnitude of speed loss or delays, in given ice conditions on certain routes.

In addition to the prediction of ship resistance in broken ice, study of the ship resistance in fragmented ice is important for the understanding of the icebreaking resistance in sheet ice. This is due to the fact that the resistance of a ship in solid sheet ice consists of a number of resistance components including the broken ice clearing component.

Figure 1.4 shows possible benefits and applications of the study of ship resistance in ice floe covered waters.

1.5 HIGHLIGHTS OF PREVIOUS WORK

Most previous work on the ship ice problem has concentrated on the study of continuous sheet ice ship resistance (icebreaking). Recently, a limited number of studies address the problem of broken ice ship resistance. In the early fifties research work in the U.S.S.R. was reported by Nogid (1959). A number of expressions were presented for calculation of ship resistance in broken ice. However, most available work is empirical in nature and is therefore limited in its application for different hull forms or ice conditions.

STUDY OF SHIPS IN ICE FLOES

Ships Following Icebreakers in Sheet Ice

Broken Ice Clearing Component Sheet Ice

Ships Sailing in Pack Ice
(Broken Ice Fields)

Cargo Vessels

Coasters

Ferries & RO/RO

Offshore Supply

Patrol Vessel

S&R

Fishing Vessel

Fig.1.4 Applications of Ship Resistance in Ice Floes

1.6 METHOD OF APPROACH

In the field of naval architecture and ship design three approaches are usually available for the problem of ship resistance predictions, namely calculation of resistance from existing model series results, conducting ship model testing in a towing tank, and performing theoretical calculations using existing formulations. The resistance of ships in broken ice covered waters has been formulated, in this thesis, based on the development of analytical and numerical models. These models calculate the magnitude of ship resistance based on the analysis of the interaction process between the ship and the ice floes. The resistance of the ship was also evaluated based on the measurements of model ship resistance in a simulated ice cover, in a towing tank. Results of the analytical, numerical and experimental approaches, developed in this thesis, are presented together with full scale test results from existing literature.

1.7 STUDY OBJECTIVES

The objective of this study is to formulate and develop methods or tools to predict the magnitude of ship resistance in broken or floe ice (Aboulazm, 1985). The calculation methods should be applicable for different ship parameters and ice cover characteristics. Resistance of ships is to be calculated for ships with different displacement, length, beam and hull shape. Also, the resistance calculation for a given ship is to be possible for different ice concentration, size, thickness and shape.

For verification, the analytical and numerical broken ice resistance calculation methods should be supported by model testing results and available full scale ship test results.

1.8 THESIS LAYOUT

This dissertation is divided into ten chapters. Chapter one covers the background to the problem of ship navigation in broken ice , objectives and applications of the study and the approach used to carry out the work. Chapter two is a review of literature conducted in the area of general ship navigation in ice and in particular ships sailing in broken ice covered waters. The review covers theoretical work, experimental model testing work and full scale field measurements in both Eastern and Western Block countries. Development of the analytical Micro Model is presented in chapter three and of the Macro Model in Chapter four. Chapter five introduces and describes the rigid discrete element method and its application in the numerical calculation of ship resistance in floe ice. The experimental investigation and the model test experiments are reported in chapters six and seven. Chapter eight contains the results of full scale ship resistance from existing literature. Results of this study are reported in Chapter nine. Discussions of the results, the conclusions of this study and recommendations for future research are presented in chapter ten.

CHAPTER 2

REVIEW OF LITERATURE

2.1 ICE NAVIGATION

Throughout the years there has always been a need for moving people and supplies to and from northern regions using the most common mode of transportation, namely, water transportation. Ships were successfully used throughout the summer in the Arctic and Subarctic regions, and until the late fall, depending on the region. Heavier more powerful ships were needed to assist in navigation in the winter, and the first icebereaking ship was built in the late eighteen century, in Germany, to assist ships in harbour navigation. Since then a large number of icebreakers and ice strengthened vessels have been designed and built for ice navigation. A comprehensive review of icebreaking development over the years, up to the mid sixties, is given by White (1965). In the last two decades, as a result of the increase in the demand for icebreakers and ice capable ships, extensive research has been conducted in the icebreaking and icebreaker design field. This research activity produced a number of major research papers, reports and dissertations which are available in the open literature, (Popov et al., 1967; Kashteljan et al 1968; Enkvist, 1972; Milano, 1972; Vance, 1974; Chu, 1974; Naegle, 1980; Hysing, 1981; Carter, 1983).

To understand and predict the performance of existing ships, or to embark on a new design for ships to operate in ice covered waters, it is necessary, not only to understand the ice forces on the ship but to estimate the thrust and power required to maintain the designed head speeds in ice.

The study of ship performance in ice covered waters follows two main streams :

- Ship structure strength requirements for ice navigation; this covered the calculation of ship-ice interaction forces and ship hull stresses, vibrations due to ice and ship hull stucture design requirements for ice navigation (scantling).
- Ship power requirements to navigate in a given ice condition at different ship speeds; this covered the areas of, analysis of the icebreaking process, ship resistance measurements and calculation, propeller design and efficiency, hull form design and optimization.

Other related areas of interest cover the problem of propeller ice interaction, ice physical and mechanical properties as related to ship navigation in ice.

The problem of ship power requirements for ice navigation has been analysed by many naval architects and researchers over the years. Three main approaches were followed to analyse and predict the resistance of a ship in ice covered waters:

- Analytical formulation of algorithms to calculate the resistance for a given ship in a given ice condition.
- Testing of ship models in simulated ice covers. This was carried out either in frozen water or in artificial materials simulating the mechanical properties of natural ice.
- Full scale ship measurements of ship power levels, thrust and speed in natural ice conditions.

In all three approaches the objective was to predict the full scale ship resistance in any percieved ice condition and to establish ship capabilities in ice. A large effort was devoted to compare, correlate and validate the results obtained, by one method with another. The best solution to the ship resistance prediction problem is obviously when the three approaches of analytical calculation, model testing and full scale testing produce comparable results. This is always the target of any study, however it is difficult to achieve in all circumstances.

Most ship resistance studies efforts have concentrated on the problem of ship resistance in continuous sheet ice. Very little attention was given to the problem of ship resistance in broken ice. Consequently very few scientific and technical materials are available in the open literature in this area. A number of researchers have concluded that more research work is needed in this important area of ice navigation in floe ice.

2.2 SHIPS IN BROKEN ICE

Prediction of ship resistance in broken ice or pack ice has become increasingly important in recent years. This is due to the following:

- The increase in ships navigating cold waters.
- The existance of large water surface area covered with broken or pack ice.
- The practice of commercial ships of following

icebreakers through sheet ice; in this case these vessels navigate in channels filled with broken ice.

- The fact that the study of broken ice resistance may support the understanding of non-breaking components of continuous sheet ice resistance.

The review of literature presented in the following sections was conducted to investigate the need for new analytical formulations or methods, for the calculation and prediction of ship resistance in broken ice.

2.3 EARLY MODEL TESTS

A large number of tests of ship models in simulated ice have been conducted in the recent past. Model testing was performed in Canada, U.S.A., U.S.S.R., Finland, Japan, Sweden, Norway and Denmark in towing tanks specially adapted to ice modelling. The great majority of these tests were conducted to measure ship resistance in continuous ice sheets. Different materials were used to simulate the material strength of ice in a model form.

Very little research attention was given to conducting ship model resistance tests in model ice covers simulating ice floe infested waters.

The first model testing in broken ice was conducted in the Soviet Union in the late Forties and early Fifties. Results of these experiments were reported to the West more than ten years after the tests were conducted. Nogid (1959) presented the modelling laws and simulation techniques for model testing in broken ice. He also reported on a model testing program conducted in the U.S.S.R. on the icebreaker "Josif Stalin" in 1950 and 1951 using models 1/29, 1/40, 1/50, 1/60, 1/100 of full scale in ice floes 6 meters (full scale) with concentration of 60%. No test results were presented by Nogid.

Bronnikov (1959) reported on a series of model tests of cargo ships, which took place between 1953 and 1955 in the towing tank of the Leningrad Shipbuilding Institute. Based on the model tests Bronnikov proposed the following relation for correlating model resistance to full scale ship:

$$R = R'(D/D')^{s}(h/h')^{m}(C/C')^{n}(d/d')^{p}[(L/B)/(L/B)']^{q} (C_b/C_b')^{r}[(B/B_1)/(B/B_1)']^{k} \qquad (2.1)$$

Where: R = Ship resistance in ice floe.

R' = Ship resistance of reference (parent) ship.

D = Ship displacement.

h = Ice thickness.

C = Ice concentration.

d = Ice floe diameter.

L = Ship length.

B = Ship beam.

C_b= Ship block coefficient.

B_1 = Ice channel width.

Parameters with comma refer to a reference ship.

The parameters s, m, n, p, q, r and k are parametric coefficients which depend on the Froude number F_r, as follows:

$$s = 1 - (m+p)/3$$

$$m = 0.267\ F_r^{-0.67}$$

$$n = 0.785\ F_r^{-0.493}$$

$$p = 1.65\ F_r^{0.42} + 0.35$$

$$q = 1.93\ F_r^{0.42} - 0.6$$

$$r = 3.24\ F_r^{2.27} + 1.25$$

$$k = 0.034\ F_r^{-1.31}$$

Dubrovin (1970) also reported on the U.S.S.R. experimental investigations of ship motion in broken ice which took place in the late Forties and early Fifties. The ship models used were for the icebreakers "Sibiz" and "Ezmack" and for the cargo ship "Lena". No description was given for these vessels and their characteristics.

To evaluate the relative magnitude of continuous icebreaking resistance components, Chu (1974) conducted ship model tests in a broken ice channel. In this type of testing, ship models were towed in a presawn track in the ice sheet. The ice sheet was sawed in a constant pattern and the ice coverage was 100 %.

In a related field a number of model testing programs were conducted to measure ship resistance in brash ice and ice clogged channels (Levine et al,1974; Kostilainen and Hanhirova, 1982; Eskola, 1983; Tatinclaux, 1984; and Kitazawa and Ettema,1985).

To study the clearing efficiency of the ship hull and the ice floe impact frequency on the rudder and screw, Herfjord and Hysing (1981) conducted a small scale model test on a bulkship in 50% and 100% broken ice in a small tank at the Ship and Ocean Laboratory in Trondheim.

2.4 FIELD MEASUREMENTS

Over the last few decades a large number of full scale ship tests were performed on ships operating in ice covered waters. The tests were conducted either during a routine ship operation schedule, or through special purpose ice expeditions. Full scale ship tests were conducted for many reasons related to environmental ice conditions, ship operation performance, ship equipment performance or to observe and document the ship-ice interaction propcess and the resulting ship hull stresses and vibrations.

The early ship field tests were conducted on icebreakers and later, as the interest in the Arctic increased and the resource development in the Arctic became more achievable, a number of other types of commercial and industrial ships were tested in ice. In the last two decades tankers, bulk carriers, cargo and offshore supply ships have been tested in differing ice conditions, varying from continuous sheet ice to ice ridges, in different areas of the world. These tests have been reported by Korri and Varsta (1979), Mookhoek et al (1981) and German et al. (1981).

Most of these tests were for measuring ship powering in sheet ice and very few tests were conducted for the sole purpose of measuring ship performance in broken ice. A limited number of full scale tests were conducted to measure ship resistance in brash ice (Vance,1980). Michailidis and Murday (1981) reported on the full scale field test of the "CCGS Franklin" where the ice cover was a combination of brash ice and broken ice. Also Ashton et al (1972) reported on similar tests in the Mississippi river with the "MV RENEE G" with and without tow barges. Results and discussions of many of these tests have been documented and published in the open literature. However, most of these tests do not cover the area of ice floe or pack ice resistance. Results of these tests are, therefore, not useful for accurate comparisons and verifications with other broken ice resistance prediction methods.

2.5 EMPIRICAL APPROACH

In the early work in the area of ship resistance in broken ice a complete analytical method to calculate the ship resistance did not exist. However, a number of equations were proposed by the Soviet researchers based on full scale observations or on limited model tests.

An empirical expression for ship resistance in broken ice was proposed by L. M. Nogid (Kashtelyan et al, 1968), based on early ship model testing. The expresssion can be written as :

$$R_i = A\ K\ D[L/B]^{1.25}[B/T]^{0.5}\ F_r^{2.0} \qquad (2.2)$$

Where: R_i = Ship resistance in broken ice.

D = Ship displacement.

L = Ship length.

B = Ship beam.

F_r = Froude number.

A = A coefficient which is a function of ice floe size and ship speed.

K = A coefficient which is a function of ice packing, ice thickness and ship speed.

The above coefficients are to be determined from graphs.

From other model experiments the following formula was proposed later:

$$R_i = \sigma_w \, L^m \, B^n \, r^k \, h^p \, (d + c \, F_r^{\,q}) \qquad (2.3)$$

Where: R_i = Ship resistance in ice floes.

σ_w = Specific weight of water.

L = Ship length.

B = Ship beam.

r = Extent of ice floe.

h = Ice floe thickness.

d = A dimensionless coefficient which depends on the degree of ice packing and on the width of channel filled with broken ice.

c = A dimensionless coefficient depending on the ice concentration.

F_r = Froude number.

The coefficients, m,n,k,p and q are empirical exponents which depend on ice concentration.

For ice floe concentration of 8/10 and for an unlimited width of the channel, Eq.2.3 has been presented by L. M. Nogid (Kashtelyan et al, 1968) as:

$$R_i = \sigma_w\ L^{0.5}\ B^{0.25}\ r^{1.2}\ h^{1.3}\ (0.4+8.0Fr^{1.35})\ (2.3a)$$

Bronnikov (1959) proposed the following relation based on a parent hull model test for ice thickness of 0.8 m and ice concentration of 8/10:

$$R_i = 977\ F_r^{1.42}(D/10920)^s(h/0.8)^m(C/0.8)^n(d/7.3)^p$$
$$[6.6/(L/B)]^q(0.65/C_b)^r[15B/B_1)]^k\ (2.4)$$

Where: F_r = Froude number.

D = Ship displacement.

h = Ice floe thickness.

C = Ice floe concentration.

d = Ice floe length.

L = Ship length.

B = Ship beam.

C_b = Block coefficient of the ship.

B_1 = Ice channel width.

s,m,n,p,q,r are empirical coefficients.

Buzuev and Ryvlin (1961) proposed a formula based on model testing:

$$R_i = C_1 \; L^{0.5} B^{0.25} [C_2 + C_3 \; F_r^{1.25}] \qquad (2.5)$$

Where: C_1, C_2, C_3 are empirical coefficients which depend on the ship hull form and ice floe condition.

Based on a field test program using "a powerful icebreaker" Buzuev and Ryvlin (1966) proposed the following empirical relation for ship resistance in small and medium ice floes :

$$R_i = h^{1.2} (C + 9.0 \; V^{1.3}) \qquad (2.6)$$

Where: R_i = Ice floe resistance in tons.

h = Ice thickness in meters.

V = Ship speed in m/s.

C = An empirical coefficient characterizing the influence of ice floe size. This coefficient is to be determined from a graph.

Kashtelyan et al, (1968) proposed the following semi-empirical formula for the estimation of ship resistance in floe ice :

$$R_i = \sigma_i \ (rh)^{1/2} \ (B/2)^2[K_1(1+2\mu \ \alpha_H \ L/B) + K_4 \ \mu\alpha(L/B)C] + K_2 \ \sigma_i \ rh \ B(\mu + \alpha_H \ Tan\alpha_o) \ F_r + K_3 \ \sigma_i \ rh \ L \ Tan^2\alpha_o \ F_r^2 \qquad (2.7)$$

Where: R_i = Pure ice resistance of a ship in broken ice.

r = Ice floe extent.

σ_i = Specific mass of ice.

h = Ice thickness.

μ = Coefficient of friction between ice and ship hull.

α = Ice waterplane area coefficient.

α_H = Waterplane area coefficient of bow area.

α_o = Half of entering angle of the bow part of the ship.

F_r = Froude number.

K_1, K_2, K_3, K_4 are empirical coefficients to be determined from graphs.

A number of formulations and approximate equations have been proposed recently in North America for fast calculation of ship resistance in broken or pack ice (Gill et al., 1981 and German et al.,1981, Vinoggradov, 1986)

In the area of brash and rubble ice ship resistance, a number of relationships and empirical expressions have been proposed. Due to the differences in ice characteristics, these expressions are presented here for general guidance. Based on model testing of a Great lake bulk carrier, Levine, Voelker and Mentz (1974) proposed the following empirical relationship for hull resistance in brash ice :

$$R = 1.29\ \sigma_i\ g\ B\ h^2 + 1.365\ V^2\ \sigma_i\ Bh \qquad (2.8)$$

Where: R = Ship resistance in brash ice.

B = Hull beam in feet.

h = Ice thickness in feet.

g = Acceleration of gravity.

σ_i= Density of ice.

Milano (1975) attempted to analyse the resistance experienced by a ship sailing in mush ice, however he neglected the friction effect and assumed that the ice pieces are displaced in a transverse direction only. These assumptions and others have been challenged by a number of researchers, and this in turn has limited the applicability of his approach.

By assuming the Mohr-Coulomb failure criterion, Mellor (1980) derived the following analytical relationship for brash ice ship resistance :

$$R = [1+2\ \mu_e(K_1 + K_2N)]B\ R_p \qquad (2.9)$$

Where: R = Ship resistance in brash ice.

$K_1 = L_1/B$

L_1 = Bow length.

$K_2 = L_2/B$

L_2 = Afterbody length in continuous contact with ice.

μ_e = Effective friction coefficient.

B = Ship beam.

N = A factor of uncertain value but less than unity.

R_p = Force per unit width of a brash ice layer in the (Rankine) passive loading state, which can be expressed as :

$$R_p = 0.5[(1+\sin\Phi)/(1-\sin\Phi)](1-n)\sigma_i g(1-\sigma_i/\sigma_w)h^2 \qquad (2.10)$$

Where: Φ = Internal friction angle of brash ice.

n = Porosity of the brash ice.

σ_i = Density of ice.

σ_w = Density of water.

Another empirical relationship for brash ice ship hull resistance has been proposed by Greisman (1981) :

$$r=(R/R_o)(h_o/h)(L_o/L)^{1.5}=1.0+23.4(v/\ gL-0.12)^{1.5} \quad (2.11)$$

Where: R_o, h_o and L_o are arbitrary values selected to normalize resistance force R, ice thickness h, and hull length L, respectively.

Based on full scale field measurements of USCGC Katmai Bay in brash ice, Vance (1980) developed the following relationships for ship resistance in brash ice of thicknesses o.46 and 1.22 meters respectively :

$$R = 0.1209\sigma_\Delta gBh^2 + 0.0622\sigma_i V^2 LB^{0.35}h^{0.65} \quad (2.12)$$

$$R = 4.4121\sigma_\Delta gBh^2 + 0.0306\sigma_i V^2 LB^{0.35}h^{0.65} \quad (2.13)$$

Where: R = Ship resistance in brash ice.

σ_Δ = Difference in density between ice and water.

σ_i = Density of ice.

Kitazawa and Ettema, 1986 developed resistance expression for ships moving with creeping speed in rubble-ice.

It is clear that the existing equations, or relationships, represent empirical expressions and they are, therefore, very limited in their applications to certain types of ships or certain ice conditions. Although some of the overall ship dimensions have been included with these expressions, no information was available on the type and shape of the hull forms used in their development.

Furthermore, a number of empirical coefficients and parameters have to be calculated from graphs which have not been provided by the Russian authors. It is also clear that a number of Russian authors have reported on the same research activities in the U.S.S.R. The relations proposed (e g Eqs 2.3, 2.5, 2.6) are very similar in nature.

The above expressions, although they may provide some guidance, are limited in their usefullness and application for the prediction of resistance for any given ship hull. The review of literature conducted has demonstrated and justified the need for a more comprehensive resistance formulation undertaken in this study.

2.6 APPLICATION OF EXISTING METHODS

The existing ship resistance prediction methods vary from empirical to semi-empirical formulations based on model and full scale results for paricular hull forms. Application of these methods on other hull forms is risky and very approximate and should be treated carefully. In addition finding the values of the relevant empirical coefficient is impossible in some cases. Nevertheless two methods have been applied on an existing hull form to develop a feeling for an "order of magnitude" of ship resistance in broken ice. The hull form used is the "CCGS Franklin", which is a Canadian R-Class Icebreaker (ship hull form characteristics are given in Table 2.1).

Results of the R-Class ship resistance in broken ice, using the empirical equation proposed by Buzuev and Ryvlin, (1966) (Eqn.2.6), are shown in Fig.2.1. The semi-empirical equation of Kashtelyan et al (1968) produced the results shown in Fig.2.2. These two methods were used because it was possible to estimate the relevant empirical coefficients. It is important to notice that both methods produced a non-zero ship resistance at zero ship speed.

2.7 SUMMARY

TABLE 1.1

Characteristics of R-Class Icebreaker

LENGTH BETWEEN PERPENDICULARS (LPP), M	87.93
LENGTH ON WATERLINE (LWL), M	93.00
WATERLINE BEAM AT MIDSHIPS, M	19.36
WATERLINE BEAM AT MAXIMUM SECTION, M	19.36
MAXIMUM WATERLINE BEAM, M	19.37
DRAFT AT MIDSHIPS, M	6.93
DRAFT AT MAXIMUM SECTION, M	6.97
DRAFT AT AFT PERPENDICULAR, M	7.16
DRAFT AT FORWARD PERPENDICULAR, M	6.71
EQUIVALENT LEVEL KEEL DRAFT, M	6.94
MAXIMUM SECTION FORWARD OF MIDSHIPS, M	-7.39
PARALLEL MIDDLE BODY, FROM, AFT OF MIDSHIPS, M	7.39
TO, FORWARD OF MIDSHIPS, M	-7.39
AREA OF MAXIMUM STATION, SQ. M	123.41
CENTER OF BUOYANCY FORWARD OF MIDSHIPS (LCB), M	-0.33
CENTER OF BUOYANCY ABOVE KEEL, M	3.88
WETTED SURFACE AREA, SQ. M	2135.52
VOLUME OF DISPLACEMENT, CU. M	7629.27
DISPLACEMENT, TONNES OF SALT WATER	7820.00
CENTER OF FLOATATION FORWARD OF MIDSHIPS (LCF), M	-0.69
CENTER OF FLOATATION ABOVE KEEL, M	6.94
AREA OF WATERLINE PLANE, SQ. M	1439.10
TRANSVERSE METACENTRIC RADIUS (BM), M	4.89
LONGITUDINAL METACENTRIC RADIUS (BML), M	96.00
CENTER OF AREA OF PROFILE PLANE FORWARD OF MIDSHIPS (CLR), M	-0.77
CENTER OF AREA OF PROFILE PLANE ABOVE KEEL, M	3.57
AREA OF PROFILE PLANE, SQ. M	562.05

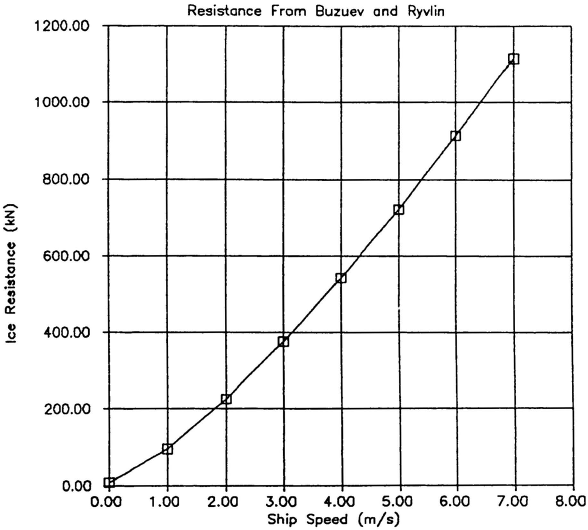

Fig.2.1 R-Class Results Using Buzuev and Ryvlin (1966)

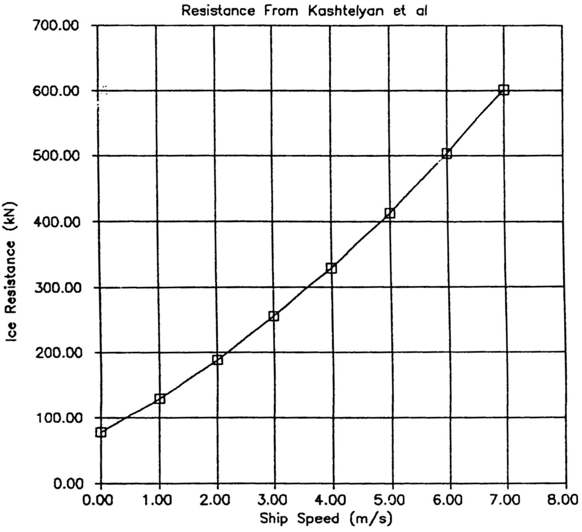

Fig.2.2 R-Class Results Using Kashtelyan et al (1968)

The review of literature conducted as a part of this study clearly indicates the shortcomings of existing methods for general predictions of ship resistance in broken ice. While there are a number of model tests conducted in broken ice, particularly in the Soviet Union, results of these tests, and others, cannot constitute a useful hull form "series" for use with new designs.

The full scale tests conducted up to the present time have been limited to continuous- or brash-ice with very limited application to the broken ice resistance problem. This is due the difference in the nature of the ice cover and due to the type of hull forms used.

Empirical formulations available in the literature are very limited in their use for resistance predictions. This is due to the fact that their development was based on limited testing of a few unique vessels. Furthermore, the characteristics of the vessels used in most studies, have not been available for any reliable interpolation for other vessels. As a result of this review, it is clear that in the existing ship ice floe resistance formulations, the correlation between the resistance expressions and the ship geometry and the ice floe properties is very limited. These expressions therefore, are not suitable for any reliable generalized use with new ship designs.

CHAPTER 3

THEORETICAL INVESTIGATION - THE MICRO MODEL

3.1 BACKGROUND

In order to accurately evaluate the magnitude of ship resistance in broken ice covered waters, under a variety of operating conditions, an analytical model is presented in this study. The model is termed the "Micro Model" as it is based initially on the interaction of an ice floe with the ship.

As the ship proceeds in the broken ice it interacts with a number of ice floes. The number of interactions will depend on the size and concentration of ice floes, the size of the ship and the speed of advance of the ship. The forces and moments of interaction of one ice floe will be analysed, and based on the possible number of floe interactions with the ship in a given time interval or distance segment in the ship's path the ship energy loss or resistance increase will be evaluated.

Although a large number of studies are available in the literature on the subject of ice impact with ships and offshore structures and the related stresses (Varsta et al., 1977; Edwards et al., 1977; Riska et al., 1983;Ghoneim et al.

,1984), to the author's knowledge no attempts were made to develop formulations for ship resistance based on the ice floe impact approach.

3.2 ASSUMPTIONS

The analytical treatment of the problem of ship resistance in broken ice has been considered by a number of researchers in the field of icebreaking to be a difficult task. This is due to the fact that a large number of parameters related to the ice, the ship and the ship-ice interaction mechanism are involved in the process. Indeed, the advance of a ship in water covered with broken ice fragments represents a challenging mechanical interaction process.

A number of assumptions have been adopted in the development of this analytical approach to enable the formulation and solution of the ship resistance equations. These assumptions concern the ice cover, the ship, the interaction process and they can be summarized as follows :

ICE COVER

The ice floes are considered to be homogeneous, isotropic, and continuous media. The thickness of the ice is small compared with its horizontal dimensions and the size of the ice floe is small compared with the ship size. Ice floes are considered as freely floating rigid bodies and no pressure exists on the ice cover. The drift velocity of the ice floes is small compared to that of the ship.

THE SHIP

The ship is considered to be rigid body, symmetrical with respect to the vertical center line plane. The center of gravity is located at the mid ship, on the centerline vertical plane. The ship is in the upright position (see Fig.3.1). Ship generated waves are assumed small.

THE INTERACTION

The ship speed remains constant during the interaction process interval. The ice ship contact occurs only at the ship's entrance and ice floes remain intact after interacting with the ship. The vibration due to impact is negligible (impact frequencies are low) as is the ship change of trim due to the impact with the ice floe.

3.3 THEORETICAL FORMULATION

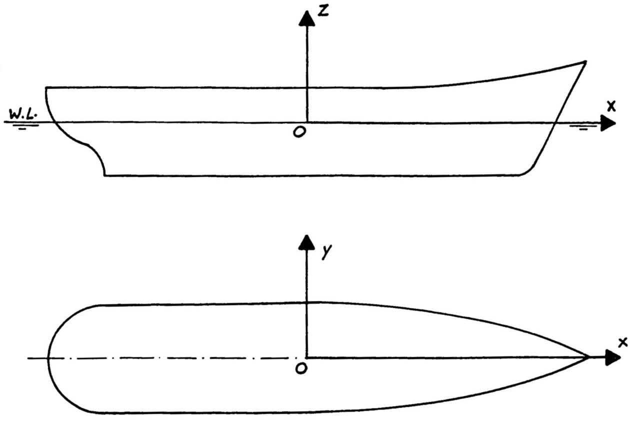

Fig.3.1 Ship Coordinate System

The interaction between the ship hull and ice floes represents an impact or collision process where ice floes are forced away from the ship's path. Classical impact theories, therefore, can be employed for this analysis. Available impact theories can be categorized into two main types, either the Saint-Venant theory or the Hertz theory (Angle, 1976). The Saint-Venant theory is based on the impact vibration approach where as the Hertz theory treats the impact as a quasi-static process and it has been adopted in this study.

Consider the ship as a freely floating body advancing at a speed V in the positive x-direction. The x, y, and z represent a right hand cartisian coordinate system with the origin O located at the waterline vertically above the ship's center of gravity. The x-axis is pointing forward, the y-axis is pointing to the port side, and the z-axis is pointing upward, as shown in Fig.3.2.

The ship with a mass M then collides with an ice floe of mass m and velocity v. As a result of the impact momentum exchange takes place where the momentum of the ship, defined as M.V will change to M.V', where V' is the new ship speed after impact.

The change of ship momentum is due to the impact force P(t) which appears during the collision duration t' with the ice floe. From the principle of conservation of momentum, the

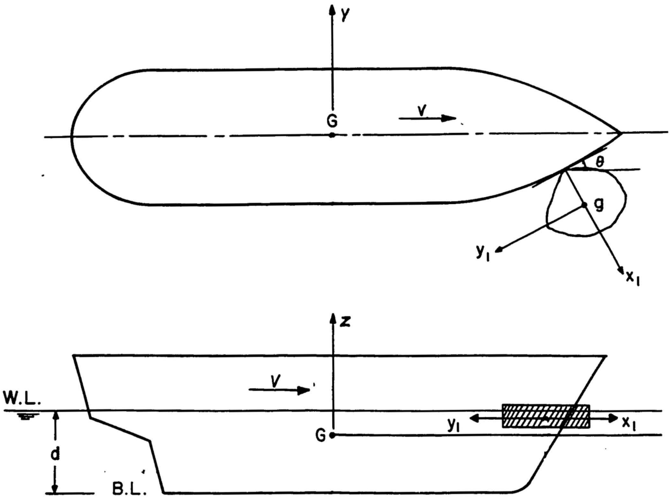

Fig.3.2 Ice Floe and Ship Impact

impulse and momentum equations for the ship, due to normal frictionless impact, can be written in vector form as:

$$M\,V \;+\; \int_0^{t'} P(t)\,dt \;=\; M\,V' \qquad (3.1)$$

Where the integration $\int_0^{t'} P(t)\,dt$ is the impulse N and t' is the duration of the impact.

For a ship colliding with an ice floe of medium size, it is possible to consider the ship as wall-sided at the region of floe impacts, furthermore, due to ship symmetry and ice floe homogenous distribution, it is possible to assume port and starboard symmetrical impacts. The ship rotational motion due to impacts can therefore, be ignored. The ship impacts and motions have therefore been restricted to the horizontal waterline plane.

For the ship and the ice floe, the conservation of linear momentum represented by Eq.3.1 may be reduced to the following relation :

$$M\,V' + m\,v' = M\,V + m\,v \qquad (3.2)$$

Where M and m are the masses of the ship and the ice floe respectively, V , v and V' , v' are the ship and ice floe speeds before and after collision respectively.

In this analysis the initial velocities of the ship and the ice floe will be known parameters. In order to find the new impact velocities the energy relation is to be used as shown in the following equation :

$$M V'^2 + m v'^2 = e (M V^2 + m v^2) \qquad (3.3)$$

Where e is known as the coefficent of restitution and it can be defined as :

$$e = (V' - v') / (V - v) \qquad (3.4)$$

From the definition of e and Eqs.3.2 and 3.3, the new impact velocities can be defined as :

$$V' = V - (1-e)(m/(M+m))(V-v) \qquad (3.5)$$

$$v' = v + (1-e)(M/(M+m))(V-v) \qquad (3.6)$$

The change in the kinetic energy of the system is, therefore;

$$\delta E = 1/2[(M V'^2 + m v'^2)-(M V^2+ m v^2)] \qquad (3.7)$$

Substituting Eqs.3.5 and 3.6 into Eq.3.7, the loss of kinetic energy due to impact can be written as :

$$E_i = 1/2 \ (Mm/(M+m))(1-e)^2(V - v)^2 \qquad (3.8)$$

A similar equation has been reached by others including Goldsmith (1960).

The occurrence of normal impacts between ice floes and the ship is limited to normal ship surfaces such as a blunt stem or pertruding appendages. The general case of sloped impact is treated in section 3.5

3.4 SHIP IMPACT WITH A NUMBER OF ICE FLOES

The analysis in Section 3.3 addresses the impact between a ship and a single ice floe. Equation 3.8 represents the ship energy loss due to such an impact. However, as the ship advances in ice floe covered water, she will continuously experience impact with a number of ice floes as shown in Fig.3.3. The number of impacts in a given time interval will depend on the ice floe concentration, the size of individual ice floes, the size of the ship, the speed of advance of the ship, and the drifting speed of the ice floes.

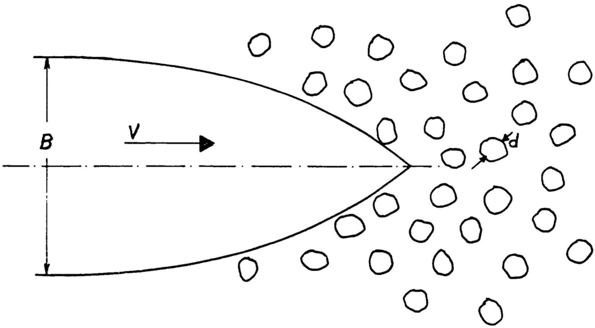

Fig.3.3 Ship Continuous Impact with Ice Floes

To study the energy loss due to continuous impact of the ship with ice floes an estimate of the number of impacts in a given time is required. The Kinetic Theory of Gases explains the phenomena of gas particle collisions and the relation between collisions and the gas pressure (Halliday and Resnick, 1966). This theory is adopted in this study to estimate the number of collisions between ice floes and an advancing ship.

Consider the ice floe to have a characteristic length d which can be defined as a/k , where a is the floe area and k is a shape factor. The ship can be considered at the entrance portion as a large circular body of diameter B. A collision between the ship and the ice floe can only take place when the centers of the two bodies approach within a distance equal to, or less than $1/2(B+d)$ of each other, as shown in Fig.3.4. The same number of collisions would result if the beam of the ship is to be increased by a distance d resulting in an effective ship beam equal to $(B+d)$. The ice floe could be reduced, in this case, to only geometrical points. As the ship of beam $(B+d)$ advances through ice, it will sweep out, in time t, a rectangle whose width is $(B+d)$ and whose length is the distance travelled in time t or $V t$. Hence, if the number of floes per unit area is n, the number of collisions in time t, N , will be :

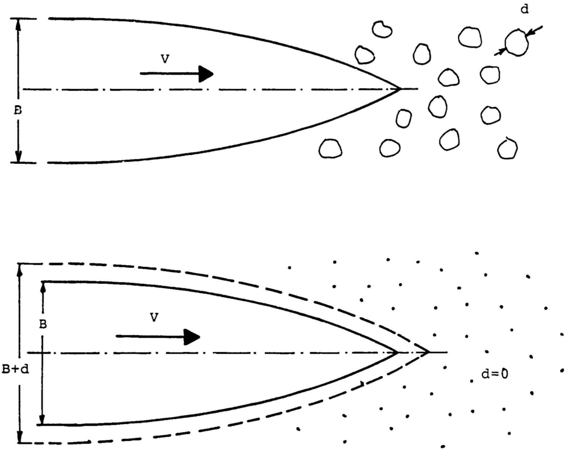

Fig.3.4 Ice Floe Ship Encounter

$$N_c = n\ (B+d)\ V\ t \tag{3.9}$$

The number of collisions per unit time or the collision frequency C_f' is :

$$C_f' = n\ (B+d)\ V \tag{3.10}$$

The average distance between collisions or the mean free path of the ship MFP, will be :

$$MFP = V\ /\ Cf' = 1/[n\ (B+d)] \tag{3.11}$$

The energy loss due to one collision has been defined in Eq.3.8. Using this equation together with Eq.3.10, the total energy loss due to continuous impacts per unit time will be :

$$E_t = (n/2)(B+d)V(Mm/(M+m))(1-e)^2(V-v)^2 \tag{3.12}$$

The energy loss of Eq.3.12 represents the ship's energy loss and thus the ship must increase it's power to maintain her original sailing speed. This loss of the ship power indicates an effective increase in the ship resistance due to sailing through ice floes. The increase of the ship resistance R_i can be written as :

$$R_i = P_i / V \qquad (3.13)$$

Where P_i is ship power loss due to ice floe impact.

The number of ice floes per unit area n in Eq.3.12 may be expressed in terms of the ice floe concentration and the size or area of individual ice floes as follows :

$$n = C / a \quad \text{or}$$

$$n = C / (kd^2) \qquad (3.14)$$

Where C is the ice concentration, a is the ice floe area and k is the shape factor for the ice floe.

Equation 3.12 can, therefore be written as :

$$E_i = C/2a \ (B+d)V(Mm/(M+m))(1-e^2)(V-v)^2 \qquad (3.15)$$

The increase of resistance due to the ship encounter with the ice floes will be :

$$R_i = C/2kd^2\ (B+d)V^2(Mm/(M+m))(1-e)^2 \qquad (3.16)$$

It is clear from Eq.3.16 that ship resistance increase due to advancing through ice floe covered waters, depends on the ice floe size and concentration, ship mass, beam and speed, and on the coefficient of restitution between the ice and the ship hull.

3.5 ICE FLOE IMPACT WITH SHIP ENTRANCE

Consider the impact between an ice floe and the ship hull at the entrance as shown in Fig.3.5. The angle θ is the incident angle which is equal to the slope of the ship's waterline at the point of impact. Angle θ_1 is the reflected angle of the ice floe after impact. The impact impulse N has two components, tangential component N_t and normal component N_n. The impulse and momentum relations for the ice floe will be :

$$-m\ V \sin\theta + N_n = m\ v' \sin\theta_1 \qquad (3.17)$$

$$m\ V \sin\theta - N_t = m\ v' \cos\theta_1 \qquad (3.18)$$

$$N_t\ r = \Omega'\ I \qquad (3.19)$$

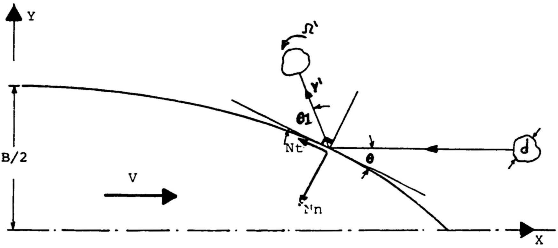

Fig.3.5 Ice Floe Impact with Ship's Entrance

Similar to Eq.3.4, the coefficient of restitution e can be defined as :

$$e = v' \sin\theta_1 / V \sin\theta \qquad (3.20)$$

For rough surfaces the coefficient of friction μ governs the relation between N_t and N_n as follows :

$$\mu = N_t / N_n \qquad (3.21)$$

By solving the above five equations (Appendix II contains the solution of the above impact equations by elimination) it is possible to obtain the reflected speeds of the ice floe. The speeds of the ice floe after impact with the ship hull will, therefore be:

$$v' = V \sin\theta\{[\cot\theta - \mu(1+e)]^2 + e^2\}^{1/2} \qquad (3.22)$$

$$\Omega' = (mr/I) V \sin\theta[\mu(1+e)] \qquad (3.23)$$

The kinetic energy of the ice floe after impact with the ship will be :

$$E_f = 1/2\ m\ v'^2 + 1/2\ I\ \Omega'^2 \qquad (3.24)$$

or
$$E_f = \tfrac{1}{2}\, m\, V^2 \sin\theta\{[\cot\theta-\mu(1+e)]^2+e^2\} + \tfrac{1}{2}(m^2/I)\, r^2 V^2 \sin\theta[\mu(1+e)]^2 \qquad (3.25)$$

The energy loss due to an impact between a single ice floe and ship entrance will therefore be :

$$E_i = \tfrac{1}{2}\, m\, V^2 \sin^2\theta(\csc^2\theta-[(\cot\theta-\mu(1+e))^2+e^2] -(mr^2/I)[\mu(1+e)]^2\} \qquad (3.26)$$

For short duration impact with negligible stress waves and vibrations, the energy loss due to impact is taken to be equal to the energy loss from the ship power. This power loss is effectively an increase of ship resistance.

Introducing the expression for the ship ice floe encounter rate Eq.3.10 and Eq.3.14, the total energy loss for constant slope entrance ship due to continous impact with ice floes will be :

$$E_t = C\, m\, V^3 \sin\theta(B+d)/(2kd^2)\{\csc^2\theta -[(\cot\theta-\mu(1+e))^2+e^2] -(mr^2/I)[\mu(1+e))^2\} \qquad (3.27)$$

From the relation between power and resistance, the power loss of Eq.3.27 can be expressed in terms of increased ship resistance as follows :

$$
\begin{aligned}
R_i = & (C\ m\ V^2 \sin^2\theta(B+d))/(2kd^2)\{\csc^2\theta \\
& -[(\cot\theta-\mu(1+e))^2+e^2] \\
& -(mr^2/I)[\mu(1+e)]^2\}
\end{aligned} \tag{3.28}
$$

The above equation represents a new analytical expression for the increase in ship resistance due to sailing in ice floe infested waters. This expression has been used by Aboulazm and Muggeridge (1989a) to calculate the resistance of the USCGC Katmai Bay and to compare with full scale field measurements reported by Vance (1981).

3.6 ANALYSIS OF REPEATED ICE FLOE IMPACTS

The previous analysis considered the situation where the ice floes collided with the ship hull only once. However, depending on ship parameters, ice parameters and the interaction mechanism, repeated impacts may occur. Repeated impacts or ice floe impinging occurs when the ship entrance collides with the same ice floe more than one time.

It is clear that the occurence of repeated impacts will be associated with increase in the ship resistance. To examine the possibility of repeated impact or ice floe inpinging, consider the momentum equations in section 3.5. These equations have been solved to find the post impact linear and angular velocities of the ice floe v' and Ω'. The same equations can be used to determine the angle of reflection of the ice floe after impact θ_1 as follows :

$$\theta_1 = \tan^{-1}[e/(\cot\theta-\mu(1+e))] \qquad (3.29)$$

After colliding with the ship hull the ice floe will have an initial linear velocity v' as defined in Eq.3.22 and a direction θ_1 as defined in Eq.3.29 and shown in Fig.3.6.

Repeated impact occurence will depend, among other things, on the magnitude and direction of the ice floe velocity after impact v', θ_1. To calculate the possibility and the number of repeated impacts it is necessary to examine the parameters v' and θ_1 and the motion response, as a function of time, of the ice floes after impact.

From the Equations for v' and θ_1, it is clear that both parameters are functions of the coefficient of restitution, the coefficient of friction and the angle of incidence. In the special case where the coefficient of restitution e is equal to zero, the impact will be completely plastic and the reflection angle will be equal to zero. This case indicates that after the impact the ice floe will remain

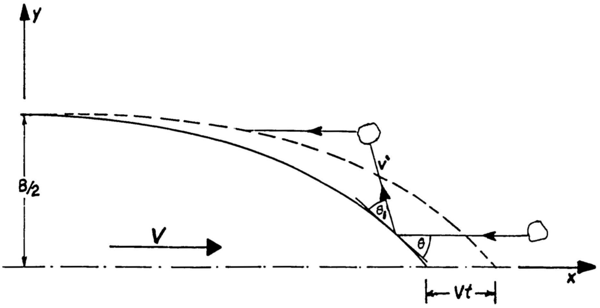

Fig.3.6 Ice Floe Repeated Impact

in contact with the ship hull and it will only slide over the hull creating friction drag.

In the general case where $0 < e < 1$ the ice floe will aquire a speed v' in the direction θ_1 after the impact. The distance travelled by the ice floe, as a function of time, after impact, can be determined from the equation of motion of the ice floe, as a floating rigid body, in sea water. This initial value problem can be represented by the following equation :

$$m' w'' + b w' + c w = -D(w,v) \qquad (3.30)$$

Where m' is the virtual mass of the ice floe (equal to the mass of ice floe plus the added mass of water) and b and c are the damping and restoring coefficients. D(w,v) is a summation of all water drag forces acting on the ice floe. The terms w'', w' and w are the acceleration, velocity and displacement of the ice floe, respectively.

For horizontal ice floe motion with no vibration, Eq.3.30 can be reduced to :

$$m' w'' = -D(w,v) \qquad (3.31)$$

The water drag forces acting on the ice floe consist of the form drag and the skin friction drag. Introducing the expressions for ice floe form drag force and skin friction drag force into Eq.3.31, the equation of motion of the ice floe, as a floating rigid body, will be :

$$m' \, w'' = -(1/2 \; \sigma_w \; A_f \; v^2 C_f + 1/2 \; \sigma_w \; A_s \; v^2 C_s \;) \qquad (3.32)$$

Where A_f is the under water area projected perpendicular to the direction of motion of the ice floe, A_s is the ice floe surface area. The parameters C_f and C_s are the form drag coefficient and skin friction coefficient respectively.

Equation 3.32 may be written as :

$$m' \; vdv/dw = -1/2\sigma_w v^2 \,(A_f C_f + A_s C_s) \qquad (3.33)$$

For relatively slow ice floe speeds, the coefficients C_s and C_f are assumed to be independent of the velocity v. Equation 3.33 can therefore, be solved by separation of variables as follows :

$$dv/v \;=\; -(\sigma_w/2m')\,(A_f C_f + A_s C_s) \;\; dw \qquad (3.34)$$

$$\text{or} \quad \int_{v'}^{v} dv/v = k' \int_{0}^{w} dw \qquad (3.35)$$

Where k' is a parameter equal to $(\sigma_w/2m')(A_fC_f+A_sC_s)$

Solution of this integral equation will yield :

$$v = v' \exp(-k'w) \qquad (3.36)$$

Equation 3.36 expresses the speed of the ice floe v after impact as a function of ice floe displacement. A similar equation can be developed for the rotational velocity of the ice floe. The linear velocity of the ice floe presented in Eqn 3.36 may be expressed in series form as:

$$v = v' [1 - (k'w) + 1/2(k'w) - \ldots\ldots] \qquad (3.37)$$

Considering the first two terms in the series, the ice floe will come to stop at the following approximate distance:

$$w = 1/k'$$

$$\text{or} \quad w = 2m'(C_fA_f+C_sA_s)/\sigma_w \qquad (3.38)$$

The time displacement relationship for the ice floe after impact with the ship can be obtained from Eq.3.36 where :

$$dw/dt = v' \exp(-k'w) \tag{3.39}$$

By separation of variables;

$$\int_0^w \exp(k'w)\,dw = \int_0^t v'dt$$

$$\text{hence } t = 1/v'k\ [\exp(kw)-1] \tag{3.40}$$

$$\text{or} \quad w = (1/k)\ \ln(v'k't+1) \tag{3.41}$$

Where w represents the distance travelled by the ice floe after impact. This distance will be reduced if the ice floe is to collide with another ice floe.

Repeated impacts between an ice floe and the ship is assumed to take place when the transverse projection of the distance travelled by the ice floe after impact from Eq.3.38 fall short of the half beam of the ship, as shown in Fig.3.7. In this case the ship will collide a second time with the floe. This collision may be repeated a number of times until

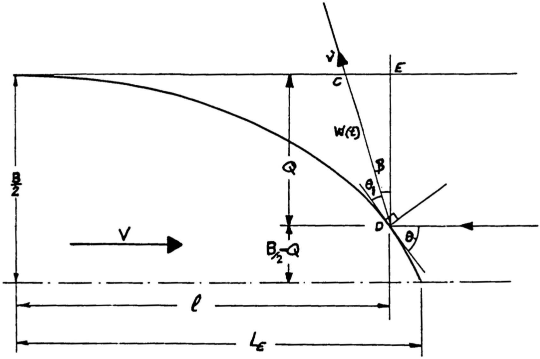

Fig.3.7 Condition for Ice Floe Repeated Impact

the ice floe is completely cleared from the path of the ship. This condition can be represented in equation form as :

$$W \cos\beta < Q+d \tag{3.42a}$$

or

$$\sin(\theta+\theta_1)/K' < Q + d \tag{3.42b}$$

Where: Q = Distance between the point of impact and the maximum beam.

β = Reflected v' direction with the y-axis. It is equal to $[\pi/2-(\theta+\theta_1)]$

3.7 EFFECT OF REPEATED IMPACTS

The occurrence of repeated impacts will result in an increase in the number of collisions per unit time, between ice floes and the ship. Consequently, the energy loss by the ship will increase as a result of repeated impacts. The number of collisions per unit time has been given in Eq.3.8 and from this equation, the number of collisions per unit length of ship projected beam, per unit time will be :

$$N' = n\ V \tag{3.43}$$

The expression for the distance W in Eq.3.42 represents the condition required for the occurrence of repeated impacts. Repeated impacts will, therefore, occur in the area described by $B+d-(2Q.\cos\beta)$ and the number of extra collisions due to repeated impacts are:

$$N'' = n\ V\ [B+d-(2Q\ \cos\beta)] \tag{3.44}$$

The percentage increase in the number of impacts will be:

$$\delta N = nV[B+d-(2Q\ \cos\beta)]\ /\ nV(B+d) \tag{3.45}$$

or

$$\delta N = (B+d-2Q\ \cos\beta)\ /\ (B+d) \tag{3.46}$$

Accounting for repeated impacts, the total number of impacts per unit time N_t will be :

$$N_t = nV(B+d)(1+\delta N) \tag{3.47}$$

The expression for the increase of ship resistance due to ice floes, Eq.3.28, has been adjusted for the number of impacts per unit time, to include repeated impacts. Resistance increase will, therefore, be :

$$R_i = (CmV^2 \sin^2\theta (B+d)(1+\delta N))/(2kd^2)\{\csc^2\theta$$
$$-[(\cot\theta - \mu(1+e))^2 + e^2]$$
$$-(mr^2/I)[\mu(1+e)]^2\} \qquad (3.48)$$

Equation (3.48) represents a new analytical model (MICRO MODEL) to calculate the ship resistance increase due to navigation in broken or pack ice. This model includes many important ice and ship parameters relevant to the process of ship advance in waters covered with broken ice.

This model can be used for the initial evaluation of ship resistance for use in new ship design and for evaluation of ship trip time for existing ships in a given ice condition in a given route.

3.8 MICRO MODEL APPLICATIONS

The theoretical basis on which the Micro Model was developed and the assumptions adopted in the development of the model have introduced some limitations on the application of the Micro Model to general ship resistance prediction. The Micro Model is suitable for ships advancing at moderate speeds in waters covered with relatively small concentrations of small ice floes. The Micro Model may not be suitable for slow ship speeds and for high ice floe concentrations due to the limited ice floe ship impacts. Numerical calculations using the Micro Model applied to the R-Class icebreaker are presented in chapter 9.

CHAPTER 4

THEORETICAL INVESTIGATION - THE MACRO MODEL

4.1 BACKGROUND

The resistance of a ship advancing through ice floe infested waters is evaluated, in this chapter, based on an analytical model which has been termed the "MACRO MODEL". This MACRO MODEL has been developed to calculate the ship resistance in broken ice covered waters based on the treatment of ice floes as an extended ice and water cover. As the ship advances through the water surface that is covered with broken ice fragments it displaces a water surface area which has previously been covered with ice fragments, and in doing so the ship has to exert work to move the ice pieces away from her path. The rate of exerting work represents the increase of ship power requirement due to ice. The increase in ship power is a direct result of ship resistance increase due to ice.

As mentioned in the previous chapter, the magnitude of the ice drag or the resistance encountered by the ship, as she sweeps ice fragments from her path, will depend on the characteristics of both the ship and the ice cover as will as the speed of the ship. The main ship characteristics influencing the magnitude of resistance are the ship beam B, the length of waterplane entrance l, the shape of the waterplane entrance curve and the coefficient of friction between the ice and ship hull surface. The major ice parameters affecting ship resistance are ice fragment size, shape and thickness and the ice type and concentration. Ship speed represents a very important parameter affecting resistance magnitude, as the speed of the ship together with the ice concentration determine the rate of ice floe encounter as well as the extent of the ice cover disturbed by the ship.

4.2 ANALYTICAL FORMULATION

This analytical model is based on the formulation of the motion and the water drag of the ice fragments as they move away from the ship's path. The assumptions presented in the previous chapter regarding the ship and the ice cover have also been adopted for this model. However for a relatively slow ship advancing in high concentration ice cover, the impact and inertia forces are assumed to be relatively small compared with the forces due to fluid drag experienced by the ice floes.

To formulate this analytical model, consider a ship advancing at a steady forward speed V in the positive x-direction in waters covered with broken ice fragments as shown in Fig.4.1. As the ship advances forward it will continuously sweep a number of ice floes away from her path, and as a result of this sweeping process the ship will be acted upon by ice forces normal and tangential to its hull. The resultant of these forces in the x-direction represents the increase in ship resistance.

4.2.1 THE ICE BOUNDARY LAYER CONCEPT

As the ship advances in broken ice covered water a finite number of ice floes will be disturbed by the ship. This distubance will cover a surface area surrounding the ship hull extending from the ship sides to an outside boundary.

It is therefore, important to introduce a new concept to define the region of ice floe disturbance and to be called the "Ice Boundary Layer", or "IBL" for short. The Ice Boundary Layer may be defined as the horizontal transverse water surface area or layer surrounding the ship where the advance of the ship is felt by the ice floes. The ice floes inside the "IBL" are therefore, disturbed and compacted by the ice floes displaced by the advancing ship. Figure 4.2 shows the ice boundary layer surrounding the advancing ship.

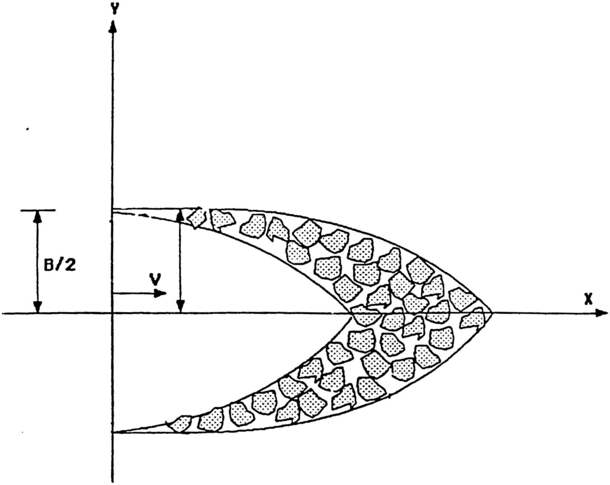

Fig.4.1 Floe Ice Displaced by the Ship

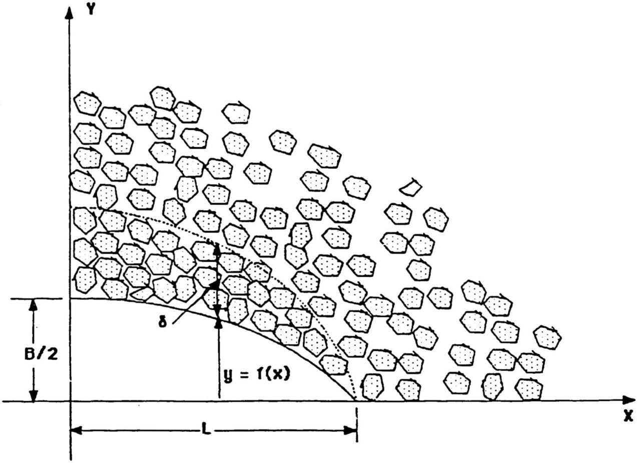

Fig.4.2 The Ice Floe Boundary Layer Concept

4.2.2 ICE BOUNDARY LAYER THICKNESS

The thickness of the ice boundary layer generated by advancing ship depends on many parameters related to the ship and to the ice floe coverage. For relatively small ice floes the thickness of ice boundary layer depends on the ice floe shape and concentration and on the increasing ship entrance beam or ship entrance slope. As a ship of beam y(x) advances, a transverse strip of unit width having ice floes of area Cy(x) will be removed and accomodated in the boundary layer voids represented by $\delta_i K_c(1-C)$. The thickness of the Ice Boundary Layer "δ_i" can, therefore, be presented by the following relation (Fig.4.3):

$$\delta_i = C\ y(x)/K_c(1-C) \tag{4.1a}$$

Where: y(x) = The beam of the ship entrance at any length x.

C = Ice concentration.

K_c = Ice floe compacting coefficient.

The ice floe compacting coefficient K_c depends on the shape of the ice fragments and on the size distribution of the ice floes. The maximum value of this coefficient is one and it occurs for relatively small ice floes. Assuming a

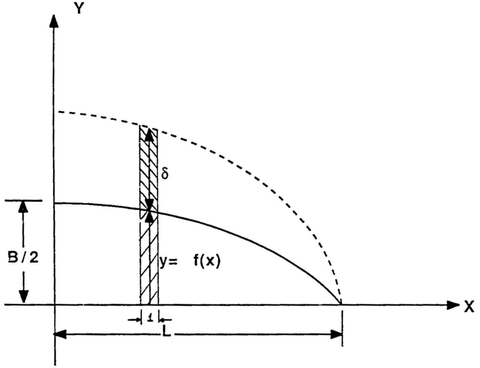

Fig. 4.3 Ice Boundary Layer Thickness

value of unity for the compacting coefficient (C=100% inside the IBL) the ice boundary layer thickness will be:

$$\delta_i = C\ y(x)/(1-C) \qquad (4.1b)$$

For a ship advancing at relatively slow speed, in moderate ice concentration, normal hull forces are mainly due to the movement of a number of ice floes against water drag inside the ice boundary layer. The hydrodynamic ice drag is due to the skin friction of the ice cover and the form drag of ice fragments. The magnitude of water drag of the ice floes, inside the boundary layer, represents a very important factor in calculating ship resistance. Based on the formulation of the ice water drag, it is possible to calculate the ice normal and tangential forces on the ship hull. The ship resistance increase due to ice can be found from the componenets of these forces in the forward x-direction.

The ship waterplane shape in the x-y plane can be presented by the curve $y=f(x)$ where maximum y equals to B/2, x=0 at the start of parallel middle body and y equals 1 at the stem. The origin, therefore, lies at the ship centerline at the start of the parallel body. The angle 0 represents the slope of the entrance curve with the x-axis where;

$$\tan\theta = y' = -dy/dx; \tag{4.2a}$$

$$\sin\theta = dy/ds \quad \text{and} \tag{4.2b}$$

$$\cos\theta = dx/ds. \tag{4.2c}$$

4.2.3 ICE ELEMENT FORCES

Consider an element of length ds on the entrance hull curve $y=f(x)$, as shown in Fig.4.4. As the ship advances in the ice cover this element will be acted upon by the normal ice force dF_n and the tangential ice force dF_t. The relationship between these forces can be taken as :

$$dF_t = \mu \, dF_n \tag{4.3}$$

Where: μ is the coefficent of friction between the ice and the ship hull surface.

In order to calculate the total normal and tangential forces on the ship entrance it is convenient to initially formulate an expression for the normal force per unit length f_n, the total force can then be obtained by the integration of f_n over the entire ship hull entance for both the port

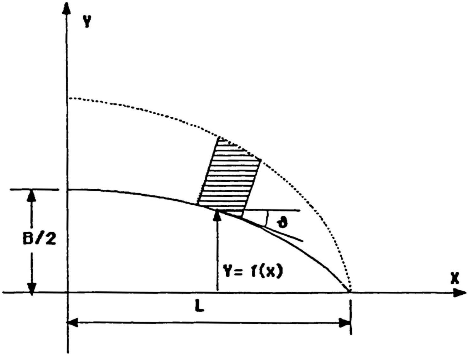

Fig.4.4 Ice Boundary Layer Surface Element

and the starboard sides.

From the fundamentals of fluid mechanics, the normal force due to water drag of an ice cover of unit width f_n can be written as :

$$f_n = 1/2\sigma_w A_s V_n^2 C_s + 1/2\sigma_w A_f V_n^2 C_f \qquad (4.4)$$

Where: A_s and A_f are the surface area and projected areas of the ice floes.

C_s and C_f are the skin friction and form drag coefficients.

σ_w is the sea water density and V_n is the ice floe velocity normal to the ship hull.

The value of normal ice force per unit waterline entrance length f_n in Eq.4.4 can be found by evaluating ice areas and speed parameters. This will be done in the following sections.

The velocity V_n represents the velocity of ice floe motion in a direction normal to the waterline entrance curve. It can be expressed as:

$$V_n = V \sin\theta$$

or $V_n = Vy'/\sqrt{y'^2+1}$ (4.5)

The angle θ represents the slope of tangent to waterline entrance curve with the x-axis. The area A_s is the horizontal surface area of ice floes contained in an element of water surface area. The water surface element has a width equal to unity and a length equal to the thickness of the ice boundary layer δ_i. For relatively small ice floes the thickness of the ice boundary layer δ_i at any ship station x can be obtained from the expression for ice boundary layer thickness (Eq.4.1).

The water surface element has, therefore, an ice area equal to;

$$dA_s = [C\ y(x)/(1-C)]\ \cos\theta.1 \quad (4.6a)$$

or $$dA_s = y(x)/[(1-C)(y'^2+1)^{1/2}] \quad (4.6b)$$

The area A_f is the frontal submerged area of ice floes contained in an element of water surface area. This area is projected perpendicular to the normal ice speed direction. For water surface area with unit width, this area can be expressed as;

$$dA_f = \sqrt{C/K}\ h_w \quad (4.7)$$

Where: C = Ice concentration.

h_w= Draft of ice floes.

K = Ice floe form factor.

The expressions developed in Eqs.4.5, 4.6 and 4.7 for V_n, A_s, A_f can be substituted in Eq.4.4, and the normal ice force per unit waterline entrance length will therefore, be:

$$f_n=1/2\sigma_w\ V^2\sin^2\theta[Cy(x)\cos\theta C_s\ /(1-C) + \surd C/K\ h_w C_f] \quad (4.8)$$

The total normal ice force acting on the ship can be obtained from Eq.4.8 by integrating f_n over the entire length of the ship entrance for the port and starboard sides:

$$F_n = f_n.ds \quad (4.9)$$

Similarly the total tangential force will be :

$$F_t = \mu\ f_n\ ds \quad (4.10)$$

4.3 SHIP RESISTANCE INCREASE

Ship resistance due to ice can, therefore, be obtained from the x-components of both the tangential and normal total ice forces (Fig.4.5), as follows :

$$R_i = 2f_n\sin\theta ds + 2\mu f_n\cos\theta ds \qquad (4.11)$$

or $$R_i = 2f_n\,(dy/ds)ds + 2\mu f_n\,(dx/ds)ds$$

$$R_i = 2f_n\,dy + 2\mu f_n\,dx \qquad (4.12)$$

Substituting the expression for f_n from Eq.4.7 into Eq.4.12;

$$R_i = \sigma_w V^2\sin^2\theta[Cy(x)\cos\theta C_s/(1-C) + \surd C/k\ h_w C_f]dy$$
$$+\sigma_w\mu V^2\sin\theta[Cy(x)\cos\theta C_s/(1-C) + \surd C/k\ h_w C_f]dx \quad (4.13)$$

Equation 4.13 represents the expression for the increase in ship resistance due to ice floes R_i. Besides the increase in resistance due to ice drag a portion of the ship's energy is used to bring the ice floes up to the normal velocity of the ship hull. This energy loss to the ship represents an added increase in ship resistance. This added resistance will be formulated in the following section.

The energy transfered to an element of ice floe cover will be :

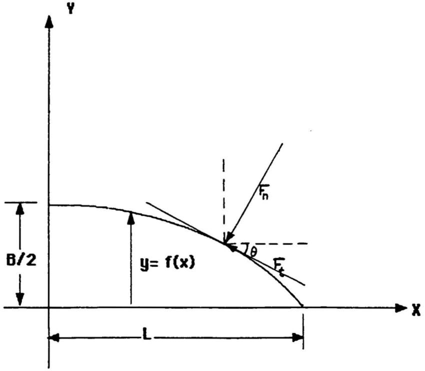

Fig.4.5 Resultant of Ice Forces on Ship Hull

$$E = 1/2\ dM\ V^2 \sin^2\theta \tag{4.14}$$

The mass of the ice floe cover element will be restricted to the inside of the ice boundary layer thickness and it can be expressed as :

$$dM = 2\ C\ y(x)\ h\ \sigma_i\ \cos\theta/(1-C)\ ds \tag{4.15}$$

The increase in ship resistance will therefore be:

$$R_i = [\sigma_i\ V^2 h\ C/(1(1-C))]\ \sin^2\theta \cos\theta y(x) ds \tag{4.16a}$$

or $$R_i = [\sigma_i\ V^2 hC/(1(1-C))]\ \sin^2\theta \cos\theta y(x) dy \tag{4.16b}$$

The resistance increase expressed in the above equation can be added to the ice resistance in Eq.4.13 and the total ice resistance will, therefore, be :

$$R_{it} = \sigma_w V^2\ \sin^2\theta[CC_s y\cos\theta/(1-C) + \surd C/k\ h_w C_f]dy$$

$$+\sigma_w V^2\ \mu\ \sin^2\theta[CC_s y\cos\theta/(1-C) + \surd C/k\ h_w C_f]dx$$

$$+\sigma_i V^2 h[C/(1(1-C)]\sin^2\theta\cos\theta\ ydy \tag{4.17}$$

Equation 4.17 represents an analytical model (MACRO MODEL) for the calculation of ship resistance in different ice conditions in the early design stages.

4.4 EFFECT OF SHIP HULL FORM

It is clear from the Macro Model expression that the resistance of a ship due to ice floe depends on ship beam, ship entrance length, ice concentration, shape of the ice floes, the square of ship speed, coefficient of friction between ice and ship hull and the shape function of the ship waterline entrance y=f(x). Also the resistance of the ship will depend on the water drag of the ice floes.

Depending on the complexity of the shape function y=f(x) representing the entrance part of the ship, ship ice floe resistance R_{it} can be calculated by performing the integrations in the right hand side of Eq.4.17 either through direct integration or numerical integration.

Appendix III contains the integration form of Eqs.4.17 for a general hull entrance shape function. This integration can be simplified for functionally defined hull geometry such as circular, parabolic or elliptic forms. For constant slope hull entrance the MACRO MODEL can be simplified to the following form :

$$R_{it} = 1/2[\sigma_w\ V^2\ \alpha^2\ l\ (\alpha+\mu)/(\alpha^2+1)].$$

$$[CC_s\ l^2\alpha/((1-c)(\alpha^2+1)^{1/2}) + 2h_w C_{fw}/C/K]$$

$$+\ 1/8\ \sigma_i\ V^2\ h\ C\ \alpha^2 B^2/((1-C)(\alpha^2+1)) \tag{4.18}$$

Equation 4.18 represents the ship resistance increase due to ice floes for constant sloped entrance ships. This equation can be used for simplified hull forms or for segmented hull entrance, as shown in Fig. 4.6. This equation represents a special case of the MACRO MODEL for use with simplified hull forms.

The Macro Model is an analytical calculation tool for the estimation of ship resistance in broken or pack ice. It is more suitable for the situation of relatively slow ship speeds and for small ice floes.

4.5 MACRO MODEL APPLICATION

The Macro Model was developed based on the formulation of the ice floe ship imposed motion and the ice floe water drag. The impact between the ice floes and the ship have been ignored. This ship ice floe interaction senario occurs only when the ship is advancing at low speed in small ice floes.

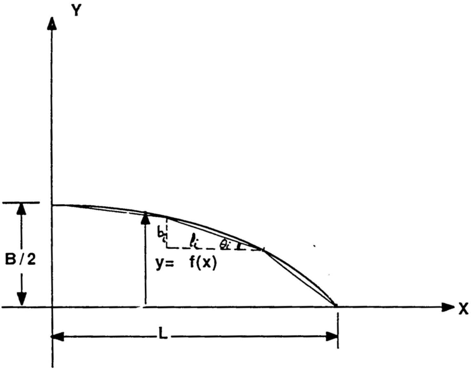

Fig.4.6 Segmented Ship Hull Entrance

Consequently the Macro Model is only applicable to relatively slow ship speeds in moderate concentrations of small ice floes. Numerical results of the Macro Model applied to existing ship hull are presented in chapter 9.

CHAPTER 5

NUMERICAL CALCULATION - DISCRETE ELEMENT METHOD

5.1 BACKGROUND

The finite element method has been used successfully for many years to study the stresses and pressures in a continuous and homogenous medium. However, for discontinuous media, or for rapid and large scale element displacement (such as a failure mechanism), the standard finite element method ceases to be accurate. For this reason, other types of discrete element methods have been developed. While the standard finite element method is based on the analysis of forces and displacements at the nodal points which exist between the elements. The discrete element method is based on the formulation of the force and displacement relationships for the centroid of each discrete element resulting from the interaction with other elements.

In the early seventies, Cundall (1971a) presented a new discrete finite element method which he called the distinct element method, to study the packing and the movements of a granular medium (rock blocks). Cundall (1971b) then developed a computer model to simulate large scale movements in blocks of rocks, based on the distinct element method. The distinct element method is basically a numerical model describing the kinematic and the dynamic behaviour of assemblies of solid blocks. The interaction of the 'blocks' is monitored contact by contact, and the motion of the blocks are modelled block by block.

Since the time that method was first proposed , several forms of distinct elements computer codes have been developed with new added features, such as, block deformation, non-linear inelastic behaviour of joints, plastic behaviour and fluid flow and fluid pressure generation in joints and voids.

Kawai (1977) proposed a new discrete method which he termed the Rigid Body Spring Method (RBSM), for the study of failure mechanisms in structures.

5.2 RESISTANCE IN BROKEN ICE: DISTINCT ELEMENT APPROACH

The two models presented in the previous chapters represent new analytical methods for the calculation of the ship resistance in broken ice or floe ice. As part of the study of ship performance in broken ice different numerical finite element approaches have been investigated for possible use for ship ice resistance calculation. It is clear that a discrete element formulation is more suitable for representing the ship advance in broken ice covers more accurately. The discrete element method has therefore, been adopted, for the first time, to simulate the ice floe motion and the interaction between the ice floes and a ship. The distinct element method has been used successfully for many years (Cundall, 1971, Cundall, 1978 and others) in the rock mechanics field.

Due to the similarity between rock blocks and ice floe formation the distinct element method is, therefore more suitable than other numerical techniques for ice floe simulation. The distinct element method has been introduced as a new numerical method for the calculation of ship resistance in pack ice by Aboulazm and Muggeridge (1989b).

The distinct element method is basically an explicit numerical technique describing the kinematic and the dynamic behaviour of discrete elements (ice blocks). The interaction of the ice floes is monitored contact by contact, and the motion of the ice floes are modelled block by block.

The equilibrium between the ice floes, the displacement and the contact forces of the floes are found through a series of calculations tracing the movements of individual ice floes. The numerical calculations of the ice floe behaviour is based on the cyclic time step marching technique. The calculations alternate between the application of Newton's second law and the force displacement law at the contact between the ice floes or the ice floes and the ship (as shown in Fig. 5.1). In these calculations ice floes experience very little deformation in comparison to their motion as rigid bodies.

A simplified distinct element formulation has therefore, been developed and presented in this thesis to simulate the interaction between advancing ship, the ice floes and the resulting ship resistance increase due to the ice fragments. The motion of the ship in the ice cover and the subsequant displacement of the ice floes have been analysed using the time step marching approach.

5.2.1 ASSUMPTIONS

In the process of interaction between the ship and a number of ice floes the following assumptions have been adopted for the distinct element formulation and the time step approach:

- The accelerations of ice floes and the ship are constant during a time step.
- During a single time step, a disturbance does not propagate from any ice floe further than its immediate neighbouring ice floes.

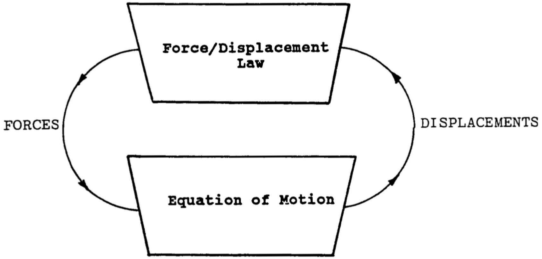

Fig.5.1 Force Displacement Relations

- The forces acting on any ice floe are only due to other floes in contact with it.
- Damping forces are constant during a time step.
- The motion resulting from forces acting on ice floe follow Newton's 2nd law.
- Deformation of any individual ice floe is much smaller than displacement of ice cover. Ice floes are therefore treated as rigid bodies.
- Overlaping is assumed mathematically (not physically) for use with time step to simulate ice floe displacement.

5.3 ICE FLOE FORCES AND DISPLACEMENTS

As the ship advances into the ice floe covered water a number of ice floes are displaced from their original positions. To analyse the displacement of the ice floes by the ship and the resulting ice forces and ship resistance increase, the case of two body interaction will be considered in more detail.

Consider the inteaction between the two ice floes i and j and their contact shown in Fig.5.2. The coordinates (x^i,y^i), (x^i,y^j) and (x^c,y^c) represent the global coordinates of the centroids of the ice floe i , the ice floe j and the contact point between the two ice floes (Cundall, 1974). The

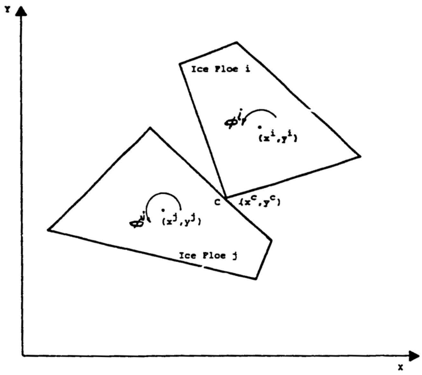

Fig.5.2 Coordinate System for Ice Floes in Contact

displacement of the contact point in the x and y directions, as shown in Fig.5.2 , will be:

$$\delta u^c_y = \delta u^i_y - \delta u^j_y + \delta\phi^i(x^c-x^i) - \delta\phi^j(x^c-x^j) \qquad (5.1)$$

$$\delta u^c_x = u^i_x - \delta u^j_x + \delta\phi^j(y^c-y^i) + \delta\phi^j(y^c-y^j) \qquad (5.2)$$

The displacement of the contact point in normal and tangential directions will be :

$$\delta u^c_s = \delta u^c_x \cos\alpha' + \delta u^c_y \sin\alpha' \qquad (5.3)$$

$$\delta u^c_n = \delta u^c_y \cos\alpha' - \delta u^c_x \sin\alpha' \qquad (5.4)$$

As shown in Fig.5.3, the normal and tangential forces at the contact point will, therefore be :

$$F^c_n = \delta u^c_n K_n \qquad (5.5)$$

$$F^c_s = \delta u^c_s K_s \qquad (5.6)$$

The forces at the centroids of the ice floes i and j due to the contact force, in the x and y directions, are shown in Fig.5.4 and they will be:

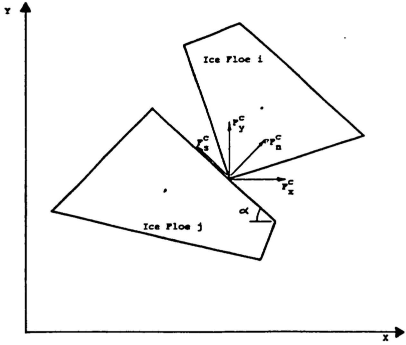

Fig.5.3 Contact Forces Between Two Ice Floes

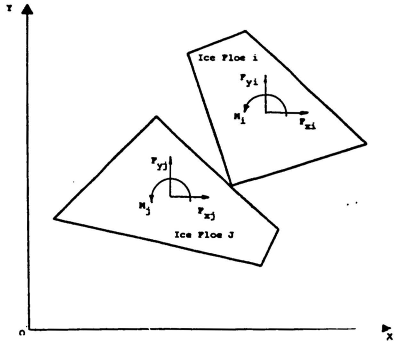

Fig.5.4 Total Forces and Moments Acting on Ice Floes

$$F^{cj}_{y} = F^{c}_{s} \sin\alpha' - F^{c}_{n} \cos\alpha' \qquad (5.7)$$

$$F^{ci}_{x} = F^{c}_{s} \cos\alpha' + F^{c}_{n} \sin\alpha' \qquad (5.8)$$

$$F^{ci}_{y} = -F^{cj}_{y} \qquad (5.9)$$

$$F^{ci}_{x} = -F^{cj}_{x} \qquad (5.10)$$

If the forces $F_{x\ load}$ and $F_{y\ load}$ represent the external forces acting on the ice floe i, the total forces and moments acting at the centoid of this ice floe will be :

$$F^{i}_{x\ sum} = \Sigma_c\ F^{ci}_{x} + F^{i}_{x\ load} \qquad (5.11)$$

$$F^{i}_{y\ sum} = \Sigma_c\ F^{ci}_{y} + F^{i}_{y\ load} \qquad (5.12)$$

$$M^{i}_{sum} = \Sigma_c\ [F^{ci}_{y}(x^{c}-x^{i}) - F^{ci}_{x}(y^{c}-y^{i})] \qquad (5.13)$$

From Newton's second law the velocities of the centroid of the ice floe i, are shown in Fig.5.5 and they can be written as :

$$u^{\cdot i}_{y} = [F^{i}_{y\ sum}\ \delta t]/m^{i} \qquad (5.14)$$

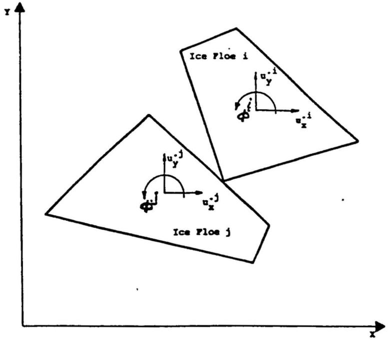

Fig.5.5 Velocities of Ice Floes

$$u^{\cdot i}_{x} = [F^{i}_{x\,sum}\ \delta t]/m^{i} \tag{5.15}$$

$$\phi^{\cdot i} = [M^{i}_{sum}\ \delta t]/I^{i} \tag{5.16}$$

The displacement of the ice floe, as shown in Fig.5.6, can therefore be calculated from :

$$u^{i}_{y} = u^{\cdot i}_{y}\ \delta t \tag{5.17}$$

$$u^{i}_{x} = u^{\cdot i}_{x}\ \delta t \tag{5.18}$$

$$\phi^{i} = \phi^{\cdot i}\ \delta t \tag{5.19}$$

The interaction between the ice the floes and the ship will involve the displacement of the ice floes and as a result forces will be generated between the ice floes and the ship hull as shown in Fig.5.7.

5.4 NUMERICAL TIME MARCHING SCHEME

The calculation cycle presented above is an explicit numerical method which marches time forward by very small time step δt. This explicit method assumes that, in one time step, a given block can only communicate with its nearest

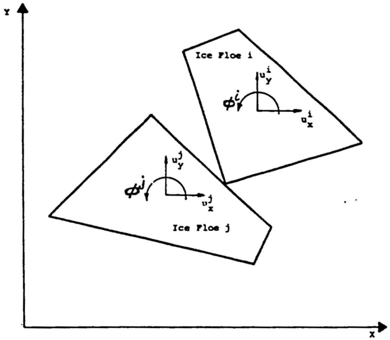

Fig.5.6 Displacements of Ice Floes

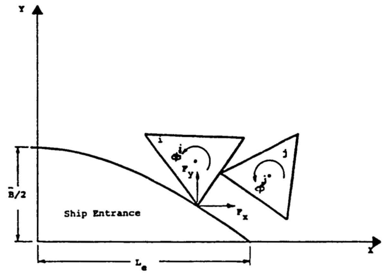

Fig.5.7 Ice Floes Interaction with Ship Entrance

neighbors.

The one cycle presented can then be repeated again and again to obtain the required displacements and forces. The displacement of each ice floe is traced and updated after each time interval through a x-y mesh of boxes as shown in Fig.5.8.

5.5 APPLICATION TO SHIP RESISTANCE PROBLEM

The analytical formulation for the distinct element method presented above has been incorporated in a computer program to facilitate the calculation of forces of a number of ice floes for a large number of time steps. The computer model "RBM" was developed in 1978 as a base line two dimensional distinct element program (complete description and listing of the RBM program is given by Cundall et al., 1978). This program has been updated, modified and extended to tackle the broken ice forces problem and the problem of ship resistance in broken ice. The new version of "RBM" has been termed "DICER" (for **D**istinct element **ICE** **R**esistance). The computer programs "RBM" as well as "DICER" are written in FORTRAN language and they consist of a number of subroutines which define and run the ice floe domain problem. The program DICER can model a large number of assemblies of arbitrary ice

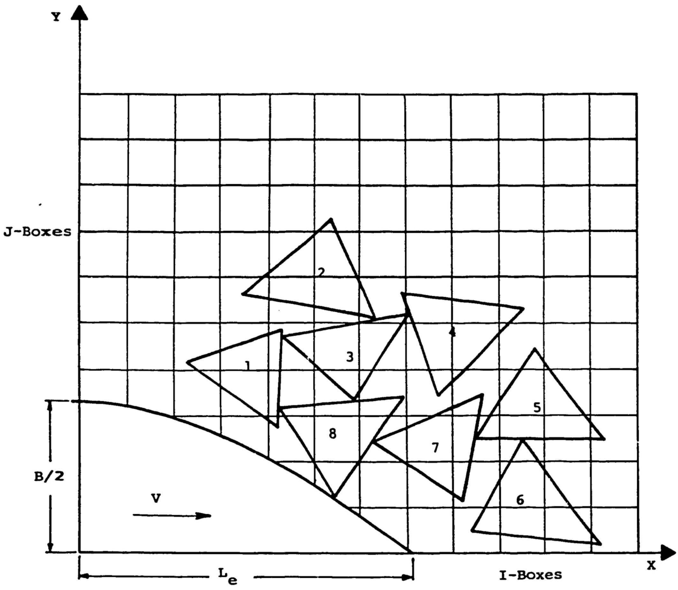

Fig.5.8 Ice Floe Ship Interaction-Distinct Element Grid

floes which may be in contact or not. Consideration of ice floe geometry, number of ice floes, number of calculation cycles and the size of time step is limited by computer memory and speed.

The distinct element approach, introduced in this thesis for the calculation of ship resistance in broken ice, can be applied on ships advancing at medium speeds in high concentration ice floes. The ice floes are to be relatively large and have straight sides with a minimum number of sides and corners.

CHAPTER 6

EXPERIMENTAL INVESTIGATION

6.1 BACKGROUND

To design or operate ships for operation in ice infested waters it is necessary to predict the performance qualities (resistance versus speed) of these ships in anticipated ice conditions. The study and prediction of ship performance in ice floe covered waters can be accomplished through a number of approaches.

As demonstrated in the previous sections it is possible to formulate analytical and numerical models to calculate the resistance of a ship in a given ice condition, however these models, as well as others, have to be validated against known results before being widely accepted as design tools.

In some cases it is possible to build and test a full-scale prototype ship to find and formulate some form of empirical relations for ship resistance predictions. This approach is clearly expensive, however, a number of existing ships operating in the ice are collecting data either during their routine operation or through designed ice expeditions.

The use of reduced scale physical models for ship study in open waters has been a valid technique for the prediction of ship resistance since the last century. Physical model testing provides a relatively inexpensive way to predict ship resistance with a minimum requirement for time and effort. However, physical modelling is limited to the accuracy of the instruments used and to the scale relations adopted.

Ship model testing in simulated ice has been conducted in a number of countries in the last half century. These model testing activities were mostly restricted to the study of ship resistance in continous ice sheets.

Very few model tests were conducted in simulated broken ice. As a part of the present ship resistance study a model test experiment in simulated broken ice has been undertaken and is described in this and the following chapter.

6.2 EXPERIMENTAL PARAMETERS

The main objective of ship model testing is to measure the resistance of the model as it advances in different simulated ice conditions. The parameters involved in model testing are numerous and they can be categorized into three main groups, namely, the ship parameters, the ice floe cover parameters and the parameters related to the interaction process between the ice floes and the ship.

The ship models used in model testing are usually geometrically scaled down from proposed prototype ship dimensions or from existing full scale vessels by a suitable scaling factor. The model will, therefore, be geometrically similar to the full scale ship. For resistance tests the model is only prepared as a bare hull with appendages (such as skegs, rudders) without the propeller.

In testing ship model resistance in simulated broken ice, the ice parameters of major importance are the size of ice floes, the thickness of ice sheet and the ice floe concentration (represented in 1/10 fractions). In this type of test the strength of the ice has a relatively minor effect due to the lack of any ice breaking, particularly when considering ice floes of moderate size. The modelling approach can be considered, in this case, as a total hydrodynamic modelling which does follow Froude model laws. No attempt, therefore, was made to model the structure properties of the ice or any ship structure flexibility.

6.3 SCALING LAWS

In order to construct and perform physical modelling experiments, it is important to adopt suitable similitude rules. Similitude rules can be obtained by three different approaches, namely dimensional analysis, similitude of forces and the general method of comparing the equations of motion and ice resistance in the model and prototype. In comparing prototype and model, three types of similitude between the model and the prototype are generally considered :

I - Geometrical similarity where the ratios of all similar length dimensions are equal to a constant.

II - Kinematic similarity where all similar particles travel geometrically similar paths and the ratios of velocities are equal to a constant.

III- Dynamic similarity where the forces on the prototype are related to the forces on the model by a constant.

A model is said to be in complete similitude if it has geometric, kinematic and dynamic similitude. This is always a goal in physical modelling and it is seldom obtained in practice due to incompatable scaling requirements. This limitation, however, does not hinder the usefullness of the results, provided the deviations are of secondary importance.

6.4 MODELLING THE SHIP IN ICE

To obtain close similarity in modelling ships in ice floes the forces involved must be examined. In the interaction process between ship and ice floes, the total ship resistance force can be broken down into components, and it can be written as :

$$R_t = R_w + R_i \tag{6.1}$$

Where: R_t = Total ship resistance

R_w = Resistance in open water

R_i = Re·istance due to ice floes.

In the case of navigation in broken ice covered waters, with small and mediam size ice blocks, breaking of ice seldom takes place. As a result resistance due to ice R_i involves only the process of pushing or sweeping of ice pieces by the ship away from her path. Consequently modelling of ships in broken ice pieces or ice floe infested waters can be taken as hydrodynamical modelling and for this type of modelling Froude scaling laws should apply.

Table (6.1) shows the similarity relations for modelling vessels in ice floe covered water. These relations describe the geometry, speed and resistance for the full scale ship as compared with the model results.

TABLE 6.1

Similarity Relations for Modelling

Length	=	L_p	=	λL_m
Beam	=	B_p	=	λB_m
Drift	=	T_p	=	λT_m
Force	=	F_p	=	$\lambda^3 F_m$
Displacement	=	∇_p	=	$\lambda^3 \nabla_m$
Velocity	=	V_p	=	$\sqrt{\lambda}_m V_m$
Time	=	t_p	=	$\sqrt{\lambda}\ t_m$
Torque	=	Q_p	=	$\lambda^4 Q_m$
Thrust	=	TH_p	=	$\lambda^3\ TH_m$
Power	=	P_{dp}	=	$\lambda^{3.5} P_{dm}$
Gravity	=	g_p	=	g_m
Thickness	=	h_p	=	λh_m
Ice Length	=	l_p	=	l_m
Coefficient of Friction	=	f_p	=	f_m
Visocosity	=	u_p	=	u_m
Density of Water	=	ρ_{wp}	=	ρ_{wm}
Density of Ice	=	ρ_{ip}	=	ρ_{im}

---

where: p = prototype, m = model, λ = geometric scale factor

The next chapter describes and reports on a model testing program conducted at the towing tank of Memorial University as part of this study.

6.5 SHIP MODEL CORRELATION

From the model testing program in simulated broken ice, it is important to use the test results to predict the resistance for existing or proposed full scale ships. This can be done through the Froude modelling approach, where the measured model resistance forces are scaled up to predict the full scale ship resistance.

As mentioned before the total model resistance measured can be divided into two major components; resistance due to open water and resistance due to ice floes. The resistance due to ice floes has been isolated by running the same ship models in open water under the exact same conditions as the ice floe tests. The resistance due to ice floes has therefore, been taken as the difference between measured total ship model resistance and measured open water resistance of the model.

Based on Froude modelling the ice resistance per unit ship mass (i.e. the specific resistance) should be the same for both the ship model and the full scale ship, i.e.:

$$R_{im}/m = R_{ip}/M \tag{6.2}$$

Where: R_{im} = Ice floe resistance of the model, which is equal to the difference between measured total resistance of the model in ice and the open water resistance of the model.

R_{ip} = Ice floe resistance of the prototype ship.

M = Mass of the full scale ship.

m = Mass of the ship model.

From the above principle the full scale ship ice resistance can, therefore, be written as:

$$R_{ip} = (R_{im})\ \lambda^3 \tag{6.3}$$

Where λ is the model scale.

To predict the total resistance of a prototype ship, the open water resistance has to be established. Open water resistance of a full scale ship can be obtained through conventional naval architecture methods such as model testing in open water, standard series calculation methods or theoretical resistance calculations. The total full scale ship resistance in ice floe covered waters will therefore be:

$$R_{tp} = R_{wp} + R_{ip} \qquad (6.4)$$

This resistance is usually termed bare hull ship resistance. To make use of this resistance magnitude in ship design and powering requirements, it is necessary to include the appendages and their effects on ship resistance. Also ship resistance allowance should be made for the type of service the ship will be engaged in. The resistance service allowance and appendage effects are necessary to reach the required ship thrust power. The ship required engine power can be calculated using conventional naval architecture methods.

It is clear that both the ship resistance and the power requirement depend on the speed of advance of the ship. It is, therefore, important to calculate the total ship resistance in the speed range required in the ship operation. Calculation of ship resistance and power are essential for the purpose of designing new ships to operate in perceived ice conditions and for the prediction of speed of advance of existing ships in given ice cover conditions.

CHAPTER 7

MODEL TESTING

7.1 BACKGROUND

As part of the study of ship resistance in broken ice, a series of model tests were conducted. This was necessary to obtain some model results and to verify the results obtained by analytical and numerical models. The model experiments were conducted at the towing tank facilities in the Faculty of Engineering at Memorial University.

In developing the model test program, the similarity relations and the model full scale correlations presented in the previous chapter have been adopted.

7.2 EXPERIMENTAL ARRANGEMENT

The ship model towing experiments were conducted in the towing tank of Memorial University of Newfoundland. Located on the North Side of the Campus of the University, the tank is built of reinforced concrete and has inside dimensions of 58.27 meters length, 4.57 meters width and 3.04 meters depth,as shown in Fig.7.1. The operating water depth

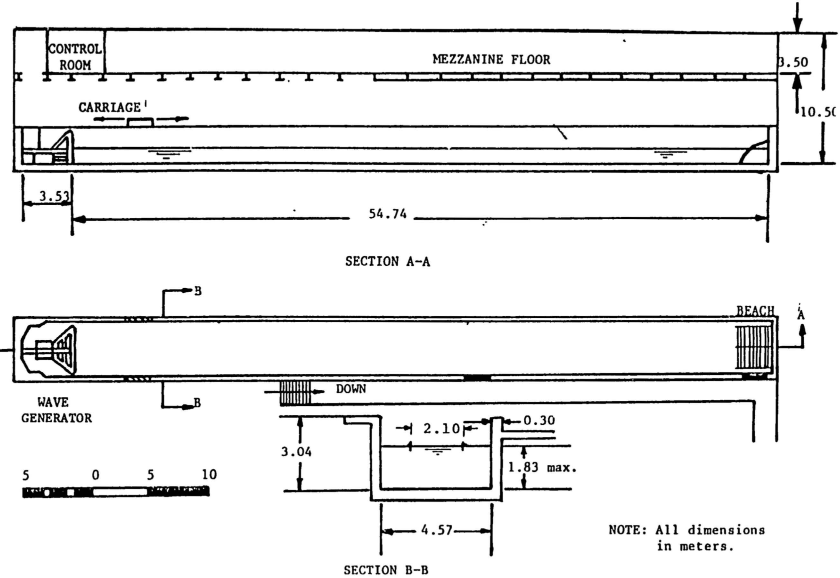

Fig.7.1 Elevation and Plan View of Wave/Towing Tank

in the tank is 1.83 meter and lower. At one end (West end), the tank is equipped with a hydraulically operated piston type wave generator where an aluminum waveboard is driven by a hydraulic actuator with 48.8 kn force capability over a 0.25 meter stroke. The wave tank has glass viewing windows installed along one side of the tank. These windows are used for observations and video recording. At the east end of the tank a wave dissipating parabolic beach is constructed of wood grids mounted on top of an aluminum frame. A complete description of the tank was given by Murray and Muggeridge (1981) before the carriage and the new beach were installed.

7.2.1 TOWING CARRIAGE

The tank is equipped with a towing carriage (see Fig.7.2). The carriage is designed to run on rails installed on the sides of the tank. Although the overall inside length of the tank is 61 meters, the wavemaker takes a length of 5 meters and the beach occupies 4.6 meters. The remaining effective length of the tank to be used for towing is approximately 49 meters. The maximum speed of the carriage is 5 m/sec, and the normal operating speed is 3 m/sec. The speed control on the carriage can achieve and repeat a steady speed of 0.05 % of set speed. Also, the carriage has

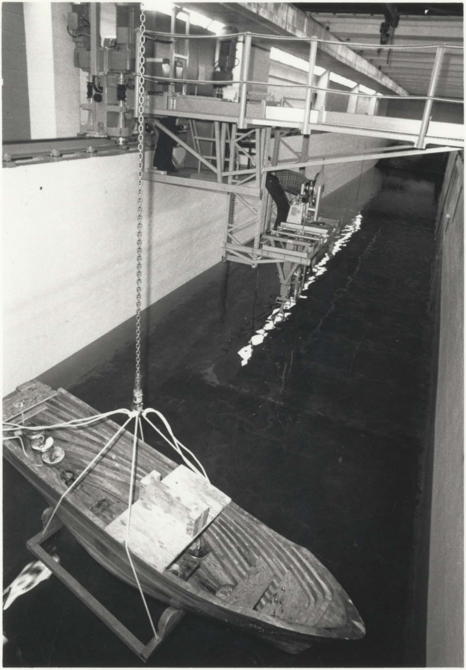

Fig.7.2 Towing Carriage

acceleration control which controls the acceleration and deceleration of the carriage in the range of 0.1 to 0.5 gravitational acceleration. The carriage can travel at very low speeds and is equipped with three types of braking systems. A complete description of the towing carrige is given by Hsiung et al (1981).

7.2.2 ICE COVER MODEL

The ice cover has been modelled to accurately simulate the interaction between ship, water and the ice. In attempting to model the ice cover researchers have used different materials such as polyethylene, polypropylene, wood, paraffin wax and frozen water (with additives) to produce artificial ice.

In this study, due to the hydrodynamic modelling approach, the model ice was constructed from fragments of wax. The wax has an average thickness of 0.0375 meters (1.5 meters full scale), an approximate relative density of 0.88 and a friction coefficient of 0.2. A large number of wax pieces were prepared and spread over the water surface to simulated a given ice concentration. Low concentration ice floes in the towing tank are shown in Fig. 7.3 and high concentration shown in Fig. 7.4. Two sizes of wax pieces were used, large

Fig.7.3 Model Ice Floe, Low Concentration

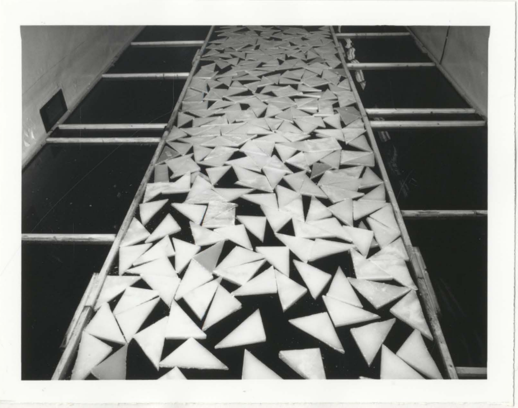

Fig.7.4 Model Ice Floe, High Concentration

fragments with an area of 0.032 square meters and small fragments with an area of 0.016 square meters. The shape of wax fragments was triangular for both sizes. As it was practically difficult to cover the whole tank with ice floes, a channel of wood was constructed to reduce the amount of ice floes or wax required. The channel extended a length of 26.0 meters, has a width of 2.13 meters (more than four times the model beam) and it was centered in the tank.

7.2.3 SHIP MODEL

Ship models are usually built to reduced scale using different materials such as wood or fiberglass. In this study the model used is a 1:40 model of the Canadian Coast Guard R-Class icebreaker which was built of fiberglass from a mould supplied by the National Research Council's Institute of Marine Dynamics. The characteristics of the R-Class model are presented by Newbury (1984).

7.3 TYPICAL TEST RUN

Before the start of the test the ice friction coefficient was measured using tilting surface method. In a typical test run thereafter, the dynamometer was calibrated then the ship model was attached to the carriage and the instruments were initialized. The ice cover was distributed over the water surface and the carriage speed and acceleration were set (see Fig. 7.5). The carriage was then driven until it reached the assigned speed the dynamometer lock pin was then removed and the resistance recording started. Before the carriage was to slow down the lock pin was inserted to stop the force recording and to prevent any damage due to carriage deceleration. The resistance measured was recorded on a strip chart recorder over the test run duration.

The model test runs were repeated for different ship speeds starting with slow speed and ending with the highest possible practical speed. Two ice floe sizes were used and the model test runs were repeated for different ice concentrations and different ship models (see Fig. 7.6 and Fig. 7.7).

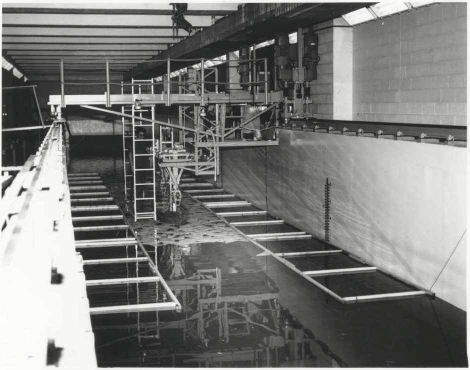

Fig.7.5 Ship Model Test Run

Fig.7.6 Ship Model Bow in Simulated Broken Ice

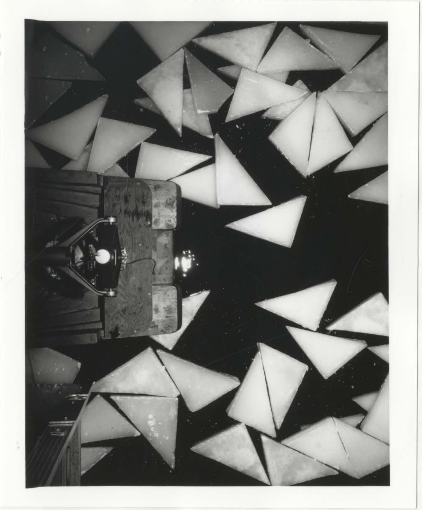

Fig.7.7 Ship Model Stern in Simulated Broken Ice

CHAPTER 8

FULL SCALE SHIP TESTS

8.1 BACKGROUND

Field measurements of ship performance in ice have been practised for some time. Ice expeditions started as early as the last century when explorers looked for the North-West Passage.

In recent years a number of ice expeditions were conducted including the famous "Manhattan Project" where the "SS Manhattan" sailed accross the North-West Passage between the Atlantic and the Pacific Oceans (Gray and Maybourn, 1981; Mookhock, Voelker and DeBord, 1981).

Most ice expeditions have a number of functions to accomplish, most important from the naval architecture view point, are the resistance and power of the ship in different ice conditions. Ice condition may vary from continuous ice sheet and ice ridges to pack or broken ice.

Field measurements of ship performance in broken ice or ice floe covered waters have not so far been treated as independent ship-in-ice testing programs. Most reported field tests were initiated for the purpose of measuring ship performance in consolidated ice sheet. All efforts were geared to measuring ship powering and speeds in different ice thicknesses, and in some cases tests were done to measure hull stresses. The reported results of ship resistance in broken ice were therefore byproducts of ship field programs. These measurements were taken as ships were steaming through different ice conditions enroute to and from the designated testing area.

A limited number of field tests in broken ice were reported in the area of navigation in broken channels created by icebreakers.

8.2 PROBLEMS WITH FULL SCALE TESTS

Although full scale ship resistance measurements represent a realistic ship resistance indication for a given ship a number of problems are associated with full scale field testing :

1. Field tests are very expensive endeavours.
2. One or two field tests are not enough to develop new methods for ship resistance predictions.
3. A number of ship hull forms have to be used with some form of systematic variation of hull form

parameters in order to develop a useful prediction method. Once again this is almost impossible due to the cost.

4. It is impossible to measure ship resistance in the field and as a result only shaft thrust can be measured. The thrust then has to be translated into ship resistance using thrust deduction factors which is usually based on open water data.
5. The speed of the ship has to be measured approximately.
6. The ice floe condition is very diffcult to define. The concentration, thickness and size of ice floes are defined in most cases based on visual observations which can be subjective.
7. It is difficult to create any variation in the ice condition, such as ice concentration or thickness. This variation is important for any parametric study.
8. The environmental forces such as current, wind and waves and their effects on the ice floes are usually ignored. These forces may have significant effects on the pressure on the ice field and therefore affect the thrust measurements.

Full scale ship resistance measurements should therefore be viewed as supportive and verifying tools for other analytical and numerical ship resistance prediction methods.

8.3 ICEBREAKERS

The available open literature indicates that most full scale ship tests were done on icebreaker type ships (Edward et al., 1972; German and Lawrence, 1975; Nobel et al., 1978; Edward et al., 1981; Vance et al., 1981; Hellman and Schwarz, 1983). This was due to the fact that ships designed to break the ice were built long before any commercial icebreaking ship. A number of icebreaker test expeditions were conducted in Europe, in the Soviet Union and in North America. These tests were conducted in lakes, oceans and/or rivers. Most full scale tests of icebreakers were performed in continuous sheet ice as such ice defined the upper limit of most icebreakers power. Very little measurements for icebreakers in broken ice have taken place.

The Canadian icebreaker "CCGS Franklin" was partly tested in fragmented ice during two probes of Lake Melville, Labrador, where measurements were made of ship performance and of the ice conditions (Michailidis and Murdey, 1981).

Although the tests did not cover a useful range of broken ice, the "CCGS Franklin" results have been used in this thesis as an example of full scale results in broken ice. The values of the thrust measured for different ship speed in different fragmented ice conditions for the CCGS Franklin are shown in Table 8.1. The ice floe conditions during the tests consisted of a combination of brash ice, small ice floes and large ice floes with different concentrations and having average thickness of 0.65 meters. No information was available on the exact size of the brash ice.

The American "USCGC Katmai Bay" was also tested in fragmented ice (Vance et al,1981). Results from these tests have been used for comparison and validation of an analytical broken ice ship resistance model (Aboulazm and Muggeridge, 1989a).

Other full scale tests were performed in brash river ice (Ashton et al, 1972). However the total number of field measurements of ship resistance in broken ice, available in the open literature is very limited.

8.4 COMMERCIAL SHIPS IN ICE

Up to recent years very few commercial ships were designed to break an ice sheet, however a number of ships were

TABLE 8.1

R-CLASS ICEBREAKER CCGS FRANKLIN
FULL SCALE TEST RESULTS

(After Michailidis and Murdey, 1981)

Ship Speed (m/s)	Thrust (kN)	Ice Condition
2.26	168.4	50% Brash, 35% 1.5-1.8 m, 15% 3.0-4.0 m.
3.91	204.2	60% Brash, 30% 1.2-1.8 m, 15% 3.0-4.6 m.
4.42	249.4	60% Brash, 25% 1.2-1.8 m, 15% 3.0-4.0 m.
5.29	318.8	70% Brash, 20% 1.2-1.8 m, 10% 1.8-2.4 m.
6.17	388.6	70% Brash, 20% 1.2-1.8 m, 10% 3.0-4.6 m.
7.30	428.5	60% Brash, 35% 1.2-1.8 m, 5% 3.0-4.6 m.
7.71	548.1	60% Brash, 40% 1.2-2.4 m.
8.22	667.6	60% Brash, 40% 1.2-2.4 m.

designed to safely navigate in light ice conditions or to follow icebreakers in heavy ice conditions. Very few results are available on the full scale total ship resistance in ice, particularly pack ice or broken ice for commercial vessels. However a number of full scale experiments were conducted on icebreaking bulk carriers and tankers (German et al, 1981; Gray and Maybourn, 1981; Makinen and Roose,1977; TDC,1981).

8.5 OTHER VESSELS

Other types of vessels have been designed and tested for operation in ice covered waters. The offshore supply vessels required for operation on the Canadian East Coast and the Beafourt Sea require special ice navigation considerations.

A number of these powerful ice classed supply vessels have been tested in ice covered waters (Ghoniem et al, 1984).

Ashton et al (1972) reported on full scale field measurements of a river push tug in the Mississippi River.

8.6 SUMMARY

Full scale ship resistance measurements in ice in general, and in broken ice in particular, represent an important research tool for understanding and predicting ship resitance in ice. However due to the cost involved in this type of testing, a very limited number of field tests have been performed.

A full scale field measurement of ship resistance in broken ice was beyond the scope of this thesis. Beside the cost and the time required to perform a successfull full scale field testing program, field testing can not easily produce a variety of broken ice conditions, such as ice floe concentration, size and concentration, to verify analytical and numerical models. Neverthless the results from available full scale testing of the R-Class icebreaker have been used for comparison with the analytical and numerical ship resistance results for very limited ice conditions.

CHAPTER 9

RESULTS AND COMPARISONS

9.1 GENERAL

The study presented in this dissertation covers different approaches to the problem of ship resistance in broken ice covered waters. Analytical solutions of this problem are been presented on the basis of two theoretical models, the MICRO MODEL and the MACRO MODEL as well as the numerical distinct element approach. Calculations using these models were carried out on a real ship hull form of the Canadian R-Class ice breaker "CCGS Franklin". Results of these calculations are presented in this chapter.

Model tests were also conducted on a 1/40 model of the "CCGS Franklin" running in simulated broken ice covered water. These tests were conducted in conjunction with the analytical and numerical formulations of the broken ice ship resistance to compare and verify the results. The results obtained from model experiments, in a variety of operating conditions, are presented in this chapter.

Although no part of this study was devoted to conducting full scale ship field tests in natural broken ice, a number of test results available in the open literature have been collected and presented in this thesis. Full scale field tests have been used for the comparison with results from both the theoretical and physical model test methods.

9.2 MICRO-MODEL RESULTS

The MICRO MODEL developed in chapter 3 has been used to predict the ship resistance in different conditions of broken ice for different ship speeds. As shown from Eq.3.48, the resistance of the ship in broken ice depends on a number of parameters related to the ship, the ice floe cover and to the interaction process. In order to make use of the model for a given ship, the numerical values of these parameters have to be established. Discussions of these parameters and their values are given in the following sections.

The coefficient of friction between the ice floes and the ship hull depends on a number of factors such as, surface roughness of the hull, type and temperature of the ice and the ice loading rate. A number of studies have been conducted to measure the coefficient of friction of ice and the effects of the above factors (Makinen, Lahti and Rimppi, 1975; Keinonen and Nyman, 1979). In this study the average value of 0.15 was taken initially for the numerical calculation. It was felt that this value represented an average value for general numerical predictions, however, for specific cases it is always possible to conduct tests to establish the coefficient of friction under a given operating environment.

The added mass coefficient of ice floes depends on a number of parameters such as the size and the draft of the ice floe and the water depth. The added mass coefficient for a variety of body shapes has been reported by Blevins (1979). For this study the added mass coefficient was taken as unity, which is an acceptable value for approximate general use.

The impact coefficient of restitution between the ice floes and the ship is an ice material property which depends on the mechanical and physical properties of the ice and on the rigidity of the ship hull. The value of this coefficient has been taken as 0.1 for this study. This value was based on the drop ball experimental work done by Yen et al (1970), and Likhomanov and Kheisin (1971).

A computer model code has been developed for the MICRO MODEL formulation and, using this program, a number of numerical ship resistance results were obtained. The resistance in broken ice was calculated for a the Canadian Coast Guard R-class ship hull form in different ice concentration. A detailed description of the R-class hull form and its characteristics is given in Table 2.1.

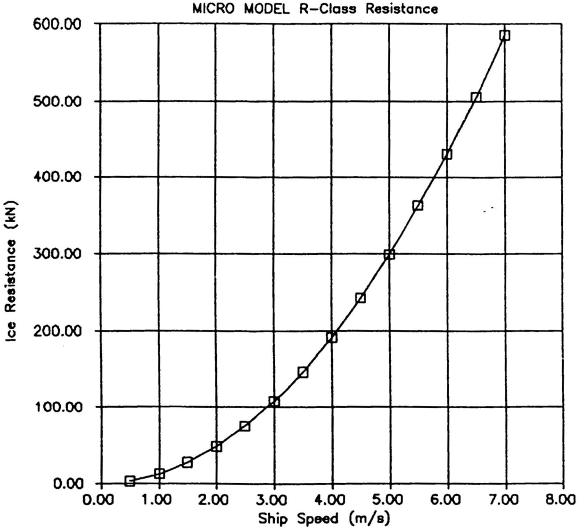

Fig.9.1 Micro Model Resistance Results for R-Class

Figure 9.1 shows the resistance results for the R-class in broken ice cover with concentration of 80%. The size of the individual ice floes has been taken as 4 meters, and the ice thickness used is 1.0 meter. The speed of the ship has been increased in steps from 0.5 meters per seconds to 7.0 meters per seconds. This speed range covers the possible operating range of this vessel in broken ice. For each specified ship speed the resistance was calculated and plotted in kN as Shown in Fig.9.1.

To study the effect of the coefficient of friction between the ship hull and the ice floe, numerical calculations of ship resistance for different friction coefficients were performed. Micro Model ship resistance results for friction coefficient of 0.1, 0.15 and 0.2 are shown in Fig. 9.2.

The effect of the coefficient of restitution e between the ice floe and the ship hull on the Micro Model ship resistance predictions is shown in Fig. 9.3. The values of e=0.05, e=0.1 and e=0.15 were used in the calculation.

9.3 MACRO-MODEL RESULTS

The resistance of a ship due to ice floes was also calculated for the "CCGS Franklin" using the Macro Model. As shown in Chapter 4 the Macro Model is based on the calculation

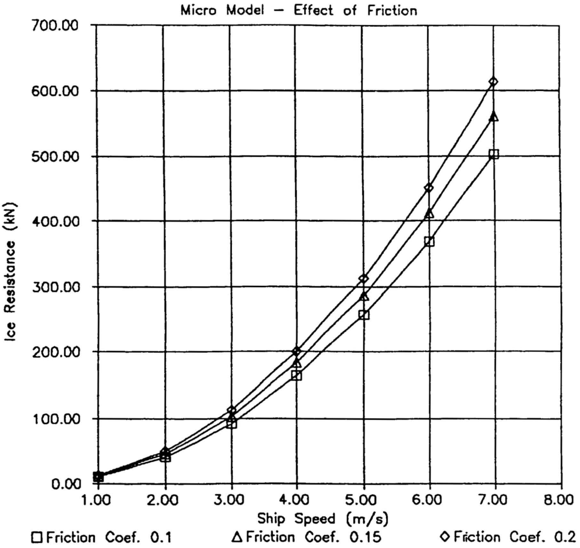

Fig.9.2 Micro Model Results for Different Friction Coefficients

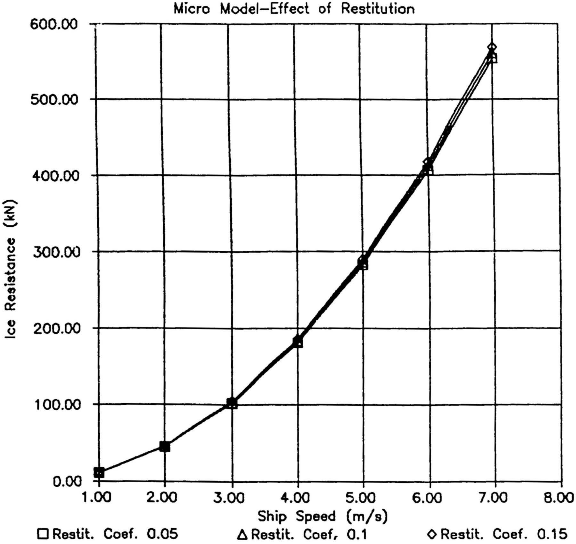

Fig.9.3 Micro Model Results for Different Restitution Coefficients

of ship resistance in broken ice due to the fluid drag of the ice floes as they are displaced by the advancing ship. Consequently the Macro Model uses, not only the ice floes and ship geometrical parameters, but a number of hydrodynamic parameters, such as the skin friction and form drag coefficients of the ice floes.

The skin friction coefficient for ice floes depend on the Reynold number which is variable with ice floe speed, however for this study it was taken as an average constant value equals to 0.1. The form drag coefficient for the ice floes, which also depends on the Reynold number, has been taken as 2.0 (Horner, 1975; Schlichting, 1979).

Calculation of ship resistance in broken ice using the Macro Model has also been performed on the R-Class icebreaker the "CCGS Franklin" in different operating conditions. Figure 9.4 shows the broken ice ship resistance of the "CCG Franklin", in kN, for different ship speeds. In the Macro Model resistance calculation of the ice floe concentration was taken as 80% and the ice floes were 1.0 meter thick.

9.4 RESULTS FROM DISTINCT ELEMENT METHOD

Numerical calculation of ship resistance in broken ice

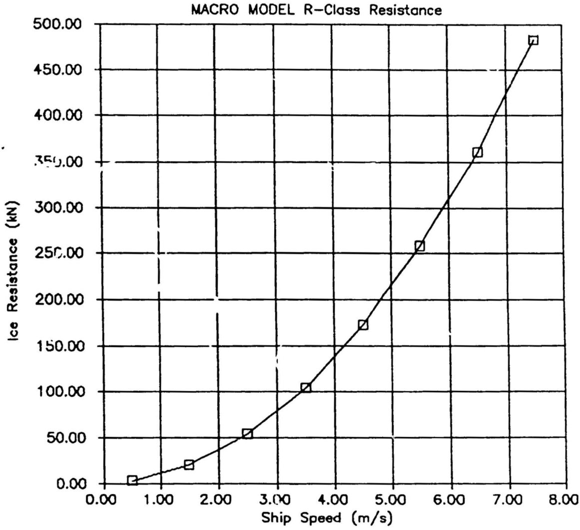

Fig.9.4 Macro Model Resistance Results for R-Class

has also been performed on the R-Class icebreaker hull form using the distinct element method in conjunction with the Micro Model. The calculation of ship resistance was performed using the program DICER for different ship speeds and for ice floe of concentration 80%, ice floe size 4 meters and ice thickness 1.0 meter. The shape of the individual ice floe was taken as triangular. The computer program DICER is written in FORTRAN and it has been installed on a Vax computer using the VMS operating system.

Calculation of ship resistance was performed for different ship speeds in the range of 1.0 m/s to 7.0 m/s. Numerical results of ship resistance in broken ice in kN using the program "DICER" are shown in Fig.9.5.

9.5 RESULTS OF MODEL EXPERIMENTS

The model test program conducted as part of this study covered a number of operating conditions. As mentioned before the model used in the tests is a 1/40 scaled model of the R-Class icebreaker tested at different ship speeds, however the ice conditions have varied in terms of ice floe size and ice floe concentration. Two sizes of ice floes have been used in the model tests, large ice floes of an area equal to 0.032 square meters and small ice floes of an area equal

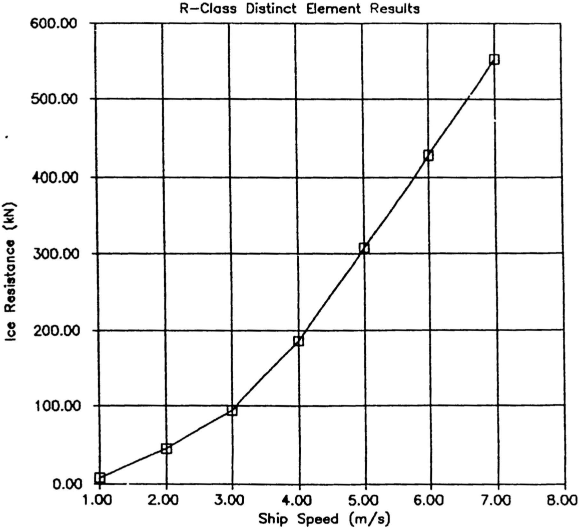

Fig.9.5 Distinct Element Results for R-Class

to 0.016 square meters. Both sizes of ice floes were made of triangular shape. The ice floe concentrations were taken as 50%, 60%, 78.7%, 85% and 95% for the small ice floes and 50%, 67.5% and 78.7% for the large ice floes.

Results of model tests obtained are presented in this thesis. Figures 9.6, 9.7 and 9.8 show the resistanc in kN for the R-class full scale ship in large ice floes with 50%, 67% and 78% concentration. The R-Class ship resistance results in small ice floes for concentrations of 50%, 60%, 78% and 85% are shown in Figs. 9.9, 9.10, 9.11 and 9.12.

The effect of ship speed on ship resistance, for different ice floe concentrations is illustrated in Fig. 9.13 for large ice floes and in Fig. 9.14 for small ice floes.

The effect of ice floe concentration on ship resistance for a given ship speed is shown in Fig. 9.15 for large ice floes and Fig. 9.16 for small ice floes.

Although only two sizes of ice floes have been used in the tests, the effect of ice floe size on ship resistance has been demonstrated, in a simplified form, in Fig. 9.17.

9.6 FULL SCALE SHIP RESULTS

Full scale data for ship resistance in ice are very limited. The most available results are for the case of ship

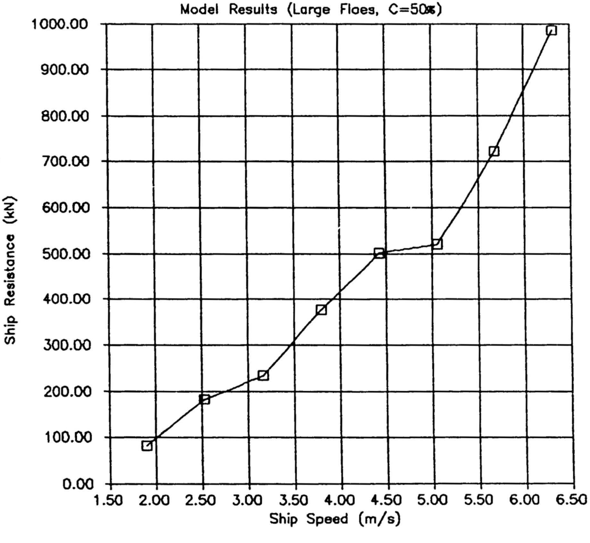

Fig.9.6 Full Scale Resistance in Broken Ice
(R-Class Model, Large Floes, C=50%)

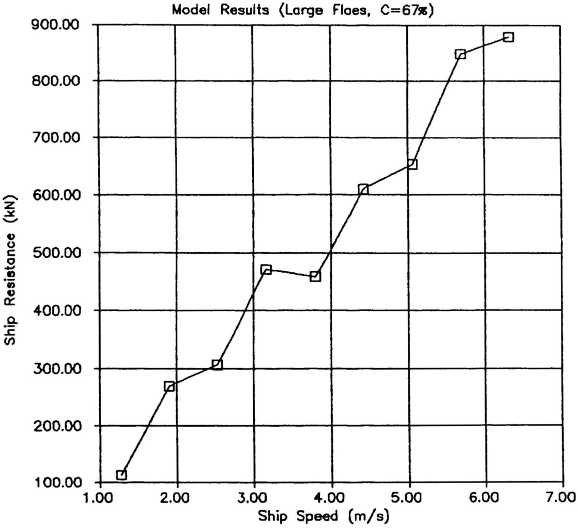

Fig.9.7 Full Scale Resistance in Broken Ice
(R-Class Model, Large Floes, C=67%)

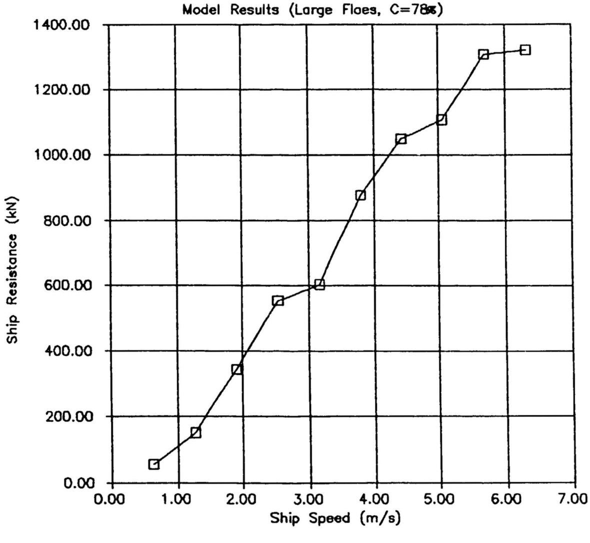

Fig.9.8 Full Scale Resistance in Broken Ice
(R-Class Model, Large Floes, C=78%)

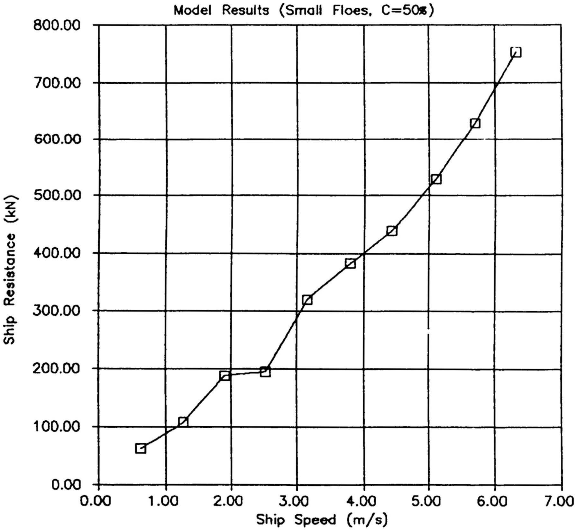

Fig.9.9 Full Scale Resistance in Broken Ice
(R-Class Model, Small Floes, C=50%)

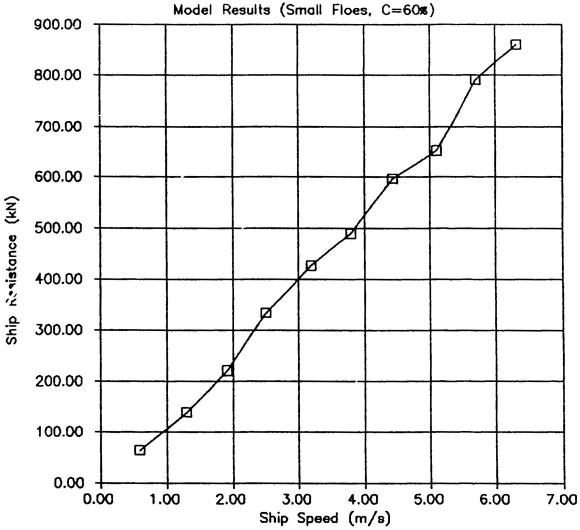

Fig.9.10 Full Scale Resistance in Broken Ice
(R-Class Model, Small Floes, C=60%)

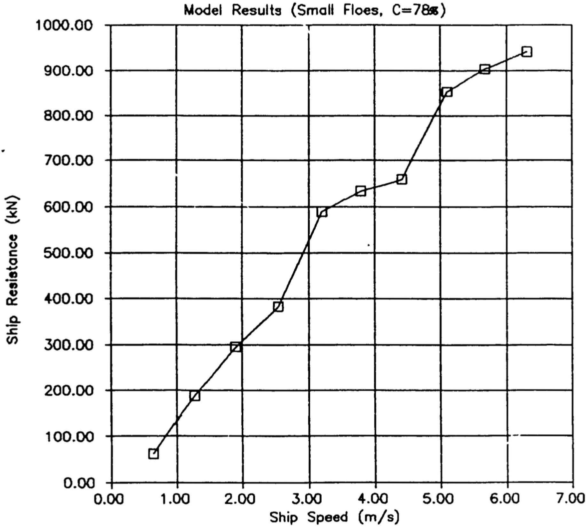

Fig.9.11 Full Scale Resistance in Broken Ice (R-Class Model, Small Floes, C=78%)

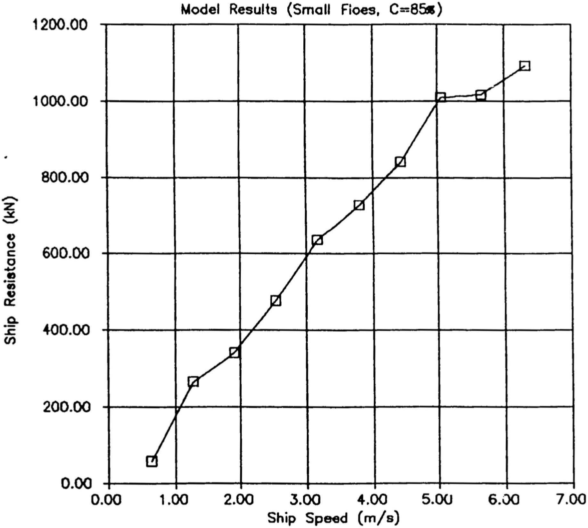

Fig.9.12 Full Scale Resistance in Broken Ice
(R-Class Model, Small Floes, C=85%)

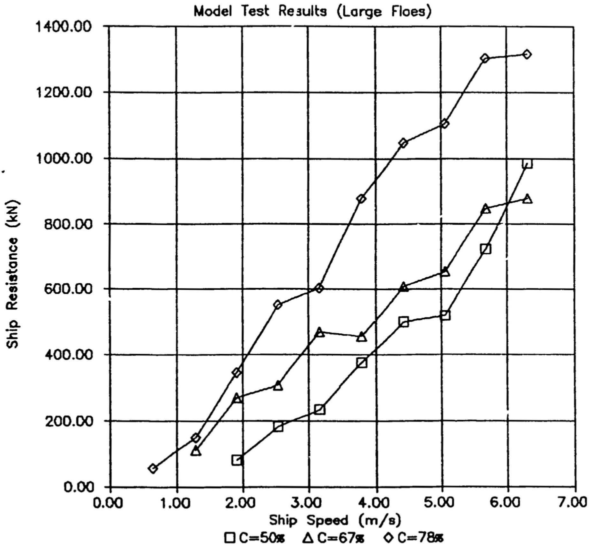

Fig.9.13 Comparative Ship Resistance (Large Floes)

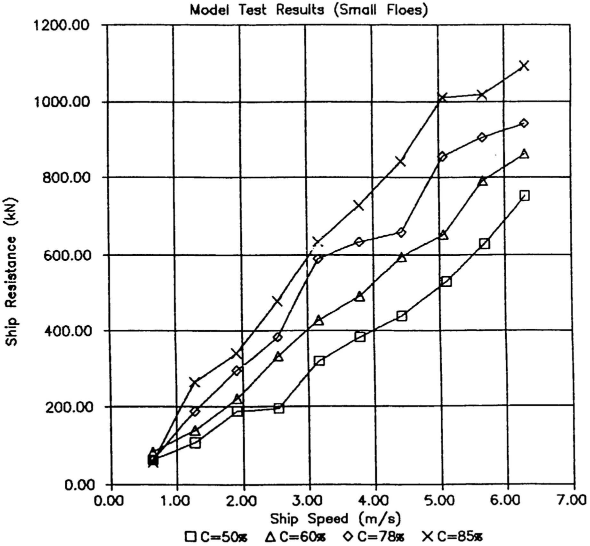

Fig.9.14 Comparative Ship Resistance (Small Floes)

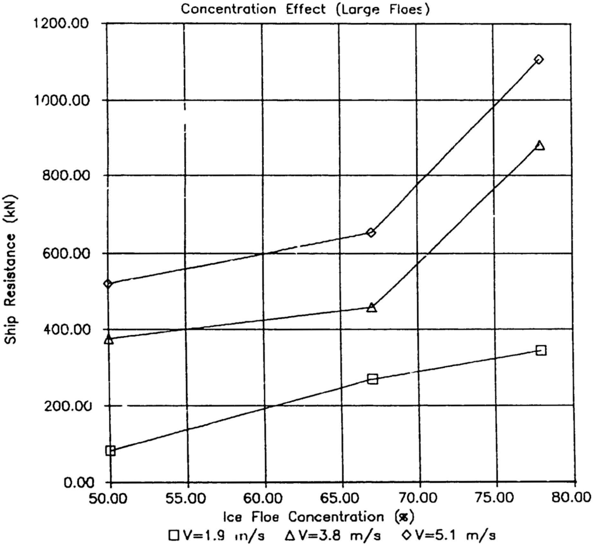

Fig.9.15 Effect of Ice Concentration on Resistance
(Large Ice Floes)

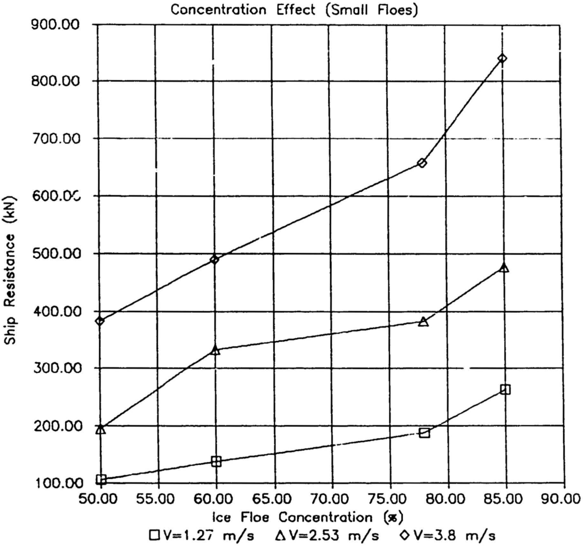

Fig.9.16 Effect of Ice Concentration on Resistance (Small Ice Floes)

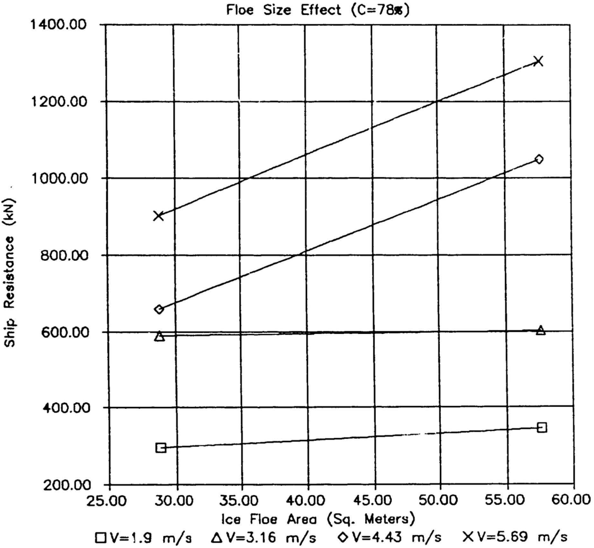

Fig.9.17 Effect of Ice Floe Size on Resistance

resistance in sheet ice. However limited full scale data are available for a few cases of ship resistance in broken or brash ice.

The resistance of the R-Class ship hull (The CCG Franklin) in the field have been measured by Michailidis and Murdy, (1981), however it is very difficult to isolate all useful parameters due to:

1. The total ship thrust was measured and not the resistance. In order to deduct the ship resistance from measured thrust one has to use a thrust deduction factor which in most cases is based on open water data.
2. The broken ice cover consisted of a combination of brash ice (with no defined size), small broken ice and large broken ice at different concentration ratios.
3. The width of the channel relative to the ship beam was not identified as the broken ice ship thrust was measured.
4. Environmental conditions which may have affected the pressure in the ice cover have not been recorded.

With all the problems associated with full scale field measurement in general and the R-Class probe in particular, the "CCGS Franklin" results obtained have been used for the purpose of general comparison with other resistance prediction methods.

The results for the R-Class total thrust, as measured in the field by Michalidis and Murdy (1981), are shown in Table 8.1. The total measured ship thrust, as related to ship speed, is presented in Fig. 9.18. The ice floe cover consists of brash ice, small ice floes and large ice floes at different percentages (Michailidis and Murdy, 1981). The results from the CCGS Franklin probe should therefore, be viewed in light of the mixed ice floe conditions.

9.7 SUMMARY

The results obtained from different analytical, numerical methods as well as from model towing tests and full scale field measurements, have shown that it is possible to approximately predict the resistance of a ship in a given broken ice condition.

The Micro Model can be useful for the prediction of ship resistance in broken ice of medium size with low concentration. In this type of ice cover the ship will collide with ice floes, with possible ship repeated impacts

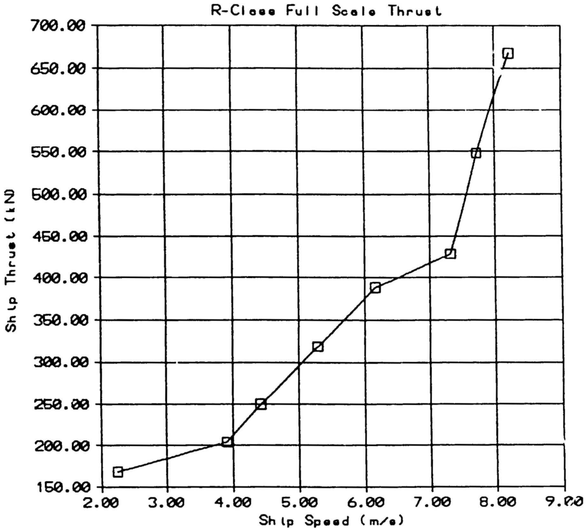

Fig.9.18 R-Class Full Scale Thrust Results
(Michailidis and Murdy, 1981)

with the same ice floes. Ship resistance increase, in this type of ice floe encounter condition, is due to ship energy loss resulting from ice floe impacts.

The Macro Model can be applied to calculate the ship resistance in relatively small ice floes of moderate concentrations. Ship resistance in this type of ice is mostly due to ice floe drag as they move away under the influence of the ship advance.

The numerical two-dimensional distinct element method can be applied to a variety of ice floe conditions. However, it is more suitable for relatively large ice floes of high concentration.

Testing of ship models in simulated ice floe covered waters is very useful tool for the prediction of ship resistance for a particular hull. However for establishing a systematic series of hull forms a large number of models have to be tested in different ice conditions.

Full scale measurements of ship resistance is an expensive and time consuming proposition. However, if it is possible, full scale tests can be used to verify analytical and numerical methods and to analyse the assumptions used with these methods.

CHAPTER 10

DISCUSSIONS, CONCLUSIONS AND RECOMMENDATIONS

10.1 GENERAL

This study was initiated to find ways and means for predicting the ship resistance due to ice floes. The study follows four main approaches, namely analytical formulation, numerical simulation, physical ship model testing in simulated ice floes and full scale test results from the existing literature. Each of these approaches is useful within a certain range of ship and ice conditions, however a number of assumptions have been adopted for the different approaches. The limitations and assumptions built into each approach should be considered when using the models developed in this thesis.

10.2 DISCUSSIONS AND CONCLUSIONS

This study has shown that it is possible to develop equations for the fast calculation of ship resistance in broken ice covered waters.

The analytical Micro Model was developed on the basis of continuous ice floe impacts with the ship, as the ship advances through ice. The model is useful for the prediction of ship resistance in the case of medium size ice floes with low concentration.

The Macro Model was developed based on the total drag of ice floes due to their displacement by the advancing ship. This model, on the other hand, will be applicable for relatively small ice floes with high concentration for slow ship speeds. The model is not applicable for very high ice floe concentration as the ice floes tend to pile up. This limitation is due to the definition of the new Ice Boundary Layer concept which has been introduced in this study.

The distinct element method has been introduced and applied to the broken ice problem. The adoption of the distinct element method for the calculation of ship resistance in broken ice is in its early developmental stages. A number of limitations are faced using this approach due to the assumptions used in the method. There is a similarity between the Micro Model and the distinct element approach and it is possible to integrate the Micro Model with a generalized distinct elemnt model in the future.

Ship model testing has been made feasible, in this study, where the ship and the ice cover were modelled to simulate actual ship navigation in ice floe covered waters.

The model results have compared favourably with existing literature results.

The ice floe resistance results for the R-Class Icebreaker derived from the analytical Micro and Macro Models, and from the numerical distinct element method, are shown in Fig. 10.1. Model test results are also shown in Fig.10.1 and they are for small ice floes and are modified from Fig.9.11 to reflect the difference in ice floe thickness (1 meter thickness instead of 1.5 meters), and the ice floe size (4 meters diameter instead of 5.8 meters). The full scale results presented in Fig.10.1 are for ship thrust data, not resistance. Furthermore, the thrust was measured in ice floe thickness between 0.65 and 0.8 meters.

The results from the analytical Micro Model and the Macro Model, as well as, the distinct element method results, have compared favourably with the results obtained from the physical model testing results.

The limited available results collected from existing full scale experiment data have shown reasonable agreements with both the theoretical model results and model testing results.

New analytical and numerical methods and tools for calculation of ship resistance in broken ice have been developed and presented in this dissertation. These methods

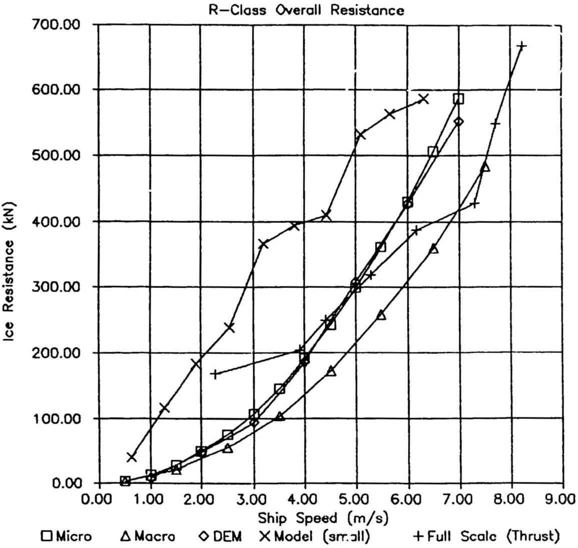

Fig.10.1 Overall Ship Resistance

have shown good potential for use with the prediction of ship resistance in ice floe covered waters. Due to some of the assumptions adopted in the analysis, the models developed are approximate and they may be used for the initial design of new ships intended for navigation in ice floe covered waters. The formulae developed here may also be important for the prediction of existing ship performance in any perceived pack ice waters.

10.3 RECOMMENDATION FOR FUTURE WORK

The ice floe ship resistance study presented in this dissertation covered different analytical, experimental and full scale grounds. Based on this study a number of recommendations can be made for future research work in the area of ship performance in pack ice. The following is a summary of research and development areas which need to be addressed in the future:

- Extension of resistance models to 3-dimensional hull representation.
- More work in the area coefficient of restitution between different types of ice and the ship hull.
- Probabilistic epresentation of ice floe size and impact

frequencies.

- Development of a generalized distinct element model, taking into account 3-dimensional effects.
- Study of the limiting ice floe size for icebreaking.
- Analysis of the effect of broken ice channel width.
- Development of model series for ice floe operation
- Optimization of hull form for ice floe navigation.
- Ship model testing in a wide range of model ice size, concentration, shape, and combination of size and shape.
- Full scale ship testing in different ice floe conditions.
- Study of model and full scale correlations in floe ice.
- More effort towards the analysis of the full scale results.

REFERENCES

Aboulazm, A. F. (1985) "Ship Resistance in Ice Floe Covered Waters - A Proposal for Doctoral Research" Unpublished Report, Fac. of Engineering, Memorial University of Newfoundland.

Aboulazm, A.F. and Muggeridge, D. (1989a) "Analytical Investigation of Ship Resistance in Broken or Pack Ice" OMAE Conf., The Hague, The Netherland, March.

Aboulazm, A. F. and Muggeridge, D. (1989b)"Ship Resistance in Pack Ice - Rigid Distinct Element Method Approach" CMIA Technical Conference, Montreal, Canada, February.

Ashton, G.D., Den HartDg, S.L. and Hanamoto, B. (1972) "Icebreaking by Tow on the Mississippi River" U.S.Army CRREL, Special Report No.192.

Blevins, R.D. (1979) "Formulas for Natural Frequency and Mode Shape" Van Nostrand Reinhold Company, New York, 528p.

Bronnikov, A.V. (1959) "Analysis of Resistance of Cargo Ships Going Through Pack Ice" Transactions of Leningrad Shipbuilding Institute No. 27, pp.13. BSRA Translation No. 1869, No date.

Buzuev, A.Ya. and Ryvlin, A.Ya. (1966) "Calculation of Ice Resistance to the Movement of an Icebreaker Under Different Ice Conditions" Problems of the Arctic and the Antarctic, Issues 29-32. Published for the National Science Foundation, Washington D.C. by the Israel Program for Scientific Translation.

Buzuev, A. and Ryvlin, A.Y. (1961) "Calculation of the Resistance Encounterd by an Icebreaker Moving Through Ice Cakes and Brash" Morskoi Flot, 21 (8) pp. 136-138, 1961.

Cammaert, A.B. and Muggeridge, D.B. (1988) "Ice Interaction with Offshore Structures", Van Nostrand Reinhold, New York, 432p.

Canadian Coast Guard, CCG (1977) "Ice Navigation in Canadian Waters" Canadian Government Report No. DSS T31-3372.

Carter, G.W. (1983) "Ship Resistance to Continous Motion in Level Ice" Transport Research and Development Center, Department of Transport, Government of Canada, Report TP-3679E, March.

Chu, F.D. (1974) "Ship Resistance in Homogeneous Ice Fields" Doctor of Technology Thesis, The Helsinki University of Technology, Otaniemi, Finland.

Cundall, P. A. (1971a) "Measurements and Analysis of Accelerations in Rock Slopes" Ph.D. Dissertation, Imperial College, London, England.

Cundall, P. A. (1971b) "A Computer Model for Simulating Progressive Large Scale Movements in Blocky Rock System" Proc. Symp. of Int. Soc. of Rock Mechanics, Vol.2, No 2, Nancy, France.

Cundall, P. A. (1974) "A Computer Program for Rock-Mass Behaviour Using Interactive Graphics for the Input and Output of Geometrical Data" U.S.Army Corp of Engineers, Report No. MRD-2-74.

Cundall, P. A. (1976) "Explicit, Finite Difference Methods in Geomechanics" Num. Meth. in Geomechanics, ASCE, New york.

Cundall, P. A. (1978) "BALL - A Program to Model Granular Media Using the Distinct Element Method" Tech. Note, Adv. Tech. Group, Dames and Moore, London, England.

Cundall, P.A., Marti, Beresford, P.T., Last, N.C. and Asgian, M.I. (1978) "Computer Modelling of Jointed Rock Masses" Dames and Moore, Los Angeles, California, U.S.A.

Daley, G.,St.John, J.W., Seibold, S. and Bayly, I.(1984) "Analysis of Extreme Ice Loads Measured on USCGC Polar Sea" Trans. of SNAME, Vol. 92.

Dubroven, O. (1970) "Calculation of Broken Ice Resistance Based on Model Testing" (Translated from Russian), Dept. of Naval Arch., Univ. of Michigan, Report No. 052.

Edwards, Y.R., Dunne,M.A., Comfort,G., Bulat,V. and Johnson,B. (1981) "Results of Full Scale Trials in Ice of CCGS Pierre Radison" SNAME Spring Meeting/Star, Ottawa, Canada, June.

Edwards, R.H. et al (1977) "Results of Full Scale Impact Load Stydies Aboard CCGS Norman McLeod Rogers" Proc. POAC77 Conf., St.John's, Canada.

Engle, P.A. (1976) "Impact Wear of Materials" Elsvier Scientific Publishing Company, Amsterdam.

Enkvist, E. (1972) "On the Ice Resistance Encountered by Ships Operating in the Continuous Mode of Ice Breaking" The Swedish Acadamy of Engineering Science in Finland Report No.24, Helsinki.

Enkvist, E. et al (1979) "The Ship-Ice Interaction" Proceeding POAC79. Trondheim, Norway, August 13-19, pp.977-1002.

Eskola, H. (1983) "Propulsion Tests of a Ship Model in Conditions Simulating an Old Ice-Clogged Channel" POAC 83, Helsinki, Finland, pp.494-506.

German, J.G., Sterling, J., Benjamin, A., Timonen, P. and Hearnshow, J.D. (1981) "Data Collection System on Board the Ice-Strengthened Cargo Vessel M.V.Arctic for Evaluation of Ship Performance in Ice" SNAME Ice Tech Symp., Ottawa, Canada.

German,J.G. and Lawrence,R.G.A. (1975) "Full Scale Testing in Ice of Three Icebreakers" SNAME Ice Tech Symp., Montreal.

Ghoneim, G.A.M., Johansson, B.M., Smyth, M.W. and Grinstead,J. (1984) "Global Ship Ice Impact Forces Determined from Full-Scale Tests and Analytical Modeling of the Icebreakers Canmar Kigoriak and Robert LeMeur" Trans. of SNAME, Vol.92.

Gill, R.J., Aboulazm, A.F., Terry, B. and Russell, W.E. (1981) "A Ship Transit Model for Passage Through Ice and its Application to the Labrador Area" SNAME Spring Meeting/Star Symp., Ottawa, Canada.

Goldsmith, W. (1960) "Impact" Edward Arnold (Publishers) Ltd, London.

Gray,W.O, Maybourn,R. (1981) "Manhattan's Arctic Venture-A Semitechnical History" SNAME Spring Meeting/Star Symp. Ottawa, Canada.

Greisman, P. (1981) "Brash Ice Behavior" U.S. Coast Guard Research and Development Center, Groton, Report No. CGRDC9/81.

Hellman,J.H. and Schwarz,J. (1983) "Performance Tests of Icebreaker Max Waldeck in Model and Full Scale" Proc. 7th POAC Conf., Helsinki, Finland.

Herfjord,K., Hysing,T.(1981) "Form Design of Vessels" Marine Structures and Ships in Ice, A Joint Norwegian Research Project, Report No.81-03.

Hoerner,S.F. (1965) "Fluid Dynamic Drag" Published by Hoerner Fluid Dynamics.

Hsiung,C.C., Friis,D., Milne,W., Peters, G.R., Weber,H.W. (1983) "New Towing Facilities at Memorial University of Newfoundland" 20th American Towing Tank Conference, Davison Laboratory, Steven Institute of Technology, Hoboken, New Jersey, August.

Hysing, T (Ed) (1981) "Design Case Part 1: Ice Transiting Vessels" Marine Structures and Ships in Ice, A Joint Norwegian Research Project, Report No. 81-02/1.

Kashteljan, V.I., Posnjak, I.I. and Ryvlin, A.Y. (1968) "Ice Resistance to Motion of a Ship" Sudostroenie, Leningrad, USSR.

Kawai, T. (1977) "A New Element Method in Discrete Structure Analysis" J. of Soc. of Naval Arch. of Japan, Vol.141.

Keinonen, A. and Nyman, T. (1979) "An Experimental Model-Scale Study on the Compressible, Frictional and Cohesive Behavior of Broken Ice Mass" Proc. IAHR Symp. on Ice Probl., Lulea, Sweden.

Kheisin, D.L. and Popove, Y.N.(ed) (1973) "Ice Navigation Quilities of Ships" CRREL Translation No. 417.

Kitazawa, T. and Ettema, R. (1986) "Rubble-Ice Resistance for Ships Moving with Creeping Speed" 5th OMAE, Tokyo.

Kitazawa, T. and Ettema, R. (1985) "Resistance to Ship Hull Motion Through Brash Ice" Cold Regions Science and Technology, 10.

Korri, P. and Varsta, P. (1979) "On the Ice Trial of 14,500 dwt Tanker in the Gulf of Bothnia" NSTM-79, Helsinki, Finland.

Korzhavin, K. (1971) "Action of Ice on Engineering Structures" CRREL Translation No. 319, Hanover, New Hampshire, U.S.A.

Kostilainen, V. and Hanhirova, K. (1982) "Model Testing of Ships in Ice-Clogged Channels" Helsinki University of Technology, Ship Hydrodynamics Laboratory, Report No. 22., Otaniemi, Finland.

Kotras, T.V., Baird, A.V. and Naegle, J.N. (1983) "Predicting Ship Performance in Level Ice" SNAME Transaction No.91.

Levine, G.H., Voelker, R.Y. and Mentz, P.B. (1974) "Advances in the Development of Commercial Ice-Transiting Ships" SNAME Annual Meeting, New York.

Likhomanov,V.A. and Kheisin, D.E. (1971) "Experimental Investigation of Solid Body Impact on Ice" Prob. of the Arctic and the Subarctic, Vol.38, pp 128-136.

Major, R.A. et al (1975) "A Model to Predict Hull-Ice Impact Loads in the St.Laurence" SNAME Ice Tech75 Symposium, Montreal, Canada, April, pp.D1-D33.

Makinen,E., Roos,R. (1977) "Ice Navigation Capabilities of Lunni-Class Icebereaker Tankers" SNAME Eastern Canadian Section, Montreal.

Makinen, E., Lahti, A. and Rimppi, M. (1975) "Influence of Friction on Ice Resistance - Search for Low Friction Surface" SNAME Ice Tech75 Symposuim, Montreal, Canada, April 9-11.

Mansour, A.M.A. (1982) "Computer Simulation of Ice Breaking by Explosion and Impact" Ph.D. Dissertation, University of Wisconson-Madison, Wisconsin, U.S.A.

Markham, W.E. (1980) "Ice Atlas : Eastern Canadian Seaboard" Atmospheric Environment Service, Department of Environment, Government of Canada.

Mellor, M. (1979) "Towing Ships Through Ice-Clogged Channels by Warping and Kedging" U.S Army CRREL Report No. 79-21.

Mellor, M. (1980) "Ship Resistance in Thick Brash Ice" Cold Regions Science and Technology No.3, pp.305-321.

Michailidis, M. and Murdey, D.C. (1981) "Performance of CCGS Franklin in Lake Melville, 1980" SNAME Ice Tech Symp., Ottawa, Canada.

Michel, B. (1978) "Ice Mechanics" Les Presses de L'Universite' Level, Quebec, Canada, 499p.

Milano, V.R. (1975) "Ship Resistance to Continuous Motion in Mush Ice" SNAME Ice Tech75 Symposium, Montreal, Canada, April 9-11, Paper B.

Milano, V.R. (1972) "Ship Resistance to Continous Motion in Ice" Ph.D. Dissertation, Steven Institute of Technology, New Jersy, U.S.A.

Mookhoek, A.D., Voelker, R.P. and DeBord, F.W. (1981) "Summary of Selected Technical Results from the Manhattan Arctic Marine Project" SNAME Ice Tech Symp., Ottawa, Canada.

Muggeridge, D.B., Murray, J.J. (1981) "Calibration of a 58m Wave Flume" Canadian Journal of Civil Engineering, Vol.8, No.4, Page 449-455.

Naegle, J.N. (1980) "Ice-Resistance Prediction and Motion Simulation for Ships in the Continuous Mode of Icebreaking" Ph.D. Dissertation, University of Michigan, Ann Arbor, Michigan, U.S.A.

Newbury,S. (1984) " R-Class Icebreaking Experiments with Model 327 and Comparison with 1:40 Scale and Full Scale Results" Institute of Marine Dynamics, NRC, Report No. LTR-SH-383.

Nobel,P.G., Allan, R.J., Dunne, M.A., Johnson, B. (1978) "Ice Effects Trials in Arctic Waters on CCGS Louis S. St. Laurant" SNAME Trans. No.86.

Nogid, L.M. (1959) "Impact of Ships With Ice" Transactions of Leningrad Shipbuilding Institute, No. 26, p.123. Also BSRA Translation No. 1867.

Nogid, L.M. (1959) "Model Representation of a Ship Going Through a Continuous Ice Field or Pack Ice" Transactions of Leningrad Ship Building Institute No.28, pp.45.

Schwarz, J. (1974) "Present Status of Icebreaker Research" Iowa Institute of Hydraulic Research, 17th American Towing Tank Conference, Pasadena, California, U.S.A., June.

Popov, Y.N. Feddeyev, O.V., Kheysin, D.Y. and Yakovlev, A.A. (1967) "Strength of Ships Sailing in Ice" Subostroyeniye, Leningrad, USSR. (English Translation).

Riska, K. et al (1983) "Ice Load and Pressure Measurements on Board I.B. Sisu" proc. POAC83 Conf., Espoo, Finland.

Riska, K. and Varsta, P. (1977) "State-Of-The-Art Review of Basic Ice Problem for a Naval Architect" Technical Research Center of Finland, Ship Laboratory, Report No. 2, p.63.

Schlichting, H. (1979) "Boundary-Layer Theory" McGraw-Hill Comp., New York.

Tatinclaux, J.C. (1984) "Model Tests on Two Models of WTGB 140-foot Icebreaker" U.S. Army CRREL Report No. 84-3.

TDC (1981) "M.V.Arctic Experimental Voyage to Lake Melville Labrador in February/March 1981" Transport Development Center Report No. TP3646E.

Tuovinen, P. (1979) "The Size Distribution of Ice Blocks in Broken Channel" Ship Hydrodynamic Laboratory, Helsinki University of Technology, Otaniemi, Finland, Report No. M-78.

Vance,G.P.,Gracewski,A.S. and Goodwin,M.J.(1981)"Full Scale Icebreaking Tests of the USCGC Katmai Bay" SNAME Ice Tech Symp., Montreal.

Vance, G.P. (1980) "Analysis of the Performance of a 140-foot Great Lakes Icebereaker: USCGC Katmai Bay" U.S. Army CRREL Report No. 80-8.

Vance, G.P. (1974) "A Modeling System for Vessels in Ice" Ph.D. Dissertation, University of Rhode Island, U.S.A.

Varsta, P. (1983) "On the Mechanics of Ice Load on Ships in Level Ice in the Baltic Sea" Thesis for the Degree of

Doctor of Technology, Helsinki University of Tecnnology, Espoo, Finland, August. Also Technical Research Center of Finland Publication No. 11.

Varsta, P. (1977) "Measurements and Analysis of Ice Induced Stresses in the Shell of an Icebreaker" Winter Navigation Research Board, Repot No. 21, Helsinki.

Vinogradov, O.C. (1986) "Simulation Methodology of Vessel-Ice Floes Interaction Problem" 5th OMAE, Tokyo.

White, R.M. (1965) "Dynamically Developed Force at the Bow of an Icebreaker" D.Sc. Thesis, Naval Architecture and Marine Engineering Department, Massachusetts Institute of Technology, Cambridge, Massachusetts, U.S.A., September.

White, R.M. (1969) "Prediction of Icebreaker Capabilities" Transactions of the Royal Institute of Naval Architects, Paper W8.

Yen,Y., Odar,F., Bracy,L. (1970) "Impact of Spheres on Ice" ASCE, Enging Mechanics Div., pp 641-652.

APPENDIX I
ICE CLASSIFICATION AND GLOSSARY
(CCG, 1977)

Bergy Bit — A large piece of floating *glacier ice*, generally showing less than 5 m. above sea level but more than 1 m. and normally about 100 – 300 sq.m. in area.

Beset — Situation of a vessel surrounded by ice and unable to move.

Bight — An extensive crescent-shaped indentation in the *ice edge*, formed either by wind or current.

Brash Ice — Accumlations of *floating ice* made up of fragments not more than 2 m. across, the wreckage of other forms of ice.

Close Pack Ice — *Pack ice* in which the concentration is 6/8 to less than 7/8 (7/10 thru 8/10). composed of *floes* mostly in contact.

Compacted Ice Edge — Close, clear-cut *ice edge* compacted by wind or current, usually on the windward side of an area of *pack ice*

Concentration — The ratio in eighths or tenths of the sea surface actually covered by ice to the total area of sea surface, both ice covered and *ice free*, at a specific location or over a defined area.

Consolidated Pack Ice — *Pack Ice* in which the concentration is 8/8 (10/10) and the floes are frozen together.

Crack — Any fracture which has not parted.

Difficult Area A general qualitative expression to indicate, in a relative manner, that the severity of ice conditions prevailing in an area are such that navigation in it is difficult.

Easy Way — A general qualitative expression to indicate, in a relative manner, that ice conditions prevailing in an area are such that navigation in it is not difficult.

Fast Ice — *Sea Ice* which forms and remains fast along the coast, where it is attached to the shore, to an *ice wall*, to an *ice front*, between shoals or grounded *icebergs*. Vertical fluctuations may be observed during changes of sea level. Fast ice may be formed in situ from sea water or by freezing of *pack ice* of any age to the shore, and it may extend a few meters or several hundred kilometers from the coast. Fast ice more than one year old may be prefixed with the appropriate age category, *old, second-year*, or *multi-year*.. If it is thicker than about 2 m. above sea level it is called an *ice shelf*.

Fast Ice Edge — The demarcation at any given time between fast *ice* and *open water*.

First-year Ice — *Sea ice* of not more than one winter's growth, developing from *young ice*; thickness from 30 cm-2m. May be subdivided into thin *first-year ice/white ice, medium first-year ice*, and *thick first-year ice*.

Flaw — A narrow separation zone between *pack ice* and fast where the pieces of ice are in chaotic state, that forms when *pack ice* shears under the effect of a strong wind or current along the *fast ice boundary* (cf. *shearing*)

Flaw Lead — A passageway between *pack ice* and *fast ice* which is navigable by surface vessels.

Floe — Any relatively flat piece of *sea ice* 20 m. or more across. Floes are subdivided according to horizontal extent as follows.

Giant – Over 10 km. across Vast – 2-10 km. across Big – 500-2,000 m. across Medium – 100-500m. across Small – 20-100 m. across

Floeberg — A massive piece of *sea ice* composed of a *hummock* or a group of *hummocks*, frozen together and separated from any ice surroundings. It may float up to 5 m. above the sea level.

Frazil Ice — Fine spicules or plates of ice suspended in water.

Glacier Berg — An irregularly shaped *iceberg*.

Glacier Ice — Ice in or originating from a *glacier*, whether on land or floating on the sea as *icebergs*, bergy bits, or growlers.

Grease Ice — A later stage of freezing than *frazil ice* when the crystals have coagulated to form a soupy layer on the surface. Grease ice reflects little light, giving the sea a matte appearance.

Grey Ice — *Young ice* 10-15 cm. thick. Less elastic than *nilas* and breaks on swell. Usually rafts under pressure.

Grounded Ice — *Floating ice* aground in shoal water (cf. *stranded ice*).

Growler — Smaller piece of ice than a *bergy bit* or *floeberg*, often transparent but appearing green or almost black in color, extending less than 1 m. above the sea surface and normally occupying an area of about 20 sq. m.

Hummocked Ice — *Sea ice* piled haphazardly one piece over another to form an uneven surface. When weathered it has the appearance of smooth hillocks.

Iceberg — A massive piece of ice of greatly varying shape, more than 5 m. above sea level, which has broken away from a *glacier*, and which may be afloat or aground. Icebergs may be described as *tabular*, dome-shaped, sloping, pinnacled, weathered, or *glacier bergs*.

Ice Blink — A whitish glare on low clouds above an accumulation of distant ice.

Icebound — A harbor, inlet, etc., is said to be icebound when navigation by ships is prevented by ice, except possibly with the assistance of an icebreaker.

Ice Boundary — The demarcation at any given time between *fast ice* and *pack ice* or between areas of *pack ice* of different *concentrations* (cf. *ice edge*).

Ice Edge — The demarcation at any given time between the open sea and *sea ice* of any kind, whether fast or drifting. It may be termed *compacted* or *diffuse* (cf. *ice boundary*).

Ice Field — Area of *pack ice* consisting of any size of *floes*, which is greater than 10 km. across (cf. *ice patch*).

Ice Free — No *sea ice* present. There may be some *ice of land origin* (cf. *open water*).

Ice Jam — An accumulation of broken *river ice* or *sea ice* caught in a narrow channel.

Ice Patch — An area of *pack ice* less than 10 km. across.

Ice Under Pressure — Ice in which deformation processes are actively occurring, hence a potential impediment or danger to shipping.

Large Fracture — More than 500 m. wide.

Lead — Any *fracture* or passageway through *sea ice* which is navigable by surface vessels.

Multi-year Ice — *Old ice* up to 3 m. or more thick which has survived at least two summers' melt. *Hummocks* smoother than in *second-year ice*, and the ice is almost salt-free. Color, where bare, is usually blue. Melt pattern consists of large interconnecting irregular *puddles* and a well-developed drainage system.

New Ice — A general term for recently formed ice which includes *frazil ice*, *grease ice*, *slush*, and *shuga*. These types of ice are composed of ice crystals which are only weakly frozen together (if at all) and have a definite form only while they are afloat.

Nilas — A thin elastic crust of ice, easily bending on waves and swell and under pressure, thrusting in a pattern of interlocking "fingers" (*finger-rafting*). Has a matte surface and is up to 10 cm. in thickness. May be subdivided into dark nilas and light nilas.

Nip — Ice is said to nip when it forcibly presses against a ship. A vessel so caught, though undamaged, is said to have been nipped.

Old Ice — *Sea ice* which has survived at least one summer's melt. Most topographic features are smoother than on *first-year ice*. May be subdivided into *second-year ice* and *multi-year ice*.

Open Pack Ice — *Pack ice* in which the ice *concentration* is 3/8 to less than 6/8 (4/10 thru 6/10) with many *leads* and *polynyas*, and the *floes* are generally not in contact with one another.

Pack Ice — Term used in a wide sense to include any area of *sea ice*, other than *fast ice*, no matter what form it takes or how it is disposed.

Rafting — Pressure processes whereby one piece of ice overrides another. Most common in *new* and *young ice* (cf. *finger rafting*).

Ram — An underwater ice projection from an *ice wall*, *ice front*, *iceberg*, or a *floe*. Its formation is usually due to more intensive melting and erosion of the unsubmerged part.

Ridge — A line or wall of broken ice forced up by pressure. May be fresh or weathered. The submerged volume of broken ice under a ridge forced downwards by pressure is termed and *ice keel*.

Rotten Ice — *Sea ice* which has become honeycombed and which is in an advanced state of disintegration.

Sea Ice — Any form of ice originating from the freezing of sea water.

Second-Year Ice — *Old ice* which has survived only one summer's melt. Because it is thicker and less dense than *first-year ice*, it stands higher out of the water. In contrast to *multi-year ice*, summer melting produces a regular pattern of numerous small *puddles*. Bare patches and puddles are usually greenish-blue.

Shore Lead — A *lead* between *pack ice* and the shore or between *pack ice* and an *ice front*.

Shuga — An accumulation of spongy white ice lumps, a few centimeters across; formed from *grease ice* or *slush* and sometimes from *anchor ice* rising to the surface.

Slush — Snow which is saturated and mixed with water on land or ice surfaces, or as a viscous floating mass in water after a heavy snowfall.

Strip — Long narrow area of *pack ice*, about 1 km. or less in width, usually composed of small fragments detached from the main mass of ice, and run together under the influence of wind, swell, or current.

Tongue — A projection of the ice edge up to several kilometers in length caused by wind or current.

Very Close Pack Ice — Pack ice in which the concentration is 7/8 to less than 8/8 (9/10 to less than 10/10).

Very Open Pack Ice — *Pack ice* in which the *concentration* is 1/8 to less than 3/8 (1/10 thru 3/10) and water preponderates over ice.

Water Sky — Dark streaks on the underside of low clouds indicating the presence of water features in the vicinity of *sea-ice*.

Young Ice — Ice in the transition stage between *nilas* and *first-year ice*, 10-30 cm. in thickness. May be subdivided into *grey ice* and *grey-white ice*.

APPENDIX II

SOLUTION OF SHIP-ICE IMPACT EQUATIONS

The equations governing the impact between an ice floe and the ship have been deveolped in Chapter 3. To obtain the post impact velocities, these equations are solved by elimination as follows:

$$-m\ V\ \sin\theta + N_n = m\ V'\ \sin\theta_1 \qquad \text{(II-1)}$$

$$m\ V\ \cos\theta\ - N_t = m\ V'\ \cos\theta_1 \qquad \text{(II-2)}$$

$$N_t\ r = \Omega'\ I \qquad \text{(II-3)}$$

$$V'\ \sin\theta_1\ = e\ V\ \sin\theta \qquad \text{(II-4)}$$

$$N_t\ = \mu\ N_n \qquad \text{(II-5)}$$

The above equations may be arranged as follows:

$$N_t\ = m\ V\ \cos\theta\ - m\ V'\ \cos\theta_1 \qquad \text{(II-6)}$$

$$N_n\ = m\ V'\ \sin\theta_1\ + m\ V\ \sin\theta \qquad \text{(II-7)}$$

From Eqn.(II-5) and dividing Egn.(II-6) by Eqn.(II-7)

$$V\cos\theta - V'\cos\theta_1\ = \mu V'\sin\theta_1 + \mu V\sin\theta \qquad \text{(II-8)}$$

or

$$V\cos\theta-\mu V\sin\theta=\mu V'\sin\theta_1+V'\cos\theta_1$$

$$V\cos\theta-\mu V\sin\theta=V'(\mu\sin\theta_1+\cos\theta_1)$$

And from Eqn.(II-4)

$$V\cos\theta-\mu V\sin\theta=(eV\sin\theta/\sin\theta_1)(\mu\sin\theta_1+\cos\theta_1)$$

$$=eV\sin\theta(\mu+\cot\theta_1)$$

or

$$V\cot\theta-\mu V=eV(\mu+\cot\theta_1)$$

$$e\cot\theta_1 = \cot\theta-\mu-e\mu$$

or

$$\cot\theta_1 = [\cot\theta-\mu(1+e)]/e \qquad \text{(II-9)}$$

Substituting Eqn.(II-9) in Eqn.(II-4)

$$V'=V\sin\theta([\cot\theta-\mu(1+e)]^2 +e^2)^{1/2} \qquad \text{(II-10)}$$

Where v' is the linear velocity of the ice floe after ship impact.

Substituting Eqn.(II-9) and Eqn.(II-10) in Eqn.(II-2);

$$N_t=mV\cos\theta-mV\sin\theta([\cot\theta-\mu(1+e)]^2+e^2)^{1/2}$$

$$[\cot\theta-\mu(1+e)]/\{[\cot\theta-\mu(1+e)]^2+e^2\}^{1/2}$$

$$N_t = mV\cos\theta-mV\sin\theta[\cos\theta/\sin\theta-\mu(1+e)]$$

$$N_t = \mu mV(1+e)\sin\theta \qquad \text{(II-11)}$$

Subistituting Eqn.(II-11) in Eqn.(II-3);

$$\Omega'=(r/I)\mu mV(1+e)\sin\theta \qquad \text{(II-12)}$$

Where Ω' is the rotational velocity of the ice floe after ship impact.

APPENDIX III

INTEGRATION OF THE MACRO MODEL EQUATION

The Macro Model presented in Chapter 4 has been developed to calculate the ship resistance in broken or pack ice based on the total ice cover. To use this model for any ship the right hand side of Eq.4.17 has to be integrated for any given hull shape function. This has been done as follows :

From Eq.4.17;

$$R_i = \sigma_w V^2 \int_0^{B/2} \sin^2\theta[C\ C_s \cos\theta\ y(x)/(1-C) + h_w C_f\ C/K]\ dy$$

$$+ \sigma_w V^2 \mu \int_0^{1} \sin^2\theta[C\ C_s \cos\theta\ y(x)/(1-C) + h_w C_f\ C/K]\ dx$$

$$+ [\sigma_i\ ChV^2/(l(1-C))] \int_0^{B/2} \sin\theta \cos\theta\ y\ dy \qquad \text{III.1}$$

$$R_i = [\sigma_w\ V^2\ C\ Cs/(1-C)] \int_0^{B/2} \sin^2\theta \cos\theta\ y(x)\ dy$$

$$+ \sigma_w\ V^2 h_w\ C_f\ \ C/K \int_0^{B/2} \sin^2\theta\ dy$$

$$+ [\sigma_w\ V^2 \mu CCs/(1-C)] \int_0^{1} \sin^2\theta \cos\theta\ y(x)\ dx$$

$$+ \sigma_w V^2 \mu h_w \; C_f . \; C/K \int_0^{l} \sin^2\theta \; dx$$

$$+ [\sigma_i hCV/(l(1-C))] \int_0^{B/2} \sin\theta \; \cos\theta \; y \; dy \qquad \text{III.2}$$

From the ship waterline entrance shape function $y=f(x)$;

$$\tan\theta = y'$$

$$\sin\theta = y'/ \; y'^2+1$$

$$\cos\theta = 1/ \; y'^2+1$$

$$y'= dy/dx \qquad \text{III.3}$$

$$R_i = [\sigma_w V^2 CC_s/(1-C)] \int_0^{B/2} [y'^2 \; y(x)/(y'^2+1)^{3/2}] \; dy$$

$$+ \sigma_w V^2 \; h \; C_f \; C/K \int_0^{B/2} [y'^2/(y'^2+1)] \; dy$$

$$+ [\sigma_w V^2 \mu \; C \; C_s/(1-C)] \int_0^{l} [y'^2 y(x)/(y'^2+1)^{3/2}] \; dx$$

$$+ \sigma_w \; V^2 \mu \; C_f \; h \;\; C/k \int_0^{l} [y'^2/(y'^2+1)] \; dx$$

$$+ [\sigma_i h \; C \; V/(l(1-C))] \int_0^{B/2} [y.y'/(y'^2+1)] \; dy \; \text{III.4}$$

If the entrance shape function y=f(x) is known, the derivative y' can be derived and the above equation can be integrated. However, if the the shape function is not known and the water line is only defined from a given lines plan, the entrance waterline in this case can be divided into straight line segments. The integration in the above equation can be evaluated for each segment, as shown in Fig. 4.6, then added for the whole waterline entrance.

For a segmented water line entrance or for constant slope entrance, the resistance R_i in the above equation can be calculated as follows :

The slope of any segment (or waterline entrance) is taken as α, where $y' = \alpha$ = constant (or $y' = B/2l$) and from the above equation;

$$R_i = [\sigma_w V^2 C C_s \, \alpha^2/((1-C)(\alpha^2+1)^{3/2})] \cdot [y^2/2]_0^{B/2}$$

$$+ \left[\sigma_w V^2 \ h \ C_f \ C/K \ \alpha^2/(\alpha^2+1)\right].\left[\ y \ \right]_0^{B/2}$$

$$+ \left[\sigma_w V^2 C \ C_s \mu \ \alpha^3/((1-C)(\alpha^2+1)^{3/2})\right].\left[\ x^2/2 \ \right]_0^{1}$$

$$+ \left[\sigma_w V^2 \ h_w C_f \mu \ \ C/K \ \alpha^2/(\alpha^2+1)\right].\left[\ x \ \right]_0^{1}$$

$$+ \left[\sigma_i h \ C \ V^2 \ \alpha/(1.(i-c)(\alpha^2+1)\right]\}.\left[\ y/2 \ \right]_0^{B/2} \qquad \text{III.5}$$

$$R_i = \left[\sigma_w V^2 CC_s \ \alpha^2/((1-C)(\alpha^2 \ {:}1)^{3/2})\right].(B^2/8)$$

$$+ \left[\sigma_w V^2 h_w \ C_f \ \ C/K \ \alpha^2/(\alpha^2+1)\right].(b/2)$$

$$+ \left[\sigma_w V^2 C \ C_s \mu \ \alpha^3/((1-C)(\alpha^2+1)^{3/2})\right].(1^2/2)$$

$$+ \left[\sigma_w V^2 \mu \ C_f \ h_w \ \ C/k \ \alpha^2/(\alpha^2+1)\right].(1)$$

$$+ \left[\sigma_i \ hCV^2/(8.1(1-C))\right]\left[\alpha/(\alpha^2+1)\right].(B^2) \qquad \text{III.6}$$

$$R_i = \left[\sigma_w V^2 C \ C_s \ \alpha^2/((1-C)(\alpha^2+1)^{3/2})\right].\left[B^2/8 \ + \ \mu 1^2\alpha/2\right]$$

$$+ \ \sigma_w V^2 h_w \ C_f \ \ C/k.\alpha^2 (B/2+\mu 1)/(\alpha^2+1)$$

$$+ \left[\sigma_i h \ CV^2/(8.1(1-C))\right].\left[\alpha/(\alpha^2+1)\right].(B^2) \qquad \text{III.7}$$

$$R_i = [\sigma_w V^2 C\ C_s\ l^2\alpha^2/((1-C)(\alpha^2+1)^{3/2}).[B^2/8 + \mu l^2\alpha/2]$$

$$+ \sigma_w\ V^2 h_w\ C_f.l.\surd C/K.\alpha^2.(\alpha+\mu)/(\alpha^2+1)$$

$$+ [\sigma_i h\ CV^2/(8.l.(1-C))].[\alpha/(\alpha^2+1)].(B^2) \qquad \text{III.8}$$

$$R_i = 1/2[\sigma_w V^2 C\ C_s.l^2.\alpha^3/((1-C)(\alpha^2+1)^{3/2})].[\alpha+\mu]$$

$$+ [\sigma_w V^2\surd C/k\ h_w\ C_f\ \alpha^2.l/(\alpha^2+1)].[\alpha+\mu]$$

$$+ \sigma_i h\ C\ V^2 B^2\alpha/(8.l(1-C)(\alpha^2+1)) \qquad \text{III.9}$$

This equation can be rearranged to yield the following:

$$R_i = 1/2[\sigma_w V^2\alpha^2 l(\alpha+\mu)/(\alpha^2+1)].[\{CC_s l^2\alpha/((1-C)(\alpha^2+1)^{1/2}\} +$$

$$2\ h_w C_f\surd C/k] \ + \ \sigma_i V^2\ hCB^2\alpha/(8.l(1-C)(\alpha^2+1)) \qquad \text{III.10}$$

This equation presents the ship resistance in broken ice for a constant slope ship entrance. It can be used either with simplified hull forms or with segmented hull of any given ship entrance.

END

28-02-91

FIN