Composability of transactions using closed nesting in software transactional memory

by

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A thesis submitted to the

School of Graduate Studies

in partial fulfilment of the

requirements for the degree of

Master of Science

Department of Computer Science

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Abstract

With the boom in the development of multi-core machines and the development of multi-threaded applications as such, concurrent programming has gained increasingly more significance than ever before. However, concurrent programming using traditional methods such as locks, mutex and monitors is not easy, as they require a programmer to predetermine the lock management scheme for each case. This approach is error-prone. Besides, it is very difficult to trace the bugs in such programs. Software transactional memory (STM) is a new technology that solves this problem by offering automatic management of locks. As such, in recent years STM has gained a lot of attention in both industry and academia. However, most of the work in STM is restricted to non-nested transactions, while the domain of nested transactions remains largely unexplored.

One of the striking features of STM is its ability to support composability of transactions through three types of nesting, namely *flat nesting, closed nesting and open nesting*. In this thesis, we study the complexities involved in designing STM protocols for *closed nested transactions*. To this end, we extend Imbs and Raynal's STM protocol [1], which is designed for non-nested transactions, to closed nested transactions. We propose several extensions, employing different modes of concurrency for subtransactions in the transaction tree : (i) serial execution (no concurrency) of subtransactions at each level; (ii) pessimistic concurrency control at all nodes; (iii) optimistic concurrency control at all nodes; and (iv) a mixture of optimistic concurrency control at some nodes while pessimistic concurrency control at other nodes in the same transaction tree.

Acknowledgements

The accomplishment of this thesis would not have been possible without the intelligent guidance and sustained support of my supervisor, Dr. Krishnamurthy Vidyasankar, at every stage of my Master's program. I am truly indebted to him. I am also thankful to faculty and staff of Computer Science Department at MUN, namely- Dr. Edward Brown, Dr. Manrique Mata-Montero, Elaine Boone, Sharon Deir, Regina Edwards, and Darlene Oliver.

I am highly grateful to Sumeet Ghosh, Karan Bhawasinka, Wagdi M. Alrawagfeh, Cristy S. Hynes and Saima Siddiqui for their sustained support and constant encouragement. Finally, I thank my parents, Dileshwar Prasad and Shanti Devi, as well as my uncle/aunty- Priya Sinha, Pradeep Sinha, Amarendra D. Singh, Rajesh Khosla- for their support all through and believing in me.

Contents

List of Tables

List of Figures

Chapter 1

Introduction

1.1 Why Software Transactional Memory?

Modern systems are often complex, dynamic and distributed in nature. As such, parallel and distributed computations have become inherent in these systems. To meet the requirements of the changing times, we have seen significant shift of paradigm in technological advancements from single core machines to multi-core machines in hardware sector. To complement the growth in the hardware sector, ever-increasing emphasis is being laid on the parallel programming paradigm to utilize the resources better for faster computations.

In concurrent programming, several threads compete to access shared data. Moreover, parallel programs execute in a non-deterministic manner and hence synchronization of concurrently executing threads is a critical issue. Otherwise, threads may read inconsistent values, or may see a thread's intermediate values of computation. If several threads modify a resource simultaneously, then the final value may not correspond to any thread's computation. Mutual exclusion, facilitated by applying locks on the shared objects, is a mechanism that prevents several threads from accessing a shared resource at the same time. However, lock-based solutions have inherent drawbacks. In case of large grained locking where the set of data controlled by a single lock is too large, the concurrency is drastically hampered, whereas in case of fine grained locking, it is very difficult to manage the locks associated with each data item. To illustrate why it is not easy for programmers to manually manage the locks in parallel programming paradigm, let us consider a simple bank account example (using the usual implementation of synchronized methods) as shown in Figure 1.1.

Consider the case where several clients execute the transfer concurrently on the same (shared) account. The initial balance in the account is 0. Each client first deposits a given amount, and then withdraws the same amount. Thus, at the end of execution of a transaction, the final value of the balance should remain 0. In a multithreaded environment, the above example presents a scenario which suffers from a race condition, i.e., the concurrent threads compete with one another to gain access to the shared object (bank account). When several threads executing concurrently try to invoke the transfer method, their operations are not synchronized. In other words, the deposit and withdraw operations of one thread can be interleaved with those of another thread. Therefore, we often encounter cases in which the final value of the balance at the end of a transaction's execution is non-zero. The non-zero value does not correspond to the expected execution of any of the threads. This will not be the case if the transactions are executed atomically in a serial fashion, i.e., the atomicity of the transactions is violated here.

One may argue that the above issue can be addressed by enclosing the deposit

Account class

```
class Account()
{
        int balance = 0;

        void withdraw(int n)
        {
                this.lock();
                balance -= n;
                this.unlock();

        }

        void deposit(int n)
        {       this.lock();
                balance += n;
                this.unlock();
        }

}
```

Client program

```
void transfer( Account acc, int amount )
{
        acc.deposit(amount);
        acc.withdraw(amount);
}
```

Figure 1.1: Bank Example 1

Client program

```
void transactional transfer( Account acc, int amount )
{
        acc.deposit(amount);
        acc.withdraw(amount);
}
```

Figure 1.2: Bank Example 2

and withdraw procedures within lock() and unlock() calls. That's right, but it again emphasizes the same point. The programmer has to worry about different lock management issues for different granularity and set of locking. Concurrent programming involving manual management of locks is more error-prone, and locks are unable to support modular programming, i.e., gluing together smaller programs to form larger programs.

Hence, there is clearly a need for a system for dynamic management of locks in a parallel programming environment. This is where Software Transactional Memory (STM) comes into the picture.

1.2 Software transactional memory (STM)

Software Transactional Memory (STM) aims at providing a mechanism for handling low-level concurrency control for accessing shared objects in a multi-threaded environment in such a way that programmers may write programs without having to worry about the underlying concurrency management [9, 17, 10, 5]. It allows programmers to denote atomic regions declaratively, and the underlying STM system provides transactional guarantees. For example, under STM, the client code for the bank account example in Figure 1.1 would change as shown in Figure 1.2. The locking of the objects is managed dynamically by STM to ensure effectively atomic execution of the set of statements within the transactional block.

1.3 Nesting in STM: transaction tree

One of the unique features of STM is the composability of the transactions [13, 14, 8, 16, 12, 18, 7]. In other words, a set of smaller transactions can be combined to form a larger transaction through nesting. The execution of nested subtransactions can be conceptually represented by a dynamic tree called transaction tree [12], in which the transactions are related by parent-child relationship. A transaction that has no parent is termed as a root transaction. A root transaction has the highest level (0). The level of a child transaction in a transaction tree is one level lower than that of its parent. Whenever a transaction t invokes a new child transaction $t^{'}$, a new node $t^{'}$ is added in the transaction tree as a child of t. When a (sub)transaction $t^{''}$ is aborted (or discarded due to an ancestor's abort), node $t^{''}$ is removed from the transaction tree. The committed transactions are retained in the transaction tree.

Nested transactions are created when an atomic region is created inside an outer atomic region. The different types of nesting are (a) *flat nesting* (b) *closed nesting*, and (c) *open nesting*.

1.3.1 Flat nesting

In flat nesting, the steps of a flat nested subtransaction are treated as if they are steps of the root transaction itself. The commit of a subtransaction is local to its parent only, i.e., its read/write sets are merged with those of its parent. The write operations of a flat nested transaction are reflected on the global objects only when their local read/write sets are propagated to the root level through the commit of the intermediate ancestors (in any), and the root transaction commits. However, when a subtransaction aborts, the entire root transaction is aborted.

1.3.2 Closed nesting

As far as the commit of a closed nested transaction is concerned, it is similar to that of a flat nested transaction. Unlike a flat nested transaction, the abort of a closed nested subtransaction does not cause the abort of its ancestors. In event of an abort, its local read/write sets are not merged with those of its parent, thereby not affecting the state of its parent.

1.3.3 Open nesting

Here, when a nested subtransaction commits, the effects of its write steps are immediately reflected on the globally shared data [15]. Thus, its changes become visible to all other transactions in the system, although its ancestors may still be executing. In case any of its ancestors aborts, compensating actions are required to undo the effects of the changes it made to globally shared data.

1.4 Motivation

So far, a number of lock-based as well as timestamp-based protocols have been proposed for efficiently supporting non-nested transactions in STM. However, very little work has been carried out in exploiting the full potential of parallel nesting. Designing a protocol for supporting parallel nesting is not trivial; there is an inherent complexity involved in contention management across different levels and guaranteeing level-wise serial execution of nested subtransactions [6, 11, 4, 16]. As such, most of the work in STM so far has been carried out for normal (non-nested) transactions. Those that consider nested transactions support only serial execution of nested subtransactions [8, 12]. Recent works [1, 2, 3, 18] go further to support parallelism at the child-level, under the restriction that the parent transaction does not execute while it has active children.

Given the complexities involved in designing STM protocols for nested transactions, our interest lies in not only obtaining a higher degree of concurrency for nested transactions but also exploring and employing various modes of concurrency for nested transactions.

Note that this thesis deals with closed nested transactions only. It is directed towards the theoretical study of developing a comprehensive insight into the complexities involved in designing STM protocols for closed nested transactions. Therefore, the term 'nested transaction' henceforth will be used to mean 'closed nested transaction', unless specified otherwise.

1.5 Contributions

The major contributions of this thesis are as follows:

- *A detailed analysis of the complexities associated with designing STM protocols for closed nested transactions:* We examine in what ways designing an STM protocol for nested transaction varies from the one for non-nested transactions. Further, we study the various cases to be considered while designing an STM protocol for nested transactions. Finally, we also provide solutions for handling the various cases.

- *A set of protocols, for closed nested transactions, employing different modes of concurrency:* We provide a set of STM protocols for closed nested transactions in an incremental mode of development. Starting with a simple protocol (SimpSTM: Chapter 4) with no concurrency at the nested level, we progress to achieving full concurrency at all the nodes of the transaction tree. Further, we also employ a mixture of optimistic and pessimistic modes of concurrency at different nodes (MxSTM, Chapter 7).

- *A system for formally proving the correctness of nested transactions:* Formally proving the correctness of the STM protocols for nested transactions is in itself a challenging task. We provide a system for constructing (mapping) and analyzing the histories produced by nested transactions and determining their correctness (level-wise serializability).

1.6 Organization of thesis

In Chapter 2, we provide the discussion of the background. In Chapter 3, we discuss the formalism used for developing the protocols for nested transactions in later chapters.

- In these two chapters, we briefly discuss some of the concepts (semantics) of an existing work [9]. We use these semantics in each of the protocols presented in this thesis. We also describe the system model as well as the formalism used. Finally, we discuss the correctness criteria and the proof system for establishing the correctness of the STM protocols.

In each of the Chapters 4, 5, 6 and 7, we present the STM protocols for closed nested transactions, following an incremental mode of development.

- In Chapter 4, we present a simple STM protocol, SimpSTM, for nested transactions, under the constraint that nested subtransactions execute in a serial fashion, one at a time. Thus, there is no concurrency at the nested level. This protocol lays the foundation for more complex protocols that are presented in later chapters. Further, it also provides a comprehensive set of proofs, that are used (referenced) in the remaining chapters (5, 6 and 7) for establishing the correctness of the protocols in these chapters.

- In Chapter 5, we present another protocol, ParSTM, that employs optimistic approach of concurrency control at the root level, and a pessimistic one at the nested level. Thus, there is some (partial) concurrency at the nested level.

- In Chapter 6, we present a protocol called HParSTM in which an optimistic approach of concurrency control is used at each node of the transaction tree, thereby offering full concurrency at all the levels. This is the first protocol, that we know of in the literature, in which the siblings can execute concurrently along with their ancestors.

- In Chapter 7, we present a protocol, MxSTM, obtained by mixing (integrating) the protocols presented in Chapters 5 (for pessimistic part) and 6 (for optimistic part). Under this protocol, in a transaction tree, at some nodes an optimistic approach of concurrency control is followed while at others pessimistic behaviour is exhibited. This means we can obtain different degrees of concurrency at different nodes (levels).

Finally in Chapter 8, we provide conclusions and directions for future work.

Chapter 2

Preliminaries: background

2.1 Transaction tree

As stated earlier, the execution of nested subtransactions can be conceptually represented by a dynamic tree called transaction tree [12], in which the transactions are related by parent-child relationship. A transaction that has no parent is termed as a *root transaction*. A root transaction has the highest level (0). A root transaction operates at the global level. The level of a child transaction in a transaction tree is one level lower than that of its parent. Whenever a transaction t invokes a new transaction t', a new node t' is added as a child of t in the transaction tree. When a (sub)transaction t'' is aborted, then the subtree rooted at t'' is removed from the transaction tree. The committed (sub)transactions are retained in the transaction tree.

The children of a node t are represented as $t_1, t_2, ..., t_k$, etc. and denoted as $children(t)$. The active (not yet committed/aborted) children of t are given by

$activeChildren(t)$. We shall denote the transaction (sub)tree rooted at node t as $transTree(t)$. Thus, $transTree(root)$ represents the entire transaction tree. Alternatively, in case t is a subtransaction, we shall invariably use $subTree(t)$ to represent $transTree(t)$. For a subtransaction t, the term $outsideTrans(t)$ is used to denote the $transTree(root)$, excluding the $transTree(t)$. A non-leaf node is composed of one or more child transaction(s) and is called a *composite* transaction. In a transaction tree, all transactions other than the *root* transaction are *nested transactions*.

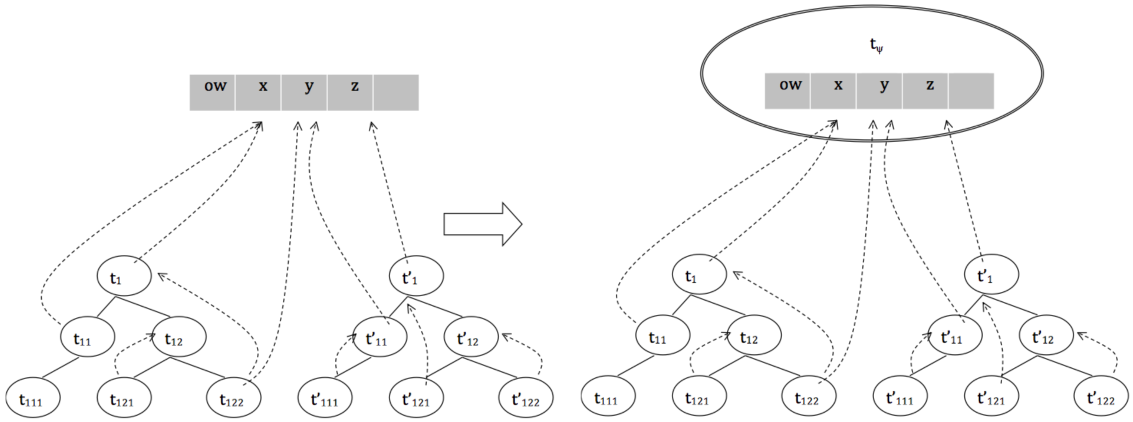
2.2 Super transaction and super tree

For sake of uniformity in notation, we would like to refer to globally shared objects in the same way as we refer to local objects of a transaction. For this, we associate all the globally shared objects with a highest level fictitious transaction called *super transaction* (denoted by t_ψ), such that all the transactions, previously referred to as root-level transactions, are now children of the super transaction. We call the resulting tree the *super tree* (see Figure 2.1).

2.3 Shared objects: global copy vs local copy

Global object $t_\psi.x$: In our model, for each shared object x, the system contains a global copy that is non-null valued and is accessible by all the transactions. It is not associated with the super transaction, and not with any transaction.

Local object $t.x$: This local copy of x associated with t is denoted by $t.x$. The object $t.x$ is accessible to t as well as its descendants in the transaction tree.



(a) Transaction trees

(b) Super tree

Figure 2.1: Transaction trees and super tree (dotted lines denote access of shared objects by nodes).

2.4 Common features of a nested transaction in our model

In our model, the common semantics associated with a closed nested transaction t in a transaction tree are as follows.

1. Transaction t maintains its own local copy of an object it reads or writes.

2. While reading an object x, t reads from its local copy $t.x$. If a local copy is not available, then it tries to read from its nearest ancestor t' having a local copy of x. In the worst case, t reads from $t_\psi.x$. The read value should be "consistent" w.r.t. t as well as each of its intermediate ancestors up to t'. (We discuss "consistency" later on.)

3. The write operations are performed in the local space.

4. The read and write operations are logged in local read set and local write set

respectively.

5. Transaction t commits only if the combined steps of its committed children and its own steps are "consistent". Otherwise it aborts.

6. If t is a non-root transaction and commits, its local read and write sets are merged with the corresponding read and write sets of its parent. When a root transaction commits, its write set values are transferred to the globally shared objects. Thus, the changes due to the write steps of a committed subtransaction t are reflected on the globally shared objects only when all the ancestors of t commit.

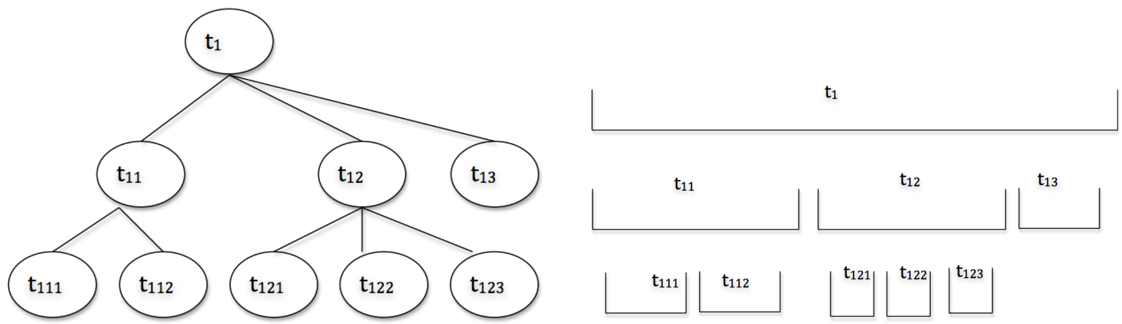
7. In case of abort of t, its read and write sets are ignored, and no changes are made to the parent's objects. Here, the results of t as well as its descendants are discarded, and are not propagated to t's ancestors. Therefore, they are not accessible to transactions in $outsideTrans(t)$.

2.5 Informal discussion about linearizability of nested transactions

Linearizability: An execution is *linearizable* if each transaction in the execution appears to have occurred at a single instant of time, called its *linearization point*, during its lifespan. In addition, no two transactions can have the same linearization point. In other words, the steps of the transactions can be ordered to obtain a *sequential history* in which transactions seem to execute serially, one after the other, in the order of their linearization points. Here, the ordering of the steps of the transactions must respect the constraint that the linearization point of each transaction lies at some point of time within its lifespan. This constraint distinguishes linearizability from serializability.

For sake of convenience of argument, we use the expression "a transaction t is linearizable" to mean that "the steps of t are such that a unique linearization point for t can be obtained w.r.t. other transactions in a (linearizable) execution".

Linearizability of nested transactions: Consider any non-leaf node t in a transaction tree. Transaction t may perform some read or write operations, or invoke a new child. Let the set of its children be denoted by S_c. Then, the entire execution of each child t_c in S_c should appear to occur atomically at t's level. Observe here that the steps of its children on t's local objects may be interleaved with one another and with t's own local operations. To accommodate this, consider each of the local read or write operations of t to have been carried out by a fictitious new child with that operation being its only operation. Then, the execution of each child should be such that it is linearizable with its sibling transactions. Similarly, at the root level, it is required that the overall execution of each root-level transaction is performed in a linearizable manner. These points are illustrated by Figure 2.2. Figure 2.2b depicts the level-wise serial (linear) execution for the transaction tree shown in Figure 2.2a.



(a) Nested transactions (b) Equivalent level-wise serial execution

Figure 2.2: Execution of nested transactions

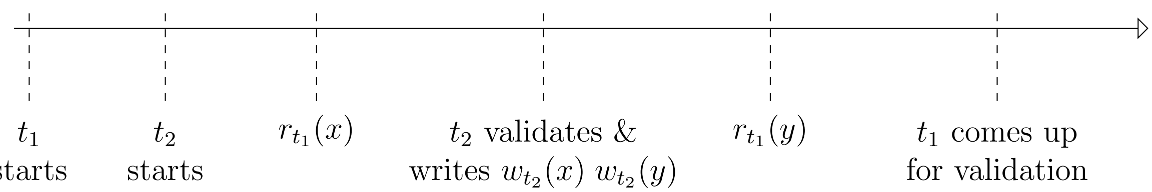


Figure 2.3: Traditional optimistic approach of concurrency

2.6 Concurrency control mechanism: a relevant study

2.6.1 Opacity

Opacity is the most widely accepted correctness criterion for STM systems [1, 9, 11]. The consistency criterion we use for our protocols is *level-wise opacity* (defined in the next chapter), based on the definition of *opacity* for non-nested transactions. Opacity states that *both committed and aborted transactions should read from a consistent state of the shared memory.* The state resulting from a linearizable execution of some committed transactions is taken to be consistent. To illustrate the idea, consider history $\widehat{\mathcal{H}_1}$ under the case of traditional optimistic approach, as depicted in Figure 2.3.

$\widehat{\mathcal{H}_1} = \langle r_{t_1}(x)\ w_{t_2}(x)\ w_{t_2}(y)\ c_{t_2}\ r_{t_1}(y) \rangle$

First, t_1 reads x. Then, t_2 modifies the values of x and y, and commits after successful validation. Next, t_1 reads y. Now, t_1 comes up for validation and fails because the value of x it read previously has been already overwritten by t_2. The x and y values that t_1 read are not from a consistent state.

According to opacity, we want aborted transactions also to read from consistent states only, i.e., in case of above example ($\widehat{\mathcal{H}_1}$), t_1 should be *forbidden* to read from y. This notion of *forbidden read* operation is elegantly captured in the protocol presented by Damien Imbs and Michel Raynal in [9]. The mechanism presented in [9] has been used in each of the algorithms proposed in this thesis. A clear understanding of this protocol will help in seeing through some of the complex protocols presented in the later chapters. We shall discuss this protocol in the next section.

2.6.2 Damien Imbs and Michel Raynal's STM Protocol

The Protocol 2.1 has been designed for non-nested transactions. (Capital letters have been used in [9] to denote transactions and shared objects.) It uses a single copy of each base object x. Each transaction has its own local copy of the base object associated with its read or write steps. (Recall that $t.x$ denotes the local copy of object x associated with transaction t, whereas $t_\psi.x$ denotes the globally shared copy.) To keep track of the conflicts between the transactions, the following control variables are used (with slightly different notations in [9] as OW, RS_X, FBD_X).

$t_\psi.ow$: a (overwritten) set that contains the ids of the transactions that read some object $t_\psi.x$ that was modified later (so, there is a conflict).

operation $read_t$ (x):
if ($t.x$ does not exist) **then**
    allocate local space for $t.x$;
    $t.lrs \leftarrow t.lrs \cup \{x\}$;
    lock $t_\psi.x$; $t.x \leftarrow t_\psi.x; t_\psi.x.rs \leftarrow t_\psi.x.rs \cup \{t\}$; unlock $t_\psi.x$;
    if ($t \in t_\psi.x.fbd$) **then** return (abort); **end if**
end if;
return(value of $t.x$)

---

operation $write_t(x, v)$:
$t.read\_only \leftarrow false$; // $t.read\_only$ is set to $true$ initially.
if ($t.x$ does not exist) **then** allocate local space for $t.x$ **end if**;
$t.x \leftarrow v$;
$t.lws \leftarrow t.lws \cup \{x\}$;

---

operation $try\_to\_commit_t$ ():
if ($t.read\_only$)
  then return(commit);
else lock all the objects in $t.lrs \cup t.lws$;
if ($t \in t_\psi.ow$) **then** release all the locks; return(abort) **end if**;
for each $x \in t.lws$ **do** $t_\psi.x \leftarrow t.x$ **end for**;
$t_\psi.ow \leftarrow t_\psi.ow \cup (\cup_{x \in t.lws} t_\psi.x.rs)$;
for each $x \in t.lws$ **do** $t_\psi.x.fbd \leftarrow t_\psi.ow; t_\psi.x.rs \leftarrow \emptyset$; **end for**;
release all the locks;
return(commit)
end if

Table 2.1: **Protocol 2.1:** D. Imbs and M. Raynal's STM Protocol [9]

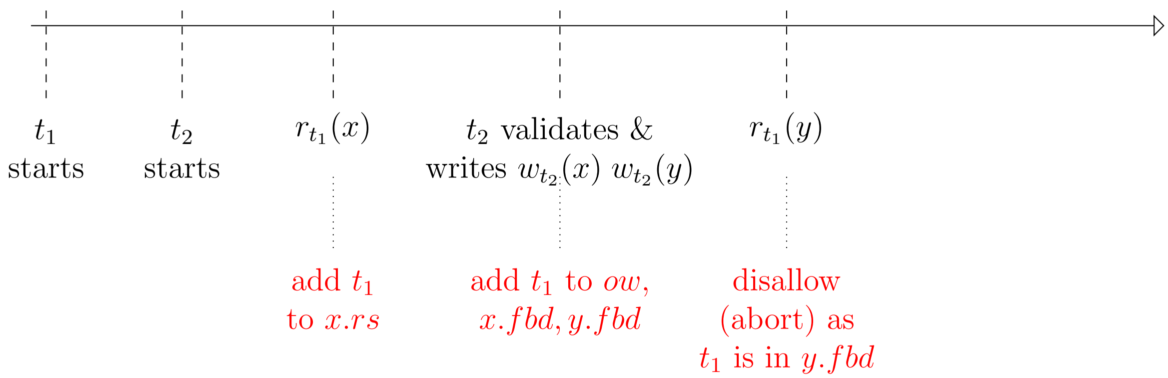


Figure 2.4: Optimistic approach under Imbs and Raynal's protocol

$t_\psi.x.rs$: a (read) set associated with each shared object $t_\psi.x$. It stores the ids of the transactions that read from the object $t_\psi.x$ since its last update. Thus, the reads are *visible* to other transactions.

$t_\psi.x.fbd$: a (forbidden) set associated with each global object $t_\psi.x$. The predicate $t \in t_\psi.x.fbd$ means that the transaction t has read an object $t_\psi.y$ that since then has been overwritten (hence $t \in t_\psi.ow$), and the overwriting of $t_\psi.y$ is such that any future read of $t_\psi.x$ by t will be invalid (i.e., the value obtained by t from $t_\psi.y$ and any value it will obtain from $t_\psi.x$ in the future cannot be mutually consistent).

To illustrate how these data structures, $t_\psi.x.rs, t_\psi.x.fbd$ and $t_\psi.ow$ prevent the *forbidden* read, consider the example of $\widehat{\mathcal{H}_1}$ again through Figure 2.4. For the following discussion, all the references to line numbers are associated with Protocol 2.1. When a transaction t_1 performs a read operation on a shared object $t_\psi.x$, it adds its id in $t_\psi.x.rs$ (line 04). Later, when another transaction t_2 modifies objects $t_\psi.x$ and $t_\psi.y$, it adds t_1 to $t_\psi.ow$ (line 17), followed by updating both $t_\psi.x.fbd$ and $t_\psi.y.fbd$ (line 18). Now, if t_1 tries to access $t_\psi.y$, it will detect the conflict by noticing its id in $t_\psi.y.fbd$, and consequently abort (due to line 05). Observe that $t_\psi.ow$ contains the ids of all the transactions that previously read any object whose value since then has been modified, and $t_\psi.x.fbd$ is updated using $t_\psi.ow$. As such, none of those transactions (in $t_\psi.x.fbd$) will be allowed to read $t_\psi.x$, or any other object that is written after the update of $t_\psi.x$. Therefore, a cyclic conflict between transactions through transitivity is not possible.

Besides, a transaction t maintains two local sets, $t.lrs$ (local read set) and $t.lws$ (local write set) to document its read/write operations (lines 03, 11). Before committing, the validation phase consists of ensuring that the transaction does not belong to set $t_\psi.ow$ (line 15). However, a transaction that has no write operation is committed immediately (lines 12-13), and is thus treated differently.

2.6.2.1 About correctness

For each transaction t, its linearization point ℓ_t, is defined as follows:

1. If a transaction t aborts, ℓ_t is placed just before t is added to the set $t_\psi.ow$ (line 17 of the $try\_to\_commit_t()$ operation of transaction that entails its abort).

2. If t is a read only transaction, ℓ_t is placed at the earliest of (1) the occurrence time of the test during its last read operation (line 05 of the $read_t(x)$ operation) and (2) the time just before t is added to $t_\psi.ow$ (if it ever is).

3. If t is an update transaction that commits, ℓ_t is placed just after the execution of line 17 by t (update of $t_\psi.ow$).

Using these linearization points, a set of proofs (listed under part (1) of Section 3.7.6) are provided in [9] to show that the history produced by this protocol is indeed linearizable. The formalism as well as the set of proofs presented in the rest of the chapters are based along the same lines as presented in [9].

In the next chapter, we provide the formalism as well as the outline of the proof system used for showing the correctness of the protocols discussed in this thesis.

Chapter 3

Computation model, base formalism, and proof outline

3.1 Computation model

Our system is similar to the one used in [9]. The computational model consists of processes, base objects, locks and atomic registers. There are n asynchronous sequential processes (i.e.,threads) denoted $p_1, ..., p_n$ that cooperate through base read/write atomic registers and locks. Each of the shared objects is protected by an individual lock. Each process is made up of a sequence of transactions, and has its own memory. The processes issue transactions one at a time.

A transaction is a sequence of read and write operations that can examine (read) and modify (write), respectively, the state of the base objects. It consists of a sequence of events that are an *operation invocation*, an *operation response*, a *commit invocation*, a *commit response*, and an *abort event*. An operation is considered *terminated* if its response event has occurred. Similarly, a transaction is considered *completed* if its commit response or abort event has occurred.

Further, each transaction satisfies the following constraints: (1) a transaction performs one operation at a time, and (2) a composite transaction must wait until all of its children have completed before entering the validation for its commit.

The set of transactions is denoted by T. The set of objects is denoted by X and the set of possible values associated with them is V.

3.2 Histories and base formalism

3.2.1 Events and histories at shared memory level

There is an event associated with each operation on a shared memory (objects, locks, sets). Let $\mathcal{H}$ be the set of all events produced by the STM system. Each access on a shared object is atomic. As such, there is a total order on the events in $\mathcal{H}$. Thus, at the shared memory level, an execution can be represented by the pair $\widehat{\mathcal{H}} = \langle \mathcal{H}, <_{\mathcal{H}} \rangle$, where $<_{\mathcal{H}}$ denotes the total ordering on its events. $\widehat{\mathcal{H}}$ is called *shared memory history*. We use the following notations (similar to those used in [9]).

- B_t marks the event associated with the beginning of transaction t, and E_t denotes its completion. E_t can be of two types A_t or C_t, where A_t is the event "abort of t", and C_t is event "commit of t".

- $AL_{t_i}(t_j.x, op)$ denotes the event (response) associated with the acquisition of the lock on $t_j.x$, issued by transaction t_i during an invocation of operation op, where op is $read_{t_i}(x)$, or $try\_to\_commit_{t_i}()$ (abbreviated in the proof as ttc).

- Similarly, we have $RL_{t_i}(t_j.x, op)$ denoting the corresponding event (response) associated with the release of the lock.

  As each access to a shared object is atomic in nature (due to use of either an atomic register or a lock), we denote each read or write operation as a single event for sake of simplicity.

- $r_{t_i}(t_j.x, v)$ denotes the read operation performed by transaction t_i on the object $t_j.x$ (i.e. object x of transaction t_j), where v is the value returned by that read operation.

- $w_{t_i}(t_j.x, v)$ denotes the write of value v by transaction t_i on the object $t_j.x$.

For sake of simplicity, in the histories discussed in this thesis, B_t has been omitted as it can be intuitively inferred to occur just before the first read/write operation of transaction t. However, C_t or A_t has been used as a single event to mark the end (commit or abort) of the transaction.

3.2.2 History at transaction level

Given an execution, let $\mathcal{H}'$ denote the set of transactions issued during that execution. The order relation between the transactions in $\mathcal{H}'$, denoted by $\rightarrow_{\mathcal{H}'}$, is defined as follows: $t_1 \rightarrow_{\mathcal{H}'} t_2$ if t_1 ends before t_2 begins, and t_1 and t_2 are concurrent if $t_1 \nrightarrow_{\mathcal{H}'} t_2$ and $t_2 \nrightarrow_{\mathcal{H}'} t_1$. Thus, at the transaction level, the execution is defined by a partial order $\widehat{\mathcal{H}'} = \langle \mathcal{H}', \rightarrow_{\mathcal{H}'} \rangle$, that is called *transaction history* [9].

The *reads from* relation between transactions, denoted $\rightarrow_{rf}$, is defined as: $t_1 \xrightarrow{t.x}_{rf} t_2$, if t_2 read a value written by t_1 in t's object $t.x$.

A *transaction history* $\widehat{\mathcal{H}^\sigma} = \langle \mathcal{H}^\sigma, \rightarrow_{\mathcal{H}^\sigma} \rangle$ is *sequential* if no two of its transactions are concurrent. Hence, in a sequential history, for $t_1 \neq t_2$, $t_1 \not\rightarrow_{\mathcal{H}^\sigma} t_2 \Leftrightarrow t_2 \rightarrow_{\mathcal{H}^\sigma} t_1$, that is, $\rightarrow_{\mathcal{H}^\sigma}$ is a *total order*. A sequential transaction history is *legal* if each of its read operations returns the value of the most recent write on the same object. A sequential transaction history $\widehat{\mathcal{H}^\sigma}$ is equivalent to a transaction history $\widehat{\mathcal{H}'}$ if (1) $\mathcal{H}^\sigma = \mathcal{H}'$ (i.e., they are made of the same transactions with the same invocations and the same responses), and the total order $\rightarrow_{\mathcal{H}^\sigma}$ respects the partial order $\rightarrow_{\mathcal{H}'}$ (i.e, $\rightarrow_{\mathcal{H}'} \subset \rightarrow_{\mathcal{H}^\sigma}$).

A transaction history $\widehat{\mathcal{H}^\lambda}$ is *linearizable* if there exists a history $\widehat{\mathcal{H}^{seq}}$ that is sequential, legal and equivalent to $\widehat{\mathcal{H}^\lambda}$.

The set of transactions that commit in $\widehat{\mathcal{H}}$ are given by *committed*$(\widehat{\mathcal{H}})$, and the aborted ones are given by *aborted*$(\widehat{\mathcal{H}})$. The set of transactions that have committed or aborted is given by *complete*$(\widehat{\mathcal{H}})$. The history restricted to committed transactions is denoted by *permanent*$(\widehat{\mathcal{H}})$ or $\Pi(\widehat{\mathcal{H}})$.

3.2.3 Level-wise history

As discussed earlier, each node t in the transaction tree has its own set of local copies of objects. These objects are accessible (shared) by t and its descendants. The formalism discussed in Sections 3.2.1 and 3.2.2 can be applied to these objects as well. Thus, at each level (node) of the super tree, we have a separate history. We call this history a *level-wise history*. Here, $\mathcal{H}_t$ denotes the set of all events associated with the objects of node t, and $<_{\mathcal{H}_t}$ gives the total order on these events. Thus, a *level-wise* *shared memory history* is given by $\widehat{\mathcal{H}_t} = \langle \mathcal{H}_t, <_{\mathcal{H}_t} \rangle$. Similarly, $\widehat{\mathcal{H}'_t} = \langle \mathcal{H}'_t, \rightarrow_{\mathcal{H}'_t} \rangle$ is used to denote a *level-wise transaction history* at t, whereas $\widehat{\mathcal{H}^\sigma_t} = \langle \mathcal{H}^\sigma_t, \rightarrow_{\mathcal{H}^\sigma_t} \rangle$ denotes a *level-wise sequential transaction history.*

3.3 Local timeline and linearization point at a level

3.3.1 Local timeline

With each transaction t in the transaction tree, we associate a notion of local timeline spanning the lifespan of that transaction, and it is denoted by τ_t. An instant of time, i, in τ_t is denoted by τ_t^i. Let t_c be a child of t. Then, the linearization point, ℓ_{t_c}, for t_c is the time in τ_t at which the entire execution of t_c can be treated to have occurred. Thus, the linearization point of a child transaction is defined within the lifespan of itself as well as its parent. The time corresponding to ℓ_{t_c} in τ_t is denoted by $\tau_t^{\ell_{t_c}}$. Similarly, as the accesses to shared object $t.x$ are atomic in nature, the times corresponding to $r_{t_c}(t.x)$ and $w_{t_c}(t.x)$ in τ_t can be denoted by $\tau_t^{r_{t_c}(x)}$ and $\tau_t^{w_{t_c}(x)}$ respectively. The timeline associated with t_ψ is taken to be infinite.

3.3.2 Linearization point at a level

To determine the linearization point of a transaction t at its parent's (t_p) level in the super tree, we do not need to consider t's steps at all the levels; we only need to take into account t's steps on its ancestors' objects.

For example, consider the transaction subtree rooted at t_{12} in Figure 3.1, and let us denote it by $TR_{t_{12}}$. For determining the linearization point for t_{12} at t_1's level,

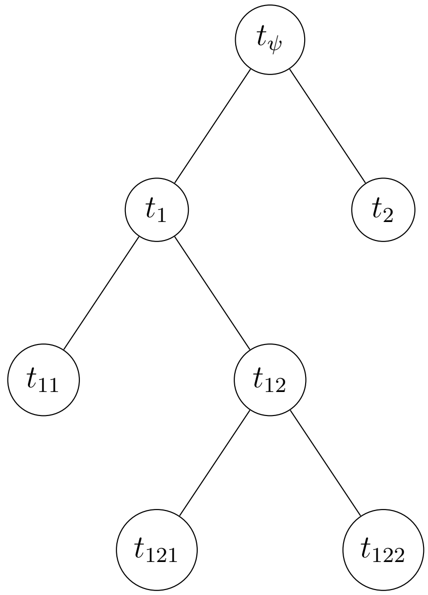


Figure 3.1: Transaction tree

we can treat $TR_{t_{12}}$ as a single transaction. The steps of the transactions within the nodes of $TR_{t_{12}}$ are local to $TR_{t_{12}}$ and therefore do not affect $TR_{t_{12}}$'s linearizability at t_1's level. The steps of $TR_{t_{12}}$ (those of t_{12}, t_{121}, t_{122}) on the objects of t_1 and t_ψ are important.

3.4 Construction of level-wise history

3.4.1 External read

While reading, if t does not have a local copy of an object x, then it tries to read the value of x from its nearest ancestor having a (non-null valued) local copy of x. Such a read operation that takes place outside the local space of t is termed as *external read* operation of t.

Example: With reference to Figure 3.1, read operations $r_{t_{122}}(t_{12}.y)$, $r_{t_{122}}(t_1.x)$ and $r_{t_{122}}(t_\psi.z)$ are external reads of t_{122}.

3.4.2 Visible read objects

A subtransaction t may have read an object x from its ancestor t', other than its parent t_p. Transaction t_p may not have a local copy $t_p.x$. When t commits, it creates $t_p.x$ using its local copy $t.x$. Thus, the value of $t_p.x$ at this point corresponds to the one obtained in the read step of t. Subsequently, this value of $t_p.x$ can be accessed by t_p and its other descendants. In other words, the read of t becomes *visible* at its parent t_p.

Example: With reference to Figure 3.1, $\widehat{\mathcal{H}_1} = \langle r_{t_{121}}(t_1.x, 0)\ c_{t_{121}}\ r_{t_{122}}(t_{12}.x, 0)\rangle$ Here, when t_{121} commits and merges with t_{12}, the value 0 of $t_{121}.x$ (that it read from $t_1.x$) is made available in its parent's corresponding local copy $t_{12}.x$. Therefore, t_{122} is able to read the value 0 from $t_{12}.x$.

Note: We can notice here that a subtransaction reads a value from the nearest local copy of the object visible to it. That nearest copy could be made available due to a visible read operation or due to a write operation.

If t aborts, then no merging of its read steps' values with those of its parent takes place. Therefore, its read steps are not visible at the parent or any ancestor's level.

3.4.3 Commit write

When a closed nested subtransaction t commits and merges with its parent t_p, for each object x that is updated in its local space, t updates $t_p.x$ with the value of $t.x$ at the time of committing. We refer to this write operation as *commit write*. Note that the value of $t.x$ at the time of t's commit corresponds to the "last write" operation that took place on $t.x$.

Again with reference to Figure 3.1,

Example: $\widehat{\mathcal{H}_2} = \langle w_{t_{121}}(t_{121}.x, 0)\ c_{t_{121}}\ w_{t_{12}}(t_{12}.x, 1)\ w_{t_{12}}(t_{12}.y, 2)\ w_{t_{122}}(t_{122}.y, 3)\ c_{t_{122}}\ c_{t_{12}}\rangle$. Observe that x is first updated by t_{121}, and then by t_{12} itself. Therefore, when t_{12} commits, the last write on $t_{12}.x$ corresponds to $w_{t_{12}}(t_{12}.x, 1)$. Similarly, the last write on $t_{12}.y$ corresponds to $w_{t_{122}}(t_{122}.y, 3)$.

Now, upon commit, t_{121}, t_{122} and t_{12} use the values corresponding to the respective last writes on their local objects x and y to update respective objects x and y of their respective parents. Therefore, $\widehat{\mathcal{H}_2}$ can be extended as follows to reflect the commit writes at the parent level.

$\widehat{\mathcal{H}_2} = \langle w_{t_{121}}(t_{121}.x, 0)\ \mathbf{w_{t_{121}}(t_{12}.x, 0)}\ c_{t_{121}}\ w_{t_{12}}(t_{12}.x, 1)\ w_{t_{12}}(t_{12}.y, 2)\ w_{t_{122}}(t_{122}.y, 3)\ \mathbf{w_{t_{122}}(t_{12}.y, 3)}\ c_{t_{122}}\ \mathbf{w_{t_{12}}(t_{1}.x, 1)}\ \mathbf{w_{t_{12}}(t_{1}.y, 3)}\ c_{t_{12}}\rangle$.

Observe that $w_{t_{121}}(t_{12}.x, 0), w_{t_{122}}(t_{12}.y, 3), w_{t_{12}}(t_1.x, 1)$ and $w_{t_{12}}(t_1.y, 3)$ shown in bold fonts here are commit writes.

3.4.4 Mapping of level-wise history

The *level wise history of events* at node t ($\widehat{\mathcal{H}_t}$, defined in Section 3.2.3) includes the following steps:

i. *read operations* performed on t's local objects by t or its committed descendants.

ii. *local write* operations by t on its local objects.

iii. *commit write* operations on t's local objects by t's committed children.

iv. *external read* operations on t's ancestors' objects by t and its committed descendants. To take into account these external read operations, we treat them as if they took place on t's corresponding objects.

Our interest lies in showing that $\widehat{\mathcal{H}_t}$ is equivalent to a sequential history of t's children and its local operations. The operations on t's objects by t's descendants may be interleaved with t's own local operations on its objects. With this end in view, $\widehat{\mathcal{H}_t}$ is obtained after employing the following transformation using a mapping function $f_m(\widehat{\mathcal{H}_t^{or}})$, where $\widehat{\mathcal{H}_t^{or}}$ is the original history of events involving node t and its descendants. We call the resulting history *mapped level-wise history* or simply *level-wise history.*

The following transformations (rules) are employed through $f_m(\widehat{\mathcal{H}_t^{or}})$:

1. *Mapping local read/write operations of t*: Each operation $op_t(t.x)$ (where op denotes read or write) performed by t on its local object $t.x$ is considered to have been successfully performed by a (different) fictitious child transaction $t^{'}$ whose only operation is $op_{t'}(t.x)$ and has "committed". For example, local operations of t in $H_t^1 = \langle r_t(t.x)\ w_t(t.x, v_1)\ w_t(t.y, v_2)\ r_t(t.x)\rangle$ are mapped respectively to $H_t^2 = \langle r_{t^1}(t.x)\ c_{t^1}\ w_{t^2}(t.x, v_1)\ c_{t^2}\ w_{t^3}(t.y, v_2)\ c_{t^3}\ r_{t^4}(t.x)\ c_{t^4}\rangle$, where t^1, t^2, t^3 and t^4 are the fictitious children of t. This way the local (read or write) steps of t also are transformed into the steps of its children.

2. *Mapping operations of t's descendants on t's objects*: Let t_c be a child of t, and t_d be a descendant of t_c. Then, the read operation performed on $t.x$ by t_d is taken to have been performed by t_c (for the reason that t_d is a part of t_c after all). For example, a history $H_t^3 = \langle r_{t_c}(t.x)\ r_{t_d}(t.y)\rangle$ is mapped to $H_t^4 = \langle r_{t_c}(t.x)\ r_{t_c}(t.y)\rangle$.

This means, in $\widehat{\mathcal{H}_t}$ we work as if all the steps on t's local objects were performed only by its children.

3. *Mapping external reads of* t *and its descendants on objects of* t*'s ancestors*: Let $t^{'}$ be an ancestor of t. Then, a read operation on $t^{'}.x$ by t_c or its descendant (t_d) is treated as if it was performed by t_c on t's object $t.x$. For example, $r_{t_d}(t^{'}.x)$ is mapped to $r_{t_c}(t.x)$. In case this read operation was performed by t itself, then we first apply step (1) to transform this operation into step of its fictitious (committed) child transaction, and then apply step (2). (This way we incorporate the external read operations made by transactions in $transTree(t)$ at the level of t's ancestors into $\widehat{\mathcal{H}_t}$.) For example, $\langle r_{t_d}(t^{'}.x)\ r_{t_c}(t^{'}.y)\ r_t(t^{'}.z)\rangle$ is mapped to $\langle r_{t_c}(t.x)\ r_{t_c}(t.y)\ r_{t^1}(t.z)\rangle$ in $\widehat{\mathcal{H}_t}$.

For better understanding consider the following example, using Figure 3.1. For sake of simplicity, we shall consider the history of the committed transactions only. We shall assume that a transaction begins just before it executes its first operation. Further, c_t and a_t denote t's commit and abort respectively.

Now, we consider the execution shown in Figure 3.2 and construct the corresponding history ($\widehat{\mathcal{H}_3}$) for it. This history is constructed by considering, at each level t, the following steps on its object $t.x$: (a) the read operations performed by t's descendants, (b) the commit-writes performed by the committed children, and (c) the local read and write operations performed by t itself. The construction of the resulting history is illustrated as follows. The external reads and commit writes by descendants at ancestors' levels have been highlighted using **bold** font to improve readability of the history. At a parent level t_i, $\left[t_j\ :\ \{\}\ \mathbf{w_{t_j}(t_i.x)}...c_{t_j}\right]$ denotes the place holder for

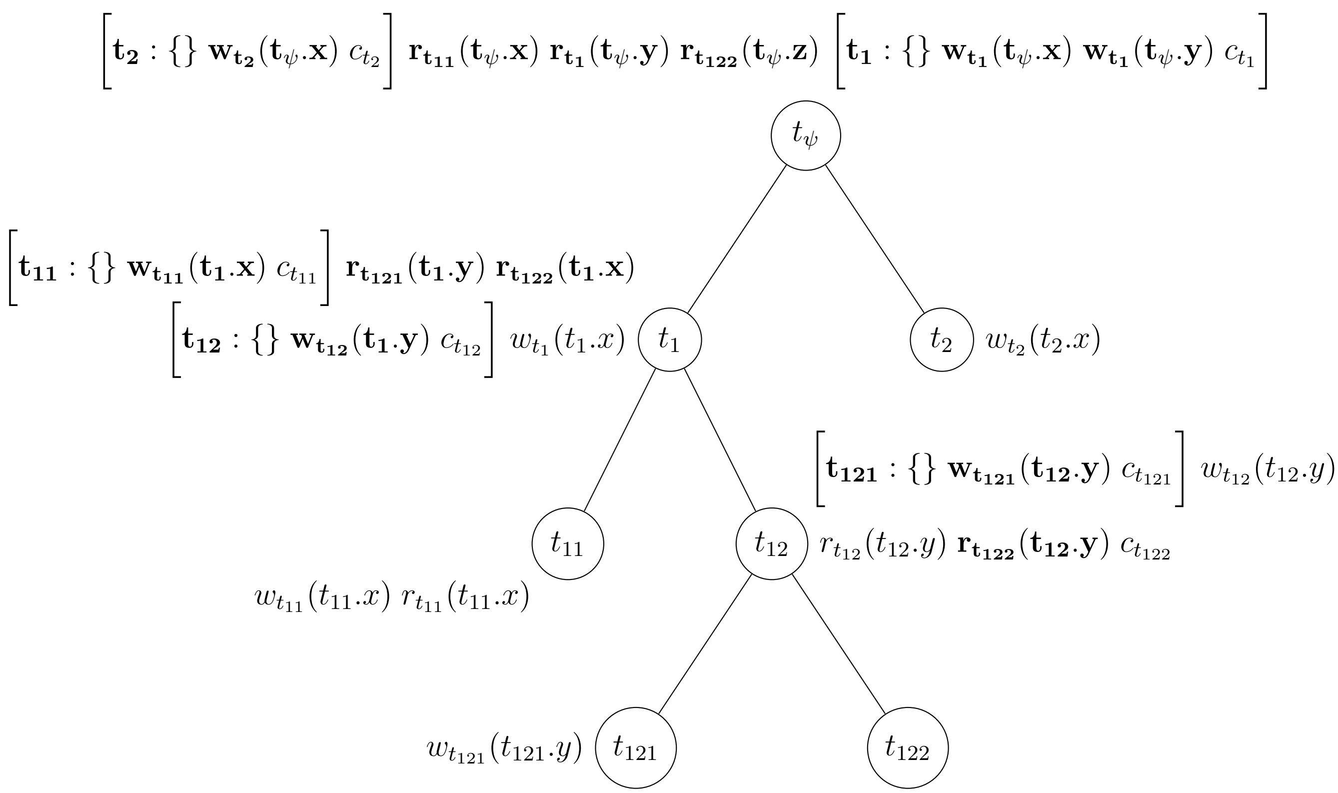


Figure 3.2: Level wise history of events (The read and write steps by descendants at a level are highlighted in **bold**.)

subsequence produced with the commit (merging of) the child t_j. Here, $\{\}$ points to the history produced at node t_j and $\mathbf{w_{t_j}(t_i.x)}$ is a commit write. The subsequence pertaining to t_j at t_i's level is completed by inserting the history produced at node t_j in place of $\{\}$. For example, $\left[t_2 : \{\}\ \mathbf{w_{t_2}(t_\psi.x)}\ c_{t_2}\right]$ at t_ψ's level is completed using $\underline{w_{t_2}(t_2.x)}\ \mathbf{w_{t_2}(t_\psi.x)}\ c_{t_2}$ (underlined part is the subsequence produced at t_2 and is put in place of $\{\}$). Following this approach in bottom to top manner, we obtain the resulting history equivalent to $\widehat{\mathcal{H}_3}$ given below.

$\widehat{\mathcal{H}_3} = \langle w_{t_2}(t_2.x)\ \mathbf{w_{t_2}(t_\psi.x)}\ c_{t_2}\ \mathbf{r_{t_{11}}(t_\psi.x)}\ \mathbf{r_{t_1}(t_\psi.y)}\ \mathbf{r_{t_{122}}(t_\psi.z)}\ w_{t_{11}}(t_{11}.x)\ r_{t_{11}}(t_{11}.x)$ $\mathbf{w_{t_{11}}(t_1.x)}\ c_{t_{11}}\ \mathbf{r_{t_{121}}(t_1.y)}\ \mathbf{r_{t_{122}}(t_1.x)}\ w_{t_{121}}(t_{121}.y)\ \mathbf{w_{t_{121}}(t_{12}.y)}\ c_{t_{121}}\ w_{t_{12}}(t_{12}.y)\ r_{t_{12}}(t_{12}.y)$ $\mathbf{r_{t_{122}}(t_{12}.y)}\ c_{t_{122}}\ \mathbf{w_{t_{12}}(t_1.y)}\ c_{t_{12}}\ w_{t_1}(t_1.x)\ \mathbf{w_{t_1}(t_\psi.x)}\ \mathbf{w_{t_1}(t_\psi.y)}\ c_{t_1}\rangle$

To show the linearizability of nested subtransactions, we construct the level-wise histories in a bottom up manner, i.e., we consider non-leaf nodes t_{12}, t_1 and t_ψ in order. The corresponding level-wise histories at different levels are as follows:

$\widehat{\mathcal{H}_{t_{12}}} = \langle \mathbf{r_{t_{122}}(t_\psi.z)}\ \mathbf{r_{t_{121}}(t_1.y)}\ \mathbf{r_{t_{122}}(t_1.x)}\ \mathbf{w_{t_{121}}(t_{12}.y)}\ c_{t_{121}}\ w_{t_{12}}(t_{12}.y)\ r_{t_{12}}(t_{12}.y)$ $\mathbf{r_{t_{122}}(t_{12}.y)}\ c_{t_{122}}\ c_{t_{12}}\ \rangle$

After Mapping:

$\widehat{\mathcal{H}_{t_{12}}} = \langle \mathbf{r_{t_{122}}(t_{12}.z)}\ \mathbf{r_{t_{121}}(t_{12}.y)}\ \mathbf{r_{t_{122}}(t_{12}.x)}\ \mathbf{w_{t_{121}}(t_{12}.y)}\ c_{t_{121}}\ w_{t_{12}^1}(t_{12}.y)\ \mathbf{c_{t_{12}^1}}\ r_{t_{12}^2}(t_{12}.y)\ \mathbf{c_{t_{12}^2}}$ $r_{t_{122}}(t_{12}.y)\ c_{t_{122}}\ \rangle$

$\Rightarrow \widehat{\mathcal{H}^\sigma_{t_{12}}} = \langle t_{121}, t_{12}^1, t_{122}, t_{12}^2\rangle$ (Recall that t_{12}^1, t_{12}^2 here denote the fictitious children of t_{12}, representing its local read and write operations.)

$\{Observation:$

$\mathbf{r_{t_{122}}(t_{12}.z)}$ due to $r_{t_{122}}(t_\psi.z)$ (Rule 3);

$\mathbf{r_{t_{121}}(t_{12}.y)}$ due to $r_{t_{121}}(t_1.y)$ (Rule 3);

$\mathbf{r_{t_{122}}(t_{12}.x)}$ due to $r_{t_{122}}(t_1.x)$ (Rule 3);

$\mathbf{w_{t_{12}^1}(t_{12}.y)\ c_{t_{12}^1}}$, $\mathbf{r_{t_{12}^2}(t_{12}.y)\ c_{t_{12}^2}}$ due to $w_{t_{12}}(t_{12}.y)$ and $r_{t_{12}}(t_{12}.y)$ respectively

(Rule 1) }

$\widehat{\mathcal{H}_{t_1}} = \langle \mathbf{r_{t_{11}}(t_\psi.x)\ r_{t_1}(t_\psi.y)\ r_{t_{122}}(t_\psi.z)\ w_{t_{11}}(t_1.x)}\ c_{t_{11}}\ \mathbf{r_{t_{121}}(t_1.y)\ r_{t_{122}}(t_1.x)\ w_{t_{12}}(t_1.y)}\ c_{t_{12}}$

$w_{t_1}(t_1.x) \rangle$

After mapping:

$\widehat{\mathcal{H}_{t_1}} = \langle \mathbf{r_{t_{11}}(t_1.x)\ r_{t_1^1}(t_1.y)\ c_{t_1^1}\ r_{t_{12}}(t_1.z)}\ w_{t_{11}}(t_1.x)\ c_{t_{11}}\ \mathbf{r_{t_{12}}(t_1.y)\ r_{t_{12}}(t_1.x)\ w_{t_{12}}(t_1.y)}\ c_{t_{12}}$

$w_{t_1^2}(t_1.x)\ c_{t_1^2}\rangle$

$\Rightarrow \widehat{\mathcal{H}^\sigma_{t_1}} = \langle t_1^1, t_{11}, t_{12}, t_1^2\rangle$

{ *Observation* :

$\mathbf{r_{t_{11}}(t_1.x)}$ due to $r_{t_{11}}(t_\psi.x)$ (Rule 3);

$\mathbf{r_{t_1^1}(t_1.y)\ c_{t_1^1}}$ due to $r_{t_1}(t_\psi.y)$ (Rule 1);

$\mathbf{r_{t_{12}}(t_1.z)}$ due to $r_{t_{122}}(t_\psi.z)$ (Rule 3);

$\mathbf{r_{t_{12}}(t_1.y)}$ due to $r_{t_{121}}(t_1.y)$ (Rule 2);

$\mathbf{r_{t_{12}}(t_1.x)}$ due to $r_{t_{122}}(t_1.x)$ (Rule 2);

$\mathbf{w_{t_1^2}(t_1.x)\ c_{t_1^2}}$ due to $w_{t_1}(t_1.x)$ (Rule 1) }

$\widehat{\mathcal{H}_{t_\psi}} = \langle w_{t_2}(t_\psi.x)\ c_{t_2}\ \mathbf{r_{t_{11}}(t_\psi.x)\ r_{t_1}(t_\psi.y)\ r_{t_{122}}(t_\psi.z)}\ w_{t_1}(t_\psi.x)\ w_{t_1}(t_\psi.y)\ c_{t_1}\rangle$

After mapping:

$\widehat{\mathcal{H}_{t_\psi}} = \langle w_{t_2}(t_\psi.x)\ c_{t_2}\ \mathbf{r_{t_1}(t_\psi.x)\ r_{t_1}(t_\psi.y)\ r_{t_1}(t_\psi.z)}\ w_{t_1}(t_\psi.x)\ w_{t_1}(t_\psi.y)\ c_{t_1}\rangle$

$\Rightarrow \widehat{\mathcal{H}^\sigma_{t_\psi}} = \langle t_2, t_1\rangle$

{*Observation* :

$\mathbf{r_{t_1}(t_\psi.x)}$ due to $r_{t_{11}}(t_\psi.x)$ (Rule 2);

$\mathbf{r_{t_1}(t_\psi.z)}$ due to $r_{t_{122}}(t_\psi.z)$ (Rule 2) }

Now that we have obtained the serial order for transactions at all the levels, the serializability of the transactions at the nested levels can be represented in the following manner. The ordering of the children, t_1 and t_2, of a transaction t can be shown by enclosing them within the $\{...\}$ brackets immediately after t, e.g., $t\{t_1, t_2\}$. Thus, serializability of the entire execution can be shown as follows: $t_2, t_1\{t_1^1, t_{11}, t_{12}\{t_{121}, t_{12}^1, t_{122}, t_{12}^2\}\}$.

Note: The composition of level-wise histories at different levels is done in the bottom to top order. The steps of the children are composed to obtain the history at the parent level.

3.5 About correctness of nested transactions

3.5.1 Avoiding cyclic conflict between transactions across levels

Considering the level-wise history individually cannot guarantee the consistency of the overall state of the transaction tree. It is quite possible that, in the level-wise histories, a subtransaction t is linearizable at its parent level, t_p. However, when t's steps are taken as part of t_p at another ancestor's level, t renders t_p non-linearizable at the higher level due to a cyclic conflict.

Hence, to ensure the correctness, the STM protocols must guarantee that there is no cyclic conflict between transactions across different levels. The various cases are discussed as follows. In this section, we only consider the partial history relevant to the discussion. *Observe that this scenario does not apply to non-nested transactions,*

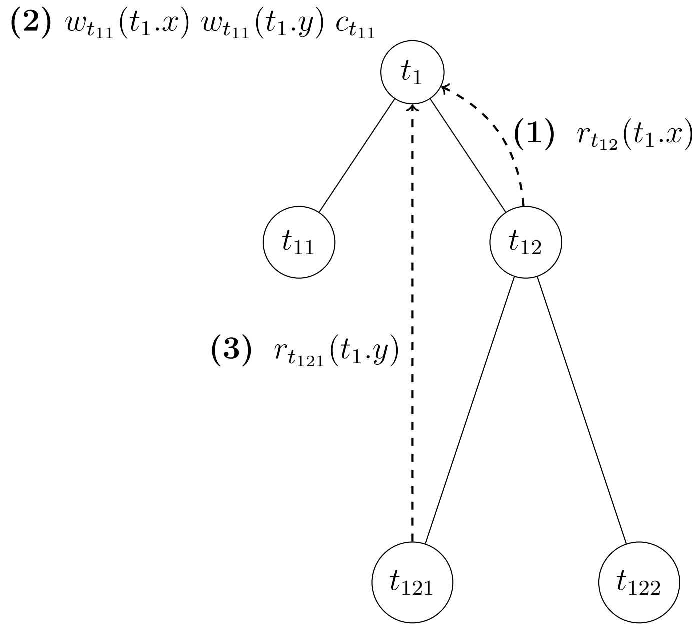


Figure 3.3: Reading a value inconsistent w.r.t. to an ancestor
(The order of events at different levels is indicated by the bold numbers in bracket.)

and is specific to nested transactions.

1. *Reading a value that is inconsistent w.r.t. an intermediate ancestor*

   Consider the partial history $\widehat{\mathcal{H}_4}$ depicted in Figure 3.3.

   $\widehat{\mathcal{H}_4} = \langle r_{t_{12}}(t_1.x), w_{t_{11}}(t_1.x), w_{t_{11}}(t_1.y), c_{t_{11}}, r_{t_{121}}(t_1.y)\rangle$

   Here t_{12} reads $t_1.x$. Then, t_{11} modifies $t_1.x$ and $t_1.y$. At this point, the value of $t_1.y$ becomes inconsistent for t_{12}. Next, t_{121} tries to read $t_1.y$. Observe here that t_{121} is a part (child) of t_{12}. If it commits, its steps would become part of its ancestor t_{12}'s steps. That means t_{121}'s step $r_{t_{121}}(t_1.y)$ would render its ancestor t_{12} non-linearizable at higher level, t_1, in the following manner.

   Imagine t_{121} commits and merges with t_{12} after $r_{t_{121}}(t_1.y)$. Now, $r_{t_{121}}(t_1.y)$ can be replaced by $r_{t_{12}}(t_1.y)$ (due to merging of steps of t_{121} with t_{12}). With this

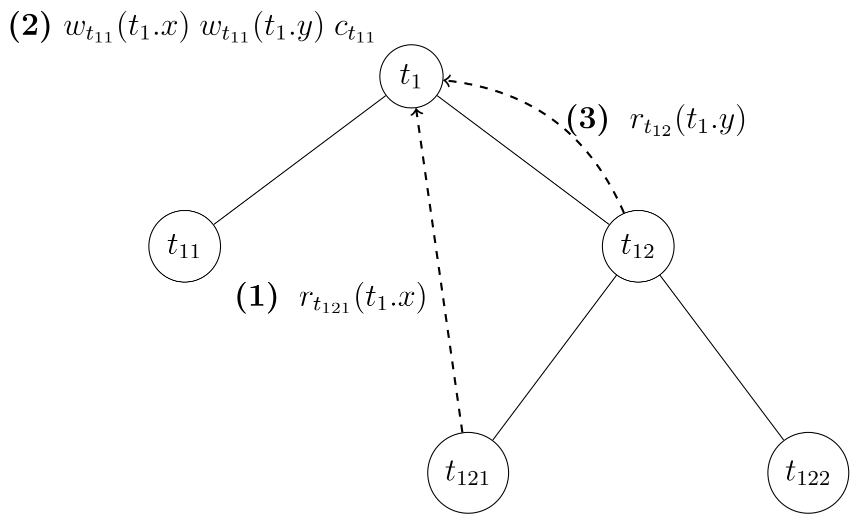


Figure 3.4: Cyclic conflict through transitivity across levels

transformation, history $\widehat{\mathcal{H}_4}$ maps to $\widehat{\mathcal{H}_{4'}} = \langle r_{t_{12}}(t_1.x), w_{t_{11}}(t_1.x), w_{t_{11}}(t_1.y), c_{t_{11}}, r_{t_{12}}(t_1.y)\rangle$. In $\widehat{\mathcal{H}_{4'}}$, observe that t_{12} is not linearizable with t_{11}. The value of $t_1.x$ read by t_{12} is modified later by t_{11}, requiring the serial order t_{12}, t_{11}. However, t_{12} read $t_1.y$ after t_{11} modified it. Thus, we get the serial order t_{11}, t_{12}. Hence, there is a cycle between t_{11} and t_{12}.

Remark: When a transaction t reads an object $t^{'}.x$ from an ancestor $t^{'}$, its value should be consistent w.r.t. not only t but also each of t's intermediate ancestors in the path from t to $t^{'}$.

2. *Reading inconsistent value through transitivity*

   Consider the following history depicted in Figure 3.4.

   $\widehat{\mathcal{H}_5} = \langle r_{t_{121}}(t_1.x)\ w_{t_{11}}(t_1.x)\ w_{t_{11}}(t_1.y)\ c_{t_{11}}\ r_{t_{12}}(t_1.y)\ ?\rangle$

   First, t_{121} reads from $t_1.x$. Later, t_{11} commits after modifying $t_1.x$ and $t_1.y$. Next, t_{12} reads $t_1.y$. At this point, considering t_{121} is a descendant (part) of t_{12}, there is cyclic conflict between t_{11} and t_{12}. The value of $t_1.x$ read by t_{121} is later

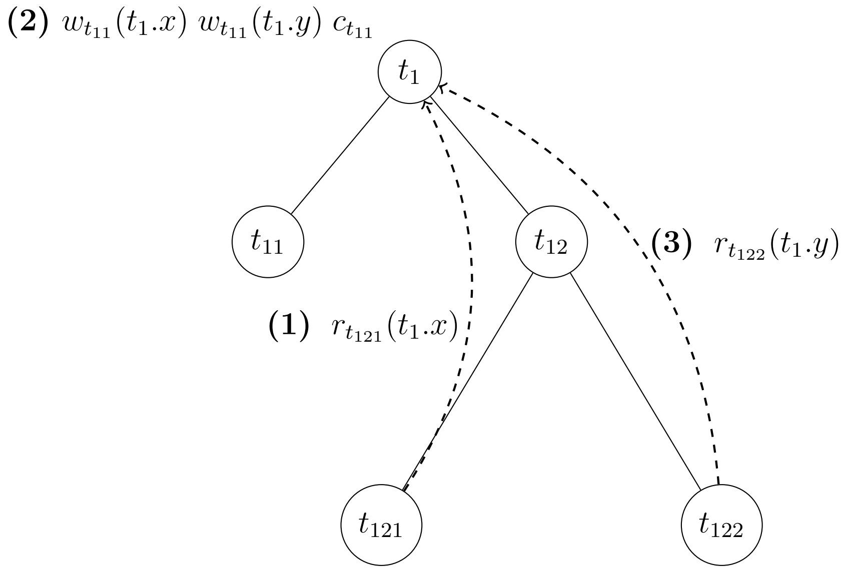


Figure 3.5: Incompatible transactions

modified by t_{11}, requiring the serial order t_{12}, t_{11}. Next, t_{12} read $t_1.y$ after t_{11} modified $t_1.y$, thus giving the serial order t_{11}, t_{12}. Hence, there is a cycle. Note that, t_{121} being a part (child) of t_{12}, if t_{121}'s read steps are merged with those of t_{12}, it brings t_{12} in cyclic conflict with t_{11} and renders it non-linearizable at t_1's level.

Remark: The STM protocol for nested transactions should ensure that no cyclic relationship occurs between transactions across different levels through transitivity.

3. *Incompatible transactions*

Consider the example depicted in Figure 3.5. The corresponding history is as follows.

$\widehat{\mathcal{H}_6} = \langle r_{t_{121}}(t_1.x), w_{t_{11}}(t_1.x), w_{t_{11}}(t_1.y), c_{t_{11}}, r_{t_{122}}(t_1.y)\rangle$

Here, t_{121} first reads from $t_1.x$. Later, t_{11} modifies $t_1.x$ and $t_1.y$. Next, t_{122}

reads $t_1.y$. Observe that t_{11} modified $t_1.x$ which was previously read by t_{121}, giving the order t_{121}, t_{11}. Next, t_{122} read $t_1.y$ after t_{11} modified it. This gives the serial order t_{11}, t_{122}. Now, note here if both t_{121} and t_{122} are allowed to commit and merge their read sets with that of t_{12}, then it brings t_{12} now in cyclic conflict with t_{11} and renders the history non-linearizable at t_1's level. Here, the two read operations, $r_{t_{121}}(t_1.x)$ and $r_{t_{122}}(t_1.y)$ are mutually incompatible at the higher level, and hence are called *incompatible read operations*, and the two transactions, t_{121} and t_{122}, are called *incompatible transactions*. (We shall revisit incompatible transactions and formally define them in Chapter 5.)

Incompatibility point

Let t and $t^{'}$ be two transactions such that $t^{'}$ is a descendant of t and $t^{'}$ is incompatible with t. Then, the incompatibility point of $t^{'}$ at t's level is defined as the earliest instant of time in τ_t at which $t^{'}$ becomes incompatible with t, and is denoted by $\tau_t^{i_{t'}}$.

Remark: The STM protocol should ensure that at any point of time, the read set of a transaction does not contain incompatible read operations. To this end, we observe the following two constraints: (i) a transaction is not allowed to perform a read operation from an ancestor that is not compatible with it, and (ii) two incompatible child transactions are not both allowed to merge (commit) with the parent.

$r_{t_{111}}(t_1.x)\ w_{t_{12}}(t_1.x)\ w_{t_{12}}(t_1.y)\ c_{t_{12}}\ r_{t_{112}}(t_1.y)$?

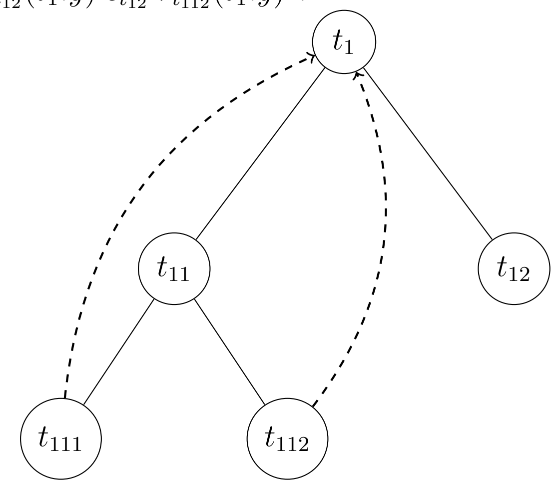


Figure 3.6: When to abort an incompatible transaction

3.5.2 When to abort an incompatible subtransaction

Consider the execution depicted in Figure 3.6 and observe that the read operations $r_{111}(t_1.x)$ and $r_{112}(t_1.y)$ are incompatible. Therefore, t_{111} and t_{112} are mutually incompatible. Incompatibility comes into picture here at the time of $r_{112}(t_1.y)$ (say τ^1). Note that the two incompatible transactions in picture here are not related by ancestor-descendant relation. Now the question here is whether we should abort one of the two incompatible transactions at time τ^1, given the case that t_{111} and t_{112} here are not related by ancestor-descendant relation at time τ^1. The answer is no.

At time τ^1, we cannot abort t_{112} because t_{111} is not part of its ancestor yet, and hence its step $r_{112}(t_1.y)$ is consistent w.r.t. its ancestors t_{11} and t_1. Similarly, we cannot abort t_{111} as t_{112} is not part of its ancestor yet. Thus, at this point, the reads of each of the subtransactions are consistent w.r.t. its ancestor. There is no point in forcefully aborting one of the subtransactions as we cannot guarantee that the other transaction will be able to commit eventually. The only requirement is that only one of the subtransactions should be allowed to commit. On commitment of either of them, the other should be aborted. Hence, deferring the determination of aborting an incompatible subtransaction until the commit phase offers the flexibility of allowing both subtransactions to continue their execution as long as they are compatible with their ancestor.

3.6 Consistency criterion: level-wise opacity

Several definitions of Opacity for non-nested transactions have been proposed [1, 9, 11]. The definition [9] that is close to the spirit of this thesis is given below.

Definition 3.1 (Opacity). *A history $\widehat{\mathcal{H}}$ is opaque if it satisfies the following properties:*

1. *The history $\Pi(\widehat{\mathcal{H}})$ is equivalent to a sequential history (where all non-concurrent transactions are ordered as in $\widehat{\mathcal{H}}$) that is legal.*

2. *All transactions that abort in complete($\widehat{\mathcal{H}}$) are invisible and their reads are consistent.*

The above definition is meant for non-nested transactions. It separates the history of committed transactions from that of the aborted ones : (i) history of committed transactions is equivalent to a sequential and legal history, and (ii) aborted transactions are *invisible* and have consistent reads. A sequential history is legal if every transaction reads an object's value that corresponds to the last transaction that updated the value of that object. An *invisible* transaction is the one that does not update the value of any base object [**?**]. As such, the results of an aborted transaction are invisible to other transactions.

Now, we extend Definition 3.1 to *level-wise opacity* for nested transactions as follows:

Definition 3.2 (**Level-wise opacity**). *A level-wise history* $\widehat{\mathcal{H}_t}$ *is opaque if it satisfies the following properties:*

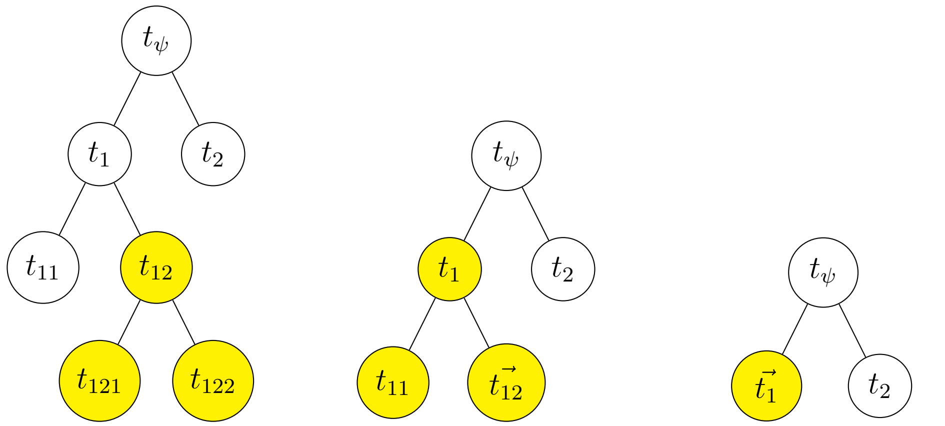
1. *The history* $\Pi(\widehat{\mathcal{H}_t})$ *is equivalent to a sequential history (where all non-concurrent transactions are ordered as in* $\widehat{\mathcal{H}_t}$*) that is legal.*

2. *For all* $t^{'} \in aborted(\mathcal{H}_t^{'})$*, (i) until the time just before the abort, the execution of* $t^{'}$ *was consistent, and (ii) after the abort,* $t^{'}$ *and its descendants are invisible for* $outsideTrans(t^{'})$*.*

The proofs for correctness of the protocols presented in this thesis are based on Definition 3.2.

3.7 Outline of the proof technique

Owing to the hierarchical structure of nested transactions, the correctness of the overall execution can be shown by considering the execution at each level of the super tree. We shall discuss the various aspects of proof system in the following sections.



(a) Considering $\widehat{\mathcal{H}_{t_{12}}}$

(b) Considering $\widehat{\mathcal{H}_{t_1}}$

(c) Considering $\widehat{\mathcal{H}_{t_\psi}}$

Figure 3.7: Bottom to top approach constructing histories and composing steps of subtransactions ($\vec{t_{12}} \Rightarrow t_{12}\{t_{121}, t_{122}\}$; $\vec{t_1} \Rightarrow t_1\{t_{11}, \vec{t_{12}}\}$)

3.7.1 Bottom up approach for constructing level-wise histories

In the nesting of transactions, the linearization point of a child transaction lies within the life span of its parent. Therefore, we show the linearizability of transactions level by level. We construct the histories in a bottom up manner using the mapping function, and in the process determine the linearizability of the transactions at each level. We illustrate the level-wise linearizability of transactions using Figure 3.7. First, we consider the history $\widehat{\mathcal{H}_{t_{12}}}$ at node t_{12} and determine the linearizability of its children t_{121} and t_{122}. Next, we consider $\widehat{\mathcal{H}_{t_1}}$, and determine the linearization points for t_{11} and t_{12}. Note that, in this case, while determining the linearization point for t_{12}, we consider the combined steps of t_{12} and its children t_{121} and t_{122}. Finally, we look at the global history through $\widehat{\mathcal{H}_{t_\psi}}$ and determine the linearization points for the root level transactions t_1 and t_2.

This way, while working through the level-wise histories, we consider the steps of

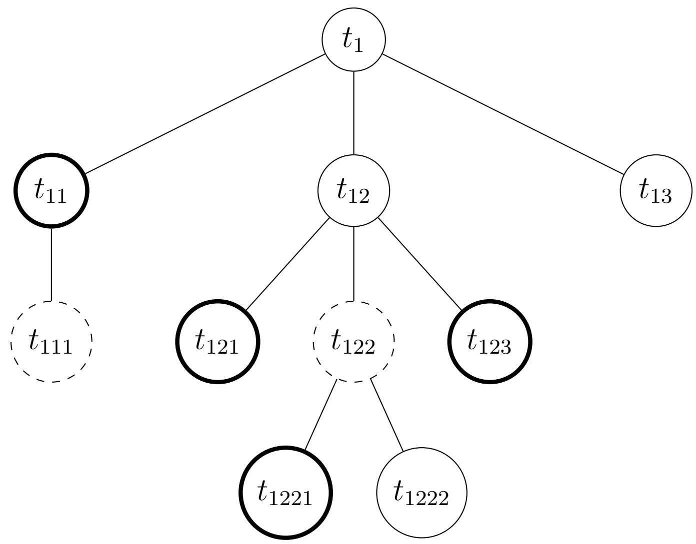


Figure 3.8: Transaction tree (thick circle:committed; thin circle:active; dotted circle:aborted)

a transaction t (that may have accessed objects from different ancestors) at different levels and show its consistency (level-wise opacity) at each level.

To show correctness, we separate the history of committed transactions from that of the aborted ones. First, we consider the history restricted to committed transactions, $\Pi(\mathcal{H}_t)$, at each level, and show the level-wise linearizability of the committed transactions at each level.

3.7.2 Level-wise history of committed transactions

The level-wise history of committed transactions at node t is denoted by $\Pi(\widehat{\mathcal{H}_t})$. To obtain $\Pi(\widehat{\mathcal{H}_t})$, consider the transaction subtree rooted at node t (i.e., $transTree(t)$). In $transTree(t)$, excepting the case of t, we prune all the aborted subtransactions, and their descendants. The resulting tree is denoted by $prunedTree(t)$. Note that t is included in $prunedTree(t)$, even if t itself is an aborted or an active transaction. For the sake of the example, treat the active nodes shown in Figure 3.8 as committed ones. Now, $prunedTree(t_1)$ comprises of nodes $t_1, t_{11}, t_{12}, t_{13}, t_{121}$ and t_{123}; $prunedTree(t_{12})$ contains t_{12}, t_{121} and t_{123}. Similarly, $prunedTree(t_{122})$ contains t_{122}, t_{1221} and t_{1222}. Note that transactions t_{111} and t_{122} (including subtree rooted at t_{122}) are not considered in $prunedTree(t_1)$ and $prunedTree(t_{12})$ as they are aborted. Note that t_{122} is not considered in $prunedTree(t_{12})$ and hence not considered in $prunedTree(t_1)$ as well.

Now, $\Pi(\widehat{\mathcal{H}_t})$ is defined to contain the steps of only the transactions in $prunedTree(t)$ on the objects of t and t's ancestors. In other words, $\Pi(\widehat{\mathcal{H}_t})$ contains only the steps of t and its committed children (after applying the mapping function). For example, $\Pi(\widehat{\mathcal{H}_{t_1}})$ contains the operations of $t_1, t_{11}, t_{12}, t_{13}, t_{121}$ and t_{123} on objects of t_1 and t_ψ, whereas $\Pi(\widehat{\mathcal{H}_{t_{12}}})$ contains the operations of t_{12}, t_{121} and t_{123} on objects of t_{12}, t_1 and t_ψ.

3.7.3 Reduction of a non-committed transaction

Let t be a non-committed (active or aborted) child of a transaction t_p in the transaction tree. Recall that a subtransaction t_c can update the values of the objects of its ancestors only upon its commit. Thus, transaction t could have only read the objects of its parent t_p or other higher level ancestors. With this end in view, given a $t \in aborted(\mathcal{H}'_{t_p})$ or $t \in active(\mathcal{H}'_{t_p})$ (where t is a child of t_p), we construct $t^\gamma = \gamma(t)$ (γ stands for "reduced") as follows. Transaction t^γ is obtained by taking into account only the external read steps performed by t (and the committed transactions that are part of t) on the objects of its ancestors (prior to its abort, if it is an aborted transaction). Its local read and write steps are discarded (as they are local to t), and it is treated as commited. In case of an aborted transaction, the corresponding abort event is replaced by a commit event. This way, t^{γ} can be viewed as a committed read only transaction at its parent level.

3.7.4 Closure (history) for a transaction

In our model for nested transactions, if a local copy of an object is not available, then a subtransaction reads from the local space of its nearest ancestor having a copy of that object. Hence, considering t being a subtransaction in a transaction tree, the consistency of read step of t, at time τ, depends upon the state of its ancestors at that time. The state of an ancestor at time τ depends upon (a) its local (read/write) and external (read) operations, and (b) steps of its committed children, until time τ. Aborted or active transactions of t or its ancestors, at time τ, do not have any bearing on the consistency of t.

Given a history $\mathcal{H}$ for a transaction tree, the closure of history (or simply *closure*) for a transaction t is denoted as $\mathcal{H}^{\mathcal{C}_t}$, and obtained in following three steps :

(i) $\mathcal{H}'$: Consider the prefix of $\mathcal{H}$ up to the last read/write operation of t.

(ii) $\mathcal{H}''$: Discard the steps of (a) aborted transactions and (b) active transactions (other than t or its ancestors) in $\mathcal{H}'$.

(iii) $\mathcal{H}^{\mathcal{C}_t}$: For the active transactions (t as well as its active ancestors), append commit events in $\mathcal{H}''$, committing each child before its parent. These transactions are thereby treated as read only committed children at their parent level.

(Note that the local writes, if any, of t and its ancestors are not mapped to corresponding commit writes at their parent level.)

Observe that a closure represents a history of committed transactions. For illustration, consider $\mathcal{H}_6$ corresponding to the execution of transaction tree in Figure 3.8.

$\widehat{\mathcal{H}_6} = \langle w_{t_{11}}(t_{11}.x)\ r_{t_{111}}(t_{11}.x)\ a_{t_{111}}\ w_{t_{11}}(t_1.x)\ c_{t_{11}}\ w_{t_1}(t_1.y)\ r_{t_{13}}(t_1.y) r_{t_{121}}(t_1.x)\ c_{t_{121}}$
$r_{t_{1221}}(t_{12}.x)\ c_{t_{1221}}\ w_{t_{122}}(t_{122}.z)\ r_{t_{1222}}(t_{122}.z)\ \mathbf{r_{t_{122}}(t_{122}.x)} \bigstar\ r_{t_{123}}(t_{12}.x)\ a_{t_{122}}\ c_{t_{123}}\rangle$

Take the case of aborted transaction t_{122}. In $\widehat{\mathcal{H}_6}$, the last (read) operation of t_{122} is $r_{t_{122}}(t_{122}.x)$. Hence, we cut $\widehat{\mathcal{H}_6}$ right after $r_{t_{122}}(t_{122}.x)$. (The cutting point is marked by $\bigstar$.) Thus, $\mathcal{H}^{\mathcal{C}_{t_{122}}}$ is as follows:

Step (i) : Cut the history $\widehat{\mathcal{H}_6}$ at $\bigstar$.

$\widehat{\mathcal{H}'_6} = \langle w_{t_{11}}(t_{11}.x)\ r_{t_{111}}(t_{11}.x)\ a_{t_{111}}\ w_{t_{11}}(t_1.x)\ c_{t_{11}}\ w_{t_1}(t_1.y)\ r_{t_{13}}(t_1.y)\ r_{t_{121}}(t_1.x)\ c_{t_{121}}$
$r_{t_{1221}}(t_{12}.x)\ c_{t_{1221}}\ w_{t_{122}}(t_{122}.z)\ r_{t_{1222}}(t_{122}.z)\ r_{t_{122}}(t_{122}.x)\ \rangle$

Step (ii) : Discard the steps of $t_{111}, t_{13}, t_{123}, t_{1222}$ as they are non-committed in $\widehat{\mathcal{H}'_6}$ and do not belong to $\{t_1, t_{12}, t_{122}\}$. (Removed part is underlined.)

$\widehat{\mathcal{H}'_6} = \langle w_{t_{11}}(t_{11}.x)\ \underline{r_{t_{111}}(t_{11}.x)\ a_{t_{111}}}\ w_{t_{11}}(t_1.x)\ c_{t_{11}}\ w_{t_1}(t_1.y)\ \underline{r_{t_{13}}(t_1.y)}\ r_{t_{121}}(t_1.x)\ c_{t_{121}}$
$r_{t_{1221}}(t_{12}.x)\ c_{t_{1221}}\ w_{t_{122}}(t_{122}.z)\ \underline{r_{t_{1222}}(t_{122}.z)}\ r_{t_{122}}(t_{122}.x)\ \rangle$

$\Rightarrow \langle w_{t_{11}}(t_{11}.x)\ w_{t_{11}}(t_1.x)\ c_{t_{11}}\ w_{t_1}(t_1.y)\ r_{t_{121}}(t_1.x)\ c_{t_{121}}\ r_{t_{1221}}(t_{12}.x)\ c_{t_{1221}}$
$w_{t_{122}}(t_{122}.z)\ r_{t_{122}}(t_{122}.x)\ \rangle$

Step (iii) : Complete non-committed transactions t_1, t_{12} and t_{122} in $\widehat{\mathcal{H}''_6}$. (Added part is underlined.)

$\widehat{\mathcal{H}_6^{\mathcal{C}_{t_{122}}}} = \langle w_{t_{11}}(t_{11}.x)\ w_{t_{11}}(t_1.x)\ c_{t_{11}}\ w_{t_1}(t_1.y)\ r_{t_{121}}(t_1.x)\ c_{t_{121}}\ r_{t_{1221}}(t_{12}.x)\ c_{t_{1221}}$

$w_{t_{122}}(t_{122}.z)\ r_{t_{122}}(t_{122}.x)\ \underline{c_{t_{122}}\ c_{t_{12}}\ c_{t_1}}\rangle$

Here, $\widehat{\mathcal{H}_6^{\mathcal{C}_{t_{122}}}}$ is history of committed transactions, comprising of the steps of $t_1, t_{11}, t_{12}, t_{121}, t_{122}$ and t_{1221}. Now, following the discussion in Section 3.4.4, the level-wise sequential history is given by $\{t_1\{t_{11}, t_{12}\{t_{121}, t_{122}\{t_{1221}\}\}\}\}$.

3.7.5 Handling aborted and active transactions

An aborted transaction is totally discarded, i.e., its read steps as well as write steps are ignored at its parent's and other ancestors' levels. However, to show the correctness of aborted transactions, we consider one aborted transaction at a time in a transaction tree. Let t_a be such an aborted transaction. Then, we obtain the closure for t_a ($\widehat{\mathcal{H}^{\mathcal{C}_{t_a}}}$) by looking at the execution of transaction tree until the time just before the abort of t_a.

Note that an active or aborted transaction only performs read operations on its ancestors' objects. Updating of the parent's objects occurs only upon its commit, provided it is an update transaction. Therefore, it is fair to treat an aborted (or active) transaction as a read only committed child at its parent's level and show that each of its (external) read steps were consistent. By construction (step (iii)) of $\widehat{\mathcal{H}^{\mathcal{C}_{t_a}}}$, t_a and its ancestors are reduced to read only committed children at their parent's level. Further, $\widehat{\mathcal{H}^{\mathcal{C}_{t_a}}}$ represents a history a committed transactions. With this end in view, the correctness of an aborted transaction follows directly from the proofs for committed transactions. We construct the level-wise history of committed transactions using $\widehat{\mathcal{H}^{\mathcal{C}_{t_a}}}$, and show the correctness in the same way as done for committed transactions.

Observe that the closure can also be used for showing the correctness of any (active) transaction at any point of its execution.

3.7.6 Summary of the proof technique

The set of proofs, based on Definition 3.2, can be divided into following three parts.

1. For committed transactions, $\Pi(\widehat{\mathcal{H}_t})$ is equivalent to a legal sequential history $\widehat{\mathcal{H}_t^\sigma}$ which is obtained by ordering the transactions in $\mathcal{H}_t^{'}$ using the definition of linearization points based on the STM protocol (discussed for the protocols in later chapters):

    (a) $\rightarrow_{\mathcal{H}_t^\sigma}$ is total order.

    (b) $\rightarrow_{\mathcal{H}_t^{'}} \subseteq \rightarrow_{\mathcal{H}_t^\sigma}$

    (c) $t_w \xrightarrow{t.x}_{rf} t_r \Rightarrow \nexists t_w^{'}$ such that $(t_w \rightarrow_{\mathcal{H}_t^\sigma} t_w^{'} \rightarrow_{\mathcal{H}^\sigma} t_r) \wedge (w_{t_w^{'}}(t.x) \in \mathcal{H}_t)$.

    (d) $t_w \xrightarrow{t.x}_{rf} t_r \Rightarrow t_w \rightarrow_{\mathcal{H}_t^\sigma} t_r$.

    Here, t_r, t_w and $t_{w'}$ are the children of node t.

2. There is no cyclic conflict between transactions across different levels:

    (a) If t_1, t_2 are incompatible children of t, then $\neg(t_1 \in \Pi(\mathcal{H}_t^{'}) \wedge t_2 \in \Pi(\mathcal{H}_t^{'})))$.

    (b) A subtransaction cannot operate after it becomes incompatible with any of its ancestors.

3. For an aborted transaction t_a:

    (a) After the abort of t_a, the results of t_a and its descendants are not visible to transactions in $outsideTrans(t_a)$.

    (b) Steps of transaction $\gamma(t_a)$ at its ancestral levels are consistent.

Chapter 4

SimpSTM: A simple STM protocol for (closed) nested transactions

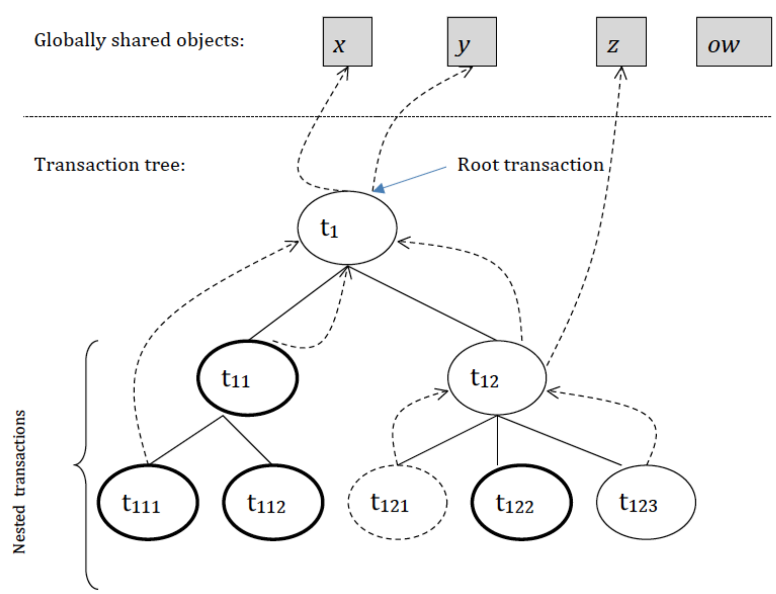


Figure 4.1: Closed nested transactions (dark circle: committed; thin circle: active; dotted circle:aborted)

The STM protocol discussed in Section 2.6.2 was designed for non-nested transactions. In this section, we extend that protocol and design a protocol (called "SimpSTM") for (closed) nested transactions. SimpSTM is designed under the constraint that the subtransactions of the nested transactions are executed in a sequential fashion. More precisely, this constraint is defined as follows:

Constraint 4.1 (Sequential execution of subtransactions). *Given a transaction tree, let t_p be any node with t_1 and t_2 as any two of its children. Then, we have either $E_{t_1} <_{\mathcal{H}_{t_p}} B_{t_2}$ or $E_{t_2} <_{\mathcal{H}_{t_p}} B_{t_1}$, denoting that a new child is invoked only after the previously created (if any) child has completed. Further, t_p does not execute any step while it has an active child.*

Technically, the steps of all the transactions in a transaction tree are executed by the same thread. When t_p invokes a child t_c, the thread previously executing the steps of t_p, executes the steps of t_c (and its descendants). Transaction t_p stays idle (waits) until t_c completes (commits or aborts) and the thread (control) is returned to t_p.

Besides the features listed in Section 2.4, the key features of SimpSTM are as follows.

- At a time, only one node in a transaction tree executes its steps. This is due to the constraint that the subtransactions in a transaction tree are executed in a sequential fashion.
- When t reads from the globally shared object $t_\psi.x$, the read value is consistent w.r.t. the entire transaction tree.

4.1 SimpSTM

4.1.1 Pseudocode

Protocol 4.1: SimpSTM

$t_\psi.ow$, a set of ids $\in T$

State of globally shared object $t_\psi.x$:
  $val \in V$
  rs and fbd, sets of ids $\in T$

Common to root and non-root nodes :

State of a transaction's local object $t.x$:
  $val \in V$

State of transaction t:
  $parent \in T$, parent's id (initially $\perp$)
  mts, set of ids $\in T$ (initially $\emptyset$)
  lws, lrs, sets of ids $\in X$ (initially $\emptyset$)

Operation $\mathbf{begin_t(t_p)}$:
$t.parent \leftarrow t_p$;
$t.mts \leftarrow \{t\}$;

Operation $\mathbf{invoke\_child_t(t_c)}$:
$begin_{t_c}(t)$;

Operation $\mathbf{read_t(x)}$:
if ($t.x$ exists) **then** return $t.x.val$;
$v = search\_parent_{t_p}(x, t, t.mts)$;
if ($v = null$) **then** $t.abort()$; **end if**
$t.x.val \leftarrow v$;
$t.lrs \leftarrow t.lrs \cup \{x\}$;
return v;

Operation $\mathbf{write_t(x, v)}$:
$t.x.val \leftarrow v$;
$t.lws \leftarrow t.lws \cup \{x\}$;

Operation $\mathbf{merge_t(t_c)}$:
for each $x \in (t_c.lws \cup t_c.lrs)$ **do**
  $t.x.val \leftarrow t_c.x.val$; **end for**
$t.lws \leftarrow t.lws \cup t_c.lws$;
$t.lrs \leftarrow t.lrs \cup t_c.lrs$;
$t.mts \leftarrow t.mts \cup t_c.mts$;

Operation $\mathbf{abort_t()}$:
return $(abort)$;

Operation $\mathbf{search\_parent_t(x, t_d, s_{mts})}$:
if ($t.x$ exists) **then** return $t.x.val$; **end if**
$s \leftarrow t.mts \cup s_{mts}$;
return $t_p.search\_parent(x, t_d, s)$;

For reading global objects (at t_ψ's level) :
* Invoked by root transaction only

Operation $\mathbf{search\_parent_{t_\psi}(x, t_d, s_{mts})}$:
lock $t_\psi.x$;
if($t_\psi.x.fbd \cap (s_{mts}) \neq \emptyset$) **then**
  unlock $t_\psi.x$; return $null$; **end if**
$v \leftarrow t_\psi.x.val$;
$t_\psi.x.rs \leftarrow t_\psi.x.rs \cup \{t_d\}$;
unlock $t_\psi.x$;
return v;

Specific to non-root node (t) :

Operation $\mathbf{try\_to\_commit_t()}$:
$s_{lrs} \leftarrow t.lrs \cup t_p.lrs$; $s_{lws} \leftarrow t.lws \cup t_p.lws$;
if ($s_{lws} \neq \emptyset \wedge s_{lrs} \neq \emptyset$) **then**
  if ($t.mts \cap t_\psi.ow \neq \emptyset$) **then**
  $t.abort()$; **end if end if**
$merge_{t_p}(t)$;
return $(commit)$;

Specific to root node (t_ρ) :

Operation $\mathbf{try\_to\_commit_{t_\rho}()}$:
if ($t_\rho.lws = \emptyset$) **then**
  return $(commit)$; **end if**
lock all the objects in $t_\rho.lws \cup t_\rho.lrs$;
if ($t_\rho.mts \cap t_\psi.ow \neq \emptyset$) **then**
  release all the locks;
  $t_\rho.abort()$; **end if**
$t_\psi.ow \leftarrow t_\psi.ow \cup (\bigcup_{x \in t_\rho.lws} t_\psi.x.rs)$;
for each $x \in t_\rho.lws$ **do**
  $t_\psi.x.val \leftarrow t_\rho.x.val$;
  $t_\psi.x.fbd \leftarrow t_\psi.ow$;
  $t_\psi.x.rs \leftarrow \emptyset$; **end for**
release all the locks;
return $(commit)$;

4.1.2 Data structures

4.1.2.1 Variable state

At the global level (t_ψ):

We use the same data structures ($t_\psi.x$ and $t_\psi.ow$) and the associated semantics for globally shared objects, as used in [9] (discussed in Section 2.6.2). There is a lock associated with each globally shared object $t_\psi.x$. The value of a base object $t_\psi.x$ is given by $t_\psi.x.val$. The set $t_\psi.ow$ is kept in an atomic register, and is not protected by a lock. Here, we assume an atomic register that can perform both reading and writing together atomically, as in *read-modify-write*. Various techniques for implementing such an atomic register have been discussed in the Section 3.4 of [9]. However, these techniques have not been discussed in this thesis to keep it within the scope. Note that only such atomic objects have been assumed throughout this thesis.

At the local level (t):

In the local space of a transaction t, a local copy of an object $t.x$ has only a value field (denoted as $t.x.val$). It does not have the pair of sets, rs and fbd, associated with the globally shared objects.

4.1.2.2 Transaction state

Each transaction t ($t \neq t_\psi$) stores the id of its parent in $parent$. The set mts (*merged transaction set*) contains the id(s) of t as well as t's descendants that have successfully merged with t. Thus, mts is used to take into account the fact that a transaction can be composed of one or more committed descendants, and therefore, the combined steps of the transactions in mts should be considered while checking the consistency of t. Further, the set lrs (*local read set*) is used to record the ids of objects read by t, whereas lws (*local write set*) is used for recording the ids of objects written by t.

4.1.3 Working of SimpSTM

In SimpSTM, the allocation of space for local copies of objects in the local space of a transaction is automatically done whenever required. The procedures of the protocol are discussed as follows.

$begin_t(t_p)$: Each transaction begins with this procedure. Here, t_p is the id of the parent transaction that invoked t. If t is a root level transaction, then t_p is t_ψ. The set $t.mts$ is initialized with $\{t\}$.

$invoke\_child_t(t_c)$: This method is used by the parent t to invoke a new child transaction t_c. Note that t invokes a new child only when it does not already have a child that is currently active.

$read_t(x)$: When t needs to read an object x, it checks its local space. If a local copy exists, then it reads from $t.x$. Otherwise, it tries to read from the local space of its parent, t_p. If t_p also does not have a local copy of $t_p.x$, then t_p, in turn, requests its own parent, and so on. This is done by calling *search_parent*, a recursive procedure, that searches from the parent level to higher level ancestors, until a local copy of x is found. In the worst case, the search leads to reading from the globally shared copy $t_\psi.x$. If the read operation is successful, then t assigns the read value to its local copy $t.x$ and adds x to $t.lrs$. In case *search_parent* returns *null*, t aborts.

$search\_parent_t(x, t_d, s_{mts})$: This method associated with t is invoked by its children to search for a local copy of an object x for the descendant t_d. Set s_{mts} is the union of $t^{'}.mts$ of each intermediate ancestor $t^{'}$ in the path from t_d to t.

Case I: Reading from non-global level ($t \neq t_\psi$)

Transaction t returns the value of its local copy x if $t.x$ exists. Otherwise, it merges its $t.mts$ with s_{mts} and invokes the *search_parent* method of its parent.

Case II: Reading from global level ($t = t_\psi$)

Here, this method is invoked by the root transaction. In this case, the globally shared object $t_\psi.x$ is locked. If none of the transactions in s_{mts} belongs to $t_\psi.x.fbd$ then, t_d is added to $t_\psi.x.rs$ and the value of $t_\psi.x$ is returned. Otherwise, *null* is returned to indicate an attempt to read an inconsistent value.

Note: In order to focus on the key concept and facilitating readability of the pseudocode, the method *search_parent* has been implemented everywhere using a recursive approach (memory overhead) instead of the iterative approach.

$write_t(x, v)$: All the writes take place in local space initially. Here, t updates the value of its local object $t.x$ to v, and then adds x to $t.lws$.

$merge_t(t_c)$: A child transaction t_c calls this method to merge its local results with its parent t. The subtransaction t_c may have read some object, from an ancestor, whose local copy is not available with its parent. A copy of each such object is created in its parent's local space. Next, for each object x in $t_c.lws$, the value of $t.x.val$ is set to $t_c.x.val$. Subsequently, the sets $t_c.lrs, t_c.lws, t_c.mts$ are merged with the corresponding sets of the parent t.

try_to_commit$_t()$: The nature of a commit process of a transaction t depends upon its type, i.e., whether t is a non-root transaction or a root transaction.

Case I: t is a non-root level transaction (t) Here we need to take into account if the merging of child transaction t can turn a read-only or a write-only parent transaction

t_p into an update transaction, and possibly make t_p inconsistent if t read some global object that has been modified. Therefore, we consider the joint local read sets and local write sets of t and t_p to check if the resultant parent is bound to be an update transaction. In that case, we first validate t's steps by ensuring that none of the ids in $t.mts$ belong to $t_\psi.ow$. Upon successful validation, t merges its local results with its parent, and commits. If the validation is not successful, then t aborts. No merging of steps takes place in the event of an abort. In case t_p is found to be a read-only or a write-only transaction, then it commits (and merges its steps with those of its parent) without having to validate its steps.

Case II: t is a root-level transaction (t_ρ)

Here, if t_ρ is a read only transaction (i.e. $t_\rho.lws = \emptyset$), it commits immediately. Otherwise, it obtains locks on all global objects whose ids are present in its local read and write sets. Next, it checks if any transaction belonging to $t_\rho.mts$ is present in $t_\psi.ow$. If yes, then the transaction releases all the locks and aborts. Otherwise, for each x present in $t_\rho.lws$, it updates $t_\psi.x.val$ using the value of its local copy $t_\rho.x$. All the ids present in $t_\psi.x.rs$ of each $t_\psi.x$ updated by t_ρ are added to $t_\psi.ow$. Next, for each $x \in t_\rho.lws$, $t_\psi.x.fbd$ is updated using $t_\psi.ow$, followed by clearing $t_\psi.x.rs$. Finally, all the locks are released and t_ρ commits.

Observe that, compared to Imbs and Raynal's Protocol 2.1, the root transaction in SimpSTM behaves differently by making use of set mts. If there are no nested sub-transactions, mts will contain the id of only the root transaction, thereby SimpSTM will work like Protocol 2.1.

$call\_abort_t()$: This method is invoked when a transaction t has to abort.

4.2 Proof of correctness

We prove that the properties (1)-(3) stated in Section 3.7.6 are satisfied. First, we consider the level-wise history of committed transactions and show that the properties (1) and (2) are satisfied. Next, we consider the aborted transactions, and show that property (3) is satisfied.

Composite transaction:

Note that for a composite transaction t_1, its consistency depends also upon the consistency of steps of its descendants that have merged with it. In other words, the steps of all the subtransactions that have successfully merged with t_1 are also represented in t_1 now. For this purpose, we shall use the notation $\widehat{t_1}$ to denote "some transaction in $t_1.mts$." Observe that $t_1.mts$ always contains t_1.

Let $\beta(\widehat{t_1}, t_2.s, \tau)$ be the predicate denoting "at time τ of an event/operation, $\widehat{t_1}$ belongs to a set s of transaction t_2". In that case, at time τ, we have $t_1.mts \cap t_2.s \neq \emptyset$.

4.2.1 Definition of linearization point

Extending the proof of Protocol 2.1 by Imbs & Raynal [9] for nested transaction, the linearization point ℓ_t of a transaction t in a transaction tree is defined within the lifespan of its parent, t_p. Depending on whether t is a root level transaction or a non-root level transaction, its linearization point is defined as follows.

Case I: t is a non-root transaction (i.e., $t \neq t_\rho$)

1. If t commits, then ℓ_t is the point at which t merges with its parent (line 51).

2. If t aborts, then ℓ_t is the point at which it performed its last successful read operation on its ancestor's object (at the time of invocation line 18).

Case II: t is a root level transaction (i.e., $t = t_\rho$)

3. If a transaction t aborts, ℓ_t is placed just before $\widehat{t}$ is added to the set $t_\psi.ow$ (line 60 of the $try\_to\_commit_t()$ operation that entails its abort).

4. If t is a committed read only transaction, ℓ_t is placed at the earliest of (1) the occurrence time of the test during its last read operation (line 40 of the *search_parent* operation) and (2) the time just before $\widehat{t}$ (any id in $t.mts$) is added to $t_\psi.ow$ (if it ever is).

5. If an update transaction t commits, ℓ_t is placed just after the execution of line 60 by t (update of $t_\psi.ow$).

Note: For any aborted transaction, if it does not have any external read operation at its level then its linearization point lies at the time of its creation (line 11).

The above definition of linearization points is used to obtain the level-wise sequential history $\widehat{\mathcal{H}_t^\sigma}$ by ordering t's children according to their linearization points. Next, we shall provide the set of proofs for properties stated in Section 3.7.6.

4.2.2 Proof for committed transactions

All the histories considered in this section are the histories restricted to committed transactions only (i.e. $\Pi(\widehat{\mathcal{H}_t}), \Pi(\widehat{\mathcal{H}'_t})$ and $\Pi(\widehat{\mathcal{H}_t^\sigma})$). For committed transactions, we show the following.

(a) $\rightarrow_{\mathcal{H}_t^\sigma}$ is total order.

(b) $\rightarrow_{\mathcal{H}'_t} \subseteq \rightarrow_{\mathcal{H}_t^\sigma}$.

(c) $t_w \xrightarrow{t.x}_{rf} t_r \Rightarrow \nexists t_w^{'}$ such that $(t_w \rightarrow_{\mathcal{H}_t^{\sigma}} t_w^{'} \rightarrow_{\mathcal{H}^{\sigma}} t_r) \wedge (w_{t_w^{'}}(t.x) \in \mathcal{H}_t)$.

(d) $t_w \xrightarrow{t.x}_{rf} t_r \Rightarrow t_w \rightarrow_{\mathcal{H}_t^{\sigma}} t_r$.

Let us recall the notations. Here, $\widehat{\mathcal{H}_t}$ denotes a level-wise shared memory history (of events) at node t in the transaction tree. Similarly, $\widehat{\mathcal{H}_t^{'}}$ denotes a level-wise transaction history (that follows partial order), whereas $\widehat{\mathcal{H}_t^{\sigma}}$ denotes a level-wise sequential transaction history (that follows total order). Further, $t_w \xrightarrow{t.x}_{rf} t_r$ denotes that t_w and t_r are the two children of t such that t_r reads from $t.x$ (t's object) the value that was written by t_w.

Lemma 4.1. $\rightarrow_{\mathcal{H}_t^{\sigma}}$ *is total order.*

Proof. Trivial from the ordering of linearization points for transactions, and Constraint 4.1. □

Lemma 4.2. $\rightarrow_{\mathcal{H}_t^{'}} \subseteq \rightarrow_{\mathcal{H}_t^{\sigma}}$.

Proof. This lemma follows from the fact that, given any transaction $t_1 \in \mathcal{H}_t^{'}$, its linearization point is placed within its lifetime. Therefore, if $t_1 \rightarrow_{\mathcal{H}_t^{'}} t_2$ (t_1 ends before t_2 begins), then $t_1 \rightarrow_{\mathcal{H}_t^{\sigma}} t_2$. □

Lemma 4.3. *If* $t_1 \in \Pi(\mathcal{H}_{\psi})$ *then* $\beta(\widehat{t_1}, t_{\psi}.ow, \tau) \Rightarrow \ell_{t_1} <_{\mathcal{H}_{\psi}} \tau$.

Proof. Note that $\mathcal{H}_{\psi}$ denotes the history produced at the global level (i.e., associated with globally shared copy of objects), and t_1 is a root level transaction. We have to show that the linearization point for t_1 cannot lie after the time τ at which $\widehat{t_1}$ has been added to $t_{\psi}.ow$. There are two cases:

- If t_1 is read-only and commits, again by construction, its linearization point ℓ_{t_1} is placed, at the latest, just before the time at which $\widehat{t_1}$ (first time a transaction in $t_1.mts$) is added to $t.ow$ (if it ever is), which proves the lemma.

- If t_1 writes and commits, its linearization point ℓ_{t_1} is placed during $try\_to\_commit()$, while t_1 holds the locks of every object of t_ψ that it has read. If $\widehat{t_1}$ was in $t_\psi.ow$ before it acquired all the locks, it would not commit (due to lines 56-59). Let us notice that $\widehat{t_1}$ can be added to $t_\psi.ow$ only by another root-level update transaction, holding a lock on the globally shared object previously read by $\widehat{t_1}$. As t_1 releases the locks just before committing (lines 65-66), it follows that ℓ_{t_1} occurs before the time at which $\widehat{t_1}$ is added to $t_\psi.ow$, which again proves the lemma. □

Lemma 4.4. $t_w \xrightarrow{t.x}_{rf} t_r \Rightarrow \nexists t'_w$ *such that* $(t_w \rightarrow_{\mathcal{H}^\sigma} t'_w \rightarrow_{\mathcal{H}^\sigma} t_r) \wedge (w_{t'_w}(t.x) \in \mathcal{H}_t)$.

Proof. We have two cases here : (1) $t \neq t_\psi$ and (2) $t = t_\psi$.

Case I: $t \neq t_\psi$ (History of non-root level transactions)

It means $\widehat{\mathcal{H}_t}$ is the history at some node other than the super transaction (t_ψ). In this case, the proof follows directly from the Constraint 4.1, which states that the subtransactions are executed in a sequential manner, and only one of the transactions in a transaction tree executes at a time. This implies that t_w was the latest transaction to modify $t.x$ before t_r was started.

Case II: $t = t_\psi$ (History of root-level transactions)

By contradiction, let us assume that there are three root-level transactions, t_w, t'_w and t_r, and a global object $t_\psi.x$ such that:

$-t_w \xrightarrow{t_\psi.x}_{rf} t_r$

$-w_{t'_w}(t_\psi.x, v') \in \mathcal{H}_{t_\psi}$

$-t_w \rightarrow_{\mathcal{H}^\sigma_{t_\psi}} t'_w \rightarrow_{\mathcal{H}^\sigma_{t_\psi}} t_r$.

As both t_w and $t_w^{'}$ write $t_\psi.x$ in shared memory, they have necessarily committed (a write in shared memory occurs only at lines 61-64 during the execution of $try\_to\_commit()$, i.e., $t_w, t_{w'} \in \Pi(\mathcal{H}_{t_\psi})$. Moreover, their linearization points ℓ_{t_w} and $\ell_{t_w'}$ occur while they hold the lock on $t_\psi.x$ (before committing), from which we have the following implications:

$t_w \rightarrow_{\mathcal{H}^{\sigma}_{t_\psi}} t_w^{'} \Leftrightarrow \ell_{t_w} <_{\mathcal{H}_{t_\psi}} \ell_{t_w'}$,

$\ell_{t_w} <_{\mathcal{H}_{t_\psi}} \ell_{t_w'} \Rightarrow RL_{t_w}(t_\psi.x, ttc) <_{\mathcal{H}_{t_\psi}} AL_{t_w'}(t_\psi.x, ttc)$

$\Rightarrow w_{t_w}(t_\psi.x, v) <_{\mathcal{H}_{t_\psi}} w_{t_w'}(t_\psi.x, v^{'})$,

$t_w \xrightarrow{t_\psi.x}_{rf} t_r \Rightarrow RL_{t_w}(t_\psi.x, ttc) <_{\mathcal{H}_{t_\psi}} AL_{t_r}(t_\psi.x, read(x))$,

Please note that access to the object $t_\psi.x$ is protected by a lock and, hence, is atomic in nature. Since t_r reads the value written by t_w, it means that $t_w \xrightarrow{t_\psi.x}_{rf} t_r \Rightarrow$ the event $r_{t_r}(t_\psi.x, v)$ follows $w_{t_w}(t_\psi.x, v)$ but precedes $w_{t_{w'}}(t_\psi.x, v^{'})$, i.e,

$(t_w \xrightarrow{t_\psi.x}_{rf} t_r) \wedge (w_{t_w}(t_\psi.x, v) <_{\mathcal{H}_{t_\psi}} w_{t_w'}(t_\psi.x, v^{'}) \Rightarrow w_{t_w}(t_\psi.x, v) <_{\mathcal{H}_{t_\psi}} r_{t_r}(t_\psi.x, v) <_{\mathcal{H}_{t_\psi}} w_{t_w'}(t_\psi.x, v^{'})$.

When a subtransaction in $\widehat{t_r}$ ($\widehat{t_r}$ accounts for the possibility that $t_\psi.x$ could be read by t_r or any of its descendants that merged with t_r) reads an object $t_\psi.x$, it always adds its id to $t_\psi.x.rs$ before releasing the lock on $t_\psi.x$ (lines 43-44). Therefore, the predicate $\beta(\widehat{t_r}, t_\psi.x.rs, RL_{\widehat{t_r}}(t_\psi.x, read(x)))$ is true ($t_\psi.x.rs$ is set to $\emptyset$ only after being added to the set $t_\psi.ow$). Using this observation, we have the following:

$r_{t_r}(t_\psi.x, v) <_{\mathcal{H}_{t_\psi}} w_{t_w'}(t_\psi.x, v^{'}) \wedge \beta(\widehat{t_r}, t_\psi.x.rs, RL_{\widehat{t_r}}(t_\psi.x, read(x)))$

$\Rightarrow \beta(\widehat{t_r}, t_\psi.x.rs, AL_{t_w'}(t_\psi.x, ttc))$,

$\beta(\widehat{t_r}, t_\psi.x.rs, AL_{t_w'}(t_\psi.x, ttc)) \wedge (w_{t_w'}(t_\psi.x, v^{'}) \in \mathcal{H}_{t_\psi}) \Rightarrow \beta(\widehat{t_r}, t_\psi.ow, \ell_{t_w'}) \Rightarrow \ell_{t_r} <_{\mathcal{H}_{t_\psi}} \ell_{t_w'} \Leftrightarrow t_r \rightarrow_{\mathcal{H}^{\sigma}_{t}} t_w^{'}$.

which proves that, contrary to the initial assumption, t_w' cannot precede t_r in the sequential transaction history $\widehat{\mathcal{H}^{\sigma}_{t_\psi}}$. □

Lemma 4.5. $t_w \xrightarrow{t.x}_{rf} t_r \Rightarrow t_w \rightarrow_{\mathcal{H}^{\sigma}_{t}} t_r$.

Proof. Again we have two cases:

Case I: $t \neq t_\psi$

Follows directly from Constraint 4.1.

Case II: $t = t_\psi$

The proof is made up of two parts. First it is shown that $t_w \xrightarrow{t_\psi.x}_{rf} t_r \Rightarrow \neg\beta(\widehat{t_r}, t_\psi.ow, \ell_{t_w})$, and then it is shown that $\neg\beta(\widehat{t_r}, t_\psi.ow, \ell_{t_w}) \wedge t_w \xrightarrow{t_\psi.x}_{rf} t_r \Rightarrow t_w \rightarrow_{\mathcal{H}^{\sigma}_{t_\psi}} t_r$.

Proof of $t_w \xrightarrow{t_\psi.x}_{rf} t_r \Rightarrow \neg\beta(\widehat{t_r}, t_\psi.ow, \ell_{t_w})$. Let us assume by contradiction that the predicate $\beta(\widehat{t_r}, t_\psi.ow, \ell_{t_w})$ is true. Due to lines 56, 60, 63, 65 we have $\beta(\widehat{t_r}, t_\psi.ow, \ell_{t_w}) \Rightarrow \beta(\widehat{t_r}, t_\psi.x.fbd, RL_{t_w}(t_\psi.x, ttc))$

If the read of $t_\psi.x$ from shared memory by t_r is before the write by t_w, we cannot have $t_w \xrightarrow{t_\psi.x}_{rf} t_r$. So, in the following we consider that the read of $t_\psi.x$ from shared memory by t_r is after its write by t_w. We have then $RL_{t_w}(t_\psi.x, ttc) <_{\mathcal{H}_{t_\psi}} AL_{\widehat{t_r}}(t_\psi.x, .read(x))$, and consequently $\beta(\widehat{t_r}, t_\psi.x.fbd, RL_{t_w}(t_\psi.x, ttc)) \Rightarrow \beta(\widehat{t_r}, t_\psi.x.fbd, AL_{t_r}(t_\psi.x, ttc))$.

As $\widehat{t_r} \in t_\psi.x.fbd$ when it locks $t_\psi.x$, it follows that read operation fails (due to lines 40-41, 37, 17-18), and consequently we cannot have $t_w \xrightarrow{t_\psi.x}_{rf} t_r$. Summarizing the previous reasoning we have $\beta(\widehat{t_r}, t_\psi.ow, \ell_{t_w}) \Rightarrow \neg(t_w \xrightarrow{t_\psi.x}_{rf} t_r)$, and taking the contrapositive we finally obtain $t_w \xrightarrow{t_\psi.x}_{rf} t_r \Rightarrow \neg\beta(\widehat{t_r}, t_\psi.ow, \ell_{t_w})$

Proof of $\neg\beta(\widehat{t_r}, t_\psi.ow, \ell_{t_w}) \wedge t_w \xrightarrow{t_\psi.x}_{rf} t_r \Rightarrow t_w \rightarrow_{\mathcal{H}^{\sigma}_{t}} t_r$. As defined earlier, the linearization point ℓ_{t_r} depends on whether t_r is a read only or an update transaction. The proof considers the two possible cases.

- If t_r is an update transaction that commits, its linearization point ℓ_{t_r} (that is defined at line 60 when it updates the set $t_\psi.ow$) occurs after its invocation of $try\_to\_commit()$. Due to this observation, the fact that t_w releases its locks after its linearization point, and $t_w \xrightarrow{t_\psi.x}_{rf} t_r$, we have $\ell_{t_w} <_{\mathcal{H}_{t_\psi}} \ell_{t_r}$, i.e., $t_w \rightarrow_{\mathcal{H}^\sigma_{t_\psi}} t_r$.

- If t_r is a read only transaction that commits, its linearization point ℓ_{t_r} is placed either before the time t_r is added to $t_\psi.ow$, or at the time of the test during its last read operation (line 40). In either case, we have $w_{t_w}(t_\psi.x, v) <_{\mathcal{H}_{t_\psi}} \ell_{t_w} <_{\mathcal{H}_{t_\psi}} RL_{t_w}(t_\psi.x, ttc) <_{\mathcal{H}_{t_\psi}} AL_{\widehat{t_r}}(t_\psi.x, read(x)) <_{\mathcal{H}_{t_\psi}} r_{t_r}(t_\psi.x, v) <_{\mathcal{H}_{t_\psi}} \ell_{t_r}$, from which we have $\ell_{t_w} <_{\mathcal{H}_{t_\psi}} \ell_{t_r}$, i.e., $t_w \rightarrow_{\mathcal{H}^\sigma_{t_\psi}} t_r$. Hence, in all cases, we have $t_w \xrightarrow{t_\psi.x}_{rf} t_r \Rightarrow t_w \rightarrow_{\mathcal{H}^\sigma_{t_\psi}} t_r$. □

Proof of non-existence of cyclic relation between transactions across levels

Cyclic conflict between transactions across levels comes into the picture when the read set of a subtransaction contains incompatible read operations. In SimpSTM, at any time only one subtransaction in a transaction tree actively executes its steps while its ancestors remain idle. The values of local objects of its ancestors, other than t_ψ, remain consistent as they do not change while t is active. Therefore, the only way cyclic conflict between transactions at different levels can occur is when a subtransaction t reads a value from a global object $t_\psi.x$ that is inconsistent for t or any of its intermediate ancestors. Hence, we prove the following.

Lemma 4.6. *When a subtransaction t reads a value from an object $t_\psi.x$, it is con-*

sistent w.r.t. t as well as each of its intermediate ancestors in the transaction tree.

Proof. The state of a transaction consists of its steps as well as the steps of its committed children. In SimpSTM, the ids of transactions that form the current state of a transaction t is given by the set $t.mts$. Before a subtransaction t tries to read from a global object $t_\psi.x$, it recursively searches the local space of each of its intermediate ancestors (lines 17, 35). In the process, the content of mts of t as well as each of its ancestors is added to set s_{mts} (lines 18, 36). Thus, by the time we come to read from $t_\psi.x$, s_{mts} contains the ids of all the non-aborted transactions currently in the transaction tree. Now, before reading from $t_\psi.x$, we check that the value of $t_\psi.x$ is consistent with each of the transactions whose id is in s_{mts} by ensuring $t_\psi.x.fbd \cap s_{mts} = \emptyset$ (line 40). The value is read only if it is consistent. Otherwise, t aborts (lines 40-41, 37, 18-19).

□

Theorem 4.1. *Every level-wise history of committed transactions, $\Pi(\widehat{\mathcal{H}^\sigma_t})$, produced by SimpSTM satisfies the level-wise opacity consistency criterion.*

Proof. The proof follows from definition of linearization points, and Lemmas 4.1 through 4.6. □

4.2.3 Proof for aborted transactions

As stated earlier (Constraint 4.1), the subtransactions are executed in a sequential manner. That means in a transaction tree, only one subtransaction commits or aborts at a time. To prove the correctness (opacity criterion) for an aborted subtransaction (say t_a), we prove the following property (2) of Definition 3.2 : (i) Until the time just before the abort of t_a, the execution of t_a was consistent, and (ii) after the abort, t_a and its descendants are invisible for $outsideTrans(t_a)$.

During the execution of the transaction tree, several nested subtransactions might abort. To prove the correctness for aborted transactions, it is important that we consider the steps of one aborted transaction at a time. This criterion is inherently captured in the construction of a closure for a transaction. To make the idea clear, consider the following history (shown in Figure 4.2):

$\widehat{\mathcal{H}_1} = \langle r_{t_1}(t_\psi.x), r_{t_{11}}(t_1.x), r_{t_{11}}(t_\psi.y), w_{t_{11}}(t_{11}.y), w_{t_2}(t_\psi.y), c_{t_2}, a_{t_{11}}, r_{t_{12}}(t_\psi.z), c_{t_{12}}, r_{t_{13}}(t_\psi.y), w_{t_{13}}(t_{13}.x), w_{t_3}(t_\psi.y), c_{t_3}, a_{t_{13}}, c_{t_1}\rangle$.

In history $\widehat{\mathcal{H}_1}$, t_1, t_{11}, t_{12} and t_{13} form parts of the same transaction tree, with t_1 being the root transaction, and t_{11}, t_{12}, t_{13} being its children. Transactions t_2 and t_3 are two other root-level transactions. Here, subtransactions t_{11} and t_{13} abort in order. Let us observe how the linearization point for t_1 (the transaction tree associated with t_1) shifts when we consider the steps of its children (in case of aborted children, we consider their *reduced* forms). Let us determine the linearization point for t_1 at the times of the aborts of t_{11} and t_{13} respectively.

At the time of abort of t_{11}*:* Since t_2 modified $t_\psi.y$ after it was read by t_{11} (descendant of t_1), t_1's linearization point, denoted by $\ell^1_{t_1}$, occurs before that of t_2. Thus, the serial order at root level is t_1, t_2.

At the time of abort of t_{13}*:* Here t_{13} is able to read the fresh value of $t_\psi.y$ written by t_2 (as t_{11} is not considered a part of t_1 at this point). Further, $t_\psi.y$ is modified by t_3 before the abort of t_{13}. Hence, linearization point for t_1 at this juncture lies at $\ell_{t_{13}}$. Consequently, the serial order of root-level transactions is t_2, t_1, t_3. Notice how the serial order of root transactions t_1 and t_2 is different in this case, owing to the shift

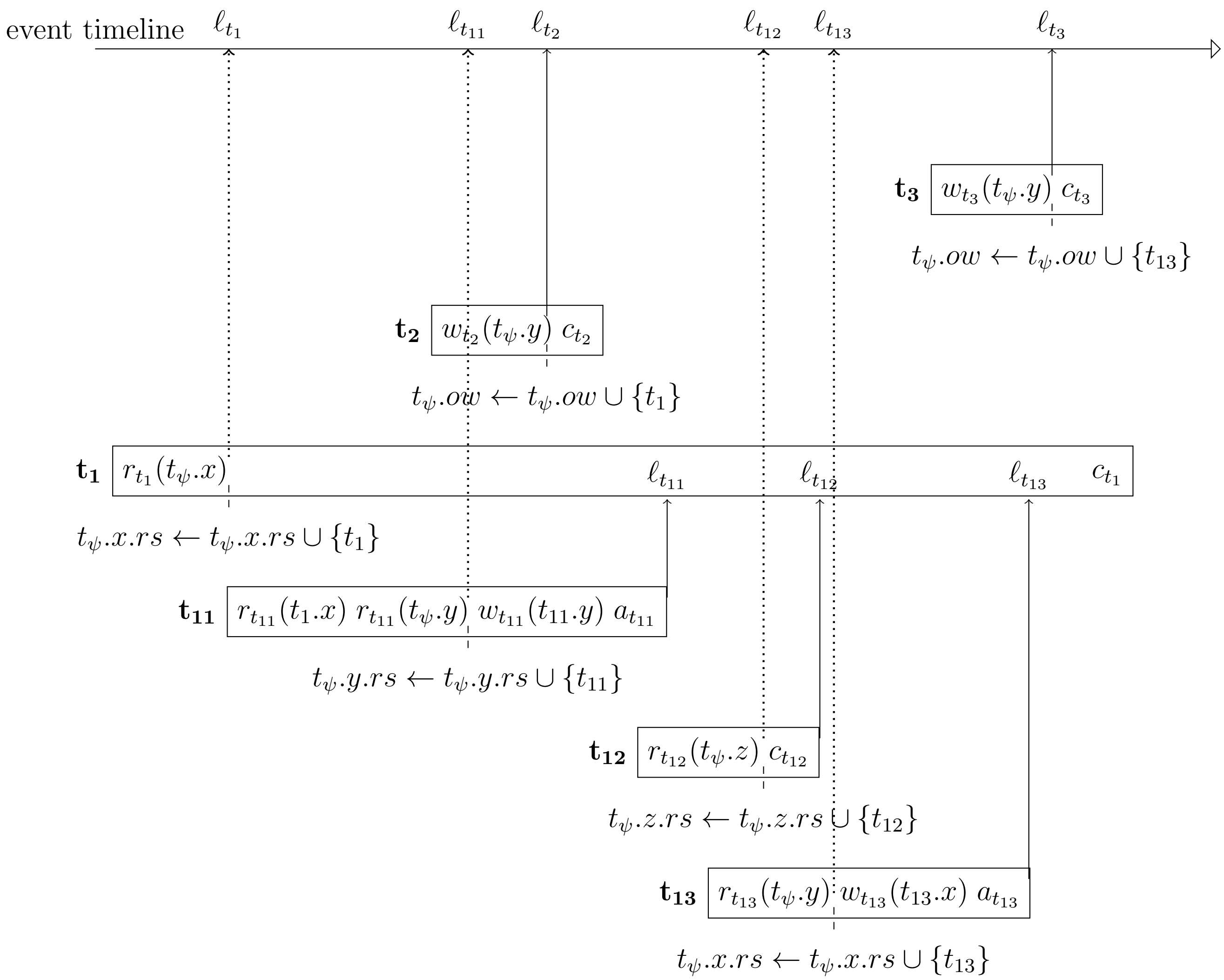


Figure 4.2: Linearization points of transactions

of linearization point of t_1.

Observation: Notice that if we consider the steps of both aborted subtransactions, t_{11} and t_{12}, then it is not possible to determine the linearization point of t_1. The operations $r_{t_{11}}(t_\psi.y)\ w_{t_2}(t_\psi.y)\ c_{t_2}$ dictates the serial order t_1, t_2, whereas $w_{t_2}(t_\psi.y)\ c_{t_2}\ a_{t_{11}}\ r_{t_{13}}(t_\psi.y)$ demands the order t_2, t_1. For this reason, we consider one aborted subtransaction at a time, and the steps of the previously aborted subtransactions are ignored (also implied by part(ii) of property(2) of Definition 3.2). Hence, the part of the $\widehat{\mathcal{H}_1}$ considered at the time of abort of t_{11} is $\widehat{\mathcal{H}_1^{\mathcal{C}_{t_{11}}}} = \langle r_{t_1}(t_\psi.x)\ r_{t_{11}}(t_1.x)\ r_{t_{11}}(t_\psi.y)\ w_{t_{11}}(t_{11}.y)$ $w_{t_2}(t_\psi.y)\ \ c_{t_2}\ \ a_{t_{11}}\rangle$, whereas the history considered at the time of abort of t_{13} is $\widehat{\mathcal{H}_1^{\mathcal{C}_{t_{13}}}} = \langle r_{t_1}(t_\psi.x)\ w_{t_2}(t_\psi.y) c_{t_2}\ r_{t_{12}}(t_\psi.z)\ c_{t_{12}}\ r_{t_{13}}(t_\psi.y)\ w_{t_{13}}(t_{13}.x)\ w_{t_3}(t_\psi.y)\ c_{t_3}\ a_{t_{13}}\rangle$. Note that the steps of the previously aborted subtransaction, t_{11}, are ignored in the latter case.

Treatment of aborted transaction:

To prove the part(i) of property (2) of Definition 3.2, we consider the state of the transaction tree, at a time ($\tau_{t_a^1}$) just before the abort of a subtransaction (say) t_a, and obtain the closure for t_a.

Now, we prove part(i) of the property(2) of Definition 3.2.

Lemma 4.7. *Until the time just before the abort of t_a, the execution of t_a was consistent, i.e., $\gamma(t_a)$ is linearizable at its ancestors' levels.*

Proof. The closure for t_a, $\widehat{\mathcal{H}^{\mathcal{C}_{t_a}}}$ represents a history of committed transactions, including all the steps of t_a. Hence, level wise histories of committed transactions can be obtained using $\widehat{\mathcal{H}^{\mathcal{C}_{t_a}}}$ and the correctness can be proved in the same way as done for the history of committed transactions in Section 4.2.2. $\square$

Next, we prove part(ii) of property(2) of Definition 3.2.

Lemma 4.8. *If $t^{'} \in aborted(\mathcal{H}_t^{'})$, then results (read and write sets) of $t^{'}$ and its descendants are not made available to its ancestors (including t).*

Proof. By construction of SimpSTM, when a subtransaction commits, it writes in the memory location of its parent only (lines 51, 26-31 for closed nested transactions, and 61-64 in case of a root level transaction whose parent is the fictitious transaction t_{ψ}). At this point, we can observe that, at the time of committing, transaction t also contains the results of its descendants (if any) that have merged with it. Before writing into the memory location of its parent, transaction t undergoes consistency checking (lines 48-49, 57-58). If the consistency checking fails at this point, then the transaction aborts (lines 50, 59) without modifying the objects of its parent. This means the results of $t^{'}$ are not propagated to t. This in turn implies that, even if t commits later, results of $t^{'}$ will not be carried upward to t's parent or other ancestors for that matter. Thus, if $t^{'}$ aborts, its results cannot be available to its ancestors. □

Lemma 4.9. *If $t^{'} \in aborted(\mathcal{H}_t^{'})$, then $t^{'}$ and its descendants (if any) are invisible to transactions $\in outsideTrans(t^{'})$.*

Proof. Again, by construction, if a local copy of an object is not available with a transaction t, then it tries to read from the local space of its ancestors only, through the recursive call to *search_parent* method (lines 18, 34-37, 38-45), not from any other transaction in the super tree. Now, given the hierarchical composition of transactions, a transaction $t^{''}$ in $outsideTrans(t^{'})$ can witness the results of $t^{'}$ only if they are made available to the (least) common ancestors of $t^{'}$ and $t^{''}$ in the super tree.

As the results of an aborted transaction are not made available to its ancestors (by Lemma 4.8), none of the transactions in $outsideTrans(t^{'})$ can see the results of $t^{'}$. Hence, $t^{'}$ is invisible to transactions in $outsideTrans(t^{'})$. □

Theorem 4.2. *If $t^{'} \in aborted(\mathcal{H}_t^{'})$, then $t^{'}$ satisfies level-wise opacity.*

Proof. The proof follows directly from Lemmas 4.7 through 4.10. □

Theorem 4.3. *Each level wise history $\widehat{\mathcal{H}_t}$ produced by SimpSTM satisfies level-wise opacity.*

Proof. The proof follows from the conjunction of Theorem 4.1 and Theorem 4.2. □

Chapter 5

ParSTM

5.1 The main idea

In Chapter 4, we saw that SimpSTM was designed with the constraint that the subtransactions in the transaction tree are executed sequentially, i.e., one after the other. As such, under SimpSTM there is no concurrent execution of the nodes in the transaction tree. So, the idea here is to relax that constraint (Constraint 4.1) and introduce some degree of concurrency among the nodes. To this end, we employ an optimistic approach at the global level, and a pessimistic approach (2PL for nested transactions) at the intra-transaction (transaction tree) level.

5.1.1 Optimistic behaviour at the global level (t_ψ):

Here, the locking of global objects (associated with t_ψ) is done in the same optimistic manner as done under SimpSTM. In other words, when a transaction t accesses a global object $t_\psi.x$ for a read or write operation, it releases the lock on $t_\psi.x$ immediately

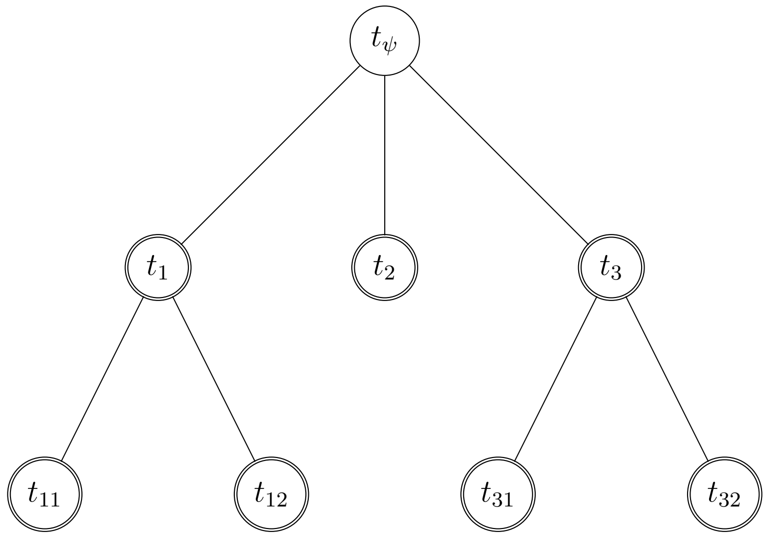


Figure 5.1: Optimistic mode of concurrency at global level (single circle) and pessimistic mode at nested level (double circle)

after the execution of its operation, i.e., t does not retain the lock on $t_\psi.x$ throughout its lifetime. This allows other transactions to access $t_\psi.x$ in the meantime.

5.1.2 Pessimistic behaviour at the nested level (p-node, t_π):

With each node t in the transaction tree, there is a local copy $t.x$ of each object x that is accessed by t or its descendants. Each of these objects is protected by a lock. To indicate the point that each node in the transaction tree exhibits pessimistic behaviour, we call it a *p-node* and denote it by t_π in general (Figure 5.1). A *p-node* denotes a transaction whose objects are accessed through a pessimistic approach of concurrency control. More precisely, when an object $t_p.x$ of a node, t_p, is locked by its child transaction t_c, then t_c retains the ownership of that lock until t_c completes its execution (commit or abort). That means, until t_c completes, neither t_p nor any other child of t_p can access (obtain the lock on) $t_p.x$. They have to wait to access $t_p.x$.

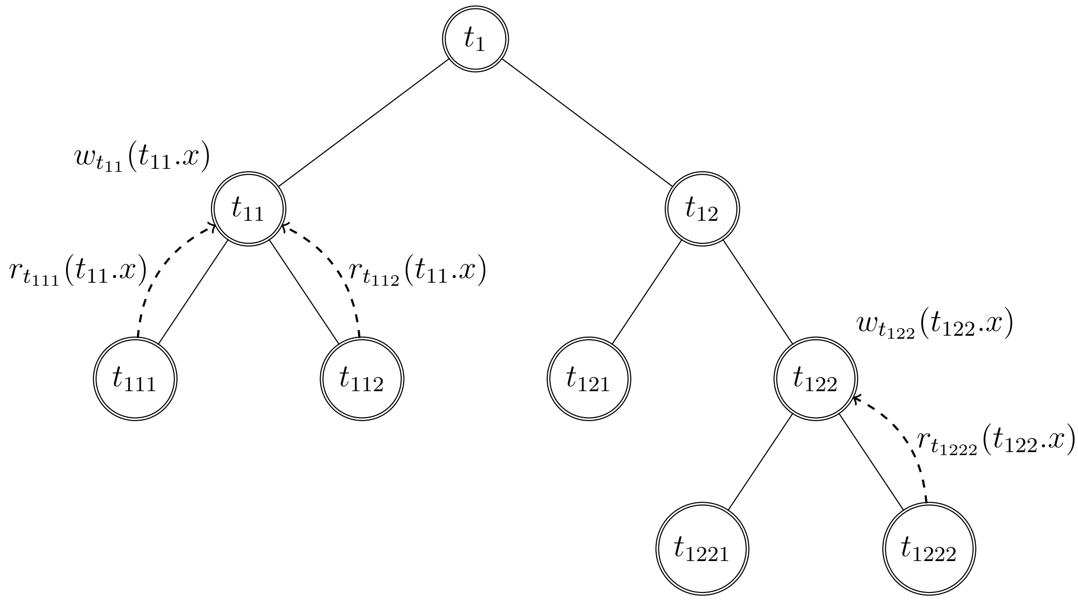


Figure 5.2: Partial concurrency

Note: When t_p itself locks an object $t_p.x$ for its local operation, it unlocks $t_p.x$ after the completion of its operation, thereby making it available for subsequent access by its descendants.

5.1.2.1 Partial concurrency at the nested level

Owing to the pessimistic approach (similar to 2PL for nested transactions) at the nested level, partial concurrency is achieved at the intra-transactional level. The different cases can be studied as follows.

(i) Same copy of object is accessed sequentially: For example, consider a node t_{11} in a transaction tree given in Figure 5.2. Transactions t_{111} and t_{112} are the two children of t_{11}. Now, t_{111} and t_{112} can access different objects of t_{11} concurrently. However, if the two children want to access the same object of t_{11}, then one of them has to wait. In another words, if t_{111} wants to access $t_{11}.x$ and t_{112} wants to access

$t_{11}.y$, then they can access the respective objects of t_{11} concurrently. If both t_{111} and t_{112} want to access the same object $t_{11}.x$, then the lock on $t_{11}.x$ is granted to only one of them, say t_{111}, at a time, and t_{112} has to wait until t_{111} terminates and releases the lock on $t_{11}.x$.

(ii) Different copies of the same object can be accessed concurrently: Now consider the case at nodes t_{11} and t_{122}. Local copies of x are created at these nodes due to their respective local writes, $w_{t_{11}}(t_{11}.x)$ and $w_{t_{122}}(t_{122}.x)$. Now, these two different local copies, $t_{11}.x$ and $t_{122}.x$, can be accessed concurrently. For example, accessing of $t_{11}.x$ by t_{111} and $t_{122}.x$ by t_{1222} can take place concurrently.

5.1.2.2 Handling deadlock situations

A deadlock situation may occur between children (descendants) when they try to lock the same set of objects of the parent (ancestor). A child node may be waiting for a lock on the parent's object. The reverse is not true as the parent does not access its child's object. Further, the parent performs one read or write operation on one local object at a time, and releases the lock on its local object as soon as the operation completes. Therefore, there is no deadlock situation between the parent and the child.

To handle the possibility of a deadlock between the children, each node in the transaction tree maintains a wait-for graph, whose access is controlled by a lock.

Protocol 5.1: management of wait-for graph, wfg_t:

$t_{own(x)} = t^{'}, \exists t^{'} : \langle x, t^{'}, t^{'} \rangle \in t.wfg$

i Operation $\mathbf{request\_lock_t(x, t_c)}$:
ii lock $t.wfg$;
iii add edge $\langle x, t_c, t_{own(x)} \rangle$ to $t.wfg$;
iv **if** ($t.wfg$ contains a cycle) **then**
v   remove $\langle x, t_c, t_{own(x)} \rangle$ from $t.wfg$;
vi   unlock $t.wfg$; return $false$; **end if**
vii unlock $t.wfg$; return $true$;

viii Operation $\mathbf{align\_request_t(x, t_c)}$:
ix lock $t.wfg$; $t_{old} \leftarrow t_{own(x)}$;
x $\langle x, t_{old}, t_{old)} \rangle \leftarrow \langle x, t_c, t_{c)} \rangle$;
xi replace all $\langle x, t^{'}, t_{old)} \rangle$ with $\langle x, t^{'}, t_c \rangle$;
xii **if** ($t.wfg$ contains a cycle) **then**
xiii   Undo the effects of lines x-xi;
xiv   remove edge $\langle x, t_c, t_{old} \rangle$;
xv   unlock $t.wfg$; return $false$; **end if**
xvi unlock $t.wfg$; return $true$;

xvii Operation $\mathbf{secure\_lock_t(x, t_c)}$:
xviii **if** ($\neg request\_lock_t(x, t_c)$) **then** return $false$; **end if**
xix lock $t.x$;
xx **if** ($\neg align\_request_t(x, t_c)$) **then**
    unlock $t.x$; return $false$; **end if**
xxi return $true$;

Please refer to Protocol 5.1 for the following discussion.

(a) Construction of wait-for graph wfg_t at a node t:

The wait-for-graph wfg_t consists of nodes and directed edges between them. The nodes in the graph represent t and its children. An entry in wfg_t is denoted by the tuple $\langle x, t_1, t_2 \rangle$. Initially, for each object x, there is an entry (owner node) $\langle x, t_{init}, t_{init} \rangle$, t_{init} is a fictitious transaction.

Meaning of a tuple $\langle x, t_1, t_2 \rangle$:

- $t_1 = t_2 = t^{'}$: Denotes a node $t^{'}$ that is the current owner of lock on $t.x$. The owner transaction in such a tuple is denoted by $t_{own(x)}$.
- $t_1 \neq t_2$: Denotes an edge from node t_1 to node t_2, indicating t_1 is waiting for a lock on $t.x$ which is currently held by t_2.

We construct the graph by considering only the edges, i.e., those entries $\langle x, t_1, t_2 \rangle$ in which $t_1 \neq t_2$. Further, to ensure the correctness (in face of concurrent execution), the access to wfg_t is protected by a lock.

(b) Managing wfg_t:

Recall that a local object x associated with node t is denoted as $t.x$. To successfully access $t.x$, a node t_c secures a lock on $t.x$ by following three steps: (i) request a lock on $t.x$, (ii) lock $t.x$, and (iii) align the requests of other transactions for locking $t.x$.

(i) request_lock$_t(x, t_c)$ *:* When a node t_c requests a lock on $t.x$, wfg_t is locked first. Let $t_{own(x)}$ be the current owner of $t.x$. We add an edge $\langle x, t_c, t_{own(x)} \rangle$. In this case, it simply indicates a node t_c is waiting for a lock on $t.x$, and other subtransactions might be trying to lock $t.x$ at the same time. Now, if addition of edge $\langle x, t_c, t_{own(x)} \rangle$ leads to a cycle in wfg_t, then we remove the edge $\langle x, t_c, t_{own(x)} \rangle$ from wfg_t and return $false$ to indicate failure (deadlock situation). Otherwise, $true$ is returned to indicate that the request does not cause deadlock and t_c waits until it obtains a lock on $t.x$.

(ii) lock $t.x$*:* A number of subtransactions might be waiting for a lock on $t.x$. When the lock on $t.x$ is released by the previous owner, t_{old}, one of the waiting transactions is selected in a non-deterministic way as the new owner of the lock.

(iii) align_request$_t(x, t_c)$*:* When t_c gets the lock, it first needs to rearrange the edges in wfg_t to reflect ownership of lock on $t.x$, and let other transactions, waiting for a lock on $t.x$, wait for t_c to release the lock. To this end, it replaces all the edges $\langle x, t', t_{old} \rangle$ with $\langle x, t', t_c \rangle$. If this change leads to a cycle in wfg_t, then it is rolled back along with removal of edge $\langle x, t_c, t_{old} \rangle$ from wfg_t, and $false$ is returned to indicate failure. If the alignment is successful, then t_c sets itself as the new owner, $t_{own(x)}$, and $true$ is returned.

(iv) secure_lock$_t(x, t_c)$*:* If *request_lock*, *lock* $t.x$ and *align_request* are completed successfully, it returns *true*. Otherwise, $false$ is returned.

While trying to secure a lock, a transaction t_c does not need to keep a lock on wfg_t all the time. Observe that the lock on wfg_t is released at the end of *request_lock* as well as *align_request*. This allows other transactions also to lock wfg_t and complete their *request_align* steps, while t_c waits for the opportunity to lock its object.

(c) Correctness of wfg_t:

To show the correctness of wait-for graph constructed this way and its management, we show the following:

i *Acyclicity of wait-for graph is maintained.*

ii *Access to wait-for graph through secure_lock method is non-blocking.*

iii *Access to wait-for graph by the owner for its local operation does not bring it in cyclic conflict with its children.*

Lemma i. *Acyclicity of wait-for graph is maintained.*

Proof. As discussed earlier, a successful acquisition of lock on parent's object $t.x$ requires the following: (i) checking that the request for the lock does not cause a cycle in wfg_t, (ii) acquiring the lock on $t.x$, and (iii) resetting the wait-for relations corresponding to the others' requests for a lock on $t.x$, ensuring that the resetting does not lead to a cycle in wfg_t.

The correctness follows from the argument that acyclic property of the wfg_t is maintained at all the time. Note that access to wfg_t is protected by a lock (lines ii, ix). This means only one transaction updates wfg_t at a time.

Initially, when none of the objects have been locked, there are only nodes $\langle x, t_{init}, t_{init} \rangle$, and no edges. This means there is no cycle in wfg_t initially. Since the request for accessing $t.x$ is made through method *secure_lock*, the first edge in wfg_t is added through method *request_lock*. Next we show that before and after the completion of methods *request_lock* and *align_request*, wfg_t is acyclic.

During *request_lock*, wfg_t is locked (line ii) and a new edge is added (line iii) to wfg_t only if it does not cause a cycle (lines $iv - vi$). This means, if wfg_t is acyclic at the starting of *request_lock*, then it is acyclic at the end of the operation as well.

Next, during *align_request*, the edges in wfg_t are rearranged (lines x-xi) only if the new arrangement does not lead to a cycle in wfg_t. Also, note that the corresponding edge $\langle x, t, t' \rangle$ is reduced to a node $\langle x, t, t \rangle$. Otherwise, if a cycle is detected, the previous state of wfg_t is restored (lines $xii - xv$). This means that the acyclic property of wfg_t is preserved after *align_request* operation.

Now, we have following conclusions: ($m, n > 0$ and denote the number of executions of a method)

(Initial state) $\rightarrow wfg_t$ is acyclic.

$\Rightarrow$ (initial state) $\rightarrow (request\_lock)^m \rightarrow wfg_t$ is acyclic.

$\Rightarrow$ (initial state) $\rightarrow (request\_lock)^m (align\_request)^n \rightarrow wfg_t$ is acyclic.

Thus, the acyclic property of wfg_t is always preserved. In other words, wfg_t ensures there is no deadlock situation (cycle). □

Lemma ii. *Access to the wait-for graph through the secure_lock method is non-blocking.*

Proof. The lemma implies that when a subtransaction t_1 accesses *secure_lock* to obtain a lock on object x, it does not prevent other subtransactions from accessing *secure_lock* until t_1's invocation of *secure_lock* has completed.

The lock on wfg_t is held only for the duration of *request_lock* and *align_request* (*lines ii-vi and ix-xvi*). After successfully calling *request_lock*, while t_1 waits for actually getting a lock on x (*line xix*), other subtransactions are free to lock wfg_t

through *request_lock* or *align_request*. This allows other transactions to validate their requests for locking objects. Similarly, upon completion of *align_request*, t_1 releases the lock on wfg_t immediately. $\square$

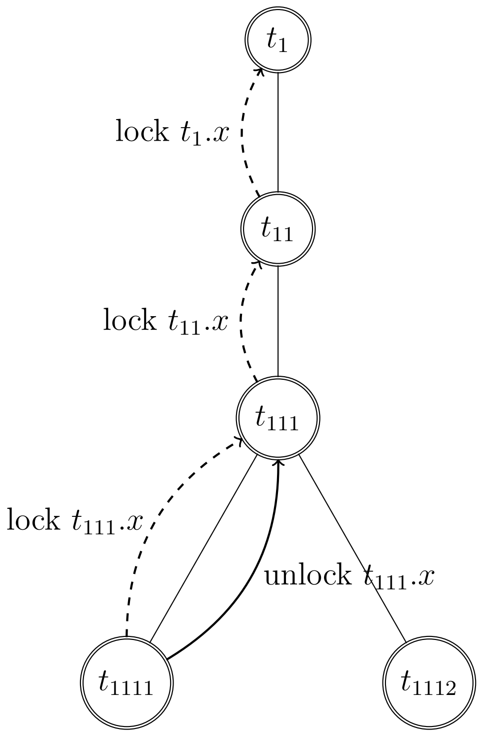
Lemma iii. *Access to wait-for graph by the owner for its local operation does not bring it in cyclic conflict with its children.*

Proof. When a transaction t tries to lock its object $t.x$ for its local read/write operation, the edge $\langle x, t, t' \rangle$, where t' is the current owner of the lock, is added in wfg_t. At that time, there is no edge $\langle y, t'', t \rangle$ for any y and t''. Since t performs only one local operation at a time, it does not have any other lock. When t gets the lock on $t.x$, in the *align_request* operation, some edges $\langle x, t''', t \rangle$ may be added, but $\langle x, t, t' \rangle$ is deleted at that time. Therefore, there will be no edge out-directed from t to any other node. Thus, t can never be in a directed cycle. $\square$

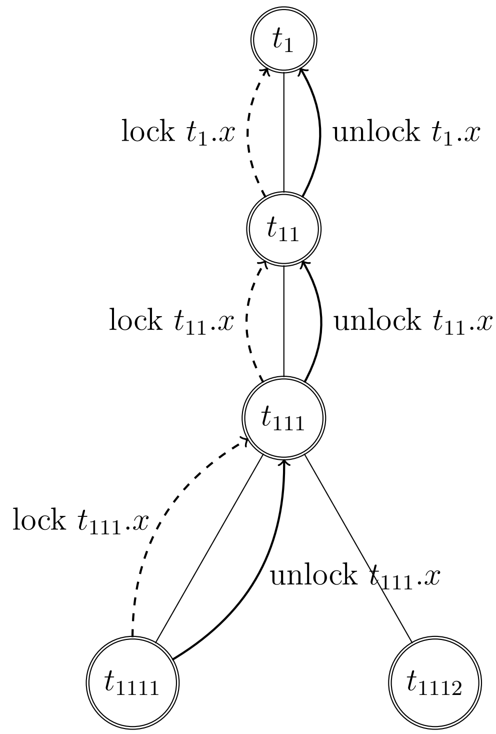
5.2 Implementing 2PL for nested transactions

Recall that each local copy of an object with a transaction is protected by a simple mutex based lock. Then, the two phase locking for nested transactions is achieved as follows:

1. When a subtransaction t reads an object $t'.x$ from an ancestor t', it (recursively) locks the object $t''.x$ at each ancestor t'' in the path from t to t' (including t'). Transaction t does not release the lock on them upon termination of its (successful) read operation.



(a) When t_{1111} commits, lock on x is released only up to the parent t_{111}



(b) When t_{1111} aborts, lock on x is released up to the original owner t_1

Figure 5.3: Implementing 2PL for nested transactions: (1) cascaded locking of x at all the ancestors up to t_1 during $r_{t_{1111}}(t_1.x)$ (shown by dotted arrows), and (2) unlocking $t_{111}.x$ only upon completion of t_{1111} (shown by solid arrow).

2. Transaction t releases locks to its ancestor(s) only upon its completion.

    Case i : If t commits, it releases lock on x only at its parent level.

    Case ii : If t aborts, lock on x is released all the way up to the ancestor $t^{'}$ from which it was originally read.

For example, consider Figure 5.3. Transaction t_{1111} recursively searches for a local copy of x with its ancestors, before finally reading from $t_1.x$, and, in the process, locks object x at each of its ancestors up to t_1. Upon successful read operation, t_{1111} does not release these locks; rather it retains them through its lifetime. That means no other transaction can obtain the lock on x at any of its ancestors up to t_1, i.e., t_{111}, t_{11} and t_1. In other words the value of x at t_{1111}'s ancestors up to t_1 cannot be changed while t_{1111} is active.

Moreover, when t_{1111} commits, it unlocks x only at its parent t_{111}, not at other ancestors. Therefore, at this point, only $t_{111}.x$ becomes available to t_{111} and its other descendants. However, if t_{1111} aborts, it unlocks x at all the ancestors up to t_1. Similarly, if t_{111} commits later on, only $t_{11}.x$ is unlocked. Otherwise, if t_{111} aborts, locks on $t_{11}.x$ as well as $t_1.x$ are released. In other words, if the subtransactions in the path from t_{1111} to t_1 commit one by one, they release the lock on x only at their respective parent level. However, if any of the subtransactions in that path abort, the lock on x is released up to t_1.

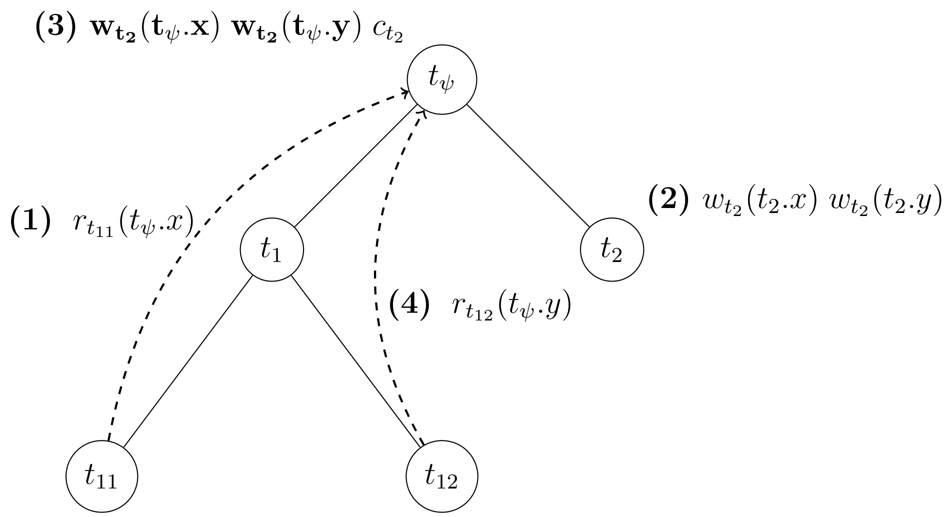


Figure 5.4: Incompatible transactions in ParSTM

5.3 Issue of incompatible read operations/transactions

We resume the discussion of incompatible transactions, referred to in Section 3.5.1. For sake of simplicity, consider a simple two-level transaction tree with t_1 as root transaction. Let t_{11} and t_{12} be the two children of t_1, and t_2 be another root transaction. Now, referring to Figure 5.4, consider the following history.

$$\widehat{\mathcal{H}_1} = \langle r_{t_{11}}(t_\psi.x)\ w_{t_2}(t_2.x)\ w_{t_2}(t_2.y)\ \mathbf{w_{t_2}(t_\psi.x)}\ \mathbf{w_{t_2}(t_\psi.y)}\ c_{t_2}\ r_{t_{12}}(t_\psi.y)\ (c_{11}\ c_{12})?\rangle$$

Recall that the lock on global objects $(t_\psi.x, t_\psi.y)$ are released soon after the read or write operations on them. Here, local copies of objects x and y are not available with transactions initially. As such, t_{11} (locks and) reads from $t_\psi.x$, adds its id to $t_\psi.x.rs$ before releasing the lock on $t_\psi.x$. Next, another root transaction t_2 modifies $t_\psi.x$ and $t_\psi.y$, consequently adding t_{11} to $t_\psi.x.fbd, t_\psi.y.fbd$ and $t_\psi.ow$ respectively. Now, t_{12} wants to read $t_\psi.y$. When t_{12} locks $t_\psi.y$, it can notice t_{11} in $t_\psi.y.fbd$. However, as t_{11} has not yet committed and become part of t_1 (t_{12}'s ancestor), it is legal for t_{12} to read $t_\psi.y$.

Now, let us look at how the steps of the children affect the linearizability of the root transaction t_1. When a closed nested transaction commits, its steps become part of the parent transaction. Following this point, consider the steps of t_{11} first to determine the linearization point of t_1. Transaction t_{11} read $t_\psi.x$ before it was modified by t_2. This implies that linearization point of t_1 precedes that of t_2, if t_{11} commits. Next, consider the steps of t_{12}. The read step $r_{t_{12}}(t_\psi.y)$ returns the value written by t_2, implying the linearization point of t_1 should be placed after that of t_2, if t_{12} commits. Hence, a contradiction (if both t_{11} and t_{12} commit). Here, the conflicting write operations of t_2 are sandwiched between the respective read operations of t_{11} and t_{12} on the global objects. Therefore, t_1 cannot be linearized at the global level ($\widehat{\mathcal{H}_\psi}$), if both t_{11} and t_{12} are allowed to commit. In other words, the two read operations, $r_{t_{11}}(t_\psi.x)$ and $r_{t_{12}}(t_\psi.y)$, are *incompatible* for t_1. Thus, *incompatible* operations and transactions are defined as follows.

Definition 5.1 (Incompatible operations and transactions). *Let t_1, t_2 be any two transactions in the super tree, and t $(t \neq t_\psi)$ be the least common ancestor of t_1 and t_2. Let $r_{t_1}(t_\psi.x)$ and $r_{t_2}(t_\psi.y)$ be successful read operations of t_1 and t_2 respectively on global objects $t_\psi.x$ and $t_\psi.y$. Then, these two read operations are incompatible if $t_1 \in t_\psi.y.fbd$ at the time of $r_{t_2}(t_\psi.y)$ or vice-versa. The two transactions, t_1 and t_2, are termed as incompatible transactions.*

Each transaction maintains an atomic object called *cm* (*consistency management*) that has two sets *its* (*incompatible transaction set*) and *mts* (*merged transaction set*). The set *its* is used to keep track of incompatible transactions, whereas *mts*, initially containing its own id, is used to keep track of descendants that have merged with it. It works as follows. Let us denote the root level transaction as t_ρ. Suppose a transaction t in $transTree(t_\rho)$ reads from the global object $t_\psi.x$. To keep a record of all the transactions in $transTree(t_\rho)$ that have read some global object, each transaction maintains a special set called vts (*visited transaction set*). Note that each transaction, before reading any global object $t_\psi.x$, adds its id to $t_\rho.vts$. Thus, the set $t_\rho.vts$ contains the ids of all the transactions in $transTree(t_\rho)$ that have read some global object. Now, upon reading $t_\psi.x$, the ids of transactions that are present in both $t_\rho.vts$ and $t_\psi.x.fbd$ are added to $t.cm.its$.

Later, when t tries to merge with its parent t_p, it ensures that t is not incompatible with t_p, by ensuring that $t_p.cm.mts \cap t.cm.its = \emptyset$ and $t_p.cm.its \cap t.cm.mts = \emptyset$.

Now, let us examine history $\widehat{\mathcal{H}_1}$ again under this scheme. Here, t_1 plays the role of the root transaction t_ρ. Initially, we have $t_{11} \in t_{11}.cm.mts, t_{12} \in t_{12}.cm.mts, t_1 \in t_1.cm.mts, t_{11}.cm.its = \emptyset$ and $t_{12}.cm.its = \emptyset$. In this history, at the time t_{12} accesses $t_\psi.y$, clearly $t_{11} \in t_\psi.y.fbd$ and $t_1.vts$. Therefore, t_{12} adds t_{11} to $t_{12}.cm.its$.

Note: To avoid concurrent merging of incompatible subtransactions, we impose the constraint that at each level only one child of a (parent) transaction can merge with it at a time. This is achieved by use of a mrg lock for merging.

If t_{11} (commits and) merges with t_1 first, then t_{11} is added to $t_1.cm.mts$. Later, when t_{12} tries to merge, it would fail (abort) as $t_1.cm.mts \cap t_{12}.cm.its = \{t_{11}\} \neq \emptyset$. Similarly, if t_{12} merges first, then t_{12} is added to $t_1.cm.mts$ and t_{11} to $t_1.cm.its$. As a result, t_{11} will not be able to commit as $t_1.cm.its \cap t_{11}.cm.mts = \{t_{11}\} \neq \emptyset$. Thus, we can ensure that two incompatible children of a transaction cannot both commit.

If we consider the previous case by replacing t_{12} with t_1 (and so replacing $r_{t_{12}}(t_\psi.y)$ with $r_{t_1}(t_\psi.y)$) in $\widehat{\mathcal{H}_1}$, the solution still works by not allowing t_{11} to commit in the first place (even if t_{11} is a read only transaction).

Remark. Observe that in $\widehat{\mathcal{H}_1}$, if t_1 (a read only transaction) commits with the results of t_{11} (i.e., $r_{t_{11}}(t_\psi.x)$), then its linearization point, at its parent t_ψ's level, lies before $w_{t_2}(t_\psi.x)$. In the alternate case, if t_1 commits with the results of t_{12} (i.e., $r_{t_{12}}(t_\psi.y)$), then t_1's linearization point lies after $w_{t_2}(t_\psi.y)$. This way, the protocol offers flexibility for read only subtransactions.

5.4 The protocol: ParSTM

5.4.1 Protocol

Protocol 5.2: ParSTM

State of global object/ set :

$t_\psi.x$ with fields
  $val \in V$
  $rs, fbd \subset T$
$t_\psi.ow \subset T$

State of response object:
$res_x(value, level, s)$:
  $val \in V$, set to $value$
  $lvl \in L$, set to $level$
  $s_{its} \subset T$, set to s

Helper methods:
Operation **check_compatibility**$(\mathbf{cm_{t_a}}, \mathbf{cm_{t_d}})$
return $((cm_{t_a}.mts \cap cm_{t_d}.its = \emptyset) \wedge (cm_{t_a}.its \cap cm_{t_d}.mts = \emptyset))$;

Operation **update_cm**$(\mathbf{cm}, \mathbf{s_m}, \mathbf{s_i})$:
$cm.mts \leftarrow cm.mts \cup s_m$,
$cm.its \leftarrow cm.its \cup s_i$;

Common to both nodes (t_π/t_ρ) ***:***
t_* denotes t_π/t_ρ.

State of local object $t_*.x$ **:**
  $val \in V$, initially $null$

State of local atomic object $t_*.cm$ **:**
  $mts \subset T$
  $its \subset T$

State of transaction t_***:**
  $parent \in T$, parent's id (t_p)
  $lvl \in L$
  $lrs, lws \subset X$
  $vts \subset T$
  $pls \subset X$
  $prs \subset X \times L$
  $wfg \subset X \times T \times T$
  mrg : lock

Operation **begin**$_{\mathbf{t_*}}(\mathbf{t_p}, \mathbf{level})$:
$t_*.parent \leftarrow t_p$;
$t_*.lvl \leftarrow level$;
$t_*.cm.mts \leftarrow \{t_*\}$;

Operation **invoke_child**$_{\mathbf{t_*}}(\mathbf{t_c})$:
$begin_{t_c}(t_*, t_*.lvl - 1)$;

Operation **unlock_parent_locks**$_{\mathbf{t_*}}(\mathbf{s})$:
for each $x : x \in (s \cap t_*.pls)$ **do**
  $t_*.pls \leftarrow t_*.pls \setminus \{x\}$;
  unlock $t_p.x$; **end for**

Operation **unlock_to_ancestors**$_{\mathbf{t_*}}(\mathbf{s})$:
$s_{anc} \leftarrow \cup_{x.lvl > t_p.lvl} s$
$t_*.unlock\_parent\_locks(s)$;
if $(s_{anc} \neq \emptyset)$ **then**
  $t_p.unlock\_to\_ancestors(s_{anc})$ **end if**

Operation **get_local_lock**$_{\mathbf{t_*}}(\mathbf{x}, \mathbf{t_1})$:
if $(\neg secure\_lock_{t_*}(x, t_1))$ **then**
  $t_1.abort()$; **end if**

Operation **get_locks**$_{\mathbf{t_*}}(\mathbf{s}, \mathbf{t_c})$:
for each $x \in s$ **do**
  $get\_local\_lock_{t_*}(x, t_c)$;
  $t_c.pls \leftarrow t_c.pls \cup \{x\}$; **end for**

Operation **write**$_{\mathbf{t_*}}(\mathbf{x}, \mathbf{v})$:
$get\_local\_lock_{t_*}(x, t_*)$;
$t_*.x.val \leftarrow v$;
unlock $t_*.x$;
$t_*.lws \leftarrow t_*.lws \cup \{x\}$;

Operation **abort**$_{\mathbf{t_*}}()$:
$t_*.unlock\_to\_ancestors(t_*.prs)$;
$t_*.unlock\_parent\_locks(t_*.pls)$;
$t_*.abort\_active\_desc()$;
return $(abort)$;

Operation **abort_active_desc**$_{\mathbf{t_*}}()$:
for each $t' \in activeChildren(t_*)$ **do**
  $t'.force\_abort()$; **end for**

Operation **force_abort**$_{\mathbf{t_*}}()$:
for each $t' \in activeChildren(t_*)$ **do**
  $t'.force\_abort()$; **end for**
return $(abort)$;

Operation **abort_incompat_desc**$_{\mathbf{t_*}}(\mathbf{s_i})$:
$s_a \leftarrow s_i \cap t_*.vts$;
if $(s_a = \emptyset)$ **then** return; **end if**
for each $(t_c \in activeChildren(t_*))$ **do**
  if $(t_c.cm.mts \cap s_a \neq \emptyset)$ **then**
    $t_c.abort()$;
  else
    $t_c.abort\_incompat\_desc(s_a)$;
  end if
end for

$t_*.vts \leftarrow t_*.vts \setminus s_a$;

Specific to non-root node (t_π) ***:***

Operation $\mathbf{read}_{\mathbf{t}_\pi}(\mathbf{x})$:
$get\_local\_lock_{t_\pi}(mrg, t_\pi)$;
$get\_local\_lock_{t_\pi}(x, t_\pi)$;
$res_x \leftarrow \phi$;
if $(t_\pi.x.val = null)$ **then**
  $res_x \leftarrow search\_parent_{t_p}(x, t_\pi, t_\pi, t_\pi.cm)$;
  if $(res_x = null)$ **then**
    $abort_{t_\pi}()$; **end if**
  $t_\pi.x.val \leftarrow res_x.val$;
  $t_\pi.lrs \leftarrow t_\pi.lrs \cup \{x\}$;
  $update\_cm(t_\pi.cm, \emptyset, res_x.s_{its})$;
  $t_\pi.prs \leftarrow t_\pi.prs \cup \langle x, res_x.lvl\rangle$;
  $t_\pi.pls \leftarrow t_\pi.pls \cup \{x\}$;
end if
unlock $t_\pi.mrg$;
$v \leftarrow t_\pi.x.val$;
unlock $t_\pi.x$;
$t_\pi.abort\_incompat\_desc(res_x.s_{its})$;
return v;

Operation $\mathbf{search\_parent}_{\mathbf{t}_\pi}(\mathbf{x}, \mathbf{t_c}, \mathbf{t_o}, \mathbf{cm_d})$:
if $(\neg secure\_lock_{t_\pi}(x, t_c))$ **then**
  return $null$; **end if**
$cm \leftarrow t_\pi.cm$;
if $(t_\pi.x.val \neq null)$ **then**
  if$(\neg check\_compatibility(cm, cm_d))$**then**
    unlock $t_\pi.x$;
    return $null$; **end if**
  $res_x \leftarrow \langle t_\pi.x.val, t_\pi.lvl, \emptyset\rangle$;
else
  $update\_cm(cm, cm_d.mts, cm_d.its)$;
  $t_\pi.vts \leftarrow t_\pi.vts \cup \{t_o\}$;
  $res_x \leftarrow search\_parent_{t_p}(x, t_\pi, t_o, cm)$;
  if $(res_x = null)$ **then**
    unlock $t_\pi.x$;
    return res_x; **end if**
  $t_\pi.prs \leftarrow t_\pi.prs \cup \langle x, res_x.lvl\rangle$;
  $t_\pi.pls \leftarrow t_\pi.pls \cup \{x\}$;
end if
return res_x;

Operation $\mathbf{try\_to\_merge}_{\mathbf{t}_\pi}(\mathbf{t_c})$:
$get\_local\_lock_{t_\pi}(mrg, t_c)$;
$s \leftarrow \cup\{x : \langle x, *\rangle \in t_c.pls\}$
$t_\pi.get\_locks(t_c.lws \setminus s, t_c)$
if$(\neg check\_compatibility(t_\pi.cm, t_c.cm))$**then**
  unlock $t_\pi.mrg$;
  $abort_{t_c}()$; **end if**
for each $x \in t_c.lws$ **do**
  $t_\pi.x.val \leftarrow t_c.x.val$; **end for**
for each $x \in t_c.lrs : t_\pi.x.val = null$ **do**
  $t_\pi.x.val \leftarrow t_c.x.val$; **end for**
$t_\pi.lws \leftarrow t_\pi.lws \cup t_c.lws$;
$t_\pi.lrs \leftarrow t_\pi.lrs \cup t_c.lrs$;
$update\_cm(t_\pi.cm, t_c.cm.mts, t_c.cm.its)$;
unlock $t_\pi.mrg$;
$t_c.unlock\_parent\_locks(t_c.pls)$;

Operation $\mathbf{try\_to\_commit}_{\mathbf{t}_\pi}()$:
$try\_to\_merge_{t_p}(t_\pi)$;
$t_\pi.abort\_incompat\_desc(t_\pi.cm.its)$;
return $(commit)$;

Specific to root node (t_ρ) ***:*** Protocol 5.3

Protocol 5.3: ParSTM (Special case of root node, t_ρ)

Operation $\mathbf{search\_parent}_{\mathbf{t}_\rho}(\mathbf{x}, \mathbf{t_c}, \mathbf{t_o}, \mathbf{cm_d})$:
if $(\neg secure\_lock_{t_\rho}(x, t_c))$ **then**
  return $null$; **end if**
$cm \leftarrow t_\rho.cm$;
if $(t_\rho.x.val \neq null)$ **then**
  if$(\neg check\_compatibility(cm, cm_d))$**then**
    unlock $t_\rho.x$;
    return $null$; **end if**
  return $res_x \leftarrow \langle t_\rho.x.val, t_\rho.lvl, \emptyset\rangle$;
else
  $s_m \leftarrow cm.mts \cup cm_d.mts$;
  $t_\rho.vts \leftarrow t_\rho.vts \cup \{t_o\}$;
  lock $t_\psi.x$;
  if $(t_\psi.x.fbd \cap s_m = \emptyset)$ **then**
    $t_\psi.x.rs \leftarrow t_\psi.x.rs \cup \{t_o\}$;
    $s_i \leftarrow t_\psi.x.fbd \cap t_\rho.vts$;
    $res_x \leftarrow \langle t_\psi.x.val, t_\rho.lvl, s_i\rangle$;
    unlock $t_\psi.x$;
    return res_x;
  else
    unlock $t_\psi.x$;
    unlock $t_\rho.x$;
    return $null$; **end if**
end if

Operation $\mathbf{read}_{\mathbf{t}_\rho}(\mathbf{x})$:
$get\_local\_lock_{t_\rho}(mrg, t_\rho)$; $s_i \leftarrow \emptyset$;
$get\_local\_lock_{t_\rho}(x, t_\rho)$;
if $(t_\rho.x.val = null)$ **then**
  lock $t_\psi.x$;
  if $(t_\psi.x.fbd \cap t_\rho.mts \neq \emptyset)$ **then**
    unlock $t_\psi.x$; $abort_{t_\rho}()$; **end if**
  $t_\psi.x.rs \leftarrow t_\psi.x.rs \cup \{t_\rho\}$;
  $t_\rho.x.val \leftarrow t_\psi.x.val$;
  $s_i \leftarrow t_\psi.x.fbd \cap t_\rho.vts$;
  $update\_cm(t_\rho.cm, \emptyset, s_i)$;
  unlock $t_\psi.x$;
  $t_\rho.lrs \leftarrow t_\rho.lrs \cup \{x\}$;
end if
unlock $t_\rho.mrg$;

$v \leftarrow t_\rho.x.val$;
unlock $t_\rho.x$
$t_\rho.abort\_incompat\_desc(s_i)$;
return v;

Operation $\mathbf{try\_to\_commit}_{\mathbf{t}_\rho}()$:
if $(t_\rho.lws = \emptyset)$ **then**
    return *commit*; **end if**
for each $t_\psi.x : x \in (t_\rho.lrs \cup t_\rho.lws)$ **do**
    lock $t_\psi.x$;
    $t_\rho.pls \leftarrow t_\rho.pls \cup \{x\}$ **end for**
if $(t_\rho.cm.mts \cap t_\psi.ow \neq \emptyset)$ **then**
    $abort_{t_\rho}()$; **end if**
$t_\psi.ow \leftarrow \cup_{x \in t_\rho.lws} t_\psi.x.rs$;
for each $x \in t_\rho.lws$ **do**
    $t_\psi.x.val \leftarrow t_\rho.x.val$
    $t_\psi.x.fbd \leftarrow t_\psi.ow$;
    $t_\psi.x.rs \leftarrow \emptyset$ **end for**
$t_\rho.unlock\_parent\_locks(t_\rho.pls)$;
return $(commit)$;

Preview of the protocol

Unlike SimpSTM, at each node t in the transaction tree the access to a local copy of an object, say $t.x$, is protected by a lock. The descendants of t have to lock $t.x$ before reading it. If the read operation fails, the lock on $t.x$ is released. However, if the read is successful, then the lock on $t.x$ is retained until the subtree containing that descendant completes (as discussed in Section 5.2).

For example, referring to Figure 5.3, say t_{1111} wants to read x, and the value for x is available with t_1, and not with any of the intermediate ancestors, t_{11} and t_{111}. As described later in the protocol, the method for reading an object x from the parent's local space is $search\_parent_{t_p}(x, ...)$, where t_p is the id of the parent. Then, t_{1111} calls $search\_parent_{t_{111}}(x, ...)$, and obtains the lock on $t_{111}.x$ in the process. Since t_{111} does not have a local copy of x with itself, t_{111} calls $search\_parent_{t_{11}}(x, ...)$ and obtains a lock on $t_{11}.x$. Similarly, t_{11} calls $search\_parent_{t_1}(x, ...)$ and locks $t_1.x$. If the read operation on $t_1.x$ is successful, then the locks on $t_1.x, t_{11}.x$ and $t_{111}.x$ are not released. However, if the read operation is unsuccessful, then the locks on these objects are released (to reflect the effect that unsuccessful read operation does not lead to locking of objects).

In alternate case, in which even t_1 does not have a local copy of object x, the $search\_parent(x, ...)$ method propagates to t_ψ, and an attempt is made to read from the global object t_ψ. Here we should observe the difference in how global objects are treated. If the read operation $t_\psi.x$ succeeds, then the lock on $t_\psi.x$ is released, but we retain the locks on the objects of the ancestors in the transaction tree, i.e., $t_{111}.x, t_{11}.x$ and $t_1.x$. In case the read is unsuccessful, then the locks on all of these objects are released.

Further, note the following. When a subtransaction t_c writes an object $t_c.x$ which has not been previously read from the parent, t_p, then a local write operation on $t_c.x$ does not entail obtaining a lock on the parent's object $t_p.x$.

5.4.2 State of shared objects

Globally shared objects: Same as used for SimpSTM in Chapter 4. Recall that, at the global level, each object $t_\psi.x$ is protected by a lock, and has the following fields: (1) val for value, (2) a set rs for storing ids of transactions that have read $t_\psi.x$, and (3) a set fbd for storing ids of transactions forbidden to access $t_\psi.x$. The set $t_\psi.ow$ is used to indicate ids of transactions that read an object whose value has been overwritten later.

Locally shared objects: A local copy $t.x$, available (locally) with a transaction t, has only the value field, and is protected by a lock.

Response object: A response object, res_x, is used as a data structure to communicate $\langle$ *value, level, a set containing incompatible transactions* $\rangle$ information across levels while reading a value of object x from higher levels.

5.4.3 State of transaction

Each transaction t keeps a reference to its parent's id using *parent* (also denoted by t_p). The information regarding the level of t is contained in lvl. The read and write steps are logged using sets lrs (*local read set*) and lws (*local write set*) respectively. The ancestors' objects on which t acquires locks are recorded in pls (*parent's lock set*). The set prs (*pessimistic read set*) records, for each object read, the object id as well as the original level from which its value was obtained. The descendants of t that have visited it are tracked though set the vts (*visited transaction set*) kept in an atomic variable. Finally, t also maintains a wait-for graph, wfg that is used by its children and t itself to detect and resolve deadlock situation while acquiring a lock on t's objects. An atomic object cm (*consistency management*) contains sets mts and its that are used to keep track of *merged* transactions and *incompatible* transactions respectively. Here also the atomic object used is an atomic register that can perform read and write together atomically, as mentioned in the Section 4.1.2.1. Further, lock mrg is used while searching for a local copy at the parent level, and during the merging of a child's sets with those of its parent.

5.4.4 Methods common to both root as well as non-root nodes (t_*)

$begin_{t_*}(t_p, level)$: Each transaction t_* begins with this method, where t_p denotes the id of its parent and $level$ denotes its level in the super tree. The set mts (merged transaction set) is initialized with t_*.

$invoke\_child_{t_*}(t_p)$: This method is used by transaction t_* to invoke a new child transaction t_c.

$unlock\_parent\_locks_{t_*}(s)$: This method is used by transaction t_* to release the locks on the parent objects in set s.

$check\_compatibility(cm_{t_a}, cm_{t_d})$: It is used to check the compatibility between two transactions using their respective consistency management objects.

$update\_cm(cm, s_m, s_i)$: It is used to perform the update of an atomic object cm, whereby contents of s_m and s_i are respectively appended to corresponding sets mts and its of cm. Owing to the atomic nature of cm, this update occurs in a single atomic step.

$get\_local\_lock_{t_*}(x, t_1)$: This method is used to obtain a lock on object $t_*.x$ for transaction t_1. In case of failure to lock $t_*.x$, transaction t_1 is aborted.

$get\_locks_{t_*}(s, t_c)$: This method is invoked by t_*'s child t_c to obtain locks on t_*'s objects in set s. If an attempt to lock $t_*.x$ using the *secure_lock* method is successful, then x is added to $t_c.pls$ (and prs) to indicate that t_c is in possession of a lock on its parent's object $t_*.x$.

$write_{t_*}(x, v)$: self-explanatory.

$abort_{t_*}()$: Upon aborting, t_* releases all the ancestral locks acquired through its own operations or those of its descendants. Next, as the children of the aborted transaction have to be aborted as well, t_* invokes the *abort_active_desc* method of all of its active children.

$abort\_active\_desc_{t_*}()$: This method is used by the parent t_* to force the abort of its active descendants. This is issued as a consequence of an abort of t_*.

$force\_abort_{t_*}()$: This method is used to abort a descendant such that it immediately returns *abort*, without having the need to release the parent locks in its posession. This invoked by an aborting ancestor.

$abort\_incompat\_desc_{t_*}(s_i)$: This method is used to abort the incompatible descendants of t_*. The set s_i contains the ids of subtransactions that are incompatible with t_*. This method propagates down the subtree rooted at t_* in cascading manner. At each of the descendants t_d we check if t_d is incompatible with t_*. If t_d is found incompatible, then we abort (discard) the subtree rooted at t_d. Otherwise, we check each of the active children of t_d iteratively.

5.4.5 Methods specific to non-root nodes (t_π)

$read_{t_\pi}(x)$: To read an object x, transaction t_π locks its local object $t_\pi.x$. If $t_\pi.x.val$ is not null, then the value of $t_\pi.x$ is returned after unlocking $t_\pi.x$. In case $t_\pi.x$ is null-valued, then t_π tries to read the value from its ancestors, using the method $search\_parent$. Before t_π invokes $search\_parent$ method, it locks $t_\pi.mrg$ to ensure that no incompatible child merges with it while it tries to read the value of x from its ancestor. Now, if the read is not successful, then t_π aborts. Otherwise, the value of $t_\pi.x$ is updated. Further, x is added to $t_\pi.pls$ to indicate the ownership of parent lock, and prs is updated to record the external read. The set $t_\pi.cm.its$ is updated with any incompatible descendant using $res_x.s_{its}$. The lock $t_\pi.mrg$ is unlocked and incompatible descendants (if any) of t_π are aborted. Object x is added to $t_\pi.lrs$. Finally, $t_\pi.x$ is unlocked and the value of $t_\pi.x$ is returned.

$search\_parent_{t_\pi}(x, t_c, t_o, cm_d)$: This method propagates recursively in a bottom-to-top manner. This method of t_π is invoked by its child transaction, t_c, to search for a local copy of object x available with t_π. The descendant originally trying to read the value of $t_\pi.x$ is t_o. First, a lock on $t_\pi.x$ is obtained. In case the value of $t_\pi.x$ is non-null, the compatibility of t_o and its ancestors up to t_c with t_π is checked. If compatible, then a response object res_x, containing $\langle t_\pi.x.val, t_\pi.lvl, \emptyset\rangle$, is returned. Otherwise, *null* is returned to indicate an unsuccessful read. Alternatively, if a local (non-null valued) copy of x is not available with t_π, then t_π forwards the search for a local copy of x to its parent node. If the res_x object obtained from the parent is null, then $t_\pi.x$ is unlocked before returning $null$. Otherwise, $\langle x, res_x.lvl\rangle$ is recorded in $t_\pi.prs$ and $t_\pi.pls$ is updated before returning res_x.

$try\_to\_merge_{t_\pi}(t_c)$: This method is invoked by the child transaction t_c. The steps are self-explanatory. However, it should be noted here that the order of steps in line 118 (obtaining mrg lock) and line 120 (locking of parent's objects) is important in order to avoid a deadlock situation between the child and the parent.

Observe that parent t_π, during it read step, obtains the lock $t_\pi.mrg$ (line 79) first and then locks its local object $t_\pi.x$ (line 80) before reading from its ancestor. Here, during the merging phase, the child transaction t_c also tries to first obtain the lock on $t_\pi.mrg$ (line 118) before object $t_\pi.x$ (in case it does, line 120). Only the transaction successful in obtaining the lock on $t_\pi.mrg$ proceeds to lock $t_\pi.x$. Note that locking of both $t_\pi.mrg$ and $t_\pi.x$ is done through *secure_lock* method to ensure any cyclic dependency is captured in the wait-for graph $t_\pi.wfg$. In case a deadlock is detected, the transaction will be aborted. After the merge is complete, locks are released up to the parent level.

$try\_to\_commit_{t_\pi}()$: self-explanatory.

5.4.6 Methods specific to root-node (t_ρ)

$search\_parent_{t_\rho}(x, t_c, t_o, cm_d)$: The behaviour of *search_parent* method associated with the root transaction (t_ρ) is somewhat different from the one associated with non-root transactions. This is due to that fact that, if a local copy of the desired object is not available with the root transaction, then it tries to read directly from the globally shared copy of the object.

First, it secures a lock on its local object $t_\rho.x$. If $t_\rho.x.val$ is non-null and t_o along with t_o's ancestors up to t_c are compatible with t_ρ, then a response object res_x containing $\langle t_\rho.x.val, t_\rho.lvl, \emptyset\rangle$, is returned. If not compatible, then the lock on $t_\rho.x$ is released and *null* is returned to indicate failure.

If $t_\rho.x$ is null-valued, then an attempt is made to read $t_\psi.x$. At this point, s_m is updated with the mts of all the nodes in the path from t_o to t_ρ (including t_ρ). Subtransaction t_o is added to $t_\rho.vts$. Now, after locking $t_\psi.x$, it is checked if any transaction in s_m is forbidden to access $t_\psi.x$ (i.e., present in $t_\psi.x.fbd$). If yes, then $t_\psi.x.rs$ is unlocked and *null* is returned to indicate failure. Otherwise, t_o is added to $t_\psi.x.rs$ before unlocking $t_\psi.x.rs$, and a response object containing $\langle t_\psi.x.val, t_\rho.lvl, s\rangle$ (s contains ids of descendants of t_o, if any, forbidden to read $t_\psi.x$) is returned.

$read_{t_\rho}(x)$: If a local copy of an object x is not available, then a non-root node calls the *search_parent* method of its parent. In contrast, a root node, having no parent, tries to read directly from the globally shared copy in that case. First, it locks the merge lock $t_\rho.mrg$ and local copy $t_\rho.x$ in order. If $t_\rho.x$ is null-valued, then it locks $t_\psi.x$. Next, it checks the consistency of its step by ensuring that no transaction in $t_\rho.cm.mts$ belongs to $t_\psi.x.fbd$. If the check fails, then $t_\psi.x$ is unlocked and t_ρ aborts.

Otherwise, t_ρ is added to $t_\psi.x.rs$, $t_\rho.x.val$ is updated using $t_\psi.x.val$, and $t_\rho.cm.its$ is updated using $t_\psi.x.fbd$ and $t_\rho.vts$, before unlocking $t_\psi.x$ and $t_\rho.mrg$. Incompatible descendants (if any) of t_ρ are aborted and x is added to $t_\rho.lrs$. Finally the value of $t_\rho.x$ is returned after unlocking it.

$try\_to\_commit_{t_\rho}()$: self-explanatory.

5.4.7 Regarding abort of a transaction and its descendants

When a transaction t in a transaction tree aborts, the execution of the entire subtree rooted at t, i.e., $subTree(t)$, has to be discarded and hence all the transactions in the $subTree(t)$ are aborted. When a subtransaction aborts, the key thing is to release the locks on objects of t's ancestors acquired by transactions in $subTree(t)$.

By construction of ParSTM, whenever t or any of its descendants obtains a lock on an object of t's ancestor, it is duly recorded at t's level (lines 113-114, 89-90 during external read; 49 during merging). Thus, when t aborts, it can act on behalf of the entire transactions in $subTree(t)$ and releases the locks of its ancestors' objects acquired by transactions in $subTree(T)$. Subsequently, when the descendants of t are forced to abort due to the abort of their ancestor t, they do not need to worry about releasing any locks in their possession (lines 58, 62, 66). This is owing to the fact that (i) an abort of t means that execution at the subtree level t is suspended anyway, and (ii) locks on objects of t's ancestors are already released by t (line 56-57).

The same strategy is followed for an abort of a transaction and its descendants in subsequent Chapters 6 and 7.

5.4.8 Optimization: abort of incompatible descendants

By construction of ParSTM, observe that during an external read or upon merging of a child, an ancestor t_a can detect early that its descendant, say t_d, is incompatible with it (due to line 88 or 130). As an incompatible child is not allowed to merge with its parent (due to line 121), it follows that t_d is bound to abort eventually when an attempt will be made to merge its execution with that of its ancestor t_a. To this end, we optimize by forcing the abort of incompatible descendants, such as t_d, the moment the ancestor t_a identifies them as incompatible (lines 95 or 135). This policy of preemptive abort of incompatible descendants by an ancestor has been followed in the protocols discussed in forthcoming Chapters 6 and 7 as well.

5.5 Consistency checking and linearization points at level t

5.5.1 Consistency checking during external read operation

When a descendant t_d tries to read a value from its ancestor t, the consistency checking (line 102 or 142) at t involves use of its local sets $t.cm$ which are not protected by any lock during this operation. During this check, it is possible that the contents of $t.cm$ may change, either due to concurrent external read of t (line 88) or merging of its child (line 130).

5.5.2 Linearization points of events in a level-wise history

The level wise event history $\widehat{\mathcal{H}_t}$ consists of the following events: (i) local read/write operations of t, (ii) external reads of t's descendants w.r.t. t, (iii) external reads of t itself, (iv) write operations due to merging of t's children t_c, and (v) commits of t's children.

Let ℓ_{op} denote the linearization point of an event. Then, the linearization points of the various events in the history are defined as follows:

i Local read/write operation of t

  (a) $read_t(t.x)$: ℓ_{op} corresponds to the time when it unlocks $t.x$ (line 94 or 177)

  (b) $write_t(t.x)$: ℓ_{op} corresponts to the time when it unlocks $t.x$ (line 53)

ii External read operation of t

  (a) $read_t(t_a.x)$: ℓ_{op} corresponds to the time just after $t.cm$ is updated (line 88 or 171)

iii External read of a descendant t_d on t's object or that of t's ancestor t_a:

  (a) $read_{t_d}(t.x)$: ℓ_{op} corresponds to the time just after t_d reads $t.cm$ for consistency checking (line 100 or 140)

  (b) $read_{t_d}(t_a.x)$: ℓ_{op} corresponds to the time just after t_d reads $t.cm$ for consistency checking (line 100 or 140)

iv Write due to merging of child t_c

  (a) $write_{t_c}(t.x)$: ℓ_{op} corresponds to the time just after t_c updates $t.x$ (line 125 or 127)

v Commit of child t_c

(a) C_{t_c} : ℓ_{op} corresponds to the time just after $t.cm$ is updated (line 130)

5.5.3 Ordering of external read/search at t with overlapping local operations of t

Let t_d be a descendant of t that performs the read operation $read_{t_d}(t.x)$ on object $t.x$. Let the events of reading the atomic object $t_2.cm$ by t_1 be denoted by $R_{t_1}(t_2.cm)$ and its local update by $W_{t_2}(t_2.cm)$. Then, the ordering of $read_{t_d}(t.x)$ during the following concurrent operation can be specified as follows:

i *External read operation* $read_t(t_a.y)$:

As update of $t.cm$ is atomic in nature, events $R_{t_d}(t.cm)$ (line 100 or 140) and $W_t(t.cm)$ (line 88 or 171) are linearizable, i.e., either $R_{t_d}(t.cm) < W_t(t.cm)$ or $R_{t_d}(t.cm) > W_t(t.cm)$. Consequently, the ordering of $read_{t_d}(t.x)$ and $read_t(t_a.y)$ follows the same order as $R_{t_d}(t.cm)$ and $W_t(t.cm)$.

ii *Local merge/commit operation* $try\_to\_merge_t(t_c)$:

Similarly, here events $R_{t_d}(t.cm)$ (line 100 or 140) and $W_t(t.cm)$ (line 130) are linearizable. We have either $R_{t_d}(t.cm) < W_t(t.cm)$ or $R_{t_d}(t.cm) > W_t(t.cm)$. Accordingly, $read_{t_d}(t.x)$ and C_{t_c} are ordered as well.

Two concurrent read operations $read_{t_{d1}}(t.x)$ and $read_{t_{d2}}(t.y)$ can be ordered arbitrarily w.r.t. to each other.

5.5.4 Linearization point of nested transaction

Definition of linearization point ℓ_t of a transaction t:

Case I: t is a non-root transaction (t_π)

1. If t commits, its linearization point, ℓ_t, lies at the time just after it updates the parent's *cm* (consistency management) object (line 130).

2. If t aborts, ℓ_t lies at the time it accessed $t_\pi.cm$ for the consistency check of its last successful read operation (lines 100 or 140).

Case II: t is a root transaction (t_ρ)

3. If t is a read only transaction that commits, ℓ_t is placed at the earliest of (1) the time of the test during its last read operation (line 150 or 166) and (2) the time just before $\hat{t}$ (any id in $t.cm.mts$) is added to $t_\psi.ow$, if it ever is (line 188).

4. If an update transaction t commits, ℓ_t is placed just after the execution of line 188 by t (update of $t_\psi.ow$).

5. If transaction t aborts, ℓ_t is placed just before $\widehat{t}$ is added to the set $t_\psi.ow$ (line 188 of the $try\_to\_commit_t()$ operation that entails its abort).

5.6 Proof

Here, the set of proofs is divided into three parts, dealing with: (1) (optimistic part) the history of committed transactions at the global level ($\Pi(\widehat{\mathcal{H}_{t_\psi}})$, (2) (pessimistic part) the level-wise history of committed transactions produced at the nodes ($\Pi(\widehat{\mathcal{H}_{t_\pi}})$) of a transaction tree, and (3) aborted transactions.

First, considering the history of committed transactions, we present the proof for the optimistic part, using the history produced at the global level ($\mathcal{H}_{t_\psi}$), and the proof for the pessimistic part using the history produced at the nodes ($\mathcal{H}_{t_\pi}$) in the transaction tree.

Note: All the line numbers under this section refer to Protocol 5.2 and 5.3 (Section 5.5).

5.6.1 Proof for committed transactions

In this section, we only consider the histories restricted to committed transactions.

Part I: History ($\widehat{\mathcal{H}_{t_\psi}}$) produced at the global level (t_ψ)

$\widehat{\mathcal{H}_{t_\psi}}$ deals with the linearizability of root transactions only. Thus, in effect, it is as good as dealing with non-nested transactions. The case of a root transaction here is similar to the one under SimpSTM. Let us observe the similarities for a root transaction t_ρ under SimpSTM and ParSTM.

- Set $t_\rho.cm.mts$ contains the ids of t_ρ and its descendants that have successfully merged with it.
- Sets $t_\rho.lrs$ and $t_\rho.lws$ are used to record read and write operations respectively by t_ρ or its descendants in $t_\rho.cm.mts$
- $t_\rho.cm.mts$ is used for consistency checking during the read operation on a global object $t_\psi.x$ ($t_\psi.x.fbd \cap t_\rho.cm.mts = \emptyset$) as well as during commit process ($t_\psi.ow \cap t_\rho.cm.mts = \emptyset$).

The correctness for the root transactions can be proved in the same way as done in Chapter 4. The only extra requirement here is the proof for the incompatible transactions. As discussed earlier, the merging of incompatible subtransactions with the parent would render the parent non-linearizable at the higher level. Hence, we need to show that incompatible transactions will not be merged together.

Lemma 5.1. *Let $\widehat{\mathcal{H}_t}$ be a level-wise history. Let t_1 and t_2 be any two distinct transactions in $\{t \cup children(t)\}$. If $r_{\widehat{t_2}}(t_\psi.x) : \beta(\widehat{t_1}, t_\psi.x.fbd, AL_{\widehat{t_2}}(t_\psi.x, read_{\widehat{t_2}}(t_\psi.x)))$, then we have (1) if $t_1, t_2 \neq t$, then $\neg(t_1 \in \Pi(\widehat{\mathcal{H}_t}) \wedge t_2 \in \Pi(\widehat{\mathcal{H}_t}))$ or (2) $t_1 = t \wedge t_2 \notin \Pi(\widehat{\mathcal{H}_t})$ or (3) $t_2 = t \wedge t_1 \notin \Pi(\widehat{\mathcal{H}_t})$.*

Proof. Here, incompatibility of transactions comes into the picture when subtransactions read from the globally shared objects.

Since t_1 and t_2 are distinct (incompatible) transactions, either both the transactions are children of t, or one of the two is t, while the other one is a child of t. First, we show that

$r_{\widehat{t_2}}(t_\psi.x) : \beta(\widehat{t_1}, t_\psi.x.fbd, AL_{\widehat{t_2}}(t_\psi.x, read_{\widehat{t_2}}(t_\psi.x))) \Rightarrow \widehat{t_1} \in t_2.cm.its.$

$r_{\widehat{t_2}}(t_\psi.x) : \beta(\widehat{t_1}, t_\psi.x.fbd, AL_{\widehat{t_2}}(t_\psi.x, read_{\widehat{t_2}}(t_\psi.x)))$ means that when subtransaction $\widehat{t_2}$ acquired lock on $t_\psi.x$ to perform a read operation, $\widehat{t_1} \in t_\psi.x.fbd$. Observe that, by construction, before releasing the lock on $t_\psi.x$, transaction $\widehat{t_2}$ adds $\widehat{t_1}$ to $\widehat{t_2}.cm.its$ (lines 152-155, 88; 171) using intersection of $t_\rho.vts$ and $t_\psi.x.fbd$ (due to line 148). Later, $\widehat{t_2}$ releases the lock on its local copy $\widehat{t_2}.x$ (lines 94, 177). Thus, we have following implications:

$r_{\widehat{t_2}}(t_\psi.x) : \beta(\widehat{t_1}, t_\psi.x.fbd, AL_{\widehat{t_2}}(t_\psi.x, read_{\widehat{t_2}}(x))) \Rightarrow \beta(\widehat{t_1}, \widehat{t_2}.cm.its, RL_{\widehat{t_2}}(t_\psi.x, read_{\widehat{t_2}}(x)))$
$\Rightarrow \widehat{t_1} \in \widehat{t_2}.cm.its$ (due to line 153-155, 88; 170-171).

Recall that $\widehat{t_2}$ denotes a (sub)transaction in $t_2.cm.mts$. If $\widehat{t_2} \neq t_2$, then it means $\widehat{t_2}$ is t_2's descendant that merged with t_2, and in the process merged its its with that of t_2 (line 130).

Therefore, $\widehat{t_1} \in \widehat{t_2}.cm.its \wedge \widehat{t_2} \in t_2.cm.mts \Rightarrow \widehat{t_1} \in t_2.cm.its$.

$\Rightarrow t_1$ and t_2 are incompatible transactions.

Now, we want to show that the two incompatible transactions, t_1 and t_2, cannot be merged together. Let us consider the first case in which t_1, t_2 are children of t. We have to show $t_1 \in \Pi(\widehat{\mathcal{H}_t}) \Rightarrow t_2 \notin \Pi(\widehat{\mathcal{H}_t})$ and vice versa.

Case I: $t_1, t_2 \in children(t)$.

Observe that, for merging with its parent, each child transaction t_c has to first obtain its parent's (t_p) lock $t_p.mrg$ (line 118). This ensures that only one child of t_p merges with it at a time. Now, we have the following two subcases to consider:

Case I(a): $AL_{t_1}(t.mrg, ttc) <_{\mathcal{H}_t} AL_{t_2}(t.mrg, ttc)$ (assuming $t_1 \in \Pi(\widehat{\mathcal{H}_t})$).

$AL_{t_1}(t.mrg, ttc) <_{\mathcal{H}_t} AL_{t_2}(t.mrg, ttc)$

$\Rightarrow RL_{t_1}(t.mrg, ttc) <_{\mathcal{H}_t} AL_{t_2}(t.mrg, ttc) \Rightarrow \beta(\widehat{t_1}, t.cm.mts, RL_{t_1}(t.mrg, ttc))$ (due to line 130)

$\Rightarrow \beta(\widehat{t_1}, t.cm.mts, AL_{t_2}(t.mrg, ttc))$

This means, when t_2 locks $t.mrg$ to merge with t, it will discover that it is incompatible with t as $t.cm.mts$ already contains $\widehat{t_1}$, i.e., $\widehat{t_1} \in t.cm.mts \cap t_2.cm.its$. Consequently t_2 aborts (due to line 121-123). Thus, $t_1 \in \Pi(\widehat{\mathcal{H}_t}) \Rightarrow t_2 \notin \Pi(\widehat{\mathcal{H}_t})$.

Case I(b): $AL_{t_2}(t.mrg, ttc) <_{\mathcal{H}_t} AL_{t_1}(t.mrg, ttc)$ (assuming $t_2 \in \Pi(\mathcal{H}_t)$).

The proof is symmetric to *Case I(a)*. $AL_{t_2}(t.mrg, ttc) <_{\mathcal{H}_t} AL_{t_1}(t.mrg, ttc)$

$\Rightarrow RL_{t_2}(t.mrg, ttc) <_{\mathcal{H}_t} AL_{t_1}(t.mrg, ttc) \Rightarrow \beta(\widehat{t_1}, t.cm.its, RL_{t_2}(t.mrg, ttc))$ (due to lines 152-155, 88; 130).

$\Rightarrow \beta(\widehat{t_1}, t.cm.its, AL_{t_1}(t.mrg, ttc))$

In this case, t_1 aborts later on (due to lines 121-123) as $t.cm.its$ already contains $\widehat{t_1}$, i.e., $t.cm.its \cap t_1.cm.mts = \widehat{t_1} \neq \emptyset$. We have $t_2 \in \Pi(\mathcal{H}_t) \Rightarrow t_1 \notin \Pi(\mathcal{H}_t)$.

Case II: Either $t_1 = t$, or $t_2 = t$.

Case II(a): $t_1 = t$.

$t_1 = t \Rightarrow t_1$ is the parent of t_2

$\Rightarrow t_1$ is an ancestor of each $\widehat{t_2} \in t_2.cm.mts$.

$\Rightarrow$ read operation $r_{\widehat{t_2}}(t_\psi.x) : \beta(\widehat{t_1}, t_\psi.x.fbd, AL_{\widehat{t_1}}(t_\psi.x, read_{\widehat{t_2}}(t_\psi.x)))$ is not possible ($failure$ due to lines 107, 155, 150; 159, 84-85).

Case II(b): $t_2 = t$.

Similar to *Case I(b)*. t_2, being the ancestor t, adds $\widehat{t_1}$ to $t.cm.its$ anyway, leading to abort of t_1 right away (due to lines 153-155, 88; 170-171; 102-104; 153-155; 83-85).

Further, observe that t might be involved in reading from its ancestor a value that is incompatible with its child that is trying to merge with it. This situation is avoided by ensuring that, while t is reading (from its ancestors), its child cannot merge with it. This is due to the fact that, during its read operation, t locks $t.mrg$ which is required to be locked by its child transaction in order to merge with t.

Thus, the conjunction of the *cases I* and *II* proves the lemma.

□

Lemma 5.1 shows that the read set of a transaction cannot contain incompatible read operations, thereby ensuring its linearizability at ancestor levels.

Theorem 5.1. *The level-wise history* $\Pi(\widehat{\mathcal{H}_{t_\psi}})$ *of committed transactions, produced at the global level, satisfies level-wise opacity.*

Proof. The proof follows from the combination of the definition of linearization points (2 and 3) for root transactions, Lemma 5.1, and on the basis of the set of proofs (Lemmas 4.3, 4.4 and 4.5) outlined for root transactions in Chapter 4.

□

Part II: History $(\widehat{\mathcal{H}_{t_\pi}})$ **produced at a node** t_π **of transaction tree**

We shall prove that the execution of the children of t_π follows the 2PL protocol for nested transactions, i.e., the children of t_π hold the lock on the parent's (t_π) objects in a pessimistic manner. They retain the lock on the parent's object until they complete (commit/ abort). In other words, we shall prove that any two children of t_π, say t_1 and t_2, can execute concurrently only if they do not operate on a common object $t_\pi.x$. We show that 2PL policy is followed by the child transaction for locking its parent's objects through the following two points.

Let node t_c be a child of node t in a transaction tree.

1. *If* t_c *successfully acquires a lock on* t*'s local object* $t.x$*, then the lock on* $t.x$ *is not released until all the transactions in the subtree rooted at* t_c *complete (commit or abort).*

2. *When* t_c *commits or aborts, it releases all the locks in its possession at its parent* (*t*) *level.*

Next, we provide the set of proofs to show that the history produced by ParSTM at the nodes of a transaction tree satisfies the above conditions.

Lemma 5.2. *Let t be a child of t_π. If t acquires a lock on t_π's object, $t_\pi.x$, then t releases the lock on $t_\pi.x$ only upon its completion.*

Proof. By construction, t acquires a lock on $t_\pi.x$ either during its read operation (lines 98, 138) or while trying to merge with the parent (line 120 during *try_to_merge*). If $t.x$ is null-valued, then t tries to obtain a value for $t.x$ by invoking the method $search\_parent_{t_\pi}$ (line 109) of the parent. Note that, in the definition of the method $search\_parent_{t_\pi}$, there is locking of object $t_\pi.x$ (lines 98, 138), but no unlocking of $t_\pi.x$ in case of a successful read operation. Unlocking is done only if the read is unsuccessful (lines 102-103, 110-111, 142-143). The only other case when t locks t_π's objects is during the validation phase (line 120, *try_to_merge* method) towards the end of t's execution. Again by construction of the protocol, the lock on $t_\pi.x$ is released upon commit (successful validation; lines 132) or on abort (line 123, 55-57). Thus, t retains the lock on $t_\pi.x$ until its completion. □

Lemma 5.3. *If t has read from $t_\pi.x$, then no other transaction, t_π or any other child of t_π, can modify $t_\pi.x$ until t completes its execution.*

Proof. If t has read $t_\pi.x$, then it means that t currently holds the (exclusive) lock on $t_\pi.x$ (lines 98, 138). By construction, any read (lines 80, 98, 138, 163) or write (lines 51, 119) operation is controlled using a lock associated with $t_\pi.x$. That means, to operate on $t_\pi.x$, t_π or its children must first acquire a lock on $t_\pi.x$. The lock on $t_\pi.x$ can be acquired by another transaction (t_π or any of its children) only after t releases the lock on $t_\pi.x$ first. Since t releases the lock on $t_\pi.x$ only upon its completion (by Lemma 5.2), it means $t_\pi.x$ cannot be modified by another transaction until t completes. $\square$

Lemma 5.4. *Let $t_1 \xrightarrow{t_\pi.x}_\alpha t_2$ be the relation defined as: t_2 accesses (the parent's object) $t_\pi.x$ after it was accessed by t_1 for its read/write operation. Then, $t_1 \xrightarrow{t_\pi.x}_\alpha t_2 \Rightarrow t_1 \rightarrow_{\mathcal{H}^\sigma_{t_\pi}} t_2$.*

Proof. We have $t_1 \xrightarrow{t_\pi.x}_\alpha t_2 \Rightarrow AL_{t_1}(t_\pi.x, r/w) <_{\mathcal{H}_{t_\pi}} AL_{t_2}(t_\pi.x, r/w)$

$\Rightarrow AL_{t_1}(t_\pi.x, r/w) <_{\mathcal{H}_{t_\pi}} RL_{t_1}(t_\pi.x, ttc) <_{\mathcal{H}_{t_\pi}} AL_{t_2}(t_\pi.x, r/w) <_{\mathcal{H}_{t_\pi}} RL_{t_2}(t_\pi.x, ttc)$ (using Lemma 5.2).

$\Rightarrow RL_{t_1}(t_\pi.x, ttc) <_{\mathcal{H}_{t_\pi}} RL_{t_2}(t_\pi.x, ttc)$

$\Rightarrow \ell_{t_1} <_{\mathcal{H}_{t_\pi}} \ell_{t_2}$

$\Rightarrow t_1 \rightarrow_{\mathcal{H}^\sigma_{t_\pi}} t_2$ $\square$

Lemma 5.5. *The level wise event history $\widehat{\mathcal{H}_t}$ at node t is linearizable.*

Proof. The proof follows from the definition of the linearization points of the events for a level wise histroy, in Section 5.2.2.

$\square$

While a descendant t_d reads or checks for a value at an ancestor t, several other operations (external reads of other transactions, t's local or external read operation, or commit of t's child) could be happening concurrently at t. Descendant t_d can become incompatible with t either due to external read of t or commit of a t's child. We shall show that the consistency checking on the value read by t_d from t is guaranteed to be correct.

Now, similar to the definition of $\beta(t_1, t.ow, \tau)$, let us define $\gamma_{inc}(t_1, t, \tau)$ to indicate that transaction t_1 becomes incompatible with transaction t at time τ. In other words, at time τ, $check\_compatibility(t_1.cm, t.cm) = false$. Using this definition, we shall prove the next lemma.

Lemma 5.6. *Let t_d be a descendant of t such that history $op_t, read_{t_d}(t.x) \in \widehat{\mathcal{H}_t}$, where op_t denotes an external read operation or a commit of t's child at time τ. Then, we show that (i) $op_t, read_{td}(t.x)$ can be ordered, and (ii) $op_t < read_{t_d}(t.x) \Rightarrow \neg\gamma_{inc}(t_d, t, \tau)$, i.e., t and t_d are not inconsistent before time τ.*

Proof. By definition of the linearization points for op_t and $read_{t_d}(t.x)$ in $\widehat{\mathcal{H}_t}$ w.r.t. access to $t.cm$, and $t.cm$ being an atomic variable, it follows $op_t, read_{t_d}(t.x)$ in $\widehat{\mathcal{H}_t}$ are linearizable, i.e., $op_t < read_{t_d}(t.x)$ in $\widehat{\mathcal{H}_t}$ or $op_t > read_{t_d}(t.x)$ in $\widehat{\mathcal{H}_t}$

To prove part (ii), let us assume by contrast that $\gamma_{inc}(t_d, t, \tau)$, i.e., at the completion of op_t at time τ, $check\_compatibility(t.cm, t_d.cm) = false$.

Now, for $op_t < read_{t_d}(t.x)$, we have the following two cases:

Case I: op_t is an external read operation of t

By definition of linearization points for events in a level wise history (Section 5.5.3), we have

$\ell_{op_t} < \ell_{read_{t_d}(t.x)}$

$\Rightarrow$ update of $t.cm$ (line 88 or 171) such that $check\_compatibility(t.cm, t_d.cm) = false$ occurs before $t.cm$ is read (line 100 or 140) for consistency check of t_d. (Recall that $t.cm$ is kept in an atomic variable.)

$\gamma_{inc}(t_d, t, \tau) < \ell_{read_{t_d}(t.x)}$

$\Rightarrow check\_compatibility(t.cm, t_d.cm) = false$ during consistency check for $read_{t_d}(t.x)$ (line 102 or 142)

$\Rightarrow read_{t_d}(t.x)$ fails (due to lines 102-104 or 142-144), i.e., $op_t < read_{t_d}(t.x)$ is not possible in this case.

Case II: op_t is commit of t's child

By definition of the linearization points for events in a level wise history (Section 5.5.3), we have

$\ell_{op_t} < \ell_{read_{t_d}(t.x)}$

$\Rightarrow$ update of $t.cm$ (line 130) such that $check\_compatibility(t.cm, t_d.cm) = false$ occurs before $t.cm$ is read (line 100 or 140) for consistency check of $read_{t_d}(t.x)$. (Recall that $t.cm$ is kept in an atomic variable.)

$\gamma_{inc}(t_d, t, \tau) < \ell_{read_{t_d}(t.x)}$

$\Rightarrow$ $check\_compatibility(t.cm, t_d.cm) = false$ during consistency check for $read_{t_d}(t.x)$ (line 102 or 142)

$\Rightarrow$ $read_{td}(t.x)$ fails (due to lines 102-103 or 142-144), i.e., $op_t < read_{td}(t.x)$ is not possible in this case also.

Hence, after analysis of the both the cases, we conclude that $op_t < read_{t_d}(t.x) \Rightarrow check\_compatibility(t.cm, t_d.cm) = true$ upon completion of op_t.

$\square$

Theorem 5.2. *Level wise transaction history of committed transactions $\Pi(\widehat{\mathcal{H}^{\sigma}_{t_\pi}})$ produced at a node t_π follows 2PL for nested transactions, and satisfies level-wise opacity.*

Proof. The proof follows from lemmas 5.2 to 5.6 that show that the execution of subtransactions in the transaction tree is based on the 2PL for nested transactions, and is linearizable (using the definition of linearization points). Thus, it satisfies level-wise opacity.

□

5.6.2 Proof for aborted transactions

Before diving into showing the correctness of aborted transactions, we prove that in case of an abort of a subtransaction, the objects locked by it are indeed released up to the respective ancestors from which they were read. To complete the scenario, we shall also show that the objects are released only up to the parent level in case of commit.

Lemma 5.7. *When a subtransaction t commits, it releases the locks only up to its parent. However, in case of its abort, it releases the locks on ancestors' objects held by t and its descendants all the way up to the respective level from which they were read.*

Proof. The locks obtained by a transaction t at an ancestor level, either through *search_parent* method or its *commit*, are recorded in $t.prs$ (lines 90, 120; 49). Proving that the locks are released only up to the parent level in case of commit is straight forward. In case of a commit, locks are released through *unlock_parent_locks* method (line 132). Observe that this method only unlocks objects at t's parent level (line 37). It does not propagate to higher levels. Thus, in case of a commit, the locks are released only up to the parent level.

In case of an abort of t, it is t's responsibility to release all the locks on its ancestors' objects held by t or its descendants. Such locks obtained by t itself are tracked during *search_parent* (line 113-114) or during *try_to_merge* (lines 120, 49). The locks obtained on behalf of t's descendants are logged during *search_parent* (lines

113-114). Thus, t is able to track all the locks held by it or its descendants on t's ancestors' objects.

Now, in case of t's abort, *unlock_to_ancestors* method is invoked (line 56) to release the locks for objects in *t.prs* at higher levels. Observe that *unlock_to_ancestors* is cascading in nature. Using the level information associated with objects in *t.prs* (*pessimistic read set*), first lock at parent level is released (line 40), and then at higher level ancestors in a cascading manner (line 42) until the required level is reached (line 39, 41). This way, locks are released all the way up to the respective level from which an object was originally read in the chain, and not just up to the parent level.

□

To show the correctness of aborted transactions, we consider one aborted transaction t_a at a time in the transaction tree and obtain its closure. The level-wise histories at different levels are obtained in the same fashion as done in Chapter 3 (refer to Section 3.7). However, we should show that the closure does not contain incompatible read operations.

Owing to the concurrent execution of transactions at the nested levels, it is quite possible that two active transactions in the transaction tree are mutually incompatible. We need to show that these two transactions are not part of t_a or any of its ancestors in the closure for t_a.

Lemma 5.8. *Let $\mathcal{H}$ denote the execution of the entire transaction tree in which t_a is an aborted subtransaction whose last operation occurs at time τ^{t_a}. Then, t_a is compatible with other transactions in the closure $\mathcal{H}^{\mathcal{C}_{t_a}}$ for t_a.*

Proof. Following the definition of the closure $\mathcal{H}^{\mathcal{C}_{t_a}}$ for t_a, let $read_{t_a}(t.x)$ be the last operation of t_a. Clearly, $read_{t_a}(t.x)$ is the last operation in $\mathcal{H}^{\mathcal{C}_{t_a}}$.

Assume that t_1 and t_2 are two incompatible transactions belonging to $\mathcal{H}^{\mathcal{C}_{t_a}}$. Let P denote the set containing t_a as well as its ancestors, and S denote all the transactions whose steps are represented in $\mathcal{H}^{\mathcal{C}_{t_a}}$. Clearly, $t_1, t_2 \in S$. In other words, t_1, t_2 either belong to P or have successfully merged with some transactions in P. Also, now we have two cases.

Case I: t_1 and t_2 are two committed descendants that have merged with a node t in P. This is not possible as at any level, two incompatible subtransactions are not allowed to merge with the parent (follows from Lemma 5.1). This implies that both t_1 and t_2 cannot be part of S (i.e., $\neg(t_1 \in S \wedge t_2 \in S)$). Hence, the contradiction.

Case II: t_1 is part of t_i and t_2 is part of t_j, where t_i, t_j are two distinct transactions in P.

Clearly, t_i and t_j have an ancestor-descendant relationship. For simplicity, let t_i be an ancestor of t_j.

Now, if $t_1 = t_i$, then t_1 can become incompatible with t_2 by performing an external read operation, say $r_{t_1}(t_p.x)$, that is incompatible with the previous read steps of t_2. By construction of ParSTM, before completion of $r_{t_1}(t_p.x)$, the incompatible descendant t_j (as it contains t_2) is aborted (line 95). Now t_a clearly being a descendant of t_i, it means the last operation of t_a (at time τ^{t_a}) occurred before $r_{t_1}(t_p.x)$. This in turn contradicts the fact that $r_{t_1}(t_p.x)$ is part of $\mathcal{H}^{\mathcal{C}_{t_a}}$, as all the steps after τ^{t_a} are discarded in $\mathcal{H}^{\mathcal{C}_{t_a}}$ (Section 3.7.4, step (i)).

Alternatively, t_i can become incompatible with t_j (containing t_2) when $\widehat{t_1}$ commits and merges with t_i. Even in this case, by construction of ParSTM, t_j will be aborted before $\widehat{t_1}$ commits (due to line 135). That means $t_1 \notin S$. Hence, the contradiction.

Note that an incompatible descendant t_j (containing t_2) cannot exist after becoming incompatible with its ancestor t_i (due to lines 85 or 135). That means all the steps of t_2 were consistent and completed before it became incompatible with t_i.

Thus, we conclude that all the steps in $\mathcal{H}^{\mathcal{C}_{t_a}}$ are compatible.

$\square$

Theorem 5.3. *The history $\Pi(\mathcal{H}^{\mathcal{C}_{t_a}})$ for an aborted transaction t_a satisfies level-wise opacity.*

Proof. Using Lemma 5.5, it has been proved that $\mathcal{H}^{\mathcal{C}_{t_a}}$ consists of compatible steps only. Moreover, recall that by construction of a closure, transaction t_a and its active ancestors are transformed into committed transactions in $\mathcal{H}^{\mathcal{C}_{t_a}}$. That means, $\Pi(\mathcal{H}^{\mathcal{C}_{t_a}})$ represents a history of committed transactions. Now the set of proofs for a history of committed transactions in Section 5.5.1 can be applied directly to $\Pi(\mathcal{H}^{\mathcal{C}_{t_a}})$. This implies that $\Pi(\mathcal{H}^{\mathcal{C}_{t_a}})$ too satisfies level-wise opacity. $\square$

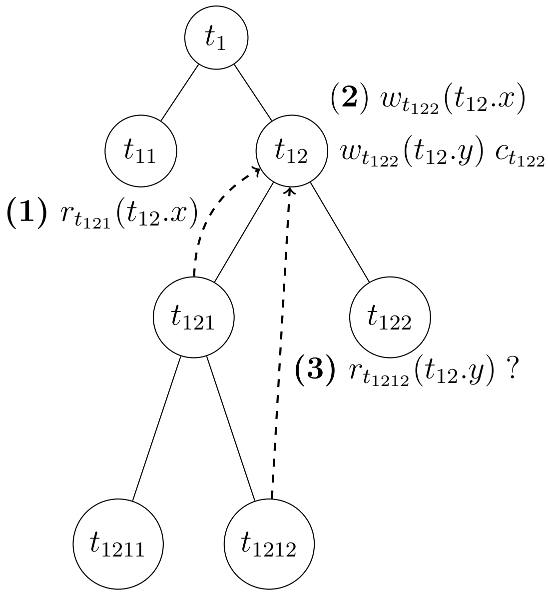
Chapter 6

HParSTM

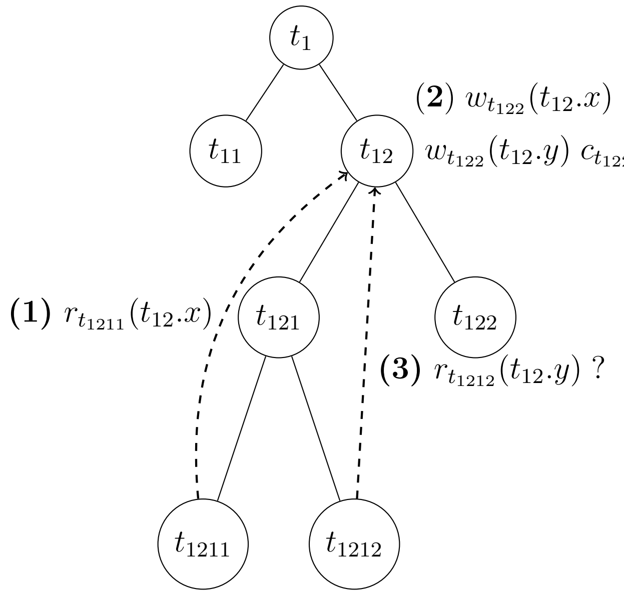
6.1 Overview of HParSTM

In the previous chapter, we discussed ParSTM that exercised pessimistic concurrency control at the nested level. In this chapter we discuss HParSTM that leverages full concurrency by allowing all transactions in a transaction tree to execute concurrently. In simplest terms, this is achieved by replicating the optimistic concurrency control mechanism discussed for non-nested transactions in Section 2.1 at every level (node) of a transaction tree.

In other words, a control variable ow (overwritten set) and lock-based local copy of objects, using sets rs (read set) and fbd (forbidden set) as control variables, are used at each level. Moreover, similar semantics are associated with the operations on these control variables. A (sub)transaction t's local copy of object x, $t.x$, is accessible to t as well as its descendants. When the descendants of t access $t.x$, they add their ids to $t.x.rs$. Later, if $t.x$ is modified by t or its children, then the ids of descendants present



(a) For history $\mathcal{H}_1$



(b) For history $\mathcal{H}_2$ and $\mathcal{H}_4$

Figure 6.1: Regarding consistency of read operations

in $t.x.rs$ are added to $t.ow$, followed by updating $x.fbd$ using $t.ow$, and clearing $t.x.rs$ (similar to Protocol given Section 2.1: lines 17-18 in Table 2.1).

6.2 Discussion of contention management

6.2.1 Standard cases

Most of the scenarios and approaches presented in this section have already been discussed in previous chapters. However, to set the context for HParSTM, we shall discuss the various cases in light of the protocol in this chapter.

6.2.1.1 Consistency checking at the time of a read operation

Each transaction t uses *mts* (*merged transaction set*) to keep track of the descendants that have merged with t. When a subtransaction t tries to read from an object $t'.x$

from its ancestor $t^{'}$'s local space, it should ensure that none of the transactions in mts of t or any of its intermediate ancestors belongs to $t^{'}.x.fbd$.

Example:

$$\mathcal{H}_1 = r_{t_{121}}(t_{12}.x), w_{t_{122}}(t_{12}.x), w_{t_{122}}(t_{12}.y)c_{122}, r_{t_{1212}}(t_{12}.y)?$$

Referring to Figure 6.1a, t_{121} (t_{1212}'s ancestor) reads from $t_{12}.x$, therefore $t_{121} \in t_{12}.x.rs$. Later, t_{122} modifies $t_{12}.x$ and $t_{12}.y$, while merging with t_{12}, resulting in $t_{121} \in t_{12}.x.fbd$, $t_{12}.y.fbd$ and $t_{12}.ow$. Later, t_{1212} tries to read $t_{12}.y$. Now, although $t_{1212} \notin t_{12}.y.fbd$, it is prevented from reading $t_{12}.y$ (which is $t_{122}.y$) as its intermediate ancestor t_{121} belongs to $t_{12}.y.fbd$.

$$\mathcal{H}_2 = r_{t_{1211}}(t_{12}.x)c_{1211}, w_{t_{122}}(t_{12}.x), w_{t_{122}}(t_{12}.y)c_{122}, r_{t_{1212}}(t_{12}.y)?$$

Consider another example using Figure 6.1b. In $\mathcal{H}_2$, instead of t_{121} reading from $t_{12}.x$, its child t_{1211} reads from $t_{12}.x$ and merges with t_{121}. In this case, when t_{1212} tries to read $t_{12}.y$, it will notice $t_{1211} \in t_{12}.y.fbd$. Now, as t_{1211} has already become part of t_{121}, it is illegal for t_{1212} to access $t_{12}.y$ and is consequently prevented.

6.2.1.2 Avoiding cyclic conflict through transitivity across levels

In the nesting of transactions, there is a possibility of cyclic conflict between transactions not only at the same level (between sibling transactions) but also between transactions at different levels (ancestors and descendants). The cyclic conflict between sibling subtransactions is automatically taken care of by the control variables at their parent's level. The cyclic conflict between transactions at different levels is

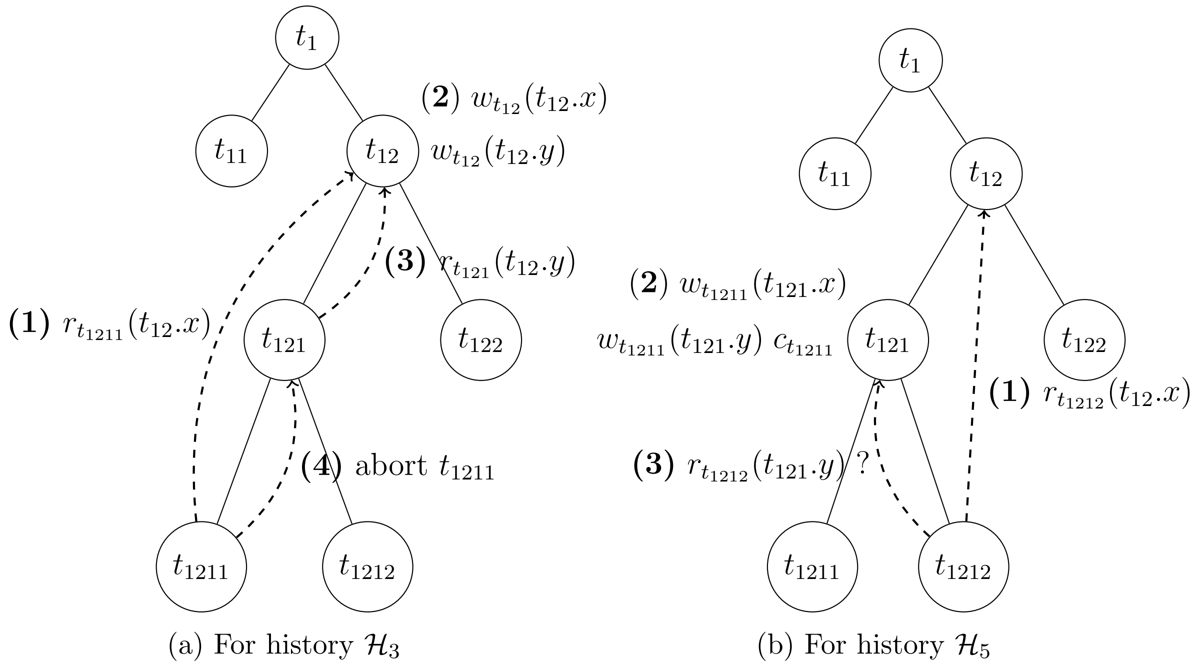


Figure 6.2: Reading from different levels

handled by aborting a subtransaction as soon as it becomes incompatible with its ancestor.

Example:

$\mathcal{H}_3 = r_{t_{1211}}(t_{12}.x), w_{t_{12}}(t_{12}.x), w_{t_{12}}(t_{12}.y), r_{t_{121}}(t_{12}.y)$?

Refer to Figure 6.2a for history $\mathcal{H}_3$. Here, first t_{1211} reads from $t_{12}.x$ ($\Rightarrow t_{1211} \in t_{12}.x.rs$). Later, t_{12} modifies $t_{12}.x$ and $t_{12}.y$ ($\Rightarrow$ $t_{1211} \in t_{12}.x.fbd$ and $t_{12}.y.fbd$). Next, t_{121} successfully reads from $t_{12}.y$ (as $t_{121} \notin t_{12}.y.fbd$) and creates its own local copy $t_{121}.y$. Observe that $r_{t_{1211}}(t_{12}.x)$ is incompatible with $r_{t_{121}}(t_{12}.y)$. Therefore, t_{121} immediately aborts t_{1211}.

6.2.1.3 Keeping track of incompatible read operations

In case of ParSTM, the incompatible read operations are introduced when subtransactions read from the global objects which are operated through optimistic concurrency control. In case of HParSTM, this may happen while reading from any ancestor in the super tree. This is due to the fact that the objects at each level in HParSTM are operated through optimistic concurrency control. As discussed in Section 5.3, here also each transaction keeps track of its incompatible transactions using set *its* (*incompatible transaction set*).

Example:

$\mathcal{H}_4 = r_{t_{1211}}(t_{12}.x), w_{t_{12}}(t_{12}.x), w_{t_{12}}(t_{12}.y), r_{t_{1212}}(t_{12}.y), (c_{1211}, c_{1212})?$

Consider the history $\mathcal{H}_4$ using Figure 6.1b. Here, $t_{1211} \in t_{12}.x.rs$. Next, t_{12}, on $w_{t_{12}}(t_{12}.x)$ and $w_{t_{12}}(t_{12}.y)$, adds t_{1211} to $t_{12}.x.fbd, t_{12}.y.fbd$ and $t_{12}.ow$. Now, t_{1212}, during $r_{t_{1212}}(t_{12}.y))$, finds $t_{1211} \in t_{12}.y.fbd$.

Here t_{1212} is incompatible with t_{1211}. Both t_{1211} and t_{1212} share the common ancestor t_{121} under t_{12}, and the conflicting write operations of t_{12} are sandwiched between the respective read operations of t_{1211} and t_{1212} on t_{12}'s objects. Therefore, t_{1212} adds t_{1211} to its *its*.

6.2.2 Special cases

In this section, we discuss the special cases presented by HParSTM as well as the solutions to address them.

6.2.2.1 Tracking overwrite at intermediate ancestor level

In ParSTM, when a subtransaction t reads an object x from its ancestor t_a, the pessimistic locking scheme ensures that the value of x is not changed at each of its intermediate ancestors, including t_a. The optimistic concurrency control in HParSTM offers no such guarantee. This poses a challenge for maintaining consistency. To illustrate the scenario, consider the following case.

Example:

$\mathcal{H}_5 = r_{t_{1212}}(t_{12}.x), w_{t_{1211}}(t_{121}.x), w_{t_{1211}}(t_{121}.y)c_{1211}, r_{t_{1212}}(t_{121}.y)?$

In history $\mathcal{H}_5$, t_{1212} first reads $t_{12}.x$. Later, t_{1211} modifies objects x and y in t_{121}'s (t_{1212}'s ancestor) local space, thus t_{1211} modifies the value of the object x previously read by t_{1212}. Now, if t_{1212} is allowed to read the value of y written by t_{1211} in t_{121}'s local space, then a cyclic conflict is established between t_{1211} and t_{1212}. As t_{1212} has read $t_{121}.y$ written by t_{1211}, the serial order should be t_{1211}, t_{1212}. This means that t_{1212} should have read a value of object $t_{121}.x$ written by t_{1211}, instead of the value of $t_{12}.x$, which is not true. Hence, the cycle.

Solution. When a subtransaction t reads an object $t^{'}.x$ from its ancestor $t^{'}$, besides adding its id to $t^{'}.x.rs$, it adds its id to read set $t^{''}.x.rs$ of the null-valued local object $t^{''}.x$ of each of the intermediate ancestors $t^{''}$ as well. Now, since t_{1212} accessed $t_{12}.x$ before t_{1211} merges with their common ancestor t_{121}, it means $t_{121}.x.rs$ already contained t_{1212} before $t_{121}.x$ is modified. Hence, after modification of $t_{121}.x$ and $t_{121}.y$, we have $t_{1212} \in t_{121}.x.fbd, t_{121}.y.fbd$ and $t_{121}.ow$. This ensues that t_{1212} is forbidden to access $t_{121}.y$ or any other object $t_{121}.z$ written along/after the write of $t_{121}.x$.

6.2.2.2 Significance of vts

$\mathcal{H}_6 = r_{t_{122}}(t_{12}.x), c_{122}, r_{t_{1211}}(t_{12}.x)w_{t_{12}}(t_{12}.x), w_{t_{12}}(t_{12}.y), r_{t_{1212}}(t_{12}.y), (c_{121}, c_{1212})$?

Consider $\mathcal{H}_6$ in which we extend $\mathcal{H}_4$ by adding $r_{t_{122}}(t_{12}.x), c_{122}$. Transaction t_{122} is a committed child of t_{12} and has read $t_{12}.x$. Subtransaction t_{1211} also has read $t_{12}.x$. Later, when t_{12} writes $t_{12}.x$ and $t_{12}.y$, both t_{122} and t_{1211} are added to $t_{12}.x.fbd$ and $t_{12}.y.fbd$. Next, t_{1212} reads $t_{12}.y$. Here, t_{122} and t_{1212} are compatible, but not t_{1211} and t_{1212}. Observe that during $r_{1212}(t_{12}.y)$, $t_{12}.y.fbd$ contains both t_{1211} and t_{122}, but t_{1212} should add only t_{1211} (not t_{121}) to its *its*. Otherwise, t_{1212} will definitely be not able to merge with t_{12}, even under a valid scenario.

This is where set *vts* comes handy. Note that t_{122} and t_{1211} belong to different subtrees under t_{12}. Differently, set $t_{121}.vts$ contains t_{1211} but not t_{122} for the same reason. Hence, t_{1212} uses $t_{121}.vts$ during $r_{t_{1212}}(t_{12}.y)$ to filter out t_{122} and add only t_{1211} to $t_{1212}.cm.its$.

6.3 Protocol

Protocol 6.1: HParSTM

State of base object x:
$val :\in V$
rs and $fbd : \subset T$

State of response object:
$res_x(value, level, s)$:
$val \in V$, set to $value$
$lvl \in L$, set to $level$
$s_{its} \subset T$, set to s

Helper methods:
Operation $check\_compatibility(cm_{t_a}, cm_{t_d})$:
return $((cm_{t_a}.mts \cap cm_{t_d}.its = \emptyset) \wedge (cm_{t_a}.its \cap cm_{t_d}.mts = \emptyset))$;

Operation $\mathbf{update\_cm(cm, s_m, s_i)}$:
$cm.mts \leftarrow cm.mts \cup s_m$,
$cm.its \leftarrow cm.its \cup s_i$;

State of local atomic object $t_*.cm$ **:**
$mts \subset T$
$its \subset T$

State of transaction t:
$parent \in T$, parent's id (t_p)
$lvl \in L$;
$pls \subset X$
$lrs, lws \subset X$
mrg : locks
mts, its, vts and $ow \subset T$

Operation $\mathbf{begin_t(t_p, level)}$:
$t.parent \leftarrow t_p$;
$t.lvl \leftarrow level$;
$t.mts \leftarrow \{t\}$;

Operation $\mathbf{invoke\_child_t(t_c)}$:
$t_c.begin(t, t.lvl - 1)$;

Operation $\mathbf{unlock\_parent\_locks_t(s)}$:
for each $x \in (s \cap t.pls)$ **do**
    $t.pls \leftarrow t.pls \setminus \{x\}$;
    unlock $t_p.x$; **end for**

Operation $\mathbf{abort_t()}$:
$t.unlock\_parent\_locks(t.pls)$;
$t.abort\_active\_desc()$;
return $(abort)$;

Operation $\mathbf{abort\_incompat\_desc_t(s_i)}$:
$s_a \leftarrow s_i \cap t.vts$;
if $(s_a = \emptyset)$ **then** return; **end if**
lock $t.mrg$;
for each $(t_c \in activeChildren(t))$ **do**
    if $(t_c.cm.mts \cap s_a \neq \emptyset)$ **then**
        $t_c.abort()$;
    else
        $t_c.abort\_incompat\_desc(s_a)$; **end if**
end for
unlock $t.mrg$;
$t.vts \leftarrow t.vts \setminus s_a$;

Operation $\mathbf{abort\_active\_desc_t}()$:
for each $t' \in activeChildren(t)$ **do**
    $t'.force\_abort()$; **end for**

Operation $\mathbf{force\_abort_t}()$:
for each $t' \in activeChildren(t)$ **do**
    $t'.force\_abort()$; **end for**
return $(abort)$;

Operation $\mathbf{get\_local\_lock_t(x)}$:
lock $t.x$;

Operation $\mathbf{get\_locks_t(s, t_c)}$:
for each $x \in s$ **do**
    $get\_local\_lock_t(x, t_c)$;
    $t_c.pls \leftarrow t_c.pls \cup \{x\}$; **end for**

Operation $\mathbf{write_t(x, v)}$:
lock $t.x$;
$t.x.val \leftarrow v$;
$t.ow \leftarrow t.ow \cup t.x.rs$;
$t.x.fbd \leftarrow t.ow$;
$t.x.rs \leftarrow \emptyset$;
unlock $t.x$;
$t.lws \leftarrow t.lws \cup \{x\}$;

Operation $\mathbf{search\_parent_t(x, t_c, t_o, cm_d)}$:
lock $t.x$;
$cm \leftarrow t.cm$;
$t.x.rs \leftarrow t.x.rs \cup \{t_o\}$;
if $(t.x.val \neq null)$ **then**
    if$(\neg check\_compatibility(cm, cm_d) \vee$
    $(t.x.fbd \cap cm_d.mts \neq \emptyset))$ **then**
        unlock $t.x$;
        return $null$; **end if**
    $s_i \leftarrow t.x.fbd \cap t_c.vts$;
    $res_x \leftarrow \langle t.x.val, t.lvl, s_i \rangle$;
    unlock $t.x$;
else
    $update\_cm(cm, cm_d.mts, cm_d.its)$;

$t.vts \leftarrow t.vts \cup \{t_o\}$;
$res_x \leftarrow search\_parent_{t_p}(x, t, t_o, cm)$;
unlock $t.x$;
end if
return res_x;

Operation $\mathbf{read_t(x)}$:
lock $t.mrg$;
lock $t.x$;
$res_x \leftarrow \phi$;
if $(t_\pi.x.val = null)$ **then**
$res_x \leftarrow search\_parent_{t_p}(x, t, t, t.cm)$;
if $(res_x = null)$ **then**
$abort_t()$; **end if**
$t.x.val \leftarrow res_x.val$;
$t.lrs \leftarrow t.lrs \cup \{x\}$;
$update\_cm(t.cm, \emptyset, res_x.s_{its})$;
end if
unlock $t.mrg$;
$v \leftarrow t.x.val$;
unlock $t.x$;
$t.abort\_incompat\_desc(res_x.s_{its})$;
return v;

Operation $\mathbf{try\_to\_merge_t(t_c)}$:
lock $t.mrg$;
$t.get\_locks(t_c.lrs \cup t_c.lws, t_c)$;
if $(\neg check\_compatibility(t.cm, t_c.cm) \vee$
$(t_c.lws \neq \emptyset \wedge (t_c.cm.mts \cap t.ow \neq \emptyset))$ **then**
unlock $t.mrg$;
$t_c.abort()$; **end if**
for each $x \in t_c.lws$ **do**
$t.x.val \leftarrow t_c.x.val$; **end for**
for each $x \in t_c.lrs : t.x.val = null$ **do**
$t.x.val \leftarrow t_c.x.val$; **end for**
$t.ow \leftarrow t.ow \cup (\cup_{x \in t_c.lws} t.x.rs)$;
for each $x \in t_c.lws$ **do**
$t.x.fbd \leftarrow t.ow$;
$t.x.rs \leftarrow \emptyset$; **end for**
$t.lrs \leftarrow t.lrs \cup t_c.lrs$;
$t.lws \leftarrow t.lws \cup t_c.lws$;
$update\_cm(t.cm, t_c.cm.mts, t_c.cm.its)$;
unlock $t.mrg$;
$t_c.unlock\_parent\_locks(t_c.pls)$;

Operation $\mathbf{try\_to\_commit_t()}$:
$t_p.try\_to\_merge(t)$;
$t_p.abort\_incompat\_desc(t.cm.its)$;
return $(commit)$;

Protocol 6.2: HParSTM (Special case of root node, t_ρ)

Operation $\mathbf{search\_parent_{t_\rho}(x, t_c, t_o, cm_d)}$:
lock $t_\rho.x$;
$cm \leftarrow t_\rho.cm$;
$t.x.rs \leftarrow t.x.rs \cup \{t_o\}$;
if $(t_\rho.x.val \neq null)$ **then**
if$(\neg check\_compatibility(cm, cm_d)) \vee$
$(t_\rho.x.fbd \cap cm_d.mts \neq \emptyset))$ **then**
unlock $t_\rho.x$;
return $null$; **end if**
$res_x \leftarrow \langle t_\rho.x.val, t_\rho.lvl, \emptyset \rangle$;
unlock $t_\rho.x$;
else
$s_m \leftarrow cm.mts \cup cm_d.mts$;
$t_\rho.vts \leftarrow t_\rho.vts \cup \{t_o\}$;
lock $t_\psi.x$;
if $(t_\psi.x.fbd \cap s_m = \emptyset)$ **then**
$t_\psi.x.rs \leftarrow t_\psi.x.rs \cup \{t_o\}$;
$s_i \leftarrow t_\psi.x.fbd \cap t_\rho.vts$;
$res_x \leftarrow \langle t_\psi.x.val, t_\rho.lvl, s_i \rangle$;
else
$res_x \leftarrow null$; **end if**
unlock $t_\psi.x$;
unlock $t_\rho.x$;
end if
return res_x

The following methods are same as in Protocol 5.3.

Operation $\mathbf{read_{t_\rho}(x)}$:
lock $t_\rho.mrg$;
lock $t_\rho.x$;
$s_i \leftarrow \emptyset$;
if $(t_\rho.x.val = null)$ **then**
lock $t_\psi.x$;
if $(t_\psi.x.fbd \cap t_\rho.cm.mts \neq \emptyset)$ **then**
unlock $t_\psi.x$;
$abort_{t_\rho}()$; **end if**
$t_\psi.x.rs \leftarrow t_\psi.x.rs \cup \{t_\rho\}$;
$t_\rho.x.val \leftarrow t_\psi.x.val$;
$s_i \leftarrow t_\psi.x.fbd \cap t_\rho.vts$;
$update\_cm(t_\rho.cm, \emptyset, s_i)$;
unlock $t_\psi.x$;
$t_\rho.lrs \leftarrow t_\rho.lrs \cup \{x\}$; **end if**
unlock $t_\rho.mrg$;
$v \leftarrow t_\rho.x.val$;
unlock $t_\rho.x$
$t_\rho.abort\_incompat\_desc(s_i)$;
return v;

Operation $\mathbf{try\_to\_commit_{t_\rho}()}$:
if $(t_\rho.lws = \emptyset)$ **then**
return $commit$; **end if**
for each $t_\psi.x : x \in (t_\rho.lrs \cup t_\rho.lws)$ **do**
lock $t_\psi.x$;

$t_\rho.pls \leftarrow t_\rho.pls \cup \{x\}$ **end for**
if $(t_\rho.cm.mts \cap t_\psi.ow \neq \emptyset)$ **then**
$abort_{t_\rho}()$; **end if**
$t_\psi.ow \leftarrow \cup_{x \in t_\rho.lws} t_\psi.x.rs$;
for each $x \in t_\rho.lws$ **do**
$t_\psi.x.val \leftarrow t_\rho.x.val$
$t_\psi.x.fbd \leftarrow t_\psi.ow$;
$t_\psi.x.rs \leftarrow \emptyset$ **end for**
$t_\rho.unlock\_parent\_locks(t_\rho.pls)$;
return $(commit)$;

6.3.1 Transaction state

A transaction t begins with $begin(t_p, level)$, where t_p denotes the id of the parent of t. The t_p is t_ψ for a root-level transaction. The set lrs (*local read set*) is used to record the objects read from the ancestors, whereas set lws (*local write set*) is used to record its write steps. The access to each copy of a base object is protected by a lock. The set ow is used to store the ids of those descendants of t that have read a value from its locally shared objects whose values have been modified since the reading. The local variable $parent$ stores the reference to t's parent t_p.

Each transaction maintains a consistency management object, cm, which consists of sets mts and its. Set mts (*merged transaction set*) contains the ids of t as well as those descendants of t whose results have been propagated to (merged with) t. A descendant t' of t is included in $t.mts$ only when t' and all of its intermediate ancestors up to t commit. The set its (*incompatible transaction set*) denotes a set of transactions that t is incompatible with and hence cannot be merged together. Further, a set pls (*parent lock set*) is used to keep track of the locks obtained on parent's objects. Further, lock mrg is used for merge and read operations.

Observe that a transaction in HParSTM does not maintain set prs as is done in ParSTM in Chapter 5. This is owing to the fact that an external read in HParSTM does not result in pessimistic locking of ancestors at multiple levels. A transaction only needs to abort the locks up to its parent level (obtained in try_to_merge operation), even in case of its abort.

6.3.2 Working of HParSTM:

In *HParSTM*, the allocation of space for local copy of an object (x) in the local space of a transaction is automatically done whenever required. For a transaction t, this typically happens when a transaction (t or its descendant) tries to obtain a lock on a local copy $t.x$ of t, and $t.x$ does not already exist. At the time of allocation of space for a local copy of object x with transaction t, the initial state of $t.x$ is: $t.x.val = null$, $t.x.rs = \emptyset$ and $t.x.fbd = \emptyset$. Further, in the text, wherever a transaction is looking for a local copy of an object to read, we mean a non-null valued copy of that object. Several steps of the protocol are self-explanatory or similar to the ones already discussed in previous chapters. We describe only the salient features. The key procedures of the protocol are discussed as follows.

$begin_t(t_p, level)$, $invoke\_child_t(t_c)$: Self-explanatory.

$unlock\_parent\_locks_t$: Set pls contains the ids of parent's objects on which t currently holds the locks. Locks on these objects are released using the method.

$abort_t()$: This method is invoked when a transaction t has to be aborted. Before aborting, transaction t releases all the locks on parent's objects in its possession. Finally, t calls the *abort()* method of its active children (if any). Observe that, in comparision to ParSTM, there is no $unlock\_to\_ancestors$ needed in this case, as there is no multi-level pessimistic locking on ancestors' objects in case of HParSTM.

$check\_compatibility_t(cm_a, cm_d)$: Discussed earlier.

$abort\_incompat\_desc_t(s_i)$: Discussed earlier.

$abort\_active\_desc_t()$: Discussed earlier.

$get\_local\_lock_t(x)$: This method is used to lock object $t.x$.

$get\_locks_t(s, t_c)$: This method is invoked by the child t_c to obtain locks on parent t's objects in set s.

$write_t(x, v)$: In order to perform a write operation in its local space, transaction t locks its local copy $t.x$, updates the value of $x.val$, adds $x.rs$ to ow before clearing $x.rs$, and updates $x.fbd$ using ow. Then, t unlocks $t.x$, followed by addition of x in $t.lws$.

$read_t(x)$: Same as the one discussed in case of ParSTM in Chapter 5, except that pls is not updated in this case as the lock on parent's object is released upon completion of the *read* operation.

$search\_parent_t(x, t_c, t_o, cm_d)$: This method is similar to the *search_parent* method discussed in Chapter 5. This method is invoked by the child transaction t_c on behalf of a descendant t_o (original descendant requiring to read value of nearest copy of x) to search for a local copy of x at its parent t's level. First, $t.x$ is locked, and t_o is added to $t.x.rs$. If $t.x$ is not null-valued, then we ensure that none of the transactions in mts of t_o and its intermediate ancestors up to t_c (1) belongs to $t.x.fbd$ and (2) is incompatible with t. Otherwise, $t.x$ is unlocked and *null* is returned to indicate failure. If the consistency check is valid, then incompatible descendants, s_i, of t_o are obtained using $t.x.fbd$ and $t_c.vts$, and value of $res_x(t.x.val, t.lvl, s_i)$ is returned, followed by unlocking of $t.x$.

In case $t.x$ is null valued, t_o is added to $t.vts$. Next, t forwards the search to its parent after updating cm_d (cumulative cm). Finally, the value obtained from the parent level is returned after unlocking $t.x$. Unlike ParSTM, local object $t.x$ is unlocked not only in case of failed search but also in case of a successful search.

$try\_to\_merge_t(t_c)$: This method is invoked by the child transaction t_c to merge its local sets with the parent t. Access control between competing children is achieved by using a mrg lock. Only one child transaction can merge at a time. For each $x \in (t_c.lrs \cup t_c.lws)$ a lock is obtained on object $t.x$. Next, for consistency, compatibility of t_c with t as well as its membership in $t.ow$ is checked. If the consistency check of t_c with t is successful, then for each object $x \in (t_c.lrs \cup t_c.lws)$, the value of $t.x$ is updated using that of $t_c.x$. Set $t.ow$ is updated by merging the cumulative content of $t.x.rs$ for each $x \in t_c.lws$. For each $x \in (t_c.lws)$, $t.x.fbd$ is updated with $t.ow$ and $t.x.rs$ is reset. The sets $t_c.lrs, t_c.lws$ and $t_c.cm$ are merged with the corresponding sets of the parent t. Finally, all the locks previously obtained by t_c are released.

$try\_to\_commit_t()$: If t is a nested transaction, then it tries to merge with its parent.

In case t is a root-level (non-nested) transaction, then the behaviour of the validation process for t is the similar to that proposed in [9] for a non-nested transaction, as already discussed in Chapters 4 and 5. When the root transaction commits, the objects in its local write set are modified globally, i.e., the change is reflected in the globally shared copy of objects available with t_ψ. The root transaction first checks if it has only read steps. If it is found to be $read\_only$, then the transaction commits immediately. Otherwise, it locks all the objects in its sets $t.lrs$ and $t.lws$. Then, it checks if any of the subtransactions in its set $t.mts$ belongs to the $t_\psi.ow$ set. If yes, then it means that the consistency of the root transaction has been compromised, and the transaction releases all the locks before aborting. Otherwise it updates the values of all the global objects in its write set, followed by updating $t_\psi.ow$ and $t_\psi.x.fbd$ for each x it writes. Finally, the root transaction releases all the locks and commits. Observe that, during the commit of a root transaction, there is no checking of incompatibility or update operations for the sets cm, lrs, lws of its parent.

6.3.3 About management of sets

The local objects/sets associated with a transaction (other than t_ψ) exist only during the lifespan of that transaction. The local memory allotted to that transaction is freed as soon as it completes. Only the globally shared objects/sets, associated with the fictitious transaction t_ψ, exist throughout the run of the STM system. As such the size of their content may grow very large if not managed over the run of the STM system. We can adopt the following approach for suppressing the ids of transactions that have been aborted or committed. When t commits or aborts, we subtract set $t.vts$ from the sets ($t_p.*.rs, t_p.*.fbd$ and $t_p.ow$) at the parent level.

6.3.4 About deadlock freedom

In general, a deadlock situation may arise between two descendants, say t_1 and t_2, of a node t if each descendant requests a lock on t's object such that it is currently held by the other descendant. In other words, say t_1 and t_2 already hold lock on $t.x$ and $t.y$ respectively. Now, if t_1 and t_2 try to lock $t.y$ and $t.x$ respectively (notice the reverse order of objects), a deadlock situation will arise where each subtransaction will be waiting **indefinitely** for a lock held by the other. Observe that for a deadlock situation to possibly occur, the participating transaction should try to lock **at least two** shared objects. Otherwise, deadlock is not possible.

Unlike ParSTM, in HParSTM the lock on an object is released soon after the external read on it by a descendant. That means when a transaction t's object is locked by its descendant for an external read, that lock/descendant cannot be an accomplice in a deadlock situation. If more than one descendant requests the lock for the same object, then only one of them gets the lock while others wait for their chance until the lock is released at the completion of the current external read operation. The only exception is when a descendant tries to merge with its parent. At that time, it may try to obtain and hold lock on more than one object of the parent. Thus, deadlock may occur when more than one child tries to merge with the parent at the same time. We prevent this case by restricting only one child to merge with the parent by using mrg lock.

6.4 Consistency checking and linearization points at level t

6.4.1 Linearization points of events in a level-wise history

The level wise event history $\widehat{\mathcal{H}_t}$ comprises of the following events: (i) local read/write operations of t, (ii) external reads of t's descendants w.r.t. t, (iii) external reads of t itself, (iv) write operations due to merging of t's child t_c, and (v) commits of t's children.

Let ℓ_{op} denote the linearization point of an event. Then, the linearization points of the various events in the history are defined as follows:

i Local read/write operation of t

(a) $read_t(t.x)$: ℓ_{op} corresponds to the time when it unlocks $t.x$ (line 103 or 172)

(b) $write_t(t.x)$: ℓ_{op} corresponds to the time when t updates $t.ow$ (line 68)

ii External read operation of t

(a) $read_t(t_a.x)$: ℓ_{op} corresponds to the time when $t.cm$ is updated (line 99)

iii External read of a descendant t_d on t's object or that of t's ancestor t_a:

(a) $read_{t_d}(t.x)$: ℓ_{op} corresponds to the time when t_d reads $t.cm$ for consistency checking (line 72 or 132)

(b) $read_{t_d}(t_a.x)$: ℓ_{op} corresponds to the time when t_d reads $t.cm$ for consistency checking (line 72 or 132)

iv Write due to commit of child t_c

(a) $write_{t_c}(t.x)$: ℓ_{op} corresponds to the time when t_c updates $t.x$ (line 114)

v Commit of child t_c

(a) C_{t_c} : ℓ_{op} corresponds to the time when $t.cm$ is updated (line 123)

6.4.2 Definition of linearization point of a transaction

The definition of linearization points of root nodes in HParSTM is similar to what is defined in Chapters 4 and 5. A comprehensive set of proofs has been furnished for the same as well in those chapters. In this chapter, we shall focus on the definition of linearization points for non-root nodes in HParSTM and provide a set of proofs for the same. With this end in view, the linearization points for non-root nodes are defined as follows.

1. If t is an update transaction that commits, its linearization point, ℓ_t, lies at the time just after it updates the parent's $t_p.ow$ and $t_p.cm$ (consistency management) object (line 123).

2. If t is a read only commited transaction, then ℓ_t is placed at the earliest of (i) the time it reads $t_p.cm$ for its last successful external read operation (lines 72, 132), and (ii) the time just before $\widehat{t}$ (any id in $t.mts$) is added to $t_p.ow$ (if it ever is) (lines 65, 117).

3. If a transaction t aborts, ℓ_t is determined as if it were a read only transaction, i.e., ℓ_t lies at the earliest of (i) the time it reads $t_p.cm$ for its last successsful external read operation (lines 72, 132), and (ii) the time just before $\widehat{t}$ (any id in $t.mts$) is added to $t_p.ow$ (if it ever is) (lines 65, 117).

Observe that in HParSTM, the linearization point of each transaction has been defined w.r.t. access/update of consistency management set $t.cm$ and $t.ow$ instead of set $t.ow$ alone as was done in [9]. This is so due to the fact that in HParSTM the consistency checking is based on combination of checking the membership in sets $t.cm$ and $t.ow$. Checking $t.ow$ alone is not sufficient.

6.5 Proof

Owing to the construction of HParSTM, the local objects available with a transaction t are operated on the same way (by t's children) as the global copies of objects. In Chapter 4, we have already outlined the proofs for the correctness for the history produced at the global level ($\mathcal{H}_{t_\psi}$). The same set of proofs can be applied here for the level-wise history produced at any node of the super tree. In other words, to draw a parallelism, at each level t the local objects available with node t can be treated as global copies of objects, and t's children as root-level transactions. The additional element we need to account for in the proofs for HParSTM is the compatibility of transactions.

6.5.1 Proof for committed transactions

Recall that, in ParSTM, the consistency at each level was achieved using locks associated with objects in a pessimistic manner. HParSTM differs in the sense that the objects at each node are accessed in an optimistic manner. Further, Unlike ParSTM where sets fbd and ow have been used only at the global level $\mathcal{H}_{t_\psi}$, HParSTM uses these sets at each level. For this reason, the proofs for $HParSTM$ are slightly different as well.

For a given level-wise history $\mathcal{H}_t$ of committed transactions produced by HParSTM, we need to prove:

1. $\rightarrow_{\mathcal{H}_t^\sigma}$ is total order.

2. $\rightarrow_{\mathcal{H}_t'} \subseteq \rightarrow_{\mathcal{H}_t^\sigma}$.

3. *If t_1, t_2 are two incompatible transactions, then they cannot merge together.*

4. $t_w \xrightarrow{t.x}_{rf} t_r \Rightarrow \nexists t'_w$ such that $(t_w \rightarrow_{\mathcal{H}^\sigma_t} t'_w \rightarrow_{\mathcal{H}^\sigma_t} t_r) \wedge (w_{t'_w}(t.x) \in \mathcal{H}_t)$.

5. $t_w \xrightarrow{t.x}_{rf} t_r \Rightarrow t_w \rightarrow_{\mathcal{H}^\sigma_t} t_r$.

Proofs of (a) $\rightarrow_{\mathcal{H}^\sigma_t}$ is total order, and (b) $\rightarrow_{\mathcal{H}'_t} \subseteq \rightarrow_{\mathcal{H}^\sigma_t}$ follows directly from the definition of linearization points discussed above.

Next, we prove the remaining parts.

Lemma 6.1. *Let $\widehat{\mathcal{H}_t}$ be a level-wise history of HParSTM. Let t_1 and t_2 be any two distinct transactions $\in \{t \cup children(t)\}$, and t' be an ancestor of t. If $r_{\widehat{t_2}}(t'.x) : \beta(\widehat{t_1}, t'.x.fbd, AL_{\widehat{t_2}}(t'.x, read_{\widehat{t_2}}(t'.x)))$, then we have (1) if $t_1, t_2 \neq t$, then $\neg(t_1 \in \Pi(\widehat{\mathcal{H}_t}) \wedge t_2 \in \Pi(\widehat{\mathcal{H}_t})$, or (2) $t_1 = t \wedge t_2 \notin \Pi(\widehat{\mathcal{H}_t})$ or (3) $t_2 = t \wedge t_1 \notin \Pi(\widehat{\mathcal{H}_t})$.*

Proof. Since t_1 and t_2 are distinct transactions, either both the transactions are children of t, or one of the two is t, while the other one is a child of t. First, we show that

$r_{\widehat{t_2}}(t'.x) : \beta(\widehat{t_1}, t'.x.fbd, AL_{\widehat{t_2}}(t'.x, read_{\widehat{t_2}}(t'.x))) \Rightarrow \widehat{t_1} \in t_2.cm.its$.

$r_{\widehat{t_2}}(t'.x) : \beta(\widehat{t_1}, t'.x.fbd, AL_{\widehat{t_2}}(t'.x, read_{\widehat{t_2}}(t'.x)))$ means that when subtransaction $\widehat{t_2}$ acquired the lock to perform a read operation on the object $t'.x$ of t's ancestor t', $\widehat{t_1} \in t'.x.fbd$. Observe that, by the construction of HParSTM, before releasing the locks on its object $t'.x$, $\widehat{t_1}$ is added to $res_x.its$ (line 79-80, 147-148, 99) using $\widehat{t_1} \in t.vts$ (due to line 84, 147) followed by releasing the lock on its own local copy of object $t_2.x$ (line 103). Thus, we have following implications:

$r_{\widehat{t_2}}(t'.x) : \beta(\widehat{t_1}, t'.x.fbd, AL_{\widehat{t_2}}(t'.x, read_{\widehat{t_2}}(x))) \Rightarrow \beta(\widehat{t_1}, res_x.its, RL_{\widehat{t_2}}(t'.x, read_{\widehat{t_2}}(x)))$

By construction $res_x.its$ is used to update $t_2.cm.its$ before unlocking $t_2.x$ (lines 79-80, 147-148, 99, 103). Thus, we have

$RL_{\widehat{t_2}}(t^{'}.x, read_{\widehat{t_2}}(x)) <_{\mathcal{H}_t} RL_{\widehat{t_2}}(t_2.x, read_{\widehat{t_2}}(x)) \Rightarrow \beta(\widehat{t_1}, \widehat{t_2}.cm.its, RL_{\widehat{t_2}}(\widehat{t_2}.x, read_{\widehat{t_2}}(x)))$

(due to line 79-80, 147-148).

Finally, $\widehat{t_1} \in \widehat{t_2}.cm.its \wedge \widehat{t_2} \in t_2.cm.mts \Rightarrow \widehat{t_1} \in t_2.cm.its$ (due to merging, $\widehat{t_2}$ is a descendant of t_2).

Now, let us consider the first case in which t_1, t_2 are children of t. We have to show $t_1 \in \Pi(\widehat{\mathcal{H}_t}) \Rightarrow t_2 \notin \Pi(\widehat{\mathcal{H}_t})$ and vice versa.

Case I: $t_1, t_2 \in children(t)$.

The $try\_to\_merge_t(t_c)$ method is invoked by the child to merge its local sets with those of its parent. Synchronization of concurrent requests from the children is achieved by means of a special lock, called merge lock (denoted, $t.mrg$). That means only one child transaction can merge with t at a time. Now, we have following two subcases to consider:

Case I(a): $AL_{t_1}(t.mrg, ttc) <_{\mathcal{H}_t} AL_{t_2}(t.mrg, ttc)$ (assuming $t_1 \in \Pi(\widehat{\mathcal{H}_t})$).

$AL_{t_1}(t.mrg, ttc) <_{\mathcal{H}_t} AL_{t_2}(t.mrg, ttc)$

$\Rightarrow RL_{t_1}(t.mrg, ttc) <_{\mathcal{H}_t} AL_{t_2}(t.mrg, ttc) \Rightarrow \beta(\widehat{t_1}, t.cm.mts, RL_{t_1}(t.mrg, ttc))$ (due to line 123)

$\Rightarrow \beta(\widehat{t_1}, t.cm.mts, AL_{t_2}(t.mrg, ttc))$

This means, when t_2 tries to merge with t, it will discover that it is incompatible with t and consequently abort (due to line 109-112). Thus, $t_1 \in \Pi(\widehat{\mathcal{H}_t}) \Rightarrow t_2 \notin \Pi(\widehat{\mathcal{H}_t})$.

Case I(b): $AL_{t_2}(t.mrg, ttc) <_{\mathcal{H}_t} AL_{t_1}(t.mrg, ttc)$ (assuming $t_2 \in \Pi(\mathcal{H}_t)$).

$AL_{t_2}(t.mrg, ttc) <_{\mathcal{H}_t} AL_{t_1}(t.mrg, ttc)$

$\Rightarrow RL_{t_2}(t.mrg, ttc) <_{\mathcal{H}_t} AL_{t_1}(t.mrg, ttc) \Rightarrow \beta(\widehat{t_1}, t.cm.its, RL_{t_1}(t.mrg, ttc))$ (due to line 123).

$\Rightarrow \beta(\widehat{t_1}, t.cm.its, AL_{t_2}(t.mrg, ttc))$

In this case, t_1 aborts later on (due to line 109-112) while trying to merge with t. We have $t_2 \in \Pi(\mathcal{H}_t) \Rightarrow t_1 \notin \Pi(\mathcal{H}_t)$.

Case II: Either $t_1 = t$, or $t_2 = t$.

Case II(a): $t_1 = t$.

$t_1 = t \Rightarrow t_1$ is the parent of t_2

$\Rightarrow t_1$ is an ancestor of each $\widehat{t_2} \in t_2.cm.mts$.

$\Rightarrow$ read operation $r_{\widehat{t_2}}(t'.x) : \beta(\widehat{t_1}, t'.x.fbd, AL_{\widehat{t_1}}(t'.x, read_{\widehat{t_1}}(t'.x)))$ is not possible ($failure$ due to lines 123, 75-78, 135-138, 95-96).

Case II(b): $t_2 = t$.

Similar to *Case I(b)*. t_2, being the ancestor t, adds $\widehat{t_1}$ to $t.its$ anyway, leading to abort of t_1 (due to lines 79-80, 88; 147-148, 99, 104).

Thus, the conjunction of the *cases I* and *II* proves the lemma. $\square$

Lemma 6.2. *Given a level-wise hstory $\widehat{\mathcal{H}_t}$, let $t_1 \in \Pi(\widehat{\mathcal{H}_t})$. Then,*

$\beta(t_1, t.ow, \tau) \Rightarrow \ell_{t_1} <_{\mathcal{H}_t} \tau$.

Proof. We have to show that the linearization point for a transaction cannot lie after the time at which its id has been added to $t.ow$. There are two cases:

- If t_1 is a read only commited transaction, then ℓ_{t_1} is placed at the earliest of (i) the time it reads $t.cm$ for its last successsful external read operation, and (ii) the time just before $\widehat{t_1}$ (any id in $t_1.mts$) is added to $t.ow$ (if it ever is), which proves the lemma.

- If t_1 writes and commits, its linearization point ℓ_{t_1} is placed during $try\_to\_commit()$, while t_1 holds the locks of every object of t that it has read. If $\widehat{t_1}$ was in $t.ow$ before it acquired all the locks, it would not commit (due to lines 110-112, 181-182). Let us notice that $\widehat{t_1}$ can be added to $t.ow$ only by t or an update child transaction of t holding a lock on a base object previously read by $\widehat{t_1}$. As t_1 releases the locks just before committing (lines 123, 125; 183, 188), it follows that ℓ_{t_1} occurs before the time at which $\widehat{t_1}$ is added to $t.ow$ (if it ever is), which again proves the lemma. □

Similarly, we shall prove the next lemma. Recall that $\gamma_{inc}(t_1, t, \tau)$ has been defined in Chapter 5 to mean that transaction t_1 becomes incompatible with transaction t at time τ .

Lemma 6.3. *Given a level-wise history $\widehat{\mathcal{H}_t}$, let $t_1 \in \Pi(\widehat{\mathcal{H}_t})$. Then,*

$\gamma_{inc}(t_1, t, \tau) \Rightarrow \ell_{t_1} <_{\mathcal{H}_t} \tau$.

Proof. The proof is similar to that of the previous lemma.

We have to show that the linearization point for a transaction t_1 cannot lie after the time at which t_1 has become incompatible with t. There are two cases:

- If t_1 is a read only committed transaction, then ℓ_{t_1} is placed at the earliest of (i) the time it reads $t.cm$ for its last successsful external read operation, and (ii) the time just before $\widehat{t_1}$ (any id in $t_1.mts$) is added to $t.ow$ (if it ever is). For a successful read operation t_1 must read $t.cm$ (line 72 or 132) before time τ at which we have $\gamma_{inc}(t_1, t, \tau)$. Otherwise, t_1 will fail the subsequent consistency check at line 75 or 135 and consequently abort, leading to an unsuccessful read operation. Thus, ℓ_{t_1} occurs before τ, which proves the lemma.

- If t_1 writes and commits, its linearization point ℓ_{t_1} is placed during $try\_to\_commit()$, while t_1 holds the lock on $t.mrg$ lock. If $\widehat{t_1}.cm$ was incompatible with $t.cm$ before it acquired lock on $t.mrg$ (line 107), it would not commit (due to failure to pass the consistency check at line 109). Let us notice that $t.cm$ is updated either during an external read of t (line 99 or 167) or by merging of its committed child (line123). Further, while $t.mrg$ is locked by a child, t cannot perform an external read operation as it must lock $t.mrg$ as well first (line 90). As t_1 releases the lock on $t.mrg$ just before committing (lines 123, 124), it follows that ℓ_{t_1} occurs before the time τ at which $\gamma_{inc}(t_1, t, \tau)$, which again proves the lemma. $\square$

Lemma 6.4. *$t_w \xrightarrow{t.x}_{rf} t_r \Rightarrow \nexists t_w'$ such that $(t_w \rightarrow_{\mathcal{H}_t^\sigma} t_w' \rightarrow_{\mathcal{H}_t^\sigma} t_r) \wedge (w_{t_w'}(t.x) \in \mathcal{H}_t)$.*

Proof. By contradiction, let us assume that there are transactions t_w, t_w' and t_r and an object $t.x$ such that:

$-t_w \xrightarrow{t.x}_{rf} t_r$

$-w_{t_w'}(t.x)v' \in \mathcal{H}_t$

$-t_w \rightarrow_{\mathcal{H}_t^\sigma} t_w' \rightarrow_{\mathcal{H}_t^\sigma} t_r$.

As both t_w and t_w' write $t.x$ in shared memory, they have necessarily committed (a write in shared memory occurs only at lines 114, 185 (and 64 in case of fictitious child transaction) during the execution of *try_to_commit*, i.e., $t_w, t_{w'} \in \Pi(\widehat{\mathcal{H}_t})$). Moreover, their linearization points ℓ_{t_w} and $\ell_{t_w'}$ occur while they hold the lock on $t.x$ (before committing), from which we have the following implications:

$t_w \rightarrow_{\mathcal{H}_t^\sigma} t_w' \Leftrightarrow \ell_{t_w} <_{\mathcal{H}_t} \ell_{t_w'}$,

$\ell_{t_w} <_{\mathcal{H}_t} \ell_{t_w'} \Rightarrow RL_{t_w}(t.x, ttc) <_{\mathcal{H}_t} AL_{t_w'}(t.x, ttc)$

$\Rightarrow w_{t_w}(t.x)v <_{\mathcal{H}_t} w_{t_w'}(t.x)v'$,

$(t_w \xrightarrow{t.x}_{rf} t_r) \wedge (w_{t_w}(t.x)v <_{\mathcal{H}_t} w_{t_w'}(t.x)v') \Rightarrow w_{t_w}(t.x)v <_{\mathcal{H}_t} r_{t_r}(t.x)v <_{\mathcal{H}_t} w_{t_w'}(t.x)v'$.

When a subtransaction in $\widehat{t_1}$ (i.e., t_1 or any of its descendants that merged with t_1) reads an object $t.x$, it always adds its id to $t.x.rs$ and to null-valued $t'.x.rs$ of each of its intermediate ancestors t' (if any) upon acquiring a lock on $t.x$ (lines 73, 133). Therefore, the predicate $\beta(\widehat{t_1}, t.x.rs, RL_{\widehat{t_1}}(t.x, read(x)))$ is true ($t.x.rs$ is set to $\emptyset$ only after being added to the set $t.ow$). Using this observation, we have the following:

$r_{t_r}(t.x)v <_{\mathcal{H}_t} w_{t_w'}(t.x)v' \wedge \beta(\widehat{t_r}, t.x.rs, RL_{\widehat{t_r}}(t.x, read(x)))$

$\Rightarrow \beta(\widehat{t_r}, t.x.rs, AL_{t_w'}(t.x, ttc))$,

$\beta(\widehat{t_r}, t.x.rs, AL_{t_w'}(t.x, ttc)) \wedge (w_{t_w'}(t.x)v' \in \mathcal{H}_t) \Rightarrow \beta(\widehat{t_r}, t.ow, \ell_{t_w'}) \Rightarrow \ell_{t_r} <_{\mathcal{H}_t} \ell_{t_w'} \Leftrightarrow$ $t_r \rightarrow_{\mathcal{H}_t^\sigma} t_w'$.

which proves that, contrary to the initial assumption, t_w' cannot precede t_r in the sequential transaction history $\widehat{\mathcal{H}_t^\sigma}$. $\square$

Similar to ParSTM discussed in Chapter 5, we also need to show that in HParSTM the value read by a descendant from its ancestor is consistent at the time of reading. Recall that a descendant t_d can become incompatible with an ancestor t either due to an external read of t or a commit of a t's child. We shall show that the consistency checking of the value read by t_d from t is guaranteed to be correct.

Lemma 6.5. *Let t_d be a descendant of t such that history $op_t, read_{t_d}(t.x) \in \widehat{\mathcal{H}_t}$, where op_t denotes an external read operation or a commit of t's child at time τ. Then, we show that (i) $op_t, read_{td}(t.x)$ can be ordered, and (ii) $op_t < read_{t_d}(t.x) \Rightarrow \neg\gamma_{inc}(t_d, t, \tau)$, i.e., t and t_d are not inconsistent before time τ.*

Proof. By definition of linearization points for op_t and $read_{t_d}(t.x)$ in $\widehat{\mathcal{H}_t}$ w.r.t. access to $t.cm$, and $t.cm$ being an atomic variable, it follows $op_t, read_{t_d}(t.x)$ in $\widehat{\mathcal{H}_t}$ are ordered according to their linearization points.

To prove part (ii), let us assume by contrast that $\gamma_{inc}(t_d, t, \tau)$, i.e., at the completion of op_t at time τ, $check\_compatibility(t.cm, t_d.cm) = false$.

Now, for $op_t < read_{t_d}(t.x)$, we have the following two cases:

Case I: op_t is an external read operation of t

By definition of linearization points for events in a level wise history (Section 6.1), we have

$\ell_{op_t} < \ell_{read_{t_d}(t.x)}$

$\Rightarrow$ update of $t.cm$ (line 99) such that $check\_compatibility(t.cm, t_d.cm) = false$ occurs before $t.cm$ is read (line 72 or 132) for consistency check of t_d. (Recall that $t.cm$ is kept in an atomic variable.)

$\tau < \ell_{read_{t_d}(t.x)}$ such that $\gamma_{inc}(t_d, t, \tau) = true$

$\Rightarrow$ *check_compatibility*$(t.cm, t_d.cm) = false$ during the consistency check for $read_{t_d}(t.x)$ (line 75 or 135)

$\Rightarrow read_{t_d}(t.x)$ fails (due to lines 75-78, 135-138, 95-96), i.e., $op_t < read_{t_d}(t.x)$ is not possible in this case.

Case II: op_t is a commit of t's child

By definition of linearization points for events in a level wise history (Section 5.5.3), we have

$\ell_{op_t} < \ell_{read_{t_d}(t.x)}$

$\Rightarrow$ update of $t.cm$ (line 123) such that *check_compatibility*$(t.cm, t_d.cm) = false$ occurs before $t.cm$ is read (line 72 or 132) for consistency check of $read_{t_d}(t.x)$ (Recall that $t.cm$ is kept in an atomic variable).

$\tau < \ell_{read_{t_d}(t.x)}$ such that $\gamma_{inc}(t_d, t, \tau) = true$

$\Rightarrow$ *check_compatibility*$(t.cm, t_d.cm) = false$ during consistency check for $read_{t_d}(t.x)$ (line 75 or 135)

$\Rightarrow read_{t_d}(t.x)$ fails (due to lines 75-78, 135-138, 95-96), i.e., $op_t < read_{t_d}(t.x)$ is not possible in this case also.

Hence, after analysis of both the cases, we conclude that $op_t < read_{t_d}(t.x) \Rightarrow \neg\gamma_{inc}(t_d, t, \tau)$ upon completion of op_t.

$\square$

Lemma 6.6. $t_w \xrightarrow{t.x}_{rf} t_r \Rightarrow t_w \rightarrow_{\mathcal{H}_t^\sigma} t_r$.

Proof. The proof is made up of two parts. First it is shown that $t_w \xrightarrow{t.x}_{rf} t_r \Rightarrow \neg\beta(\widehat{t_r}, t.ow, \ell_{t_w})$, and then it is shown that $\neg\beta(\widehat{t_r}, t.ow, \ell_{t_w}) \wedge t_w \xrightarrow{t.x}_{rf} t_r \Rightarrow t_w \rightarrow_{\mathcal{H}_t^\sigma} t_r$.

Proof of $t_w \xrightarrow{t.x}_{rf} t_r \Rightarrow \neg\beta(t_r, t.ow, \ell_{t_w})$. Let us assume by contradiction that the predicate $\beta(\widehat{t_r}, t.ow, \ell_{t_w})$ is true. Due to lines 66, 119 (or 186) we have $\beta(\widehat{t_r}, t.ow, \ell_{t_w}) \Rightarrow \beta(\widehat{t_r}, t.x.fbd, RL_{t_w}(t.x, ttc))$

If the read of $t.x$ from shared memory by t_r is before the write by t_w, we cannot have $t_w \xrightarrow{t.x}_{rf} t_r$. So, in the following we consider that the read of $t.x$ from shared memory by t_r is after its write by t_w. We have then $RL_{t_w}(t.x, ttc) <_{\mathcal{H}_t} AL_{\widehat{t_r}}(t.x, read(x))$, and consequently $\beta(\widehat{t_r}, t.x.fbd, RL_{t_w}(t.x, ttc)) \Rightarrow$
$\beta(\widehat{t_r}, t.x.fbd, AL_{t_r}(t.x, ttc))$.

As $\widehat{t_r} \in t.x.fbd$ when it locks $t.x$, it follows that the read operation fails at line 75-76, 135-136 (or 145) and consequently we cannot have $t_w \xrightarrow{t.x}_{rf} t_r$. Summarizing the previous reasoning we have $\beta(\widehat{t_r}, t.ow, \ell_{t_w}) \Rightarrow \neg(t_w \xrightarrow{t.x}_{rf} t_r)$, and taking the contrapositive we finally obtain $t_w \xrightarrow{t.x}_{rf} t_r \Rightarrow \neg\beta(\widehat{t_r}, t.ow, \ell_{t_w})$

Proof of $\neg\beta(\widehat{t_r}, t.ow, \ell_{t_w}) \wedge t_w \xrightarrow{t.x}_{rf} t_r \Rightarrow t_w \rightarrow_{\mathcal{H}_t^\sigma} t_r$. As defined earlier, the linearization point ℓ_{t_r} depends on whether t_r is a read only or an update transaction. The proof considers the two possible cases.

- If t_r is an update transaction that commits, its linearization point ℓ_{t_r} (that is defined at line 123 after it updates the set $t.ow$ and $t.cm$) occurs while merging ($try\_to\_commit()$). Due to this observation, the fact that t_w releases its locks after its linearization point, and $t_w \xrightarrow{t.x}_{rf} t_r$, we have $\ell_{t_w} <_{\mathcal{H}_t} \ell_{t_r}$, i.e., $t_w \rightarrow_{\mathcal{H}_t^\sigma} t_r$.

- If t_r is a read only transaction that commits, its linearization point ℓ_{t_r} is placed just before the earliest time at which it is added to $t.ow$ (lines 65, 117), or at the time it accesses $t.cm$ for its read operation (lines 72 or 132). In the latter case, we have $w_{t_w}(t.x)v <_{\mathcal{H}_t} \ell_{t_w} <_{\mathcal{H}_t} RL_{t_w}(t.x, ttc) <_{\mathcal{H}_t} AL_{\widehat{t_r}}(t.x, read(x)) <_{\mathcal{H}_t} r_{t_r}(t.x)v <_{\mathcal{H}_t} \ell_{t_r}$ from which we have $\ell_{t_w} <_{\mathcal{H}_t} \ell_{t_r}$, i.e., $t_w \rightarrow_{\mathcal{H}_t^\sigma} t_r$. Hence, in all cases, we have $t_w \xrightarrow{t.x}_{rf}$

$t_r \Rightarrow t_w \rightarrow_{\mathcal{H}_t^\sigma} t_r$. $\square$

Theorem 6.1. *Every level-wise history of committed transactions,* $\Pi(\widehat{\mathcal{H}_t^\sigma})$*, produced by HParSTM satisfies the level-wise opacity consistency criterion.*

Proof. The proof follows from the definition of linearization points, and Lemmas 6.1 through 6.4. $\square$

6.5.2 Proof for aborted transactions

The correctness of aborted transaction is established in the same way as done in Chapter 5. For each aborted transaction t_a we consider its closure $\widehat{\mathcal{H}^{\mathcal{C}_{t_a}}}$ (Section 3.7.4). Observe that $\widehat{\mathcal{H}^{\mathcal{C}_{t_a}}}$ represents a history of committed transactions. Next, using $\widehat{\mathcal{H}^{\mathcal{C}_{t_a}}}$, we construct the level history at each of the ancestors of t_a and prove the consistency at each level, as done for the history of committed transactions.

Chapter 7

MxSTM

7.1 The main idea

The idea here is to have nodes employing different (optimistic/ pessimistic) concurrency control mechanism in the transaction tree (super tree) for STM. There are two types of nodes: *p-nodes* and *o-nodes*. The local objects of a *p-node* are operated on in a pessimistic manner. On the other hand, the local objects of an *o-node* are operated on in an optimistic manner. Thus, on the basis of the management of the locks, we have two types of objects in the picture.

7.1.1 About nesting of transactions

In the nesting of transactions, a node can be a *p-node* or an *o-node*. Depending upon the type of node (*p-type* or *o-type*), we get different behaviours (pessimistic or optimistic) at that node. Also, depending upon the various combinations of *p-type* and *o-type* nodes, we can obtain different degrees of concurrency in a subtree. For

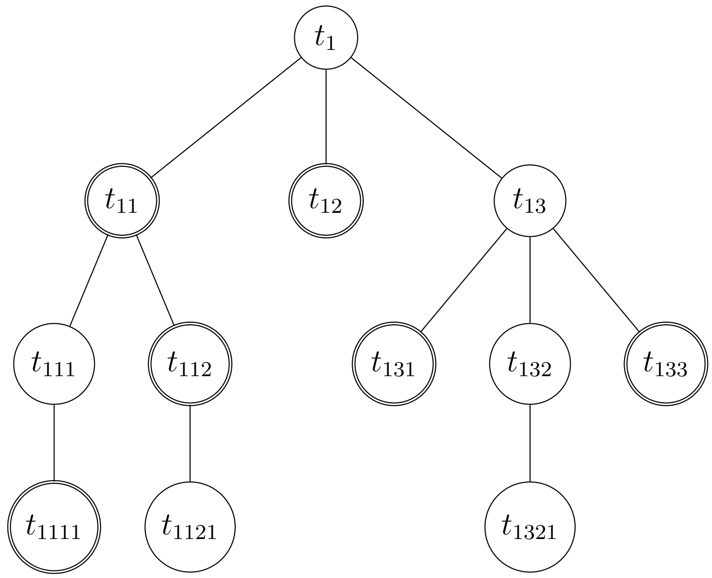


Figure 7.1: Zones of different modes of concurrency in nested transactions (single circle: *o-node*; double circle: *p-node*)

example, consider node t_{13} in Figure 7.1. Node t_{13} is an *o-node*. Two of its children, t_{131} and t_{133}, are *p-nodes*, whereas the other child, t_{132}, is an *o-node*. Also consider the path from t_1 to t_{1121}. Nodes t_1 and t_{1121} are *o-nodes*, whereas the intermediate nodes, t_{11} and t_{112}, are *p-nodes*.

7.1.1.1 Behaviour of a *p-node*

As stated earlier, a *p-node* denotes a transaction which employs a pessimistic approach to concurrency control. More precisely, when an object x (denoted as $t_p.x$) of a *p-node*, t_p, is locked by its child transaction t_c, then t_c retains the ownership of that lock until t_c completes its execution (commit/abort). That means, t_p or any other child of t_p wanting a lock on $t_p.x$ now has to wait until t_c releases the lock on $t_p.x$ upon its completion. However, it should be noted that if it is a local operation of t_p, then it releases the lock on its local object after its read/write operations. This way, the pessimistic approach offers limited concurrency. Since a child may try to lock more than one objects at a time during its normal operation, there is a possibility of a deadlock situation between the children. Hence, a *p-node* maintains a wait-for graph, whose access is controlled by a lock, to detect and resolve this deadlock situation between its children.

7.1.1.2 Behaviour of an *o-node*

An *o-node* denotes a transaction that applies optimistic concurrency control mechanism. Let t_p be an *o-node*, and t_c be one of its children. Here, when t_c acquires a lock on $t_p.x$ for its read operation, it does not retain the lock throughout its lifetime, rather it releases the lock immediately after the termination of its operation. Thus, $t_p.x$ becomes available for use by t_p or its children. Hence, there is greater degree of concurrency offered by an *o-node*.

7.2 Design challenge

In previous chapters, we already designed the protocols for emulating the behaviours of *p-node* and *o-node*. We shall denote an *o-node* by t_ω, and a *p-node* by t_π. Each of the nodes in HParSTM is an *o-node*, and the (non-root) nodes in ParSTM is a *p-node*. Hence, the job here is to integrate the two protocols, ParSTM and HParSTM, to obtain the desired result in a correct fashion.

As discussed before, the set of ancestors of a node t in the transaction tree can be a combination of *o-node*s and *p-nodes*. This introduces a challenge of ensuring that the locks of *p-nodes* and *o-nodes* are managed in the right fashion, especially while

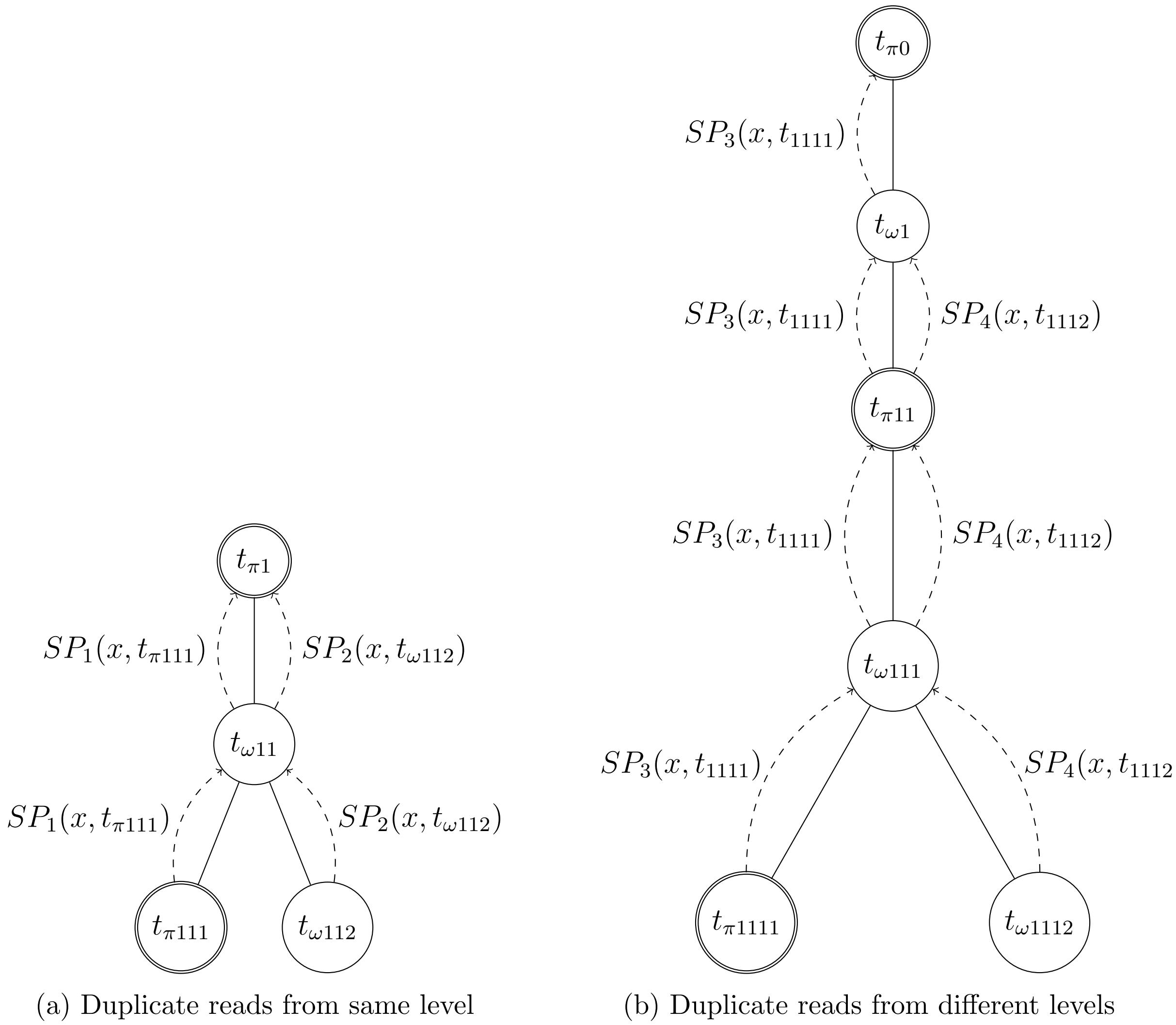


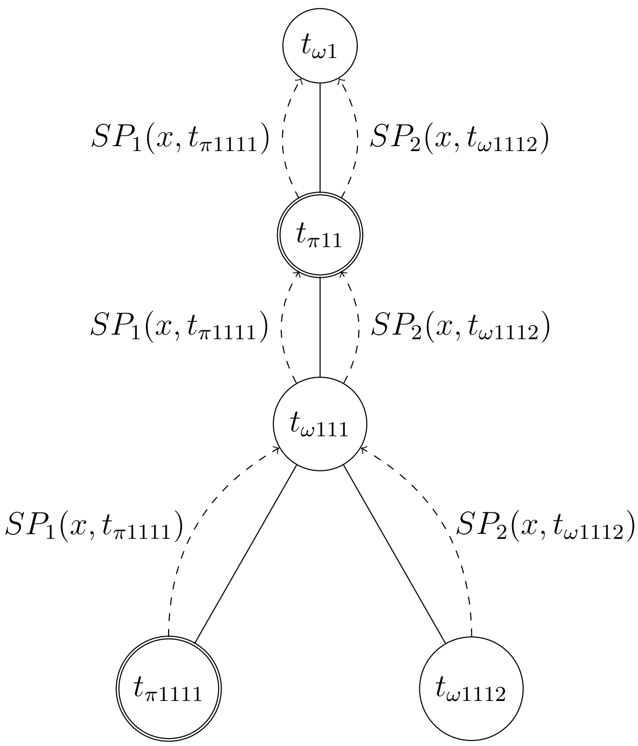
Figure 7.2: Duplicate reads (SP: search_parent)

reading from the ancestors.

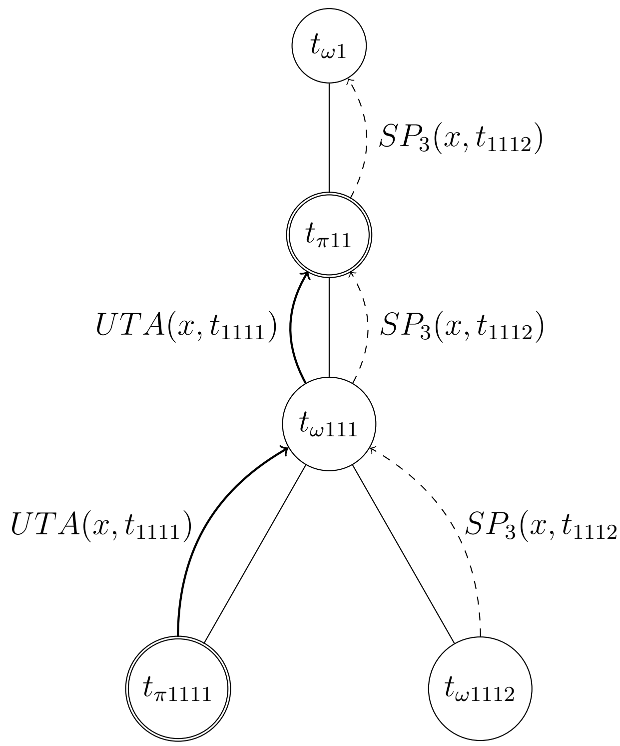
7.2.1 Handling special cases for MxSTM

7.2.1.1 Issue of duplicate request at a *p-node*

In MxSTM, the interleaving of *o-nodes* and *p-nodes* introduces a unique case (challenge), not faced in any of the previous protocols. If a *p-node*, $t_{\pi 1}$, has an *o-node*,



(a) Duplicate reads by $t_{\pi 1111}$ and $t_{\omega 1112}$



(b) Abort of $t_{\pi 1111}$

Figure 7.3: Handling release of locks in case of abort of a transaction with duplicate reads (SP: search_parent; UTA: unlock_to_ancestors)

$t_{\omega 11}$, as one of its children, then $t_{\pi 1}$ may get more than one request for reading same object through $t_{\omega 11}$ on behalf of $t_{\omega 11}$'s descendants.

To explain the case, let us refer to Figure 7.2. In Figure 7.2a, $t_{\pi 1}$ and $t_{\pi 111}$ are *p-nodes* whereas rest of the nodes are *o-nodes*. Let us say that the value of an object x is only available with $t_{\pi 1}$.

Step 1: Now, $t_{\pi 111}$ reads object $t_{\pi 1}.x$ through propagation of *search_parent* request through its ancestors $t_{\omega 11}$ and $t_{\pi 1}$ in order. Recall that $t_{\pi 1}$ being a *p-node*, its objects are accessed in pessimistic manner, i.e., $t_{\omega 11}$ retains the lock on its parent's object $t_{\pi 1}.x$. However, the lock on the object of the *o-node* $t_{\omega 11}.x$ is released after the operation.

Step 2: Now, suppose $t_{\omega 112}$ wants to read x and invokes *search_parent* procedure. The request propagates up to $t_{\omega 11}$ (because $t_{\omega 11}.x$ was unlocked after completion of external read by $t_{\pi 111}$ and is available for locking again). Observe that $t_{\omega 11}$ already holds the lock on $t_{\pi 1}.x$ owing to previous external read operation on $t_{\pi 1}.x$. At this point the question is how should the second *search_parent* request for x from $t_{\omega 11}$ be handled at $t_{\pi 1}$?

As $t_{\omega 11}$ already holds the lock on $t_{\pi 1}.x$, it is logical to argue that $t_{\omega 11}$ should foward the request *search_parent* to $t_{\pi 1}$ and should return the value of $t_{\pi 1}.x$, without trying to lock it again. However, this behavior is something new and not witnessed in ParSTM. In ParSTM, the object of a *p-node* could be read by only one transaction at a time. In this case, the value of object $t_{\pi 1}.x$ is read by $t_{\pi 111}$ and $t_{\omega 112}$ at the same time. In other words, $t_{\pi 1}.x$ has been read more than once by its descendant $t_{\omega 11}$. In Figure 7.2a, both $t_{\pi111}$ and $t_{\omega112}$ read from the same level $t_{\pi1}$. However, it is possible that the two duplicate reads obtain values from different levels, as shown in Figure 7.2b. This can happen when the value becomes available later at an intermediate *o-node* ancestor, owing to update of x due to its local write or merging of its child.

Now, let us refer to Figure 7.3 and take the case of an abort of one of the transactions that performed the duplicate reads earlier. In Figure 7.3a transactions $t_{\pi1111}$ and $t_{\omega1112}$ perform duplicate reads on $t_{\omega1}.x$ through $t_{\omega111}$. Next, in Figure 7.3b, one of the two transactions, say $t_{\pi1111}$, aborts. In ParSTM, when a subtransaction aborts, it releases the lock up to the original ancestor from which it read the value. This behavior creates a problem here. Transaction $t_{\omega111}$ should not release the lock on $t_{\pi11}.x$ as it is read by another descendant $t_{\omega1112}$ which is still active. The lock on $t_{\pi11}.x$ should be released only in one of the following three cases:

a. Both $t_{\pi1111}$ and $t_{\omega1112}$ abort.

b. Transaction $t_{\omega111}$ commits.

c. Transaction $t_{\omega111}$ aborts.

7.2.1.2 Solution

Each node (*o-node* as well as *p-node*) maintains a data structure called prs (*pessimistic read set*) containing objects x_{pr} that has a field *trc* (*total read count*) to keep track of *total number of pessimistic reads*, and a dictionary *lcs* (*level count set*) of $\langle lvl, rc\rangle$ key-value pairs, where lvl is the level of the farthest pessimistic node through which object x was read, and rc is the number of reads of x from that level. For the sake of simplicity, we shall denote an entry ($\langle lvl, rc\rangle$) in $t.prs.x_{pr}.lcs$ corresponding to level l such that $l = lvl$ as $t.prs.x_{pr}(l)$.

Further, each object x_{pr} in prs is individually lockable. Thus, the locks on two different objects, say x_{pr} and y_{pr}, can be concurrently obtained.

Working:

While reading a value of x by a descendant t_d from an ancestor t_a if there happens to be a *p-node* (if any), t_π, in the path from t_a to t_d (top to bottom order), then $t.prs.x_{pr}.trc$ is incremented by 1 at each descendant t of t_π in the path from t_π to t_d (excluding t_π). Conversely, when t_d aborts, then $t.prs.x_{pr}.trc$ decremented by 1 at each intermediate node in the path from t_d to t_π (excluding t_π) and if $t.prs.x_{pr}.trc$ equals 0 and t holds the lock on its parent's object $t_p.x$, then t releases the lock on $t_p.x$.

7.2.2 Comparing MxSTM with ParSTM and HParSTM

The methods in HParSTM and ParSTM were carefully designed so that they can be integrated to form MxSTM, with the smallest number of changes in the individual methods. The idea was to keep the code modular, easy to understand and reusable. A comparative study of MxSTM against HParSTM and ParSTM reveals that most of the code from HParSTM and ParSTM has been reused as is. However, a few adjustments had to be made in order to handle the special cases unique to MxSTM.

7.2.2.1 Changes w.r.t. both ParSTM and HParSTM

res_x: Object res_x has been updated to include two additional boolean fields, namely *is_pread* (*Is Pessimistic Read*) and *is_plocked* (*Is Parent object Locked*). Thus, res_x has following structure:

res_x:{

i	$val \in V$	iv	$is\_pread \in boolean$
ii	$lvl \in L$	v	$is\_plocked \in boolean$
iii	$its \in T$		

}

$read_{t_*}(x)$: *Read* method has been updated to include tracking of external read through a pessimistic ancestor (if any).

prs: In ParSTM (Chapter 5), prs was defined as a set containing $\langle x, level \rangle$ pairs. Here, prs is extended to contain more complex objects x_{pr}, as discussed in Section 7.2.1.2.

$unlock\_to\_ancestors$: This method too has been updated to take into consideration the external 'pessimistic read count' of an object before unlocking it at the parent level (if needed).

7.2.2.2 Specific changes w.r.t. HParSTM

$search\_parent$: This method has been updated to handle duplicate requests of its descendants that propagate to its ancestor that is a *p-node*, by tracking their pessimsitic read counts at its level. Other methods are the same as in HParSTM.

$unlock\_to\_ancestors$: Unlike HParSTM, an *o-node* t_ω here has to participate in the release of locks up to higher level ancestors, in case some transaction in $subTree(t_\omega)$, through their external read operations, obtained pessimistic locks on objects of t_ω's pessimistic ancestors.

abort: Similarly, *abort* involves releasing the locks not only to the parent level, but also up to higher level ancestors, if needed.

7.2.2.3 Specific changes w.r.t. ParSTM

search_parent: This method has been updated similiarly as done for HParSTM. Other methods are the same as in ParSTM.

vts: In MxSTM, not only the root node but also the intermediate ancestors can employ optimistic concurrency control (*o-nodes*). As such, *vts* of a child *p-node* can be used by its parent (an *o-node*) to update *its*.

7.2.2.4 New methods

We introduce the following methods in MxSTM to manage the external read count on ancestors object.

trc_incre (Total read count Increment): To increment the total read count of external pessimistic read using a lock.

trc_decre (Total read count Decrement): To decrement the total read count of external pessimistic read using a lock.

prc_incre (Pessimistc read count Increment): To increment the external pessimistic read count of an object from a level.

prc_decre (Pessimistc read count Decrement): To decrement the external pessimistic read count of an object at a level and release the lock on the parent's object accordingly.

7.3 Protocol

7.3.1 Pseudocode

Protocol 7.1: MxSTM

Common to both* o-node *and* p-node *:

t_* denotes t_π/t_ω.

State of pessimistic read object x_{pr}:
  $trc \in N$, default 0
  $lcs \subset L \times N$:
    $lvl \in L$
    $rc \in N$, default 0

Operation $\mathbf{diff\_lrc(x1_{pr}, x2_{pr}, l2)}$:
for each $(\langle l, n\rangle \in \cup_{lvl \geq l2} x2_{pr}.lcs)$ **do**
    $x1_{pr}(l).rc \leftarrow x1_{pr}(l).rc - n$;
    $x1_{pr}.trc \leftarrow x1_{pr}.trc - n$; **end for**
$x1_{pr}.lcs \leftarrow x1_{pr}.lcs \setminus \{\cup_{rc=0} x1_{pr}.lcs\}$;

State of transaction t_*:
  $parent \in T$, parent's id (t_p)
  $type \in \{$ *o-node, p-node* $\}$
  $lrs, lws \subset X$
  $cm\{mts, its\}$ and $vts \subset T$
  $pls \subset X$
  mrg : lock
  $prs \subset X_{pr}$: individual items lockable

Operation $\mathbf{begin_{t_*}(t_p, level, type)}$:
$t_*.parent \leftarrow t_p$;
$t_*.lvl \leftarrow level$;
$t_*.cm.mts \leftarrow \{t_*\}$;
$t_*.type \leftarrow type$;

Operation $\mathbf{invoke\_child_{t_*}(t_c, type)}$:
$begin_{t_c}(t_*, t_*.lvl - 1, type)$;

Operation $\mathbf{unlock\_parent\_locks_{t_*}(s)}$:
for each $x \in (s \cap t_*.pls)$ **do**
    unlock $t_p.x$;
    $t_*.pls \leftarrow t_*.pls \setminus \{x\}$; **end for**

Operation $\mathbf{unlock\_to\_ancestors_{t_*}(s)}$:
$s_{anc} \leftarrow \cup_{x_{pr}.trc>0 \wedge MaxLevel(x_{pr})>t_p.lvl} s$
$t_*.prc\_decre(s)$;
if $(s_{anc} \neq \emptyset)$ **then**
    $t_p.unlock\_to\_ancestors(s_{anc})$; **end if**

Operation $\mathbf{trc\_incre_{t_*}(x)}$:
lock $t_*.prs.x_{pr}$;
$t_*.prs.x_{pr}.trc \leftarrow t_*.prs.x_{pr}.trc + 1$;
unlock $t_*.prs.x_{pr}$;

Operation $\mathbf{trc\_decre_{t_*}(x)}$:
lock $t_*.prs.x_{pr}$;
$t_*.prs.x_{pr}.trc \leftarrow t_*.prs.x_{pr}.trc - 1$;
if $(t_*.prs.x_{pr}.trc = 0$
$\wedge x \in t_*.pls)$ **then**
    unlock $t_p.x$;
    $t_*.pls \leftarrow t_*.pls \setminus \{x\}$; **end if**
unlock $t_*.prs.x_{pr}$;

Operation $\mathbf{prc\_incre_{t_*}(x, l)}$:
lock $t_*.prs.x_{pr}$;
$t_*.prs.x_{pr}(l).rc \leftarrow t_*.prs.x_{pr}(l).rc + 1$;
unlock $t_*.prs.x_{pr}$;

Operation $\mathbf{prc\_decre_{t_*}(s_{prs})}$:
for each $(x_{pr} \in s_{prs})$ **do**
    lock $t_*.prs.x_{pr}$
    $diff\_lrc(t_*.prs.x_{pr}, x_{pr}, t_*.lvl)$;
    if $(t_*.prs.x_{pr}.trc = 0$
    $\wedge x \in t_*.pls)$ **then**
      unlock $t_p.x$;
      $t_*.pls \leftarrow t_*.pls \setminus \{x\}$; **end if**
    unlock $t_*.prs.x_{pr}$;
end for

Operation $\mathbf{get\_locks_{t_*}(s, t_c)}$:
for each $x \in s$ **do**
    $t_*.get\_local\_lock(x, t_c)$;
    $t_c.pls \leftarrow t_c.pls \cup \{x\}$; **end for**

Operation $\mathbf{abort_{t_*}}()$:
$t_*.abort\_active\_desc()$;
$t_*.unlock\_to\_ancestors(t_*.prs)$;
$t_*.unlock\_parent\_locks(t_*.pls)$;
return $(abort)$;

Operation $\mathbf{abort\_active\_desc_{t_*}}()$:
for each $t^{'} \in activeChildren(t_*)$ **do**
    $t^{'}.force\_abort()$; **end for**

Operation $\mathbf{force\_abort_{t_*}}()$:
for each $t^{'} \in activeChildren(t_*)$ **do**

$t'.force\_abort()$; **end for**
return $(abort)$;

Operation $\mathbf{abort\_incompat\_desc_{t_*}(s_i)}$:
$s_a \leftarrow s_i \cap t_*.vts$;
if $(s_a = \emptyset)$ **then** return; **end if**
for each $(t_c \in activeChildren(t_*))$ **do**
 if $(t_c.cm.mts \cap s_a \neq \emptyset)$ **then**
  $t_c.abort()$;
 else
  $t_c.abort\_incompat\_desc(s_a)$;
 end if
end for
$t_*.vts \leftarrow t_*.vts \setminus s_a$;

Operation $\mathbf{read_{t_*}(x)}$:
$t_*.get\_local\_lock(mrg, t_*)$;
$t_*.get\_local\_lock(x, t_*)$;
$res_x \leftarrow \phi$;
if $(t_*.x.val = null)$ **then**
  $trc\_incre_{t_*}(x)$;
  $f \leftarrow x \in t_*.pls$;
  $res_x \leftarrow search\_parent_{t_p}(x, t_*, t_*, t_*.cm, f)$;
  if $(res_x = null)$ **then**
   $trc\_decre_{t_*}(x)$;
   $abort_{t_*}()$; **end if**
  $t_*.x.val \leftarrow res_x.val$;
  $t_*.lrs \leftarrow t_*.lrs \cup \{x\}$;
  $update\_cm(t_*.cm, \emptyset, res_x.s_{its})$;
  if $(\neg f \wedge res_x.is\_plocked)$ **then**
   $t_*.pls \leftarrow t_*.pls \cup \{x\}$; **end if**
  if $(\neg res_x.is\_pread)$ **then**
   $t_*.trc\_decre(x)$
  else
   $t_*.prc\_incre(x, res_x.lvl)$; **end if**
end if
$v \leftarrow t_*.x.val$;
unlock $t_*.x$;
$t_*.abort\_incompat\_desc(res_x.s_{its})$;
unlock $t_*.mrg$;
return v;

Operation $\mathbf{try\_to\_commit_{t_*}()}$:
$try\_to\_merge_{t_p}(t_*)$;
return $(commit)$;

Specific to p-node (t_π) ***:***

State of object x **:**
 $val \in V$

State of transaction t_π**:**
 $wfg \subset X \times T \times T$: lockable

Operation $\mathbf{get\_local\_lock_{t_\pi}(x, t_1)}$:
if $(\neg secure\_lock_{t_\pi}(x, t_1))$ **then**
  $t_1.abort()$; **end if**

Operation $\mathbf{release\_lock_{t_\pi}(x, isLocked)}$:
if $(isLocked)$ **then**
  unlock $t_\pi.x$; **end if**

Operation $\mathbf{write_{t_\pi}(x, v)}$:
lock $t_\pi.x$;
$t_\pi.x.val \leftarrow v$;
unlock $t_\pi.x$;
$t_\pi.lws \leftarrow t_\pi.lws \cup \{x\}$;

Operation $\mathbf{search\_parent_{t_\pi}(x, t_c, t_o, cm_d, f)}$:
if $(\neg f)$ **then**
  if $(\neg secure\_lock_{t_\pi}(x, t_c))$ **then**
   return $null$; **end if**
end if
$cm \leftarrow t_\pi.cm$;
// Check if value is locally available
if $(t_\pi.x.val \neq null)$ **then**
 if$(\neg check\_compatibility(cm, cm_d))$**then**
   $release\_lock_{t_\pi}(x, \neg f)$;
   return $null$; **end if**
  $res_x \leftarrow \langle t_\pi.x.val, t_\pi.lvl, \emptyset, true, true\rangle$;
  return res_x;
end if
// Otherwise, try to read from its parent
$update\_cm(cm, cm_d.mts, cm_d.its)$;
$t_\pi.vts \leftarrow t_\pi.vts \cup \{t_o\}$;
$t_\pi.trc\_incre(xl)$;
$f2 \leftarrow x \in t_\pi.pls$;
$res_x \leftarrow search\_parent_{t_p}(x, t_\pi, t_o, cm, f2)$;
if $(res_x = null)$ **then**
  $release\_lock_{t_\pi}(x, \neg f)$;
  $t_\pi.trc\_decre(x)$;
  return $null$; **end if**
if $(\neg res_x.is\_pread)$ **then**
  $t_\pi.trc\_decre(x)$;
else
  $t_\pi.prc\_incre(x, res_x.lvl)$; **end if**
if $(res_x.is\_plocked \wedge \neg f2)$ **then**
  $t_\pi.pls \leftarrow t_\pi.pls \cup \{x\}$; **end if**
$res_x.is\_pread \leftarrow true$;
$res_x.is\_plocked \leftarrow true$;
return res_x;

Operation $\mathbf{try\_to\_merge_{t_\pi}(t_c)}$
$t_\pi.get\_local\_lock(mrg, t_c))$;
$s \leftarrow \cup\{x : x \in t_c.pls\}$
$t_\pi.get\_locks(t_c.lws \setminus s, t_c)$
if$(\neg check\_compatibility(t_\pi.cm, t_c.cm))$**then**
  unlock $t_\pi.mrg$;
  $t_c.abort()$; **end if**
for each $x \in t_c.lws$ **do**
  $t_\pi.x.val \leftarrow t_c.x.val$; **end for**
for each $x \in t_c.lrs : t_\pi.x.val = null$ **do**
  $t_\pi.x.val \leftarrow t_c.x.val$; **end for**
$t_\pi.lws \leftarrow t_\pi.lws \cup t_c.lws$;
$t_\pi.lrs \leftarrow t_\pi.lrs \cup t_c.lrs$;
$update\_cm(t_\pi.cm, t_c.cm.mts, t_c.cm.its)$;
$t_c.unlock\_parent\_locks(t_c.pls)$;
$t_\pi.abort\_incompat\_desc(res_x.s_{its})$;
unlock $t_\pi.mrg$;

Specific to o-node (t_ω) :

State of base object x**:**
$val \in V$
rs and $fbd \subset T$

State of transaction t_ω**:**
$ow \subset T$

Operation $\mathbf{get\_local\_lock_{t_\omega}(x, t_1)}$:
lock $t_\omega.x$;

Operation $\mathbf{write_{t_\omega}(x, v)}$
lock $t_\omega.x$;
$t_\omega.x.val \leftarrow v$;
$t_\omega.ow \leftarrow t_\omega.ow \cup t_\omega.x.rs$;
$t_\omega.x.fbd \leftarrow t_\omega.ow$;
$t_\omega.x.rs \leftarrow \emptyset$;
unlock $t_\omega.x$;
$t_\omega.lws \leftarrow t_\omega.lws \cup \{x\}$;

Operation $\mathbf{search\_parent_{t_\omega}(x, t_c, t_o, cm_d, f)}$:
lock $t_\omega.x$;
$cm \leftarrow t_\omega.cm$;
$t_\omega.x.rs \leftarrow t_\omega.x.rs \cup \{t_o\}$;
// Check if value is locally available
if ($t_\omega.x.val \neq null$) **then**
  if($\neg check\_compatibility(cm, cm_d) \vee$
  $(t_\omega.x.fbd \cap cm_d.mts \neq \emptyset)$) **then**
    unlock $t_\omega.x$;
    return $null$; **end if**
  $s_i \leftarrow t_\omega.x.fbd \cap t_c.vts$;
  $res_x \leftarrow \langle t_\omega.x.val, t_\omega.lvl, s_i, false, false\rangle$;
  unlock $t_\omega.x$;
  return res_x;
end if
// Otherwise, try to read from its parent
$update\_cm(cm, cm_d.mts, cm_d.its)$;
$t_\omega.vts \leftarrow t_\omega.vts \cup \{t_o\}$;
$t_\omega.trc\_incre(x)$;
$f2 \leftarrow x \in t_\omega.pls$;
$res_x \leftarrow search\_parent_{t_p}(x, t_\omega, t_o, cm, f2)$;
if ($res_x = null$) **then**
  $t_\omega.trc\_decre(x)$;
  unlock $t_\omega.x$;
  return $null$; **end if**
if ($\neg res_x.is\_pread$) **then**
  $t_\omega.trc\_decre(x)$;
else
  $t_\omega.prc\_incre(x, res_x.lvl)$
end if
if ($res_x.is\_plocked \wedge \neg f2$) **then**
  $t_\omega.pls \leftarrow t_\omega.pls \cup \{x\}$; **end if**
$res_x.is\_plocked \leftarrow false$;
unlock $t_\omega.x$;
return res_x;

Operation $\mathbf{try\_to\_merge_{t_\omega}(t_c)}$:
$t_\omega.get\_local\_lock(mrg, t_c)$);
$t_\omega.get\_locks(t_c.lrs \cup t_c.lws, t_c)$;
if ($\neg check\_compatibility(t_\omega.cm, t_c.cm) \vee$
  $(t_c.lws \neq \emptyset \wedge (t_c.cm.mts \cap t_\omega.ow \neq \emptyset)$)**then**
  unlock $t_\omega.mrg$;
  $t_c.abort()$; **end if**
for each $x \in (t_c.lws)$ **do**
  $t_\omega.x.val \leftarrow t_c.x.val$; **end for**
for each $x \in t_c.lrs : t_\omega.x.val = null$ **do**
  $t_\omega.x.val \leftarrow t_c.x.val$; **end for**
$t_\omega.ow \leftarrow t_\omega.ow \cup (\cup_{x \in t_c.lws} t_\omega.x.rs)$;
for each $x \in t_c.lws$ **do**
  $t_\omega.x.fbd \leftarrow t_\omega.ow$;
  $t_\omega.x.rs \leftarrow \emptyset$; **end for**
$t_\omega.lrs \leftarrow t_\omega.lrs \cup t_c.lrs$;
$t_\omega.lws \leftarrow t_\omega.lws \cup t_c.lws$;
$update\_cm(t_\omega.cm, t_c.cm.mts, t_c.cm.its)$;
$t_c.unlock\_parent\_locks(t_c.pls)$;
$t_\omega.abort\_incompat\_desc(res_x.s_{its})$;
unlock $t_\omega.mrg$;

Special case of root node, t_ρ ***:***
if (t_ρ is a **p-node**): use **Protocol 5.3**
if (t_ρ is a **o-node**): use **Protocol 6.2**

7.3.2 State of pessimistic read object, x_{pr}, and helper methods

x_{pr}: The pessimistic read object, x_{pr} consists of *trc* (*total read count*), and *lcs* (*level count set*) which is a hashtable of $\langle lvl, rc\rangle$ key-value pairs, where *lvl* denotes the level of the highest level *p-node* ancestor through which the value of x was obtained, and *rc* (*read count*) is the number of times the value was read through that level. An entry in *lcs* corresponding to a level l is denoted by $x_{pr}(l)$. Thus, $x_{pr}(l).lvl$ and $x_{pr}(l).rc$ denote the corresponding *lvl* and *rc* of that entry respectively.

$diff\_lrc(x1_{pr}, x2_{pr}, l2)$: This helper method is used to decrement the level-wise read count of $x1_{pr}$ by the read count of the corresponding level-wise entry in $x2_{pr}$. The parameter $l2$ is used to filter out entries in $x2_{pr}.lcs$ whose level is lower than the level to which $x1_{pr}$ belongs.

7.3.3 Methods common to ***o-node*** and ***p-node*** (t_*)

The common methods have already been discussed in previous chapters. Therefore, we discuss only the main methods and leave out the discussion of other (self-explanatory) methods.

$begin_{t_*}(t_p, level, ntype)$: Each transaction t_* begins with this method, where t_p denotes the id of its parent, *level* is the level of the node in the super tree, and *ntype* is the type of the transaction, *p-type* or *o-type*. Here, *ntype* has been introduced for enabling easy identification of the type of a transaction.

$unlock\_to\_ancestors_{t_*}(s)$: In $MxSTM$, the *unlock_to_ancestors* method uses the method *prc_decre* to decrement the level-wise read count at each level, and release the pessimistic lock on the parent's objects accordingly.

$trc\_incre_{t_*}(x)$: As discussed before.

$trc\_decre_{t_*}(x)$: As discussed before.

$prc\_incre_{t_*}(x, l)$: As discussed before.

$prc\_decre_{t_*}(s)$: As mentioned before, this method is used to decrement the level-wise read count of object $t_*.prs.x_{pr}$ for each object x_{pr} in s_{prs} using an individual lock on $t_*.prs.x_{pr}$. Upon decrementing, if the resulting $t_*.prs.x_{pr}.trc$ is 0 then the lock on parent's object $t_p.x$ is released. This method is used by an aborting subtransaction to release the pessimistic lock on its ancestor's objects.

$abort_{t_*}()$: Before aborting, transaction t_* releases the locks on its ancestor's objects obtained by t_* or its descendants, followed by forcefully aborting its active descendants.

$abort\_incompat\_desc_{t_*}()$: This method is used by an ancestor to abort its incompatible descendants.

$read_{t_*}(x)$: To read from its local copy of its object, $t_*.x$, transaction t_* locks $t_*.x$. If $t_*.x$ is null-valued, then t_* invokes the method $search\_parent(x)$ to get the value from its parent's local copy of x.

Before invoking the $search\_parent$ method of its parent, it first increments $t_\pi.prs.x_{pr}.trc$. This ensures that, if it does already possess a lock on the parent's object, then that lock is not released in the meantime due to the abort of a descendant. This is important as in this case t_π assumes that it will continue to hold the lock on $t_p.x$ and informs its parent not to lock $t_p.x$ while invoking $t_p.search\_parent$. If res_x returned from $t_p.search\_parent$ is $null$, then $t_\pi.prs.x_{pr}.trc$ is decremented to reset it to its previous value. Alternatively, if $res_x.is\_plocked$ is $true$, then the lock on the parent's object was retained in the process and therefore t_π adds x to $t_\pi.pls$. Further, if $res_x.is\_pread$ is $true$, then $t_\pi.prs.x_{pr}(res_x.lvl)$ is incremented. The objects read by t_* are recorded in its local read set, $t_*.lrs$. The compatibility set cm is updated accordingly. At the end of the operation, the lock on $t_*.x$ is released.

$try\_to\_commit_{t_*}()$: A (nested) transaction t_* merges its local read/write sets with those of its parent, and updates the value of the local objects of its parent.

7.3.4 State of local objects and methods associated with *p-node* (t_π)

State of local object x : In the case of a *p-node*, local object x has only the value field, and is protected by a lock.

State of a p-node (t_π): The id of the parent of a transaction t_π is denoted by $parent$. The sets, lrs (local read set) and lws (local write set) record the objects read and written respectively by t_π. A *p-node* also maintains a wait-for gragh, wfg, for its children, to detect and resolve deadlock situation among them. The set prs is used to keep track of the level-wise external pessimistic read count.

$write_{t_\pi}(x, v)$: self-explanatory.

$search\_parent_{t_\pi}(x, t_c, ...)$: The call to this method is cascaded in nature, and is invoked by the child node t_c to obtain a lock on object $t_\pi.x$ of its parent, t_π. In case of a duplicate request where t_c already holds a lock on $t_\pi.x$, $t_\pi.x$ need not be locked. Now, if t_π does not have a local value for $t_\pi.x$, then it tries to obtain the value from its own parent ($t_\pi.parent$), before returning that value to its child t_c. Like the *read* operation, this method also, in case of forwarding the request to its parent, updates its $prs.x_{pr}$ to ensure correct checking and recording of ownership on its parent's lock. If $res_x.is\_plocked$ returned from its parent is *true*, then it indicates that the lock on the parent's object was retained in the process and therefore t_π adds x to $t_\pi.pls$. Finally, if the res_x needs to be passed further down, the t_π sets $res_x.is\_pread$ and $res_x.is\_plocked$ to *true* to indicate to its child that the value was read through a *p-node* (i.e., t_π) and a pessimistic lock on its parent was retained in the process.

$try\_to\_merge_{t_\pi}(t_c)$: This method is invoked by the child node to merge its results (values of objects, read set, write set, etc.) with the parent. Please note that, in case the child transaction writes a local object x without having previously read its value from the parent, then it does not possess the lock on that object of the parent. Hence, at the time of merging, locks on such objects of the parent are obtained. Next, the value of all the objects, belonging to *lws* and *lrs* of the child node, is updated using the local copy of the child node. Finally, the compatibility object *cm* and sets *lrs* and *lws* of the child node are merged with those of its parent, before releasing the locks.

7.3.5 State of local objects and methods associated with *o-node* (t_ω)

State of local object x : Here, a local object x has three components, namely value field (*val*), read set (*rs*) and forbidden set (*fbd*). Each local copy is protected by a lock.

State of an o-node (t_ω): Here, a transaction uses the *parent* field to store the id of its parent. Further, sets, *lrs* and *lws*, are used to record the objects read and written respectively by t_ω. The set *ow* (overwritten set) denotes the of those children of t_ω, that read an object $t_\omega.x$ which has been modified (overwritten) later. An *o-node* also maintains the set prs to keep track of the level-wise external pessimistic read count.

$write_{t_\omega}(x, v)$: First, $t_\omega.x$ is locked. The value of $t_\omega.x$ is updated. The ids of all the children that previously read $t_\omega.x$ are added to $t_\omega.ow$, followed by updating $t_\omega.x.fbd$ using $t_\omega.ow$, and clearing $t_\omega.x.rs$. Finally, $t_\omega.x$ is unlocked, and x is added to $t_\omega.lws$.

$search\_parent_{t_\omega}(x, t_c, ...)$: Similar to the $search\_parent(x)$ method of a *p-node*, this method is invoked by the child node. If the value of the requested object is *null*, then the value is obtained from the parent by invoking the $search\_parent(x)$ of its own parent (recursive call). Unlike in case of a *p-node*, before returning the value of the object to the child node (t_c), it is checked if it is legal for t_c to read $t_\omega.x$ by checking for the membership of t_c in $t_\omega.x.fbd$ as well as doing a compatibility check. If the validation is not successful, then t_c is aborted. Otherwise, t_c is added to $t_\omega.x.rs$ before unlocking $t_\omega.x$ and returning the value. Similar to the operation for *p-nodes*, $t_\omega.prs.x_{pr}$ is updated accordingly. Note that in the case of an *o-node*, it only sets $res_x.is\_plocked$ to $false$ to inform its child that a pessimistic lock was not obtained at its parent level in the process.

$try\_to\_merge_{t_\omega}(t_c)$: This method is invoked by the child node, t_c. First, all the objects belonging to $t_c.lrs \cup t_c.lws$ in the parent's (t_ω's) local space are locked, followed by locking $t_\omega.mrg$. The validation for t_c consists of ensuring $t_c.lws \neq \emptyset$ and $t_c \notin t_\omega.ow$ as well as checking for compatibility. If the validation is not successful, then t_c is aborted. Otherwise, the value of the parent's objects that are present in $t_c.lrs$ and $t_c.lws$ are updated using t_c's local copy of those objects. Next, the ids of transactions in $t_\omega.x.rs$ of each $t_\omega.x$ that is modified are added to $t_\omega.ow$. The set $t_\omega.ow$ is used to update $t_\omega.x.fbd$ of each $t_\omega.x$ that is modified, followed by clearing $t_\omega.x.rs$. If t_ω is not the super transaction, then $t_c.lrs$ and $t_c.lws$ are merged with the corresponding sets of the parent, t_ω.

7.3.6 About deadlock freedom

To show that MxSTM is deadlock- free, we shall examine *p-nodes* and *o-nodes*. Observe that, by construction (also, refer to discussion in Section 5.1.2.2 and Section 6.3.4), any access to local objects of a *p-node* is duly recorded in its *wait-for-graph, wfg*, to prevent any deadlock situation. Thus, no deadlock is possible involving objects of *p-nodes*. In other words, locks on such objects cannot be held for an indefinite period of time.

In case of an *o-node*, say t_ω, locks on its objects are released soon after the completion of the external read on them. However, when the object $t_\omega.x$ is locked as part of the *search_parent* operation that escalates up to a t_ω's ancestor, say $t_{\pi 1}$, that is a *p-node*, then $t_{\pi 1}$'s local object $t_{\pi 1}.x$ may not be immediately available. In that case, the lock on $t_\omega.x$ will be held until $t_{\pi 1}.x$ becomes available. As the lock on $t_{\pi 1}.x$ cannot be held indefinitely (based on the discussion earlier about *p-nodes*), it follows that the lock on $t_\omega.x$ cannot be held forever. In the meantime, while $t_\omega.x$ is locked, any other request for $t_\omega.x$ (for a read or commit of a child node) will ensue in a wait that is deadlock-free. Thus, locking of t_ω's objects for external reads on them is free from deadlocks.

During commit, deadlock freedom is ensured by allowing only one child to merge with the parent, by using the parent's *mrg* lock. In the case of a *p-node*, the *wait-*

for-graph is used in addition for this purpose.

7.4 Correctness

7.4.1 Definition of linearization points of events

7.4.1.1 At a *p-node* t_π:

Let ℓ_{op} denote the linearization point of an event. Then, the linearization points of the various events in the history are defined as follows:

i Local read/write operation of t_π

(a) $read_{t_\pi}(t_\pi.x)$: ℓ_{op} corresponds to the time when it unlocks $t_\pi.x$ (line 110)

(b) $write_{t_\pi}(t_\pi.x)$: ℓ_{op} corresponds to the time when it unlocks $t_\pi.x$ (line 130)

ii External read operation of t_π

(a) $read_{t_\pi}(t_a.x)$: ℓ_{op} corresponds to the time just after $t_\pi.cm$ is updated (line 101)

iii External read of a descendant t_d on t_π's object or that of t_π's ancestor t_a:

(a) $read_{t_d}(t_\pi.x)$: ℓ_{op} corresponds to the time just after t_d reads $t_\pi.cm$ for consistency checking (line 137)

(b) $read_{t_d}(t_a.x)$: ℓ_{op} corresponds to the time just after t_d reads $t_\pi.cm$ for consistency checking (line 137)

iv Write due to commit of child t_c

(a) $write_{t_c}(t_\pi.x)$: ℓ_{op} corresponds to the time just after t_c updates $t_\pi.x$ (line 171) v Commit of child t_c

(a) C_{t_c} : ℓ_{op} corresponds to the time when $t_\pi.cm$ is updated (line 176)

7.4.1.2 At an *o-node* t_ω:

i Local read/write operation of t_ω

(a) $read_{t_\omega}(t_\omega.x)$: ℓ_{op} corresponds to the time when it unlocks $t_\omega.x$ (line 110)

(b) $write_{t_\omega}(t_\omega.x)$: ℓ_{op} corresponds to the time when t_ω updates $t_\omega.ow$ (line 190)

ii External read operation of t_ω

(a) $read_{t_\omega}(t_a.x)$: ℓ_{op} corresponds to the time when $t_\omega.cm$ is updated (line 101)

iii External read of a descendant t_d on t_ω's object or that of t_ω's ancestor t_a:

(a) $read_{t_d}(t_\omega.x)$: ℓ_{op} corresponds to the time when t_d reads $t_\omega.cm$ for consistency checking (line 197)

(b) $read_{t_d}(t_a.x)$: ℓ_{op} corresponds to the time when t_d reads $t_\omega.cm$ for consistency checking (line 197)

iv Write due to commit of child t_c

(a) $write_{t_c}(t_\omega.x)$: ℓ_{op} corresponds to the time when t_c updates $t_\omega.x$ (line 236)

v Commit of child t_c

(a) C_{t_c} : ℓ_{op} corresponds to the time just after $t_\omega.cm$ is updated by t_c (line 245).

7.4.2 Definition of linearization point of a transaction t

7.4.2.1 At ***p-node*** (i.e., parent t_p of t is a ***p-node***) :

1. If t commits, its linearization point, ℓ_t, lies at the time just after it updates the parent's cm (consistency management) object (line 176).

2. If t aborts, ℓ_t coincides with the last time when t reads $t_p.cm$ for its successsful external read operation at t_p's level (line 137).

7.4.2.2 At ***o-node*** (i.e., parent t_p of t is an ***o-node***):

1. If t is an update transaction that commits, its linearization point, ℓ_t, lies at the time just after it updates the parent's ow and cm (consistency management) object (line 245).

2. If t is a read only committed transaction, then ℓ_t is placed at the earliest of (i) the time it reads $t_p.cm$ for its last successsful external read operation (line 197), and (ii) the time just before $\widehat{t}$ (any id in $t.cm.mts$) is added to $t_p.ow$ (if it ever is) (lines 190, 239).

3. If a transaction t aborts, ℓ_t is determined as if it were a read only transaction, i.e., ℓ_t lies at the earliest of (i) the time it reads $t_p.cm$ for its last successsful external read operation (line 197), and (ii) the time just before $\widehat{t}$ (any id in $t.cm.mts$) is added to $t_p.ow$ (if it ever is). (lines 190, 239).

This definition of linearization points should be used when we consider the various level-wise histories of MxSTM to establish the correctness.

7.4.3 Proof

To show that the protocols presented in ParSTM and HParSTM have been integrated in a correct fashion to obtain MxSTM, we show that the following two points indeed hold true: (1) at *o-nodes*, the lock management is done in an optimistic fashion, whereas (2) at *p-nodes*, the locks are managed in a pessimistic manner (2PL for nested transactions).

Lemma 7.1. *In MxSTM, the locks associated with an o-node (t_ω) are managed in an optimistic manner, whereas those associated with a p-node (t_π) are operated in a pessimistic manner.*

Proof. Let us take the case of an *o-node*, t_ω, first. Consider an object $t_\omega.x$. We need to show that lock on $t_\omega.x$ is retained only for the duration of the read/write operation, not for the entire lifespan of the transaction. We consider all the methods (cases) in which $t_\omega.x$ is locked, and show that $t_\omega.x$ in unlocked at end of each of these methods.

- $read_{t_*}(t_*.x)$: $t_*.x$ locked at line 90, and released at line 110.
- $search\_parent_{t_\omega}(x, t_c, t_o, cm_d, f)$: $t_\omega.x$ locked at line 196, and released at line 202, 206, 216 or 226.
- $write_{t_\omega}(x, v)$: $t_\omega.x$ locked at line 188, and released at line 193.
- $try\_to\_merge_{t_\omega}(t_c)$: $t_\omega.x$ locked at line 230, and released at line 246 (or 234 due to abort).

Next, we show that when the lock on $t_\pi.x$ is obtained by t_π's child t_c in a successful read/write operation, it is released by t_c only upon its completion (commit/abort). The $read_{t_\pi}(x)$ and $write_{t_\pi}(x, v)$ methods are used for local read/write operations of t_π. As such, t_π releases the lock immediately after its read/write operation on its local object $t_\pi.x$.

Let us now consider the corresponding methods of t_π in which $t_\pi.x$ is locked by its children.

- $search\_parent_{t_\pi}(x, t_c, t_o, cm_d, f)$: $t_\pi.x$ locked at line 134, but the object is not unlocked in case of successful read. Unlocking is done only if the read step is unsuccessful (lines 140 or 151).

- $try\_to\_merge_{t_\pi}(t_c)$: $t_\pi.x$ locked at line 166. This operation is performed by a transaction during its commit phase which marks the end of the lifespan of the transaction. The child transaction t_c releases all the locks on its parent's objects only upon successful merging/committing (line 177) or upon aborting (line 169).

Observe that, when a subtransaction performs an external read on an object $t_\pi.x$ of *p-node* t_π, it holds the lock on that object until it terminates. Thus, the lock obtained during an external read is held for the lifespan of the descendant transaction. On the other hand, in case of an *o-node* t_ω, the lock on $t_\omega.x$ is released upon completion of the external read operation. Here, such a lock is held only until the external read operation completes.

$\square$

Next, we show that MxSTM handles incompatible transactions correctly.

Lemma 7.2. *MxSTM ensures correct checking of incompatible transactions at each level.*

Proof. Checking for incompatibility of transactions is done using the sets, *mts* and *its*. The incompatibility of transactions is introduced due to reading from and update of objects at higher level *o-nodes*. Hence, we need to show that (1) when a subtransaction t reads an object $t_\omega.x$ from an ancestor t_ω, t is added to $t_\omega.x.rs$, irrespective of the type of t or its intermediate ancestors, and (2) the sets, *cm.mts* and *cm.its*, are updated at every level.

The recursive method $search\_parent_{t_*}(x, t_c, t_o, cm_d, f)$ that is used by a subtransaction to read from ancestors, has the same signature for both types of nodes. As such, the id of the subtransaction, t_o, originally initiating the method is propagated upwards. If the value is successfully returned from an *o-node* (ancestor, t_ω), then t_o is added to $t_\omega.x.rs$ (line 198). When $t_\omega.x$ is updated, t_o is added to $t_\omega.x.fbd$ and $t_\omega.ow$. This ensures that later, when some other subtransaction t_d that is incompatible with t_o, reads from t_ω, the set $t_d.cm.its$ is updated correctly.

Further, observe that every node t_* (*o-node* as well as *p-node*) maintains the sets, $t_*.cm.its$ and $t_*.cm.mts$. The set $t_*.cm.mts$ is updated at the time of merge operation (lines 176, 245), whereas $t_*.cm.its$ is updated while reading from an ancestor (line 101) and during the merge process (lines 176, 245).

□

Next, we shall show that the protocol handles duplicate requests at a *p-node* correctly. First, we shall show that at any level (node), for a given object x, only one request can come from the same child. Then, we shall show that at a *p-node*, t_π, any subsequent duplicate request for accessing $t_\pi.x$ from a child t_c that already retains lock $t_\pi.x$ is handled correctly. Correctness here means (i) as t_c already holds the lock for $t_\pi.x$, it should be allowed to access $t_\pi.x$ directly, and (ii) in case the subsequent call fails for some reason, $t_\pi.x$ should not be unlocked in the current operation context, to preserve the state of previous successful call.

Lemma 7.3. *At any level (node) t, only one request for external read on an object $t.x$ can come from the same child t_c at a time.*

Proof. As per the statement of the lemma, note that the types of t and t_c do not matter and the lemma is applicable in general. Further, we know that the external read is performed through the *search_parent* method.

Observe that before invoking $t.search\_parent$, the lock on $t_c.x$ needs to be obtained (line 90 for t_c's own read; line 134 in case of *p-node*; line 196 in case of *o-node*). In case t_c is a *p-node*, then looking at line 133 one can argue that $t_c.x$ is not locked at line 133 in all cases. However, it should be noted that the exception at line 133 is only applied for a descendant t_d of t_c that already has a lock on $t_c.x$. Thus, the statement that only one transaction holds the lock on $t_c.x$ at a time is true.

Now, one may question if two or more concurrent invocations for reading x can come from t_d? The answer is 'no'. To put the answer in perspective, let us look at the behaviour of concurrent requests for external reads. The invocation of *search_parent* is initiated by the *read* operation. Let us consider two descendants, t_{d1} and t_{d2} of t_d such that t_d is their least common ancestor. Both the descendants want to read x. For the sake of the argument, let us also assume that requests of t_{d1} and t_{d2} reach t_d concurrently. Now, based on the earlier discussion, only one of the two descendants can possibly obtain the lock on $t_d.x$ first. That means the request of only one of the descendants will be propagated forward beyond t_d while the other request has to wait. That means only one request for external read on an object x can come from the same descendant.

Thus, following this line of argument, we conclude that only one request for an external read on $t.x$ can come from a child t_c at a time. $\square$

Lemma 7.4. *At a p-node, duplicate request by a child is allowed in a non-blocking manner.*

Proof. Let t_π be a *p-node* and t_c be its child that has already read $t_\pi.x$ through the *search_parent* method. We shall show that the subsequent request by t_c to read $t_\pi x$ is allowed, given that t_c is consistent with t_π. Observe that t_c may be required to make the duplicate read at t_π for its own purpose or on behalf of its descendant.

Let R_o and R_d denote the original and duplicate requests respectively. Based on Lemma 7.3, we have $R_o < R_d$, i.e., the duplicate request occurs after the original request has completed. Now, given that $R_o < R_d$, we have

$x \in t_c.pls$ during R_d (due to lines 142, 158-159)

$\Rightarrow$ t_c does not have to wait for lock during R_d (due to lines 94, 133)

Now, if t_c is compatible with t_π (line 139) during R_d, then t_c can read $t_\pi.x$ (line 142)

Thus, t_c can make duplicate requests at its parent level without having to wait for a lock on $t_\pi.x$.

Further observe that, as no new request for locking t_π's object is made, the duplicate request does not alter the existing wait-for dependency for locks at t_π's level. Hence, a deadlock scenario does not arise in this case.

□

Lemma 7.5. *Sets pls and prs are managed correctly during an external read.*

Proof. Set *pls* (*parent lock set*) is used to denote whether a subtransaction currently holds a lock on an object of its parent, where as *prs* (*pessimistic read set*) is used to track the number of read count and the level of the highest level pessimistic node whose object was locked during an external read involving this transaction.

Let us consider a subtransaction t with its parent t_p.

Case pls:

First, for *pls*, let us show that an object x is added to $t.pls$ at a transaction t only if t retains the lock on its parent's object $t_p.x$ upon a successful external read. By construction of the Protocol, x is added to $t.pls$ only if the response object res_x from its parent t_p has its property *is_plocked* (*Is Parent Locked*) set to *true* (lines 102-103; 158-159; 223-224).

Now, if t_p is an *p-node*, then $res_x.is\_plocked$ is set to *true* only in case of a successful external read operation (line 142, 161) at t_p. Otherwise, t_p returns *null* which does not lead to adding x to $t.pls$ (lines 96-98; 150-153; 214-217).

On the other hand, if t_p is an *o-node*, $res_x.is\_plocked$ is set to $false$ even in case of a successful external read. Consequently, t_c does not add x to $t_c.pls$ while reading from an *o-node*.

Case prs:

Let t_d be a descendant that reads x from its ancestor t_a. Let t_π be the highest level *p-node* in the path from t_a to t_d. Then, we show that:

(a) In case of a successful read, $t.prs$ is updated (increment $t.prs.x_{pr}(t_\pi.lvl).rc$ by 1) at each descendant t of t_π in the path from t_π to t_d.

(b) Conversely, when t_d aborts, $t.prs.x_{pr}(t_\pi.lvl).rc$ is decremented by 1 at each intermediate ancestor t in the path from t_d to t_π, excluding t_π.

Proof for part(a):

Before invoking *search_parent* of the parent t_p, the invoking child transaction t optimistically increments its $t.prs.x_{pr}.trc$ by 1 (lines 93, 147, 211). By construction of *search_parent* in $MxSTM$, observe that if the response object res_x passes through a *p-node*, t_π, then $res_x.is\_pread$ is set to *true* (line 160). Thereafter, all the subsequent descendants of t_π in the path from t_d increment $prs.x_{pr}(t_\pi.lvl).rc$ by 1 (line 107, 157, 221). If res_x is *null* or $res_x.is\_pread$ is $false$, then $t.prs.x_{pr}.trc$ is decremented by 1 to compensate for the optimistic increment made to $t.prs.x_{pr}.trc$ at the begining of the invocation (line 97, 105, 152, 155, 215, 219). Thus, we see that prs is correctly updated during external read operation.

Proof for part(b):

Now, we shall show that when t_d aborts, $t.prs.x_{pr}$ is decremented at each of the ancestors in the path from t_d to t_π, excluding t_π. Observe that the entries in $t_d.prs.x_{pr}$ contains the *read count* as well as level of t_π. Upon aborting, t_d invokes $unlock\_to\_ancestors$ with its prs. Observe that the recursive operation $unlock\_to\_ancestors$ propagates upward up to t_π's child in the path from t_d to t_π (lines 31, 34). At each level t, the entries in $t.prs.x_{pr}$ are decremented by an amount equal to the corresponding level-wise entries in $t_d.prs.x_{pr}$ (line 32, 54, 6-10).

□

Lemma 7.6. *MxSTM guarantees safety during duplicate external read and unlock_to_ancestors* *occurring at the same time at a level.*

Proof. Given parent t_p of node t_c is a *p-node*, if $x \in t_c.pls$ holds true at the time of invocation of $search\_parent_{t_p}$, then invocation of $unlock\_to\_ancestor$ at t_c does not alter the relation $x \in t_c.pls$ while $search\_parent_{t_p}$ is executing.

In other words, if a descendant t_d of t_c previously read x through t_p then the lock on $t_p.x$ is retained by t_c and x is added to $t_c.pls$. In that case, $x \in t_c.pls$ evaluates to *true* at the time of the subsequent invocation of $t_p.search\_parent(x)$. Now, we have to show that invocation of $t_p.unlock\_to\_ancestors(x, ...)$ due to abort of t_d, after the invocation of $t_p.search\_parent(x)$ by t_c, does not unlock $t_p.x$. This is important because this invocation $t_p.search\_parent(x)$ assumes that t_c already holds the lock on $t_p.x$ and does not try to lock $t_p.x$ for t_c.

By the design of the Protocol, following Lemma 7.5, if $x \in t_c.pls$ holds *true* during a *search_parent* invocation, then it also means $t_c.prs.x_{pr}.trc > 0$. Let $t_c.prs.x_{pr}.trc = n_1$. Then, we have $n_1 > 0$. Observe that before invoking $t_p.search\_parent(x)$, t_c invokes $t_c.trc\_incre(x)$ to increment $t_c.prs.x_{pr}.trc$ by 1. Conversely, the execution of $t_c.unlock\_to\_ancestors(x, ...)$ invokes $t_c.prc\_decre(s)$ to decrement $t_c.prs.x_{pr}.trc$ by a count $\leq n_1$. Thus, given $t_c.trc\_incre(x) < t_c.prc\_decre(s)$, we have:

Before execution of $t_c.trc\_incre(x)$, $t_c.prs.x_{pr}.trc = n_1$

After execution of $t_c.trc\_incre(x)$, $t_c.prs.x_{pr}.trc = n_1 + 1 = n_2$

Before execution of $t_c.prc\_decre(\{\langle x, l, n\rangle\})$, $t_c.prs.x.rc = n_2$

After execution of $t_c.prc\_decre(\{\langle x, l, n\rangle\})$, $t_c.prs.x.rc = n_3$

In the worst case scenario, n could be as large as n_1, but even then we have:

$n_3 = n_2 - n \geq n_2 - n_1 \geq 1 > 0$

Since the critrerion for unlocking $t_p.x$ is that $t_c.prs.x_{pr}.trc = 0$, $n_3 \geq 1 \neq 0 \Rightarrow t_p.x$ is not unlocked during the execution of $t_c.unlock\_to\_ancestors(x, ...)$, i.e., $x \in t_c.pls$ still holds true.

□

Lemma 7.7. *Duplicate external read operations at a level obtaining values from different ancestral levels are logged correctly.*

Proof. Let $R1$ be the first external read operation by a descendant t_{d1} of an *o-node*, t_ω, on its ancestor t_π. Observe that the intermediate ancestors of t_ω up to t_π could be a combination of *o-nodes* and *p-nodes*. Let $t_{\omega'}$ be such an intermediate ancestor such that $t_{\omega'}.x$ becomes non-null after $R1$ has completed. This can happen either due to a local write by $t_{\omega'}$ or a commit of its child. Let $R2$ be the subsequent external read operation by t_ω's another descendant, t_{d2}, such that it obtains the value from $t_{\omega'}.x$. Further, let $t_{\pi'}$ be the highest level *p-node* in the path from $t_{\omega'}$ to t_{d2}. Then we have to show that $t_\omega.prs.x_{pr}.lcs$ contains $\langle t_\pi.lvl, 1\rangle$ as well as $\langle t_{\pi'}.lvl, 1\rangle$.

By construction of MxSTM, during $R1$, $\langle t_\pi.lvl, 1\rangle$ is added to $t_\omega.prs.x_{pr}.lcs$ (due to lines 142 or 160, 221, 49). Similarly, when $R2$ completes, $\langle t_{\pi'}.lvl, 1\rangle$ is added to $t_\omega.prs.x_{pr}.lcs$ (due to lines 205, 160, 221, 49).

□

Lemma 7.8. *At a level t_c, if the highest level p-node $t_{\pi'}$ registered by a duplicate external read operation $R2$ is different from the p-node t_π registered by the original read operation $R1$, then the level t from which value has been read by $R2$ is an o-node* *and it lies between t_π and $t_{\pi'}$.*

Proof. Here $R1$ is the original external read operation through a subtransaction t_c such that the value was read from an ancestor t_a and the highest level *p-node* registered in the process is t_π. $R2$ is the subsequent duplicate external read operation such that it registers $t_{\pi'}$ as the highest level *p-node*. Node t is the ancestor from which $R2$ obtains its value. Then, we show that (a) t is not an ancestor of t_a, (b) t is an *o-node*, and (c) t lies between t_π and $t_{\pi'}$ in the ancestral path.

Proof of part (a): As $R1$ read from t_a, it means t_a is non-null valued. That means that invocation of *search_parent* during $R2$ cannot go beyond t_a. Hence, t cannot be an ancestor of t_a.

Further, t cannot be t_a as in that case, the highest level *p-node* registered by $R2$ would be same as the one registered by $R1$. Hence, t is a descendant of t_a.

Proof of part (b):

If t is an *p-node*, then $t.x$ is locked due to the pessimistic locking of $t.x$ for the original read operation $R1$. As such, $t.x$ cannot be updated to have a new value. Also observe that the original external read operation propagated to t_π because $t.x$ was null-valued initially. As $t.x$ remains null-valued, $R2$ cannot read from $t.x$. However, if t is an *o-node*, $t.x$ can be updated in the mean time to have a new value. This means, t is an *o-node*.

Proof of part (c):

By contrast, assume that t lies between t_a and t_π. Then, the *p-node* registered by $R2$ would be t_π itself (due to lines 205, 160, 221, 49). That means t cannot be an ancestor of t_π. The only alternative then is that t is a descendant of t_π. Further, for

$t_{\pi'}$ to be registered as the highest level *p-node*, $t_{\pi'}$ has to lie in the ancestral path up to t. That means, t is an ancestor of $t_{\pi'}$. In other words, t lies in the ancestral path between t_π and $t_{\pi'}$.

□

Lemma 7.9. *An abort of a subtransaction releases the lock up to the appropriate level.*

Proof. Let t_d be a subtransaction that has performed an external read operation through its ancestor t_π that is a *p-node*. Observe that the lock on t_π is retained in the process as it is a *p-node*. Now, if $\widehat{t_d}$ (t_d or a transaction containing t_d in its mts) aborts, then the idea is that the lock on $t_\pi.x$ should be released.

Observe that upon successful external read operation, subtransaction t_d records the level of the ancestor t_a it read x from. If there is a *p-node* ancestor t_π involved in the path from t_a to t_d, then level information $x_{pr}\langle t_\pi.lvl, n\rangle$ is captured in set prs at each descendant of t_π in the path from t_π to t_d. Later, if any of these descendants, say t, aborts, then t invokes $unlock\_to\_ancestors$ using $x_{pr}\langle t_\pi.lvl, n\rangle$ in $t.prs$. As, at intermediate ancestor level t, we have $t_\pi.lvl > t.lvl$, the $unlock\_to\_ancestors$ invocation is propagated up to immediate child of t_π in the path from t_π to t_d (due to line 31, 33-34). During $unlock\_to\_ancestors$, at the intermediate level t, $t.prs.x_{pr}(t_\pi.lvl).rc$ is decremented by n and if resulting $t.prs.x_{pr}.trc$ is 0 and t holds the lock on its parents object $t_p.x$, then $t_p.x$ is unlocked.

Thus we see that upon the abort of a subtransaction, it releases the lock up to the appropriate level. □

Lemma 7.10. *A failed duplicate request at a p-node does not alter the state of the* *original request.*

Proof. As in Lemma 7.4, let us consider a *p-node*, t_π, with its child t_c such that t_c is an *o-node* and already retains the lock on $t_\pi.x$ owing to its previous invocation of $t_\pi.search\_parent$ on behalf its descendant t_{d1}. Now, suppose t_c invokes $t_\pi.search\_parent(x, t_c, ...)$ again on behalf of its another descendant t_{d2} such that the request fails at t_π's level.

Typically, in the *search_parent* method of a *p-node*, the lock $t_\pi.x$ is released in case of failure of the operation (lines 140, 151). As a duplicate request does not participate in locking $t_\pi.x$ (lines 148-149, 212-213), unlocking $t_\pi.x$ in case of failure would invalidate the expectation that the lock on $t_\pi.x$ should be retained on behalf of the prior original request by t_{d1}. The failure of t_{d2}'s attempt for an external read should have no effect on the state of t_{d1}. This is ensured by unlocking $t_\pi.x$ only if the lock on $t_\pi.x$ was obtained in the context of the current operation (lines 94-95, 148-149, 212-213, 125-126). As the lock on $t_\pi.x$ is not obtained in the case of a duplicate request, no unlocking of $t_\pi.x$ is done in the case of failure either. Thus, the state of the original request is preserved.

□

Lemma 7.11. *Abort of descendants is non-blocking.*

Proof. Abort of descendants is initiated by an ancestor in a top-to-bottom manner in a transaction tree (lines 70-72, 74-75). The methods used for this puprose are - *abort, force_abort, abort_incompat_desc.* Observe that no lock is obtained in any of these methods. Therefore, abort of descendants is non-blocking. □

7.4.3.1 History ($\widehat{\mathcal{H}_{t_\omega}}$) produced at an ***o-node*** (t_ω)

The history $\mathcal{H}_{t_\omega}$ produced at an *o-node* t_ω is similar to the history produced at a node t by HParSTM (Chapter 6). By construction, the objects of t_ω in MxSTM are accessed in the same way as in HParSTM (as discussed already in Section 7.1.1.2). Observe that, MxSTM has been designed in a way that we do not check the types of transactions involved while accessing the objects. It is automatically taken care of, by keeping the signature of the methods (e.g., *search_parent*, etc) intact but modifying their definitions, wherever required, accordingly for *o-node* and *p-node*. Further, recall that the linearization point of a child depends upon the type of its parent, not its own type (Section 7.4.1). Thus, when we consider the history at t_ω, the type of its children does not matter. Its child could be a *p-node* or *o-node*. This means that the level-wise history at an *o-node* in MxSTM can be constructed in the same way as in case of a node in HParSTM.

While constructing the level-wise history at a node t_ω, in bottom-to-top manner in a transaction tree (Section 3.4 and 3.7), we concern ourselves only with the objects of t_ω, and treat the subtree rooted at a t_ω's child as single transaction. We do not need to care about the composition of transactions in that subtree, or the type of that child .

The set of proofs used for HParSTM can be directly applied to show the correctness for $\mathcal{H}_{t_\omega}$.

Next, we show that the history produced at an *p-node*, t_π, in MxSTM is same as that produced at a node in ParSTM.

7.4.3.2 History ($\widehat{\mathcal{H}_{t_\pi}}$) produced at a ***p-node*** (t_π)

Similarly, by construction (Section 7.1.1.1), we observe that the methods associated with t_π node in MxSTM are similar to the methods defined for a non-root node in ParSTM (Chapter 5). In other words, the objects associated with t_π in MxSTM are operated in the same fashion as are the objects of a non-root node in ParSTM. Therefore, the history $\mathcal{H}_{t_\pi}$, produced at a node t_π by MxSTM, is similar to the history $\mathcal{H}_t$ produced locally at a (pessimistic) node t by ParSTM. Hence, the set of proofs used for $\mathcal{H}_t$ can be applied for $\mathcal{H}_{t_\pi}$.

Chapter 8

Conclusion and future work

This thesis provides a comprehensive study into the complexities involved in designing STM protocols for *closed nested* transactions. Compared to non-nested transactions, nested transactions pose a set of new problems unique to them that need to be treated differently. To this end, we provide a formalism for closed nested transactions and extend the definition of *opacity* used for non-nested transactions to define *level-wise opacity* as a consistency criterion for nested transactions. In addition, we describe a model for mapping the execution of nested transactions to obtain *level-wise histories* in a transaction tree. We also provide a framework for formally proving the correctness of STM protocols for nested transactions. This framework can be used for establishing the correctness of other STM protocols for nested transactions.

Furthermore, we design a set of four STM protocols (SimpSTM, ParSTM, HParSTM, and MxSTM) for closed nested transactions. These protocols offer different modes of concurrency. Starting with SimpSTM, a simple protocol which offers no concurrency at the nested level (subtransactions are executed serially), we progress to ParSTM which uses *pessimistic concurrency control* scheme at nested level and offers partial concurrency: two subtransactions in the transaction tree can execute concurrently as long as they do not try to access the same object; otherwise they execute sequentially. Next, we obtain full concurrency in HParSTM by employing *optimistic concurrency control* mechanism at each node of the transaction tree. Finally, we combine ParSTM and HParSTM to obtain a *hybrid* protocol, MxSTM, in which some nodes operate under optimistic concurrency control while others under pessimistic concurrency control mechanism. The protocols ParSTM and HParSTM are carefully crafted in a modular way, using shared interface, such that the two can be easily integrated to obtain MxSTM. Special cases have been duly discussed and addressed.

In future, it would be interesting to implement and test the protocols against standard benchmarks to analyze their performance, especially under varying levels of nesting. Further, MxSTM can be very useful in developing new applications where different degrees of concurrency can be employed at different levels. For example, the level where most of the child threads are read only, *o-node* (optimistic approach) can be used, and the one where the frequency of updates by children is high, *p-node* (pessimistic approach) can be employed.

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