Advanced Control of Managed Pressure Drilling

by

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Abstract

Automation of managed pressure drilling (MPD) enhances the safety and increases efficiency of drilling and that drives the development of controllers and observers for MPD. The objective is to maintain the bottom hole pressure (BHP) within the pressure window formed by the reservoir pressure and fracture pressure and also to reject kicks. Practical MPD automation solutions must address the nonlinearities and uncertainties caused by the variations in mud flow rate, choke opening, friction factor, mud density, etc. It is also desired that if pressure constraints are violated the controller must take appropriate actions to reject the ensuing kick. The objectives are addressed by developing two controllers: a gain switching robust controller and a nonlinear model predictive controller (NMPC). The robust gain switching controller is designed using $H_{\infty}$ loop shaping technique, which was implemented using high gain bumpless transfer and 2D look up table. Six candidate controllers were designed in such a way they preserve robustness and performance for different choke openings and flow rates. It is demonstrated that uniform performance is maintained under different operating conditions and the controllers are able to reject kicks using pressure control and maintain BHP during drill pipe extension. The NMPC was designed to regulate the BHP and contain the outlet flow rate within certain tunable threshold. The important feature of that controller is that it can reject kicks without requiring any switching and thus there is no scope for shattering due to switching between pressure and flow control. That is achieved by exploiting the constraint handling capability of NMPC. Active set method was used for computing control inputs. It is demonstrated that NMPC is able to contain kicks and maintain BHP during drill pipe extension.
To Amma & Bapu
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Co-Authorship Statement

I, Anirudh Nandan, hold primary author status for all the Chapters in this thesis. However, each manuscript is co-authored by my supervisor and co-researcher, whose contributions have facilitated the development of this work as described below.

- **Anirudh Nandan, Syed Imtiaz, Stephen Butt "Robust Gain Switching Control of Constant Bottom hole Pressure Drilling," submitted to a journal.**
  
  Statement: I am the primary author and carried out the design and development of the presented controllers. I drafted the manuscript and incorporated the comments of the co-authors in the final manuscript. Co-authors helped in conceiving the idea and selection of appropriate techniques, and test cases.

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  Statement: I am the primary author and carried out the design and development of the presented controllers. I drafted the manuscript and incorporated the comments of the co-author in the final manuscript. The co-author helped in conceiving the idea and selection of appropriate techniques.
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1.1 Motivation

In the coming decades, the voracious appetite for oil and the concern for energy security will drive hydrocarbon exploration in pristine environments like the Arctic and ultra deep waters. Safety enhancements in drilling are essential to protect those fragile environments from contamination, as there is an ever-present danger of reservoir influxes called kicks while drilling. For an event free drilling, the bottom hole pressure BHP must be maintained within the pressure window between reservoir and fracture pressures. If the reservoir pressure exceeds the BHP, a kick will be encountered and on the other hand if the BHP exceeds the fracture pressure, drilling fluid will be lost to the formation. During pipe extension operations there is an enhanced danger of encountering a kick because of the loss of pressure in the well. An unmitigated kick may lead to a blow-out which can be catastrophic for example, the Macondo incident in the Gulf of Mexico where valuable lives were lost and cost British Petroleum Plc 41 billion USD (Forbes), and resulted in massive degradation of the environment. The reduction of non-productive time (NPT) spent on handling kicks is also crucial because projects in remote locations will be cost intensive.

Precise and fast control of the BHP can be achieved through MPD as it uses back pressure devices like choke to actively manage the BHP. In MPD, kick rejection is performed by increasing the back pressure and during pipe extension operations the
variations in BHP are mitigated through appropriate manipulation of back pressure. In a manually controlled MPD system, the outcomes are highly dependent on the skill and dexterity of the operator. In recent years there is a drive to apply automatic control techniques to drilling for BHP regulation and kick rejection with the aim of making MPD safer and efficient. The MPD process is nonlinear and has many uncertainties in the system due to variations in mud density, viscosity, frictional loss, flow rate, and choke opening. Practical control systems must be robust and tolerant to variations in operating conditions and system uncertainties to attenuate the kick by steering the BHP back into the pressure window. Also in MPD systems, certain states like the bottom hole flow rate and disturbances like kick flow rate and reservoir pressure are not measured, which requires estimation of these states and disturbances.

In the recent years researchers have developed many different controllers for MPD systems. These controllers vary in control strategy, for example, pressure control Godhavn et al. 2011 for BHP tracking vs flow control Hauge et al. 2012, 2013 for kick attenuation. Controllers ranging from proportional-integral-derivative (PID), internal model control (IMC), model predictive control (MPC), nonlinear model predictive control (NMPC) have been developed for MPD. Those controllers were shown to be effective under different situations. A detailed review of these controllers is provided in Chapter 2. It will be clear from the review that the robustness of the controller has received less focus for this system with many uncertainties. Also the potential of NMPC was not fully exploited. In this thesis the focus is on those issues and two controllers are proposed for the MPD system.
1.2 Objective

The goal of the research is to develop controllers that deliver consistent performance irrespective of the operating conditions, robust to parametric variations in mud density, frictional factor etc. The controller should also deliver superior performance during normal drilling operations and during drilling pipe extension. Kicks must be quickly attenuated and BHP must be maintained within the pressure window. We propose a robust gain switching controller and a nonlinear model predictive controller (NMPC) to achieve the above mentioned goals.

1.3 Structure of the thesis

The rest of the thesis is organized as follows: Chapter 2 provides a review of literature on drilling automation and estimation; major contributions to this field are summarized and the gaps in research are identified. Chapter 3 presents the development of robust gain switching controller for pressure regulation and the results of simulation studies are presented. A new design of NMPC for BHP regulation and reservoir influx mitigation is presented in Chapter 4. Finally, Chapter 5 concludes this thesis by highlighting the contributions of this thesis and few recommendations are made for future work.
Chapter 2

Literature Review

In this chapter the major contributions towards the control and automation of drilling are reviewed and gaps in research are identified. But first a brief overview of conventional drilling and MPD are provided. The bibliography of articles referred in this chapter is presented at the end of this thesis.

2.1 Conventional drilling

The hydrocarbons trapped in rocks hundreds of meters below the sea level is extracted by drilling wells using a rotating bit. To assist the removal of cuttings a drilling fluid often referred as mud is pumped into the well at a pressure \( p_p \) and a flow rate \( q_p \). The mud flows through the drill sting, passes through the nozzles of the bit, then flows through the annulus and then passes through a shaker where impurities are removed before flowing back to the mud pit which is open to the atmosphere. Bourgoyne Jr et al. (1986) gives a comprehensive account of conventional drilling. A schematic depiction of conventional drilling is shown in Figure 2.1. In overbalanced drilling techniques a positive pressure difference is maintained in the well to prevent kicks which occur when well pressure is less than the reservoir pressure \( p_{res} \). The drilling mud helps in maintaining overbalanced conditions. In conventional drilling, the bottom hole pressure \( p_{bh} \) is the sum of frictional pressure in annulus \( p_f \) and hydrostatic head \( p_h \), given by Equation (2.1). Frictional pressure will be absent when there is no mud flow hence to maintain overbalanced conditions during pipe extension.
Figure 2.1: Schematic depiction of conventional drilling

(during which the mud flow rate will be ramped down to zero) mud density must be chosen such that the hydrostatic pressure is higher than the formation pressure, given by Condition (2.2).

\[ p_{bh} = p_h + p_f, \]  \hspace{1cm} (2.1)  
\[ p_h > p_{res}, \]  \hspace{1cm} (2.2)  
\[ p_h = \rho gh \]  \hspace{1cm} (2.3)  

where \( \rho \) is mud density, \( g \) is acceleration due to gravity, and \( h \) is true vertical depth at bottom hole.
In conventional drilling when a kick is encountered drilling has to be stopped and a heavier mud is pumped into the well to re-establish Condition (2.2) and that is a major drawback of conventional drilling as stopping of drilling contributes to non-productive time (NPT). It is hard to drill reservoirs with narrow pressure windows using conventional drilling technique as frictional loss has large uncertainty and the sum of hydrostatic pressure and frictional pressure can easily exceed the fracture pressure \( p_{frac} \), shown in Figure 2.2. In order to overcome these drawbacks MPD was introduced.

### 2.2 Managed pressure drilling

MPD is a closed-path drilling technique in which the mud flowing out of the annulus passes through a choke which provides a back pressure \( p_c \) as opposed to flowing to the atmospheric pressure \( p_o \) as is the case in conventional drilling. In MPD the
bottom hole pressure $p_{bh}$ is the sum of back pressure $p_b$, hydrostatic pressure $p_h$, and frictional pressure $p_f$ given by Equation (2.4). The BHP can be controlled with ease by manipulating the mud flow rate or back pressure and thereby greatly improving the efficacy of well pressure management.

$$p_{bh} = p_h + p_f + p_b$$

(2.4)

The International Association of Drilling Contractors (IADC) defines MPD as follows: “Managed Pressure Drilling (MPD) is an adaptive drilling process used to precisely control the annular pressure profile throughout the wellbore. The objectives are to ascertain the downhole pressure environment limits and to manage the annular hydraulic pressure profile accordingly”. Some variants of MPD are as follows:

i. Constant bottom hole pressure drilling (CBHP)

ii. Pressurized mud cap drilling

iii. Dual gradient drilling

Typically, back pressure in MPD is controlled manually. In order to improve safety and precision of the process, automation of MPD has been considered. This thesis is focused on automation of constant bottom hole pressure drilling (CBHP).

### 2.2.1 Constant bottom hole pressure drilling

In CBHP, the BHP tracks a target which is marginally greater than the reservoir pressure and lesser than the fracture pressure, also often referred to as “walking the line”. In CBHP the sum of three pressure components (back pressure, hydrostatic pressure, and frictional pressure) is maintained marginally higher than the reservoir pressure. Usually the mud density is chosen in such a way that the hydrostatic pressure
is less than reservoir pressure, \((p_h < p_{res})\), and the back pressure \((p_b)\) is manipulated to bring the BHP above the reservoir pressure.

\[ p_h < p_{res} \quad \text{(2.5)} \]

In an abnormal situation, when the drill enters a pressurized zone the reservoir pressure becomes greater than the BHP. This situation is known as kick. A kick can be rejected by commanding a higher back pressure as that will directly translate into higher BHP. This can be done easily by manipulating the choke valve without stopping the drilling. An ideal kick attenuation in MPD system is shown in Figure 2.4.

During drilling pipe extension, the effect of the loss in the frictional pressure can be compensated by an appropriate increase in the back pressure, ideal pressure management during pipe extension is shown in Figure 2.5. Typically, mature reservoirs have narrow pressure windows and they are often called undrillable wells. Such reservoirs
can be drilled using CBHP drilling because in this technique hydrostatic pressure is deliberately chosen in a way the sum of hydrostatic pressure and frictional pressure is less than reservoir pressure and back pressure “the third component” is manipulated nimbly so that the BHP always remains within the pressure window, schematic depiction of pressure trajectories in CBHP is shown in Figure 2.3.
2.2.2 Control layers in managed pressure drilling

Control of MPD is hierarchical in nature. Control layers as suggested by (Breyholtz et al., 2010b) are shown in Figure 2.6. The rate of penetration of the drill, energy consumed by the actuators, mud flow rate, and the pressure profile of the well are some of the parameters which have to be optimized with the goal of performing event free drilling. The optimizer forms the topmost layer and it generates pressure and flow rate setpoints for the MPD system. A dynamic controller manipulates the actuators to achieve the target pressures and flow rates. At the lowest level, controllers
realize actuator outcomes by manipulating valve opening and pump rpm in choke and mud pump respectively. In most of the MPD systems real-time measurements of BHP is not available. Some of the recent MPD systems are equipped with BHP measurements. Typically these measurements are transmitted by tele-metry, which is highly unreliable, noisy, and adds delay to the measurements. In the absence of BHP measurements or when they have significant delay, using the topside measurements (such as pump pressure, choke pressure, and choke flow rate) the BHP must be estimated. In state-feedback controllers, the bit flow rate has to be estimated as it is an unmeasured state. The heave induced movement of the drilling rig also produces considerable fluctuations in the well pressures which was addressed in (Mahdianfar et al., 2012), (Landet et al., 2013), (Nikoofard et al., 2013), and (Albert et al., 2015). The focus of this thesis is on developing robust, high performance solutions for pressure control during drilling and pipe extension as well as kick mitigation. Faults such as kicks, leaks, and blocked nozzles have to be detected and diagnosed using the measurements of the outputs and the estimates of the unmeasured states. The optimizer takes corrective actions with the help of measurements, estimated states, and the knowledge of the faults. All these layers together achieve the overarching goal of automated pressure management in a drilling well.

2.3 Automatic control of drilling

There are broadly two approaches to automatic control of MPD systems, namely flow control and pressure control; each has its own merits and demerits. In flow control, the difference in the in/out flow rates is regulated. The objective is to achieve zero flux as that translates into zero kick. Flow control offers the best solution for kick mitigation but when there is no kick a flow controller will simply be tracking the
reservoir pressure; it is often desired that BHP tracks a setpoint which is greater than reservoir pressure. On the other hand, a pressure controller tracks a BHP setpoint which is the prime objective of CBHP type drilling but when a kick occurs it manages it by expanding the valve opening to relieve pressure and thereby bringing the reservoir fluids to the surface. Below we provide a detailed review of literature on pressure and flow control.

2.3.1 Pressure control

In pressure regulated MPD, either the BHP or the choke pressure is regulated by manipulating choke opening and/or mud flow. MPD pressure controllers track BHP when reliable real-time measurements of BHP are available otherwise they track choke pressure and in that case choke pressure setpoint is deduced from BHP estimates. The pressure controllers which have been developed for MPD fall under three broad categories: Proportional-Integral-Derivative (PID) controllers, nonlinear controllers, and model predictive controllers (MPC).

Godhavn et al. (2010) designed a PID controller for choke pressure tracking. The PID tuning parameters were obtained from first order transfer function models which were derived from ordinary differential equations (ODE). Controller was tested for pressure regulation during drill pipe extension sequence as well as during surge and swab scenarios. During the pipe extension sequence, the variations in the annular frictional loss was compensated by dynamically manipulating the choke pressure. They suggested gain scheduling for dealing with nonlinearity, and high integral gain to minimize offset during severe variations in mud pump flow rate.

Siahaan et al. (2012) designed an adaptive PID controller using the unfalsified procedure for MPD. In unfalsified procedure, the adaptive controller chooses PID tuning parameters from a set of candidate parameters by using the measurement data and
a cost function. The candidate parameter which gives best performance and stability is automatically chosen. Knowledge of the system model is not required for designing this kind of controller. The stability and performance can be improved by expanding the parameter set but at the expense of computational power. The controller was tested for rejecting a ramp disturbance on mud flow rate which typically occurs during drilling pipe extension. This controller offers desirable properties such as stability, performance, and independence from tuning. However, implementing this controller can prove to be difficult. Also, this is a non-standard controller, most control engineers might not have the requisite skills to design and maintain this kind of controller.

Carlsen et al. (2013) devised an automatic well control sequence which is similar to the conventional pressure management. In this method, when a kick is encountered drilling is stopped, pressure is stabilized, and reservoir fluid is circulated out. A PID controller, an internal model controller (IMC) controller, and an MPC were designed and tested for handling large gas kicks. The PID controller manipulated the choke opening to control either the choke pressure or the BHP depending on the control objective. The IMC and MPC controllers were configured as a multiple input – multiple output (MIMO) controllers, they controlled pump pressure, choke pressure or bottom hole pressure by manipulating the mud pumping rate and the choke opening. All the controllers were implemented using first order process models. It was found that MIMO IMC and MPC controllers deliver superior performance and stability. The MPC cost function was formulated to follow an output target and to minimize input usage. The robustness of MPC improved with increase in prediction horizon. Since single phase models were used the efficacy of the controllers in handling gas kicks was rather limited.

Nygaard and Nævdal (2006) pioneered the application of nonlinear model predictive control (NMPC) to drilling. The key contributions were the development of a control
relevant model for a 2-phase flow drilling well and the development of an NMPC for underbalanced drilling (UBD) well. The objective of NMPC was to minimize the deviation of BHP from its target by manipulating the choke opening. The mud pumping rate and the rate of drilling were treated as disturbances. Levenberg–Marquardt algorithm was used for control input optimization. A PI controller was also designed for the sake of evaluating the performance of NMPC. The controllers were tested for pressure regulation during drilling pipe extension operation – on a 2-phase simulation model – during which the mud pump is ramped down, held at no flow condition for some time before being ramped up. It was shown that both NMPC and PI were superior to manual choke control but PI requires retuning if operating conditions change. NMPC handled nonlinearity effortlessly and also achieved superior BHP regulation owing to the fairly detailed model of UBD well and input optimization. The designed controller was not tested for kick rejection.

Breyholtz et al. (2009) focused on regulating BHP during pipe connection sequence and downlinking. The process of activating the directional drilling unit is called downlinking. During downlinking operations mud pulses are sent, resulting in BHP fluctuations. An NMPC was designed for rejecting the fluctuations in BHP by coordinating the use of choke opening, back pump flow rate, and main mud pump flow rate. The NMPC used single shooting multi-step quasi-Newton method. The optimization was performed in a hierarchical fashion. The objective was not only to track a BHP setpoint but also choke opening, mud pump flow rate or back pump flow rate targets. If holistic optimization is infeasible, tracking of BHP will be given priority over input targets. For evaluating the performance of NMPC, a PI controller was also designed. One of the strengths of NMPC is its ability to work with more than one degree of freedom and it was exploited in this work. The BHP was estimated by using an adaptive model based nonlinear observer developed by (Stamnes et al., 2008). In
order to obtain accurate estimates of BHP, real-time parametric estimation of the frictional model and the mud density were also performed. This work significantly advanced the technique of handling BHP fluctuations during drilling pipe connection and downlinking but it did not address the handling of kicks.

Godhavn et al. (2011) summarized the core objective of MPD control as BHP set-point tracking. The control objective was to regulate the choke pressure in such a way the error in BHP was minimized. The BHP was regulated by using a nonlinear controller and a BHP estimator which also estimates frictional loss. The observer developed by (Stamnes et al., 2008) was also used in this work as well. A model based choke pressure regulator was designed using feedback linearization technique which computes the choke opening by inverting the nonlinear model of the choke. The designed controller and BHP estimator were tested on a test rig called Ullrigg in Stavanger, Norway. Through this work the disadvantages of simple PID controllers were addressed. However, implementation of nonlinear controllers might prove to be difficult as they are incompatible with most industrial control setups. This work again focused only on drill pipe extension, kick rejection was not addressed.

(Breyholtz et al., 2010a, 2011) developed a MPC for regulating BHP in a dual-gradient drilling (DGD) system. The control objectives were regulation of BHP and hook position by manipulating mud pump, subsea pump, and drill string velocity. A nonlinear MPC model was used and control inputs were computed using single-shooting multi-step quasi-Newton method. Control was tested on a detailed model of drilling called WeMod. A simple model for MPC design was fitted using data generated from the detailed model, and it was suggested that such a fitting can be performed in real case by using rich measurement data. The performance of the controller in regulating the BHP during drill string movement was tested and it delivered good performance. The focus of the work was on rejecting disturbances due to drill string movement on BHP,
problems like kicks and drilling pipe extension were not considered.

As a drilling program progresses, intermittently casings will be run into the well. Casings are essential to prevent reservoir influxes and well collapse. The uncased section is called open well and the point where casing meets the open well is called shoe. With objective of controlling the well pressure profile, sometimes it is desired that one controls both the BHP and pressure at the shoe. Møgster et al. (2013) developed a linear MPC for controlling the pressure at the BHP and at the casing shoe. The MPC manipulated the pump flow rate and the choke opening in order to control both the pressures. Controller was developed by using linear first order transfer function models of the system and implemented using Statoil’s SEPTIC software. The innovation of this work was in controlling pressure at two points but its effectiveness in dealing with kicks and pipe extension was not demonstrated.

### 2.3.2 Flow control

A nonlinear flow controller which utilizes feedback linearization technique was developed in (Hauge et al., 2012, 2013). In feedback linearization the control input is computed from the inverse of the nonlinear model. The controller minimizes the in/out flux flow rate in the annular control volume by minimizing the error between the outlet flow rate and bit flow. In order to realize that controller, bit flow rate was estimated along with kick flow rate and kick location. The controller was tested on a multi-phase simulation model called OLGA. The controller was able to attenuate kicks, and the estimator was able to estimate the magnitude of the kicks and their locations. As a consequence of flow control, the controller tracks the reservoir pressure as that is the only way to achieve zero in/out flux. During pipe extension BHP must be regulated and flow controllers are unsuitable for that.

Zhou et al. (2011) developed a control solution which acts as a pressure controller dur-
ing normal drilling operations is switched to flow control during kick handling using a switching logic, this switching strategy overcomes the above mentioned limitation of flow controllers. During kick handling, the controller tracks the estimate of bit flow rate and to accomplish that a passivity based nonlinear observer for bit flow rate was developed. Passivity based nonlinear observers are developed by injecting the error innovation term into the dynamic equations of the system. For example, bit flow rate is estimated by injecting the error in pump pressure into the dynamic equation of bit flow rate. While the controller handles a kick, simultaneously the reservoir pressure and kick flow rate are estimated using passivity based nonlinear observers. Using the estimate of reservoir pressure, a BHP setpoint is chosen to resume pressure control. After a dwell time, controller switches from flow control mode to pressure control mode. When another kick is encountered — if the magnitude of the kick is greater than a threshold — the controller automatically switches to kick handling mode (i.e. flow control mode). Zhou and Nygaard (2011a) developed a similar pressure-flow switching type controller for dual-gradient drilling system.

Pedersen et al. (2013) developed an MPC for regulating the BHP and exit flow rate of an underbalanced drilling (UBD) well. In UBD, hydrocarbons are produced while drilling because of negative pressure difference between the BHP and the reservoir pressure. In UBD it is desirable to regulate the BHP and also the exit flow rate to prevent large amounts of reservoir fluids from gushing out. The simultaneous control of BHP and exit flow rate is achieved by manipulating the choke opening and pump flow rate. MPC was developed by using first order with time delay (FOTD) models of the $2 \times 2$ system. Controller was implemented using Statoil’s SEPTIC software and tested using WeMod simulation package. Simulations for pipe extension scenario and handling of a large gas bubble were shown and the controller was able to track the BHP and flow rate setpoints. It was suggested that the controller can be improved if
better models which account for 2-phase nature of the system is used.

2.4 Estimators and observers

In MPD systems, typically the topside pressures such as pump pressure and choke pressure, and flow rates such as pump flow rate and choke flow rate are measured. In sophisticated MPD systems the BHP is measured but its measurements generally have a time-delay and prone to be noisy hence its estimation might be necessary. The bit flow rate is typically an unmeasured state and kick flow rate is an unmeasured disturbance. For successful MPD operation the knowledge of BHP is essential to enable appropriate BHP setpoint selection hence reservoir pressure must be estimated. Parameters like frictional factors and geometry of the well are uncertain and their estimation will help in improving robustness of MPD control and will improve BHP estimates. Several observers to address some or several of the above mentioned issues have been developed and some of them were discussed in conjunction with controllers in previous subsections. Here, few other contributions to MPD state and parameter observation are reviewed.

2.4.1 Kalman type observers

Lorentzen et al. (2003) developed an ensemble of extended Kalman filter (EKF) for tuning first principles based 2-phase flow model and it was applied to drilling. This work is an early example of applying Kalman type filters to drilling. A detailed 2-phase model of drilling was developed and using ensemble EKF the model was tuned for better pressure prediction. Ensemble EKF was preferred over least squares methodology because the former is better suited for online tuning while the latter is suitable for offline post-processing. Despite the advantages of ensemble EKF, rou-
tinely applying this observer to real systems will be difficult because of difficulties in linearizing complex nonlinear model.

A new kind of Kalman filter called Unscented Kalman filter (UKF) does not require linearization of model, has fast computational times, and is very well suited for nonlinear systems Wan et al. (2000). A kalman filter for MPD which uses a simpler model with superior convergence properties will be more practical. Gravdal et al. 2010 developed an UKF for MPD model calibration. Since the frictional pressure drop is time varying and uncertain because of its dependency of viscosity, density, well geometry etc., continuous update of MPD flow models will lead to robust control of BHP. An UKF based on 2-phase drilling model was developed and was tested against synthetic measurements which were intentionally corrupted by adding noise, real data obtained from pressure while drilling (PWD) data from a North Sea high pressure (HP)-high temperature (HT) well, and a two-phase case involving a gas kick. The frictional factors in drill string and annulus, pump pressure, and bottom hole pressure were estimated in each of the case while ramping up and ramping down the mud flow rate. It was shown that the calibrated model improved pressure prediction considerably.

Mahdianfar et al. (2013) developed an UKF for joint estimation of states and unknown parameters. The filter requires only topside measurements like pump pressure and choke pressure. The frictional flow model and geometry terms were augmented with unknown parameters. The unmeasured bit flow rate was estimated along with the unknown parameters. The filter was tested on a detailed simulation model, it was able to estimate frictional factors, well geometry, and bit flow rate by only using topside measurements. This filter has the potential to improve the precision and performance of model based state feedback MPD controllers.
2.4.2 Nonlinear observers and moving horizon estimators

Any uncertainty in hydrostatic pressure due to mud density variations or uncertainty in frictional pressure due to frictional factor variations will directly affect the BHP estimates, as BHP is a sum of hydrostatic pressure, frictional pressure and back pressure. BHP can be estimated robustly by using adaptive estimators in which the model parameters are updated continuously. Hasan (2014, 2015) designed an explicit feedback law by using backstepping method which relies on change of variables using Volterra operator to deduce the transformation kernel. The MPD system was modelled using hyperbolic partial differential equations (PDE). In order to implement the control law estimates of the unknown parameters and states are required which are obtained using backstepping observers. A salient feature of this observer is its ability to estimate bottom hole pressure by using only topside measurements. The observers adapts to the system by using parameter update laws. The observer was validated using a field scale experimental flow loop.

Hasan and Imsland (2014) developed a moving horizon estimator (MHE) for MPD. The MPD system was described using an infinite dimensional partial differential equations (PDE) model. The PDE model was converted into high dimensional ordinary differential equations (ODE) model using early lumping approach in which the infinite dimensional system is discretized along a geometric dimension using a non-uniform grid. The resulting ODE model can be represented in state space. In MHE, states are estimated by solving a least squares minimization problem. The optimization was implemented using gradients and line search. The estimator was tested on a field scale flow loop.

Zhou and Nygaard (2011b) designed an adaptive nonlinear observer for estimating friction factor and mud density in the annulus as well as the flow rate through the
drill bit. In order to develop the observer simple first principles based model of MPD was developed. Using the topside pressure measurements and models of drill string and annulus the flow rate at the bit, mud density, and frictional loss were estimated using parameter update laws. The exponential stability of the observers was established using Lyapunov analysis. The observer was tested on a horizontal flow loop. Stamnes et al. (2011a,b) developed a nonlinear adaptive observer which uses several delayed observers to get robust estimates of unknown parameters and hence improved state estimates but at the expense of computational load. The uncertainties in frictional loss and hydrostatic pressure were modelled as multiplicative uncertainties which were estimated online using parameter update laws. The estimator takes present measurements and past measurements into account which greatly improve the convergence properties. The observer was tested on offshore well data and observer gains were tuned for fast estimation of unknown parameters.

Li et al. (2012, 2011) developed a method for fast estimation of BHP. The observer consists of three components: a state predictor, an update law, and a low pass filter for smoothing the updates. The drill string pressure dynamics is used for BHP estimation as the knowledge of drill string parameters is more reliable. Using topside pressure measurements the bit flow rate is estimated using a predictor and update law and its estimates are filtered using a low pass filtered before ultimately estimating the BHP. The error in BHP estimate can be reduced by increasing the adaptive gain but drill string parameters are assumed to be known, if there is strong parametric uncertainty the BHP estimates might not be reliable. Observer was tested through simulations using a nonlinear model of the well.
2.4.3 Fault detection

Drilling is prone to several faults which can compromise the integrity of the equipment or of the well and ultimately leading to loss of lives and environmental degradation. Therefore, it is important to detect faults so that corrective measures can be taken. Apart from kicks, some of the other faults that can occur are: leaks in which the drilling fluid flows into the reservoir and thereby rupturing the walls of the well; plugging of the bit nozzles; pack offs in which cuttings accumulate around the drill string; ruptured drill string (drill string washout) in which mud flow into the annulus without reaching the bit this could lead to loss of drill string (Willersrud et al., 2015a). Zhou et al. 2011 developed a nonlinear observer for kick detection and quantification. The observer was designed by exploiting the passivity of the MPD system, error in topside pressure measurements were injected into system equations to estimate the unknown kick flow rate. Hauge et al. 2013 developed a model based nonlinear observer which based on the excitation provided by the in/out flux in flow rate detects kicks, quantifies kick flow rate and isolates the kick location. Willersrud et al. 2015b,c developed a model based fault detection and isolation method for detecting the above mentioned faults. Model based observers using the excitation provided by measured errors estimate states and parameters. The signal generated by model is compared with measurements to generate residuals. Statistical changes are detected using generalized likelihood ratio test (GLRT) to generate alarms when faults are detected. GLRT with multivariate t-distribution first isolates the type of incident and then its location. A drawback of this method is that it requires a stable adaptive observer for residual generation and that is overcome by using analytical redundancy relations (ARR) in Willersrud et al. 2015a. ARR can be formed directly using system equations and they also have the capacity to detect faulty actuators and sensors. The fault
detection and isolation methods developed in Willersrud et al. 2015a,b,c were tested on an field scale experimental flow loop.

2.5 Conclusions

From the discussion in the previous section we see that MPD is highly nonlinear system with many uncertainties. Parameters like choke opening and mud flow rate also contribute to nonlinearity of the MPD process. The MPD process has many sources of uncertainties, variations in well geometry, mud density, mud viscosity, frictional factor, etc. System uncertainties must be addressed thoroughly to ensure stability and uniform performance. Easily implementable control structures are desired for rapid propagation of automation technologies in the drilling industry.

Therefore, there is need for developing a robust nonlinear controllers with easily implementable structure for addressing uncertainties and nonlinearities of the MPD system. The two principal approaches to controlling MPD are pressure control and flow control. Pressure control offers the ability to track an overbalanced BHP but it lets reservoir fluids to flow to the surface. On the other hand flow control is effective in containing kicks but is not capable of tracking BHP. Comprehensive MPD control solutions will harness the benefits of both pressure and flow control. A pressure-flow switching controller which relies on a switching logic had been developed in the past (Zhou et al., 2011) but there is potential to develop controller which avoids explicit switching.

In this thesis we propose to address these issues through implementation of two controllers: robust gain switching controller and nonlinear model predictive controller (NMPC). In robust gain switching controller we develop multiple $H_{\infty}$ loop shaping controllers for maintaining uniform tracking performance and stability for va-
riety of operating conditions. In NMPC we exploit the constraint handling capability of NMPC in a way that the controller tracks BHP during normal drilling and contains kick within certain threshold when it occurs.
Chapter 3
Robust Gain Switching Control of Constant Bottom hole Pressure Drilling

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Preface

This paper has been submitted to the Journal of Process Control and it is currently under review. A version of this paper was presented in Oceans’15 IEEE/MTS conference held at St. John’s, Newfoundland and Labrador. The lead author performed the necessary literature review, developed the controller, implemented the observer, generated the results, and wrote the manuscript. The co-authors Dr. Imtiaz and Dr. Butt helped in identifying the gap in research, supervising the research, and editing the manuscript.
Abstract

Automation of managed pressure drilling is crucial in order to enhance safety. This process is highly nonlinear and the system varies considerably with changes in drilling conditions. In this work we have analysed the effect of various operating conditions on plant parameters and designed a controller which will deliver consistent performance for different working conditions and will also be robustly stable. The control objectives of robustness and good performance are achieved by using multiple robust loop shaping controllers. Based on choke opening and mud flow rate, an appropriate controller is selected by utilizing a gain schedule. An observer for estimation of the reservoir pressure is also implemented so that an appropriate bottom hole pressure setpoint can be selected to maintain overbalanced conditions.

3.1 Introduction

Managed pressure drilling (MPD) is being used increasingly due to strict safety regulations and also to drill wells with narrow pressure window. According to Malloy et al. (2009), if the pressure in the well is either actively or passively managed, it can be called MPD and in such systems over balanced condition is maintained at all times. Effective control of an MPD system can be achieved by using automatic controllers. In Hauge et al. (2013) a flow controller was developed to regulate the outlet flow rate and thereby regulating the bottom hole pressure. Flow controllers are very effective in preventing reservoir fluids from reaching the surface as they regulate in/out flow difference. Under normal conditions, the control objective is to track the bottom hole pressure setpoint and a pressure controller is suitable for that purpose. The pressure controllers developed for drilling which are available in the literature range
from simple PI/PID controllers to advanced nonlinear model predictive (NMPC) controllers. In Godhavn et al. (2010) a simple PID to control drilling system pressures was discussed. Controller performance was demonstrated for a single operating condition. A nonlinear model predictive controller was developed in Nygaard and Nævdal (2006) to maintain bottom hole pressure under fluctuating pump flow rates and results were compared to a simple PI controller with feed forward. It was shown that the performance of the PI controller deteriorated when working conditions deviated. In Breyholtz et al. (2010, 2011) MPCs were designed to manipulate flow rates and hook position in order to achieve certain bottom hole pressure targets. An $L_1$ adaptive pressure controller which works in conjunction with an estimator was presented in Li et al. (2009, 2011). A mixed pressure and flow control approach was taken in Zhou et al. (2011). The controller acts as a pressure regulator during normal operation but switches to a flow regulator when a kick is underway. Similar switching strategy was used to control dual-gradient drilling, a variant of MPD in Zhou and Nygaard (2011). Constant bottom hole pressure drilling (CBHP) is another variant of MPD in which the down hole pressure is maintained near a target. Constant pressure is achieved by the use of dynamic annular pressure in addition to the hydrostatic pressure offered by the mud. There have been few successful implementations of closed loop CBHP drilling systems which are presented in Roes et al. (2006); Fredericks et al. (2008).

For a nonlinear system, nonlinear controllers can deliver optimal performance but implementation of such controllers require additional customization and control experts on site for uninterrupted operation. Also the performance of nonlinear controllers can degrade drastically under parametric uncertainty. Our objective is to exploit the available SISO control loop structure in most MPD systems and develop a simple controller. If a simple controller can deliver consistent performance for a wide range of operating conditions, there will be wide spread adoption of automatic control in
drilling. Hence we propose

- a gain switching controller in which gain is selected based on two parameters by using appropriate gain schedule,

- the controller ensures $H_{\infty}$ stability for various parametric uncertainty in the system.

![Figure 3.1: Schematic depiction of CBHP drilling](image)

**3.2 System description**

The drill string and annulus form the two prominent control volumes of the drilling system as shown in Figure 3.1. The system consists of a main pump which supplies
the drilling mud at a pressure $p_p$ and volumetric flow rate $q_p$ and an additional back pressure pump which discharges mud at a lower volumetric flow rate $q_b$. The pump pressure $p_p$ is given by (3.1). A choke at the exit of the annulus control volume provides a back pressure $p_c$ and mud flows through it at a volumetric flow rate $q_c$. The choke pressure $p_c$ is given by (3.2). The drilling model which we have considered is based on the detailed model presented in Kaasa et al. (2012). It was utilized in Hauge et al. (2013) to design an observer to estimate in/out flux and unknown states, and in Imsland and Kaasa (2012) to design an observer to estimate the BHP.

\[
\dot{p}_p = \frac{\beta_d}{V_d} (q_p - q_{bit}) \tag{3.1}
\]
\[
\dot{p}_c = \frac{\beta_a}{V_a} (q_{bit} - q_c + q_b + q_k) \tag{3.2}
\]
\[
\dot{q}_{bit} = \frac{1}{M} (p_p - p_c - p_{f_d} - p_{f_a} - (\rho_a - \rho_d)gh_t) \tag{3.3}
\]
\[
p_{bh} = p_c + p_{f_a} + \rho_a gh_t \tag{3.4}
\]
\[
p_{bh} = p_p - p_{f_d} + \rho_d gh_t \tag{3.5}
\]
\[
q_k = K_{pi}(p_{res} - p_{bh}) \tag{3.6}
\]
\[
q_c = u_c C_d A_o \sqrt{\frac{2(p_c - p_o)}{\rho_a}} \tag{3.7}
\]
\[
p_{f_d} = \frac{32 \rho f_d q_p |q_p L_d|}{\pi^2 D_d^5} \tag{3.8}
\]
\[
p_{f_a} = \frac{32 \rho f_a q_{bit} |q_{bit} L_a|}{\pi^2 (D_a - D_d)(D_a^2 - D_d^2)^2} \tag{3.9}
\]

The bottom hole pressure $p_{bh}$ is the sum of choke pressure, annular frictional pressure, and the hydrostatic pressure given by (3.4). Alternatively, $p_{bh}$ can be measured through the drill string control volume given by (3.5). Due to inaccuracies in frictional loss models, both the derived measurements might be unequal. In this paper, $p_{bh}$ will always be measured through the annulus. The frictional losses are a function of the actual length of control volumes (measured depth), the mud flow rates, mud
density and viscosity while the hydrostatic pressure is a function of the true vertical depth \(h_t\) and mud density. The frictional pressure drops in drill string and annulus are modelled using Equations (3.8) & (3.9) respectively, as in (Hauge et al., 2012). In this work, we assume a steady state reservoir model as described by Equation (3.6). The choke model is given by Equation (3.7) and a comprehensive discussion on chokes can be found in Merritt (1967). \(u_c \in [0, 1]\) is the choke opening, \(V_d\) and \(V_a\) are the volumes of drill string and annulus control volumes, \(\beta_d\) and \(\beta_a\) are their respective bulk moduli, and \(q_{bit}\) is the mud flow rate at the bit given by Equation (3.3).

### 3.3 Models for multiple linear controller design

The nonlinearity of the MPD system is addressed by developing multiple linear controllers. Within each operating range uncertainties in mud density and flow rate are addressed by developing a robust controller. In this section, the effect of flow rate, choke opening, and mud density on the variation of plant gains and time constants will be discussed. Eventually, a suitable gain envelope will be formed in order to search for appropriate controller gains. The following assumptions are made while deriving simple first order process models:

- Mud density remains constant \((\rho_d = \rho_a)\)
- There is no kick \((q_k = 0)\)

The first order transfer function models of MPD are obtained by using the method presented in (Godhavn et al., 2010). The MPD system is written as a linearized first order process between the choke pressure and choke opening, given by (3.10).

\[
\Delta p_c = \frac{a \Delta u_c}{T_p s + 1}, \tag{3.10}
\]
where $a$ and $T_p$ are the gain and time constants of the process respectively. The gain of the process is obtained by partially differentiating the steady state equation for the choke pressure and the time constant is obtained from the dynamic equation for the choke pressure. The steady state choke pressure is given by (3.11) which obtained by rearranging (3.7).

$$p_c = \frac{\rho_a q_c^2}{2u_c^2 C_d^2 A_o^2} + p_o$$  \hfill (3.11)

At an operating point ‘0’, the gain $a$ and the time constant $T_p$ can be computed using Equations (3.12) and (3.13).

$$a = \left. \frac{\partial p_c}{\partial u_c} \right|_0 = -\frac{\rho_a q_c^2}{C_d^2 A_o^2 u_c^3}$$  \hfill (3.12)

$$T_p = \left. -\frac{1}{\frac{\partial p_c}{\partial p_c}} \right|_0 = \frac{V_a \rho_a q_c}{\beta_a u_c^3 C_d^2 A_o^2}$$  \hfill (3.13)

3.3.1 Effect of choke opening and flow rate

![Figure 3.2: Effect of choke opening when $\rho_a = 1000 \text{ kg/m}^3$, $q_c = 350 \text{ gpm}$](image)

During the course of a drilling program choke and mud pump must be operated at different operating points in the range $[0, 1]$ and $[0, q_{pn}]$ respectively, where $q_{pn}$ is the
nominal resting value of the mud flow rate. An appropriate nominal flow rate is chosen by the operator for facilitating the transport of cuttings, providing lubrication etc. The process between the choke opening and the choke pressure is inversely acting hence a decrease in choke opening will result in an increase in the choke pressure. According to Equation (3.12) gain is linearly dependent on the mud density and is nonlinearly dependent on the choke opening and the mud flow rate. According to Equation (3.13) the time constant is nonlinearly dependent on the choke opening and is linearly dependent on the mud density and mud flow rate. At steady state

\[ q_c = q_p + q_b \]

hence the gain of the process depends only on the total in flow rate \( Q_s = q_p + q_b \).

![Figure 3.3: Effect of flow rate when \( \rho_a = 1000 \text{ kg/m}^3 \), \( u_c = 30\% \)](image)

**3.3.2 Parameter envelope**

It is clear from the earlier discussions that choke opening and flow rate are the principal contributors of nonlinearity in the plant gain, therefore we will tackle nonlinearity by developing a gain schedule dependent on those two parameters and these scheduling variables can be measured with relative ease. On the other hand, mud density changes are hard to measure hence it will be treated as an uncertain parameter. It will
be shown that the developed robust controller is able to handle uncertainty in mud density. In Figure 3.4, the plant gain is plotted as a function of main pump flow rate and choke opening for various mud densities. The top most surface (first surface) corresponds to the plant with mud density $1000 \text{ kg/m}^3$. The second to fifth gain surfaces correspond to plants with mud densities of $1100 \text{ kg/m}^3$, $1200 \text{ kg/m}^3$, $1300 \text{ kg/m}^3$, and $1400 \text{ kg/m}^3$ respectively. The controller will be designed for the following conditions: mud density of $1300 \text{ kg/m}^3$, mud flow rates $\in [200 \text{ gpm}, 450 \text{ gpm}]$, and choke opening $\in [25\%, 60\%]$.

### 3.4 Controller design

The objective is to use as few linear controllers as possible to robustly stabilize all the plants of the class given by Equation (3.14) and to always have a closed loop settling time of 10s or better:

$$G(s) = \frac{K(u_c, q_s, \rho)}{\tau(u_c, q_s, \rho) s + 1} \quad (3.14)$$

where $u_c \in [25\%, 60\%]$, $q_s \in [200 \text{ gpm}, 450 \text{ gpm}]$, $\rho \in [1000 \text{ kg/m}^3, 1600 \text{ kg/m}^3]$ are the choke opening, total mud flow rate, and mud density respectively. A single $H_{\infty}$ loop shaping controller will be designed and the pre-compensator will be switched,
according to operating conditions, in order to prevent performance deterioration. The transfer function models given by (3.14) relate the choke opening and choke pressure. The choke pressure setpoint will be derived from the BHP model given by (3.5) and frictional loss model will be treated as known in this work.

### 3.4.1 Robust controller design

![Figure 3.5: A particular partition of gain surface](image)

Let $G_N$ be the nominal plant of an arbitrary gain partition shown in Figure 3.5 with gain $K_N$ and time constant $\tau_N$. $W_N$ is a pre-compensator given by Equation (3.15):

$$W_N = \frac{1.256}{s(s + 1.256)}G_N^{-1}$$

(3.15)

MPD is an open loop stable process and so are its first order models. We develop the robust controller following the loop shaping procedure proposed by McFarlane and Glover (1989). The methodology is also described in Skogestad and Postlethwaite (2007). Here we describe the theory for completeness. The pre-compensator incorporates the inverse of the nominal plant, a low pass filter for good noise attenuation, and an integrator. The shaped nominal plant is given by Equation (3.16):

$$G_S = G_NW_N$$

(3.16)
The shaped nominal plant $G_S$ has a left co-prime factorization (LFT) as follows:

$$G_S = \tilde{M}^{-1}\tilde{N} \quad (3.17)$$

The robust controller $G_r$ must stabilize a class of perturbed plants given by Equation (3.18):

$$G_P = \{(M + \Delta M)^{-1}(N + \Delta N) : \|[\Delta N \Delta M]\|_\infty < \epsilon\} \quad (3.18)$$

where $\epsilon > 0$ is the stability margin, $\gamma_{min}$ is the minimum achievable $H_\infty$ norm from the perturbation to the input and output, $\epsilon_{max}$ is the maximum achievable stability margin and is given by Equation (3.19):

$$\gamma_{min} = \epsilon_{max}^{-1} = \{1 - ||[N \ M]^2||_H^{-1/2} = (1 + \rho(XZ))^{1/2} \quad (3.19)$$

where $\rho$ is the maximum eigenvalue.

If $A, B, C, \text{ and } D$ are the minimal state space realization of $G_S$, then $Z$ and $X$ are unique positive definite solutions of the algebraic Riccati Equation. According to Glover and McFarlane (1989), the robust controller $G_r$ can be obtained for a $\gamma > \gamma_{min}$ using Equation (3.20):

$$G_r = \begin{bmatrix} A + BF + \gamma^2(L^T)^{-1}ZC^T(C + DF) & \gamma^2(L^T)^{-1}ZC^T \\ B^TX & -D^T \end{bmatrix} \quad (3.20)$$

where $F = -S^{-1}(D^TC + B^TX)$ and $L = (1 - \gamma^2)I + XZ$.

Using $\gamma = 2.2373 > \gamma_{min} = 1.7210$ we obtain the robust controller given by Equation (3.21) with $\epsilon = 0.4470$. The following controller was obtained by using MATLAB.
function coprimeunc presented in Skogestad and Postlethwaite (2007):

\[ G_r = \frac{-2.332 s - 3.14}{s^2 + 3.802 s + 5.76} \]  \hspace{1cm} (3.21)

### 3.4.2 Gain surface partitioning

In Section 3.4.1, a robust controller was designed for the nominal plant \( G_N \) of an arbitrary gain surface partition. The plant parameters of that nominal plant are given by Equations (3.22) and (3.23):

\[
K_N = \frac{K_1 + K_2}{2} ; \quad |K_1| > |K_N| > |K_2| \hspace{1cm} (3.22)
\]

\[
\tau_N = \frac{\tau_1 + \tau_2}{2} ; \quad \tau_1 > \tau_N > \tau_2 \hspace{1cm} (3.23)
\]

where \( K_1, K_2 \) and \( \tau_1, \tau_2 \) are the extreme gains and time constants of the considered arbitrary gain partition. \( K_N \) and \( \tau_N \) are mean gain and time constant. Writing \( K_1 \) and \( K_2 \) in terms of \( K_N \) we get:

\[
K_1 = a_1 K_N ; \quad a_1 > 1 \hspace{1cm} (3.24)
\]

\[
K_2 = a_2 K_N ; \quad 0 < a_2 < 1 \hspace{1cm} (3.25)
\]

From Equations (3.22), (3.24), and (3.25) the following relation is derived:

\[
a_1 + a_2 = 2 \hspace{1cm} (3.26)
\]

The ratio between the extreme case gains \( K_1 \) and \( K_2 \) is given by:

\[
R_K = \frac{a_1}{a_2} \hspace{1cm} (3.27)
\]
where $R_K$ is inversely proportional to the required number of gain partitions and hence the number of controllers. Assuming that $\tau_1 \approx \tau_2$ and using Equations (3.24) and (3.25), the complementary transfer functions of $G_N$, $G_1$ and $G_2$ are as follows:

\[
T_N = \frac{||G_r||_0 G_N W_N}{1 - G_r G_N W_N} = \frac{0.6847s^2 + 2.603s + 3.944}{s^4 + 5.058s^3 + 10.54s^2 + 10.16s + 3.944} \tag{3.28}
\]

\[
T_1 = \frac{||G_r||_0 G_1 W_N}{1 - G_r G_1 W_N} = \frac{a_1(0.6847s^2 + 2.603s + 3.944)}{s^4 + 5.058s^3 + 10.54s^2 + 7.21s + a_1(2.95s + 3.944)} \tag{3.29}
\]

\[
T_2 = \frac{||G_r||_0 G_2 W_N}{1 - G_r G_2 W_N} = \frac{a_2(0.6847s^2 + 2.603s + 3.944)}{s^4 + 5.058s^3 + 10.54s^2 + 7.21s + a_2(2.95s + 3.944)} \tag{3.30}
\]

From the above equations we get the following condition

\[\bar{\sigma}(T_1) \geq \bar{\sigma}(T_N) \geq \bar{\sigma}(T_2) \tag{3.31}\]

where $\bar{\sigma}$ is the maximum amplitude ratio in the frequency domain.

If $0 < a_2 << 1$ and consequently $a_1 >> 1$ and $R_K >> 1$, only a few controllers will be required but the closed loop performance of $G_2$ will be very poor and $\bar{\sigma}(T_1)$ will be large. If $a_2 \approx 1$ and consequently $a_1 \approx 1$ and $R_K \approx 1$, numerous controllers will be required but the response of $T_2$ will be fast and $\bar{\sigma}(T_1)$ will be small. Hence optimal $a_1$ and $a_2$ must be found which will minimize the required number of controllers, subject to the following constraints:

i. settling time of $T_1, T_2 < 10s$

ii. $||T_1||_{\infty} = 1$ and $||T_2||_{\infty} = 1$

The optimal $a_2$ is likely to result in a $T_2$ response just satisfying the constraint (i). It can be seen in Table 3.1 when $a_2 = 0.715$, $H_\infty$ norm of $T_1$ is 1 and settling time (98% of steady state) of $T_2$ is 9.6s, both the constraints are just met but results in 8 partitions. In order to reduce the number of required controllers, we soften the con-
Figure 3.6: Response of nominal and worst case plants

Figure 3.7: Magnitude of nominal and worst case closed loop transfer functions

straights. When $a_2 = 0.656$ we get 6 partitions, which results in a negligible increase in $||T_1||_{\infty}$ and the performance constraint is violated by 1s which is acceptable. The response of $G_N$, $G_1$, $G_2$ for a step change in reference using $W_N$ and $G_r$ is shown in Figure 3.6. The magnitude of $T_N$, $T_1$, $T_2$ in the frequency domain is shown in Figure 3.7. Nominal plants for every gain surface partition were computed and their respective pre-compensators were designed and are presented in Table 3.2. The gain surface was partitioned in such a way that the worst case gain ratio in each of the
partition is always $R_K = 2.049$ and it was shown in that case all plants contained in such partitions will be robustly stable and meet the performance criteria. Controllers were designed for a mud density of 1300 kg/m$^3$, according to (3.12) for an increase of 300 kg/m$^3$ in density the gain will increase by $a_1 = 1.2308$ times and for a decrease of 300 kg/m$^3$ in density the gain will decrease by $a_2 = 0.7692$ times (when other parameters remain constant). Then the worst case gain ratio possible due to density variations is $R_K = 1.6$ which is less than the maximum allowed ratio of 2.049. Therefore all plants of the class given by Equation (3.14) have been robustly stabilized. In order to perform the search, the gain surface was descretized. The choke value was incremented by 1% and the flow rate was incremented by 10 gpm. The algorithm used to partition the gain surface is presented in Appendix A.

Table 3.1: Finding the optimal $a_2$ value

| $a_1$ | $a_2$ | $R_K$ | $||T_1||_\infty$ | $T_2$ settling time (95%) | $T_2$ settling time (98%) | No. of partitions |
|-------|-------|-------|------------------|---------------------------|---------------------------|------------------|
| 1.285 | 0.715 | 1.797 | 1.00            | 7.54                      | 9.60                      | 8                |
| 1.330 | 0.670 | 1.985 | 1.02            | 8.32                      | 10.66                     | 7                |
| 1.340 | 0.660 | 2.030 | 1.02            | 8.50                      | 10.90                     | 7                |
| **1.344** | **0.656** | **2.049** | **1.02** | **8.56** | **11.00** | **6** |
| 1.350 | 0.650 | 2.077 | 1.03            | 8.69                      | 11.16                     | 6                |
| 1.403 | 0.597 | 2.350 | 1.06            | 9.80                      | 12.62                     | 5                |

3.4.3 Implementing the gain schedule controller

The proposed gain scheduling scheme is based upon two parameters. At every control cycle, the controller needs to evaluate its operating region based upon what it selects as the appropriate pre-compensator. The gain schedule is implemented using a lookup table. Every combination of choke opening and mud flow rate has an appropriate pre-compensator associated with it. The resulting gain schedule is shown
in Figure 3.8. Every operating region is assigned a number and their corresponding
pre-compensators are given in Table 3.2. For lower flow rates \( Q_s < 200 \text{ gpm} \) compensator 5 is assigned and for lower choke openings \( u_c < 25\% \) compensator 6 is assigned. While ramping down the choke has to close as quickly as possible, good
tracking performance is not required hence only one compensator is used for low flow
rate conditions. To facilitate the smooth transfer from one pre-compensator to an-
other we make use of a bumpless transfer technique. At a given time, only one of
the six pre-compensators will be active and the other pre-compensators will be track-
Figure 3.9: Pre-compensator switching

The required overbalanced pressure $\Delta_{bh}$ given by:

$$ p_{bh}^{set} = \hat{p}_{res} + \Delta_{bh} $$  \hspace{1cm} (3.33)
where $\hat{p}_{\text{res}}$ is the reservoir pressure estimate. According to Equation (3.2), when there is a kick there will be a change in choke pressure. The proposed observer is based on a modified version of the observer designed in Zhou et al. (2011). The original observer was designed for an MPD system which had a flow controller to handle kicks. In the present case, we implement a CBHP controller. Therefore, the proposed modification was necessary. Let us consider variable $V_1$ which is dimensionally equal to volume but based on pressure dynamics:

$$V_1 = \frac{V_a}{\beta_a} p_c + \frac{V_d}{\beta_d} p_p.$$  (3.34)

Then the derivative of $V_1$ obtained using (3.1) and (3.2)

$$\dot{V}_1 = q_p + q_b + q_k - q_c.$$  (3.35)

Using (3.6) we get:

$$\dot{V}_1 = q_p + q_b + K_{pi}(p_{\text{res}} - p_{bh}) - q_c.$$  (3.36)

but the productivity index might not be known so we use $K_o$. Then the observer is:

$$\dot{V}_1 = q_p + q_b + K_o(\hat{p}_{\text{res}} - p_{bh}) - q_c + l(V_1 - \dot{V}_1).$$  (3.37)

The update law for reservoir pressure is given by:

$$\dot{\hat{p}}_{\text{res}} = \gamma(V_1 - \dot{V}_1).$$  (3.38)
where $\gamma$ is a positive adaptation gain. In order to understand error dynamic the following variable are introduced as in Zhou et al. (2011):

$$
\dot{\tilde{V}}_2 = V_1 - \tilde{V}_2, \quad (3.39)
$$

$$
\dot{\tilde{p}}_{res} = p_{res} - \hat{p}_{res}. \quad (3.40)
$$

From Equations (3.2) and (3.6) we get:

$$
\dot{\tilde{V}}_1 = -l\tilde{V}_1 + K_o\tilde{p}_{res} + (K_{pi} - K_o)(p_{res} - p_{bh}) \quad (3.41)
$$

$$
\dot{\tilde{p}}_{res} = -\gamma\tilde{V}_1. \quad (3.42)
$$

The error $\tilde{V}_1$ is driven by $(K_{pi} - K_o)$ and $(p_{res} - p_{bh})$. In Zhou et al. (2011) the error converged to 0 because $(p_{res} - p_{bh}) \to 0$ due to flow control during a kick. But in our case, due to continued pressure control, the error $\tilde{V}_1$ is driven by $(K_{pi} - K_o)$. Therefore the operator must use a $K_o$ value which is higher than the estimate of $K_{pi}$ in order to get a slightly higher estimate of $p_{res}$ during a kick and use that value to revise the setpoint and reject the kick quickly. It will be shown in simulations that this strategy is quicker than relying on flow control to reject a kick. The details on kick flow estimator is not provided here and can be found in Zhou et al. (2011).

### 3.6 Simulations

Simulation studies were carried out to demonstrate performance and compare it with another existing MPD controllers. The ability of the controller to track a slow ramp on the setpoint and its capability to deliver consistent performance for different operating conditions is demonstrated. The designed controller is compared with a PI controller. The performance of the controller for pressure regulation during pipe ex-
tension sequence is demonstrated and a method to reject kicks using pressure control is demonstrated. The values of the parameters used in simulations are presented in Table 3.3. Physical properties of mud like density and normally used mud flow rates can be found in Bourgoyne Jr et al. (1986).

3.6.1 Setpoint tracking and robustness

![Figure 3.10: Bottom hole pressure during ramp setpoint tracking](image)

A case of active drilling is simulated in this section. The initial measured depth of the well is 3000 m and the TVD is 3000 m. A vertical well is drilled at the rate of 6 m/hr. A mud flow rate of 400 gpm was chosen and the mud density was 1227 kg/m$^3$. The bottom hole pressure is shown in Figure 3.10. The density of reservoir fluid is 1382 kg/m$^3$. Till 3092 m Controller 3 was active and then Controller 4 became active because the operating conditions moved from Region 3 to Region 4 of the gain schedule. The switching of controllers did not induce any transients or instability because of the high gain bumpless transfer. The bottom hole pressure setpoint was pore pressure plus 1 bar overbalance. Next we show the setpoint tracking performance of the controller under parametric
uncertainty. Mud density was kept constant at $1300\ kg/m^3 (\approx 10.85\ ppg)$ and the flow rate was varied from $350\ gpm$ to $450\ gpm$ in increments of $50\ gpm$. The BHP setpoint was revised from $490\ bar$ to $493\ bar$ and Figures 3.11 and 3.12 show that the controller delivered consistent performance for all the cases. Next simulations were performed under three different conditions: $1350\ kg/m^3$ and $370\ gpm$; $1300\ kg/m^3$ and $370\ gpm$; $1250\ kg/m^3$ and $450\ gpm$. Figures 3.13 and 3.14 show that the controller delivered
consistent performance for all the cases.

A PI controller was designed for good performance at mid range choke pressure (10 bar – 20 bar) and reasonable performance at lower choke pressures using IMC tuning relations presented in Seborg et al. (2006). Keeping the mud density constant at 1300 kg/m$^3$ and flow rate at 350 gpm, the choke pressure setpoint was progressively increased by steps of 4 bar from 4 bar to 32 bar. As shown in Figures 3.15
Figure 3.15: Comparing PI and robust gain switching controllers at lower pressures (< 20 bar)

and 3.16, the robust gain switching controller delivers consistent performance while the PI controller is considerably sluggish at lower choke pressures and at higher pressure response is quicker however there is considerable overshoot.
3.6.2 Pressure regulation during pipe extension sequence

Here the pipe extension sequence is simulated. The controller has to trap the pressure by closing down the choke when there is loss of frictional pressure due to ramping down of pump flow rate during pipe extension sequence. In this case a mud density of 1300 kg/m³ was used and the mud pump was ramped down from 400 gpm to 0 gpm in 120s. The controller had to track a BHP setpoint of 490 bar and it responds by
Figure 3.19: Choke opening during pipe extension sequence

closing the choke. After a dwell period of 60s, the mud pump is ramped back to 400 gpm in 120s. The maximum offset was 1 bar and maximum over shot was 3 bar. The BHP was well within the pressure window at all times.

3.6.3 Kick attenuation

Figure 3.20: Bottom hole pressure during kick attenuation

In order to reject a kick in CBHP drilling, the bottom hole setpoint has to be revised.
In this simulation, we use a mud density of 1300 kg/m$^3$ and a mud pump flow rate of 400 gpm. The initial pore pressure is 476 bar at a TVD of 3500 m and the MD of the well is 3500 m. A kick is introduced at 1500s of the simulation. Using the reservoir pressure estimate, the setpoint is revised at 3060 s to the new reservoir pressure of 484 bar plus an overbalance pressure of 4 bar. A noise of ±0.1 bar is introduced in bottom hole pressure measurements and random process noise was also introduced. Figure 3.20 shows that after revising the setpoint, the bottom hole pressure settles in less than 40s and it can be seen that the pressure revision is faster than the method proposed in Zhou et al. (2011) in which a simulation under similar pressure revision conditions was shown. During the first 5 min of kick attenuation in Zhou et al. (2011) the controller acts as a flow controller and that time window is a tunable parameter. After 5 min the pressure setpoint is revised to a new overbalanced pressure and it is during that time our proposed controller offers better performance. Flow control is useful for accurate estimation of reservoir pressure but it is considerably slower than a pressure controller. Figure 3.21 shows that the actual kick and the estimate kick and it is rejected almost instantly after setpoint revision. The control input is shown in Figure 3.22.

### 3.7 Conclusion

In this chapter, a gain switching $H_\infty$ robust controller is presented for a CBHP type MPD process. The salient contributions of this work are as follow:

- Two parameter based gain switching is performed. The appropriate controller will be selected based on the total flow rate and choke opening.

- Due to controller switching and the $H_\infty$ robust controller, the closed loop response is always robustly stable and the controller is able to deliver consistent
When the operating conditions change, the controller is able to choose the appropriate controller. Even though the controller was originally designed for a 1300 kg/m$^3$ mud it was found that it was able to meet all control requirements even when mud density was changed by $\pm 50$ kg/m$^3$. The controller was able to meet performance criteria consistently under severe parametric uncertainty introduced by variations in
mud flow rate and choke opening. The proposed controller is able to achieve the revised bottom hole pressure setpoint in order to reject a kick in approximately 30s. Faster kick attenuation can be achieved if a better observer designed specifically for CBHP MPD is implemented. The controller is also found to operate well under noisy measurements. The designed controller is also able to track a BHP setpoint during pipe extension sequence with minimal offset. Thus the designed controller proves to be very versatile for bottom hole pressure setpoint tracking purposes.

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<td>–</td>
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Bibliography


Chapter 4

Nonlinear Model Predictive Control of Managed Pressure Drilling

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Preface

This paper has been submitted to the ISA Transactions. A version of this paper has been submitted to DYCOPS’16 to be held in Trondheim, Norway. The lead author performed the necessary literature review, developed the controller, implemented the observer, generated the results, and wrote the manuscript. The co-author Dr. Imtiaz helped in identifying the gap in research, supervising the research, and editing the manuscript.
Abstract

Mitigation of abnormal events like kicks while drilling is important not only to improve safety but also to enhance efficiency of the drilling process. Managed pressure drilling (MPD) is a closed loop drilling technology which enhances drilling safety and enables fast mitigation of kicks. In constant bottomhole pressure (CBHP) MPD it is required that the controller tracks bottomhole pressure (BHP) during normal drilling and during drill pipe extension. The performance of kick mitigation can be improved if the controller sacrifices BHP tracking for kick mitigation automatically. We propose a new design of nonlinear model predictive controller (NMPC) for automatic kick mitigation which tracks BHP when there is no kick and contains the outlet flow rate within certain tunable threshold when a kick occurs. This is achieved by exploiting constraint handling capability of NMPC. The NMPC is based on output feedback control architecture and employs offset-free formulation proposed in (Morari and Maeder, 2012). NMPC uses active set method for computing control inputs. We demonstrate that the NMPC is able to track BHP with minimal offset during drill pipe extension and contains kicks within a threshold when they occur.

4.1 Introduction

Managed pressure drilling (MPD) is an overbalanced drilling technique (Malloy et al., 2009) in which the bottom hole pressure is regulated by employing an automated choke manifold. The precise control of bottom hole pressure by MPD is enabling drilling of so-called undrillable wells in which pressure window is very narrow and ensuring the safer handling of reservoir influxes. One of the major safety issues in drilling is when the bottom hole pressure $p_{bh}$ becomes less than the reservoir pressure.
there will be an influx of reservoir fluids, commonly referred as kick. If a kick is unmitigated, large quantities of reservoir fluids may flow to the rig surface endangering the lives of rig workers and the environment. MPD can also recover the system quickly from any abnormal situation, thus (Vieira et al., 2008) reported that without MPD it took 65 days to drill a particular well but while using MPD it took only 45 days.

Automation of MPD has the potential to reduce NPT further and automation usually leads to enhanced safety. Automated MPD solutions range from automating conventional well control methods to model based control of pressure at different points and drilling fluid flow rate. A review of computer control in managed pressure drilling can be found in (Nikolaou, 2013). Godhavn et al. 2010 developed a simple PID controller to track choke pressure setpoint. The controller demonstrated good performance for pipe extension sequence. A nonlinear controller for BHP regulation was designed in (Godhavn et al., 2011) using feedback linearization technique. These controllers, however were not configured for kick mitigation. In order to improve safety during such operations, in (Carlsen et al., 2013) PI, IMC, and MPC pressure controllers were designed to automate kick handling sequence. In (Nandan et al., 2014), a robust $H_{\infty}$ loop shaping controller was designed for handling variations in mud density, well length, and mud flow rate. For severe changes in the flow rate and choke opening, gain switching robust controller was suggested. The advantage offered by pressure control is its ability to track a BHP setpoint but during a kick, continued pressure setpoint tracking will not attenuate a kick (Zhou et al., 2011). Flow controllers have been designed for kick handling.

Feedback linearised flow controllers were presented in (Hauge et al., 2012) and (Hauge et al., 2013). The choke opening was used to regulate the exit flow rate and thereby the in/out flux. In/out flux and bit flow rate estimators were also presented in (Hauge
et al., 2012, 2013). Santos et al. 2003 developed a well control method by comparing the in/out flow rates for detecting kicks and subsequently kick was mitigated by manipulating the back pressure. Flow control is an effective strategy for suppressing kicks, but under normal condition the control objective is to track the bottom hole pressure trajectory. Zhou et al. 2011 implemented a switching controller which works as a pressure controller during normal operation and as a flow controller while handling kicks. A nonlinear passivity based observer was developed to estimate kick magnitude and reservoir pressure. A nonlinear pressure/flow switching controller was designed for dual gradient drilling (DGD) in (Zhou and Nygaard, 2011). DGD is a variant of MPD in which mud of varying density is used and as a result the hydrostatic pressure is piece-wise linear.

Model predictive controller (MPC) and nonlinear MPC design have also been considered for MPD, they are well suited for MPD because of their ability to handle constraints and nonlinearity. An NMPC scheme for control of underbalanced drilling (UBD) was developed in (Nygaard and Nævdal, 2006). BHP was regulated by computing optimal choke opening in receding horizon fashion. A two phase model of drilling well was used to model UBD well drilling. Breyholtz et al. 2009 used NMPC to coordinate pump flow rate and choke opening in order to control BHP. The controller was evaluated for pressure regulation during pipe extension sequence, however mitigation of kicks were not considered. Breyholtz et al. 2010, 2011 considered linear MPC of DGD, the focus of the study was on optimal movement of drill string in order to minimize pressure variations. The hook position and bottom hole pressure were controlled by manipulating the drill string velocity and main pump and subsea pump flow rates. Controller performance was demonstrated in presence of noise and uncertainty. Møgster et al. 2013 implemented linear MPC to control the bottom hole pressure and the pressure at the casing shoe by manipulating the mud flow rate.
and choke opening. The controller was implemented using Statoil’s in-house MPC software SEPTIC. The controller regulated BHP and casing shoe pressure but its effectiveness in dealing with kicks and severe drop in pumping rate was not studied. Pedersen et al. 2013 implemented MPC on UBD system by using First Order Plus Time Delay (FOPTD) models. The bottom hole pressure and return flow rate were regulated by manipulating the choke opening and mud pump flow rate. Regulating outlet flow is useful in UBD as it allows hydrocarbons to come to the surface during drilling. Since MPD does not allow hydrocarbons to flow to the surface, regulating the outflow is usually not an objective for MPD control. The above literature review clearly shows MPC/NMPC have been used successfully to UBD and DGD systems. The application of MPC/NMPC to MPD system is very limited. Moreover, NMPC applications do not exploit its full potential (i.e., constraint handling capability).

In this paper, we present a new design of NMPC for MPD application which implements the philosophy of switched pressure/flow control by cleverly employing the constraints of NMPC. The NMPC operates as a pressure controller which tracks BHP under normal drilling conditions. The controller acts more like a flow controller when a kick occurs and contains the kick within a tunable threshold.

A brief description of MPD is furnished in Section 4.2 which is followed by the design of the controller and optimization scheme in Section 4.3 which includes discussion on constraint and cost function design. In Section 4.4 details on the implemented observer to estimate bit flow rate, kick flow rate, and reservoir pressure are provided. The simulation results are presented in Section 4.5. Finally, Section 4.6 concludes this chapter.
4.2 System Description

The MPD process consists of two control volumes, the drill string and the annulus. The schematic representation of MPD process is shown in Figure 4.1. The drilling mud is pumped into the drill string under pump pressure $p_p$ and at flow rate $q_p$. The mud exits the drill string through the drill bit at a flow rate $q_{bit}$. The drilling mud then flows through the annulus control volume and through a choke at pressure $p_c$ and flow rate $q_c$. The pump pressure, choke pressure, and bit flow rate are given by Equations (4.1), (4.2), and (4.3) respectively; $\beta_d$ and $\beta_a$ are bulk moduli of mud in drill string and annulus respectively; $\rho_d$ and $\rho_a$ are the mud densities in the drill
string and annulus respectively; $V_d$ and $V_a$ are the volumes of the drill string and the annulus respectively; $f_d$ and $f_a$ are frictional loss coefficients in the drill string and the annulus respectively; and $M$ is a mass like property. The pressure at the bottom hole $p_{bh}$ is given by Equation (4.4). The flow through the choke is given by Equation (4.5) where $u_c \in [0, 1]$ is the choke opening. The kick flow rate $q_k$ is given by Equation (4.6). Due to the addition of reservoir fluids and cuttings in the annulus, generally mud density changes when mud flows from the drill string into the annulus and that induces pressure changes equal to $(\rho_a - \rho_d)gh_t$. Frictional loss and mud density are major sources of uncertainty and when there is a kick it acts as a persistent disturbance. Detailed derivation of the model can be found in (Kaasa et al., 2012).

\[
\dot{p}_p = \frac{\beta_d}{V_d} (q_p - q_{bit})
\]
\[
\dot{p}_c = \frac{\beta_a}{V_a} (q_{bit} - q_c + q_b + q_k)
\]
\[
\dot{q}_{bit} = \frac{1}{M} (p_p - p_c - f_d q_p^2 - f_a q_{bit}^2 - (\rho_a - \rho_d)gh_t)
\]
\[
p_{bh} = p_c + p_f + \rho_a gh_t
\]
\[
q_c = u_c C_d A_o \sqrt{\frac{2(p_c - p_o)}{\rho_a}}
\]
\[
q_k = K_{ps}(p_{res} - p_{bh})
\]

### 4.3 Controller Design

The core elements of an NMPC are the cost function, prediction model, state constraints, and input constraints. In this section the design of each of those elements is explained.
4.3.1 Prediction model

The state space model \( f \) represents the nominal three state model of managed pressure drilling described by the Equations (4.1), (4.2), and (4.3). The predicted states are given by Equation (4.7).

\[
x(k + T) = x(k) + \int_{k}^{k+T} f(x(\tau))d\tau,
\]

where \( x(k) \) is the current state and \( T \) is the sampling time.

In order to design an offset free NMPC we utilize the results presented in (Morari and Maeder, 2012) and (Rawlings and Mayne, 2009). As a first step, disturbance model and a disturbance integrator is incorporated in the prediction model and the resulting model is called an augmented model represented by \( f \). The augmented state space model consists of the state space \( f_{\text{aug}} \) along with the disturbance model \( d \) given by Equation (4.8) and during state prediction the initial estimate of the disturbance is held constant given by Equation (4.9), so it acts like a step disturbance. The prediction model is numerically integrated using explicit Runge-Kutta 4,5 formula also known as Dormand-Prince pair. The predicted state is given by Equation (4.8).

\[
x(k + T) = x(k) + \int_{k}^{k+T} f_{\text{aug}}(x(\tau), d(k))d\tau,
\]

\[
d(k + T) = d(k),
\]

where \( q_k(k) \) is given by Equation (4.6). And the predicted output is given by

\[
y(k + T) = g_{\text{aug}}(x(\tau), d(k)),
\]

where \( g_{\text{aug}} \) is the augmented output model.
### 4.3.2 Optimization scheme

The cost function minimizes the error between the equilibrium state targets $\bar{x}$ and the system states $x(k)$; the equilibrium input target $\bar{u}$ and the current input $u(k)$ to achieve offset free tracking of the reference $r(k) = p_{ref}$.

$$J = \min_{\bar{u}} \sum_{k=k}^{k+m} (\hat{x}(\kappa) - \bar{x})^T \lambda_1 (\hat{x}(\kappa) - \bar{x}) + \lambda_2 (u(\kappa) - \bar{u})^2$$ \hspace{1cm} (4.11)

where $\lambda_1 \in \mathbb{R}^{3 \times 3}$ and $\lambda_2 \in \mathbb{R}$ are cost function weights and $m$ is the prediction horizon.

The optimization problem is constrained by the following constraints

\begin{align*}
\hat{d}(0) &= \hat{q}_k, \hspace{1cm} (4.12) \\
\bar{x} &= f_{aug}(\bar{x}, \bar{u}, \hat{d}(k)), \hspace{1cm} (4.13) \\
r(k) &= g_{aug}(\bar{x}, \hat{d}(k)). \hspace{1cm} (4.14) \\
\hat{x} \in X, \bar{x} \in X_{NL}, u(k) \in U, \hspace{1cm} (4.15) \\
\bar{x} \in X, \bar{u} \in U, \hspace{1cm} (4.16)
\end{align*}

where the state and input constraint sets $X$ and $U$ are given by

\begin{align*}
X := & \begin{bmatrix} p_{p_{\text{min}}} \leq p_p \leq p_{p_{\text{max}}} \\
p_{c_{\text{min}}} \leq p_c \leq p_{c_{\text{max}}} \\
q_{\text{bit}_{\text{min}}} \leq q_{\text{bit}} \leq q_{\text{bit}_{\text{max}}} \end{bmatrix}, \hspace{1cm} (4.17) \\
U := & \begin{bmatrix} u_{c_{\text{min}}} \leq u_c \leq u_{c_{\text{max}}} \end{bmatrix} \hspace{1cm} (4.18)
\end{align*}

The control objective is not only to track a bottom hole pressure setpoint but also to contain the reservoir influx within certain threshold and in order to achieve that we
include a nonlinear state constraint given by following equation

\[ X_{NL} = \left[ 0 \leq (q_c - \bar{q}_{bit}) \leq \epsilon \right] \] (4.19)

where \( \bar{q}_{bit} \) is the equilibrium state target for the bit flow rate. By adding this nonlinear constraint the annular discharge is constrained within a tunable threshold \( \epsilon \).

To achieve offset free tracking mere incorporation of disturbance model is insufficient; along with augmenting the model with disturbance, equilibrium state targets which will reject the disturbance must be generated with the help of the augmented model (Morari and Maeder, 2012). Moreover, our objective is to track an output reference \( r(k) = p_{ref} \) and that requires the computation of relevant state and input targets denoted by \( \bar{x} \) and \( \bar{u} \) respectively and they are computed by solving the equilibrium Equations (4.13) and (4.14) which are implemented as equality constraints in the optimization algorithm.

## 4.4 Observer

The proposed NMPC structure relies on unmeasured state \( q_{bit} \) and disturbance \( q_k \) which is dependent on the reservoir pressure \( p_{res} \). We use an observer based on the observers proposed by (Zhou et al., 2011) to estimate these parameters. We present the observer for the sake of completeness. The bit flow rate is estimated using the dynamic equation of the pump pressure and pump pressure measurements \( (p_p) \). The error in pump pressure is injected in Equation (4.1) to get the estimated pump pressure \( (\hat{p}_p) \), Equation (4.20). The bit flow rate is estimated with the help of a parameter
update law Equation (4.21).

\[
\dot{p}_p = \frac{\beta_d}{V_d} (p - \dot{q}_{bit} + l_1 (p_p - \dot{p}_p)) \quad (4.20)
\]

\[
\dot{q}_{bit} = -\gamma_1 (p_p - \dot{p}_p) \quad (4.21)
\]

The reservoir flow (kick) can be estimated by detecting changes in the flow rates. When there is no kick the in/out flux must be zero i.e. the difference between the choke flow rate and the sum of pump flow rate and kick flow rate must be zero. When there is a kick, during transience that quantity will not be equal to zero and that error can be used for estimating kick flow rate. In order to estimate \( q_{res} \) a new variable \( q_1 \) is introduced given by Equation (4.22). The time derivative of \( q_1 \) is given by Equation (4.23) obtained using Equations (4.1) and (4.2).

\[
q_1 = \frac{V_a}{\beta_a} p_c + \frac{V_d}{\beta_d} p_p \quad (4.22)
\]

\[
\dot{q}_1 = q_p + q_{res} - q_c \quad (4.23)
\]

Using the derived measurement \( q_1 \) reservoir flow estimator is formed and it is driven by the dynamics in topside pressure measurements \( p_p \) and \( p_c \) and they are given by Equations (4.24) and (4.25).

\[
\dot{\hat{q}}_1 = q_p + \hat{q}_{res} - q_c + l_2 (q_1 - \hat{q}_1) \quad (4.24)
\]

\[
\dot{\hat{q}}_{res} = \gamma_2 (q_1 - \hat{q}_1) \quad (4.25)
\]

The reservoir pressure can be estimated by using a parameter update law and assuming a reservoir model. In order to estimate \( p_{res} \) Equation (4.23) is modified using Equation (4.6) resulting in Equation (4.26) as in (Zhou et al., 2011). \( q_2 \) is estimated
by injecting error in the derived measurement $q_1$ and given by Equation (4.27) and the reservoir pressure $p_{res}$ is estimated using Equation (4.28). Generally, only inaccurate estimates of the productivity index $K_{pi}$ will be available and hence it is replaced by a tuning parameter $K_o$.

\[
\dot{q}_2 = q_p + K_{pi}(\hat{p}_{res} - p_{bh}) - q_c \tag{4.26}
\]

\[
\dot{q}_2 = q_p + K_o(\hat{p}_{res} - p_{bh}) - q_c + l_3(q_1 - \hat{q}_2) \tag{4.27}
\]

\[
\dot{p}_{res} = \gamma_3(q_1 - \hat{q}_2) \tag{4.28}
\]

### 4.5 Simulation Results

In this section, we present the results of simulations to test the designed controller on the model described in Section 4.2. The NMPC controller was implemented in MATLAB using the function provided in (Grüne and Pannek, 2011) as a template. The developed NMPC scheme uses sequential discretization technique for solving finite horizon optimization problem and active set method for computing optimal control actions. The values of plant parameters used for simulation is given in Table 4.1. The controller tuning parameters are given in Table 4.3. The state and input constraints used for all simulations is provided in Table 4.4.

Even though the settling time for a step change in pressure setpoint is approximately 40s the dynamics of the system is significantly faster during kick rejection and pressure regulation during pipe connection sequence. Hence, a prediction/control horizon of less than 40s should be sufficient. Through trial and error, it was found that a horizon of more than 24s did not improve performance. The required prediction/control horizon of can be obtained either through fast sampling (say 1s) and many samples (24 samples) or through slow sampling (say 6s) and fewer samples (4 samples). The
first approach is impractical because if controller has to work with too many samples
the computation time will exceed sampling time (i.e. \( > 1s \)) due to large memory
usage. Hence in this work a sampling time of 6s and horizon of 4 samples was chosen.
The computation time was \(< 1s\) which is a fraction of the chosen sampling time (6s).
In the next sections we simulate different scenarios to demonstrate controller perfor-
mance and robustness under noise and plant uncertainty.

![Graphs](image)

(a) Bottom hole pressure  (b) Choke opening

(c) Kick estimate  (d) Value of flow constraint

Figure 4.2: Kick handling and pressure setpoint revision (nominal case)
4.5.1 Outlet flow constrained pressure regulation

The initial bottom hole pressure setpoint is $p_{ref} = 480$ bar. In this simulation mud is pumped at the rate of 1200 LPM. A kick is encountered at 120s, and that leads to violation of the flow constraint threshold of $\epsilon = 10$, as shown in Figure 4.2d. The controller responds by constricting the choke as shown in Figure 4.2b and that causes an increase in $p_{bh}$. Due to the increase in pressures, the reservoir pressure estimator is able to estimate the new reservoir pressure as shown in Figure 4.2a. It is to be noted that the controller is not tracking the reservoir pressure which can be possible only by resorting to complete flow control, instead it gives up pressure tracking in order to satisfy flow constraints. Using the new reservoir estimate, $p_{ref}$ is revised to 475 bar at 252s. Eventually due to overbalanced conditions the kick is completely rejected. The parameters used for tuning the observer is provided in Table 4.2.

Table 4.1: Values of well parameters used in simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_a$</td>
<td>89.9456</td>
<td>m$^3$</td>
</tr>
<tr>
<td>$V_d$</td>
<td>25.5960</td>
<td>m$^3$</td>
</tr>
<tr>
<td>TVD</td>
<td>3500</td>
<td>m</td>
</tr>
<tr>
<td>$M$</td>
<td>$8.04 \times 10^8$</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\beta_a$</td>
<td>$2.3 \times 10^9$</td>
<td>Pa</td>
</tr>
<tr>
<td>$\beta_d$</td>
<td>$2.3 \times 10^9$</td>
<td>Pa</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>1300</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$\rho_d$</td>
<td>1300</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$f_d$</td>
<td>$1.65 \times 10^{10}$</td>
<td>s$^2$/m$^6$</td>
</tr>
<tr>
<td>$f_a$</td>
<td>$2.08 \times 10^9$</td>
<td>s$^2$/m$^6$</td>
</tr>
<tr>
<td>$C_d$</td>
<td>0.6</td>
<td>–</td>
</tr>
<tr>
<td>$A_o$</td>
<td>$2 \times 10^{-3}$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>$p_o$</td>
<td>$1.013 \times 10^5$</td>
<td>Pa</td>
</tr>
<tr>
<td>$K_{pi}$</td>
<td>$6.133 \times 10^{-9}$</td>
<td>m$^2$/(sPa)</td>
</tr>
</tbody>
</table>
Here we test the ability of the designed controller to track a bottom hole setpoint and to contain the reservoir influx within a threshold. In this simulation the initial bottom hole pressure setpoint is $p_{ref} = 470$ bar and mud is pumped at the rate of 1500 LPM. A kick is encountered at 120s, and that leads to violation of the flow constraint as shown in Figure 4.3d, initially a threshold of $\epsilon = 10$ LPM was chosen.
Table 4.2: Values of observer parameters used in simulations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_1$</td>
<td>$1 \times 10^{-7}$</td>
<td>$-$</td>
</tr>
<tr>
<td>$l_2$</td>
<td>0.2</td>
<td>$-$</td>
</tr>
<tr>
<td>$l_3$</td>
<td>0.2</td>
<td>$-$</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>$2 \times 10^{-6}$</td>
<td>$-$</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>0.005</td>
<td>$-$</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>$5 \times 10^6$</td>
<td>$-$</td>
</tr>
<tr>
<td>$K_o$</td>
<td>$4.9066 \times 10^{-9}$</td>
<td>m³/(s Pa)</td>
</tr>
</tbody>
</table>

Table 4.3: Controller tuning parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_1$</td>
<td>diag[0,1,0]</td>
<td>$-$</td>
</tr>
<tr>
<td>$\lambda_2$</td>
<td>1000</td>
<td>$-$</td>
</tr>
<tr>
<td>$m$</td>
<td>4</td>
<td>$-$</td>
</tr>
<tr>
<td>$T$</td>
<td>6</td>
<td>s</td>
</tr>
</tbody>
</table>

The controller responds by closing down the choke as shown in Figure 4.3b and that causes an increase in bottom hole pressure, $p_{bh}$ shown in Figure 4.3a. Using the new reservoir estimate, $p_{ref}$ is revised to 475 bar at 252s. The flow constraints are relaxed ($\epsilon = 100$) during the setpoint revision for faster revision as shown in Figure 4.3d. Due to constraint softening, the setpoint is revised in under 30s.

Table 4.4: State and input constraints

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_p^{min}$</td>
<td>$8 \times 10^5$</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_p^{max}$</td>
<td>$150 \times 10^5$</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_c^{min}$</td>
<td>$8 \times 10^5$</td>
<td>Pa</td>
</tr>
<tr>
<td>$p_c^{max}$</td>
<td>$50 \times 10^5$</td>
<td>Pa</td>
</tr>
<tr>
<td>$q_{bit}^{min}$</td>
<td>$-0.002$</td>
<td>m³/s</td>
</tr>
<tr>
<td>$q_{bit}^{max}$</td>
<td>0.0283</td>
<td>m³/s</td>
</tr>
<tr>
<td>$u_c^{min}$</td>
<td>0</td>
<td>%</td>
</tr>
<tr>
<td>$u_c^{max}$</td>
<td>100</td>
<td>%</td>
</tr>
</tbody>
</table>
Figure 4.4: Kick handling and pressure setpoint revision under plant-model mismatch and measurement noise

### 4.5.3 Robustness under plant-model mismatch

A measurement noise of 0.1 bar is added to pressure measurements and plant-model mismatch is introduced by augmenting state equations with random noise. The robustness of controller is tested by tracking a higher pressure setpoint with lower mud flow rate, forcing the controller to work at lower choke opening. The initial bottom hole pressure setpoint is $p_{ref} = 480$ bar and mud is pumped at the rate of 1200 LPM leading to a lower choke opening. A kick is encountered at 120s, leading to viola-
tion of flow constraint. Unlike the previous case the constraint does not settle at that threshold value due to noise. It can be seen in Figure 4.4d the differential flow \((q_c - \hat{q}_{bit})\) occasionally violates the threshold during kick handling but the controller acts to nudge it back to the acceptable region. Therefore, reasonable noise and plant uncertainty does not affect the flow constraint handling considerably. During normal setpoint tracking the noise and plant uncertainty does not affect the controller as shown in (i.e., \(0 - 120s\)) Figure 4.4d. With the help of the new reservoir pressure estimate, the setpoint is revised. Flow constraint is again relaxed during setpoint revision. Bottom hole pressure and drilling pressure window are shown in Figure 4.4a. Choke opening is shown in Figure 4.4b. Kick flow rate is shown in Figure 4.4c. This NMPC being a state feedback controller the estimate of the bit flow rate is required and it is shown in Figure 4.5.

4.5.4 Controller performance during pipe extension sequence

We test the ability of the controller to track bottom hole pressure \(p_{bh}\) set point \(p_{ref}\) during pipe extension sequence. In order to get the best performance during pipe
extension sequence the flow constraint must be switched off as the objective is solely to regulate BHP. Typically in pipe extension sequence the mud pump flow rate is ramped down from a nominal value to 0 $LPM$ in approximately 60 s to 120 s. While performing pipe extension there will be no mud flow, then mud pump is ramped up from no flow to a nominal value. To test the NMPC we used a similar sequence as shown in Figure 4.6d, the mud pump flow rate is ramped down at 60s from 1500 $LPM$ to 0 $LPM$ in 60s; between 120 s and 180 s there is no flow in the system; starting at 180 s mud pump is ramped back to 1500 $LPM$ in 60s. The setpoint to be tracked is

Figure 4.6: Bottom hole pressure tracking during pipe extension sequence
\( p_{\text{ref}} = 470 \text{ bar} \). A measurement noise of 0.1 \text{ bar} is added to topside pressure measurements. The bottom hole pressure \( p_{\text{bh}} \) is shown in Figure 4.6a. The controller responds by closing down the choke in order to trap the pressure as shown in Figure 4.6b. The initial overshoot is because of back flow, whenever there is a negative change in pump flow rate there will be momentary increase in pressure but eventually pressure will decrease. In order to maintain a constant \( p_{\text{bh}} \), the choke pressure \( p_c \) has to increase (shown in Figure 4.6c) to compensate for the loss in frictional pressure drop.

### 4.6 Conclusion

In this article a nonlinear model predictive controller for pressure regulation and reservoir flow containment was presented. The control objectives were achieved by penalizing the deviation of BHP from the setpoint and enforcing hard constraints on the in/out flow rate flux. The controller was designed as an output feedback controller which regulates bottom hole pressure by manipulating the choke opening. Equilibrium state references were generated by using the dynamic model of MPD and disturbance model was incorporated in the prediction model for offset free output tracking. It was shown that in the event of a kick, reservoir influx was contained by the controller within the allowed threshold. It was also shown that the controller is able to perform well in presence of measurement and model uncertainty. The flow constraint is not severely affected due to measurement noise and it was shown constraint softening lead to considerably improved performance. The controller is also able to regulate bottom hole pressure during severe loss in mud flow rate which typically occurs during the pipe extension sequence. The controller is able to work under different mud flow rates and choke opening without any deterioration in performance. In the future we will be treating frictional losses as uncertain parameters and utilize parameter estimation.
for updating the model of the plant and the proposed controller will be tested on an experimental set up which is under construction.
Bibliography


Chapter 5

Conclusion

5.1 Conclusion

The objective of thesis was to develop advanced controllers for constant bottomhole pressure (CBHP) managed pressure drilling (MPD) systems that can deliver good tracking performance during drilling and regulate the pressure during drill pipe extension. Also the controller should be able to successfully mitigate abnormal events such as kicks to keep the process safe. We developed two controllers to attain these objectives: (i) a robust gain switching $H_{\infty}$ loop shaping controller, (ii) a nonlinear model predictive controller (NMPC).

The bottomhole pressure (BHP) of an MPD system is a highly nonlinear function of mud flow rate and choke opening, and an MPD system has many sources of uncertainties. In order to develop the robust gain switching controller, the nonlinearity of the system was mapped for choke opening and mud flow rate. The resulting 2-dimensional (2-D) map was partitioned into several regions so that individual controllers can be designed for each of those regions. The partitioning of the map was performed with the goal of using as few controllers as possible while maintaining certain performance and robustness requirements. Choke pressure settling time of 10s was the performance requirement and $H_{\infty}$ norm of 1 was the stability requirement. This partitioning resulted in 6 compensators. To improve robustness in the presence of parametric uncertainty arising out of mud density variations, an $H_{\infty}$ loop shaping controller was
developed. To enable smooth switching between different compensators, high gain bumpless transfer technique was used. A method for kick rejection using pressure control was developed. The controller was tested for BHP tracking during drilling and drill pipe extension; kick rejection was also tested. The controller is able to reject kicks in approximately 1 minute and maintains a uniform closed loop choke pressure settling time of 10 s.

Kicks are best rejected by flow control in which the exit mud flow rate is regulated but during normal drilling operations and during drill pipe extension, a BHP setpoint must be tracked. That leads to a situation where the controller has to be switched from flow control to pressure control either manually or by using some switching logic. Such switching is generally prone to shattering in the presence of noise. In the NMPC a unique method for kick rejection has been developed. The controller tracks BHP in the absence of kicks but sacrifices BHP tracking and contains kick flow rate when a kick occurs. That is achieved by clever use of the constraint handling capability of the NMPC. The NMPC uses active set method for computing inputs. The NMPC was tested for maintaining BHP during drill pipe extension and kick rejection. The controller was able to contain the outlet flow rate within a threshold of 10 LPM in less than 1 minute. The controller performed well in the presence of plant and measurement noise.

5.2 Future work

The controllers developed in the thesis were tested on a simple numerical model of drilling which consists of two control volumes. The advantages and limitations of the controllers can be better appreciated if they are tested on an experimental model of drilling. A flow loop is under construction at Memorial University of Newfound-
land which will be used for testing these controllers in the future. Real-time NMPC computation is difficult and time consuming and that could be a potential implementation issue. Nonlinear observers were used for estimating kick flow rate, reservoir pressure, and bit flow rate in this thesis. The performance of the nonlinear observers can deteriorate in the presence of noise and plant model mismatch. The estimation of unmeasured quantities like mud density, friction factor, area of choke opening, etc. will lead to better control solutions. These limitations of nonlinear observers can be addressed by developing an unscented Kalman filter (UKF) for state, parameter, and unknown input estimation. UKF based unknown input observer (UIO) has the potential for estimating the unknown kick inflow rate accurately in the presence of noise and plant model mismatch. A UKF based UIO will be developed in the future.
Appendix A

Gain surface partitioning algorithm

The gain surface was partitioned for computing nominal plants and to develop a gain schedule, using the following algorithm

Load $GF$ \{the surface fit\}

$C = \{20, 21, 22, ..., 60\}$ \{choke openings\}

$F = \{200, 210, 220, ..., 450\}$ \{flow rates\}

$R$ \{The ratio between highest and lowest gain in a partition\}

\[
\text{for } Ci = 0 \textbf{ to } \text{length}(C) \textbf{ do}
\]

\[
\text{for } Fj = 0 \textbf{ to } \text{length}(F) \textbf{ do}
\]

\[
GFF(Ci, Fj) = GF(C(Ci), F(Fj))
\]

end for

end for

$\text{cond} = 1$

$n = 0$

$Gmin = \text{min}(abs(GFF))$

\[\textbf{while } \text{cond} == 1 \textbf{ do}\]

\[n = n + 1\]

\[\textbf{for } Ci = 0 \textbf{ to } \text{length}(C) \textbf{ do}\]

\[\textbf{for } Fj = 0 \textbf{ to } \text{length}(F) \textbf{ do}\]

\[\text{if } Gmin \leq abs(GF(Ci, Fj)) \textbf{ and } abs(GF(Ci, Fj)) \leq R \times Gmin \textbf{ then}\]

\[GFF(Ci, Fj) = GF(Ci, Fj)\]
else
    
    \( GFF(C_i, F_j) = 0 \)
    
end if

end for

end for

\[
K_N = \frac{G_{\text{min}} + \max(\text{abs}(GFF))}{2}
\]

for \( C_i = 0 \) to \( \text{length}(C) \) do
    
for \( F_j = 0 \) to \( \text{length}(F) \) do
        
if \( GFF(C_i, F_j) \neq 0 \) then

    \( GN(C_i, F_j, n) = K_N \)

else

    \( GN(C_i, F_j, n) = 0 \)

end if

end for

end for

\[
G_{\text{min}} = \min(\text{abs}(GFF))
\]

if \( G_{\text{min}} \geq \min(\text{abs}(GFF)) \) then

    \( \text{cond} = 0 \)

else

    \( \text{cond} = 1 \)

end if

end while

\[
GNOM = \sum_{i=1}^{n} GN(C_i, F_j, i) \quad \{ \text{The resulting gain schedule} \}
\]


Landet, I. S., Pavlov, A., and Aamo, O. M. Modeling and control of heave-induced


