PERFORMANCE EVALUATION OF THE PROPULSION SYSTEM FOR THE AUTONOMOUS UNDERWATER VEHICLE "C-SCOUT"

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PERFORMANCE EVALUATION OF THE PROPULSION SYSTEM FOR THE AUTONOMOUS UNDERWATER VEHICLE " C-SCOUT "

By

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A thesis submitted to the School of Graduate Studies
in partial fulfillment of the requirements for the degree of
Master of Engineering

Faculty of Engineering and Applied Science

Memorial University of Newfoundland

August 2003

St John's, Newfoundland, Canada

Your file Votre référence
ISBN: 0-612-99122-9
Our file Notre référence
ISBN: 0-612-99122-9

ABSTRACT

This thesis begins with a study of AUV energy systems and endurance. A comparative study of a number of high energy density systems is carried out and based on this study, a few high energy density systems are selected for possible use on the AUV "C-SCOUT". The optimum operating speed and the endurance limits of "C-SCOUT" are then calculated. Next, two powering prediction methods (the “ITTC 1978” method and the “Load-Varying Self-Propulsion Test” method) are compared, in order to estimate the powering performance of “C-SCOUT”. Subsequently an account of the resistance, self-propulsion and bollard pull tests conducted on "C-SCOUT", is included. Areas discussed include test setup, test procedure, test results and problems faced. Finally this thesis takes a look at propeller design. A high efficiency propeller is designed for "C-SCOUT" using Eckhardt and Morgan’s propeller design method.

ACKNOWLEDGMENTS

First of all I would like to thank God for giving me an opportunity to spend two wonderful years studying for my masters degree in the beautiful city of St John's, Newfoundland.

I would then like to thank my supervisors Dr Neil Bose and Dr Christopher Williams for their guidance, support and help throughout my program. Thank you gentlemen – a student could not have asked for better supervisors.

I would also like to gratefully acknowledge the financial support provided to me through the NSERC Strategic Project "Offshore Environmental Engineering Using Autonomous Underwater Vehicles".

Thanks are also due to the National Research Council Canada – Institute for Marine Dynamics, St John's for the technical support that was provided during the in-water tests, for the office space that was provided to me and for the library facilities that I was given access to.

I would also like to thank my parents and sister for their constant support and help. Thanks are also due to my good friends in St John's – Tim and Kristina Churchill and the Swamidas's.

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LIST OF ABBREVIATIONS AND SYMBOLS

Symbol	Definition

Chapter 2

V	Vehicle Forward Speed (m/s)
P_E	Effective Power (W)
P_H	Hotel Load (W)
P_T	Total Power Required by the AUV (W)
E_T	Total Energy Provided by the Energy System (W-hr)
R	Vehicle Resistance (N)
C_D	Drag Coefficient of the AUV
ρ	Mass Density of Water (kg/m^3)
A	Cross Sectional Area of the Vehicle (m/s)
R	Range (km)
P_d	Delivered Power (W)
P_m	Motor Mechanical Power (W)
P_e	Motor Electrical Power (W)
P_h	Hotel Load (W)
P_t	Total Power Required by Vehicle at a Particular Forward Speed (W)
t	Total Time for a Mission at a Particular Forward Speed (hr)
E	Total Energy Required by the Vehicle at a Particular Forward Speed (W-hr)
t'	Total Time for a Mission at a Particular Forward Speed (Using the Current Energy System) (hr)
D	Max. Distance the Vehicle Can Travel at Sea (Using the Current Energy System) (km)
t"	Total Time for a Mission at a Particular Forward Speed (Using a High Energy Density System) (hr)
D"	Max. Distance the Vehicle Can Travel at Sea (Using a High Energy Density System) (km)
Ag-Zn	Silver Zinc Batteries
Li-ion	Lithium-ion Batteries
Li-ion P	Lithium-ion Polymer Batteries
$Al\text{-}O_2$	Aluminium Oxygen Semi-Fuel Cell
PEM	Proton Exchange Membrane Fuel Cell
SPE	Solid Polymer Electrolyte
Ni-Cd	Nickel Cadmium Battery
$Pb\text{-}H_2SO_4$	Lead Acid Batteries
CCDE	Closed Cycle Diesel Engine
NA	Data Not Available
ISE	International Submarine Engineering Ltd.
HL	Hotel Load

Chapter 3

V_M	Model Speed (m/s)
V_S	Ship (Full Scale Vehicle Speed) Speed (m/s)
R_{TM}	Total Resistance of Model (N)

R_{TS}	Total Resistance of Ship (N)
C_{TM}	Coefficient of Total Resistance of the Model
C_{TS}	Coefficient of Total Resistance of the Ship
C_{FM}	Coefficient of Frictional Resistance of the Model
C_{FS}	Coefficient of Frictional Resistance of the Ship
C_{RM}	Coefficient of Residuary Resistance of the Model
C_{RS}	Coefficient of Residuary Resistance of the Model
C_A	Correlation Allowance
C_{AA}	Air Resistance Coefficient
k	Form Factor
k_M	Model Form Factor
k_S	Ship Form Factor
ρ_S	Density of Sea Water (kg/m^3)
ρ_M	Density of Fresh Water (kg/m^3)
S_S, S	Wetted Surface Area of the Ship (m^2)
S_M	Wetted Surface Area of the Model (m^2)
Rn	Propeller Reynolds Number
R_{nco}	Local Reynolds Number at 0.75 Radius (Propeller)
Fn	Froude Number
T^0	Water Temperature (^{0}C)
J_0	Open Water Advance Coefficient
J	Behind Ship Advance Coefficient
T_M	Model Thrust (N)
T_S	Ship Thrust (N)
Q_M	Model Torque (N-m)
Q_S	Ship Torque (N-m)
n_M	Model Propeller Speed (rpm or rps)
n_S	Ship Propeller Speed (rpm or rps)
K_{TOM}	Open Water Thrust Coefficient for the Model
K_{QOM}	Open Water Torque Coefficient for the Model
K_{TBM}	Behind Ship Thrust Coefficient for the Model
K_{QBM}	Behind Ship Torque Coefficient for the Model
K_{FDM}	Tow Force Coefficient for the Model
K_{FD}	Tow Force Coefficient for the Model as a Function of J^2
K_R	Resistance Coefficient
K_{TS}	Thrust Coefficient for the Ship
K_{QS}	Torque Coefficient for the Ship
t	Thrust Deduction Fraction (ITTC 1978 Method)
t^*	Thrust Deduction Fraction (Load Varying Self-Propulsion Test Method)
C_{DM}	Drag Coefficient of the Model Propeller
C_{DS}	Drag Coefficient of the Ship Propeller

δC_D	Difference in the Model / Ship Propeller Drag Coefficients
δK_T	Correction Factor for K_T
δK_Q	Correction Factor for K_Q
P	Propeller Pitch (m)
D	Propeller Diameter (m)
q	Propeller Blade Chord Length (m)
Z	Number of Propeller Blades
m	Propeller Blade Thickness (m)
k_P	Propeller Blade Roughness
w_{TM}	Effective Wake Fraction for the Model using the Thrust Identity
w_T	Effective Wake Fraction using the Thrust Identity
w_Q	Effective Wake Fraction using the Torque Identity
w_{TS}	Effective Wake Fraction for the Ship using the Thrust Identity
C_{VS}	Coefficient of Viscous Resistance of the Ship
C_{VM}	Coefficient of Viscous Resistance of the Model
η_0	Open Water Efficiency of the Propeller (%)
η_D	Propulsive Efficiency of the Propeller (%)
η_H	Hull Efficiency (%)
η_R	Relative Rotative Efficiency (%)
F	Tow Force (N)
$F = F_D$	Tow Force corresponding to Ship Self-Propulsion point (N)
F= 0	Tow Force corresponding to Model Self-Propulsion point (N)
$F_{T=0}$	Total Resistance of the Model under Idling Propeller Conditions (N)
C_{FD}	Skin Friction Correction Coefficient
dF	Differential Tow Force (N)
dT	Differential Thrust (N)
T	Thrust (N)
Q	Torque (N-m)
T_M^*	Model Thrust at Ship Self-Propulsion Point (N)
T_S^*	Ship Thrust at Ship Self-Propulsion Point (N)
T_M^{**}	Model Thrust at Model Self-Propulsion Point (N)
m	Slope
c	Intercept on the y axis
λ	Scale Ratio (D_S / D_M)
J_M	Advance Ratio for the Model
J_S	Advance Ratio for the Ship
P_D	Delivered Power (W)
P_E	Effective Power (W)
ITTC	International Towing Tank Conference
x	Non-Dimensional Radius of the Propeller
BM	Behind Model
BS	Behind Ship

Chapter 4

F	Measured Tow Force (N)
$F_{T=0}$	Value of the Tow Force when Propeller Thrust is Zero (N)
T	Measured Propeller Thrust (N)
T_S	Propulsor Thrust at Self-Propulsion Point (N)
T_P	Propeller Thrust (N)
T_t	Total Thrust (N)
T_P	Propeller Thrust (N)
t^*	Thrust Deduction Fraction (Load Varying Self-Propulsion Test Method)
a	Augment in Resistance
V	Vehicle Speed (m/s)
ρ	Mass Density of Water (kg/m^3)
n	Propeller Speed (rpm or rps)
D	Propeller Diameter (m)
K_{tp}	Propeller Thrust Coefficient
K_{td}	Duct Thrust Coefficient
K_{tt}	Total (Propeller + Duct) Thrust Coefficient
K_q	Propeller Torque Coefficient
K_{fd}	Tow Force Coefficient
K_{ts}	Propeller Thrust Coefficient as a Function of J^2
J	Advance Coefficient
P_d	Delivered Power (W)
P_e	Effective Power (W)
h_d	Propulsive Efficiency of the Propeller (%)
C_d	Drag Coefficient
Re	Reynolds Number
Fr	Froude Number

Chapter 5

D	Propeller Dia. (m)
R	Propeller Radius (m)
r	Radial Distance (measured from the centreline of the propeller) (m)
r_h	Hub Radius (m)
X	Non - Dimensional Radius
X_h	Non - Dimensional Radius of the Propeller Hub
P	Section Pitch (m)
L	Section Chord Length (m)
Z	Number of Blades
ρ	Mass Density of Water (kg/m^3)
ρ_0	Mass Density of Manganese Bronze (kg/m^3)
g	Acceleration due to Gravity (m/s^2)
C_{th}	Thrust Loading Coefficient

C_T	Thrust Loading Coefficient (Calculated using the Speed of Advance)
C_{TS}	Thrust Loading Coefficient (Calculated using the Vehicle Speed)
C_{Ti}	Ideal Thrust Loading Coefficient (Calculated using the Speed of Advance)
C_{TSi}	Ideal Thrust Loading Coefficient (Calculated using the Vehicle Speed)
C_P	Power Coefficient (Calculated using the Speed of Advance)
C_{PS}	Power Coefficient (Calculated using the Vehicle Speed)
C_{Pi}	Ideal Power Coefficient (Calculated using the Speed of Advance)
C_{PSi}	Ideal Power Coefficient (Calculated using the Vehicle Speed)
P_d	Delivered Power (W)
T	Thrust (N)
Q	Torque (N-m)
η_0	Propeller Efficiency in Open Water (%)
η_b	Propeller Efficiency when fitted behind the Vehicle (%)
η_i	Ideal Propeller Efficiency (%)
n	Propeller Speed (rpm or rps)
V	Vehicle Speed (m/s)
V_A	Speed of Advance of the Propeller (m/s)
A_0	Propeller Disc Area (m^2)
A_E	Propeller Expanded Area (m^2)
A_E / A_0	Expanded Area Ratio
a	Rotational Inflow Factor
a'	Axial Inflow Factor
C_L	Coefficient of Lift
C_{Li}	Coefficient of Lift at the Ideal Angle of Attack
C_D	Coefficient of Drag
A.R	Aspect Ratio
α	Angle of Attack (^{0})
α_i	Ideal Angle of Attack (^{0})
β	Hydrodynamic Pitch Angle (uncorrected for Induced Velocities) (^{0})
β_i	Hydrodynamic Pitch Angle (corrected for Induced Velocities) (^{0})
Φ	Geometric Pitch Angle (^{0})
U / 2	Induced Velocity (m/s)
U_t / 2	Tangential Component of the Induced Velocity (m/s)
U_a / 2	Radial Component of the Induced Velocity (m/s)
V_r	Resultant Inflow Velocity to the Propeller (m/s)
ω	Angular Velocity of the Propeller (rad/s)
J	Advance Coefficient
λ	Advance Ratio (calculated using the Speed of Advance)
λ_S	Advance Ratio (calculated using the Vehicle Speed)
λ_i	Advance Ratio in an Ideal Fluid
Γ	Circulation
K	Goldstein Factor

dL	Differential Lift (N)
dD	Differential Drag (N)
ε	Ratio of the Differential Drag by the Differential Lift
dr	Differential Radial Distance (m)
dx	Differential Non-Dimensional Radius
dT_i	Differential Ideal Thrust (N)
dQ_i	Differential Ideal Torque (N-m)
dC_{TS}	Differential Thrust Loading Coefficient (Calculated using the Speed of Advance) (N)
dC_{TSi}	Differential Thrust Loading Coefficient (Calculated using the Vehicle Speed) (N)
w_x	Wake Fraction at Non-Dimensional Radius x
w_0	Effective Wake Fraction
m_x	Maximum Section Camber (m)
t_x	Maximum Section Thickness (m)
t_0	Blade Thickness at the Root Section (m)
t_{Tip}	Blade Thickness at the Tip (m)
m_x / L	Camber Ratio
t_x / L	Thickness Ratio
t_x / D	Radial Distribution of Maximum Thickness
σ_x	Section Cavitation Number
P	Pressure due to the Head of Water (N/m^2)
P_a	Atmospheric Pressure (N/m^2)
h '	Depth of Submergence (m)
tdc	Top Dead Centre Position
K_1	Camber Correction Factor
K_2	Camber Correction Factor
K_3	Correction Factor (dependent on the shape of the Meanline)
K_4	Correction Factor (dependent on the shape of the Meanline)
α_1	Pitch Correction Factor (rad)
α_2	Pitch Correction Factor (rad)
α_b	Bound Vortex Contribution to the Pitch Correction Factor α_2 (rad)
α_f	Free Vortex Contribution to the Pitch Correction Factor α_2 (rad)
α_i'	Difference between β_i & β (rad)
α_0	Angle of Zero Lift of Meanline (rad)
μ	Angular Position of the Blade (^{0})
G	Non-Dimensional Circulation per Blade
θ	Lerbs factor (used to determine h)
h	Lerbs factor (used to determine α_f)
K_0	Empirical Factor used in Keller's Formula
X_U	Abscissa of a point on the Blade Section Upper Surface (m)
Y_U	Ordinate of a point on the Blade Section Upper Surface (m)
X_L	Abscissa of a point on the Blade Section Lower Surface (m)

Y_L	Ordinate of a point on the Blade Section Lower Surface (m)
Y_t	Thickness Distribution Ordinate (m)
Y_c	Meanline Ordinate (m)
$\tan \theta$	Meanline Slope
X'	Chordwise Position
M_T	Moment due to Thrust (N-m)
M_Q	Moment due to Torque (N-m)
x_0	Non-Dimensional Radius of the Section being analyzed
I_{X0}	Moment of Inertia of the Root Section about an axis that is parallel to the nose -tail line & passes through the centroid of the section (m^4)
I_{Y0}	Moment of Inertia of the Root Section about an axis that is perpendicular to the nose - tail line & passes through the centroid of the section (m^4)
M_{X0}	Bending Moment about an axis that is parallel to the nose -tail line & passes through the centroid of the section (N-m)
M_{Y0}	Bending Moment about an axis that is perpendicular to the nose -tail line & passes through the centroid of the section (N-m)
m	Mass of one blade (kg)
A_X	Blade Sectional Area (m^2)
$A_{X=0.2}$	Blade Sectional Area at the Non-Dimensional Radius X = 0.2 (m^2)
X_C	Longitudinal C.G of a Blade (m)
F_C	Centrifugal Force (N)
$\sigma_{L.E}$	Total Stress at the Leading Edge of the Root Section (N/m^2)
$\sigma_{T.E}$	Total Stress at the Trailing Edge of the Root Section (N/m^2)
$\sigma_{S.B}$	Total Stress at the point of maximum thickness on the Suction Back of the Root Sec. (N/m^2)
X_1	Distance from the Neutral Axis to the Leading Edge (along the X axis) (m)
X_2	Distance from the Neutral Axis to the Trailing Edge (along the X axis) (m)
X_3	Distance from the Neutral Axis to the point of maximum thickness on the Suction Back (m)
Y_1	Distance from the Neutral Axis to the Leading Edge (along the Y axis) (m)
Y_2	Distance from the Neutral Axis to the Trailing Edge (along the Y axis) (m)
Y_3	Distance from the Neutral Axis to the point of maximum thickness on the Suction Back (along the Y axis) (m)
δ_X	Blade Tip Deflection about the X0 axis (mm)
δ_Y	Blade Tip Deflection about the Y0 axis (mm)
X0	An axis that is parallel to the nose -tail line & passes through the section centroid
Y0	An axis that is perpendicular to the nose -tail line & passes through the section centroid
L_0	Blade Length (m)
E	Modulus of Elasticity of the Blade Material (N/m^2)
S.M	Simpson's Multiplier

LIST OF APPENDICES

Chapter 1

Introduction

1.1 Introduction

Out of all the planets in the solar system, ours is the only "water" planet. Approximately 70 percent of our planet's surface is covered with water. Yet the oceans are the most alien part of our earth. The innermost depths of the oceans are still shrouded in mystery, a dangerous and alien place where humans fear to go. It is a world of silence, darkness and immense pressure. It is also a world that contains a rich diversity of ecological systems; castles of coral, forests of sponges, fleets of whales, eels, seahorses and other marine life in immeasurable variety. The oceans are also a relatively untapped resource base for minerals and energy. As land based resources deplete, mankind will undoubtedly turn to the oceans to supply their needs. In fact the process has already begun with the setting up of offshore oil and gas platforms. In places like the Grand Banks of Newfoundland, overfishing by foreign vessels has dramatically depleted fish stocks.

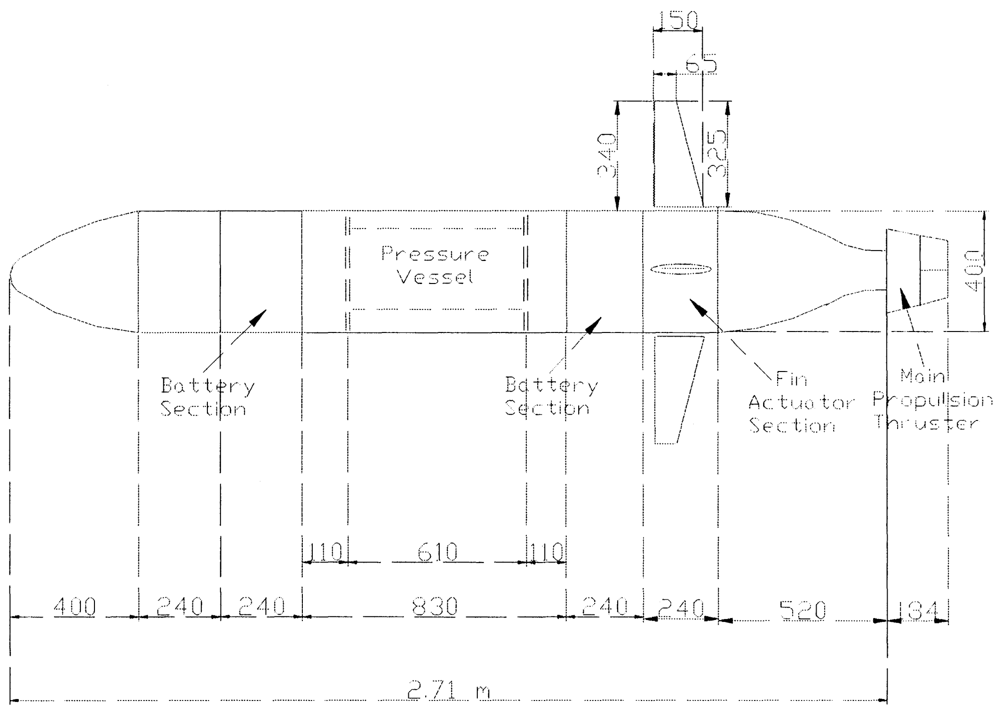
As ocean exploration heads into deeper waters, Autonomous Underwater Vehicles (AUVs) are increasingly emerging as a commercially viable technology. An AUV is a self-propelled underwater robot capable of carrying out pre-programmed tasks without human intervention. During the course of its mission, an AUV can be hundreds of miles away from any support vessel and completely out of contact with any external human input for hours or days on end. These missions include oceanographic sampling, environmental monitoring, iceberg profiling, pipeline tracking, mine detection, searching for lost wrecks etc [Ref. 1-4].

For decades AUVs were either expensive military systems or very low cost academic research tools. They lacked the necessary brain (high power computers) and the brawn (high energy density systems). However, with recent advances made in the field of computers, communication and energy systems, the necessary brains and brawn have finally begun to arrive. The functionality and the endurance of AUVs have greatly increased, thus making them commercially attractive tools for deep sea exploration.

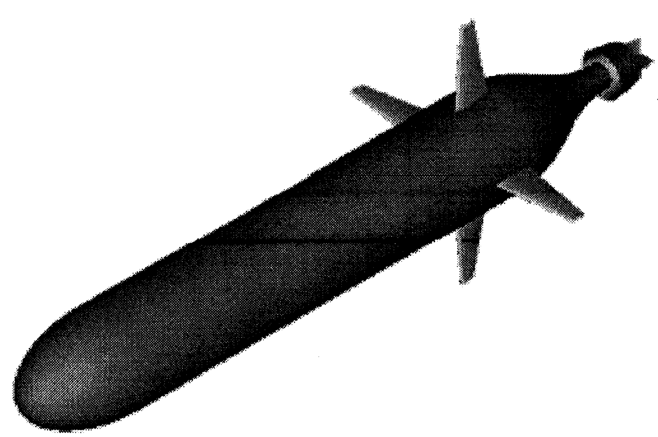
According to an analysis conducted by C&C Technologies Inc. [Ref. 1-1], in depths greater than about 400 meters, an AUV is more cost effective than a deep towed system. In fact the deeper the water and the larger the number of line turns, the greater the AUV cost savings. For rectangular survey areas, an AUV is 10 percent less expensive and requires only 50 percent of the time needed for a deep tow survey. For linear surveys such as pipelines and cable routes, an AUV is 15 percent less expensive and requires only 40 percent of the time needed for a deep tow survey.

1.2 The AUV "C-SCOUT"

The Autonomous Underwater Vehicle "C-SCOUT" is being jointly developed by the National Research Council Canada – Institute for Marine Dynamics (NRC-IMD) and the Ocean Engineering Research Center at Memorial University of Newfoundland (MUN-OERC) with the support of several Canadian companies and universities. The acronym "C-SCOUT" stands for Canadian Self Contained Off-the-shelf Underwater Testbed. "C-SCOUT" is being developed as part of a larger NSERC Strategic Project entitled "Offshore Environmental Engineering Using Autonomous Underwater Vehicles". The project is concerned with the development of AUVs for environmental monitoring missions, in order to assess the impact of discharges from offshore oil and gas operations. In its current configuration , the vehicle is torpedo shaped, with an overall length of 2.7 metres and a diameter of 0.40 metres. The vehicle is streamlined in shape with an ellipsoidal nose, cylindrical mid-body and a cubic spline tail [Figs. 1.1 and 1.2].

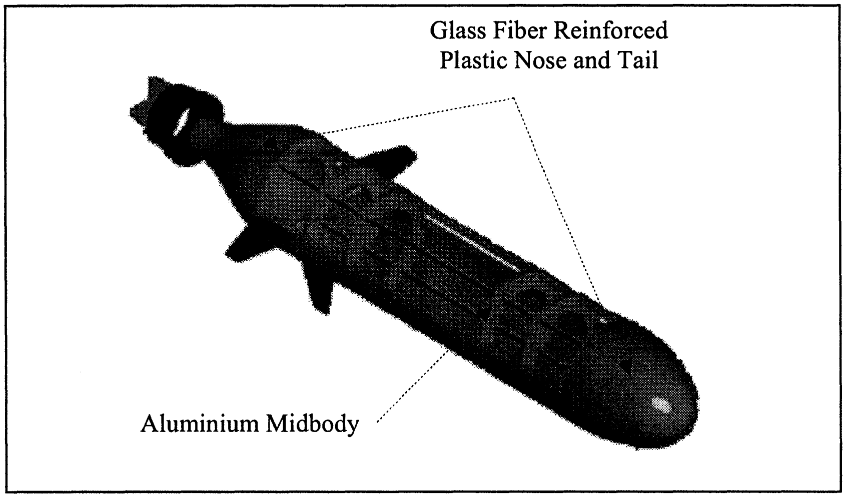


The "C-SCOUT" Vehicle (Baseline Version): Fig. 1.1
(all dimensions in mm, unless specified otherwise)

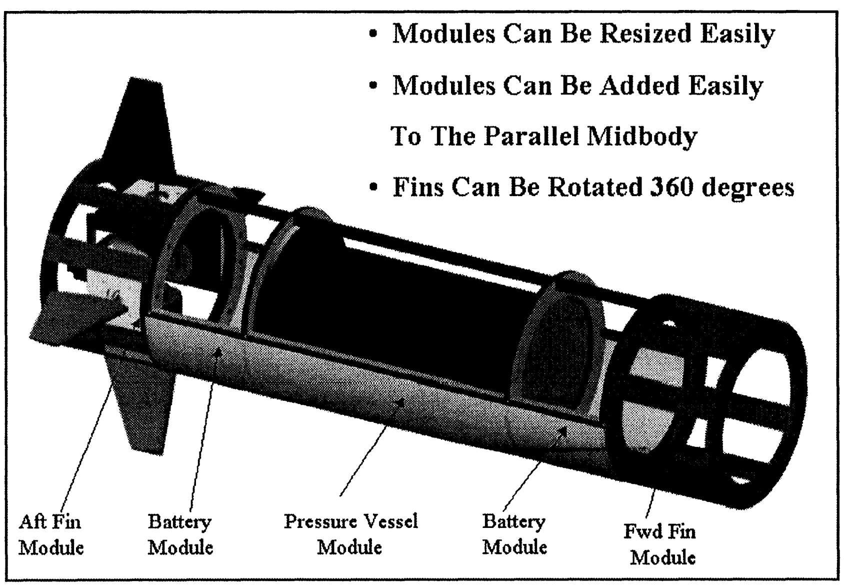


The "C-SCOUT" Vehicle (Baseline Version): Fig. 1.2

The vehicle has been designed and constructed using modular sections, so that it can be easily adapted to a wide variety of configurations, depending on its mission. These modules include the nose module, the forward and aft battery modules, the pressure vessel module, the aft fin module and the tail module [Figs. 1.3 and 1.4].

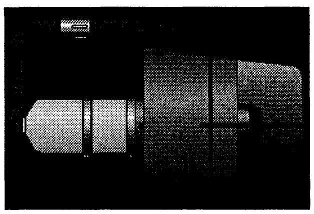


A Look Inside the Vehicle: Fig 1.3



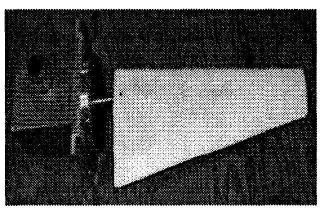
Modules: Fig. 1.4

The vehicle is propelled using a 0.75 kW (1 HP) ducted thruster (Tecnadyne Model 1020 Thruster) [Fig 1.5]. Four fins (NACA-0012 sections) have been provided for maneuvering the vehicle in the horizontal and vertical planes [Fig 1.5]. The vehicle electronics are housed in a sealed pressure vessel [Fig. 1.6] and the electronics are based on a Distributed Embedded Modular Architecture (DEMA) [Ref. 1-3].



Main Propulsion Thruster

- Tecnadyne Thruster (Model 1020)

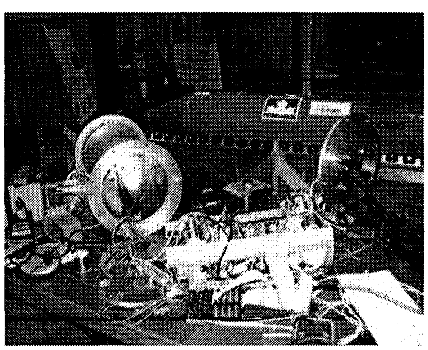


Fin and Fin Actuator

- NACA-0012 Sections
- Matsushita DC Motor (Model GMX-6MP012A)

Main Propulsion Thruster: Fig. 1.5

Vehicle Electronics



- Distributed Embedded Modular Architecture (DEMA)
- Consists of a Series of Independent Microcontroller Modules
- Modules Communicate With Each Other Over an Internal Serial Bus

Vehicle Electronics: Fig. 1.6

1.3 Thesis Objectives

The primary objectives of this thesis work were to:

- Do a comparative study of available energy systems for AUVs and subsequently recommend a few high energy density systems for "C-SCOUT".
- Determine the optimum vehicle speed for minimum energy consumption and to determine the endurance limits of "C-SCOUT" using the current energy system on board the vehicle and also using a high energy density system that could possibly be used on the vehicle.
- Evaluate the performance of the existing propulsion system of "C-SCOUT" by conducting resistance and propulsion tests in a towing tank.
- Design a higher efficiency propeller for "C-SCOUT".

1.4 Thesis Outline

Chapter 2 (AUV Energy Systems and Endurance): This chapter starts with a survey of a number of current AUVs, in order to determine what their endurance limits are, what their operating speeds are and what energy systems they have on board. Subsequently this chapter includes a comparative study of a number of energy systems currently being used by AUVs. Thereafter the optimum operating speed of "C-SCOUT" for minimum energy consumption and the endurance limits of "C-SCOUT" are calculated.

Chapter 3 (Powering Prediction): In this chapter, two powering prediction methods (the "ITTC 1978" method and the "Load-Varying Self-Propulsion Test" method) are compared, in order to estimate the powering performance of "C-SCOUT". This chapter explains the theory behind the analysis methods used in Chapter 4.

Chapter 4 (Resistance and Propulsion Tests): This chapter gives an account of the resistance and propulsion tests that were carried out on "C-SCOUT". Areas discussed include test setup, test procedure, test results and problems faced during the course of the tests.

Chapter 5 (Design of a High Efficiency Propeller for "C-SCOUT"): The chapter starts with an explanation as to why "C-SCOUT" needs a high efficiency propeller. Subsequently the high efficiency propeller envisioned for "C-SCOUT" is described and the factors that make this propeller highly efficient, are explained. Thereafter the propeller design process is explained and the calculations are worked out.

Chapter 6 (Conclusions and Recommendations): This chapter briefly states the conclusions drawn by the author at the end of his thesis work and the recommendations he would like to make.

Chapter 2

AUV Energy Systems and Endurance

Introduction

The endurance of an AUV is principally governed by the amount of energy stored on board the vehicle. Endurance refers to the range (distance) or the time that an AUV can travel at sea, while working on a mission, given a fixed amount of energy stored on board the vehicle. Over the last few decades, one of the biggest barriers to commercial development of AUVs has been the lack of high energy density, atmosphere independent energy systems. The Lead Acid battery, which was developed at the turn of the last century, has been the dominant subsea energy source for decades. Reasons for its continued use include low cost, ruggedness, ease of use and the fact that Lead Acid technology is well understood. However, the biggest drawback of this battery is its low energy density. Consequently vehicles using Lead Acid batteries have relatively low endurance.

Recent advances in energy systems have led to the development of new, high energy density technologies such as fuel cells, Li-ion polymer batteries etc, all of which show considerable promise. Over the next decade or so, as these high energy density technologies are developed and perfected, the endurance of AUVs is expected to greatly increase, thus making them commercially viable tools for offshore exploration. This chapter explores the present day AUV energy systems market and then recommends a few high energy density systems that could be possibly used on "C-SCOUT". Subsequently the optimum vehicle speed (from energy considerations) and the vehicle endurance limits are worked out.

2.1 AUV Comparison

The author conducted a survey of a few present day AUVs, in order to get an idea about the most popular energy systems being used currently. The results are tabulated in Table 2.1.

Table 2.1

AUV Comparison

Key

Ni-Cd : Nickel Cadmium

Al-O$_2$: Aluminium Oxygen

Ag-Zn : Sliver Zinc

Li-ion : Lithium ion

Pb-H$_2$SO$_4$: Lead Acid

PEM : Proton Exchange Membrane

CCDE : Closed Cycle Diesel Engine

N.A : (Data) Not Available

S. No.	**AUV**	**Speed** (knots)	**Endurance** (hrs or km)	**Energy Storage / Supply Device**
1	**ARCS** ISE Ltd, Canada	5.5 knots (maximum) 4 knots (cruising)	36 km – 1 Ni-Cd 72 km – 2 Ni-Cd 235 km – Al-O$_2$	1 or 2 Ni-Cd battery units (10 kW-hr each) Al-O$_2$ semi-fuel cells (100 kW-hr)
2	**AURORA** ISE Ltd, Canada	3.5 knots (maximum) 1.5 – 2 knots (cruising)	750 km	Li-ion battery units
3	**THESEUS** ISE Ltd, Canada	4 knots (cruising)	780 km	Ag-Zn battery units (360 kW-hr)
4	**HUGIN 3000** Kongsberg Simrad AS, Norway	4 knots (cruising)	6-8 hrs – Ni-Cd 36 hrs – Al-O$_2$	Ni-Cd battery units (3 kW-hr) Al-O$_2$ semi-fuel cells (18 kW-hr)

S. No.	AUV	Speed (knots)	Endurance (hrs or km)	Energy Storage / Supply Device
5	**DeepC** Federal Ministry of Education and Research, Germany	6 knots (maximum) 0.5 – 4 knots (cruising)	400 km 60 hrs	PEM fuel cells
6	**HS-AUTOSUB** Halliburton Subsea, UK	2-4 knots (cruising)	170 km 24 hrs	Li-ion battery units
7	**AUTOSUB** Southampton Oceanography Centre, UK	2-4 knots (cruising)	500 km 144 hrs	Alkaline primary battery units
8	**SEA ORACLE** Bluefin Robotics Corporation, US	4 knots (maximum) 3 knots (cruising)	20 hrs	Ag-Zn battery units
9	**OKPO 6000** Daewoo Heavy Industries, Korea	3 knots (maximum)	10 hrs	Ag-Zn battery units
10	**MARIDAN 600** Maridan ApS Denmark	4 knots (maximum) 3 knots (cruising)	36 km 10 hrs	Pb-H_2SO_4 battery units
11	**XP-21** Applied Remote Technology Inc, (Raytheon), US	5 knots (maximum)	N.A	Pb-H_2SO_4 battery units Ag-Zn battery units Al-O_2 semi-fuel cells
12	**OCEAN VOYAGER II** Florida Atlantic University, US	5 knots (maximum) 3 knots (cruising)	50-100 km	Pb-H_2SO_4 battery units Ag-Zn battery units

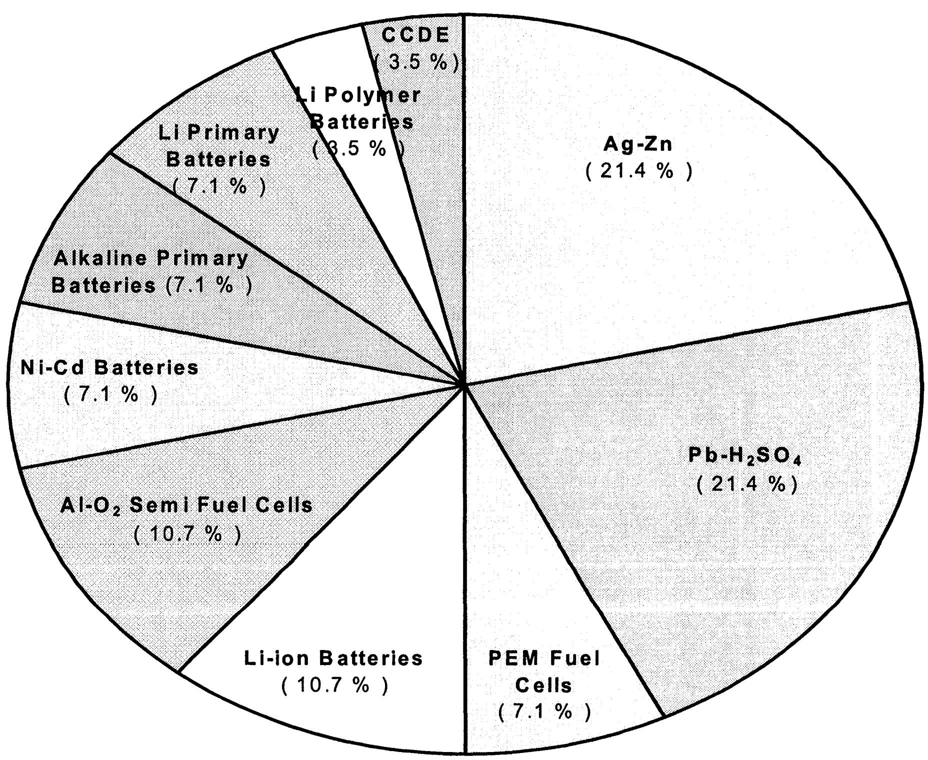
13	**CARIBOU** (ODYSSEY III) MIT, US	4 knots (maximum) 3 knots (cruising)	20 hrs	Li-ion Polymer battery units
14	**XANTHOS** (ODYSSEY IIc) MIT, US	1 – 3 knots (cruising)	22 km 4 hrs	Ag-Zn battery units (1.2 kW-hr)
15	**CETUS** Lockheed Martin / MIT, US	5 knots (maximum) 1.5 – 2.5 knots (cruising)	20 - 40 km	Pb-H_2SO_4 battery units
16	**ABE** WHOI, US	2 knots (maximum) 1 knot (cruising)	50 km	Pb-H_2SO_4 battery units (1 kW-hr) Alkaline primary battery units (2.2 kW-hr) Li primary battery units (10 kW-hr)
17	**REMUS** WHOI, US	5 knots (maximum) 3 knot (cruising)	46.3 km	Pb-H_2SO_4 battery units (400 W-hr)
18	**URASHIMA** JAMSTEC, Japan	4 knots (maximum) 3 knot (cruising)	80 km 300 km, 54 hrs	Li-ion battery units PEM fuel cells
19	**AQUA EXPLORER 2** KDD Labs, Japan	2 knots (maximum) 1 knot (cruising)	24 hrs	Li primary battery units (3870 W-hr)
20	**R-One Robot** URA Labs / Mitsui Eng. and Shipbuilding Co, Japan	3.6 knots (maximum) 2 knot (cruising)	120 km 25 hrs	CCDE (60 kW-hr)

Based on the above survey, the most popular energy systems currently on board AUVs are:

6) Silver Zinc batteries
7) Lead Acid batteries
8) Lithium-ion batteries
9) Aluminium-Oxygen semi-fuel cells
10) PEM fuel cells

1) Nickel-Cadmium batteries
2) Lithium Primary batteries
3) Alkaline Primary batteries
4) Lithium Polymer batteries
5) Alkaline Primary batteries

The percentage breakup is shown in Fig. 2.1. The percentage refers to the percentage of vehicles using these systems. The pie chart below is by no means exhaustive – it merely reflects the trend being followed by current-day AUVs.



Percentage Distribution of AUV Energy Systems: Fig. 2.1

2.2 Comparison of AUV Energy Systems

A study was conducted to determine the energy densities and the advantages / disadvantages of energy systems currently being employed by AUVs. The comparative energy densities are tabulated in Table 2.2. In this table, "primary cells" refer to those cells which are not rechargeable and "secondary cells" refer to those cells that are rechargeable.

Table 2.2

Energy Density Comparison of AUV Energy Systems

S. No.	Classification	Energy Storage / Supply Device	Gravimetric Energy Density (W-hr / kg)
1	Secondary cell	Lead Acid	30 to 45
2	Secondary cell	Nickel-Cadmium	30 to 55
3	Primary cell	Alkaline AAA AA C D	100 120 102 110
4	Secondary cell	Silver-Zinc	110 to 150
5	Secondary cell	Lithium-ion	130 to 170
6	Secondary cell	Lithium-ion Polymer	130 to 170
7	Thermal engine	Closed Cycle Diesel engine	125
8	Thermal engine	Stirling engine	175
9	Semi-fuel cell	Aluminium-Oxygen	260 to 280
10	Primary cell	Lithium-Thionyl Chloride	450
11	Fuel cell	PEM	300 to 1030

2.2.1 Lead Acid Batteries

Lead Acid batteries are the primary source of power in conventional Diesel electric submarines and manned submersibles. Over the past few decades, they were the primary source of power for AUVs. A Lead Acid battery basically consists of three active components; a positive electrode (lead dioxide), a negative electrode (pure lead) and an electrolyte (dilute sulphuric acid). The latest class of Lead Acid batteries (maintenance free batteries) use a gel instead of liquid sulphuric acid as the electrolyte.

The advantages of Lead Acid batteries include:

- Low cost
- Rugged / Durable
- Easy to maintain
- Well understood

The disadvantages of Lead Acid batteries include:

- Low energy density
- Very heavy and require a large volume
- High discharge rates
- Moderate charge / discharge cycle life (≅ 250 cycles)
- Loss of capacity at low temperatures
- Give off hydrogen gas when charging / discharging
- Lead (a heavy metal) is toxic and poses disposal problems
- Long charging time (≅ 8 hours)

2.2.2 Nickel-Cadmium Batteries

Nickel-Cadmium batteries have marginally higher energy densities than Lead Acid batteries. A typical Nickel-Cadmium battery consists of a nickel hydroxide cathode and a cadmium anode, both of which are immersed in potassium hydroxide electrolyte.

The advantages of Nickel-Cadmium batteries include:

- Greater energy density than Lead Acid batteries
- Good discharge performance (a more uniform output with respect to Lead Acid battery)
- Cold temperatures do not degrade their performance significantly
- Evolves no gases during charging / discharging
- High charge / discharge cycle life (≅ 800 cycles)
- Rugged / Durable

The disadvantages of Nickel-Cadmium batteries include:

- More expensive than Lead Acid batteries
- Displays a "Memory Effect" *
- Gives little warning before being completely discharged and becoming dead.
- Cadmium (a heavy metal) is toxic and poses disposal problems
- Long charging time (≅ 8 hours)

* Memory Effect: When a battery has been recharged to a particular level several times consecutively, then the battery remembers this level and whenever it hits this level the next time, then it acts as if it is discharged, despite the fact that much of its capacity remains.

2.2.3 Silver-Zinc Batteries

Silver-Zinc batteries have energy densities that are approximately three to four times greater than Lead Acid / Nickel-Cadmium batteries. Due to their high energy density and availability, these batteries are widely used in current day AUVs. A typical Silver-Zinc battery consists of a porous silver oxide cathode and a zinc anode, both of which are immersed in potassium hydroxide electrolyte.

The advantages of Silver-Zinc batteries include:

- High energy density (110 to 150 W-hr / kg)
- Good discharge performance (a more uniform output with respect to Lead Acid battery)
- Cold temperatures do not degrade their performance significantly

The disadvantages of Silver-Zinc batteries include:

- Expensive batteries
- Low charge / discharge cycle life (≅ 40 to 50 cycles)
- Substantial drop in energy density near the end of its cycle life
- Low wet (in-water) standard life (2 to 18 months)
- Difficult to determine the state of charge
- Long recharging process. Need precise charging. Batteries need to be charged individually
- Outgassing prevents charging in an enclosed space

2.2.4 Lithium-ion Batteries

Lithium-ion batteries were introduced into the market in the early 1990's, principally for use in the communication and computer markets. However their high energy densities made them an ideal choice for use on board AUVs. These batteries essentially consist of a lithium cobalt oxide or lithium manganese oxide cathode and an anode constructed by dissolving lithium as ions into carbon. Lithium salt is used as the electrolyte. Some of the companies currently manufacturing these batteries include Yardney Technical Products Inc. (USA), Ultralife (USA), Valence Technology Inc. (USA) and Saft (France). The US Navy (Naval Surface Warfare Center, MD) in collaboration with Honeywell Power Sources Center (USA), is researching Lithium-ion batteries as a possible replacement for Silver Zinc batteries, for powering the next generation of naval underwater vehicles.

The advantages of Lithium-ion batteries include:

- High energy density (130 to 170 W-hr / kg)
- High charge / discharge cycle life ($\cong$ 1000 cycles)
- Require no maintenance
- Long battery life (6 to 8 years)
- Hermetically sealed – no outgassing
- Excellent discharge performance
- Can operate over a range of temperatures.
- No "Memory Effect"

The disadvantages of Lithium-ion batteries include:

- Expensive batteries

- May demonstrate a drop in performance if the charging voltages are not controlled carefully. Needs charge control circuitry.

2.2.5 Lithium-ion Polymer Batteries

Lithium-ion Polymer batteries are an improvement over the conventional Lithium-ion batteries. Both batteries have a similar electrochemistry, however in the Lithium-ion Polymer batteries, the electrolyte is immobilized (plasticized or solidified), creating a matrix of ion conductive polymers. Since the electrolyte is in a solid form, therefore the cell no longer requires a metal / plastic casing to hold the liquid electrolyte. The cells are instead packed in thin foils and this helps save weight and greatly enhances design flexibility. Individual cells can be stacked together to create a battery of optimum dimensions that can make best use of the internal volume of an AUV. Some of the companies manufacturing these batteries include Yardney Technical Products Inc. (USA), Ultralife (USA), Valence Technology Inc. (USA) and Saft (France). These batteries are principally used in the communication and computer markets. They are also used in the electric car market. The high energy density of these batteries, coupled with their design flexibility, make them an ideal choice for powering AUVs.

The advantages of Lithium-ion Polymer batteries include:

- High energy density (130 to 170 W-hr / kg)
- Packed in foils. No casing required.
- Design flexibility – individual cells can be stacked together to form battery packs of any size or shape.
- High charge / discharge cycle life (≅ 600 cycles)
- Requires no maintenance

- Long battery life (5 years)
- No outgassing
- Solid electrolyte used – eliminates the possibility of leakages. Can be used under ambient pressure conditions (does not need to be installed in a pressure vessel)
- Excellent discharge performance
- Can operate over a range of temperatures.
- No "Memory Effect"

The disadvantages of Lithium-ion Polymer batteries include:

- Expensive batteries
- May demonstrate a drop in performance if the charging voltages are not controlled carefully. Needs charge control circuitry.

2.2.6 Fuel Cells

Fuel cells convert the chemical energy of a fuel and an oxidant into a low voltage direct current (DC), via electrochemical reactions similar to those associated with conventional secondary batteries. However unlike the latter devices, the fuel cell does not consume materials that form an integral part of its structure. Hence they can operate for as long as they are fed with a suitable fuel and an oxidant and the reaction products are removed. In theory any fuel and oxidant can be used in a fuel cell, but for almost all practical purposes the Hydrogen-Oxygen fuel cell is the only viable option for underwater applications. A typical Hydrogen-Oxygen fuel cell consists of two porous electrodes separated by an ion-conducting electrolyte. The fuel electrode is the anode, the oxidant electrode is the cathode. The two electrodes are connected to an external load. Hydrogen and oxygen chemically react with the electrodes and a current flow is established [Ref 2-1].

Fuel cells are generally classified by the electrolyte they use. There are five basic types of fuel cells:

- Alkaline
- Phosphoric Acid
- Molten Carbonate
- Monolithic Solid Oxide
- Proton Exchange Membrane (PEM) or Solid Polymer Electrolyte (SPE)

Molten Carbonate (operating temperature $\cong$ 650 ^{0}C), Monolithic Solid Oxide ($\cong$ 1000 ^{0}C) and Phosphoric Acid ($\cong$ 180 ^{0}C) fuel cells operate at very high temperatures and are hence unsuitable for use on underwater vehicles. Alkaline and PEM fuel cells have attracted the most attention for underwater applications. However, Alkaline cells cannot tolerate carbon dioxide contamination (a by-product of the reforming process used for extracting hydrogen from hydrogen rich compounds) and hence they need pure hydrogen fuel. PEM fuel cells on the other hand are more tolerant to carbon dioxide contamination. They can therefore make use of hydrogen obtained by reforming hydrogen-rich compounds. Also PEM fuel cells (≅ 80 ^{0}C) operate at a lower temperature than Alkaline fuel cells (≅ 120 ^{0}C). These two factors have given PEM fuel cells the edge over Alkaline fuel cells.

Fuel cell development work is in progress in many laboratories in Canada, USA, Japan and other countries around the world. Siemens AG (Germany) is currently building four 120 kW fuel cell power modules for German / Italian naval submarines. The Advanced Research Projects Agency (ARPA) of the Department of Defense (USA) is sponsoring a program to expand the capability of a UUV, by replacing the current Silver-Zinc batteries on board the vehicle with a 10 kW PEM fuel cell developed by International Fuel Cells Corporation (USA). The Japan Marine Science and Technology Center (JAMSTEC) has developed a prototype Solid Polymer Electrolyte (SPE) fuel cell for use on an AUV; trial results indicate that the fuel cell has an energy density of 1030 W-hr / kg [Ref 2-2].

Energia Space Corporation, Moscow (Russia) has developed Hydrogen-Oxygen fuel cells for its space shuttle program. The company offers a modified version of its fuel cell for undersea applications. The specifications of this fuel cell are an energy density of 300 W-hr / kg, capacity of 30 kW-hr and a cell life of 10,000 hours. Ballard Power Systems (Canada) is currently marketing a portable PEM fuel cell module for surface applications (Nexa Power Module). The unit produces 1200 W of power and has a cell life of 1500 hours. The unit could possibly be modified for subsea use.

The advantages of fuel cells include:

- Much higher energy densities than conventional batteries (300 to 1030 W-hr / kg): Present battery systems have limited energy densities. To obtain more energy using conventional batteries, additional batteries need to be added. Additional batteries mean an increase in energy with a corresponding increase in weight of the battery system. Consequently the energy density of the battery system remains more or less constant irrespective of the number of batteries added. Fuel cells on the other hand need only additional hydrogen and oxygen to supply additional energy. The size / weight of the fuel cell unit remains constant. Therefore as additional hydrogen / oxygen is added to the system, the energy density of the system increases.
- Silent and vibration-free operation – no mechanical / combustion processes involved
- Clean exhaust (no NO_X and other hydrocarbon emissions).
- More efficient (50 to 70 %) than Diesel Engines / Turbines – no mechanical losses
- Can maintain a high efficiency over a range of loads (30 to 100 % loads)
- Capable of instant start up: As long as hydrogen and oxygen are present, fuel cells can produce power whether they are fully "warmed-up" or not.
- Responds instantly to electrical load changes – the system does not need to wait for controllers or pumps to change settings in response to load changes.
- Rapid recharging possible: The fuel cell can be replenished by simply filling up the hydrogen and oxygen tanks and emptying the produced-water tank.

The disadvantages of fuel cells include:

- Costly equipment: A PEM fuel cell stack is extremely expensive (≅ US \$ 6000 per kW) due to the high cost membranes and gas-diffusion electrodes present in the cells.
- Hydrogen storage issues: Oxygen is normally stored on board either in gaseous form under pressure or in liquid form under cryogenic conditions. Onboard storage of hydrogen however poses a problem. While hydrogen (boiling point: 15^0C and density: 0.085 kg / m^3) is very light in weight, it is also quite voluminous when stored in gaseous form. Therefore storing it in sufficient quantities is difficult in a volume-limited submersible. Another option is cryogenic storage. However this requires a complex control system to ensure safety. The best solution to the problem at present seems to be onboard extraction of hydrogen by reforming hydrogen-rich compounds such as methanol, ammonia, hydrazine etc. The issue still remains unresolved and work is in progress at different laboratories around the world to overcome this problem.

2.2.7 Semi-Fuel Cells

In a semi-fuel cell, the oxidant (cathode) is pumped into the cell, but the fuel is supplied in solid form within the cell as a metal anode. Electrical power is produced by the reaction between the metal anode and the oxidant. The anode is usually made from lithium or aluminium and is consumed during the energy conversion process [Ref. 2-1]. The energy densities of the semi-fuel cells presently available in the market, lie in the range of 260 to 280 W-hr / kg. Aluminium-Oxygen semi-fuel cells are currently being used by the HUGIN 3000 and XP-21 AUVs. International Submarine Engineering Ltd. (Canada) tested a 2.3 kW, 100 kW-hr Aluminium-Oxygen semi-fuel cell developed by Fuel Cell Technologies (Canada), on their ARCS AUV. This semi-fuel cell had an energy density of 260 W-hr / kg. It used 36 kg of aluminium anodes, 30 kg of oxygen (stored in gaseous form at 4000 psi) and needed two hours to refuel.

The advantages of a semi-fuel cell over a fuel cell include:

- The semi-fuel cell needs only approximately half the quantity of oxygen that a fuel cell needs in order to produce one unit of electricity. This is because its cell voltage is approximately double that of a fuel cell.
- Since the "fuel" in a semi-fuel cell is the aluminium or lithium anode, therefore Hydrogen storage or its on-board generation is not required. The associated safety hazards posed by hydrogen are also eliminated.
- Semi-fuel cells are cheaper than fuel cells. This is because the high cost membranes and gas-diffusion electrodes present in fuel cells are not required in semi-fuel cells.
- Semi-fuel cells operate at much lower temperatures ($\cong 25\ ^0C$) as compared to fuel cells ($\cong 80\ ^0C$).

The disadvantages of a semi-fuel cell include:

- Waste disposal problem – the expended aluminium / lithium anodes are difficult to dispose off.

2.2.8 Closed Cycle Diesel Engines (CCDE)

In a CCDE, a conventional Diesel Engine operates in a closed cycle. The exhaust outlet of the engine is connected to the inlet side of the engine through an exhaust gas processing unit. In this unit, the exhaust gas is cooled down and the combustion products are removed. The combustion products are stored in onboard tanks and are discharged on return to base. The cleaned and cooled exhaust gas is re-circulated to the engine as a synthetic atmosphere, after mixing it with replenishment oxygen. Since the CCDE operates in a closed cycle, it is therefore independent of the external atmosphere and consequently the vehicle can dive to any depth without altering engine performance.

The URA labs at the University of Tokyo and Mitsui Engineering and Shipbuilding Corporation Ltd. (Japan) have jointly developed an AUV (R-One Robot) that uses a CCDE. This CCDE has an output of 5 kW and a capacity of 60 kW-hr. At a speed of 2 knots, this gives the vehicle an endurance limit of 25 hours / 120 kms. The R-One Robot completed sea trials between the years 1996-98. In October 2000 the vehicle was sent on a full mission and it explored the underwater volcano Teisi Knoll off the coast of Japan. Test results from this mission indicated that the CCDE functioned quite reliably. In June 1989, the German company Bruker launched an experimental 50 tonne manned submersible (Seahorse KD) that was equipped with a CCDE. This engine could operate in dual mode; when the vehicle surfaced it would operate in an open cycle mode (breathing air) and when the vehicle dived it would operate in a closed cycle mode (breathing a synthetic atmosphere).

CCDEs are ideally suited for AUVs which embark on long range missions, since the additional oxygen and fuel required for these missions give the CCDE a higher energy density than conventional batteries. For short range missions, the energy density of CCDE is comparable to conventional batteries (since the engine, the exhaust gas processing unit and other associated equipment are quite heavy and voluminous).

2.2.9 Stirling Engines

The Stirling Engine is an external combustion engine that operates on a closed regenerative thermodynamic cycle. It differs in three fundamental ways from a conventional internal combustion engine:

- The working gas (helium) operates in a closed circuit between two cylinders
- The pistons are double acting
- A regenerator is provided for extracting / adding heat to the working gas.

A simple Stirling Engine consists of two pistons enclosed in two cylinders. The space above each piston is termed the "hot" space and the space below each piston is termed the "cold" space. The working gas which is enclosed between the two pistons, moves continuously back and forth between the hot and cold spaces through a regenerator. The regenerator either extracts heat or adds heat to the working gas, depending on whether the gas is moving from a hot space to a cold space or from a cold space to a hot space, respectively. The two pistons are mechanically linked to each other in order to synchronize volumetric variation between the two cylinders.

The power output of the engine can be controlled by altering the pressure of the working gas. The combustion pressure of the Stirling Engine can also be raised above ambient, to that of the surrounding water pressure (up to a maximum of 300 m), so that the engine can operate submerged without the need for an exhaust gas processing unit.

The first Stirling Engine installed in a subsea vehicle was that installed in the 1200 tonne Swedish naval submarine “Nacken” between the periods 1988-90, by the Swedish companies United Stirling AS and Kockums. Two 75 kW Stirling Engines were installed in the “Nacken” and these engines improved the subsea endurance of the submarine by a factor of five to eight. Stirling Engines were also installed in the French commercial submarine “SAGA-1” in 1990. Kockums is currently developing a prototype AUV energy hull sub-section (0.99 m internal diameter and 3.5 m long) comprising of a four cylinder Stirling Engine, a liquid oxygen tank, a hydrocarbon fuel tank and the associated control system. The power output of this system is 5 to 15 kW, the capacity of the system is 600 kW-hr and the energy density of the system is 175 W-hr / kg.

The advantages of the Stirling Engine over the CCDE include:

- Low noise and vibration levels
- Low cyclic torque variations
- Does not need an exhaust gas processing unit

The disadvantages of the Stirling Engine include:

- They are slow starting engines and cannot be started instantly

Like CCDE, Stirling Engines are ideally suited for AUVs which embark on long range missions, since the additional oxygen and fuel required for these missions give the Stirling Engines a higher energy density than conventional batteries. For short range missions, the energy density of Stirling Engines is comparable to conventional batteries (since the engine and other associated equipment are quite heavy and voluminous).

2.2.10 Lithium Primary Batteries

Primary batteries are expendable (one-shot) devices that cannot be reused once they have been discharged. The Lithium-Thionyl Chloride primary battery has a very high energy density (450 W-hr / kg). However it is not commonly used on AUVs since it suffers from the following problems:

- The batteries are always active. They have the capability to violently release energy (explode) under certain circumstances and thus pose a safety hazard.
- The net energy cost ($ per kW-hr) works out to be very high, since the batteries have to be frequently replaced.

2.2.11 Selection of a High Energy Density System for "C-SCOUT"

The factors to be kept in mind while selecting a suitable energy system for any underwater vehicle are:

- High energy density (W-hr / kg)
- Low / moderate cost
- High reliability
- Easy to recharge and maintain
- Low start up time
- Non hazardous / safe
- Good wet / dry shelf life
- Environmentally friendly

At present no underwater energy system can perfectly satisfy all these requirements simultaneously. Therefore it is necessary to find a system that offers the best compromise.

After studying the energy systems on board current AUVs (section 2.1) and after evaluating different energy systems (section 2.2), the author selected four high energy density systems that could possibly be used on "C-SCOUT". These are:

- Silver-Zinc batteries
- Lithium-ion Polymer batteries
- Aluminium-Oxygen semi-fuel cells
- PEM fuel cells

Out of these four systems, Silver-Zinc batteries and Lithium-ion Polymer batteries are immediate solutions. Aluminium-Oxygen semi-fuel cells and PEM fuel cells are long-term solutions (to be employed once outstanding issues such as the onboard storage / generation of hydrogen, the waste disposal problem of expended aluminium / lithium anodes, etc., have been resolved).

The currently available Stirling engines, Closed Cycle Diesel engines and their associated systems are bulky and occupy a large proportion of the vehicle length. To use such systems on board "C-SCOUT" will require the length / diameter of the vehicle to be modified. Also their energy densities are not much higher than Silver-Zinc and Lithium-ion Polymer batteries. Owing to these two reasons, they have not been considered by the author.

Lithium-Thionyl Chloride primary batteries are unsafe for use and hence they have not been considered as a possible option. Alkaline primary batteries have moderately high energy densities, but the cost to produce 1 kW-hr of energy using these cells is extremely high (refer to Table 2.3 on the next page) and hence they have not been considered.

Table 2.3

The Cost of Producing 1 kW-hr of Energy [Ref. 2-13]

S.No.	Energy System	Cost per kW-hr (US $)
1	Alkaline primary battery (AAA)	890
2	Alkaline primary battery (AA)	330
3	Alkaline primary battery (C)	180
4	Alkaline primary battery (D)	90
5	Nickel-Cadmium battery	7.5
6	Lead Acid battery	8.5
7	Lithium-ion battery	24
8	Internal Combustion engine	0.3
9	Fuel cell	4.1
10	Electricity from the grid	0.1

2.3 Optimum Speed of the "C-SCOUT" AUV

Next, the author worked out the optimum speed of "C-SCOUT" for the most effective energy consumption. For this calculation, the vehicle was assumed to have a range of 50 km. Results from the self-propulsion tests (see Chapter 4) were then used to provide a relationship between delivered power (P_d) and the vehicle forward speed (V). This delivered power was then converted to motor mechanical power (P_m) by assuming 2 % losses in the bearings and magnetic coupling of the Tecnadyne 1020 thruster.

The motor mechanical power (P_m) was then converted to motor electrical power (P_e) by assuming a motor efficiency of 90 % (to account for iron and friction losses in the motor) [Ref 2-14]. At each forward speed, the motor electrical power (P_e) was then added to the hotel load (P_h), in order to give the total power requirement of the vehicle at that speed (P_t). The total time (t) required for a mission (at a particular vehicle speed) was then calculated by dividing the range (50 km) by the vehicle speed. The time (t) was then multiplied with the total power requirement of the vehicle at that speed (P_t), in order to give the total energy requirement (E) of the vehicle at that speed (see Appendix A for the calculations). These calculations were carried out at hotel loads of 100, 150, 200 and 250 W.

The total energy requirement (ordinate) was then plotted against the vehicle forward speed (abscissa) [Figs. 2.2 to 2.5]. From the plots it can be seen that the optimum speed of "C-SCOUT" at which it consumes the minimum energy and has the maximum range, is around 2 m/s. Similar results are obtained for vehicle ranges other than 50 km. It should be noted that the optimum speed is independent of the vehicle range. (Refer to Eqn. 2.5). Table 2.4 shows the amount of energy required by "C-SCOUT" when traveling at an optimum speed of 2 m/s.

Table 2.4

Energy Required by "C-SCOUT"
(Vehicle Speed = 2 m/s)

S.No.	Hotel Load (W)	Required Energy (kW-hr)
1	100	1.8
2	150	2.2
3	200	2.5
4	250	2.9

Analytical Expression for the Optimum Speed of "C-SCOUT"

The total power (P_T) required by an AUV is the sum of the effective power (P_E) and the Hotel Load (P_H). The effective power (P_E) is the power required to propel the vehicle and it is equal to the product of the vehicle resistance (R) and the vehicle forward speed (V). The hotel load (P_H) is defined as the power required by the vehicle for all other purposes apart from propulsion.

$$\boxed{P_T = P_E + P_H = R^* V + P_H} \quad \text{W} \qquad \ldots..2.1$$

Now the vehicle resistance (R) can be expressed as:

$$\boxed{R = 1/2\ C_D\ \rho A V^2} \quad \text{N} \qquad \ldots..2.2$$

where "C_D" is the drag coefficient of the vehicle, "ρ" is the mass density of water, "A" is the cross sectional area of the vehicle hull and "V" is the forward speed of the vehicle.

Substituting Eqn. 2.2 into Eqn. 2.1:

$P_T = R^* V + P_H = 1/2\ C_D\ \rho A V^3 + P_H$

If E_T is the total energy provided by the energy system (W-hr), then the time (t) for which the vehicle can travel at sea is:

$$\boxed{t = E_T / P_T} \quad \text{hr} \qquad \text{.....2.3}$$

The range (R) of the vehicle is:

$$\boxed{R = 3.6\ V * t = 3.6\ V * E_T / (\ 1/2\ C_D\ \rho A V^3 + P_H)} \quad \text{km} \qquad \text{.....2.4}$$

where the factor 3.6 is used for converting the vehicle speed in m/s to km/hr.

The optimum speed at which the vehicle range is maximized, can be determined by differentiating Eqn. 2.4 with respect to speed (V) and then equating it to zero (the coefficient of drag is assumed to be a constant). Differentiation leads to the following expression:

$1/2\ C_D\ \rho A V^3 + P_H = 3/2\ C_D\ \rho A V^3$

on simplifying the expression we get:

$P_H = C_D\ \rho A V^3$

from which the optimum speed of the vehicle for maximum range can be expressed as:

$$\boxed{V = (\ P_H / C_D\ \rho\ A\ )^{1/3}} \qquad \text{.....2.5}$$

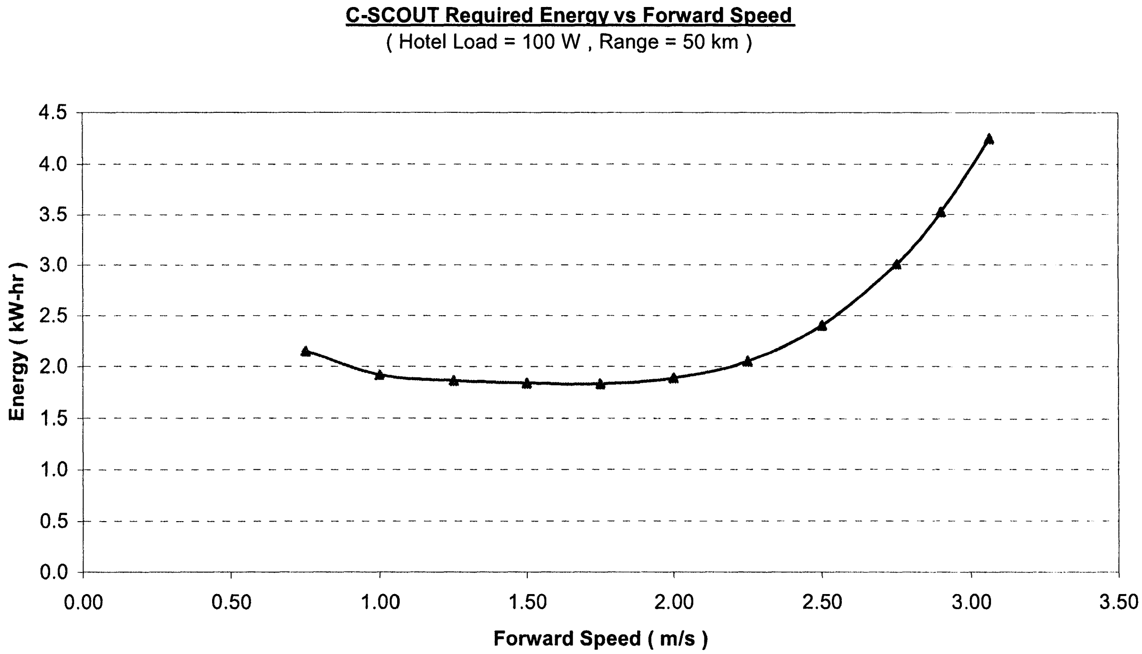
If Eqn. 2.5 is substituted into Eqn. 2.1, it can be shown that at the optimum vehicle speed for maximum range, the hotel load (P_H) is 2/3 times the total power (P_T) required by the vehicle and the effective power (P_E) is 1/3 times the total power (P_T) required by the vehicle.

The value of the optimum vehicle speed for maximum vehicle range, can then be calculated by substituting the values of the hotel load, drag coefficient, mass density of water and the cross sectional area of the vehicle, into Eqn. 2.5. Table 2.5 tabulates the optimum speeds at different hotel loads.

Table 2.5

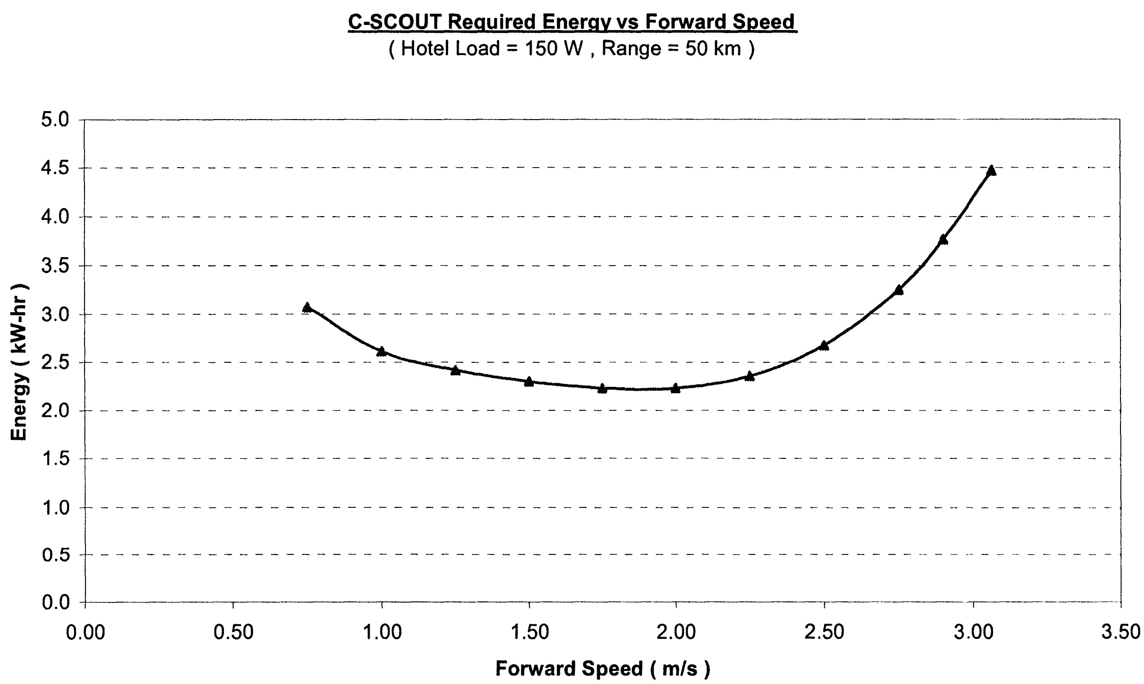
Optimum Speed of C-SCOUT

S.No.	Hotel Load (W)	Optimum Speed (m/s)
1	100	1.6
2	150	1.8
3	200	2.0
4	250	2.1



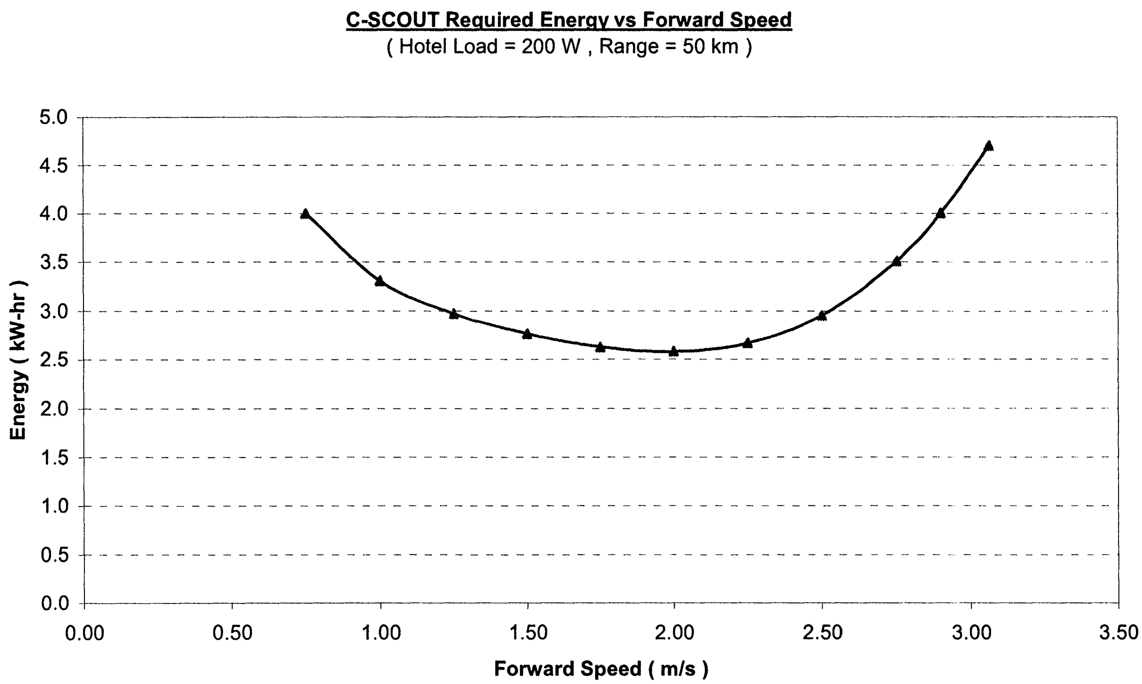
C-SCOUT Required Energy vs Forward Speed
(Hotel Load = 100 W , Range = 50 km)
Energy (kW-hr)
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.00
0.50
1.00
1.50
2.00
2.50
3.00
3.50
Forward Speed (m/s)

Fig 2.2



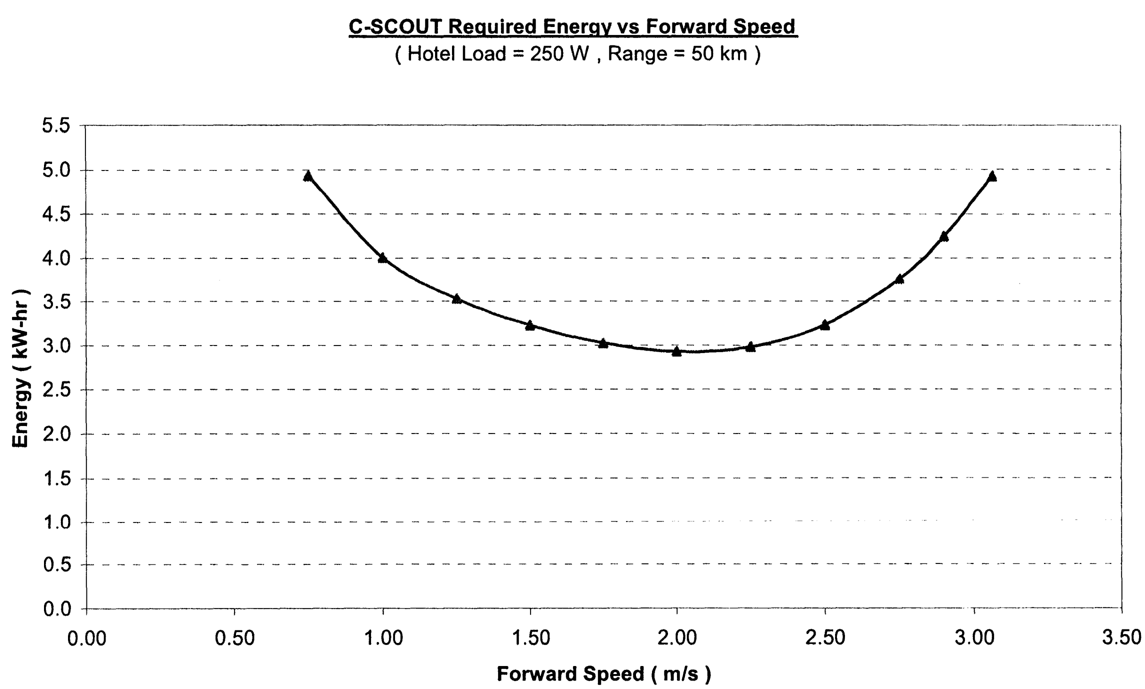
C-SCOUT Required Energy vs Forward Speed
(Hotel Load = 150 W , Range = 50 km)
Energy (kW-hr)
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.00
0.50
1.00
1.50
2.00
2.50
3.00
3.50
Forward Speed (m/s)

Fig 2.3



C-SCOUT Required Energy vs Forward Speed
(Hotel Load = 200 W , Range = 50 km)
Energy (kW-hr)
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.00
0.50
1.00
1.50
2.00
2.50
3.00
3.50
Forward Speed (m/s)

Fig 2.4



C-SCOUT Required Energy vs Forward Speed
(Hotel Load = 250 W , Range = 50 km)
Energy (kW-hr)
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.00
0.50
1.00
1.50
2.00
2.50
3.00
3.50
Forward Speed (m/s)

Fig 2.5

2.4 <u>Endurance of the "C-SCOUT" AUV</u>

As the next step, the endurance limits of "C-SCOUT" were calculated. First the endurance limits using the present energy system on board the vehicle were calculated (Section 2.4.1). Subsequently the endurance limits using a few higher energy density systems were calculated (Section 2.4.2). These calculations were carried out at hotel loads of 100, 150, 200 and 250 W.

2.4.1 Endurance Using the Present Energy System

The present energy system on board "C-SCOUT" consists of six Lead Acid batteries. Two of these batteries supply 312 W-hr of energy (each) and weigh 9.25 kg (each). The remaining four batteries supply 379.2 W-hr of energy (each) and weigh 11 kg (each). The total energy supply available on board the vehicle is therefore 2 x 312 + 4 x 379.2 = 2140.8 W-hr. The total weight of all the batteries is 2 x 9.25 + 4 x 11 = 62.50 kg. The energy density of the battery system is therefore 2140.8 / 62.50 = 34.25 W-hr / kg.

At each forward speed, the total time required for a mission (t') was then calculated by dividing the total energy supply available on board (2140.8 W-hr) by the total power requirement of the vehicle at that speed (P_t). At each forward speed, the maximum distance the vehicle can possibly travel at sea (D) was then calculated by multiplying the time (t') with the vehicle forward speed (V) (see Appendix A for the calculations).

The time (t'), distance (D) (ordinate) was then plotted against the vehicle forward speed (abscissa) [Figs. 2.6 to 2.9]. Table 2.6 tabulates the vehicle endurance at different hotel loads.

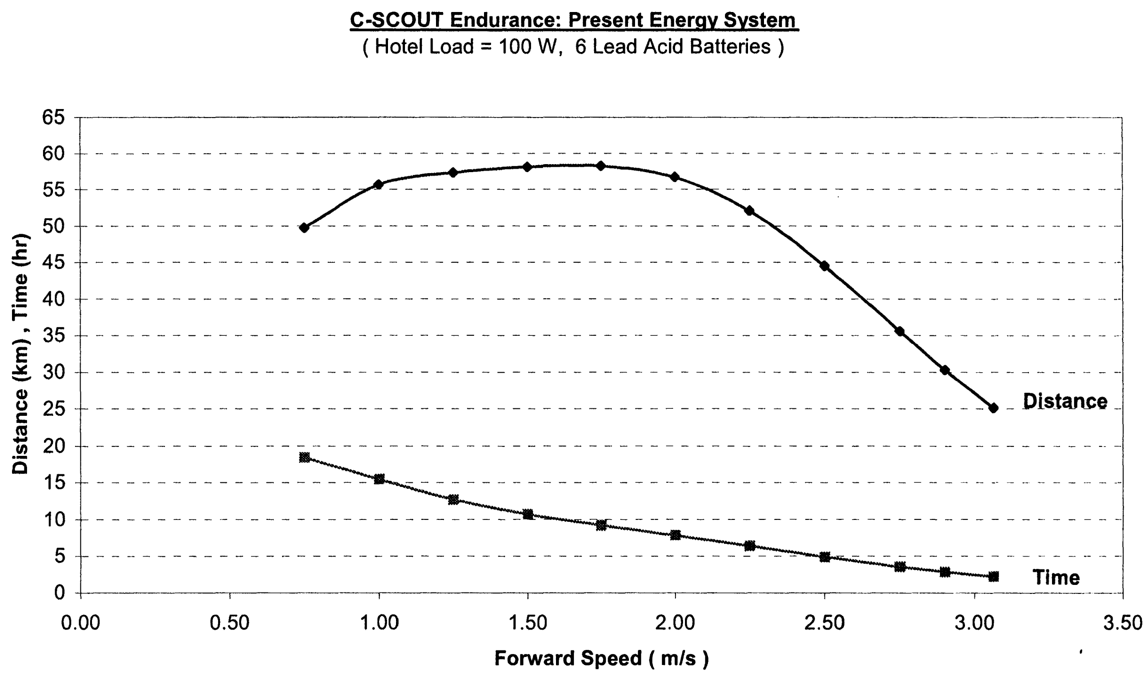

C-SCOUT Endurance: Present Energy System
(Hotel Load = 100 W, 6 Lead Acid Batteries)
65
60
55
50
45
40
35
30
25
20
15
10
5
0
Distance (km) , Time (hr)
Distance
Time
0.00
0.50
1.00
1.50
2.00
2.50
3.00
3.50
Forward Speed (m/s)

Fig. 2.6

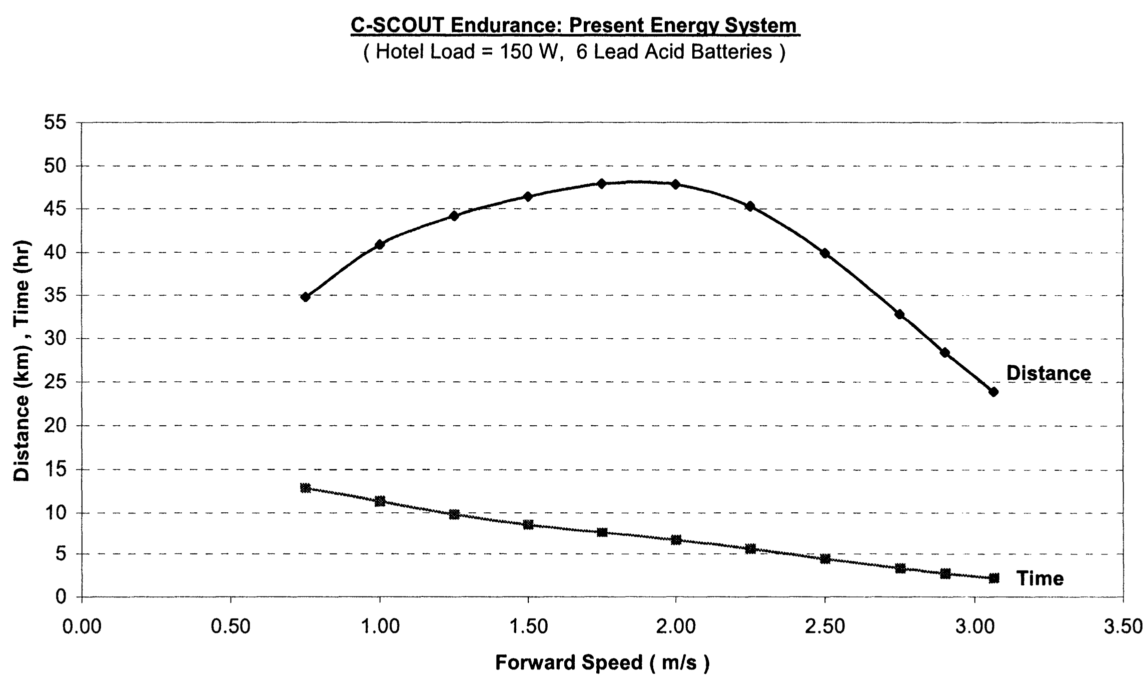

C-SCOUT Endurance: Present Energy System
(Hotel Load = 150 W, 6 Lead Acid Batteries)
55
50
45
40
35
30
25
20
15
10
5
0
Distance (km) , Time (hr)
Distance
Time
0.00
0.50
1.00
1.50
2.00
2.50
3.00
3.50
Forward Speed (m/s)

Fig. 2.7

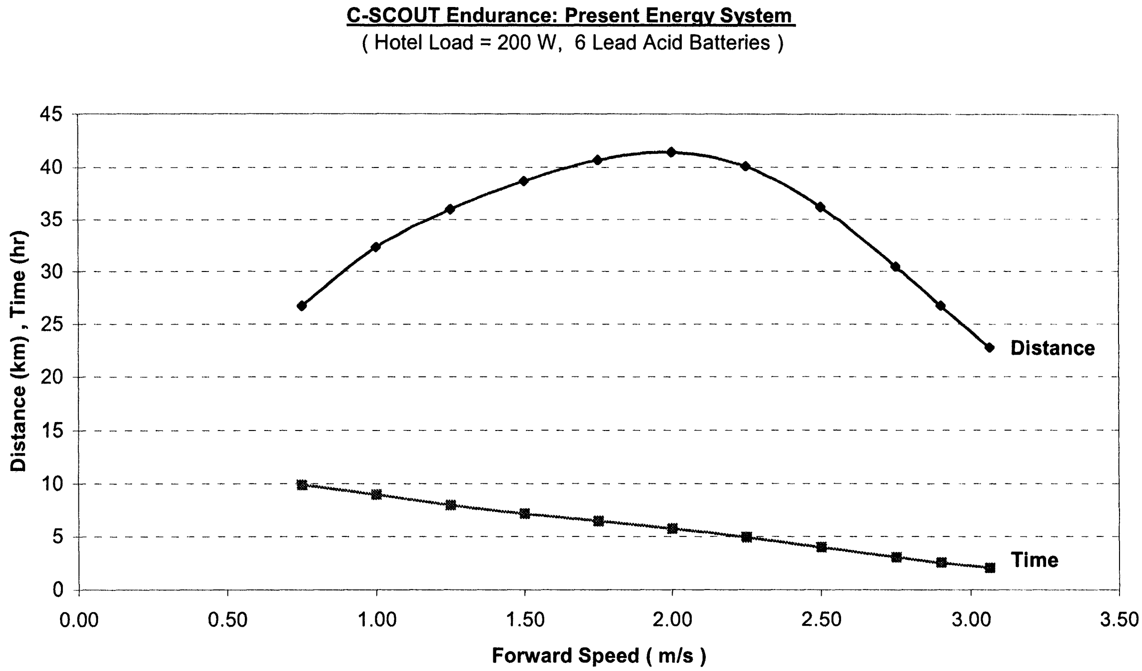

C-SCOUT Endurance: Present Energy System
(Hotel Load = 200 W, 6 Lead Acid Batteries)
Distance (km) , Time (hr)
45
40
35
30
25
20
15
10
5
0
Distance
Time
0.00
0.50
1.00
1.50
2.00
2.50
3.00
3.50
Forward Speed (m/s)

Fig. 2.8

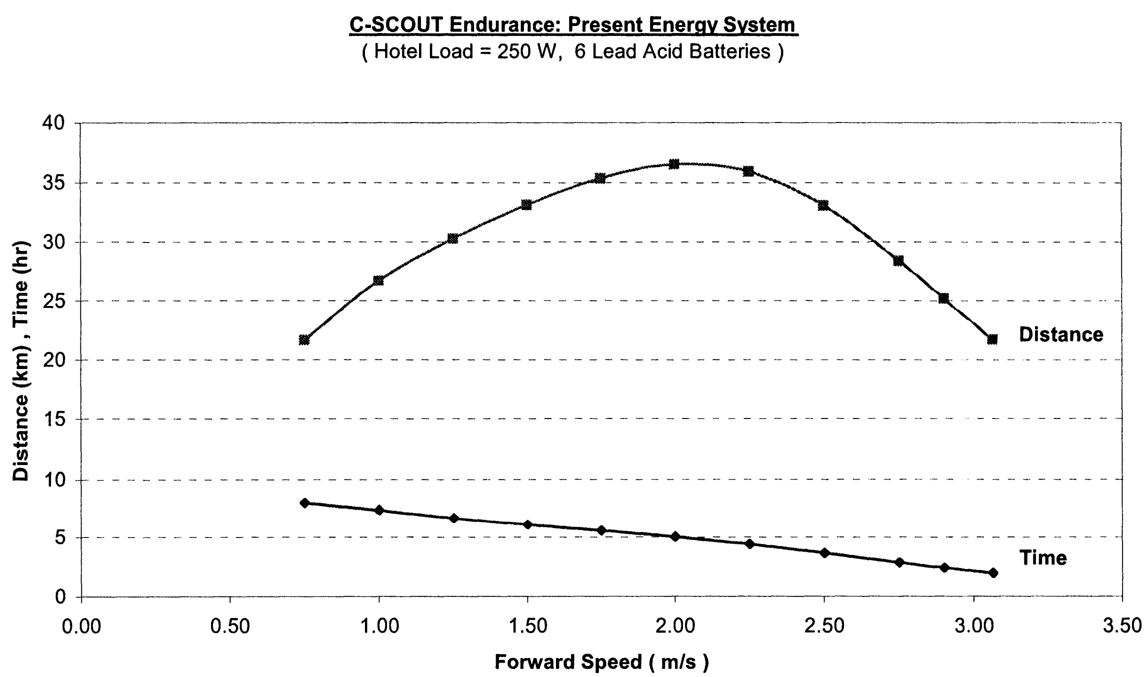

C-SCOUT Endurance: Present Energy System
(Hotel Load = 250 W, 6 Lead Acid Batteries)
Distance (km) , Time (hr)
40
35
30
25
20
15
10
5
0
Distance
Time
0.00
0.50
1.00
1.50
2.00
2.50
3.00
3.50
Forward Speed (m/s)

Fig. 2.9

2.4.2 Endurance Using a High Energy Density System

The current energy system on board “C-SCOUT” has a low energy density (34.25 W-hr / kg). If the endurance of the vehicle is to be increased, then it is essential that this system be replaced with a system of higher energy density. The vehicle endurance limits using the four high energy density systems selected in Section 2.2.11 were calculated (see Appendix A for the calculations) and plotted [Figs. 2.10 to 2.17], using a procedure similar to that used in section 2.4.1, For these calculations, the energy density of the selected systems were taken as:

- Silver Zinc batteries: 110 W-hr / kg
- Lithium-ion Polymer batteries: 130 W-hr / kg
- Aluminium Oxygen semi-fuel cells: 260 W-hr / kg
- PEM fuel cells: 300 W-hr / kg

Also, for these calculations, the total onboard energy of an energy system having equivalent weight to that of the current energy system on board □C-SCOUT□ (62.5 kg) was obtained by multiplying the above energy densities with a weight of 62.5 kg.

Plots [Figs. 2.10 to 2.17] show that maximum vehicle endurance can be obtained by using PEM fuel cells. This is followed by Aluminium Oxygen semi-fuel cells, Lithium-ion Polymer batteries and Silver Zinc batteries respectively. The benefit of using a high energy density system is an improvement in the endurance of the vehicle. For example if the vehicle travels at a speed of 2 m/s and has a hotel load of 250 Watt [Fig. 2.16], then use of PEM fuel cells leads to an endurance limit that is nine times that obtained using Lead Acid batteries. Similarly use of Aluminium Oxygen semi-fuel cells, Lithium-ion Polymer batteries and Silver Zinc batteries leads to an endurance limit that is eight, four and three times respectively, that obtained using Lead Acid batteries. With a hotel load of 250 Watt, the current endurance limit of “C-

SCOUT" is a distance of 36 km. This goes up to 320 km, 277 km, 139 km and 117 km if PEM fuel cells, Aluminium Oxygen semi-fuel cells, Lithium-ion Polymer Batteries and Silver Zinc Batteries respectively, are used instead of the Lead Acid battery. Table 2.6 tabulates the vehicle endurance at different hotel loads, at a speed of 2 m/s.

Table 2.6

"C-SCOUT" Endurance
(Vehicle Speed = 2 m/s)

S.No.	**Hotel Load** (W)	***Lead Acid Battery***	***Silver Zinc Battery***	***Li-ion Polymer Battery***	***Aluminium Oxygen Semi-Fuel Cell***	***PEM Fuel Cell***
		Range (km)	**Range** (km)	**Range** (km)	**Range** (km)	**Range** (km)
1	100	57	182	215	430	496
2	150	48	154	182	363	419
3	200	41	133	157	314	363
4	250	36	117	139	277	320

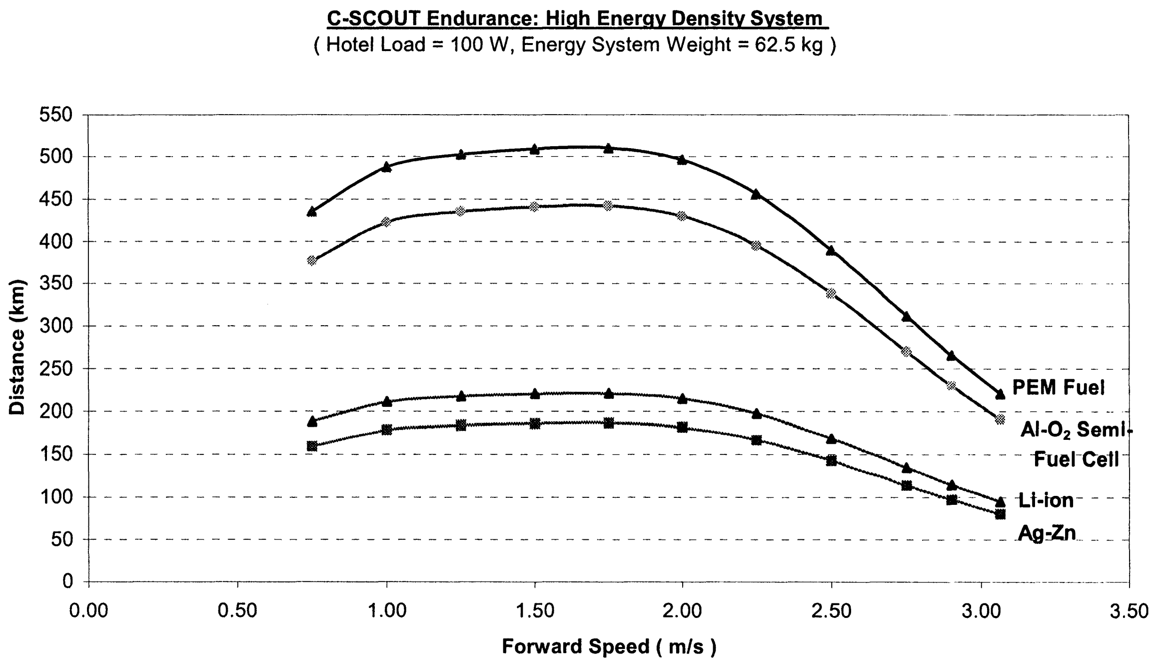


Fig. 2.10

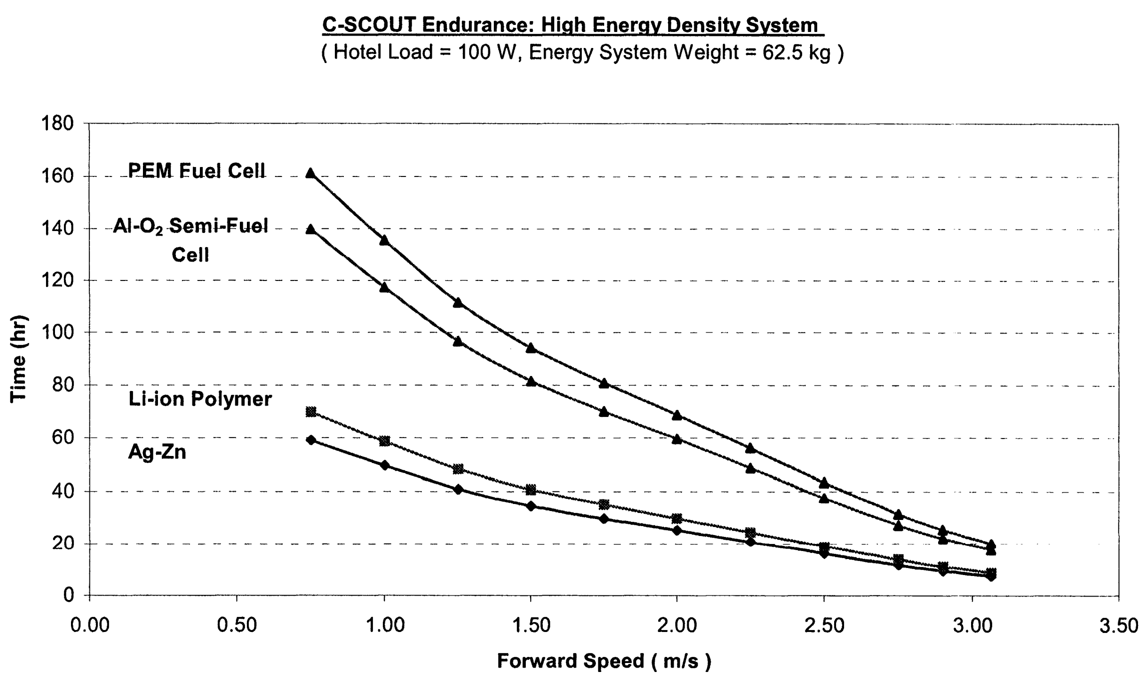


Fig. 2.11

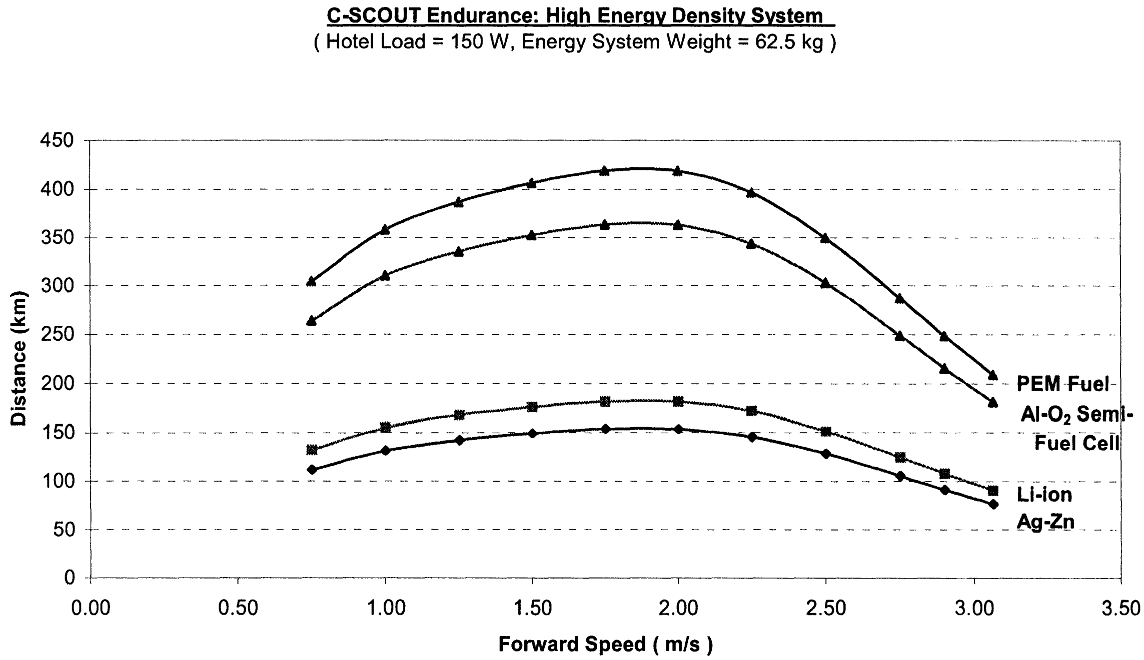


Fig. 2.12

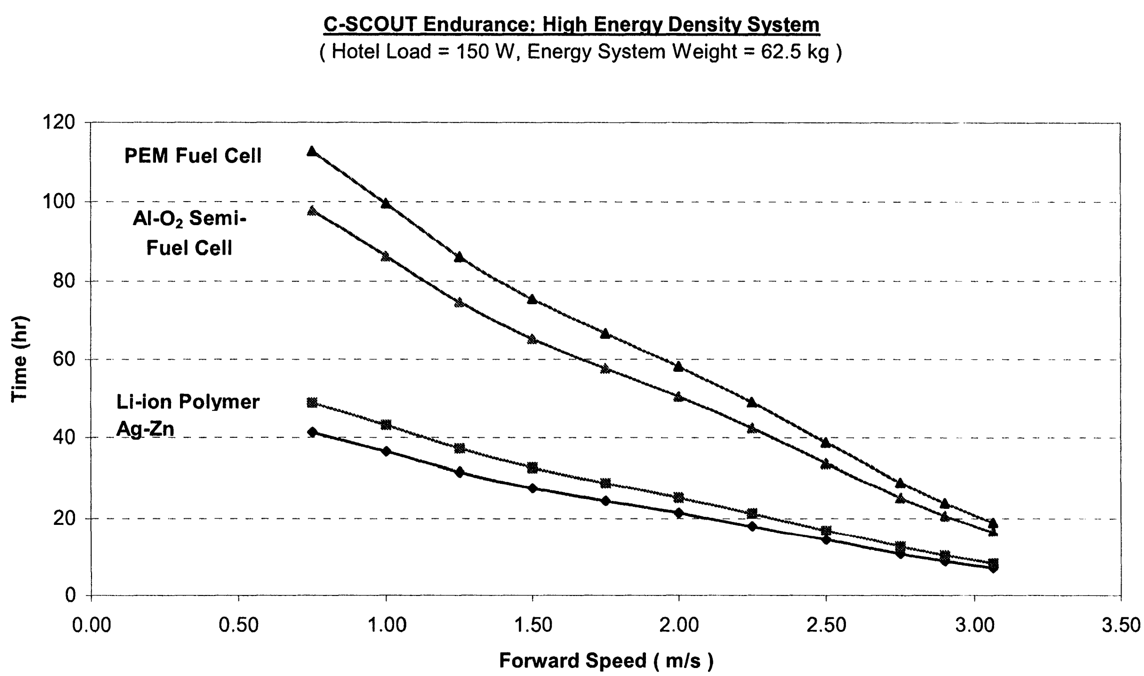


Fig. 2.13

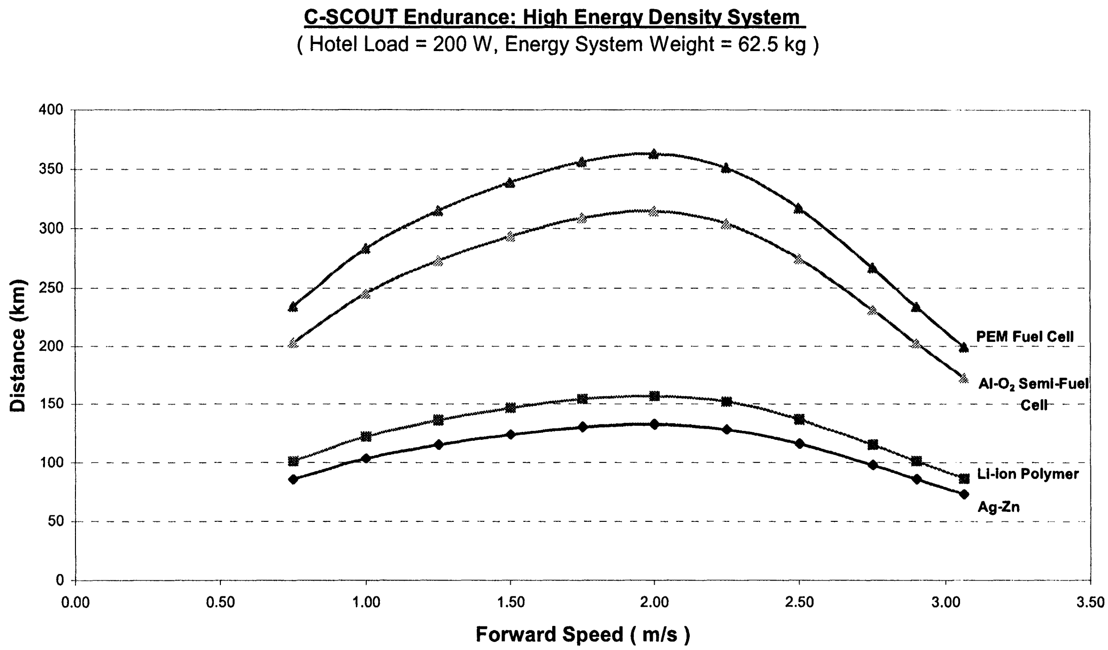


Fig. 2.14

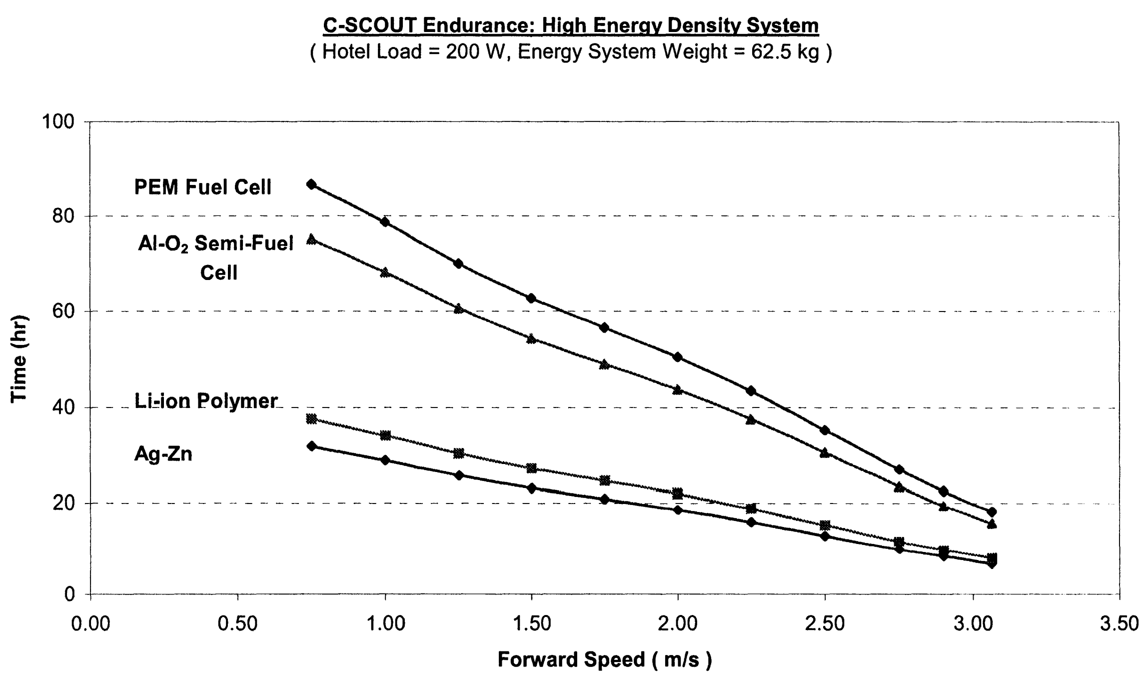


Fig. 2.15

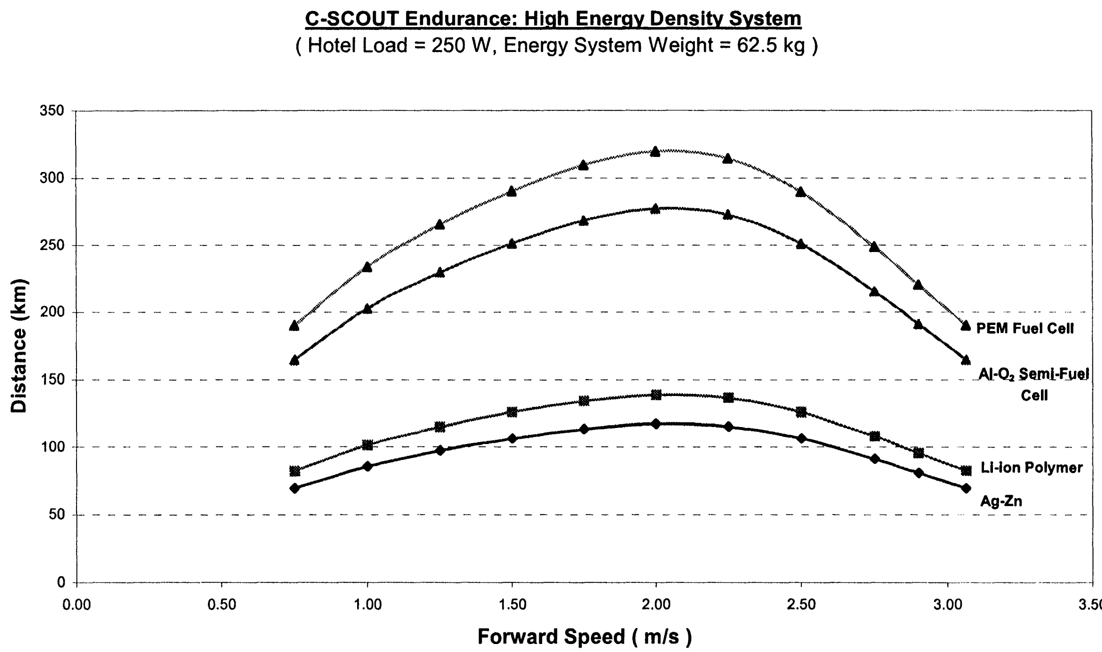


Fig. 2.16

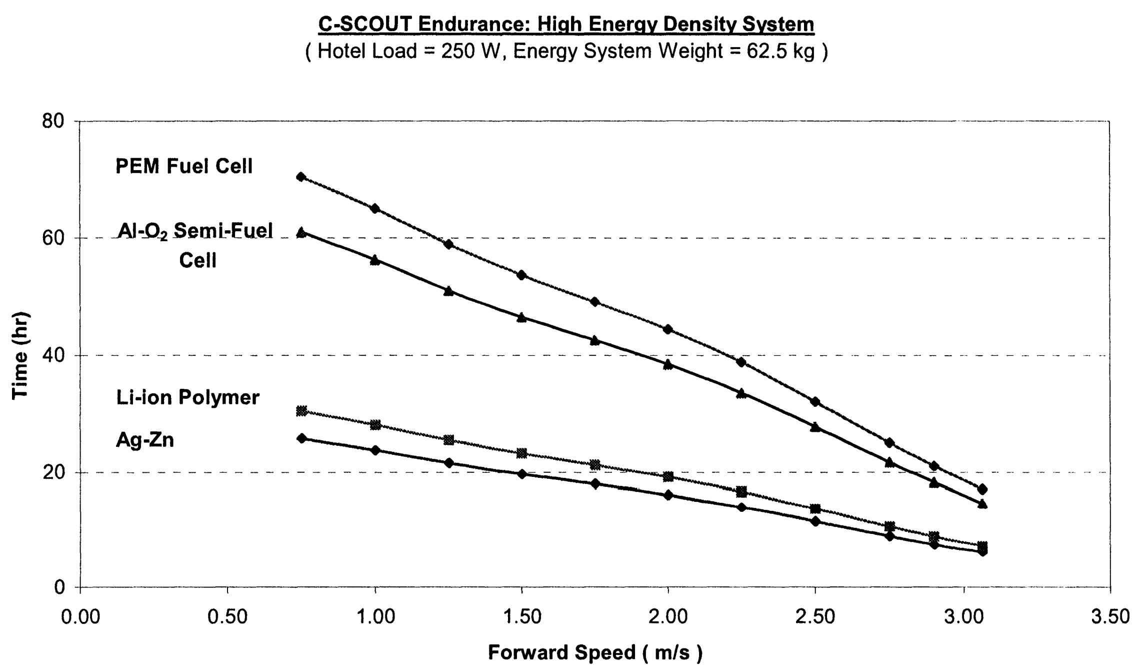


Fig. 2.17

2.5 Conclusions

With advent of higher energy density battery systems such as Lithium-ion Polymer batteries, Aluminium-Oxygen semi-fuel cells, PEM fuel cells etc, the days of the Lead Acid battery as the dominant energy source for AUVs are numbered. As technology develops, it is hoped that AUVs with the ability to circumnavigate the globe on a single charge of energy, will be a possibility in the future.

Chapter 3

Powering Prediction for Ships / Submersibles

Powering prediction for ships / submersibles based on the ITTC 1978 method makes use of experimental data from tests conducted on the hull and propulsion system in isolation of each other. This means that the interaction effects between the hull and propulsion system have to be accounted for approximately. This can lead to inaccuracies in a powering prediction, especially in cases when compound propulsors such as ducted propellers, podded propellers, etc., are used. To overcome these deficiencies, Kracht [Ref 3-2] and Schmiechen [Ref 3-1] proposed a powering prediction method that is based solely on one set of experiments; load-varying self-propulsion tests. In these tests, the hull and propulsion system are treated as one physical unit. Therefore interaction effects between the hull and propulsion system are taken into consideration and this leads to more accurate results in cases where there are strong interaction effects, although scale effects still lead to small errors. The level of hull-propulsor interaction can generally be gauged by looking at values of the thrust deduction fraction. Values of thrust deduction fraction less than 0.1 indicate that the hull-propulsor interaction is not particularly strong, whereas values around 0.3 indicate strong hull-propulsor interaction.

Preliminary work on powering prediction using load-varying self-propulsion tests, was carried out at MARIN, Netherlands [Ref 3-3] and the Berlin Model Basin (VWS), Germany [Ref 3-2]. This chapter compares powering prediction for ships / submersibles based on the ITTC 1978 method with that based on the load-varying self-propulsion test method and states the salient points of each method.

In the marine industry, whenever powering prediction figures are required for new ships / submersibles, it is a standard practice to first construct a scaled-down, geometrically-similar model of the ship / submersible, then test it in water and subsequently extrapolate the model scale results to full scale results. For in-water tests on ship models, the model floats on the surface of water. For in-water tests on submersible models, the model is submerged below the surface of water. In this chapter, the term "model" (denoted by the subscript "M") refers to a scaled-down geometrically-similar model of the actual ship / submersible. The term "ship" (denoted by the subscript "S") refers to the actual full scale ship / submersible.

3.1 Powering Prediction Using the ITTC 1978 Performance Prediction Method

The ITTC 1978 Performance Prediction Method is currently the industry standard for powering prediction of marine vehicles. This method essentially requires that three independent tests be carried out on a scaled-down geometrically-similar model of the marine vehicle, in order to predict the performance of the actual vehicle. These three tests are – the resistance test, the propeller open water test and the self-propulsion test.

3.1.1 Resistance Tests

Resistance tests [Ref 3-10, 3-6] can be further categorized as standard resistance tests and low-speed resistance tests.

a) Standard Resistance Tests

The basic purpose of the standard resistance tests is to determine the drag force (resistance) of a scaled-down, geometrically-similar, bare-hull model of the marine vehicle. The measured drag force is then extrapolated to full scale in order to obtain the drag of the actual vehicle. During a standard resistance test, the model is towed by a towing carriage over a range of speeds (normally 5% of the speed range below the minimum speed at which reliable data is required to 5 % above the highest speed required). The following parameters are recorded during a test:

- Model speed (V_M)
- Resistance (R_{TM})
- Trim angle
- Sinkage
- Water temperature (T^0)

The raw data are then non-dimensionalized into resistance coefficients which are then extrapolated to full scale:

$$C_{TM} = \frac{R_{TM}}{\frac{1}{2}\rho_M V_M^2 S_M}$$ Coefficient of total resistance of the model 3.1

$$C_{FM} = \frac{0.075}{(\log_{10}Rn_M - 2)^2}$$ Coefficient of frictional resistance of the model (based on the 1957 model-ship correlation line) 3.2

$$C_{RM} = C_{TM} - (1 + k)\,C_{FM}$$ Coefficient of residuary resistance of the model 3.3

$$C_{RM} = C_{RS}$$ 3.4

$$k_M = k_S = k$$ 3.5

$$C_{TS} = (1+k)\,C_{FS} + C_{RS} + C_A + C_{AA}$$ Coefficient of total resistance of the ship 3.6

$$R_{TS} = C_{TS}\,\tfrac{1}{2}\rho_S V_S^2 S_S$$ Total resistance of the full scale vehicle 3.7

$$P_E = R_{TS} V_S$$ Effective power 3.8

It should be noted that the value of the coefficient of frictional resistance has traditionally been estimated using the ITTC 1957 model–ship correlation line. However it is physically more correct to use a true turbulent flat-plate friction line such as that formulated by Schlichting [Ref 3-8] or by Grigson [Ref 3-9].

b) Low-Speed Resistance Tests

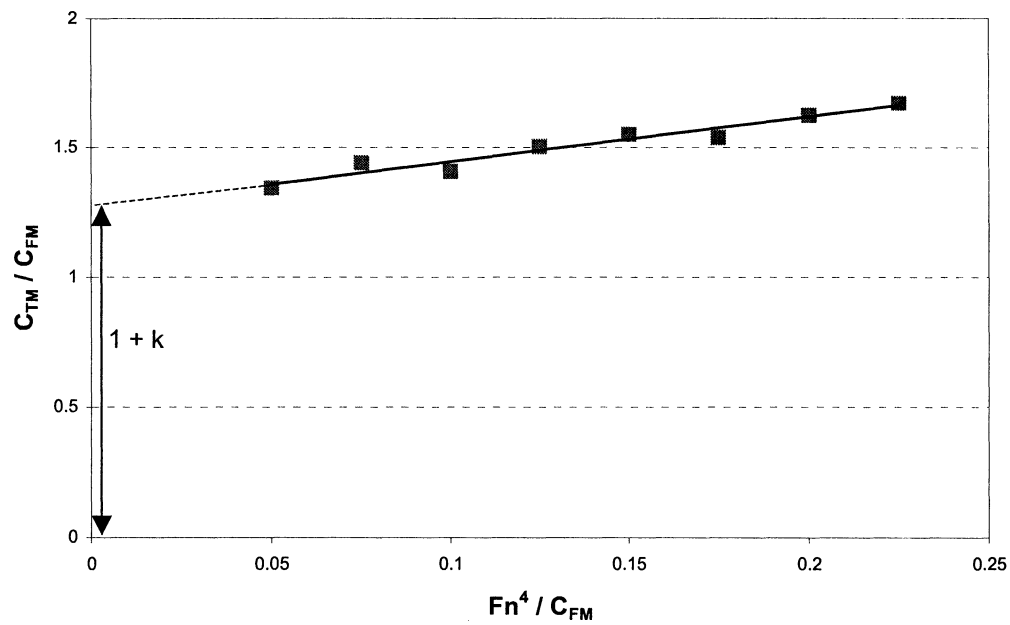
The basic purpose of the low-speed resistance tests is to determine the form factor (k) of the scaled-down geometrically-similar model of the marine vehicle. The form factor accounts for the three dimensional form of the marine vehicle. It is used to convert the two dimensional value of the coefficient of frictional resistance (C_F) into a three dimensional value (1+k) C_F. The form factor is considered to be invariant with Reynolds number (Rn), hence $k_M = k_S = k$. The low speed resistance tests are carried out between the Froude numbers (Fn) of 0.12 to 0.20, so as to preclude any wave making resistance effects. The parameters recorded include:

- Model speed (V_M)
- Resistance (R_{TM})
- Water temperature (T^0)

The ITTC 1978 method uses a procedure similar to Prohaska's method [Ref 3-7] for analyzing the test results. At each towing speed, the C_{FM} and C_{TM} values are calculated. A plot is then made with C_{TM} / C_{FM} as the ordinate and Fn^n / C_{FM} as the abscissa [Fig 3.1]. A straight line is fitted through the data points. The equation of this line is of the form:

$$\boxed{C_{TM} / C_{FM} = (1+k) + C\,(Fn^n / C_{FM})} \qquad \text{.....3.9}$$

where: "1 + k" is the intercept on the C_{TM} / C_{FM} axis when Fn = 0 , "C" is the slope of the straight line, "n" is a power that lies between 4 and 6 , The value of "k" is then determined from the value of the intercept.



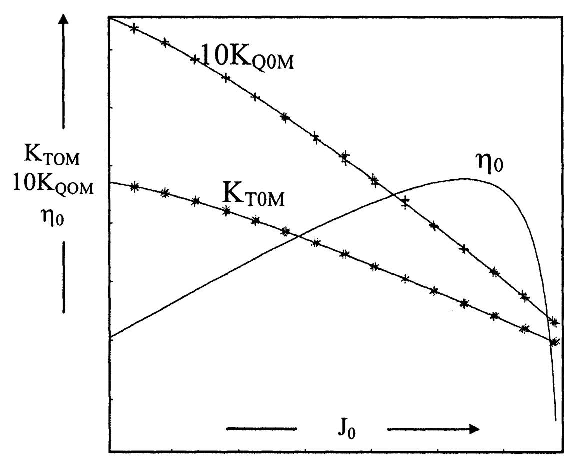
Form Factor: Fig 3.1

3.1.2 Propeller Open Water Tests

The basic purpose of the propeller open water test [Refs 3-10, 3-6] is to evaluate the performance of a scaled-down, geometrically-similar model of the actual propeller, when working under homogeneous inflow conditions. The model propeller is fitted onto an open water propeller boat and towed by the towing carriage at different speeds. In a typical open water test program, the propeller is rotated at a constant speed and the carriage speed is systematically varied so as to cover a range of advance coefficients from $J_0 = 0$ to the J_0 corresponding to $K_{T0} = 0$. This corresponds to the range of operation in which the propeller drives the vehicle in the ahead direction (the first quadrant). The parameters recorded include:

- Carriage speed (V_A)
- Propeller thrust (T_M)
- Propeller torque (Q_M)
- Propeller rotational speed (n_M)
- Water temperature (T^0)

The raw data are then non-dimensionalized into the advance coefficient (J_0), thrust coefficient (K_{TOM}) and the torque coefficient (K_{Q0M}) [Fig 3.2].



Open Water Test: Fig. 3.2

$$J_0 = \frac{V_A}{n_M D_M} \quad \text{.....3.10}$$

$$K_{Q0M} = \frac{Q_M}{\rho_M n_M^2 D_M^5} \quad \text{.....3.12}$$

$$K_{TOM} = \frac{T_M}{\rho_M n_M^2 D_M^4} \quad \text{.....3.11}$$

The difference in the Reynolds number at which the model propeller is tested and at which the actual ship propeller runs, causes a change in the frictional coefficient over the surface of the blades. This is corrected using the correction factors δK_T and δK_Q [Ref 3-6]. The corrected values K_{TS} and K_{QS} represent the non-dimensional coefficients for the full scale propeller and these are used for determining the self-propulsion point of the ship propeller.

$$K_{TS} = K_{T0M} - \delta K_T \qquad .....3.13$$

$$K_{QS} = K_{Q0M} - \delta K_Q \qquad .....3.14$$

where:

$$\delta K_T = -\delta C_D \, 0.3 \, \frac{P}{D} \, \frac{qZ}{D} \qquad .....3.15$$

$$\delta K_Q = \delta C_D \, 0.25 \, \frac{qZ}{D} \qquad .....3.16$$

$$\delta C_D = C_{DM} - C_{DS} \qquad .....3.17$$

$$C_{DM} = 2\left(1 + \frac{2m}{q}\right)\left[\frac{0.044}{(R_{nco})^{1/6}} - \frac{5}{(R_{nco})^{2/3}}\right] \qquad .....3.18$$

$$C_{DS} = 2\left(1 + \frac{2m}{q}\right)\left[1.89 + 1.62 \log \frac{q}{k_P}\right]^{-5/2} \qquad .....3.19$$

q = Chord length of the propeller blade

m = Maximum thickness of the propeller blade

P / D = Pitch to diameter ratio of the propeller blade

R_{nco} is the local Reynolds number at x = 0.75 (should be at least $2 * 10^5$)

k_P = Blade roughness

3.1.3 Self-Propulsion Tests

The basic purpose of this test [Ref 3-10, 3-6] is to evaluate the propulsive performance of a scaled down geometrically similar model of the vehicle, when it is being self-propelled. This test tries to model as accurately as possible the actual vehicle operating conditions. Hence the propulsion system and all the appendages are fitted in place and the propeller operates in a non-uniform wake field behind the vehicle. The model is free to heave and pitch, but is restrained in its longitudinal symmetry plane. Two different types of self-propulsion tests are currently in use:

a) Load-varying self-propulsion tests

b) Speed-varying self-propulsion tests

a) Load-Varying Self-Propulsion Tests

In a load-varying self-propulsion test, the model is towed at a number of different speeds, so as to cover the entire speed range of the vehicle. At each vehicle speed, the propeller rotational speed is systematically varied so as to cover a wide range of propeller loadings – from the towed region, through the self-propulsion point, to the overload region. At each vehicle speed, 5 to 7 values of propeller rotational speeds are used for covering a wide range of propeller loadings. The parameters recorded include:

- Model speed (V_M)
- Propeller thrust (T_M)
- Propeller torque (Q_M)
- Propeller rotational speed (n_M)
- Tow force (F)
- Water temperature (T^0)

The raw data are then non-dimensionalized into the advance coefficient (J), thrust coefficient (K_{TBM}), torque coefficient (K_{QBM}) and the tow force coefficient (K_{FDM}).

$$\boxed{J = \frac{V_M}{n_M D_M}} \qquad \text{.....3.20}$$

$$\boxed{K_{TBM} = \frac{T_M}{\rho_M n_M^2 D_M^4}} \qquad \text{.....3.21}$$

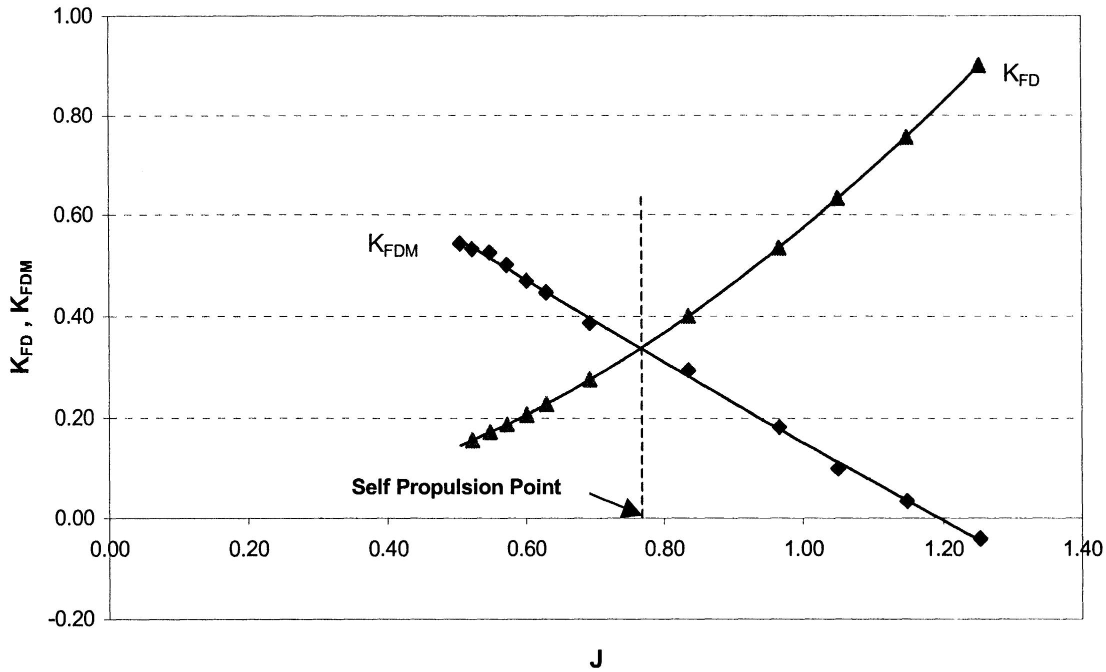
$$\boxed{K_{QBM} = \frac{Q_M}{\rho_M n_M^2 D_M^5}} \qquad \text{.....3.22}$$

$$\boxed{K_{FDM} = \frac{F}{\rho_M n_M^2 D_M^4}} \qquad \text{.....3.23}$$

Now since the load-varying self-propulsion test is not conducted at the exact self-propulsion point, but instead at a number of points above and below this point, therefore the exact self-propulsion point needs to be interpolated. For this purpose the tow force coefficient (K_{FDM}) is plotted against the advance coefficient (J) [Fig 3.3]. On this plot, an additional curve which represents the tow force coefficient (K_{FD}) as a function of J^2 is also plotted.

$$\boxed{K_{FD} = \frac{C_{FD} S_M}{2 D_M^2} J^2} \qquad \text{.....3.24}$$

This additional curve basically represents that value of towing force which is equal to the towing force at ship self-propulsion point (F_D) (see section 3.1.4 for a note on F_D). The intersection of the curves K_{FDM} and K_{FD} represents the ship self-propulsion point. At this point the values of K_{TBM}, K_{QBM} and J are read off.



Self-Propulsion Point: Fig 3.3

To determine the thrust deduction fraction (t) at the ship self-propulsion point, an additional curve of the non-dimensional coefficient K_R is inserted into the above plot. K_R is defined as:

$$\boxed{K_R = \frac{R_{TM}}{\rho_M \, n_M^2 \, D_M^4}} \qquad \text{.....3.23}$$

The value of K_R at the ship self-propulsion point is then read off. The thrust deduction fraction is then calculated using the expression:

$$\boxed{t = \frac{K_{TBM} - K_R}{K_{TBM}}} \qquad \text{.....3.24}$$

which is essentially the non-dimensional form of the equation

$$\boxed{t = \frac{T_M^* - R_{TM}}{T_M^*}}$$

where T_M^* is the model thrust at ship self-propulsion point and R_{TM} is the total resistance of the model.

Since the thrust deduction fraction is assumed to be independent of propeller loading and scale effects, therefore:

$$\boxed{t_{Model} = t_{Ship} = t}$$

Next to determine the effective wake fraction, the K_{TBM}, K_{QBM} and the K_{T0M}, K_{Q0M} curves are plotted together. The effective wake fraction is then determined by either considering a thrust identity (constant thrust) or a torque identity (constant torque). The effective wake fraction determined using a thrust identity is denoted by w_T and that determined by using a torque identity is denoted by w_Q. For a thrust identity, a straight line representing the value of K_{TBM} at the self-propulsion point, is drawn [Fig 3.4]. At the point where this line intersects the K_{T0M} and K_{TBM} curves, the values of J_0 and J are read off. The effective wake fraction is then calculated using the expression:

$$\boxed{w_{TM} = \frac{J - J_0}{J}} \qquad \text{.....3.25}$$

A similar procedure is used to determine the effective wake fraction using a torque identity. Generally w_T is somewhat larger than w_Q. Use of the thrust identity wake fraction is generally preferred in the analysis of propulsion factors.

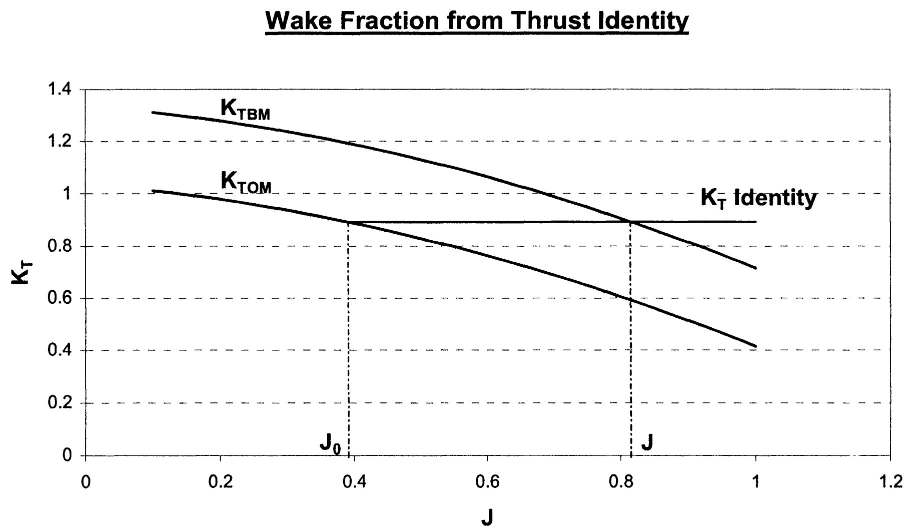


Fig 3.4

Unlike the thrust deduction fraction, the scale effect on wake fraction is large and cannot be ignored. Therefore in the ITTC 1978 method, the model scale wake fraction (w_{TM}) is extrapolated to the full scale (w_{TS}) using the relation:

$$w_{TS} = (t + 0.04) + (w_{TM} - t - 0.04)\,(C_{VS} / C_{VM}) \qquad \text{.....3.26}$$

where the viscous resistance coefficients are expressed as:

$$C_{VS} = (1 + k)\,C_{FS} + C_A \qquad \text{.....3.27}$$

$$C_{VM} = (1 + k)\,C_{FM} \qquad \text{.....3.28}$$

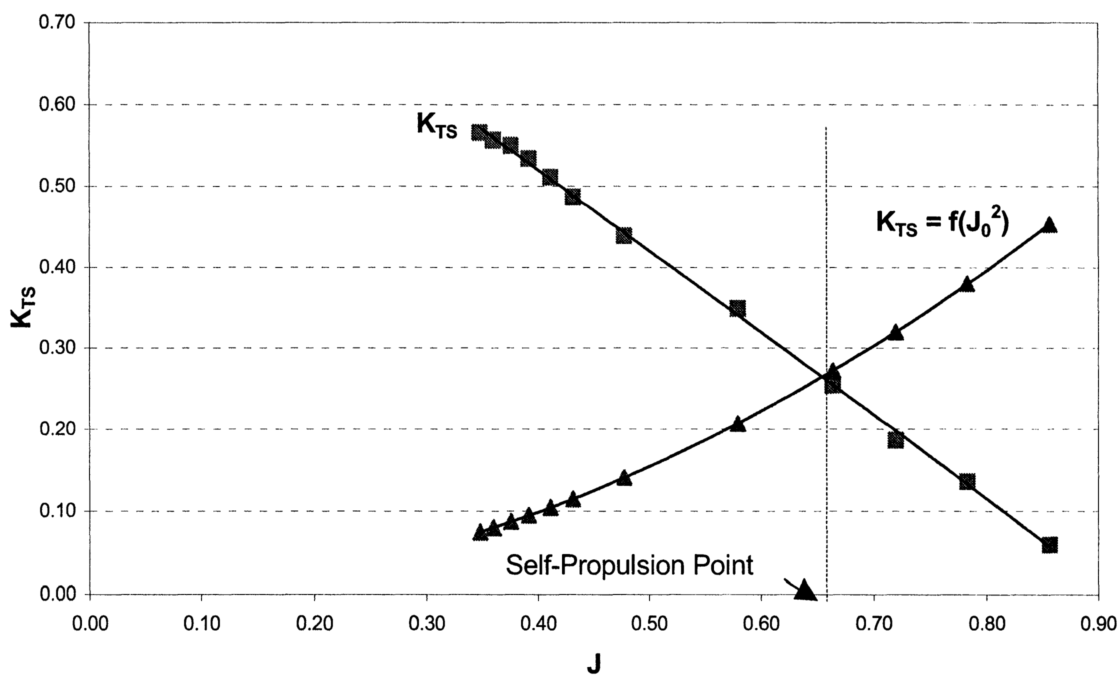
Next the relative rotative efficiency ($\eta_R = \eta_B / \eta_0$) is calculated using the relation:

$$\eta_R = \frac{K_{Q0M}}{K_{QBM}} \qquad \text{.....3.29}$$

where K_{Q0M} and K_{QBM} are determined at the advance ratio (J) corresponding to ship self-propulsion point.

Next, in order to determine the self-propulsion point (propeller operating point) of the full scale vehicle, a K_{TS}, K_{QS} vs J plot is made [Section 3.1.2]. An additional curve of K_{TS} as a function of J_0^2 [Eqn. 3.30] is also plotted [Fig 3.5].

$$\boxed{K_{TS} = \frac{S}{2\,D^2} \frac{C_{TS}}{(1-t)(1-w_{TS})^2} J_0^2} \qquad \text{.....3.30}$$



Self Propulsion Point of the Full Scale Vehicle: Fig 3.5

The intersection of the two K_{TS} curves represents the self-propulsion point (propeller operating point) of the full scale ship [Fig 3.5]. At this point, the values of J, K_{TS} and K_{QS} for the full scale ship are read off and are used to calculate the following power prediction factors:

$$\boxed{n_S = \frac{V_S}{J\,D_S}} \qquad .....3.31$$

$$\boxed{Q = \rho_S\, n_S^2\, D_S^5\, K_{QS}} \qquad .....3.32$$

$$\boxed{P_D = 2\,\pi\,\rho_S\, n_S^3\, D_S^5\, K_{QS}} \qquad .....3.33$$

$$\boxed{P_E = R_{TS}\, V_S} \qquad .....3.34$$

$$\boxed{\eta_0 = \frac{J_0\, K_{TS}}{2\,\pi\, K_{QS}}} \qquad .....3.35$$

$$\boxed{\eta_H = \frac{1 - t}{1 - w_{TS}}} \qquad .....3.36$$

$$\boxed{\eta_D = P_E / P_D} \qquad .....3.37$$

b) Speed-Varying Self-Propulsion Tests

In a speed-varying self-propulsion test, the model is towed at a number of different speeds so as to cover the entire speed range of the vehicle. At each vehicle speed, the propeller rotational speed is adjusted such that the towing force (F) becomes equal to the towing force at ship self-propulsion point (F_D) (see section 3.1.4 for a note on F_D). The model is then said to be operating at the ship self-propulsion point. At this point the following parameters are recorded:

- Model speed (V_M)
- Propeller thrust (T_M)
- Propeller torque (Q_M)
- Propeller rotational speed (n_M)
- Tow force (F)
- Water temperature (T^0)

The raw data are then converted to non-dimensional coefficients, which are then analyzed in a manner similar to that for the load-varying self-propulsion tests.

Comparison of the Load-Varying and Speed-Varying Self-Propulsion Tests

Speed-varying tests are less time consuming and cost less than load-varying tests. This is because speed-varying tests require a fewer number of runs in the tow tank. However, the propulsion factors derived from speed-varying tests are less realistic than those obtained from load-varying tests. In a load-varying test, the model propeller is subjected to a range of towing forces, at each vehicle speed. Therefore these tests are able to properly investigate the influence of propeller loading on the propeller-hull interaction effects. Hence the propulsion factors obtained are more realistic. In a speed-varying test, the model propeller is subjected to only one specific towing force (F_D). Therefore load dependent interaction effects between the propeller and the hull cannot be investigated by this method. Due to the above mentioned reasons, load-varying self-propulsion tests were employed for evaluating the propulsive performance of "C-SCOUT" (see chapter 4).

3.1.4 Towing Force at Ship Self-Propulsion Point (F_D)

From the resistance test results: $C_{TS} = (1+k)\,C_{FS} + C_{RS} + C_A + C_{AA}$

where: $C_{RM} = C_{TM} - (1 + k)\,C_{FM}$ and $C_{RM} = C_{RS}$

Substituting and solving:

$$\boxed{C_{TS} = C_{TM} - [\,(1+k)\,(C_{FM} - C_{FS}) - C_A - C_{AA}\,]} \quad \text{.....3.38}$$

This basically shows that the coefficient of total resistance of the ship (C_{TS}) is smaller than that of the model (C_{TM}) by the skin friction correction (C_{FM} - C_{FS}) times (1+k) minus the correlation allowance (C_A) and the coefficient of air resistance (C_{AA}). Therefore in a self-propulsion test, at the model self-propulsion point, the model propeller is actually working at a higher value of the thrust loading coefficient ($C_{th} = T_M / (½\, \rho\, D^2\, V_A^2)$) than the full scale ship propeller. Due to this extra loading, the model propeller's efficiency is lower than that of the ship's propeller. To compensate for this difference, the self-propulsion point for the model is instead defined at the ship self-propulsion point. At this point, the towing force is:

$$\boxed{F_D = ½\, \rho\, S_M\, V_M^2\, C_{FD}} \quad \text{.....3.39}$$

where:

$$\boxed{C_{FD} = (1+k)\,(C_{FM} - C_{FS}) - C_A - C_{AA}} \quad \text{.....3.40}$$

This ensures that at the self-propulsion point, the model propeller and the ship propeller are both working at the same thrust loading coefficient and that both propellers are working under dynamically similar conditions.

3.2 Powering Prediction Using Load-Varying Self-Propulsion Tests Alone

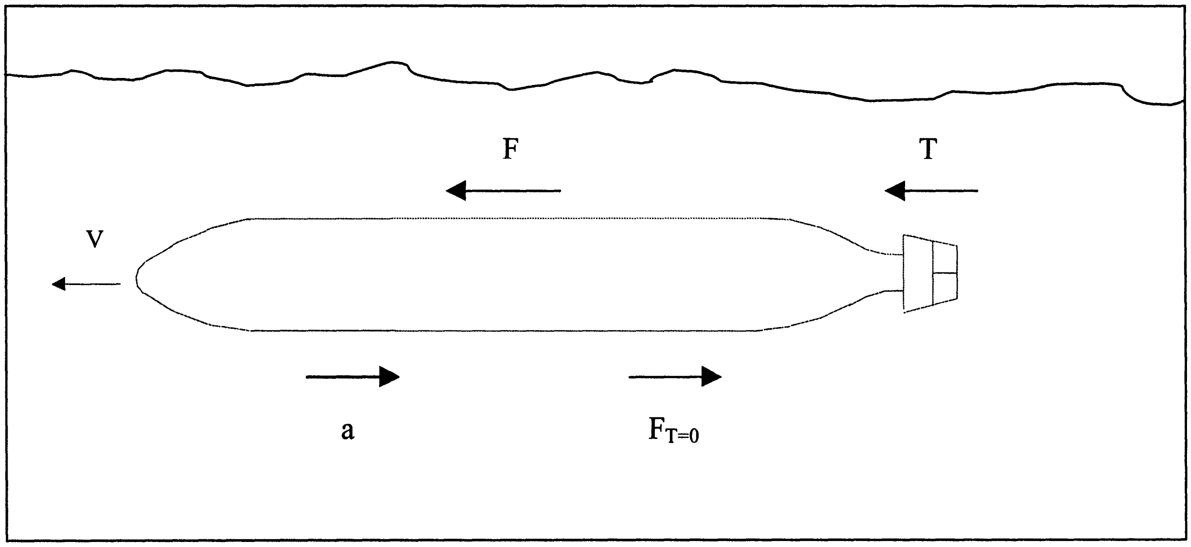
3.2.1 Rationale

The ITTC 1978 Performance Prediction Method gives fairly accurate powering estimates for cases when either there is a weak interaction between the propulsor and the vehicle hull [Ref 3-5] or when the vehicle is of low / moderate power [Ref 3-2]. In cases when there is a strong interaction between the propulsor and the hull, the powering estimates using the ITTC 1978 method are questionable.

Strong propulsor-hull interaction occurs in cases when compound propulsors such as ducted propellers, podded propellers, etc., are used. Unlike conventional propellers, compound propulsors can strongly modify the flow pattern around the hull. This means that methods such as the ITTC 1978 method, which make use of results from tests on the vehicle hull and propulsor in isolation, actually ignore the effect of an altered flow pattern around the hull, consequently leading to inaccuracies in the powering estimates.

Kracht [Ref 3-2] and Schmiechen [Ref 3-1], in an effort to overcome these shortcomings, suggested that powering prediction be based on only one set of experiments; load-varying self-propulsion tests. In these tests, the hull and propulsor are treated as one physical unit. Preliminary work on this concept was carried out by Kracht [Ref 3-2], Schmiechen [Ref 3-1], Holtrop [Ref 3-3], Bose and Molloy [Ref 3-4], etc. The following section discusses the theory behind this method and highlights the salient points of this method.

3.2.2 Theory



Forces Acting on a Submerged Vehicle: Fig. 3.6

Tow Force – Thrust Equation

In a load-varying self-propulsion test [Fig 3.6], consider the case when the vehicle is moving ahead at constant speed (V), under steady-state conditions. The following forces act on the body:

F : Towing force

T : Propeller thrust

$F_{T=0}$: Total resistance of the vehicle under unloaded / idling propeller conditions

t^* : Thrust deduction fraction (calculated using self-propulsion test results alone)

a : Augment in resistance ($a = t^* T$)

Equilibrium of forces leads to the relationship:

$T + F = F_{T=0} + a$

Substituting the value of "a" and rearranging the terms:

$T - t^* T = F_{T=0} - F$

or

$$T(1 - t^{*}) = F_{T=0} - F$$

or

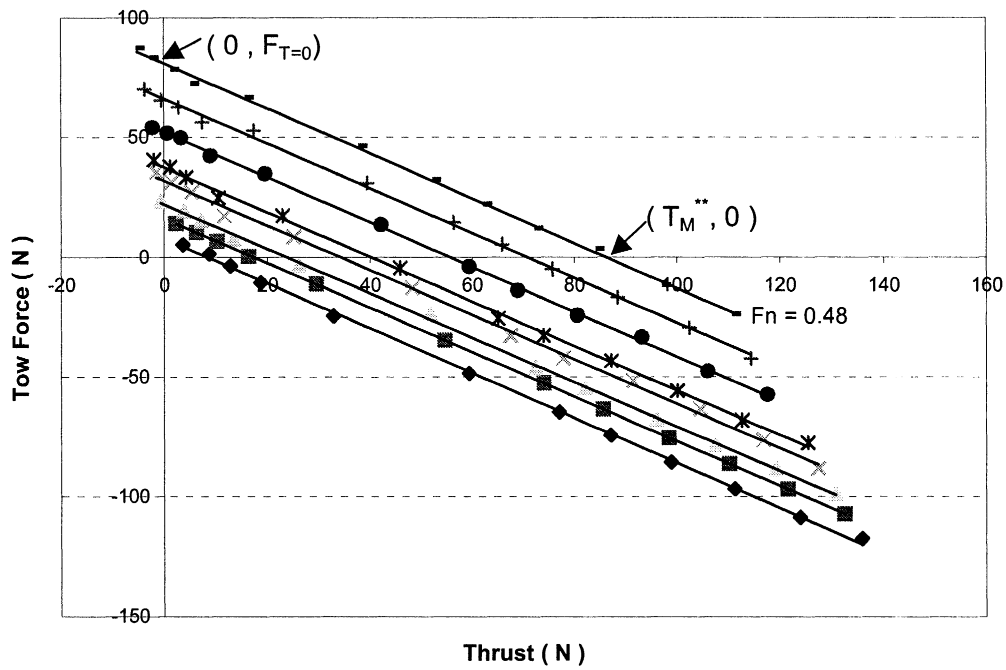
$$\boxed{F = -(1 - t^{*})\,T + F_{T=0}} \qquad \text{Tow force – thrust equation} \qquad \text{.....3.41}$$

For most marine vehicles, the thrust deduction fraction (t^{*}) is independent of propeller (thrust) loading. Assuming $F_{T=0}$ is constant for variable propeller loading, differentiation of the tow force – thrust equation [Eqn. 3.41] with respect to thrust, leads to :

$$\boxed{\frac{dF}{dT} = -(1 - t^{*}) = \text{Constant (for variable propeller loading)}} \qquad \text{.....3.42}$$

This basically shows that the tow force – thrust equation [Eqn. 3.41] is linear over a range of propeller loadings. It therefore represents a straight line, having an equation of the form $y = mx + c$, where "m" is the slope of the line and "c" is the intercept it makes on the y axis [Fig 3.7].

The linear relationship between tow force (F) and thrust (T) generally holds good for all kinds of hull forms and propulsors. Only for the fullest hull forms with relatively small propulsors, could there be a tendency for a slight departure from linearity [Ref 3-3].



Tow Force vs Thrust: Fig. 3.7

Thrust Deduction Fraction (t^*)

From the tow force – thrust plot [Fig 3.7], the slope of each straight line can be seen to be:

Slope = $\dfrac{-F_{T=0}}{T_M^{**}}$ where T_M^{**} is the model thrust at model self-propulsion point.

Also as calculated previously, the slope of the line is also equal to:

$$\frac{dF}{dT} = -(1 - t^*) = \text{Slope}$$

Equating the two:

$$\frac{F_{T=0}}{T_M^{**}} = -(1 - t^*) = \frac{dF}{dT}$$

or

$$\boxed{t^* = \frac{1 - F_{T=0}}{T_M^{**}} = 1 + \frac{1}{dT\,/\,dF}}$$ 3.43

Also from the ITTC 1978 method [Eqn 3.24], the thrust deduction fraction (t) is defined as:

$$\boxed{t = \frac{T_M^* - R_{TM}}{T_M^*}}$$

From the above two equations, it can be observed that in order to calculate the thrust deduction fraction (t) as defined by ITTC 1978, the results from two independent tests (resistance and self-propulsion tests) are needed. On the other hand, in order to calculate the thrust deduction fraction (t^*) as defined by the load-varying self-propulsion test method, results from only one test are needed. The forces $F_{T=0}$ and T_M^{**} that are used for calculating t^* belong to the same running condition. Therefore no falsification of the value of t^* is possible, on account of usage of test results from tests that are not performed under the exactly same conditions. Hence the value of the thrust deduction fraction (t^*) as determined by the load-varying self-propulsion test method, is likely to be more realistic than the value of the thrust deduction fraction (t) as determined by the ITTC 1978 method.

3.2.3 Method of Analyzing the Test Results

1) *Determining the Model Propulsor Thrust (T_M^*) at Ship Self-Propulsion Point ($F = F_D$)*

After conducting a load varying self-propulsion test, the test results are used to make a plot between tow force (ordinate) and thrust (abscissa), at each Froude number at which the tests were conducted [Fig 3.7]. The relationship between tow force and thrust is linear for this submersible ("C-SCOUT"). Using regression analysis, straight lines are fitted through the data points and the equation of each line is determined. The equation of each line is of the form:

$y = mx + c$

where "m" is the slope and "c" the intercept that the line makes on the y axis. From the discussion in the previous section, we know that the equation of each line can also be written as:

$F = - (1 - t^*) T + F_{T=0}.$

By comparing the two equations, the value of t^* and $F_{T=0}$ can be determined:

$t^* = 1 + m$

$F_{T=0} = c$

The values of t^* and $F_{T=0}$ are then substituted into the tow force – thrust equation [Eqn 3.41]. Now in order to determine the propulsor thrust at the ship self-propulsion point (T_M^*), we need to substitute that value of tow force that exists at the ship self-propulsion point (F_D) [Sec 3.1.4], into the tow force – thrust equation. The value of the propulsor thrust at ship self-propulsion point (T_M^*) is then obtained as:

$$\boxed{T_M^* = \frac{F_{T=0} - F_D}{1 - t^*}}$$

where:

$$\boxed{F_D = \tfrac{1}{2}\, \rho\, S_M\, V_M^2\, C_{FD}}$$

$$\boxed{C_{FD} = (1+k)\,(C_{FM} - C_{FS}) - C_A - C_{AA}}$$

The values of "C_A" and "C_{AA}" are calculated using databases of model test facilities (possibly created using regression analysis of results from previously conducted tests). The values of the frictional coefficients "C_{FM}" and "C_{FS}" are determined using a turbulent flat plate friction line such as that formulated by Schlichting [Ref 3-8] or by Grigson [Ref 3-9], instead of the traditional ITTC 1957 model – ship correlation line. The value of the form factor " k " is determined using low-speed load-varying self-propulsion tests [Ref 3-3].

2) *Determining the Ship Propulsor Thrust (T_S^*) at Ship Self-Propulsion Point ($F = F_D$)*

The model propulsor thrust (T_M^*) is then extrapolated to the ship propulsor thrust (T_S^*), using the relationship:

$$\boxed{T_S^* = T_M^*\, \lambda^3\, \rho_S / \rho_M} \qquad .....3.45$$

where λ is the scale ratio ($\lambda = D_S / D_M$).

The above relationship is derived by dividing the expression for ship propulsor thrust by that for model propulsor thrust and by assuming that Froude scaling is valid for speed ($V_S / V_M = \lambda^{1/2}$) and that the advance coefficient (J) and the thrust coefficients (K_{TBS} and K_{TBM}) are the same for the model and the ship.

3) *Determining the Non-Dimensional Coefficients K_{TBM}, K_{QBM}, J_M and Converting them to their Full Scale Values K_{TS}, K_{QS}, J_S*

The raw data are then non-dimensionalized into the advance coefficient (J_M), thrust coefficient (K_{TBM}), torque coefficient (K_{QBM}) and the tow force coefficient (K_{FDM}).

$$J_M = \frac{V_M}{n_M D_M} \qquad \text{.....3.46}$$

$$K_{QBM} = \frac{T_M}{\rho_M n_M^2 D_M^4} \qquad \text{.....3.47}$$

$$K_{TBM} = \frac{Q_M}{\rho_M n_M^2 D_M^5} \qquad \text{.....3.48}$$

These model coefficients then need to be converted to their full scale values K_{TS}, K_{QS}, J_S. This is achieved by performing the following two corrections:

a) Correction for Change in the Frictional Coefficient Over the Surface of the Propeller Blades

The difference in the propeller Reynolds numbers (Rn) at which the model is tested and at which the actual ship operates, causes a change in the frictional coefficient over the surface of the propeller blades. This can be corrected for using the correction factors [Ref 3-6]:

$$K_{TS} = K_{TBM} - \delta K_T$$

$$K_{QS} = K_{QBM} - \delta K_Q$$

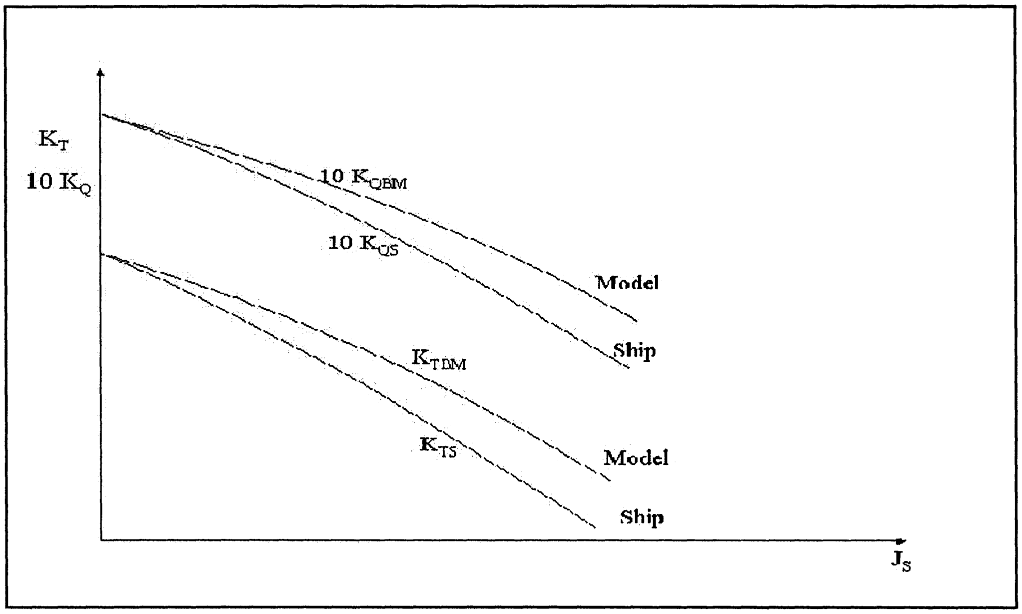
The values of the correction factors δK_T and δK_Q are determined using the expressions given in section 3.1.2. However it should be noted that these correction factors are now applied to the propeller coefficients in the behind condition (unlike the ITTC 1978 method, where they are applied to the open water propeller coefficients). These corrections cause a vertical shift (downwards) in the K_{TS} and K_{QS} curves [Fig 3.8].

b) *Correction for Wake Scaling*

Since the model is tested at a much lower propeller Reynolds number than the actual ship (due to practical constraints at the model testing facilities), therefore the inflow velocity (V_A) at the model's propeller is lower than that at the ship's propeller. Hence the effective wake fraction at the ship's propeller is lower than that at the model's propeller. Therefore at the thrust (T_M^*) corresponding to the ship self-propulsion point, the thrust loading coefficient (C_{TH}) of the propeller is higher than that of the ship. This causes the model propeller's efficiency to drop to a value lower than that of the ship's propeller. To correct for this effect, the advance coefficient of the full scale propeller is corrected using the expression [Ref 3-3]:

$$\boxed{J_S = J_M \ (1 - w_M) \,/\, (1 - w_S)} \qquad .....3.49$$

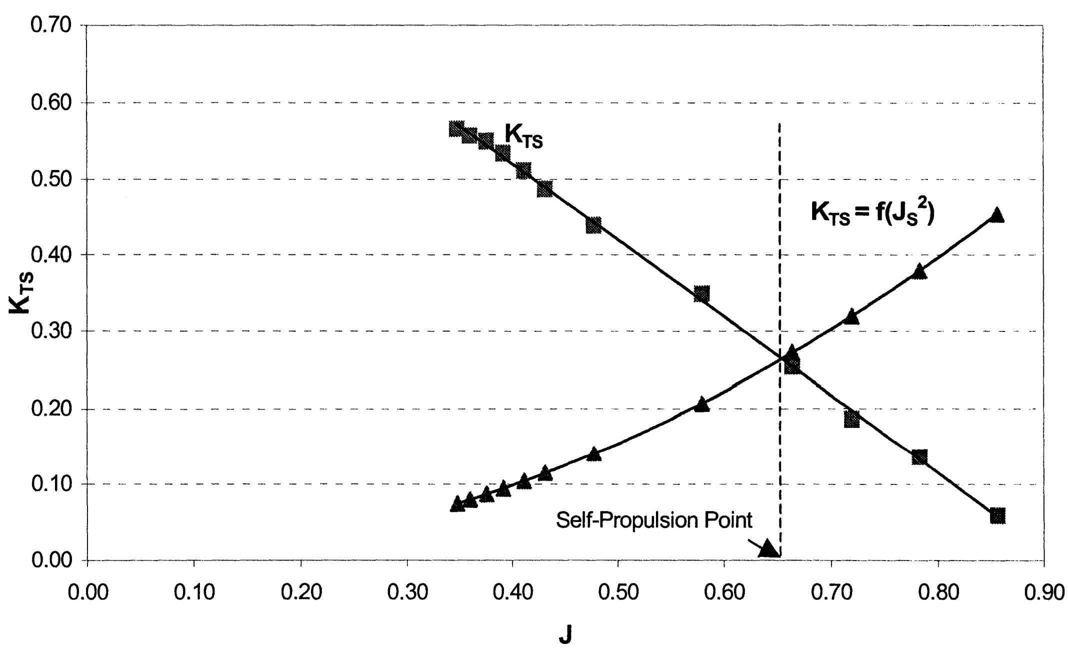
This correction causes the K_{TS} and K_{QS} curves to shift towards the left [Fig 3.8]. The value of the scaling factor "$(1 - w_M)/(1 - w_S)$" required for this correction, can be estimated using databases of model testing facilities (for similar ships / submersibles).



Shift in K_T , 10 K_Q Curves: Fig. 3.8

4) *Determining the Self-Propulsion Point of the Full Scale Vehicle*

A plot is made between K_{TS}, K_{QS} vs J_S. On this plot an additional curve of K_{TS} as a function of J_S^2 is plotted [Fig 3.9]:



Self-Propulsion Point of the Full Scale Vehicle: Fig. 3.9

where:

$$K_{TS} = \frac{T_S^*}{\rho_S\ D_S^2\, V_S^2}\ J_S^2 \qquad .....3.50$$

The intersection of the two K_{TS} curves gives the ship self-propulsion point (propeller operating point of the ship). At this point the values of K_{QS} and J_S are read off and are used to calculate the power prediction quantities of the full scale vehicle.

5) *Determining the Power Prediction Factors of the Full Scale Vehicle*

These are calculated using the following expressions

$$n_S = \frac{V_S}{J_S\, D_S} \qquad .....3.51$$

$$Q = \rho_S\, n_S^2\, D_S^5\, K_{QS} \qquad .....3.52$$

$$P_D = 2\,\pi\,\rho_S\, n_S^3\, D_S^5\, K_{QS} \qquad .....3.53$$

$$P_E = ½\, C_{TS}\, \rho_S\, S_S\, V_S^3 \qquad .....3.54$$

$$\eta_D = P_E / P_D \qquad .....3.55$$

3.3 "C-SCOUT" Tests

The propulsive performance of the autonomous underwater vehicle "C-SCOUT" was assessed by using the load-varying self-propulsion test method (see Chapter 4). For these tests, a full-scale version of the "C-SCOUT" vehicle was used. This vehicle was outfitted with a propulsion system and submerged 0.91 meters below the surface of water. The test results were analyzed using the method described in section 3.2.3. A comparative analysis of the propulsive performance of "C-SCOUT" using the ITTC 1978 method, could not be attempted, since results from one of the three tests required by this method (the propeller open water test), were not available.

3.4 Testing Prototype Vessels and Scale Models

If a scale model of "C-SCOUT" had been used for these tests, instead of the actual vehicle (prototype vessel), then the following would have to be considered:

1) When testing a scale model, at model self-propulsion point, the model propeller is actually working at a higher value of the thrust loading coefficient, than the propeller of the prototype vessel. To compensate for this difference, the self-propulsion point of the model is instead defined at the self-propulsion point of the prototype vessel. For a detailed explanation, please refer to section 3.1.4.

2) Due to practical constraints at model testing facilities, the scale model is tested at a much lower Reynolds number than the prototype vessel. This causes the prototype vessel to have a smaller wake fraction and a higher propeller entrance velocity, which in turn results in the model propeller having a lower efficiency than the full scale propeller. To compensate for this effect, the advance coefficient of the full scale propeller is corrected.

This correction causes the K_{TS} and K_{QS} curves to shift towards the left [Fig. 3.8]. For a detailed explanation, please refer to section 3.2.3 3.b.

3) The difference in the Reynolds number at which the model is tested and at which the prototype vessel operates, causes a change in the frictional coefficient over the surface of the propeller blades. This is corrected for using correction factors, which then cause a vertical shift in the K_{TS} and K_{QS} curves [Fig. 3.8]. For a detailed explanation, please refer to section 3.2.3 3.a.

3.5 Conclusions

With the increasing use of non-conventional / compound propulsors, there has been an increased demand for an accurate power prediction method. The accuracy of the power prediction results provided by the ITTC 1978 method has not been very satisfactory when compound propulsors have been used [Ref 3-5]. This is because the ITTC 1978 method makes use of results from tests in which the hull and propulsor are tested in isolation of each other, thus ignoring interaction effects between the hull and propulsor. For accurate results, it is therefore preferable to employ the load-varying self-propulsion test method for such cases.

Chapter 4

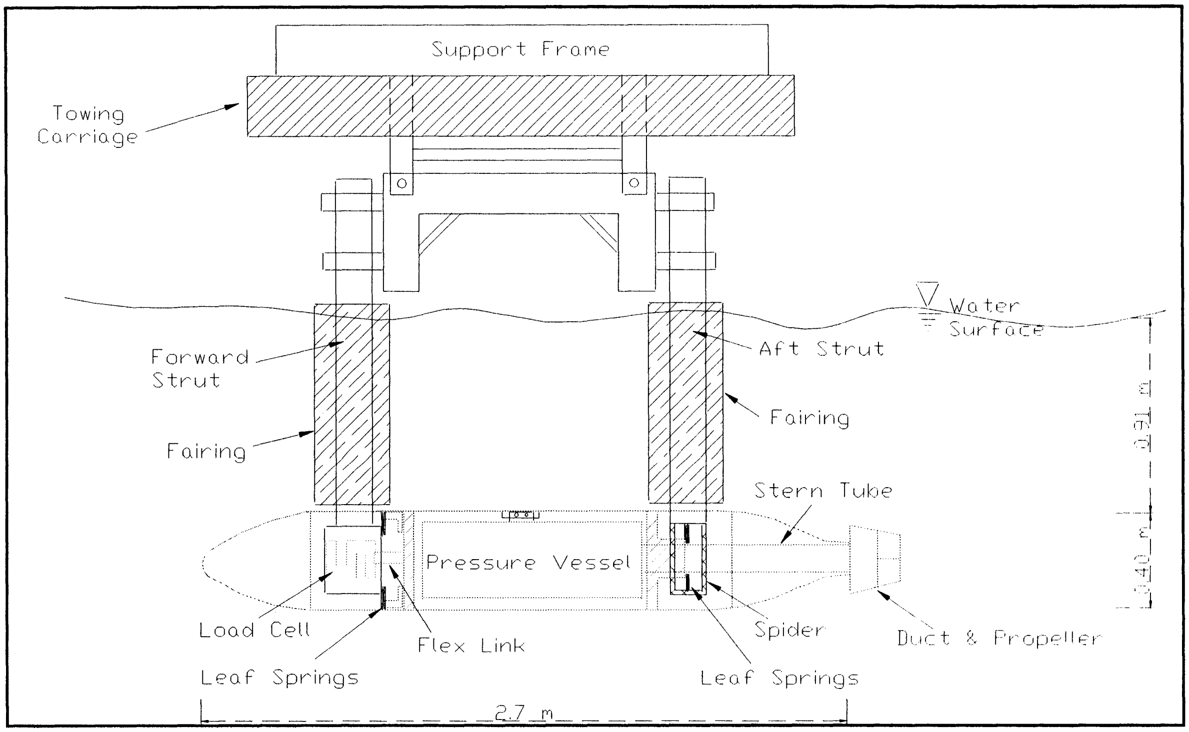
Resistance and Propulsion Tests

Introduction

All AUV designers strive to improve the endurance of their AUVs. Endurance refers to the range (distance) or time that an AUV can travel at sea, while working on a mission, given a fixed amount of energy which is stored on board the vehicle. Endurance is a measure of the overall efficiency of an AUV. Two components that contribute to this overall efficiency are resistance and propulsion. The lower the vehicle resistance, the higher its endurance. Similarly, the higher the efficiency of the propulsion system, the higher is the vehicle endurance. In order to improve the efficiency of the propulsion system and reduce the resistance of the vehicle, one must first be able to quantify what their values are, hence resistance and self-propulsion tests are one step in gaining this information. Resistance, self-propulsion and bollard-pull tests were conducted, on a full scale version of the "C-SCOUT" vehicle, at the towing tank (54.7 x 4.57 x 2.5 m) of the Ocean Engineering Research Centre (OERC) at Memorial University.

4.1 Test Setup

A bare-hull version of the "C-SCOUT" vehicle (2.70 m long, 0.40 m diameter) was outfitted with a propulsion system for these tests. The vehicle was immersed 0.91 m below the water surface [Figs. 4.1 and 4.2]. Two faired vertical struts, one at each end of the vehicle, were used to support the vehicle in the water. The top ends of these struts were attached to a support frame, which was attached to the towing carriage. The bottom ends of the struts were attached to the vehicle forward and aft through flexible supports.



Sketch of Test Setup: Fig. 4.1

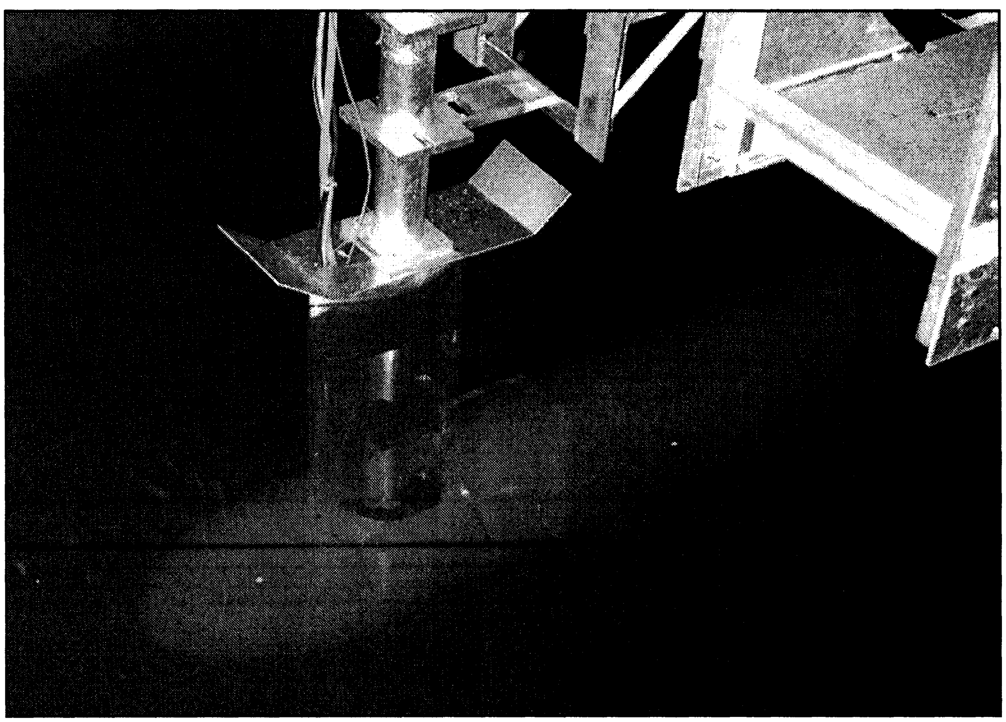


Photo of Test Setup: Fig. 4.2

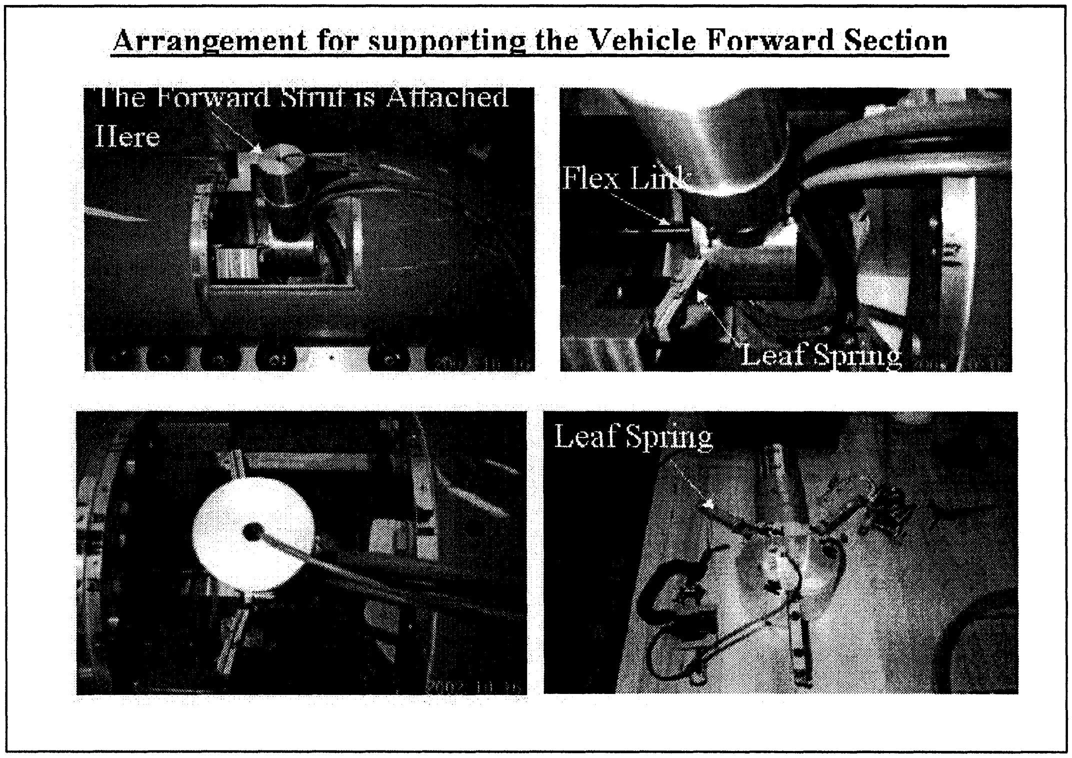
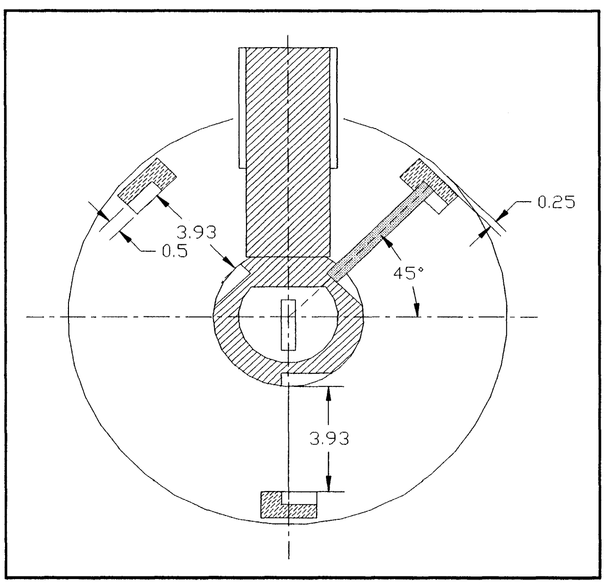
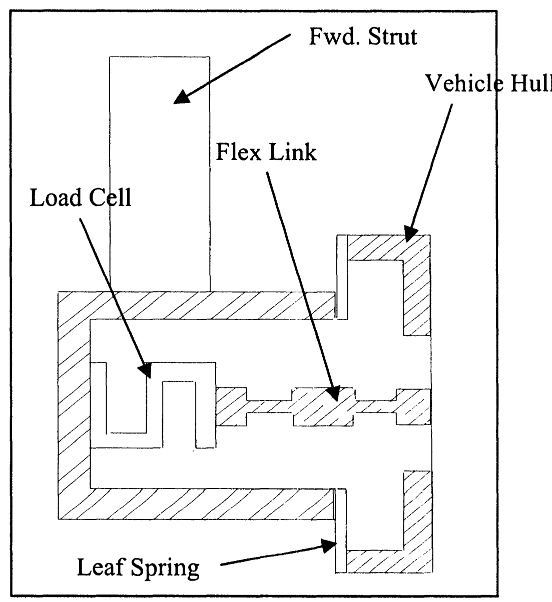


Fig. 4.3



Forward Flexible Support: Fig 4.4

(all dimensions in inches)



Forward Flexible Support: Fig 4.5

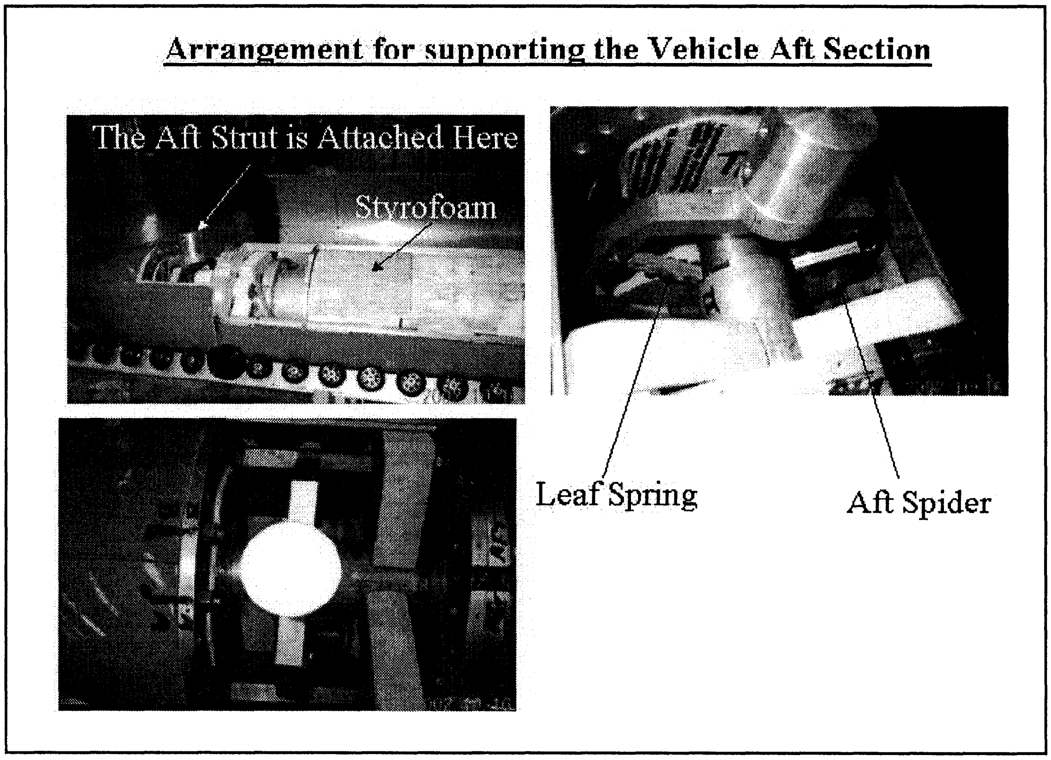
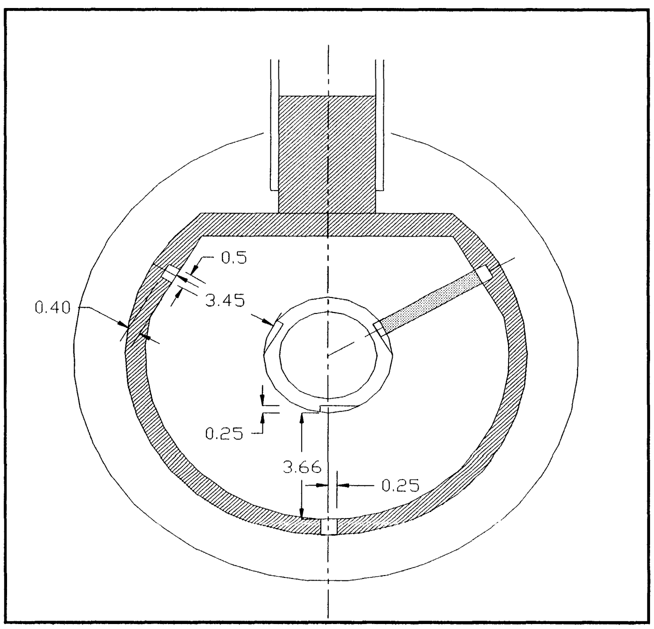
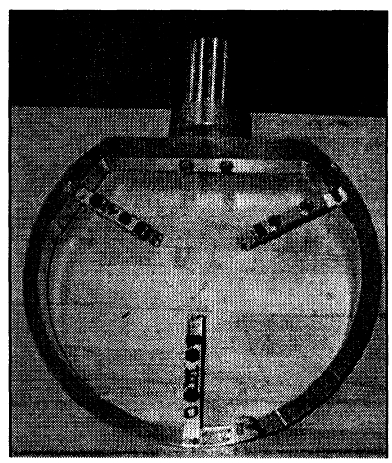


Fig. 4.6

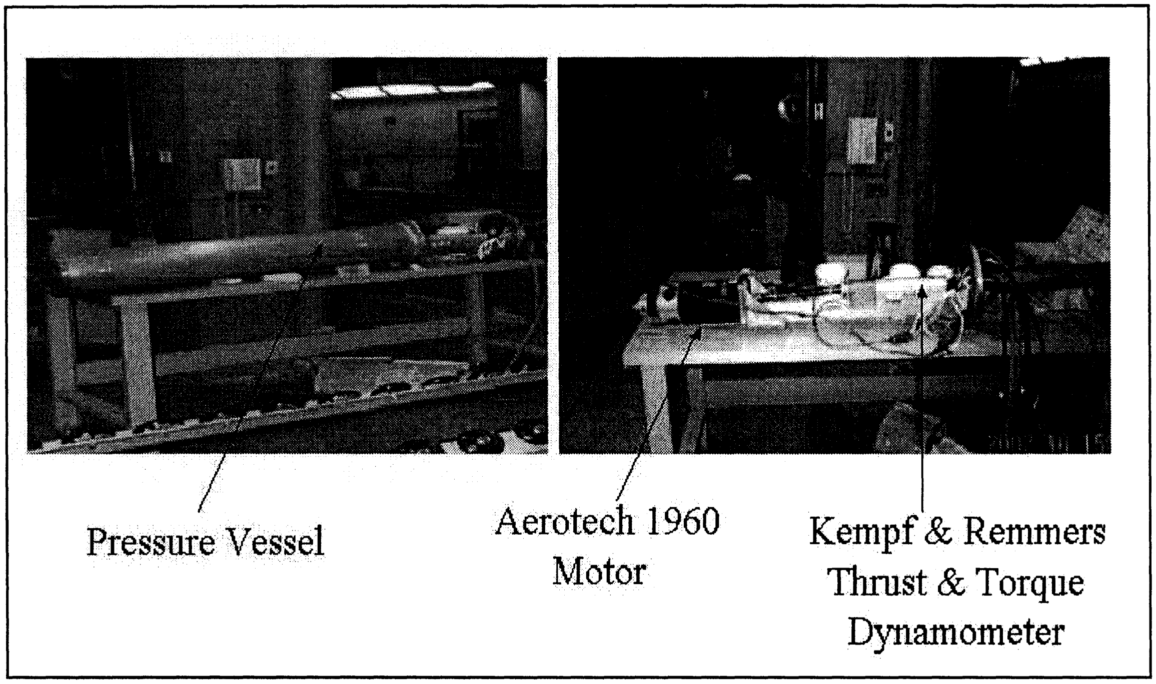


Aft Flexible Support: Fig. 4.7

(all dimensions in inches)



Aft Flexible Support: Fig. 4.8



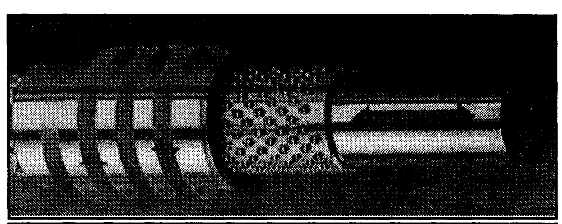
Pressure Vessel: Fig. 4.9

Each of these flexible supports had three leaf springs mounted in a Y-shaped configuration [Figs. 4.3 to 4.8]. For the forward flexible support [Figs. 4.3 to 4.5], the outer end of each leaf spring was attached to the vehicle hull; the inner end of each leaf spring was attached to the load cell housing. For the aft flexible support [Figs. 4.6 to 4.8], the outer end of each leaf spring was attached to a ring, which was in turn attached to the aft strut; the inner end of each leaf spring was attached to the vehicle hull. These leaf springs constrained the vehicle to move only forward and aft, relative to the struts.

The forward flexible support housed a single-axis load cell (450 N capacity). This load cell was attached between the forward strut and the hull, in order to measure the force required to tow the vehicle through the water. Inside the vehicle, in the free space between the forward and aft flexible supports, a pressure vessel was fitted [Fig. 4.9]. This pressure vessel housed the main propulsion motor (Aerotech model 1960 DC motor) and a thrust and torque dynamometer (Kempf and Remmers model 25). The propulsion motor had an built-in tachometer for measuring the rotational speed of the propeller shaft. This motor was used for the self-propulsion tests and is not the motor used on the regular vehicle for propulsion.

The pressure vessel was charged with nitrogen gas (15 kPa), so as to keep the internal pressure of the cylinder slightly higher than the outside water pressure and thereby preclude the possibility of water ingress into the pressure vessel, in case of leakage at the stern tube seal. Nitrogen gas was supplied to the pressure vessel using a portable nitrogen bottle placed on the towing carriage; a flexible hose supplied gas from the bottle to the pressure vessel.

The stern tube housed the propulsion shafting. The hardened steel propeller shaft was coupled to the thrust and torque dynamometer at the inboard end and had the propeller mounted on its outboard end. The shaft was supported by three linear rotary bearings [Fig. 4.10], fitted at equal distances apart, inside the stern tube. The linear rotary bearings permitted the shaft to rotate, as well as to slide freely in the axial direction. This ensured that all thrust developed by the propeller was transmitted directly to the dynamometer and that there was no substantial loss of measured thrust due to over-tight bearings.



Linear Rotary Bearing: Fig. 4.10

At the location where the propeller shaft penetrated the cover plate of the pressure vessel, two lip seals (fitted back to back) were used for sealing purposes. The propeller, hub and accelerating duct used for these tests were identical to that of the commercially available Tecnadyne model 1020 thruster, used on the underwater vehicle "C-SCOUT".

An Omega OM3 series signal conditioner was used for amplifying and filtering the analogue output signals from the different transducers (load cell, dynamometer, tachometer). A DaqBoard 2000 data acquisition card, fitted into the PCI slot of a Pentium desktop computer, was used as the data acquisition system. This card was controlled and operated by the DaqView data acquisition software. The DaqBoard 2000 card had a built-in 16 bit, 200 kHz analog-to-digital converter, which converted the continuous analogue signals from the signal conditioner into digital signals. The acquired data were processed and stored on the hard drive of the computer in ASCII format.

4.2 Test Procedure

The test procedure involved the following steps:

- Calibrating the transducers
- Preparing the vehicle
- Attaching the vehicle to the struts
- Performing the in-situ tests / checks
- Performing the resistance and propulsion tests

4.2.1 Calibrating the Transducers

The load cell and the dynamometer were mechanically calibrated by subjecting them to known loads and torques [Figs. 4.11 and 4.14] and then noting the output voltages of these transducers. The output voltages (abscissa) were then plotted against the applied loads or torques (ordinate) and straight lines were fitted through the data points using regression analysis [Fig. 4.12, 4.13 and 4.15]. The equations of these straight lines were then used for directly converting the output voltages of the transducers into engineering units (N, N-m, etc.), during the course of the actual tests.

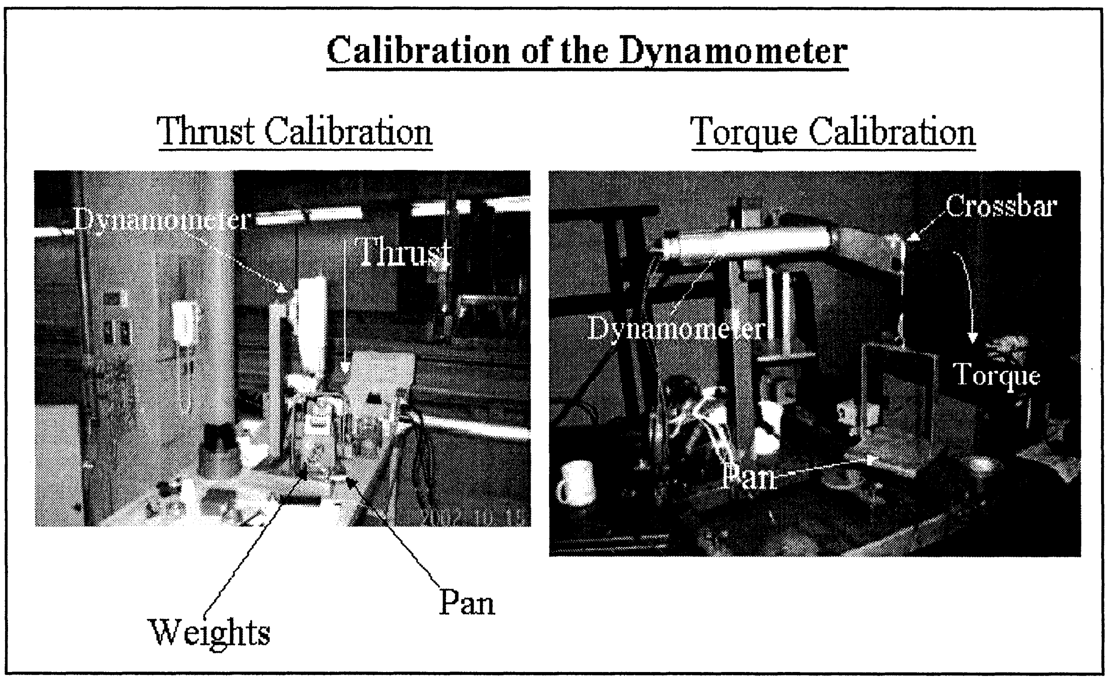


Fig. 4.11

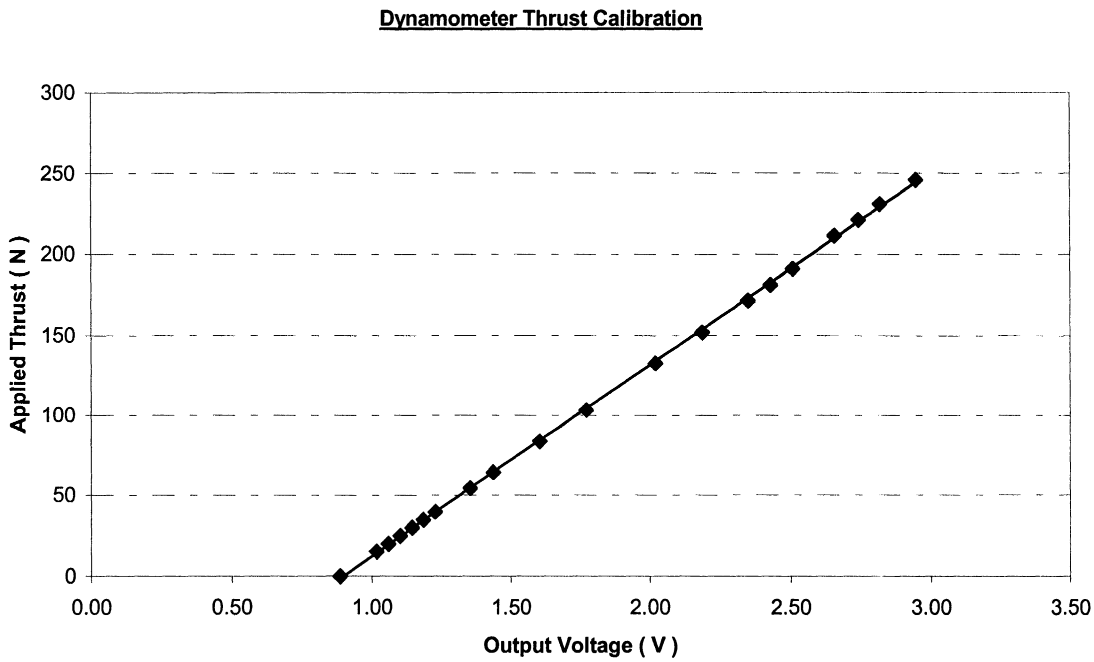


Fig. 4.12

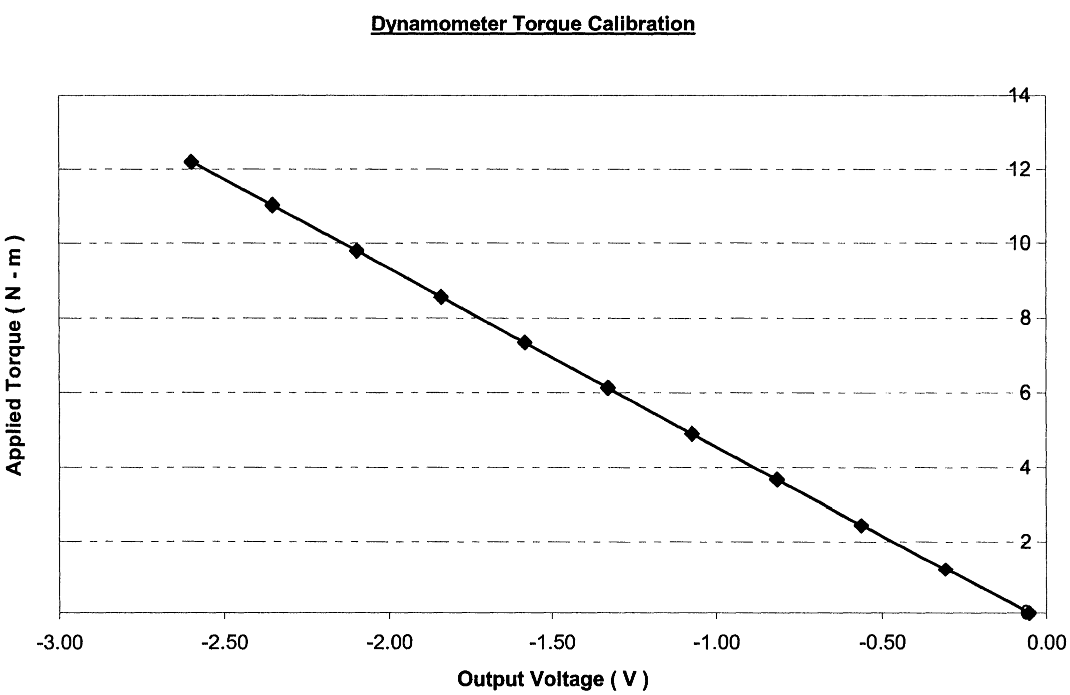


Fig. 4.13

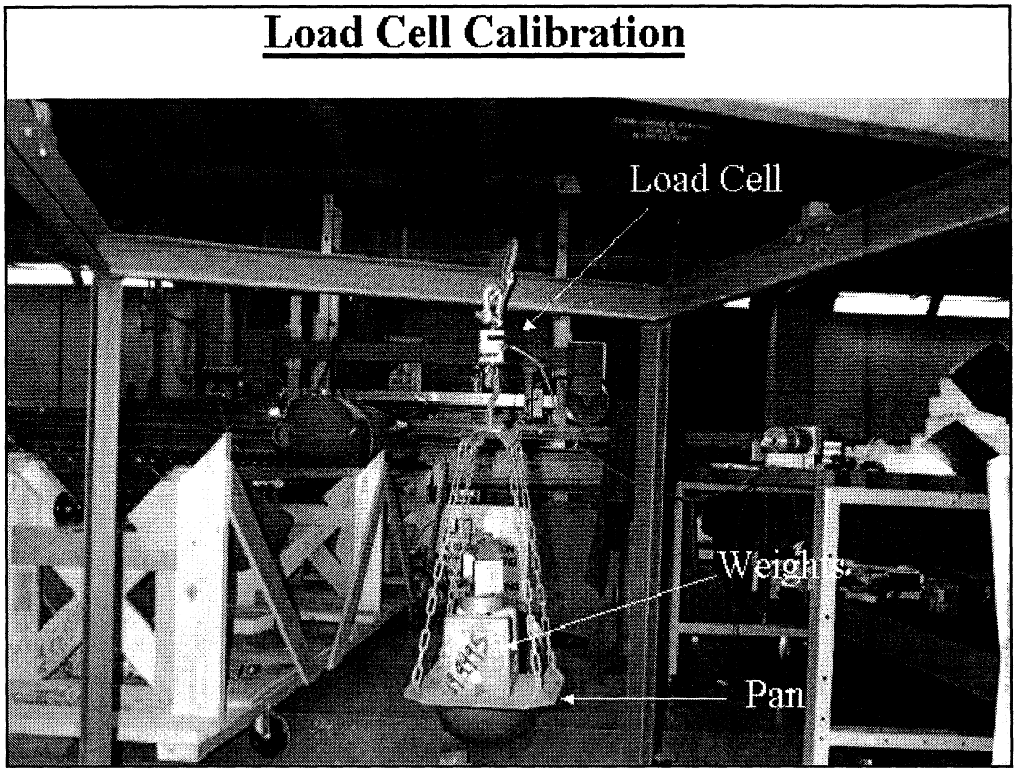


Fig. 4.14

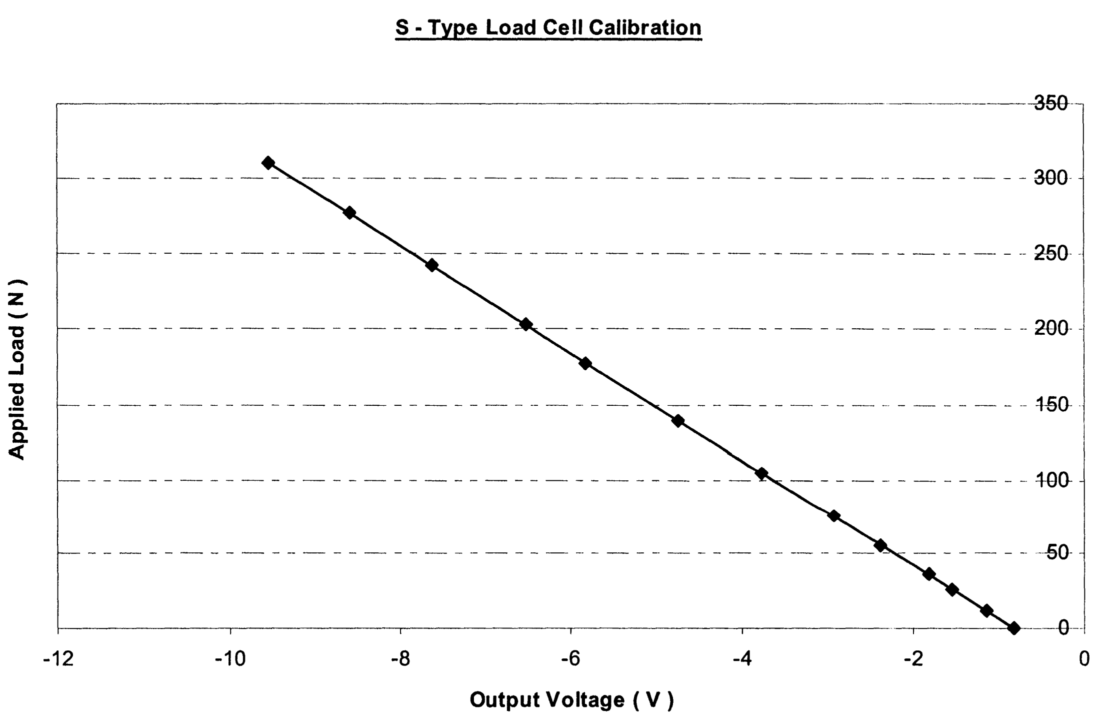


Fig. 4.15

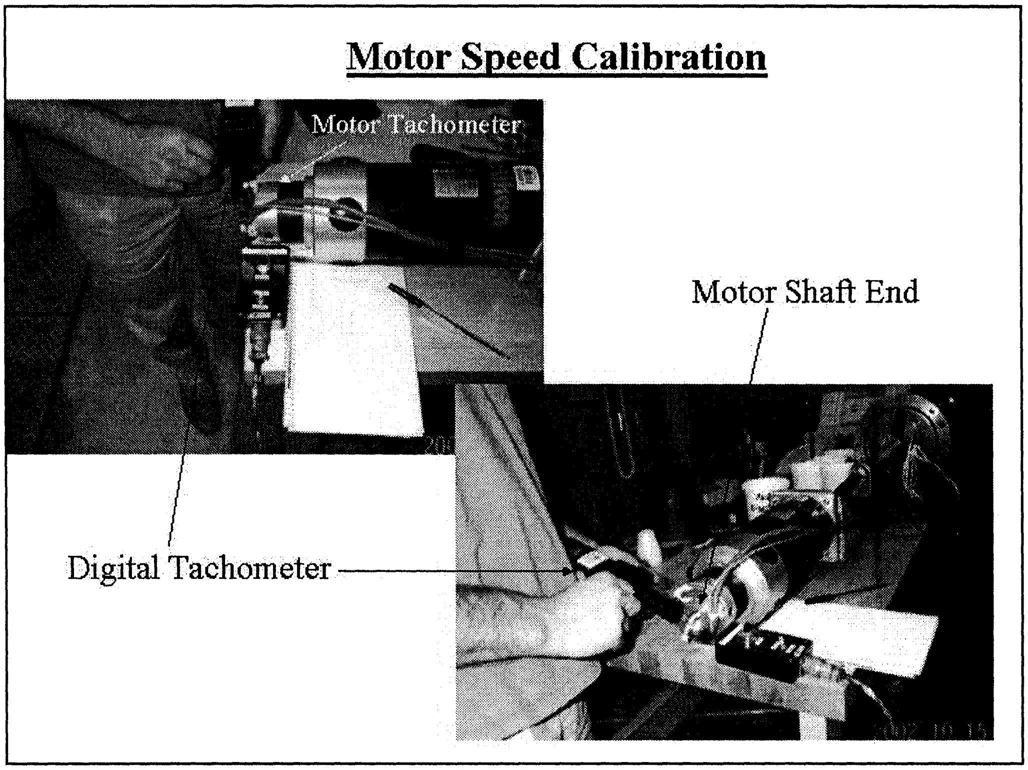


Fig. 4.16

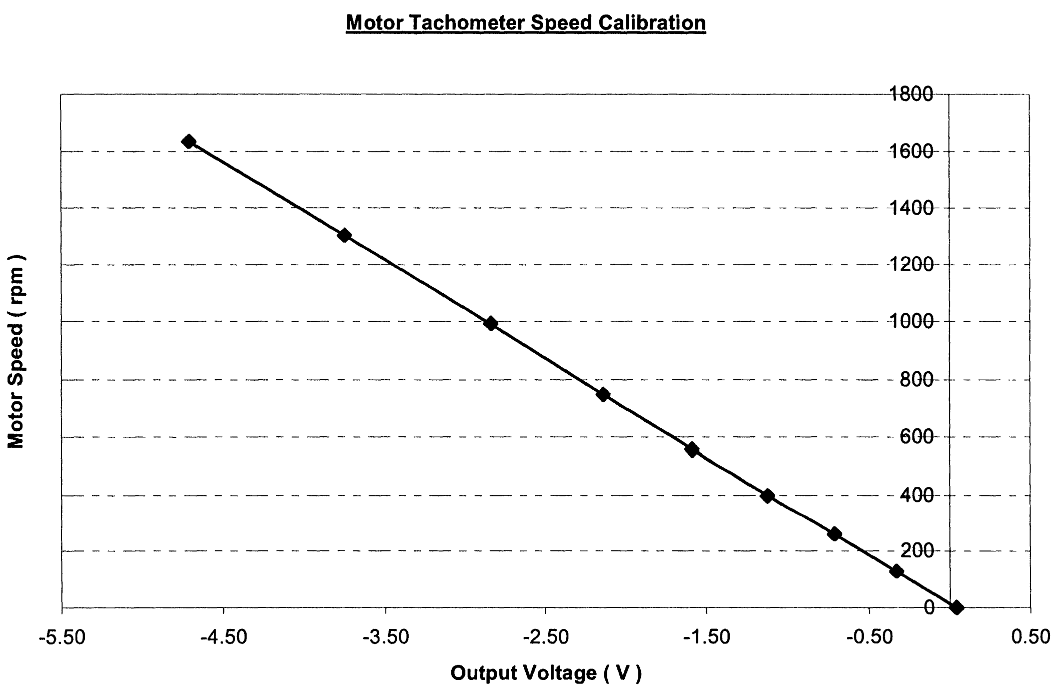


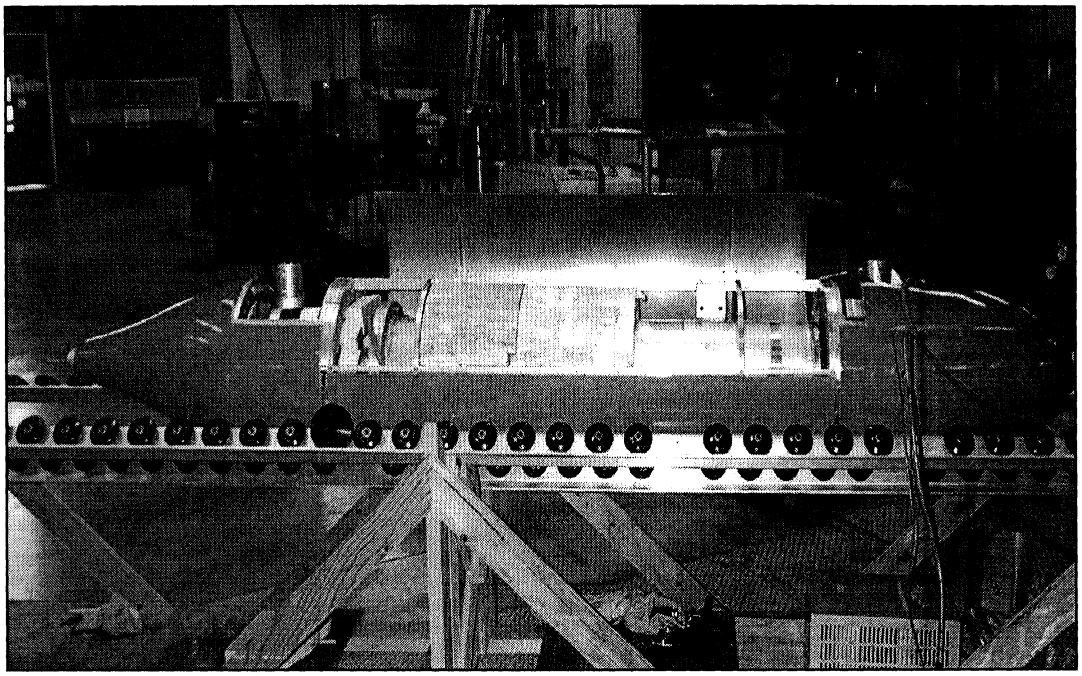
Fig. 4.17

The propulsion motor tachometer [Fig 4.16] was calibrated by running the motor at a number of speeds and recording the output voltage of the tachometer; the motor speed was measured using a portable optical tachometer. The output voltages (abscissa) of the motor tachometer were then plotted against the motor speed (ordinate) and a straight line was fitted through the data points using regression analysis [Fig. 4.17]. The equation of this straight line was then used for directly converting the output voltages of the tachometer into engineering units (rpm), during course of the actual tests. All calibrations were carried out using the same data acquisition system that was used for the actual tests. This prevents possible errors due to different electronic circuits being used.

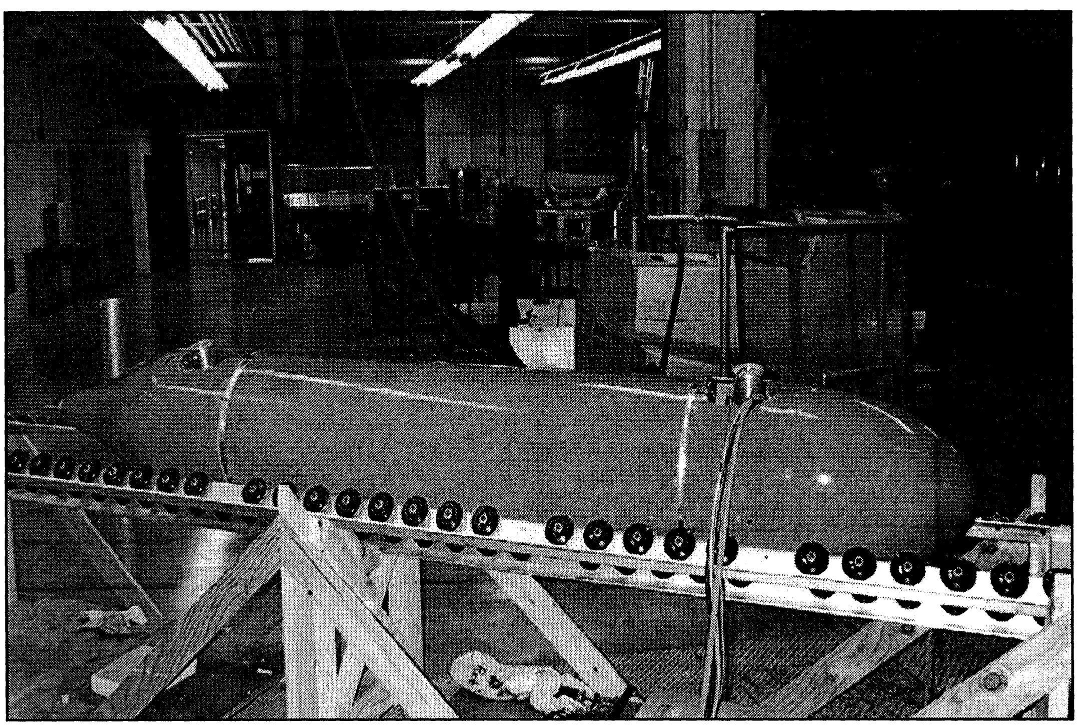
4.2.2 Preparing the Vehicle

Once the transducers were calibrated, they were then fitted into the vehicle. The dynamometer was fitted onto a frame using four Allen bolts. This frame was used for supporting the dynamometer and the propulsion motor [Fig. 4.9 and 4.16]. The live end of the dynamometer was coupled to the propeller shaft and the dead end of the dynamometer was coupled to the motor shaft. The frame was then inserted into the pressure vessel and the pressure vessel was then sealed.

The pressure vessel was then lifted and placed inside the vehicle hull and bolted in place. The waterproof cables (for supplying power to the motor and for transmitting the transducer output voltages to the data acquisition system) were then plugged into the



Preparing the Vehicle: Fig. 4.18



The Prepared Vehicle: Fig. 4.19

waterproof connectors on the pressure vessel. The nitrogen hose was then fitted and clamped in place using a jubilee clip [Fig. 4.18]. The space between the pressure vessel and the vehicle hull was then filled with styrofoam blocks, in order to give the vehicle additional buoyancy and hence ensure neutral buoyancy in water. The forward and aft flexible supports were then fitted in place. Subsequently the hinged upper half of the vehicle hull (skin) was closed and screwed in place [Fig. 4.19].

4.2.3 Attaching the Vehicle to the Struts

The vehicle was then lifted using an overhead crane and placed in the water. Subsequently the vehicle was attached to the struts [Figs. 4.20 to Fig. 4.25].

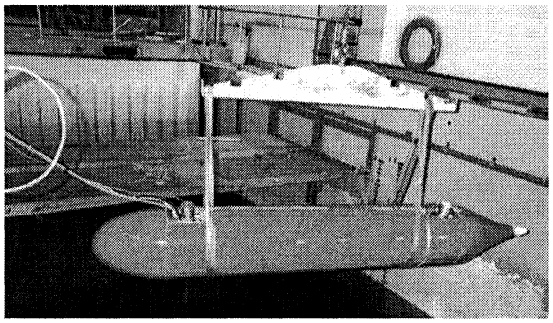


Fig. 4.20

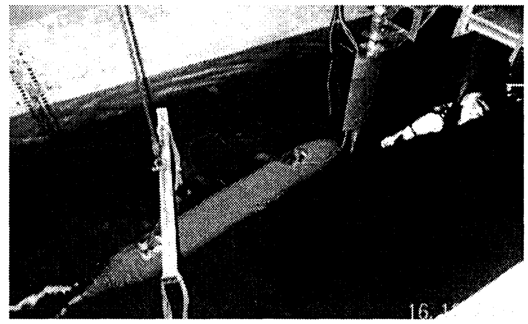


Fig. 4.21

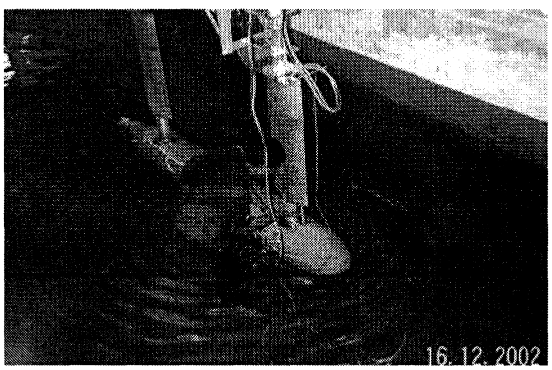


Fig. 4.22

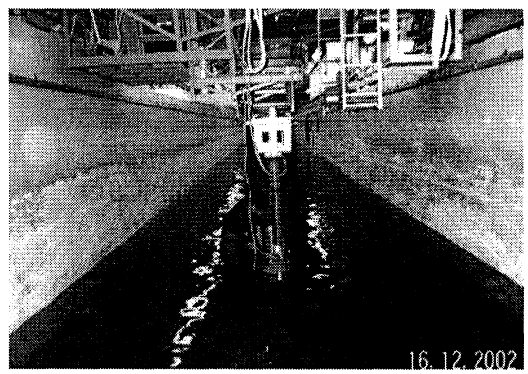


Fig. 4.23

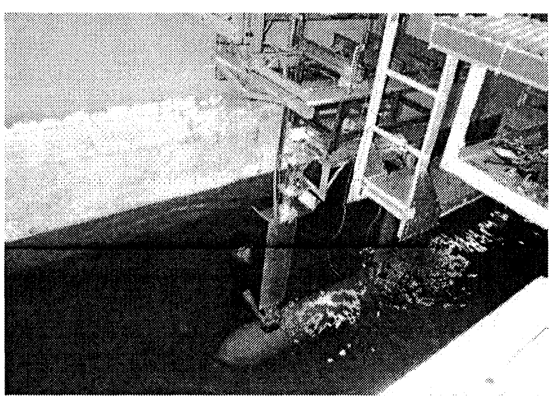


Fig. 4.24

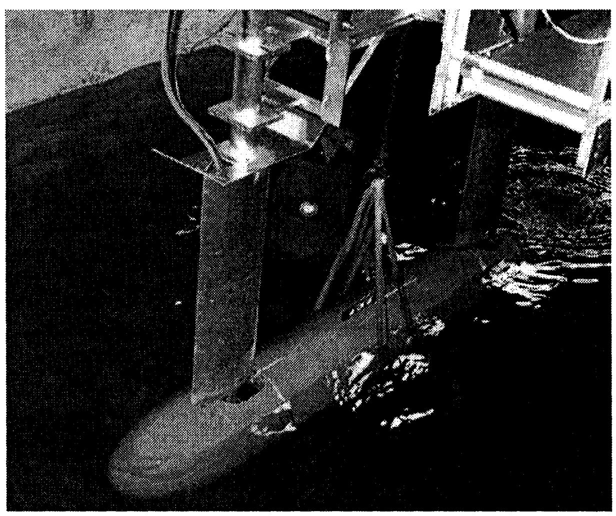
Fig. 4.25

4.2.4 Performing the In-Situ Tests / Checks

Subsequently the in-situ tests and checks were performed. These included an alignment check of the vehicle and leaf springs, an axial pull check and shaft friction torque tests.

Alignment Check of the Vehicle / Leaf Springs

The vehicle trim was adjusted by raising / lowering the forward and aft struts, so as to ensure that the vehicle was at level trim [Fig. 4.26]. The leaf springs were aligned by rotating the forward / aft struts clockwise / anticlockwise by a small amount, so as to ensure that all the springs lay in the same vertical plane.



Vehicle Alignment Check: Fig. 4.26

Axial Pull Check

The (tow force) load cell readings were then checked by applying a series of axial loads on the vehicle. The measured and applied loads were found to correspond accurately.

Shaft Friction Torque Tests

For performing the shaft friction torque tests, the propeller and duct were removed and a dummy hub was fitted in place. The propeller shaft was rotated at a number of different speeds and the friction torque measured by the dynamometer was noted. A plot was then made between the shaft friction torque (ordinate) and the shaft speed (abscissa). [Fig. 4.27].

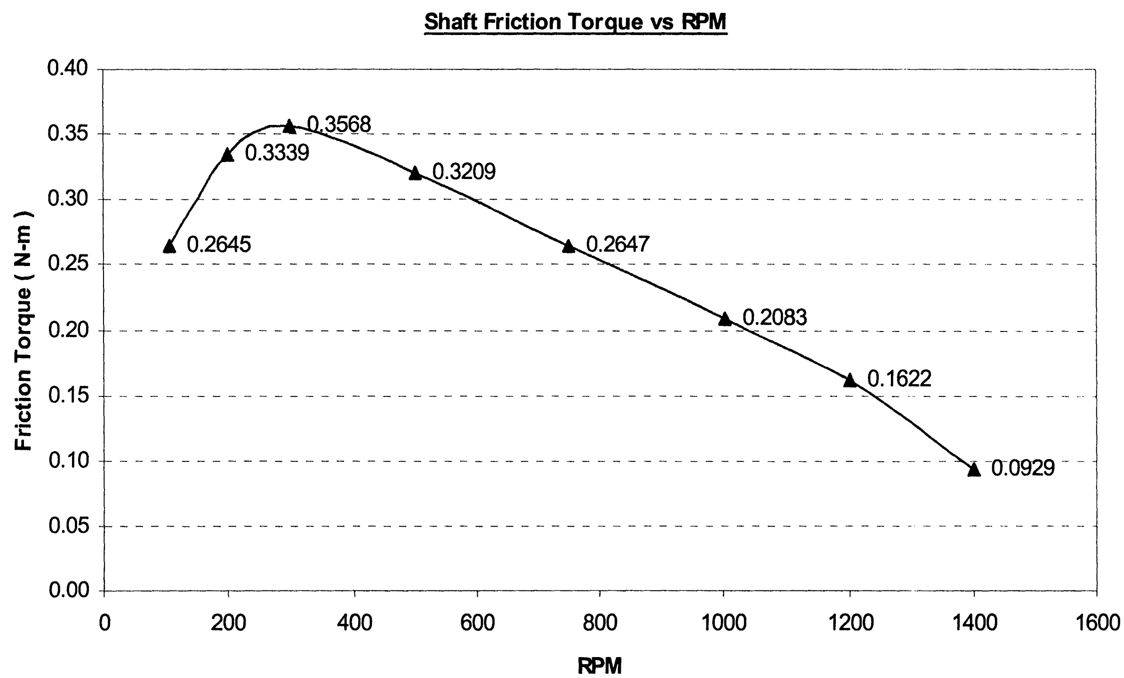


Fig. 4.27

4.2.5 Performing the Resistance and Propulsion Tests

After completing the in-situ tests / checks, the propeller and duct were fitted in place, and the tank water level was raised to 6.3 ft. Subsequently the resistance and propulsion tests were performed [Ref. 4-1]. The test plan included the following tests:

a) Load-varying self-propulsion tests

b) Bollard-pull tests

c) Bare-hull resistance tests

For the resistance tests, the bare-hull of the vehicle was towed at a series of constant speeds, in both the forward and reverse directions, and the vehicle resistance was noted. For the bollard-pull tests, the towing carriage was kept stationary and the propeller was rotated at a number of different speeds. At each rotational speed of the propeller, the tow force, the propeller thrust, torque and rotational speed were noted.

For the self-propulsion tests, the vehicle was towed at a combination of towing speeds and propeller shaft speeds, in the forward direction; here the vehicle speed, tow force, propeller thrust, torque and the propeller rotational speed were measured, with and without the duct. By performing these different types of tests, one can isolate the effects of the hull, propeller and duct and thus obtain a measure of the overall system efficiency, as well as an insight into where further improvements in efficiency can be obtained.

4.3 Test Results

4.3.1 Load-Varying Self-Propulsion Tests

One of the primary objectives of these tests was to evaluate the propulsive performance of the vehicle, using load-varying self-propulsion tests. The load-varying self-propulsion tests included two phases. Phase I tests were performed with only the propeller fitted onto the vehicle. Phase II tests were performed with both the propeller and duct fitted onto the vehicle.

Each of the two sets of tests were conducted using the following speeds of the towing carriage: 0.8, 1.1, 1.3, 1.6, 1.8, 2.1, 2.3 and 2.5 m/s. At each towing speed, the propeller shaft speed was systematically varied so as to cover the entire range of propeller loadings. Each test used the following shaft speeds: 500, 550, 600, 650, 750, 900, 1000, 1050, 1100, 1150, 1200 and 1250 RPM. At shaft speeds below 500 RPM, the thrusts and torques generated by the propeller were too small in magnitude to be measured accurately, hence no tests were conducted below 500 RPM.

The results from Phases I and II were then utilized to determine the duct thrust, using the procedure described in section 4.3.2. The duct thrust was then added to the propeller thrust in order to obtain the total thrust of the propulsor.

For the Phase I tests, the tow force (ordinate) was plotted against the propeller thrust (abscissa) as shown in Figure 4.28. For the Phase II tests, the tow force (ordinate) was plotted against the total propulsor thrust (abscissa) as shown in Figure 4.29.

Both plots were found to be linear and hence they satisfied the linearity axiom for load-varying self-propulsion tests [Refs. 4-2 and 4-3]. Using regression analysis, straight lines were fitted through the data points. Each straight line represents a particular vehicle speed (V) at which the self-propulsion test was conducted.

Phase I Tests

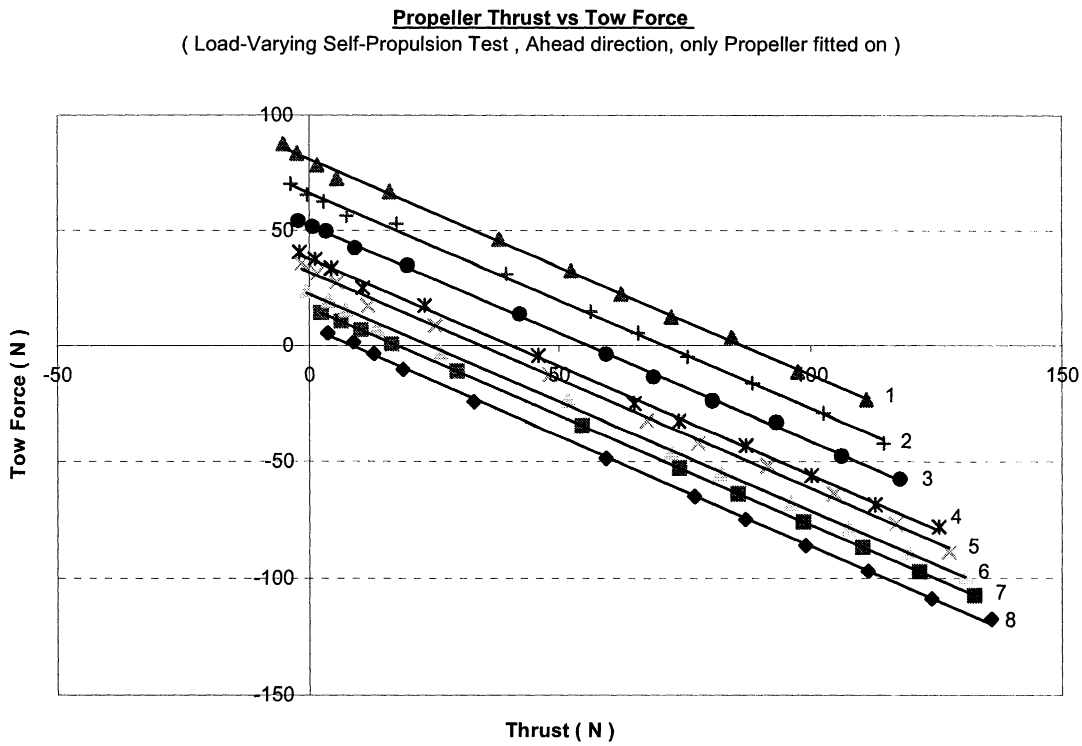


Fig. 4.28

Table 4.1

Equations of the Straight Lines

S.No.	Vehicle Speed	Equation
1	V = 2.5 m/s	y = -0.9369x + 80.875
2	V = 2.3 m/s	y = -0.9341x + 66.219
3	V = 2.1 m/s	y = -0.9358x + 52.164
4	V = 1.8 m/s	y = -0.9327x + 37.544
5	V = 1.6 m/s	y = -0.9329x + 31.978
6	V = 1.3 m/s	y = -0.9307x + 22.203
7	V = 1.1 m/s	y = -0.9319x + 16.302
8	V = 0.8 m/s	y = -0.9392x + 7.9354

y = Tow Force , x = Thrust

Phase II Tests

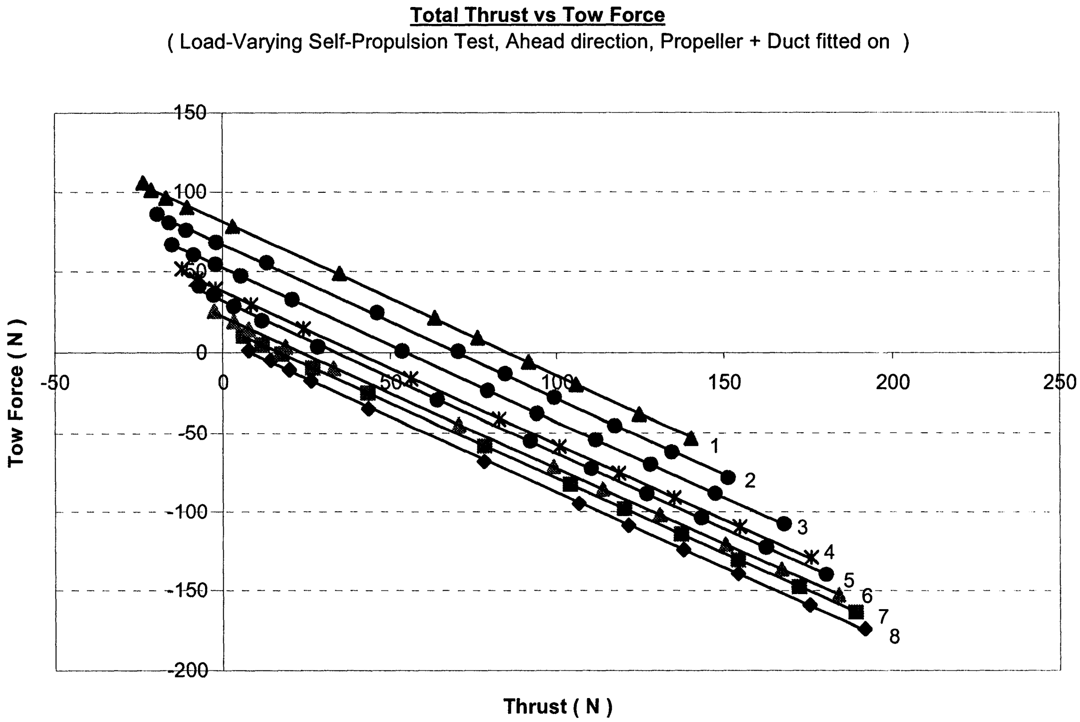


Fig. 4.29

Table 4.2

Equations of the Straight Lines

S.No.	Vehicle Speed	Equation
1	V = 2.5 m/s	y = -0.9554x + 81.46
2	V = 2.3 m/s	y = -0.9554x + 66.907
3	V = 2.1 m/s	y = -0.9561x + 52.395
4	V = 1.8 m/s	y = -0.9551x + 38.033
5	V = 1.6 m/s	y = -0.9532x + 32.217
6	V = 1.3 m/s	y = -0.9508x + 22.343
7	V = 1.1 m/s	y = -0.9493x + 15.984
8	V = 0.8 m/s	y = -0.9545x + 7.722

y = Tow Force , x = Thrust

The equation of each straight line in Fig. 4.28 and Fig. 4.29 is of the form:

$$\boxed{F = -(1 - t^{*})\cdot T + F_{T=0}} \qquad \ldots\ldots 4.1$$

where 'F' is the measured tow force, 'T' is the measured propulsor thrust, and '$F_{T=0}$' is the value of the tow force when the propulsor thrust is zero, that is, the intercept on the ordinate axis of Fig. 4.28 and Fig. 4.29. From these equations, the vehicle resistance under idling propeller conditions, $F_{T=0}$, and the thrust deduction fraction (t^*) were determined; see Tables 4.3 and 4.4. Since the full scale "C-SCOUT" vehicle was used for these tests, therefore the self-propulsion point of the vehicle is that point when the tow force (F) becomes equal to zero. Substituting F = 0 in the above equation, the values of the propulsor thrust, at each self-propulsion point (T_s), were determined; see Tables 4.3 and 4.4.

The measured values of the vehicle speed, propeller rotational speed, propeller torque, propeller thrust and tow force were then used to form non-dimensional thrust, torque, tow force and advance coefficients [see Appendix B for test results], according to the following definitions. For the Phase I tests, these coefficients are the propeller thrust coefficient 'K_{tp}', propeller torque coefficient 'K_q', tow force coefficient 'K_{fd}' and advance coefficient 'J'. For the Phase II tests, these coefficients are the propeller thrust coefficient 'K_{tp}', duct thrust coefficient 'K_{td}', total thrust coefficient 'K_{tt}', propeller torque coefficient 'K_q', tow force coefficient 'K_{fd}' and advance coefficient 'J'. Here 'ρ' is the mass density of water, 'n' is the propeller rotational speed, 'D' is the propeller diameter and 'V' is the vehicle (towed) speed.

$\boxed{K_{td} = T_d / \{\rho.n^2.D^4\}}$ - Thrust coefficient for the duct 4.2

$\boxed{K_{tp} = T_p / \{\rho.n^2.D^4\}}$ - Thrust coefficient for the propeller 4.3

$\boxed{K_{tt} = K_{tp} + K_{td}}$ - Thrust coefficient for the propeller + duct 4.4

$\boxed{K_q = Q / \{\rho.n^2.D^5\}}$ - Torque coefficient for the propeller 4.5

$\boxed{K_{fd} = F / \{\rho.n^2.D^4\}}$ - Tow Force Coefficient 4.6

$\boxed{J = V / \{n.D\}}$ - Advance Coefficient for the propeller 4.7

For the Phase I tests, these coefficients were plotted as curves of K_{tp} vs J, $10K_q$ vs J and K_{fd} vs J, as shown in Fig. 4.30 to Fig 4.37. For the Phase II tests, these coefficients were plotted as curves of K_{tp} vs J, K_{td} vs J, K_{tt} vs J, $10K_q$ vs J and K_{fd} vs J, as shown in Fig. 4.38 to Fig. 4.45. The self-propulsion point in these plots can also be determined by inserting an additional curve of thrust coefficient 'K_{ts}' as a function of J^2 into the plots. The coefficient 'K_{ts}' is defined as:

$\boxed{K_{ts} = T_s.\, J^2 / \{\rho.D^2.V^2\}}$ 4.8

Where 'T_s' is the propulsor thrust at self-propulsion point. For the Phase I tests, the intersection of the 'K_{ts}' curve and the 'K_{tp}' curve represents the self-propulsion point of the vehicle [Fig. 4.30 to Fig. 4.37]. For the Phase II tests, the intersection of the 'K_{ts}' curve and the 'K_{tt}' curve represents the self-propulsion point of the vehicle [Fig. 4.38 to Fig. 4.45]. Note that in each figure, the curve of 'K_{fd}' crosses the J-axis, indicating that the tow force is zero at the self-propulsion point.

Load-Varying Self-Propulsion Test
(V = 0.8 m / s , Ahead Direction , Only Propeller Fitted on)

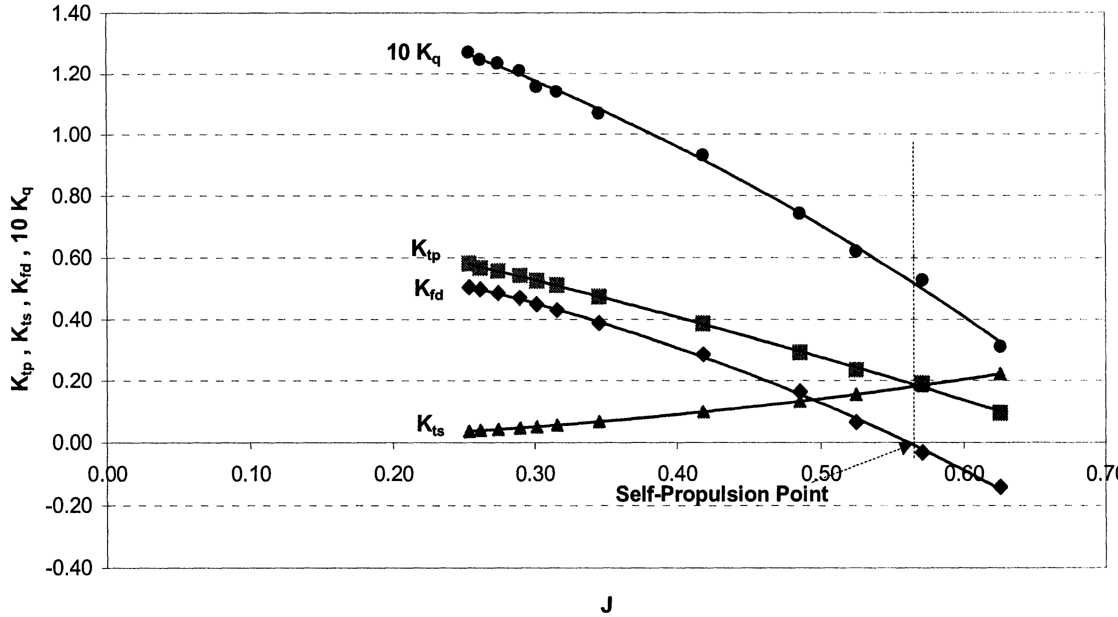


Fig. 4.30

Load-Varying Self-Propulsion Test
(V = 1.1 m / s , Ahead Direction , Only Propeller Fitted on)

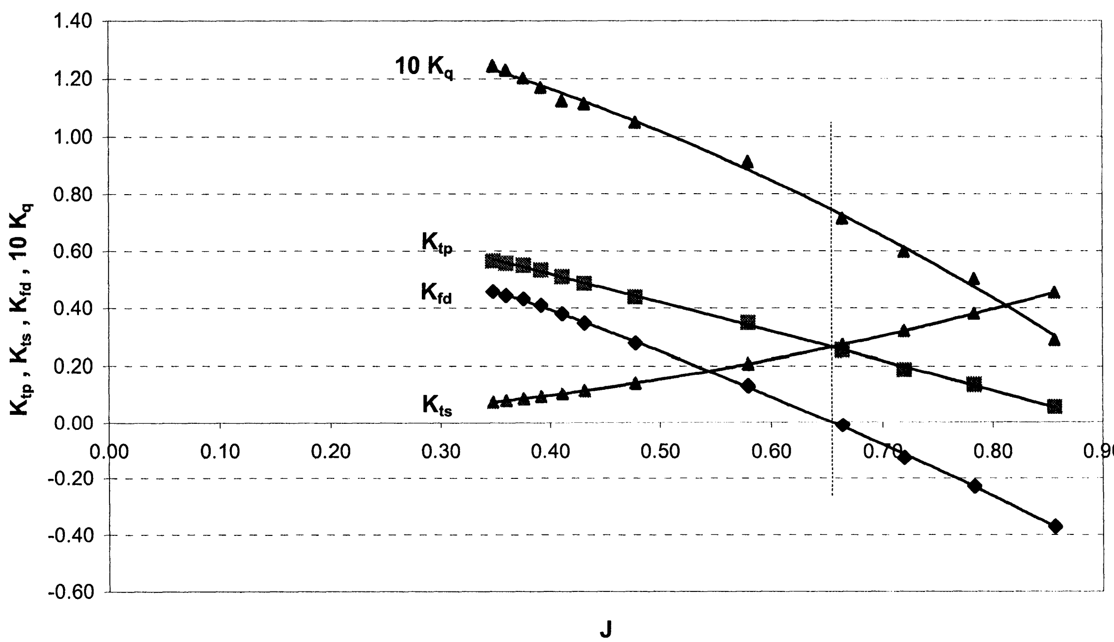


Fig. 4.31

Load-Varying Self-Propulsion Test
(V = 1.3 m / s , Ahead Direction , Only Propeller Fitted on)

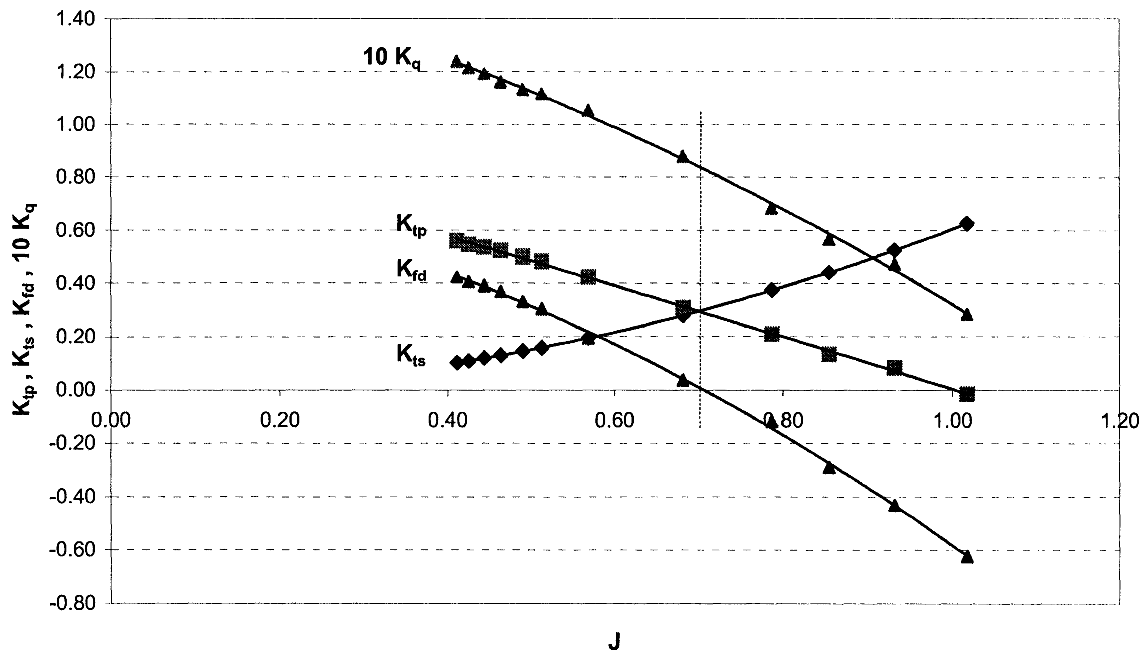


Fig. 4.32

Load-Varying Self-Propulsion Test
(V = 1.6 m / s , Ahead Direction , Only Propeller Fitted on)

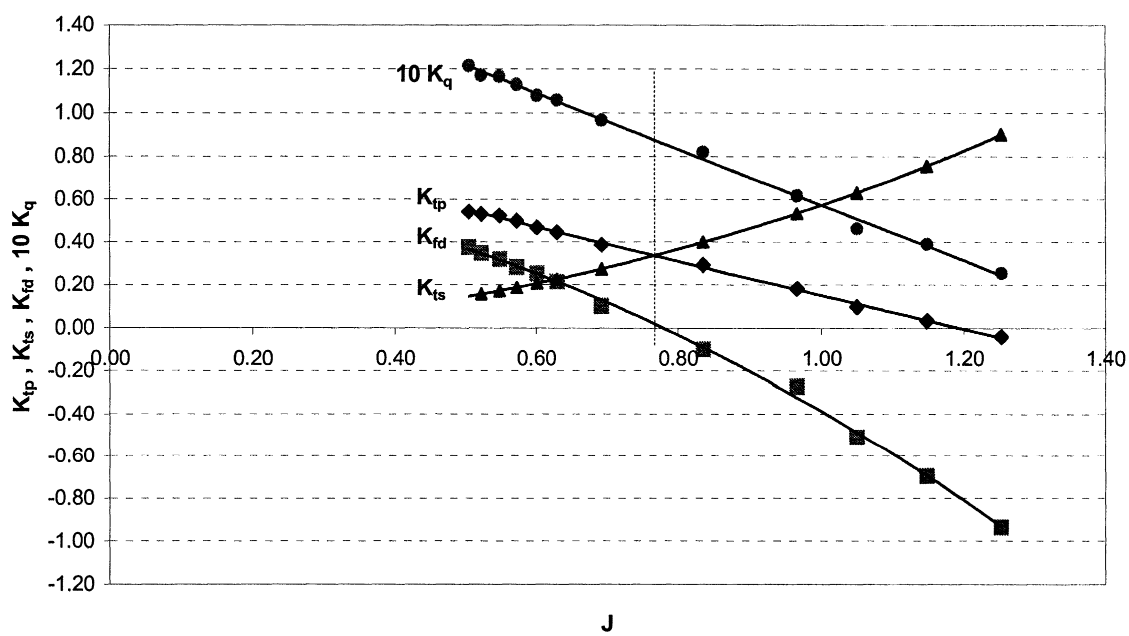


Fig. 4.33

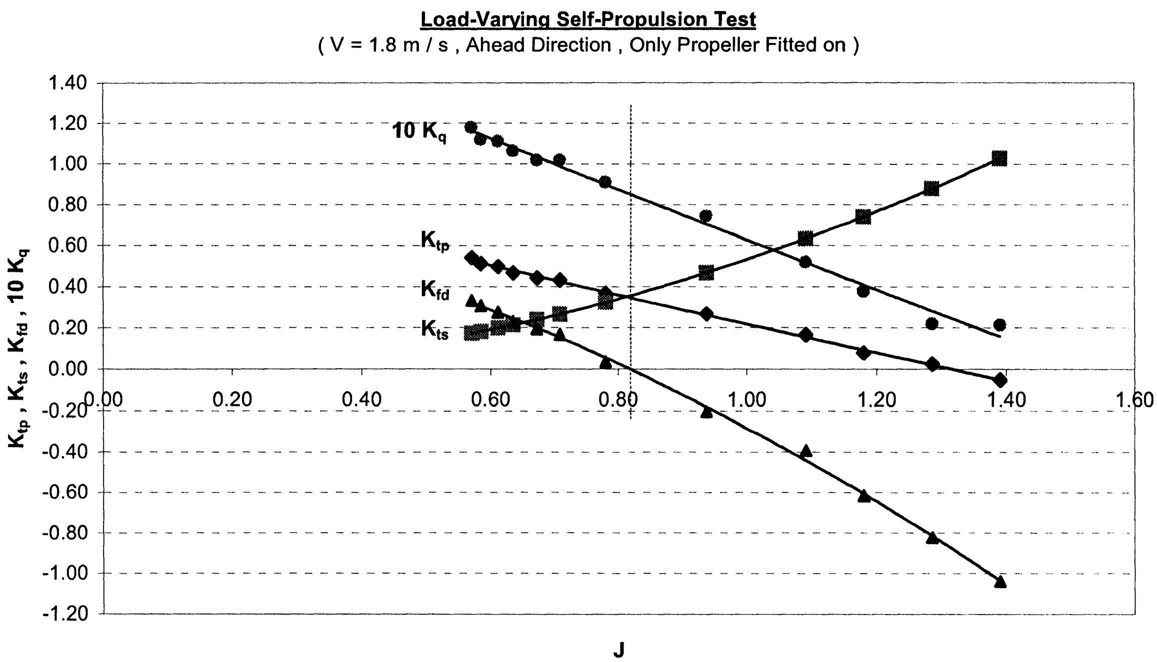


Fig. 4.34

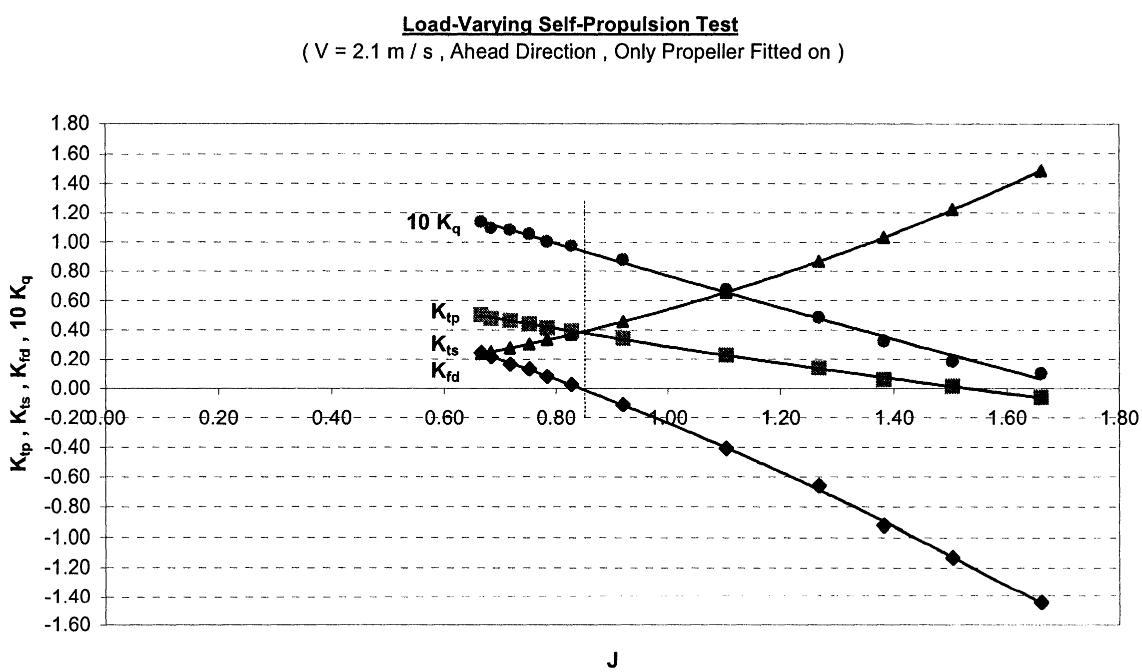


Fig. 4.35

Load-Varying Self-Propulsion Test
(V = 2.3 m / s , Ahead Direction , Only Propeller Fitted on)

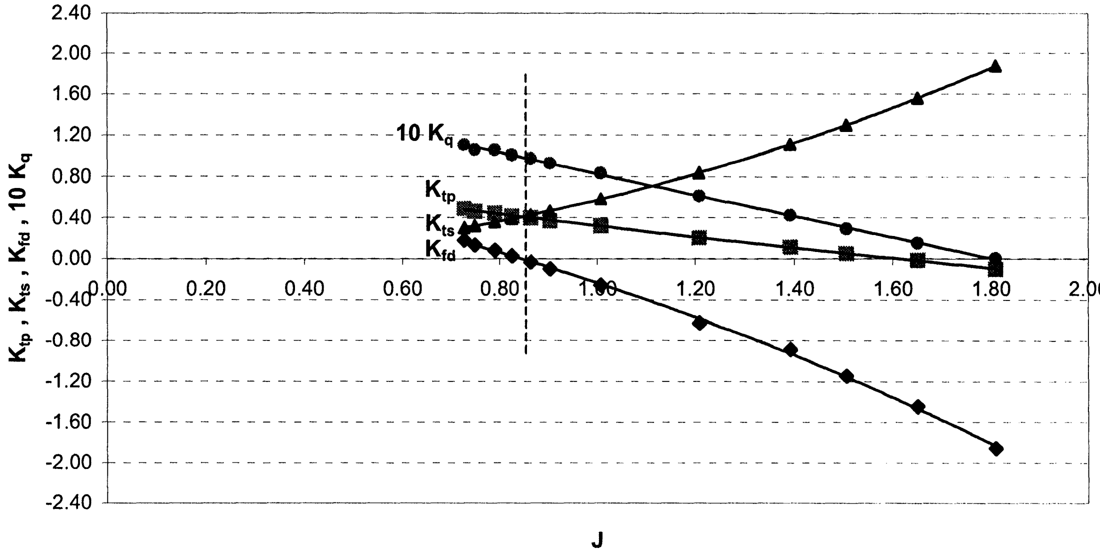


Fig 4.36

Load-Varying Self-Propulsion Test
(V = 2.5 m / s , Ahead Direction , Only Propeller Fitted on)

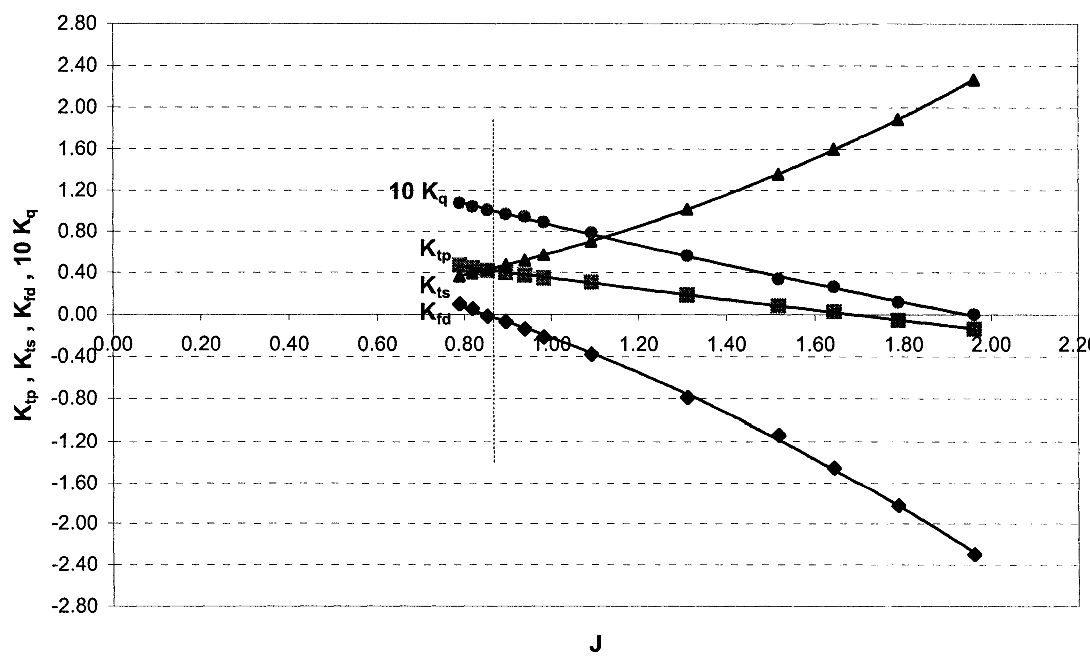


Fig. 4.37

Load-Varying Self-Propulsion Test
(V = 0.8 m / s , Ahead Direction , Propeller + Duct Fitted on)

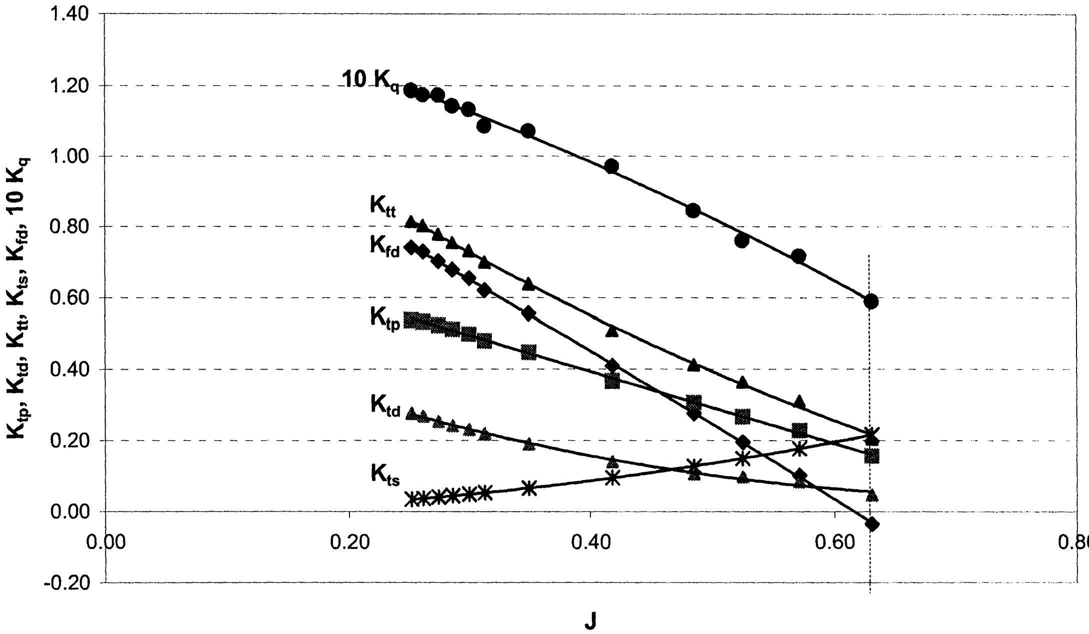


Fig. 4.38

Load-Varying Self-Propulsion Test
(V = 1.1 m / s , Ahead Direction , Propeller + Duct Fitted on)

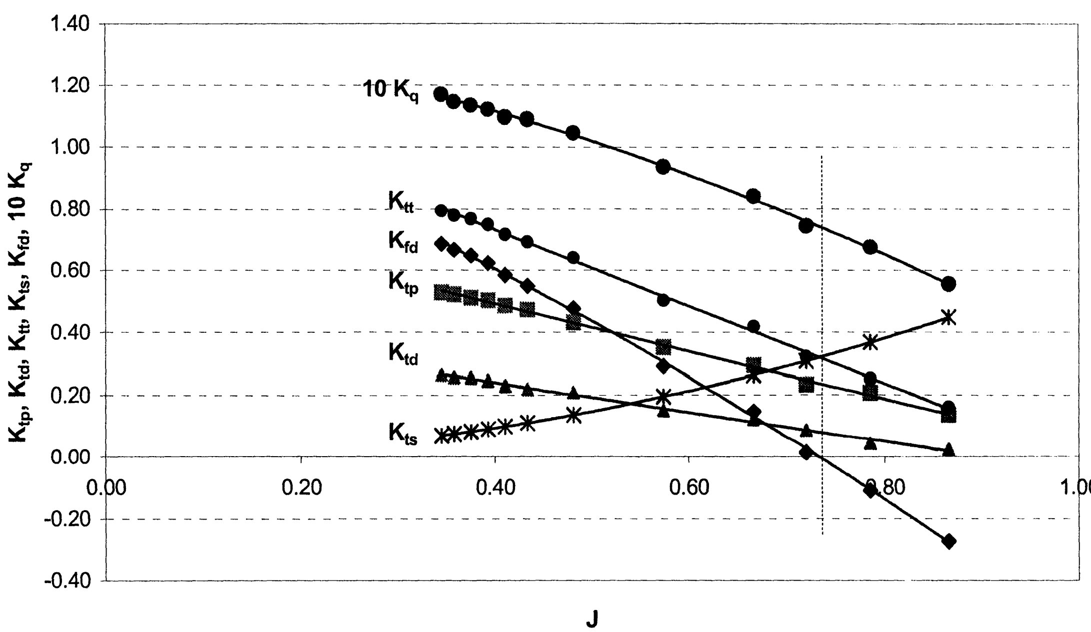


Fig. 4.39

Load-Varying Self-Propulsion Test
(V = 1.3 m / s , Ahead Direction , Propeller + Duct Fitted on)

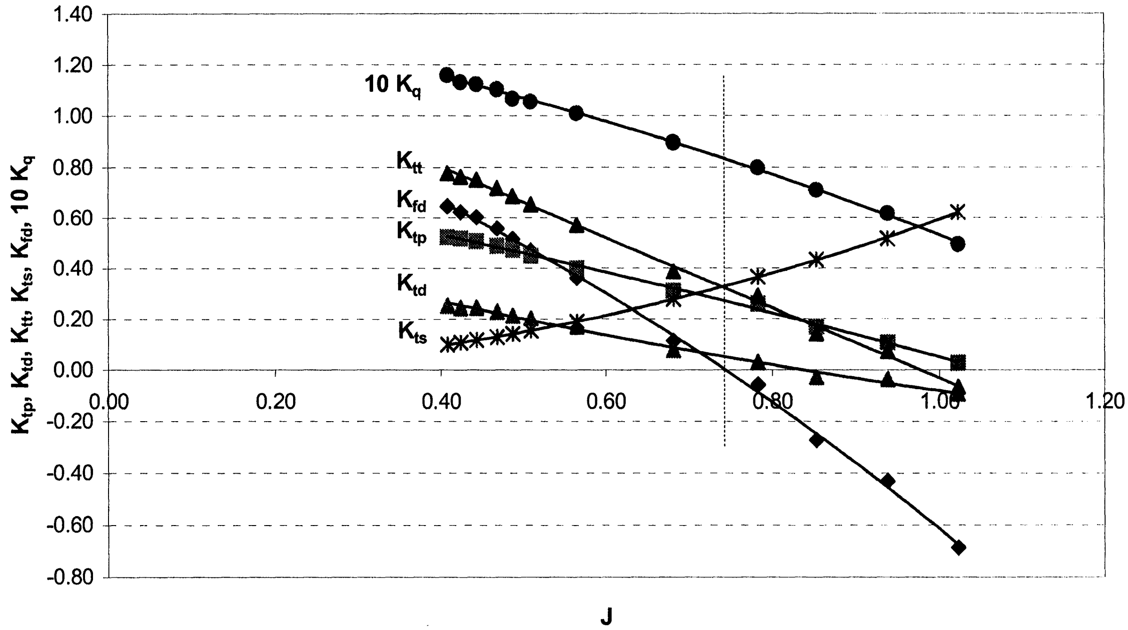


Fig. 4.40

Load-Varying Self-Propulsion Test
(V = 1.6 m / s , Ahead Direction , Propeller + Duct Fitted on)

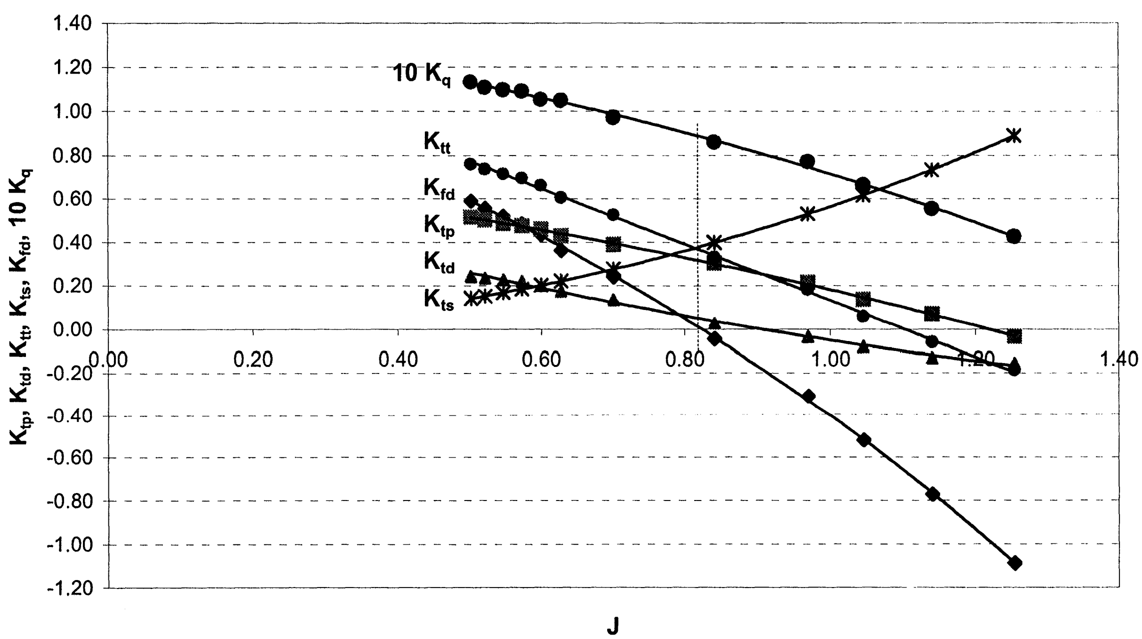


Fig. 4.41

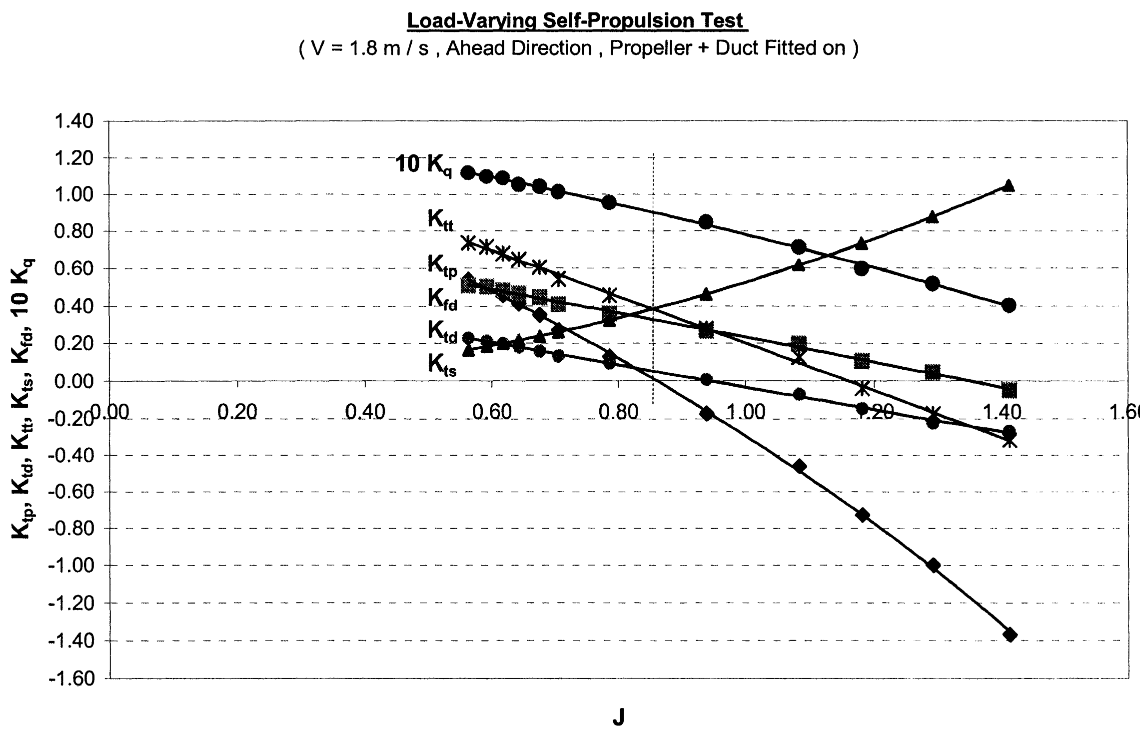


Fig. 4.42

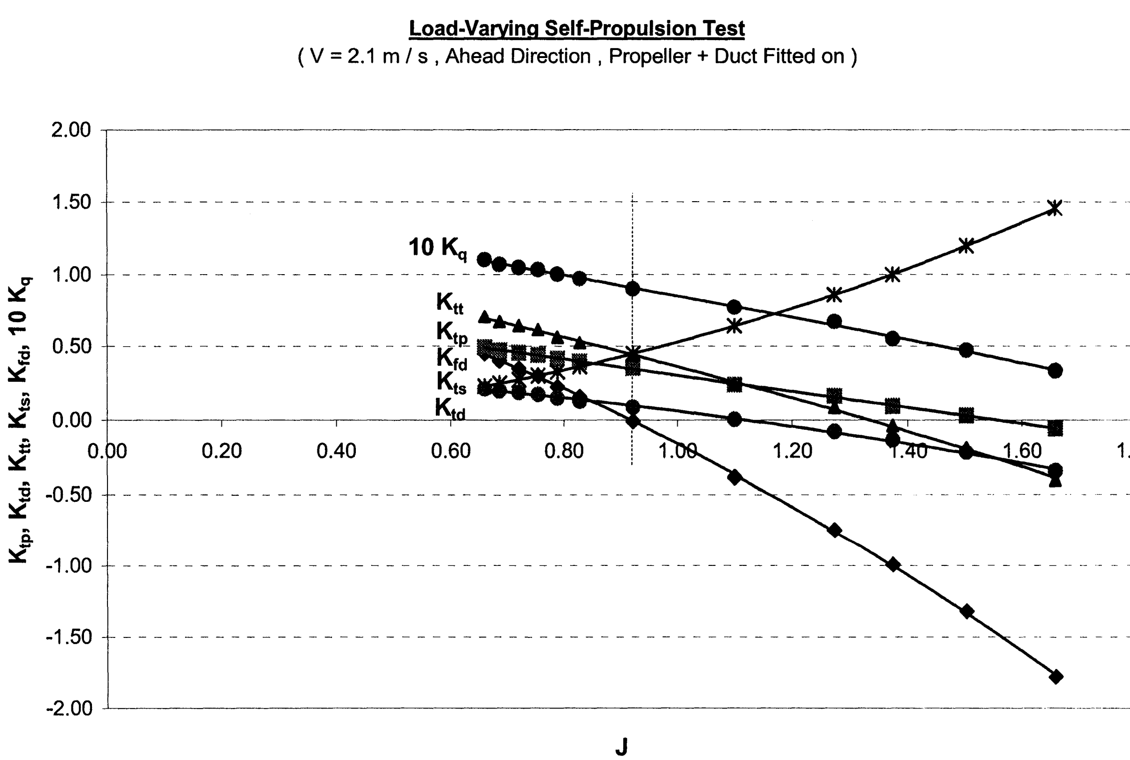


Fig. 4.43

Load-Varying Self-Propulsion Test
(V = 2.3 m / s , Ahead Direction , Propeller + Duct Fitted on)

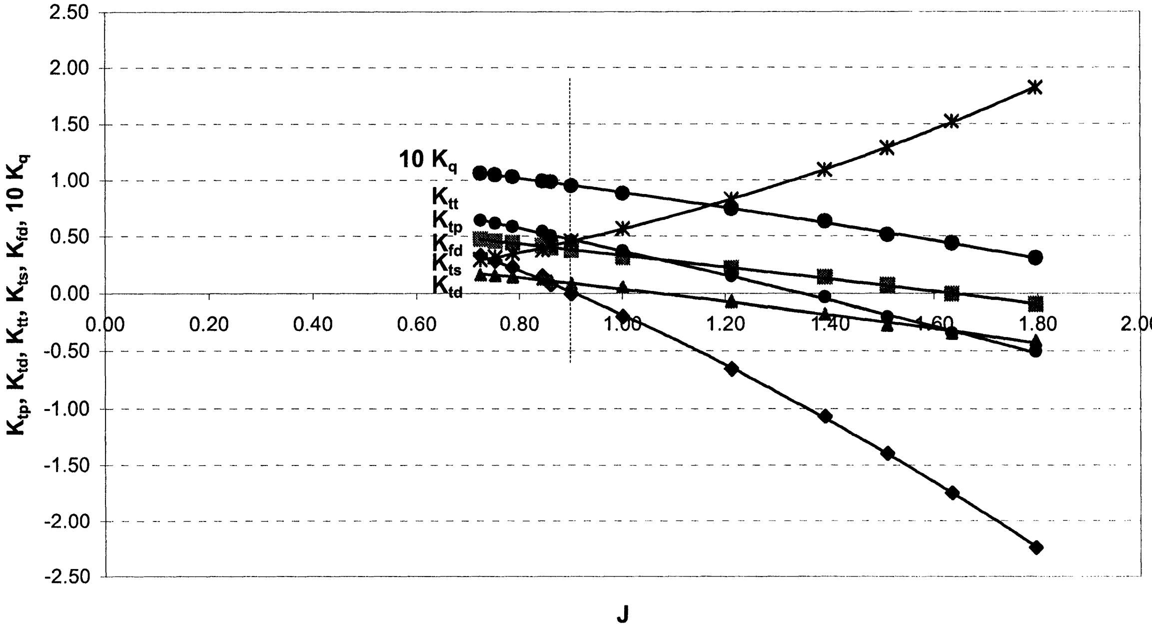


Fig. 4.44

Load-Varying Self-Propulsion Test
(V = 2.5 m / s , Ahead Direction , Propeller + Duct Fitted on)

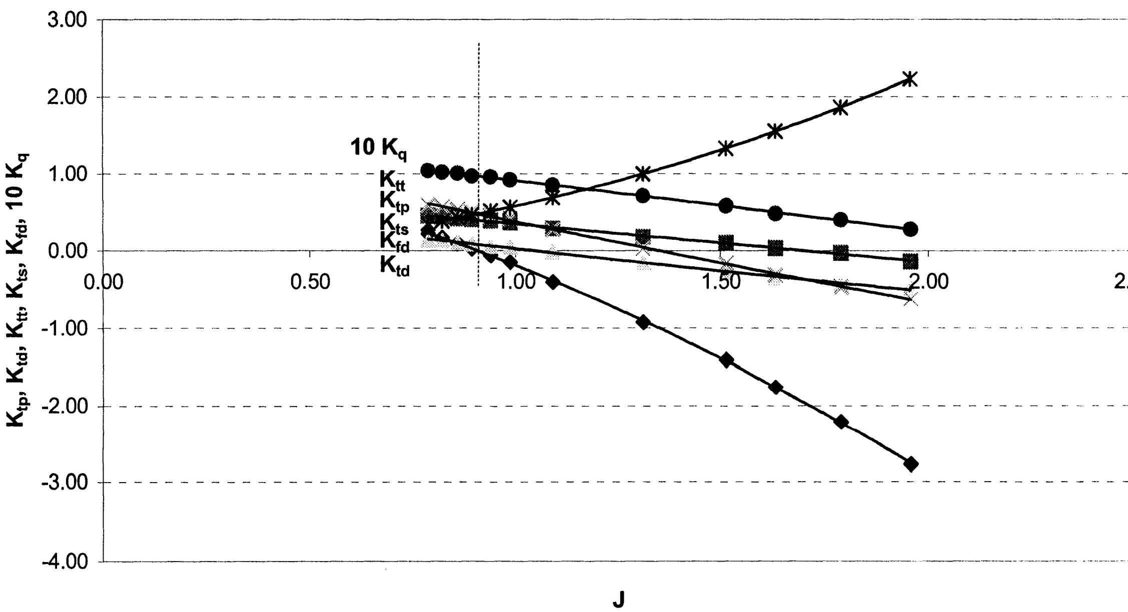


Fig. 4.45

From the self-propulsion point, the propeller torque coefficient '$10K_q$' and the advance coefficient 'J' were read off [see Appendix B]. These were then used to determine the propeller rotational speed 'n' and the delivered power 'P_d' at the self-propulsion point, using the following expressions.

$$\boxed{n = V / \{J \cdot D\}} \qquad \text{.....4.9}$$

and

$$\boxed{P_d = 2\pi\rho n^3 D^5 K_q} \qquad \text{.....4.10}$$

The effective power 'P_e' of the vehicle was then calculated from

$$\boxed{P_e = V \cdot F_{T=0}} \qquad \text{.....4.11}$$

Subsequently the propulsive efficiency 'η_d' of the propulsion system was calculated from

$$\boxed{\eta_d = P_e / P_d} \qquad \text{.....4.12}$$

The results for the Phase I and Phase II tests are tabulated in Tables 4.3 and 4.4 respectively. For the Phase I tests, Fig. 4.46 shows a plot of delivered power and effective power vs forward speed and Fig. 4.47 shows a plot of propulsive efficiency vs forward speed. Similarly for the Phase II tests, Fig. 4.48 shows a plot of delivered power and effective power vs forward speed and Fig. 4.49 shows a plot of propulsive efficiency vs forward speed. '$F_{T=0}$' for the Phase I and Phase II tests, are compared with 'R' from the bare hull resistance tests, in Table 4.5.

Table 4.3 (Phase I)

Self-Propelled Condition, Ahead Direction, Only Propeller Fitted on

S. No.	V	n	$F_{T=0}$	t^*	T_S	P_d	P_e	η_d
	(m/s)	(rpm)	(N)		(N)	(W)	(W)	(%)
1	2.5	1161	80.8	0.0631	86.3	382.0	202.8	53.0
2	2.3	1074	66.2	0.0659	70.8	291.3	152.8	52.4
3	2.1	984	52.1	0.0642	55.7	215.2	109.8	51.0
4	1.8	875	37.5	0.0673	40.2	138.0	67.7	49.1
5	1.6	824	31.9	0.0671	34.2	117.1	51.2	43.7
6	1.3	733	22.2	0.0693	23.8	79.0	28.9	36.5
7	1.1	664	16.3	0.0681	17.4	52.4	17.9	34.3
8	0.8	557	7.9	0.0608	8.4	20.9	6.3	30.4

Table 4.4 (Phase II)

Self-Propelled Condition, Ahead Direction, Propeller + Duct Fitted on

S. No.	V	n	$F_{T=0}$	t^*	T_S	P_d	P_e	η_d
	(m/s)	(rpm)	(N)		(N)	(W)	(W)	(%)
1	2.5	1089	81.4	0.0446	85.2	300.4	204.4	68.0
2	2.3	1004	66.9	0.0446	70.0	227.2	154.8	68.1
3	2.1	921	52.3	0.0439	54.8	172.6	110.4	63.9
4	1.8	832	38.0	0.0449	39.8	124.3	68.6	55.2
5	1.6	777	32.2	0.0468	33.7	100.0	51.7	51.6
6	1.3	694	22.3	0.0492	23.4	66.7	29.1	43.6
7	1.1	617	15.9	0.0507	16.8	44.0	17.6	39.9
8	0.8	500	7.7	0.0455	8.0	17.6	6.1	35.1

Table 4.5

Comparison of $F_{T=0}$ and R

			Propeller fitted on		***Propeller + Duct fitted on***	
S.No	**V**	**R**	$F_{T=0}$	$F_{T=0}$ / R	$F_{T=0}$	$F_{T=0}$ / R
	(m/s)	(N)	(N)		(N)	
1	0.8	8.0	7.9	1.0	7.7	1.0
2	1.1	13.9	16.3	1.2	15.9	1.1
3	1.3	18.7	22.2	1.2	22.3	1.2
4	1.6	27.2	31.9	1.2	32.2	1.2
5	1.8	33.8	37.5	1.1	38.0	1.1
6	2.1	46.4	52.1	1.1	52.3	1.1
7	2.3	60.9	66.2	1.1	66.9	1.1
8	2.5	76.9	80.8	1.1	81.4	1.1

Delivered and Effective Power vs Forward Speed
(Load-Varying Self-Propulsion Tests, Ahead Direction, Only Propeller Fitted on)

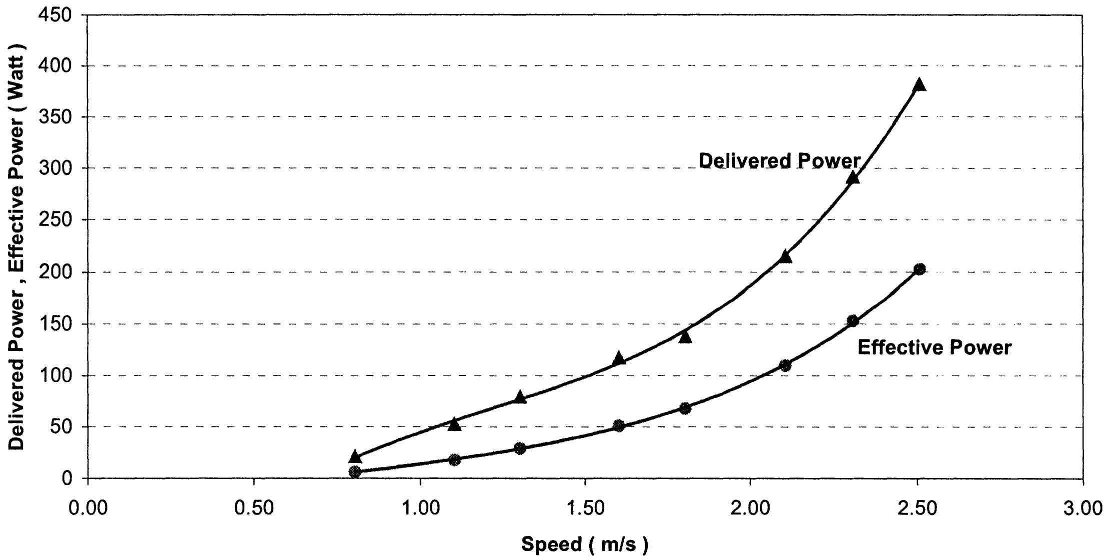


Fig. 4.46

Propulsive Efficiency vs Forward Speed
(Load-Varying Self-Propulsion Tests, Ahead Direction, Only Propeller Fitted on)

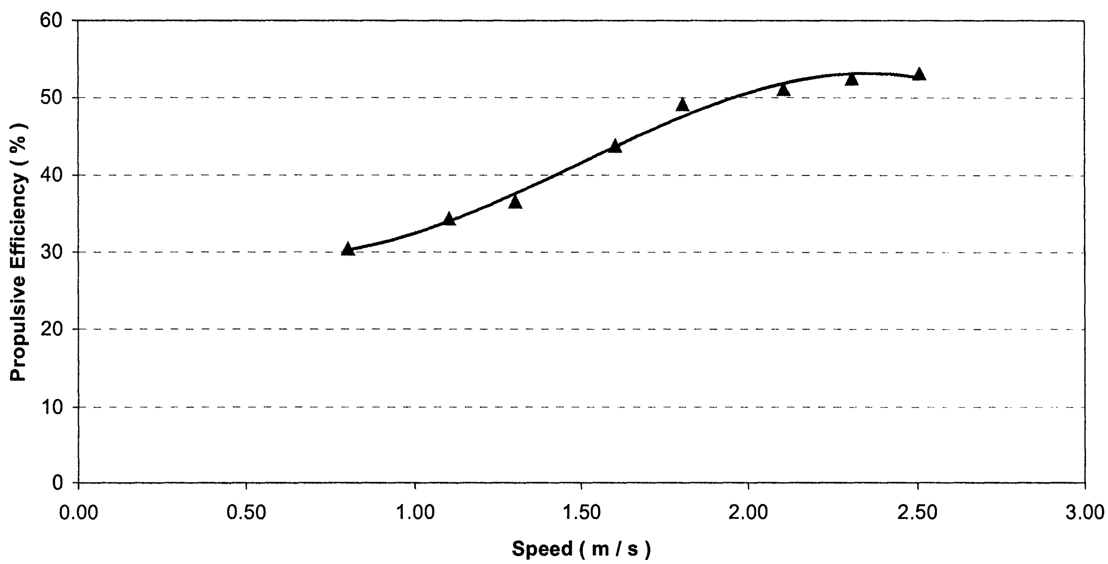


Fig. 4.47

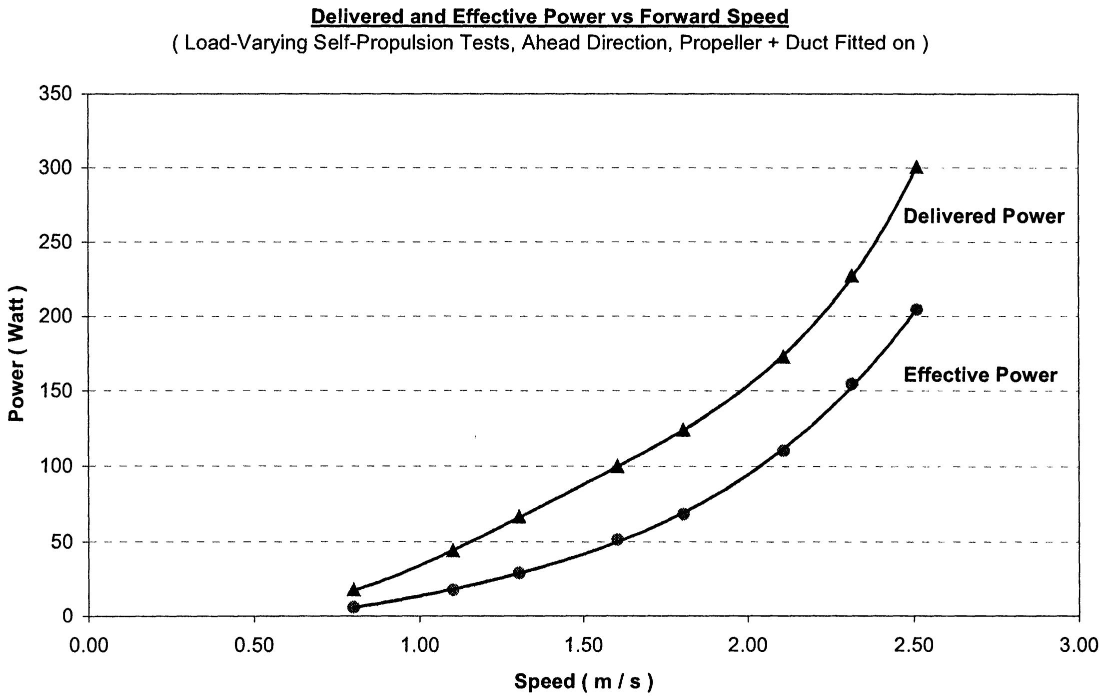


Fig. 4.48

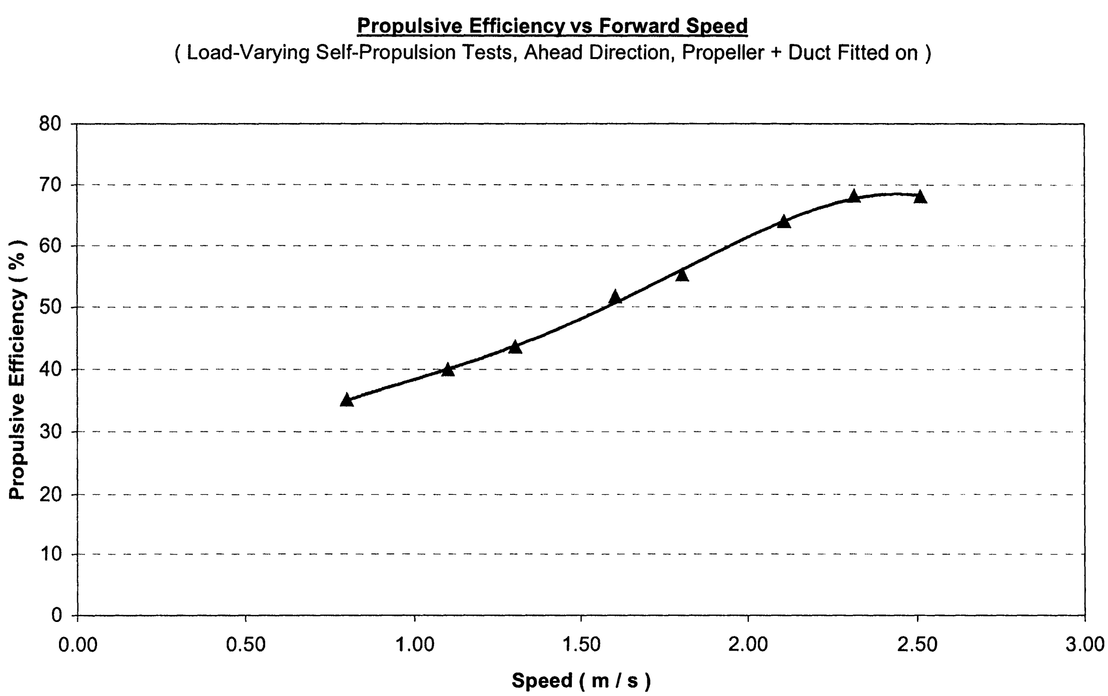
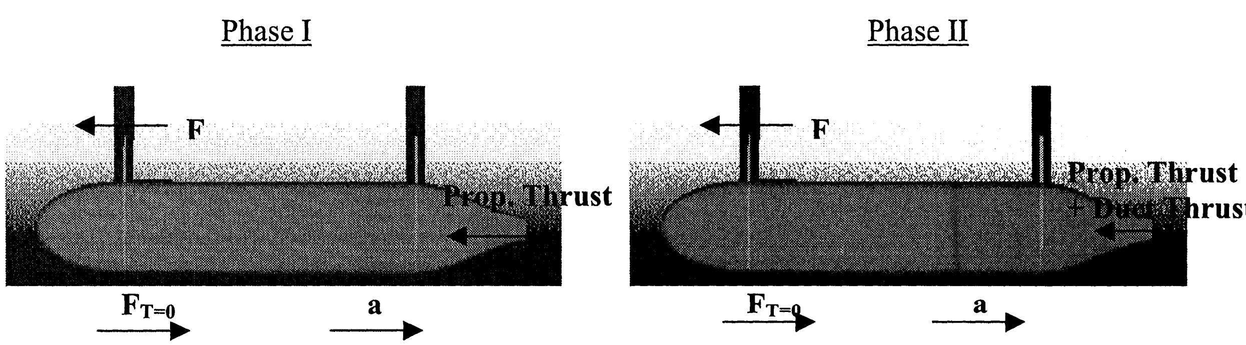


Fig. 4.49

4.3.2 Duct Thrust Measurement Method

The test setup for the self-propulsion tests was used for the duct thrust measurements. In this case direct measurement of duct thrust is required, so the duct needs to be supported by its own load cell, which is in turn supported by a rigid structure. In a conventional test where the vehicle is attached rigidly to the towing struts, this load cell would be attached to the aft strut or to the support frame.

But for these tests, since the vehicle was not attached rigidly to the towing struts, the load cell for measuring duct thrust could no longer be attached to the aft strut or to the support frame, but instead would have had to be directly attached to the vehicle hull. Such an arrangement invariably causes the flow pattern around the aft end of the vehicle to get altered, thereby changing the test results.



Forces Acting on the Vehicle: Fig. 4.50

Further, the entire arrangement for supporting the vehicle using flexible supports was already quite complicated and fitting an additional load cell and its support structure was not found to be practical. Therefore it was decided not to measure the duct thrust directly, but instead to infer the duct thrust from the results from the tests of Phases I and II.

Fig. 4.50 shows the forces acting on the vehicle during the course of Phases I and II. The towing force at zero propulsor thrust ($F_{T=0}$) is essentially the same in both the cases (see Table 4.3 and 4.4). If the augment in resistance (a thrust-deduction effect of propulsor) is assumed to be same in both the cases, then the duct thrust can be inferred using the following expression:

$$\boxed{\text{Duct Thrust} = [\text{Tow Force(II)- Propeller Thrust(II)}] - [\text{Tow Force(I)- Propeller Thrust(I)}]} \quad \ldots\ldots 4.13$$

Using the above expression, the duct thrust was inferred at each propeller speed for which a self-propulsion test was conducted. This duct thrust was then added to the propeller thrust, in order to obtain the total thrust generated by the propulsor, at each propeller speed for which a self-propulsion test was conducted [see Appendix B].

4.3.3 Bollard-Pull Tests

The bollard-pull of a vehicle is essentially a measure of the greatest towing force the vehicle can exert on a towline, while the vehicle is stationary (zero forward speed). The vehicle was fitted with both the propeller and the duct for these tests. Bollard-pull tests were conducted by keeping the vehicle stationary and then systematically varying the propeller speed. At each propeller speed, the following parameters were recorded: propeller speed, propeller thrust, shaft torque, tow force and water temperature. The recorded data were then used to make plots of tow force versus propeller speed [Fig. 4.51], and of delivered power versus propeller speed [Fig. 4.52]. [see Appendix B for test results].

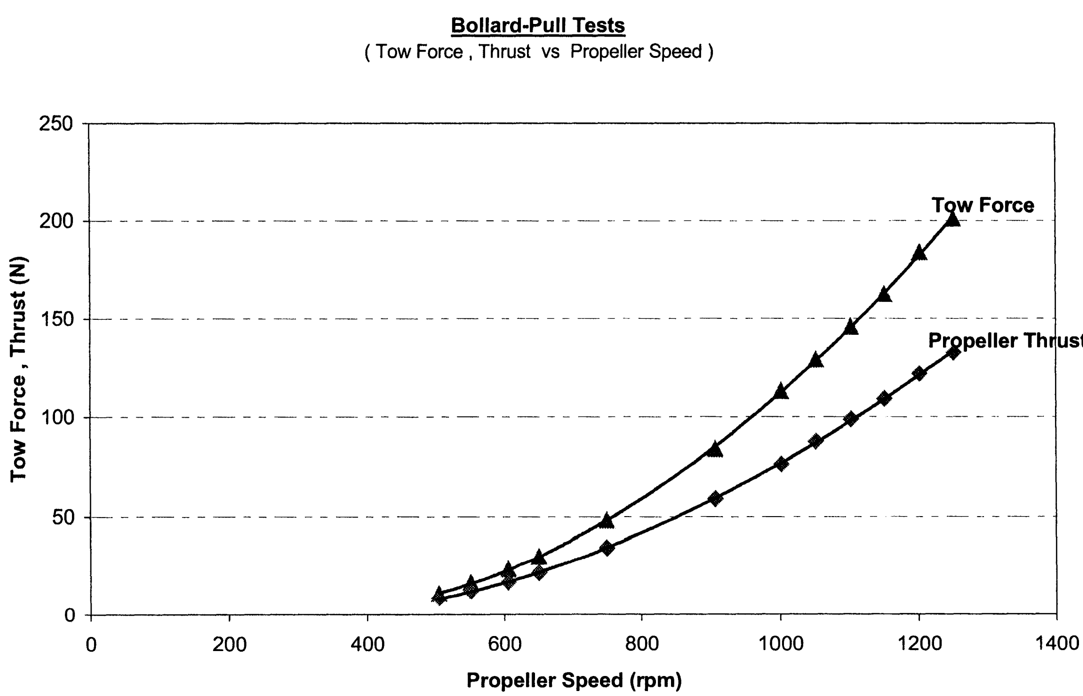


Fig. 4.51

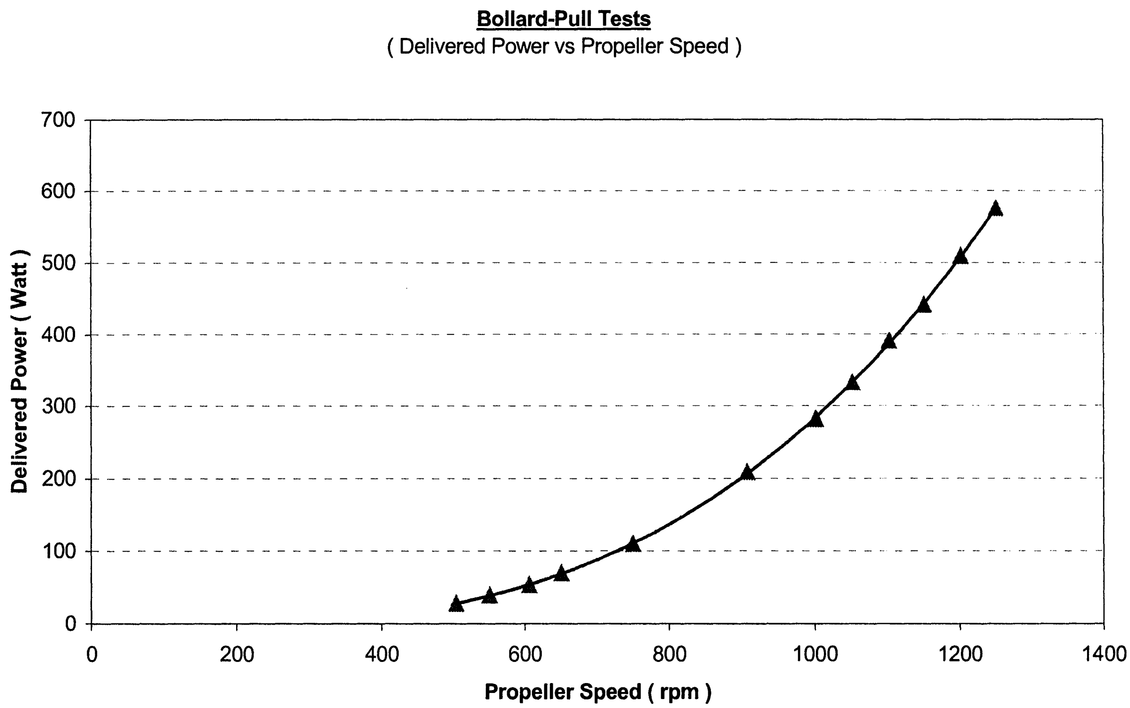


Fig. 4.52

4.3.4 Bare-Hull Resistance Tests

Resistance tests were conducted in order to determine the drag (resistance) of the bare-hull of the vehicle. The propeller and duct were removed for these tests. The vehicle was towed over a range of speeds in the ahead as well as astern directions. The parameters recorded included the carriage speed, vehicle resistance and the water temperature. Tests were not conducted at speeds below 0.6 m/s, since the measured resistance at speeds below 0.6 m/s was too small to be reliably measured by the load cell. For the astern direction, the tests were limited to a maximum speed of 2 m/s, since the towing carriage could go no faster than 2 m/s when traveling backwards.

The measured data [see Appendix B for test results] were then used to create plots of vehicle resistance versus vehicle speed, for both the ahead and astern conditions, Fig. 4.53 and Fig 4.54.

Since the depth of the towing tank was limited, therefore the vehicle could only be submerged to a centreline depth of 0.91 m (2.2 body diameters) below the free surface of water. As a rule of thumb, the body needs to be submerged at least five body diameters below the surface of water, in order to completely preclude free surface (wave making) effects [Ref 4-4]. Consequently at towing speeds above 2 m/s, a significant amount of wave making resistance showed up in the data. This is indicated by a sharp jump in the vehicle resistance above 2 m/s, Fig. 4.53. Below towing speeds of 2 m/s, the vehicle resistance is predominantly viscous and there is a uniform parabolic-type increase in resistance with increasing forward speeds.

The vehicle bare-hull resistances in the ahead and astern directions were compared, Fig. 4.55. The resistance values (towing forces) were close to each other at lower speeds. With increasing speeds, the difference in resistance increased and then stabilized to a more or less constant difference of about 10.7 percent.

The vehicle bare-hull resistance in the ahead direction (R) was also compared with the magnitude of the towing force at zero propulsor thrust ($F_{T=0}$); Fig 4.56 and Table 4.5. The values of R and $F_{T=0}$ were close to each other at lower speeds. At higher speeds the difference in R and $F_{T=0}$ stabilized to a more or less constant value of about 9.8 percent.

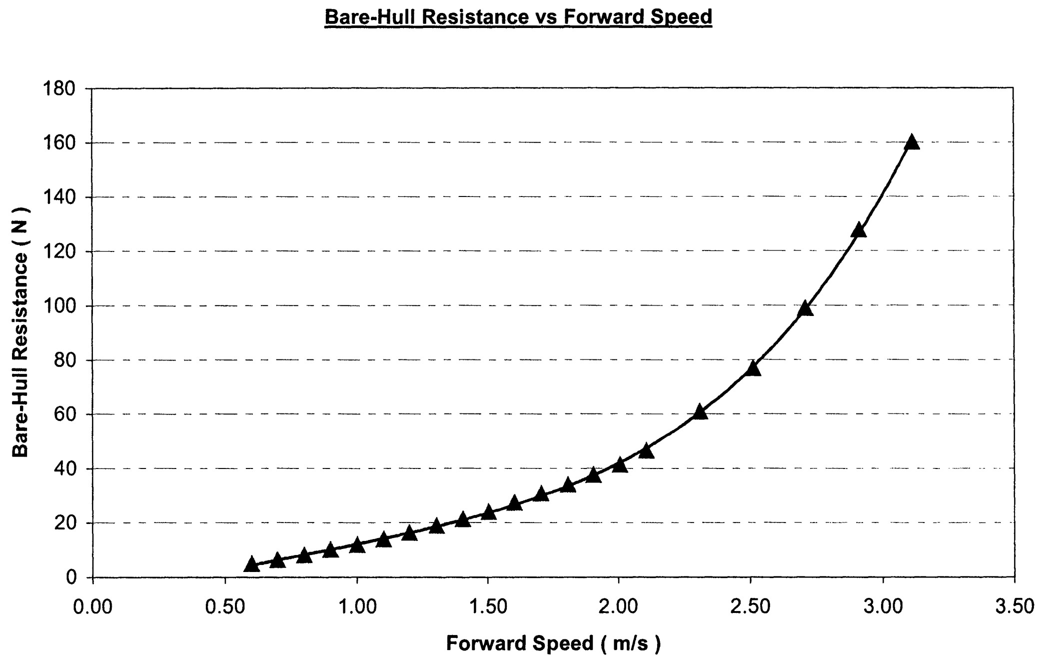


Fig. 4.53

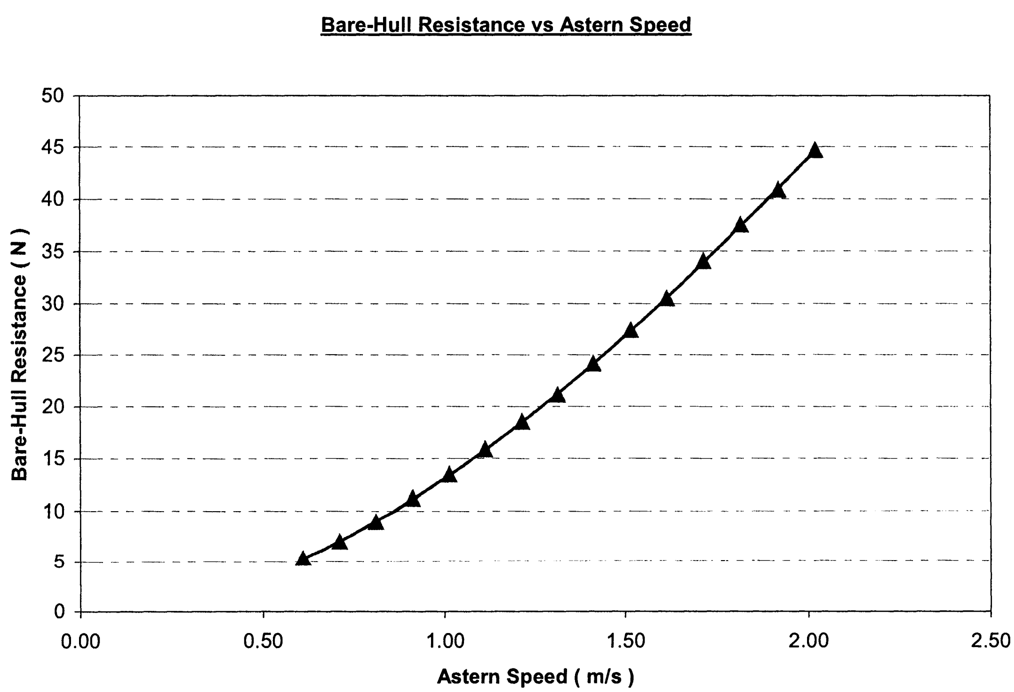


Fig. 4.54

Comparison of Ahead and Astern Bare-Hull Resistances

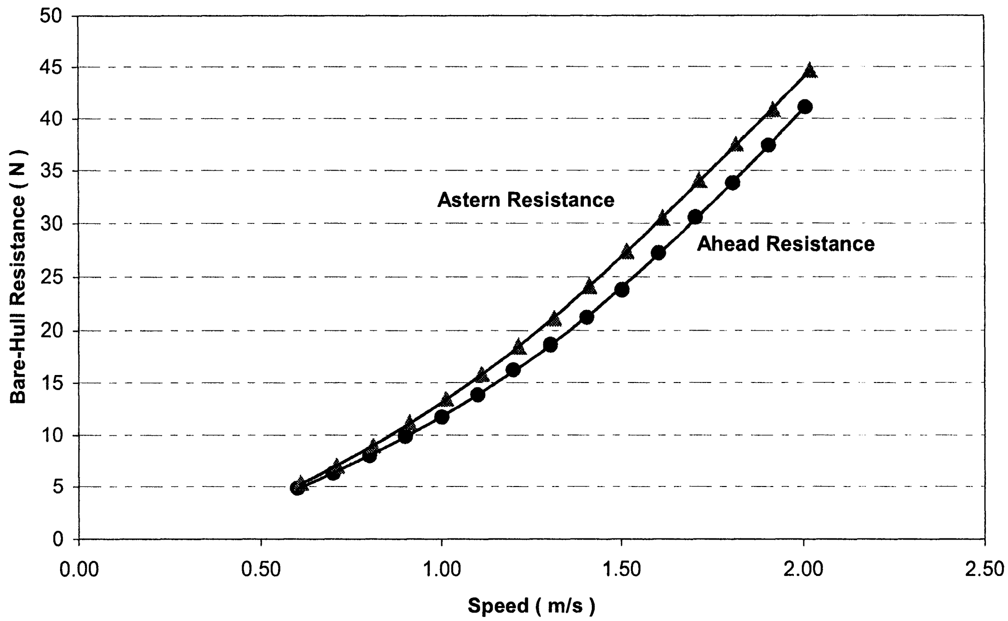


Fig. 4.55

Resistance Tests
(Comparison of R and $F_{T=0}$)
($F_{T=0}$ for "Propeller + Duct Fitted on" Case)

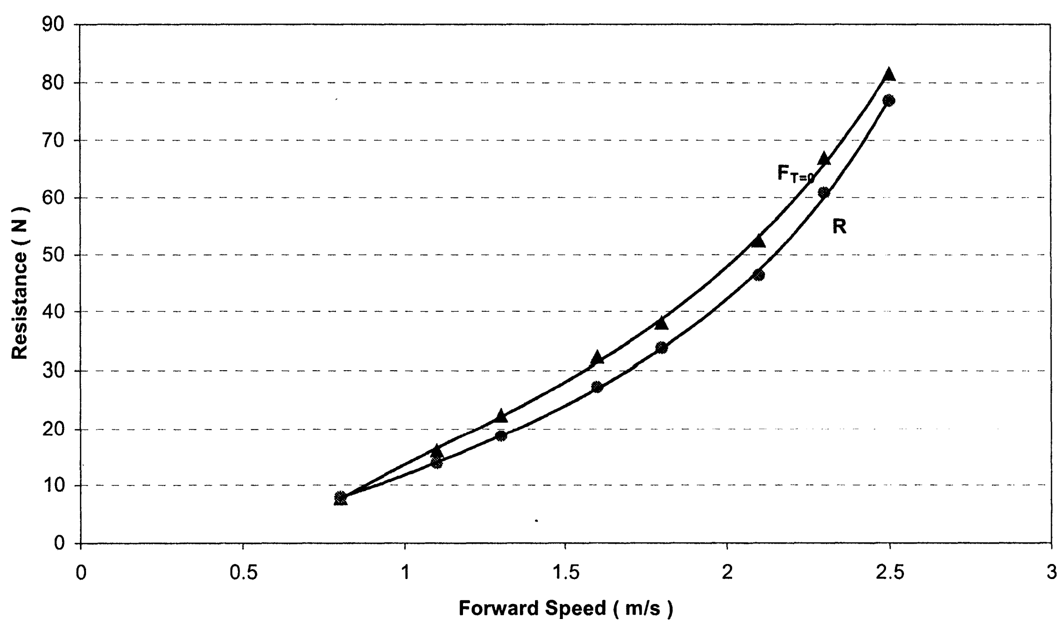


Fig. 4.56

The data were then non-dimensionalized into a drag coefficient 'C_d'; the frontal area $\pi D^2/4$ of the vehicle was used for calculating the drag coefficient. The drag coefficient 'C_d' was then plotted against the vehicle forward speed and astern speed; see Fig. 4.57 and Fig 4.58. In Fig 4.57, at speeds above 2 m/s, the rise of 'C_d' indicates a higher than V^2 dependence of resistance on speed (V). The data were also non-dimensionalized into a drag coefficient 'C_d' using the wetted surface area (3.009 m^2) of the vehicle. This drag coefficient 'C_d' was then plotted against the vehicle forward speed and astern speed; see Fig. 4.59 and Fig 4.60.

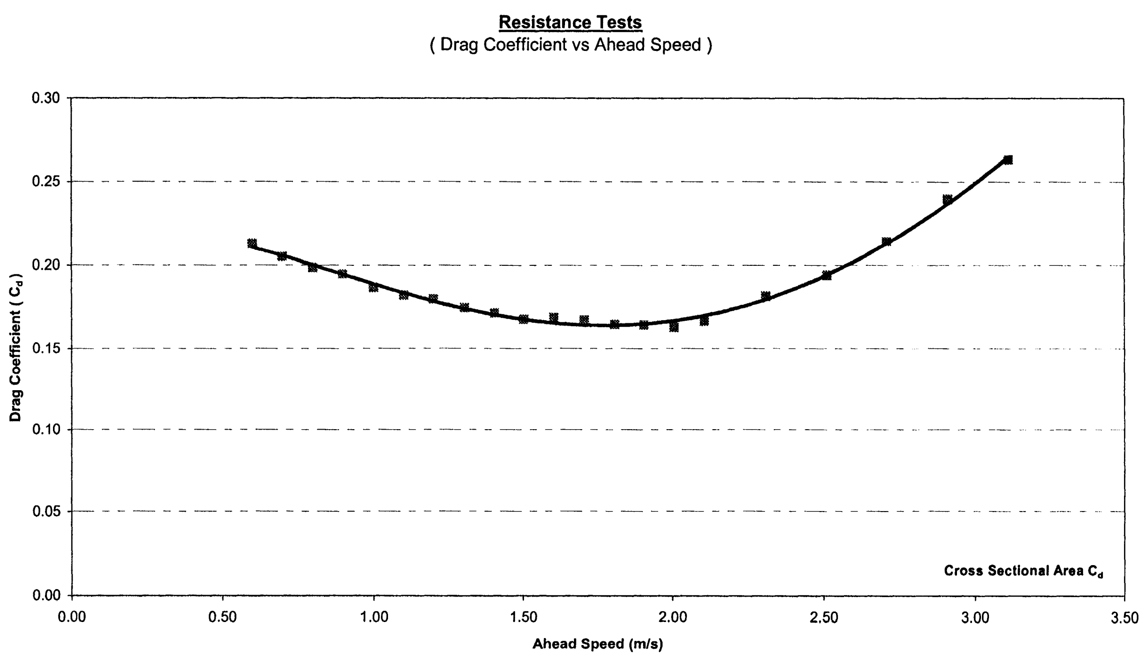


Fig. 4.57

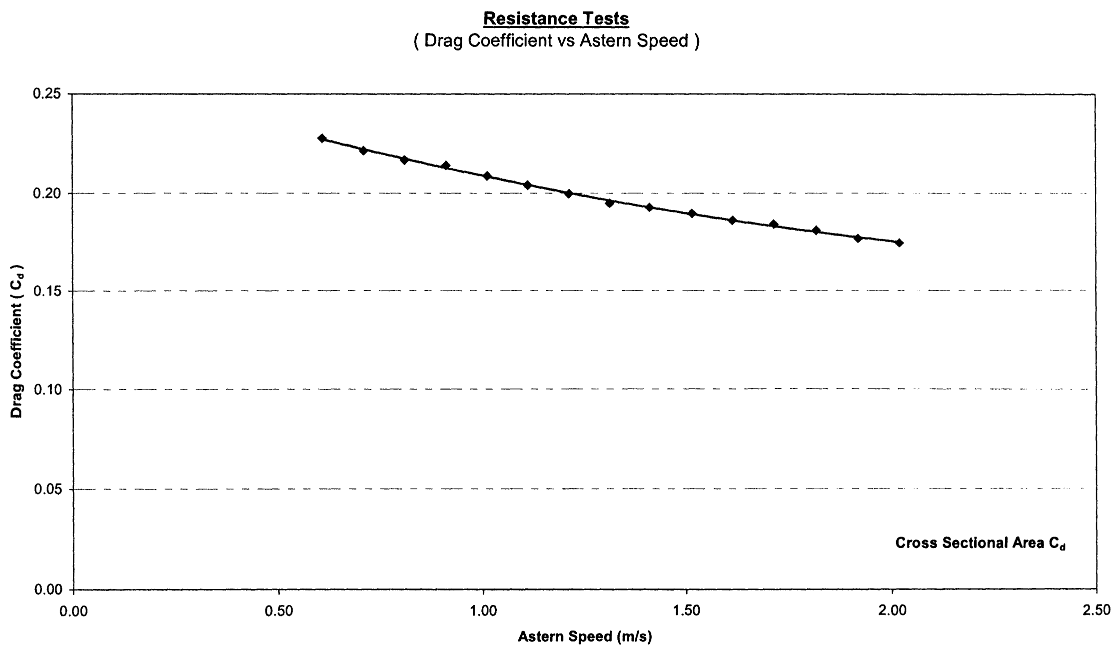


Fig. 4.58

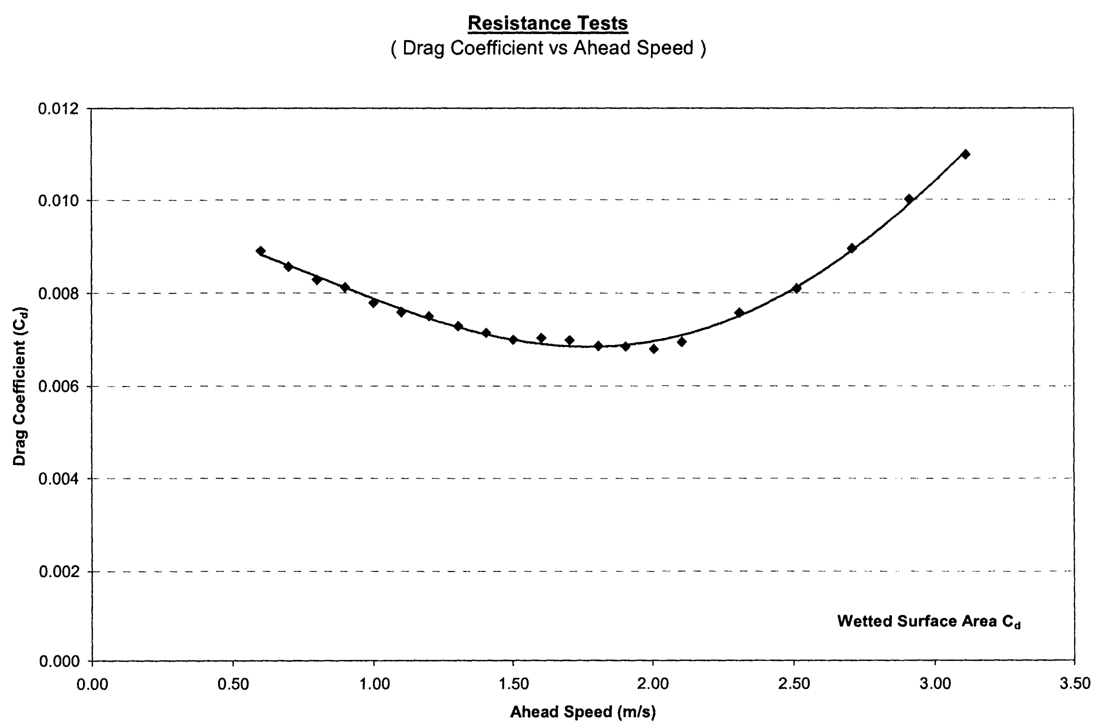


Fig. 4.59

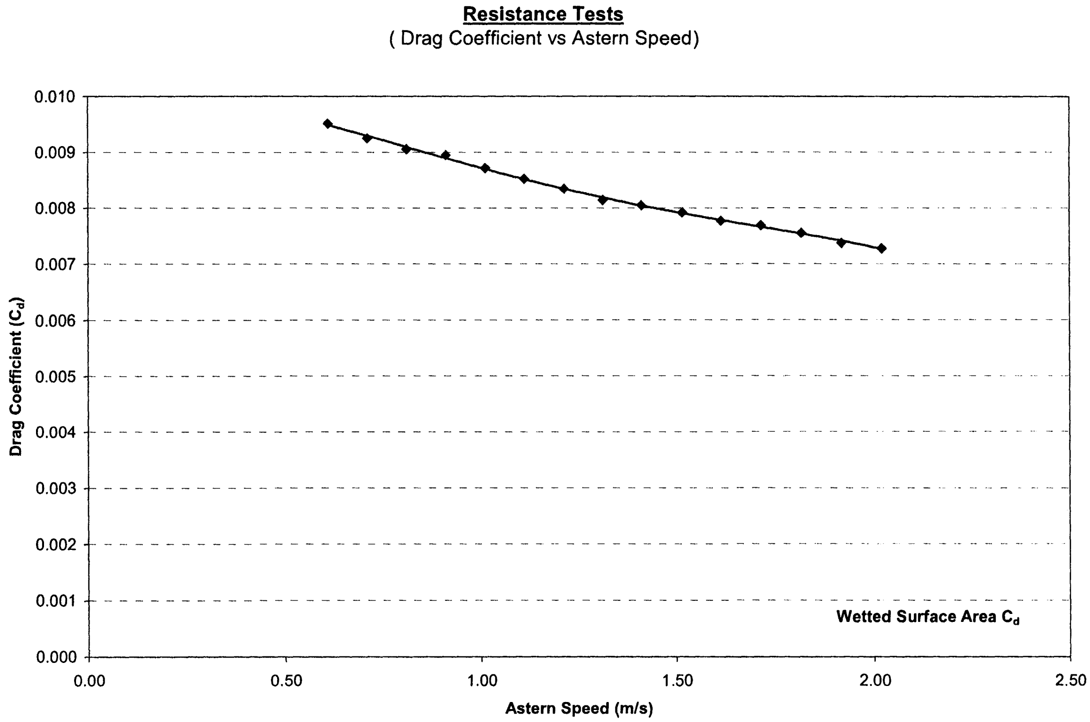


Fig. 4.60

4.4 Uncertainty Analysis

An uncertainty analysis was carried out in order to ascertain the overall level of uncertainty in the test measurements. The analysis was carried out according to the ITTC recommended procedure 7.5-02-03-01.2 Rev.00 (Propulsion, Performance Uncertainty Analysis, Example for Propulsion Test) and 7.5-02-02-02 Rev.01 (Testing and Extrapolation Methods, Resistance Uncertainty Analysis, Example for Resistance Test). Results of the uncertainty analysis are tabulated in Appendix B. These results are summarised in Table 4.6 on the following page.

Table 4.6

Summary of the Uncertainty Analysis

Load Varying Self-Propulsion Test (Propeller Fitted on)		
Page No.	***Term***	***Uncertainty***
B 14 to B 18	n	The uncertainty in the value of 'n' (U_n) is about 0.039 rpm and is essentially the same for different forward speeds (V).
B 19 to B 23	V	For different forward speeds (V), the uncertainty in the value of 'V' (U_V) is about 0.003 m/s.
B 24 to B 28	Q	For different forward speeds (V), the uncertainty in the value of 'Q' (U_Q) is about 0.008 N-m.
B 29 to B 33	T_P	For different forward speeds (V), the uncertainty in the value of 'T_P' (U_T) is about 0.25 N.
B 34 to B 38	F	For different forward speeds (V), the uncertainty in the value of 'F' (U_F) is about 0.27 N.
B 39 to B 43	J	For different forward speeds (V), the uncertainty in the value of 'J' (U_J) is less than or equal to 0.002.
B 44 to B 47	K_q	For different forward speeds (V), the uncertainty in the value of 'K_q' (U_{Kq}) is less than or equal to 0.002.
B 48 to B 51	K_{tp}	For different forward speeds (V), the uncertainty in the value of 'K_{tp}' (U_{Ktp}) is less than or equal to 0.02.
Load Varying Self-Propulsion Test (Propeller + Duct Fitted on)		
Page No.	***Term***	***Uncertainty***
B 52 to B 56	n	The uncertainty in the value of 'n' (U_n) is about 0.039 rpm and is essentially the same for different forward speeds (V).
B 57 to B 61	V	For different forward speeds (V), the uncertainty in the value of 'V' (U_V) is about 0.003 m/s.
B 62 to B 66	Q	For different forward speeds (V), the uncertainty in the value of 'Q' (U_Q) is about 0.008 N-m.
B 67 to B 71	T_t	For different forward speeds (V), the uncertainty in the value of 'T_P' (U_T) is about 0.25 N.
B 72 to B 76	F	For different forward speeds (V), the uncertainty in the value of 'F' (U_F) is about 0.27 N.
B 77 to B 81	J	For different forward speeds (V), the uncertainty in the value of 'J' (U_J) is less than or equal to 0.002.
B 82 to B 85	K_q	For different forward speeds (V), the uncertainty in the value of 'K_q' (U_{Kq}) is less than or equal to 0.002.
B 86 to B 89	K_{tt}	For different forward speeds (V), the uncertainty in the value of 'K_{tt}' (U_{Ktp}) is less than or equal to 0.02.

Table 4.6 (continued)

Summary of the Uncertainty Analysis

Bare-Hull Resistance Test		
Page No.	***Term***	***Uncertainty***
B 90 to B 92	V (Ahd.)	For different forward speeds (V), the uncertainty in the value of 'V' (U_V) is less than or equal to 0.004 m/s.
	V (Ast.)	For different astern speeds (V), the uncertainty in the value of 'V' (U_V) is less than 0.004 m/s.
B 93 to B 95	R (Ahd.)	For different forward speeds (V), the uncertainty in the value of 'R' (U_R) is about 0.27 N.
	R (Ast.)	For different astern speeds (V), the uncertainty in the value of 'R' (U_R) is about 0.27 N.
B 96 to B 98	C_d (Ahd.)	For different forward speeds (V), the uncertainty in the value of 'C_d' (U_{Cd}) is less than 0.03.
	C_d (Ast.)	For different astern speeds (V), the uncertainty in the value of 'C_d' (U_{Cd}) is less than 0.03.

Bollard-Pull Test		
Page No.	***Term***	***Uncertainty***
B 99 to B 100	n	For different propeller rotational speeds (n), the uncertainty in the value of 'n' (U_n) is about 0.039 rpm.
B 101 to B 102	Q	For different propeller rotational speeds (n), the uncertainty in the value of 'Q' (U_Q) is about 0.008 N-m.
B 103 to B 104	T_P	For different propeller rotational speeds (n), the uncertainty in the value of 'T_P' (U_T) is about 0.25 N.
B 105 to B 106	F	For different propeller rotational speeds (n), the uncertainty in the value of 'F' (U_F) is about 0.27 N.

4.5 Problems Faced During the Tests

i) Leaf Spring Breakage: In a conventional self-propulsion test, a braking arrangement is generally provided, in order to prevent forces of large magnitude from acting on the (tow force) load cell during the acceleration / deceleration periods. During a test run, when the vehicle accelerates / decelerates, the brakes are engaged. This ensures that all the towing forces are transmitted directly from the struts to the hull (the load cell is bypassed). Once the vehicle reaches a constant speed, the brakes are disengaged and subsequently all the towing forces are transmitted from the strut to the hull via the load cell. For the self-propulsion tests conducted on "C-SCOUT", no braking arrangement was provided. Consequently at high speeds (> 3 m/s), when the vehicle was being accelerated / decelerated for a run, forces of large magnitude acted on the forward and aft flexible supports, leading to spring breakage. To prevent spring breakage, the maximum test speed was therefore limited to 3 m/s.

ii) Stern Tube Bearing Seizure: The stern tube initially had two bearings (oil-lite bushings), one fitted at each end of the stern tube. During the course of the tests, the aft bearing seized up. The two oil-lite bearings were subsequently replaced with three linear rotary bearings [Fig. 4.10] and no further instances of bearing seizure occurred.

iii) Difficulty in Accessing Components: In a conventional self-propulsion test with a ship model, the model floats on the surface of the water and hence all components are easily accessible. For the tests conducted on "C-SCOUT", the vehicle was submerged in water, making it very difficult to access components in case something malfunctioned.

4.6 Conclusions

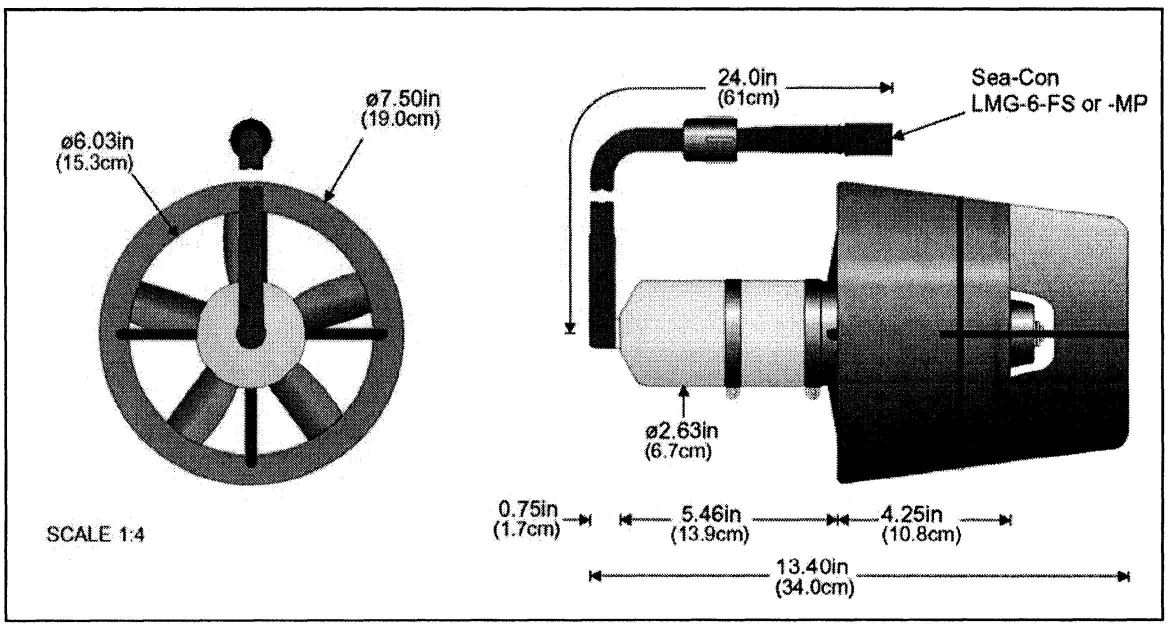
These resistance and propulsion experiments with "C-SCOUT", constitute a first step in evaluating the vehicle performance. They provide a wealth of information about the performance of the propulsion system over a range of vehicle speeds and propeller loadings. This data will be used for optimizing the propulsion system and the hull form of the vehicle.

Chapter 5

Design of a High Efficiency Propeller for "C-SCOUT"

5.1 The Need for a High Efficiency Propeller

The amount of stored energy available on board an AUV is limited. Hence all AUVs operate on a tight energy budget. The propulsion system of an AUV is one of the major consumers of energy. Therefore a high efficiency propeller implies lower energy consumption, which then translates into increased vehicle endurance and a reduction in vehicle operating costs. The AUV "C-SCOUT" currently uses a 1 HP Tecnadyne (model 1020) thruster unit as its main propulsion system. The unit [Fig 5.1] consists of a brushless DC motor mounted inside a waterproof casing, a magnetic coupling, propeller and an accelerating duct.



Tecnadyne (Model 1020) Thruster: Fig 5.1 [Ref 5-13]

Table 5.1 compares the performance of the Tecnadyne thruster unit with a proposed high efficiency propeller that was designed for "C-SCOUT" by the author (see Section 5.4). The results for the Tecnadyne thruster were obtained from the propulsion tests conducted on "C-SCOUT" (see Chapter 4). From the table, it can be observed that in order to move "C-SCOUT" at a speed of 2 m/s, the Tecnadyne thruster unit requires a delivered power of 153 Watt. In contrast, the proposed high efficiency propeller needs only 115 Watt for the same job. Use of the high efficiency propeller therefore leads to a gain of 20 % in terms of propulsive efficiency and a savings of 38 Watt in terms of energy consumption.

Table 5.1

Tecnadyne 1020 Thruster vs High Efficiency Propeller (at a Forward Speed of 2 m/s)

S.No	**Propulsor**	**Propeller Diameter (D)** cm	**Propeller Speed (n)** rpm	**Delivered Power (P_d)** Watt	**Propulsive Efficiency (η_d)** %
1	*Tecnadyne 1020*				
1.a	Propeller alone	15.2	953	187	50
1.b	Propeller + Duct	15.2	890	153	62
2	*High efficiency propeller*	50.0	300	115	82

The low efficiency of the Tecnadyne thruster unit is the consequence of using a small diameter, high rpm, heavily-loaded propeller. Heavily-loaded blades are less efficient and are more susceptible to cavitation. The small diameter of the propeller means that it has to rotate at a higher rpm to produce the same amount of thrust, than a propeller of larger diameter. This leads to increased rotational energy losses and a corresponding drop in propeller efficiency. Due to the above mentioned reasons, it can be clearly seen that use of the Tecnadyne thruster unit means low propeller efficiencies and high energy consumption.

5.2 High Efficiency Propeller Concept

The high efficiency propeller envisioned for "C-SCOUT" basically consists of a large diameter (0.5 m), slender, fine-bladed, slow-turning (300 rpm), wake-adapted, single-screw propeller with two blades [Fig 5.6]. The advantage of such a propeller is a potential propeller efficiency of 80 to 90 %.

5.2.1 Reasons for the High Efficiency of the Propeller

1) Large Diameter

a) Low Thrust Loading: From propeller momentum theory it can be shown that the ideal efficiency (η_i) of a propeller is given by [Ref 5-14]:

$$\boxed{\eta_i = \frac{2}{1 + (1 + C_{th})^{1/2}}} \quad \text{.....5.1}$$

where the thrust loading coefficient (C_{th}) is defined as:

$$\boxed{C_{th} = \frac{T}{\frac{1}{2}\,\rho\, V_A^2\, A_0}} \quad \text{.....5.2}$$

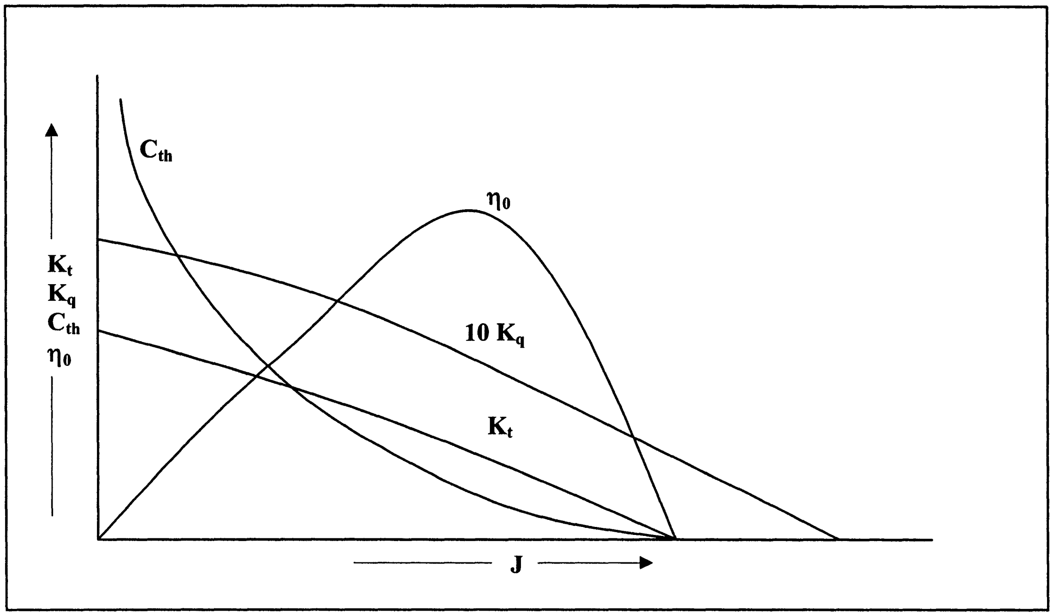
T = Propeller thrust (N)

ρ = Density of water (kg/m^3)

V_A = Speed of advance of the propeller (m/s)

A_0 = Propeller disc area (m^2)

From the above expression and from Fig 5.2, it can be seen that the lower the thrust loading, the higher the propeller efficiency. A low thrust loading can be achieved by increasing the propeller disc area (by increasing the propeller diameter).



K_t, K_q, η_o, C_{th} vs J: Fig 5.2 [Ref 5-14]

b) High Mass Flow: The thrust generated by a propeller is governed by Newton's second law. That is, propeller thrust is equal to the rate of change of momentum (mass times axial induced velocity) of the mass of water that the propeller accelerates. Now there are two ways that a propeller can produce a particular amount of thrust. Either it can accelerate a large mass of water at low induced velocities (using a large diameter, slow rpm propeller) or it can accelerate a small mass of water at high induced velocities (using a small diameter, high rpm propeller). From propeller momentum theory, it can be proved that the former method of thrust generation is much more efficient than the latter method. According to this theory, the ideal efficiency (η_i) of a propeller can also be expressed as [Ref 5-14]:

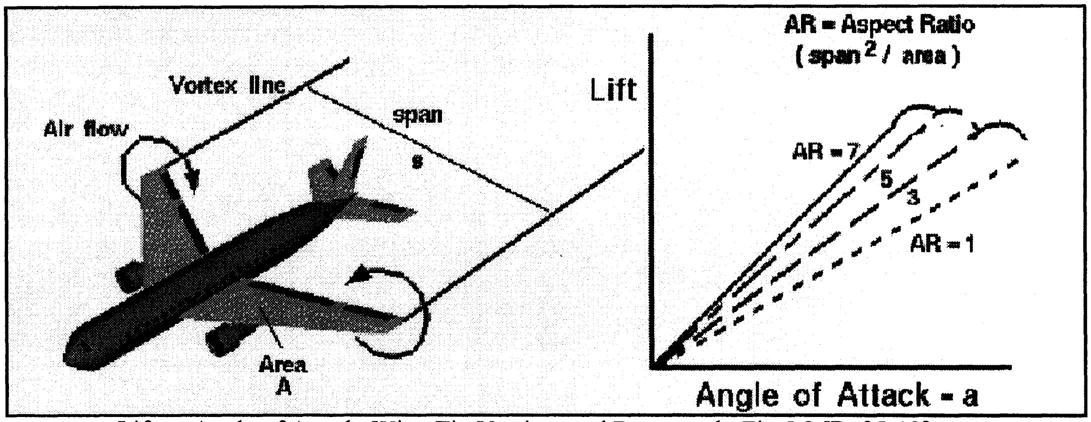
$$\boxed{\eta_i = \frac{1 - a'}{1 + a}} \qquad .....5.3$$

where “a’ ” is the rotational inflow factor and “a” is the axial inflow factor. The factors “a” and “a’ ” are used for calculating the axial induced velocity (aV_A) and the rotational induced velocity ($a'\omega$). From the above expression, it can be seen that the propeller efficiency can be maximized by minimizing the factors “a” and “a’ ”, ie., by minimizing the axial and rotational induced velocities. Therefore one of the objectives while designing a high efficiency propeller is to maximize the mass flow and minimize the induced velocities. A large diameter propeller by virtue of its large disc area, can accelerate a large mass of water while imparting low induced velocities. Consequently such a propeller is extremely efficient.

2) High Aspect Ratio Blades: High efficiency propeller blades have a large span and a short chord length (high aspect ratio), conceptually similar to the wings of a glider. The efficiency of a propeller blade is directly proportional to the blade lift to drag ratio. Therefore the higher the lift/drag ratio, the greater is the propeller efficiency [Ref 5-12].

$$\boxed{\eta_{blade} \propto \frac{\text{Lift}}{\text{Drag}} \propto \frac{C_L}{C_D}} \qquad \text{.....5.4}$$

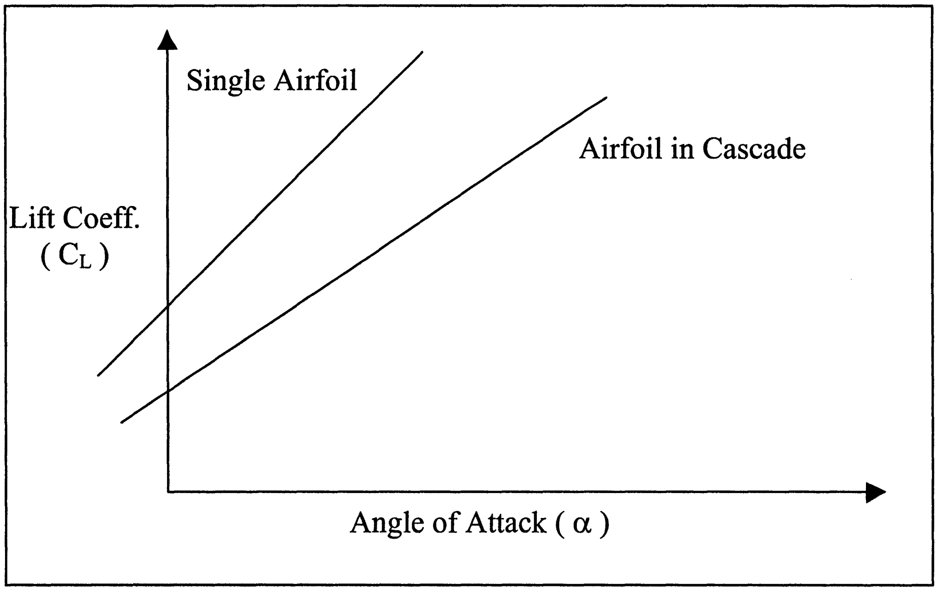
At the same angle of attack, high aspect ratio blades have a higher coefficient of lift (C_L) and a lower coefficient of drag (C_D) than low aspect ratio blades and this leads to increased propeller efficiency. This can be explained as follows. Any lifting surface such as a propeller blade generates lift due to a pressure differential across its two faces. At the blade tip, due to this pressure differential, a certain amount of spillage occurs. Fluid from the high pressure side rushes into the low pressure side – this sets up blade tip vortices and these vortices lead to a downwash of fluid behind the blades [Fig 5.3]. This downwash causes the blades to develop a local induced angle of attack – which then leads to a reduction in lift generated by the blades and an additional drag force to act on the blades (known as induced drag).



Lift vs Angle of Attack, Wing Tip Vortices and Downwash: Fig 5.3 [Ref 5-12]

The effect of tip vortices and downwash is quite pronounced on low aspect ratio blades and this effect reduces as the aspect ratio of the blades increases. Hence high aspect ratio blades have higher lift/drag ratios than blades of lower aspect ratios and consequently high aspect ratio blades are more efficient.

3) Use of a Minimum Number of Blades: In general, for the same advance coefficient (J), as the number of blades on the propeller increases, its efficiency drops. This is due to the cascade effect [Ref 5-11]. The cascade effect occurs when a series of lifting surfaces (blades) work in sufficient proximity to each other, so as to mutually alter the flow regime around each other. Downwash from the first blade affects the second blade, causing its lift to drop [Fig 5.4] and its drag to increase. Downwash from the second blade then affects the third blade and so on. The lift/drag ratio of individual blades drops and consequently the propeller efficiency drops. Ideally a propeller with one blade is the most efficient, but getting such a propeller to balance is difficult. Hence a propeller with two blades is the next logical choice. Among all the practically feasible multi-bladed propellers, a two bladed propeller is least affected by the cascade effect.



C_L vs α (Cascade Effect): Fig 5.4 [Ref 5-11]

4) Use of a Wake-Adapted Propeller: The high efficiency propeller is designed to be wake-adapted. In a wake-adapted propeller, the propeller pitch matches the inflow conditions, ie., at each section of the blade, the blade geometric pitch angle (Φ) is set such that the angle of attack is equal to the ideal angle of attack (α_i):

$$\boxed{\alpha_i = \Phi - \beta_i} \qquad .....5.5$$

where α_i is the ideal angle of attack and β_i is the hydrodynamic pitch angle. The ideal angle of attack is defined as that angle of attack at which flow enters the leading edge smoothly [Ref 5-19]. Smooth entry prevents a low pressure peak near the leading edge of the blade, thus preventing flow separation (and the accompanying increase in drag) and hence maximizing propeller efficiency and avoiding potential cavitation.

5) Use of Cambered Airfoil Sections: The high efficiency propeller blade sections consist of a superposition of a NACA a=0.8 meanline with a NACA 16-015 airfoil thickness distribution

[Fig 5-7]. This family of airfoil sections in general has a high lift/drag ratio and hence it maximizes propeller efficiency. Airfoil sections can generate lift either by having an angle of attack or by having section camber (asymmetry). The high efficiency propeller blade sections generate most of their lift due to section camber. Consequently blade sections have a very small angle of attack ($= \alpha_i$). At small angles of attack, the lift/drag ratio is maximum and this ensures maximum propeller efficiency. Also since the propeller blade is lightly loaded, the lift required to be generated by individual blade sections is low. This ensures that the camber required by individual blade sections is low – this minimizes drag losses.

6) Use of Fine (Thin) Blades with Low Expanded Area Ratio: The minimum expanded area ratio (A_E / A_O) is determined by cavitation considerations and the minimum blade thickness (t) is determined by structural strength considerations. The lower the expanded area ratio and the lower the blade thickness, lower is the blade drag and hence the propeller efficiency is higher. The high efficiency propeller has a very low expanded area ratio of 0.11 and a thickness ratio (t_x / D) of 0.0094 at 0.7R. Hence the blades generate low drag and have high efficiency.

7) Use of Low Propeller Rotational Speeds: The higher the rotational speed of a propeller, the higher the rotational energy losses and consequently the lower the propeller efficiency. As a general rule, the propeller rotational speed is inversely proportional to the propeller diameter. Use of a large diameter propeller therefore permits the use of low propeller speeds and hence minimizes the rotational energy losses. Also for a given vehicle speed (V) and propeller diameter (D), there exists a particular propeller rotational speed (n) and a corresponding value of the advance coefficient (J) at which the propeller efficiency (η) is maximum. Any further increase in the propeller rotational speed, leads to a reduction in the advance coefficient and hence efficiency. High propeller rotational speeds therefore implies a low value of J and a propeller that operates in a region of low η. Hence use of optimum propeller rotational speeds ensures maximum efficiency.

Thus as explained in the above sections, the key features that give the high efficiency propeller its high efficiency are: a large diameter, large mass flow, low thrust loading, high aspect ratio blades, minimum number of blades, low blade expanded area ratio, low blade thickness, a wake-adapted pitch distribution and lift development using section camber alone.

5.2.2 The Disadvantages of the High Efficiency Propeller

1) Blade Damage Issues: The high efficiency propeller has a diameter (0.5m) (see section 5.4.1) that is 25 % greater than the hull diameter (0.4 m). This is not expected to pose much of a problem in the open seas, where the vehicle is expected to spend much of its mission time. However in shallow waters, in the vicinity of other objects, the vehicle needs to be maneuvered with care, so as to preclude the possibility of blade damage.

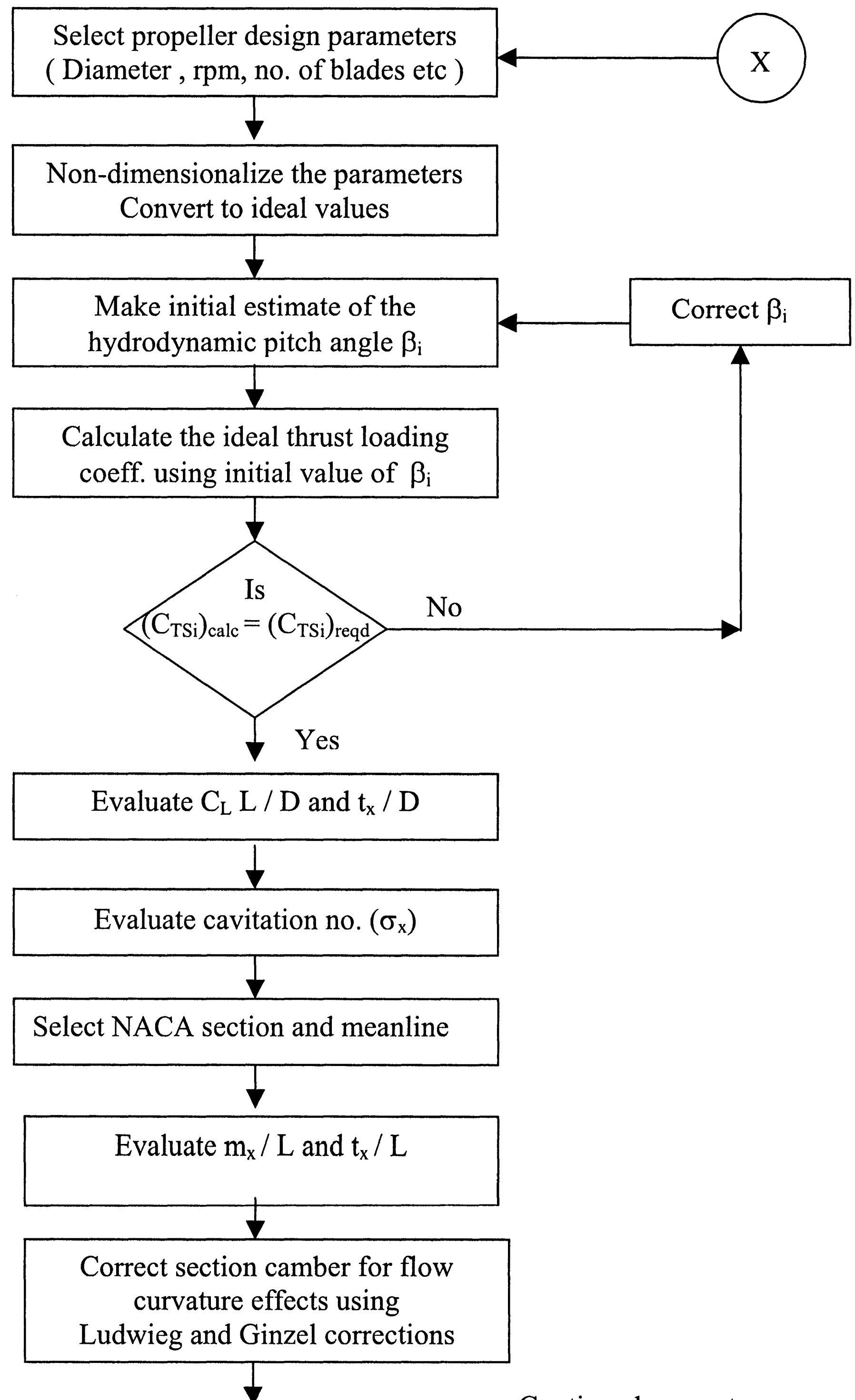
2) Structural Strength Issues: Large diameter blades imply increased moment arms and hence greater stress levels at the blade root section. This calls for additional blade thickness to withstand the stresses. However, since the vehicle operates at low speeds (2 to 4 knots), therefore the thrust and torque loads on the blades are quite low. Consequently the blade stresses are also quite low.

5.3 Design Methodology

The high efficiency propeller was designed using Eckhardt and Morgan's propeller design method [Ref 5-1] for a wake-adapted propeller. This method is based on the lifting line theory of Hill [Ref 5-2] along with modifications made by Lerbs [Ref 5-3] to account for the radial variation in the wake around the propeller disc. Once the propeller has been designed by lifting line theory, Eckhardt and Morgan then used Ludwieg and Ginzel's correction factors [Ref 5-4] to modify the section camber (in order to account for flow curvature effects). Then they also modified the section pitch using Lerb's h-factor method [Ref 5-5] (in order to account for viscosity, ideal angle of attack and lifting surface effect). Subsequently the blade strength is checked using the cantilever beam theory developed by Morgan [Ref 5-6].

Eckhardt and Morgan's propeller design method thus complements lifting line theory with lifting surface theory and is a fairly accurate method for designing lightly loaded propellers of moderate / high aspect ratios. Figure 5.10 shows a flowchart for the steps involved in systematically designing a propeller using Eckhardt and Morgan's method.

Flowchart for Eckhardt and Morgan's Propeller Design Method



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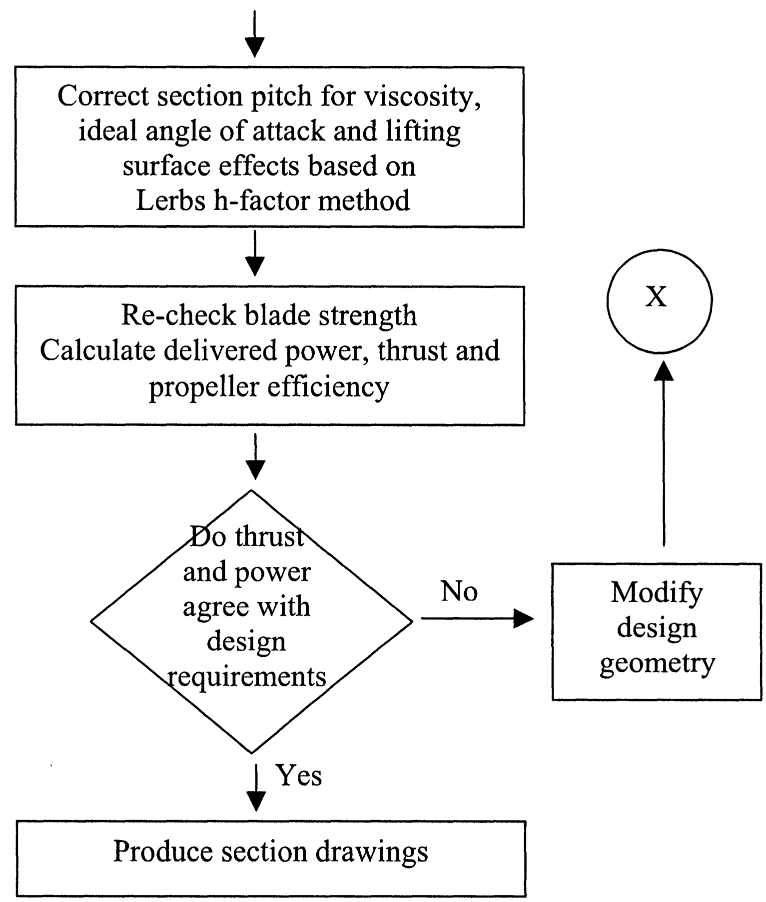


Flowchart for Eckhard and Morgan's Method: Fig 5.5

5.4 High Efficiency Propeller Design: Steps

5.4.1 Selection of the Propeller Design Parameters

a) Vehicle Speed (V): Results from the self-propulsion tests (Chapter 4) indicated that the optimum vehicle speed at which the maximum propulsive efficiency was achieved, lay in the region of 2 to 2.5 m/s (≅ 4 to 5 knots). Also the vehicle speed for optimum energy consumption was found to be around 2 m/s (see Chapter 2). Apart from this, a study of the maximum / cruising speeds of a number of present day AUVs (see Table 2.1, Chapter 2) indicated that the maximum speeds of most AUVs lay in the region of 4 to 5 knots (≅ 2 to 2.5 m/s) and that the cruising speeds of most AUVs lay in the region of 3 to 4 knots (≅ 1.5 to 2 m/s). Based on these facts, a speed of 2 m/s (≅ 4 knots) was chosen as the design speed for the high efficiency propeller.

b) Propeller Diameter (D): The propeller was envisioned to have a large diameter, but the question of "how large was large enough" arose. Since the propeller was not a standard series propeller, therefore no charts were available for optimizing the diameter – rpm combination. Therefore, to get an idea about what diameter propeller was needed, the thrust loading coefficient (C_{th}) and the ideal efficiencies (η_i) for a number of diameters ranging from 0.15 m to 0.70 m were calculated [Table 5.2]. Based on these results, a diameter of 0.5 m was selected. This diameter while not being too large, provided an ideal efficiency of over 96 %.

Table 5.2

Propeller Diameter vs Ideal Efficiency

D	C_{th}	η_{IDEAL}
m		%
0.70	0.06	98.3
0.65	0.07	98.1
0.60	0.09	97.7
0.55	0.11	97.3
0.50	0.14	96.7
0.45	0.18	95.8
0.40	0.24	94.4
0.35	0.35	92.3
0.30	0.55	89.0
0.25	0.94	83.5
0.20	1.80	74.7
0.15	4.16	61.1

c) Propeller Rotational Speed (n): Based on the results of the self-propulsion tests (Chapter 4), for the speed range of 2 to 2.5 m/s (speed range for maximum propulsive efficiency), the advance coefficient (J) was found to lie between 0.8 and 0.9. The advance coefficient J = 0.8 is an efficient operating regime for most propellers [Ref 5-20]. Therefore this was selected as the design J for the high efficiency propeller. At this value of J, for a vehicle speed of 2 m/s and a propeller diameter of 0.5 m, the propeller speed works out to be 300 rpm. The high efficiency propeller is expected to be driven by a small DC motor (0.15 kw capacity or smaller). A large variety of DC motors of this capacity / speed range are currently available in the market. Therefore propeller / DC motor matching should not be a problem.

d) Thrust (T) and Thrust Deduction Fraction (t): The amount of thrust required to propel the vehicle at a speed of 2 m/s (T = 52 N) and the value of the thrust deduction fraction (t = 0.065) were obtained from the self-propulsion test results (Chapter 4).

e) Radial Distribution of the Wake Fraction (w_x): In the absence of detailed wake survey information on "C-SCOUT", this information was extrapolated from wake survey tests carried out on a similar body of revolution [Ref 5-15].

f) Number of Blades (Z): Since the propeller was being designed for maximum efficiency, this required minimization of the cascade effect. Hence a minimum number of blades (Z = 2) were selected.

g) Blade Material: Manganese bronze was selected as the blade material owing to its high tensile and compressive strength, good machinability, corrosion resistance and a good ability to withstand fatigue failure. Other materials with similar properties, which could be used instead, include Naval Brass and Nickel Aluminium Bronze.

5.4.2 Calculation of the Non-dimensional Parameters (C_T and λ):

Using the selected propeller design parameters, the values of C_T , C_{TS} , λ , λ_S were calculated using Eqns. 5.6, 5.7, 5.8 and 5.9 [Ref 5-1] (See section 5.5.1 for the calculations):

$$\boxed{C_T = \frac{T}{\frac{1}{2}\rho \frac{D^2 \pi}{4} V_A^2}} \quad \text{.....5.6}$$

$$\boxed{C_{TS} = \frac{T}{\frac{1}{2}\rho \frac{D^2 \pi}{4} V^2}} \quad \text{.....5.7}$$

$$\boxed{l = \frac{V_A}{\pi n D}} \quad \text{.....5.8}$$

$$\boxed{\lambda_S = \frac{V}{\pi n D}} \quad \text{.....5.9}$$

Since Eckhardt and Morgan's method is based on potential flow theory, it is therefore necessary to change the real fluid thrusts (C_T and C_{TS}) into ideal fluid thrusts (C_{Ti} and C_{TSi}). This was achieved using Eqns. 5.10 and 5.11 [Ref 5-1] (See section 5.5.1 for the calculations):

$$\boxed{C_{Ti} = \frac{C_T}{1 - 2\,\varepsilon\lambda_i}} \quad .....5.10 \qquad \text{where} \qquad \boxed{\lambda_i = x \tan \beta_i} \quad .....5.11$$

Since ε and λ_i are initially unknown, it was assumed that the value of C_{Ti} is 3 % greater than C_T. Similarly the value of C_{TSi} was assumed to be 3 % greater than C_{TS}. This value of C_{TSi} is the required (design) value and is denoted as $(C_{TSi})_{\text{required}}$.

5.4.3 Determination of the Hydrodynamic Pitch Angle (β_i):

The next step involves determination of the hydrodynamic pitch angle β_i . An initial estimate of β_i was made using Lerbs's expression [Eqn. 5.12] for optimum wake-adapted propellers [Ref 5-1] (See the spreadsheet in section 5.5.1 for the calculations. Refer to the first row: "hydrodynamic pitch angle calculation – first iteration"):

$$\boxed{\tan \beta_i = \frac{\lambda_s \, (1 - \omega_0)^{1/2} \, (1 - \omega_x)^{1/2}}{X \qquad \eta_i}} \quad .....5.12$$

The ideal efficiency (η_i) required for use in the above expression, was determined using a set of curves developed by Kramer [Ref 5-1] for optimum propellers operating in open water. Since the propeller that is being designed is a non-optimum wake-adapted propeller, therefore the value of tan β_i obtained using Lerbs's expression for optimum wake-adapted propellers [Eqn. 5.12], gives only a first estimate of the actual value of tan β_i . Using this value of tan $\beta_{i,}$

the elemental ideal thrust loading coefficient for each section was computed using Hill's expression [Eqn 5.13]:

$$dC_{TSi} / dx = 8\,K\,X \; \frac{U_t}{2\,V} \left(\frac{X}{\lambda} - \frac{U_t}{2V} \right) \qquad \text{.....5.13}$$

where U_t / 2V was obtained using [Eqn 5.14] and β was obtained using [Eqn 5.15]:

$$\frac{U_t}{2\,V} = (1 - \omega_x) \; \frac{\sin\beta_i \, \sin(\beta_i - \beta)}{\sin\beta} \qquad \text{.....5.14}$$

$$\tan\beta = \frac{\lambda_s}{X}(1 - \omega_x) \qquad \text{.....5.15}$$

The elemental thrust loading coefficients [Eqn. 5.21] were then integrated over the length of the blade in order to obtain the "calculated" value of the ideal thrust loading coefficient. The "required" value of C_{Tsi} was then compared with the "calculated" value of C_{TSi}. Since there was a small difference between the two, the value of tan β_i obtained using the first estimate was corrected using the expression [Ref 5-1]:

$$(\tan\beta_i)_{corrected} = (\tan\beta_i)_{previous} \left[1 + \frac{(C_{TSi})_{required} - (C_{TSi})_{calculated}}{5\,(C_{TSi})_{required}} \right] \qquad \text{.....5.16}$$

Using this corrected value of tan β_I , the values of the ideal elemental thrust loading coefficients were again calculated and integrated over the blade length, so as to obtain $(C_{TSi})_{calculated}$. (See section 5.5.1 for the calculations. Refer to the second row: "hydrodynamic pitch angle calculation – second iteration"). The values of $(C_{TSi})_{calculated}$ and $(C_{TSi})_{required}$ were again compared and were found to converge (no difference existed between the two). At this point (of convergence) the correct value of tan β_i and hence the hydrodynamic pitch angle β_i was obtained.

5.4.4 Calculation of the Section Chord Lengths (L):

Cavitation criterion (Keller's formula) was used to determine the minimum expanded area ratio (A_E / A_O) required, at a depth of submergence of 1 m and a water temperature of 2 ^{0}C. Keller's formula states that:

$$\boxed{\frac{A_E}{A_O} = \left[\frac{1.3 + 0.3\,Z}{(P_0 - P_v)\,D^2} \right] T + K_0} \qquad \text{.....5.17}$$

where:

$P_0 = P_A + \rho\, g\, h' = 1.1110 * 10^5 \ N / m^2$

$P_v = 588.399 \ N / m^2 \ (0\ ^0C)$

$Z = 2$

$T = 52.26$ N

$D = 0.5$ m

$K_0 = 0.1$

Substituting and solving, the minimum A_E / A_O needed to prevent cavitation was found to be 0.1036. Substituting the value of A_E / A_O = 0.1036 into Eqn. 5.18, the chord lengths at different sections were determined:

$$\boxed{\frac{A_E}{A_O} = \frac{2\,Z}{\pi} \int_{x_h}^{1} \frac{L}{D}\, dx} \qquad \text{.....5.18}$$

The calculated chord lengths were then adjusted so as to obtain a final A_E / A_O of 0.1114. (See Appendix C for the calculations).

5.4.5 Calculation of the Blade Thickness Ratio (t_x / L):

A linearly decreasing blade thickness ratio (t_x / L), varying from 15 % at the root section (x = 0.2) to 4.5 % at the section x = 0.9, was selected. Blade stresses for this thickness distribution were checked using cantilever beam theory and were found to be much lower than the yield strength of the blade material (See section 5.4.12.e and section 5.5.2 for the strength calculations).

5.4.6 Calculation of the Section Lift-Length-Diameter Coefficient (C_L L / D):

The next step involves calculation of the lift-length-diameter coefficient (C_L L / D) at each section. This was calculated using Hill's expression [Ref 5-1]:

$$\boxed{C_L \cdot \frac{L}{D} = \frac{4\pi}{Z} \; \frac{X K \;\; U_t / 2V}{\frac{X}{\lambda} - \frac{U_t}{2V}} \; \cos \beta_i} \qquad \ldots\ldots 5.19$$

The section lift coefficient (C_L) was then obtained by dividing C_L L / D by L / D (obtained from section 5.4.4). (See the spreadsheet in section 5.5.1 for the calculations. Refer to the fourth row: "coefficient of lift calculation").

5.4.7 Selection of the Blade Section Profile (Meanline and Thickness Distribution):

For most marine vehicles, blade section profiles of airfoil shape are the usual choice, since these profiles have a high lift / drag ratio, thus making them extremely efficient. A section profile of airfoil shape was therefore selected for the high efficiency propeller. The section profile was built by combining a NACA a=0.8 meanline with a NACA 16-015 thickness distribution [Fig 5.7]. The NACA a = 0.8 meanline is a common choice for propeller applications. It has uniform chordwise loading for 80 % of its chord length from the leading edge, followed by linear attenuation in the loading. This type of loading is quite realistic in real fluids, whereas uniform chordwise loading (a = 1.0 meanline) is not. The a = 0.8 meanline is able to develop nearly 100 % of its theoretical lift in viscous flow. The a = 1.0 meanline in comparison is able to develop only about 74 % of its theoretical lift in viscous flow. The NACA 16 series airfoils are particularly well suited for propeller applications. Their thickness distribution was developed in order to produce a shape that generated very low induced velocities and hence had minimum energy losses. These airfoils have the location of maximum thickness at 0.5 chord length.

Method of Combining the Meanline and Thickness Distribution:

The ordinates and slopes for the a = 0.8 meanline, for a design lift coefficient (C_{Li}) of unity, were obtained from Ref 5-16. The ordinates and slopes for meanlines having lift coefficients other than unity, is obtained by multiplying the values of the ordinates and slopes for the case when $C_{Li} = 1$, with the required lift coefficient. The ordinates for the NACA 16-015 thickness distribution were again obtained from Ref 5-16. This thickness distribution was laid out perpendicular to the meanline (See Appendix C for the calculations). The abscissas and the ordinates of points on the upper (X_U, Y_U) and lower (X_L, Y_L) faces of the section, were obtained using the following expressions [Ref 5-16]:

$$\boxed{X_U = X' - Y_t \sin\theta} \quad \text{.....5.20}$$

$$\boxed{Y_U = Y_c + Y_t \cos\theta} \quad \text{.....5.21}$$

$$\boxed{X_L = X' + Y_t \sin\theta} \quad \text{.....5.22}$$

$$\boxed{Y_L = Y_c - Y_t \cos\theta} \quad \text{.....5.23}$$

where:

X' : chordwise position

Y_c : meanline ordinate

$\tan\theta$: meanline slope

Y_t : thickness distribution ordinate

5.4.8 Calculation of the Section Cavitation Numbers (σ_x):

The cavitation number (σ_x) at each section was then calculated using Eqn. 5.24 [Ref 5-1]. The propeller shaft centerline was assumed to be submerged 1 m below the surface of water and the water temperature was assumed to be 2 ^{0}C. In Eqn. 5.24, the pressure due to the head of water (P) was determined using $P = \rho\, g\, [\, h - x.\, D/2\,]$, where "h" is the shaft submergence and "P_a" is the atmospheric pressure. Eqn. 5.24 calculates the cavitation number at each section of a propeller blade which is in the top dead center (tdc) position. In the tdc position, the propeller blade experiences the least static head, coupled with a mean dynamic head.

$$\boxed{\sigma_x = \frac{(P + P_a)\sin^2\beta}{\frac{1}{2}\rho V^2 \cos^2(\beta - \beta_i)(1 - \omega_x)^2}} \qquad \text{.....5.24}$$

The calculated values of the section cavitation numbers were found to be considerably higher than the minimum cavitation numbers given in the incipient cavitation charts for the NACA a = 0.8 meanline and the NACA 16 thickness distribution [Ref 5-1] (See the spreadsheet in section 5.5.1 for the calculations. Refer to the third row: "cavitation number calculation"). This basically shows that the expanded area ratio (A_E / A_O) can be reduced still further. However, A_E / A_O was not reduced any further. The reason for this is that reduction in A_E / A_O leads to a corresponding reduction in the braking effect of the propeller when running at astern rotation to stop the vehicle [Ref 5-17]. Also a reduction in A_E / A_O leads to a reduction in blade strength. Since the expanded area ratio was already quite small ($A_E / A_O = 0.11$), it was not considered wise to reduce it any further.

5.4.9 Calculation of the Section Camber Ratios (m_x / L) and the Camber Corrections:

For the NACA a = 0.8 meanline, at $C_{Li} = 1$, $m_x / L = 0.0679$. As explained in section 5.4.7, since the meanline ordinates vary linearly with the coefficient of lift, therefore the camber ratio can be expressed as:

$$\boxed{m_x / L = 0.0679\, C_L} \qquad \text{.....5.25}$$

The section camber ratios were then calculated by substituting the appropriate values of the lift coefficient (C_L) into the above expression. The calculated values of the camber ratios (m_x / L) are two-dimensional values operating in rectilinear flow. They then need to be corrected for flow curvature effects. Ludwieg and Ginzel's correction factors K_1 and K_2 [Ref 5-4] were used to account for flow curvature effects. The corrected camber ratio is given by the relation:

$$\boxed{\left(\frac{m_x}{L}\right)_{corrected} = K_1 K_2 \frac{m_x}{L}} \qquad \text{.....5.26}$$

where the correction factor K_1 = function (λ_i , A_E/A_0) and K_2 = function (x , A_E/A_0). (See the spreadsheet in section 5.5.1 for the calculations. Refer to the fifth row: "camber correction calculation").

5.4.10 Calculation of the Section Pitch Distributions (P / D) and the Pitch Corrections:

Once the section cambers have been corrected, the section pitches need to be corrected for viscosity, for ideal angle of attack of the meanline and for change in curvature over the chord length. Corrections for viscosity and for ideal angle of attack of the meanline are combined into an additional angle of attack α_1, expressed as [Ref 5-1]:

$$\boxed{\alpha_1 = K_3\, C_L} \qquad \text{.....5.27}$$

where "K_3" is a correction factor that depends on the shape of the meanline.

Correction for change in curvature over the chord length is made using Lerbs h-factor method. This correction is necessary since Ludwieg and Ginzel's camber correction was based on flow curvature effects at the mid-point of the section. Experiments with propellers however showed that they were underpitched with this correction. Lerbs defined an additional angle of attack (α_2) to account for this change in curvature over the chord length. This additional angle of attack is defined as [Ref 5-1]:

$$\boxed{\alpha_2 = (\alpha_b + \alpha_f) - (\alpha_i' + \alpha_0)} \qquad \text{.....5.28}$$

where α_b is the bound vortex contribution, defined by the expression [Ref 5-1]:

$$\boxed{\alpha_b = \frac{\sin \beta i}{2} \sum \left[\left(\frac{L}{D} \sin \mu - 0.7 \cos \beta i \cos \mu \right) \int_{x_h}^{1} \frac{G}{(P/R)^3}\, dx \right]}$$

$$\text{.....5.29}$$

where :

μ = angular position of the blade

G = Non-dimensional circulation per blade [Ref 5-1]

$$\boxed{G = \frac{2\,X\,K}{Z} \frac{U_t}{2V} \frac{1}{(1 - \omega_x)}} \qquad \text{.....5.30}$$

$$(P/R)^3 = \left[X^2 + (L/D)^2 + 0.49 - 2(L/D \cos\mu \cos\beta i + 0.7 \sin\mu)X \right]^{3/2}$$

.....5.31

α_f is the free vortex contribution, defined by the expression [Ref. 5-1]:

$$\alpha_f = \alpha_i' \frac{2}{1 + \cos^2 \beta_i (2/h - 1)}$$

.....5.32

where [Ref 5-1]:

$$\alpha_i' = \beta_i - \beta$$

.....5.33

The parameter h is a function of θ , which is defined as [Ref 5-1] :

$$\theta = \arctan\left(\frac{0.7}{\sin \beta_i} \frac{D}{L} \right)$$

.....5.34

α_0 is defined by the expression [Ref 5-1] :

$$\alpha_0 = K_4 C_L$$

.....5.35

where "α_0" is the angle of zero lift of the meanline and "K_4" is a factor that depends on the shape of the meanline [Ref 5-1].

The value of α_1 was determined using Eqn. 5.27 (See the spreadsheet in section 5.5.1 for the calculations. Refer to the sixth row: "pitch correction (α_1) calculation"). Thereafter the values of α_b , α_f , α_i and α_0 were determined using Eqns. 5.29 to 5.35. Subsequently the value of α_2 was determined using Eqn. 5.28 (See the spreadsheet in section 5.5.1 for the calculations. Refer to the seventh row: "pitch correction (α_2) calculation").

The pitch corrections α_1 and α_2 were then used in the following expression [Ref 5-1], so as to obtain the final corrected pitch distribution at each section (See the spreadsheet in section 5.5.1 for the calculations. Refer to the eighth row: "final P/D ratio calculation"):

$$\boxed{P/D = \pi X \tan(\beta_i + \alpha_1)\left(1 + \frac{\Delta P/D}{P/D}\right)} \quad \text{.....5.36}$$

where [Ref 5-1]:

$$\boxed{1 + \frac{\Delta P/D}{P/D} = \frac{\tan(\beta_i + \alpha_2)_{0.7R}}{\tan(\beta_i)_{0.7R}}} \quad \text{.....5.37}$$

5.4.11 Calculation of the Thrust Loading Coefficient (C_{TS}), Power Coefficient (C_{PS}), Delivered Power (P_d) and Propeller Efficiency (η_b):

Subsequently the thrust loading coefficient (C_{TS}) & the power coefficient (C_{PS}) of the propeller (in a real fluid) were calculated using the expressions [Ref 5-1]:

$$\boxed{C_{TS} = \int_{x_h}^{1} (1 - \varepsilon \tan \beta_i) \frac{dC_{TSi}}{dx} . dx} \quad \text{.....5.38}$$

$$\boxed{C_{PS} = \int_{x_h}^{1} \frac{x}{\lambda_S} (\tan \beta_i + \varepsilon) \frac{dC_{TSi}}{dx} . dx} \quad \text{.....5.39}$$

where [Ref 5-1]:

$$\boxed{\varepsilon \cong \frac{0.008}{C_L}} \quad \text{.....5.40}$$

The delivered power (P_d) required by the propeller was then determined using [Ref 5-1]:

$$\boxed{P_d = \tfrac{1}{2}\, \rho\, \pi\, R^2\, V^3\, C_{PS}} \qquad \text{.....5.41}$$

The efficiency (η_b) of the propeller was subsequently determined using [Ref 5-1]:

$$\boxed{\eta_b = C_{TS} / C_{PS} \times 100} \;\; \% \qquad \text{.....5.42}$$

(See the spreadsheet in section 5.5.1 for the calculations. Refer to the ninth row: "final thrust coefficient / power coefficient calculation").

5.4.12 Verification of the Blade Strength Using Cantilever Beam Theory:

The blade stresses at the root section ($x = 0.2$) were then calculated using cantilever beam theory [Ref 5.6]. The principal stresses acting on the blade section are the bending stresses due to propeller thrust and propeller torque and the tensile stress due to centrifugal force. Since the high efficiency propeller has no skew or rake, therefore the stress components due to centrifugal bending and due to the out of plane stresses, are very small and can therefore be neglected.

a) Calculation of the Bending Moments M_{X0} and M_{Y0}:

First of all, the bending moments due to thrust (M_T) and torque (M_Q) were calculated using the expressions [Ref 5.6] (See the spreadsheet in section 5.5.2 for the calculations):

$$\boxed{M_T = \frac{1}{2}\rho \frac{\pi R^3}{Z} V^2 \int_{x_h}^{1} (x - x_0)(1 - \varepsilon \tan \beta_i) \frac{dC_{TSi}}{dx} \, . \, dx} \qquad \text{.....5.43}$$

$$\boxed{M_Q = \frac{1}{2}\rho \frac{\pi R^3}{Z} V^2 \int_{x_h}^{1} (x - x_0)(\tan \beta_i + \varepsilon) \frac{dC_{TSi}}{dx} \, . \, dx} \qquad \text{.....5.44}$$

where "x_0" is the non-dimensional radius of the section being analyzed.

The moments M_T and M_Q were then resolved into two components [Fig 5.9]: M_{X0} about an axis parallel to the nose-tail line and passing through the centroid of the blade section; M_{Y0} about an axis perpendicular to the nose-tail line and passing through the centroid of the blade section.

The following expressions were used for obtaining M_{X0} and M_{Y0} [Ref 5.6]:

$$\boxed{M_{X0} = M_T \cos\phi + M_Q \sin\phi} \qquad .....5.45$$

$$\boxed{M_{Y0} = M_T \sin\phi - M_Q \cos\phi} \qquad .....5.46$$

where ϕ is the geometric pitch angle of the blade section, defined as [Ref 5.6]:

$$\boxed{\phi = \arctan\left[\frac{1}{x\,\pi}\,\frac{P}{D}\right]} \qquad .....5.47$$

b) Calculation of the Moment of Inertias (I_{X0} and I_{Y0}) and the Position of the Centroid (X_{CG} and Y_{CG}) of the Root Section:

For the root section, the position of the centroid (X_{CG} and Y_{CG}) and the moments of inertia about an axis parallel to the nose-tail line and passing through the centroid of the blade section (I_{X0}) and about an axis perpendicular to the nose-tail line and passing through the centroid of the blade section (I_{Y0}), were calculated using Simpson's rule (See the spreadsheet in Appendix C for the calculations) [Fig 5.8].

c) Calculation of the Blade Mass (m) and the Blade Longitudinal Center of Gravity (X_c):

The blade mass was then calculated using the expression [Ref 5.11] (See the spreadsheet in Appendix C for the calculations):

$$\boxed{m = \rho_0 \int_{r_h}^{R} A_x \, dr} \qquad .....5.48$$

where:

r_h : Hub radius

R : Tip radius

A_x : Blade section area

ρ_0 : Density of manganese bronze (blade material)

The blade mass calculated using the above expression was increased by a factor of 2.5 % in order to account for the additional mass of the blade fillets [Ref 5-11]. Blade section area (A_x) was calculated from positions x = 0.2 to x = 1.0 using the following expression[Ref 5.11]:

$$\boxed{A_x = \int_0^L t\, dl} \qquad \text{.....5.49}$$

where:

t : Blade section thickness

L : Chord length

The position of the longitudinal center of gravity (X_c) of one blade was then calculated using the expression [Ref 5.11]:

$$\boxed{X_c = \frac{\int_{x_h}^{1} A_x\, x\, dx}{\int_{x_h}^{1} A_x\, dx}} \qquad \text{.....5.50}$$

d) Calculation of the Centrifugal Force (F_C) Acting on One Blade:

The centrifugal force acting on one blade was then calculated using the expression [Ref 5.11] (See the spreadsheet in section 5.5.2 for the calculations):

$$\boxed{F_C = 2\,\pi^2\, m\, X_c\, D\, n^2} \qquad \text{.....5.51}$$

where:

m : Blade mass

X_c : Blade LCG

D : Propeller diameter

n : Propeller speed (rps)

e) Calculation of the Stresses at the Root Section:

The total stresses acting on the root section were then calculated at three critical points on the root section – at the leading edge ($\sigma_{L.E}$), at the trailing edge ($\sigma_{T.E}$) and at the point of maximum thickness on the suction back ($\sigma_{S.B}$), using the following expressions [Fig 5.9] (See the spreadsheet in section 5.5.2 for the calculations) [Ref 5.6]:

$$\boxed{\sigma_{L.E} = \frac{Y_1\, M_{X0}}{I_{X0}} - \frac{X_1\, M_{Y0}}{I_{Y0}} + \frac{F_c}{A_{x=0.2}}} \qquad \text{.....5.52}$$

$$\boxed{\sigma_{T.E} = \frac{Y_2\, M_{X0}}{I_{X0}} + \frac{X_2\, M_{Y0}}{I_{Y0}} + \frac{F_c}{A_{x=0.2}}} \qquad \text{.....5.53}$$

$$\boxed{\sigma_{S.B} = -\frac{Y_3\, M_{X0}}{I_{X0}} + \frac{X_3\, M_{Y0}}{I_{Y0}} + \frac{F_c}{A_{x=0.2}}} \qquad \text{.....5.54}$$

where:

+ ve sign : Tensile stress

- ve sign : Compressive stress

X_1 : Distance from the neutral axis to the leading edge (along the x axis) [Fig 5.9]

X_2 : Distance from the neutral axis to the trailing edge (along the x axis) [Fig 5.9]

X_3 : Distance from the neutral axis to the point of max. thickness on the suction back (along the x axis) [Fig 5.9]

Y_1 : Distance from the neutral axis to the leading edge (along the y axis) [Fig 5.9]

Y_2 : Distance from the neutral axis to the trailing edge (along the y axis) [Fig 5.9]

Y_3 : Distance from the neutral axis to the point of max. thickness on the suction back (along the y axis) [Fig 5.9]

$A_{x=0.2}$: Blade section area at section x = 0.2

The magnitudes of the calculated stresses were as follows:

$\sigma_{L.E}$ = 6.2 MPa (Tensile)

$\sigma_{T.E}$ = 7.3 MPa (Tensile)

$\sigma_{S.B}$ = 8.3 MPa (Compressive)

These stresses were much lower than the yield strength (138 MPa), the ultimate tensile (414 MPa) and compressive strengths (159 MPa) of manganese bronze.

f) Calculation of the Blade Tip Deflection:

The blade tip deflections δ_x (about the X0 axis) and δ_y (about the Y0 axis) were calculated using beam theory (See the spreadsheet in section 5.5.2 for the calculations):

$$\boxed{\delta_x = \frac{M_{X0} L_0^2}{2 E I_{X0}}} \qquad .....5.55$$

$$\boxed{\delta_y = \frac{M_{Y0} L_0^2}{2 E I_{Y0}}} \qquad .....5.56$$

where:

E : Modulus of elasticity of blade material (Manganese Bronze)

L_0 : Blade length

The magnitudes of the deflections were:

$\delta_x = 0.38$ mm

$\delta_y = 0.004$ mm

2 Bladed Large Diameter Open Screw Propeller for C-SCOUT

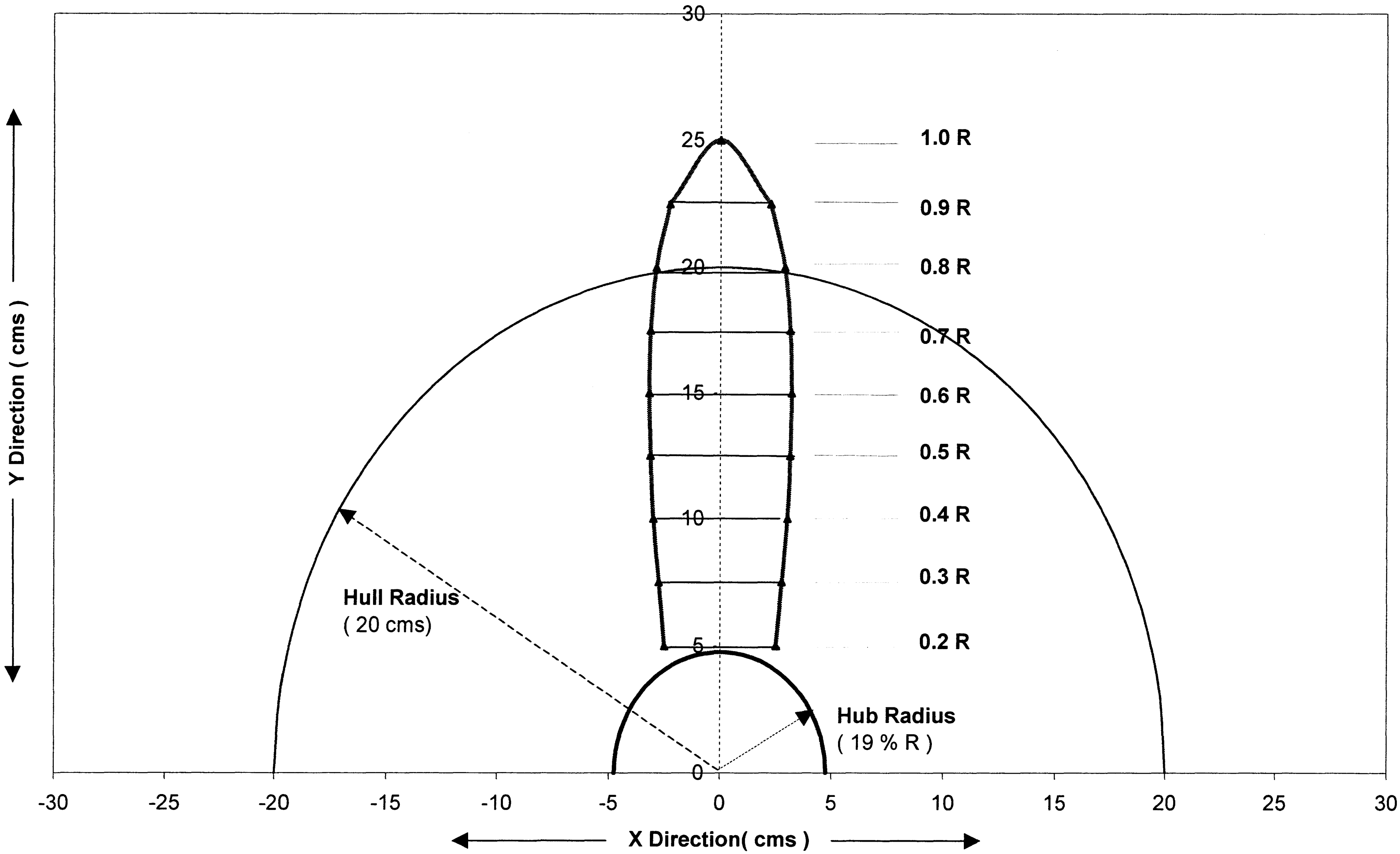
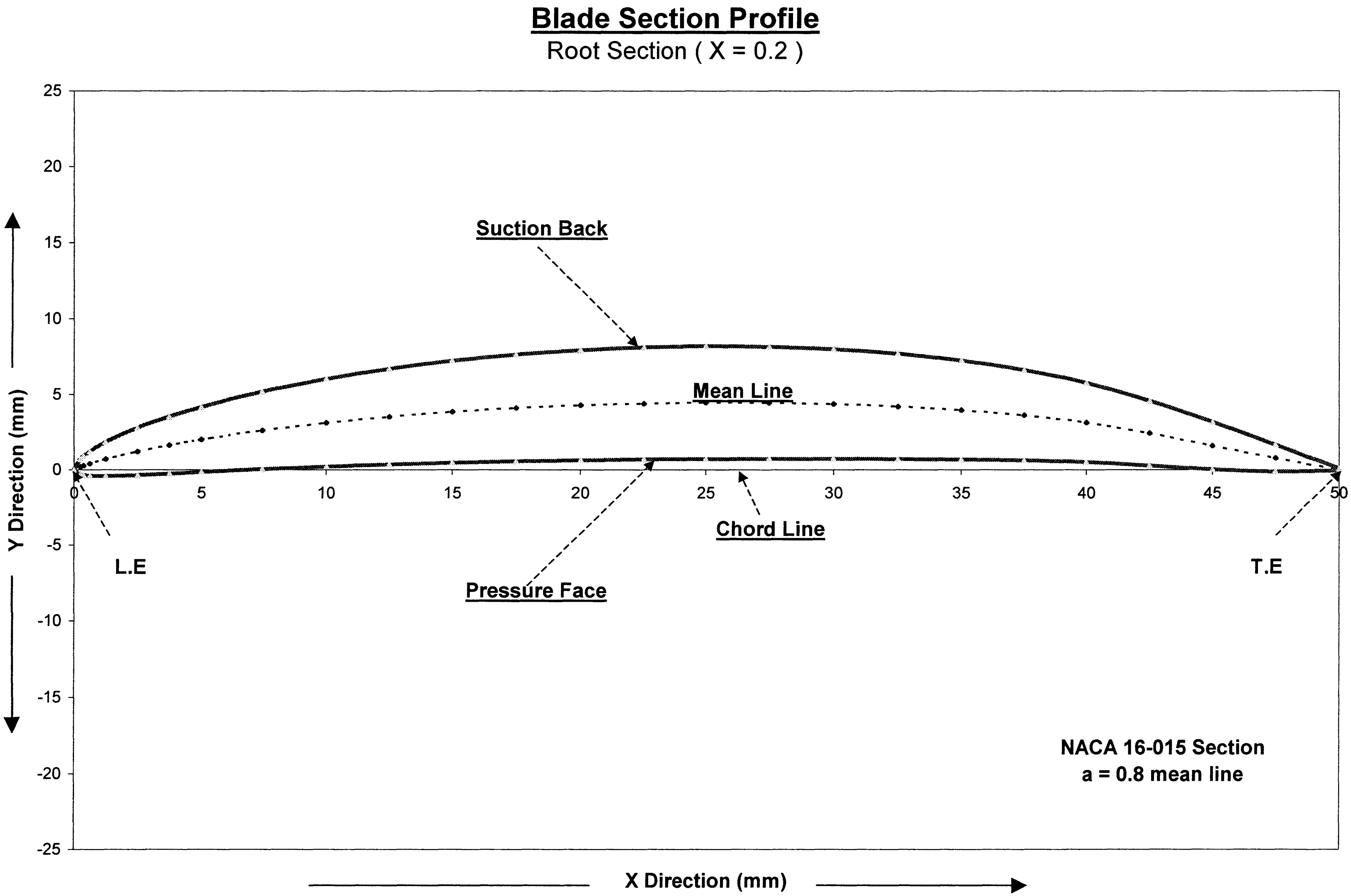


Fig. 5.6



Blade Section Profile
Root Section (X = 0.2)
Suction Back
Mean Line
Chord Line
Pressure Face
L.E
T.E
NACA 16-015 Section
a = 0.8 mean line
Y Direction (mm)
X Direction (mm)
25
20
15
10
5
0
-5
-10
-15
-20
-25
0
5
10
15
20
25
30
35
40
45
50

Fig. 5.7

Moment of Inertia Calculation

Blade Root Section (X = 0.2)

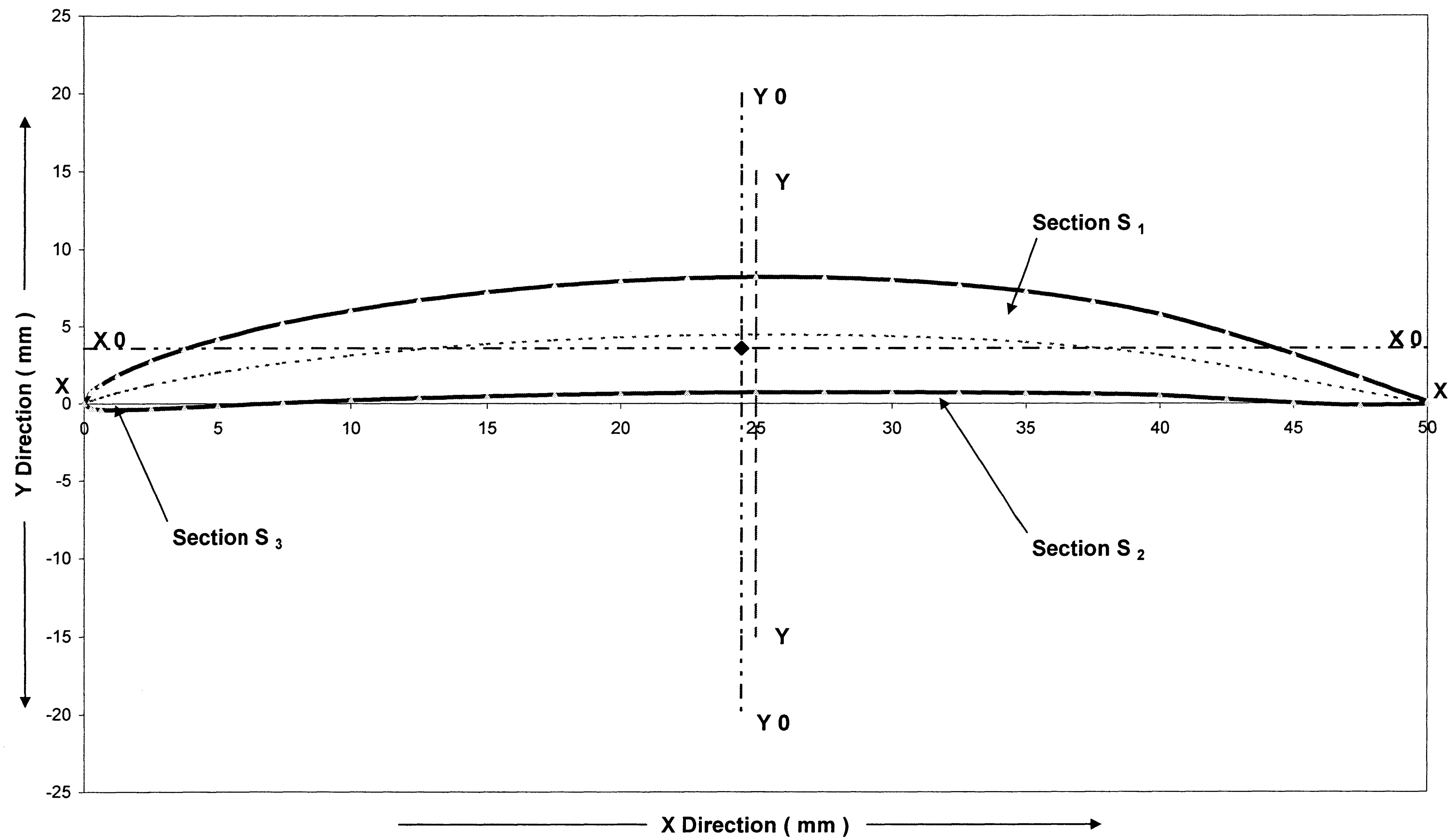
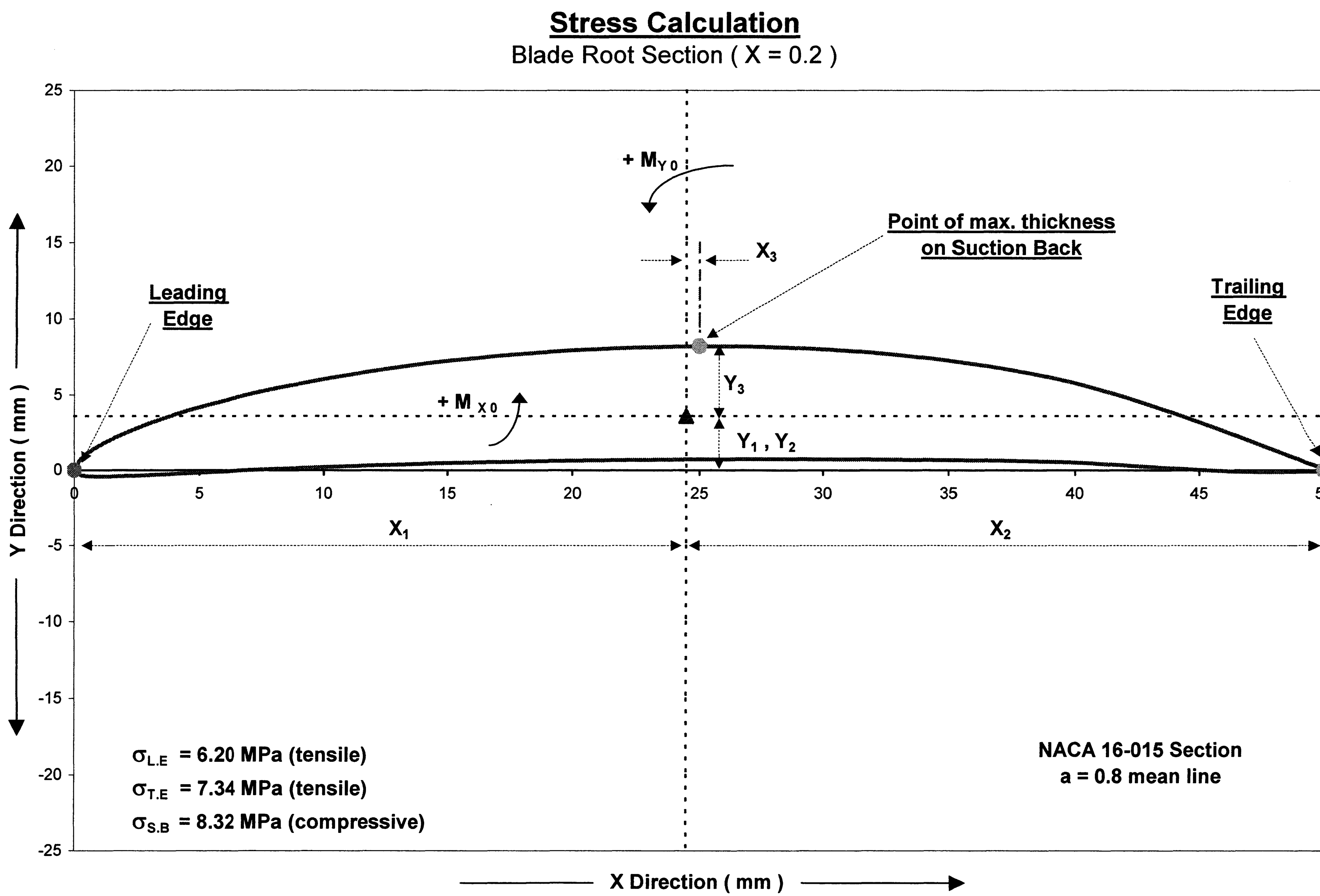


Fig. 5.8



Stress Calculation
Blade Root Section (X = 0.2)
+ M_{Y0}
Point of max. thickness on Suction Back
X_3
Leading Edge
Trailing Edge
Y_3
+ M_{X0}
Y_1 , Y_2
X_1
X_2
Y Direction (mm)
$\sigma_{L.E}$ = 6.20 MPa (tensile)
$\sigma_{T.E}$ = 7.34 MPa (tensile)
$\sigma_{S.B}$ = 8.32 MPa (compressive)
NACA 16-015 Section
a = 0.8 mean line
X Direction (mm)

Fig. 5.9

5.5 High Efficiency Propeller Design: Calculations

5.5.1 Propeller Design Calculations

Vehicle Speed (V)	2.00	m/s		
Speed of Advance (V_a)	1.94	m/s		
Thrust Ded. Fraction (t)	0.0653			
Wake Fraction (W_e)	0.0297			
Propeller Speed (N)	300	rpm	5.00	rps
Propeller Dia. (D)	0.50	m		
Number of Blades (Z)	2			
Propeller Thrust	52.26	N		
Prop.Shaft Submergence	1.00	m		

Non-Dimensional Parameters (Using Speed of Advance)

Advance Coefficient (J)	0.7762
Advance Ratio (λ)	0.2470
Thrust Coefficient (C_T)	0.1413
Ideal Thrust Coefficient (C_{Ti})	0.1455
Kramer's Ideal Efficiency (η_i)	0.9400

Non-Dimensional Parameters (Using Vehicle Speed)

Advance Coefficient (J_s)	0.8000
Advance Ratio (λ_S)	0.2545
Thrust Coefficient (C_{TS})	0.1330
Ideal Thrust Coefficient (C_{TSi})	0.1370

Hydrodynamic Pitch Angle (β_i) Calculation - First Iteration (Row 1)

x	1 - w_x	tan β	β	sin β	tan β_i	β_i	sin β_i	sin (β_i -β)	1 / λ_i	κ	U_t / 2 V	dC_{TSi}	S.M	Product
0.2	0.5835	0.7426	0.6387	0.5962	1.0187	0.7947	0.7136	0.1553	4.9080	1.0000	0.1085	0.1175	1	0.1175
0.3	0.7005	0.5943	0.5362	0.5109	0.7441	0.6397	0.5970	0.1033	4.4794	0.9695	0.0846	0.2153	4	0.8611
0.4	0.7987	0.5083	0.4702	0.4531	0.5960	0.5374	0.5119	0.0672	4.1949	0.9381	0.0606	0.2749	2	0.5497
0.5	0.8782	0.4471	0.4204	0.4081	0.4999	0.4636	0.4472	0.0432	4.0005	0.9011	0.0415	0.2878	4	1.1512
0.6	0.9390	0.3983	0.3791	0.3701	0.4308	0.4068	0.3956	0.0277	3.8689	0.8529	0.0278	0.2650	2	0.5299
0.7	0.9703	0.3528	0.3392	0.3327	0.3754	0.3591	0.3514	0.0199	3.8059	0.7804	0.0204	0.2431	4	0.9723
0.8	0.9878	0.3143	0.3045	0.2998	0.3314	0.3200	0.3146	0.0155	3.7720	0.6712	0.0160	0.2154	2	0.4307
0.9	1.0000	0.2828	0.2756	0.2722	0.2964	0.2881	0.2842	0.0125	3.7489	0.5093	0.0131	0.1686	4	0.6745
1.0	1.0000	0.2545	0.2493	0.2467	0.2667	0.2607	0.2577	0.0114	3.7489	0.0000	0.0119	0.0000	1	0.0000
												Integrated C_{TSi}		0.1762
												Required C_{TSi}		0.1370

Hydrodynamic Pitch Angle (β_i) Calculation - Second Iteration (Row 2)

Correction Factor for β_i 0.98215

x	1 - w_x	tan β	β	sin β	tan β_i	β_i	sin β_i	sin (β_i -β)	1 / λ_i	κ	U_t / 2 V	$dC_{TS\,i}$	S.M	Product
0.2	0.5835	0.7426	0.6387	0.5962	1.0006	0.7857	0.7073	0.1464	4.9972	1.0000	0.1013	0.1110	1	0.1110
0.3	0.7005	0.5943	0.5362	0.5109	0.7309	0.6311	0.5901	0.0948	4.5608	0.9700	0.0767	0.1967	4	0.7866
0.4	0.7987	0.5083	0.4702	0.4531	0.5853	0.5296	0.5052	0.0593	4.2711	0.9404	0.0528	0.2413	2	0.4826
0.5	0.8782	0.4471	0.4204	0.4081	0.4910	0.4564	0.4408	0.0360	4.0732	0.9053	0.0341	0.2386	4	0.9545
0.6	0.9390	0.3983	0.3791	0.3701	0.4231	0.4003	0.3897	0.0212	3.9392	0.8584	0.0209	0.2016	2	0.4031
0.7	0.9703	0.3528	0.3392	0.3327	0.3687	0.3532	0.3459	0.0140	3.8751	0.7867	0.0141	0.1702	4	0.6808
0.8	0.9878	0.3143	0.3045	0.2998	0.3255	0.3147	0.3095	0.0101	3.8406	0.6777	0.0103	0.1404	2	0.2808
0.9	1.0000	0.2828	0.2756	0.2722	0.2911	0.2833	0.2795	0.0076	3.8171	0.5147	0.0078	0.1025	4	0.4099
1.0	1.0000	0.2545	0.2493	0.2467	0.2620	0.2562	0.2534	0.0070	3.8171	0.0000	0.0072	0.0000	1	0.0000
												Integrated $C_{TS\,i}$		0.1370
												Required $C_{TS\,i}$		0.1370

Cavitation Number Calculation (Row 3)

x	P + P_A	1 - w_x	$\sin^2\beta$	$\cos^2(\beta - \beta_i)$	σ	0.8σ
0.2	110616.32	0.5835	0.3554	0.9786	59.0124	47.2100
0.3	110371.15	0.7005	0.2610	0.9910	29.6254	23.7003
0.4	110125.99	0.7987	0.2053	0.9965	17.7830	14.2264
0.5	109880.82	0.8782	0.1666	0.9987	11.8825	9.5060
0.6	109635.65	0.9390	0.1369	0.9996	8.5189	6.8151
0.7	109390.49	0.9703	0.1107	0.9998	6.4330	5.1464
0.8	109145.32	0.9878	0.0899	0.9999	5.0287	4.0230
0.9	108900.15	1.0000	0.0741	0.9999	4.0332	3.2266
1.0	108654.99	1.0000	0.0609	1.0000	3.3060	2.6448

Coefficient of Lift Calculation (Row 4)

x	κ	U_t / 2 V	cos β i	C_L * L / D	L / D	C_L
0.2	1.0000	0.1013	0.7069	0.1316	0.1000	1.3161
0.3	0.9700	0.0767	0.8074	0.1027	0.1100	0.9340
0.4	0.9404	0.0528	0.8630	0.0709	0.1200	0.5912
0.5	0.9053	0.0341	0.8976	0.0452	0.1250	0.3614
0.6	0.8584	0.0209	0.9210	0.0267	0.1275	0.2096
0.7	0.7867	0.0141	0.9383	0.0168	0.1250	0.1341
0.8	0.6777	0.0103	0.9509	0.0107	0.1150	0.0930
0.9	0.5147	0.0078	0.9601	0.0062	0.0900	0.0690
1.0	0.0000	0.0072	0.9674	0.0000	0.0000	0.0000

Camber Correction Calculation (Row 5)

x	C_L	Initial m_x / L	λ_i	k_1	k_2	Corrected m_x / L
0.2	1.3161	0.0894	0.2001	0.86	1.00	0.0769
0.3	0.9340	0.0634	0.2193	0.86	1.10	0.0600
0.4	0.5912	0.0401	0.2341	0.87	1.30	0.0454
0.5	0.3614	0.0245	0.2455	0.88	1.50	0.0324
0.6	0.2096	0.0142	0.2539	0.90	1.75	0.0224
0.7	0.1341	0.0091	0.2581	0.90	1.80	0.0148
0.8	0.0930	0.0063	0.2604	0.91	1.80	0.0103
0.9	0.0690	0.0047	0.2620	0.91	1.80	0.0077
1.0	0.0000	0.0000	0.2620	0.00	0.00	0.0000

$$(m_x / L)_{Corrected} = (m_x / L)_{Initial} * k_1 * k_2$$

Pitch Correction (α_1) Calculation (Row 6)

x	β_i	α_1	$\beta_i + \alpha_1$	$\tan(\beta_i + \alpha_1)$	Initial P / D
0.2	0.7857	0.0264	0.8121	1.0549	0.6631
0.3	0.6311	0.0187	0.6499	0.7600	0.7163
0.4	0.5296	0.0119	0.5414	0.6014	0.7557
0.5	0.4564	0.0073	0.4637	0.5001	0.7855
0.6	0.4003	0.0042	0.4045	0.4281	0.8069
0.7	0.3532	0.0027	0.3559	0.3717	0.8175
0.8	0.3147	0.0019	0.3165	0.3275	0.8232
0.9	0.2833	0.0014	0.2846	0.2926	0.8273
1.0	0.2562	0.0000	0.2562	0.2620	0.8230

$$(P / D)_{Initial} = \pi * x * \tan (\beta_i + \alpha_1)$$

Pitch Correction (α_2) Calculation (Row 7)

Blade 1

μ	90°	1.5714 rads
cos μ	0	
sin μ	1	
cos β_i	0.9383	(at X = 0.7)
sin β_i	0.3459	(at X = 0.7)
L / D	0.1250	(at X = 0.7)
A	0.0217	

x	$(P/R)^3$	$G/(P/R)^3$	S. M.	Product
0.2	0.1369	0.2537	1	0.2537
0.3	0.0736	0.4326	4	1.7303
0.4	0.0344	0.7237	2	1.4475
0.5	0.0131	1.3386	4	5.3545
0.6	0.0041	2.7854	2	5.5708
0.7	0.0020	4.0626	4	16.2503
0.8	0.0041	1.3731	2	2.7461
0.9	0.0132	0.2758	4	1.1031
1.0	0.0344	0.0000	1	0.0000

Integral 1.1485

$(\alpha_b)_{\text{Blade 1}}$ 0.0249

Blade 2

μ	270°	4.7143 rads
cos μ	0	
sin μ	-1	
cos β_i	0.9383	(at X = 0.7)
sin β_i	0.3459	(at X = 0.7)
L / D	0.1250	(at X = 0.7)
A	-0.0218	

x	$(P/R)^3$	$G/(P/R)^3$	S. M.	Product
0.2	0.7501	0.0463	1	0.0463
0.3	1.0233	0.0311	4	0.1245
0.4	1.3566	0.0183	2	0.0367
0.5	1.7558	0.0100	4	0.0401
0.6	2.2270	0.0052	2	0.0103
0.7	2.7762	0.0029	4	0.0115
0.8	3.4094	0.0017	2	0.0033
0.9	4.1326	0.0009	4	0.0035
1.0	4.9518	0.0000	1	0.0000

Integral 0.0092

$(\alpha_b)_{\text{Blade 2}}$ -0.0002

$\alpha_b = (\alpha_b)_{\text{Blade 1}} + (\alpha_b)_{\text{Blade 2}} =$ 0.0247

α_i 0.0140

α_o 0.0186

ϕ 86.4654

h 1.1

α_f 0.0163

$\alpha_2 = \alpha_b + \alpha_f - \alpha_i - \alpha_o =$ 0.0083

Pitch Correction = $[\tan(\beta_i + \alpha_2)]_{0.7R} \,/\, [\tan(\beta_i)]_{0.7R}$ 1.0258

Final P/D Ratio Calculation (Row 8)

x	Corrected P / D
0.2	0.68
0.3	0.73
0.4	0.78
0.5	0.81
0.6	0.83
0.7	0.84
0.8	0.84
0.9	0.85
1.0	0.84

$(P / D)_{Corrected} = (P / D)_{Initial}$ * Pitch Correction

Final Thrust Coefficient Calculation (Row 9)

ε	$1 - \varepsilon \tan \beta_i$	$dC_{TS\,i} / dx * dx$	SM	Product
0.0061	0.9939	0.1110	1	0.1103
0.0086	0.9937	0.1967	4	0.7817
0.0135	0.9921	0.2413	2	0.4787
0.0221	0.9891	0.2386	4	0.9441
0.0382	0.9839	0.2016	2	0.3966
0.0597	0.9780	0.1702	4	0.6658
0.0861	0.9720	0.1404	2	0.2729
0.1159	0.9663	0.1025	4	0.3961
0.0000	0.0000	0.0000	1	0.0000
			C_{TS}	0.1349

Final Power Coefficient Calculation (Row 10)

x / λ_S	$\tan \beta_i + \varepsilon$	$dC_{TS\,i} / dx * dx$	SM	Product
0.7857	1.0066	0.1110	1	0.0878
1.1786	0.7394	0.1967	4	0.6855
1.5714	0.5989	0.2413	2	0.4541
1.9643	0.5132	0.2386	4	0.9621
2.3571	0.4613	0.2016	2	0.4383
2.7500	0.4283	0.1702	4	0.8019
3.1429	0.4115	0.1404	2	0.3632
3.5357	0.4070	0.1025	4	0.5899
3.9286	0.2620	0.0000	1	0.0000
			C_P	0.1461

Propeller Efficiency	η_b	92.32	%
Propeller Thrust	T	52.99	N
Propeller Torque	Q	3.65	N - m
Delivered Power	P_d	114.79	Watt

% Difference $C_{TS\,(calc.)}$ & $C_{TS\,(design)}$	1.3715	%
% Difference C_{TSi} & C_{TS}	1.5564	%

Final Parameters for the Designed Propeller

Propeller Diameter	D	0.50	m
Number of Blades	Z	2	
Expanded Area Ratio	A_E / A_O	0.11	
Speed of Rotation	n	300	rpm
Vehicle Speed	V	2.01	m / s
Propeller Thrust	T	52.99	N
Propeller Torque	Q	3.65	N - m

Delivered Power	P_d	114.79	Watt
Effective Power	P_e	94.17	Watt
Propeller Efficiency	η_b	92.32	%
Propulsive Efficiency	η_d	82.04	%

x	P / D	L / D	t_x / D	t_x / L	m_x / L
0.2	0.68	0.1000	0.0150	0.1500	0.0769
0.3	0.73	0.1100	0.0149	0.1350	0.0600
0.4	0.78	0.1200	0.0144	0.1200	0.0454
0.5	0.81	0.1250	0.0131	0.1050	0.0324
0.6	0.83	0.1275	0.0115	0.0900	0.0224
0.7	0.84	0.1250	0.0094	0.0750	0.0148
0.8	0.84	0.1150	0.0069	0.0600	0.0103
0.9	0.85	0.0900	0.0041	0.0450	0.0077
1.0	0.84	0.0000	0.0030	0.0000	0.0000

P Pitch
D Diameter
L Chord Length
t_x Maximum Section Thickness
m_x Maximum Section Camber

5.5.2 Blade Root Section Stress Calculation

Moment due to Thrust (M_T)

X	$X - X_0$	$1 - \varepsilon \tan \beta_i$	$dC_{TSi} / dx * dx$	SM	Product
0.2	0	0.9940	0.1128	1	0.0000
0.3	0.1	0.9937	0.1967	4	0.0782
0.4	0.2	0.9921	0.2413	2	0.0957
0.5	0.3	0.9891	0.2386	4	0.2832
0.6	0.4	0.9839	0.2016	2	0.1586
0.7	0.5	0.9780	0.1702	4	0.3329
0.8	0.6	0.9720	0.1404	2	0.1638
0.9	0.7	0.9663	0.1025	4	0.2773
1	0.8	0.0000	0.0000	1	0.0000

Integral	0.0463	
M_T	2.1404	N-m

Moment due to Torque (M_Q)

X	$X - X_0$	$\tan \beta_i + \varepsilon$	$dC_{TSi} / dx * dx$	SM	Product
0.2	0	1.0065	0.1128	1	0.0000
0.3	0.1	0.7394	0.1967	4	0.0582
0.4	0.2	0.5989	0.2413	2	0.0578
0.5	0.3	0.5132	0.2386	4	0.1469
0.6	0.4	0.4613	0.2016	2	0.0744
0.7	0.5	0.4283	0.1702	4	0.1458
0.8	0.6	0.4115	0.1404	2	0.0693
0.9	0.7	0.4070	0.1025	4	0.1168
1	0.8	0.2620	0.0000	1	0.0000

Integral	0.0223	
M_Q	1.0307	N-m

Geometric Pitch Angle	Φ	0.8320	Radians	47.6673	Degrees
	cos Φ	0.6734			
	sin Φ	0.7392			
Resultant Moment about the X Axis	M_{X0}	2.2033	N-m	$(M_{X0} = M_T \cdot \cos\Phi + M_Q \cdot \sin\Phi)$	
Resultant Moment about the Y Axis	M_{Y0}	0.8882	N-m	$(M_{Y0} = M_T \cdot \sin\Phi - M_Q \cdot \cos\Phi)$	
Moment of Inertia about the X Axis	I_{X0}	1.20E-09	m^4		
Moment of Inertia about the Y Axis	I_{Y0}	3.91E-08	m^4		
Blade L.C.G	X_c	0.5065			
Blade Mass (I blade)	**m**	0.3756	Kg		
Blade Profile Area at X = 0.2	$A_{x=0.2}$	276.0935	mm^2		
Centrifugal Force	F_c	46.9732	N	$(F_c = 2 * \pi^2 * m * x_c * D * n^2)$	
Distance from the N.A to the Pressure Face (at the L.E / T.E)	$Y_1 = Y_2$	3.5867	mm		
Distance from the N.A to the Suction Back	Y_3	4.6268	mm		
Distance from the N.A to the Leading Edge	X_1	24.4766	mm		
Distance from the N.A to the Trailing Edge	X_2	25.5234	mm		
Distance from the N.A to the Max. Thickness Section	X_3	0.5234	mm		

Calculated Stress at the Blade Root Section (X = 0.2)

Stress at the Leading Edge $\sigma_{L.E} = (Y_1 * M_{X0}) / I_{X0} - (X_1 * M_{Y0}) / I_{Y0} + F_c / A$

($\sigma_{L.E}$)	6.21E+06	N / m²	6.2061	MN / m²	(MPa)	(Tensile)

Stress at the Trailing Edge $\sigma_{T.E} = (Y_2 * M_{X0}) / I_{X0} + (X_2 * M_{Y0}) / I_{Y0} + F_c / A$

($\sigma_{T.E}$)	7.34E+06	N / m²	7.3412	MN / m²	(MPa)	(Tensile)

Stress at point of max thickness on the Suction Back $\sigma_{S.B} = -(Y_3 * M_{X0}) / I_{X0} + (X_3 * M_{Y0}) / I_{Y0} + F_c / A$

($\sigma_{S.B}$)	-8.32E+06	N / m²	-8.3211	MN / m²	(MPa)	(Compressive)

Propeller Blade Tip Deflection

Blade Tip Deflection about the X axis $\delta_x = (M_{X0} * L^2) / (2 * E * I_{X0})$

δ_x 0.3808 mm

Blade Tip Deflection about the Y axis $\delta_Y = (M_{Y0} * L^2) / (2 * E * I_{Y0})$

δ_y 0.0047 mm

Manganese Bronze (Blade Material)

(Properties)

Tensile Strength	414	MPa
Compr. Strength	159	MPa
Yield Strength	138	MPa
Mod. of Elasticity	96.53	GPa

5.6 Conclusions

A high efficiency propeller was designed (calculations shown in Section 5.5) using the procedure described in Section 5.4. Since the high efficiency propeller leads to a gain of 20 % in terms of propulsive efficiency and a savings of 38 Watt in terms of energy consumption, it provides an attractive solution to the problem of minimizing energy consumption and maximizing propulsive efficiency. Long term benefits of this propeller include increased vehicle endurance and reduced operating costs.

Chapter 6

Conclusions and Recommendations

6.1 Conclusions

With the introduction of high energy density systems and compact high power computers, AUV technology has finally started coming of age. The brains (compact high power computers) and the brawn (high energy density systems) that AUVs lacked for a long time are finally becoming available commercially, thus making AUV technology commercially viable. The next decade or so promises to be an exciting time for AUV technology.

Selecting the best high energy density system for an AUV is a complex techno-economic problem. There are quite a few high energy density systems available in the market today for AUVs. After doing a comparative study of a number of high energy density systems available in the market today, the author selected four systems that could possibly be used on "C-SCOUT". Two of these systems (Lithium-ion Polymer batteries and Silver Zinc batteries) are immediate solutions. The other two systems (Aluminium Oxygen semi-fuel cells and PEM fuel cells) are long term solutions. These systems have energy densities that are much higher than those offered by Lithium-ion Polymer batteries and Silver Zinc batteries. However, the technology still needs some research and development in order to solve critical issues like the onboard storage / generation of hydrogen etc.

The benefit of using a high energy density system is an improvement in the endurance of the vehicle. For example if the vehicle travels at a speed of 2 m/s and has a hotel load of 250 Watt, then the use of PEM fuel cells leads to an endurance limit that is nine times that obtained using Lead Acid batteries. Similarly the use of Aluminium Oxygen semi-fuel cells, Lithium-ion Polymer batteries and Silver Zinc batteries leads to an endurance limit that is eight, four and three times respectively, that obtained using Lead Acid batteries. With a hotel load of 250 Watt, the current endurance limit of "C-SCOUT" is a distance of 36.5 km. This goes up to 320 km, 277 km, 139 km and 117 km if PEM fuel cells, Aluminium Oxygen semi-fuel cells, Lithium-ion Polymer batteries and Silver Zinc batteries respectively, are used instead of the Lead Acid battery.

Resistance, load-varying self-propulsion and bollard pull tests were conducted on "C-SCOUT" by the author, at the towing tank of the Ocean Engineering Research Center at Memorial University of Newfoundland in December 2002. A bare hull version of the "C-SCOUT" vehicle was outfitted with a propulsion system for these tests. The results from these tests provided a wealth of information about the performance of the propulsion system over a range of vehicle speeds and propeller loadings.

The maximum propulsive efficiency of the propulsion system (without the duct fitted on) was found to be around 53 %. This increased to around 68 % once the duct was fitted on. The tow force – thrust plots [Fig 4.28 and Fig 4.29] were found to be linear. Linearity of these plots indicates that the thrust deduction fraction (t^*) is essentially constant over a range of propeller loadings. The average value of the thrust deduction fraction was found to be equal to 0.06 (without the duct fitted on) and 0.04 (with the duct fitted on).

The vehicle resistance in the ahead and astern directions were found to be close to each other at lower speeds. At higher speeds, the difference in the ahead and astern resistance initially increased and then stabilized to a more or less constant difference of around 10.7 %. At vehicle speeds below 2 m/s, the vehicle resistance was found to be predominantly viscous. At speeds above 2 m/s, a significant amount of wave-making resistance was noted. Results obtained from the resistance and propulsion tests will be used for optimizing the propulsion system and the hull form of the "C-SCOUT" vehicle.

A high efficiency propeller was designed for "C-SCOUT" using Eckhardt and Morgan's propeller design method. This propeller is essentially a large diameter (0.5 m), slow-turning (300 rpm), slender, fine-bladed ($t_x/D = 0.0094$ at 0.7R), wake-adapted, single-screw propeller with two blades. The propeller blades have a low expanded area ratio ($A_E/A_0 = 0.11$). The blade section profile consists of a superposition of a NACA a=0.8 meanline with a NACA 16-015 thickness distribution. The propeller has a propulsive efficiency of 82 %. At a forward speed of 2 m/s, use of this propeller instead of the current Tecnadyne 1020 thruster, will lead to a gain of 20 % in terms of propulsive efficiency and a savings of 38 Watt in terms of energy consumption.

6.2 Recommendations

At the end of his thesis work, the author would like to make the following recommendations for the "C-SCOUT" project:

- In order to improve vehicle endurance, the low energy density Lead Acid batteries that are being currently used to power "C-SCOUT", need to be replaced with a high energy density system.
- The optimum operating speed of the vehicle from energy considerations is 2 m/s (3.8 knots). It is therefore recommended that the vehicle be run at or near this speed, while cruising on a mission at sea.
- For future in-water captive-model tests on "C-SCOUT" in a towing tank, it is essential that a braking mechanism be provided, in order to prevent forces of large magnitude from acting on the load cells during the acceleration / deceleration periods of the tests.
- In order to preclude wave-making effects during future in-water tests, it is recommended that the vehicle be submerged at least five body diameters (2 meters) below the surface of water.
- In order to achieve further improvements in vehicle endurance, it is recommended that the current Tecnadyne 1020 thruster be replaced with a high efficiency propulsion system.

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Appendix A

Calculations for Chapter 2

Vehicle Endurance Calculations

50 km, 100 W

V	P_d	P_m	P_e	P_h	P_t	t	E
m/s	W	W	W	W	W	hr	W-hr
0.75	14.47	14.62	16.24	100.00	116.24	18.52	2.15
1.00	34.21	34.55	38.39	100.00	138.39	13.89	1.92
1.25	60.52	61.13	67.92	100.00	167.92	11.11	1.87
1.50	88.11	89.00	98.89	100.00	198.89	9.26	1.84
1.75	117.14	118.32	131.47	100.00	231.47	7.94	1.84
2.00	153.21	154.75	171.95	100.00	271.95	6.94	1.89
2.25	207.38	209.47	232.75	100.00	332.75	6.17	2.05
2.50	296.16	299.16	332.39	100.00	432.39	5.56	2.40
2.75	441.53	445.99	495.54	100.00	595.54	5.05	3.01
2.90	567.08	572.81	636.46	100.00	736.46	4.79	3.53
3.06	746.00	753.54	837.27	100.00	937.27	4.53	4.25

t'	D
hr	km
18.42	49.73
15.47	55.69
12.75	57.37
10.76	58.12
9.25	58.27
7.87	56.68
6.43	52.11
4.95	44.56
3.59	35.59
2.91	30.35
2.28	25.19

50 km, 150 W

V	P_d	P_m	P_e	P_h	P_t	t	E
m/s	W	W	W	W	W	hr	W-hr
0.75	14.47	14.62	16.24	150.00	166.24	18.52	3.08
1.00	34.21	34.55	38.39	150.00	188.39	13.89	2.62
1.25	60.52	61.13	67.92	150.00	217.92	11.11	2.42
1.50	88.11	89.00	98.89	150.00	248.89	9.26	2.30
1.75	117.14	118.32	131.47	150.00	281.47	7.94	2.23
2.00	153.21	154.75	171.95	150.00	321.95	6.94	2.24
2.25	207.38	209.47	232.75	150.00	382.75	6.17	2.36
2.50	296.16	299.16	332.39	150.00	482.39	5.56	2.68
2.75	441.53	445.99	495.54	150.00	645.54	5.05	3.26
2.90	567.08	572.81	636.46	150.00	786.46	4.79	3.77
3.06	746.00	753.54	837.27	150.00	987.27	4.53	4.48

t'	D
hr	km
12.88	34.77
11.36	40.91
9.82	44.21
8.60	46.45
7.61	47.92
6.65	47.88
5.59	45.31
4.44	39.94
3.32	32.83
2.72	28.42
2.17	23.91

50 km, 200 W

V m/s	P_d W	P_m W	P_e W	P_h W	P_t W	t hr	E W-hr	t' hr	D km
0.75	14.47	14.62	16.24	200.00	216.24	18.52	4.00	9.90	26.73
1.00	34.21	34.55	38.39	200.00	238.39	13.89	3.31	8.98	32.33
1.25	60.52	61.13	67.92	200.00	267.92	11.11	2.98	7.99	35.96
1.50	88.11	89.00	98.89	200.00	298.89	9.26	2.77	7.16	38.68
1.75	117.14	118.32	131.47	200.00	331.47	7.94	2.63	6.46	40.69
2.00	153.21	154.75	171.95	200.00	371.95	6.94	2.58	5.76	41.44
2.25	207.38	209.47	232.75	200.00	432.75	6.17	2.67	4.95	40.07
2.50	296.16	299.16	332.39	200.00	532.39	5.56	2.96	4.02	36.19
2.75	441.53	445.99	495.54	200.00	695.54	5.05	3.51	3.08	30.47
2.90	567.08	572.81	636.46	200.00	836.46	4.79	4.01	2.56	26.72
3.06	746.00	753.54	837.27	200.00	1037.27	4.53	4.70	2.06	22.76

50 km, 250 W

V m/s	P_d W	P_m W	P_e W	P_h W	P_t W	t hr	E W-hr	t' hr	D km
0.75	14.47	14.62	16.24	250.00	266.24	18.52	4.93	8.04	21.71
1.00	34.21	34.55	38.39	250.00	288.39	13.89	4.01	7.42	26.72
1.25	60.52	61.13	67.92	250.00	317.92	11.11	3.53	6.73	30.30
1.50	88.11	89.00	98.89	250.00	348.89	9.26	3.23	6.14	33.13
1.75	117.14	118.32	131.47	250.00	381.47	7.94	3.03	5.61	35.36
2.00	153.21	154.75	171.95	250.00	421.95	6.94	2.93	5.07	36.53
2.25	207.38	209.47	232.75	250.00	482.75	6.17	2.98	4.43	35.92
2.50	296.16	299.16	332.39	250.00	582.39	5.56	3.24	3.68	33.08
2.75	441.53	445.99	495.54	250.00	745.54	5.05	3.77	2.87	28.43
2.90	567.08	572.81	636.46	250.00	886.46	4.79	4.25	2.42	25.21
3.06	746.00	753.54	837.27	250.00	1087.27	4.53	4.93	1.97	21.71

Comparison of Different High Energy Density Systems

50 km, 100 W

V	P_t	Ag-Zn		Li-ion P		$Al-O_2$		PEM	
m/s	W	t" (hr)	D" (km)	t" (hr)	D" (km)	t" (hr)	D" (km)	t" (hr)	D" (km)
0.75	116.24	59.14	159.69	69.90	188.73	139.80	377.45	161.30	435.52
1.00	138.39	49.68	178.84	58.71	211.36	117.42	422.72	135.49	487.75
1.25	167.92	40.94	184.24	48.39	217.74	96.77	435.47	111.66	502.47
1.50	198.89	34.57	186.66	40.85	220.60	81.70	441.20	94.27	509.08
1.75	231.47	29.70	187.12	35.10	221.14	70.20	442.29	81.01	510.33
2.00	271.95	25.28	182.02	29.88	215.11	59.75	430.23	68.95	496.42
2.25	332.75	20.66	167.36	24.42	197.79	48.84	395.57	56.35	456.43
2.50	432.39	15.90	143.10	18.79	169.12	37.58	338.23	43.36	390.27
2.75	595.54	11.54	114.29	13.64	135.07	27.29	270.13	31.48	311.69
2.90	736.46	9.34	97.46	11.03	115.18	22.07	230.36	25.46	265.80
3.06	937.27	7.34	80.89	8.67	95.60	17.34	191.19	20.00	220.61

50 km, 150 W

V	P_t	Ag-Zn		Li-ion P		$Al-O_2$		PEM	
m/s	W	t" (hr)	D" (km)	t" (hr)	D" (km)	t" (hr)	D" (km)	t" (hr)	D" (km)
0.75	166.24	41.36	111.66	48.87	131.96	97.75	263.92	112.79	304.53
1.00	188.39	36.49	131.38	43.13	155.26	86.26	310.53	99.53	358.30
1.25	217.92	31.55	141.97	37.28	167.78	74.57	335.56	86.04	387.18
1.50	248.89	27.62	149.16	32.65	176.28	65.29	352.57	75.33	406.81
1.75	281.47	24.43	153.88	28.87	181.86	57.73	363.72	66.62	419.68
2.00	321.95	21.35	153.75	25.24	181.71	50.47	363.41	58.24	419.32
2.25	382.75	17.96	145.49	21.23	171.95	42.46	343.90	48.99	396.80
2.50	482.39	14.25	128.27	16.84	151.59	33.69	303.17	38.87	349.82
2.75	645.54	10.65	105.43	12.59	124.60	25.17	249.21	29.05	287.55
2.90	786.46	8.74	91.26	10.33	107.86	20.66	215.71	23.84	248.90
3.06	987.27	6.96	76.79	8.23	90.76	16.46	181.51	18.99	209.44

50 km, 200 W

V	P_t	Ag-Zn		Li-ion P		$Al\text{-}O_2$		PEM	
m/s	W	t" (hr)	D" (km)	t" (hr)	D" (km)	t" (hr)	D" (km)	t" (hr)	D" (km)
0.75	216.24	31.79	85.84	37.57	101.45	75.15	202.90	86.71	234.11
1.00	238.39	28.84	103.82	34.08	122.70	68.17	245.40	78.65	283.15
1.25	267.92	25.66	115.47	30.33	136.47	60.65	272.93	69.98	314.92
1.50	298.89	23.00	124.21	27.18	146.79	54.37	293.59	62.73	338.75
1.75	331.47	20.74	130.67	24.51	154.43	49.02	308.85	56.57	356.37
2.00	371.95	18.48	133.08	21.84	157.28	43.69	314.56	50.41	362.95
2.25	432.75	15.89	128.68	18.78	152.08	37.55	304.16	43.33	350.96
2.50	532.39	12.91	116.22	15.26	137.35	30.52	274.70	35.22	316.96
2.75	695.54	9.88	97.86	11.68	115.65	23.36	231.29	26.96	266.88
2.90	836.46	8.22	85.81	9.71	101.41	19.43	202.82	22.42	234.02
3.06	1037.27	6.63	73.09	7.83	86.38	15.67	172.76	18.08	199.34

50 km, 250 W

V	P_t	Ag-Zn		Li-ion P		$Al\text{-}O_2$		PEM	
m/s	W	t" (hr)	D" (km)	t" (hr)	D" (km)	t" (hr)	D" (km)	t" (hr)	D" (km)
0.75	266.24	25.82	69.72	30.52	82.40	61.04	164.79	70.43	190.15
1.00	288.39	23.84	85.82	28.17	101.42	56.35	202.85	65.02	234.06
1.25	317.92	21.62	97.31	25.56	115.00	51.11	230.01	58.98	265.40
1.50	348.89	19.71	106.41	23.29	125.76	46.58	251.51	53.74	290.21
1.75	381.47	18.02	113.54	21.30	134.19	42.60	268.37	49.15	309.66
2.00	421.95	16.29	117.31	19.26	138.64	38.51	277.29	44.44	319.94
2.25	482.75	14.24	115.36	16.83	136.33	33.66	272.66	38.84	314.61
2.50	582.39	11.80	106.24	13.95	125.56	27.90	251.12	32.19	289.75
2.75	745.54	9.22	91.29	10.90	107.89	21.80	215.78	25.15	248.98
2.90	886.46	7.76	80.97	9.17	95.69	18.33	191.38	21.15	220.82
3.06	1087.27	6.32	69.73	7.47	82.41	14.95	164.82	17.25	190.17

Appendix B

Resistance and Propulsion Test Results

<u>Load Varying Self - Propulsion Test</u> (Propeller fitted on)

V (m/s)	n (rpm)	Q (N-m)	F (N)	T_P (N)	J	K_{fd}	K_{tp}	K_{ts}	$10 * K_q$	Self-Propulsion Point		
0.8007	503.8949	0.1810	5.3638	3.6262	0.6256	-0.1410	0.0953	0.2221	0.3122	**J**	0.5680	
0.8012	552.6343	0.3680	1.4334	8.7409	0.5708	-0.0313	0.1910	0.1846	0.5277	**V**	0.8029	m/s
0.8027	602.3573	0.5143	-3.5438	12.8134	0.5246	0.0652	0.2357	0.1554	0.6207	**10 K_q**	0.5077	
0.8034	651.5623	0.7198	-10.4367	18.6583	0.4854	0.1641	0.2933	0.1328	0.7425	**K_q**	0.0508	
0.8005	753.4148	1.2080	-24.3516	32.8282	0.4183	0.2863	0.3860	0.0993	0.9319	**K_{tp}**	0.1821	
0.8012	913.8563	2.0427	-48.5726	59.3485	0.3452	0.3881	0.4743	0.0675	1.0711	**Q**	0.3590	N-m
0.8037	1002.5757	2.6202	-64.7292	77.0108	0.3156	0.4298	0.5113	0.0561	1.1415	**T**	8.4491	N
0.8045	1051.5807	2.9224	-74.4405	87.1072	0.3012	0.4492	0.5257	0.0510	1.1572	**R**	7.9354	N
0.8098	1102.1417	3.3564	-85.7547	98.8701	0.2893	0.4711	0.5432	0.0464	1.2100	**P_d**	20.9319	W
0.8032	1154.0417	3.7562	-96.8046	111.2684	0.2740	0.4851	0.5576	0.0423	1.2351	**P_e**	6.3712	W
0.8015	1207.3702	4.1532	-108.6784	123.8401	0.2614	0.4975	0.5669	0.0387	1.2476	**η_d**	30.4378	%
0.8022	1246.6728	4.5121	-117.3754	135.8194	0.2533	0.5040	0.5832	0.0363	1.2713			

V (m/s)	n (rpm)	Q (N-m)	F (N)	T_P (N)	J	K_{fd}	K_{tp}	K_{ts}	$10 * K_q$	Self-Propulsion Point		
1.1030	507.2079	0.1699	14.2900	2.2849	0.8562	-0.3707	0.0593	0.4538	0.2892	**J**	0.6538	
1.1027	554.5345	0.3535	10.5355	6.2834	0.7829	-0.2286	0.1364	0.3796	0.5034	**V**	1.1033	m/s
1.1035	603.8746	0.4982	6.8977	10.2080	0.7194	-0.1262	0.1868	0.3201	0.5983	**10 K_q**	0.7470	
1.1032	654.2447	0.7005	0.5784	16.3454	0.6639	-0.0090	0.2548	0.2727	0.7166	**K_q**	0.0747	
1.1044	750.3733	1.1746	-11.0514	29.4461	0.5794	0.1310	0.3490	0.2073	0.9135	**K_{tp}**	0.2645	
1.1041	910.3564	1.9868	-34.4575	54.5330	0.4775	0.2775	0.4391	0.1409	1.0498	**Q**	0.7530	N-m
1.1030	1006.6678	2.5799	-52.5912	73.9689	0.4314	0.3463	0.4871	0.1152	1.1148	**T**	17.4932	N
1.1037	1056.9924	2.8694	-63.4548	85.6126	0.4111	0.3790	0.5114	0.1045	1.1247	**R**	16.3020	N
1.1031	1108.8989	3.2875	-75.4534	98.4729	0.3916	0.4095	0.5344	0.0949	1.1707	**P_d**	52.4151	W
1.1031	1156.0328	3.6710	-86.3969	110.2023	0.3757	0.4314	0.5503	0.0874	1.2029	**P_e**	17.9867	W
1.1029	1206.4133	4.0899	-96.9218	121.4830	0.3599	0.4444	0.5570	0.0802	1.2305	**η_d**	34.3158	%
1.1034	1249.3622	4.4395	-107.1228	132.4165	0.3477	0.4580	0.5661	0.0748	1.2455			

Load Varying Self - Propulsion Test (Propeller fitted on)

V (m/s)	n (rpm)	Q (N-m)	F (N)	T_P (N)	J	K_{fd}	K_{tp}	K_{ts}	10 * K_q	Self-Propulsion Point		
1.3031	504.2337	0.1657	23.7086	-0.5806	1.0174	-0.6223	-0.0152	0.6262	0.2854	**J**	0.6999	
1.3022	550.4535	0.3278	19.5828	3.8393	0.9314	-0.4313	0.0846	0.5254	0.4738	**V**	1.3025	m/s
1.3025	600.5634	0.4670	15.5347	7.2149	0.8539	-0.2874	0.1335	0.4414	0.5670	**10 K_q**	0.8406	
1.3035	652.2352	0.6650	7.4959	13.4575	0.7868	-0.1176	0.2111	0.3742	0.6845	**K_q**	0.0841	
1.3036	752.8192	1.1361	-3.1778	26.1866	0.6817	0.0374	0.3084	0.2809	0.8778	**K_{tp}**	0.2966	
1.3022	901.8296	1.9541	-23.6329	51.7382	0.5685	0.1939	0.4245	0.1958	1.0521	**Q**	1.0305	N-m
1.3022	1000.1110	2.5466	-45.8753	72.3597	0.5126	0.3061	0.4828	0.1592	1.1149	**T**	23.8562	N
1.3023	1046.2973	2.8254	-54.5532	82.1375	0.4900	0.3326	0.5007	0.1454	1.1302	**R**	22.2030	N
1.3020	1105.7242	3.2400	-67.4871	96.1106	0.4636	0.3684	0.5246	0.1302	1.1604	**P_d**	79.0985	W
1.3023	1155.3342	3.6311	-78.2728	107.4396	0.4438	0.3913	0.5372	0.1193	1.1912	**P_e**	28.9203	W
1.3022	1206.0095	4.0298	-88.6396	119.2426	0.4251	0.4067	0.5471	0.1095	1.2133	η_d	36.5624	%
1.3024	1248.0299	4.4059	-98.8778	130.6479	0.4109	0.4237	0.5598	0.1022	1.2387			

V (m/s)	n (rpm)	Q (N-m)	F (N)	T_P (N)	J	K_{fd}	K_{tp}	K_{ts}	10 * K_q	Self-Propulsion Point		
1.6032	503.9995	0.1484	35.6003	-1.5261	1.2523	-0.9353	-0.0401	0.9006	0.2559	**J**	0.7664	
1.6046	550.3443	0.2710	31.5343	1.5354	1.1479	-0.6948	0.0338	0.7553	0.3918	**V**	1.6038	m/s
1.6035	601.4574	0.3833	27.4347	5.3738	1.0496	-0.5061	0.0991	0.6324	0.4639	**10 K_q**	0.8758	
1.6038	653.7645	0.6033	17.3544	11.6456	0.9658	-0.2710	0.1818	0.5352	0.6181	**K_q**	0.0876	
1.6040	755.9224	1.0703	8.5633	25.1105	0.8354	-0.1000	0.2933	0.4003	0.8202	**K_{tp}**	0.3370	
1.6038	911.6408	1.8340	-12.6824	48.1265	0.6926	0.1018	0.3865	0.2753	0.9663	**Q**	1.3576	N-m
1.6044	1003.7944	2.4389	-32.5581	67.4672	0.6293	0.2156	0.4469	0.2270	1.0599	**T**	34.2780	N
1.6042	1051.3377	2.7228	-42.0001	77.7648	0.6007	0.2536	0.4695	0.2070	1.0787	**R**	31.9780	N
1.6043	1104.0239	3.1449	-51.6517	91.4829	0.5721	0.2828	0.5009	0.1877	1.1299	**P_d**	117.1749	W
1.6034	1152.5333	3.5384	-63.4763	104.5737	0.5477	0.3189	0.5254	0.1722	1.1665	**P_e**	51.2868	W
1.6036	1210.1129	3.9176	-76.3438	116.7686	0.5217	0.3479	0.5322	0.1562	1.1715	η_d	43.7695	%
1.6030	1250.8055	4.3454	-88.3925	127.3434	0.5046	0.3770	0.5432	0.1462	1.2163			

Load Varying Self - Propulsion Test (Propeller fitted on)

V (m/s)	n (rpm)	Q (N-m)	F (N)	T_P (N)	J	K_{fd}	K_{tp}	K_{ts}	$10 * K_q$	Self-Propulsion Point		
1.8042	510.9808	0.1265	40.6558	-1.9385	1.3901	-1.0391	-0.0495	1.0288	0.2122	**J**	0.8119	
1.8034	552.4233	0.1528	37.6344	1.1898	1.2853	-0.8230	0.0260	0.8803	0.2193	**V**	1.8051	m/s
1.8068	602.7853	0.3126	33.4536	4.2981	1.1801	-0.6144	0.0789	0.7393	0.3767	**10 K_q**	0.8602	
1.8056	651.5334	0.5023	24.9715	10.5564	1.0911	-0.3926	0.1660	0.6328	0.5181	**K_q**	0.0860	
1.8042	757.8054	0.9753	17.3566	22.9946	0.9373	-0.2017	0.2672	0.4678	0.7437	**K_{tp}**	0.3506	
1.8051	911.1136	1.7271	-4.5070	45.8331	0.7800	0.0362	0.3685	0.3236	0.9111	**Q**	1.5051	N-m
1.8048	1002.7594	2.3395	-25.1929	65.0234	0.7086	0.1672	0.4316	0.2672	1.0189	**T**	40.2530	N
1.8061	1056.9214	2.6019	-32.6326	73.8682	0.6728	0.1950	0.4413	0.2405	1.0200	**R**	37.5440	N
1.8051	1118.8065	3.0425	-43.1904	87.2252	0.6352	0.2303	0.4650	0.2146	1.0644	**P_d**	138.0146	W
1.8043	1161.1239	3.4240	-55.8141	100.0789	0.6118	0.2763	0.4954	0.1993	1.1121	**P_e**	67.7707	W
1.8059	1215.6599	3.7826	-68.1665	112.6564	0.5849	0.3078	0.5087	0.1818	1.1208	**η_d**	49.1040	%
1.8057	1246.0544	4.1864	-77.6279	125.3448	0.5705	0.3337	0.5388	0.1730	1.1807			

V (m/s)	n (rpm)	Q (N-m)	F (N)	T_P (N)	J	K_{fd}	K_{tp}	K_{ts}	$10 * K_q$	Self-Propulsion Point		
2.1091	500.1371	0.0592	54.2008	-2.2244	1.6603	-1.4461	-0.0593	1.4872	0.1037	**J**	0.8427	
2.1056	551.5545	0.1283	51.6384	0.7034	1.5030	-1.1328	0.0154	1.2228	0.1847	**V**	2.1062	m/s
2.1068	600.3476	0.2676	49.7643	3.3349	1.3816	-0.9215	0.0618	1.0322	0.3251	**10 K_q**	0.9442	
2.1067	654.3489	0.4750	42.3233	9.0348	1.2675	-0.6597	0.1408	0.8688	0.4858	**K_q**	0.0944	
2.1101	752.6505	0.8776	34.8043	19.5316	1.1038	-0.4100	0.2301	0.6567	0.6784	**K_{tp}**	0.3842	
2.1068	901.9876	1.6341	13.6338	42.0655	0.9196	-0.1118	0.3451	0.4572	0.8795	**Q**	2.0877	N-m
2.1070	1002.6789	2.2376	-3.9622	59.4139	0.8273	0.0263	0.3944	0.3700	0.9746	**T**	55.7426	N
2.0959	1052.8833	2.5437	-13.7635	68.8805	0.7837	0.0829	0.4147	0.3356	1.0048	**R**	52.1640	N
2.1064	1103.4297	2.9378	-24.0542	80.5944	0.7516	0.1318	0.4418	0.3055	1.0566	**P_d**	215.2089	W
2.1073	1156.4853	3.3080	-33.2420	93.1518	0.7174	0.1659	0.4648	0.2781	1.0831	**P_e**	109.8674	W
2.1066	1214.1716	3.6923	-47.5793	106.0173	0.6831	0.2154	0.4799	0.2523	1.0968	**η_d**	51.0515	%
2.1060	1246.6579	4.0446	-57.4544	117.5641	0.6651	0.2467	0.5048	0.2394	1.1396			

Load Varying Self - Propulsion Test (Propeller fitted on)

V (m/s)	n (rpm)	Q (N-m)	F (N)	T_P (N)	J	K_{fd}	K_{tp}	K_{ts}	10 * K_q	Self-Propulsion Point		
2.3079	502.2690	0.0030	70.1815	-3.6889	1.8090	-1.8566	-0.0976	1.8753	0.0052	J	0.8459	
2.3069	550.3896	0.1071	65.5744	-0.4585	1.6502	-1.4446	-0.0101	1.5617	0.1549	V	2.3076	m/s
2.3074	603.5423	0.2439	62.5675	2.8578	1.5052	-1.1463	0.0524	1.2988	0.2932	10 K_q	0.9832	
2.3070	652.5464	0.4126	56.4648	7.3859	1.3919	-0.8849	0.1158	1.1110	0.4243	K_q	0.0983	
2.3084	752.0456	0.7863	52.8606	17.3533	1.2085	-0.6237	0.2048	0.8365	0.6088	K_{tp}	0.4101	
2.3081	901.6673	1.5450	31.0591	39.3701	1.0078	-0.2550	0.3232	0.5819	0.8322	Q	2.5899	N-m
2.3082	1006.0527	2.1446	14.6680	56.3185	0.9033	-0.0967	0.3713	0.4674	0.9279	T	70.8906	N
2.3077	1052.1662	2.4494	5.4600	65.8199	0.8635	-0.0329	0.3968	0.4273	0.9689	R	66.2190	N
2.3074	1101.3796	2.7844	-4.9423	75.6556	0.8248	0.0272	0.4162	0.3900	1.0051	P_d	291.3985	W
2.3075	1150.4349	3.1913	-16.4586	88.4838	0.7897	0.0830	0.4462	0.3575	1.0559	P_e	152.8059	W
2.3077	1213.5677	3.5556	-29.2323	102.4455	0.7487	0.1325	0.4642	0.3212	1.0572	η_d	52.4388	%
2.3068	1249.1721	3.9343	-42.1765	114.3675	0.7270	0.1804	0.4891	0.3032	1.1041			

V (m/s)	n (rpm)	Q (N-m)	F (N)	T_P (N)	J	K_{fd}	K_{tp}	K_{ts}	10 * K_q	Self-Propulsion Point		
2.5100	504.0554	0.0003	87.5695	-5.1534	1.9605	-2.3002	-0.1354	2.2674	0.0006	J	0.8502	
2.5086	552.5634	0.0817	83.4748	-2.4533	1.7874	-1.8245	-0.0536	1.8868	0.1172	V	2.5077	m/s
2.5072	601.0760	0.2188	78.4785	1.5344	1.6422	-1.4496	0.0283	1.5945	0.2651	10 K_q	1.0199	
2.5095	651.4524	0.3268	72.4764	5.3858	1.5166	-1.1397	0.0847	1.3574	0.3372	K_q	0.1020	
2.5020	751.7646	0.7249	66.8172	15.9565	1.3103	-0.7890	0.1884	1.0193	0.5617	K_{tp}	0.4272	
2.5092	905.0740	1.4758	46.3855	37.9313	1.0915	-0.3779	0.3090	0.7033	0.7889	Q	3.1408	N-m
2.5081	1004.8353	2.0551	32.5307	52.3894	0.9827	-0.2150	0.3463	0.5706	0.8913	T	86.3219	N
2.5087	1052.4015	2.3871	22.3437	62.3939	0.9385	-0.1346	0.3760	0.5201	0.9438	R	80.8750	N
2.5087	1103.3557	2.6963	12.3667	72.3930	0.8952	-0.0678	0.3969	0.4732	0.9699	P_d	382.0881	W
2.5023	1154.7567	3.0789	3.6930	84.3833	0.8531	-0.0185	0.4223	0.4320	1.0111	P_e	202.8115	W
2.5092	1207.1602	3.4660	-11.5045	97.3348	0.8183	0.0527	0.4458	0.3953	1.0415	η_d	53.0798	%
2.5091	1252.5791	3.8650	-23.5327	110.9502	0.7886	0.1001	0.4719	0.3672	1.0788			

Load Varying Self - Propulsion Test (Duct + Propeller fitted on)

V (m/s)	n (rpm)	Q (N-m)	F (N)	T_p (N)	T_t (N)	T_d (N)	J	K_{fd}	K_{tp}	K_{td}	K_{tt}	K_{ts}	10 * K_q	Self-Propulsion Point		
0.8002	499.5767	0.3364	1.3073	5.8645	7.6425	1.7779	0.6306	-0.0350	0.1568	0.0475	0.2044	0.2163	0.5902	J	0.6309	
0.8004	551.4566	0.4972	-4.6450	10.3820	14.1975	3.8156	0.5714	0.1019	0.2278	0.0837	0.3116	0.1775	0.7159	V	0.8007	m/s
0.8012	600.4575	0.6260	-10.5645	14.4121	19.7185	5.3064	0.5253	0.1955	0.2668	0.0982	0.3650	0.1497	0.7603	10 K_q	0.5900	
0.8018	650.4466	0.8160	-17.5455	19.4810	26.1852	6.7042	0.4853	0.2768	0.3073	0.1058	0.4130	0.1276	0.8446	K_q	0.0590	
0.8002	753.2179	1.2582	-34.8750	31.2554	43.2349	11.9795	0.4183	0.4102	0.3677	0.1409	0.5086	0.0952	0.9712	K_{tt}	0.2163	
0.8002	902.9860	1.9967	-68.0707	54.7882	78.0332	23.2450	0.3489	0.5571	0.4484	0.1903	0.6387	0.0662	1.0723	Q	0.3363	N-m
0.8003	1008.5085	2.5232	-94.6789	73.2083	106.5934	33.3850	0.3124	0.6212	0.4804	0.2191	0.6994	0.0531	1.0864	T	8.0900	N
0.8005	1052.3455	2.8657	-108.5758	82.9812	121.3940	38.4129	0.2995	0.6543	0.5001	0.2315	0.7315	0.0488	1.1331	R	7.7220	N
0.8014	1104.1946	3.1813	-123.8928	93.6252	137.7978	44.1727	0.2857	0.6781	0.5125	0.2418	0.7542	0.0443	1.1426	P_d	17.6053	W
0.8002	1150.2549	3.5448	-139.0968	103.7442	154.0467	50.3024	0.2739	0.7016	0.5233	0.2537	0.7770	0.0408	1.1732	P_e	6.1826	W
0.8008	1206.8237	3.9061	-159.0113	116.5481	175.1871	58.6390	0.2612	0.7286	0.5340	0.2687	0.8027	0.0371	1.1745	η_d	35.1180	%
0.8006	1252.7469	4.2519	-174.2846	126.4966	191.4733	64.9767	0.2516	0.7411	0.5379	0.2763	0.8142	0.0344	1.1864			

V (m/s)	n (rpm)	Q (N-m)	F (N)	T_p (N)	T_t (N)	T_d (N)	J	K_{fd}	K_{tp}	K_{td}	K_{tt}	K_{ts}	10 * K_q	Self-Propulsion Point		
1.1006	500.6686	0.3193	10.2786	5.2064	6.1254	0.9190	0.8655	-0.2737	0.1386	0.0245	0.1631	0.4483	0.5579	J	0.7029	
1.1022	552.7477	0.4722	4.9858	9.5730	11.5778	2.0047	0.7851	-0.1089	0.2091	0.0438	0.2529	0.3678	0.6769	V	1.1018	m/s
1.1031	603.4646	0.6209	-0.7660	12.7872	17.6112	4.8240	0.7197	0.0140	0.2343	0.0884	0.3227	0.3086	0.7466	10 K_q	0.7840	
1.1032	651.6550	0.8161	-9.4535	18.7681	26.6374	7.8693	0.6665	0.1486	0.2949	0.1237	0.4186	0.2646	0.8416	K_q	0.0784	
1.1005	754.9726	1.2195	-24.9334	30.1015	43.0749	12.9734	0.5739	0.2919	0.3524	0.1519	0.5043	0.1971	0.9369	K_{tt}	0.2951	
1.1026	901.8126	1.9412	-58.3065	52.6797	78.2263	25.5467	0.4814	0.4785	0.4323	0.2096	0.6419	0.1382	1.0452	Q	0.6818	N-m
1.1014	1000.0880	2.4885	-82.6355	71.0720	103.9908	32.9188	0.4336	0.5514	0.4742	0.2196	0.6939	0.1123	1.0895	T	16.8376	N
1.1017	1056.9566	2.7980	-98.0144	81.5514	120.2376	38.6862	0.4104	0.5855	0.4872	0.2311	0.7183	0.1006	1.0968	R	15.9840	N
1.1013	1104.4112	3.1236	-114.1839	92.2256	137.1587	44.9331	0.3926	0.6247	0.5046	0.2458	0.7505	0.0921	1.1214	P_d	44.0797	W
1.1020	1156.9473	3.4722	-130.2701	102.7820	154.0702	51.2882	0.3750	0.6495	0.5125	0.2557	0.7682	0.0839	1.1360	P_e	17.6106	W
1.1015	1212.9133	3.8531	-147.3841	115.6347	172.0909	56.4562	0.3575	0.6686	0.5246	0.2561	0.7807	0.0764	1.1469	η_d	39.9518	%
1.1011	1258.8191	4.2369	-163.3845	125.9249	188.8362	62.9113	0.3444	0.6881	0.5303	0.2650	0.7953	0.0709	1.1709			

Load Varying Self - Propulsion Test (Duct + Propeller fitted on)

V (m/s)	n (rpm)	Q (N-m)	F (N)	T_p (N)	T_t (N)	T_d (N)	J	K_{fd}	K_{tp}	K_{td}	K_{tt}	K_{ts}	10 * K_q	Self-Propulsion Point		
1.3033	501.8541	0.2851	25.8684	1.0075	-2.4715	-3.4789	1.0224	-0.6855	0.0267	-0.0922	-0.0655	0.6227	0.4957	J	0.7399	
1.3116	550.7550	0.4280	19.5634	4.8992	3.2712	-1.6280	0.9376	-0.4304	0.1078	-0.0358	0.0720	0.5170	0.6179	V	1.3035	m/s
1.3018	601.3456	0.5866	14.6455	9.1934	7.6691	-1.5242	0.8523	-0.2703	0.1697	-0.0281	0.1415	0.4337	0.7103	10 K_q	0.8367	
1.3022	654.7460	0.7812	3.6444	16.6810	18.6342	1.9533	0.7830	-0.0567	0.2597	0.0304	0.2901	0.3658	0.7980	K_q	0.0837	
1.3025	752.1400	1.1578	-9.6546	26.3389	32.8566	6.5177	0.6818	0.1139	0.3107	0.0769	0.3876	0.2772	0.8962	K_{tt}	0.3260	
1.3028	907.7795	1.9016	-44.6663	49.4971	70.4953	20.9982	0.5650	0.3617	0.4008	0.1701	0.5709	0.1903	1.0105	Q	0.9193	N-m
1.3020	1006.9377	2.4451	-71.4491	68.2528	99.0206	30.7678	0.5091	0.4703	0.4492	0.2025	0.6518	0.1547	1.0560	T	23.4992	N
1.3014	1051.7237	2.6985	-85.4106	78.4238	113.7191	35.2953	0.4872	0.5153	0.4732	0.2129	0.6861	0.1418	1.0683	R	22.3430	N
1.3116	1103.5535	3.0711	-101.8478	89.1648	130.8195	41.6547	0.4679	0.5581	0.4886	0.2283	0.7169	0.1288	1.1043	P_d	66.7978	W
1.3007	1155.5678	3.4310	-120.5716	101.3690	150.4082	49.0392	0.4431	0.6026	0.5066	0.2451	0.7517	0.1174	1.1251	P_e	29.1250	W
1.3006	1208.2135	3.7775	-136.2495	113.4923	166.7611	53.2687	0.4238	0.6229	0.5188	0.2435	0.7624	0.1074	1.1332	η_d	43.6018	%
1.3020	1256.7456	4.1836	-152.6671	123.8806	183.7284	59.8478	0.4079	0.6451	0.5234	0.2529	0.7763	0.0993	1.1599			

V (m/s)	n (rpm)	Q (N-m)	F (N)	T_p (N)	T_t (N)	T_d (N)	J	K_{fd}	K_{tp}	K_{td}	K_{tt}	K_{ts}	10 * K_q	Self-Propulsion Point		
1.6030	503.1755	0.2466	41.2862	-1.2270	-7.0565	-5.8296	1.2542	-1.0882	-0.0323	-0.1537	-0.1860	0.8909	0.4265	J	0.8138	
1.6053	554.4423	0.3896	35.5623	3.2810	-2.6691	-5.9501	1.1399	-0.7720	0.0712	-0.1292	-0.0579	0.7338	0.5550	V	1.6051	m/s
1.6045	604.3557	0.5547	28.4763	7.5364	3.2865	-4.2499	1.0452	-0.5203	0.1377	-0.0777	0.0600	0.6176	0.6650	10 K_q	0.8936	
1.6063	652.4344	0.7494	19.6455	13.8101	11.6498	-2.1603	0.9693	-0.3080	0.2165	-0.0339	0.1826	0.5299	0.7710	K_q	0.0894	
1.6063	752.2439	1.1132	3.7317	25.8394	28.1917	2.3523	0.8407	-0.0440	0.3047	0.0277	0.3325	0.3986	0.8615	K_{tt}	0.3741	
1.6059	901.5599	1.8053	-29.3272	47.4539	64.0440	16.5901	0.7013	0.2408	0.3896	0.1362	0.5258	0.2775	0.9726	Q	1.2304	N-m
1.6056	1006.7175	2.4295	-55.1247	65.2205	91.9317	26.7112	0.6279	0.3630	0.4295	0.1759	0.6054	0.2226	1.0498	T	33.7987	N
1.6063	1054.3192	2.6763	-72.6503	76.7941	110.3025	33.5084	0.5998	0.4362	0.4610	0.2012	0.6622	0.2029	1.0543	R	32.2170	N
1.6059	1102.5010	3.0303	-88.4574	86.8346	126.9004	40.0658	0.5735	0.4857	0.4768	0.2200	0.6967	0.1856	1.0917	P_d	100.0958	W
1.6050	1154.9706	3.3480	-104.0080	97.6091	143.2220	45.6129	0.5471	0.5203	0.4883	0.2282	0.7165	0.1691	1.0991	P_e	51.7120	W
1.6054	1211.5007	3.7189	-122.4283	110.6533	162.3490	51.6957	0.5217	0.5567	0.5031	0.2351	0.7382	0.1537	1.1095	η_d	51.6626	%
1.6019	1257.0444	4.0895	-139.6617	122.2759	180.1017	57.8258	0.5017	0.5898	0.5164	0.2442	0.7606	0.1427	1.1333			

Load Varying Self - Propulsion Test (Duct + Propeller fitted on)

V (m/s)	n (rpm)	Q (N-m)	F (N)	T_p (N)	T_t (N)	T_d (N)	J	K_{fd}	K_{tp}	K_{td}	K_{tt}	K_{ts}	10 * K_q	Self-Propulsion Point		
1.8051	503.4015	0.2345	51.8995	-1.8453	-12.0461	-10.2007	1.4117	-1.3668	-0.0486	-0.2686	-0.3172	1.0487	0.4053	J	0.8544	
1.8061	550.5344	0.3599	45.3553	2.2818	-7.9307	-10.2125	1.2916	-0.9987	0.0502	-0.2249	-0.1746	0.8768	0.5200	V	1.8054	m/s
1.8054	601.7534	0.4952	39.4746	5.8363	-2.0996	-7.9358	1.1812	-0.7275	0.1076	-0.1463	-0.0387	0.7339	0.5988	10 K_q	0.9029	
1.8043	655.6402	0.7021	29.6365	12.8210	8.3160	-4.5049	1.0835	-0.4601	0.1990	-0.0699	0.1291	0.6182	0.7152	K_q	0.0903	
1.8049	756.7032	1.1097	14.8382	23.0704	24.0023	0.9320	0.9391	-0.1729	0.2689	0.0109	0.2797	0.4641	0.8487	K_{tt}	0.3840	
1.8053	903.5768	1.7817	-16.0784	44.1607	56.0137	11.8530	0.7866	0.1314	0.3610	0.0969	0.4579	0.3255	0.9556	Q	1.4270	N-m
1.8053	1007.3473	2.3494	-41.3141	62.1857	82.5484	20.3627	0.7056	0.2717	0.4090	0.1339	0.5429	0.2619	1.0138	T	39.8209	N
1.8062	1051.7675	2.6355	-58.6353	74.1854	100.6055	26.4201	0.6761	0.3537	0.4475	0.1594	0.6069	0.2402	1.0433	R	38.0330	N
1.8057	1105.5646	2.9403	-75.5367	85.3344	118.5649	33.2305	0.6430	0.4124	0.4659	0.1814	0.6474	0.2174	1.0534	P_d	124.3640	W
1.8064	1151.0459	3.2937	-91.0393	95.7116	135.0686	39.3570	0.6179	0.4586	0.4821	0.1982	0.6803	0.2006	1.0886	P_e	68.6635	W
1.8051	1199.5015	3.6086	-109.2754	108.5271	154.5146	45.9875	0.5925	0.5069	0.5034	0.2133	0.7167	0.1847	1.0983	η_d	55.2117	%
1.8046	1261.8668	4.0584	-128.6862	121.0877	175.6173	54.5296	0.5630	0.5393	0.5075	0.2285	0.7360	0.1669	1.1161			

V (m/s)	n (rpm)	Q (N-m)	F (N)	T_p (N)	T_t (N)	T_d (N)	J	K_{fd}	K_{tp}	K_{td}	K_{tt}	K_{ts}	10 * K_q	Self-Propulsion Point		
2.1082	500.4856	0.1922	66.8995	-2.1356	-15.0436	-12.9080	1.6584	-1.7824	-0.0569	-0.3439	-0.4008	1.4600	0.3360	J	0.9012	
2.1078	552.4532	0.3309	60.4425	1.3531	-8.6422	-9.9953	1.5021	-1.3216	0.0296	-0.2186	-0.1890	1.1983	0.4748	V	2.1087	m/s
2.1093	604.4664	0.4607	54.4746	5.2497	-2.1310	-7.3807	1.3738	-0.9950	0.0959	-0.1348	-0.0389	1.0009	0.5522	10 K_q	0.9232	
2.1056	651.4342	0.6493	47.4534	10.3091	5.3703	-4.9388	1.2725	-0.7463	0.1621	-0.0777	0.0845	0.8618	0.6700	K_q	0.0923	
2.1084	754.9317	1.0051	32.8382	20.4760	20.7011	0.2251	1.0995	-0.3845	0.2398	0.0026	0.2424	0.6417	0.7723	K_{tt}	0.4310	
2.1095	901.1852	1.6707	1.0784	42.8728	53.4900	10.6172	0.9216	-0.0089	0.3523	0.0872	0.4395	0.4503	0.9008	Q	1.7891	N-m
2.1082	1002.9008	2.2317	-23.4953	59.8464	79.1802	19.3338	0.8276	0.1559	0.3971	0.1283	0.5254	0.3636	0.9716	T	54.8007	N
2.1098	1053.7204	2.5445	-37.6353	69.3011	94.0054	24.7043	0.7883	0.2262	0.4165	0.1485	0.5650	0.3294	1.0035	R	52.3950	N
2.1103	1101.8507	2.8646	-54.5367	80.1924	111.6413	31.4489	0.7540	0.2998	0.4408	0.1729	0.6137	0.3012	1.0332	P_d	172.6572	W
2.1101	1153.6637	3.1899	-70.0393	90.9845	128.0193	37.0348	0.7201	0.3512	0.4562	0.1857	0.6419	0.2748	1.0495	P_e	110.4875	W
2.1091	1210.1050	3.5803	-88.2754	103.8501	147.2988	43.4488	0.6862	0.4023	0.4733	0.1980	0.6713	0.2497	1.0707	η_d	63.9924	%
2.1086	1258.3035	3.9869	-107.6862	117.4283	167.6550	50.2267	0.6597	0.4539	0.4950	0.2117	0.7067	0.2310	1.1027			

Load Varying Self - Propulsion Test (Duct + Propeller fitted on)

V (m/s)	n (rpm)	Q (N-m)	F (N)	T_p (N)	T_t (N)	T_d (N)	J	K_{fd}	K_{tp}	K_{td}	K_{tt}	K_{ts}	10 * K_q	Self-Propulsion Point		
2.3113	506.5541	0.1820	86.1252	-3.5334	-19.3516	-15.8182	1.7964	-2.2400	-0.0919	-0.4114	-0.5033	1.8214	0.3106	J	0.9079	
2.3012	554.4124	0.3087	80.4523	-0.1165	-15.8623	-15.7458	1.6341	-1.7468	-0.0025	-0.3419	-0.3444	1.5205	0.4398	V	2.3142	m/s
2.3106	602.4889	0.4268	75.8979	4.0222	-10.8438	-14.8660	1.5099	-1.3954	0.0739	-0.2733	-0.1994	1.2875	0.5149	10 K_q	0.9400	
2.3087	653.7372	0.6197	68.2312	9.2811	-1.9037	-11.1847	1.3904	-1.0655	0.1449	-0.1747	-0.0297	1.0936	0.6349	K_q	0.0940	
2.3127	751.5777	0.9597	55.4110	18.7622	13.1329	-5.6292	1.2115	-0.6547	0.2217	-0.0665	0.1552	0.8274	0.7440	K_{tt}	0.4641	
2.3075	907.7627	1.6567	24.5844	38.9545	45.9201	6.9655	1.0008	-0.1991	0.3155	0.0564	0.3719	0.5672	0.8804	Q	2.1617	N-m
2.3104	1009.7375	2.2082	0.7728	57.7942	70.3932	12.5991	0.9008	-0.0051	0.3783	0.0825	0.4608	0.4584	0.9484	T	70.0300	N
2.3089	1055.8389	2.5040	-13.1249	66.6765	84.5472	17.8707	0.8609	0.0786	0.3992	0.1070	0.5061	0.4192	0.9836	R	66.9070	N
2.3716	1105.4558	2.7615	-27.7563	76.7109	99.2872	22.5764	0.8446	0.1516	0.4189	0.1233	0.5422	0.3824	0.9896	P_d	227.2621	W
2.3101	1156.8492	3.1357	-45.5095	88.9061	117.4706	28.5645	0.7862	0.2269	0.4433	0.1424	0.5858	0.3492	1.0260	P_e	154.8345	W
2.3079	1207.4927	3.4799	-62.1596	100.1417	134.4150	34.2733	0.7525	0.2845	0.4584	0.1569	0.6152	0.3205	1.0452	η_d	68.1304	%
2.3092	1255.9025	3.8185	-78.5473	112.6834	151.0913	38.4079	0.7239	0.3323	0.4768	0.1625	0.6393	0.2963	1.0601			

V (m/s)	n (rpm)	Q (N-m)	F (N)	T_p (N)	T_t (N)	T_d (N)	J	K_{fd}	K_{tp}	K_{td}	K_{tt}	K_{ts}	10 * K_q	Self-Propulsion Point		
2.5094	504.8586	0.1628	105.9464	-5.1789	-23.5927	-18.4138	1.9569	-2.7740	-0.1356	-0.4821	-0.6177	2.2324	0.2797	J	0.9078	
2.5104	552.4243	0.2813	101.2323	-1.3845	-21.2282	-19.8438	1.7891	-2.2138	-0.0303	-0.4340	-0.4642	1.8645	0.4036	V	2.5100	m/s
2.5067	604.5795	0.4035	96.3633	1.8110	-16.9244	-18.7354	1.6324	-1.7594	0.0331	-0.3421	-0.3090	1.5567	0.4834	10 K_q	0.9736	
2.5107	653.2312	0.5696	90.3483	6.9611	-10.5697	-17.5308	1.5132	-1.4130	0.1089	-0.2742	-0.1653	1.3334	0.5845	K_q	0.0974	
2.5093	753.2477	0.9274	78.3736	15.4460	2.9669	-12.4791	1.3115	-0.9218	0.1817	-0.1468	0.0349	1.0028	0.7157	K_{tt}	0.4802	
2.5099	906.0469	1.6051	49.0260	35.9381	34.7210	-1.2172	1.0906	-0.3986	0.2922	-0.0099	0.2823	0.6931	0.8562	Q	2.6345	N-m
2.5126	1002.7932	2.1197	21.5911	55.1685	63.3290	8.1605	0.9865	-0.1433	0.3661	0.0542	0.4203	0.5658	0.9231	T	85.2600	N
2.5100	1052.4234	2.4384	9.2021	64.9951	76.2347	11.2396	0.9390	-0.0554	0.3916	0.0677	0.4593	0.5137	0.9641	R	81.4600	N
2.5104	1106.7229	2.7268	-5.5593	74.6193	91.4780	16.8588	0.8930	0.0303	0.4066	0.0919	0.4984	0.4645	0.9749	P_d	300.4342	W
2.5101	1152.3582	3.0710	-19.5704	83.9508	105.8466	21.8958	0.8576	0.0984	0.4219	0.1100	0.5319	0.4285	1.0127	P_e	204.4646	W
2.5109	1204.5756	3.4002	-38.1235	95.8461	124.7560	28.9099	0.8207	0.1753	0.4408	0.1330	0.5738	0.3921	1.0261	η_d	68.0564	%
2.5096	1256.7444	3.7705	-53.2054	108.0306	140.1545	32.1239	0.7862	0.2248	0.4565	0.1357	0.5922	0.3603	1.0454			

Duct Thrust Extrapolation

only Prop. on					Prop. + Duct on					only Prop on			Prop + Duct on	only Prop on	
V (m/s)	n (rpm)	J	F (N)	T_p (N)	V (m/s)	n (rpm)	J	F (N)	T_p (N)	J	F (N)	T_p (N)	$F - T_p$ (N)	$F - T_p$ (N)	T_d (N)
0.8007	503.8949	0.6256	5.3638	3.6262	0.8002	502.5735	0.6269	1.3073	5.8645	0.6269	5.3288	3.6210	-7.1718	-8.9498	1.7779
0.8012	552.6343	0.5708	1.4334	8.7409	0.8004	551.4566	0.5714	-4.6450	10.3820	0.5714	1.6898	7.8627	-5.7370	-9.5525	3.8156
0.8027	602.3573	0.5246	-3.5438	12.8134	0.8012	600.4575	0.5253	-10.5645	14.4121	0.5253	-4.0378	13.1919	-3.8476	-9.1541	5.3064
0.8034	651.5623	0.4854	-10.4367	18.6583	0.8018	650.4466	0.4853	-17.5455	19.4810	0.4853	-10.2344	18.8742	-1.9355	-8.6397	6.7042
0.8005	753.4148	0.4183	-24.3516	32.8282	0.8002	753.2179	0.4183	-34.8750	31.2554	0.4183	-23.9146	32.2745	3.6196	-8.3599	11.9795
0.8012	913.8563	0.3452	-48.5726	59.3485	0.8002	902.9860	0.3489	-68.0707	54.7882	0.3489	-47.5126	57.4752	13.2825	-9.9625	23.2450
0.8037	1002.5757	0.3156	-64.7292	77.0108	0.8003	1008.5085	0.3124	-94.6789	73.2083	0.3124	-67.3279	79.2424	21.4706	-11.9145	33.3850
0.8045	1051.5807	0.3012	-74.4405	87.1072	0.8005	1052.3455	0.2995	-108.5758	82.9812	0.2995	-76.1428	88.9610	25.5946	-12.8182	38.4129
0.8098	1102.1417	0.2893	-85.7547	98.8701	0.8014	1104.1946	0.2857	-123.8928	93.6252	0.2857	-86.7294	100.6345	30.2676	-13.9050	44.1727
0.8032	1154.0417	0.2740	-96.8046	111.2684	0.8002	1150.2549	0.2739	-139.0968	103.7442	0.2739	-96.9863	111.9361	35.3526	-14.9499	50.3024
0.8015	1207.3702	0.2614	-108.6784	123.8401	0.8008	1206.8237	0.2612	-159.0113	116.5481	0.2612	-109.1949	125.3707	42.4632	-16.1758	58.6390
0.8022	1246.6728	0.2533	-117.3754	135.8194	0.8006	1252.7469	0.2516	-174.2846	126.4966	0.2516	-119.4611	136.6498	47.7880	-17.1887	64.9767
V (m/s)	n (rpm)	J	F (N)	T_p (N)	V (m/s)	n (rpm)	J	F (N)	T_p (N)	J	F (N)	T_p (N)	$F - T_p$ (N)	$F - T_p$ (N)	T_d (N)
1.1030	507.2079	0.8562	14.2900	2.2849	1.1006	500.6686	0.8655	10.2786	5.2064	0.8655	14.1086	2.2954	-15.4850	-16.4040	0.9190
1.1027	554.5345	0.7829	10.5355	6.2834	1.1022	552.7477	0.7851	4.9858	9.5730	0.7851	11.1953	5.3682	-14.5588	-16.5635	2.0047
1.1035	603.8746	0.7194	6.8977	10.2080	1.1031	603.4646	0.7197	-0.7660	12.7872	0.7197	6.0708	10.7744	-12.0212	-16.8452	4.8240
1.1032	654.2447	0.6639	0.5784	16.3454	1.1032	651.6550	0.6665	-9.4535	18.7681	0.6665	0.8604	16.3236	-9.3146	-17.1839	7.8693
1.1044	750.3733	0.5794	-11.0514	29.4461	1.1005	754.9726	0.5739	-24.9334	30.1015	0.5739	-11.4573	29.5988	-5.1681	-18.1415	12.9734
1.1041	910.3564	0.4775	-34.4575	54.5330	1.1026	901.8126	0.4814	-58.3065	52.6797	0.4814	-33.4364	53.3562	5.6268	-19.9198	25.5467
1.1030	1006.6678	0.4314	-52.5912	73.9689	1.1014	1000.0880	0.4336	-82.6355	71.0720	0.4336	-52.1842	73.5396	11.5635	-21.3553	32.9188
1.1037	1056.9924	0.4111	-63.4548	85.6126	1.1017	1056.9566	0.4104	-98.0144	81.5514	0.4104	-63.9686	86.1918	16.4630	-22.2232	38.6862
1.1031	1108.8989	0.3916	-75.4534	98.4729	1.1013	1104.4112	0.3926	-114.1839	92.2256	0.3926	-74.4169	97.3917	21.9583	-22.9748	44.9331
1.1031	1156.0328	0.3757	-86.3969	110.2023	1.1020	1156.9473	0.3750	-130.2701	102.7820	0.3750	-86.1232	109.9234	27.4881	-23.8001	51.2882
1.1029	1206.4133	0.3599	-96.9218	121.4830	1.1015	1212.9133	0.3575	-147.3841	115.6347	0.3575	-99.2352	123.9419	31.7494	-24.7068	56.4562
1.1034	1249.3622	0.3477	-107.1228	132.4165	1.1011	1258.8191	0.3444	-163.3845	125.9249	0.3444	-110.1865	135.6382	37.4596	-25.4517	62.9113
V (m/s)	n (rpm)	J	F (N)	T_p (N)	V (m/s)	n (rpm)	J	F (N)	T_p (N)	J	F (N)	T_p (N)	$F - T_p$ (N)	$F - T_p$ (N)	T_d (N)
1.3031	504.2337	1.0174	23.7086	-0.5806	1.3033	501.8541	1.0224	25.8684	1.0075	1.0224	23.9228	-0.5259	-26.8759	-23.3969	-3.4789
1.3022	550.4535	0.9314	19.5828	3.8393	1.3116	550.7550	0.9376	19.5634	4.8992	0.9376	19.9850	2.8496	-24.4626	-22.8346	-1.6280
1.3025	600.5634	0.8539	15.5347	7.2149	1.3018	601.3456	0.8523	14.6455	9.1934	0.8523	14.2409	8.0737	-23.8389	-22.3146	-1.5242
1.3035	652.2352	0.7868	7.4959	13.4575	1.3022	654.7460	0.7830	3.6444	16.6810	0.7830	8.5663	13.7124	-20.3254	-22.2786	1.9533
1.3036	752.8192	0.6817	-3.1778	26.1866	1.3025	752.1400	0.6818	-9.6546	26.3389	0.6818	-2.8693	26.0713	-16.6843	-23.2020	6.5177
1.3022	901.8296	0.5685	-23.6329	51.7382	1.3028	907.7795	0.5650	-44.6663	49.4971	0.5650	-26.9059	52.7349	-4.8308	-25.8290	20.9982
1.3022	1000.1110	0.5126	-45.8753	72.3597	1.3020	1006.9377	0.5091	-71.4491	68.2528	0.5091	-46.0278	73.5993	3.1963	-27.5715	30.7678
1.3023	1046.2973	0.4900	-54.5532	82.1375	1.3014	1051.7237	0.4872	-85.4106	78.4238	0.4872	-55.4250	83.7335	6.9868	-28.3085	35.2953
1.3020	1105.7242	0.4636	-67.4871	96.1106	1.3116	1103.5535	0.4679	-101.8478	89.1648	0.4679	-64.7226	93.6943	12.6830	-28.9717	41.6547
1.3023	1155.3342	0.4438	-78.2728	107.4396	1.3007	1155.5678	0.4431	-120.5716	101.3690	0.4431	-78.2864	108.1230	19.2026	-29.8366	49.0392
1.3022	1206.0095	0.4251	-88.6396	119.2426	1.3006	1208.2135	0.4238	-136.2495	113.4923	0.4238	-90.2450	120.7565	22.7572	-30.5116	53.2687
1.3024	1248.0299	0.4109	-98.8778	130.6479	1.3020	1256.7456	0.4079	-152.6671	123.8806	0.4079	-101.0751	132.1363	28.7865	-31.0613	59.8478

Duct Thrust Extrapolation

only Prop. on					Prop. + Duct on					only Prop on			Duct on	Prop on	
V (m/s)	n (rpm)	J	F (N)	T_p (N)	V (m/s)	n (rpm)	J	F (N)	T_p (N)	J	F (N)	T_p (N)	F - T_p (N)	F - T_p (N)	T_d (N)
1.6032	503.9995	1.2523	35.6003	-1.5261	1.6030	503.1755	1.2542	41.2862	-1.2270	1.2542	35.6755	-1.4458	-40.0592	-34.2297	-5.8296
1.6046	550.3443	1.1479	31.5343	1.5354	1.6053	554.4423	1.1399	35.5623	3.2810	1.1399	31.6231	1.2701	-38.8433	-32.8932	-5.9501
1.6035	601.4574	1.0496	27.4347	5.3738	1.6045	604.3557	1.0452	28.4763	7.5364	1.0452	25.3902	6.3726	-36.0127	-31.7628	-4.2499
1.6038	653.7645	0.9658	17.3544	11.6456	1.6063	652.4344	0.9693	19.6455	13.8101	0.9693	19.7697	11.5256	-33.4556	-31.2953	-2.1603
1.6040	755.9224	0.8354	8.5633	25.1105	1.6063	752.2439	0.8407	3.7317	25.8394	0.8407	8.5430	23.3804	-29.5711	-31.9234	2.3523
1.6038	911.6408	0.6926	-12.6824	48.1265	1.6059	901.5599	0.7013	-29.3272	47.4539	0.7013	-12.2246	46.9413	-18.1267	-34.7168	16.5901
1.6044	1003.7944	0.6293	-32.5581	67.4672	1.6056	1006.7175	0.6279	-55.1247	65.2205	0.6279	-31.4537	68.2607	-10.0958	-36.8070	26.7112
1.6042	1051.3377	0.6007	-42.0001	77.7648	1.6063	1054.3192	0.5998	-72.6503	76.7941	0.5998	-41.1811	78.8333	-4.1438	-37.6522	33.5084
1.6043	1104.0239	0.5721	-51.6517	91.4829	1.6059	1102.5010	0.5735	-88.4574	86.8346	0.5735	-51.7748	90.2178	1.6228	-38.4430	40.0658
1.6034	1152.5333	0.5477	-63.4763	104.5737	1.6050	1154.9706	0.5471	-104.0080	97.6091	0.5471	-63.9740	103.1880	6.3989	-39.2140	45.6129
1.6036	1210.1129	0.5217	-76.3438	116.7686	1.6054	1211.5007	0.5217	-122.4283	110.6533	0.5217	-77.4263	117.3470	11.7750	-39.9207	51.6957
1.6030	1250.8055	0.5046	-88.3925	127.3434	1.6019	1257.0444	0.5017	-139.6617	122.2759	0.5017	-89.3107	129.7507	17.3858	-40.4400	57.8258

V (m/s)	n (rpm)	J	F (N)	T_p (N)	V (m/s)	n (rpm)	J	F (N)	T_p (N)	J	F (N)	T_p (N)	F - T_p (N)	F - T_p (N)	T_d (N)
1.8042	510.9808	1.3901	40.6558	-1.9385	1.8051	503.4015	1.4117	51.8995	-1.8453	1.4117	40.6569	-0.8034	-50.0542	-39.8534	-10.2007
1.8034	552.4233	1.2853	37.6344	1.1898	1.8061	550.5344	1.2916	45.3553	2.2818	1.2916	37.9281	-0.5035	-47.6371	-37.4246	-10.2125
1.8068	602.7853	1.1801	33.4536	4.2981	1.8054	601.7534	1.1812	39.4746	5.8363	1.1812	32.3158	5.0592	-45.3109	-37.3750	-7.9358
1.8056	651.5334	1.0911	24.9715	10.5564	1.8043	655.6402	1.0835	29.6365	12.8210	1.0835	26.4884	11.4642	-42.4575	-37.9525	-4.5049
1.8042	757.8054	0.9373	17.3566	22.9946	1.8049	756.7032	0.9391	14.8382	23.0704	0.9391	15.9131	22.9275	-37.9086	-38.8405	0.9320
1.8051	911.1136	0.7800	-4.5070	45.8331	1.8053	903.5768	0.7866	-16.0784	44.1607	0.7866	-3.9124	43.8477	-28.0823	-39.9353	11.8530
1.8048	1002.7594	0.7086	-25.1929	65.0234	1.8053	1007.3473	0.7056	-41.3141	62.1857	0.7056	-22.5514	63.7857	-20.8716	-41.2343	20.3627
1.8061	1056.9214	0.6728	-32.6326	73.8682	1.8062	1051.7675	0.6761	-58.6353	74.1854	0.6761	-31.4680	73.4382	-15.5501	-41.9702	26.4201
1.8051	1118.8065	0.6352	-43.1904	87.2252	1.8057	1105.5646	0.6430	-75.5367	85.3344	0.6430	-43.1497	86.1779	-9.7977	-43.0282	33.2305
1.8043	1161.1239	0.6118	-55.8141	100.0789	1.8064	1151.0459	0.6179	-91.0393	95.7116	0.6179	-53.3702	97.3995	-4.6723	-44.0293	39.3570
1.8059	1215.6599	0.5849	-68.1665	112.6564	1.8051	1199.5015	0.5925	-109.2754	108.5271	0.5925	-64.9756	110.2148	0.7483	-45.2392	45.9875
1.8057	1246.0544	0.5705	-77.6279	125.3448	1.8046	1261.8668	0.5630	-128.6862	121.0877	0.5630	-80.2297	127.1608	7.5985	-46.9311	54.5296

V (m/s)	n (rpm)	J	F (N)	T_p (N)	V (m/s)	n (rpm)	J	F (N)	T_p (N)	J	F (N)	T_p (N)	F - T_p (N)	F - T_p (N)	T_d (N)
2.1091	500.1371	1.6603	54.2008	-2.2244	2.1082	500.4856	1.6584	66.8995	-2.1356	1.6584	54.0240	-2.1681	-64.7639	-51.8559	-12.9080
2.1056	551.5545	1.5030	51.6384	0.7034	2.1078	552.4532	1.5021	60.4425	1.3531	1.5021	52.4925	-0.6922	-61.7956	-51.8003	-9.9953
2.1068	600.3476	1.3816	49.7643	3.3349	2.1093	604.4664	1.3738	54.4746	5.2497	1.3738	48.0280	4.3156	-59.7243	-52.3436	-7.3807
2.1067	654.3489	1.2675	42.3233	9.0348	2.1056	651.4342	1.2725	47.4534	10.3091	1.2725	43.6156	9.2080	-57.7625	-52.8237	-4.9388
2.1101	752.6505	1.1038	34.8043	19.5316	2.1084	754.9317	1.0995	32.8382	20.4760	1.0995	33.6565	19.8827	-53.3142	-53.5393	0.2251
2.1068	901.9876	0.9196	13.6338	42.0655	2.1095	901.1852	0.9216	1.0784	42.8728	0.9216	14.5964	39.9720	-43.9512	-54.5684	10.6172
2.1070	1002.6789	0.8273	-3.9622	59.4139	2.1082	1002.9008	0.8276	-23.4953	59.8464	0.8276	-3.1288	58.8136	-36.3511	-55.6849	19.3338
2.0959	1052.8833	0.7837	-13.7635	68.8805	2.1098	1053.7204	0.7883	-37.6353	69.3011	0.7883	-12.9237	69.2938	-31.6658	-56.3701	24.7043
2.1064	1103.4297	0.7516	-24.0542	80.5944	2.1103	1101.8507	0.7540	-54.5367	80.1924	0.7540	-22.8534	79.9580	-25.6557	-57.1046	31.4489
2.1073	1156.4853	0.7174	-33.2420	93.1518	2.1101	1153.6637	0.7201	-70.0393	90.9845	0.7201	-34.1337	92.1136	-20.9452	-57.9800	37.0348
2.1066	1214.1716	0.6831	-47.5793	106.0173	2.1091	1210.1050	0.6862	-88.2754	103.8501	0.6862	-47.0057	106.0292	-15.5747	-59.0234	43.4488
2.1060	1246.6579	0.6651	-57.4544	117.5641	2.1086	1258.3035	0.6597	-107.6862	117.4283	0.6597	-58.2606	118.2295	-9.7421	-59.9688	50.2267

Duct Thrust Extrapolation

only Prop. on					Prop. + Duct on					only Prop on			Duct on	Prop on	
V (m/s)	n (rpm)	J	F (N)	T_p (N)	V (m/s)	n (rpm)	J	F (N)	T_p (N)	J	F (N)	T_p (N)	$F - T_p$ (N)	$F - T_p$ (N)	T_d (N)
2.3079	502.2690	1.8090	70.1815	-3.6889	2.3113	506.5541	1.7964	86.1252	-3.5334	1.7964	70.1007	-3.3271	-82.5918	-66.7736	-15.8182
2.3069	550.3896	1.6502	65.5744	-0.4585	2.3012	554.4124	1.6341	80.4523	-0.1165	1.6341	65.5989	-1.0089	-80.3358	-64.5900	-15.7458
2.3074	603.5423	1.5052	62.5675	2.8578	2.3106	602.4889	1.5099	75.8979	4.0222	1.5099	61.8298	3.2243	-79.9201	-65.0541	-14.8660
2.3070	652.5464	1.3919	56.4648	7.3859	2.3087	653.7372	1.3904	68.2312	9.2811	1.3904	58.2501	8.0775	-77.5123	-66.3276	-11.1847
2.3084	752.0456	1.2085	52.8606	17.3533	2.3127	751.5777	1.2115	55.4110	18.7622	1.2115	51.1627	17.3812	-74.1732	-68.5439	-5.6292
2.3081	901.6673	1.0078	31.0591	39.3701	2.3075	907.7627	1.0008	24.5844	38.9545	1.0008	31.9903	38.5141	-63.5389	-70.5045	6.9655
2.3082	1006.0527	0.9033	14.6680	56.3185	2.3104	1009.7375	0.9008	0.7728	57.7942	0.9008	14.1489	57.0171	-58.5670	-71.1660	12.5991
2.3077	1052.1662	0.8635	5.4600	65.8199	2.3089	1055.8389	0.8609	-13.1249	66.6765	0.8609	4.6684	66.7538	-53.5516	-71.4223	17.8707
2.3074	1101.3796	0.8248	-4.9423	75.6556	2.3716	1105.4558	0.8446	-27.7563	76.7109	0.8446	0.3371	71.1938	-48.9546	-71.5309	22.5764
2.3075	1150.4349	0.7897	-16.4586	88.4838	2.3101	1156.8492	0.7862	-45.5095	88.9061	0.7862	-17.6066	89.5678	-43.3966	-71.9611	28.5645
2.3077	1213.5677	0.7487	-29.2323	102.4455	2.3079	1207.4927	0.7525	-62.1596	100.1417	0.7525	-29.8499	102.1053	-37.9821	-72.2554	34.2733
2.3068	1249.1721	0.7270	-42.1765	114.3675	2.3092	1255.9025	0.7239	-78.5473	112.6834	0.7239	-41.4538	113.9978	-34.1361	-72.5440	38.4079

V (m/s)	n (rpm)	J	F (N)	T_p (N)	V (m/s)	n (rpm)	J	F (N)	T_p (N)	J	F (N)	T_p (N)	$F - T_p$ (N)	$F - T_p$ (N)	T_d (N)
2.5100	504.0554	1.9605	87.5695	-5.1534	2.5094	504.8586	1.9569	105.9464	-5.1789	1.9569	87.4743	-5.1206	-100.7675	-82.3537	-18.4138
2.5086	552.5634	1.7874	83.4748	-2.4533	2.5104	552.4243	1.7891	101.2323	-1.3845	1.7891	83.6727	-3.6687	-99.8478	-80.0041	-19.8438
2.5072	601.0760	1.6422	78.4785	1.5344	2.5067	604.5795	1.6324	96.3633	1.8110	1.6324	77.7464	1.6925	-98.1743	-79.4389	-18.7354
2.5095	651.4524	1.5166	72.4764	5.3858	2.5107	653.2312	1.5132	90.3483	6.9611	1.5132	73.2244	6.5542	-97.3094	-79.7786	-17.5308
2.5020	751.7646	1.3103	66.8172	15.9565	2.5093	753.2477	1.3115	78.3736	15.4460	1.3115	65.0928	16.2477	-93.8196	-81.3405	-12.4791
2.5092	905.0740	1.0915	46.3855	37.9313	2.5099	906.0469	1.0906	49.0260	35.9381	1.0906	48.2725	35.4745	-84.9641	-83.7470	-1.2172
2.5081	1004.8353	0.9827	32.5307	52.3894	2.5126	1002.7932	0.9865	21.5911	55.1685	0.9865	32.8107	52.1094	-76.7596	-84.9201	8.1605
2.5087	1052.4015	0.9385	22.3437	62.3939	2.5100	1052.4234	0.9390	9.2021	64.9951	0.9390	23.1745	62.2624	-74.1972	-85.4368	11.2396
2.5087	1103.3557	0.8952	12.3667	72.3930	2.5104	1106.7229	0.8930	-5.5593	74.6193	0.8930	11.9390	73.9797	-69.0600	-85.9187	16.8588
2.5023	1154.7567	0.8531	3.6930	84.3833	2.5101	1152.3582	0.8576	-19.5704	83.9508	0.8576	1.7912	84.4850	-64.3804	-86.2762	21.8958
2.5092	1207.1602	0.8183	-11.5045	97.3348	2.5109	1204.5756	0.8207	-38.1235	95.8461	0.8207	-10.2948	96.9273	-57.7226	-86.6325	28.9099
2.5091	1252.5791	0.7886	-23.5327	110.9502	2.5096	1256.7444	0.7862	-53.2054	108.0306	0.7862	-23.1296	110.0787	-54.8252	-86.9491	32.1239

Bare Hull Resistance Tests

Ahead

V	Re	Fn	R	C_d	C_d
(m/s)			(N)	CSA	WSA
0.6010	1.36E+06	0.1168	4.8372	0.2131	0.0089
0.6987	1.59E+06	0.1358	6.2947	0.2052	0.0086
0.8003	1.82E+06	0.1555	7.9859	0.1985	0.0083
0.8990	2.04E+06	0.1747	9.8816	0.1946	0.0081
1.0015	2.27E+06	0.1946	11.7594	0.1866	0.0078
1.1016	2.50E+06	0.2140	13.8754	0.1820	0.0076
1.2007	2.72E+06	0.2333	16.2767	0.1797	0.0075
1.3048	2.96E+06	0.2535	18.6795	0.1746	0.0073
1.4054	3.19E+06	0.2731	21.2504	0.1712	0.0072
1.5037	3.41E+06	0.2922	23.8043	0.1676	0.0070
1.6040	3.64E+06	0.3117	27.2382	0.1685	0.0070
1.7060	3.87E+06	0.3315	30.5770	0.1672	0.0070
1.8091	4.10E+06	0.3515	33.8079	0.1644	0.0069
1.9061	4.32E+06	0.3704	37.4270	0.1640	0.0068
2.0064	4.55E+06	0.3899	41.1379	0.1626	0.0068
2.1073	4.78E+06	0.4095	46.4036	0.1663	0.0069
2.3105	5.24E+06	0.4489	60.8839	0.1815	0.0076
2.5122	5.70E+06	0.4881	76.8940	0.1939	0.0081
2.7087	6.15E+06	0.5263	98.8119	0.2143	0.0090
2.9112	6.61E+06	0.5657	127.6270	0.2397	0.0100
3.1129	7.06E+06	0.6049	160.1392	0.2630	0.0110

Astern

V	Re	Fn	R	C_d	C_d
(m/s)			(N)	CSA	WSA
0.6088	1.38E+06	0.1183	5.3066	0.2279	0.0095
0.7089	1.61E+06	0.1377	6.9982	0.2216	0.0093
0.8097	1.84E+06	0.1573	8.9347	0.2169	0.0091
0.9113	2.07E+06	0.1771	11.1765	0.2142	0.0089
1.0126	2.30E+06	0.1968	13.4500	0.2088	0.0087
1.1124	2.52E+06	0.2161	15.8659	0.2041	0.0085
1.2146	2.76E+06	0.2360	18.5111	0.1997	0.0083
1.3139	2.98E+06	0.2553	21.1376	0.1949	0.0081
1.4123	3.20E+06	0.2744	24.1367	0.1926	0.0080
1.5161	3.44E+06	0.2946	27.3797	0.1896	0.0079
1.6148	3.66E+06	0.3138	30.4808	0.1860	0.0078
1.7155	3.89E+06	0.3333	34.0387	0.1841	0.0077
1.8173	4.12E+06	0.3531	37.5215	0.1808	0.0076
1.9185	4.35E+06	0.3728	40.8561	0.1767	0.0074
2.0195	4.58E+06	0.3924	44.6591	0.1743	0.0073

CSA Cross Sectional Area
WSA Wetted Surface Area

Bollard Pull Tests

(Propeller + Duct fitted on)

n	Q	F	T_p	P_d
rpm	N-m	N	N	Watt
503.9033	0.5283	-11.3075	8.9647	27.8915
550.4767	0.6797	-17.3445	12.5679	39.1967
605.9664	0.8448	-24.1935	17.6148	53.6326
650.5832	1.0205	-30.4523	22.3550	69.5562
750.1102	1.3944	-48.3194	35.0872	109.5780
907.7347	2.1970	-84.1484	59.2718	208.9271
1001.8816	2.6913	-113.0899	76.4800	282.4713
1052.1447	3.0135	-129.0183	87.9141	332.1587
1102.7669	3.3844	-145.7299	98.9344	390.9927
1151.0962	3.6596	-162.4119	109.1874	441.3201
1201.9108	4.0432	-183.5970	121.8579	509.0934
1250.9090	4.3910	-200.7750	132.5731	575.4300

Load Varying Self-Propulsion Test (Propeller fitted on)

Uncertainty Analysis (Propeller Rotational Speed)

V	Vehicle Velocity (m/s)	
n	Propeller Rotational Speed (rpm)	
C	Number of Counted Pulses in Time dt	
dt	Time (s)	
S.D	Standard Deviation	
P_n	Precision Error for n	$P_n = 2 \times S.D$
B_C	Bias Error for C	
B_{dt}	Bias Error for dt	
B_n	Bias Error for n	$B_n = ((\delta n / \delta C \; B_C)^2 + (\delta n / \delta dt \; B_{dt})^2)^{1/2}$
U_n	Total Error for n	$U_n = (B_n^2 + P_n^2)^{1/2}$
% Unc.	Percentage Uncertainty for n	$\% \text{ Unc.} = (U_n / n) \times 100$

Uncertainty Analysis (Rotational Speed)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	n (rpm)	P_n	B_C δn/δC	B_{dt} δn/δdt	B_n	U_n	% Unc.
0.8007	503.8949	0.0004	0.0391	-0.0009	0.0391	0.0391	0.0078
0.8012	552.6343	0.0005	0.0391	-0.0010	0.0391	0.0391	0.0071
0.8027	602.3573	0.0005	0.0391	-0.0011	0.0391	0.0391	0.0065
0.8034	651.5623	0.0005	0.0391	-0.0013	0.0391	0.0391	0.0060
0.8005	753.4148	0.0004	0.0391	-0.0016	0.0391	0.0391	0.0052
0.8012	913.8563	0.0005	0.0391	-0.0020	0.0391	0.0391	0.0043
0.8037	1002.5757	0.0005	0.0391	-0.0023	0.0391	0.0392	0.0039
0.8045	1051.5807	0.0005	0.0391	-0.0024	0.0392	0.0392	0.0037
0.8098	1102.1417	0.0004	0.0391	-0.0025	0.0392	0.0392	0.0036
0.8032	1154.0417	0.0005	0.0391	-0.0027	0.0392	0.0392	0.0034
0.8015	1207.3702	0.0005	0.0391	-0.0028	0.0392	0.0392	0.0032
0.8022	1246.6728	0.0005	0.0391	-0.0029	0.0392	0.0392	0.0031

V (m/s)	n (rpm)	P_n	B_C δn/δC	B_{dt} δn/δdt	B_n	U_n	% Unc.
1.1030	507.2079	0.0006	0.0391	-0.0009	0.0391	0.0391	0.0077
1.1027	554.5345	0.0006	0.0391	-0.0010	0.0391	0.0391	0.0071
1.1035	603.8746	0.0006	0.0391	-0.0011	0.0391	0.0391	0.0065
1.1032	654.2447	0.0006	0.0391	-0.0013	0.0391	0.0391	0.0060
1.1044	750.3733	0.0006	0.0391	-0.0016	0.0391	0.0391	0.0052
1.1041	910.3564	0.0006	0.0391	-0.0020	0.0391	0.0391	0.0043
1.1030	1006.6678	0.0007	0.0391	-0.0023	0.0391	0.0392	0.0039
1.1037	1056.9924	0.0006	0.0391	-0.0024	0.0392	0.0392	0.0037
1.1031	1108.8989	0.0006	0.0391	-0.0026	0.0392	0.0392	0.0035
1.1031	1156.0328	0.0006	0.0391	-0.0027	0.0392	0.0392	0.0034
1.1029	1206.4133	0.0006	0.0391	-0.0028	0.0392	0.0392	0.0032
1.1034	1249.3622	0.0006	0.0391	-0.0029	0.0392	0.0392	0.0031

Uncertainty Analysis (Rotational Speed)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	n (rpm)	P_n	B_C $\delta n/\delta C$	B_{dt} $\delta n/\delta dt$	B_n	U_n	% Unc.
1.3031	504.2337	0.0006	0.0391	-0.0009	0.0391	0.0391	0.0078
1.3022	550.4535	0.0006	0.0391	-0.0010	0.0391	0.0391	0.0071
1.3025	600.5634	0.0007	0.0391	-0.0011	0.0391	0.0391	0.0065
1.3035	652.2352	0.0007	0.0391	-0.0013	0.0391	0.0391	0.0060
1.3036	752.8192	0.0007	0.0391	-0.0016	0.0391	0.0391	0.0052
1.3022	901.8296	0.0007	0.0391	-0.0020	0.0391	0.0391	0.0043
1.3022	1000.1110	0.0007	0.0391	-0.0023	0.0391	0.0392	0.0039
1.3023	1046.2973	0.0007	0.0391	-0.0024	0.0392	0.0392	0.0037
1.3020	1105.7242	0.0007	0.0391	-0.0025	0.0392	0.0392	0.0035
1.3023	1155.3342	0.0007	0.0391	-0.0027	0.0392	0.0392	0.0034
1.3022	1206.0095	0.0007	0.0391	-0.0028	0.0392	0.0392	0.0032
1.3024	1248.0299	0.0007	0.0391	-0.0029	0.0392	0.0392	0.0031

V (m/s)	n (rpm)	P_n	B_C $\delta n/\delta C$	B_{dt} $\delta n/\delta dt$	B_n	U_n	% Unc.
1.6032	503.9995	0.0008	0.0391	-0.0009	0.0391	0.0391	0.0078
1.6046	550.3443	0.0008	0.0391	-0.0010	0.0391	0.0391	0.0071
1.6035	601.4574	0.0008	0.0391	-0.0011	0.0391	0.0391	0.0065
1.6038	653.7645	0.0009	0.0391	-0.0013	0.0391	0.0391	0.0060
1.6040	755.9224	0.0008	0.0391	-0.0016	0.0391	0.0391	0.0052
1.6038	911.6408	0.0008	0.0391	-0.0020	0.0391	0.0391	0.0043
1.6044	1003.7944	0.0008	0.0391	-0.0023	0.0391	0.0392	0.0039
1.6042	1051.3377	0.0008	0.0391	-0.0024	0.0392	0.0392	0.0037
1.6043	1104.0239	0.0008	0.0391	-0.0025	0.0392	0.0392	0.0035
1.6034	1152.5333	0.0008	0.0391	-0.0027	0.0392	0.0392	0.0034
1.6036	1210.1129	0.0008	0.0391	-0.0028	0.0392	0.0392	0.0032
1.6030	1250.8055	0.0009	0.0391	-0.0030	0.0392	0.0392	0.0031

Uncertainty Analysis (Rotational Speed)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	n (rpm)	P_n	B_C $\delta n/\delta C$	B_{dt} $\delta n/\delta dt$	B_n	U_n	% Unc.
1.8042	510.9808	0.0010	0.0391	-0.0009	0.0391	0.0391	0.0077
1.8034	552.4233	0.0010	0.0391	-0.0010	0.0391	0.0391	0.0071
1.8068	602.7853	0.0009	0.0391	-0.0011	0.0391	0.0391	0.0065
1.8056	651.5334	0.0009	0.0391	-0.0013	0.0391	0.0391	0.0060
1.8042	757.8054	0.0010	0.0391	-0.0016	0.0391	0.0391	0.0052
1.8051	911.1136	0.0010	0.0391	-0.0020	0.0391	0.0391	0.0043
1.8048	1002.7594	0.0010	0.0391	-0.0023	0.0391	0.0392	0.0039
1.8061	1056.9214	0.0010	0.0391	-0.0024	0.0392	0.0392	0.0037
1.8051	1118.8065	0.0010	0.0391	-0.0026	0.0392	0.0392	0.0035
1.8043	1161.1239	0.0010	0.0391	-0.0027	0.0392	0.0392	0.0034
1.8059	1215.6599	0.0010	0.0391	-0.0029	0.0392	0.0392	0.0032
1.8057	1246.0544	0.0010	0.0391	-0.0029	0.0392	0.0392	0.0031

V (m/s)	n (rpm)	P_n	B_C $\delta n/\delta C$	B_{dt} $\delta n/\delta dt$	B_n	U_n	% Unc.
2.1091	500.1371	0.0011	0.0391	-0.0009	0.0391	0.0391	0.0078
2.1056	551.5545	0.0012	0.0391	-0.0010	0.0391	0.0391	0.0071
2.1068	600.3476	0.0011	0.0391	-0.0011	0.0391	0.0391	0.0065
2.1067	654.3489	0.0011	0.0391	-0.0013	0.0391	0.0391	0.0060
2.1101	752.6505	0.0011	0.0391	-0.0016	0.0391	0.0391	0.0052
2.1068	901.9876	0.0012	0.0391	-0.0020	0.0391	0.0392	0.0043
2.1070	1002.6789	0.0012	0.0391	-0.0023	0.0391	0.0392	0.0039
2.0959	1052.8833	0.0011	0.0391	-0.0024	0.0392	0.0392	0.0037
2.1064	1103.4297	0.0011	0.0391	-0.0025	0.0392	0.0392	0.0036
2.1073	1156.4853	0.0011	0.0391	-0.0027	0.0392	0.0392	0.0034
2.1066	1214.1716	0.0011	0.0391	-0.0028	0.0392	0.0392	0.0032
2.1060	1246.6579	0.0011	0.0391	-0.0029	0.0392	0.0392	0.0031

Uncertainty Analysis (Rotational Speed)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	n (rpm)	P_n	B_C $\delta n/\delta C$	B_{dt} $\delta n/\delta dt$	B_n	U_n	% Unc.
2.3079	502.2690	0.0013	0.0391	-0.0009	0.0391	0.0391	0.0078
2.3069	550.3896	0.0012	0.0391	-0.0010	0.0391	0.0391	0.0071
2.3074	603.5423	0.0013	0.0391	-0.0011	0.0391	0.0391	0.0065
2.3070	652.5464	0.0013	0.0391	-0.0013	0.0391	0.0391	0.0060
2.3084	752.0456	0.0013	0.0391	-0.0016	0.0391	0.0391	0.0052
2.3081	901.6673	0.0013	0.0391	-0.0020	0.0391	0.0392	0.0043
2.3082	1006.0527	0.0013	0.0391	-0.0023	0.0391	0.0392	0.0039
2.3077	1052.1662	0.0013	0.0391	-0.0024	0.0392	0.0392	0.0037
2.3074	1101.3796	0.0013	0.0391	-0.0025	0.0392	0.0392	0.0036
2.3075	1150.4349	0.0013	0.0391	-0.0027	0.0392	0.0392	0.0034
2.3077	1213.5677	0.0013	0.0391	-0.0028	0.0392	0.0392	0.0032
2.3068	1249.1721	0.0013	0.0391	-0.0029	0.0392	0.0392	0.0031

V (m/s)	n (rpm)	P_n	B_C $\delta n/\delta C$	B_{dt} $\delta n/\delta dt$	B_n	U_n	% Unc.
2.5100	504.0554	0.0015	0.0391	-0.0009	0.0391	0.0391	0.0078
2.5086	552.5634	0.0014	0.0391	-0.0010	0.0391	0.0391	0.0071
2.5072	601.0760	0.0015	0.0391	-0.0011	0.0391	0.0391	0.0065
2.5095	651.4524	0.0015	0.0391	-0.0013	0.0391	0.0391	0.0060
2.5020	751.7646	0.0015	0.0391	-0.0016	0.0391	0.0391	0.0052
2.5092	905.0740	0.0014	0.0391	-0.0020	0.0391	0.0392	0.0043
2.5081	1004.8353	0.0015	0.0391	-0.0023	0.0391	0.0392	0.0039
2.5087	1052.4015	0.0015	0.0391	-0.0024	0.0392	0.0392	0.0037
2.5087	1103.3557	0.0014	0.0391	-0.0025	0.0392	0.0392	0.0036
2.5023	1154.7567	0.0015	0.0391	-0.0027	0.0392	0.0392	0.0034
2.5092	1207.1602	0.0015	0.0391	-0.0028	0.0392	0.0392	0.0032
2.5091	1252.5791	0.0015	0.0391	-0.0030	0.0392	0.0392	0.0031

Load Varying Self-Propulsion Test (Propeller fitted on)

Uncertainty Analysis (Velocity)

V	Vehicle Velocity (m/s)	
D	Diameter of Carriage Wheel (m)	
C	Number of Counted Pulses in Time dt	
dt	Time (s)	
S.D	Standard Deviation	
P_V	Precision Error for V	$P_V = 2 \times S.D$
B_C	Bias Error for C	
B_D	Bias Error for D	
B_{dt}	Bias Error for dt	
B_V	Bias Error for V	$B_V = ((\delta V / \delta C\ B_C)^2 + (\delta V / \delta D\ B_D)^2 + (\delta V / \delta dt\ B_{dt})^2)^{1/2}$
U_V	Total Error for V	$U_V = (B_V^2 + P_V^2)^{1/2}$
% Unc.	Percentage Uncertainty for V	% Unc. = $(U_V / V) \times 100$

Uncertainty Analysis (Velocity)
Load Varying Self - Propulsion Test (Propeller fitted on)

V (m/s)	P_V	$B_C\ \delta V/\delta C$	$B_D\ \delta V/\delta D$	$B_{dt}\ \delta V/\delta dt$	B_V	U_V	% Unc
0.8007	0.0003	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4413
0.8012	0.0004	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4418
0.8027	0.0003	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4400
0.8034	0.0003	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4394
0.8005	0.0004	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4419
0.8012	0.0004	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4418
0.8037	0.0003	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4397
0.8045	0.0004	0.0035	0.0002	-0.00005	0.0035	0.0035	0.4397
0.8098	0.0002	0.0035	0.0002	-0.00005	0.0035	0.0035	0.4356
0.8032	0.0003	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4402
0.8015	0.0003	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4405
0.8022	0.0004	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4412

V (m/s)	P_V	$B_C\ \delta V/\delta C$	$B_D\ \delta V/\delta D$	$B_{dt}\ \delta V/\delta dt$	B_V	U_V	% Unc
1.1030	0.0004	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3223
1.1027	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3231
1.1035	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3231
1.1032	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3229
1.1044	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3221
1.1041	0.0004	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3219
1.1030	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3227
1.1037	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3228
1.1031	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3235
1.1031	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3227
1.1029	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3230
1.1034	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3231

Uncertainty Analysis (Velocity)
Load Varying Self - Propulsion Test (Propeller fitted on)

V (m/s)	P_V	$B_C\ \delta V/\delta C$	$B_D\ \delta V/\delta D$	$B_{dt}\ \delta V/\delta dt$	B_V	U_V	% Unc
1.3031	0.0005	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2744
1.3022	0.0006	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2751
1.3025	0.0006	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2748
1.3035	0.0006	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2751
1.3036	0.0005	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2743
1.3022	0.0005	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2744
1.3022	0.0006	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2751
1.3023	0.0006	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2748
1.3020	0.0005	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2747
1.3023	0.0005	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2744
1.3022	0.0005	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2746
1.3024	0.0005	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2746

V (m/s)	P_V	$B_C\ \delta V/\delta C$	$B_D\ \delta V/\delta D$	$B_{dt}\ \delta V/\delta dt$	B_V	U_V	% Unc
1.6032	0.0006	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2251
1.6046	0.0007	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2256
1.6035	0.0007	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2260
1.6038	0.0007	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2254
1.6040	0.0007	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2252
1.6038	0.0006	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2250
1.6044	0.0007	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2261
1.6042	0.0007	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2254
1.6043	0.0006	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2249
1.6034	0.0007	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2260
1.6036	0.0007	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2257
1.6030	0.0007	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2255

Uncertainty Analysis (Velocity)

Load Varying Self - Propulsion Test (Propeller fitted on)

V (m/s)	P_V	$B_C\ \delta V/\delta C$	$B_D\ \delta V/\delta D$	$B_{dt}\ \delta V/\delta dt$	B_V	U_V	% Unc
1.8042	0.0008	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2030
1.8034	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2041
1.8068	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2032
1.8056	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2031
1.8042	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2037
1.8051	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2039
1.8048	0.0008	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2029
1.8061	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2030
1.8051	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2034
1.8043	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2040
1.8059	0.0008	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2028
1.8057	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2031

V (m/s)	P_V	$B_C\ \delta V/\delta C$	$B_D\ \delta V/\delta D$	$B_{dt}\ \delta V/\delta dt$	B_V	U_V	% Unc
2.1091	0.0010	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1766
2.1056	0.0010	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1764
2.1068	0.0010	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1773
2.1067	0.0010	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1771
2.1101	0.0010	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1765
2.1068	0.0011	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1776
2.1070	0.0010	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1765
2.0959	0.0010	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1777
2.1064	0.0010	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1773
2.1073	0.0010	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1767
2.1066	0.0010	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1771
2.1060	0.0011	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1776

Uncertainty Analysis (Velocity)
Load Varying Self - Propulsion Test (Propeller fitted on)

V (m/s)	P_V	$B_C\ \delta V/\delta C$	$B_D\ \delta V/\delta D$	$B_{dt}\ \delta V/\delta dt$	B_V	U_V	% Unc
2.3079	0.0011	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1636
2.3069	0.0012	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1642
2.3074	0.0012	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1641
2.3070	0.0012	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1646
2.3084	0.0011	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1637
2.3081	0.0011	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1632
2.3082	0.0011	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1635
2.3077	0.0012	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1643
2.3074	0.0011	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1630
2.3075	0.0011	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1635
2.3077	0.0011	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1638
2.3068	0.0012	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1644

V (m/s)	P_V	$B_C\ \delta V/\delta C$	$B_D\ \delta V/\delta D$	$B_{dt}\ \delta V/\delta dt$	B_V	U_V	% Unc
2.5100	0.0013	0.0035	0.0008	-0.00029	0.0036	0.0038	0.1532
2.5086	0.0012	0.0035	0.0008	-0.00029	0.0036	0.0038	0.1529
2.5072	0.0013	0.0035	0.0008	-0.00029	0.0036	0.0039	0.1543
2.5095	0.0013	0.0035	0.0008	-0.00029	0.0036	0.0038	0.1533
2.5020	0.0013	0.0035	0.0008	-0.00028	0.0036	0.0039	0.1543
2.5092	0.0013	0.0035	0.0008	-0.00029	0.0036	0.0038	0.1531
2.5081	0.0013	0.0035	0.0008	-0.00029	0.0036	0.0038	0.1533
2.5087	0.0013	0.0035	0.0008	-0.00029	0.0036	0.0039	0.1537
2.5087	0.0013	0.0035	0.0008	-0.00029	0.0036	0.0038	0.1531
2.5023	0.0013	0.0035	0.0008	-0.00028	0.0036	0.0039	0.1546
2.5092	0.0013	0.0035	0.0008	-0.00029	0.0036	0.0039	0.1542
2.5091	0.0013	0.0035	0.0008	-0.00029	0.0036	0.0039	0.1536

Load Varying Self-Propulsion Test (Propeller fitted on)

Uncertainty Analysis (Torque)

V	Vehicle Velocity (m/s)	
Q	Propeller Torque (N-m)	
F	Calibrating Weight (N)	
R	Lever Arm (m)	
P_Q	Precision Error for Q	$P_Q = 2 \times S.D$
S.D	Standard Deviation	
S.E.E	Standard Error Estimate	
AD	Analog to Digital Converter	
B_F	Bias Error for F	
B_R	Bias Error for R	
B_1	Bias Error due to Calibration	$B_1 = ((\delta Q / \delta F\ B_F)^2 + (\delta Q / \delta R\ B_R)^2)^{1/2}$
B_2	Bias Error due to Data Acquisition	$B_2 = 2 \times S.E.E$
B_3	Bias Error due to Data Reduction	$B_3 = (AD_{error} \times AD_{range}) / (AD_{accuracy})$
B_Q	Bias Error for Q	$B_Q = (B_1^2 + B_2^2 + B_3^2)^{1/2}$
U_Q	Total Error for Q	$U_Q = (B_Q^2 + P_Q^2)^{1/2}$
% Unc.	Percentage Uncertainty for Q	% Unc. = $(U_Q / Q) \times 100$

Uncertainty Analysis (Torque)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	Q (N-m)	P_Q	$B_F\ \delta Q/\delta F$	$B_R\ \delta Q/\delta R$	B_1	B_2	B_3	B_Q	U_Q	% Unc.
0.8007	0.1810	0.0007	0.00001	0.00014	0.0001	0.0076	0.0007	0.0076	0.0077	4.2361
0.8012	0.3680	0.0007	0.00002	0.00029	0.0003	0.0076	0.0007	0.0076	0.0077	2.0855
0.8027	0.5143	0.0007	0.00003	0.00041	0.0004	0.0076	0.0007	0.0076	0.0077	1.4930
0.8034	0.7198	0.0007	0.00004	0.00058	0.0006	0.0076	0.0007	0.0077	0.0077	1.0687
0.8005	1.2080	0.0008	0.00006	0.00097	0.0010	0.0076	0.0007	0.0077	0.0077	0.6403
0.8012	2.0427	0.0008	0.00010	0.00163	0.0016	0.0076	0.0007	0.0078	0.0078	0.3841
0.8037	2.6202	0.0008	0.00013	0.00210	0.0021	0.0076	0.0007	0.0079	0.0080	0.3037
0.8045	2.9224	0.0008	0.00015	0.00234	0.0023	0.0076	0.0007	0.0080	0.0080	0.2746
0.8098	3.3564	0.0008	0.00017	0.00269	0.0027	0.0076	0.0007	0.0081	0.0081	0.2424
0.8032	3.7562	0.0008	0.00019	0.00300	0.0030	0.0076	0.0007	0.0082	0.0082	0.2196
0.8015	4.1532	0.0008	0.00021	0.00332	0.0033	0.0076	0.0007	0.0083	0.0084	0.2015
0.8022	4.5121	0.0008	0.00023	0.00361	0.0036	0.0076	0.0007	0.0084	0.0085	0.1882

V (m/s)	Q (N-m)	P_Q	$B_F\ \delta Q/\delta F$	$B_R\ \delta Q/\delta R$	B_1	B_2	B_3	B_Q	U_Q	% Unc.
1.1030	0.1699	0.0008	0.00001	0.00014	0.0001	0.0076	0.0007	0.0076	0.0077	4.5211
1.1027	0.3535	0.0008	0.00002	0.00028	0.0003	0.0076	0.0007	0.0076	0.0077	2.1736
1.1035	0.4982	0.0008	0.00002	0.00040	0.0004	0.0076	0.0007	0.0076	0.0077	1.5437
1.1032	0.7005	0.0009	0.00004	0.00056	0.0006	0.0076	0.0007	0.0077	0.0077	1.0996
1.1044	1.1746	0.0009	0.00006	0.00094	0.0009	0.0076	0.0007	0.0077	0.0077	0.6589
1.1041	1.9868	0.0009	0.00010	0.00159	0.0016	0.0076	0.0007	0.0078	0.0078	0.3950
1.1030	2.5799	0.0009	0.00013	0.00206	0.0021	0.0076	0.0007	0.0079	0.0080	0.3085
1.1037	2.8694	0.0009	0.00014	0.00230	0.0023	0.0076	0.0007	0.0080	0.0080	0.2796
1.1031	3.2875	0.0009	0.00016	0.00263	0.0026	0.0076	0.0007	0.0081	0.0081	0.2472
1.1031	3.6710	0.0009	0.00018	0.00294	0.0029	0.0076	0.0007	0.0082	0.0082	0.2243
1.1029	4.0899	0.0009	0.00020	0.00327	0.0033	0.0076	0.0007	0.0083	0.0084	0.2044
1.1034	4.4395	0.0009	0.00022	0.00355	0.0036	0.0076	0.0007	0.0084	0.0085	0.1909

Uncertainty Analysis (Torque)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	Q (N-m)	P_Q	$B_F\ \delta Q/\delta F$	$B_R\ \delta Q/\delta R$	B_1	B_2	B_3	B_Q	U_Q	% Unc.
1.3031	0.1657	0.0009	0.00001	0.00013	0.0001	0.0076	0.0007	0.0076	0.0077	4.6400
1.3022	0.3278	0.0009	0.00002	0.00026	0.0003	0.0076	0.0007	0.0076	0.0077	2.3471
1.3025	0.4670	0.0009	0.00002	0.00037	0.0004	0.0076	0.0007	0.0076	0.0077	1.6484
1.3035	0.6650	0.0009	0.00003	0.00053	0.0005	0.0076	0.0007	0.0077	0.0077	1.1596
1.3036	1.1361	0.0010	0.00006	0.00091	0.0009	0.0076	0.0007	0.0077	0.0077	0.6820
1.3022	1.9541	0.0010	0.00010	0.00156	0.0016	0.0076	0.0007	0.0078	0.0079	0.4019
1.3022	2.5466	0.0010	0.00013	0.00204	0.0020	0.0076	0.0007	0.0079	0.0080	0.3126
1.3023	2.8254	0.0010	0.00014	0.00226	0.0023	0.0076	0.0007	0.0080	0.0080	0.2840
1.3020	3.2400	0.0010	0.00016	0.00259	0.0026	0.0076	0.0007	0.0081	0.0081	0.2508
1.3023	3.6311	0.0010	0.00018	0.00290	0.0029	0.0076	0.0007	0.0082	0.0082	0.2266
1.3022	4.0298	0.0011	0.00020	0.00322	0.0032	0.0076	0.0007	0.0083	0.0084	0.2075
1.3024	4.4059	0.0011	0.00022	0.00352	0.0035	0.0076	0.0007	0.0084	0.0085	0.1926

V (m/s)	Q (N-m)	P_Q	$B_F\ \delta Q/\delta F$	$B_R\ \delta Q/\delta R$	B_1	B_2	B_3	B_Q	U_Q	% Unc.
1.6032	0.1484	0.0011	0.00001	0.00012	0.0001	0.0076	0.0007	0.0076	0.0077	5.1970
1.6046	0.2710	0.0011	0.00001	0.00022	0.0002	0.0076	0.0007	0.0076	0.0077	2.8459
1.6035	0.3833	0.0011	0.00002	0.00031	0.0003	0.0076	0.0007	0.0076	0.0077	2.0151
1.6038	0.6033	0.0011	0.00003	0.00048	0.0005	0.0076	0.0007	0.0077	0.0077	1.2822
1.6040	1.0703	0.0012	0.00005	0.00086	0.0009	0.0076	0.0007	0.0077	0.0078	0.7260
1.6038	1.8340	0.0012	0.00009	0.00147	0.0015	0.0076	0.0007	0.0078	0.0079	0.4286
1.6044	2.4389	0.0012	0.00012	0.00195	0.0020	0.0076	0.0007	0.0079	0.0080	0.3268
1.6042	2.7228	0.0012	0.00014	0.00218	0.0022	0.0076	0.0007	0.0079	0.0080	0.2948
1.6043	3.1449	0.0012	0.00016	0.00252	0.0025	0.0076	0.0007	0.0080	0.0081	0.2584
1.6034	3.5384	0.0012	0.00018	0.00283	0.0028	0.0076	0.0007	0.0081	0.0082	0.2326
1.6036	3.9176	0.0012	0.00020	0.00313	0.0031	0.0076	0.0007	0.0083	0.0083	0.2129
1.6030	4.3454	0.0012	0.00022	0.00348	0.0035	0.0076	0.0007	0.0084	0.0085	0.1951

Uncertainty Analysis (Torque)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	Q (N-m)	P_Q	$B_F\ \delta Q/\delta F$	$B_R\ \delta Q/\delta R$	B_1	B_2	B_3	B_Q	U_Q	% Unc.
1.8042	0.1265	0.0012	0.00001	0.00010	0.0001	0.0076	0.0007	0.0076	0.0077	6.1087
1.8034	0.1528	0.0012	0.00001	0.00012	0.0001	0.0076	0.0007	0.0076	0.0077	5.0619
1.8068	0.3126	0.0012	0.00002	0.00025	0.0003	0.0076	0.0007	0.0076	0.0077	2.4751
1.8056	0.5023	0.0012	0.00003	0.00040	0.0004	0.0076	0.0007	0.0076	0.0077	1.5421
1.8042	0.9753	0.0012	0.00005	0.00078	0.0008	0.0076	0.0007	0.0077	0.0078	0.7970
1.8051	1.7271	0.0013	0.00009	0.00138	0.0014	0.0076	0.0007	0.0078	0.0079	0.4551
1.8048	2.3395	0.0013	0.00012	0.00187	0.0019	0.0076	0.0007	0.0079	0.0080	0.3404
1.8061	2.6019	0.0013	0.00013	0.00208	0.0021	0.0076	0.0007	0.0079	0.0080	0.3080
1.8051	3.0425	0.0013	0.00015	0.00243	0.0024	0.0076	0.0007	0.0080	0.0081	0.2667
1.8043	3.4240	0.0013	0.00017	0.00274	0.0027	0.0076	0.0007	0.0081	0.0082	0.2399
1.8059	3.7826	0.0013	0.00019	0.00303	0.0030	0.0076	0.0007	0.0082	0.0083	0.2199
1.8057	4.1864	0.0013	0.00021	0.00335	0.0034	0.0076	0.0007	0.0083	0.0084	0.2017

V (m/s)	Q (N-m)	P_Q	$B_F\ \delta Q/\delta F$	$B_R\ \delta Q/\delta R$	B_1	B_2	B_3	B_Q	U_Q	% Unc.
2.1091	0.0592	0.0006	0.00000	0.00005	0.0000	0.0076	0.0007	0.0076	0.0077	12.9350
2.1056	0.1283	0.0013	0.00001	0.00010	0.0001	0.0076	0.0007	0.0076	0.0077	6.0373
2.1068	0.2676	0.0013	0.00001	0.00021	0.0002	0.0076	0.0007	0.0076	0.0078	2.8976
2.1067	0.4750	0.0013	0.00002	0.00038	0.0004	0.0076	0.0007	0.0076	0.0078	1.6340
2.1101	0.8776	0.0014	0.00004	0.00070	0.0007	0.0076	0.0007	0.0077	0.0078	0.8874
2.1068	1.6341	0.0014	0.00008	0.00131	0.0013	0.0076	0.0007	0.0077	0.0079	0.4814
2.1070	2.2376	0.0014	0.00011	0.00179	0.0018	0.0076	0.0007	0.0078	0.0080	0.3559
2.0959	2.5437	0.0014	0.00013	0.00203	0.0020	0.0076	0.0007	0.0079	0.0080	0.3154
2.1064	2.9378	0.0014	0.00015	0.00235	0.0024	0.0076	0.0007	0.0080	0.0081	0.2762
2.1073	3.3080	0.0014	0.00017	0.00265	0.0027	0.0076	0.0007	0.0081	0.0082	0.2481
2.1066	3.6923	0.0015	0.00018	0.00295	0.0030	0.0076	0.0007	0.0082	0.0083	0.2252
2.1060	4.0446	0.0015	0.00020	0.00324	0.0032	0.0076	0.0007	0.0083	0.0084	0.2083

Uncertainty Analysis (Torque)

Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	Q (N-m)	P_Q	B_F $\delta Q/\delta F$	B_R $\delta Q/\delta R$	B_1	B_2	B_3	B_Q	U_Q	% Unc.
2.3079	0.0030	0.0002	0.00000	0.00000	0.0000	0.0076	0.0007	0.0076	0.0076	253.9565
2.3069	0.1071	0.0015	0.00001	0.00009	0.0001	0.0076	0.0007	0.0076	0.0078	7.2595
2.3074	0.2439	0.0015	0.00001	0.00020	0.0002	0.0076	0.0007	0.0076	0.0078	3.1928
2.3070	0.4126	0.0015	0.00002	0.00033	0.0003	0.0076	0.0007	0.0076	0.0078	1.8880
2.3084	0.7863	0.0015	0.00004	0.00063	0.0006	0.0076	0.0007	0.0077	0.0078	0.9938
2.3081	1.5450	0.0016	0.00008	0.00124	0.0012	0.0076	0.0007	0.0077	0.0079	0.5106
2.3082	2.1446	0.0016	0.00011	0.00172	0.0017	0.0076	0.0007	0.0078	0.0080	0.3722
2.3077	2.4494	0.0016	0.00012	0.00196	0.0020	0.0076	0.0007	0.0079	0.0080	0.3283
2.3074	2.7844	0.0016	0.00014	0.00223	0.0022	0.0076	0.0007	0.0080	0.0081	0.2916
2.3075	3.1913	0.0017	0.00016	0.00255	0.0026	0.0076	0.0007	0.0081	0.0082	0.2576
2.3077	3.5556	0.0017	0.00018	0.00284	0.0029	0.0076	0.0007	0.0081	0.0083	0.2339
2.3068	3.9343	0.0017	0.00020	0.00315	0.0032	0.0076	0.0007	0.0083	0.0084	0.2142

V (m/s)	Q (N-m)	P_Q	B_F $\delta Q/\delta F$	B_R $\delta Q/\delta R$	B_1	B_2	B_3	B_Q	U_Q	% Unc.
2.5100	0.0003	0.0001	0.00000	0.00000	0.0000	0.0076	0.0007	0.0076	0.0076	2231.3629
2.5086	0.0817	0.0014	0.00000	0.00007	0.0001	0.0076	0.0007	0.0076	0.0078	9.5018
2.5072	0.2188	0.0017	0.00001	0.00018	0.0002	0.0076	0.0007	0.0076	0.0078	3.5786
2.5095	0.3268	0.0017	0.00002	0.00026	0.0003	0.0076	0.0007	0.0076	0.0078	2.3976
2.5020	0.7249	0.0018	0.00004	0.00058	0.0006	0.0076	0.0007	0.0077	0.0079	1.0839
2.5092	1.4758	0.0018	0.00007	0.00118	0.0012	0.0076	0.0007	0.0077	0.0079	0.5372
2.5081	2.0551	0.0018	0.00010	0.00164	0.0016	0.0076	0.0007	0.0078	0.0080	0.3897
2.5087	2.3871	0.0018	0.00012	0.00191	0.0019	0.0076	0.0007	0.0079	0.0081	0.3379
2.5087	2.6963	0.0018	0.00013	0.00216	0.0022	0.0076	0.0007	0.0079	0.0081	0.3017
2.5023	3.0789	0.0018	0.00015	0.00246	0.0025	0.0076	0.0007	0.0080	0.0082	0.2672
2.5092	3.4660	0.0018	0.00017	0.00277	0.0028	0.0076	0.0007	0.0081	0.0083	0.2404
2.5091	3.8650	0.0019	0.00019	0.00309	0.0031	0.0076	0.0007	0.0082	0.0084	0.2185

Load Varying Self-Propulsion Test (Propeller fitted on)

Uncertainty Analysis (Thrust)

V	Vehicle Velocity (m/s)	
T_P	Propeller Thrust (N)	
P_T	Precision Error for T_P	$P_T = 2 \times S.D$
S.D	Standard Deviation	
S.E.E	Standard Error Estimate	
AD	Analog to Digital Converter	
B_1	Bias Error due to Calibration	$B_1 = 0.00005 \times T_P$
B_2	Bias Error due to Data Acquisition	$B_2 = 2 \times S.E.E$
B_3	Bias Error due to Data Reduction	$B_3 = (AD_{error} \times AD_{range}) / (AD_{accuracy})$
B_T	Bias Error for T_P	$B_T = (B_1^2 + B_2^2 + B_3^2)^{1/2}$
U_T	Total Error for T_P	$U_T = (B_T^2 + P_T^2)^{1/2}$
% Unc.	Percentage Uncertainty for T_P	% Unc. = $(U_T / T_P) \times 100$

Uncertainty Analysis (Thrust)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	T_P (N)	P_T	B_1	B_2	B_3	B_T	U_T	% Unc.
0.8007	3.6262	0.0085	0.0002	0.2487	0.0182	0.2494	0.2495	6.8803
0.8012	8.7409	0.0087	0.0004	0.2487	0.0182	0.2494	0.2495	2.8544
0.8027	12.8134	0.0088	0.0006	0.2487	0.0182	0.2494	0.2495	1.9472
0.8034	18.6583	0.0086	0.0009	0.2487	0.0182	0.2494	0.2495	1.3372
0.8005	32.8282	0.0089	0.0016	0.2487	0.0182	0.2494	0.2495	0.7601
0.8012	59.3485	0.0089	0.0030	0.2487	0.0182	0.2494	0.2495	0.4204
0.8037	77.0108	0.0092	0.0039	0.2487	0.0182	0.2494	0.2495	0.3240
0.8045	87.1072	0.0095	0.0044	0.2487	0.0182	0.2494	0.2496	0.2865
0.8098	98.8701	0.0094	0.0049	0.2487	0.0182	0.2494	0.2496	0.2524
0.8032	111.2684	0.0096	0.0056	0.2487	0.0182	0.2494	0.2496	0.2243
0.8015	123.8401	0.0094	0.0062	0.2487	0.0182	0.2494	0.2496	0.2016
0.8022	135.8194	0.0097	0.0068	0.2487	0.0182	0.2494	0.2496	0.1838

V (m/s)	T_P (N)	P_T	B_1	B_2	B_3	B_T	U_T	% Unc.
1.1030	2.2849	0.0095	0.0001	0.2487	0.0182	0.2494	0.2495	10.9209
1.1027	6.2834	0.0098	0.0003	0.2487	0.0182	0.2494	0.2495	3.9714
1.1035	10.2080	0.0100	0.0005	0.2487	0.0182	0.2494	0.2496	2.4447
1.1032	16.3454	0.0102	0.0008	0.2487	0.0182	0.2494	0.2496	1.5268
1.1044	29.4461	0.0103	0.0015	0.2487	0.0182	0.2494	0.2496	0.8475
1.1041	54.5330	0.0106	0.0027	0.2487	0.0182	0.2494	0.2496	0.4577
1.1030	73.9689	0.0108	0.0037	0.2487	0.0182	0.2494	0.2496	0.3375
1.1037	85.6126	0.0109	0.0043	0.2487	0.0182	0.2494	0.2496	0.2916
1.1031	98.4729	0.0111	0.0049	0.2487	0.0182	0.2494	0.2496	0.2535
1.1031	110.2023	0.0109	0.0055	0.2487	0.0182	0.2494	0.2496	0.2265
1.1029	121.4830	0.0111	0.0061	0.2487	0.0182	0.2494	0.2497	0.2055
1.1034	132.4165	0.0113	0.0066	0.2487	0.0182	0.2494	0.2497	0.1886

Uncertainty Analysis (Thrust)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	T_P (N)	P_T	B_1	B_2	B_3	B_T	U_T	% Unc.
1.3031	-0.5806	0.0090	0.0000	0.2487	0.0182	0.2494	0.2495	-42.9741
1.3022	3.8393	0.0117	0.0002	0.2487	0.0182	0.2494	0.2496	6.5018
1.3025	7.2149	0.0118	0.0004	0.2487	0.0182	0.2494	0.2496	3.4599
1.3035	13.4575	0.0117	0.0007	0.2487	0.0182	0.2494	0.2496	1.8549
1.3036	26.1866	0.0119	0.0013	0.2487	0.0182	0.2494	0.2496	0.9533
1.3022	51.7382	0.0123	0.0026	0.2487	0.0182	0.2494	0.2497	0.4826
1.3022	72.3597	0.0121	0.0036	0.2487	0.0182	0.2494	0.2497	0.3450
1.3023	82.1375	0.0125	0.0041	0.2487	0.0182	0.2494	0.2497	0.3040
1.3020	96.1106	0.0128	0.0048	0.2487	0.0182	0.2494	0.2497	0.2598
1.3023	107.4396	0.0127	0.0054	0.2487	0.0182	0.2494	0.2497	0.2324
1.3022	119.2426	0.0129	0.0060	0.2487	0.0182	0.2494	0.2498	0.2095
1.3024	130.6479	0.0131	0.0065	0.2487	0.0182	0.2494	0.2498	0.1912

V (m/s)	T_P (N)	P_T	B_1	B_2	B_3	B_T	U_T	% Unc.
1.6032	-1.5261	0.0130	-0.0001	0.2487	0.0182	0.2494	0.2497	-16.3610
1.6046	1.5354	0.0137	0.0001	0.2487	0.0182	0.2494	0.2497	16.2641
1.6035	5.3738	0.0143	0.0003	0.2487	0.0182	0.2494	0.2498	4.6478
1.6038	11.6456	0.0145	0.0006	0.2487	0.0182	0.2494	0.2498	2.1448
1.6040	25.1105	0.0143	0.0013	0.2487	0.0182	0.2494	0.2498	0.9947
1.6038	48.1265	0.0147	0.0024	0.2487	0.0182	0.2494	0.2498	0.5190
1.6044	67.4672	0.0148	0.0034	0.2487	0.0182	0.2494	0.2498	0.3703
1.6042	77.7648	0.0150	0.0039	0.2487	0.0182	0.2494	0.2498	0.3213
1.6043	91.4829	0.0152	0.0046	0.2487	0.0182	0.2494	0.2499	0.2731
1.6034	104.5737	0.0155	0.0052	0.2487	0.0182	0.2494	0.2499	0.2390
1.6036	116.7686	0.0154	0.0058	0.2487	0.0182	0.2494	0.2499	0.2140
1.6030	127.3434	0.0157	0.0064	0.2487	0.0182	0.2494	0.2499	0.1963

Uncertainty Analysis (Thrust)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	T_P (N)	P_T	B_1	B_2	B_3	B_T	U_T	% Unc.
1.8042	-1.9385	0.0161	-0.0001	0.2487	0.0182	0.2494	0.2499	-12.8898
1.8034	1.1898	0.0158	0.0001	0.2487	0.0182	0.2494	0.2499	21.0001
1.8068	4.2981	0.0163	0.0002	0.2487	0.0182	0.2494	0.2499	5.8138
1.8056	10.5564	0.0164	0.0005	0.2487	0.0182	0.2494	0.2499	2.3672
1.8042	22.9946	0.0165	0.0011	0.2487	0.0182	0.2494	0.2499	1.0868
1.8051	45.8331	0.0167	0.0023	0.2487	0.0182	0.2494	0.2499	0.5453
1.8048	65.0234	0.0167	0.0033	0.2487	0.0182	0.2494	0.2499	0.3844
1.8061	73.8682	0.0169	0.0037	0.2487	0.0182	0.2494	0.2499	0.3384
1.8051	87.2252	0.0172	0.0044	0.2487	0.0182	0.2494	0.2500	0.2866
1.8043	100.0789	0.0173	0.0050	0.2487	0.0182	0.2494	0.2500	0.2498
1.8059	112.6564	0.0175	0.0056	0.2487	0.0182	0.2494	0.2500	0.2219
1.8057	125.3448	0.0176	0.0063	0.2487	0.0182	0.2494	0.2500	0.1995

V (m/s)	T_P (N)	P_T	B_1	B_2	B_3	B_T	U_T	% Unc.
2.1091	-2.2244	0.0173	-0.0001	0.2487	0.0182	0.2494	0.2499	-11.2370
2.1056	0.7034	0.0110	0.0000	0.2487	0.0182	0.2494	0.2496	35.4817
2.1068	3.3349	0.0183	0.0002	0.2487	0.0182	0.2494	0.2500	7.4971
2.1067	9.0348	0.0182	0.0005	0.2487	0.0182	0.2494	0.2500	2.7672
2.1101	19.5316	0.0185	0.0010	0.2487	0.0182	0.2494	0.2500	1.2802
2.1068	42.0655	0.0186	0.0021	0.2487	0.0182	0.2494	0.2501	0.5944
2.1070	59.4139	0.0187	0.0030	0.2487	0.0182	0.2494	0.2501	0.4209
2.0959	68.8805	0.0190	0.0034	0.2487	0.0182	0.2494	0.2501	0.3631
2.1064	80.5944	0.0192	0.0040	0.2487	0.0182	0.2494	0.2501	0.3103
2.1073	93.1518	0.0193	0.0047	0.2487	0.0182	0.2494	0.2501	0.2685
2.1066	106.0173	0.0196	0.0053	0.2487	0.0182	0.2494	0.2502	0.2360
2.1060	117.5641	0.0196	0.0059	0.2487	0.0182	0.2494	0.2502	0.2128

Uncertainty Analysis (Thrust)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	T_P (N)	P_T	B_1	B_2	B_3	B_T	U_T	% Unc.
2.3079	-3.6889	0.0205	-0.0002	0.2487	0.0182	0.2494	0.2502	-6.7823
2.3069	-0.4585	0.0119	0.0000	0.2487	0.0182	0.2494	0.2496	-54.4470
2.3074	2.8578	0.0200	0.0001	0.2487	0.0182	0.2494	0.2502	8.7531
2.3070	7.3859	0.0207	0.0004	0.2487	0.0182	0.2494	0.2502	3.3876
2.3084	17.3533	0.0208	0.0009	0.2487	0.0182	0.2494	0.2502	1.4419
2.3081	39.3701	0.0209	0.0020	0.2487	0.0182	0.2494	0.2502	0.6356
2.3082	56.3185	0.0210	0.0028	0.2487	0.0182	0.2494	0.2502	0.4443
2.3077	65.8199	0.0210	0.0033	0.2487	0.0182	0.2494	0.2503	0.3802
2.3074	75.6556	0.0212	0.0038	0.2487	0.0182	0.2494	0.2503	0.3308
2.3075	88.4838	0.0215	0.0044	0.2487	0.0182	0.2494	0.2503	0.2829
2.3077	102.4455	0.0219	0.0051	0.2487	0.0182	0.2494	0.2504	0.2444
2.3068	114.3675	0.0220	0.0057	0.2487	0.0182	0.2494	0.2504	0.2189

V (m/s)	T_P (N)	P_T	B_1	B_2	B_3	B_T	U_T	% Unc.
2.5100	-5.1534	0.0231	-0.0003	0.2487	0.0182	0.2494	0.2504	-4.8592
2.5086	-2.4533	0.0215	-0.0001	0.2487	0.0182	0.2494	0.2503	-10.2016
2.5072	1.5344	0.0209	0.0001	0.2487	0.0182	0.2494	0.2502	16.3072
2.5095	5.3858	0.0232	0.0003	0.2487	0.0182	0.2494	0.2504	4.6497
2.5020	15.9565	0.0234	0.0008	0.2487	0.0182	0.2494	0.2504	1.5696
2.5092	37.9313	0.0237	0.0019	0.2487	0.0182	0.2494	0.2505	0.6604
2.5081	52.3894	0.0236	0.0026	0.2487	0.0182	0.2494	0.2505	0.4781
2.5087	62.3939	0.0238	0.0031	0.2487	0.0182	0.2494	0.2505	0.4015
2.5087	72.3930	0.0238	0.0036	0.2487	0.0182	0.2494	0.2505	0.3460
2.5023	84.3833	0.0239	0.0042	0.2487	0.0182	0.2494	0.2505	0.2969
2.5092	97.3348	0.0241	0.0049	0.2487	0.0182	0.2494	0.2506	0.2574
2.5091	110.9502	0.0243	0.0055	0.2487	0.0182	0.2494	0.2506	0.2259

Load Varying Self-Propulsion Test (Propeller fitted on)

Uncertainty Analysis (Tow Force)

V	Vehicle Velocity (m/s)	
F	Tow Force (N)	
P_F	Precision Error for F	$P_F = 2 \times S.D$
S.D	Standard Deviation	
S.E.E	Standard Error Estimate	
AD	Analog to Digital Converter	
B_1	Bias Error due to Calibration	$B_1 = 0.00005 \times F$
B_2	Bias Error due to Data Acquisition	$B_2 = 2 \times S.E.E$
B_3	Bias Error due to Load Cell Misalignment	$B_3 = F - (\cos 0.25^0 \times F)$
B_4	Bias Error due to Data Reduction	$B_4 = (AD_{error} \times AD_{range}) / (AD_{accuracy})$
B_F	Bias Error for F	$B_F = (B_1^2 + B_2^2 + B_3^2 + B_4^2)^{1/2}$
U_F	Total Error for F	$U_F = (B_F^2 + P_F^2)^{1/2}$
% Unc.	Percentage Uncertainty for F	% Unc. = $(U_F / F) \times 100$

Uncertainty Analysis (Tow Force)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	F (N)	P_F	B_1	B_2	B_3	B_4	B_F	U_F	% Unc.
0.8007	5.3638	0.0092	0.0003	0.2693	0.0001	0.0054	0.2694	0.2695	5.0249
0.8012	1.4334	0.0085	0.0001	0.2693	0.0000	0.0054	0.2694	0.2695	18.8015
0.8027	-3.5438	0.0091	-0.0002	0.2693	0.0000	0.0054	0.2694	0.2695	-7.6054
0.8034	-10.4367	0.0092	-0.0005	0.2693	-0.0001	0.0054	0.2694	0.2695	-2.5825
0.8005	-24.3516	0.0094	-0.0012	0.2693	-0.0002	0.0054	0.2694	0.2695	-1.1068
0.8012	-48.5726	0.0096	-0.0024	0.2693	-0.0005	0.0054	0.2694	0.2695	-0.5549
0.8037	-64.7292	0.0095	-0.0032	0.2693	-0.0006	0.0054	0.2694	0.2696	-0.4164
0.8045	-74.4405	0.0097	-0.0037	0.2693	-0.0007	0.0054	0.2694	0.2696	-0.3621
0.8098	-85.7547	0.0099	-0.0043	0.2693	-0.0008	0.0054	0.2694	0.2696	-0.3144
0.8032	-96.8046	0.0101	-0.0048	0.2693	-0.0009	0.0054	0.2694	0.2696	-0.2785
0.8015	-108.6784	0.0102	-0.0054	0.2693	-0.0010	0.0054	0.2694	0.2696	-0.2481
0.8022	-117.3754	0.0102	-0.0059	0.2693	-0.0011	0.0054	0.2694	0.2696	-0.2297

V (m/s)	F (N)	P_F	B_1	B_2	B_3	B_4	B_F	U_F	% Unc.
1.1030	14.2900	0.0116	0.0007	0.2693	0.0001	0.0054	0.2694	0.2696	1.8868
1.1027	10.5355	0.0111	0.0005	0.2693	0.0001	0.0054	0.2694	0.2696	2.5589
1.1035	6.8977	0.0099	0.0003	0.2693	0.0001	0.0054	0.2694	0.2695	3.9078
1.1032	0.5784	0.0080	0.0000	0.2693	0.0000	0.0054	0.2694	0.2695	46.5925
1.1044	-11.0514	0.0115	-0.0006	0.2693	-0.0001	0.0054	0.2694	0.2696	-2.4396
1.1041	-34.4575	0.0116	-0.0017	0.2693	-0.0003	0.0054	0.2694	0.2696	-0.7825
1.1030	-52.5912	0.0117	-0.0026	0.2693	-0.0005	0.0054	0.2694	0.2696	-0.5127
1.1037	-63.4548	0.0118	-0.0032	0.2693	-0.0006	0.0054	0.2694	0.2696	-0.4249
1.1031	-75.4534	0.0118	-0.0038	0.2693	-0.0007	0.0054	0.2694	0.2697	-0.3574
1.1031	-86.3969	0.0119	-0.0043	0.2693	-0.0008	0.0054	0.2694	0.2697	-0.3121
1.1029	-96.9218	0.0120	-0.0048	0.2693	-0.0009	0.0054	0.2694	0.2697	-0.2782
1.1034	-107.1228	0.0121	-0.0054	0.2693	-0.0010	0.0054	0.2694	0.2697	-0.2518

Uncertainty Analysis (Tow Force)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	F (N)	P_F	B_1	B_2	B_3	B_4	B_F	U_F	% Unc.
1.3031	23.7086	0.0131	0.0012	0.2693	0.0002	0.0054	0.2694	0.2697	1.1375
1.3022	19.5828	0.0130	0.0010	0.2693	0.0002	0.0054	0.2694	0.2697	1.3771
1.3025	15.5347	0.0130	0.0008	0.2693	0.0001	0.0054	0.2694	0.2697	1.7360
1.3035	7.4959	0.0129	0.0004	0.2693	0.0001	0.0054	0.2694	0.2697	3.5976
1.3036	-3.1778	0.0128	-0.0002	0.2693	0.0000	0.0054	0.2694	0.2697	-8.4862
1.3022	-23.6329	0.0130	-0.0012	0.2693	-0.0002	0.0054	0.2694	0.2697	-1.1411
1.3022	-45.8753	0.0132	-0.0023	0.2693	-0.0004	0.0054	0.2694	0.2697	-0.5879
1.3023	-54.5532	0.0131	-0.0027	0.2693	-0.0005	0.0054	0.2694	0.2697	-0.4944
1.3020	-67.4871	0.0134	-0.0034	0.2693	-0.0006	0.0054	0.2694	0.2697	-0.3997
1.3023	-78.2728	0.0135	-0.0039	0.2693	-0.0007	0.0054	0.2694	0.2697	-0.3446
1.3022	-88.6396	0.0134	-0.0044	0.2693	-0.0008	0.0054	0.2694	0.2697	-0.3043
1.3024	-98.8778	0.0138	-0.0049	0.2693	-0.0009	0.0054	0.2694	0.2698	-0.2728

V (m/s)	F (N)	P_F	B_1	B_2	B_3	B_4	B_F	U_F	% Unc.
1.6032	35.6003	0.0155	0.0018	0.2693	0.0003	0.0054	0.2694	0.2698	0.7579
1.6046	31.5343	0.0152	0.0016	0.2693	0.0003	0.0054	0.2694	0.2698	0.8556
1.6035	27.4347	0.0151	0.0014	0.2693	0.0003	0.0054	0.2694	0.2698	0.9834
1.6038	17.3544	0.0150	0.0009	0.2693	0.0002	0.0054	0.2694	0.2698	1.5546
1.6040	8.5633	0.0144	0.0004	0.2693	0.0001	0.0054	0.2694	0.2698	3.1501
1.6038	-12.6824	0.0151	-0.0006	0.2693	-0.0001	0.0054	0.2694	0.2698	-2.1273
1.6044	-32.5581	0.0152	-0.0016	0.2693	-0.0003	0.0054	0.2694	0.2698	-0.8287
1.6042	-42.0001	0.0154	-0.0021	0.2693	-0.0004	0.0054	0.2694	0.2698	-0.6424
1.6043	-51.6517	0.0155	-0.0026	0.2693	-0.0005	0.0054	0.2694	0.2698	-0.5224
1.6034	-63.4763	0.0156	-0.0032	0.2693	-0.0006	0.0054	0.2694	0.2698	-0.4251
1.6036	-76.3438	0.0157	-0.0038	0.2693	-0.0007	0.0054	0.2694	0.2699	-0.3535
1.6030	-88.3925	0.0157	-0.0044	0.2693	-0.0008	0.0054	0.2694	0.2699	-0.3053

Uncertainty Analysis (Tow Force)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	F (N)	P_F	B_1	B_2	B_3	B_4	B_F	U_F	% Unc.
1.8042	40.6558	0.0174	0.0020	0.2693	0.0004	0.0054	0.2694	0.2699	0.6640
1.8034	37.6344	0.0171	0.0019	0.2693	0.0004	0.0054	0.2694	0.2699	0.7172
1.8068	33.4536	0.0171	0.0017	0.2693	0.0003	0.0054	0.2694	0.2699	0.8068
1.8056	24.9715	0.0168	0.0012	0.2693	0.0002	0.0054	0.2694	0.2699	1.0808
1.8042	17.3566	0.0169	0.0009	0.2693	0.0002	0.0054	0.2694	0.2699	1.5550
1.8051	-4.5070	0.0171	-0.0002	0.2693	0.0000	0.0054	0.2694	0.2699	-5.9887
1.8048	-25.1929	0.0171	-0.0013	0.2693	-0.0002	0.0054	0.2694	0.2699	-1.0714
1.8061	-32.6326	0.0172	-0.0016	0.2693	-0.0003	0.0054	0.2694	0.2699	-0.8272
1.8051	-43.1904	0.0174	-0.0022	0.2693	-0.0004	0.0054	0.2694	0.2699	-0.6250
1.8043	-55.8141	0.0175	-0.0028	0.2693	-0.0005	0.0054	0.2694	0.2699	-0.4837
1.8059	-68.1665	0.0177	-0.0034	0.2693	-0.0006	0.0054	0.2694	0.2700	-0.3960
1.8057	-77.6279	0.0178	-0.0039	0.2693	-0.0007	0.0054	0.2694	0.2700	-0.3478

V (m/s)	F (N)	P_F	B_1	B_2	B_3	B_4	B_F	U_F	% Unc.
2.1091	54.2008	0.0203	0.0027	0.2693	0.0005	0.0054	0.2694	0.2701	0.4984
2.1056	51.6384	0.0202	0.0026	0.2693	0.0005	0.0054	0.2694	0.2701	0.5231
2.1068	49.7643	0.0202	0.0025	0.2693	0.0005	0.0054	0.2694	0.2701	0.5428
2.1067	42.3233	0.0199	0.0021	0.2693	0.0004	0.0054	0.2694	0.2701	0.6382
2.1101	34.8043	0.0198	0.0017	0.2693	0.0003	0.0054	0.2694	0.2701	0.7761
2.1068	13.6338	0.0196	0.0007	0.2693	0.0001	0.0054	0.2694	0.2701	1.9810
2.1070	-3.9622	0.0193	-0.0002	0.2693	0.0000	0.0054	0.2694	0.2701	-6.8159
2.0959	-13.7635	0.0195	-0.0007	0.2693	-0.0001	0.0054	0.2694	0.2701	-1.9622
2.1064	-24.0542	0.0196	-0.0012	0.2693	-0.0002	0.0054	0.2694	0.2701	-1.1228
2.1073	-33.2420	0.0199	-0.0017	0.2693	-0.0003	0.0054	0.2694	0.2701	-0.8125
2.1066	-47.5793	0.0201	-0.0024	0.2693	-0.0005	0.0054	0.2694	0.2701	-0.5677
2.1060	-57.4544	0.0204	-0.0029	0.2693	-0.0005	0.0054	0.2694	0.2702	-0.4702

Uncertainty Analysis (Tow Force)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	F (N)	P_F	B_1	B_2	B_3	B_4	B_F	U_F	% Unc.
2.3079	70.1815	0.0242	0.0035	0.2693	0.0007	0.0054	0.2694	0.2705	0.3854
2.3069	65.5744	0.0239	0.0033	0.2693	0.0006	0.0054	0.2694	0.2704	0.4124
2.3074	62.5675	0.0235	0.0031	0.2693	0.0006	0.0054	0.2694	0.2704	0.4322
2.3070	56.4648	0.0234	0.0028	0.2693	0.0005	0.0054	0.2694	0.2704	0.4789
2.3084	52.8606	0.0234	0.0026	0.2693	0.0005	0.0054	0.2694	0.2704	0.5115
2.3081	31.0591	0.0229	0.0016	0.2693	0.0003	0.0054	0.2694	0.2703	0.8704
2.3082	14.6680	0.0260	0.0007	0.2693	0.0001	0.0054	0.2694	0.2706	1.8450
2.3077	5.4600	0.0225	0.0003	0.2693	0.0001	0.0054	0.2694	0.2703	4.9506
2.3074	-4.9423	0.0224	-0.0002	0.2693	0.0000	0.0054	0.2694	0.2703	-5.4691
2.3075	-16.4586	0.0227	-0.0008	0.2693	-0.0002	0.0054	0.2694	0.2703	-1.6424
2.3077	-29.2323	0.0229	-0.0015	0.2693	-0.0003	0.0054	0.2694	0.2703	-0.9248
2.3068	-42.1765	0.0232	-0.0021	0.2693	-0.0004	0.0054	0.2694	0.2704	-0.6411

V (m/s)	F (N)	P_F	B_1	B_2	B_3	B_4	B_F	U_F	% Unc.
2.5100	87.5695	0.0276	0.0044	0.2693	0.0008	0.0054	0.2694	0.2708	0.3093
2.5086	83.4748	0.0275	0.0042	0.2693	0.0008	0.0054	0.2694	0.2708	0.3244
2.5072	78.4785	0.0274	0.0039	0.2693	0.0007	0.0054	0.2694	0.2708	0.3450
2.5095	72.4764	0.0271	0.0036	0.2693	0.0007	0.0054	0.2694	0.2708	0.3736
2.5020	66.8172	0.0268	0.0033	0.2693	0.0006	0.0054	0.2694	0.2707	0.4052
2.5092	46.3855	0.0266	0.0023	0.2693	0.0004	0.0054	0.2694	0.2707	0.5836
2.5081	32.5307	0.0263	0.0016	0.2693	0.0003	0.0054	0.2694	0.2707	0.8320
2.5087	22.3437	0.0261	0.0011	0.2693	0.0002	0.0054	0.2694	0.2706	1.2112
2.5087	12.3667	0.0258	0.0006	0.2693	0.0001	0.0054	0.2694	0.2706	2.1881
2.5023	3.6930	0.0245	0.0002	0.2693	0.0000	0.0054	0.2694	0.2705	7.3241
2.5092	-11.5045	0.0257	-0.0006	0.2693	-0.0001	0.0054	0.2694	0.2706	-2.3520
2.5091	-23.5327	0.0260	-0.0012	0.2693	-0.0002	0.0054	0.2694	0.2706	-1.1500

Load Varying Self-Propulsion Test (Propeller fitted on)

Uncertainty Analysis (Advance Coefficient)

V	Vehicle Velocity (m/s)	
n	Propeller Rotational Speed (rpm)	
D	Propeller Diameter (m)	
J	Advance Coefficient	
P_V	Precision Error for V	
B_V	Bias Error for V	
P_J	Precision Error for J	
P_n	Precision Error for n	
B_n	Bias Error for n	
B_D	Bias Error for D	
B_J	Bias Error for J	$B_J = (\delta J / \delta V \; B_V)^2 + (\delta J / \delta n \; B_n)^2 + (\delta J / \delta D \; B_D)^2)^{1/2}$
U_J	Total Error for J	$U_J = (B_J^2 + P_J^2)^{1/2}$
$\% U$	Percentage Uncertainty for J	$\% U = (U_J / J) \times 100$

Uncertainty Analysis (Advance Coefficient)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	P_V	B_V	n (rpm)	P_n	B_n	B_D	B_V $\delta J/\delta V$	B_n $\delta J/\delta n$	B_D $\delta J/\delta D$	B_J	P_J	U_J	J	% U
0.8007	0.0003	0.0035	503.8949	0.0004	0.0391	0.0001	4.58E-05	-8.09E-07	-6.84E-06	4.63E-05	0.0005	0.0005	0.6256	0.0863
0.8012	0.0004	0.0035	552.6343	0.0005	0.0391	0.0001	4.18E-05	-6.73E-07	-6.24E-06	4.23E-05	0.0006	0.0006	0.5708	0.1053
0.8027	0.0003	0.0035	602.3573	0.0005	0.0391	0.0001	3.83E-05	-5.68E-07	-5.74E-06	3.88E-05	0.0006	0.0006	0.5246	0.1098
0.8034	0.0003	0.0035	651.5623	0.0005	0.0391	0.0001	3.54E-05	-4.86E-07	-5.31E-06	3.58E-05	0.0006	0.0006	0.4854	0.1147
0.8005	0.0004	0.0035	753.4148	0.0004	0.0391	0.0001	3.06E-05	-3.62E-07	-4.57E-06	3.10E-05	0.0006	0.0006	0.4183	0.1361
0.8012	0.0004	0.0035	913.8563	0.0005	0.0391	0.0001	2.53E-05	-2.46E-07	-3.77E-06	2.55E-05	0.0006	0.0006	0.3452	0.1821
0.8037	0.0003	0.0035	1002.5757	0.0005	0.0391	0.0001	2.30E-05	-2.05E-07	-3.45E-06	2.33E-05	0.0006	0.0006	0.3156	0.1777
0.8045	0.0004	0.0035	1051.5807	0.0005	0.0392	0.0001	2.20E-05	-1.87E-07	-3.29E-06	2.22E-05	0.0006	0.0006	0.3012	0.1941
0.8098	0.0002	0.0035	1102.1417	0.0004	0.0392	0.0001	2.10E-05	-1.71E-07	-3.16E-06	2.12E-05	0.0005	0.0005	0.2893	0.1734
0.8032	0.0003	0.0035	1154.0417	0.0005	0.0392	0.0001	2.00E-05	-1.55E-07	-3.00E-06	2.02E-05	0.0006	0.0006	0.2740	0.2208
0.8015	0.0003	0.0035	1207.3702	0.0005	0.0392	0.0001	1.91E-05	-1.41E-07	-2.86E-06	1.93E-05	0.0006	0.0006	0.2614	0.2128
0.8022	0.0004	0.0035	1246.6728	0.0005	0.0392	0.0001	1.85E-05	-1.33E-07	-2.77E-06	1.87E-05	0.0006	0.0006	0.2533	0.2480

V (m/s)	P_V	B_V	n (rpm)	P_n	B_n	B_D	B_V $\delta J/\delta V$	B_n $\delta J/\delta n$	B_D $\delta J/\delta D$	B_J	P_J	U_J	J	% U
1.1030	0.0004	0.0035	507.2079	0.0006	0.0391	0.0001	4.56E-05	-1.10E-06	-9.36E-06	4.66E-05	0.0007	0.0007	0.8562	0.0834
1.1027	0.0005	0.0035	554.5345	0.0006	0.0391	0.0001	4.17E-05	-9.20E-07	-8.56E-06	4.26E-05	0.0008	0.0008	0.7829	0.0980
1.1035	0.0005	0.0035	603.8746	0.0006	0.0391	0.0001	3.83E-05	-7.76E-07	-7.87E-06	3.91E-05	0.0008	0.0008	0.7194	0.1105
1.1032	0.0005	0.0035	654.2447	0.0006	0.0391	0.0001	3.54E-05	-6.61E-07	-7.26E-06	3.61E-05	0.0008	0.0008	0.6639	0.1225
1.1044	0.0005	0.0035	750.3733	0.0006	0.0391	0.0001	3.08E-05	-5.03E-07	-6.34E-06	3.15E-05	0.0008	0.0008	0.5794	0.1333
1.1041	0.0004	0.0035	910.3564	0.0006	0.0391	0.0001	2.54E-05	-3.42E-07	-5.22E-06	2.60E-05	0.0008	0.0008	0.4775	0.1627
1.1030	0.0005	0.0035	1006.6678	0.0007	0.0391	0.0001	2.30E-05	-2.80E-07	-4.72E-06	2.35E-05	0.0008	0.0008	0.4314	0.1893
1.1037	0.0005	0.0035	1056.9924	0.0006	0.0392	0.0001	2.19E-05	-2.54E-07	-4.50E-06	2.24E-05	0.0008	0.0008	0.4111	0.1938
1.1031	0.0005	0.0035	1108.8989	0.0006	0.0392	0.0001	2.09E-05	-2.31E-07	-4.28E-06	2.13E-05	0.0008	0.0008	0.3916	0.2024
1.1031	0.0005	0.0035	1156.0328	0.0006	0.0392	0.0001	2.00E-05	-2.12E-07	-4.11E-06	2.04E-05	0.0008	0.0008	0.3757	0.2088
1.1029	0.0005	0.0035	1206.4133	0.0006	0.0392	0.0001	1.92E-05	-1.95E-07	-3.94E-06	1.96E-05	0.0008	0.0008	0.3599	0.2087
1.1034	0.0005	0.0035	1249.3622	0.0006	0.0392	0.0001	1.85E-05	-1.82E-07	-3.80E-06	1.89E-05	0.0008	0.0008	0.3477	0.2241

Uncertainty Analysis (Advance Coefficient)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	P_V	B_V	n (rpm)	P_n	B_n	B_D	B_V δJ/δV	B_n δJ/δn	B_D δJ/δD	B_J	P_J	U_J	J	% U
1.3031	0.0005	0.0035	504.2337	0.0006	0.0391	0.0001	4.60E-05	-1.31E-06	-1.11E-05	4.73E-05	0.0008	0.0008	1.0174	0.0824
1.3022	0.0006	0.0035	550.4535	0.0006	0.0391	0.0001	4.21E-05	-1.10E-06	-1.02E-05	4.34E-05	0.0008	0.0008	0.9314	0.0882
1.3025	0.0006	0.0035	600.5634	0.0007	0.0391	0.0001	3.86E-05	-9.27E-07	-9.34E-06	3.97E-05	0.0009	0.0009	0.8539	0.1033
1.3035	0.0006	0.0035	652.2352	0.0007	0.0391	0.0001	3.56E-05	-7.86E-07	-8.60E-06	3.66E-05	0.0009	0.0009	0.7868	0.1173
1.3036	0.0005	0.0035	752.8192	0.0007	0.0391	0.0001	3.08E-05	-5.90E-07	-7.46E-06	3.17E-05	0.0009	0.0009	0.6817	0.1252
1.3022	0.0005	0.0035	901.8296	0.0007	0.0391	0.0001	2.57E-05	-4.11E-07	-6.22E-06	2.65E-05	0.0009	0.0009	0.5685	0.1535
1.3022	0.0006	0.0035	1000.1110	0.0007	0.0391	0.0001	2.32E-05	-3.34E-07	-5.61E-06	2.39E-05	0.0009	0.0009	0.5126	0.1804
1.3023	0.0006	0.0035	1046.2973	0.0007	0.0392	0.0001	2.22E-05	-3.06E-07	-5.36E-06	2.28E-05	0.0009	0.0009	0.4900	0.1830
1.3020	0.0005	0.0035	1105.7242	0.0007	0.0392	0.0001	2.10E-05	-2.74E-07	-5.07E-06	2.16E-05	0.0009	0.0009	0.4636	0.1942
1.3023	0.0005	0.0035	1155.3342	0.0007	0.0392	0.0001	2.01E-05	-2.51E-07	-4.85E-06	2.07E-05	0.0009	0.0009	0.4438	0.1945
1.3022	0.0005	0.0035	1206.0095	0.0007	0.0392	0.0001	1.92E-05	-2.30E-07	-4.65E-06	1.98E-05	0.0009	0.0009	0.4251	0.2080
1.3024	0.0005	0.0035	1248.0299	0.0007	0.0392	0.0001	1.86E-05	-2.15E-07	-4.49E-06	1.91E-05	0.0009	0.0009	0.4109	0.2191

V (m/s)	P_V	B_V	n (rpm)	P_n	B_n	B_D	B_V δJ/δV	B_n δJ/δn	B_D δJ/δD	B_J	P_J	U_J	J	% U
1.6032	0.0006	0.0036	503.9995	0.0008	0.0391	0.0001	4.62E-05	-1.62E-06	-1.37E-05	4.82E-05	0.0010	0.0010	1.2523	0.0794
1.6046	0.0007	0.0036	550.3443	0.0008	0.0391	0.0001	4.23E-05	-1.36E-06	-1.26E-05	4.42E-05	0.0011	0.0011	1.1479	0.0927
1.6035	0.0007	0.0036	601.4574	0.0008	0.0391	0.0001	3.87E-05	-1.14E-06	-1.15E-05	4.04E-05	0.0011	0.0011	1.0496	0.1012
1.6038	0.0007	0.0036	653.7645	0.0009	0.0391	0.0001	3.56E-05	-9.63E-07	-1.06E-05	3.72E-05	0.0011	0.0011	0.9658	0.1136
1.6040	0.0007	0.0036	755.9224	0.0008	0.0391	0.0001	3.08E-05	-7.20E-07	-9.14E-06	3.22E-05	0.0011	0.0011	0.8354	0.1261
1.6038	0.0006	0.0036	911.6408	0.0008	0.0391	0.0001	2.56E-05	-4.96E-07	-7.57E-06	2.67E-05	0.0010	0.0010	0.6926	0.1435
1.6044	0.0007	0.0036	1003.7944	0.0008	0.0391	0.0001	2.32E-05	-4.09E-07	-6.88E-06	2.42E-05	0.0011	0.0011	0.6293	0.1709
1.6042	0.0007	0.0036	1051.3377	0.0008	0.0392	0.0001	2.22E-05	-3.73E-07	-6.57E-06	2.31E-05	0.0010	0.0011	0.6007	0.1748
1.6043	0.0006	0.0036	1104.0239	0.0008	0.0392	0.0001	2.11E-05	-3.38E-07	-6.26E-06	2.20E-05	0.0010	0.0010	0.5721	0.1737
1.6034	0.0007	0.0036	1152.5333	0.0008	0.0392	0.0001	2.02E-05	-3.10E-07	-5.99E-06	2.11E-05	0.0011	0.0011	0.5477	0.1938
1.6036	0.0007	0.0036	1210.1129	0.0008	0.0392	0.0001	1.93E-05	-2.82E-07	-5.71E-06	2.01E-05	0.0011	0.0011	0.5217	0.2067
1.6030	0.0007	0.0036	1250.8055	0.0009	0.0392	0.0001	1.86E-05	-2.64E-07	-5.52E-06	1.94E-05	0.0011	0.0011	0.5046	0.2173

Uncertainty Analysis (Advance Coefficient)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	P_V	B_V	n (rpm)	P_n	B_n	B_D	B_V δJ/δV	B_n δJ/δn	B_D δJ/δD	B_J	P_J	U_J	J	% U
1.8042	0.0008	0.0036	510.9808	0.0010	0.0391	0.0001	4.58E-05	-1.77E-06	-1.52E-05	4.83E-05	0.0013	0.0013	1.3901	0.0918
1.8034	0.0009	0.0036	552.4233	0.0010	0.0391	0.0001	4.23E-05	-1.52E-06	-1.41E-05	4.46E-05	0.0014	0.0014	1.2853	0.1058
1.8068	0.0009	0.0036	602.7853	0.0009	0.0391	0.0001	3.88E-05	-1.28E-06	-1.29E-05	4.09E-05	0.0013	0.0013	1.1801	0.1092
1.8056	0.0009	0.0036	651.5334	0.0009	0.0391	0.0001	3.59E-05	-1.09E-06	-1.19E-05	3.78E-05	0.0013	0.0013	1.0911	0.1155
1.8042	0.0009	0.0036	757.8054	0.0010	0.0391	0.0001	3.09E-05	-8.06E-07	-1.03E-05	3.25E-05	0.0014	0.0014	0.9373	0.1452
1.8051	0.0009	0.0036	911.1136	0.0010	0.0391	0.0001	2.57E-05	-5.58E-07	-8.53E-06	2.71E-05	0.0014	0.0014	0.7800	0.1780
1.8048	0.0008	0.0036	1002.7594	0.0010	0.0391	0.0001	2.33E-05	-4.61E-07	-7.75E-06	2.46E-05	0.0013	0.0013	0.7086	0.1843
1.8061	0.0009	0.0036	1056.9214	0.0010	0.0392	0.0001	2.21E-05	-4.15E-07	-7.36E-06	2.33E-05	0.0013	0.0013	0.6728	0.1938
1.8051	0.0009	0.0036	1118.8065	0.0010	0.0392	0.0001	2.09E-05	-3.71E-07	-6.95E-06	2.20E-05	0.0013	0.0013	0.6352	0.2051
1.8043	0.0009	0.0036	1161.1239	0.0010	0.0392	0.0001	2.01E-05	-3.44E-07	-6.69E-06	2.12E-05	0.0014	0.0014	0.6118	0.2246
1.8059	0.0008	0.0036	1215.6599	0.0010	0.0392	0.0001	1.92E-05	-3.14E-07	-6.40E-06	2.03E-05	0.0013	0.0013	0.5849	0.2207
1.8057	0.0009	0.0036	1246.0544	0.0010	0.0392	0.0001	1.88E-05	-2.99E-07	-6.24E-06	1.98E-05	0.0013	0.0013	0.5705	0.2312

V (m/s)	P_V	B_V	n (rpm)	P_n	B_n	B_D	B_V δJ/δV	B_n δJ/δn	B_D δJ/δD	B_J	P_J	U_J	J	% U
2.1091	0.0010	0.0036	500.1371	0.0011	0.0391	0.0001	4.71E-05	-2.16E-06	-1.82E-05	5.05E-05	0.0015	0.0015	1.6603	0.0896
2.1056	0.0010	0.0036	551.5545	0.0012	0.0391	0.0001	4.27E-05	-1.78E-06	-1.64E-05	4.58E-05	0.0015	0.0015	1.5030	0.1002
2.1068	0.0010	0.0036	600.3476	0.0011	0.0391	0.0001	3.92E-05	-1.50E-06	-1.51E-05	4.21E-05	0.0015	0.0015	1.3816	0.1117
2.1067	0.0010	0.0036	654.3489	0.0011	0.0391	0.0001	3.60E-05	-1.26E-06	-1.39E-05	3.86E-05	0.0015	0.0015	1.2675	0.1184
2.1101	0.0010	0.0036	752.6505	0.0011	0.0391	0.0001	3.13E-05	-9.56E-07	-1.21E-05	3.35E-05	0.0015	0.0015	1.1038	0.1361
2.1068	0.0011	0.0036	901.9876	0.0012	0.0391	0.0001	2.61E-05	-6.65E-07	-1.01E-05	2.80E-05	0.0016	0.0016	0.9196	0.1709
2.1070	0.0010	0.0036	1002.6789	0.0012	0.0391	0.0001	2.35E-05	-5.38E-07	-9.05E-06	2.52E-05	0.0015	0.0015	0.8273	0.1854
2.0959	0.0010	0.0036	1052.8833	0.0011	0.0392	0.0001	2.24E-05	-4.86E-07	-8.57E-06	2.39E-05	0.0015	0.0015	0.7837	0.1935
2.1064	0.0010	0.0036	1103.4297	0.0011	0.0392	0.0001	2.13E-05	-4.45E-07	-8.22E-06	2.29E-05	0.0015	0.0015	0.7516	0.2034
2.1073	0.0010	0.0036	1156.4853	0.0011	0.0392	0.0001	2.04E-05	-4.05E-07	-7.85E-06	2.18E-05	0.0015	0.0015	0.7174	0.2052
2.1066	0.0010	0.0036	1214.1716	0.0011	0.0392	0.0001	1.94E-05	-3.67E-07	-7.47E-06	2.08E-05	0.0015	0.0015	0.6831	0.2242
2.1060	0.0011	0.0036	1246.6579	0.0011	0.0392	0.0001	1.89E-05	-3.48E-07	-7.27E-06	2.02E-05	0.0015	0.0015	0.6651	0.2325

Uncertainty Analysis (Advance Coefficient)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	P_V	B_V	n (rpm)	P_n	B_n	B_D	B_V δJ/δV	B_n δJ/δn	B_D δJ/δD	B_J	P_J	U_J	J	% U
2.3079	0.0011	0.0036	502.2690	0.0013	0.0391	0.0001	4.71E-05	-2.35E-06	-1.98E-05	5.11E-05	0.0017	0.0017	1.8090	0.0941
2.3069	0.0012	0.0036	550.3896	0.0012	0.0391	0.0001	4.30E-05	-1.95E-06	-1.80E-05	4.67E-05	0.0017	0.0017	1.6502	0.1029
2.3074	0.0012	0.0036	603.5423	0.0013	0.0391	0.0001	3.92E-05	-1.63E-06	-1.65E-05	4.25E-05	0.0017	0.0017	1.5052	0.1144
2.3070	0.0012	0.0036	652.5464	0.0013	0.0391	0.0001	3.63E-05	-1.39E-06	-1.52E-05	3.93E-05	0.0018	0.0018	1.3919	0.1264
2.3084	0.0011	0.0036	752.0456	0.0013	0.0391	0.0001	3.15E-05	-1.05E-06	-1.32E-05	3.41E-05	0.0018	0.0018	1.2085	0.1452
2.3081	0.0011	0.0036	901.6673	0.0013	0.0391	0.0001	2.62E-05	-7.29E-07	-1.10E-05	2.85E-05	0.0017	0.0017	1.0078	0.1685
2.3082	0.0011	0.0036	1006.0527	0.0013	0.0391	0.0001	2.35E-05	-5.86E-07	-9.88E-06	2.55E-05	0.0017	0.0017	0.9033	0.1911
2.3077	0.0012	0.0036	1052.1662	0.0013	0.0392	0.0001	2.25E-05	-5.36E-07	-9.44E-06	2.44E-05	0.0017	0.0017	0.8635	0.1993
2.3074	0.0011	0.0036	1101.3796	0.0013	0.0392	0.0001	2.15E-05	-4.89E-07	-9.02E-06	2.33E-05	0.0017	0.0017	0.8248	0.2024
2.3075	0.0011	0.0036	1150.4349	0.0013	0.0392	0.0001	2.06E-05	-4.48E-07	-8.64E-06	2.23E-05	0.0017	0.0017	0.7897	0.2167
2.3077	0.0011	0.0036	1213.5677	0.0013	0.0392	0.0001	1.95E-05	-4.03E-07	-8.19E-06	2.11E-05	0.0017	0.0017	0.7487	0.2323
2.3068	0.0012	0.0036	1249.1721	0.0013	0.0392	0.0001	1.89E-05	-3.80E-07	-7.95E-06	2.05E-05	0.0018	0.0018	0.7270	0.2449

V (m/s)	P_V	B_V	n (rpm)	P_n	B_n	B_D	B_V δJ/δV	B_n δJ/δn	B_D δJ/δD	B_J	P_J	U_J	J	% U
2.5100	0.0013	0.0036	504.0554	0.0015	0.0391	0.0001	4.72E-05	-2.53E-06	-2.14E-05	5.19E-05	0.0019	0.0019	1.9605	0.0991
2.5086	0.0012	0.0036	552.5634	0.0014	0.0391	0.0001	4.31E-05	-2.11E-06	-1.95E-05	4.73E-05	0.0019	0.0019	1.7874	0.1066
2.5072	0.0013	0.0036	601.0760	0.0015	0.0391	0.0001	3.96E-05	-1.78E-06	-1.80E-05	4.35E-05	0.0020	0.0020	1.6422	0.1219
2.5095	0.0013	0.0036	651.4524	0.0015	0.0391	0.0001	3.65E-05	-1.52E-06	-1.66E-05	4.01E-05	0.0019	0.0019	1.5166	0.1284
2.5020	0.0013	0.0036	751.7646	0.0015	0.0391	0.0001	3.16E-05	-1.14E-06	-1.43E-05	3.48E-05	0.0020	0.0020	1.3103	0.1529
2.5092	0.0013	0.0036	905.0740	0.0014	0.0391	0.0001	2.63E-05	-7.87E-07	-1.19E-05	2.89E-05	0.0019	0.0019	1.0915	0.1758
2.5081	0.0013	0.0036	1004.8353	0.0015	0.0391	0.0001	2.37E-05	-6.38E-07	-1.07E-05	2.60E-05	0.0020	0.0020	0.9827	0.1991
2.5087	0.0013	0.0036	1052.4015	0.0015	0.0392	0.0001	2.26E-05	-5.82E-07	-1.03E-05	2.48E-05	0.0020	0.0020	0.9385	0.2089
2.5087	0.0013	0.0036	1103.3557	0.0014	0.0392	0.0001	2.16E-05	-5.30E-07	-9.79E-06	2.37E-05	0.0019	0.0019	0.8952	0.2144
2.5023	0.0013	0.0036	1154.7567	0.0015	0.0392	0.0001	2.06E-05	-4.82E-07	-9.33E-06	2.26E-05	0.0020	0.0020	0.8531	0.2347
2.5092	0.0013	0.0036	1207.1602	0.0015	0.0392	0.0001	1.97E-05	-4.43E-07	-8.95E-06	2.17E-05	0.0020	0.0020	0.8183	0.2483
2.5091	0.0013	0.0036	1252.5791	0.0015	0.0392	0.0001	1.90E-05	-4.11E-07	-8.62E-06	2.09E-05	0.0020	0.0020	0.7886	0.2524

Load Varying Self-Propulsion Test (Propeller fitted on)

Uncertainty Analysis (Torque Coefficient)

V	Vehicle Velocity (m/s)	
n	Propeller Rotational Speed (rpm)	
D	Propeller Diameter (m)	
Q	Propeller Torque (N-m)	
ρ	Density of Water (kg/m^3)	
K_q	Torque Coefficient	
P_q	Precision Error for Q	
B_q	Bias Error for Q	
P_{Kq}	Precision Error for K_q	
P_n	Precision Error for n	
B_n	Bias Error for n	
B_ρ	Bias Error for ρ	
B_D	Bias Error for D	
B_{Kq}	Bias Error for K_q	$B_{Kq} = (\delta K_q / \delta Q\ B_q)^2 + (\delta K_q / \delta \rho\ B_\rho)^2 + (\delta K_q / \delta n\ B_n)^2 + (\delta K_q / \delta D\ B_D)^2)^{1/2}$
U_{Kq}	Total Error for K_q	$U_{Kq} = (B_{Kq}^2 + P_{Kq}^2)^{1/2}$
% U	Percentage Uncertainty for K_q	$\% U = (U_{Kq} / K_q) \times 100$

Uncertainty Analysis (Torque Coefficient)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	n (rpm)	Q (N-m)	P_q	B_q	P_n	B_n	B_ρ	B_D	$B_q\ \delta K_q/\delta Q$	$B_\rho\ \delta K_q/\delta\rho$	$B_n\ \delta K_q/\delta n$	$B_D\ \delta K_q/\delta D$	B_{Kq}	P_{Kq}	U_{Kq}	Kq	% U
0.8007	503.8949	0.1810	0.0007	0.0076	0.0004	0.0391	0.0578	0.0001	3.66E-07	-5.01E-10	-1.35E-09	-2.85E-08	3.67E-07	0.0008	0.0008	0.0312	2.6347
0.8012	552.6343	0.3680	0.0007	0.0076	0.0005	0.0391	0.0578	0.0001	3.04E-07	-8.47E-10	-2.07E-09	-4.81E-08	3.08E-07	0.0009	0.0009	0.0528	1.6231
0.8027	602.3573	0.5143	0.0007	0.0076	0.0005	0.0391	0.0578	0.0001	2.56E-07	-9.97E-10	-2.24E-09	-5.66E-08	2.63E-07	0.0009	0.0009	0.0621	1.3897
0.8034	651.5623	0.7198	0.0007	0.0077	0.0005	0.0391	0.0578	0.0001	2.19E-07	-1.19E-09	-2.48E-09	-6.77E-08	2.30E-07	0.0009	0.0009	0.0742	1.1880
0.8005	753.4148	1.2080	0.0008	0.0077	0.0004	0.0391	0.0578	0.0001	1.65E-07	-1.50E-09	-2.69E-09	-8.49E-08	1.86E-07	0.0009	0.0009	0.0932	0.9516
0.8012	913.8563	2.0427	0.0008	0.0078	0.0005	0.0391	0.0578	0.0001	1.14E-07	-1.72E-09	-2.55E-09	-9.76E-08	1.50E-07	0.0009	0.0009	0.1071	0.8493
0.8037	1002.5757	2.6202	0.0008	0.0079	0.0005	0.0391	0.0578	0.0001	9.58E-08	-1.83E-09	-2.48E-09	-1.04E-07	1.41E-07	0.0009	0.0009	0.1142	0.7933
0.8045	1051.5807	2.9224	0.0008	0.0080	0.0005	0.0392	0.0578	0.0001	8.78E-08	-1.86E-09	-2.39E-09	-1.05E-07	1.37E-07	0.0009	0.0009	0.1157	0.7899
0.8098	1102.1417	3.3564	0.0008	0.0081	0.0004	0.0392	0.0578	0.0001	8.11E-08	-1.94E-09	-2.39E-09	-1.10E-07	1.37E-07	0.0009	0.0009	0.1210	0.7618
0.8032	1154.0417	3.7562	0.0008	0.0082	0.0005	0.0392	0.0578	0.0001	7.50E-08	-1.98E-09	-2.33E-09	-1.13E-07	1.35E-07	0.0010	0.0010	0.1235	0.7846
0.8015	1207.3702	4.1532	0.0008	0.0083	0.0005	0.0392	0.0578	0.0001	6.95E-08	-2.00E-09	-2.25E-09	-1.14E-07	1.33E-07	0.0010	0.0010	0.1248	0.7616
0.8022	1246.6728	4.5121	0.0008	0.0084	0.0005	0.0392	0.0578	0.0001	6.61E-08	-2.04E-09	-2.22E-09	-1.16E-07	1.33E-07	0.0010	0.0010	0.1271	0.7689

V (m/s)	n (rpm)	Q (N-m)	P_Q	B_Q	P_n	B_n	B_ρ	B_D	$B_q\ \delta K_q/\delta Q$	$B_\rho\ \delta K_q/\delta\rho$	$B_n\ \delta K_q/\delta n$	$B_D\ \delta K_q/\delta D$	B_{Kq}	P_{Kq}	U_{Kq}	Kq	% U
1.1030	507.2079	0.1699	0.0008	0.0076	0.0006	0.0391	0.0578	0.0001	3.61E-07	-4.64E-10	-1.24E-09	-2.64E-08	3.62E-07	0.0010	0.0010	0.0289	3.4340
1.1027	554.5345	0.3535	0.0008	0.0076	0.0006	0.0391	0.0578	0.0001	3.02E-07	-8.08E-10	-1.97E-09	-4.59E-08	3.06E-07	0.0010	0.0010	0.0503	1.9791
1.1035	603.8746	0.4982	0.0008	0.0076	0.0006	0.0391	0.0578	0.0001	2.55E-07	-9.61E-10	-2.15E-09	-5.45E-08	2.61E-07	0.0010	0.0010	0.0598	1.7119
1.1032	654.2447	0.7005	0.0009	0.0077	0.0006	0.0391	0.0578	0.0001	2.18E-07	-1.15E-09	-2.38E-09	-6.53E-08	2.27E-07	0.0011	0.0011	0.0717	1.4847
1.1044	750.3733	1.1746	0.0009	0.0077	0.0006	0.0391	0.0578	0.0001	1.66E-07	-1.47E-09	-2.65E-09	-8.33E-08	1.86E-07	0.0011	0.0011	0.0914	1.1517
1.1041	910.3564	1.9868	0.0009	0.0078	0.0006	0.0391	0.0578	0.0001	1.14E-07	-1.69E-09	-2.51E-09	-9.57E-08	1.49E-07	0.0011	0.0011	0.1050	1.0288
1.1030	1006.6678	2.5799	0.0009	0.0079	0.0007	0.0391	0.0578	0.0001	9.49E-08	-1.79E-09	-2.41E-09	-1.02E-07	1.39E-07	0.0011	0.0011	0.1115	0.9867
1.1037	1056.9924	2.8694	0.0009	0.0080	0.0006	0.0392	0.0578	0.0001	8.68E-08	-1.81E-09	-2.31E-09	-1.02E-07	1.34E-07	0.0011	0.0011	0.1125	0.9499
1.1031	1108.8989	3.2875	0.0009	0.0081	0.0006	0.0392	0.0578	0.0001	7.99E-08	-1.88E-09	-2.30E-09	-1.07E-07	1.33E-07	0.0011	0.0011	0.1171	0.9074
1.1031	1156.0328	3.6710	0.0009	0.0082	0.0006	0.0392	0.0578	0.0001	7.45E-08	-1.93E-09	-2.26E-09	-1.10E-07	1.33E-07	0.0011	0.0011	0.1203	0.9154
1.1029	1206.4133	4.0899	0.0009	0.0083	0.0006	0.0392	0.0578	0.0001	6.94E-08	-1.98E-09	-2.22E-09	-1.12E-07	1.32E-07	0.0011	0.0011	0.1231	0.8753
1.1034	1249.3622	4.4395	0.0009	0.0084	0.0006	0.0392	0.0578	0.0001	6.56E-08	-2.00E-09	-2.17E-09	-1.14E-07	1.31E-07	0.0011	0.0011	0.1245	0.8868

V (m/s)	n (rpm)	Q (N-m)	P_Q	B_Q	P_n	B_n	B_ρ	B_D	$B_q\ \delta K_q/\delta Q$	$B_\rho\ \delta K_q/\delta\rho$	$B_n\ \delta K_q/\delta n$	$B_D\ \delta K_q/\delta D$	B_{Kq}	P_{Kq}	U_{Kq}	Kq	% U
1.3031	504.2337	0.1657	0.0009	0.0076	0.0006	0.0391	0.0578	0.0001	3.65E-07	-4.58E-10	-1.23E-09	-2.60E-08	3.66E-07	0.0011	0.0011	0.0285	3.8693
1.3022	550.4535	0.3278	0.0009	0.0076	0.0006	0.0391	0.0578	0.0001	3.07E-07	-7.61E-10	-1.87E-09	-4.32E-08	3.10E-07	0.0011	0.0011	0.0474	2.2954
1.3025	600.5634	0.4670	0.0009	0.0076	0.0007	0.0391	0.0578	0.0001	2.58E-07	-9.10E-10	-2.05E-09	-5.17E-08	2.63E-07	0.0011	0.0011	0.0567	2.0035
1.3035	652.2352	0.6650	0.0009	0.0077	0.0007	0.0391	0.0578	0.0001	2.19E-07	-1.10E-09	-2.28E-09	-6.24E-08	2.28E-07	0.0012	0.0012	0.0684	1.7122
1.3036	752.8192	1.1361	0.0010	0.0077	0.0007	0.0391	0.0578	0.0001	1.65E-07	-1.41E-09	-2.53E-09	-8.00E-08	1.83E-07	0.0012	0.0012	0.0878	1.3178
1.3022	901.8296	1.9541	0.0010	0.0078	0.0007	0.0391	0.0578	0.0001	1.17E-07	-1.69E-09	-2.54E-09	-9.59E-08	1.51E-07	0.0012	0.0012	0.1052	1.1292
1.3022	1000.1110	2.5466	0.0010	0.0079	0.0007	0.0391	0.0578	0.0001	9.61E-08	-1.79E-09	-2.42E-09	-1.02E-07	1.40E-07	0.0012	0.0012	0.1115	1.0692
1.3023	1046.2973	2.8254	0.0010	0.0080	0.0007	0.0392	0.0578	0.0001	8.85E-08	-1.81E-09	-2.35E-09	-1.03E-07	1.36E-07	0.0012	0.0012	0.1130	1.0584
1.3020	1105.7242	3.2400	0.0010	0.0081	0.0007	0.0392	0.0578	0.0001	8.02E-08	-1.86E-09	-2.28E-09	-1.06E-07	1.33E-07	0.0012	0.0012	0.1160	1.0549
1.3023	1155.3342	3.6311	0.0010	0.0082	0.0007	0.0392	0.0578	0.0001	7.45E-08	-1.91E-09	-2.24E-09	-1.09E-07	1.32E-07	0.0012	0.0012	0.1191	1.0055
1.3022	1206.0095	4.0298	0.0011	0.0083	0.0007	0.0392	0.0578	0.0001	6.93E-08	-1.95E-09	-2.19E-09	-1.11E-07	1.31E-07	0.0013	0.0013	0.1213	1.0746
1.3024	1248.0299	4.4059	0.0011	0.0084	0.0007	0.0392	0.0578	0.0001	6.57E-08	-1.99E-09	-2.16E-09	-1.13E-07	1.31E-07	0.0013	0.0013	0.1239	1.0681

Uncertainty Analysis (Torque Coefficient)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	n (rpm)	Q (N-m)	P_Q	B_Q	P_n	B_n	B_ρ	B_D	$B_q\ \delta K_q/\delta Q$	$B_\rho\ \delta K_q/\delta \rho$	$B_n\ \delta K_q/\delta n$	$B_D\ \delta K_q/\delta D$	B_{Kq}	P_{Kq}	U_{Kq}	Kq	% U
1.6032	503.9995	0.1484	0.0011	0.0076	0.0008	0.0391	0.0578	0.0001	3.66E-07	-4.11E-10	-1.10E-09	-2.33E-08	3.66E-07	0.0013	0.0013	0.0256	5.2245
1.6046	550.3443	0.2710	0.0011	0.0076	0.0008	0.0391	0.0578	0.0001	3.07E-07	-6.29E-10	-1.55E-09	-3.57E-08	3.09E-07	0.0013	0.0013	0.0392	3.4096
1.6035	601.4574	0.3833	0.0011	0.0076	0.0008	0.0391	0.0578	0.0001	2.57E-07	-7.45E-10	-1.68E-09	-4.23E-08	2.60E-07	0.0014	0.0014	0.0464	2.9419
1.6038	653.7645	0.6033	0.0011	0.0077	0.0009	0.0391	0.0578	0.0001	2.18E-07	-9.92E-10	-2.05E-09	-5.63E-08	2.25E-07	0.0014	0.0014	0.0618	2.3104
1.6040	755.9224	1.0703	0.0012	0.0077	0.0008	0.0391	0.0578	0.0001	1.64E-07	-1.32E-09	-2.36E-09	-7.48E-08	1.80E-07	0.0014	0.0014	0.0820	1.7319
1.6038	911.6408	1.8340	0.0012	0.0078	0.0008	0.0391	0.0578	0.0001	1.14E-07	-1.55E-09	-2.30E-09	-8.81E-08	1.44E-07	0.0014	0.0014	0.0966	1.4351
1.6044	1003.7944	2.4389	0.0012	0.0079	0.0008	0.0391	0.0578	0.0001	9.51E-08	-1.70E-09	-2.30E-09	-9.66E-08	1.36E-07	0.0014	0.0014	0.1060	1.3345
1.6042	1051.3377	2.7228	0.0012	0.0079	0.0008	0.0392	0.0578	0.0001	8.74E-08	-1.73E-09	-2.23E-09	-9.83E-08	1.32E-07	0.0014	0.0014	0.1079	1.3139
1.6043	1104.0239	3.1449	0.0012	0.0080	0.0008	0.0392	0.0578	0.0001	8.02E-08	-1.81E-09	-2.23E-09	-1.03E-07	1.31E-07	0.0014	0.0014	0.1130	1.2422
1.6034	1152.5333	3.5384	0.0012	0.0081	0.0008	0.0392	0.0578	0.0001	7.46E-08	-1.87E-09	-2.20E-09	-1.06E-07	1.30E-07	0.0014	0.0014	0.1166	1.2198
1.6036	1210.1129	3.9176	0.0012	0.0083	0.0008	0.0392	0.0578	0.0001	6.86E-08	-1.88E-09	-2.11E-09	-1.07E-07	1.27E-07	0.0014	0.0014	0.1172	1.2336
1.6030	1250.8055	4.3454	0.0012	0.0084	0.0009	0.0392	0.0578	0.0001	6.52E-08	-1.95E-09	-2.12E-09	-1.11E-07	1.29E-07	0.0015	0.0015	0.1216	1.2205

V (m/s)	n (rpm)	Q (N-m)	P_Q	B_Q	P_n	B_n	B_ρ	B_D	$B_q\ \delta K_q/\delta Q$	$B_\rho\ \delta K_q/\delta \rho$	$B_n\ \delta K_q/\delta n$	$B_D\ \delta K_q/\delta D$	B_{Kq}	P_{Kq}	U_{Kq}	Kq	% U
1.8042	510.9808	0.1265	0.0012	0.0076	0.0010	0.0391	0.0578	0.0001	3.56E-07	-3.41E-10	-9.02E-10	-1.93E-08	3.56E-07	0.0015	0.0015	0.0212	7.2415
1.8034	552.4233	0.1528	0.0012	0.0076	0.0010	0.0391	0.0578	0.0001	3.04E-07	-3.52E-10	-8.62E-10	-2.00E-08	3.05E-07	0.0016	0.0016	0.0219	7.2300
1.8068	602.7853	0.3126	0.0012	0.0076	0.0009	0.0391	0.0578	0.0001	2.56E-07	-6.05E-10	-1.36E-09	-3.43E-08	2.58E-07	0.0015	0.0015	0.0377	4.0886
1.8056	651.5334	0.5023	0.0012	0.0076	0.0009	0.0391	0.0578	0.0001	2.19E-07	-8.32E-10	-1.73E-09	-4.72E-08	2.24E-07	0.0015	0.0015	0.0518	2.9799
1.8042	757.8054	0.9753	0.0012	0.0077	0.0010	0.0391	0.0578	0.0001	1.63E-07	-1.19E-09	-2.13E-09	-6.78E-08	1.76E-07	0.0016	0.0016	0.0744	2.1486
1.8051	911.1136	1.7271	0.0013	0.0078	0.0010	0.0391	0.0578	0.0001	1.14E-07	-1.46E-09	-2.17E-09	-8.30E-08	1.41E-07	0.0016	0.0016	0.0911	1.7848
1.8048	1002.7594	2.3395	0.0013	0.0079	0.0010	0.0391	0.0578	0.0001	9.51E-08	-1.64E-09	-2.21E-09	-9.29E-08	1.33E-07	0.0016	0.0016	0.1019	1.5865
1.8061	1056.9214	2.6019	0.0013	0.0079	0.0010	0.0392	0.0578	0.0001	8.62E-08	-1.64E-09	-2.10E-09	-9.30E-08	1.27E-07	0.0016	0.0016	0.1020	1.5650
1.8051	1118.8065	3.0425	0.0013	0.0080	0.0010	0.0392	0.0578	0.0001	7.79E-08	-1.71E-09	-2.07E-09	-9.70E-08	1.24E-07	0.0016	0.0016	0.1064	1.4957
1.8043	1161.1239	3.4240	0.0013	0.0081	0.0010	0.0392	0.0578	0.0001	7.32E-08	-1.79E-09	-2.08E-09	-1.01E-07	1.25E-07	0.0016	0.0016	0.1112	1.4717
1.8059	1215.6599	3.7826	0.0013	0.0082	0.0010	0.0392	0.0578	0.0001	6.76E-08	-1.80E-09	-2.01E-09	-1.02E-07	1.23E-07	0.0016	0.0016	0.1121	1.4596
1.8057	1246.0544	4.1864	0.0013	0.0083	0.0010	0.0392	0.0578	0.0001	6.53E-08	-1.90E-09	-2.06E-09	-1.08E-07	1.26E-07	0.0017	0.0017	0.1181	1.4026

V (m/s)	n (rpm)	Q (N-m)	P_Q	B_Q	P_n	B_n	B_ρ	B_D	$B_q\ \delta K_q/\delta Q$	$B_\rho\ \delta K_q/\delta \rho$	$B_n\ \delta K_q/\delta n$	$B_D\ \delta K_q/\delta D$	B_{Kq}	P_{Kq}	U_{Kq}	Kq	% U
2.1091	500.1371	0.0592	0.0006	0.0076	0.0011	0.0391	0.0578	0.0001	3.71E-07	-1.66E-10	-4.50E-10	-9.45E-09	3.71E-07	0.0013	0.0013	0.0104	12.0880
2.1056	551.5545	0.1283	0.0013	0.0076	0.0012	0.0391	0.0578	0.0001	3.05E-07	-2.97E-10	-7.27E-10	-1.68E-08	3.06E-07	0.0017	0.0017	0.0185	9.4342
2.1068	600.3476	0.2676	0.0013	0.0076	0.0011	0.0391	0.0578	0.0001	2.58E-07	-5.22E-10	-1.18E-09	-2.96E-08	2.59E-07	0.0018	0.0018	0.0325	5.3884
2.1067	654.3489	0.4750	0.0013	0.0076	0.0011	0.0391	0.0578	0.0001	2.17E-07	-7.80E-10	-1.61E-09	-4.43E-08	2.22E-07	0.0017	0.0017	0.0486	3.5688
2.1101	752.6505	0.8776	0.0014	0.0077	0.0011	0.0391	0.0578	0.0001	1.65E-07	-1.09E-09	-1.96E-09	-6.18E-08	1.76E-07	0.0018	0.0018	0.0678	2.5971
2.1068	901.9876	1.6341	0.0014	0.0077	0.0012	0.0391	0.0578	0.0001	1.16E-07	-1.41E-09	-2.12E-09	-8.02E-08	1.41E-07	0.0018	0.0018	0.0880	2.0410
2.1070	1002.6789	2.2376	0.0014	0.0078	0.0012	0.0391	0.0578	0.0001	9.49E-08	-1.56E-09	-2.11E-09	-8.88E-08	1.30E-07	0.0018	0.0018	0.0975	1.8630
2.0959	1052.8833	2.5437	0.0014	0.0079	0.0011	0.0392	0.0578	0.0001	8.67E-08	-1.61E-09	-2.08E-09	-9.16E-08	1.26E-07	0.0018	0.0018	0.1005	1.7891
2.1064	1103.4297	2.9378	0.0014	0.0080	0.0011	0.0392	0.0578	0.0001	7.98E-08	-1.70E-09	-2.08E-09	-9.63E-08	1.25E-07	0.0018	0.0018	0.1057	1.7117
2.1073	1156.4853	3.3080	0.0014	0.0081	0.0011	0.0392	0.0578	0.0001	7.35E-08	-1.74E-09	-2.04E-09	-9.87E-08	1.23E-07	0.0018	0.0018	0.1083	1.6546
2.1066	1214.1716	3.6923	0.0015	0.0082	0.0011	0.0392	0.0578	0.0001	6.76E-08	-1.76E-09	-1.97E-09	-1.00E-07	1.21E-07	0.0018	0.0018	0.1097	1.6829
2.1060	1246.6579	4.0446	0.0015	0.0083	0.0011	0.0392	0.0578	0.0001	6.49E-08	-1.83E-09	-1.99E-09	-1.04E-07	1.23E-07	0.0019	0.0019	0.1140	1.6248

Uncertainty Analysis (Torque Coefficient)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	n (rpm)	Q (N-m)	P_Q	B_Q	P_n	B_n	B_ρ	B_D	$B_q\ \delta K_q/\delta Q$	$B_\rho\ \delta K_q/\delta\rho$	$B_n\ \delta K_q/\delta n$	$B_D\ \delta K_q/\delta D$	B_{Kq}	P_{Kq}	U_{Kq}	Kq	% U
2.3079	502.2690	0.0030	0.0002	0.0076	0.0013	0.0391	0.0578	0.0001	3.68E-07	-8.38E-12	-2.26E-11	-4.76E-10	3.68E-07	0.0013	0.0013	0.0005	248.1630
2.3069	550.3896	0.1071	0.0015	0.0076	0.0012	0.0391	0.0578	0.0001	3.07E-07	-2.49E-10	-6.11E-10	-1.41E-08	3.07E-07	0.0019	0.0019	0.0155	12.4668
2.3074	603.5423	0.2439	0.0015	0.0076	0.0013	0.0391	0.0578	0.0001	2.55E-07	-4.71E-10	-1.06E-09	-2.67E-08	2.56E-07	0.0020	0.0020	0.0293	6.7772
2.3070	652.5464	0.4126	0.0015	0.0076	0.0013	0.0391	0.0578	0.0001	2.18E-07	-6.81E-10	-1.41E-09	-3.87E-08	2.22E-07	0.0020	0.0020	0.0424	4.6865
2.3084	752.0456	0.7863	0.0015	0.0077	0.0013	0.0391	0.0578	0.0001	1.65E-07	-9.77E-10	-1.76E-09	-5.55E-08	1.74E-07	0.0020	0.0020	0.0609	3.3531
2.3081	901.6673	1.5450	0.0016	0.0077	0.0013	0.0391	0.0578	0.0001	1.16E-07	-1.34E-09	-2.01E-09	-7.58E-08	1.38E-07	0.0020	0.0020	0.0832	2.4310
2.3082	1006.0527	2.1446	0.0016	0.0078	0.0013	0.0391	0.0578	0.0001	9.41E-08	-1.49E-09	-2.01E-09	-8.46E-08	1.27E-07	0.0021	0.0021	0.0928	2.2106
2.3077	1052.1662	2.4494	0.0016	0.0079	0.0013	0.0392	0.0578	0.0001	8.66E-08	-1.56E-09	-2.00E-09	-8.83E-08	1.24E-07	0.0020	0.0020	0.0969	2.0939
2.3074	1101.3796	2.7844	0.0016	0.0080	0.0013	0.0392	0.0578	0.0001	7.98E-08	-1.61E-09	-1.99E-09	-9.16E-08	1.21E-07	0.0021	0.0021	0.1005	2.0541
2.3075	1150.4349	3.1913	0.0017	0.0081	0.0013	0.0392	0.0578	0.0001	7.40E-08	-1.70E-09	-2.00E-09	-9.62E-08	1.21E-07	0.0021	0.0021	0.1056	1.9894
2.3077	1213.5677	3.5556	0.0017	0.0081	0.0013	0.0392	0.0578	0.0001	6.73E-08	-1.70E-09	-1.90E-09	-9.63E-08	1.18E-07	0.0021	0.0021	0.1057	2.0061
2.3068	1249.1721	3.9343	0.0017	0.0083	0.0013	0.0392	0.0578	0.0001	6.44E-08	-1.77E-09	-1.92E-09	-1.01E-07	1.19E-07	0.0021	0.0021	0.1104	1.9393

V (m/s)	n (rpm)	Q (N-m)	P_Q	B_Q	P_n	B_n	B_ρ	B_D	$B_q\ \delta K_q/\delta Q$	$B_\rho\ \delta K_q/\delta\rho$	$B_n\ \delta K_q/\delta n$	$B_D\ \delta K_q/\delta D$	B_{Kq}	P_{Kq}	U_{Kq}	Kq	% U
2.5100	504.0554	0.0003	0.0001	0.0076	0.0015	0.0391	0.0578	0.0001	3.66E-07	-9.47E-13	-2.54E-12	-5.37E-11	3.66E-07	0.0015	0.0015	0.0001	2478.4485
2.5086	552.5634	0.0817	0.0014	0.0076	0.0014	0.0391	0.0578	0.0001	3.04E-07	-1.88E-10	-4.61E-10	-1.07E-08	3.04E-07	0.0020	0.0020	0.0117	17.2520
2.5072	601.0760	0.2188	0.0017	0.0076	0.0015	0.0391	0.0578	0.0001	2.57E-07	-4.26E-10	-9.58E-10	-2.42E-08	2.58E-07	0.0023	0.0023	0.0265	8.5581
2.5095	651.4524	0.3268	0.0017	0.0076	0.0015	0.0391	0.0578	0.0001	2.19E-07	-5.41E-10	-1.12E-09	-3.07E-08	2.21E-07	0.0023	0.0023	0.0337	6.7359
2.5020	751.7646	0.7249	0.0018	0.0077	0.0015	0.0391	0.0578	0.0001	1.65E-07	-9.02E-10	-1.62E-09	-5.12E-08	1.73E-07	0.0023	0.0023	0.0562	4.1171
2.5092	905.0740	1.4758	0.0018	0.0077	0.0014	0.0391	0.0578	0.0001	1.15E-07	-1.27E-09	-1.90E-09	-7.19E-08	1.35E-07	0.0023	0.0023	0.0789	2.9021
2.5081	1004.8353	2.0551	0.0018	0.0078	0.0015	0.0391	0.0578	0.0001	9.41E-08	-1.43E-09	-1.93E-09	-8.12E-08	1.24E-07	0.0023	0.0023	0.0891	2.5887
2.5087	1052.4015	2.3871	0.0018	0.0079	0.0015	0.0392	0.0578	0.0001	8.64E-08	-1.52E-09	-1.95E-09	-8.60E-08	1.22E-07	0.0023	0.0023	0.0944	2.4229
2.5087	1103.3557	2.6963	0.0018	0.0079	0.0014	0.0392	0.0578	0.0001	7.93E-08	-1.56E-09	-1.91E-09	-8.84E-08	1.19E-07	0.0023	0.0023	0.0970	2.3687
2.5023	1154.7567	3.0789	0.0018	0.0080	0.0015	0.0392	0.0578	0.0001	7.32E-08	-1.62E-09	-1.91E-09	-9.21E-08	1.18E-07	0.0023	0.0023	0.1011	2.3201
2.5092	1207.1602	3.4660	0.0018	0.0081	0.0015	0.0392	0.0578	0.0001	6.78E-08	-1.67E-09	-1.88E-09	-9.49E-08	1.17E-07	0.0024	0.0024	0.1042	2.2914
2.5091	1252.5791	3.8650	0.0019	0.0082	0.0015	0.0392	0.0578	0.0001	6.39E-08	-1.73E-09	-1.88E-09	-9.83E-08	1.17E-07	0.0024	0.0024	0.1079	2.2150

Load Varying Self-Propulsion Test (Propeller fitted on)

Uncertainty Analysis (Thrust Coefficient)

V	Vehicle Velocity (m/s)	
n	Propeller Rotational Speed (rpm)	
T_P	Propeller Thrust (N)	
D	Propeller Diameter (m)	
ρ	Density of Water (kg/m^3)	
K_{tp}	Thrust Coefficient for Propeller	
P_T	Precision Error for T_P	
B_T	Bias Error for T_P	
P_{Ktp}	Precision Error for K_{tp}	
P_n	Precision Error for n	
B_n	Bias Error for n	
B_ρ	Bias Error for ρ	
B_D	Bias Error for D	
B_{Ktp}	Bias Error for K_{tp}	$B_{Ktp} = ((\delta K_{tp} / \delta T_P\ B_T)^2 + (\delta K_{tp} / \delta\rho\ B_\rho)^2 + (\delta K_{tp} / \delta n\ B_n)^2 + (\delta K_{tp} / \delta D\ B_D)^2)^{1/2}$
U_{Ktp}	Total Error for K_{tp}	$U_{Ktp} = (B_{Ktp}^2 + P_{Ktp}^2)^{1/2}$
% U	Percentage Uncertainty for K_{tp}	$\% U = (U_{Ktp} / K_{tp}) \times 100$

Uncertainty Analysis (Thrust Coefficient)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	n (rpm)	T_P (N)	P_T	B_T	P_n	B_n	B_ρ	B_D	$B_T\ \delta K_{tp}/\delta T_P$	$B_\rho\ \delta K_{tp}/\delta\rho$	$B_n\ \delta K_{tp}/\delta n$	$B_D\ \delta K_{tp}/\delta D$	B_{Ktp}	P_{Ktp}	U_{Ktp}	K_{tp}	% U
0.8007	503.8949	3.6262	0.0085	0.2494	0.0004	0.0391	0.0578	0.0001	1.82E-06	-1.53E-09	-4.11E-09	-6.95E-08	1.82E-06	0.0085	0.0085	0.0953	8.9298
0.8012	552.6343	8.7409	0.0087	0.2494	0.0005	0.0391	0.0578	0.0001	1.51E-06	-3.07E-09	-7.51E-09	-1.39E-07	1.52E-06	0.0087	0.0087	0.1910	4.5613
0.8027	602.3573	12.8134	0.0088	0.2494	0.0005	0.0391	0.0578	0.0001	1.27E-06	-3.78E-09	-8.50E-09	-1.72E-07	1.29E-06	0.0088	0.0088	0.2357	3.7397
0.8034	651.5623	18.6583	0.0086	0.2494	0.0005	0.0391	0.0578	0.0001	1.09E-06	-4.71E-09	-9.78E-09	-2.14E-07	1.11E-06	0.0086	0.0086	0.2933	2.9366
0.8005	753.4148	32.8282	0.0089	0.2494	0.0004	0.0391	0.0578	0.0001	8.14E-07	-6.20E-09	-1.11E-08	-2.81E-07	8.62E-07	0.0089	0.0089	0.3860	2.3088
0.8012	913.8563	59.3485	0.0089	0.2494	0.0005	0.0391	0.0578	0.0001	5.54E-07	-7.61E-09	-1.13E-08	-3.46E-07	6.53E-07	0.0089	0.0089	0.4743	1.8796
0.8037	1002.5757	77.0108	0.0092	0.2494	0.0005	0.0391	0.0578	0.0001	4.60E-07	-8.21E-09	-1.11E-08	-3.73E-07	5.92E-07	0.0092	0.0092	0.5113	1.8016
0.8045	1051.5807	87.1072	0.0095	0.2494	0.0005	0.0392	0.0578	0.0001	4.18E-07	-8.44E-09	-1.09E-08	-3.83E-07	5.67E-07	0.0095	0.0095	0.5257	1.8093
0.8098	1102.1417	98.8701	0.0094	0.2494	0.0004	0.0392	0.0578	0.0001	3.81E-07	-8.72E-09	-1.07E-08	-3.96E-07	5.49E-07	0.0094	0.0094	0.5432	1.7324
0.8032	1154.0417	111.2684	0.0096	0.2494	0.0005	0.0392	0.0578	0.0001	3.47E-07	-8.95E-09	-1.05E-08	-4.07E-07	5.35E-07	0.0096	0.0096	0.5576	1.7241
0.8015	1207.3702	123.8401	0.0094	0.2494	0.0005	0.0392	0.0578	0.0001	3.17E-07	-9.10E-09	-1.02E-08	-4.13E-07	5.21E-07	0.0094	0.0094	0.5669	1.6602
0.8022	1246.6728	135.8194	0.0097	0.2494	0.0005	0.0392	0.0578	0.0001	2.98E-07	-9.36E-09	-1.02E-08	-4.25E-07	5.19E-07	0.0097	0.0097	0.5832	1.6654

V (m/s)	n (rpm)	T_P (N)	P_T	B_T	P_n	B_n	B_ρ	B_D	$B_T\ \delta K_{tp}/\delta T_P$	$B_\rho\ \delta K_{tp}/\delta\rho$	$B_n\ \delta K_{tp}/\delta n$	$B_D\ \delta K_{tp}/\delta D$	B_{Ktp}	P_{Ktp}	U_{Ktp}	K_{tp}	% U
1.1030	507.2079	2.2849	0.0095	0.2494	0.0006	0.0391	0.0578	0.0001	1.80E-06	-9.52E-10	-2.54E-09	-4.32E-08	1.80E-06	0.0095	0.0095	0.0593	16.0554
1.1027	554.5345	6.2834	0.0098	0.2494	0.0006	0.0391	0.0578	0.0001	1.50E-06	-2.19E-09	-5.34E-09	-9.94E-08	1.51E-06	0.0098	0.0098	0.1364	7.1992
1.1035	603.8746	10.2080	0.0100	0.2494	0.0006	0.0391	0.0578	0.0001	1.27E-06	-3.00E-09	-6.72E-09	-1.36E-07	1.27E-06	0.0100	0.0100	0.1868	5.3625
1.1032	654.2447	16.3454	0.0102	0.2494	0.0006	0.0391	0.0578	0.0001	1.08E-06	-4.09E-09	-8.46E-09	-1.86E-07	1.10E-06	0.0102	0.0102	0.2548	4.0103
1.1044	750.3733	29.4461	0.0103	0.2494	0.0006	0.0391	0.0578	0.0001	8.21E-07	-5.60E-09	-1.01E-08	-2.54E-07	8.60E-07	0.0103	0.0103	0.3490	2.9566
1.1041	910.3564	54.5330	0.0106	0.2494	0.0006	0.0391	0.0578	0.0001	5.58E-07	-7.05E-09	-1.05E-08	-3.20E-07	6.43E-07	0.0106	0.0106	0.4391	2.4182
1.1030	1006.6678	73.9689	0.0108	0.2494	0.0007	0.0391	0.0578	0.0001	4.56E-07	-7.82E-09	-1.05E-08	-3.55E-07	5.78E-07	0.0108	0.0108	0.4871	2.2212
1.1037	1056.9924	85.6126	0.0109	0.2494	0.0006	0.0392	0.0578	0.0001	4.14E-07	-8.21E-09	-1.05E-08	-3.73E-07	5.57E-07	0.0109	0.0109	0.5114	2.1349
1.1031	1108.8989	98.4729	0.0111	0.2494	0.0006	0.0392	0.0578	0.0001	3.76E-07	-8.58E-09	-1.05E-08	-3.90E-07	5.42E-07	0.0111	0.0111	0.5344	2.0798
1.1031	1156.0328	110.2023	0.0109	0.2494	0.0006	0.0392	0.0578	0.0001	3.46E-07	-8.84E-09	-1.04E-08	-4.01E-07	5.30E-07	0.0109	0.0109	0.5503	1.9839
1.1029	1206.4133	121.4830	0.0111	0.2494	0.0006	0.0392	0.0578	0.0001	3.18E-07	-8.94E-09	-1.01E-08	-4.06E-07	5.16E-07	0.0111	0.0111	0.5570	1.9952
1.1034	1249.3622	132.4165	0.0113	0.2494	0.0006	0.0392	0.0578	0.0001	2.96E-07	-9.09E-09	-9.87E-09	-4.13E-07	5.08E-07	0.0113	0.0113	0.5661	1.9986

V (m/s)	n (rpm)	T_P (N)	P_T	B_T	P_n	B_n	B_ρ	B_D	$B_T\ \delta K_{tp}/\delta T_P$	$B_\rho\ \delta K_{tp}/\delta\rho$	$B_n\ \delta K_{tp}/\delta n$	$B_D\ \delta K_{tp}/\delta D$	B_{Ktp}	P_{Ktp}	U_{Ktp}	K_{tp}	% U
1.3031	504.2337	-0.5806	0.0090	0.2494	0.0006	0.0391	0.0578	0.0001	1.82E-06	2.45E-10	6.56E-10	1.11E-08	1.82E-06	0.0090	0.0090	-0.0152	-59.2044
1.3022	550.4535	3.8393	0.0117	0.2494	0.0006	0.0391	0.0578	0.0001	1.53E-06	-1.36E-09	-3.34E-09	-6.17E-08	1.53E-06	0.0117	0.0117	0.0846	13.8531
1.3025	600.5634	7.2149	0.0118	0.2494	0.0007	0.0391	0.0578	0.0001	1.28E-06	-2.14E-09	-4.83E-09	-9.73E-08	1.29E-06	0.0118	0.0118	0.1335	8.8538
1.3035	652.2352	13.4575	0.0117	0.2494	0.0007	0.0391	0.0578	0.0001	1.09E-06	-3.39E-09	-7.03E-09	-1.54E-07	1.10E-06	0.0117	0.0117	0.2111	5.5519
1.3036	752.8192	26.1866	0.0119	0.2494	0.0007	0.0391	0.0578	0.0001	8.16E-07	-4.95E-09	-8.90E-09	-2.25E-07	8.46E-07	0.0119	0.0119	0.3084	3.8650
1.3022	901.8296	51.7382	0.0123	0.2494	0.0007	0.0391	0.0578	0.0001	5.68E-07	-6.82E-09	-1.02E-08	-3.10E-07	6.47E-07	0.0123	0.0123	0.4245	2.9019
1.3022	1000.1110	72.3597	0.0121	0.2494	0.0007	0.0391	0.0578	0.0001	4.62E-07	-7.75E-09	-1.05E-08	-3.52E-07	5.81E-07	0.0121	0.0121	0.4828	2.5107
1.3023	1046.2973	82.1375	0.0125	0.2494	0.0007	0.0392	0.0578	0.0001	4.22E-07	-8.04E-09	-1.04E-08	-3.65E-07	5.58E-07	0.0125	0.0125	0.5007	2.5003
1.3020	1105.7242	96.1106	0.0128	0.2494	0.0007	0.0392	0.0578	0.0001	3.78E-07	-8.42E-09	-1.03E-08	-3.82E-07	5.38E-07	0.0128	0.0128	0.5246	2.4437
1.3023	1155.3342	107.4396	0.0127	0.2494	0.0007	0.0392	0.0578	0.0001	3.46E-07	-8.62E-09	-1.01E-08	-3.92E-07	5.23E-07	0.0127	0.0127	0.5372	2.3677
1.3022	1206.0095	119.2426	0.0129	0.2494	0.0007	0.0392	0.0578	0.0001	3.18E-07	-8.78E-09	-9.88E-09	-3.99E-07	5.10E-07	0.0129	0.0129	0.5471	2.3612
1.3024	1248.0299	130.6479	0.0131	0.2494	0.0007	0.0392	0.0578	0.0001	2.97E-07	-8.99E-09	-9.77E-09	-4.08E-07	5.05E-07	0.0131	0.0131	0.5598	2.3437

Uncertainty Analysis (Thrust Coefficient)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	n (rpm)	T_P (N)	P_T	B_T	P_n	B_n	B_ρ	B_D	$B_T\ \delta K_{tp}/\delta T_P$	$B_\rho\ \delta K_{tp}/\delta\rho$	$B_n\ \delta K_{tp}/\delta n$	$B_D\ \delta K_{tp}/\delta D$	B_{Ktp}	P_{Ktp}	U_{Ktp}	K_{tp}	% U
1.6032	503.9995	-1.5261	0.0130	0.2494	0.0008	0.0391	0.0578	0.0001	1.82E-06	6.44E-10	1.73E-09	2.92E-08	1.82E-06	0.0130	0.0130	-0.0401	-32.4782
1.6046	550.3443	1.5354	0.0137	0.2494	0.0008	0.0391	0.0578	0.0001	1.53E-06	-5.43E-10	-1.34E-09	-2.47E-08	1.53E-06	0.0137	0.0137	0.0338	40.5633
1.6035	601.4574	5.3738	0.0143	0.2494	0.0008	0.0391	0.0578	0.0001	1.28E-06	-1.59E-09	-3.58E-09	-7.23E-08	1.28E-06	0.0143	0.0143	0.0991	14.4461
1.6038	653.7645	11.6456	0.0145	0.2494	0.0009	0.0391	0.0578	0.0001	1.08E-06	-2.92E-09	-6.04E-09	-1.33E-07	1.09E-06	0.0145	0.0145	0.1818	7.9882
1.6040	755.9224	25.1105	0.0143	0.2494	0.0008	0.0391	0.0578	0.0001	8.09E-07	-4.71E-09	-8.43E-09	-2.14E-07	8.37E-07	0.0143	0.0143	0.2933	4.8841
1.6038	911.6408	48.1265	0.0147	0.2494	0.0008	0.0391	0.0578	0.0001	5.56E-07	-6.20E-09	-9.22E-09	-2.82E-07	6.24E-07	0.0147	0.0147	0.3865	3.8089
1.6044	1003.7944	67.4672	0.0148	0.2494	0.0008	0.0391	0.0578	0.0001	4.59E-07	-7.17E-09	-9.68E-09	-3.26E-07	5.63E-07	0.0148	0.0148	0.4469	3.3166
1.6042	1051.3377	77.7648	0.0150	0.2494	0.0008	0.0392	0.0578	0.0001	4.18E-07	-7.54E-09	-9.72E-09	-3.42E-07	5.41E-07	0.0150	0.0150	0.4695	3.1992
1.6043	1104.0239	91.4829	0.0152	0.2494	0.0008	0.0392	0.0578	0.0001	3.79E-07	-8.04E-09	-9.87E-09	-3.65E-07	5.27E-07	0.0152	0.0152	0.5009	3.0384
1.6034	1152.5333	104.5737	0.0155	0.2494	0.0008	0.0392	0.0578	0.0001	3.48E-07	-8.44E-09	-9.92E-09	-3.83E-07	5.18E-07	0.0155	0.0155	0.5254	2.9539
1.6036	1210.1129	116.7686	0.0154	0.2494	0.0008	0.0392	0.0578	0.0001	3.16E-07	-8.54E-09	-9.57E-09	-3.88E-07	5.00E-07	0.0154	0.0154	0.5322	2.8980
1.6030	1250.8055	127.3434	0.0157	0.2494	0.0009	0.0392	0.0578	0.0001	2.96E-07	-8.72E-09	-9.46E-09	-3.96E-07	4.94E-07	0.0157	0.0157	0.5432	2.8946

V (m/s)	n (rpm)	T_P (N)	P_T	B_T	P_n	B_n	B_ρ	B_D	$B_T\ \delta K_{tp}/\delta T_P$	$B_\rho\ \delta K_{tp}/\delta\rho$	$B_n\ \delta K_{tp}/\delta n$	$B_D\ \delta K_{tp}/\delta D$	B_{Ktp}	P_{Ktp}	U_{Ktp}	K_{tp}	% U
1.8042	510.9808	-1.9385	0.0161	0.2494	0.0010	0.0391	0.0578	0.0001	1.77E-06	7.96E-10	2.11E-09	3.61E-08	1.77E-06	0.0161	0.0161	-0.0495	-32.5520
1.8034	552.4233	1.1898	0.0158	0.2494	0.0010	0.0391	0.0578	0.0001	1.51E-06	-4.18E-10	-1.02E-09	-1.90E-08	1.51E-06	0.0158	0.0158	0.0260	60.8484
1.8068	602.7853	4.2981	0.0163	0.2494	0.0009	0.0391	0.0578	0.0001	1.27E-06	-1.27E-09	-2.84E-09	-5.76E-08	1.27E-06	0.0163	0.0163	0.0789	20.6820
1.8056	651.5334	10.5564	0.0164	0.2494	0.0009	0.0391	0.0578	0.0001	1.09E-06	-2.66E-09	-5.53E-09	-1.21E-07	1.10E-06	0.0164	0.0164	0.1660	9.8974
1.8042	757.8054	22.9946	0.0165	0.2494	0.0010	0.0391	0.0578	0.0001	8.05E-07	-4.29E-09	-7.66E-09	-1.95E-07	8.28E-07	0.0165	0.0165	0.2672	6.1864
1.8051	911.1136	45.8331	0.0167	0.2494	0.0010	0.0391	0.0578	0.0001	5.57E-07	-5.92E-09	-8.79E-09	-2.69E-07	6.18E-07	0.0167	0.0167	0.3685	4.5411
1.8048	1002.7594	65.0234	0.0167	0.2494	0.0010	0.0391	0.0578	0.0001	4.60E-07	-6.93E-09	-9.36E-09	-3.15E-07	5.57E-07	0.0167	0.0167	0.4316	3.8766
1.8061	1056.9214	73.8682	0.0169	0.2494	0.0010	0.0392	0.0578	0.0001	4.14E-07	-7.09E-09	-9.08E-09	-3.22E-07	5.24E-07	0.0169	0.0169	0.4413	3.8360
1.8051	1118.8065	87.2252	0.0172	0.2494	0.0010	0.0392	0.0578	0.0001	3.69E-07	-7.47E-09	-9.04E-09	-3.39E-07	5.02E-07	0.0172	0.0172	0.4650	3.7043
1.8043	1161.1239	100.0789	0.0173	0.2494	0.0010	0.0392	0.0578	0.0001	3.43E-07	-7.95E-09	-9.29E-09	-3.61E-07	4.98E-07	0.0173	0.0173	0.4954	3.4983
1.8059	1215.6599	112.6564	0.0175	0.2494	0.0010	0.0392	0.0578	0.0001	3.13E-07	-8.17E-09	-9.11E-09	-3.71E-07	4.85E-07	0.0175	0.0175	0.5087	3.4453
1.8057	1246.0544	125.3448	0.0176	0.2494	0.0010	0.0392	0.0578	0.0001	2.98E-07	-8.65E-09	-9.41E-09	-3.93E-07	4.93E-07	0.0176	0.0176	0.5388	3.2720

V (m/s)	n (rpm)	T_P (N)	P_T	B_T	P_n	B_n	B_ρ	B_D	$B_T\ \delta K_{tp}/\delta T_P$	$B_\rho\ \delta K_{tp}/\delta\rho$	$B_n\ \delta K_{tp}/\delta n$	$B_D\ \delta K_{tp}/\delta D$	B_{Ktp}	P_{Ktp}	U_{Ktp}	K_{tp}	% U
2.1091	500.1371	-2.2244	0.0173	0.2494	0.0011	0.0391	0.0578	0.0001	1.85E-06	9.53E-10	2.58E-09	4.33E-08	1.85E-06	0.0173	0.0173	-0.0593	-29.2102
2.1056	551.5545	0.7034	0.0110	0.2494	0.0012	0.0391	0.0578	0.0001	1.52E-06	-2.48E-10	-6.08E-10	-1.13E-08	1.52E-06	0.0111	0.0111	0.0154	71.6773
2.1068	600.3476	3.3349	0.0183	0.2494	0.0011	0.0391	0.0578	0.0001	1.28E-06	-9.91E-10	-2.23E-09	-4.50E-08	1.28E-06	0.0183	0.0183	0.0618	29.6928
2.1067	654.3489	9.0348	0.0182	0.2494	0.0011	0.0391	0.0578	0.0001	1.08E-06	-2.26E-09	-4.68E-09	-1.03E-07	1.08E-06	0.0182	0.0182	0.1408	12.9479
2.1101	752.6505	19.5316	0.0185	0.2494	0.0011	0.0391	0.0578	0.0001	8.16E-07	-3.69E-09	-6.64E-09	-1.68E-07	8.33E-07	0.0185	0.0185	0.2301	8.0547
2.1068	901.9876	42.0655	0.0186	0.2494	0.0012	0.0391	0.0578	0.0001	5.68E-07	-5.54E-09	-8.32E-09	-2.52E-07	6.21E-07	0.0186	0.0186	0.3451	5.4009
2.1070	1002.6789	59.4139	0.0187	0.2494	0.0012	0.0391	0.0578	0.0001	4.60E-07	-6.33E-09	-8.55E-09	-2.88E-07	5.42E-07	0.0187	0.0187	0.3944	4.7509
2.0959	1052.8833	68.8805	0.0190	0.2494	0.0011	0.0392	0.0578	0.0001	4.17E-07	-6.66E-09	-8.57E-09	-3.02E-07	5.15E-07	0.0190	0.0190	0.4147	4.5902
2.1064	1103.4297	80.5944	0.0192	0.2494	0.0011	0.0392	0.0578	0.0001	3.80E-07	-7.09E-09	-8.71E-09	-3.22E-07	4.98E-07	0.0192	0.0192	0.4418	4.3537
2.1073	1156.4853	93.1518	0.0193	0.2494	0.0011	0.0392	0.0578	0.0001	3.46E-07	-7.46E-09	-8.75E-09	-3.39E-07	4.84E-07	0.0193	0.0193	0.4648	4.1587
2.1066	1214.1716	106.0173	0.0196	0.2494	0.0011	0.0392	0.0578	0.0001	3.14E-07	-7.71E-09	-8.61E-09	-3.50E-07	4.70E-07	0.0196	0.0196	0.4799	4.0908
2.1060	1246.6579	117.5641	0.0196	0.2494	0.0011	0.0392	0.0578	0.0001	2.98E-07	-8.11E-09	-8.82E-09	-3.68E-07	4.73E-07	0.0196	0.0196	0.5048	3.8889

Uncertainty Analysis (Thrust Coefficient)
Load Varying Self-Propulsion Test (Propeller fitted on)

V (m/s)	n (rpm)	T_P (N)	P_T	B_T	P_n	B_n	B_ρ	B_D	$B_T\ \delta K_{tp}/\delta T_P$	$B_\rho\ \delta K_{tp}/\delta\rho$	$B_n\ \delta K_{tp}/\delta n$	$B_D\ \delta K_{tp}/\delta D$	B_{Ktp}	P_{Ktp}	U_{Ktp}	K_{tp}	% U
2.3079	502.2690	-3.6889	0.0205	0.2494	0.0013	0.0391	0.0578	0.0001	1.83E-06	1.57E-09	4.22E-09	7.11E-08	1.83E-06	0.0205	0.0205	-0.0976	-21.0482
2.3069	550.3896	-0.4585	0.0119	0.2494	0.0012	0.0391	0.0578	0.0001	1.53E-06	1.62E-10	3.99E-10	7.36E-09	1.53E-06	0.0120	0.0120	-0.0101	-118.4513
2.3074	603.5423	2.8578	0.0200	0.2494	0.0013	0.0391	0.0578	0.0001	1.27E-06	-8.41E-10	-1.88E-09	-3.82E-08	1.27E-06	0.0200	0.0200	0.0524	38.2765
2.3070	652.5464	7.3859	0.0207	0.2494	0.0013	0.0391	0.0578	0.0001	1.09E-06	-1.86E-09	-3.85E-09	-8.44E-08	1.09E-06	0.0207	0.0207	0.1158	17.9174
2.3084	752.0456	17.3533	0.0208	0.2494	0.0013	0.0391	0.0578	0.0001	8.17E-07	-3.29E-09	-5.92E-09	-1.49E-07	8.31E-07	0.0208	0.0208	0.2048	10.1790
2.3081	901.6673	39.3701	0.0209	0.2494	0.0013	0.0391	0.0578	0.0001	5.69E-07	-5.19E-09	-7.79E-09	-2.36E-07	6.16E-07	0.0209	0.0209	0.3232	6.4796
2.3082	1006.0527	56.3185	0.0210	0.2494	0.0013	0.0391	0.0578	0.0001	4.57E-07	-5.96E-09	-8.03E-09	-2.71E-07	5.31E-07	0.0210	0.0210	0.3713	5.6663
2.3077	1052.1662	65.8199	0.0210	0.2494	0.0013	0.0392	0.0578	0.0001	4.18E-07	-6.37E-09	-8.20E-09	-2.89E-07	5.08E-07	0.0210	0.0210	0.3968	5.3021
2.3074	1101.3796	75.6556	0.0212	0.2494	0.0013	0.0392	0.0578	0.0001	3.81E-07	-6.68E-09	-8.22E-09	-3.03E-07	4.87E-07	0.0212	0.0212	0.4162	5.1027
2.3075	1150.4349	88.4838	0.0215	0.2494	0.0013	0.0392	0.0578	0.0001	3.49E-07	-7.16E-09	-8.44E-09	-3.25E-07	4.77E-07	0.0215	0.0215	0.4462	4.8276
2.3077	1213.5677	102.4455	0.0219	0.2494	0.0013	0.0392	0.0578	0.0001	3.14E-07	-7.45E-09	-8.33E-09	-3.38E-07	4.62E-07	0.0219	0.0219	0.4642	4.7261
2.3068	1249.1721	114.3675	0.0220	0.2494	0.0013	0.0392	0.0578	0.0001	2.96E-07	-7.85E-09	-8.53E-09	-3.57E-07	4.64E-07	0.0220	0.0220	0.4891	4.5062

V (m/s)	n (rpm)	T_P (N)	P_T	B_T	P_n	B_n	B_ρ	B_D	$B_T\ \delta K_{tp}/\delta T_P$	$B_\rho\ \delta K_{tp}/\delta\rho$	$B_n\ \delta K_{tp}/\delta n$	$B_D\ \delta K_{tp}/\delta D$	B_{Ktp}	P_{Ktp}	U_{Ktp}	K_{tp}	% U
2.5100	504.0554	-5.1534	0.0231	0.2494	0.0015	0.0391	0.0578	0.0001	1.82E-06	2.17E-09	5.83E-09	9.87E-08	1.82E-06	0.0231	0.0231	-0.1354	-17.0992
2.5086	552.5634	-2.4533	0.0215	0.2494	0.0014	0.0391	0.0578	0.0001	1.51E-06	8.61E-10	2.11E-09	3.91E-08	1.51E-06	0.0215	0.0215	-0.0536	-40.1848
2.5072	601.0760	1.5344	0.0209	0.2494	0.0015	0.0391	0.0578	0.0001	1.28E-06	-4.55E-10	-1.02E-09	-2.07E-08	1.28E-06	0.0210	0.0210	0.0283	73.9228
2.5095	651.4524	5.3858	0.0232	0.2494	0.0015	0.0391	0.0578	0.0001	1.09E-06	-1.36E-09	-2.82E-09	-6.17E-08	1.09E-06	0.0232	0.0232	0.0847	27.4470
2.5020	751.7646	15.9565	0.0234	0.2494	0.0015	0.0391	0.0578	0.0001	8.18E-07	-3.03E-09	-5.45E-09	-1.37E-07	8.29E-07	0.0234	0.0234	0.1884	12.4443
2.5092	905.0740	37.9313	0.0237	0.2494	0.0014	0.0391	0.0578	0.0001	5.64E-07	-4.96E-09	-7.42E-09	-2.25E-07	6.08E-07	0.0237	0.0237	0.3090	7.6835
2.5081	1004.8353	52.3894	0.0236	0.2494	0.0015	0.0391	0.0578	0.0001	4.58E-07	-5.56E-09	-7.49E-09	-2.52E-07	5.23E-07	0.0236	0.0236	0.3463	6.8289
2.5087	1052.4015	62.3939	0.0238	0.2494	0.0015	0.0392	0.0578	0.0001	4.17E-07	-6.04E-09	-7.77E-09	-2.74E-07	4.99E-07	0.0238	0.0238	0.3760	6.3424
2.5087	1103.3557	72.3930	0.0238	0.2494	0.0014	0.0392	0.0578	0.0001	3.80E-07	-6.37E-09	-7.83E-09	-2.89E-07	4.78E-07	0.0238	0.0238	0.3969	6.0082
2.5023	1154.7567	84.3833	0.0239	0.2494	0.0015	0.0392	0.0578	0.0001	3.47E-07	-6.78E-09	-7.96E-09	-3.08E-07	4.64E-07	0.0239	0.0239	0.4223	5.6701
2.5092	1207.1602	97.3348	0.0241	0.2494	0.0015	0.0392	0.0578	0.0001	3.17E-07	-7.16E-09	-8.04E-09	-3.25E-07	4.54E-07	0.0241	0.0241	0.4458	5.4172
2.5091	1252.5791	110.9502	0.0243	0.2494	0.0015	0.0392	0.0578	0.0001	2.95E-07	-7.58E-09	-8.20E-09	-3.44E-07	4.53E-07	0.0243	0.0243	0.4719	5.1588

Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

Uncertainty Analysis (Propeller Rotational Speed)

V	Vehicle Velocity (m/s)	
n	Propeller Rotational Speed (rpm)	
C	Number of Counted Pulses in Time dt	
dt	Time (s)	
S.D	Standard Deviation	
P_n	Precision Error for n	$P_n = 2 \times S.D$
B_C	Bias Error for C	
B_{dt}	Bias Error for dt	
B_n	Bias Error for n	$B_n = ((\delta n / \delta C \; B_C)^2 + (\delta n / \delta dt \; B_{dt})^2)^{1/2}$
U_n	Total Error for n	$U_n = (B_n^2 + P_n^2)^{1/2}$
% Unc.	Percentage Uncertainty for n	% Unc. = $(U_n / n) \times 100$

Uncertainty Analysis (Rotational Speed)
Load Varying Self-Propulsion Test (Prop.+ Duct fitted on)

V (m/s)	n (rpm)	P_n	$B_C\ \delta n/\delta C$	$B_{dt}\ \delta n/\delta dt$	B_n	U_n	% Unc.
0.8002	499.5767	0.0005	0.0391	-0.0009	0.0391	0.0391	0.0078
0.8004	551.4566	0.0005	0.0391	-0.0010	0.0391	0.0391	0.0071
0.8012	600.4575	0.0005	0.0391	-0.0011	0.0391	0.0391	0.0065
0.8018	650.4466	0.0005	0.0391	-0.0013	0.0391	0.0391	0.0060
0.8002	753.2179	0.0005	0.0391	-0.0016	0.0391	0.0391	0.0052
0.8002	902.9860	0.0005	0.0391	-0.0020	0.0391	0.0391	0.0043
0.8003	1008.5085	0.0005	0.0391	-0.0023	0.0391	0.0392	0.0039
0.8005	1052.3455	0.0005	0.0391	-0.0024	0.0392	0.0392	0.0037
0.8014	1104.1946	0.0005	0.0391	-0.0025	0.0392	0.0392	0.0035
0.8002	1150.2549	0.0005	0.0391	-0.0027	0.0392	0.0392	0.0034
0.8008	1206.8237	0.0005	0.0391	-0.0028	0.0392	0.0392	0.0032
0.8006	1252.7469	0.0005	0.0391	-0.0030	0.0392	0.0392	0.0031

V (m/s)	n (rpm)	P_n	$B_C\ \delta n/\delta C$	$B_{dt}\ \delta n/\delta dt$	B_n	U_n	% Unc.
1.1006	500.6686	0.0006	0.0391	-0.0009	0.0391	0.0391	0.0078
1.1022	552.7477	0.0006	0.0391	-0.0010	0.0391	0.0391	0.0071
1.1031	603.4646	0.0007	0.0391	-0.0011	0.0391	0.0391	0.0065
1.1032	651.6550	0.0007	0.0391	-0.0013	0.0391	0.0391	0.0060
1.1005	754.9726	0.0007	0.0391	-0.0016	0.0391	0.0391	0.0052
1.1026	901.8126	0.0007	0.0391	-0.0020	0.0391	0.0391	0.0043
1.1014	1000.0880	0.0007	0.0391	-0.0023	0.0391	0.0392	0.0039
1.1017	1056.9566	0.0007	0.0391	-0.0024	0.0392	0.0392	0.0037
1.1013	1104.4112	0.0007	0.0391	-0.0025	0.0392	0.0392	0.0035
1.1020	1156.9473	0.0007	0.0391	-0.0027	0.0392	0.0392	0.0034
1.1015	1212.9133	0.0007	0.0391	-0.0028	0.0392	0.0392	0.0032
1.1011	1258.8191	0.0007	0.0391	-0.0030	0.0392	0.0392	0.0031

Uncertainty Analysis (Rotational Speed)
Load Varying Self-Propulsion Test (Prop.+ Duct fitted on)

V (m/s)	n (rpm)	P_n	B_C $\delta n/\delta C$	B_{dt} $\delta n/\delta dt$	B_n	U_n	% Unc.
1.3033	501.8541	0.0008	0.0391	-0.0009	0.0391	0.0391	0.0078
1.3116	550.7550	0.0007	0.0391	-0.0010	0.0391	0.0391	0.0071
1.3018	601.3456	0.0007	0.0391	-0.0011	0.0391	0.0391	0.0065
1.3022	654.7460	0.0008	0.0391	-0.0013	0.0391	0.0391	0.0060
1.3025	752.1400	0.0008	0.0391	-0.0016	0.0391	0.0391	0.0052
1.3028	907.7795	0.0008	0.0391	-0.0020	0.0391	0.0391	0.0043
1.3020	1006.9377	0.0008	0.0391	-0.0023	0.0391	0.0392	0.0039
1.3014	1051.7237	0.0007	0.0391	-0.0024	0.0392	0.0392	0.0037
1.3116	1103.5535	0.0008	0.0391	-0.0025	0.0392	0.0392	0.0035
1.3007	1155.5678	0.0008	0.0391	-0.0027	0.0392	0.0392	0.0034
1.3006	1208.2135	0.0008	0.0391	-0.0028	0.0392	0.0392	0.0032
1.3020	1256.7456	0.0008	0.0391	-0.0030	0.0392	0.0392	0.0031

V (m/s)	n (rpm)	P_n	B_C $\delta n/\delta C$	B_{dt} $\delta n/\delta dt$	B_n	U_n	% Unc.
1.6030	503.1755	0.0009	0.0391	-0.0009	0.0391	0.0391	0.0078
1.6053	554.4423	0.0009	0.0391	-0.0010	0.0391	0.0391	0.0071
1.6045	604.3557	0.0009	0.0391	-0.0011	0.0391	0.0391	0.0065
1.6063	652.4344	0.0009	0.0391	-0.0013	0.0391	0.0391	0.0060
1.6063	752.2439	0.0009	0.0391	-0.0016	0.0391	0.0391	0.0052
1.6059	901.5599	0.0009	0.0391	-0.0020	0.0391	0.0391	0.0043
1.6056	1006.7175	0.0009	0.0391	-0.0023	0.0391	0.0392	0.0039
1.6063	1054.3192	0.0009	0.0391	-0.0024	0.0392	0.0392	0.0037
1.6059	1102.5010	0.0009	0.0391	-0.0025	0.0392	0.0392	0.0036
1.6050	1154.9706	0.0009	0.0391	-0.0027	0.0392	0.0392	0.0034
1.6054	1211.5007	0.0009	0.0391	-0.0028	0.0392	0.0392	0.0032
1.6019	1257.0444	0.0009	0.0391	-0.0030	0.0392	0.0392	0.0031

Uncertainty Analysis (Rotational Speed)
Load Varying Self-Propulsion Test (Prop.+ Duct fitted on)

V (m/s)	n (rpm)	P_n	$B_C\ \delta n/\delta C$	$B_{dt}\ \delta n/\delta dt$	B_n	U_n	% Unc.
1.8051	503.4015	0.0010	0.0391	-0.0009	0.0391	0.0391	0.0078
1.8061	550.5344	0.0011	0.0391	-0.0010	0.0391	0.0391	0.0071
1.8054	601.7534	0.0011	0.0391	-0.0011	0.0391	0.0391	0.0065
1.8043	655.6402	0.0011	0.0391	-0.0013	0.0391	0.0391	0.0060
1.8049	756.7032	0.0011	0.0391	-0.0016	0.0391	0.0391	0.0052
1.8053	903.5768	0.0011	0.0391	-0.0020	0.0391	0.0391	0.0043
1.8053	1007.3473	0.0010	0.0391	-0.0023	0.0391	0.0392	0.0039
1.8062	1051.7675	0.0011	0.0391	-0.0024	0.0392	0.0392	0.0037
1.8057	1105.5646	0.0011	0.0391	-0.0025	0.0392	0.0392	0.0035
1.8064	1151.0459	0.0011	0.0391	-0.0027	0.0392	0.0392	0.0034
1.8051	1199.5015	0.0011	0.0391	-0.0028	0.0392	0.0392	0.0033
1.8046	1261.8668	0.0011	0.0391	-0.0030	0.0392	0.0392	0.0031

V (m/s)	n (rpm)	P_n	$B_C\ \delta n/\delta C$	$B_{dt}\ \delta n/\delta dt$	B_n	U_n	% Unc.
2.1082	500.4856	0.0012	0.0391	-0.0009	0.0391	0.0391	0.0078
2.1078	552.4532	0.0012	0.0391	-0.0010	0.0391	0.0391	0.0071
2.1093	604.4664	0.0013	0.0391	-0.0011	0.0391	0.0391	0.0065
2.1056	651.4342	0.0013	0.0391	-0.0013	0.0391	0.0391	0.0060
2.1084	754.9317	0.0012	0.0391	-0.0016	0.0391	0.0391	0.0052
2.1095	901.1852	0.0012	0.0391	-0.0020	0.0391	0.0392	0.0043
2.1082	1002.9008	0.0012	0.0391	-0.0023	0.0391	0.0392	0.0039
2.1098	1053.7204	0.0012	0.0391	-0.0024	0.0392	0.0392	0.0037
2.1103	1101.8507	0.0013	0.0391	-0.0025	0.0392	0.0392	0.0036
2.1101	1153.6637	0.0013	0.0391	-0.0027	0.0392	0.0392	0.0034
2.1091	1210.1050	0.0013	0.0391	-0.0028	0.0392	0.0392	0.0032
2.1086	1258.3035	0.0013	0.0391	-0.0030	0.0392	0.0392	0.0031

<u>Uncertainty Analysis</u> (Rotational Speed)
Load Varying Self-Propulsion Test (Prop.+ Duct fitted on)

V (m/s)	**n** (rpm)	P_n	$B_C\ \delta n/\delta C$	$B_{dt}\ \delta n/\delta dt$	B_n	U_n	**% Unc.**
2.3113	506.5541	0.0013	0.0391	-0.0009	0.0391	0.0391	0.0077
2.3012	554.4124	0.0014	0.0391	-0.0010	0.0391	0.0391	0.0071
2.3106	602.4889	0.0013	0.0391	-0.0011	0.0391	0.0391	0.0065
2.3087	653.7372	0.0014	0.0391	-0.0013	0.0391	0.0391	0.0060
2.3127	751.5777	0.0013	0.0391	-0.0016	0.0391	0.0391	0.0052
2.3075	907.7627	0.0014	0.0391	-0.0020	0.0391	0.0392	0.0043
2.3104	1009.7375	0.0014	0.0391	-0.0023	0.0391	0.0392	0.0039
2.3089	1055.8389	0.0014	0.0391	-0.0024	0.0392	0.0392	0.0037
2.3716	1105.4558	0.0014	0.0391	-0.0025	0.0392	0.0392	0.0035
2.3101	1156.8492	0.0014	0.0391	-0.0027	0.0392	0.0392	0.0034
2.3079	1207.4927	0.0014	0.0391	-0.0028	0.0392	0.0392	0.0032
2.3092	1255.9025	0.0014	0.0391	-0.0030	0.0392	0.0392	0.0031

V (m/s)	**n** (rpm)	P_n	$B_C\ \delta n/\delta C$	$B_{dt}\ \delta n/\delta dt$	B_n	U_n	**% Unc.**
2.5094	504.8586	0.0015	0.0391	-0.0009	0.0391	0.0391	0.0077
2.5104	552.4243	0.0015	0.0391	-0.0010	0.0391	0.0391	0.0071
2.5067	604.5795	0.0015	0.0391	-0.0011	0.0391	0.0391	0.0065
2.5107	653.2312	0.0015	0.0391	-0.0013	0.0391	0.0391	0.0060
2.5093	753.2477	0.0015	0.0391	-0.0016	0.0391	0.0391	0.0052
2.5099	906.0469	0.0015	0.0391	-0.0020	0.0391	0.0392	0.0043
2.5126	1002.7932	0.0015	0.0391	-0.0023	0.0391	0.0392	0.0039
2.5100	1052.4234	0.0015	0.0391	-0.0024	0.0392	0.0392	0.0037
2.5104	1106.7229	0.0015	0.0391	-0.0025	0.0392	0.0392	0.0035
2.5101	1152.3582	0.0016	0.0391	-0.0027	0.0392	0.0392	0.0034
2.5109	1204.5756	0.0015	0.0391	-0.0028	0.0392	0.0392	0.0033
2.5096	1256.7444	0.0016	0.0391	-0.0030	0.0392	0.0392	0.0031

Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

Uncertainty Analysis (Velocity)

V	Vehicle Velocity (m/s)	
D	Diameter of Carriage Wheel (m)	
C	Number of Counted Pulses in Time dt	
dt	Time (s)	
S.D	Standard Deviation	
P_V	Precision Error for V	$P_V = 2 \times S.D$
B_C	Bias Error for C	
B_D	Bias Error for D	
B_{dt}	Bias Error for dt	
B_V	Bias Error for V	$B_V = ((\delta V / \delta C\ B_C)^2 + (\delta V / \delta D\ B_D)^2 + (\delta V / \delta dt\ B_{dt})^2)^{1/2}$
U_V	Total Error for V	$U_V = (B_V^2 + P_V^2)^{1/2}$
% Unc.	Percentage Uncertainty for V	% Unc. = $(U_V / V) \times 100$

Uncertainty Analysis (Velocity)
Load Varying Self - Propulsion Test (Prop. + Duct fitted on)

V (m/s)	P_V	$B_C\ \delta V/\delta C$	$B_D\ \delta V/\delta D$	$B_{dt}\ \delta V/\delta dt$	B_V	U_V	% Unc
0.8002	0.0004	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4421
0.8004	0.0003	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4417
0.8012	0.0004	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4426
0.8018	0.0004	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4415
0.8002	0.0004	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4429
0.8002	0.0004	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4421
0.8003	0.0004	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4420
0.8005	0.0004	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4422
0.8014	0.0004	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4419
0.8002	0.0004	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4432
0.8008	0.0004	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4426
0.8006	0.0004	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4424

V (m/s)	P_V	$B_C\ \delta V/\delta C$	$B_D\ \delta V/\delta D$	$B_{dt}\ \delta V/\delta dt$	B_V	U_V	% Unc
1.1006	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3237
1.1022	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3230
1.1031	0.0006	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3238
1.1032	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3232
1.1005	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3242
1.1026	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3231
1.1014	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3240
1.1017	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3231
1.1013	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3240
1.1020	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3235
1.1015	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3239
1.1011	0.0006	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3243

<u>Uncertainty Analysis</u> (Velocity)

Load Varying Self - Propulsion Test (Prop. + Duct fitted on)

V (m/s)	P_V	$B_C\ \delta V/\delta C$	$B_D\ \delta V/\delta D$	$B_{dt}\ \delta V/\delta dt$	B_V	U_V	% Unc
1.3033	0.0006	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2751
1.3116	0.0006	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2739
1.3018	0.0007	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2765
1.3022	0.0006	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2756
1.3025	0.0007	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2761
1.3028	0.0006	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2758
1.3020	0.0007	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2765
1.3014	0.0007	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2766
1.3116	0.0006	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2734
1.3007	0.0007	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2765
1.3006	0.0006	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2760
1.3020	0.0007	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2768

V (m/s)	P_V	$B_C\ \delta V/\delta C$	$B_D\ \delta V/\delta D$	$B_{dt}\ \delta V/\delta dt$	B_V	U_V	% Unc
1.6030	0.0007	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2255
1.6053	0.0008	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2262
1.6045	0.0007	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2261
1.6063	0.0008	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2261
1.6063	0.0008	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2264
1.6059	0.0007	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2254
1.6056	0.0007	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2259
1.6063	0.0007	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2256
1.6059	0.0007	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2256
1.6050	0.0007	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2260
1.6054	0.0008	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2262
1.6019	0.0008	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2272

Uncertainty Analysis (Velocity)

Load Varying Self - Propulsion Test (Prop. + Duct fitted on)

V (m/s)	P_V	$B_C\ \delta V/\delta C$	$B_D\ \delta V/\delta D$	$B_{dt}\ \delta V/\delta dt$	B_V	U_V	% Unc
1.8051	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2039
1.8061	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2041
1.8054	0.0008	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2028
1.8043	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2037
1.8049	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2034
1.8053	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2042
1.8053	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2031
1.8062	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2035
1.8057	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2039
1.8064	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2041
1.8051	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2039
1.8046	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2037

V (m/s)	P_V	$B_C\ \delta V/\delta C$	$B_D\ \delta V/\delta D$	$B_{dt}\ \delta V/\delta dt$	B_V	U_V	% Unc
2.1082	0.0010	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1772
2.1078	0.0010	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1767
2.1093	0.0011	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1776
2.1056	0.0011	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1777
2.1084	0.0010	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1772
2.1095	0.0011	0.0035	0.0007	-0.00023	0.0036	0.0038	0.1779
2.1082	0.0011	0.0035	0.0007	-0.00023	0.0036	0.0038	0.1783
2.1098	0.0010	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1771
2.1103	0.0011	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1773
2.1101	0.0011	0.0035	0.0007	-0.00023	0.0036	0.0038	0.1779
2.1091	0.0011	0.0035	0.0007	-0.00023	0.0036	0.0038	0.1782
2.1086	0.0011	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1774

Uncertainty Analysis (Velocity)
Load Varying Self - Propulsion Test (Prop. + Duct fitted on)

V (m/s)	P_V	$B_C\ \delta V/\delta C$	$B_D\ \delta V/\delta D$	$B_{dt}\ \delta V/\delta dt$	B_V	U_V	% Unc
2.3113	0.0011	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1629
2.3012	0.0012	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1657
2.3106	0.0012	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1648
2.3087	0.0012	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1643
2.3127	0.0011	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1635
2.3075	0.0013	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1655
2.3104	0.0012	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1640
2.3089	0.0013	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1657
2.3716	0.0013	0.0035	0.0008	-0.00027	0.0036	0.0038	0.1613
2.3101	0.0012	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1648
2.3079	0.0013	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1661
2.3092	0.0013	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1654

V (m/s)	P_V	$B_C\ \delta V/\delta C$	$B_D\ \delta V/\delta D$	$B_{dt}\ \delta V/\delta dt$	B_V	U_V	% Unc
2.5094	0.0013	0.0035	0.0008	-0.00029	0.0036	0.0039	0.1541
2.5104	0.0013	0.0035	0.0008	-0.00029	0.0036	0.0038	0.1529
2.5067	0.0014	0.0035	0.0008	-0.00029	0.0036	0.0039	0.1545
2.5107	0.0013	0.0035	0.0008	-0.00029	0.0036	0.0039	0.1534
2.5093	0.0014	0.0035	0.0008	-0.00029	0.0036	0.0039	0.1546
2.5099	0.0013	0.0035	0.0008	-0.00029	0.0036	0.0039	0.1535
2.5126	0.0013	0.0035	0.0008	-0.00029	0.0036	0.0039	0.1536
2.5100	0.0013	0.0035	0.0008	-0.00029	0.0036	0.0039	0.1540
2.5104	0.0013	0.0035	0.0008	-0.00029	0.0036	0.0039	0.1537
2.5101	0.0014	0.0035	0.0008	-0.00029	0.0036	0.0039	0.1543
2.5109	0.0014	0.0035	0.0008	-0.00029	0.0036	0.0039	0.1545
2.5096	0.0013	0.0035	0.0008	-0.00029	0.0036	0.0039	0.1540

Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

Uncertainty Analysis (Torque)

V	Vehicle Velocity (m/s)	
Q	Propeller Torque (N-m)	
F	Calibrating Weight (N)	
R	Lever Arm (m)	
P_Q	Precision Error for Q	$P_Q = 2 \times S.D$
S.D	Standard Deviation	
S.E.E	Standard Error Estimate	
AD	Analog to Digital Converter	
B_F	Bias Error for F	
B_R	Bias Error for R	
B_1	Bias Error due to Calibration	$B_1 = ((\delta Q / \delta F\ B_F)^2 + (\delta Q / \delta R\ B_R)^2)^{1/2}$
B_2	Bias Error due to Data Acquisition	$B_2 = 2 \times S.E.E$
B_3	Bias Error due to Data Reduction	$B_3 = (AD_{error} \times AD_{range}) / (AD_{accuracy})$
B_Q	Bias Error for Q	$B_Q = (B_1^2 + B_2^2 + B_3^2)^{1/2}$
U_Q	Total Error for Q	$U_Q = (B_Q^2 + P_Q^2)^{1/2}$
% Unc.	Percentage Uncertainty for Q	$\% \text{ Unc.} = (U_Q / Q) \times 100$

Uncertainty Analysis (Torque)
Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

V (m/s)	Q (N-m)	P_Q	$B_F\ \delta Q/\delta F$	$B_R\ \delta Q/\delta R$	B_1	B_2	B_3	B_Q	U_Q	% Unc.
0.8002	0.3364	0.0008	0.0000	0.0003	0.0003	0.0076	0.0007	0.0076	0.0077	2.2836
0.8004	0.4972	0.0008	0.0000	0.0004	0.0004	0.0076	0.0007	0.0076	0.0077	1.5463
0.8012	0.6260	0.0008	0.0000	0.0005	0.0005	0.0076	0.0007	0.0077	0.0077	1.2293
0.8018	0.8160	0.0008	0.0000	0.0007	0.0007	0.0076	0.0007	0.0077	0.0077	0.9447
0.8002	1.2582	0.0008	0.0001	0.0010	0.0010	0.0076	0.0007	0.0077	0.0077	0.6156
0.8002	1.9967	0.0009	0.0001	0.0016	0.0016	0.0076	0.0007	0.0078	0.0078	0.3930
0.8003	2.5232	0.0009	0.0001	0.0020	0.0020	0.0076	0.0007	0.0079	0.0079	0.3149
0.8005	2.8657	0.0009	0.0001	0.0023	0.0023	0.0076	0.0007	0.0080	0.0080	0.2800
0.8014	3.1813	0.0009	0.0002	0.0025	0.0026	0.0076	0.0007	0.0080	0.0081	0.2546
0.8002	3.5448	0.0009	0.0002	0.0028	0.0028	0.0076	0.0007	0.0081	0.0082	0.2313
0.8008	3.9061	0.0009	0.0002	0.0031	0.0031	0.0076	0.0007	0.0083	0.0083	0.2126
0.8006	4.2519	0.0009	0.0002	0.0034	0.0034	0.0076	0.0007	0.0084	0.0084	0.1979

V (m/s)	Q (N-m)	P_Q	$B_F\ \delta Q/\delta F$	$B_R\ \delta Q/\delta R$	B_1	B_2	B_3	B_Q	U_Q	% Unc.
1.1006	0.3193	0.0009	0.0000	0.0003	0.0003	0.0076	0.0007	0.0076	0.0077	2.4103
1.1022	0.4722	0.0010	0.0000	0.0004	0.0004	0.0076	0.0007	0.0076	0.0077	1.6312
1.1031	0.6209	0.0010	0.0000	0.0005	0.0005	0.0076	0.0007	0.0077	0.0077	1.2420
1.1032	0.8161	0.0010	0.0000	0.0007	0.0007	0.0076	0.0007	0.0077	0.0077	0.9466
1.1005	1.2195	0.0010	0.0001	0.0010	0.0010	0.0076	0.0007	0.0077	0.0078	0.6364
1.1026	1.9412	0.0010	0.0001	0.0016	0.0016	0.0076	0.0007	0.0078	0.0079	0.4047
1.1014	2.4885	0.0010	0.0001	0.0020	0.0020	0.0076	0.0007	0.0079	0.0080	0.3198
1.1017	2.7980	0.0010	0.0001	0.0022	0.0022	0.0076	0.0007	0.0080	0.0080	0.2868
1.1013	3.1236	0.0010	0.0002	0.0025	0.0025	0.0076	0.0007	0.0080	0.0081	0.2594
1.1020	3.4722	0.0011	0.0002	0.0028	0.0028	0.0076	0.0007	0.0081	0.0082	0.2360
1.1015	3.8531	0.0011	0.0002	0.0031	0.0031	0.0076	0.0007	0.0082	0.0083	0.2155
1.1011	4.2369	0.0011	0.0002	0.0034	0.0034	0.0076	0.0007	0.0084	0.0084	0.1988

Uncertainty Analysis (Torque)
Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

V (m/s)	Q (N-m)	P_Q	$B_F\ \delta Q/\delta F$	$B_R\ \delta Q/\delta R$	B_1	B_2	B_3	B_Q	U_Q	% Unc.
1.3033	0.2851	0.0010	0.0000	0.0002	0.0002	0.0076	0.0007	0.0076	0.0077	2.7024
1.3116	0.4280	0.0011	0.0000	0.0003	0.0003	0.0076	0.0007	0.0076	0.0077	1.8030
1.3018	0.5866	0.0011	0.0000	0.0005	0.0005	0.0076	0.0007	0.0076	0.0077	1.3173
1.3022	0.7812	0.0012	0.0000	0.0006	0.0006	0.0076	0.0007	0.0077	0.0078	0.9926
1.3025	1.1578	0.0012	0.0001	0.0009	0.0009	0.0076	0.0007	0.0077	0.0078	0.6725
1.3028	1.9016	0.0012	0.0001	0.0015	0.0015	0.0076	0.0007	0.0078	0.0079	0.4145
1.3020	2.4451	0.0012	0.0001	0.0020	0.0020	0.0076	0.0007	0.0079	0.0080	0.3262
1.3014	2.6985	0.0012	0.0001	0.0022	0.0022	0.0076	0.0007	0.0079	0.0080	0.2976
1.3116	3.0711	0.0013	0.0002	0.0025	0.0025	0.0076	0.0007	0.0080	0.0081	0.2644
1.3007	3.4310	0.0013	0.0002	0.0027	0.0028	0.0076	0.0007	0.0081	0.0082	0.2394
1.3006	3.7775	0.0013	0.0002	0.0030	0.0030	0.0076	0.0007	0.0082	0.0083	0.2201
1.3020	4.1836	0.0013	0.0002	0.0033	0.0034	0.0076	0.0007	0.0083	0.0084	0.2017

V (m/s)	Q (N-m)	P_Q	$B_F\ \delta Q/\delta F$	$B_R\ \delta Q/\delta R$	B_1	B_2	B_3	B_Q	U_Q	% Unc.
1.6030	0.2466	0.0011	0.0000	0.0002	0.0002	0.0076	0.0007	0.0076	0.0077	3.1292
1.6053	0.3896	0.0013	0.0000	0.0003	0.0003	0.0076	0.0007	0.0076	0.0078	1.9894
1.6045	0.5547	0.0013	0.0000	0.0004	0.0004	0.0076	0.0007	0.0076	0.0078	1.3990
1.6063	0.7494	0.0013	0.0000	0.0006	0.0006	0.0076	0.0007	0.0077	0.0078	1.0372
1.6063	1.1132	0.0013	0.0001	0.0009	0.0009	0.0076	0.0007	0.0077	0.0078	0.7008
1.6059	1.8053	0.0013	0.0001	0.0014	0.0014	0.0076	0.0007	0.0078	0.0079	0.4368
1.6056	2.4295	0.0014	0.0001	0.0019	0.0019	0.0076	0.0007	0.0079	0.0080	0.3290
1.6063	2.6763	0.0014	0.0001	0.0021	0.0021	0.0076	0.0007	0.0079	0.0080	0.3006
1.6059	3.0303	0.0014	0.0002	0.0024	0.0024	0.0076	0.0007	0.0080	0.0081	0.2682
1.6050	3.3480	0.0014	0.0002	0.0027	0.0027	0.0076	0.0007	0.0081	0.0082	0.2452
1.6054	3.7189	0.0014	0.0002	0.0030	0.0030	0.0076	0.0007	0.0082	0.0083	0.2235
1.6019	4.0895	0.0014	0.0002	0.0033	0.0033	0.0076	0.0007	0.0083	0.0084	0.2060

Uncertainty Analysis (Torque)
Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

V (m/s)	Q (N-m)	P_Q	$B_F\ \delta Q/\delta F$	$B_R\ \delta Q/\delta R$	B_1	B_2	B_3	B_Q	U_Q	% Unc.
1.8051	0.2345	0.0012	0.0000	0.0002	0.0002	0.0076	0.0007	0.0076	0.0077	3.2962
1.8061	0.3599	0.0014	0.0000	0.0003	0.0003	0.0076	0.0007	0.0076	0.0078	2.1588
1.8054	0.4952	0.0014	0.0000	0.0004	0.0004	0.0076	0.0007	0.0076	0.0078	1.5708
1.8043	0.7021	0.0015	0.0000	0.0006	0.0006	0.0076	0.0007	0.0077	0.0078	1.1098
1.8049	1.1097	0.0015	0.0001	0.0009	0.0009	0.0076	0.0007	0.0077	0.0078	0.7050
1.8053	1.7817	0.0015	0.0001	0.0014	0.0014	0.0076	0.0007	0.0078	0.0079	0.4438
1.8053	2.3494	0.0015	0.0001	0.0019	0.0019	0.0076	0.0007	0.0079	0.0080	0.3405
1.8062	2.6355	0.0015	0.0001	0.0021	0.0021	0.0076	0.0007	0.0079	0.0081	0.3058
1.8057	2.9403	0.0015	0.0001	0.0024	0.0024	0.0076	0.0007	0.0080	0.0081	0.2764
1.8064	3.2937	0.0015	0.0002	0.0026	0.0026	0.0076	0.0007	0.0081	0.0082	0.2494
1.8051	3.6086	0.0015	0.0002	0.0029	0.0029	0.0076	0.0007	0.0082	0.0083	0.2301
1.8046	4.0584	0.0015	0.0002	0.0032	0.0033	0.0076	0.0007	0.0083	0.0084	0.2079

V (m/s)	Q (N-m)	P_Q	$B_F\ \delta Q/\delta F$	$B_R\ \delta Q/\delta R$	B_1	B_2	B_3	B_Q	U_Q	% Unc.
2.1082	0.1922	0.0011	0.0000	0.0002	0.0002	0.0076	0.0007	0.0076	0.0077	4.0146
2.1078	0.3309	0.0016	0.0000	0.0003	0.0003	0.0076	0.0007	0.0076	0.0078	2.3571
2.1093	0.4607	0.0016	0.0000	0.0004	0.0004	0.0076	0.0007	0.0076	0.0078	1.6933
2.1056	0.6493	0.0016	0.0000	0.0005	0.0005	0.0076	0.0007	0.0077	0.0078	1.2038
2.1084	1.0051	0.0016	0.0001	0.0008	0.0008	0.0076	0.0007	0.0077	0.0078	0.7803
2.1095	1.6707	0.0016	0.0001	0.0013	0.0013	0.0076	0.0007	0.0078	0.0079	0.4738
2.1082	2.2317	0.0016	0.0001	0.0018	0.0018	0.0076	0.0007	0.0078	0.0080	0.3588
2.1098	2.5445	0.0016	0.0001	0.0020	0.0020	0.0076	0.0007	0.0079	0.0081	0.3171
2.1103	2.8646	0.0017	0.0001	0.0023	0.0023	0.0076	0.0007	0.0080	0.0081	0.2842
2.1101	3.1899	0.0017	0.0002	0.0026	0.0026	0.0076	0.0007	0.0081	0.0082	0.2578
2.1091	3.5803	0.0017	0.0002	0.0029	0.0029	0.0076	0.0007	0.0082	0.0083	0.2326
2.1086	3.9869	0.0017	0.0002	0.0032	0.0032	0.0076	0.0007	0.0083	0.0084	0.2119

Uncertainty Analysis (Torque)

Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

V (m/s)	Q (N-m)	P_Q	$B_F\ \delta Q/\delta F$	$B_R\ \delta Q/\delta R$	B_1	B_2	B_3	B_Q	U_Q	% Unc.
2.3113	0.1820	0.0010	0.0000	0.0001	0.0001	0.0076	0.0007	0.0076	0.0077	4.2314
2.3012	0.3087	0.0017	0.0000	0.0002	0.0002	0.0076	0.0007	0.0076	0.0078	2.5367
2.3106	0.4268	0.0017	0.0000	0.0003	0.0003	0.0076	0.0007	0.0076	0.0078	1.8360
2.3087	0.6197	0.0017	0.0000	0.0005	0.0005	0.0076	0.0007	0.0077	0.0078	1.2655
2.3127	0.9597	0.0018	0.0000	0.0008	0.0008	0.0076	0.0007	0.0077	0.0079	0.8201
2.3075	1.6567	0.0018	0.0001	0.0013	0.0013	0.0076	0.0007	0.0077	0.0079	0.4797
2.3104	2.2082	0.0018	0.0001	0.0018	0.0018	0.0076	0.0007	0.0078	0.0080	0.3639
2.3089	2.5040	0.0018	0.0001	0.0020	0.0020	0.0076	0.0007	0.0079	0.0081	0.3232
2.3716	2.7615	0.0018	0.0001	0.0022	0.0022	0.0076	0.0007	0.0079	0.0081	0.2951
2.3101	3.1357	0.0018	0.0002	0.0025	0.0025	0.0076	0.0007	0.0080	0.0082	0.2628
2.3079	3.4799	0.0018	0.0002	0.0028	0.0028	0.0076	0.0007	0.0081	0.0083	0.2395
2.3092	3.8185	0.0019	0.0002	0.0031	0.0031	0.0076	0.0007	0.0082	0.0084	0.2208

V (m/s)	Q (N-m)	P_Q	$B_F\ \delta Q/\delta F$	$B_R\ \delta Q/\delta R$	B_1	B_2	B_3	B_Q	U_Q	% Unc.
2.5094	0.1628	0.0011	0.0000	0.0001	0.0001	0.0076	0.0007	0.0076	0.0077	4.7396
2.5104	0.2813	0.0018	0.0000	0.0002	0.0002	0.0076	0.0007	0.0076	0.0078	2.7899
2.5067	0.4035	0.0019	0.0000	0.0003	0.0003	0.0076	0.0007	0.0076	0.0079	1.9517
2.5107	0.5696	0.0019	0.0000	0.0005	0.0005	0.0076	0.0007	0.0076	0.0079	1.3846
2.5093	0.9274	0.0019	0.0000	0.0007	0.0007	0.0076	0.0007	0.0077	0.0079	0.8530
2.5099	1.6051	0.0019	0.0001	0.0013	0.0013	0.0076	0.0007	0.0077	0.0080	0.4973
2.5126	2.1197	0.0020	0.0001	0.0017	0.0017	0.0076	0.0007	0.0078	0.0081	0.3803
2.5100	2.4384	0.0020	0.0001	0.0020	0.0020	0.0076	0.0007	0.0079	0.0081	0.3331
2.5104	2.7268	0.0020	0.0001	0.0022	0.0022	0.0076	0.0007	0.0079	0.0082	0.3002
2.5101	3.0710	0.0020	0.0002	0.0025	0.0025	0.0076	0.0007	0.0080	0.0083	0.2690
2.5109	3.4002	0.0020	0.0002	0.0027	0.0027	0.0076	0.0007	0.0081	0.0083	0.2455
2.5096	3.7705	0.0020	0.0002	0.0030	0.0030	0.0076	0.0007	0.0082	0.0085	0.2243

Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

Uncertainty Analysis (Total Thrust)

V	Vehicle Velocity (m/s)	
T_t	Total Thrust (N)	
P_T	Precision Error for T_t	$P_T = 2 \times S.D$
S.D	Standard Deviation	
S.E.E	Standard Error Estimate	
AD	Analog to Digital Converter	
B_1	Bias Error due to Calibration	$B_1 = 0.00005 \times T_t$
B_2	Bias Error due to Data Acquisition	$B_2 = 2 \times S.E.E$
B_3	Bias Error due to Data Reduction	$B_3 = (AD_{error} \times AD_{range}) / (AD_{accuracy})$
B_T	Bias Error for T_t	$B_T = (B_1^2 + B_2^2 + B_3^2)^{1/2}$
U_T	Total Error for T_t	$U_T = (B_T^2 + P_T^2)^{1/2}$
% Unc.	Percentage Uncertainty for T_t	% Unc. $= (U_T / T_t) \times 100$

Uncertainty Analysis (Total Thrust)
Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

V (m/s)	T_t (N)	P_T	B_1	B_2	B_3	B_T	U_T	% Unc.
0.8002	7.6425	0.0090	0.0004	0.2487	0.0182	0.2494	0.2495	3.2648
0.8004	14.1975	0.0092	0.0007	0.2487	0.0182	0.2494	0.2495	1.7575
0.8012	19.7185	0.0091	0.0010	0.2487	0.0182	0.2494	0.2495	1.2654
0.8018	26.1852	0.0094	0.0013	0.2487	0.0182	0.2494	0.2495	0.9529
0.8002	43.2349	0.0095	0.0022	0.2487	0.0182	0.2494	0.2495	0.5772
0.8002	78.0332	0.0097	0.0039	0.2487	0.0182	0.2494	0.2496	0.3198
0.8003	106.5934	0.0098	0.0053	0.2487	0.0182	0.2494	0.2496	0.2342
0.8005	121.3940	0.0098	0.0061	0.2487	0.0182	0.2494	0.2496	0.2056
0.8014	137.7978	0.0099	0.0069	0.2487	0.0182	0.2494	0.2496	0.1812
0.8002	154.0467	0.0100	0.0077	0.2487	0.0182	0.2495	0.2497	0.1621
0.8008	175.1871	0.0102	0.0088	0.2487	0.0182	0.2495	0.2497	0.1425
0.8006	191.4733	0.0104	0.0096	0.2487	0.0182	0.2495	0.2498	0.1304

V (m/s)	T_t (N)	P_T	B_1	B_2	B_4	B_T	U_T	% Unc.
1.1006	6.1254	0.0100	0.0003	0.2487	0.0182	0.2494	0.2496	4.0741
1.1022	11.5778	0.0108	0.0006	0.2487	0.0182	0.2494	0.2496	2.1557
1.1031	17.6112	0.0110	0.0009	0.2487	0.0182	0.2494	0.2496	1.4172
1.1032	26.6374	0.0112	0.0013	0.2487	0.0182	0.2494	0.2496	0.9370
1.1005	43.0749	0.0115	0.0022	0.2487	0.0182	0.2494	0.2496	0.5795
1.1026	78.2263	0.0117	0.0039	0.2487	0.0182	0.2494	0.2497	0.3191
1.1014	103.9908	0.0116	0.0052	0.2487	0.0182	0.2494	0.2497	0.2401
1.1017	120.2376	0.0119	0.0060	0.2487	0.0182	0.2494	0.2497	0.2077
1.1013	137.1587	0.0121	0.0069	0.2487	0.0182	0.2494	0.2497	0.1821
1.1020	154.0702	0.0124	0.0077	0.2487	0.0182	0.2495	0.2498	0.1621
1.1015	172.0909	0.0124	0.0086	0.2487	0.0182	0.2495	0.2498	0.1452
1.1011	188.8362	0.0125	0.0094	0.2487	0.0182	0.2495	0.2498	0.1323

Uncertainty Analysis (Total Thrust)
Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

V (m/s)	T_t (N)	P_T	B_1	B_2	B_4	B_T	U_T	% Unc.
1.3033	-2.4715	0.0128	-0.0001	0.2487	0.0182	0.2494	0.2497	-10.1023
1.3116	3.2712	0.0130	0.0002	0.2487	0.0182	0.2494	0.2497	7.6328
1.3018	7.6691	0.0133	0.0004	0.2487	0.0182	0.2494	0.2497	3.2560
1.3022	18.6342	0.0134	0.0009	0.2487	0.0182	0.2494	0.2497	1.3401
1.3025	32.8566	0.0134	0.0016	0.2487	0.0182	0.2494	0.2497	0.7600
1.3028	70.4953	0.0135	0.0035	0.2487	0.0182	0.2494	0.2497	0.3543
1.3020	99.0206	0.0138	0.0050	0.2487	0.0182	0.2494	0.2498	0.2523
1.3014	113.7191	0.0137	0.0057	0.2487	0.0182	0.2494	0.2498	0.2197
1.3116	130.8195	0.0138	0.0065	0.2487	0.0182	0.2494	0.2498	0.1910
1.3007	150.4082	0.0139	0.0075	0.2487	0.0182	0.2495	0.2499	0.1661
1.3006	166.7611	0.0142	0.0083	0.2487	0.0182	0.2495	0.2499	0.1499
1.3020	183.7284	0.0143	0.0092	0.2487	0.0182	0.2495	0.2499	0.1360

V (m/s)	T_t (N)	P_T	B_1	B_2	B_4	B_T	U_T	% Unc.
1.6030	-7.0565	0.0140	-0.0004	0.2487	0.0182	0.2494	0.2497	-3.5392
1.6053	-2.6691	0.0135	-0.0001	0.2487	0.0182	0.2494	0.2497	-9.3558
1.6045	3.2865	0.0138	0.0002	0.2487	0.0182	0.2494	0.2497	7.5988
1.6063	11.6498	0.0145	0.0006	0.2487	0.0182	0.2494	0.2498	2.1440
1.6063	28.1917	0.0147	0.0014	0.2487	0.0182	0.2494	0.2498	0.8860
1.6059	64.0440	0.0148	0.0032	0.2487	0.0182	0.2494	0.2498	0.3901
1.6056	91.9317	0.0149	0.0046	0.2487	0.0182	0.2494	0.2498	0.2718
1.6063	110.3025	0.0152	0.0055	0.2487	0.0182	0.2494	0.2499	0.2265
1.6059	126.9004	0.0153	0.0063	0.2487	0.0182	0.2494	0.2499	0.1969
1.6050	143.2220	0.0155	0.0072	0.2487	0.0182	0.2495	0.2499	0.1745
1.6054	162.3490	0.0158	0.0081	0.2487	0.0182	0.2495	0.2500	0.1540
1.6019	180.1017	0.0160	0.0090	0.2487	0.0182	0.2495	0.2500	0.1388

Uncertainty Analysis (Total Thrust)
Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

V (m/s)	T_t (N)	P_T	B_1	B_2	B_4	B_T	U_T	% Unc.
1.8051	-12.0461	0.0170	-0.0006	0.2487	0.0182	0.2494	0.2499	-2.0748
1.8061	-7.9307	0.0168	-0.0004	0.2487	0.0182	0.2494	0.2499	-3.1512
1.8054	-2.0996	0.0163	-0.0001	0.2487	0.0182	0.2494	0.2499	-11.9017
1.8043	8.3160	0.0170	0.0004	0.2487	0.0182	0.2494	0.2499	3.0054
1.8049	24.0023	0.0172	0.0012	0.2487	0.0182	0.2494	0.2499	1.0413
1.8053	56.0137	0.0173	0.0028	0.2487	0.0182	0.2494	0.2500	0.4463
1.8053	82.5484	0.0172	0.0041	0.2487	0.0182	0.2494	0.2500	0.3028
1.8062	100.6055	0.0174	0.0050	0.2487	0.0182	0.2494	0.2500	0.2485
1.8057	118.5649	0.0175	0.0059	0.2487	0.0182	0.2494	0.2500	0.2109
1.8064	135.0686	0.0175	0.0068	0.2487	0.0182	0.2494	0.2501	0.1851
1.8051	154.5146	0.0177	0.0077	0.2487	0.0182	0.2495	0.2501	0.1619
1.8046	175.6173	0.0178	0.0088	0.2487	0.0182	0.2495	0.2501	0.1424

V (m/s)	T_t (N)	P_T	B_1	B_2	B_4	B_T	U_T	% Unc.
2.1082	-15.0436	0.0192	-0.0008	0.2487	0.0182	0.2494	0.2501	-1.6624
2.1078	-8.6422	0.0190	-0.0004	0.2487	0.0182	0.2494	0.2501	-2.8936
2.1093	-2.1310	0.0184	-0.0001	0.2487	0.0182	0.2494	0.2500	-11.7327
2.1056	5.3703	0.0189	0.0003	0.2487	0.0182	0.2494	0.2501	4.6565
2.1084	20.7011	0.0191	0.0010	0.2487	0.0182	0.2494	0.2501	1.2081
2.1095	53.4900	0.0192	0.0027	0.2487	0.0182	0.2494	0.2501	0.4676
2.1082	79.1802	0.0192	0.0040	0.2487	0.0182	0.2494	0.2501	0.3159
2.1098	94.0054	0.0193	0.0047	0.2487	0.0182	0.2494	0.2501	0.2661
2.1103	111.6413	0.0194	0.0056	0.2487	0.0182	0.2494	0.2502	0.2241
2.1101	128.0193	0.0196	0.0064	0.2487	0.0182	0.2494	0.2502	0.1954
2.1091	147.2988	0.0199	0.0074	0.2487	0.0182	0.2495	0.2503	0.1699
2.1086	167.6550	0.0203	0.0084	0.2487	0.0182	0.2495	0.2503	0.1493

Uncertainty Analysis (Total Thrust)
Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

V (m/s)	T_t (N)	P_T	B_1	B_2	B_4	B_T	U_T	% Unc.
2.3113	-19.3516	0.0218	-0.0010	0.2487	0.0182	0.2494	0.2503	-1.2934
2.3012	-15.8623	0.0217	-0.0008	0.2487	0.0182	0.2494	0.2503	-1.5779
2.3106	-10.8438	0.0213	-0.0005	0.2487	0.0182	0.2494	0.2503	-2.3079
2.3087	-1.9037	0.0180	-0.0001	0.2487	0.0182	0.2494	0.2500	-13.1324
2.3127	13.1329	0.0215	0.0007	0.2487	0.0182	0.2494	0.2503	1.9057
2.3075	45.9201	0.0218	0.0023	0.2487	0.0182	0.2494	0.2503	0.5451
2.3104	70.3932	0.0219	0.0035	0.2487	0.0182	0.2494	0.2503	0.3556
2.3089	84.5472	0.0221	0.0042	0.2487	0.0182	0.2494	0.2504	0.2961
2.3716	99.2872	0.0223	0.0050	0.2487	0.0182	0.2494	0.2504	0.2522
2.3101	117.4706	0.0225	0.0059	0.2487	0.0182	0.2494	0.2504	0.2132
2.3079	134.4150	0.0226	0.0067	0.2487	0.0182	0.2494	0.2505	0.1863
2.3092	151.0913	0.0228	0.0076	0.2487	0.0182	0.2495	0.2505	0.1658

V (m/s)	T_t (N)	P_T	B_1	B_2	B_4	B_T	U_T	% Unc.
2.5094	-23.5927	0.0232	-0.0012	0.2487	0.0182	0.2494	0.2504	-1.0615
2.5104	-21.2282	0.0229	-0.0011	0.2487	0.0182	0.2494	0.2504	-1.1796
2.5067	-16.9244	0.0228	-0.0008	0.2487	0.0182	0.2494	0.2504	-1.4795
2.5107	-10.5697	0.0225	-0.0005	0.2487	0.0182	0.2494	0.2504	-2.3687
2.5093	2.9669	0.0206	0.0001	0.2487	0.0182	0.2494	0.2502	8.4330
2.5099	34.7210	0.0238	0.0017	0.2487	0.0182	0.2494	0.2505	0.7214
2.5126	63.3290	0.0239	0.0032	0.2487	0.0182	0.2494	0.2505	0.3956
2.5100	76.2347	0.0240	0.0038	0.2487	0.0182	0.2494	0.2505	0.3286
2.5104	91.4780	0.0242	0.0046	0.2487	0.0182	0.2494	0.2506	0.2739
2.5101	105.8466	0.0244	0.0053	0.2487	0.0182	0.2494	0.2506	0.2368
2.5109	124.7560	0.0245	0.0062	0.2487	0.0182	0.2494	0.2506	0.2009
2.5096	140.1545	0.0248	0.0070	0.2487	0.0182	0.2494	0.2507	0.1789

Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

Uncertainty Analysis (Tow Force)

V	Vehicle Velocity (m/s)	
F	Tow Force (N)	
P_F	Precision Error for F	$P_F = 2 \times S.D$
S.D	Standard Deviation	
S.E.E	Standard Error Estimate	
AD	Analog to Digital Converter	
B_1	Bias Error due to Calibration	$B_1 = 0.00005 \times F$
B_2	Bias Error due to Data Acquisition	$B_2 = 2 \times S.E.E$
B_3	Bias Error due to Load Cell Misalignment	$B_3 = F - (\cos 0.25^0 \times F)$
B_4	Bias Error due to Data Reduction	$B_4 = (AD_{error} \times AD_{range}) / (AD_{accuracy})$
B_F	Bias Error for F	$B_F = (B_1^2 + B_2^2 + B_3^2 + B_4^2)^{1/2}$
U_F	Total Error for F	$U_F = (B_F^2 + P_F^2)^{1/2}$
% Unc.	Percentage Uncertainty for F	% Unc. = $(U_F / F) \times 100$

Uncertainty Analysis (Tow Force)
Load Varying Self-Propulsion Test (Duct + Prop. fitted on)

V (m/s)	F (N)	P_F	B_1	B_2	B_3	B_4	B_F	U_F	% Unc.
0.8002	1.3073	0.0095	0.0001	0.2693	0.0000	0.0054	0.2694	0.2695	20.6176
0.8004	-4.6450	0.0107	-0.0002	0.2693	0.0000	0.0054	0.2694	0.2696	-5.8037
0.8012	-10.5645	0.0107	-0.0005	0.2693	-0.0001	0.0054	0.2694	0.2696	-2.5518
0.8018	-17.5455	0.0109	-0.0009	0.2693	-0.0002	0.0054	0.2694	0.2696	-1.5365
0.8002	-34.8750	0.0110	-0.0017	0.2693	-0.0003	0.0054	0.2694	0.2696	-0.7730
0.8002	-68.0707	0.0112	-0.0034	0.2693	-0.0006	0.0054	0.2694	0.2696	-0.3961
0.8003	-94.6789	0.0113	-0.0047	0.2693	-0.0009	0.0054	0.2694	0.2696	-0.2848
0.8005	-108.5758	0.0113	-0.0054	0.2693	-0.0010	0.0054	0.2694	0.2697	-0.2484
0.8014	-123.8928	0.0115	-0.0062	0.2693	-0.0012	0.0054	0.2694	0.2697	-0.2177
0.8002	-139.0968	0.0114	-0.0070	0.2693	-0.0013	0.0054	0.2695	0.2697	-0.1939
0.8008	-159.0113	0.0116	-0.0080	0.2693	-0.0015	0.0054	0.2695	0.2697	-0.1696
0.8006	-174.2846	0.0117	-0.0087	0.2693	-0.0017	0.0054	0.2695	0.2698	-0.1548

V (m/s)	F (N)	P_F	B_1	B_2	B_3	B_4	B_F	U_F	% Unc.
1.1006	10.2786	0.0124	0.0005	0.2693	0.0001	0.0054	0.2694	0.2697	2.6234
1.1022	4.9858	0.0107	0.0002	0.2693	0.0000	0.0054	0.2694	0.2696	5.4070
1.1031	-0.7660	0.0090	0.0000	0.2693	0.0000	0.0054	0.2694	0.2695	-35.1850
1.1032	-9.4535	0.0124	-0.0005	0.2693	-0.0001	0.0054	0.2694	0.2697	-2.8524
1.1005	-24.9334	0.0125	-0.0012	0.2693	-0.0002	0.0054	0.2694	0.2697	-1.0815
1.1026	-58.3065	0.0125	-0.0029	0.2693	-0.0006	0.0054	0.2694	0.2697	-0.4625
1.1014	-82.6355	0.0126	-0.0041	0.2693	-0.0008	0.0054	0.2694	0.2697	-0.3264
1.1017	-98.0144	0.0128	-0.0049	0.2693	-0.0009	0.0054	0.2694	0.2697	-0.2752
1.1013	-114.1839	0.0128	-0.0057	0.2693	-0.0011	0.0054	0.2694	0.2697	-0.2362
1.1020	-130.2701	0.0129	-0.0065	0.2693	-0.0012	0.0054	0.2694	0.2698	-0.2071
1.1015	-147.3841	0.0132	-0.0074	0.2693	-0.0014	0.0054	0.2695	0.2698	-0.1831
1.1011	-163.3845	0.0135	-0.0082	0.2693	-0.0016	0.0054	0.2695	0.2698	-0.1652

Uncertainty Analysis (Tow Force)
Load Varying Self-Propulsion Test (Duct + Prop. fitted on)

V (m/s)	F (N)	P_F	B_1	B_2	B_3	B_4	B_F	U_F	% Unc.
1.3033	25.8684	0.0149	0.0013	0.2693	0.0002	0.0054	0.2694	0.2698	1.0429
1.3116	19.5634	0.0149	0.0010	0.2693	0.0002	0.0054	0.2694	0.2698	1.3790
1.3018	14.6455	0.0148	0.0007	0.2693	0.0001	0.0054	0.2694	0.2698	1.8420
1.3022	3.6444	0.0132	0.0002	0.2693	0.0000	0.0054	0.2694	0.2697	7.4001
1.3025	-9.6546	0.0146	-0.0005	0.2693	-0.0001	0.0054	0.2694	0.2698	-2.7941
1.3028	-44.6663	0.0148	-0.0022	0.2693	-0.0004	0.0054	0.2694	0.2698	-0.6040
1.3020	-71.4491	0.0149	-0.0036	0.2693	-0.0007	0.0054	0.2694	0.2698	-0.3776
1.3014	-85.4106	0.0150	-0.0043	0.2693	-0.0008	0.0054	0.2694	0.2698	-0.3159
1.3116	-101.8478	0.0153	-0.0051	0.2693	-0.0010	0.0054	0.2694	0.2699	-0.2650
1.3007	-120.5716	0.0152	-0.0060	0.2693	-0.0011	0.0054	0.2694	0.2699	-0.2238
1.3006	-136.2495	0.0156	-0.0068	0.2693	-0.0013	0.0054	0.2695	0.2699	-0.1981
1.3020	-152.6671	0.0159	-0.0076	0.2693	-0.0015	0.0054	0.2695	0.2699	-0.1768

V (m/s)	F (N)	P_F	B_1	B_2	B_3	B_4	B_F	U_F	% Unc.
1.6030	41.2862	0.0173	0.0021	0.2693	0.0004	0.0054	0.2694	0.2699	0.6538
1.6053	35.5623	0.0171	0.0018	0.2693	0.0003	0.0054	0.2694	0.2699	0.7590
1.6045	28.4763	0.0171	0.0014	0.2693	0.0003	0.0054	0.2694	0.2699	0.9479
1.6063	19.6455	0.0168	0.0010	0.2693	0.0002	0.0054	0.2694	0.2699	1.3738
1.6063	3.7317	0.0135	0.0002	0.2693	0.0000	0.0054	0.2694	0.2697	7.2274
1.6059	-29.3272	0.0172	-0.0015	0.2693	-0.0003	0.0054	0.2694	0.2699	-0.9204
1.6056	-55.1247	0.0173	-0.0028	0.2693	-0.0005	0.0054	0.2694	0.2699	-0.4897
1.6063	-72.6503	0.0174	-0.0036	0.2693	-0.0007	0.0054	0.2694	0.2700	-0.3716
1.6059	-88.4574	0.0174	-0.0044	0.2693	-0.0008	0.0054	0.2694	0.2700	-0.3052
1.6050	-104.0080	0.0176	-0.0052	0.2693	-0.0010	0.0054	0.2694	0.2700	-0.2596
1.6054	-122.4283	0.0178	-0.0061	0.2693	-0.0012	0.0054	0.2694	0.2700	-0.2206
1.6019	-139.6617	0.0179	-0.0070	0.2693	-0.0013	0.0054	0.2695	0.2701	-0.1934

<u>Uncertainty Analysis</u> (Tow Force)
Load Varying Self-Propulsion Test (Duct + Prop. fitted on)

V (m/s)	F (N)	P_F	B_1	B_2	B_3	B_4	B_F	U_F	% Unc.
1.8051	51.8995	0.0194	0.0026	0.2693	0.0005	0.0054	0.2694	0.2701	0.5204
1.8061	45.3553	0.0193	0.0023	0.2693	0.0004	0.0054	0.2694	0.2701	0.5954
1.8054	39.4746	0.0193	0.0020	0.2693	0.0004	0.0054	0.2694	0.2701	0.6841
1.8043	29.6365	0.0191	0.0015	0.2693	0.0003	0.0054	0.2694	0.2700	0.9112
1.8049	14.8382	0.0190	0.0007	0.2693	0.0001	0.0054	0.2694	0.2700	1.8199
1.8053	-16.0784	0.0192	-0.0008	0.2693	-0.0002	0.0054	0.2694	0.2701	-1.6796
1.8053	-41.3141	0.0193	-0.0021	0.2693	-0.0004	0.0054	0.2694	0.2701	-0.6537
1.8062	-58.6353	0.0195	-0.0029	0.2693	-0.0006	0.0054	0.2694	0.2701	-0.4606
1.8057	-75.5367	0.0197	-0.0038	0.2693	-0.0007	0.0054	0.2694	0.2701	-0.3576
1.8064	-91.0393	0.0197	-0.0046	0.2693	-0.0009	0.0054	0.2694	0.2701	-0.2967
1.8051	-109.2754	0.0198	-0.0055	0.2693	-0.0010	0.0054	0.2694	0.2702	-0.2472
1.8046	-128.6862	0.0202	-0.0064	0.2693	-0.0012	0.0054	0.2694	0.2702	-0.2100

V (m/s)	F (N)	P_F	B_1	B_2	B_3	B_4	B_F	U_F	% Unc.
2.1082	66.8995	0.0217	0.0033	0.2693	0.0006	0.0054	0.2694	0.2703	0.4040
2.1078	60.4425	0.0216	0.0030	0.2693	0.0006	0.0054	0.2694	0.2702	0.4471
2.1093	54.4746	0.0215	0.0027	0.2693	0.0005	0.0054	0.2694	0.2702	0.4961
2.1056	47.4534	0.0215	0.0024	0.2693	0.0005	0.0054	0.2694	0.2702	0.5695
2.1084	32.8382	0.0213	0.0016	0.2693	0.0003	0.0054	0.2694	0.2702	0.8229
2.1095	1.0784	0.0181	0.0001	0.2693	0.0000	0.0054	0.2694	0.2700	25.0347
2.1082	-23.4953	0.0212	-0.0012	0.2693	-0.0002	0.0054	0.2694	0.2702	-1.1500
2.1098	-37.6353	0.0214	-0.0019	0.2693	-0.0004	0.0054	0.2694	0.2702	-0.7180
2.1103	-54.5367	0.0215	-0.0027	0.2693	-0.0005	0.0054	0.2694	0.2702	-0.4955
2.1101	-70.0393	0.0217	-0.0035	0.2693	-0.0007	0.0054	0.2694	0.2703	-0.3859
2.1091	-88.2754	0.0219	-0.0044	0.2693	-0.0008	0.0054	0.2694	0.2703	-0.3062
2.1086	-107.6862	0.0221	-0.0054	0.2693	-0.0010	0.0054	0.2694	0.2703	-0.2510

Uncertainty Analysis (Tow Force)
Load Varying Self-Propulsion Test (Duct + Prop. fitted on)

V (m/s)	F (N)	P_F	B_1	B_2	B_3	B_4	B_F	U_F	% Unc.
2.3113	86.1252	0.0249	0.0043	0.2693	0.0008	0.0054	0.2694	0.2706	0.3141
2.3012	80.4523	0.0246	0.0040	0.2693	0.0008	0.0054	0.2694	0.2705	0.3362
2.3106	75.8979	0.0244	0.0038	0.2693	0.0007	0.0054	0.2694	0.2705	0.3564
2.3087	68.2312	0.0242	0.0034	0.2693	0.0007	0.0054	0.2694	0.2705	0.3964
2.3127	55.4110	0.0241	0.0028	0.2693	0.0005	0.0054	0.2694	0.2705	0.4881
2.3075	24.5844	0.0238	0.0012	0.2693	0.0002	0.0054	0.2694	0.2704	1.1000
2.3104	0.7728	0.0185	0.0000	0.2693	0.0000	0.0054	0.2694	0.2700	34.9381
2.3089	-13.1249	0.0238	-0.0007	0.2693	-0.0001	0.0054	0.2694	0.2704	-2.0603
2.3716	-27.7563	0.0239	-0.0014	0.2693	-0.0003	0.0054	0.2694	0.2704	-0.9743
2.3101	-45.5095	0.0240	-0.0023	0.2693	-0.0004	0.0054	0.2694	0.2704	-0.5943
2.3079	-62.1596	0.0243	-0.0031	0.2693	-0.0006	0.0054	0.2694	0.2705	-0.4351
2.3092	-78.5473	0.0245	-0.0039	0.2693	-0.0007	0.0054	0.2694	0.2705	-0.3444

V (m/s)	F (N)	P_F	B_1	B_2	B_3	B_4	B_F	U_F	% Unc.
2.5094	105.9464	0.0279	0.0053	0.2693	0.0010	0.0054	0.2694	0.2709	0.2557
2.5104	101.2323	0.0278	0.0051	0.2693	0.0010	0.0054	0.2694	0.2708	0.2675
2.5067	96.3633	0.0278	0.0048	0.2693	0.0009	0.0054	0.2694	0.2708	0.2811
2.5107	90.3483	0.0276	0.0045	0.2693	0.0009	0.0054	0.2694	0.2708	0.2997
2.5093	78.3736	0.0274	0.0039	0.2693	0.0007	0.0054	0.2694	0.2708	0.3455
2.5099	49.0260	0.0270	0.0025	0.2693	0.0005	0.0054	0.2694	0.2707	0.5522
2.5126	21.5911	0.0269	0.0011	0.2693	0.0002	0.0054	0.2694	0.2707	1.2538
2.5100	9.2021	0.0269	0.0005	0.2693	0.0001	0.0054	0.2694	0.2707	2.9418
2.5104	-5.5593	0.0267	-0.0003	0.2693	-0.0001	0.0054	0.2694	0.2707	-4.8691
2.5101	-19.5704	0.0269	-0.0010	0.2693	-0.0002	0.0054	0.2694	0.2707	-1.3833
2.5109	-38.1235	0.0271	-0.0019	0.2693	-0.0004	0.0054	0.2694	0.2707	-0.7101
2.5096	-53.2054	0.0273	-0.0027	0.2693	-0.0005	0.0054	0.2694	0.2708	-0.5089

Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

Uncertainty Analysis (Advance Coefficient)

V	Vehicle Velocity (m/s)	
n	Propeller Rotational Speed (rpm)	
D	Propeller Diameter (m)	
J	Advance Coefficient	
P_V	Precision Error for V	
B_V	Bias Error for V	
P_J	Precision Error for J	
P_n	Precision Error for n	
B_n	Bias Error for n	
B_D	Bias Error for D	
B_J	Bias Error for J	$B_J = (\delta J / \delta V\ B_V)^2 + (\delta J / \delta n\ B_n)^2 + (\delta J / \delta D\ B_D)^2)^{1/2}$
U_J	Total Error for J	$U_J = (B_J^2 + P_J^2)^{1/2}$
% U	Percentage Uncertainty for J	$\% U = (U_J / J) \times 100$

Uncertainty Analysis (Advance Coefficient)
Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

V (m/s)	P_V	B_V	n (rpm)	P_n	B_n	B_D	B_V δJ/δV	B_n δJ/δn	B_D δJ/δD	B_J	P_J	U_J	J	% U
0.8002	0.0004	0.0035	499.5767	0.0005	0.0391	0.0001	4.62E-05	-8.22E-07	-6.90E-06	4.67E-05	0.0006	0.0006	0.6306	0.0954
0.8004	0.0003	0.0035	551.4566	0.0005	0.0391	0.0001	4.19E-05	-6.75E-07	-6.25E-06	4.23E-05	0.0006	0.0006	0.5714	0.1090
0.8012	0.0004	0.0035	600.4575	0.0005	0.0391	0.0001	3.85E-05	-5.70E-07	-5.74E-06	3.89E-05	0.0007	0.0007	0.5253	0.1328
0.8018	0.0004	0.0035	650.4466	0.0005	0.0391	0.0001	3.55E-05	-4.86E-07	-5.31E-06	3.59E-05	0.0006	0.0006	0.4853	0.1296
0.8002	0.0004	0.0035	753.2179	0.0005	0.0391	0.0001	3.07E-05	-3.62E-07	-4.57E-06	3.10E-05	0.0006	0.0006	0.4183	0.1491
0.8002	0.0004	0.0035	902.9860	0.0005	0.0391	0.0001	2.56E-05	-2.52E-07	-3.82E-06	2.59E-05	0.0006	0.0006	0.3489	0.1814
0.8003	0.0004	0.0035	1008.5085	0.0005	0.0391	0.0001	2.29E-05	-2.02E-07	-3.42E-06	2.32E-05	0.0006	0.0006	0.3124	0.1922
0.8005	0.0004	0.0035	1052.3455	0.0005	0.0392	0.0001	2.19E-05	-1.86E-07	-3.28E-06	2.22E-05	0.0006	0.0006	0.2995	0.2046
0.8014	0.0004	0.0035	1104.1946	0.0005	0.0392	0.0001	2.09E-05	-1.69E-07	-3.12E-06	2.11E-05	0.0006	0.0006	0.2857	0.2242
0.8002	0.0004	0.0035	1150.2549	0.0005	0.0392	0.0001	2.01E-05	-1.55E-07	-3.00E-06	2.03E-05	0.0007	0.0007	0.2739	0.2488
0.8008	0.0004	0.0035	1206.8237	0.0005	0.0392	0.0001	1.91E-05	-1.41E-07	-2.86E-06	1.93E-05	0.0007	0.0007	0.2612	0.2560
0.8006	0.0004	0.0035	1252.7469	0.0005	0.0392	0.0001	1.84E-05	-1.31E-07	-2.75E-06	1.86E-05	0.0006	0.0006	0.2516	0.2484

V (m/s)	P_V	B_V	n (rpm)	P_n	B_n	B_D	B_V δJ/δV	B_n δJ/δn	B_D δJ/δD	B_J	P_J	U_J	J	% U
1.1006	0.0005	0.0035	500.6686	0.0006	0.0391	0.0001	4.62E-05	-1.13E-06	-9.46E-06	4.72E-05	0.0008	0.0008	0.8655	0.0922
1.1022	0.0005	0.0035	552.7477	0.0006	0.0391	0.0001	4.19E-05	-9.25E-07	-8.59E-06	4.28E-05	0.0008	0.0008	0.7851	0.1020
1.1031	0.0006	0.0035	603.4646	0.0007	0.0391	0.0001	3.84E-05	-7.77E-07	-7.87E-06	3.92E-05	0.0009	0.0009	0.7197	0.1225
1.1032	0.0005	0.0035	651.6550	0.0007	0.0391	0.0001	3.55E-05	-6.67E-07	-7.29E-06	3.63E-05	0.0009	0.0009	0.6665	0.1309
1.1005	0.0005	0.0035	754.9726	0.0007	0.0391	0.0001	3.07E-05	-4.96E-07	-6.28E-06	3.13E-05	0.0009	0.0009	0.5739	0.1487
1.1026	0.0005	0.0035	901.8126	0.0007	0.0391	0.0001	2.57E-05	-3.48E-07	-5.26E-06	2.62E-05	0.0008	0.0008	0.4814	0.1754
1.1014	0.0005	0.0035	1000.0880	0.0007	0.0391	0.0001	2.31E-05	-2.83E-07	-4.74E-06	2.36E-05	0.0009	0.0009	0.4336	0.2040
1.1017	0.0005	0.0035	1056.9566	0.0007	0.0392	0.0001	2.19E-05	-2.53E-07	-4.49E-06	2.24E-05	0.0009	0.0009	0.4104	0.2109
1.1013	0.0005	0.0035	1104.4112	0.0007	0.0392	0.0001	2.10E-05	-2.32E-07	-4.29E-06	2.14E-05	0.0009	0.0009	0.3926	0.2253
1.1020	0.0005	0.0035	1156.9473	0.0007	0.0392	0.0001	2.00E-05	-2.12E-07	-4.10E-06	2.04E-05	0.0009	0.0009	0.3750	0.2369
1.1015	0.0005	0.0035	1212.9133	0.0007	0.0392	0.0001	1.91E-05	-1.93E-07	-3.91E-06	1.95E-05	0.0009	0.0009	0.3575	0.2473
1.1011	0.0006	0.0035	1258.8191	0.0007	0.0392	0.0001	1.84E-05	-1.79E-07	-3.77E-06	1.88E-05	0.0009	0.0009	0.3444	0.2514

Uncertainty Analysis (Advance Coefficient)
Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

V (m/s)	P_V	B_V	n (rpm)	P_n	B_n	B_D	$B_V\ \delta J/\delta V$	$B_n\ \delta J/\delta n$	$B_D\ \delta J/\delta D$	B_J	P_J	U_J	J	% U
1.3033	0.0006	0.0035	501.8541	0.0008	0.0391	0.0001	4.62E-05	-1.33E-06	-1.12E-05	4.76E-05	0.0010	0.0010	1.0224	0.0948
1.3116	0.0006	0.0035	550.7550	0.0007	0.0391	0.0001	4.21E-05	-1.11E-06	-1.03E-05	4.34E-05	0.0010	0.0010	0.9376	0.1029
1.3018	0.0007	0.0035	601.3456	0.0007	0.0391	0.0001	3.86E-05	-9.24E-07	-9.32E-06	3.97E-05	0.0010	0.0010	0.8523	0.1146
1.3022	0.0006	0.0035	654.7460	0.0008	0.0391	0.0001	3.54E-05	-7.79E-07	-8.56E-06	3.65E-05	0.0010	0.0010	0.7830	0.1273
1.3025	0.0007	0.0035	752.1400	0.0008	0.0391	0.0001	3.08E-05	-5.91E-07	-7.46E-06	3.17E-05	0.0010	0.0010	0.6818	0.1477
1.3028	0.0006	0.0035	907.7795	0.0008	0.0391	0.0001	2.56E-05	-4.06E-07	-6.18E-06	2.63E-05	0.0010	0.0010	0.5650	0.1814
1.3020	0.0007	0.0035	1006.9377	0.0008	0.0391	0.0001	2.30E-05	-3.30E-07	-5.57E-06	2.37E-05	0.0010	0.0010	0.5091	0.2004
1.3014	0.0007	0.0035	1051.7237	0.0007	0.0392	0.0001	2.21E-05	-3.02E-07	-5.33E-06	2.27E-05	0.0010	0.0010	0.4872	0.2063
1.3116	0.0006	0.0035	1103.5535	0.0008	0.0392	0.0001	2.10E-05	-2.77E-07	-5.12E-06	2.16E-05	0.0010	0.0010	0.4679	0.2104
1.3007	0.0007	0.0035	1155.5678	0.0008	0.0392	0.0001	2.01E-05	-2.50E-07	-4.85E-06	2.07E-05	0.0011	0.0011	0.4431	0.2376
1.3006	0.0006	0.0035	1208.2135	0.0008	0.0392	0.0001	1.92E-05	-2.29E-07	-4.63E-06	1.98E-05	0.0010	0.0010	0.4238	0.2389
1.3020	0.0007	0.0035	1256.7456	0.0008	0.0392	0.0001	1.85E-05	-2.12E-07	-4.46E-06	1.90E-05	0.0010	0.0010	0.4079	0.2570

V (m/s)	P_V	B_V	n (rpm)	P_n	B_n	B_D	$B_V\ \delta J/\delta V$	$B_n\ \delta J/\delta n$	$B_D\ \delta J/\delta D$	B_J	P_J	U_J	J	% U
1.6030	0.0007	0.0036	503.1755	0.0009	0.0391	0.0001	4.63E-05	-1.62E-06	-1.37E-05	4.83E-05	0.0011	0.0011	1.2542	0.0900
1.6053	0.0008	0.0036	554.4423	0.0009	0.0391	0.0001	4.20E-05	-1.34E-06	-1.25E-05	4.39E-05	0.0012	0.0012	1.1399	0.1061
1.6045	0.0007	0.0036	604.3557	0.0009	0.0391	0.0001	3.86E-05	-1.13E-06	-1.14E-05	4.02E-05	0.0011	0.0011	1.0452	0.1086
1.6063	0.0008	0.0036	652.4344	0.0009	0.0391	0.0001	3.57E-05	-9.68E-07	-1.06E-05	3.73E-05	0.0012	0.0012	0.9693	0.1216
1.6063	0.0008	0.0036	752.2439	0.0009	0.0391	0.0001	3.10E-05	-7.29E-07	-9.19E-06	3.23E-05	0.0012	0.0012	0.8407	0.1435
1.6059	0.0007	0.0036	901.5599	0.0009	0.0391	0.0001	2.58E-05	-5.07E-07	-7.67E-06	2.70E-05	0.0011	0.0011	0.7013	0.1626
1.6056	0.0007	0.0036	1006.7175	0.0009	0.0391	0.0001	2.31E-05	-4.07E-07	-6.87E-06	2.41E-05	0.0011	0.0012	0.6279	0.1832
1.6063	0.0007	0.0036	1054.3192	0.0009	0.0392	0.0001	2.21E-05	-3.71E-07	-6.56E-06	2.31E-05	0.0011	0.0011	0.5998	0.1870
1.6059	0.0007	0.0036	1102.5010	0.0009	0.0392	0.0001	2.11E-05	-3.40E-07	-6.27E-06	2.20E-05	0.0012	0.0012	0.5735	0.2065
1.6050	0.0007	0.0036	1154.9706	0.0009	0.0392	0.0001	2.02E-05	-3.09E-07	-5.98E-06	2.10E-05	0.0012	0.0012	0.5471	0.2158
1.6054	0.0008	0.0036	1211.5007	0.0009	0.0392	0.0001	1.92E-05	-2.81E-07	-5.71E-06	2.01E-05	0.0012	0.0012	0.5217	0.2258
1.6019	0.0008	0.0036	1257.0444	0.0009	0.0392	0.0001	1.85E-05	-2.61E-07	-5.49E-06	1.93E-05	0.0012	0.0012	0.5017	0.2430

Uncertainty Analysis (Advance Coefficient)
Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

V (m/s)	P_V	B_V	n (rpm)	P_n	B_n	B_D	B_V δJ/δV	B_n δJ/δn	B_D δJ/δD	B_J	P_J	U_J	J	% U
1.8051	0.0009	0.0036	503.4015	0.0010	0.0391	0.0001	4.65E-05	-1.83E-06	-1.54E-05	4.90E-05	0.0014	0.0014	1.4117	0.0984
1.8061	0.0009	0.0036	550.5344	0.0011	0.0391	0.0001	4.25E-05	-1.53E-06	-1.41E-05	4.48E-05	0.0014	0.0014	1.2916	0.1097
1.8054	0.0008	0.0036	601.7534	0.0011	0.0391	0.0001	3.89E-05	-1.28E-06	-1.29E-05	4.10E-05	0.0014	0.0014	1.1812	0.1172
1.8043	0.0009	0.0036	655.6402	0.0011	0.0391	0.0001	3.57E-05	-1.08E-06	-1.18E-05	3.76E-05	0.0014	0.0014	1.0835	0.1284
1.8049	0.0009	0.0036	756.7032	0.0011	0.0391	0.0001	3.09E-05	-8.09E-07	-1.03E-05	3.26E-05	0.0014	0.0014	0.9391	0.1484
1.8053	0.0009	0.0036	903.5768	0.0011	0.0391	0.0001	2.59E-05	-5.68E-07	-8.60E-06	2.73E-05	0.0014	0.0014	0.7866	0.1840
1.8053	0.0009	0.0036	1007.3473	0.0010	0.0391	0.0001	2.32E-05	-4.57E-07	-7.72E-06	2.45E-05	0.0013	0.0013	0.7056	0.1891
1.8062	0.0009	0.0036	1051.7675	0.0011	0.0392	0.0001	2.22E-05	-4.20E-07	-7.39E-06	2.34E-05	0.0014	0.0014	0.6761	0.2080
1.8057	0.0009	0.0036	1105.5646	0.0011	0.0392	0.0001	2.12E-05	-3.80E-07	-7.03E-06	2.23E-05	0.0014	0.0014	0.6430	0.2230
1.8064	0.0009	0.0036	1151.0459	0.0011	0.0392	0.0001	2.03E-05	-3.50E-07	-6.76E-06	2.14E-05	0.0015	0.0015	0.6179	0.2367
1.8051	0.0009	0.0036	1199.5015	0.0011	0.0392	0.0001	1.95E-05	-3.23E-07	-6.48E-06	2.05E-05	0.0014	0.0014	0.5925	0.2395
1.8046	0.0009	0.0036	1261.8668	0.0011	0.0392	0.0001	1.85E-05	-2.91E-07	-6.16E-06	1.95E-05	0.0014	0.0014	0.5630	0.2525

V (m/s)	P_V	B_V	n (rpm)	P_n	B_n	B_D	B_V δJ/δV	B_n δJ/δn	B_D δJ/δD	B_J	P_J	U_J	J	% U
2.1082	0.0010	0.0036	500.4856	0.0012	0.0391	0.0001	4.70E-05	-2.16E-06	-1.81E-05	5.05E-05	0.0016	0.0016	1.6584	0.0949
2.1078	0.0010	0.0036	552.4532	0.0012	0.0391	0.0001	4.26E-05	-1.77E-06	-1.64E-05	4.57E-05	0.0016	0.0016	1.5021	0.1051
2.1093	0.0011	0.0036	604.4664	0.0013	0.0391	0.0001	3.89E-05	-1.48E-06	-1.50E-05	4.18E-05	0.0017	0.0017	1.3738	0.1219
2.1056	0.0011	0.0036	651.4342	0.0013	0.0391	0.0001	3.61E-05	-1.27E-06	-1.39E-05	3.87E-05	0.0017	0.0017	1.2725	0.1318
2.1084	0.0010	0.0036	754.9317	0.0012	0.0391	0.0001	3.12E-05	-9.49E-07	-1.20E-05	3.34E-05	0.0016	0.0016	1.0995	0.1431
2.1095	0.0011	0.0036	901.1852	0.0012	0.0391	0.0001	2.61E-05	-6.67E-07	-1.01E-05	2.80E-05	0.0016	0.0016	0.9216	0.1767
2.1082	0.0011	0.0036	1002.9008	0.0012	0.0391	0.0001	2.35E-05	-5.38E-07	-9.05E-06	2.52E-05	0.0017	0.0017	0.8276	0.2001
2.1098	0.0010	0.0036	1053.7204	0.0012	0.0392	0.0001	2.23E-05	-4.88E-07	-8.62E-06	2.40E-05	0.0016	0.0016	0.7883	0.2053
2.1103	0.0011	0.0036	1101.8507	0.0013	0.0392	0.0001	2.14E-05	-4.47E-07	-8.25E-06	2.29E-05	0.0016	0.0016	0.7540	0.2184
2.1101	0.0011	0.0036	1153.6637	0.0013	0.0392	0.0001	2.04E-05	-4.08E-07	-7.88E-06	2.19E-05	0.0017	0.0017	0.7201	0.2344
2.1091	0.0011	0.0036	1210.1050	0.0013	0.0392	0.0001	1.95E-05	-3.70E-07	-7.50E-06	2.09E-05	0.0017	0.0017	0.6862	0.2501
2.1086	0.0011	0.0036	1258.3035	0.0013	0.0392	0.0001	1.87E-05	-3.43E-07	-7.22E-06	2.01E-05	0.0016	0.0016	0.6597	0.2496

Uncertainty Analysis (Advance Coefficient)
Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

V (m/s)	P_V	B_V	n (rpm)	P_n	B_n	B_D	B_V δJ/δV	B_n δJ/δn	B_D δJ/δD	B_J	P_J	U_J	J	% U
2.3113	0.0011	0.0036	506.5541	0.0013	0.0391	0.0001	4.67E-05	-2.31E-06	-1.96E-05	5.07E-05	0.0017	0.0017	1.7964	0.0958
2.3012	0.0012	0.0036	554.4124	0.0014	0.0391	0.0001	4.27E-05	-1.92E-06	-1.79E-05	4.63E-05	0.0019	0.0019	1.6341	0.1136
2.3106	0.0012	0.0036	602.4889	0.0013	0.0391	0.0001	3.93E-05	-1.63E-06	-1.65E-05	4.26E-05	0.0018	0.0018	1.5099	0.1201
2.3087	0.0012	0.0036	653.7372	0.0014	0.0391	0.0001	3.62E-05	-1.39E-06	-1.52E-05	3.93E-05	0.0018	0.0018	1.3904	0.1295
2.3127	0.0011	0.0036	751.5777	0.0013	0.0391	0.0001	3.15E-05	-1.05E-06	-1.32E-05	3.42E-05	0.0017	0.0017	1.2115	0.1440
2.3075	0.0013	0.0036	907.7627	0.0014	0.0391	0.0001	2.61E-05	-7.19E-07	-1.09E-05	2.83E-05	0.0019	0.0019	1.0008	0.1882
2.3104	0.0012	0.0036	1009.7375	0.0014	0.0391	0.0001	2.34E-05	-5.82E-07	-9.85E-06	2.54E-05	0.0018	0.0018	0.9008	0.2001
2.3089	0.0013	0.0036	1055.8389	0.0014	0.0392	0.0001	2.24E-05	-5.32E-07	-9.42E-06	2.43E-05	0.0019	0.0019	0.8609	0.2169
2.3716	0.0013	0.0036	1105.4558	0.0014	0.0392	0.0001	2.14E-05	-4.99E-07	-9.24E-06	2.34E-05	0.0019	0.0019	0.8446	0.2213
2.3101	0.0012	0.0036	1156.8492	0.0014	0.0392	0.0001	2.05E-05	-4.44E-07	-8.60E-06	2.22E-05	0.0019	0.0019	0.7862	0.2381
2.3079	0.0013	0.0036	1207.4927	0.0014	0.0392	0.0001	1.96E-05	-4.07E-07	-8.23E-06	2.13E-05	0.0019	0.0019	0.7525	0.2520
2.3092	0.0013	0.0036	1255.9025	0.0014	0.0392	0.0001	1.88E-05	-3.77E-07	-7.92E-06	2.04E-05	0.0019	0.0019	0.7239	0.2602

V (m/s)	P_V	B_V	n (rpm)	P_n	B_n	B_D	B_V δJ/δV	B_n δJ/δn	B_D δJ/δD	B_J	P_J	U_J	J	% U
2.5094	0.0013	0.0036	504.8586	0.0015	0.0391	0.0001	4.71E-05	-2.53E-06	-2.14E-05	5.18E-05	0.0020	0.0020	1.9569	0.1028
2.5104	0.0013	0.0036	552.4243	0.0015	0.0391	0.0001	4.31E-05	-2.11E-06	-1.96E-05	4.74E-05	0.0020	0.0020	1.7891	0.1104
2.5067	0.0014	0.0036	604.5795	0.0015	0.0391	0.0001	3.94E-05	-1.76E-06	-1.79E-05	4.32E-05	0.0021	0.0021	1.6324	0.1259
2.5107	0.0013	0.0036	653.2312	0.0015	0.0391	0.0001	3.64E-05	-1.51E-06	-1.65E-05	4.00E-05	0.0020	0.0020	1.5132	0.1302
2.5093	0.0014	0.0036	753.2477	0.0015	0.0391	0.0001	3.16E-05	-1.14E-06	-1.43E-05	3.47E-05	0.0021	0.0021	1.3115	0.1566
2.5099	0.0013	0.0036	906.0469	0.0015	0.0391	0.0001	2.63E-05	-7.85E-07	-1.19E-05	2.89E-05	0.0020	0.0020	1.0906	0.1820
2.5126	0.0013	0.0036	1002.7932	0.0015	0.0391	0.0001	2.37E-05	-6.42E-07	-1.08E-05	2.61E-05	0.0020	0.0020	0.9865	0.2041
2.5100	0.0013	0.0036	1052.4234	0.0015	0.0392	0.0001	2.26E-05	-5.82E-07	-1.03E-05	2.48E-05	0.0020	0.0020	0.9390	0.2142
2.5104	0.0013	0.0036	1106.7229	0.0015	0.0392	0.0001	2.15E-05	-5.27E-07	-9.77E-06	2.36E-05	0.0020	0.0020	0.8930	0.2271
2.5101	0.0014	0.0036	1152.3582	0.0016	0.0392	0.0001	2.06E-05	-4.86E-07	-9.38E-06	2.27E-05	0.0021	0.0021	0.8576	0.2413
2.5109	0.0014	0.0036	1204.5756	0.0015	0.0392	0.0001	1.98E-05	-4.45E-07	-8.97E-06	2.17E-05	0.0021	0.0021	0.8207	0.2520
2.5096	0.0013	0.0036	1256.7444	0.0016	0.0392	0.0001	1.89E-05	-4.09E-07	-8.60E-06	2.08E-05	0.0021	0.0021	0.7862	0.2616

Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

Uncertainty Analysis (Torque Coefficient)

V	Vehicle Velocity (m/s)	
n	Propeller Rotational Speed (rpm)	
D	Propeller Diameter (m)	
Q	Propeller Torque (N-m)	
ρ	Density of Water (kg/m^3)	
K_q	Torque Coefficient	
P_q	Precision Error for Q	
B_q	Bias Error for Q	
P_{Kq}	Precision Error for K_q	
P_n	Precision Error for n	
B_n	Bias Error for n	
B_ρ	Bias Error for ρ	
B_D	Bias Error for D	
B_{Kq}	Bias Error for K_q	$B_{Kq} = (\delta K_q / \delta Q\ B_q)^2 + (\delta K_q / \delta\rho\ B_\rho)^2 + (\delta K_q / \delta n\ B_n)^2 + (\delta K_q / \delta D\ B_D)^2\ )^{1/2}$
U_{Kq}	Total Error for K_q	$U_{Kq} = (B_{Kq}^2 + P_{Kq}^2)^{1/2}$
% U	Percentage Uncertainty for K_q	$\% U = (U_{Kq} / K_q) \times 100$

Uncertainty Analysis (Torque Coefficient)
Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

V (m/s)	n (rpm)	Q (N-m)	P_q	B_q	P_n	B_n	B_ρ	B_D	$B_q\ \delta K_q/\delta Q$	$B_\rho\ \delta K_q/\delta\rho$	$B_n\ \delta K_q/\delta n$	$B_D\ \delta K_q/\delta D$	B_{Kq}	P_{Kq}	U_{Kq}	Kq	% U
0.8002	499.5767	0.3364	0.0008	0.0076	0.0005	0.0391	0.0578	0.0001	3.72E-07	-9.48E-10	-2.57E-09	-5.38E-08	3.76E-07	0.0009	0.0009	0.0590	1.5807
0.8004	551.4566	0.4972	0.0008	0.0076	0.0005	0.0391	0.0578	0.0001	3.06E-07	-1.15E-09	-2.82E-09	-6.52E-08	3.13E-07	0.0010	0.0010	0.0716	1.3444
0.8012	600.4575	0.6260	0.0008	0.0077	0.0005	0.0391	0.0578	0.0001	2.58E-07	-1.22E-09	-2.75E-09	-6.93E-08	2.67E-07	0.0010	0.0010	0.0760	1.2914
0.8018	650.4466	0.8160	0.0008	0.0077	0.0005	0.0391	0.0578	0.0001	2.20E-07	-1.36E-09	-2.82E-09	-7.70E-08	2.33E-07	0.0010	0.0010	0.0845	1.1574
0.8002	753.2179	1.2582	0.0008	0.0077	0.0005	0.0391	0.0578	0.0001	1.65E-07	-1.56E-09	-2.80E-09	-8.85E-08	1.87E-07	0.0009	0.0009	0.0971	0.9771
0.8002	902.9860	1.9967	0.0009	0.0078	0.0005	0.0391	0.0578	0.0001	1.16E-07	-1.72E-09	-2.58E-09	-9.77E-08	1.52E-07	0.0010	0.0010	0.1072	0.9292
0.8003	1008.5085	2.5232	0.0009	0.0079	0.0005	0.0391	0.0578	0.0001	9.45E-08	-1.74E-09	-2.34E-09	-9.90E-08	1.37E-07	0.0010	0.0010	0.1086	0.9146
0.8005	1052.3455	2.8657	0.0009	0.0080	0.0005	0.0392	0.0578	0.0001	8.76E-08	-1.82E-09	-2.34E-09	-1.03E-07	1.35E-07	0.0010	0.0010	0.1133	0.8924
0.8014	1104.1946	3.1813	0.0009	0.0080	0.0005	0.0392	0.0578	0.0001	8.03E-08	-1.83E-09	-2.25E-09	-1.04E-07	1.32E-07	0.0010	0.0010	0.1143	0.9011
0.8002	1150.2549	3.5448	0.0009	0.0081	0.0005	0.0392	0.0578	0.0001	7.49E-08	-1.88E-09	-2.22E-09	-1.07E-07	1.31E-07	0.0011	0.0011	0.1173	0.9008
0.8008	1206.8237	3.9061	0.0009	0.0083	0.0005	0.0392	0.0578	0.0001	6.89E-08	-1.89E-09	-2.12E-09	-1.07E-07	1.27E-07	0.0011	0.0011	0.1174	0.9072
0.8006	1252.7469	4.2519	0.0009	0.0084	0.0005	0.0392	0.0578	0.0001	6.48E-08	-1.90E-09	-2.06E-09	-1.08E-07	1.26E-07	0.0011	0.0011	0.1186	0.8896

V (m/s)	n (rpm)	Q (N-m)	P_Q	B_Q	P_n	B_n	B_ρ	B_D	$B_q\ \delta K_q/\delta Q$	$B_\rho\ \delta K_q/\delta\rho$	$B_n\ \delta K_q/\delta n$	$B_D\ \delta K_q/\delta D$	B_{Kq}	P_{Kq}	U_{Kq}	Kq	% U
1.1006	500.6686	0.3193	0.0009	0.0076	0.0006	0.0391	0.0578	0.0001	3.71E-07	-8.96E-10	-2.42E-09	-5.08E-08	3.74E-07	0.0011	0.0011	0.0558	2.0185
1.1022	552.7477	0.4722	0.0010	0.0076	0.0006	0.0391	0.0578	0.0001	3.04E-07	-1.09E-09	-2.66E-09	-6.17E-08	3.11E-07	0.0011	0.0011	0.0677	1.6923
1.1031	603.4646	0.6209	0.0010	0.0077	0.0007	0.0391	0.0578	0.0001	2.56E-07	-1.20E-09	-2.69E-09	-6.80E-08	2.64E-07	0.0012	0.0012	0.0747	1.5757
1.1032	651.6550	0.8161	0.0010	0.0077	0.0007	0.0391	0.0578	0.0001	2.20E-07	-1.35E-09	-2.81E-09	-7.67E-08	2.33E-07	0.0012	0.0012	0.0842	1.4310
1.1005	754.9726	1.2195	0.0010	0.0077	0.0007	0.0391	0.0578	0.0001	1.64E-07	-1.50E-09	-2.70E-09	-8.54E-08	1.85E-07	0.0012	0.0012	0.0937	1.2700
1.1026	901.8126	1.9412	0.0010	0.0078	0.0007	0.0391	0.0578	0.0001	1.17E-07	-1.68E-09	-2.52E-09	-9.53E-08	1.51E-07	0.0012	0.0012	0.1045	1.1570
1.1014	1000.0880	2.4885	0.0010	0.0079	0.0007	0.0391	0.0578	0.0001	9.60E-08	-1.75E-09	-2.37E-09	-9.93E-08	1.38E-07	0.0012	0.0012	0.1090	1.1354
1.1017	1056.9566	2.7980	0.0010	0.0080	0.0007	0.0392	0.0578	0.0001	8.66E-08	-1.76E-09	-2.26E-09	-1.00E-07	1.32E-07	0.0013	0.0013	0.1097	1.1458
1.1013	1104.4112	3.1236	0.0010	0.0080	0.0007	0.0392	0.0578	0.0001	8.01E-08	-1.80E-09	-2.21E-09	-1.02E-07	1.30E-07	0.0013	0.0013	0.1121	1.1179
1.1020	1156.9473	3.4722	0.0011	0.0081	0.0007	0.0392	0.0578	0.0001	7.39E-08	-1.82E-09	-2.14E-09	-1.04E-07	1.27E-07	0.0013	0.0013	0.1136	1.1208
1.1015	1212.9133	3.8531	0.0011	0.0082	0.0007	0.0392	0.0578	0.0001	6.81E-08	-1.84E-09	-2.06E-09	-1.05E-07	1.25E-07	0.0013	0.0013	0.1147	1.1076
1.1011	1258.8191	4.2369	0.0011	0.0084	0.0007	0.0392	0.0578	0.0001	6.41E-08	-1.88E-09	-2.03E-09	-1.07E-07	1.25E-07	0.0013	0.0013	0.1171	1.0737

V (m/s)	n (rpm)	Q (N-m)	P_Q	B_Q	P_n	B_n	B_ρ	B_D	$B_q\ \delta K_q/\delta Q$	$B_\rho\ \delta K_q/\delta\rho$	$B_n\ \delta K_q/\delta n$	$B_D\ \delta K_q/\delta D$	B_{Kq}	P_{Kq}	U_{Kq}	Kq	% U
1.3033	501.8541	0.2851	0.0010	0.0076	0.0008	0.0391	0.0578	0.0001	3.69E-07	-7.96E-10	-2.14E-09	-4.52E-08	3.72E-07	0.0013	0.0013	0.0496	2.5341
1.3116	550.7550	0.4280	0.0011	0.0076	0.0007	0.0391	0.0578	0.0001	3.06E-07	-9.92E-10	-2.44E-09	-5.63E-08	3.12E-07	0.0013	0.0013	0.0618	2.0871
1.3018	601.3456	0.5866	0.0011	0.0076	0.0007	0.0391	0.0578	0.0001	2.57E-07	-1.14E-09	-2.57E-09	-6.47E-08	2.65E-07	0.0013	0.0013	0.0710	1.8237
1.3022	654.7460	0.7812	0.0012	0.0077	0.0008	0.0391	0.0578	0.0001	2.17E-07	-1.28E-09	-2.65E-09	-7.27E-08	2.29E-07	0.0014	0.0014	0.0798	1.7935
1.3025	752.1400	1.1578	0.0012	0.0077	0.0008	0.0391	0.0578	0.0001	1.65E-07	-1.44E-09	-2.59E-09	-8.17E-08	1.84E-07	0.0014	0.0014	0.0896	1.5943
1.3028	907.7795	1.9016	0.0012	0.0078	0.0008	0.0391	0.0578	0.0001	1.15E-07	-1.62E-09	-2.42E-09	-9.21E-08	1.47E-07	0.0015	0.0015	0.1010	1.4521
1.3020	1006.9377	2.4451	0.0012	0.0079	0.0008	0.0391	0.0578	0.0001	9.46E-08	-1.70E-09	-2.28E-09	-9.62E-08	1.35E-07	0.0014	0.0014	0.1056	1.3611
1.3014	1051.7237	2.6985	0.0012	0.0079	0.0007	0.0392	0.0578	0.0001	8.73E-08	-1.72E-09	-2.21E-09	-9.74E-08	1.31E-07	0.0014	0.0014	0.1068	1.3517
1.3116	1103.5535	3.0711	0.0013	0.0080	0.0008	0.0392	0.0578	0.0001	8.01E-08	-1.77E-09	-2.18E-09	-1.01E-07	1.29E-07	0.0015	0.0015	0.1104	1.3343
1.3007	1155.5678	3.4310	0.0013	0.0081	0.0008	0.0392	0.0578	0.0001	7.39E-08	-1.81E-09	-2.12E-09	-1.03E-07	1.26E-07	0.0015	0.0015	0.1125	1.3361
1.3006	1208.2135	3.7775	0.0013	0.0082	0.0008	0.0392	0.0578	0.0001	6.84E-08	-1.82E-09	-2.04E-09	-1.03E-07	1.24E-07	0.0015	0.0015	0.1133	1.3321
1.3020	1256.7456	4.1836	0.0013	0.0083	0.0008	0.0392	0.0578	0.0001	6.42E-08	-1.86E-09	-2.01E-09	-1.06E-07	1.24E-07	0.0015	0.0015	0.1160	1.2996

Uncertainty Analysis (Torque Coefficient)

Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

V (m/s)	n (rpm)	Q (N-m)	P_Q	B_Q	P_n	B_n	B_ρ	B_D	$B_q\ \delta K_q/\delta Q$	$B_\rho\ \delta K_q/\delta\rho$	$B_n\ \delta K_q/\delta n$	$B_D\ \delta K_q/\delta D$	B_{Kq}	P_{Kq}	U_{Kq}	Kq	% U
1.6030	503.1755	0.2466	0.0011	0.0076	0.0009	0.0391	0.0578	0.0001	3.67E-07	-6.85E-10	-1.84E-09	-3.89E-08	3.69E-07	0.0014	0.0014	0.0427	3.3324
1.6053	554.4423	0.3896	0.0013	0.0076	0.0009	0.0391	0.0578	0.0001	3.02E-07	-8.91E-10	-2.17E-09	-5.06E-08	3.07E-07	0.0016	0.0016	0.0555	2.8904
1.6045	604.3557	0.5547	0.0013	0.0076	0.0009	0.0391	0.0578	0.0001	2.55E-07	-1.07E-09	-2.39E-09	-6.06E-08	2.62E-07	0.0016	0.0016	0.0665	2.3565
1.6063	652.4344	0.7494	0.0013	0.0077	0.0009	0.0391	0.0578	0.0001	2.19E-07	-1.24E-09	-2.57E-09	-7.03E-08	2.30E-07	0.0016	0.0016	0.0771	2.0830
1.6063	752.2439	1.1132	0.0013	0.0077	0.0009	0.0391	0.0578	0.0001	1.65E-07	-1.38E-09	-2.49E-09	-7.85E-08	1.83E-07	0.0016	0.0016	0.0861	1.8772
1.6059	901.5599	1.8053	0.0013	0.0078	0.0009	0.0391	0.0578	0.0001	1.16E-07	-1.56E-09	-2.35E-09	-8.86E-08	1.46E-07	0.0016	0.0016	0.0973	1.6597
1.6056	1006.7175	2.4295	0.0014	0.0079	0.0009	0.0391	0.0578	0.0001	9.46E-08	-1.69E-09	-2.27E-09	-9.57E-08	1.35E-07	0.0016	0.0016	0.1050	1.5351
1.6063	1054.3192	2.6763	0.0014	0.0079	0.0009	0.0392	0.0578	0.0001	8.68E-08	-1.69E-09	-2.18E-09	-9.61E-08	1.30E-07	0.0016	0.0016	0.1054	1.5182
1.6059	1102.5010	3.0303	0.0014	0.0080	0.0009	0.0392	0.0578	0.0001	8.02E-08	-1.75E-09	-2.15E-09	-9.95E-08	1.28E-07	0.0017	0.0017	0.1092	1.5219
1.6050	1154.9706	3.3480	0.0014	0.0081	0.0009	0.0392	0.0578	0.0001	7.38E-08	-1.76E-09	-2.07E-09	-1.00E-07	1.24E-07	0.0017	0.0017	0.1099	1.5091
1.6054	1211.5007	3.7189	0.0014	0.0082	0.0009	0.0392	0.0578	0.0001	6.79E-08	-1.78E-09	-1.99E-09	-1.01E-07	1.22E-07	0.0016	0.0016	0.1110	1.4849
1.6019	1257.0444	4.0895	0.0014	0.0083	0.0009	0.0392	0.0578	0.0001	6.40E-08	-1.82E-09	-1.96E-09	-1.03E-07	1.22E-07	0.0017	0.0017	0.1133	1.4782

V (m/s)	n (rpm)	Q (N-m)	P_Q	B_Q	P_n	B_n	B_ρ	B_D	$B_q\ \delta K_q/\delta Q$	$B_\rho\ \delta K_q/\delta\rho$	$B_n\ \delta K_q/\delta n$	$B_D\ \delta K_q/\delta D$	B_{Kq}	P_{Kq}	U_{Kq}	Kq	% U
1.8051	503.4015	0.2345	0.0012	0.0076	0.0010	0.0391	0.0578	0.0001	3.67E-07	-6.51E-10	-1.75E-09	-3.69E-08	3.68E-07	0.0016	0.0016	0.0405	3.9179
1.8061	550.5344	0.3599	0.0014	0.0076	0.0011	0.0391	0.0578	0.0001	3.07E-07	-8.35E-10	-2.05E-09	-4.74E-08	3.10E-07	0.0018	0.0018	0.0520	3.3923
1.8054	601.7534	0.4952	0.0014	0.0076	0.0011	0.0391	0.0578	0.0001	2.57E-07	-9.61E-10	-2.16E-09	-5.46E-08	2.63E-07	0.0018	0.0018	0.0599	3.0128
1.8043	655.6402	0.7021	0.0015	0.0077	0.0011	0.0391	0.0578	0.0001	2.17E-07	-1.15E-09	-2.37E-09	-6.52E-08	2.26E-07	0.0018	0.0018	0.0715	2.5113
1.8049	756.7032	1.1097	0.0015	0.0077	0.0011	0.0391	0.0578	0.0001	1.63E-07	-1.36E-09	-2.44E-09	-7.73E-08	1.81E-07	0.0018	0.0018	0.0849	2.1398
1.8053	903.5768	1.7817	0.0015	0.0078	0.0011	0.0391	0.0578	0.0001	1.16E-07	-1.53E-09	-2.30E-09	-8.71E-08	1.45E-07	0.0018	0.0018	0.0956	1.9296
1.8053	1007.3473	2.3494	0.0015	0.0079	0.0010	0.0391	0.0578	0.0001	9.43E-08	-1.63E-09	-2.19E-09	-9.24E-08	1.32E-07	0.0018	0.0018	0.1014	1.7648
1.8062	1051.7675	2.6355	0.0015	0.0079	0.0011	0.0392	0.0578	0.0001	8.71E-08	-1.68E-09	-2.16E-09	-9.51E-08	1.29E-07	0.0018	0.0018	0.1043	1.7562
1.8057	1105.5646	2.9403	0.0015	0.0080	0.0011	0.0392	0.0578	0.0001	7.95E-08	-1.69E-09	-2.07E-09	-9.60E-08	1.25E-07	0.0018	0.0018	0.1053	1.7505
1.8064	1151.0459	3.2937	0.0015	0.0081	0.0011	0.0392	0.0578	0.0001	7.42E-08	-1.75E-09	-2.06E-09	-9.92E-08	1.24E-07	0.0019	0.0019	0.1089	1.7123
1.8051	1199.5015	3.6086	0.0015	0.0082	0.0011	0.0392	0.0578	0.0001	6.90E-08	-1.76E-09	-1.99E-09	-1.00E-07	1.22E-07	0.0019	0.0019	0.1098	1.6904
1.8046	1261.8668	4.0584	0.0015	0.0083	0.0011	0.0392	0.0578	0.0001	6.34E-08	-1.79E-09	-1.93E-09	-1.02E-07	1.20E-07	0.0019	0.0019	0.1116	1.6811

V (m/s)	n (rpm)	Q (N-m)	P_Q	B_Q	P_n	B_n	B_ρ	B_D	$B_q\ \delta K_q/\delta Q$	$B_\rho\ \delta K_q/\delta\rho$	$B_n\ \delta K_q/\delta n$	$B_D\ \delta K_q/\delta D$	B_{Kq}	P_{Kq}	U_{Kq}	Kq	% U
2.1082	500.4856	0.1922	0.0011	0.0076	0.0012	0.0391	0.0578	0.0001	3.71E-07	-5.39E-10	-1.46E-09	-3.06E-08	3.72E-07	0.0016	0.0016	0.0336	4.8014
2.1078	552.4532	0.3309	0.0016	0.0076	0.0012	0.0391	0.0578	0.0001	3.04E-07	-7.62E-10	-1.87E-09	-4.33E-08	3.08E-07	0.0020	0.0020	0.0475	4.1880
2.1093	604.4664	0.4607	0.0016	0.0076	0.0013	0.0391	0.0578	0.0001	2.54E-07	-8.87E-10	-1.98E-09	-5.03E-08	2.59E-07	0.0020	0.0020	0.0552	3.6544
2.1056	651.4342	0.6493	0.0016	0.0077	0.0013	0.0391	0.0578	0.0001	2.19E-07	-1.08E-09	-2.23E-09	-6.11E-08	2.28E-07	0.0021	0.0021	0.0670	3.0653
2.1084	754.9317	1.0051	0.0016	0.0077	0.0012	0.0391	0.0578	0.0001	1.64E-07	-1.24E-09	-2.22E-09	-7.04E-08	1.78E-07	0.0020	0.0020	0.0772	2.5743
2.1095	901.1852	1.6707	0.0016	0.0078	0.0012	0.0391	0.0578	0.0001	1.16E-07	-1.45E-09	-2.17E-09	-8.21E-08	1.42E-07	0.0020	0.0020	0.0901	2.2202
2.1082	1002.9008	2.2317	0.0016	0.0078	0.0012	0.0391	0.0578	0.0001	9.48E-08	-1.56E-09	-2.11E-09	-8.85E-08	1.30E-07	0.0020	0.0020	0.0972	2.0872
2.1098	1053.7204	2.5445	0.0016	0.0079	0.0012	0.0392	0.0578	0.0001	8.66E-08	-1.61E-09	-2.07E-09	-9.15E-08	1.26E-07	0.0020	0.0020	0.1004	2.0409
2.1103	1101.8507	2.8646	0.0017	0.0080	0.0013	0.0392	0.0578	0.0001	7.99E-08	-1.66E-09	-2.04E-09	-9.42E-08	1.24E-07	0.0021	0.0021	0.1033	2.0093
2.1101	1153.6637	3.1899	0.0017	0.0081	0.0013	0.0392	0.0578	0.0001	7.36E-08	-1.69E-09	-1.98E-09	-9.56E-08	1.21E-07	0.0021	0.0021	0.1050	2.0048
2.1091	1210.1050	3.5803	0.0017	0.0082	0.0013	0.0392	0.0578	0.0001	6.78E-08	-1.72E-09	-1.93E-09	-9.76E-08	1.19E-07	0.0021	0.0021	0.1071	1.9840
2.1086	1258.3035	3.9869	0.0017	0.0083	0.0013	0.0392	0.0578	0.0001	6.36E-08	-1.77E-09	-1.91E-09	-1.00E-07	1.19E-07	0.0021	0.0021	0.1103	1.9117

Uncertainty Analysis (Torque Coefficient)
Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

V (m/s)	n (rpm)	Q (N-m)	P_Q	B_Q	P_n	B_n	B_ρ	B_D	$B_q\ \delta K_q/\delta Q$	$B_\rho\ \delta K_q/\delta\rho$	$B_n\ \delta K_q/\delta n$	$B_D\ \delta K_q/\delta D$	B_{Kq}	P_{Kq}	U_{Kq}	Kq	% U
2.3113	506.5541	0.1820	0.0010	0.0076	0.0013	0.0391	0.0578	0.0001	3.62E-07	-4.99E-10	-1.33E-09	-2.83E-08	3.63E-07	0.0017	0.0017	0.0311	5.3828
2.3012	554.4124	0.3087	0.0017	0.0076	0.0014	0.0391	0.0578	0.0001	3.02E-07	-7.06E-10	-1.72E-09	-4.01E-08	3.05E-07	0.0022	0.0022	0.0440	5.0144
2.3106	602.4889	0.4268	0.0017	0.0076	0.0013	0.0391	0.0578	0.0001	2.56E-07	-8.27E-10	-1.86E-09	-4.69E-08	2.60E-07	0.0022	0.0022	0.0515	4.2501
2.3087	653.7372	0.6197	0.0017	0.0077	0.0014	0.0391	0.0578	0.0001	2.18E-07	-1.02E-09	-2.11E-09	-5.79E-08	2.25E-07	0.0022	0.0022	0.0635	3.4534
2.3127	751.5777	0.9597	0.0018	0.0077	0.0013	0.0391	0.0578	0.0001	1.65E-07	-1.19E-09	-2.15E-09	-6.78E-08	1.79E-07	0.0022	0.0022	0.0744	2.9463
2.3075	907.7627	1.6567	0.0018	0.0077	0.0014	0.0391	0.0578	0.0001	1.14E-07	-1.41E-09	-2.11E-09	-8.02E-08	1.40E-07	0.0022	0.0022	0.0880	2.5544
2.3104	1009.7375	2.2082	0.0018	0.0078	0.0014	0.0391	0.0578	0.0001	9.35E-08	-1.52E-09	-2.04E-09	-8.64E-08	1.27E-07	0.0022	0.0022	0.0948	2.3665
2.3089	1055.8389	2.5040	0.0018	0.0079	0.0014	0.0392	0.0578	0.0001	8.61E-08	-1.58E-09	-2.03E-09	-8.96E-08	1.24E-07	0.0022	0.0022	0.0984	2.2774
2.3716	1105.4558	2.7615	0.0018	0.0079	0.0014	0.0392	0.0578	0.0001	7.91E-08	-1.59E-09	-1.95E-09	-9.02E-08	1.20E-07	0.0023	0.0023	0.0990	2.2840
2.3101	1156.8492	3.1357	0.0018	0.0080	0.0014	0.0392	0.0578	0.0001	7.31E-08	-1.65E-09	-1.93E-09	-9.35E-08	1.19E-07	0.0023	0.0023	0.1026	2.2499
2.3079	1207.4927	3.4799	0.0018	0.0081	0.0014	0.0392	0.0578	0.0001	6.78E-08	-1.68E-09	-1.88E-09	-9.52E-08	1.17E-07	0.0023	0.0023	0.1045	2.2006
2.3092	1255.9025	3.8185	0.0019	0.0082	0.0014	0.0392	0.0578	0.0001	6.34E-08	-1.70E-09	-1.84E-09	-9.66E-08	1.16E-07	0.0023	0.0023	0.1060	2.1885

V (m/s)	n (rpm)	Q (N-m)	P_Q	B_Q	P_n	B_n	B_ρ	B_D	$B_q\ \delta K_q/\delta Q$	$B_\rho\ \delta K_q/\delta\rho$	$B_n\ \delta K_q/\delta n$	$B_D\ \delta K_q/\delta D$	B_{Kq}	P_{Kq}	U_{Kq}	Kq	% U
2.5094	504.8586	0.1628	0.0011	0.0076	0.0015	0.0391	0.0578	0.0001	3.64E-07	-4.49E-10	-1.20E-09	-2.55E-08	3.65E-07	0.0019	0.0019	0.0280	6.6512
2.5104	552.4243	0.2813	0.0018	0.0076	0.0015	0.0391	0.0578	0.0001	3.04E-07	-6.48E-10	-1.59E-09	-3.68E-08	3.07E-07	0.0024	0.0024	0.0404	5.8369
2.5067	604.5795	0.4035	0.0019	0.0076	0.0015	0.0391	0.0578	0.0001	2.54E-07	-7.76E-10	-1.74E-09	-4.41E-08	2.58E-07	0.0024	0.0024	0.0483	5.0596
2.5107	653.2312	0.5696	0.0019	0.0076	0.0015	0.0391	0.0578	0.0001	2.18E-07	-9.38E-10	-1.94E-09	-5.33E-08	2.24E-07	0.0024	0.0024	0.0585	4.1475
2.5093	753.2477	0.9274	0.0019	0.0077	0.0015	0.0391	0.0578	0.0001	1.64E-07	-1.15E-09	-2.06E-09	-6.52E-08	1.77E-07	0.0025	0.0025	0.0716	3.4324
2.5099	906.0469	1.6051	0.0019	0.0077	0.0015	0.0391	0.0578	0.0001	1.15E-07	-1.37E-09	-2.05E-09	-7.80E-08	1.39E-07	0.0025	0.0025	0.0856	2.8641
2.5126	1002.7932	2.1197	0.0020	0.0078	0.0015	0.0391	0.0578	0.0001	9.46E-08	-1.48E-09	-2.00E-09	-8.41E-08	1.27E-07	0.0025	0.0025	0.0923	2.6785
2.5100	1052.4234	2.4384	0.0020	0.0079	0.0015	0.0392	0.0578	0.0001	8.66E-08	-1.55E-09	-1.99E-09	-8.79E-08	1.23E-07	0.0025	0.0025	0.0964	2.5601
2.5104	1106.7229	2.7268	0.0020	0.0079	0.0015	0.0392	0.0578	0.0001	7.89E-08	-1.57E-09	-1.92E-09	-8.88E-08	1.19E-07	0.0025	0.0025	0.0975	2.5730
2.5101	1152.3582	3.0710	0.0020	0.0080	0.0016	0.0392	0.0578	0.0001	7.35E-08	-1.63E-09	-1.91E-09	-9.23E-08	1.18E-07	0.0025	0.0025	0.1013	2.4813
2.5109	1204.5756	3.4002	0.0020	0.0081	0.0015	0.0392	0.0578	0.0001	6.80E-08	-1.65E-09	-1.85E-09	-9.35E-08	1.16E-07	0.0025	0.0025	0.1026	2.4522
2.5096	1256.7444	3.7705	0.0020	0.0082	0.0016	0.0392	0.0578	0.0001	6.32E-08	-1.68E-09	-1.81E-09	-9.53E-08	1.14E-07	0.0026	0.0026	0.1045	2.4414

Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

Uncertainty Analysis (Thrust Coefficient)

V	Vehicle Velocity (m/s)	
n	Propeller Rotational Speed (rpm)	
T_t	Total Thrust (N)	
D	Propeller Diameter (m)	
ρ	Density of Water (kg/m^3)	
K_{tt}	Total Thrust Coefficient	
P_T	Precision Error for T_t	
B_T	Bias Error for T_t	
P_{Ktt}	Precision Error for K_{tt}	
P_n	Precision Error for n	
B_n	Bias Error for n	
B_ρ	Bias Error for ρ	
B_D	Bias Error for D	
B_{Ktt}	Bias Error for K_{tt}	$B_{Ktt} = ((\delta K_{tt} / \delta T_t\ B_T)^2 + (\delta K_{tt} / \delta\rho\ B_\rho)^2 + (\delta K_{tt} / \delta n\ B_n)^2 + (\delta K_{tt} / \delta D\ B_D)^2)^{1/2}$
U_{ktt}	Total Error for K_{tt}	$U_{Ktt} = (B_{Ktt}^2 + P_{Ktt}^2)^{1/2}$
% U	Percentage Uncertainty for K_{tt}	$\% U = (U_{Ktt} / K_{tt}) \times 100$

Uncertainty Analysis (Thrust Coefficient)
Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

V (m/s)	n (rpm)	T_t (N)	P_T	B_T	P_n	B_n	B_ρ	B_D	$B_t\ \delta K_{tt}/\delta T_t$	$B_\rho\ \delta K_{tt}/\delta\rho$	$B_n\ \delta K_{tt}/\delta n$	$B_D\ \delta K_{tt}/\delta D$	B_{Ktt}	P_{Ktt}	U_{Ktt}	K_{tt}	% U
0.8002	499.5767	7.6425	0.0090	0.2494	0.0005	0.0391	0.0578	0.0001	1.85E-06	-3.28E-09	-8.88E-09	-1.49E-07	1.86E-06	0.0090	0.0090	0.2044	4.4103
0.8004	551.4566	14.1975	0.0092	0.2494	0.0005	0.0391	0.0578	0.0001	1.52E-06	-5.00E-09	-1.23E-08	-2.27E-07	1.54E-06	0.0092	0.0092	0.3116	2.9575
0.8012	600.4575	19.7185	0.0091	0.2494	0.0005	0.0391	0.0578	0.0001	1.28E-06	-5.86E-09	-1.32E-08	-2.66E-07	1.31E-06	0.0091	0.0091	0.3650	2.4977
0.8018	650.4466	26.1852	0.0094	0.2494	0.0005	0.0391	0.0578	0.0001	1.09E-06	-6.63E-09	-1.38E-08	-3.01E-07	1.13E-06	0.0094	0.0094	0.4130	2.2790
0.8002	753.2179	43.2349	0.0095	0.2494	0.0005	0.0391	0.0578	0.0001	8.15E-07	-8.17E-09	-1.47E-08	-3.71E-07	8.95E-07	0.0095	0.0095	0.5086	1.8702
0.8002	902.9860	78.0332	0.0097	0.2494	0.0005	0.0391	0.0578	0.0001	5.67E-07	-1.03E-08	-1.54E-08	-4.66E-07	7.34E-07	0.0097	0.0097	0.6387	1.5209
0.8003	1008.5085	106.5934	0.0098	0.2494	0.0005	0.0391	0.0578	0.0001	4.55E-07	-1.12E-08	-1.51E-08	-5.10E-07	6.83E-07	0.0098	0.0098	0.6994	1.4029
0.8005	1052.3455	121.3940	0.0098	0.2494	0.0005	0.0392	0.0578	0.0001	4.18E-07	-1.17E-08	-1.51E-08	-5.33E-07	6.78E-07	0.0098	0.0098	0.7315	1.3412
0.8014	1104.1946	137.7978	0.0099	0.2494	0.0005	0.0392	0.0578	0.0001	3.79E-07	-1.21E-08	-1.49E-08	-5.50E-07	6.68E-07	0.0099	0.0099	0.7542	1.3142
0.8002	1150.2549	154.0467	0.0100	0.2495	0.0005	0.0392	0.0578	0.0001	3.50E-07	-1.25E-08	-1.47E-08	-5.66E-07	6.66E-07	0.0100	0.0100	0.7770	1.2887
0.8008	1206.8237	175.1871	0.0102	0.2495	0.0005	0.0392	0.0578	0.0001	3.18E-07	-1.29E-08	-1.45E-08	-5.85E-07	6.66E-07	0.0102	0.0102	0.8027	1.2723
0.8006	1252.7469	191.4733	0.0104	0.2495	0.0005	0.0392	0.0578	0.0001	2.95E-07	-1.31E-08	-1.42E-08	-5.94E-07	6.63E-07	0.0104	0.0104	0.8142	1.2786

V (m/s)	n (rpm)	T_t (N)	P_T	B_T	P_n	B_n	B_ρ	B_D	$B_t\ \delta K_{tt}/\delta T_t$	$B_\rho\ \delta K_{tt}/\delta\rho$	$B_n\ \delta K_{tt}/\delta n$	$B_D\ \delta K_{tt}/\delta D$	B_{Ktt}	P_{Ktt}	U_{Ktt}	K_{tt}	% U
1.1006	500.6686	6.1254	0.0100	0.2494	0.0006	0.0391	0.0578	0.0001	1.84E-06	-2.62E-09	-7.07E-09	-1.19E-07	1.85E-06	0.0100	0.0100	0.1631	6.1438
1.1022	552.7477	11.5778	0.0108	0.2494	0.0006	0.0391	0.0578	0.0001	1.51E-06	-4.06E-09	-9.94E-09	-1.84E-07	1.52E-06	0.0108	0.0108	0.2529	4.2781
1.1031	603.4646	17.6112	0.0110	0.2494	0.0007	0.0391	0.0578	0.0001	1.27E-06	-5.18E-09	-1.16E-08	-2.35E-07	1.29E-06	0.0110	0.0110	0.3227	3.4149
1.1032	651.6550	26.6374	0.0112	0.2494	0.0007	0.0391	0.0578	0.0001	1.09E-06	-6.72E-09	-1.40E-08	-3.05E-07	1.13E-06	0.0112	0.0112	0.4186	2.6807
1.1005	754.9726	43.0749	0.0115	0.2494	0.0007	0.0391	0.0578	0.0001	8.11E-07	-8.10E-09	-1.45E-08	-3.68E-07	8.91E-07	0.0115	0.0115	0.5043	2.2840
1.1026	901.8126	78.2263	0.0117	0.2494	0.0007	0.0391	0.0578	0.0001	5.68E-07	-1.03E-08	-1.55E-08	-4.68E-07	7.37E-07	0.0117	0.0117	0.6419	1.8257
1.1014	1000.0880	103.9908	0.0116	0.2494	0.0007	0.0391	0.0578	0.0001	4.62E-07	-1.11E-08	-1.51E-08	-5.06E-07	6.86E-07	0.0116	0.0116	0.6939	1.6748
1.1017	1056.9566	120.2376	0.0119	0.2494	0.0007	0.0392	0.0578	0.0001	4.14E-07	-1.15E-08	-1.48E-08	-5.24E-07	6.68E-07	0.0119	0.0119	0.7183	1.6598
1.1013	1104.4112	137.1587	0.0121	0.2494	0.0007	0.0392	0.0578	0.0001	3.79E-07	-1.20E-08	-1.48E-08	-5.47E-07	6.66E-07	0.0121	0.0121	0.7505	1.6151
1.1020	1156.9473	154.0702	0.0124	0.2495	0.0007	0.0392	0.0578	0.0001	3.46E-07	-1.23E-08	-1.45E-08	-5.60E-07	6.58E-07	0.0124	0.0124	0.7682	1.6170
1.1015	1212.9133	172.0909	0.0124	0.2495	0.0007	0.0392	0.0578	0.0001	3.14E-07	-1.25E-08	-1.40E-08	-5.69E-07	6.50E-07	0.0124	0.0124	0.7807	1.5909
1.1011	1258.8191	188.8362	0.0125	0.2495	0.0007	0.0392	0.0578	0.0001	2.92E-07	-1.28E-08	-1.38E-08	-5.80E-07	6.49E-07	0.0125	0.0125	0.7953	1.5740

V (m/s)	n (rpm)	T_t (N)	P_T	B_T	P_n	B_n	B_ρ	B_D	$B_t\ \delta K_{tt}/\delta T_t$	$B_\rho\ \delta K_{tt}/\delta\rho$	$B_n\ \delta K_{tt}/\delta n$	$B_D\ \delta K_{tt}/\delta D$	B_{Ktt}	P_{Ktt}	U_{Ktt}	K_{tt}	% U
1.3033	501.8541	-2.4715	0.0128	0.2494	0.0008	0.0391	0.0578	0.0001	1.84E-06	1.05E-09	2.83E-09	4.77E-08	1.84E-06	0.0128	0.0128	-0.0655	-19.5798
1.3116	550.7550	3.2712	0.0130	0.2494	0.0007	0.0391	0.0578	0.0001	1.52E-06	-1.16E-09	-2.84E-09	-5.25E-08	1.52E-06	0.0130	0.0130	0.0720	18.0904
1.3018	601.3456	7.6691	0.0133	0.2494	0.0007	0.0391	0.0578	0.0001	1.28E-06	-2.27E-09	-5.11E-09	-1.03E-07	1.28E-06	0.0133	0.0133	0.1415	9.4101
1.3022	654.7460	18.6342	0.0134	0.2494	0.0008	0.0391	0.0578	0.0001	1.08E-06	-4.66E-09	-9.63E-09	-2.11E-07	1.10E-06	0.0134	0.0134	0.2901	4.6271
1.3025	752.1400	32.8566	0.0134	0.2494	0.0008	0.0391	0.0578	0.0001	8.17E-07	-6.22E-09	-1.12E-08	-2.83E-07	8.65E-07	0.0134	0.0134	0.3876	3.4627
1.3028	907.7795	70.4953	0.0135	0.2494	0.0008	0.0391	0.0578	0.0001	5.61E-07	-9.17E-09	-1.37E-08	-4.16E-07	6.99E-07	0.0135	0.0135	0.5709	2.3688
1.3020	1006.9377	99.0206	0.0138	0.2494	0.0008	0.0391	0.0578	0.0001	4.56E-07	-1.05E-08	-1.41E-08	-4.75E-07	6.59E-07	0.0138	0.0138	0.6518	2.1206
1.3014	1051.7237	113.7191	0.0137	0.2494	0.0007	0.0392	0.0578	0.0001	4.18E-07	-1.10E-08	-1.42E-08	-5.00E-07	6.52E-07	0.0137	0.0137	0.6861	1.9997
1.3116	1103.5535	130.8195	0.0138	0.2494	0.0008	0.0392	0.0578	0.0001	3.80E-07	-1.15E-08	-1.41E-08	-5.23E-07	6.46E-07	0.0138	0.0138	0.7169	1.9281
1.3007	1155.5678	150.4082	0.0139	0.2495	0.0008	0.0392	0.0578	0.0001	3.46E-07	-1.21E-08	-1.42E-08	-5.48E-07	6.49E-07	0.0139	0.0139	0.7517	1.8524
1.3006	1208.2135	166.7611	0.0142	0.2495	0.0008	0.0392	0.0578	0.0001	3.17E-07	-1.22E-08	-1.37E-08	-5.56E-07	6.40E-07	0.0142	0.0142	0.7624	1.8655
1.3020	1256.7456	183.7284	0.0143	0.2495	0.0008	0.0392	0.0578	0.0001	2.93E-07	-1.25E-08	-1.35E-08	-5.66E-07	6.38E-07	0.0143	0.0143	0.7763	1.8447

<u>Uncertainty Analysis</u> (Thrust Coefficient)
Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

V (m/s)	n (rpm)	T_t (N)	P_T	B_T	P_n	B_n	B_ρ	B_D	$B_t\ \delta K_{tt}/\delta T_t$	$B_\rho\ \delta K_{tt}/\delta\rho$	$B_n\ \delta K_{tt}/\delta n$	$B_D\ \delta K_{tt}/\delta D$	B_{Ktt}	P_{Ktt}	U_{Ktt}	K_{tt}	% U
1.6030	503.1755	-7.0565	0.0140	0.2494	0.0009	0.0391	0.0578	0.0001	1.83E-06	2.99E-09	8.03E-09	1.36E-07	1.83E-06	0.0140	0.0140	-0.1860	-7.5424
1.6053	554.4423	-2.6691	0.0135	0.2494	0.0009	0.0391	0.0578	0.0001	1.50E-06	9.30E-10	2.27E-09	4.22E-08	1.50E-06	0.0135	0.0135	-0.0579	-23.3545
1.6045	604.3557	3.2865	0.0138	0.2494	0.0009	0.0391	0.0578	0.0001	1.27E-06	-9.64E-10	-2.16E-09	-4.38E-08	1.27E-06	0.0138	0.0138	0.0600	23.0258
1.6063	652.4344	11.6498	0.0145	0.2494	0.0009	0.0391	0.0578	0.0001	1.09E-06	-2.93E-09	-6.08E-09	-1.33E-07	1.09E-06	0.0145	0.0145	0.1826	7.9542
1.6063	752.2439	28.1917	0.0147	0.2494	0.0009	0.0391	0.0578	0.0001	8.17E-07	-5.34E-09	-9.60E-09	-2.42E-07	8.52E-07	0.0147	0.0147	0.3325	4.4300
1.6059	901.5599	64.0440	0.0148	0.2494	0.0009	0.0391	0.0578	0.0001	5.69E-07	-8.44E-09	-1.27E-08	-3.83E-07	6.86E-07	0.0148	0.0148	0.5258	2.8198
1.6056	1006.7175	91.9317	0.0149	0.2494	0.0009	0.0391	0.0578	0.0001	4.56E-07	-9.72E-09	-1.31E-08	-4.41E-07	6.35E-07	0.0149	0.0149	0.6054	2.4656
1.6063	1054.3192	110.3025	0.0152	0.2494	0.0009	0.0392	0.0578	0.0001	4.16E-07	-1.06E-08	-1.37E-08	-4.83E-07	6.38E-07	0.0152	0.0152	0.6622	2.2990
1.6059	1102.5010	126.9004	0.0153	0.2494	0.0009	0.0392	0.0578	0.0001	3.80E-07	-1.12E-08	-1.38E-08	-5.08E-07	6.35E-07	0.0153	0.0153	0.6967	2.2001
1.6050	1154.9706	143.2220	0.0155	0.2495	0.0009	0.0392	0.0578	0.0001	3.47E-07	-1.15E-08	-1.35E-08	-5.22E-07	6.27E-07	0.0155	0.0155	0.7165	2.1670
1.6054	1211.5007	162.3490	0.0158	0.2495	0.0009	0.0392	0.0578	0.0001	3.15E-07	-1.19E-08	-1.33E-08	-5.38E-07	6.24E-07	0.0158	0.0158	0.7382	2.1439
1.6019	1257.0444	180.1017	0.0160	0.2495	0.0009	0.0392	0.0578	0.0001	2.93E-07	-1.22E-08	-1.32E-08	-5.55E-07	6.27E-07	0.0160	0.0160	0.7606	2.1070

V (m/s)	n (rpm)	T_t (N)	P_T	B_T	P_n	B_n	B_ρ	B_D	$B_t\ \delta K_{tt}/\delta T_t$	$B_\rho\ \delta K_{tt}/\delta\rho$	$B_n\ \delta K_{tt}/\delta n$	$B_D\ \delta K_{tt}/\delta D$	B_{Ktt}	P_{Ktt}	U_{Ktt}	K_{tt}	% U
1.8051	503.4015	-12.0461	0.0170	0.2494	0.0010	0.0391	0.0578	0.0001	1.82E-06	5.09E-09	1.37E-08	2.31E-07	1.84E-06	0.0170	0.0170	-0.3172	-5.3689
1.8061	550.5344	-7.9307	0.0168	0.2494	0.0011	0.0391	0.0578	0.0001	1.53E-06	2.80E-09	6.89E-09	1.27E-07	1.53E-06	0.0168	0.0168	-0.1746	-9.6398
1.8054	601.7534	-2.0996	0.0163	0.2494	0.0011	0.0391	0.0578	0.0001	1.28E-06	6.21E-10	1.40E-09	2.82E-08	1.28E-06	0.0163	0.0163	-0.0387	-42.2202
1.8043	655.6402	8.3160	0.0170	0.2494	0.0011	0.0391	0.0578	0.0001	1.08E-06	-2.07E-09	-4.28E-09	-9.41E-08	1.08E-06	0.0170	0.0170	0.1291	13.1930
1.8049	756.7032	24.0023	0.0172	0.2494	0.0011	0.0391	0.0578	0.0001	8.07E-07	-4.49E-09	-8.03E-09	-2.04E-07	8.33E-07	0.0172	0.0172	0.2797	6.1605
1.8053	903.5768	56.0137	0.0173	0.2494	0.0011	0.0391	0.0578	0.0001	5.66E-07	-7.35E-09	-1.10E-08	-3.34E-07	6.57E-07	0.0173	0.0173	0.4579	3.7861
1.8053	1007.3473	82.5484	0.0172	0.2494	0.0010	0.0391	0.0578	0.0001	4.56E-07	-8.72E-09	-1.17E-08	-3.96E-07	6.04E-07	0.0172	0.0172	0.5429	3.1738
1.8062	1051.7675	100.6055	0.0174	0.2494	0.0011	0.0392	0.0578	0.0001	4.18E-07	-9.74E-09	-1.26E-08	-4.43E-07	6.09E-07	0.0174	0.0174	0.6069	2.8724
1.8057	1105.5646	118.5649	0.0175	0.2494	0.0011	0.0392	0.0578	0.0001	3.78E-07	-1.04E-08	-1.27E-08	-4.72E-07	6.05E-07	0.0175	0.0175	0.6474	2.7086
1.8064	1151.0459	135.0686	0.0175	0.2494	0.0011	0.0392	0.0578	0.0001	3.49E-07	-1.09E-08	-1.29E-08	-4.96E-07	6.07E-07	0.0175	0.0175	0.6803	2.5775
1.8051	1199.5015	154.5146	0.0177	0.2495	0.0011	0.0392	0.0578	0.0001	3.21E-07	-1.15E-08	-1.30E-08	-5.23E-07	6.14E-07	0.0177	0.0177	0.7167	2.4743
1.8046	1261.8668	175.6173	0.0178	0.2495	0.0011	0.0392	0.0578	0.0001	2.90E-07	-1.18E-08	-1.27E-08	-5.37E-07	6.10E-07	0.0178	0.0178	0.7360	2.4230

V (m/s)	n (rpm)	T_t (N)	P_T	B_T	P_n	B_n	B_ρ	B_D	$B_t\ \delta K_{tt}/\delta T_t$	$B_\rho\ \delta K_{tt}/\delta\rho$	$B_n\ \delta K_{tt}/\delta n$	$B_D\ \delta K_{tt}/\delta D$	B_{Ktt}	P_{Ktt}	U_{Ktt}	K_{tt}	% U
2.1082	500.4856	-15.0436	0.0192	0.2494	0.0012	0.0391	0.0578	0.0001	1.85E-06	6.44E-09	1.74E-08	2.92E-07	1.87E-06	0.0192	0.0192	-0.4008	-4.7994
2.1078	552.4532	-8.6422	0.0190	0.2494	0.0012	0.0391	0.0578	0.0001	1.51E-06	3.03E-09	7.43E-09	1.38E-07	1.52E-06	0.0190	0.0190	-0.1890	-10.0751
2.1093	604.4664	-2.1310	0.0184	0.2494	0.0013	0.0391	0.0578	0.0001	1.27E-06	6.25E-10	1.40E-09	2.84E-08	1.27E-06	0.0184	0.0184	-0.0389	-47.3867
2.1056	651.4342	5.3703	0.0189	0.2494	0.0013	0.0391	0.0578	0.0001	1.09E-06	-1.36E-09	-2.82E-09	-6.16E-08	1.09E-06	0.0189	0.0189	0.0845	22.4321
2.1084	754.9317	20.7011	0.0191	0.2494	0.0012	0.0391	0.0578	0.0001	8.11E-07	-3.89E-09	-6.98E-09	-1.77E-07	8.30E-07	0.0191	0.0191	0.2424	7.8944
2.1095	901.1852	53.4900	0.0192	0.2494	0.0012	0.0391	0.0578	0.0001	5.69E-07	-7.06E-09	-1.06E-08	-3.20E-07	6.53E-07	0.0192	0.0192	0.4395	4.3766
2.1082	1002.9008	79.1802	0.0192	0.2494	0.0012	0.0391	0.0578	0.0001	4.60E-07	-8.44E-09	-1.14E-08	-3.83E-07	5.98E-07	0.0192	0.0192	0.5254	3.6620
2.1098	1053.7204	94.0054	0.0193	0.2494	0.0012	0.0392	0.0578	0.0001	4.16E-07	-9.07E-09	-1.17E-08	-4.12E-07	5.86E-07	0.0193	0.0193	0.5650	3.4228
2.1103	1101.8507	111.6413	0.0194	0.2494	0.0013	0.0392	0.0578	0.0001	3.81E-07	-9.85E-09	-1.21E-08	-4.47E-07	5.88E-07	0.0194	0.0194	0.6137	3.1679
2.1101	1153.6637	128.0193	0.0196	0.2494	0.0013	0.0392	0.0578	0.0001	3.47E-07	-1.03E-08	-1.21E-08	-4.68E-07	5.83E-07	0.0196	0.0196	0.6419	3.0599
2.1091	1210.1050	147.2988	0.0199	0.2495	0.0013	0.0392	0.0578	0.0001	3.16E-07	-1.08E-08	-1.21E-08	-4.89E-07	5.83E-07	0.0199	0.0199	0.6713	2.9707
2.1086	1258.3035	167.6550	0.0203	0.2495	0.0013	0.0392	0.0578	0.0001	2.92E-07	-1.13E-08	-1.22E-08	-5.15E-07	5.92E-07	0.0203	0.0203	0.7067	2.8782

Uncertainty Analysis (Thrust Coefficient)
Load Varying Self-Propulsion Test (Prop. + Duct fitted on)

V (m/s)	n (rpm)	T_t (N)	P_T	B_T	P_n	B_n	B_ρ	B_D	$B_t\ \delta K_{tt}/\delta T_t$	$B_\rho\ \delta K_{tt}/\delta \rho$	$B_n\ \delta K_{tt}/\delta n$	$B_D\ \delta K_{tt}/\delta D$	B_{Ktt}	P_{Ktt}	U_{Ktt}	K_{tt}	% U
2.3113	506.5541	-19.3516	0.0218	0.2494	0.0013	0.0391	0.0578	0.0001	1.80E-06	8.08E-09	2.16E-08	3.67E-07	1.84E-06	0.0218	0.0218	-0.5033	-4.3396
2.3012	554.4124	-15.8623	0.0217	0.2494	0.0014	0.0391	0.0578	0.0001	1.50E-06	5.53E-09	1.35E-08	2.51E-07	1.52E-06	0.0217	0.0217	-0.3444	-6.3135
2.3106	602.4889	-10.8438	0.0213	0.2494	0.0013	0.0391	0.0578	0.0001	1.27E-06	3.20E-09	7.19E-09	1.45E-07	1.28E-06	0.0213	0.0213	-0.1994	-10.7051
2.3087	653.7372	-1.9037	0.0180	0.2494	0.0014	0.0391	0.0578	0.0001	1.08E-06	4.77E-10	9.88E-10	2.17E-08	1.08E-06	0.0181	0.0181	-0.0297	-60.7239
2.3127	751.5777	13.1329	0.0215	0.2494	0.0013	0.0391	0.0578	0.0001	8.18E-07	-2.49E-09	-4.49E-09	-1.13E-07	8.26E-07	0.0215	0.0215	0.1552	13.8828
2.3075	907.7627	45.9201	0.0218	0.2494	0.0014	0.0391	0.0578	0.0001	5.61E-07	-5.97E-09	-8.91E-09	-2.71E-07	6.23E-07	0.0218	0.0218	0.3719	5.8739
2.3104	1009.7375	70.3932	0.0219	0.2494	0.0014	0.0391	0.0578	0.0001	4.53E-07	-7.40E-09	-9.92E-09	-3.36E-07	5.64E-07	0.0219	0.0219	0.4608	4.7624
2.3089	1055.8389	84.5472	0.0221	0.2494	0.0014	0.0392	0.0578	0.0001	4.15E-07	-8.13E-09	-1.04E-08	-3.69E-07	5.55E-07	0.0221	0.0221	0.5061	4.3747
2.3716	1105.4558	99.2872	0.0223	0.2494	0.0014	0.0392	0.0578	0.0001	3.78E-07	-8.71E-09	-1.07E-08	-3.95E-07	5.47E-07	0.0223	0.0223	0.5422	4.1206
2.3101	1156.8492	117.4706	0.0225	0.2494	0.0014	0.0392	0.0578	0.0001	3.45E-07	-9.41E-09	-1.10E-08	-4.27E-07	5.50E-07	0.0225	0.0225	0.5858	3.8486
2.3079	1207.4927	134.4150	0.0226	0.2494	0.0014	0.0392	0.0578	0.0001	3.17E-07	-9.88E-09	-1.11E-08	-4.49E-07	5.50E-07	0.0226	0.0226	0.6152	3.6802
2.3092	1255.9025	151.0913	0.0228	0.2495	0.0014	0.0392	0.0578	0.0001	2.93E-07	-1.03E-08	-1.11E-08	-4.66E-07	5.51E-07	0.0228	0.0228	0.6393	3.5732

V (m/s)	n (rpm)	T_t (N)	P_T	B_T	P_n	B_n	B_ρ	B_D	$B_t\ \delta K_{tt}/\delta T_t$	$B_\rho\ \delta K_{tt}/\delta \rho$	$B_n\ \delta K_{tt}/\delta n$	$B_D\ \delta K_{tt}/\delta D$	B_{Ktt}	P_{Ktt}	U_{Ktt}	K_{tt}	% U
2.5094	504.8586	-23.5927	0.0232	0.2494	0.0015	0.0391	0.0578	0.0001	1.81E-06	9.92E-09	2.66E-08	4.50E-07	1.87E-06	0.0232	0.0232	-0.6177	-3.7635
2.5104	552.4243	-21.2282	0.0229	0.2494	0.0015	0.0391	0.0578	0.0001	1.51E-06	7.45E-09	1.83E-08	3.38E-07	1.55E-06	0.0230	0.0230	-0.4642	-4.9438
2.5067	604.5795	-16.9244	0.0228	0.2494	0.0015	0.0391	0.0578	0.0001	1.26E-06	4.96E-09	1.11E-08	2.25E-07	1.28E-06	0.0229	0.0229	-0.3090	-7.3953
2.5107	653.2312	-10.5697	0.0225	0.2494	0.0015	0.0391	0.0578	0.0001	1.08E-06	2.65E-09	5.50E-09	1.21E-07	1.09E-06	0.0225	0.0225	-0.1653	-13.6404
2.5093	753.2477	2.9669	0.0206	0.2494	0.0015	0.0391	0.0578	0.0001	8.15E-07	-5.60E-10	-1.01E-09	-2.54E-08	8.15E-07	0.0207	0.0207	0.0349	59.1906
2.5099	906.0469	34.7210	0.0238	0.2494	0.0015	0.0391	0.0578	0.0001	5.63E-07	-4.53E-09	-6.77E-09	-2.06E-07	6.00E-07	0.0238	0.0238	0.2823	8.4486
2.5126	1002.7932	63.3290	0.0239	0.2494	0.0015	0.0391	0.0578	0.0001	4.60E-07	-6.75E-09	-9.12E-09	-3.06E-07	5.53E-07	0.0239	0.0239	0.4203	5.6981
2.5100	1052.4234	76.2347	0.0240	0.2494	0.0015	0.0392	0.0578	0.0001	4.17E-07	-7.37E-09	-9.49E-09	-3.35E-07	5.35E-07	0.0240	0.0240	0.4593	5.2351
2.5104	1106.7229	91.4780	0.0242	0.2494	0.0015	0.0392	0.0578	0.0001	3.77E-07	-8.00E-09	-9.80E-09	-3.63E-07	5.24E-07	0.0242	0.0242	0.4984	4.8651
2.5101	1152.3582	105.8466	0.0244	0.2494	0.0016	0.0392	0.0578	0.0001	3.48E-07	-8.54E-09	-1.00E-08	-3.88E-07	5.21E-07	0.0244	0.0244	0.5319	4.5963
2.5109	1204.5756	124.7560	0.0245	0.2494	0.0015	0.0392	0.0578	0.0001	3.19E-07	-9.21E-09	-1.04E-08	-4.18E-07	5.26E-07	0.0245	0.0245	0.5738	4.2783
2.5096	1256.7444	140.1545	0.0248	0.2494	0.0016	0.0392	0.0578	0.0001	2.93E-07	-9.51E-09	-1.03E-08	-4.32E-07	5.22E-07	0.0248	0.0248	0.5922	4.1960

Bare Hull Resistance Test

Uncertainty Analysis (Velocity - Ahead / Astern Directions)

V	Vehicle Velocity (m/s)	
D	Diameter of Carriage Wheel (m)	
C	Number of Counted Pulses in Time dt	
dt	Time (s)	
S.D	Standard Deviation	
P_V	Precision Error for V	$P_V = 2 \times S.D$
B_C	Bias Error for C	
B_D	Bias Error for D	
B_{dt}	Bias Error for dt	
B_V	Bias Error for V	$B_V = ((\delta V / \delta C\ B_C)^2 + (\delta V / \delta D\ B_D)^2 + (\delta V / \delta dt\ B_{dt})^2)^{1/2}$
U_V	Total Error for V	$U_V = (B_V^2 + P_V^2)^{1/2}$
% Unc.	Percentage Uncertainty for V	$\% \text{ Unc.} = (U_V / V) \times 100$

Uncertainty Analysis (Velocity)
Bare Hull Resistance Test (Ahead Direction)

V (m/s)	P_V	B_C δV/δC	B_D δV/δD	B_{dt} δV/δdt	B_V	U_V	% Unc
0.6010	0.0002	0.0035	0.0001	-0.00002	0.0035	0.0035	0.5860
0.6987	0.0002	0.0035	0.0001	-0.00004	0.0035	0.0035	0.5045
0.8003	0.0003	0.0035	0.0001	-0.00005	0.0035	0.0035	0.4415
0.8990	0.0003	0.0035	0.0002	-0.00006	0.0035	0.0035	0.3935
1.0015	0.0004	0.0035	0.0002	-0.00008	0.0035	0.0035	0.3539
1.1016	0.0004	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3227
1.2007	0.0005	0.0035	0.0003	-0.00011	0.0035	0.0036	0.2968
1.3048	0.0005	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2741
1.4054	0.0006	0.0035	0.0004	-0.00013	0.0035	0.0036	0.2550
1.5037	0.0006	0.0035	0.0004	-0.00015	0.0035	0.0036	0.2389
1.6040	0.0006	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2250
1.7060	0.0007	0.0035	0.0005	-0.00018	0.0036	0.0036	0.2128
1.8091	0.0008	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2024
1.9061	0.0009	0.0035	0.0006	-0.00020	0.0036	0.0037	0.1930
2.0064	0.0009	0.0035	0.0006	-0.00022	0.0036	0.0037	0.1844
2.1073	0.0010	0.0035	0.0007	-0.00023	0.0036	0.0037	0.1767
2.3105	0.0012	0.0035	0.0008	-0.00026	0.0036	0.0038	0.1638
2.5122	0.0013	0.0035	0.0008	-0.00029	0.0036	0.0039	0.1535
2.7087	0.0014	0.0035	0.0009	-0.00031	0.0036	0.0039	0.1447
2.9112	0.0015	0.0035	0.0010	-0.00034	0.0037	0.0040	0.1360
3.1129	0.0016	0.0035	0.0011	-0.00037	0.0037	0.0040	0.1297

Uncertainty Analysis (Velocity)
Bare Hull Resistance Test (Astern Direction)

V (m/s)	P_V	$B_C\ \delta V/\delta C$	$B_D\ \delta V/\delta D$	$B_{dt}\ \delta V/\delta dt$	B_V	U_V	% Unc
0.6088	0.0002	0.0035	0.0001	-0.00002	0.0035	0.0035	0.5787
0.7089	0.0002	0.0035	0.0001	-0.00004	0.0035	0.0035	0.4974
0.8097	0.0003	0.0035	0.0002	-0.00005	0.0035	0.0035	0.4363
0.9113	0.0004	0.0035	0.0002	-0.00007	0.0035	0.0035	0.3883
1.0126	0.0004	0.0035	0.0002	-0.00008	0.0035	0.0035	0.3499
1.1124	0.0005	0.0035	0.0003	-0.00009	0.0035	0.0036	0.3197
1.2146	0.0005	0.0035	0.0003	-0.00011	0.0035	0.0036	0.2935
1.3139	0.0006	0.0035	0.0004	-0.00012	0.0035	0.0036	0.2723
1.4123	0.0006	0.0035	0.0004	-0.00013	0.0035	0.0036	0.2539
1.5161	0.0006	0.0035	0.0004	-0.00015	0.0035	0.0036	0.2368
1.6148	0.0007	0.0035	0.0005	-0.00016	0.0036	0.0036	0.2238
1.7155	0.0008	0.0035	0.0005	-0.00018	0.0036	0.0036	0.2120
1.8173	0.0009	0.0035	0.0006	-0.00019	0.0036	0.0037	0.2019
1.9185	0.0009	0.0035	0.0006	-0.00020	0.0036	0.0037	0.1922
2.0195	0.0010	0.0035	0.0006	-0.00022	0.0036	0.0037	0.1836

Bare Hull Resistance Test

Uncertainty Analysis (Resistance - Ahead / Astern Directions)

V	Vehicle Velocity (m/s)	
R	Bare Hull Resistance (N)	
P_R	Precision Error for R	$P_R = 2 \times S.D$
S.D	Standard Deviation	
S.E.E	Standard Error Estimate	
AD	Analog to Digital Converter	
B_1	Bias Error due to Calibration	$B_1 = 0.00005 \times R$
B_2	Bias Error due to Data Acquisition	$B_2 = 2 \times S.E.E$
B_3	Bias Error due to Load Cell Misalignment	$B_3 = R - (\cos 0.25^0 \times R)$
B_4	Bias Error due to Data Reduction	$B_4 = (AD_{error} \times AD_{range}) / (AD_{accuracy})$
B_R	Bias Error for R	$B_R = (B_1^2 + B_2^2 + B_3^2 + B_4^2)^{1/2}$
U_R	Total Error for R	$U_R = (B_R^2 + P_R^2)^{1/2}$
% Unc.	Percentage Uncertainty for R	% Unc. = $(U_R / R) \times 100$

Uncertainty Analysis (Bare Hull Resistance - Ahead)

V (m/s)	R (N)	P_R	B_1	B_2	B_3	B_4	B_R	U_R	% Unc.
0.6010	4.8372	0.0088	0.0002	0.2693	0.0000	0.0054	0.2694	0.2695	5.5717
0.6987	6.2947	0.0092	0.0003	0.2693	0.0001	0.0054	0.2694	0.2695	4.2818
0.8003	7.9859	0.0099	0.0004	0.2693	0.0001	0.0054	0.2694	0.2695	3.3753
0.8990	9.8816	0.0109	0.0005	0.2693	0.0001	0.0054	0.2694	0.2696	2.7282
1.0015	11.7594	0.0114	0.0006	0.2693	0.0001	0.0054	0.2694	0.2696	2.2927
1.1016	13.8754	0.0119	0.0007	0.2693	0.0001	0.0054	0.2694	0.2696	1.9432
1.2007	16.2767	0.0125	0.0008	0.2693	0.0002	0.0054	0.2694	0.2697	1.6567
1.3048	18.6795	0.0138	0.0009	0.2693	0.0002	0.0054	0.2694	0.2697	1.4440
1.4054	21.2504	0.0145	0.0011	0.2693	0.0002	0.0054	0.2694	0.2698	1.2694
1.5037	23.8043	0.0154	0.0012	0.2693	0.0002	0.0054	0.2694	0.2698	1.1334
1.6040	27.2382	0.0164	0.0014	0.2693	0.0003	0.0054	0.2694	0.2699	0.9908
1.7060	30.5770	0.0181	0.0015	0.2693	0.0003	0.0054	0.2694	0.2700	0.8829
1.8091	33.8079	0.0193	0.0017	0.2693	0.0003	0.0054	0.2694	0.2701	0.7988
1.9061	37.4270	0.0198	0.0019	0.2693	0.0004	0.0054	0.2694	0.2701	0.7217
2.0064	41.1379	0.0210	0.0021	0.2693	0.0004	0.0054	0.2694	0.2702	0.6568
2.1073	46.4036	0.0221	0.0023	0.2693	0.0004	0.0054	0.2694	0.2703	0.5825
2.3105	60.8839	0.0247	0.0030	0.2693	0.0006	0.0054	0.2694	0.2705	0.4443
2.5122	76.8940	0.0264	0.0038	0.2693	0.0007	0.0054	0.2694	0.2707	0.3520
2.7087	98.8119	0.0274	0.0049	0.2693	0.0009	0.0054	0.2694	0.2708	0.2741
2.9112	127.6270	0.0280	0.0064	0.2693	0.0012	0.0054	0.2694	0.2709	0.2123
3.1129	160.1392	0.0291	0.0080	0.2693	0.0015	0.0054	0.2695	0.2711	0.1693

Uncertainty Analysis (Bare Hull Resistance - Astern)

V (m/s)	R (N)	P_R	B_1	B_2	B_3	B_4	B_R	U_R	% Unc.
0.6088	5.3066	0.0094	0.0003	0.2693	0.0001	0.0054	0.2694	0.2695	5.0792
0.7089	6.9982	0.0098	0.0003	0.2693	0.0001	0.0054	0.2694	0.2695	3.8516
0.8097	8.9347	0.0104	0.0004	0.2693	0.0001	0.0054	0.2694	0.2696	3.0171
0.9113	11.1765	0.0114	0.0006	0.2693	0.0001	0.0054	0.2694	0.2696	2.4123
1.0126	13.4500	0.0122	0.0007	0.2693	0.0001	0.0054	0.2694	0.2696	2.0048
1.1124	15.8659	0.0130	0.0008	0.2693	0.0002	0.0054	0.2694	0.2697	1.6997
1.2146	18.5111	0.0132	0.0009	0.2693	0.0002	0.0054	0.2694	0.2697	1.4569
1.3139	21.1376	0.0141	0.0011	0.2693	0.0002	0.0054	0.2694	0.2697	1.2761
1.4123	24.1367	0.0148	0.0012	0.2693	0.0002	0.0054	0.2694	0.2698	1.1177
1.5161	27.3797	0.0163	0.0014	0.2693	0.0003	0.0054	0.2694	0.2699	0.9856
1.6148	30.4808	0.0172	0.0015	0.2693	0.0003	0.0054	0.2694	0.2699	0.8856
1.7155	34.0387	0.0194	0.0017	0.2693	0.0003	0.0054	0.2694	0.2701	0.7934
1.8173	37.5215	0.0198	0.0019	0.2693	0.0004	0.0054	0.2694	0.2701	0.7199
1.9185	40.8561	0.0211	0.0020	0.2693	0.0004	0.0054	0.2694	0.2702	0.6613
2.0195	44.6591	0.0224	0.0022	0.2693	0.0004	0.0054	0.2694	0.2703	0.6053

Bare Hull Resistance Test

Uncertainty Analysis (Drag Coefficient - Ahead / Astern Directions)

V	Vehicle Velocity (m/s)	
R	Bare Hull Resistance (N)	
D	Propeller Diameter (m)	
A	Cross Sectional Area of the Vehicle Hull (m^2)	
ρ	Density of Water (kg/m^3)	
C_d	Drag Coefficient (Cross Sectional Area)	
P_{Cd}	Precision Error for C_d	
P_R	Precision Error for R	
B_R	Bias Error for R	
P_R	Precision Error for R	
B_V	Bias Error for V	
P_V	Precision Error for V	
B_A	Bias Error for A	
B_ρ	Bias Error for ρ	
B_{Cd}	Bias Error for C_d	$B_{Cd} = (\delta C_d / \delta R\ B_R)^2 + (\delta C_d / \delta \rho\ B_\rho)^2 + (\delta C_d / \delta A\ B_A)^2 + (\delta C_d / \delta V\ B_V)^2)^{1/2}$
U_{Cd}	Total Error for C_d	$U_{Cd} = (B_{Cd}^2 + P_{Cd}^2)^{1/2}$
% Unc.	Percentage Uncertainty for C_d	% Unc. = $(U_{Cd} / C_d) \times 100$

Uncertainty Analysis (Drag Coefficient)
Bare Hull Resistance Test (Ahead Direction)

V (m/s)	R (N)	P_R	B_R	P_V	B_V	B_ρ	B_A	$B_R\ \delta C_d/\delta R$	$B_\rho\ \delta C_d/\delta\rho$	$B_A\ \delta C_d/\delta A$	$B_V\ \delta C_d/\delta V$	B_{Cd}	P_{Cd}	U_{Cd}	C_d	% Unc.
0.6010	4.8372	0.0088	0.2694	0.0002	0.0035	0.0578	0.0028	0.0119	-1.23E-05	-0.0048	-0.0025	0.0130	0.0088	0.0157	0.2131	7.3783
0.6987	6.2947	0.0092	0.2694	0.0002	0.0035	0.0578	0.0028	0.0088	-1.19E-05	-0.0046	-0.0021	0.0101	0.0092	0.0137	0.2052	6.6654
0.8003	7.9859	0.0099	0.2694	0.0003	0.0035	0.0578	0.0028	0.0067	-1.15E-05	-0.0044	-0.0017	0.0082	0.0099	0.0129	0.1985	6.4846
0.8990	9.8816	0.0109	0.2694	0.0003	0.0035	0.0578	0.0028	0.0053	-1.13E-05	-0.0043	-0.0015	0.0070	0.0109	0.0130	0.1946	6.6818
1.0015	11.7594	0.0114	0.2694	0.0004	0.0035	0.0578	0.0028	0.0043	-1.08E-05	-0.0042	-0.0013	0.0061	0.0114	0.0129	0.1866	6.9345
1.1016	13.8754	0.0119	0.2694	0.0004	0.0035	0.0578	0.0028	0.0035	-1.05E-05	-0.0041	-0.0012	0.0055	0.0119	0.0132	0.1820	7.2283
1.2007	16.2767	0.0125	0.2694	0.0005	0.0035	0.0578	0.0028	0.0030	-1.04E-05	-0.0040	-0.0011	0.0051	0.0125	0.0135	0.1797	7.5182
1.3048	18.6795	0.0138	0.2694	0.0005	0.0035	0.0578	0.0028	0.0025	-1.01E-05	-0.0039	-0.0009	0.0047	0.0138	0.0146	0.1746	8.3731
1.4054	21.2504	0.0145	0.2694	0.0006	0.0035	0.0578	0.0028	0.0022	-9.90E-06	-0.0038	-0.0009	0.0045	0.0145	0.0152	0.1712	8.8873
1.5037	23.8043	0.0154	0.2694	0.0006	0.0035	0.0578	0.0028	0.0019	-9.69E-06	-0.0037	-0.0008	0.0043	0.0154	0.0160	0.1676	9.5270
1.6040	27.2382	0.0164	0.2694	0.0006	0.0036	0.0578	0.0028	0.0017	-9.74E-06	-0.0038	-0.0007	0.0042	0.0164	0.0169	0.1685	10.0353
1.7060	30.5770	0.0181	0.2694	0.0007	0.0036	0.0578	0.0028	0.0015	-9.67E-06	-0.0037	-0.0007	0.0041	0.0181	0.0186	0.1672	11.1032
1.8091	33.8079	0.0193	0.2694	0.0008	0.0036	0.0578	0.0028	0.0013	-9.51E-06	-0.0037	-0.0006	0.0039	0.0193	0.0197	0.1644	11.9714
1.9061	37.4270	0.0198	0.2694	0.0009	0.0036	0.0578	0.0028	0.0012	-9.48E-06	-0.0037	-0.0006	0.0039	0.0198	0.0202	0.1640	12.3332
2.0064	41.1379	0.0210	0.2694	0.0009	0.0036	0.0578	0.0028	0.0011	-9.41E-06	-0.0036	-0.0006	0.0038	0.0211	0.0214	0.1626	13.1578
2.1073	46.4036	0.0221	0.2694	0.0010	0.0036	0.0578	0.0028	0.0010	-9.62E-06	-0.0037	-0.0006	0.0039	0.0221	0.0225	0.1663	13.5177
2.3105	60.8839	0.0247	0.2694	0.0012	0.0036	0.0578	0.0028	0.0008	-1.05E-05	-0.0040	-0.0006	0.0042	0.0248	0.0251	0.1815	13.8385
2.5122	76.8940	0.0264	0.2694	0.0013	0.0036	0.0578	0.0028	0.0007	-1.12E-05	-0.0043	-0.0006	0.0044	0.0265	0.0268	0.1939	13.8426
2.7087	98.8119	0.0274	0.2694	0.0014	0.0036	0.0578	0.0028	0.0006	-1.24E-05	-0.0048	-0.0006	0.0049	0.0274	0.0278	0.2143	12.9822
2.9112	127.6270	0.0280	0.2694	0.0015	0.0037	0.0578	0.0028	0.0005	-1.39E-05	-0.0053	-0.0006	0.0054	0.0280	0.0285	0.2397	11.9035
3.1129	160.1392	0.0291	0.2695	0.0016	0.0037	0.0578	0.0028	0.0004	-1.52E-05	-0.0059	-0.0006	0.0059	0.0291	0.0297	0.2630	11.3066

<u>Uncertainty Analysis</u> (Drag Coefficient)
Bare Hull Resistance Test (Astern Direction)

V (m/s)	R (N)	P_R	B_R	P_V	B_V	B_ρ	B_A	$B_R\ \delta C_d/\delta R$	$B_\rho\ \delta C_d/\delta\rho$	$B_A\ \delta C_d/\delta A$	$B_V\ \delta C_d/\delta V$	B_{Cd}	P_{Cd}	U_{Cd}	C_d	% Unc.
0.6088	5.3066	0.0094	0.2694	0.0002	0.0035	0.0578	0.0028	0.0116	-1.32E-05	-0.0051	-0.0026	0.0129	0.0094	0.0160	0.2279	7.0090
0.7089	6.9982	0.0098	0.2694	0.0002	0.0035	0.0578	0.0028	0.0085	-1.28E-05	-0.0049	-0.0022	0.0101	0.0098	0.0141	0.2216	6.3523
0.8097	8.9347	0.0104	0.2694	0.0003	0.0035	0.0578	0.0028	0.0065	-1.25E-05	-0.0048	-0.0019	0.0084	0.0104	0.0133	0.2169	6.1475
0.9113	11.1765	0.0114	0.2694	0.0004	0.0035	0.0578	0.0028	0.0052	-1.24E-05	-0.0048	-0.0017	0.0072	0.0114	0.0135	0.2142	6.3135
1.0126	13.4500	0.0122	0.2694	0.0004	0.0035	0.0578	0.0028	0.0042	-1.21E-05	-0.0047	-0.0015	0.0064	0.0122	0.0138	0.2088	6.5869
1.1124	15.8659	0.0130	0.2694	0.0005	0.0035	0.0578	0.0028	0.0035	-1.18E-05	-0.0046	-0.0013	0.0059	0.0130	0.0142	0.2041	6.9761
1.2146	18.5111	0.0132	0.2694	0.0005	0.0035	0.0578	0.0028	0.0029	-1.15E-05	-0.0045	-0.0012	0.0054	0.0133	0.0143	0.1997	7.1741
1.3139	21.1376	0.0141	0.2694	0.0006	0.0035	0.0578	0.0028	0.0025	-1.13E-05	-0.0043	-0.0010	0.0051	0.0141	0.0150	0.1949	7.7198
1.4123	24.1367	0.0148	0.2694	0.0006	0.0035	0.0578	0.0028	0.0022	-1.11E-05	-0.0043	-0.0010	0.0049	0.0148	0.0156	0.1926	8.1118
1.5161	27.3797	0.0163	0.2694	0.0006	0.0035	0.0578	0.0028	0.0019	-1.10E-05	-0.0042	-0.0009	0.0047	0.0163	0.0170	0.1896	8.9671
1.6148	30.4808	0.0172	0.2694	0.0007	0.0036	0.0578	0.0028	0.0016	-1.08E-05	-0.0041	-0.0008	0.0045	0.0173	0.0178	0.1860	9.5900
1.7155	34.0387	0.0194	0.2694	0.0008	0.0036	0.0578	0.0028	0.0015	-1.06E-05	-0.0041	-0.0008	0.0044	0.0194	0.0199	0.1841	10.8294
1.8173	37.5215	0.0198	0.2694	0.0009	0.0036	0.0578	0.0028	0.0013	-1.05E-05	-0.0040	-0.0007	0.0043	0.0198	0.0203	0.1808	11.2285
1.9185	40.8561	0.0211	0.2694	0.0009	0.0036	0.0578	0.0028	0.0012	-1.02E-05	-0.0039	-0.0007	0.0042	0.0211	0.0215	0.1767	12.1563
2.0195	44.6591	0.0224	0.2694	0.0010	0.0036	0.0578	0.0028	0.0011	-1.01E-05	-0.0039	-0.0006	0.0041	0.0224	0.0228	0.1743	13.0691

Bollard Pull Test (Prop. + Duct fitted on)

Uncertainty Analysis (Propeller Rotational Speed)

n	Propeller Rotational Speed (rpm)	
C	Number of Counted Pulses in Time dt	
dt	Time (s)	
S.D	Standard Deviation	
P_n	Precision Error for n	$P_n = 2 \times S.D$
B_C	Bias Error for C	
B_{dt}	Bias Error for dt	
B_n	Bias Error for n	$B_n = ((\delta n / \delta C \; B_C)^2 + (\delta n / \delta dt \; B_{dt})^2)^{1/2}$
U_n	Total Error for n	$U_n = (B_n^2 + P_n^2)^{1/2}$
% Unc.	Percentage Uncertainty for n	% Unc. = $(U_n / n) \times 100$

<u>Uncertainty Analysis</u> (Rotational Speed)
Bollard Pull Test (Prop. + Duct fitted on)

n (rpm)	**P_n**	**B_C $\delta n/\delta C$**	**B_{dt} $\delta n/\delta dt$**	**B_n**	**U_n**	**% Unc.**
503.9033	0.0004	0.0391	-0.0009	0.0391	0.0391	0.0078
550.4767	0.0005	0.0391	-0.0010	0.0391	0.0391	0.0071
605.9664	0.0005	0.0391	-0.0011	0.0391	0.0391	0.0065
650.5832	0.0005	0.0391	-0.0013	0.0391	0.0391	0.0060
750.1102	0.0005	0.0391	-0.0016	0.0391	0.0391	0.0052
907.7347	0.0005	0.0391	-0.0020	0.0391	0.0391	0.0043
1001.8816	0.0005	0.0391	-0.0023	0.0391	0.0392	0.0039
1052.1447	0.0004	0.0391	-0.0024	0.0392	0.0392	0.0037
1102.7669	0.0005	0.0391	-0.0025	0.0392	0.0392	0.0036
1151.0962	0.0005	0.0391	-0.0027	0.0392	0.0392	0.0034
1201.9108	0.0005	0.0391	-0.0028	0.0392	0.0392	0.0033
1250.9090	0.0005	0.0391	-0.0030	0.0392	0.0392	0.0031

Bollard Pull Test (Prop. + Duct fitted on)

Uncertainty Analysis (Propeller Torque)

n	Propeller Rotational Speed (rpm)	
Q	Propeller Torque (N-m)	
F	Calibrating Weight (N)	
R	Lever Arm (m)	
P_Q	Precision Error for Q	$P_Q = 2 \times S.D$
S.D	Standard Deviation	
S.E.E	Standard Error Estimate	
AD	Analog to Digital Converter	
B_F	Bias Error for F	
B_R	Bias Error for R	
B_1	Bias Error due to Calibration	$B_1 = ((\delta Q / \delta F\ B_F)^2 + (\delta Q / \delta R\ B_R)^2)^{1/2}$
B_2	Bias Error due to Data Acquisition	$B_2 = 2 \times S.E.E$
B_3	Bias Error due to Data Reduction	$B_3 = (AD_{error} \times AD_{range}) / (AD_{accuracy})$
B_Q	Bias Error for Q	$B_Q = (B_1^2 + B_2^2 + B_3^2)^{1/2}$
U_Q	Total Error for Q	$U_Q = (B_Q^2 + P_Q^2)^{1/2}$
% Unc.	Percentage Uncertainty for Q	% Unc. = $(U_Q / Q) \times 100$

Uncertainty Analysis (Propeller Torque)
Bollard Pull Test (Prop. + Duct fitted on)

n (rpm)	Q (N-m)	P_Q	B_F $\delta Q/\delta F$	B_R $\delta Q/\delta R$	B_1	B_2	B_3	B_Q	U_Q	% Unc.
503.9033	0.5283	0.0006	0.00003	0.00042	0.0004	0.0076	0.0007	0.0076	0.0077	1.4519
550.4767	0.6797	0.0006	0.00003	0.00054	0.0005	0.0076	0.0007	0.0077	0.0077	1.1300
605.9664	0.8448	0.0007	0.00004	0.00068	0.0007	0.0076	0.0007	0.0077	0.0077	0.9105
650.5832	1.0205	0.0007	0.00005	0.00082	0.0008	0.0076	0.0007	0.0077	0.0077	0.7552
750.1102	1.3944	0.0007	0.00007	0.00112	0.0011	0.0076	0.0007	0.0077	0.0077	0.5557
907.7347	2.1970	0.0007	0.00011	0.00176	0.0018	0.0076	0.0007	0.0078	0.0079	0.3581
1001.8816	2.6913	0.0007	0.00013	0.00215	0.0022	0.0076	0.0007	0.0079	0.0080	0.2960
1052.1447	3.0135	0.0007	0.00015	0.00241	0.0024	0.0076	0.0007	0.0080	0.0080	0.2668
1102.7669	3.3844	0.0008	0.00017	0.00271	0.0027	0.0076	0.0007	0.0081	0.0081	0.2404
1151.0962	3.6596	0.0008	0.00018	0.00293	0.0029	0.0076	0.0007	0.0082	0.0082	0.2245
1201.9108	4.0432	0.0008	0.00020	0.00323	0.0032	0.0076	0.0007	0.0083	0.0083	0.2061
1250.9090	4.3910	0.0008	0.00022	0.00351	0.0035	0.0076	0.0007	0.0084	0.0084	0.1924

Bollard Pull Test (Prop. + Duct fitted on)

Uncertainty Analysis (Propeller Thrust)

n	Propeller Rotational Speed (rpm)	
T_P	Propeller Thrust (N)	
P_T	Precision Error for T_P	$P_T = 2 \times S.D$
S.D	Standard Deviation	
S.E.E	Standard Error Estimate	
AD	Analog to Digital Converter	
B_1	Bias Error due to Calibration	$B_1 = 0.00005 \times T_P$
B_2	Bias Error due to Data Acquisition	$B_2 = 2 \times S.E.E$
B_3	Bias Error due to Data Reduction	$B_3 = (AD_{error} \times AD_{range}) / (AD_{accuracy})$
B_T	Bias Error for T_P	$B_T = (B_1^2 + B_2^2 + B_3^2)^{1/2}$
U_T	Total Error for T_P	$U_T = (B_T^2 + P_T^2)^{1/2}$
% Unc.	Percentage Uncertainty for T_P	% Unc. = $(U_T / T_P) \times 100$

Uncertainty Analysis (Propeller Thrust)
Bollard Pull Test (Prop. + Duct fitted on)

n (rpm)	T_P (N)	P_T	B_1	B_2	B_3	B_T	U_T	% Unc.
503.9033	8.9647	0.0080	0.0004	0.2487	0.0182	0.2494	0.2495	2.7829
550.4767	12.5679	0.0082	0.0006	0.2487	0.0182	0.2494	0.2495	1.9851
605.9664	17.6148	0.0085	0.0009	0.2487	0.0182	0.2494	0.2495	1.4164
650.5832	22.3550	0.0083	0.0011	0.2487	0.0182	0.2494	0.2495	1.1160
750.1102	35.0872	0.0086	0.0018	0.2487	0.0182	0.2494	0.2495	0.7111
907.7347	59.2718	0.0087	0.0030	0.2487	0.0182	0.2494	0.2495	0.4210
1001.8816	76.4800	0.0088	0.0038	0.2487	0.0182	0.2494	0.2495	0.3263
1052.1447	87.9141	0.0092	0.0044	0.2487	0.0182	0.2494	0.2496	0.2839
1102.7669	98.9344	0.0091	0.0049	0.2487	0.0182	0.2494	0.2496	0.2523
1151.0962	109.1874	0.0093	0.0055	0.2487	0.0182	0.2494	0.2496	0.2286
1201.9108	121.8579	0.0096	0.0061	0.2487	0.0182	0.2494	0.2496	0.2048
1250.9090	132.5731	0.0095	0.0066	0.2487	0.0182	0.2494	0.2496	0.1883

Bollard Pull Test (Prop. + Duct fitted on)

Uncertainty Analysis (Tow Force)

n	Propeller Rotational Speed (rpm)	
F	Tow Force (N)	
P_F	Precision Error for F	$P_F = 2 \times S.D$
S.D	Standard Deviation	
S.E.E	Standard Error Estimate	
AD	Analog to Digital Converter	
B_1	Bias Error due to Calibration	$B_1 = 0.00005 \times F$
B_2	Bias Error due to Data Acquisition	$B_2 = 2 \times S.E.E$
B_3	Bias Error due to Load Cell Misalignment	$B_3 = F - (\cos 0.25^0 \times F)$
B_4	Bias Error due to Data Reduction	$B_4 = (AD_{error} \times AD_{range}) / (AD_{accuracy})$
B_F	Bias Error for F	$B_F = (B_1^2 + B_2^2 + B_3^2 + B_4^2)^{1/2}$
U_F	Total Error for F	$U_F = (B_F^2 + P_F^2)^{1/2}$
% Unc.	Percentage Uncertainty for F	% Unc. = $(U_F / F) \times 100$

Uncertainty Analysis (Tow Force)
Bollard Pull Test (Prop. + Duct fitted on)

n (rpm)	F (N)	P_F	B_1	B_2	B_3	B_4	B_F	U_F	% Unc.
503.9033	-11.3075	0.0087	-0.0006	0.2693	-0.0001	0.0054	0.2694	0.2695	-2.3834
550.4767	-17.3445	0.0089	-0.0009	0.2693	-0.0002	0.0054	0.2694	0.2695	-1.5539
605.9664	-24.1935	0.0089	-0.0012	0.2693	-0.0002	0.0054	0.2694	0.2695	-1.1140
650.5832	-30.4523	0.0095	-0.0015	0.2693	-0.0003	0.0054	0.2694	0.2695	-0.8851
750.1102	-48.3194	0.0092	-0.0024	0.2693	-0.0005	0.0054	0.2694	0.2695	-0.5578
907.7347	-84.1484	0.0098	-0.0042	0.2693	-0.0008	0.0054	0.2694	0.2696	-0.3204
1001.8816	-113.0899	0.0102	-0.0057	0.2693	-0.0011	0.0054	0.2694	0.2696	-0.2384
1052.1447	-129.0183	0.0100	-0.0065	0.2693	-0.0012	0.0054	0.2694	0.2696	-0.2090
1102.7669	-145.7299	0.0103	-0.0073	0.2693	-0.0014	0.0054	0.2695	0.2697	-0.1850
1151.0962	-162.4119	0.0105	-0.0081	0.2693	-0.0015	0.0054	0.2695	0.2697	-0.1661
1201.9108	-183.5970	0.0111	-0.0092	0.2693	-0.0017	0.0054	0.2695	0.2698	-0.1469
1250.9090	-200.7750	0.0110	-0.0100	0.2693	-0.0019	0.0054	0.2696	0.2698	-0.1344

Appendix C

Calculations for Chapter 5

C.1 Expanded Area Ratio & Chord Length Calculation

X	t_x / L	t_x / D	L / D	S.M	Product
0.20	0.1500	0.0150	0.1000	1	0.1000
0.30	0.1350	0.0149	0.1100	4	0.4400
0.40	0.1200	0.0144	0.1200	2	0.2400
0.50	0.1050	0.0131	0.1250	4	0.5000
0.60	0.0900	0.0115	0.1275	2	0.2550
0.70	0.0750	0.0094	0.1250	4	0.5000
0.80	0.0600	0.0069	0.1150	2	0.2300
0.90	0.0450	0.0041	0.0900	4	0.3600
1.00	0.0000	0.0030	0.0000	1	0.0000

Integral 0.0875

A_E / A_O 0.1114

C.2 Blade Root Section (X = 0.2) Profile Calculation

NACA Mean Line (a = 0.8)

C_L 1.00

x / c (%)	y_c / c (%)	dy_c / dx (%)
0.0000	0.0000	0.0000
0.5000	0.2870	0.4854
0.7500	0.4040	0.4493
1.2500	0.6160	0.4036
2.5000	1.0770	0.3410
5.0000	1.8410	0.2772
7.5000	2.4830	0.2387
10.0000	3.0430	0.2105
15.0000	3.9850	0.1689
20.0000	4.7480	0.1373
25.0000	5.3670	0.1110
30.0000	5.8630	0.0878
35.0000	6.2480	0.0663
40.0000	6.5280	0.0460
45.0000	6.7090	0.0261
50.0000	6.7900	0.0062
55.0000	6.7700	-0.0143
60.0000	6.6440	-0.0361
65.0000	6.4050	-0.0601
70.0000	6.0370	-0.0879
75.0000	5.5140	-0.1231
80.0000	4.7710	-0.1841
85.0000	3.6830	-0.2392
90.0000	2.4350	-0.2558
95.0000	1.1630	-0.2490
100.0000	0.0000	-0.2039

NACA Mean Line (a = 0.8)

C_L 1.3147

x / c (ratio)	y_c / c (ratio)	dy_c / dx (ratio)
0.0000	0.0000	0.0000
0.0050	0.0038	0.0064
0.0075	0.0053	0.0059
0.0125	0.0081	0.0053
0.0250	0.0142	0.0045
0.0500	0.0242	0.0036
0.0750	0.0326	0.0031
0.1000	0.0400	0.0028
0.1500	0.0524	0.0022
0.2000	0.0624	0.0018
0.2500	0.0706	0.0015
0.3000	0.0771	0.0012
0.3500	0.0821	0.0009
0.4000	0.0858	0.0006
0.4500	0.0882	0.0003
0.5000	0.0893	0.0001
0.5500	0.0890	-0.0002
0.6000	0.0873	-0.0005
0.6500	0.0842	-0.0008
0.7000	0.0794	-0.0012
0.7500	0.0725	-0.0016
0.8000	0.0627	-0.0024
0.8500	0.0484	-0.0031
0.9000	0.0320	-0.0034
0.9500	0.0153	-0.0033
1.0000	0.0000	-0.0027

NACA 16-015 Airfoil

t_x / L 0.15

x / c (%)	y_t / c (%)
0.0000	0.0000
0.5000	1.0321
0.7500	1.2591
1.2500	1.6155
2.5000	2.2584
5.0000	3.1400
7.5000	3.7927
10.0000	4.3250
15.0000	5.1736
20.0000	5.8339
25.0000	6.3597
30.0000	6.7754
35.0000	7.0932
40.0000	7.3189
45.0000	7.4545
50.0000	7.5000
55.0000	7.4514
60.0000	7.2946
65.0000	7.0129
70.0000	6.5898
75.0000	6.0086
80.0000	5.2527
85.0000	4.3055
90.0000	3.1504
95.0000	1.7708
100.0000	0.1500

NACA 16-015 Airfoil + NACA Mean Line (a = 0.8)

t_x / L 0.15 C_L 1.3147

x / c (ratio)	y_t / c (ratio)	y_c / c (ratio)	dy_c / dx (ratio)
0.0000	0.0000	0.0000	0.0000
0.0050	0.0103	0.0038	0.0064
0.0075	0.0126	0.0053	0.0059
0.0125	0.0162	0.0081	0.0053
0.0250	0.0226	0.0142	0.0045
0.0500	0.0314	0.0242	0.0036
0.0750	0.0379	0.0326	0.0031
0.1000	0.0433	0.0400	0.0028
0.1500	0.0517	0.0524	0.0022
0.2000	0.0583	0.0624	0.0018
0.2500	0.0636	0.0706	0.0015
0.3000	0.0678	0.0771	0.0012
0.3500	0.0709	0.0821	0.0009
0.4000	0.0732	0.0858	0.0006
0.4500	0.0745	0.0882	0.0003
0.5000	0.0750	0.0893	0.0001
0.5500	0.0745	0.0890	-0.0002
0.6000	0.0729	0.0873	-0.0005
0.6500	0.0701	0.0842	-0.0008
0.7000	0.0659	0.0794	-0.0012
0.7500	0.0601	0.0725	-0.0016
0.8000	0.0525	0.0627	-0.0024
0.8500	0.0431	0.0484	-0.0031
0.9000	0.0315	0.0320	-0.0034
0.9500	0.0177	0.0153	-0.0033
1.0000	0.0015	0.0000	-0.0027

x	Chord Line Abscissa	x_u	Upper Surface Abscissa	$x_u = x - y_t \sin\phi$
L or c	Chord Length	y_u	Upper Surface Ordinate	$y_u = y_c + y_t \cos\phi$
y_t	Thickness Distribution Ordinate	x_L	Lower Surface Abscissa	$x_L = x + y_t \sin\phi$
y_c	Mean Line Ordinate	y_L	Lower Surface Ordinate	$y_L = y_c - y_t \cos\phi$
dy_c / dx	Mean Line Slope			

Calculating the Upper & Lower Surface Abscissa & Ordinates

x/c	y_t/c	y_c/c	dy_c/dx	$\sin\Phi$	$\cos\Phi$	x_u/c	y_u/c	x_L/c	y_L/c
0.0000	0.0000	0.0000	0.0000	0.00000	1.00000	0.0000	0.0000	0.0000	0.0000
0.0050	0.0103	0.0038	0.0064	0.00638	0.99998	0.0049	0.0141	0.0051	-0.0065
0.0075	0.0126	0.0053	0.0059	0.00591	0.99998	0.0074	0.0179	0.0076	-0.0073
0.0125	0.0162	0.0081	0.0053	0.00531	0.99999	0.0124	0.0243	0.0126	-0.0081
0.0250	0.0226	0.0142	0.0045	0.00448	0.99999	0.0249	0.0367	0.0251	-0.0084
0.0500	0.0314	0.0242	0.0036	0.00364	0.99999	0.0499	0.0556	0.0501	-0.0072
0.0750	0.0379	0.0326	0.0031	0.00314	1.00000	0.0749	0.0706	0.0751	-0.0053
0.1000	0.0433	0.0400	0.0028	0.00277	1.00000	0.0999	0.0833	0.1001	-0.0032
0.1500	0.0517	0.0524	0.0022	0.00222	1.00000	0.1499	0.1041	0.1501	0.0007
0.2000	0.0583	0.0624	0.0018	0.00181	1.00000	0.1999	0.1208	0.2001	0.0041
0.2500	0.0636	0.0706	0.0015	0.00146	1.00000	0.2499	0.1342	0.2501	0.0070
0.3000	0.0678	0.0771	0.0012	0.00115	1.00000	0.2999	0.1448	0.3001	0.0093
0.3500	0.0709	0.0821	0.0009	0.00087	1.00000	0.3499	0.1531	0.3501	0.0112
0.4000	0.0732	0.0858	0.0006	0.00060	1.00000	0.4000	0.1590	0.4000	0.0126
0.4500	0.0745	0.0882	0.0003	0.00034	1.00000	0.4500	0.1627	0.4500	0.0137
0.5000	0.0750	0.0893	0.0001	0.00008	1.00000	0.5000	0.1643	0.5000	0.0143
0.5500	0.0745	0.0890	-0.0002	-0.00019	1.00000	0.5500	0.1635	0.5500	0.0145
0.6000	0.0729	0.0873	-0.0005	-0.00047	1.00000	0.6000	0.1603	0.6000	0.0144
0.6500	0.0701	0.0842	-0.0008	-0.00079	1.00000	0.6501	0.1543	0.6499	0.0141
0.7000	0.0659	0.0794	-0.0012	-0.00116	1.00000	0.7001	0.1453	0.6999	0.0135
0.7500	0.0601	0.0725	-0.0016	-0.00162	1.00000	0.7501	0.1326	0.7499	0.0124
0.8000	0.0525	0.0627	-0.0024	-0.00242	1.00000	0.8001	0.1153	0.7999	0.0102
0.8500	0.0431	0.0484	-0.0031	-0.00314	1.00000	0.8501	0.0915	0.8499	0.0054
0.9000	0.0315	0.0320	-0.0034	-0.00336	0.99999	0.9001	0.0635	0.8999	0.0005
0.9500	0.0177	0.0153	-0.0033	-0.00327	0.99999	0.9501	0.0330	0.9499	-0.0024
1.0000	0.0015	0.0000	-0.0027	-0.00268	1.00000	1.0000	0.0015	1.0000	-0.0015

Converting to mm

x	y_c	x_u	y_u	x_L	y_L
(mm)	(mm)	(mm)	(mm)	(mm)	(mm)
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.2500	0.1887	0.2467	0.7047	0.2533	-0.3274
0.3750	0.2656	0.3713	0.8951	0.3787	-0.3640
0.6250	0.4049	0.6207	1.2127	0.6293	-0.4028
1.2500	0.7080	1.2449	1.8372	1.2551	-0.4212
2.5000	1.2102	2.4943	2.7802	2.5057	-0.3598
3.7500	1.6322	3.7440	3.5286	3.7560	-0.2641
5.0000	2.0003	4.9940	4.1628	5.0060	-0.1622
7.5000	2.6196	7.4943	5.2064	7.5057	0.0328
10.0000	3.1211	9.9947	6.0381	10.0053	0.2042
12.5000	3.5280	12.4954	6.7079	12.5046	0.3482
15.0000	3.8541	14.9961	7.2418	15.0039	0.4664
17.5000	4.1072	17.4969	7.6538	17.5031	0.5606
20.0000	4.2912	19.9978	7.9507	20.0022	0.6318
22.5000	4.4102	22.4987	8.1375	22.5013	0.6830
25.0000	4.4635	24.9997	8.2135	25.0003	0.7135
27.5000	4.4503	27.5007	8.1760	27.4993	0.7246
30.0000	4.3675	30.0017	8.0148	29.9983	0.7202
32.5000	4.2104	32.5028	7.7168	32.4972	0.7039
35.0000	3.9685	35.0038	7.2634	34.9962	0.6736
37.5000	3.6247	37.5049	6.6290	37.4951	0.6204
40.0000	3.1363	40.0064	5.7626	39.9936	0.5099
42.5000	2.4211	42.5068	4.5738	42.4932	0.2683
45.0000	1.6007	45.0053	3.1759	44.9947	0.0255
47.5000	0.7645	47.5029	1.6499	47.4971	-0.1209
50.0000	0.0000	50.0002	0.0750	49.9998	-0.0750

Selecting 21 Stations

x	y_c
(mm)	(mm)
0.0000	0.0000
2.5000	1.2102
5.0000	2.0003
7.5000	2.6196
10.0000	3.1211
12.5000	3.5280
15.0000	3.8541
17.5000	4.1072
20.0000	4.2912
22.5000	4.4102
25.0000	4.4635
27.5000	4.4503
30.0000	4.3675
32.5000	4.2104
35.0000	3.9685
37.5000	3.6247
40.0000	3.1363
42.5000	2.4211
45.0000	1.6007
47.5000	0.7645
50.0000	0.0000

x_u	y_u
(mm)	(mm)
0.0000	0.0000
2.4943	2.7802
4.9940	4.1628
7.4943	5.2064
9.9947	6.0381
12.4954	6.7079
14.9961	7.2418
17.4969	7.6538
19.9978	7.9507
22.4987	8.1375
24.9997	8.2135
27.5007	8.1760
30.0017	8.0148
32.5028	7.7168
35.0038	7.2634
37.5049	6.6290
40.0064	5.7626
42.5068	4.5738
45.0053	3.1759
47.5029	1.6499
50.0002	0.0750

x_L	y_L
(mm)	(mm)
0.0000	0.0000
2.5057	-0.3598
5.0060	-0.1622
7.5057	0.0328
10.0053	0.2042
12.5046	0.3482
15.0039	0.4664
17.5031	0.5606
20.0022	0.6318
22.5013	0.6830
25.0003	0.7135
27.4993	0.7246
29.9983	0.7202
32.4972	0.7039
34.9962	0.6736
37.4951	0.6204
39.9936	0.5099
42.4932	0.2683
44.9947	0.0255
47.4971	-0.1209
49.9998	-0.0750

C.3 **Moment of Inertia & Centroid Position Calculation (Blade Root Section)**

1) Calculating the M.I & Centroid Position about the Y-Y axis

a) Section S_1 - about the Y-Y Axis

Station	x (mm)	y_u (mm)	S.M	L (mm)	y_u * S.M	y_u * S.M * L	y_u * S.M * L^2
1	0.00	0.0000	1	25	0.0000	0.0000	0.0000
2	2.50	2.7802	4	22.5	11.1207	250.2168	5629.8774
3	5.00	4.1628	2	20	8.3257	166.5133	3330.2667
4	7.50	5.2064	4	17.5	20.8255	364.4457	6377.7994
5	10.00	6.0381	2	15	12.0762	181.1425	2717.1373
6	12.50	6.7079	4	12.5	26.8316	335.3945	4192.4307
7	15.00	7.2418	2	10	14.4836	144.8358	1448.3580
8	17.50	7.6538	4	7.5	30.6151	229.6132	1722.0991
9	20.00	7.9507	2	5	15.9014	79.5068	397.5342
10	22.50	8.1375	4	2.5	32.5499	81.3747	203.4367
11	25.00	8.2135	2	0	16.4269	0.0000	0.0000
12	27.50	8.1760	4	-2.5	32.7041	-81.7602	204.4004
13	30.00	8.0148	2	-5	16.0296	-80.1479	400.7394
14	32.50	7.7168	4	-7.5	30.8673	-231.5049	1736.2866
15	35.00	7.2634	2	-10	14.5267	-145.2674	1452.6741
16	37.50	6.6290	4	-12.5	26.5159	-331.4485	4143.1064
17	40.00	5.7626	2	-15	11.5252	-172.8780	2593.1697
18	42.50	4.5738	4	-17.5	18.2952	-320.1653	5602.8932
19	45.00	3.1759	2	-20	6.3517	-127.0344	2540.6870
20	47.50	1.6499	4	-22.5	6.5996	-148.4913	3341.0537
21	50.00	0.0750	1	-25	0.0750	-1.8750	46.8748
					293.8723	160.3921	40067.3542

x Chord Line Abscissa
y_u Upper Surface Ordinate
L Lever Arm
S.M Simpson's Multplier

	A_1	293.8723	mm^2
Position of Centroid from the Y-Y Axis	d_1	0.5458	mm
M.I of section S_1 about the Y-Y Axis	$M.I_{Y\text{-}Y S1}$	40067.3542	mm^4

b) **Section S_2 - about the Y-Y Axis**

Station	x	y_L	S.M	L	y_L * S.M	y_L * S.M * L	y_L * S.M * L^2
	(mm)	(mm)		(mm)			
1	10.00	0.2042	1	15	0.2042	3.0629	45.9433
2	12.50	0.3482	4	12.5	1.3928	17.4098	217.6225
3	15.00	0.4664	2	10	0.9328	9.3279	93.2789
4	17.50	0.5606	4	7.5	2.2423	16.8173	126.1297
5	20.00	0.6318	2	5	1.2636	6.3179	31.5893
6	22.50	0.6830	4	2.5	2.7319	6.8297	17.0742
7	25.00	0.7135	2	0	1.4269	0.0000	0.0000
8	27.50	0.7246	4	-2.5	2.8985	-7.2462	18.1154
9	30.00	0.7202	2	-5	1.4404	-7.2019	36.0095
10	32.50	0.7039	4	-7.5	2.8157	-21.1179	158.3846
11	35.00	0.6736	2	-10	1.3471	-13.4715	134.7150
12	37.50	0.6204	4	-12.5	2.4815	-31.0189	387.7364
13	40.00	0.5099	2	-15	1.0198	-15.2974	229.4616
14	42.50	0.2683	4	-17.5	1.0732	-18.7818	328.6818
15	45.00	0.0255	1	-20	0.0255	-0.5095	10.1906
					19.4135	-45.7332	1529.1106

x Chord Line Abscissa
y_L Lower Surface Ordinate
L Lever Arm
S.M Simpson's Multplier

	A_2	19.4135	mm^2
Position of Centroid from the Y-Y Axis	d_2	-2.3557	mm
M.I of section S_2 about the Y-Y Axis	$M.I_{Y-YS2}$	1529.1106	mm^4

c) **Section S_3 - about the Y-Y Axis**

Station	x	y_L	S.M	L	y_L * S.M	y_L * S.M * L	y_L * S.M * L^2
	(mm)	(mm)		(mm)			
1	0.00	0.0000	1	25	0.0000	0.0000	0.0000
2	2.50	0.3598	4	22.5	1.4392	32.3813	728.5803
3	5.00	0.1622	1	20	0.1622	3.2430	64.8600
					1.3344	29.6870	661.2003

x Chord Line Abscissa
y_L Lower Surface Ordinate
L Lever Arm
S.M Simpson's Multplier

	A_3	1.3344	mm^2
Position of Centroid from the Y-Y Axis	d_3	22.2468	mm
M.I of section S_3 about the Y-Y Axis	$M.I_{Y-YS3}$	661.2003	mm^4

Sections S_1, S_2, S_3 combined - about the Y-Y Axis

Position of Centroid from the Y-Y Axis (d_x)		
$d_x = (A_1 * d_1 - A_2 * d_2 + A_3 * d_3) / (A_1 - A_2 + A_3)$	0.5234	mm
Position of Centroid from the origin (x = 0)	24.4766	mm
M.I of the complete section about the Y-Y axis		
$M.I_{Y\text{-}Y} = M.I_{S1} - M.I_{S2} + M.I_{S3}$	39199.4439	mm^4
M.I of the complete section about an axis parallel to the Y-Y Axis & passing through the Centroid		
$I_{Y0} = M.I_{Y\text{-}Y} - A \cdot d_x^2$	39123.8955	mm^4

2) Calculating the M.I & Centroid Position about the X-X axis

a) Section S_1 - about the X-X Axis

Station	x (mm)	y_u (mm)	S.M	y_u * S.M	y_u^2 * S.M	y_u^3 * S.M
1	0.00	0.0000	1	0.0000	0.0000	0.0000
2	2.50	2.7802	4	11.1207	30.9177	85.9571
3	5.00	4.1628	2	8.3257	34.6584	144.2770
4	7.50	5.2064	4	20.8255	108.4250	564.5004
5	10.00	6.0381	2	12.0762	72.9169	440.2782
6	12.50	6.7079	4	26.8316	179.9831	1207.3067
7	15.00	7.2418	2	14.4836	104.8870	759.5700
8	17.50	7.6538	4	30.6151	234.3210	1793.4401
9	20.00	7.9507	2	15.9014	126.4268	1005.1795
10	22.50	8.1375	4	32.5499	264.8735	2155.3994
11	25.00	8.2135	2	16.4269	134.9220	1108.1767
12	27.50	8.1760	4	32.7041	267.3890	2186.1766
13	30.00	8.0148	2	16.0296	128.4737	1029.6894
14	32.50	7.7168	4	30.8673	238.1978	1838.1320
15	35.00	7.2634	2	14.5267	105.5131	766.3807
16	37.50	6.6290	4	26.5159	175.7730	1165.1939
17	40.00	5.7626	2	11.5252	66.4151	382.7236
18	42.50	4.5738	4	18.2952	83.6782	382.7267
19	45.00	3.1759	2	6.3517	20.1722	64.0639
20	47.50	1.6499	4	6.5996	10.8887	17.9653
21	50.00	0.0750	1	0.0750	0.0056	0.0004
				293.8723	995.3491	4749.2050

	A_1'	293.8723	mm^2
Position of Centroid from the X-X Axis	d_1'	3.3870	mm
M.I of section S_1 about the X-X Axis	$M.I_{X\text{-}X\ S1}$	4749.2050	mm^4

x	Chord Line Abscissa
y_u	Upper Surface Ordinate
S.M	Simpson's Multplier

b) **Section S_2 - about the X-X Axis**

Station	x (mm)	y_L (mm)	S.M	y_L * S.M	y_L^2 * S.M	y_L^3 * S.M
1	10.00	0.2042	1	0.2042	0.0417	0.0085
2	12.50	0.3482	4	1.3928	0.4850	0.1689
3	15.00	0.4664	2	0.9328	0.4350	0.2029
4	17.50	0.5606	4	2.2423	1.2570	0.7046
5	20.00	0.6318	2	1.2636	0.7983	0.5044
6	22.50	0.6830	4	2.7319	1.8658	1.2743
7	25.00	0.7135	2	1.4269	1.0181	0.7263
8	27.50	0.7246	4	2.8985	2.1003	1.5219
9	30.00	0.7202	2	1.4404	1.0373	0.7471
10	32.50	0.7039	4	2.8157	1.9821	1.3952
11	35.00	0.6736	2	1.3471	0.9074	0.6112
12	37.50	0.6204	4	2.4815	1.5395	0.9551
13	40.00	0.5099	2	1.0198	0.5200	0.2652
14	42.50	0.2683	4	1.0732	0.2880	0.0773
15	45.00	0.0255	1	0.0255	0.0006	0.0000
				19.4135	5.9484	2.5452

x Chord Line Abscissa
y_L Lower Surface Ordinate
S.M Simpson's Multplier

	A_2'	19.4135	mm^2
Position of Centroid from the X-X Axis	d_2'	0.3064	mm
M.I of section S_2 about the X-X Axis	$M.I_{X\text{-}X\,S\,2}$	2.5452	mm^4

c) **Section S_3 - about the X-X Axis**

Station	x (mm)	y_L (mm)	S.M	y_L * S.M	y_L^2 * S.M	y_L^3 * S.M
1	0.00	0.0000	1	0.0000	0.0000	0.0000
2	2.50	0.3598	4	1.4392	0.5178	0.1863
3	5.00	0.1622	1	0.1622	0.0263	0.0043
				1.3344	0.2267	0.0529

x Chord Line Abscissa
y_L Lower Surface Ordinate
S.M Simpson's Multplier

	A_3'	1.3344	mm^2
Position of Centroid from the X-X Axis	d_3'	-0.1699	mm
M.I of section S_3 about the X-X Axis	$M.I_{X\text{-}X\,S\,3}$	0.0529	mm^4

Sections S_1, S_2, S_3 combined - about the X-X Axis

Position of Centroid from the X-X Axis (d_y)		
$d_y = (A_1' * d_1' - A_2' * d_2' + A_3' * d_3') / (A_1' - A_2' + A_3')$	3.5867	mm
M.I of the complete section about the X-X axis		
$M.I_{X\text{-}X} = M.I_{X\text{-}X\,S\,1} - M.I_{X\text{-}X\,S\,2} + M.I_{X\text{-}X\,S\,3}$	4746.7127	mm^4
M.I of the complete section about an axis parallel to the X-X Axis & passing through the Centroid		
$I_{X\,0} = M.I_{X\text{-}X} - A \cdot d_y^2$	1198.8900	mm^4

M.I & Centroid Position - Final Values

$X_{Centroid}$	24.4766	mm
$Y_{Centroid}$	3.5867	mm
$I_{Y\,0}$	39123.8955	mm^4
$I_{X\,0}$	1198.8900	mm^4

C.4 Blade Mass & Longitudinal Position of Centroid (X_c) Calculation

Blade Mass

X	r_0 m	A_x mm^2
0.2000	0.0500	276.0935
0.3000	0.0750	300.6658
0.4000	0.1000	318.0597
0.5000	0.1250	301.9773
0.6000	0.1500	269.2947
0.7000	0.1750	215.6980
0.8000	0.2000	146.0535
0.9000	0.2250	67.0907
1.0000	0.2500	0.0000

Station	A_x m^2	S.M	A_x * S.M
1	2.76E-04	1	0.0003
2	3.01E-04	4	0.0012
3	3.18E-04	2	0.0006
4	3.02E-04	4	0.0012
5	2.69E-04	2	0.0005
6	2.16E-04	4	0.0009
7	1.46E-04	2	0.0003
8	6.71E-05	4	0.0003
9	0.00E+00	1	0.0000
			4.40E-05

Blade Vol. 4.40E-05 m^3 (1 blade)

Density 8320 Kg / m^3 (Manganese Bronze)

Mass 0.3664 Kg (1 blade)

Total Mass 0.375562 Kg (accounting for Fillet Mass)

A_x Blade Sectional Area
r_0 Blade Section Radial Position
S.M Simpson's Multiplier

Longitudinal Position of the Blade Centroid (X_c)

Station	X	A_x mm^2	S.M	A_x * S.M	A_x * X	A_x * X * S.M
1	0.2	276.0935	1	276.0935	55.2187	55.2187
2	0.3	300.6658	4	1202.6633	90.1997	360.7990
3	0.4	318.0597	2	636.1194	127.2239	254.4478
4	0.5	301.9773	4	1207.9091	150.9886	603.9545
5	0.6	269.2947	2	538.5894	161.5768	323.1536
6	0.7	215.6980	4	862.7922	150.9886	603.9545
7	0.8	146.0535	2	292.1069	116.8428	233.6855
8	0.9	67.0907	4	268.3629	60.3816	241.5266
9	1.0	0.0000	1	0.0000	0.0000	0.0000
				176.1546		89.2247

X_c = 0.5065

Sectional Area (A_X) Calculation

X = 0.2

Station	x mm	t mm	S.M	t * S.M
1	0.00	0.0000	1	0.0000
2	2.50	3.1400	4	12.5600
3	5.00	4.3250	2	8.6500
4	7.50	5.1736	4	20.6944
5	10.00	5.8339	2	11.6678
6	12.50	6.3597	4	25.4388
7	15.00	6.7754	2	13.5508
8	17.50	7.0932	4	28.3728
9	20.00	7.3189	2	14.6378
10	22.50	7.4545	4	29.8180
11	25.00	7.5000	2	15.0000
12	27.50	7.4514	4	29.8056
13	30.00	7.2946	2	14.5892
14	32.50	7.0129	4	28.0516
15	35.00	6.5898	2	13.1796
16	37.50	6.0086	4	24.0344
17	40.00	5.2527	2	10.5054
18	42.50	4.3055	4	17.2220
19	45.00	3.1504	2	6.3008
20	47.50	1.7708	4	7.0832
21	50.00	0.1500	1	0.1500
			A_X	276.0935 mm^2

X = 0.3

Station	x mm	t mm	S.M	t * S.M
1	0.00	0.0000	1	0.0000
2	2.75	3.1086	4	12.4344
3	5.50	4.2818	2	8.5635
4	8.25	5.1219	4	20.4875
5	11.00	5.7756	2	11.5511
6	13.75	6.2961	4	25.1844
7	16.50	6.7076	2	13.4153
8	19.25	7.0223	4	28.0891
9	22.00	7.2457	2	14.4914
10	24.75	7.3800	4	29.5198
11	27.50	7.4250	2	14.8500
12	30.25	7.3769	4	29.5075
13	33.00	7.2217	2	14.4433
14	35.75	6.9428	4	27.7711
15	38.50	6.5239	2	13.0478
16	41.25	5.9485	4	23.7941
17	44.00	5.2002	2	10.4003
18	46.75	4.2624	4	17.0498
19	49.50	3.1189	2	6.2378
20	52.25	1.7531	4	7.0124
21	55.00	0.1485	1	0.1485
			A_X	300.6658 mm^2

Sectional Area (A_X) Calculation

X = 0.4

Station	x mm	t mm	S.M	t * S.M
1	0.00	0.0000	1	0.0000
2	3.00	3.0144	4	12.0576
3	6.00	4.1520	2	8.3040
4	9.00	4.9667	4	19.8666
5	12.00	5.6005	2	11.2011
6	15.00	6.1053	4	24.4212
7	18.00	6.5044	2	13.0088
8	21.00	6.8095	4	27.2379
9	24.00	7.0261	2	14.0523
10	27.00	7.1563	4	28.6253
11	30.00	7.2000	2	14.4000
12	33.00	7.1533	4	28.6134
13	36.00	7.0028	2	14.0056
14	39.00	6.7324	4	26.9295
15	42.00	6.3262	2	12.6524
16	45.00	5.7683	4	23.0730
17	48.00	5.0426	2	10.0852
18	51.00	4.1333	4	16.5331
19	54.00	3.0244	2	6.0488
20	57.00	1.7000	4	6.7999
21	60.00	0.1440	1	0.1440

A_X 318.0597 mm^2

X = 0.5

Station	x mm	t mm	S.M	t * S.M
1	0.0000	0.0000	1	0.0000
2	3.1250	2.7475	4	10.9900
3	6.2500	3.7844	2	7.5688
4	9.3750	4.5269	4	18.1076
5	12.5000	5.1047	2	10.2093
6	15.6250	5.5647	4	22.2590
7	18.7500	5.9285	2	11.8570
8	21.8750	6.2066	4	24.8262
9	25.0000	6.4040	2	12.8081
10	28.1250	6.5227	4	26.0908
11	31.2500	6.5625	2	13.1250
12	34.3750	6.5200	4	26.0799
13	37.5000	6.3828	2	12.7656
14	40.6250	6.1363	4	24.5452
15	43.7500	5.7661	2	11.5322
16	46.8750	5.2575	4	21.0301
17	50.0000	4.5961	2	9.1922
18	53.1250	3.7673	4	15.0693
19	56.2500	2.7566	2	5.5132
20	59.3750	1.5495	4	6.1978
21	62.5000	0.1313	1	0.1313

A_X 301.9773 mm^2

Sectional Area (A_X) Calculation

X = 0.6

Station	x mm	t mm	S.M	t * S.M
1	0.0000	0.0000	1	0.0000
2	3.1875	2.4021	4	9.6084
3	6.3750	3.3086	2	6.6173
4	9.5625	3.9578	4	15.8312
5	12.7500	4.4629	2	8.9259
6	15.9375	4.8652	4	19.4607
7	19.1250	5.1832	2	10.3664
8	22.3125	5.4263	4	21.7052
9	25.5000	5.5990	2	11.1979
10	28.6875	5.7027	4	22.8108
11	31.8750	5.7375	2	11.4750
12	35.0625	5.7003	4	22.8013
13	38.2500	5.5804	2	11.1607
14	41.4375	5.3649	4	21.4595
15	44.6250	5.0412	2	10.0824
16	47.8125	4.5966	4	18.3863
17	51.0000	4.0183	2	8.0366
18	54.1875	3.2937	4	13.1748
19	57.3750	2.4101	2	4.8201
20	60.5625	1.3547	4	5.4186
21	63.7500	0.1148	1	0.1148
			A_X	269.2947 mm^2

X = 0.7

Station	x mm	t mm	S.M	t * S.M
1	0.0000	0.0000	1	0.0000
2	3.1250	1.9625	4	7.8500
3	6.2500	2.7031	2	5.4063
4	9.3750	3.2335	4	12.9340
5	12.5000	3.6462	2	7.2924
6	15.6250	3.9748	4	15.8993
7	18.7500	4.2346	2	8.4693
8	21.8750	4.4333	4	17.7330
9	25.0000	4.5743	2	9.1486
10	28.1250	4.6591	4	18.6363
11	31.2500	4.6875	2	9.3750
12	34.3750	4.6571	4	18.6285
13	37.5000	4.5591	2	9.1183
14	40.6250	4.3831	4	17.5323
15	43.7500	4.1186	2	8.2373
16	46.8750	3.7554	4	15.0215
17	50.0000	3.2829	2	6.5659
18	53.1250	2.6909	4	10.7638
19	56.2500	1.9690	2	3.9380
20	59.3750	1.1068	4	4.4270
21	62.5000	0.0938	1	0.0938
			A_X	215.6980 mm^2

Sectional Area (A_X) Calculation

X = 0.8

Station	x mm	t mm	S.M	t * S.M
1	0.0000	0.0000	1	0.0000
2	2.8750	1.4444	4	5.7776
3	5.7500	1.9895	2	3.9790
4	8.6250	2.3799	4	9.5194
5	11.5000	2.6836	2	5.3672
6	14.3750	2.9255	4	11.7018
7	17.2500	3.1167	2	6.2334
8	20.1250	3.2629	4	13.0515
9	23.0000	3.3667	2	6.7334
10	25.8750	3.4291	4	13.7163
11	28.7500	3.4500	2	6.9000
12	31.6250	3.4276	4	13.7106
13	34.5000	3.3555	2	6.7110
14	37.3750	3.2259	4	12.9037
15	40.2500	3.0313	2	6.0626
16	43.1250	2.7640	4	11.0558
17	46.0000	2.4162	2	4.8325
18	48.8750	1.9805	4	7.9221
19	51.7500	1.4492	2	2.8984
20	54.6250	0.8146	4	3.2583
21	57.5000	0.0690	1	0.0690
			A_X	146.0535 mm^2

X = 0.9

Station	x mm	t mm	S.M	t * S.M
1	0.00	0.0000	1	0.0000
2	2.25	0.8478	4	3.3912
3	4.50	1.1678	2	2.3355
4	6.75	1.3969	4	5.5875
5	9.00	1.5752	2	3.1503
6	11.25	1.7171	4	6.8685
7	13.50	1.8294	2	3.6587
8	15.75	1.9152	4	7.6607
9	18.00	1.9761	2	3.9522
10	20.25	2.0127	4	8.0509
11	22.50	2.0250	2	4.0500
12	24.75	2.0119	4	8.0475
13	27.00	1.9695	2	3.9391
14	29.25	1.8935	4	7.5739
15	31.50	1.7792	2	3.5585
16	33.75	1.6223	4	6.4893
17	36.00	1.4182	2	2.8365
18	38.25	1.1625	4	4.6499
19	40.50	0.8506	2	1.7012
20	42.75	0.4781	4	1.9125
21	45.00	0.0405	1	0.0405
			A_X	67.0907 mm^2