A REPORT ON A UNIT-DESIGNED TO DEVELOP RAPID AND ACCURATE CALCULATIONS IN HIGH SCHOOL MATHEMATICS

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ABSTRACT

The purpose of this paper was to develop a unit on rapid and accurate calculations for use in the high school mathematics program, to assess its suitability within the existing program, and to evaluate the benefits to be derived from it. In order to do this, the experimenter considered four questions:

1. Can students attain competence with mathematical principles used in short-cut methods of thinking in computation?

2. Will students become more agile in handling mental computation as a result of this unit?

3. What is the effect of the unit on student attitudes?

4. What are the attitudes of teachers towards the experimental materials?

The study was essentially a non-comparative one, in that no control group was used. The study consisted of a seventeen-lesson unit taught to 218 Grade X mathematics students enrolled in six classes at Prince of Wales Collegiate High School. The materials for the unit were developed by the experimenter.

To determine the students' achievement in the unit, experimenter-made tests were administered.

The Connelly Taxonomized Attitude Questionnaire was given as a pretest and as a posttest in order to determine the effect of the unit on student attitudes towards mathematics. Teacher records were also assessed to determine the students' attitudes to the materials in the

unit. To determine how the teachers felt towards the materials, an experimenter-made questionnaire was given to each of the five teachers who taught the experimental materials.

Analysis of the test results and subsequent oral questioning of the students showed that many of them failed to achieve mastery of the short-cuts presented. A dependent t-test for means was performed on the pretest-posttest attitude scores. A t-value of -1.5827 indicated that there was no significant change in the attitudes of students towards mathematics at the .10 level of significance during the teaching of the experimental materials.

Responses to the questionnaire indicated that teachers were favourable towards the materials in the unit. They recommended the materials to other teachers, and they felt the materials had value for both the terminating and Honours students in mathematics as well as for the average-ability students. They further strongly recommended that this material be included at all grade levels.

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CHAPTER I

INTRODUCTION

Background

The changes in the mathematics curriculum during the decade of the 1960's were so extensive and so far-reaching that they can best be described as a revolution in the teaching of mathematics.

Studies such as that of Brownell (1947) were the impetus for many research studies leading to the adoption of programs designed to promote understanding of the structure of our number system. The result has been a program which has been referred to in general as the "New Mathematics Program."

As the decade of the 1960's came to a close, children who had been introduced to the new mathematics program in kindergarten and first grade were now in junior and senior high school. This became a period when careful evaluation was needed. How effective had the new mathematics program been in maintaining a balance between the new concepts and understandings and the competence needed in computation?

There have been questions raised concerning the level of computational skills attained by these students. Concern has also been expressed that because of the emphasis the new courses place on ideas and concepts, little time has been left for the development of the fundamental skills in arithmetic.

Computation is a vital part of mathematics. Developing concepts at the expense of developing computational skills is not a satisfactory approach. It is through computation that a student may first develop some insights into mathematics and learn to function as a knowledgeable citizen. While it is undoubtedly true that no one can function without a clear understanding of concepts, it is a fallacy to maintain that concepts alone provide the student with mathematical facility. If the child is to become facile in handling number situations that he meets in everyday experiences, he must be provided with instruction and practice to develop such skills. Attempts have been made to achieve understanding in arithmetic so that pupils will not arrive in upper grades as slaves to a routine of rules. However, if students are to be able to perform calculations rapidly and accurately, time should be allotted to mental arithmetic in the mathematics program.

Purpose of the Study

The purpose of this study was to develop a unit on rapid and accurate calculation for use in the grade-ten matriculation mathematics program. The unit included processes for learning short-cuts for adding, subtracting, multiplying, and dividing one- and two-digit numbers and fractions, without the aid of paper and pencil. The mental computation exercises provided opportunities for the utilization of meanings pupils had acquired as well as opportunities for the development of more mature understandings of basic principles and number relationships.

Statement of the Problem

In order to investigate the merits of the inclusion of a unit in computation as an integral part of the mathematics program, the experimenter considered the following questions:

1. Can students attain competence with the mathematical principles used in short-cut methods of thinking in computation?

2. Do learners exposed to short, frequent periods of mental arithmetic become adept at handling two-digit calculation without the use of pencil and paper?

3. What is the effect of the unit on student attitudes toward mathematics?

4. What are the attitudes of teachers toward the materials in the unit?

CHAPTER II

REVIEW OF THE LITERATURE

The revolution in school mathematics was necessitated primarily because of a lack of understanding of concepts on the part of mathematics students. Concern is now being expressed that there may have been a tendency to emphasize concept building at the expense of mathematical skills in many classroom situations. However, computation is a vital part of mathematics. Developing concepts at the expense of developing computation skills is not a satisfactory approach.

It is the opinion of the investigator that computational skills must be present, honed and polished like an artisan's best tools, if the student is to be able to apply concepts to problem solving or any other mathematical endeavour. This is a concern that has been expressed by educators and non-educators alike.

Depth of Concern

This topic is not just a subject for intellectual or pedagogical debate. Widespread public concern has been expressed about the apparent sacrifice of computational skills on the altar of the development of concepts.

This concern about students' lack of computational skills is not new: it is not an effect of the introduction of the "New Mathematics." Two decades before the new mathematics was widely implemented, the educator Lyda (1947) expressed his concern that arithmetic skills possessed by students of average and above-average ability were distressingly low. In order to ascertain why the scores were so low, Lyda randomly selected a small sample of seventh, eighth, ninth, tenth, and eleventh grade students, and closely analyzed both the mental processes and the written series of operations they had employed. The following arithmetic inabilities and shortcomings were noted: inability to analyze a problem; inability to outline a method of attack; manipulation of figures without understanding; absence of a check on reasonableness of answers; inability to perform accurately operations using whole numbers, fractions, and decimals; inability to reduce fractions to lowest terms and to find what fractional part one number is of another.

The public, as well as pedagogues, is concerned with this problem, and this concern is not abating now that the new mathematics is well established. The question of "Why Johnny Can't Add" has received much attention. Articles in newspapers have called attention to a lack of computational ability as measured by achievement tests among children instructed in modern mathematics programs. Writing in The Wall Street Journal, Martin (1973) said that many of these kids can't add, subtract, multiply, or divide. The Bergen Record published Walcott's (1973) statement that what was lost sight of was the need for children to learn mathematically how to add, subtract, multiply, and divide.

Replies have been offered to these criticisms. The suggestion has been made that the supposed lack of computational skills is more apparent than real. Many teachers put forth the thesis that the studies on which these dismal conclusions rest are not valid. They suggest that the students can add, subtract, multiply, and divide well. They put forward for consideration the possibility that there are circumstances operating in these tests that are reducing the scores. Are the results of these tests in fact answering the question, "Can students in the new mathematics really compute?"

Leonard J. Garigliano (1975) did a thorough analysis on the type of test whose results evoked such distress as that expressed in The Wall Street Journal and the Bergen Record. He concluded that there well might be no foundation in fact for the dismay which the press had expressed. Garigliano first suggested that the apparently dreadful results were more the result of the time of testing than of a real lack of students' ability. The tests were administered in October, shortly after the students returned from a long summer vacation. He also pointed out that the mechanics of the test taken were designed to test speed rather than real ability. In other words, the test was a speed rather than a power test. Garigliano further cautioned that the construction and norming of this test were suspect.

One factor that the press and concerned public opinion has not yet become aware of is that some tests indicate that students in the new mathematics can compute as well as people who went through the old system. During the 1972-73 academic year, the National Assessment of Educational Progress conducted its first assessment in mathematics. These results were summarized in The Mathematics Teacher (October, 1975). Representative national samples of nine year olds, thirteen year olds, seventeen year olds, and adults between the ages of twenty-six and thirty-five in the United States were assessed to determine their levels of attainment in mathematical concepts and skills. For an exercise to be included in this assessment it had to be related to an educational objective considered important by mathematicians and laymen, and accepted by mathematics educators as a desirable teaching goal in most schools.

The results showed that the thirteen-year-olds could do about as well as adults on most computational tasks, and the seventeen-year-olds could do better. If the criticism of the new mathematics program were correct, we would expect the reverse.

Despite these findings, the report of the National Assessment of Educational Progress makes it very clear that improvements are needed. The sample population showed weaknesses in percents, decimals, fractions, problem-solving abilities, as well as in other areas.

Relevance in the Computer Age

It is difficult, if not impossible, to assess the importance of criticism unless one has a clear idea of how important the subject under discussion is. Medieval philosophers debated hotly such issues as "How many angels can sit on the head of a pin?" Argument and counterargument flew, while tempers became short and blood pressures soared. Today we view this uproar as ludicrous.

The twentieth century has been described as "The Age of the Computer." The debate on the need for proficiency in computational skills has been considered irrelevant. It has been stated that computers will eradicate students' need for a high level of computational skills. The issue with which this paper is concerned would appear as ridiculous as the subject of the ancient philosophers' discussions if mathematicians agreed with these views. However, many mathematics educators feel that the twentieth century human being needs to attain a reasonable degree of computational skill in order to live successfully in our Western culture.

This is not to deny that the impact of the computer has been enormous, and this paper now examines its effect on the subject.

Soon everyone who faces an arithmetic problem will be able to call on a low-cost electronic calculator as an aid. This development has led some people to question the high instructional and testing priority currently assigned to speed and accuracy in arithmetic computation.

The Mathematics Editorial Panel of the National Council of Teachers of Mathematics posed seven issues directly related to this question to a sample of teachers, mathematicians, and laymen. The results as reported by Carpenter et al. (1975) are summarized below:

1. Sixty-eight percent agreed that speed and accuracy in arithmetic computation are still major goals of elementary and junior high school teaching today.

2. Eighty-four percent agreed that speed and accuracy in arithmetic computation are still essential for a large segment of business and industrial workers, and intelligent consumers.

3. Forty-eight percent agreed that the pending adoption of metric measurement implies that computation with rational numbers should place emphasis on decimal fractions.

4. Forty-eight percent agreed that in the face of declining arithmetic computation test scores, the energies of mathematics instruction should be concentrated on these skills until achievement reaches the level of mastery.

5. Sixty-one percent agreed that weakness in computational skills acts as a significant barrier to the learning of mathematics theory and its application. They felt that it is through arithmetical

example that the student gets the feel of what theory and application are about.

6. Twenty-eight percent agreed that every seventh grade mathematics student should be provided with an electronic calculator for his personal use through secondary school. However, they felt caution should be exercised to ensure that the student does not become too dependent on it.

7. Ninety-six percent agreed that availability of calculators will permit treatment of more realistic applications of mathematics, thereby increasing student motivation.

There is, then, a place for the electronic calculator in the teaching of mathematics. Its best role, however, lies in facilitating the development of computational skills, not in replacing them.

Kenneth J. Travers (1969) enunciated the thesis that lack of computational skills can be a real stumbling block for low achievers. Travers feels that electronic calculators can play a large part in ameliorating this difficulty.

Travers claims that never before have so many students been deprived of so much mathematics because of the computation barrier. Deliberate attempts will continually have to be made to de-emphasize computation where it is a barrier and to re-emphasize the "big ideas" for all students, not only those select few who have always been fed on the cream of the curriculum. Available instructional materials are geared to the more-able students who usually experience little trouble with computation. But less-able students should be able to profit from the school mathematics revolution and the use of calculators.

Gaslin (1975) showed in his study done in 1971 that the calculator provides a means by which low-ability students can compute with rational numbers. The ninth-grade students used in that study exhibited transfer of the skill obtained by operating on positive rational numbers to ordering rational numbers and combining rational numbers involving more than one operation.

Gaslin supports Travers' viewpoint that the computer will open the doors to mathematical endeavours that would otherwise be closed to lower-ability students. He stated that the use of calculators would allow some topics to be included in the curriculum for slow students which would otherwise have to be dropped because of the arduous computation involved. Some examples are finding area, volume, ratio-proportion, and evaluation of polynomial expressions.

The benefits of utilizing calculators in a mathematics program, then, are significant. Travers (1969) concludes with the observation that once the calculator has set up a receptivity in the students, the teacher can and should go back to teach computation skills to the students who understand the concepts to which these skills apply.

Teachers Want New Mathematics Program to Re-emphasize Computational Skills: The Pendulum

Bell (1974) gives specific directions for mathematical literacy and competence for our future citizens. He enunciates a short and limited list of what is really vital as a minimum residue for "Everyman" from his school mathematics experience. This list includes: the efficient and informed use of computation algorithms; confident, ready, and informed use of estimates and approximations, including such things as number sense, rapid and accurate calculations with one- or two-digit numbers, and approximate calculation via positive and negative powers of ten.

All citizens have basic mathematical needs, as Bell states. A committee was established in 1970 by the National Council of Teachers of Mathematics. It was asked to set up guidelines for the mathematical needs of all people. It recognized that there are three aspects to any mathematics program: the aspect designed for students who will study mathematics as a discipline; the aspect designed for students who will use mathematics as an important tool in their post-secondary education; and the aspect designed for those students who need mathematics for effective citizenship and personal living. However, this committee narrowed its terms of reference to the minimum, doing skills needed by the enlightened citizen.

The committee's conclusions correlated closely with Bell's. It felt that the following proficiencies were vital: facility with numbers and numerals such as expressing a rational number using decimal notation, and representing very large and very small numbers using scientific notation; operations and properties using rational numbers; solving problems involving percent and estimating results; and computing perimeters of polygons, areas of rectangles, triangles, and circles, as well as measurement, probability and statistics.

Long and Heir (1973) surveyed the importance which a random sample of 260 vocational instructors placed on 66 basic mathematical skills. These 66 skills were obtained from standard mathematical texts or remedial texts, and were reviewed for appropriateness by selected teacher educators.

Fifty-five of the 66 skills had half or more than half of the teachers indicating that they were essential. Each skill was deemed essential by at least 25 percent of the sample and 18 of the skills were deemed essential by 75 percent. The skills most essential to vocational education concerns include the four fundamental operations with whole numbers, basic uses of fractions and decimals, rule reading, and rounding numbers. Fifty-one percent of the skills were identified by more than 25 percent of the teachers as requiring remedial attention.

Hosford (1973) maintains that the modern mathematics program used in schools beginning in the late 1950's and early 1960's, emphasized the discipline of mathematics and its structure and beauty. But, the discipline of mathematics has another component that needs and deserves similar attention: the skills of computation.

Hosford quotes three small studies which show that teachers are ready for a re-emphasis and open valuing of computational skills on the part of all students. Hosford advocates a "Right-to-Figure" movement. This is based on the belief that computational skills must be developed with or without many of the understandings and concepts usually associated with modern mathematics; preferably with, but if necessary, alone.

Hosford (1973) made use of 34 written performance objectives for mathematics programs developed by the State Department of Education in New Mexico for the public schools. Each mathematics teacher participating in a three-day state-wide workshop was asked to consider himself as though he were a member of a curriculum committee charged with creating a final course in high school mathematics for those students planning the minimum possible course work in mathematics. Results showed that the highest ratings were consistently given by the teachers to those objectives dealing with the four fundamental operations on natural and rational numbers, including decimals and percents. Just as noticeable and significant were the topics receiving no positive evaluation: five of the six geometry topics; probability and statistics; and anything to do with sets or set language. The course developed for all students by these teachers would be aimed almost entirely at the skills aspect.

In his second study, Hosford (1973) turned his attention to elementary school teachers of mathematics. A list of 27 common objectives for all students completing the sixth grade was synthesized from several sources. Twenty-eight teachers individually sorted the objectives according to the valued importance of achievement by the end of grade six. The most important objectives found by these teachers were ranked as follows: basic factors of the four operations; operations with whole numbers; solving word problems; operations with fractions and mixed numbers; and rounding and place value. Here, again, the study showed that teachers felt computational skills made up the most important aspect of the mathematics program.

The two studies cited above looked at mathematics for all students. Hosford (1973) also examined mathematics for the talented high school student. This question was attacked by asking what course should be offered to such students in their final year. Thirty-five mathematics teachers at the same state-wide conference in New Mexico, referred to earlier, were asked this question. The results of this examination were not so definitive as those of Hosford's two studies mentioned previously. The student's right to select what to study received almost half of the first-choice votes. However, in subsequent discussion with teachers, self-directed study was interpreted as a study that would center around an examination of the many processes, materials, and curiosities of computation.

The New Mathematics: A Change for the Better

Testing has shown consistently that the change to the new mathematics has been a change for the better. Beckman (1969) showed that students enrolled in the New Math program had gained an entire year on students who had come through the old system.

He based his testing on mathematical literacy as defined by the National Council of Teachers of Mathematics Commission on Post-War Plans (1940). This group defined mathematical literacy as 29 competencies, including: computation with whole numbers, common fractions; decimals; percents; and signed numbers. Beckman administered a test based on these criteria in 1950 to a group of students who were, of course, educated in the traditional mathematics program. He gave the same test in 1965 to students who were enrolled in the new mathematics. Comparison of the results showed the mathematical competencies of the 1965 students were as good in the fall of the ninth grade as they had been in the spring of the ninth grade for the students tested in 1950. In other words, students had progressed almost a full year under the new system.

Skills and Concepts not Mutually Exclusive

As we have seen, there are many strengths to the new mathematics program. One of its best points is the undisputed improvement in clarity of concepts which students are now acquiring. Most of the controversy surrounding the new mathematics centers on a lack of facility in computational skills. Many studies have shown that mathematics teaching is most successful where there is a satisfactory blend of theory and practice.

Not all people agree on the specific role of the understanding of concepts as a part of a broad instructional program. There is disagreement concerning the temporal relationship between the "how" of a computational process and its "why." Should theory come before practice, or should practice come first, with theory evolving from it?

Weaver (1950) addressed this problem. He maintained that as a result of meaningful experiences many children frequently discover and formulate algorithms themselves. On the other hand, when learning is mechanical or authoritative, transfer of learning is inhibited.

Weaver's argument supports the conclusion of Brownell and Mosher (1949). A study in third grade subtraction showed that when subtraction using two-digit numbers was clearly understood and rationalized by children, there was more significant transfer of skills and concepts to subtraction with three-digit numbers than there was when the initial instruction was by rote learning. Subsequent sub-skills in borrowing were learned and taught more easily and effectively when the initial instructional experiences were rational and meaningful.

Therefore, it is clear that informed opinion maintains that clear concepts should be developed before any stress and emphasis is placed on computational skills. Nonetheless, a sound mathematics program must provide adequate time and opportunity for the development of these skills.

A study by Miller (1970) showed the superiority of a modified traditional program over a modern program in arithmetic. His work was based on 137 students in three classes of college freshmen. The modified traditional group was given materials developed by the author which included many of the concepts of modern texts supplemented by a large number of problems for practice. The modern group used a modern textbook, without supplements. Both groups were tested on four areas: addition and subtraction; multiplication and division; fractions; and percentages. In all four areas the modified traditional group was superior to the modern group. One of the important factors in the increased gains of the modified traditional group was the greater number of examples of the basic concepts and important principles compared to a limited number of examples offered the modern group.

Daily homework assignments of a reinforcing nature are a significant factor in raising the achievement level of learning in the area of arithmetic computation. A study by Koch (1965) involving sixth-grade students, showed that homework was of value in increasing students' skills and abilities. Three classes were given the same arithmetic development, but were required to do homework for different periods of time. The first group was given a homework assignment designed to take 30 minutes. The second group was given a homework assignment similar in content, but shortened to require only 15 minutes. The third group was given no homework assignment at all. The results showed that homework does lead to increased achievement in computation. However, this homework must be of reasonable length. The group with only 15 minutes of homework showed little improvement over the non-homework group. The gains were made by the 30-minute homework group. Koch freely admits that his study is somewhat compromised by his small sample and the teacher variables. However, he feels his findings are nonetheless valid.

Koch's study points out that adequate time is needed for developing and practising computational skills. The question now arises: Will providing this time in the mathematics program take away from time needed for developing concepts? Ashlock and Herman (1970) report a study by the Cincinnati Public School System which indicates the answer is no.

Three groups were used in the Ashlock and Herman study. The first was the control group which had no remediation. The second and third groups received remedial work during class time. The second group's remedial work was programmed instruction, based on individual needs as previously diagnosed. The third group received remedial work during the first five to ten minutes of class time. The method of remediation used here was a study exercise technique presented by the teacher.

The Ashlock and Herman study showed that time taken from regular class for remediation not only increased the computational skills of these groups, but did so without any loss in the developing of concepts and reasoning ability.

A study by Schall (1973) supports this finding. He took 399 fifth-graders from 14 classrooms and worked with them using sequences in mental arithmetic from a programmed booklet. The students grew in ability to do mental arithmetic and their attitudes improved. Although time was taken from the arithmetic class period, no significant differences were found between the experimental group and control groups in performance on a standardized arithmetic achievement test.

Why be Concerned with Computation Skills in High School

Computation is introduced to students in elementary school. One may well ask why the high school mathematics program should be concerned with it. We must remember that students forget techniques unless they are practiced.

Suppes and Ihrke (1967) made a most important discovery in their longitudinal project of teaching an accelerated program in elementary school mathematics to gifted children. The amount of forgetting that developed in relation to basic skills was so great that they devised a series of daily drills of 20 examples to help these students retain their basic arithmetical skills, even though the main curriculum work consisted of extensive supplementary material. Such practice can be assumed to be even more important with less-talented students.

Again, this principle is even more vital in high school, where teachers are so involved with presenting the material prescribed by the curriculum that they tend to neglect to recapitulate basic skills. Lyda (1947) capsuled this point when he recommended that every mathematics teacher should consider himself a teacher of arithmetic. Unless students review, they forget.

Review and drill acquired a bad name in the traditional mathematics program because too often drill preceded and even replaced the development of concepts. Ausubel (1965) points out that the role of drill in educational theory tends to be minimized because it is regarded as rote, mechanical, passive, old-fashioned, and psychologically unnecessary for the learning process. On the other hand, it is well established that learning increases in proportion to repetition of practice. The importance of over learning for long-term retention is accepted by educators. Ausubel states that practice can result in meaningful mastery if the learner possesses the necessary background concepts.

Bruner's (1969) theory of learning is predicated on the image of the spiral curriculum. The concepts developed in elementary school form the structure on which high school teachers will build. The high school teacher must spiral back, review, and drill these concepts and skills, and then build and develop them. The words 'review' and 'drill' are used here in the contexts Hoover (1970) defined. By 'review' he meant a more basic function of developing new associations and relationships from previously learned concepts. 'Drill' means to extend or polish skill learning. This broadening of basic learnings not only increases the likelihood of varied applications but, as Bruner points out, these concepts provide a basic structure which prevents memory loss from blocking students' recall of necessary details when they are needed. Because studies have shown that concepts and principles are mastered more by pupils in new mathematics than they were by students in the traditional mathematics programs, it would not now be an onerous job for the high school teacher to review and drill.

In selecting learning materials for high school grades we cannot afford to ignore the newer approach based on patterns and structures. Neither can we ignore the demand for increased competencies in mathematical skills necessary for effective functioning in today's society.

Much has been done in recent years to strengthen the arithmetic curriculum on the elementary level. Attempts have been made to achieve understanding in arithmetic so that pupils will not arrive in the upper grades slaves to a routine of rules and manipulations which have no real meaning for them. Also the principle of social utility, widely accepted as a basis of curriculum construction, holds that the curriculum should include experiences which are similar to and which prepare the child for, dealing with problems as they are really met in everyday activities. In everyday experiences the child meets situations that require facility in computation, and unless the child is provided with instruction and practice to develop such skill he is not very likely to become facile in handling number situations mentally.

Many educators have stated that mental arithmetic can accomplish the two-fold aim of instruction described above: increased computational skills and adequate attention to concepts. Gane (1962) stated that a regular and carefully planned program of mental arithmetic provides a realistic preparation for the everyday use of arithmetic which the child encounters out of school. Gane goes on to point out that improvement in mental arithmetic tends to improve ability in all phases of arithmetic. Beberman (1959) stated that mental arithmetic is one of the best ways of helping children become independent of techniques which are usually learned by strict memorization. It encourages children to discover computational short-cuts and thus gain deeper insights into the number system.

Many studies have supported these statements. Flourney (1959) studied methods used by pupils in grade six in performing each of the four fundamental processes without the use of paper and pencil. The more than 150 pupils wrote explanations of their ways of thinking before and after instruction in mental arithmetic procedures. Upon comparison, it was found that before instruction, 85 percent of the pupils used the same procedure to solve the problem mentally as they would if they were working with paper and pencil. After instruction in "short-cut" methods of thinking, there was a widespread change to the use of other ways of thinking when not using paper and pencil. A significant gain in mean score was noted on a mental arithmetic test which had been administered before instruction began and after the instruction in varied ways of thinking arithmetically had been given. The same investigator (1954) concluded that a sample of intermediate grade pupils who had finished a program in mental computation showed significant growth in mental arithmetic and in problem solving. A study by Payne (1965) showed that experimental classes which had 20 minutes of mental activities three days a week during their regular arithmetic period, improved their accuracy and increased their speed in mental computation. Test results showed that there was no decrease in performance on a standardized paper-and-pencil arithmetic achievement test when compared to the control group. This is significant because time was taken from the arithmetic period of the experimental pupils for special instruction in mental computation.

A study by Schall (1973) shows that short exercises in mental arithmetic do seem to be a worthwhile addition to the traditional pencil-and-paper oriented mathematics classroom. Students apparently enjoy mental arithmetic as the attitude score for all mental arithmetic groups was elevated.

It is generally recognized that mental arithmetic must be built upon a thorough understanding and familiarity with the basic mathematical principles and relationships which govern the intelligent use of any mathematics. Vaughin (1957) states that in addition to being an interest-catching device, mental arithmetic has great value in helping students learn the fundamental combinations used in the operations of addition, subtraction, multiplication, and division.

Considering its importance and usefulness there seems to be too little space and time allotted to it in the curriculum. It must not be something incidental to the arithmetic program, but should be incorporated as an integral part of it because of its important function of improving and solidifying understandings of numbers and number relationships, and giving opportunities for practice in applying these relationships. A program which includes mental arithmetic can reasonably be expected to contribute to an understanding of mathematics and to help build desirable attitudes toward arithmetic and its applications.

Need for and Significance of the Study

In recent attempts to identify the mathematical needs of "Everyman," mathematics educators have emphasized the importance of computational skill. Bell (1974) gives specific directions for mathematical literacy and competence for our future citizens. He enunciates a short and limited list of what is really vital as a minimum residue for "Everyman" from his school mathematics experience. This list includes such things as number sense and rapid and accurate calculation with one- or two-digit numbers. A committee was established in 1970 by the National Council of Teachers of Mathematics to set up guidelines for the mathematical needs of all people. Their conclusions correlated closely with Bell's.

Flourney (1959) points out that most uses of mathematics appear not to require paper and pencil and that we ought to be focusing more instruction on this need. Even in an age when most students have access to a pocket calculator, there should be no doubt of the importance of skills. In a recent survey of teachers, mathematicians, and laymen conducted by the editorial panel of The Mathematics Teacher (October, 1975) 68 percent agreed that speed and accuracy are major goals of mathematics education.

The lack of computational skills has been a subject of much concern in the past couple of years. There is conflicting evidence as to whether students are worse at computation than they used to be. But one thing is certain: students are weaker than they ought to be. It seems highly probable that pupils in secondary schools are capable of becoming adept in handling everyday activities. However, they are likely to perform much below their level of ability in these situations unless schools provide them with such arithmetic experiences.

Every teacher of mathematics should consider himself a teacher of arithmetic. The development and verification of computational short-cuts based on the mathematical properties of numbers should prove not only beneficial to students but exciting to them. To a great extent it can remove computational barriers so students can begin to enjoy the underlying mathematics.

CHAPTER III

PROCEDURES

Introduction

The study was essentially a non-comparative one, in that no control group was used. The study consisted of a 17-lesson unit to 218 grade ten mathematics students enrolled in six classes at Prince of Wales Collegiate High School. The materials for the unit were developed by the experimenter. In determining the students' achievement in the unit, experimenter-made tests were administered.

The Connelly Taxonomized Attitude Questionnaire was given as a pretest and as a posttest in order to determine the effect of the unit on student attitudes towards mathematics. Teacher records were also assessed to determine the students' attitudes to the materials in the unit.

To determine how the teachers felt towards the materials, an experimenter-made questionnaire was given to each of the five teachers who taught the experimental materials.

Definition of Terms

Computation -- This term refers to the ability to perform the fundamental operations of addition, subtraction, multiplication, and division with two-digit whole numbers and fractions.

Achievement -- This term refers to ability to calculate mentally.

Level-Two Mathematics -- This refers to an academic program with materials considered to be appropriate for a wide range of pupil ability. It is recognized for post-secondary or technical programs.

Honours Program -- A program for the best mathematics students.

Scope and Limitations

This study was primarily concerned with the development of a unit to increase student ability to calculate mentally. In assessing the merits of such a unit, the study was essentially a non-comparative one in that no control group was used. Nevertheless the experimenter established certain procedures to help insure the validity of the findings. For example in answering question (1): Can students attain competence with the mathematical principles used in short-cut methods of thinking in computation?, extensive in-depth interviews were conducted with the students and teachers participating in this study. To answer question (2): Will students become more agile in handling mental computation as a result of the unit?, student quizzes were closely analyzed to establish whether their lost marks were the result of inaccuracies or the result of their failure to implement the short-cuts and consequent lack of time. In addition a representative sampling of students was questioned orally to establish to what degree they were employing the short-cuts.

The study was limited to approximately 200 grade-ten level-II students in six classes at Prince of Wales Collegiate, St. John's. This population included primarily students of average ability. There was a small percentage of below-average students. The unit was introduced to the students as a part of the mathematics program and taught by their regular mathematics teacher.

Instructional Unit

The unit which was taught to these six classes consisted of materials developed by the experimenter. The development of rapid and accurate calculation procedures were based on mathematical principles. The important mathematical laws and principles which underlie the intelligent use of mental arithmetic (written arithmetic as well) are the distributive principle, associative principle, commutative principle, and ratio idea.

Relationships for drill on multiplication facts were for the most part based on the distributive principle as illustrated in $5 \times 63 = 5 \times (60 + 3) = (5 \times 60) + (5 \times 3) = 300 + 15 = 315$. The associative principle was applied to addition and to multiplication. For example: $75 + 28 = 75 + (25 + 3) = (75 + 25) + 3 = 100 + 3 = 103$; $12 \times 25 = (3 \times 4) \times 25 = 3 \times (4 \times 25) = 3 \times 100 = 300$. The commutative principle was applied to addition and multiplication as follows: (a) $8 + 76 = 76 + 8$; (b) $5\frac{1}{2} \times 12 = 12 \times 5\frac{1}{2}$. This principle was used when convenience demanded the reversing of addends or factors. The ratio idea referred to the fact that both the numerator and the denominator of a fraction can be multiplied or divided by the same non-zero number without changing the value of the fractions. For example:

(a) $72 \div 18 = 8 \div 2$

(b) $10 \div 2\frac{1}{2} = 20 \div 5$

By extending the ratio idea, pupils were shown how to deal with near multiples of ten. For example $15 \times 19 = 15 \times (20 - 1) = (15 \times 20) - 15$. Further scope was added to the ratio idea in examples such as 24×25.

A possible solution was $24 \times 100 = 2400$ and $2400 \div 4 = 600$.

Subtraction was taught using the equal additions method. Students were led to see that $47 - 28$ could be calculated more rapidly if it was seen as $49 - 30$.

Because numbers that end in five are encountered frequently in daily living, the short-cut method for squaring two-digit numbers that end in five was introduced. The students were shown how to modify and adapt this short-cut. For example, they were led to see that 34×36 could be more efficiently dealt with if seen as $(35 - 1) \times (35 + 1)$.

The materials were used to form a 17-lesson unit taught over a period of ten weeks by the five volunteer teachers and the experimenter. The unit was presented in 15-minute lessons emphasizing computation without paper and pencil. Within a day or two following each lesson, the students were given a short five-to-ten-minute period to practice the skills introduced in the previous lesson or lessons. After completion, the correct answers were given by the teachers. The teachers were given instruction on the materials in the unit to familiarize them with the short-cuts employed.

Evaluation Procedures

In an attempt to answer the question (1): Can students attain competence with the mathematical principles used in the short-cut methods of thinking on computation?, the experimenter relied on teacher observation while presenting the unit. Teachers were asked to informally record the oral explanations of students as they explained how they arrived at their answers.

In attempting to answer the question (2): Do learners exposed to a short, frequent period of mental arithmetic become adept at handling two-digit calculations without the use of pencil and paper?, the experimenter constructed achievement tests which were used periodically throughout the unit. These tests were completed by the students without the aid of pencil-and-paper calculations and in a time limit that could only be attained by students who were facile with numbers. The time allotted was decided upon by giving a similar test to a sample of students who the teachers felt were highly profiting by the unit.

In order to answer the question (3): What is the effect of the unit on student attitudes?, the experimenter administered as a pretest and posttest, the Connelly Taxonomized Attitude Scale using Objective II items of that scale.

In an attempt to answer the question (4): What are the attitudes of teachers toward the experimental materials?, an experimenter-made questionnaire was administered at the end of the unit.

Since the experimenter was in daily contact with the five volunteer teachers, the informal observations and daily comments of these teachers were recorded throughout the presentation of the unit by the experimenter.

CHAPTER IV

ANALYSIS OF DATA

Student Achievement

The first question to be answered was whether students would attain competence with the mathematical principles used in the short-cut methods of thinking. In spite of an initial apparent familiarity with the mathematical principles which formed the basis of this unit, further work indicated that students did not have the mastery of these principles which their facility in talking about them would indicate. Students could verbalize easily and effortlessly the mathematical principles involved. Despite this evidence, teachers soon questioned how complete the students' understanding of these principles really was. There were two bases for this doubt: first, the students did not approach given problems with these principles in mind; second, the type of mistakes they made indicated strongly that they did not, in fact, really understand the principles. This was particularly true of the distributive property. For example, students did not know how to apply this property to multiplying 35 by 6. Even when rewritten as $(30 + 5) \times 6 = 180 + 30$, students failed to see how the partial sum of 30 was obtained.

It was interesting to note that students immediately used a pencil-and-paper approach to these operations without any critical thinking. They did not even consider the possibility of computing the answer without writing something. This was true not only at the beginning of the unit but, for a large percent, was also true at the end.

Three factors are seen as contributing to this result. First, the heavy demands of the existing program severely restricted the amount of time available for this unit. To adequately teach these short-cuts based on mathematical principles and relations took more time than had been allotted because the students had not previously been exposed to any work in mental arithmetic and because teachers had to spend much more time than had been foreseen in teaching the application of these ideas, concepts, and principles to aid rapid and accurate calculations. Second, the attitude of many of the students was adversely affected by their knowledge that this unit would not count in their year-end results. Third, they were unable to adapt to the new approach which differed from the pencil-and-paper approach which had become ingrained over the past ten years. They nearly all relied on the algorithms developed in elementary school and when questioned why these work, very few could explain how they had been developed.

As the unit came to an end, teachers felt that the students were beginning to see that there was something meaningful and potentially very helpful to be learned here. They began to think critically and to perceive that here was something that could make their work quicker and easier. They seemed to be at the point where they did not reach automatically for paper and pencil when faced with a mathematical problem. It was a further source of encouragement that there were some students who not only gained these insights, but began to develop some short-cuts themselves.

When the unit was being planned, it was felt that more-than-adequate time had been allotted. However, once the actual teaching began, teachers found they needed much more time than had been scheduled to develop the short-cuts and to drill them adequately. Consequently, time originally allotted for practice had to be taken for explanation and teaching, and, as is to be expected, teachers were unable to ensure the use of these short-cuts. Indeed, even the students who seemed to be gaining most from the unit, began to revert to the slower, more laborious, paper and pencil method they had used previously.

Because teachers were unanimous in noting that this unit did help the students attain competence with mathematical principles, they expressed regret that students had not been exposed to this aspect of arithmetic from at least grade seven. Indeed, this aspect of arithmetic is almost totally neglected.

Assessing the unit to see whether students became more agile in handling mental computations posed a dilemma for the experimenter. Many of the students achieving higher scores were proficient in computation using paper and pencil methods, and the number used were such that the operations could be computed quickly by them in the old ways. This is especially true of the whole number section. For example, when asked to add 98 and 36, the student added the ones and then the tens rather than looking from right to left and applying the associative property. Consequently, analysis of the test results led the experimenter to question whether these higher scores were the result of use of the short-cuts or efficient use of the algorithms.

Whole Numbers

Addition and subtraction of whole numbers will be analyzed first. In the practice session following the lesson on addition, students completed a set of 16 exercises in approximately one and one-half minutes with a very high degree of accuracy. These exercises, and all others to which reference is made in this paper, can be found in the Appendix. Teacher observation indicated, however, that many students were not employing the short-cuts, but had reverted to the traditional methods. When teachers discussed this with the students, a frequent response was that since they were so proficient with two-digit numbers with the better-known methods, why should they learn new ways?

The first subtraction sheet was passed out without any previous instruction. Teachers made two observations immediately. First, the students were very slow. Second, no student attempted to apply a mathematical principle designed to make their work less onerous and quicker. Immediately after instruction, however, they did start using the short-cuts and in the practice session many of the students completed the 16 exercises in approximately two minutes with a high degree of accuracy.

At the end of that portion of the unit dealing with addition and subtraction, the experimenter designed a test consisting of exercises very similar to those used in the practice sessions of the unit. Twenty-five exercises were given with a time limit of three minutes. This time limit was selected because it was felt that only those students employing the short-cuts could finish the test with a high degree of accuracy. In addition, classroom experience indicated that since they could complete 16 exercises in 1.5 minutes, 25 exercises in three minutes was reasonable.

The test results showed 18.5 percent of the students scored between 92 and 100. Twenty-three percent scored between 80 and 88. Thirty-four and one-half percent scores between 52 and 76. Twenty-three and one-half percent failed to achieve a score of 50. These results tend to indicate that 40 percent of the students were using the short-cuts developed, and that, on the other hand, 23.5 percent were unable to pass even after instruction in short-cuts and with the simple numbers employed.

This caused the experimenter to do two things. First, the actual test sheets were collected and examined to ascertain whether marks had been lost because of inaccuracy or because of failure to attempt exercises. As the scores lowered, more and more items had been left unattempted. That indicated these students were not employing the short-cuts to aid rapid and accurate calculations. Second, a sample of the students drawn from the full range of results was orally tested with these operations. Many of the students who had attained high scores had done so using the old methods at which they were very proficient. Oral testing of the students with low scores showed that they sometimes employed the short-cuts and sometimes used the old methods. Their general mathematical weakness was such that they did not understand the principles taught in developing the short-cuts and they were very slow in using the algorithms previously developed. Consequently, their weakness in rapid and accurate calculation was not alleviated by the short-cuts.

As a point of interest, the experimenter compared these results with those achieved by a selected group of Honours students who had received no instruction. Fifty-three and one-half percent of them scored over 80. On the other hand, only 4.4 percent failed to achieve a score of 50.

Attention is now directed to multiplication and division of whole numbers. Instruction in short-cuts in multiplication was begun with the FOIL method, which is outlined in the unit. For example: 21 x 16 =

(20 + 1) (10 + 6). The students were asked to apply the distributive property to these factors and add the partial sums. Many students questioned this method. They took a very long time to complete the exercises using it. As a result, seeing no advantage, they resisted the practice which would have speeded up this process.

Reaction was enthusiastic, on the other hand, about short-cuts which allowed them to increase their speed immediately and work accurately with less effort. Students were fascinated with, and quickly adopted, the following short-cuts: using numbers that were near multiples of 10, such as 99; multiplying by 15 and by 11; squaring numbers that end in five, and extensions of that rule. After students had been introduced to one or two of the short-cuts, many began to develop their own short-cuts and to use extensions of the rules.

In the practice sessions most of the students could complete 12 exercises in three minutes. However, many of the students who did not complete these exercises failed to do so because they did not know the two-factor multiplication facts that can be formed by using the numbers zero through nine.

The great interest shown in short-cuts in multiplication virtually evaporated in division, except for the students who were thoroughly enjoying and benefiting from the unit. A lot of students continued to use the old methods and therefore continued to take a very long time to complete their practice exercises. The time allotted for division was not sufficient to allow teachers to overcome this resistance.

At the end of this section a test consisting of 25 items similar to those used in the practice sessions was administered. A time limit of ten minutes was selected because it was felt that only those students employing the short-cuts could finish the test with a high degree of accuracy. In addition, classroom experience indicated that this was a reasonable time limit.

The test results showed 15 percent of the students scored between 92 and 100. Thirty and one-half percent scored between 80 and 88. Forty percent scored between 52 and 76. Fourteen and one-half percent failed to achieve a score of 50. These results tend to indicate that 45.5 percent of the students were using the short-cuts developed and that, on the other hand, 14.5 percent were unable to pass even after instruction in short-cuts and with the simple numbers employed.

Further analysis of the test sheets supported the teachers' observations that many more students employed the special multiplication short-cuts previously mentioned than employed short-cuts in the operation of division.

As a point of interest, the experimenter again compared these results with those achieved by a selected group of Honours students. These students had received instruction only in the use of one short-cut: squaring numbers that end in five. Fifty-one and one-half percent scored over 80. On the other hand, 14 percent failed to achieve a score of 50. It is interesting to note that these Honours students did employ many short-cuts, in division as well as in multiplication. Further, only the students who were in the lower segment of this group failed to achieve a score of 50.

Fractions

Teachers involved in this unit were astounded that very few grade-ten students—the product of ten years of new mathematics, which heavily stresses understanding—did apply the principles on which the short-cuts were based to the operations with whole numbers. The situation was even more serious when the unit dealt with fractions. The students found this part of the unit very difficult. Their performance when dealing with the operations using fractions was so weak that it was almost impossible to develop the short-cuts within the time allotted.

A feeling of dissatisfaction was general among teachers as soon as attention shifted to fractions. During the teaching sessions students employed the short-cuts with simple fractions. While they were somewhat slower in dealing with mixed numbers, their interest in, and application of, short-cuts remained high. However, after a time interval of only one or two days they lost all facility with these operations. This indicated that they had been operating on the basis of rote learning, rather than understanding.

At the end of this section on addition and subtraction, a test consisting of ten items similar to those used in the practice sessions was administered. A time limit of six minutes was selected because it was felt this was ample. No student scored 100. No student scored 90. Only nine percent of the students scored 80. On the other hand, 40 percent of the students failed to achieve a score of 50.

Many students failed to complete the test and the consensus of the teachers was that they could not have done so even with twice the time. They did not have a mastery of the algorithms developed for fractions to fall back on, as they had had when dealing with whole numbers. Pressure of time forced teachers to move to the operations of multiplication and division with fractions even though they were not at all satisfied with what they had done with addition and subtraction.

Many students experienced difficulty applying the distributive property to operations involving fractions. Further, many students could not transfer the pattern of squaring a binomial and multiplying the factors of the difference of two squares to squaring numbers such as 15½ and multiplying 24½ by 25½.

At the end of this session a test was given on the four operations involving fractions. The test comprised ten items similar to those used in the practice session. The time limit was six minutes. Six and one-half percent of the students achieved a score of 100. The same percent achieved a score of 90. Seven and one-half percent achieved a score of 80. Forty and one-half percent failed to achieve a score of 50. These results indicate that only approximately 20 percent of the students could complete the operations using short-cuts. On the other hand, over 40 percent could not achieve a score of 50 even after instruction.

As a point of interest, the experimenter again compared these results with those achieved by a selected group of Honours students who had had no previous instruction. No student achieved a score of 100. Three percent achieved a score of 90. Nine percent achieved a score of 80. Twenty-four percent failed to achieve a score of 50.

It was blatantly obvious to the teachers that both Level II and Honours students were very weak in operations involving fractions. Students realized this themselves. They were more receptive to anything that could help them deal with fractions, such as this unit's short-cuts, than they had been where whole numbers were concerned. Teachers felt a real regret that they were unable to give adequate treatment to this area, because of the benefits that could have derived by the students.

Student Attitudes

The third question this paper posed asked what was the effect of the unit on students' attitudes. This was analyzed on two bases. First, an opinionnaire was given to see if student attitude toward mathematics had changed as a result of the presentation of the unit. Second, teacher records of students' reaction to the unit were studied.

The opinionnaire used to measure student attitudes was the Connelly Taxonomized Attitude scale designed by Dr. R. Connelly of Memorial University of Newfoundland. The reliability coefficient for the opinionnaire is .87. There are 16 items on this instrument, each with five possible responses: strongly agree, agree, no opinion, disagree strongly, disagree. Items 1-6 were negatively stated and items 7-16 were positively stated.

The responses were scored as follows:

	Items 1-6	Items 7-16
Strongly agree	1	5
Agree	2	4
No opinion	3	3
Disagree	4	2
Strongly disagree	5	1

The highest possible score was 80 indicating a most positive attitude towards mathematics. The lowest possible score was 16 indicating a most negative attitude toward mathematics. A score of 48 would be considered neutral.

The opinionnaire was administered immediately prior to the teaching of the unit and immediately after the end of the unit. Students' scores were included if and only if the student completed the opinionnaire at both times.

A dependent t-test for means was performed on the set of difference scores from these opinionnaires, a t-value of -1.5827 indicated that there was no significant change in attitudes towards mathematics at the .10 level of significance during the period in which the experimental materials were taught.

Student reaction was extremely diverse. Nearly all the students who had done well in their regular mathematics program quickly mastered the short-cuts, became proficient in their use, developed short-cuts on their own, and in general thoroughly enjoyed the unit. Many of the students who had done poorly in their regular mathematics program could not become proficient in the use of short-cuts in the practice time allotted. Consequently they received no reinforcement and did not display positive attitudes. However, there were some traditionally weak students who grasped the short-cuts quickly, realized the benefits to be derived from their use, and readily adopted them. Teachers saw a carry-over of these positive reactions to the regular mathematics program.

Three of the five participating teachers reported no negative class attitudes, with only one or two individual exceptions. Indeed, one teacher reported that a class which had been indifferent and generally unmotivated all year became alert, industrious and competitive during this unit.

Negative reactions were reported by two teachers. They gave different reasons for them. One teacher indicated that the negative reaction she encountered was the result of the students' realization that no credit was to be given for this unit. The other teacher stated that the negative attitude was prominent at the beginning of the unit when whole numbers were being dealt with. Students saw no need for short-cuts because they were quite satisfied with their ability to compute using familiar algorithms. Once the unit started to deal with fractions, however, negative attitudes tended to disappear as the students perceived the benefits to be derived from short-cuts.

The diversity of reaction mentioned above could be seen also in student comment which ranged from, "Let's do some computation . . . time passes quickly," to "Oh no, not again!"

Teacher Attitudes

To answer the fourth question, which asked what were the attitudes of teachers towards the experimental materials, a questionnaire containing six items was given to each teacher who taught the unit.

On the form, teachers were asked to indicate whether they enjoyed teaching the materials in the unit, whether they would consider including them as a part of the high school mathematics program in the future, and whether they would recommend this unit to other teachers. They were also asked if they considered the material in this unit beneficial for average-ability mathematics students at high school. In addition, they were asked if the materials would have any value for students who would terminate their study of mathematics at the end of high school and for students enrolled in the Honours program. A copy of the questionnaire appears in Appendix C.

Response to question (1) indicated that all teachers enjoyed teaching the unit and displayed a positive attitude towards the materials in it. However, three teachers qualified their responses. One said, "I found the introduction of the initial concepts frustrating, not because of the material in the unit, but because students are weak. It was enlightening to see how little they understood." Another comment was, "Students' weakness in the application of basic mathematical principles was so great that it prohibited in-depth treatment of the unit." Another teacher stated, "I would have enjoyed it more if the unit had not been so rushed and if it could have been presented over a longer period of time."

All teachers, with one exception, said that they would consider including this unit as a part of the high school mathematics program in the future. The dissenting teacher recommended strongly that the unit be included at the elementary level. The other teachers recommended the introduction of this unit at the junior high level, and the extension of it with increasing difficulty at the senior high school level. One teacher suggested that the unit should be included in the regular program through the year so that students could get more practice over a longer period of time.

All teachers would recommend this unit to other teachers. This was especially true if the unit were to be developed throughout the elementary, junior, and senior high school programs as an integral part of the curriculum, rather than as a segmented unit done in a period of ten weeks. One teacher recommended that the application of these principles and the encouraging of mental computation be done in the elementary and junior high school through activities for fun and learning, such as games.

All teachers agreed that the material in this unit is beneficial for average-ability mathematics students at the high school level. They further felt that the materials would have value for both the terminal students and the students enrolled in the Honours program. Two teachers felt strongly that it would have great value for the terminal students. All teachers felt that the Honours students would quickly pick up the short-cuts and would enjoy the unit. One teacher added that for these students this unit would be more appropriate in junior high school.

From the analysis of the responses to the teacher questionnaire several generalizations can be made: first, the teachers enjoyed teaching the materials; second, the teachers would recommend the materials to other teachers; third, the materials are beneficial for the average-ability mathematics students at all grade levels; fourth, the materials have value for both the terminal students and Honours students in mathematics.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

The purpose of this paper was to develop a unit on rapid and accurate calculations for use in the high school mathematics program, to assess its suitability within the existing program, and to evaluate the benefits to be derived from it. In order to do this, the experimenter considered four questions:

1. Can students attain competence with mathematical principles used in short-cut methods of thinking in computation?

2. Will students become more agile in handling mental computation as a result of this unit?

3. What is the effect of the unit on teacher attitudes?

4. What are the attitudes of teachers towards the experimental materials?

Analysis of the test results and subsequent oral questioning of the students showed that many of them failed to achieve mastery of the short-cuts presented.

A dependent t-test for means was performed on the pretest, post-test attitude scores. A t-value of -1.5827 indicated that there was no significant change in the attitudes of students towards mathematics at the .10 level of significance during the teaching of the experimental materials.

Student reaction to the experimental materials was varied and diverse. Nearly all students who mastered the short-cuts and became proficient in their use enjoyed the unit. On the other hand, many of the students who did not become proficient in the use of short-cuts in the practice time allotted did not display positive attitudes.

Response to the teacher questionnaire was favourable towards the materials in the unit. Teachers recommended the materials to other teachers and felt the materials had value for both the terminating and Honours students in mathematics, as well as for average students. They further strongly recommended that this material be included at all grade levels.

Conclusions

The present study indicated that perhaps schools tend to place too much emphasis upon pencil and paper work, and that students set down figures and compute without doing adequate critical thinking.

The crucial point which many authors seem to indicate is that the benefits of theory in mathematics eliminate the need for a great amount of computational drill. But this is not necessarily so. For example, once the distributive law is understood it does not necessarily follow that the computation of problems in multiplication is mastered with little or no further need for practice. The experience of the unit shows that even after students had understood the principles, much practice time was needed before the students became efficient in their use.

Much has been done in recent years to strengthen the arithmetic curriculum on the elementary level. Attempts have been made to achieve understanding in arithmetic and consequently the ability to think independently and to solve problems in the most efficient manner, rather than through mindless rote. If students are to do this, they must be prepared from elementary school on. This ability cannot be acquired overnight, as has been illustrated by this unit.

It can be seen even from the relatively small percent of students for whom improvement seemed apparent that a unit like this would be beneficial for developing and improving number sense. This growth could perhaps be experienced by a larger percent of the students if the unit were expanded and made an integral part of the mathematics program.

This unit has suggested that while theory is important, so is practice. As evidenced by comments made by teachers and students, planned mental arithmetic experiences were favourably received as a total part of the mathematics program. This suggests that mental arithmetic is an excellent vehicle for improving computational skills at all levels.

Recommendations

Based upon the results of this study, the following recommendations are made.

1. A sequential program in mental computation should be a part of the mathematics program at all grade levels.

2. Work in arithmetic should not be restricted to either the paper and pencil approach or to mental computation. Many circumstances call for parts of both procedures. But in all work, thinking should be based on a good understanding of mathematical ideas, concepts, and principles. To gain this end, pupils must be given an opportunity both to learn and to practice. Because textbooks do not provide adequately for this development, teachers must take the initiative and accept the

responsibility to provide such material themselves. In addition, teachers should be alert to seize every opportunity to incorporate these experiences into the program whenever an opening arises.

3. Mental arithmetic for its own sake is rejected. Its value lies in the fact that it provides practice for the application of mathematical principles and relationships, and facilitates the students' use of numbers. The objective is not to create something that will perform a calculation quickly and accurately without thinking. If that were so, it would be cheaper and more efficient to make machines than to educate people.

4. Teachers should continue to emphasize concepts but not at the expense of the development of computational skills. Computation is a vital part of mathematics. Therefore new and interesting ways should be devised to ensure practice in the basic skills which are necessary at each developmental level.

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APPENDIX A

Instructions to Teachers

Instructional Materials

Tests

Instructions to Teachers

Each teacher was provided with the set of materials to be used in the study. The rationale of, and the need for, the study were explained.

The experimenter met periodically with the five teachers and presented the materials to them in the same manner in which these were to be presented to the students. Teachers were given exercises similar to those in the instructional package and were asked to explain how they arrived at their answers. In this manner they derived the short-cuts used in the study. Teachers were asked to present these short-cuts used by the experimenter; however, they were to encourage students to develop other short-cuts and to discuss the mathematical principles these exercises exemplified.

All teachers were asked to record any observations they felt were pertinent to the study. These included: reactions of students to the materials; the extent to which they developed short-cuts themselves; the extent to which students actually employed the short-cuts, as opposed to algorithms previously learned; the time the students took to complete the practice exercises; and the degree of accuracy the students achieved.

Teachers were asked to administer tests within a given time limit. The results of these tests were discussed, giving teachers input into the teaching of the unit.

Instructional Materials

A. One and Two Digit Whole Numbers

Part I - Addition

Objective: By the end of this unit students should be able to use the commutative and associative laws in rearranging numbers in an addition problem to facilitate the adding.

LESSON ONE

Example one:

$$
\begin{aligned}
75 + 77 &= (70 + 5) + (70 + 7) \\
&= (70 + 70) + (5 + 7) \\
&= 140 + 12 \\
&= 152
\end{aligned}
$$

Example two:

$$
\begin{aligned}
97 + 18 &= 97 + (3 + 15) \\
&= (97 + 3) + 15 \\
&= 100 + 15 \\
&= 115
\end{aligned}
$$

Other examples:

1. 16 + 28	6. 39 + 46
2. 19 + 17	7. 49 + 63
3. 93 + 17	8. 65 + 96
4. 26 + 33	9. 27 + 38
5. 99 + 76	10. 28 + 39

Practice Exercises

A. Find the answers to the following exercises by mental computation:

1. 48 + 27	9. 48 + 40
2. 75 + 36	10. 76 + 16
3. 32 + 24	11. 64 + 28
4. 27 + 24	12. 47 + 47
5. 25 + 18	13. 55 + 26
6. 50 + 34	14. 68 + 24
7. 27 + 39	15. 35 + 35
8. 37 + 17	16. 66 + 25

B. Find the answers to the following exercises by mental computation:

1. 38 + 23	9. 27 + 24
2. 48 + 63	10. 97 + 18
3. 99 + 73	11. 38 + 26
4. 56 + 32	12. 18 + 26
5. 73 + 18	13. 97 + 96
6. 57 + 39	14. 83 + 26
7. 66 + 19	15. 37 + 12
8. 25 + 18	16. 36 + 44

LESSON TWO

Example one:

$$\begin{aligned} 8 + 23 + 92 &= (8 + 23) + 92 \\ &= (23 + 8) + 92 \\ &= 23 + (8 + 92) \\ &= 23 + 100 \\ &= 123 \end{aligned}$$

Example two:

$$\begin{aligned} 97 + 36 + 13 &= (90 + 10 + 30) + (7 + 3) + 6 \\ &= 130 + 10 + 6 \\ &= 146 \end{aligned}$$

Other examples:

1. 18 + 27 + 82
2. 25 + 18 + 75
3. 35 + 28 + 65
4. 23 + 18 + 25 + 22 + 17

Practice Exercises

A. Find the answers to the following using mental computation:

1. 16 + 19 + 84
2. 18 + 25 + 82
3. 38 + 62 + 91
4. 53 + 24 + 47
5. 25 + 83 + 15
6. 63 + 5 + 37
7. 8 + 26 + 92 + 75
8. 17 + 16 + 22 + 23 + 14 + 38

B. Find the answers to the following using mental computation:

1. 72 + 79 + 28
2. 83 + 54 + 17
3. 39 + 64 + 71
4. 27 + 38 + 73
5. 93 + 23 + 7
6. 97 + 36 + 13
7. 25 + 13 + 6 + 27 + 35 + 4
8. 23 + 75 + 18 + 27 + 25 + 32

Part II - Subtraction

Objective: By the end of this unit students should be able to apply the equal addition property: $a - b = (a + c) - (b + c)$ in a subtraction problem to facilitate the subtracting

LESSON THREE

Example one:

$$46 - 19 = (46 + 1) - (19 + 1) = 47 - 20 = 27$$

Example two:

$$45 - 31 = (45 - 1) - (31 - 1) = 44 - 30 = 14$$

Other examples:

1. 42 - 18	6. 42 - 33
2. 53 - 27	7. 92 - 17
3. 87 - 59	8. 84 - 15
4. 48 - 33	9. 45 - 17
5. 83 - 39	10. 97 - 25

Practice Exercises

A. Find the answers to the following using mental computation

1. 77 - 49	9. 98 - 28
2. 97 - 38	10. 37 - 18
3. 68 - 27	11. 33 - 26
4. 33 - 26	12. 48 - 23
5. 52 - 29	13. 41 - 19
6. 67 - 18	14. 86 - 35
7. 77 - 32	15. 77 - 38
8. 87 - 36	16. 26 - 13

B. Find the answers to the following using mental computation:

1. 86 - 15	6. 66 - 27
2. 97 - 36	7. 88 - 39
3. 46 - 17	8. 73 - 29
4. 97 - 48	9. 92 - 18
5. 78 - 69	10. 83 - 29

Part III - Multiplication

Objective: 1. By the end of this unit the students should be able to use the commutative, associative, and distributive properties to facilitate the multiplying of numbers.

2. By the end of this unit the students should be able to apply the short-cut method of squaring a number that ends in five and be able to modify and adapt other problems to lend itself to this rule.

LESSON FOUR

Example one:

$$\begin{aligned} 3 \times 12 &= 3 \times (10 + 2) \\ &= (3 \times 10) + (3 \times 2) \\ &= 30 + 6 \\ &= 36 \end{aligned}$$

Other examples:

1. 8 x 26	6. 6 x 16
2. 5 x 73	7. 8 x 74
3. 3 x 13	8. 2 x 97
4. 4 x 93	9. 8 x 25
5. 3 x 68	10. 3 x 76

Practice Exercises

A. Find the answers to the following by mental computation:

1. 6 x 27	7. 2 x 99
2. 8 x 73	8. 7 x 27
3. 5 x 33	9. 4 x 18
4. 4 x 53	10. 8 x 35
5. 9 x 31	11. 7 x 63
6. 3 x 79	12. 6 x 17

B. Find the answers to the following by mental computation:

1. 7 x 23	7. 9 x 17
2. 9 x 57	8. 2 x 97
3. 8 x 15	9. 5 x 32
4. 3 x 37	10. 4 x 73
5. 6 x 18	11. 8 x 65
6. 4 x 48	12. 9 x 99

LESSON FIVE

Foil Rule: $(10a + b)(10c + d) = 100ac + 10ad + 10bc + bd$

Example one:

$$
\begin{aligned}
23 \times 16 &= (20 + 3)(10 + 6) \\
&= (20 \times 10) + (20 \times 6) + (3 \times 10) + (3 \times 6) \\
&= 200 + 120 + 30 + 18 \\
&= 368
\end{aligned}
$$

Other examples:

1. 11 x 72	4. 16 x 32
2. 18 x 98	5. 25 x 13
3. 45 x 18	6. 14 x 14

Practice Exercises

A. Find the answers to the following using mental computation:

1. 12 x 36	6. 18 x 85
2. 23 x 18	7. 31 x 19
3. 16 x 72	8. 13 x 18
4. 32 x 22	9. 16 x 32
5. 21 x 45	10. 28 x 42

B. Find the answers to the following using mental computation:

1. 12 x 42	6. 24 x 25
2. 13 x 12	7. 62 x 11
3. 10 x 72	8. 14 x 19
4. 18 x 15	9. 72 x 21
5. 15 x 36	10. 34 x 23

LESSON SIX

Other methods to consider and which could be used as a check on the foil methods.

Example one:

$$\begin{aligned} 45 \times 12 &= 45 \times (2 \times 6) \\ &= (45 \times 2) \times 6 \\ &= 90 \times 6 \\ &= 540 \end{aligned}$$

Example two:

$$\begin{aligned} 15 \times 26 &= (10 + 5) \times 26 \\ &= (10 \times 26) + (\tfrac{1}{2} \times 10 \times 26) \\ &= 260 + 130 \\ &= 390 \end{aligned}$$

LESSON SEVEN

Squares of numbers that end in five:

Example: $35^2 = 30 \times 40 + 25$

$= 1200 + 25$

$= 1225$

Pattern: $35^2 = (3 \times 10)\ ((3 \times 1) \times 10)$ + product of units digit

$(30 + 5)(30 + 5) = 30^2 + 2 \times 5 \times 30 + 25$

$= 30^2 + 10 \times 30 + 25$

$= 30\ (30 + 10) + 25$

$= 30 \times 40 + 25$

Other examples:

1. 65^2
2. 15^2
3. 85^2
4. 45^2

Practice Exercises

A. Find the answers to the following using mental computation:

1. 75×75
2. 35×35
3. 65×65
4. 15×15
5. 55^2
6. 95^2
7. 45^2
8. 25^2
9. 85^2

LESSON EIGHT

Adapting the short-cut method of squaring numbers than end in five to examples such as:

1. $35 \times 36 = 35\ (35 + 1)$

$= 35^2 + 35$

$= 1225 + 35$

$= 1260$

Example three:

$$99 \times 45 = (100 - 1) \times 45$$
$$= 4500 - 45$$
$$= 4450$$

Example four:

$$16 \times 25 = (16 \times 100) \div 4$$
$$= 1600 \div 4$$
$$= 400$$

Example five:

$$11 \times 63 = 6 \times 100 + (6 + 3) \times 10 + 3$$
$$= 600 + 90 + 3$$
$$= 693$$

Practice Exercises

A. Find the answers to the following using mental computation:

1. 35 x 12	5. 13 x 15
2. 15 x 42	6. 99 x 28
3. 20 x 36	7. 25 x 12
4. 62 x 11	8. 18 x 15

B. Find the answers to the following by mental computation:

1. 25 x 36	6. 99 x 15
2. 12 x 15	7. 29 x 11
3. 14 x 32	8. 75 x 12
4. 99 x 25	9. 89 x 31
5. 13 x 17	10. 54 x 11

2. $36 \times 34 = (35 + 1) \times (35 - 1)$

$= 35^2 - 1$

$= 1225 - 1$

$= 1224$

Practice Exercises

A. Find the answers to the following using mental computation:

1. 45 x 45	6. 24 x 26
2. 45 x 46	7. 55 x 56
3. 45 x 44	8. 84 x 86
4. 25 x 26	9. 74 x 75
5. 44 x 46	10. 64 x 65

B. Find the answers to the following using mental computation:

1. 15 x 16	6. 24 x 26
2. 14 x 16	7. 95 x 96
3. 75 x 76	8. 95 x 94
4. 35 x 34	9. 54 x 56
5. 84 x 85	10. 65 x 66

LESSON NINE

Example one:

$4 \times 17 \times 25 = 4 \times (25 \times 17)$

$= (4 \times 25) \times 17$

$= 100 \times 17$

$= 1700$

Example two:

$$15 \times 10 \times 15 = 15 \times (15 \times 10)$$
$$= (15 \times 15) \times 10$$
$$= 225 \times 10$$
$$= 2250$$

Other examples:

1. 5 x 39 x 2	3. 35 x 15 x 35
2. 5 x 45 x 20	4. 5 x 18 x 20

Practice Exercises

A. Find the answers to the following using mental computation:

1. 4 x 6 x 25	5. 5 x 79 x 20
2. 5 x 25 x 5	6. 15 x 5 x 15
3. 7 x 35 x 5	7. 2 x 89 x 50
4. 2 x 38 x 50	8. 16 x 10 x 14

Part IV - Division

Objective: At the end of this unit students should be able to apply the ratio idea that the numerator and denominator of a fraction can both be multiplied or divided by the same number in a division problem to facilitate the dividing.

LESSON TEN

Example one:

$$625 \div 25 = 1250 \div 50 = 2500 \div 100 = 25$$

Example two:

$96 \div 12 = 48 \div 6 = 8$

Other Examples:

1. $72 \div 18$
2. $19 \div 5$
3. $64 \div 24$
4. $81 \div 27$

Practice Exercises

A. Find the answers to the following using mental computation:

1. $84 \div 14$
2. $96 \div 24$
3. $75 \div 25$
4. $77 \div 21$
5. $56 \div 16$
6. $72 \div 48$
7. $99 \div 18$
8. $52 \div 26$
9. $84 \div 42$
10. $90 \div 45$

B. Fractions and Mixed Numbers

Part I - Addition

Objective: By the end of this unit students should be able to mentally compute the addition of simple fractions and mixed numbers.

LESSON ELEVEN

Example one:

$$\frac{5}{8} + \frac{2}{3} = \frac{3 \times 5 + 8 \times 2}{8 \times 3} = \frac{15 + 16}{24} = \frac{31}{24}$$

Example two:

$$\frac{7}{8} + \frac{1}{5} = \frac{7 \times 5 + 8 \times 1}{8 \times 5} = \frac{35 + 8}{40} = \frac{43}{40}$$

Other examples:

1. $\frac{4}{5} + \frac{1}{3}$

2. $\frac{7}{9} + \frac{1}{8}$

3. $\frac{15}{16} + \frac{1}{14}$

4. $\frac{3}{8} + \frac{5}{6}$

Practice Exercises

A. Find the answers to the following using mental computation:

1. $\frac{5}{6} + \frac{2}{3}$

2. $\frac{1}{8} + \frac{4}{5}$

3. $\frac{1}{7} + \frac{5}{6}$

4. $\frac{2}{9} + \frac{7}{8}$

5. $\frac{5}{11} + \frac{1}{12}$

6. $\frac{6}{7} + \frac{1}{4}$

7. $\frac{5}{12} + \frac{2}{7}$

8. $\frac{1}{15} + \frac{15}{16}$

9. $\frac{14}{17} + \frac{1}{16}$

10. $\frac{2}{9} + \frac{3}{5}$

B. Find the answers to the following using mental computation:

1. $\frac{3}{8} + \frac{2}{7}$

2. $\frac{5}{6} + \frac{8}{9}$

3. $\frac{3}{25} + \frac{25}{26}$

4. $\frac{1}{34} + \frac{1}{36}$

5. $\frac{1}{15} + \frac{15}{16}$

6. $\frac{7}{10} + \frac{11}{16}$

7. $\frac{5}{7} + \frac{13}{14}$

8. $\frac{3}{26} + \frac{5}{24}$

9. $\frac{1}{7} + \frac{9}{10}$

10. $\frac{5}{8} + \frac{6}{11}$

LESSON TWELVE

Example one:

$$8\frac{1}{3} + 7\frac{2}{3} = (8 + 7) + (\frac{1}{3} + \frac{2}{3})$$
$$= 15 + 1$$
$$= 16$$

Example two:

$$16\frac{3}{5} + 17\frac{1}{4} = (16 + 17) + (\frac{3}{5} + \frac{1}{4})$$
$$= 33 + \frac{17}{20}$$
$$= 33\frac{17}{20}$$

Other examples:

1. $16\frac{3}{5} + 17\frac{3}{5}$
2. $12\frac{1}{6} + 20\frac{1}{8}$
3. $8\frac{1}{3} + 7\frac{5}{6}$
4. $8\frac{1}{15} + 9\frac{14}{16}$

A. Find the answers to the following using mental computation:

1. $20\frac{1}{5} + 13\frac{3}{5}$
2. $18\frac{1}{6} + 19\frac{5}{6}$
3. $25\frac{5}{7} + 26\frac{3}{5}$
4. $5\frac{7}{8} + 10\frac{1}{6}$
5. $18\frac{4}{5} + 72\frac{3}{7}$
6. $6\frac{5}{8} + 5\frac{1}{7}$
7. $3\frac{1}{4} + 2\frac{1}{5}$
8. $8\frac{3}{5} + 2\frac{1}{6}$
9. $3\frac{1}{8} + 6\frac{3}{7}$
10. $5\frac{1}{4} + 10\frac{1}{5}$

B. Find the answers to the following using mental computation:

1. $5\frac{1}{2} + 5\frac{1}{2}$
2. $11\frac{7}{10} + 11\frac{3}{10}$
6. $85\frac{1}{5} + 15\frac{3}{4}$
7. $3\frac{5}{24} + 2\frac{1}{26}$

3. $5\frac{3}{4} + 5\frac{1}{4}$

4. $8\frac{1}{3} + 6\frac{7}{8}$

5. $20\frac{1}{15} + 2\frac{15}{16}$

8. $7\frac{3}{5} + 10\frac{1}{8}$

9. $26\frac{7}{8} + 14\frac{9}{10}$

10. $16\frac{2}{3} + 16\frac{7}{8}$

Part II - Subtraction

Objective: At the end of this unit students should be able to mentally compute the subtraction of simple fractions and two-digit mixed numbers.

LESSON THIRTEEN

Example one:

$$\frac{1}{3} - \frac{1}{5} = \frac{1 \times 5 - 3 \times 1}{3 \times 5} = \frac{5 - 3}{15} = \frac{2}{15}$$

Example two:

$$\frac{14}{15} - \frac{1}{16} = \frac{14 \times 16 + 15 \times 1}{15 \times 16} = \frac{224 - 15}{240} = \frac{209}{240}$$

Practice Exercises

A. Find the answers to the following using mental computation:

1. $\frac{1}{4} - \frac{1}{5}$

2. $\frac{1}{6} - \frac{1}{8}$

3. $\frac{5}{8} - \frac{1}{4}$

4. $\frac{6}{7} - \frac{1}{3}$

5. $\frac{25}{26} - \frac{1}{25}$

6. $\frac{5}{8} - \frac{1}{5}$

7. $\frac{9}{10} - \frac{1}{6}$

8. $\frac{11}{24} - \frac{1}{25}$

LESSON FOURTEEN

Example one:

$$\begin{aligned} 8\tfrac{3}{4} - 6\tfrac{1}{3} &= (8 - 6) + (\tfrac{3}{4} - \tfrac{1}{3}) \\ &= 2 + \frac{5}{12} \\ &= 2\frac{5}{12} \end{aligned}$$

Example two:

$$3\frac{1}{14} - 2\frac{1}{16} = (3 - 2) + (\frac{1}{14} - \frac{1}{16})$$
$$= 1 + \frac{2}{224}$$
$$= 1\frac{1}{112}$$

Practice Exercises

A. Find the answers to the following using mental computation:

1. $5\frac{5}{8} - 3\frac{1}{8}$
2. $9\frac{5}{6} - 2\frac{1}{4}$
3. $11\frac{4}{5} - 8\frac{1}{6}$
4. $8\frac{3}{14} - 3\frac{1}{16}$
5. $12\frac{2}{3} - 10\frac{1}{8}$
6. $7\frac{3}{25} - 3\frac{1}{24}$
7. $18\frac{5}{7} - 3\frac{1}{4}$
8. $5\frac{1}{94} - 3\frac{1}{95}$

Part III - Multiplication

Objectives: By the end of this unit students should be able to mentally compute the multiplication of one and two digit mixed numbers.

LESSON FIFTEEN

Example one:

$$20 \times 3\frac{3}{4} = 20 \times (3 + \frac{3}{4})$$
$$= (20 \times 3) + (20 \times \frac{3}{4})$$
$$= 60 + 15$$
$$= 75$$

Example two:

$$5 \times 5\tfrac{3}{4} = 5 \times (5 + \tfrac{3}{4})$$
$$= (5 \times 5) + (5 \times \tfrac{3}{4})$$
$$= 25 + 3\tfrac{3}{4}$$
$$= 28\tfrac{3}{4}$$

Practice Exercises

A. Find the answers to the following using mental computation:

1. $6 \times 6\frac{2}{3}$
2. $9 \times 11\frac{1}{3}$
3. $26 \times 2\frac{2}{13}$
4. $35 \times 35\frac{1}{34}$
5. $15 \times 19\frac{1}{3}$
6. $9 \times 99\frac{1}{6}$
7. $25 \times 25\frac{1}{6}$
8. $10 \times 11\frac{1}{7}$
9. $12 \times 7\frac{3}{7}$
10. $15 \times 1\frac{1}{16}$

LESSON SIXTEEN

Example one:

$$15\tfrac{1}{2} \times 15\tfrac{1}{2} = (15 + \tfrac{1}{2})\ (15 + \tfrac{1}{2})$$
$$= 15^2 + 2 \times 15 \times \tfrac{1}{2} + \tfrac{1}{2}^2$$
$$= 225 + 15 + \tfrac{1}{4}$$
$$= 240\tfrac{1}{4}$$

Example two:

$$14\tfrac{1}{2} \times 15\tfrac{1}{2} = (15 - \tfrac{1}{2})\ (15 + \tfrac{1}{2})$$
$$= 15^2 - (\tfrac{1}{2})^2$$
$$= 225 - \tfrac{1}{4}$$
$$= 224\tfrac{3}{4}$$

Practice Exercises

A. Find the answers to the following using mental computation:

1. $7\frac{1}{2} \times 7\frac{1}{2}$
2. $12\frac{1}{2} \times 11\frac{1}{2}$
3. $9\frac{1}{3} \times 9\frac{1}{3}$
4. $35\frac{1}{2} \times 34\frac{1}{2}$
5. $10\frac{1}{5} \times 10\frac{1}{5}$
6. $8\frac{1}{8} \times 8\frac{1}{8}$
7. $5\frac{1}{2} \times 4\frac{1}{2}$
8. $25\frac{1}{2} \times 24\frac{1}{2}$

Part IV - Division

Objective: By the end of this lesson students should be able to apply the ratio idea that the numerator and denominator of a fraction can both be multiplied or divided by the same number in a division problem to facilitate the dividing.

LESSON SEVENTEEN

Example one:

$21 \div 3\frac{1}{2} = 42 \div 7 = 6$

Example two:

$10\frac{1}{2} \div 1\frac{1}{2} = 21 \div 3 = 7$

Other examples:

1. $15 \div 2\frac{1}{2}$
2. $44 \div 5\frac{1}{2}$
3. $12 \div 1\frac{1}{3}$
4. $6\frac{1}{4} \div 1\frac{1}{4}$

Practice Exercises

Find the answers to the following using mental computation:

1. $18 \div 1\frac{1}{2}$
2. $10 \div 1\frac{1}{4}$
6. $4 \div 1\frac{1}{3}$
7. $19 \div 9\frac{1}{4}$

3. $21 \div 10\frac{1}{2}$

4. $10\frac{1}{2} \div 3\frac{1}{2}$

5. $3\frac{3}{5} \div 1\frac{1}{5}$

8. $6 \div 1\frac{1}{5}$

9. $7\frac{1}{3} \div 1\frac{5}{6}$

10. $6\frac{2}{3} \div 3\frac{1}{3}$

Tests

Test one

Find the answers to the following using mental computation.

Time: 3 minutes.

1. 89 + 34 =	14. 58 + 87 =
2. 94 + 38 =	15. 49 + 81 =
3. 76 - 31 =	16. 77 + 94 =
4. 93 + 26 + 37 + 74 =	17. 93 - 28 =
5. 19 ± 73 =	18. 56 + 82 =
6. 99 - 43 =	19. 88 + 36 + 99 + 44 + 12 + 11 =
7. 89 - 67 =	20. 86 + 33 + 14 =
8. 99 + 36 =	21. 63 - 37 =
9. 14 + 53 =	22. 59 + 37 =
10. 66 - 39 =	23. 36 + 67 + 33 + 94 =
11. 91 - 43 =	24. 31 - 19 =
12. 87 - 19 =	25. 50 + 73 =
13. 18 + 93 + 17 + 82 + 93 =	

Test two

Find the answers to the following using mental computation.

Time: 10 minutes.

1. 38 x 15 =	14. 32 x 25 =
2. 19 x 25 =	15. 29 x 31 =
3. 85 x 85 =	16. 25 x 75 =
4. 99 x 31 =	17. 95 x 96 =

5. 63 x 11 =

6. 25 x 26 =

7. 21 x 35 =

8. 74 x 75 =

9. 98 x 39 =

10. 45 x 45 =

11. 75 x 11 =

12. 34 x 36 =

13. 87 x 15 =

18. 15 x 10 x 15 =

19. 35 x 2 x 35 =

20. 25 x 36 x 4 =

21. 169 ÷ 26 =

22. 18 ÷ 5 =

23. 144 ÷ 24 =

24. 72 ÷ 27 =

25. 77 ÷ 21 =

Test three

Find the answers to the following using mental computation.

Time: 6 minutes

1. $26\frac{3}{8} + 36\frac{1}{5} =$

2. $36\frac{11}{16} - 25\frac{1}{14} =$

3. $99\frac{4}{5} - 38\frac{1}{3} =$

4. $15 \times 37\frac{1}{3}$

5. $18 \times 5\frac{1}{7} =$

6. $36 \times 34\frac{1}{9} =$

7. $(15\frac{1}{2})^2 =$

8. $24\frac{1}{2} \times 25\frac{1}{2}$

9. $99\frac{1}{14} + 37\frac{1}{15} =$

10. $92\frac{4}{5} - 38\frac{1}{10} =$

APPENDIX B

Student Questionnaire

Name:__________________________

Class:__________________________

In the space provided write your name and class. This is NOT a test and will not be used in any way to produce a grade for you. The items on this instrument are statements about mathematics. For each item select a response which best describes your impression of the statement and place your response in the space provided at the left. The response choices are:

A -- Strongly agree
B -- Agree
C -- No opinion
D -- Disagree
E -- Strongly disagree

____ 1. I have nothing but contempt for mathematics.

____ 2. I regard mathematics as a lasting tribute to man's ignorance.

____ 3. I feel under a great strain in a mathematics class.

____ 4. Mathematics makes me feel as though I'm lost in a jungle.

____ 5. Mathematics makes me feel uncomfortable.

____ 6. Mathematics is mainly pencil pushing.

____ 7. The very existence of humanity depends on mathematics.

____ 8. Mathematics may be compared to a tree, ever putting forth new branches.

____ 9. Mathematics is a subject which I have enjoyed studying in school.

____ 10. My general attitude toward mathematics is favourable.

____ 11. I feel mathematics is the greatest means for increasing the world's knowledge.

____ 12. Mathematics is stimulating to me.

____ 13. Working with various mathematical topics is fun.

____ 14. I see nothing wrong with learning a variety of mathematical topics.

____ 15. I feel mathematics helps make other subjects easier to understand.

____ 16. Mathematics fascinates me.

APPENDIX C

Teacher Questionnaire

1. Did you enjoy teaching the materials in this unit?

2. Do you feel that the material in this unit is beneficial for average ability mathematics students at highschool level?

3. Do you feel that the materials would have any value for students who will terminate their study of mathematics at the end of highschool?

4. Do you feel that it would be advantageous for students who enroll in the honours courses at highschool level to study this unit?

5. Would you consider including this as a part of the highschool mathematics program in the future? If so at what level(s)?

6. Would you recommend the material in this unit to other teachers?