APPLICATION OF GRAPH THEORETIC CONCEPTS IN SHORT CIRCUIT ANALYSIS OF POWER SYSTEMS

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by

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ABSTRACT

The ever increasing size of electrical power systems necessitates systematic and comprehensive methods for their analysis. This report is concerned with the application of graph theoretic concepts in power system analysis, with particular reference to short circuits. It is demonstrated that the application of graph concepts provides a very elegant method for the formulation and analysis of power systems problems.

The formulation is based on a decomposition of the power systems into a number of subsystems. The subsystems are then combined according to their physical interconnection pattern - so that an appropriate mathematically equivalent system representation is available for the system as a whole. Such a method is considered to be particularly suitable for the analysis of large power systems.

The formulation of the Z_{BUS} and Y_{BUS} models for a typical transmission system is given. Also, as an example, the multi-terminal representation for a typical system containing an auto-transformer is derived. The method of combining different subsystems, such as the transmission systems and the generator systems, is demonstrated. Building algorithms, using the concepts of multi-terminal representations, are developed such that the impedance matrix (Z_{BUS}) of a network can be formulated in stages - by adding one element at a time. Separate cases for (i) addition of a branch, not mutually coupled (ii) addition of a link, not mutually coupled (iii) addition of a mutually coupled group, are illustrated. The symbolic formulations of these cases are

discussed. Numerical examples to illustrate the use of these algorithms are worked out. Finally, an attempt is made to obtain the short circuit currents and voltages of a typical power system, by application of graph theoretic principles. Separate cases for three phase faults, as well as for two simultaneous faults, are considered. In each of these cases, numerical examples which illustrate the formulation procedure and analysis are worked out.

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CHAPTER I

INTRODUCTION

The ever increasing size of power systems together with the associated complexities require systematic and comprehensive methods of analysis for evaluation of their performance and viability. Even in the early days of the industry this need was recognized and much effort was directed towards developing solution techniques for power system problems. The main problems frequently encountered are short circuit analysis, load flows and the transient stability. Until the 1950's, the D.C. and A.C. network analysers were popularly being used for such power system simulation.

However, particularly since the last decade, the availability of digital computers has provided a powerful tool at the disposal of power system engineers for the purpose. It has provided the capability for performing the time consuming and complex mathematical calculations very fast. Thus the merits and demerits of various strategies in planning and operation can be evaluated efficiently. The use of digital computers for power system simulation has indirectly provided added impetus to the mathematical modelling of the system from its interconnection pattern.

It is theoretically possible to develop a single model to deal with all the principal aspects of the analysis. Such an approach however is not always practical though it is a desirable goal to strive for. Because of the many diverse and often conflicting physical considerations, the simulation of power systems has essentially been specifically oriented towards the type of study concerned, viz., short circuit study, load flow, or the transient stability study. It is advantageous to consider each type of study in its own right, carefully emphasising the significant factors that are pertinent to the study while discarding those that may have negligible effect on it.

The various mathematical models that are developed for the solution of electrical networks depend on

(i) Kirchoff's current and voltage law

(ii) terminal equations of the components comprising the network

and (iii) the interconnection pattern of the components.

The mesh, node and the state equations which are usually formulated for the solution of networks follow simply as a result of substitution procedures between these basic sets of equations. When we deal with large networks, as is typically the case with power systems, topological methods using the tools of graph theory are found to be extemely useful and amenable for eventual computer simulation. The analysis is based on a decomposition of the power system into three main subsystems:

(i) the Generator system,

(ii) the Transmission system,

(iii) subsystems pertaining to the loads.

Each of the subsystems can be represented by a model which is appropriate to the particular study involved. For example, for the short circuit study, a generator can be represented by a voltage source in series with the internal impedance of the machine whereas in transient stability studies a more complex model of the synchronous machine may be necessary.

Similarly, for short circuit studies, the line charging of the transmission lines are neglected whereas for load flow studies they are included in the model. The various subsystems are interconnected with each other at the buses. Using graph theoretic concepts one can obtain what is called 'Multi-terminal representation' of each of the subsystems at the buses. Once such a representation is obtained, the modelling becomes very systematic and conceptually elegant. Procedures for solution for a specific type of study can then be spelled out in a manner which can be easily simulated on digital computers.

This study concerns itself with the use of linear graph theory in short circuit studies. Specifically, the contribution concerns (i) derivation of multi-terminal representations for networks; (ii) the derivation of the $\underline{Z}_{BUS}$ algorithms; (iii) analysis of simultaneous faults and (iv) illustration of (i), (ii) and (iii) for a typical system. Most of the literature of the building algorithms centers around 'heuristic' concepts such as 'injected currents', 'fictitious nodes' etc. Consequently, it is difficult to distinguish between the concepts involved and the actual algorithm developed. It is demonstrated in this report that a rigorous way to derive the building algorithm is through the concept of multi-terminal representation. The other contribution in the report concerns itself with the analysis of simultaneous faults. It is shown that the concept of multi terminal representation lends itself very easily to such types of studies. A brief preview of the various chapters now follows:

In Chapter II, the concept of multi-terminal representation is briefly reviewed and illustrated by some numerical examples. In

Chapter III, this concept is used to develop the $\underline{Z}_{BUS}$ algorithm including mutual couplings. In Chapter IV, the analysis of short circuits and simultaneous faults are studied. Some of the results arrived at through extensive algebraic manipulation previously are shown to follow easily in a graph theoretic treatment. Finally, in Chapter V the work is summarized and suggestions for future work in the area are indicated.

CHAPTER II

MULTI TERMINAL REPRESENTATION OF POWER NETWORKS

2.1 Introduction

In the study of power systems we are often interested in obtaining a proper mathematical representation of the entire system or one of its subsystems at the buses. In graph theoretic language this is known as 'Multi-Terminal Representation' or M.T.R. in the abbreviated form.[4] In this approach a system is viewed as an assembly of subsystems, each of which can be independently characterized by a mathematical model. This model consists of a terminal tree graph and a set of terminal equations in the complementary current and voltage variables. Such a description is very useful when we are to solve a large system as an interconnection of several subsystems. An elementary knowledge of graph theory is necessary for the material that follows. It is not however intended to cover these in this report and references [1 - 3] are being suggested as an introduction to these concepts.

2.2 Tellegen's Theorem [4]

The proof of the 'Multi Terminal Representation' rests on a theorem known as 'Tellegen's Theorem' which is stated below:

"If $\underline{V}(t)$ and $\underline{I}(t)$ are the voltage and current matrices associated with an oriented graph G of an arbitrary lumped network, then

$$\underline{V}^T(t) \ \underline{I}(t) = 0, \qquad .. \ 2.1$$

provided that the entries $\underline{V}(t)$ and $\underline{I}(t)$ correspond to the same ordering of elements in G".

Stated in the above form, Tellegen's Theorem implies the concept of power invariance.

Proof:

Consider the cutset and circuit equations respectively of the graph G for a formulation tree T.

$$\underline{\mathcal{A}}\ \underline{I}(t) = 0 \qquad .. \quad 2.2$$

and $$\underline{\mathcal{B}}\ \underline{V}(t) = 0 \qquad .. \quad 2.3$$

It is known [1] that the current vector $\underline{I}(t)$ can be expressed as a linear combination of the co-tree current variables $\underline{I}_C(t)$ only:

$$\underline{I}(t) = \underline{\mathcal{B}}^T\ \underline{I}_C(t) \qquad .. \quad 2.4$$

and the voltage vector $\underline{V}(t)$ can be expressed as a linear combination of tree voltage variables $\underline{V}_T(t)$:

$$\underline{V}(t) = \underline{\mathcal{A}}^T\ \underline{V}_T(t) \qquad .. \quad 2.5$$

Substituting equation (4) and (5) in (1)

$$\underline{V}_T^{\,T}(t)\ \underline{I}(t) = \underline{V}_T^{\,T}(t)\,\underline{\mathcal{A}}\,\underline{\mathcal{B}}^T\ \underline{I}_C(t)$$

$$= \underline{V}^T(t)(\underline{\mathcal{A}}\,\underline{\mathcal{B}}^T)\ \underline{I}_C(t) \qquad .. \quad 2.6$$

Substituting equation 2.4 and 2.5 in 2.1, we get

$$\underline{\mathcal{A}}\,\underline{\mathcal{B}}^T = 0 \qquad .. \quad 2.7$$

we have from equation 2.6

$$\underline{V}^T(t)\ \underline{I}(t) = 0\ . \qquad .. \quad 2.8$$

It will be noted that the theorem is dependent solely on the orthogonality of the cutset and circuit vectors. In other words, the result is a direct consequence of the graph equations only. Since these equations are quite independent of the terminal equations, Tellegen's Theorem is valid for any lumped parameter network, whether it may be linear or non-linear, active or passive, time-invariant or time-varying. This, thus, confirms the concept of power invariance in any network.

In power systems, since the majority of the studies are concerned with the network in A.C. steady state, we are restricting our attention to these forms of voltages and currents only. This means that the components of $\underline{V}$ and $\underline{I}$ are phasors. The following corollaries of the Tellegen's theorem will be to our interest since these correspond to the invariance of total complex power in the network.

$$\underline{V}^{*T} \, \underline{I} = 0 \qquad .. \; 2.9a$$

$$\underline{V}^{T} \, \underline{I}^{*} = 0 \qquad .. \; 2.9b$$

The asterisk (*) denotes the complex conjugate operation.

2.3 Theorem on Multi Terminal Representation [4,7]

"The performance characteristics of a n-terminal network are completely specified by a set of (n-1) terminal equations in (n-1) pairs of oriented complementary current and voltage variables $\underline{V}$ and $\underline{I}$ identified by an arbitrarily chosen tree graph which is designated as a terminal graph".

Under the steady state conditions, the equivalence of a n-terminal network consisting of passive components only is established by (n-1) terminal equations of the form

$$\underline{V}_T = \underline{Z}_T \, \underline{I}_T$$

$$\underline{I}_T = \underline{Y}_T \, \underline{V}_T$$

Where $\underline{V}_T$ and $\underline{I}_T$ are the voltage and current vectors associated with the terminal graph of the n-terminal component and the $\underline{Z}_T$ and $\underline{Y}_T$ matrices represent the impedance and admittance parameter matrices. Components of $\underline{V}$ and $\underline{I}$ are phasors and components of $\underline{Z}_T$ and $\underline{Y}_T$ are complex numbers.

Proof of the Theorem

Let $\underline{V}_N$ and $\underline{I}_N$ be the voltage and current vectors associated with the graph N of the n-terminal component. Superpose a tree graph on N; this augmented tree is designated by the symbol A and let the voltage and current vectors associated with the graph A be $\underline{V}_A$ and $\underline{I}_A$ respectively. Now, let us consider the graph which is the union of N and A, namely $N \cup A$; the voltage and current vectors of this graph can be represented in the partitioned form

$$\underline{V} = \begin{bmatrix} \underline{V}_N \\ \hline \underline{V}_A \end{bmatrix} \quad \text{and} \quad \underline{I} = \begin{bmatrix} \underline{I}_N \\ \hline \underline{I}_A \end{bmatrix} \qquad .. \ 2.10$$

From the corollary of Tellegen's theorem, we have

$$\underline{V}^T \underline{I}^* = 0$$

i.e.

$$\begin{bmatrix} \underline{V}_N^T & \underline{V}_A^T \end{bmatrix} \begin{bmatrix} \underline{I}_N^* \\ \hline \underline{I}_A^* \end{bmatrix} = 0 \qquad .. \ 2.11$$

or finally

$$\underline{V}_A^T \underline{I}_A^* = -\underline{V}_N^T \underline{I}_N^* \qquad .. \ 2.12$$

Our purpose is to determine an equivalent graph which has the same power associated with it as the network graph N, defined by the inner product $\underline{V}_N^T \underline{I}_N^*$. This is easily identified when we introduce a set of complementary variables $\underline{V}_T$, $\underline{I}_T$ such that

$$\underline{V}_T = \underline{V}_A \quad \text{and} \quad \underline{I}_T = -\underline{I}_A \qquad .. \ 2.13$$

Thus equation 2.12 reduces to

$$\underline{V}_T^T \underline{I}_T^* = \underline{V}_N^T \underline{I}_N^* \qquad .. \ 2.14$$

From equation 2.13, it is evident that the variables $\underline{V}_T$ and $\underline{I}_T$ correspond to a tree graph which is topologically identical to the augmented tree graph A. We shall refer to this new tree graph as the terminal graph T. If the tree graph A consists of only voltage drivers or current drivers, the augmented network whose graph is given by N $\cup$ A can be solved to obtain a set of (n-1) terminal equations in the variables $\underline{V}_A$ and $\underline{I}_A$ either in impedance or admittance form. In terms of the new set of variables $\underline{V}_T$ and $\underline{I}_T$, these equations will appear as

$$\underline{V}_T = \underline{Z}_T \ I_T \qquad .. \ 2.15$$

or

$$\underline{I}_T = \underline{Y}_T \ \underline{V}_T \qquad .. \ 2.16$$

Equations 2.15 or 2.16 along with the terminal graph constitute a multi-terminal representation of the network. It is an exact mathematical equivalent of the original network because all the network voltages are determined by $\underline{V}_T$ from the circuit equations of the augmented graph N $\cup$ A. The network voltages constitute a cotree set of voltages with respect to the terminal graph. Once these are known, the network currents are determined from the terminal equations of the individual components.

It may be noted that the terminal representation of a n-terminal component is not unique. Equation 2.12 is valid for any augmented tree. Furthermore, the terminal equations for a given tree can be either in the form of Equation 2.15 or 2.16 or indeed in any mixed form.

2.4 An analytical derivation of parameter matrices of a network [4]

The admittance matrix corresponding to a Lagrangian terminal tree graph with common vertex identified by the ground bus is denoted as $\underline{Y}_{BUS}$. If the terminal graph is not restricted to a Lagrangian tree, the corresponding admittance matrix is denoted as $\underline{Y}_{BR}$. In the study of power systems, it is customary to designate a bus as the reference bus. The reference bus is usually the ground bus and all bus voltages are generally measured with reference to the ground bus. However, any other bus also could be designated as a reference bus, although we may lose the physical understanding of the Y parameters that are involved in the formulation of the problem. If all load, line charging and other shunt paths are neglected and if the ground bus is chosen as the reference vertex of the Lagrangian tree, then the terminal representation of a transmission network in the impedance form (i.e. $\underline{Z}_{BUS}$) does not exist. This is because of the fact that the ground bus forms an isolated vertex. If the augmented tree graph corresponds to current sources, we cannot choose a tree and yet have all specified current sources in the cotree. However $\underline{Y}_{BUS}$ does exist and the matrix in this case will be singular and hence indefinite. The rigorous proof of this is contained in reference [4]. With shunt paths neglected, if for

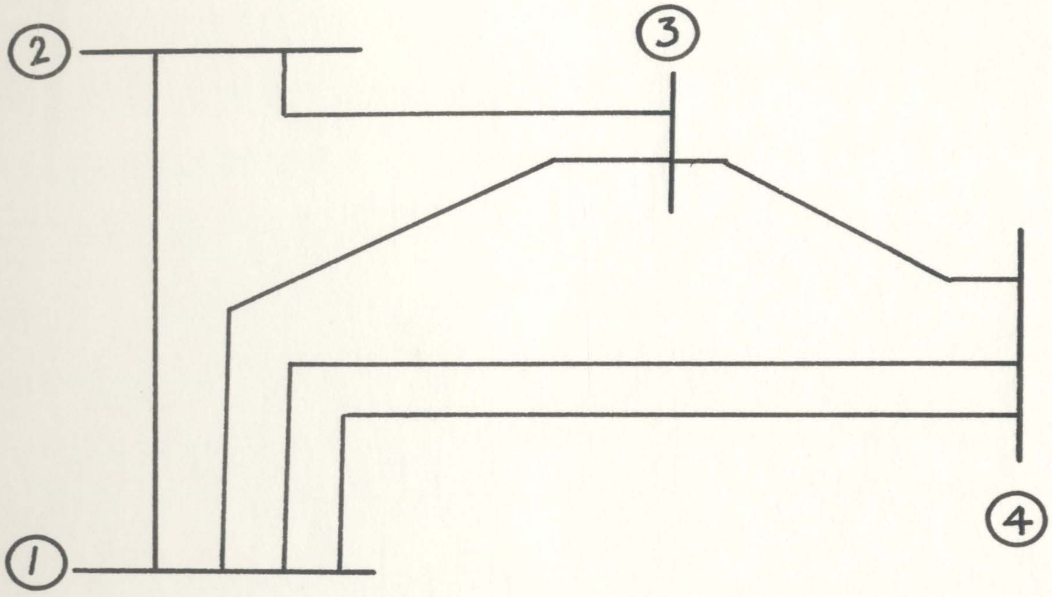


FIG. 1. TRANSMISSION NETWORK

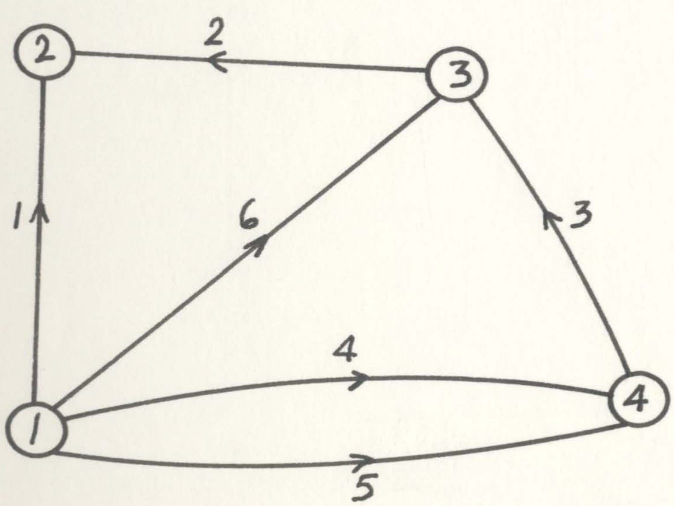


FIG. 2. ORIENTED GRAPH

the connected portion of the graph, a tree other than the Lagrangian tree is chosen as the terminal graph, then the corresponding Z representation is $\underline{Z}_{BR}$. Whereas, if a Lagrangian tree is chosen with slack bus as reference vertex, it is denoted by $\underline{Z}_{BN}$.

In the literature [5] the symbol $\underline{Z}_{BUS}$ is used whenever the terminal graph is a Lagrangian tree whether the ground bus is isolated or not. Such a terminology may tend to create a confusion. For our formulation, distinction will be made in the various cases as described above. We shall illustrate two specific formulations $\underline{Y}_{BUS}$ and $\underline{Z}_{BN}$ for a typical transmission system. The symbolic formulation of this is contained in reference [4].

2.5 An example for $\underline{Z}_{BN}$ formulation.

Let us consider a transmission network as shown in Fig. 1 whose line data are given in Table I.

Table I

Impedances

Element Number	Self		Mutual	
	Bus Code	Impedance	Bus Code	Impedance
1	1 - 2	j.10		
2	2 - 3	j.30		
3	3 - 4	j.50		
4	1 - 4 (1)	j.15	1 - 4 (1)	j.1
5	1 - 4 (2)	j.15	1 - 4 (1)	j.1
6	1 - 3	j.25		

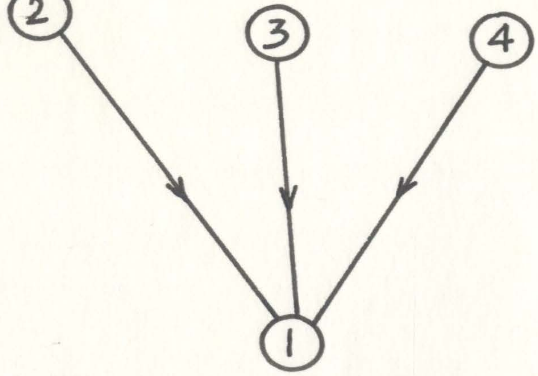


FIG. 3. DESIRED TERMINAL GRAPH

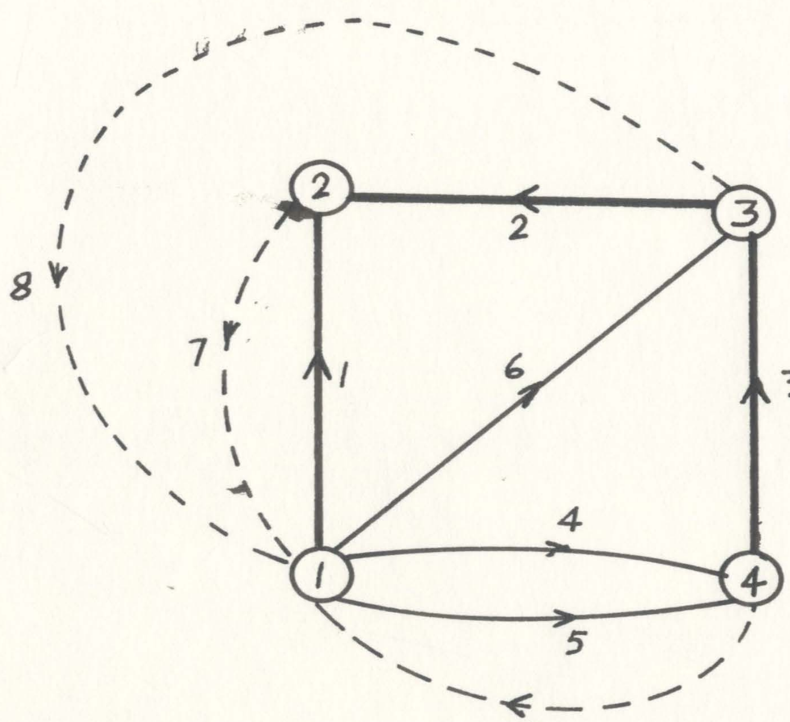


FIG. 4. AUGMENTED TERMINAL GRAPH

The oriented graph is shown in Fig. 2. In the first instance we desire to obtain the $\underline{Z}_{BN}$ of the network. Let (1) be the reference vertex. It is desired to obtain the multi-terminal representation in the impedance form corresponding to the terminal graph shown in Fig. 3. Since in this case we are interested in $\underline{Z}_{BN}$, all the buses are augmented by current sources whose graph is topologically identical to the terminal graph in Fig. 3 and the augmented graph as shown in Fig. 4 is obtained. A formulation tree T(1, 2, 3) is chosen. The circuit equations corresponding to the formulation tree are:

$$\left[\begin{array}{ccc:cccccc} -1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{c} V_1 \\ V_2 \\ V_3 \\ \hdashline V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{array}\right] = 0 \qquad \text{.. 2.17}$$

Equation 2.17 is re-arranged as follows:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ V_7 \\ V_8 \\ V_9 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 \\ 1 & -1 & -1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} \qquad .. \ 2.18$$

The cutset equations corresponding to the same formulation tree are:

$$\left[\begin{array}{ccc|cccccc} 1 & 0 & 0 & 1 & 1 & 1 & -1 & -1 & -1 \\ 0 & 1 & 0 & -1 & -1 & -1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{c} I_1 \\ I_2 \\ I_3 \\ \hline I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \\ I_9 \end{array}\right] = 0 \qquad .. \ 2.19$$

Equation 2.19 can be rewritten as follows:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & -1 & -1 \\ 1 & 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \\ I_9 \end{bmatrix} \quad .. \ 2.20$$

The terminal equations of the transmission network, as given in Table I, are written in the matrix form as follows:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} j.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & j.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & j.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & j.15 & j.1 & 0 \\ 0 & 0 & 0 & j.1 & j.15 & 0 \\ 0 & 0 & 0 & 0 & 0 & j.25 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} \quad .. \ 2.21$$

The mutual impedance between elements 4 and 5 is due to the physical proximity of the transmission lines involved. The fact that the elements are mutually coupled, does not alter the graph, but this is reflected in the terminal equations. For the orientations shown in Fig. 2, the mutual coupling term will be positive in the terminal equations.

Now, substituting equation 2.21 into equation 2.18, we get

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ V_7 \\ V_8 \\ V_9 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 \\ 1 & -1 & -1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} j.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & j.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & j.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & j.15 & j.1 & 0 \\ 0 & 0 & 0 & j.1 & j.15 & 0 \\ 0 & 0 & 0 & 0 & 0 & j.25 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} \qquad .. \; 2.22$$

Again substituting Equation 2.20 into Equation 2.22 , we get

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ V_7 \\ V_8 \\ V_9 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -1 & -1 & 0 & 0 \\ 1 & -1 & -1 & 0 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 & -1 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} j.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & j.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & j.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & j.15 & j.1 & 0 \\ 0 & 0 & 0 & j.1 & j.15 & 0 \\ 0 & 0 & 0 & 0 & 0 & j.25 \end{bmatrix} \begin{bmatrix} -1 & -1 & -1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & -1 & -1 \\ 1 & 1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \\ I_9 \end{bmatrix}$$

or .. 2.23

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ V_7 \\ V_8 \\ V_9 \end{bmatrix} = \begin{bmatrix} -j1.05 & -j1.0 & -j.4 & j.1 & j.4 & j.9 \\ -j1.0 & -j1.05 & -j.4 & j.1 & j.4 & j.9 \\ -j\,.4 & -j\,.4 & -j.65 & j.1 & j.4 & j.4 \\ j\,.1 & j\,.1 & j.1 & -j.1 & -j.1 & -j.1 \\ j\,.4 & j\,.4 & j.4 & -j.1 & -j.4 & -j.4 \\ j\,.9 & j\,.9 & j.4 & -j.1 & -j.4 & -j.9 \end{bmatrix} \begin{bmatrix} I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \\ I_9 \end{bmatrix} \qquad .. \; 2.24$$

From the first three equations of Equation 2.24 , we get

$$\begin{bmatrix} j1.05 & j1.0 & j.4 \\ j1.0 & j1.05 & j.4 \\ j\ .4 & j\ .4 & j.65 \end{bmatrix} \begin{bmatrix} I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} j.1 & j.4 & j.9 \\ j.1 & j.4 & j.9 \\ j.1 & j.4 & j.4 \end{bmatrix} \begin{bmatrix} I_7 \\ I_8 \\ I_9 \end{bmatrix}$$

or

$$\begin{bmatrix} I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} j1.05 & j1.0 & j.4 \\ j1.0 & j1.05 & j.4 \\ j\ .4 & j\ .4 & j.65 \end{bmatrix}^{-1} \begin{bmatrix} j.1 & j.4 & j.9 \\ j.1 & j.4 & j.9 \\ j.1 & j.4 & j.4 \end{bmatrix} \begin{bmatrix} I_7 \\ I_8 \\ I_9 \end{bmatrix}$$

$$= \begin{bmatrix} .03 & .12 & .47 \\ .03 & .12 & .47 \\ .11 & .50 & .10 \end{bmatrix} \begin{bmatrix} I_7 \\ I_8 \\ I_9 \end{bmatrix} \qquad ..\ 2.25$$

Substituting Equation 2.25 into the last three equations of 2.24, we get

$$\begin{bmatrix} V_7 \\ V_8 \\ V_9 \end{bmatrix} = \begin{bmatrix} j.1 & j.1 & j.1 \\ j.4 & j.4 & j.4 \\ j.9 & j.9 & j.4 \end{bmatrix} \begin{bmatrix} .03 & .12 & .47 \\ .03 & .12 & .47 \\ .11 & .50 & .10 \end{bmatrix} \begin{bmatrix} I_7 \\ I_8 \\ I_9 \end{bmatrix} + \begin{bmatrix} -j.1 & -j.1 & -j.1 \\ -j.1 & -j.4 & -j.4 \\ -j.1 & -j.4 & -j.9 \end{bmatrix} \begin{bmatrix} I_7 \\ I_8 \\ I_9 \end{bmatrix}$$

$$\begin{bmatrix} j.027 & j.072 & j.104 \\ j.072 & j.\ 30 & j.\ 42 \\ j.104 & j.\ 42 & j.\ 89 \end{bmatrix} \begin{bmatrix} I_7 \\ I_8 \\ I_9 \end{bmatrix} + \begin{bmatrix} -j.1 & -j.1 & -j.1 \\ -j.1 & -j.4 & -j.4 \\ -j.1 & -j.4 & -j.9 \end{bmatrix} \begin{bmatrix} I_7 \\ I_8 \\ I_9 \end{bmatrix}$$

$$= \begin{bmatrix} -j.073 & -j.028 & j.004 \\ -j.028 & -j.\ 1 & j.02 \\ j.004 & j.02 & -j.01 \end{bmatrix} \begin{bmatrix} I_7 \\ I_8 \\ I_9 \end{bmatrix} \qquad ..\ 2.26$$

Since the bus currents corresponding to the terminal graph described in Fig.3 are negatives of I_7, I_8 and I_9, $\underline{Z}_{BN}$ of the transmission network is given by Eq. 2.27.

$$\underline{Z}_{BN} = \begin{bmatrix} j.073 & j.028 & -j.004 \\ j.028 & j.1 & -j.02 \\ -j.004 & -j.02 & j.1 \end{bmatrix} \qquad ..\ 2.27$$

2.6 An example for $\underline{Y}_{BUS}$ formulation.

For $\underline{Y}_{BUS}$ formulation, let us, as an example, consider the same transmission network as shown in Fig. 1. By taking inverses of the line impedance datas, the admittance of the elements are obtained as in Table II.

TABLE II

Admittances

Element Number	Self		Mutual	
	Bus Code	Admittance	Bus Code	Admittance
1	1 - 2	-j10		
2	2 - 3	-j 3.3		
3	3 - 4	-j 2.0		
4	1 - 4 (1)	-j12	1 - 4 (1)	j8
5	1 - 4 (2)	-j12	1 - 4 (2)	j8
6	1 - 3	-j 4.0		

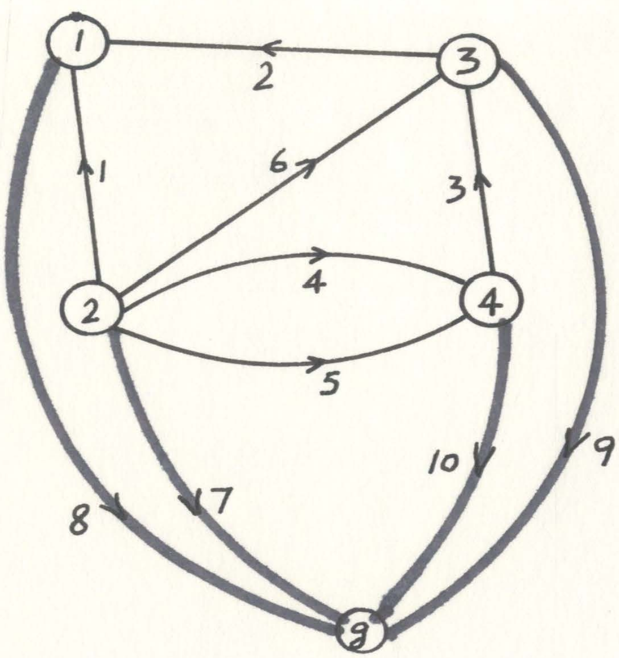


FIG. 5. AUGMENTED TERMINAL GRAPH

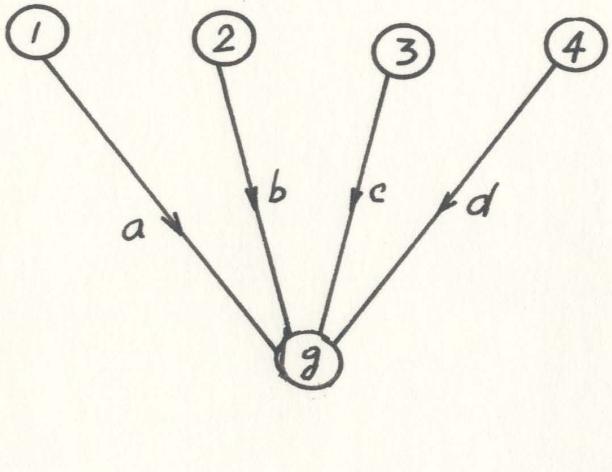


FIG. 6. DESIRED TERMINAL GRAPH

Since $\underline{Y}_{BUS}$ representation is desired, the oriented system graph is augmented with voltage sources with reference to the ground bus and the augmented graph as in Fig. 5 is derived. Our desired terminal graph is as shown in Fig. 6.

The cutset equations with respect to the formulation tree T(7,8,9,10) are

$$\left[\begin{array}{cccc|cccccc} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1 & -1 & 0 \end{array}\right] \left[\begin{array}{c} I_7 \\ I_8 \\ I_9 \\ I_{10} \\ \hline I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{array}\right] = 0 \qquad .. \ 2.28$$

From Eq. 2.28 we get

$$\begin{bmatrix} I_7 \\ I_8 \\ I_9 \\ I_{10} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & -1 & -1 & -1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} \qquad .. \ 2.29$$

The terminal equations of the transmission network, as detailed in Table II can be written in the following form.

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} -j10 & 0 & 0 & 0 & 0 & 0 \\ 0 & -j3.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -j2.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -j12 & j8 & 0 \\ 0 & 0 & 0 & j8 & -j12 & 0 \\ 0 & 0 & 0 & 0 & 0 & -j4.0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} \qquad .. \ 2.30$$

Substituting Eq. 2.30 into Eq. 2.29 we get

$$\begin{bmatrix} I_7 \\ I_8 \\ I_9 \\ I_{10} \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & -1 & -1 & -1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -j10 & 0 & 0 & 0 & 0 & 0 \\ 0 & -j3.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -j2.0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -j12 & j8 & 0 \\ 0 & 0 & 0 & j8 & -j12 & 0 \\ 0 & 0 & 0 & 0 & 0 & -j4.0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} \qquad .. \ 2.31$$

Again the circuit equations, for the formulation tree (7,8,9,10) are

$$\begin{bmatrix} -1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_7 \\ V_8 \\ V_9 \\ V_{10} \\ \hline V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = 0 \qquad .. \; 2.32$$

Rearranging Eq. 2.32 we get

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_7 \\ V_8 \\ V_9 \\ V_{10} \end{bmatrix} \qquad .. \; 2.33$$

Substituting Eq. 2.33 into Eq. 2.31

$$\begin{bmatrix} I_7 \\ I_8 \\ I_9 \\ I_{10} \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & -1 & -1 & -1 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -j10 & 0 & 0 & 0 & 0 & 0 \\ 0 & -j3.3 & 0 & 0 & 0 & 0 \\ 0 & 0 & -j2 & 0 & 0 & 0 \\ 0 & 0 & 0 & -j12 & j8 & 0 \\ 0 & 0 & 0 & j8 & -j12 & 0 \\ 0 & 0 & 0 & 0 & 0 & -j4 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} V_7 \\ V_8 \\ V_9 \\ V_{10} \end{bmatrix}$$

or

$$\begin{bmatrix} I_7 \\ I_8 \\ I_9 \\ I_{10} \end{bmatrix} = \begin{bmatrix} j22 & -j10 & -j4 & -j8 \\ -j10 & j13.3 & -j3.3 & 0 \\ -j4 & -j\,3.3 & j9.3 & -j2 \\ -j8 & 0 & -j2 & j10 \end{bmatrix} \begin{bmatrix} V_7 \\ V_8 \\ V_9 \\ V_{10} \end{bmatrix} \qquad .. \; 2.34$$

However, for the augmented graph (Fig. 5) and the desired terminal graph (Fig. 6), it will be noted that

$$\begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \end{bmatrix} = - \begin{bmatrix} I_7 \\ I_8 \\ I_9 \\ I_{10} \end{bmatrix} \; ; \text{ and } \; \begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \end{bmatrix} = \begin{bmatrix} V_7 \\ V_8 \\ V_9 \\ V_{10} \end{bmatrix}$$

therefore, the $\underline{Y}_{BUS}$ matrix is

$$\underline{Y}_{BUS} = \begin{bmatrix} -j22 & j10 & j4 & j8 \\ j10 & -j13.3 & j3.3 & 0 \\ j4 & j3.3 & -j9.3 & j2 \\ j8 & 0 & j2 & -j10 \end{bmatrix} . \qquad .. \; 2.35$$

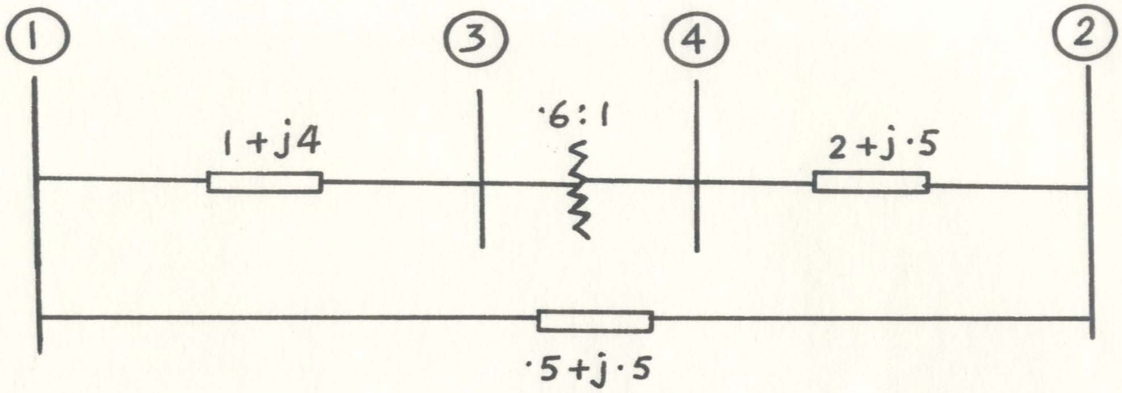


FIG. 7. TRANSMISSION SYSTEM WITH AUTO-TRANSFORMER

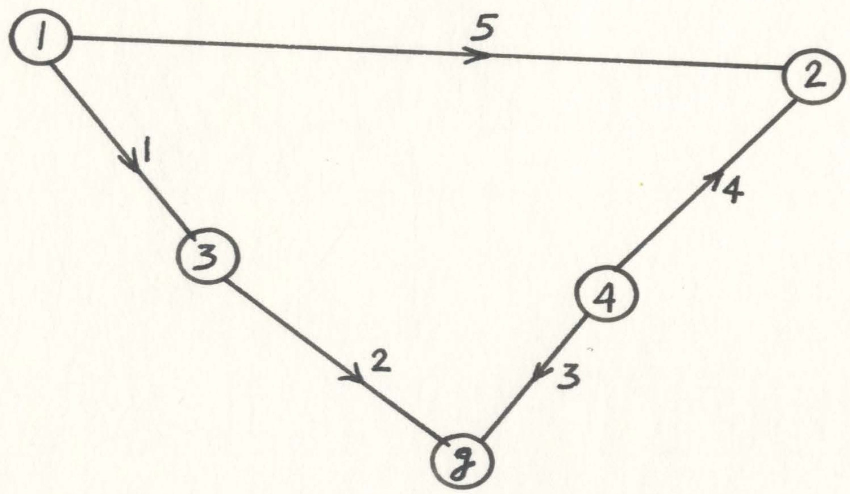


FIG. 8. ORIENTED GRAPH

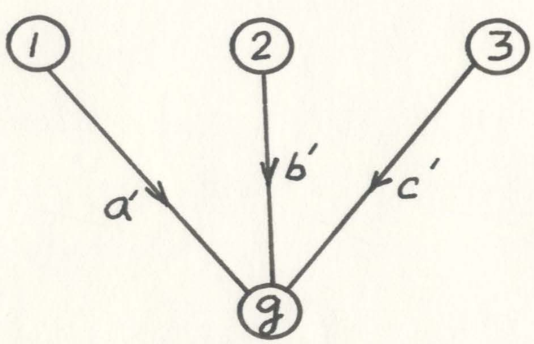


FIG. 9. DESIRED TERMINAL GRAPH

2.7 Derivation of Multi-terminal Representation for Systems Containing Auto-Transformers.

As has been discussed earlier, the multi-terminal representation provides an adequate basis to describe a n-terminal component. A multi-terminal representation can be derived for the various components of the power system e.g. generators, transformers, transmission lines, etc. When the M.T.R.s of these individual components are interconnected in accordance with their physical connections, a M.T.R. of the complete system can again be derived at the desired buses. As an example we shall describe the derivation of $\underline{Z}_{BUS}$ of a system containing an auto-transformer. For this purpose let us consider a system [12] as described in Fig. 7. Fig.8 shows the oriented graph of the system. The component terminal equations, as obtained from the data in the system diagram are given by equations 2.36 and 2.37, where 'a' is the turns ratio which in this case is 0.6.

$$\begin{bmatrix} V_1 \\ V_5 \\ V_4 \end{bmatrix} = \begin{bmatrix} Z_1 & 0 & 0 \\ 0 & Z_5 & 0 \\ 0 & 0 & Z_4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_5 \\ I_4 \end{bmatrix} = \begin{bmatrix} (1+j4) & 0 & 0 \\ 0 & (.5+j.5) & 0 \\ 0 & 0 & (2+j.5) \end{bmatrix} \begin{bmatrix} I_1 \\ I_5 \\ I_4 \end{bmatrix} \quad .. \ 2.36$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & -\frac{1}{a} \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} = \begin{bmatrix} .6 & 0 \\ 0 & -.166 \end{bmatrix} \begin{bmatrix} V_3 \\ I_3 \end{bmatrix} \quad .. \ 2.37$$

It is desired to obtain a terminal representation in the $\underline{Z}_{BUS}$ form corresponding to the terminal graph as shown in Fig. 9. For this

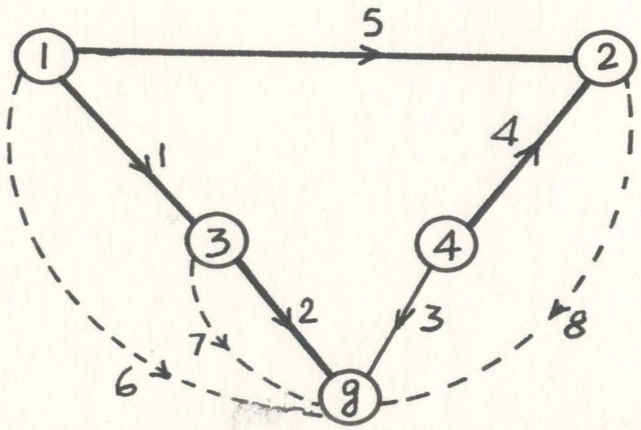


FIG. 10. ORIENTED AUGMENTED TERMINAL GRAPH

purpose the buses are augmented by current sources and the augmented graph as described in Fig. 10 results. The circuit and the cutset equations with respect to the formulation tree T(2, 1, 4, 5) are given by Eq. 2.38 and 2.39 respectively.

$$\left[\begin{array}{cccc:cccc} -1 & -1 & -1 & 1 & 1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & -1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \left[\begin{array}{c} V_2 \\ V_1 \\ V_4 \\ V_5 \\ \hdashline V_3 \\ V_6 \\ V_7 \\ V_8 \end{array}\right] = 0 \qquad .. \ 2.38$$

$$\left[\begin{array}{cccc:cccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 & -1 \end{array}\right] \left[\begin{array}{c} I_2 \\ I_1 \\ I_4 \\ I_5 \\ \hdashline I_3 \\ I_6 \\ I_7 \\ I_8 \end{array}\right] = 0 \qquad .. \ 2.39$$

Rearranging Eq. 2.38, we get

$$\begin{bmatrix} V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} V_2 + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_4 \\ V_5 \end{bmatrix} \qquad .. \ 2.40$$

Substituting for V_3 from Eq. 2.37 into the first equation of Eq. set 2.30, we get

$$-V_1 - V_4 + V_5 = V_2 - V_3 = \frac{a-1}{a} V_2 \qquad .. \ 2.41$$

Substituting Eq. 2.41 into Eq. 2.40

$$\begin{bmatrix} V_6 \\ V_7 \\ V_8 \end{bmatrix} = \frac{a}{a-1} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} [-1 \quad -1 \quad 1] \begin{bmatrix} V_1 \\ V_4 \\ V_5 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_4 \\ V_5 \end{bmatrix}$$

$$= \frac{a}{a-1} \begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_4 \\ V_5 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_4 \\ V_5 \end{bmatrix} \qquad .. \ 2.42$$

Now from the cutset Eq. 2.39, we get

$$\begin{bmatrix} I_1 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} -1 & -1 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_3 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix}$$

or

$$\begin{bmatrix} I_1 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} I_3 + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix} \qquad .. \; 2.43$$

Substituting for I_2 from Eq. 2.37 into the first equation of the Eq. set 2.39, we get

$$I_2 + I_3 = -(I_6 + I_7 + I_8)$$

or

$$I_3 = -\frac{a}{a-1}(I_6 + I_7 + I_8) \qquad .. \; 2.44$$

Substituting Eq. 2.44 into Eq. 2.43; we get

$$\begin{bmatrix} I_1 \\ I_4 \\ I_5 \end{bmatrix} = \frac{a}{a-1} \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} [-1 \;\; -1 \;\; -1] \begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix}$$

$$= \frac{a}{a-1} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix} \qquad .. \; 2.45$$

Again substituting Eq. 2.36 into Eq. 2.42, we get

$$\begin{bmatrix} V_6 \\ V_7 \\ V_8 \end{bmatrix} = \frac{a}{a-1} \begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} Z_1 & 0 & 0 \\ 0 & Z_4 & 0 \\ 0 & & Z_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_4 \\ I_5 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} Z_1 & 0 & 0 \\ 0 & Z_4 & 0 \\ 0 & & Z_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_4 \\ I_5 \end{bmatrix}$$

.. 2.46

substituting Equation 2.45 into Equation 2.46

$$\begin{bmatrix} V_6 \\ V_7 \\ V_8 \end{bmatrix} = \frac{a}{a-1}\begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} Z_1 & 0 & 0 \\ 0 & Z_4 & 0 \\ 0 & 0 & Z_5 \end{bmatrix}\left\{ \frac{a}{a-1}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix}\begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix} \right.$$

$$\left. + \begin{bmatrix} -1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix} \right\} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} Z_1 & 0 & 0 \\ 0 & Z_4 & 0 \\ 0 & 0 & Z_5 \end{bmatrix}\left\{ \frac{a}{a-1}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix}\begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix} \right.$$

$$\left. + \begin{bmatrix} -1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix} \right\} \qquad .. \; 2.47$$

or

$$\begin{bmatrix} V_6 \\ V_7 \\ V_8 \end{bmatrix} = \frac{a^2}{(a-1)^2}\begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} Z_1 & 0 & 0 \\ 0 & Z_4 & 0 \\ 0 & 0 & Z_5 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix}\begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix}$$

$$+ \frac{a}{a-1}\begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}\begin{bmatrix} Z_1 & 0 & 0 \\ 0 & Z_4 & 0 \\ 0 & 0 & Z_5 \end{bmatrix}\begin{bmatrix} -1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix}$$

$$+ \frac{a}{a-1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} Z_1 & & \\ & Z_4 & \\ & & Z_5 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix}$$

$$+ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} Z_1 & & \\ & Z_4 & \\ & & Z_5 \end{bmatrix} \begin{bmatrix} -1 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix} \qquad .. \; 2.48$$

Equation 2.48 can be written in a form

$$\underline{V} = \left[- \alpha^2 \underline{A}\,\underline{Z}\,\underline{A}^T - \alpha\,\underline{A}\,\underline{Z}\,\underline{B}^T - \alpha\,\underline{B}\,\underline{Z}\,\underline{A}^T - \underline{B}\,\underline{Z}\,\underline{B}^T \right] \underline{I}$$

$$= - \left[(\alpha\,\underline{A} + \underline{B})\,\underline{Z}\,(\alpha\,\underline{A}^T + \underline{B}^T) \right] \underline{I} \qquad .. \; 2.49$$

where

$$\underline{V} = \begin{bmatrix} V_6 \\ V_7 \\ V_8 \end{bmatrix} ; \quad \alpha = \frac{a}{a-1} \qquad \underline{A} = \begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix} ; \qquad \underline{Z} = \begin{bmatrix} Z_1 & & \\ & Z_4 & \\ & & Z_5 \end{bmatrix}$$

On simplification of Eq. 2.48 and substituting for numerical values of α and $\underline{Z}$ into Eq. 2.48, we get

$$\begin{bmatrix} V_6 \\ V_7 \\ V_8 \end{bmatrix} = \left(\frac{a}{a-1}\right)^2 \begin{bmatrix} -Z_1 -Z_4 -Z_5 & -Z_1 -Z_4 -Z_5 & -Z_1 -Z_4 -Z_5 \\ -Z_1 -Z_4 -Z_5 & -Z_1 -Z_4 -Z_5 & -Z_1 -Z_4 -Z_5 \\ -Z_1 -Z_4 -Z_5 & -Z_1 -Z_4 -Z_5 & -Z_1 -Z_4 -Z_5 \end{bmatrix} \begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix}$$

$$+ \frac{a}{a-1} \begin{bmatrix} Z_1 & 0 & Z_1+Z_5 \\ Z_1 & 0 & Z_1+Z_5 \\ Z_1 & 0 & Z_1+Z_5 \end{bmatrix} \begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix}$$

$$+ \begin{bmatrix} Z_1 & Z_1 & Z_1 \\ 0 & 0 & 0 \\ Z_1+Z_5 & Z_1+Z_5 & Z_1+Z_5 \end{bmatrix} \begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix} + \begin{bmatrix} -Z_1 & 0 & -Z_1 \\ 0 & 0 & 0 \\ -Z_1 & 0 & -Z_1-Z_5 \end{bmatrix} \begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix}$$

$$\begin{bmatrix} V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} -11.82-j27.25 & -9.32-j17.25 & -12.57-j28 \\ -9.32-j17.25 & -7.82-j11.25 & -10.07-j18.0 \\ -12.57-j28.0 & -10.07-j18.0 & -13.82-j29.25 \end{bmatrix} \begin{bmatrix} I_6 \\ I_7 \\ I_8 \end{bmatrix} \quad .. \ 2.50$$

However, since $\begin{bmatrix} V_6 \\ V_7 \\ V_8 \end{bmatrix} = - \begin{bmatrix} V_a' \\ V_b' \\ V_c' \end{bmatrix}$

we get

$$\begin{bmatrix} V_a' \\ V_b' \\ V_c' \end{bmatrix} = \begin{bmatrix} 11.82+j27.25 & 9.32+j17.25 & 12.57+j28.0 \\ 9.32+j17.25 & 7.82+j11.25 & 10.07+j18.0 \\ 12.57+j28 & 10.07+j18.0 & 13.82+j29.25 \end{bmatrix} \begin{bmatrix} I_a' \\ I_b' \\ I_c' \end{bmatrix}$$

$$\text{or } \underline{Z}_{BUS} = \begin{bmatrix} 11.82+j27.25 & 9.32+j17.25 & 12.57+j28 \\ 9.32+j17.25 & 7.82+j11.25 & 10.07+j18.0 \\ 12.57+j28 & 10.07+j18 & 13.82+j29.25 \end{bmatrix} \quad .. \; 2.51$$

These results are in conformity with those in reference [12].

2.8 Conclusion.

In this chapter, multi-terminal representation has been derived for transmission systems with and without off-nominal transformers. In short circuit studies we are mostly interested in obtaining $\underline{Z}_{BN}$ of the transmission systems. This is because, in short circuit studies, from physical considerations all loads, line charging, etc., are neglected, and all transformers are assumed to have nominal tap ratios. We have also noted in this chapter that for deriving the multi terminal representation, manipulation and inversion of matrices are required. For large systems particularly, one would naturally like to avoid such matrix manipulations. In the next chapter we shall describe a procedure such that the matrix manipulation is reduced to a minimum in the derivation of $\underline{Z}_{BN}$.

CHAPTER III

BUILDING ALGORITHM FOR $\underline{Z}_{BUS}$ MATRICES

3.1 Introduction

The ever increasing size of the present day power systems impose a great practical limitation on their analysis by the conventional computing techniques. For many utilities, systems having 2000 or more buses are not uncommon. Solution of such large networks require a very large amount of core storage and the computational time becomes prohibitively high. Although larger computers are being made available and faster computing techniques are being developed, it is also true that the power systems network size is also increasing at a fast rate. With the present day emphasis on interconnection and power pooling arrangements amongst the utilities, it appears, this trend is likely to continue. For solution of networks, whether large or small, as has been discussed in Chapter II, manipulation and inversion of matrices are necessary. Matrix inversion and manipulation for even small networks involve a fair amount of storage and computation time. To obviate this, considerable amount of work has been done [5,8,11], in obtaining $\underline{Z}_{BUS}$ and $\underline{Y}_{BUS}$ by a step by step or algorithmic approach. The underlying principle in this approach is the formulation of the appropriate impedance or admittance matrix of a network in stages i.e. by adding one element to the network at a time. This, in other words, amounts to a repeated application of the concept of multi-terminal representation. This multi-stage process eliminates the need for inversion of large matrices. Such algorithms developed by other researchers [5,8,9,10,11], rely on intitutive or conventional concepts, such as, injection of currents, column elimination etc. Conceptually, these arguments are at times difficult to comprehend fully or are rather involved in terms of symbologies encountered.

In this chapter an attempt is made to develop the $\underline{Z}_{BUS}$ algorithm using the concept of multi-terminal representation. It is felt that this provides a proper conceptual framework for a study of power networks since it relies less on heuristic arguments. On the basis of development in this chapter, extensions can be made to include features for modification of the Z-matrix.

3.2 $\underline{Z}_{BUS}$ algorithms - some preliminary comments.

Power system networks contain, as a rule, a large number of transmission lines, some of which are mutually coupled. The number of mutually coupled lines depends on various factors e.g. size of the system, availability of right-of-ways, the need to install parallel lines, etc. It is possible that there may exist several coupled groups in the system - a coupled group being defined as one where each line is coupled with at least another line in the group. If the line list is ordered such that the elements belonging to a coupled group appear together then the primitive impedance matrix can be cast in the form:

$$\underline{Z} = \left[\begin{array}{c:c:c:c} \underline{Z}_1 & \underline{0} & \cdot\cdot & \underline{0} \\ \hdashline \underline{0} & \underline{Z}_2 & \cdot\cdot & \underline{0} \\ \hdashline \cdot & \cdot & \cdot\cdot & \cdot \\ \cdot & \cdot & \cdot\cdot & \cdot \\ \hdashline \underline{0} & \underline{0} & \cdot\cdot & \underline{Z}_n \end{array}\right]$$

Where $\underline{Z}_1$ corresponds to the lines which are uncoupled and hence is diagonal. $\underline{Z}_2 \ldots \underline{Z}_n$ are submatrices corresponding to the coupled groups and therefore they are not in diagonal form. Such a classification is helpful in handling large systems. In this chapter the algorithm for building the $\underline{Z}_{BN}$-matrix will be discussed. The general formulation is first carried out in symbolic terms. The actual algorithm will be restricted to the case where the coupled group contains no more than two lines. For medium to small scale utilities, this can be considered as adequate. In assembling the Z-matrix the uncoupled lines are processed first and then the coupled groups are processed. By appropriate coding it is not difficult to list the lines in this manner. For the uncoupled lines the following cases are considered.

i) Addition of a branch creating a new node.

ii) Addition of a link which does not create a new node.

In case (i) the modified Z-matrix will be increased in size by one. In case (ii) the order of the new Z-matrix remains the same. In processing coupled lines, the symbolic formulation is indicated when a coupled group of any order is added to an existing network. In the

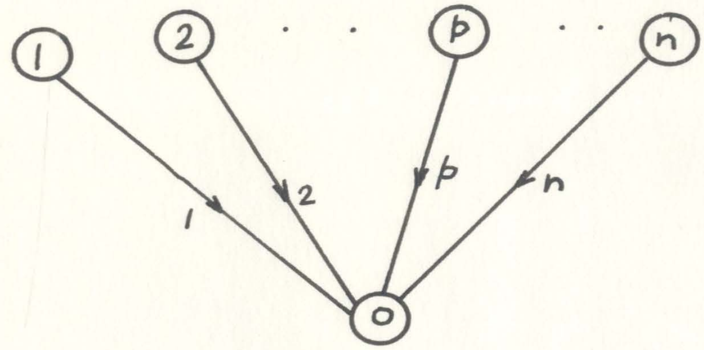


FIG. 11. TERMINAL GRAPH OF ORIGINAL SYSTEM

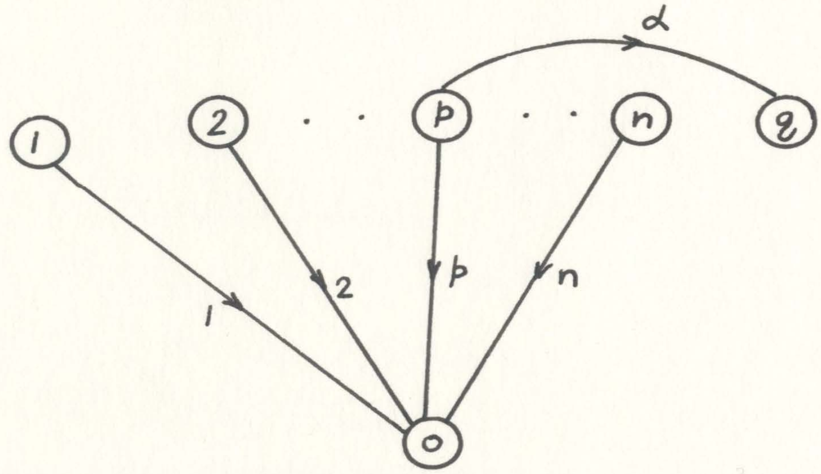


FIG. 12. SYSTEM GRAPH WITH ELEMENT α ADDED

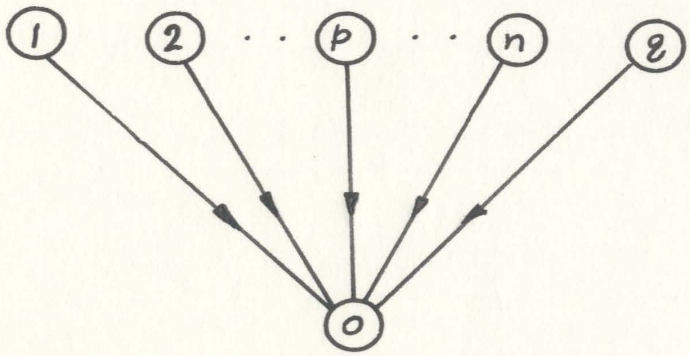


FIG. 13. DESIRED TERMINAL GRAPH OF MODIFIED SYSTEM

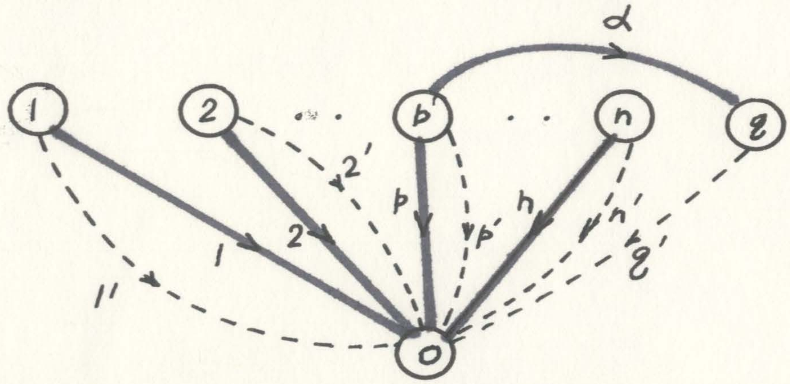


FIG. 14. AUGMENTED SYSTEM GRAPH

specific instances when (i) two coupled lines create two new nodes and (ii) two coupled lines constitute links, the nature of the modified Z-matrix will be exhibited.

3.3 Case I: Addition of a branch, not mutually coupled to an existing network.

Let us consider a transmission system having a terminal representation

$$\underline{V}_{BUS(0)} = \underline{Z}_{BUS(0)} \underline{I}_{BUS(0)}$$

corresponding to the Lagrangian tree as shown in Fig. 11. The reference vertex is the node 0 . Let a new element α be added from node p to a new bus denoted by q . Thus the system graph with the added element α will be as in Fig. 12. In the case of α , the orientation can be chosen arbitrarily. However, for consistency in later representation we shall assume the orientation to be from p to q . Thus our aim is to derive $\underline{Z}_{BUS(m)}$, which represents the $\underline{Z}_{BUS}$ of the modified network. The terminal graph representation of $\underline{Z}_{BUS(m)}$ will be of the form as shown in Fig. 13.

To derive the Z-representation of the system, the terminal graph as well as the system graph is augmented with current sources. The augmented system graph is shown in Fig. 14. The formulation tree is shown in thick lines, corresponding to elements 1, 2 ... n, α . Since the added element is a branch - which creates a new node - it is apparent that the order of the $\underline{Z}_{BUS(m)}$ matrix would be increased by one. Let the $\underline{Z}_{BUS}$ of original network be denoted by:

$$\underline{Z}_{BUS(0)} = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1p} & \cdots & z_{1n} \\ z_{21} & z_{22} & \cdots & z_{2p} & \cdots & z_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ z_{p1} & z_{p2} & \cdots & z_{pp} & \cdots & z_{pn} \\ \vdots & \vdots & & \vdots & & \vdots \\ z_{n1} & z_{n2} & \cdots & z_{pn} & \cdots & z_{nn} \end{bmatrix} \qquad .. \quad 3.1$$

Let $z_{\alpha\alpha}$ represent the self-impedance of the element α . The circuit and cutset equation with respect to the formulation tree of the augmented graph in Fig. 14 would then be given by Eq. 3.2 and 3.3 respectively.

$$\left[\begin{array}{ccccccc:c:c} & & & -\underline{U} & & & & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} & \underline{U} \\ \hdashline 0 & \cdot & \cdot & -1_p & \cdot & \cdot & 0 & 1 & \end{array}\right] \left[\begin{array}{c} \underline{V}_T \\ V_\alpha \\ \hdashline \underline{V}_C \end{array}\right] = 0 \qquad .. \quad 3.2$$

$$\left[\begin{array}{c:ccc:c} \underline{U} & & \underline{U} & & \begin{matrix} 0 \\ \vdots \\ 1_p \\ \vdots \\ 0 \end{matrix} \\ \hdashline & 0 & \cdots & 0 & -1 \end{array}\right] \left[\begin{array}{c} \underline{I}_T \\ I_\alpha \\ \hdashline \underline{I}_C \end{array}\right] = 0 \qquad .. \quad 3.3$$

Where

$$\begin{bmatrix} \underline{V}_T \\ \text{----} \\ V_\alpha \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ V_p \\ \cdot \\ \cdot \\ V_n \\ \text{----} \\ V_\alpha \end{bmatrix} \quad ; \quad \underline{V}_C = \begin{bmatrix} V_1{}' \\ V_2{}' \\ \\ V_p{}' \\ \\ V_n{}' \\ V_q{}' \end{bmatrix}$$

and

$$\begin{bmatrix} \underline{I}_T \\ I_\alpha \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ I_p \\ \cdot \\ \cdot \\ I_n \\ I_\alpha \end{bmatrix} \quad ; \quad \underline{I}_C = \begin{bmatrix} I_1{}' \\ I_2{}' \\ \\ I_p{}' \\ \\ I_n{}' \\ I_q{}' \end{bmatrix}$$

The notation 1_p is used to indicate the position 1 in the p^{th} row or column as the case may be.

The component equations are

$$\begin{bmatrix} \underline{V}_T \\ V_\alpha \end{bmatrix} = \left[\begin{array}{c|c} \underline{Z}_{BUS(0)} & \underline{0} \\ \hline \underline{0} & z_{\alpha\alpha} \end{array}\right] \begin{bmatrix} \underline{I}_T \\ I_\alpha \end{bmatrix} \qquad .. \quad 3.4$$

From equation 3.2 we express $\underline{V}_C$ in terms of $\underline{V}_T$ and V_α and making use of equation 3.4 and 3.3 we finally get

$$\underline{V}_C = - \left[\begin{array}{c|c} -\underline{U} & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} \\ \hline 0 \, ..-1_p .. 0 & 1 \end{array}\right] \left[\begin{array}{c|c} \underline{Z}_{BUS(0)} & \underline{0} \\ \hline \underline{0} & z_{\alpha\alpha} \end{array}\right] \left[\begin{array}{c|c} -\underline{U} & \begin{matrix} 0 \\ \vdots \\ -1_p \\ \vdots \end{matrix} \\ \hline 0 \, ..0 \, .. \, 0 & 1 \end{array}\right] \begin{bmatrix} \underline{I}_C \end{bmatrix}$$

$$= \left[\begin{array}{c|c} \underline{Z}_{BUS(0)} & \begin{matrix} z_{1p} \\ \vdots \\ z_{np} \end{matrix} \\ \hline z_{p1} \cdots z_{pn} & z_{pp} + z_{\alpha\alpha} \end{array}\right] \begin{bmatrix} \underline{I}_C \end{bmatrix} \qquad .. \quad 3.5$$

Since the bus currents corresponding to the terminal graph of the modified systems are negative of $\underline{I}_C$, we conclude that

$$\underline{Z}_{BUS(m)} = \left[\begin{array}{c|c} \underline{Z}_{BUS(0)} & \begin{matrix} z_{1p} \\ \vdots \\ z_{np} \end{matrix} \\ \hline z_{p1} \cdots z_{pn} & z_{pp} + z_{\alpha\alpha} \end{array}\right] \qquad .. \quad 3.6$$

Since $\underline{Z}_{BUS(0)}$ is a symmetric matrix, it is trivially noted that $z_{pi} = z_{ip}$ $(i = 1, 2 \ldots n)$. Thus it is noted that when a new branch is added, to determine the new bus impedance matrix, only the calculation of one additional row or column is required.

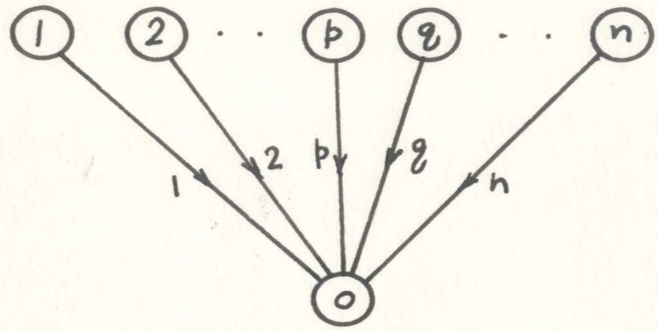


FIG. 15. TERMINAL GRAPH OF TRANSMISSION NETWORK

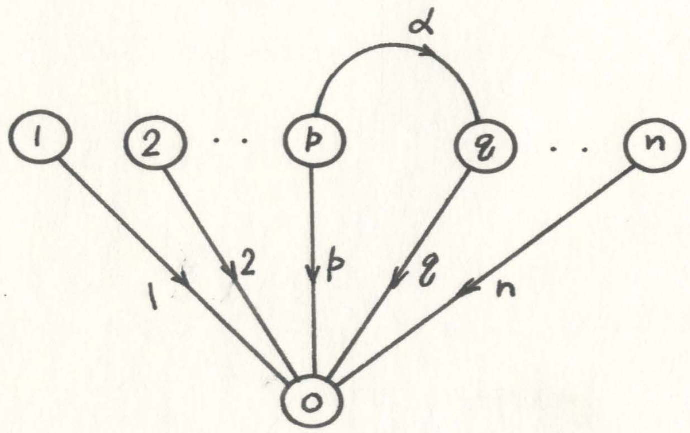


FIG. 16. MODIFIED SYSTEM GRAPH

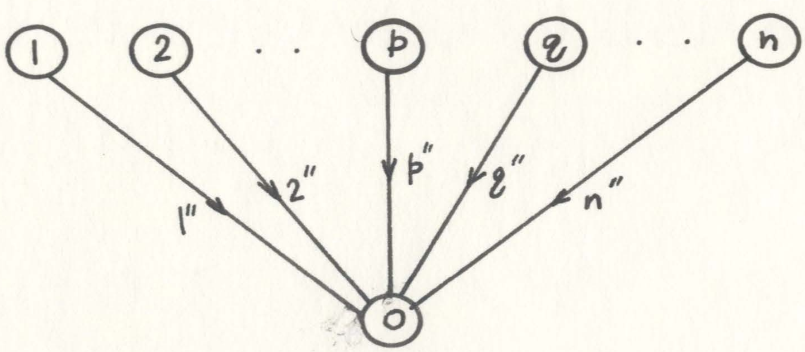
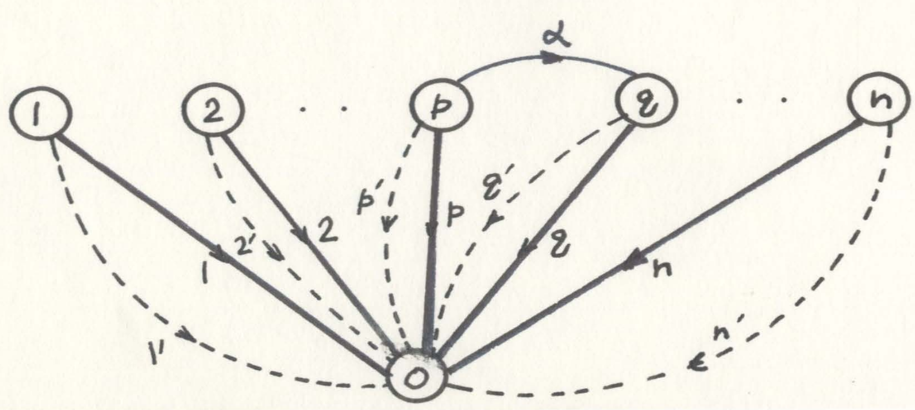


FIG. 17. TERMINAL GRAPH OF MODIFIED SYSTEM



3.4 Case II: Addition of a Link, Not Mutually Coupled to an Existing Network.

The algorithm for obtaining $\underline{Z}_{BUS(m)}$ of a network which is extended by a link, can be derived following the same procedure as in the previous case. However, in this case, since the added element is a link and thus no new node has been created, the order of $\underline{Z}_{BUS(m)}$ matrix would be the same as $\underline{Z}_{BUS(0)}$.

Let the terminal graph as shown in Fig. 15 correspond to $\underline{Z}_{BUS(0)}$. Let a new element be added between nodes p and q with an orientation from p to q. Thus the new system graph as shown in Fig. 16 results. We desire to derive $\underline{Z}_{BUS(m)}$ to represent the impedance matrix of the modified network, the terminal graph for which is shown in Fig. 17.

Since a Z representation is desired, the system graph of Fig. 16 is augmented with current sources as shown in Fig. 18. The formulation tree is shown in thick lines. Let $z_{\alpha\alpha}$ represent the self-impedance of the added element and

$$\underline{Z}_{BUS(0)} = \begin{bmatrix} z_{11} & z_{12} & \cdot\ \cdot & z_{1p} & z_{1q} & \cdot\ \cdot & z_{1n} \\ z_{21} & z_{22} & \cdot\ \cdot & z_{2p} & z_{2q} & \cdot\ \cdot & z_{2n} \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ z_{p1} & z_{p2} & \cdot\ \cdot & z_{pp} & z_{pq} & \cdot\ \cdot & z_{pn} \\ z_{q1} & z_{q2} & \cdot\ \cdot & z_{qp} & z_{qq} & \cdot\ \cdot & z_{qn} \\ \vdots & \vdots & & \vdots & \vdots & & \vdots \\ z_{n1} & z_{n2} & \cdot\ \cdot & z_{np} & z_{nq} & \cdot\ \cdot & z_{nn} \end{bmatrix} \qquad .. \quad 3.7$$

The circuit and the cutset equation with respect to the formulation tree of the augmented graph in Fig. 18 in this instance are given by Eq. 3.8 and 3.9 respectively.

$$\left[\begin{array}{c:c:c} 0 \; . \; . \; -1_p \; 1_q \; . \; . \; 0 & 1 & 0 \; . \; . \; . \; 0 \\ \hdashline & 0 & \\ -\underline{U} & \vdots & \underline{U} \\ & 0 & \end{array}\right] \left[\begin{array}{c} \underline{V}_T \\ \hline V_\alpha \\ \hline \underline{V}_C \end{array}\right] = 0 \qquad .. \quad 3.8$$

$$\left[\begin{array}{c:c:c} & 0 & \\ & \cdot & \\ & \cdot & \\ \underline{U} & 1_p & \underline{U} \\ & -1_q & \\ & \cdot & \\ & \cdot & \\ & 0 & \end{array}\right] \left[\begin{array}{c} \underline{I}_T \\ \hline I_\alpha \\ \hline \underline{I}_C \end{array}\right] = 0 \qquad .. \quad 3.9$$

Where

$$\underline{V}_T = \begin{bmatrix} V_1 \\ V_2 \\ \cdot \\ \cdot \\ V_p \\ V_q \\ \cdot \\ \cdot \\ V_n \end{bmatrix} ; \qquad \underline{I}_T = \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ I_p \\ I_q \\ \cdot \\ \cdot \\ I_n \end{bmatrix}$$

$$\begin{bmatrix} V_\alpha \\ \underline{V}_C \end{bmatrix} = \begin{bmatrix} V_\alpha \\ \hline V_1' \\ V_2' \\ \cdot \\ \cdot \\ V_p' \\ V_q' \\ \cdot \\ \cdot \\ V_n' \end{bmatrix} ; \qquad \begin{bmatrix} I_\alpha \\ \underline{I}_C \end{bmatrix} = \begin{bmatrix} I_\alpha \\ \hline I_1' \\ I_2' \\ \cdot \\ \cdot \\ I_p' \\ I_q' \\ \cdot \\ \cdot \\ I_n' \end{bmatrix}$$

Performing the substitution procedures of the chord formulation method, we get

$$\begin{bmatrix} 0 \\ \underline{V}_C \end{bmatrix} = - \underline{B}_1 \, \underline{Z} \, B_1^T \begin{bmatrix} I_\alpha \\ \underline{I}_C \end{bmatrix} \qquad .. \; 3.10$$

where $\underline{B}_1 = \left[\begin{array}{c|c} 0 \; . \; . \; -1_p \; 1_q \; . \; . \; 0 & 1 \\ \hline & 0 \\ & \cdot \\ & \cdot \\ -\underline{U} & 0 \end{array}\right]$

Performing the triple matrix product and substituting for I_α from the top equation into the bottom set of equations and finally noting that $\underline{I}_C$ is the negative of the bus currents, the new terminal equations as follows are obtained.

$$\underline{V}_{BUS} = \underline{Z}_{BUS(m)} \; \underline{I}_{BUS} \; . \qquad .. \; 3.11$$

Where

$$\underline{Z}_{BUS(m)} = \underline{Z}_{BUS(0)} + \frac{1}{\hat{z}} \begin{bmatrix} z_{p1}-z_{q1} \\ z_{p2}-z_{q2} \\ \vdots \\ z_{pp}-z_{qp} \\ z_{pq}-z_{qq} \\ \vdots \\ z_{pn}-z_{qn} \end{bmatrix} \left[(z_{p1}-z_{q1}) \; (z_{p2}-z_{q2}) .. (z_{pp}-z_{pq}) \; (z_{pq}-z_{qq}) .. (z_{pn}-z_{q} \right. \quad .. \; 3.12$$

where $\hat{z} = 2 z_{pq} - z_{pp} - z_{qq} - z_{\alpha\alpha}$.

The algorithms in both Case I and Case II for adding a branch and a link not coupled to the existing network agree with the results in the literature [5] .

3.5 Addition of a Mutually Coupled Group.

As was mentioned earlier, while assembling a Z_{BUS} matrix, for the sake of convenience it is desirable to arrange the lines belonging to a mutually coupled group consecutively. For utilities in urban areas several lines may comprise a coupled group. However, for small and medium size utilities covering a scattered area, a coupled group of two lines only is not uncommon. In the developments that follow the addition of an arbitrary number of mutually coupled lines is considered which corresponds to branches as well as links. The symbolic formulation is then illustrated for the specific case of two lines in a group.

Let the partial network have a Z representation

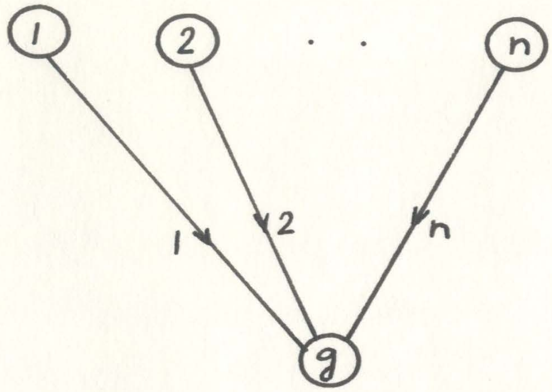


FIG. 19. TERMINAL GRAPH OF PARTIAL NETWORK

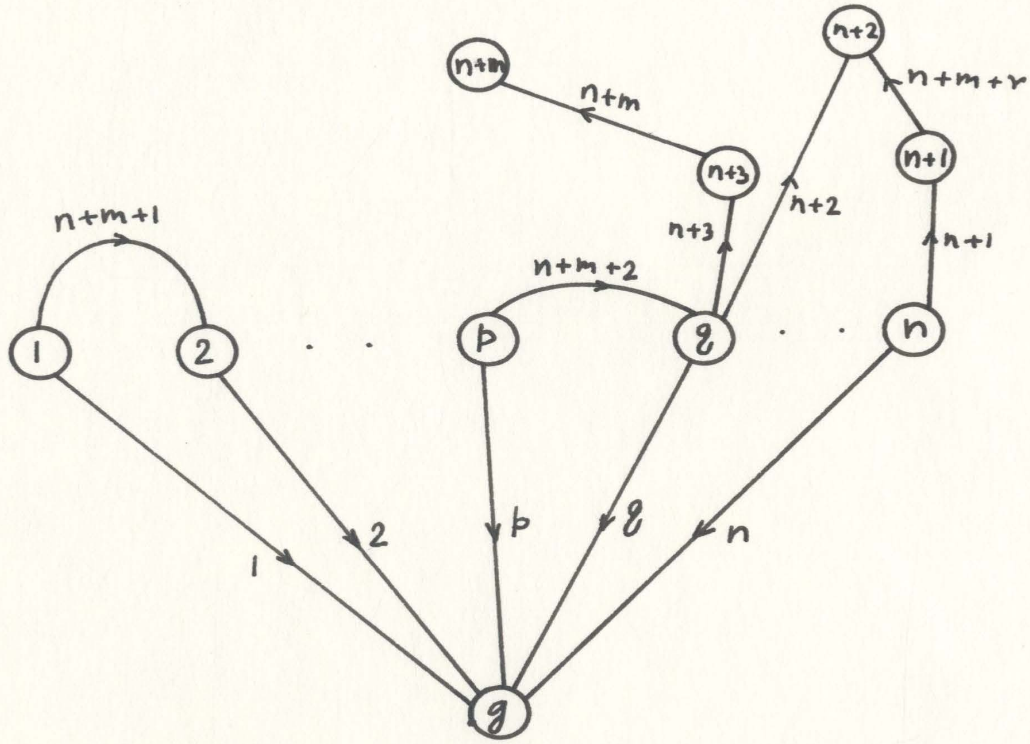


FIG. 20. SYSTEM GRAPH OF MODIFIED SYSTEM

$$\underline{V}_{BUS(0)} = \underline{Z}_{BUS(0)}\ \underline{I}_{BUS(0)}$$

where the terminal graph correspond to Fig. 19. Let a mutually coupled group consisting of (m+r) lines be added such that with the superposing of the oriented graph corresponding to these lines, there will be (n+m) nodes in total. Let the numbering of the elements be such that elements (n+1) ... (n+m) are branches and thus form new nodes (n+1) ... (n+m); whereas elements (n+m+1) ... (n+m+r) are links which form no new nodes. The overall graph is shown in Fig. 20.

The new $\underline{Z}_{BUS}$ denoted by $\underline{Z}_{BUS(m)}$ is a (n+m) order matrix. Following the standard procedure, the (n+m) nodes are augmented by current sources, the corresponding elements being denoted by 1', 2', ... n', .. (n+m)'. The circuit equations corresponding to the formulation tree will be given by

$$\begin{bmatrix} \underline{B}_{11} & \underline{B}_{12} & \vdots & \underline{U} & \underline{0} \\ & & \vdots & & \\ \underline{B}_{21} & \underline{B}_{22} & \vdots & \underline{0} & \underline{U} \end{bmatrix} \begin{bmatrix} \underline{V}_{T1} \\ \underline{V}_{T2} \\ \hline \underline{V}_{C1} \\ \underline{V}_{C2} \end{bmatrix} = 0 \qquad .. \quad 3.13$$

where $\underline{V}_{T1} = \begin{bmatrix} V_1 \\ \cdot \\ \cdot \\ \cdot \\ V_n \end{bmatrix}$; $\underline{V}_{T2} = \begin{bmatrix} V_{n+1} \\ \cdot \\ \cdot \\ \cdot \\ V_{n+m} \end{bmatrix}$

and

$$\underline{V}_{C1} = \begin{bmatrix} V_{n+m+1} \\ \cdot \\ \cdot \\ \cdot \\ V_{n+m+r} \end{bmatrix} ; \qquad \underline{V}_{C2} = \begin{bmatrix} V_1' \\ V_2' \\ \cdot \\ \cdot \\ \cdot \\ V_{n+m}' \end{bmatrix}$$

The component equations are

$$\underline{V}_{T1} = \underline{Z}_{BUS(0)} \underline{I}_{T1}$$

$$\begin{bmatrix} \underline{V}_{T2} \\ \underline{V}_{C1} \end{bmatrix} = \begin{bmatrix} \underline{Z}_M \end{bmatrix} \begin{bmatrix} \underline{I}_{T2} \\ \underline{I}_{C1} \end{bmatrix}$$

where $\underline{Z}_M$ is the impedance matrix of the mutually coupled group.

The chord formulation will yield as the penultimate result (details omitted)

$$\begin{bmatrix} \underline{0} \\ \underline{U} \end{bmatrix} \underline{V}_{C2} = - \left[\begin{array}{c|c} \underline{B}_{11} & \underline{B}_{12} \quad \underline{U} \\ \underline{B}_{21} & \underline{B}_{22} \quad \underline{0} \end{array}\right] \left[\begin{array}{c|c} \underline{Z}_{BUS(0)} & \underline{0} \\ \hline \underline{0} & \underline{Z}_M \end{array}\right] \left[\begin{array}{cc} \underline{B}_{11}^T & \underline{B}_{21}^T \\ \hline \underline{B}_{12}^T & \underline{B}_{22}^T \\ \underline{U} & \underline{0} \end{array}\right] \begin{bmatrix} \underline{I}_{C1} \\ \underline{I}_{C2} \end{bmatrix} \quad .. \ 3.14$$

In equation 3.14, the partitioning of the pre and post multiplying matrices has been done to indicate conformity in the matrix multiplication.

Thus, if $\underline{\beta}_{12} = [\ \underline{B}_{12} \ \vdots \ \underline{U}\ ]$; and $\underline{\beta}_{22} = [\ \underline{B}_{22} \ \vdots \ \underline{0}\ ]$

then the triple product of equation 3.14 will result in the following

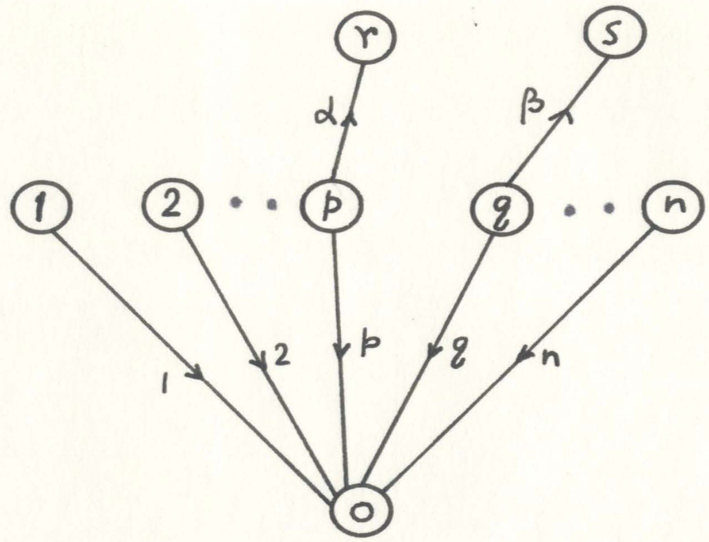


FIG. 21. SYSTEM GRAPH OF MODIFIED SYSTEM

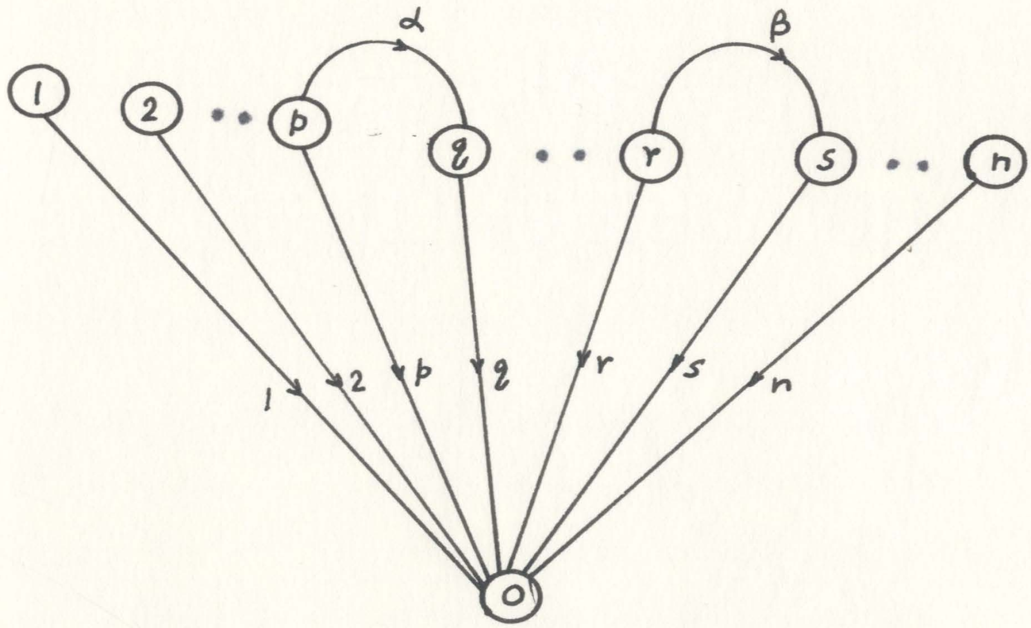


FIG. 22. SYSTEM GRAPH OF MODIFIED SYSTEM

$$\begin{bmatrix} \underline{O} \\ \underline{U} \end{bmatrix} \underline{V}_{C2} = - \left[\begin{array}{c:c} \underline{B}_{11}\underline{Z}_{BUS(0)}\underline{B}_{11}^{T}+\underline{\beta}_{12}\underline{Z}_{M}\underline{\beta}_{12}^{T} & \underline{B}_{11}\underline{Z}_{BUS(0)}\underline{B}_{21}^{T}+\underline{\beta}_{12}\underline{Z}_{M}\underline{\beta}_{22}^{T} \\ \underline{B}_{21}\underline{Z}_{BUS(0)}\underline{B}_{11}^{T}+\underline{\beta}_{22}\underline{Z}_{M}\underline{\beta}_{12}^{T} & \underline{B}_{21}\underline{Z}_{BUS(0)}\underline{B}_{21}^{T}+\underline{\beta}_{22}\underline{Z}_{M}\underline{\beta}_{22}^{T} \end{array}\right] \begin{bmatrix} \underline{I}_{C1} \\ \underline{I}_{C2} \end{bmatrix} \quad .. \ 3.15$$

$$= - \left[\begin{array}{c:c} \underline{Z}_1 & \underline{Z}_2 \\ \underline{Z}_2^{T} & \underline{Z}_3 \end{array}\right] \left[\begin{array}{c} \underline{I}_{C1} \\ \hdashline I_{C2} \end{array}\right] \quad .. \ 3.16$$

From the first equation of equation set 3.16

$$\underline{Z}_1 \underline{I}_{C1} = - \underline{Z}_2 \underline{I}_{C2}$$

or

$$\underline{I}_{C1} = - \underline{Z}_1^{-1} \underline{Z}_2 \underline{I}_{C2}$$

therefore

$$\underline{V}_{C2} = - \left[-\underline{Z}_2^{T} \underline{Z}_1^{-1} \underline{Z}_2 + \underline{Z}_3 \right] I_{C2}$$

Noting the final change of signs, with respect to $\underline{I}_{C2}$, $\underline{Z}_{BUS(m)}$ is obtained as

$$\underline{Z}_{BUS(m)} = \left[\underline{Z}_3 - \underline{Z}_2^{T} \underline{Z}_1^{-1} \underline{Z}_2 \right] \quad .. \ 3.17$$

In the above, it will be noted that the matrix inversion required is of the same order as the number of links being added.

3.6 Special Cases of Addition of a Mutually Coupled Group of Lines.

Let two elements α and β be added to the existing network at nodes p and q respectively to form new nodes r and s as shown in Fig. 21. Let elements α and β be mutually coupled having a terminal relation:

$$\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} \quad \begin{bmatrix} z_{\alpha\alpha} & z_{\alpha\beta} \\ z_{\alpha\beta} & z_{\beta\beta} \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix}$$

Let $z_{\alpha\alpha}$ = self impedance of element α

$z_{\beta\beta}$ = self impedance of element β

$z_{\alpha\beta}$ = mutual impedance between elements α and β.

Elements α and β are assumed to be oriented from p to r and q to s respectively.

For this case r = 0, m = 2. Consequently there are no submatrices corresponding to $\underline{B}_{11}$ and $\underline{B}_{12}$ and

$$[\ \underline{B}_{21} \ \vdots\ \underline{B}_{22}\ ] = m \left[\begin{array}{ccccc|cc} & & -\underline{U} & & & \underline{0} & \\ \hline 0 & .. & -1_p & 0 & ..\ 0 & 1 & 0 \\ 0 & .. & 0 & -1_q & ..\ 0 & 0 & 1 \end{array}\right] \quad ; \quad \underline{Z}_M = \begin{bmatrix} z_{\alpha\alpha} & z_{\alpha\beta} \\ z_{\alpha\beta} & z_{\beta\beta} \end{bmatrix}$$

Carrying out the algebraic manipulation

$$\underline{Z}_{BUS(m)} = \left[\begin{array}{ccc|cc} & & & z_{1p} & z_{1q} \\ & & & \cdot & \cdot \\ & & & \cdot & \cdot \\ & \underline{Z}_{BUS(0)} & & z_{np} & z_{nq} \\ \hline z_{p1} & \cdots & z_{pn} & z_{pp}+z_{\alpha\alpha} & z_{pq}+z_{\alpha\beta} \\ z_{q1} & \cdots & z_{qn} & z_{pq}+z_{\alpha\beta} & z_{qq}+z_{\beta\beta} \end{array}\right]$$

Case (ii)

Let the two elements added be such that they constitute links. Let p , q , r , s be the existing nodes and let α be added from p to q and β from r to s . The system graph with the added elements will thus be given by Fig. 22.

In this case there are no elements corresponding to T_2 and hence $m = 0$, $r = 2$ consequently there are no submatrices corresponding to $\underline{B}_{12}$ and $\underline{B}_{22}$. Also, in this case

$$\underline{B}_{11} = \begin{array}{c} \begin{matrix} 1 & & & p & q & & & r & s & & & n \end{matrix} \\ \begin{bmatrix} 0 & . & . & -1 & 1 & . & . & 0 & 0 & . & . & 0 \\ 0 & . & . & 0 & 0 & . & . & -1 & 1 & . & . & 0 \end{bmatrix} \end{array} ; \quad \underline{B}_{21} = -\underline{U}$$

Thus $\underline{Z}_{BUS(m)} = \begin{bmatrix} \underline{Z}_{BUS(0)} \end{bmatrix} +$

$$\underbrace{\begin{bmatrix} z_{p1}-z_{q1} & z_{r1}-z_{s1} \\ \vdots & \\ z_{pn}-z_{qn} & z_{rn}-z_{sn} \end{bmatrix}}_{\underline{Z}_{p-q,\ r-s}} \begin{bmatrix} 2z_{pq}-z_{pp}-z_{qq}-z_{\alpha\alpha} & z_{ps}+z_{qr}-z_{pr}-z_{qs}-z_{\alpha\beta} \\ & \\ z_{ps}+z_{qr}-z_{pr}-z_{qs}-z_{\alpha\beta} & 2z_{rs}-z_{rr}-z_{ss}-z_{\beta\beta} \end{bmatrix}^{-1} -\underline{Z}^{T}_{p-q,r-s}$$

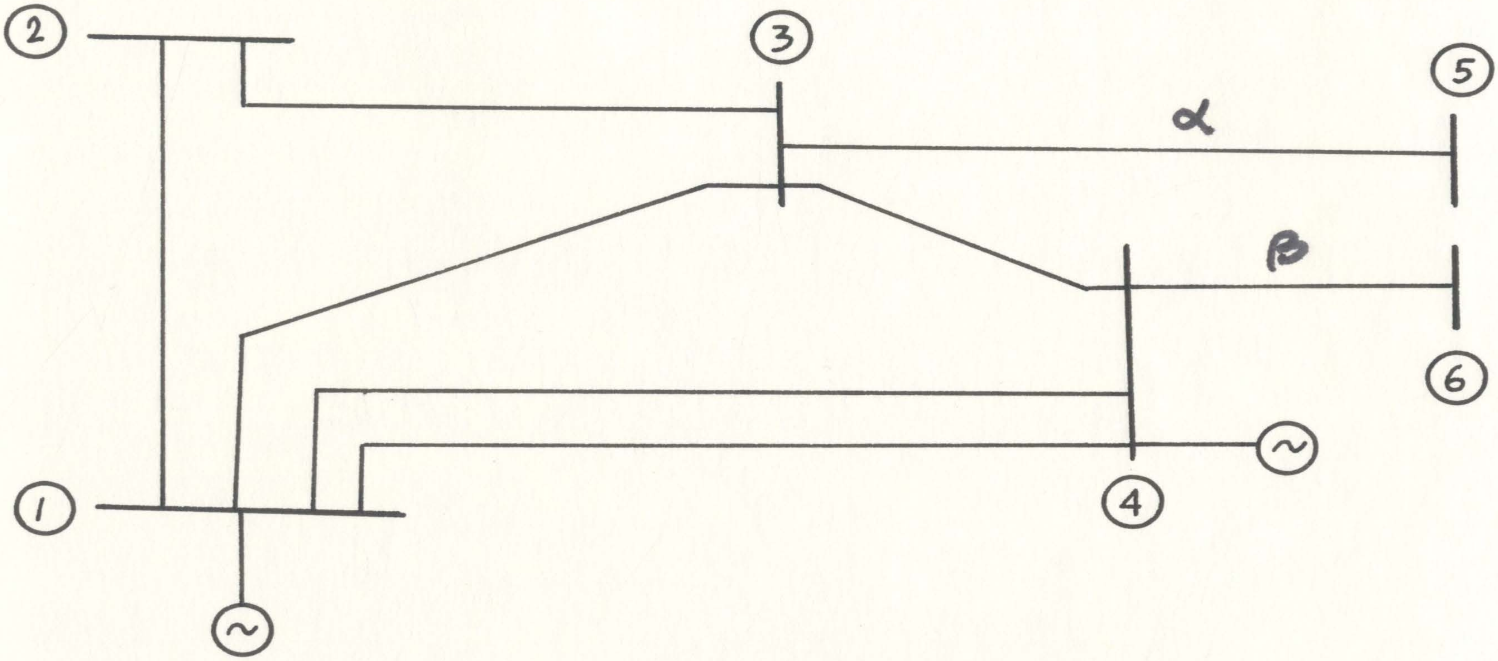


FIG. 23. POWER SYSTEM WITH ELEMENTS α AND β ADDED

3.7 Numerical Examples.

We shall now consider two numerical examples to illustrate the special cases as discussed in Section 3.6.

Case I

Let α and β be two transmission lines added to a Power system as shown in Fig. 23. Let the component equations for this element be given by

$$\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \begin{bmatrix} j.15 & j.05 \\ j.05 & j.1 \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix}$$

and let $\underline{Z}_{BUS(0)}$ for the original power system be given by

$$\underline{Z}_{BUS(0)} = \begin{bmatrix} j.050 & j.049 & j.055 & j.048 \\ j.049 & j.121 & j.082 & j.051 \\ j.055 & j.082 & j.158 & j.033 \\ j.048 & j.051 & j.033 & j.055 \end{bmatrix}$$

In order to derive $\underline{Z}_{BUS(m)}$, we can assume $p = 3$ and $q = 4$. Thus $\underline{Z}_{BUS(m)}$ can be derived by inspection as follows

$$\underline{Z}_{BUS(m)} = \left[\begin{array}{cccc|cc} j.050 & j.049 & j.055 & j.048 & j.055 & j.048 \\ j.049 & j.121 & j.082 & j.051 & j.082 & j.051 \\ j.055 & j.082 & j.158 & j.033 & j.158 & j.033 \\ j.048 & j.051 & j.033 & j.055 & j.033 & j.055 \\ \hline j.055 & j.082 & j.158 & j.033 & j.158+j.15 & j.033+j.05 \\ j.048 & j.051 & j.033 & j.055 & j.033+j.05 & j.055+j.1 \end{array}\right]$$

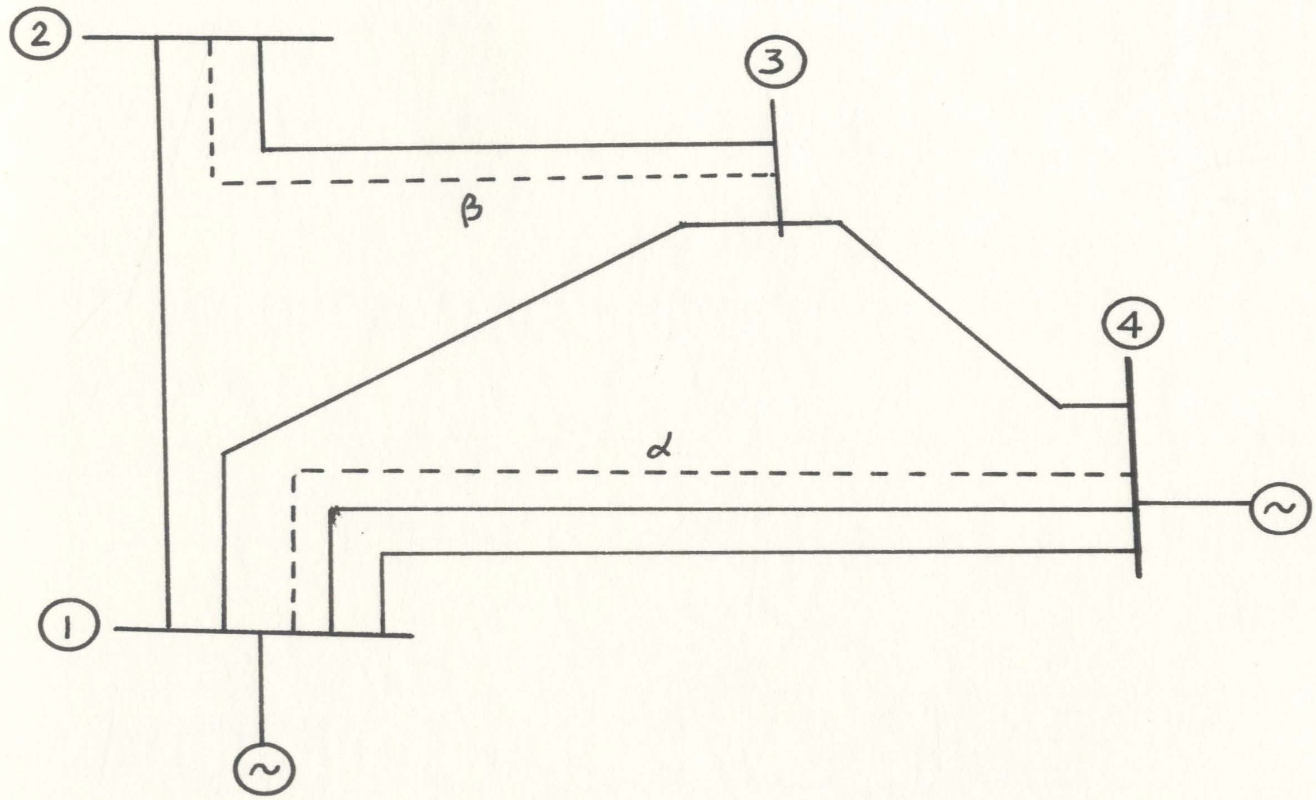


FIG. 24. POWER SYSTEM WITH ELEMENTS α AND β ADDED

$$\underline{Z}_{BUS(m)} = \begin{bmatrix} j.050 & j.049 & j.055 & j.048 & j.055 & j.048 \\ j.049 & j.121 & j.082 & j.051 & j.082 & j.051 \\ j.055 & j.082 & j.158 & j.033 & j.158 & j.033 \\ j.048 & j.051 & j.033 & j.055 & j.033 & j.055 \\ j.055 & j.082 & j.158 & j.033 & j.308 & j.083 \\ j.048 & j.051 & j.033 & j.055 & j.083 & j.155 \end{bmatrix}$$

Case II

In this example let us assume that α and β are links as shown in Fig. 24.

Let the component equations for the elements α and β be given by

$$\begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = \begin{bmatrix} j.15 & j.1 \\ j.1 & j.30 \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \end{bmatrix}$$

In this case $p = 1, \quad q = 4,$

$r = 2, \quad s = 3.$

Therefore $\underline{Z}_{p-q,r-s}$ will be given by

$$\underline{Z}_{p-q,r-s} = \begin{bmatrix} j.050-j.048 & j.049-j.055 \\ j.049-j.051 & j.121-j.082 \\ j.055-j.033 & j.082-j.158 \\ j.048-j.055 & j.051-j.033 \end{bmatrix} = \begin{bmatrix} j.002 & -j.006 \\ -j.002 & j.039 \\ j.022 & -j.076 \\ -j.007 & j.018 \end{bmatrix}$$

$$2\, z_{pq} - z_{pp} - z_{qq} - z_{\alpha\alpha} = 2xj.048-j.050-j.055-j.15 = -j.159$$

$$z_{ps} + z_{qr} - z_{pr} - z_{qs} - z_{\alpha\beta} = j.055+j.051-j.049-j.033-j.05 = -j.026$$

$$2\, z_{rs} - z_{rr} - z_{ss} - z_{\beta\beta} = 2x\ .082-j.121-j.158-j.1 = j.215$$

Thus
$$\begin{bmatrix} 2z_{pq}-z_{pp}-z_{qq}-z_{\alpha\alpha} & z_{ps}+z_{qr}-z_{pr}-z_{qs}-z_{\alpha\beta} \\ z_{ps}+z_{qr}-z_{pr}-z_{qs}-z_{\alpha\beta} & 2z_{rs}-z_{rr}-z_{ss}-z_{\alpha\beta} \end{bmatrix}^{-1}$$

$$= \begin{bmatrix} -j.159 & -j.026 \\ -j.026 & -j.215 \end{bmatrix}^{-1} = \begin{bmatrix} j6.3 & -j.775 \\ -j\ .775 & j4.7 \end{bmatrix}$$

Therefore $\underline{Z}_{BUS(m)} = \underline{Z}_{BUS(0)}$

$$+ \begin{bmatrix} j.002 & -j.006 \\ -j.002 & j.039 \\ j.022 & -j.076 \\ -j.007 & j.018 \end{bmatrix} \begin{bmatrix} j6.3 & -j\ .775 \\ -j\ .775 & j4.7 \end{bmatrix} \begin{bmatrix} j.002-j.002 & j.022-j.007 \\ -j.006\ j.039 & -j.076\ j.18 \end{bmatrix}$$

$$= \begin{bmatrix} j.050 & j.049 & j.055 & j.048 \\ j.049 & j.121 & j.082 & j.051 \\ j.055 & j.082 & j.158 & j.033 \\ j.048 & j.051 & j.033 & j.055 \end{bmatrix} + \begin{bmatrix} -j.001 & j.0010 & -j\ 0027 & j.0006 \\ +j.001 & -j.007 & -j\ 014 & j\ 003 \\ -j.0027 & -j\ 014 & -j.033 & j.008 \\ j.0006 & -j.003 & j.008 & -j.0012 \end{bmatrix}$$

$$= \begin{bmatrix} j.049 & j.050 & j.052 & j.048 \\ j.050 & j.114 & j.068 & j.054 \\ j.052 & j.068 & j.125 & j.041 \\ j.048 & j.054 & j.041 & j.054 \end{bmatrix}$$

3.8 Conclusion.

In this chapter the concept of multi-terminal representation has been applied in the development of $\underline{Z}_{BUS}$ algorithm. The generality of the algorithm has been demonstrated in as much as that this can be applied while adding a coupled as well as an uncoupled group of lines to a network at a time. Numerical examples have been worked out to demonstrate the applicability of the algorithm in a practical power system. In the next chapter, the short circuit analysis of power systems is taken up.

CHAPTER IV

SHORT CIRCUIT ANALYSIS

4.1 Introduction

A fault proof power system is neither practical, nor economical. Modern power systems are designed and constructed to ensure a high degree of reliability, but still faults do occur due to various reasons, such as, equipment failures, lightning, falling of trees on transmission lines etc. As such, proper safeguards against these contingencies are imperative for the successful operation of a system. Short circuit currents, as a rule, will exceed considerably the rated current of the affected installation and can disturb the operation of a power system considerably. In fact the short circuits can not only lead to the damage of the affected installation, but also often develop situations when the successful operation of an entire power system could be in jeopardy. As such a knowledge of the conditions existing on a power system during such short circuits is essential. Such information is used to design an adequate system of protection and operation strategy.

In electrical terms, the common form of short circuits would fall into one or the other of the following categories:

(i) Three Phase.

(ii) Line to Ground.

(iii) Line to Line to Ground.

(iv) Line to Line.

(v) Three Phase to Ground.

These different types of faults will, in general, affect the power systems differently. The values of currents and voltages that are obtained during these different types of faults are of interest to the power system engineers - since these influence the application of circuit breakers, stresses on electrical machinery, schemes of protection/control etc. However, in practice it is not generally necessary to analyse for each of the above types of faults since generally we are interested only in the worst case. As such 3 phase and the line to ground faults are of primary interest to power systems engineers.

In this section an attempt will be made to obtain solution of a power system under short circuit conditions by the application of graph theoretic principles. The symbolic formulation for the short circuit study would be discussed first. The formulation will subsequently be applied to a simplified Newfoundland Power System for the calculation of short circuit current at the different buses. Only symmetrical three phase faults will be considered.

4.2 Physical Assumptions.

The short circuit analysis essentially consists of the steady state solution of a linear system. As such the following physical assumptions are relevant to the short circuit studies as applicable to a power system.

i) The generating system is represented by a voltage source behind the internal impedance of the machine. The value of the internal impedance is taken as the transient or sub-transient reactance.

This is because a few cycles after the initiation of the fault the relevant protective gear is expected to respond to the fault and isolate the faulted circuit by the operation of circuit breakers. As such it is generally desired to obtain the quantity of currents and voltages for this period only. Depending on the purpose of the study, either the transient or the sub-transient reactances can be used. For example, if the study is being done to determine the rating of circuit breakers, the sub-transient reactance shall be used, whereas if it is desired to determine the minimum fault current for a relay operation, the transient reactance shall be used.

ii) The normal load, line charging, capacitances and other shunt connections to ground are neglected. This is based on the fact that the short circuit currents, in general, considerably exceed such load currents, etc., and as such will introduce an error of little significance.

iii) The resistance of the generators, transformers, etc., are neglected. For transmission lines, if the resistance is smaller than the reactances by a factor of five or more, then the resistances are neglected. From a practical point of view this does not introduce any error of significance since the values are generally very small. On the other hand this simplifies the arithmetic. However, from a mathematical point of view, such resistance, can be taken into account if so desired.

iv) All transformers are considered at their nominal taps. It is assumed that the impedance of the transformers do not change with the change of taps. Considering that the taps of small percentage values of the main winding are generally encountered, this does not introduce much error.

Some of the above physical assumptions which are used to simplify the resulting mathematical model are conflicting in nature with one another. For instance the neglect of the resistance will yield a value of the calculated short circuit current greater than the actual short circuit current. But on the other hand, neglecting of the line charging will yield a calculated value which is less than the actual value of short circuit current. On the whole the simplified mathematical model based on the above mentioned simplifying assumptions will predict the actual line flows within the limits of acceptable engineering approximations.

4.3 Derivation of the Terminal Representation for a Generator-Transmission System.

A prerequisite for the short circuit study through graph theoretic concepts is to obtain a multi-terminal representation of the generator system. For this purpose the generator system and the transmission system will at the first instance be considered separately. The transmission system will then be augmented by the generator system to obtain a general formulation. At this point it is important to mention that the terminal representation of the transmission system in the impedance form does not exist if the ground bus is chosen as the reference vertex. This is particularly of significance in short circuit studies, since as mentioned earlier, all shunt connections to ground such as loads, line charging etc.,

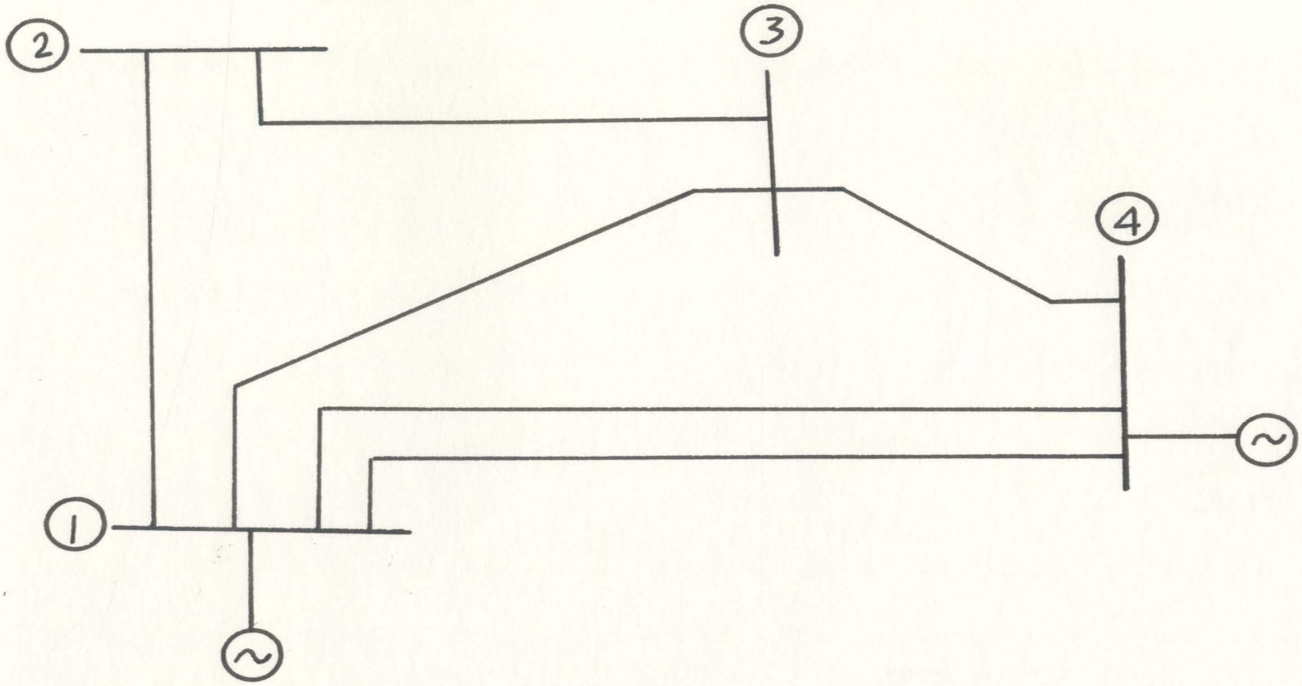


FIG. 25. GENERATOR - TRANSMISSION SYSTEM
(Simplified and slightly modified
Newfoundland Power System).

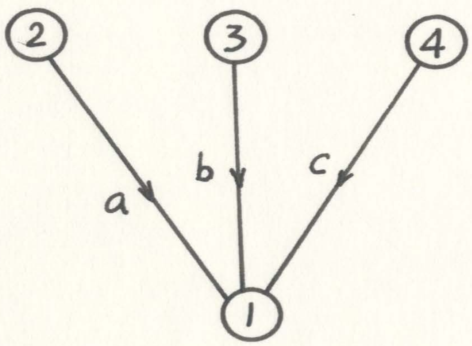


FIG. 26. TERMINAL GRAPH OF TRANSMISSION SYSTEM

in this case are neglected and thus the ground forms an isolated vertex. Hence we use a Z matrix representation for the transmission network with reference to an arbitrary bus of the system. To be specific, $\underline{Z}_{BN}$ is the multi terminal representation that will be used.

4.4 Example

We will illustrate the derivation of the terminal representation of generator-transmission system through an example. Let us consider the transmission system as described in Fig. 1 in Chapter II. Let two sets of generators be connected at buses 1 and 4 and thus let Fig. 25 represent the combined generator-transmission network.

For the transmission system, following the procedure discussed earlier, the mutli-terminal representation corresponding to the terminal graph in Fig. 26, can be obtained in the form

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \\ Z_{21} & Z_{22} & Z_{23} \\ Z_{31} & Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad .. \quad 4.1$$

The numerical values of the above Z-matrix, as given by Eq. 2.27 in Chapter II are

$$\underline{Z}_{BN} = \begin{bmatrix} j.073 & j.028 & -j.004 \\ j.028 & j.01 & -j.02 \\ -j.004 & -j.02 & j.1 \end{bmatrix}$$

For short circuit studies the generator system is represented by voltage sources connected behind their internal impedances. The

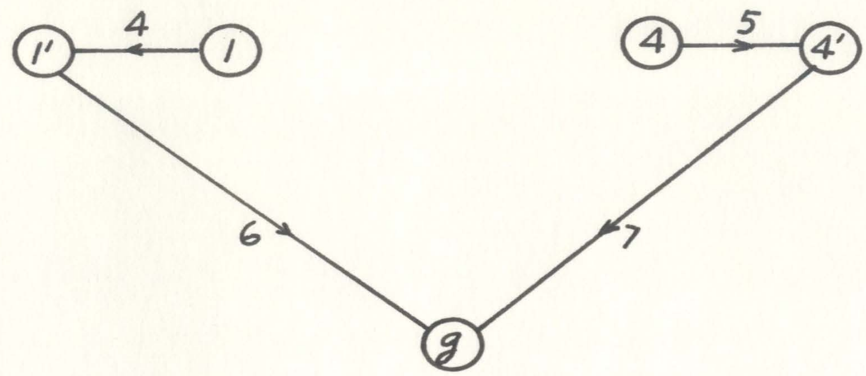


FIG. 27. TERMINAL GRAPH OF GENERATOR SYSTEM

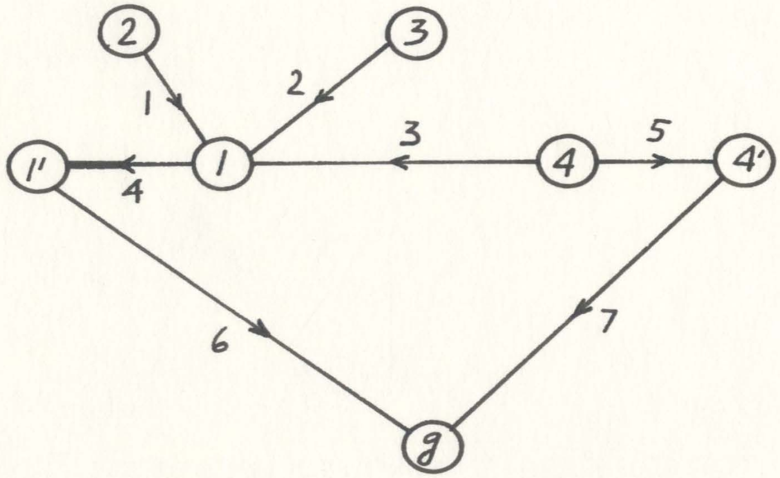


FIG. 28. COMBINED GENERATOR - TRANSMISSION SYSTEM

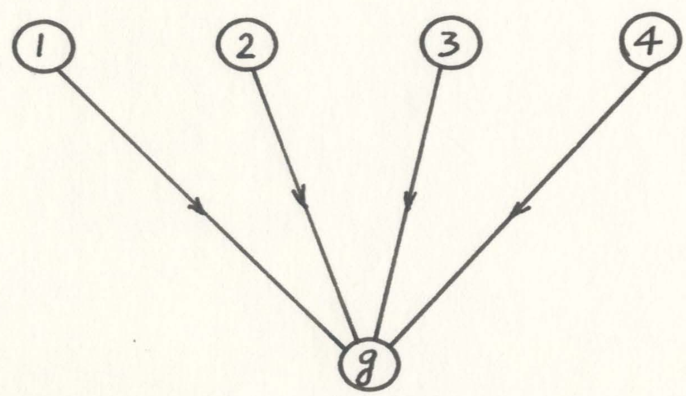


FIG. 29. TERMINAL GRAPH OF COMBINED SYSTEM

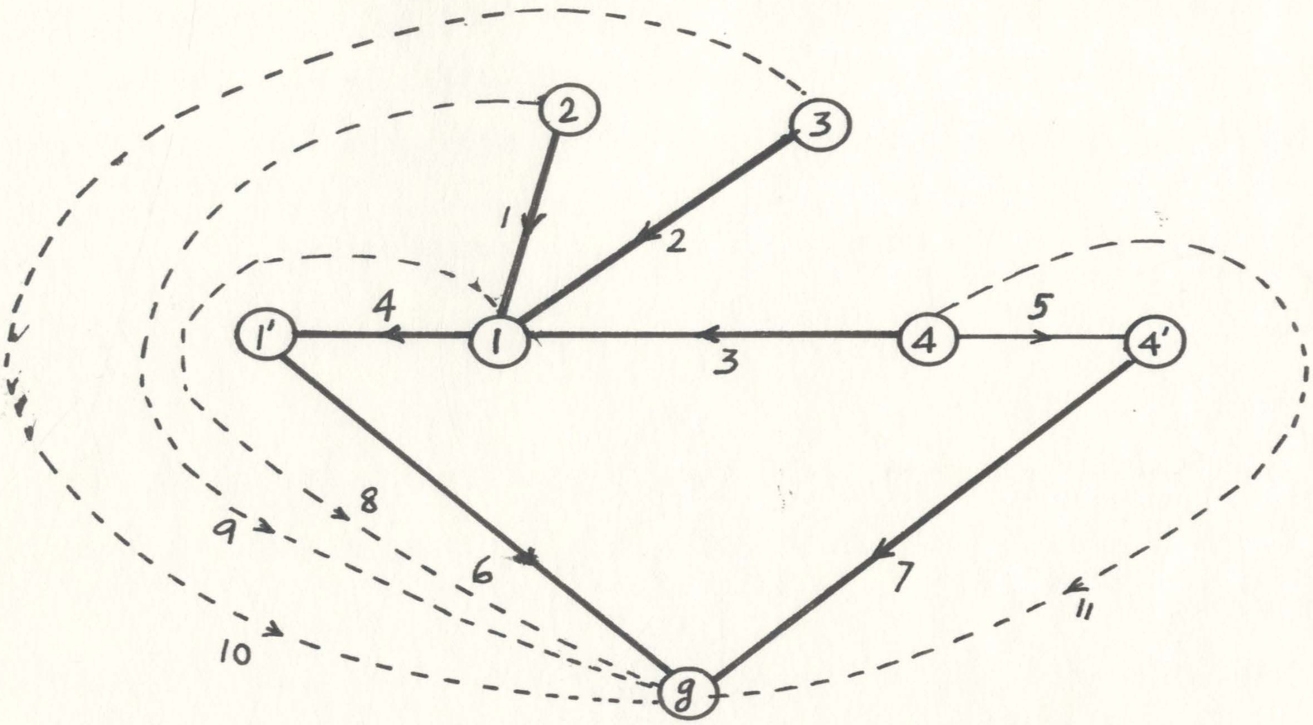


FIG. 30. AUGMENTED TERMINAL GRAPH OF COMBINED SYSTEM

terminal graph of the generator system can thus be represented by Fig. 27. The generator system when superimposed on the transmission system will result in a system graph as shown in Fig. 28. In this combined graph, elements 4 and 5 correspond to the reactances of the generators and the elements 6 and 7 are voltage sources behind the reactances. The reactance as discussed earlier could be either the transient or the sub-transient depending on a particular study.

A multi-terminal representation of the combined system can again be derived with reference to the ground bus g . Since for the purpose of the short circuit studies, we are interested in the voltages and currents at the external buses only (i.e. at nodes 1, 2, 3, & 4) these buses will be retained in the multi-terminal representation. The terminal graph of the combined generator-transmission system as shown in Fig. 29 thus results. In general terms, this terminal representation will be of the form

$$\underline{V} = \underline{Z}\,\underline{I} + \underline{V}_o$$

where $\underline{V}$ and $\underline{I}$ are the vectors of the bus voltage and current matrices; $\underline{Z}$ is the terminal impedance matrix corresponding to Fig. 29, and $\underline{V}_o$ is the matrix representing the voltage sources.

In order to obtain the terminal equations in the impedance form corresponding to the terminal graph of Fig. 27, we consider the augmented graph as in Fig. 30, for the combined generator-transmission system. The nodes corresponding to external buses have been augmented by current sources for this purpose. To obtain the multi-terminal representation of the generator-transmission system, the procedure as described in Chapter II will be followed.

By manipulation of the circuit and cutset equations with respect to the formulation tree T(6, 7, 4, 1, 2, 3) in Fig. 28 we get (details omitted)

$$\begin{bmatrix} 0 \\ V_8 \\ V_9 \\ V_{10} \\ V_{11} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} V_6 \\ V_7 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} Z_4 & 0 & 0 & 0 & 0 \\ 0 & Z_{11} & Z_{12} & Z_{13} & 0 \\ 0 & Z_{21} & Z_{22} & Z_{23} & 0 \\ 0 & Z_{31} & Z_{32} & Z_{33} & 0 \\ 0 & 0 & 0 & 0 & Z_5 \end{bmatrix} \begin{bmatrix} -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I \\ I \\ I \\ I \\ I \end{bmatrix}$$

.. 4.2

In this case it will be noted that V_6 and V_7 are known voltage sources and further we assume $V_6 = V_7$. Thus it can be shown that:

$$\begin{bmatrix} V_8 \\ V_9 \\ V_{10} \\ V_{11} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} V_6 \\ V_7 \end{bmatrix} + \begin{bmatrix} Z_4+Z_5+Z_{33} \end{bmatrix}^{-1} \begin{bmatrix} -Z_4 \\ -Z_4-Z_{13} \\ -Z_4-Z_{23} \\ -Z_4-Z_{33} \end{bmatrix} \begin{bmatrix} -Z_4 & -Z_4-Z_{13} & -Z_4-Z_{23} & -Z_4-Z_{33} \end{bmatrix} \begin{bmatrix} I_8 \\ I_9 \\ I_{10} \\ I_{11} \end{bmatrix}$$

$$+ \begin{bmatrix} -Z_4 & -Z_4 & -Z_4 & -Z_4 \\ -Z_4 & -Z_4-Z_{11} & -Z_4-Z_{12} & -Z_4-Z_{13} \\ -Z_4 & -Z_4-Z_{12} & -Z_4-Z_{22} & -Z_4-Z_{23} \\ -Z_4 & -Z_4-Z_{13} & -Z_4-Z_{23} & -Z_4-Z_{33} \end{bmatrix} \begin{bmatrix} I_8 \\ I_9 \\ I_{10} \\ I_{11} \end{bmatrix}$$

.. 4.3

or

$$\begin{bmatrix} V_8 \\ V_9 \\ V_{10} \\ V_{11} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} V_6$$

$$+ \frac{1}{Z_4+Z_5+Z_{33}} \left\{ \begin{bmatrix} Z_4^{\,2} & Z_4(Z_4+Z_{13}) & Z_4(Z_4+Z_{23}) & Z_4(Z_4+Z_{33}) \\ Z_4(Z_4+Z_{13}) & (Z_4+Z_{13})^2 & (Z_4+Z_{13})(Z_4+Z_{23}) & (Z_4+Z_{13})(Z_4+Z_{33}) \\ Z_4(Z_4+Z_{23}) & (Z_4+Z_{13})(Z_4+Z_{23}) & (Z_4+Z_{23})^2 & (Z_4+Z_{23})(Z_4+Z_{33}) \\ Z_4(Z_4+Z_{33}) & (Z_4+Z_{13})(Z_4+Z_{33}) & (Z_4+Z_{23})(Z_4+Z_{33}) & (Z_4+Z_{33})^2 \end{bmatrix} \right.$$

$$\left. \begin{bmatrix} Z_4 & Z_4 & Z_4 & Z_4 \\ Z_4 & Z_4+Z_{11} & Z_4+Z_{12} & Z_4+Z_{13} \\ Z_4 & Z_4+Z_{12} & Z_4+Z_{22} & Z_4+Z_{23} \\ Z_4 & Z_4+Z_{13} & Z_4+Z_{23} & Z_4+Z_{33} \end{bmatrix} \right\} \begin{bmatrix} I_8 \\ I_9 \\ I_{10} \\ I_{11} \end{bmatrix} \qquad .. \quad 4.4$$

Thus, noting the change of signs, the multi terminal representation of the generator-transmission system can be represented in the form

$$\underline{V}_{BUS} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} V_6 + \left[\underline{Z} \right] \underline{I}_{BUS} \qquad .. \quad 4.5$$

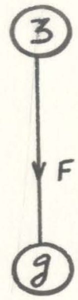


FIG. 31. TERMINAL GRAPH OF FAULT.

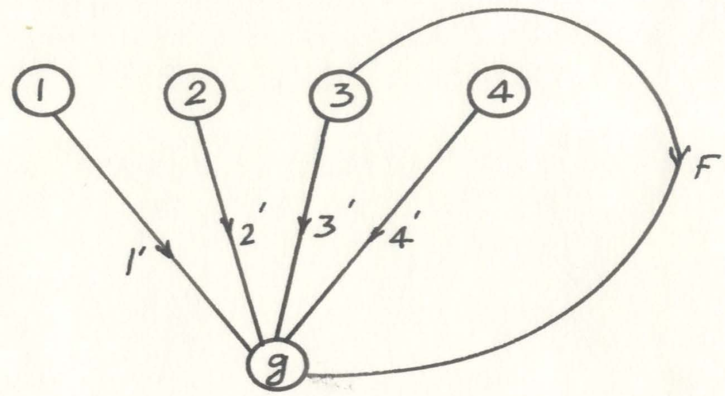


FIG. 32. TERMINAL GRAPH OF FAULTED SYSTEM.

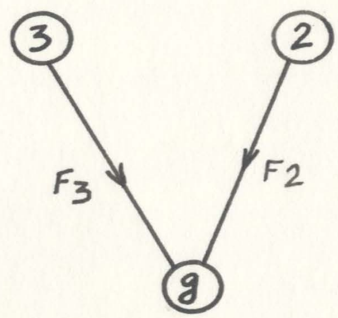


FIG. 33. TERMINAL GRAPH OF TWO FAULTS.

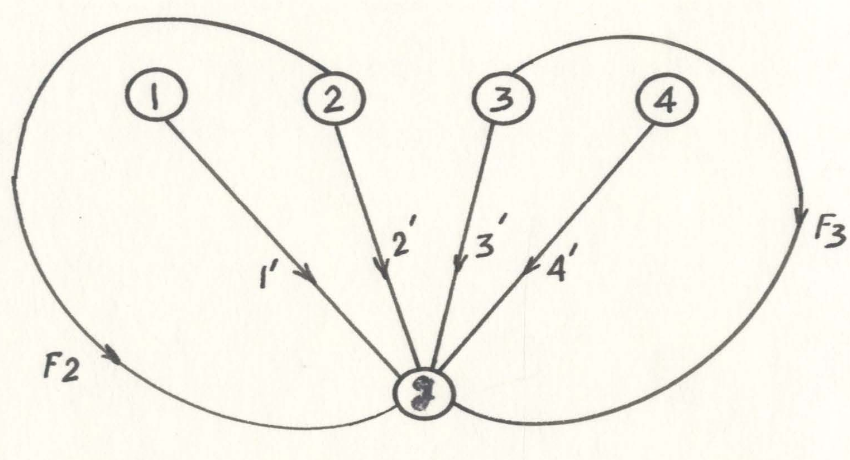


FIG. 34. TERMINAL GRAPH OF FAULTED SYSTEM.

where $\underline{V}_{BUS} \begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \end{bmatrix} = \begin{bmatrix} V_8 \\ V_9 \\ V_{10} \\ V_{11} \end{bmatrix}$ and $\underline{I}_{BUS} \begin{bmatrix} I_1' \\ I_2' \\ I_3' \\ I_4' \end{bmatrix} = - \begin{bmatrix} I_8 \\ I_9 \\ I_{10} \\ I_{11} \end{bmatrix}$

In symbolic form equation 4.5 can be written as

$$\underline{V}_{BUS} = \underline{V}_{BUS(0)} + \underline{Z} \ \underline{I}_{BUS} \qquad .. \ 4.6$$

Having obtained the MTR of the generator-transmission system we shall continue with this example to arrive at the solution of the faulted system.

Let us assume, for our example, that bus 3 of Generator-Transmission system is faulted. Let the fault impedance be represented by Z_F. Thus the terminal graph of this fault will be represented by Fig. 31 and the corresponding terminal graph of this fault will be given by

$$V_F = Z_F \ I_F \ . \qquad .. \ 4.7$$

The terminal graph of the generator-transmission system when augmented by the fault system would be given by Fig. 32. This network can be solved for the fault currents as follows:

From the cutset and circuit equation for the formulation tree T(1', 2', 3', 4') we get

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -I_F \\ 0 \end{bmatrix} \qquad .. \ 4.8$$

and $V_3' = V_F = Z_F I_F$ .. 4.9

Again from Eq. 4.4 and 4.5

$$V_3' = V_6 + \frac{1}{Z_4+Z_5+Z_{33}} \left[(Z_4+Z_{23})^2 + (Z_4+Z_{22}) \right] I_3'$$

$$= V_6 - Z_X I_F \qquad .. \; 4.10$$

Thus from Eq. 4.9 and 4.10

$$Z_F I_F = V_6 + Z_X I_3' = V_6 - Z_x I_F$$

or

$$I_F = \frac{V_6}{Z_F+Z_x} \qquad .. \; 4.11$$

As a numerical example for this solution of a faulted system, let us assume the following values

$$Z_F = j0.1 \text{ p.u.}$$

$$Z_4 = j0.21 \text{ p.u.}$$

$$Z_5 = j0.65 \text{ p.u.}$$

$$V_6 = 1 + j0 \text{ p.u.}$$

Also for the transmission system of Fig. 1,

$$\underline{Z}_{BN} = j \begin{bmatrix} .073 & .028 & -.004 \\ .028 & .1 & -.02 \\ .004 & -.02 & .01 \end{bmatrix}$$

Thus if bus 3 of the Power System as in Fig. 25 is faulted, substituting the appropriate numerical values in equation 4.4, we get

$$\begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} V_6 + \begin{bmatrix} j.050 & j.049 & j.055 & j.048 \\ j.049 & j.121 & j.082 & j.051 \\ j.055 & j.082 & j.158 & j.033 \\ j.048 & j.051 & j.033 & j.055 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} \quad .. \; 4.12$$

Thus substituting Eq. 4.8 into Eq. 4.12

$$V_3' = (1+j0) + \begin{bmatrix} j.055 & j.082 & j.158 & j.033 \end{bmatrix} \begin{bmatrix} I_1' \\ I_2' \\ I_3' \\ I_4' \end{bmatrix} \quad .. \; 4.13$$

or $V_3' = (1+j0) - j.158 \times I_F$

but $V_3' = Z_F \, I_F$

Therefore $$I_F = \frac{1+j0}{j.158+j.1} = - j3.88 \text{ p.u.} \quad .. \; 4.14$$

Now combining Eq. 4.8 and 4.12 and substituting for I_F, we get

$$\begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} V_6 + \begin{bmatrix} j.050 & j.049 & j.055 & j.048 \\ j.049 & j.121 & j.082 & j.051 \\ j.055 & j.082 & j.158 & j.033 \\ j.048 & j.051 & j.033 & j.055 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ I_3' \\ 0 \end{bmatrix}$$

$$\text{or} \quad \begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} (1+j0) + \begin{bmatrix} j.055 \\ j.082 \\ j.158 \\ j.033 \end{bmatrix} -j3.88$$

$$\text{or} \quad \begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \end{bmatrix} = \begin{bmatrix} .788 \\ .68 \\ .39 \\ .872 \end{bmatrix} \qquad .. \ 4.15$$

This represents the bus voltages at buses 1 , 2 , 3 and 4 when bus 3 is faulted through an impedance Z_F = .1 p.u. Since we have all the bus voltages, the individual line voltages can be determined be considering a graph, where these set of voltages are superimposed on the original generator-transmission system as shown in Fig. 35. From the circuit equation of the formulation tree T(1', 2', 3', 4',9 ,10) we get

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \\ V_9' \\ V_{10}' \end{bmatrix} \qquad .. \ 4.16$$

Taking $V_9 = V_{10} = 1+j0$, and substituting for V_1', V_2', V_3' and V_4' we get

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} .108 \\ -.29 \\ .408 \\ -.084 \\ -.084 \\ .398 \\ -.212 \\ -.128 \end{bmatrix} \qquad .. \; 4.17$$

Again from the component equations, we get

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} j.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & j.3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & j.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & j.15 & j.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & j.1 & j.15 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & j.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -j.065 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -j.21 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix} \qquad .. \; 4.18$$

or

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix} = \begin{bmatrix} j.1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & j.3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & j.5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & j.15 & j.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & j.1 & j.15 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & j.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -j.015 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -j.21 \end{bmatrix}^{-1} \begin{bmatrix} .108 \\ -.29 \\ .482 \\ -.084 \\ -.084 \\ .398 \\ -.212 \\ -.128 \end{bmatrix} \quad .. \ 4.19$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix} = \begin{bmatrix} j1.08 \\ -j\,.96 \\ j\,.96 \\ j\,.33 \\ j\,.33 \\ j1.58 \\ -j3.3 \\ -j\,.6 \end{bmatrix} \quad .. \ 4.20$$

4.5 Simultaneous Faults.

4.5.1 Introduction

Simultaneous faults on power systems are not uncommon. Such faults may occur at random or due to other physical reasons such as lightning flash-overs at two locations, or as a result of a ground on one location raising the voltage on the sound phase so that a flash over occurs at a second point, etc. Though relatively infrequent, analysis of such simultaneous faults are important enough to justify their detailed analysis. Relay systems giving satisfactory operation for a fault at a single location may fail to respond correctly to simultaneous faults.

Methods are available to analyse such faults by the conventional analytical methods. However, such methods use cumbersome algebraic manipulations and are time consuming. In this section, an attempt will be made to analyse such faults by the application of graph theoretic concepts. It will be observed that once the multi-terminal representation of a network is available, the solution of simultaneous faults as well can be obtained easily.

4.5.2 An illustration

We shall discuss the solution of simultaneous faults with the same example as in Fig. 25. The multi-terminal representation of this system is given by Fig. 29 and the terminal equation 4.4. Let us assume that two faults occur simultaneously at buses 2 and 3 . From Eq. 4.12 we note the $\underline{Z}_{BN}$ for this combined generator-transmission system is given by

$$\underline{Z}_{BUS} = \begin{bmatrix} j.050 & j.049 & j.055 & j.048 \\ j.049 & j.121 & j.082 & j.051 \\ j.055 & j.082 & j.158 & j.033 \\ j.048 & j.051 & j.033 & j.055 \end{bmatrix}$$

Thus if bus 2 and bus 3 are simultaneously faulted, the augmented terminal graph of the complete faulted system would be given by Fig. 34

This network can again be solved following the procedure as in Section 4.3. For this numerical example let $Z_{F1} = .1$ p.u.

$Z_{F2} = .15$ p.u.

In this case, from the cutset equation, and circuit equations, we get

$$\begin{bmatrix} I_1' \\ I_2' \\ I_3' \\ I_4' \end{bmatrix} = \begin{bmatrix} 0 \\ -I_{F1} \\ -I_{F2} \\ 0 \end{bmatrix}$$

$$V_2' = V_{F1} = Z_{F1}\, I_{F1}$$

$$V_3' = V_{F2} = Z_{F1}\, I_{F2}$$

Again from Eq. 4.4 and 4.5, we get

$$\begin{bmatrix} V_2' \\ V_3' \end{bmatrix} = \begin{bmatrix} 1+j0 \\ 1+j0 \end{bmatrix} + \begin{bmatrix} j.049 & j.121 & j.082 & j.051 \\ j.055 & j.082 & j.158 & j.033 \end{bmatrix} \begin{bmatrix} 0 \\ -I_{F1} \\ -I_{F2} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+j0 \\ 1+j0 \end{bmatrix} + \begin{bmatrix} j.121 & j.082 \\ j.082 & j.158 \end{bmatrix} \begin{bmatrix} -I_{F1} \\ -I_{F2} \end{bmatrix}$$

or

$$\begin{bmatrix} Z_{F1}\, I_{F1} \\ Z_{F2}\, I_{F2} \end{bmatrix} = \begin{bmatrix} 1+j0 \\ 1+j0 \end{bmatrix} - \begin{bmatrix} j.121 & j.082 \\ j.082 & j.158 \end{bmatrix} \begin{bmatrix} I_{F1} \\ I_{F2} \end{bmatrix}$$

$$\text{or} \quad \begin{bmatrix} .1 \; I_{F1} \\ .15 \; I_{F2} \end{bmatrix} = \begin{bmatrix} 1+j0 \\ 1+j0 \end{bmatrix} - \begin{bmatrix} j.121 & j.082 \\ j.082 & j.158 \end{bmatrix} \begin{bmatrix} I_{F1} \\ I_{F2} \end{bmatrix}$$

Solving for I_{F1} and I_{F2} we get

$$\begin{bmatrix} I_{F1} \\ I_{F2} \end{bmatrix} = -j \begin{bmatrix} 3.64 \\ 2.34 \end{bmatrix} \text{p.u.}$$

Now, substituting for I_{F1} and I_{F2} in Eq. 4.4

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} V_6 + \begin{bmatrix} j.050 & j.049 & j.055 & j.048 \\ j.049 & j.121 & j.082 & j.051 \\ j.055 & j.082 & j.158 & j.033 \\ j.048 & j.051 & j.033 & j.055 \end{bmatrix} \begin{bmatrix} 0 \\ -j3.64 \\ -j2.34 \\ 0 \end{bmatrix}$$

$$\text{or} \quad \begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} (1+j0) + \begin{bmatrix} j.049 & j.055 \\ j.121 & j.082 \\ j.082 & j.158 \\ j.051 & j.033 \end{bmatrix} \begin{bmatrix} -j3.64 \\ -j2.34 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} .693 \\ .368 \\ .332 \\ .59 \end{bmatrix} \text{p.u.} \qquad .. \; 4.21$$

This represents the bus voltage at bus 1, 2, 3 and 4 for simultaneous faults at bus 2 and bus 3. From this, the individual line voltages

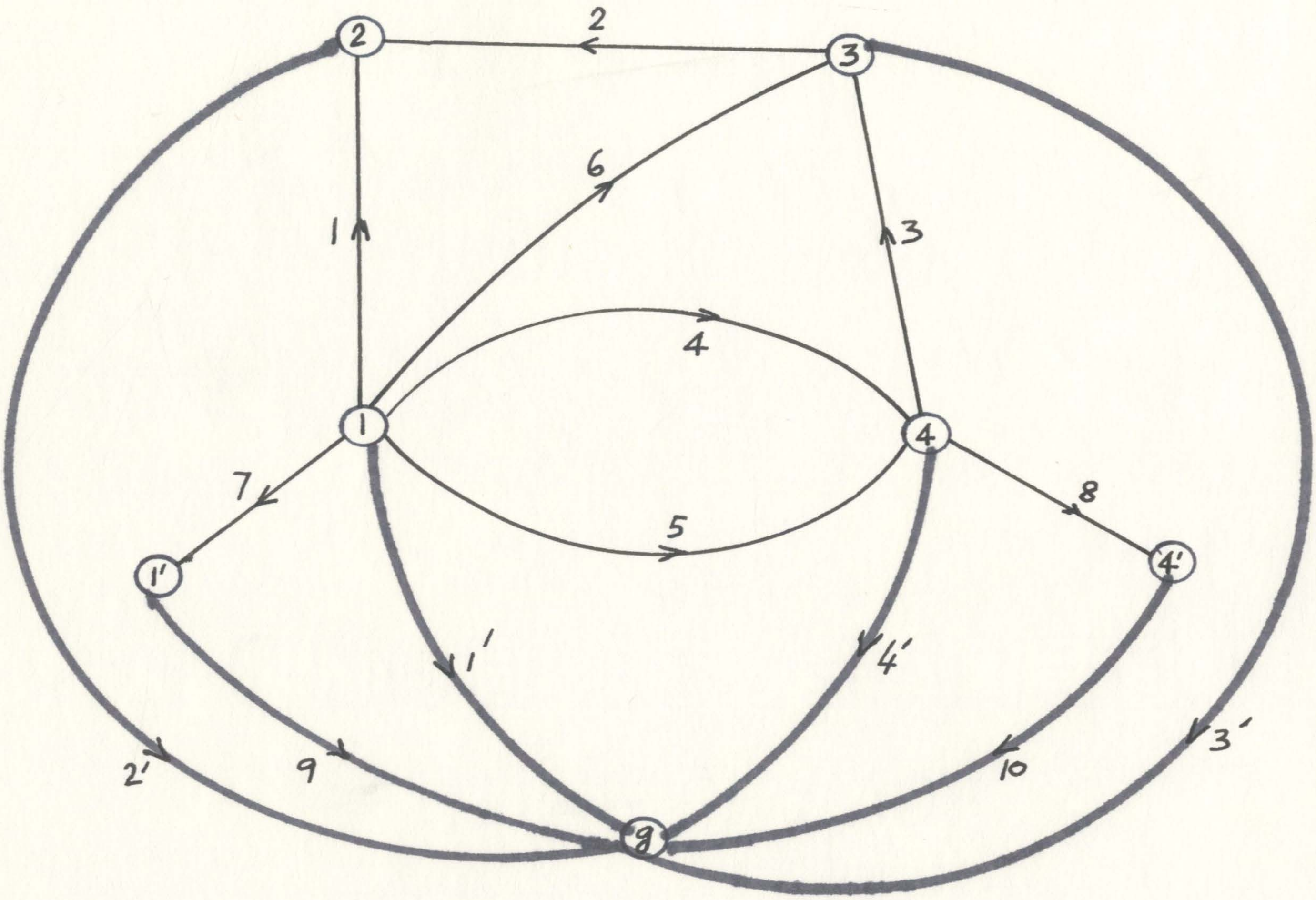


FIG. 35. UNION OF BUS VOLTAGE GRAPH WITH SYSTEM GRAPH.

can be determined by considering a graph, where these set of voltages are superimposed on the original generator-transmission system as in Fig. 35. From the circuit equation of the formulation tree $T(1', 2', 3', 4', 9, 10)$ we get

$$\begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \\ V_5' \\ V_6' \\ V_7' \\ V_8' \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} V_1' \\ V_2' \\ V_3' \\ V_4' \\ V_9' \\ V_{10}' \end{bmatrix}$$

Taking $V_9 = V_{10} = 1+j0$ and substituting for V_1', V_2', V_3', V_4' leads to

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} .325 \\ -.036 \\ .258 \\ .103 \\ .103 \\ .361 \\ -.307 \\ -.41 \end{bmatrix} \qquad \text{.. 4.22}$$

Substituting equation 4.22 in the component equations gives

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \end{bmatrix} = \begin{bmatrix} j3.25 \\ -j\ .12 \\ j\ .516 \\ j\ .34 \\ j\ .34 \\ j1.45 \\ -j4.7 \\ -j1.96 \end{bmatrix}$$

Thus we observe that solution of simultaneous faults as well can conveniently be obtained by applying the same concept as for single point faults. This procedure, it will be appreciated, can further be extended to cover simultaneous faults at n-number of buses.

4.6 Conclusion

In this chapter the concept of multi-terminal representation is applied to the short circuit analysis of a typical power system for a single three phase to ground fault as well as two simultaneous faults. The generality of the procedure is demonstrated.

CHAPTER V

CONCLUSION

In this report an attempt has been made to explain the principle of Multi-Terminal Representation as applicable to power networks. It is then applied to develop multi-terminal representations of transmission systems containing off-nominal transformers as well. This concept was then extended to develop a building algorithm, so that an appropriate network equivalence at the buses can be derived conveniently by a step-by-step procedure. Such a procedure would be of particular advantage when analysing large networks. Finally, the concept of the multi terminal representation was used for a typical power systems analysis - namely, the short circuit study. It is felt that this provides an easy, universal and conceptually elegant method for the short circuit study. Analysis of simultaneous faults can also be carried out without any difficulty. Concepts applied in this work have been extended for load flow studies [7] and further work consists in extending the application to transient stability studies. It is also felt that such a method would be very amenable to solution by the use of modern digital computers. Further work in the development of a computer programme for the short circuit study is also suggested. On the whole, the application of graph theoretic techniques for power system analysis seems to offer much promise since it naturally lends itself to computer simulation.

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