MODEL PREDICTIVE CONTROL OF A MULTIVARIABLE SOIL HEATING PROCESS

PRODYUT KUMER ROY

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By

[illegible], B.Sc. Engg.

A Thesis Submitted to the School of Graduate Studies in Partial Fulfillment

of the Requirements for the Degree of Master of Engineering

FACULTY OF ENGINEERING AND APPLIED SCIENCE

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

JULY 2008

ST. JOHN'S, NEWFOUNDLAND, CANADA

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Abstract

Multivariable control has been a challenging research area in process control, particularly for dynamically coupled and nonlinear time varying process systems. Since 1960, various multivariable control techniques have been proposed in the literature to address these issues. Out of these techniques Model Predictive Control (**MPC**) based control methodologies has received considerable attention during last few decades.

The aim of this thesis is to provide a comprehensive analysis of different MPC techniques that can be used for a wider class of multivariable process systems. MPC schemes use a model to predict the future behavior of the process to be controlled and the control move that provides the minimum future error is chosen to drive the system. The model employed in the MPC scheme is generally a linear model. The representation of the linear model in two different forms, parametric form or weighting sequence form, has developed two popular and widely accepted MPC techniques, such as Generalized Predictive Control (**GPC**) and Dynamic Matrix Control (**DMC**) based MPC techniques. Although the GPC representation is the most advanced form of MPC, the DMC technique is popular in industrial applications. The strict linear representation of the process model in the above MPC schemes is insufficient to provide better response results against nonlinear and time varying systems. To overcome this issue, two approaches are incorporated: (a) adaptive MPC design and (b) fuzzy modeling. The adaptive structure uses an online parameter identification technique using the Recursive Least Squares (**RLS**) method. The fuzzy MPC system uses the Takagi-Sugeno (**TS**) type fuzzy rule based model structure. Each rule of the TS system represents a local linear model of the

process. This particular feature is exploited to extract the linearaized parameters of the fuzzy model in order to define an adaptive fuzzy MPC system using the RLS technique. The performances of the two adaptive MPC schemes are verified against a simulated multivariable nonlinear soil heating process system. The control objective is to maintain a desired temperature profile of the soil heating system, while tracking the temperatures outputs at three different locations in the soil sample. Three heaters are located at the outer surface of the soil cell and considered as point heat sources in the model. The soil heating system is modeled using the general purpose ABAQUS finite element program and is dynamically linked with the FORTRAN based control code to achieve a realistic simulation. In order to show the effectiveness, the performances of the proposed control schemes are compared against the tracking performances of the linear model-based non-adaptive MPC techniques. A decoupled multivariable PID control scheme is also developed in this study to justify the superiority of the MPC based control strategies. The simulations results suggest the superior performance of the proposed adaptive MPC schemes against other linear MPC techniques.

Acknowledgements

This thesis is submitted for the degree of Masters of Science in Engineering and Applied Science at Memorial University of Newfoundland. First of all, I would like to express my gratitude to my supervisor here at Memorial University of Newfoundland, Dr. George Mann for his excellent guidance and for always finding time to discuss my work. I would have never finished this thesis without his constant support, encouragement, stimulating advices and patient guidance. All of these helped me a lot in maintaining my confidence to continue my research. I would also like to thank Dr.B. C. Hawlader in C-CORE for helping me to develop the finite element model based simulation strategy. The main funding sources for this research, Atlantic Innovation Fund (AIF) and Natural Science and Natural Sciences and Engineering Research Council of Canada (**NSERC**), are gratefully acknowledged. I would also like to thank Dr.V. Masek and Dr. Siu O'Young, for providing me with valuable comments and critics about this research. Finally I am grateful to my sister Krishna Rani Roy in St. John's, who has been constantly behind me during last few years and made this journey possible.

Glossary

Symbols

z^{-1}	Backward shift operator
$(.)^T$	Transpose of $(.)$
$diag\,(x_1,\cdots,x_n)$	Diagonal matrix with diagonal elements equal to $x_1,\cdots,x_n$
$y_i(k+p/k)$	Predicted value of $y_i(k+p)$ with available information at instant k
$y_i^{forced}(k+p)$	Forced response part of the prediction scheme
$y_i^{free}(k+p)$	Free response part of the prediction scheme
m	Number of input variables
n	Number of output variables
R	Number of fuzzy rules
j	Index for input variables
i	Index for output variables
r	Index for fuzzy rules
P	Prediction horizon
M	Control horizon
g	Non-adaptive step response coefficient parameter
$\hat{g}$	Online adapted step response coefficient parameter
$\mathbf{G}$	Multivariable dynamic matrix
$\hat{\mathbf{G}}$	Multivariable adaptive dynamic matrix

$\mathbf{Y}$	Vector of predicted outputs for prediction horizon
$\hat{\mathbf{Y}}$	Vector of adaptive predicted outputs for prediction horizon
$\Delta\mathbf{U}$	Vector of future control increments for the control horizon
$\Delta\hat{\mathbf{U}}$	Vector of adaptive future control increments for the control horizon
$\mathbf{F}$	Vector of free response profile
$\hat{\mathbf{F}}$	Vector of adaptive free response profile
$\mathbf{W}$	Vector of future references
δ	Weighted factor for future prediction error
λ	Weighted factor for future control increments
$\mathbf{D}$	Decoupling transfer function network matrix
$\mathbf{H}$	Multivariable MIMO transfer function model
$\mathbf{y}_n$	Vector of n number of process output
$\mathbf{u}_m$	Vector of m number of process input
$d_i(k)$	Measurable disturbance at the instant k due to plant-model mismatch
$y_i^m(k)$	Measured output from the process at the instant k
$y_i(0)$	Initial value of the i^{th} output
J	Objective function for MPC optimization
e_i^f	Feedback filter modeling error
$\mathbf{A}(z^{-1})$	Process left polynomial matrix
$\mathbf{B}(z^{-1})$	Process right polynomial matrix
ξ	Random noise vector

k_d^{ij}	Dead time of the process expressed in sampling time units
$\boldsymbol{\theta}_i$	Linear parametric vector for MPC formulation
$\boldsymbol{\varphi}_i$	Regression vector for online adaptation
μ	Forgetting factor of the RLS estimator
$\hat{a}_l^i(r), \hat{b}_l^{ij}(r)$	Linear local model parameters of the r^{th} fuzzy rule
$\hat{\boldsymbol{\theta}}_i$	Fuzzy rule consequent parameter matrix
$\boldsymbol{\beta}_i$	Vector for the degree of fulfillment of the fuzzy rules
$\hat{\boldsymbol{\theta}}_i^r$	Linear parameter vector of the r^{th} fuzzy rule for adaptation
σ	Tolerance limit of the parameter estimator

Acronyms

AMPC	Adaptive Model Predictive Control
CARIMA	Controlled Auto-Regressive Integrated Moving Average
DMC	Dynamic Matrix Control
EHAC	Extended Horizon Adaptive Control
EPSAC	Extended Prediction Self-Adaptive Control
FEM	Finite Element Model
GPC	Generalized Predictive Control
IMC	Internal Model Control
ISE	Integral Square Error
LQ	Linear Quadratic
MIMO	Multi-Input Multi-Output
MISO	Multiple-Input Single-Output
MMAC	Multiple Model Adaptive Control
MPHC	Model Predictive Heuristic Control
MPC	Model Predictive Control
NMPC	Nonlinear Model Predictive Control
PID	Proportional Integral Derivative
PPSC	Pan-Atlantic Petroleum System Consortium
QP	Quadratic Programming
RLS	Recursive Least Squares
SISO	Single-Input Single-Output
SQP	Sequential Quadratic Programming

TS	Takagi-Sugeno
Z-N	Ziegler-Nichol

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Chapter 1

Introduction

1.1 Research Motivation

Control applications of process industries in general present many challenging problems for control researchers. The nonlinear dynamic behavior, uncertain and time varying parameters, long time delays, non-minimum phase and unmeasured disturbance are known to be the most challenging issues in the field of process control [1]. In the past few decades, control of these systems has received considerable attention in both academia and in the process industries. A significant number of researches have been carried out in the control industry to solve these problems [2]. Among them, Model Predictive Control (**MPC**) based control strategy has received considerable attention in the control community [3], and has been regarded as one of appealing and attractive approaches for multivariable process control practice. Some new and very promising results of MPC schemes in the literature also allow one to think that this control technique will experience greater expansion within this community [2]. The main reason for this success can be attributed to the fact that MPC is the most general way of posing the process control problems in the time domain approach [3]. Thus the general formulation of MPC provides the opportunity to integrate its applications with optimal control, stochastic control, intelligent control, and multivariable control, and also with different types of advanced adaptive model identification strategies [3]–[7]. Considering these advantages, various MPC based techniques have been developed and being widely received by the academic world and also by the process industries [3]. However, this open methodology also provides a respectful challenge for further extensions among the present researchers, and therefore motivates them to develop new MPC based control solutions that can effectively be implemented in process industries [2], [3]. The present research under the Pan-Atlantic Petroleum System Consortium (**PPSC**) project aims to present different MPC based control techniques that can be used to control a wide class of nonlinear multivariable process operation in a flexible way. This study also aims to serve as a guideline outlining how to implement MPC for multivariable and dynamically coupled process systems. The study attempts to show how intelligent control techniques, such as fuzzy logic control is effectively incorporated to obtain superior performance of the MPC against traditional linear MPC approach.

1.2 Overview of Model Predictive Control

1.2.1 Introduction to MPC

MPC is a methodology that refers to a class of computer control algorithms in which an explicit process model is used for predicting the future behaviour of a dynamical process over an extended time horizon. At each control interval, MPC algorithm optimizes the future output behaviour in order to predict an open-loop sequence of values corresponding to the manipulated variables of the process. Different authors have provided the so-called standard definition of the MPC in the following way,

- MPC is a descriptive name for a class of computer control schemes for the explicit prediction of future plant behaviour. It computes the appropriate control action required to drive the predicted output as close to target values as possible [5], [8].

- MPC is an optimization-based strategy that uses a plant model to predict the effect of a control action on the plant [7].

In general, MPC consists of a broad range of control methods having one common feature; the controller is based on the prediction of the future system behavior by using a process model. Therefore the idea that appearing in greater or lesser degree in all the predictive control family can be summarized as follows, [2] – [10]:

- Explicit use of a model to predict the future of process output.
- Control sequence calculation based on the minimization of a certain definite objective function.
- Moving horizon strategy or receding horizon strategy, i.e. at each instant, the horizon is displaced towards the future, which involves the application of the first control signal of the sequence calculation at each step.

Because of the explicit use of the process model and the optimization approach, MPC can be applied to handle a wide class of complex processes, *e.g.* multivariable, non-minimum phase, open-loop unstable, nonlinear processes with long time delay [3], [11]. It can also deal with constraints efficiently. All these features make the MPC application more interesting among the present researchers. There are many applications of predictive control successfully in use at the present time, not only in the process industries but also application to the control of a diversity of processes ranging from robot manipulators to clinical anesthesia [3], [11]. The satisfactory performance of these applications shows the capacity of the MPC to achieve highly efficient control systems, which is able to operate during long periods of time with hardly any intervention [3].

1.2.2 MPC Strategy

The methodology of all the controllers belonging to the MPC family is characterized by the following 'moving horizon or receding horizon strategy', illustrated in Fig.1.1 [12]. A discrete-time setting is assumed in which current time is labeled as time step k. At the present time, k a model is used to predict the future behavior of the process output, y (.) for $k = 1, \cdots, P$ based on past and current control and process variables and on the optimal future control moves of the manipulated variables over a horizon, P. An objective function based on the difference between predicted output and set point sequence (desired output) is minimized to obtain optimum values for manipulated variables moves u (.) over the control horizon of M control moves ($M \leq P$). Although M moves are optimized, only the first move is implemented. After the move u (.) at the step k is implemented, the feedback measurement at the next time step, k+1 for y (.) is obtained. A correction for model error is performed, since the measured output will, in general, not be equal to the model predicted value. A new optimization problem is then solved again, over the prediction horizon of P steps by adjusting M control moves.

A popular analogy that is often used to explain this concept is the control mechanisms that comes into play when driving a car [3], [13]. The driver knows the desired trajectory for a finite horizon and by taking into account the car characteristics (mental model of the car) decides which control actions (accelerator, brakes and steering) to take in order to follow the desired trajectory.

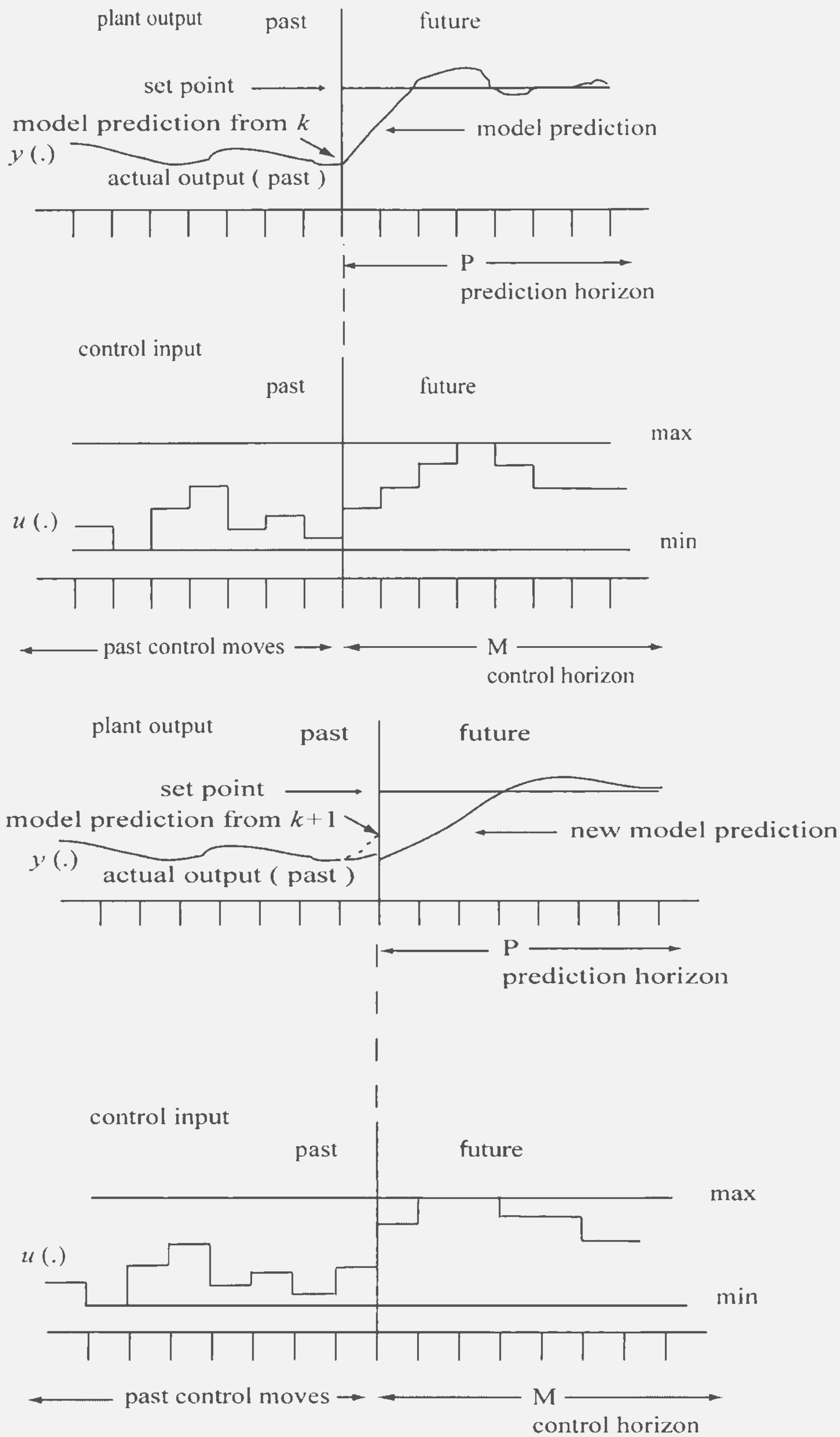

Fig. 1.1 Moving horizon strategy

Only the first control actions are taken at each instant, and the procedure is again repeated for the next control decisions in a receding horizon fashion. Notice that when using classical control schemes, such as PID control, the control actions are taken based on the past errors and the future prediction is completely disregarded.

The basic structure for implementing the MPC strategy can be described by using Fig. 1.2 [3], [14]. A dynamic model of the system is used to predict the future plant outputs based on past and current values and on the optimal future control actions. These actions are calculated by the optimizer taking into account the cost function (where the future tracking error is considered) as well as the constraints.

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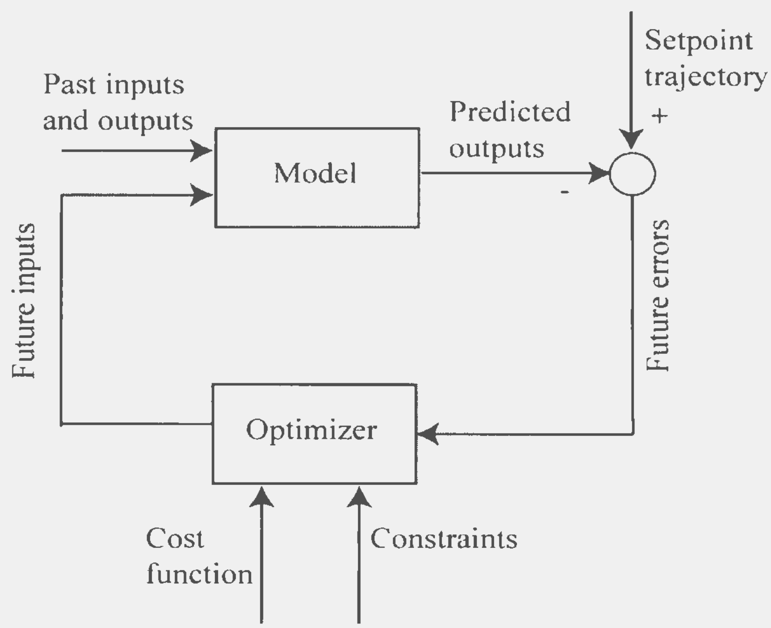


Fig.1.2 Basic structure of MPC

The process model plays an important role in the MPC based control strategy. A complete design should include the necessary mechanisms for obtaining the best possible model from the process. The obtained model must be capable of capturing the process dynamics so as to precisely predict the future outputs as well as being simple to implement and to understand. Therefore, this model is mostly chosen as a linear model. The main advantage of choosing the linear model is that it results in a linear prediction model. The common control objective used in MPC is the Linear Quadratic (**LQ**) objective function. An LQ objective, a linear prediction model and linear constraints gives rise to a so-called Quadratic Programming (**QP**) optimization problem in the MPC based control scheme. In general the QP optimization problem can be solved within a finite number of numerical operations [3]. The QP problem makes the resulting MPC algorithm robust for process control. On the other side, if the process model is allowed to be nonlinear then in general the prediction model will be nonlinear. This leads to a convex quadratic program to a non-convex nonlinear problem during optimization, which usually solved by employing Sequential Quadratic Programming (**SQP**) technique [2], [3]. It is much more difficult to solve. Furthermore, in this situation there is no guarantee that the global optimum can be found, especially in real time control, when the optimum has to be obtained in a prescribed period of time.

The general expression of such an objective function to solve the optimization problem can be written as,

$$J = \sum_{p=N_1}^{P} \delta(p)[y(k+p/k) - w(k+p)]^2 + \sum_{p=1}^{M} \lambda(p)[\Delta u(k+p-1)]^2$$

where $\Delta u(k+p-1)$ denotes the change of the control signal, N_1 is the minimum cost horizon, $y(k+p/k)$ is the p-steps ahead future prediction with available information at instant k, and $w(k+p)$ is the future setpoint sequences. The coefficient $\delta(p)$ and

$\lambda(p)$ are the weighted term of the predicted error and the control effort during optimization.

1.2.3 Historical Perspective on MPC

The origination of MPC can be traced back to as early as 1960; where open-loop optimal control was a research topic of significant interest during that time. The idea of moving horizon or receding horizon control, which is the basic core behind all MPC algorithms, was proposed by Propoi, 1963[15], within the frame of open-loop optimal feedback control system. After that, Lee and Markus, 1967 [16] anticipated current MPC practice in their optimal control text in the following way:

"*One technique for obtaining a feedback controller synthesis from knowledge of open-loop controllers is to measure the current control process state and then compute very rapidly for the open-loop control function. The first portion of this function is then used during a short time interval, after which a new measurement of the process state is made and a new open-loop control function is computed for this new measurement. The procedure is then repeated*".

This idea was impractical during that time due to lack of sophisticated hardware and computerized set-up. It was also desirable to derive a closed form control law that could be implemented with computational equipments available at reasonable cost. The true birth of MPC using the above idea was introduced in industry after the publications the seminal paper by Richalet et al. (1976) in which Model Predictive Heuristic Control (**MPHC**) was presented [17], [18]. Later the publication of Cutler and Ramaker, 1979 [19] introduced Dynamic Matrix Control (**DMC**) based control strategy. A dynamic linear weighting sequence process model is explicitly used in both algorithms in order to predict the effect of the future control actions on the output. The control actions are determined by minimizing the predicted error subject to operational constraints during optimization. The optimization is repeated at each sampling period with up to date information about the process. These formulations were both heuristic and algorithmic in nature and took the advantages of increasing potential of the digital computers at that time. Besides these two algorithms, there exist a large variety of MPC algorithms. All of these algorithms use the same underlying idea of predictive control; an explicit model, the moving horizon strategy and the computation of the control signal by optimizing the predicted output. The only difference between them is that they use different types of disturbances and plant models of the true systems for prediction. Detailed literature surveys of these techniques are presented in [3] – [5].

1.3 Present Research

1.3.1 Problem Description

Most industrial processes are multivariable, and have many variables that have to be controlled (outputs) and have many manipulated variables (inputs) to drive the plant. In some cases, a change in one manipulated variable mainly affects the corresponding controlled variable and each of the input-output pair can be considered as a Single-Input Single-Output (**SISO**) system which can easily be controlled by independent control loops. In many cases, when one of the manipulated variables is changed, it not only affects the corresponding controlled variables, but also influences the other controlled variables of the process systems. These interactions between process variables may result in a poor performance of the controlled process or even instability [20]. Multivariable decoupling theory based control technique can be used to counteract these interactions and to simplify the multivariable process into several SISO processes [20] – [23]. However, an effective decoupling network is very difficult to achieve for processes with nonlinear complex dynamics and long dead times [3], [22].

The present study considers MPC based control schemes as a solution to overcome these problems for a multivariable process system. In order to achieve this objective a soil heating system having three inputs and three outputs is considered. Fig. 1.3 shows the schematic view of the soil heating process.

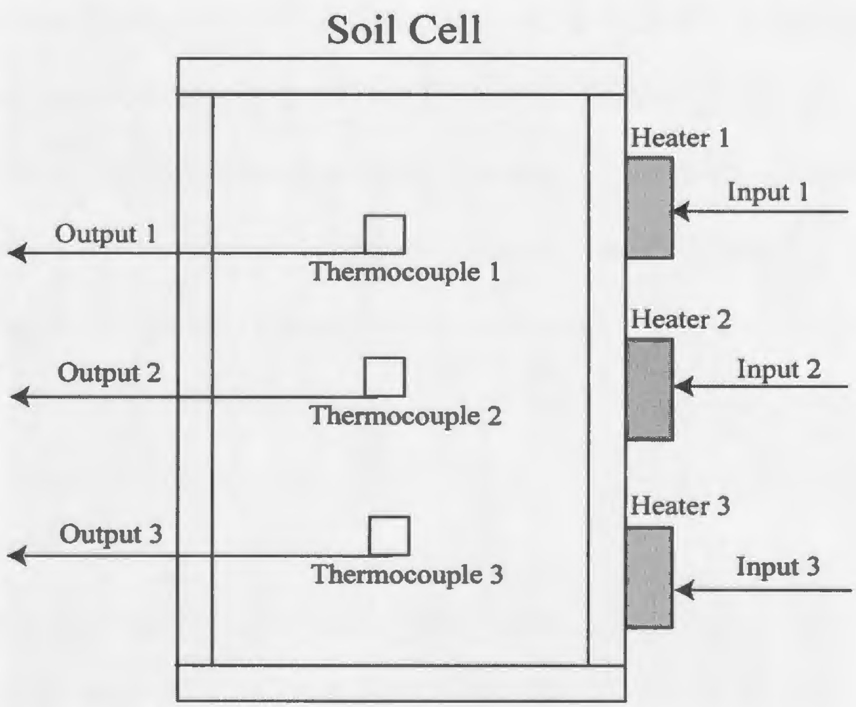


Fig.1.3 Soil heating process

The overall system comprises three heating sources (heater 1, 2 and 3) as inputs (input1, input 2, and input 3) and three thermocouples (thermocouple 1, 2 and 3) for temperature measurement at three different locations as outputs (output1, output 2, and output 3) in a cylindrical soil cell. The control objective of this process is to achieve temperature set points at the chosen locations in the cell.

The process is multivariable and there will be interactions between input and output variables of the system. This interaction can cause oscillations and even instability [20]. The system exhibits a dead time. That means, the output responds to the input after some delayed sample timed interval. It is well known that the process with dead time is difficult to control, because of the phase lag introduced by the dead time in the closed loop. However, a time-variant nonlinear transient heat transfer occurs through the soil cell that makes the process more complex and nonlinear. Because of these reasons any classical control algorithm will not give satisfactory results. MPC is one of the effective techniques to solve these types of multivariable control problems explicitly by considering the nonlinear interactions and time-lag (dead time) between inputs and outputs variables of the process when the control law is developed. However, the resulting control law in MPC is linear, easy to implement, and at the same time its tuning methodology is relatively simple [14].

1.3.2 Present Research Issues

The application of linear model based MPC schemes for the control of nonlinear processes is one of the most interesting issues in the current research on MPC [2]. At present, a major part of MPC applications in process industries stems from the use of the linear response model and has shown improved control performance against the classical PID based control performance. However, when these linear MPC schemes are applied to the nonlinear and time variant processes, the application of the linear model based controllers are limited to a relatively small operating range. Hence the capabilities of the linear model based controllers will degrade when the operation level moves away from the original design level of operation [24]. Nonlinear model can be employed with MPC to solve these problems [2], [25]. But these algorithms generally lead to the use of computationally intensive nonlinear optimization techniques that make industrial applications almost impossible [2], [9], [26], [27]. Recursive adaptation on the linear model parameters can be used to overcome these problems more efficiently by re-identifying the process with its moves into a different operating region. It can also maintain a precise control performance over a wider operational range [2], [24]. Apart from the adaptive model based MPC, fuzzy model based MPC techniques can also be used to handle a wide class of nonlinear process control problems. Although there are many representations in fuzzy systems for empirical modeling, the Takagi-Sugeno (**TS**) type fuzzy model is the popular one for nonlinear approximation where several local linear models for different operating conditions are identified and combination of these local models through fuzzy logic representations results in an approximate nonlinear model for the wide operation range [28]. To avoid nonlinear optimization in MPC, different instantaneous linearization technique can be employed for linear model extraction from the nonlinear fuzzy model [29]. Online adaptation to the fuzzy local linear models can also be used for better performance of controlled processes against time variant process dynamics [2], [9], [11]. The present study considers these issues to develop new multivariable MPC algorithms that can be used to support the present soil heating process system, as well as a wide class of multivariable processes in control industries.

1.3.3 Research Contributions

This thesis represents an attempt to highlight the above issues related to the linear MPC approach for nonlinear processes and intends to provide the contributions in the following three aspects:

1. The most popular DMC based MPC strategy is used to develop a high performance Adaptive MPC (**AMPC**) technique for a wide class of nonlinear multivariable process systems. A decoupled PID controller and a non-adaptive linear model based MPC scheme are also developed to confirm the superiority of the proposed adaptive MPC system. The proposed system uses a recursive parameters identification strategy for online adaptation on the linear model and to cope against the nonlinearity, parameter uncertainty, and time variant process dynamics.
2. The popular TS type fuzzy model based adaptive MPC strategy for a class of nonlinear time variant multivariable process systems is proposed and compared against the performance of two linear model based MPC systems. To avoid the nonlinear optimization, the proposed MPC algorithm utilizes a linear model extracted from the nonlinear fuzzy model at every time step and is used for linear MPC formulation. Online adaptation of the fuzzy scheme is also employed to handle the time variant behavior and parameters uncertainty that always exists on process systems.

3. The performances of the proposed controllers are verified against the above multivariable soil heating process. For the verifications, the soil heating system is modelled using the general-purpose ABAQUS finite element program. A dynamic control simulation is performed while linking these control algorithms into the ABAQUS finite element analysis using a FORTRAN based user-defined subroutine program. Thus a more realistic approach is employed for verification by developing the dynamic simulation model using the general-purpose ABAQUS finite element program.

1.4 Short Outline of the Thesis

This thesis consists of 6 chapters. The contents of these chapters are briefly outlined below:

Chapter 2: Process Description and Finite Element Modeling

This chapter introduces the soil heating process with its original hardware configuration. The finite element analysis of this process and the dynamic control simulation approach is presented using the general-purpose ABAQUS finite element program where the control algorithm is linked into the ABAQUS finite element program by a user-defined subroutine.

Chapter 3: Multivariable Dynamic Matrix Control

A DMC based MPC algorithm has been developed in this chapter to control a highly coupled multivariable process system. The performance of the proposed MPC controller has been compared with a multivariable PID controller, where three decoupled PID controllers have been implemented simplifying the Multi-Input-Multi-Output (**MIMO**) system to three SISO systems.

Chapter 4: Adaptive Model Predictive Control

In this chapter an AMPC strategy for a coupled multivariable process system has been developed. The proposed AMPC is developed using the DMC strategy to realize the basic predictive control structure and Recursive Least Squares (**RLS**) method for online identification of the model parameters.

Chapter 5: Adaptive Fuzzy Model Based Predictive Control

This chapter presents a novel approach to design an adaptive fuzzy model based MPC algorithm for controlling a nonlinear multivariable process system. The proposed system uses TS type fuzzy model structure. The system recognizes the active fuzzy rules, which are recursively adapted for handling the time variant behavior of the process.

Chapter 6: Concluding Remarks

This chapter starts out by discussing the conclusions of the work presented in this thesis. Then there is a discussion on suitable direction for further research.

Chapter 2

Process Description and Finite Element Modeling

2.1 Introduction

Mann (1999) [30] used the soil heating process system in the INCA laboratory to verify his control algorithms. The present study considers a different and more realistic approach to verify the proposed control logics with the soil heating system. A finite element analysis is performed on the soil cell and a model structure is built to mimic the nonlinear soil heating process dynamics. The finite element modelling technique allows representing the soil heating process dynamics based on the law of physics and the problem of defining a single mathematical equation for this nonlinear process is eliminated. Thus representing the finite element based soil heating process model as plant, a more realistic approach is developed in the study for verification. The application of the process will be used in C-CORE at Memorial University of Newfoundland for studying transport properties and moisture migration of soil under different gravitational conditions.

2.2 Hardware Configuration of the Soil Heating Process

The schematic view of the soil heating process and its hardware configuration is presented in Fig. 2.1 [30].

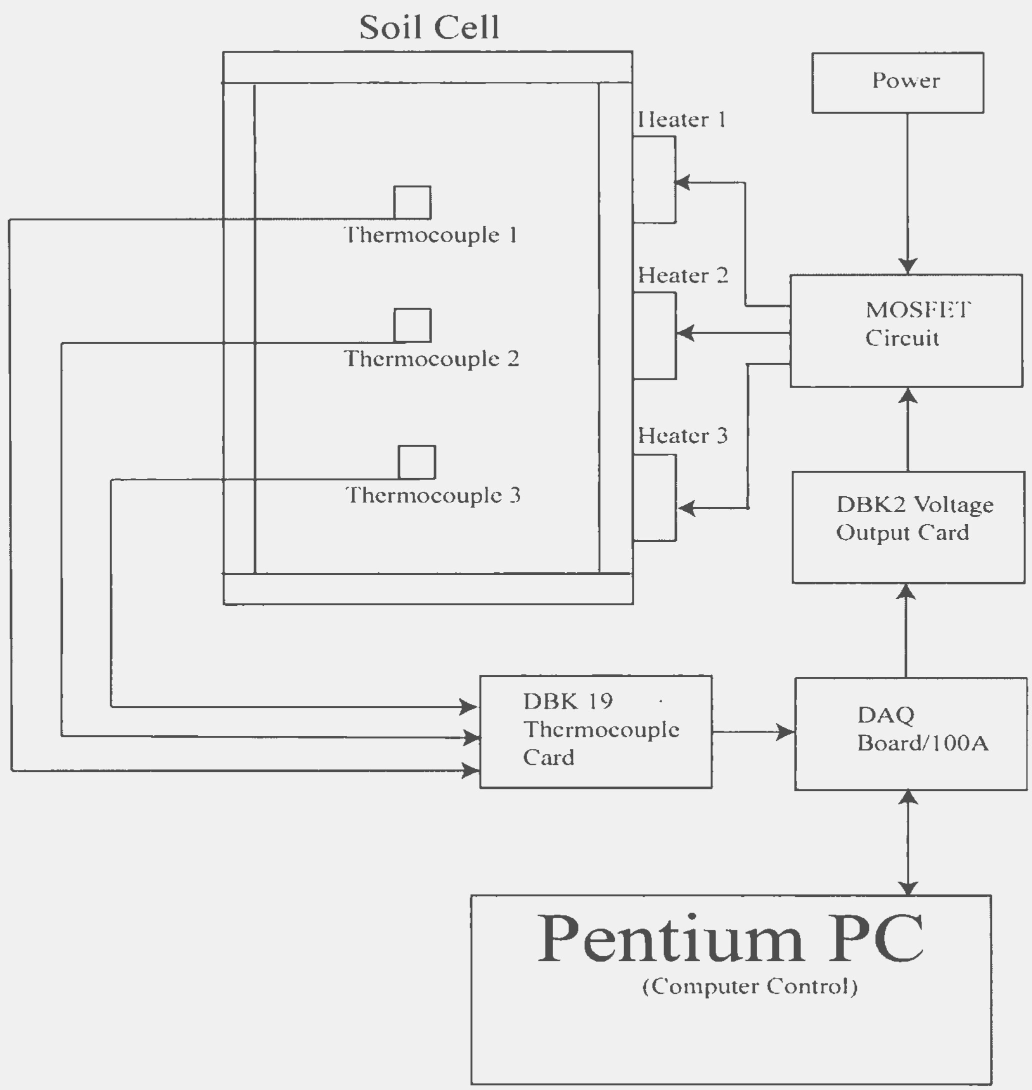


Fig. 2.1 Hardware configuration of the soil heating process adopted from [30]

Three heaters (heater1, 2 and 3) as inputs located at the peripheral of the cylindrical container supply heat to the system and three thermocouples (thermocouple 1, 2 and 3) located along the centreline of the soil cylinder measure the temperatures as outputs at three different locations. The system comprises a metal cylindrical container of 305mm height and 152mm diameter filled with dry sand.

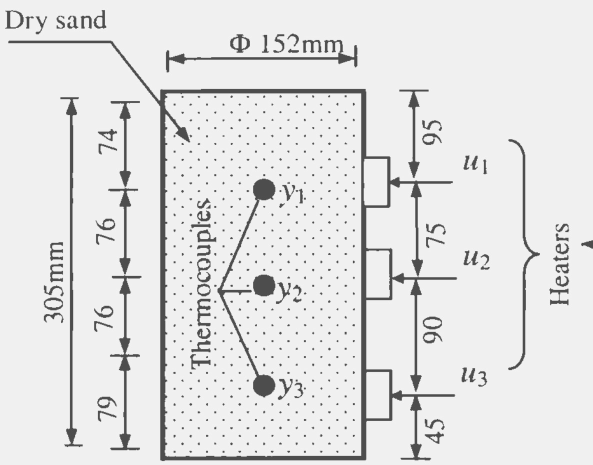
2.3 Finite Element Model (FEM) of the Soil Heating System

In the experiment as shown in Fig. 2.1 the soil used inside the cylinder is dry sand. Therefore conduction is only the mode of heat transfer through this medium. The general heat conduction equation for this medium can be written as:

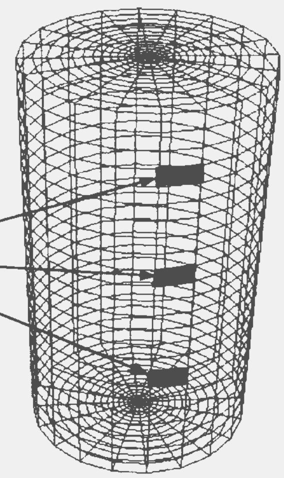
$$\nabla^2 T = \frac{\rho c}{K}\frac{\partial T}{\partial t}$$

where ∇ is the Laplacian operator, T is temperature, ρ is the density, c is the specific heat and K is the thermal conductivity of the soil. The above equation is a nonlinear differential equation under given boundary condition. The development of a closed-form solution for the three-dimensional nonlinear transient heat transfer is mathematically a complex problem and can become intractable. Therefore, the analysis can be performed numerically using the ABAQUS/Standard-6.3 finite element based program. ABAQUS/Standard-6.3 finite element numerical code has the capability to analyze heat transfer through a body for given boundary conditions. This numerical code has been used in the study to model the system. The schematic view of the soil cylinder for finite element analysis and the corresponding finite element descretization of the cell using 7200 eight nodded brick shape small elements are presented in Fig. 2.2(a) and (b), respectively.

The heaters u_1, u_2 and u_3 in the finite element model are modelled using concentrated heat flux (q_c). The concentrated heat flux is the amount of heat flux applied at each node.



(a) Schematic view



(b) Finite element model

Fig.2.2 Finite element analysis in the soil heating process

The performance of the heater depends on the area of the medium it covered. Therefore, the energy (E) from the heater is related to q_c as:

$$E = \sum_N q_c$$

where N is the number of node covered by the heater. The temperature of the thermocouples y_1, y_2 and y_3 is simply the nodal point temperature at the desired location. The thermal properties of soil are obtained from [31]: $\rho = 1600$ kg/m^3, c = 0.2 Cal/kg°C and K = 1.9 W/m°C. It is assumed that the top and bottom surfaces of the system are insulated and heat dissipation occurs only through the cylindrical surface. The cylindrical surface temperature is assumed to be always at a constant room temperature of 20°C. However, the free heat transfers within the soil system makes this process a coupled multi-variable heating system.

2.4 Verification of Finite Element Analysis

2.4.1 Finite Element Analysis through a Semi-infinite Medium

In order to verify the performance of the finite element modeling a one-dimensional transient heat transfer for a semi-infinite medium has been analyzed. This is considered because closed-form solution is available for this problem. Figure 2.3 (a) shows a section of finite element mesh. Energy (E=0.2 Watt) is supplied from the top of a rectangular bar (10 mm x 10 mm) through four nodes (see the arrows).

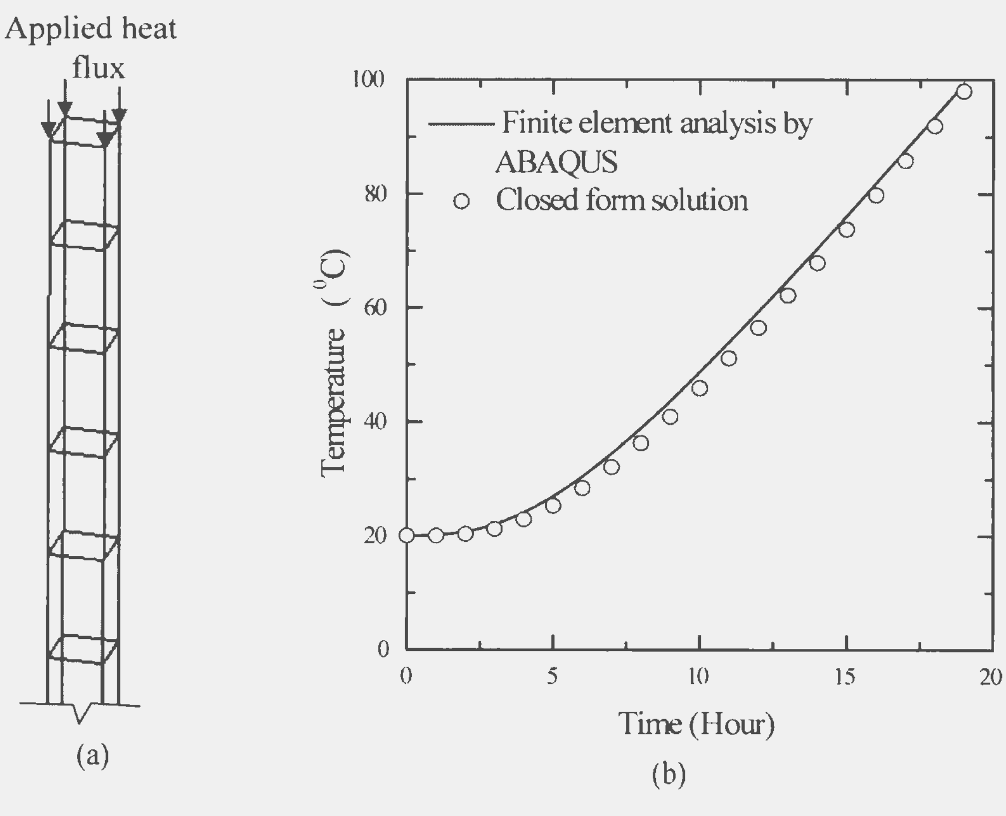


Fig. 2.3 Verification of finite element analysis

The side of the bar is insulated to model one-dimensional heat flow. Initial temperature is considered as the room temperature 20°C. The solid line in Fig 2.3 (b) shows the increase in temperature with time for an element at 500 mm from the heat supply. The closed-form solution for this problem is given by [31]:

$$T(x,t) = T_i + \frac{2q}{k}\sqrt{\frac{\alpha t}{\pi}}exp\left(\frac{-x^2}{4\alpha t}\right) - \frac{qx}{k}\operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

where, T is the temperature; x is the distance from the top where the heater supplies heat; q is the heat flux (= E/A), t is the time, erfc is the complementary error function and α is the thermal diffusivity ($=K/c\rho$). The prediction using this closed form solution is also shown in Fig. 2.3 (b) as dotted line, which is much closed to the finite element prediction. That is, modeling of the heater using concentrated heat flux is reasonably accurate. A time-step of 0.6 minute has been used in this analysis, which will be used in heat transfer analysis of the present soil heating system in the following section.

2.4.2 Verification of FEM based Soil Heating Process

As it is extremely complex to develop a closed-form solution for a three-dimensional nonlinear heat transfer, an alternative approach is used for the verification of the finite element based soil heating process model. In general if the cylindrical soil cell of the real soil heating system as shown in Fig.2.1 is surrounded with a nonconductive metal then for a constant heat supply the system will reasonably behave as an open-loop integrating process. This is because; there is no radiation of heat through the external surface of the cell. As a result the system is conserving heat energy with time inside a closed cylindrical cell. In other case, if the outer surface of the soil cell is surrounded by a non-insulated metal then for steady state heat transfer an open-loop stable temperature profile may reasonably be found. Mann (1999) [30] designed his experiment with a conductive metal in the outer surface of the soil cell such that heat can always radiate through the outer surface of the soil cell. A stable open-loop response was found from the experiment.

The finite element based soil heating model dynamics is verified with the above conditions. The dimensions used for this analysis in the soil cell are shown in Fig. 2.4. A heater is placed on the surface of the soil cylinder at a depth of 91 mm from the top. Three thermocouples on the centerline of soil cylinder at different depths (A, B, C) measure the temperature change. Note that top and bottom surfaces in the cell are assumed to be insulated in the analysis.

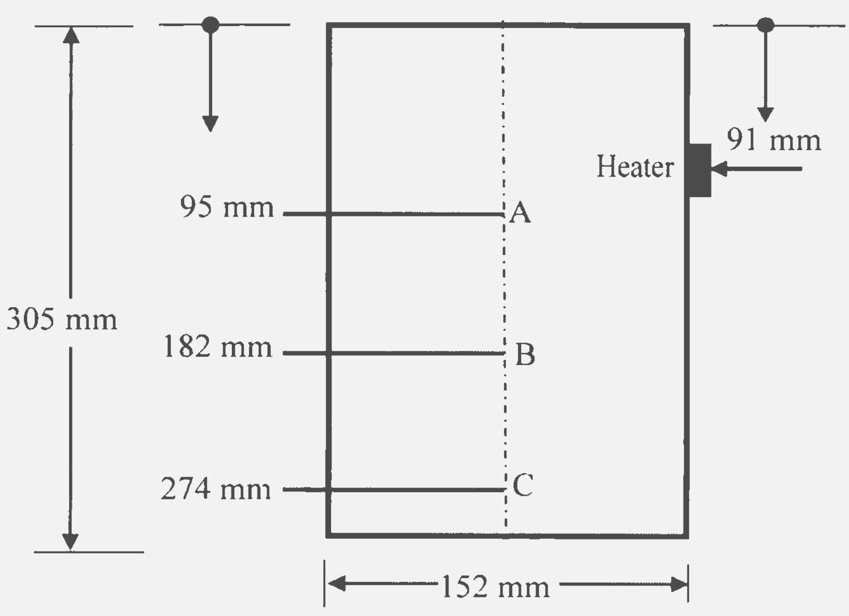


Fig. 2.4 Dimension of the system for verification

Simulations have been performed employing two types of boundary conditions on the finite element based cylindrical soil surface. The conditions are given by,

1. Cylindrical surface is non-insulated and
2. Cylindrical surface is insulated.

1. Non-insulated Cylindrical Surface

In the simulation a boundary condition for heat dissipation through the external surface is considered in the finite element based heat transfer analysis. A constant energy of 40 Watts heats from the heater is supplied to the soil cell and the boundary temperature is maintained at a constant room temperature, 20°C. The increase in temperature at three locations, A, B and C of the thermocouples are shown in Fig. 2.5. It can be seen from the Fig.2.5 that a stable temperature profile is achieved after 3 hours. After this time a steady state heat transfer occurs in the system.

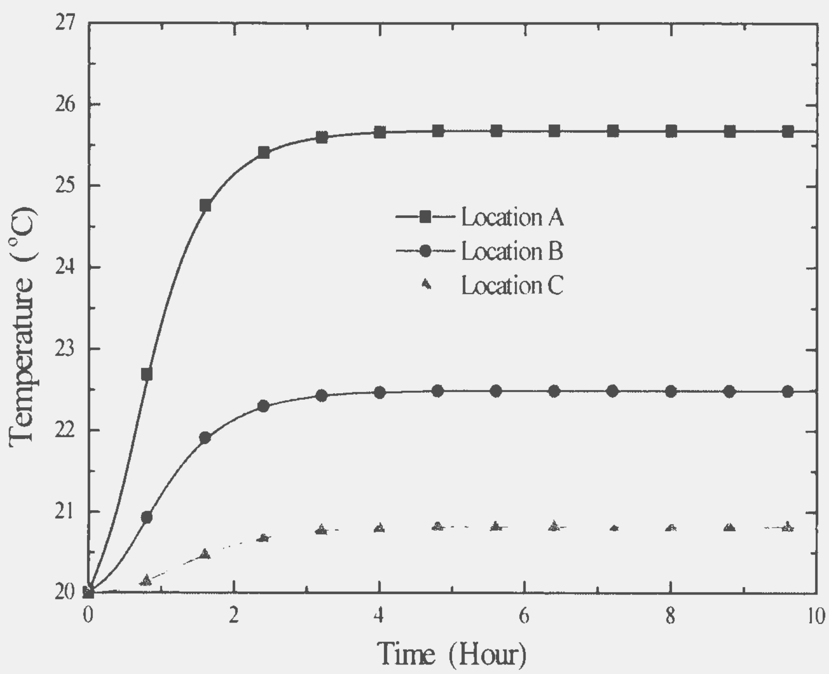


Fig. 2.5 Increase in temperature for non-insulated cylindrical surface

2. Insulated Cylindrical Surface

A number of simulations were carried out with the insulated boundary surface on the finite element based soil cell. The increase in temperatures at the locations A, B and C of thermocouples for different energy supply from the heater (4 Watt, 8 Watt and 12 Watt) are shown in Fig. 2.7. The higher the energy from the heater, the faster is the temperature increase. Simulation results justify the system is integrating with the insulated boundary condition.

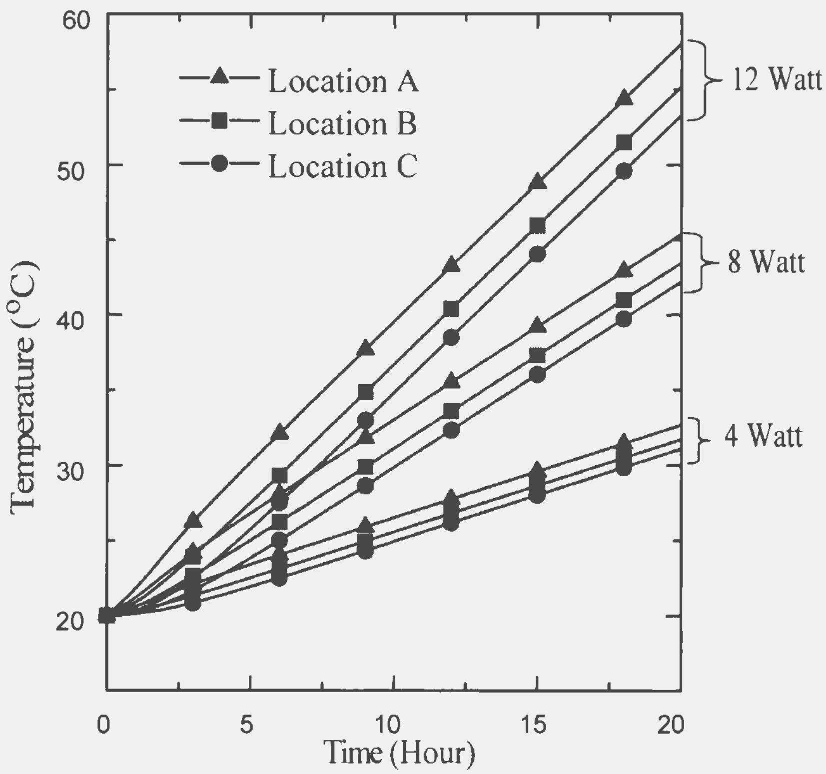


Fig. 2.6 Increase in temperature for insulated cylindrical surface

2.5 Dynamic Control Simulation

After modelling the soil heating process dynamics using ABAQUS finite element code a user-defined subroutine (**UMATHT**) has been developed to link the proposed control algorithms dynamically with the finite element based soil heating process dynamics. Thus linking the control code into the user subroutine, following Internal Model Control (**IMC**) based control simulation is performed. The schematic view for this dynamic simulation is presented in Fig.2.7.

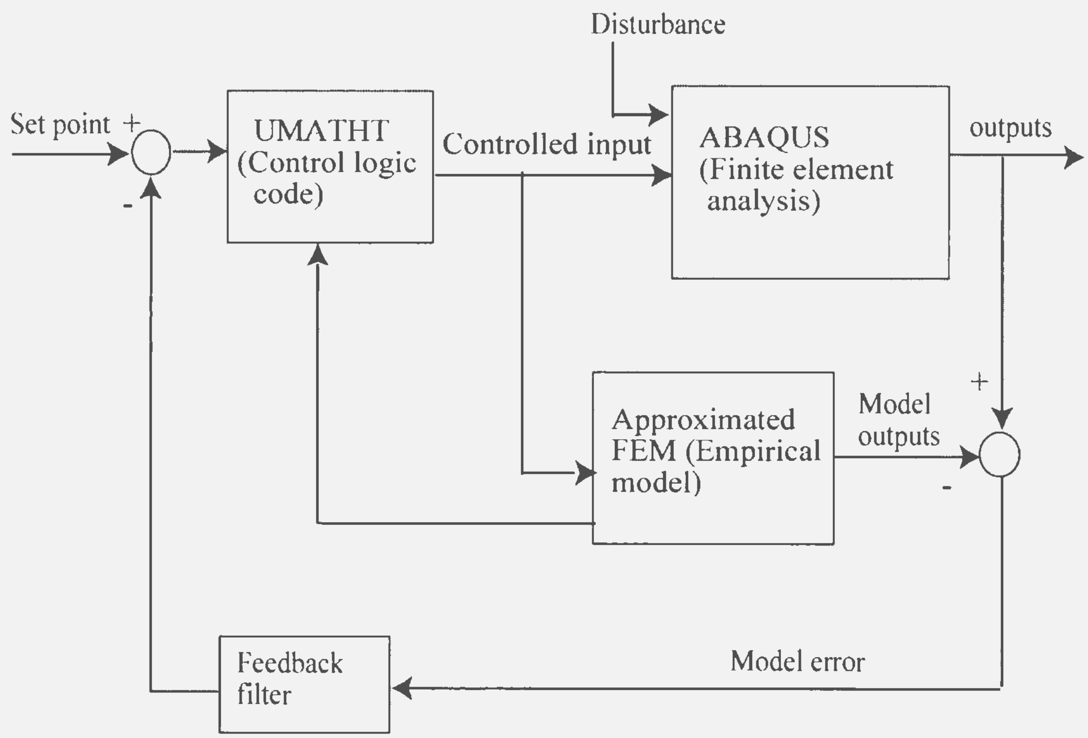


Fig.2.7 Finite element based dynamic control simulation structure

In the approach the control law is written in FORTRAN into the user subroutine and the approximated model (empirical model) of the FEM is obtained offline from the FEM based simulated open-loop input-output data.

2.6 Summary

A finite element based dynamic control simulation technique is presented for the present soil heating process. The present study considers this technique as a tool to verify the proposed controller's performance. In this control approach it is possible to perform the online modification of process parameters for robust analysis, such as heat transfer from the surface, surface insulation, variation of soil properties at different locations, disturbance due to power shut down, for simulations with least effort. However, this program has the capability to calculate the element state variables at each step, which can directly be used for updating the control variables dynamically while using the integrated control law.

Chapter 3

Multivariable Dynamic Matrix Control

3.1 Introduction

MPC has recently become one of the effective and better control methods for handling a wide class of multivariable process systems. Although there are many forms of MPC algorithms available in the literature, the DMC is arguably the most popular form of MPC algorithm currently used for multivariable control problems in the process industries [4], [24], [32]. The DMC technique is simple and relatively easy to implement using common intuition and heuristics [2], [33]. A large part of DMC's appeal is drawn from an intuitive use of a linear finite step response model of the process, a quadratic performance objective over a finite prediction horizon and optimal manipulated input moves computed as the solution to a linear least squares optimization problem.

The aim of this chapter is to present a simple design and implementation technique for a multivariable DMC controller. To verify the performance, the proposed control system is simulated against the finite element based soil heating system. Writing the FORTRAN code based on DMC strategy into the user-subroutine, a continuous control simulation is performed at each sampling instance with the dynamics of the process obtained from FEM based soil heating process. In order to compare the DMC output results the performance of a classical multivariable decoupled PID controller network is also verified where three decoupled PID controllers have been implemented simplifying the MIMO system to three SISO systems. The simulation results show that the proposed

DMC controller able to outperform the decoupled PID control system. Better results are also shown when a high disturbance is applied at different level of operation.

3.2 Background

Cutler and Ramaker [19] presented details of an unconstrained multivariable MPC algorithm, which they named DMC at the 1979 National AIChE meeting, and at the 1980 Joint Automatic Control Conference. In a companion paper at the 1980 meeting Prett and Gillette [34] described an application of DMC technology to a Fluidized Catalytic Cracking Unit (**FCCU**) reactor/regenerator in which the algorithm was modified to handle the nonlinearities and constraints. A significant number of DMC algorithms have been proposed in the literature to handle a wide class of multivariable process control problems. Qin and Badgwell [4] reported about 600 successful applications of DMC in the process industries. Apart from the DMC another form of MPC that has also rapidly gained acceptance in the control community is the Generalized Predictive Control (**GPC**) [35] based multivariable control strategy. The GPC employs a linear Controlled Auto-Regressive Integrated Moving Average (**CARIMA**) model of the process, which allows a rigorous mathematical treatment of the predictive control paradigm. The GPC performance objective is very similar to that of DMC but the optimization is achieved via recursion on the Diophantine identity [36].

3.3 Formulation of Multivariable Non-Adaptive DMC

As a starting point, consider the following discrete-time linear step response model for an n-output, m-input multivariable process. The i^{th} process output at the time instant k is described by,

$$y_i(k) = y_i(0) + \sum_{j=1}^{m}\sum_{l=1}^{\infty} g_l^{ij}\Delta u_j(k-l) \tag{3.1}$$

where $y_i(0)$ is the initial condition of the i^{th} output y_i, Δu_j is the change in the j^{th} manipulated input, g_l^{ij} is the l^{th} unit step response coefficient of the i^{th} output that corresponds to j^{th} input. The response coefficient g_l^{ij} in equation (3.1) converges to the steady-state gain if, and only if, the system is stable. For such a stable processes the model can be decomposed and re-expressed as,

$$y_i(k) = y_i(0) + \sum_{j=1}^{m}\sum_{l=1}^{N_{ij}} g_l^{ij}\Delta u_j(k-l) + \sum_{j=1}^{m}\sum_{l=N_{ij}+1}^{\infty} g_l^{ij}\Delta u_j(k-l) \tag{3.2}$$

where N_{ij} is the process stable time in samples. If the process is asymptotically stable, the coefficients g_l^{ij} of the step response leads to a constant value after N_{ij} sampling periods, so it can be considered that

$$g_{N_{ij}+1}^{ij} = g_{N_{ij}+2}^{ij} = \cdots = g_{\infty}^{ij} = g_{ss}^{ij} \approx 0 \text{ and}$$

$$\sum_{j=1}^{m}\sum_{l=N_{ij}+1}^{\infty} \Delta u_j(k-l) = u_j(k-N_{ij}-1),$$

where g_{ss}^{ij} is the steady state step response coefficients.

Thus the model in equation (3.2) can be written in the following form,

$$y_i(k) = y(0) + \sum_{j=1}^{m}\sum_{l=1}^{N_{ij}} g_l^{ij}\Delta u_j(k-l) \tag{3.3}$$

The cornerstone of the DMC algorithm is the response model in equation (3.3) that predicts the output, $y_i(k+p/k)$; p sampling instants ahead of the current time instant, k and is given by,

$$y_i(k+p/k) = y_i(0) + \underbrace{\sum_{j=1}^{m}\sum_{l=1}^{p} g_l^{ij}\Delta u_j(k+p-l)}_{\text{effect of current \& future moves}} + \underbrace{\sum_{j=1}^{m}\sum_{l=p+1}^{N_{ij}-1} g_l^{ij}\Delta u_j(k+p-l)}_{\text{effect of past moves}}. \qquad (3.4)$$

To cope with the unmeasured disturbances and inaccuracies due to plant-model mismatch a current disturbance measurement, d_i is estimated with equation (3.4) through the prediction horizon, $p = 1,\ldots,P$ and is written as

$$y_i(k+p/k) = y_0 + \sum_{j=1}^{m}\sum_{l=1}^{p} g_l^{ij}\Delta u_j(k+p-l) + \sum_{j=1}^{m}\sum_{l=p+1}^{N_{ij}-1} g_l^{ij}\Delta u_j(k+p-l) + d_i(k). \qquad (3.5)$$

Since the future values of $d_i(k+p)$ are not available an estimate of the future disturbance is used. In the absence of any additional knowledge of $d_i(k+p)$ over future sampling instants, the predicted disturbance is assumed to be equal to that estimated at the current time instant. Therefore

$$d_i(k) = y_i^m(k) - \left[y_i(0) + \sum_{j=1}^{m}\sum_{l=1}^{N_{ij}} g_l^{ij}\Delta u_j(k-l) \right] \qquad (3.6)$$

where y_i^m is the current measurement of the i^{th} output.

The prediction model in equation (3.5) can be represented as

$$y_i(k+p/k) = \sum_{j=1}^{m}\sum_{l=1}^{p} g_l^{ij}\Delta u_j(k+p-l) + f_i(k+p). \qquad (3.7)$$

Where,

$$f_i(k+p) = y_i^m(k) - \left[y_i(0) + \sum_{j=1}^{m} \sum_{l=1}^{N_{ij}} g_l^{ij} \Delta u_j(k-l) \right] + y_i(0) + \sum_{j=1}^{m} \sum_{l=1}^{N_{ij}} g_{p+l}^{ij} \Delta u_j(k-l) \qquad (3.8)$$

is defined as the free response profiles of the i^{th} output, because this part of the response does not depend on the future control actions.

Now using the prediction model described in equation (3.7) the future step predictions through the prediction horizon, P with M future control actions, can be expressed in the following form,

$$y_i(k+1/k) = g_1^{i1} \Delta u_1(k) + \cdots + g_1^{ij} \Delta u_j(k) + \cdots + g_1^{im} \Delta u_m(k) + f_i(k+1)$$

$$\begin{aligned} y_i(k+2/k) = {} & g_1^{i1} \Delta u_1(k+1) + g_2^{i1} \Delta u_1(k) + \cdots + g_1^{ij} \Delta u_j(k+1) + g_2^{ij} \Delta u_j(k) + \\ & \cdots + g_1^{im} \Delta u_m(k+1) + g_2^{im} \Delta u_m(k) + f_i(k+2) \end{aligned}$$

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$$\begin{aligned} y_i(k+M/k) = {} & g_1^{i1} \Delta u_1(k+M-1) + \cdots + g_M^{i1} \Delta u_1(k) + \cdots + g_1^{ij} \Delta u_j(k+M-1) + \cdots \\ & + g_M^{ij} \Delta u_j(k) + \cdots + g_1^{im} \Delta u_m(k+M-1) + \cdots + g_M^{im} \Delta u_m(k) + f_i(k+M) \end{aligned}$$

$$\begin{aligned} y_i(k+M+1/k) = {} & g_1^{i1} \underbrace{\Delta u_1(k+M)}_{=0} + \cdots + g_{M+1}^{i1} \Delta u_1(k) + \cdots + g_1^{ij} \underbrace{\Delta u_j(k+M)}_{=0} + \cdots \\ & + g_{M+1}^{ij} \Delta u_j(k) + \cdots + g_1^{im} \underbrace{\Delta u_m(k+M)}_{=0} + \cdots + g_{M+1}^{im} \Delta u_m(k) + f_i(k+M) \end{aligned}$$

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$$\begin{aligned} y_i(k+P/k) = {} & g_1^{i1} \underbrace{\Delta u_1(k+P-1)}_{=0} + \cdots + g_P^{i1} \Delta u_1(k) + \cdots + g_1^{ij} \underbrace{\Delta u_j(k+P-1)}_{=0} + \cdots \\ & + g_P^{ij} \Delta u_j(k) + \cdots + g_1^{im} \underbrace{\Delta u_m(k+P-1)}_{=0} + \cdots + g_P^{im} \Delta u_m(k) + f_i(k+P). \end{aligned}$$

These above set of future predictions for the i^{th} output can be written in the following compact matrix-vector form,

$$\mathbf{Y}_i = \mathbf{G}_i^R \Delta\mathbf{U} + \mathbf{F}_i . \tag{3.9}$$

Where,

$$\mathbf{Y}_i = \left[y_i(k+1/\ k), \cdots, y_i(k+P/\ k) \right\}$$

$$\mathbf{F}_i = \left[f_i(k+1), \cdots, f_i(k+P) \right\} \text{ and}$$

$$\Delta\mathbf{U} = \left[\Delta\mathbf{U}_1, \cdots, \Delta\mathbf{U}_m \right\}^T \text{ where}$$

$$\Delta\mathbf{U}_j = \left[\Delta u_j(k), \cdots, \Delta u_j(k+M) \right] .$$

The matrix $\mathbf{G}_i^R$ in equation (3.9) is the dynamic response coefficient matrix of the i^{th} output related to, $j = 1, \cdots, m$ inputs of the process, and is given by

$$\mathbf{G}_i^R = \left[\mathbf{G}_{i1}, \cdots, \mathbf{G}_{im} \right\}_{P \times mM} ,$$

where

$$\mathbf{G}_{ij} = \begin{bmatrix} g_1^{ij} & 0 & . & . & . & 0 \\ g_2^{ij} & g_1^{ij} & 0 & . & . & 0 \\ . & . & . & . & . & . \\ g_M^{ij} & g_{M-1}^{ij} & . & . & . & g_1^{ij} \\ . & . & . & . & . & . \\ g_P^{ij} & g_{P-1}^{ij} & . & . & . & g_{P-M+1}^{ij} \end{bmatrix}_{P \times M} . \tag{3.10}$$

The predicted equation in (3.9) can be combined for, $i = 1, \cdots, n$ in the following compact form,

$$\mathbf{Y} = \mathbf{G}\Delta\mathbf{U} + \mathbf{F} \tag{3.11}$$

while defining,

$$\mathbf{Y} = \left[\mathbf{Y}_1, \cdots, \mathbf{Y}_n \right]^T$$

$$\mathbf{F} = \left[\mathbf{F}_1, \cdots, \mathbf{F}_n\right]^T \text{ and}$$

$$\mathbf{G} = \left[\mathbf{G}_1^R, \cdots, \mathbf{G}_n^R\right]^T_{nP\times mM}.$$

The objective of the control is to determine the current and future control moves $\Delta u_j(k+p-l)$ in equation (3.7) such that the predicted output profiles $y_i(k+p/k)$ for $p=1,\ldots,P$ will drive as close to the setpoint sequences, $w_i(k+p)$ as possible in a least square sense with a penalty of M control moves of Δu_j. To do this following cost function is selected:

$$J = \sum_{i=1}^{n}\sum_{p=1}^{p}\delta_i(p)[w_i(k+p) - y_i(k+p/k)]^2 + \sum_{j=1}^{m}\sum_{p=1}^{M}\lambda_i(p)[\Delta u_j(k+p-1)]^2 \tag{3.12}$$

or the same can be expressed in matrix form as,

$$\begin{aligned} J = \boldsymbol{\delta}_1[\mathbf{W}_1 - \mathbf{Y}_1]^T[\mathbf{W}_1 - \mathbf{Y}_1] + \boldsymbol{\lambda}_1[\Delta\mathbf{U}_1]^T[\Delta\mathbf{U}_1] + \cdots + \boldsymbol{\delta}_i[\mathbf{W}_i - \mathbf{Y}_i]^T[\mathbf{W}_i - \mathbf{Y}_i] + \\ \boldsymbol{\lambda}_j[\Delta\mathbf{U}_j]^T[\Delta\mathbf{U}_j] + \cdots + \boldsymbol{\delta}_n[\mathbf{W}_n - \mathbf{Y}_n]^T[\mathbf{W}_n - \mathbf{Y}_n] + \boldsymbol{\lambda}_m[\Delta\mathbf{U}_m]^T[\Delta\mathbf{U}_m] \end{aligned}.$$

It can be written in the following compact form

$$J = \boldsymbol{\delta}[\mathbf{W} - \mathbf{Y}]^T[\mathbf{W} - \mathbf{Y}] + \boldsymbol{\lambda}[\Delta\mathbf{U}]^T[\Delta\mathbf{U}]. \tag{3.13}$$

Where $\mathbf{W}$ is the setpoint vector and is given by,

$\mathbf{W} = \left[\mathbf{W}_1, \cdots, \mathbf{W}_n\right]$, with the i^{th} setpoint trajectory as

$$\mathbf{W}_i = \left[w_i(k+1), \cdots, w_i(k+P)\right]$$

and the matrices $\boldsymbol{\delta}$ and $\boldsymbol{\lambda}$ are the diagonal matrices of controlled variables weights and manipulated variables weights of the MIMO DMC with dimensions ($nP \times nP$) and ($mM \times mM$), respectively and are given by,

$$\boldsymbol{\delta} = \left[diag(\boldsymbol{\delta}_1, \cdots, \boldsymbol{\delta}_n) \right]$$

$$\boldsymbol{\lambda} = \left[diag(\boldsymbol{\lambda}_1, \cdots, \boldsymbol{\lambda}_m) \right].$$

The diagonal terms of $\boldsymbol{\delta}$ and $\boldsymbol{\lambda}$ also the diagonal matrices of the i^{th} controlled variables weights and j^{th} manipulated variables weights with dimensions ($P \times P$) and ($M \times M$) respectively. These matrices are given by,

$$\boldsymbol{\delta}_i = \left[diag\,(\delta_i(1), \cdots, \delta_i(P)) \right]$$

$$\boldsymbol{\lambda}_j = \left[diag\,(\lambda_j(1), \cdots, \lambda_j(M)) \right].$$

Now using equation (3.11), the cost function in equation (3.13) can be expressed as,

$$J = \boldsymbol{\delta}[\mathbf{W} - \mathbf{G}\Delta\mathbf{U} - \mathbf{F}]^T [\mathbf{W} - \mathbf{G}\Delta\mathbf{U} - \mathbf{F}] + \boldsymbol{\lambda}[\Delta\mathbf{U}]^T [\Delta\mathbf{U}]$$

or

$$J = \boldsymbol{\delta}\mathbf{W}^T\mathbf{W} + \boldsymbol{\delta}\Delta\mathbf{U}^T\mathbf{G}^T\mathbf{G}\Delta\mathbf{U} + 2\boldsymbol{\delta}\Delta\mathbf{U}^T\mathbf{G}^T\mathbf{F} + \boldsymbol{\delta}\mathbf{F}^T\mathbf{F} - 2\boldsymbol{\delta}\Delta\mathbf{U}^T\mathbf{G}^T\mathbf{W} - 2\mathbf{F}^T\mathbf{W} + \boldsymbol{\lambda}[\Delta\mathbf{U}]^T[\Delta\mathbf{U}] \tag{3.14}$$

Minimizing J in equation (3.14) with respect to $\Delta\mathbf{U}$ yields:

$$\frac{\partial J}{\partial \Delta\mathbf{U}} = 2\boldsymbol{\delta}\mathbf{G}^T\mathbf{G}\Delta\mathbf{U} + 2\boldsymbol{\delta}\mathbf{G}^T\mathbf{F} - 2\boldsymbol{\delta}\mathbf{G}^T\mathbf{W} - 2\mathbf{F}^T\mathbf{W} + 2\boldsymbol{\lambda}\Delta\mathbf{U} = 0$$

This leads to the following unconstrained close loop DMC control law,

$$\Delta\mathbf{U} = (\mathbf{G}^T\boldsymbol{\delta}\mathbf{G} + \boldsymbol{\lambda}\mathbf{I})^{-1}\mathbf{G}^T\boldsymbol{\delta}(\mathbf{W} - \mathbf{F}) \tag{3.15}$$

or $$\Delta\mathbf{U} = \mathbf{K}(\mathbf{W} - \mathbf{F}).$$

where $\mathbf{K} = (\mathbf{G}^T\boldsymbol{\delta}\mathbf{G} + \boldsymbol{\lambda}\mathbf{I})^{-1}\mathbf{G}^T\boldsymbol{\delta}$ is the MPC gain matrix and the term $(\mathbf{W} - \mathbf{F})$ is the vector of future predicted error over P sampling instants due to the effect of past inputs. The only control moves $\Delta u_1(k), \cdots, \Delta u_m(k)$ in equation (3.15) are applied to the plant at current step and whole procedure is repeated in each sampling step. The implementation of the control law with the process can be described by Fig.3.1 [3].

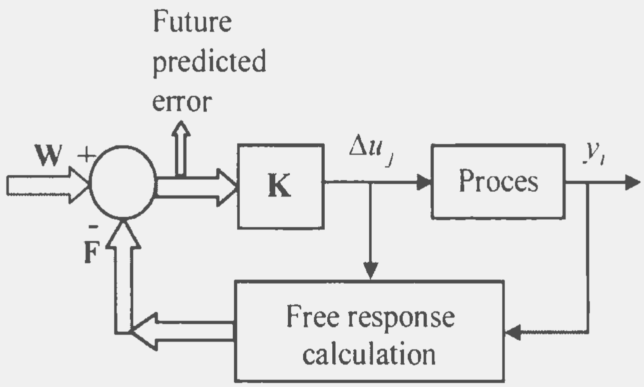


Fig.3.1 MPC law

The control move $\Delta\mathbf{U}$ is zero if there are no future predicted errors, that is, $(\mathbf{W}-\mathbf{F})=\mathbf{0}$ then the control objective is fulfilled only with the free evolution of the process. In other case, the increment in the control actions is computed by optimizing the future prediction errors. This control law provides the feedback of the measured disturbance at every sampling instance. The other schemes such as GPC assume a model for noise and disturbances and employ the information through a series of filters to predict future disturbances profiles. However, the control actions computed in equation (3.15) still has limitations when implementing to the real process due to the saturation limit of the actuators. Constraints on the incremental values of the manipulated variables can be imposed for the safety reasons. Typical constraints which generally implement in the control process can be applied with this scheme for the safety purpose, [37]

$$u_j^{\min} - u_j(k-1) \le \Delta u_j(k) \le u_j^{\max} - u_j(k-1) \tag{3.16}$$

where $u_j^{\min}$ and $u_j^{\max}$ are the minimum and maximum limits of the j^{th} manipulated input variable.

3.4 Application of DMC to the Soil Heating Process

The above DMC based control algorithm is applied to the finite element based multivariable soil heating process. To identify the process model for DMC, open-loop step input test simulations are performed on the finite element based soil heating model. In the simulations a constant open-loop control signal, u_j is applied to the finite element based process model and responses of the corresponding outputs (y_1, y_2 and y_3) are measured to model the finite element system. Analyzing the open-loop step responses following linear step response model is obtained for the i^{th} output [3] and is given by,

$$y_i(k) = y_i(0) + \sum_{j=1}^{3}\sum_{l=1}^{N_{ij}} g_l^{ij} \Delta u_j(k-l),\ i = 1,\cdots,3 \tag{3.17}$$

Note that, the sampling interval is chosen 0.6 minute and the corresponding values of N_{ij} for the process models are,

$$N_{11} = 40 \quad N_{12} = 50 \quad N_{13} = 55$$

$$N_{21} = 35 \quad N_{22} = 50 \quad N_{23} = 45$$

$$N_{31} = 30 \quad N_{32} = 50 \quad N_{33} = 55.$$

The complete control structure of the proposed control system with FEM of the soil cell (plant) is shown in Fig.3.2. To cope with the model-plant mismatch due to process uncertainties and to avoid the steady state control error, the proposed scheme is implemented within an IMC structure (Fig. 3.2). A feedback filter is introduced into the control scheme and the filtered modeling error is utilized to modify the setpoint. For this purpose, the following first-order low-pass filter is used [2]:

$$e_i^f(k) = K_f\left(y_i(k) - y_i^m(k)\right) + \left(1 - K_f\right) e_i^f(k-1) \tag{3.18}$$

where K_f is the adjustable filter parameter, $K_f \in [0, 1]$. The feedback filter is able to filter out the measurement noise and stabilize the loop by reducing the loop gain.

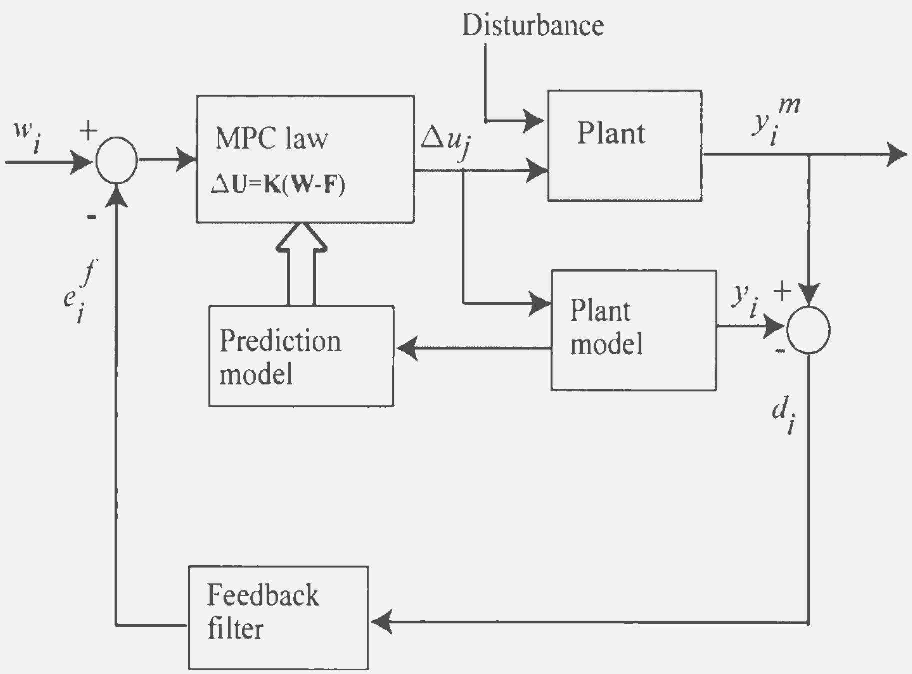


Fig. 3.2 Non-adaptive MPC structure for the soil heating process

The overall control strategy can be described in the following simple steps:

1. Identify the step response model described in equation (3.17) in offline and placed in parallel with the FEM as shown in Fig.3.2
2. Measure the current disturbance ($d_i(k)$) due to plant-model mismatch using equation (3.6) and add with the prediction model shown in equation (3.7).

3. Predict the output in the future steps and develop the control law present in equation (3.15).
4. Apply the control inputs ($\Delta u_1(k),\cdots,\Delta u_3(k)$) to both the plant and the model.
5. Go to step 2 and repeat

3.5 Decoupled Multivariable Control Scheme

In the control theory, a decoupler is defined as a device, which eliminates the interaction between manipulated and controlled variables by changing all the manipulated variables in such a manner that only the desired controlled variables will be changed. A decoupling network design problem for a multivariable process is presented in this section. The design uses the method shown by Westpha (1995) [23]. The general structure for an n-output, m-input ($n \times m$) multivariable process system with decoupling network is shown in Fig.3.3.

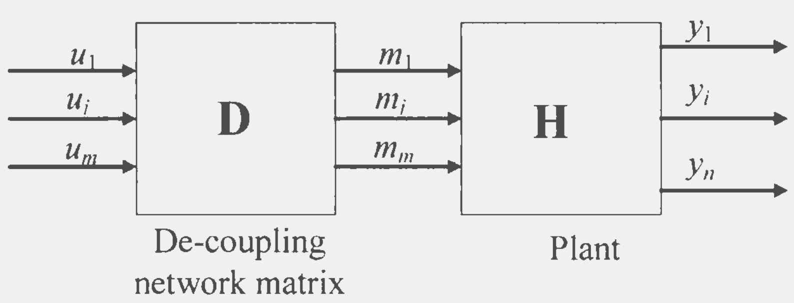


Fig. 3.3 $n \times m$ Decoupled MIMO process

In the figure, **H** is defined as the process dynamic transfer function matrix and **D** is the desired decoupling matrix, and is given by,

$$\mathbf{D} = \begin{bmatrix} D_{11} & \cdots & D_{1m} \\ \vdots & \vdots & \vdots \\ D_{i1} & \cdots & D_{im} \\ \vdots & \vdots & \vdots \\ D_{n1} & \cdots & D_{nm} \end{bmatrix} \text{ and} \tag{3.19}$$

$$\mathbf{H} = \begin{bmatrix} H_{11} & \cdots & H_{1m} \\ \vdots & \vdots & \vdots \\ H_{i1} & \cdots & H_{im} \\ \vdots & \vdots & \vdots \\ H_{n1} & \cdots & H_{nm} \end{bmatrix} . \tag{3.20}$$

Where D_{ij} and H_{ij} are the decoupled transfer function model and plant dynamic transfer function model for the i^{th} output corresponding to j^{th} input. The aim is to design the decoupling network matrix **D** such that u_1 affects only y_1, u_2 affects only $y_2, \cdots$, and u_m affects only y_n, respectively at steady state.

The design equations can be summarized using the following matrix notation,

$$\mathbf{y}_n = \mathbf{Hm} \tag{3.21}$$

$$\mathbf{m} = \mathbf{Du}_m . \tag{3.22}$$

Where $\mathbf{y}_n$, $\mathbf{u}_m$, and $\mathbf{m}$ are the vector of process outputs, manipulated inputs to the de-coupler and manipulated inputs to the process, respectively and are given by,

$$\mathbf{y}_n = [y_1, \cdots, y_n]^T$$

$$\mathbf{u}_m = [u_1, \cdots, u_m]^T$$

$$\mathbf{m} = [m_1, \cdots, m_m]^T .$$

Thus from equations (3.21) and (3.22) results the following form,

$$\mathbf{y}_n = \mathbf{HDu}_m . \tag{3.23}$$

In order to affect decoupled control the coupling that exists in equation (3.23) is deliberately removed using the following procedure. Modify equation (3.23) as,

$$\mathbf{y}_n = \mathbf{L}\mathbf{u}_m \tag{3.24}$$

where $\mathbf{L}$ is the diagonal matrix redefined while modifying the process and decoupler matrices as given by,

$$\mathbf{L} = \mathbf{H}_M \mathbf{D}_M \,. \tag{3.25}$$

The matrix $\mathbf{H}_M$ is the modified process transfer function matrix with all off diagonal elements in equation (3.19) is zero, (i.e. for $i \neq j$, $\mathbf{H}_M(i, j) = 0$, else $\mathbf{H}_M(i, j) = \mathbf{H}(i, j)$) and $\mathbf{D}_M$ is the modified decoupling network matrix with all diagonal elements in equation (3.20) are equal to 1 (i.e. for $i = j$, $\mathbf{D}_M(i, j) = 1$, else $\mathbf{D}_M(i, j) = \mathbf{D}(i, j)$).

Using equations (3.23) to (3.25) the following relation is established.

$$\mathbf{D} = \mathbf{H}^{-1}\mathbf{H}_M \mathbf{D}_M \tag{3.26}$$

or it can also be written using the equation (3.25) as,

$$\mathbf{D} = \mathbf{H}^{-1}\mathbf{L} \,.$$

The model in equation (3.26) defines the decoupler for a ($n \times m$) multivariable process. Solving the equations in (3.26) for the present (3×3) soil heating process the desired decoupling transfer functions model can be obtained. After designing the decoupling network, it is possible to implement three PID controllers (defining as PID_1, PID_2 and PID_3) to control and tune the output 1 (y_1), output 2 (y_2) and output 3 (y_3), separately.

The complete schematic diagram of the finite element based soil heating process (FEM) with decoupled PID control network is shown in Fig. 3.4.

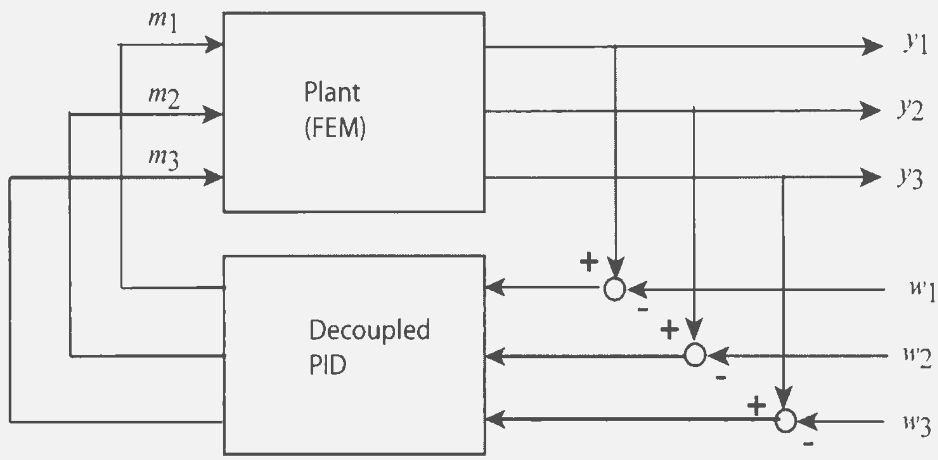


Fig. 3.4 Decoupled PID controller for the soil heating process

The internal structure of the decoupled PID controllers in Fig.3.4 is presented in Fig.3.5. The variables e_1, e_2 and e_3 in the figure are defined as the feedback error corresponds to output 1 (y_1), output 2 (y_2) and output 3 (y_3), respectively.

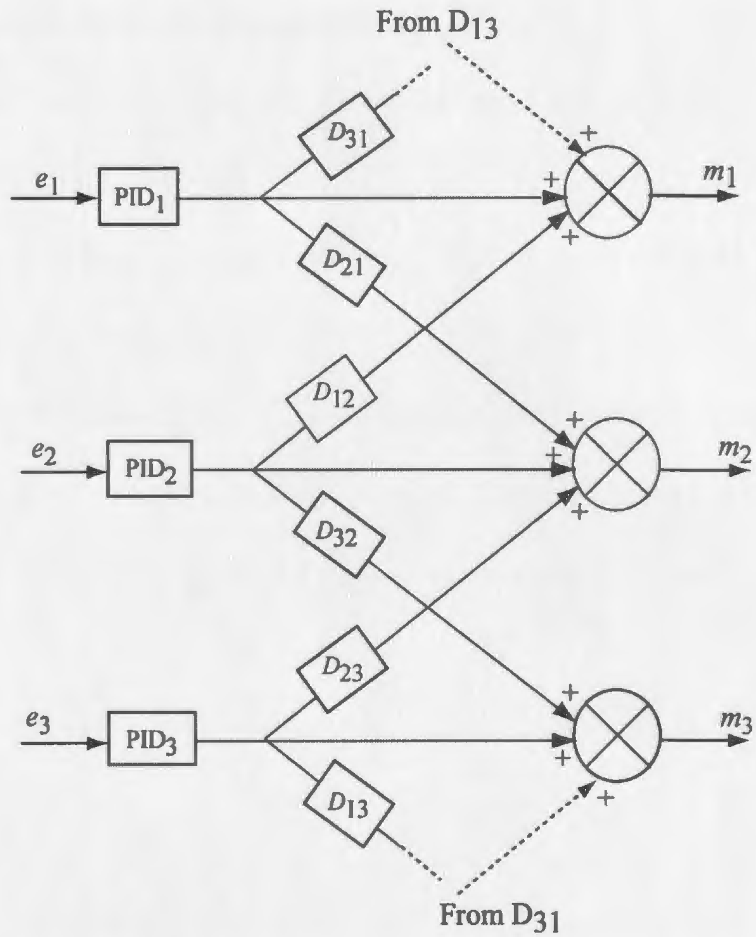


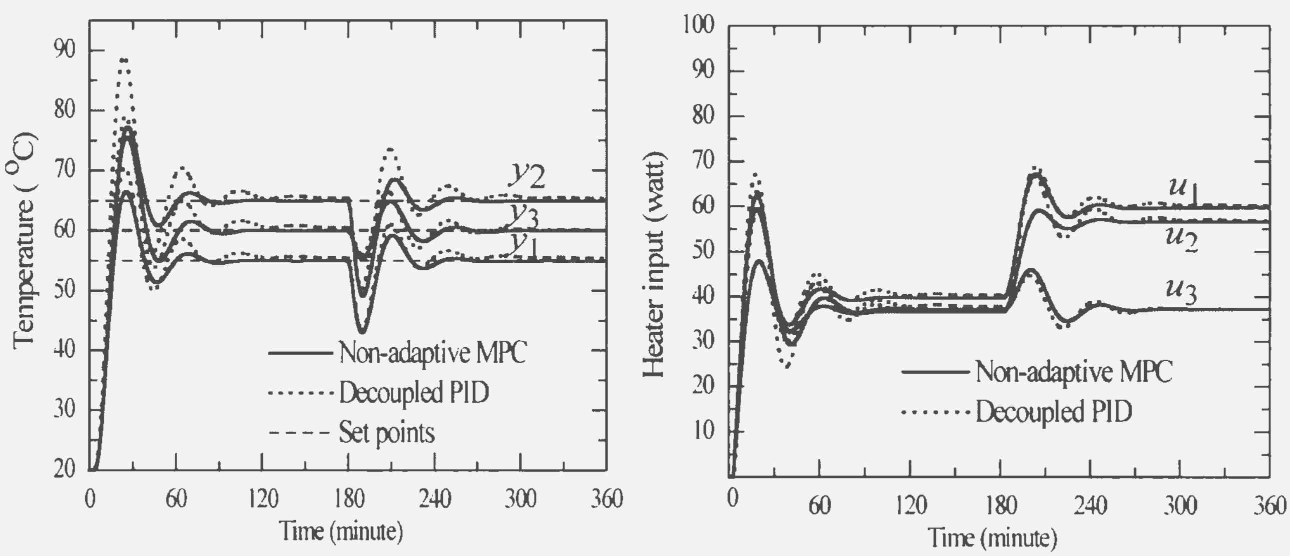
Fig. 3.5 Internal structure of decoupled PID controller

3.6 Simulation Results and Comparisons

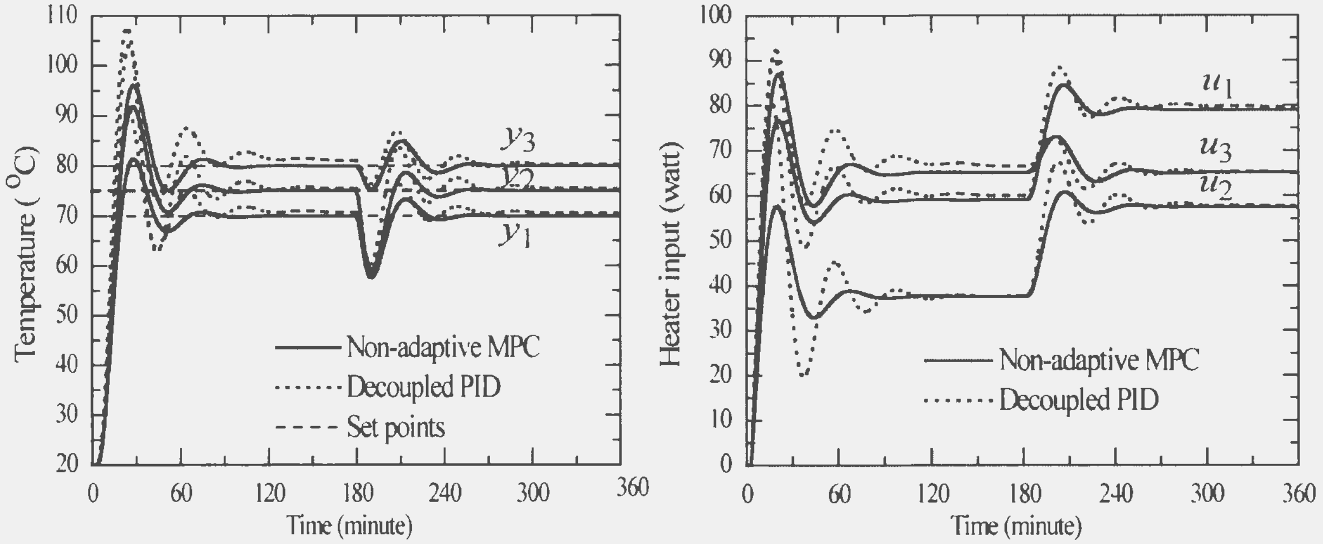
The objective of the proposed non-adaptive MPC scheme is to make the process outputs follow the desired outputs with lower overshoot, smaller settling time, minimum heat (Watt) distribution, better load disturbance characteristics and least tracking error. To achieve the goals two fixed temperature set-point tracking experiments and a variable set-point tracking experiment were carried out and compared with the PID control performance. Simulation results and discussions are presented in the following subsections:

3.6.1 Simulations with Fixed Setpoint Temperature

Simulations were carried out for the non-adaptive MPC scheme with the set point temperatures 55°C, 65°C, 60°C and 70°C, 75°C, 80°C for output 1, 2 and 3 (y_1, y_1 and y_3), respectively and compared against the decoupled PID based control performance. The simulation results of both schemes are presented in Fig.3.6. For the simulations the initial temperature of the FEM outputs was considered at 20°C and the boundary temperature was kept fixed at 25°C. A high negative disturbance of heat is applied constantly to both control system at 180minutes, while the responses are at steady state.



(a) Simulation with set points temperatures 55°C, 65°C and 60°C



(b) Simulation with set points temperatures $70^{\circ}C$, $75^{\circ}C$ and $80^{\circ}C$

Fig. 3.6 Comparisons of MPC and PID against fixed setpoint sequence

3.6.2 Simulation with Variable Setpoint Temperature

For better comparisons the simulation performance of the both control schemes against a variable setpoint sequence was carried out. The simulations were performed for 540 minute where the set points for all three outputs were changed after every 90 minute interval. The applied set point sequences for all outputs are,

For output 1: $55^{\circ}C$ $70^{\circ}C$ $90^{\circ}C$ $75^{\circ}C$ $65^{\circ}C$ $55^{\circ}C$

For output 2: $65^{\circ}C$ $80^{\circ}C$ $90^{\circ}C$ $85^{\circ}C$ $75^{\circ}C$ $65^{\circ}C$

For output 3: $60^{\circ}C$ $75^{\circ}C$ $90^{\circ}C$ $75^{\circ}C$ $70^{\circ}C$ $60^{\circ}C$.

The simulation results and comparisons are presented in Fig. 3.7.

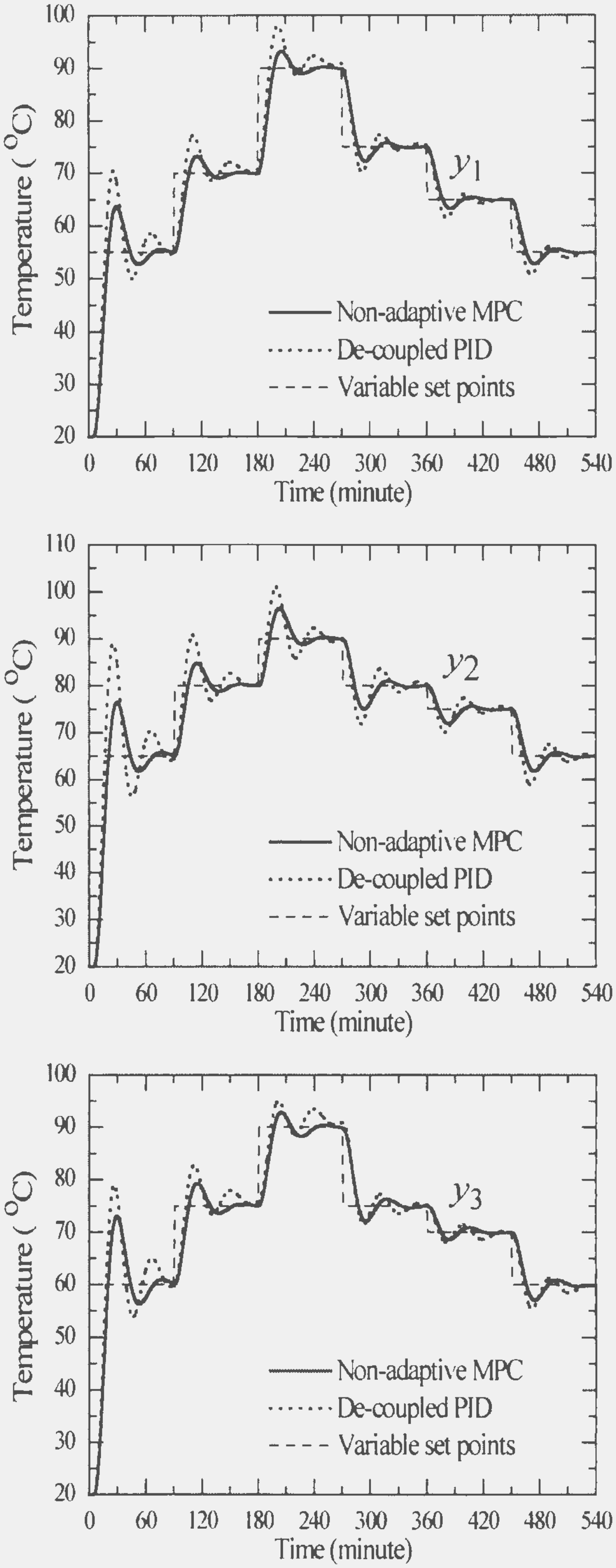


Fig. 3.7 Comparison of MPC and PID against variable setpoint sequences

3.6.3 Tuning Strategy and Performance Analysis

For the comparisons, following tuning parameters were used (shown in Table. 3.1) in the MPC scheme to obtain the better performances:

Table: 3.1 Tuning parameters for MPC

Controller	Prediction Horizon (P)	Control Horizon (M)	Weights for Control Variables δ_i	Weights for Manipulated Variables λ_j	Sampling Interval T (minute)
MPC	20	10	1.08, 1.10, 1.12	0.1, 0.1, 0.1	0.6

The PID tuning parameters are chosen from Ziegler Nichol (**Z-N**) formula and are listed in Table 3.2.

Table: 3.2 Decoupled PID gains

PID Controllers	Proportional gain K_p	Integral gain K_i	Derivative gain K_d
PID_1	12	0.45	0.01
PID_2	10	0.14	0.02
PID_3	14	0.61	0.05

Moreover the simulation was carried out for the both schemes with the following constraints on the manipulated inputs,

$$0 \le u_1(k), u_2(k), u_3(k) \le 120$$

$$-20 \le \Delta u_1(k), \Delta u_2(k), \Delta u_3(k) \le 20$$

The comparison of simulation results presented in Fig.3.6 concludes the following performance Table 3.3:

Table: 3.3 Performance comparisons of MPC and PID

Controller	Overshoot (%)	Settling Time (minute)	Absolute Steady State Error
MPC	20.9, 18.9, 26.1	79.8, 81.6, 82.2	0.12, 0. 23, 0.19
PID	27.1, 35.9, 30.7	80.4, 82.6, 83.4	0.82, 0.73, 0.89

From the performance table it is clear that the non-adaptive MPC controller shows better set point tracking performance compare to the PID control scheme. More importantly, the input heat distribution performance of the proposed MPC is more linear compared to the PID scheme. More over, the load disturbance performance of the proposed MPC system is satisfactory.

In general, the conventional PID controllers show acceptable performance especially for SISO plant systems. However, for the MIMO soil heating process, performance of PID controller is shown unsatisfactory performance because it is unable to overcome the high coupling effect and dead time that exist in the heating process system. The MPC controller has the ability to counteract these coupling effects and dead time when the control law is developed. The only drawback of MPC is the model inaccuracies [37], [38]. To cope with model inaccuracies, the current measurement is implemented to correct the predicted output profiles. This is a form of feedback control, which assures accuracy of the MPC and robustness against model inaccuracies [2]. By doing this, the general stability and robustness is increased. Tuning of weighting factors, λ_j and δ_i are important in the close-loop system behavior [36]. A proper selection of these weighting factors makes the close-loop system stable. The prediction horizon (*P*) and control horizon (*M*) also play an important role in MPC performance. Typically, the prediction horizon is in the range of 20–50 samples ahead, and the control horizon is 25–35% of the prediction horizon [38]. In order to improve the controller performance it is necessary to increase the prediction horizon, control horizon and also the number of past control actions N_{ij} taken into account when calculating the free response. This becomes evident when dealing with systems of relatively high dead time and having complexity in number of outputs and inputs.

3.7 Summary

A multivariable DMC based MPC scheme is presented and simulated with the finite element based MIMO soil heating process. The control structure and implementation technique proposed in this chapter is also directly applicable to GPC based multivariable MPC scheme. The result shows the high performance nature of the proposed MPC system against the general decoupled PID control systems for a coupled MIMO system. Although PID systems are quite satisfactory for SISO process systems, their performance for coupled MIMO systems are quite limited. The decoupled PID system however provides the opportunity to implement the general SISO based PID tuning schemes for MIMO systems. This study can be extended to use the general PID schemes while using IMC structures. The only limitation for the implementation of the proposed MPC scheme is that in the case of highly nonlinear or variable dead time system the application of this linear model based controller is limited to relatively a small operating region.

Specifically, if the computations are based entirely on the model prediction, the accuracy of the model has significant effect on the performance of the closed loop system [39], [24]. Hence the performance of the DMC based MPC will become unsatisfactory when the operation level shifted from it's the original design level of operation [24]. On line adaptive identification of the plant model [2], [33] or multiple model adaptive strategy [24] can maintain the performance of the controller over a wide range of operating level.

Chapter 4

Adaptive Model Predictive Control

4.1 Introduction

In most applications of the classical MPC techniques, the process is modeled over its operating range by extracting a linear approximated model. The extracted linear model must be capable of predicting the future outputs, must be simple to implement, cost effective to simulate and easy to understand. For a linear process system the approximation provides improved control performance against other linear control systems, such as PID control. However, for nonlinear process systems such a linear prediction can be justified only for a limited control region where the linearization has been performed. Moreover reliable linear models for nonlinear processes cannot be easily obtained by using the conventional approaches based on physical modelling or linear system identification [9].

The Nonlinear Model Predictive Control (**NMPC**) systems therefore have been emerged as an alternative solution [4], [26], [40] to address the nonlinear effects. Computational and design complexities that exist in traditional NMPC sometimes limit their applications for real time control systems, particularly for fast multi-variable processes. In addition, the non-convexity of the cost function makes the nonlinear optimization in NMPC rather complex [41]. On the other hand, in linear model based MPC the optimization is carried out in the form of a structured convex quadratic program resulting in a unique optimal solution. Moreover, several reliable standard solution packages are available for linear optimisation [4].

Alternatively it is possible to develop the traditional linear MPC with an online identification of the model parameters of the linear model while allowing the system to be adaptive for each linearized control region of the process. When the adaptation is performed at each control time instance, the region can be made as small as possible, where the application of a linear model can be easily justified. This will indirectly compensate the necessity of employing a nonlinear model to develop the MPC law. Moreover this successive linear adaptation reduces the NMPC based optimization problem to a linear optimization problem at each sampling step. The present study investigates this strategy of using a linear RLS technique for updating the linear model parameters recursively. The adapted parameters at every sampling step are applied to compute the MPC based control law with a predefined optimization procedure.

The form of adaptive linear model identification strategy with MPC, defined as indirect AMPC scheme has generated a considerable interest among the researchers [2], [33], [42] -- [53]. Among them, most of researchers [2], [42] – [50], [53] have considered GPC based system to develop the adaptive model based control law. The GPC based adaptive technique has the ability to solve the long-standing control problems such as variable dead time, open-loop unstable and non-minimum phase systems and is regarded as the most advanced form of AMPC system [2], [3], [33]. However, due to the involvement of the regressive process model, the GPC based adaptive controller has been shown to be sensitive against the prediction model [44], [54] and any mismatch of process parameters may lead to instability [33], [44]. The DMC is the widely used control algorithm in the chemical process industry [2], [5], [24]. The derivation of the DMC law is based on a weighting sequence model such as linear step response or finite impulse model, both of which are easier to obtain and convenient to solve the MPC problem. The only drawback of the weighting sequence model is that it requires many parameters to describe the process, which could reduce the computational efficiency when the recursive adaptation is performed. The involvement of lesser parameters for model identification in the GPC makes the model adaptation more attractive than DMC model based recursive adaptive systems [33].

Multiple Model Adaptive Control (**MMAC**) based strategy [24] can be applied to overcome the above limitations in the DMC based AMPC scheme. This scheme uses number of different non-adaptive DMC controllers and each controller posses its own linear weighting sequence model to describe the process dynamics applicable for a chosen operational level. Adaptation of multiple DMC models for each expected operational point will provide an alternative solution to the recursive DMC model adaptation. However, the modelling in MMAC requires analysis of plant data for each discretized operational level. Also, this MMAC scheme is not applicable when the gain of the process changes its sign during operation [24].

The present study considers the DMC based AMPC strategy to develop a parametric input-output model based adaptive DMC system where it takes the advantages of GPC based recursive adaptive systems. The chosen soil heating process dynamics is modelled using the general purpose ABAQUS/Standard finite element program. Writing a FORTRAN code based on adaptive control strategy into a user-subroutine, a continuous simulation is performed where the model parameters are updated at each sampling instance with the dynamics of the process obtained from FEM.

This chapter intends to provide the following contributions. First, the most popular DMC based MPC is used to develop a high performance AMPC system for a coupled MIMO process system. Although there are similar approaches with different MPC strategy are reported in the past [43] – [51] the applications were limited to SISO systems. Finally, the chapter provides a comprehensive study and comparisons of the AMPC against the non-adaptive MPC scheme. Thus, validity of the proposed algorithm is adequately justified.

4.2 Background

In the literature, there are two distinct architectures in MPC that have been formulated in the field of adaptive control: direct and indirect adaptive control scheme [2]. The direct adaptive control scheme adapts the control law based on a performance measure while the indirect scheme continuously adapt the linear model of the process and the adapted model is then used to synthesize the control law using a predefined optimization procedure. There are different types of direct adaptive MPC strategy available in the literature. Such as, Peterka's Predictor Based Self-Tuning Control [55], Ydstie's Extended Horizon Adaptive Control (**EHAC**) [56] and De Keyser's Extended Prediction Self Adaptive Control (**EPSAC**) [57]. In the predictor based self tuning control scheme the control law is developed minimizing the most recent predicted values of a quadratic criterion for a given control horizon. The EHAC method tries to keep the future predicted output close to the reference at a period of time after process delay. The prediction is calculated by the recursion of Diophantine identity. In the EPSAC a constant control signal starting from the present moment using a sub-optimal predictor instead of solving the Diophantine equation. The above developments are essentially limited to SISO systems and their extension to the MIMO case suffers some limitations as highlighted by Garcia *et al.* (1989) in [5]. Also, the extensive applications of the aforementioned methods are not reported elsewhere. On the other hand, the indirect adaptive MPC scheme can be employed for multivariable process in a straightforward way. In the past several articles proposed various indirect adaptive control mechanisms for controlling nonlinear processes [58], [59]. A popular approach for adaptive MPC is to linearize the nonlinear analytical model (a model based on the law of physics) at each sampling instance and the linearized model is employed to develop the control law [60] – [62]. Analytical models are difficult to obtain due to the underlying physics and chemistry of the process, and are often too complex to employ directly in the optimization calculation. Others [63], [64] have used the nonlinear analytical model to obtain linear state space models at different operating levels. These models are then weighted using a Bayesian estimator at each sampling instance to obtain an adapted internal process model. Another adaptive strategy uses gain and time constant schedule for updating the process model [65], [66]. An extension of this method is to use multiple linear local models to update the process model [24]. Linear models that described the system at various operating points are developed based on plant measurements. Past researchers in [67] have illustrated that linear models can be combined in order to obtain an approximation of the process that approaches its true behavior. As the accuracy of this approximation depends on the number of small linear models, these models have to be developed using reliable plant data at each level of operation [24]. Moreover, this scheme is not applicable to handle the non-minimum phase systems. Recursive parameter estimation on the linear model is another popular approach for adaptation in the MPC based control scheme [42] – [53]. In general, recursive formulations update the parameters of the process model as new plant measurements become available at each sampling instance. However this estimation schemes have well known problems including: convergence problems if the data does not contain sufficient and persistent excitation, inaccurate model parameters influence measured disturbances or noise influence the measurements, and sensitivity to process dead times and high noise levels [24].

4.3 Formulation of Multivariable AMPC

In the proposed AMPC scheme, an online RLS parameter identification strategy is introduced to perform the online adaptation and a parametric input-output model is extracted to formulate the proposed adaptive MPC scheme. The formulation of the MPC scheme is nearly same as that of DMC but the output predictions is computed recursively at each sampling step from the adapted linear parametric input-output model parameters. The proposed scheme is presented in the following subsections:

4.3.1 Formulation of RLS Scheme

A general m-input and n-output multivariable linear system can be used to approximate a local operating region of a non-oscillating process by the following discrete-time MIMO parametric input-output model [68].

$$\mathbf{A}(z^{-1})\mathbf{y}_n(k) = \mathbf{B}(z^{-1})\mathbf{u}_m(k-1) + \boldsymbol{\xi}(k) \tag{4.1}$$

where $\mathbf{y}_n(k)$ and $\mathbf{u}_m(k-1)$ is n dimensional and m dimensional process output and input vector, expressed in equations (3.21) and (3.22) respectively, and $\boldsymbol{\xi}(k)$ is n dimensional random noise vector, and is given by,

$$\boldsymbol{\xi}(k) = [\xi_1(k), \cdots, \xi_n(k)]^T .$$

The matrix $\mathbf{A}(z^{-1})$ is $n \times n$ monic polynomial matrix and $\mathbf{B}(z^{-1})$ is an $n \times m$ polynomial matrix defined as,

$$\mathbf{A}(z^{-1}) = \begin{bmatrix} A_1(z^{-1}) & \cdots & 0 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 0 & \cdots & A_n(z^{-1}) \end{bmatrix}$$

and

$$\mathbf{B}(z^{-1}) = \begin{bmatrix} z^{-k_d^{11}} B_{11}(z^{-1}) & \cdots & z^{-k_d^{1m}} B_{1m}(z^{-1}) \\ \cdot & \cdots & \cdot \\ \cdot & \cdots & \cdot \\ z^{-k_d^{n1}} B_{n1}(z^{-1}) & \cdots & z^{-k_d^{nm}} B_{nm}(z^{-1}) \end{bmatrix}.$$

The i^{th} row diagonal element of $\mathbf{A}(z^{-1})$ is,

$$A_i(z^{-1}) = 1 + a_1^i z^{-1} + \cdots + a_{n_A^i}^i z^{-n_A^i},$$

and the $(i,j)^{\text{th}}$ element of $\mathbf{B}(z^{-1})$ is given by,

$$B_{ij}(z^{-1}) = b_0^{ij} + b_1^{ij} z^{-1} + \cdots + b_{n_B^{ij}}^{ij} z^{-n_B^{ij}} .$$

where n_A^i and n_B^{ij} are the degree of polynomial of A_i and B_{ij}, respectively and z^{-1} is the unit time delay operator. The delay time k_d^{ij} for the i^{th} process output is expressed in sampling time unit and is given by the relation, $\tau_d^{ij} = Tk_d^{ij}$ where the dead time τ_d^{ij} is the integer multiple of the sampling time T.

The model in equation (4.1) for the i^{th} output, ignoring the noise term, $\xi_i(k)$ can be represented in the following form [64]:

$$y_i(k) = -\sum_{l=1}^{n_A^i} a_l^i y_i(k-l) + \sum_{j=1}^{m} \sum_{l=0}^{n_B^{ij}} b_l^{ij} u_j(k-l-k_d^{ij}-1) \tag{4.2}$$

The model parameters in equation (4.2) are updated at each sampling time for achieving the adaptation. Thus, based on the system model in equation (4.2), the estimated vector of the linear model parameters is defined as:

$$\boldsymbol{\theta}_i = [\mathbf{A}_i \quad \mathbf{B}_{i1}, \cdots, \mathbf{B}_{im}]^T,$$

where the elements of $\boldsymbol{\theta}_i(k)$ are,

$$\mathbf{A}_i = \left[a_1^i, \cdots, a_{n_A^i}^i\right] \qquad \text{and} \qquad \mathbf{B}_{ij} = \left[b_0^{ij}, \cdots, b_{n_B^{ij}}^{ij}\right].$$

The RLS technique [23], [64] for updating the above model parameter vector can be summarized as follows. The new parameter estimate can be expressed as,

$$\underbrace{\boldsymbol{\theta}_i(k)}_{\text{new estimate}} = \underbrace{\boldsymbol{\theta}_i(k-1)}_{\text{old estimate}} + \underbrace{\frac{\mathbf{P}_i(k-1)\boldsymbol{\varphi}_i^T(k)}{\mu + \boldsymbol{\varphi}_i(k)\mathbf{P}_i(k-1)\boldsymbol{\varphi}_i^T(k)}}_{\text{correcting vector}} \times \underbrace{[\underbrace{y_i^m(k)}_{\text{new measurement}} - \underbrace{\boldsymbol{\varphi}_i(k)\boldsymbol{\theta}_i(k-1)}_{\text{one step ahead prediction of the new measurement}}]}_{\text{estimated error, } d_i} . \tag{4.3}$$

The covariance matrix $\mathbf{P}_i(k)$ is updated while using the following

$$\mathbf{P}_i(k) = \frac{1}{\mu}\left(\mathbf{P}_i(k-1) - \frac{\mathbf{P}_i(k-1)\boldsymbol{\varphi}_i^T(k)\boldsymbol{\varphi}_i(k)\mathbf{P}_i(k-1)}{1 + \boldsymbol{\varphi}_i(k)\mathbf{P}_i(k-1)\boldsymbol{\varphi}_i^T(k)}\right). \tag{4.4}$$

Where the components of the regression vector $\boldsymbol{\varphi}_i(k)$ is,

$$\boldsymbol{\varphi}_i(k) = \left[\tilde{\mathbf{Y}}_i(k) \quad \tilde{\mathbf{U}}_1(k), \cdots, \tilde{\mathbf{U}}_m(k)\right]$$

are $\tilde{\mathbf{Y}}_i(k) = \left[-y_i(k-1),\cdots,-y_i(k-n_A^i)\right]$ and $\tilde{\mathbf{U}}_j(k) = \left[u_j(k-k_d^{ij}-1),\cdots,u_j(k-k_d^{ij}-n_B^{ij})\right]$, and $\mathbf{P}_i(k)$ is a symmetric matrix with $\mathbf{P}_i(0) = \alpha\mathbf{I}$. The coefficient α is a real large number, ($\alpha >> 100$), and μ is the forgetting factor. In order to provide more weighting to the recent data the forgetting factor is restricted within the $0.95 \leq \mu \leq 1$ [23]. The implementation strategy of the above adaptation technique can be described by using Fig.4.1.

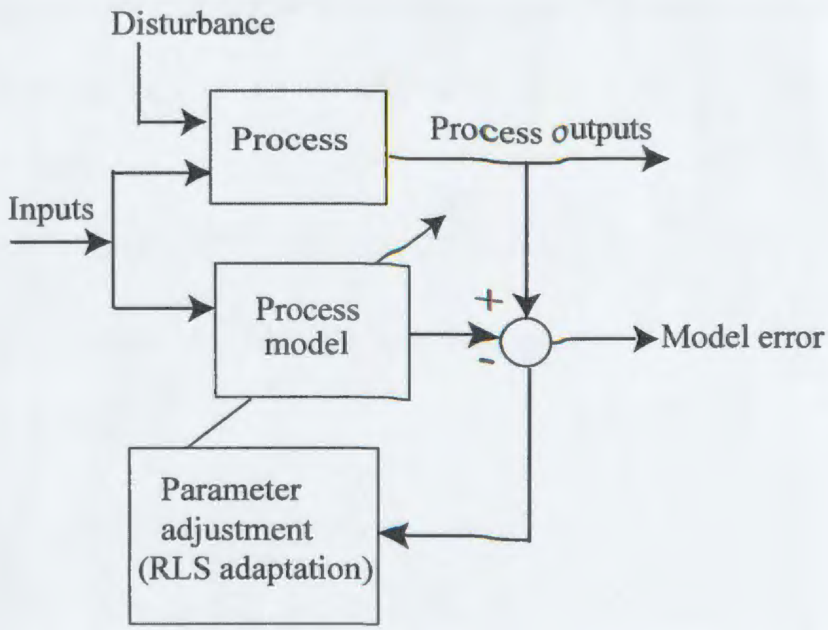


Fig. 4.1 Block diagram of RLS adaptation technique

4.3.2 AMPC Controller Design

The cornerstone of the MPC algorithm is based on the system response model as shown in equation (4.2), that predicts the outputs ($y_i(k+p/k)$) over p sampling instances ahead of the current time instant, k. These predictions through the prediction horizon (P) are based on the natural divisions of the system response where the output can be decomposed into forced and free terms together with the measurement of the current disturbance. The prediction is thus formulated as [2]:

$$y_i(k+p/k) = y_i^{forced}(k+p) + y_i^{free}(k+p) + d_i(k) \qquad (p = 1,\cdots,P). \tag{4.5}$$

The current disturbance $d_i(k)$ is assumed to be constant throughout the prediction horizon and is computed as:

$$d_i(k) = y_i^m(k) - y_i(k). \tag{4.6}$$

The RLS scheme present in equations (4.3) and (4.4) executed at each sampling instant to update the current model parameter vector $\boldsymbol{\theta}_i(k)$ with the measurement of the current disturbance $d_i(k)$. The approximation of equation (4.5) with the updated model parameters is then given by,

$$\hat{y}_i(k+p/k) = \hat{y}_i^{forced}(k+p) + \hat{y}_i^{free}(k+p). \tag{4.7}$$

Where the forced output terms $\hat{y}_i^{forced}(k+p)$ are estimated at the current step with the updated model parameters as,

$$\hat{y}_i^{forced}(k+p) = \sum_{j=1}^{m}\sum_{r=1}^{p} \hat{g}_r^{ij} \Delta\hat{u}_j(k+p-r)$$

where, $$\hat{g}_r^{ij} = -\sum_{l=1}^{n_A^i} a_l^i \hat{g}_{r-l}^{ij} + \sum_{l=0}^{n_B^{ij}} b_l^{ij}$$

is the r^{th} element of the adapted unit linear step response model parameter corresponding to i^{th} output to j^{th} input, in which when $r \le k_d^{ij}$, $\hat{g}_r^{ij} = 0$ and $\Delta\hat{u}_j(k+p-r)$ are the unknown current and future input moves. The free response term $\hat{y}_i^{free}(k+p)$ is inferred as the future response of the system provided that the system input will be maintained at a constant value. In other words,

$$u_j(k-1)=\hat{u}_j(k)=\cdots=\hat{u}_j(k+P).$$

The free response predictions can be estimated recursively from the updated linear model parameters as follows. For convenience, the free response term is expressed while using,

$$\hat{y}_i^{free}(k+p)=\hat{f}_i(k+p).$$

Therefore

$$\hat{f}_i(k+p)=\sum_{l=1}^{n_A^i} a_l^i \hat{f}_i(k+p-l)+\sum_{j=1}^{m}\sum_{l=1}^{n_B^{ij}} b_{l-1}^{ij} u_j(k-l-k_d^{ij}+p) \tag{4.8}$$

The initial conditions for equation (4.8) are the predicted output at the current time and is given by,

$$\hat{f}_i(k)=\hat{f}_i(k-1)=\cdots=\hat{y}_i(k).$$

Hence, the prediction form in equation (4.7) can be written as:

$$\hat{y}_i(k+p/k)=\underbrace{\sum_{j=1}^{m}\sum_{r=1}^{p}\hat{g}_r^{ij}\Delta\hat{u}_j(k+p-r)}_{\hat{y}_i^{forced}}+\underbrace{\hat{f}_i(k+p)}_{\hat{y}_i^{free}} \tag{4.9}$$

The above set of predictions through the prediction horizon can be written in the following compact form,

$$\hat{\mathbf{Y}}=\hat{\mathbf{G}}\Delta\hat{\mathbf{U}}+\hat{\mathbf{F}} \tag{4.10}$$

where, $\hat{\mathbf{G}}$ is the adapted multi-variable dynamic matrix expressed as,

$$\hat{\mathbf{G}}=\left[\hat{\mathbf{G}}_{ij}\right]_{nP\times mM}$$

and the elements are given by:

$$\hat{\mathbf{G}}_{ij} = \begin{bmatrix} \hat{g}_1^{ij} & 0 & . & . & . & 0 \\ \hat{g}_2^{ij} & \hat{g}_1^{ij} & 0 & . & . & 0 \\ . & . & . & . & . & . \\ \hat{g}_M^{ij} & \hat{g}_{M-1}^{ij} & . & . & . & \hat{g}_1^{ij} \\ . & . & . & . & . & . \\ \hat{g}_P^{ij} & \hat{g}_{P-1}^{ij} & . & . & . & \hat{g}_{P-M+1}^{ij} \end{bmatrix}_{P\times M} .$$

The vectors $\hat{\mathbf{Y}}$, $\Delta\hat{\mathbf{U}}$ and $\hat{\mathbf{F}}$ are the adapted predicted output, optimal control input and the free response vector, respectively and given by the following expressions,

$$\hat{\mathbf{Y}} = \left[\hat{\mathbf{Y}}_1, \cdots, \hat{\mathbf{Y}}_n\right]^T,$$

$$\Delta\hat{\mathbf{U}} = \left[\Delta\hat{\mathbf{U}}_1, \cdots, \Delta\hat{\mathbf{U}}_m\right]^T \text{ and}$$

$$\hat{\mathbf{F}} = \left[\hat{\mathbf{F}}_1, \cdots, \hat{\mathbf{F}}_n\right]^T$$

where,

$$\hat{\mathbf{Y}}_i = \left[\hat{y}_i(k+1/k), \cdots, \hat{y}_i(k+P/k)\right]$$

$$\Delta\hat{\mathbf{U}}_j = \left[\Delta\hat{u}_j(k), \cdots, \Delta\hat{u}_j(k+M)\right] \text{ and}$$

$$\hat{\mathbf{F}}_i = \left[\hat{f}_i(k+1), \cdots, \hat{f}_i(k+P)\right].$$

The objective is to determine the control moves $\Delta\hat{u}_j(k+p-r)$ in equation (4.9) such that the predicted output $\hat{y}_i(k+p/k)$ will drive as close to the set-point. To do this the following cost function is selected.

$$J = \sum_{i=1}^{n}\sum_{p=1}^{P}\delta_i(p)[w_i(k+p) - \hat{y}_i(k+p/k)]^2 + \sum_{j=1}^{m}\sum_{p=1}^{M}\lambda_j(p)[\Delta\hat{u}_j(k+p-1)]^2 \qquad (4.11)$$

The equation (4.11) can be written in the following compact form

$$J = \boldsymbol{\delta}[\mathbf{W} - \hat{\mathbf{Y}}]^T[\mathbf{W} - \hat{\mathbf{Y}}] + \boldsymbol{\lambda}[\Delta\hat{\mathbf{U}}]^T[\Delta\hat{\mathbf{U}}] \qquad (4.12)$$

Combining equations (4.12) and (4.10) yields:

$$J = \boldsymbol{\delta}[\mathbf{W} - \hat{\mathbf{G}}\Delta\hat{\mathbf{U}} - \hat{\mathbf{F}}]^T[\mathbf{W} - \hat{\mathbf{G}}\Delta\hat{\mathbf{U}} - \hat{\mathbf{F}}] + \boldsymbol{\lambda}[\Delta\hat{\mathbf{U}}]^T[\Delta\hat{\mathbf{U}}] \tag{4. 13}$$

After minimizing equation (4.13) with respect to $\Delta\hat{\mathbf{U}}$, the following closed-loop solution of unconstrained adaptive multi-variable control law is obtained,

$$\Delta\hat{\mathbf{U}} = \hat{\mathbf{K}}(\mathbf{W} - \hat{\mathbf{F}}). \tag{4.14}$$

Where the adapted MPC gain matrix, $\hat{\mathbf{K}} = (\hat{\mathbf{G}}^T\boldsymbol{\delta}\hat{\mathbf{G}} + \boldsymbol{\lambda}\mathbf{I})^{-1}\hat{\mathbf{G}}^T\boldsymbol{\delta}$ and the term $(\mathbf{W} - \hat{\mathbf{F}})$ is the vector of future predicted error over P sampling instances due to the effect of past inputs. The only control moves, $\Delta\hat{u}_1(k), \cdots, \Delta\hat{u}_m(k)$ in equation (4.14) are applied to the plant at current step and whole procedure is repeated in each sampling interval.

4.4 Application of AMPC to the Soil Heating Process

The above AMPC scheme is applied to the FEM based soil heating process. The complete control structure of AMPC system with FEM of the soil cell (plant) is shown in Fig.4.2. To cope with the model-plant mismatch due to process uncertainties and to avoid the steady state control error, the proposed AMPC scheme also implemented within an IMC structure as shown in Fig.4.2 [9]. The RLS identification technique is incorporated in the IMC structure to realize the adaptive control.

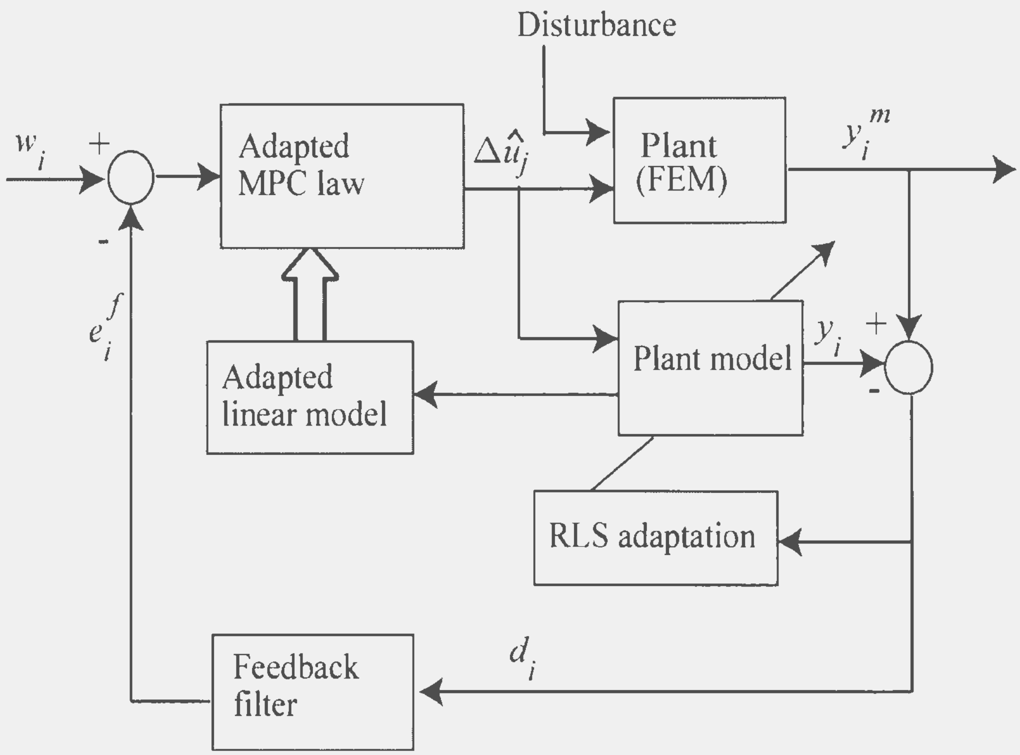


Fig. 4.2 Block diagram of the AMPC scheme

The overall AMPC scheme can be realized while implementing the following simple steps at every sampling interval:

1. Identify the initial model parameter vector $\boldsymbol{\theta}_i(0)$ in offline.
2. Measure the current disturbance ($d_i(k)$) in equation (4.6) and update the model parameter vector ($\boldsymbol{\theta}_i(k)$) using the RLS scheme shown in equations (4.3) and (4.4).
3. Employ the updated model parameters to build the prediction model ($\hat{y}_i(k+p/k)$) described in equation (4.9) and compute the MPC based control law given in equation (4.14).
4. Apply the control moves, $\Delta\hat{u}_1(k),\cdots,\Delta\hat{u}_m(k)$ to both the plant and the model.
5. Go to step 2 and repeat.

4.5 Linear Model Identification from the Soil Heating Process

A linear parametric input-output model is extracted from the FEM based soil heating process dynamics. The RLS online identification scheme is then incorporated with the extracted model to perform the online adaptation in the proposed MPC system. The identification and validation of the extracted linear model with RLS adaptation are presented in the following subsections:

4.5.1 Identification of Non-Adaptive Linear Model

To implement the RLS adaptation scheme described in equations (4.3) and (4.4) in the present soil heating process, it is necessary to identify the initial parametric input-output model parameter vector, $\boldsymbol{\theta}_i(0)$ from the process. To do this, a simulation is performed by applying a constant open-loop control signal $u_j = 18$ Watt to the FEM of the soil cell and responses of the corresponding outputs (y_1, y_2 and y_3) are thus modelled. Analysing the responses using "plant reaction curve" method [3], [14] the following 3×3 MIMO discrete transfer function model is obtained.

$$\mathbf{y}_n(k) = \mathbf{H}\mathbf{u}_m(k-1) \tag{4.15}$$

Where,

$$\mathbf{H}=\left[H_{ij}\right]_{3\times3}=\begin{bmatrix} \dfrac{0.00317\ z^{-1}}{1-0.89\,z^{-1}}z^{-1} & \dfrac{0.000357\ z^{-1}}{1-0.97\,z^{-1}}z^{-2} & \dfrac{0.000017\ z^{-1}}{1-0.94\,z^{-1}}z^{-6} \\ \dfrac{0.00127\ z^{-1}}{1-0.91\,z^{-1}}z^{-2} & \dfrac{0.00354\ z^{-1}}{1-0.88\,z^{-1}}z^{-1} & \dfrac{0.00021\ z^{-1}}{1-0.94\,z^{-1}}z^{-3} \\ \dfrac{0.000101\ z^{-1}}{1-0.931\,z^{-1}}z^{-6} & \dfrac{0.00157\ z^{-1}}{1-0.89\,z^{-1}}z^{-3} & \dfrac{0.00296\ z^{-1}}{1-0.895z^{-1}}z^{-1} \end{bmatrix} \tag{4.16}$$

represents the MIMO discrete transfer function matrix. The element H_{ij} is the plant open-loop discrete transfer function model relating i^{th} output to j^{th} input of the plant. Note that, the sampling interval is chosen 0.6 minute while building the model. The model in equation (4.15) can be rewritten in the following parametric input-output model form [3]:

$$\begin{bmatrix} A_1(z^{-1}) & 0 & 0 \\ 0 & A_2(z^{-1}) & 0 \\ 0 & 0 & A_3(z^{-1}) \end{bmatrix} \mathbf{y}_n(k)$$
$$= \begin{bmatrix} z^{-k_d^{11}} B_{11}(z^{-1}) & z^{-k_d^{12}} B_{12}(z^{-1}) & z^{-k_d^{13}} B_{13}(z^{-1}) \\ z^{-k_d^{21}} B_{21}(z^{-1}) & z^{-k_d^{22}} B_{22}(z^{-1}) & z^{-k_d^{23}} B_{23}(z^{-1}) \\ z^{-k_d^{31}} B_{31}(z^{-1}) & z^{-k_d^{32}} B_{23}(z^{-1}) & z^{-k_d^{33}} B_{33}(z^{-1}) \end{bmatrix} \mathbf{u}_m(k-1) \quad . \tag{4.17}$$

Solving the equations (4.15) and (4.17) the polynomial matrices $\mathbf{A}(z^{-1})$ and $\mathbf{B}(z^{-1})$ are obtained and the elements are thus given by,

$$A_1(z^{-1}) = 1 - 2.8z^{-1} + 2.60z^{-2} - 0.81z^{-3},$$

$$A_2(z^{-1}) = 1 - 2.73z^{-1} + 2.48z^{-2} - 0.752z^{-3},$$

$$A_3(z^{-1}) = 1 - 1.878z^{-1} + 0.952z^{-2} - 0.072z^{-3},$$

$$B_{11}(z^{-1}) = 0.00317 - 0.00605z^{-1} + 0.00288z^{-2}, \qquad k_d^{11} = 1,$$

$$B_{12}(z^{-1}) = 0.000357 - 0.000653z^{-1} + 0.000296z^{-2}, \qquad k_d^{12} = 2,$$

$$B_{13}(z^{-1}) = 0.000017 - 0.0000316z^{-1} + 0.0000147z^{-2}, \qquad k_d^{13} = 6,$$

$$B_{21}(z^{-1}) = 0.00127 - 0.00231z^{-1} + 0.001041z^{-2}, \qquad k_d^{21} = 2,$$

$$B_{22}(z^{-1}) = 0.0035 - 0.0065z^{-1} + 0.00301z^{-2}, \qquad k_d^{22} = 1,$$

$$B_{23}(z^{-1}) = 0.00021 - 0.000376z^{-1} + 0.000168z^{-2}, \qquad k_d^{23} = 3,$$

$$B_{31}(z^{-1}) = 0.0001 - 0.00018z^{-1} + 0.00086z^{-2}, \qquad k_d^{31} = 6,$$

$$B_{32}(z^{-1}) = 0.00157 - 0.00287z^{-1} + 0.0013z^{-2}, \qquad k_d^{32} = 3,$$

$$B_{33}(z^{-1}) = 0.003 - 0.00295z^{-1} + 0.000242z^{-2}, \qquad k_d^{33} = 1.$$

The model in equation (4.17) is employed as the initial model for the RLS scheme to perform the online adaptation and can be written in the following vector form as shown in equations (4.3) and (4.4).

Define the initial model parameter vector for the i^{th} output as, $\boldsymbol{\theta}_i(k-1) = \boldsymbol{\theta}_i(0)$. Therefore,

$$\boldsymbol{\theta}_1(0) = [-2.8, 2.60, -0.81, 0.00317, -0.00605, 0.00288, 0.000357, -0.000653, 0.000296, \\ 0.000017, -0.0000316, 0.0000147]^T$$

$$\boldsymbol{\theta}_2(0) = [-2.73, 2.48, -0.752, 0.00127, -0.00231, 0.001041, 0.0035, -0.0065, 0.00301, \\ 0.00021, -0.000376, 0.000168]^T$$

$$\boldsymbol{\theta}_3(0) = [-1.87, 0.952, -0.072, 0.0001, -0.00018, 0.00086, 0.00157, -0.00287, 0.0013, \\ 0.003, -0.00295, 0.000242]^T$$

and the corresponding components vectors are given by,

$$\boldsymbol{\varphi}_1(k) = [-y_1(k-1), -y_1(k-2), -y_1(k-3), u_1(k-2), u_1(k-3), u_1(k-4), \\ u_2(k-3), u_2(k-4), u_2(k-5), u_3(k-7), u_3(k-8), u_3(k-9)]$$

$$\boldsymbol{\varphi}_2(k) = [-y_2(k-1), -y_2(k-2), -y_2(k-3), u_1(k-3), u_1(k-4), u_1(k-5), \\ u_2(k-2), u_2(k-3), u_2(k-4), u_3(k-4), u_3(k-5), u_3(k-6)]$$

and

$$\boldsymbol{\varphi}_3(k) = [-y_3(k-1), -y_3(k-2), -y_3(k-3), u_1(k-7), u_1(k-8), u_1(k-9), \\ u_2(k-4), u_2(k-5), u_2(k-6), u_3(k-2), u_3(k-3), u_3(k-4)] \; .$$

Now the tracking performance of the approximated linear parametric model is validated by comparing the open-loop step response characteristics of the model with the FEM responses. Fig.4.3 shows the open-loop step responses of the linear model and corresponding FEM results.

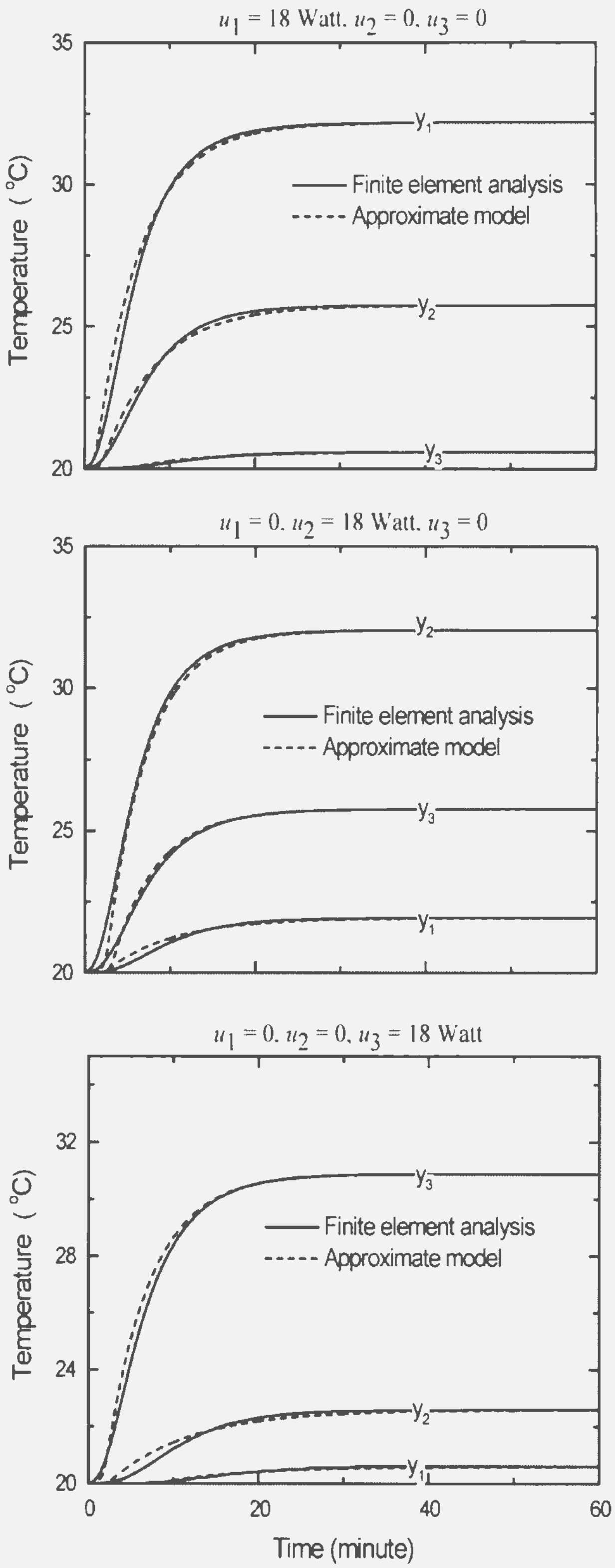
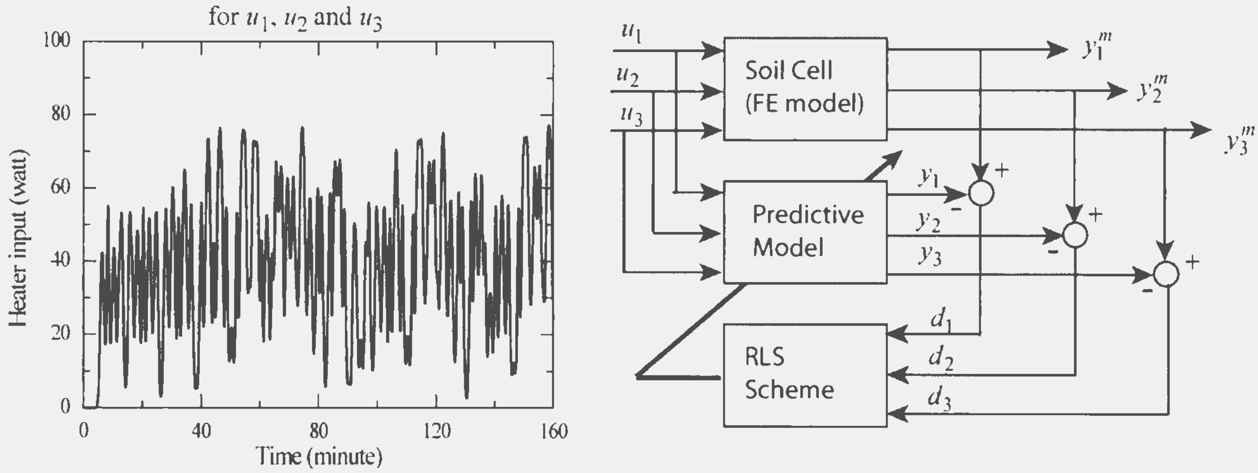


Fig. 4.3 Open-loop step responses of the soil heating model

Simulation results (Fig. 4.3) justify the accuracy of the extracted linear model to perform the control simulations.

4.5.2 Comparison of Tracking Performance

The performance of the approximated linear model in equation (4.17) with the RLS adaptation scheme is verified against variable input sequences. The input profile and the simulation set-up are presented in Fig.4.4 (a) and Fig.4.4 (b), respectively. The temperature outputs of the adaptive model (y_1, y_2 and y_3) and the FEM (y_1^m, y_2^m and y_3^m) outputs are measured at each step with the sampling interval 0.6 minute.



(a) Variable input sequence (b) Simulation set-up

Fig. 4.4 Simulation with variable input sequences

The difference between the adaptive model outputs and the FEM outputs (tracking error) are computed and compared with the non-adaptive linear model based tracking performance. The tracking errors of both adaptive and non-adaptive scheme are presented in Fig.4.5.

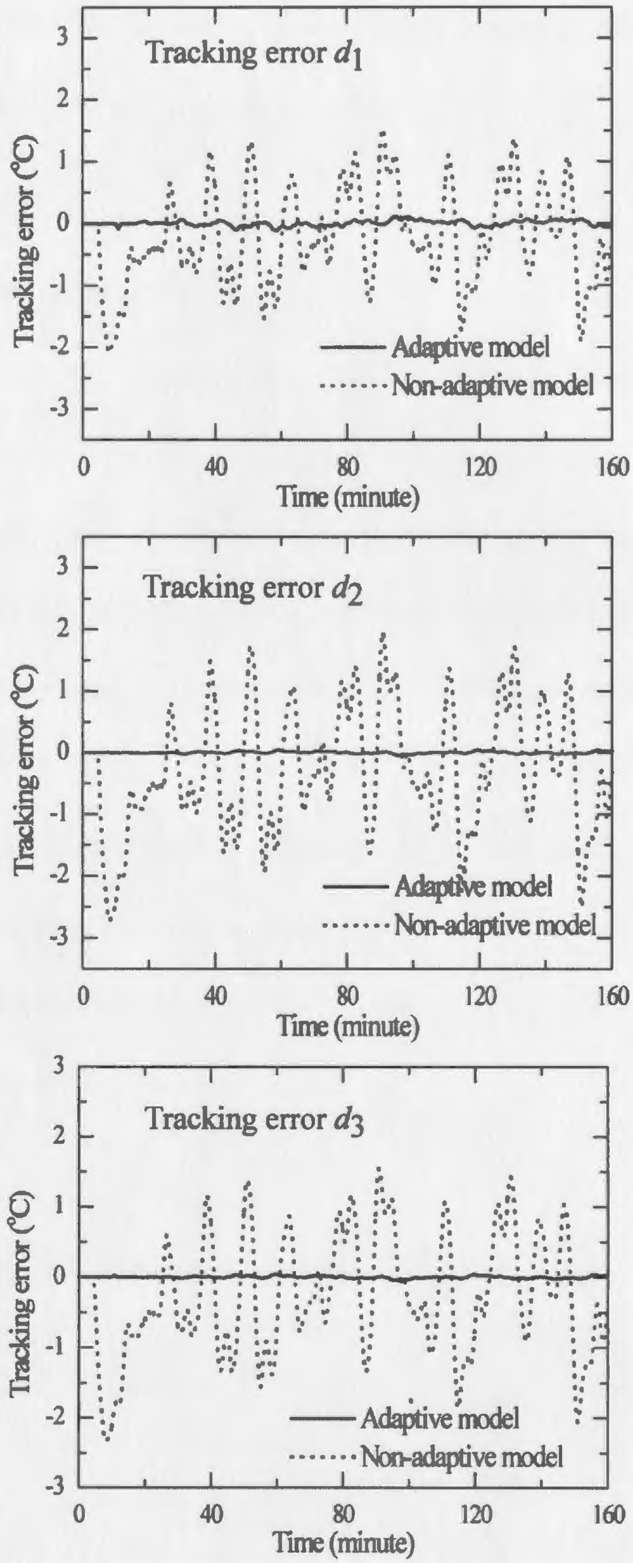


Fig. 4.5 Tracking error with variable input sequences

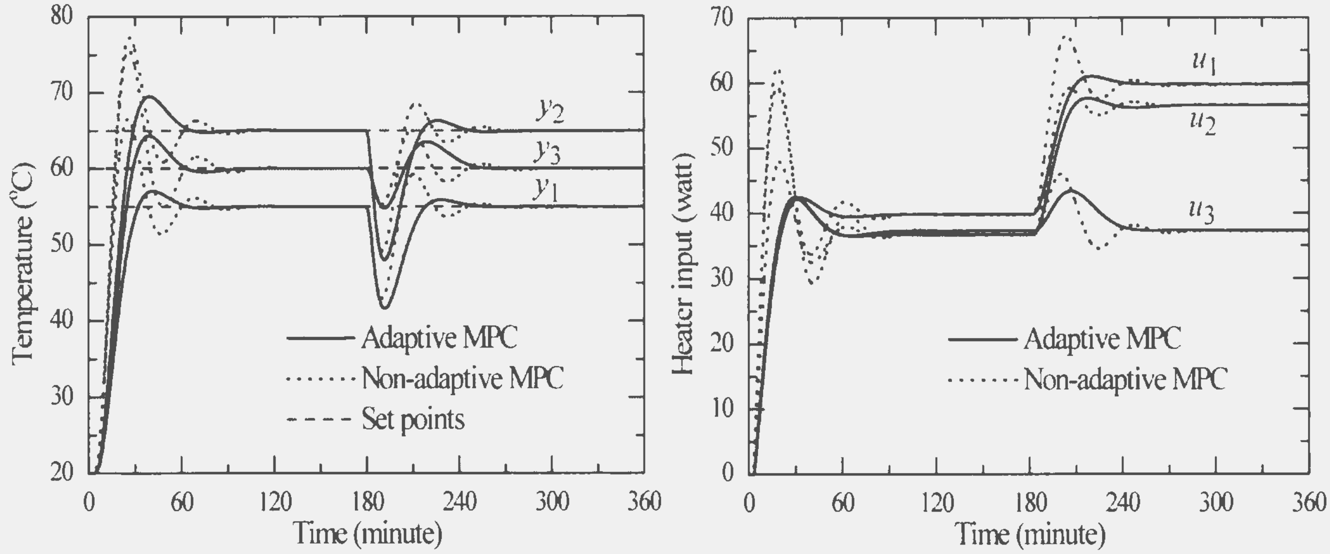
Simulation results justify the accuracy of the online adaptive scheme against the non-adaptive model in the present soil heating process.

4.6 Control Simulations

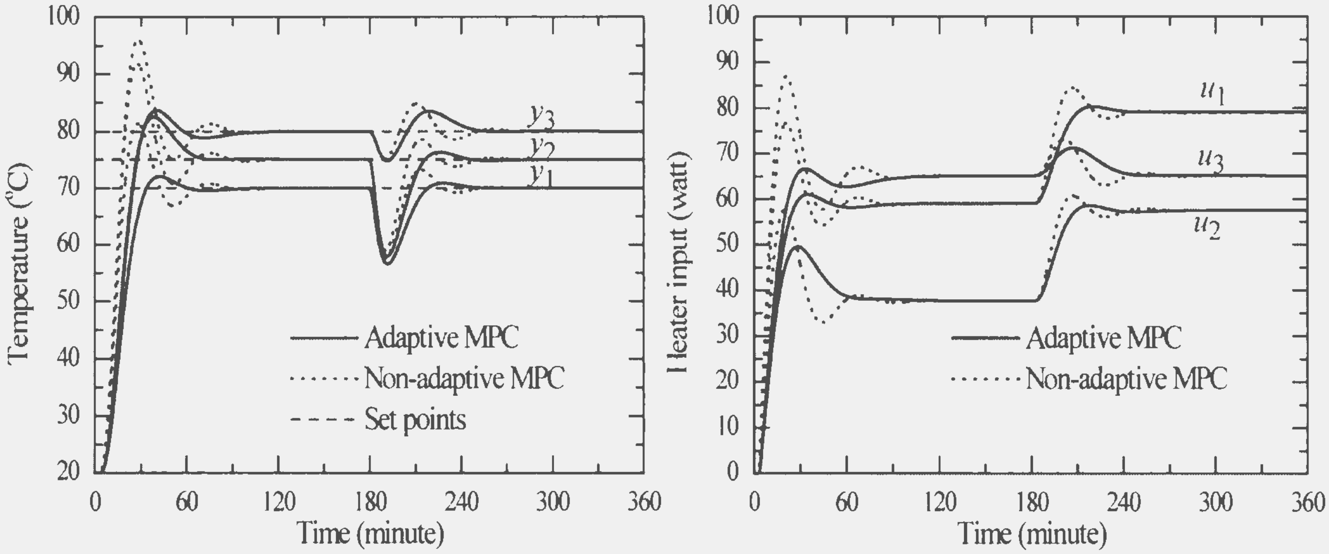
The control objective of this exercise is to achieve precise temperature tracking with lower overshoot, smaller settling time, minimum heat distribution, better load disturbance characteristics and least tracking error. To achieve the goal two temperature set-point tracking experiments and a variable set-point tracking experiment were carried out for the proposed AMPC scheme and compared with the non-adaptive model based tracking performance. The output set-point temperatures are chosen as 55°C, 65°C, 60°C and 70°C, 75°C, 80°C for output 1, 2 and 3, respectively. A high negative disturbance of heat is also applied constantly to both the control system at 180 minute, while the responses are at steady state. The simulation results are presented in Fig. 4.6. The tuning parameters chosen for the two schemes are shown in Table 4.1.

Table 4.1 Tuning parameters for AMPC and MPC

Controller	Prediction Horizon (P)	Control Horizon (M)	Weights for Control Variables δ_i	Weights for Manipulated Variables λ_j	Sampling Interval T (minute)
AMPC	15	5	1.12,1.05,1.15	0.1,0.1,0.1	0.6
MPC	20	10	1.08,1.10,1.12	0.1,0.1,0.1	0.6



(a) Simulation performance with the set point temperatures 55°C, 65°C, 60°C



(b) Simulation performance with the set point temperatures 70°C, 75°C, 80°C

Fig. 4.6 AMPC and non-adaptive MPC against fixed set point temperature

For better comparisons the same systems were also simulated for tracking the same variable set point sequences and the results are presented in Fig. 4.7.

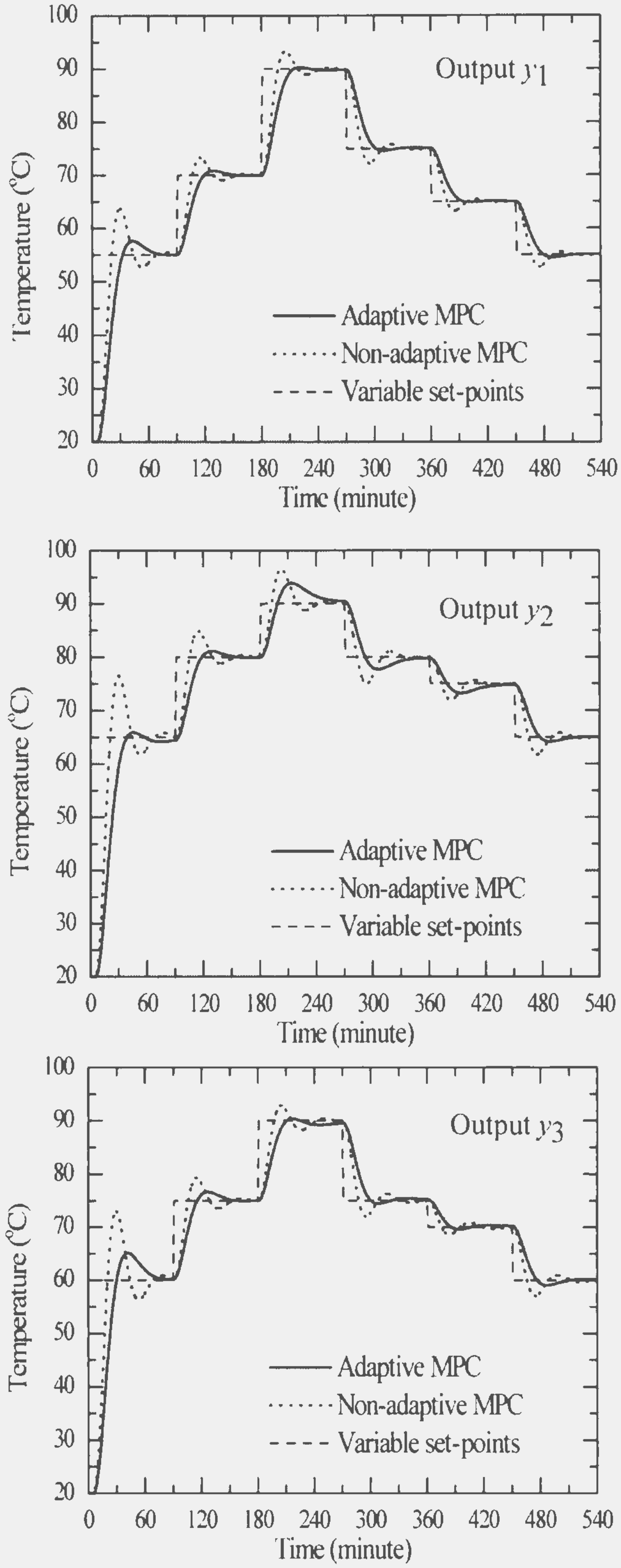


Fig. 4.7 AMPC and non-adaptive MPC against variable set point temperature

The simulation results clearly show that the AMPC outperforms the non-adaptive MPC with lower overshoot, smaller settling time, minimum heat distribution and better load disturbance characteristics. The better performance of the AMPC is mainly due to the fact that its control law given in equation (4.14) is computed at every sampling step with the adapted linear model parameters. This in turn will update the process dynamic matrix $\mathbf{G}$ and the free response vector $\mathbf{F}$ of the model more precisely. Both of them are important for achieving accurate control performance. In the non-adaptive MPC case, the control law is implemented with the pre-estimated linear model parameters where the precision of the estimates of $\mathbf{G}$ and $\mathbf{F}$ are limited to a chosen operating range. With the change of operating regions resulted in more overshoot and undershoot as compared to the response characteristics with the AMPC.

4.7 Summary

An online adaptive model identification strategy for MPC has been developed, analyzed and implemented systematically in this chapter. The application and benefits of the proposed AMPC strategy over the non-adaptive MPC strategy are also demonstrated through several simulations. The simulation results show that the proposed AMPC system has the better capability to overcome the nonlinear and coupling effects of the process system and therefore able to produce accurate tracking performance against the desired output temperature profile. The tracking performance of the adaptive model also indicates that the proposed linear adaptation is a computationally efficient alternative to NMPC systems. The linearization of the nonlinear system at every sampling instance allows a higher resolution in achieving different linearized models and as a result the recursive parameter estimation method able to handle the process variations much efficiently than in a traditional MPC system. More importantly, the load disturbance performance of the proposed AMPC system was satisfactory. Hence the proposed AMPC system provides a useful and relatively simple alternative when non-adaptive MPC fails to produce better response against nonlinear process dynamics. The only drawback of the scheme is that, as the adapted parameters of the linear model are applied to develop the MPC control law, any erroneous parameter estimations may result in undesirable changes in the control signal. When RLS technique is employed for time varying processes the process and estimator mismatch will cause the covariance matrix to increase. Under those circumstances when a fixed forgetting factor is used all the past elements in the covariance matrix will contribute towards the estimation. Particularly at the steady state this may leads to an exponential growth of the covariance matrix and may result unstable control performance [33]. To overcome the problem it is possible to adapt the forgetting factor recursively based on the information content in the present data [2].

Chapter 5

Adaptive Fuzzy Model Based Predictive Control

5.1 Introduction

For complex and highly nonlinear process systems, a suitable identification method for obtaining an accurate empirical process model is quite challenging. There are several techniques existing in the literature to address this issue. Among them, fuzzy logic based model identification technique is considered to be an appropriate tool for nonlinear process modelling and can be incorporated with the traditional MPC schemes for formulating an effective control law. Although there are many representations of fuzzy schemes for nonlinear process modeling, fuzzy model of the TS type is the most convenient form to use in MPC systems. It has the ability to approximate the complex nonlinear systems in a parametric form and can directly be used for solving the general MPC problem [11]. Further, this technique allows the complex high dimensional nonlinear modeling problem to be decomposed into a set of simpler linear local models to represent small operating regions defined by the fuzzy boundaries. Fuzzy inference is used to interpolate the fuzzy outputs of the local models in a smooth fashion to generate an approximated nonlinear fuzzy model. Therefore TS type fuzzy model based MPC strategy for nonlinear process recently generated considerable interests among the present researchers [2], [9], [11], [69] – [82]. In many cases [2], [69] – [78] the parameters of the linear model extracted from the nonlinear fuzzy model are used for linear MPC formulation at every sampling instant. In other cases, such as in [9], [11], [79] – [82], a nonlinear optimization problem is solved at every step with the nonlinear fuzzy model. The accuracy of the extracted linear parameters depends on the process dynamic behavior described by the fuzzy local models. However, when the process is time-variant the predefined linear local models will become applicable only for a small operating region. Introducing more local models may increase the accuracy but may lead to over-fitting and heavy computational burden. The online adaptation of the fuzzy local models can be employed to overcome the nonlinear time-variant process dynamics. The adaptation is usually achieved by using a RLS parameter estimation technique [2], [69], [78], [79] A single linear model based AMPC technique can also be applied to handle such process system [2]. The adaptation of a single process model over wider operational range may result in a transient error, and this may in turn lead into undesirable behavior of the controlled process [2].

The present study investigated the above strategy and developed an adaptive TS type fuzzy model based MPC scheme to control and maintain the temperature profile of the coupled nonlinear multivariable soil heating system. To handle the parameter uncertainty and time variant behavior of the process, an iterative RLS parameter estimation technique for adaptive performance of the local models is introduced. An online linearization technique is adopted to extract the linear parameters from the nonlinear TS type fuzzy model in formulating the linear MPC scheme. The proposed scheme formulates the MPC strategy using general DMC structure.

This chapter summarizes the contributions in two aspects. Firstly, the most popular DMC based MPC is formulated with the TS type adaptive fuzzy model structure and its robustness is verified with the coupled nonlinear MIMO soil heating process system. Finally, this chapter provides a comprehensive study and comparisons of the proposed fuzzy model based MPC strategy against two linear MPC schemes. Thus, validity of the proposed algorithm is adequately justified.

5.2 Background

In the literature there are many successful applications of MPC using TS type fuzzy model have been reported in [2], [9], [11], [69] – [82]. All the approaches discussed in these papers can generally be classified into two groups: 1) methods using directly the nonlinear fuzzy model in the optimization procedure [9], [11], [79] – [82], and 2) methods using a linearized model instead of directly using the nonlinear fuzzy one [2], [69] – [78].

The use of the nonlinear fuzzy model directly in MPC is motivated by the possibility to improve the control performance by improving the prediction accuracy. But it leads to a non-convex optimization (nonlinear optimization) problem which must be solved at each sampling period. This hampers the application to fast processes where iterative optimization techniques cannot be properly used duc to short sampling time. However, in the literature there are several nonlinear optimization techniques available that can be used to solve this problem. Among them the most straightforward way of minimizing the TS model in NMPC is to use the Nedler–Mead method or sequential quadratic programming technique [9]. These algorithms, however, require significant computing power which may be a serious obstacle for real-time implementation. Moreover, the algorithm can be trapped in a local minimum, which may result in undesirable control actions and poor performance of the controlled process [9]. The Branch and Bound (**B&B**) search method is another popular approach for the nonlinear optimization and have been widely used to solve the TS model based NMPC problem, as for example in [9], [11], and [80]. In these approaches, the control space is discretized and the optimization problem is reduced to searching the best control action in the discrete space using the B&B optimal search technique. This discretization of the control space introduces a tradeoff between the number of discrete alternatives and the computational complexity [81]. Additional problems introduced by this discretization are the oscillations (chattering) at steady state and slow step responses [81]. To address these issues the authors in [81] proposed an alternative solution to overcome these problems by using a fuzzy predictive filter to construct the discrete control alternatives. This filter is represented as an adaptive set of control actions multiplied by a gain factor and keeps the number of necessary alternatives low during the optimization and hence increases the control performance.

Another approach for NMPC optimization is the sequential technique which can be used to solve this NMPC problem [2]. This algorithm involves the iterative solution of the nonlinear model equation as an inner loop and to evaluate the objective function with an optimization code in the outer loop. The optimization and the model solution are executed iteratively until the desired accuracy is achieved. An alternative to this sequential solution is to solve the optimization problem and the model equation simultaneously [2]. This simultaneous technique involves the transformation of the dynamic model into algebraic equations using weighted residual techniques.

On the other hand the linearized TS model based MPC is generally used to avoid the NMPC optimization problems. The primary advantage of the successive linearization on the TS model is that the NMPC problem is reduced to linear MPC problem at every sampling step. There are several linearization techniques available in the present literature for linearizing the TS type fuzzy model. For example in [2] and [74], a linear step response model is extracted from the TS model for linear MPC formulation. Therefore, the resulting optimization problem is convex, and is solved using a pre-defined linear optimization procedure. Several authors, such as in [2], [75] applied Jacobian linearization technique on the zero order TS type fuzzy model for extracting linear parametric model parameters of the process around the current operating point. Another popular approach is proposed in [71] – [73] where the TS model is linearized by converting into a linear state space form and the control signal is obtained by solving a constrained quadratic optimization problem. To account for errors introduced by the linearization, an iterative optimization scheme is also proposed with this method in [72] where the quadratic problem provides a search direction toward the minimum of the optimization problem. In [76] and [77], a different and interesting technique is incorporated to identify the linear model for MPC formulation. This method involves a search technique in the fuzzy space to identify the most active rule with the highest membership degree. The linear model of the corresponding rule consequent is then applied directly to solve the linear optimization problem. A common approach of linearizing the fuzzy model about the current operating point is by weighting the nonlinear fuzzy model parameters at each sampling instant [69], [70], [78]. The resulted model is obtained in an adaptive parametric form and is employed online for the linear

MPC formulation. Better accuracy of these weighted linear parameters is generally achieved by using more fuzzy partitions in the antecedent fuzzy space [69].

In literature other than TS model, fuzzy relational model based MPC law has also been developed in [83] [84] to handle the nonlinear process systems. This scheme is computationally more complex and requires large computational effort in nonlinear optimization [2], [11], [75]. Also, the further extensions of the aforementioned methods are not reported elsewhere. Apart from these fuzzy MPC schemes, the simplest way to control a nonlinear process is by using the inverse of a fuzzy singleton model (a special case of the TS model) and use it in an open-loop (feed-forward) configuration [2], [9], [11], [85]. The obtained inverse model is used as a controller and under special conditions stable control can be guaranteed for minimum phase systems [11]. This type of control can only be applied if the inverse of a fuzzy model exists. Since this is a feed-forward configuration, ideal control configuration can not be directly applied in practice because the model never a perfect mapping of the system. So any model-plant mismatch results the system unstable. Moreover in this scheme there is always a possibility to violate the constraints limit for the computed optimum inverse model input to the process. So to check the constraints limit at every step a nonlinear B & B optimization is required to solve to find the best optimal solution [9], [11].

5.3 Formulation of TS Type Adaptive Fuzzy MPC

In the proposed adaptive fuzzy MPC scheme, an online RLS parameter identification strategy is incorporated with the TS type fuzzy model to perform the online adaptation of the fuzzy local models and an online linearization technique is employed to extract the linear parametric input-output model to formulate the proposed linear MPC based control scheme. The formulation of the MPC scheme is nearly same as that of DMC but the output predictions is computed recursively at each sampling step from the extracted linear parametric input-output model parameters. The proposed scheme is presented in the following subsections:

5.3.1 TS Fuzzy Model for MIMO Process

An n-output, m-input nonlinear process can be approximately modeled by a set of coupled Multiple Input Single Output (**MISO**) models. For the i^{th} output the decomposed model at the time instant k can be described as,

$$y_i(k) = f\left(\boldsymbol{\varphi}_i(k)\right) \tag{5.1}$$

where, $\boldsymbol{\varphi}_i(k)$ is the regression vector for the i^{th} output , expressed in equation (4.3) for RLS adaptation and $f(.)$ is a nonlinear function used for nonlinear approximation.

The unknown nonlinear function $f(.)$ in equation (5.1) can be approximated by using the TS type fuzzy model. The model comprises a number of logical rules for the approximation where each rule possesses nonlinear process variables in the antecedent space and piecewise linear function in the consequent space. The antecedents of fuzzy rules divide the input space into a number of fuzzy regions while the consequent functions approximate the local behavior of the process. The general rule base TS model for the approximation is expressed in the following form. The r^{th} rule is defined as,

L_r^i : **If** $y_i(k\text{-}1)$ is $C_1^i(r)$ **and** .... **and** $y_i(k - n_A^i)$ is $C_{n_A}^i(r)$

and $u_1(k - k_d^{i1} - 1)$ is $D_{11}^i(r)$ **and** ...

and $u_1(k-k_d^{i1}-n_B^{i1})$ is $D^i_{1n_B^{i1}}(r)$ **and** ...

and $u_m(k-k_d^{im}-1)$ is $D^i_{m1}(r)$ **and** ...

and $u_m(k-k_d^{im}-n_B^{im})$ is $D^i_{mn_B^{im}}(r)$ **then**

$$y_i^r(k)=\sum_{l=1}^{n_A^i}\hat{a}_l^i(r)y_i(k-l)+\sum_{j=1}^{m}\sum_{l=0}^{n_B^{ij}}\hat{b}_l^{ij}(r)u_j(k-l-k_d^{ij}-1)\,. \tag{5.2}$$

Where, r is the rule index, $C_1^i(r),\cdots,C_{n_A}^i(r)$ and $D_{11}^i(r),\cdots,D^i_{1n_B^{i1}}(r),\cdots,D^i_{mn_B^{im}}(r)$ are the antecedent fuzzy sets representing the fuzzy subspace in which the implications L_r^i for $r=1,\cdots,R$ can be applied for reasoning, $y_i^r(k)$ presents the local linear model for the r^{th} rule consequent and $\hat{a}_l^i(r)$ and $\hat{b}_l^{ij}(r)$ are the consequent linear model parameters of the r^{th} rule.

The consequent parameters in equation (5.2) for all R rules can be expressed using a parameter matrix $\hat{\mathbf{\Theta}}_i$ and is given by,

$$\hat{\mathbf{\Theta}}_i=\begin{bmatrix}\hat{\mathbf{A}}_i(1) & \hat{\mathbf{B}}_{i1}(1) & \cdots & \hat{\mathbf{B}}_{im}(1)\\ \vdots & \vdots & \vdots & \vdots\\ \hat{\mathbf{A}}_i(r) & \hat{\mathbf{B}}_{i1}(r) & \cdots & \hat{\mathbf{B}}_{im}(r)\\ \vdots & \vdots & \vdots & \vdots\\ \hat{\mathbf{A}}_i(R) & \hat{\mathbf{B}}_{i1}(R) & \cdots & \hat{\mathbf{B}}_{im}(R)\end{bmatrix} \tag{5.3}$$

with

$$\hat{\mathbf{A}}_i(r)=\left[\hat{a}_1^i(r),\cdots,\hat{a}_{n_A^i}^i(r)\right]$$

$$\hat{\mathbf{B}}_{ij}(r)=\left[\hat{b}_0^{ij}(r),\cdots,\hat{b}_{n_B^{ij}}^{ij}(r)\right]$$

where $\hat{\mathbf{\Theta}}_i$ is defined as the consequent parameter matrix. Now using the TS defuzzification technique overall fuzzy model output can be written as,

$$y_i^f(k) = \boldsymbol{\beta}_i \hat{\boldsymbol{\Theta}}_i \boldsymbol{\varphi}_i \tag{5.4}$$

where $\boldsymbol{\beta}_i$ is the vector of the membership grades (degree of fulfillment) assigned to each of the R rule implications at every sampling step and is given by,

$$\boldsymbol{\beta}_i = \left[\beta_1^i, \cdots, \beta_R^i\right]. \tag{5.5}$$

The elements of $\boldsymbol{\beta}_i$ are given by:

$$\beta_r^i = \frac{\xi_r^i}{\sum_{r=1}^{R} \xi_r^i}$$

where ξ_r^i is the degree of fulfillment of the r^{th} rule in the antecedent space. This is obtained while applying the t-norm fuzzy operation to the r^{th} rule and is given by [69] and [76],

$$\xi_r^i = \prod_{l=1}^{n_A} \mu[C_l^i(r)].\prod_{j=1}^{m} \prod_{l=1}^{n_B^{ij}} \mu[D_{jl}^i(r)]$$

where $\mu[.]$ is the grade of the membership estimated from the antecedent variables.

Although the fuzzy model consists of a number of piecewise linear models, the overall model output in equation (5.4) is nonlinear. To formulate the linear MPC strategy with an analytical approach, a simple method of linearizing the fuzzy model about the current operating point is used at each sampling instant. The weighted linear parameters due to the fuzzy inference are then given by [69]:

$$\boldsymbol{\theta}_i = [\mathbf{A}_i \quad \mathbf{B}_{i1}, \cdots, \mathbf{B}_{im}] = \boldsymbol{\beta}_i . \hat{\boldsymbol{\Theta}}_i \tag{5.6}$$

where $\mathbf{A}_i = \left[a_1^i, \cdots, a_{n_A^i}^i\right]$ and

$$\mathbf{B}_{ij} = \left[b_0^{ij}, \cdots, b_{n_b^{ij}}^{ij}\right].$$

The vector $\boldsymbol{\theta}_i$ denotes the weighted linear fuzzy model parameter vector, which are computed at every sampling step and employed with the MPC scheme to develop the control law.

5.3.2 Formulation of MPC Scheme

A linear model based MPC strategy is formulated in this section. The linear parameters in equation (5.6) can be used to build the following parametric input-output model,

$$\hat{y}_i(k) = \sum_{l=1}^{n_A^i} a_l^i y_i(k-l) + \sum_{j=1}^{m} \sum_{l=0}^{n_B^{ij}} b_l^{ij} u_j(k-l-k_d^{ij}-1)\,. \qquad (5.7)$$

The model in equation (5.7) can be applied to predict the future steps output ($\hat{y}_i(k+p/k)$) for p sampling instants ahead of the current time instant, k. The prediction through the prediction horizon, P is based on the natural divisions of the system responses into free and forced terms and the measurement of the current disturbance due to model-plant mismatch. The prediction is thus formulated as [3]:

$$\hat{y}_i(k+p/k) = \hat{y}_i^{forced}(k+p) + \hat{y}_i^{free}(k+p) + d_i(k) \qquad (p = 1, \cdots, P). \qquad (5.8)$$

The current disturbance $d_i(k)$ in equation (5.8) is assumed to be constant throughout the prediction horizon and is computed as:

$$d_i(k) = [y_i^m(k) - \hat{y}_i(k)]$$

where, $\hat{y}_i(k)$ is the i^{th} linear fuzzy model output expressed in equation (5.7) . The forced outputs term $\hat{y}_i^{forced}(k+p)$ in equation (5.8) can be estimated from the current operating point as:

$$\hat{y}_i^{forced}(k+p) = \sum_{j=1}^{m} \sum_{q=1}^{p} \hat{g}_q^{ij} \Delta \hat{u}_j(k+p-q)$$

where, $$\hat{g}_q^{ij} = -\sum_{l=1}^{n_A^i} a_l^i \hat{g}_{q-l}^{ij} + \sum_{l=0}^{n_B^{ij}} b_l^{ij}$$

is the q^{th} element of the extracted unit linear step response model parameter corresponding to i^{th} output to j^{th} input, in which when $q \le k_d^{ij}$, $\hat{g}_q^{ij} = 0$ and $\Delta\hat{u}_j(k+p-q)$ are the unknown current and future input moves. The free response term $\hat{y}_i^{free}(k+p)$ in equation (5.8) is inferred as the future response of the system provided that the system input will be maintained at a constant value through the prediction horizon *P*. In other words,

$$u_j(k-1) = \hat{u}_j(k) = \cdots = \hat{u}_j(k+P)$$

The free response predictions with disturbance can be estimated recursively from the updated linear model parameters as follows. For convenience, the free response term is expressed while using,

$$\hat{y}_i^{free}(k+p) = \hat{f}_i(k+p).$$

Therefore,

$$\hat{f}_i(k+p) = \sum_{l=1}^{n_A^i} a_l^i \hat{f}_i(k+p-l) + \sum_{j=1}^{m}\sum_{l=1}^{n_B^{ij}} b_{l-1}^{ij} u_j(k-l-k_d^{ij}+p) + d_i(k) \tag{5.9}$$

The initial conditions for equation (5.9) are the predicted output at the current time and is given by,

$$\hat{f}_i(k) = \hat{f}_i(k-1) = \cdots = \hat{y}_i(k).$$

Hence the prediction form in equation (5.8) can be written as,

$$\hat{y}_i(k+p/k) = \underbrace{\sum_{j=1}^{m}\sum_{q=1}^{P} \hat{g}_q^{ij}\Delta\hat{u}_j(k+p-q)}_{\hat{y}_i^{forced}} + \underbrace{\hat{f}_i(k+p)}_{\hat{y}_i^{free}} \tag{5.10}$$

Now employ the prediction form in equation (5.10) with the following cost function for optimization,

$$J = \sum_{i=1}^{n}\sum_{p=1}^{P}\delta_i(p)[w_i(k+p) - \hat{y}_i(k+p/k)]^2 + \sum_{j=1}^{m}\sum_{p=1}^{M}\lambda_j(p)[\Delta\hat{u}_j(k+p-1)]^2 \qquad (5.11)$$

Solving the equation in (5.11) the following optimal fuzzy model based unconstrained adapted multivariable MPC law is obtained and is given by,

$$\Delta\hat{\mathbf{U}} = (\hat{\mathbf{G}}^T\boldsymbol{\delta}\hat{\mathbf{G}} + \boldsymbol{\lambda}\mathbf{I})^{-1}\hat{\mathbf{G}}^T\boldsymbol{\delta}(\mathbf{W} - \hat{\mathbf{F}}) = \hat{\mathbf{K}}(\mathbf{W} - \hat{\mathbf{F}}) \qquad (5.12)$$

Where, $\hat{\mathbf{K}} = (\hat{\mathbf{G}}^T\boldsymbol{\delta}\hat{\mathbf{G}} + \boldsymbol{\lambda}\mathbf{I})^{-1}\hat{\mathbf{G}}^T\boldsymbol{\delta}$ is the MPC gain matrix and the term $(\mathbf{W} - \hat{\mathbf{F}})$ is the vector of future predicted error over P sampling instants due to the effect of past inputs. The only control moves $\Delta\hat{u}_1(k),\cdots,\Delta\hat{u}_m(k)$ in equation (5.12) are applied to the plant at current step and whole procedure is repeated in each sampling step.

5.3.3 RLS Adaptation of the Fuzzy Model

For online adaptation of the TS model, the rule premises are kept fixed and only the linear model parameters of the active rule consequents are adapted. Thus the overall model output in equation (5.4) at every sampling step, is the sum of the contributions of all the adapted active rules.

Define the estimated parameter vector of the r^{th} excited rule as:

$$\hat{\boldsymbol{\theta}}_i^r(k) = \left[\hat{\mathbf{A}}_i(r) \quad \hat{\mathbf{B}}_{i1}(r),\cdots,\hat{\mathbf{B}}_{im}(r)\right]^T \qquad (5.13)$$

The RLS technique for updating the local model parameter vector can be summarized as [2]:

$$\underbrace{\hat{\boldsymbol{\theta}}_i^r(k)}_{\text{new estimate}} = \underbrace{\hat{\boldsymbol{\theta}}_i^r(k-1)}_{\text{old estimate}} + \underbrace{\frac{\mathbf{P}_i^r(k-1)\boldsymbol{\varphi}_i^T(k)\beta_r^i}{\lambda_i^r + \beta_r^i\boldsymbol{\varphi}_i(k)\mathbf{P}_i^r(k-1)\boldsymbol{\varphi}_i^T(k)}}_{\text{correcting vector}} \times \underbrace{[\underbrace{y_i^m(k)}_{\substack{\text{new} \\ \text{measurement}}} - \underbrace{\boldsymbol{\varphi}_i(k)\hat{\boldsymbol{\theta}}_i^r(k-1)}_{\substack{\text{one step ahead} \\ \text{prediction of the} \\ \text{new measurement}}}]}_{\text{estimated error}, d_i^r} \qquad (5.14)$$

and the covariance matrix $\mathbf{P}_i^r(k)$ is updated as follows:

$$\mathbf{P}_i^r(k) = \frac{1}{\lambda_i^r}\left(\mathbf{P}_i^r(k-1) - \frac{\beta_r^i\mathbf{P}_i^r(k-1)\boldsymbol{\varphi}_i^T(k)\boldsymbol{\varphi}_i(k)\mathbf{P}_i^r(k-1)}{\lambda_i^r + \beta_r^i\boldsymbol{\varphi}_i(k)\mathbf{P}_i^r(k-1)\boldsymbol{\varphi}_i^T(k)}\right). \qquad (5.15)$$

Where $\mathbf{P}_i^r(k)$ is a symmetric matrix with $\mathbf{P}_i^r(0) = \alpha\mathbf{I}$, α is a real large number and λ_i^r is the scalar forgetting factor of the r^{th} rule adaptation and the range chosen for λ_i^r is $0.80 \le \lambda_i^r \le 1$.

The estimator in equations (5.14) and (5.15) will work well if the process is consistently excited with the estimated error (d_i^r). But when the control becomes perfect with little excitation error, the estimator windup problem may occur. This will drastically change the model parameters with noises and in turn make the overall system unstable. To avoid the estimator windup a dead zone or tolerance limit (σ) is introduced with the adaptive scheme where the updating of parameters is stopped when the estimated error is sufficiently small considering to the noise level.

$$d_i^r(k) = \begin{cases} y_i^m(k) - \boldsymbol{\varphi}_i(k)\hat{\boldsymbol{\theta}}_i^r(k-1) \\ 0 \quad if \left|y_i^m(k) - \boldsymbol{\varphi}_i(k)\hat{\boldsymbol{\theta}}_i^r(k-1)\right| \le |\sigma| \end{cases} .$$

5.3.4 Consequent Parameter Identification

The initial parameters of the consequent models (local models) can be identified offline using the linear least squares parameter estimation technique. This technique involves the available inputs-output data samples from the process and arranges them in the following matrix form,

$$\mathbf{Y}_N^i = \begin{bmatrix} y_i(1) \\ \vdots \\ y_i(n) \\ \vdots \\ y_i(N) \end{bmatrix} \text{ and } \boldsymbol{\varphi}_N^i = \begin{bmatrix} \boldsymbol{\varphi}_i(1) \\ \vdots \\ \boldsymbol{\varphi}_i(n) \\ \vdots \\ \boldsymbol{\varphi}_i(N) \end{bmatrix}$$

where $\boldsymbol{\varphi}_N^i$ is the regression matrix and $\boldsymbol{\varphi}_i(n)$ is the regression vector of the n^{th} data sample, $\mathbf{Y}_N^i$ is the output data vector and the index, $n = 1, \cdots, N$ represent the number of available data samples. Now build the weighted matrix, $\mathbf{B}_r^i$ for the r^{th} rule as,

$$\mathbf{B}_r^i = \left[diag(\beta_r^i(1), \cdots, \beta_r^i(n), \cdots, \beta_r^i(N) \right]$$

where $\beta_r^i(n)$ is the truth value of the n^{th} data samples in the antecedent space.

The parameters of all the rule consequents can be estimated globally within one least squares problem and is given by,

$$\min_{\tilde{\boldsymbol{\theta}}_i} \frac{1}{N} (\mathbf{Y}_N^i - \tilde{\boldsymbol{\varphi}}_N^i \tilde{\boldsymbol{\theta}}_i)^T (\mathbf{Y}_N^i - \tilde{\boldsymbol{\varphi}}_N^i \tilde{\boldsymbol{\theta}}_i). \tag{5.16}$$

Where $\tilde{\boldsymbol{\varphi}}_N^i$ denotes the matrix composed of matrices $\mathbf{B}_r^i$ and $\boldsymbol{\varphi}_N^i$ as follows

$$\tilde{\boldsymbol{\varphi}}_N^i = \left[\mathbf{B}_1^i \boldsymbol{\varphi}_N^i, \cdots, \mathbf{B}_R^i \boldsymbol{\varphi}_N^i \right]$$

and the $\tilde{\boldsymbol{\theta}}_i$ vector is given by,

$$\tilde{\boldsymbol{\theta}}_i = \left[(\hat{\boldsymbol{\theta}}_i^1)^T, \cdots, (\hat{\boldsymbol{\theta}}_i^R)^T \right]^T.$$

The optimal estimate of equation (5.16) is,

$$\tilde{\boldsymbol{\theta}}_i = [(\tilde{\boldsymbol{\varphi}}_N^i)^T \tilde{\boldsymbol{\varphi}}_N^i]^{-1} (\tilde{\boldsymbol{\varphi}}_N^i)^T \mathbf{Y}_N^i \tag{5.17}$$

which define the consequent parameters of all the fuzzy rules and can be used as the initial model in equations (5.14) and (5.15) to perform the online adaptation. The analysis of the computational complexity shows that the computational load cubically increases with the number of rules. Due to this cubic complexity, the global parameter estimation becomes computationally expensive for fuzzy systems with many rules. The local parameter estimation approach does not estimate all the rules parameters simultaneously. This approach uses a set of local estimation criteria for separately identify the parameters of each local model and is given by,

$$\min_{\hat{\boldsymbol{\theta}}_i^r} \frac{1}{N} (\mathbf{Y}_N^i - \boldsymbol{\varphi}_N^i \hat{\boldsymbol{\theta}}_i^r)^T \mathbf{B}_r^i (\mathbf{Y}_N^i - \boldsymbol{\varphi}_N^i \hat{\boldsymbol{\theta}}_i^r) \tag{5.18}$$

The weighted least squares estimate of the r^{th} rule consequent is then,

$$\hat{\boldsymbol{\theta}}_i^r = [(\boldsymbol{\varphi}_N^i)^T \mathbf{B}_r^i \boldsymbol{\varphi}_N^i]^{-1} (\boldsymbol{\varphi}_N^i)^T \mathbf{B}_r^i \mathbf{Y}_N^i . \tag{5.19}$$

5.3.5 Control Strategy of Adaptive Fuzzy MPC Scheme

The above adaptive fuzzy model based predictive control mechanism consists of following simple steps at every sampling instant:

1) Identify the initial fuzzy local model parameters in offline using the least squares technique described in equations (5.18) and (5.19).
2) Measure the antecedents variables in equation (5.2) and fuzzify to build the vector of weights ($\boldsymbol{\beta}_i$) in equation (5.5) for each output.
3) Update the consequents parameter vector ($\hat{\boldsymbol{\theta}}_i^r(k)$) in equation (5.13) of each

rule using the RLS estimation technique described in equations (5.14) and (5.15).

4) Build the consequent model parameters matrix ($\hat{\mathbf{\Theta}}_i$) in equation (5.3) with the updated consequent parameters of each rule.

5) Compute the weighted linear model parameter vector ($\mathbf{\theta}_i$) using the linearization scheme described in equation (5.6).

6) Employ the weighted parameters to build the linear model described in equation (5.7) and compute the MPC control law in equation (5.12).

7) Apply the control inputs ($\Delta\hat{u}_1(k),\cdots,\Delta\hat{u}_m(k)$) to both the plant and the model.

8) Go to step 2 and repeat.

The complete control structure is shown in Fig.5.1. To cope with the model-plant mismatch due to process uncertainties and to avoid the steady state control error, the proposed scheme is also implemented within an IMC structure (Fig. 5.1). A feedback filter is introduced into the control scheme and the filtered modeling error is utilized to modify the setpoint [2]. For this purpose, the following first-order low-pass filter is used:

$$e_i^f(k) = K_f \underbrace{\left(y_i^m(k) - \hat{y}_i(k)\right)}_{d_i} + \left(1 - K_f\right) e_i^f(k-1)$$

where K_f is the adjustable filter parameter, $K_f \in [0,1]$. The feedback filter is able to filter out the measurement noise and stabilize the loop by reducing the loop gain.

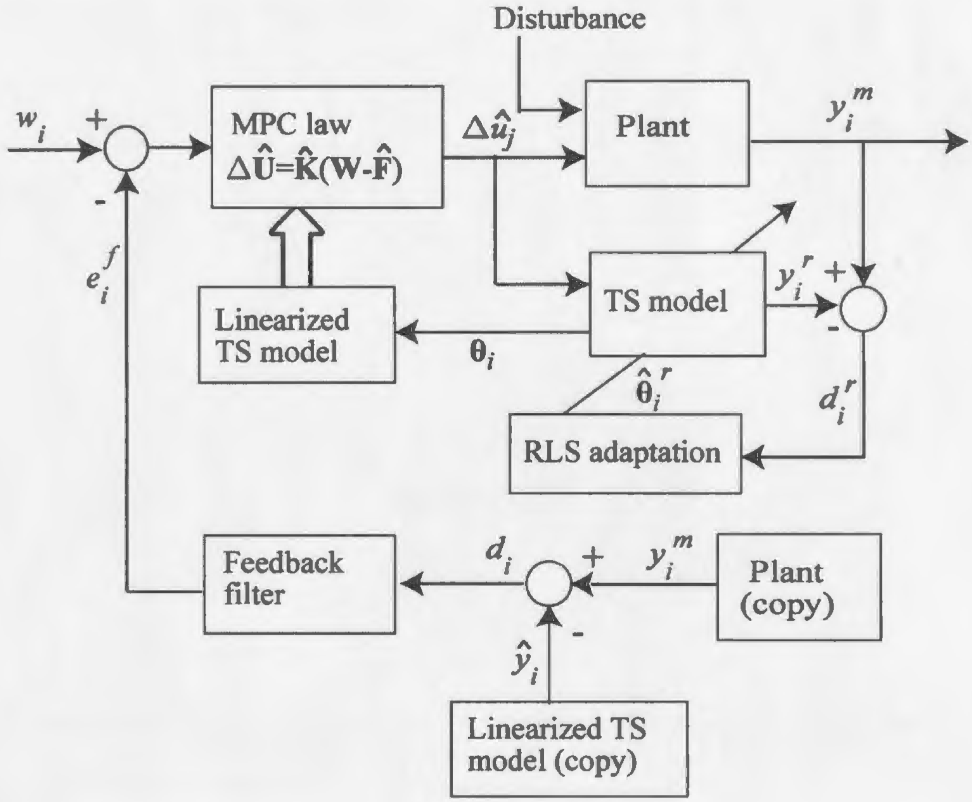


Fig.5.1 Adaptive fuzzy model based predictive control scheme

5.4 Identification of Fuzzy Model for the Soil Heating Process

The parameters of the fuzzy local models were identified from the simulated open-loop FEM inputs/output data pair, using the method of least squares. Simulation was carried out employing five rules for each output and the output variable $y_i(k-1)$ was considered as the antecedent variable in the input space. The partition in the antecedent space for $y_i(k-1)$ is presented in Fig.5.2.

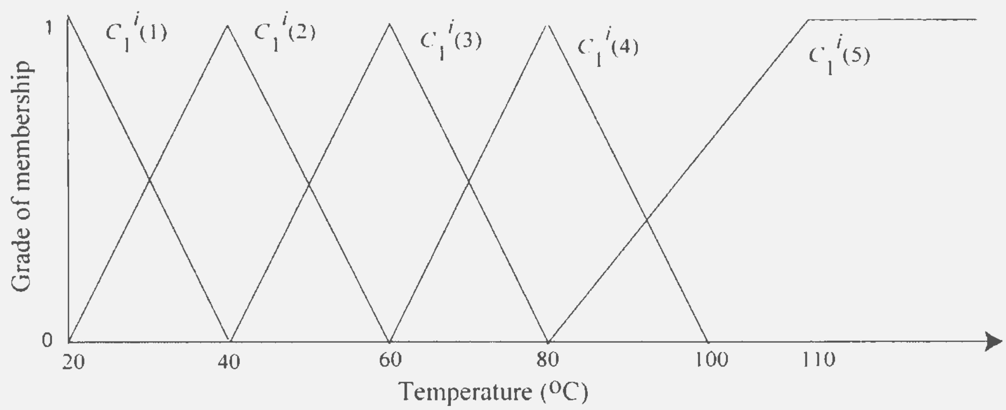


Fig.5.2 Fuzzy portioning in the input space

The offline identified rule base TS model for the present soil heating process are,

For output 1

L_1^1 : if y_1 $(k\text{-}1)$ is $C_1^1(1)$ then

$$y_1^1(k) = 2.52y_1(k-1) + 0.0031u_1(k-2) + 0.00014u_2(k-3) + 0.000023u_3(k-7)$$

L_2^1 : if y_1 $(k\text{-}1)$ is $C_1^1(2)$ then

$$y_1^2(k) = 2.42y_1(k-1) + 0.0021u_1(k-2) + 0.00016u_2(k-3) + 0.000013u_3(k-7)$$

L_3^1 : if y_1 $(k\text{-}1)$ is $C_1^1(3)$ then

$$y_1^3(k) = 2.54y_1(k-1) + 0.0023u_1(k-2) + 0.00034u_2(k-3) + 0.000053u_3(k-7)$$

L_4^1 : if y_1 $(k\text{-}1)$ is $C_1^1(4)$ then

$$y_1^4(k) = 2.52y_1(k-1) + 0.0041u_1(k-2) + 0.00014u_2(k-3) + 0.000023u_3(k-7)$$

L_5^1 : if y_1 $(k\text{-}1)$ is $C_1^1(5)$ then

$$y_1^5(k) = 2.52y_1(k-1) + 0.0033u_1(k-2) + 0.00016u_2(k-3) + 0.000028u_3(k-7)$$

For output 2

L_1^2 : if $y_2\,(k\text{-}1)$ is $C_1^2(1)$ then

$$y_2^1(k) = 2.72\,y_2(k-1) + 0.0011\,u_1(k-3) + 0.034u_2(k-2) + 0.0023u_3(k-4)$$

L_2^2 : if $y_2\,(k\text{-}1)$ is $C_1^2(2)$ then

$$y_2^2(k) = 2.72\,y_2(k-1) + 0.0015\,u_1(k-3) + 0.044u_2(k-2) + 0.0021u_3(k-4)$$

L_3^2 : if $y_2\,(k\text{-}1)$ is $C_1^2(3)$ then

$$y_2^3(k) = 2.73\,y_2(k-1) + 0.0015\,u_1(k-3) + 0.037u_2(k-2) + 0.0023u_3(k-4)$$

L_4^2 : if $y_2\,(k\text{-}1)$ is $C_1^2(4)$ then

$$y_2^4(k) = 2.74\,y_2(k-1) + 0.0021\,u_1(k-3) + 0.031u_2(k-2) + 0.0021u_3(k-4)$$

L_5^2 : if $y_2\,(k\text{-}1)$ is $C_1^2(5)$ then

$$y_2^5(k) = 2.73\,y_2(k-1) + 0.0018\,u_1(k-3) + 0.031u_2(k-2) + 0.0022u_3(k-4)$$

and for output 3

L_1^3 : if $y_3\,(k\text{-}1)$ is $C_1^3(1)$ then

$$y_3^1(k) = 2.62\,y_3(k-1) + 0.0021\,u_1(k-7) + 0.031u_2(k-4) + 0.0022u_3(k-2)$$

L_2^3 : if $y_3\,(k\text{-}1)$ is $C_1^3(2)$ then

$$y_3^2(k) = 2.62\,y_3(k-1) + 0.0021\,u_1(k-7) + 0.032u_2(k-4) + 0.0024u_3(k-2)$$

L_3^3 : if $y_3\,(k\text{-}1)$ is $C_1^3(3)$ then

$$y_3^3(k) = 2.64\,y_3(k-1) + 0.0015\,u_1(k-7) + 0.039u_2(k-4) + 0.0021u_3(k-2)$$

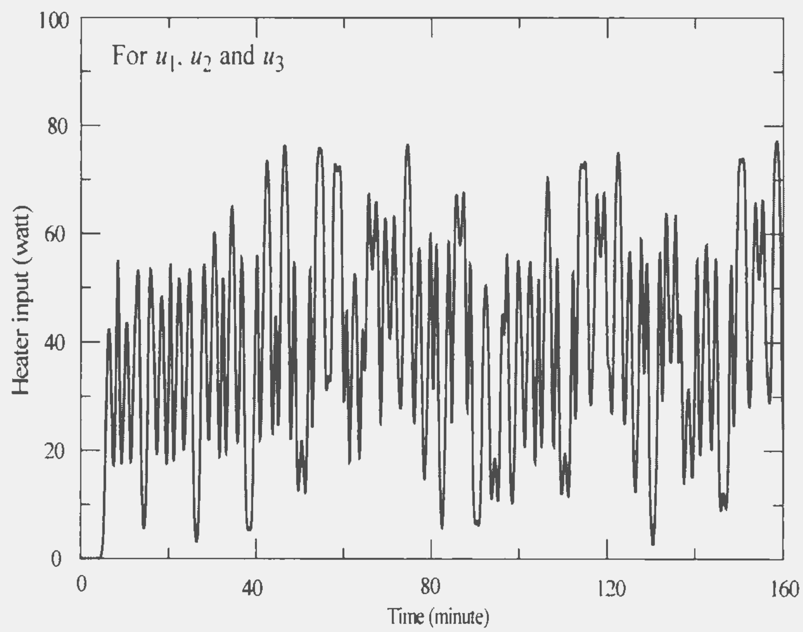
L_4^4 : if $y_3\,(k\text{-}1)$ is $C_1^3(4)$ then

$$y_3^4(k) = 2.61y_3(k-1) + 0.0013u_1(k-7) + 0.032u_2(k-4) + 0.0021u_3(k-2)$$

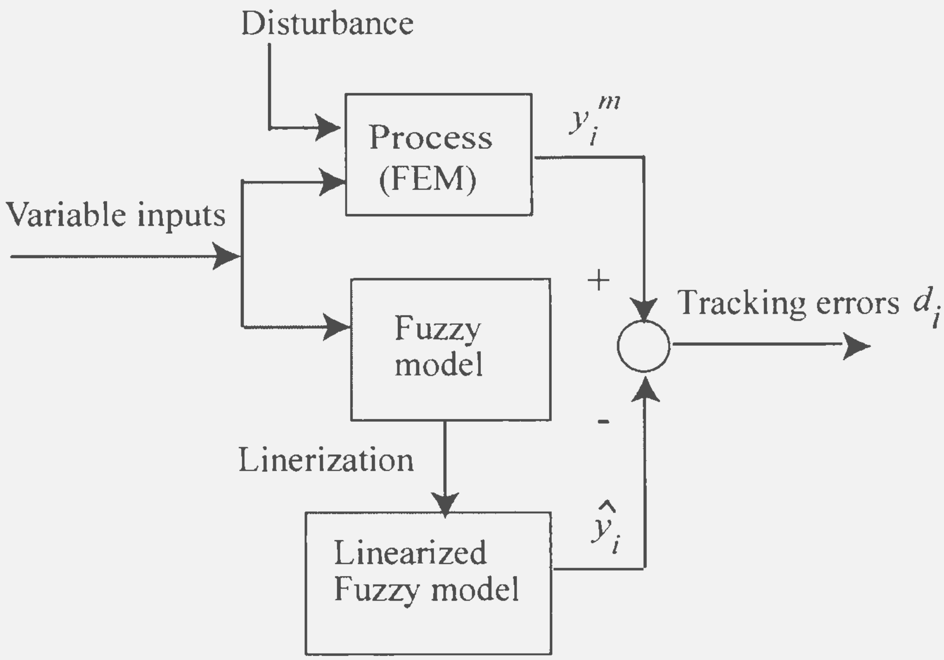
L_5^3 : if $y_3\,(k\text{-}1)$ is $C_1^3(5)$ then

$$y_3^5(k) = 2.62y_3(k-1) + 0.0012u_1(k-7) + 0.033u_2(k-4) + 0.0021u_3(k-2)\,.$$

To validate the tracking performance of the identified fuzzy model against variable inputs sequences an Integral Square Error (**ISE**) based performance index was used. The input profiles in u_1, u_2 and u_3 for validation and the simulation set-up, is presented in Fig.5.3. In the simulation the temperature outputs of the linearized fuzzy model ($\hat{y}_1(k)$, $\hat{y}_2(k)$ and $\hat{y}_3(k)$) and the FEM ($y_1^m(k)$, $y_2^m(k)$ and $y_3^m(k)$) are measured at each step with the sampling interval 0.6 minute.



(a) Variable inputs profile for validation



(b) Simulation set-up

Fig. 5.3 Validation of fuzzy model against variable input sequences

The difference between the linearized fuzzy model outputs and the FEM outputs (tracking error) are computed and compared with the non-adaptive linear step response model (DMC model) based tracking performance. The tracking errors of both schemes are presented in Fig.5.4.

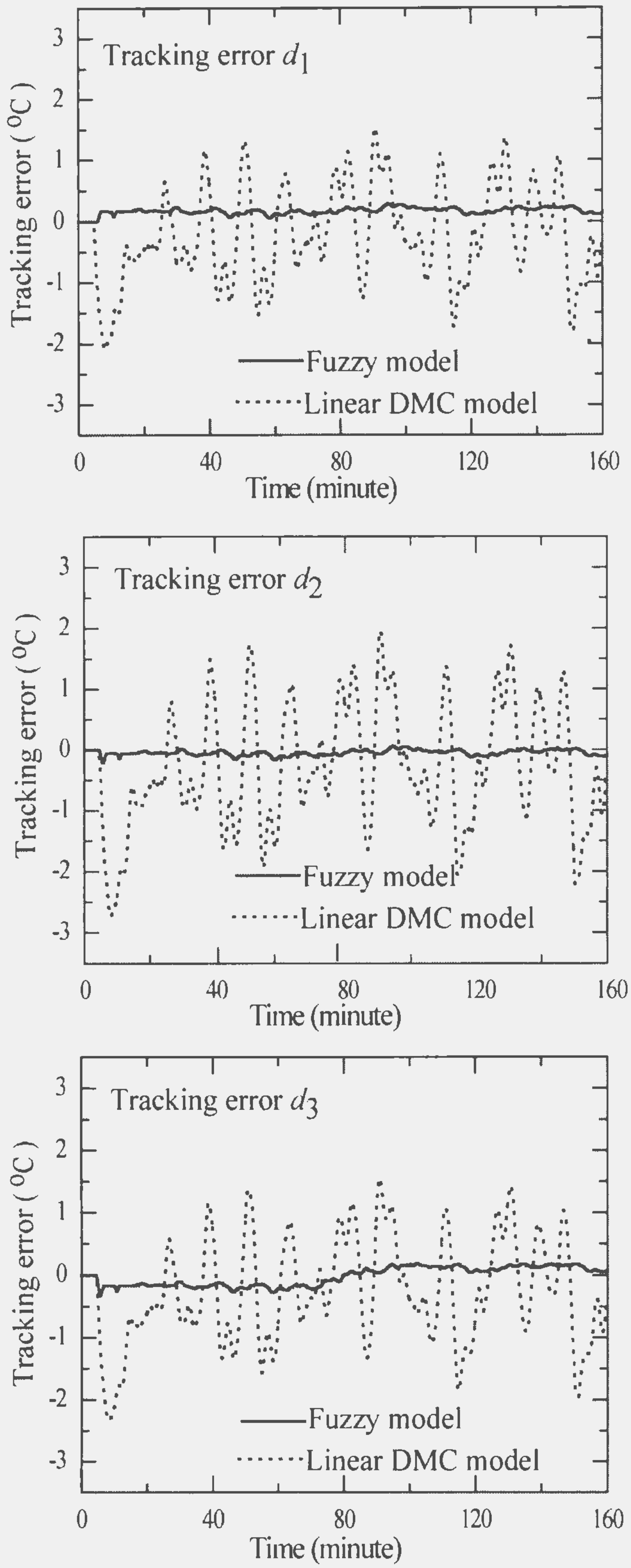


Fig.5.4 Tracking Performance of fuzzy model over linear DMC model

Simulation results justify the accuracy of the linearized fuzzy model against the linear DMC model based tracking performance.

5.5 Control Simulations

The above fuzzy model based MPC scheme and two non-adaptive linear model based classical MPC schemes were applied with the finite element based soil heating process model. The control objective of this exercise is to achieve precise temperature tracking with lower overshoot, smaller settling time, minimum heat distribution, better load disturbance characteristics and least tracking error. The tuning parameters chosen for all the schemes are shown in Table 5.1.

Table 5.1 Tuning parameters for fuzzy MPC and linear MPC

Controller	Prediction Horizon (*P*)	Control Horizon (*M*)	Weights for Control Variables δ_i	Weights for Manipulated Variables λ_j	Sampling Interval *T* (minute)
Fuzzy MPC	15	5	1.62,1.46,1.48	0.15,0.10,0.14	0.6
DMC/GPC	20	10	1.08,1.10,1.12	0.10,0.10,0.10	0.6

The simulation results and the comparisons are presented in the following subsections.

5.5.1 Comparisons of Non-Adaptive Fuzzy MPC over Linear MPC

Simulation was carried out for the non-adaptive fuzzy model based MPC scheme with the setpoint temperatures 55°C, 65°C, 60°C for output 1, 2 and 3, respectively and compared against the proposed (described in chapter 3) linear DMC model based non-adaptive MPC scheme based tracking performance. A GPC model (CARIMA model) based classical MPC technique is also applied in the system to confirm the superiority of the proposed fuzzy model based MPC system. The comparison of simulation results are presented in Fig.5.5 and Fig.5.6, respectively. A high negative disturbance of heat is applied constantly to both control systems when the simulation time reaches 180 minutes.

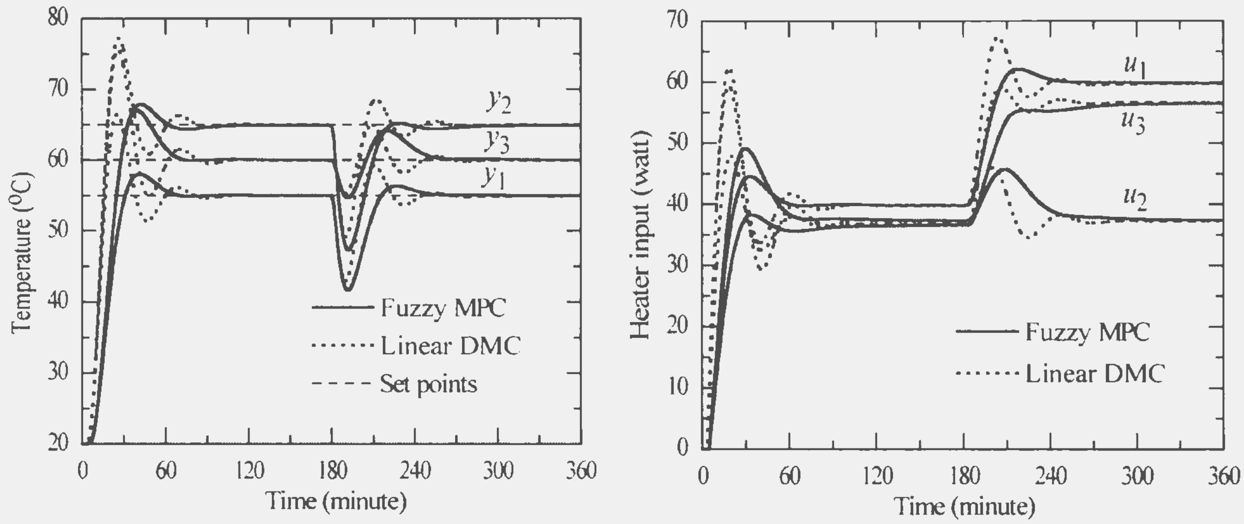


Fig.5.5 Linear DMC model based MPC over fuzzy MPC

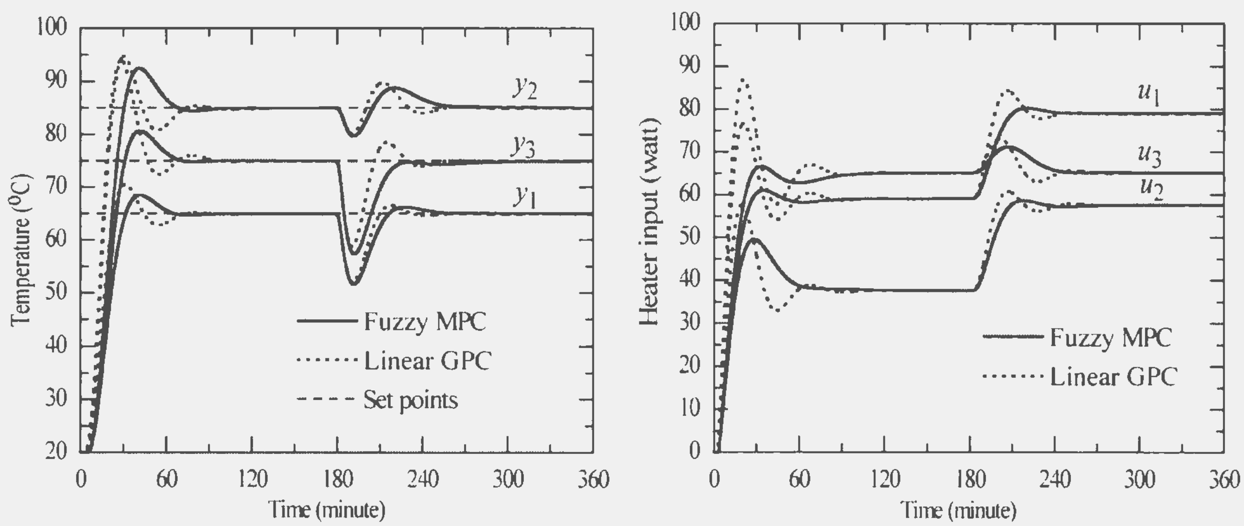


Fig.5.6 Linear GPC model based MPC over fuzzy MPC

The simulation results clearly show the non-adaptive fuzzy MPC outperforms the linear MPC schemes with respect to lower overshoot, smaller settling time, minimum heat distribution and better load disturbance characteristics. In general, the control law in non-adaptive MPC is implemented with the pre-estimated linear model parameters. As such, its ability to compensate the nonlinear process dynamics over wide operating range is limited. The non-adaptive fuzzy MPC, on the other hand, is based on multiple fuzzy models, excites different local models and rules as the operation level changes and hence shown better performance.

5.5.2 Comparison of Adaptive and Non-Adaptive Fuzzy MPC

To prove the importance of employing online adaptation on the fuzzy model against time variant process dynamics, the performance of the adaptive fuzzy MPC scheme was compared against the above non-adaptive fuzzy MPC system. To make the present process time variant the boundary temperature of the soil surface and also the specific heat and thermal conductivity of the soil were varied with time. Simulations were carried out for both schemes against the same variable setpoint sequences and the results are presented in Fig.5.7. The adaptive fuzzy system used a total of three rules for each output and the parameters of each active rule are adapted at every sampling state. A low number of rules are chosen mainly to produce better control performance and also to reduce the computational burden. The tolerance limit or the dead zone (σ) of the estimator was chosen 0.01 for all the outputs.

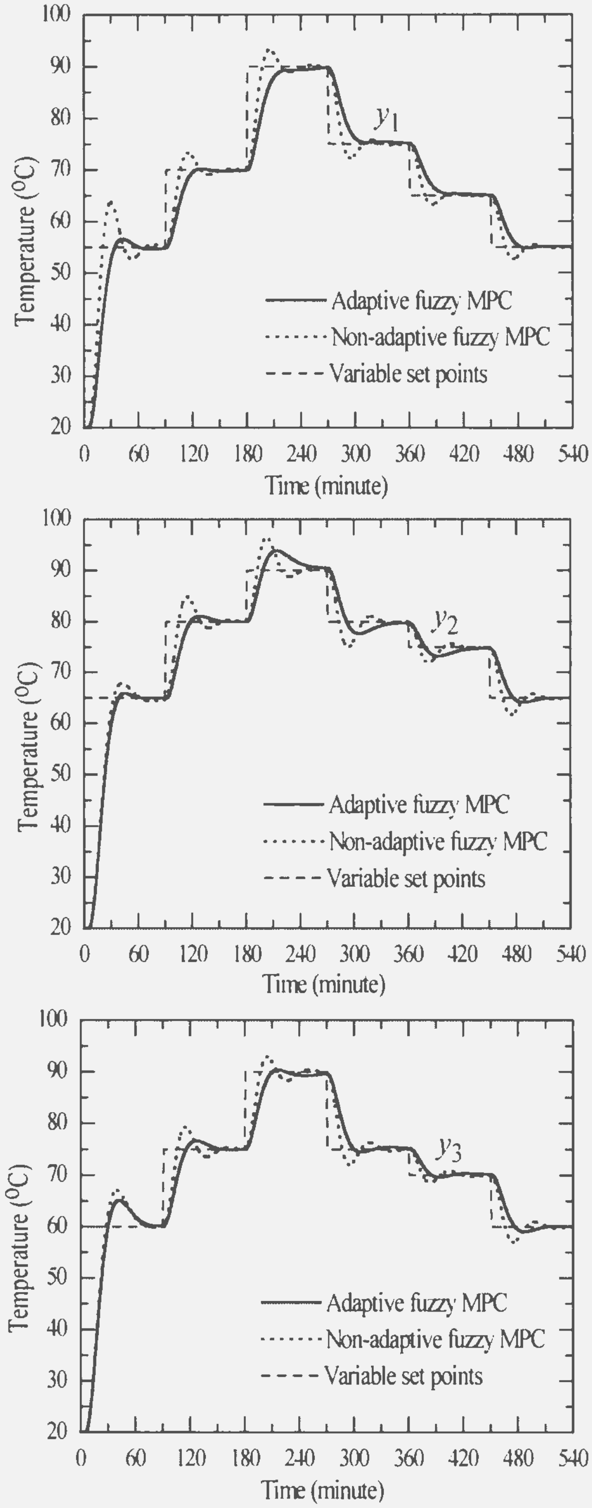


Fig.5.7 Comparison between adaptive and non-adaptive fuzzy MPC

The inconsistent control performance of the non-adaptive fuzzy scheme indicates that even with the multi-model, the time variant process dynamics may not be completely modeled due to process nonlinearity introduced due to changes in operating set points. This suggests the inclusion of online adaptation scheme with the non-adaptive fuzzy model. The online adaptation scheme allows fine-tuning of the local model parameters with the variation of the time variant nonlinear process dynamics and which in turn update the weighted model parameters in equation (5.6) more precisely to establish an improved and adaptive control law. The results show the superiority of the rule adaptation against the non-adaptive fuzzy system, particularly when tracking different set points in the time varying process dynamics.

5.6 Summary

A TS type fuzzy model based MPC strategy for a MIMO process system has been developed, analyzed and implemented systematically in this chapter. The application and benefits of the proposed strategy over linear MPC was also demonstrated through simulations. The simulation results reveal that the proposed fuzzy control system has the better capability to overcome the nonlinear and coupling effects of the process system and is therefore able to produce accurate tracking performance against the desired output temperature profile. The inclusion of adaptation on the fuzzy local models also indicates the superiority of the proposed scheme against the time variant process system. More importantly, the load disturbance performance of the proposed fuzzy control system was satisfactory. Hence the proposed system provides a useful and relatively simple alternative when non-adaptive linear MPC fails to a produce better response against nonlinear process dynamics. During control, the online adapted fuzzy model was linearized at each sampling instance employing a simple linearization technique. Subsequently the linear DMC based MPC scheme was formulated in accordance with the extracted linear parameters from the fuzzy model. In this way, the advantages of both fuzzy modelling and the existence of analytical solution in the case of linear DMC are combined. Moreover this fuzzy model with the linearized scheme provides the opportunity to implement other general linear MPC techniques in a straightforward way. The involvement of large number of rules in the TS type fuzzy system representing many local models may lead to over fitting. With linear output memberships the control surface may become more linear with larger rules. Under those circumstances, the online parameters estimation technique becomes less robust and ineffective, particularly at the boundaries between the fuzzy memberships functions [69]. To address this issue the adaptive fuzzy MPC system was chosen with three fuzzy rules for each output. In comparison to the fuzzy relational model based predictive control [81] and or inverse fuzzy model based control [9], [82] for nonlinear process, the presented approach is much simpler to implement, requires less computational effort and therefore suitable for real time process with faster dynamics. The only drawback of the scheme is that, as the adapted parameters of the local models are applied to develop the MPC control law, any erroneous parameters estimation may result in undesirable changes in the control signal resulting in a poor control performance. Typically, such situation occurs at the beginning of the adaptation process, particularly during the transient period the unexpected response characteristics due to load adjustments may lead to erroneous estimation. Using the equality and inequality constraints on the parameters of the local models, it is possible to avoid unrealistic model parameters that could result poor control performance [2].

Chapter 6

Concluding Remarks

6.1 Conclusions

The main objective of this thesis was to develop a computationally efficient multivariable MPC strategy for a broad class of nonlinear process systems. To fulfill this objective a number of multivariable MPC techniques were developed. The application and benefits of these techniques were demonstrated through the simulations performances against the finite element based highly coupled nonlinear multivariable soil heating process.

In chapter 3, formulation of the multivariable non-adaptive DMC based MPC strategy was presented and addressed the high performance behaviour of the multivariable MPC system against the general decoupled PID based multivariable control systems. To overcome the limitations mentioned in using the non-adaptive DMC, a multivariable AMPC scheme was developed and presented in chapter 4. The superior performance of the AMPC scheme was justified through the comparisons of several simulation results against the non-adaptive DMC based MPC technique. The comparative simulation results show that the proposed multivariable AMPC system has the better capability to overcome the nonlinear and coupling effects of the soil heating process and therefore able to produce accurate tracking performance against the desired output temperature profile. In chapter 5 the TS type adaptive and non-adaptive fuzzy model based MPC strategy were introduced, analyzed and implemented systematically. The advantages of using the fuzzy model in MPC for nonlinear processes were also justified by comparing the control performance against the two linear model based MPC strategies (classical DMC and GPC strategies).

Finally a sound analysis of these comparisons, control techniques and implementation strategies concludes that the proposed multivariable AMPC system and the multivariable adaptive fuzzy MPC scheme have the better capability for exhibiting satisfactory performance. Hence these two schemes represent an effective and relatively simple technique to handle a large class of nonlinear multivariable process systems.

6.2 Recommendations for Future Research

Some recommendations for further studies are outlined below:

- All the multivariable adaptive MPC schemes proposed in this study were based on the DMC based MPC structure. However, the GPC is another popular form of MPC strategy widely used for multivariable process control. So to integrate these methodologies with the general GPC based control structure will provides a better performance analysis of the proposed controllers for further study.
- In this thesis the tuning of all the DMC schemes were performed by using the general DMC tuning rules described in [2], [14], [38]. However, in literature such as in [24], [36] proposed an adaptive strategy on the tuning parameters for the implementation of DMC scheme. So inclusion of these tuning rules in the present DMC techniques may offer better performance.
- The nonlinear based predictive control or NMPC is the most challenging research issue among the present MPC researchers. The use of the nonlinear model directly in MPC can improve the control performance by improving the prediction

accuracy. So to develop an effective NMPC technique is another line of research on this study for future studies.

- In this study for online adaptation on the process model, the RLS based parameter estimation technique was used. But in literature there are several parameter identification techniques available for online adaptation on the process model, as for example, the instrumental variable method, maximum likelihood estimation, the bootstrap method and the sequential correlation method, are the most common techniques for online parameter estimation [68]. So to compare the accuracy of the RLS technique for online adaptation these methods also need to be considered.
- The applications of all the proposed schemes were verified in this study against the soil heating process system which has relatively slow dynamics. A sampling time 0.6 min was used in the study which is sufficient enough to solve the MPC law before that defined sampling time. But in the process industries there are many processes where the process must be sampled before few micro seconds. In order to prove the effectiveness of the proposed schemes it is essential to verify them against a process with fast dynamics.
- Although the finite element based soil heating process exhibits the true process dynamics but for better justification the proposed control systems should be verified against the real soil heating process system.
- Finally a vast theoretical work on the proposed schemes involving the stability issues is necessary for future studies.

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