TRANSFORMATIONS IN HIGH SCHOOL GEOMETRY: AN EXPERIMENTAL STUDY

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TRANSFORMATIONS IN HIGH SCHOOL GEOMETRY: AN EXPERIMENTAL STUDY

by

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ABSTRACT

The purpose of this study was to determine the feasibility of introducing a unit on transformations to the tenth-grade geometry program. To do this the experimenter considered five questions.

1. Can students attain competence with the mathematical concepts unique to the transformation approach?

2. How does student achievement on the transformation unit compare with student achievement on topics in the current geometry program?

3. What is the effect of the unit on student attitudes towards mathematics?

4. What are the attitudes of teachers towards the experimental materials?

5. Can the unit be taught effectively without special teacher training?

The study is essentially a non-comparative study in that no control group was used. The study consisted of presenting a fourteen lesson unit on transformations to 208 grade ten geometry students in six classes at Beaconsfield High School. The materials for the unit were taken from chapters four and five of a text by Coxford and Usiskin, <u>Geometry: A Transformation Approach</u>.

To determine the students' achievement on the unit, two tests were administered. The first was given at the end of the eighth lesson and the other was given at the end of the fourteenth lesson. Both of these tests were constructed by the experimenter and was designed to test whether the

behavioral objectives for the unit had been achieved. The Connelly Taxonomized Attitude Questionnaire was given as a pre-test, post-test to determine the effect of the experimental unit on student attitudes towards mathematics. To determine how the teachers felt towards the materials an experimenter-made questionnaire was given to each of the five teachers who taught the experimental materials.

A dependent t-test for means was performed on the average score for the experimental unit and the average score computed to represent the students' achievement on topics in the regular program of studies. A t-value of 10.717 indicated that there was a significant difference between the two sets of scores at the .05 level of significance. Since the mean for the experimental unit was higher than the mean for the regular topics it seemed reasonable to assume that students achieved considerably higher on the experimental unit.

A dependent t-test for means was also performed on the pre-test, post-test attitude scores. A t-value of .772 indicated that there was no significant change in the attitudes of students towards mathematics at the .10 level of significance during the teaching of the experimental materials.

The responses to the teacher questionnaire were analyzed and the results showed that teachers had favorable attitudes towards the materials. They recommended them highly to other teachers and they indicated that the materials were easy to read and most appropriate for the average student. They also indicated that the unit had value for both the terminating and continuing students in mathematics.

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TABLE OF CONTENTS

	Page
ABSTRACT	ii
ACKNOWLEDGEMENTS	iv
INTRODUCTION	1
Statement of the Problem	1
Need for and Significance of the Study	1
Scope and Limitations	2
Sample	3
REVIEW OF THE LITERATURE	4
Proposed Changes	5
Transformations: An Historical Background	7
Experimental Studies	10
PROCEDURE AND TEST INSTRUMENTS	14
ANALYSIS OF DATA	16
Student Achievement	16
Student Attitudes	17
Teacher Attitudes	18
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS	21
Summary	21
Conclusions	23
Recommendations	24
REFERENCES	25

APPENDICES

	Page
A. BEHAVIORAL OBJECTIVES	28
B. UNIT TESTS	31
C. STUDENT QUESTIONNAIRE	36
D. TEACHER QUESTIONNAIRE	38

INTRODUCTION

Statement of the Problem

This study was performed to determine the feasibility of introducing a unit on transformations to the tenth grade geometry program.

To do this the experimenter considered the following questions:

1. Can students attain competence with the mathematical concepts unique to the transformation approach?

2. How does student achievement on the transformation unit compare with student achievement on topics in the current geometry program?

3. What is the effect of the unit on student attitudes towards mathematics?

4. What are the attitudes of teachers towards the experimental materials?

5. Can the unit be taught effectively without special teacher training?

Need for and Significance of the Study

In attempts to reform the geometry curriculum in recent years, a number of groups and individuals have presented objectives for the teaching of geometry in high school. One objective which has been listed by several individuals and groups centers around the topic of transformations. Adler (1968) suggests that we include an introduction to the role of transformations of space, while Allendoerfer (1969) suggests that we provide an understanding of the basic facts about transformations. The Cambridge

Conference on School Mathematics in its 1963 report, Goals for School Mathematics, suggests that transformations be introduced early in the high school mathematics program. In addition, it recommends that experiments be conducted to determine the feasibility of teaching plane geometry through transformations. Several studies have been conducted in the United States in the area of transformations. However, it appears that no such studies have been conducted in Newfoundland or in other parts of Canada. Indeed, if we hope to introduce the topic of transformations to the geometry program in Newfoundland schools, there is a need to conduct studies to test the feasibility of such a venture.

The present study is considered to be significant to the extent that it assesses the feasibility of introducing transformations into the grade ten geometry program. The results could be useful to curriculum planning committees when planning future geometry programs and the study could serve as a basis for further studies using other population samples.

Scope and Limitations

This study was essentially a non-comparative study in that no control group was used. The study consisted of presenting a fourteen-lesson unit on transformations to 208 geometry students in six classes at Beaconsfield High School, St. John's in an attempt to answer the questions previously listed in the problem. Because of the newness of the experimental materials, it was necessary to seek volunteer teachers to use them. Four teachers other than the experimenter volunteered to teach the experimental materials to six classes in which they were teaching geometry. All but one of these teachers are mathematics majors. No teacher had previous experience with the teaching of transformations.

Sample

The sample consisted of six intact grade ten geometry classes at Beaconsfield High School, St. John's. The 208 students represent approximately 90 percent of the total grade ten population enrolled in the academic program during the year 1974-75. The sample included primarily students of average and above average mathematical ability. There was a small percentage of below average students. The classes were not randomly selected. The classes selected were those currently being taught by the teachers who volunteered to teach the experimental materials.

REVIEW OF THE LITERATURE

The geometry courses based on Euclid's axioms and postulates has been a topic of much discussion in recent years and many individuals suggest that it be replaced. Howard Fehr (1972) is one of those individuals who says that Euclid must go. He claims that the only reason for the survival of Euclid's geometry in past years was because it was the only subject available at the secondary school level to introduce students to an axiomatic development of mathematics. Fehr also points out that recent advances in algebra, probability theory and analysis have made it possible to use Euclidean topics in an elementary and simple manner to introduce axiomatic structure. In addition, much more important useful geometric knowledge has been developed in the last century that is not reflected in the current geometry course.

Many critics have attacked Euclid on the grounds that he made errors and that his work contained flaws. Meder (1958) points out that Euclid's development of geometry contains many serious omissions. For example, he failed to introduce any concept of order among points on a line and the concept of betweeness. Adler (1968) also points out this fault of Euclid, as well as the fact that Euclid failed to realize the need for undefined terms. Consequently Euclid was forced to use circular reasoning in proofs. An example of this is as follows: The proof of the isosceles triangle theorem depends upon bisecting the vertex angle. The isosceles triangle theorem is used to prove the SSS congruence which in turn is used to justify the angle bisection.

According to Felix Klein, as reported by Meder (1958), Euclid's intention was not to create a geometry to produce mathematicians, but philosophers. "Euclid intended," said Klein, "to write an introduction to mathematics in a manner that would serve acceptably as a preparation for general philosophical studies" (p. 578). To accomplish this, he pushed aside practical applications and placed emphasis on deduction and logic, not on geometric thinking. He wrote for scholars, not school children and it is unlikely that a discussion with such objectives can be made suitable for school use.

Proposed Changes

Mathematics teachers throughout the world have tried to overcome the inadequacies of the geometry courses based on Euclid's elements by introducing a variety of changes into the courses of study. Irving Adler (1968) listed ten such changes which are as follows:

1. The use of modified versions of the axioms introduced by David Hilbert to correct the defects in the logical structure of Euclid's elements.
2. The simultaneous development of plane and solid geometry.
3. The early introduction of metric ideas such as length of segments, angle measure and area of plane figures.
4. Reliance on the properties of the real number system, as proposed by Birkhoff.
5. The introduction of co-ordinate geometry.
6. The use of vector methods.
7. The use of those transformations of the plane called isometries that leave the distance between points unchanged.
8. The inclusion of some non-Euclidean geometry.
9. The development of Euclidean space as a vector space with an inner product as proposed by Dieudonné.
10. The development of the Euclidean plane as a co-ordinatized affine plane, with the real number system used as the set of

co-ordinates on a line and with a perpendicularity relation introduced in the plane. (p. 226).

Howard F. Fehr (1973), in the thirty-sixth yearbook of the National Council of Teachers, suggested that Euclid should be replaced with a study from many approaches. He stated that geometry should enter every year of the study of mathematics, and it should grow in depth and complexity until it becomes embedded in all the other areas of mathematical study -- linear algebra, analysis and applied mathematics -- until it becomes a way of thinking. The six approaches that he suggests are:

1. Physical informal -- working with drawings, paper folding, and apparatus to gain an intuitive feeling for geometric figures in Euclidean two- and three-space.

2. Synthetic - axiomatic -- considering the affine plane with a minimum of axioms and with finite as well as infinite models.

3. Co-ordinate - analytic -- assigning co-ordinates to the affine plane and subsequently to Euclidean two- and three-space and using the usual algebraic techniques.

4. Transformations -- studying mappings of the plane and three-space and eventually relating the study to group structure and the algebra of matrices.

5. Vectors -- examining the algebra of points in two- and three-affine space and introducing the inner product for the study of perpendicularity and Euclidean space.

6. Vector spaces and linear algebra -- building axiomatic or dimensional vector space and its linear algebra. (p. 378)

Each of these six approaches is to be made not as an isolated bit of study but in a spiral ascent in which one returns to each procedure at a higher level of abstraction. This approach, Fehr points out, is already being used in many schools in Europe, notably England, Belgium, and the Nordic countries.

Both Adler and Fehr include transformations in their proposals. Adler suggests the use of transformations of the plane. Fehr suggests that students study mappings of the plane and three-space and eventually relate this to group structures and the algebra of matrices. In addition to Adler and Fehr there are many other individuals who have supported the idea of inserting transformations into the high school geometry program; for example Max Jeger (1966), William Betz (1933), Kelley and Ladd (1965), Fischer and Hyden (1965), Jurgensen et al (1969), and Eccles (1972).

These individuals have stated many good reasons for introducing transformations into the high school, four of which are as follows:

1. Mathematically, transformations are convenient examples of 1-1 onto functions and hence lead easily to an elementary study of groups.
2. Certain transformations are useful in analysis.
3. Much of the study of advanced geometry is done with the aid of transformations.
4. Transformations provide unity to tenth grade geometry and the means by which the formal definitions of congruence similarity and symmetry are closely related to previous intuitive ideas.

Transformations: An Historical Background

It is customary, says Usiskin (1974), to think that transformations are new to geometry and new to secondary school texts. This is far from the truth, for there has existed a vast literature illustrating ways in which transformations can be used in elementary geometry.

After Euclid's elements, the first influential geometry in the Western World was written by the French mathematician Legendre (Usiskin, 1974). There is no use of transformations in the 'Elements of Euclid,' and Legendre did little other than to describe them. Since the texts of Euclid and Legendre formed the basis for geometry texts in the United States until the 1950's, transformations were not evident in most state texts. There were, however, two high school texts written in the United States before the 1930's using transformations. They were written by G. A. Hill (1887) and John W. Young (1915), but they were not widely used.

In Europe, the situation was different due to the work of Felix Klein (1939). In 1872, he discovered a way of characterizing all possible geometries using geometric transformations. This discovery had a threefold effect: (1) Geometries such as affine geometry became subjects of study in themselves; (2) Transformations became a fundamental concept in geometry -- the means by which geometries could be created and compared; and (3) Because Klein lived for over fifty years after the first announcement of his discovery, transformations became widely used in Germany and a few other European countries.

In France, G. Meray (1874) seems to have been the first to make use of transformations in an elementary geometry text written in 1874. Similar approaches were made by Jacques Hadamard (1906), by Philippe Andre (1908) and Bourlet (1928). In Germany, texts were written by Henrici and Treullein (1891), by W. Leitzmann (1923), and K. Fladt (1928).

In the 1960's, there were several tenth grade geometry texts published in the United States that included content relating to transformations, but few, if any, were devoted entirely to the transformational approach. Most texts included a chapter or two as part of their program. A text by Rosenberg and Johnson (1968), Geometry: A Dimensional Approach, was one such text that included about four pages of material on transformations. Another was a text by Paul Jergensen et al (1969), Modern School Mathematics: Geometry. He included a short end-of-the-book chapter.

Another which treated transformations more extensively was a text by Paul J. Kelley and Norman E. Ladd (1965), Geometry. A text by Moise and Downs (1964), Geometry, also included an end-of-the-book chapter on transformations.

In the 1960's in Europe, the programs of most countries included geometry based on geometric transformations. In the USSR, the program began at the age equivalent of seventh grade. In England, Denmark, France, West Germany, and Holland, transformations were considered important in the study of geometry (Usiskin, 1974). It seems reasonable to conclude that the impact of transformations on the geometry curriculum of the '60's was far greater in Europe than in the United States.

In the 1970's, several texts were published in the United States which treated transformations, but to my knowledge, only two of these used transformations as a central role. One text by Frank M. Eccles (1971), An Introduction to Transformational Geometry, was designed as a supplement to the regular course. The other text is by Coxford and Usiskin (1971), Geometry: A Transformation Approach, which was designed for the average student in grade ten. The most notable of these two is the text by Coxford and Usiskin. The first three chapters of this text follow the standard introduction to sets and real numbers, points, lines, planes, etc. Reflections are introduced in the fourth chapter. Chapter five deals with composition of reflections. Translations, rotations and glide-reflections are defined as products of reflections. Characterization of quadrilaterals by their lines of symmetry is given in chapter eight. Size transformations are defined in chapter thirteen and additional materials relating matrices and transformations are given in chapters twenty and twenty-one.

This text is being used in parts of the United States at present.

In Canada, the text is being used to a small degree in the province of Manitoba.

Experimental Studies

A number of studies have been carried out in the area of transformations. Several of these have been comparative experimental studies, but others have been theoretical or descriptive in nature and based on teacher or student opinions.

Probably the most notable and the most widely researched study was one by Usiskin (1970) titled, The Effects of Teaching Euclidean Geometry via Transformations on Student Achievement and Attitudes in Tenth Grade Geometry. The research conducted sought to study the effects of a transformation approach on achievement and attitudes of tenth grade students. In addition, informal observations were made regarding the amount of guidance needed by teachers in order to teach the experimental materials and regarding the attitudes of teachers towards the experimental materials. The conclusions drawn as a result of the study were that the average students can learn, and reasonably average teachers can teach a geometry course in which the mathematical development is based on transformations.

As a follow up to this study, Kort (1971) carried out research in tenth grade geometry which compared the effects of the transformations approach and a non-transformation approach on retention of geometry and on transfer in eleventh grade mathematics. Both the transformation and the non-transformation approach geometry were essentially Euclidean. One hundred and eighty-four students from a large urban high school participated in the study. The major conclusion that he made was that tenth grade geometry can be taught via transformations with possible advantages for retention of geometry and transfer in eleventh grade mathematics. The primary implication was that tenth grade geometry should be changed to extensively utilize transformations only if subsequent mathematics courses are altered to capitalize on a background in geometric transformations.

Jerome H. Solheim (1971) performed a study to determine the effects of transformations of the plane on the attitudes of secondary school geometry students. The experimental group consisted of 56 high school students in three high schools, while the control group consisted of 107 high school students in six high schools. Both groups were taught for a period of five weeks. Solheim found that there was no significant change in the attitude of the control group during the experimental period. The unit on transformations did, however, have a significant adverse affect upon the mean attitude on the experimental group. Results of a questionnaire completed by the teachers of the experimental unit indicated that it would have been desirable to have more completely integrated the content of the unit into the content of the regular course. Also, informal observations indicated that both groups showed approximately the same level of achievement.

Alton Thorpe Olson (1970) developed a unit in plane geometry through transformations and used it to test the feasibility of presenting such materials to high school sophomores. The unit consisted of thirty lessons and included topics on parallelism, perpendicularity and congruent triangles. Other topics which revealed the algebraic structure of the transformations were also included. The unit was taught in eight intact classes in three southwestern Wisconsin high schools. Olson found that the total population did not measure up to the criterion level he had set (70 percent of the students getting 50 percent or better), but the top I.Q.

quartiles did reach the criterion level as did the group defined by A or B in previous mathematics classes. Generally, the high ability students enjoyed the challenge while the lower ability students did not. Olson therefore concluded that the materials were most appropriate for high ability students.

Clifford Tremblay (1972) developed a unit of motion geometry for the junior high level. He developed the unit subjecting it to successive classroom trials, revising the materials as indicated by the trials and then he wrote a final version of the unit. Tests based on the objectives of the unit were administered to the students in three classroom trials of the unit. The low error rates on the tests indicated that the students on these trials had acquired to a satisfactory extent, the understanding developed in the unit.

Michael J. Hoban (1959) performed a non-comparative study of a unit on transformations in the seventh grade. The unit was developed for talented seventh graders by the Secondary School Mathematics Curriculum Improvement Study Group (SSMCIS) which is directed by Howard Fehr. The main purpose of the study was to determine whether the unit was appropriate for the chosen population of 240 students in nine classes at suburban schools in the New York Metropolitan area. Six of the nine classes mastered each of the cognitive objectives set and therefore were considered to have mastered the material in the unit. Hoban concluded then that the material was quite appropriate for his chosen population.

One last study worthy of note is a thesis by Thomas W. Shilgalis (1971) who did a critical analysis of the transformation approach. His study was theoretical in the sense that no experiment was involved. Shilgalis advocated the transformation approach to geometry. He claimed that transformations capitalize on and expand the students' understanding of the function concept and also helps students to develop a helpful pattern of attack for solving problems or doing proofs. He said that new topics such as invariance, symmetry, matrices and groups arise naturally in a study of transformations. He believes that transformations should assume a central rather than a peripheral role and feels that teachers of the material must know the material well before attempting to teach it.

From the above studies, one could not say with certainty that the transformation approach is better than the traditional approach. Indications are that the transformation approach is as "good" as the traditional one, but then one cannot make valid judgment based on such a small amount of research. The fact that only three of these studies mentioned above were comparative research studies indicates the need for more research in this area. The comments and observations of the other individuals suggest that transformations have a place in the curriculum, is teachable, and can be taught by teachers without much further training.

PROCEDURE AND TEST INSTRUMENTS

The unit on transformations taught to these six classes consisted of two parts: (1) Reflections, and (2) Transformations. The experimental materials are found in chapters four and five of the text, Geometry: A Transformation Approach by Coxford and Usiskin (1971). The sections used were 4.1, 4.2, 4.3, 4.5, 4.7, 5.1, 5.2, and 5.3. The materials from these eight sections were used to form a fourteen-lesson unit which was taught over a period of approximately four weeks by the four volunteer teachers and the experimenter. Below is a list of the topics and the suggested time allocation for each topic.

1. Reflection Defined (2 periods)
2. Reflecting Sets of Points (1 period)
3. Reflecting Collinear Sets of Points (2 periods)
4. Orientation and Reflection of Polygons (2 periods)
5. Notation for Reflections (1 period)
6. What is a Transformation? (2 periods)
7. Composites of Transformations (2 periods)
8. Rotations (2 periods)

In addition to the fourteen periods allocated for the lessons, two periods were also used to administer two achievement tests. The first of these tests was given at the end of the eighth lesson and the other was given at the end of the fourteenth lesson. Both students and teachers were supplied with a copy of the materials. The teachers were also supplied with a solution manual and guide.

To answer the first question stated previously in the problem, "Can students attain competence with the concepts unique to the transformation approach?", the experimenter first constructed a set of behavioral objectives for the experimental unit (See Appendix A). Then he constructed two achievement tests, designed to test whether or not these objectives had been reached.

To answer the second question, "How does student achievement on the transformation unit compare with student achievement on topics in the current geometry program?", a dependent t-test was performed on two variables. The first variable was an average grade for each student for the regular geometry topics completed prior to the study. The second variable was a grade found by averaging the scores on the two achievement tests on transformations.

To answer the third question, "What is the effect of the experimental unit on students' attitudes towards mathematics?", the experimenter administered as a pre-test, post-test, the Connelly Taxonomized Attitude Scale using Objective II items of that scale (See Appendix C).

To answer the fourth question, "What are the attitudes of teachers toward the experimental materials?", an experimenter-made questionnaire was given to the teachers at the end of the experimental period.

To answer question five, "Can the unit be taught effectively without special teacher training?", the experimenter considered the results of the above tests, and questionnaires. In addition, informal observations and comments of the teachers were used.

ANALYSIS OF DATA

Student Achievement

The two tests used to measure the achievement of students on the experimental materials were constructed by the experimenter. Each item on the test was designed to test whether the behavioral objectives as stated in Appendix A had been met. Points were assigned to each item on each test with the total possible score on each being 100 points. After both tests were scored, an average score was found for each student. This score represented the student's achievement on the transformation unit.

The scores on the experimental tests indicate that the students achieved well with respect to the mathematical concepts unique to transformations. A mean grade of 82.918 out of 100 indicates that the students attained a high degree of competence with the experimental materials.

For each student who participated in the study a score was obtained to represent the student's achievement on the topics in the regular geometry course completed prior to the study. Each score was found by averaging the achievement scores on the four topics: Congruence, Similarity, Quadrilaterals and Parallel Lines, and the Right Triangle.

A dependent t-test for the means was performed on the scores for the regular topics and the scores for the experimental unit. A t-value of 10.717 indicated that there was a significant difference between the two sets of scores at the .05 level of significance. Since the mean for the experimental scores was greater than the mean for the regular topics, it seems reasonable to conclude that students achieved considerably higher on the experimental materials. This suggests an affirmative answer to question two previously listed in the problem.

Student Attitudes

The opinionnaire used to measure attitudes was the Connelly Taxonomized Attitude Scale designed by Dr. R. Connelly of Memorial University of Newfoundland. The reliability coefficient for the opinionnaire is .87. There are sixteen items on this instrument, each with five possible responses: strongly agree, agree, no opinion, disagree, strongly disagree. Items 1-6 were negatively stated and items 7-16 were positively stated.

The responses were scored as follows:

	Items 1-6	Items 7-16
Strongly agree	1	5
Agree	2	4
No opinion	3	3
Disagree	4	2
Strongly disagree	5	1

The highest possible score was 80 indicating a most positive attitude towards mathematics. The lowest possible score was 16 indicating a most negative attitude towards mathematics. A score of 48 would be considered neutral.

The opinionnaire was administered twice (before and after the teaching of the experimental materials). Students' scores were included if and only if the student completed the opinionnaire at both times.

A dependent t-test for means was performed on the set of difference scores from these opinionnaires. A t-value of 0.772 indicated that there was no significant change in attitudes towards mathematics at the .10 level of significance, during the period in which the experimental materials were taught.

Teacher Attitudes

A short questionnaire form of ten items was given to each teacher who taught the experimental materials. On the form teachers were asked to indicate their opinions regarding certain aspects of the experimental materials. They were also asked questions as to whether they used the teacher's guide and solution manual and whether they consulted other teachers. Also included were questions which asked their opinion of the value of the materials to students who would terminate their study of mathematics at the end of high school, and to students who would continue post-secondary studies in mathematics. A copy of the questionnaire appears in Appendix D.

Responses to question one indicated that four teachers thought the materials were appropriate for the average student while one teacher indicated that the below average student could learn the materials reasonably well. In question two, three teachers indicated that the materials were easier to read than other geometry texts while two teachers indicated that the materials were at the same reading level.

One teacher enjoyed teaching the materials better than other geometry topics and four said they enjoyed teaching them as well as the other topics. Three teachers said they would recommend the materials to other teachers and two said they would highly recommend the materials.

Responses to question five indicated that teachers had positive attitudes toward the materials. All teachers indicated that prior to teaching they did not think they would have any trouble with the mathematics of the unit. The responses to questions 6-8 indicated that two teachers used the teachers manual and solution key for each section while three teachers indicated they seldom used it. No teachers indicated that they spent more time preparing the lessons than for other topics. Two teachers said that less time was required while three teachers said that about the same time was required.

All teachers thought that the materials had value for both the terminal student in mathematics and the continuing student. In questions 9-10 four teachers indicated that it would definitely be advantageous for students who will pursue courses in post-secondary mathematics to study transformations while one teacher said maybe. Three teachers felt that the materials definitely had value for the terminal student while two teachers indicated maybe.

In the section for other comments one teacher said that the students enjoyed learning the materials. He felt that as a result interest was revived in his class in the regular geometry topics. The department head who was one of the experimental teachers indicated that he was quite pleased with the materials and that if time permits he plans to include the unit in the geometry program next year.

From the analysis of the responses to the teacher questionnaire several generalizations can be made: (1) The teachers enjoyed teaching the materials as well as the regular topics; (2) The teachers would recommend the materials to other geometry teachers; (3) The materials are easily read and are most appropriate for the average students; (4) No more time is required to prepare the lessons on transformations than to prepare lessons on regular topics; and (5) The materials have some value for both the terminating and continuing student in mathematics.

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

The purpose of this study was to determine the feasibility of introducing a unit on transformations to the tenth grade geometry program. To do this the experimenter considered five questions.

1. Can students attain competence with the mathematical concepts unique to the transformation approach?

2. How does student achievement on the transformation unit compare with student achievement on topics in the current geometry program?

3. What is the effect of the unit on student attitudes towards mathematics?

4. What are the attitudes of teachers towards the experimental materials?

5. Can the unit be taught effectively without special teacher training?

The study is essentially a non-comparative study in that no control group was used. The study consisted of presenting a fourteen-lesson unit on transformations to 208 grade ten geometry students in six classes at Beaconsfield High School. The materials for the unit were taken from chapters four and five of a text by Coxford and Usiskin, Geometry: A Transformation Approach.

To determine the students' achievement on the unit, two tests were administered. The first was given at the end of the eighth lesson and the other was given at the end of the fourteenth lesson. Both of these tests were constructed by the experimenter and were designed to test whether the behavioral objectives for the unit had been achieved. The Connelly Taxonomized Attitude Questionnaire was given as a pre-test, post-test to determine the effect of the experimental unit on student attitudes towards mathematics. To determine how the teachers felt towards the materials an experimenter-made questionnaire was given to each of the five teachers who taught the experimental materials.

A dependent t-test for means was performed on the average score for the experimental unit and the average score computed to represent the students' achievement on topics in the regular program of studies. A t-value of 10.717 indicated that there was a significant difference between the two sets of scores at the .05 level of significance. Since the mean for the experimental unit was higher than the mean for the regular topics it seemed reasonable to assume that students achieved considerably higher on the experimental unit.

A dependent t-test for means was also performed on the pre-test, post-test attitude scores. A t-value of .772 indicated that there was no significant change in the attitudes of students towards mathematics at the .10 level of significance during the teaching of the experimental materials.

The responses to the teacher questionnaire were analyzed and the results showed that teachers had favorable attitudes towards the materials. They recommended them highly to other teachers and they indicated that the materials were easy to read and most appropriate for the average student. They also indicated that the unit had value for both the terminating and continuing students in mathematics.

Conclusions

The conclusions in this report are made relative to the questions asked in the "Problem" section of the report.

Question 1. Can students attain competence with the mathematical concepts unique to the transformation approach?

Conclusion: The mean score of 82.918 on the achievement tests indicates that students can attain competence with these concepts.

Question 2. How does student achievement on the transformation unit compare with student achievement on topics in the current geometry program?

Conclusion: When a dependent t-test was performed on achievement scores associated with the regular program and the experimental unit, a t-value of 10.717 was found. We can conclude therefore that students achieved significantly higher on the experimental unit.

Question 3. What is the effect of the unit on student attitudes towards mathematics?

Conclusion: A t-value for means of .772 showed that there was no significant change in attitude towards mathematics during the experimental period.

Question 4. What are the attitudes of teachers towards the experimental materials?

Conclusion: The teachers indicated that they favored the materials. They recommended highly the materials to other teachers and indicated that the materials were easily read. The materials are most appropriate for the average student.

Question 5. Can the unit be taught effectively without special teacher training?

Conclusion: We can conclude very confidently that teachers need no special training. The fact that students achieved well and even significantly higher than on regular topics confirms this conclusion. The analysis of the teacher opinionnaire also gives evidence to support the conclusion.

Recommendations

From the above study we have concluded that concepts unique to the transformation approach can be taught to average students by teachers who have had no special training in the area of transformations. No attempt was made to teach applications of the concepts to congruence or similarity since the time budgeted to the experimenter at school did not permit its inclusion in the experimental unit.

To anyone who decides to include a unit on transformations in the grade ten geometry program the experimenter would recommend that you consult the text by Coxford and Usiskin, Geometry; A Transformation Approach. The experimenter would also recommend that in addition to the sections from that text used in this study, that sections on application to congruence and similarity be included.

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APPENDIX A

BEHAVIORAL OBJECTIVES

Objectives Section 1.1

1. Given a point and the reflecting line the student should be able to locate the reflection image of the point over the line using either a compass or a protractor.

2. Given a point and its reflection image the student should be able to construct the reflecting line using straight edge and compass.

3. Given the co-ordinates of a point and the co-ordinates of its image, the student should be able to graph the points and locate the reflecting line.

4. Given an equation of a line and the co-ordinates of a point the student should be able to graph the line and the point and locate the image of the point and name the co-ordinates of the image of the point over the line.

Objectives Section 1.2

1. The student should be able to explain generally, how to find the reflection image of a set of points.

2. When asked the question, "Can a point have more than one reflection image," the student should be able to answer correctly and state the postulate which supports his answer.

3. Given a set of points and a line, the student should be able to sketch the reflection image of the set of points over that line.

Objectives Section 1.3

1. Given a segment, line, or ray, the student should be able to draw the reflection image of each set of points in a line.

2. The student should be able to identify from a list those properties which are preserved by reflection.

3. The student should be able to explain the statement: "Reflections preserve collinearity, betweeness, and distance."

Objectives Section 1.4

1. When given a polygon and a reflecting line, the student should be able to draw the image of the polygon over the line.

2. The student should be able to explain how the orientation of a polygon and the orientation of its reflection image are related.

3. Given a polygon with the vertices named, the student should be able to state whether the polygon and its reflection image are clockwise or counterclockwise oriented.

Objectives Section 1.5

1. The student should be able to interpret the notation for reflections and apply it to find the reflection images of points and sets of points (including lines, rays, segments, and polygons).

Objectives Section 2.1

1. The student should be able to recall the definition of a transformation.

2. The student should be able to state at least one reason why transformations are important.

3. Given diagrams of images and preimages the student should be able to recognize if the transformations preserve collinearity, orientation, distance, and angle measure.

Objectives Section 2.2

1. The student should be able to interpret the notation for composition of reflections.

2. The students should be able to define a translation.

3. Given a diagram of a figure and two parallel lines the student should be able to draw the image of the figure under a translation.

Objectives Section 2.3

1. The student should be able to define a rotation in terms of a composite of reflections.

2. Given a point and the magnitude of a rotation, the student should be able to locate the image of the point under the given rotation.

3. Given a diagram of a figure and two non-parallel lines, the student should be able to draw the image of the figure under a rotation.

4. Given a figure and the image of the figure under a rotation and the center of the rotation, the student should be able to determine the angle of rotation.

5. The student should be able to define a half-turn.

6. The student should be able to determine which properties are preserved.

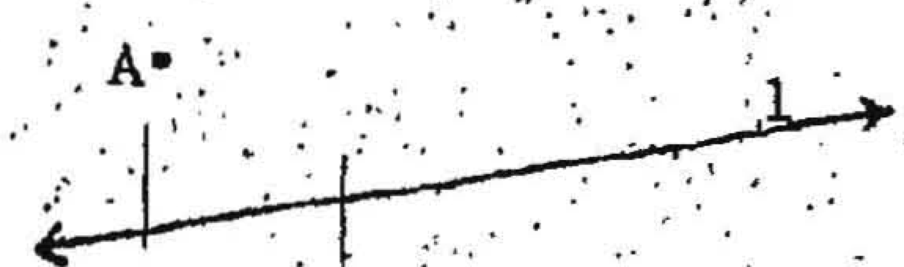
APPENDIX B

UNIT TESTS

Unit Test — Chapter 1

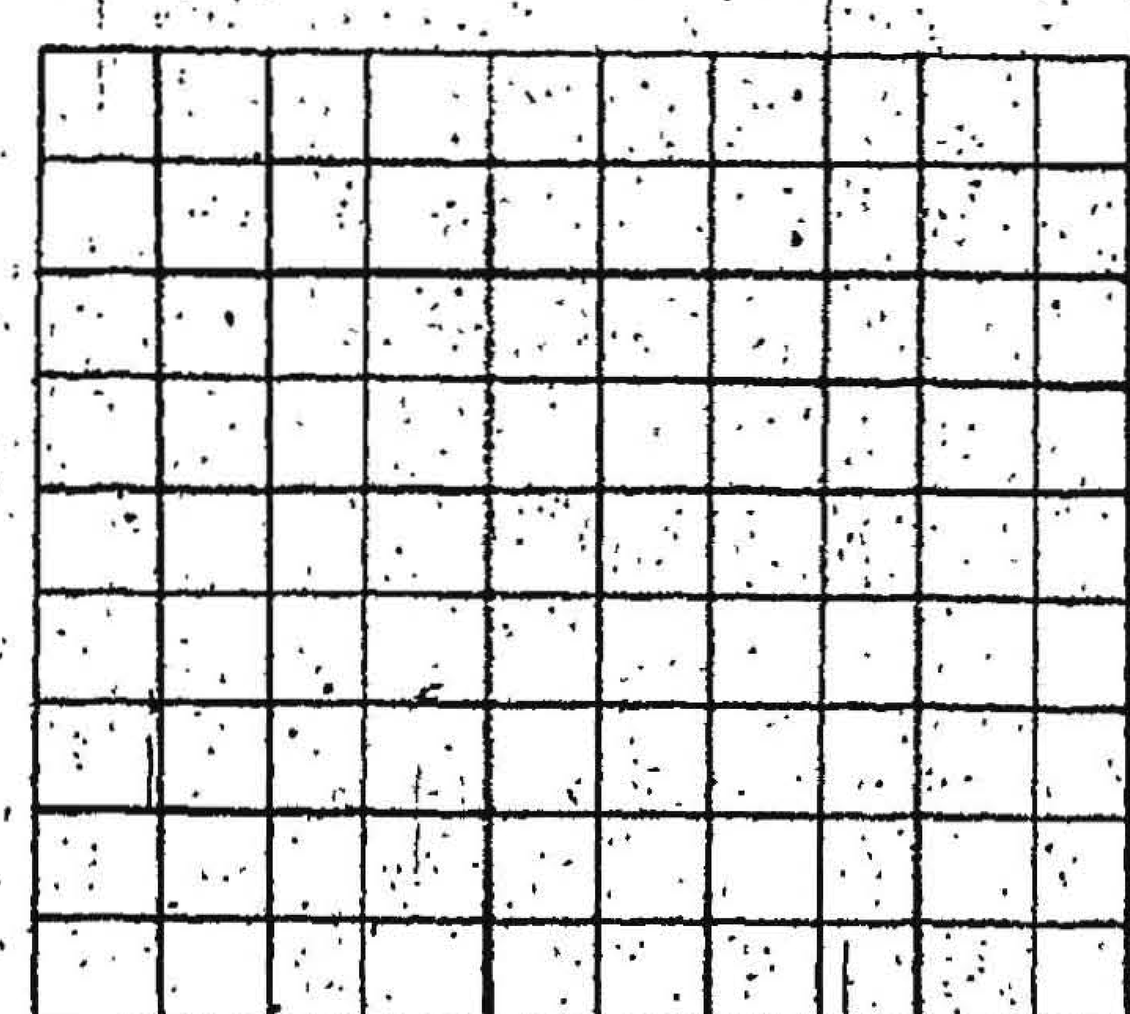
Value   Students are required to answer ALL questions. You must answer the questions in the space provided.

(10) 1. Given the point A and the reflecting line l, locate the image of point A over the line l, using either a compass or a protractor.



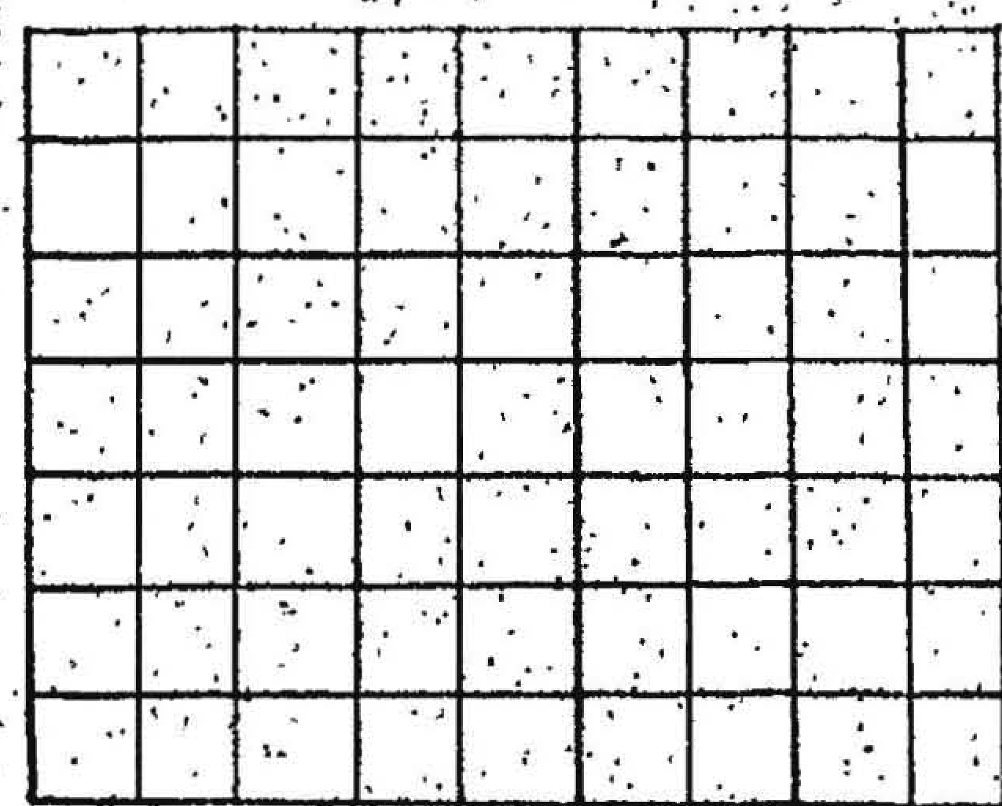
(10) 2. Find the reflecting line so that the reflection image of point A is B.

(10) 3. Graph the two points whose co-ordinates are A(4,1) and B($^{-}3$,0) and locate the reflecting line.



Value

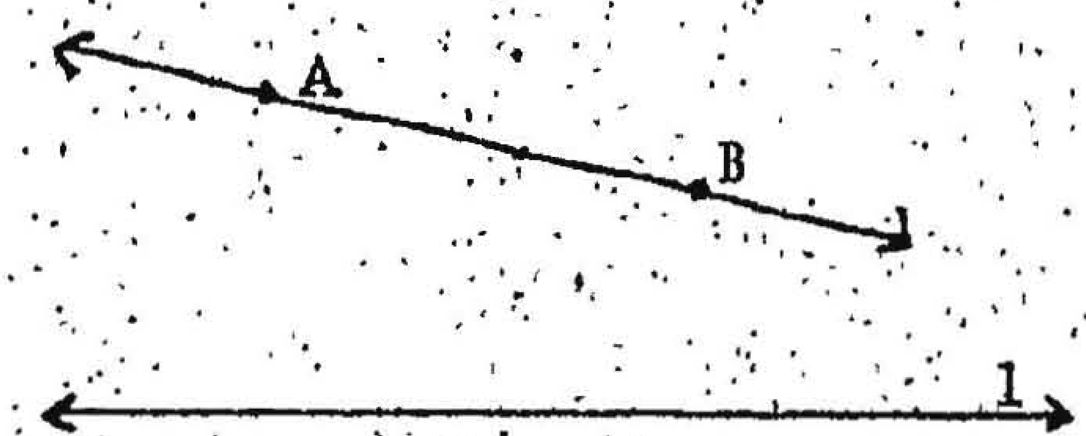
(10) 4. Graph the line described by the equation $x=3$ and the point whose co-ordinates are $(^-3,4)$ and locate the image of the point and name the co-ordinates.



(6) 5. Explain how you can find the reflection image of a set of points.

(6) 6. Can a point have more than one reflection image? State the postulate which supports your answer.

(10) 7. In the following drawing, find the reflection image of line AB in line 1.



(8) 8. Which of the following properties are preserved by any reflection?

(a) angle measure ______

(b) betweenness ______

(c) distance ______

(d) orientation ______

(e) none of these ______

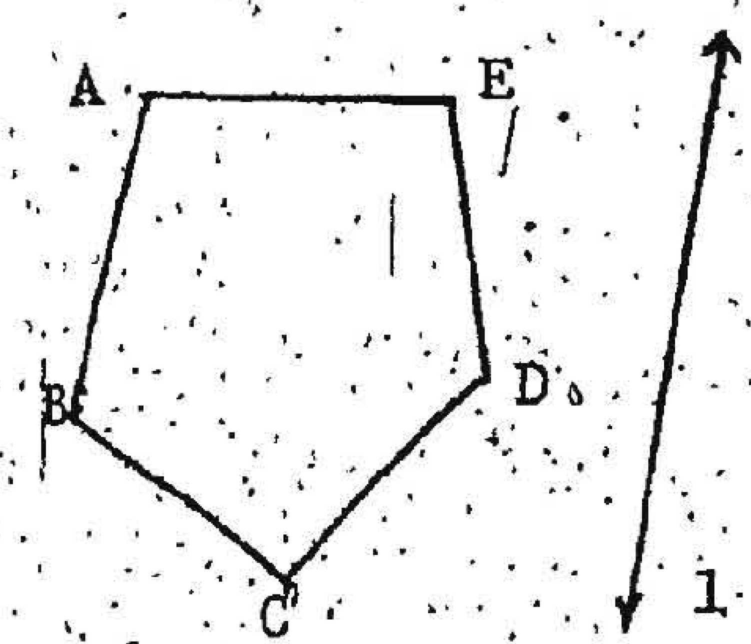
(6) 9. Explain what is meant by the statement, "Reflections preserve collinearity."

____

____

____

(12) 10. In the following find r(ABCDE) over l.



(6) 11. Explain how the orientation of a polygon and the orientation of its reflection image are related.

____

____

____

____

(6) 12. True or False.

_______ (a) In question 10 above, polygon ABCDE is clockwise oriented.

_______ (b) In question 10 above, the reflection image A'B'C'D'E' is clockwise oriented.

Unit Test -- Chapter 2

Value Students are required to answer ALL questions.

(20) 1. Define briefly the following terms:

(a) transformation ______________________

(b) translation ______________________

(c) rotation ______________________

(d) half-turn ______________________

(6) 2. State one reason why transformations called reflections are important.

____

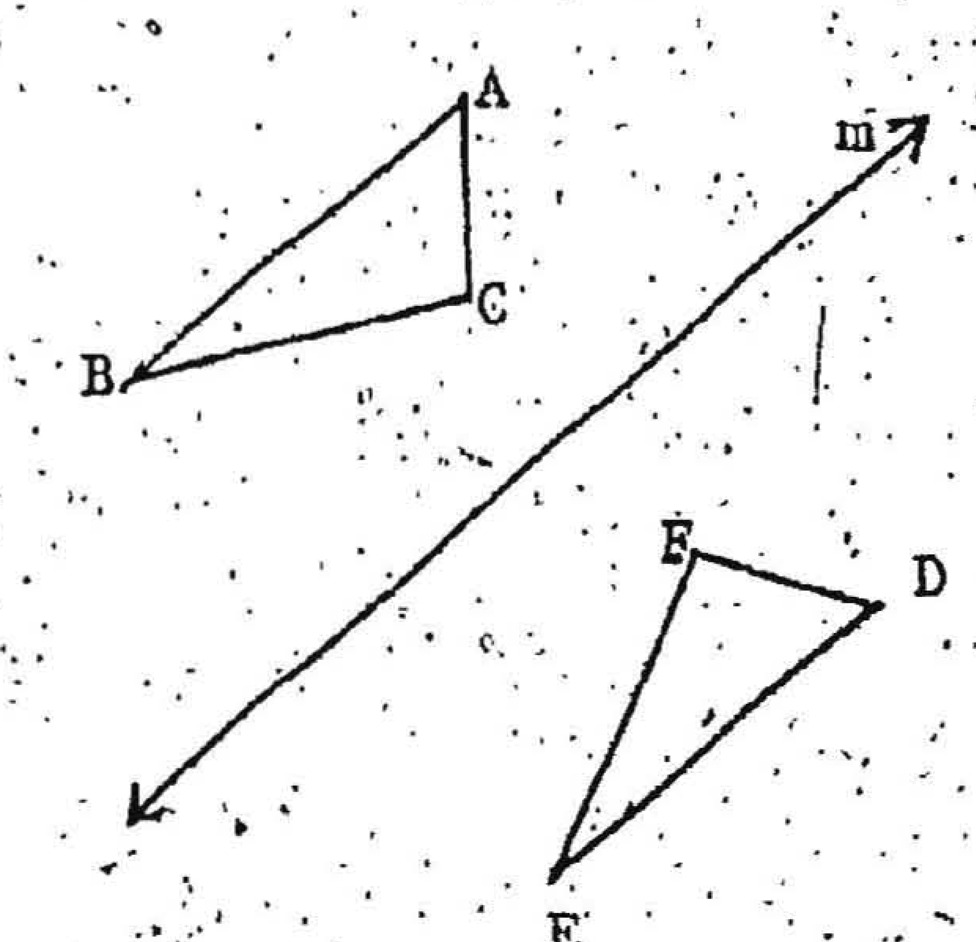
(12) 3. In the diagram at the right, which of the following properties are preserved by the transformation?

(a) collinearity ________

(b) orientation ________

(c) distance ________

(d) angle measure ________

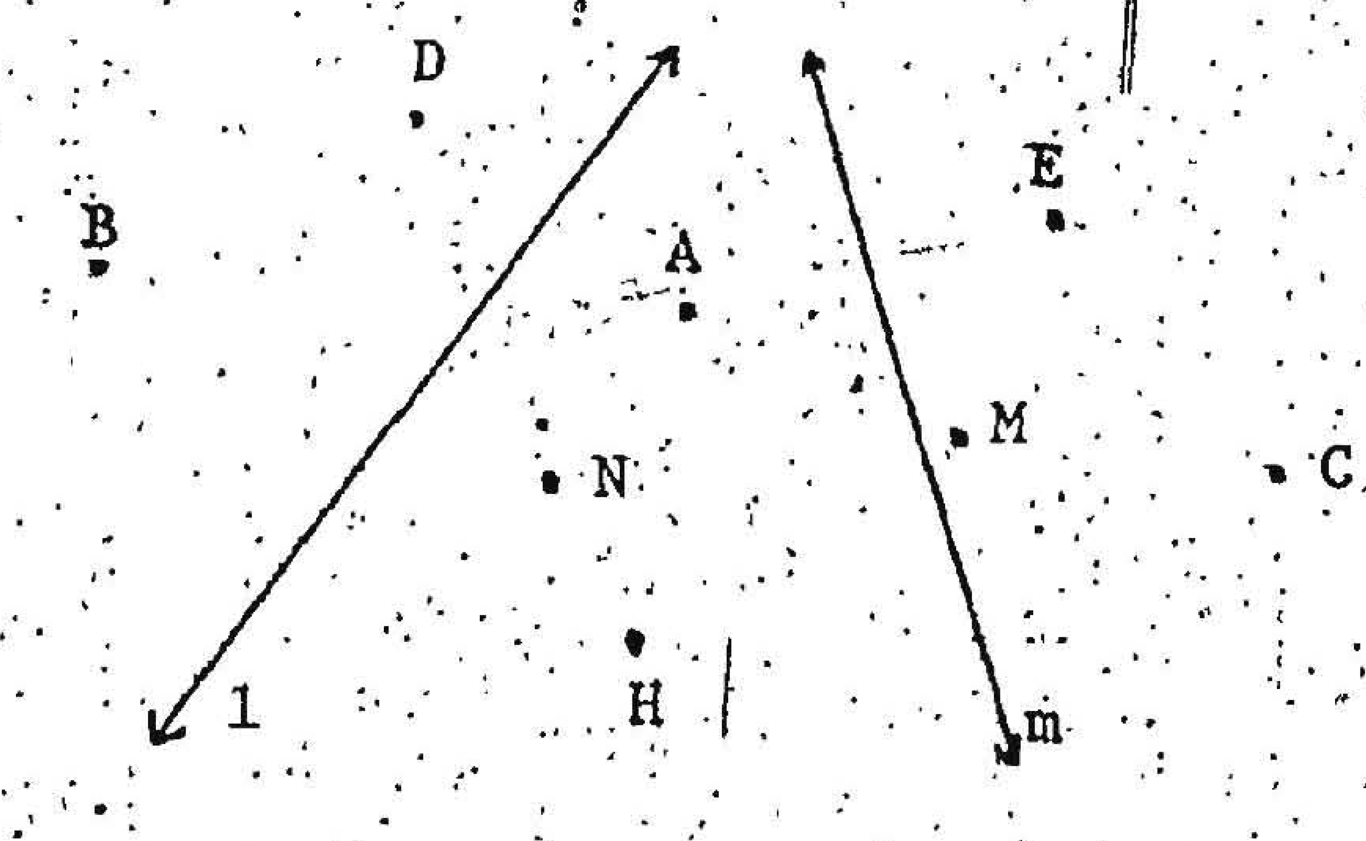


(10) 4. In the diagram at the right, find

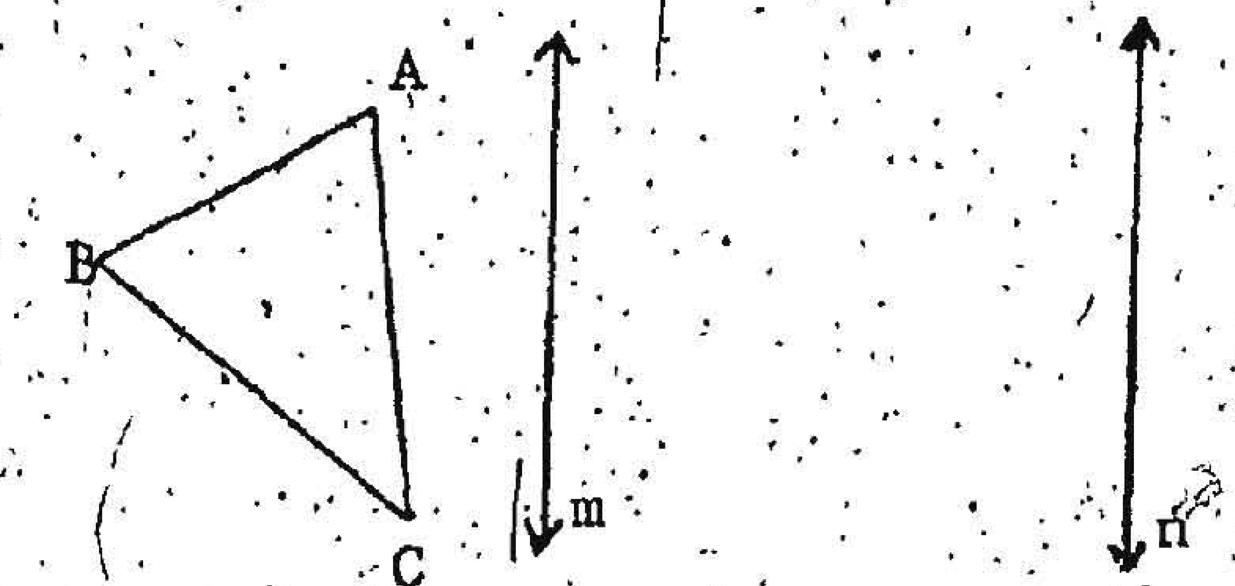
(a) $r_1(B)$ ________

(b) $r_m(r_1(B))$ ________

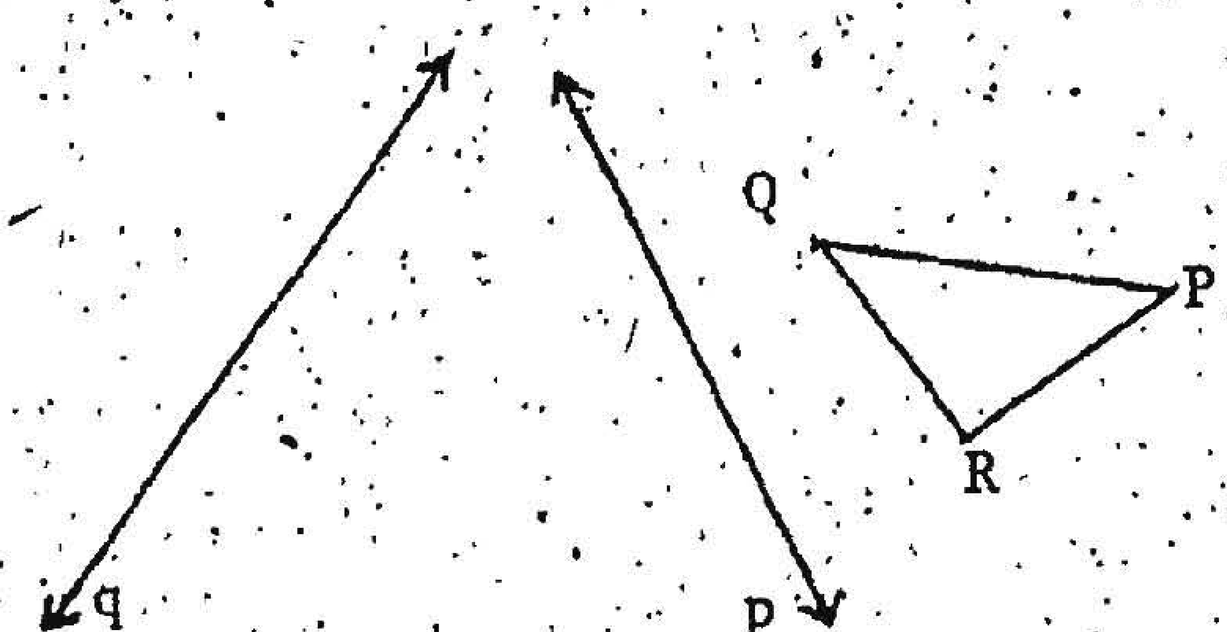
(c) $r_m \circ r_1(D)$ ________



(16) 5. Draw the image of $\triangle ABC$ under the translation $r_n \circ r_m(\triangle ABC)$



(16) 6. In the figure below, draw the image of $\triangle PQR$ under the rotation $r_q \circ r_p(\triangle PQR)$



(8) 7. What is the magnitude of the rotation in the following diagram?

X • Center

(12) 8. Name the properties that are preserved by rotations.

____

____

____

APPENDIX C

STUDENT QUESTIONNAIRE

Name: ____________________

Class: ____________________

In the space provided write your name and class. This is not a test and will not be used in any way to produce a grade for you. The items on this instrument are statements about mathematics. For each item select a response which best describes your impression of the statement and place your response in the space provided at the left. The response choices are:

A -- strongly agree

B -- agree

C -- no opinion

D -- disagree

E -- strongly disagree

_____ 1. I have nothing but contempt for mathematics.

_____ 2. I regard mathematics as a lasting tribute to man's ignorance.

_____ 3. I feel under great strain in a mathematics class.

_____ 4. Mathematics makes me feel as though I'm lost in a jungle.

_____ 5. Mathematics makes me feel uncomfortable.

_____ 6. Mathematics is mainly pencil pushing.

_____ 7. The very existance of humanity depends on mathematics.

_____ 8. Mathematics may be compared to a great tree, ever putting forth new branches.

_____ 9. Mathematics is a subject which I have enjoyed studying in school.

_____ 10. I feel mathematics is the greatest means for increasing world's knowledge.

_____ 11. Mathematics is stimulating to me.

_____ 12. Working with various mathematical topics is fun.

_____ 13. I see nothing wrong with learning a variety of mathematical topics.

_____ 14. I feel mathematics makes other subjects easier to understand.

_____ 15. Mathematics fascinates me.

_____ 16. My general attitude towards mathematics is favorable.

APPENDIX D

TEACHER QUESTIONNAIRE

1. In general, I feel that the material is written for

   _____ (a) the above average geometry student

   _____ (b) the average geometry student

   _____ (c) the below average geometry student

   in my school.

2. Compared with other mathematics textbooks I have taught, this material is

   _____ (a) easier to read and understand

   _____ (b) at about the same level

   _____ (c) harder to read and understand

3. Compared with other topics in the prescribed textbook, I enjoyed teaching the experimental materials

   _____ (a) less than

   _____ (b) as well as

   _____ (c) better than

4. I

   _____ (a) would strongly recommend

   _____ (b) would recommend

   _____ (c) am indifferent to

   _____ (d) would not recommend

   this material to other geometry teachers.

5. When you first began to teach this material did you feel you would have troubles with the mathematics?

_____ (a) definitely

_____ (b) somewhat

_____ (c) not really

6. How often did you use the teacher's manual and solution key?

_____ (a) for each section

_____ (b) often

_____ (c) seldom

_____ (d) never

7. How often did you have to consult with other teachers because you had trouble understanding the materials yourself?

_____ (a) often

_____ (b) sometimes

_____ (c) seldom

_____ (d) never

8. Compared with other new topics that you have taught, how much time was required to prepare the lessons?

_____ (a) less time

_____ (b) about the same

_____ (c) more time

_____ (d) considerably more

9. Do you feel that it would be advantageous for students who will pursue courses in post secondary mathematics to study transformations?

_____ (a) not at all

_____ (b) maybe

_____ (c) yes, definitely

10. Do you feel that the materials would have any value for students who will terminate their study of mathematics at the end of high school.

_____ (a) not at all

_____ (b) maybe

_____ (c) yes, definitely

OTHER REMARKS: