A STUDY INVESTIGATING TWO HIERARCHICAL MODELS RELATING TO THE ATTAINMENT OF THE CONCEPT OF MOMENTUM

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BY

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ABSTRACT

The present study is concerned with a theoretical and empiracal examination and extension of an earlier study by Raven (1967). Raven attempted to determine whether the concept of momentum followed a logical sequence, as determined from a logical analysis of the momentum concept, or a psychological sequence, as suggested by Piagetian research. The objections to Raven's study are threefold. Firstly, Raven based his hierarchy on only one task for each step in the hierarchy, with the exception of the concept of momentum where two tasks are used. Secondly, it is suggested that some of Raven's tasks do not test what is purported. Thirdly, Raven's results imply the superiority of a psychological as opposed to a logical model in the development of the momentum concept. It will be argued that Raven's logical hierarchy was inadequately developed and hence does not allow a meaningful conclusion, either with respect to the specific concept or to the relationship between psychological and logical hierarchies.

The purpose of this study is to investigate Raven's (1967) claim that the development of the concept of momentum by young children follows a psychological rather than a logical progression. The psychological hierarchy involved is that used by Raven. The logical hierarchy is different from Raven's because of the investigator's belief that Raven's logical hierarchy was inadequately constructed, thereby biasing his findings.

Data were obtained by the group-testing of 197 subjects from grades one to eight in a St. John's school. Two statistical tests, namely the White and Clark 'test of inclusion' and the 'ordering-theoretic' method, were used to analyse the data. The results of this analysis do not support Raven's contention that young children develop an understanding of the concept of momentum in accordance with a psychologically derived hierarchy, rather than a logical hierarchy. Instead, with little change, a logical hierarchy hypothesized by the present investigator is substantiated.

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THE PROBLEM

Introduction to the Problem

Although curriculum developers generally agree that there is a definite need to establish appropriate sequencing for the development of content there is, in fact, little agreement regarding how content should be sequenced. Sequel (1966), in her study of curriculum development over a span of roughly fifty years from 1890 to 1940, notes that curriculum development was not recognized as a specialist field until about 1940, and that it was during this period that educators began to realize the haphazard fashion in which subject-matter content was being compiled. Because of an increase in knowledge and the demands placed on text-book writers by the public, the subject specialities were becoming overcrowded and disjointed and displayed little organization or sequence. According to Sequel, the general course of thought and the nature and value of the curriculum-making process that eventually led to the recognition of curriculum development as a specialist field evolved from the contributions made by such influential educators as Charles and Frank McMurray, Franklin Bobbitt, Werrett Wallace Charter, Harold Rugg and Hollis Caldwell, each of whom enjoyed periods of influence fairly evenly distributed over this time period. Perhaps, however, the most influential of all in its impact on the sequencing of content was the work of John Dewey (1916). Dewey's five-step "scientific method" did much to influence the structure of science courses and to describe the processes of science during the early part of this century. More recently, other educators such as Tyler (1950), Bruner (1960), Taba (1962), Ausubel (1964), Schwab (1966) and Gagne (1977) have influenced the curriculum field in an effort to make its contents more meaningful.

Posner and Strike (1976), in dealing with methods of sequencing content, claim that before one can answer the prescriptive question "how should content be sequenced?" one must first find the answer to the descriptive question "how can content be sequenced?" There are many possible alternatives in the sequencing of content material such as concepts or skills. Posner and Strike propose a framework within which these sequencing alternatives and their implications for education can be discussed. Five distinct categories of sequencing principles, namely world-related, concept-related, learning-related, inquiry-related and utilization-related, each with a number of sub-categories, are suggested. The present study is concerned with some aspects of the learning-related category.

Shulman and Tamir (1973), in their review of science education of the previous decade, describe as "revolutionary" the changes and developments that have occurred in this field during that period. Contributions from psychologists, scientists, science educators and teachers have resulted in a greater diversity of science programs, differing in scope, content and structure. Perhaps most significant of all in their impact on science curricula and the learner are the influences of the major learning theorists whose theories serve as the basis of many of the science curricula, particularly at the elementary level. Robert

Gagne and Jean Piaget are perhaps the most evident in their influence on the structure of science programs. Two major American elementary school science curricula, Science--A Process Approach (S-APA) and the Science Curriculum Improvement Study (SCIS), have been influenced by the learning hierarchy model of Gagne and by Piaget's stage development theory, respectively. Despite this, few researchers have attempted to compare the effectiveness of these two models as an aid to facilitating concept development in science. A study by Raven (1967) involved a comparison of these two models with respect to a particular science concept, the concept of momentum. The present study involves a theoretical and empirical replication and extension of Raven's study.

Learning Hierarchies

Research into learning hierarchies began in 1962 when Gagne attempted to teach seven children how to find formulas for sums of terms in number series. He derived a network of elements which he called a hierarchy of knowledge by asking the question "What would the individual have to be able to do in order that he can attain successful performance on this task, provided he is given only instructions?" The results of this study led Gagne to believe that if valid learning hierarchies represented the sequence in which skills are learned then they would be valuable tools in the construction of learning programs leading to the acquisition of problem-solving skills and knowledge in general (Gagne, 1965).

Since that time when the term "hierarchy" was first used to describe a theory of learning there has been a continuing interest in the application of Gagne's hierarchy theory to problems in instruction and evaluation and, spontaneously but independently, by psychologists in studying sequences of cognitive and psychological development (Resnick, 1973). Three different interpretations of hierarchy theory, developed in accordance with the theoretical backgrounds and interests of the investigators, are described by Resnick. They are as follows:

1. Learning psychologists and instructional designers tend to define hierarchies in terms of asymmetrical transfer relationships between two or more tasks. Thus two tasks are considered to be hierarchically related if (a) one task is easier to learn than the other, and (b) learning the simpler task first produces positive transfer to learning the more complex task. For example, learning to count is demonstrably easier than learning to add.

2. Two tasks can also be said to be hierarchically related when (a) one task is more difficult to perform than the other, and (b) anyone who can perform the more complex task can reliably be expected to perform the simpler one.
   This second definition of a hierarchy has greatly interested testing and evaluation specialists, particularly those concerned with designing diagnostic or placement tests for individualized educational programs.

3. Developmental psychologists have employed the concept of hierarchy to explain the occurrence of invariant sequences in the acquisition of concepts and logical structures as well as in physical and psychosocial development. "Stage" theories of development, such as Piaget's, are hierarchical theories in that they propose that an individual can reach a higher stage of development only by passing through a fixed series of lower stages.

The Gagnean Hierarchical Model of Learning

The cumulative learning model as described by Gagne (1965) represents the structuring of learning material by beginning with extremely simple levels of tasks, such as discriminations, and gradually progressing to much more complex tasks through positive transfer of training. Although Gagne's emphasis upon hierarchies of learning as a description of how learning takes place is still the basis of his model many changes are apparent. This is particularly evident from the first to third editions of his book Conditions of Learning (1967, 1970, 1977). The most recent publication reveals that significant changes in his model have developed since its first conception particularly with regard to the restriction of hierarchies to certain areas of learning and to the amount of content that may be covered by a hierarchy. His concept of what a learning hierarchy should be has become much clearer and more concise.

From his studies of the learning process, instructional treatment of content, assessment techniques used and the need to relate the instructional procedures of one subject to those of another, Gagne (1972) strongly indicates the need to recognize five domains of learning. These include the learning of motor skills, verbal information, intellectual skills, cognitive strategies and attitudes. None of these domains can cut across boundaries to completely include skills from other domains as the manner in which each develops is not identical. Gagne (1972) suggests that these domains may require different conditions for learning. Only the learning of intellectual skills is suggested to be hierarchical. For efficient learning to take place the learner must possess or be able to learn the prerequisite capabilities or skills necessary for the terminal task. Each domain is important in everyday school work but substantial development in each of the other domains requires the prior learning of relevant intellectual skills, where an intellectual skill means "knowing how" to demonstrate a capability as opposed to "knowing that" about something. This, plus the fact the domain of intellectual skills represents a large part of school learning, indicates the importance of identification of hierarchies of such skills.

Gagne (1970) distinguishes eight distinct types of learning which are related hierarchically, the particular arrangement being as represented in Figure 1. These eight types belong to the domain of intellectual skills. Although all these varieties of learning apply to school instruction, most instruction in school subjects is concerned with the learning and use of concepts and rules and with problem-solving. The four lower levels, in particular, may be applicable to only the very young child.

According to Gagne (1962) the learning hierarchy may be developed by asking the question "What must the learner be able to do if he is to achieve a particular new capability?" and asking the same question of each higher capability. The resulting hierarchy may be linear or branched and if branched several capabilities may be considered directly prerequisite to the next higher one. For example, it may be hypothesized that for a learner to apply his knowledge of momentum to a novel problem in a meaningful way he must first understand the concepts of mass and speed and how they relate to one another in a problem on momentum. For learners to be able to apply their understanding of momentum to a similar problem represents the possession of an intellectual skill. A mere statement of what momentum is is an example of verbalized knowledge, while the manipulation of the apparatus represents the domain of motor skills. The learner's approach to a novel problem on momentum involves the use of particular cognitive strategies. Finally the feeling the learner gets from his involvement with the subject represents the attitude domain.

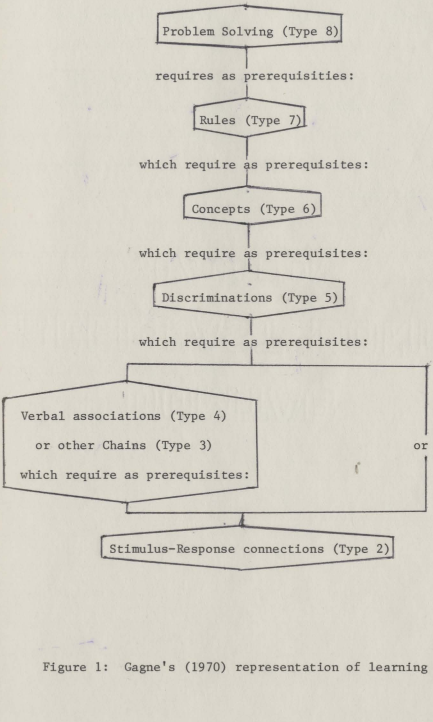


Figure 1: Gagne's (1970) representation of learning types

The basic functional unit of a learning hierarchy consists of a pair of intellectual skills, one subordinate to the other. The subordinate skill being identified as such because it is known to contribute to the learning of the superordinate skill (Gagne, 1970). For example, it may be hypothesized that a student attempting a novel problem on momentum must first be able to conserve mass and speed before he can manipulate them proportionately in a problem on momentum. It is only when the subordinate skills representing conservation of mass and speed have been mastered that the learning of the related higher level skill be facilitated. If the subordinate skill has not been mastered there will be no facilitation of the learning of the higher level skill. According to Gagne, for a learning hierarchy to be valid it must identify a set of intellectual skills that are ordered in a manner indicating substantial amounts of positive transfer from those lower level skills to connected ones of higher position.

Piaget's Theory of Intellectual Development

According to Piaget (1964), intellectual development is prerequisite to learning. Hence, learning occurs as a function of development rather than explaining it. For Piaget, intellectual development is an evolutionary process involving the formation of a set of intellectual structures progressively constructed and differentiated by continuous interaction between the subject and the external world. Piaget explains the development of these intellectual structures in terms of an invariant progression by each individual through four main stages, namely, the sensori-motor, pre-operational, concrete-operational and formal-operational, respectively, each of which is qualitatively distinct from the others.

Three concepts are central to Piaget's theory of intellectual development. These he calls cognitive structure, cognitive function and cognitive content. Progression through the stages of development is seen as being governed by sets of cognitive structures which undergo qualitative change during development. The key difference between each successive stage is structural. For Piaget, these cognitive structures, which refer to the pattern of cognition during development, are real but their existence can only be inferred from a study of the cognitive content of each stage of development. According to Brainerd (1978), Piaget finds these structures in the reasoning and problem-solving behavior of infants, children and adolescents. Piaget's cognitive structures grow and change during the course of development, building upon the structures of each previous stage until a new stage is reached.

The second concept, cognitive function, is said to underlie all changes in structure and is a process which remains constant for all ages. Intellectual development is said to occur as a function of two invariants: organization and adaption. Piaget believes that cognitive activity is guided by these two functional invariants. The actual process whereby structural change does take place is called equilibration. According to Piaget organization and adaption are complementary. They are not separate processes but rather two distinct sides of the same process. Organization is characterized by forethought and afterplanning and displaying intelligent behavior in coherent and discernible patterns. Adaption, the second functional invariant, is divided into two complementary processes, assimilation and accommodation. Assimilation refers to the interpretation of incoming information into a form that is acceptable to the current levels of development. The individual simply attempts "to make some sense" of the information. The higher the level of development the more sense that can be made of the information. Accommodation on the other hand refers to the changing of the structures in order to assimilate the information. As previously mentioned, cognitive structures accommodate by a special process called equilibration. When the limit to which information can be assimilated by the cognitive structures is attained disequilibrium ensues and changes to the structures must occur if learning is to take place.

Cognitive content, the third central concept, simply refers to the raw and observable behaviors that we call intelligence. It, like cognitive structure, changes as a function of experience and structural reorganization. What we know about cognitive structure and cognitive function is the result of our observations and measurement of cognitive content.

Schemes are examples of abstract cognitive structures through which information is assimilated. They are the basic units of abstract cognitive structures. Brainerd (1978) distinguishes two types, sensori-motor and cognitive. The second type represents the abstract cognitive structures of later development while the sensori-motor schemes describe intelligence during the first few years of life. In their simplest form schemes seem to be nothing more than a predictable sequence of responses to a given stimulus, for example, sucking or grasping. However, as development proceeds they grow into formal cognitive structures.

According to Piaget, each of his stages must satisfy a certain set of criteria. The first criterion is that qualitative change must occur in cognitive content. The second is that every child must pass through the same stages in exactly the same order. For example, the sensori-motor stage must precede the pre-operational stage all of the time, not just on the average and stage two must precede stage three (concrete operational) while the third stage must precede the fourth stage (formal operational). This notion of invariance, however, refers only to the order in which the stages emerge and not to the ages at which they appear. The ages at which each of the four stages appears may vary widely depending upon such factors as maturation, experience and the social environment in which the child is raised. The third requirement states that each stage builds upon the previous one. Early cognitive structures form the foundation of the higher level cognitive structures. Finally, although Nagy and Griffiths (1979) suggest that experimental evidence for this is weak, all the structures that characterize a given stage must be consolidated into a uniform whole before an individual can proceed to the next stage. They should emerge in unison and not in any sequence within a particular stage.

A Combined Approach

The roles of learning and development as espoused by Robert Gagne and Jean Piaget may seem to differ considerably. Strauss (1972) suggests that there may be some broad generalizations with which both theorists would seem to agree such as (1) a child develops intellectual capabilities, (2) capabilities are a product of a child's interaction with his environment, and (3) capability acquisition is sequential. However, due to their different philosophical views of man and accordingly their conceptions of what constitutes a psychological act, he believes that it is almost impossible to reconcile the theories of both men. According to Strauss, Piaget sees the child as an active constructor of his intellectual structures and intellectual development as evolution through stages or organized mental structures of which the component parts are operations. For Piaget, learning is the result of the application of acquired operations to a wider variety of objects and events, while the child's structural development determines the concepts he can learn. Thus, in this instance, learning is viewed as being subordinate to development. On the other hand, Strauss sees Gagne as taking an entirely opposite view. According to Strauss, Gagne considers the child to be a relatively passive recipient. His thought mirrors the logic inherent in his environment. For Gagne, learning represents the cumulative effects of discrimination, generalization and transfer. Development is subordinate to learning. In contrasting the two theories, Strauss claims that Gagne sees learning as occurring in an incremental or cumulative fashion where change is quantitative in nature while Piaget regards change as both qualitative and quantitative in nature, qualitative between stage growth and quantitative within stage intellectual growth.

Griffiths (1979) disagrees with Strauss' claim of irreconcilable differences between the two theoretical positions and suggests that they may not be as disparate as Strauss would lead us to believe. He suggests that Gagne's claim that the learner develops the appropriate cognitive strategies by first acquiring a wide variety of intellectual skills, which are then generalizable to more complex situations, takes away somewhat from Strauss' position that the two theorists differ markedly with regard to their definitions of the form of intellectual capabilities. Further, Griffiths suggests that Gagne's cognitive strategies may closely resemble Piaget's generalized intellectual structures.

With regard to Strauss' claim that Gagne's learner is a passive recipient, copying reality, as opposed to Piaget's student who is an active participant in the development of his intellectual structures, Griffiths feels that Strauss is correct. Because of the part intellectual skills play in Gagne's learning theory there is a need to carefully control their learning. In this respect a child may indeed be copying reality. However, Griffiths argues that Gagne's distinction between the domains of learning, especially that of cognitive strategies, ameliorates Strauss' claim. When a student is given a novel problem he in both cases becomes an active constructor of his own problem-solving strategies. Griffiths maintains that the difference between the two theorists in this case may be the degree of control imposed upon the learner.

Griffiths advocates that neither model should be used in ignorance of the other; that the difference underlying the theoretical positions of the two models should not be sufficient cause to exclude either one from situations where it may put the learner at an advantage. The particular task at hand should dictate the application of either theory or both. The learner's understanding of prerequisite capabilities and his intellectual structures should be considered in the light of the proposed problem. Griffiths suggests that if some school content is observed empirically to be hierarchical in nature then there is no reason why this should not be taken into account in structuring the curriculum. Similarly, if this content requires intellectual structures which are typically absent until individuals reach some particular age, this should also be considered in structuring the arrangement of curriculum content.

Phillips (1971) takes a somewhat different approach to the use of Piaget's theory in curriculum development. He states that Piaget's theory as it presently exists has produced disappointing results in the development of science curricula, mostly due to its extremely broad structure and its lack of prerequisite sequencing necessary for detailed curriculum specifications and prescriptions. He notes that investigations attempting to explore the substructure of prerequisites of certain concepts have been relatively few, and that before Piaget's theory can be truly helpful in curriculum development, studies explorative of the fine structure of concepts must be undertaken. Phillips attempted to do this with respect to the concept of displacement volume. This study is reported in detail in Chapter 2. Raven's (1967) study upon the development of the concept of momentum is one of the few other examples of substructure exploration. In this study Raven attempted to determine whether the concept of momentum followed a logical sequence, as determined from a logical analysis of the momentum concept, or a psychological sequence, as suggested by the research of Brunswick (1947) and Smedslund (1964). The present study is concerned with a theoretical and empirical examination and extension of Raven's study. Before discussing the need for such a study, key technical terms will be defined in the section which follows.

Definition of Terms

Capability: the ability to perform certain specific functions under specified conditions, e.g. a capability might be the ability to calculate the amount of work done in lifting an object to a specified height.

Gagne-type task analysis: deriving a hierarchy by asking Gagne's question ("What must the learner be able to do in order to learn this new element, given only instructions?") of each element in turn, from the terminal element downward. All connections that seem reasonably possible are included in the hypothesized hierarchy.

Intellectual skill: knowing "how" as contrasted with knowing "that" of information (Gagne, 1974). For example, knowing how to derive and demonstrate the equation $p = mv$ rather than just being able to define momentum.

Intuition: knowing the meaning of a concept without understanding the relationship between its component parts; e.g. with regard to speed, the child may focus his attention on such factors as which car passed another or which car arrived first rather than on the distance-travelled-over-time relationship.

Learning hierarchy: an arrangement of intellectual skills in which skills are related to others in subordinate-superordinate relationships, such that the subordinate skill in each pair is logically necessary for the learning of the superordinate skill and exhibits transfer of learning to the superordinate skill.

Logical hierarchy: a special case of a learning hierarchy. The two may or may not be identical. In a logical hierarchy skills are related by logical necessity but are not necessarily related in terms of transfer.

Momentum: the product of the mass and velocity of a body.

Possession of the momentum concept: being able to solve a numerical problem demonstrating an understanding of the relationship p = mv.

Psychological hierarchy: a hierarchy derived from a theoretical, and perhaps related empirical, consideration of the mental structures of the individuals comprising a population. In the present case, Raven used research based on the Piagetian model to derive his psychological hierarchy.

Qualitative skill: knowing how to derive and demonstrate a capability in a qualitative manner. For example, a child may describe the speed of a car as "faster than" or "slower than" another car or the weight of an object as "heavier than" or "lighter than" another object rather than assigning numerical values such as "twice as fast" or "twice as heavy."

Quantitative skill: knowing how to derive and demonstrate a capability in a quantitative manner. For example, the child may respond that the red car travelled "twice as fast" as the green car, not just "faster."

Speed: For this study velocity and speed are equated to mean rate of movement. The problem of vector and scalar quantities is ignored for present purposes. Speed is the distance travelled over a given amount of time.

Subordinate skills: the prerequisite capability(ies) necessary to perform the next step in a learning hierarchy. For example, the child must demonstrate an understanding of the concepts of mass and speed before he can manipulate them proportionately in a problem on momentum.

The Need for the Study

Belanger (1969), in reviewing learning studies of that decade in science education, noted the growing awareness of Piaget's work in the construction of science courses but indicated that there was a need for studies "investigating the fine structure and schema interaction" for specific science tasks. Phillips (1971) also indicates concern about the application of Piaget's theory to the development of science curricula because it does not provide a fine structure for the development of science concepts. Robertson and Richardson (1975) point out that "while much is now being made of the hierarchical structure within a science in curriculum projects and the stages or levels of cognitive development in learning theory, little research evidence exists in relation to such basic questions as (a) are science concepts attained in particular hierarchical sequence, and (b) is the conservation of a derived quantity in physics dependent upon the prior conservation of the fundamental quantities--mass, length, and time?" Raven's (1967) study, which focuses upon the development of the concept of momentum, is one such study that attempts to explore the substructure of a concept.

There is, however, a further need for examining Raven's particular hierarchical sequence, for several reasons. Firstly, Raven bases his hierarchy on only one task for each step in the hierarchy, with the exception of the concept of momentum where two tasks are used. If any one of these tasks was inappropriate the sequence of steps in the hierarchy could be different. According to White and Clark (1973) a minimum of two tasks per step should be used to allow for consideration of measurement error in validation of hierarchical relationships. Secondly, as will be discussed later there is reason to believe that some of Raven's tasks do not test what is purported. Thirdly, Raven's results imply the superiority of a psychological as opposed to a logical model in the development of the momentum concept. It will be argued that Raven's logical hierarchy was inadequately developed and hence does not allow a meaningful conclusion, either with respect to the specific concept or to the relationship between psychological and logical hierarchies in general. Griffiths (1979) suggests that the hypothesized hierarchy may not be a hierarchy at all, just the components of the developmental sequence re-arranged in a "logical" order. Further, he suggests that the steps involved in the hierarchy are very large and a more precisely defined hierarchy may yield different results. This, as will be discussed later, constitutes part of the task of the present study. Griffiths further points out that a learner's progression through a learning hierarchy involves the *mastery* of component skills but a child at the intuitive level may not have any understanding of the component parts. For these reasons it is not surprising to see Raven's logical hierarchy rejected in favor of the psychological sequence.

Purpose of the Study

The purpose of this study is to investigate Raven's (1967) claim that the development of the concept of momentum by young children follows a psychological rather than a logical progression. The psychological hierarchy involved is that used by Raven. The logical hierarchy used is different from Raven's, because of this investigator's belief that Raven's logical hierarchy was inadequately constructed, thereby biasing his findings. In the present study it is considered that a minimal understanding of momentum must involve the interaction between mass and speed. To this extent the tasks used by this investigator reflect an extension of Raven's study to a minimal understanding of momentum.

Research Questions

(1) Is Raven's developmental hierarchy substantiated

(a) when tasks the same or equivalent to his are applied to a new sample?

(b) by new tasks testing the same stated skills, when applied to the new sample?

(2) Can a more valid learning hierarchy be identified, leading to the same terminal skill, from a Gagne-type task analysis of the momentum concept?

Delimitations of the Study

Restriction of the sample to one school within the St. John's area and to one class per grade from grades one to eight represents a severe delimitation. Although the total number of subjects tested may disclose the existence of a hierarchy, the difference in intellectual ability and background for each class may have been sufficiently different to obscure gradual development of the concept, if such development exists. The testing of a much larger sample of subjects of different intellectual levels from the same grades may have disclosed pertinent information with regard to the organization and development of certain common misconceptions among class levels, perhaps due to the acquisition of skills or to the intellectual levels themselves.

A further delimiting factor results from the small sample taken from grades one and two. This, to some extent, may have affected the results obtained for the qualitative portion of the hierarchy and hence prevent valid comparisons with similar studies. Both of these factors may be seen as affecting the generalizability of the results.

The restriction of the study to one particular topic in physics may be considered another delimiting factor. Any superiority of the learning hierarchy model or the developmental model as a guide to the learning of science concepts may not be generalizable to other concepts in science. Finally, any hierarchy identified may not represent the only valid hierarchy for the concept under study.

Limitations of the Study

One potentially serious limitation of this study is concerned with the method of presentation of the tasks involved. In this particular study, in contrast to Raven's, the tasks were presented in group form rather than individual interviews. Also the subjects' responses, except for grades one and two, were written. Thus the investigator was forced to rely upon the subject's writing skills. An attempt was made to compensate for this fault by selecting at random five students from each class and testing them on the same skills to determine the consistency of their responses between group and individual administration.

A further limitation might be that although the results of this study may indicate the sequence of particular skills and the order in which they may possibly appear in a child's development, it is not possible to conclude that the best or only hierarchy has been found for this particular concept. Nor would failure to identify a valid hierarchy indicate that it is not possible to identify one.

Summary

The problem of sequencing content has been discussed and two models of sequencing derived from the Gagnean and Piagetian theories of learning have been proposed as possible alternatives to the problem. Also the possibility of a combined approach of these two theories to learning and instruction has been suggested and the need for more studies involving more fine structure in the concepts investigated has been stressed. Raven's study is an example of such an investigation. In a study of the development of the concept of momentum, Raven provided evidence in support of a psychological hierarchy over a logical hierarchy. The present author argues that Raven's study involved an inadequate learning hierarchy, as well as some questionable test items. Therefore, Raven's general conclusion may be unfounded. This study proposes to test the findings of Raven's study against the construction of a new logical hierarchy.

Chapter 2

RELATED RESEARCH

Learning Hierarchies

Up to this date, most validated learning hierarchies have been in the area of science and mathematics. Gagne's own hierarchies contained arithmetic, algebraic and geometric skills. In science, perhaps the best known and most extreme attempt to apply Gagne's hierarchical model has been Science--A Process Approach (S-APA), a complete K-6 general science program, developed by the American Association for the Advancement of Science. The outcome of this program was to be the integration of hundreds of skills which the learner was expected to possess at the end of grade six.

In a study which shows much support for the cumulative learning model, Wiegand (1969) focused upon a logical analysis of a variation of Piaget's inclined plane task, which involved deriving the relationship between the height and weight of a car on an inclined plane, the weight of a block, and the distance it was pushed when struck by the car. This task was analyzed to provide a hypothesized hierarchy of intellectual skills which was then subjected to empirical test. Piaget's inclined plane task served as a test of transfer. The study was designed to test whether the performance of Piaget's final task could be accounted for on the basis of a cumulative learning model. Thirty students (14 boys and 16 girls) who failed both a pretest for the final task and also the transfer task participated in the study. Subjects were assigned to one of three treatment groups representing demonstration-test-retest, test-retest, and test, respectively.

Wiegand found that children who could not perform either the final task or the transfer task did so quite readily when they were taught the subordinate capabilities between the first and second presentation of both tasks. The demonstration had no significant effect on the performance and the initial test did not enable subjects to perform either the final or transfer task except when they had already attained the needed subskills as revealed by their performance on the test. The retest of subordinate capabilities failed in the initial test appeared sufficient to enable subjects to acquire the hypothesized subordinate skills. The results of this study indicate that the development of intellectual skills occurs through the cumulative effect of learning subordinate capabilities rather than by the adaptation of structures of intellectual growth. The results appear to be consistent with a view of intellectual development that contrasts with that of Piaget.

Resnick (1973) notes that this study poses a challenge to the cognitive-development point of view, showing that acquisition of subordinate tasks leads to the acquisition not only of the terminal task itself but of a logically similar transfer task. Carroll (1973) on the other hand argues that it was not the attainment of the prerequisite skills that was so important here but the fact the attention given to the subordinate skills helped the students recognize their applicability to the criterion task and to follow an analytical procedure that at first was not evident to them. Carroll suggests that what is involved here is more of a general competence in analytical skills gained by exposure to the subordinate skills and that the teaching of the subordinate skills might not be the most important factor.

Almy (1973) criticizes Wiegand's study, as well as others, because of the fact that the child may be moving through the steps of a hierarchy at a fairly fast pace "capturing one rule of procedure after another and failing to grasp any intuitive conception of what it is about." He may not be creating any procedure for himself, or in Piaget's terms developing cognitive structures. Also, Almy argues, the success of the testing procedure may be contingent on the background of experiences of the students. In interpreting the success of the testing procedure, the Piagetian developmentalist would like to know something about the environmental influences of the subjects as well as their developmental level. According to this view, only in this way can the results be meaningful.

In a well executed study by Okey and Gagne (1970) a program on solubility product calculations was selected and 15 subordinate skills were derived by a Gagne-type task analysis. Four different tests were used to measure student performance: a pretest and posttest on the criterion task and a pretest and posttest on the subordinate skills in the learning hierarchy. The equivalence of these tests was determined in a separate investigation by submitting pairs of items to students. Items meeting the criterion of 80% pass or fail on both questions were selected for the final form of the tests. The sample consisted of 135 tenth, eleventh and twelfth grade chemistry students in five chemistry classes. Two equal groups were randomly selected from each class. Approximately 7 class periods of 50 minutes each were required for a treatment group to take the test and complete the learning program. The first group completed the unit while the second group was involved in an unrelated chemistry unit. The second group then completed the revised unit. A significant difference in the level of performance was confirmed for the second group as compared to the first. The researchers thus concluded, in accordance with the cumulative learning model, that adding instruction leading to improved performance on subordinate skills in a science learning task significantly improved performance on the criterion task.

Despite the attractiveness of the study Griffiths (1979) notes that the skills involved were not defined as precisely as they might have been. In some cases one subordinate skill might encompass a wide range of outcomes. Griffiths further criticizes the study for the fact that the percentage of individuals successful on subordinate skills was less than desirable. For example, for each of nine out of fifteen subordinates skills, less than 80 percent of the experimental group were successful. For four of these skills less than 40 percent were successful. Griffiths argues that the lack of these subordinate skills for individual subjects was not investigated, nor were specific transfer effects between skills. As a result, the validity of the hierarchy in terms of both its psychometric and transfer characteristics may be less encouraging than the results imply.

Developmental Hierarchies

Although a review of the literature reveals that a significant amount of research has been done concerning Gagne's learning hierarchy model and Piaget's developmental stage theory there is very little evidence of research investigating the "fine structure" of specific science tasks. Belanger (1969), in his review of learning studies in science eudcation, states that research of this kind is urgently needed to make Piaget's very general description of concrete operations useful in curriculum development. Piaget's developmental sequence would probably be quite useful in curriculum development except for the fact that he is concerned with the "generalized knower" and as Belanger has noted "the specific characteristics of a seven year old differ extensively from those of a twelve year old." He suggests that researchers "who have particular talents in the techniques of empirics could make significant contributions to Piagetian studies by investigating the fine structure and reporting what happens in detail within the stage of concrete operations between seven and twelve years of age for very specific science tasks." Belanger cites Raven's analysis of the sequence of concepts necessary for understanding momentum as that kind of research so badly needed to make Piaget's very general description of concrete operations useful in curriculum development.

From a content analysis of the momentum construct Raven (1967) predicted that the components of momentum should be acquired in a logical sequence: *conservation of matter* ⟶ *speed* ⟶ *proportional use of mass and speed with momentum held constant* ⟶ *momentum*. However, based upon Piaget's findings and the research reported in the literature the following psychological sequence was developed: *momentum* ⟶ *conservation of matter* ⟶ *proportional use of mass and speed with momentum held constant* ⟶ *speed*.

One hundred and sixty primary school children (ages 5, 6, 7 and 8 years) were asked to solve a task for each of the above skills. The results of the study supported the psychological sequence and were in agreement with Piaget's description of thinking in the pre-operational and concrete stages. Although a sharp demarcation of concrete performance between consecutive ages was not observed, a hierarchical ordered series of successes on test items increasing as a function of age was found.

Raven concluded that although a child may understand neither the word momentum nor the momentum equation, the primary school child may possess the concept of momentum without understanding its subcomponents, e.g. speed, mass, duration. As will be argued later in this chapter, the meaning attributed by Raven to possession of the momentum concept may be critical in assessing the appropriateness of his findings.

In a study by Phillips (1971) a model for examining some of the "fine structure" within the child's interpretation of the concept of displacement volume was presented. The conservation tasks used were related to six levels of a proposed hierarchical model for the attainment of concepts leading up to the conservation of displacement volume. Two different methods of task presentation, objective presentation and graphic presentation, were used. The results showed that five of the six tested levels did scale in the anticipated order and that no significant difference (at the .05 level) was obtained between the two methods of task presentation.

The fact that 100 of the 120 subjects interviewed were successful in these tasks leading up to the attainment of the concept of displacement volume in the hierarchical sequence proposed, may hold some important implications for teaching. According to Phillips, it may be possible to use this sequence in the teaching of the concept of displacement volume.

Robertson and Richardson (1975) carried out a study on a stratified random sample of 200 boys and girls drawn from grades 7-10 to determine if scientific concepts are attained in a particular hierarchical sequence and if the conservation of a derived quantity in physics is dependent upon the prior conservation of the fundamental quantities mass, length and time. Using Piaget's criterion of 75 percent for assigning the conservation of a quantity to an age level the authors found that this criterion was exceeded by boys and girls at grade 7 for the concepts of mass, weight-force, length, distance, speed (straight tunnels) and speed (concentric circles). Both boys and girls conserved vertical height at grade 8, and also time at grade 9. However, at the grade 9 level only boys conserved volume. The conservation of speed before time was contrary to what was predicted, suggesting that the conservation of speed is not dependent on the prior conservation of time. In general, the authors contended that the results of their study provide evidence concerning possible hierarchical structures for learning physics.

Bass and Montague (1972) applied Piaget's findings to the construction of learning hierarchies and instructional materials for the problem of equilibrium in the balance and equilibrium of a cart on an inclined plane. The results supported the learning hierarchy for the first task but not for the inclined plane task, in each case with the same sample of ninth grade students. Bass and Montague felt that this study helped to substantiate their beliefs that curriculum developers need fine-structured studies of developmental sequences to supplement Piaget's analyses, and that Gagne-type task analysis procedures could profitably be used in conjunction with Piaget's developmental sequences in the construction of learning hierarchies.

Methods Used to Validate Learning Hierarchies

White (1973) notes that learning hierarchies based upon Gagne's methods generally have faulty designs. Almost all of the studies suffered from one or more of the following weaknesses: small sample size, imprecise specification of component elements, the use of only one question per element, and the placing of tests at the end of the learning program or even the omission of instruction altogether. According to White, these flaws and the lack of a test of hierarchical dependence which takes account of errors of measurement mean that no meaningful quantitative conclusion has been reached about the validity of even one step in any hierarchy derived to that time.

Throughout the history of Gagne's learning hierarchy model, investigators have been plagued by the lack of reliable methods for determining whether each connection in a learning hierarchy is valid or not. Indexes such as Gagne and Paradise's proportion positive transfer and the five variants of it proposed by Walbesser, Guttman's coefficient of reproducibility and the phi correlation coefficient, according to White (1974), have all proved unsatisfactory. White states that Gagne and Paradise's indexes are of no use because the index can take values close to zero even if there is no hierarchical relationship between the skills or even if they are independent of one another. Also, the index takes no account of errors of measurement and lacks a sampling distribution.

In some investigations more than one question was used for each skill. This leads to difficulties in deciding how many questions correct constitutes a pass for each skill. Again according to White, Walbesser's five indexes which were intended to overcome some of these weaknesses failed to do so and instead created others. These indexes are measures of how easy the subject found the subject matter, not of the validity of the hierarchical relations among skills. Guttman's (1944) coefficient of reproducibility which has been used for testing the validity of hierarchies was applied for another reason. It applies to the hierarchy as a whole and not to the individual connections within it. One incorrect connection could lead to a rejection of the whole hierarchy. Capie and Jones' (1971) phi-correlation coefficient advocates the establishment of a hierarchy by calculating the phi-correlation coefficient for each pair of skills and, where the coefficients are significantly different from zero, placing the skills in order of difficulty. According to White, the criteria of this method are necessary but not sufficient conditions for a valid hierarchy. Use of these criteria alone can lead to a hierarchy which contains superfluous skills and superfluous connections between the skills. White and Clark (1973) have developed a method which attempts to overcome some of these shortcomings. This method as well as another recent method, the ordering-theoretic method, will be discussed in detail in the next section.

White (1974) makes the following recommendations for improvement of practice in the identification and validation of learning hierarchies.

1. Define in behavioral terms, the element which is to be the pinnacle of the learning hierarchy.

2. Derive the hierarchy by asking Gagne's question (What must the learner be able to do in order to learn this new element, given only instruction?) of each element in turn, from the pinnacle element downward. Include all connections that seem reasonably possible, since the validation process can only destroy postulated connections, not create them. Avoid verbalized elements, they can be included in the instructions.

3. Check the reasonableness of the postulated hierarchy with experienced teachers and subject-matter experts.

4. Invent possible divisions of the elements of the hierarchy, so that very precise definitions are obtained.

5. Carry out an investigation of whether the invented divisions do in fact represent different skills. One way of doing this is to write two or more questions for each division and give them to a sample of Ss. Where-ever any subjects are observed to answer correctly the set of questions for one division, while answering incorrectly the set for another, the divisions are taken to be separate skills. White has given a description of the practical arrangement of such an investigation.

6. Write a learning program for the elements, embedding in it test questions for the element. The questions for an element should follow immediately after the frames that teach the element. There must be two or more questions for each element to allow for an estimate of their reliability.

7. Have at least 150 Ss, suitably chosen, work through the program, answering the questions as they come to them.

8. Analyze the results to see whether any of the postulated connections between elements should be rejected. A suitable test of a hierarchical relationship has been developed by White and Clark. The hypotheses compared in the test are Ho: the proportion of the population from which the sample was drawn who can learn higher elements without the lower element is zero. The test provides estimates of the probabilities of the observed results given that Ho is true or given specific values of the proportion under Ha.

9. Remove from the hierarchy all connections for which the probability under Ho is small, say 0.05 or less.

According to White these changes in hierarchical methodology should lead to a sound basis for the validation of future learning hierarchies.

Griffiths (1979), however, maintains that White's model is lacking in several respects. Firstly, the White and Clark test represents a psychometric approach to hierarchy validation. Hence, according to Griffiths, any hierarchy validated in this manner does not necessarily imply transfer leading to greater learning of the superordinate skill(s). Secondly, Griffiths argues that White's recommendation of a programmed instruction format restricts the applicability of the learning hierarchy to only one mode of instruction, and that if there is a generalized hierarchy its structure should still exist for other modes of instruction. Thirdly, Griffiths suggests that the testing of subordinate skills should be carried out after as well as during the instructional period, and that the primary psychometric test be made on the former. Finally, he recommends the use of a test of positive transfer as well as psychometric validation.

Methods of empirical validation procedures differ according to the definition of a learning hierarchy accepted by an investigator and by the intended use of his findings. Griffiths and Cornish (1978) have grouped the methods which have been used to validate learning hierarchies into two classes, those which reflect the transfer properties of hierarchies and those which reflect the notion of a relatively inviolate sequence, respectively. These authors concentrate on several methods of the second group, namely the 'ordering-theoretic' method (Bart and Krus, 1973; Airasian and Bart, 1975), the 'test of inclusion' (White and Clark, 1973) and a method suggested by Dayton and Macready (1976). The first two focus upon comparisons of pairs of skills while the third method considers the hierarchy as a whole. For the purpose of this study only the first two will be considered.

'Ordering-Theoretic' Method

The validity of a hierarchy is determined by considering the relationship between the pairs of elements in it. The following contingency table will help to explain the operation of this method. In this table ✓ denotes possession and X denotes non-possession of a skill while the letters A, B, C, D represent the observed frequencies in the appropriate cells. High A and C values tend to be supportive of a hierarchical relationship while high D values tend to deny the relationship. The 'ordering-theoretic' method focuses upon whether an arbitrary pre-specified tolerance level for D is exceeded. If it is, no hierarchical connection is considered to exist.

		UPPER SKILL	
		X	✓
LOWER SKILL	✓	B	A
	X	C	D

Figure 2: Data matrix for the 'ordering-theoretic' method

This test is applied to all possible combinations of pairs of skills in the hierarchy from which a composite hierarchy is identified. Griffiths and Cornish (1978), however, state that this method is deterministic, and does not take into account errors of measurement. No test is provided to determine the statistical confidence with which each stated hierarchical relationship can be claimed to exist.

White and Clark Test of Inclusion

This is a more sophisticated test for the comparison of pairs of skills in a learning hierarchy. The test can only be applied when two or more questions are used for each skill. When only one question is used, no estimate of the size of errors of measurement is possible. The test focuses on the number of subjects who answer incorrectly all the questions for the lower skill of a pair and correctly all the questions for the hypothesized higher skill. When this number exceeds a critical value the connection is judged invalid. The critical value is determined by the reliability of the questions and by the value specified for the probability of wrongly rejecting the null hypothesis that the connection is hierarchical (White and Clark, 1973).

The matrix representing two questions per skill is shown in Figure 3.

SKILL II (UPPER) QUESTIONS CORRECT

		0	1	2
SKILL I (LOWER) QUESTIONS CORRECT	2			
	1			
	0			

Figure 3: Data matrix for the White and Clark Test

A skill-by-skill matrix of scores is then formed. The scores ranging from zero to two to three, as appropriate. The cell representing a score of zero on the lower skill and the maximum possible on the upper skill is used to test the hierarchical relationship. This cell is assumed to contain those subjects most likely to possess the upper skill and lacking the lower one. The basis of the method is to test the null hypothesis that there will be no entries in the critical cell, other than those representing errors of measurement. The probability that the observed frequency does not violate the null hypothesis is calculated by using the marginal totals. For the case of two questions per skill the probability that a member of the sample will be found in the critical cell is

$$P_{\Theta_2} = P_o(1-\Theta_b)^2\Theta_d{}^2 + P_1(1-\Theta_a)^2\Theta_d{}^2 2 + P_{II}(1-\Theta_b)^2\Theta_c{}^2 + P_B(1-\Theta_a)^2\Theta_c{}^2$$

where

P_{Θ} = the proportion of the population with neither skill.

P_B = the proportion of the population with both skills.

P_I = the proportion of the population with skill I only.

P_{II} = the proportion of the population with skill II only.

Θ_a = the probability of someone with skill I answering correctly any skill I question.

Θ_b = the probability of someone without skill I answering correctly any skill I question.

$\Theta_c \Theta_d$ = are the corresponding probabilities for skill II.

To make the estimate of P_{O_2} as large as possible and hence reduce the possibility of Type I error, Θ_b is assumed to be zero and Θ_d is assumed to equal one. That is, it is assumed that all subjects with one skill I question correct really possessed skill I and all those with one skill II question correct lacked the skill. Modifications can be made to the derivation above to accommodate three questions per skill. The same procedure can be used with one and two percent exceptions in addition to those representing errors of measurement, resulting in hierarchies of substantial rather than absolute levels of hierarchical dependence (Griffiths and Cornish, 1978).

Advantages and Disadvantages of the Two Methods

Of the two methods discussed for comparison of pairs of skills the White and Clark (1973) test of inclusion is easily the more sophisticated. While the 'ordering-theoretic' method is much simpler to use and has been applied recently in several studies it is conceptually less pleasing than the White and Clark test. The 'ordering-theoretic' method does not take into account errors of measurement and no test is provided to determine the statistical confidence which can be attached to the existence of each accepted hierarchical relationship. Because the 'ordering-theoretic' method is deterministic, whereas the White and Clark test is probablistic, the primary test for the hierarchical dependencies within the present study will be the White and Clark test.

Neither the White and Clark test nor the 'ordering-theoretic' method considers the hierarchy as a whole, so it is possible that in combining the results of analysing the skills in pairs a different hierarchy may be arrived at than when the hierarchy is considered as a unit (Griffiths, 1979).

Studies Relating to the Attainment of the Momentum Concept

The concept of linear momentum was chosen by Raven because the development of the mass and speed concept had been studied extensively in the past. The following sections will deal with studies relating to the attainment of these concepts and how they may interact to lead to a formal understanding of the concept of momentum.

Conservation of Substance

Piaget's findings suggest that concepts of quantity develop in three stages, with the final stage earmarked by the discovery of conservation (Elkind, 1961). Both Lovell and Ogilvie (1960) and Elkind (1961) found that children first conserve matter at ages 7-8 years, in agreement with Piaget's earlier findings. In Piaget's view, thinking from 4-7 years of age is largely dependent upon perception. During this period thinking tends to be determined by centering on one aspect of a situation with other apsects ignored. But from 7-8 years of age the child is able to break away from the influences of perception and is increasingly able to apply logical thought to practical problems and concrete situations.

Piaget suggests that conservation concepts develop in three stages, representing, in turn, non-conservation, inconsistent conservation and conservation. To him children in stage one have only a general impression of quantity but are capable of judging crude weight, volume and mass differences. For example, in the 'sausage' experiment they give non-conservation responses because of their perception that the sausage is different from the ball. They do not concentrate on the transformation but rather judge quantity by single dimensions, e.g. length, width.

Children in the second of the above stages have a differentiated impression of quantity and are unable to judge quantity differences according to pairs of dimensions (short-wide, long-narrow). In the 'sausage' experiment children at this stage tend to give non-conservation responses because to them the sausage is more in length and less in width. They are unable to resolve the contradiction and when pressed judge the quantity by a single dimension.

In the third stage they are able to conserve, as their answers indicate that the perceived transformation can be cancelled (the sausage is rolled back into a ball) or that the perceived differences can be equated (what the sausage gained in length it lost in width) and therefore the quantity is the same. Piaget attributes the initial appearance of the conservation of mass at ages 7-8 years to the development of the mental operations of logical multiplication and equation of differences.

Conservation of Speed

With regards to the conservation of speed Piaget (1957) again suggests that the child at first is under the influence of his perceptions and that his first notion of speed is based on the intuitive realization that of two bodies in motion in the same direction the one that passes the other has the greater speed. This intuition may be much easier for the child to acquire than the distance-duration relationship, probably because the notion of order is easier to grasp than the concept of intervals or measurement. Passing is nothing but a change of order of the two objects. According to Piaget, young children also judge speed from the point of arrival of objects quite independent of distance covered. Thus "faster" means arriving "in front of" or "before." Lovell, Kellett and Moorhouse (1962) found that 75 percent of the children at about 9 years of age had attained an intuitive concept of speed and were able to coordinate distance/time relationships.

Proportionality

In Raven's study the subjects ranging in ages from 5 to 8 years were presented with the problem of varying mass and speed proportionately. According to Piaget (1958), an understanding of proportions does not appear until substage IIIA (typically 11 to 12 years of age or older). Piaget theorizes that in an investigation in which a system of proportions comes into play, "before the subject arrives at the calculation of numerical relations he isolates an anticipatory scheme for qualitative proportionality. This scheme, simply a logical one at first, leads to the discovery of metrical proportions." This step represents the quantitative extension of a qualitative notion. In Piaget's balance problem this idea of qualitative proportionality is evidenced by some of the subjects' reasoning, e.g. "The larger the distance the smaller the weight. They go together." Accordingly, the subjects used in Raven's study may have had only an intuitive or qualitative notion of proportionality, not a quantitative one. This may also be evidenced by some of their reasoning, "heavier things have more push and lighter things have less push." The children used in this study probably did not possess the metrical proportionality necessary to solve these problems other than in a qualitative and perhaps intuitive manner.

A Critical Review of Raven's Study

Raven's Definition of Momentum

Formally momentum is defined as $p = mv$ where the momentum is directly proportional to the body's mass and velocity. It is suggested that in Raven's study the subjects did not necessarily have an understanding of either the word momentum or the momentum equation. However, the manner in which any subject used the words to describe this specific phenomenon was taken as an indication of his understanding of momentum and its component parts. For example, when the words "push" or "pull" were used as a verb they were taken to mean "exert a force" but when used as a noun they were considered equivalent to the word momentum.

Raven's Tasks

In Raven's study all 160 subjects were tested on six different tasks. Each task was presented to each child two times consecutively followed by the criterion question. The tasks were administered in random order to individual children to control for the effects of learning. The six tasks presented were described as (1) *conservation of matter*, (2) *speed*, (3) *proportional use of matter with momentum held constant*, (4) *proportional use of speed with momentum held constant*, (5) *first momentum task*, (6) *second momentum task*. These tasks comprised the steps of both the psychological and logical hierarchies in different sequences.

A number of criticisms have been made in Chapter 1 concerning Raven's study. However, perhaps one of the strongest is that he used only one task per step in the hierarchy and that if either of the tasks in any pair under test was inappropriate the whole sequence could be altered. Another important criticism would be that although Raven claims to have derived his logical hierarchy by a method similar to a Gagne-type task analysis, the resulting sequence of steps does not bear any resemblance to a Gagne-type learning hierarchy, nor are his tasks suitable for testing the skills of such a learning hierarchy. This weakness is particularly evident in Raven's tasks testing proportionality. Raven suggests that his subjects understood how to manipulate mass and speed proportionately and could predict mass and speed variables in various tasks. However, as mentioned earlier, these children solved proportionality problems in a qualitative rather than a quantitative fashion. Hence, their understanding may only have been intuitive.

It is suggested that in Raven's proportionality tasks too much was left to the subject's perceptual experiences and not enough to his logical thought processes. In both tasks specifically involving proportionality the child was given an opportunity to manipulate the apparatus and then asked a series of questions, rather than as a non-participating observer asked to logically derive the correct solutions. The tasks required only an intuitive or qualitative understanding of the specific skills involved and this is not sufficient for identifying the skilled performance of the components of a valid learning hierarchy.

Raven's tasks on proportionality also may be described as crude or unsophisticated methods which do not readily lend themselves to the formation of logical proportions and their numerical solutions. For example, two tennis balls, one with weights added, were pushed by the subject at two shoe boxes positioned at equal distances from the starting point. The subject was then asked what he would have to do to the weight of one of the balls to make both boxes move the same distance. He was then asked to explain his answer. It could be argued that the subject's answer reflects only a general understanding of the interaction of mass and speed and not in a manner demonstrating metrical proportionality. Perhaps a task using two collision carts, a series of weights regularly increasing by a fixed multiple and two wooden blocks of the same weight situated at the end of a track, would be a more appropriate apparatus. Such a design might encourage the student to reason logically and, where possible, to apply numerical solutions to the task.

As will be discussed later in Chapter 3, for the very young child understanding of momentum may be related to his sensory experiences, for example the effect of a physical push on an object. Mass and speed are not separate concepts but exist as one in the push exerted on the object. Later these variables become separated and the very young child learns to manipulate them proportionately, first one variable at a time (mass, speed) then two variables (mass and speed) together. The quantitative stage does not follow until the child has correctly internalized these skills. This author argues that the "physical push" is representative of the type of skill found in Raven's hierarchy but not of the skilled performance found in a true logical hierarchy. Therefore, it is incorrect for Raven to claim the superiority of the psychological hierarchy over the logical hierarchy if the logical hierarchy is not truly a logical hierarchy.

Chapter 3

DESIGN, INSTRUMENTATION AND PROCEDURE

The present chapter will explore the testing and validity of Raven's psychological hierarchy followed by a discussion of the steps involved in the identification and validation of a new logical hierarchy. The sample, instrumentation and procedure, together with the rationale for each decision made will be described in the sections that follow.

Raven's Psychological Hierarchy

The purpose of the study carried out by Raven (1967) was to determine the validity of the sequence of skills postulated as being necessary for an understanding of the concept of momentum. Two hierarchies, a psychological hierarchy and a logical hierarchy, were proposed. The results of the study supported the existence of the psychological hierarchy rather than the 'logical' hierarchy. Raven's psychological hierarchy was constructed from an analysis of psychological studies relating to the major concepts necessary for an understanding of the concept of momentum. Six tasks were identified as comprising the steps of the psychological hierarchy. These same tasks also comprised the steps of the logical hierarchy, but in a different sequence. The six tasks presented were described as (1) *conservation of matter*, (2) *speed*, (3) *the proportional use of matter with momentum held constant*, (4) *the proportional use of speed with momentum held constant*, (5) *first momentum task*, (6) *second momentum task*. The tasks were presented two times consecutively in random order to individual children, followed by the criterion question. The present investigations, in part, tests the validity of Raven's psychological hierarchy with tasks the same or equivalent to his when applied to a new sample. The tasks for Raven's psychological hierarchy may be more precisely stated as representing the following skills. These skills, hierarchically arranged, are presented in Figure 4.

H *Speed*

The subject will be able to tell whether two cars, moving through two tunnels of unequal length, but starting and stopping at the same time, are moving at the same speed or at different speeds.

B *Proportional Use of Speed with Momentum Held Constant*

Given two carts of different weights and two blocks of the same weight placed equidistant from the two carts, the subject will be able to tell whether one of the carts will have to go faster or slower than, or the same speed as, the other to move the block it strikes the same distance as the block struck by the other cart.

D *Proportional Use of Matter with Momentum Held Constant*

Given two carts moving at different speeds and two blocks of the same weight placed equidistant from the two carts, the subject will be able to tell which of the two carts will have to be made heavier in order to move both blocks the same distance.

E *Conservation of Matter*

The subject will be able to tell whether two identical balls of clay have the same amount of material when rolled into

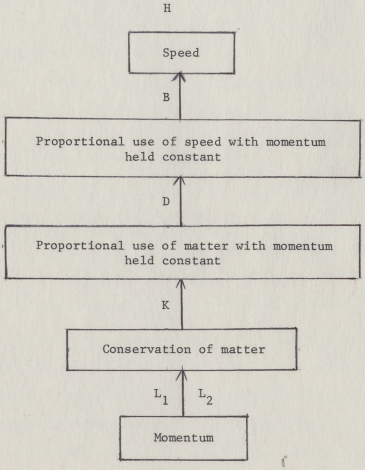


Figure 4: Raven's Psychological Hierarchy

different shapes.

L_1 *First Momentum Task*

The subject will be able to predict which of two carts is the heavier after observing the interaction of two moving carts.

L_2 *Second Momentum Task*

The subject will be able to predict which of two carts is the faster after observing the interaction of two moving carts.

Construction of the Logical Hierarchy

The present investigator argues that Raven's 'logical' hierarchy was not appropriately derived. Hence, it may be misleading to say that a psychological model is superior to a logical model for the development of the concept of momentum. White (1973) claims that Raven's method of validating his hierarchy was by testing whether one element was more difficult than another. This, he points out, is not a valid procedure: the existence of a hierarchical relationship between two elements implies a difference in their difficulties, but the converse is not necessarily true. For example, although it may be more difficult to learn how to solve second order differential equations than to learn how to spell "hierarchy," it is not impossible to learn the harder task first. Raven's logical hierarchy was a linear one in which he proposed that *conservation of matter* was learned first, followed by *speed, proportional use of matter with momentum held constant, proportional use of speed with momentum held constant* and finally, the *terminal task* representing understanding of momentum. Raven appears to have asked only in a very general way what should follow what. As a result, some of Raven's 'logical' connections (e.g. *conservation of matter* before *speed*) do not appear to be logical. The present study, it is hoped, provides a more detailed answer to the same question. In this study a different, more finely structured logical hierarchy was derived by a Gagne-type task analysis.

A task analysis of the terminal skill, representing understanding of momentum, led to the identification of the following skills:

Terminal Skill

A. *Formal Momentum Task*

Given a collision between two identical bodies of equal weight and the initial speed of one of these bodies, calculate the speed of the other body after collision.

Qualitative Skills
(Not hierarchically arranged)

B. *Proportional Use of Speed with Momentum Held Constant*

Given two carts of different weights and two blocks of the same weight placed equidistant from the two carts, the subject will be able to tell whether one of the carts will have to go faster or slower than or the same speed as the other to move the block it strikes the same distance as the block struck by the other cart.

C. *Proportional Use of Speed with Matter Held Constant*

Given two carts of equal weight moving at different speeds, two blocks of the same weight placed equidistant from the two carts and the distance travelled by one of the blocks when struck by one of the carts, the subject will be able to tell whether the second block when struck by the second cart will move farther than, not as far as or the same distance as the first block.

D. *Proportional Use of Matter with Momentum Held Constant*

Given two carts of equal weight moving at different speeds and two blocks of the same weight placed equidistant from the two carts, the subject will be able to tell which of the two carts will have to be made heavier in order to move both blocks the same distance.

E. *Proportional Use of Matter with Speed Held Constant*

Given two carts of unequal weight moving at the same speed, two blocks of the same weight placed equidistant from the two carts and the distance travelled by one of the blocks when struck by one of the carts, the subject will be able to tell whether the second block when struck by the second cart will move farther than, not as far as or the same distance as the first block.

Quantitative Skills
(Not hierarchically arranged)

F. *Proportional Use of Speed with Momentum Held Constant*

Given the proportional relationship between the masses of two carts striking two blocks of equal weight, the subject will be able to determine the proportional relationship between the speeds of the two carts if the blocks are to be moved the same distance.

G. *Proportional Use of Speed with Matter Held Constant*

Given two carts of equal weight moving at different speeds, two blocks of the same weight placed equidistant from the two carts and the distance travelled by one of the blocks when struck by one of the carts, the subject will be able to calculate the distance travelled by the second block when struck by the second cart.

H. *Speed*

Given two cars passing through two tunnels of unequal but known lengths and having the same starting and stopping time, the subject will be able to determine whether the cars are travelling at the same speed or at different speeds.

I. *Proportional Use of Matter with Momentum Held Constant*

Given the proportional relationship between the speeds of two carts striking two blocks of equal weight the subject will be able to determine the proportional relationship between the masses of the two carts if the blocks are to be moved the same distance.

J. *Proportional Use of Matter with Speed Held Constant*

Given two carts of unequal weight moving at the same speed, two blocks of the same weight placed equidistant from the two carts and the distance travelled by one of the blocks when struck by one of the carts, the subject will be able to calculate the distance travelled by the second block when struck by the second cart.

K. *Conservation of Matter*

The subject will be able to tell whether two identical balls of clay have the same amount of material when rolled into other shapes.

These skills arranged in hierarchical order are illustrated in Figure 5.

In this investigation, the students were tested on their ability to solve momentum problems quantitatively as well as qualitatively and hence to demonstrate a more complete understanding of the concept of momentum. For grades one, two and three, however, subjects were

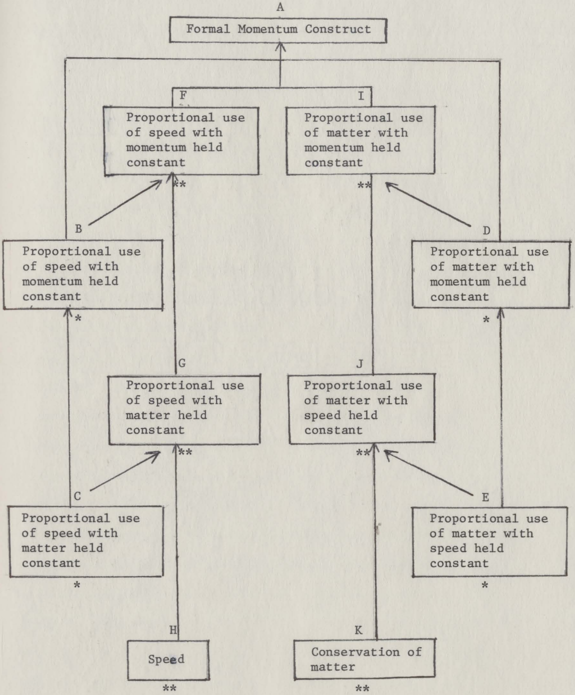


Figure 5: Hypothesized Logical Hierarchy

Note: *Qualitative Skill **Quantitative Skill required to answer only tasks representing the qualitative portion of the hierarchy, it being assumed from an examination of the curriculum at these grades that quantitative test items would not yet be meaningful to them. Subjects for this study were selected from grades kindergarten to three and were only required to solve problems in a qualitative manner. For Raven's tasks the concept of momentum required little or no formal meaning. Words like "push" or "pull" were taken as a meaning for the word momentum depending on how they were used. It is argued at this point, however, that since a complete learning hierarchy leading to the concept of momentum necessarily implies skilled quantitative performance it was not possible for Raven to construct a learning hierarchy which could be validated on a sample of children only up to grade three. If his work was correct we might expect the qualitative responses he described to be replicated with the younger and less well intellectually developed members of the sample of this present study. However, his study could not refer properly to the failure of the "logical" hierarchy because it could not be adequately tested on his restricted sample.

For the very young child, understanding of momentum may be related to sensory experience of natural phenomena. The word "push" as mentioned, depending on how it was used, may be an elementary form of the concept of momentum. It is suggested by the present author that in the very early stages mass and speed are not manipulated separately but exist as one in the push exerted on an object. Later, the child may develop a qualitative understanding of mass and speed and be able to manipulate them separately. However, it is suggested that before the child will learn to make comparisons between two different objects he will independently manipulate the mass and speed of various objects. For example, in moving something heavy the young child may realize that a heavy object requires a greater push, moves slowly, does not travel a great distance, but at a given speed exerts a greater force on another object; whereas the opposite is found to be true for something light. With regard to speed, the child begins to realize that the faster you push something the further it will move an object. At this stage, proportionality refers to a qualitative response such as "more than," "less than," "faster than," "slower than" and does not as yet represent numerical proportionality. It is suggested that it is not until later that the young child truly learns to manipulate these variables proportionately, and further, that the quantitative stage does not follow until the child has correctly internalized these skills. Failure to do so may result in misconceptions being internalized which result in the child's inability to understand and perform higher order tasks.

In the qualitative stage the child will have only an intuitive understanding of mass and speed. For example, with regard to mass the child may be easily distracted by such variables as shape and size. The very young child may have the misconception that large objects are heavy and small objects are light. It is only when the child realizes that the mass of an object is not dependent on its size that he can manipulate mass proportionately in a qualitative manner. With regards to speed the young child may focus his attention on such factors as which car arrived first or whether one car passed another, rather than on the distance-travelled-over-time relationship. In order to consistently solve a speed problem the subject must be able to solve the distance/time relationship. Hence, for this investigator speed is quantitative in nature and is placed in the quantitative portion of the hierarchy. For this reason, it is also reasonable to expect speed to be more difficult than all the other tasks in Raven's psychological hierarchy. It is not until the child has a more advanced understanding of mass and speed that he can enter into the quantitative stage. No qualitative tasks were used for either mass or speed because they would have involved only the identification of which of two objects is heavier or which of two objects is moving faster. It was considered that such skills would have been developed by this age.

It must be understood that in the qualitative portion of the logical hierarchy the meaning of the word proportionality has a different meaning than when used in the quantitative portion. As mentioned previously, qualitative proportionality may refer to a "more than," "less than" relationship while in the quantitative proportionality an exact numerical relationship must be expressed.

The design of this hierarchy was discussed with other science educators to determine its reasonableness and changes were made regarding some of the steps in the hierarchy. As mentioned previously, Raven's hierarchy was a linear one and represented five fairly large steps. In the present study, an attempt was made to minimize step size in order to avoid missing important skills. Numerical skills, although they were a necessary part of the quantitative portion of the hierarchy, were, nevertheless, kept as simple as possible. Attention, instead, was paid to the logic inherent in the responses. Answers were in the written form and all work was shown. Responses requiring mere verbalization of recalled information were not encouraged, as such responses do not necessarily imply the acquisition of a skill, rather the memorization of a definition or fact.

Sample

The sample consisted of 197 subjects selected by class from grades one to eight in a St. John's school. Classes were selected randomly. There were 93 boys and 104 girls. The number of students per grade, the mean age and the standard deviation for each class are given in Table 1. Only one school, with a population of approximately 940 students from grades K-11, was involved. The population of the school represented a variety of socioeconomic backgrounds.

Procedure

The tasks testing the skills within the hypothesized hierarchy were piloted with one class of grade five students in a St. John's school. After feedback was obtained from the test data and from consultation with teachers, changes were made with regard to the appropriateness of the reading level to the grades being tested and to the test items themselves. Further discussion with educators in this field resulted in refining certain skills within the hierarchy and deleting others.

The procedure involved taking classes intact. Each class was brought to the testing area by the classroom teacher, who then left. The purpose of the study was then explained to the students who were assured that they were not being given an examination and that the results of the study would not be used by their school for placement or

Table 1

Grade, Number of Subjects, Mean Age, Standard Deviation

Grade	Number of Subjects	Mean Age	Standard Deviation
1	10	6.93	.38
2	11	8.25	.64
3	24	8.98	.44
4	31	10.06	.60
5	28	11.43	1.57
6	32	12.32	.71
7	29	13.19	.88
8	32	14.25	.89

N = 197

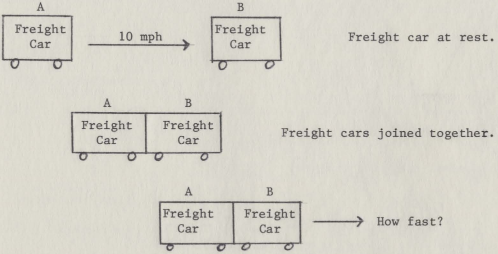
grading purposes. The order in which the grades were tested was 8, 6, 4, 2, 7, 5, 3,1 to prevent as much as possible immediate contact between similar age groups. The students were also asked not to discuss the questions or activities with fellow students after the testing period. Classes were taken on the average of three times each for 50-minute periods. However, there was no limit on the test in time. Therefore, some classes may have completed the study in two periods while others may have needed four. Test booklets were placed face down on each student's desk prior to the testing time and students were required to turn to a particular test item only at the direction of the investigator. Each question had to be completed by all students in the class before the next one could be attempted. There was a total of 13 questions with at least two parts each, each part being considered as a separate item. All questions with their individual sections were demonstrated by the investigator and all students were required to work out their answers on the question booklets provided. For grades three to eight all answers were written on the question booklet which contained both the qualitative and quantitative sections. However, grades one and two were provided only the qualitative portion of the booklet as the quantitative section required skills that were considered beyond their experience. Also, because of the expectation of limited writing skills for students in these two grades, all responses to the questions were written down by the investigator and an assistant. For this reason the number of subjects taken from grade one had to be limited to ten and for grade two, eleven.

Two weeks after all subjects had completed the tasks a sample of five students from each of the classes already tested were retested for the same skills, to check for consistency of response between group and individual administration.

Test Instrument

All skills present in the hypothesized hierarchy were tested by at least two questions per skill as prescribed by White (1973) and were randomly dispersed throughout the question sheet. A total of 11 skills were tested. Each class progressed through the question booklet in the same order but not necessarily the same rate. They were allowed as much time as they needed to complete the questions. The booklet contained a graphic illustration of each test item. Each item was demonstrated with appropriate physical apparatus and read aloud by the investigator. Every effort was made to insure that each subject understood the problem. The subjects were asked if they understood what they had to do. If it was requested, or if some subject appeared to be having difficulty, the whole demonstration was repeated. The subjects were then given an opportunity to work out their solutions to the test item in the question booklet. For each answer a justification was required. The correctness of the response was *in all cases* judged on the basis of correct answer and correct explanation. A copy of the question booklet appears in Appendix A. The following is an example of one of the problems the subjects were required to complete to demonstrate possession of the terminal skill representing momentum:

A freight car moving along a track collides with another freight car which is at rest. If the freight cars are of equal mass and are coupled by the collision, how does the speed of the coupled cars compare with the initial speed of the single moving freight car? The speed of the single car was 10 miles per hour.



What was the speed of the coupled freight cars?____________________

Why do you think so?____

____

Chapter 4

Analysis of Results

Introduction

The subjects for this study were children from grades one to eight in a St. John's school. All classes were presented with the test items in the same order. The overall test is essentially a collection of two-item tests, one test per skill. The responses to these tests served as the basis for testing the validity of the links between skills within the hypothesized hierarchy. Therefore, the validity and reliability of each test is of much importance. This chapter will begin with a discussion of the parameters of the tests, followed by the results of applying the White and Clark test of inclusion to the data. Although the White and Clark test was used as the main test of the data, the "ordering-theoretic" method was used to determine the probable strength of a connection and to diagnose problems with respect to specific skills. A specially written computer program (Cornish, R., 1978) for the White and Clark test of inclusion was used. All other statistical procedures were performed using the SPSS 300 statistical package (Nie et al, 1975).

Validity of the Test Items

To ensure good construct validity the test items and the hypothesized hierarchy were examined by two science educators. Also the tasks testing the skills within the hypothesized hierarchy were piloted with one class of grade five students in a St. John's school. In a few cases changes were made to test items, either because it was thought that they failed to test the skill or because they were worded inappropriately.

Reliability of the Test Items

The White and Clark test, although not requiring a mastery decision, requires that the items testing a particular skill should exhibit low inter-item variance. Given the fact that the overall test really represents a number of two-item tests, conventional reliability statistics are not meaningful. In order to provide some evidence of reliability the phi correlation was used as an index of the degree of correlation between two items testing the same skill. Ideally, a correlation coefficient of one between two test items testing the same skill should be obtained. However, in practice such a perfect correlation is seldom found as individual items, while representing the same domain, may not be identical in structure or presentation. Moreover, the values of phi coefficients are affected by the marginal totals of the contingency tables from which they are determined. Hence, while perfect correlations could not be expected, phi coefficients significantly greater than zero were considered necessary between both items testing the same skill. The values obtained are represented in Table 2. These values may indicate either the strength or weakness of a particular relationship between test items testing the same skill. The value of N varies between tests partly because some subjects were absent for a particular testing session or because some subjects' responses were difficult to interpret and thus were treated as having missed that test item. Also, as previously mentioned, 20 of the subjects from grades one and two were not administered the quantitative portion of the test as an inspection of the curricula for these grade levels indicated that

Table 2

Phi Coefficients Between Items Testing the Same Skill

Skill	Test Items	Number of Subjects	Phi Coefficient	Significance Level
First Momentum Task L_1	01,02	189	.58	.001*
Second Momentum Task L_2	03,04	186	.23	.01 *
Formal Momentum Task A	05,06	165	.24	.01
Proportional Use of Matter with Speed Held Constant (Quantitative) J	07,08	167	.60	.001
Proportional Use of Matter with Speed Held Constant (Qualitative) E	09,10	188	.77	.001
Proportional Use of Matter with Momentum Held Constant (Qualitative) D	11,12	187	.86	.001*
Proportional Use of Matter with Momentum Held Constant (Quantitative) I	13,14	167	.74	.001
Proportional Use of Speed with Matter Held Constant (Qualitative) C	15,16	187	.40	.001
Proportional Use of Speed with Matter Held Constant (Quantitative) G	17,18	169	.58	.001
Conservation of Matter K	19,20	189	.68	.001*
Speed H	21,22	189	.68	.001*
Proportional Use of Speed with Momentum Held Constant (Qualitative) B	23,24	186	.79	.001*
Proportional Use of Speed with Momentum Held Constant (Quantitative) F	25,26	167	.75	.001

Note: *Skill used by both Raven and present investigator.

the skills necessary for the performance of these tasks may not yet be developed. The values of the phi coefficients reported in Table 2 are almost all significantly different from zero at the .001 level of confidence. However, several anomalies are apparent and these will be discussed.

The phi coefficient for test items 3 and 4 representing Raven's second momentum task was relatively low. This, in part, could be due to the fact that although both test items were representative of the skill 'momentum' they were not identical and thus subjects' responses to these test items differed. The low phi coefficient for test items 5 and 6 could be accounted for on the basis that of the 36 subjects getting test item 5 correct and of the 42 subjects getting test item 6 correct only 16 of these subjects got both test items correct. Of these 16 subjects, however, 14 subjects were from grades 7 and 8. This may indicate that subjects below these grade levels may not yet have completely acquired the skills necessary to perform this task, possibly due to the lack of appropriate experiences within the curricula or the stage of intellectual development of the subjects.

Further evidence of reliability was sought by re-testing a small proportion of the subjects on an individual basis, in order to provide some evidence of whether the group-testing procedure yielded results similar to those which would have been obtained by individual testing. In this regard two weeks after the initial testing five subjects were selected from each of the previously tested classes and retested on two of the same test items in individual sessions. Consideration was given to using statistical tests based upon contingency tables (e.g. chi square, Cohen K) to determine the significance of the relationship between the test and retest data for each item. However, the potential for distortion of marginal totals because of the small number of students who could be retested rendered such tests less meaningful. Hence, the less theoretically pleasing method of reporting percentage agreement was adopted. These data are presented in Table 3.

For the most part agreement is generally reasonable. The potential for learning in the first testing situation and the small number often involved mitigate against more substantial agreement. Test items 17 and 25, because of the low percentage of agreement between the test and retest, could be considered unreliable. However, it could be due to random fluctuation within the sample. Because of constraints within the school only a small number of subjects were available for the retest and these may not have been representative of the larger sample. For this reason these items were not discarded from the test. For the same reason not all test items could be retested. However, with the exception of those eliminated during the field test an attempt was made to retest one test item per skill.

Tests Applied to the Data

The modified form of the White and Clark test of inclusion was used to determine the validity of the connections between pairs of skills in the hypothesized hierarchy at the 00, 01 and 02 levels of stringency. The literature suggests that substantial rather than absolute hierarchical dependency is a suitable test for the validity of the connections between skills. White (1974) prefers the absolute criterion of no exceptions other than those attributable to errors of measurement.

Table 3

Percentage of Agreement Between Test and Retest

Test Item	Number of Subjects	Percentage Agreement Test/Retest
5	10	90%
7	10	70%
9	23	52%
11	13	69%
13	10	60%
15	21	95%
17	9	44%
19	15	100%
21	9	67%
23	14	71%
25	9	44%
26	5	80%

Linke (1975) suggests a 2% criterion, while Beeson (1977) allows 5% exceptions in addition to those representing measurement error. The investigator prefers the recommendation of Griffiths (1979) that the level of stringency be relaxed until the point is reached when the number of bi-directional connections increases. The data in the present study, as in Griffiths' (1979), indicate that this occurs when a 2% criterion is applied. However, for the readers' interest the data obtained at the 00, 01 and 02 levels are presented. In comparison to other tests available at this time the White and Clark test appears to be the most sophisticated and most appropriate test for determining the existence of these hierarchical connections. This test makes use of more information, does not require a mastery decision and provides a more rigorous statistical test. It is applied to all pairs of skills within the hierarchy as suggested by the data or where there is the possibility of a logical connection. However, in cases where N is less than 100 the power of the test becomes relatively small. In such instances the 'ordering-theoretic' method was used as the primary test of the data. As in the case of the White and Clark test all possible logical connections were considered. Based on Griffiths' (1979) findings, hierarchical connections were tested allowing successively for one, two and five percent exceptions. In comparing the results of applying the 'ordering-theoretic' method and the White and Clark test to the same data, Griffiths found that the results of applying the 'ordering-theoretic' method at the 1% level is most similar to the results of applying the White and Clark test at the 00 level. At the 2% level the result of using the 'ordering-theoretic' method is most similar to that for the White and Clark test at the 01 level. Finally, at the 5% level, the result of using the 'ordering-theoretic' method is most similar to that for the White and Clark test at the 02 level. For Griffiths (1979), little or no difference existed in the hierarchies suggested at each level of stringency when the 'ordering-theoretic' method was used in place of the White and Clark test. Griffiths found that particularly at the least stringent level the 'ordering-theoretic' method may be substituted for the White and Clark test of inclusion.

Research question one asks "Is Raven's developmental hierarchy substantiated (a) when tasks the same or equivalent to his are applied to a new sample? (b) by new tasks testing the same skills when applied to a new sample?" Research question two asks "Can a valid logical hierarchy leading to understanding of the concept of momentum be identified?"

Whether the answer to research question one is affirmative or negative, research question two will be considered because, as was pointed out in Chapter 2, Raven's logical hierarchy was incorrectly constructed and hence his claim that his psychological hierarchy was superior to his logical hierarchy was unfounded.

Application of the 'Ordering-Theoretic' Method to the Test Data for Raven's Psychological Hierarchy

According to Raven, children go through the following concept sequence: *momentum* ⟶ *conservation of matter* ⟶ *proportional use of mass with momentum held constant* ⟶ *proportional use of speed with momentum held constant* ⟶ *speed*. The responses of 45 subjects from grades 1, 2 and 3 were analyzed to determine if this sequence was substantiated by the data. The number of subjects for this portion of the study was small because of the age of the subjects and the type of testing procedure employed. For this reason the 'ordering-theoretic' method was used as the primary test of the data because as previously mentioned the power of the White and Clark test becomes very small when N is less than 100. The test was applied to all pairs of skills in the order indicated by Raven to be appropriate. The results of applying the 'ordering-theoretic' method to the data are represented in Table 4. The table contains the total number of entries in the critical cells for both the upper and lower skills. The data are presented with the total number of exceptions for the upper skill over the total number of exceptions for the lower skill with the levels at which the connection is accepted. Because Raven's psychological hierarchy is a linear one while the logical hierarchy hypothesized by the present investigator is a branched one and some of the tasks are the same, the letters used to designate the skills in Raven's psychological hierarchy are as follows: *speed* (H), *proportional use of speed with momentum held constant* (B), *proportional use of matter with momentum held constant* (D), *conservation of matter* (K), *first momentum task* (L_1), *second momentum task* (L_2).

The data in Table 4 indicate that the only agreement with Raven's findings were that *speed* (H) was superordinate to the skills *proportional use of speed with momentum held constant* (B) and *conservation of matter* (K). However, with respect to the skill *speed* (H) being superordinate to the skills *proportional use of matter with momentum held constant* (B) and *conservation of matter* it may be argued that *speed* is a quantitative skill while Raven's other skills were qualitative in

Table 4

Ordering-Theoretic Method at Three Levels of Exceptions for Raven's Psychological Hierarchy: Number of Exceptions for Upper and Lower Skill and Level at which Connection is Valid (Grades 1, 2 and 3)

Upper Skill / Lower Skill	Number of Subjects	Number of Exceptions for Upper/Lower Skill	Level of Exceptions 1%	2%	5%
H/B	43	2/8			1
H/D	43	5/5			
H/K	45	1/13			1
H/L_1	45	4/8			
H/L_2	44	3/11			
B/D	43	9/4			
B/K	43	6/13			
B/L_1	43	6/9			
B/L_2	42	5/9			
D/K	43	4/17			
D/L_1	43	5/10			
D/L_2	42	4/13			
K/L_1	45	8/3			
K/L_2	44	8/7			
L_1/L_2	44	3/5			

Note: 1) The reader's understanding of this and similar tables may be aided by an illustrate example. For skills H and B, 2 subjects out of 43 exhibited skill B but not H, while 8 out of 43 exhibited skill H but not skill B. The connection is accepted at the 5% level of exceptions.

2) 1 = validated connection, blank = non-validated connection. In column 3, in each case the first number represents the number of exceptions for the connection in the direction shown. The second number represents the corresponding number when the direction of the connection is reversed.

H - Speed

B - Proportional Use of Speed with Momentum Held Constant

D - Proportional Use of Matter with Momentum Held Constant

K - Conservation of Matter

L_1 - First Momentum Task

L_2 - Second Momentum Task

nature, thus it is reasonable to expect *speed* to be superordinate to these tasks. Children at this age level may not be able to solve quantitative problems regarding the distance-travelled-over-time relationship but may be able to manipulate variables proportionately but in a qualitative manner. Hence, research question one has been answered negatively. Only one out of the five hierarchical relationships found by Raven was substantiated.

Although the application of the 'ordering-theoretic' method to the present data indicates that Raven's psychological hierarchy is not supported by the findings of this study an examination of the data does suggest the order of difficulty indicated by Raven to be correct. For example, *speed* (H) was found to be more difficult than all the other skills in the hierarchy. The *proportional use of speed with momentum held constant* (B), although not superordinate to the *proportional use of matter with momentum held constant* (D), was found to be more difficult than all the other skills hypothesized by Raven. The *proportional use of matter with momentum held constant* (D) was found to be more difficult than the skills *conservation of matter* (K) and the *first* (L_1) and *second* (L_2) *momentum tasks*. However, the skill *conservation of matter* (K) was not found to be more difficult than either the *first* (L_1) or *second* (L_2) *momentum tasks*. These data indicate support for White's (1973) contention that the order of difficulty of Raven's tasks, while consistent with a hierarchy does not necessarily imply the existence of a hierarchy. Hence, Raven's psychological hierarchy is not supported by the present data. In addition, the comments presented earlier suggesting that Raven's logical hierarchy is faulty indicate the need to consider a new logical hierarchy. The application of the White and Clark test to the data for the new logical hierarchy is presented in the section which follows.

Application of the White and Clark Test to
the Test Data for the Logical Hierarchy

In response to research question two, the White and Clark test was applied to the test data at three levels of stringency to determine if a more valid logical hierarchy derived by a Gagne-type task analysis could be identified. According to the hypothesized hierarchy all skills should be subordinate to the terminal skill representing momentum, e.g. the *proportional use of matter with speed held constant* (E) should be subordinate to the *proportional use of matter with momentum held constant* (D) which should be subordinate to the *formal momentum construct* (A). Four of the skills, *speed* (H), *conservation of matter* (K), *proportional use of speed with momentum held constant* (B) and the *proportional use of matter with momentum held constant* (D), were included in Raven's hierarchy. The results of applying the White and Clark test at the 00, 01 and 02 levels of stringency to the qualitative and quantitative portions of the hierarchy are indicated in Tables 5 and 6, respectively. The results of applying the 'ordering-theoretic' method at the 1%, 2% and 5% levels of exceptions are presented in the same tables for the readers' interest.

For the qualitative portion of the hierarchy concerned with speed the *terminal skill* (A) was not found to be superordinate to the skill *proportional use of speed with momentum held constant* (B) but was superordinate to the skill *proportional use of speed with matter held*

Table 5

Application of the White and Clark Test at Three Levels of Stringency and the 'Ordering-Theoretic' Method at Three Levels of Exceptions to the Qualitative Test Data: Number of Exceptions for Upper and Lower Skills and Level at which Connection is Valid (complete sample)

Upper Skill / Lower Skill	Number of Subjects	Number of Exceptions	Level of Exceptions White & Clark Test 00	01	02	'Ordering-Theoretic' Meth 1%	2%	5%
A/B	158	8/27		1	1			
A/C	161	0/63	1	1	1	1	1	1
B/C	182	4/75		1	1			1
A/D	164	5/42	1	1	1			1
A/E	164	3/42	1	1	1		1	1
D/E	186	14/34						

Note: 1 = validated connection; blank = non-validated connection

In column 3, in each case the first number represents the number of entries in the critical cell for the connection in the direction shown. The second number represents the corresponding number when the direction of the connection is reversed.

A - Formal Momentum Task

B - Proportional Use of Speed with Momentum Held Constant

C - Proportional Use of Speed with Matter Held Constant

D - Proportional Use of Matter with Momentum Held Constant

E - Proportional Use of Matter with Speed Held Constant

Table 6

Application of the White and Clark Test at Three Levels of Stringency and the 'Ordering-Theoretic' Method at Three Levels of Exceptions to the Quantitative Test Data: Number of Exceptions for Upper and Lower Skills and Level at which Connection is Valid (complete sample)

Upper Skill / Lower Skill	Number of Subjects	Number of Exceptions	Level of Exceptions White & Clark Test 00	01	02	'Ordering-Theoretic' 1%	2%	5%
A/F	158	9/5						
A/G	162	2/26	1	1	1		1	1
A/H	104	0/13	1	1	1	1	1	1
F/G	164	3/8	1	1	1		1	1
F/H	109	2/23	1	1	1		1	1
G/H	108	5/10	1	1	1			1
A/I	163	6/15	1	1	1			1
A/J	164	3/14	1	1	1		1	1
A/K	158	0/77	1	1	1	1	1	1
I/J	166	14/22						
I/K	161	3/92		1	1		1	1
J/K	160	1/75	1	1	1	1	1	1

Note: 1 = validated connection; blank = non-validated connection

In column 3, in each case, the first number represents the number of entries in the critical cell for the connection in the direction shown. The second number represents the corresponding number when the direction is reversed.

A - Formal Momentum Task

F - Proportional Use of Speed with Momentum Held Constant

G - Proportional Use of Speed with Matter Held Constant

H - Speed

I - Proportional Use of Matter with Momentum Held Constant

J - Proportional Use of Matter with Speed Held Constant

K - Conservation of Matter

constant (C) at the 00 level of stringency. However, at both the 01 and 02 levels it was found to be superordinate to the *proportional use of speed with momentum held constant* (B) and to the *proportional use of speed with matter held constant* (C) in the order hypothesized by the investigator. For the corresponding quantitative portion of the hierarchy the *terminal skill* (A) was found to be superordinate to the *proportional use of speed with matter held constant* (G) and *speed* (H) in the hypothesized order at all three levels of stringency. However, while the *proportional use of speed with momentum held constant* (F) was found to be superordinate to the *proportional use of speed with matter held constant* (G) and *speed* (H) at all three levels of stringency, it was not subordinate to the *terminal skill* (A) at any level. The number of exceptions in the critical cell when considering A and F, respectively, were not sufficiently different to indicate a connection in either direction. Retrospective examination of the test items suggests that while F may be logically subordinate to skill A, the difficulties inherent in the test items are similar. Hence, in the sense of a psychometric hierarchical dependency, no connection is observed. This does not deny the possibility of a hierarchical dependency in the transfer sense. Unfortunately the scope of the present study does not allow for a test of this .

For the qualitative portion of the hierarchy concerned with matter the *terminal skill* (A) was superordinate to the skills *proportional use of matter with momentum held constant* (D) and the *proportional use of matter with speed held constant* (E) at all three levels of stringency. However, the *proportional use of matter with momentum held constant* (D) was not superordinate to the *proportional use of matter with speed held constant* (E) at any of the three levels of stringency. Although the connection between the skills D and E is not accepted by the White and Clark test at any of the three levels of stringency the difference between the number in the critical cell for these skills is substantial enough to suggest that the hypothesized order may have merit. However, because the test for a hierarchical relationship was not statistically significant the connection between these skills was eliminated from the hierarchy.

Correspondingly the quantitative skills representing the *proportional use of matter with momentum held constant* (I) and the *proportional use of matter with speed held constant* (J) were both subordinate to the *terminal skill* (A) but neither was subordinate to the other at any level of stringency for the White and Clark test. As in the case of the qualitative skills, the data is consistent with the hypothesized direction. However, for the quantitative skills the strength of the hierarchical connection is much weaker. This could be due in part to the fact that those subjects who were unable to perform the qualitative skills were also unable to perform the corresponding quantitative skills and others may have failed to succeed because of their lack of numerical skills.

At the 00 level of stringency only the quantitative skill *proportional use of matter with speed held constant* (J) was found to be superordinate to the *conservation of matter* (K). However, at both the 01 and 02 levels of stringency the skills *proportional use of matter with momentum held constant* (I) and the *proportional use of matter with* *speed held constant* (J) were both found to be superordinate to the *conservation of matter* (K).

To summarize, with few exceptions, the responses of the subjects tested exhibited the hierarchical pattern indicated in the logical hierarchy hypothesized by the investigator, with the exception of the connections between the skills *proportional use of speed with momentum held constant* (F) and the *formal momentum construct* (A), the qualitative skills *proportional use of matter with momentum held constant* (D) and the *proportional use of matter with speed held constant* (E) and the parallel quantitative skills *proportional use of matter with momentum held constant* (I) and the *proportional use of matter with speed held constant* (J). Because the quantitative skill *proportional use of speed with momentum held constant* (F) was not found to be subordinate to the *terminal skill* (A) at any level of stringency and did not indicate any direction, this skill was eliminated from the quantitative portion of the hierarchy. Further, although the connections between the qualitative skills *proportional use of matter with momentum held constant* (D) and the *proportional use of matter with speed held constant* (E) and the parallel quantitative skills I and J were in the hypothesized direction, they were not statistically significant and therefore were eliminated from the hierarchy. The qualitative and quantitative hierarchies that emerge from the application of the White and Clark test at the 01 and 02 levels of stringency are represented in Figures 6 and 7, respectively.

The emergence of different hierarchies at different levels of stringency poses the question of which hierarchy is the more appropriate. Griffiths (1979) argues that although it would seem that the more strin-

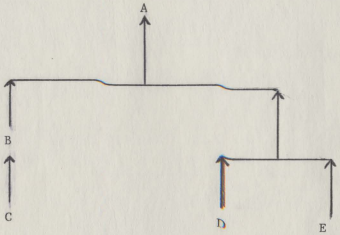


Figure 6: Hierarchy One from application of the White and Clark Test at the 01 and 02 level to the qualitative test data

A - Formal Momentum Task

B - Proportional Use of Speed with Momentum Held Constant

C - Proportional Use of Speed with Matter Held Constant

D - Proportional Use of Matter with Momentum Held Constant

E - Proportional Use of Matter with Speed Held Constant

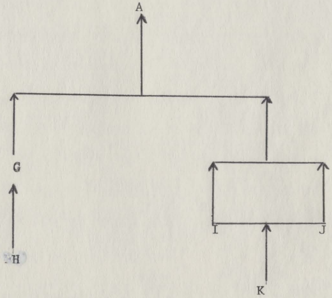


Figure 7: Hierarchy Two from application of the White and Clark test at the 01 and 02 levels to the quantitative test data

A - Formal Momentum Task

G - Proportional Use of Speed with Momentum Held Constant

H - Speed

I - Proportional Use of Matter with Momentum Held Constant

J - Proportional Use of Matter with Speed Held Constant

K - Conservation of Matter

gent the test the more certain one can be of the validity of the hierarchy, the hierarchy established at the less stringent level may be the most informative. At too strict a stringency level the hierarchy may become too small to be of practical use. Griffiths argues further that in the absence of any set criteria the optimum stringency level may be that at which the number of uni-directional connections begins to decrease. In the present study, the consistency of fit at the 01 and 02 levels of stringency suggests that analysis should stop at this point.

Application of the White and Clark Test to the Test Data for the Qualitative-Quantitative Connections within the Hypothesized Hierarchy

The connections between the qualitative and quantitative portions of the hierarchy were analysed to determine whether the acquisition of the quantitative skills was dependent upon the prior acquisition of the qualitative skills. This analysis was performed for both the qualitative and quantitative skills representing the *proportional use of speed with momentum held constant*, the *proportional use of speed with matter held constant*, the *proportional use of matter with momentum held constant* and the *proportional use of matter with speed held constant*. The results of this analysis are presented in Table 7.

The application of the White and Clark test to the data at both the 00 and 01 levels of stringency indicates that the performance of the quantitative skills representing the *proportional use of speed with matter held constant* and the *proportional use of matter with speed held constant* are dependent upon the prior acquisition of the parallel quantitative skills. Further analysis of the data at the 02 level of strin-

Table 7

Application of the White and Clark Test at Three Levels of Stringency and the 'Ordering-Theoretic' Method at Three Levels of Exceptions to the Qualitative-Quantitative Connections within the Hypothesized Hierarchy: Number of Exceptions for Upper and Lower Skills and Level at which Connection is Valid

Upper Skill / Lower Skill	Number of Subjects	Number of Exceptions	Level of Exceptions White & Clark Test 00	01	02	'Ordering-Theoretic' Meth 1%	2%	5%
F/B	167	7/26			1			1
G/C	167	2/42	1	1	1		1	1
I/D	167	10/39						
J/E	167	3/35	1	1	1		1	1

Note: 1 - validated connection, blank = non-validated connection

In column 3, in each case, the first number represents the number of entries in the critical cell for the connection in the direction shown. The second number represents the corresponding number when the direction of the connection is reversed.

F = Proportional Use of Speed with Momentum Held Constant (Quantitative

B = Proportional Use of Speed with Momentum Held Constant (Qualitative)

G = Proportional Use of Speed with Matter Held Constant (Quantitative)

C = Proportional Use of Speed with Matter Held Constant (Qualitative)

I = Proportional Use of Matter with Momentum Held Constant (Quantitative)

D = Proportional Use of Matter with Momentum Held Constant (Qualitative)

J = Proportional Use of Matter with Speed Held Constant (Quantitative)

E = Proportional Use of Matter with Speed Held Constant (Qualitative) gency also confirms this connection for the qualitative and quantitative skills representing *proportional use of speed with momentum held constant* but not for the skills representing *proportional use of matter with momentum held constant*. Examination of the data by the 'ordering-theoretic' method, represented in Table 8, reveals that 6% of the subjects tested for the skill *proportional use of matter with momentum held constant* were able to perform the quantitative skill without the prior acquisition of the qualitative skill. A total of 23.4% were able to perform the qualitative skill but not the quantitative skill. Although this connection is not significant at any level of stringency for the White and Clark test or at the 5% level of exceptions for the 'ordering-theoretic' method and is not included in the hierarchy, the data is consistent with the direction hypothesized by the investigator, that the performance of the quantitative skills representing the *proportional use of matter with momentum held constant* is dependent upon the successful performance of the corresponding qualitative skill. A total of 40.1% of the subjects tested were unsuccessful for both the qualitative and quantitative tasks representing this skill. This may indicate that a large percentage of the subjects tested did not possess an understanding of proportionality or that essential subskills leading to the attainment of this concept may not be represented within the hypothesized hierarchy. These results, in connection with those from the qualitative and quantitative data, are expressed graphically in the hierarchy represented in Figure 8.

Table 8

Percentage of Subjects Getting Both Test Items Correct or Incorrect for the Qualitative-Quantitative Connections within the Hypothesized Hierarchy

Upper Skill / Lower Skill	Number of Subjects	Number of Test Items Correct for Upper Skill / Number of Test Items Incorrect for Lower Skill 2/0	0/2	0/0	2/2
F/B	167	4.2%	15.6%	52.1%	10.2%
G/C	167	1.2%	25.1%	7.2%	34.1%
I/D	167	6.0%	23.4%	40.1%	13.8%
J/E	167	1.8%	21.0%	29.3%	20.4%

Note: 2/0 - Percentage of subjects getting both test items correct for upper skill while getting neither test item correct for the lower skill

0/2 - Percentage of subjects getting neither test item correct for the upper skill while getting both test items correct for the lower skill

0/0 - Percentage of subjects getting neither test item correct for either the upper or lower skill

2/2 - Percentage of subjects getting both test items correct for both the upper and lower skills

F - Proportional Use of Speed with Momentum Held Constant (Quantitative)

B - Proportional Use of Speed with Momentum Held Constant (Qualitative)

G - Proportional Use of Speed with Matter Held Constant (Quantitative)

C - Proportional Use of Speed with Matter Held Constant (Qualitative)

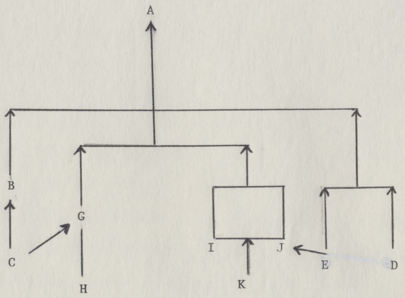


Figure 8: Hierarchy Three from application of the White and Clark test at the 01 and 02 levels to the test data

A - Formal Momentum Task

B - Proportional Use of Speed with Momentum Held Constant (Qualitative)

C - Proportional Use of Speed with Matter Held Constant (Qualitative)

G - Proportional Use of Speed with Matter Held Constant (Quantitative)

H - Speed

D - Proportional Use of Matter with Momentum Held Constant (Qualitative)

I - Proportional Use of Matter with Momentum Held Constant (Quantitative)

E - Proportional Use of Matter with Speed Held Constant (Qualitative)

J - Proportional Use of Matter with Speed Held Constant (Quantitative)

K - Conservation of Matter

Subject Misconceptions Relating to Specific Skills Pertaining to the Attainment of the Skill Conservation of Momentum

In response to test items representing particular skills it was found that some of the subjects consistently answered certain test items incorrectly according to a particular misconception. Further analysis of these misconceptions may provide additional insight as to how a child arrives at an understanding of the proportionality relationship between mass and speed in problems on momentum. To demonstrate an understanding of conservation of momentum subjects must be able to vary mass and speed proportionately. Those subjects who did not possess this skill failed to understand that in those tasks testing for conservation of momentun the product of the mass and speed of one cart must equal the product of the mass and speed of the second. Some subjects concentrated on manipulating the mass of the cart while others manipulated the speed but not in a proportional manner. For example, in the formal momentum task 30% of those subjects who responded incorrectly thought that because the two freight cars were coupled by a collision, their weight was doubled and correspondingly their speed was doubled. In the second formal momentum task, in which one freight car collided with a second and itself came to a rest, 49% of the subjects responded that because of the collision the speed of the second car was halved. This again supports the argument for the first momentum task that those subjects did not possess an appropriate concept of conservation of momentum and thus compensated incorrectly.

In the task representing the skill *proportional use of matter with momentum held constant*, 35% of the subjects for the qualitative test items and 32% for the quantitative test items, when asked "which cart would have to be made heavier in order to get both blocks to move the same distance," incorrectly compensated by making the faster cart heavier to slow it down to the same speed as the other cart. From the responses it would appear that the subjects believed that if both carts went the same speed they would push two blocks of equal weight the same distance regardless of the weights of the carts.

For the qualitative items testing the skill *proportional use of matter with speed held constant*, 10% of the subjects responded that because both carts moved at the same speed they would move two blocks the same distance, regardless of the different weights of the two carts. For the quantitative test items for this skill, 8% of the subjects responded in the same manner. For the same skill for the qualitative test items, 26% of the subjects disregarded the interactive effects of mass and speed for both carts. For the quantitative test items, 14% of the subjects answered similarly, e.g. "If a cart is heavy it will move slower and push the block a shorter distance," or "If a cart is light it will move faster and push the block a greater distance," rather than comparing the products of the mass and speed for both carts. A total of 46% of the subjects tested for the qualitative portion of the skill *proportional use of speed with momentum held constant*, and 28% of those tested on the parallel quantitative items, incorrectly compensated by making the heavier cart faster rather than making the lighter cart faster to give them the same momentum. Fifteen percent stated that if both carts were at the same speed they would push two blocks of equal weight the same distance regardless of the weight of the carts. For the qualitative items testing the skill *proportional use of speed with matter held constant*, 6% responded that if both carts weighed the same they would move two blocks of equal weight the same distance.

Summary

Data obtained by presenting subjects in grades one to eight with a number of tasks relating to the concept of momentum were analysed by application of the Griffiths and Cornish (1978) modification of the White and Clark test of inclusion and the 'ordering-theoretic' method. Because of the small number of subjects involved and because the 'ordering-theoretic' method at the 5% level of exceptions was found to reveal results quite similar to those obtained by the White and Clark test at the 02 level of stringency, the 'ordering-theoretic' method was applied to the test data for subjects in grades 1, 2 and 3 for Raven's psychological hierarchy. Data obtained from subjects in grades one to eight for the new logical hierarchy were analysed by the White and Clark test of inclusion.

The data obtained in this study do not support Raven's contention that young children develop an understanding of the concept momentum in accordance with a psychologically derived hierarchy, rather than a logical hierarchy. Although an examination of the data does reveal the general order of difficulty hypothesized by Raven to be correct, only one out of five of the hierarchical relationships was significant at the 5% level of exceptions for the 'ordering-theoretic' method. With little change, a logical hierarchy hypothesized by the present investigator is substantiated.

Although the application of the White and Clark test at three levels of stringency does not support the hierarchy in its entirety, the best fit of the data to the hypothesized hierarchy occurs at both the 01 and 02 levels of stringency. While the skill *proportional use of speed with momentum held constant* (F) was found to be superordinate to the *proportional use of speed with matter held constant* (G) and to the understanding of *speed* (H), it was not found to be subordinate to the *formal momentum construct* (A). For this reason, this skill was eliminated from the quantitative portion of the hierarchy. The skill *proportional use of matter with speed held constant* was not subordinate to the skill *proportional use of matter with momentum held constant* at either the 01 or 02 levels of stringency for either the qualitative or quantitative portion of the hierarchy. Although the data is consistent with the direction hypothesized by the investigator the connection was not statistically significant and therefore was eliminated from the hierarchy. With the exception of the connection between the qualitative and quantitative skills, representing the *proportional use of matter with momentum held constant*, the application of the White and Clark test to the data at three levels of stringency indicates that the successful completion of the quantitative skills for the *proportional use of speed with momentum held constant*, the *proportional use of speed with matter held constant* and the *proportional use of matter with speed held constant* depends upon the prior acquisition of the qualitative skills.

Chapter 5

SUMMARY, IMPLICATIONS AND RECOMMENDATIONS

Summary

The original purpose of this study was to investigate Raven's findings as to the superiority of a psychologically derived hierarchy over a logically derived hierarchy. Although Raven claims that his logical hierarchy was constructed by a Gagne-type task analysis, the hierarchy itself appears to be nothing more than a rearrangement of the components of his psychological hierarchy and thus was incorrectly conceived. Therefore, an attempt was made to construct a new logical hierarchy by beginning with the terminal element, *conservation of momentum*, and determining the prerequisite skills necessary to perform this task. A study of the components of the concept of momentum reveals that many of the prerequisites necessary for understanding momentum, e.g. proportionality and speed, are of central importance in the study of science and mathematics in school and are often a source of difficulty. The order in which these skills develop is of importance for the understanding of other difficult concepts. A logical hierarchy was proposed by this investigator and modified after field testing and discussion with other teachers. The test items, as well, were piloted and modified after feedback was obtained. Two test items correct served as the test of possession for each skill.

Many methods used to validate learning hierarchies have come under much criticism, resulting in modifications and newer methods. These, as well, are currently coming under criticism and are being extensively analysed and modified. Two tests were applied to the data to determine the validity of the hypothesized hierarchy. These tests were the White and Clark 'test of inclusion' and the 'ordering-theoretic' method, developed by Bart and others. In cases where a connection between skills was rejected, the 'ordering-theoretic' method was used to determine if there was any substantial difference between the upper and lower skills to indicate a meaningful, if non-significant, connection. As a result of this procedure the hypothesized hierarchy could not be accepted in its entirety, but the hierarchy that emerged from the analysis of the data by the White and Clark test at the 02 level of stringency and by the 'ordering-theoretic' method at the 5% level of exceptions was generally in the order hypothesized by the investigator. The best arrangement of these skills is represented in Figure 8. This hierarchy was considered to represent a psychometrically valid hierarchy. The connection between the *terminal task* (A) representing momentum and the skill *proportional use of speed with momentum held constant* was eliminated from the hypothesized hierarchy because of insufficient evidence to indicate a connection between these skills in either direction. Also, the skill *proportional use of matter with speed held constant* was not found to be subordinate to the skill *proportional use of matter with momentum held constant* for either the qualitative or quantitative portions of the hierarchy. Hence, because no statistically significant hierarchical dependency was observed for either the qualitative or quantitative skills the connections between these skills were eliminated from the hierarchy.

Although the results obtained from application of the White and

Clark test and the 'ordering-theoretic' method were not identical, they did show substantial agreement. The White and Clark test appears to be the more rigorous test for the identification of a connection between skills. However, where the White and Clark test simply accepted or rejected a connection at three levels of stringency, the 'ordering-theoretic' method gave additional information suggesting meaningful but statistically insignificant connections between skills.

Implications

The fact that the subjects used for this study were taken from one school in a St. John's location may tend to limit the generalizability of the results of this study. However, because of the type of research involved, the similarity of school curricula at this level and teacher training and experience, there is no reason to suspect that the sample used was atypical. Hence, the study is considered to have implications for the arrangement of instruction for the momentum concept, the diagnosing of problems with respect to particular skills relating to the learning of the concept of momentum, methodology of learning hierarchy validation and the relationship between Gagnean and Piagetian theory.

With respect to sequencing of instruction leading to the understanding of the momentum concept several implications may be stated. A major implication is that a number of intellectual skills have been identified, each of which is a necessary prerequisite to a formal understanding of the momentum concept. The actual arrangement of these skills is represented in Figure 8. Several aspects of the validated hierarchy are of interest. Firstly, an understanding of the meaning of speed is necessary before speed can be manipulated proportionately.

Secondly, possession of the skill *conservation of matter* is essential before mass can be manipulated proportionately in problems relating to momentum. Thirdly, the development of proportional use of speed may not necessarily parallel the development of the proportional use of matter. Fourthly, a formal understanding of the conservation of momentum depends upon the prior development of the proportional use of mass and speed. Finally, the qualitative understanding of these skills precedes their quantitative development.

There is substantial disagreement with Raven's findings that children go through the following sequence for the concept of momentum: *momentum* ⟶ *conservation of matter* ⟶ *proportional use of mass and speed with momentum held constant* ⟶ *speed*. The data in this present study support the need to develop an understanding of the use of *speed*, the *conservation of matter* and the *proportional use of mass and speed* before the learner can demonstrate a *formal* understanding of momentum.

An analysis of the incorrect responses by subjects to test items for particular skills indicates that a large number of subjects consistently responded to tasks for specific skills according to a particular misconception. Most of these misconceptions centered around the subjects' lack of understanding of the concept of proportionality and its relationship between mass and speed. For these particular test items many subjects incorrectly compensated mass for speed or speed for mass in solving problems on proportionality. It may be speculated that some of these misconceptions may be due to the subjects' intellectual level, the type of curricula they have been exposed to or their experiences at that age. However, further analysis of the sequence of prerequisite skills within the hierarchy leading up to this central concept may be necessary to reveal additional subskills that are essential for an understanding of proportionality.

With regards to the quantitative test items for specific skills some subjects responded to some of the quantitative items in a qualitative manner but not to others. This could be due to the fact that some children may be in transition between different stages of intellectual thought and may revert back to a lower level when presented with a novel problem.

Although a number of hierarchy validation methods are presently available, many are still in the stages of being revised. Up to this date no one method is available that will guarantee the arrangement of intellectual skills within a hierarchy with any great degree of certainty. Comparison of the data by several methods, while acknowledging the limitations of each model, is perhaps one of the safest means of hierarchy validation. Two methods applied in this study, the White and Clark test of inclusion and the 'ordering-theoretic' method, offer some promise in this regard. Although the White and Clark test is superior to the 'ordering-theoretic' method in that it allows for errors of measurement with different degrees of stringency, does not require a mastery decision and provides an appropriate test of significance, both methods consider skills in pairs. However, the 'ordering-theoretic' method is very simple to apply and may not yield results significantly different to those obtained from the application of the White and Clark test. The importance of the investigation, however, may dictate which is the most appropriate method. If the relationship between a particular pair or pairs of skills is of major importance, the White and Clark test may be the better method.

For the present study, the White and Clark test, because it is the most sophisticated method, was used primarily as the main test of the data. The 'ordering-theoretic' method, because of its acknowledged limitations, was applied to the data to indicate where the possibility of a connection between skills did exist and the strength of the connection. The 'ordering-theoretic' method at the 5% level of exceptions, however, did yield results similar to those of the White and Clark test at the 02 level of stringency. This evidence supports the findings of Griffiths (1979) that the analysis of data by both methods may yield results that are not significantly different.

The final implication of the present study relates to the relationship between the Gagnean theory and the Piagetian theory of intellectual development. Although the results of this study indicate that the attainment of an understanding of the concept of momentum is explained on the basis of a logical hierarchy as opposed to a psychological hierarchy, this does not mean that the Gagnean hierarchy model cannot be used in conjunction with the Piagetian model of intellectual development. For this study a qualitative hierarchy was derived to test subjects' qualitative skills and a parallel quantitative hierarchy was derived to test subjects' quantitative skills. The responses to the test items on proportionality for both hierarchies indicate that most subjects could not solve the tasks for the quantitative skills without first successfully solving the tasks for the qualitative skills. This suggests that qualitative proportionality precedes quantitative proportionality and is in agreement with Piaget's study of this concept. As previously mentioned in Chapter 4, a large number of subjects were unable to solve problems on proportionality. This possibly could be due to the fact that many subjects were probably not at the formal operational stage of development. By deriving a hierarchy in conjunction with Piaget's stage theory of intellectual development the prerequisite capabilities leading up to the attainment of such concepts as proportions could perhaps be more appropriately identified.

Suggestions for Further Research

1. The hierarchical model of learning should be applied to other concepts in science, to determine if learning of these concepts can also be accounted for on the basis of a cumulative learning model.

2. The relationship between the Gagnean theory and the Piagetian theory of learning should be further investigated to determine if each can be used in conjunction with the other.

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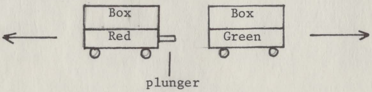
Appendix A

Test Instrument and Administration Procedure

All tasks are described in the order in which they appear in the question booklet.

L_1 *First Momentum Task*

This was one of the tasks used by Raven to determine if the subject had attained an understanding of the concept of momentum. Two collision carts (Macalaster Scientific Corporation) were used in this experiment. One collision cart was covered with green construction paper, the other red. The carts were placed front to back so that when the plunger was released the carts would move at the same speed in opposite directions. The subjects were shown how the carts worked and how they interacted. A number of identical boxes were presented and the subjects were told that each box might contain one weight, two weights or no weights. The investigator then placed a box on each cart and caused them to interact by pushing the plunger. The carts were stopped by bricks placed equidistant from their starting points. Hence, the subject was forced to focus on which car was stopped first and not on the distance travelled by the carts. After the demonstration the subjects were directed to go to question one in their question booklet, read the question, choose the correct response and then explain their answers in the space provided. If any student did not understand the demonstration it was repeated and the question was read aloud for him. Two separate tasks were presented as a test of this skill. However, in the second task the boxes contained a different combination of weights.



The box with the greatest number of weights is:

a) the red box

b) the green box

c) they both have the same number of weights

Why do you think so?

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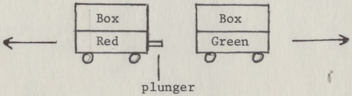
2.



The box with the greatest number of weights is the:

a) red box

b) green box

c) they both have the same number of weights

Why do you think so?

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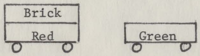
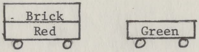
L_2 *Second Momentum Task*

This was the second task used by Raven to determine if the subject had attained the skills necessary for an understanding of the concept of momentum. This task used collision carts (Macalaster Scientific Corporation) and bricks instead of tennis balls and shoe boxes as in Raven's study. The subjects were shown how the carts interacted when they were pushed at each other. The carts were situated so that the rod in front of one cart would push against the back of the other cart. When the moving cart hit the stationary cart the energy was transferred from the moving cart to the stationary cart. The initial momentum of the moving cart was approximately equal to the momentum of the second cart after it was struck. The investigator then directed the subjects to the appropriate question in the question booklet and instructed the subjects to underline the answer they thought was correct and to explain it. Two tasks were used as a test of this skill.

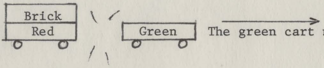
3.



A brick is placed on the red cart.



The red cart is pushed at the green cart.



The green cart moves away.

Will the green cart move:

a) faster than the red cart before it hit the green cart?

b) slower than the red cart before it hit the green cart?

c) the same speed as the red cart before it hit the green cart?

d) Cannot tell from what I am given.

Why do you think so?

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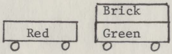
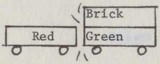
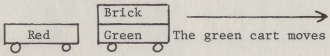
4. 

A brick is placed on the green cart.



The red cart is pushed at the green cart.



The green cart moves away.

Will the green cart move:

a) faster than the red cart before it hit the green cart?

b) slower than the red cart before it hit the green cart?

c) the same speed as the red cart before it hit the green cart?

d) cannot tell from what I am given.

Why do you think so?

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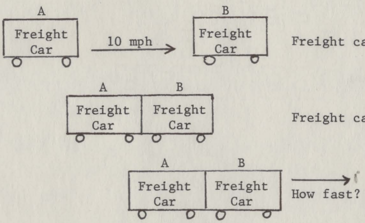
A *Formal Momentum Construct*

These two tasks were presented to determine if the subjects had a formal understanding of the momentum construct. The tasks were explained and demonstrated by the investigator with the use of two collision carts

(Macalaster Scientific Corporation). Each task was presented separately. If necessary the demonstration was repeated until all subjects were satisfied. The subjects were then directed to the appropriate test item in the question booklet where the problem was presented together with a graphic representation of the task. They were then instructed to place the answer of their choice in the space provided and to explain it. Successful completion of both tasks served as the test of this skill.

5. A freight car moving along a track collides with another freight car which is at rest. If the freight cars are of equal mass and are coupled by the collision, how does the speed of the coupled cars compare with the initial speed of the single moving freight car? The speed of the single car was 10 miles per hour.



Freight car at rest.

Freight cars joined together.

How fast? ________________

What was the speed of the coupled freight cars? ______________

Why do you think so?

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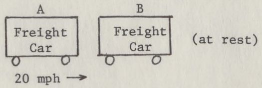
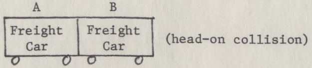
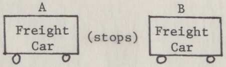
6. A single freight car, moving with a speed of 20 miles per hour makes a head-on collision with another freight car of the same mass. As a result the moving freight car comes to a rest but the second freight car moves away. What will be the speed of the second freight car? ____________________







What is the speed of this car?

What will be the speed of the second freight car? ________________

Why do you think so?

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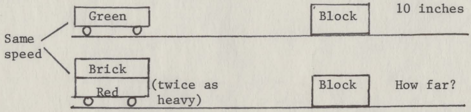
J *Proportional Use of Matter with Speed Held Constant (Quantitative)*

Two collision carts (Macalaster Scientific Corporation), two wooden blocks of the same size and weight placed equidistant from the two carts and a brick equivalent in size and weight to one of the carts were used in a test of this skill. Each task was demonstrated and explained separately by the investigator. In the demonstration both carts (one being proportionately heavier than the other) were pushed at the same speed towards the blocks. Given the distance travelled by one of the blocks the subjects had to numerically solve the distance travelled by the second block. When it was evident that all subjects understood the problem, they were directed to the appropriate test item in the question booklet where they had to place their answer in the space provided and explain it. Successful completion of both tasks served as a test of this skill.

7.



If the green cart pushes a block 10 inches, how far will the red cart push a block of the same weight if it is twice as heavy and moves at the same speed as the green cart? ____________

Why do you think so?

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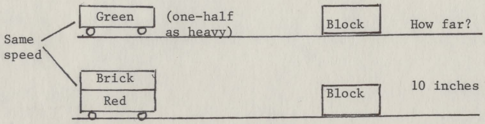
8.

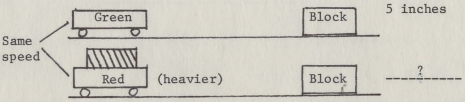


If the red cart pushes a block 10 inches, how far will the green cart push a block of the same weight, if it is only one-half as heavy as the red cart? ___________

Why do you think so?

E *Proportional Use of Matter with Speed Held Constant (Qualitative)*

Two collision carts (Macalaster Scientific Corporation), two wooden blocks of the same size and weight placed equidistant from the two carts and a series of different sized weights were used in a test of this skill. The presentation of the tasks for this skill was similar to that for the previous skill. However, for these tasks subjects were required to solve the problems in a qualitative manner (e.g. a greater distance, a shorter distance), underline the answer of their choice and then explain it. Successful completion of both tasks served as a test of this skill.

9. 

The green cart pushes a block 5 inches. If the red cart is heavier and moves at the same speed as the green cart, will it push a block of the same weight:

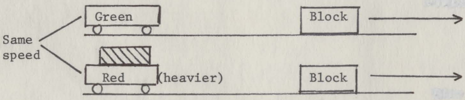
a) the same distance as the green cart?

b) a greater distance than the green cart?

c) a shorter distance than the green cart?

d) cannot tell

Why do you think so?

10. 

The red cart pushes a block 5 inches. If the green cart is lighter but moves at the same speed as the red cart, will it push a block of the same weight:

a) the same distance as the red cart?

b) a greater distance than the red cart?

c) a shorter distance than the red cart?

d) cannot tell

Why do you think so?

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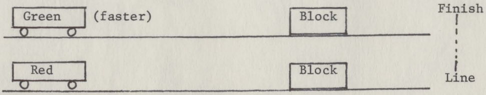
D *Proportional Use of Matter with Momentum Held Constant (Qualitative)*

Two collision carts (Macalaster Scientific Corporation), two wooden blocks of the same size and weight placed equidistant from the two carts and a series of weights of the same size and weight were used instead of the two tennis balls and shoe boxes used by Raven for these tasks. As for all other tasks the experimenter manipulated the apparatus while the subjects watched. In each task the experimenter pushed both carts at different speeds towards the blocks. The subjects were then asked which of the two carts, while moving at their respective speeds, would have to be made heavier in order to get both blocks to move the same distance. The subjects were then directed to the apparatus test item in the question booklet. When all subjects understood the problem they were asked to underline the answer of their choice and to explain it. Successful completion of both tasks served as a test of this skill.

11.



The green cart is moving faster than the red cart. In order to get both blocks to move the same distance which cart will I have to make heavier?

a) the red cart?

b) the green cart?

c) it does not make any difference

d) cannot tell

Why do you think so?

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____

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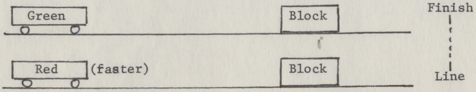
12.



The red cart is moving faster than the green cart. In order to get both blocks to move the same distance which cart will I have to make heavier?

a) the red cart

b) the green cart

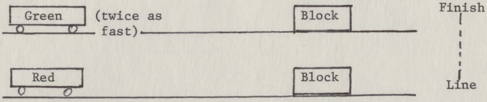
c) it does not make any difference

d) cannot tell

Why do you think so?

I *Proportional Use of Matter with Momentum Held Constant (Quantitative)*

This task was carried out in a manner similar to the previous one except for the fact that its purpose was to demonstrate numerical proportionality.

13. 

With the green cart moving twice as fast as the red cart, what must I do to the weight of one of the carts to get both blocks to move the same distance?

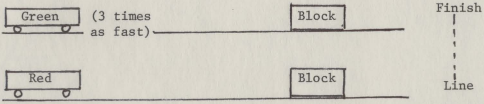
a) make the green cart twice as heavy as the red cart

b) make the red cart twice as heavy as the green cart

c) it does not make any difference about the weight

d) cannot tell

Why do you think so?

14. 

With the green cart moving three times as fast as the red cart, what must I do to the weight of one of the carts to get both blocks to move the same distance?

a) make the green cart three times as heavy as the red cart

b) make the red cart three times as heavy as the green cart

c) it does not make any difference about the weight

d) cannot tell

Why do you think so?

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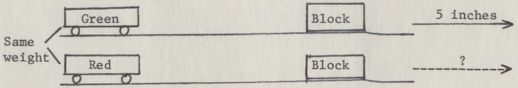
____

____

C *Proportional Use of Speed with Matter Held Constant (Qualitative)*

Two collision carts (Macalaster Scientific Corporation), two wooden blocks of the same size and weight placed equidistant from the two carts and a series of different sized weights were used in a test of this skill. Each task was presented separately; however, both tasks involved the same procedure. In the demonstration the experimenter pushed both carts at different speeds towards the blocks. Given the distance travelled by one of the blocks the subjects had to state whether the distance travelled by the second block was greater than, less than or the same as the other block. When all subjects understood the problem they were directed to the appropriate test item in the question booklet and asked to underline the answer of their choice and to explain it. Successful completion of both tasks served as a test of this skill.

15. Both the green and red carts weigh the same.



If the green cart bumps a block and moves it 5 inches, will the red cart, which is moving faster than the green cart, move another block of the same weight:

a) a greater distance than the green cart does?

b) a shorter distance than the green cart does?

c) the same distance as the green cart does?

Why do you think so?

___

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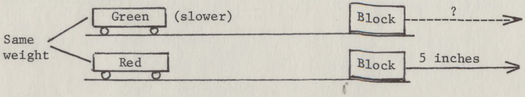
16. Both the green and red carts weigh the same.



If the red cart bumps a block and moves it 5 inches, will the green cart, if it is moving slower than the red cart, move another block of the same weight:

a) a greater distance than the red cart does?

b) a shorter distance than the red cart does?

c) the same distance as the red cart does?

Why do you think so?

___

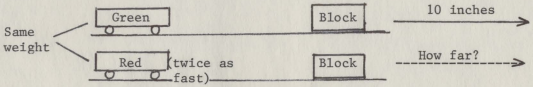
___

___

G *Proportional Use of Speed with Matter Held Constant (Quantitative)*

The procedure for this task was carried out in a manner similar to the previous one except that this problem focused on a numerical solution to the proportionality problem.

17. Both the green and red carts weigh the same.



If the green cart pushes a block 10 inches, how far will the red cart push another block of the same weight if it moves twice as fast as the green cart? ____________

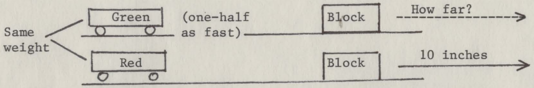
Why do you think so?

____

____

____

18. Both the green and red carts weigh the same.



If the red cart pushes a block 10 inches, how far will the green cart push a block of the same weight, if it moves only one-half as far as the red cart? ____________

Why do you think so?

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K *Conservation of Matter*

This task was demonstrated to the whole class. The experimenter took two balls of clay, one blue and the other green, and explained to the subjects that both balls of clay contained the same amount of clay and weighed the same. He then took the green ball and rolled it into the shape of a hot dog. The investigator then directed the subjects' attention to the question booklet and re-read aloud the question for them. The subjects were then instructed to underline the answer they thought was correct and to explain it in the space provided.

19. If you roll the green ball into a hot dog, will the blue ball have:

a) more clay than the green hot dog?

b) the same amount of clay as the green hot dog?

c) less clay than the green hot dog?

Why do you think do?

____

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In the second conservation task the investigator took two more clay balls, one blue and the other green, and told the subjects that both balls had the same amount of clay and weighed the same. He then took the blue clay ball and broke it up into tiny pieces. Again the subjects were directed to the question booklet. The subjects were then told to underline the answer they thought was correct and to explain it.

20. If you take the blue clay ball and break it up into smaller pieces and arrange the pieces in a row, will the amount of blue clay present be:

a) more than in the green clay ball?

b) the same amount as in the green clay ball?

c) less than in the green clay ball?

Why do you think so?

H *Speed*

The task used to test for an understanding of this concept was first used by Piaget to determine if the child had a formal understanding of speed, involving a distance/time relationship, or whether the child had only an intuitive understanding of speed. In this experiment the subjects were shown two tubes, one 60 inches long by 5 inches wide, the other 42 inches long by 5 inches wide. The subjects were told that one tube was longer than the other and both tubes were held up for their inspection. Each tube contained a car, one red and the other green, attached to two rods of the same length. The investigator then explained to the subjects that both cars would start at the same starting line and travel through the tunnels so that they would come out at the same time. After placing a cardboard blind between himself and the subjects to shield his hand movements from the subjects, the investigator then moved to the end of the table and grabbed hold of the two rods. The investigator then pushed the cars through the tunnels, making sure they started at the same time and came out at the same time. When all subjects did agree that the cars did appear at the same time they were directed to question 21 in the question booklet and told to underline and explain the answer they thought was correct.

21. Did the cars travel:

a) at the same speed?

b) at different speeds?

Why do you think so?

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This task used the same materials that were used in the first experiment on speed. However, in this task the tunnels were placed so that the cars had the same starting point but a different finish line. As before the carts were pushed so that they appeared at the ends of their respective tunnels at the same time. If all subjects did not agree that both cars appeared at the same time the demonstration was performed again. They were then referred to the appropriate question in the question booklet and told to underline and explain the answer they thought was correct.

22. Did the cars travel:

a) at the same speed?

b) at different speeds?

Why do you think so?

____

____

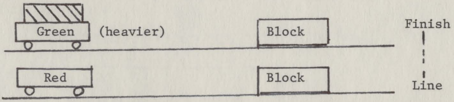
____

B *Proportional Use of Speed with Momentum Held Constant (Qualitative)*

Two collision carts (Macalaster Scientific Corporation), two wooden blocks of the same size and weight placed equidistant from the two carts and a series of weights were used for this skill instead of the two tennis balls and shoe boxes as used in Raven's tasks concerning proportionality. The investigator manipulated the apparatus while the subjects watched and responded to the question in the test booklet.

Two collision carts, one red and the other green, were positioned at the same starting point equidistant from the two blocks. A weight was added to one of the carts. The subjects were then asked which cart would have to be made faster in order to get both blocks to move the same distance. After all subjects indicated that they understood the problem they were referred to the appropriate question in the test booklet and asked to underline and explain the answer of their choice.

23. Suppose I add a weight to the green cart so as to make it heavier than the red cart.



In order to get both blocks to move the same distance, which cart will I have to make move faster?

a) green cart?

b) red cart?

c) both carts should move the same speed

d) cannot tell

Why do you think so?

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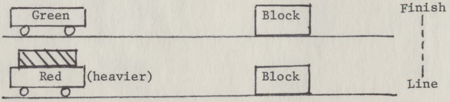
24. Suppose I add some weights to the red cart so as to make it heavier than the green cart.



In order to get both blocks to move the same distance, which cart will I have to make move faster?

a) green cart

b) red cart

c) both carts should move the same speed

d) cannot tell

Why do you think so?

____

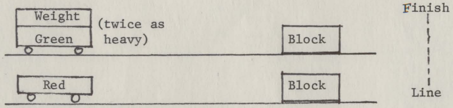
____

____

F *Proportional Use of Speed with Momentum Held Constant (Quantitative)*

The tasks for this skill were carried out in a manner similar to those for the previous one, except for the fact that the tasks required the subjects to respond in a quantitative manner, e.g. twice as fast, rather than in a qualitative manner, e.g. faster than.

25. Suppose I add one weight to the green cart so as to make it twice as heavy as the red cart.



What must I do to the speed of one of the carts to make both blocks move the same speed?

a) make the red cart move twice as fast as the green cart

b) make the green cart move twice as fast as the red cart

c) make both carts move at the same speed

d) cannot tell

Why do you think so?

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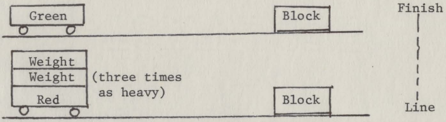
26. Suppose I add two weights to the red cart so as to make it three times as heavy as the green cart.



What must I do to the speed of one of the carts to make both blocks move the same distance?

a) make the red cart move three times as fast as the green cart

b) make the green cart move three times as fast as the red cart

c) make both carts move at the same speed

d) cannot tell

Why do you think so?

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Appendix B

Subjects' Responses to Test Items

	Test Item	Number of Correct Responses	Number of Incorrect Responses	Total Number of Responses	Number of Missing Responses
First Momentum Tasks	01	128	69	197	0
	02	125	64	189	8
Second Momentum Tasks	03	107	79	186	11
	04	139	47	186	11
Formal Momentum Tasks	05	36	129	165	32
	06	41	124	167	30
Proportional Use of Matter with Speed Held Constant (Quantitative)	07	60	107	167	30
	08	48	119	167	30
Proportional Use of Matter with Speed Held Constant (Qualitative)	09	107	81	188	9
	10	97	92	189	8
Proportional Use of Matter with Momentum Held Constant (Qualitative)	11	82	105	187	10
	12	79	108	187	10
Proportional Use of Matter with Momentum Held Constant (Quantitative)	13	79	108	187	10
	14	44	123	167	30
Proportional Use of Speed with Matter Held Constant (Qualitative)	15	145	42	187	10
	16	149	38	187	10
Proportional Use of Speed with Matter Held Constant (Quantitative)	17	95	74	169	28
	18	73	96	169	28
Conservation of Matter	19	165	24	189	8
	20	154	35	189	8
Speed	21	77	112	189	8
	22	76	113	189	8
Proportional Use of Speed with Momentum Held Constant (Qualitative)	23	74	112	186	11
	24	63	123	186	11
Proportional Use of Speed with Momentum Held Constant (Quantitative)	25	32	135	167	30
	26	31	136	167	30

Male - 93; Female - 104; Total Number of Subjects - 197