TRANSMISSION COST CALCULATION FOR RESTRUCTURED ELECTRIC POWER SYSTEMS









Transmission Cost Calculation

for Restructured Electric Power Systems

By

Yi Meng, B. Eng.

A thesis submitted to the School of Graduate Studies

in partial fulfillment of the requirements for the

degree of Master of Engineering

Faculty of Engineering and Applied Science

Memorial University of Newfoundland

May 2007

St. John's

Newfoundland

Canada



Library and Archives Canada

Published Heritage Branch

395 Wellington Street Ottawa ON K1A 0N4 Canada Bibliothèque et Archives Canada

Direction du Patrimoine de l'édition

395, rue Wellington Ottawa ON K1A 0N4 Canada

> Your file Votre référence ISBN: 978-0-494-31270-4 Our file Notre référence ISBN: 978-0-494-31270-4

NOTICE:

The author has granted a nonexclusive license allowing Library and Archives Canada to reproduce, publish, archive, preserve, conserve, communicate to the public by telecommunication or on the Internet, loan, distribute and sell theses worldwide, for commercial or noncommercial purposes, in microform, paper, electronic and/or any other formats.

The author retains copyright ownership and moral rights in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

AVIS:

L'auteur a accordé une licence non exclusive permettant à la Bibliothèque et Archives Canada de reproduire, publier, archiver, sauvegarder, conserver, transmettre au public par télécommunication ou par l'Internet, prêter, distribuer et vendre des thèses partout dans le monde, à des fins commerciales ou autres, sur support microforme, papier, électronique et/ou autres formats.

L'auteur conserve la propriété du droit d'auteur et des droits moraux qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

In compliance with the Canadian Privacy Act some supporting forms may have been removed from this thesis.

While these forms may be included in the document page count, their removal does not represent any loss of content from the thesis.



Conformément à la loi canadienne sur la protection de la vie privée, quelques formulaires secondaires ont été enlevés de cette thèse.

Bien que ces formulaires aient inclus dans la pagination, il n'y aura aucun contenu manquant.

Abstract

In the present open access restructured power system market, it is necessary to develop an appropriate pricing scheme that can provide useful economic information about transmission costs to market participants, such as generation companies, transmission companies and customers. The estimation and allocation of the transmission costs in the transmission pricing scheme is a challenging task for power utilities. In this thesis, different transmission cost calculation and allocation techniques corresponding to various components of the transmission costs are discussed. Transmission service costs are determined based on participants' actual usages on transmission networks using the usage-based method. Using locational marginal price (LMP) method, transmission congestion costs are calculated based on participants' usages and the differences in locational marginal prices. The usage-based method is also used to determine transmission loss costs. A comprehensive transmission pricing scheme using a power flow tracing method and LMP method is proposed, in which the transmission service costs, congestion costs and loss costs are considered and energy transaction information is provided. Case studies using different power system models are presented throughout the thesis to illustrate the application and effectiveness of the studied methods.

Acknowledgements

I would like to thank and express my sincere gratitude and appreciation to my supervisor Dr. Benjamin Jeyasurya for his constant advice, encouragement and guidance during all stages of this research.

I take the opportunity to express my appreciation and thanks to my wife Lisha and my parents for their love, support and encouragement. Without them, this work would never have come in to existence.

Special thanks and appreciation are given to the Natural Sciences and Engineering Research Council of Canada and to Memorial University of Newfoundland for the financial support, which made this research possible. Thanks are also given to the Faculty of Engineering at Memorial University for providing the resources to carry out this research.

Contents

Abstract	i
Acknowledgem	ents ii
Contents	
List of Figures	viii
List of Tables	Х
List of Abbrevi	ations and Symbols xvi

1	Introduction		
	1.1 Objective of Research	2	
	1.2 Organization of Thesis	3	

2 Introduction to Restructured Electricity Transmission Market and

Transmission Pricing Studies	6
2.1 Introduction	6
2.2 Traditional Vertically Integrated Power System	7
2.3 Restructured Power System	10
2.3.1 Competition in Restructured Power System	10
2.3.2 Restructuring and Deregulation in Power Systems	13

2.3.3	Open Access in Restructured Power System	16
2.4 Con	mponents of Restructured Power System	17
2.4.1	Generation Companies (Gencos)	18
2.4.2	Transmission Owners Companies (Transcos)	18
2.4.3	Distribution Companies (Discos)	19
2.4.4	Independent System Operator (ISO)	19
2.5 Tra	ding Model in Restructured Power System Market	20
2.5.1	Bilateral Trading and Multilateral Trading Model	21
2.5.2	Pool Trading Model	22
2.6 Res	structured Transmission System and Market	23
2.7 Intr	roduction to Transmission Pricing Scheme	25
2.8 Cor	mponents of Transmission Cost	27
2.8.1	Transmission service cost	27
2.8.2	Transmission Congestion Cost	28
2.8.3	Transmission Loss Cost	29
2.9 Sun	nmary	31

3	Transmission Service Cost Calculation and Allocation		
	3.1 Introduction	32	
	3.2 MW-mile Method	33	
	3.3 Calculation of Participants' Usages Using Different Methods	38	
	3.3.1 Distribution Factors Method	40	

3.3.2	Bialek's Tracing Method	49
3.3.3	Kirschen's Tracing Method	56
3.4 Cas	e Study	64
3.5 Sur	nmary	70

4	Transmission Congestion Cost Calculation and Allocation		.72
	4.1	Introduction	.72
	4.2	Locational Marginal Price (LMP) Method	.76
	4.3	Case Study	.84
	4.4	Summary	.88

5	Transmission Loss Cost Calculation and Allocation	90
	5.1 Introduction	90
	5.2 pro rata and Z-bus Method	94
	5.2.1 pro rata Method	94
	5.2.2 Z-bus Method	95
	5.3 Power Flow Tracing Method	100
	5.4 Case Study	103
	5.5 Summary	106
6	Comprehensive Transmission Pricing Scheme	107
	6.1 Introduction	107

6.2	Gen	eral Calculation Formulae of Transmission Service, Co	ongestion and
	Loss	s Cost Using a Power Flow Tracing Method	109
6	.2.1	Calculation of Transmission Service Cost	109
6	.2.2	Calculation of Transmission Congestion Cost	113
6	.2.3	Calculation of Transmission Loss Cost	116
6.3	Opti	mal Power Dispatch	118
6.4	Prop	oosed Comprehensive Transmission Pricing Scheme	121
6.5	Case	e Study	125
6.6	Sum	imary	
7 Con	nclusi	ons and Future Work	139
7.1	Sum	mary of the Research and Contribution of the Thesis	139
7.2	Reco	ommendations for Future Work	141
References	6		142
Appendix A	A:	Data of the 6-bus system	148
Appendix]	B:	Data of the IEEE 24-bus system	150
Appendix	C:	DC Power Flow Formulation	153
Appendix]	D:	Transmission Service Cost Case Study Results	155
Appendix]	E:	Transmission Loss Cost Case Study Results	161

Appendix F: Comprehensive Transmission Pricing Scheme Case Study Results

.

List of Figures

Fig. 2.1	Traditional Vertically Integrated Power System
Fig. 2.2	Block Diagram of Traditional Vertically Integrated Power Utility
Fig. 2.3	Block Diagram of Competition in Generation Companies
Fig. 2.4	Block Diagram of Restructured Power Utilities
Fig. 2.5	Restructured Power System 15
Fig. 2.6	Bilateral and Multilateral Trading Model
Fig. 2.7	Pool Trading Model
Fig. 2.8	Transmission Market Structure Model I 24
Fig. 2.9	Transmission Market Structure Model II
Fig. 2.10	Block Diagram of Transmission Service Cost
Fig. 2.11	Block Diagram of Transmission Congestion Cost
Fig. 2.12	Block Diagram of Transmission Loss Cost
Fig. 3.1	Block Diagram of Usage-based Method
Fig. 3.2	Base Case Operating Conditions of the 6-bus System
Fig. 3.3	Single Line Diagram of the 3-bus System I
Fig. 3.4	Proportional Sharing Principle Example 50

Fig. 3.5 Simplified State Graph for the 6-bus System I – Generator Contributions.. 57

3.6 Simplified State Graph for the 6-bus System II – Load Contributions	58
3.7 Single Line Diagram of the IEEE 24-bus System	65
3.8 Simplified State Graph for the IEEE 24-bus System– Generator Contribution	ms
	67

Fig. 4.1	Block Diagram of Transmission Congestion Cost	. 73
Fig. 4.2	Block Diagram of Incremental (Marginal) Pricing Method	. 74
Fig. 4.3	Single Line Diagram of the 3-bus System II	. 77
Fig. 4.4	3-bus System II Study under Unconstrained Condition	. 77
Fig. 4.5	3-bus System II Study under Constrained Condition	. 78
Fig. 4.6	LMP Calculation Example Using Generation Shift Factor Method	. 81
Fig. 4.7	LMP Calculation Using Kirschen's Method for the 3-bus System II	. 82

Fig. 5.1	Illustration of Transmission Loss Cost	91
----------	----------------------------------------	----

Fig. 6.1	Block Diagram I of the Proposed Pricing Scheme 108
Fig. 6.2	Single Line Diagram of the 3-bus System III 111
Fig. 6.3	State Graph of the Generator Contributions for the 3-bus System III 112
Fig. 6.4	State Graph of the Load Contributions for the 3-bus System III 112
Fig. 6.5	Block Diagram II of the Comprehensive Transmission Pricing Scheme 122
Fig. A.1	Single Line Diagram of the 6-bus System
Fig. B.1	Single Line Diagram of the IEEE 24-bus System

List of Tables

Table 3.1 C	Comparison of Different Service Cost Calculation Methods
Table 3.2 A	Assumed Service Cost of Transmission Lines of the 6-bus System
Table 3.3 C	Contributions of Generators to Line Flows Using GGDFs Method for the
6	5-bus System
Table 3.4 S	Service Costs Allocated to Generators Using GGDFs Method for the 6-bus
S	System
Table 3.5 C	Contributions of Loads to Line Flows Using GLDFs Method for the 6-bus
S	System
Table 3.6 S	Service Costs Allocated to Loads Using GLDFs Method for the 6-bus System
••	
Table 3.7 C	Contributions of Generators to Line Flows Using Bialek's Method for the
6	5-bus System
Table 3.8 S	Service Costs Allocated to Generators Using Bialek's Method for the 6-bus
S	System
Table 3.9 C	Contributions of Loads to Line Flows Using Bialek's Method for the 6-bus
S	System
Table 3.10 S	Service Costs Allocated to Loads Using Bialek's Method for the 6-bus System

Table 3.11	Contributions of Generators to Line Flows Using Kirschen's Method for the
	6-bus System
Table 3.12	Service Costs Allocated to Generators Using Kirschen's Method for the 6-bus
	System
Table 3.13	Contributions of Loads to Line Flows Using Kirschen's Method for the 6-bus
	System
Table 3.14	Service Costs Allocated to Loads Using Kirschen's Method for the 6-bus
	System
Table 3.15	Result Comparison Using Various Usages Calculation Methods for the 6-bus
	System
Table 3.16	Comparison of Different Usages Calculation Methods
Table 3.17	Assumed Service Costs of Transmission Lines of the IEEE 24-bus System 66
Table 3.18	Comparison of Service Costs Allocated to Generations Using Various Usages
	Calculation Methods for the 24-bus System
Table 4.1	Contributions of Generators to Line Flows for the 3-bus System II
Table 4.2	Generator Marginal Prices for the 24-bus System
Table 4.3	Locational Marginal Price of Each Bus for the 24-bus System
Table 4.4	Transmission Congestion Cost of Each Line for the 24-bus System
Table 4.5	Transmission Congestion Costs Allocated to Generations for the 24-bus
	System
Table 4.6	Transmission Congestion Costs Allocated to Loads for the 24-bus System 88

Table 5.1	Transmission Loss Cost Allocation Using Z-bus Method for the 6-bus System
Table 5.2	Transmission Loss Allocation to Generators Using Kirschen's Method for the
	6-bus System
Table 5.3	Transmission Loss Allocation to Loads Using Kirschen's Method for the
	6-bus System 101
Table 5.4	Transmission Loss Cost Allocation Using Kirschen's Method for the 6-bus
	System
Table 6.1	Contributions of Generations and Loads to Line Flows for the 3-bus System
	Ш
Table 6.2	Contributions of Generations and Loads to Line Losses for the 3-bus System
	III
Table 6.3	Loss Cost of Each Line Responding to Generators for the 3-bus System III
Table 6.4	Loss Costs Allocated to Loads for the 3-bus System III 118
Table 6.5	Total Transmission Costs Allocated to Generations and Loads for the 3-bus
	System III
Table 6.6	Bilateral Transaction Data for the 24-bus System 125
Table 6.7	Multilateral Transaction Data for the 24-bus System
Table 6.8	Pool Transaction Data for the 24-bus System
Table 6.9	Optimization Dispatch Results for Generations for the 24-bus System 127

Table 6.10	Optimization Dispatch Results for Loads for the 24-bus System 128
Table 6.11	Optimization Dispatch Results for Line Flows and Losses for the 24-bus
	System
Table 6.12	Voltage Magnitude of Each Bus for the 24-bus System
Table 6.13	Generator Marginal Prices for the 24-bus System
Table 6.14	Assumed Service Costs of Transmission Line for the 24-bus System 131
Table 6.15	Locational Marginal Price of Each Bus for the 24-bus System
Table 6.16	Transmission Costs Allocated to Generations for the 24-bus System 134
Table 6.17	Transmission Costs Allocated to Loads for the 24-bus System 134
Table 6.18	Revenues of Generation Companies for the 24-bus System
Table 6.19	Payments of Customers for the 24-bus System 137
Table A.1	Generation and Load Details of the 6-bus System
Table A.2	Line Characteristics of the 6-bus System
Table B.1	Generation and Load Details of the IEEE 24-bus System 151
Table B.2	Generation Fuel Cost Coefficients of the IEEE 24-bus System 151
Table B.3	Line Characteristics of the IEEE 24-bus System
Table D.1	Contributions of Generators to Line Flows Using GGDFs Method for the
	24-bus System

Table D.2	2 Contributions of Generators to Line Flows Using Bialek's Method for the	
	24-bus System	
Table D.3	Contributions of Generators to Line Flows Using Kirschen's Method for the	
	24-bus System	
Table D.4	Transmission Service Cost Calculation and Allocation Using GGDFs Method	
	for the 24-bus System 158	
Table D.5	Transmission Service Cost Calculation and Allocation Using Bialek's Method	
	for the 24-bus System 159	
Table D.6	Transmission Service Cost Calculation and Allocation Using Kirschen's	
	Method for the 24-bus System	
	•	
Table E.1	Transmission Losses Allocated to Buses Using Z-bus Method for the 24-bus	
	System	
Table E.2	Transmission Loss Costs Allocated to Generations Using pro rata Method for	
	the 24-bus System 161	
Table E.3	Transmission Loss Costs Allocated to Loads Using pro rata Method for the	
	24-bus System	
Table E.4	Contribution of Each Generator to Line Losses Using Kirschen's Method for	
	the 24-bus System 162	
Table E.5.1	Contribution of Each Load to Line Losses Using Kirschen's Method (I) for the	
	24-bus System	

s.-

Table E.7	Transmission Loss Costs Allocated to Loads Using Kirschen's Method f	
	24-bus System	

Table F.1	Contribution of Each Generator to Line Flows for the 24-bus System 166
Table F.2.1	Contribution of Each Load to Line Flows (I) for the 24-bus System 167
Table F.2.2	Contribution of Each Load to Line Flows (II) for the 24-bus System 168
Table F.3	Contribution of Each Generator to Line Losses for the 24-bus System 169
Table F.4.1	Contribution of Each Load to Line Losses (I) for the 24-bus System 170
Table F.4.2	Contribution of Each Load to Line Losses (II) for the 24-bus System 171

List of Abbreviations and Symbols

А	: Generation shift distribution factors matrix
A_d	: Bialek's downstream distribution matrix
A_{u}	: Bialek's upstream distribution matrix
ALMP	: Average locational marginal price
С	: Generalized load distribution factors matrix
C ^s	: Transmission service cost
C^{C}	: Transmission congestion cost
C^{L}	: Transmission loss cost
D	: Generalized generation distribution factors matrix
Discos	: Distribution companies
F	: Active power flow
FERC	: Federal Energy Regulator Commission
FTR	: Firm transmission right
G	: Generator
Gencos	: Generation companies
GGDFS	: Generalized generation distribution factors
GLDFS	: Generalized load distribution factors

GSDFs	: Generation shift distribution factors
ISO	: Independent system operator
L	: Load
LMP	: Locational marginal price
MWMILE	: MW-mile Value
OPF	: Optimal power flow
Р	: Active power
Pref	: Preferred
p.u.	: Per unit
Q	: Reactive power
R	: Transmission line resistance
TC	: Total transmission cost
Transcos	: Transmission owners companies
V	: Voltage
W_{G_l}	: Generator marginal price
х	: Transmission line reactance
Y	: Bus admittance matrix
θ	: Bus voltage angle
\$/hr	: Dollar per hour

Chapter 1

Introduction

The traditional vertically integrated power industry is undergoing significant changes [1-7]. Functions and ownerships of generation, transmission and distribution are unbundled and separated from the traditional power system structure. The competition among generations is allowed to supply the economical energy and customers have more options to choose their suppliers. Pursuing the economical goal in order to increase revenues or reduce costs becomes a new objective for all market participants.

The transmission system is an essential facility in power industry, because it is the electrical highway through which electricity flows and every participant has to use it. It is composed of the integrated transmission network that was owned and controlled by traditional utilities before. Now it can be considered as an independent transmission company. The transmission company under the restructured and competitive environment should provide services through non-discriminatory open access to all generations and customers.

Since power suppliers and customers should be charged a price for the transmission services, transmission cost is the recovery cost of the services that reflects actual usages on transmission networks corresponding to generations and customers. All power market participants require knowledge of associated transmission costs to make correct economic and engineering decisions for upgrading and expanding of generation, transmission and distribution facilities.

It is necessary to develop a transmission pricing scheme that can provide useful and precise information to market participants through the calculation and allocation of the transmission costs. The pricing scheme should compensate transmission companies fairly for providing transmission services, estimate costs due to congestion problems, determine loss costs, allocate entire transmission costs reasonably among all transmission users and display participants' revenues and costs.

1.1 Objective of Research

Even though many methods using complex algorithms have been proposed, the estimation and allocation of transmission costs in a power system is a challenging task. Little work has been performed for estimating all transmission costs in a pricing scheme. The purpose of the research presented in this thesis is to create an advanced pricing scheme that determines and allocates transmission service cost, congestion cost and loss cost using effective methods. In comparison with other approaches, it is easier to

understand and implement for power utilities to determine transmission costs. The principal goals of this research are summarized as follows:

- 1. To recognize and define the components of the transmission cost under restructured power system markets.
- 2. To implement and compare transmission service cost calculation and allocation methods.
- 3. To study transmission congestion problem and determine transmission congestion cost.
- 4. To evaluate and allocate transmission loss cost using different methods.
- 5. To propose a transmission pricing scheme to determine all transmission costs and provide energy transaction information.
- 6. For each of the above goals, use suitable power system models and perform case studies to demonstrate the effectiveness of the different methods studied.

1.2 Organization of Thesis

Chapter 2 presents background information of the traditional vertically integrated power system and restructured power system. The main differences in the system structure and techniques between the two systems are described, while the changes and new challenges of the power system restructuring are also given. Subsequently, the restructured transmission system and market based on various tracing models are introduced. The requirements and objective of the transmission pricing scheme are presented and the components of the transmission cost are defined.

In Chapter 3, the discussion and comparison about transmission service costs calculation and allocation methods are presented. An overview of a usage-based method and three usage calculation methods is given. Their general formulae are set up and the principal features are highlighted. The methods are implemented in Matlab and PowerWorld Simulator and tested using a 6-bus power system and the IEEE 24-bus power system. Results from various methods are discussed and compared.

The calculation and allocation of transmission congestion costs are presented in Chapter 4. After defining locational marginal price (LMP), the relationship between congestion costs and LMP is described. The principle and calculation procedure using LMP method to determine the congestion costs is presented. The determination of LMP values using two different methods is also given. The studied methods are tested using the IEEE 24-bus power system.

In Chapter 5, transmission loss allocation and loss cost calculation are presented. The loss allocation using a well known Z-bus method is presented. A new method using power flow tracing method for the loss allocation and cost calculation is proposed in this chapter. The methods are tested on different power system models and a comparison between the two methods is presented.

Chapter 6 proposes a comprehensive transmission pricing scheme using a power flow tracing method and locational marginal price (LMP) method, which can determine all components of transmission costs and calculate the revenues and costs of participants about energy transactions. The general formulae used for all systems in the scheme are presented and highlighted. A useful strategy: optimal power dispatch for managing pricing scheme is introduced. The detailed procedure of the proposed scheme is illustrated and described. The proposed scheme is implemented and tested using the IEEE 24-bus system.

Chapter 7 gives the conclusions of the thesis and highlights the contribution of this research. Suggestions for future research are given.

Chapter 2

Introduction to Restructured Electricity Transmission Market and Transmission Pricing Studies

2.1 Introduction

Since the development of a transmission pricing scheme in the restructured transmission system becomes essential, it is necessary to understand the main changes and new challenges for restructuring the power industry. The objective of this chapter is to introduce the background information corresponding to the restructured transmission system and transmission pricing studies.

Initially, the traditional vertically integrated power system and restructured power system will be described respectively. The principal differences in the system structure and techniques between the two power systems are highlighted. The important changes and new challenges of the power system restructuring are also given. The restructured transmission system and market based on various tracing models are introduced. After describing the requirements and objectives of the transmission pricing scheme, the components of transmission cost are presented.

2.2 Traditional Vertically Integrated Power System

For 120 years after electricity was commercialized in 1878, electric power systems around the world have been physically and operationally very similar. The normal functions of these systems are electricity generation, system operations, electricity transmission and distribution.

Thousands of generation plants, including oil, coal, nuclear and wind generations, are responsible for producing and supplying electricity to customers all over the world. Transmission and distribution systems are used to transport electricity. Transmission networks serve large areas, and distribution systems are used for local customers. As Fig 2.1 shows, Generator A and B supply electrical power to customer E and F through the transmission network and distribution system C.

Since the typical organization of the traditional power system was vertically integrated prior to power system restructuring and deregulation, the first characteristic is that traditional power utilities were incorporating all functions mentioned above [1-7]. All service functions, including electricity generation, transmission and distribution, were bundled. A single company in each area built and owned its generations (G), transmission network (T) and distribution systems (D), as Fig. 2.2 presents. They typically produced, transported, and retailed the electricity and operated the whole system. Taking the system shown in Fig 2.1 as the example, generation A and B, the transmission network and distribution systems belong to the same company.



Fig. 2.1 Traditional Vertically Integrated Power System



Fig. 2.2 Block Diagram of Traditional Vertically Integrated Power Utility [7]

The second characteristic is monopoly. The vertical integration was almost always accompanied by a legal monopoly within a service area. Only one company could provide electricity to customers in that area and only national or local electric utility was permitted to produce, transmit, distribute and sell electrical power. In most countries, the government or a government owned company had the monopoly while private companies dominated the industry in some countries, such as the Unite States, Spain and Germany.

The utilities had to supply electricity for the needs of all customers in their service areas based on obligation instead of the profit. Their business and operation had to conform to guidelines and rules set down by government regulators.

Another characteristic is that the monopoly company generally limits customer's choice of supplier by legislation rather than by the wishes of the customer. For example, in Fig. 2.1, customer E and F must be supplied by generator A only, and generator B has to supply power to the local customer based on legislation, even though generator B can provide cheaper electricity. In addition, electricity prices were also regulated since these vertically integrated utilities had monopolies in their own areas. The government guaranteed that regulated rates would provide the electric utilities with a "reasonable" or "fair" profit.

2.3 **Restructured Power System**

During the last 10 years, the traditional vertically integrated power industry has been undergoing significant changes [1-7]. The characteristics of the restructured power systems include the appearance of competition, system restructuring and deregulation, and transmission system open access.

2.3.1 Competition in Restructured Power System

Why do we need competition in power system? The competition can certainly force all market participants to be aware of their own profits and rights, which mean revenues and costs in the economic term. For generations, every supplier wants to raise the market prices to achieve maximum revenues, as the competition can benefit customers who can expect to have the following [4]:

- Low electricity prices;
- Reliable services;
- Fairly predictable bills;

When the competition among generations is allowed to supply economical electrical energy, customers have more options to choose their suppliers, as Fig.2.3 presents. Thus competition can fulfill the main objective, which is to significantly reduce the costs of power charged to consumers. As shown in Fig. 2.1, the customer E and F can choose either generator A or B based on their prices under the competitive environment.



Fig. 2.3 Block Diagram of Competition in Generation Companies

The costs of generations will be also reduced by driving prices through market forces and more competitions. When customers are allowed to choose a provider for electricity transactions, the generations have to face the price competition. The generations must improve economic efficiency to reduce their costs because of the forces of market competition, in order to maintain or increase their revenues.

Another effect from the competition is that market risks are assigned to utilities instead of customers. The market risks include market demands and prices, technological change rendering plants economically obsolete, management decisions about maintenance, staffing and investment. Under the traditional regulation system, customers generally take most of the risks. If new technology is invented and applied, customers have to continue to pay (more) for the old technology. Moreover, if demands turn out to be less than anticipated, electricity prices have to rise to cover the cost of excess capacity so that customers have to pay more. These risks are transferred to power utilities under the competition. They will pay for mistakes or profit from good decisions and management. Since the utilities also take the risk of the change in technology, they have strong responsibility to choose the best and reliable technology. For the risk of changes in market demands and prices, the utilities need to take responsibility to be flexible in their building plans and watch the market constantly.

Among three components of the power system, the generation is the major candidate for being considered competitive. The competition can be guaranteed by establishing the competitive environment in which generations provide different energy prices based on the market instead of internal coordination and government rules. Customers can choose to buy from various suppliers and change the supplier as they wish. In addition, more generations are allowed to enter the region where only legitimate generations that belong to traditional utilities can provide electricity before.

The transmission network and distribution systems will remain the natural monopoly because they could not economically provide competing services. Although there have been a few cases where some isolated lines (peripheral to the network) were sold to investors who made profits from these lines, this is not really competition in transmission. All competitors (generations) and customers still require non-discriminatory transmission access.

12

2.3.2 Restructuring and Deregulation in Power Systems

Restructuring is defined as changing existing companies, separating some functions and combing others, and sometimes creating new companies. The aim of the restructuring in power systems is to prevent discriminatory behaviour in energy market, or to create more competitors, or to consolidate transmission over a wide region. The separation of the functions of the traditional system is the principal objective.

On April 24, 1996, the Federal Energy Regulator Commission (FERC) in the United States issued Final Rule 888 [8] that required power utilities to provide a separable and reliable service to customers. Based on the order, the three components of the traditional power systems and their services should be unbundled and separated.

Unbundling means that various tasks, which are normally carried out within the traditional organization, should be identified and separated so that these tasks can be open to competition for profits. The generation part in the traditional utility will be spit up into a sufficiently large number of smaller independent competing generating companies. The role of these companies is only to produce and sell energy to customers. These generation utilities will no longer have a monopoly, small business will be free to choose and buy power from cheaper sources.

The transmission networks are also subject to the form of "unbundling" to become independent transmission companies. Transmission companies become the mechanisms in which electricity exchanges and transactions happen. Generations and customers will both be obligated to deliver or wheel power over transmission networks
for fees that should be the same cost rate for all participants. The last component of the corporate unbundling is the creation of independent distribution companies whose role is to provide low-voltage, normally radial service to individual industrial, commercial or residential customers.



Fig. 2.4 Block Diagram of Restructured Power Utilities

Fig. 2.4 illustrates the structure of the restructured power system. In comparison with the traditional system shown in Fig. 2.2, the traditional vertically integrated company is separated to numerous smaller independent companies.

For example, Fig. 2.5 shows that generator A and B, distribution C and D, and transmission system in a traditional electrical company. These components are unbundled to create new independent generation company A and B, distribution company C and D, transmission system company. Respectively, the above companies will produce, deliver and distribute electricity to customers in the same service region.



Fig. 2.5 Restructured Power System

Restructuring in electricity industry will create new business opportunities where new companies selling new products and services will appear, consumers will have alternatives in buying electricity services, and new technologies will develop.

The regulation in the traditional power industry is about controlling prices of monopoly suppliers and restricting entry to the market. The standard definition of deregulation for restructured power systems is to remove controls on prices and entry of competing suppliers. The deregulation in electrical industry comprises several changes. For the supplier, barriers of the entry to the old regime are removed. For transmission systems, the control and operation are separated from traders. Trading arrangement will depend on market change and customers' wish instead of obligations and government guidelines. A free floating and more flexible price also replaces the regulated price. California was the first state in the United States to implement the restructuring and deregulation of the power industry in 1996 [3]. The entire restructured system comprises three utilities owning transmission systems and generations. The distribution systems are controlled and operated by several distribution companies. Although the transmission networks are owned by the utilities, the independent system operator (CAISO) is taking responsibility to control the transmission networks. The generations will bid to enter the energy spot market, and the fluctuating electricity price will be based on the spot market operated by the ISO.

In Canada, Ontario Hydro was responsible for Ontario's electrical energy industry [3]. In 2000, the Ontario Hydro was split into several companies. Generation was handled by Ontario Power Generation, while Hydro One owned the transmission and distribution system. The Independent Electricity Market Operator was given responsibility both for organizing the spot market where electricity would be traded, and for ensuring open access to the transmission system. The energy price bought and sold on the spot market will be set by market forces instead of government legislation, and more generations were encouraged to enter the market.

2.3.3 Open Access in Restructured Power System

FERC issued Rule 888 and 889 [8-9] claimed: "Transmission open access promoting wholesale competition through open access non-discriminatory transmission services by public utilities". It means that the transmission system under competitive and unbundling situation should provide services and technique information through non-discriminatory open access to all generations and customers.

Transmission system is an essential facility that every participant has to use. Transmission open access means that everyone gets the same deal, with no discrimination in the opportunity to use or in the cost to use them. Competition in energy production requires open access to the transmission networks so that any competitor can use them.

As the competition and open access have brought into the market and the industry structure has been restructured and deregulated, the advantages of the restructured electrical power system can be summarized as the cost reduction of energy production and distribution, the elimination of inefficiencies, and the increase of customer choices.

2.4 Components of Restructured Power System

The key structural components representing various segments of the electricity market are generation companies (Gencos), transmission owners companies (Transcos), distribution companies (Discos) and independent system operator (ISO) [1-7]. Other components include retail companies (Retailcos), scheduling coordinators (Scs), power exchange (PX), aggregators, brokers, marketers and customers. Depending on the structure and the regulatory framework, some of these components may be consolidated together, or may be unbundled.

2.4.1 Generation Companies (Gencos)

Gencos that are the owners of the generation plants in most cases are responsible for operating and maintaining these generating plants. These companies are formed once the generation parts are split from traditional power utilities.

The objective of Gencos is to maximize profit in the restructured system. A Genco may offer electrical power at several locations that will ultimately be delivered through Transcos and Discos to customers. Gencos have opportunities to provide electricity to customers who sign sales contracts or to sell electricity to Power Exchange Pool.

The prices of Gencos are not regulated and they should treat other market participants fairly. In contrast, transmission open access allows Gencos to access the transmission network without distinction.

2.4.2 Transmission Owners Companies (Transcos)

Transcos are the owners of the transmission system that is the most crucial element in the electrical market. The secure and efficient operation of the transmission system is the key to be efficient in the market. Transcos are responsible for delivering electricity from Gencos to Discos and customers. It is composed of the integrated network that was owned and controlled by traditional utilities before. Now it becomes independent and provides open access to all participants and radial connections that join generating units and large customers to network.

The basic objective of transmission open access is that Transcos are regulated to provide non-discriminatory services with fair costs for all market participants. Transcos play an important role of building, owing, maintaining and operating the transmission system in a certain geographical region to provide services for maintaining the overall reliability of the electrical system. In North America, some Transcos are under the control of the regional ISO.

2.4.3 Distribution Companies (Discos)

The responsibility of a Disco is to supply electricity from Gencos and Transcos to customers in a certain geographical region through its facilities. It is the same as the responsibility of the distribution segment of a traditional utility. However, it will be restricted to maintain and operate distribution networks only. The Disco will build and own distribution networks connected to transmission systems and customers, respond to distribution network outages and power quality concerns, and support voltages.

2.4.4 Independent System Operator (ISO)

The appearance of the ISO is one of most significant changes in restructured power system. Since the control of the transmission grid cannot be guaranteed without the independent operator, it is necessary to develop an independent operational control mechanism: ISO for the electricity market. The first ISO was established in California in 1996 and this concept was recognized by FERC and many electricity utilities in the whole world soon. The characteristic of ISO is the absolute independence of any market participants, such as Gencos, Transcos, Discos and all customers. It provides non-discriminatory open access to all transmission system users that comply with the FERC issued Order 888. The ISO is responsible for administrating transmission tariffs, maintaining the system security, coordinating maintenance scheduling and matching electricity supply with demand.

Hence, the ISO has the authority to commit and dispatch some or all system generators and curtail loads for maintaining the system security. It can remove transmission violation and balance supply and demand. In addition, the ISO ensures that proper economic signals, which can encourage efficient use and motivate investment, are sent to all market participants. For example, the ISO will develop short-run or long-run schedule and transmission pricing schemes.

2.5 Trading Model in Restructured Power System Market

The restructured electricity markets provide three trading options for participants. They can schedule energy transactions based on bilateral trading model or multilateral (as group) trading model. They also can buy and sell energy through the pool trading model (centralized trading based on bidding transactions) [1, 3-5 and 10].

2.5.1 Bilateral Trading and Multilateral Trading Model

Since bilateral trade means the commerce between two market participants, electricity suppliers and consumers would independently arrange power transaction with each other based on their own financial demand in the bilateral trading model. Owing to the free market competition, this model provides an opportunity for consumers to choose the least expensive generators with promoting economic efficiency.



Fig. 2.6 Bilateral and Multilateral Trading Model [10]

Fig. 2.6 shows the example of the bilateral trading model. Generator 1 will supply energy to load 1 based on the bilateral contract. Several possible trading options for market participants can be observed. Sellers may have one load to supply as well as loads may only buy the power from only one generator. Loads may also buy the power from more than one generator and generators can deliver electricity to several loads in order to optimize their performance. The main disadvantage of the bilateral model is the difficult arrangement and management for the ISO because of decentralized decision-making. Multilateral trading model is a generalization of bilateral transactions where a group of energy producers and buyers are put together to form a balance transaction based on a series of contracts. In Fig. 2.6 all generations or all loads can compose a group of suppliers or buyers respectively; their electricity transactions are based on multilateral trading contracts. In practice, bilateral and multilateral transactions often coexist.

2.5.2 Pool Trading Model

In contrast with the bilateral and multilateral model, the direct transaction between generators and customers are not allowed within the pool model. All trading behaviors happen within a centralized marketplace (the pool), which is operated by the ISO and other mechanisms authorized by the ISO. Transaction price, quantity bids and offers from generation and consumption will be submitted to the pool operators, as Fig. 2.7 shows. Based on those data, the operators select the bids and offers that optimally clear the market while respecting the security constrains imposed by the transmission network.

The system operator plays a much more active role in pool trading model than it does in the bilateral and multilateral model. The shortcoming of the pool model is that all transactions have to be controlled and dispatched by the operators. The more expensive electricity may be assigned to some customers who could not choose the cheaper suppliers based on their wishes.



Fig. 2.7 Pool Trading Model [10]

In this thesis, the trading model that incorporates bilateral, multilateral and pool trading model will be applied since this comprehensive model can eliminate all shortcoming of the above individual models and provide more options for participants.

2.6 Restructured Transmission System and Market

As one of the restructured electricity power markets, the transmission market has been undergoing rapid and irreversible changes since the 1990s [1-7]. Restructured transmission system offers open access to all power suppliers and customers and organizes the competition on an equitable and transparent basis.

Based on the market trading models presented in 2.5, there are two basic structure models for the transmission market [10-14], as shown in Fig. 2.8 and 2.9. Model I is a hybrid structure model, in which either pool or bilateral transactions can be observed. Not

only does the ISO play a vital role among all market tractions since the ISO determines the market rules and operation, participants (normally generations and loads) can also sign trading contracts with each other. However, all electricity transmitted from Gencos to Discos and customers through Transcos in this model should always be based on an appropriate and efficient transmission pricing schemes determined by the ISO.



Fig. 2.8 Transmission Market Structure Model I [11]



Fig. 2.9 Transmission Market Structure Model II [11]

Model II is based on the bilateral trading model among market participants instead of the control and management of the ISO. The transactions in this model do not allow any modifications unless all participants agree to adjust those bilateral contracts and proposed transaction should not violate any constraints. The transmission market structure of Model I will be applied in this thesis, since the ISO can provide more flexible management and market operation when changes occurs, such as the growth of customer demand and cost variation.

In the restructured transmission market, the power suppliers and customers should be charged a price for the recovery costs of transmission services in either structure Model I or II, and Transcos should profit for providing the services. However, how to calculate the recovery costs for the transmission services and allocate the costs to each market participant in a fair and appropriate basis in the complicated system is certainly a challenge for Transcos and the ISO. A transmission pricing scheme that can estimate transmission costs is required.

2.7 Introduction to Transmission Pricing Scheme

When discussing transmission pricing scheme, it is necessary to define transmission service and pricing [10]: "The transmission function will facilitate a competitive electricity market by impartially providing energy transportation service to all energy buyers and sellers, while fairly recovering the cost of providing those service". All users, including Gencos, Discos and customers, should pay to Transcos for using transmission networks to trade electricity. The principal objective of a transmission pricing scheme is to determine and analyse transmission costs and provide some economic signals to each market participant, such as the revenues or costs of Gencos, Transcos and customers. The transmission pricing scheme is very useful because a proper transmission pricing scheme could meet revenue expectations for Transcos, promote an efficient operation of electricity markets, encourage investment in optimal locations of generations and transmission lines, and adequately reimburse owners of transmission assets in a competitive environment.

The calculation and allocation of transmission costs are the primary functions for the pricing scheme [10-14]. The pricing scheme normally comprises the simple and transparent derivation of charges including different kinds of transmission costs, transmission service rates and fair and practical transmission cost allocation scheme for all participants in the market.

The requirements for transmission pricing scheme are as follows [10]:

- To compensate grid companies fairly for providing transmission services
- To solve congestion problems without violating security constraints and calculate the costs related to the congestions.
- To determine costs due to transmission line losses
- To allocate transmission costs reasonably among all transmission users, both native load and third party
- To maintain the reliability of the transmission grid
- To display the revenues and costs of market participants

2.8 Components of Transmission Cost

The study presented in this thesis considers three components for the transmission cost: transmission service cost, transmission congestion cost and transmission loss cost. Only service cost and congestion cost have been mentioned in most references [10-15]. However, the pricing scheme should include loss cost since it can accurately reflect all related transmission costs. A simple 2- bus power system is used to illustrate the different components of the transmission cost.

2.8.1 Transmission service cost

Transmission service cost is defined as the fixed transmission cost or embedded cost that covers the transmission revenue requirement of transmission owners. It is the direct cost of providing transmission services for the recovery of past capital transmission networks investment [10, 14-15].



Fig. 2.10 Block Diagram of Transmission Service Cost

Fig. 2.10 shows that generator A and customer C will be charged the transmission service cost for 100 MW electricity delivered from A to C. For this operating scenario, generator B does not have to pay.

2.8.2 Transmission Congestion Cost

Transmission congestion cost is the charge for the incremental electrical power delivery through the constrained transmission networks. It includes operating cost for generation redispatch and transmission transaction rescheduling, reinforcement cost for capital costs of new transmission facilities and opportunity cost for benefits caused by transaction planning of utilities due to operational constraints [10, 14-15].

As Fig. 2.11 presents, when the demand of customer C is increased to 120MW, congestion occurs since the capacity of the transmission line is 100 MW. The more expensive generator B has to be brought into the market to supply extra energy to customer C. Generator A, B and customer C will pay congestion costs to the transmission system owner for the dispatching operational cost and extra transaction costs because of the congestion.



Fig. 2.11 Block Diagram of Transmission Congestion Cost

Since Gencos and customers will pay transmission congestion costs to Transcos, the costs can be considered the "investment" in the transmission network to improve its capability and reduce the congestion.

2.8.3 Transmission Loss Cost

Transmission loss cost is the recovery cost of electricity transmission losses due to transmission line resistances. Fig. 2.12 shows that the power flow from generation A to customer C loses 2 MW in the transmission line. Generator A should be compensated and customer C should be charged for the energy loss. Customers will pay transmission loss costs to Gencos as loss compensations.



Fig. 2.12 Block Diagram of Transmission Loss Cost

Some references [1, 10] claim that the loss cost should not be considered the component of transmission cost and it is unfair for those participants being charged. However, increasing utilities and researchers pay more attention to the study of transmission loss cost, since the transmission losses become significant for large and complicated transmission networks. Generation companies have to increase their outputs to counteract line losses to satisfy customer's demands. The transmission losses have

already influenced the revenues and efficiency of Gencos. Gencos should achieve compensations for energy delivery losses and customer should pay transmission costs to Gencos as loss compensations. In this thesis, the loss cost will be considered as one of components of transmission cost and determined in the pricing scheme.

As a result, the total transmission cost in the pricing scheme is given by:

$$TC_t = C_t^S + C_t^C + C_t^L \tag{2.1}$$

where

 TC_t = Total transmission cost of the transaction t

 C_i^s = Transmission service cost of the transaction t

- C_t^c = Transmission congestion cost of the transaction t
- C_t^L = Transmission loss cost of the transaction t

For generation companies, C_i^s and C_i^c are charges paid to transmission companies and C_i^L is the revenue from customers as the compensation for transmission loss cost. For transmission companies, C_i^s and C_i^c are revenues while C_i^L is equal to zero since they are not related to loss costs. For customers, all components of the cost are payments. C_i^s and C_i^c are paid to transmission companies and C_i^L is paid to generation companies. The development of the transmission pricing to evaluate and allocate transmission costs in a power system has received widespread attention from researcher. There is significant on-going research into the calculation and allocation of transmission costs.

2.9 Summary

This chapter has provided an overview on power industry restructuring in order to build an appropriate transmission pricing scheme that fits the restructured power market. After describing and comparing the characteristics of the traditional vertically integrated power system and the restructured and unbundled power system, the various components of the restructured power system were presented.

Based on the changes of the restructured power system, three trading models for market users were illustrated with some examples. The objective and requirements of the transmission pricing scheme were presented. This chapter also defined and confirmed the components of the transmission cost, such as transmission service, congestion and loss cost. The concepts shown in this chapter are used to study the calculation of all transmission costs and to develop an effective transmission pricing scheme in this thesis. Various pricing studies will be discussed in the subsequent chapters.

Chapter 3

Transmission Service Cost Calculation and Allocation

3.1 Introduction

In the present restructured transmission power system market under open access, a proper transmission pricing scheme built by the independent system operator (ISO) becomes necessary for generations, transmissions, distributions and all customers. In order to establish the efficient pricing scheme, the recovery of transmission service costs must be properly estimated and allocated to a market participant.

References [15-22] reviewed many methods for the calculation and allocation of transmission service costs. Some have been already used widely by electrical utilities, while others are still in development stages. Among these methods, the usage-based approach is considered a general, simple and accurate method to determine and allocate the transmission service costs. The basic principle of the usage-based method is to estimate and distribute the service costs based on actual usages of participants on transmission lines, as shown in Fig. 3.1.

This chapter focuses on studying a usage-based method: MW-mile method. It is used to determine the service costs of transmission lines and allocate the costs to participants based on their usages on networks. However, MW-mile method cannot determine participants' usages. Thus different methods, including distribution factors method, Bialek's tracing method and Kirschen's tracing method, are also presented in this chapter to estimate participants' usages used for the service cost allocation. The goal is to investigate the contributions of generators or loads to line flows. Case studies using different test power systems are presented to illustrate and compare these methods.



Fig. 3.1 Block Diagram of Usage-based Method

3.2 MW-mile Method

Postage-stamp rate method and contract path method were applied to estimate service costs in utilities before. However, the two methods are principally based on some artificial operational assumptions, such as fixed service rate, regulated cost and fixed transmission path. They could not provide accurate economic information and reflect the actual system operation, regardless of the actual service usages of participants and network conditions.

As the first usage-based cost strategy proposed for the recovery of transmission service costs, MW-mile method is used in many utilities recently that will reflect actual energy delivery of each participant through transmission lines, when considering real network conditions. The aim is to precisely determine and assign service costs based on the actual use on transmission networks for the users. This method focuses on studying some important factors, such as the magnitude of the actual usages of users on each line power flow, the path of transmission, the length and the unit service cost of each line. The length and unit cost are applied to calculate service costs and the cost allocation is based on the magnitudes of actual usages and the flow path. The comparison of postage-stamp rate, contract path and MW-mile methods is shown in Table 3.1.

Method	Characteristics	Drawbacks
Postage Stamp	 Transmission service costs are determined by peak demand, transaction power and stamp rate Assuming the entire transmission system is used and charge is fixed Based on the average system cost 	Ignorance of the impact of any particular transaction on actual system operation, such as: supply and delivery points, distance of delivery and distribution of generations and loads
Contract Path	 Suppliers and customers agree on a contract path for transmission path Cost based on transaction electrical energy through assumed path 	Since the contract path is fictitious and is not based on real network situation, it does not reflect actual power flow of system and provide wrong economic results.
MW-mile	 Considering the actual network conditions using power flow analysis It is an usage method that reflect actual transaction power flow Based on actual usages of power flows, path, length and unit cost of transmission line 	Congestion problems still could not be solved.

 Table 3.1
 Comparison of Different Service Cost Calculation Methods

The functions of the MW-mile method include service cost calculation and allocation. Initially, the service cost will be calculated after transmission companies confirm the length (in miles) and the service cost per unit (in \$/mile hr) of each transmission line. Hence, the service cost of line m-n is expressed as:

$$C_{m-n}^{S} = L_{m-n}C_{m-n}$$
(3.1)

where

 L_{m-n} = length of line m-n

 C_{m-n} = service cost per unit of line m-n

The total service cost of a particular transmission system is given by:

$$C_{total}^{S} = \sum_{all\ lines} C_{m-n}^{S} \tag{3.2}$$

Two power system models are considered in this chapter: a simple 6-bus system [23] and a 24-bus system [24]. The details and parameters of the two systems are given in Appendix A and B. Fig 3.2 illustrates the basic operating condition of the 6-bus system obtained using PowerWorld Simulator [25].



Fig. 3.2 Base Case Operating Conditions of the 6-bus System [23]

Line	Length (mile)	Cost per unit (\$ / mile hr)	Service Cost (\$/h)
1-2	20	3.0	60
1-4	40	5.0	200
1-5	30	4.0	120
2-3	50	5.0	250
2-4	30	3.0	90
2-5	50	3.0	150
2-6	50	4.6	230
3-5	20	4.0	80
3-6	20	5.5	110
4-5	60	4.5	270
5-6	40	4.25	170
Total	-	-	1730

Table 3.2 Assumed Service Cost of Transmission Lines of the 6-bus System

Table 3.2 presents the assumed transmission line length and service cost per unit in the 6-bus system. The service cost per unit of each line will be determined by individual transmission companies based on transmission markets and their economic strategies. For example, the length of line 2-4 L= 30 miles, and the cost per unit of the line C = 3.0 \$/ mile-hr. Thus, the service cost for line 2-4 is given by:

$$C_{2-4}^{s} = 30 \times 3 = 90$$
 \$/ hr

The total service cost of the 6-bus system is 1730 \$/hr. Subsequently, all line service costs are distributed to participants based on their actual usages. These usages can be represented by MW-mile values. The MW-mile value of user t on line m-n is expressed as:

$$MWMILE_{t,m-n} = c_{m-n}L_{m-n}MW_{t,m-n}$$
(3.3)

If the actual usage of generator G_1 on line 2-4 is 20 MW, the MW-mile value of G_1 on line 2-4 is

$$MWMILE_{G_{1,2-4}} = 30 \times 3 \times 20 = 1800 \$ \cdot MW / hr$$

Then, all MW-mile values of user t are accumulated to reflect the total actual usage of users on the network as follows [17]:

$$MWMILE_{t} = \sum_{all\ lines} c_{k} L_{k} MW_{t,k}$$
(3.4)

where

 $MW_{t,k} =$ flow in line k, due to user t

The service cost allocated to the user t is based on the proportion between the total MW-mile value of the user and the sum of the MW-mile values of all users as follows [15]:

$$C_{t}^{S} = C_{total}^{S} \times \frac{\sum_{all \ lines}}{\sum_{all \ users \ all \ lines}} c_{k} L_{k} M W_{t,k}$$
(3.5)

The determination of the usages of participants is a key step to allocate service costs. Reference [15] proposed that the actual usage of a user was achieved using dc power flow formulation presented in Appendix C [23]. In this study, these usages are determined using distribution factors method, or Bialek's tracing method, or Kirschen's tracing method.

3.3 Calculation of Participants' Usages Using Different Methods

An important objective in the application of usage calculation methods is to accurately determine the actual usage of users on the transmission line power flows. The operators should investigate which generators are supplying a particular load and what is each generator's (load's) contribution to the individual line power flow. As an example, consider the system shown in Fig. 3.3, where the total generation and load of a 3-bus system are 130 MW. Generator G_1 and G_2 provide energy to load L_2 and L_3 . All line losses are ignored.



Fig. 3.3 Single Line Diagram of the 3-bus System I

Since the demand at bus 3 (80 MW) is supplied by G_1 and G_2 , the power flow $P_{\text{line1-3}}$ (60 MW) and $P_{\text{line2-3}}$ (20 MW) are delivered from generators to L_3 through the line 1-3 and line 2-3. Thus, G_1 and G_2 are contributors on these two line flows. The aim of the study is to estimate the actual usages on the two lines, when determining the contributions of G_1 and G_2 to the power flow $P_{\text{line1-3}}$ and $P_{\text{line2-3}}$. For example, if generator G_1 and G_2 are assumed to contribute 11.42 MW and 8.58 MW respectively to $P_{\text{line2-3}}$, these contributions are considered the actual usages of G_1 and G_2 on line 2-3.

Based on the contributions and transmission service costs of different transmission lines, the ISO can allocate the costs to various generators and loads. The study in this thesis focuses on the analysis of active power since all transmission costs are only related to active power. This thesis does not discuss the reactive power allocation.

3.3.1 Distribution Factors Method

Distribution factors based on dc power flow theory have extensively been used for security and contingency analysis [23, 26]. These factors can approximately reflect the changes in transmission line flows corresponding to the changes of generation/load values. They are used as a tool to investigate the actual usages of participants for allocating transmission service costs [15, 17].

The distribution factors include *Generation Shift Distribution Factors* (GSDFs) and *Generalized Generation/Load Distribution Factors* (GGDFs/GLDFs). GSDFs are used to determine GGDFs and GLDFs. GGDFs and GLDFs are used to estimate participants' usages based on the determination of the contributions of generators and loads to line power flows.

3.3.1.1 Generation Shift Distribution Factors (GSDFs)

Since GSDF is associated with the line flow changes due to the changes in generations, it provides an approach to trace the contributions of generations on the incremental line flows. The factor is defined as [23]:

$$\Delta F_{l-k} = A_{l-k,i} \Delta G_i \tag{3.6}$$

where

$$\Delta F_{l-k}$$
 = the change in active power flow between buses l and k

 $A_{l-k,i}$ = GSDF factor of a line joining buses *l* and *k* corresponding to change in generator at bus *i*

 ΔG_i = the change in generation at bus *i*, with the reference bus excluded.

The factor determines the increase or decrease of the transmission flow caused by generators. It is based on the selection of the reference bus and the operational conditions of the system. From the reactance matrix [X] by dc power flow, GSDFs matrix [A] can be expressed as [23]:

$$A_{li} = \frac{1}{x_l} (X_{ni} - X_{mi})$$
(3.7)

where

 X_{ni} = *n*th element from the reactance matrix [X] by dc power flow

 X_{mi} = *m*th element from the reactance matrix [X] by dc power flow

 x_l = line reactance for line l

The reactance matrix [X] for the 6-bus system shown in Fig. 3.2 is presented below:

	0	0	0	0	0	0 -
	0	0.0941	0.0805	0.0630	0.0643	0.0813
V	0	0.0805	0.1659	0.0590	0.0908	0.1290
Λ =	0	0.0630	0.0590	0.1009	0.0542	0.0592
	0	0.0643	0.0908	0.0542	0.12215	0.0893
	0	0.0813	0.1290	0.0592	0.0893	0.1633

Using equation 3.7, GSDFs matrix [A] shown below can be determined (reference at bus 1). The row values of the matrix are the factors A of 11 lines corresponding to each bus. For example, A (1, 2) = (0 - 0.941) / 0.2 = -0.4705 is the value of line 1-2 for bus 2.

$$A = \begin{bmatrix} 0 & -0.4706 & -0.4026 & -0.3149 & -0.3217 & -0.4064 \\ 0 & -0.3149 & -0.2949 & -0.5044 & -0.2711 & -0.2960 \\ 0 & -0.2145 & -0.3026 & -0.1807 & -0.4072 & -0.2976 \\ 0 & 0.0544 & -0.3416 & 0.0160 & -0.1057 & -0.1907 \\ 0 & 0.3115 & 0.2154 & -0.3790 & -0.1013 & 0.2208 \\ 0 & 0.0993 & -0.0342 & 0.0292 & -0.1927 & -0.0266 \\ 0 & 0.0642 & -0.2422 & 0.0189 & -0.1246 & -0.4090 \\ 0 & 0.0622 & 0.2890 & 0.0183 & -0.1207 & 0.1526 \\ 0 & -0.0077 & 0.3695 & -0.0023 & 0.0150 & -0.3433 \\ 0 & -0.0034 & -0.0795 & 0.1166 & -0.1698 & -0.0752 \\ 0 & -0.0565 & -0.1273 & -0.0166 & 0.1096 & -0.2467 \end{bmatrix}$$

3.3.1.2 Generalized Generation Distribution Factors (GGDFs)

While GSDFs focus on studying the incremental change of line flows due to the change in generations, GGDFs are often directly applied to estimate the contribution of each generator to line flows. GGDFs depend on line parameters and system condition rather than on the choice of the reference bus. GGDF is defined as [15]:

$$F_{l-k} = \sum_{i=1}^{N} D_{l-k,j} G_i$$
 (3.8)

where

$$D_{l-k,j} = D_{l-k,r} + A_{l-k,j}$$
(3.9)

$$D_{l-k,r} = \left\{ F_{l-k}^{0} - \sum_{\substack{i=1\\i \neq r}}^{N} A_{l-k,r} G_i \right\} / \sum_{i=1}^{N} G_i$$
(3.10)

and

 F_{l-k} = total active power flow between buses l and k

 F_{l-k}^0 = power flow between buses *l* and *k* from the previous iteration

 $D_{l-k,j} = \text{GGDF}$ of a line between buses l and k corresponding to generator at bus j

 $D_{l-k,r}$ = GGDF of a line between buses l and k corresponding to generator at bus r

 G_i = total generation at bus *i*

Taking the same 6-bus system as an example, based on the above GSDFs matrix [A], GGDFs matrix [D] is given below.

$$D = \begin{bmatrix} 0.3495 & -0.1211 & -0.0531 \\ 0.3505 & 0.0356 & 0.0556 \\ 0.2927 & 0.0783 & -0.0098 \\ 0.0949 & 0.1493 & -0.2466 \\ 0.0181 & 0.3296 & 0.2335 \\ 0.0563 & 0.1556 & 0.0221 \\ 0.1714 & 0.2356 & -0.0708 \\ -0.0079 & 0.0543 & 0.2811 \\ 0.0975 & 0.0898 & 0.4670 \\ 0.0418 & 0.0384 & -0.0377 \\ 0.0556 & -0.0009 & -0.0717 \end{bmatrix}$$

In the matrix [D], the rows refer to the values of a particular transmission line related to different generators while the columns represent the values of various lines corresponding to the same generator. For example, the GGDF of line 3 (row 3), between buses 1 and 5, corresponding to generator 1 (column 1) is $D_{1-5, G1}$ (= 0.2927). Based on the D matrix, the contribution of Generator 1, 2 and 3 to the active power flow P_{ij} of each network line can be determined. Table 3.3 shows the results of the contributions of generators to line flows in the 6-bus system using GGDFs method. For example, the power flow P_{ij} = 28.66 MW on line 1-2 and it is assigned to each generator based on equation 3.8 as follows:

G₁:
$$G_1 \times D_{1-2,G_1} = 108.45 \times 0.3495 = 37.902 MW$$

G₂:
$$G_2 \times D_{1-2,G_2} = 50 \times -0.1211 = -6.057 MW$$

G₃:
$$G_3 \times D_{1-2,G_3} = 60 \times -0.0531 = -3.185 MW$$

Line k	P _{ij} (MW)	G ₁ (MW)	G ₂ (MW)	G ₃ (MW)
1-2	28.66	37.902	-6.057	-3.185
1-4	43.13	38.012	1.781	3.338
1-5	35.07	31.747	3.913	-0.590
2-3	2.97	10.297	7.470	-14.797
2-4	32.46	1.9677	16.481	14.012
2-5	15.22	6.111	7.780	1.329
2-6	26.12	18.588	11.780	-4.248
3-5	18.72	-0.857	2.714	16.864
3-6	43.09	10.578	4.491	28.021
4-5	4.19	4.533	1.919	-2.262
5-6	1.68	6.027	-0.045	-4.302
Total	251.31	164.905	52.225	34.181

Table 3.3 Contributions of Generators to Line Flows Using GGDFs Method for the 6-bus System

The contribution of each generator to the individual line flow will be used to allocate transmission service costs to participants using MW-mile method. The contributions shown in Table 3.3 are applied to distribute service costs to generator G_1 , G_2 , and G_3 in the 6-bus system. The results are shown in Table 3.4. The sum of MW-mile values related to G_1 is 25109.86 \$ MW /hr, and the total MW-mile value of the system is 47602.04 \$ MW/hr. Based on equation 3.5, the service cost allocated to G_1 is given by:

$$C_{G_{1}}^{S} = C_{total}^{S} \times \frac{\sum_{all \ lines} c_{k} L_{k} M W_{G_{1},k}}{\sum_{all \ users \ all \ lines} c_{k} L_{k} M W_{G_{1},k}}$$
$$= 1730 \times \frac{25109.86}{47602.04} = 912.57 \ \text{//hr}$$

Tuble 5.4 Service Cosis Allocalea lo Generalors Osing GGDT's Melliou for the 0-bas s	ıble 3.4	Service Costs A	llocated to General	ors Using GGDF.	s Method for the 6-bus Sy
--------------------------------------------------------------------------------------	----------	-----------------	---------------------	-----------------	---------------------------

	Line Cost	Ct-Lt-MW11	CH + MW21	CLL+MW2+
Line k	(¢/h)			
	(\$/nr)	/ G ₁	/ G ₂	/ 03
1-2	60	2274.10	363.42	191.08
1-4	200	7602.30	356.11	667.54
1-5	120	3809.70	469.50	70.77
2-3	250	2574.20	1867.40	3699.10
2-4	90	177.10	1483.30	1261.00
2-5	150	916.57	1167.00	199.38
2-6	230	4275.30	2709.40	977.07
3-5	80	68.59	217.09	1349.10
3-6	110	1163.60	493.96	3082.30
4-5	270	1223.80	518.07	610.62
5-6	170	1024.60	7.58	731.39
Total	1730	25109.86	9652.83	12839.35
$\sum \sum c_k L_k MW_{t,k}$			47602.04	
0	C_t^{S} (\$/hr)	912.57 350.81 46		466.62

As a result, the total service cost of 1730 \$/hr is allocated to generators G_1 (912.57\$/hr), and G_2 (350.81\$/hr) and G_3 (466.62\$/hr) respectively.

3.3.1.3 Generalized Load Distribution Factors (GLDFs)

In comparison with GGDFs used for the cost allocation corresponding to generations, GLDFs are normally applied to trace the contributions of loads on line flows. GLDF is expressed as [15]:

$$F_{l-k} = \sum_{i=1}^{N} C_{l-k,j} L_j$$
(3.11)

where

$$C_{l-k,j} = C_{l-k,r} - A_{l-k,j}$$
(3.12)

$$C_{l-k,r} = \left\{ F_{l-k}^{0} + \sum_{\substack{j=1\\j\neq r}}^{N} A_{l-k,j} L_{j} \right\} / \sum_{j=1}^{N} L_{j}$$
(3.13)

and

$$F_{l-k} = \text{total active power flow between buses } l \text{ and } k$$

$$F_{l-k}^{0} = \text{power flow between buses } l \text{ and } k \text{ from the previous iteration}$$

$$C_{l-k,j} = \text{GLDF of a line between buses } l \text{ and } k \text{ corresponding to load at bus } j$$

$$C_{l-k,r} = \text{GLDF of a line between buses } l \text{ and } k \text{ due to the load at reference bus } r$$

$$L_{j} = \text{total load at bus } j$$

Taking the same 6-bus system as an example, based on the above GSDFs matrix [A], GLDFs matrix [C] is given as follows:

	0.1037	0.1105	0.1952
	0.3526	0.1193	0.1442
	0.0526	0.2790	0.1694
	- 0.0953	0.0264	0.1114
	0.5146	0.0343	- 0.0852
C =	- 0.0201	0.2018	0.0357
	- 0.0664	0.0771	0.3625
	0.0876	0.2266	- 0.0467
	0.0973	0.0800	0.4383
	- 0.1395	0.1470	0.0523
	- 0.0266	- 0.1529	0.2035

The rows of matrix [C] are values of the same transmission line corresponding to different loads while the columns are values of various lines for the same load. For example, the GLDF of line 3 (row 3), between buses 1 and 5, corresponding to the load 6 (column 3) is $C_{1-5, L6}$ (= 0.1694). Based on the C matrix, the contribution of load 4, 5 and 6 to the active power flow P_{ij} of each network line can be determined. Table 3.5 shows the results of the contributions of loads to line flows in the 6-bus system using GLDFs method.

Line k	P _{ij} (MW)	$L_4(MW)$	$L_5(MW)$	$L_6(MW)$
1-2	28.66	7.258	7.737	13.666
1-4	43.13	24.682	8.352	10.096
1-5	35.07	3.680	19.531	11.859
2-3	2.97	-6.673	1.8471	7.7954
2-4	32.46	36.022	2.404	-5.966
2-5	15.22	-1.406	14.126	2.500
2-6	26.12	-4.649	5.396	25.372
3-5	18.72	6.132	15.861	-3.272
3-6	43.09	6.809	5.600	30.681
4-5	4.19	-9.763	10.290	3,663
5-6	1.68	-1.864	-10.699	14.243
Total	251.31	108.94	101.84	129.11

Table 3.5 Contributions of Loads to Line Flows Using GLDFs Method for the 6-bus System

For instance, the power flow $P_{ij} = 28.66$ MW on line 1-2 is allocated to L₄ based on equation 3.11 as follows:

L4:
$$L_4 \times C_{1-2,L_4} = 70 \times 0.1037 = 7.259 MW$$

The contribution of each load to the individual line flow will be used to allocate transmission service costs to loads using MW-mile method. The results are shown in Table 3.6.

Lingk	Line Cost	$c_k L_k MW_{4,k}$	$c_k L_k MW_{5,k}$	$c_k L_k MW_{6,k}$
Line K	(\$/hr)	/ L4	/ L5	/ L ₆
1-2	60	435.47	464.20	819.93
1-4	200	4936.40	1670.40	2019.20
1-5	120	441.63	2343.70	1423.00
2-3	250	1668.10	461.78	1948.80
2-4	90	3241.90	216.39	536.93
2-5	150	210.91	2118.80	375.06
2-6	230	1069.20	1241.10	5835.60
3-5	80	490.53	1268.90	261.79
3-6	110	749.02	615.96	3374.90
4-5	270	2636.10	2778.20	989.13
5-6	170	316.89	1818.90	2421.40
Total	1730	16196.15	14998.43	20005.74
ΣΣ	$\sum \sum c_k L_k MW_{t,k}$		51200.32	
0	C_t^{S} (\$/hr)	547.25 506.78 675.		675.97

Table 3.6 Service Costs Allocated to Loads Using GLDFs Method for the 6-bus System

The sum of MW-mile values (absolute values) corresponding to L_4 is 16196.15 \$ MW /hr, and the total MW-mile value is 51200.32 \$ MW/hr. Based on equation 3.5, the service cost allocated to L_4 is given by:

$$C_{L_4}^{S} = C_{total}^{S} \times \frac{\sum_{all \ lines} c_k L_k M W_{L_4,k}}{\sum_{all \ users \ all \ lines} c_k L_k M W_{L_4,k}}$$
$$= 1730 \times \frac{16196.15}{51200.32} = 547.25 \, \text{\%/hr}$$

The total service cost 1730 \$/hr is allocated to L_4 (547.25\$/hr), L_5 (506.78\$/hr) and L_6 (675.97\$/hr) respectively.

Although the calculation process is not complicated, the drawback of the distribution factors method is its inaccuracy because GSDFs matrix [A], GGDFs matrix [D] and GLDFs matrix [C] are based on dc power flow model. Additional security analysis should be adopted simultaneously to prevent the security violations that may occur under actual operational conditions.

3.3.2 Bialek's Tracing Method

The aim of the tracing method is to investigate the contributions of generators or loads to line flows and determine the actual usages of participants based on an important assumption: proportional sharing principle [19-20]. This principle claims that the nodal inflows will be shared proportionally between the nodal outflows. Thus, the proportion of the inflow through a particular node allocated to particular generators is the same as the proportion of the outflow allocated to the same generators. The principle is illustrated in Fig. 3.4.

Transmission line m-i power inflow through node i is $P_i = 60MW$, of which 60% is assumed to be supplied by generator 1 and 40% by generator 2. Hence the 20MW
outflow of line i-n is allocated to generator 1 as $20 \times 60\% = 12$ MW and to generator 2 as $20 \times 40\% = 8$ MW. Similarly the 40MW outflow of line i-o consists of the contribution of $40 \times 60\% = 24$ MW from generator 1 and the contribution of $40 \times 40\% = 16$ MW from generator 2.



Fig. 3.4 Proportional Sharing Principle Example

Based on the assumed principle, Bialek's tracing method can easily discover how much power flows are associated with a specific generator or load in order to determine their usages on the network. Bialek's tracing method has two versions: upstream looking algorithm and downstream looking algorithm [19]. The upstream looking algorithm will trace generators' contributions to flows, and conversely, the downstream looking algorithm will distribute power flows to individual load. For the upstream algorithm, the contribution of individual generator to every line flow is expressed as [19]:

$$P_{ij}^{g} = \sum_{k=1}^{n} D_{ij,k}^{G} P_{Gk}; \quad j \in \alpha_{i}^{u}$$
(3.14)

where

$$P_i^g = \sum_{j \in \alpha_i^u} \left| P_{ij}^g \right| + P_{Gi}; \quad i = 1, 2, ..., n$$
(3.15)

$$\begin{bmatrix} A_{u} \end{bmatrix}_{ij} = \begin{cases} 1 & i = j \\ -\frac{\left|P_{ji}\right|}{P_{j}} & j \in \alpha_{i}^{u} \\ 0 & otherwise \end{cases}$$
(3.16)

$$D_{ij,k}^{G} = \frac{P_{ij}^{g} \left[A_{u}^{-1} \right]_{ik}}{P_{i}^{g}} \cong \frac{P_{ij} \left[A_{u}^{-1} \right]_{ik}}{P_{i}}$$
(3.17)

and

 P_{ij}^{s} = an unknown gross line flow in line *i*-*j*

 $P_i^{\,g}$ = an unknown gross nodal power flow through node *i*

 A_{u} = upstream distribution matrix

 P_{Gk} = generation in node k

 α_i^u = set of buses supplying directly bus *i*

 $D_{ij,k}^{G}$ = topological distribution factors

Using the above equations, the distribution matrix A_{μ} of the 6-bus system is presented below:

$$A_{\mu} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -0.2646 & 1 & 0 & 0 & 0 \\ 0 & -0.038 & 1 & 0 & 0 \\ -0.4033 & -0.4226 & 0 & 1 & 0 & 0 \\ -0.3283 & -0.1992 & -0.2999 & -0.054 & 1 & 0 \\ 0 & -0.3390 & -0.7001 & 0 & -0.0219 & 1 \end{bmatrix}$$

	∫ 1	0	0	0	0	0
	0.2646	1	0	0	0	0
A ⁻¹	0.0101	0.038	1	0	0	0
$A_u =$	0.5151	0.4226	0	1	0	0
	0.4118	0.2334	0.2999	0.054	1	0
	0.1058	0.3707	0.7067	0.0012	0.0219	1

Table 3.7 presents the actual usage of individual generator G_1 , G_2 and G_3 on all lines using Bialek's method. In comparison with the results using GGDFs method, only positive values appear. Some line power flows are not allocated to all generators. Generator G_1 supplies all of the power flow 28.66 MW through line1-2, while the transmission power in line 3-5 (18.72 MW) is allocated to all three generators.

Line k	P _{ij} (MW)	$G_1(MW)$	$G_2(MW)$	G ₃ (MW)
1-2	28.66	28.66	0	0
1-4	43.13	43.13	0	0
1-5	35.07	35.07	0	0
2-3	2.97	1.11	1.86	0
2-4	32.46	12.12	20.34	0
2-5	15.22	5.68	9.54	0
2-6	26.12	9.75	16.37	0
3-5	18.72	0.34	0.56	17.82
3-6	43.09	0.77	1.30	41.02
4-5	4.19	3.06	1.13	0
5-6	1.68	1.01	0.26	0.41
Total	251.31	140.70	51.36	59.25

Table 3.7 Contributions of Generators to Line Flows Using Bialek's Method for the 6-bus System

Based on the transmission power flow allocation, the service costs are assigned to all generators using MW-mile method. The results are shown in Table 3.8. The sum of

the entire transmission service cost is 1730 \$/hr, and this charge is allocated to generator G_1 (1019.30 \$/hr), G_2 (406.51 \$/hr) and G_3 (304.19 \$/hr).

Line k	Line Cost	$c_k L_k MW_{1,k}$	$c_k L_k MW_{2,k}$	$c_k L_k MW_{3,k}$
	(\$/hr)	/ G1	/ G ₂	/ G3
1-2	60	1719.60	0	0
1-4	200	8626.00	0	0
1-5	120	4208.40	0	0
2-3	250	277.22	465.28	0
2-4	90	1090.70	1830.70	0
2-5	150	852.37	1430.60	0
2-6	230	2243.00	3764.60	0
3-5	80	26.91	45.16	1425.50
3-6	110	85.15	142.92	4511.80
4-5	270	826.89	304.41	0
5-6	170	172.26	43.82	69.52
Total	1730	20128.51	8027.50	6006.88
$\sum \sum c_k L_k M W_{t,k}$			34162.39	
0	C_t^{S} (\$/hr)	1019.30	406.51	304.19

 Table 3.8
 Service Costs Allocated to Generators Using Bialek's Method for the 6-bus System

The contribution of individual load to every line flow using the downstream algorithm is given by [19]:

$$P_{ij}^{l} = \sum_{k=1}^{n} D_{ij,k}^{L} P_{Lk}; \quad j \in \alpha_{i}^{d}$$
(3.18)

where

$$P_i^l = \sum_{l \in \alpha_i^d} \left| P_{il}^l \right| + P_{Li}; \quad i = 1, 2, ..., n$$
(3.19)

$$\begin{bmatrix} A_d \end{bmatrix}_{il} = \begin{cases} 1 & i = l \\ -\frac{\left|P_{ij}\right|}{P_l} & l \in \alpha_i^d \\ 0 & otherwise \end{cases}$$
(3.20)

$$D_{ij,k}^{L} = \frac{P_{ij}^{l} \left[A_{d}^{-1} \right]_{ik}}{P_{i}^{l}} \cong \frac{P_{ij} \left[A_{d}^{-1} \right]_{ik}}{P_{i}}$$
(3.21)

and

 P_{il}^{l} = an unknown gross line flow in line *i*-*l*

 P_i^l = an unknown gross nodal power flow through node *i*

 A_d = downstream distribution matrix

$$P_{Lk}$$
 = load in node k

 α_i^d = set of nodes supplied directly from node *i*

 $D_{ij,k}^{L}$ = topological distribution factors

Taking the 6-bus system as the example, Table 3.9 presents the actual usage of individual load L_4 , L_5 and L_6 on all lines using Bialek's method. The line flow (28.66 MW) on line 1-2 is allocated to all three loads, and the flow (43.09 MW) on line 3-6 is assigned to L_6 only.

	and the second se			
Line k	P _{ij} (MW)	L ₄ (MW)	L ₅ (MW)	$L_6(MW)$
1-2	28.66	11.45	6.53	10.68
1-4	43.13	40.74	2.33	0.06
1-5	35.07	0	34.26	0.81
2-3	2.97	0	0.88	2.09
2-4	32.46	30.65	1.77	0.04
2-5	15.22	0	14.87	0.35
2-6	26.12	0	0	26.12
3-5	18.72	0	18.29	0.43
3-6	43.09	0	0	43.09
4-5	4.19	0	4.09	0.10
5-6	1.68	0	0	1.68
Total	251.31	82.84	83.02	85.45

Table 3.9 Contributions of Loads to Line Flows Using Bialek's Method for the 6-bus System

Table 3.10 Service Costs Allocated to Loads Using Bialek's Method for the 6-bus System

T · 1	Line Cost	$c_k L_k MW_{1,k}$	$c_k L_k MW_{2,k}$	$c_k L_k MW_{3,k}$
Line k	(\$/hr)	/ L4	/ L5	/ L ₆
1-2	60	687.07	391.89	640.63
1-4	200	8148.20	466.76	11.01
1-5	120	0	4111.40	96.99
2-3	250	0	219.77	522.73
2-4	90	2759.60	158.08	3.73
2-5	150	0	2230.40	52.62
2-6	230	0	0	6007.60
3-5	80	0	1463.10	34.52
3-6	110	0	0	4739.90
4-5	270	0	1105.20	26.07
5-6	170	0	0	285.60
Total	1730	11594.87	10146.60	12421.39
$\sum \sum c_k L_k MW_{t,k}$		34162.86		
0	C_{t}^{s} (\$/hr)	587.16	513.82	629.02

Based on the contributions of loads to all flows, the service costs are assigned to all loads respectively. The results are shown in Table 3.10. The sum of the entire transmission service cost is also 1730 \$/hr, and this charge is allocated to L_4 (587.16 \$/hr), L_5 (513.82 \$/hr) and L_6 (629.02 \$/hr).

However, Pan, et al [17] have indicated that the proportional sharing assumption for developing the formulae of Bialek's algorithm will cause minor errors. The errors will become significant if all lines are under heavily loaded conditions.

3.3.3 Kirschen's Tracing Method

Based on the same proportional sharing principle, Kirschen's tracing method can provide solutions to the question of how much of power flows and losses are contributed by each generator or load. These contributions represent the actual usages of participants for the service cost allocation. Some concepts, such as domains, commons, links and state graph, are used in Kirschen's method [20].

The domain of a particular generator is defined as the set of buses supplied by that generator. For example, for the 6-bus test system shown in Fig. 3.2, the domain of generator 1 includes all the buses while the domain of generator 2 encompasses bus 2, 3, 4, 5, 6 and the domain of generator 3 only includes bus 3, 5, 6.

The concept of commons is the set of adjacent buses supplied by the same set of generators. For instance, the 6-bus system contains three commons:

1. common1: Bus 1 supplied by G_1 ;

- 2. common2: Bus 2, 4 supplied by G_1 and G_2 ;
- 3. common 3: Bus 3, 5, 6 supplied by G_1, G_2 and G_3

In addition, links are branches connecting commons. For the 6-bus system, there are seven links as follows:

- 1. line 1-2 and 1-4 are links connecting common 1 and common 2;
- 2. line 2-3, 2-5, 2-6, 4-5 are links connecting common 2 and common 3;
- 3. line 1-5 is the link connecting common1 and common 3.

Based on those definitions, a power system can be simplified to a state graph with power flows between commons. Taking the 6-bus system as an example, Fig 3.5 shows a clear state graph that can represent the entire system. It includes three commons and three links, and presents generations and loads at commons.



Fig. 3.5 Simplified State Graph for the 6-bus System I – Generator Contributions

Kirschen's method also has two versions to trace the contributions from generations and loads respectively. Fig 3.5 presents the graph used to determine generations' contributions (upstream), while the contributions of loads can be estimated using another graph (downstream), as shown in Fig 3.6.



Fig. 3.6 Simplified State Graph for the 6-bus System II – Load Contributions

The recursive calculation procedure for tracing the contribution of generators (loads) to commons, links and loads (generators) is applied in this method. It is expressed as [20]:

$$F_{ijk} = C_{ij}F_{jk} \tag{3.22}$$

$$I_k = \sum_j F_{jk} \tag{3.23}$$

$$C_{jk} = \frac{\sum_{j} F_{ijk}}{I_k}$$
(3.24)

where

 C_{ij} = contribution of generator (load) *i* to the load (generator) and the outflow of common *j*

 C_{ik} = contribution of generator (load) *i* to the load (generator) and the outflow of common *k*

- F_{jk} = flow on the link between commons j and k
- F_{ijk} = flow on the link between commons *j* and *k* due to generator (load) *i*
- I_k = inflow of common k

From the above equations, the contributions of generators or loads to line flows that reflect their usages on the lines can be determined. Table 3.11 presents the contributions of generators G_1 , G_2 and G_3 to the power flow of each transmission line in the 6-bus system. For example, the power flows of line 1-2, 1-4 and 1-5 are allocated to G_1 , and G_2 contributes to line 2-3, 2-4, 2-5, 2-6 and 4-5. In addition, only line 3-5, 3-6 and 5-6 are assigned to all three generators.

Line k	P _{ij} (MW)	G ₁ (MW)	G ₂ (MW)	G ₃ (MW)
1-2	28.66	28.66	0	0
1-4	43.13	43.13	0	0
1-5	35.07	35.07	0	0
2-3	2.97	1.76	1.21	0
2-4	32.46	19.14	13.32	0
2-5	15.22	8.97	6.25	0
2-6	26.12	15.40	10.72	0
3-5	18.72	8.30	2.60	7.82
3-6	43.09	19.11	5.97	18.01
4-5	4.19	2.47	1.72	0
5-6	1.68	0.74	0.24	0.70
Total	251.31	182.74	42.04	26.53

Table 3.11 Contributions of Generators to Line Flows Using Kirschen's Method for the 6-bus System

Kirschen's method shares many functions and features with Bialek's tracing method, since all tracing methods are based on the proportional sharing principle. Thus, from Table 3.7 and 3.11, the power flow allocation results of many columns are identical or similar.

Based on the contributions shown in Table 3.11, Table 3.12 shows the service costs allocated to generators using MW-mile method for the 6-bus system. The total system MW-mile value is equal to 34162.90 \$ MW/hr, which is close to the result using Bialek's method. The costs allocated to the generator G₁, G₂ and G₃ are 1274.12 \$/hr, 317.84 \$/hr and 138.04 \$/hr.

Lingk	Line Cost	$c_k L_k MW_{1,k}$	$c_k L_k MW_{2,k}$	$c_k L_k MW_{3,k}$
	(\$/hr)	/ G1	/ G ₂	/ G3
1-2	60	1719.60	0	0
1-4	200	8626.00	0	0
1-5	120	4208.40	0	0
2-3	250	437.70	304.80	0
2-4	90	1722.17	1199.23	0
2-5	150	1345.84	937.17	0
2-6	230	3541.48	2466.12	0
3-5	80	664.04	207.72	625.85
3-6	110	2101.67	657.42	1980.80
4-5	270	666.90	464.40	0
5-6	170	126.64	39.61	119.35
Total	1730	25160.43	6276.47	2726.00
ΣΣ	$c_k L_k MW_{t,k}$	34162.90		
0	C_t^{S} (\$/hr)	1274.12	317.84	138.04

Table 3.12 Service Costs Allocated to Generators Using Kirschen's Method for the 6-bus System

The contributions of loads to line flows for the service cost allocation to loads can be estimated using Kirschen's method. The results are given in Table 3.13. Since the demands of L_5 and L_6 are the same and they belong to the same common, their contributions are equivalent. Hence, their service costs are also the same.

			the second s	
Line k	P _{ij} (MW)	L ₄ (MW)	L ₅ (MW)	L ₆ (MW)
1-2	28.66	16.90	5.88	5.88
1-4	43.13	25.45	8.84	8.84
1-5	35.07	0	17.54	17.54
2-3	2.97	0	1.49	1.49
2-4	32.46	19.15	6.65	6.65
2-5	15.22	0	7.61	7.61
2-6	26.12	0	13.06	13.06
3-5	18.72	0	9.36	9.36
3-6	43.09	0	21.55	21.55
4-5	4.19	0	2.10	2.10
5-6	1.68	0	0.84	0.84
Total	251.31	61.51	94.90	94.90

Table 3.13 Contributions of Loads to Line Flows Using Kirschen's Method for the 6-bus System

Based on the actual usages of loads on lines shown in Table 3.13, Table 3.14 shows the service costs allocated to loads using MW-mile method. The total system MW-mile value is 34162.91 \$ MW/hr corresponding to the total service cost 1730 \$/hr. The costs allocated to the load L₄, L₅ and L₆ are 396.38 \$/hr, 666.81 \$/hr and 666.81 \$/hr.

Lingle	Line Cost	$c_k L_k MW_{1,k}$	$c_k L_k MW_{2,k}$	$c_k L_k MW_{3,k}$
Line k	(\$/hr)	/ L4	/ L ₅	/ L ₆
1-2	60	1014.56	352.52	352.52
1-4	200	5089.34	1768.33	1768.33
1-5	120	0	2104.20	2104.20
2-3	250	0	371.25	371.25
2-4	90	1723.63	598.89	598.89
2-5	150	0	1141.50	1141.50
2-6	230	0	3003.80	3003.80
3-5	80	0	748.80	748.80
3-6	110	0	2369.95	2369.95
4-5	270	0	565.65	565.65
5-6	170	0	142.80	142.80
Total	1730	7827.53	13167.69	13167.69
ΣΣ	$\Sigma\Sigma c_k L_k MW_{t,k}$ 34162.91			
0	C_t^{S} (\$/hr)	396.38	666.81	666.81

Table 3.14 Service Costs Allocated to Loads Using Kirschen's Method for the 6-bus System

Table 3.15 Result Comparison Using Various Usages Calculation Methods for the 6-bus System

Service Cost Allocation	Participant	Distribution Factors Method (\$/hr)	Bialek's Method (\$/hr)	Kirschen's Method (\$/hr)
	G1	912.57	1019.30	1274.12
Costs allocated to generations (\$/hr)	G ₂	350.81	406.51	317.84
	G ₃	466.62	304.19	138.04
	L ₄	547.25	587.16	396.38
Costs allocated to loads (\$/hr)	L_5	506.78	513.82	666.81
	L ₆	675.97	629.02	666.81

Table 3.15 presents different service cost allocation results of the 6-bus system using various methods. The costs are assigned to generators and loads respectively. Significant differences among the results using various methods can be observed. For example, the transmission service cost of generator G_1 using Kirschen's method (1274.12 \$/hr) is higher than the others while the result using GGDFs method is smallest.

It is very difficult to judge which results are more accurate because there is not a standard for the transmission cost studies. However, the results using Kirschen's method precisely reflect the actual system condition. Since G_1 supplies the most energy to all buses and delivers energy through all lines, it is allocated the highest service costs. In contrast, G_3 is assigned the lowest cost because it only supplies power to bus 5 and bus 6, and delivers power through line 3-5 and line 3-6 only.

For loads, L_5 and L_6 are distributed the highest costs, since they are supplied by all three generators and the demand power are delivered through most lines. Conversely, L_4 is supplied by G_1 and G_2 , and the demand only flows on line 1-4 and line 2-4. The service cost allocated to L_4 is the lowest.

Bialek's method also approximately reflects the actual system situation except that the cost allocated to L_4 is more than the cost allocated to L_5 . Unlike Kirschen's method, the results using distribution factors method does not illustrate the system condition. In addition, Kirschen's method is the simplest method when considering the calculation procedure. The comparison of three methods is presented in Table 3.16.

Method	Characteristics	Drawbacks
Distribution Factors Method	 A method that uses distribution factors: Generation Shift Distribution Factors (GSDFs), Generalized Generations / Loads Distribution Factors (GGDFs/GLDFs) To evaluate the relationships between transmission line flows and the generation/load values Can be only used for active power flow 	It is inaccurate since it is based on dc power flow model. Additional security analysis should be adopted simultaneously to prevent the contingency that probably occurs under actual operational condition.
Bialek's Tracing Method	 A method of tracing the power flow of electricity in meshed electrical networks May be applied to assess how much of the real and reactive power output from a particular generator goes to a particular load Proportional sharing principle assumption To provide the chance to trace the line flow for its origins in the meshed network 	Minor errors may be incurred when the lines are heavily loaded due to the assumptions used in the problem formulation.
Kirschen's Tracing Method	 Another tracing method to estimate the contribution to power flow for generators and loads Proportional sharing principle assumption to provide the chance to trace the real and reactive line flow for its origins Based on some important concepts, such as domains, commons and links 	The option for choosing slack bus causes different results.

Table 3.16 Comparison of Different Usages Calculation Methods

3.4 Case Study

The second case study based on the IEEE 24-bus system [24] is presented in this section to illustrate the calculation and allocation of transmission service costs using MW-mile method and different usage calculation methods. This power system consists of 10 generators, 17 loads and 38 branches. Fig 3.7 shows the single line diagram of this system. The detailed generations, loads and line parameters are given in Appendix B. The

total generation is 2951.1 MW, the total load is 2850 MW and the total line loss is 101.1 MW.



Fig. 3.7 Single Line Diagram of the IEEE 24-bus System [24]

Since the principle and calculation procedure for allocating service costs to either generators or loads are the same, this case study only determines and allocates service costs to each generator to compare the applications of different methods. Initially, distribution factors method, Bialek's and Kirschen's tracing method are applied to determine the contributions of generators to line flows. The power flows of all lines are obtained using full AC power flow program and PowerWorld Simulator based on generations and loads shown in Table B.2. Subsequently, service costs corresponding to each line are allocated to each generator based on their contributions using MW-mile method. The assumed service costs of lines are given in Table 3.17. All results are shown in Appendix D.

Line	Cost (\$/h)	Line	Cost (\$/h)
1-2	60	11-13	80
1-3	200	11-14	30
1-5	180	12-13	130
2-4	200	12-23	190
2-6	400	13-23	150
3-9	180	14-16	210
3-24	250	15-16	50
4-9	220	15-21	90
5-10	100	15-24	40
6-10	160	16-17	80
7-8	180	16-19	40
8-9	240	17-18	260
8-10	300	17-22	150
9-11	120	18-21	40
9-12	180	19-20	30
10-11	150	20-23	20
10-12	100	21-22	150

Table 3.17 Assumed Service Costs of Transmission Lines of the IEEE 24-bus System

a) Calculation of Usages (Contributions) of Generators

Table D.1 shows the contributions of 10 generators on the active power flow P_{ij} of individual transmission lines using GGDFs allocation method. One can find that the power flow of each line is allocated to every generator although some values are positive

and others are negative. For example, generator G_1 , G_{15} , G_{16} , G_{18} , G_{21} , G_{22} and G_{23} contribute positive power on line 1-2, and the contributions of G_2 , G_7 and G_{13} to the same line flow are negative.

The results of the contributions of generators using Bialek's method are presented in Table D.2. Significant differences occur, since all generators do not simultaneously contribute to the same line power flow P_{ij} . Generator G_{21} and G_{22} contribute most to transmission line flows while the contribution from G_7 is equal to zero.



Fig. 3.8 Simplified State Graph for the IEEE 24-bus System – Generator Contributions

Based on the concept of the 'commons' in Kirschen's tracing method, the 24-bus system consists of 14 commons for tracing the contributions of generators, as shown in Fig. 3.8. According to the state graph, Table D.3 presents the results of generations' contributions using Kirschen's method. No negative values are observed and all generators do not contribute to the same line flow.

Tables D.1, D.2 and D.3 certainly show significant differences, especially between GGDFs and the two tracing methods. However, as Tables 3.3, 3.7 and 3.11 show, the smaller differences can be observed for the 6-bus test system. It demonstrates that a complicated system would cause significant differences when using different methods. In Table D.1, both positive and negative contributions of individual generator to the same line flow appear due to GGDFs matrix [D]. These contributions cannot explicitly reflect the actual usages of generators on lines. In contrast, the results using tracing methods are more direct and transparent.

From Tables D.2 and D.3, the contribution results using different tracing methods are the same or very close in many columns because both tracing methods are based on the same proportional sharing principle. However, very different results can be observed from the remaining columns. The reason is that Bialek's method traces the contribution from each generator to every single line, and Kirschen's method identifies the contribution of each generator to a broader area, named as common, which may include a large number of internal lines and buses.

In addition, the selection of the root common when using Kirschen's method also causes the difference. G_{22} and G_{23} belong to the highest common generators because they

contribute most on line flows. Conversely, G_1 , G_2 and G_7 , which belong to the lowest common generators, contribute less or nothing to line flows.

b) Transmission Service Cost Calculation and Allocation

Based on above actual usages corresponding to generators, MW-mile method is applied to determine the transmission service cost of each generator. Table D.4 presents the cost allocation results using GGDFs method while the cost allocation results using the two tracing methods are given in Table D.5 and D.6.

As Tables D.4, D.5 and D.6 show, different methods give the different results of the transmission cost allocation using contrasting power flow allocation methods. For example, the total system MW-mile value using GGDFs method is 1332649 \$ MW/hr, which is much higher than the value 686951 \$ MW/hr using Bialek's method and 763694 \$ MW/hr using Kirschen's method.

Table 3.18 presents the service costs allocated to each generator only using different usages calculation methods. A case study for transmission service cost calculation and allocation to generators and loads simultaneously will be presented in Chapter 6. As discussed in 3.3.3, the results using the tracing methods can reflect the actual system conditions, including generations and demands. For example, G_{23} is allocated the greatest cost (1280.42 \$/hr) using Kirschen's method because its output is highest. In contrast, since G_7 does not supply any power to the system and is a local generator, the cost allocated to G_7 is zero. However, G_7 is still assigned some costs using GGDF method, although it does deliver power through any line.

Table 3.18 Comparison of Service Costs Allocated to Generations Using Various Usages Calculation

Comorator	Distribution Factors	Bialek's Method	Kirschen's Method
Generator	Method (\$/hr)	(\$/hr)	(\$/hr)
G ₁	541.21	78.07	70.18
G ₂	551.01	89.43	132.08
G ₇	16.81	0	0
G ₁₃	929.56	804.76	787.81
G ₁₅	197.97	210.05	170.60
G ₁₆	206.58	267.99	218.17
G ₁₈	538.95	398.38	589.20
G ₂₁	475.10	730.32	683.34
G ₂₂	524.92	1358.38	1028.20
G ₂₃	977.89	1022.62	1280.42
Total	4960.00	4960.00	4960.00

Methods for the 24-bus System

3.5 Summary

It is essential to develop an appropriate method to calculate and allocate transmission service costs in a transmission pricing scheme. A usage-based method: MW-mile method was described in this chapter. The motivation of MW-mile method is to determine each line service cost based on its length and cost per unit, and then allocate these costs to each participant based on their actual usages on lines.

The participants' usages can be determined using different methods, including distribution factor method, Bialek's and Kirschen's tracing method. The usages can be represented by the contributions of generators or loads to line flows. Based on the contributions, the service costs are assigned to individual users. The numerical example and case study based on a 6-bus system and the IEEE 24-bus system demonstrated the effectiveness of those methods to determine transmission service costs. The comparison of three usage calculation methods was also presented.

The study indicated that the service cost allocation results are significantly different using various methods. The results using distribution factors method might not be accurate since it is based on dc power flow model, and it cannot reflect actual system conditions. Both tracing power flow methods provided the results that reflect the system conditions. Since Kirschen's method is the simplest to apply, MW-mile method and Kirschen's method will be used in the proposed pricing scheme to calculate and allocate transmission service costs.

However, these methods cannot estimate transmission congestion cost. It will be useful to develop a method that can determine and allocate the cost related to network congestion problems. A method based on locational marginal prices to calculate the transmission congestion costs is presented in the next chapter.

Chapter 4

Transmission Congestion Cost Calculation and Allocation

4.1 Introduction

In recent years there has been increased interest in studying transmission congestion cost and management in restructured transmission power market. The principal objective is to appropriately solve electrical power delivery congestion problems without transmission networks security violation. Another aim is to accurately determine transmission costs caused by the congestion and fairly allocate the costs to each market participant.

In power systems, congestion is defined as the condition where overloads occur in transmission networks [1-2, 15]. The unexpected change in customers' demands and uncoordinated transactions are the main reasons that cause transmission congestion. The consequence of the congestion is that the customer might not receive the additional

power from a desired generator and more expensive generators have to be brought on line to supply the additional demand in the markets.

For example, a preferred energy transaction between generator A and customer C is 120 MW in a 2-bus system, as shown in Fig 4.1. However, congestion occurs because the capacity of the transmission line is 100 MW. Thus, generator A can only supply 100 MW to customer C. The more expensive generator B has to be brought into the market to supply extra energy (20MW) to customer C to satisfy its demand.



Fig. 4.1 Block Diagram of Transmission Congestion Cost

Energy prices and transmission pricing are highly affected by the transmission congestion. The revenue of generator A is decreased and the payment of the customer C is increased. Congestion also causes extra operational costs for the generation redispatch by operators. Since the congestion is caused by the energy transactions of Gencos and customers, they should be charged as the congestion compensation on the transmission system. These charges are considered as transmission congestion costs, which are due to the deployment of higher-priced generators and extra redispatch operations caused by the congestion. Krause [10] and Einhorn, et al [11] claim that congestion costs should be allocated to customers only. However, it is unfair for the customers because the congestion is caused by generations and customers together. Congestion costs should be allocated to each market participant. These costs from suppliers and buyers should be paid to the transmission company as "investment", which can be used to expand and improve the transmission network to eliminate congestion problems. The investment will guarantee future benefits for market participants.

Many approaches have been developed and applied in the restructured power system markets to measure congestion costs and allocate the costs to transmission system users. The common methods include cost of out-of-merit dispatch, locational marginal price, usage charges of zonal lines and physical transmission rights [1-3, 10, 15]. This thesis focuses on the study of a transmission congestion calculation and allocation method referred to as the Incremental (Marginal) pricing method [10].



Fig. 4.2 Block Diagram of Incremental (Marginal) Pricing Method [10]

Incremental (Marginal) pricing method, as shown in Fig. 4.2, is an accurate method derived from marginal cost theory to recover congestion costs when energy delivery congestion occurs. Marginal cost is defined as the additional cost caused by incremental demand through a constrained transmission line [10].

Incremental (Marginal) pricing method comprises the following: Short-run Incremental (marginal) cost method when transmission system capacity is fixed; and Long-run Incremental (marginal) cost method while the line capacities can be expanded. Since most transmission system capacities are fixed, the study presented here is based on the Short-run Incremental (marginal) cost method.

The Short-run Incremental (marginal) cost method incorporates locational marginal price (LMP) method, firm transmission rights (FTRs) strategy and Zonal-based pricing method to solve and manage congestion problems. Since the goal of this chapter is to calculate and allocate transmission congestion costs, only the study of LMP method is presented here. After describing the relationship between congestion costs and LMP, the principle and calculation procedure of LMP method are introduced. The determination of LMP values using different methods is highlighted. Case study using the IEEE 24-bus system is presented to illustrate the implementation of the studied methods.

4.2 Locational Marginal Price (LMP) Method

LMP method is generally used to determine nodal (bus) marginal price to display price differences among different buses when transmission line capacity constraints are considered. Locational marginal price is defined as the marginal cost for supplying the next increment of electricity at a specific bus, when considering generation marginal costs and the capacity limits of transmission lines [27]. It is the cost to serve the next new MW of load at a particular location based on generation costs while observing all transmission limits.

After determining bus marginal prices by LMP method, generators will sell electricity and obtain revenues based on the LMPs of the generation buses, and loads will buy electricity at the LMPs of the load buses.

On-going research indicates that the transmission congestion is always related to the LMPs, since the congestion will cause the difference of the LMPs. A simple 3-bus system is taken as the example to illustrate the relationship between the LMP and the transmission congestion [27]. Fig. 4.3 presents the data of generators, loads and transmission lines of a 3-bus system under study. Load 2 (150 MW) and Load 3 (250 MW) are supplied by Generator 1 and 2. The generation marginal price of generator 1 is 10 \$/MWh and the generation marginal price of generator 2 is 20 \$/MWh.



Fig. 4.3 Single Line Diagram of the 3-bus System II

Fig. 4.4 shows the generation dispatches and power flows of the system when transmission line limits are ignored. Since the generation marginal price of generator G_1 is cheaper than G_2 , only generator G_1 is assigned to supply the additional power demand at each bus. Thus, this generation price set by G_1 will be considered the LMP of each bus and LMP1 = LMP2 = LMP3 = 10 \$/MWh under unconstraint condition.



Fig. 4.4 3-bus System II Study under Unconstrained Condition

In contrast, Fig. 4.5 presents the case study under constrained condition when all line capacity limits are 200 MW. Congestion occurs because the line flow on line 1-3 should be under 200MW. Both generator G_1 and G_2 have to be re-dispatched to supply energy, although the generation price at G_2 is more expensive than G_1 .



Fig. 4.5 3-bus System II Study under Constrained Condition

Different LMP values appear when the congestion appears. For bus 1, the incremental demand will only be supplied by G_1 and the LMP at bus 1 is equal to the marginal price of G_1 : LMP1 = 10 \$/MWh. Since the energy flow of line 1-3 has already reached its limit, the local generator G_2 at bus 3 has to turn on to satisfy the new increasing demand at bus 3. Thus G_2 becomes the marginal units for bus 3 and LMP at bus 3 is equal to the generation price of G_2 : LMP3 = 20 \$/MWh.

 G_1 is not the sole generator to provide the incremental demand in bus 2 through line 1-2, since the increasing power flow on line 1-2 from G_1 to load 2 will cause the capacity limit violation on line 1-3. To avoid the violation of line 1-3, G_2 must supply part of energy for the additional demand at bus 2. Hence, bus 2 obtains energy from both G_1 and G_2 , and LMP2 will be based on the contributions and generation costs of G_1 and G_2 . Assuming both G_1 and G_2 contribute 50% power flow for the additional demand at bus 2, the LMP at bus 2 is given by:

$$LMP2 = (0.5 \times 10 \text{ /MWh}) + (0.5 \times 20 \text{ /MWh}) = 15 \text{ /MWh}$$

From the above examples, the LMP at each bus will be same when no congestion is considered. In contrast, the LMP value will be different once congestions occur. These differences can definitely reflect the transmission congestion problems and should be used to calculate congestion costs. The LMP can act as a price indicator of transmission congestion problems and be used in energy markets as an elementary part of transmission pricing scheme.

To estimate congestion costs, the LMP values should be determined first. Two methods for the calculation of LMPs are described in this section. Reference [15, 27] presented a generation shift factor method to determine the LMPs. It claimed that LMP_i comprise three components at any bus i: marginal generation price at the reference bus (LMP_i^{ref}) , marginal losses cost (LMP_i^{loss}) and congestion cost (LMP_i^{cong}) . Decomposing these three components LMP_i can be expressed as follows [15]:

$$LMP_{i} = LMP_{i}^{ref} + LMP_{i}^{loss} + LMP_{i}^{cong}$$

$$\tag{4.1}$$

$$LMP_i^{loss} = (DF_i - 1) \times LMP_i^{ref}$$
(4.2)

$$LMP_{i}^{cong} = -\sum_{k \in K} GSF_{ik} \beta_{k}$$

$$\tag{4.3}$$

where

 DF_i = delivery factor of bus *i* relative to the reference bus *i*

 GSF_{ik} = generation shift factor for bus *i* on line *k*

 β_k = constraint cost of line k

K = set of congested transmission lines

The delivery factor DF_i reflects the energy loss on a specific line when electricity is delivered from a particular bus to another bus over the line. For example, if 1 MW is sent from bus 1 but only 0.9 MW is delivered to bus 2 through line 1-2 (line loss = 0.1 MW), $DF_i = 0.9 / 1 = 0.9$ when bus 2 is the reference bus. This value depends on the choice of the reference bus. The detailed introduction and calculation of the generation shift factor has been presented in Chapter 3. The constraint cost β_k can be expressed as [15]:

$$\beta_{k} = \frac{\text{Reduction in total cost}}{\text{Change in constraint's flow}}$$
(4.4)

Fig. 4.6 shows a two-bus system as an example. The generation marginal prices of G1 and G2 are 10 \$/MWh and 15 \$/MWh respectively. Regardless of the transmission line loss, the $DF_{1-1} = DF_{1-2} = 1$. If bus 1 is considered the reference bus, the generator shift factor is given by:



Fig. 4.6 LMP Calculation Example Using Generation Shift Factor Method

The LMP at bus 1 is given by:

 $LMP_{1}^{ref} = 10 \,\$/hr$ $LMP_{1}^{loss} = (DF_{1-1} - 1) \times LMP_{i}^{ref} = (1 - 1) \times 10 = 0$ $LMP_{1}^{cong} = -GSF_{11}\beta = -(0) \times 5 = 0$ $LMP_{1} = LMP_{1}^{ref} + LMP_{1}^{loss} + LMP_{1}^{cong} = 10 + 0 + 0 = 10 \,\$/hr$

For the LMP at bus 2:

$$LMP_{1}^{ref} = 10 \/hr$$

$$LMP_{1}^{loss} = (DF_{2-1} - 1) \times LMP_{i}^{ref} = (1 - 1) \times 10 = 0$$

$$LMP_{1}^{cong} = -GSF_{21}\beta = -(-1) \times 5 = 5 \/hr$$

$$LMP_{1} = LMP_{1}^{ref} + LMP_{1}^{loss} + LMP_{1}^{cong} = 10 + 5 + 0 = 15 \/hr$$

Another method used to calculate LMPs is Kirschen's tracing method, which investigates the contributions of generators to power flows. Since the power flow tracing method also discovers the changes in line flows for changes in generations, the LMP of each bus can be determined based on the contributions of generators on power flows and generator marginal prices.

The first step is to determine all marginal generators that supply the incremental power demand on each bus. For the buses connecting marginal generators in a power system, the LMP value of a particular bus is equal to the marginal price of the particular generator at the bus. For other buses without marginal generators, the LMP of a particular bus depends on the contributions of marginal generators to line power flows corresponding to the bus.



Fig. 4.7 LMP Calculation Using Kirschen's Method for the 3-bus System II

Taking the 3-bus system shown in Fig. 4.3 as the example, the total generation and demand are 500 MW, as Fig 4.7 presents. The contributions of generators to line flows using Kirschen's tracing method is shown in Table 4.1.

Link k	Power flow (MW)	Power flow allocated to Generator G ₁ (MW)	Power flow allocated to Generator $G_2(MW)$	
1-2	211.52	211.52	0	
1-3	198.47	198.47	0	
2-3	38.50	26.49	12.01	

Table 4.1 Contributions of Generators to Line Flows for the 3-bus System II

At bus 1 and bus 3, the incremental powers will be supplied by G_1 and G_2 respectively. Thus, the LMP1 = 10 \$/hr (marginal price of G_1) and LMP3 = 20\$/hr (marginal price of G_2). For bus 2, the incremental power will be supplied by G_1 and G_2 together. The LMP is based on the contributions from Table 4.1 and marginal prices of generators as follows:

$$LMP_{2} = 10 \times [(211.52 + 26.49)/(211.52 + 38.50)] + 20 \times [12.01/(211.52 + 38.5)]$$

= 10.48 \$/ MWh

Once the LMP of each bus is determined, the congestion cost of the transmission line can be expressed as follows [27]:

$$C_{m-n}^{c} = f_{m-n} (LMP_n - LMP_m)$$
(4.5)

where

 f_{m-n} = power flow on line *m*-*n*

 LMP_n , $LMP_m = LMP$ at bus m or n

The congestion cost for line 1-3 in the 3-bus system shown in Fig. 4.7 is given:

$$C_{1-3}^{c} = f_{1-3}(LMP_3 - LMP_1) = 198.47 \times (15 - 10) = 992.35 \ \text{/} MWh$$

4.3 Case Study

A case study based on the IEEE 24-bus system is presented in this section to demonstrate the calculation and allocation of transmission congestion costs using the LMP method. The IEEE 24-bus system is shown in Fig. 3.7. The system generations, loads and line parameters are presented in Appendix B. Based on generation fuel cost coefficients shown in Table B.3, the generator marginal price of each generator is given in Table 4.2.

Generator	Marginal Price (\$/MWh) 28.04		
G ₁			
G ₂	28.04		
G ₇	30.08		
G ₁₃	27.44		
G ₁₅	23.10		
G ₁₆	23.10		
G ₁₈	19.00		
G ₂₁	19.00		
G ₂₂	19.01		
G ₂₃	16.60		

Table 4.2 Generator Marginal Prices for the 24-bus System

The LMP at each bus is determined using Kirschen's method, and the results are presented in Table 4.3. For the buses with generators, the LMP is equal to the marginal price of the generator at the bus. For example, the LMP at bus 23 is 16.60 \$/hr (marginal price of G_{23}). For other buses, the increment demands will be supplied by all generators together. Thus, their LMPs are based on the contributions of generators to line flows. The contributions shown in Table D.3 are used to estimate the LMPs. For example, the power

flows related to bus 5 are assigned to all generations except for G_2 and G_7 . Hence, the LMP at bus 5 is given by:

$$LMP_{5} = [28.04 \times 42.795 + 23.1 \times (1.706 + 0.34) + 19 \times (3.662 + 1.063) + 19.01 \times (1.318 + 3.325) + 27.44 \times 8.075 + 23.1 \times 1.19 + 19 \times 1.658 + 16.6 \times 7.068]/(50.17 + 21.25) = 25.30 \ \$/MWh$$

Bus	Locational Marginal Price (\$/MWh)	Bus	Locational Marginal Price (\$/MWh)
1	28.04	13	27.44
2	28.04	14	20.17
3	19.95	15	23.10
4	24.25	16	23.10
5	25.30	17	19.00
6	23.54	18	19.00
7	30.08	19	18.49
8	21.64	20	16.60
9	21.82	21	19.00
10	21.82	22	19.01
11	21.43	23	16.60
12	20.55	24	19.95

Table 4.3 Locational Marginal Price of Each Bus for the 24-bus System

The congestion cost of each line can be calculated based on (4.5). The results are given in Table 4.4. As an example, the flow on line 1-3 is 26.07 MW and the LMP difference between bus 1 and bus 3 is 8.086 \$/MWh. Thus, the congestion cost related to line 1-3 is: $26.07 \times 8.086 = 210.80$ \$/hr.
Lingle	Dower Flow (MW)	LMP difference	Congestion Cost
Line k	rower riow (INIW)	(\$/MWh)	(\$/h)
1-2	18.65	0	0
1-3	26.07	8.086	210.80
1-5	50.17	2.738	137.37
2-4	31.13	3.794	118.11
2-6	41.56	4.503	187.14
3-9	35.77	1.868	66.818
3-24	243.27	0	0
4-9	43.87	2.424	106.34
5-10	21.25	3.480	73.95
6-10	96.04	1.715	164.71
7-8	123.30	8.436	1040.16
8-9	157.67	0.178	28.07
8-10	151.91	0.178	27.04
9-11	166.78	0.391	65.21
9-12	182.46	1.276	232.82
10-11	227.98	0.391	89.14
10-12	244.91	1.276	312.51
11-13	208.10	6.009	1250.5
11-14	190.38	1.264	240.64
12-13	181.53	6.894	1251.5
12-23	252.45	3.946	996.17
13-23	187.28	10.84	2030.12
14-16	389.70	2.933	1143.00
15-16	78.370	0	0
15-21	493.66	4.10	2024.00
15-24	246.93	3.146	776.84
16-17	359.01	4.095	1470.10
16-19	96.08	4.610	442.93
17-18	185.27	0.005	0.93
17-22	178.41	0.005	0.89
18-21	118.72	0	0
19-20	85.22	1.89	161.07
20-23	213.70	0	0
21-22	218.49	0.01	2.18
Total	-	-	14651.02

Table 4.4Transmission Congestion Cost of Each Line for the 24-bus System

The total congestion cost corresponding to all transmission lines is 14651.02 \$/hr. These costs should be fairly allocated to generators and customers. Usage-based methods, including GGDF, Bialek's and Kirschen's tracing method, can be used for the allocation of the congestion costs. In this case study, the contributions of generators (loads) to line flows using Kirschen's method are used to allocate congestion costs to generators and loads.

Table 4.5 presents the results of the congestion costs allocated to generators. For example, G_1 is assigned the greatest congestion cost (5236.10 \$/hr), since it produces the highest energy to the system. In contrast, G_7 only provides electricity to the local customer at bus 7, and its usage on the transmission system is zero. Thus, the congestion cost allocated to G_7 is zero.

Generator	Transmission Congestion Cost (\$/hr)
G1	145.87
G ₂	271.67
G ₇	0
G ₁₃	2175.30
G ₁₅	332.57
G ₁₆	551.42
G ₁₈	1608.40
G ₂₁	2321.10
G ₂₂	2008.50
G ₂₃	5236.10
Total	14651.03

Table 4.5 Transmission Congestion Costs Allocated to Generations for the 24-bus System

The congestion costs allocated to loads are shown in Table 4.6. The cost related to load L_{13} (265MW) is 1257.7 \$/hr. For L_{20} , the congestion cost is zero because the LMPs at bus 20 and bus 23 are the same.

Load	Transmission Congestion Cost (\$/hr)
L ₁	198.86
L ₂	18.254
L ₃	790.34
L ₄	593.76
L ₅	473.88
L ₆	1098.20
L ₇	1990.00
L ₈	1339.90
L9	1373.80
L ₁₀	1525.90
L ₁₃	1257.70
L ₁₄	990.88
L ₁₅	1392.90
L ₁₆	511.32
L ₁₈	1.08
L ₁₉	1094.20
L ₂₀	0
Total	14651.06

Table 4.6 Transmission Congestion Costs Allocated to Loads for the 24-bus System

4.4 Summary

The calculation of transmission congestion costs is an important part of the transmission pricing scheme. It is a controversial research topic in today's power system restructuring. In this chapter, the congestion cost calculation using LMP method was presented.

The relationship between LMP and transmission congestion was illustrated with examples. Two different methods, i.e. generation shift factor method and Kirschen's method were used to determine LMP values. The procedure of the congestion cost calculation and allocation using LMP methods was also given. The case study based on the IEEE 24-bus power system model was presented to demonstrate the effectiveness of the studied methods. LMP method was used to calculate transmission congestion costs, when Kirschen's method was applied to determine LMP values.

As LMP method cannot estimate transmission service and loss costs, the next chapter will discuss the calculation of transmission loss costs. Chapter 6 will also present a comprehensive transmission pricing scheme that can determine all costs.

Chapter 5

Transmission Loss Cost Calculation and Allocation

5.1 Introduction

Transmission loss is that part of electrical power delivered on a particular transmission line lost due to the line resistance. Transmission loss in a line i is defined by [2]:

$$P_{Loss} = I^2 \times R_i \tag{5.1}$$

where

I = the current through line i

 R_i = the resistance of line *i*

Fig. 5.1 shows that the power flow from generator A to customer C will lose 2 MW in the transmission line when the transaction between A and C is 100 MW. The output of the generator is equal to the sum of the transmission loss and the customer demand:



Fig. 5.1 Illustration of Transmission Loss Cost

Every transaction between generations and customers through networks causes some transmission losses and the losses are actually quite significant in a large network. Normally the amount of the power losses in a standard system represents approximately 5% of the produced energy. It means million dollars every year, and the loss costs have already influenced benefits and efficiency of generation companies. It is necessary to determine the loss costs and allocate to participants since generation companies must obtain compensations for the losses of produced energy and customers should pay for the loss costs to generations as loss compensation.

The allocation of transmission loss costs among the generators and customers is a challenging and contentious issue in a fully deregulated system. It is a procedure for subdividing the system transmission losses into fractions, the costs of which then become the responsibility of individual users of the power system (Gencos, Discos and Customers). The loss allocation does not affect generation levels or power flows; however, it does modify the distribution of revenues and payments at the network buses among suppliers and consumers. Gencos should know the compensations they can

achieve and customers should understand their loss payments. The procedure includes the loss allocation and cost allocation.

The difficulty presented when selecting a loss allocation method is the absence of a standard means for comparing the different methods. Therefore, based on "fair and equitable" practice, any loss allocation algorithm should have most of the desirable properties stated below [28]:

- To be simple to understand and implement;
- To be consistent with power flow solution;
- To be able to promote efficient market operation, where the losses are reflected by network usage and the relative position of the bus in the network;
- To avoid volatility and provide appropriate economic signals.

Many loss allocation methods have been proposed in the literature. Most of the existing loss allocation methods are divided into the following: *pro rata* [28-32], proportional sharing [19-21], incremental transmission loss (ITL) [29-33], and loss formula method (Z-bus) [34].

In [24], the *pro rata* technique is used to allocate the system losses by considering only the active generation or load of each participant, but not the location of the generation or load in the network. This technique is being used in England, Spain, and Brazil. The losses allocated to a generator (customer) are proportional to the corresponding level of energy generation (consumption). This method is simple to understand and implement. However, it "ignores" the network situation. In addition, it is unfair for the load located near the generating bus since it is allocated more losses. Proportional sharing method is also called power flow tracing method discussed in Chapter 3. Based on the assumption that the power injections are proportionally shared among the outflows of each bus, this method can determine the contributions of generators or loads to the line power flows. According to the same contribution ratio, each line loss will be allocated to each generator or load.

For ITL, incremental transmission loss coefficients (sensitivities) are applied to assign losses to generators and demands in relation to bus injections. However, this method depends on the slack bus. The ITL coefficient of the slack bus is always defined to be zero, thus the slack bus is allocated no losses. Furthermore, ITL coefficient can be either positive or negative and this may be interpreted as cross subsidies.

The Z-bus method [34] uses the Z matrix of the system to obtain a "natural division" of losses among the system buses. This method uses the current rather than power injections. Although this approach yields negative losses sometimes, only the absolute values are used, and consequently, the allocations must be normalized. In addition, this method only allocates the losses to each bus instead of generators or loads. *pro rata* has to be performed to find the allocation to each participant after assigning losses to each bus.

Two different methods are presented in this chapter to determine and allocate transmission loss costs. A combination method that allocates losses using Z-bus method and calculates loss costs using *pro rata* is described. Another loss cost calculation and allocation method using a power flow tracing method is also given. Case study based on the IEEE 24-bus system is presented to compare the results using different methods.

5.2 pro rata and Z-bus Method

5.2.1 pro rata Method

The *pro rata* allocation method is the simplest loss allocation method. It assigns losses based on a comparison of the level of the power injected/consumed by a specific generator or load to the total power generated or consumed in the system. Starting from a solved power flow solution, losses are systematically distributed based on the real power injected or consumed at each node, as shown in (5.2) and (5.3) [30].

$$L_{Gi} = \frac{P_{Loss}}{x} \frac{P_{Gi}}{P_G}$$
(5.2)

$$L_{Di} = \frac{P_{Loss}}{x} \frac{P_{Di}}{P_D}$$
(5.3)

Equations 5.2 and 5.3 represent the *pro rata* allocation of losses to the generator at bus *i* and load at bus *j*. P_G is the total real power generated in the system while P_{Gi} is the total MW output of the generators at bus *i*. P_D is the total real power consumed and P_{Dj} is the real power consumed by loads of bus *j*. P_{loss} is the system transmission power losses. The multiplying factor *x* can be used to weight the distribution of system losses towards either of the market participants. Most companies allocate 50% of losses to the demands and 50% to the generators.

From the above equations, it is clear that this method relies on the power injections or consumptions at buses and is independent of the network topology. Losses are distributed across all buses, according to their level of generation or consumption only. Two loads in different locations but with identical demands will be allocated the same level of loss, irrespective of their comparative proximity to system generation. No incentive is provided for placing generation closer to load centers, a practice which usually leads to reduced system losses. In addition, the *pro rata* method is also unable to trace power flows, making it difficult to justify the different allocations.

For the losses allocated to the load and generator located at the same bus, the equations are given by:

$$L_{Gi} = L_i \frac{P_{Gi}}{P_{Gi} + P_{Di}}$$
(5.4)

$$L_{Di} = L_i \frac{P_{Di}}{P_{Gi} + P_{Di}}$$
(5.5)

where

 P_{Gi} = the total MW output of the generators at bus *i*. P_{Di} = the real power consumed by loads of bus *i*

 L_i = power losses allocated to bus *i*

5.2.2 Z-bus Method

The Z-bus loss allocation method uses the equations of electric circuits without any simplification. It is based on expressing the total system losses in simple manner related directly to the equations describing a solved load flow condition. Provided all generators and loads are represented as current injections into the system, total losses can be expressed by the Z-bus matrix formulation as [34]:

$$P_{loss} = \Re\left\{\sum_{i=1}^{n} I_i^* \left(\sum_{j=1}^{n} Z_{ij} I_j\right)\right\}$$
(5.6)

where

Z = Z-bus matrix of the network (can be obtained as the inverse of bus admittance matrix)

I = vector of complex bus current injections

Since Z = R + jX, this can be re-written in a more useful form with the resistance matrix R and the reactance matrix X as:

$$P_{loss} = \Re\left\{\sum_{i=1}^{n} I_i^* \left(\sum_{j=1}^{n} R_{ij} I_j\right)\right\} + \Re\left\{\sum_{i=1}^{n} I_i^* \left(\sum_{j=1}^{n} j X_{ij} I_j\right)\right\}$$
(5.7)

In a network that can be represented by a symmetrical impedance matrix, the second component (reactance X) in (5.7) sums to zero. The proof [34] is given by:

Since:
$$\Re\{(I^*)^T ZI\} = \Re\{(I^*)^T Z^* I^*\} = \Re\{(I^*)^T (Z^*)^T I\}$$

$$\Rightarrow \Re\{(I^*)^T (R+jX)I\} = \Re\{(I^*)^T (R-jX)^T I\}$$

Since Z is a symmetrical matrix:

$$\Rightarrow \Re\{(I^*)^T(jX)I\} = -\Re\{(I^*)^T(jX)^TI\}$$

Therefore: $\Re\{(I^*)^T(jX)I\}=0$

Thus, the total system losses can be expressed as [34]:

$$L_{i} = \Re\left\{\sum_{i=1}^{n} I_{i}^{*}\left(\sum_{j=1}^{n} R_{ij}I_{j}\right)\right\}$$
(5.8)

The total losses of the system are given by:

$$P_{loss} = \sum_{i=1}^{n} L_i \tag{5.9}$$

It is apparent from (5.8) and (5.9) that the total system losses are now distributed to all buses in the system. This distribution is dependent upon both the size of the current injection at the bus and also the position of the bus within the network. The losses are technically justifiable and the loss formula can be used by individual market participants to adjust their operational strategies to reduce their allocated loss. In addition, as the formula shows how losses relate to network topology, it might be possible to identify system conditions that could be adjusted to improve overall network behavior.

Taking the 6-bus system shown in Fig. 3.2 as an example, the real part of the Z-bus matrix of the system can be determined by running an AC power flow program as follows.

$$[R] = \Re\{[Z]\} = \Re\{[[Y]]^{-1}\}$$

$$= \begin{bmatrix} 0.022 & -0.027 & -0.008 & 0.005 & -0.004 & -0.008 \\ -0.027 & 0.010 & 0.000 & 0.000 & -0.005 & -0.001 \\ -0.008 & 0.000 & 0.017 & -0.007 & -0.004 & 0.009 \\ 0.005 & 0.000 & -0.007 & 0.022 & -0.006 & -0.008 \\ -0.004 & -0.005 & -0.004 & -0.006 & 0.012 & -0.004 \\ -0.008 & 0.000 & 0.009 & -0.008 & -0.004 & 0.018 \end{bmatrix}$$

The bus current based on bus voltage and admittance matrix, it is given by:

$$[I_i] = [Y][V_i] = \begin{bmatrix} -0.744 - j0.811 \\ 0.357 - j0.786 \\ 0.080 - j1.058 \\ 0.005 + j0.924 \\ 0.054 + j0.996 \\ 0.229 + j1.000 \end{bmatrix}$$

The transmission losses allocated to each bus using (5.8) is given by:

$$[L_i] = \begin{bmatrix} 2.744\\ 1.071\\ 1.507\\ 0.992\\ 1.428\\ 0.702 \end{bmatrix}$$

Bus 1 is assigned the greatest loss (2.744 MW) and the loss allocated to bus 6 is the smallest (0.702 MW). In addition, the sum of losses allocated is 8.45 MW. Using PowerWorld Simulator with full AC power flow method, the total losses of the 6-bus system is 8.43 MW. Two results are almost same. It demonstrates that the calculation using Z-bus method is very accurate.

The purpose in this chapter is to find transmission loss costs allocated to all participants. Although the losses can be distributed to buses accurately, it is impossible to know how the transmission loss cost is distributed to the specific participant. Hence, the loss at each bus still needs be allocated to each generator or load. Since there is a generator or load at each bus only, losses at buses can easily be distributed to each generator G_1 , G_2 and

 G_3 are assigned the loss 2.744, 1.071 and 1.507 MW, while the losses distributed to load L_4 , L_5 and L_6 are 0.992, 1.428 and 0.702 MW.

Assuming the marginal price of all generators is \$20/MWh, the transmission loss cost corresponding to individual generator or load is shown in Table 5.1.

 $G_1({\rm hr})$ -2.744×20=-54.88 $L_4(\text{hr})$ 0.992×20=19.85 $-1.071 \times 20 = -21.43$ $G_2(\text{hr})$ $L_5(\text{hr})$ $1.428 \times 20 = 28.57$ -1.507×20=-30.13 $G_3(\text{hr})$ 0.702×20=14.05 $L_6(\text{/hr})$ Total(\$/hr) -106.44 Total(\$/hr) 62.47

Table 5.1 Transmission Loss Cost Allocation Using Z-bus Method for the 6-bus System

As seen in Table 5.1, the loss cost to generator G_1 is the highest since the output of generator G_1 is the highest too. Generator G_2 provides the least electricity to the system and the loss cost to G_2 is the smallest. Even though all demands of customers are identical, the loss cost of each load is different. In addition, the negative values of loss costs of generators represent some revenues for generators as the loss compensation. In contrast, the loss costs of loads are positive values, which mean payments to generations from loads.

However, the amount (-106.44 \$/hr) of the loss compensation allocated to generators is not equal to the total payment (62.47 \$/hr) from loads. It indicates that generators may not gain enough loss compensation from the loads. This is the main shortcoming for using Z-bus and *pro rata* method. Although it is accurate to allocate losses to buses using these methods, it cannot keep the balance of the loss cost allocation between generators and loads.

5.3 **Power Flow Tracing Method**

The purpose of power flow tracing method is to find the contributions of generators or loads on each transmission line power flow and use the contributions to assign transmission costs, as discussed in Chapter 3. The transmission losses can also be assigned to each system participant based on the same contribution proportions found by power flow tracing method. Power flow tracing method includes Bialek's and Kirschen's method [19-21]. Kirschen's method is used in this chapter.

Starting from a solved power flow solution, each transmission line loss under a particularly operational condition is given and distributed using Kirschen's method. After obtaining these contributions from generators and loads, the transmission costs corresponding to each participant can be calculated based on the marginal prices of generators.

Using the same example of the 6-bus system shown in Fig 3.2, Table 5.2 and 5.3 present the results of the transmission losses allocated to individual generator or load using Kirschen's power flow tracing method. For generators, generator G_1 is assigned most of the loss (5.905MW) while G_2 and G_3 are allocated 1.53 and 0.995 MW. For loads, load L_5 and L_6 are allocated the same loss while the loss of 2.17 MW is given to load L_4 .

Line k	L _{ij} (MW)	G ₁ (MW)	G ₂ (MW)	G ₃ (MW)
1-2	0.93	0.93	0	0
1-4	1.12	1.12	0	0
1-5	1.12	1.12	0	0
2-3	0.04	0.024	0.016	0
2-4	1.64	0.96	0.68	0
2-5	0.56	0.33	0.23	0
2-6	0.62	0.36	0.26	0
3-5	1.23	0.54	0.17	0.52
3-6	1.07	0.47	0.15	0.45
4-5	0.04	0.024	0.016	0
5-6	0.06	0.027	0.008	0.025
Total	8.43	5.905	1.53	0.995

Table 5.2 Transmission Loss Allocation to Generators Using Kirschen's Method for the 6-bus System

Table 5.3 Transmission Loss Allocation to Loads Using Kirschen's Method for the 6-bus System

Line k	L _{ij} (MW)	$L_4(MW)$	$L_5(MW)$	L ₆ (MW)
1-2	0.93	0.55	0.19	0.19
1-4	1.12	0.66	0.23	0.23
1-5	1.12	0	0.56	0.56
2-3	0.04	0	0.02	0.02
2-4	1.64	0.96	0.34	0.34
2-5	0.56	0	0.28	0.28
2-6	0.62	0	0.31	0.31
3-5	1.23	0	0.62	0.61
3-6	1.07	0	0.53	0.54
4-5	0.04	0	0.02	0.02
5-6	0.06	0	0.03	0.03
Total	8.43	2.17	3.13	3.13

The transmission loss costs allocated to generators and loads are shown in Table 5.4 based on the contributions from Table 5.2 and 5.3 when all generation marginal prices are assumed to be 20/MWh. Generator G₁ achieves 54.88 \$/hr as the loss

compensation of produced energy, which is greater than using the Z-bus method. The load L_4 pays 21.7 \$/hr that is higher than using Z-bus method too.

Furthermore, the loss costs allocated to the generating participants and consumers must be specified arbitrarily. The typical proportion is 50% although some companies allocate all losses to customers. Here, the ratio of 50% that represent the generators and loads will equally share the losses is applied since it is fair and equitable for every market participant. Table 5.4 shows that the compensation payments from loads are the same as the revenues generators should obtain.

Table 5.4 Transmission Loss Cost Allocation Using Kirschen's Method for the 6-bus System

$G_1({hr})$	$-5.905 \times 20/2 = -59.05$	$L_4($/hr)$	2.17×20/2=21.70
G ₂ (\$/hr)	-1.53×20//2=-15.30	L ₅ (\$/hr)	3.13×20/2=31.30
G ₃ (\$/hr)	-9.95×20/2=-9.95	$L_6($/hr)$	3.13×20/2=31.30
Total(\$/hr)	-168.6/2=-84.3	Total(\$/hr)	168.6/2=84.3

In comparison with the results using Z-bus and *pro rata* method shown in Table 5.1, the results using Kirschen's method are quite different. For example, the loss cost of G_1 using Kirschen's method is -59.05 \$/hr that is higher than the value using Z-bus method. For load L_4 , the loss cost using power flow tracing method is 21.70 \$/hr that is higher too. The total loss costs allocated to generators or loads using Kirschen's method are same, while generators cannot obtain enough loss compensations from loads using Z-bus and *pro rata* method.

5.4 Case Study

A case study for the IEEE 24-bus system shown in Fig. 3.7 is presented to demonstrate the implementation of the two methods used to calculate and allocate transmission loss costs. The comparison of different results using the combination method and Kirschen's power flow tracing method is also given. The details of the IEEE 24-bus system are presented in Appendix B. The case study comprises two parts. The fist one is to determine the transmission loss costs of generators and loads using Z-bus and *pro rata* method. The second one is to apply Kirschen's method to find loss costs. All results are shown in Appendix E.

a) Z-bus and *pro rata* method

Initially, Z-bus method is applied to determine the loss allocation on each bus based on the system resistance matrix and bus voltage. The results are shown in Table E.1. The amount of transmission losses is 107.1 MW, which is identical to the value from PowerWorld Simulator. Some values of losses allocated to buses are negative. For example, the loss at bus 3 is -1.909 MW. As mentioned above, these negative values will be transferred to be positive because all losses assigned to each participant should be absolute values.

However, the total losses of the system will be increased when these negative values become positive. For example, the total loss of the 24-bus system is increased to 141.2 MW after negative sign changes. Consequently, the total loss cost will be

significantly increased too. It is another disadvantage for using Z-bus method to find the loss cost.

Since some buses comprise both generator and load, the losses assigned to each bus still need to be distributed to individual generator or load using *pro rata* method, based on (5.4) and (5.5). The average system marginal price of electricity is 18.56 \$/MWh based on each generator marginal price shown in Table C.2. The loss costs allocated to generators and loads are given in Table E.2 and E.3.

The loss cost of generator G_{21} is -379.48 \$/hr. G_{21} can obtain 379.48 \$/hr from the loads as the loss compensation. In contrast, the loss cost allocated to the load L_7 is \$314.07/hr. This represents the loss cost that L_7 should pay to generators as the compensation. However, the total loss cost on generators (-1433.69 \$/hr) is not equal to the total cost on loads (1179.29 \$/hr). It indicates that generators may not obtain enough loss compensation from the loads.

b) Kirschen's Method

Table E.4 and E.5 present how each transmission line loss is assigned to every generator or load. The contributions of generators or load to losses are based on the same sharing proportions of power flows shown in Chapter 3 using Kirschen's tracing power flow method. For example, the loss (2.28 MW) of line 12-13 is only allocated to G_{13} (1.174 MW) and G_{23} (1.106 MW). However, this loss is assigned to the load $L_4 - L_{10}$ and L_{13} .

The loss costs corresponding to particular generators are shown in Table E.6 based on the contributions from Table E.4. The costs are determined by each generator marginal price instead of the average price. It can absolutely improve the calculation accuracy. The loss allocated to G_1 is 0.784 MW and the marginal price of G_1 is 28.04 \$/MWh. The revenue of G_1 as the loss compensation is given by:

$$C_{G1}^{L} = \frac{0.784 \times 28.04}{2} = 10.99 \ hr$$

The same quantity of the loss costs distributed to generators is allocated to loads based on the contributions of loads to losses shown in Table E.5. The results are presented in Table E.7. The loss cost to load L_7 is the highest (131.52 \$/hr) and load L_{20} only is assigned 1.13 \$/hr as the loss compensation payment.

In comparison with the results using Z-bus and *pro rata* method, the results using Kirschen's method are quite different. For example, the loss cost of G_{21} using Kirschen's method is -127.28 \$/hr which is smaller than the value using Z-bus method. For load L₇, the loss cost using power flow tracing method is 131.52 \$/hr that is smaller too. The total loss costs allocated to generators or loads using Kirschen's method are almost the same because the assumption that generators and loads equally share the losses is given. Using the power flow tracing method can achieve reasonable results for loss cost allocation.

5.5 Summary

This chapter has presented and compared two methods to determine the transmission loss cost. Using the system resistance matrix, it is easy to find the loss allocation to each bus using Z-bus method. Then, loss costs of generators and loads can be calculated using *pro rata* method. A new method using tracing power flow contributions has been presented. Loss costs of each participant can be determined based on contributions of generators or loads to transmission line losses using the power flow tracing method.

Case study indicates that both methods are easy to implement. However, the negative results of the losses allocated to buses appeared using Z-bus method. The total loss costs were unexpectedly increased. The compensations generators obtain did not match the loss payments from loads. This will be unfair for generators. Using the power flow tracing method, all problems were solved when generators and loads were assumed to be assigned the same losses. The power flow tracing method will be used to calculate the transmission loss cost in the proposed comprehensive transmission pricing scheme to be presented in Chapter 6.

Chapter 6

Comprehensive Transmission Pricing Scheme

6.1 Introduction

Chapter 3 introduced the usage-based method for the calculation and allocation of transmission service costs. The determination of congestion costs using LMP method was presented in Chapter 4. Chapter 5 presented the calculation and allocation of loss costs using different methods. However, these methods cannot estimate the entire transmission cost when an individual method is applied.

The shortcoming of the usage-based methods is that they ignore the impact of any particular transaction on actual system operations and transmission congestion problems for the additional incremental demand. These methods are often applied under normal operational conditions without the careful consideration of the transmission network security constraints. The transmission service and loss costs could not be reflected and recovered using the Incremental method, since the main goal of this method is to eliminate the transmission congestion problem. Reference [15] presented a transmission pricing scheme that estimates service and congestion costs, irrespective of the calculation of loss costs. Furthermore, the pricing scheme did not provide enough economic information about energy transactions. In this chapter, a new comprehensive transmission pricing scheme is proposed and described, which investigates all three components of transmission costs. In this scheme, the transmission service cost and loss cost will be determined using Kirschen's power flow tracing method and the calculation of the congestion cost is calculated using locational marginal price (LMP) method.



Fig. 6.1 Block Diagram I of the Proposed Pricing Scheme

Fig. 6.1 provides a block diagram of the proposed scheme. The purpose is to trace the actual contributions of generators (loads) to each line flow and loss using Kirschen's tracing method, and then all components of transmission costs can be calculated and allocated simultaneously based on the contributions. This method can also be applied to estimate the locational marginal price (LMP) used for the congestion cost calculation instead of the generation shift factor method [15, 23]. In addition, this scheme will investigate the energy transaction revenues or payments of market participants.

This chapter is organized as follows: general formulae for calculating service, loss and congestion costs using Kirschen's method and LMP method are presented first. A useful strategy, optimal power flow (OPF) used for power dispatch in the pricing scheme is introduced. The proposed transmission pricing scheme is outlined and described. Case study based on the IEEE 24-bus system is presented to illustrate the proposed scheme.

6.2 General Calculation Formulae of Transmission Service, Congestion and Loss Cost Using a Power Flow Tracing Method

In this section, the calculations of three components of the transmission costs using Kirschen's power flow tracing method are presented. In addition, the estimation of the LMPs using tracing method is also given.

6.2.1 Calculation of Transmission Service Cost

The detailed calculation of transmission service costs using tracing method is presented in Chapter 3. Here only general equations are given. Using Kirschen's Tracing Method, the contribution of each generator (load) on each line flow can be determined. Then the transmission service costs will be assigned to each participant based on the contributions.

Let $f_{m-n,G_i}(f_{m-n,D_i})$ refer to the contribution of each generator (load) at bus *i* to each line flow f_{m-n} , D_{m-n} is the length of line *m-n* in miles, and R_{m-n} represents the required transmission service cost per unit length of line *m-n* (\$/mile hr). The service cost for line *m-n* corresponding to generator (load) $G_i(D_i)$ is given by [15]:

$$C_{m-n,G_i}^{S} = \frac{f_{m-n,G_i} D_{m-n} R_{m-n}}{f_{m-n}}$$
(6.1)

$$C_{m-n,D_i}^{S} = \frac{f_{m-n,D_i} D_{m-n} R_{m-n}}{f_{m-n}}$$
(6.2)

If $Z_{m-n} = D_{m-n}R_{m-n}$ is the required transmission service cost of line *m*-*n* in \$/hr, the payment of $G_i(D_i)$ for the service cost of all lines is as follows:

$$C_{G_{i}}^{S} = \sum_{all \ lines} C_{m-n,G_{i}}^{S}$$

$$= \sum_{all \ lines} \frac{f_{m-n,G_{i}} D_{m-n} R_{m-n}}{f_{m-n}}$$

$$= \sum_{all \ lines} \frac{f_{m-n,G_{i}} Z_{m-n}}{f_{m-n}}$$
(6.3)

$$C_{D_{i}}^{S} = \sum_{all \ lines} C_{m-n,D_{i}}^{S}$$

$$= \sum_{all \ lines} \frac{f_{m-n,D_{i}}D_{m-n}R_{m-n}}{f_{m-n}}$$

$$= \sum_{all \ lines} \frac{f_{m-n,D_{i}}Z_{m-n}}{f_{m-n}}$$
(6.4)

The total payment by all participating generators (loads) for transmission service cost is:

$$C_{Gt}^{S} = \sum_{j \in S_{G}} \sum_{all \ lines} \frac{f_{m-n,G_{i}} Z_{m-n}}{f_{m-n}}$$
(6.5)

$$C_{D_{t}}^{S} = \sum_{j \in S_{G}} \sum_{all \ lines} \frac{f_{m-n,D_{i}} Z_{m-n}}{f_{m-n}}$$
(6.6)



Fig. 6.2 Single Line Diagram of the 3-bus System III

A simple 3-bus system is considered as an example to illustrate the procedure of the service cost calculation. Fig. 6.2 shows generations (177.29 MW), loads (170MW), generator marginal costs, line flows and losses in this system.

From Kirschen's tracing method, the system consists of two commons. The state graphs of the contributions from generation and loads are shown in Fig. 6.3 and Fig. 6.4 respectively.



Fig. 6.3 State Graph of the Generator Contributions for the 3-bus System III



Fig. 6.4 State Graph of the Load Contributions for the 3-bus System III

The absolute and relative contribution matrices of generators and loads can be obtained as:

$$A_{G} = \begin{bmatrix} 157.29 & 157.29 \\ 0 & 20 \end{bmatrix}; \quad R_{G} = \begin{bmatrix} 1.0000 & 0.8872 \\ 0.0000 & 0.1128 \end{bmatrix}$$
$$A_{L} = \begin{bmatrix} 110 & 110 \\ 0 & 60 \end{bmatrix}; \quad R_{L} = \begin{bmatrix} 1.0000 & 0.6471 \\ 0.0000 & 0.3529 \end{bmatrix}$$

Based on the contribution matrices, the contributions of generators and loads to each line flow are shown in Table 6.1.

Table 6.1	Contributions of	f Generations and	Loads to Line	Flows for the	: 3-bus System III

Line k	P _{ij} (MW)	G ₁ (MW)	G ₂ (MW)	L ₂ (MW)	L ₃ (MW)
1-2	67.21	67.21	0	23.72	43.49
1-3	90.08	90.08	0	0	90.08
2-3	24.50	21.74	2.76	0	24.50

Assuming that the transmission service costs of all lines are 100 \$/hr, these costs are allocated to each generator and load based on contributions as follows:

For loads:

L₂ Payment: $100 \cdot 0.3529 + 0 + 0 = 35.29$ / hr

L₃ Payment: $100 \cdot 0.6471 + 100 + 100 = 264.71$ / hr

Total service cost for loads = 300 \$/hr

For generators:

 G_1 Payment: $100 + 100 + 100 \cdot 0.8872 = 288.72$ / hr

G₂ Payment: $0+0+100 \cdot 0.1128 = 11.28$ / hr

Total service cost for generations = 300 /hr

6.2.2 Calculation of Transmission Congestion Cost

From Chapter 4, the transmission congestion cost is principally based on the actual power flow through the congested transmission line and the difference in locational marginal prices (LMPs) between the source buses and sink buses. The key is to

estimate the contributions of generations or loads to each line flow and the LMP value of each bus. The tracing method is applied to calculate these contributions and LMPs.

Let $f_{m-n,G_i}(f_{m-n,D_i})$ be the contribution of a generator (G_i) or load (D_i) at bus *i* to a line flow between bus *m* and *n*. The congestion costs that are allocated to the generator (load) are presented below [15]:

$$C_{G_j}^c = \sum_{j \in S_G} f_{m-n,G_j} \times (LMP_n - LMP_m)$$
(6.7)

$$C_{D_i}^C = \sum_{j \in S_G} f_{m-n,D_i} \times (LMP_n - LMP_m)$$
(6.8)

As discussed in Chapter 4, the contributions of generators using Kirschen's method can also be used to determine locational marginal prices (LMPs). The first step is to determine all marginal generators that supply the incremental power demand on each bus. For the buses connecting marginal generators in a power system, the LMP value of a particular bus is equal to the marginal price of the particular generator connected to the bus. For other buses without marginal generators, the LMP of a particular bus depends on the contributions of marginal generators to line power flows corresponding to the bus.

Let $f_{m-n,i}^{G_j}$ refer to the contribution of each marginal generator j to each line flow $f_{m-n,i}$ corresponding to bus i, and W_{G_i} represent the generator marginal price of generator G_i (\$/MWh). The LMP at bus i is given by:

$$LMP_{i} = \sum_{all \ generators} W_{G_{j}} \frac{\sum f_{m-n,i}^{G_{j}}}{\sum f_{m-n,i}}$$
(6.9)

For the 3-bus system example shown in Fig. 6.2, the LMP of bus 1 (2) is equal to the marginal price of generator 1 (2) since generator 1 (2) will supply the incremental 1MW demand at bus 1 (2). Therefore, the LMPs of bus 1 and bus 2 are 10 \$/MWh and 20 \$/MWh respectively. On the other hand, the incremental 1MW demand at bus 3 will be provided by both generator 1 and 2. The LMP of bus 3 is given by:

 $LMP_3 = 10 \times [(90.78 + 21.74)/(90.78 + 24.5)] + 20 \times [2.76/(90.78 + 24.5)] = 10.24 \text{/} MWh$

Using (6.7) and (6.8), the congestion costs of the 3-bus test system allocated to loads and generations are as follows:

For loads:

L₂ Payment: $23.72 \cdot (20 - 10) + 0 \cdot (10.24 - 10) + 0 \cdot (20 - 10.24) = 237.20 \text{/} hr$

L₃ Payment: $43.49 \cdot (20 - 10) + 90.08 \cdot (10.24 - 10) + 24.50 \cdot (20 - 10.24) = 695.64$ / hr

Total congestion cost for loads = 932.84 \$/hr

For generators:

G₁ Payment: $67.21 \cdot (20 - 10) + 90.08 \cdot (10.24 - 10) + 21.74 \cdot (20 - 10.24) = 905.90$ / hr

G₂ Payment: $0 \cdot (20 - 10) + 0 \cdot (10.24 - 10) + 2.76 \cdot (20 - 10.24) = 26.94$ / hr

Total congestion cost for generations = 932.84 \$/hr

The total congestion charges allocated to loads and generations are the same.

6.2.3 Calculation of Transmission Loss Cost

The principle and procedure of loss cost calculation and allocation, shown in Chapter 5, are similar to the calculation of the service cost. It is also based on the contribution of each generator (load) on each line flow using the tracing method.

Let $L_{m-n,G_i}(L_{m-n,D_i})$ refer to the contribution of each generator (load) at bus *i* to each line loss L_{m-n} , and W_{G_i} represent the generator marginal cost unit of generator G_i (\$/MWh). The loss cost for line *m-n* corresponding to generator G_i is given by:

$$C_{m-n,G_i}^L = \sum_{all \ generators} L_{m-n,G_i} W_{G_i}$$
(6.10)

Since generators and load should equally share the loss cost and loads will pay these costs to generations, the payment of D_i for the loss cost of the line *m*-*n* is as follows:

$$C_{m-n,D_i}^{L} = \frac{1}{2} C_{m-n,G_i} \frac{L_{m-n,D_i}}{L_{m-n}}$$
(6.11)

The payment of D_i for the loss cost of all lines is given by:

$$C_{D_{i}}^{L} = \sum_{all \ lines} C_{m-n,D_{i}}$$

$$= \sum_{all \ lines} \frac{1}{2} C_{m-n,G_{i}} \frac{L_{m-n,D_{i}}}{L_{m-n}}$$
(6.12)

The total payment by all participating customers for transmission loss cost is:

$$C_{Dt}^{L} = \sum_{j \in S_{G}} \sum_{all \ lines} \frac{1}{2} C_{m-n,G_{i}} \frac{L_{m-n,D_{i}}}{L_{m-n}}$$
(6.13)

Taking the same system shown in Fig. 6.1 as the example, the contribution of individual generator (load) for each line loss are shown in Table 6.2 based on the same absolute and relative contribution matrices of generators and loads using Kirschen's method.

Line k	L _{ij} (MW)	G ₁ (MW)	G ₂ (MW)	L_2 (MW)	L ₃ (MW)
1-2	2.71	2.71	0	0.96	1.75
1-3	4.10	4.10	0	0	4.10
2-3	0.47	0.42	0.05	0	0.47

Table 6.2 Contributions of Generations and Loads to Line Losses for the 3-bus System III

Based on (6.10), the loss cost of each line corresponding to generators is given in Table 6.3. The total loss cost is 73.3 \$/hr. However, generations should equally share this cost with loads. Thus loss costs allocated to generator 1 and 2 are 36.2 \$/hr and 0.5 \$/hr. It means that generators can obtain these loss compensations from loads. Table 6.4 presents results of loss cost allocated to loads using (6.13). Load 2 and 3 will pay 4.8 \$/hr and 31.9 \$/hr to generations.

Line k	Loss cost responding to	Loss cost responding	Loss Cost
	G ₁ (\$/hr)	to G ₂ (\$/hr)	(\$/hr)
1-2	2.71x10=27.1	0	27.1
1-3	4.10x12=41.0	0	41.0
2-3	0.42x10=4.2	0.05x20=1.0	5.2
Total	72.3/2=36.2	1.0/2=0.5	73.3/2=36.7

Table 6.3 Loss Cost of Each Line Responding to Generators for the 3-bus System III

Line k	Loss Cost (\$/hr)	L ₂ (\$/hr)	L ₃ (\$/hr)
1-2	27.1	9.6	17.5
1-3	41.0	0	41.0
2-3	5.2	0	5.2
Total	73.3/2=36.7	9.6/2=4.8	63.7/2=31.9

Table 6.4 Loss Costs Allocated to Loads for the 3-bus System III

The total transmission costs allocated to generators and loads are given in Table 6.5. The loss costs of generators are negative because the loss costs are considered as loss compensations from loads. G_1 is assigned the highest transmission cost (1158.42 \$/hr) because it provides the greatest electricity (157.29MW) to customers through transmission lines. Since G_2 only supplies 20 MW to customers, it is allocated the lowest cost (37.72 \$/hr). As L_3 (110 MW) is supplied by G_1 and G_2 through all lines, the cost allocated to L_3 is the second highest (992.25 \$/hr). Hence, the results are reasonable because they exactly reflect the actual system conditions.

Generators and loads	Transmission Service Cost C_t^s (\$/hr)	Transmission Congestion Cost C_t^C (\$/hr)	Transmission Loss Cost C_t^L (\$/hr)	Total Costs TC_i (\$/hr)
G ₁	288.72	905.90	-36.2	1158.42
G ₂	11.28	26.94	-0.5	37.72
L ₂	35.29	237.20	4.8	277.29
L ₃	264.71	695.64	31.9	992.25

Table 6.5 Total Transmission Costs Allocated to Generations and Loads for the 3-bus System III

 G_1 and G_2 pay service and congestion costs to the transmission company when they obtain the loss compensations from loads. L_2 and L_3 pay service and congestion costs to the transmission company, and pay loss costs to generators.

6.3 Optimal Power Dispatch

After receiving the transaction schedule from participants, the ISO or other operators in power utilities should check the feasibility of the schedule. It means that the transaction demands and generations of the schedule should satisfy all system security constraints, including bus voltage, the output limitation of generators and transmission line capacities. Once any congestions or violations occur, the operators have to redispatch generator outputs and line flows.

Since economy is an essential goal for the energy market, the economic power dispatch is necessary. Optimal power flow (OPF) method is considered an effective tool used for power dispatch in the proposed scheme. OPF was introduced as an extension of conventional economic dispatch in the beginning of 1960s [35]. The purpose is to optimize a certain objective function while satisfying a set of physical and operational constraints imposed by equipment limitation and security requirements.

In this research, the optimal objective focuses on minimizing the total fuel cost of the generators. Since the fuel cost function of generators typically uses cubic cost model, the objective function of OPF is expressed as [36]:

Minimize:
$$F = \sum_{i=1}^{n} (\gamma_i + \alpha_i P_{Gi} + \beta P_{Gi}^2)$$
 (6.14)

where

F = the total fuel cost

 P_{Gi} = the output real power of generator *i*

 $\alpha_i, \beta_i, \gamma_i$ = the cost coefficients of generator *i*

The control variables in this study include equality and inequality constraints. The basic power flow equations are considered the equality constraints. They are expressed as:

$$\sum_{i=1}^{m} P_i = 0 \tag{6.15}$$

$$\sum_{i=1}^{m} Q_i = 0 ag{6.16}$$

where

 P_i = the active power flow at bus *i*

 Q_i = the reactive power flow at bus i

The inequality constraints contain limits on control variables namely, generator active and reactive power outputs, limitation of bus voltage (magnitude and angle), and limitation of line flow capacities. They are given by:

$$P_{G_i}^{\min} \le P_{G_i} \le P_{G_i}^{\max} \tag{6.17}$$

$$Q_{G_i}^{\min} \le Q_{G_i} \le Q_{G_i}^{\max} \tag{6.18}$$

$$P_{ij}^{\min} \le P_{ij} \le P_{ij}^{\max} \tag{6.19}$$

$$Q_{ij}^{\min} \le Q_{ij} \le Q_{ij}^{\max} \tag{6.20}$$

$$v_i^{\min} \le v_i \le v_i^{\max} \tag{6.21}$$

$$\delta_i^{\min} \le \delta_i \le \delta_i^{\max} \tag{6.22}$$

where

 P_{Gi} = the real power output of generator *i*

 Q_{Gi} = the reactive power output of generator *i*

 P_{ij} = the active power flow between bus i and j

 Q_{ij} = the reactive power flow between bus i and j

 v_i = the voltage magnitude at bus i

 δ_i = the voltage angle at bus *i*

6.4 Proposed Comprehensive Transmission Pricing Scheme

In this section, a comprehensive transmission pricing scheme is proposed [37] and described. Fig. 6.5 presents the outline of the proposed scheme in which Kirschen's power flow tracing method is applied. The scheme includes seven steps as follows:


Fig. 6.5 Block Diagram II of the Comprehensive Transmission Pricing Scheme.

Step 1: Submit Transaction Data by Market Participants

During this step, all participants should submit their transaction data to the ISO or other operators of utilities for a particular schedule. The data may comprise initial pool, bilateral and multilateral power trading schedules, generation limits and load limits.

Step 2: Check Feasibility of the Initial Transaction Schedule

Operators should check all transaction data submitted by participants and judge whether the desired schedule causes any transmission network congestions and constraint violations.

Step 3: Redispatch Generations Using OPF and Load Shedding

If congestions or violations occur, the generations in the initial transaction schedules should be re-dispatched to solve all congestion problems by the operators using optimal power dispatch strategy and the necessary load shedding of customer demands. If the initial schedules are accepted without any constrained violations, the process could directly enter the next step and there are no congestion costs.

Step 4: Calculate Contributions of Generators (Loads) to Line Flows and Losses

Using Kirschen's tracing method, the contribution of each generator (load) on each line flow and loss is determined. These contributions can be applied by operators to calculate transmission charges.

Step 5: Determine Locational Marginal Price

Based on the contributions of generators to line flows and generation marginal prices, the LMP of each bus should be calculated to reflect congestion costs.

Step 6:Calculate Transmission Service, Congestion and Loss Cost, and Allocate to Each Participant

Based on the contributions from step 4, the operators will determine all transmission costs: transmission service and loss costs using MW-mile method, congestion costs using LMP method. These costs will be allocated to each generator or load based on the contributions of generators and loads to line flows.

Step 7: Determine Revenues and Payments of Market Participants

After determining all transmission cost, the economic data about energy transactions for each participant should be provided. The data include detailed revenues and costs for generation companies (Gencos), revenues for the transmission company (Transcos) and payments from customers.

6.5 Case Study

In this section, the proposed comprehensive transmission pricing scheme is applied using the IEEE 24-bus system shown in Fig. B.1. The details of the system are given in Appendix B. Bus 7 is the reference bus. The voltage limit of each bus is from 0.94 to 1.06 p.u..

In order to simulate the practical transmission system market, the transaction models of the 24-bus system include all three types: bilateral, multilateral, and pool trading models. Tables 6.6 to 6.8 provide an initial transaction schedule at a particular hour based on the three models.

Bus	Туре	Min (MW)	Max (MW)	Pref (MW)
18	G	0	400	300
18	D	0	300	300
23	G	0	660	150
20	D	0	150	150

Table 6.6 Bilateral Transaction Data for the 24-bus System

Table 6.7Multilateral Transaction Data for the 24-bus System

Bus	Туре	Min (MW)	Max (MW)	Pref (MW)
21	G	0	400	200
22	G	0	300	200
23	G	0	660	200
14	D	0	200	200
15	D	0	350	200
19	D	0	200	200

Bus	Туре	Min (MW)	Max (MW)	Pref (MW)
1	G	0	192	170
2	G	0	192	170
7	G	0	300	30
13	G	0	591	400
15	G	0	215	215
16	G	0	155	155
18	G	0	400	100
21	G	0	400	200
22	G	0	300	100
23	G	0	660	310
1	D	0	100	100
2	D	0	100	100
3	D	0	150	150
4	D	0	50	50
5	D	0	50	50
6	D	0	150	200
7	D	0	100	100
8	D	0	150	150
9	D	0	200	200
10	D	0	200	200
13	D	0	300	300
15	D	0	350	150
16	D	0	100	100

Table 6.8 Pool Transaction Data for the 24-bus System

In Table 6.6, G_{18} and G_{23} sign bilateral contracts with load L_{18} and L_{20} respectively. Thus, the electricity transaction prices for L_{18} and L_{20} depend on the generator marginal prices of G_{18} and G_{23} . Table 6.7 presents multilateral transaction data, in which load L_{14} , L_{15} and L_{16} are supplied by generator G_{21} , G_{22} and G_{23} together. The electricity price of the multilateral transactions is the average value of the marginal prices of the three generators. The pool transaction data is shown in Table 6.8. Generations and customers who join the pool will sell and purchase electricity based on an average market price of the whole pool.

The desired transaction schedule is submitted to the ISO or other system operators for feasible checking, in order to examine congestion and contingency. After running power flow program and contingency analysis, the congestion is found to occur on line 6 -10 as follows:

Line 6-10: line flow: 213.8 MVA > line limit: 200 MVA (107%)

In order to solve this congestion problem, the optimal power dispatch strategy with necessary load shedding is applied. With the constraints of generation outputs, line capacities and voltage magnitudes, the OPF program using Matlab [38] and Matpower [39] is used to look for the optimal dispatch results. The best results are obtained when the load L_6 is decreased to 150 MW. The results are shown in Tables 6.9-6.10.

Bus	Min (MW)	Max (MW)	Dispatch Results (MW)	Transaction (MW)
1	0	192	160.7	Pool: 160.7
2	0	192	165.2	Pool: 165.2
7	0	300	71.9	Pool: 71.9
13	0	591	384.1	Pool: 384.1
15	0	215	215	Pool: 215
16	0	155	155	Pool: 155
10	0	400	400	Pool: 100
10	0	400	400	Bilateral: 300
21	0	400	400	Pool: 200
21	0	400	400	Multilateral: 200
22	0	200	200	Pool: 100
	0	500	500	Multilateral: 200
				Pool: 310
23	0	660	660	Multilateral: 200
				Bilateral: 150
		Total Generation	2911.9	

Table 6.9 Optimization Dispatch Results for Generations for the 24-bus System

Table 6.9 presents generation redispatch results, and demand results are shown in Table 6.10. The total generation (2911.9 MW) is larger than the total load (2850 MW) since the total system transmission loss is 61.9 MW.

Bus	Min (MW)	Max (MW)	Dispatch Results (MW)	Transaction (MW)		
1	0	100	100	Pool: 100		
2	0	100	100	Pool: 100		
3	0	150	150	Pool: 150		
4	0	50	50	Pool: 50		
5	0	50	50	Pool: 50		
6	0	150	150	Pool: 150		
7	0	100	100	Pool: 100		
8	0	150	150	Pool: 150		
9	0	200	200	Pool: 200		
10	0	200	200	Pool: 200		
13	0	300	300	Pool: 300		
14	0	250	250	Pool: 50		
14	0	230	230	Multilateral:200		
15	0	250	250	Pool: 150		
15	0	550	550	Multilateral:200		
16	0	100	100	Pool: 100		
18	0	300	300	Multilateral:300		
19	0	200	200	Pool: 200		
20	0	150	150	Bilateral: 150		
	Total Load 2850					

Table 6.10 Optimization Dispatch Results for Loads for the 24-bus System

Table 6.11 presents the line flows and losses corresponding to the redispatch transaction schedule. The line flow on line 1-2 is 19.72 MW and the loss is 0.01 MW. No transmission lines are forced to carry the maximum limit. The total transmission loss is equal to the difference between the total generation and load in Tables 6.9 and 6.10.

Line	Limit (MVA)	Pij (MW)	Pji (MW)	Loss (MW)
1-2	175	19.72	-19.71	0.01
1-3	175	-12.40	12.51	0.11
1-5	175	52.42	-53.37	0.95
2-4	175	30.75	-30.40	0.35
2-6	175	54.14	-51.37	2.77
3-9	175	40.91	-40.21	0.7
3-24	400	-203.41	204.48	1.07
4-9	175	-19.59	19.79	0.2
5-10	175	2.42	-1.95	0.47
6-10	200	-98.63	100.25	1.63
7-8	200	-28.11	29.00	0.89
8-9	175	-94.39	99.15	4.76
8-10	175	-84.61	87.92	3.31
9-11	400	-131.52	131.97	0.45
9-12	400	-147.20	147.77	0.57
10-11	400	-184.70	185.52	0.82
10-12	400	-201.47	201.52	0.95
11-13	500	-150.75	152.39	1.64
11-14	500	-166.74	168.37	1.63
12-13	500	-123.33	124.39	1.06
12-23	500	-226.90	233.34	6.44
13-23	500	-192.71	196.67	3.96
14-16	500	-368.36	375.40	7.04
15-16	500	105.69	-105.43	0.26
15-21	500×2	-224.13×2	227.25×2	3.12×2
15-24	500	207.56	-204.48	3.08
16-17	500	-336.5	340.17	3.67
16-19	500	121.51	-121.04	0.47
17-18	500	201.31	-200.61	0.7
17-22	500	-139.55	142.09	2.54
18-21	500×2	-50.65×2	50.73×2	0.08×2
19-20	500×2	-39.48×2	39.61×2	0.13×2
20-23	500×2	-114.61×2	115.00×2	0.39×2
21-22	500	-155.92	157.94×2	2.02
			Total Loss	61.91

Table 6.11 Optimization Dispatch Results for Line Flows and Losses for the 24-bus System

Table 6.12 shows that voltage magnitudes of buses are within limits (0.94-1.06 p.u.) without any violations.

Bus	Voltage Magnitude (p.u.)	Bus	Voltage Magnitude (p.u.)
1	0.947	13	1.027
2	0.946	14	0.982
3	0.944	15	1.008
4	0.933	16	1.011
5	0.965	17	1.021
6	1.018	18	1.025
7	1.000	19	1.015
8	0.962	20	1.030
9	0.957	21	1.024
10	1.005	22	1.039
11	0.987	23	1.042
12	0.996	24	0.969

Table 6.12 Voltage Magnitude of Each Bus for the 24-bus System

Tables 6.13 and 6.14 present some system economic data. Based on the generation fuel cost coefficients shown in Table C.2 and redispatch output results, the individual generator marginal price is determined and shown in Table 6.13. For example, the price of G_{23} is the cheapest while the highest price is from G_7 . The marginal prices are used to determine LMPs, congestion costs and loss costs.

Generator	Marginal Price (\$/MWh)
G ₁	28.21
G ₂	28.30
_G ₇	31.44
G ₁₃	27.68
G ₁₅	24.30
G ₁₆	23.10
G ₁₈	19.00
G ₂₁	19.00
G ₂₂	18.00
	16.60

Table 6.13 Generator Marginal Prices for the 24-bus System

Table 6.14 provides the assumed service cost data of each transmission line at a particular hour. The service costs will be allocated to generators and loads based on their usages.

Line	Cost (\$/h)	Line	Cost (\$/h)
1-2	25	11-13	80
1-3	360	11-14	70
1-5	150	12-13	80
2-4	240	12-23	170
2-6	330	13-23	150
3-9	200	14-16	70
3-24	150	15-16	30
4-9	180	15-21	40
5-10	150	15-24	90
6-10	100	16-17	45
7-8	110	16-19	40
8-9	280	17-18	25
8-10	280	17-22	180
9-11	140	18-21	25
9-12	140	19-20	35
10-11	140	20-23	20
10-12	140	21-22	120

Table 6.14 Assumed Service Costs of Transmission Line for the 24-bus System

With the optimization results, the next step is to trace the contribution of each generator or load to line flows and losses using Kirschen's power flow tracing method. As shown in Chapter 3, the IEEE 24 bus system comprises 14 commons. Using the upstream and downstream approach, the proportion of each power line flow corresponding to every generator or load can be estimated. Then, the contributions of generators or loads to the flows are obtained based on the proportions.

Tables F.1 and F.2 give the contribution results of generators and loads to power flows respectively. Generator G_{23} is assigned 65.026 MW and 58.833 MW comes from generator G_{13} for the power flow of line 12-13. Table F.2 shows that the line flow 19.715 MW on line 1-2 is distributed to L_2 (10.745 MW), L_4 (2.248 MW) and L_6 (6.722 MW) respectively.

The contributions of generators or loads to transmission losses can also be calculated using the same principle. The allocation results for generators and loads are given in Table F.3 and F.4. All loss on line 17-22 is distributed to G_{22} while the loss on 15-21 is assigned to every load except for L_{13} and L_{20} .

The next step is to calculate the locational marginal price of individual buses in order to estimate the congestion costs. Using (6.9), the LMPs are obtained based on the contributions of generators to line flows and the generator marginal prices. The results are presented in Table 6.15. For example, the LMP of bus 1 is equal to the generator marginal price of G_1 at bus 1. For bus 3, only generators G_{15} , G_{21} and G_{22} contribute to power flows in the transmission lines connected to bus 3. Based on (6.9), the LMP at bus 3 is given by:

$$LMP_{3} = 24.3 \times (4.06 + 13.06 + 65.67) / (12.45 + 40.46 + 203.5) + 19.00 \times (6.03 + 19.63 + 98.71) / (12.45 + 40.46 + 203.5) + 18.00 \times (2.42 + 7.87 + 39.57) / (12.45 + 40.46 + 203.5) = 20.51 \ \$ / MWh$$

Bus	Locational Marginal Price (\$/MWh)	Bus	Locational Marginal Price (\$/MWh)
1	28.21	13	27.68
2	28.30	14	20.18
3	20.51	15	24.30
4	25.55	16	23.10
5	27.41	17	18.58
6	23.76	18	19.00
7	31.44	19	18.77
8	21.39	20	16.60
9	21.57	21	19.00
10	21.56	22	18.00
11	21.19	23	16.60
12	20.14	24	20.51

Table 6.15 Locational Marginal Price of Each Bus for the 24-bus System

Transmission service and loss costs are determined and allocated to each generator and load based on the contributions obtained by Kirschen's tracing method. Congestion costs are calculated by the contributions and differences in the LMP values of various buses. Tables 6.16 and 6.17 show the results of the different costs allocated to generators and loads respectively.

	Transmission	Transmission	Transmission	Total Costs
Generator	Service Cost	Congestion Cost	Loss Cost	TC,
	C_{t}^{s} (\$/hr)	\tilde{C}_{t}^{C} (\$/hr)	C_{t}^{L} (\$/hr)	(\$/hr)
G ₁	92.80	73.07	-16.92	148.95
G ₂	199.84	288.88	-39.42	449.30
G ₇	0	0	0	0
G ₁₃	646.37	1398.30	-86.01	1958.66
G ₁₅	245.65	463.89	-31.99	677.55
G ₁₆	161.78	506.46	-36.17	632.07
G ₁₈	333.92	1692.40	-66.31	1960.01
G ₂₁	562.17	2711.10	-92.96	3180.31
G ₂₂	744.55	1921.80	-98.32	2568.03
G ₂₃	1397.92	4688.80	-151.92	5934.80
Total	4385.00	13744.70	-620.02	17509.68

Table 6.16 Transmission Costs Allocated to Generations for the 24-bus System

As shown in Table 6.16, generator G_{23} is allocated the highest transmission cost since it is the largest energy supplier (660 MW/hr) in the 24-bus system. Generator G_7 is a local generator that does not provide energy to the system through the transmission network so that the total cost allocated to generator G_7 is zero. It demonstrates that the more power electricity suppliers deliver through the transmission network, the more money they have to pay for the service. In addition, transmission service and congestion cost values of generators are positive because they are charges paid to transmission owners from generations. The loss costs of generations are negative since the loss cost is a kind of "revenue compensations" of energy losses from loads.

	Transmission	Transmission	Transmission	Total Costs
Load	Service Cost	Congestion Cost	Loss Cost	$TC_{\rm s/hr}$
	C_t^{3} (\$/hr)	C_t° (\$/hr)	C_{t}^{L} (\$/hr)	
L ₁	31.69	91.82	1.93	125.44
L ₂	5.78	10.56	0.27	16.61
L ₃	170.69	753.08	26.39	950.16
L ₄	186.21	391.05	27.54	604.80
L ₅	98.46	120.97	20.02	239.45
L ₆	554.66	1165.30	82.19	1802.15
L ₇	122.93	512.32	20.66	655.91
L ₈	491.14	1163.10	57.76	1712.00
L9	657.39	1558.71	77.30	2293.40
L ₁₀	657.39	1558.71	77.30	2293.40
L ₁₃	274.50	1460.80	38.20	1773.50
L ₁₄	236.52	1146.80	49.23	1432.55
L ₁₅	397.70	1754.10	61.50	2213.30
L ₁₆	118.30	573.53	24.63	716.46
L ₁₈	154.94	91.81	16.12	262.87
L ₁₉	217.20	1391.60	37.50	1646.30
L ₂₀	9.50	0	1.48	10.98
Total	4385.00	13744.26	620.02	18749.28

Table 6.17 Transmission Costs Allocated to Loads for the 24-bus System

All transmission costs allocated to loads are shown in Table 6.17. The results are positive because they are payments from loads. Service and congestion costs will be paid to transmission network owners and loss costs will be considered compensations paid to generation companies.

This case study illustrates that the proposed transmission pricing scheme can provide economical signals to each market participant about energy transactions. Let the 10 generators represent 10 generation companies, 17 loads refer to 17 customers, and all transmission lines belong to the transmission company A in the 24-bus system. Table 6.18 shows the revenues of generation companies. Table 6.19 gives the payments of customers.

In Table 6.18 and Table 6.19, B refers to the bilateral transaction, M is the multilateral transaction, and P refers to the pool transaction. For the bilateral transaction model, power suppliers and customers will sell and purchase electricity based on the marginal price of the individual contracted generator. For example, the payment from load L_{18} to generator G_{18} is $300 \times 19.00 = 5700$ \$/h. For the multilateral transaction model, the price is based on the average marginal price of all contracted generators. Thus, load L_{14} , L_{15} and L_{19} purchase electricity based on the price: (19+18+16.6)/3 = 17.87 \$/MWh. For participants in the pool model, they use the average price (23.11 \$/MWh) of the entire system.

Table 6.18 provides the detailed revenues and costs corresponding to each generation company. The payments to the transmission company A include service costs and congestion costs assigned to Gencos. The loss compensations from customers are

loss costs allocated to each generation company. The sale revenue of generation company G_{23} is 13229 \$/hr (660 MW) and the transmission loss compensation from customers is 151.92 \$/hr. In contrast, the payment, which reflects the transmission service and congestion costs to the transmission company A, is -6086.70 \$/hr. The net revenue of G_{23} is the sum of the sale revenue, the loss compensation and the payment to transmission company A (8614.62 \$/hr).

	01.0.	Energy	Sale Loss Compensation		Payment to	Net	
Generator	Sale Price	Sale	Revenue	from	Transmission	Revenue	
	(\$/MWh)	(MWh)	(\$/hr) Customers(\$/hr)		company A (\$/hr)	(\$/hr)	
1	23.11 (P)	160.7 (P)	3714.50	16.92	-165.87	3565.55	
2	23.11 (P)	165.2 (P)	3820.80	39.42	-488.72	3371.50	
7	23.11 (P)	10 (P)	231.11	0	0	231.11	
13	23.11 (P)	384.1(P)	8878.30	86.01	-2044.70	6919.61	
15	23.11 (P)	215 (P)	4969.60	31.99	-709.54	4292.05	
16	23.11 (P)	155 (P)	3582.80	36.17	-668.23	2950.74	
	23.11 (P)	100 (P)	0011.50		2 02 (2 2		
18	19.00 (B)	300 (B)	8011.50	66.31	-2026.30	6051.51	
	23.11 (P)	200 (P)	0106.20	00.00	2072.20	5015.96	
21	17.87 (M)	200 (M)	8196.30	92.96	-32/3.30		
	23.11 (P)	100 (P)	5004.00	09.22	2666.20	2216.02	
22	17.87 (M)	200 (M)	5884.80	98.32	-2000.30	3310.82	
	23.11 (P)	310 (P)					
23	17.87 (M)	200 (M)	13229.00	151.92	-6086.70	7294.22	
	16.60 (B)	150 (B)					
Total	-	-	60516.31	620.02	-18129.66	43006.67	

Table 6.18 Revenues of Generation Companies for the 24-bus System

Table 6.19 shows the payments of customers. Load L_1 pays 123.51 \$/hr to the transmission company A as transmission service and congestion costs. The payment to

generation companies includes two parts: one is the energy transaction payment (2311.50 \$/hr for 100 MW), and another is 1.93 \$/hr that is the loss cost allocated to L₁ as the loss compensation. The total payment assigned to L₁ is 2436.94 \$/hr. For the transmission company A, the total revenue of this transaction schedule is 36258.92 \$/hr (18129.66 \$/hr from Gencos and 18129.26 \$/hr from customers).

	Payment to	Pa	Payment to Generation Companies								
Load	Transmission Company A (\$/hr)	Energy Purchase (MW)	Purchase Price (\$/MWh)	Energy Payment (\$/hr)	Loss Compensation (\$/hr)	Payment (\$/hr)					
1	123.51	100 (P)	23.11 (P)	2311.50	1.93	2436.94					
2	16.34	100 (P)	23.11 (P)	2311.50	0.27	2328.11					
3	923.77	150 (P)	23.11 (P)	3467.20	26.39	4417.36					
4	577.26	50 (P)	23.11 (P)	1155.70	27.54	1760.50					
5	219.43	50 (P)	23.11 (P)	1155.70	20.02	1395.15					
6	1719.96	150 (P)	23.11 (P)	3467.20	82.19	5269.35					
7	635.25	100 (P)	23.11 (P)	2311.50	20.66	2967.41					
8	1654.24	150 (P)	23.11 (P)	3467.20	57.76	5179.20					
9	2216.10	200 (P)	23.11 (P)	4622.90	77.30	6916.30					
10	2216.10	200 (P)	23.11 (P)	4622.90	77.30	6916.30					
13	1735.30	300 (P)	23.11 (P)	6934.40	38.20	8707.90					
14	1383.32	50 (P) 200 (M)	23.11 (P) 17.87 (M)	3573.30	49.23	5005.85					
15	2151.80	150 (P) 200 (M)	23.11 (P) 17.87 (M)	7040.50	61.50	9253.80					
16	691.83	100 (P)	23.11 (P)	2311.50	24.63	3027.96					
18	246.75	300 (B)	19.00 (B)	5700.00	16.12	5962.87					
19	1608.80	200 (M)	17.87 (M)	3573.30	37.50	5219.60					
20	9.50	150 (B)	16.60 (B)	2490.00	1.48	2500.98					
Total	18129.26	2850	-	60516.30	620.02	79265.58					

Table 6.19 Payments of Customers for the 24-bus System

6.6 Summary

In the restructured environment, it is necessary to develop and use the reasonable and fair transmission pricing scheme that can calculate and allocate all transmission costs. In order to flexibly manage transmission costs and solve shortcomings of previous methods, a comprehensive transmission pricing scheme based on a power flow tracing method was presented in this chapter.

An overview and the general formulae of the calculation of transmission service, congestion and loss costs using Kirschen's method and LMP method were given. The optimal power dispatch strategy based on OPF used in the proposed scheme was described. The detailed process of the proposed pricing scheme has been explained. A case study using the IEEE 24-bus system was presented to illustrate the effectiveness of this scheme.

In comparison with previous research on the transmission pricing, the proposed pricing scheme is simple to understand and implement. All three components of the transmission cost namely, transmission service cost, transmission congestion cost and transmission losses cost can be determined and allocated to market participants using the scheme. Based on different energy transaction types, the scheme can provide the detailed economical information of energy transactions. The scheme has also proposed the estimation of locational marginal price using the tracing method for the calculation of congestion costs.

Chapter 7

Conclusions and Future Work

In this thesis, the generalized analysis and calculation of transmission costs have been presented and described using usage-based methods and incremental (marginal) methods. In order to easily manage transmission costs and solve congestion and loss problems, a comprehensive transmission pricing scheme using a power flow tracing method and LMP method to calculate and allocate transmission costs has been developed. Studies on different power system models have been performed to illustrate the effectiveness of the different techniques.

7.1 Summary of the Research and Contribution of the Thesis

The main contributions of this thesis can be summarized as follows:

- Different components of transmission costs in the restructured power systems, including transmission service costs, congestion costs and loss costs, have been described.
- 2. A usage-based method and three usage calculation methods, used for the calculation and allocation of the transmission service costs, have been implemented and compared.
- 3. The cause and effect of the transmission congestion costs have been presented and highlighted. An effective congestion cost calculation method has been introduced and implemented. An approach for the determination of the locational marginal price based on a power flow tracing method has been developed.
- 4. Power flow tracing method has been used in the determination of the transmission loss costs.
- 5. A comprehensive transmission pricing scheme to determine all transmission costs and provide energy transaction data using the tracing method and LMP method has been developed. Optimal power dispatch strategy is used in the scheme. This can be considered as the most significant contribution of this thesis.

Case studies have been presented throughout the thesis to illustrate the evaluation of the different components of transmission cost in a deregulated electric power system. The results of the work presented in the thesis show that the proposed approach and pricing scheme provide both simplicity and reasonableness in the calculation and allocation of transmission cost. This advantage gives power utilities abundant economical information about energy transactions under the competitively restructured power market.

7.2 **Recommendations for Future Work**

The analysis methods used in the thesis provide both qualitative and quantitative insight into the study of transmission costs. However, it is difficult to judge whether these methods can fit all power systems under complicated and different operational conditions. On-going research indicates that there is no generalized method and the selection of methods is based on the particular characteristics of the network. As a result, the effectiveness of the proposed pricing scheme needs further investigation based on practical power systems under different operational conditions.

In order to improve the proposed scheme, more studies on other advanced methods are necessary. For example, AC power flow method, which uses sensitivity indices derived from AC power flow model to estimate the usages of users, can improve the accuracy in cost determination [15, 17].

Firm transmission right (FTR) and optimal power dispatch with prioritization load shedding are considered as powerful tools used in the pricing scheme [40-43]. They involve complicated relationship among market participants. Further studies on these strategies may be useful.

References

- L. L. Lai, *Power System Restructuring and Deregulation*, John Wiley & Sons, Inc., London, 2001.
- [2] S. Stoft, *Power System Economics*, IEEE Press, Inc., Piscataway, 2002.
- [3] M. Shahidehpour and M. Alomoush, *Restructured Electrical Power Systems*, Marcel Dekker, Inc., New York, 2001.
- [4] S. Hunt, *Making Competition Work in Electricity*, John Wiley & Sons, Inc., New York, 2002.
- [5] M. Ilic, F. Galiana and L. Fink, Power systems Restructuring: Engineering and Economics, Kluwer Academic Publishers, Inc., Boston, 1998.
- [6] K. Bhattacharya, M.H.J. Bollen and J.E. Daalder, *Operation of Restructured Power Systems*, Kluwer Academic Publishers, Inc., Boston, 2001.
- [7] L. Philipson and H. L. Wills, *Understanding Electric Utilities and De-Regulation*, Taylor & Francis Group, Boca Raton, 2006.
- [8] Federal Energy Regulator Commission (FERC), Order-No.888, April 24, 1996.
- [9] Federal Energy Regulator Commission (FERC), Order-No.889, April 24, 1996.

- [10] T. Krause, "Evaluation of Transmission Pricing Methods for Liberalized Markets a Literature Survey", EEH internal report, Zurich, July 2003.
- [11] M. Einhorn and R. Siddiqi, *Electricity Transmission Pricing and Technology*, Kluwer Academic Publishers, Inc., Boston, 1996.
- [12] F. Lévêque, Transport Pricing of Electricity Networks, Kluwer Academic Publishers, Inc., Boston, 2003.
- [13] D. Shirmohammadi, "Evaluation of Transmission Network Capacity Use for Wheeling Transactions", IEEE Trans. on Power Systems, Vol.11, No.2, October 1996, pp. 1002-1008.
- [14] D. Shirmohammadi, C. Rajagopalan, E. R. Alward and C. L. Thomas, "Cost of Transmission Transactions: An Introduction", IEEE Trans. on Power Systems, Vol. 6, No. 3, August 1991, pp.1006-1016.
- [15] M. Shahidehpour, H. Yamin and Z. Li, *Market Operations in Electric Power Systems*, John Wiley & Sons, Inc., New York, 2002.
- [16] J. Lima, "Allocation of Transmission Fixed Charges: an overview", IEEE Trans. on Power Systems, Vol.11, No.3, August 1996, pp. 1409-1418.
- [17] J. Pan, Y. Teklu and S. Rahman, "Review of Usage-Based Transmission Cost Allocation Methods under Open Access", IEEE Trans. on Power Systems, Vol.15, No.4, November 2000, pp. 1218-1224.

- [18] P. Wei, B. Yuan, Y. Ni, and F. Wu, "Power Flow tracing for Transmission Open Access", International Conference on Electric Utility Deregulation and Restructuring and Power Technologies, City University, London, 2000.
- [19] J. Bialek, "Tracing the flow of electricity", IEE Proc. on. Generation, Transmission and Distribution, Vol.143, No.4, July 1996, pp.313-320.
- [20] D. Kirschen, R. Allan and G. Strbac, "Contributions of Individual Generators to Loads and Flow", IEEE Trans. on Power Systems, Vol.12, No.1, February 1997, pp.52-60.
- [21] G. Strbac, D. Kirschen and S. Ahmed, "Allocating Transmission System Usage on the Basis of Traceable Contributions of Generations and Loads to Flows", IEEE Trans. on Power Systems, Vol.13, No. 2, May 1997, pp.527-534.
- [22] D. Kirschen and G. Strbac, *Fundamentals of Power System Economics*, John Wiley & Sons, Inc., New York, 2004.
- [23] A. J. Wood and B. F. Wollenberg, *Power Generation, Operation and Control*, John Wiley & Sons, Inc., New York, 1996.
- [24] IEEE Committee Report, "IEEE Reliability Test System", IEEE Trans. on Power Apparatus and Systems, PAS-98, Nov/Dec 1979, pp.2047 -2054.
- [25] PowerWorld Simulator 8.0, PowerWorld Corporation, Champaign, 2002.

- [26] W.Y.Ng, "Generalized generation distribution factors for power system security evaluations", IEEE Trans. on Power Apparatus and Systems, Vol.Pas-100, No.3, March 1981, pp. 1001-1005.
- [27] Website http://www.pjm.com: Locational Marginal pricing, 2006.
- [28] A. M. Silva and J. G. Costa, "Transmission Loss Allocation: Part I Single Energy Market", IEEE Trans. on Power Systems, Vol.18, No.4, November 2003, pp.1389-1393.
- [29] J. J. Gonzalez and P. Basagoiti, "Spanish Power Exchange Market and Information System, Design Concepts, and Operating Experience", Proc. IEEE PICA Conf., Santa Clara, CA, May 1999, pp.245-252.
- [30] V. S. Lim, D. F. McDonald and T. K. Saha, "Comparative Distribution Of System Losses to Market Participants using Different Loss Allocation Methods", 2005 IEEE/PES Transmission and Distribution Conference & Exhibition: Asia and Pacific, Dalian, China, November 2005, pp.1-7.
- [31] A. J. Conejo, J. M. Arroyo, N. Alguacil and A. L. Guijarro, "Transmission Loss Allocation: A Comparison of Different Practical Algorithms", IEEE Trans. on Power Systems, Vol.17, No.3, August 2002, pp.571-576.
- [32] D. A. Lima and A. Padilha-Feltrin, "Comparison of Algorithms for Loss in Transmission Networks Considering Power Redispatch", 2004 IEEE/PES Transmission and Distribution Conference & Exhibition: Latin America, November 2004, pp.212-217.

- [33] F. D. Galiana, A. J. Conejo and I. Kockar, "Incremental Transmission Loss Allocation Under Pool Dispatch", IEEE Trans. on Power Systems, Vol.17, No.1, February 2002, pp.26-33.
- [34] A. J. Conejo, F. D. Galiana and I. Kockar, "Z-bus Loss Allocation", IEEE Trans. on Power Systems, Vol.16, No.1, February 2001, pp.105-109.
- [35] M. E. EI-Hawary, "Optimal Power Flow: Solution Techniques, Requirements and Challenges," IEEE Special Publication 96 TP 111-0, 1996.
- [36] J. A. Momoh, *Electric Power System Applications of Optimization*, Marcel Dekker, Inc., New York, 2001.
- [37] Y. Meng and B. Jeyasurya, "Investigation of Transmission Cost Allocation Using a Power Flow Tracing Method", accepted for the presentation at the IEEE Power Engineering Society General Meeting, Tampa, Florida, USA, June, 2007.
- [38] Matlab 7.0, the MathWorks, Inc., Natick, 2004.
- [39] Matpower 3.0.0, Power System Engineering Research Center, Comell University, 2005.
- [40] M. Alomoush and M. Shahidehpour, "Fixed transmission rights for zonal congestion management", IEE Proceedings - Generation, Transmission and Distribution, Vol.146, No.5, September 1999, pp. 465-477.

- [41] M. Alomoush and M. Shahidehpour, "Generalized model for fixed transmission rights auction", Electric Power System Research, Vol.54, No.3, June 2000, pp. 207-220.
- [42] R. Fang and A. David, "Optimal Dispatch under Transmission Contracts", IEEE Trans. on Power Systems, Vol.14, No.2, May 1999, pp. 732-737.
- [43] R. Fang and A. David, "Transmission Congestion Management in an Electricity Market", IEEE Trans. on Power Systems, Vol.14, No.3, August 1999, pp. 877-883.

Appendix A: Data of the 6-bus system

Appendix A contains the information of the 6-bus system [23] discussed in the thesis. The single line diagram is shown in Fig. A.1. The generations, loads and line characteristics are presented in Table A.1 and A.2 respectively.



Fig. A.1 Single Line Diagram of the 6-bus System [23]

1 abie 11.1 Ocher and Doug Delans of the O bis System	Table A.1	Generation and Load D	Details of the 6-bus Syst	em
-------------------------------------------------------	-----------	-----------------------	---------------------------	----

Bus	Gene	ration	Bus		Load
1	108.45MW	23.25Mvar	4	70MW	70Mvar
2	50MW	86.71Mvar	5	70MW	70Mvar
3	60MW	98.85Mvar	6	70MW	70Mvar

Line No.	From Bus	To Bus	Resistance (p.u.)	Reactance (p.u.)	Line Charging (p.u.)
1	1	2	0.1	0.2	0.02
2	1	4	0.05	0.2	0.02
3	1	5	0.08	0.3	0.03
4	2	3	0.05	0.25	0.03
5	2	4	0.05	0.1	0.01
6	2	5	0.1	0.3	0.02
7	2	6	0.07	0.2	0.025
8	3	5	0.12	0.26	0.025
9	3	6	0.02	0.1	0.01
10	4	5	0.2	0.4	0.04
11	5	6	0.1	0.3	0.03

Table A.2	Line	<i>Characteristics</i>	of the	6-bus	System
-----------	------	------------------------	--------	-------	--------

* All characteristics in p.u are based on 100 MVA

•

Appendix B: Data of the IEEE 24-bus system

Appendix B contains the information of the IEEE 24-bus system [24] discussed in the thesis. The single line diagram is shown in Fig. B.1. The generations, loads, generation fuel cost coefficients and line characteristics are presented in Table B.1, B.2, and B.3 respectively.



Fig. B.1 Single Line Diagram of the IEEE 24-bus System [24]

Bus	Gene	ration	Bus	Lo	ad
1	152MW	60Mvar	1	108MW	22Mvar
2	152MW	44.94Mvar	2	97MW	20Mvar
7	4MW	120Mvar	3	180MW	37Mvar
13	472MW	160Mvar	4	74MW	15Mvar
15	155MW	80Mvar	5	71 M W	14Mvar
16	155MW	80Mvar	6	136MW	28Mvar
18	400MW	132.64Mvar	7	125MW	25Mvar
21	400MW	91.73Mvar	8	171MW	35Mvar
22	401.14MW	-25.98Mvar	9	175MW	36Mvar
23	660MW	139.03Mvar	10	195MW	40Mvar
			13	265MW	54Mvar
			14	194MW	39Mvar
			15	317 M W	64Mvar
			16	100MW	20Mvar
			18	333MW	68Mvar
			19	181MW	37Mvar
			20	128MW	26Mvar

Table B.1 Generation and Load Details of the IEEE 24-bus System

Bus	В	С
1	25.0	0.01
2	25.0	0.01
7	30.0	0.01
13	18.0	0.01
15	20.0	0.01
16	20.0	0.01
18	15.0	0.005
21	15.0	0.005
22	15.0	0.005
23	10.0	0.005

 Table B.2
 Generation Fuel Cost Coefficients of the IEEE 24-bus System

Line No.	From Bus	To Bus	Resistance (p.u.)	Reactance (p.u.)	Line charging (p.u.)
1	1	2	0.0026	0.0139	0.4611
2	1	3	0.0546	0.2112	0.0572
3	1	5	0.0218	0.0845	0.0229
4	2	4	0.0328	0.1267	0.0343
5	2	6	0.0497	0.192	0.052
6	3	9	0.0308	0.119	0.0322
7	3	24	0.0023	0.0839	0
8	4	9	0.0268	0.1037	0.0281
9	5	10	0.0228	0.0883	0.0239
10	6	10	0.0139	0.0605	2.459
11	7	8	0.0159	0.0614	0.0166
12	8	9	0.0427	0.1651	0.0447
13	8	10	0.0427	0.1651	0.0447
14	9	11	0.0023	0.0839	0
15	9	12	0.0023	0.0839	0
16	10	11	0.0023	0.0839	0
17	10	12	0.0023	0.0839	0
18	11	13	0.0061	0.0476	0.0999
19	11	14	0.0054	0.0418	0.0879
20	12	13	0.0061	0.0476	0.0999
21	12	23	0.0124	0.0966	0.203
22	13	23	0.0111	0.0865	0.1818
23	14	16	0.005	0.0389	0.0818
24	15	16	0.0022	0.0173	0.0364
25	15	21	0.0063	0.049	0.103
26	15	21	0.0063	0.049	0.103
27	15	24	0.0067	0.0519	0.1091
28	16	17	0.0033	0.0259	0.0545
29	16	19	0.003	0.0231	0.0485
30	18	17	0.0018	0.0144	0.0303
31	17	22	0.0135	0.1053	0.2212
32	18	21	0.0033	0.0259	0.0545
33	18	21	0.0033	0.0259	0.0545
34	19	20	0.0051	0.0396	0.0833
35	19	20	0.0051	0.0396	0.0833
36	20	23	0.0028	0.0216	0.0455
37	20	23	0.0028	0.0216	0.0455
38	21	22	0.0087	0.0678	0.1424

Table B.3 Line Characteristics of the IEEE 24-bus System

* All characteristics in p.u are based on 100 MVA

.

Appendix C: DC Power Flow Formulation

The Newton–Raphson power flow method is the most robust power flow algorithm used in practice. However, the drawback to its use is that the terms in the Jacobian matrix must be recalculated after each iteration and then the entire set of equations must be resolved each time. Since thousands of power flows are often run for power flow studies, ways to speed up this process have been sought.

DC power flow is a linearized version of the load flow problem based on the some assumptions. One is that all line conductances are negligible. For example: Gij ≈ 0 , where Gij is the conductance of the line connecting bus i and j. Furthermore, all angular differences are assumed small. This implies that sin $\theta \approx \theta$, where θ is in radians. Another assumption is that all voltages remain constant at their nominal values, for example, at 1.0 p.u. The implication of above assumptions is that only real power equations are considered with no line losses.

Given these assumptions, the real power injection equation can be simplified as follows [23]:

$$P_i = \sum_{j \in k(i)} (-B_{ij})(\theta_i - \theta_j)$$
(C.1)

where

 P_i = the real power injection in bus *i*

 B_{ii} = the susceptance of the lines connecting buses *i* and *j*

 θ_i, θ_j = bus *i* and bus *j* angular in radians

From the above equation, the power flow on each line using DC power flow theory is given by [23]:

$$P_{ik} = \frac{1}{X_{ik}} (\theta_i - \theta_j) \tag{C.2}$$

where

 P_{ik} = the power flow on line *i*-*k*

 X_{ik} = the line reactance for line *i*-*k*

The DC power flow program in this thesis for the power flow calculations are based on the above theory and equation C.1, C.2.

Appendix D:Transmission Service Cost Case Study Results

Appendix D contains the results discussed in Chapter 3, which are about transmission usages calculation and transmission service cost allocation for the IEEE 24-bus system using GGDFs, Bialek's and Kirchen's methods. These results are shown in Table D.1, D.2, D.3, D.4, D.5 and D.6 respectively.

Linek	Pij			Transn	nission Pow	ver Flow A	llocation to	Generators	s (MW)		
	(MW)	G ₁	G ₂	G ₇	G ₁₃	G ₁₅	G ₁₆	G ₁₈	G ₂₁	G ₂₂	G ₂₃
1-2	18.65	65.69	-77.68	-0.07	-2.30	3.61	2.51	7.95	8.37	8.04	2.51
1-3	26.07	39.16	35.75	0.17	7.43	-6.63	-4.36	-14.28	-15.17	-14.47	-1.53
1-5	50.17	44.20	38.98	-0.18	-14.26	0.02	-1.16	-1.41	-0.95	-1.33	-13.76
2-4	31.13	33.59	37.90	-0.10	-9.66	-1.79	-2.13	-5.05	-4.91	-5.04	-11.69
2-6	41.56	27.06	31.38	-0.11	-8.31	0.25	-0.50	-0.28	0.01	-0.23	-7.71
3-9	35.77	4.21	1.86	-0.47	-29.01	9.75	4.92	19.17	21.04	19.54	-15.24
3-24	243.27	47.98	46.92	0.99	76.90	-3.10	4.00	0.84	-1.92	0.37	70.29
4-9	43.87	34.24	38.56	-0.08	-7.62	-1.12	-1.47	-3.32	-3.18	-3.30	-8.84
5-10	21.25	42.72	37.49	-0.22	-18.89	-1.50	-2.67	-5.33	-4.87	-5.26	-20.23
6-10	96.04	29.87	34.18	-0.03	0.40	3.11	2.37	7.10	7.39	7.17	4.48
7-8	123.3	6.14	6.14	4.16	19.08	6.27	6.27	16.17	16.17	16.21	26.68
8-9	157.67	11.28	10.99	2.24	26.13	7.27	7.59	19.16	19.03	19.18	34.81
8-10	151.91	4.46	4.75	2.18	22.75	8.79	8.46	22.26	22.39	22.34	33.54
9-11	166.78	28.45	29.33	0.95	9.78	7.77	5.08	16.72	17.76	16.94	34.01
9-12	182.46	27.05	27.85	0.89	-2.38	14.01	11.85	33.46	34.30	33.69	1.75
10-11	227.98	44.43	44.17	1.13	24.40	7.40	6.01	17.37	17.91	17.50	47.66
10-12	244.91	43.10	42.75	1.07	12.44	13.70	12.84	34.28	34.62	34.42	15.68
11-13	208.1	29.18	29.22	0.78	-111.40	26.23	25.99	67.39	67.48	67.57	5.68
11-14	190.38	43.90	44.47	1.30	146.18	-10.86	-14.70	-32.80	-31.31	-32.63	76.82
12-13	181.53	31.70	31.88	0.88	-89.79	15.30	14.12	38.01	38.47	38.19	62.78
12-23	252.45	38.79	39.07	1.09	100.91	12.76	10.91	30.62	31.34	30.82	-43.87
13-23	187.28	26.14	26.36	0.74	162.94	6.11	4.69	13.99	14.54	14.12	-82.36
14-16	389.7	54.16	54.74	1.57	178.07	-0.39	-4.23	-5.78	-4.29	-5.54	121.40
15-16	78.37	-2.63	-3.49	-0.29	-66.60	80.78	-39.09	59.42	106.01	67.56	-123.30
15-21	493.66	59.56	59.36	1.52	171.29	80.25	52.22	-35.03	-84.38	-43.58	232.46
15-24	246.93	-22.73	-21.67	-0.32	1.51	28.84	21.74	65.61	68.37	66.24	39.34
16-17	359.01	46.22	46.42	1.27	157.19	27.61	55.65	-86.60	-37.25	-78.35	226.85
16-19	96.08	-3.34	-3.83	-0.21	-72.59	43.95	47.21	117.47	116.20	117.55	-266.32
17-18	185.27	27.83	28.01	0.78	98.23	11.52	35.73	-132.37	-63.99	36.09	143.44
17-22	178.41	18.63	18.66	0.50	59.71	16.34	20.16	46.40	27.38	-113.81	84.45
18-21	118.72	3.80	3.97	0.14	23.60	-12.98	11.23	204.39	-127.23	-27.31	39.10
19-20	85.22	-3.90	-4.39	-0.23	-74.33	43.38	46.64	116.00	114.73	116.07	-268.75
20-23	213.7	2.72	2.23	-0.05	-53.78	50.12	53.39	133.41	132.14	133.53	-240.02
21-22	218.49	22.47	22.44	0.58	67.91	25.57	21.75	61.75	80.77	-178.77	94.00
Total	5546.58	906.13	764.76	22.56	805.93	512.33	423.03	826.70	616.99	373.54	294.11

Table D.1 Contributions of Generators to Line Flows Using GGDFs Method for the 24-bus System

	Pij			Transr	nission Pov	ver Flow A	llocation to	Generator	s (MW)	·····	
	(MW)	G ₁	G ₂	G ₇	G ₁₃	G ₁₅	G ₁₆	G ₁₈	G ₂₁	G ₂₂	G ₂₃
1-2	18.65	15.902	0	0	0	0.636	0	0	1.3586	0.7537	0
1-3	26.07	0	0	0	0	6.0331	0	0	12.888	7.1493	0
1-5	50.17	42.777	0	0	0	1.7109	0	0	3.6547	2.0274	0
2-4	31.13	2.9162	27.71	0	0	0.1166	0	0	0.2492	0.1382	0
2-6	41.56	3.8932	36.994	0	0	0.1557	0	0	0.3326	0.1845	0
3-9	35.77	0	0	0	0	8.2779	0	0	17.683	9.8093	0
3-24	243.27	0	0	0	0	56.413	0	0	120.51	66.85	0
4-9	43.87	0	0	0	13.291	1.2242	2.3024	2.1879	3.0332	4.4156	17.416
5-10	21.25	0	0	0	7.1041	0.1517	1.2411	1.1794	0.5495	1.778	9.2461
6-10	96.04	0	0	0	32.107	0.6855	5.6094	5.3304	2.4833	8.0359	41.788
7-8	123.3	0	0	0	39.252	2.1842	6.8295	6.4899	5.9063	11.383	51.255
8-9	157.67	0	0	0	47.768	4.3998	8.2748	7.8633	10.902	15.87	62.593
8-10	151.91	0	0	0	50.782	1.0843	8.872	8.4308	3.9277	12.71	66.094
9-11	166.78	0	0	0	62.142	2.4684	20.198	19.193	8.9416	28.935	24.901
9-12	182.46	0	0	0	54.49	0	0	0	0	0	127.97
10-11	227.98	0	0	0	84.945	3.3742	27.609	26.236	12.223	39.553	34.039
10-12	244.91	0	0	0	73.141	0	0	0	0	0	171.77
11-13	208.1	0	0	0	148.57	0	0	0	0	0	59.533
11-14	190.38	0	0	0	0	5.8937	48.225	45.826	21.349	69.086	0
12-13	181.53	0	0	0	129.6	0	0	0	0	0	51.932
12-23	252.45	0	0	0	0	0	0	0	0	0	252.45
13-23	187.28	0	0	0	0	0	0	0	0	0	187.28
14-16	389.7	0	0	0	0	12.064	98.714	93.805	43.701	141.42	0
15-16	78.37	0	0	0	0	18.136	0	0	38.742	21.492	0
15-21	493.66	0	0	0	0	0	0	0	317.52	176.14	0
15-24	246.93	0	0	0	0	57.144	0	0	122.07	67.717	0
16-17	359.01	0	0	0	0	0	0	140.99	26.95	191.07	0
16-19	96.08	0	0	0	0	2.9744	24.338	23.127	10.774	34.866	0
17-18	185.27	0	0	0	0	0	0	142.83	27.3	15.145	0
17-22	178.41	0	0	0	0	0	0	0	0	178.41	0
18-21	118.72	0	0	0	0	0	0	0	76.36	42.36	0
19-20	85.22	0	0	0	0	0	0	0	0	0	85.22
20-23	213.7	0	0	0	0	0	0	0	0	0	213.7
21-22	218.49	0	0	0	0	0	0	0	0	218.49	0
Total	5546.58	65.49	64.70	0	743.19	185.13	252.21	523.49	889.40	1365.78	1457.19

Table D.2 Contributions of Generators to Line Flows Using Bialek's Method for the 24-bus System

1	Pij			Transn	nission Pov	ver Flow A	llocation to	Generators	s (MW)		
Line K	(MW)	G ₁	G ₂	G ₇	G ₁₃	G ₁₅	G ₁₆	G ₁₈	G ₂₁	G ₂₂	G ₂₃
1-2	18.65	15.908	0	0	0	0.6341	0	0	1.3614	0.746	0
1-3	26.07	0	0	0	0	6.0482	0	0	12.879	7.1432	0
1-5	50.17	42.795	0	0	0	1.7058	0	0	3.6624	2.0068	0
2-4	31.13	2.926	27.706	0	0	0.115	0	0	0.2499	0.1369	0
2-6	41.56	3.9066	36.988	0	0	0.1538	0	0	0.3337	0.1829	0
3-9	35.77	0	0	0	0	8.2986	0	- 0	17.67	9.801	0
3-24	243.27	0	0	0	0	56.439	0	0	120.18	66.656	0
4-9	43.87	0	0	0	16.671	0.7019	2.4567	3.4219	2.1935	2.7199	15.705
5-10	21.25	0	0	0	8.075	0.34	1.19	1.6575	1.0625	1.3175	7.6075
6-10	96.04	0	-0	0	36.495	1.5366	5.3782	7.4911	4.802	5.9545	34.382
7-8	123.3	0	0	0	46.854	1.9728	6.9048	9.6174	6.165	7.6446	44.141
8-9	157.67	0	0	0	59.915	2.5227	8.8295	12.298	7.8835	9.7755	56.446
8-10	151.91	0	0	0	57.726	2.4306	8.507	11.849	7.5955	9.4184	54.384
9-11	166.78	0	0	0	63.376	2.6685	9.3397	13.009	8.339	10.34	59.707
9-12	182.46	0	0	0	93.967	0	0	0	0	0	88.493
10-11	227.98	0	0	0	86.632	3.6477	12.767	17.782	11.399	14.135	81.617
10-12	244.91	0	0	0	126.13	0	0	0	0	0	118.78
11-13	208.1	0	0	0	107.17	0	0	0	0	0	100.93
11-14	190.38	0	0	0	0	5.9018	48.166	66.823	25.511	43.978	0
12-13	181.53	0	0	0.	93.488	0	0	0	0	0	88.042
12-23	252.45	0	0	0	0	0	0	0	0	0	252.45
13-23	187.28	0	0	0	0	0	0	0	0	0	187.28
14-16	389.7	0	0	0	0	12.081	98.594	136.78	52.22	90.021	0
15-16	78.37	0	0	0	0	18.182	0	0	38.715	21.473	0
15-21	493.66	0	0	0	0	0	0	0	317.42	176.24	0
15-24	246.93	0	0	0	0	57.288	0	0	121.98	67.659	0
16-17	359.01	0	0	0	0	0	0	205.71	39.491	113.81	0
16-19	96.08	0	0	0	0	2.9785	24.308	33.724	12.875	22.194	0
17-18	185.27	0	0	0	0	0	0	106.16	20.38	58.731	0
17-22	178.41	0	0	0	0	0	0	0	0	178.41	0
18-21	118.72	0	0	0	0	0	0	0	76.337	42.383	0
19-20	85.22	0	0	0	0	0	0	0	0	0	85.22
20-23	213.7	0	0	0	0	0	0	0	0	0	213.7
21-22	218.49	0	0	0	0	0	0	0	0	218.49	0
Total	5546.58	65.54	64.69	0	796.50	185.65	226.44	626.33	910.71	1181.4	1488.9

Table D.3 Contributions of Generators to Line Flows Using Kirschen's Method for the 24-bus System

•
System

Line k	Line Cost (\$/hr)	ckLk MW1,k /G1	ckLk MW2,k /G ₂	ckLk MW7,k /G7	ckLk MW13,k /G ₁₃	ckLk MW15,k G ₁₅	ckLk MW16,k /G ₁₆	ckLk MW18,k /G ₁₈	ckLk MW21,k /G ₂₁	ckLk MW22,k /G ₂₂	ckLk MW23,k /G ₂₃
1-2	60	3941.6	4660.5	4.2735	138	216.43	150.87	477.02	502.5	482.58	150.8
1-3	200	7832.1	7150.5	34.094	1485.1	1325.9	871.69	2857	3033.5	2894.4	305.25
1-5	180	7956.9	7016.6	31.798	2566.8	4.1085	208.04	253.16	170.7	239.65	2476.8
2-4	200	6717.4	7580.2	19.644	1931.2	357.74	426.97	1009.3	982.36	1007.2	2337.3
2-6	400	10824	12551	42.315	3325	100.2	198.37	112.63	3.4233	93.004	3083.7
3-9	180	757.79	335.58	85.407	5221.7	1755.6	886.41	3449.9	3787.7	3516.5	2743.8
3-24	250	11995	11730	246.96	19224	773.95	1001.1	209.59	480.32	91.781	17574
4-9	220	7533.5	8482.6	17.809	1676	246.29	322.45	730.29	700.69	727.04	1944.2
5-10	100	4271.5	3749.1	21.585	1888.6	149.62	267.48	532.65	486.84	526.12	2022.8
6-10	160	4778.7	5469.5	5.1106	64.226	497.93	378.5	1136.5	1182.9	1147.3	716.05
7-8	180	1106.1	1106.1	749.11	3434.7	1127.9	1127.9	2910.7	2910.7	2918	4802.7
8-9	240	2707	2637.1	536.88	6270.4	1743.7	1822.7	4598	4567.3	4604.2	8353.6
8-10	300	1338.2	1425.6	653.16	6824.9	2635.6	2536.8	6678.7	6717.1	6702	10061
9-11	120	3413.8	3519.2	113.43	1173.9	932.62	610.18	2005.9	2131.2	2032.4	4081.1
9-12	180	4868.9	5012.7	159.84	428.53	2521.1	2132.5	6023	6174	6064	315.17
10-11	150	6665.2	6624.9	169.58	3660.6	1110	901.29	2605	2686.1	2625.4	7148.9
10-12	100	4310	4275.3	107.5	1244.1	1370	1283.7	3428.2	3461.7	3442.5	1568.1
11-13	80	2334.3	2337.2	62.183	8912.3	2098.3	2078.9	5390.8	5398.4	5405.6	454.48
11-14	30	1316.9	1334.1	39.107	4385.5	325.86	440.91	983.97	939.25	978.76	2304.6
12-13	130	4121	4143.8	114.36	11673	1988.5	1835.9	4941.8	5001.2	4964.4	8161.1
12-23	190	7370	7422.5	207.53	19173	2423.9	2072.8	5818.7	5955.2	5856.7	8334.8
13-23	150	3921.7	3953.5	111.43	24442	915.77	703.18	2099	2181.6	2118.4	12354
14-16	210	11374	11494	330.49	37394	82.469	887.84	1214.1	901.11	1163.5	25494
15-16	50	131.54	174.58	14.597	3330	4038.9	1954.6	2971.2	5300.7	3378.2	6165.1
15-21	90	5360.5	5342.4	136.38	15416	7222.8	4700	3152.9	7594.5	3922.6	20921
15-24	40	909.25	866.76	12.936	60.362	1153.7	869.73	2624.3	2734.7	2649.8	1573.6
16-17	80	3697.5	3713.6	101.47	12575	2209.2	4451.6	6927.8	2979.8	6268	18148
16-19	40	133.63	153.16	8.5711	2903.6	1757.9	1888.5	4698.8	4648.1	4701.9	10653
17-18	260	7236.1	7281.3	202.12	25539	2995.9	9291	34416	16638	9383.8	37296
17-22	150	2794.3	2798.4	74.598	8956.2	2450.6	3023.4	6960.7	4106.8	17071	12668
18-21	40	152.01	158.96	5.7996	944.12	519.31	449.17	8175.6	5089.3	1092.2	1564
19-20	30	117	131.65	6.8699	2229.8	1301.3	1399.2	3480	3441.9	3482.1	8062.6
20-23	20	54.354	44.585	1.0969	1075.5	1002.5	1067.8	2668.3	2642.9	2670.6	4800.4
21-22	150	3370.5	3366.3	87.632	10187	3835.8	3263	9262.3	12116	26815	14100
Total	4960	145412	148044	4516	249753	53191	55505	144804	127649	141037	262739
ΣΣckLl	kMWt,k					13	32649				
C ^s t (\$/hr)	541.21	551.01	16.81	929.56	197.97	206.58	538.95	475.10	524.93	977.89

.

System

Line k	Line Cost	ckLk MW1,k	ckLk MW2,k	ckLk MW7,k	ckLk MW13,k	ckLk MW15,k	ckLk MW16,k	ckLk MW18,k	ckLk MW21,k	ckLk MW22,k	ckLk MW23,k
	(\$/hr)	/G ₁	/G ₂	/G ₇	/G ₁₃	G15	/G16	/G ₁₈	/G ₂₁	/G ₂₂	/G ₂₃
1-2	60	954.1	0	0	0	38.16	0	0	81.516	45.22	0
1-3	200	8555.4	0	0	0	342.18	0	0	730.95	405.49	0
1-5	180	524.91	4987.8	0	0	20.994	0	0	44.847	24.878	0
2-4	200	778.64	7398.8	0	0	31.143	0	0	66.525	36.904	0
2-6	400	0	0	0	0	2413.2	0	0	5155	2859.7	0
3-9	180	0	0	0	0	1490	0	0	3182.9	1765.7	0
3-24	250	0	0	0	9813	546.05	1707.4	1622.5	1476.6	2845.8	12814
4-9	220	0	0	0	2924	269.32	506.52	481.34	667.31	971.43	3831.5
5-10	100	0	0	0	4776.8	439.98	827.48	786.33	1090.2	1587	6259.3
6-10	160	0	0	0	1136.7	24.269	198.58	188.71	87.913	284.49	1479.4
7-8	180	0	0	0	5779.3	123.4	1009.7	959.48	446.99	1446.5	7521.9
8-9	240	0	0	0	12188	260.23	2129.3	2023.4	942.64	3050.4	15862
8-10	300	0	0	0	18643	740.53	6059.3	5758	2682.5	8680.5	7470.4
9-11	120	0	0	0	10193	404.91	3313.1	3148.4	1466.7	4746.3	4084.7
9-12	180	0	0	0	9808.3	0	0	0	0	0	23035
10-11	150	0	0	0	10971	0	0	0	0	0	25765
10-12	100	0	0	0	14857	0	0	0	0	0	5953.3
11-13	80	0	0	0	10368	0	0	0	0	0	4154.6
11-14	30	0	0	0	0	176.81	1446.7	1374.8	640.48	2072.6	0
12-13	130	0	0	0	0	7428.8	0	0	15869	8803.2	0
12-23	190	0	0	0	0	3445.9	0	0	7361	4083.4	0
13-23	150	0	0	0	0	1809.6	14807	14071	6555.2	21212	0
14-16	210	0	0	0	0	624.63	5110.9	4856.8	2262.6	7321.9	0
15-16	50	0	0	0	0	0	0	7049.6	1347.5	9553.4	0
15-21	90	0	0	0	0	0	0	12854	2457	1363	0
15-24	40	0	0	0	0	0	0	0	0	0	3408.8
16-17	80	0	0	0	0	0	0	0	25402	14091	0
16-19	40	0	0	0	0	0	0	0	3054.4	1694.4	0
17-18	260	0	0	0	0	0	0	0	0	46387	0
17-22	150	0	0	0	0	0	0	0	0	32774	0
18.21	40	0	0	0	0	0	0	0	0	0	10098
10.20	30	0	0	0	0	0	0	0	0	0	5618.4
20.22	20	0	0	0	0	0	0	0	0	0	4274
20-23	150	0	0	0	0	8462	0	0	18076	10028	0
Total	4060	10813	12387	0	111457	29092	37116	55174	101147	188133	141630
	4700			L			26051				
	KIVI WI,K	78.07	89.43	0	804 76	210.05	267.99	398 38	730 32	1358 38	1022.62
	фШ)	, 5.07	57.45	· · · ·	004.10		201.00	570.50	, 30.32	1000.00	1022.02

System
bysicin

Line	Line Cost	ckLk MW1,k	ckLk MW2,k	ckLk MW7,k	ckLk MW13,k	ckLk MW15,k	ckLk MW16,k	ckLk MW18,k	ckLk MW21,k	ckLk MW22,k	ckLk MW23,k
к	(\$/hr)	/G1	/G ₂	/G7	/G ₁₃	G ₁₅	/G ₁₆	/G18	/G ₂₁	/G ₂₂	/G ₂₃
1-2	60	954.51	0	0	0	38.046	0	0	81.687	44.76	0
1-3	200	0	0	0	0	1209.6	0	0	2575.7	1428.6	0
1-5	180	7703.1	0	0	0	307.04	0	0	659.23	361.22	0
2-4	200	585.24	5541.1	0	0	23.036	0	0	49.995	27.394	0
2-6	400	1562.7	14795	0	0	61.509	0	0	133.49	73.146	0
3-9	180	0	0	0	0	1493.8	0	0	3180.7	1764.2	0
3-24	250	0	0	0	0	14110	0	0	30044	16664	0
4-9	220	0	0	0	3667.5	154.42	540.48	752.81	482.57	598.39	3455.2
5-10	100	0	0	0	807.5	34	119	165.75	106.25	131.75	760.75
6-10	160	0	0	0	5839.2	245.86	860.52	1198.6	768.32	952.72	5501.2
7-8	180	0	0	0	8433.7	355.1	1242.9	1731.1 [°]	1109.7	1376	7945.5
8-9	240	0	0	0	14380	605.45	2119.1	2951.6	1892	2346.1	13547
8-10	300	0	0	0	17318	729.17	2552.1	3554.7	2278.7	2825.5	16315
9-11	120	0	0	0	7605.2	320.22	1120.8	1561.1	1000.7	1240.8	7164.9
9-12	180	0	0	0	16914	0	0	0	0	0	15929
10-11	150	0	0	0	12995	547.15	1915	2667.4	1709.9	2120.2	12243
10-12	100	0	0	0	12613	0	0	0	0	0	11878
11-13	80	0	0	0	8573.7	0	0	0	0	0	8074.3
11-14	30	0	0	0	0	177.05	1445	2004.7	765.33	1319.3	0
12-13	130	0	0	0	12153	0	0	0	0	0	11445
12-23	190	0	0	0	0	0	0	0	0	0	47966
13-23	150	0	0	0	0	0	0	0	0	0	28092
14-16	210	0	0	0	0	2536.9	20705	28725	10966	18904	0
15-16	50	0	0	0	0	909.09	0	0	1935.7	1073.7	0
15-21	90	0	0	0	0	0	0	0	28568	15861	0
15-24	40	0	0	0	0	2291.5	0	0	4879.3	2706.4	0
16-17	80	0	0	0	0	0	0	16457	3159.3	9104.5	0
16-19	40	0	0	0	0	119.14	972.33	1349	514.99	887.78	0
17-18	260	0	0	0	0	0	0	27602	5298.7	15270	0
17-22	150	0	0	0	0	0	0	0	0	26762	0
18-21	40	0	0	0	0	0	0	0	3053.5	1695.3	0
19-20	30	0	0	0	0	0	0	0	0	0	2556.6
20-23	20	0	0	0	0	0	0	0	0	0	4274
21-22	150	0	0	0	0	0	. 0	0	0	32774	0
Total	4960	10806	20337	0	121299	26268	33592	90720	105214	158312	197147
ΣΣckI	kMWt.k					70	53694	L <u> </u>	L	·	
C ^s t	(\$/hr)	70.18	132.08	0	787.81	170.60	218.17	589.20	683.34	1028.20	1280.42

Appendix E: Transmission Loss Cost Case Study Results

Appendix E contains the results discussed in Chapter 5, which are about transmission loss allocation and loss cost calculation for the IEEE 24-bus system using Z-bus, pro rata and Kirschen's method. The results are shown in Table E.1, E.2, E.3, E.4, E.5, E.6 and E.7 respectively.

Table E.1 Transmission Losses Allocated to Buses Using Z-bus Method for the 24-bus System

Bus	Loss (MW)	Bus	Loss (MW)	Bus	Loss (MW)
1	0.516	9	0.409	17	-0.178
2	0.302	10	2.283	18	3.715
3	-1.909	11	0.264	19	-5.618
4	1.286	12	0.234	20	-4.231
5	0.933	13	2.651	21	20.441
6	4.281	14	-2.223	22	26.116
7	17.459	15	-5.869	23	22.827
8	15.286	16	1.915	24	0.259

Table E.2 Transmission Loss Costs Allocated to Generations Using pro rata Method for the 24-bus System

Generator	Transmission Loss cost (\$/hr)
G ₁	-5.60
G ₂	-3.42
G ₇	-10.05
G ₁₃	-31.52
G ₁₅	-35.78
G ₁₆	-21.61
G ₁₈	-37.63
G ₂₁	-379.48
G ₂₂	-484.83
G ₂₃	-423.77
Total	-1433.69

Load	Transmission Loss cost (\$/hr)	Load	Transmission Loss cost (\$/hr)
	3.98	L ₁₀	42.38
L ₂	2.18	L ₁₃	17.70
L_3	35.44	L ₁₄	41.278
L_4	23.87	L ₁₅	73.17
L ₅	17.31	L ₁₆	13.94
L ₆	79.47	L ₁₈	31.38
L ₇	314.07	L ₁₉	104.29
L ₈	283.77	L ₂₀	78.54
Lo	7.59	Total	1170.29

Table E.3 Transmission Loss Costs Allocated to Loads Using pro rata Method for the 24-bus System

Table E.4 Contribution of Each Generator to Line Losses Using Kirschen's Method for the 24-bus System

Lingk	Lij		Transmission Loss Allocation to Generators (MW)									
LINCK	(MW)	G ₁	G ₂	G ₇	G ₁₃	G ₁₅	G ₁₆	G ₁₈	G ₂₁	G ₂₂	G ₂₃	
1-2	0.01	0.01	0	0	0	0.0003	0	0	0.0007	0.0004	0	
1-3	1.76	0	0	0	0	0.4083	0	0	0.8694	0.4822	0	
1-5	0.69	0.5885	0	0	0	0.0234	0	0	0.0503	0.0276	0	
2-4	1.1	0.103	0.979	0	0	0.004	0	0	0.009	0.004	0	
2-6	0.89	0.084	0.7921	0	0	0.003	0	0	0.007	0.004	0	
3-9	0.49	0	0	0	0	0.114	0	0	0.242	0.134	0	
3-24	1.62	0	0	0	0	0.376	0	0	0.8	0.444	0	
4-9	0.9	0	0	0	0.342	0.014	0.05	0.07	0.045	0.056	0.322	
5-10	0.14	0	0	0	0.053	0.002	0.008	0.011	0.007	0.009	0.05	
6-10	2.29	0	0	0	0.87	0.037	0.128	0.179	0.115	0.142	0.819	
7-8	4.6	0	0	0	1.748	0.074	0.258	0.356	0.23	0.285	1.647	
8-9	13.85	0	0	0	5.263	0.222	0.776	1.08	0.693	0.859	4.958	
8-10	12.06	0	0	0	4.582	0.193	0.675	0.941	0.603	0.748	4.318	
9-11	0.76	0	0	0	0.289	0.012	0.043	0.059	0.038	0.047	0.272	
9-12	0.91	0	0	0	0.467	0	0	0	0	0	0.441	
10-11	1.37	0	0	0	0.521	0.022	0.077	0.107	0.068	0.085	0.49	
10-12	1.54	0	0	0	0.793	0	0	0	0	0	0.745	
11-13	3.07	0	0	0	1.581	0	0	0	0	0	1.489	
11-14	2.25	0	0	0	0	0.07	0.569	0.79	0.302	0.52	0	
12-13	2.28	0	0	0	1.174	0	0	0	0	0	1.106	
12-23	8.48	0	0	0	0	0	0	0	0	0	8.48	
13-23	3.96	0	0	0	0	0	0	0	0	0	3.96	
14-16	8.4	0	0	0	0	0.26	2.125	2.948	1.126	1.94	0	
15-16	0.15	0	0	0	0	0.035	0	0	0.074	0.041	0	
15-21	7.86	0	0	0	0	0	0	0	5.054	2.806	0	
15-24	4.69	0	0	0	0	1.088	0	0	2.317	1.285	0	
16-17	4.36	0	0	0	. 0	0	0	2.498	0.48	1.382	0	
16-19	0.31	0	0	0	0	0.01	0.078	0.109	0.042	0.072	0	
17-18	0.66	0	0	0	0	0	0	0.378	0.073	0.209	0	
17-22	4.32	0	0	0	0	0	0	0	0	4.32	0	
18-21	0.24	0	0	0	0	0	0	0	0.154	0.086	0	
19-20	0.28	0	0	0	0	0	0	0	0	0	0.28	
20-23	0.7	0	0	0	0	0	0	0	0	0	0.7	
21-22	4.16	0	0	0	0	0	0	0	0	4.16	0	
Total	101.4	0.785	1.771	0	17.686	2.967	4.787	9.529	13.397	20.148	30.08	

	Lij		Transmission Loss Allocation to Loads (MW)									
Line k	(MW)	L ₁	L ₂	L ₃	L ₄	L ₅	L ₆	L ₇	L ₈	L9	L ₁₀	
1-2	0.01	0	0.0057	0	0.0015	0	0.0027	0	0	0	0	
1-3	1.76	1.0754	0.1056	0	0.0281	0.4998	0.0510	0	0	0	0	
1-5	0.69	0	0	0	0	0.69	0	0	0	0	0	
2-4	1.1	0	0	0	0.3872	0	0.7128	0	0	0	0	
2-6	0.89	0	0	0	0.3132	0	0.5767	0	0	0	0	
3-9	0.49	0	0	0	0.0294	0.0127	0.0539	0.0731	0.1014	0.1038	0.1156	
3-24	1.62	0.0405	0.0032	0.4568	0.0097	0.0243	0.0210	0.0162	0.0340	0.0340	0.0372	
4-9	0.9	0	0	0	0.3168	0	0.5832	0	0	0	0	
5-10	0.14	0	0	0	0	0.14	0	0	0	0	0	
6-10	2.29	0	0	0	0.8068	0	1.4839	0	0	0	0	
7-8	4.6	0	0	0	0	0	0	4.6	0	0	0	
8-9	13.85	0	0	0	0.831	0.3601	1.5235	2.0637	2.8669	2.9362	3.2686	
8-10	12.06	0	0	0	0.7236	0.3135	1.3266	1.7969	2.4964	2.5567	2.8462	
9-11	0.76	0	0	0	0.0456	0.0197	0.0836	0.1132	0.1573	0.1611	0.1793	
9-12	0.91	0	0	0	0.0546	0.0236	0.1001	0.1355	0.1883	0.1929	0.2147	
10-11	1.37	0	0	0	0.0822	0.0356	0.1507	0.2041	0.2835	0.2904	0.3233	
10-12	1.54	0	0	0	0.0924	0.0400	0.1694	0.2294	0.3187	0.3264	0.3634	
11-13	3.07	0	0	0	0.1842	0.0798	0.3377	0.4574	0.6354	0.6508	0.7245	
11-14	2.25	0	0	0	0.135	0.0585	0.2475	0.3352	0.4657	0.477	0.531	
12-13	2.28	0	0	0	0.0957	0.0410	0.1778	0.2394	0.3328	0.342	0.3807	
12-23	8.48	0	0	0	0.3561	0.1526	0.6614	0.8904	1.2381	1.272	1.4162	
13-23	3.96	0	0	0	0.1663	0.0712	0.3088	0.4158	0.5781	0.594	0.6613	
14-16	8.4	0	0	0	0.168	0.0756	0.3024	0.4116	0.5712	0.588	0.6468	
15-16	0.15	0	0	0	0.003	0.0013	0.0054	0.0075	0.0102	0.0105	0.0115	
15-21	7.86	0.1965	0.0157	2.2165	0.0471	0.1179	0.1021	0.0786	0.1650	0.1650	0.1807	
15-24	4.69	0.1172	0.0093	1.3226	0.0281	0.0703	0.0609	0.0469	0.0984	0.0984	0.1078	
16-17	4.36	0	0	0	0.0872	0.0392	0.1569	0.2136	0.2964	0.3052	0.3357	
16-19	0.31	0	0	0	0	0	0	0	0	0	0	
17-18	0.66	0	0	0	0.0066	0.0033	0.0132	0.0165	0.0231	0.0237	0.0264	
17-22	4.32	0	0	0	0.0432	0.0216	0.0864	0.108	0.1512	0.1555	0.1728	
18-21	0.24	0	0	0	0.0024	0.0012	0.0048	0.006	0.0084	0.0086	0.0096	
19-20	0.28	0	0	0	0	0	0	0	0	0	0	
20-23	0.7	0	0	0	0.0196	0.0084	0.0371	0.0497	0.0686	0.0707	0.0791	
21-22	4.16	0.0832	0.0083	0.9484	0.0291	0.0540	0.0582	0.0540	0.0998	0.0998	0.1081	
Total	101.4	1.513	0.148	4.94	5.093	2.956	9.400	12.563	11.19	11.463	12.741	

Table E.5.1 Contribution of Each Load to Line Losses Using Kirschen's Method (1) for the 24-bus System

Linok	LHAND	Transmission Loss Allocation to Loads (MW)						
Line K		L ₁₃	L ₁₄	L ₁₅	L ₁₆	L ₁₈	L ₁₉	L ₂₀
1-2	0.01	0	0	0	0	0	0	0
1-3	1.76	0	0	0	0	0	0	0
1-5	0.69	0	0	0	0	0	0	0
2-4	1.1	0	0	0	0	0	0	0
2-6	0.89	0	0	0	0	0	0	0
3-9	0.49	0	0	0	0	0	0	0
3-24	1.62	0	0.0680	0.805	0.0356	0	0.0340	0
4-9	0.9	0	0	0	0	0	0	0
5-10	0.14	0	0	0	0	0	0	0
6-10	2.29	0	0	0	0	0	0	0
7-8	4.6	0	0	0	0	0	0	0
8-9	13.85	0	0	0	0	0	0	0
8-10	12.06	0	0	0	0	0	0	0
9-11	0.76	0	0	0	0	0	0	0
9-12	0.91	0	0	0	0	0	0	0
10-11	1.37	0	0	0	0	0	0	0
10-12	1.54	0	0	0	0	0	0	0
11-13	3.07	0	0	0	0	0	0	0
11-14	2.25	0	0	0	0	0	0	0
12-13	2.28	0.6703	0	0	0	0	0	0
12-23	8.48	2.4931	0	0	0	0	0	0
13-23	3.96	1.1642	0	0	0	0	0	0
14-16	8.4	0	2.8056	0	1.4448	0	1.386	0
15-16	0.15	0	0.0501	0	0.0258	0	0.0247	0
15-21	7.86	0	0.3301	3.906	0.1729	0	0.1650	0
15-24	4.69	0	0.1969	2.330	0.1031	0	0.0984	0
16-17	4.36	0	1.4562	0	0.7499	0	0.7194	0
16-19	0.31	0	0	0	0	0	0.31	0
17-18	0.66	0	0.1148	0	0.0587	0.3174	0.0567	0
17-22	4.32	0	0.7473	0	0.3844	2.0779	0.3715	0
18-21	0.24	0	0.0415	0	0.0213	0.1154	0.0206	0
19-20	0.28	0	0	0	0	0	0.28	0
20-23	0.7	0.1386	0	0	0	0	0.0924	0.1358
21-22	4.16	0	0.2787	1.672	0.1456	0.3868	0.1414	0
Total	101.4	4.466	6.089	8.714	3.142	2.898	3.700	0.136

Table E.5.2 Contribution of Each Load to Line Losses Using Kirschen's Method (II) for the 24-bus System

Generator	Transmission Loss cost (\$/hr)
G ₁	-10.99
G ₂	-24.83
G ₇	0
G ₁₃	-242.65
G ₁₅	-34.27
G ₁₆	-55.29
G ₁₈	-90.53
G ₂₁	-127.28
G ₂₂	-191.51
G ₂₃	-249.67
Total	-1027.01

Table E.6 Transmission Loss Costs Allocated to Generations Using Kirschen's Method for the 24-bus System

Table E.7 Transmission Loss Costs Allocated to Loads Using Kirschen's Method for the 24-bus System

Load	Transmission Loss cost (\$/hr)	Load	Transmission Loss cost (\$/hr)
L ₁	14.96	L ₁₀	130.70
L ₂	1.49	L ₁₃	38.94
L ₃	47.83	L ₁₄	59.64
L ₄	55.47	L ₁₅	84.30
L ₅	32.32	L ₁₆	30.77
L ₆	102.32	L ₁₈	27.54
L ₇	131.52	L ₁₉	35.78
L ₈	114.79	L ₂₀	1.13
L9	117.58	Total	1027.09

Appendix F: Comprehensive Transmission Pricing Scheme Case Study Results

Appendix F contains the results discussed in Chapter 6. The total transmission cost allocation and calculation for the IEEE 24-bus system using the proposed transmission pricing scheme are given. The results are shown in Table F.1, F.2, F.3 and F.4 respectively.

Linek	Pij		Transmission Power Flow Allocation to Generators (MW)								
LINCK	(MW)	G ₁	G ₂	G ₇	G ₁₃	G ₁₅	G ₁₆	G ₁₈	G ₂₁	G ₂₂	G ₂₃
1-2	19.715	18.296	0	0	0	0.453	0	0	0.69	0.276	0
1-3	12.45	0	0	0	0	4.009	0	0	6.026	2.415	0
1-5	52.89	49.082	0	0	0	1.217	0	0	1.8512	0.740	0
2-4	30.58	3.027	27.326	0	0	0.078	0	0	0.098	0.0489	0
2-6	52.755	5.223	47.142	0	0	0.137	0	0	0.169	0.084	0
3-9	40.56	0	0	0	0	13.06	0	0	19.631	7.869	0
3-24	203.95	0	0	0	0	65.672	0	0	98.712	39.566	0
4-9	19.69	0	0	0	6.616	0.63	1.201	1.634	1.240	1.063	7.305
5-10	2.18	0	0	0	0.732	0.07	0.133	0.181	0.137	0.118	0.809
6-10	99.44	0	0	· 0°	33.412	3.182	6.066	8.254	6.2657	5.370	36.892
7-8	28.56	0	0	0	9.596	0.914	1.742	2.371	1.799	1.542	10.596
8-9	96.77	0	0	0	32.515	3.097	5.903	8.032	6.097	5.226	35.902
8-10	86.27	0	0	0	28.987	2.761	5.263	7.16	5.435	4.659	32.006
9-11	131.75	0	0	0	44.268	4.216	8.037	10.935	8.300	7.115	48.879
9-12	147.49	0	0	0	70.058	0	0	0	0	0	77.432
10-11	185.11	0	0	0	62.197	5.924	11.292	15.364	11.662	9.9959	68.676
10-12	202	0	0	0	95.95	0	0	0	0	0	106.05
11-13	151.58	0	0	0	72.001	0	0	0	0	0	79.58
11-14	167.56	0	0	0	0	9.551	43.398	58.981	24.966	30.663	0
12-13	123.86	0	0	0	58.833	0	0	0	0	0	65.026
12-23	230.12	0	0	0	0	0	0	0	0	0	230.12
13-23	194.69	0	0	0	0	0	0	0	0	0	194.69
14-16	371.88	0	0	0	0	21.197	96.317	130.9	55.41	68.054	0
15-16	105.56	0	0	0	0	33.99	0	0	51.091	20.479	0
15-21	451.38	0	0	0	0	0	0	0	322.29	129.09	0
15-24	206.02	0	0	0	0	66.338	0	0	99.714	39.968	0
16-17	338.33	0	0	0	0	0	0	210.78	38.231	89.319	0
16-19	121.28	0	0	0	0	6.913	31.412	42.691	18.071	22.194	0
17-18	200.69	0	0	0	0	0	0	125.03	22.678	52.982	0
17-22	140.82	0	0	0	0	0	0	0	0	140.82	0
18-21	101.38	0	0	0	0	0	0	0	72.385	28.995	0
19-20	79.1	0	0	0	0	0	0	0	0	0	79.1
20-23	229.62	0	0	0	0	0	0	0	0	0	229.62
21-22	159.94	0	0	0	0	0	0	0	0	159.94	0
Total	4786.0	75.628	74.468	0	515.16	243.41	210.76	622.31	872.94	868.6	1302.7

Table F.1 Contribution of Each Generator to Line Flows for the 24-bus System

	Pij		Transmission Power Flow Allocation to Loads(MW)									
Line k	(MW)	Li	L ₂	L ₃	L4	L ₅	Lø	L ₇	L ₈	L,	L ₁₀	
1-2	19.715	. 0	10.745	0	2.248	0	6.7228	0	0	0	0	
1-3	12.45	7.209	0.772	0	0.162	3.8221	0.4855	0	0	0	0	
1-5	52.89	0	0	0	0	52.89	0	0	0	0	0	
2-4	30.58	0	0	0	7.645	0	22.935	0	0	0	0	
2-6	52.755	0	0	0	13.189	0	39.566	0	0	0	0	
3-9	40.56	0	0	0	1.744	0.1216	5.1511	1.663	8.6798	11.6	11.6	
3-24	203.95	2.243	0.224	46.501	0.958	1.2441	2.8553	0.8769	4.6908	6.3224	6.3224	
4-9	19.69	0	0	0	4.923	0	14.768	0	0	0	0	
5-10	2.18	0	0	0	0	2.18	0	0	0	0	0	
6-10	99.44	0	0	0	24.86	0	74.58	0	0	0	0	
7-8	28.56	0	0	0	0	0	0	28.56	0	0	0	
8-9	96.77	0	0	0	4.161	0.2903	12.29	3.9676	20.709	27.676	27.676	
8-10	86.27	0	0	0	3.709	0.2588	10.956	3.5371	18.462	24.673	24.673	
9-11	131.75	0	0	0	5.665	0.3952	16.732	5.4017	28.194	37.68	37.68	
9-12	147.49	0	0	0	6.342	0.4424	18.731	6.0471	31.563	42.182	42.182	
10-11	185.11	0	0	0	7.96	0.5553	23.509	7.5895	39.614	52.941	52.941	
10-12	202	0	0	0	8.686	0.606	25.654	8.282	43.228	57.772	57.772	
11-13	151.58	0	0	0	6.518	0.4547	19.251	6.2148	32.438	43.352	43.352	
11-14	167.56	0	0	0	7.2051	0.5026	21.28	6.87	35.858	47.922	47.922	
12-13	123.86	0	0	0	3.3442	0.2477	9.9088	3.2204	16.473	22.171	22.171	
12-23	230.12	0	0	0	6.2132	0.4602	18.41	5.9831	30.606	41.191	41.191	
13-23	194.69	0	0	0	5.2566	0.3893	15.575	5.0619	25.894	34.85	34.85	
14-16	371.88	0	0	0	4.4626	0.3718	13.388	4.4626	22.685	30.122	30.122	
15-16	105.56	0	0	0	1.2667	0.1055	3.8002	1.2667	6.4392	8.5504	8.5504	
15-21	451.38	4.965	0.497	102.91	2.1215	2.7534	6.3193	1.9409	10.382	13.993	13.993	
15-24	206.02	2.266	0.227	46.973	0.9682	1.2567	2.8843	0.8858	4.7385	6.3866	6.3866	
16-17	338.33	0	0	0	4.06	0.3383	12.18	4.06	20.638	27.405	27.405	
16-19	121.28	0	0	0	0	0	0	0	0	0	0	
17-18	200.69	0	0	0	1.2443	0.1003	3.8131	1.2643	6.4221	8.6297	8.6297	
17-22	140.82	0	0	0	0.8731	0.0704	2.6756	0.8872	4.5062	6.0553	6.0553	
18-21	101.38	0	0	0	0.6285	0.0506	1.9262	0.6386	3.2442	4.3593	4.3593	
19-20	79.1	0	0	0	0	0	0	0	0	0	0	
20-23	229.62	0	0	0	4.1332	0.2296	11.94	3.9035	19.747	26.636	26.636	
21-22	159.94	1.44	0.16	29.749	0.7997	0.8156	2.3831	0.7037	3.9345	5.31	5.31	
Total	4786.0	18.123	12.624	226.14	141.35	70.954	420.67	113.29	439.15	587.78	587.78	

Table F.2.1 Contribution of Each Load to Line Flows (1) for the 24-bus System

Lingh	D" AMD	Transmission Power Flow Allocation to Loads (MW)										
	P1j (MW)	L ₁₃	L ₁₄	L ₁₅	L ₁₆	L ₁₈	L ₁₉	L ₂₀				
1-2	19.715	0	0	0	0	0	0	0				
1-3	12.45	0	0	0	0	0	0	0				
1-5	52.89	0	0	0	0	0	0	0				
2-4	30.58	0	0	0	0	0	0	0				
2-6	52.755	0	0	0	0	0	0	0				
3-9	40.56	0	0	0	0	0	0	0				
3-24	203.95	0	11.115	108.3	5.5678	0	6.7303	0				
4-9	19.69	0	0	0	0	0	0	0				
5-10	2.18	0	0	0	0	0	0	0				
6-10	99.44	0	0	0	0	0	0	0				
7-8	28.56	0	0	0	0	0	0	0				
8-9	96.77	0	0	0	0	0	0	0				
8-10	86.27	0	0	0	0	0	0	0				
9-11	131.75	0	0	0	0	0	0	0				
9-12	147.49	0	0	0	0	0	0	0				
10-11	185.11	0	0	0	0	0	0	0				
10-12	202	0	0	0	0	0	0	0				
11-13	151.58	• 0	0	0	0	0	0	0				
11-14	167.56	0	0	0	0	0	0	0				
12-13	123.86	46.324	0	0	0	0	0	0				
12-23	230.12	86.065	0	0	0	0	0	0				
13-23	194.69	72.814	0	0	0	0	0	0				
14-16	371.88	0	126.44	0	63.22	0	76.607	0				
15-16	105.56	0	35.89	0	17.945	0	21.745	0				
15-21	451.38	0	24.6	239.6	12.323	0	14.896	0				
15-24	206.02	0	11.228	109.4	5.6243	0	6.7987	0				
16-17	338.33	0	115.03	0	57.516	0	69.696	0				
16-19	121.28	0	0	0	0	0	121.28	0				
17-18	200.69	0	36.124	0	18.062	94.525	21.875	0				
17-22	140.82	0	25.348	0	12.674	66.326	15.349	0				
18-21	101.38	0	18.248	0	9.1242	47.75	11.05	0				
19-20	79.1	0	0	0	0	0	79.1	0				
20-23	229.62	56.027	0	0	0	0	27.784	52.583				
21-22	159.94	0	12.395	69.46	6.2057	13.755	7.5172	0				
Total	4786.00	261.23	416.42	526.8	208.26	222.36	480.43	52.583				

Table F.2.2 Contribution of Each Load to Line Flows (11) for the 24-bus System

.

	Lij		Transmission Loss Allocation to Generators (MW)									
Line K	(MW)	G ₁	G ₂	G ₇	G ₁₃	G ₁₅	G16	G ₁₈	G ₂₁	G ₂₂	G ₂₃	
1-2	0.01	0.0092	0	0	0	0.0002	0	0	0.0003	0.0001	0	
1-3	0.11	0	0	0	0	0.0354	0	0	0.0532	0.0213	0	
1-5	0.95	0.8816	0	0	0	0.0218	0	0	0.0332	0.0133	0	
2-4	0.35	0.0346	0.3125	0	0	0.0009	0	0	0.0011	0.0005	Ó	
2-6	2.77	0.2742	2.4736	0	- 0	0.0072	0	0	0.0088	0.0044	0	
3-9	0.7	0	0	0	0	0.2254	0	0	0.3388	0.1358	0	
3-24	1.07	0	0	0	0	0.3445	0	0	0.5178	0.2075	0	
4-9	0.2	0	0	0	0.0672	0.0064	0.0122	0.0166	0.0126	0.0108	0.0742	
5-10	0.47	0	0	0	0.1579	0.0150	0.0286	0.0390	0.0296	0.0253	0.1743	
6-10	1.63	0	0	0	0.5476	0.0521	0.0994	0.1352	0.1026	0.0880	0.6047	
7-8	0.89	· 0	0	0	0.2990	0.0284	0.0542	0.0738	0.0560	0.0480	0.3301	
8-9	4.76	0	0	0	1.5994	0.1523	0.2903	0.3950	0.2998	0.2570	1.766	
8-10	3.31	0	0	0	1.1122	0.1059	0.2019	0.2747	0.2085	0.1787	1.228	
9-11	0.45	0	0	0	0.1512	0.0144	0.0274	0.0373	0.0283	0.0243	0.1669	
9-12	0.57	0	0	0	0.2707	0	0	0	0	0	0.2992	
10-11	0.82	0	0	0	0.2755	0.0262	0.0500	0.0680	0.0516	0.0442	0.3042	
10-12	0.95	0	0	0	0.4512	0	0	0	0	0	0.4987	
11-13	1.64	0	0	0	0.779	0	0	0	0	0	0.861	
11-14	1.63	0	0	0	0	0.0929	0.4221	0.5737	0.2428	0.2982	0	
12-13	1.06	0	0	0	0.5035	0	0	0	0	0	0.5565	
12-23	6.44	0	0	0	0	0	0	0	0	0	6.44	
13-23	3.96	0	0	0	0	0	0	0	0	0	3.96	
14-16	7.04	0	0	0	0	0.4012	1.8234	2.4781	1.049	1.2883	0	
15-16	0.26	0	0	0	0	0.0837	0	0	0.1258	0.0504	0	
15-21	6.24	0	0	0	0	0	0	0	4.4554	1.7846	0	
15-24	3.08	0	0	0	0	0.9917	0	0	1.4907	0.5975	0	
16-17	3.67	0	0	0	0	0	0	2.2864	0.4147	0.9688	0	
16-19	0.47	0	0	0	0	0.0267	0.1217	0.1654	0.0700	0.0860	0	
17-18	0.7	0	0	0	0	0	0	0.4361	0.0791	0.1848	0	
17-22	2.54	0	0	0	0	0	0	0	0	2.54	0	
18-21	0.16	0	0	0	0	0	0	0	0.1142	0.0457	0	
19-20	0.26	0	0	0	0	0	0	· 0	0	0	0.26	
20-23	0.78	0.0092	0	0	0	0.0002	0	0	0.0003	0.0001	0	
21-22	2.02	0	0	0	0	0.0354	0	0	0.0532	0.0213	0	
Total	61.96	1.1998	2.7862	0	6.2146	2.633	3.1316	6.9798	9.7847	10.924	18.304	

 Table F.3
 Contribution of Each Generator to Line Losses for the 24-bus System

1	Lij		Transmission Loss Allocation to Loads(MW)								
Line k	(MW)	L ₁	L ₂	L ₃	L ₄	L ₅	L ₆	L ₇	L ₈	L9	L ₁₀
1-2	0.01	0	0.0054	0	0.0011	0	0.0034	0	0	0	0
1-3	0.11	0.0636	0.0068	0	0.0014	0.0337	0.0042	0	0	0	0
1-5	0.95	0	0	0	0	0.95	0	0	0	0	0
2-4	0.35	0	0	0	0.0875	0	0.2625	0	0	0	0
2-6	2.77	0	0	0	0.6925	0	2.0775	0	0	0	0
3-9	0.7	0	0	0	0.0301	0.0021	0.0889	0.0287	0.1498	0.2002	0.2002
3-24	1.07	0.0117	0.0011	0.2439	0.0050	0.0065	0.0149	0.0046	0.0246	0.0331	0.0331
4-9	0.2	0	0	0	0.05	0	0.15	0	0	0	0
5-10	0.47	0	0	0	0	0.47	0	0	0	0	0
6-10	1.63	0	0	0	0.4075	0	1.2225	0	0	0	0
7-8	0.89	0	0	0	0	0	0	0.89	0	0	0
8-9	4.76	0	0	0	0.2046	0.0142	0.6045	0.1951	1.0186	1.3614	1.3614
8-10	3.31	0	0	0	0.1423	0.0099	0.4203	0.1357	0.7084	0.9466	0.9466
9-11	0.45	0	0	0	0.0193	0.0013	0.0571	0.0184	0.0963	0.1287	0.1287
9-12	0.57	0	0	0	0.0245	0.0017	0.0723	0.0233	0.1219	0.1630	0.1630
10-11	0.82	0	0	0	0.0352	0.0024	0.1041	0.0336	0.1754	0.2345	0.2345
10-12	0.95	0	0	0	0.0405	0.0028	0.1206	0.0389	0.2033	0.2717	0.2717
11-13	1.64	0	0	0	0.0705	0.0049	0.2082	0.0672	0.3509	0.4690	0.4690
11-14	1.63	0	0	0	0.0700	0.0048	0.2070	0.0668	0.3488	0.4661	0.4661
12-13	1.06	0	0	0	0.0286	0.0021	0.0848	0.0275	0.1409	0.1897	0.1897
12-23	6.44	0	0	0	0.1738	0.0128	0.5152	0.1674	0.8565	1.1528	1.1528
13-23	3.96	0	0	0	0.1069	0.0079	0.3168	0.1029	0.5266	0.7088	0.7088
14-16	7.04	0	0	0	0.0844	0.0070	0.2534	0.0844	0.4294	0.5702	0.5702
15-16	0.26	0	0	0	0.0031	0.0002	0.0093	0.0031	0.0158	0.0210	0.0210
15-21	6.24	0.0686	0.0068	1.4227	0.0293	0.0380	0.0873	0.0268	0.1435	0.1934	0.1934
15-24	3.08	0.0338	0.0033	0.7022	0.0144	0.0187	0.0431	0.0132	0.0708	0.0954	0.0954
16-17	3.67	0	0	0	0.0440	0.0036	0.1321	0.0440	0.2238	0.2972	0.2972
16-19	0.47	0	0	0	0	0	0	0	0	0	0
17-18	0.7	0	0	0	0.0043	0.0003	0.0133	0.0044	0.0224	0.0301	0.0301
17-22	2.54	0	0	0	0.0157	0.0012	0.0482	0.0160	0.0812	0.1092	0.1092
18-21	0.16	0	0	0	0.0009	8e-005	0.0030	0.0010	0.0051	0.0068	0.0068
19-20	0.26	0	0	0	0	0	0	0	0	0	0
20-23	0.78	0	0.0054	0	0.0011	0	0.0034	0	0	0	0
21-22	2.02	0.0636	0.0068	0	0.0014	0.0337	0.0042	0	0	0	0
Total	61.96	0.1961	0.0257	2.7446	2.4129	1.6083	7.196	2.0159	5.8315	7.8071	7.8071

Table F.4.1 Contribution of Each Load to Line Losses (1) for the 24-bus System

		Transmission Loss Allocation to Loads (MW)										
Line k	Lij (MW)	L ₁₃	L ₁₄	L ₁₅	L ₁₆	L ₁₈	L ₁₉	L ₂₀				
1-2	0.01	0	. 0	0	0	0	0	0				
1-3	0.11	0	0	0	0	0	0	0				
1-5	0.95	0	0	0	0	0	0	0				
2-4	0.35	0	0	0	0	0	0	0				
2-6	2.77	0	0	0	0	0	0	0				
3-9	0.7	0	0	0	0	0	0	0				
3-24	1.07	· 0	0.0583	0.5681	0.0292	0	0.0353	0				
4-9	0.2	0	0	0	0	0	0	0				
5-10	0.47	0	0	0	0	0	0	0				
6-10	1.63	0	0	0	0	0	0	0				
7-8	0.89	0	0	0	0	0	0	0				
8-9	4.76	0	0	0	0	0	0	0				
8-10	3.31	0	0	0	0	0	0	0				
9-11	0.45	0	0	0	0	0	0	0				
9-12	0.57	0	0	0	0	0	0	0				
10-11	0.82	0	0	0	0	0	0	0				
10-12	0.95	0	0	0	0	0	0	0				
11-13	1.64	0	0	0	0	0	0	0				
11-14	1.63	0	0	0	0	0	0	0				
12-13	1.06	0.3964	0	0	0	0	0	0				
12-23	6.44	2.4086	0	0	0	0	0	0				
13-23	3.96	1.481	0	0	0	0	0	0				
14-16	7.04	0	2.3936	0	1.1968	0	1.4502	0				
15-16	0.26	0	0.0884	0	0.0442	0	0.0535	0				
15-21	6.24	0	0.3400	3.3134	0.1703	0	0.2059	0				
15-24	3.08	0	0.1678	1.6355	0.0840	0	0.1016	0				
16-17	3.67	0	1.2478	0	0.6239	0	0.7560	0				
16-19	0.47	0	0	0	0	0	0.47	0				
17-18	0.7	0	0.126	0	0.063	0.3297	0.0763	0				
17-22	2.54	0	0.4572	0	0.2286	1.1963	0.2768	0				
18-21	0.16	0	0.0288	0	0.0144	0.0753	0.0174	0				
19-20	0.26	0	0	0	0	0	0.26	0				
20-23	0.78	0.1903	0	0	0	0	0.0943	0.1786				
21-22	2.02	0	0.1565	0.8772	0.07836	0.1737	0.0949	0				
Total	61.96	4.4764	5.0646	6.3944	2.5329	1.7751	3.8926	0.1786				

Table F.4.2 Contribution of Each Load to Line Losses (II) for the 24-bus System

•







