AIRPORT INVESTMENT PLANNING IN NEWFOUNDLAND

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ABSTRACT

Efficient transportation planning by both public and private agencies requires an estimate of future travel demands. These estimates are dependent upon both the transportation system components and the socio-economic variables in a region. The basic problem in transportation planning is to convert existing transportation system data and socio-economic data into future travel demands. This type of planning requires a logical planning framework and a rigourous and consistent modelling technique.

In a developmental situation, the airplane is a most desirable mode of travel because of its independence of expensive surface routes. Air travel demand for short haul trips is highly dependent upon airport location and accessibility. Before implementing any changes in airport location, the planners must be able to evaluate the consequences of various alternatives.

This thesis describes the criteria involved in airport investment planning and outlines a logical framework for regional airport location planning. The mathematical modelling technique used in this framework is based on linear graph theory. The model

and framework is intended to aid the planner in deriving traffic consequences in relation to specific investments in the Newfoundland air network.

The systematic analysis of the regional airport location problem provides for a thorough understanding of the role of transportation in stimulating future public and private investments.

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CHAPTER 1

INTRODUCTION

In the dynamic environment which accompanies a developing region a statement of transportation investment requirements is essential in order to make maximum use of a limited available capital. The scope of the Canadian planning problems in economic terms indicates that annually over twenty percent of the Gross National Product has been attributed to transportation (1) [1]. Canada's economic and social well being is dependent on transportation. In fact, Owen (2) points out that the Canadian people are the most mobile (on a per capita basis) in the world.

The importance of air transportation to the social and economic life of a region stems from its dual nature as an economic resource and as a productive entity. As an economic resource it makes possible other social and economic developments to occur at separate locations. As a productive entity, which competes in the markets for resources, it generates a large and rapidly rising flow

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[1]. Numbers in parenthesis refer to references at the end of this thesis.

of incomes on its own account. In its resource role, air transportation increases the efficiency with which both human and material resources are used.

The unique contribution of air transportation is speed. Speed makes it possible to accomplish more in a given period of time than could otherwise be done, and speed makes it possible to do things that otherwise could not be done at all.

Many forecasts of demand, especially of air travel, have proven to be inaccurate. In some cases this has resulted in lost revenues due to inadequate or inconvenient facilities (eg. Toronto International Airport). In other cases it has resulted in opportunity costs by providing a facility in which the designed capacity is never reached (eg. Gander International Airport).

The primary purpose of this study was to develop a methodology in regional airport planning in order to overcome the above deficiencies. It is intended that this methodology be a logical series of component analysis that can be applied to any regional airport system.

The first portion of this study discusses in general terms the criteria for airport investment. It emphasizes those variables which have specific application to the Newfoundland situation. The next section discusses the framework and indicates how it is applied to Newfoundland. The mathematical model, based on linear graph theory, demonstrates its applicability to simulating physical systems of this type.

The results of the models calibration are compared with actual origin destination statistics. Changes were then made in the system components and the new travel volumes were then calculated. It was also demonstrated how an increase or decrease in the relative economic standing of a city affects the travel demand model.

CHAPTER 2

CRITERIA FOR AIRPORT INVESTMENT

2.1 AIRLINE TRAFFIC TRENDS

In approaching the airport investment problem it is useful to begin by considering two rather different ways in which the airplane can be used. Its primary use is a relatively high speed, low cost scheduled common carrier. However, it also has potential in developmental situations where its independence of surface routes is its most significant characteristic. The first two of these characteristics have contributed heavily to the very rapid growth in world-wide scheduled air carrier movements of passengers and commodities in recent years (4).

The development of aircraft with substantially lower operating costs, coupled with increasing inter-airline competition on major domestic and international routes has led to a decline in average fares. Presently, average airline fares per passenger-mile are roughly one-half to one-third of the vehicle-mile cost of the typical American automobile (3). This relatively low cost which can compete favourably with other modes of travel is one of the major factors in this rapid growth in commercial airline traffic.

With this growth in traffic has come two problems related to each other and to the problem of airport investment in a developed urban economy: air traffic congestion and airport noise. This noise constitutes a cost to most of those persons who perceive it on the ground. For the most part, however, compensation has not been paid, and the costs have not been charged to those using the airplanes. The lower fares are, at least in part, the result of letting the major cost of this noise pollution to fall on the general public and remain external to the air transportation industry. It must be remembered, however, that these large high-performance jet aircraft are the result of the increase in demand for air travel and not the cause of it.

2.2 APPLICATION TO ECONOMIC DEVELOPMENT

Besides its ability to carry large numbers of persons at high speed and relatively low cost, the substantial independence of the airplane from ground facilities holds potential for its use in regional and national economic development. A wide variety of small modern aircraft have the capability of operating from short grass or dirt runways, or use floats or skiis for operating on water or snow. Although as a transportation vehicle the airplane is an expensive mode of carrying heavy or bulky cargo, its independence of surface routes frequently makes it the most economical transportation system in undeveloped regions.

This is of particular importance in the Province of Newfoundland for a number of reasons:

1. The Island. The fact that part of the Province is an island places air travel in a more favourable position than other modes of travel with respect to mainland connections.
2. Labrador. Although part of the mainland, Labrador can be considered to be an island as it is not connected to the mainland by road. This situation will probably exist far beyond the present planning horizon.
3. Provincial location. No matter what transportation developments occur in the future, Newfoundland will always be about a thousand miles from the economic heart of Canada.
4. The economic situation. The low economic standing of Newfoundland with respect to the rest of Canada places prime importance on the fact that aircraft are independent of expensive ground routes which would unnecessarily tie up investment capital.

The linking together of isolated communities is of prime importance in Newfoundland for both social and political reasons. One need only look at a Provincial road map to see the unacceptable isolation that still exists. At present the airplane and helicopter are the best and most economical vehicles to fulfill governmental and health services in these isolated communities.

2.3 AIRPORT INVESTMENT OBJECTIVES

In the early stages of the development of aviation needs criteria led to few, if any real economic distortions. Newfoundland was on the transatlantic air map to fulfill a service function as a refueling station and a defense function as an air force base. These needs resulted in the construction of five major airports in Newfoundland:

1. Gander;
2. Torbay;
3. Stephenville;
4. Argentia;
5. Goose Bay.

As aviation technology advanced and economic development promoted, more air services were demanded and four service airports were constructed:

1. Deer Lake (Corner Brook);

2. St. Anthony (Grenfell Mission);

3. Churchill Falls (construction);

4. Wabush (mining).

Other social requirements (i. e. health, welfare, recreation, etc.) resulted in the construction of many small grass or dirt air strips and sea plane bases. This left Newfoundland with fifty-two airports (fig. 2.1) which is far more than could be considered adequate. In fact M. W. S. Eagles of Air Canada (5) stated that when considering the area and population, there are far more airports in Newfoundland than in any other comparable area in North America.

With the development of long range jet aircraft in the late fifties, it no longer became necessary to use Newfoundland as a refueling station for transatlantic flights. Defense needs changed as well with the use of long range missiles. This development has all but rendered the airplane as an obsolete method of defense. The problem now is how to make the best use of the existing airports to further economic and social development.

The basic objective in the developmental situation in Newfoundland is to maximize the rate of growth of per capita income. Implementation of this objective requires a criterion which searches for the sequence of investments which will yield the highest per capita growth rate. In selecting investments to maximize the growth rate, one must evaluate the effect of each investment on the pace of

Figure 2.1 Existing Airports in Newfoundland (20)

economic development. The best investment is not the one with the highest marginal productivity, but the one which stimulates the maximum subsequent investments.

The gains and costs of improving or providing for further air transportation facilities must be compared with the other transportation and communications available. The conclusion which this implies for airport investments in a developing region are largely negative. Airport investment to promote development should not be undertaken unless it can be definitely demonstrated that the investment will lead to a sequence of larger future investments than will other use of the resources. It is felt that not many airports will pass this test.

The above basic developmental objective, when considered in the context of airport investment can be evaluated under the following specific objectives:

1. Economic efficiency objectives;
2. Economic development objectives;
3. Social development objectives;
4. Level of Service objectives.

2.3.1 ECONOMIC EFFICIENCY OBJECTIVES

Economic efficiency will be obtained when the difference between the design capacity and the actual passenger and aircraft utilization of the facility is small. Inefficiency of an airport will be encountered in one of three ways:

1. The airport may have inadequate capacity in the sense that costly queing delays build up during busy periods of the day.
2. It may be inadequate in the technical sense that it fails to comply with minimum requirements - for example, in runway length or strength, etc. - of the aircraft to be operated on it.
3. It may be inefficient in the sense that the design capacity is never (or rarely) reached. This leads to lost opportunity costs in that the capital invested in an oversized airport could have been used on another project with a higher economic return.

It appears that most of Newfoundland's airport inefficiencies are of the third type. In order to minimize these inefficiencies it is necessary to determine existing and future passenger and freight demands. It is felt that the methodology and mathematical modelling outlined in the following chapters will enable the optimal planning of airport investments to meet these demands.

Once these demands are determined, primary and secondary routes can be designated and appropriate technology specified for both the passenger and freight demands. Determining the required service and investment for these demands is of primary importance in order to choose the best alternative of all transportation and communication investment schemes.

At present, there is no airport in Newfoundland that has a problem of too many aircraft movements to make it deficient in capacity. However, Deer Lake is deficient in runway configuration and in air terminal space. Before any investment in additional capacity at Deer Lake is made, one must be able to determine the resulting air traffic patterns and an estimate of future demand. Further, if the Deer Lake Airport were brought up to an acceptable D.O.T. standard for regularly scheduled aircraft, how would this affect the utility of the Stephenville Airport? Would it be economically advantageous to make Deer Lake the major airport for the West Coast and to phase out Stephenville? Should both airports continue to serve the area?

Another area of interest is the underutilization of Gander Airport. Should Gander continue its efforts to stay on the trans-atlantic air way, or should Torbay try to fulfill this function? These capacity problems can be considered using the mathematical model developed in Chapter 4.

2.3.2 ECONOMIC DEVELOPMENT OBJECTIVES

In undertaking investments to further either regional or national economic development, the objective is typically one of maximizing the growth in per capita output. A true developmental airport is one in which the increased accessibility, made available by the airport, is expected to stimulate a chain of developments with either regional or national significance. These may be characterized by the Canadian Department of Transport as "development" airports, which are "those for which scheduled services are not necessarily foreseen, but the establishment of which would contribute greatly or be essential to the exploitation of natural resources" (4). For example, Wabush and Churchill Falls in their initial stages.

Investment of this type can require extremely complex and largely unproven criteria. In these cases, one may be much less interested in the conventionally defined "net benefit" of the airport investment itself than in the chain of development investments which the airport investment sets in motion. However, it must be demonstrated in advance that these subsequent investments will follow and that they will economically justify the initial investment.

Airports to promote economic development may be required to satisfy the following criteria:

1. Provide access to natural resources - for example, mineral (Wabush, Buchans, etc.) and

recreational (hunting, tuna fishing, etc.).

2. Provide access to markets - for example, the delivery of fresh fish and lobster to the prime markets of Canada (Toronto, Montreal, etc.).
3. Provide for the trans-shipment and distribution of raw materials, perishable goods, mail, etc.

Any or all of the above may be required under planned economic development. It must be remembered, however, that a transportation facility by itself will develop nothing. There must already exist other social and economic pressures in the environment that will beneficially influence new investment when the airport is put into an area. If these pressures are non-existent, then investment will be required into other sectors of the economy as well as the airport facility.

2.3.3 SOCIAL DEVELOPMENT OBJECTIVES

A potential impact of airport investment which is probably not adequately measured by the willingness-to-pay of direct users is the social impact of more closely relating communities together by reducing travel time among them. The linking together of isolated communities is predominately a factor in developing economics and regions. The role of breaking down the isolation that stems from inaccessibility is a broad social and political function.

Airports provided in this type of situation may be characterized by what the Canadian Department of Transport has called "remote area" airports. These are "small airstrips essential to the maintenance of administrative or health services in isolated communities where other transportation is infrequent, unreliable or non-existent" (4).

Development situations of this type will require airport investment analysis of only a rather trivial type. Once the higher level decision has been made to provide a community with air transportation, one needs only to minimize the combined cost of the required airport and corresponding aircraft technology. Small aircraft capable of operating with either floats or skiis would probably be the most economical in the Newfoundland situation. Helicopters could be used during the freeze-thaw periods of the year on a stand-by or emergency basis.

Equally important with this type of service would require investment into an adequate communication system which could coordinate flights so that a community would know when to expect flights. It would also be necessary in cases of medical emergencies and weather reports.

2.3.4 LEVEL OF SERVICE OBJECTIVES

In any actual investment decision, the decision group will typically have several objectives in mind which they attempt to realize simultaneously. Some of these are legitimate economic objectives and some are not. But even those objectives which are not strictly economic will frequently be important to some persons in the investment decision group and will thus affect the decision. If a good decision is to be made, it is essential that all objectives and values affecting it be identified and made as explicit as possible.

One major objective, particularly for investment decision in a metropolitan area, is the minimization or reduction of airport noise. At the subsonic speeds of the present turbine airplanes noise is a problem only in terminal areas where aircraft are approaching or departing airports and are operating at relatively low altitudes. The noise constitutes a technological external cost. It is technological in that it affects either utility or production function of activities in the airport vicinity, and it is external in that it is realized neither by the airlines nor the airport operator.

It is frequently impossible to control land use around airports. In many cases controls on land development have been non-existent or totally inadequate. Even where control has been possible, it has often been by-passed in favour of land development and speculation.

Some progress has been made at the airplane, but these gains have been expensive and small in magnitude compared with the problem.

Thus, at least at present, the airport noise problem is effectively without a technological, planning, or economic solution. At the same time the problem grows worse as traffic volume and the number of aircraft operations increase. It appears that the noise problem constitutes a serious block to the development of additional airport capacity in a metropolitan area.

In the technical sense, the cost of airport noise is incommensurable with other economic values of airport development. It is possible to measure the noise effects of alternative investment proposals and to relate the measured noise levels to probable reactions of persons engaged in avarious activities (8). But as in the value of time problem, we lack a well defined cost function relating noise levels to dollar values which may then be added to other investment costs and compared with the expected economic gains of the investment.

The problems of airport noise in Newfoundland are, at present, minimal, if not non-existent. It is felt that in the foreseeable future there will not be a significant increase in the problem of noise pollution. However, there may be a decrease in the allowable tolerance level of a community, which is impossible to predict. With this probable increase in public aversion to airport noise, it is clearly necessary to include it as a separate term in the investment objective function. The weight which noise will be given in the objective function will undoubtedly vary with persons in the decision group. In a developing economy, this noise function should be considered to be of negligible importance. However, it is strongly recommended that the Government enforce some appropriate zoning legislation in the vicinity of all existing airports in the hope that the problem will not be aggravated by hodge-podge development.

Estimating the value of passenger time saved is a rather different matter. The concept of a delay is not uniquely defined; but for present purposes we will define a delay as the difference between scheduled and actual arrival time of the aircraft. The value of delay to the passenger lies in the use to which it can be put and this in turn depends on the actual options available for using the time. A delay of 15 - 30 minutes may be of little importance to some people and of great importance to others.

It may be of little importance to a person at one time and of great importance to him at another. Little is known about air passenger's delay-cost function when it is viewed in terms of lost opportunities. It is probably at or close to zero for delays of a few minutes, rising rapidly and then possibly flattening out again for very large delays. But all that one can say in general terms is that delay cost functions for individual passengers are non-linear and vary substantially for different passengers and at different times for the same passenger. Further, the passenger, his employer, and his wife may place quite different valuations on particular delays.

Any aircraft delays in Newfoundland are usually attributed to bad weather conditions and not to deficient airport capacity. Any investment to upgrade an airport in Newfoundland to all weather landing capacity must be economically justified with the gains made by a decrease in aircraft operating costs, in passenger time saved and in inconvenience. .

Access benefits may be defined as the difference in time and other access costs for the geographic distribution of air transportation service with and without the proposed investments, summed over all future users of the service. The concern here is with the net access benefits which result from an airport investment, and not the benefits resulting from investment in improved access modes.

Revenue gains will result from the reaction of passengers and shippers to decreases in access costs. The primary revenue benefit to the airlines will come from new traffic which has been induced by the lower access costs and times.

There are a number of potential cost impacts from airport investments which change air transportation service patterns. In analyzing them it is useful to make a distinction between an airport investment which adds a new service point, and an investment which adds capacity at an existing service point.

In the first case one must analyze both changes in travel times and distances and changes in average waiting times. When an airport is placed in a region which has had no direct service before, passengers have the choice of using the closer airport with relatively few schedules or a larger, more remote airport with higher schedule frequency. If the frequency of schedules is significantly different between the two airports, average waiting time may be large enough to offset gains in access times. Therefore, in evaluating the access advantages of a new service point, one must not overlook the potentially lower schedule frequency and increase in average waiting times.

In the second case, if the airport is to be placed in a terminal area to supplement existing airport capacity, the problem is a more general one of the optimum location of the additional capacity. As mentioned previously, the second case does not apply in Newfoundland.

Thus, in evaluating any solution of the West Coast Airports, one must consider that two airports will have a longer average waiting time and a lower frequency of aircraft movements at both airports. If only one airport serves the region, there will be shorter average waiting times and higher frequencies, but there will also be greater access costs to some areas. However, if Deer Lake were chosen to be the airport to serve the region, the largest percentage of the users (those in Corner Brook) would have a lower access cost. This problem will be dealt with in greater detail later.

2.4 IMPACTS OF AIRPORT INVESTMENT

The income redistributional effects of airport investment will usually be measured in terms of changes in regional income, per capita income, regional employment, or changes in population. Airports are frequently held out to be a major factor in industrial location decisions (9). Most studies of the probable economic impact of proposed new airports are grossly incomplete because they fail to consider the cost impacts of the expected economic growth (3). These would include the increased loads on and cost of a wide range of public services and activities, the incident of changes in real property costs, the loss of business of competing transportation modes, and so forth.

When income redistributional effects can be traced directly to an airport investment, they should be included in the total investment analysis.

It should also be determined before the investment is made, whether or not airport investment is the most efficient way of achieving income redistribution. It is felt that this would not be the most efficient means in Newfoundland.

A second impact, not totally dissociated from the first, is the impact of an airport investment on the pattern of community growth. These impacts arise from a complex of technological and economic external costs and gains generated by the airport. If properly controlled so as to minimize the detrimental affects of noise while exploiting the economic externalities to achieve community objectives, airport investment can be used as a positive force in community development. The potential, and need, for such manipulation of airport investments will increase markedly with the advant of scheduled V/STOL operations in metropolitan areas. The important point is to note the potential of airport investment for affecting urban development, and to urge that this potential be explicitly recognized in airport investment criteria.

In Canada, where airports are for the most part Federally owned and operated, the government has its own organizational objectives of enterprise size and growth. Investment typically increases the size of the airport itself, and with it the size of the operating organization. Investment in runways, terminal buildings, land acquisition, and the like constitute physical, public evidence that the airport operation is becoming increasingly important to the economic well-being and stature of the community. As airports become "big business" they can justify larger staffs, with higher salaries and more prestige at the top of the hierarchy. Airport management is not unmindful of this impact of the investment decision.

However, it must be remembered that airport services are demanded, not as final products, but as services required for the production of air transportation. As demand for air transportation shifts within a region, it is important not to let previous incorrect decisions in airport locations and sizing dictate all future investment decisions.

If, however, the social viewpoint is partitioned into an "airline" and a "community" viewpoint, the decision process is changed somewhat. From the airline point of view, the airport investment is worthwhile as long as the additional costs the airline must pay in landing fees and terminal space are less than its gains in operating costs and potential additional revenues attracted by the improved equipment. Passenger time savings will be largely irrelevant from the airline point of view except as it affects demand and gross revenues. From the viewpoint of the community, passenger time savings will be a real gain, but not one which necessarily affects those making the investment decision. Communities have frequently decided to improve airports at general community expense simply to "get on the air map" or to "get on the jet map". A community image value becomes relevant to the decision and places the airlines in a strong bargaining position. Since the political decision makers must avoid having to publicly acknowledge a "technically inadequate airport", the airlines can press the point that the proposed airport investment will yield major benefits to the community and not by the airlines. So the investment decision is frequently not independent of the pricing decision from the airlines viewpoint.

2.5 SUMMARY

1. The rapid growth in commercial airline traffic was discussed with its associated problems in urban areas.
2. The characteristic of the substantial independence of the airplane from ground facilities was related to Newfoundland's unique developmental problems.
3. The needs criteria which led to the development of Newfoundland's existing airport system were shown to be no longer valid. The present objective of maximizing the rate of growth of per capita income was considered in the context of specific airport investment objectives.
4. The effects of airport investment on community and economic growth were presented from both the community and airlines point of view.

CHAPTER 3

A FRAMEWORK FOR AIRPORT INVESTMENT PLANNING IN NEWFOUNDLAND

3.1 THE NEWFOUNDLAND PROBLEM

3.1.1 Economic Situation

In relation to the rest of the country, the province of Newfoundland is outside the mainstream of Canadian economic life. There are a number of factors that point to Newfoundland's low economic standards (14) and these include:

1. Lower per capita incomes than the Canadian average.
2. Low net value of production per employee in many industries.
3. High reliance of the economy on the primary industries. It is important to note that the highest rate of capital formation is generally on the secondary manufacturing industries (15).
4. High number of non-Newfoundland industrial ownership. This tends to discourage the retention of profits in New-

foundland, further reducing the capital formation necessary.

3.1.2 Relationship to Regional Goals

In terms of the total Canadian economy, the Newfoundland situation is unacceptable. The problem stated in general terms, is how to bring Newfoundland up to the existing Canadian economic and social standards. Investment in airport planning will not achieve this goal. However, efficient airport planning and utilization will allow other sectors of the economy to minimize their business and goods air transport costs and, in some cases, will make new market areas available.

The Pushie Report (16) recommends the following social and economic goals that should be actively pursued and are closely related to the broad goals of national policy (17):

1. A high rate of economic growth.
2. Full employment (defined by the Economic Council of Canada to be 97% employed).
3. A reasonable stability of prices.
4. A reasonably equitable distribution of rising incomes.
5. The achievement of an environment, both social and economic, that will encourage further economic development.

The Commission (16) recommends specifically that: "Inasmuch as a balance must be struck between conflicting policies that some of these goals could suggest - particularly in the short term, the Commission recommends that particular emphasis should be placed on sustaining a high rate of economic growth."

In cases of regional development, the public sector must supply or stimulate those social or economic forces that are lacking in the environment, in order to achieve these goals. This generally requires public investment into a number of areas.

Specific objectives for investment in a developing economy have been identified (18) and include:

1. Maximize efficiency in investment; i. e. maximize economic returns.
2. Redistribution of incomes and maximization of other secondary benefits.

Transportation investment can be conceptualized as the mid point on a scale of influence on attitudes and influence on productivity (Fig. 3. 1) and may be seen as a necessary, but not a sufficient condition to stimulate economic development (19). In Newfoundland's case, we see that it has an abundance of fairly well distributed airports (Fig. 2. 1), but we still do not have optimum utilization, nor rapid economic growth. Thus, in general terms, transportation will develop nothing by itself.

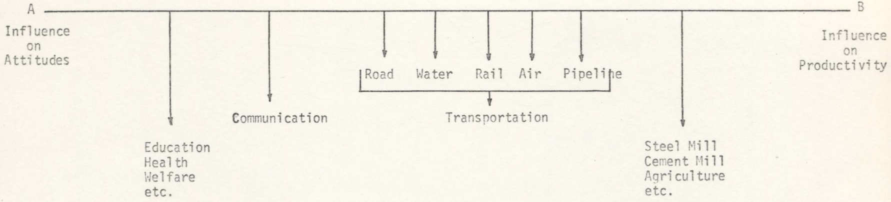

Figure 3.1 Investment Options and Attitudes of a Regional Government. (1)

3.1.3 Objectives of Transportation

A statement of Newfoundland's transportation needs require that terms of reference be established within which the existing system can be examined. Since transportation must contribute to regional goals, thus, the terms of reference should be related to the regional goals. Pearson (14) suggests some appropriate transportation objectives:

1. The transportation system must provide the opportunity to stimulate development. This objective within iteslf includes -

    a) The provision of access to resources.

    b) The provision of access to markets.

2. The transportation system should provide the private sector with the opportunity to achieve economies of location in relation to employment, resources and markets.

3. The transportation system should provide people with the ability to carry out social activities at different locations.

4. The transportation system should provide people with the ability to maximize their opportunities for employment.

5. A reasonable level of service in terms of the transport-

ation system should be established.

6. Investment in the transportation system should maximize returns.

It is felt that these objectives should be pursued actively to achieve an efficient and integrated transportation system. The air network should be considered as an important component in the total transportation system and, as such, should attempt to be complimentary to the other modes, and not merely supplementary, especially in the developmental situation that exists.

Newfoundland's unique situation in that it is an island located approximately 1000 miles east of the economic heart of Canada, places the airplane in a very favourable position to compliment the road, rail and sea networks. Because of the goal of increasing the economic activity of Newfoundland and the impossibility of good road and rail connections, and time consuming sea connections to the Mainland, it is highly desirable that good air transportation links between the main centres of the Island and the Mainland exist. To obtain quality of service on these air links, the demand for air services must be sufficient to warrant the use of high performance aircraft operating at high service frequencies (21). This requires the concentration of air service demand as much as is possible.

3.1.4 The Air Networks Role in Development

In a region such as Newfoundland, where the economy is based on the extraction of raw materials and the population distributed so sparsely, the role of the air network is relatively clear:

1. To provide adequate and reliable passenger transportation to and from the major economic centres of Canada.
2. To provide for transportation that is essential to the maintenance of administrative or health services in isolated communities where other transportation is infrequent, unreliable or non-existent.
3. To provide for the accessibility of a regions resources, made available by airport location, that is expected to stimulate a chain of development with either regional or national significance.
4. To provide the necessary cargo space to obtain rapid delivery of perishable foodstuffs and machine parts that are not available in the region.

Same day return jet service is available from St. John's to Halifax, Montreal and Toronto. Fig. 2.1 indicates the numerous airports located in the Province and demonstrates the accessibility of almost all portions of the region. Air cargo space is readily available for both intra and inter-provincial flights and, in fact, is under utilized (5).

Thus, all four of these primary roles are, at present, being adequately fulfilled in Newfoundland. The only possible exception being No. 2, as there are still a few small outports which cannot be reached at all during a part of the year. It is felt to be unnecessary to provide airports in these areas for such a limited use. It would be much more economical to have available, one or two helicopters, which could be used only for emergency medical services to these areas.

Once these primary developmental air network roles are fulfilled, the airport network is immediately faced with another problem - that is underutilization. Once the capital investment has been made to provide an airport, the objectives may change slightly. As a region's economy develops, the air travel demand may shift from one area to another within the region. When this occurs, the investment objective of maximizing economic returns should still apply and it may become necessary to abandon some airports and build up others which were previously not used very much. If an airport becomes obsolete in either a technical sense (i. e. outdated equipment, physical deterioration of runways, etc.) or a demand sense (lack of demand of passengers, cargo or aircraft), then its operation should be terminated. An airport should not be operated as a welfare institution. However, if the community still requires air service to fulfill role two (above), investment in a sea plane base and heliport would probably be more economical than continued maintenance of runways. The savings made from decreased operation and maintenance costs could be invested in another fashion which would be more beneficial to the area.

In any existing airport network there is always an efficiency objective of maximizing economic returns, which generally means to maximize efficient use of existing facilities. Where underutilization occurs, investment is generally required, not in the transportation sector, but in the social and productive sectors of a region. Some Government subsidy changes may make air transportation more competitive, but it is felt that it would not make a significant difference in utilization.

Some examples of investment in productive activity which would increase utilization are:

1. Investment in fresh fish and lobster pounds and depots which would have large storage capacity enabling a constant supply to Montreal, Toronto, Boston, etc.
2. Investment in advertising and promotion of Newfoundland's fish products which currently are only being marketed locally, i. e. cod tongues, smoked caplin, etc.
3. Investment in tourist areas presently being exploited only minimally, i. e. Bonne Bay (scenery, fishing, camping, etc. in summer and skiing in winter), tuna fishing in

Notre Dame Bay and Conception Bay, etc.

4. Investment in advertising and promotion of these tourist areas (see 3 above).

5. Investment in production and promotion of films of Newfoundland's culture and history, i. e. Farley Mowat's writings.

It is felt that investment in the above areas would have relatively rapid beneficial effects on transportation utilization and on economic activity.

Some examples of investment into social attitudes which should increase utilization are:

1. Investment into educational facilities which would give Newfoundlanders a broader view of opportunities and potential markets available for their goods and services to people in and outside of the Province.

2. Investment into cultural facilities which would promote and preserve Newfoundland's culture, increase Newfoundlanders' appreciation of their own culture and induce "cultural exchange" with other regions.

Investment into the above areas would not show any immediate results. However, it is felt that it is necessary in order to create a long range attitude towards social communication which should be a desirable aspect of Newfoundland's development process.

Once the decision is made to invest in an area to promote development, there is the problem of determining what facilities are needed and how much capital is required. This requires that one can determine the consequences of the investment on the developmental process. The following section outlined a framework for planning and evaluating airport investments in Newfoundland.

3.2 THE FRAMEWORK

Figure 3.2 attempts to structure the regional airport investment planning process into a logical framework of interconnected analysis.

3.2.1 Airport Investment Objectives

Before any realistic airport location planning can be done, one must identify exactly which objectives have the highest priority. The following, in order of importance, represent the objectives to be pursued in planning the air network for Newfoundland (according to the author):

1. Economic Development
2. Economic Efficiency
3. Social Community
4. Level of Service Objectives

These have all been discussed in detail in Chapter 2. It is

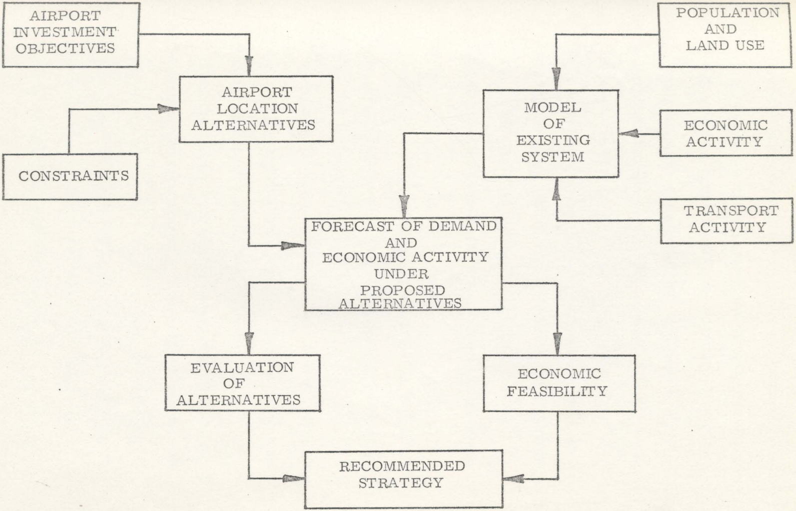

FIG. 3.2 A FRAMEWORK FOR PLANNING AND EVALUATING REGIONAL AIRPORT LOCATION ALTERNATIVES

felt that any future investment for economic development, as discussed in section 2. 3. 2 would be made into an area that at present has little or no population. Its purpose would be to give access to a natural resource for the exploitation and development by private enterprise. For example, some mining or hydro power development like Wabush or Churchill Falls. It is felt that the airport investment should be borne solely by the private enterprise seeking the resource. If the project is economically feasible only through Government subsidy for an airport, it is felt that the investment capital required could give a better return in another developmental project.

Level of service objectives, as discussed in section 2. 3. 4, should be given a high priority only in a relatively well developed economy.

Social community objectives are important in a developing region and are at present being reasonably fulfilled. For further discussion of this topic, see sections 2. 3. 3 and 3. 1. 4.

Therefore, the basic objective to be pursued in the present Newfoundland situation is number 2 - Economic Efficiency. In more detail, this means to minimize the sum of the capital and operating costs of a regional airport system and the ground transportation costs of passengers, consistent with satisfactory ground access times of existing and potential users.

3.2.2 Constraints

The planning and evaluating of a system of regional airport locations, with respect to the economic efficiency objective, requires two distinct phases:

1. The determination that an urban area should or should not have an airport capable of handling high performance aircraft.
2. If the above is affirmative, then there is the problem of optimum location with respect to the city.

Pearson (26) derives the traffic consequences of urban airport location alternatives. The constraints to be considered in this second phase are -

1. Capital
2. Land Use
3. Existing Airports
4. Airspace Utilization
5. Climate
6. Noise Limitations

The problem considered in this work is that of the first phase. The constraints to be considered are closely associated with the above; the primary constraint being available capital. In order to achieve rapid economic development, one has to plan the use of available capital in such a way as to maximize the rate of increase of per capita production and income. It is unlikely that further investment in new airport locations will make optimum use of the minimal available capital for development. However, there are many political and social problems involved because airports, of the type considered here, are a Federal responsibility. Therefore, money destined for airport expansion cannot be taken and reallocated Provincially.

Basically, the constraints involved for this study are the following:

1. Capital
2. Political
3. Physical
4. Existing Airports
5. Climate

They are listed in order of importance. In the near future an airport located in a relatively foggy area will be equipped with all weather landing devices. Existing airport investment is actually a book value and should not play too large a role in future investment. The physical constraint refers to existing topographical and soil conditions.

3.2.3 Review of Existing Data

Population

The population of Newfoundland is approximately 495,000.

Of these, 465,000 live on the Island of Newfoundland and 30,000 live in Labrador (22).

The population distribution is summarized in Figure 3.3.

The enclosed areas on the overlay represent the most concentrated areas of population and labour force. The remainder of the Island's population is generally sparsely distributed along the coastline. Table 3.1 summarizes the population and employment data into six regions which are served by regularly scheduled air services. Although St. Anthony is served by bi-weekly flights, it has been omitted here for three reasons:

1. Less than 300 passenger trips were recorded between June 1969 and June 1970 (24).
2. The airport consists of a simple dirt strip which is closed for long periods during the freeze - thaw time of year.
3. The airport's function was considered to be more of the social community type, as discussed in section 2.3.3, and therefore not subject to the same economic relationships used for modelling in the other regions of the Province.

It may be noted that the total population in Table 3.1 is less than that indicated above. The reason being that there are a number of areas that do not have access to a regularly serviced airport, i.e. Burgeo, Harbour Breton, the east coast of the Northern Peninsula,

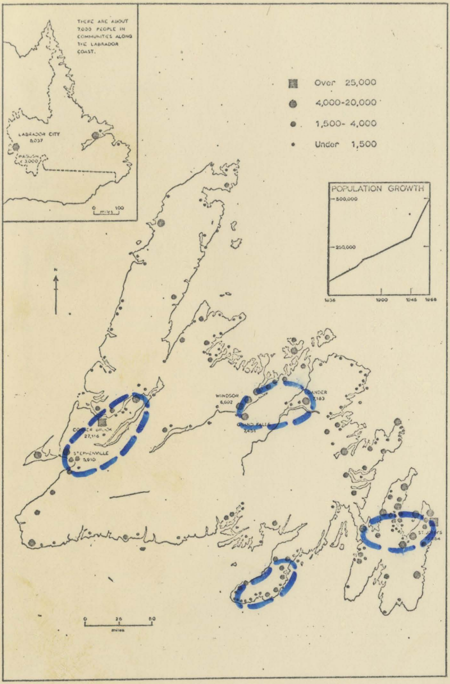

Figure 3.3 Population Distribution (16)

TABLE 3.1

SUMMARY OF POPULATION & EMPLOYMENT FIGURES IN NEWFOUNDLAND (22)

REGION	CODE	POPULATION	SERVICE INDUSTRY	MANUFACTURING INDUSTRY	MINING & FORESTRY
St. John's	004	247,337	58,104	3,626	3,441
Gander	005	94,900	15,280	2,433	3,215
Corner Brook	006	96,580	17,360	3,234	2,268
Goose Bay	007	8,300	1,950	10	230
Churchill Falls	008	3,200	2,500	--	---
Wabush	009	11,500	2,700	14	320
Total		462,817	97,894	9,317	9,474
Average		67,732	14,197	1,339	1,405

the coast of Labrador, etc. However, these areas account for only 6% of the Province's population.

The trend in Newfoundland in the past few years has been towards centralization and consolidation. It is anticipated that any significant future expansion of the population will probably be in the areas indicated by the overlay in Figure 3. 3. Thus, the relative importance of these areas will increase. An area of the Province that may become a major growth area is around Marystown on the Burin Peninsula. The population now on the Southern Burin is about 25, 000. However, once the Burin Highway is paved, this area will have, not good accessibility to Torbay, but at least it will be accessible. There may be enough demand in this area for weekly or bi-weekly flights. This would require minimal investment as there already is an airstrip at Frenchmans Cove.

It is believed that any significant increase in passenger air travel in Newfoundland will be the result of:

1. An increase of population concentration.
2. An increase of productive economic activity.

It is further believed that any significant increase in population concentration will be the result of an increase in economic development.

Land Use

When one considers that there are about 156,000 square miles in the Province of Newfoundland (43,000 on the Island and 113,000 on Labrador), it may be more appropriate to talk about the non use of land. However, unused land will not generate a demand for air traffic.

Any significant increase or decrease in agricultural activity will not affect the demand for air travel significantly. The same would also be true of the inshore fishery and of the existing tourist land use, i. e. provincial parks.

Since almost all air passengers originate and terminate in urban areas (3, 23) it would seem that a significant change in air travel demand would accompany an increase in urban concentration and urban land use. Therefore, the centralization trend would be beneficial to the air transportation systems utilization.

Economic Activity

Newfoundland's economy is dependent upon two basic industries: the primary industries and the service industries. The primary industries consist of mining, fishing and forestry. Of these the mining industry's net value of production is more than the others combined (Table 3.2). Table 4.1 indicates that the number of annual business trips taken by those employed in the mining industry is about 1.3 times that the number of employees. As can be seen in Table 4.2, this is a rather high ratio. Thus, an increase in the mining industry's operation in Newfoundland would have a beneficial

TABLE 3.2

NET VALUE OF PRODUCTION AND EMPLOYMENT

IN NEWFOUNDLAND (ESTIMATE 1965) (14)

	Industry		Employment Number	Employment Percent		Net Value of Production ($000)	Net Value of Production Percent
1.	Mining		5,000	4.5		101,563	17.0
2.	Forestry	Total	6,700	6.0		58,826	9.7
		Primary	3,700	3.5		21,060	3.5
		Processing	3,000	2.5		37,766	6.2
3.	Fishing	Total	22,075	19.5		33,585	5.6
		Primary	18,776	16.5		22,468	3.6
		Processing	3,300	3.0		11,117	2.0
4.	Agriculture		1,850	2.0		3,800	0.6
5.	Manufacturing		3,600	3.0		28,000	4.7
6.	Construction		7,700	7.0		84,991	14.1
7.	Transportation, Communication		13,900	12.5			
8.	Wholesale, Retail Trade		19,200	17.0	57.5 (items 7–11)	281,153	47.0
9.	Finance		1,500	1.0			
10.	Government Serv.		11,300	10.0			
11.	Professional Serv.		19,000	17.0			

effect on the air travel demand. Also indicated in Table 4. 2 is that the forestry and fishery service industries have a trip to employee ratio of 1. 17; thus, an increase in activity in these sectors would have a beneficial effect on air travel demand. However, the fishery services indicated by this Table would not refer to the inshore fishery as it exists here in Newfoundland. A shift in the fishery to large scale trawlers and fish plants would increase economic productivity and produce more jobs that would require business travel. The success of Ocean Fisheries (Trepasery), Hants Harbour and Harbour Breton bear out this fact.

The service industries occupy a disproportionate amount of the labour force - 76%. An increase in this percentage is undesirable. It is felt that as much encouragement as is possible should be given to people to make a shift to the secondary industries, as it is here that the fastest rate of capital formation occurs (15). Some possibilities may be in the manufacturing of childrens hockey sticks, wooden and paper matches, sheep farming, wool, and knitting mills, etc. Encouragement should also be given to companies to do as much of their engineering and architectural work as is possible within Newfoundland. A high rate of capital formation is associated with these services as is a high rate of annual business trips. Both of these facts would be beneficial to the air network.

It is also anticipated that the proposed oil refinery at Come-by-Chance will generate a significant increase in air travel demand. (See Table 4.2)

At present, the tourist industry does not contribute significantly to the air network's utilization. However, some potential areas for further exploitation would be:

1. The tuna fishing in Notre Dame Bay and Conception Bay, is one of the best areas in the world. (This is not a well known fact at present.)
2. In the winter months, Bonne Bay could have some of the finest skiing east of the Rockies. (This is also a little known fact.)

Further study along these lines is required to determine the exact economic potential of these areas. Promotion of the Bonne Bay skiing potential would increase the air networks utilization during the slack winter months.

Existing Transportation System

The major components of the transportation system in Newfoundland are the water, rail, road and air networks.

The water transport network's basic function is to supply the Island with bulk commodities (foodstuffs, automobiles, hardware, etc.). Whether or not the water transport system is operating at high or low efficiency will not affect the air network, as the airplane cannot compete economically with the boat for bulk commodities. The airplane can complement this bulk commodity service by supplying perishable foodstuffs to the Island and fresh lobster and salmon to the Mainland. At present, this is not being done as well as it might be.

The rail network ceased all passenger operations in the summer of 1969 and is now only distributing and collecting bulk goods to and from the Island's interior. It is felt that even this function will decrease in importance as the road system improves and with the implementation of automated container handling at the seaports for truck distribution. On this basis, the airplane cannot compete with the train either.

The only major area on the Island that still lacks highway access is the South Coast between Rose Blanche and Bay d'Espoir. The majority of the Island's population lives near the Trans Canada Highway or has excellent access to it. This means that the highest concentration of people (St. John's, Gander, Grand Falls and Corner Brook) have excellent road connections.

For passenger trips of less than 300 miles, the road offers a high degree of competition with the airplane (1, 3). Therefore, the private automobile and the bus will divert much of the travel demand from the airplane on the St. John's to Gander - Grand Falls area trip (200 - 260 miles), and the Gander - Grand Falls area to Corner Brook area (160 - 200 miles). For the St. John's to Corner

Brook trip (425 miles), the airplane offers the best mode of passenger transport. However, the fact that the Deer Lake airport is over 30 miles from Corner Brook detracts from the advantages of air travel. The Stephenville airport is over 60 miles from Corner Brook and therefore even less attractive. It is felt that efficient same day return service from St. John's to Corner Brook, with good limousine service at Deer Lake, would attract more demand for air travel. Efficient car rental service at all airports would further enhance air travel, especially for business trips.

When the Burin road is paved, the Marystown area will be less than 200 road miles from both St. John's and Gander. Thus, with this paving, the major areas of the Island will have excellent road transportation and fairly good airport accessibility.

Labrador is in a different transport position. There are virtually no roads or reasonable sea transportation. The railroad to Wabush and Schefferville supplies bulk goods and withdraws iron ore. Almost all of Goose Bay's vital supplies are brought in by air (24). Therefore, all passenger transportation in Labrador (excluding snowmobile) is done by air. This situation will continue at least into the foreseeable future (22).

The airway network is illustrated in Figure 3.4. Air Canada presently supplies trunk service between Gander and London, St. John's and Halifax and St. John's and Montreal. Regional carrier

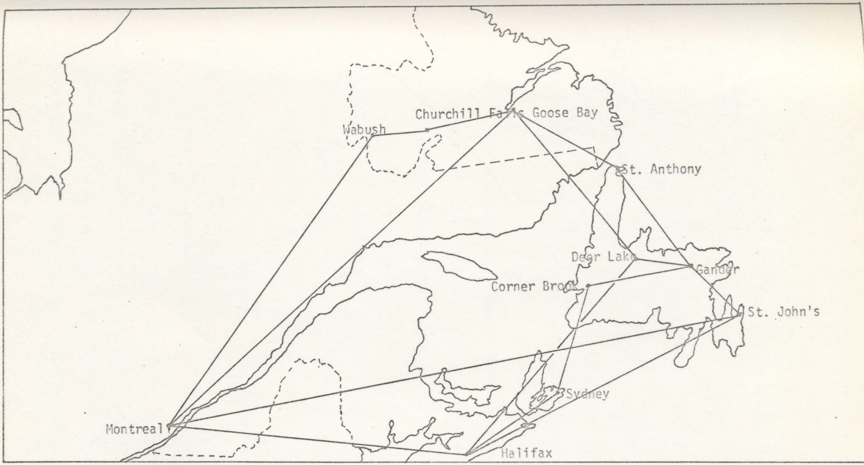

Figure 3.4 Newfoundland Airway Network

service is supplied by Air Canada, Eastern Provincial Airways and Quebecair. These airlines connect St. John's, Gander, Deer Lake, Stephenville, Goose Bay, Churchill Falls, Wabush and, bi-weekly, St. Anthony.

It is anticipated that the future will see Air Canada supplying only trunk services and E. P. A. taking over all regional services in Newfoundland. This would mean a more integrated air service with less duplication of services and thus, a more economical service. However, this may also mean that only one airport would serve the Western Shore of Newfoundland. Any decisions regarding this possibility will require a detailed study of the alternatives available and will no doubt cause numerous political problems.

The origin-destination statistics for the year 1969 are indicated in Table 3. 3. This information is the summation of data collected from Air Canada, E. P. A. and extrapolations from the Air Transport Board Statistics, D. O. T. , Ottawa, 1967.

3. 2. 4 Airport Location Alternatives

The planner, having considered the existing data, objectives and constraints, must then propose a system of airport locations that he feels would result in a more efficient system. That is, more efficient in satisfying the objectives than does the present system. In order to compare alternate systems, the present system should

		001	002	003	004	005	006	007	008	009	TOTAL
Montreal	001	——	——	——	28,978	6,335	6,212	3,367	3,668	7,839	56,399
Halifax	002	——	——	——	22,966	5,605	4,453	686	4	9	33,723
Sydney	003	——	——	——	3,220	1,740	840	40	--	30	5,970
St. John's	004	28,978	22,966	3,220	--	11,629	10,772	3,714	5,547	3,815	90,642
Corner Brook	005	6,335	5,605	1,740	11,629	--	1,466	1,056	1,705	1,451	30,987
Gander	006	6,212	4,453	840	10,772	1,466	--	1,408	2,075	1,891	29,045
Goose Bay	007	3,367	686	40	3,715	1,056	1,408	--	656	781	11,709
Churchill Falls	008	3,668	4	--	5,547	1,705	2,075	656	--	253	14,908
Wabush	009	7,839	9	30	3,815	1,451	1,891	781	253	--	16,069

Table 3.3 1969 Origin-Destination Statistics (Figures indicate number of round trips between cities) (5,24,25)

be modelled, tested and evaluated. This will be discussed more fully in the next chapter.

3.2.5 Model of Existing System

The existing system must be modelled in such a way as to describe mathematically the relationships between the economic and social environment and the resulting air travel demand. This will be further detailed and developed in the following chapter.

3.2.6 Forecast

Forecasts of the expected natural increase in air travel demand using the present system must be made so that meaningful comparisons of alternate proposals can be determined. Forecasts due to induced business and changes in system components will also be discussed in the next chapter.

3.2.7 Evaluation of Alternatives

A step-by-step comparison of all relevant variables for each alternative must be made. A consensus must be arrived at among the planning and decision group as to the relative importance of each variable considered. A decision matrix can then be set up which would rank the alternatives in numerical order. This numerical ordering, combined with the net present value of each alternative (section 3.2.8) would aid the decision group to select the best solution from among many alternatives.

3.2.8 Economic Feasibility

Once an acceptable alternative is determined various costs and benefits must be assessed for each system. There are a number of methods to determine the economic feasibility of engineering projects, however Whol (13) and Pearson (1) suggest that the net present value method is the most meaningful. Costs germane to this method include:

1. Overall construction costs including support facilities such as new connecting roadways.
2. Operating costs including salaries, overhead and maintenance for the planned life of the project.
3. Operating revenues for the life of the project.

Benefits germane to the framework include:

1. Decreased cost and access time to airports.
2. Decreased airplane operating costs.
3. Increase in economic return on investment.
4. Increased revenues from demand due to optimal location of airport system.

The above costs and benefits should be considered using an appropriate interest rate. Whol (13) has suggested that the interest rate is easily incorporated by calculating the net present value. This reduces all future monies to present day terms. The net present value is obtained from:

$$NPV\ (c) = \sum_{o}^{t} \frac{Ck}{(1 + i)^{k}} - \sum_{o}^{t} \frac{Rk}{(1 + i)^{k}} \qquad (3.1)$$

where:

t	= number of years of the project (design life)
NPV (c)	= net present value of costs
c	= total costs occurring in year K
i	= interest rate
R	= revenues occurring in year K

3.2.9 Recommend Strategy

Having completed the planning framework a decision must be made as to which alternative is to be implemented. A detailed planning and design strategy can then be drawn up.

The next chapter develops the modelling techniques to be used and demonstrates its applicability. It must be remembered, however, that this thesis only outlines the planning framework and indicates how it can be applied. It does not profess to be a complete engineering and economic report on the Newfoundland Airport System, but only indicates a logical methodology for such a report.

3.3 SUMMARY

1. The Newfoundland situation was discussed in general terms with emphasis on its economic problems. Regional goals were presented with specific reference to transportation objectives.
2. The air networks role in economic and social development indicated that investment may be required in sectors other than the transportation sector. Some examples were given.
3. A framework for planning and evaluating regional airport location alternatives was illustrated and shown to be a logical step-by-step framework of interconnected analysis.
4. Each step of the framework was briefly discussed to expand on the type of work involved in each step. The linear graph modelling techniques was left to the next chapter for discussion.

CHAPTER 4

APPLICATION OF LINEAR GRAPH THEORY

4.1 DEMAND MODELLING

The modelling of travel demands requires that statistically reliable relationships are developed between aggregates of individuals and their trip making characteristics. The basic agent is the individual whose preferences towards transport modes and his concept or perception of any given mode is constantly changing. Further, since travel potential attributes (1) may not be measurable, it is difficult to derive an individual's economic demand curve. Therefore, it is highly unlikely that perfectly reliable predictors of travel demand could be made.

At the aggregate level, many variables have been identified which characterize certain segments of the travel market (1). The travel characteristic variables describe either socio-economic travel components or physical measurements of the travel network. The former include income, occupation, perception of transport system variables and perception of destination attraction. The latter include travel cost, travel time, frequency of departure and capacity of supply.

The manner in which the components are postulated to interact describes the travel demand model. The relationship of the system's components to travel demand is as follows:

1. The total travel potential is given by the willingness to exchange time and money to make a trip to any destination on the travel network.
2. The economic demand is given as level of demand which is the interaction of the willingness to exchange money for the perceived system parameters (which are equal to the user travel benefits).
3. The equilibrium demand is given by the modal distribution on the system to all destinations. This is the interaction of the economic level of demand with the total 'price' of travel.

The above simulation technique may best be described as an interdependent system of socio-economic and physical components. The equilibrium demand depends not only on the economic demand but also on the travel potential. All of the variables which describe the three demand components are dependent on each other.

One technique to model total systems is linear graph analysis (1, 33). Linear graph analysis requires individual components to be modelled separately in terms of complimentary pressure and flow variables. The imposition of their interconnection pattern then yields a model for the entire system. The procedure is analytic in form and theory and provides a consistent and rigorous approach for modelling systems. Further, these techniques have been applied to socio-economic systems including traffic networks (1, 27, 28).

Linear graph analysis is used in this thesis to develop a set of linear equations which characterize the flows on every link (origin, physical network or destination) of the Newfoundland air network by the individual and system interdependent components of travel demand. Since every link is mathematically described, the equilibrium demand components of generation, distribution, modal distribution and assignment are completed simultaneously.

In order to construct the travel demand model, each system component is described as a terminal graph. A set of terminal graphs connected at the vertices to form a one-to-one correspondence with the physical system is called a systems graph. It is then necessary to derive both the chord and branch formulation equations of the system. Both the formulation methods can be found in References 1 and 33. Brief outlines of both formulations are shown in Appendix A.

The chord formulation requires that link resistances, destination city attractions and known origin area flows be measured. Solution of the chord equations, cut set equations and terminal equations yields the unknown link flows and pressures.

Branch fomulations models can be constructed using the derived origin area pressures associated with origin flows, which will quantify the generation characteristics associated with changes in the system. The pressure associated with the origin area is that which is 'used up' in travel (i. e. it sums to zero around a circuit). It is a function of the origin area flow and the equivalent resistance of the system.

Linear graph analysis requires that the following requirements be met:

1. The individual system components must be quantitatively describable by two fundamental variables. These variables are a "y" or flow variable, and a complimentary "x", or pressure variable which causes flow.
2. The components are connected at their ends (vertices) to yield a model for the entire system. The interconnected model must satisfy the two generalized Kirchoff laws. The first law states that the algebraic sum of all flows (y) at a vertex is zero. The second law states that the algebraic sum of all pressures around any closed loop of the system must be zero.
3. The flow and pressure variables must be related by a linear or non-linear function.

The "y" variable for the intercity travel network is person trips per year. This satisfies the first Kirchoff law while eliminating the necessity of modelling for storage within the system. That is, all air travellers are assumed to return to their origin over the yearly period.

The "x" variable is postulated to be a value measure used by the travellers in making a trip and a choice of mode. It is analogous to the portion of the travel potential of an origin area which is used up as a trip is made, and thus is that pressure which caused flow. The "x" variable is not a measure of the total perceived value of making trips, but rather the measurable perceived total cost of making the trip.

The reasoning for the above postulates is as follows:

1. If it is believed that the making of a trip and the choice of mode can be simulated with a reasonable degree of accuracy, then it follows that there is some underlying process made by the traveller in making such a choice.
2. The traveller will act as a free agent and attempt to optimize his degree of satisfaction.
3. Relating points 1. and 2. above to a value measurement used in travel allows the origin pressure to dissipate as the trip is made, thus satisfying the second Kirchoff law.

The application of the second Kirchoff postulate to traffic networks as described in this thesis requires an assumption which is:

The perceived cost (pressure) of a trip from any particular origin to all destinations is a constant.

The reason for this assumption are:

1. Each origin is modelled with its own unique travel pressure.
2. The origin and travel links form a closed loop with each destination.

The computer program developed by Pearson (1) and reproduced in Appendix B will be used to solve the system of equations involved in the air network systems graph for Newfoundland (Figure 4.1).

4.2 SYSTEM IDENTIFICATION

The system examined in this model is shown in Figure 4.1. It includes six areas of the province and three external Mainland connections:

001 Montreal

002 Halifax

003 Sydney

004 St. John's

005 Corner Brook

006 Gander

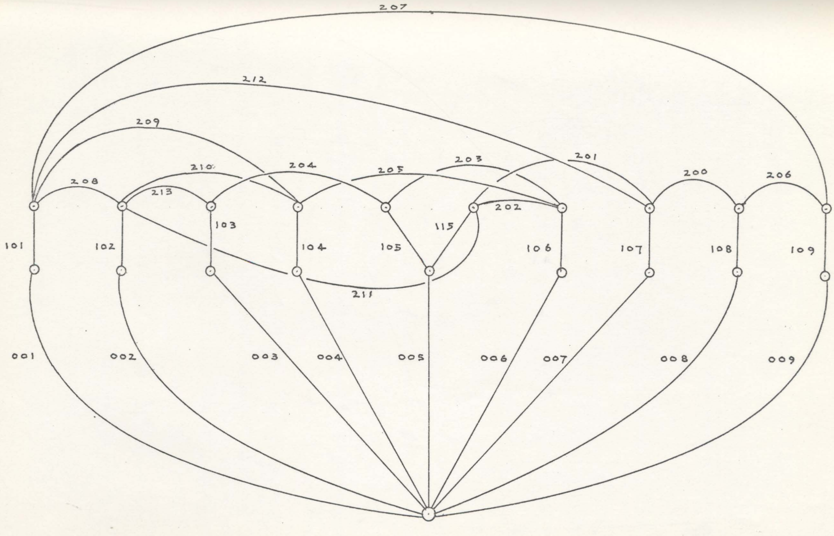

Figure 4.1 Systems Graph for the Newfoundland Airway System

007 Goose Bay

008 Churchill Falls

009 Wabush

Although St. Anthony is served by bi-weekly flights, it has been omitted here for three reasons:

1. Less than 300 passenger trips were recorded between June 1969 and June 1970 (24).
2. The airport consists of a single dirt strip which is closed for long periods during the freeze - thaw time of year.
3. The airport's function was considered to be more of the social community type as discussed in section 2. 3. 3 and therefore not subject to the same economic relationships used for modelling in the other regions of the province.

The components of the system include origin cities, destination cities and all non-stop airway links between the origin and destination cities. The origin and destination cities are shown as links 001 to 009 in Figure 4. 1. The origin or destination 'city' included all major centres of population served in the region. For example, link 006 indicates the city of Gander, however, this also includes Lewisporte, Grand Falls, Windsor, Bishop's Falls, etc.

Links 001 to 109 and link 115 refer to the airports serving a particular region and is a measure of the airport access and egress costs and times. These links also include an empirically derived constant which reflects the trade-offs made by travellers in choosing a mode of travel for distances under 300 miles.

Links 200 to 213 refer to the non-stop air routes between origin and destination cities. Included in these links is a measure of a traveller's resistance to travel, measured in terms of travel time, travel cost and frequency of departure.

4.3 MEASUREMENT OF ATTRACTIONS

A study by Air Canada (29) provided partial information on the trip making characteristics of various employment types. The survey included business trips by all modes over 250 miles in length (one way). The results of the Air Canada study are summarized in appropriate categories for this study in Tables 4.1 and 4.2. The categories are: service industries, manufacturing industries and mining and forestry industries. It was felt that due to the nature of the inshore fishery and the low scale farming in Newfoundland, that these industries would not contribute significantly to passenger air travel demand and therefore were left out of this classification. In order to make use of this data, the following assumptions are made:

1. Industries not included in the survey would, on the average, exhibit similar trip making characteristics when they were classified into the three broad industrial groups.

TABLE 4.1

SUMMARY OF TRIP MAKING CHARACTERISTICS OF VARIOUS INDUSTRIAL TYPES (29)

CODE	INDUSTRY	ANNUAL BUSINESS TRIPS	TOTAL EMPLOYEES
301	General Building Contractors	3,000	4,648
302	Business Forms	92	325
303	Trucking	337	1,737
304	Water Transport	1,133	1,582
305	Communications & Newspapers	1,688	1,830
306	Groceries	348	1,632
307	Beverages & Tobacco	984	1,125
308	Department Stores	1,207	2,036
309	Personal Credit Institutes	292	497
310	Mortgage Companies	335	1,638
311	Security Brokers	428	461
312	Life Insurance Companies	350	807
313	Business Services	830	816
314	Legal Services	307	595
315	Labour Unions	1,036	297
316	Engineering & Architectural Services	2,983	1,302
317	Accounting	674	1,040
318	Forestry & Fishery Services	445	381
319	Mining	3,555	2,740
320	Petroleum & Natural Gas	3,015	2,272
321	Paper & Paper Products	440	1,101
322	Lumber	332	2,059
323	Metal Industries	1,276	2,529
324	Ship Building	64	1,220
325	Electrical Supplies	705	962
326	Machine Supplies	1,944	2,733

TABLE 4.2

INDUSTRIAL CLASSIFICATION & AVERAGE ANNUAL TRIP MAKING CHARACTERISTICS (29)

SERVICE				MINING & FORESTRY				MANUFACTURING			
CODE	TRIPS	EMPLOYEES	TRIPS / EMPLOYEES	CODE	TRIPS	EMPLOYEES	TRIPS / EMPLOYEES	CODE	TRIPS	EMPLOYEES	TRIPS / EMPLOYEES
301	3,000	4,648	.65	318	445	381	1.17	323	1,276	2,529	.50
302	92	325	.28	319	3,555	2,740	1.30	324	64	1,220	.05
303	337	1,737	.19	320	3,015	2,272	1.33	325	705	962	.73
304	1,133	1,582	.71	321	440	1,101	.40	326	1,944	2,733	.72
305	1,688	1,830	.92	322	332	2,059	.16				
306	348	1,632	.21								
307	984	1,125	.87								
308	1,207	2,036	.59								
309	292	497	.59								
310	335	1,638	.20								
311	428	461	.93								
312	350	807	.43								
313	830	816	1.02								
314	307	595	.52								
315	1,036	297	3.50								
316	2,983	1,302	2.29								
317	674	1,040	.65								
	16,023	22,368	13.84		7,777	8,553	4.36		3,989	7,444	2.00
$\frac{\text{Trips}}{\text{Empl.}}$ = .72		Ave. $\frac{\text{(Trips)}}{\text{(Empl.)}}$ = .82		$\frac{\text{Trips}}{\text{Empl.}}$ = .91		Ave. $\frac{\text{(Trips)}}{\text{(Empl.)}}$ = .87		$\frac{\text{Trips}}{\text{Empl.}}$ = .54		Ave. $\frac{\text{(Trips)}}{\text{(Empl.)}}$ = .50	
Average of Above Row = .77				= .89				= .52			

2. The trip making characteristics would reflect the attraction of the industry group.

3. The choice of mode does not depend on the attraction of a region but only on the trip length and the accessibility.

Since both business and pleasure trips are included in this study, it is felt to be necessary to include a term for the population of a region as well. Thus, it is postulated that the attraction of a region in relative terms is given by:

$$Ak = \phi k \left(B_p \frac{Pk}{P_{k,\ ave}} + B_s \frac{Sk}{S_{k,\ ave}} + B_m \frac{Mk}{M_{k,\ ave}} + B_{Man} \frac{Man_k}{Man_{k,\ ave}}\right) \quad 4.3.$$

where:

A_k	=	the relative attraction of a region
ϕ_k	=	a calibration constant
B_p' B_s' B_m' B_{man}	=	trip attraction characteristics of each classification which are derived below
P_k' S_k' M_k' Man_k	=	the characteristics of each region, respectfully, the population, and the employees in the service, mining and forestry and manufacturing industries
Pk_{ave}' Sk_{ave}' Mk_{ave}' $Mank_{ave}$	=	the characteristics of an average city in the network

The B coefficients were derived partly from the Air Canada data shown in Table 4.1. Table 4.2 shows a breakdown of the employment types into mining and forestry, service and manufacturing industries. The average number of trips per employee for each of the three industrial types was calculated and is shown in Table 4. 2 Also calculated was the ratio of trips to employees for each employment type within the industrial classification. These ratios were also averaged and then averaged again with the total average. For example, in the service industry, there are a total of 16, 023 annual trips and 22, 368 employees.

$$\frac{16,023}{22,368} = .72$$

The sum of the individual employment type ratios is 13. 84. Thus, the average is

$$\frac{13.84}{17} = .82$$

Thus, the final average is

$$\frac{.72 + .82}{2} = .77$$

Similarly, the final average of the mining and forestry industry = . 89 and the manufacturing industry = . 52. Since this study is also considering pleasure trips, it was decided to add a term in the attraction function that was related to the population in the region. A number of studies (23, 30) indicate the approximate percentage of air pleasure trips in Canada. Considering these figures and also the fact that Newfoundland is an island, it was felt that a 25% figure would be appropriate. Thus, 75% is made up of business trips relating to the three industrial types.

Thus:

$$B_p = .25$$

$$B_s = .75 \left(\frac{.77}{.77 + .89 + .52} \right)$$

$$= .28$$

$$B_m = .75 \left(\frac{.89}{2.18} \right)$$

$$= .30$$

$$B_{man} = .75 \left(\frac{.52}{2.18} \right)$$

$$= .17$$

Therefore, the attraction function is -

$$A_k = \phi \left(.25 \frac{P}{P,_{ave}} + .30 \frac{M}{M,_{ave}} + .28 \frac{S}{S,_{ave}} + .17 \frac{Man}{Man,_{ave}} \right) \quad 4.3.1$$

Table 3.1 summarizes the population, service industry employment, manufacturing industry employment, and the mining and forestry industry employment within each region in Newfoundland being considered.

The city of St. John's is shown below as a sample calculation. (Initially ϕ is assumed = 1.0).

$$A_k = \left(.25 \times \frac{247,337}{67,732} + .30 \times \frac{3,441}{9,305} + .28 \frac{58,104}{14,197} + .17 \frac{3,262}{1,339} \right)$$

$$= 3.266$$

The results of this calculation on all regions is summarized in Table 4. 3. In the case of Churchill Falls, ϕ_k was taken as being 3. 0 because it is estimated that the work force changes about 3 times per year.

The attraction figures for Montreal, Halifax and Sydney were selected on a different basis, because they are really external to the system being considered. Initially, Halifax was taken to have the same attraction as St. John's (3. 266). Sydney was assigned a value of 0. 300 arbitrarily. Montreal was assigned a value of 13. 06, or four times the attraction of St. John's.

The regions served by the airports were considered to be those shown in Figure 4. 2. It was assumed that the Montreal attraction included a factor for Toronto, Ottawa and points west. Halifax included Boston and the rest of the Maritimes.

4. 4 MEASUREMENT OF ACCESS LINK RESISTANCES

The airport access and egress link resistances (101 to 110) for air passengers were calculated considering travel costs, travel times, passenger processing times [1] and passenger insurance time [2]. The resistances were calculated from equation 4. 4. 1.

$$R = M \left(\frac{c}{m}\right) + k \left(td + M \left(\frac{t}{m}\right)\right) \qquad (4.4.1)$$

---

1. Process time includes time from arrival at airport to check-in, check-in time, departure weighing time and aircraft loading time. (This is assumed to be 30 minutes.)

TABLE 4.3

SUMMARY OF RELATIVE ATTRACTIONS FOR

REGIONS IN THE NEWFOUNDLAND AIR NETWORK

CODE	AREA	ATTRACTION
001	Montreal	13.06
002	Halifax	3.266
003	Sydney	.300
004	St. John's	3.266
005	Gander	1.643
006	Corner Brook	1.596
007	Goose Bay	.123
008	Churchill Falls	.183
009	Wabush	.167

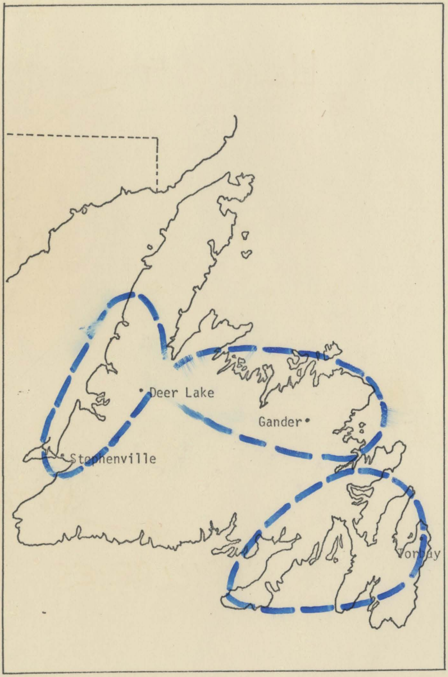

Figure 4.2(a) Areas Regularly Serviced by Scheduled Aircraft in Newfoundland

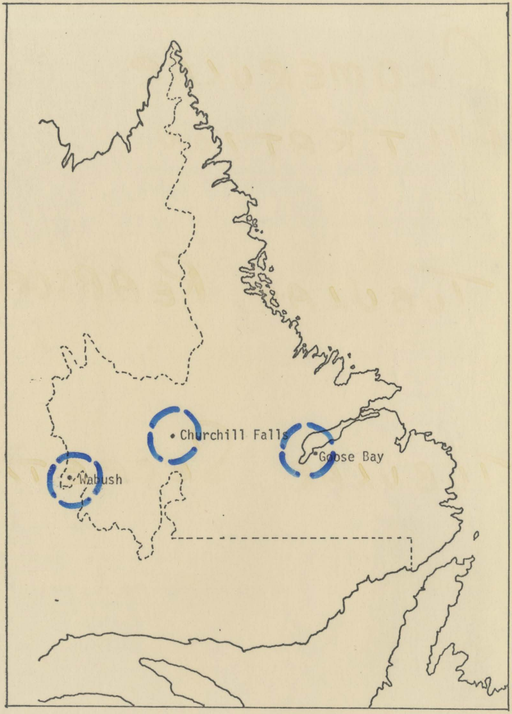

Figure 4.2(b) Areas Regularly Serviced by Scheduled Aircraft in Labrador

2. Insurance time is allocated to account for unseen delays. (This is also assumed to be 30 minutes.)

where:

R	=	Resistance to travel
M	=	No. of miles on link
$\frac{c}{m}$	=	Cost in cents per mile for access vehicle operation
K	=	A constant defining the perceived travel time costs in cents per mile
td	=	delay time at airport (passenger processing & insurance time)
$\frac{t}{m}$	=	Length of time to cross one mile of the airport access link

The choice of the value of K is discussed thoroughly in reference (1). The access travel costs are taken as 12 cents per mile (1).

The resistance calculation for link 104, the airport access and egress for Torbay airport is given below:

The major areas served by the airport were assumed to be: St. John's, Carbonear area and the Burin Peninsula. The first step requires that the distance and time to the airport be weighted by the relative population and employment figures.

Centre	Distance to Airport	Time to Airport	Weighted Time	Weighted Distance
St. John's	8	15	11	6
Burin	201	360	21	14
Carbonear	72	110	29	16
		Total	51	36

Thus, the total weighted access time is 51 minutes and the total weighted access distance is 36 miles. Therefore:

$$R = 36 \times 12 + 10\ (60 + 51)$$

$$= 1542$$

The access and egress resistances are summarized in Table 4.4. For airports within 300 miles of each other, it was necessary to modify the resistances of the airport access links. Of course, this only applies to the areas that are connected by road. Within this trip length range, there is considerable competition between air and road travel. For a detailed discussion of this measure, see reference (1).

4.5 MEASUREMENT OF AIRWAY LINK RESISTANCES

The airway link resistances (links 200 to 213) were calculated considering air fares, route travel times and scheduled departure frequency. The resistances were calculated using equation (4.5.1).

$$R = C + K\ J_m \qquad (4.5.1)$$

where:
- R = resistance to flow
- C = cost in cents to travel on any air link
- K = a constant defining the perceived travel time costs in cents per mile
- J_m = mean journey time (1)

TABLE 4.4

SUMMARY OF RESISTANCE VALUES FOR

ACCESS AND EGRESS

CODE	ACCESS/ EGRESS	ACCESS FOR < 300 MILES	REMARKS
101 Montreal	1160	99999	From Halifax & Sydney
102 Halifax	1410	99999	From Montreal & Sydney
103 Sydney	970	99999	From Montreal & Halifax
104 Torbay	1542	8,000	From Gander
105 Stephenville	1878	20,000	From Gander
115 Deer Lake	1370	2,000	From Gander
106 Gander	1838	20,000 8,000	From Corner Brook and St. John's (resp.)
107 Goose Bay	772		
108 Churchill Falls	740		
109 Wabush	686		

The airway link resistances are summarized in Table 4.5.

A description of how these attractions and resistances are used in the computer program developed by Pearson (1) is given in Appendix B. An outline of both the chord and branch formulations of linear graph theory is given in Appendix A.

4.6 CALIBRATION OF THE MODEL

There are a number of variables in both the attraction equation (eq. 4.3.1) and the resistance equations (eq. 4.4.1) and (eq. 4.5.1) that require calibration. For example, in the attraction equation, ϕ_k was set equal to .001 to avoid numerical problems with matrix inversion. ϕ_k also takes on an empirical value to account for variables not considered in the regression equation (eq. 4.3.1). The term in the attraction equation representing a factor for pleasure trips was empirically derived and requires further calibration. In the resistance equations, k was assumed to equal 10 (representing a time value of $6.00 per hour) (1, 31). The value of ground travel costs was taken as 12 cents per mile (32) and is subject to personal and regional perception of ground travel costs.

It is felt that a more detailed attraction function with more up-to-date statistics on business activities and their distribution within Newfoundland would provide more meaningful and accurate results. Resistance functions could also include factors representing

TABLE 4.5

SUMMARY OF RESISTANCES FOR THE

NEWFOUNDLAND AIRWAY NETWORK

CODE	LINK			RESISTANCE
200	Churchill Falls	-	Goose Bay	9,760
201	Deer Lake	-	Goose Bay	12,040
202	Gander	-	Deer Lake	9,100
203	Gander	-	Stephenville	9,600
204	Stephenville	-	Sydney	4,870
205	Gander	-	St. John's	4,360
206	Wabush	-	Churchill Falls	9,260
207	Montreal	-	Wabush	13,220
208	Halifax	-	Montreal	6,540
209	Montreal	-	St. John's	15,360
210	Halifax	-	St. John's	12,200
211	Halifax	-	Deer Lake	11,500
212	Montreal	-	Goose Bay	23,020
213	Sydney	-	Halifax	4,900

the time of day of departures and same day return capabilities, etc.

The dynamic environment within Newfoundland as a developing region also means that flight routings and schedules have changed rapidly in 1969, which alters the results of the model as well. All of these factors could be considered on further study and would aid in the development of a more comprehensive model. However, the purpose of this study is to demonstrate that this planning and modelling technique is applicable for this type of planning problem.

The actual origin-destination data shown in Table 3. 3 is reproduced here for convenience. Table 4. 6 shows the results of preliminary calibrations of the chord formulation equations. In comparing these results one can see that all results are in the right order of magnitude and that many are entirely representative of reality. However, further calibration is necessary in order to obtain reliable results on all links.

The computer program requires a matrix inversion step which demands full core of the IBM 360 model 40 computer. This is a time consuming procedure and taxes the research funds available to the limit. However, it is felt that these results demonstrate the validity of this modelling technique.

4. 7 SUMMARY

1. The problems involved in travel demand modelling were

		001	002	003	004	005	006	007	008	009	TOTAL
Montreal	001	——	——	——	28,978	6,335	6,212	3,367	3,668	7,839	56,399
Halifax	002	——	——	——	22,966	5,605	4,453	686	4	9	33,723
Sydney	003	——	——	——	3,220	1,740	840	40	--	30	5,970
St. John's	004	28,978	22,966	3,220	--	11,629	10,772	3,714	5,547	3,815	90,642
Corner Brook	005	6,335	5,605	1,740	11,629	--	1,466	1,056	1,705	1,451	30,987
Gander	006	6,212	4,453	840	10,772	1,466	--	1,408	2,075	1,891	29,045
Goose Bay	007	3,367	686	40	3,715	1,056	1,408	--	656	781	11,709
Churchill Falls	008	3,668	4	--	5,547	1,705	2,075	656	--	253	14,908
Wabush	009	7,839	9	30	3,815	1,451	1,891	781	253	--	16,069

Table 3.3 1969 Origin-Destination Statistics (Figures indicate number of round trips between cities) (5,24,25)

TABLE 4.6

RESULTS OF PRELIMINARY CALIBRATION OF LINEAR GRAPH SIMULATION TECHNIQUE

CALCULATED ONE YEAR TOTAL ROUND TRIPS

		1	2	3	4	5	6	7	8	9	TOTAL
Montreal	1	-----	-----	-----	27,127	6,604	4,558	4,927	4,654	8,351	56,399
Halifax	2	-----	-----	-----	20,020	6,523	4,568	1,019	754	661	33,723
Sydney	3	-----	-----	-----	2,807	1,878	979	126	90	74	5,970
St. John's	4	30,319	17,624	5,652	-----	19,535	10,636	3,129	2,149	1,586	90,642
Corner Brook	5	4,242	4,785	4,207	10,327	-----	2,384	2,789	1,712	538	30,987
Gander	6	5,612	5,681	3,123	7,033	4,560	-----	1,555	1,041	435	29,045
Goose Bay	7	3,846	1,053	185	1,973	1,456	1,039	-----	1,351	802	11,709
Churchill Falls	8	4,099	386	127	2,785	2,258	2,668	1,370	-----	1,211	14,908
Wabush	9	9,545	777	118	2,509	1,811	1,755	1,098	452	-----	16,069

discussed. The basic principles of linear graph theory were presented along with brief discussions of both the branch and chord formulations.

2. The Newfoundland air network was identified graphically and discussed.
3. Regional attractions were developed and presented.
4. Access link resistances were calculated.
5. Airway link resistances were calculated.
6. A brief discussion of the problems involved in model calibration was presented.

CHAPTER 5

APPLICABILITY OF FRAMEWORK AND LINEAR GRAPH MODEL

5.1 RESULTS

The results of the model development and actual origin - destination data were presented at the end of Chapter 4. Comparison of selected origin - destination pairs indicate that discrepancies are small in actual numbers or are small on a percentage basis. For example, on the Wabush to Churchill Falls run, actual figures are 253 round trips per year, compared to a calculated figure of 452, the difference being only 200 per year. On the St. John's to Montreal run the figures are 28,978/30,319 for a difference of 1,341, however, this represents less than a 5% difference. It is felt that the results in general are representative of actual data. Further model calibration, which was limited due to expensive computer costs and a meager budget, would show even better results.

5.2 ADVANTAGES OF LINEAR GRAPH

The travel demand simulation procedure described in previous sections does not differ significantly from existing models with respect to the following (1):

1. There are coefficients of the model that must be calibrated.
2. The calibration constants are assumed to remain constant over the planning horizon.
3. The origin flow values (or travel potentials) for the planning horizon must be estimated through some type of regression formulation.

However, the chord formulation model (1, 33) does offer some advantages and these include:

1. Generation, distribution, and assignment are considered as interdependent, and are completed simultaneously for each origin.
2. The model considers the competitive attractions of all destinations on the system with respect to each origin.
3. The travel links are mathematically described by their time and cost parameters.
4. The model quantifies the interconnections of many of the variables relating to demand. The variable and interconnection measurements are achieved at the aggregate level.
5. The model is analytic; therefore no iterations or balancing procedures are required to determine the travel

volumes on each link.

The chord formulation technique (1, 33) is used to solve for the unknown flows and the unknown pressures. By substituting these derived pressures associated with the origin flows into the branch formulation (1, 33) it is possible to quantify trip generation characteristics associated with changes in the system. The pressure associated with the origin area is that which is used up in travel (i. e. it sums to zero around a circuit). It is a function of the origin area flow and the equivalent resistance of the system.

The branch formulation permits the trade-offs made by a passenger in choosing a mode of travel to be quantified at the aggregate level.

The chord formulation served to calibrate and check the coefficients of k and to derive the origin area travel potentials. No attempt was made to relate the travel potentials to the origin area socio-economic characteristics because of the limited number of origin areas. The derived travel potentials were used as pressure drivers in a branch formulation of the system shown in Figure 3. 6. Branch formulation models were constructed for both the St. John's and Corner Brook origins. Changes were then made in the cost and time parameters of airway links 202 and 205 and access link 115.

As would be anticipated, a decrease in airfare and travel time produces air increase in traffic volumes [1]. Further, a large decrease in access time and cost on link 115, produces a significant increase in travel volumes. The conclusion being, that a non-stop flight between St. John's and Corner Brook with relatively easy airport accessibility at Corner Brook would show a significant increase in air travel demand.

Part of the Gander areas relative attractiveness for air trips is accounted for by the fact that the airport is used as a trans Atlantic air stop. If Gander were to be phased off of the trans Atlantic air map and St. John's were to be used instead, there would be a change in the relative attractiveness of the two areas. There would be an increase in the attraction of St. John's and a decrease in the attraction of Gander. Branch formulation models for the entire Newfoundland air network were constructed with these changes made. As was anticipated, there was a decrease in travel volumes to the Gander area and an increase in volume to St. John's.

Changes in the socio-economic status of a region will be reflected in the airway network by a corresponding change in travel demand. For example, an increase in the labour force of Corner

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[1]. The decrease in travel time represents a non-stop flight from St. John's to Corner Brook.

Brook would have an affect on the terms in the attraction equation (3.8.1). The relative attractiveness of Corner Brook would then be increased and show a corresponding increase in the air travel demand. Further, if investment into Bonne Bay skiing and resort potential were made, there would also be an increase in the relative attraction of this area. This would mean, as expected, an increase in the air travel demand.

Thus, it is demonstrated that this modelling technique is sensitive to, and shows corresponding changes in the air travel demand when:

1. Changes are made in the airway network (time, costs, routes and scheduling).
2. Investment is made into an airport to change or expand on its functions.
3. Investment is made into an improved airport location with respect to a particular city.
4. Changes in the labour force of a particular area are made.
5. Investment is made into an area to expand or introduce a new economic activity.

In view of the above discussion, the linear graph model used in this thesis offers a number of advantages over the existing methods of demand simulation. These advantages include:

1. The linear graph technique considers the interconnections of travel potential, economic demand and equilibrium demand.
2. The technique requires that the demand components are individually described and then interconnected. The procedure is probably more closely related to the actual demand phenomena than interactions suggested by other models.
3. The resultant system of linear equations simultaneously generates, distributes, and assigns traffic volumes.
4. The linear graph model is analytic, there is only one solution required to obtain generation, distribution and assignment.
5. The linear graph technique considers the competition between modes.
6. The linear graph technique considers the attraction competition between destinations.
7. The number of variable estimates are much less with the linear graph model than with existing models.

5.3 AREA OF APPLICATION

In general terms, the applicability of the planning framework (fig. 3.2) and linear graph theory has been demonstrated. With respect to the problem of Newfoundland's economic development, both the modelling and the framework have specific application in a number of areas:

1. Tourism and Recreation - tuna fishing, moose hunting, salmon fishing, and the possibility of skiing in the Bonne Bay area, etc.
2. Industrial development - lobster marketing and local fish products, etc.
3. Transportation Economic Efficiency - west coast airports, Gander, etc.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 CONCLUSIONS

1. Newfoundland's economic situation is unacceptably below the standards set by the rest of Canada (16).

2. The needs criteria which determined the development of the present airports in Newfoundland no longer exist.

3. There are a number of economic deficiencies in the airport locations presently being used in Newfoundland.

4. The air network should play an integral part in a multi-modal transport system policy for economic development in Newfoundland.

5. The framework for planning outlined in this thesis appears to be a logical approach to regional airport investment planning in Newfoundland.

7. The linear graph simulation techniques appears to be the most appropriate modelling technique for this type

of planning problem.

6.2 RECOMMENDATIONS

1. It is recommended that this thesis form the basis for continuing the study on regional airport location planning in Newfoundland.

2. Further study is required in the development of more appropriate attraction and resistance functions.

3. A travel potential equation should be developed to relate economic activity directly to travel pressure.

4. The linear graph technique allows for exact calibration of the travel data and should be accomplished. (Note: this was not completed due to a lack of available computer time).

5. It is recommended that the following areas be considered in the immediate future in further specific studies:

   1. Gander Airport
   2. The West Coast Airports
   3. Fresh fish and live lobster marketing depots.

6. The author anticipates airport maintenance problems at both Argentia and Goose Bay when the United States Air Force completes its withdrawal. It is recom-

mended that studies be conducted to determine the outcome of these facilities if they are turned over to Canadian or Newfoundland authorities as was Stephenville.

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APPENDIX "A"

APPENDIX 'A'

Elements of Linear Graph Theory

The elements of linear graph theory are discussed in detail in "Networks and Systems" by Peter H. O. N. Roe, Addison-Wesley Publishing Company, Reading, Mass. 1966

The appendix summarizes linear graph theory beginning with the definitions of the cut set and circuit equations.

Cut-Set and Circuit Equations

The interconnection model is described by two basic postulates. One, the Kirchoff vertex postulate states that at any vertex

$$\sum_{i=1}^{e} a_i y_i = 0 \qquad (A.1)$$

where e = number of elements,

y_i = through variable of ith element

a_i = 0 if the ith element is not incident at the Vth vertex

a_i = 1 if the ith variable is oriented away from the Vth vertex

a_i = -1 if the ith variable is oriented towards the Vth vertex

The other, known as the circuit postulate states that for any circuit in the systems graph:

$$\sum_{i=1}^{e} b_i x_i = 0 \qquad (A.2)$$

where e = the number of elements in the graph,

x_i = across variable of the ith element,

b_i = 0 if the ith element is not in the jth circuit,

b_i = 1 if the orientation of the ith element is the same as the orientation chosen for the jth circuit

b_i = -1 if the orientation of the ith element is opposite to that of the jth circuit.

A fundamental circuit of a graph for any selected tree is the set of circuits formed by each chord and its unique tree branch. The number of independent circuit equations is given by the product of the circuit matrix of the across variables.

$$[B_{11} \quad u] \begin{bmatrix} X_b \\ \\ X_c \end{bmatrix} = 0 \qquad (A.3)$$

where B_{11} = a coefficient matrix corresponding to the branches,

u = a unit matrix corresponding to the chords,

X_b = a column matrix of the branches,

X_c = a column matrix of the chords.

The fundamental circuit matrix $B = [B_{11} \quad u]$ is defined by $B = b_{ij}$

where b_{ij} = 1 if the element j is in the circuit i and the orientation of the circuit and element coincide,

b_{ij} = -1 if the element j is in the circuit i and the orientation do not coincide,

b_{ij} = 0 if the element j is not in circuit i.

The order of the matrix is $(e - v + 1)$, e

where e = the number of elements, and

v = the number of vertices.

The fundamental set of cut-sets with respect to a tree is the cut-sets formed by each branch of the tree and all chords of the tree for which the fundamental circuit contains that branch. The number of independent cut-set equations is given by the matrix product of the cut-set matrix and the column matrix of the through variables

$$[u \quad a_{12}] \begin{bmatrix} Y_b \\ Y_c \end{bmatrix} = 0 \tag{A.4}$$

where u = a unit matrix corresponding to the branches of a tree,

a_{12} = a coefficient matrix corresponding to the chords,

Y_b = a column matrix of the branch through variables,

Y_c = a column matrix of the chord through variables.

The fundamental cut-set matrix $a = [u \quad a_{12}]$ is defined by

$a = a_{ij}$

where a_{ij} = 1 if the element j is in the cut-set i and the element orientation of the branch in the cut-set coincides,

a_{ij} = -1 if the element j is in the cut-set with the opposite orientation of the branch,

a_{ij} = 0 if the element j is not in the cut-set.

The order of the matrix is $(V - 1)$, e.

Further, it may be shown that [33]:

$$aB^T = 0 \text{ or } Ba^T = 0,$$

thus $$a_{12} = -B_{11}^T \text{ and } B_{11} = -G_{12}^T \tag{A.5}$$

where a = fundamental cut-set matrix,

B^T = transpose of fundamental tranpose matrix,

a_{12} = coefficient matrix corresponding to the chord through variables,

B_{12} = coefficient matrix corresponding to the branch across variables.

The fundamental cut-set matrix corresponding to a tree can be written from the fundamental circuit matrix of the same tree and vice-versa.

The Branch Formulation

The branch formulation requires that the given pressure drivers be placed in the branches and that the given flow variables be placed in the chords. It is also required that the terminal equations be given explicitly in the through variables.

From any selected tree, the fundamental cut-set can be expressed as:

$$\begin{bmatrix} u & o & A_{11} & A_{12} \\ o & u & A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} Y_{b(s)} \\ Y_b \\ Y_c \\ Y_{c(s)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{A.6}$$

where $Y_{b(s)}$ = a column matrix of specified flows in the branches,

Y_b = a column matrix of unknown branch flows,

Y_c = a column matrix of unknown chord flows,

$Y_{c(s)}$ = a column matrix of known chord flows.

The terminal relations are expressed in terms of the through variables:

$$X_{b(s)} = [F_{1(s)}]$$

$$Y_{c(s)} = [F_{2(s)}]$$

$$\begin{bmatrix} Y_b \\ Y_c \end{bmatrix} = \begin{bmatrix} G_{11(s)} & G_{12(s)} \\ G_{21(s)} & G_{22(s)} \end{bmatrix} \begin{bmatrix} X_b \\ X_c \end{bmatrix} \qquad \text{(A.7)}$$

where G_{ij} represent coefficient matrices.

Expanding (A.6) to separate the terms containing known pressures or flows, Equation (A.8) is obtained:

$$\begin{bmatrix} u \\ 0 \end{bmatrix} [Y_{b(s)}] + \begin{bmatrix} 0 & A_{11} \\ u & A_{21} \end{bmatrix} \begin{bmatrix} Y_b \\ Y_c \end{bmatrix} + \begin{bmatrix} A_{12} \\ A_{22} \end{bmatrix} [Y_{c(s)}] = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \text{(A.8)}$$

The terminal equations (A.7) are substituted into the relationship (A.8) to obtain:

$$\begin{bmatrix} U \\ 0 \end{bmatrix} [Y_{b(s)}] + \begin{bmatrix} 0 & A_{11} \\ u & A_{21} \end{bmatrix} \begin{bmatrix} G_{11(s)} & G_{12(s)} \\ G_{21(s)} & G_{22(s)} \end{bmatrix} \begin{bmatrix} X_b \\ X_c \end{bmatrix} + \begin{bmatrix} A_{12} \\ A_{22} \end{bmatrix} [Y_{c(s)}] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

..... (A.9)

The fundamental circuit equations can be written as:

$$\begin{bmatrix} B_{11} & B_{12} & u & 0 \\ B_{21} & B_{22} & 0 & u \end{bmatrix} \begin{bmatrix} X_{b(s)} \\ X_b \\ X_c \\ X_{c(s)} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{A.10}$$

This equation can be rewritten in the form:

$$\begin{bmatrix} X_b \\ X_c \end{bmatrix} = \begin{bmatrix} 0 & u \\ -B_{11} & -B_{12} \end{bmatrix} \begin{bmatrix} X_{b(s)} \\ X_b \end{bmatrix} \tag{A.11}$$

Substituting into (A.9) and recalling the relationships (A.5), the following is obtained:

$$\begin{bmatrix} u \\ 0 \end{bmatrix} [Y_{b(s)}] + \begin{bmatrix} 0 & A_{11} \\ u & A_{21} \end{bmatrix} \begin{bmatrix} G_{11(s)} & G_{12(s)} \\ G_{21(s)} & G_{22(s)} \end{bmatrix} \begin{bmatrix} 0 & u \\ A_{11}^T & A_{21}^T \end{bmatrix} \begin{bmatrix} X_{b(s)} \\ X_b \end{bmatrix} +$$

$$\begin{bmatrix} A_{12} \\ A_{22} \end{bmatrix} [Y_{c(s)}] = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{A.12}$$

or

$$\begin{bmatrix} u \\ 0 \end{bmatrix} [Y_{b(s)}] + \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} X_{b(s)} \\ X_b \end{bmatrix} + [Y_{c(s)}] = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{A.13}$$

where Z_{11}, ..., are coefficient matrices of the matrix triple product.

Noting that $[X_b]$ appears in the second set of equations only, Equation (A.12) may be rewritten as:

$$[U \quad A_{21}] \begin{bmatrix} G_{11(s)} & G_{12(s)} \\ G_{21(s)} & G_{22(s)} \end{bmatrix} \begin{bmatrix} U \\ A_{21}^{T} \end{bmatrix} [X_b] + [A_{22}] [Y_{c(s)}] = 0 \tag{A.14}$$

These equations can be solved for unknown branch pressures only. The number of unknowns are the non-specified branch pressures or across variables and these number $(v - p - n_x)$ where n_x is the number of specified voltages, p the number of parts of the graph and v the number of vertices. The remaining unknowns are solved from the cut-set, circuit and terminal matrices.

The Chord Formulation

This formulation requires that the given across variables be placed in the branches and that the given through variables be placed in the chords. The terminal relationships are given explicitly in the across variables.

The fundamental circuit equations can be written as:

$$\begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix} [X_{b(s)}] + \begin{bmatrix} B_{12} & u \\ B_{22} & 0 \end{bmatrix} \begin{bmatrix} X_b \\ X_c \end{bmatrix} + \begin{bmatrix} 0 \\ u \end{bmatrix} [X_{c(s)}] = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{A.15}$$

The terminal relationships in terms of the across variables are:

$$X_{b(s)} = F_{1(s)}$$

$$Y_{c(s)} = F_{2(s)}$$

$$\begin{bmatrix} X_b \\ X_c \end{bmatrix} = \begin{bmatrix} R_{11(s)} & R_{12(s)} \\ R_{21(s)} & R_{22(s)} \end{bmatrix} \begin{bmatrix} Y_b \\ Y_c \end{bmatrix} \qquad (A.16)$$

The fundamental cut-set equations can be written as:

$$\begin{bmatrix} Y_b \\ Y_c \end{bmatrix} = \begin{bmatrix} -A_{21} & -A_{22} \\ u & 0 \end{bmatrix} \begin{bmatrix} Y_c \\ Y_{c(s)} \end{bmatrix} \qquad (A.17)$$

Substituting (A.16) and (A.17) into (A.15) and recalling the relationships (A.5) the following is obtained:

$$\begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix} [X_{b(s)}] + \begin{bmatrix} B_{12} & u \\ B_{22} & 0 \end{bmatrix} \begin{bmatrix} R_{11(s)} & R_{12(s)} \\ R_{21(s)} & R_{22(s)} \end{bmatrix} \begin{bmatrix} B_{12}^T & B_{22}^T \\ u & 0 \end{bmatrix} \begin{bmatrix} Y_c \\ Y_{c(s)} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ u \end{bmatrix} [X_{c(s)}] = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (A.18)$$

Equation (A.18) can be written in terms of unknown chord flows only by separating the equations into two sets:

$$[B_{12} \quad u] \begin{bmatrix} R_{11(s)} & R_{12(s)} \\ R_{21(s)} & R_{22(s)} \end{bmatrix} \begin{bmatrix} B_{12}^T \\ u \end{bmatrix} [Y_c] + [B_{11}] [X_{b(s)}] = [0]$$

..... (A.19)

The number of unknown chord through variables is given by $(e - v + p - n_y)$ where e is the number of elements and n_y the specified through variables. The remaining unknowns are solved from the cut-set, circuit and terminal matrices.

Example of the Branch Formulation

Consider the system graph shown in Figure A.1(b), with the selected tree containing elements 1, 5 and 10. Further, let it be assumed that the across variable or travel propensity of element 1 is known, as are the flows through elements 8 and 9.

The fundamental cut-set equations can be written as:

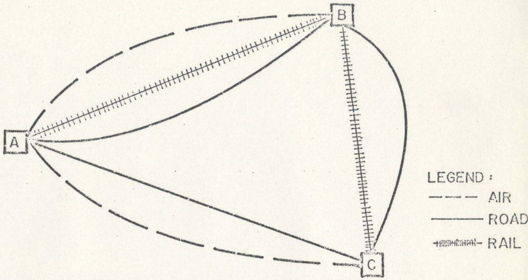
$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} [Y_1] + \begin{bmatrix} 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Y_5 \\ Y_{10} \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_6 \\ Y_7 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} Y_8 \\ Y_9 \end{bmatrix} = [0] \quad \ldots\ldots (A.20)$$

The terminal relations explicit in flow variables can be written as:

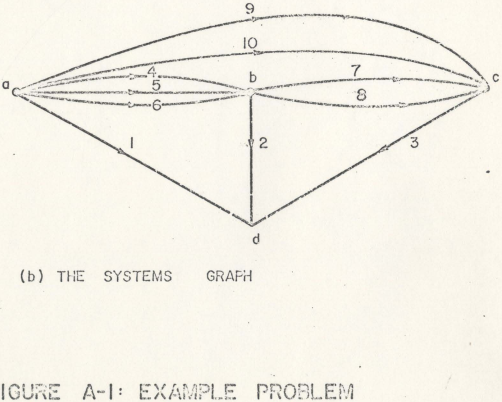
$$X_1 = F_{1(s)}$$

$$-Y_8 = F_{2(s)}$$

$$Y_9 = F_{3(s)}$$



(a) AN INTERCITY TRANSPORTATION SYSTEM



(b) THE SYSTEMS GRAPH

FIGURE A-1: EXAMPLE PROBLEM

$$\begin{bmatrix} Y_5 \\ Y_{10} \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_6 \\ Y_7 \end{bmatrix} = \begin{bmatrix} G_5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_{10} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & A_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & G_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & G_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & G_7 \end{bmatrix} \begin{bmatrix} X_5 \\ X_{10} \\ X_2 \\ X_3 \\ X_4 \\ X_6 \\ X_7 \end{bmatrix} \qquad (A.21)$$

The fundamental circuit equations reduce to:

$$\begin{bmatrix} X_5 \\ X_{10} \\ X_2 \\ X_3 \\ X_4 \\ X_6 \\ X_7 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_5 \\ X_{10} \end{bmatrix} \qquad (A.22)$$

Substituting (A.21) and (A.22) into (A.20), the branch equations are obtained:

$$\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} Y_1 + \begin{bmatrix} 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} G_5 & & & & & & \\ & G_{10} & \emptyset & & & & \\ & & A_2 & & & & \\ & & & A_3 & & & \\ & & \emptyset & & G_4 & & \\ & & & & & G_6 & \\ & & & & & & G_7 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_5 \\ X_{10} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} Y_8 \\ Y_9 \end{bmatrix} = [\bar{0}] \qquad (A.23)$$

The set of equations containing only unknown branch pressure or across variables is written as:

$$\begin{bmatrix} Z_{22} & Z_{23} \\ Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} X_5 \\ X_{10} \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} Y_8 \\ Y_9 \end{bmatrix} = [\bar{0}] \qquad (A.24)$$

where Z_{ij} are coefficients of the matrix triple product.

The system of equations has been reduced to two equations and two unknowns. The flow from city A on the system is completely described. Remaining unknowns are solved by substitution into (A.20), (A.21) and (A.22).

Example of the Chord Formulation

Again consider Figure A.1(b). The fundamental circuit equations can be written as:

$$\begin{bmatrix}1\\1\\0\\0\\0\\0\\0\end{bmatrix}[X_1] + \begin{bmatrix}1&0&1&0&0&0&0\\0&1&0&1&0&0&0\\-1&0&0&0&1&0&0\\-1&0&0&0&0&1&0\\1&-1&0&0&0&0&1\\1&-1&0&0&0&0&0\\0&-1&0&0&0&0&0\end{bmatrix}\begin{bmatrix}X_5\\X_{10}\\X_2\\X_3\\X_4\\X_6\\X_7\end{bmatrix} + \begin{bmatrix}0&0\\0&0\\0&0\\0&0\\0&0\\1&0\\0&1\end{bmatrix}\begin{bmatrix}X_8\\X_9\end{bmatrix} = [\bar{0}]$$

.... (A.25)

The terminal relationships in terms of across variables are given as:

$$X_1 = F_{1(s)}$$

$$Y_8 = F_{8(s)}$$

$$Y_9 = F_{9(s)}$$

$$\begin{bmatrix}X_5\\X_{10}\\X_2\\X_3\\X_4\\X_6\\X_7\end{bmatrix} = \begin{bmatrix}R_5&&&&&&\\&R_{10}&&&\emptyset&&\\&&J_2&&&&\\&&&J_3&&&\\&\emptyset&&&R_4&&\\&&&&&R_6&\\&&&&&&R_7\end{bmatrix}\begin{bmatrix}Y_5\\Y_{10}\\Y_2\\Y_3\\Y_4\\Y_6\\Y_7\end{bmatrix}$$

(A.26)

The cut-set equations can be reduced to the relationship:

$$\begin{bmatrix} Y_5 \\ Y_{10} \\ Y_2 \\ Y_3 \\ Y_4 \\ Y_6 \\ Y_7 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & -1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} Y_2 \\ Y_3 \\ Y_4 \\ Y_6 \\ Y_7 \\ Y_8 \\ Y_9 \end{bmatrix} \qquad (A.27)$$

The substitution of (A.26) and (A.27) into (A.25) yields the chord formulation equations:

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} [X_1] + \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} R_5 & & & & & & \\ & R_{10} & & & \emptyset & & \\ & & J_2 & & & & \\ & & & J_3 & & & \\ & & & & R_4 & & \\ & & & & & R_6 & \\ & & & & & & R_7 \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & -1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & -1 & -1 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} Y_2 \\ Y_3 \\ Y_4 \\ Y_6 \\ Y_7 \\ Y_8 \\ Y_9 \end{bmatrix}$$

/Continued ....

$$+ \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} X_8 \\ X_9 \end{bmatrix} = [\bar{0}] \qquad (A.28)$$

The set of equations is unknown chord flows is written as:

$$\begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} [X_1] + \begin{bmatrix} H_{11} & H_{12} & H_{13} & H_{14} & H_{15} \\ H_{21} & H_{22} & H_{23} & H_{24} & H_{25} \\ H_{31} & H_{32} & H_{33} & H_{34} & H_{35} \\ H_{41} & H_{42} & H_{43} & H_{44} & H_{45} \\ H_{51} & H_{52} & H_{53} & H_{54} & H_{55} \end{bmatrix} \begin{bmatrix} Y_2 \\ Y_3 \\ Y_4 \\ Y_6 \\ Y_7 \end{bmatrix} = [0] \qquad \ldots\ldots (A.29)$$

where H_{ij} are coefficients of the matrix triple product.

The set of equations was reduced to five equations in five unknowns.

APPENDIX "B"

APPENDIX "B"

PROGRAMME FOR A NUMERICAL SOLUTION OF A LINEAR GRAPH SYSTEM

Programme Terminology:

M	=	number of nodes in the systems graph.
N	=	number of links which have no flows specified.
L	=	number of links having flow (or pressure) specified.
CHOICE	=	parameter allowing user to choose formulation to be used; if = 1: branch formulation = 2: chord formulation
CONS	=	calibration constant applied to the attractions.
ITER	=	parameter allowing user to state if iteration is desired; if = 1: iteration desired = 0: no iteration desired
NP	=	parameter indicating if previously specified systems description applies to problem which follows; if = 1: read in new description = 0: old description applies
AIN(M,N+L)	=	complete systems graph description consisting of: if CHOICE = 1.0 - CUT-SET MATRIX, and if CHOICE = 2.0 - CIRCUIT MATRIX
BIN(N+L)	=	terminal matrix of resistances and attractions
LN(L)	=	vector indentifying the link(s) for which flow or pressure is specified
TYP(N+L)	=	vector indicating the type of terminal matrix entry; if = 1.0: resistance = 2.0: attraction
D(L)	=	value of flow(s) or pressure(s) specified.
A(M,N)	=	description of non-specified part of the system. Consists of all entries of (AIN) except the vector(s) corresponding to the specified links

APPENDIX "B" (continued)

C(M,L) = placement vector(s) for the specified link(s). Consists of all vectors excluded from (A).

B(N,M) = terminal matrix corresponding to matrix (A). Consists of all (BIN) entries, excluding those corresponding to specified links, placed along the diagonal and zero entries elsewhere. The attractions and resistances are converted to a form suitable for use in either the "branch" or the "chord" formulation.

CY(N+L) = vector specifying all correct flows (or pressures) for use during the iterative process. If iteration is not specified this vector must not be read in.

```
          C
          C
          C
001             DIMENSION A(90,90),B(90,90),TYP(90),C(90,10),D(10),AT(90
               1W(90),LN(10),AIN(90,90),BIN(90),KOUT(2,90),AOUT(90),X(90)
002             DIMENSION AA(10,90)
003             COMMON A,B,TYP,C,D,AT,W,LN,AIN,BIN,KOUT,AOUT,AA
004           1 READ (5,1001,END=100)M,N,L,CHOICE,CONS,ITER
005             NL=N+L
006             NITER=0
007             NPASS=1
          C
008             CALL RDONE(M,N,L,CHOICE,CONS,NL)
009         101 IF(NITER.GT.0) GO TO 10
010             READ(5,1102)NP
011             IF(NP.EQ.1) GO TO 1
012             CALLRDTWO(L)
          C
013          10 CALL ASSIGN(M,N,L,CHOICE,NITER)
014             IF(CHOICE.EQ.1.0) GO TO 2
          C
          C     INITIAL SOLUTION
          C
015           2 CALL ANSOLV(M,N,L,X,CHOICE,ITER,NITER)
          C
          C     COMPLETED SOLUTION
          C
016             IF(CHOICE.EQ.2.0) GO TO 111
017             CALL BRANCH(M,N,L,X,ITER,NITER)
018             IF(ITER.EQ.0) NITER=0
019             GO TO 101
020         111 CALL CHORD(M,N,L,X,ITER,NITER)
021             IF(ITER.EQ.0) NITER=0
022             GO TO 101
          C
          C     FORMATS
          C
023        1001 FORMAT( 3I4,2F10.0, I4)
024        1102 FORMAT(I1)
          C
025         100 STOP
026             END
```

```
      SUBROUTINE RDONE(M,N,L,CHOICE,CONS,NL)
      DIMENSION A(90,90),B(90,90),TYP(90),C(90,10),D(10),AT(9
     1W(90),LN(10),AIN(90,90),BIN(90),KOUT(2,90),AOUT(90),AA(
      COMMON A,B,TYP,C,D,AT,W,LN,AIN,BIN,KOUT,AOUT,AA
      DO 53 I=1,M
   53 READ(5,1100)(AIN(I,J),J=1,NL)
      READ(5,1101)(BIN(I),I=1,NL)
      READ(5,1100)(TYP(I),I=1,NL)
      WRITE (6,1020)
      DO 54 I=1,M
      IF(N.GT.10) GO TO 600
      WRITE (6,1003) (AIN(I,J),J=1,NL)
      GO TO 54
  600 K=0
      DO 601 J=1,NL
      IF(AIN(I,J).EQ.0.0) GO TO 601
      K=K+1
      KOUT(1,K)=I
      KOUT(2,K)=J
      AOUT(K)=AIN(I,J)
  601 KK=K
      WRITE(6,1025)(AOUT(K),KOUT(1,K),KOUT(2,K),K=1,KK)
   54 CONTINUE
      WRITE (6,1021)
      WRITE (6,1003) (BIN(I),I=1,NL)
      WRITE(6,1022)CONS
      IF(CHOICE.EQ.2.0) GO TO 4
      WRITE (6,1009)
      DO 3 I=1,NL
      IF(TYP(I).EQ.2.0) BIN(I)=BIN(I)*CONS
    3 IF(TYP(I).EQ.1.0) BIN(I)=1.0/BIN(I)
      GO TO 7
    4 DO 5 I=1,NL
      IF(TYP(I).EQ.2.0) BIN(I)=BIN(I)*CONS
    5 IF(TYP(I).EQ.2.0) BIN(I)=1.0/BIN(I)
      WRITE(6,1010)
    7 DO 6 I=1,N
      DO 6 J=1,N
    6 B(I,J)=0.0
C
C     FORMATS
C
 1003 FORMAT(// 5X,  8F15.2 )
 1009 FORMAT (//// ' BRANCH FORMULATION' )
 1010 FORMAT (//// ' CHORD FORMULATION ' )
 1020 FORMAT (//'1INPUT  AIN   '//)
 1021 FORMAT (//' INPUT  BIN   '//)
 1022 FORMAT( //' ATTRACTION CALIBRATION CONSTANT =', F10.6)
 1025 FORMAT (5(F15.2,1X,1H(,I2,1H,I2,1H)))
 1100 FORMAT(15F5.2)
 1101 FORMAT(15F5.2)
C
      RETURN
C
```

```
              C
              C
0049                ENTRY RDTWO(L)
0050                READ(5,1000        )(LN(I),I=1,L)
0051                READ(5,1103)(D(I),I=1,L)
0052                WRITE(6,1023)
0053                DO 501 I=1,L
0054            501 WRITE(6,1024)LN(I)
              C
              C       FORMATS
              C
0055           1000 FORMAT (2514 )
0056           1023 FORMAT (1H1,' FEEDER LINK(S)  ')
0057           1024 FORMAT (10I10)
0058           1103 FORMAT(8F15.6)
              C
0059                RETURN
0060                END
```

```
      SUBROUTINE ASSIGN(M,N,L,CHOICE,NITER)
      DIMENSION A(90,90),B(90,90),TYP(90),C(90,10),D(10),AT(90,
     1W(90),LN(10),AIN(90,90),BIN(90),KOUT(2,90),AOUT(90),AA(1
      COMMON A,B,TYP,C,D,AT,W,LN,AIN,BIN,KOUT,AOUT,AA
      DO 30 I=1,M
      DO 30 J=1,N
      A(I,J)=AIN(I,J)
      LJ=L+J
      DO 31 K=1,L
   31 IF(J.GE.LN(K)) A(I,J)=AIN(I,LJ)
   30 CONTINUE
      IF(CHOICE.EQ.2.0) GO TO 501
  501 DO 32 I=1,M
      DO 32 J=1,L
   32 C(I,J)=AIN(I,LN(J))
      DO 38 I=1,N
      B(I,I)=BIN(I)
      IL=I+L
      DO 39 J=1,L
   39 IF(I.GE.LN(J)) B(I,I)=BIN(IL)
   38 CONTINUE
      DO 2 I=1,M
      DO 2 J=1,N
    2 AT(J,I)= A(I,J)
      IF(NITER.GT.0) GO TO 10
      WRITE(6,1018)
      DO 60 I=1,M
   60 WRITE(6,1003) (C(I,J),J=1,L)
      WRITE(6,1019)
      WRITE(6,1003)  (D(I),I=1,L)
      IF (CHOICE.EQ.1.0) WRITE (6,1014)
      IF (CHOICE.EQ.2.0) WRITE (6,1015)
      DO 50 I=1,M
      IF(N.GT.10) GO TO 602
      WRITE (6,1003) (A(I,J),J=1,N)
      GO TO 50
  602 K=0
      DO 603 J=1,N
      IF(A(I,J).EQ.0.0) GO TO 603
      K=K+1
      KOUT(1,K)=I
      KOUT(2,K)=J
      AOUT(K)=A(I,J)
  603 KK=K
      WRITE(6,1025)(AOUT(K),KOUT(1,K),KOUT(2,K),K=1,KK)
   50 CONTINUE
      WRITE (6,1016)
      DO 51 I=1,N
   51 WRITE (6,1003) B(I,I)
      WRITE (6,1017)
      DO 52 I=1,N
      IF(N.GT.10) GO TO 604
      WRITE (6,1003) (AT(I,J),J=1,M)
      GO TO 52
```

```
0054          604 K=0
0055              DO 605 J=1,M
0056              IF(AT(I,J).EQ.0.0) GO TO 605
0057              K=K+1
0058              KOUT(1,K)=I
0059              KOUT(2,K)=J
0060              AOUT(K)=AT(I,J)
0061          605 KK=K
0062              WRITE(6,1025)(AOUT(K),KOUT(1,K),KOUT(2,K),K=1,KK)
0063           52 CONTINUE
            C
            C       FORMATS
            C
0064         1014 FORMAT (//' CUT-SET MATRIX'//)
0065         1015 FORMAT (//' CIRCUIT MATRIX'//)
0066         1003 FORMAT(// 5X,  8F15.4 )
0067         1016 FORMAT (//' TERMINAL MATRIX...DIAGONAL ENTRIES ONLY'//)
0068         1017 FORMAT (//' TRANSPOSED  A '//)
0069         1018 FORMAT (//' PLACEMENT MATRIX  C '//)
0070         1019 FORMAT (//' SPECIFIED VALUE(S)'//)
0071         1025 FORMAT (5(F15.2,1X,1H(,I2,1H,,I2,1H)))
            C
0072           10 RETURN
0073              END
```

```
      SUBROUTINE ANSOLV(M,N,L,X,CHOICE,ITER,NITER)
      DIMENSION A(90,90),B(90,90),TYP(90),C(90,10),D(10),AT(90,
     1W(90),LN(10),AIN(90,90),BIN(90),KOUT(2,90),AOUT(90),AA(1
      DIMENSION X(90),LL(10)
      COMMON A,B,TYP,C,D,AT,W,LN,AIN,BIN,KOUT,AOUT,AA
      DO 9 I=1,M
      DO 8 J=1,N
      W(J)=0.0
      DO 8 K=1,N
    8 W(J)=W(J)+A(I,K)*B(K,J)
      DO 9 K=1,N
    9 A(I,K)=W(K)
      DO 11 I=1,M
      DO 10 J=1,M
      W(J)=0.0
      DO 10 K=1,N
   10 W(J)=W(J)+A(I,K)*AT(K,J)
      DO 11 K=1,M
   11 A(I,K)=W(K)
      IF(M.GT.10) GO TO 6
      IF((ITER.EQ.1).AND.(NITER.GT.0)) GO TO 6
      WRITE(6,1002)
      DO 90 I=1,M
   90 WRITE(6,1003)(A(I,J),J=1,M)
C
C     VECTOR OF GIVEN VALUES
C
    6 DO 12 I=1,M
      W(I)=0.0
      DO 12 J=1,L
   12 W(I)=W(I)+C(I,J)*D(J)
      IF(CHOICE.EQ.2.0) GO TO 24
      DO 1 I=1,L
      DO 1 J=1,M
    1 W(J)=W(J)-A(J,LN(I))*D(I)
      MSOLV=M-L
      DO 7 K=1,L
      DO 7 I=1,M
    7 AA(K,I)=A(LN(K),I)
      DO 2 I=1,M
      DO 2 J=1,MSOLV
      DO 2 K=1,L
    2 IF(J.GE.LN(K)) A(I,J)=A(I,J+L)
      DO 3 I=1,MSOLV
      DO 3 J=1,MSOLV
      DO 3 K=1,L
      IF(I.GE.LN(K)) GO TO 4
      GO TO 3
    4 A(I,J)=A(I+L,J)
      W(I)=W(I+L)
    3 CONTINUE
      GO TO 30
   24 DO 25 I=1,L
      LL(I)=LN(I)+M-N-L
```

```
      DO 25 J=1,M
   25 W(J)=W(J)-A(J,LL(I))*D(I)
      MSOLV=M-L
      DO 26 K=1,L
      DO 26 I=1,M
   26 AA(K,I)=A(LL(K),I)
      DO 27 I=1,M
      DO 27 J=1,MSOLV
      DO 27 K=1,L
   27 IF(J.GE.LL(K)) A(I,J)=A(I,J+L)
      DO 28 I=1,MSOLV
      DO 28 J=1,MSOLV
      DO 28 K=1,L
      IF(I.GE.LL(K)) GO TO 29
      GO TO 28
   29 A(I,J)=A(I+L,J)
      W(I)=W(I+L)
   28 CONTINUE
C
C     SOLUTION OF EQUATIONS
C
C     NOTE  VARIABLE SCALE IS INCORPORATED TO SCALE THE INPUTS
C           SUBROUTINE MYGSDL IN ORDER TO PREVENT EXPONENT OVEF
C           FLOWS WHICH MAY OCCUR WHEN USING THE CHORD FORMULAT
      SCALE=100.0
      DO 200 I=1,MSOLV
      W(I)=W(I)/SCALE
      DO 200 J=1,MSOLV
  200 A(I,J)=A(I,J)/SCALE
   30 CALL MYGSDL(A,MSOLV,W)
   23 DO 5 I=1,MSOLV
    5 X(I)=W(I)
      IF(CHOICE.EQ.2.0) GO TO 31
      DO 20 I=1,MSOLV
      DO 21 J=1,L
      IF(I.LT.LN(J)) GO TO 21
      K=I+L
      W(K)=X(I)
   21 CONTINUE
   20 CONTINUE
      DO 22 I=1,L
      LP=LN(I)
   22 W(LP)=D(I)
      GO TO 35
   31 DO 32 I=1,MSOLV
      DO 33 J=1,L
      IF(I.LT.LL(J)) GO TO 33
      K=I+L
      W(K)=X(I)
   33 CONTINUE
   32 CONTINUE
      DO 34 I=1,L
   34 W(LL(I))=D(I)
   35 DO 15 I=1,M
```

```
0099          15  X(I)=W(I)
0100              IF(ITER.EQ.0) WRITE(6,1005)
0101              IF((ITER.EQ.1).AND.(NITER.EQ.0)) WRITE(6,1005)
            C
            C       FORMATS
            C
0102          1002 FORMAT (///' MATRIX TRIPLE PRODUCT' )
0103          1003 FORMAT(// 5X,   8F15.2 )
0104          1005 FORMAT (/// ' SOLUTIONS' )
            C
0105              RETURN
0106              END
```

```
      SUBROUTINE BRANCH(M,N,L,X,ITER,NITER)
      DIMENSION A(90,90),B(90,90),TYP(90),C(90,10),D(10),AT(90
     1W(90),LN(10),AIN(90,90),BIN(90),KOUT(2,90),AOUT(90),AA(1
     2X(90),WW(90)
      COMMON A,B,TYP,C,D,AT,W,LN,AIN,BIN,KOUT,AOUT,AA
      IF((ITER.EQ.1).AND.(NITER.GT.0)) GO TO 1
      WRITE(6,1006)
    1 DO 91 I=1,M
      IF((ITER.EQ.1).AND.(NITER.GT.0)) GO TO 91
      WRITE(6,1008)I,W(I)
   91 CONTINUE
      III=M
      NP=N-III+L
      DO 33 I=1,NP
      DO 33 J=1,M
   33 AT(I,J)=AIN(J,I+M)
      DO 35 I=1,NP
      SUM=0.0
      DO 34 J=1,M
   34 SUM=SUM+AT(I,J)*W(J)
      W(I+III)=SUM
   35 X(I+III)=W(I+III)
      IF((ITER.EQ.1).AND.(NITER.GT.0)) GO TO 2
      DO 36 I=1,NP
      LL=I+III
   36 WRITE(6,1008)LL,W(LL)
      DO 3 I=1,M
    3 WW(I)=W(I)
      N1=N+1
      DO 37 I=1,N1
   37 W(I)=W(I)*BIN(I)
      DO 5 K=1,L
      SUM=0.0
      DO 4 I=1,M
    4 SUM = SUM + AA(K,I)*WW(I)
      LT=LN(K)
    5 W(LT)=SUM
      DO 6 I=1,N1
    6 X(I)=W(I)
      WRITE(6,1011)
      DO 40 I=1,N1
   40 WRITE(6,1012)I,W(I)
C
C       FORMATS
C
 1006 FORMAT (// ' UNKNOWN PRESSURES' )
 1008 FORMAT (/ ' X(',I3,' ) =', F15.2 )
 1011 FORMAT (///' ELEMENT FLOWS')
 1012 FORMAT (/ ' Y (',I3,' ) =', F15.2 )
C
    2 RETURN
      END
```

```
      SUBROUTINE CHORD(M,N,L,X,ITER,NITER)
      DIMENSION A(90,90),B(90,90),TYP(90),C(90,10),D(10),AT(90,
     1W(90),LN(10),AIN(90,90),BIN(90),KOUT(2,90),AOUT(90),X(90)
      DIMENSION AA(10,90)
      COMMON A,B,TYP,C,D,AT,W,LN,AIN,BIN,KOUT,AOUT,AA
      IF((ITER.EQ.1).AND.(NITER.GT.0)) GO TO 1
   41 WRITE(6,1007)
    1 DO 92 I=1,M
      II=I+N-M+L
      W(II)=X(I)
      IF((ITER.EQ.1).AND.(NITER.GT.0)) GO TO 92
      WRITE(6,1012)II,W(II)
   92 CONTINUE
      DO 93 I=1,M
      II=I+N-M+L
   93 X(II)=W(II)
      NP=N-M+L
      DO 47 I=1,NP
      DO 47 J=1,M
   47 AT(I,J)=AIN(J,I)
      DO 43 I=1,NP
      SUM=0.0
      DO 42 J=1,M
   42 SUM=SUM+AT(I,J)*W(J+N-M+L)
      W(I)=SUM
   43 X(I)=W(I)
      IF((ITER.EQ.1).AND.(NITER.GT.0)) GO TO 2
      DO 44 I=1,NP
   44 WRITE(6,1012)I,W(I)
      N1=N+1
      DO 45 I=1,N1
   45 W(I)=W(I)*BIN(I)
      DO 5 K=1,L
      SUM=0.0
      DO 4 I=1,M
      IIK=I+N+L-M
    4 SUM=SUM+AA(K,I)*X(IIK)
      LT=LN(K)
    5 W(LT)=SUM
      WRITE(6,1013)
      DO 46 I=1,N1
   46 WRITE(6,1008)I,W(I)
C
C        FORMATS
C
 1007 FORMAT (// ' UNKNOWN FLOWS' )
 1008 FORMAT (/ ' X(',I3,' ) =', F15.2 )
 1012 FORMAT (/ ' Y (',I3,' ) =', F15.2 )
 1013 FORMAT (/// ' ELEMENT PRESSURES')
C
    2 RETURN
      END
```

```
      SUBROUTINE MYGSDL(AIN,MA,B)
C
C     GAUSS-SEIDEL METHOD FOR SOLUTION OF LINEAR EQUATIONS
C      AIN=GIVEN MATRIX  B=VECTOR OF ABSOLUTE TERMS  MA=MATRIX D
C      ITERATION IS INITIALIZED INTERNALLY WITH /NE'S AS A FIRST
C       ITERATION HALTS WHEN SUCCESSIVE ITERATIONS AGREE WITHIN
C       MAX NO OF ITERATIONS(M) =50
C
      DIMENSION AIN(90,90),B(90),A(90),C(90),D(90),OD(90,90)
      M=1000
      LSC=0
      TOL=0.01
      DO 8 I=1,MA
      A(I)=1.0
      C(I)=0.0
      D(I)=0.0
      DO 8 J=1,MA
      OD(I,J)=0.0
      IF(J.EQ.I) GO TO 1
      IF(J.NE.I) GO TO 2
    1 D(I)=AIN(I,I)
      GO TO 8
    2 OD(I,J)=AIN(I,J)
    8 CONTINUE
    6 IF(LSC.EQ.M) GO TO 7
      LSC=LSC+1
      DO 3 I=1,MA
      SUM=0.0
      DO 15 J=1,MA
   15 SUM=SUM+OD(I,J)*A(J)
      A(I)=(B(I)-SUM)/D(I)
    3 CONTINUE
      DO 4 I=1,MA
      TEST=ABS((A(I)-C(I))/A(I))
      IF(TEST.GT.TOL) GO TO 19
    4 CONTINUE
      GO TO 10
   19 DO 5 I=1,MA
    5 C(I)=A(I)
      GO TO 6
    7 WRITE(6,102)
      GO TO 10
  102 FORMAT(//' NO CONVERGENCE AFTER 50 ITERATIONS'//)
   10 DO 20I=1,MA
   20 B(I)=A(I)
   11 RETURN
      END
```