A STUDY OF SOME OF THE MOST DIFFICULT TOPICS IDENTIFIED IN TEACHING MATHEMATICS IN GRADES FOUR TO EIGHT IN THREE ELEMENTARY SCHOOLS

JOHN PATRICK McGRATH

A Study of Some of the Most Difficult Topics Identified in Teaching Mathematics in Grades Four to Eight in Three Elementary Schools

by

John Patrick McGrath, B.A.(Ed.), B.A.

(C)

A Report submitted in partial fulfillment
of the requirements for the degree of
Master of Education

Department of Curriculum and Instruction
Memorial University of Newfoundland
August 1977

St. John's Newfoundland

ABSTRACT

This study attempted to make use of the expertise and experience of teachers to identify some of the most difficult topics in teaching mathematics in grades four to eight in three particular elementary schools. It also sought teachers' opinions as to why the difficulties exist, asked the teachers to share their classroom-tested techniques for overcoming these difficulties, and then attempted to determine trends of difficulties across grade levels.

The procedure involved classroom visits, teacher interviews, and two questionnaires administered to teachers at each grade level. The investigator observed five grade four classes, seven grade fives, ten grade sixes, eight grade sevens and nine grade eight classes. He worked with thirty-eight teachers who are directly involved with the teaching of elementary school mathematics.

The first questionnaire listed topics appropriate to each grade level and asked teachers to rate them from 1 - no difficulty, to 5 = extreme difficulty. Using the results of the first questionnaire, a list of approximately eight of the most difficult topics was prepared for each grade level. On a second questionnaire, teachers were presented with these eight or so topics and asked to select the three most difficult. They were also asked to indicate the specific aspects of the difficulties, their reasons for the difficulties, and suggestions for dealing with them.

The resulting information was analysed for each grade level and trends across grade levels were determined.

Topics such as word problems, division, multiplication number facts, fractions, and geometry were indicated as major problem areas at all grade levels. Difficulties with such topics as place value, work with other bases, per cent, and areas and volumes were specific to particular grade levels.

Reasons for the difficulties ranged from very general to very specific. Lack of reading comprehension skills was often given as a reason for the difficulty with word problems, whereas, not having multiplication number facts mastered was often given as one reason for the difficulty with long division.

Teaching techniques offered ranged from general suggestions such as making greater use of manipulative materials, to specific suggestions such as allowing the use of multiplication fact cards for work with division. Some suggestions were very explicit in that they outlined step-by-step procedures for dealing with certain problem areas.

ACKNOWLEDGEMENTS

The writer is grateful for all the assistance and cooperation he received in conducting and reporting this project.

He is deeply indebted to the principals, assistant principals, and teachers of mathematics at St. Patrick's Hall School, St. Bonaventure's School, and Holy Cross School.

Special thanks is expressed to Rev. Brother J.P. McHugh, principal of Brother Rice High School, for his encouragement and for arranging the writer's teaching schedule to accommodate the study.

Appreciation is expressed to the professional staff at the school board office for their encouragement and cooperation in this project.

The writer also wishes to express sincere thanks to Dr. Glyn Wooldridge, Dr. Lionel Pereira-Mendoza, and Rev. Brother G.R. Bellows, CFC, for the improvements resulting from their excellent suggestions.

A special word of thanks goes to the writer's wife, Eleanor, for typing the manuscript, and for her patience and encouragement.

TABLE OF CONTENTS

LIST OF TABLES

CHAPTER I

INTRODUCTION

Background

At the close of the 1975 school year, it was decided by the administration and staff of Brother Rice High School in St. John's to request Rev. Brother G.R. Bellows, CFC, to carry out a complete evaluation of the school. The evaluation was completed during the month of February, 1976, and a report was presented to the teachers on February 23, 1976. One of the observations in Brother Bellows' report was that while coordination between various academic departments within the high school could be readily achieved, the same could not be said about program coordination between Brother Rice High School and its "feeder" schools: St. Patrick's Hall School, St. Bonaventure's School, and Holy Cross School. Brother Bellows referred to this group as the Brother Rice Family of Schools and recommended that a program of subject coordination from kindergarten to grade eleven be eventually planned, adopted, and implemented. He suggested that department heads at Brother Rice High School play a major role in this necessary endeavour.

This project, then, evolved from Brother Bellows' evaluation and represented the first step towards opening up an avenue of communication with the teachers of mathematics in the elementary schools that feed into Brother Rice High School. In the course of the project, the investigator, Mathematics Department Head at Brother Rice High School, became acquainted with every teacher in grades four to eight in the three "feeder" schools as well as with their principals and vice-principals. The investigator was also invited into thirty-nine different classrooms to hold discussions with the teachers and to see the students involved in their mathematics activities. Thus, the project made a beginning at providing elementary mathematics teachers with the opportunity to exchange problems and ideas with colleagues in the Brother Rice Family of Schools.

Statement of the Problem

During the late fifties, there was a tremendous upsurge of interest and projects aimed at the improvement of mathematics education in our schools. New mathematics programs were designed, experimented with and introduced into the school systems in order to help students to understand mathematics rather than just to learn it by rote. Later, students became involved with the manipulation of materials and teachers were encouraged to use guided discovery in order to help students actively formulate the structure of mathematics. In order to bring new developments in mathematics to the elementary school student, some topics were moved down into the lower grades and, in some cases, drill exercises for acquiring computational skills were de-emphasized. While there were a few educators who were more cautious in accepting the merits of the new mathematics, most felt that students were apparently understanding and enjoying the subject more.

However, as one would expect, the new mathematics programs did not solve all of the difficulties in teaching mathematics. In some instances, they created a few new ones. With the emphasis on understanding, some people became concerned that students were not getting enough practice in computational skills. Another difficulty involved poor reading skills which had been considered a hindrance to learning traditional mathematics and continued to pose problems in learning modern mathematics. Also, with the new topics and new methods associated with new mathematics, many teachers lacked the necessary preparation to teach the subject satisfactorily. In addition to these general problems, teachers found that with the introduction of new topics and the moving of topics down to the lower grades, many students were experiencing difficulty in learning certain concepts. Number systems in bases other than base ten, the closure property, using place holders and problem solving were just a few of the specific concepts that students had difficulty in comprehending.

At Brother Rice High School, the mathematics teachers noticed for some time that students came to high school without having mastered many of the basic concepts and skills of elementary school mathematics. It appeared, then, that local elementary school teachers were also experiencing difficulties in working with the new mathematics programs. It was thought that there might be problem areas at specific grade levels or, perhaps, problems that ran through the grades without being dealt with effectively. If these difficulties were identified, the areas for concentration in coordinating the mathematics program through the system might become obvious. Furthermore, apart from what needed to be improved, the way to effect the improvement might be indicated by the people who are most familiar with the situation, the elementary teachers themselves.

Objectives of the Study

This study, then, attempted to identify some of the main problems associated with teaching mathematics in the elementary grades. In particular, the study involved grades four to eight in three Roman Catholic boys' schools: St. Patrick's Hall School, St. Bonaventure's School, and Holy Cross School. It sought teachers' opinions as to why the difficulties exist and then asked the teachers to share their classroom-tested techniques for overcoming these difficulties.

Answers were sought for the following questions:

1. What topics at specific grade levels seem to cause the most trouble for students?

2. What, in the teachers' opinions, are the main reasons for these difficulties?

3. What classroom tested techniques have teachers developed for overcoming these difficulties?

4. Are there any trends with difficulties through the grades?

CHAPTER II

REVIEW OF THE LITERATURE

The four questions posed in this study pointed to three main areas of concentration. They were: the curriculum; the instruction; the learning. While this study dealt mainly with difficulties connected with the curriculum, questions two and three involving reasons for difficulties and suggestions for teaching techniques also pointed to problems involving learning and instructional theories.

Consequently, a review of the literature was carried out first to determine the aims and directions of the new mathematics with respect to curriculum, learning, and instruction. It was felt that this information would bring into focus the rationale for the mathematics education program now in use in Newfoundland schools. Secondly, looking at the same three areas of curriculum, learning, and instruction, an attempt was made to determine from the literature what difficulties arose in teaching the new mathematics in the elementary schools.

The New Mathematics - Aims and Directions

With the reform projects of the fifties and sixties on the teaching of mathematics (Nichols, 1968), there developed a phenomenon that became known as the "new mathematics". Many lay people used the term but not many could explain its meaning. Indeed, it would be very difficult to give a precise explanation of the new or modern mathematics. However, some insights to the subject would probably be gained from a review of some of the aims presented in the earlier stages and the new directions in mathematics education which followed.

Curriculum

Good Mathematics for Younger Children

Many educators who were calling for improvements in the school mathematics program in the fifties and sixties began questioning the notion of postponing certain mathematical concepts until children are ready to learn them. Bruner (1960, p. 12) maintained that our schools may be wasting precious years by postponing the teaching of many important subjects on the grounds that they are too difficult. In continuing the argument Bruner (1960) stated:

> We begin with the hypothesis that any subject can be taught effectively in some intellectually honest form to any child at any stage of development. It is a bold hypothesis and an essential one in thinking about the nature of a curriculum. No evidence exists to contradict it; considerable evidence is being amassed that supports it (p. 33).

Gibb (1959) was apparently convinced that the child should be introduced to some sophisticated mathematics at an earlier age. She said:

> We need experiences in our arithmetic classes which will enable children to begin forming mathematical ideas of number groups, quantity and space, and experiences which will lend

> themselves to the habits of mathematical thinking: order, a whole and its parts, and the combination of wholes to make another whole. In so doing, let us keep clearly in mind the critical points of development of mathematical understanding (p. 137).

The Curriculum Advocated at the Cambridge Conference

In an attempt to describe the kind of curriculum envisaged by the advocates of the modern mathematics program, a brief look at the Cambridge Conference of 1963 was in order. Under the direction of Martin and Gleason of Harvard, twenty-five mathematicians and scientists met to consider what the modern school mathematics curriculum should become by the year 1990. This distinguished group maintained that it was possible to move topics down through the grades by a new organization of the subject matter and by abandonment of drill for drill's sake (Adler, 1966, p. 180).

In grades kindergarten to two the Cambridge Report, Goals for School Mathematics, proposed the teaching of the number line, inequalities, physical interpretations of addition and multiplication, and the discovery by the children of the commutative nature of addition and multiplication. It also suggested the use of a box as a placeholder, an introduction to Cartesian coordinates, the early use of sets, and the concept of function. The report recommended that each child have abundant opportunity to manipulate suitable physical objects.

The Cambridge Report suggested that, in grades three to six, pupils learn about the commutative, associative, and distributive laws and the arithmetic of signed numbers. Other topics to be taught included rational and irrational numbers, polar coordinates, vectors, truth tables, open sentences and the solution of simple equations.

Learning

With the reorganization of content and the moving down of some topics into the lower grades, what did the new mathematics do to improve learning? The trend was to be away from rote memorization of facts and emphasis on learning by understanding. One way that this was to be accomplished was by structuring the content so that recurring patterns were used to unify the subject. Adler (1966) stated:

> I would go beyond this conclusion, however, and argue that the inclusion of such "advanced" concepts in the elementary school course of study not only does not make it more difficult to teach, but actually makes it easier to teach. The modern ideas of mathematical structure make learning easier because they simplify and unify what the children have to learn (p. 183).

In an attempt to explain to parents how modern mathematics facilitated learning, Petronia (1966) said:

> Essentially, then, it implies a reorganization of traditional subject matter, some new content, and a decidedly new teaching approach, all within the framework of the unifying concepts that form the basis of the structure of mathematics as a whole. Its main objective and primary concern is to have the student understand why he does what he does. Its approach is one of investigation and discovery. It aims to make

mathematics intellectually exciting and challenging (p. 469).

Jones and Coxford (1970, p. 49) noted that as early as the mid-1930's Brownell had been advocating an emphasis on understanding as the key to learning mathematics. Brownell's advice, then, was now finding wide acceptance within the framework of the new mathematics movement.

Instruction

Besides the emphasis on the structure of mathematics, the attempt to foster understanding also encouraged the "discovery" approach as the main teaching method. Thus, another pedagogical approach from much earlier times finally achieved respectability. For not only had Warren Colburn suggested this method in his textbook of 1821, but good teachers through the ages have been guiding pupils to discover things for themselves (Willoughby, 1968, p. 13).

In describing the discovery approach of the Greater Cleveland Mathematics Program, Gundlach (1961) said:

> Problem situations are presented to students as if they had not been explored already by the great geniuses of the past and present. These situations are presented in such a manner that pattern discovery has a good chance of taking place almost spontaneously; then students are guided to a point where the established symbolism for the rediscovered pattern seems almost a necessity (p. 192).

In discussing the Cambridge Conference on School

Mathematics, Weaver (1964) noted:

> It was therefore felt that the design of imaginative problem sequences involving combinations of routine techniques and 'discovery' procedures was a matter of greatest importance in curriculum development (p. 209).

<u>Summary</u>

The purpose of this section was to bring into focus the rationale for the mathematics education program now in use in Newfoundland schools. Thus, a review of the literature about the aims and directions of the new mathematics highlighted definite trends in mathematics education that emerged during the past twenty years. These trends emerged by considering three aspects of the new mathematics: curriculum, learning, and instruction.

In the area of curriculum, the trend was to move more of the "real" mathematics down into the lower grades. Younger children were introduced to more sophisticated mathematics.

It was thought that learning the new mathematics would be facilitated by an emphasis on understanding rather than rote learning. Also, the formalized structure of the content was intended to unify the subject and, thus, make it easier to learn.

The instructional technique that was strongly advocated was the discovery method. Here, the student was to be actively involved with the manipulation of materials designed to help him discover certain facts for himself.

The New Mathematics - Areas of Difficulty

Using as a guide the aims and directions of the new mathematics with respect to curriculum, learning, and instruction, the investigator again turned to the literature. His primary aim was to determine what mathematics educators considered to be problems arising in the new curriculum. However, he also sought out discussions of difficulties with the learning and instructional theories of modern mathematics.

While the investigator's project attempted to get at some of the specific topics that cause the most trouble in the teaching of elementary school mathematics, a review of the literature revealed, for the most part, problems that were more general in nature. The more general of the difficulties reported here involved learning and instruction, but, a few were specific curriculum difficulties.

Curriculum

Non-Decimal Numeration Systems

Hoping to increase the child's understanding of our base ten number system, curriculum innovators introduced number systems other than base ten. Fehr (1966, p. 84) maintained that, while it seemed like a reasonable approach, experience was proving it wasn't working.

Higgins (1972) carried out an investigation of the effects of non-decimal numeration instruction. He concluded:

> The data for this study suggest that the implementation of non-decimal numeration as a vehicle of instruction in elementary mathematics curriculum seems to have no effect on mathematical understanding of base ten (p. 296).

The National Advisory Committee on Mathematics Education (NACOME) Report (1975) stated:

> Research on arithmetic difficulties of young children shows that inadequate understanding of our decimal place-value numeration system is one of the most common barriers to improved computation (p. 19).

Solving Word Problems

In modern mathematics programs, even children in the primary grades are introduced to word problems. However, despite its early introduction a great number of students have trouble with word problems right up through the school system.

Henney (1971, p. 224) maintained that although children work with "story problems" from kindergarten on, the stumbling block with problem solving appears at approximately the fourth grade level. This stumbling block, she suggested, occurs because the language, sentence structure, and vocabulary become more complex at that level and the difficulty with problem solving essentially originates as a problem with reading.

She offered one interesting suggestion for teachers:

> If reading-skills instruction is given during mathematics instruction, certain reading skills learned during the developmental reading period will be reinforced and learning will more likely

be seen as functional (p. 225).

One might well ask: is the ability to solve problems a gift that only certain mathematically talented students possess or are there in fact techniques that teachers can use to develop problem solving skills in children? A review of the literature and research clearly indicated that there are techniques available to teachers which have proven successful in fostering problem solving skills, for example, Polya (1957), Dahmus (1970), Sowder (1972), Bassler, Beers and Richardson (1975), Pereira-Mendoza (1975).

Teacher Preparation

One problem mentioned in the literature which is not specifically a curriculum problem is that of teacher preparation. However, the investigator included it in this section because it is a problem which was directly related to the change in the curriculum. The new programs required teachers who were familiar with their content and structure in order to ensure that the mathematics taught in the schools was consistent with the philosophy of the new mathematics movement. As Gundlach (1961) stated it:

> A most serious problem was, and still is, to provide a sufficient number of teachers with the background necessary to create such challenging and exciting classroom situations. In addition to being a good teacher, this requires substantial subject-matter mastery, considerably more, in most cases, than college preparation (p. 192).

Teachers in the elementary schools still do not receive the assistance necessary to prepare them to teach the new mathematics programs effectively. Green (1976) carried out a study to identify broad areas of concern in teaching elementary mathematics. From his study involving approximately 700 elementary school teachers, Green concluded:

> 1. Elementary school teachers have very real and genuine concerns, and many unanswered questions about teaching modern elementary school mathematics.
> 2. Elementary school teachers need more help in understanding the purposes of methods of teaching, use of textbooks and materials, and content of modern mathematics.
> 3. Present college and university courses and inservice training programs are not meeting many of the needs of elementary school teachers and teacher trainees (p. 102).

Learning

As mentioned earlier, learning the new mathematics was to be facilitated by structuring the content as a unified body of knowledge. Some educators expected that too great an emphasis on formal structure might, indeed, make learning more difficult.

Brownell (1959) had a word of caution on the influence of the reform projects in this respect. He stated:

> As a matter of fact, their influence could be too great if their activities should produce a sterile arithmetic. By this term, I mean arithmetic as the pure science of number, taught with little regard for the ability of children to learn it or for the social purposes to be served. This danger, however, is unlikely to become a reality, for forces are available within the schools to counteract it (p. 44).

One aim of the new projects was that the mathematics taught in the schools be "correct" mathematics. Consequently, most new programs recommended precise use of mathematical language and symbols. For example, the distinction between number (the idea) and numeral (the name or symbol for the idea) was very popular for quite some time.

Fehr (1966) reacted to the distinction this way:

> In this connection, also, it is nonsense to stress the distinction between numeral and number. Of course, numbers are abstract, and numerals are names or symbols for the abstract, but, in general, no misconception arises in using the two words synonymously (p. 84).

Willoughby (1968, p. 13) doubted that such a consistent and careful distinction between objects and their names really serves a useful purpose.

Henry (1971) suggested that quite often, the source of a child's difficulties with arithmetic is his inability to read the notation. He stated:

> The symbols that we use in mathematics to denote the number of elements in a set, to denote certain operation, and to denote various relations are concise, efficient, and usable. But if a child cannot understand the meaning of these symbols, they say little to him (p. 37).

Schoen (1974, p. 236) maintained that the excessive use of symbols is a source of the difficulty children often have with sets in elementary mathematics.

The NACOME Report (1975, p. 20) stated that, in fact, most teachers have returned to a much more modest emphasis on symbols and precision of expression.

That the undue emphasis on a formal approach is still a vital issue in elementary school mathematics was indicated by Rappaport (1976):

> The mathematics curriculum should be very flexible. In the early grades children should be given the opportunity to grope, explore, experiment; to try out manipulative devices; and to learn to cope with real life situations that arise from the environment. Formal arithmetic learning should be kept at a minimum and informal learning experiences should be encouraged (p. 345).

Instruction

Problems with the Discovery Approach

Modern mathematics encourages teachers to use the discovery approach. Indeed, many teachers who have used this method successfully can get very excited about it as one method of teaching. However, as many educators have observed, there are problems with it. Just because students are presented with activities and ideas does not guarantee that they will discover the intended outcome. Corle (1964) offered this comment:

> No teacher can afford to make discovery her single method of teaching. There are facts that every child must know, and bringing this information to a child's attention may require directive teaching procedures (p. 246).

There is even a danger that an activity will become an end in itself with the student oblivious of the fact that he is supposed to be experiencing insights and drawing conclusions. Adler (1966) therefore emphasized, "Discovery in the classroom must be guided discovery, and often it should be cooperative discovery" (p. 184).

Perhaps the teacher could insure that the activity does not become an end in itself by making use of behavioural objectives. Such a practice would force the teacher to specify, at least to himself, the intended outcome of the activity.

Problems with Allowing for Individual Differences

The NACOME Report (1975) revealed that some new mathematics programs now are beginning to offer varying content for different student audiences. The report implied that this practice is occurring at the high school level. The trend is to focus on broad structural concepts and heuristic methods in dealing with the more capable students and to emphasize computational skill and problems of everyday life and specialized trades with the slower students.

Different content for different ability groups was certainly not recommended by the early advocates of modern mathematics. In proposing correct mathematics and sound pedagogy for all students, Begle (1958) presented five points:

> 1. No one can predict exactly which mathematical skills will be important and useful in the future.
> 2. No one can predict exactly what career any particular student will choose when he leaves school.
> 3. Teaching which emphasizes understanding, insight, and imagination, without neglecting basic skills, is the best for all students, whatever their ability, and makes the best

> preparation for any vocation that uses mathematics.
> 4. An understanding of the role of mathematics in our society is essential for intelligent citizenship.
> 5. Any normal individual can appreciate some, at least, of the beauty and power of mathematics, and this appreciation is an important part of a civilized person's cultural background (pp. 616-617).

Many advocates of the new mathematics programs suggested that the elementary school teacher could provide for different ability levels in two ways. The first was by selecting appropriate material from acceptable content areas. The second was to vary teaching approaches and methodology.

Gibb (1965) maintained:

> Providing a differentiated curriculum for individual children carries with it the responsibility of making appropriate selections of materials and teaching methods suitable not only for different age levels but also for different maturity levels at the same age (p. 21).

To provide for individual differences Johnson (1967, p. 188) suggested a variety of approaches for teaching mathematics. They were: 1. Laboratory lessons; 2. Learning games; 3. Audiovisual presentations; 4. Classes directed by the pupils; 5. Discovery lessons; 6. Enrichment lessons; 7. Individualized instruction.

Coping with the varying ability levels of students, then, is clearly indicated in the literature as a problem with which the elementary school mathematics teacher is definitely faced. Knowing what material to include and what to omit and when and how to vary teaching strategies presents the teacher with quite a challenge and a need for lots of assistance.

Summary

The literature about the difficulties in teaching elementary school mathematics, then, for the most part does not involve specific difficulties with the content area. There is, however, quite a discussion of problems in more general terms.

With respect to curriculum, one specific area of difficulty mentioned was the non-decimal numeration system. Children, apparently, are having trouble working in other bases, and, contrary to what was expected, it does not seem to improve work with base ten. Another specific difficulty in the area of curriculum involves working with word problems. This becomes more serious at the grade four level and is perhaps related to reading problems. One problem that is directly related to curriculum is teacher preparation. By moving more sophisticated mathematics down into the lower grades, a problem arose in preparing teachers who, for the most part, are not mathematics majors.

In the area of learning, a problem arose with the structure of the content. Children apparently had more difficulty because they found it too formal and, perhaps, were not ready for such a precise treatment.

One problem related to instruction was with the discovery approach. There was a danger that students and teachers would become so involved in the activities that they would forget about what was to be discovered. Another instructional problem involved allowing for individual differences. The original view was that all students of varying abilities could handle the same content if the teaching approach was varied. This approach, apparently, is slowly being rejected.

Thus, a review of the literature about elementary school mathematics highlights the issues and problems that have emerged during the past twenty years. These issues and problems have been primarily of a general nature. The point of departure of this study is to examine the more specific teaching and learning problems associated with elementary school mathematics in grades four to eight in three specific schools.

CHAPTER III

PROCEDURES AND INSTRUMENTS

As mentioned in Chapter I,[1] this project originated from a recommendation that each department in Brother Rice High School should work more closely with the teachers in the three feeder schools: St. Patrick's Hall School, St. Bonaventure's School, and Holy Cross School. The study involves the difficulties associated with the teaching and learning of mathematics in these particular elementary schools. Again, the main objective was to make a beginning at working with the elementary school teachers on subject coordination. The intention was to make use of the expertise and experience of the teachers in an attempt to find answers for the following questions:

1. What topics at specific grade levels seem to cause the most trouble for students?

2. What, in the teachers' opinions, are the main reasons for these difficulties?

3. What classroom tested techniques have teachers developed for overcoming these difficulties?

4. Are there any trends with difficulties through the grades?

In early September, the investigator met with each of the three school principals and explained the proposed project. Assuring him of their personal cooperation, the principals arranged meetings with their respective staffs at which he again explained the project and sought the cooperation and approval of the teachers. All of the teachers were very open and receptive to the study and seemed eager to cooperate.

The investigator then proceeded to go out to the schools once a week for a period of approximately six months. From September to February he spent one full day in each school every third week in rotation.

During these visits, he held informal discussions with teachers and was invited into almost all the mathematics classes from grades four to eight in the three schools. The investigator observed five different grade four classes, seven grade fives, ten grade sixes, eight grade sevens and nine grade eights. He worked with thirty-eight teachers who are directly involved with the teaching of elementary school mathematics.

In January, 1977, a questionnaire was presented to teachers at each grade level (See Appendix A). First of all, the questionnaire asked for the following teacher information: the number of years teaching mathematics; the number of years teaching mathematics at the stated grade level; the number of professional courses (one semester) in teaching mathematics; and the number of University courses (one semester) in mathematics.

Secondly, the questionnaire listed the topics for each grade level to be taught during the year and asked teachers to check off the topics that they would cover.

Teachers were also asked to rate these topics on a scale of one to five indicating the degree of difficulty they expected pupils to experience with each topic. The rating scale used was as follows:

1 - no difficulty;

2 - moderate difficulty - cleared up very quickly;

3 - serious difficulty - usually taken care of;

4 - very serious difficulty - some never grasp at this level;

5 - extreme difficulty - most do not grasp at this level.

A mean rating for each topic was obtained. Then by using a mean rating greater than or equal to three, the investigator made a list of specific topics at each grade level which would present the most difficulty in teaching.

In March, 1977, a second questionnaire was delivered to each teacher (See Appendix B). This questionnaire listed the most difficult topics for each grade level as determined by the first questionnaire. Teachers were asked to select the <u>three</u> most troublesome topics and then explain what specific aspects of each of the three topics gave students the most trouble. Secondly, teachers were asked to try to explain why these topics were the most difficult. Finally, teachers were asked to suggest techniques for dealing with these troublesome topics.

From the completed second questionnaire, the investigator listed all the troublesome topics mentioned by the teachers at each grade level in their three choices. An attempt was then made to find a consensus as to the specific aspects of the difficulty with each topic and the main reasons for the difficulty. Samples of suggested teaching techniques were selected for each grade level. Finally, the investigator compared difficulties across grade levels in order to determine possible trends.

CHAPTER IV

RESULTS

What do the teachers identify as the most difficult concepts and skills involved in teaching modern elementary school mathematics? What reasons do they have to offer as to why the difficulties occur? What suggestions or techniques do they have to offer for dealing with these difficulties? Are there any trends across the various grade levels with respect to difficulties?

This section attempts to answer the above questions for three specific elementary schools. It presents a report of the data obtained from the teachers of mathematics in grades four to eight in St. Patrick's Hall School, St. Bonaventure's School, and Holy Cross School.

First, Table I includes some factual information on the preparation and experience of the teachers involved in the study. Then, Table II presents the list of topics for each grade level which received a difficulty rating greater than or equal to three on the first questionnaire. Since the investigator could not deal adequately with all of the difficulties obtained from the first questionnaire, he asked teachers on the second questionnaire to select from the stated list the three most difficult topics. In this way, there was a slight reduction in the number of topics to deal with. All topics selected for each grade level on the second questionnaire are also listed in Table II. Next, taking each grade level separately, each topic selected on the second questionnaire is discussed in terms of specifics, reasons, and suggested teaching techniques. Finally, Table III highlights the occurrence of difficulties across various grade levels which is discussed in some detail.

TABLE I

Teacher Experience and Preparation Indicated On the First Questionnaire

Grade Level	FOUR	FIVE	SIX	SEVEN	EIGHT
Number of Teachers Responding	5	7	8	5	7
Average Number of Years Teaching Mathematics	13	13.3	8	7	4.4
Average Number of Years Teaching Mathematics at This Grade Level	8	6.1	4.4	4	3
Average Number of Professional Courses (One Semester) in Teaching Mathematics	1.8	1.9	1.8	1.6	1
Average Number of University Courses (One Semester) in Mathematics	1.8	2.1	2.6	4.6	1.7

TABLE II

Most Difficult Topics As Selected By Teachers

GRADE	First Questionnaire: Topics With Difficulty Rating Greater Than Or Equal To Three	Second Questionnaire: All Topics Mentioned By Teachers When Selecting Three Most Difficult
FOUR	Word Problems Multiplication Number Facts Geometry Estimation Multiplication With Two and Three Digit Numbers Long Division Fractions	Word Problems. Long Division Place Value Multiplication Number Facts Multiplication With Two and Three Digit Numbers
FIVE	Work With Other Bases Word Problems Informal Use of The If-Then Deduction Statement Geometry Division Algorithm Fractions Decimals	Word Problems Division Algorithm Fractions Work With Other Bases
SIX	Expanded Notation Using Exponents Extension of the Division Algorithm Short Division Word Problems Number Theory (GCF, LCM, Divisibility) Geometry Fractions Rational Number Operations Decimals	Word Problems Fractions Division Geometry GCF and LCM
SEVEN	Estimating Division Geometry Greatest Common Factors Subtraction With Mixed Numerals Averages Division of Decimals Per Cent Word Problems	Word Problems Per Cent Geometry Division of Decimals Division Estimating Averages (of fractions)
EIGHT	Word Problems Solving Per Cent Problems Per Cent of Increase and Decrease Interest Integers (graphing sentences) Exponents-Powers of Powers and Scientific Notation Similar Figures-Finding Missing Measures Real Numbers Solving Equations	Word Problems Solving Per Cent Problems Per Cent of Increase and Decrease Integers (graphing sentences) Rational Numbers Solving Equations Areas and Volumes

ANALYSIS BY GRADE LEVEL OF PROBLEM AREAS IDENTIFIED BY TEACHERS

GRADE FOUR

Word Problems

Specifics Reported by Teachers

Pupils cannot determine the correct operation.

Reasons Given by Teachers

1. Pupils are lacking in reading comprehension skills.
2. There is not enough opportunity for problem solving through working with concrete materials.
3. Children are entered too early into the abstract.

Techniques Suggested by Teachers

1. Provide opportunities for pupils to manipulate concrete materials to coincide with the problems they are reading.
2. Instead of always working from the word problem to the operation, try working in the reverse order. That is, first, have the child combine three objects and two objects to get five objects. Secondly, have him write and verbalize the fact $3 + 2 = 5$. Third, ask him to make up a word problem which makes use of that fact. For example, if I have three pennies and I receive two more pennies then I have five pennies.
3. Provide lots of practice of the type where the teacher reads a problem aloud and all the pupils are required

to do is state the operation to be used.

4. Provide continuous practice with phrases such as "gave away", "more than", "altogether", "in each" etc. Such phrases may not yet really be assimilated by the child to a point where he can use them with ease and complete understanding.

Long Division

Specifics Reported by Teachers

Pupils have difficulty in coming up with an estimate each time.

Reasons Given by Teachers

1. Pupils receive insufficient experience with estimating.
2. The multiplication number facts required for division are not readily at the pupils' fingertips.

Techniques Suggested by Teachers

1. Use the "old fashioned" method from the very beginning. Show the pupils the process of successive estimation and maybe let them try a few that way. However, when they are learning the skill it should be the algorithm.
2. After the division is carried out, have pupils check by multiplication. This reinforces the connection between division and multiplication.
3. If lack of multiplication number facts seems to be a

GRADE FOUR (continued)

problem, permit the pupils to use a multiplication fact card.

Place Value

Specifics Reported by Teachers

Pupils have difficulty in arranging numbers in vertical form in proper position for the operation.

Reasons Given by Teachers

Children have not been given enough practice in thinking in groups. They see individual digits rather than groups of one's, groups of ten's, groups of hundred's etc.

Techniques Suggested by Teachers

1. Stress the concept of grouping by tens. Work as much as possible in the concrete stage using objects and materials that can be readily obtained such as pencils, sticks and even the pupils themselves.
2. Provide practice in reading numbers with similar digits, for example, 009, 090, 900 etc.

Multiplication Number Facts

Specifics Reported by Teachers

Pupils have difficulty memorizing the multiplication tables and therefore, do not have these facts when required.

GRADE FOUR (continued)

Reasons Given by Teachers

It is perhaps too early to require the quick recall of multiplication number facts for some pupils at the grade four level.

Techniques Suggested by Teachers

1. Allow pupils to use a multiplication fact card through grade four and well into grade five. Require quick recall of multiplication number facts by the end of grade five.
2. Require memorization of multiplication facts in grade four. Pupils should see the connection between multiplication and repeated addition. The commutative property should be used. Perhaps games to aid memorization could be used.

(Note that there is disagreement among teachers as to when multiplication tables should be mastered).

Multiplication With Two and Three Digit Numbers

Specifics Reported by Teachers

Pupils have difficulty in determining what two numbers are actually being multiplied. For example:

$$\begin{array}{r} 346 \\ \underline{\times\ 27} \end{array}$$

7 X 6 - This presents no problem; 7 X 40 - Pupils don't realize that it really is 7 X 40.

GRADE FOUR (continued)

Also, pupils have difficulty keeping the one's under the one's place and the ten's under the ten's place etc.

Reasons Given by Teachers

Pupils lack the basic multiplication facts and the ability to recognize the place value of each numeral.

Techniques Suggested by Teachers

One suggestion is as follows:

a. Write out the number to be multiplied at the side;

b. Draw lines which clearly define place values;

c. Multiply across, starting at the top;

d. Add.

		3	4	6	
		X	2	7	
			4	2	= 7 X 6
		2	8	0	= 7 X 40
	2	1	0	0	= 7 X 300
		1	2	0	= 20 X 6
		8	0	0	= 20 X 40
+	6	0	0	0	= 20 X 300
	9	3	4	2	

GRADE FIVE

Word Problems

Specifics Reported by Teachers

Pupils cannot determine the correct operation.

Reasons Given by Teachers

1. The present group of students are generally one to two years behind in their reading levels.
2. Word problems require a level of reasoning ability that most pupils at this age have not reached yet.

Techniques Suggested by Teachers

1. Have the pupils make up their own word problems using things that are familiar to them, such as, money, hockey cards, and games they play.
2. Provide practice for the development of the comprehension skills necessary to solve problems. For example, "five more than", "five greater than", "was increased by" etc. Pupils should be drilled on the meaning of such phrases.
3. Make the problems more concrete. Pose problems which involve actual manipulation of materials in the classroom. The transfer from local types of problems to textbook problems will come naturally to the pupils.

GRADE FIVE (continued)

Division Algorithm

Specifics Reported by Teachers

Pupils have trouble with each step.

Reasons Given by Teachers

1. More emphasis is needed in memorizing the multiplication number facts.
2. Pupils have difficulty with estimating.

Techniques Suggested by Teachers

1. Concentrate on the "old" method.
2. Emphasize the connection between multiplication and division by having the pupils check by multiplication.

Fractions

Specifics Reported by Teachers

Most difficulty occurs when the denominators are different. Pupils have trouble finding the lowest common denominator. Pupils also have trouble with equivalent fractions.

Reasons Given by Teachers

1. There is not enough opportunity for pupils to work with concrete materials. The notion of fractions is too abstract.
2. Pupils fail to grasp that two fractions with different numerators and denominators can represent the same number.

GRADE FIVE (continued)

Techniques Suggested by Teachers

1. Bring in colored paper pie plates for pupils to cut up. Leave one colored plate intact while cutting others in varying sections, one cut in fourths, another cut in eighths and so on. Pupils could see that 2/8 of one plate could fit in ¼ etc.
2. Use coins in the classroom to deal with fractions as parts of a dollar. Some pupils seem surprised to learn why a quarter is called a quarter.
3. Use physical objects that pupils can separate into parts.

Work With Other Bases

Specifics Reported by Teachers

Pupils have difficulty writing a number in base ten as a number in base four.

Reasons Given by Teachers

The problem may lie with the pupil's perception of a written numeral. The written numeral "25", for example, is one symbol for one group of objects. It does not come naturally to the student to perceive "25" as two groups of ten and five ones. He sees one group of twenty-five.

Techniques Suggested by Teachers

1. Work with other bases does not really strengthen knowledge of base ten. Therefore, concentrate on base ten.

GRADE FIVE (continued)

2. Work with base ten first and then move into other bases. One method is to use popsicle sticks and proceed on a course of discovery about base ten numerals. The method is to give any number of sticks more than ten and have students group sticks in tens. They begin to see how a numeral represents a grouping. Having mastered this grouping procedure, pupils are introduced to base four. They learn to count in base four first and then proceed with some basic operations.

GRADE SIX

Word Problems

Specifics Reported by Teachers

1. Pupils often do not seem to grasp what they are asked to do.
2. They cannot determine what operation to use.

Reasons Given by Teachers

1. Pupils are lacking in reading comprehension skills.
2. There may be a problem with the level of reasoning ability required.
3. As teachers, we assume that if a pupil can add 3 + 2 = 5 then he should be able to do a word problem which requires the same operation. It just doesn't happen that way. Working with word problems requires certain specific skills in reading and translation which must be taught.

Techniques Suggested by Teachers

1. Teach the reading skills necessary for Mathematics.
2. Find problems that the pupils can do. Nothing succeeds like success.
3. Provide written directions for games that pupils must read in order to learn to play the game.
4. Make up a sequence of tasks that pupils have to perform by following written directions.

GRADE SIX (continued)

5. Find toys that have to be assembled by following instructions.

Fractions

Specifics Reported by Teachers

There is a lot of difficulty with subtraction of mixed numerals.

Reasons Given by Teachers

1. Many students who find fractions difficult do not really understand that fractions are "parts of wholes". They cannot think in parts.
2. The basic concept of a fraction seems to be too abstract for many students.

Techniques Suggested by Teachers

1. Spend more time with concrete objects, for example, diagrams, blocks etc. Too much time is being used to teach the different operations when, to many pupils, a fraction is a vague, abstract idea.
2. Get students involved in cutting a "whole" piece of paper into various "parts".

Division

Specifics Reported by Teachers

Pupils are not sure how to deal with the remainder in combining it with the next digit.

GRADE SIX (continued)

Reasons Given by Teachers

There may be too much concentration on the mechanics of the operation as opposed to the basic concept of division. Also, many pupils are still not completely familiar with the multiplication tables.

Techniques Suggested by Teachers

One suggestion:

a. Begin with very simple division problems and look at these in some detail. For example: $3\overline{)6}^{\,2}$ because $3 \times 2 = 6$. Stress the point that if a group of six objects were sub-divided into groups of three objects, there would be two of these groups.

b. Review basic concepts such as in (a) above before proceeding into the more difficult division problems.

c. Stress the actual mechanics of the division process. It is not sufficient for a pupil to know why something works, he must also know how to perform the operation.

d. If lack of multiplication facts is hindering the division, go back to drilling on multiplication facts.

Geometry

Specifics Reported by Teachers

All aspects of Geometry seem to cause trouble at this level.

GRADE SIX (continued)

Reasons Given by Teachers

Most pupils don't do very much Geometry even up to Grade six. The few who do some Geometry in Grade six have trouble with it perhaps because it is new to them.

Techniques Suggested by Teachers

1. Provide pupils with lots of "doing" experiences rather than listening experiences. They should be involved with measuring angles with a protractor, drawing triangles with given dimensions, measuring doors and desks. These students are also mature enough to be making basic constructions with compass and straight-edge.
2. Drawing is a very important part of the study of Geometry. Make sure pupils get lots of practice in drawing the basic Geometric figures. They should also be trying some of the three-dimensional drawings.

Greatest Common Factor (GCF) and Least Common Multiple (LCM)

Specifics Reported by Teachers

The problem is that these two topics are often mixed up.

Reasons Given by Teachers

1. One reason for the difficulty could be that these topics are only introductory at this level and not really meant for mastery.

GRADE SIX (continued)

2. Pupils do not really get to use the concepts in other related problems and therefore fail to grasp the concept.

Techniques Suggested by Teachers

One suggestion:

Use simple examples to show how GCF and LCM are used in problems. For example: LCM of 6 and 8 is 24 and GCF of 6 and 8 is 2.

a. Add: $\frac{1}{6} + \frac{3}{8}$ LCM = 24

$= \frac{4}{24} + \frac{9}{24}$

$= \frac{13}{24}$

b. Simplify: $\frac{6}{8}$ GCF = 2

$= \frac{\cancel{2} \times 3}{\cancel{2} \times 4}$

$= \frac{3}{4}$

GRADE SEVEN

Word Problems

Specifics Reported by Teachers

Pupils often have trouble understanding what is being asked for and determining what operation to use.

Reasons Given by Teachers

1. Students at this level still find it difficult to abstract.
2. Students have not yet developed patterns or systematic approaches to problem solving.
3. Poor treatment of this topic in the textbook.

Techniques Suggested by Teachers

1. Work with simple problems at first, dealing with things close to home. For example, ask pupils to write down the cost of their exercise, textbook and ruler. Ask them for the total cost; the cost if they had two of each; the cost if they had two exercise books, one pencil, and one ruler.
2. Have pupils follow a definite procedure for each problem such as the following:
   a. Read the problem carefully, making sure you know what it says.
   b. What are you required to do?
   c. Which of the facts do you need to use?
   d. What operation is required?

e. Solve the problem.

f. Check your answer to see if it makes sense.

Per Cent

Specifics Reported by Teachers

The difficulty is not with the concept itself but with certain types of problems. For example: a. 6% of $n = 48$; b. Change $\frac{1}{2}\%$ to a decimal.

Reasons Given by Teachers

1. This topic involves many of the concepts that pupils have already had some difficulty with and they may not be too enthusiastic about the topic. Involved are reading skills, division of decimals, and fractions. In addition to this, the notion of an equation has to be used before pupils have really been introduced to it.
2. There is too much time spent on working with per cents. Students lose interest.

Techniques Suggested by Teachers

1. Stress and insist that pupils follow a certain format for each type of per cent problem.
2. Find examples of problems that pupils often get confused and present them together so that pupils can see how they are alike and how they are different. For example: 15 is 25% of what number? What number is 15% of 60?

GRADE SEVEN (continued)

3. Another suggestion is to emphasize the ratio aspect, that is, 25% means $\frac{25}{100}$. Students could be encouraged to bring in per cent statements from newspapers and have the class interpret the meanings.

Geometry

Specifics Reported by Teachers

Pupils have difficulty in working with definitions. The difficulty is not with learning the definitions but with being able to determine when one object fits a definition and another does not.

Reasons Given by Teachers

Students have very little exposure to Geometry prior to Grade Seven. Thus, a bottleneck occurs. Students become bombarded with definitions, concepts, and drawings all at once whereas these should have been introduced gradually up through the grades.

Techniques Suggested by Teachers

1. Use concrete materials as much as possible.
2. Make sure that pupils have lots of practice in working with definitions but not memorizing them per se. That is, given any number of figures they should be able to classify them as rectangle, square, parallelogram etc.
3. Have pupils cut figures out of paper and cardboard.

GRADE SEVEN (continued)

4. Have pupils collect examples of various shapes in pictures etc. and bring them to the classroom.

Division of Decimals

Specifics Reported by Teachers

1. There is a problem with moving the decimal in the divisor.
2. Pupils have trouble with adding zeros to the dividend.
3. Pupils have trouble with putting zeros in the quotient immediately after the decimal, for example .0013.

Reasons Given by Teachers

There seems to be some mystery or magic connected with moving the decimal points. The students apparently have no idea of what they are really doing.

Techniques Suggested by Teachers

A suggested approach:

a. Review the relation of division to fractions. For example: $2\overline{)6}$ means the same as $\frac{6}{2}$.

b. Review the idea of equivalent fractions. Stress the idea of a fraction maintaining the same value provided the numerator and denominator are multiplied by the same number. For example: $\frac{2}{3} = \frac{14}{21}$.

GRADE SEVEN (continued)

c. Given the division problem $2.56\overline{).128}$, draw the connection to fractions and show that this is the same as $\frac{.128}{2.56}$.

d. Remind pupils that numerator and denominator may be multiplied by the same number and the fraction will maintain its value. Therefore, $\frac{.128}{2.56} \times \frac{100}{100} = \frac{12.8}{256}$.

This may be written as $256\overline{)12.8}$.

Now, the pupil should realize that there is no magic involved in moving the decimal, and that what is really happening is that numerator and denominator of the equivalent fraction are being multiplied by the same number.

Division

Specifics Reported by Teachers

Students become easily confused at each step of the operation.

Reasons Given by Teachers

1. The approach of easing the pupils into the "old" method of long division by means of getting them to learn the estimating method first has led to much confusion.
2. Some pupils still have not mastered the multiplication number facts.

GRADE SEVEN (continued)

Techniques Suggested by Teachers

1. Drill students on the multiplication number facts. How this is done depends on the ingenuity of the teacher. Some teachers recommend games involving multiplication while others recommend competitions similar to a spelling bee.
2. Drill students on quick division facts in order to help them get over the big hurdle of division, finding the first correct number. The drill could be carried out orally or on paper and, for the sake of this drill, the pupil need not worry about the remainder. For example: Thirty into ninety-five goes three times with some remainder. Twenty-five into one hundred and twenty goes four times with some remainder.
3. Concentrate on the same method of long division up through the grades and it should not be the method of using rough estimates. Pupils should be taught the most efficient method from the beginning.

Estimating

Specifics Reported by Teachers

There is a problem estimating with two and three digit numbers under the basic operations.

Reasons Given by Teachers

1. Students seem too casual in their attitude toward

estimating. They apparently feel that an answer can't be too important if it's only an estimate.

2. Students seldom estimate except in exercises which instruct them to do so. We, as teachers, have to incorporate this skill in teaching problem solving. Like any skill, it has to be practiced in order to be maintained.

Techniques Suggested by Teachers

1. Show the students that we think estimating is an important topic. The suggestion here is that we ourselves not be too casual with the topic and not let the pupils become careless in their estimating. The topic still calls for good work habits, pride in work, and correct method.

2. Make estimating an integral part of exercises and problems in order to get children to practice this skill. Before assigning a number of problems or an exercise set, it might be useful to conduct an oral exercise having certain students estimate the answer before doing exercises. Estimates can be recorded on the board and compared with actual answers afterwards.

Averages of Fractions

Specifics Reported by Teachers

The problem here is with dividing a fraction by a whole number.

GRADE SEVEN (continued)

Reasons Given by Teachers

Pupils have difficulty grasping the idea that dividing by a number is the same as multiplying by the reciprocal of the number.

Techniques Suggested by Teachers

One suggestion:

Review the notion that $\frac{6}{5} = 6 \times \frac{1}{5}$ or $\frac{n}{3} = n \times \frac{1}{3}$. Therefore,

$$\frac{\frac{2}{3}}{5} = \frac{2}{3} \times \frac{1}{5} = \frac{2}{15}.$$

GRADE EIGHT

Word Problems

Specifics Reported by Teachers

The problem here is with understanding what is being sought and determining the correct operation.

Reasons Given by Teachers

1. Most word problems involve situations with which pupils are not familiar or are not interested in.
2. Students have not yet developed a facility with translating from a word problem to a mathematical sentence.

Techniques Suggested by Teachers

1. Discuss specific problem solving techniques with students who are weak in this area. Have students solve problems before the class and then discuss their approach and steps taken.
2. Assign worksheets regularly throughout the year as this topic deserves a lot of extra attention.
3. Make up word problems that involve situations that are of interest to the pupils.

Solving Per Cent Problems

Specifics Reported by Teachers

There is confusion with three types of problems. For example:

GRADE EIGHT (continued)

a. 5 per cent of 16 is what number?

b. 20 is what per cent of 200?

c. 15 is 20 per cent of what number?

Reasons Given by Teachers

One major reason is that pupils fail to translate a per cent problem into a ratio problem.

Techniques Suggested by Teachers

One suggestion:

Teach this unit after the units on similar figures and solving equations have been completed. This unit may then be related to ratio and proportion. First, emphasize the ratio aspect of per cent. For example, 5% means $\frac{5}{100}$. The three types of per cent problems may then be expressed as proportions.

Example: a. 5% of 16 is what number?

$$\frac{5}{100} = \frac{x}{16}$$

b. 20 is what per cent of 200?

$$\frac{20}{200} = \frac{x}{100}$$

c. 15 is 20 per cent of what number?

$$\frac{15}{x} = \frac{20}{100}$$

GRADE EIGHT (continued)

Per Cent of Increase and Decrease

Specifics Reported by Teachers

Pupils are not sure if the relationship is between the amount of increase and original amount or between the amount of increase and new amount.

Reasons Given by Teachers

In general, the problem still seems to be related to not viewing per cent as a ratio, that is, it involves a failure to comprehend the problem as a ratio of the increase to the original amount.

Techniques Suggested by Teachers

One suggestion:

Emphasize per cent as a ratio so that the pupil has to find the two things that make up the ratio. Thus, the pupil can more easily see that the per cent of increase becomes the ratio of amount of the increase to the original amount.

Integers (Graphing Sentences)

Specifics Reported by Teachers

Pupils have problems with setting up a table of values.

Reasons Given by Teachers

Students fail to see relationships between number sentences such as $x - 2 = -3$ and $x = -3 + 2$. This leads to problems in setting up tables of values.

GRADE EIGHT (continued)

Techniques Suggested by Teachers

1. Postpone the unit on graphing until the section on solving equations is completed. Number sentences can then be more easily rearranged to facilitate setting up a table of values. For example, $y + 2 = x$ becomes $y = x - 2$ and a table can be set up by giving values to x and finding the corresponding values for y.

   For example:

x	x - 2	y
-1		
0		
1		

2. In setting up the table of values, give the values to the variable which is involved with an operation. In this way, it is unnecessary to rearrange the equation.

   For example:

   (a) $4y = x$

x	4y	y
		-1
		0
		1

   (b) $y = x + 2$

x	x + 2	y
-1		
0		
1		

Rational Numbers

Specifics Reported by Teachers

The difficulty involves having to combine the rules learned for fractions and those for integers in the four operations.

GRADE EIGHT (continued)

Reasons Given by Teachers

Pupils have not yet really mastered addition, subtraction, multiplication and division of fractions and they are still not sure of signed numbers even when working with integers.

Techniques Suggested by Teachers

One suggestion:

Provide lots of drill in the operations of rational numbers by the use of "grids". For example:

b

	a * b	x $-\frac{7}{9}$	÷ $\frac{3}{16}$	+ $\frac{11}{2}$	- $\frac{5}{8}$
	$1\frac{1}{4}$				
a	$-\frac{5}{8}$				
	$-\frac{4}{6}$				
	$\frac{5}{10}$				

GRADE EIGHT (continued)

Solving Equations

Specifics Reported by Teachers

Students are not sure what steps to follow to get the variable by itself on one side of the equation.

Reasons Given by Teachers

There seems to be some difficulty in grasping the addition principle and multiplication principle. The pupils have trouble knowing what principle to apply and when to apply it.

Techniques Suggested by Teachers

1. Teach the addition and multiplication principles in connection with additive inverses and multiplicative inverses. For example, given the equation $x - 5 = 7$, the object is to get x by itself on the left hand side. The question is, then, whether we need the additive inverse of 5 or the multiplicative inverse. Since 5 is subtracted from x we need the additive inverse. In the equation, $2x = 6$, the x is obviously multiplied by 2. Therefore, we need the multiplicative inverse of 2.
2. Relate the teaching of equations to the idea of perfectly balanced scales. Thus, in order to maintain the balance, whatever you do to one side, you must also do to the other side.

GRADE EIGHT (continued)

Areas and Volumes

Specifics Reported by Teachers

Students have difficulty with the concepts themselves rather than with the computation involved.

Reasons Given by Teachers

1. One reason for the difficulty here may be that the concepts are abstract. They are not quite sure what is being measured.
2. Also, the vocabulary and measurements in this section are quite unfamiliar to the student.
3. The difficulty here may also be related to the fact that most students have not been given sufficient exposure to figures and shapes up through the grades because of the geometry sections not being covered. In effect, the students have not been challenged enough; so, when relatively tougher topics are approached, the students' resources are not equal to the task.

Techniques Suggested by Teachers

One suggestion:

Make use of graph paper when teaching the concept of area. Figures can be drawn on the graph paper and then the area can readily be found in square centimetres. This procedure could also be related to fractions and per cents

GRADE EIGHT (continued)

by shading in part of a figure on graph paper and considering the ratio of the shaded portion to the whole figure. Relate the concept of volume to the metric unit on capacity. Containers are now readily available for comparing capacity in cubic centimetres. Experiences with such exercises should make the concepts of area and volume more meaningful to the student and thus facilitate the numerical computation of area and volume.

Trends Across Grade Levels

From the previous statement of difficulties for each grade level it is interesting to notice how some difficulties appear at only one grade level, others are present for two or three grade levels and then cease to be a problem, and some difficulties seem to persist over all the grade levels. This part of the report looks at such trends in the hope that they may point to ways to coordinate and improve the mathematics program in the system. Table III lists the major difficulties and indicates the grade levels at which they occur.

Word Problems

Table III clearly indicates that teachers consider the solving of word problems to be a major stumbling block for students. Furthermore, it presents difficulties from grade four to grade eight.

Division

Division appears to cause difficulties for students in grades four to seven. This difficulty appears to clear up in grade eight. The investigator offers two possible reasons why division does not seem to be a problem in grade eight. First, the multiplication tables are generally mastered by this time. Second, by grade eight most teachers are insisting on the most efficient method of long division which gets away from the introductory method of making successive estimates.

TABLE III

Areas Of Difficulty At Various Grade Levels

GRADE	FOUR	FIVE	SIX	SEVEN	EIGHT
Word Problems	X	X	X	X	X
Division	X	X	X	X	
Place Value	X				
Multiplication Number Facts	X	R	R	R	
Multiplication With Two and Three Digit Numbers	X				
Fractions	0	X	X	0	X
Work With Other Bases		X			
Geometry	NT	NT	X	X	R
GCF and LCM			X	0	0
Per Cent			NT	X	X
Division of Decimals			0	X	
Estimating	0	R	0	X	
Averages (of fractions)				X	
Solving Per Cent Problems			NT	X	X
Per Cent of Increase and Decrease				0	X
Integers (graphing sentences)			NT	NT	X
Solving Equations					X
Areas and Volumes				NT	X

0 - mentioned only in first questionnaire.
X - mentioned in first and second questionnaire.
R - mentioned in the Reasons section.
 - included in the questionnaire but not mentioned.
 - not included and not mentioned.
NT - not taught by most of the teachers.

Place Value

The table indicates that place value seems to be a problem at the grade four level. However, it does not seem to cause any difficulty at the other grade levels.

Multiplication Number Facts

The multiplication tables apparently cause trouble for students from grades four to grade seven. Most of the students have the tables mastered by grade eight.

Multiplication With Two and Three Digit Numbers

Pupils at the grade four level have some difficulty multiplying with two and three digit numbers. This could perhaps be partly attributed to the difficulty at this grade level with multiplication tables and place value.

Fractions

Table III highlights fractions as another topic that causes trouble at all the grade levels studied. Therefore, the student is lacking in this skill when he enters the high school.

Work With Other Bases

Of the five grade levels studied, only grade five indicates a problem when students work with bases other than ten. The investigator got the impression when visiting the schools that most teachers at other grade levels don't bother too much with this topic. They don't seem to consider it very important.

Geometry

It is obvious from the table that geometry is considered by the teachers in the study to be a major source of difficulty for students. This difficulty seems to occur at all five grade levels. However, many teachers indicated to the investigator that very little geometry is taught before grade seven.

Greatest Common Factor (GCF) and Least Common Multiple (LCM)

Teachers at grades six, seven, and eight mentioned GCF and LCM as sources of difficulty for students. Most likely, this problem continues into high school.

Per Cent

Table III clearly indicates that all aspects of this topic cause a lot of trouble for students at the grade seven and grade eight levels. It should be noted that on the first questionnaire teachers indicated that per cent and solving per cent problems are not usually taught in grade six. It is possible that the omission of this introductory material in grade six may contribute to the difficulties experienced in grades seven and eight. Furthermore, it is suggested by teachers that the problem with per cent might also be partly solved by a more careful sequencing of material. This topic, they maintain, should be taught after equations, ratios and similar figures.

Division of Decimals

Division which involves the use of decimals is a problem at the grade six and grade seven level. It does not seem to be a problem in grade eight. However, the fact that division itself is not indicated as a problem in grade eight may be related to division of decimals not being a problem in grade eight.

Estimating

The table also points to estimating as being a source of difficulty for students in grades four to seven. It is not indicated as a problem in grade eight.

Averages of Fractions

As was seen earlier, fractions present a problem to students at all five of the grade levels studied. The specific topic involving averages of fractions is apparently introduced only at the grade seven level. As might be expected, this topic involving fractions also causes difficulty for students.

Integers (Graphing Sentences)

Pupils have trouble with integers as they occur in graphing sentences at the grade eight level. However, as the table indicates, teachers indicated that they usually leave out the section on integers in grade six and seven. Again, this lack of introductory exposure to integers may explain to some extent the problem with the topic in grade eight.

Solving Equations

Table III indicates that teachers in this study feel that pupils in grade eight have considerable trouble with solving equations. While students are certainly exposed to solving equations in the earlier grades the table does not indicate a similar problem for grades four to seven. The reason for solving equations not being mentioned as a problem in the earlier grades may partly be that the earlier approach is much more intuitive. In the earlier grades students see an equation where the variable is usually represented by a place holder and they try to determine almost by trial and error what number fits into the place holder. In grade eight, however, students are introduced to very specific techniques for solving equations and this is where they have the difficulty.

Areas and Volumes

Teachers at the grade eight level apparently think that areas and volumes is a topic that many pupils have trouble with. They offer some worthwhile reasons as to why this trouble occurs. However, in this case also, the table indicates that this topic is usually left out in the previous grade. Again, the fact that students did not receive the necessary introductory treatment of areas and volumes in the previous grade may be a contributing factor to the problem in grade eight.

Summary

This section has identified the most difficult concepts and skills involved in teaching mathematics in grades four to eight in three specific elementary schools. In addition, the teachers' reasons to explain these difficulties were cited as well as their suggested teaching techniques to help overcome student difficulties. Then, an attempt was made to identify trends of student difficulties across grade levels which might suggest areas of better subject coordination.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The data presented in the previous section of this report clearly highlights the teachers' views on the major difficulties in grades four to eight mathematics in the selected schools. A careful study of that research data prompts the investigator to draw certain conclusions.

Word Problems

One of the most serious difficulties in teaching mathematics in grades four to eight in the three schools studied is solving word problems. Modern mathematics educators hoped that the increased emphasis on understanding would improve ability in problem solving. The teachers involved in this study state that this is just not happening partly because the students have a serious problem with reading. However, most teachers believe that it is an over-simplification to explain the word problem difficulty as just another example of a reading problem and then to do nothing about it. Granted, there are general reading problems involved, but, what is more important is that the specific reading skills related to problem solving probably have to be <u>taught</u>. Students do not acquire them simply by doing more problems. This view is consistent with the views of Henney (1971) mentioned earlier in the literature section.

Long Division

Students also have considerable difficulty with long division up through the grades. The teachers involved in this study identified the problem with long division to be the method of successive estimates introduced at the grade four level. While this method could be used at the beginning to facilitate understanding of the process, it would be better if students were taught the most efficient method of long division and were to acquire this skill in grade four. Presently, many students only acquire the skill of using successive estimates and find it difficult to change later on. Indeed, it is a common occurrence to see a student in high school still using this method of long division.

Multiplication Number Facts

Another source of the problem with long division is students' lack of facility with the multiplication number facts. Pupils have considerable trouble with memorizing the multiplication tables, a drill exercise which begins in grade three and is carried through to grade four. The investigator noticed a degree of inconsistency with respect to the memorization of multiplication tables. Some teachers believe that the multiplication tables should be mastered by all students by the end of grade four or even earlier. Other teachers don't insist on the memorization of the tables per se but maintain that students learn them gradually by using multiplication fact cards. Some teachers feel that the multiplication tables need not be mastered until grade five or grade six.

Estimating

Estimating is a source of the problem with long division in grades four and five and continues to be a problem in the other grades. Generally speaking, the use of estimating to determine the reasonableness of answers is not being developed as a practical skill in the schools studied.

Geometry

Another of the goals of modern mathematics is that Geometry become an essential part of the child's experience very early in his school career. In most cases in the schools studied, this goal is not being achieved. Its realization depends entirely on the teacher. If he or she feels that it is worthwhile to do some Geometry and if there is enough time, then the students may be taught a little of it. However, most teachers indicate that they just don't have the time because of all the other topics that have to be covered. They further indicate that the amount of material in the Geometry sections is just so overwhelming that they are not sure what to cover and, as a result, end up doing very little or none at all.

It appears to the investigator that the problem with Geometry is one example of a broader issue mentioned earlier in the literature, making provision for individual differences. It seems to be quite common for the individual teacher to decide that he does not have sufficient time to cover this or that topic or that his class is not bright enough to handle this or that topic. While the approach of selecting appropriate material is not necessarily bad and there is no intention to find fault with the teachers, it may result in gaps in the child's experiences that can lead to more serious problems in later grades.

Place Value and Work With Other Bases

Place value still seems to be causing trouble at the grades four and five level. Some of the teachers believe that this lack of understanding is having an effect on the acquisition of some of the computational skills such as division and multiplying with two and three digit numbers. Modern mathematics originally involved extensive use of work with other bases in order to increase understanding of place value and the base ten number system. However, the hoped for benefits of working with other bases do not seem to be materializing in the schools studied. Some of the teachers believe that work with other bases does not really improve the students' knowledge of the base ten number system. It is interesting to note that what the teachers said about place value and work with other bases is consistent with the findings of the NACOME Report.

Fractions and Decimals

Fractions and decimals are cited as a source of considerable difficulty up through the grades. Teachers attributed the trouble with these topics to the complexity of the concepts themselves. The teachers stated that students have difficulty working with parts of things rather than the whole. They recommend the use of manipulative materials in teaching these topics. This and other references of teachers to the importance of using manipulative materials in the classroom is certainly consistent with the philosophy of the new mathematics.

Per Cent

Apparently, one of the most difficult topics at the grade seven and eight level is per cent. One reason offered by teachers is that the trouble with this topic is directly related to the trouble that students have with fractions and decimals. Teachers recommend the treatment of per cent as a ratio.

Area and Volume

The teaching of area and volume is quite often mentioned by teachers as a problem at the grade eight level. This problem, they suggest, may be due partly to the lack of attention given to Geometry up through the grades. From Table III, the investigator also suggested that part of the problem may be attributed to area and volume being left out by many teachers in grade seven.

Solving Equations

Solving equations causes a considerable amount of difficulty for students at the grade eight level. Students have trouble applying the addition and multiplication principles. This weakness could be related to a lack of skill with additive and multiplicative inverses.

Recommendations

As stated earlier, this project evolved from a need for coordination between the various academic departments at Brother Rice High School and its "feeder" schools: St. Patrick's Hall School, St. Bonaventure's School, and Holy Cross School. Therefore, in this section the investigator makes certain recommendations which are based on the study and are directed to the schools involved and the school board. The investigator also makes recommendations for further study.

Schools

When Brother G.R. Bellows carried out an evaluation of Brother Rice High School in February, 1976, one of his recommendations concerned not only Brother Rice but also the three schools involved in this study. He recommended that as soon as practicable and feasible, school representatives meet with the administrators and subject area coordinators in the Brother Rice Family of Schools with a view to closer planning and coordination of school policies and instructional programs. This investigator strongly supports such a recommendation and, based on the results of this study, can see how such coordination would benefit not only the system but also the individual school. More important than that, however, the student would benefit directly in that many of the difficulties encountered, at least in the area of mathematics, could be eliminated by a better coordination of the program.

Consequently, this investigator recommends that the teachers of mathematics in St. Patrick's Hall School, St. Bonaventure's School, and Holy Cross School strive for coordination of the mathematics program within their own schools as well as within the network of the Brother Rice Family of Schools. The following recommendations could serve as guidelines for any initial efforts to coordinate the mathematics program within this system network or within an individual school.

1. It is recommended that a permanent working committee be established to work out a systematic plan for the coordination of the mathematics program in the Brother Rice Family of Schools.
2. It is recommended that all mathematics teachers establish a common approach to the teaching of long division. The common approach might take the form of the following: Use the method of successive estimates at the beginning but decide on a definite time by which pupils should have mastered the most efficient method. This could be by the end of grade four or some period in grade five.
3. It is recommended that teachers study different approaches to facilitate the memorization of multiplication number facts and decide on a definite time by which students should have the tables mastered.

The agreed upon policy here might take the form of one of the following:

a. All students should have multiplication tables mastered by the end of a specified grade;

b. Although students are involved in exercises designed for memorization of the multiplication tables as early as grade three, it is not imperative that students master the tables until the end of grade four or maybe even grade five. However, the use of multiplication fact cards should be permitted at the discretion of the teacher until the time for mastery arrives.

4. It is recommended that Geometry be an essential part of the schools' curriculum and that it be included in the mathematics curriculum at all grade levels.

5. It is recommended that teachers stock all kinds of readily available manipulative materials such as sticks, paper plates, blocks, and graph paper in the classrooms.

School Board

The Provincial Mathematics Curriculum Committee is presently involved drawing up a detailed statement of minimum objectives for mathematics. This statement would specify skills and levels of mastery for each grade level. The Roman Catholic School Board for St. John's is contributing to this effort by making recommendations to the Committee. When these objectives become available, the school board intends to have a committee of teachers study them and discuss ways in which the schools may make the best use of them. In light of this study, this investigator views such a statement of objectives as invaluable in helping the teachers determine what material must be mastered by various stages of the mathematics program. Such an approach will be a great asset in clearing up many of the difficulties in Mathematics. Recommendations 1 and 2 below refer to this proposed statement of objectives.

1. It is recommended that in adapting a statement of minimum objectives to our school situation, the school board ensure:
   a. that concepts and skills of Geometry are specified for each grade level;
   b. that the important place of estimating be clearly indicated;
   c. that the treatment of fractions and decimals be clearly indicated especially in light of the introduction of the Metric system.
2. It is recommended that the school board attempt to involve a teacher from St. Patrick's Hall, St. Bonaventure's School, and Holy Cross School on the committee of teachers that is to study this statement of minimum objectives when it becomes available.
3. It is recommended that the school board assist the schools to organize workshops designed specifically to deal with the problem areas indicated in this study. Priority should be given to planning workshops in:

a. Developing techniques for teaching reading skills in mathematics with specific attention being paid to the teaching of word problems;

b. Teaching Geometry in the elementary school;

c. The use of manipulative materials in teaching such topics as word problems, fractions, Geometry, per cent, area and volume, and work with other bases.

4. It is recommended that the school board consider all the possibilities of teacher exchanges within the system. As a mathematics department head whose experience has been limited to high school, the investigator especially enjoyed visiting the elementary schools and observing teacher and students working with mathematics at that level. Provision could be made for the visitation of other high school teachers to the elementary schools and vice versa. Teachers from both levels could exchange positions for one year. A teacher with special expertise could be on loan to a school for one year. Indeed, the possibilities seem endless.

Further Study

The investigator found it necessary to limit the study to the extent that only grades four to eight in three specific schools were involved. Also, only the major difficulties were discussed. There are several other related areas of study that could be profitably investigated.

1. A similar study could be conducted in kindergarten to grade three in the same three schools or in the high

schools in the system.

2. Another study similar to this might concentrate on the major difficulties in teaching mathematics at one particular grade level.
3. A future approach could involve an in depth study of one of the main problem areas emerging from this investigation. For example, an investigator could trace the treatment of word problems from kindergarten to grade eleven.
4. A similar study could also be extended to take in a much broader sample. Perhaps the questionnaires used in a larger sample of other schools and results compared with those obtained in this study.
5. While this study simply reported information on teacher experience and preparation, a future study might compare the difficulties listed by teachers with varying degrees of experience and preparation.

REFERENCES

REFERENCES

Adler, I. The Cambridge conference report: blueprint or fantasy? The Arithmetic Teacher, 1966, 13, 179-186.

Bassler, O.C., Beers, M.I., and Richardson, L.I. Comparison of two instructional strategies for teaching the solution of verbal problems. Journal for Research in Mathematics Education, 1975, May, 170-177.

Begle, E.G. The school mathematics study group. Mathematics Teacher, 1958, 51, 616-618.

Brownell, A. Arithmetic in 1970. The National Elementary Principal, 1959, 39, 42-45.

Bruner, J.S. The process of education. Cambridge, Massachusetts: Harvard University Press, 1960.

Corle, C.G. The new mathematics. The Arithmetic Teacher, 1964, 11, 242-247.

Dahmus, M.E. How to teach verbal problems. School Science and Mathematics, 1970, 70, 121-138.

Fehr, H.F. Sense and nonsense in a modern school mathematics program. The Arithmetic Teacher, 1966, 13, 83-91.

Gibb, E.G. Should we challenge elementary school mathematics? Mathematics Teacher, 1959, 52, 136-137.

Gibb, E.G. Basic objectives of the program. In New Directions in Mathematics. Washington, D.C.: Association for Childhood Education International, 1965, 17-21.

Green, R.W. Analysis of questions teachers ask about teaching modern mathematics in the elementary school. The Arithmetic Teacher, 1976, 23, 98-102.

Gundlach, B.H. The Greater Cleveland mathematics program. The Arithmetic Teacher, 1961, April, 192-195.

Henney, M. Improving mathematics verbal problem-solving ability through reading instruction. The Arithmetic Teacher, 1971, 18, 223-228.

Henry, B. Why can't Johnny cipher? The Arithmetic Teacher, 1971, 18, 37-39.

Higgins, J.E. An investigation of the effects of non-decimal numeration instruction on mathematical understanding. School Science and Mathematics, 1972, 72, 293-297.

Johnson, D.A. Next steps in school mathematics. *The Arithmetic Teacher*, 1967, 14, 185-189.

Jones, P.S., and Coxford, A.F., Jr. Mathematics in the evolving schools. In *A History of Mathematics Education in the United States and Canada*. Thirty-second Yearbook of the NCTM. Washington, D.C.: NCTM, 1970.

Nichols, E.D. The many forms of revolution. *NASSP Bulletin*, 1968, 52, 16-37.

Overview and Analysis of School Mathematics Grades K - 12. A report prepared by the National Advisory Committee on Mathematical Education (NACOME). Washington, D.C.: Conference Board of the Mathematical Sciences, 1975.

Pereira-Mendoza, L. The Effect of Teaching Heuristics on the Ability of Grade Ten Students to Solve Novel Problems. Unpublished Doctoral Dissertation, University of British Columbia, Canada, 1975.

Petronia, Sister Mary. A letter to parents about the "new mathematics". *The Arithmetic Teacher*, 1966, 13, 468-473.

Polya, G. *How to solve it*. New York: Doubleday and Co., 1957.

Rappaport, D. Mathematics - for whom? *The Arithmetic Teacher*, 1976, 23, 343-345.

Schoen, H.L. Some difficulty in the language of sets. *The Arithmetic Teacher*, 1974, 21, 236-237.

Sowder, L. Teaching problem-solving: Our lip-service objective? *School Science and Mathematics*, 1972, 72, 113-116.

Weaver, J.F. The Cambridge conference on school mathematics. *The Arithmetic Teacher*, 1964, 11, 207-210.

Willoughby, S.S. What is the new mathematics? *NASSP Bulletin*, 1968, 52, 4-15.

APPENDIX A

FIRST QUESTIONNAIRE

January 10, 1977

To: The teachers of mathematics in
Grades 4 - 8 at St. Pat's, St. Bon's,
and Holy Cross.

From: John McGrath

Re: Internship Project in elementary school
mathematics.

Dear Fellow Teachers:

I would like to take this opportunity to wish you all the best in the new year and to thank you for the co-operation you have already given me in my internship project.

As you know, this internship involves, in part, trying to determine some of the concepts and skills in Grades 4 to 8 mathematics which are the most difficult to teach. So far, you have given me a pretty good idea of the general problem areas. However, at this point, I have to try to zero in on some specifics.

Consequently, I have listed the contents of the various grade level texts from which your course work is selected and I will be asking you to check off topics covered and rate them as to degree of difficulty.

I realize that this is no small request, so I am relying on your generousity as well as your expertise. Perhaps, if you took a few minutes out of your day and did one page at a time, the burden might not seem so great.

I will not ask you to sign the rating sheets but I will be asking for some teacher information. This is necessary for general information in the study but will certainly not be used in any derogatory way.

While I included Metric topics, I realize that it is perhaps too early to determine what concepts will be the most difficult to teach. Most of you will probably not want to rate these topics even though you might be teaching them.

Thanks again for your encouragement and support.

John McGrath

Mathematics Internship Project — Grade FOUR

Number of years teaching mathematics ______

Number of years teaching mathematics at this grade level ______

Number of professional courses (one semester) in teaching mathematics ______

Number of University courses (one semester) in mathematics ______

Directions:

a. Place a check mark (✓) next to all the topics and sub-topics that you will attempt to cover this year (include introductory topics as well as topics for mastery).

b. Referring to the scale below, circle a number from 1 to 5 after each check mark indicating, in your opinion, the degree of the difficulty pupils will experience, with each topic.

Rating scale:

1 - no difficulty;

2 - moderate difficulty - cleared up very quickly;

3 - serious difficulty - usually taken care of;

4 - very serious difficulty - <u>some</u> never grasp at this level;

5 - extreme difficulty - <u>most</u> do not grasp at this level.

Chapter 1

Place Value	( )	1 2 3 4 5
Sets, numbers, and numerals	( )	1 2 3 4 5
Grouping by tens	( )	1 2 3 4 5
Reading numerals with two to seven digits	( )	1 2 3 4 5
Inequalities	( )	1 2 3 4 5
Introduction of billions and trillions	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5

Chapter 2

Addition and Subtraction	( )	1 2 3 4 5
Sets	( )	1 2 3 4 5
Addition and Subtraction concepts	( )	1 2 3 4 5
Equations and solutions	( )	1 2 3 4 5
Inverse relation	( )	1 2 3 4 5
Number line	( )	1 2 3 4 5
Basic principles	( )	1 2 3 4 5
Number facts through 18	( )	1 2 3 4 5
Column addition of two-and three-digit numbers	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5
Using the basic principles	( )	1 2 3 4 5
Reasoning	( )	1 2 3 4 5
Addition and regrouping	( )	1 2 3 4 5
Subtraction with regrouping	( )	1 2 3 4 5
Money-the dollar and decimal-point notation	( )	1 2 3 4 5

Geometry Unit 1

Simple closed curves	( )	1 2 3 4 5
Circles and points	( )	1 2 3 4 5
Circles and pairs of points	( )	1 2 3 4 5
Three points and a circle	( )	1 2 3 4 5

Chapter 3

Multiplication and Division	( )	1 2 3 4 5
Number in a set	( )	1 2 3 4 5
Number of equivalent sets	( )	1 2 3 4 5
Multiplication and division concept	( )	1 2 3 4 5
Repeated addition and repeated subtraction	( )	1 2 3 4 5
Number-line games	( )	1 2 3 4 5
Basic principles	( )	1 2 3 4 5
Number facts through 81	( )	1 2 3 4 5
Multiplication table	( )	1 2 3 4 5
Inverse relation	( )	1 2 3 4 5
Product sets	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5

Chapter 4

Measurement	( )	1 2 3 4 5
Length	( )	1 2 3 4 5
arbitrary units	( )	1 2 3 4 5
inches and centimetres	( )	1 2 3 4 5
inches, feet, yards, and miles	( )	1 2 3 4 5
Area	( )	1 2 3 4 5
counting squares	( )	1 2 3 4 5
approximation	( )	1 2 3 4 5
Volume	( )	1 2 3 4 5
counting cubes	( )	1 2 3 4 5
liquid	( )	1 2 3 4 5
Perimeter	( )	1 2 3 4 5
of polygons	( )	1 2 3 4 5
comparison with area	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5

Metric System

Linear Measurement	( )	1 2 3 4 5
millimetre	( )	1 2 3 4 5
perimeter	( )	1 2 3 4 5
Area Measurement	( )	1 2 3 4 5
square metre	( )	1 2 3 4 5
Volume measurement	( )	1 2 3 4 5
cubic centimetre	( )	1 2 3 4 5
Capacity Measurement	( )	1 2 3 4 5
millilitre	( )	1 2 3 4 5
Mass Measurement	( )	1 2 3 4 5
gram and kilogram relationship	( )	1 2 3 4 5
Temperature Measurement	( )	1 2 3 4 5
Celsius reference points	( )	1 2 3 4 5
Time Measurement	( )	1 2 3 4 5
24 hour clock	( )	1 2 3 4 5
reading clocks to nearest second	( )	1 2 3 4 5

Chapter 5

Special Products and Quotients	( )	1 2 3 4 5
Products that are multiples of 10, 100 and 1000	( )	1 2 3 4 5
Special missing factors	( )	1 2 3 4 5
Quotients	( )	1 2 3 4 5
Using the basic principles	( )	1 2 3 4 5
Reasoning	( )	1 2 3 4 5
A summary of the basic principles	( )	1 2 3 4 5

Geometry Unit 2

Tangents to a circle	( )	1 2 3 4 5
Central and inscribed angles	( )	1 2 3 4 5
Inscribed circles	( )	1 2 3 4 5
Circumscribed circles	( )	1 2 3 4 5

Chapter 6

Estimation	( )	1 2 3 4 5
Estimating sums	( )	1 2 3 4 5
Estimating differences	( )	1 2 3 4 5
Estimating products	( )	1 2 3 4 5
Estimating missing factors	( )	1 2 3 4 5
Estimating quotients	( )	1 2 3 4 5
Special attention to estimates leading to development of the division algorithm	( )	1 2 3 4 5
Estimation in word problems	( )	1 2 3 4 5

Chapter 7

Multiplying	( )	1 2 3 4 5
Use of the multiplication-addition principle.	( )	1 2 3 4 5
Estimation	( )	1 2 3 4 5
Inequalities	( )	1 2 3 4 5
Products involving factors with two and three digits	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5
Volume and area	( )	1 2 3 4 5

Geometry Unit 3

Edges and faces of cubes	( )	1 2 3 4 5
Patterns for cubes	( )	1 2 3 4 5
Triangular pyramids	( )	1 2 3 4 5
Cylinders and cones	( )	1 2 3 4 5

Chapter 8

Dividing	( )	1 2 3 4 5
Estimation	( )	1 2 3 4 5
Inverse relation	( )	1 2 3 4 5
Quotients and remainders	( )	1 2 3 4 5
Repeated subtraction	( )	1 2 3 4 5
Inequalities	( )	1 2 3 4 5
Reasoning	( )	1 2 3 4 5
Long-division process (two-digit divisors)	( )	1 2 3 4 5
Averages	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5

Chapter 9

Number Theory	( )	1 2 3 4 5
Odd and even numbers	( )	1 2 3 4 5
Multiples and factors	( )	1 2 3 4 5
Common factors and greatest common factors	( )	1 2 3 4 5
Prime numbers	( )	1 2 3 4 5
Function game	( )	1 2 3 4 5

Co-ordinate Geometry Unit 4

Using co-ordinates	( )	1 2 3 4 5
Graphing number pairs	( )	1 2 3 4 5
Graphing functions	( )	1 2 3 4 5
Negative numbers and graphing	( )	1 2 3 4 5

Chapter 10

Fractions	( )	1 2 3 4 5
Fractions and number pairs	( )	1 2 3 4 5
Fractions and sets	( )	1 2 3 4 5
Fractions and parts of an object	( )	1 2 3 4 5
Equivalent fractions	( )	1 2 3 4 5
Sets of equivalent fractions	( )	1 2 3 4 5
Fractions and measurement	( )	1 2 3 4 5
A check (definition) for equivalent fractions	( )	1 2 3 4 5
Lower, higher, and lowest terms	( )	1 2 3 4 5
Improper fractions	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5

Chapter 11

Rational Numbers	( )	1 2 3 4 5
Fractions and numbers	( )	1 2 3 4 5
On the number line	( )	1 2 3 4 5
Names for rational numbers	( )	1 2 3 4 5
Equality of rational numbers.	( )	1 2 3 4 5
Inequalities for rational numbers.	( )	1 2 3 4 5
Rational numbers greater than one	( )	1 2 3 4 5
Addition of rational numbers (intuitive)	( )	1 2 3 4 5
Whole numbers and rational numbers	( )	1 2 3 4 5
Mixed numerals	( )	1 2 3 4 5
Use in linear measurement	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5

Mathematics Internship Project GRADE FIVE

Number of years teaching mathematics ______

Number of years teaching mathematics at this grade level ______

Number of professional courses (one semester) in teaching mathematics ______

Number of University courses (one semester) in mathematics ______

Directions:

a. Place a check mark (✓) next to all the topics and sub-topics that you will attempt to cover this year (include introductory topics as well as topics for mastery);

b. Referring to the scale below, circle a number from 1 to 5 after each check mark indicating, in your opinion, the degree of the difficulty pupils will experience with each topic.

Rating scale:

1 - no difficulty;

2 - moderate difficulty - cleared up very quickly;

3 - serious difficulty - usually taken care of;

4 - very serious difficulty - some never grasp at this level;

5 - extreme difficulty - most do not grasp at this level.

Chapter 1

Place Value and Inequalities	( )	1 2 3 4 5
General concept of place value	( )	1 2 3 4 5
The abacus	( )	1 2 3 4 5
Thousands and millions	( )	1 2 3 4 5
Introduction of billions, trillions, and so on	( )	1 2 3 4 5
Inequalities for large numbers	( )	1 2 3 4 5
Reading of large numbers	( )	1 2 3 4 5
Expanded notation	( )	1 2 3 4 5
Work with other bases	( )	1 2 3 4 5
Roman numerals	( )	1 2 3 4 5
Other types of numerals (Egyptian, Greek, East Arabic)	( )	1 2 3 4 5
Evolution of Hindu-Arabic numerals	( )	1 2 3 4 5

Chapter 2

Equations and Operations	( )	1 2 3 4 5
Number facts	( )	1 2 3 4 5
Functions	( )	1 2 3 4 5
Emphasis on the conceptual meanings of addition, subtraction, multiplication, and division	( )	1 2 3 4 5
Emphasis on the inverse relationships between addition and subtraction and between multiplication and division	( )	1 2 3 4 5
Basic principles for multiplication and addition of whole numbers	( )	1 2 3 4 5
The multiplication-addition (distributive) principle	( )	1 2 3 4 5
Multiplication and repeated addition, division and repeated subtraction	( )	1 2 3 4 5
Number-line games	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5

Chapter 3

Reasoning	( )	1 2 3 4 5
Use of the basic principles	( )	1 2 3 4 5
Informal use of the if-then deduction statement	( )	1 2 3 4 5
Special products and quotients necessary for the development of the division and multiplication algorithms	( )	1 2 3 4 5
Mental computation	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5
Set concepts	( )	1 2 3 4 5

Chapter 4

Estimation	( )	1 2 3 4 5
Rounding off in estimation	( )	1 2 3 4 5
Estimation of sums, differences, products, and quotients	( )	1 2 3 4 5
Special attention to estimation in preparation for the division algorithm	( )	1 2 3 4 5
Estimation in word problems	( )	1 2 3 4 5

Chapter 5

Geometry and Measurement	( )	1 2 3 4 5
An introduction to the more abstract nature of geometry	( )	1 2 3 4 5
Congruence of segments	( )	1 2 3 4 5
Length	( )	1 2 3 4 5
Congruence of angles	( )	1 2 3 4 5
Angle Measure	( )	1 2 3 4 5
Constructions	( )	1 2 3 4 5
Parallel and perpendicular lines	( )	1 2 3 4 5
Quadrilaterals	( )	1 2 3 4 5
Perimeter of polygons	( )	1 2 3 4 5
Area	( )	1 2 3 4 5

Chapter 6

Computing	( )	1 2 3 4 5
The addition algorithm	( )	1 2 3 4 5
The subtraction algorithm	( )	1 2 3 4 5
The multiplication algorithm	( )	1 2 3 4 5
The division algorithm (through two-digit divisors and three-digit quotients)	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5
Reasoning exercises using addition	( )	1 2 3 4 5
Averages	( )	1 2 3 4 5
Time, rate, and distance exercises	( )	1 2 3 4 5
Additional emphasis on estimation	( )	1 2 3 4 5
Short division	( )	1 2 3 4 5
Area and perimeter	( )	1 2 3 4 5
Multiplication, division, and money	( )	1 2 3 4 5
Short stories	( )	1 2 3 4 5

Chapter 7

Number Theory	( )	1 2 3 4 5
Factors	( )	1 2 3 4 5
Multiples	( )	1 2 3 4 5
Factor trees	( )	1 2 3 4 5
Prime numbers	( )	1 2 3 4 5
Complete factorization of a number	( )	1 2 3 4 5
Prime factorization	( )	1 2 3 4 5
Set theoretic concepts, including union and intersection	( )	1 2 3 4 5
Greatest common factor	( )	1 2 3 4 5
Least common multiple	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5

Chapter 8

Fractions	( )	1 2 3 4 5
Fractions and number pairs	( )	1 2 3 4 5
Improper fractions	( )	1 2 3 4 5
Equivalent fractions	( )	1 2 3 4 5
Sets of equivalent fractions	( )	1 2 3 4 5
Building sets of equivalent fractions	( )	1 2 3 4 5
Checking for equivalence of fractions	( )	1 2 3 4 5
Higher and lower terms	( )	1 2 3 4 5
Lowest terms	( )	1 2 3 4 5
Skills for reducing to lowest terms	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5

Chapter 9

Rational Numbers	( )	1 2 3 4 5
Transition from the study of fractions to the study of numbers	( )	1 2 3 4 5
Rational numbers related to sets of equivalent fractions	( )	1 2 3 4 5
Naming rational numbers	( )	1 2 3 4 5
Rational numbers on the number line	( )	1 2 3 4 5
Equality for rational numbers	( )	1 2 3 4 5
Inequalities for rational numbers	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5

Chapter 10

Addition and Subtraction of Rational Numbers	( )	1 2 3 4 5
Using the number line to illustrate addition and subtraction	( )	1 2 3 4 5
Using parts of objects to illustrate addition and subtraction	( )	1 2 3 4 5
Whole numbers and rational numbers	( )	1 2 3 4 5
Mixed numerals	( )	1 2 3 4 5
Skills for finding the sum of two rational numbers	( )	1 2 3 4 5
Relationship between subtraction of rational numbers and addition of rational numbers	( )	1 2 3 4 5
Skills for finding differences of rational numbers	( )	1 2 3 4 5
Least-common-denominator method for finding the sum and difference of two rational numbers	( )	1 2 3 4 5
Basic principles for addition of rational numbers	( )	1 2 3 4 5
Sums and differences using mixed numerals	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5

Chapter 11

Decimals	( )	1 2 3 4 5
Decimals and place value	( )	1 2 3 4 5
Decimal notation for tenths, hundredths, and thousandths	( )	1 2 3 4 5
Addition and subtraction of rational numbers using decimal notation	(/)	1 2 3 4 5
Relationship between dollar notation and decimal notation	( )	1 2 3 4 5
Decimals and metric units	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5

Chapter 12

Geometry and Measurement	( )	1 2 3 4 5
Congruent triangles	( )	1 2 3 4 5
Copying triangles	( )	1 2 3 4 5
Area of triangles	( )	1 2 3 4 5
Space figures	( )	1 2 3 4 5
Volume	( )	1 2 3 4 5
Surface area	( )	1 2 3 4 5

Chapter 13

Multiplication and Division of Rational Numbers	( )	1 2 3 4 5
Regions and multiplication	( )	1 2 3 4 5
The number line and multiplication	( )	1 2 3 4 5
Basic principles for rational numbers	( )	1 2 3 4 5
Introduction of the concept of division of rational numbers through the inverse relation between division and multiplication	( )	1 2 3 4 5
Fractions using rational numbers	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5

Chapter 14

Co-ordinate Geometry	( )	1 2 3 4 5
Graphs on the number line	( )	1 2 3 4 5
Co-ordinates	( )	1 2 3 4 5
Graphing functions	( )	1 2 3 4 5
Integers and graphing	( )	1 2 3 4 5

Chapter 15

A New Mathematical System	( )	1 2 3 4 5
Clock arithmetic	( )	1 2 3 4 5
Addition, subtraction, and multiplication on a "twelve-clock"	( )	1 2 3 4 5
A "four-clock"		
Basic principles in clock arithmetic	( )	1 2 3 4 5
Metric Concepts	( )	1 2 3 4 5
Linear Measurement	( )	1 2 3 4 5
Multiples and sub-multiples of metre	( )	1 2 3 4 5
Speed in km/h	( )	1 2 3 4 5
Map Scale	( )	1 2 3 4 5
Area Measurement	( )	1 2 3 4 5
Apply area formulas for rectangles and squares	( )	1 2 3 4 5
Volume Measurement	( )	1 2 3 4 5
Multiples and sub-multiples of cubic metre	( )	1 2 3 4 5
Capacity Measurement	( )	1 2 3 4 5
Multiples and sub-multiples of litre	( )	1 2 3 4 5
Mass Measurement	( )	1 2 3 4 5
Multiples and sub-multiples of gram	( )	1 2 3 4 5
Temperature Measurement	( )	1 2 3 4 5
Celsius thermometer	( )	1 2 3 4 5
Time Measurement	( )	1 2 3 4 5
Leap Year	( )	1 2 3 4 5
Decade	( )	1 2 3 4 5
Century	( )	1 2 3 4 5
Time Zones	( )	1 2 3 4 5
Travel schedules	( )	1 2 3 4 5

Mathematics Internship Project

Grade SIX

Number of years teaching mathematics ______

Number of years teaching mathematics at this grade level ______

Number of professional courses (one semester) in teaching mathematics ______

Number of University courses (one semester) in mathematics ______

Directions:

a. Place a check mark (✓) next to all the topics and sub-topics that you will attempt to cover this year (include introductory topics as well as topics for mastery);

b. Referring to the scale below, circle a number from 1 to 5 after each check mark indicating, in your opinion, the degree of the difficulty pupils will experience with each topic.

Rating scale:

1 - no difficulty;

2 - moderate difficulty - cleared up very quickly;

3 - serious difficulty - usually taken care of;

4 - very serious difficulty - some never grasp at this level;

5 - extreme difficulty - most do not grasp at this level.

Chapter 1

Place Value and Number Bases	( )	1 2 3 4 5
The general concept of place value	( )	1 2 3 4 5
Reading and writing large numbers	( )	1 2 3 4 5
Inequalities	( )	1 2 3 4 5
Exponents and powers of ten	( )	1 2 3 4 5
Approximation and rounding	( )	1 2 3 4 5
Expanded notation using exponents	( )	1 2 3 4 5
Other systems of numeration	( )	1 2 3 4 5

Chapter 2

Whole numbers	( )	1 2 3 4 5
Basic number facts	( )	1 2 3 4 5
Relationships between operations	( )	1 2 3 4 5
General concepts for the operations	( )	1 2 3 4 5
Writing equations	( )	1 2 3 4 5
Basic principles for whole numbers	( )	1 2 3 4 5
Special products and quotients	( )	1 2 3 4 5
Work with function concepts	( )	1 2 3 4 5

Chapter 3

Computing	( )	1 2 3 4 5
Review of the addition, subtraction, and multiplication algorithms	( )	1 2 3 4 5
Estimation	( )	1 2 3 4 5
Extension of the division algorithm	( )	1 2 3 4 5
Short division	( )	1 2 3 4 5
Word problems requiring use of the algorithms	( )	1 2 3 4 5
Averages	( )	1 2 3 4 5
Money problems	( )	1 2 3 4 5
Time, rate, and distance	( )	1 2 3 4 5

Chapter 4

Number Theory	( )	1 2 3 4 5
Factors	( )	1 2 3 4 5
Multiples	( )	1 2 3 4 5
Factor trees	( )	1 2 3 4 5
Prime numbers	( )	1 2 3 4 5
Prime factorization of a number	( )	1 2 3 4 5
Concepts of divisibility	( )	1 2 3 4 5
Greatest common factor	( )	1 2 3 4 5
Least common multiple	( )	1 2 3 4 5
Clock arithmetic	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5

Chapter 5

Geometry and Measurement	( )	1 2 3 4 5
Introduction to the abstract nature of geometry	( )	1 2 3 4 5
Congruence of segments, angles, and triangles	( )	1 2 3 4 5
Measuring segments and angles	( )	1 2 3 4 5
Simple closed curves	( )	1 2 3 4 5
Polygons	( )	1 2 3 4 5
Area and perimeter of polygons	( )	1 2 3 4 5
Constructions	( )	1 2 3 4 5
The Pythagorean Theorem	( )	1 2 3 4 5
Parallel and Perpendicular lines	( )	1 2 3 4 5

Chapter 6

Fractions and Rational Numbers	( )	1 2 3 4 5
Fractions and number pairs	( )	1 2 3 4 5
Equivalent fractions	( )	1 2 3 4 5
Sets of equivalent fractions	( )	1 2 3 4 5
Lowest terms	( )	1 2 3 4 5
Transition from fractions to numbers	( )	1 2 3 4 5
Rational numbers on the number line	( )	1 2 3 4 5
Equality and inequality for rational numbers	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5

Chapter 7

Addition and Subtraction of Rational Numbers	( )	1 2 3 4 5
Illustrations using the number line	( )	1 2 3 4 5
Illustrations using parts of objects	( )	1 2 3 4 5
Least common denominators	( )	1 2 3 4 5
Mixed numerals and improper fractions	( )	1 2 3 4 5
The relationship between addition and subtraction of rational numbers	( )	1 2 3 4 5
Basic principles for addition of rational numbers	( )	1 2 3 4 5
Basic skills for finding sums and differences of rational numbers	( )	1 2 3 4 5
Exercises on functions	( )	1 2 3 4 5
Word problems	( )	1 2 3 4 5

Chapter 8

Multiplication and Division of Rational Numbers	( )	1 2 3 4 5
Regions and multiplication	( )	1 2 3 4 5
Illustrations with the number line	( )	1 2 3 4 5
Basic principles for multiplication of rational numbers	( )	1 2 3 4 5
The relationship between multiplication and division of rational numbers	( )	1 2 3 4 5
Development of the rules and basic skills for multiplying two rational numbers	( )	1 2 3 4 5
The distributive principle	( )	1 2 3 4 5
Word problems using rational numbers	( )	1 2 3 4 5
Number-line games with rational numbers	( )	1 2 3 4 5
Division of rational numbers	( )	1 2 3 4 5
Exercises on functions	( )	1 2 3 4 5
Estimation	( )	1 2 3 4 5

Chapter 9

Ratio	( )	1 2 3 4 5
The general concept of ratio	( )	1 2 3 4 5
Using ratio to compare sets	( )	1 2 3 4 5
Using ratio in measurement	( )	1 2 3 4 5
Scale drawings	( )	1 2 3 4 5
Word problems using ratio concepts	( )	1 2 3 4 5

Chapter 10

Decimals	( )	1 2 3 4 5
Decimals and place value	( )	1 2 3 4 5
Fractions and decimals	( )	1 2 3 4 5
Inequalities with decimal notation	( )	1 2 3 4 5
Addition and subtraction with decimal notation	( )	1 2 3 4 5
Approximation using decimal and money notation	( )	1 2 3 4 5
Multiplication and division with decimal notation	( )	1 2 3 4 5
Estimation involving decimal notation	( )	1 2 3 4 5
Introduction to the concept of repeating decimals	( )	1 2 3 4 5
Scientific notation	( )	1 2 3 4 5
Word problems using decimal notation	( )	1 2 3 4 5

Chapter 11

Geometry and Measurement	( )	1 2 3 4 5
The concept of a circle	( )	1 2 3 4 5
Circumference of a circle	( )	1 2 3 4 5
Area of a circle	( )	1 2 3 4 5
Emphasis upon some elementary concepts of space geometry	( )	1 2 3 4 5
The five regular solids	( )	1 2 3 4 5
Constructions	( )	1 2 3 4 5
Surface area	( )	1 2 3 4 5
Volume	( )	1 2 3 4 5
Geometric illusions	( )	1 2 3 4 5

Chapter 12

Percent	( )	1 2 3 4 5
Definition of percent	( )	1 2 3 4 5
Introduction of the notation for percent	( )	1 2 3 4 5
Word problems using the idea of percent	( )	1 2 3 4 5
Graphs and diagrams showing percent concepts	( )	1 2 3 4 5
Percent and ratio	( )	1 2 3 4 5
Interest	( )	1 2 3 4 5
Percents in equations	( )	1 2 3 4 5

Chapter 13

Integers	( )	1 2 3 4 5
Integers on the number line	( )	1 2 3 4 5
A variety of physical models to illustrate integers	( )	1 2 3 4 5
Basic principles for addition of integers	( )	1 2 3 4 5
Basic operations with integers	( )	1 2 3 4 5
Integer inequalities	( )	1 2 3 4 5

Chapter 14

Graphing		
Graphs of numbers on the number line	( )	1 2 3 4 5
Establishing the usual co-ordinate system	( )	1 2 3 4 5
Graphing points on the plane by relating points to pairs of numbers	( )	1 2 3 4 5
Graphs of functions	( )	1 2 3 4 5
Line graphs	( )	1 2 3 4 5

Chapter 15

Probability	( )	1 2 3 4 5
Experiments with just two possible outcomes	( )	1 2 3 4 5
An experiment with multiple outcomes	( )	1 2 3 4 5
"Equally likely" outcomes	( )	1 2 3 4 5
The language of chance and probability	( )	1 2 3 4 5

Metric Concepts	( )	1 2 3 4 5
Linear Measurement	( )	1 2 3 4 5
Indirect linear measurement	( )	1 2 3 4 5
Area Measurement	( )	1 2 3 4 5
Square kilometre	( )	1 2 3 4 5
Hectare	( )	1 2 3 4 5
Discovery of other area formulas	( )	1 2 3 4 5
Volume Measurement	( )	1 2 3 4 5
Cubic metre	( )	1 2 3 4 5
Formula for rectangular solids	( )	1 2 3 4 5
Interrelationship of volume, capacity and mass	( )	1 2 3 4 5
Capacity Measurement	( )	1 2 3 4 5
Interrelationship of volume, capacity and mass	( )	1 2 3 4 5
Mass Measurement	( )	1 2 3 4 5
Interrelationship of volume, capacity and mass	( )	1 2 3 4 5
Metric tonne	( )	1 2 3 4 5
Temperature Measurement	( )	1 2 3 4 5
Celsius thermometer	( )	1 2 3 4 5
Time Measurement	( )	1 2 3 4 5
Numeric dating	( )	1 2 3 4 5

Mathematics Internship Project — Grade SEVEN

Number of years teaching mathematics ______

Number of years teaching mathematics at this grade level ______

Number of professional courses (one semester) in teaching mathematics ______

Number of University courses (one semester) in mathematics ______

Directions:

a. Place a check mark (✓) next to all the topics and sub-topics that you will attempt to cover this year (include introductory topics as well as topics for mastery);

b. Referring to the scale below, circle a number from 1 to 5 after each check mark indicating, in your opinion, the degree of the difficulty pupils will experience with each topic.

Rating scale:

1 - no difficulty;

2 - moderate difficulty - cleared up very quickly;

3 - serious difficulty - usually taken care of;

4 - very serious difficulty - <u>some</u> never grasp at this level;

5 - extreme difficulty - <u>most</u> do not grasp at this level.

Chapter 1

Whole Numbers and Operations	( )	1 2 3 4 5
Whole Number Names	( )	1 2 3 4 5
Expanded Numerals	( )	1 2 3 4 5
Exponential Notation	( )	1 2 3 4 5
Sentences About Numbers	( )	1 2 3 4 5
Multiplication and Division	( )	1 2 3 4 5
Related Sentences	( )	1 2 3 4 5
Operations	( )	1 2 3 4 5
Division By Zero	( )	1 2 3 4 5
Associative Property	( )	1 2 3 4 5
Distributive Property	( )	1 2 3 4 5

Chapter 2		
Computations With Whole Numbers	( )	1 2 3 4 5
Addition	( )	1 2 3 4 5
Multiplication	( )	1 2 3 4 5
Subtraction	( )	1 2 3 4 5
Estimating	( )	1 2 3 4 5
Division	( )	1 2 3 4 5
Square Roots	( )	1 2 3 4 5
Broken-Line Graphs	( )	1 2 3 4 5
Pictographs	( )	1 2 3 4 5
Chapter 3		
Geometry	( )	1 2 3 4 5
Rays	( )	1 2 3 4 5
Lines	( )	1 2 3 4 5
Planes	( )	1 2 3 4 5
Curves	( )	1 2 3 4 5
Angles	( )	1 2 3 4 5
Polygons	( )	1 2 3 4 5
Diagonals of a Polygon	( )	1 2 3 4 5
Reflections	( )	1 2 3 4 5
Rotations	( )	1 2 3 4 5
Congruent Figures	( )	1 2 3 4 5
Chapter 4		
Factoring and Primes	( )	1 2 3 4 5
Factors	( )	1 2 3 4 5
Multiples	( )	1 2 3 4 5
Divisibility	( )	1 2 3 4 5
Prime and Composite Numbers	( )	1 2 3 4 5
Sieve of Eratosthenes	( )	1 2 3 4 5
Factorizations	( )	1 2 3 4 5
Prime Factorizations	( )	1 2 3 4 5
Greatest Common Factors	( )	1 2 3 4 5
Even and Odd Numbers	( )	1 2 3 4 5
Chapter 5		
Multiplication and Division	( )	1 2 3 4 5
Fractional Numerals	( )	1 2 3 4 5
Numbers of Arithmetic	( )	1 2 3 4 5
Multiplication	( )	1 2 3 4 5
Equivalent Fractional Numerals	( )	1 2 3 4 5
Comparing Numbers	( )	1 2 3 4 5
Simplifying	( )	1 2 3 4 5
Square Roots	( )	1 2 3 4 5
Division	( )	1 2 3 4 5

Chapter 6

Addition and Subtraction	( )	1 2 3 4 5
Least Common Multiples	( )	1 2 3 4 5
Addition	( )	1 2 3 4 5
Subtraction	( )	1 2 3 4 5
Addition With Mixed Numerals	( )	1 2 3 4 5
Subtraction With Mixed Numerals	( )	1 2 3 4 5
Mixed Numerals In Multiplication and Division	( )	1 2 3 4 5
Averages	( )	1 2 3 4 5

Chapter 7

Figures on a Plane	( )	1 2 3 4 5
One-, Two-, and Three-Dimensional Figures	( )	1 2 3 4 5
Rulers	( )	1 2 3 4 5
Standard Measures	( )	1 2 3 4 5
Sentences About Measures	( )	1 2 3 4 5
Perimeters	( )	1 2 3 4 5
Measuring Angles	( )	1 2 3 4 5
The Protractor	( )	1 2 3 4 5
Classification of Angles	( )	1 2 3 4 5
Angles of a Triangle	( )	1 2 3 4 5
Classification of Triangles	( )	1 2 3 4 5
Perpendicular Lines	( )	1 2 3 4 5
Parallel Lines	( )	1 2 3 4 5
Quadrilaterals	( )	1 2 3 4 5

Chapter 8

Decimal Numerals	( )	1 2 3 4 5
Decimals	( )	1 2 3 4 5
Addition	( )	1 2 3 4 5
Subtraction	( )	1 2 3 4 5
Rounding Numbers	( )	1 2 3 4 5
Multiplication	( )	1 2 3 4 5
Multiplying by Powers of 10	( )	1 2 3 4 5
Division	( )	1 2 3 4 5
Decimal Names for Fractions	( )	1 2 3 4 5
Dividing by Powers of 10	( )	1 2 3 4 5

Chapter 9

Per Cent Notation	( )	1 2 3 4 5
The Per Cent Symbol	( )	1 2 3 4 5
Finding Per Cents	( )	1 2 3 4 5
Number Sentences and Per Cents	( )	1 2 3 4 5
Per Cent of Increase or Decrease	( )	1 2 3 4 5
Decimal, Fractional, and Per Cent Equivalents	( )	1 2 3 4 5

Chapter 10		
Putting Per Cents to Work	( )	1 2 3 4 5
Ratio	( )	1 2 3 4 5
Rounding Numbers for Shopping	( )	1 2 3 4 5
Interest	( )	1 2 3 4 5
Finding Rates	( )	1 2 3 4 5
Savings Accounts	( )	1 2 3 4 5
Discount	( )	1 2 3 4 5
Commission	( )	1 2 3 4 5
100% Bar Graphs	( )	1 2 3 4 5
Circle Graphs	( )	1 2 3 4 5
Probability	( )	1 2 3 4 5
Chapter 11		
Measures	( )	1 2 3 4 5
Length	( )	1 2 3 4 5
English and Metric Units	( )	1 2 3 4 5
Tolerances	( )	1 2 3 4 5
Capacity	( )	1 2 3 4 5
Weight	( )	1 2 3 4 5
Mass	( )	1 2 3 4 5
Time	( )	1 2 3 4 5
The 24-Hour Clock	( )	1 2 3 4 5
Speed	( )	1 2 3 4 5
Chapter 12		
Areas and Volumes	( )	1 2 3 4 5
Perimeter Formulas	( )	1 2 3 4 5
Circumference	( )	1 2 3 4 5
Areas of Rectangles and Squares	( )	1 2 3 4 5
Sweeps	( )	1 2 3 4 5
Areas	( )	1 2 3 4 5
Pythagorean Property	( )	1 2 3 4 5
Volumes	( )	1 2 3 4 5
Chapter 13		
Integers	( )	1 2 3 4 5
Integers	( )	1 2 3 4 5
Addition	( )	1 2 3 4 5
Properties of Addition	( )	1 2 3 4 5
Subtraction	( )	1 2 3 4 5
Absolute Value	( )	1 2 3 4 5
Number Line Graphs	( )	1 2 3 4 5
Ordered Pairs	( )	1 2 3 4 5
Graphing Sentences	( )	1 2 3 4 5

Metric Concepts	( )	1 2 3 4 5
Linear Measurement	( )	1 2 3 4 5
Area Measurement	( )	1 2 3 4 5
Volume Measurement	( )	1 2 3 4 5
Capacity Measurement	( )	1 2 3 4 5
Mass Measurement	( )	1 2 3 4 5
Temperature Measurement	( )	1 2 3 4 5
Time Measurement	( )	1 2 3 4 5

Mathematics Internship Project — Grade EIGHT

Number of years teaching mathematics ________

Number of years teaching mathematics at this grade level ________

Number of professional courses (one semester) in teaching mathematics ________

Number of University courses (one semester) in mathematics ________

Directions:

a. Place a check mark ( ) next to all the topics and sub-topics that you will attempt to cover this year (include introductory topics as well as topics for mastery);

b. Referring to the scale below, circle a number from 1 to 5 after each check mark indicating, in your opinion, the degree of the difficulty pupils will experience with each topic.

Rating scale:

1 - no difficulty;

2 - moderate difficulty - cleared up very quickly;

3 - serious difficulty - usually taken care of;

4 - very serious difficulty - some never grasp at this level;

5 - extreme difficulty - most do not grasp at this level.

Chapter 1

Computations and Flow Charts	( )	1 2 3 4 5
Whole Numbers and Operations	( )	1 2 3 4 5
Algorithms	( )	1 2 3 4 5
Nomograph	( )	1 2 3 4 5
Multiplication	( )	1 2 3 4 5
Computers	( )	1 2 3 4 5
Flow Charting	( )	1 2 3 4 5
Division	( )	1 2 3 4 5

Chapter 2

Fractional Numbers	( )	1 2 3 4 5
Numbers of Arithmetic	( )	1 2 3 4 5
Finding Common Denominators	( )	1 2 3 4 5
Addition	( )	1 2 3 4 5
Subtraction	( )	1 2 3 4 5
Mixed Numerals	( )	1 2 3 4 5
Multiplication	( )	1 2 3 4 5
Division	( )	1 2 3 4 5

Chapter 3

Congruences and Constructions	( )	1 2 3 4 5
Congruent Figures	( )	1 2 3 4 5
Rotations and Reflections	( )	1 2 3 4 5
Comparing Figures	( )	1 2 3 4 5
Compass and Straightedge	( )	1 2 3 4 5
Angles	( )	1 2 3 4 5
Bisecting	( )	1 2 3 4 5
Parallel and Perpendicular Lines	( )	1 2 3 4 5
Constructing Perpendiculars	( )	1 2 3 4 5
Polygons	( )	1 2 3 4 5
Regular Polygons and the Circle	( )	1 2 3 4 5

Chapter 4

Decimal Numerals	( )	1 2 3 4 5
Decimal Numerals	( )	1 2 3 4 5
Comparing Numbers	( )	1 2 3 4 5
Addition	( )	1 2 3 4 5
Subtraction	( )	1 2 3 4 5
Rounding Numbers	( )	1 2 3 4 5
Estimating	( )	1 2 3 4 5
Multiplication	( )	1 2 3 4 5
Multiplying by Powers of 10	( )	1 2 3 4 5
Division	( )	1 2 3 4 5
Fractional Numerals to Decimals	( )	1 2 3 4 5
Decimal Equivalents	( )	1 2 3 4 5

Chapter 5

Working with Per Cents ( ) 1 2 3 4 5

Per Cents	( )	1 2 3 4 5
Changing to Per Cents	( )	1 2 3 4 5
The Decimal Point	( )	1 2 3 4 5
Ratio	( )	1 2 3 4 5
Finding Per Cents	( )	1 2 3 4 5
Solving Per Cent Problems	( )	1 2 3 4 5
Per Cent of Increase and Decrease	( )	1 2 3 4 5
Interest	( )	1 2 3 4 5
Discount	( )	1 2 3 4 5
Commission	( )	1 2 3 4 5

Chapter 6

Integers ( ) 1 2 3 4 5

Integers	( )	1 2 3 4 5
The Number Line	( )	1 2 3 4 5
Addition	( )	1 2 3 4 5
Subtraction	( )	1 2 3 4 5
Inverses in Subtraction	( )	1 2 3 4 5
Multiplication	( )	1 2 3 4 5
Division	( )	1 2 3 4 5
Absolute Value	( )	1 2 3 4 5
Number Line Graphs	( )	1 2 3 4 5
Graphing Integer Pairs	( )	1 2 3 4 5
Graphing Sentences	( )	1 2 3 4 5

Chapter 7

Exponential Notation ( ) 1 2 3 4 5

Exponential Notation	( )	1 2 3 4 5
0 to 1 as Exponents	( )	1 2 3 4 5
Multiplying	( )	1 2 3 4 5
Dividing	( )	1 2 3 4 5
Negative Integers as Exponents	( )	1 2 3 4 5
Powers of Powers	( )	1 2 3 4 5
Expanded Notation	( )	1 2 3 4 5
Powers of Ten	( )	1 2 3 4 5
Scientific Notation	( )	1 2 3 4 5

Chapter 8

Areas and Volumes	( )	1 2 3 4 5
Quadrilaterals	( )	1 2 3 4 5
Area	( )	1 2 3 4 5
Areas of Rectangular Regions	( )	1 2 3 4 5
Areas of Parallelograms	( )	1 2 3 4 5
Areas of Triangles	( )	1 2 3 4 5
Areas of Trapezoids	( )	1 2 3 4 5
Areas of Circular Regions	( )	1 2 3 4 5
Prisms and Surface Area	( )	1 2 3 4 5
Areas of Cylinders	( )	1 2 3 4 5
Volumes of Prisms and Cylinders	( )	1 2 3 4 5
Dimensions in Sports	( )	1 2 3 4 5

Chapter 9

Rational Numbers	( )	1 2 3 4 5
Rational Numbers	( )	1 2 3 4 5
The Number Line	( )	1 2 3 4 5
Inverses and Absolute Value	( )	1 2 3 4 5
Multiplication	( )	1 2 3 4 5
Reciprocals	( )	1 2 3 4 5
Division	( )	1 2 3 4 5
Addition	( )	1 2 3 4 5
Subtraction	( )	1 2 3 4 5
Square Roots	( )	1 2 3 4 5
Decimal Numerals for Rational Numbers	( )	1 2 3 4 5

Chapter 10

Similar Figures	( )	1 2 3 4 5
Congruent Figures	( )	1 2 3 4 5
Similar Figures	( )	1 2 3 4 5
Comparing Figures	( )	1 2 3 4 5
Ratios and Proportions	( )	1 2 3 4 5
Finding Missing Measures	( )	1 2 3 4 5

Chapter 11

Real Numbers	( )	1 2 3 4 5
Non-Repeating Decimals	( )	1 2 3 4 5
Real Numbers	( )	1 2 3 4 5
The Real Number Line	( )	1 2 3 4 5
The Pythagorean Property	( )	1 2 3 4 5
Approximating Square Roots	( )	1 2 3 4 5
Graphing Equations	( )	1 2 3 4 5
Curved Graphs	( )	1 2 3 4 5

Chapter 12

Solving Equations	( )	1 2 3 4 5
Solutions of Equations	( )	1 2 3 4 5
The Addition Principle	( )	1 2 3 4 5
The Multiplication Principle	( )	1 2 3 4 5
Equations with Subtraction	( )	1 2 3 4 5
Equations with Variables on Both Sides	( )	1 2 3 4 5

Chapter 13

Probability and Statistics	( )	1 2 3 4 5
Probability	( )	1 2 3 4 5
Compound Probability	( )	1 2 3 4 5
Frequency Distribution	( )	1 2 3 4 5
Mean	( )	1 2 3 4 5
Mode	( )	1 2 3 4 5
Median	( )	1 2 3 4 5
Broken-Line Graphs	( )	1 2 3 4 5
Bar Graphs	( )	1 2 3 4 5
Circle Graphs	( )	1 2 3 4 5

Metric Concepts	( )	1 2 3 4 5
Linear Measurement	( )	1 2 3 4 5
Area Measurement	( )	1 2 3 4 5
Volume Measurement	( )	1 2 3 4 5
Capacity Measurement	( )	1 2 3 4 5
Mass Measurement	( )	1 2 3 4 5
Temperature Measurement	( )	1 2 3 4 5
Time Measurement	( )	1 2 3 4 5

APPENDIX B

SECOND QUESTIONNAIRE

March 18, 1977

To: The teachers of Mathematics in Grades 4 - 8 at St. Pat's, St. Bon's, and Holy Cross.

From: John McGrath

Re: Internship Project in elementary school mathematics.

Dear Fellow Teachers:

I have now reached the final phase of my internship, which, needless to say, has taken much longer than I expected. I was very pleased with the cooperation I received from you with respect to the questionnaire I sent out in January. From that I was able to pin-point some of the specific topics in elementary school mathematics which are often the most troublesome.

My task now is to seek your opinions as to why certain topics cause the most trouble and ask you to share your ideas and techniques for dealing with these problem areas.

I thought at first that I could accomplish this by meeting with teachers again on an individual basis. However, while I do intend to follow up by meeting with some of you, I feel that the most efficient way to collect the necessary information is by means of another questionnaire.

Again I find myself imposing on your generousity but you will, I hope, be able to share in the benefits of receiving other teachers' ideas which I will eventually pass on to you in a handbook.

I would appreciate it very much if you would share your ideas no matter how simple you might think they are. If you have come up with anything which helps your pupils to cope with a difficult skill or concept then other teachers might be able to use it too.

Thanks again for your cooperation.

John McGrath

Mathematics Internship Project — Grade Four

Teacher's name (voluntary): ______________________________

A. The following topics have been rated by teachers as the most difficult for this grade level:

- Word problems
- Multiplication-number facts through 81 (tables)
- Geometry
- Estimation
- Multiplication with two and three digit numbers
- Long Division
- Fractions

B. If there are any other topics which you consider to be of equal difficulty or more difficult than the above, please list them here.

____

____

____

C. From the above list, including any which you might have added, please select THREE topics which you consider to be the most troublesome and, at the same time, try to zero in on the specific troublesome parts. List the three topics here.

1. ______________________________
2. ______________________________
3. ______________________________

In the space below, would you please try to explain what specific aspects of each of the three topics seem to give the most trouble.

1. ______________________________

____

2. ______________________________

____

3. ______________________________

____

D. What, in your opinion, are the <u>reasons</u> for the difficulty with those topics?

Topic number 1. ______________________________

Reasons:

Topic number 2. ______________________________

Reasons:

Topic number 3. ______________________________

Reasons:

E. What techniques or approaches can you suggest to other teachers for dealing with these troublesome topics? Please include ideas which you have developed yourself, read about, or received from other teachers. Your ideas would be greatly appreciated and, no matter how simple or elaborate, somebody else might find them very helpful.

Suggestions for dealing with topic 1. ______________________

Suggestions for dealing with topic 2. ____________________

Suggestions for dealing with topic 3. ____________________

Mathematics Internship Project | Grade Five

Teacher's name (voluntary) ______________________________

A. The following topics have been rated by teachers as the most difficult for this grade level:

- Work with other bases
- Word problems
- Informal use of the if-then deduction statement
- Geometry
- Division algorithm
- Fractions
- Decimals

B. If there are any other topics which you consider to be of equal difficulty or more difficult than the above, please list them here.

____

____

____

C. From the above list, including any which you might have added, please select THREE topics which you consider to be the most troublesome and, at the same time, try to zero in on the specific troublesome parts. List the three topics here.

1. ______________________________

2. ______________________________

3. ______________________________

In the space below, would you please try to explain what specific aspects of each of the three topics seem to give the most trouble.

1. ______________________________

____

2. ______________________________

____

3. ______________________________

____

D. What, in your opinion, are the <u>reasons</u> for the difficulty with these topics?

Topic number 1. ______________________________

Reasons:

Topic number 2. ______________________________

Reasons:

Topic number 3. ______________________________

Reasons:

E. What techniques or approaches can you suggest to other teachers for dealing with these troublesome topics? Please include ideas which you have developed yourself, read about, or received from other teachers. Your ideas would be greatly appreciated and, no matter how simple or elaborate, somebody else might find them very helpful.

Suggestions for dealing with topic 1. ______________________

Suggestions for dealing with topic 2. ______________________

Suggestions for dealing with topic 3. ______________________

Mathematics Internship Project — Grade Six

Teacher's name (voluntary): ______________________________

A. The following topics have been rated by teachers as the most difficult for this grade level:

- Expanded notation using exponents
- Extension of the division algorithm
- Short division
- Word problems
- Number theory (GCF, LCM, divisibility)
- Geometry
- Fractions
- Rational number operations
- Decimals

B. If there are any other topics which you consider to be of equal difficulty or more difficult than the above, please list them here.

____

____

____

C. From the above list, including any which you might have added, please select THREE topics which you consider to be the most troublesome and, at the same time, try to zero in on the specific troublesome parts. List the three topics here.

1. ______________________________
2. ______________________________
3. ______________________________

In the space below, would you please try to explain what specific aspects of each of the three topics seem to give the most trouble.

1. ______________________________

____

2. ______________________________

____

3. ______________________________

____

D. What, in your opinion, are the <u>reasons</u> for the difficulty with these topics?

Topic number 1. ____________________________

Reasons:

Topic number 2. ____________________________

Reasons:

Topic number 3. ______________________________

Reasons:

E. What techniques or approaches can you suggest to other teachers for dealing with these troublesome topics? Please include ideas which you have developed yourself, read about, or received from other teachers. Your ideas would be greatly appreciated and, no matter how simple or elaborate, somebody else might find them very helpful.

Suggestions for dealing with topic 1. ______________________

Suggestions for dealing with topic 2. ______________________

Suggestions for dealing with topic 3. ______________________

Mathematics Internship Project Grade Seven

Teacher's name (voluntary): ______________________________

A. The following topics have been rated by teachers as the most difficult for this grade level:

Estimating
Division
Geometry
Greatest common factors
Subtraction with mixed numerals
Averages
Division of decimals
Per Cent
Word Problems

B. If there are any other topics which you consider to be of equal difficulty or more difficult than the above, please list them here.

____

____

____

C. From the above list, including any which you might have added, please select THREE topics which you consider to be the most troublesome and, at the same time, try to zero in on the specific troublesome parts. List the three topics here.

1. ______________________________

2. ______________________________

3. ______________________________

In the space below, would you please try to explain what specific aspects of each of the three topics seem to give the most trouble.

1. ______________________________

____

2. ______________________________

____

3. ______________________________

____

D. What, in your opinion, are the <u>reasons</u> for the difficulty with these topics?

Topic number 1. ______________________________

Reasons:

Topic number 2. ______________________________

Reasons:

Topic number 3. ______________________________

Reasons:

E. What techniques or approaches can you suggest to other teachers for dealing with these troublesome topics? Please include ideas which you have developed yourself, read about, or received from other teachers. Your ideas would be greatly appreciated and, no matter how simple or elaborate, somebody else might find them very helpful.

Suggestions for dealing with topic 1. ______________________

Suggestions for dealing with topic 2. ____________________

Suggestions for dealing with topic 3. ____________________

Mathematics Internship Project   Grade Eight

Teacher's name (voluntary) ____________________

A. The following topics have been rated by teachers as the most difficult for this grade level:

Word Problems
Solving Per Cent Problems
Percent of Increase and Decrease
Interest
Integers (graphing sentences)
Exponents - powers of powers and scientific notation
Similar figures - finding missing measures
Real numbers
Solving equations

B. If there are any other topics which you consider to be of equal difficulty or more difficult than the above, please list them here.

____

____

____

C. From the above list, including any which you might have added please select THREE topics which you consider to be the most troublesome and, at the same time, try to zero in on the specific troublesome parts. List the three topics here.

1. ____________________

2. ____________________

3. ____________________

In the space below, would you please try to explain what specific aspects of each of the three topics seem to give the most trouble.

1. ____________________

____

2. ____________________

____

3. ____________________

____

D. What, in your opinion, are the <u>reasons</u> for the difficulty with these topics?

Topic number 1. ____________________

Reasons:

Topic number 2. ____________________

Reasons:

Topic number 3. ______________________________

Reasons:

E. What techniques or approaches can you suggest to other teachers for dealing with these troublesome topics? Please include ideas which you have developed yourself, read about, or received from other teachers. Your ideas would be greatly appreciated and, no matter how simple or elaborate, somebody else might find them very helpful.

Suggestions for dealing with topic 1. ______________

Suggestions for dealing with topic 2. ______________________

Suggestions for dealing with topic 3. ______________________