REDUCTION OF OFFSHORE PLATFORM RESPONSE WITH A LIQUID VIBRATION ABSORBER AND SEISMIC RESPONSE OF ELEVATED LIQUID STORAGE TANKS

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SENG CHEOK LEE
REDUCTION OF OFFSHORE PLATFORM RESPONSE WITH
A LIQUID VIBRATION ABSORBER
and
SEISMIC RESPONSE OF ELEVATED LIQUID STORAGE TANKS

by

Seng Cheok Lee, B.Sc.(C.Eng)

A Thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Engineering

Faculty of Engineering and Applied Science
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St. John's
Newfoundland, Canada
Author

Seng Cheok Lee

Approved by:

Supervisor

D. V. Reddy

Dr. D. V. Reddy, Professor
Faculty of Engineering and
Applied Science

Internal Examiner

External Examiner
To My Family
ABSTRACT

The thesis is divided into two parts - I and II.

I  Reduction of Offshore Platform Response with a Liquid Vibration Absorber

The LVA consists of a liquid-filled cylindrical container mounted near the deck level of the platform. During wave excitation, the liquid in the container swirls into an oscillating motion which interacts with the platform motion to produce a reduction in the platform response. The reduction in the platform response is largely due to energy dissipation through damping of the liquid, and to a smaller extent the inertia of the liquid. In this investigation, the effectiveness of the LVA in reducing the dynamic response of the offshore platform model is studied. The finite element programme for dynamic analysis of two-dimensional fixed offshore platforms, developed by DuVall (1), has been extended by Glacel (2) to include damping effects and a tuned mass damper. This study describes further modifications of the work with an additional option of replacing the actual spring mass model by an LVA. The relatively rigid container wall is discretised by finite beam elements. The work of Housner (3) is employed to account for the liquid sloshing loads on the container wall, while the liquid damping is based on a semi-empirical formulation by Stephens et al (4). The structural response of the model is determined for a digitised wave height spectrum, with and without the LVA in operation. The platform is analysed to determine the LVA effectiveness, and the variation of the system response with the LVA parameters. The various parameters considered are, cylinder radius,
liquid height, liquid mass, frequency, and damping. Methods of frequency tuning, and damping device mechanisms are discussed.

II Seismic Response of Elevated Liquid Storage Tanks

The structure under consideration consists of a liquid storage tank mounted on an axisymmetric pedestal. The tank is a thin elastic cylinder with an axisymmetric dome-top and a conical base which is relatively rigid. A finite element model is presented for the seismic analysis of the structure. The shell mass and stiffness matrices are generated by using the computer code SAMMSOR-II (56). The procedure of Shaaban and Nash (45) is used to generate the added liquid mass matrix which accounts for the hydrodynamic effect on the tank wall. A digitised acceleration of an earthquake is provided as the ground excitation input, and the displacement response of the whole system determined by mode superposition.
ACKNOWLEDGEMENTS

The author is very thankful to his supervisor, Professor D. V. Reddy, for his excellent guidance, valuable suggestions and encouragement at every stage of the thesis. Appreciation is expressed to Dr. M. Arockiasamy and Dr. W. Bobby for valuable ideas and enthusiastic support. The encouragement of Dean Dempster, Faculty of Engineering and Applied Science, and Dean Aldrich, School of Graduate Studies, is gratefully acknowledged. Special thanks are due to Gary Somerton for his help in the computing work. The graduate fellowship award by the University, and the partial support by the Natural Sciences and Engineering Research Council Canada under Grant No. 85619 of Dr. D. V. Reddy are gratefully acknowledged.
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<td>D</td>
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<td>I(\xi)</td>
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$M_0$  Impulsive mass of fluid in a cylindrical container

$M_1$  Convective mass of fluid in a cylindrical container

$N(\xi)$  Axial force at an arbitrary point along the element due to the dead loads above that point

$P$  Wave load vector

$R$  Radius of cylindrical container

$r$  Pile radius

$S(\omega)$  Wave spectral density

$s$  Depth of baffle below mean free surface level

$t$  Time in seconds

$u$  Fluid particle velocity

$w$  Baffle width

$x$  Displacement vector

$x_{st}$  Static displacement of main mass in a two DOF system

$z$  Upward vertical distance from the seabed

$\bar{z}$  Amplitude of sloshing liquid at tank wall

$D$  Flexural plate rigidity

$\lambda$  Wave length

$\sigma^2$  Variance of wave amplitude

$\sigma_0$  Standard deviation of deck displacement without the LVA

$\sigma_x$  Standard deviation of deck displacement with LVA

$\sigma_n$  Normalised standard deviation of deck displacement, $\sigma_x/\sigma_0$

$\omega, \Omega$  Wave Frequency

$\omega_a$  Absorber frequency, or frequency of sloshing liquid

$\omega_S$  Fundamental frequency of offshore platform

$\nu$  Kinematic viscosity

$\nu$  Poisson's ratio
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<td>$\eta$</td>
<td>Wave amplitude</td>
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<td>$\rho$</td>
<td>Density of liquid</td>
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<td>$\rho_m$</td>
<td>Mass per unit area of membrane</td>
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<td>$\rho_p$</td>
<td>Density of plate material</td>
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<td>$\mu$</td>
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<td>$F_g$</td>
<td>Foundation force vector</td>
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<td>$v$</td>
<td>Shell displacement in circumferential direction</td>
</tr>
<tr>
<td>$\vec{v}$</td>
<td>Velocity vector of liquid</td>
</tr>
<tr>
<td>$W$</td>
<td>Work done on liquid</td>
</tr>
<tr>
<td>$w$</td>
<td>Shell displacement in radial direction</td>
</tr>
<tr>
<td>$Y$</td>
<td>Generalised displacement vector</td>
</tr>
<tr>
<td>$z$</td>
<td>Upward vertical distance from the mean free surface level</td>
</tr>
<tr>
<td>$\nabla$</td>
<td>Laplacian operator</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Velocity potential of liquid</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Mode shape vector</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Liquid density</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Frequency of the liquid-filled elevated tank</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Damping ratio of the liquid-filled elevated tank</td>
</tr>
</tbody>
</table>
PART ONE

REDUCTION OF OFFSHORE PLATFORM RESPONSE WITH

A LIQUID VIBRATION ABSORBER
INTRODUCTION

It has become a necessary guideline that the fundamental frequency of the offshore platform be no less than 0.2 Hz owing to the fact that waves in the frequency range below this value have the greatest energy content. To satisfy this condition, more structural material is required, consequently increasing the cost of the platform. A possible solution of reducing the offshore platform response with considerably less cost is to install a tuned mass damper near the deck level. The tuned mass damper, whose function is to absorb energy is essentially a spring-mass-dashpot system. When suitably damped, it can be an effective device in reducing the dynamic response of the platform. The tuned mass damper has proven to be successful for tall buildings in reducing the structural response to wind excitation. The extension of the tuned mass damper concept to an offshore platform was investigated by Glaceel (2); analysis of the effectiveness of the mass damper indicated a considerable reduction in the platform response to wave loading. One setback to the use of the tuned mass damper is that a reduction of the platform response is achieved at the expense of large deflections and stresses in the damper spring. More important is the fact that a large free travel space required by the damper on the overcrowded platform can be expensive in terms of space. It is suggested that an alternative for the application of the mass damper concept is the use of a liquid vibration absorber (LVA). It is expected that the LVA will be less costly to install and maintain than the actual spring-mass damper.
The main advantage of the LVA over the actual spring-mass damper is that the amplitude of the latter is in the horizontal direction, while that of the former is in the vertical direction, overcoming large horizontal space requirements.
CHAPTER 2

LITERATURE REVIEW

The topics reviewed are the wave amplitude spectrum, analysis of wave forces on structures, response analysis of offshore structures to ice forces, and the application of the tuned mass damper concept in civil engineering.

2.1 Wave Force Analysis

Several wave theories have been developed, depending on the sea conditions. In relatively deep water, Stok's Fifth Order theory described by Skjelbreia and Hendrickson (5) is commonly used to describe steep nonlinear waves. It has been found in deep waters, that predictions of water particle velocities and acceleration using Airy Linear theory, but with integration of the forces up to the actual water surface, give results which do not differ greatly from predictions based on Stok's Fifth Order theory.

For determination of wave forces, the Morison equation is sufficient for structural members of relatively small diameter - $D/\lambda < 0.2$, provided there is no wave scattering due to the local influence of neighbouring components. The evaluation of the inertial force term in the Morison equation requires the inertial coefficient, $C_I$, for the particular structural component shape and the corresponding direction of the acceleration vector. Values of $C_I$ for various simple geometrical shapes have been given by Myers et al (6). The drag force coefficient, $C_D$, was recommended as 0.6 by Evans (7) and Hudspeth (8) for data obtained from the Gulf of Mexico. For slender
structural members in deep waters, the drag force may be omitted without losing significant accuracy.

2.2 Wave Spectrum

The sea water surface is commonly described as a stationary, ergodic Gaussian, or normal process with zero mean (9). This process can be described by representing the sea at any place and time by the wave spectrum $S(\omega)$ which has the property

$$\sigma^2 = E(\eta^2) = 2 \int_0^\infty S(\omega) d\omega \quad (2.1)$$

where

$S(\omega) =$ wave spectrum in $m^2 \cdot s$,

$\sigma^2 =$ variance of wave amplitude,

and

$E(\eta^2) =$ mean square of wave amplitude.

Several empirical derived wave spectra representing the sea conditions are available. The Pierson-Moskowitz (10), which is the most widely used, is given by

$$S(\omega) = \frac{g^2 \alpha_s}{\omega^5} \beta^{\omega/\omega'} e^{-\beta (\omega/\omega')} \quad (2.2)$$

where

$g = 9.81 \text{ m/s}^2$,

$\alpha_s = 8.1 \times 10^{-3}$,

$\beta = 0.74$,

$\omega' = g/u$,

and

$u =$ windspeed in m/s.

The P-M spectrum used in this work corresponds to seas generated by
120 km/hr storms.

2.3 Ice Forces on Structures

Although not within the scope of the thesis, an environmental loading of particular concern to the expanding interests on offshore oil industry in Newfoundland is that due to moving ice.

Comprehensive studies were carried out during the period 1930 - 60 by Korzhavin (11) and Zubov (12) for the determination of ice pressures on structures in rivers. Blenkarn and Knapp (13) discussed the ice conditions and maximum ice forces in the Grand Banks off the coast of Newfoundland. Kopaigorodski et al (14) made model studies to determine the mean ice pressures and variation of the values around the means, and concluded that sheets with small h/d (i.e. thickness to indentor width) ratios fail by instability while shear failures occur for sheets with large h/d ratios.

The dynamic response of an offshore monopod at Cook Inlet, Alaska, subjected to current-driven ice loads for dynamic response has been studied by Blumberg and Strader (15). Reddy, Cheema and Swamidas (16) developed response spectra for ice forces and used these for analysing the response analysis of a framed tower taking into account the three-dimensionality. Swamidas and Reddy (17) analysed an offshore monopod tower considering ice-structure interaction by the finite element method. A detailed literature review on the development of ice engineering is well described in Ref. 16.

2.4 Application of the Tuned Mass Damper Concept

Literature on the study of a vibration absorber dates back to 1909, when Fahm advocated the absorber to reduce the dynamic motions
of ships (18). The motion of the main system is reduced when the inertial force of the absorber interacts with the exciting force. In addition to reducing the motion of the main mass by its inertial effects, it also reduces the system response by acting as an energy dissipating device when provided with adequate damping. Morrow et al (19) showed that, under a white noise input the absolute displacement of the mass could be reduced with an absorber of adequate damping. Gupta and Chandrasekaran (20) studied the use of absorbers to limit structural response to earthquake excitations. Wirsching and Yao (21) developed analogue computer simulations of structures with absorbers, subjected to non-stationary earthquake-like excitations, and indicated reduction in internal loads for certain multistorey structures. Crandall and Mark (22) analysed a single-degree-of-freedom system subjected to a white noise base acceleration. Wirsching and Campbell (23) generalised their study to a multi-degree-of-freedom system and optimised the absorber parameters to minimise the relative motions of the system. McNamara (24) studied the application of the tuned mass damper to reduce wind-induced structural response of buildings for an elastic range, and obtained the response reductions for various damper parameters. In a study on the TMD application to offshore platforms, Glacel (2), reported significant reduction in the platform response, and an increase in fatigue life of steel-jacketed platforms.
CHAPTER 3

THEORETICAL FORMULATION

3.1 The Offshore Platform

The offshore platform considered in the analysis is an axisymmetric tower with the base fixed to the ocean bed, and supporting a load representing the deck, as shown in Fig. 3.1. The matrix equation which is used to model the dynamic motions of the platform in response to wave forces is

\[ M \ddot{x} + [K + KG] x = P(z, x, t) \]  \hspace{1cm} (3.1)

where \( M \), \( K \), and \( KG \) are the mass, stiffness, and geometric stiffness matrices, and \( P \) represents the vector of wave loads imposed on the structure. The finite element programme of Ref. 1, which is used to analyse the dynamic response of the offshore platform described above, is based on the formulation of two-dimensional beam elements.

3.2 Beam Element

Consider a non-uniform straight beam segment as shown in Fig. 3.2 having two degrees of freedom at each node, horizontal translation \((x_1, x_3)\) and rotation \((x_2, x_4)\). The displacement functions chosen, satisfying the nodal and internal continuity requirements are the cubic hermitian polynomials given as:

\[ x = \psi_1(\xi)x_1 + \psi_2(\xi)x_2 + \psi_3(\xi)x_3 + \psi_4(\xi)x_4 \]  \hspace{1cm} (3.2)

where

\[ \psi_1(\xi) = 1 - 3\xi^2 + 2\xi^3, \]  \hspace{1cm} (3.3a)

\[ \psi_2(\xi) = 3\xi^2 - 2\xi^3, \]  \hspace{1cm} (3.3b)
Fig. 3.1 Fixed Offshore Platform with a Tuned Mass Damper

Fig. 3.2 Beam Element
\[
\psi_3(\xi) = L(\xi - 2\xi^2 + \xi^3), \quad (3.3c) \\
\psi_4(\xi) = L(\xi^3 - \xi^2), \quad (3.3d)
\]

and

\[
\xi = z/L \quad (3.3e)
\]

The elements \( m_{ij} \) of the mass matrix, \( M \), are determined from the definition

\[
m_{ij} = \int_0^L m(\xi)\psi_i(\xi)\psi_j(\xi)\,d\xi \quad (3.4)
\]

where \( m(\xi) \) is the mass per unit length at an arbitrary point along the element. The elements \( k_{ij} \) of the stiffness matrix, \( K \), are given by the definition

\[
k_{ij} = \frac{E}{L} \int_0^L I(\xi)\psi''_i(\xi)\psi''_j(\xi)\,d\xi \quad (3.5)
\]

where

\( E \) is the Young's modulus,

and

\( I(\xi) \) is the moment of inertia at any point along the element.

The elements \( k_{ij} \) of the geometric stiffness matrix, \( KG \), are given by the definition

\[
k_{ij} = \frac{1}{L} \int_0^L N(\xi)\psi'_i(\xi)\psi'_j(\xi)\,d\xi \quad (3.6)
\]

where

\( N(\xi) \) is the axial force at an arbitrary point along the element due to the dead loads above that point.

A five-point scheme of the Gaussian quadrature integration is used to obtain the terms of Eqs. 3.4, 3.5 and 3.6. Detailed formulation of the matrices \( M \), \( K \), and \( KG \) is given by Ref. 1.
3.3 Damped Equations of Motion

If structural damping is considered, Eq. 3.1 becomes

\[ M \ddot{x} + CV \dot{x} + [CH + (K + KG)]x = P \]  \hspace{1cm} (3.7)

where

\[ CV = \alpha M + \beta K \] \hspace{1cm} (3.8)

Rayleigh Viscous Damping

in which \( \alpha \) and \( \beta \) are the viscous damping coefficients, and

\[ CH = i2\gamma(K + KG) \] \hspace{1cm} (3.9)

Hysteretic Damping

in which \( \gamma \) is the hysteretic damping coefficient

Replacing \([K + KG]\) by \( K \), Eq. 3.7 becomes

\[ M \ddot{x} + [\alpha M + \beta K]x + (1 + i2\gamma)Kx = P \] \hspace{1cm} (3.10)

3.4 Wave Forces

From Airy's wave theory the fluid particle acceleration, \( \frac{du}{dt} \), is given by

\[ \frac{du}{dt} = \frac{1}{2i\omega} \frac{\cosh(kz)}{\sinh(kh)} e^{i\omega t}, \] \hspace{1cm} (3.11)

where

\( H = \) wave height from tip to trough,
\( k = \) wave number, \( k = 2\pi/\lambda \),
\( \lambda = \) wave length,
\( h = \) water depth from ocean floor to still water surface,
\( z = \) distance from ocean floor upwards,

and

\( \omega^2 = kgtanh(kh). \)

The Morison equation is used to obtain the loads imposed by ocean
waves on the platform. For deep waters, the drag force term is omitted, and the horizontal force \( dP(z) \) on a differential length of a single pile, \( dz \), is given by

\[
dP(z) = C_I \rho_0 r^2 \frac{du}{dt} \, dz
\]  

(3.12)

where

\[
C_I = \text{inertial coefficient},
\]

\[
\rho = \text{density of water},
\]

and

\[
r = \text{pile radius}.
\]

The total wave force on the pile between two points \( z_1 \) and \( z_2 \), where \( z_1 < z_2 < z_{\text{surface}} \) is given by

\[
P = \int_{z_1}^{z_2} dP = \int_{z_1}^{z_2} \left( C_I \rho_0 r^2 \frac{du}{dt} \right) \, dz
\]

(3.13)

3.5 Determination of Platform Response

The displacement vector \( \mathbf{x} \) of the platform can be defined as

\[
\mathbf{x} = \mathbf{A} \mathbf{P}.
\]

(3.14)

where

\( \mathbf{A} \) is a coefficient matrix to be determined,

and

\( \mathbf{P} \) is the wave load vector, as defined in 3.1.

Observing the nature of Eqs. 3.11 and 3.13, \( \mathbf{P} \) can be written as \( \mathbf{P} e^{i\omega t} \), and Eq. 3.10 gives

\[
(-\omega^2 \mathbf{M} + i\omega \mathbf{K} + \mathbf{K} + i2\gamma \mathbf{K} + i\omega \mathbf{P}) \mathbf{A} \mathbf{P} e^{i\omega t} = \mathbf{P} e^{i\omega t}
\]

(3.15)
and

\[ A = \left[ -\omega^2 M + i\omega M + K + i2\gamma K + i\omega\beta K \right]^{-1} \] (3.16)

A set of vectors, \( P(\omega_i) \), can be obtained from a set of wave amplitudes \( H(\omega_i) \), which is used to excite the structural model. Consequently, the corresponding set of displacement vectors, \( x(\omega_i) \) are obtained from Eq. 3.14.

3.6 Wave Amplitude Spectrum

\( H(\omega_i) \) may be obtained from a condensed spectrum represented by a finite number of wave frequencies, \( \omega_i \). This condensed spectrum is obtained by determining the area between \( \omega - d\omega \) and \( \omega + d\omega \) of the Pierson-Moskowitz wave energy spectrum (as shown in Fig. 3.3a) around a specified frequency \( \omega \), taking the square root of that area, and assigning that value to the frequency. This is done for all frequencies specified, forming a histogram of frequencies, \( \omega_i \), versus the equivalent wave amplitudes, \( \eta_i \), as shown in Fig. 3.3b. The variance of the wave amplitudes can be represented by the sum of the squares of the discretised wave amplitudes,

\[ \sigma^2 = \sum_{i=1}^{N} \left( \eta(\omega_i) \right)^2, \] (3.17)

where \( N \) is the total number of discretised wave amplitudes.

3.7 Modelling of the LVA

The method of modelling the sloshing loads of a liquid with a free surface in an accelerating container, (Fig. 3.4a), developed by Ref. 3, is used in this study. When a container containing a liquid of mass \( M \) is accelerated in a horizontal direction, a certain fraction of the liquid is forced to participate in this motion as if
Fig. 3.3a Pierson-Moskowitz Wave Energy Spectrum

Fig. 3.3b Discretised Wave Amplitude
it were a solid mass, \( M_0 \), that is rigidly attached to the tank at a height, \( H_0 \), from the bottom of the container as shown in Fig. 3.4b. This mass is referred to as the impulsive mass. The motion of the tank also induces oscillations into the liquid exerting an oscillating force on the tank, similar to that exerted by a solid mass, \( M_1 \), oscillating horizontally against a restraining spring as shown in Fig. 3.4b. This mass is referred to as the convective mass since it involves the motion of the liquid. A detailed formulation of the equivalent spring-mass system is given by Ref. 3, and the resulting terms expressing the mechanism of liquid sloshing in an upright liquid-filled cylinder are

\[
M_0 = \frac{1}{\sqrt{3}} M \left( \frac{d}{R} \right) \left( \tanh(\sqrt{3} \frac{R}{d}) \right), \\
H_0 = \frac{3}{8} d \left( 1 + \frac{4}{3} \left( \sqrt{3} \frac{R}{d} \right) / \tanh(\sqrt{3} \frac{R}{d}) - 1 \right), \\
M_1 = M \frac{1}{4} \left( \frac{11}{12} \right) \sqrt{\frac{27}{8} \frac{R}{d}} \tanh(\sqrt{\frac{27}{8} \frac{d}{R}}), \\
H_1 = d \left( 1 - \cosh(\sqrt{\frac{27}{8} \frac{d}{R}}) - \frac{135}{88} / \sqrt{\frac{27}{8} \frac{d}{R}} \sinh(\sqrt{\frac{27}{8} \frac{d}{R}}) \right),
\]

and

\[
\omega^2 = \frac{g}{R} \frac{27}{8} \tanh(\sqrt{\frac{27}{8} \frac{d}{R}})
\]

where

\( \omega \) = fundamental sloshing frequency,

and

\( M_0, H_0, M_1, H_1, R \) and \( d \) are defined in Fig. 3.4.

These expressions include fluid pressures on the bottom of the container. Although expressions used by Ref. 3 involved some approximations, the results given by his theory were shown to be
within 2.5% of the classical theory, and considered sufficiently accurate for the present investigation.

3.8 Damping of Sloshing Liquids

Viscous damping of sloshing liquids is associated with the dissipation of energy during oscillations, resulting in the decrease in the amplitude of successive oscillations. This decreasing amplitude can be described by the damping ratio as

$$\xi = \frac{1}{2\pi} \frac{\text{Amplitude of any oscillation}}{\text{Amplitude one cycle later}} \quad (3.19)$$

Viscous damping depends on the container radius, $R$, liquid depth, $d$, and kinematic viscosity, $\nu$. Owing to the fact that liquid sloshing is essentially a nonlinear phenomenon, few theories are available for predicting the damping of liquid motion in a cylinder. A semi-empirical equation for evaluating the damping ratio of a liquid in a cylindrical container was obtained by Ref. 4, and given as

$$\xi = 0.56\nu^{\frac{1}{2}} R^{-3\nu} g^{-1^\nu} \tanh(1.84 \frac{d}{R}) (1 + 2(1 - \frac{d}{R})\text{csch}(3.68 \frac{d}{R})) \quad (3.20)$$

The damping ratios of a few liquids based on Eq. 3.20 are given in Table 3.1. A few liquids have relatively high kinematic viscosities, giving $\xi_a$ (for the same cylinder geometry and liquid depth) as high as 1%, but the practicability of using them is not known.

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Cyl. Radius $R$ (m)</th>
<th>Liquid Depth $d$ (m)</th>
<th>Kin. Viscosity $\nu$ at 100°C</th>
<th>Damp. Ratio $\xi$ at 100°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>6</td>
<td>1.6</td>
<td>$1 \times 10^{-6}$ m²/s</td>
<td>0.0001</td>
</tr>
<tr>
<td>Glycerin</td>
<td>6</td>
<td>1.6</td>
<td>$3 \times 10^{-3}$ m²/s</td>
<td>0.005</td>
</tr>
</tbody>
</table>
3.9 The Absorber Cylinder

The cylinder is divided into two finite beam elements with lengths depending on $H_0$ and $H_1$ as shown in Fig. 3.5a,b. When the liquid-filled cylinder is mounted on the offshore platform, five additional degrees of freedom, $x_1^i$, $x_2^i$, $x_3^i$, $x_4^i$, and $x_a$, as shown in Fig. 3.4b, are added. The response of the cylinder wall, does not have any significant effect on the dynamic motion of the platform. Hence, the flexibility of the cylinder wall need not be considered.

3.10 Relation of Liquid Mass and Frequency with Cylinder Geometry

The sloshing frequency of a liquid in a cylinder depends on the radius, $R$, and the ratio, $d/R$. Fig. 3.6, which is based on Eq. 3.18, shows two sets of curves, one joining the coordinates \( \left( \frac{d}{R}, R \right) \) corresponding to the same sloshing frequencies, and the other corresponding to the same total liquid mass in the cylinder. For the range of values $0 < \frac{d}{R} < 0.9$, an increase in the sloshing frequency, $\omega_a$, is predominantly due to an increase in $d/R$, while for the range $\frac{d}{R} > 0.9$, the increase is predominantly due to a decrease in the radius, $R$. Fig. 3.6 will be used later in the analyses for variation of absorber parameters.

Fig. 3.7 shows the $M_0/M$ and $M_1/M$ values for varying $d/R$ values. It can be seen from the figure that $M_0/M$ increases with increasing $d/R$, while $M_1/M$ decreases. The value $M_1/M$ is of some significance, as it corresponds to the oscillating mass of the vibration absorber. It may be advantageous, therefore, to choose a suitable $d/R$ ratio in order to provide a larger convective (or oscillating) mass $M_1$, but a change in $d/R$ ratio can affect other parameters; for example, if the
Fig. 3.4 Housner's Theory.  
(a) Liquid-Filled Cylinder  
(b) Equivalent Mechanical System

Fig. 3.5 Cases of Mechanical Systems

(a) Case I, $H_0 < H_1$  
(b) Case II, $H_1 < H_0$
Fig. 3.6 Mass-Frequency-Dimension Chart
Fig. 3.7 Variation of $M_0$ and $M_1$ with $d/R$
liquid mass is kept constant and d/R reduced by a certain amount, three parameters are affected:

a) Increase in convective mass, \( M_1 \),

b) Decrease in sloshing frequency of liquid in the cylinder (as can be inferred from Fig. 3.6),

c) Decrease in the impulsive mass, \( M_0 \), causing an increase in the deck mass, and consequently reducing the fundamental frequency of the platform very slightly.
CHAPTER 4
APPLICATION OF LVA TO AN OFFSHORE PLATFORM

4.1 Offshore Platform Response without an Absorber

The 366 m offshore platform of Ref. 2 is used as a model in this analysis, the fundamental frequency of which is 0.2 Hz. For convenience, the structural data of the platform are shown in Table 4.1. The model is first excited without the absorber by wave loading obtained from the discretized Pierson-Moskowitz wave energy spectrum (corresponding to 120 km/hr storm). The output is given as a series of steady-state displacements, rotation, acceleration at each nodal point, and strain at the midpoint of each element. Curve A of Fig. 4.1 gives the deck displacements corresponding to the wave amplitude input, the displacements being normalised to the standard deviation of the deck displacements. At some point on the displacement-frequency curve, the deck amplitude rises sharply reaching a peak at the wave frequency equalling the fundamental frequency of the offshore platform. This is the resonant frequency; however at this frequency, the amplitude of the platform is prevented from becoming infinitely large by structural damping.

4.2 Offshore Platform Response with an LVA

The model is excited by wave loading, but with an LVA mounted at the deck level. The container radius and liquid height are chosen from Fig. 3.6 such that the natural frequency of the absorber, $\omega_a$, is almost equal to the fundamental frequency, $\omega_s$, of the platform. The programme is run for three different conditions of liquid as shown in Table 4.2. Curve B of Fig. 4.1 shows the deck displacement for
<table>
<thead>
<tr>
<th>Table 4.1 Structural Data for the Offshore Platform</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of Platform</td>
</tr>
<tr>
<td>Water Depth</td>
</tr>
<tr>
<td>Base,</td>
</tr>
<tr>
<td>External Radius</td>
</tr>
<tr>
<td>Internal Radius</td>
</tr>
<tr>
<td>Platform,</td>
</tr>
<tr>
<td>External Radius</td>
</tr>
<tr>
<td>Internal Radius</td>
</tr>
<tr>
<td>Deck Mass</td>
</tr>
<tr>
<td>Deck Inertia</td>
</tr>
<tr>
<td>Total Structural Mass</td>
</tr>
<tr>
<td>Modulus of Elasticity</td>
</tr>
<tr>
<td>Fundamental Frequency</td>
</tr>
<tr>
<td>Viscous Damping</td>
</tr>
<tr>
<td>$\alpha$</td>
</tr>
<tr>
<td>$\beta$</td>
</tr>
<tr>
<td>Hysteretic Damping</td>
</tr>
<tr>
<td>$\gamma$</td>
</tr>
</tbody>
</table>
Run 1 (i.e. with water as the absorber liquid). The vibration of the platform deck at the old resonant frequency (0.2 Hz) is reduced to almost zero, and two new peaks are defined in the vicinity, at 0.185 Hz, and at 0.215 Hz. These two peaks are the new resonant frequencies of the system. The standard deviation of the deck displacement is reduced by 9.4%. The reduction in the standard deviation (SD) or the root mean square (RMS) value is an indication of the performance of the absorber system. The procedure for Run 2 is the same, but water is replaced with glycerin in the LVA. The effect, as shown in Curve C of Fig. 4.1, is seen to further decrease the vibration around the new resonant peaks (0.185 Hz, 0.215 Hz). The overall effect on the deck displacement is to reduce the standard deviation, $\sigma_x$, by 12.6%. This greater reduction in $\sigma_x$ is due due to a larger damping ratio in the glycerin than in water. Run 3 is a case where the damping is increased further to $\xi_a = 0.05$, and the further reduction in the deck displacement response is indicated in Curve D of Fig. 4.1; the reduction in the standard deviation of the deck response is 19%. The reduction in the standard deviation of the top element strain is 26.2%.

4.3 Mechanism of Vibration Absorption in a Two-Mass System

The subject of forced vibration of a two-mass model as shown in Fig. 4.2 may be found in a standard text on vibration, and it can be seen that a similar behaviour can be obtained, as in the case of the offshore platform. The two new resonant frequencies depend only on the mass ratio, $\mu$ (where $\mu = m/M$), and are determined by the equations

$$\left(\frac{\Omega}{\omega_a}\right)^2 = (1 + \mu/2) \pm \sqrt{\mu + \mu^2/4}$$  \hspace{1cm} (4.1)
Fig. 4.1 Normalised Deck Displacement Versus Wave Frequency
Fig. 4.2 a) Amplitudes of Forced Vibration of a SDOF System for Different Degrees of Damping. b) Amplitudes of Mass M for Various Values of Absorber Damping and Damping of the Main Spring.
where
\[ u = \frac{m}{M}, \]
\[ f = \frac{\omega_a}{\omega_s}, \]
\[ \omega_s = \text{frequency of the main mass}, \]
\[ \ddot{g} = \frac{\Omega}{\omega_s}, \]
\[ \xi = \text{damping ratio of the main spring}, \]

and
\[ \xi_a = \text{damping ratio of the absorber spring}. \]

Den Hartog (18) showed in his study of the dynamic motion of a two-mass model, that the displacement of the main mass is given as
\[ \frac{x}{x_{st}} = \sqrt{\frac{(2\xi_a \ddot{g})^2 + (\ddot{g}^2 - f^2)^2}{(2\xi_a \ddot{g})^2 (\ddot{g}^2 - 1 + \mu \ddot{g}^2)^2 + (\mu f^2 \ddot{g}^2 - (\ddot{g}^2 - 1)(\ddot{g}^2 - f^2))^2}} \quad (4.2) \]

The tuning of the absorber to reduce vibration of the main mass is obtained from
\[ f = \frac{1}{1 + \mu} \quad (4.3) \]

Brock (25) and Den Hartog (18) obtained an expression for optimum damping for the same model as follows:
\[ \xi_a^2 = \frac{3\mu}{8(1 + \mu)^3} \quad (4.4) \]

The results of Refs. 2, 18 and 25 will be used later as guidelines for predicting the optimum absorber parameters required to minimise the offshore platform response to wave loading.

4.4 Comparison of Response Reductions

To compare the performance of the LVA with that of the tuned mass damper of Ref. 2, the original programme of Ref. 2 including a tuned
mass damper is used. The inputs to his programme, as shown in Table 4.3, are the equivalent spring-mass-dashpot parameters of the LVA used in 4.2. The comparison of the deck displacement response between the two cases is shown in Fig. 4.3. With the LVA, a larger reduction in the standard deviation of deck response is noted; \( \sigma_x \) for the deck response with LVA is 2% smaller than that with a tuned mass damper. This small difference is likely to be due to the additional overturning moment caused by the elevations \( H_1 \) and \( H_0 \), of the masses \( M_1 \) and \( M_0 \) respectively, increasing the effectiveness of the LVA.
Table 4.2  Damping Ratio of LVA

<table>
<thead>
<tr>
<th>Run</th>
<th>Cyl. Dimensions</th>
<th>Type</th>
<th>Liquid Properties</th>
<th>Density</th>
<th>Damp. Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R</td>
<td>d</td>
<td></td>
<td>ν</td>
<td>ρ</td>
</tr>
<tr>
<td>1</td>
<td>5.6 m</td>
<td>1.6 m</td>
<td>Water</td>
<td>1x10⁻⁶ m²/s</td>
<td>1.00 Mg/m³</td>
</tr>
<tr>
<td>2</td>
<td>5.6 m</td>
<td>1.6 m</td>
<td>Glycerin</td>
<td>3x10⁻³ m²/s</td>
<td>1.25 Mg/m³</td>
</tr>
<tr>
<td>3</td>
<td>5.6 m</td>
<td>1.6 m</td>
<td>Water with Damp Device</td>
<td>1x10⁻⁶ m²/s</td>
<td>1.00 Mg/m³</td>
</tr>
</tbody>
</table>

Table 4.3  Equivalent Mass Damper Parameters

<table>
<thead>
<tr>
<th>LVA</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cylinder Radius, R</td>
<td>5.5 m</td>
</tr>
<tr>
<td>Liquid Depth, d</td>
<td>1.6 m</td>
</tr>
<tr>
<td>Kin. Viscosity, ν</td>
<td>10⁻⁶ m²/s</td>
</tr>
<tr>
<td>Liquid Density, ρ</td>
<td>1 Mg/m³</td>
</tr>
<tr>
<td>Damper Mass, M₁</td>
<td>97.8 Mg</td>
</tr>
<tr>
<td>Mass Damper of Ref. 2</td>
<td>Mass, M₀, added to Deck Mass</td>
</tr>
<tr>
<td>Damper Frequency, ωₐ</td>
<td>0.2003 Hz</td>
</tr>
<tr>
<td>Damping Ratio, ωₐ</td>
<td>0.0001</td>
</tr>
</tbody>
</table>
Fig. 4.3 Comparison of the Performance of LVA with Glacel's Damper
5.1 Parameters Affecting Platform Response

The response of the platform with an LVA depends on the parameters $\mu$, $f$, $\bar{g}$, and $\xi_a$

where

$$
\mu = \frac{\text{Mass of Liquid in Tank}}{\text{Structural Mass of Platform}} = \frac{\rho \pi R^2 d}{M},
$$

$$
f = \frac{\text{Sloshing Frequency of Absorber}}{\text{Fundamental Frequency of Platform}} = \frac{\omega_a}{\omega_s},
$$

$$
\bar{g} = \frac{\text{Wave Frequency}}{\text{Fundamental Frequency of Platform}} = \frac{\Omega}{\omega_s},
$$

and

$$
\xi_a = \text{Damping Ratio of Absorber}.
$$

For convenience, $\mu$ will be referred to as the mass ratio, $f$ the absorber frequency, $\bar{g}$ the wave frequency, and $\omega_a$ the absorber damping ratio. All the four parameters are in dimensionless form.

5.2 Response Reduction with an LVA (without Damping Device)

To find the optimum absorber performance, the mass ratio, $\mu$, and the absorber frequency, $f$, are varied by varying the the tank radius, $R$, and the depth to radius ratio, $d/R$.

5.2.1 Liquid Mass Variation

The platform is analysed, with glycerin as the absorber fluid, the absorber frequency, $f$, held constant at 1.0, and the mass ratio,
μ, varied. In order to vary μ without offsetting the specified value of f, appropriate values of R and d/R are read from Fig. 3.6. The deck displacement responses obtained from the analyses for three different values of μ are shown in Fig. 5.1; the displacements are normalised to the SD of the deck displacement response without the absorber. The minimum responses for all the three cases occur at the wave frequency, $\bar{g} = f$. Of the three cases, $\mu = 0.013, 0.027$ and $0.053$, the smallest SD value of the deck displacement response is obtained for $\mu = 0.013$, for which the two resonant peaks of the displacement-frequency curve are almost at the same level.

The procedure is repeated for more intermediate values of μ. Fig. 5.2 shows the deck displacement-mass ratio curves, each of which corresponds to a particular wave frequency, $\bar{g}_i$. The dominant wave frequencies relevant to the structural model are those in the region around unity as shown in Fig. 5.2 i.e. $\bar{g} = 0.85$ to 1.05. The curves corresponding to these frequencies are indicated by the broken line in Fig. 5.2.

The entire series of simulations are repeated with $f = 1.08$. Fig. 5.3 presents plots of $\sigma_n - \mu$ curves for the cases $f = 1, 1.08$, where $\sigma_n$ is the SD of deck displacement response normalised to the SD of deck displacement response without an LVA. The curves indicate considerable fluctuation of $\sigma_n$ with varying $\mu$ in both cases. This, as will be shown later, is due to the absence of adequate damping in the liquid.
Fig. 5.1 Normalised Deck Displacement Versus Wave Frequency; No Damping Device
Fig. 5.2 Normalised Deck Displacement Versus Mass Ratio
Fig 5.3 Normalised SD of Deck Displacement Versus Mass Ratio: Without Damping Device
5.2.2 Frequency Variation

The absorber frequency, $f$, is varied, keeping the mass ratio, $\mu$, constant at 0.013, and the fluid viscosity the same as in 5.2.1. The appropriate absorber dimensions are obtained from Fig. 3.6 in order to keep $\mu$ constant while varying $f$. The deck displacement response for three different values of $f$, 0.8, 1.0 and 1.08, are shown in Fig. 5.4. The resonant peaks exhibit shifts as the absorber frequency, $f$, is increased or decreased. For $f < 1$, the right resonant peak is the dominant one. As $f$ tends to unity, the right peak falls while the left one rises; at $f = 1.0$, the peaks are at the same level and the minimum SD for the displacement response is obtained. As $f$ is increased further, the left peak continues to rise while the right one falls. Theoretically, as $f$ approaches infinity, the frequency of the absorber will be too large to affect the response of the structure and the right resonant peak disappears while the left peak changes in magnitude, and shifts in position along the wave frequency axis until it is almost similar to the original resonant frequency peak (without absorber).

Fig. 5.5 shows plots similar to Fig. 5.2, but for varying values of $f$ instead of $\mu$. The dominant wave frequencies relevant to the structural model are again shown to be around unity, i.e. $\bar{g} = 0.9$ to 1.05, and the curves corresponding to these frequencies are indicated by the broken line in Fig. 5.5. It can be inferred from the minimum points for the curves (indicated by crosses), that the response of the platform to each particular wave frequency, $\bar{g}_i$, is minimum at the absorber frequency value, $f_i = \bar{g}_i$; this result can be used to optimise the effectiveness of the absorber by shifting the absorber frequency.
Absorber Fluid: Glycerin

$\mu = 0.013$

$\bar{g} \rightarrow$

Fig. 5.4 Normalised Deck Displacement Versus Wave Frequency; No Damping Device
Absorber Fluid: Glycerin

μ = 0.013

\( \bar{g} = 0.95 \)  
\( \bar{g} = 0.9 \)  
\( \bar{g} = 1.1 \)  
\( \bar{g} = 0.85 \)  
\( \bar{g} = 1.0 \)  
\( \bar{g} = 1.05 \)  
\( \bar{g} = 1.10 \)

\( f + \)

Fig. 5.5 Normalised Deck Displacement Versus Absorber Frequency
into the range of dominant wave frequencies relevant to the platform model, i.e. 0.9 - 1.05. Fig. 5.6 shows the $\sigma_n - f$ curves for the mass levels $\mu = 0.013, 0.027$. Again, considerable fluctuation of $\sigma_n$ is noted as the absorber frequency, $f$, is varied, owing to inadequate damping.

5.3 LVA Application with a Damping Device

To increase energy dissipation in the absorber, the damping ratio is increased from 0.005 to 0.05 by introducing a damping device in the absorber, as discussed in 5.4.

5.3.1 Mass Variation

The mass ratio, $\mu$, is varied, and the absorber frequency, $f$, kept constant at 0.8, 1.0, and 1.08. Fig. 5.7 shows curves of normalised SD of deck displacement response, $\sigma_n$, versus the mass ratio, $\mu$. Each of the curves indicated decreasing $\sigma_n$ with increasing $\mu$. For $f = 1.0$ and $\mu = 0.027$, the reduction in the SD value of the deck displacement response is 22%. As $\mu$ is increased, the percentage of reduction in the SD value becomes smaller.

5.3.2 Frequency Tuning

Now, the absorber frequency, $f$, is varied, keeping the mass ratio, $\mu$, constant at 0.013, 0.027, and 0.04. Plots of $\sigma_n - f$ curves corresponding to the specified mass ratios are presented in Fig. 5.8. For each case of $\mu$, an optimum absorber frequency, $f_{\text{opt.}}$, is obtained. Table 5.1 shows the values of $f_{\text{opt.}}$, and the reduction in the SD of the deck displacement response for each case of $\mu$. The optimum frequency values, $f_{\text{opt.}}$, as seen from Table 5.1, approach unity as
Fig. 5.6 Normalised SD of Deck Displacement Versus Absorber Damping: Without Damping Device
Fig. 5.7 Normalised SD of Deck Displacement Versus Mass Ratio: With Damping Device
Fig. 5.8 Normalised SD of Deck Displacement Versus Absorber Frequency: With Damping Device
\( \mu \) tends to zero. This behaviour shows consistency with Eq. 4.3 which was obtained from the two-mass system formulation of Ref. 18. The reduction in the SD value of the top element strain in Case 3 is 32.8 %.

<table>
<thead>
<tr>
<th>Case</th>
<th>Mass Ratio, ( \mu )</th>
<th>( f_{opt} )</th>
<th>Reduction in SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.013</td>
<td>0.989</td>
<td>18.8 %</td>
</tr>
<tr>
<td>2</td>
<td>0.027</td>
<td>0.952</td>
<td>23.5 %</td>
</tr>
<tr>
<td>3</td>
<td>0.040</td>
<td>0.940</td>
<td>25.0 %</td>
</tr>
</tbody>
</table>

5.3.3 Optimum Damping

To determine the optimum damping, the damping ratio of the absorber, \( \xi_a \), is varied with \( f = 1.0 \) and \( \mu = 0.013 \). The \( \sigma_n - \xi_a \) curve is presented in Fig. 5.9. As \( \xi_a \) increases, \( \sigma_n \) decreases initially, then reaching a minimum at an optimum damping, \( \xi_a (\text{optimum}) \), and the gradually increases. For \( f = 1.0 \) and \( \mu = 0.013 \), the optimum damping, \( \xi_a (\text{optimum}) = 0.07 \).

5.4 Damping Devices

Adequate damping of liquid motion may be provided by using fixed baffles attached to the cylinder wall as shown in Fig. 5.10. The baffles may be perforated with the advantage of considerable lightness, and their position along the vertical axis of the cylinder can be controlled. The main effect is the change of first-mode sloshing into a rotary motion. The liquid oscillating in its fundamental mode produces a wave having its maximum amplitude at the cylinder wall.
Fig. 5.9 Optimum Absorber Damping
The direction of the liquid motion near the wall is almost vertical and thus normal to the baffle located a small distance, \( s \), beneath the free surface. Energy dissipation results from the baffle resistance on the wave motion, which damps the liquid. The damping of the liquid free surface oscillations in a cylinder by a flat solid ring baffle has been predicted theoretically by Miles (26) in the following form:

\[
\xi_a = 2.83 e^{-4.6s/R} \left( \frac{2w}{R} - \left( \frac{w}{R} \right)^2 \right) 1.5 \left( \frac{z}{R} \right)^{0.5} \quad (5.1)
\]

where \( R \), \( s \), \( w \), and \( z \) are related to the tank geometry as shown in Fig. 5.10. Figs. 5.11 a,b are based on Eq. 5.1 from which \( \xi_a \) is obtained with the given parameters \( R \), \( s \), \( w \), and \( z \). Fig. 5.11 shows that damping decreases exponentially with increasing \( s/R \), indicating that damping is greatest when the baffle is located at the mean free surface level. For the given parameters, \( w/R = 0.1 \) and \( z/R \), with the baffle attached at the mean free surface level, \( \xi_a = 0.075 \). For ring baffles, a damping ratio as high as 0.2 to 0.3 is possible. The lowering of sloshing frequencies due to the introduction of a baffle must be taken into account; nevertheless, an increase in damping in the absorber with the use of a ring baffle reduces the structural response, as it increases the rate of energy dissipation in the structural system.

### 5.5 Frequency Tuning Devices

The natural frequencies of the sloshing liquid in a cylinder can be controlled without adjusting the cylinder geometry and liquid height by using movable devices which are either immersed in the liquid or just cover the liquid surface. Siekmann and Chang (27) made
Fig. 5.10 Damping Device
Fig. 5.11a Damping Factor of LVA as a Function of Baffle Depth for $z/R = 0.05$
Fig. 5.11b  Damping Factor of LVA as a Function of Baffle Depth for $z/R = 0.1$
a theoretical study on the control of natural frequencies of a sloshing liquid in an upright cylindrical container. The liquid domain is divided into two regions, \( \Phi_1 \) and \( \Phi_2 \), by means of an elastic mat which is either a membrane or a plate as shown in Fig. 5.12. The mat is attached to the tank wall at a height \( h \), such that the edge of the mat, though movable, will remain at this height during sloshing. For the case where the membrane covers the liquid surface, the fundamental frequency is given by Ref. 27 as

\[
\omega^2 = \frac{\rho g k + \tau k^3}{\rho \cosh(kd) + \rho_m k \sinh(kd)} \sinh(kd) \tag{5.2}
\]

where

- \( \rho \) = density of fluid,
- \( g \) = gravitational acceleration,
- \( \tau \) = tensile force per unit length in membrane,
- \( \rho_m \) = mass per unit area of membrane,
- \( k \) = constant, \( k = 1.84/R \),

and

- \( R \) = cylinder radius.

For the case of an elastic plate covering the free surface,

\[
\omega^2 = \frac{D k^5 + \rho g k}{\rho_p k \sinh(kd) + \rho \cosh(kd)} \sinh(kd) \tag{5.3}
\]

where

- \( D \) = flexural plate rigidity, \( D = \frac{E t^3}{12(1-\nu^2)} \),
- \( E \) = Young's modulus,
Fig. 5.12 Frequency Tuning Membrane
\[ v = \text{Poisson's ratio}, \]
\[ \rho_p = \text{density of plate material}, \]
and
\[ t = \text{plate thickness}. \]

Figs. 5.13 a, b, and c are based on Eq. 5.2, giving the sloshing frequencies for the given parameters \( \rho, \tau, \rho_m, d \) and \( R \). Given the parameters, \( \rho = 1000 \text{ kg/m}^3, d = 1.3 \text{ m}, \) and \( R = 6 \text{ m} \), the natural sloshing frequency as obtained from Fig. 3.6 is \( 1.08 \text{ rad/s} \). With the application of an elastic membrane (\( \tau = 30 \text{ kN/m}^3, \rho_m = 20 \text{ kg/m}^3 \)) covering the free surface of the liquid, the new sloshing frequency, as obtained from Fig. 5.13c, is \( \omega_a = 1.2 \text{ rad/s} \).

It is of interest to note that the membrane, besides increasing the frequency of the liquid, also increases the damping in the liquid motion. In this respect, the membrane may prove to be effective in reducing the response of the structure if the right damping and frequency can be achieved simultaneously. Another point to note is that by introducing a membrane or a plate, the sloshing force tends to be suppressed, and a penalty is involved in that part of the convective mass, \( M_1 \), that is converted into the impulsive mass. This has the effect of reducing the effectiveness of the absorber (as the oscillating liquid mass is smaller), and increasing the deck mass.
Fig. 5.13a  Change of Sloshing Frequency Using an Elastic Membrane with \( \tau = 10 \) kN/m
Fig. 5.13b Change of Sloshing Frequency Using an Elastic Membrane with $\tau = 20$ kN/m
Fig. 5.13c Change of Sloshing Frequency Using an Elastic Membrane with $\tau = 30$ kN/m
CHAPTER 6

CONCLUSIONS AND DISCUSSION

6.1 Conclusion

6.1.1 Absorber without Damping Device

i) When the LVA is mounted on the platform model, the platform response was reduced. The response reduction is dependent on the liquid mass and frequency, and the damping ratio of the liquid motion (in another sense, dependent on the cylinder radius, the depth to radius ratio, and the liquid viscosity and density). The reduction of the standard deviation of the deck displacement response for a 5.5 m - radius cylinder with glycerin filled to depth 1.6 m is 12.6 %.

ii) The platform response fluctuates in both cases - varying liquid mass and sloshing frequency; hence the optimum mass and frequency for minimum response cannot be defined.

6.1.2 Absorber with Damping Device

i) When the absorber fluid is damped with a damping device, the response reduction is significantly greater. For \( f \) (absorber frequency) = 1.0, and \( \mu \) (mass ratio) = 0.013, the optimum damping, \( \xi_a \) (optimum) = 0.07.

ii) The platform response reduces with an increase in liquid mass, but as the liquid mass is increased further in the subsequent simulations, the percentage of response reduction is smaller.

iii) An optimum absorber frequency can be obtained for each value of liquid mass. For the mass ratio, \( \mu = 0.04 \), the optimum absorber
frequency, \( f = 0.94 \); the corresponding reduction in the standard deviation of the deck displacement response is 25%.

iv) The reduction in the standard deviation in the top element strain for \( \mu = 0.04, f = 0.94, \) and \( \xi_a = 0.05 \), is 32.8%. The reduction in the strain response increases the fatigue life of the structure.

6.2 Discussion

The standard deviation (SD or \( \sigma \)) of the deck displacement is indicative of the overall structural response, and the SD of the element strain indicates the stress response of the member. The reduction in the SD of member stresses reduces the probability of exceeding the ultimate stress limit of the member. Moreover, if fatigue is the mode of failure, the number of cycles to failure is increased when the SD of member stress levels is reduced (28, 29). The reduction of the deck acceleration is important from the consideration of human comfort and instrumentation.

The advantages of the LVA over the tuned mass damper are:

i) It is less costly in installation and maintenance,

ii) As the LVA is cylindrical, it can function in any horizontal direction,

iii) The horizontal liquid motion is restricted by the cylinder wall, hence less platform space is required for the absorber operation and

iv) The liquid mass can be easily controlled by pumping operations to and from the base of the platform.
PART TWO

SEISMIC RESPONSE OF ELEVATED LIQUID STORAGE TANKS
INTRODUCTION

Investigations of the seismic behaviour of elevated liquid storage structures have been motivated by numerous events of earthquake damage and destruction of elevated water tanks in places of high seismicity. Housner (3) proposed a method of determining the hydrodynamic pressures in a rigid fluid container subject to horizontal accelerations. This enabled the representation of the elevated liquid storage tank by a two-mass system, a method commonly used in succeeding investigations on the seismic response analysis of elevated water tanks. Although the two-mass system is a reasonable representation for the elevated water tank, study of the seismic behaviour appears to be adequate only for the supporting structure. A report by Hill and Biggs (30) indicated that although most of the earthquake damage to water towers occurred in the supporting structure, some were due to the structural inadequacy of the storage tank itself. Veletsos and Yang (31) reported that the hydrodynamic effects in flexible tanks may be larger than those in rigid tanks of the same dimensions. The current seismic design coefficients for water storage towers are of a significantly higher order than those for buildings. This possible conservatism may be a reflection of uncertainties (e.g. those reported by Refs. 30 and 31) in the stress behaviour induced by hydrodynamic effects.

The majority of elevated water tanks have been in the 600 m³ class or smaller. More recently, tank sizes up to 12000 m³ (Fig. 7.1e) have been constructed, most of them in steel or concrete. The
supporting structure may consist of a frame (Fig. 7.1a), a multicolumn assembly (Fig. 7.1b), or an axisymmetric pedestal (Figs. 7.1c,d,e). As many elevated water tanks of today are much larger in dimensions than those previously constructed, more sophisticated method of analysis are required to determine their seismic behaviour. The considerable need for predicting the response of the larger liquid storage tower structures, incorporating detailed shell behaviour of the tank, has initiated the studies described in this report. The work is restricted to the formulation of a procedure for seismic analysis of axisymmetric water towers, an extension of the work by Balendra and Nash (32) on the seismic response analysis of ground-supported liquid storage tanks. Much of the information is presented without proof, and mathematical details may be followed up through the quoted references. A study of the soil-structure interaction effect is beyond the scope of this work.
Fig. 7.1 Types of Water Tanks
8.1 Dynamic Analysis of Ground-Supported Cylindrical Tanks

Early studies of the forced vibration of contained liquids go back to 1950; significant and theoretical investigations are those of Jacobsen (33), and Jacobsen and Ayre (34) which dealt with the seismic behaviour of rigid cylindrical tanks. Ref. 33 calculated the forces and moments induced by the fluid inside a cylindrical tank subjected to horizontal acceleration. Ref. 34 conducted experiments on liquid-filled cylindrical tanks, with damped harmonic excitation of tank bases. The work was followed by Housner's (3) formulation of simplified expressions for obtaining the hydrodynamic pressures developed in rigid ground-supported tanks, subjected to horizontal acceleration. Satisfactory agreement between the behaviour of petroleum storage tanks in the 1964 Alaskan earthquake and the predictions based on the work of Ref. 3, has been demonstrated by Shepherd (35). During the past decade, there has been a rapid development in the subject related to the sloshing behaviour of liquids in fuel tanks of liquid propellant rockets. In the field of aerospace technology, Lindholm, Kana and Abramson (36), Chu (37), Kana and Chu (38), DiMaggio and Bleich (39), and Fung, Sechler and Kaplan (40) made valuable contributions to the response analysis of pressurized cylinders containing liquids. Luk (41) presented a development of a finite element model, and some numerical examples for liquid sloshing problems involving both rigid and axisymmetric containers. Edwards (42) investigated coupled interaction between the
elastic walls of a cylindrical tank and the contained liquid, and simulated the liquid-filled tank with ground motion by numerical integration. Bauer and Siekmann (43) analysed the general case of hydroelastic coupled oscillations of a partially-filled liquid container with a flexible bottom and an elastic side wall.

More recently, Wu, Mouzakis, Nash and Colonell (44), Shaaban and Nash (45), and Balendra and Nash (32) made significant contributions to the advancement on the seismic analysis of ground-supported liquid-filled cylindrical tanks. Ref. 44 presented an analytical formulation and developed a computer programme for determination of natural frequencies of free vibration of an elastic circular shell partially filled with liquid. Ref. 45 developed a finite element computer programme for the time-history analysis of seismic response of ground supported liquid-filled tank, while Ref. 32 expanded the work to include an axisymmetric dome top. Recent experimental investigations by Clough and Clough (46) on the seismic behaviour of ground-supported liquid-filled cylindrical tanks have indicated significant cross-sectional distortion of elastic tanks caused by uplifting of the base, and geometric imperfections of the side wall, which had not been included in theoretical predictions.

8.2 Seismic Response of Elevated Water Tanks

Literature related to the dynamic behaviour of elevated water tanks dates back to as early as 1936, when Carder (47) tested steel water tanks, by a series of pull-back tests on the supporting towers, to study the effects of the foundation conditions and tower motion in seismic events. Extensive model studies were undertaken by Ruge (48)
with the object of deriving data useful for the seismic design of water tanks. Many of the later investigations used the method of determining hydrodynamic pressures proposed by Ref. 3. Both Cloud (49) and Blume (50) have reported reasonable correlation between the observed behaviour of water tanks and the vibrational parameters obtained from the method of Ref. 3. Chandrasekaran and Krishna (51) have reported an analytical investigation of the seismic behaviour of reinforced concrete water towers. Simplification of the elevated tank with a two-mass idealization have also been used in the work of Sonobe and Nishikawa (52), Ifrim and Bratu (53), Garcia (54), and Shepherd (55). Ref. 55 studied the free vibrations of an axisymmetric prestressed concrete elevated water tank, and carried out a simple pull-back test to substantiate the validity of the theoretical model.
9.1 Shell Mass and Stiffness Matrices

The mass and stiffness matrices, $M_s$ and $K_s$, for the axisymmetric shell structure are generated by using the finite element computer code SAMMSOR-II (56). The programme idealises the shell of revolution by curved ring elements, developed by Stricklin, Navaratna and Pian (57). The displacements of an element in the meridional, circumferential, and normal directions, denoted by $u$, $v$, and $w$ respectively, are represented by a Fourier representation in the circumferential direction as follows:

$$\sum_m u_m(z,t) \cos m\theta,$$  \hspace{1cm} (9.1a)

$$\sum_m v_m(z,t) \sin m\theta,$$  \hspace{1cm} (9.1b)

and

$$\sum_m w_m(z,t) \cos m\theta,$$  \hspace{1cm} (9.1c)

where $u_m$, $v_m$, and $w_m$ are the generalised displacements for the $m$th circumferential harmonic.

9.2 Equations of Motion of Liquid

Fig. 9.1 shows the coordinate system $(r, \theta, z)$ adopted in defining the equations governing the liquid motion. For an inviscid, incompressible liquid, the governing equations of motion are the Laplace equation

$$\nabla^2 p(r, \theta, z) = 0,$$  \hspace{1cm} (9.2)

and the Bernoulli Equation
Fig. 9.1 Coordinate System
where \( \phi(r, \theta, z) \) is the velocity potential, \( p \) the liquid pressure, \( \rho_f \) the liquid density, and \( g \) the gravitational acceleration.

9.3 Boundary Conditions

At the free surface, the boundary equation may be expressed as

\[
\frac{\partial^2 \phi}{\partial t^2} \bigg|_{z=0} + g \frac{\partial \phi}{\partial z} \bigg|_{z=0} = 0 \tag{9.4}
\]

At the elastic cylindrical tank wall, the boundary condition expressing the liquid solid-interaction is given as

\[
\frac{\partial \phi}{\partial r} \bigg|_{r=R} = \frac{\partial W}{\partial t} \bigg|_{r=R} \tag{9.5}
\]

At the rigid tank bottom, the boundary condition is written as

\[
\frac{\partial \phi}{\partial z} \bigg|_{z=-H} = 0 \tag{9.6}
\]

9.4 Coupled Liquid-Tank Interaction

The variational functional for the liquid is given as

\[
I = \int_{t_1}^{t_2} \left( T - \Pi - W \right) dt \tag{9.7}
\]

where \( T \), \( \Pi \), and \( W \) are the kinetic and potential energies, and the work done on the liquid respectively. Hsiung and Weingarten (58) obtained \( T \) by integration over the liquid volume \( V \), \( \Pi \) by integration over the free surface \( F \), and \( W \) by integration over the liquid-tank interface \( \Sigma \), as follows:
\[ T = \frac{\rho_f}{2} \int_V \nabla \phi \cdot \nabla \phi \, dV , \quad (9.8a) \]

\[ \Pi = \frac{1}{2} \int_F \xi (\rho_f g \xi) \, dS , \quad (9.8b) \]

and

\[ W = \int_\Sigma \rho_f \left( \frac{\partial w}{\partial t} \right) \phi \, dS \quad (9.8c) \]

where \( \rho_f \) is the liquid density, \( \xi \) the liquid elevation from the mean free surface level, and \( \nabla \phi \) the velocity vector \( \vec{v} \) given by

\[ \vec{v} = \text{grad} \phi \quad (9.9) \]

It has been shown by Ref. 58 that the functional involving the governing equations 9.2 and 9.3, together with the boundary equations 9.4, 9.5 and 9.6, can be expressed in the form

\[ I = \frac{1}{2} \int_V \nabla \rho \cdot \nabla \rho \, dV - \frac{g}{2} \int_F \left( \frac{\partial \rho}{\partial t} \right)^2 \, dS - \rho_f \int_\Sigma \rho \frac{\partial^2 w}{\partial t^2} \, dS \quad (9.10) \]

Ref. 32 discretised the liquid in the cylindrical tank into annular elements of rectangular cross-section, and expressed Eq. 9.10 in terms of element nodal pressure vector, \( \mathbf{p} \), and the shell nodal displacement vector, \( \mathbf{U} \), giving the variational functional in matrix form as follows:

\[ I = \frac{1}{2} \mathbf{p}^T \mathbf{K}_f \mathbf{p} - \frac{1}{2} \mathbf{p}^T \mathbf{M}_f \mathbf{\ddot{p}} - \rho_f \mathbf{p} \mathbf{S} \mathbf{\ddot{U}} \quad (9.11) \]

where \( \mathbf{K}_f \), \( \mathbf{M}_f \), and \( \mathbf{S} \) are the liquid stiffness, liquid mass, and coupling force matrices respectively. Minimizing the variational functional by the Euler-Lagrange procedure gives the following matrix equation

\[ \mathbf{K}_f \mathbf{p} + \mathbf{M}_f \mathbf{\ddot{p}} - \rho_f \mathbf{S} \mathbf{\ddot{U}} = 0 \quad (9.12) \]

The equations for the dynamic fluid pressure on the shell is given by (59):
By using Eqs. 9.12 and 9.13, and neglecting the free surface pressure of the liquid, the equations of motion representing the free vibration of the coupled liquid-tank system were obtained by Ref. 32 as follows:

\[
M_s \ddot{U} + K_s U + S^T P = 0
\]  

(9.13)

in which \( [M_s + A] \) is the modified mass matrix accounting for the hydrodynamic effect of the liquid. The matrix, \( A \), generated by the computer code, FLUID, developed by Ref. 32 is given by:

\[
A = S^T K_f S
\]  

(9.14)

9.5 Response Analysis of an Elevated Water Tank

The tower in Fig. 7.1c is the example problem for the analysis. The mass and stiffness matrices of the elevated liquid-filled tank are generated based on the theory outlined above, while the mass and stiffness matrices of the outer shell of the supporting pedestal are generated separately by the computer code of Ref. 56. The inner tube, carrying the piping connections, and enclosed by the spiral staircase, contributes a negligible amount to the stiffness of the supporting structure, but in view of its concentricity with the outer shell, its stiffness can be added to the stiffness of the outer shell to obtain the combined stiffness of the supporting structure. The sloshing due to the liquid part filling the conical base of the tank is assumed negligible, and the water in this region is treated as a solid mass.

After assembling the matrices of the tank and supporting structure to form the overall matrices \( M \) and \( K \), and imposing the boundary conditions at the foundation (fixed in this case), the frequencies, \( \omega \), and the mode shapes, \( \phi \), of the structure are obtained...
first by matrix decomposition using Choleski's method, and then solved by a standard eigenvalue subroutine.

The Rayleigh damping matrix is given by:

$$C = a_0 M + a_1 K,$$  \hspace{1cm} (9.16)$$

where

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = 2.0 \begin{bmatrix} 1/\omega_1 & \omega_1 \\ 1/\omega_2 & \omega_2 \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}$$

The partitioned matrix equation of motion may be written as

$$\begin{bmatrix} M & M_c \\ M_c^T & M_g \end{bmatrix} \begin{bmatrix} \ddot{U}_t \\ \ddot{U}_g \end{bmatrix} + \begin{bmatrix} C & C_c \\ C_c^T & C_g \end{bmatrix} \begin{bmatrix} \dot{U}_t \\ \dot{U}_g \end{bmatrix} + \begin{bmatrix} K & K_c \\ K_c^T & K_g \end{bmatrix} \begin{bmatrix} U_t \\ U_g \end{bmatrix} = \begin{bmatrix} 0 \\ F_g \end{bmatrix},$$

\hspace{1cm} (9.17)

in which the lower segment of Eq. 9.17 pertains to the base nodal displacements, and the upper segment to the non-base nodal displacements. $M_c$, $C_c$, and $K_c$ are the coupling effects between the base and non-base nodes. The total displacements, $U_t$, of the off-base nodes can be expressed as the sum of the pseudostatic component, $U_s$, and the dynamic component, $U$, as follows:

$$U_t = U_s + U$$

\hspace{1cm} (9.18)

The vector, $U_s$, is considered to be developed through rigid body displacements resulting from $U_g$, and the relationship is given by

$$K_c U_g + K U_s = 0$$

\hspace{1cm} (9.19)

This can be rewritten as

$$U_s = R U_g,$$

\hspace{1cm} (9.20)
The equations for the off-base elements in Eq. 9.17 can be written as
\[ M \dddot{U} + C \ddot{U} + K U = -[M R + M_c] \dddot{U}_g - [C R + C_c] \dddot{U}_g \]  
(9.21)
The terms on the right-hand side are the effective earthquake forces, and if the velocity dependent term, \([C R + C_c] \ddot{U}_g\), is neglected*, Eq. 9.21 would reduce to
\[ M \dddot{U} + C \ddot{U} + K U = -[M R + M_c] \dddot{U} = P_{\text{eff}} \]  
(9.22)
where \(P_{\text{eff}}\) is the effective force vector. Eq. 9.22 can be decoupled by substituting \(U = \phi Y\), and pre-multiplying the result by \(\phi^T\); the individual equation for the \(i\)th mode is obtained as follows:
\[ \dddot{Y}_i + 2 \xi_i \omega_i \ddot{Y}_i + \omega_i^2 Y_i = P_i/M_i \]  
(9.23)
where \(P_i = \phi_i^T P_{\text{eff}}\), and \(M_i = \phi_i^T M \phi_i\).

\(Y(t)\) for the mode number specified is found by Duhamel integration, from which \(U(t)\) is obtained. By omitting the terms \([C_g + C_c R]\) and \([K_g + K_c R]\) from Eq. 9.17, the reactions at the foundations can be given as
\[ F_g = [M_g + M_c R] \dddot{U}_g + M_c \dddot{U} + C_c \dddot{U} + K_c U \]  
(9.24)
Consequently, the stress resultants, \(N_s\), \(N_\theta\) and \(N_{s\theta}\), and the stress couples, \(M_s\), \(M_\theta\) and \(M_{s\theta}\), are computed at all the nodes for the specified circumferential angles \(\theta\). The programme of Ref. 32 is modified to include damping, and generalise its applicability to elevated liquid storage tanks.

* The damping contribution to the effective earthquake forces is small, and is usually neglected (60).
10.1 Structural Data

The dimensions and properties of the elevated tank are shown in Fig. 10.1. The number of elements employed is 35 as shown in Fig. 10.2, and the first eight modes are shown in the main response analysis.

10.2 Criterion for Selecting Damping Ratio

The damping ratio for liquid motion is relatively small, ranging from 0.0001 to 0.005, while the structural damping ratio is much larger in value as shown in Table 10.1.

<table>
<thead>
<tr>
<th>Type of Structural Material</th>
<th>Damping Ratio, $\xi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>0.005 to 0.02</td>
</tr>
<tr>
<td>Concrete</td>
<td>0.05 to 0.07</td>
</tr>
<tr>
<td>Masonry</td>
<td>0.15 to 0.40</td>
</tr>
</tbody>
</table>

At present, there is no definite rule on which to base the computation of the damping ratios for the whole elevated tank system containing liquid. The damping ratio for the overall system depends on the amount of influence from the liquid motion, as well as the motion of the supporting structure. When the tank is either empty or completely full, no sloshing is involved, and the first and second mode damping ratios, $\xi_1$ and $\xi_2$, are both chosen as 0.05 for the structure to
Fig. 10.1 Dimensions and Material Properties

MATERIAL PROPERTIES

\[ E_{\text{mat}} = 1.38 \times 10^7 \text{kN/m}^2 \]
\[ E_{\text{con}} = 1.38 \times 10^7 \text{kN/m}^2 \]
\[ G = 4.83 \times 10^6 \text{kN/m}^2 \]
\[ \rho_{\text{con}} = 220 \text{ kN/m}^3 \]
\[ \rho_{\text{liq}} = 98 \text{ kN/m}^3 \]

Fig. 10.2 Node Numbering
be analysed. When partially filled with water, the first mode involves sloshing almost entirely, whereas the second mode essentially involves the motion of the supporting structure. Hence, it seems reasonable to use $\xi_1 = 0.001$ and $\xi_2 = 0.05$ for partially filled elevated tanks. From the damping ratios chosen, the appropriate damping coefficients, $a_0$ and $a_1$ can be determined.

10.3 Load Input and Simulation

The ground acceleration depends on the level of seismic resistance sought. For the present example, the digitised S69E ground acceleration of the 1952 TAFT earthquake with a 0.2g peak amplitude is provided as the input to the programme. The first circumferential mode of the structure is excited, giving a series of displacement-time responses. The stress resultants and couples at the midpoint of each element, and the foundation reactions are also obtained. The procedure is repeated for different cases of liquid depths. The analysis is also carried out for an empty elevated tank, with weight equal to the weight of the liquid-filled tank to simulate the no-sloshing case. The four different cases considered in the analysis of the tower are shown in Table 10.2.

<table>
<thead>
<tr>
<th>Case</th>
<th>Water Depth, d</th>
<th>$\xi_1$</th>
<th>$\xi_2$</th>
<th>Liquid Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>7.25 m</td>
<td>0.001</td>
<td>0.05</td>
<td>Include Sloshing</td>
</tr>
<tr>
<td>II</td>
<td>11.00 m</td>
<td>0.001</td>
<td>0.05</td>
<td>Include Sloshing</td>
</tr>
<tr>
<td>III</td>
<td>11.00 m</td>
<td>0.05</td>
<td>0.05</td>
<td>Include Sloshing</td>
</tr>
<tr>
<td>IV</td>
<td>11.00 m</td>
<td>0.05</td>
<td>0.05</td>
<td>Sloshing Suppressed</td>
</tr>
</tbody>
</table>
10.4 Results and Discussion

The predicted free vibration frequencies, as shown in Table 10.3, decreased significantly as the water depth is increased. This is due to the effect of increasing the mass near the top of the structure as the water depth is increased. It is also noted that the case including sloshing (III) gives higher frequencies than the no-sloshing case (IV); this may be due to the considerably larger 'impulsive' mass (referred to in 3.7 as the liquid mass which moves in unison with the structure) in the latter.

Displacement plots, Figs. 10.4 a, b, c and d, show that the maximum radial displacements for all the four cases occur at time $t = 3.4 \, \text{s}$. The differences between the responses for the cases considered are small as the supporting structure is relatively stiff. The displacement responses obtained for Case II ($\xi_1 = 0.001, \xi_2 = 0.05$) are larger than those for Case III ($\xi_1 = 0.05, \xi_2 = 0.05$) due to the larger first mode damping in the latter. This suggests that larger dynamic response of the water tank can result from possible influence on the first mode motion by the sloshing of liquid, which introduces smaller damping ratios.

The maximum values of the foundation are also obtained at $t = 3.4 \, \text{s}$ and shown in Table 10.4. From Table 10.4, the results show that for most instances, Case III (sloshing included) gives larger responses than for Case IV (sloshing not included) at the tank wall, and smaller responses at the supporting structure, indicating a possible amplification of the stress response in the tank region caused by liquid sloshing.

Case I gives larger displacements than Case II at the tank wall,
Fig. 10.3 S69E Ground Acceleration of the 1952 TAFT Earthquake
Fig. 10.4a Radial Displacement Response of Node 15 for Case I: Water Depth of 7.25 m, $\xi_1 = 0.001$, $\xi_2 = 0.05$. 

Displacement (mm)
Fig. 10.4b Radial Displacement Response of Node 15 for Case II: Water Depth of 11.0 m, $\xi_1 = 0.001$, $\xi_2 = 0.05$. 
Fig. 10.4c  Radial Displacement Response of Node 15 for Case III: Water Depth of 11.0 m, $\xi_1 = 0.05$, $\xi_2 = 0.05$. 
Fig. 10.4d Radial Displacement Response of Node 15 for Case IV: Water Depth of 11.0 m, $\xi_1 = 0.05$, $\xi_2 = 0.05$, Strohing Suppressed.
suggesting that the larger liquid volume does not necessarily cause
greater response to ground excitation.

The stresses obtained at the tank wall are at their maximum at
time $t = 12.1$ s. Table 10.5 presents the stress resultants at the
midpoints of Elements 15 and 17 at $t = 12.1$ s for the cases considered.
The results show that the base of the tower has the largest force
resultants at any instant. As far as the tank wall is concerned, the
bottom of the storage tank is the most stressed region. The total
axial moment at the midpoints of Elements 17 (near tank bottom) and
and 35 (tower base) can be computed by multiplying $M_s$ (axial moment
per unit length) at $\theta = 0^\circ$, by $\pi R$. These can be checked with the
critical buckling moment expression used by Ref. 35,

$$M_{cr} = EI \left\{ \frac{0.6}{1 + 0.004 \frac{E}{f_y}} \right\}$$  \hspace{1cm} (10.1)

where

$E = \text{Young's modulus},$

$I = \text{section modulus},$

$t = \text{cylinder thickness},$

$R = \text{cylinder radius},$

and

$f_y = \text{structural yield stress}.$

The check indicated significant conservatism in the structure for the
present level of seismic resistance sought.
Table 10.3  Comparison of Frequencies (Hz) of the Water Tower

<table>
<thead>
<tr>
<th>Mode Case</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.503</td>
<td>2.99</td>
<td>14.05</td>
<td>20.97</td>
<td>26.30</td>
<td>27.34</td>
<td>30.91</td>
<td>31.88</td>
</tr>
<tr>
<td>II</td>
<td>0.440</td>
<td>2.99</td>
<td>14.03</td>
<td>19.82</td>
<td>22.83</td>
<td>27.24</td>
<td>28.93</td>
<td>30.93</td>
</tr>
<tr>
<td>III</td>
<td>0.372</td>
<td>2.92</td>
<td>14.03</td>
<td>18.61</td>
<td>22.15</td>
<td>26.04</td>
<td>27.30</td>
<td>30.93</td>
</tr>
<tr>
<td>IV</td>
<td>0.301</td>
<td>2.04</td>
<td>13.44</td>
<td>15.83</td>
<td>18.45</td>
<td>19.61</td>
<td>21.50</td>
<td>25.25</td>
</tr>
</tbody>
</table>

Table 10.4  Foundation Reactions, and Radial Displacements at Nodes 15 and 17 for $\theta = 0^\circ$, at $t = 3.4$ s.

<table>
<thead>
<tr>
<th>Case</th>
<th>Axial Force kN/m</th>
<th>Tang. Force kN/m</th>
<th>Rad. Force kN/m</th>
<th>Axial Moment kNm/m</th>
<th>Node w mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>89.1</td>
<td>6.4</td>
<td>20.8</td>
<td>-9.7</td>
<td>15 -7.44</td>
</tr>
<tr>
<td>II</td>
<td>84.9</td>
<td>6.2</td>
<td>19.8</td>
<td>-9.2</td>
<td>15 -7.05</td>
</tr>
<tr>
<td>III</td>
<td>80.7</td>
<td>5.9</td>
<td>18.9</td>
<td>-8.8</td>
<td>15 -6.68</td>
</tr>
<tr>
<td>IV</td>
<td>76.6</td>
<td>5.8</td>
<td>17.8</td>
<td>-8.3</td>
<td>15 -6.33</td>
</tr>
</tbody>
</table>

Table 10.5  Tank Stresses at Midpoints of Elements 15 and 17 for $\theta = 0^\circ$, at $t = 12.1$ s

<table>
<thead>
<tr>
<th>Case</th>
<th>Element</th>
<th>$N_s$ kN/m</th>
<th>$N_\theta$ kN/m</th>
<th>$M_s$ kNm/m</th>
<th>$M_\theta$ kNm/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>15</td>
<td>5.7</td>
<td>-2.3</td>
<td>0.26</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>12.6</td>
<td>-40.3</td>
<td>-1.05</td>
<td>-0.35</td>
</tr>
<tr>
<td>II</td>
<td>15</td>
<td>5.7</td>
<td>2.7</td>
<td>0.34</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>12.7</td>
<td>-37.6</td>
<td>-0.97</td>
<td>-0.33</td>
</tr>
<tr>
<td>III</td>
<td>15</td>
<td>5.5</td>
<td>2.6</td>
<td>0.32</td>
<td>0.10</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>12.1</td>
<td>-36.0</td>
<td>-0.93</td>
<td>-0.33</td>
</tr>
</tbody>
</table>
CHAPTER 11

CONCLUSION AND DISCUSSION

11.1 Conclusions

The following conclusions are drawn from this study:

i) Free vibration frequencies of the elevated water tank decrease as the water depth is increased.

ii) Elevated water tank frequencies are higher when sloshing is permitted.

iii) The liquid sloshing effect tends to cause relatively larger stresses at the tank wall and smaller values at the supporting structure.

iv) When partially filled with water, the first mode damping ratio of the elevated tank system may be dominated by the damping ratio of the liquid motion. This causes a considerable reduction in the system damping ratio, consequently amplifying the response of the structure to ground excitation.

11.2 Discussion

Although the theory used in this method is rigorous, the procedure is straightforward for seismic response analysis of axisymmetric liquid storage structures. However, the discretisation of the structure into elements requires judgment, particularly where there are high gradients in the profile. For partially filled elevated tanks, it may be desirable to use a larger number of mode shapes. The operating conditions of the tank (e.g. constant head supply or partially full conditions), and the level of seismic
resistance sought are necessary pre-analysis specifications.

The effect of the tilting action of the tank base has to be considered for thin-walled tanks (e.g. steel tanks) for high seismicity. This can partly be done by generalising the formulation to include higher circumferential harmonics in the simulation of load excitation. The work can be extended to include the conical shape of the tank bottom. Nonlinear effects of liquid sloshing can also be considered.
BIBLIOGRAPHY


45. Shaaban, S. H. and Nash, W. A., 'Finite Element Analysis of a Seismically Excited Cylindrical Storage Tank, Ground Supported, and Partially Filled with Liquid', University of Massachusetts Report to the National Science Foundation, 1976


This computer program is a modified version of the program originally developed by DuVall, and later extended by Glaceel. The program consists of the dynamic analysis of an axisymmetric offshore tower carrying a deck mass, subjected to digitised wave spectral input. The available options are:

1) Dynamic analysis without vibration absorber
2) Dynamic analysis with a tuned mass damper
3) Dynamic analysis with a liquid vibration absorber

```
IMPLICIT REAL*8(A-H,O-Z)
DOUBLE PRECISION MASS,LENGTH
COMPLEX A,B
DIMENSION TOWER(20), FREQ(100), MASS(200), STIFF(200), A(200), B(50), DIS(50), VEL(50), ACC(50), R(50), S(50), H(50), PHASE(50), Q(500), P(50)
DIMENSION EL(2)
DATA IR, IW/5, 6/
CALL ASMEL (NELEM, NEG, MBW, *HEIGHT, REB, RET, RIB, RIT, E, RO, DMASS, DINNER, STIFF, MASS, KQ, *NRIGID, IR, IW, DEPTH, KD, DM, DK, KM, LENGTH, DA, DB, TM, TM0, EL, ET, THICK, RAD)
N_FREQ=NFREQ
STOP
END
IMPLICIT REAL*8(A-H.O-Z)
COMMON/H/T/M, HIL
DIMENSION EL(2)
DIMENSION TOWER(20), FREQ(1), H(1), PHASE(1)
C
READ(IR, 1) TOWER, NELEM, N_FREQ, N_RIGID, KQ, KD, KM, KNW
WRITE(IW, 2) TOWER, NELEM, N_FREQ
IF(KQ .NE. 0) WRITE(IW, 50)
50 FORMAT(/5X, 'EARTHQUAKE OPTION IN EFFECT')
IF(KD .GT. 0) WRITE(IW, 51)
51 FORMAT(/5X, 'EARTHQUAKE OPTION IN EFFECT')
IF(KD .LT. 0) WRITE(IW, 52)
52 FORMAT(/5X, 'LIQUID SLOSHING DAMPER IN EFFECT')
C
C !!! READ OVERALL DIMENSIONS
READ(IR, 3) HEIGHT, Z0, REB, RET, RIB, RIT, DEPTH
WRITE(IW, 4) HEIGHT, Z0, REB, RET, RIB, RIT, DEPTH
C !!! READ MATERIAL PROPERTIES
READ(IR, 3) E, RO
```
WRITE(IW,5)E,R
## READ ALPHA,BETAV,BETAH FOR DAMPING. IF NO DAMPING READ IN ZERO VAL
READ (IR,3) ALPHA,BETAV,BETAH
WRITE(IW,21)ALPHA,BETAV,BETAH
## READ DECK LOAD
READ (IR,3) D MASS ,DINER
WRITE(IW,6) D MASS ,DINER
IF(KD.EQ.0)GO TO 22
IF(KD.LT.0)GO TO 23
READ(IR,41)DM,OMEGA,BETAD
DK=OMEGA **2 * DM
WRITE(IW,42)DK,DM,OMEGA,BETAD
FORMAT(3F8.4)
42 FORMAT(4X, 'DAMPING PARAMETERS'/2X, 'SPRING CONSTANT=' , F10.4/2X,
* 'DAMPER MASS =', F8.4/2X, 'DAMPER FREQUENCY(RAD/SEC) =', F8.4/
*2X, 'PERCENT CRITICAL DAMPING =', F8.4)
GO TO 22
23 CONTINUE
READ(IR,3)R,HL,ROL,ET,BETAD,THICK
WRITE(IW,1000)R,HL,ROL,ET,THICK
1000 FORMAT(2X,5E20.8)
CALL SLOSH(R,HL,ROL,THICK)
DO 11 I=1,NFREQ
11 READ (IR,3)(FREQ(I),H(I),I=1,N)
WRITE(IW,9)(FREQ(I),H(I),I=1,N)
RETURN
## READ FREQUENCIES
IF(NFREQ.LE.0)GO TO 10
## READ FREQUENCIES FOR STATIC RESPONSE IN FREQUENCY DOMAIN
N = -NFREQ
READ (IR,3)(FREQ(I),H(I),I=1,N)
WRITE(IW,7)(FREQ(I),H(I),I=1,N)
RETURN
## READ TIME AND TIME INTERVAL PLUS CONDENSED SPECTRUM PARAMETERS
## FOR DYNAMIC ANALYSIS
10 IF(NFREQ.LE.0)GO TO 12
READ(IR,3)DT,TIME
WRITE(IW,8)(DT,TIME)
DO 11 I=1,NFREQ
11 READ (IR,3)(FREQ(I),H(I),PHASE(I)
WRITE(IW,9)(FREQ(I),H(I),PHASE(I),I=1,NFREQ)
12 RETURN
 FORMAT(20A4/,7I4)
2 FORMAT(11//'5X,20A4'/,
* 5X,'ELEMENTS'/,5X,15,'FREQUENCIES'/)
3 FORMAT(8F10.2)
4 FORMAT(/'5X, 'OVERALL DIMENSIONS'/
*5X,'TOWER HEIGHT ',F8.2,' M. '/
*5X,'CAISSON HEIGHT ', F8.2,' M. '/
*5X,'EXTERNAL RADIUS AT THE BOTTOM ', F7.2,' M. '/
*5X, 'EXTERNAL RADIUS AT THE TOP' F7.2, M.*/
*5X, 'INTERNAL RADIUS AT THE BOTTOM' F7.2, M.*/
*5X, 'INTERNAL RADIUS AT THE TOP' F7.2, M.*/
*5X, 'DEPTH OF WATER' F8.2, M.*/
FORMAT('///5X, 'MATERIAL PROPERTIES'///
*5X, 'E =', E10.3/5X,'R0 =', E10.3/*)
FORMAT('///5X, 'DECK MASS ', E12.4/*
*
FORMAT('///5X, 'DECK INERTIA', E12.4/*
FORMAT('///5X, 'CONDENSED SPECTRUM PARAMETERS /*///3X, 'FREQ', 8X,
*WAVE HEIGHT ///(3X, 2F15.4))
FORMAT('///5X, 'TIME INTERVAL AND TOTAL TIME ///5X,
*DELTA T = ', F7.3/5X, 'TIME = ', F7.3/*
FORMAT('///5X, 'CONDENSED SPECTRUM PARAMETERS ///13X,
*FREQ', 8X, 'WAVE HEIGHT', 3X, 'PHASE ANGLE ///(3X, 3F15.4))
FORMAT('///5X, 'DAMPING COEFFICIENTS ///5X, 'ALPHA=', E10.3/5X,
*betaV = ', E10.3/*
FORMAT('///5X, 'ALPHA=', E10.3/5X, 'BETAH=', E10.3/*)
END
SUBROUTINE SLOSH(R, HL, ROL, G, TM, TM0, TM1, EL, OMEGA)
IMPLICIT REAL*8(A-H, Q-Z)
COMMON/HT/HOL, H1L
DIMENSION EL(2)
TM=3·14159*R*R*HL*RGL
A=R/HL
B=HL/R
F1=DCOSH(1·732*A)
F2=DSINH(1·732*A)
F3=DCOSH(1·837*B)
F4=DSINH(1·837*B)
TM0=TM/(1·732*A)*DSINH(1·732*A)/DCOSH(1·732*A)
TM1=0·21*TM*1·837*A*F4/F3
HOL=0·375*HL*(1·0+1·33*(1·732*A*F1/F2-1·0))
HIL=HL*(1·0-(F3-1·534)/(1·837*B*F4))
OMEGA=(G/R)*1·837*F4/F3
OMEGA=D SQRT(OMEGA)
IF(HIL.LE.HCL)GO TO 1
EL(1)=HOL
EL(2)=HIL-HOL
GO TO 3
IF(HIL.EQ.HCL)GO TO 2
EL(1)=HIL
EL(2)=HOL-HIL
GO TO 3
EL(1)=HOL
EL(2)=0.1
CONTINUE
RETURN
END
SUBROUTINE ASMBL (NELE, NEQ, MBW,
*HEIGHT, RED, RET, RIB, RIT, E, RO, D MASS, DINER, STIFF, MASS, KQ,
*NRI GO, IR, IV, DEPTH, KD, DM, DO, KM, LENGHT, DA, DB, TM, TM0, EL, ET, THICK, R)
IMPLICIT REAL*8(A-H, Q-Z)
DOUBLE PRECISION MASS, LENGHT, MG
COMMON/HT/HOL, H1L
DIMENSION STIFF(200), MASS(200), A(15), AMAS(19, 19), STIF(19, 19)
DIMENSION EL(2)
INITIALIZE
NEQ = 2*(NELEM + 1)
IF(KQ*NEQ,0) NEQ=NEQ+2
IF(KD*NEQ,0) NEQ=NEQ+1
IF(KD*LT,0) NEQ=NEQ+4
MBW=5
LIM=NEQ*MBW
DO 10 I=1,LIM
STIFF(1)=0.0
10 MASS(I) =0.0

COMPUTE ELEMENT MATRICES AND ASSEMBLE
DN=NELEM
LENGT = HEIGHT/DN
DA=(RET-REB)/DN
DB=(RIB-REB)/DN
A1 = REB
B1=RIB
EMASS=DMASS
IF(KD*GT,0) EMASS=EMASS + DM
IF(KD*LT,0) EMASS=EMASS+TM

DO 12 N=1,NELEM
A2 = A1 + DA
B2 = B1 + DB
CALL SUBK (E,A1,B1,A2,B2,LENGT,A)
CALL ADD (NEQ,STIFF,N,A)
CALL SUBM (RG,A1,B1,A2,B2,LENGT,EMASS,A)
CALL ADD (NEQ,MASS,N,A)
A1 = A2
B1 = B2
A11=R+THICK
B11=R
K=0
N1=NELEM+1
N2=NELEM+2
DO 201 N=N1,N2
K=K+1
DIS =EL(K)
CALL SUBK(ET,A11,B11,A11,B11,DIS,A)
CALL ADD(NEQ,STIFF,N,A)

ADD DECK MASS
I=2 * NELEM
MASS(I+1) = MASS(I+1) + D MASS
MASS(I+2) = MASS(I+2) + DINER
IF(KD*GE,0) GO TO 1
IF(HIL*GE,HIL) MASS(I+3)=TM0
IF(HIL*LT,HIL) MASS(I+5)=TM0
CONTINUE
A1 = REB
B1 = RIB
DO 14 M=1,NELEM
A2 = A1 + DA
B2 = B1 + DB
CALL SUBKRG(RD,REB,RIJ,A1,B1,A2,B2,LENHT,EMASS,M,A)
CALL ADD(NEQ,STIFF,M,A)

A1 = A2
B1 = B2
IF (KD.EQ.0) GO TO 40
IF (KD.GT.0) GO TO 38
## MODIFY MATRICES FOR MASS DAMPER
IF (HL.LT.HOL) GO TO 39
GO TO 38
CONTINUE
I = NEQ - 4
J = NEQ
K = 5*NEQ - 4
GO TO 41
CONTINUE
I = NEQ - 2
J = NEQ
K = 3 * NEQ - 2
CONTINUE
STIFF(I) = STIFF(I) + DK
STIFF(J) = DK
STIFF(K) = -DK
MASS(J) = DM
CONTINUE
IF (KM.EQ.0) GO TO 43
## PRINT OUT MASS AND STIFFNESS MATRICES
WRITE(IW,29)
FORMAT(/10X,*MASS MATRICES/*/)
DO 100 I = 1,NEQ
DO 75 J = 1,NEQ
AMAS(I,J) = 0.0
100 CONTINUE
DO 101 I = 1,NEQ
K = I + 4
DO 76 J = I,K
IJ = (J - I) * NEQ + I
AMAS(I,J) = MASS(IJ)
101 CONTINUE
DO 102 I = 1,NEQ
K = I + 1
L = I + 4
DO 77 J = K,L
AMAS(J,I) = AMAS(I,J)
102 CONTINUE
DO 30 I = 1,NEQ
WRITE(IW,27)(AMAS(I,J),J = 1,NEQ)
 FORMAT(/1X,15E8.1)
30 CONTINUE
WRITE(IW,32)
FORMAT(/10X,*STIFFNESS MATRIX/*/)
DO 103 I = 1,NEQ
DO 78 J = 1,NEQ
STIF(I,J) = 0.0
103 CONTINUE
DO 104 I = 1,NEQ
K = I + 4
DO 79 J=I,K
IJ=(J-I)*NEQ+I
STIF(I,J)=STIFF(IJ)
CONTINUE
DO 105 I=1,NEQ
K=I+1
L=I+4
DO 80 J=K,L
STIF(J,I)=STIF(I,J)
CONTINUE
DO 28 I=1,NEQ
WRITE(IW,27)(STIF(I,J),J=1,NEQ)
FORMAT(/11E10.3)
CONTINUE
43 RETURN
END
SUBROUTINE SUBK (E,A1,B1,A2,B2,LENGHT,STIFF)
IMPLICIT REAL*8(A-H,O-Z)
DOUBLE PRECISION MASS,LENGHT
DIMENSION A(5),W(5),F(4),STIFF(15)
DATA A,W/0.0,0.0538469D0,0.0538469D0,-0.906180D0,0.906180D0,
*0.568889D0,2*0.478629D0,2*0.236927D0/
DO 1 I=1,15
STIFF(I)=0.
DO 2 N=1,5
X=A(N)
X=0.5*(X+1.0)
F(1)=12.0*X-6.0
F(3)=-F(1)
F(2)=(-4.0+6.0*X)*LENGHT
F(4)=(-2.0+6.0*X)*LENGHT
AA=A1*(1.0-X)+A2*X
B=B1*(1.0-X)+B2*X
C=0.7853981*(AA**2-B**2)/LENGHT**3/2.0*W(N)*E
IJ=0
DO 2 J=1,4
DO 2 I=1,J
IJ=IJ+1
STIFF(IJ)=STIFF(IJ)+C*F(I)*F(J)
RETURN
END
SUBROUTINE ADD (NEQ,A,N,B)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(1),B(1)
NN=2*(N-1)
KL=0
DO 10 J=1,5
DO 10 I=1,J
II=J-I
IJ=NEQ*II+NN+I
KL=KL+1
A(IJ)=A(IJ)+B(KL)
RETURN
END
SUBROUTINE SUBM (RG,A1,B1,A2,B2,LENGHT,EMASS,MASS)
IMPLICIT REAL*8(A-H,O-Z)
DOUBLE PRECISION MASS, LENGHT
DIMENSION A(5), W(5), F(4), MASS(15)
DATA A, W/0.0, 0.0, 0.0, 0.0, 0.0/,
* 0.56888900, 2*0.47862900, 2*0.23692700/
DO 1 I=1,15
MASS(I)=0
DO 2 N=1,5
X=A(N)
X=X*0.5*(X+1.0)
F(3)=X*X*(3.0 - 2.0*X)
F(1)=1.0 - F(3)
F(4)=X * X * LENGHT * (X-1.0)
F(2)=X * LENGHT * (1.0 - 2.0*X + X*X)
AA = A1 * (1.0 - X) + A2 * X
B = B1 * (1.0 - X) + B2 * X
C=3.14159265 * (AA * AA - B*B) * RO * LENGHT / 2.0 * W(N)
EMASS = EMASS + C
IJ=0
DO 2 J=1,4
DO 2 I=1,J
IJ=IJ+1
MASS(IJ)=MASS(IJ) + C * F(I) * F(J)
RETURN
END

SUBROUTINE SUBKG(RO, REB, RIB, A1, B1, A2, B2, LENGHT, EMASS, M, STIFG)
IMPLICIT REAL*8(A-H, O-Z)
DOUBLE PRECISION MASS, LENGHT
DIMENSION A(5), W(5), F(4), STIFG(15)
DATA A, W/0.0, 0.0, 0.0, 0.0, 0.0/,
* 0.56888900, 2*0.47862900, 2*0.23692700/
DM=M-1
DO 1 I=1,15
STIFG(I)=0.
DO 2 N=1,5
X=A(N)
X=X*0.5*(X+1.0)
F(1)=X*X*(X-1.0)
F(2)=(1.0 - 4.0*X + 3.0*X*X) * LENGHT
F(3)=-F(1)
F(4)=(3.0*X - 2.0) * LENGHT * X
A3 = (A2-A1) * X + A1
B3 = (B2-B1) * X + B1
QMASS=RO * 3.14159265 * (DM + X) * LENGHT / 3.0 * ((REB*REB
*+A3*REB+A3*A3)-(RIB*RIB+B3*RIB+B3*B3))
P=9.81*(EMASS - QMASS) / LENGHT * W(N) / 2.0
IJ=0
DO 2 J=1,4
DO 2 I=1,J
IJ=IJ+1
STIFG(IJ)=STIFG(IJ) + P*F(I)*F(J)
RETURN
END

IMPLICIT REAL*8(A-H, O-Z)
DOUBLE PRECISION MASS, LENCEHT
COMPLEX*16 A, B, C, F, T, R, Q
DIMENSION TOWER(20), STIFF(I), MASS(I), A(I), B(15), P(4), S(I), US(15), 
* VS(15), HS(15), SS(5), FREQ(I), H(1)
J = NELEM + 1
DO 100 I = 1, J
US(I) = 0.
VS(I) = 0.
HS(I) = 0.
SS(I) = 0.
USD = 0.
HSD = 0.
DO 99 L = 1, NFREQ
W = 5.28318531 * FREQ(L)
W2 = W * W

## FORM COEFFICIENT MATRIX
LIM = NEQ * MBW
D = W * BETAV + 2.0 * BETAH
G = DCMPLX(1.0*D, D)
F = DCMPLX(-W2, W*ALPHA)
DO 1 I = 1, LIM
A(I) = G * STIFF(I) + F * MASS(I)

IF(KD_EQ_0) GO TO 15
CD = W * 2.0 * BETAD * DSQRT(DK) * DSQRT(DM)
T = DCMPLX(0.0, CD - D*DK)
R = DCMPLX(0.0, -CD + D*DK)
X = D*DK + W*ALPHA*DM
Q = DCMPLX(0.0, CD - X)
A(NEQ-2) = A(NEQ-2) + T
A(NEQ) = A(NEQ) + Q
A(3*NEQ-2) = A(3*NEQ-2) + R
CONTINUE

## FORM LOAD VECTOR
DO 2 I = 1, NEQ
B(I) = 0.
DN = NELEM
LENHT = HEIGHT / DN
DA = (RET - REB) / DN
A1 = REB
DO 4 N = 1, NELEM
A2 = A1 + DA
CALL SUPP(N, A1, A2, LENHT, ZG, DEPTH, W2, P)
J = 2*(N-1)
DO 3 I = 1, 4
J = J + 1
C = P(I)*H(L)
B(J) = B(J) + C
A1 = A2

## IMPOSE DISPLACEMENT BOUNDARY CONDITIONS
IF(NRIGO_EQ_0) GO TO 9
DO 5 I = 1, LIM, NEQ
A(I) = 0.0
A(I+1)=0.
B(1)=0.0
B(2)=0.0

SOLVE SYSTEM OF EQUATIONS
CALL SOLVE(0,IW,NEQ,MB,1,A,B,LIM,NELEM,RET,RET,LENGTH,DA,DB,S)

PRINT OUT STEADY-STATE RESPONSE
GO TO 103
IF(KNW*NEQ)GO TO 103
REQ=FREQ(L)
WRITE(IW,6)TOWER,REQ
CONTINUE
12=NEQ
IF(KD*EQ.0)GO TO 10
HACC=-CDABS(B(I2))*W2
Y=CDABS(B(I2) - B(I2-2))
USD=USD + Y**2
HSD = HSD + HACC**2
GO TO 101
IF(KNW*NEQ)GO TO 101
WRITE(IW,11)Y,HACC
1 FORMAT(13X, E15.5, 21X, E15.5)
1 CONTINUE
12=I2-1
IF(KD*LT.0)I2=I2-4
CONTINUE
NNODE=NELEM+1
DO 7 I=1,NNODE
J=NNODE-I+1
HACC=-(CDABS(B(I2-1)) * W2)
U=CDABS(B(I2-1))
V=CDABS(B(I2))
US(J)=US(J) + U**2
VS(J)=VS(J) + V**2
HS(J)=HS(J) + HACC**2
U=U/6.6402
IF(KNW*NEQ)GO TO 102
IF(I*EQ.1)WRITE(IW,8)J,U,V,HACC
72 CONTINUE
I2=I2-2
FORMAT('1' /SX,20A4/SX,'*STEADY-STATE RESPONSE AT *F8.3,' CPS*'
**/SX,'NODE',6X,'DISPLACEMENT',10X,'ROTATION',10X,'ACCELERATION'
*/ )
FORMAT(SX,14,3X,E15.5)
IF(KNW*NEQ)GO TO 104
WRITE(IW,12)REQ
04 CONTINUE
2 FORMAT(/**/SX,'*ELEMENT STRAINS AT *F8.3,' CPS*/'SX,'ELEMENT',
*6X,'STRAIN' / )
DO 13 I=1,NELEM
SS(I)=SS(I) + S(I)**2
IF(KNW*NEQ)GO TO 105
WRITE(IW,14)I,S(I)
14 FORMAT(SX,14,3X,E15.5)
13 CONTINUE
CONTINUE
CONTINUE

COMPUTE AND PRINT RESPONSE STANDARD DEVIATIONS

WRITE(IW,107)
FORMAT(('1'/'SX, 'RESPONSE STANDARD DEVIATIONS'//'SX, 'NODE', 6X, *
'DISPLACEMENT', 1DX, 'ROTATION', 1DX, 'ACCELERATION' /)
IF(KD.EQ.0)GO TO 108
USD=DSQRT(USD)
HSD=DSQRT(HSD)
WRITE(IW,109)USD,HSD

FORMAT(5X, 'DAMPER', 2X, E15.5, 21X, E15.5)
CONTINUE
J=NELEM+1
DO 110 I=1, J
US(I)=DSQRT(US(I))
VS(I)=DSQRT(VS(I))
HS(I)=DSQRT(HS(I))
WRITE(IW,8) I, US(I), VS(I), HS(I)
WRITE(IW,111)

FORMAT(//////5X, 'STRAIN STANDARD DEVIATIONS'///5X, 'ELEMENTS', 5X, *
'STRAIN' /)
DO 112 I=1, NELEM
SS(I)=DSQRT(SS(I))
WRITE(IW,14) I, SS(I)
RETURN

FUNCTION RK(W2, G, DEPTH)
IMPLICIT REAL,8(A-H,0-Z)
RK=W2/G
IF(W2.EQ.0.0) RETURN
IF(RK*DEPTH.LT.10.0) GO TO 8
RK=10.0/DEPTH
RETURN
A2=0.0
DK=(2.0-RK)/10.0
A1=A2
B=RK*DEPTH
A2=W2-RK*G*DSINH(B)/DCOSH(B)
IF(A2.EQ.0.0) RETURN
RK=RK+DK
IF(A1/A2 .GE. 0.0) GO TO 10
RK=RK-1.5*DK
DK=DK/2.0
IF(DK/RK .LT. 1.0E-4) RETURN
B=RK*DEPTH
A3=W2-RK*G*DSINH(B)/DCOSH(B)
IF(A3.EQ.0.0) RETURN
DK=DK/2.0
IF(A3/A1 .GT. 0.0) GO TO 14
RK=RK-DK
A2=A3
GO TO 12
RK=RK+DK
A1=A3
GO TO 12
END
FUNCTION DCOSH(A)
IMPLICIT REAL*8(A-H,O-Z)
B=DEXP(A)
DCOSH=0.5*(B + 1.0/B)
RETURN
END

FUNCTION DSINH(A)
IMPLICIT REAL*8(A-H,O-Z)
B=DEXP(A)
DSINH=0.5*(B-1.0/B)
RETURN
END

SUBROUTINE SUBP(N,A1,A2,LENGHT,ZO,DEPTH,W2,P)
IMPLICIT REAL*8(A-H,O-Z)
DOUBLE PRECISION MASS,LENGHT,K
DIMENSION A(5),W(5),P(4)
DATA A,W/0.000,-0.538469D0,0.538469D0,-0.906180D0,0.906180D0,
*0.568889D0,2*0.478629D0,2*0.236927D0/
DATA CI,RO,G/2.0D0,1.0D0,9.81D0/
K=RK(W2,G,DEPTH)
DO 1 I=1,4
P(I)=0.0
Z1=ZO*LENGHT*DFLOAT(N-1)
B=K*DEPTH
C=3.1415926*CI*RO*W2/DSINH(B)
DO 2 I=1,5
X=A(I)
X=0.5*(X+1.0)
RADIUS=A1+(1.0-X)+A2*X
Z=Z1*LENGHT*X
IF(Z.GT.DEPTH)RETURN
B=K*Z
F=C*RADIUS**2*DCOSH(B)*W(I)/2.0*LENGHT
H=X*X*(3.0-2.0*X)
P(1)=P(1)+(1.0-H)*F
P(2)=P(2)+(X*LENGHT*((1.0-2.0*X+X*X))*F
P(3)=P(3)+H*F
P(4)=P(4)+X*X*LENGHT*(X-1.0)*F
CONTINUE
RETURN
END

SUBROUTINE SOLVE(IO,IW,NEQ,MBW,NLS,A,B,LIM,NELEM,RET,RIT,LENGHT,*DA,DB,S)
IMPLICIT REAL*8(A-H,O-Z)
DOUBLE PRECISION MASS,LENGHT
COMPLEX*16 A,B,C,D
DIMENSION A(1),B(15)
DIMENSION S(1)

# REDUCTION OF A, ORIGINAL ARRAY IS DESTROYED
IF(IO.EQ.2)GO TO 20
NRD=NEQ-1
DO 18 I=1,NRD
D=A(I)
IF(CDABS(D).EQ.0.0D0)GO TO 18
IJ=I
DO 16 J=2,MBW

IJ=IJ+NEQ
IF(CDABS(A(IJ)) .EQ. 0.0D0) GO TO 16
C=A(IJ)/D
IK=IJ
JK=I+J-1
DO 14 K=J,MBW
A(JK)=A(JK)-C*A(IK)
IK=IK+NEQ
JK=JK+NEQ
CONTINUE
CONTINUE
IF(10.EQ.1) RETURN

! REDUCTION OF B, ORIGINAL ARRAY IS DESTROYED
NRE=NEQ-1
DO 26 I=1,NRE
D=A(I)
IF(CDABS(D) .EQ. 0.0D0) GO TO 26
IJ=I

DO 24 J=2,MBW
IJ=IJ+NEQ
IF(CDABS(A(IJ)) .EQ. 0.0D0) GO TO 24
C=A(IJ)/D
IK=I
JK=I+J-1
DO 22 K=1,NLS
B(JK)=B(JK)-C*B(IK)
IK=IK+NEQ
JK=JK+NEQ
CONTINUE
CONTINUE

## BACKSUBSTITUTION
I=NEQ
IF(CDABS(A(I)) .EQ. 0.0D0) GO TO 34
IK=I
DO 32 K=1,NLS
B(IK)=B(IK)/A(I)
IK=IK+NEQ
I=I-1
IF(I,EQ,0)GO TO 40
IJ=I

DO 38 J=2,MBW
IJ=IJ+NEQ
IF(CDABS(A(IJ)) .EQ. 0.0D0) GO TO 38
IK=I
JK=I+J-1
DO 36 K=1,NLS
B(IK)=B(IK) - A(IJ) * B(JK)
IK=IK+NEQ
JK=JK+NEQ
CONTINUE
GO TO 30

## SOLVE FOR STRAINS
I=(NELEM + 1)*2
DO 39 K=1,NELEM
J=NELEM - K + 1
N=K-1
D=RET+(DFLOAT(N)*DA)+(DA/2.0) - RIT -(DFLOAT(N)*DB) - (DB/2.0)
S(J)=CDABS((B(I)-B(I-2)))* D/(2.0*LENGT)
I=I-2
CONTINUE
RETURN
END

****** SAMPLE DATA *********

EEL-JACKETED OFFSHORE PLATFORM - 305M IN DEPTH
4 -31 1 0 -1 0 0
366.0 0.0 33.75 9.0 33.6 8.9 305.0
390000.0 1.83
0.0302 0.0043 0.015
4180.0 2432760.0
7.2874 2.6963 1.0020390000.0 0.0500 0.03
0.030 0.830 0.040 2.300 0.050 4.000 0.060
0.070 3.000 0.080 2.400 0.090 2.000 0.100
0.110 1.300 0.120 0.950 0.130 0.700 0.140
0.150 0.340 0.160 0.295 0.170 0.250 0.180
0.190 0.195 0.200 0.170 0.210 0.150 0.220
0.230 0.110 0.240 0.098 0.250 0.090 0.260
0.270 0.090 0.280 0.095 0.290 0.098 0.300
0.310 0.095 0.320 0.095 0.330 0.095
This program is an extension of Program FLUID and FREQMOD of Shaaban and Nash, and Balendra and Nash, for the free vibration analysis of ground-supported liquid storage tanks. In this work, the application is extended to elevated liquid storage tanks.
DO 20 I=1, NDFS
DO 20 J=1, NDFS
ADM(I,J)=0.0
DO 20 K=1, MM

20 ADM(I,J)=ADM(I,J)+SCT(I,K)*SC(K,J)
PRINT 1
1 FORMAT(25(2H**))
WRITE(10) NHR
WRITE(10) NDFS
WRITE(10) ((ADM(I,J), J=1, NDFS), I=1, NDFS)
PRINT 17, NHR
17 FORMAT(30X, 'ADDED MASS MATRIX FOR CIRCUMFERENTIAL WAVE=' , I4)
PRINT 11, ((ADM(I,J), J=1, NDFS), I=1, NDFS)
11 FORMAT(10(2X, E10.4))
60 CONTINUE
REWIND 10
CALL FREMOD
END

SUBROUTINE FLGEN(DENF, R, WH, XM, NN, MM, NDFS, IBAND, MMD, NDFSD, LINEAR, Q FS, SC, INDEX)
DIMENSION FS(LINEAR), SC(MMD, NDFSD)
DIMENSION FK(4, 4), FF(2, 4), N(4)
DX=R/FLOAT(NN)
DY=WH/FLOAT(MM)
A=DX*0.5
B=DY*0.5
DO 10 I=1, LINEAR
10 FS(I)=0.0

TRANSFORMATION FROM A SQUARE MATRIX TO A BANDED MATRIX
(K,L) = (K,J), J=L-K+1
TRANSFORMATION FROM A BAND TO A LINEAR ARRAY
LFS=(K-1)*IBAND + J

NN1=NN-1
MM1=MM-1
IF(INDEX .EQ. 1) NNX=NN-1
IF(INDEX .EQ. 2) NNX=NN
DO 1000 I=1, NNX
IF(INDEX .EQ. 1) XO=FLOAT(I)*DX+A
IF(INDEX .EQ. 2) XO=FLOAT(I-1)*DX+A
CALL FSTIF(A, B, XO, FK, XM, DENF)
DO 1000 J=1, MM1
N(1)=(I-1)*MM+J
N(2)=I*MM+J
N(3)=N(2)+1
N(4)=N(1)+1
DO 55 II=1, 4
K=N(II)
IPAST=K*IBAND-IBAND
DO 51 JJ=1, 4
IF(N(JJ) .LT. N(II) ) GO TO 51
L=N(JJ)-K+1
LFS=IPAST+L
FS (LFS)=FS (LFS)+FK(II, JJ)
51 CONTINUE
55 CONTINUE
CONTINUE AT(1), B(10E+07), C(95), D(1000)
DO = 10100 MI = 1, NNX
IF(INDEX EQ. 1) XO = FLOAT(I) * DX + A
IF(INDEX EQ. 2) XO = FLOAT(I-1) * DX + A
CALL FSTIF(A, B, XO, FK, XM, DENF)
N(I) = MMX(NI+1) LT. I, 10-10) A(IN+1) = 1.0
N(2) = (I + 1) * MMX
DO = 65 I = 1, 2
K = N(II) * 1, NEG
15 IPAST = K * IBAND - IBAND) / A(INL+1)
16 DO = 61 JJ = 1, 2
IF(N(JJ) LT. N(II) ) GO TO 61
L = N(JJ) = K+1 NB
LFS = IPAST + L
10 FS(LFS) = FS(LFS) + FK(II, JJ)
61 CONTINUE = 2, NB
65 CONTINUE
010 CONTINUE I = 1) GO TO 30
IF(INDEX EQ. 2) GO TO 76
XO = A.I = 1, NB
CALL FSTIF(A, B, XO, FK, XM, DENF)
DO = 1020 J = 1, MM1
20 N(2) = J = A(I+J-1) = C(L) * A(INL+K)
N(3) = J+1, NO) GO TO 26
DO = 75 I = 2, 3
K = N(II) * 1, NEG
25 IPAST = IBAND*K-IBAND) - C(L) # B(NON, IB)
26 DO = 71 JJ = 2, 3
30 IF(N(JJ) LT. N(II) ) GO TO 71
L = N(JJ) = K+1
LFS = IPAST + L
FS(LFS) = FS(LFS) + FK(II, JJ)
71 CONTINUE I = 1, NB
75 CONTINUE 5, LINEAR SEQUENCE
020 CONTINUE I = 1, NEG
J = MM1 I = 1, MM
70 IPAST = J * IBAND - IBAND
LFS = IPAST + 1 NO
75 FS(LFS) = FS(LFS) + FK(2, 2)
76 DO = 40 J = 1, MM1
40 DO = 40 J = 1, NDIFSD
40 SC(J+L) = 0.0
CALL FFORCE(R, B, FF) 60
DO = 3000 J = 1, MM1
NI = (J-1) * 2
DO = 205 JJ = 1, 4 GO TO 50
L = NI + JJ = A(INL+K) # MII
50 SC(J+L) = SC(J+L) + FF(1, JJ)
205 SC(J+L) = SC(J+L) + FF(2, JJ)
500 CONTINUE
NI = (MM-1) # 2 MM
80 SC(MM, NI+1) = SC(MM, NI+1) + FF(1, 1)
100 SC(MM, NI+2) = SC(MM, NI+2) + FF(1, 2)
RETURN
END
SUBROUTINE BINV(A, B, NN, NB, NEQ, MM, NEQD, MMD)
DIMENSION A(1), B(MMD, NEQD), C(35), D(1000)
ND=NN-MM
N=0
5 N=N+1
NL=(N-1)*NB
IF(ABS(A(NL+1)).LT. 1.0E-10) A(NL+1)=1.0
IF(N .LE. ND) GO TO 16
NCON=N-ND
DO 15 IB=1, NEQ
5 B(NCON,IB)=B(NCON,IB)/A(NL+1)
6 CONTINUE
IF(N .EQ. NN) GO TO 45
DO 10 K=2, NB
C(K)=A(NL+K)
10 A(NL+K)=A(NL+K)/A(NL+1)
DO 30 L=2, NB
I=N+L-1
IF(NN .LT. 1) GO TO 30
J=0
IL=(I-1)*NB
DO 20 K=L, NB
J=J+1
20 A(IL+J)=A(IL+J)-C(L)*A(NL+K)
IF(N .LE. ND) GO TO 26
ICON=1-ND
DO 25 IB=1, NEQ
25 B(ICON,IB)=B(ICON,IB)-C(L)*B(NCON,IB)
26 CONTINUE
30 CONTINUE
GO TO 5
N= NO. OF EQU.
L= NO. OF UNKNOWN
K= SEQUENTIAL NO. OF UNKNOWN IN THE BAND
NL+K=LFS ... LINEAR SEQUENCE
55 DO 100 IB=1, NEQ
DO 70 II=1, MM
70 D(II+ND)=B(II, IB)
DO 75 II=1, ND
75 D(II)=0.0
N=NN
40 N=N-1
NL=(N-1)*NB
IF(N .EQ. 0) GO TO 60
DO 50 K=2, NB
L=N+K-1
IF(NN .LT. L) GO TO 50
D(N)=D(N)-A(NL+K)*D(L)
50 CONTINUE
GO TO 40
50 CONTINUE
DO 80 II=1, MM
80 B(II, IB)=D(II+ND)
00 CONTINUE
RETURN
END
SUBROUTINE FFORDER(R, B, FF)
DIMENSION FF(2,4)
PI =3.14159
DO 10 I=1,4
DO 10 J=1,4
FF(I,J)=0.0
V=PI*R*B
BV=V*B
FF(1,1)=0.7*V
FF(1,2)=BV/5.0
FF(1,3)=0.3*V
FF(1,4)=-2.0*BV/15.0
FF(2,1)=0.3*V
FF(2,2)=2.0*BV/15.0
FF(2,3)=0.7*V
FF(2,4)=-BV/5.0
RETURN
END

SUBROUTINE FSTIF(A,B,X0,FK,XM,DENF)
DIMENSION A1(4,4),A2(4,4),A3(4,4),FK(4,4)
DO 12 I=1,4
DO 12 J=1,4
A1(I,J)=0.
A2(I,J)=0.
A3(I,J)=0.
V1=X0*B/A/6.
A1(1,1)=2.0*V1
A1(2,2)=2.0*V1
A1(3,3)=2.0*V1
A1(4,4)=2.0*V1
A1(1,2)=-2.0*V1
A1(2,1)=-2.0*V1
A1(3,4)=-2.0*V1
A1(4,3)=-2.0*V1
A1(1,3)=-1.0*V1
A1(3,1)=-1.0*V1
A1(2,4)=-1.0*V1
A1(4,2)=-1.0*V1
A1(2,3)=V1
A1(3,2)=V1
A1(1,4)=V1
A1(4,1)=V1
V2=X0*A/B/6.
A2(1,1) = (2.0-A/X0)*V2
A2(4,4) = (2.0-A/X0)*V2
A2(1,3) = V2
A2(3,1) = V2
A2(2,4) = V2
A2(4,2) = V2
A2(2,2) = (2.0+A/X0)*V2
A2(3,3) = (2.0+A/X0)*V2
A2(1,4) = -(2.0-A/X0)*V2
A2(4,1) = -(2.0-A/X0)*V2
A2(2,3) = -(2.0+A/X0)*V2
A2(3,2) = -(2.0+A/X0)*V2
A2(1,2) = V2
A2(2,1) = V2
A2(3,4) = V2
A2(4,3) = V2
3 V3 = 6/A/A/12.
IF(A*EQ.XO) XO = XO+.001
E1 = ((A+XO)*(A+XO)*ALOG((XO+A)/(XO-A))-2*A*(2*A+XO))*V3
E2 = ((A-XO)*(A-XO)*ALOG((XO+A)/(XO-A))+2*A*(2*A-XO))*V3
E3 = ((A-XO)*(A-XO)*ALOG((XO+A)/(XO-A))+2*A*XO)*V3
IF(A*EQ.XO) XO = XO-.001
A3(1,1) = 2*E1
A3(4,4) = 2*E1
A3(2,2) = 2*E2
A3(3,3) = 2*E2
A3(1,2) = 2*E3
A3(2,1) = 2*E3
A3(3,4) = 2*E3
A3(4,3) = 2*E3
A3(1,3) = E3
A3(3,1) = E3
A3(2,4) = E3
A3(4,2) = E3
A3(1,4) = E1
A3(4,1) = E1
A3(2,3) = E2
A3(3,2) = E2
DO 10 I = 1,4
DO 10 J = 1,4
RETURN
END
SUBROUTINE FREMOD
COMMON/CONST/NH,NELEMS,NNODES,NSIZE,NEQ
COMMON/ADD/FLUIDH,NHC
COMMON/BC/NBC
COMMON/WE/LOWEST
DIMENSION FNU1(50),FNU2(50),E1(50),E2(50),G(50),T(50),SINE(51),
COSINE(51),SINM(50),COSM(50),R(50),PH(50),PHP(50),ARCL(50)
DIMENSION AL(167),CHECK(8,8),RO(51),Z(51),COMMENT(20),JUNK(20)
DIMENSION D(144,145),GA(920),IHARM(5)
INTEGER CLFR,CLCL,CLSM
DATA CLFR,CLCL,CLSM/*CLFR','CLCL','CLSM*/
CARD 1
READ 5,NT,NMODE
5 FORMAT(515)
CARD 2
READ 6,FLUIDH,NHC,NHARM,LOWEST
FORMAT(F10.4,15,I5,I5)
CARD 3
READ 5,(IHARM(1),I=1,NHARM)
CARD 4
READ 7,NBC
7 FORMAT(A4)
REWIND NT
READ(NT) NCARDS,JUNK
IF(NCARDS.EQ.0.0) GO TO 120
DO 110 K = 1, NCARDS
10 READ(NT)(COMMENT(J), J=1,20)
PRINT 401,(COMMENT(J), J=1,20)
01 FORMAT(2X,20A4)
20 READ(NT) NHP,NELEMS,JUNK
D0140 I=1,NELEMS
READ(NT)((CHECK(I,J),I=1,8),J=1,8),(AL(I),I=1,166)
40 CONTINUE
NNODES=NELEMS+1
NEO=4*NNODES
READ(NT)(FNU1(I),I=1,NELEMS),(FNU2(I),I=1,NELEMS),(E1(I),I=1,NELEMS)
$),(E2(I),I=1,NELEMS),(G(I),I=1,NELEMS),(T(I),I=1,NELEMS)
D0 404 I=1,NELEMS
PRINT 402,FNU1(I),FNU2(I),E1(I),E2(I),G(I),T(I)
02 FORMAT(2F8.3,3E12.3,E13.4)
04 CONTINUE
D0160 I=1,NELEMS
IF(I.EQ.NELEMS) GO TO150
READ(NT) R(I),PH(I),PHP(I),ARCL(I),SINE(I),COSINE(I)
GO TO160
150 READ(NT) R(I),PH(I),PHP(I),ARCL(I),SINE(I),COSINE(I),SINE(I+1),
& COSINE(I+1)
160 CONTINUE
READ(NT) (RO(I),I=1,NNODES),(Z(I),I=1,NNODES)
D0170 I=1,NELEMS
COSM(I)=COS(PH(I))
SINM(I)=SIN(PH(I))
170 CONTINUE
NSIZE=10+26*NELEMS
D0 180 IH=1,NHP
D0 172 JH=1,NHARM
IF(IH-1.EQ.IHARM(JH)) GO TO 175
172 CONTINUE
READ(NT) (GA(I),I=1,NSIZE)
READ(NT) (GA(I),I=1,NSIZE)
GO TO 180
175 PRINT 176 , IHARM(JH)
176 FORMAT(4X,2X,*CIRCUMFERENTIAL WAVE=' ,I5)
NH=IHARM(JH)
CALL AMSMAT(D,NT)
5 FORMAT(1X,12(1X,E9.2))
NO=NEQ-4
CALL EIGEN(D,NG,NMODE)
180 CONTINUE
RETURN
END
SUBROUTINE AMSMAT(D,NT)
COMMON/CONST/NH,NELEMS,NNODES,NSIZE,NEQ
COMMON/ADD/FLUIDH,NHC
DIMENSION D(144,145),BSX(920),BMX(920)
READ(NT)(BSX(I),I=1,NSIZE)
READ(NT)(BMX(I),I=1,NSIZE)
D0 1 I=1,NEQ
D0 1 J=1,NEQ
1 D(I,J)=0.0
CALL ADDMASS(D,BMX)
NEQ1=NEQ+1
DO 6 I=1,NEQ
   I=I+1
DO 6 J=I,NEQ1
6  D(I,J)=0.0
M=0
DO 2 I=1,6
   DO 2 J=1,1
   M=M+1
   D(J,I+1)=BSX(M)
2  CONTINUE
L=5
K=0
DO 3 I=9,NEQ
   DO 3 J=L,1
      M=M+1
      D(J,I+1)=BSX(M)
3  CONTINUE
K=K+1
4  IF(K.NE.4) GO TO 3
   L=L+4
   K=0
3  CONTINUE
4  IF(M.NE.NSIZE) STOP
RETURN
END

SUBROUTINE ADDMASS(D,BMX)
COMMON/CONS/NH,NELEMS,NNODES,NSIZE,NEQ
COMMON/ADD/FLUIDH,NHC
COMMON/WE/LOWEST
DIMENSION D(144,145),ADM(50,50),BMX(920)
5  IF(FLUIDH.EQ.0.0) GO TO 80
   REWIND 10
   DO 50 I=1,NHC
      READ(10) NHR
      READ(10) NDFS
      READ(10),(ADM(K,J),J=1,NDFS),K=1,NDFS)
50  IF(NHR.EQ.NH) GO TO 80
   PRINT 60,NH
60  FORMAT(2X,'ADDED MASS MATRIX FOR CIRCUMFERENTIAL WAVE=',I5,' IS NOT $ FOUND')
   STOP
80  K=1
   DO 90 I=1,8
      DO 90 J=1,1
         D(I,J)=BMX(K)
      K=K+1
90  CONTINUE
   L=5
   M=0
   DO 110 I=9,NEQ
      DO 110 J=L,1
         D(I,J)=BMX(K)
      K=K+1
   M=M+1
   IF(M.NE.4) GO TO 100
L=L+4
M=0
10 CONTINUE
  IF(FLUIDH.EQ.0.0) GO TO 25
  DO 24 I=1,NEQ
  DO 24 J=1,1
  24 D(J,I)=D(I,J)
  IF(NH.NE.1) RETURN
  REWIND 11
  WRITE(11)((D(I,J),J=1,NEQ),I=1,NEQ)
  REWIND 13
  WRITE(13)((D(I,J),J=1,NEQ),I=1,NEQ)
  REWIND 13
  RETURN
DDING THE ADM MATRIX WITH SHELL MASS MATRIX AUX.
NWET=LOWEST+1
L=NWET
L1=L+1
DO 6 I=1,NDFS,2
   I2=4*L-1
   N=1
   DO 5 J=1,I,2
      J2=(L1-N)*4-1
      I21=I2+1
      J21=J2+1
      TEMP=I2
      TEMPJ=J2
      IF(I2*GT*J2) GO TO 11
      I2=J2
      J2=TEMP
11   D(I2,J2)=D(I2,J2)+ADM(I,J)
      I2=TEMP
      J2=TEMPJ
      TEMP=I21
      TEMPJ=J21
      IF(I21*GT*J21) GO TO 12
      I21=J21
      J21=TEMP
12   D(I21,J21)=D(I21,J21)+ADM(I+1,J)
      I21=TEMP
      J21=TEMPJ
      TEMP=I21
      TEMPJ=J21
      IF(J,EQ,1) GO TO 13
      I21=J21
      J21=TEMP
13   D(I21,J21)=D(I21,J21)+ADM(I+1,J+1)
      I21=TEMP
      J21=TEMPJ
      IF(J,EQ,1) GO TO 15
      TEMP=12
      TEMPJ=J21
      IF(I2,GT,J21) GO TO 14
      I2=J21
      J21=TEMP
14   D(I2,J21)=D(I2,J21)+ADM(I,J+1)
12 = TEMP
J21 = TEMPJ
15 N = N + 1
5 CONTINUE
L = L - 1
6 CONTINUE
DO 16 I = 1, NEQ
DO 16 J = 1, I
16 D(J, I) = D(I, J)

IF IT IS DESIRED TO DETERMINE THE RESPONSE OF THE FLUID SOLID SYSTEM
DUE TO BASE EXCITATION THEN MATRIX AUX FOR NAR = 1 MUST BE SAVED IN TAPE
IF(NHR .NE. 1) GO TO 120
REWIND 11
WRITE(11)((D(I, J), J = 1, NEQ), I = 1, NEQ)
REWIND 11
REWIND 13
WRITE(13)((D(I, J), J = 1, NEQ), I = 1, NEQ)
REWIND 13
120 CONTINUE
RETURN
END
SUBROUTINE EIGEN(D, ND, NMODE)
COMMON/BC/NBC
DIMENSION D(144, 145), V1(144), V2(144)
$,X(140, 10), OMEGA(10)
INTEGER CLFR, CLCL, CLSM
DATA CLFR, CLCL, CLSM/'CLFR', 'CLCL', 'CLSM'/
PRE-EIGENVALUE CHOLESKY REDUCTIONS
INA = 1
ND1 = ND + 1
DO 76 MA = 1, ND
DO 76 MAS = MA, ND
MA1 = MA + 1
MAS1 = MAS + 1
GASH = D(MA, MAS1)
GISH = D(MAS, MA)
MASH = 1
76 IF(MA - MASH) 77, 77, 78
78 GASH = GASH - D(MASH, MA1) * D(MASH, MAS1)
GISH = GISH - D(MA, MASH) * D(MAS, MASH)
MASH = MASH + 1
GO TO 79
77 IF(MAS - MA) 81, 81, 119
81 IF(GISH) 118, 82, 82
118 GISH = 0.
82 IF(GASH) 83, 84, 84
83 GASH = 0.
84 DIAG1 = SQRT(GASH)
DIAG2 = SQRT(GISH)
IF(DIAG1 .EQ. 0.) GO TO 85
119 D(MA, MAS1) = GASH / DIAG1
85 IF(DIAG2 .EQ. 0.) GO TO 86
D(MAS, MA) = GISH / DIAG2
86 CONTINUE
76 CONTINUE
FORM U/UL
DO 87 MA=1,ND
DO 87 MAS=MA,ND
MAS1=MAS+1
GASH=D(MAS,MA)
MASH=MA
MASH=MASH+1
IF(MAS-MASH) 88,89,89
GASH=GASH-D(MA,MASH)*D(MASH-1,MAS1)
GO TO 91
D(MA,MAS1)=GASH/D(MAS,MAS1)
CONTINUE
MULTIPLICATION TO GET (U*ULE-1*ULTE-1*UT)
DO 92 MA=1,ND
DO 92 MAS=MA,ND
MAS1=MAS+1
GASH=0.
DO 93 MAS=MAS1,ND1
GASH=GASH+D(MA,MASH)*D(MAS,MASH)
CONTINUE
D(MA,MAS1)=GASH
CONTINUE
MODE=NMODE
PU 1.0 IN V1 FROM 1 TO ND AND ITERATIVE
DO 94 I=1,ND
V1(I)=1.
NUMIT=1
ALAM2=0.
DO 95 I=1,ND
I1=I+1
GASH=0.
DO 96 J=1,I
GASH=GASH+V1(J)*D(J,I1)
CONTINUE
IF(I-ND) 97,98,98
DO 99 J=I1,ND
GASH=GASH+V1(J)*D(I,J+1)
CONTINUE
V2(I)=GASH
ALAM2=ALAM2+GASH*GASH
CONTINUE
ALAMB=SQRT(ALAM2)
SIGSQ=0.
DO 101 I=1,ND
GASH=V2(I)/ALAMB
GAS=V1(I)-GASH
SIGSQ=SIGSQ+GAS*GAS
V1(I)=GASH
CONTINUE
ZT=1./10.**12
NUMIT=NUMIT+1
IF(SIGSQ-ZT)102,102,103
IF(NUMIT-ZT)102,102,102
CONTINUE
PRINT 11
PRINT 104,NUMIT
FORMAT(' NO OF ITERATIONS=',13,/)

104
TO MULTIPLY (UE-1)*(U*X)

I=ND

109 GASH=V1(I)

J=ND

107 IF(J-I) 105,105,106

106 GASH=GASH-V2(J)*D(J,I)

J=J-1

GO TO 107

105 V2(I)=GASH/D(I,I)

I=I-1

108 PRINT 995,IN

OMEGA IN CYCLE/SEC

OMEGA(INA)=SQRT(1./ALAMB)/2./3.1415927

PRINT 112,OMEGA(INA)

RES=0.0

PRINT 12

1 FORMAT(4E14.8)

INODE=1

200 DO 300 I=1,ND,4

PRINT 111,INODE,V2(I),V2(I+1),V2(I+2),V2(I+3)

INODE=INODE+1

300 CONTINUE

111 FORMAT(2X,I5,5X,E16.8,5X,E16.8,5X,E16.8,5X,E16.8)

DO 210 I=1,ND

X(I,INA)=V2(I)

210 CONTINUE

260 PRINT 111,INODE,RES,RES,RES,RES

DO 113 I=1,ND

DO 113 J=I,ND

J1=J+1

113 D(I,J1)=D(I,J1)-ALAMB*V1(I)*V1(J)

INA=INA+1

MODE=MODE-1

CHANGING TO NEXT MODE

IF(MODE) 114,114,115

114 REWIND 14

IF(NBC.EQ. CLCL)ND=ND+4

IF(NBC.EQ. CLSM)ND=ND+3

WRITE(14)NM

DO 116 K=1,NM

WRITE(14) K,OMEGA(K)

DO 116 J=1,ND,4

WRITE(14) X(J,K),X(J+1,K),X(J+2,K),X(J+3,K)

116 CONTINUE

REWIND 14

RETURN

995 FORMAT(//,10X,'MODE NO.=',I3)

11 FORMAT(//,20X,25(2H-))

12 FORMAT(30X,'MODE SHAPE',/,'2X,'NODE',15X,'U',20X,'V',20X,'W',20X,'D

$\omega$/D$^2$)

112 FORMAT(//,10X,'NATURAL FREQUENCY=',E20.10,'IN CYCLES/SEC.')

2 FORMAT(8E16.8)

END

GO.FT10F001 DD UNIT=SYSDA,SPACE=(CYL,(1,1))

GO.FT11F001 DD UNIT=SYSDA,SPACE=(CYL,(1,1))
This program is an extension of Program RESPONSE of Shaaban and Nash, and Balendra and Nash, for the response analysis of cylindrical ground-supported liquid storage tanks subject to earthquake excitation. In this work damping effects of the structural system is included.

```
COMMUN/GEOM/FNU1(50),FNU2(50),E1(50),E2(50),G(50),I(50),SINE(51),
$COSINE(51),SINM(50),COSM(50),R(50),PH(50),PHP(50),ARCL(50)
COMMUN/CHALS/AL(167),CHECK(8,8)
COMMUN/THEtas/THeta(20),NTHeta
COMMUN/HARM/NHP,IHarm(5)
COMMUN/RZ/Ru(51),Z(51)
COMMUN/CONST/Nh,NElements,Nnodes,NSize,NEQ
COMMUN/US/0(204)
DIMENSION D(144,145),BSTF(140,4),STBH(4,4),JMASS(140,4),BBM(4,4),
$X(140,10),UME(50),GM(10,10)
$BACC(4),EFM(140,4),GP(50),PEF(140),PIN(50),SUM1(50),SUM2(50)
$UDD(144),A(50),ADD(50),ADD(4),FB1(4),FB2(4),FB3(4),FB(4)
$GA(2001),COMENT(20),JUNK(20)
DIMENSION AD(10),UD(144),RCRD(10),FB4(4),BDMP(140,4),OMED(10)
DIMENSION FREQ(2,2),ENVFSE(2,2),COEF(2)
DIMENSION SUM(10)
```
READ 10, NT, RADIUS
NT IS THE TAPE IN WHICH THE STIFFNESS AND MASS MATRICES OF THE SHELL
ARE STORED BY SAMMSOR
10 FORMAT(15,F10.4)
AR D 2
READ 20, ND1, ND2, ND3
20 FORMAT(5I5)
AR D 3
READ 30, NTHETA
30 FORMAT(I5)
C A RD 4
READ 40, (THETA(I), I=1, NTHETA)
40 FORMAT(8F10.4)
C A RD 5
READ 50, NH, (1HARM(I), I=1, NH)
50 FORMAT(2I5)
C A RD 6
READ 60, PEAK, TTOTAL, DT
60 FORMAT(3F10.4)
C A RD 7
READ 70, TSTART, TEND, DT
70 FORMAT(3F10.4)
REWIND NT
READ(NT) NCARDS, JUNK
IF(NCARDS*EQ.0.0) GO TO 120
DU110 K=1, NCARDS
110 READ(NT) (COMENT(J), J=1, 20)
PRINT 115, (COMENT(J), J=1, 20)
115 FORMAT(2X, 20A4)
120 READ(NT) NHP, NELEMS, JUNK
PI=3.14159
RAD=PI/180.0
DO130 I=1, NTHETA
THETA(I)=THETA(I)*RAD
130 CONTINUE
DO140 II=1, NELEMS
READ(NT) (((CHECK(I, J), I=1, 8), J=1, 8), (AL(I), I=1, 166)
140 CONTINUE
N NODES=NELEMS+1
NEQ=4*NNODES
READ(NT) (FNU1(I), I=1, NELEMS), (FNU2(I), I=1, NELEMS), (E1(I), I=1, NELEMS)
$),(E2(I), I=1, NELEMS), (G(I), I=1, NELEMS), (T(I), I=1, NELEMS)
DO160 I=1, NELEMS
IF(I, EQ, NELEMS) GO TO 150
READ(NT) R(I), PH(I), PHP(I), ARCL(I), SINE(I), COSINE(I)
GO TO 160
150 READ(NT) R(I), PH(I), PHP(I), ARCL(I), SINE(I), COSINE(I), SINE(I+1),
$ COSINE(I+1)
160 CONTINUE
READ(NT) (RO(I), I=1, NNODES), (Z(I), I=1, NNODES)
DO170 I=1, NELEMS
COSM(I)=COS(PH(I))
SINM(I)=SIN(PH(I))
170 CONTINUE
NSIZE=10+26*NELEMS
SINCE ONLY H A R=1 WILL BE EXCITED MUST SKIP K $ M MATRICES OF HAR=0
READ(NT) (GA(I), I=1, NSIZE)
READ(NT)(GA(I), I=1, NSIZE)
CALL EXTRACT(D, NEQ, NT)
READ(13) N3, N4, NFREE
READING THE FREQUENCIES & MODES FOR HARMONIC=1 FROM TEPE 14
REWIND 14
READ(14) NEV
DO 180 I=1, NEV
READ(14) K, OME(I)
CONVERTING THE FREQUENCIES INTO RAD/SEC.
OME(I)=OME(I)*2.0*PI
DO 180 J=1, N4, 4
READ(14) X(J, I), X(J+1, I), X(J+2, I), X(J+3, I)
180 CONTINUE
REWIND 14
READ(13) ((D(I, J), J=1, N4), I=1, N4)
NOW D IS THE K MATRIX OF NON BASE NODES
<B IS DENOTED AS BSTF
READ(13) ((BSTF(I, J), J=1, 4), I=1, N4)
<BB IS DENOTED AS STBB
READ(13) ((STBB(I, J), J=1, 4), I=1, 4)
READ(13) ((D(I, J), J=1, N4), I=1, N4)
NOW D IS THE M MATRIX OF NON BASE NODES
MB IS DENOTED AS BMASS
READ(13) ((BMASS(I, J), J=1, 4), I=1, N4)
MBB IS DENOTED AS BBM
READ(13) ((BBM(I, J), J=1, 4), I=1, 4)
REWIND 13
READ 2, (RCRD(I), I=1, NEV)
FORMAT(10F8.4)
DO 77 J=1, NEV
7 OME(I)=OME(I)*SQRT(1.0-RCRD(J)*RCRD(J))
FREQ(1, I)=1.0/OME(I)
FREQ(1, 2)=OME(1)
FREQ(2, 1)=1.0/OME(2)
FREQ(2, 2)=OME(2)
DET=FREQ(1, 1)*FREQ(2, 2)-FREQ(2, 1)*FREQ(1, 2)
ENVRSE(1, 1)=FREQ(2, 2)/DET
ENVRSE(1, 2)=-FREQ(1, 2)/DET
ENVRSE(2, 2)=FREQ(1, 1)/DET
ENVRSE(2, 1)=-FREQ(2, 1)/DET
COEF(1)=2.0*((ENVRSE(1, 1)*RCRD(1) +ENVRSE(1, 2)*RCRD(2))
COEF(2)=2.0*((ENVRSE(2, 1)*RCRD(1) +ENVRSE(2, 2)*RCRD(2))
DO 3110 I=1, N4
DO 3110 J=1, 4
110 BDMPI(I, J)=COEF(1)*BMASS(I, J)+COEF(2)*BSTF(I, J)
PRINT 3, COEF(1), COEF(2)
FORMAT(1X, 2F20.10)
CALL MODAN(D, X, GM, N4, NEV)
DO 200 I=1, 4
DO 190 K=1, N4, 4
BBM(I, 1)=BBM(I, 1)+BMASS(K, 1)
BBM(I, 2)=BBM(I, 2)+BMASS(K+1, I)
BBM(I, 3)=BBM(I, 3)+BMASS(K+2, I)
BBM(I, 4)=BBM(I, 4)+BMASS(K+3, I)
190 CONTINUE
00 CONTINUE
CALL EFFM SS(D,BMASS,N4,EFFM)
NTIME=ITOTAL/DT+1.0001
EAK IS G=384.0 IN/SEC/SEC
ACCELERATIONS ARE NORMALIZED BY G
HE ACCELERATION RECORD GENERATED BY PSEQGN IS STORED IN TAPE 20
RE WIND 9
READ(9)(GA(I),I=1,NTIME)
1 FORMAT(9F9.6)
RE WIND 9
05 FORMAT(10E10.4)
PRINT 210,ITOTAL
10 FORMAT(/10X,'TANK IS EXCITED BY AN ARTIFICIAL EARTH QUAKE APPLIED
$FOR*$,F10.4,'IN./SEC/SEC',/)
PRINT 215,PEAK
15 FORMAT(/10X,'MAX. GROUND ACCELERATION=',F10.4,'IN./SEC/SEC',/)
BACC(1)=0.0
BACC(2)=-PEAK
BACC(3)=PEAK
BACC(4)=0.0
DD220 I=1,N4
PEF(I)=0.0
DD220 J=1,4
120 PEF(I)=PEF(I)+EFFM(I,J)*BACC(J)
DD230 I=1,NEV
SUM1(I)=0.0
SUM2(I)=0.0
SUM(I)=0.0
GP(I)=0.0
DD230 J=1,N4
230 GP(I)=GP(I)+X(J,I)*PEF(J)
NSTART=TSTART/DT + 1.0001
NEND=TEND/DT+1.0001
NDT=TD T/DT
DPLM X1=0.0
DPLM X2=0.0
DPLM X3=0.0
ISTART=2
RE WIND 8
RE WIND 9
RE WIND 10
WRITE(8) TDT
WRITE(9)TDT
WRITE(10)TDT
DO 320 IT=NSTART,NEND,NDT
ITIME=IT-1
TIME=FLOAT(ITIME)*DT
CALL DUHAML(GA,GP,TIME,ITIME,DT,NEV,OME,OMED,RCRD,PIN,AD,GM,SUM,
*ISTART)
ISTART=IT
A IS THE DISPLACEMENT IN MODAL COORDINATES
U IS THE NODAL DISPLACEMENTS
ADD IS THE ACC. IN MODAL COORDINATES
UDD IS THE ACC. OF THE NON BASE NODES
DD270 I=1,NEV
A(I)=PIN(I)/(GM(I,1)*OMED(I) )
ADD(I) = GP(I) * GA(IT) / GM(I, I) - OME(I) * OME(I) * A(I) - 2.0 * OME(I) * RCRD(I) * IAD(I)
DO 275 I = 1, NEQ
U(I) = 0.0
UD(I) = 0.0
UDD(I) = 0.0
275 CONTINUE
DO 280 I = 1, N4
DO 280 J = 1, NEV
U(I) = U(I) + X(I, J) * A(J)
UD(I) = UD(I) + X(I, J) * ADD(J)
UDD(I) = UDD(I) + X(I, J) * ADD(J)
280 CONTINUE
IF (ABS (DPLM X1) * GT. ABS (U(ND1))) GO TO 291
DPLM X1 = U(ND1)
 TMEM X1 = TIME
C 291 WRITE (8) U(ND1)
291 CONTINUE
IF (ABS (DPLM X2) * GT. ABS (U(ND2))) GO TO 292
DPLM X2 = U(ND2)
 TMEM X2 = TIME
C 292 WRITE (9) U(ND2)
292 CONTINUE
IF (ABS (DPLM X3) * GT. ABS (U(ND3))) GO TO 293
DPLM X3 = U(ND3)
 TMEM X3 = TIME
C 293 WRITE (10) U(ND3)
293 CONTINUE
C BASE REACTIONS
BACC(1) = 0.0
BACC(2) = - PEAK * GA(IT)
BACC(3) = PEAK * GA(IT)
BACC(4) = 0.0
C FB1 = MBT * UDD
C FB2 = KBT * U
C FB3 = (MBB + MBT * I) * BACC
C FB = FB1 + FB2 + FB3 IS THE BASE REACTION
DO 310 I = 1, 4
FB1(I) = 0.0
FB2(I) = 0.0
FB3(I) = 0.0
FB4(I) = 0.0
DO 300 J = 1, N4
FB1(I) = FB1(I) + BMASS(J, I) * UDD(J)
FB4(I) = FB4(I) + BDMP(J, I) * UD(J)
IF (J, GT, 4) GO TO 300
JK = N4 - 4 + J
FB2(I) = FB2(I) + BSTF(JK, I) * U(JK)
FB3(I) = FB3(I) + BBM(I, J) * BACC(J)
300 CONTINUE
305 FB(I) = (FB1(I) + FB2(I) + FB3(I) + FB4(I)) / (PI * RADIUS)
310 CONTINUE
CALL PRINT (TIME, U, FB, N4, NEQ)
CALL STRESS
320 CONTINUE
C REWIND 8
SUBROUTINE EXTRACT(D, NEQ, NT)
DIMENSION D(144, 145)
NEQ = NEQ
N4 = NFREE - 4
N3 = NFREE - 3
REWIND 13
WRITE(13) N3, N4, NFREE
CALL READ DY(D, NT)
C PARTITIONING THE STIFFNESS MATRIX
C WRITING K MATRIX IN TAPE15
WRITE(13)((D(I, J), J = 1, N4), I = 1, N4)
C WRITE KB
WRITE(13)((D(I, J), J = N3, NFREE), I = 1, N4)
C WRITE KBB
WRITE(13)((D(I, J), J = N3, NFREE), I = N3, NFREE)
REWIND 11
READ(11)((D(I, J), J = 1, NEQ), I = 1, NEQ)
C WRITE M
WRITE(13)((D(I, J), J = 1, N4), I = 1, N4)
C WRITE MB
WRITE(13)((D(I, J), J = N3, NFREE), I = 1, N4)
C WRITE MBB
WRITE(13)((D(I, J), J = N3, NFREE), I = N3, NFREE)
REWIND 13
RETURN
END
SUBROUTINE READDY(D, NT)
C READING THE MODIFIED MASS MATRIX (M+ADM)
REWIND NT
READ(11)((D(I, J), J = 1, NEQ), I = 1, NEQ)
C WRITE M
WRITE(13)((D(I, J), J = 1, N4), I = 1, N4)
C WRITE MB
WRITE(13)((D(I, J), J = N3, NFREE), I = 1, N4)
C WRITE MBB
WRITE(13)((D(I, J), J = N3, NFREE), I = N3, NFREE)
REWIND 13
RETURN
END
SUBROUTINE READ D Y(D, NT)
C READING THE STIFFNESS MATRIX STORED IN TAPE NT
COMMON/CONST/NH, NELEMS, NNODES, NSIZE, NEQ
DIMENSION D(144, 145), BX(920)
NFREE = NEQ
DO 1 I = 1, NFREE
DO 1 J = 1, NFREE
1 D(I, J) = 0.0
C READS AN ARRAY INTO A SQUARE MATRIX
READ(NT)(BX(I), I = 1, NSIZE)
M = 0
DO 2 I = 1, 8
DO 2 J = 1, 1
M = M + 1
D(I, J) = BX(M)
2 D(J, I) = D(I, J)
L = 5
K = 0
DO 3 I = 9, NFREE
3
DO 4 J=L+1
M=M+1
D(I,J)=BX(M)
4 D(J,I)=D(I,J)
K=K+1
IF(K.NE.41 GO TO 3
L=L+4
K=0
3 CONTINUE
IF(M.NE.NSIZE) STOP
RETURN
END
SUBROUTINE MODAN(D,X,GM,N4,NEV)
DIMENSION D(144,145),X(140,10),XM(140,10),GM(10,10)
DO 10 I=1,N4
DO 10 J=1,NEV
XM(I,J)=0.0
DO 10 K=1,N4
XM(I,J)=XM(I,J)+D(I,K)*X(K,J)
10 CONTINUE
E=XT*A*X
DO 20 I=1,NEV
DO 20 J=1,NEV
GM(I,J)=0.0
DO 20 K=1,N4
GM(I,J)=GM(I,J)+X(K,I)*XM(K,J)
20 CONTINUE
RETURN
END
SUBROUTINE EFFMS(D,BMASS,N4,EFFM)
DIMENSION D(144,145),EFFM(140,4),BMASS(140,4)
DIS THE MASS MATRIX OF NON BASE NODES
TO FORM MI MATRIX
DO 10 I=1,N4
10 DO 20 J=1,4
EFFM(I,J)=0.0
DO 20 K=1,N4,4
EFFM(I,1)=EFFM(I,1)+D(I,K)
EFFM(I,2)=EFFM(I,2)+D(I,K+1)
EFFM(I,3)=EFFM(I,3)+D(I,K+2)
EFFM(I,4)=EFFM(I,4)+D(I,K+3)
20 CONTINUE
DO 30 I=1,N4
DO 30 J=1,4
EFFM(I,J)=-(EFFM(I,J)+BMASS(I,J))
30 CONTINUE
RETURN
END
SUBROUTINE DUHAML(GA,GP,TIME,NT,DT,M,OME,OMED,RCRD,PIN,AD,GM,SUM,*ISTART)
DIMENSION GA(2001),GP(10),OME(10),RCRD(10),PIN(10),AD(10)
*GM(10,10),OMED(10)
DIMENSION SUM(10)
DIMENSION SINE(10),EXPO(10)
DO 15 J=1,M
DO 10 IT=1START,NT
TA=FLOAT(IT)*DT
SINE(J)=SIN(OMED(J)*(TIME-TA))
EXP0(J)=EXP(-RCRD(J)*OME(J)*(TIME-TA))
SUM(J)=SUM(J)+GA(IT)*EXP0(J)*SINE(J)
IF(IT,EQ,NT)AD(J)=GA(IT)*EXP0(J)*SINE(J)*GP(J)/(GM(J,J)*OMED(J))
10 CONTINUE
15 PIN(J)=SUM(J)*GP(J)*DT RETURN
END
SUBROUTINE PRINT(T,U,FB,N4,NEQ)
DIMENSION U(204),FB(4)
PRINT 100,T
100 FORMAT(20X,25(2H--),/,'TIME=',F10.4,/) PRINT 200
200 FORMAT(/,2X,'NODE',10X,'U',18X,'V',18X,'W',14X,'DW/DZ')
RES=0.0
JK=1
DO 240 I=1,NEQ,4
IF(JK,NE,15)GO TO 239
IF(JK,EQ,15)WRITE(7,245)U(I+2)
245 FORMAT(E20.5)
PRINT 250,JK,U(I),U(I+1),U(I+2),U(I+3)
239 CONTINUE
IF(JK,NE,17)GO TO 240
PRINT 250,JK,U(I),U(I+1),U(I+2),U(I+3)
240 JK=JK+1
250 FORMAT(3X,13.5X,E12.4,5X,E12.4,5X,E12.4,5X,E12.4)
PRINT 400
400 FORMAT(/,25X,'REACTION AT THE BASE',/) PRINT 500,FB(1),FB(2),FB(3),FB(4)
500 FORMAT(10X,'AXIAL FORCE(NS)=',E12.4,'POUND/INCH',/,'4X,'TANGENTIAL $FORCE(NS)=',E12.4,'POUND/INCH',/,'9X,'RADIAL FORCE(NR)=',E12.4,'PO
$UND/INCH',/,'9X,'AXIAL MOMENT(MS)=',E12.4,'POUND INCH/INCH')
RETURN
END
SUBROUTINE STRESS
COMMON/CONST/NH,NELEMS,NNODES,NSIZE,NEQ
COMMON/THETAS/THETA(20),NTHETA
COMMON/HARM/NHP,IHARM(5)
COMMON/GEOM/FNU1(50),FNU2(50),E1(50),E2(50),G(50),T(50),SINE(51), $COSINE(51),SINM(50),COSM(50),R(50),PH(50),PHP(50),ARCL(50)
COMMON/GCD/CC1,CC2,DD1,DD2,GG1,GG2
COMMON/US/U(204)
PRINT 100
DO 200 I1=1,NELEMS
CALL STRAIN(I1)
DO 400 I=1,NTHETA
ESU=0.0
ETU=0.0
E13U=0.0
E23U=0.0
CHIS=0.0
CHIT=0.0
CHIST=0.0
CTHIS=0.0
CHIT=0.0

XIHI=I*HARM(IH)
CS=COS(XIHI*THETA(I))
SN=SIN(XIHI*THETA(I))

K=4*(I1-1)+NEQ*(I1-1)

CALCULATION OF LINEAR STRAIN
ESU=ESU+E5(IH)*CS
ETU=ETU+ET(IH)*CS
ESTU=ESTU+EST(IH)*SN
E13U=E13U+E13(IH)*CS
E23U=E23U+E23(IH)*SN

CALCULATION OF CHANGE OF CURVATURE
UB3=-U(K+1)*SINE(I1)+U(K+3)*COSINE(I1)
UB7=-U(K+5)*SINE(I1+1)+U(K+7)*COSINE(I1+1)
CHIS=COS(XIHI*THETA(I))

THETA1=THETA(I)*180./3.14159

IF(I1.NE.15)GO TO 400

PRINT 700,I1,THETA1,STRNS,STRNT,STRNST,STRMS,STRMT,STRMST

CONTINUE

FORMAT(E20.5)

EPS=ESU
EPST=ETU
EPS=ESTU

CALCULATION OF MID SURFACE STRAINS

EPS60=EPS
EPS70=EPST

CALCULATION OF STRESS & MOMENT RESULTANTS

STRNS=CC1*EPS+FNU1(I1)*CC1*EPST
STRNT=CC2*EPS+FNU1(I1)*CC2*EPST

CONTINUE

FORMAT(E20.5)

PRINT 700,I1,THETA1,STRNS,STRNT,STRNST,STRMS,STRMT,STRMST

CONTINUE

FORMAT(/,25X,'FORCE RESULTANTS',26X,'MOMENT RESULTANTS'
$ '1X,'N(S)',10X,'N(T)',8X,'N(ST)',10X,'M(S)',8X,'M(T)',10X,'$'"'M(ST)',/,'1X,'ELEM THETA',/,'1X,'NO',4X,'(DEG)'

CONTINUE

FORMAT(I4,F8.4,6(1X,E12.4))

CONTINUE

FORMAT(I4,F8.4,8(1X,E12.4))
RETURN
END

SUBROUTINE STRAIN(I1)
COMMON/CONST/NH,NELEMS,NNODES,NSIZE,NEQ
COMMON/HARM/NHP,IHARM(5)
COMMON/EE$/ES(5),ET(5),EST(5),E13(5),E23(5)
COMMON/GEOM/FNU1(50),FNU2(50),E1(50),E2(50),G(50),T(50),SINE(51),
$COSINE(51),SINM(50),COSM(50),R(50),PH(50),PHN(50),ARCL(50)
COMMON/RZ/RO(51),Z(51)
COMMON/US/U(204)
COMMON/GCD/CC1,CC2/DD1,DD2,GG1,GG2

DIMENSION E23Q1(5),E23Q3(5),E23Q5(5),E23Q7(5),ESTQ1(5),ESTQ3(5),
$ESTQ5(5),ESTQ7(5),ETQ2(5),ETQ6(5)

COMPUTES STRAINS FOR AN ELEMENT
WRITTEN FOR ANY HARMONIC ASSUMING THE DISPLACEMENTS ARE ARRANGED IN A S
ROW FROM HARMONIC=0 TO HARMONIC=NH
IN PROGRAM RESP ONLY HARMONIC=1 IS EXCITED THUS THE ARRAY U HAS DISP
HARMONIC=1 ONLY

FN=1.-FNU1(I1)*FNU2(I1)
CC1=E1(11)*T(I1)/FN
CC2=E2(11)*T(I1)/FN
GG1=G(I1)*T(I1)
GG2=G(I1)*T(I1)**3./12.0
DD1=E1(I1)*T(I1)**3.(/12.*FN)
DD2=E2(I1)*T(I1)**3.(/12.*FN)
Jl=JI
J11=JI1+1

DR0=RO(J11)-RO(J1)
DZ=Z(J11)-Z(J1)

ARL=SQRT(DRO*DR0+DZ*DZ)
SIPH=DR0/ARL
COPH=DZ/ARL
RM=(RO(J1)+RO(J11))/2.0
R2I=1.0/(2.0*RM)
ARCLI=1.0/ARL

ETQ3=R2I
ETQ7=R2I
E23Q2=-R2I*COPH
E23Q6=E23Q2
E13Q1=ARCLI*SIPH
E13Q3=-ARCLI*COPH
E13Q5=-ARCLI*SIPH
E13Q7=ARCLI*COPH

ESTQ2=-SIPH*R2I-ARCLI
ESTQ6=-SIPH*R2I+ARCLI
ESQ1=E13Q3
ESQ3=-E13Q1
ESQ5=E13Q7
ESQ7=-E13Q5

CO2R=COPH*R2I
SL2R=SIPH*R2I
CL2R=COPH*R2I
S02R=SIPH*R2I
DO 300 IH=1,NH
K=IHARM(IH)

XK=K
E23Q1(IH)=SO2R*XK
E23Q3(IH)=-CO2R*XK
E23Q5(IH)=SL2R*XK
E23Q7(IH)=-CL2R*XK
EST01(IH)=E23Q3(IH)
EST03(IH)=-E23Q1(IH)
EST05(IH)=E23Q7(IH)
EST07(IH)=E23Q5(IH)
ETQ2(IH)=R2I*XK
ETQ6(IH)=ETQ2(IH)

COMPUTE ET, ES, EST, E13, E23
KK=NEQ*(IH-1)+4*(II-1)
KK1=KK+1
KK2=KK+2
KK3=KK+3
KK5=KK+5
KK6=KK+6
KK7=KK+7

ET(IH)=ETQ2(IH)*U(KK2)+ETQ3*U(KK3)+ETQ6(IH)*U(KK6)+ETQ7*U(KK7)
ES(IH)=ESQ1*U(KK1)+ESQ3*U(KK3)+ESQ5*U(KK5)+ESQ7*U(KK7)

300 CONTINUE
RETURN

/*GO.*FT09F001 DD DSN=F3011600,INPT3,UNIT=DISK,*/
/* VOL=SER=MUN004,DISP=OLD*/
/*GO.*FT12F001 DD DSN=F3011600,DATSA2,UNIT=DISK,*/
/* VOL=SER=MUN003,DISP=OLD*/
/*GO.*FT11F001 DD DSN=F3011600,DATMT5,UNIT=DISK,*/
/* VOL=SER=MUN003,DISP=OLD*/
/*GO.*FT13P001 DD UNIT=SYSDA,SPACE=(CYL,(1,1))*/
/*GO.*FT14F001 DD DSN=F3011600,DATFR5,UNIT=DISK,*/
/* VOL=SER=MUN003,DISP=OLD*/
/*GO.*SYSIN DD */
12 280.000
51 63 127
1 0.0000
1 1
384.00000 12.5000 0.01
0.1000 12.5000 0.1000
0.0001 0.05
/*/*