PERCEPTIONS OF CHANGE IN SCHOOL MATHEMATICS

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Perceptions of Change in School Mathematics

submitted by:
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Abstract

This qualitative study examines the perceived effects that exposure to a new learning environment might have on a group of students and their mathematics teacher, especially as these perceptions relate to working together to solve mathematical problems. Five academic high school students, randomly grouped together to solve mathematical problems related to Coordinate Geometry, were observed for a thirteen-class period. Following the classroom observations, three of the five students agreed to be interviewed and to solve problems individually, in pairs, and as a group. As well, their mathematics teacher was interviewed. All interviews and problem solving sessions were audio taped.

The four research questions were: **1)** How do these students feel about their new learning environment, especially with respect to how they solved mathematical problems while working together? **2)** What are their teacher's perceptions with respect to her students as a result of such an experience? **3)** What are my own perceptions of the experience, especially with respect to how the students solved problems and worked together? **4)** What were some factors that appeared significant for the students in this mathematics class?

Factors such as student attendance, discipline, culture and language surfaced as influencing the ability of these students to work together to solve mathematical problems in their new learning environment. The role of homework in a cooperative learning environment also emerged as an issue for future research considerations.

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Contents

Chapter 1 Introduction

Mathematics educational reform has been the focus of numerous endeavours by classroom teachers. Many of these have come as a result of the directives endorsed by the National Council of Teachers of Mathematics in the document entitled *Curriculum & Evaluation Standards for School Mathematics,* usually referred to as the *Standards* (NCTM, 1989). The *Standards* presents fourteen curriculum standards for senior high school mathematics. The first two of these standards are mathematics as problem solving and mathematics as communication.

For teachers who have traditionally employed a lecture style approach, developing and initiating these two standards often poses a challenge. Typically with this approach, the teacher lectures for most of the class, writing notes or examples on a blackboard. Students are then presented with similar "problems" from their text while the teacher monitors the class and helps students who are encountering difficulty. Students work relatively independently for the remainder of the class. The class ends with the assignment of homework, usually in the form of "more of the same" problems, only to resume the same way the next class. Little time is devoted to allowing students to actually solve problems or to work cooperatively in class.

This study deals with the students of a high school mathematics teacher who is attempting to move towards the recommendations of the *Standards* especially with respect to utilizing cooperative learning as a form of communication and means to develop problem solving abilities among her students. This teacher, who had typically employed a lecture approach in her classes, experimented, for the first time, with an approach where students were encouraged to work together to solve mathematical problems as a primary component of their learning environment for a thirteen-day period. While it is admittedly impossible to study the process of change in such a short timetable, attention in this study is directed toward the effort to change (Wolcott, 1994). Therefore perceptions from all parties involved regarding the experience are the focus of this study. In particular, the study focuses upon the perceptions of the students and their teacher, especially as these perceptions relate to the students' problem solving ability as a result of such an experience.

This study deals with the realities of the mathematics classroom and the impact that changes to the learning environment might have on both the teacher and the students. It is therefore contextual in nature and inductive in its approach. During the course of the study, issues such as discipline, attendance, culture, and language emerged as unexpected influences on the degree of success with solving mathematical problems that such a group of students might experience.

Since the focus of this study is on a group of high school students who were encouraged to work cooperatively to solve mathematical problems, operational definitions of both problem solving and cooperative learning will be provided to clarify the meanings employed in this study. In addition to the brief descriptions given below, the literature review will develop the natures of both cooperative learning and problem solving more fully.

1.1 Operational Definition of Problem Solving

Establishing a precise definition of the term problem solving is not an easy task, especially considering that it can be viewed in a very broad and all-encompassing sense (Goldin, 1982). In fact "it can mean different things to different people at the same time and different things to the same person at different times" (Branca, 1980, p.3). While this is true, it is important to establish at least a working definition so that parameters of the study can be set (Schoenfeld, 1992).

According to Pólya, a pioneer in the field of problem solving, "to solve a problem is to find a way where no way is known offhand, to find a way out of a difficulty, to find a way around an obstacle, to attain a desired end, that is not immediately attainable, by

appropriate means" (Pólya, 1980, p.1). Put more succinctly, it can be considered "the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation" (Rudnick & Krulik, 1982, p.171). This is the operational definition of problem solving employed throughout this study. The problems in this study were presented to students in two contexts: via various worksheets during mathematics class and novel problems presented during problem solving sessions that took place afterwards.

1.2 Operational Definition of Cooperative Learning

Many terms have been used synonymously with cooperative learning. These include group work, small groups, and collaborative learning. Cooperative learning has been described as an environment whereby "students interact to explore ideas, synthesize knowledge, justify assertions, share discoveries, and talk through difficulties" (Lambdin, 1993, p. 48). Although defining cooperative learning is not as challenging a task as problem solving, *establishing* a cooperative learning environment in the mathematics classroom may require a paradigm shift for some teachers. This is especially true for teachers who have long employed the *traditional* methods of teaching mathematics, often referred to as "chalk and talk". This study deals

specifically with a teacher who is trying to move from traditional methods towards a cooperative learning environment with the students in her mathematics class.

According to Webb (1991), the "optimum small group setting is one in which students freely admit what they do and do not understand, and consistently give each other opportunities to demonstrate their level of understanding" (p. 386). This is the operational definition of cooperative learning employed in this study. While this may not reflect what actually transpired in the study, it was the goal.

1.3 Research Objectives

Through this inquiry I hope to come to some understanding of what immersing a group of high school students in a new learning environment in the mathematics class that includes problem solving might mean both to them and their teacher. In particular, I intend to describe the perceptions that a group of high school students, as well as their teacher, have regarding the students' problem solving ability as a result of being part of a new learning environment. My own perceptions of the experience will also be included, thus adding an additional dimension.

The primary intent of this study is to concentrate on the perceptions of a group of students and their teacher about their problem solving ability as a result of being exposed to a new learning environment where they are encouraged to work together to solve mathematical problems.

The main research questions are:

1. How do these students feel about their new learning environment, especially with respect to how they solved mathematical problems while working together?

2. What are their teacher's perceptions with respect to her students as a result of such an experience?

3. What are my own perceptions of the experience, especially with respect to how the students solved problems and worked together?

4. What were some factors that appeared significant for the students in this mathematics class?

1.4 Significance of the Study

The significance of this study is twofold. First, while there is already a vast amount of existing research in the area of problem solving in mathematics, much of it relates to the student as an individual rather than as member of a group exerting a combined effort to solve a problem. More attention needs to be directed toward problem solving in groups and whole classes (Davidson & Kroll, 1991; Lester, 1994). This study explores what effect placing a group of high school students in a new learning environment might have on their perceived ability to solve mathematical problems.

An additional recommendation for research on problem solving is to obtain knowledge through problem solving episodes of a specific group of problem solvers for a wide variety of problems instead of for a specific set of problems over a variety of problem solvers (Lester, 1980). "Such information will not only add to the body of knowledge but also significantly aid in the design of research on problem solving instruction" (Lester, 1980, p.318). It is from this perspective that this study stems.

Secondly, much of the existing research focuses primarily on the cognitive aspects of problem solving. The intent of this study is to concentrate upon the perceptions of a group of students and their teacher as these perceptions relate to the students' problem

solving ability after experiencing a learning environment based on working together while solving mathematical problems. Affective components are an underrepresented theme of research in problem solving (Lester, 1994; McLeod, 1993; Schoenfeld, 1982; Taplin, 1994). Thus, directing attention to the effect of affect may help provide some insights in this area of problem solving research, which may then highlight the interrelationship between affect and cognition in the learning of mathematics. In turn, attention to both affective components and cognition may be necessary in developing positive attitudes toward mathematics (Adams, 1989).

1.5 Limitations of the Study

The primary limitation of this study is the inability to generalize or rather transfer (Henwood & Pidgeon, 1993, p. 27) the results of this study to other groups of students or classrooms. The insights gained by examining a particular group of students in a particular class at a particular time cannot be generalized beyond that experience. However, they may produce some useful exemplars of the perceptions that some students may have about their problem solving as a result of such an experience. The contextual dimension this study provides may illuminate the perceptual impact on a group of high school students while participating in an environment where problem

solving and cooperative learning are encouraged. Hertz-Lazarowitz (1992) suggests, "It is impossible to understand, interpret, or generalize if the full story of what is going on in the classroom is lacking". It is hoped that studies such as this one, though limited on their own, may be able to contribute in uncovering the full story.

Chapter 2 Review of Selected Literature

Since cooperative learning and problem solving encompass such wide domains, the literature available on these subjects is abundant. Therefore, this review, which is divided into two parts, focuses only on aspects of cooperative learning and problem solving that are relevant to and are addressed in this study. The first part will deal with cooperative learning. Specifically, it looks at the interconnected roles of cooperative learning environments, affective factors (in particular perceptions and attitudes) and beliefs, as these relate to mathematical problem solving ability. Also included here is the relationship that may exist between cooperative learning and such noncognitive issues as attendance, discipline, culture and language.

The second part will describe two sets of heuristics for mathematical problem solving. These heuristics are included here because a discussion pertaining to the degree to which the students employed such strategies as they solved novel mathematical problems together is included in a subsequent chapter.

2.1 Cooperative Learning

Cooperative learning is not a new idea; it has been around for centuries (Johnson & Johnson, 1995; Sharan, 1990) and has provided many varied learning opportunities. A wide range of educational settings from primary to post secondary employ cooperative learning and it can be physically facilitated in diverse ways, including Jigsaw, Student Team Achievement Divisions (STAD) and Teams-Games-Tournaments (TGT) (Smith, Williams, & Wynn, 1995). The literature review will focus on the role of cooperative learning in the mathematics classroom to enhance the mathematical problem solving abilities of students.

In a recent study conducted by Lambdin that focused on group problem solving, it was found that "peers working together are able to solve problems that they are incapable of solving on their own" (Lambdin, 1993, p.50). Similar results echoed by Noddings (1985), Mwerinde & Ebert (1995), Hart (1993) and Johnson & Johnson (1990) suggest that problem solving ability is facilitated in cooperative learning groups. Findings of several other studies indicate that, "small group interactions can give rise to learning opportunities that do not typically arise in traditional classroom interactions" (Cobb & Whitenack, 1996, p.215). Dees (1990) reports that several studies appear to indicate that the "greatest advantage of the cooperative method may be seen when students

engage in complex tasks, such as concept learning and problem solving" (p.163). Johnson & Johnson (1990) see cooperative learning to be essential in *all* aspects of mathematics education and advocate that:

> "When done correctly, cooperative learning tends to promote higher achievement, greater motivation, more positive relationships among students, more positive attitudes towards the subject area and the teacher, greater self-esteem and psychological health, greater social skills, and many other important instructional outcomes." (pp. 122-123)

However, simply moving desks together does not necessarily give rise to cooperative learning as there are many ways in which unskilled groups can be nonproductive and hence unsuccessful (Johnson & Johnson, 1990; Johnson, Johnson, & Smith, 1995). Educators cannot assume that students know how to be cooperative; instruction must be provided, especially for older students, to ensure that students are not merely working individually within their groups but are truly working together (Dees, 1990). A group's success is contingent on the healthy interaction among its members. Members must possess certain social skills such as leadership, shared decision making, effective communication and conflict management (Ventimiglia, 1995, p.31). Teachers must "provide an environment in which students can practice and refine their growing ability to communicate mathematical thought processes and strategies"

(Davidson, 1990, p.5).

It appears that use of a cooperative learning environment may also positively influence a variety of nonacademic issues of education. These include attendance, discipline, language, and tolerance of different cultures. The long-term use of heterogenous groups, for example, tends to improve student attendance (Johnson, Johnson, & Smith, 1995; Slavin, 1983). Cooperative learning has also been credited with improving student behaviour in the classroom by increasing students' time on task, motivation, and attention. Students who may normally be considered disruptive may become more involved in what is going on in the class and feel the influence of their peers (Reid, Forrestal, & Cook, 1990, p. 72).

The use of cooperative learning in the classroom also provides a forum for dealing with diversity and acceptance both in culture and ability level and improves self-esteem, interpersonal relations, and race relations (Hertz-Lazarowitz, 1992; Ventimiglia, 1995). When students are placed in ethnically mixed cooperative learning groups, a new basis of perceived similarity is created, aiding in the development of cross-ethnic friendships (Slavin, 1983). Cooperative groups also facilitate language development in that classes are structured so that language is acquired in the process of acquiring the curriculum (Kagan, 1990, p. 203).

Factors such as students'self-perceptions and attitudes, issues in problem solving that have been somewhat neglected in past research attempts, have recently been the subjects of investigations (Lester, Garofalo, & Kroll, 1989; McLeod, 1989; Silver, 1987). In a study conducted by Lester, Garofalo, and Kroll (1989) it was suggested that the beliefs a person holds about his or her ability to do mathematics and to solve problems are dominant forces in shaping that person's behaviour when doing mathematical tasks. Taylor (1993), reporting on Vygotsky's influence on existing theories of attitude development in mathematics education, says that attitudes are immersed in culture and that "a person's attitude contributes to the image of her or himself as a learner in the classroom" (Taylor, 1993, p. 9). A recent study performed by Pajares and Kranzler (1995) concurred. They produced results that demonstrated that students' self-efficacy beliefs about their mathematical capability have strong direct effects on mathematical problem solving performance (Pajares & Kranzler, 1995).

A later study by Artzt (1996) focused on the interplay of cognitive processes demonstrated in a cooperative learning environment and on the perceptions held by the group members about their problem solving abilities. This study indicated that the perceptions of the group members about themselves and each other seemed to affect the level of success achieved by the group (Artzt, 1996).

Another issue that arises in problem solving research relates to the beliefs that students hold about problem solving and mathematics in general. The majority of students believe that mathematics is primarily a solitary activity that relies heavily on memorization; that there is only one right approach to solve every mathematical problem; and that problems should be solved, if at all, in a few minutes (Schoenfeld, 1987; Schoenfeld, 1992; Silver, 1987). By employing a cooperative learning environment in the mathematics classrooms, these beliefs may be challenged. When students work in groups, they may realize that it is often not only possible to have more than one answer, but also that there is more than one way to arrive at an answer (Fernandez, Hadaway, & Wilson, 1994; McLeod, 1993).

Through discussions of mathematical problems with their peers, students begin to realize that frustration is a normal part of solving a problem, as are joy and satisfaction, and that managing and monitoring are necessary skills for successful problem solving (Fernandez, Hadaway, & Wilson, 1994; Hart, 1993; McLeod, 1993; Schoenfeld, 1992). It seems that cooperative learning processes may be particularly helpful for those who have not had much success in mathematics (Dees, 1990). Such students often report that, "being able to talk to other students about their fears and difficulties makes mathematics easier" (Dees, 1990, p.186). Cooperative learning has

also been shown to reduce anxiety and stress and increase more effective coping strategies (Johnson, Johnson, & Smith, 1995).

Incorporating student writing in the mathematics classroom can also enhance problem solving. Journal writing, for example, can be an effective means to encourage students to reflect on the problem solving experience (Taplin, 1994). Although many students have difficulty writing about mathematics, it is believed that providing ample opportunity to do so helps to minimize these difficulties (Fernandez, Hadaway, & Wilson, 1994). Also, by combining writing activities with cooperative learning strategies, it is felt that the effectiveness of both is enhanced (Schoenfeld, 1987). The more students are encouraged to communicate with their teacher and each other through such diverse methods as writing and cooperative learning, the better the quality of instruction (Artzt, 1994).

In an overview of research in problem solving for the past twenty-five years, Lester (1994) identifies several areas he feels require more attention. One of these areas involves looking at what actually takes place in classrooms that are centered on problem solving. Another issue suggested for consideration is that research should focus on groups and whole classes rather than individuals (Lester, 1994). Davidson and Kroll (1991) echo this sentiment and state "to date, a relatively small percentage

of the studies have attempted to study the interactions that take place during cooperative work to determine how various academic, social, or psychological effects are produced" (p.363). It has also been suggested that research should look at instructional grouping from a variety of perspectives (Owens, 1995, p. 173).

2.2 Problem Solving Strategies

Since one of the themes of this study relates to how the group collectively solved or attempted to solve their assigned problems, attention must be given to problem solving strategies that are considered to be well accepted. This section briefly describes two strategies for problem solving that have been created and advocated by two different sources. The first set of heuristics was developed almost a half a century ago by a pioneer in mathematical problem solving, George Pólya (1957), in his work entitled *How to Solve It*. The second set of strategies developed more recently by John Mason, Leone Burton, and Kaye Stacey (1982) in their collaborative book entitled *Thinking Mathematically*.

Pólya proposed a four step plan for solving problems. These phases are entitled *Understanding the Problem*; *Devising a Plan*; *Carrying Out the Plan*; and *Looking*

Back. During the first phase, *Understanding the Problem*, the solver has to clearly understand what is expected and restate the problem in his or her own words. This getting acquainted phase can probably best be described by a series of questions such as the following: What is the unknown? What are the data? What are the conditions? In the *Devising a Plan* stage the aim is to plan, at least in outline, what is necessary to obtain the unknown. Looking for a related problem or one with the same unknown is offered as an appropriate starting point. This stage, according to Pólya, may be a long and tortuous one. The next phase, *Carrying Out the Plan*, deals with putting into action the strategies conceived in the previous stage. This is considered a less difficult task as long as the correctness of each step is ensured as a solution is being obtained.

Finally, the fourth phase, *Looking Back,* is described as an opportunity to reconsider and re-examine the results and the strategy. A key outcome of this phase is the consolidation of knowledge, thus making it reusable for future situations. Many people who consequently miss the opportunity to enhance their problem solving ability, unfortunately, often neglect this step.

Mason, Burton, and Stacey advocate a three-phase plan for problem solving - *Entry*, *Attack*, and *Review*. The *Entry* phase entails assessing both what is known and what is needed to know to solve the problem. This stage involves the reading and

understanding of the problem, identifying relevant skills and facts, employing analogous questions and sorting information. Skills also employed at the entry level may include diagrams, notation, images, or symbols.

Once the solver has a clear understanding of the problem, he or she moves on to the *Attack* stage, which involves any effort made to resolve the problem. This phase may also give rise to what Mason, Burton, and Stacey refer to as "Stuck" or "AHA!" situations. The authors refer to being "Stuck" as an "honourable and positive state" (Mason, Burton, and Stacey, 1982, p. 49) and offer possible responses. Actions such as staring at a blank page, growing tense, or getting frustrated are all common, yet negative, reactions to being stuck while attempting to solve a mathematical problem. Mason et al. offer proactive methods for overcoming this state including rereading the problem, summarizing what is known, or looking for alternate interpretations. This may lead to an "AHA!", which, as the name might suggest, is any form of insight into the problem and how it may be solved.

The final phase, the *Review* phase, is signalled by when a solution is procured or the solver is ready to concede. Here work is checked, reflection takes place, and the problem may be extended to a more general problem. Emphasis on the review phase is essential to develop thinking for future problem solving.

This brief overview of Pólya's four stage and Mason, Burton, and Stacey's three phase approaches to problem solving provides examples of well accepted heuristics to mathematical problem solving. Although these two approaches were developed independently, there are similarities between them. The first stages *Understanding the Problem* (Pólya) and *Entry* (Mason, Burton, and Stacey) and final stages *Looking Back* (Pólya) and *Review* (Mason, Burton, and Stacey) are similar in nature and design.

Chapter 3 Research Methods

3.1 Context of the Study

As with many other qualitative inquires, I began this investigation with a general interest in gaining some understanding (Stainback & Stainback, 1988) of what a new learning environment might mean for a group of students and their teacher. Of particular interest were the participants' perceptions of the experience. Together with Dr. David Reid of Memorial University of Newfoundland and Ms. Gwen Hewitt, the teacher involved, we mapped out a unit on Coordinate Geometry intending to incorporate problem solving tasks and cooperative learning methods as the primary learning environment. In my description of the context during which this study took place, I shall move from a general overview to a more specific depiction of the experience.

This study originated in a Level I (Grade 10) Academic Mathematics 1300 class, taught by Ms. Gwen Hewitt. The class was from a public high school in St. John's, Newfoundland and consisted of approximately twenty students. The word "approximately" is used here because; due to student absenteeism there did not appear

to be a definite number of students in attendance. It covered a span of thirteen mathematics periods. Appendix A provides an overview of each class. In addition, I conducted individual interview and problem solving sessions with each of the students in the focus group; problem solving sessions with various pairs of these students; one problem solving session with all three students; and an interview with Ms. Hewitt.

The first two classes involved having the students, in their assigned groups, work on novel mathematical problems and the remaining eleven classes focused on the unit of Coordinate Geometry. The topics covered were: coordinates, lines, slope, y-intercept, equations and parallel and intersecting lines. This unit was chosen primarily because it was a required unit in the curriculum and because it was felt that it would be conducive to students working together. Students were given a series of worksheets containing questions that were intended to assist them in making connections between graphs of linear functions and their equations. Appendix B contains copies of the worksheets.

The premise was that students working together ought to be able to make the necessary connections on their own without the traditional lecture style of teaching. Students were not given specific instruction about how to complete the worksheets, but were left to figure out how to answer the questions within their groups. In this way the

worksheets did present real problems for these students. In the last two classes students were given individual and group tests based on the material covered. Copies of these tests can also be found in Appendix B.

While Dr. Reid and Ms. Hewitt moved from group to group and sometimes addressed the entire class, I remained with one group, the focus group, for the duration of the thirteen classes. In this way, Reid[1], and to some degree Ms. Hewitt[2] observed and interacted with the members of the class (including the focus group) during the unit on Coordinate Geometry. During this time, I observed the students in the focus group closely and made extensive field notes. Although I tried to remain a nonparticipant observer, there were times when I interacted with the students, either to clarify the students' thinking, or to respond to a question posed by one of the students.

This study is centered on the focus group that was randomly formed by the mathematics teacher, Ms. Hewitt, and randomly selected by me, during the first class of the study. The students and group were selected randomly because I was interested

[1] Dr. Reid had his own research agenda with the class. See Blackmore, Cluett, & Reid, 1996 for a more detailed description of his (and Ms. Hewitt's) involvement.

[2] Ms. Hewitt was not considered a *researcher* in her own class, in the traditional sense of the term. Her primary role remained as teacher, although her interpretation of the experience was central to the study.

in investigating a "typical group" of academic high school students, and not a group that may have been formed because of special qualities possessed by any of the students. My thinking here was that this might very well be the approach taken by a classroom teacher. The five students who comprised this focus group shall be referred to as Valerie, Heather, Bob, Sean and Jeff. These students were typical of those found in a Level I academic mathematics class. They were all of average intelligence, though none were overly enthusiastic about math. Their rate of absenteeism may have been higher than normal. All but one student, Bob, was the average age for Grade 10. Bob, who was two years older than the others, had previously taken a Basic mathematics course and was now repeating Math 1300. Four of the five students were native Newfoundlanders while the fifth student, Heather, was from Taiwan and had been in the province for approximately six months at the time of the study.

After the classroom observations, Sean and Jeff declined any further involvement in the study. It is also noteworthy that because of student absenteeism, members of the focus group were placed with members of other groups at various times during the study. Therefore, other students from outside the focus group, namely Tom, Mike, Mark, and Craig will be included in discussions periodically.

This study employed a multiple of data sources: field notes made during classroom

observations of the focus group; individual interviews and problem solving sessions with Valerie, Heather, and Bob; problem solving sessions with various configurations of members of the focus group; and an interview with Ms. Gwen Hewitt. All interviews and problem solving sessions were audiotaped. Appendices C, D, and E contain the transcripts of interviews with students, the interview with the teacher, and problem solving sessions respectively. As well, documents such as student worksheets, individual and group tests, and Ms. Hewitt's journal entries were also employed in the study.

The problem solving sessions and interviews took place approximately six weeks after the classroom experience. The time delay was due to several factors. Permission from all parties involved took longer to obtain than expected. Also, there was a two-week school break during which no meetings could be arranged. As well, coordinating the student sessions proved more difficult than initially anticipated.

3.2 Methodology

As suggested by Croll (1986), the particular problem being studied should determine the research technique. This study represents a qualitative inquiry and is therefore

inductive in its approach (Maxwell, 1996). At the core of the inquiry is my attempt to understand (and describe) the experience for the participants in a contextual situation (Scott, 1996; Maxwell, 1996; Croll, 1986; Henwood & Pidgeon, 1993) and is open and emergent in nature (Glesne & Peshkin, 1992).

Qualitative inquiry has been described as a translation of culture that aspires to understand the participants' world and attempts to translate the text of lived actions into a meaningful account (Glesne & Peshkin, 1992). It is essentially "constructing data out of experience" (Wolcott, 1994, p. 13). However, as Wolcott (1994) states, the real mystique of qualitative inquiry lies not with the process of gathering data but in the process of interpreting the data.

Qualitative inquiry is the umbrella term for a variety of "philosophical orientations to interpretative research" (Glesne & Peshkin, 1992, p. 9). Such orientations include ethnography, grounded theory, case study, and critical research. This study employs case studies of the participants and has its roots in grounded theory (Glaser & Strauss, 1967) in that the descriptions relating to the students, teacher, and their problem solving sessions described here are discovered through the data. The research questions were not developed prior to the study but rather emerged and were refined as the data was collected. The intention was to develop descriptions rather than theories

of what the experience might have meant to the participants.

Through the grounded theory theoretical framework, I began my quest with a general notion of investigating (Stainback & Stainback, 1988) the perceptions of a group of students and their teacher regarding their new learning environment and its potential impact on the students' problem solving ability. A multitude of concepts, descriptions, and categories emerged from the data through systematic inspection (Glaser & Strauss, 1967; Henwood & Pidgeon, 1993; Glesne & Peshkin, 1992) during the course of my exploration. In this way theory was treated more as a process than an end result of the inquiry (Glaser & Strauss, 1967). At first, the flow of data was flexible and intense. As I attempted to make sense of the data through coding where the data is categorized for comparison, similarities began to emerge and patterns evolve. This aspect of grounded theory is critical in order for any emerging, evolving theory to, as Glaser & Strauss (1967) suggest, "fit" the data.

In order to establish or at the very least check for trustworthiness of my data, I utilized a multiple-data-collection method (Glesne & Peshkin, 1992). This study employed three primary sources of data collection. Participant and non-participant observation resulted in extensive field notes from both the classroom and subsequent problem solving sessions. Individual interviews with each of the three students and their

mathematics teacher were conducted. These interviews were audiotaped and field notes were recorded. As well, documents, including student worksheets, individual and group tests based on the classes and journal entries made by the mathematics teacher, Ms. Hewitt, which recorded her thoughts and emotions for the thirteen classes were used.

This method of collecting data from a variety of sources is referred to as triangulation (Croll, 1986; Glesne & Peshkin, 1992; Stainback & Stainback, 1988). In this way, I attempted to understand and to describe the experience from the perspective of the students, their teacher and myself. Looking at the same experience from a variety of perspectives adds to our understanding of the classroom processes (Croll, 1986).

This study is similar to other qualitative, interpretative works (some of which are described below) based on case study analysis of episodes, many of which occur in the context of a mathematics classroom, while investigating various aspects of cooperative learning. Allowing the study to take place in its natural setting enables the researcher to regard a classroom as a "setting in which opportunities to learn arise for researchers as well as for teachers and students" (Cobb, Yackel, & Wood, 1992, p.101).

Ongoing work by Paul Cobb and associates (Cobb, Yackel, & Wood, 1992; Cobb,

1995; Cobb & Whitenack, 1996) involves extensive observations of students from a second grade mathematics class engaged in problem solving using a cooperative learning mode. The classes were videotaped and subjects were interviewed individually.

A study conducted by Lambdin (1993) explored the issue of how working cooperatively might affect mathematical problem solving, in particular, what types of monitoring moves and roles might have occurred among students as they worked together. The data for this study was accrued through videotapes of problem solving sessions of three pairs of college age women over a nine-month period and individual interviews of the same subjects during which they solved mathematical problems.

An intensive case study by Goos and Galbraith (1996) examined the nature and quality of interactions between students working on application problems, focusing on metacognitive strategies. The subjects were two secondary school students who were selected because they demonstrated the ability to verbalize and reflect on their thinking. Data was collected via videotaped problem solving sessions, questionnaires and retrospective interviews of the pair.

A case study conducted by Dees (1990) investigated how and why the cooperative

method helps students in mathematics. In her work, college students working cooperatively to solve mathematical problems were observed and subsequently interviewed.

Aspects of my study have similarities with the works described above. However there are differences that might appear to be subtleties on the surface but may prove to be significant. The subjects here are high school students, not primary or college students. These students were heterogeneously and randomly placed in groups by their teacher to work together on material from their curriculum during regular mathematics class. The focus group was randomly selected by me during the introductory session and not based on any particular criteria or ability.

Although no formal intervention on the teacher's part was made to either prepare the students for their new learning environment or to assist them in how to work cooperatively, the first two classes were devoted to solving mathematical problems where the students were encouraged to but not specifically instructed to work cooperatively. The study lasted for thirteen classes, covering approximately one unit and thus was not longitudinal in nature. Students were asked to make journal entries three times during the Coordinate Geometry unit and were required to do group and individual tests at the completion of the unit. The individual interviews and problem

solving sessions that followed, approximately six weeks later, were conducted during the students' lunch break. The interview with the teacher took place after school approximately two months after the unit on Coordinate Geometry had concluded.

3.3 Post-Unit Problems

The problems were presented to the students in three formats - one problem that was to be solved individually during the individual interviews, two problems that were to be solved in pairs during the pairs' problem solving sessions, and one final problem that was to be solved as a group during the group problem solving session.

Individual Problem

As I interviewed Jeff, Heather, and Bob individually, I asked them each to solve the following problem which shall be referred to as the Chicken and Pig problem.

Chicken and Pig Problem

Tom and Sue saw some chickens and pigs in a barnyard. Tom said,

"There are 18 chickens and pigs." Sue said, "Yes, and altogether they have 52 legs." How many chickens and how many pigs did they see?

This problem was chosen for several reasons. Firstly, I felt that this problem was not overly challenging mathematically for the students as no complex algebraic computations were involved and no advanced previous mathematical knowledge was necessary. Secondly, the solution to this problem could be attained through a variety of approaches. Possible solutions include: using trial and error where the student might guess what number times two and another number times four sums to 52; using a system of linear equations; or drawing a chart or table to compare the number of legs with the number of animals. A third reason is a personal interest in process problems. I have spent some time delving into how problems such as the Chicken and Pig problem have been used in research on problem solving.

Pairs Problems

In subsequent problem solving sessions students worked in pairs. Heather and Valerie were the first pair followed by Heather and Bob. Scheduling problems prevented a session with Bob and Valerie. During both of these sessions the following problems

referred to as the Lizard and the Jug problems respectively, were presented to the students to solve cooperatively.

Lizard Problem

Aaron collects lizards, beetles, and worms. He has more worms than lizards and beetles together. Altogether in the collection there are twelve heads and twenty-six legs. How many lizards does Aaron have?

Jug Problem

How can you carry exactly four litres of water from a river if only a three litre jug and a five litre jug are available?

I chose the Lizard problem because it was similar to the Chicken and Pig problem from the previous session and hoped that students might recognize this and make connections to it. I discovered this problem in the book *Thinking Mathematically* by Mason, Burton and Stacey (1982), which concentrates on providing its readers with strategies for solving a variety of mathematical problems. Though more challenging than the Chicken and Pig problem in that it had three variables, I felt that the mathematics involved would not be too challenging for the pair. Like the Chicken and Pig problem, there were several possible solution paths that could be pursued. Perhaps the most significant reason for choosing this problem was that I felt it was conducive

for discussion and working cooperatively.

The Jug problem was given to the students to solve because I felt it was different enough from the first two problems to elicit more opportunities for discussion. The problem did not rely on numerical manipulations. Rather, it focused on logical thinking and its solution required an oral explanation. The first pair of students, Heather and Valerie, unfortunately did not have sufficient time to analyse the problem.

Group Problem

During the third and final session, attended by Heather, Valerie and Bob, the following problem was given. This problem shall be referred to as the Licence problem.

Licence Problem

The licence plate on my car contains five different digits. Although my brother installed it upside-down, it still shows a five-digit number. The only thing is, the new number is 63 783 more than the old number. What is the original licence plate number?

I discovered this problem in an article by Lambdin (1993) which described a study

about how people working together monitor themselves and what roles they take while solving a mathematical problem. I found the study and the problem to be relevant to my pursuits and felt that working in cooperation would enhance solving this problem with peers. Deciding which digits are possible in the licence plate, ones that are numbers upside-down, might be a good starting point for discussion among the group.

3.4 Participants

Since the students in the focus group, namely, Valerie, Heather and Bob, comprise the core of the study, a description of each is included to help the reader gain a better understanding of them. A description of the mathematics teacher, Ms. Gwen Hewitt, is also provided for the reader's benefit.

Heather

Heather, originally from Taiwan, had been living in Canada for approximately six months at the time of the study. The English language was a problem for her. She did not know, for example, many of the terms in the Lizard or Jug problems and needed

clarification about what the problems were actually asking. Heather was hesitant and far from fluent in English. She possessed an electronic translator but she used it more when dealing with the written material. I did not see her use it during the unit on Coordinate Geometry, the interview, or the problem solving sessions that followed.

Heather was a quiet, pleasant girl. She was fifteen years old at the time of the study, which is the average age for a Level I student. She was probably the most enthusiastic member of the focus group and perhaps the most capable. Heather appeared to be confident in her mathematical ability, although the apparent language and cultural barrier may have hampered her demonstration of this ability.

Bob

Bob is a native Newfoundlander and the oldest student in the focus group. Although he was enrolled in a Level I math class, he was a Level III student. Normally this would have been his senior year in high school, but because of his situation in mathematics he may require another year or possibly two in order to obtain enough

credits to graduate[3]. After completing Grade nine, Bob had taken and failed Math 1300 but then took a Basic math course and was now repeating Math 1300.

Bob was not overly assertive or confident, but he was not a passive person either. He appeared to be a friendly and easygoing person. Bob, perhaps because of his age, seemed more serious and perhaps more mature than many of the other students in the class.

Valerie

Valerie is a native Newfoundlander and was fifteen years old when the study was conducted. Therefore, she was an average age for a Level I student. Valerie could be described as a quiet, diligent girl. At first, it appeared Valerie might have had the least to offer to her classmates in terms of cooperating with them to solve mathematical problems. However, as time went on this impression proved to be false. Valerie possessed more ability than she had given herself credit for. She did not seem to be very confident in her problem solving ability but was very keen (in an

[3] In Newfoundland, students require four credits in mathematics to graduate. Math 1300 constitutes three credits, so one or more courses would be required. Acceptance to most postsecondary institutions would require a further additional course in mathematics (either Math 3200 or Math 3201).

unassuming way) on taking on whatever task was presented to her.

Ms. Gwen Hewitt

The mathematics teacher, Ms. Hewitt, although an experienced teacher, had not had much experience conducting her classes where her students were grouped together while solving problems. She had typically employed the traditional approach where she introduced a topic though lecture, presented notes and sample problems and then assigned similar tasks for her students to solve individually. Consequently this new learning environment was just as new for her as it was for her students.

Unfortunately Ms. Hewitt felt that the students in the class did not know what was expected of them, something she would try to rectify next time by focusing on group behaviours. During her interview, she made an interesting comment, putting it all into perspective:

> "We didn't just ask them to work in groups and do the same kind of work they've always done. We asked them to work in groups and do a totally different method of learning, studying, thrown at them all at the one time and I don't know if they had time to adjust to any of it before we plunged in. That's why I think things went so well."

Chapter 4 Field Notes

This section contains summaries of field notes from two different sources: field notes of the students in the focus group during the thirteen classes of Coordinate Geometry, and field notes made during the problem solving sessions and interviews that followed. The worksheets and tests referred to in this section can be found in Appendix B and transcripts of the problem solving sessions can be found in Appendix E.

4.1 Field Notes of Focus Group in Class

Class 1 Valerie, Heather, Bob, Sean, and Jeff present

During the first twenty minutes or so of the first class, students were given a quiz on material they had just covered. After the quiz was completed, students moved to their assigned groups. There was some shuffling of groups to account for absent students. Then problem number one was given.

Problem 1:

Given $69 \times 64 = 4416$

and $96 \times 46 = 4416$ Why?

The students in the focus group did not discuss the problem very much and seemed to work relatively independently. Heather soon found another pair of numbers that worked and continued to look for other pairs. Just before the class ended, Valerie recognized that the digits of the numbers were reversed.

Class 2 Heather, Valerie, Bob, and Jeff present

Students continued with the problem from last class. Bob suggested to the others that the numbers' digits were multiplied although he had difficulty getting his point across until he used an example. He wrote: 93×26

39×62

Bob: See, 9×2 is the same as 3×6, so when you switch the digits, you get the same.

The others readily agreed with Bob's explanation and after a few moments, during which it appeared that they tried it with another pair of numbers, they seemed satisfied to move onto the second problem.

Problem 2:

What is the value of x if $4^{20} + 4^{20} = 2^x$?

The students worked independently again. Valerie tried to use her calculator to work it out and soon realized it was futile. Heather worked on her own until Bob asked what she had written. She showed him:

$$4^{20} + 4^{20} = 4^{40}$$
$$2^x = 4^{40}$$
$$x = 80$$

They all appeared happy with this. When Jeff asked her how she got 80 in the last line, Heather could not explain herself very well, which led to a discussion about whether x could actually be a number. Valerie remarked that she could not remember the rules for exponents and turned to me for help. I asked them "What would happen if it had been $x^{20} + x^{20}$?". $2x^{20}$ was the consensus among the group. I then asked them "How can you use this information?" They continued for a while, basically on their own. The teacher then came over to the group and saw that Bob had written $4^2 + 4^2 = 2^x$.

Ms. Hewitt: What can you do with that?
Bob: Well $4^2 = 16$...
Ms. Hewitt: Bring this as far as you can.

Unfortunately he didn't get much of an opportunity to pursue this because within a couple of minutes the class was called together to discuss the solutions of the two

problems and the remainder of the class was spent summarizing solutions. The students in the focus group did not contribute to the class discussion.

Class 3 Heather, Valerie, Bob, and Jeff present

The third class marked the actual beginning of the unit on Coordinate Geometry. Ms. Hewitt, by means of introduction, related the story of Descartes and the fly on the ceiling. Then the first worksheet consisting of six problems was distributed to the students. The task involved matching tables of values and equations with their corresponding graphs. Jeff quickly found a solution to the first question and explained to Valerie and Heather that he just checked the x's and y's from the two endpoints and found the set that worked. The girls seemed satisfied with this explanation and Valerie turned and explained it to Bob. They then went on to answer the next question independently and compared answers.

When they started question number three, Bob was confused by the fact that the "answers are expressions", and not tables of values; and that these expressions contained an x but no y. "Where is the y?" he asked Valerie. She tried to explain it but he said that he found it was "a bit fuzzy". Then Heather attempted to explain it and Bob asked her to write it out fully.

She wrote:

$$-\tfrac{1}{2}(-2) - 1$$
$$1 - 1$$
$$0$$

Bob: Where did the -2 come from in the first line?
Heather: From the table.
Bob: Ok... but isn't 1 times -1 (in the second line) equal to -1?
Heather: No, that's subtract, not multiply.
Bob: Oh, ok.

Bob seemed content with this, although it didn't answer his question about the missing *y*. The group members then set out again to work independently on number four. Soon the boys gave up what they were working on and looked at what Valerie and Heather were doing. When Valerie got an answer, she explained to Jeff and Bob that she had taken an "ordered pair from the table of values (-2, -5)" and replaced the *x* in both equations with -2 and worked out her answer as follows:

$(-2)^2 + 2$	$-(-2)^2 + 2(-2) + 3$
$4 + 2$	$-4 + -4 + 3$
6	-5
incorrect *y* value	correct *y* value

Both Jeff and Bob recorded the answer on their own sheets and Jeff also suggested that they check an ordered pair from the graph to see if it fit the expression.

Since there were only about ten minutes remaining in class Ms. Hewitt asked the entire class to discuss how they were doing. She called on Valerie to explain how her group had matched a table of values with a graph to which Valerie simply replied, "We just tried the first and last coordinates". While the class discussion was taking place, Jeff and Bob kept working on question number five from the sheet.

Class 4 Bob, Heather, Valerie, and Jeff present

Class started with a brief review and Ms. Hewitt asked Heather to explain how they got their answer to question three. Heather's response was "You pick x values and put them into expression to get y ". Ms. Hewitt continued to explain how graphs of quadratics were different from linear graphs since parabolas were also found on the worksheet. Then the students were instructed to continue with question five. Once again, the students in the focus group started to work independently. Heather got an answer first and showed it to everyone. Bob remarked "Boy, I wouldn't be able to get that by myself". They all sat in silence for a few moments and then set off to work again on their own. Bob, looking at what Valerie had written asked, "How did you get these points?". When Valerie hesitated, Jeff interrupted and said, "Whenever you see an x, put the value in and work it out. Don't use x, just -5".

The group finished the worksheet and while waiting for other groups to finish, they were requested to make a journal entry: "Explain how a graph, table of values and equations are related". Students were given about ten minutes to respond. Bob, Heather, Valerie and Jeff all seemed to struggle with this and it took several minutes before they actually started to write anything. Valerie commented to me "this is the hardest part" to which I replied, "That's because you're not used to it".

Shortly thereafter, the second worksheet was distributed. This worksheet consisted of six graphs for which the students were required to find corresponding equations. Bob remarked, "Gee, this is a whole lot of work." He then started with ordered pairs but stopped to check something in his textbook. Jeff and Valerie both looked back to the first worksheet. Jeff wrote $y = x^2$ but couldn't get it to work. Bob looked on and said it should be double, not squared, but wrote out $y = \frac{1}{2} x$ instead. Just as the bell rang, Bob realized his mistake and changed it to $y = 2x$. Valerie, Heather, and Jeff were confused but said they'd look at it next class.

Class 5 Bob, Valerie, Heather, and Jeff present

Students picked up where they left off last class, although they appeared hesitant to start. Since only one of the group members could find the worksheet, they had to

share and I was curious whether this would encourage collaboration. Heather looked through her textbook. Valerie wrote out a table of values but then erased it. Bob thought that x was half of y and wrote $y = 2x$. The others looked on but didn't seem to know what to do to determine if Bob was right, so I suggested that they try ordered pairs from the line to see if they worked in the equation. After they each tried this, it was agreed that question one was correct.

After moving on to question two, Bob initiated discussion by offering his ordered pairs.

> Bob: y is always 3 more than x. Do you agree with this?
> Heather: I got $y = 3x$.
> Bob: Is that what you got?
> Heather: No, I don't think so.

A debate that might have taken place to determine which equation was correct did not occur, but rather discussion seemed to be curtailed by this exchange. There was an awkward silence that followed; the students appeared to just stop, uncertain about what to do. I intervened and suggested that they could perhaps verify points. After a couple of minutes of working independently, the following exchange took place.

> Heather: Is the answer $y = x + 3$?
> Jeff: No, it can't be because 2 gives you -1, not 5.
> Bob: Maybe it's minus 3 instead... yeah that's right.

No attempt was made to verify; nor did further discussion transpire. The students, nonetheless, seemed encouraged by the events and went on to question three.

> Jeff: I looked at rise over run. Question three and four pass through zero, so it's going to be the same as question one, a $2x$ or something, not an add.
> Heather: How about this one: $y = x - 1$?
> Bob/Jeff: Yeah, that's right.

In question four, Jeff suggested that, "y is half x". When Bob noticed that was negative, Jeff replied, "It doesn't matter, just change the sign". Heather cautiously proposed "$y =$ -½ x" and both Jeff and Bob agreed. Not bothering to check whether this was correct, they moved on to question five, where they appeared to encounter difficulty. Bob was soon confused about what to do with the point (-1, -1). Jeff responded, "don't bother about that one" and continued working on the problem. The other three students sat and watched for a few moments, at which time I interceded and suggested they try something. Bob, who was still stuck on what to do with (-1, -1), began to work with Jeff as Valerie looked on. Heather, working by herself, came up with $y = 3x + 2$ and showed the others. Valerie, Bob and Jeff accepted her answer and appeared ready to move on.

Just as they were about to start on question six, Ms. Hewitt called the entire class

together to discuss how the groups were doing. She randomly asked Jeff to go to the board and explain his group's strategy for question one. Jeff, who had been a driving force within his own group, experienced difficulty explaining and basically said that they had just picked $2x$ "out of the air". Bob came to his rescue and added that they found all the y's were double the x's.

Following this discussion students were asked to finish the worksheet for tomorrow and were given the last ten minutes or so to make a second journal entry: "How do you find the equation of a line given just the graph?".

Class 6 Heather present, along with three other members of the class - Tom, Mike, and Mark

Because of poor weather conditions, only four students were present in class. Ms. Hewitt grouped these four together and went over the last two questions from the worksheet, questions five and six. The teacher led the group discussion and offered suggestions for finding equations, even to the point of discussing slope intercept form and parameters m and b. Heather remained quiet during the entire class, although it did appear that she was following the discussion.

Class 7 Valerie and Heather present

Students started class by finishing questions five and six from the worksheet previously given out in class. Heather, in explaining to Valerie what was discussed yesterday in her absence, said "If $x = 0$ and $y =$ -3, then the equation is $y = x$ times something minus three". Things seemed to go smoothly and quietly for a short time, until Valerie encountered difficulty picking out the y-intercept from the table of values. She said that she understood it while Heather was explaining it but when she started on her own she soon got stuck and needed help. Heather had difficulty verbalizing what she had done.

A teacher led class discussion on writing equations ensued, resulting in summary points being written on the board. During most of the class the girls remained quiet and worked relatively independently.

Class 8 Valerie and Heather present, and worked with two other class members, Craig and Tom

Since the only group members present today were Valerie and Heather, they were placed with Craig and Tom, students from another group, who were in a similar

position in their group. Students were given the third worksheet to work on together. The objective of this worksheet, which contained nine problems, was to find equations for pairs of intersecting or parallel lines.

From the outset, there existed a different atmosphere within the group. Craig's enthusiastic initial response to the assigned work was "Ok, what have we here...we've got to find two equations, not just one". This seemed to "break the ice" and the students joined together, perhaps for the first time, to analyse what was expected of them. Tom soon realized that their task was not as difficult as they first thought because the lines were on the same slant. Valerie seemed more in synch with what was being discussed and even volunteered information from time to time. Her involvement in today's work had noticeably increased, while Heather continued to work on her own as she had done in previous classes.

After working on questions one through four, Valerie came to a realization and said to the others "The first part is always the same and the second part is always different". Tom responded with "Ok, then the slant is the same and the slope is the same". Today's class seemed to be one in that at least three students were working together and making progress.

Class 9 Heather, Bob, and Sean present

Today the groups reported to the class their strategies for writing equations. Questions on intersecting lines were then assigned to the groups. Sean, who had been absent for several days, was quite behind the others. Heather attempted to explain to Sean and Bob how to write equations but had difficulty, even with Ms. Hewitt's help. Soon they quietened down and worked on their own.

As time went on, it not only appeared that the students were working by themselves, I also had the impression that they were working at three different levels. Heather seemed to have a firm grasp of the concepts but had difficulty explaining them. Bob seemed to work at a more basic level, requiring a table of values to assist him. Sean was still at the introductory level and spent several minutes just finding ordered pairs. Not much interaction took place throughout the class, as it appeared that the students were unable to help each other and perhaps unable to ask each other for help.

Class 10 Heather, Valerie, Bob, and Sean present

At the beginning of class the groups reported on equations for intersecting lines. The students in the focus group, however, seemed disinterested in the discussion. Sean had

his head down on his desk, Bob was in a daze and Valerie doodled on her exercise book. The teacher then drew four examples on the board and asked various questions about them. Students were then asked to make a third journal entry: "Summarize what you know about slope". No conversation took place within the focus group while they completed this entry.

The final worksheet was then distributed. This fourth worksheet, containing seven questions, was a review of topics such as equations, slope and y-intercept. The students appeared weary and didn't start right away. It is possible that this lethargy may be attributed to the perceived heavy workload. After a few unproductive minutes, I felt obligated to intercede and help to get them started. However, not much enthusiasm was sparked and the remainder of the class was pretty quiet and uneventful.

Class 11 Heather, Bob, Valerie, and Sean present

Today's class was spent reviewing the material covered thus far in Coordinate Geometry as group and individual tests were to be given during the next two classes. The teacher led most of the discussion; consequently there was not much to report from the focus group.

Class 12 Bob, Valerie, Heather, and Sean present

A group test containing four questions was administered today. The teacher did not give time allotments for the questions on the test; students were expected to pace themselves while answering the questions together. The times indicated below reflect the amount of time the students in the focus group spent on each of the test questions. It was interesting to observe the students' attempts to answer these questions, not only because there seemed to be more pressure to produce today but because each of the questions required an explanation. This was something that the students seemed to avoid in the past.

Question #1 (students spent four minutes on this question)

Bob: How can you tell if lines are parallel?
Sean: I don't know. I wouldn't want to have to draw them out.
Heather drew both lines on scrap paper while the others watched.
Valerie: Parallel because the rise and run are both the same.
Bob: That's good enough.

Question #2 (students spent three minutes on this question)

Sean: I don't understand that. (reads the question again)
Heather: (starts to draw the line) No, it's not.
Bob: Why not?
Sean: Because it doesn't look like one (laughs).

Bob: Let's go on to the next one and come back after.

Question #3 (students spent six minutes on this question)

Nothing happened for a few minutes, until Heather and Valerie started to plot the points.

Heather: No, it's not a line.
Valerie: The coordinates don't go in a line, I think.
Students appeared stuck again.
Bob: Just write down I don't know.

Question #4 (students spent 21 minutes on this question)

Sean: You know the point on the y-axis is the same for A and B.
Bob: But it's not even a point.
Heather: It's 2.5.

Each student wrote something down. Heather got frustrated and scribbled out what she had written.

Sean: The points she (the teacher) gave us are not anywhere near the line.... Wouldn't you just have to find the slope of B? . . . That's not going to work.
Gwen: What are you looking for?
Bob: Equation of B.
Sean: Can you take it from here, rise 2 and run 4?
Bob: I got 1/2.
Sean: Ok. 2/4 breaks down to 1/2.
Gwen: Did you explain it?
Sean: We did - rise / run.

Gwen: Is rise/run anything legitimate?

Sean: Rise/run gives the slope in equation.

Gwen: Ok.

Valerie: Well we know the y-intercept is the same as line A and we got slope now. It can't be much more than that.

Bob: We know what we're doing; it's just hard to write it out.

They finally agreed to write $y = \frac{1}{2}x + -5/2$ stating their reasoning the rise/run is $\frac{1}{2}$ and the y-intercept is the same as line A.

Sean: Shouldn't we be given the equation for A?

Bob/Valerie/Heather: We are.

Bob: Parallel lines have same slope, right?

Sean: Then we just need to find the y-intercept.

Valerie: It's given to us, (0, 8).

Bob: Now to explain it...

Heather: Both lines have same rise/run because they're parallel.

Valerie: They're just at the y-axis at different places.

Bob: Alright, we're done.

Valerie: Wait, we got to go back to question number 2.

With only about three minutes left in class they looked back at question number two but they were not really focused on it. Sean put on his coat and put his pen in his jacket while Bob, Valerie and Heather discussed the problem briefly.

Bob: What can we say here?

Heather: It doesn't fit.

Bob: Just say points don't fit the line.

Valerie: That's good.

At this point all the students were finishing and the students in the focus group appeared satisfied with their effort.

Class 13 Heather, Valerie, Bob, and Sean present

Students were given individual tests to work on in this class; consequently there was not much opportunity to observe interactions among the members of the focus group.

4.2 Field Notes from Problem Solving Sessions

Chicken and Pig Problem

The Chicken and Pig problem was given to Heather, Bob, and Valerie as part of their individual interviews. These occurred during separate sessions. Hence no collaboration took place among them. The problem is restated here:

Chicken and Pig Problem

Tom and Sue saw some chickens and pigs in a barnyard. Tom said, "There are 18 chickens and pigs." Sue said, "Yes, and altogether they have 52 legs." How many chickens and how many pigs did they see?

Heather read the problem and asked for clarification on the number of legs that a chicken has. She was also confused over the wording of the problem as she thought that there were eighteen chickens rather than eighteen animals in total.

Once she understood the problem, Heather didn't hesitate in establishing a strategy. She immediately set up a system of equations and arrived at a correct answer within a few minutes, although she didn't check her answer. There was no attempt to Review or to Look Back until I inquired if she was sure of her answer.

When Bob was given the same problem on another day, his initial response was "I don't know how to go about it". He was confused by the wording of the problem and like Heather, thought there was eighteen chickens. After a brief discussion with me about how he might start, Bob figured that "You can get a bunch of answers for that, you can get different ones". He then set out to find one solution by trial and error and solved the problem after a few minutes. Bob then appeared to check the accuracy of his answer by substituting the values he got back into the problem and seemed satisfied that he was correct. At this point he still believed that other solutions were possible but could not find any that worked.

Valerie read the problem and worked quietly for several minutes. She, like the others, confused the number of chickens with the number of animals in total. Once given proper directions Valerie offered a guess. Her response of nine pigs and nine chickens was incorrect. After realizing the mistake, she tried several combinations until finally trying eight pigs and ten chickens. However, when she told me her answer, she said, "There are thirty-two pigs and twenty chickens", not realizing that she gave the number of legs instead. She apparently had not checked her answer, or if she had, had not done so successfully.

Lizard Problem

The lizard problem was given to Heather and Valerie in the first problem solving session, and then subsequently to Heather and Bob in the second problem solving session. The problem is restated here:

Lizard Problem

Aaron collects lizards, beetles, and worms. He has more worms than lizards and beetles together. Altogether in the collection there are twelve heads and twenty-six legs. How many lizards does Aaron have?

When Heather and Valerie began the Lizard problem, Heather asked numerous

questions, mostly concerning vocabulary used in the problem. It took some time to clarify the problem because both students seemed confused by the wording of the problem. They then worked on their own and not much interaction took place for several minutes. Heather set up a system of equations using three variables with two equations, while Valerie wrote something and then quickly erased it. There didn't seem to be any evidence of progress so I suggested that we move on to the next problem, a problem that shall be referred to as the Jug problem.

The second session, in which Heather and Bob tried the Lizard problem, was an interesting contrast to the first one. The only question asked was to clarify the number of legs beetles have. After a long silence, Bob announced that he "got it" and proceeded to explain his solution of trial and error to Heather who said that she understood the explanation.

Jug Problem

The Jug problem was administered in the same manner as the Lizard problem. The problem is restated here:

Jug Problem

How can you carry exactly four litres of water from a river if only a three litre jug and a five litre jug are available?

Heather and Valerie were the first pair to try this problem. As with the Lizard problem, confusion over vocabulary ensued which had to be addressed before the problem could be analysed. Unfortunately, before the students really had a chance to look at the problem, the bell rang and the students' pursuits were interrupted.

When Heather and Bob were presented with the Jug problem, Bob's immediate response was "This doesn't even make sense". After some prompting on my part, they appeared to make an effort. This problem seemed to invoke discussion, however Heather did not appear to be open or able to do so. Most of the conversation that took place during this session was between Bob and myself. They did manage to solve it though, and were even able to explain their thinking to Valerie during the final problem solving session.

Licence Problem

The licence problem was given to all three students in the third and final problem solving session. The problem is restated here:

Licence Problem

The licence plate on my car contains five different digits. Although my brother installed it upside-down, it still shows a five-digit number. The only thing is, the new number is 63 783 more than the old number. What is the original licence plate number?

Shortly after reading the problem, Bob repeated the same sentiment as earlier and said "I don't have a clue". Then, after a long period of silence, Bob initiated discussion by asking his partners "Do either of you know anything?". Heather recognized that only certain digits would work, ones that look like numbers upside down, and the three students spent some time getting all possible digits. They soon settled in and tried the different digits on their own. Shortly thereafter, Bob started to lose interest and began tapping on the desk while Valerie and Heather appeared to be writing and erasing numbers.

After a period of what I considered unproductive time, (I considered it unproductive time because the students were just sitting there and were not talking or writing. It appeared to me that they were each stuck), I felt that they might require some assistance, so I intervened and suggested that they systematically substitute the possible digits into the licence plate number and add it to the digits of 63783. Once

they had a strategy, albeit not of their own doing, they were off. The students seemed to become, perhaps for the first time, interested in solving the problem and actually worked together in making suggestions. At one point they even overcame a dilemma they encountered when Valerie asked, "What happens when you carry?" Eventually the problem was solved and a brief discussion ensued.

Chapter 5 Analysis

Prior to this experience, the students in this class, like many other mathematics students, were accustomed to the *traditional style* of mathematics classes. The teacher taught a topic at the board while the students sat in rows and observed the lesson. After the lecture the students were then expected to practice the skill previously demonstrated. Little dialogue or discussion transpired among the students and the teacher's role then switched to monitoring the students' progress and keeping them on track and on task. Homework was then assigned and the class typically ended only to resume the same way the next day.

Consequently these students had little exposure to working with one another within the classroom setting especially in a setting where they are expected to find things out for themselves. As a means to provide students with a period of adjustment to the new approach, students were given two problems to work on with their group. After two classes, the unit on Coordinate Geometry commenced. Students within their group were given worksheets containing graphs of lines and were asked to make connections between the graphs and various components, namely, tables of values, slopes, and equations.

From a cooperative learning point of view, this approach is far from ideal, especially when we consider the type of students in the class. Most of these students would require much time and support before they were truly able to work with one another in a productive manner (Johnson & Johnson, 1990). However, as all high school math teachers are aware, time is a critical variable. Most high school math teachers, including Ms. Hewitt, are ever conscious of the time necessary to cover the required material. During her interview Ms. Hewitt said, "time is one of your biggest problems and it's really played by ear in a unit like this". It is therefore understandable why only two classes were devoted to orienting the students to their new environment.

It is unrealistic, however, for teachers to place students into groups and expect them to automatically cooperate. The ability to consult, discuss, question, and plan does not occur naturally by simply rearranging the layout of the classroom. Dees (1990) reported that students, especially weaker and less confident ones, would require much help and exposure before they truly are able to learn cooperatively. As I observed the focus group over a period of thirteen classes, I felt they were still working independently and merely verifying their answers with each other. Little discussion took place during the "problem solving" itself. Perhaps, as was suggested by Artzt (1996), their lack of self confidence undermined their ability to work cohesively as a group. In fact they often consulted with me to verify their theory, to get them started

or to bail them out when they got stuck. When one student arrived at an answer, the others accepted it readily. It seemed that getting an answer, any answer was the goal. Often the answer finally given was not the correct answer.

During the group test, this attitude was also evident. It seemed that as long as someone could offer an answer, the rest were satisfied. Test taking skills appeared to be weak among the focus group. The students did not pace themselves and in fact for the most part, they seemed oblivious of time. They did not appear to employ any apparent strategy. Of course it is noteworthy that these students have probably never collaborated on a test before; so coordinating four minds on such a task is not an easy one to accomplish. Even Ms. Hewitt admitted that some students in the class were "thrown for a loop" by the group test.

While the focus in the classroom was on Coordinate Geometry, my interest in the experience went beyond whether the group could find the slope of a given line. My interest lay in how the students worked together to solve problems presented to them in class, since the abilities to cooperate and solve problems are important life skills transcending mathematics class. In particular, I wondered what perceived effect the exposure to this environment might have on the students' abilities to solve problems while working together. The analysis and discussion presented here will be related to

the research questions posed for this study. These questions are:

1. How do these students feel about their new learning environment, especially with respect to how they solved mathematical problems while working together?

2. What are their teacher's perceptions with respect to her students as a result of such an experience?

3. What are my own perceptions of the experience, especially with respect to how the students solved problems and worked together?

4. What were some factors that appeared significant for the students in this mathematics class?

5.1 Research Questions

1. How do these students feel about their new learning environment, especially with respect to how they solved mathematical problems while working together?

Heather, Valerie, and Bob all agreed that the experience of working with each other on problems during the unit on Coordinate Geometry was a worthwhile one and each offered interesting perspectives as to why they felt this way. Valerie told me during her interview that working this way made her "look at it better because we had more time and we had to figure it out without Miss telling us exactly what to do". Heather said that, "If we work in a group, students can think more, I think". Bob echoed this sentiment when he said "the other way she's (the teacher) teaching and she's showing you, this is *how* you solve it, but like that way you had to figure out, ok, *how do I* solve it? You had to figure out how to solve it and then solve it. It made me think more."

Another point that all three students were in agreement with was the fact that group members took some time to adjust to each other. Bob felt that working with the same people for an extended period of time was beneficial because, as he said, "you get to know the students" and thus are more willing and able to ask questions. Valerie

shared this viewpoint and felt that after getting used to the other group members, she would get confused if she went to other class members for help. Heather, however, believed that working with the group for an extended period of time might have slowed the pace in the course.

Neither Valerie nor Heather felt that the experience altered their opinion about coming to math class. Bob, on the other hand, found that it changed how he felt about coming to math class and said that working with a group was not as boring and "definitely less stressful". They all agreed that they preferred working with their classmates but for different reasons. Valerie, for example, explained that "you get more ideas from people and if you don't understand, they'll explain it to you and how to do it". Bob found that it was "easier to understand and you don't have to ask the teacher as much". Heather, early in her interview, reported that the traditional method was her preferred learning environment because in her country, students were not usually placed in groups and that the traditional way was "a better way to learn". Later, however, she admitted that working in groups held some merit in that "you can talk and for me I can learn English... and everyone has their own opinion".

It is interesting to note that following the conclusion of the Coordinate Geometry unit, students in this class continued to work with their classmates on mathematical

problems during class. This time, however, the students chose their own groups. Bob chose to work with another boy named Stephen, although Sean from the focus group continued to sit with Bob and Stephen whenever he was present. Bob, who stated during his interview that he felt that groups which are comprised of friends would not be productive because too much time would be spent talking and wasting time, said that he considered Stephen to be "a friend here but not outside of class" and regarded it to be a beneficial arrangement. Heather chose to work with Lucy, another Taiwanese student and said that she found this arrangement more comfortable because, as she said, "we are the same kind". She also said that if the teacher asked her to work with other students in the class, she would not find it as difficult because she now has experience working with others, although she admits that she would not voluntarily move to another group. Valerie chose to work with two classmates whom she considered friends but said that she did not find any significant differences between working with this group and the focus group.

2. What are their teacher's perceptions with respect to her students as a result of such an experience?

Although Ms. Hewitt was an experienced teacher, she said that she found herself

occupying a very different position during the study than she would normally occupy. She felt very much like a novice teacher, unsure of her next step. She found herself transformed from a purveyor of knowledge to a facilitator even though she admitted finding herself occasionally reverting back to her old ways. In an early journal entry, Ms. Hewitt wrote, "true investigative teaching requires a constant awareness of one's every action, at least in the early stages of adopting this method".

For Ms. Hewitt the experience was both exhausting and stressful, yet during her interview she repeated her enthusiasm for trying another unit based on the same methodology. She felt that the shift in focus from her as teacher to the students was important. All too often teachers have the underlying philosophy that they are supposed to fix the problems or errors encountered by students rather than allowing them to find things out for themselves. She also felt that through this experience she realized that her students were able to think much better than she thought they could.

Ms. Hewitt found the entire project to be a learning experience for her. Not only did she learn more about her students and how they thought; she also learned about herself and her teaching style. She wrote in her journal "I'm so used to hearing the answers a certain way that I'm not always listening to and accepting the responses when presented in somewhat convoluted reasoning or from a different perspective" and later

"I realize I've gotten into a rut of seeing things in a narrow manner and the students' richness of reasoning are exhilarating for me".

She admitted that she was surprised to discover the amount of knowledge that her students in this class possessed - more than she had given them credit for. She found this very encouraging. She also found that, through this experience, she gained some respect for the fact that they do think much better than she had thought.

Ms. Hewitt did not feel that any long-term effect on problem solving abilities per se had occurred as a result of placing the students in their new learning environment, primarily because the unit had been a relatively short one. Since the conclusion of the cooperative learning session, many things in the class had reverted back to the norm, despite the fact that the students were still being encouraged to work together. She believed that a more intensive, long-term experience might have had a more permanent impact. It did, however, as she put it "break the ice" allowing students in the class to become more comfortable with one another. She also observed effects on related issues in her students such as cooperation, confidence, and group dynamics.

With respect to the students in the focus group, Ms. Hewitt had some interesting descriptions and impressions of what effect working together on a problem solving

unit might have had on them. She felt that Heather was a good worker but kept herself very distant and didn't have much of an understanding about cooperative work. In fact she viewed Heather as "a bit more rigid than the rest of the group". Bob, she felt was very unsure of himself during the unit on Coordinate Geometry and would not be considered to be a great student. Valerie was described by Ms. Hewitt as a quiet girl who "never opened her mouth from September to December". Their teacher also described the remaining two group members, Jeff and Sean. Jeff, she believed, made a positive contribution when he was present, whereas Sean was described as being antagonistic. Both of these students also had a high absenteeism rate during the entire school year including the unit on Coordinate Geometry. Consequently, Ms. Hewitt did not consider that either of these students had benefited much from working cooperatively in class. In fact their frequent absences made the group shaky according to her and never gave the group a comfortable feeling.

Ms. Hewitt did feel that for at least two members of the focus group, Valerie and Bob, there appeared to be some positive effect. Valerie, for example, "has really come a long way . . . she has got a new confidence in herself". In fact, Ms. Hewitt described Valerie as more of a leader in her current group and she mentioned that other students often seek Valerie's assistance to solve problems. According to the teacher, the exposure to working with her peers has done something for Valerie and it seemed as

though she had "blossomed" with regard to her ability to explain and question. Bob also appeared to have demonstrated some improvement since the Coordinate Geometry unit and is now described as a much better worker and contributor. In fact, Ms. Hewitt asserted that Bob had "come right out of his shell". Bob, now working with another student in the class, had really come a long way.

Ms. Hewitt felt that the experience may have caused a dramatic improvement for one particular student named Craig, who was not in the focus group but had been placed with Valerie and Heather in class eight. Craig, once considered a weak student, was now described by his teacher as analytical; "looking for the problem in the problem" and now could not seem to get enough assignments or homework. The change in his interest level and involvement seemed to be sparked during the time of the study and had continued since then. It is unclear if this change is a result of Craig actually improving or if the change in learning environment simply better matched his learning style.

When asked about any perceived roles that may have evolved within the focus group, she remarked that she noticed that "the one who knew how to do it became the all" and quite often they left it up to Heather to work it out for the group. Ms. Hewitt felt that the other students kind of knew that she was good, so they let her go and did not trust

themselves.

3. What are my own perceptions of the experience, especially with respect to how the students solved problems and worked together?

The thirteen classes during which the students solved problems (related to Coordinate Geometry) demonstrated their lack of experience with both problem solving and working cooperatively. The first two classes, intended to provide the students with some sort of orientation to both of these areas, did not offer much assistance to the students as to how they might solve problems together. During these classes, two problems were presented to the students. In the subsequent eleven classes the focus became the completion of a series of worksheets designed to have the students explore topics such as: equations of lines, slope, y-intercept, and parallel and intersecting lines. I shall describe my perceptions of the students by comparing their strategies for solving problems with those offered by Mason, Burton, and Stacey (1982).

Typically, the students read the problem silently and then started to work independently to produce a strategy and ultimately a solution. It is difficult to ascertain what was done at the *Entry* phase, other than reading the problem, since little

or no discussion took place among the students. The *Attack* phase varied. The most common methods for solving the problem appeared to be trial and error, sometimes combined with strategies employed in previous questions (i.e. producing a table of values or ordered pairs). Occasionally, students referred to other sources as in the fourth class when Bob checked his textbook for assistance. Usually no obvious attempt was made to *review* a solution once one had been obtained. The students did not consider extensions or generalizations of the problems during any class.

It was interesting to watch how the students responded to being "Stuck". Typically, when a student did not know what to do, he or she either sat and waited for someone else to get an answer or looked on as another group member continued to work. Sometimes they inquired about what that person was doing. Often they sought my assistance for explanations of the problem or to get them started. They did not appear to look for an alternate approach or reread the problem as suggested by Mason, Burton, and Stacey (1982). This perhaps demonstrates the belief that many traditionally taught mathematics students have that the teacher or other expert can (and should) alleviate the problematic situation for them.

During the problem solving sessions that followed the unit on Coordinate Geometry, the students in the focus group were asked to solve four problems that were unrelated

to the topic in class. They were each given the Chicken and Pig problem to solve individually. Then in pairs, Valerie and Heather followed by Bob and Heather, were presented with two problems, the Lizard and the Jug problems. Finally, as a group, the three students were asked to solve the Licence problem. My comments will again be presented in a way to compare the problem solving strategies of these students with those advocated by Mason, Burton, and Stacey (1982).

The *Entry* phase of the Chicken and Pig problem appeared to cause the students some difficulty. They all, for example, misunderstood the problem and thought that there were eighteen chickens instead of eighteen animals in total. Heather needed additional clarification of the vocabulary. Once she understood the problem, she immediately set up a system of equations and quickly solved the problem. Bob, when presented with the same problem, responded that he didn't know how to do it. Once he seemed to understand what was involved, his *attack* was one of trial and error. Valerie encountered similar difficulty as the others. Her strategy was trying one combination, a combination, which did not work. She then appeared to be stuck and did not try anything. I prompted her by asking if she had tried other possibilities, wherein she then tried several before hitting on one that worked.

None of the students appeared to *review* their work or check their solution, all passing

over their solutions immediately. Heather and Bob had both successfully solved the problem but Valerie's solution of 32 pigs and 20 chickens was flawed. She didn't realize until pointed out by me that she had found the number of legs, not the number of animals. Figure 1 summarizes the strategies employed by the students.

Figure 1 Chicken and Pig Problem

	Entry	Attack	Review
Heather	- read problem - asked for clarification	- set up a system of equations	- none
Bob	- read problem - said "I don't know how to do it." - asked for clarification	- trial and error	- none
Valerie	- read problem - asked for clarification	- tried one combination - appeared to be stuck and required assistance	- none

In the second session the Lizard and Jug problems were given to students in pairs to solve.

After Valerie and Heather read the Lizard problem, much clarification was required for Heather to understand what the problem entailed. She clearly had difficulty with the English language. Once they seemed content with understanding the problem, they set

off independently. Heather tried employing a system of equations again and Valerie tried trial and error. Neither student was successful in solving the problem, yet they each continued with their initial strategy for quite some time. They did not consult with or assist each other in attempting to solve the problem.

When Heather[4] and Bob attempted to solve the same problem in the next session, there was only one question asked. "How many legs does a beetle have?". Heather had spent some time with this problem in an earlier session and had asked numerous questions concerning the problem at that time. In fact no conversation took place between the two students at all. Bob and Heather set off on their own for a while until it appeared that Bob had experienced an AHA! Thus Bob was able to successfully solve the problem. He explained his *attack* as trial and error, although he didn't *review* his solution. Figure 2 summarizes the students' approaches to the Lizard problem.

[4] Heather was asked if she would like to attempt the same problem from her earlier session with Valerie. Since she expressed an interest in retrying it, I let her and Bob look at it together during their session.

Figure 2 Lizard Problem

	Entry	Attack	Review
Heather and Valerie	- read problem - asked for clarification	- worked independently - system of equations / trial and error	- none
Bob and Heather	- read problem - only one question asked "How many legs does a beetle have?"	- Heather appeared inactive (perhaps stuck) - Bob used trial and error	- none

When Valerie and Heather were given the Jug problem, a similar discussion took place over the vocabulary and the problem itself. Unfortunately, time ran out before the two had much of a chance to tackle the problem. Bob and Heather were also presented with this problem during their session. Bob's initial response was "This doesn't even make sense." His initial confusion was over the nature of the problem; thus once he understood that the jugs had no markings and the objective was to produce exactly four litres, he was more optimistic. Again the pair set off working independently, during which time Bob checked with me on numerous occasions to verify his thinking. I had not realized until later when I listened to the audiotape that I had been so involved with the solution. Figure 3 summarizes the approaches employed by the students in solving the Jug problem.

Figure 3 Jug Problem

	Entry	Attack	Review
Heather and Valerie	- read problem - difficulty understanding the problem	- worked independently - time ran out before a solution was obtained	- did not finish problem
Bob and Heather	- read problem - Bob: "This doesn't even make sense."	- worked independently - conversation which took place was between Bob and me - Bob experienced an AHA! which appeared to help him to solve the problem	- none - both students were able to explain it to Valerie in the next session

The Licence problem proved to be an interesting one, from a problem solving perspective. Bob expressed his usual comment about not knowing what to do. After reading the problem, and seemingly understanding what was involved, it appeared that none of the three students could offer an attack to solve this problem. Heather made a comment about the numbers looking like a number upside-down but no discussion ensued. In fact they were silent for quite a while. I then decided to assist them by building on Heather's point. Once they had a plan, they worked together by trying the possible digits in an attempt to piece the number together. They cooperated with each other and offered a solution of 26783, which was incorrect. They had obviously not checked their solution because neither 2 nor 7 were possible digits. After rethinking where they had gone wrong they did manage to arrive at a correct number. This is

summarized in Figure 4.

Figure 4 Licence Problem

	Entry	Attack	Review
Heather, Bob and Valerie	- read problem - Bob: "I don't have a clue."	- needed help developing strategy - worked together once strategy was clear	- none

During the problem solving sessions, Bob was the only student able to solve three of the problems; the Chicken and Pig, Lizard, and Jug problems. As a result Bob was quite pleased with himself for his successes. Yet, he was the first student to admit defeat when the problems were presented to him. His initial reactions to the problems include: "I don't know how to go about it" in reference to the Chicken and Pig problem and "This doesn't even make sense" in reference to the Jug problem. He had a similar response to the Licence problem and said, "I don't have a clue". Bob seemed to need help getting started and typically admitted to being stuck. It is unclear whether this because he did not truly understand the problem or because he just didn't know where to begin and perhaps figured I was the best person to get him started. He did not appear to possess any skills for initiating problem solving, except to ask for help. Bob was not alone, of course, but was most vocal about enlisting help.

My perceptions of the students' problem solving abilities differs from those of the students and their teacher in that I observed the students' actions for the entire time, thus allowing me to offer a more continuous interpretation of what transpired. I shall offer descriptions of Heather, Bob, and, Valerie as I perceived them in their mathematics class and in the sessions that followed.

Heather was not the most assertive person and it is not clear whether this trait is culture or personality related. Throughout the Coordinate Geometry unit and subsequent problem solving sessions, she willingly shared her answers but not her explanations. More accurately, she probably felt that she was unable to share her ideas. In class five, for example, Bob and Heather briefly discussed their ideas on question two. Bob suggested that "y is always three more than x" and asked Heather if she agreed to which she replied "I got $y = 3x$". When questioned on this, Heather backed down and changed her answer rather than have a discussion about which answer might be right. I remembered at the time that awkwardness followed this exchange. It seemed to me that the students did not know how to respond to each other, either because they were unsure of themselves and their own thinking, or were unable to relate to each other on that level. When they later arrived at the correct answer, their rapport seemed to be restored and they appeared relieved and happy to be making progress.

During her interview, Heather admitted that she liked math and felt that she was good at it but was less confident in her problem solving abilities. She did, however, have obstacles such as language and culture. Judging from her comments, Heather did not have too much exposure to challenging a teacher in class and had little or no experience with collaborating with classmates about mathematics.

I would not describe Bob as assertive or confident but he was not complacent either. Perhaps being older than the other members of the group and not wanting to fail again caused him to feel less intimidated than Helen or Valerie when it came to asking questions, although the gender issue has not been ruled out. Bob told me that he did not mind math and in fact he admitted that he liked it when he understood it and described himself as an average problem solver, but math is definitely not in Bob's plan for the future. His interests lay in a musical band that he and some of his friends had started, and he planned to pursue a musical career. Math and perhaps school were somewhat of necessary evils for him.

As I observed the students working in class my initial impression of Valerie was that she was the student least able to offer much to her peers. However, as time went on, my opinion changed. During the eighth class, for example, when Valerie and Heather

were the only members from the focus group present, Ms. Hewitt had placed them with two other students, Craig and Tom, from another group. I observed a different Valerie that day. She was much more part of the group, more willing to share her thoughts while Heather remained withdrawn. Perhaps Valerie felt more comfortable with Craig and Tom and therefore more willing to contribute. Again when solving a problem with Heather afterwards, she seemed more focused. As I reread my notes, I realized that her contributions were more significant than they first appeared. It may have been her quiet nature and unassuming manner that influenced my initial impressions of Valerie.

4. What were some factors that appeared significant for the students in this mathematics class?

During the course of the study, several factors appeared significant for the students in this mathematics class and in particular for the students in the focus group. These factors include as attendance, language and culture, discipline, and the role of homework. These themes will be discussed here in more detail.

Attendance

Although there were to be five students in the group, there was only one class in which all five were in attendance. In fact, only Heather attended all thirteen classes while Jeff and Sean were present for only five classes. The other two students, Bob and Valerie, were each present for a total of nine classes. Ms. Hewitt admitted that "there was such high absenteeism in that class" which caused disruptions in the flow of the group. For example, Sean was absent for eight consecutive classes and his return caused the group to all but fall apart. He was several steps behind the other students, and confusion resulted as the others attempted to help him catch up. Progress was hampered, to say the least. There were also three classes in which only Heather or Heather and Valerie from the focus group were present and were consequently placed with other groups.

Any group of students who are being exposed to a new learning environment needs much support. A group of students, especially a weaker and less confident group, also needs to have consistency in order to allow the process to flow and a cooperative atmosphere to flourish. The chronic absenteeism with this group is exemplary of a typical class, although it may be on the high side of normal. It is, unfortunately, one area in which teachers and other group members have little or no control. As Gwen

said "You can only fight that battle so much". It is very difficult for any group of people to maintain its productivity if any of its members are absent for extended periods of the time. Poor attendance is a barrier that stops, or at least slows down, a group's progress. During her interview Ms. Hewitt felt it made the group shaky and said:

> "When somebody's coming in and out all the time, it doesn't give the group a comfortable feeling. They don't build up habits of you do this and I'll do that, you know the discussion isn't there. As soon as a new party is in, the discussion is curtailed just a little bit. You always got that awkwardness."

Even with the problem solving sessions and interviews which took place later, only Valerie, Heather, and Bob were available to attend these sessions.

The long-term use of heterogeneous groups has been reported to improve attendance according to Johnson, Johnson, & Smith (1995) and Slavin (1983). This study, perhaps because of its short duration did not show any positive effect on the attendance of any member of the focus group.

Language & Culture

Four of the five members of the focus group were native Newfoundlanders. The fifth student, Heather was originally from Taiwan and had lived in Canada for approximately six months at the time the study was conducted. Consequently, communicating and understanding instructions in English presented some difficulty for her. Within the group she tended to be more withdrawn and unwilling to share ideas. I do not believe it was because she did not wish to share as much as a feeling of being unable to share. She was very self-conscious of her English and usually did not volunteer ideas or other input. However she willingly shared her answers with other group members who accepted them without hesitation or challenge.

The issue of language also arose during subsequent interviews and problem solving sessions. When Heather was presented with the Chicken and Pig problem she had to verify with me that chickens have two feet but then mistakenly thought that pigs also have two feet. I assumed when I decided to use these problems that terms such as "pig", "chicken", "jug", and "collect" would require no explanation and would be readily understood by all students. This assumption proved to be premature. How many other things are assumed in a math class to be understood by all our students and in the same way? For example we ask students about parallel lines and assume that all

our students not only have an understanding of the term "parallel" but also have roughly the same understanding of the concept. Thinking about Heather made me wonder how many students sit in mathematics classes every day not knowing what we are talking about and not willing, or able, to admit it or perhaps not even interested enough to want to know.

Within the group setting, Heather did not ask her peers to explain terms or to verify meanings for her. It is not clear whether this was because she understood all that was being said or because she did not feel comfortable asking the others to explain. Heather admitted to me during her interview that she asked fewer questions because, as she said, "I think my English is not so well". It seems almost ironic to me that a student like Heather, who is probably in most need of asking questions of her peers, is also the student who probably feels most unable to ask these questions.

It was not only Heather who had difficulty verbalizing and this was especially true when whole class discussions were conducted to summarize points. Heather, Valerie, and, Jeff each had been asked by the teacher in various class discussions to report on their group's thinking and explain their answers to a particular question. This appeared to be difficult for them even though each of the students had seemingly followed the group discussion as the particular questions were being addressed. It

seemed as if mathematics was a foreign language to some of the students in the class who had great difficulty expressing what they did and why they did it. For example, when Jeff was asked during class five to explain how his group had worked out one of the problems, his response was that they had basically picked the answer "out of the air". He experienced great difficulty in explaining his reasoning, yet I noted at the time he appeared to be quite comfortable explaining himself within the group environment. In fact I considered Jeff to be a driving force within his group and as one who had contributed significantly to the solution of many of the questions. Clearly this indicates that students need constant practice in expressing their ideas and thoughts.

Students were also asked to make several journal entries during the unit on Coordinate Geometry. Although the only samples of their writing included in this study are the group and individual tests, Ms. Hewitt indicated that the writing done in their journals was weak. During the group test the difficulty experienced by the students to express themselves was evident as they struggled to get their ideas down on paper.

Besides language, culture may also have played a role in cooperative learning in this mathematics class. Heather said during her interview that she was now working with another Taiwanese classmate named Lucy who had been placed in a different group

during the unit on Coordinate Geometry. She said that she was more comfortable working with Lucy, not only because they both spoke Taiwanese but also because "we are the same kind" and "in my country my math is like teach, it's a different way".

Discipline

In a typical mathematics class there exists a wide variety of students including, as most high school teachers can testify, students whose behaviour is less than desirable. During the course of this study I was impressed by the students' on-task behaviour and in fact, their behaviour in general. Of course I observed this class for only a small portion of time and realize that the newness of the learning environment and/or the presence of outsiders may have influenced the students' behaviour. Ms. Hewitt, knowing the students much better, pointed out that classroom discipline might be a potential problem for establishing and encouraging cooperative learning environments within some of the groups in this class. Sean, a member of the focus group, for example had been a bit of a problem in the past. Ms. Hewitt found him to be antagonistic towards her and in fact the situation had deteriorated and the rapport between them worsened. Although Sean attended only five classes during the unit on Coordinate Geometry, his teacher felt that other members of the group, especially

Heather, would be intimidated by him. While I did not witness this myself as I observed the group, I did notice that Sean's presence after a prolonged absence seemed to cause the group's efforts to become counterproductive.

Of course, students like Sean exist in almost all math classes. What impact does his presence have on successful implementation of a cooperative learning environment? Is there an optimal group in which he can be placed? Or will whatever group he is placed with have to suffer through? Will placing students with a history of disciplinary problems in a cooperative learning setting diffuse the situation and improve the behaviour? It has been suggested that the use of cooperative learning may actually improve student behaviour in that their motivation and attention is improved in such an environment and that potentially disruptive students may become more involved in learning and be influenced by their peers (Reid, Forrestal, & Cook, 1990).

Role of Homework

One obvious omission in the entire problem solving session was the setting of homework. Usually homework plays a prominent role in mathematics courses and is typically assigned daily. Teachers generally feel that having students practice the

skills taught in class helps to solidify them for the students. If it serves no other purpose, the old adage that "practice makes perfect" would certainly justify the setting of homework. In the thirteen classes on Coordinate Geometry, homework was rarely assigned. It had not been built into the plan, therefore when assigned it appeared to be an afterthought and was not checked the next day. This leads to a multitude of questions. How can homework be incorporated into a cooperative learning approach based on problem solving? How can students who have spent an entire class working together in an attempt to solve a particular problem be expected to practice it that night? The role that homework plays within a cooperative learning environment within the mathematics classroom may be another issue worth pursuing in future endeavours of problem solving research.

Chapter 6 Summary

As a classroom teacher, and thus not an experienced researcher, I had not before had the opportunity to examine so closely a group of students for such a length of time. From this perspective, it was a fascinating experience for me. For thirteen mathematics classes and six lunchtime interviews and problem solving sessions I observed a group of students as they struggled to cope with each other as well as the tasks they were given. Never before in my fifteen-year teaching career had I been given the chance to sit and freely observe students, not to judge them, but for the pure enjoyment of watching them learn. My experience with the students described in this study has left me with a richer understanding of how a group of students might actually solve problems together in mathematics class.

From the onset, I must admit to not being surprised at finding the students to be weak academically (or at least mathematically) and possessing low confidence in their own ability. These tenets are rooted in my own experience with students, as I too have taught students like those in the Mathematics 1300 class described in this study. I have also experimented with cooperative problem solving in my classes. Usually my role, like Ms. Hewitt's, is played out as a facilitator and monitor of group interactions

and progress.

For the duration of this study the students in the focus group attempted to solve problems presented to them. However, as time went on, the students did not appear to gain confidence in their ability to solve mathematical problems, either individually or as a group. Perhaps, as was suggested by Artzt (1996), the students' feelings of inability undermined their work as a group. They appeared to experience difficulty throughout in discussing and expressing their mathematical ideas, and working together. It appeared that the students in the focus group remained, as Dees (1990) described, working individually within their groups rather than truly working together. This was not surprising considering the little experience that these students had with cooperative learning in mathematics. In addition, very little time was devoted at the onset of the study to familiarize these students to working together, nor were any strategies presented to them.

Besides the lack of experience, several other factors appeared to influence the students' ability to solve mathematical problems together. Student attendance surfaced as one such factor that hampered the group's ability to work effectively. Only once during the thirteen classes were all five of the group members present and two members were present in only five of the thirteen classes. Consequently, the

likelihood of the students developing a positive rapport with each other was lessened. Awkwardness among the students seemed to remain and continue during the problem solving sessions that followed. It has been suggested that the long-term use of cooperative learning improves attendance (Johnson, Johnson, & Smith, 1995; Slavin, 1983). This study, considering its length did not appear to have any positive effects on student attendance. What implications does this have for teachers who prefer to use cooperative learning as a teaching/learning approach for shorter sessions or units?

Another dimension affecting the ability of these students to work together to solve mathematical problems was the dynamics of the group. One student had been described as a discipline problem who, according to his teacher, was thought to be intimidating to other group members. Another student's cultural background appeared to separate her from the other group members. A third student had been described as very quiet. These characteristics, coupled with the attendance of some members during the study, seemed to dictate the degree to which the group interacted. Also, as Artzt (1996) suggested, perhaps the perceptions the group members held of themselves and each other affected their ability to work as a group.

As reported by three of the group members, as well as their teacher, the class had continued to work in groups after the conclusion of the study and all had voluntarily

chosen to work with different students. This seemed to be a more beneficial arrangement for each of the focus group members. The decision of which students to group together, and for how long are important considerations that may greatly affect the productivity of the group.

A final issue that emerged from this study is the role of homework in such an learning environment. How can homework be incorporated effectively into an environment where students have spent the entire class period working cooperatively solving mathematical problems? This issue does not appear to be addressed in previous research and perhaps ought to be considered for future endeavours.

Through this study, from observations in class and during the problem solving sessions and interviews with the students and their teacher, it is evident to me that these students, though weak academically and not very self-assured, have made progress in becoming more capable problem solvers. With increased exposure to learning environments such as the one described in this study, and through efforts from teachers like the one portrayed here, these students are capable of becoming competent problem solvers.

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Appendix A Overview of classes

February 1 to February 15, 1996.

Note:

- Because of scheduling/time requirements for Mathematics 1300, students sometimes have mathematics two periods in one day.
- Classes were 55 minutes in length.
- Copies of worksheets and tests can be found in Appendix B.

Class 1 **Feb. 1**

Students were assigned to groups and were presented with the following problems unrelated to coordinate geometry.

Problem 1:

Given $69 \times 64 = 4416$
and $96 \times 46 = 4416$ Why?

Problem 2:

What is the value of x in $4^{20} + 4^{20} = 2^x$?

Class 2 **Feb. 2**

Students continued on the problems from previous class followed by a class discussion about the solutions.

Class 3 **Feb. 2**

Teacher introduced the topic of coordinate geometry by conveying the story of Descartes seeing a fly on the ceiling and seeking a way to pinpoint its exact position. Students given worksheet #1 on matching graphs and ordered pairs and instructed to work on questions 1 through 4.

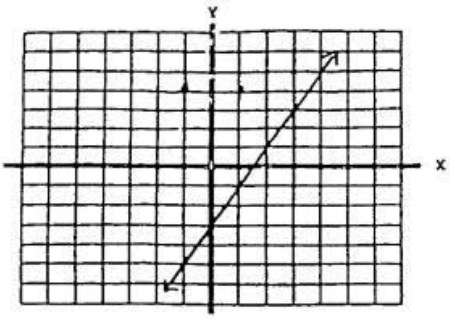
Class 4	**Feb. 5**	Students continued to work on worksheet from class 3 finished it by completing questions 5 and 6. Students also wrote journal entry #1. Worksheet #2 was then distributed to students. Journal entry #1: *Explain how a graph, table of values and equation are related.*
Class 5	**Feb. 6**	Students worked on material from previous class and wrote journal entry #2. Students then worked on sheet in finding equation of lines from graphs. Journal entry #2: *How do you find the equation of a line given just the graph?*
Class 6	**Feb. 7**	Only four students present in class because of poor weather conditions. These students worked together as a group to complete questions 5 and 6 from previously assigned worksheet.
Class 7	**Feb. 8**	Whole class discussion held concerning the relationship between the table of values of line and its equation. Summary points were produced and were written on the board.
Class 8	**Feb. 8**	Worksheet #3 (parallel and intersecting lines) was distributed.
Class 9	**Feb. 9**	Students continued working on worksheet #3.

Class 10	**Feb. 13**	Worksheet #4 (slope, y-intercept and graphing) was given to groups. Students also asked to write journal entry #3. Journal entry #3: *Summarize what you know about slope.*
Class 11	**Feb. 14**	Students finished worksheet and class summary of material covered.
Class 12	**Feb. 14**	Group test.
Class 13	**Feb. 15**	Individual test.

Worksheet #1

Which table of values matches the drawing on the left? Explain why.

1.



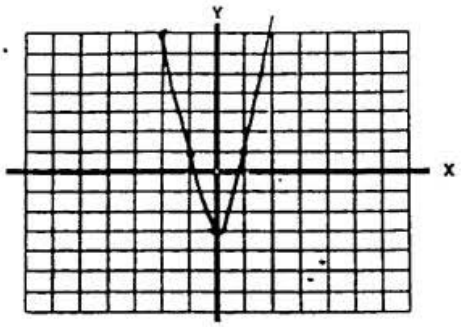
a)

x	-1	0	1	3
y	-3	-2	2	3

b)

x	-1	1	3	4
y	-5	-1	3	5

2.



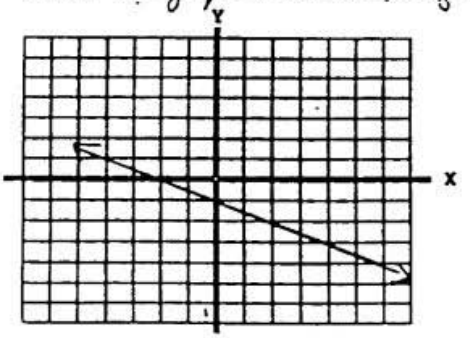
a)

x	-2	0	1	2
y	0	3	-1	8

b)

x	-2	-1	0	1
y	7	1	-3	1

3. For the graph and table of values given, which expression describes the graph? Explain why.

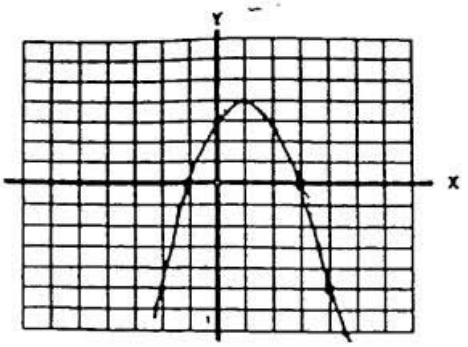


x	-2	-1	2
y	0	$-\frac{1}{2}$	-2

a) $-\frac{1}{2}x - 1$

b) $x + 3$

4.



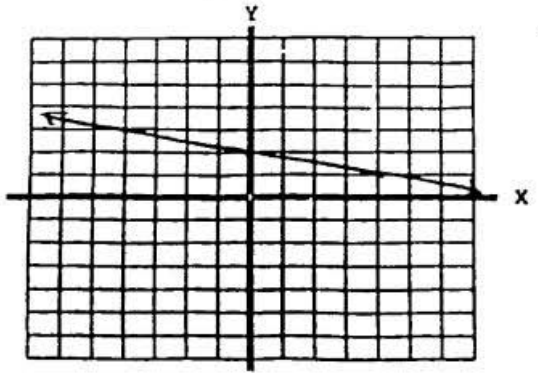
x	-2	0	3	4
y	-5	3	0	-5

a) $x^2 + 2$

b) $-x^2 + 2x + 3$

Which expression describes the graph on the left. Explain why.

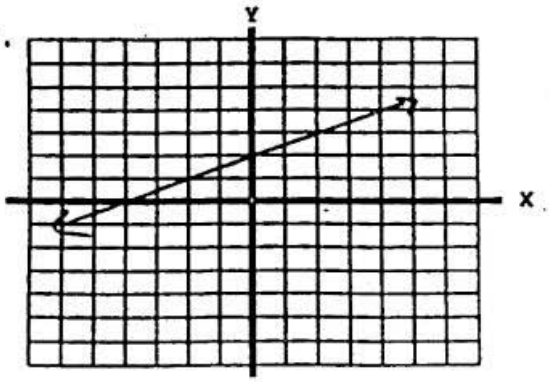
5.



a) $-\frac{1}{4}x + 2$

b) $2x - 4$

6.

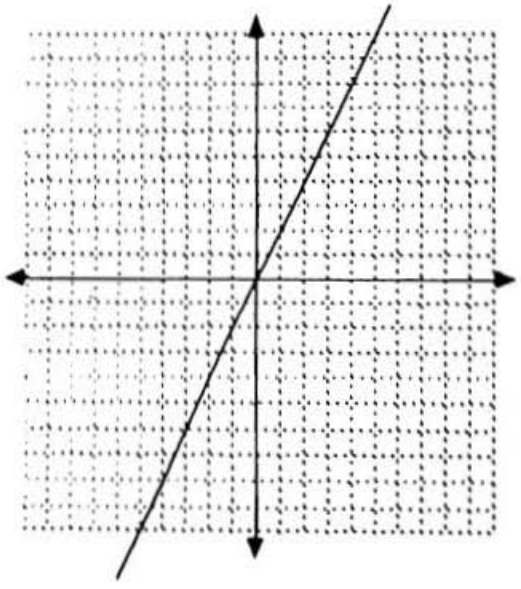


a) $x^2 - 7x + 5$

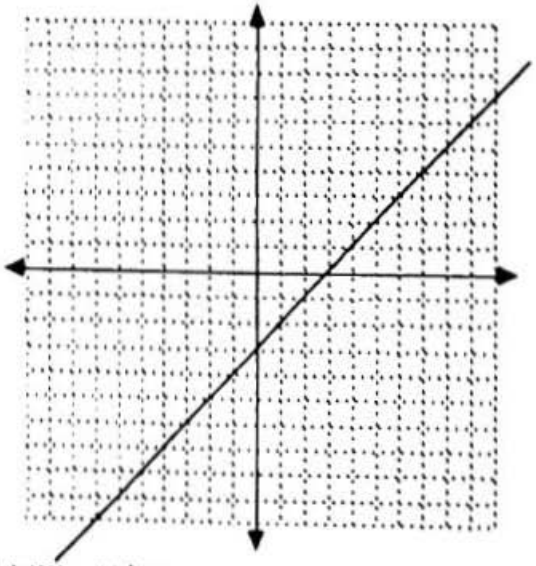
b) $\frac{1}{2}x + 2$

Find an equation for each of these lines:

1



2.



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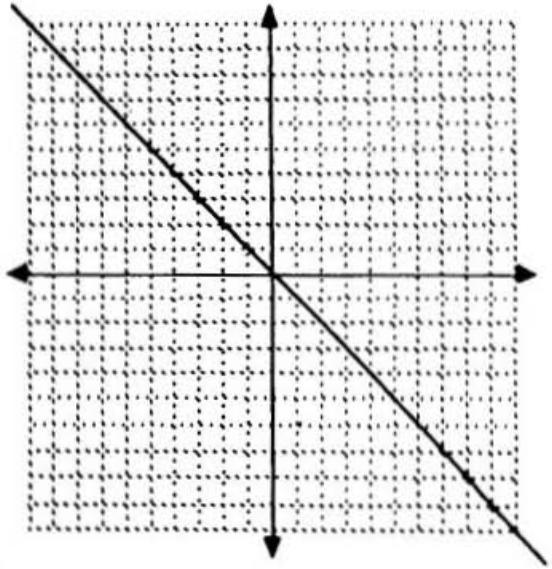

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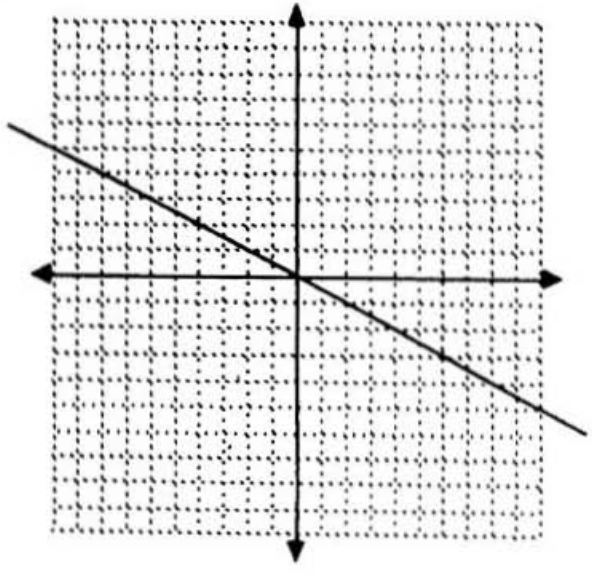
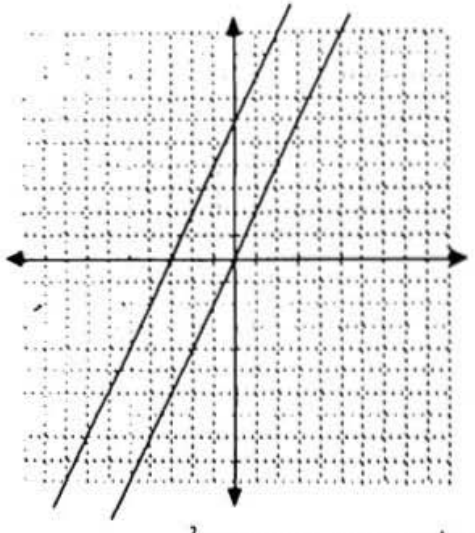


6.

5

A. For each pair of parallel lines, find the equations of the lines, and say what is the same about the equations and what is different. Explain why.

1.



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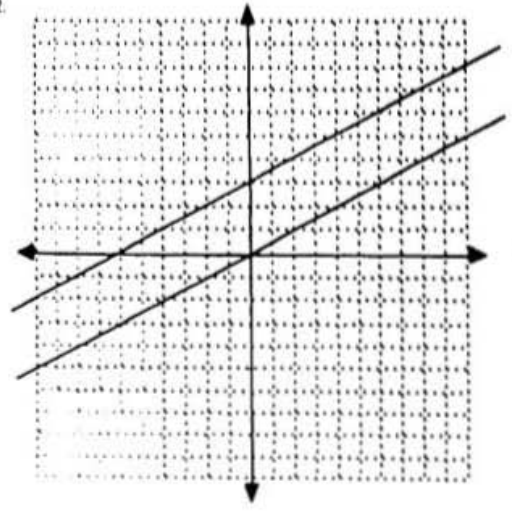

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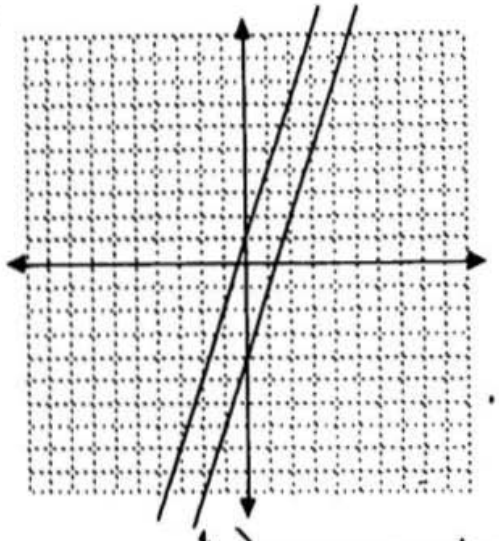

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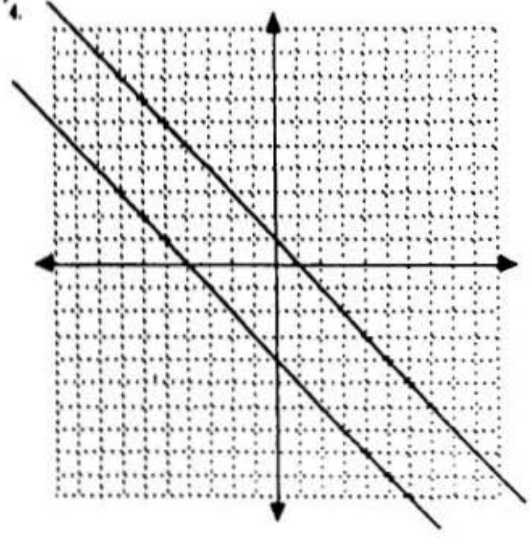
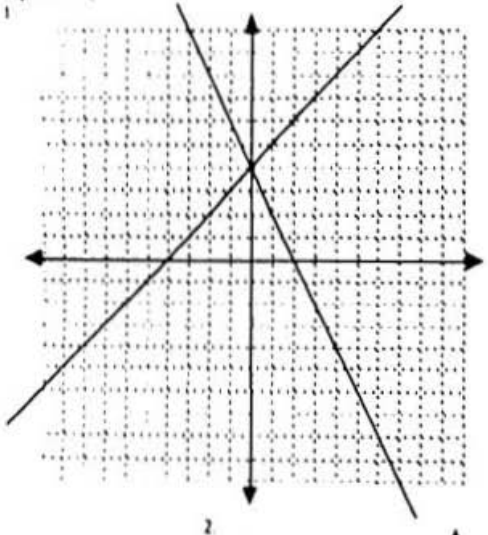


B For each pair of lines which pass through the y-axis in the same place, find the equations of the lines, and say what is the same about the equations and what is different. Explain why

1



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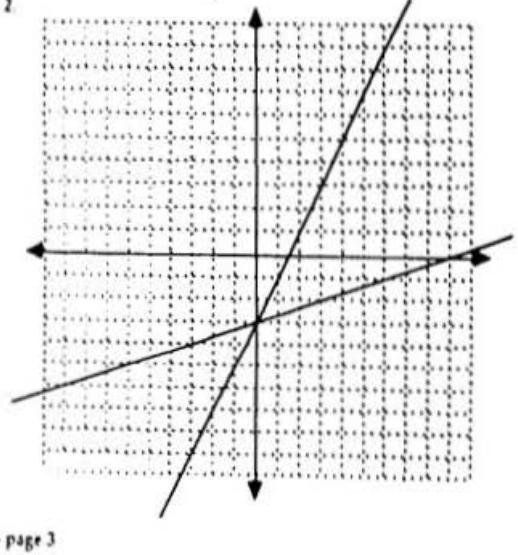

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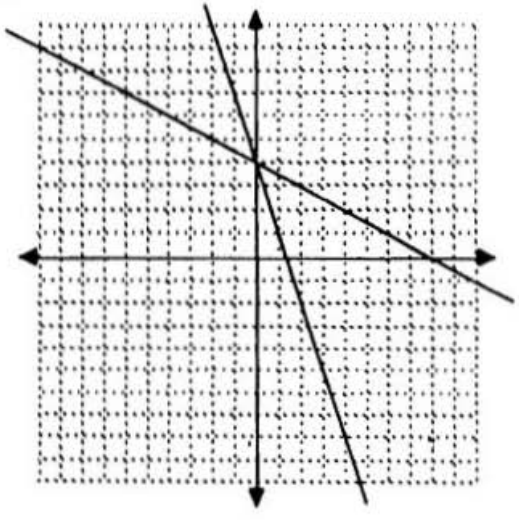

C. Without drawing the graphs, make up equations for two parallel lines. Say how you know they are parallel.

D. Without drawing the graphs, make up equations for two lines which go through the same point on the y-axis. Say how you know they go through the same point, and what point they go through.

Worksheet #4

A. Find the equation of each line. Say what the slope and y-intercept is for each equation.

1.

Equation: y=

Slope=

y-intercept=

2.

Equation: y=

Slope=

y-intercept=

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3.

Equation: y=

Slope=

y-intercept=

4.

Equation: y=

Slope=

y-intercept=

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B. Explain how you found the slopes for the equations in part A.

C. Make a graph of each of these equations. (Use a separate sheet of graph paper.) Use any shortcuts you can think of. Describe your shortcuts.

1. $y = -2x - 3$

2. $y = \frac{2}{3}x + 2$

D. For each equation write an equation for a parallel line through the point given.

1. $y = -2x - 3$ (0, 4)

2. $y = \frac{2}{3}x + 2$ (0, -12)

Math 1300 -- Group test

Select one member of your group to do all the writing. Write your final answers of this paper. Hand in any scrap paper you use to do your work.

1. Are the graphs of $y = \frac{1}{3}x + 7$ and $y = \frac{1}{3}x - 9$ parallel lines? Explain why or why not.

2. Is the point (12, 14) on the line whose equation is $y = 3x - 20$? Explain why or why not.

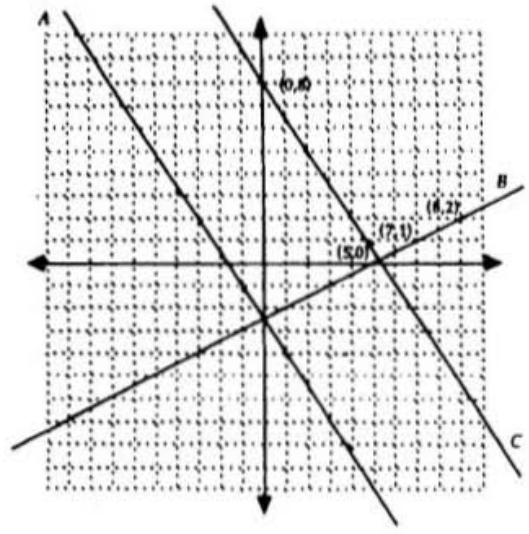
3. Is this the table of values for a line?

x	0	1	2	3
y	-2	1	5	7

If so, what is the equation of the line?
If not, why not?

4. The equation of line A is:

$$y = \frac{-2}{3}x - \frac{1}{3}$$



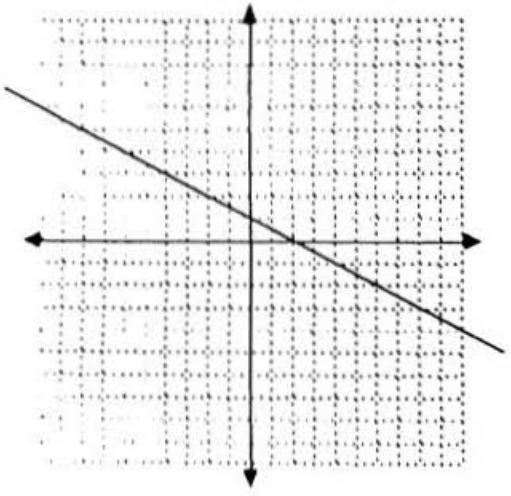
If line B crosses the y-axis at the same place as line A, what is the equation of line B? Explain how you know.

If line C is parallel to line A, what is the equation of line C? Explain how you know.

Math 1300 —Individual test

Hand in any scrap paper you use to do your work.

1 Find the equation of this line Explain how you found the equation.



2 Do the graphs of $y = 3x - 2$ and $y = 3x + 2$ cross each other? If so, where? If not, why not?

3. Do the graphs of $y = -2x + 3$ and $y = 2x + 3$ cross each other? If so, where? If not, why not?

4. What is the slope of the line through the points (1,1) and (4,5)? Explain how you found the answer.

5. Are the points (0, 3), (1, 7), (2, 12) all on the same line? If so, what is the equation of the line? If not, why not?

6. What is the equation of the line whose slope is 12 and whose y-intercept is (0, 93)?

Math 1300 — Group test. Bob, Heather, Valerie & Sean

Select one member of your group to do all the writing. Write your final answers on this paper. Hand in any scrap paper you use to do your work.

1. Are the graphs of $y = \frac{3}{4}x + 7$ and $y = \frac{3}{4}x - 9$ parallel lines? Explain why or why not.

Yes. because the $\frac{rise}{run}$ is the same.

2. Is the point (12, 14) on the line whose equation is $y = 3x - 20$? Explain why or why not.

Because the co-ordinates don't fit the same line.

3. Is this the table of values for a line? No

x	0	1	2	3
y	-2	1	5	7

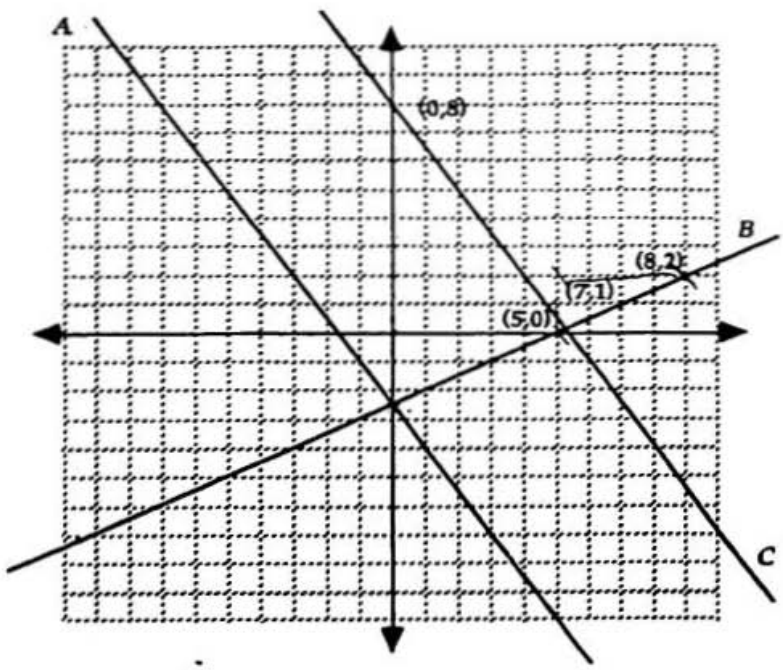
If so, what is the equation of the line?
If not, why not?

Because the co-ordinates are not on the same slope

Group A

4. The equation of line ***A*** is:

$y = \frac{-3}{2}x - \frac{5}{2}$



If line ***B*** crosses the y-axis at the same place as line ***A***, what is the equation of line ***B***? Explain how you know.

We found $\frac{rise}{run}$ and the y-inter was same for both line A & B

$y = \frac{1}{2}x - \frac{5}{2}$

If line ***C*** is parallel to line ***A***, what is the equation of line ***C***? Explain how you know.

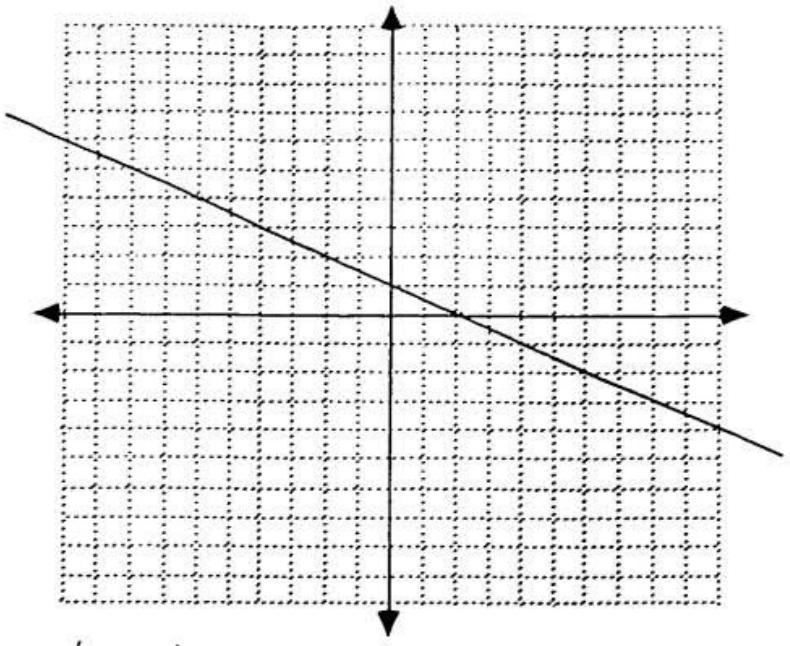
the equation of line C?

$y = -\frac{3}{2}x + 8$

Both lines have the same $\frac{rise}{run}$.

Hand in any scrap paper you use to do your work.

1. Find the equation of this line Explain how you found the equation.



$y = \frac{1}{2}x + 1$

I found the slope and the y-intercept and put them in the equation. I found the slope by picking two points and counting up for the rise and over for the run which gave me the slope.

2. Do the graphs of $y = 3x - 2$ and $y = 3x + 2$ cross each other? If so, where? If not, why not?

No They are parallel lines because they have the same slope.

3. Do the graphs of $y = -2x + 3$ and $y = 2x + 3$ cross each other? If so, where? If not, why not?

They cross at (0,3) because they have the same y-intercepts.

4. What is the slope of the line through the points (1,1) and (4,5)? Explain how you found the answer.

The slope is $\frac{4}{3}$ or $1\frac{1}{3}$ I drew a diagram and calculate $\frac{\text{Rise}}{\text{Run}}$ to find the slope.

5. Are the points (0, 3), (1, 7), (2, 12) all on the same line? If so, what is the equation of the line? If not, why not?

yes, $y = 4x + 3$

6. What is the equation of the line whose slope is 12 and whose y-intercept is (0, 93)?

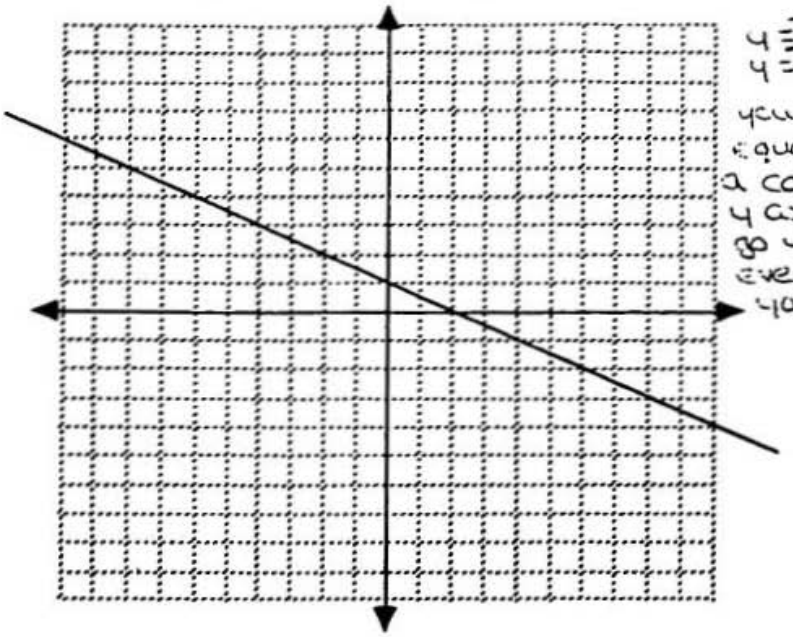
$y = 12x + 93$

Valerie

Math 1300 —Individual test

Hand in any scrap paper you use to do your work.

1. Find the equation of this line. Explain how you found the equation.



$y = -2/4\,x + 1$
$y = -1/2\,x + 1$

you would find this equation by you pick a co-ordinate on the y axis. Then you would go up to your next even point by using your $\frac{\text{rise}}{\text{run}}$ method.

2. Do the graphs of $y = 3x - 2$ and $y = 3x + 2$ cross each other? If so, where? If not, why not?

yes they do not cross each other. They cross each other at positive infinity.

3. Do the graphs of $y = -2x + 3$ and $y = 2x + 3$ cross each other? If so, where? If not, why not?

yes. They would meet a positive three

4. What is the slope of the line through the points (1,1) and (4,5)? Explain how you found the answer.

The slope is $\frac{5-1}{4-1} = \frac{4}{3}$

I found my answer by, I plotted the points in a graph and started my y-intercept. From there I went up 4 and over 3 to the next even points and that gave me the slope.

5. Are the points (0, 3), (1, 7), (2, 12) all on the same line? If so, what is the equation of the line? If not, why not?

yes they are on the same line.

$y = 4/1x + 3$

6. What is the equation of the line whose slope is 12 and whose y-intercept is (0, 93)?

$y = 12x + 0$

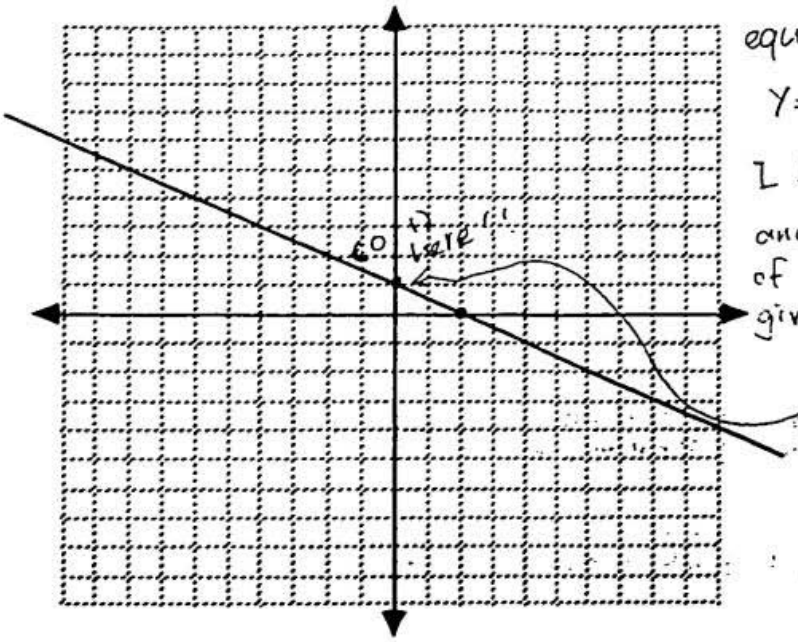
$y = 12x + 93$

Math 1300 —Individual test

Heather

Hand in any scrap paper you use to do your work.

1. Find the equation of this line. Explain how you found the equation.



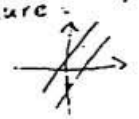
equation:

$y = -\frac{1}{2}x + 1$

I found the $\frac{\text{rise}}{\text{run}}$ and the second part of the equation is given.

2. Do the graphs of $y = 3x - 2$ and $y = 3x + 2$ cross each other? If so, where? If not, why not?

picture - sorry!

No. because they are like →  their slope is the same

3. Do the graphs of $y = -2x + 3$ and $y = 2x + 3$ cross each other? If so, where? If not, why not?

Yes. on (0, 3)

4. What is the slope of the line through the points (1,1) and (4,5)? Explain how you found the answer.

① $\frac{4}{3}x$

② draw the line out and count.

5. Are the points (0, 3), (1, 7), (2, 12) all on the same line? If so, what is the equation of the line? If not, why not?

No, I checked with all three points of the equation but (2, 12) doesn't fit.

6. What is the equation of the line whose slope is 12 and whose y-intercept is (0, 93)?

equation of the line:

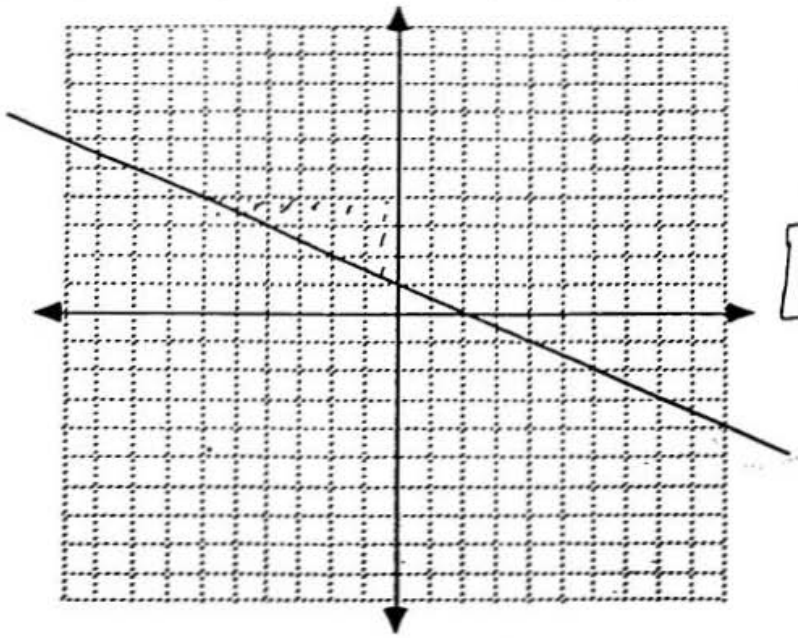
$Y = 12X + 93$

Math 1300 —Individual test

Bobo

Hand in any scrap paper you use to do your work.

1. Find the equation of this line. Explain how you found the equation.

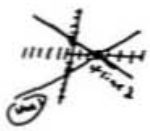


$y = ?x + 1$

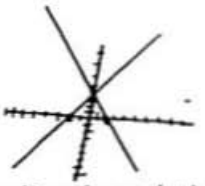
$y = \frac{3}{6}x + 1$

$y = \frac{1}{2}x + 1$

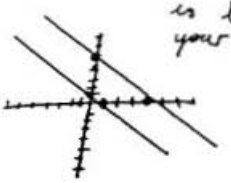
2. Do the graphs of $y = 3x - 2$ and $y = 3x + 2$ cross each other? If so, where? If not, why not? yes they cross each other. The reason for this is because both lines share the same co-ordinates on the x axis. ex.



3. Do the graphs of $y = -2x + 3$ and $y = 2x + 3$ cross each other? If so, where? If not, why not? yes, these lines cross each other on the y axis



4. What is the slope of the line through the points $(\overset{x}{1},\overset{y}{1})$ and $(\overset{x}{4},\overset{y}{5})$? Explain how you found the answer. 1 and 5. The reason for this is because your y axis is always your slope.



5. Are the points (0, 3), (1, 7), (2, 12) all on the same line? If so, what is the equation of the line? If not, why not? no they are not. They are not because the coordinates don't the same for just one line

6. What is the equation of the line whose slope is 12 and whose y-intercept is (0, 93)?

$y = 0x + 93$

Appendix C Transcript of Interviews with Students

Interview with Heather **March 27, 1996**

00:00 **D:** This is the first interview and it's with Heather. And Heather I'm going to ask you questions in a couple of different categories, I'm just going to ask you about the experience that we had about a month ago in Ms. Hewitt's math class and then I might ask you just some general questions. Feel free to say whatever you want, take your time, don't worry at all about it. Ok? Now do you remember much about the experience, about the little problem solving session that we had, gee must be mid February I guess.

H: Yeah.

D: If you don't it's no problem. I'm just going to ask you and see how much you remember. First of all Heather, how did you feel about the experience in general?

H: Mm. Good.

D: It was a good experience?

H: Yes. You can like talk and find out a way to do the questions.

1:00 **D:** Ok. How did you feel about working with the group for an extended period of time, because it was, I think, 13 classes that you were with the same group? How did you feel about that?

H: Um. It was good. It was like slower, because you can talk and like help somebody. I think slower.

D: Ok. So you felt if it wasn't in a group that it wouldn't been as slow. Is that what you're saying? Ok, which kind of leads me to my next question. How would you feel that this unit on Coordinate Geometry would have gone if Ms. Hewitt just taught it?

H: Just taught it?

D: Yeah.

H: Well I think, I think it's better if she just taught it.

D: Ok, why do you think that?

2:00 **H:** Because in my country the teacher don't, like we don't, we're not like in this group and you talk about this.

D: Ok.

H: It's a better way to learn. It's quickly.

D: Ok, so you feel that's a better way to learn for you.

H: Yeah, but like you have good side and bad side.

D: Ok, what's the good side?

H: Good side? If you work in a group like you can talk and for me I can learn English.

D: Ok.

H: Like everyone have different opinion.

D: Ok, and the bad side?

H: Slower.

D: It's slower, Ok.

H: Yeah, if teacher just taught it, it would be quickly.

D: Would you learn as much that way I wonder? Or would it make a difference in how children or how students learn things?

H: Yeah, if we work in a group, students can think more, I think.

3:00 **D:** Ok.

H: And if teacher just taught it like taught it and you just learn it you don't think how others work before. You don't know the, ah...how do you say?

D: Material, subject or....

H: Yeah, I think.

D: Ok. Was this experience with that group more difficult or stressful or less difficult or less stressful. Did it alter your feelings of being in math class? Being in the group, not related to Coordinate Geometry, just how you felt about coming to math class. Did it make a difference knowing that you were going to come and be in a group or did it not make any difference at all? Did you find it more difficult to think about it or more stressful or less stressful or less difficult knowing that you were going to come and work with students as opposed to Miss teaching you and so on?

4:00 **H:** No, I don't think there is a difference.

D: You didn't find a difference? Ok. Um. How do you feel this experience has affected your ability to solve problems?

H: I don't know what you say.

D: Solve problems?

H: You say it a simple way?

D: Oh sure, Ok. Do you think that because you spent 13 classes or so working with students, other students, working on problems, cause that's what you were doing, Ms. Hewitt wasn't teaching you, you were solving the problems. Do you think this has affected your ability to solve problems. You think that you can now solve problems better or not so well because of this experience. Or do you think it didn't make a whole lot of difference?

H: Not a lot.

5:00 **D:** Not a lot of difference. Ok. And in the group that you were in, I'm just looking over where you guys were and I'm picturing where you physically were, what do you think your role was in the group?

H: Role?

D: Your role. What do you think your function or your purpose, what you did in the group, that made a difference, or what part did you have in solving the problems? Did you serve any, did you feel you had a certain role to do or certain thing that was your responsibly or something that you felt comfortable with doing in the class, in that group.

H: Working in the group is like everybody can, like you have a question everybody can talk about it and ah...

D: But do you think you in particular as a member of the four or five or however many were there, three or whatever, did you feel a certain thing in the group?

H: No.

6:00 **D:** No? Ok. So how did you feel that would fit in the group then?

H: Fine.

D: Fine? Ok. Did you feel that people asked you more questions or you asked more questions or anything?

H: No, I think I asked less questions.

D: You think you asked less questions? Ok, why do you think you asked less questions?

H: Huh?

D: Why did you ask less questions?

H: I don't know, cause I don't know them and I think my English is not so well.

D: But you said it was an opportunity for you to learn English.

H: Yeah.

D: So it would have been a good chance for you to ask. So you didn't feel like you knew the students in the class so you asked few questions.

H: Uh huh, yeah.

D: Ok.

7:00 **H:** And because I learned, I learned math now the teacher is teaching the math I learned before so I do it more.

D: Ok so you had been taught this before so this is kind of a review for you?

H: Yeah.

D: Ok. That was some of the questions I wanted to ask you about the experience. Is there anything else you wanted to mention about the classes together? Anything that comes to mind that you'd like be to be aware of?

H: I think in the future if students can work in a group it's like you have to put a teacher right beside the group.

D: Ok.

H: Like you, so we have a problem we can ask, if we have like a question we can't do.

D: Sure, right. That would be great if you could have a teacher for every 4 or 5. That would be marvellous. I would love that. Ok. Now I'm going to ask you some questions about what's happened since that

experience. Since we finished up with the unit on Coordinate Geometry or when Dr. Reid and myself were here, and just to see if you got any feelings on what's happened since. Now I gather from speaking to Ms. Hewitt that you have been working in groups and has this been ongoing? Have you guys been working in groups everyday?

8:00 **H:** Yeah.

D: Are you still with the same group?

H: No.

D: Who do you usually work with?

H: Lucy.

D: So you and Lucy usually work together?

H: Yeah.

D: How is this working? Yourself and Lucy working together?

H: Good.

D: Ok, why?

H: Ah, because our languages are the same so we can talk more.

D: So do you speak in English when you and Lucy work together?

H: No.

D: So is that helping your English at all?

H: No, but I... I don't know.

9:00 **D:** Ok, do you find that working with Lucy is more comfortable than when you're with the other people in the group?

H: Yes.

D: So does it make a difference in terms of how you can actually work on a problem?

H: Yeah.

D: Do you behave differently working with Lucy than you did with the group?

H: Yeah I think so.

D: In what why would you say you were work differently?

H: Like we, like I don't know how to say...

D: Take your time.

10:00 **H:** Like I'm more comfortable with work with her because we are the same kind, like we come from Taiwan. When I'm working like with the group, the group I work before, I have to think about how to say English that kind of things so I don't. If I have questions I'll ask but I don't, I don't know the English I just keep it and ask my friend after class.

D: So you find it difficult sometimes to express or communicate with people who are from Newfoundland. So working with Lucy, you find, because the language is not a problem, do you find it easier to solve problems or work at the math with her?

H: Yeah.

D: And other than the language is there anything else you find working with Lucy different than working with the group?

11:00 **H:** Mm... There's a problem like our math like I'm Taiwanese and in my country my math is like teach ah, it's a different way. Like we work in different ways so sometimes I don't understand but like there's a student in my class, Sherry Lee and she asked me a question I told her

but I told her do it in another way and she don't understand like that because we been teach in a different way.

D: So you think it's probably from because your mathematical background, that you're taught a different way, that you don't find it as easy to talk to people from here.

H: Yeah.

D: Do you prefer working in a group or by yourself?

H: Me, ah, well now I think working with group.

12:00 **D:** Working with the group, ok. Working with someone like Lucy you mean?

H: Yeah.

D: If Miss were to put you with someone else, do you think that you would find it a bit easier now after having gone through that session on the unit on Coordinate Geometry.

H: Yeah, I think.

D: What do you think would be different if Miss changed the seating and said now you've got to sit with another group of students?

H: I think it's more comfortable because I've been working with people before so now if she changed the seat or changed the people I think would not be difficult.

D: Would you, on your own, move with somebody else and start working with other people?

H: Just me?

13:00 **D:** Yeah, or you and Lucy move with other people, on your own, without Miss telling you?

H: No, I don't think so.

D: No you'd rather, you would stay and just work as a pair? Ok, ah... do you like math?

H: Yeah.

D: Do you, do you feel that you're good at math?

H: Yeah I think so.

D: Do you find that the math you are learning in this environment a lot different from what you've been used to?

H: What do you mean?

D: Do you think that the math you learned when you were in Taiwan is, I know you said you were taught a different way, but is the math itself a lot the same as what you're learning here?

H: In Taiwan the math is more difficult.

D: More difficult, ok. You said that you have only been here for about a half a year. When you left can you remember what you were doing in math when you left Taiwan?

H: Yeah.

D: What were you doing, what topics?

14:00

H: I don't know how to say in English.

D: You want to just describe it to me and maybe I can help you out.

H: You know, you have to prove like this...(Draws on paper) if you have another this one, you have to prove it is the same.

D: Two congruent triangles, congruent?

H: Yeah I think.

D: Ok so you were doing geometry proofs and that kind of thing. Ok, how would you rate your problem solving ability? If I gave you a problem now and I said, "Will you work on that for me", how would you feel about doing that, first of all? Would you, would it make you uncomfortable or would it bother you or would you, you know, say, "Oh I want to get at it, let me solve this problem", or would you just take it and say, "Oh well I'll just try it"?

15:00 **H:** Yeah, I would just take it and try it.

D: Do you feel that you are good at solving problems?

H: No I don't think so, I just, you know if somebody teach me how to do it I can do it but if nobody teach me and gave me the new problem to solve, I don't think I can do that.

D: Now I'm going to ask you a question, will you solve a problem for me, and if I gave you a problem now would you try it?

H: Yeah I'll try.

D: Ok and what I'll ask you to do when you're solving this is as much as you can, just so I'll know what's going on, just think out loud or just say whatever you want or write down whatever you want and I'll just try to keep an eye on you and see what you're doing and if you got any questions, you can certainly ask me about it. Ok? If there's any words that you want me to explain. Now if you want you can talk loud enough so we can get it on tape just to find out how you're doing.

16:00

H: *(Reads the problem silently)* Chicken has two feet, right?

17:00 **D:** I think so, yeah... What are you thinking about there?

H: What?

D: What are you doing there?

18:00 **H:** Chicken, two feet. Every chicken have two. *(Works on problem again)*

D: Now, you want to just talk me through what you decided to do or how you went about it?

H: Sure, cause there's 18 chickens and...

D: There's 18 chickens and pigs.

H: Yeah and pigs and every chicken has two feet?

D: Ok, now there's 18 animals all together.

H: Oh, ok not just chickens?

D: No.

H: There's pigs too?

19:00 **D:** Yeah, 18 animals.

H: Oh ok, sorry.

D: That's ok, it's probably how the way it's worded on the paper. So that's gonna change your thinking?

H: Yeah.

20:00 **D:** Ok now, want to explain to me what you did?

H: Chicken plus pigs are 18.

D: Ok.

H: So "X" is chickens and "Y" is pigs and every chicken has two feet and pigs have four feet.

D: You just went ahead and solved the systems of equations, did you?

H: Yeah.

D: When you got your answer, as soon as you wrote it down you passed it over to me. Did you check your answer then or did you think about it or did you figure because it came out to be nice numbers that it must be right?

21:00 **H:** I check, ok...*(checks it over)* checked.

D: Ok good. Ok thanks Heather. How did you find that problem?

H: Easy I think.

D: Is it? Did you guys learn how to solve the systems of equations in this course? Or you're doing it now. So then it registered with you to go ahead and do it. Good. Is there anything else that you wanted to mention to me before we finish up? Anything that you have any questions on or anything you'd like to know about what I'm doing or anything like that?

H: Are you going to like play the tape in front of the class?

22:00 **D:** No, no, not at all. That tape, everything about this conversation and about what we've done before is going to be strictly confidential. I will probably let like Dr. Reid listen to it or if there's somebody else who's interested in my study might hear it but other than that, it won't be, you won't hear it on the radio tomorrow or anything you know but that's it. Anything else, anything you want to know about?

H: Will you come back or just...?

D: After we finish our problem solving session? No, I probably won't be back.

H: Just doing a project?

D: This is for my thesis, for my masters. I'm a teacher so I'll be going back teaching after Easter.

H: Ok. In this school?

D: No, I'm at another school, so that's where I'll go back in my old position. I appreciate your time that you've given me today Heather and thank-you very much.

23:00 **H:** Sure.

00:00

D: This interview is going to be with Bob, and Bob I'm going to ask you questions in a couple of categories. Ok?

B: Alright.

D: So if you could just answer whatever's on your mind and take it easy and don't worry about it. First of all I'm going to ask you do you remember that experience that we had when myself and Dr. Reid were in Ms. Hewitt's math class and you guys were working over there in that side of the classroom as a group?

B: Yeah.

D: Do you remember that? Most of the questions relate to that, at least the first questions anyway. Let me ask you how you felt about the whole experience, just in general, any feelings that you had because of that in anyway?

B: It was good. It was a good idea. A lot of people who don't like, catch on as easy find it like there's just people helping you. It's easier to understand and cause you don't have to ask the teacher as much. Like some people are reluctant to ask the teacher. It's easier that way.

D: Would you consider yourself one of the people reluctant to ask the teacher?

B: No.

D: So for you then.

1:00

B: I used to be though.

D: But you're not any more. If you got a question, you're letting Miss or Sir know.

B: Yes.

D: How do you feel about working with the group for an extended period of time?

B: I didn't mind at all.

D: With the same group and so on as you know, how do you feel about that?

B: It's better to get used to a group cause then you get to know the people and like you know, just starting off you probably don't know the people as well and you don't really talk to them right but if you are with them for longer you'll be able to ask them more.

D: Ok. I'm going to ask you this kind of question later on but I'll ask it to you now when it comes to mind. Do you think if they had been friends of yours in the group that it would have made a difference in your perspective?

B: It wouldn't been any good.

D: Wouldn't be any good? Why is that?

B: Cause there would be too much fooling around and we'd get nothing done.

D: How would you feel or how do you think the unit on Coordinate Geometry would have gone if Ms. Hewitt just taught it? I know that
2:00 you guys are working in groups now since then but before that you guys were pretty well in rows and Ms. Hewitt taught and you guys took notes or whatever. Do you think it would have changed how you felt your understanding of the unit went?

B: Yeah, cause if we were together you would find there was a lot of people who didn't know it and didn't bother to do it right. But then you get like some of the smart people in class like on tables and that you can just find out how to do it and that. Usually people just do badly and they won't learn it at all like they won't bother to go learn it after it's over, after they fail a test.

D: So you feel that if she just stood up and taught it that it wouldn't be learned as well?

B: Oh I would have probably yeah but I say there's people who wouldn't.

D: Did you think that the experience, being with the group for that length of time was more difficult or stressful or less difficult or stressful? Just thinking about coming to math class knowing that you're going to be in a group, did that change your attitude about coming to class?

B: It was less. It was definitely less stressful.

D: Less stressful ok. Why do you think it was less stressful?

3:00 **B:** I don't know...it's just; it's not as boring either. You know how you say, "Ah I'm going to math class now." It's not the same thing as every other class.

D: Do you remember much about what you learned in that unit on Coordinate Geometry?

B: Oh yeah.

D: Do you? Have you gone back to it at all since then or have you gone on to...

B: A little bit, not really though.

D: I was going to ask you tell me what but I think due to time constraints I might leave that for now, maybe when we're with the group we might talk a little about that. How do you feel that your problem solving ability has been affected by this because the way it was set up or Miss didn't teach very much she kind of gave you the problems to solve among your groups, to figure things out. Do you think that the experience affected your ability to look at any math problem and solve it?

4:00 **B:** Yeah, cause the other way like she's teaching and she's showing you, ok this is *how* you solve it, but like that way you had to figure out, ok,

how do I solve it? You had to figure out how to solve it and then solve it. It made me think more.

D: So you think then problems you were given after that, you were better able to solve?

B: Yeah.

D: What do you think your role was in the group, thinking about the group of three, four, five, depending on what day it was, do you think that you served a certain role or function in that group?

B: It depends on like I might have known something they didn't know or they might know something I didn't know. So everybody like knows certain things, everybody doesn't know everything.

D: Ok, but was there anything that you felt that you usually contributed to the group coming from your understanding of what's going on? Did you feel like you had a specific role?

B: Not really.

5:00 **D:** No, no, just wondering if you felt that you were at a certain responsibility in the group even though nobody talked about responsibility or who was going to do what but just maybe you might have felt it. I'm going to ask you a few questions now about what you've done or what you've gone through since you finished that experience and as I said before my understanding is that you're still working in groups, are you in the same group, do you work with the same people?

B: No, there's two different people.

D: Ok.

B: Actually yeah, it's a whole different group.

D: A whole different group that you're working with?

B: Yeah.

D: And how is this group working or how do you feel about this group?

B: It's good. Well mainly it's me and another guy who like just do most of the stuff. Like we don't talk much but if I don't know something I'll ask him, if he don't know something he'll ask me right, mostly just me and him.

D: Do you feel that this is working better than the group that you were set up with originally in terms of you getting an understanding of what's going on?

B: Yeah, I find it more comfortable than before.

D: Is this person a friend of yours?

B: No, well he is now, sort of.

6:00 **D:** Ok, so it wasn't because, remember you said before being with a friend would make a difference?

B: Yeah, I'm just a friend here but not outside of class.

D: Ok. Do you prefer working in a group or by yourself?

B: Groups, definitely.

D: Anything you want to elaborate on that?

B: Just the same reason. You can get the help, like you know it's a lot easier to get help.

D: Do you like math Bob?

B: When I understand it I do but..

D: Overall, when someone says the word math how does that make you

feel?

B: I don't mind it.

D: Don't mind, doesn't bother you one way or the other?

B: No not at all.

D: Do you like coming to math or do you like doing math?

B: I don't dislike it. I don't mind it.

D: How would you rate your problem solving ability? Would you say that you're a good problem solver, average or...

B: I'd say average.

7:00 **D:** If someone gave you a problem to do how would you feel about it?

B: First I'd go "Oh great" *(laughs)* I don't know, it would probably take me awhile to kick in. I wouldn't mind.

D: How about if I gave you a problem to try now?

B: Sure.

D: What I'll do is I'll just give you this, you can read it, if you got any questions you can let me know. I'll lend you my pen as well, or would you rather a pencil?

B: It doesn't matter.

D: Ok, here you go. If you got any questions you can let me know and while you're solving it I'd like for you just to say anything that comes to mind or things that you're thinking about when you're doing that.

8:00 **B:** *(Works on the problem for a few moments.)* Can I use a calculator?

D: Sure.

9:00 **B:** *(Continues working.)* I don't know how to go about it.

D: Try your best, if you can figure out any way it doesn't make a difference whatever way you want. Start with what you know.

B: Does this mean there's 18 chickens and pigs or 18 chickens and pigs together?

D: Right, 18 animals, I guess I should have said.

B: I don't know. There's no way you can determine because they're both equal numbers right, two and four?

10:00 **D:** Ok.

B: And to get an answer for that you use both things you also get more answers.

D: So what are the two and four representing?

B: The legs.

D: Ok, so chickens got two legs and pigs got four. So, you can do it a number of different ways. You can ask yourself well just by guessing or by trial and error can you figure out how many chickens and how many pigs would give you eighteen altogether?

B: You can get a bunch of answers for that, you can get like different ones.

D: Ok, just find one for me and just see if they'll work.

11:00 **B:** *(Continues working.)*

D: Ok will that do the trick? Will that give you, it says there's how many altogether?

12:00 **B:** You need two more so just take away one from this, eight chickens, ok but... Now it's eight pigs and ten chickens.

D: Ok, you said there were a bunch of different answers so would that be one answer of a lot?

B: Yes, I'm sure there's more than that.

13:00 **D:** There's more than that? Ok. Want to find another one for me?

B: *(Continues to work on problem.)* Four pigs and eighteen chickens.

D: You want to check again because there's only going to be eighteen altogether.

14:00 **B:** That's right, I keep forgetting about that. I can't think of another one. I don't know.

D: In fact given this information, there's eighteen animals altogether and there's fifty-two legs between all the animals, there would only be really one way of looking at it, which is what you came up with there, eight pigs and ten chickens. I just thought I'd see if you'd, you know, come up with, rationalize it, and there would be another answer. Good, good job. Before we finish up, one last question. Have you had much experience in like other math courses with working in groups or it's mostly been teacher telling you and you guys doing it?

15:00 **B:** Well last year we had like a small math class so like we weren't really in groups but basically it was the same thing cause there wasn't many people. He (last year's teacher) didn't mind us talking. Anyway like you're sitting by each other so it's the same thing.

D: Ok but the teacher did explain to you how to do it then you went ahead and did it?

B: Oh yeah.

D: Where as in this case you guys were trying to figure out how to do it and Miss didn't tell you a whole lot. So it was a little bit different I guess, was it?

B: Yeah.

D: You got any questions for me anything that you want to ask me about or...

B: No I can't think of anything off the top of my head.

16:00 **D:** Oh, ok. Well I appreciate your time and not only with the session earlier but now making time during lunchtime to let me interview you.

00:00 **D:** Valerie, I'm going to ask you questions in a couple of categories and feel free to say whatever is on your mind, don't worry about it and just take your time. Ok?

V: Ok.

D: First I'm going to ask you about the experience we had with Ms. Hewitt's class, when Dr. Reid and I were in and I was sitting with your group and I know it was a while ago but hopefully you'll remember it, what went on and so on. Do you remember anything? Ok. How did you feel about that experience in general?

V: I thought it was good. You got to work with other people. And like some people don't really talk to different people in the class. You got to know other people also.

D: Ok, so you found working with the group was helpful to you?

V: Uh huh.

D: Do you think it was helpful to most people in class, do you think most people felt that way?

V: Yes.

D: How do you feel about working with the same group for an extended period of time, because there were about 13 classes that you worked together?

V: Good.

D: Ok. Why did you think it was good that you worked with the same group for a length of time?

1:00 **V:** Cause you most always know what everyone was thinking like I got to know them after a while. If you went to different people you would get

confused.

D: How do you think the unit would have gone if Ms. Hewitt had just taught it? The way she had done maybe back in September or October, do you think it would have made a difference?

V: Probably. Cause this stuff (the unit on Coordinate Geometry) you had to figure out on your own but the other stuff, Miss just gave it to you.

D: Which way do you think would have been better in this particular case?

V: I'm not sure.

D: Ok. Just think about how you did or how you found math before we started this because you guys were pretty well in rows and then after we started it, did it make a difference in terms of how you feel you understood what was going on?

V: Yeah, I understood it better.

D: You understood it better?

V: Yeah.

D: When you were with the group?

V: Uh huh.

D: Why do you think you understood it better that way?

2:00

V: I don't know, just ideas from other people and helping.

D: Was this experience more difficult or stressful or less difficult or less stressful because of the way it was presented to you? Do you think you know just thinking about coming to Math class, knowing that you're going to be in a group, make any difference about how you felt about coming to Math class?

V: No.

D: Didn't make any difference?

V: No.

D: You didn't see any, you know, say "Oh yeah, so we're in a group today so that means...anything"?

V: No just the only difficult part was trying to figure out the answers.

D: Ok. Do you remember much about what you learned in the Coordinate Geometry?

V: *(hesitates)* No.

D: Don't remember? Ok. I was going to ask the others to kind of explain what they thought or remembered or whatever but because of time I decided not to but maybe when we meet the second time and do some problem solving I might address that a little bit then.

V: Ok.

3:00 **D:** How do you feel your problem solving ability was affected by this experience, if at all? Because that's what it was, right? Miss didn't tell you how to do anything, she just said here's some sheets, you figure it out and you guys had to work together to come up with the answers. Do you think that made you look at problems differently or affected your ability to solve problems?

V: Yeah I think it made me look at it better because we had more time and *we* had to figure it out without Miss telling us exactly what to do. Go right to it.

D: So how would that have made you a better problem solver then before because of that do you think?

V: I don't know.

D: You don't think that it had any negative effect on it, you just think

because you guys had to figure it out yourself that you look at math problems differently?

V: Yeah.

D: What do you think your role in that group was, did you think that you had a certain role or that you contributed in a certain way or you felt that it was your responsibility to do something in the group whenever you guys came up with something?

4:00 **V:** No, just like everyone worked together, no one had a particular role.

D: Ok. How do you think the group worked together? Do you think they worked together well?

V: Yeah.

D: Ok. In what regard? Why do you feel it went well?

V: It was good because everybody like helped each other and if you wanted, like one person would figure it out, they'd tell you and explain it.

D: Even though it was kind of a quiet group people didn't talk a whole lot in the group but you felt it still helped you to understand what's going on? Now I understand you guys are still working in groups?

V: Yeah.

D: And are you still with the same group?

V: No we picked our own groups.

5:00 **D:** And what group are you with now, you don't have to name names or anything but like are they people you worked with in the past or friends or people in your math class last year or just anybody?

V: No, before we started the group work, before all that I was with a couple of friends and now I'm in the same group again except for one

person.

D: Ok, so there's how many in the group then?

V: There's three.

D: There's three of you working in the group and you work the same way as you did when we did the unit on Coordinate Geometry? Does Miss explain it now on the board and then and you guys just practice it or kind of figure out the particular question that you're asked?

V: Yeah.

D: Ok, so it's a little bit different then?

V: Yeah.

D: How do you feel that's working?

V: Good.

D: Is it?

V: Uh huh.

D: You find it better than working on your own or find it better than working with the group you were with before?

V: I think that it's a bit better now where I'm used to Miss explaining it to us.

D: And do you think that the group you're with now makes a difference because they're friends?

6:00

V: No not really, I mean if one of us gets it wrong we tell the other one what they did wrong right, like what they did wrong or I have this and you have that and we'll explain it to them.

D: Ok so there's no difference in the group that you're with now really or

is there?

V: No.

D: No. Ok. Would you prefer working in a group all the time or would you prefer working by yourself or would you like a bit of both?

V: I think in a group is better.

D: You like working in a group better?

V: Yeah.

D: Ok why do you think overall no matter what group you're with or whatever, why do it make a difference with the group than working by yourself?

V: Because you get more ideas from other people and if you don't understand they'll explain it to you how to do it.

D: Ok. Do you like math?

V: Yeah.

D: Did you always like math?

V: Yeah I guess so.

7:00 **D:** These are just some questions I'm just trying to find out how you feel about it. How would you rate your problem solving ability? If I gave you a problem, let's say, and asked you to solve it, would you say you're a good problem solver, not so good, average, poor, haven't got a clue, what would you say?

V: Average.

D: About average?

V: Yeah.

D: Ok. If I gave you a problem to solve now would you solve it?

V: I don't know.

D: Would you try?

V: Yeah.

D: Ok. That's the problem there, you can read it, if you got any questions let me know and if there's something that, you want to double check and I'll ask you to try, if anything you can think of just say it out loud just so we can get it on tape just to find out how that old brain is ticking.

8:00

V: Oh, Ok. *(Starts to read the problem silently.)*

D: I'll just leave you to it and I won't look over your shoulder.

V: Ok. *(Continues working.)*

9:00 **D:** Any problems?

V: No.

D: Ok. What do you know about the problem, what are you told?

V: There are 18 chickens and pigs.

D: So there's 18 animals. I guess I should have said that. And...

V: There are 52 legs.

D: So you want to find out how many chickens and how many pigs right?

10:00 **V:** Is that 18 chickens and 18 pigs or both.

D: Eighteen animals altogether.

V: Altogether?

D: Yeah.

11:00 **V:** *(Tries the problem again.)*

D: Just maybe take a guess at how many just to see if that might help you to solve it. Guess how many pigs and how many chickens.

V: I tried that there - nine and nine.

D: And that didn't work out?

V: No cause the number is higher than 52.

D: Ok, so that was the number of chickens was it? That was the number of pigs was it?

V: And that's the number of chickens.

12:00 **D:** Ok so maybe if nine and nine didn't work maybe try adjusting one of them to see if it did work maybe that may help.

V: Ok. *(Tries again.)*

D: Maybe try in between those two because you went from nine up to eleven.

V: Uh huh.

13:00 **D:** When you did it you started bringing these numbers up, the pig numbers up, you went from nine to eleven to ten and the chicken numbers went down.

V: So I'll try the chicken one going up now.

14:00 **D:** Ok. . . so you got an answer?

V: Yeah, there are 32 pigs and 20 chickens.

D: Ah, there are only 18 animals altogether. You're so close, that was how many legs they had right cause you multiplied that by 4 and that by 2.

15:00 **V:** 8 pigs and 10 chickens.

D: Right on, good job. Ok. Is that the only answer you can think of or might there be another possible solution?

V: No. I think that would be it.

D: You think that would be it, Ok, good, good job. I'm going to turn the tape off now but before I do I want to thank you for giving your time today and letting me interview you and see how you're thinking about things. Ok?

16:00 **V:** No problem, you're welcome.

Appendix D Interview with Teacher

Interview with Ms. Gwen Hewitt **April 16, 1996**

0:00 **D:** First of all I'd ask you about the experience in general if you could remember back then. Overall positive from your point of view?

G: Yeah, I thought it was, I well I guess you'd have to say it was positive because I'd like to do it again in another unit. I would not want to plan it myself because I think it's a really demanding task to plan it, you know, you have to be real careful about the kind of questioning.

D: Sure, oh yeah.

G: And if you're going to lead them or have them investigate. So I won't take upon myself to plan a unit but I certainly would like to have a unit done in conjunction with somebody else. Yeah, I thought it was really valuable but I found it really tiring for me and I said that a dozen times. Real stressful just because it was a totally different position as a teacher than one we occupy.

1:00 **D:** That's right.

G: And it was exhausting partly because you held yourself back from telling things. I was always dealing with how shall I say this, how shall I put this and I just found it really wearing like that. Generally, I liked it generally. I would like to do more of it, I tell you I guess the thing that I found good about it - the focus was on the students and not on me and I was really surprised how much they knew even though it wasn't solid, like you went around you discovered things that they knew, and I think that was one of the downfalls of it, it didn't ever get, what's the word, not reinforced, but didn't seem to sit firmly with them. But they were coming up with all the things and I was real surprised. I didn't expect them to know as much as they did. Or to glean as much as they did.

D: I know, they really kept on task throughout. I mean they really did get into it.

2:00 **G:** But it was disappointing when I saw the tests and saw how poorly some

of them, well I shouldn't say poorly, but how much they dropped back from the knowledge that I thought they had because and I think that is something to say about them and their study habits but also about the you know, like it wasn't firmly grounded.

D: How do you feel students' approach to problem solving has changed in any way, if at all, because of this experience?

3:00 **G:** Long term wise, since then?

D: Yeah.

G: I don't know if their approach has changed much more because I mean one month, from one month's activities to switch over back what their norm is, I don't know if you can expect a long term change because there are lots of things I have reverted back to, you know.

D: But even during the course of the study?

G: During the course of the study?

D: Yeah.

G: Ah, gosh first I thought they were really...how did I see them? I wrote down a couple of things. In the beginning, they were all concerned about using the right language and they were more concerned about how to state it in some way that was wonderful. They weren't looking at that...they thinking they were looking for something that was technical, had to be said in the right way. And as far as their problem solving, I think that they were waiting for me to give the answers because the first couple of days I could feel the frustration level mounting with me because what is this when they were used to me
4:00 telling them and I didn't tell them and I'd send them back to it and some of them didn't like that because that's not the way the game gets played. So that changed. Ah, I think after we got into it after three of four days I remember at one point writing down "I don't know if she's going to blow...soon" if they're going to get real...cause they were getting agitated with me after a couple of days and I found it hard to watch their confusion and not try to fix it too. After a while they started

to...where else did they have to look if I wasn't going to give in and once they got a success with it, just one little bit of success, then it was ok we go on, now we go on. And maybe there was something there, now whether it was all trial and error type of problem solving. Once they got the hang of something successful out of this then they were willing to do further with it. That's how I saw it anyway.

5:00 **D:** Now you kind of answered this one before, but I'll ask it anyway in case there is something you want to add...How did you feel this experience has affected your teaching in general? Again, you kind of touched on it before but there is something you might have registered because of this experience that has affected you since.

G: Ah, how did it affect me? I guess...I don't think they...they are pretty weak the students are, but I think I've gained a little bit of respect for the fact that they do think much better I think they can think. Unfortunately I still think a lot of their problem is a lot of the students problems comes from the fact that they don't put enough effort in. That whole study made me realize there is knowledge in them and we're just not getting them to express it. I don't even know if they know it, so that
6:00 gave me a little better feeling because I didn't feel everything was so futile in terms of what I was teaching, it sort of enlightened me a bit about their capabilities and I liked some of the things I saw from the group working so I carried it over. Just the business of having students working together and dumping something on them and saying now you figure this out. I still do a little bit from time to time, not on new topics and things, just for instance, something from review which isn't, I suppose the same principal. I like the idea of them working together. When you start talking about it, then it's like they learn more from talking to somebody else. There are students who are doing much better this term, like really better. One that was failing last term, 30, 40,
7:00 just couldn't get it up to the 50 mark and he's had more 70's and 80's in the past two months and it's because he's channelled into a different way for thinking. From my perspective it certainly made me feel like pursuing some of it, the activities, because I can see results. The group work, I pursued that still.

D: Compared to other topics that you've done in 1300 either before or since, do you feel that the Coordinate Geometry was understood any differently by your students because of the experience. Do you think

that they understood it more so then memorize it?

G: I think they needed more practice with it. We always say, "Well if you discover it...". I don' know, we let them. I mean they had a minimum practice compared to what the other classes had. I think they do have some genuine understanding a little better then the other groups but the practice part of it, I noticed when I gave another test awhile later I brought some questions back from that unit and it was gone again. It was there but it wasn't the same. They didn't have the same amount, like it was bits and pieces of that knowledge remaining. They still didn't have all of it pulled together and I thought they could have used a bit more practice. First I thought once you discovered it, it's there but from follow up they still need a bit more practice if it's just to make it sit in their own conscious memory. Some of it wasn't conscious memory for them. They were doing things and weren't really looking at it and summarizing what they had done on their own, even though they tried to do it with them. I don't think that it was truly recognition on their part.

8:00

9:00

D: In that unit you tended to look at isolated things. I don't think you even got to the point of pulling it all together other than the test at the end of it. Some of that might have made a difference.

G: I do think a lot more practice skills because there's a lot of little things that came out of there you know that I would touch on here and there. Weeks after I'd say, remember this, remember we did that, that came out of this so there was a lot more things that were beyond that.

10:00 **D:** I asked you this before and you kind of answered it, would you take this approach again in the future with other groups?

G: Yeah I liked it and I wouldn't mind doing some work like this with advanced Math but I would like to do it on an enrichment topic because then you could whip through some of the enrichment things very quickly. I don't know why the enrichment but I think it would be a real nice addition to that group and you could move fairly quickly, so I thought that would be something because a lot of the enrichment topics you don't get to simply for time and with advanced groups, my advanced groups do discuss a lot anyway and even though they weren't the group that I did this with they're all in groups now by virtue of the

11:00

fact that the classroom is in groups. All these kids, I've seen a whole lot of difference in working together. There's three's and four's that have sent themselves up some of them have moved off to another group and there's pockets of four's that are really working well together and they have a real system going between them. I think that if you gave them a project now even at this point after they've been working slipshod together and you delineated this is what you have to do, they would fly because they have a little bit of time just working. I think it would be good for basic math groups too. Simply because to me the basic groups want to be the problem solvers. They want to get in it and look at it. They want you to give them rules and regulations but if you give them anything on the board that says, I'd call it fun, and they have to think about it or they got to size it up and come up with something. That's the crowd that I find the best. I give then the NCTM questions, you know these monthly calendars, and these are the kids that say, "Come on give us another one." Now I have to choose them because the basic math students don't have a lot of the skills but they seem to be the ones that discuss them. It amazes me because to me that's the group that wants the rules and argues "Just give me the rule, tell me what I got to do and I'll do it". Yet they're the ones that seem to, their minds or their interest is piqued by all these different things so I think if you could get them and work it in slowly you know on a project then if you through something at them that they really couldn't cope with then you'd lose them pretty fast. We've talked about doing it with the basic class in a topic next year.

12:00 **D:** I was going to ask you some questions now on the focus group, Valerie, Heather, Bob, Sean and Jeff were the focus group even though I only spoke to three of them afterwards, Valerie, Heather and Bob. Sean and Jeff never materialized. My interest in all of this is how they felt they worked together and working in a group as opposed to doing their own thing. It was kind of interesting when I interviewed them and so on afterwards. I'd just like to hear what you got to say about some of the things they answered. If you can remember the group and I know it's hard, to pick out one group and talk about it. What were your overall perceptions about how this group worked together? Anything that you noticed about them working as a group maybe even compared to now when they're working with others?

13:00 **G:** I remember that group, Heather kept herself very distant. She's quiet

anyway, I don't know if language is the factor. She's almost anti-social with the group, a pleasant girl, but didn't understand the concept of working with a group. If she did understand it she was either too shy or wasn't willing to play that game. Bob, you know it's funny, I think Bob is much better working now than he was with the group because he was very unsure of himself in his own abilities and a couple of times I remember going around and saying "Well tell me what you got?" and it was like, no she's figuring it out and you got it right. If I told him that much well it's fine, go with it, or something to that effect I think happened one day. It's kind of hard for me to go back and see it at that point and now. At that point I don't think he was contributing as much but he's become a whole lot different in contributing since then. I think that has something to do with it.

14:00

D: I was going to ask you if you felt it was a worthwhile experience for the students?

G: I do think that there were two people in the group who didn't because of the fact that they were in and out, in and out. It made others uncomfortable because they all didn't know each other and Sean's coming in and out and when he came in, well you sat there with them. I don't know but I had the general feeling that when he's here he decides he's going to do his thing. It's almost like that peer pressure of you don't want to tell him to stop bugging you but it's sort of like everyone

15:00

just draws back a little bit because he isn't doing what you're doing, he isn't playing the game and they've been doing this for a couple of days and they wouldn't dear speak and say, "Look get in or get out." He's towing them away because they almost feel intimidated or something by his behaviour and don't speak out. Jeff, he's very bright. When he was with the group he was very contributive but his absenteeism! He just doesn't have the desire.

D: Is he still as absent as much?

G: As much? I think he's been here twice, three times since January. You can only fight that battle so much. That's unfortunate for that group because that made that group shaky and I think that happens when

16:00

somebody's coming in and out all the time, it doesn't give the group a comfortable feeling. They don't build up habits of, you do this and I'll do that, you know the discussion isn't there. As soon as a new party is

in the discussion is obviously curtailed just a little bit. You always got that awkwardness.

D: I remember one class I think it was when Sean showed up after being out for about six or seven periods and it was just total disruption. I must say I was really surprised at how much everyone was set back by just this one person coming in and not knowing what's going on.

G: And another thing, there's been some discipline problems with him and he's made it a point to be antagonistic with me and he works it. So I would think that they were probably a bit anxious about what he was going to do and draw them into it. We haven't had a very good rapport
17:00 all year and it doesn't matter what's happened, it doesn't matter what I do, it always turns out the same, there is always some way now it's gotten more and more blatant. He doesn't come to school any more. Two weeks, three weeks, that's unfortunate you know. I think they sensed that, they knew that was an issue. He would be belligerent no matter what you asked. If he asked for help and I went to give it to him I would hear the comments, snarky remarks as I walked away so that so made some of the kids uncomfortable. Not in the group I know it wasn't happening at that time but I'm just saying that kind of background was there. For Heather that would be major intimidation, her culture, her manner certainly would not create waves like that. She's very quiet. She's a good worker but unfortunately I don't think she knows too much about cooperative work.

D: When I interviewed Heather she said she enjoyed the experience, she found it worthwhile. When I asked if they preferred you just taught the unit or this way she was the only one who said she would prefer if you
18:00 teach it because that's what she's used to. She's only been here less than a year and she said where she came from teachers don't do that it, it was up to you and you had to learn it. So she would have preferred that which I found interesting.

G: But in the same breath I noticed that as we were going through the work she would come in with it written up, and reasons and explanations, real text book explanations and I said where did you come up with this? She said Sandy. You see Sandy had been in one of my advanced classes and had already done this work so Sandy was just rhyming it off the reasons from the text and she was just memorizing it. She learned the rules and applied them. I could see she didn't want to do it. She

wanted a set method to follow. She was a bit more rigid than the rest in that group.

19:00 **D:** Even the language was a bit of a problem when I gave them a couple of problems to solve, the things you take for granted like jar, Valerie had a problem explaining the terms to her. She couldn't get past; she couldn't even approach the problem because she didn't know what the problem was asking her. We tend to take for granted that they know what we're talking about.

G: Did she have her translator with her?

D: No I don't think so.

G: She has a computerized translator with her and just uses the word. When we have exams and class tests she hasn't had a problem getting things done. Lately now we're working on proofs and she hasn't had too much of a problem yet. She will be given extra time in an exam to verify the meanings of words. In group work it could be a pain because you can't work right away. It takes her longer to interpret.

20:00

D: What about Valerie, we never said much about her?

G: Valerie and Maria are working together still in a group and Valerie, now they worked together before and I let them. Valerie has really come along, when I say that I know I can't tell you what her marks are but I can see a big difference in the questioning that she does to me. The understanding and her responses in the class, she is responsive whereas before she sat back and never opened her mouth from September to December. So it definitely has done something for her. I think she has got a new confidence in herself because she sits there and they'll ask me to come done and she'll say "See I said that" or someone will say "You said that Valerie" and Valerie had it done that way and just held it off to the side. She's still not confident but she's explaining and they verified at times that's what Valerie said, that's what Valerie's explanation was. I do find that she has blossomed in that respect. The discussion aspect seems to get her because she didn't seem to do that well at the time of the group.

21:00

D: No she was pretty quiet. She didn't really offer much.

G: Now she's in a different group and there's three girls but they even get up and go around the room, which is not where I would have seen her. She and Maria work real well together.

D: Was Maria in that group? Was that her?

G: Oh that's right. I thought she was in the same group. She was up in another one. Yeah, you're right.

D: Did you see any of them in that particular group have an individual role. Did you think they any of them took on an individual role when it came to solving a problem that one of them was the reader or the recorder or whatever?

22:00 **G:** No I didn't see much of that. Actually what I saw was the one that knew how to do it became the all and that's probably because we didn't take the time to teach them. I'm surprised at how little they know about cooperating and literally cooperating. You can write it down, all hands can be attentive but they all don't take an active role, they don't have the task that they think they have to do. They left it to Heather to work it out and Heather crawled inside herself, worked it out and handed it over. There was no more communication than that and some of that I think was not language related because she could have got to work with somebody it was pretty much telling how to do it and they kind of knew that she was good so let her go and didn't trust themselves. I don't think there was a whole lot of inter-play. Again that was more at the beginning.

23:00 **D:** No, I got to say I could totally concur what you are saying. I noticed that even when I was interviewing them and I happened to have pairs, like I had Valerie and Heather one time then I had Bob and Heather another time and then the three of them and even then I noticed the same kind of behaviour they would just do it themselves they had no sense of cooperating or sharing their ideas or even questioning what's going on. They always start with their own little thing and then compared the answer that they might have had.

G: When I say that they're starting to work in groups they're doing that now. They'll sit and someone might say, "How would you go about it?" and they might discuss a few things to get them started and they'll go their own way and come back and I still see that. Even though that's a bit more group work than they were doing before. Cause before it was show me you're answer when you get it done but now there's a little bit...

24:00

D: Which I guess goes to show probably the most interesting thing I've seen is you'll put them in groups and assume they'll work; that they got some sense of what are you doing or what do you think of this or whatever, but they tended to do their own little thing and the answer is what they think important. Even when I interviewed them they said you know checking the answer and it's not the process of what you do to get the answer or how you go about it.

G: It's answer oriented I will grant you and some of them like this morning, now they're on a kick, they all want to do it the same way in a group, that's the latest thing I've noticed. They started working together and thinking when they were in the project, following through on a solution, when they started to work together the ideas were coming. Now they're at the point where if one person's idea is out there, ok, we'll go with that one and pursue it until we get the answer that way.

25:00

D: So they tend not to question it, they tend to still accept it?

G: Yeah. Whoever starts it off it doesn't matter, individual thinking. It's not even like thinking, mainly it's doing. They don't sit back and think about this for a few minutes then talk about it. It's think and do. If somebody could talk about it long enough they'd understand what that person was saying but they're so conscious about doing. Putting things on paper, you know?

D: Getting something to show you that they've done something.

G: The doing of it, there's times that somebody will say something and it goes right on nobody sees it or hears it or anything else. To me that was

the most valuable thing that was said there, it might even give you a short cut way about going about this but because it was verbal and not something that was shown as a step on paper they just ignored it.

26:00 **D:** I find that an eye opening thing for me because you tend to assume that you put them in a group and they're talking, lots of times they may be talking about something but it's not necessarily what they should be talking about or the best way a group should work.

G: I do think that's one big thing. I don't know but I wouldn't mind taking a few days, if I was to put them in groups next year, do a couple of days of kind of behaviour that should take place in groups.

D: Even since I've gone back and I've only been back two weeks now but I guess I wasn't really aware of it before never really thought about it but now I notice when they're in their little group they tend to do the same thing. It's like getting the answer and whoever offers something first, go with that. Very infrequently do they challenge themselves.

27:00 **G:** I was just going to say it's not even "I'll work it out and you work it out and we'll compare". It's like whoever gets it first that's what we're going with. Easy calculation and as quick as we can.

D: And I noticed it when they were doing their group test, there was no kind of strategy at all. Nothing like "you two do question one, we'll do question two." They didn't even consider time. I remember they just went on and however long it took them to do anything and then they realized they had forgotten a question at the end of it and went back to question two and there was no time left. They had not planned on giving five minutes at the end of it.

G: You're right. There definitely isn't any planning. Now I didn't get to watch them all closely. I would walk in and see things moving or somebody would push somebody to move but I didn't get to watch the sequence of events. But you're right I don't think was very much planning- I know there wasn't very much planning. That's not the downfall of this project, that's the downfall of not knowing how to work cooperatively. I don't know but we would have seen a whole lot of different results maybe further along cause they don't really know what's expected of them. We can say talk about it, but what does talk

28:00

about it mean? Choose somebody to write but it's almost like "Ok she writes but I don't write anything. I don't ever put the pen to the paper". That I think was the biggest flaw, if you want to call it a flaw, because they just did not know.

D: We tend to assume that they can do it when....

G: You would think that they would want to work with their peers like that, but that's not real comfortable.

D: That's right especially if it's a group that weren't real familiar with each other. I think it was Bob who commented that he wouldn't want to be in a group with only his friends because he said they wouldn't do anything but he wouldn't want to be in a group where he didn't know anybody cause he felt he would be too held back by that barrier. Where he didn't know the people well enough to actually talk about it so it's a fine line between who you put them with and who they know.

29:00 **G:** That's right. Strangely enough some of these people have regrouped themselves with the oddest compatriots. I can't believe it. Bob was not a great student up to that point but has kept himself with one other person (Stephen) in a group and a fairly smart person. They get into to a bit of trouble with me now and then because they talk so much but they discuss and a couple of tests his come out with hundreds since then. So there's something bouncing off him and this other person now whether the other person doesn't seem to be held back but I think there's something bouncing back and forth then. He's obviously comfortable enough to get something from the other person, he's not intimidated. That other person knows a whole lot more then he does but there's no intimidation there, so something is taking place.

D: He did comment that he felt pretty good now about what was going on. I guess since back in February he has been working with this same
30:00 person and he said the group he was with he felt pretty good about and felt comfortable. I would like to see them work with different people to see how it would go.

G: And there is a big difference. I think some of it was sparked by that. We didn't just ask them to work in groups and do the same kind of work they've always done. We asked them to work in groups and do

totally different method of learning, studying, thrown at them all at the one time and I don't know if they had time to adjust to any of it before they were plunged in. That's why I felt like things went so well. Considering what I was expecting and what I was seeing my nerves were thinking this wasn't going to go but they're not going to take this and because of all the newness of it, you know you throw that much change a someone that quickly, and I thought that they really did cooperate with me.

31:00 **D:** But even like my presence in that group I'm sure had I not been there they might have felt more free to talk about things and not so afraid to say something wrong because quite often they would say "Is that right Miss?" Sometimes I couldn't be quiet and I'd say things but I tried to stay back as much as I could but they tended to first check it out with me. Whereas if I wasn't there they'd only have each other to discuss things with. I must say, it was eye-opening for me to sit down cause I'm like you, when I'm teaching I don't sit with one group and watch them from beginning to end and see how they get that and what they do and so on. You tend to skip around to everybody and keep everybody on track. It was interesting to see. I was still noticing even when I did the problem solving up in that room afterwards how long the silence went on. I gave them a problem that they had problems with it that they couldn't get at first. I gave them help with it and they didn't discuss anything they just read their question and just sat there. Finally after about five minutes I felt like I had to throw something at them to get them thinking. The group that they're with is really tricky. How do
32:00 you know who they're going to work best with?

G: I think at this level there was a split of friends at that time and they channelled themselves right back together since then and yes they get on my nerves for talking but they get more work done. You know there's four buddies together and they're getting more work done. A couple of them in the group are spinning right along and the other couple are leaning on the others and I've said to them, "This is not fair, not to me, it's not fair to him because his taking what you have and you're letting him do it. If you're going to give it to him at least explain it to him." So there's some of that happening there but they have set themselves off in groups and some of them move around a little bit but not very much. They've put themselves in a group and stayed there.

33:00

D: Are you surprised with who, I know you said there were a few strange combinations, they came up with?

G: Well Bob and Sean were one strange one that I couldn't believe got together. He still sits in on that group. When he comes you can see the tension and the disruption there. Not that they make a racket but he does, he sits and talks through the whole class. Actually he came in last week and did it and I asked him to stop and I said, "If you don't stop I'll ask you to leave and not sit with this group because I'm not having you disrupt what's become a really good thing." I said to Bob and Stephen "If he is bugging tell him to shut up and leave you alone". With that two of them said, "Shut up and leave us alone." Before I had it out of my mouth it was like they just wanted the opportunity to say bug off and give us a break. I gave them the permission to say it, it got a laugh out of the class but the message was there. I wasn't expecting it when I said tell him to shut up and leave you alone and with that the two of them turned in unison and said it. The message went out loud and clear, you're not going to play this, we don't want you around, and we're doing the work. See they would never say that one on one to him.

34:00

D: Something like that could make such a big difference, a hindrance or help in such small settings. There are so many factors to consider when you're in a group and working with somebody else. It's human nature, if you're working with a group you almost rather work with people you get along with not somebody who's going to make you feel like two cents every time you say something. You certainly don't mind having a laugh or having a chat anyway as you're working but I guess we tend to think that they're working and that's it don't have any comments outside or they're not being productive.

35:00

G: They sit there and I know they talk about everything else and I'll get on them every now and then and they'll say Miss we got that done we're almost to the end we're just talking, we got the hard part done this is only easy what we're doing and they're talking through that. Fair enough. There was such high absenteeism in that class. There's four that I had split up because they were friends and they've put themselves back together and they work much better. So what can I say? They're buddies.

D: Have you seen anything since that they may have developed some kind of pattern that they take roles on themselves in their particular group not if they've chosen their own group but if they're generally working with the same people? Is there anything with any of the groups that you've noticed?

36:00 **G:** I think they've become leaders. One of my weakest students has become a leader in the group and he's changed. You know I can say there's one person I have seen change more than anybody and that's Craig. Craig has become analytical. He's looking for the problem in the problem, you know what I mean? Before it was all a mystery to him but now he's looking at it like I can figure this out, there's something in here that I know. He's always taking that attitude, he's on a roll, and he can't get enough homework or assignments. Every week he asks are you going to take a quiz, are you going to take my book in? It's real encouraging with him but for some reason he's connected in to something. Whatever it is that has challenged him to think?

D: Actually there was one time when the two girls were here and three of the guys weren't and they got moved over with these two (Craig and Tom) and I noticed even a difference then because it was kind of a sense of well, "How we gonna do this?". That was kind of the introductory comment from him or the other fellow that was there.

37:00 **G:** They do talk it through and they'll argue you can't do that but the only thing is they'll all start saying what are you writing down. Today I was asking them to do something for me and I said you can't all write down the same thing you got to do a variation on it so then they helped each other come up with an equation. So there's definitely cooperation, I'm not sure if it's a product by the book kind of thing but there's definitely leaders. Valerie has become more of a leader in some ways in the group that she's in. I don't think Maria has the algebra skills, Valerie doesn't seem to make the same mistakes but that's a different group, Valerie has moved elsewhere and sort of taken a new role. Bob has come right out of his shell.

D: It's like a positive experience then afterwards even then.

G: Whether it broke the ice in the room or something I don't know but

there were a lot of really weak students in here. I thought they were really weak and they're still really weak but I've seen some lot of improved work and getting work done.

38:00 **D:** I should ask you about the attitude because I know generally in Math 1300 it's not really high on their priorities of getting their work done or being that interested in it.

G: Positive as far as in the class and working I can't ask for a lot more, they're pretty good like that. Homework is not a joy they don't like to do it generally as a group. They don't kill themselves on the study aspect and I think that's where it falls down. Those kids did all that work in class with the project but I don't know how much studying they did on their own, how much they took that home and wondered about it. That's a flaw that we generally got anyway. That never changes; I don't think that's going to change with any one project or major event.

D: Yeah, homework is a big issue, isn't it?

39:00 **G:** I think one thing that struck me about it was it took us a long time to get through a relatively small amount of work. We didn't do a lot of practice and I think if I did it again I would like to have a whole lot more practice outside just so it becomes not a game of we did this in class we don't have anything to do at home. So I'd like to build something in to it whether it's assignments or whatnot but again we were on a short term project so if you're doing it for a unit of work you'd have a lot more thinking, you'd build things in. But I do think that aspect of it has to be there to be taking seriously somewhat even if just for that. The other side of it in the practice element needs to be there.

D: Yeah. I was going to say I noticed too when we were doing it homework wasn't offered and it was the nature of the way it was designed it wasn't built in to it.

G: If we could build it in to individual practice then come back tomorrow and discuss your individual practice even that sort of thing. Somehow I just think that it takes away a little bit of the seriousness of it because I set homework every night with other things now all of a sudden it's

changed. Another thing was...

D: While you're thinking I'll ask you another question. You said about the pace that it took so long. Do you find the pace now more back on track? Now even though they are still working in groups do you find that they've picked up?

G: Yeah but I'm directing the pace. I thought about it and I thought, I
40:00 wonder would we be better off chucking, throwing out some of the topics that we did. If we were doing this type of teaching every unit, you know time is one of your biggest problems and it's really played by ear in a unit like this. I thought if we were to decide there were two or three units we were going to do in the year that are exploratory, investigative or problem solving or what ever you want to call it, then I thought if we were to do that would we not be better off to take a smaller range of things to cover. I think there was a certain amount of comfortableness with some of the things we did definitely this group got that the other group didn't get. Neither group has stayed with it for a very long time. I think we need to practice with the study group to get it reinforced again but there are more things in that unit that they didn't spend a lot of time on that they're better on than the other crowd. I got a feeling it's because of the way it was done. I just think if we chopped
41:00 out some of the topics and went more for everybody get these done really well. That gives you a whole great place to start another year. You're not going in with a whole miss mash fifteen or twenty items, objectives and each one piddly, a shaky understanding. If we took six out of the fifteen and they went in next year in twenty minutes into a class say these kids are pretty solid, in this what a way to go I mean what a nice place to start. You know that they got a firm ground in this, they're comfortable and you're comfortable to go on and start new. What we do every year there's a repetition, add a bit more and add a bit more but I wonder if you could almost do a quick review, a rerun through it.

D: When they first learned it if they had to have had a solid grasp of it and an understanding of what it was because they do a fair bit of it in grade nine. Even though it's review for a lot of them it's like the first time they've seen it.

G: It doesn't matter to them and that's the strange thing about it. This group seemed to me even later on when I tested there's a lot more things that they have understanding of whether they mastered it is

42:00 another story but they had better understanding, a better grip on it then the other class. The other students I taught I would have to say remember this and they're looking at me and then I'd have to take it a step further and give them another hint and tell them something else. With this group I did have to give them some hints but I found they could come back and tell me odds and ends, different things and yes we remember this and can't you do this and can't you do something else they offered so to me was much more beneficial than I thought. That's why I felt like I still got to pry it out so they're not real good with it but if they had more practice maybe they'd get better at it.

D: Last thing, they're writing. You did get them to do a few journal entries. Any improvement in their writing skills you think because of that or still a foreign concept?

G: I was just going to say I don't think you could see a whole lot of difference in writing from beginning to end. I think there were two I gave and that's too short a period to see any growth in writing at least by my perspective.

43:00 **D:** Sure and they haven't done much since then have they?

G: No we haven't done journal entries. I've asked them to do a few things on the board since then but not explanatory things on paper. They do this now tell me why this is so, what you did and why you did it. That's fine orally but putting it on paper is a whole different ball game. I was going to say before I think there was a lot more on task behaviour on their part. I don't know if you know if you noticed a couple of times I wrote it done students tell me "my brain is tired, let's take a break". If you think about it, we were always passing this out, now you got this one done, here's another one. Even though they got on a roll and wanted it after a while I could see this was real demanding and it's tiring for them. Somebody said one day, "Miss my brain is tired. Can we take a break?" The on task behaviour killed them because they were not used to it. They screw around a lot of time in class and when that project was on the go I don't know if it was competition because we got to get on the sheet or they're going to get on ahead of us. What are we

doing wrong, they're getting ahead of us? There was some of this little bit of a rush; we figured this out let's get on to the next one. Once they got going on it I saw that. Then I saw a couple of times students were getting tired.

D: Well they were kept going. I know lots of times there is dead time in math class where students might be tapping their fingers waiting for other students to finish but they (the students in this class) were kept going.

44:00 **G:** When we finished off one thing even if there was ten minutes left I gave them the sheet to go on and take home tonight well they'd get started on it. So they didn't have any break and I kind of liked that aspect of it the fact that there was so much time on task. It was just a nice pleasant surprise for math class.

D: That's the end of my questions. Is there anything else you want to comment on that you never asked? Well, I must say I really appreciate your time.

G: Was that ok? Did that give you what you need?

D: I think so. I've got to start thinking about it all now and start writing it.

G: If you want to phone me and alter anything or ask me anything you can do that anytime you come up with something.

Appendix E Transcript of Problem Solving Sessions

Session #1 Heather and Valerie **April 2, 1996**

Problem: *Aaron collects lizards, beetles, and worms. He has more worms than lizards and beetles together. Altogether in the collection there are twelve heads and twenty-six legs. How many lizards does Aaron have?*

0:00 **H:** What does this mean?

V: What?

H: What does this mean?

V: Ok, the first sentence? Where he collects them, say like you collect teddy bears or something like that. He collects lizards, beetles and insects, he has them all.

V: He has three kinds of things?

1:00 **H:** What's this, what's this, what's this?

V: That's all different types.

H: Is it kind of animal?

V: Yeah it's a reptile. It's all different kinds of animals like bugs and stuff.

H: Do you know how many heads and legs they have?

V: Twelve heads and twenty-six legs.

H: Each of them.

V: Altogether, like all of them together.

H: Each of them, like how many heads? I mean each one.

2:00 **V:** How many heads does each one have? Miss, is that just like one or say three of them got four or five each?

D: Well you got to figure out how many of each he's got. There's 12 heads altogether and 26 legs. So you don't know how many of each he has.

V: Do you understand?

H: No. *(Laughs)* Like how many heads and legs does each one have?

V: That's what we got to figure out. Say this one has four and this one has five and this one has three.

3:00 **D:** I think Helen is a asking how many heads each lizard has and how many legs each lizard has.

H: Do you know how many heads and legs they have?

4:00 **D:** I guess we can assume that each one of them has one head. And a lizard would probably have how many legs?

V: Two or four. Yeah, four.

D: And a beetle would have how many legs?

V/H: Four.

D: Six, actually.

5:00 **V:** Worms don't have any legs they just slither *(laughs)*.

(The students work silently.)

7:00 **D:** Why don't you check with each other and see if you're making any progress?

V: Well if there's 26 legs, we got to figure out how many legs a worm got. A worm doesn't have any legs.

D: That's right.

V: So there's 13 legs. Not including the worm. Like divided by the two of them. That's what I got. They all have heads. Divide all them by three and you have four.

H: Each lizard has four legs and each beetle has six? Altogether there is twenty-six legs.

(The students continue to work independently.)

V: Where did you get fourteen to?

11:00

H: This minus this. Oh. *(Starts to erase what she had written.)*

D: You're looking at it with x's and y's and z's. Ok. Because of time maybe if we get a chance in a minute we'll look at this one. Would you mind if I gave you another problem? We'll just leave that one for now and I'll come back to it just because I want to see you do a couple of different ones.

Problem: *How can you carry exactly four litres of water from a river if only a three litre jug and a five litre jug are available?*

13:00 **D:** Try this one.

V: I seen a problem like this on a movie. Something like that on a movie.

D: Oh, Ok.

H: What's that? Jug?

V: Jug? Like a bottle.

H: Oh. What's "available"?

V: Like you can use them, they're there. A three litre jug and a five litre jug are there for you.

H: Are there for you? You already have?

V: Yeah, you already have them there.

H: What does this mean? You have a three litre and five litre?

V: That's how much it can hold.

14:00 **H:** So you can hold like eight litre?

V: Ok, yeah, but you want four.

D: Maybe if you just explain it to her she might understand it more.

V: Ok. You got to take four litres of water from a river and you only got three litres and a five litre but you can use two of these to get exactly four litres.

H: So you get two cans of water. One is three litres and one is four litres.

15:00 **V:** Yeah, you got to figure out a way to carry four.

V: Well if you pour two litres from these two and keep them.

H: Three x plus five x equals four.

18:00 **V:** Well, do it whatever way you want I guess.

V: So you can get eight litres no problem. How can you four?

H: Say you put two in here and two in here. You got two bottles and one is three litre one is five litre.

V: You got to carry four litres. Um, you could throw one away.

(Bell rings.)

D: Gee, we didn't get as far as we had hoped to.

00:00 **D:** I'm here with Heather and Bob now and hopefully Valerie will show up now in a minute. We are going to start with the problem that I gave Heather and Valerie yesterday about the lizards and that kind of thing and I'm going to ask you to have a look at it. Heather has already read it so she's probably a bit further ahead than Bob but you can just chat about whatever comes to mind.

B: How many legs do beetles have?

D: Beetles? Six. Feel free to talk about it.

(Students work silently.)

5:00 **B:** Got it! Seven worms, three beetles and two lizards. Worms got no legs and there's seven of them, three beetles, that comes to eighteen and two lizards with four legs comes to eight. Is that right?

D: Yeah, you're right.

B: Whoo hoo!

D: You want to explain that how you came up with that?

B: All I did was there was twelve altogether so worms had no legs so it didn't matter about worms. So I tried six and I just did it by trial and error. I just tried to two of these numbers to equal up to twenty-six. All the three numbers together had to equal up to twelve.

D: Good job, Ok.

B: Ching!

D: Does that make sense to you Heather, the explanation of it?

6:00 **H:** Yeah.

D: I'll leave that one then. Good stuff. Bob's on a roll.

B: I got the one from last time too. That's two in a row.

D: Now this is the one I gave out yesterday and we didn't get very far with this one at all. The only problem is the one you got is a little bit smeared so you might not make it out as well. This one I'd like you to talk about this one together and see if you can come up with.

Problem: *How can you carry exactly four litres of water from a river if only a three litre jug and a five litre jug are available?*

(Bob reads the problem aloud.)

7:00 **B:** This doesn't even make sense. You could take a five litre jug and fill it up four fifths of the way.

D: You want exactly four litres. There's no markings on the jug at all. All you got is a five litre jug and a three litre jug and you want to measure out exactly four.

H: So like half five and half three equals.

D: How would you measure exactly half? That's the problem, measuring so they're accurate.

B: You can't.

8:00 **D:** Can you use the combination of the two of them so you can somehow come up with it?

B: But it will never be exact. I don't see how that's possible.

9:00 **D:** Just think about it for a minute and see what you can come up with something.

B: I know. Fill up the five litre jug all the way, fill up the three litre jug all the way which leaves exactly two and throw that away. Ah, I'll do it again. You know what I'm saying? Ok you have a five litre jug, so fill this up all the way and dump it in here (into the three litre jug) and it will leave you exactly two litres and then do it again.

D: What would you do with the two litres?

B: The two litres?

D: Yeah, you don't have anywhere to put it. You are certainly on the right track.

B: Drink it. *(laughs)*

10:00 **D:** That won't work. *(laughs)* Talk it out cause Heather might have thought of something.

B: I can get three litres exactly but I don't know how to get the last litre.

11:00 **D:** Just say what you've gotten so far.

B: Fill up the five litre jug and you've got the three litre jug. So you dump the five litre jug in the three litre jug and you got exactly two left in the five litre jug.

D: Ok.

B: Um.

12:00 **D:** So this is a better one to keep cause it's two litres so maybe if you emptied that one, throw that one out, then what could you do with that two litres?

B: Put it in the other one.

D: Ok, put it in there.

B: Then fill this up again but then you got nowhere to dump it to get

exactly the same.

D: How much is in that one?

B: This one here is just one. That's what I figured out. I figured out how to get three.

H: You got two here, so you just do the same.

13:00 **D:** Ok, so the two that's in there you put into the three litre one so now you got two litres in there and it's a three litre jar and now that's empty right? So if you fill that one up again then you got five litres and you've got two in here, how much more can that one hold?

H/B: One.

D: So if you more that one over there and fill that one up how much have you got left over there?

H: Oh!

14:00 **B:** Ok. Yeah!

D: That's it. I'm sure there's other ways but you guys had it just about straighten out so it was just a matter of throwing it out at each other. I'm not gonna give you the third one, you guys got math right now. You got anything that... you got a test or anything. I wonder would Ms. Hewitt give me ten minutes of the three of yours' time. I would like to see, talk to the three of you at one time and give you one problem to try to see if the three of you, cause that was my intent to see a group of people as opposed to one or two. Maybe I'll see if she wouldn't mind doing that.

00:00 **D:** I'm here with the three of them now finally. Ms. Hewitt graciously allowed us to have ten or fifteen minutes of class time to get one more done anyway. I'm gonna start by putting Bob on the spot because Bob being the mathematical genius that he is solved both those problems. You want to explain it to Valerie who was not here lunch time. We don't want to give him a swelled head or anything hey.

B: Ok, this one here. There were twelve heads which meant there was twelve of them altogether, twenty-six legs. Worms have no legs so you can use worms for the numbers to all equal twelve. I fit the beetles and the lizards together to equal twenty-six and add on the number of worms.

1:00 **V:** Oh.

B: Three beetles is eighteen legs, two lizards comes to eight equals twenty-six and that's five, add on seven gives you twelve.

D: Both of you (Bob and Heather) can explain this because both of you got a handle on this one now. That's the other one about the water jugs.

B: Do you want to explain it?

H: No *(laughs)* you.

B: There is a five litre jug and a three litre jug. If you fill up the five litre jug and dump it in the three litre jug, it takes three out of this but leaves exactly two right?

V: Yeah.

B: So then you put ... what is it again?

H: You put the two litres into that one.

2:00 **B:** Yeah, put the two litres in there then you fill up this again and dump this in here with the two litres already in it. I took one off

the five litres here so it gives you four litres here.

V: Oh.

B: That's it.

D: Ok. I'm going give you one more problem. When you're doing this one pretend I'm not here, ok? Ignore me completely. Just discuss it among yourselves and see how you do and then I'll ask you some questions about it. There you go, have fun.

Problem: *The licence plate on my car contains five different digits. Although my brother installed it upside-down, it still shows a five-digit number. The only thing is, the new number is 63 783 more than the old number. What is the original licence plate number?*

(Students read the problem silently.)

5:00 **B:** I don't have a clue.

V: It's upside down.

B: No, this is how much more. How we supposed to find that out?

(Students work quietly.)

8:00 **B:** Do either of you know anything?

H: These numbers are upside down and like this is six and this nine maybe this number is a one?

B: Ok.

H: Is this right?

V: What, the answer you mean?

H: No, like ah, number upside down?

9:00 **D:** You've got to pick numbers that would look the same if they were turned upside down or look like a number if they were turned upside down. Ok, so that's certainly one thing you could consider.

D: Everybody understands the problem, do they? What you're asked to find?

V/B: Yeah.

11:00 **D:** Ok. A few minutes ago, Heather suggested that there are only certain numbers that you could use so what would be the numbers that would look the same upside down do you think?

B: Three ... six... and nine.

D: I think unless the threes looked like this, so they probably wouldn't look the same but the rest of them I would say would be ok. So now you work with that?

14:00 **V:** That didn't work. *(after trying for awhile)*

D: Any progress? Also keep in mind that 0, 1, 6, 8, and 9 are the only digits that the licence plate can have, right, as well as the upside down number and it's five digits. Bob got an idea do you?

B: I'm not sure.

V: So if you got... wait now.

D: I'm just going help you a little bit. Suppose that this is the original licence plate and add to that, what numbers... 63 783 you get the new licence plate which can only have those numbers again. So maybe you can start piecing in the digits that got to be.

H: Can you only have one number once?

16:00 **D:** You can have them more than once but they can only be those numbers cause there the only ones that look like numbers turned upside down.

H: Ok.

D: Does that make sense what I got written there? Does that help you come up with it? Maybe the three of you can just work on it together on this piece of paper. Which of these numbers when you add three to it will get one of these numbers?

B: Three...or six.

17:00 **B:** The first one is obvious. The first one has to be one. Oh, it could be zero.

H: This minus this is this?

D: So start here. What number would this have to be? Could it be zero? Zero plus three is three, three we can't have. Could it be a one?

B: No.

D: Could it be a six?

B: Yes.

D: Could it be an eight?

B: No...yeah.

D: Eight and three is eleven so that would be ok too, right? So it could be a six or an eight. Just take either one of them and could it be a nine?

18:00 **B:** No.

D: Do the same thing with the next digit and see if it works out.

B: Now it makes sense. It's just two for most of them.

D: One of you guys try the six and Valerie try the eight and see if it works out.

B: Tried zero. This has to be a one.

H: Yeah. What about this one?

B: It could be a six.

20:00 **D:** When a number that starts with zero that would leave you with a four digit number.

V: 2 6 7 8 and 3.

D: And that works?

B/V: Um, yeah.

21:00 **D:** But can one of the numbers be a three? It can only be those right?

V: These ones here?

D: If three was allowed like if it was one of those there it would look the same upside down. If the three was allowed then maybe there would be different answers but that's one way of doing it there's probably hundreds of other ways of doing it. Because of that snag probably go back and say try the six or try the eight.

B: So if you carry that wouldn't that be ten?

22:00 **V:** What happens when you carry?

H: That would be ok, just carry.

D: What other number could you use to add to seven to give you one of those?

V: I tried eight.

H: What number is that?

D: Six is the one that leads to having a zero in the front.

H: This one minus this is three. One minus eight is three.

D: The original licence plate would be this one and the new one is 63 783 more than that one.

23:00 **V:** You could use nine.

B: Yeah, nine.

V: Wait now, if you had a nine here, it would be six and six and three is ten carrying the one.

B: Oh, yeah.

V: I can't figure out what number goes here.

B: One.

24:00 **V:** Does that work?

B: Yup!

D: Good. Now I had to give you a hard one for the third one but three of your heads were there. What I'm really interested in is not any of these answers or problems whatever is just how you guys worked together as a group. Now before I let you go is there anything that you found about working in a group? I know you said when I interviewed you individually that you found working with other people helpful but in these three examples did being in a group help you?

B: It helped me.

D: How did it help you?

B: It's just easier when there's more people thinking.

25:00 **D:** But when you had the question first off, when you were first given this problem you all went on for several minutes and didn't even talk about it.

B: We didn't have any ideas. *(laughs)*

D: So if you don't have any ideas it doesn't make any difference if you are in a group or by yourselves?

B: Someone got to have the idea first.

V: To get started.

B: You can't join our brains together.

D: How did you feel about this problem when you first got it?

B: I didn't know where to start.

D: Ok. Did it throw you for a loop compared to the other two, for example, or to ones you've had in class?

B: Yes. You weren't given as much information.

26:00 **V:** This one was more confusing.

D: I just picked that one because it's interesting to me, even though I knew it would be more difficult. That's why I wanted the three of you together to have a look at that one. The other two you guys didn't have much time yesterday at lunch so you didn't really get into them. I'm mostly interested in how you feel about you know. Are you self-conscious about saying something that may be wrong?

V: No.

B: I'm not self-conscious at all. *(laughs)*

D: No, so it wouldn't bother you at all saying something that was completely wrong that people would laugh at?

B: I wouldn't care less.

D: Cause I noticed where you were so quiet I was wondering if you were afraid to say something that someone would say "that's retarded" or something like that. That doesn't bother you? You wouldn't be afraid to say, "Let's find out the person's age" or something off the wall like that?

B: No, not at all.

D: Ok. Is there anything that anyone wants to say before I let you go back to class? I really appreciate the time you've given me and you've given me a lot of insight into how people work in groups even though as a classroom teacher I watch all the time I don't ever sit down and actually focus on one group. I am usually running around the classroom literally. So I really appreciate the opportunity to let me observe you guys solving some problems and getting some insight as to where you're coming from. Now you can rush back to class. Thanks very much.

H/B/V: You're welcome.