PAPER ONE: LEARNING TO PLAY, PLAYING TO LEARN: USING ELECTRONIC GAMES AS EDUCATIONAL TOOLS.

PAPER TWO: "WHAT'S A JOURNAL LIKE YOU DOING IN A CLASS LIKE THIS?": WRITING IN MATHEMATICS CLASS.

PAPER THREE: AN INTRODUCTION TO ETHNOMATHEMATICS

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Paper One:	**Learning to Play, Playing to Learn: Using Electronic Games as Educational Tools.**
Paper Two:	***"What's a Journal Like You Doing in a Class Like This?":*** **Writing in Mathematics Class.**
Paper Three:	**An Introduction to Ethnomathematics.**

By

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Folio Abstract

The first paper of this folio examines the use of electronic games as educational tools. A psychological view of why kids prefer certain video games is given. The research indicates that goals play an important role in the popularity and educational effectiveness of electronic games. This report also examines some of the benefits and detriments of using electronic games for educational purposes. Finally, existing educational practices are analyzed and the focus turns to how we can use what we learn from and about electronic games to enhance and invigorate our everyday teaching techniques.

The second paper in this folio reports on the importance of journal writing in the mathematics classroom. The relationships among writing in mathematics class, communication, comprehension, problem-solving and mathematics anxiety are explored. This paper also presents some practical ideas for the implementation of journals in the mathematics classroom. The topics of what to write and when to write are discussed in depth. The benefits of journal writing for the teachers' and students' mathematical experiences are also addressed.

The third paper in this folio is an introduction of the field of ethnomathematics to the mathematics educator. The field of ethnomathematics is fairly new, yet is extremely diverse. In an effort to comprehend ethnomathematics, a retrospective view is taken and the origin of ethnomathematics is explored. It is seen that the definition of ethnomathematics is dependent on several variables. In order to further understand this area of research, a model of classification for ethnomathematic studies is given. Also, the profession of ethnomathematician is defined, in so much as a 'definition' can be stated and is relevant. Finally, some thought is given to the more practical aspects of ethnomathematics in today's classrooms.

Foreword

While so much time is spent and effort is exerted to produce a professional final product. the personal aspect of graduate work is often overlooked. I would like to take this opportunity to explain why I chose these research topics and to share my personal goals for this paper folio.

One of my goals for this graduate work was to explore ideas that really interested me. This paper folio, unlike many others, is not a set of related papers on one topic. Rather, it is a set of scholarly papers on three topics of great personal and professional interest. I am grateful to the Associate Dean, Dr. Linda Phillips, for having the insight and the flexibility to allow me to pursue these interests and to share them with the professional community.

I have always been curious about the connections between video-games and mathematics, and this paper folio provived an opportunity to explore this interest. While I have learned a lot about the educational aspects of video-games, I have also gained a greater understanding of some aspects of education in general -- the role of goals in motivation. for example.

The idea for the paper on journal writing in mathematics class stemmed from my own love of English and my desire to combine writing and mathematics. Of the three papers it is perhaps the most practical and has already been made available on the internet (see address below).

The concept for the third paper was stumbled upon somewhat accidentally. While researching another topic, I literally found ethnomathematics by opening a journal to the wrong page. Having spent many of my school years moving to various places and cultures, I had already developed an interest in the connections between culture and education. However, I was unaware of the scope of the ideas surrounding ethnomathematics and researching this paper was extremely interesting.

One of my personal goals for this paper folio was to produce a final product which could be easily read and understood, especially by other teachers. I feel that I've accomplished this goal as this work has already been read and used by colleagues. While it has not been easy to compose this paper folio, I am quite proud of the result. I know that on many levels I am now a better educator.

Note:

Paper Two: *"What's a Journal Like You Doing in a Class Like This?": Writing in Mathematics Class* is available at: http://www.ucs.mun.ca/~mathed/t/rc/jour/journal.htm

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Paper One

Learning to Play, Playing to Learn: Using Electronic Games as Educational Tools.

> If you want to see the future of education, don't watch children in the average classroom. Watch children play a video game. You'll see them engaged, excited, interacting and learning -- even if it's only about how to get to the next level of the game.
>
> (Smith, 1995, p.1)

Introduction

A growing number of educators and people in related fields see electronic games as part of the learning process of mathematics as well as of other subject areas. While some educators, parents and students question the use of electronic games for educational purposes, many people believe that these games combine two key ingredients -- intrinsic motivation and computer-based interaction -- making them potentially the most powerful educational tools ever invented (Loftus and Loftus, 1983). One demonstration of this belief was the formation of the E-GEMS (Electronic Games for Education in Mathematics and Science) project. "The E-GEMS project is a collaborative effort by scientists, educators, and professional video game and educational software developers who have come together to do research on and develop teaching materials that integrate video games and computer-based explorations with existing classroom practices." (E-GEMS, 1997).

In this era of technological immersion, examining the use of computer technology in the classroom -- specifically in the form of games -- makes sense. Electronic games help to increase and maintain interest, even during the most tedious of topics. Many professionals argue that if the fun and excitement of video games could be harnessed for learning, then much of the drudgery and tedium now associated with school might be alleviated (Silvern, 1986). Overall, positive attitudes toward learning from computers have been found (Schumaker, Bembry and Young, 1995).[1] Students like computer games because usually the games (a) allow self-pacing, (b) provide gentle and consistent correction and reinforcement, and (c) give immediate feedback (Lever, Sherrod and Bransford, 1989).[2]

Video games are the first medium to combine visual dynamics with an active participatory role for the player. It is this active role that is an important part of the appeal of computer games. So important is this active role that it has become the focal point of many studies. For example, the UBC E-GEMS (University of British Columbia E-GEMS) research team focuses on the human-computer interaction and issues associated with learning in an electronic game environment (E-GEMS, 1997). Greenfield (1984) also found a predictable pattern common to research on the appeal of computer games: children are attracted to activities that let them become personally involved. At the zoo, for example, they would prefer the petting section -- with pigeons, squirrels and rabbits with which they could interact -- to the more exotic animals behind bars.

Games can be an extremely powerful resource in the acquisition of mathematical strategies and concepts. These strategies may include the skills of (a) approximating, (b) simplifying tasks, (c) identifying patterns, (d) reasoning and (e) producing and testing hypotheses (Oldfield, 1991). Students may learn geometric properties, spatial relations and in-depth problem-solving strategies. In fact, the National Council of Teachers of Mathematics [NCTM] (1989) has stated that computer-based explorations of two and three dimensional space are to receive increased attention. Also, the

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1 Research conducted with 294 ninth and tenth grade students from a large suburban school district in North Central Texas.

2 Participants in this research were 94 fifth grade students from public schools in a metropolitan area. They were participants in an on-going study involving *LOGO*.

NCTM suggests that reasoning in spatial contexts and pattern identification as a problem-solving tool are to be emphasized in the curriculum. Students should be given opportunities to visualize and manipulate three-dimensional figures in order to develop spatial skills fundamental to everyday life and to many careers. Electronic games certainly provide the opportunities advocated by the NCTM as stated above.

The purpose of this paper is to examine the educational benefits of playing electronic games as well as to acknowledge some of the possible detrimental effects. The reasons for using electronic games for educational purposes will be explored. Electronic games are not created equal and a classification system for the games will be offered. The motivating aspects of these games are explored in an attempt to understand more about improving the learning environment. Finally, the discussion turns to how educators can use what is learned from and about electronic games to enhance and invigorate their everyday teaching techniques.

Why Are Video Games Ideal Vehicles For Learning?

Electronic games are great motivators! Video games are fun! Nickerson (Nickerson and Zodhiates, 1988, p. 304) said, "If the engaging properties of computer-based games can be turned to educational advantage, the leverage that is realized could be great". Bright, Harvey and Wheeler (1985) questioned *why* electronic games are fun:

> Since students seem to spend a great deal of time playing video games, it ought to be possible to incorporate games of this type into school instruction to teach a wide range of content at a variety of taxonomic

> levels. However, a clearer understanding of the components of effective games is needed before this is possible. (p. 131)

Firstly, video games have the dynamic visual element of television, but they are also interactive (Greenfield, 1984). When students watch television, they are passive observers, but when they play an electronic game, they actively participate in the world created by the game. Games tend to focus attention more effectively partly because they involve the student actively, rather than passively (Avedon and Sutton-Smith, 1979). It is this active participation that helps to create a positive learning environment.

Malone[3] (1981) questioned why video games are so captivating. His research shows that immediate scoring and audio and visual effects had high correlations with game popularity. Video games usually have incredible colours, audio and visual effects, and fabulous graphics to capture and hold a player's attention. The games are continually responsive and they provide instantaneous reinforcement. Sedighian and Sedighian[4] (1996) had similar findings:

> At the initial stage of our research several of the computer games which we installed for the children had minimal sensory stimuli. Many children were not particularly approving of these games because they had no fancy graphics, their images were in black and white, their animations were very simple, their sound effects were primitive, and they had no background music. We have found that for children such sensory stimuli add to the fun of playing the game and make the learning of mathematics more enjoyable and memorable -- as one of the

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3 The research involved interviewing 65 students from kindergarten to eighth grade. All students had been playing computer games for at least two months. Some students had played computer games for more than two years.

4 This research was conducted over a period of more than two years. The subjects were about 50 sixth and seventh grade students.

students put it, 'they add flavour to mathematics'. (p. 6)

Sedighian and Sedighian (1997) recommended that the issue of beauty and aesthetics in the design of interactive learning environments be taken seriously and further investigated, especially in environments designed for children. Others such as Laurillard and Taylor (1994) and Chaffin, Maxwell and Thompson, (1982), agree that high quality sound and visuals are important for sustaining motivation and attention.

Another common feature among popular games was found to be the variability of the difficulty level (Malone, 1981). The games are motivating when they provide an appropriate difficulty level as chosen by the player. According to Malone, other important features include the elements of choice and randomness in the game as well as the involvement of other people, both cooperatively and competitively. However, in his research, Malone found the most important factor determining game popularity was whether or not the game had a goal.

Usually, video games are goal driven. The presence of a goal is often the single most important factor in determining the popularity of games (Greenfield, 1984; Malone, 1981). In fact, Provenzo, (1991) applying Malone's then decade old criteria to the incredibly popular game *Super Mario Bros. 2*, found that the clear presentation of a goal was still the number one motivating factor. Sedighian and Sedighian (1996) found that the children with whom they worked enjoyed computer-based mathematical games because there was a distinct goal in the game and a sense of accomplishment when that goal was reached. Sedighian and Sedighian (1996) also reported that "children expressed a sense of frustration with school mathematics because it often involved completing a set of worksheet-type problems that did not provide them with meaningful goals" (p. 4). Goals that are specific, moderately difficult, and likely to be reached tend to enhance motivation and persistence (Schunk, 1991).

Malone (1981) suggests there are three major ingredients in the student-electronic game experience which make the games such ideal vehicles for learning. These key ingredients are:

- challenge
- fantasy
- curiosity.

For a game to be challenging, it should have a goal whose outcome is uncertain. If a player is either certain to reach or certain not to reach the goal, the game is unlikely to be challenging. Nawrocki and Winner[5] (1983) found that winning while remaining challenged was a key motivating aspect. Sedighian and Sedighian (1996) found that the children asked for challenges which corresponded to their individual abilities. Also, the children studied by Sedighian and Sedighian (1996) particularly liked games that progressively became more challenging, but often became bored quickly with repetitive activities.

Challenge makes it necessary to have intrinsic motivation. Good goals in video games are those which are also personally meaningful (Malone, 1981). A game is popular when the goals and challenges presented engage the player's self-esteem. Success in any challenging activity makes people feel better about themselves. Malone found that another factor affecting the motivational value of a game was that the performance feedback be presented in a way that minimizes the possibility of damage to the self-esteem. Sedighian and Sedighian (1996) reported that the children they studied could recover from their mistakes during the game without too great a penalty and

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5 The authors conducted the research by observing players at a large video game arcade as well as playing some games themselves.

therefore did not feel threatened by making mistakes. Rather, mistakes were seen as 'stepping stones' to future success.

Malone (1981) defines fantasy as evoking mental images -- actual physical images or social situations. He suggests that computer game fantasies almost certainly derive some of their appeal from the emotional needs they help satisfy. While it is difficult to ascertain what needs people have and how those needs are being met, it is clear that different people find different fantasies appealing. Piaget, too, felt that playing in fantasy worlds was a major contributor to the development of skills in children (Woolfolk, 1993).

The final key ingredient which will be addressed here is curiosity. A characteristic of intrinsically motivating environments is that they stimulate and satisfy curiosity. When playing a video game, curiosity has to be invoked in order to maintain the interest and attention of the player. Malone (1981) claims that environments can evoke a learner's curiosity by being neither too complex nor too simple with respect to the learner's knowledge. Finding optimally complex environments is the eternal challenge for the electronic game developers of the future.

To summarize, video games are ideal vehicles for learning because they: (a) are dynamic and interactive, (b) hold the player's attention, (c) are continually responsive, (d) have variable levels of difficulty, (e) have specific goals, (f) are suitably challenging, (g) can satisfy emotional needs through the use of fantasy, and (h) are intrinsically motivating because they stimulate and satisfy curiosity.

Types of Electronic Games

The UBC E-GEMS research objectives include studying which game formats can be used to carry math-science educational content, which formats are most attractive to students, and which formats are most conducive to learning (E-GEMS, 1997). The research literature classifies electronic games into three main categories :

- Educational games
- Games of construction or simulation
- Arcade style games.

In general, the purpose of most educational games on the market is to practice a specific skill. That is, the student practices a certain skill which has already been understood. Usually, the focus is on the skill, not on the metacognitive features that allow for the learning of that particular skill. For this reason, educational games are often seen as "electronic worksheets". While practice is often necessary, a game similar to what students could do with pencils and paper will do little to enhance the learning process.

Most educational games are limited to practice and fail to evoke the sustained excitement and interest usually associated with video games. They are useful, but educators should limit the use of these types of games so the games do not quickly outlive their usefulness (Silvern, 1986).

Games of construction or simulation, according to Piaget, are not really games at all but approximations of intelligent thought (Silvern, 1986). These games allow thought processes to be seen through visible materials. Silvern (1986) states that Piaget noted that games of construction and simulation are activities which cross from play to intelligent adaptation. Games of construction are about manipulating knowledge. They allow for creativity from the player, a quality often missing from electronic games. These games are useful to mathematics teachers on a variety of levels as will be discussed later.

A distinction needs to be made between arcade style games and educational games. The focus of arcade style games is often entertainment, not learning. The focus of educational games is learning or practice, but usually not entertainment. However, it is the rule-governed nature of arcade style games which makes them suitable for educational purposes.

Beneficial Effects of Using Electronic Games as Educational Tools

This section of the paper will elaborate on games of construction or simulation, and arcade style games, and some of the potential educational benefits of each.

Perhaps the most famous game of construction, in the field of mathematics, is *LOGO*. *LOGO* is a programming language developed primarily by Seymour Papert. He felt that children could learn to use computers in a masterful way and that learning to use computers could change the way they learned everything else (Papert, 1980).

On a cognitive level, *LOGO* helps to develop problem-solving skills. Yelland[6] (1995) described *LOGO* as a rich, learning environment characterized by active participation and the construction of ideas. Her research indicated that the *LOGO* environment is a problem-solving environment. Students were engaged in metacognitive activities characterized by a strong engagement with the assigned task.

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6 This study was conducted with 60 children in a suburb of Brisbane, Australia. The mean age of the children was seven years and three months. While all the children had used a computer in school, none of them had any previous experience with *LOGO*.

Allocco et al.[7] (1992) state that *LOGO* teaches students to break down problems and to solve each component one at a time. Students learn to express themselves clearly and precisely and to organize their thoughts sequentially. Specifically addressing the field of mathematics -- *LOGO* involves "doing" mathematics (Allocco et al., 1992). While there is no significant difference in the acquisition of mathematical knowledge between *LOGO* and non-*LOGO* students. *LOGO* students have been found to be more effective in problem-solving, and applying mathematical knowledge. The *LOGO* students scored higher on figure classification, quantitative reasoning, and spatial visualization tests. The *LOGO* students also showed significant improvements in geometric skills (Allocco et al., 1992).

Games involving real-life simulation are also very popular. These reality simulators offer a variety of educational benefits. "These games encourage exploring, experimenting, and taking risks. Students can develop and test ideas, and they can discover what happens when principles are applied to a situation" (Maricopa Center for Learning and Instruction [MCLI], 1998a, p. 1). Some of the most popular simulation games, currently and from the recent past, include (a) *SimCity 2000* and *Civilization*, two games which simulate real-life; (b) *After Burner* and *Stealth Fighter ATF*, two flight simulators; and (c) *Myst* and *Riven*, two adventure simulation games.

SimCity 2000 and *Civilization* are inherently problem-solving software. The player is involved in informed decision making and learns about cause and effect relationships. Decisions are made about population growth and control, environmental concerns, and other fiscal decisions such as infrastructures, taxes, etc.. "What really makes SimCity so useful in education is that it is a dynamic model, and not a predetermined set of events" (MCLI, 1998c, p.1). Bilan (1992) states,

7 This report is a review of various research projects conducted by the seven authors.

"Simulations approach reality with a greater degree of clarity and practicality than 'content' found in the current curriculum and classroom. Therefore, they have the potential to introduce a greater degree of realism, individuality and personal responsibility to the learning process" (p. 7). These types of games increase awareness and give students experience with simultaneously manipulating several interrelated variables.

The flight simulators, *After Burner* and *Stealth Fighter ATF* are multi-tasking experiences. The player is the pilot and is shown, on the screen, the actual cockpit of the aircraft. When flying the plane, the player has to monitor and act on information from a multitude of sources -- the radar, altimeter, fuel gauge, speedometer, weapons systems, targeting systems, warning systems and the artificial horizon, to name a few. Strategy use, efficient decision making, and effective three-dimensional spatial manipulations are essential to being successful with these games.

Most recently, the competitive market of simulation games has been led by *Myst* and its sequel, *Riven*. These two games have enjoyed unprecedented success in the gaming realm. The reasons for this success differ among computer game aficionados, but already research is being conducted into the educational benefits of *Myst* and *Riven*. MCLI (1998b) reports, "Myst has set an entirely new standard for computer games. The puzzles you encounter will be solved by logic. Navigating space and time in this world requires information gathering, critical thinking, and intelligent exploration" (p. 1). MCLI (1998b) describes possible educational benefits of using *Myst* as: (a) developing hypotheses, (b) outlining solutions and alternative strategies, and (c) emphasizing cooperation.

As previously mentioned, the rule-governed nature of arcade style games makes them suitable for educational use. The student needs several skills in order to learn the rules of a game.

Silvern (1986) summarizes these skills as: (a) trial and error techniques, (b) pattern generation, (c) rule generation, (d) hypothesis testing, (e) ability to form generalizations, (f) estimation, (g) decision making, and (h) organizing information. These are the skills that are more often associated with problem-solving. It would be correct to surmise that one major advantage of arcade style games is they permit the practice of skills which are often not explicitly taught in the curriculum. Video games provide practice in problem-solving.

The skill of a problem solver in selecting relevant ideas from the repertoire of ideas presented is not a trivial skill (Allen and Ross, 1975). This skill is demonstrated by most successful video game players, yet is rarely actually taught. Most video games have one common characteristic: problem-solving under time constraints. When an arcade style game is viewed as a source of problems presented in rapid-fire order, then the reasoning processes used to solve the problems are the same processes essential for effective learning. The challenge for educators is to teach students to become cognizant of the strategies they are using and to apply these strategies to other problem-solving situations. According to Lochhead (1981) these mental routines can only be 'debugged' and improved if we are sufficiently aware of their components to examine them.

Cognitive flexibility, defined as the ability to generate several categories of possible solutions to a problem, is sometimes referred to as the most critical aspect of creative thinking (Doolittle, 1995). Cognitive flexibility is vital to efficient problem solving, both in and out of the mathematics classroom. This metacognitive skill can be developed and refined through the use of interactive electronic games. Doolittle's[8] (1995) research indicated promising evidence that

8 Research was conducted with college students and studied the effectiveness of using computer games, riddles and word associations to develop cognitive flexibility.

electronic games could enhance creative thinking and other forms of critical thinking.

Another component of problem-solving, and learning in general, is that of parallel processing. Parallel processing refers to taking in information from several sources simultaneously. Parallel processing is also required in skillful video game playing. For instance, to be a good player of *PacMan*, the video-game craze of the eighties, you had to keep track of *PacMan*, the four ghosts, the four energizers, where you were in the maze, and the time remaining. Today's electronic games have even more information sources which must be dealt with simultaneously. Therefore, it is conceivable that being an accomplished player of video-games could in fact help in the learning process.

Video games also allow a variety of movements. They allow the manipulation of objects in Euclidean space such as rotations, translations, and arranging objects in sequence. As Piaget suggested, (as cited in Gagnon, 1985) learning takes place through the manipulation of physical objects, which leads to the processes of conceptualization, internalization, and the temporal sequencing of operations. Simulating actual physical properties through the use of video games might form the basis of spatial cognitive processing in some students. Also, the literature reviewed contains numerous studies where students with a high degree of spatial visualization are also high achievers in mathematics (Guay and McDaniel, 1977[9]; Lowery, Fitzgerald and Powers, 1982,[10] as cited in Lowery and Knirk, 1982; Siemankowski and MacKnight, 1971[11]).

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9 Study conducted using 90 students from grades two to seven.

10 Given as a secondary source since referenced in Lowery and Knirk (1982) as an unpublished manuscript.

11 This research was conducted over four years at State University College at Buffalo, New York.

Over a decade ago, Gagnon[12] (1985) claimed that video games may be the next generation of media to produce subtle changes in mental skills. She researched the effects of video-games on spatial skills. Her results indicated that video-game players were using two and three dimensional symbolic representations of space. Similarly, Lowery and Knirk (1982) describe the game *Space Invaders* as requiring the ability to simultaneously coordinate horizontal and vertical axes and anticipate the intersecting of imaginary lines. These types of skills are useful in geometric applications.

Another popular game is *Castle Wolfenstein*, a complex maze-based game. The mazes are presented in three dimensions and are meant to be storeys in a prison. The aspect of spatial integration is essential when playing this game. The player needs to keep track of which maze lies beyond which door, which storeys of the prison have already been visited and which have not yet been reached. By manipulating themselves and other objects presented in these games and others, the players might be internalizing a set of intuitive rules of geometry and physics, as well as comprehending pictorial representations of space and spatial relationships (Gagnon, 1985).

Possible Detrimental Effects of Using Electronic Games as Educational Tools

Along with the educational benefits of electronic game playing, there are possible detriments as well. Many researchers argue that while the intrinsically appealing aspects of arcade style games motivate and help students in the learning process, these same games may have significant negative

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12 Research was conducted with 58 graduate and undergraduate students.

implications for learning disabled students (Christensen and Gerber, 1990).[13]

It is the potentially competing cognitive demands of the game that cause little or no positive learning effects to occur. Nawrocki and Winner (1983) claim that for instructional purposes, the overuse of colour could act as a distracter from other more vital cues. Learning disabled students are often distracted by the game context itself , such as the audio and visual effects and the time constraints. They cannot easily attend *only* to the immediate task and not have their attention inhibited by the actual gaming aspects.

Christensen and Gerber (1990) report that using game formats means risking interference with the development of automaticity. They suggest that it might not be wise to disguise drill and practice as arcade style games for use by learning disabled students. Their results indicated that software designers need to focus on cognitive, behavioural and other instructional features rather than elaborate motivational embellishments irrelevant to the instructional task when creating software for learning disabled students.

Another negative aspect of video games is centred around the ongoing debate about violence. Do video games teach violence? A substantial body of evidence indicates that viewing excessive violence on the screen is associated with aggression and violent behaviour (Loftus and Loftus, 1983). However, researchers have found that two-player aggressive video games, whether cooperative or competitive, reduce the level of aggression. It is suggested that the most harmful aspect of violent video games is that they are solitary in nature (Greenfield, 1984). A two-player aggressive game seems to provide a cathartic or releasing effect for aggression.

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13 Study was conducted using 30 students with learning disabilities and an equal number of students without learning disabilities. All students were from five elementary schools in a small city school district in southern California. The average age was ten years, two months. They varied from grades three to six.

Also associated with the negative side of electronic games are the myriad of health problems. Perhaps the most common would be headaches and eye problems due to prolonged exposure to a monitor. It is indeed strange that we allow ourselves and our students to sit much closer to a computer monitor or video game screen than we would allow to a television screen. Just over a decade and a half ago, Loftus and Loftus (1983) reported that repeated video game button-bashing, paddle-twisting, and joy-stick pushing could cause skin, joint and muscle problems. Today these ailments are regrettably well known under the collective name of repetitive motion injuries. One of the most common is carpal-tunnel syndrome (CTS), a malady most commonly associated with extensive keyboard use.

Educationally, we are concerned about the often addictive nature of video games. We want the games to be used for educational purposes as well as for entertainment. However, will students become so addicted that they expect regular classroom instruction to be both educational and entertaining? More importantly, is this expectation so outrageous? How will using video games as educational tools change our practices as educators? It is not the change itself that is necessarily negative. Rather, it is the inability or refusal of educators to change that might produce negative results. Many students accustomed to the challenge and motivational aspects of an electronic game may easily grow tired and bored of "traditional" schooling methods and indubitably, their academic performance will suffer.

Conclusion

> Only television can reach a truly mass audience...Producers ...should realize that they have a responsibility to educate the public, not just entertain it.
>
> (Hawking, 1993, p. 29-30)

Can the same be said in reference to the creators of electronic games? Is it not society's responsibility to ensure that video games are designed and used to their fullest educational potential without losing their distinctive, motivational qualities?

Our society is in direct need of the skills that are developed through experience with the electronic media. Since most people already receive most of their information from a screen -- not from print -- there is an immense need for sophisticated viewing skills (Greenfield, 1984). Similarly, the majority of our current and future occupations will involve computers in some aspect. When will we begin to train the next technologically advanced generation?

Already, video games are most children's first experience in interacting with a computer. But what of the future? Loftus and Loftus (1983) -- who in the early nineteen-eighties speculated that many video game futurists were not taking into account the human cognitive system -- recommended two possible directions for the future of electronic games. First, designers must endow games specifically designed for learning, ie., educational games, with the ultra-motivating characteristics of the arcade style games. That is, create games specifically for educational practices, but make them entertaining as well.

The first recommendation given above by Loftus and Loftus (1983) is clearly affirmed by

Edwards (1992) who proposed the following for learning in an interactive computer environment: Instead of directly teaching mathematical properties, these properties are incorporated into a game situation, in which a learner must use them to solve a problem. In order to effectively use the properties, the learner must understand them. This understanding is built through a trial and error process. Comparisons and revisions are continuously made until the learner has gained a sufficient understanding of the properties to succeed in the game.

The research of Sedighian and Sedighian (1996) also affirms the first suggestion made by Loftus and Loftus (1983). Sedighian and Sedighian (1996) state the following:

> One of the most important aspects of motivating children to learn mathematics is to understand their needs. The need to know mathematics in order to create new technology or to understand other subjects is a remote need for children. This can be amended by placing children in situations in which learning mathematics becomes a tangible need. Playing games is a tangible need for children. When playing well-designed CBMGs [computer-based mathematical game] (i.e., ones in which the mathematics is used as a continual and natural part of the game rather than as incidental diversions from the main activity), children gradually develop the need to learn the embedded mathematical content in order to satisfy their need to play the game. (p. 2)

The second recommendation made by Loftus and Loftus (1983) is to somehow make arcade style games more specifically educational. Smith (1995) echoes this concern stating that the producers of games must tie the content of their products to quality information and the educational standards now being developed and implemented. Smith elaborates, saying that the industry must develop educational devices from comparatively low-priced game hardware and software, thereby dramatically lowering the costs of such technologies.

What can these electronic games teach us about our own instructional practices? Maybe this competition with technology is the rude awakening that the educational machine has needed. An educational system that capitalized on the motivational aspects of games would have a chance of much greater success (Greenfield. 1984). One major concern already addressed in this paper is that of the addiction being carried over to the regular classroom and the consequences thereof. However. perhaps the most valuable thing we can learn is not how to make games less addictive. but how to make other learning experiences, particularly school. more so (Greenfield. 1984).

The attraction of our students to video games tells us as educators a lot about our own instructional methods. As teachers we are competing for our students' attention and interest with a variety of other influences. We have to be dynamic and stimulating in order to stay competitive. We also have to learn to have fun! Almost all classes could use more laughter to alleviate the tension or tedium that is usually present. Our positive attitudes may become contagious and addictive!

We should try to make our lessons, regardless of subject area, more relevant to real life. Of course this may be difficult to accomplish with some topics in the curriculum, but the effort must be put forth. We have to learn to evoke our students' curiosity. Maybe we should be more creative in our lesson planning, perhaps allowing for more discovery of knowledge.

We have to learn to set meaningful, specific goals in our mathematics classes. These goals must be challenging while potentially attainable. We should also try to give our students more chances to be creative and constructive learners, as opposed to their often passive stance. This could be done by simply having each student contribute one possible question for their test or more literally, having the students actually create something -- for example, a bridge made from wooden sticks for an exercise in physical properties. As with interactive electronic games, it is this active participation that enhances a learning environment.

While the various changes educators could make to enhance the classroom environment are too numerous to mention, these modifications are not the only focal point. What is also important is that teachers, parents and students see change and variety as positive entities omnipresent in their lives. While we are learning to play, we should also be playing to learn.

> If we accept that we cannot prevent science and technology from changing our world, we can at least try to ensure that the changes they make are in the right directions.
>
> (Hawking, 1993, p. 28)

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Paper Two

"What's a Journal Like You Doing in a Class Like This?": Writing in Mathematics Class.

Introduction

Many educators, students and parents question the place of journal writing in the mathematics classroom. The very idea of writing in a mathematics course is often met with incredulous looks and remarks of disbelief. "The teachers who are the most difficult to convince of the worth of writing as a normal part of their curriculum are mathematics teachers. The idea is so foreign to the majority of mathematics teachers at every level that even the mere suggestion is often met with derisive looks, rolled eyes, and a shaking head" (McIntosh, 1991, p. 423).

However, the case for journal writing in mathematics class has been presented, tested, reshaped, and has been proven beneficial for both the teachers and the students. For example, Grossman, Smith and Miller[1] (1993) conducted a study to examine the possibility that a specific relationship existed between mathematical achievement and the ability to write about mathematics. Their results indicated that there was a strong relationship between the two variables: "Students in the study who were skilled in their ability to write about mathematics also did well in the class as measured by overall course grade" (p. 4). Bell and Bell[2] (1985) had similar results stating, "students who wrote about their activities performed better as mathematical problem-solvers than those who did not" (p. 219). Linn[3] (1987) found that students scored higher on tests while they were writing in journals.

Stewart and Chance (1995) establish how journal writing supports the *Professional*

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1 This research was conducted at Georgia State University -- a metropolitan university with a diverse population of 25,000 students. Seventy first-year students participated in the study.

2 This research was conducted at a high school in Aiken, South Carolina. Two ninth grade general mathematics classes participated in the study.

3 This research was conducted at St. Joseph Academy in St. Augustine Florida. St. Joseph Academy is a small Catholic high school in a rural community. The geometry students involved were all 15 or 16 years old.

Standards for Teaching Mathematics as published by the National Council of Teachers of Mathematics [NCTM] in 1991. Stewart and Chance make the connections between journal writing and some of the standards such as: (a) teaching based on a knowledge of students' understanding, interests, experiences, and learning styles: (b) listening to students' ideas, asking students to express their ideas in writing, and deciding when and how to react and respond to students' ideas and comments; and (c) providing an opportunity for students to respond to and question the teacher, to make connections, and to communicate mathematically and otherwise.

Borasi and Rose[4] (1989) explain:

> Journal writing in fact introduces new important dimensions in the mathematics classroom: by writing in the journals, students make use of *writing* as a learning tool in the context of mathematics: by reading students' journals, teachers access a wealth of information usually unavailable to them; and by commenting on students' entries, responding to specific questions and posing new ones, teachers engage in a unique and continuous dialogue with each individual student throughout the course. In turn, each of these elements has the potential to provide a variety of benefits for mathematics education. (p. 362)

The issues mentioned above will be included in the discourse of this paper. This paper will report on the importance of journal writing in the mathematics classroom. The relationships among writing in mathematics class, comprehension, problem-solving and mathematics anxiety will be explored. The topics of what to write and when to write will be discussed in depth.

Getting started --for teachers and for students -- is often the most difficult aspect of journal

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4 Borasi and Rose (1989) base their views on their research conducted at a small four-year liberal arts college in the United States. Twenty-three students, representing a wide variety of mathematical backgrounds and abilities, kept journals as part of their mathematics course.

writing and this paper will present some practical ideas on the implementation of journals in the mathematics classroom. Time management, evaluative practices, and how to help problem writers are a few of the subjects which will be handled in a pragmatic way to best inform the mathematics teacher. The benefits of journal writing for the teachers' and students' mathematical experiences will also be addressed.

The Importance of Writing in Mathematics

> It is difficult to want to engage in learning when one cannot communicate in the same sophisticated mathematical language that others have adopted as their private language. Adding English, a common language for communication, to the mathematics class has the possibility of allowing more students to enter the dialogue and find success in mathematical thinking. (Birken, 1989, p. 35)

Theorists and researchers such as Britton (1970), Polyani (1969) and Vygotsky (1962) have long argued for the use of written language in an exploratory or heuristic way so that students may make greater sense of their own learning (as cited in Gordon and MacInnis, 1993). Thought, and the expression of thought in the form of writing, are inextricably intertwined. While this concept is familiar in the English classroom, it is a foreign idea to many mathematics classes.

When students engage in writing, they undertake a number of activities including planning, reflecting, organizing, modifying, and evaluating (Shield and Swinson, 1997). The very action of expressing thought in writing forces a slowing of the thought processes. Thus the cognitive action becomes more deliberate and precise. Bell and Bell (1985) state:

> Writing is self-paced and review-oriented, for it produces a more-or-less permanent artifact that encourages students to reprocess their thoughts by rereading and re-examining their misconceptions. Thus more than reading, which consists of decoding language cues, and speaking which is transitory and disallows reviewing, writing -- an encoding activity -- provides the contexts that encourage learning and thinking. (p. 213 - 214)

The formal education of many people has included the segregation of writing and mathematics. A student is often 'allowed' to be good in one and relatively poor in the other and still remain in everyone's eyes as a good student, intelligent, even stunningly bright (Gopen and Smith, 1990). However, the changing times have brought a revolution to the classrooms of mathematics and other sciences. With more 'across the curriculum' ideas and approaches, writing is finding its niche in almost every subject. Tobias (1989) reports that writing as, "a means of self-exploration in mathematics and science learning does seem to achieve two important goals: It provides classroom-based specific feedback, and it gives students opportunity to and experience in identifying and trying to unravel their own misconceptions" (p. 54).

The role of writing and its relation to learning mathematics is currently recognized as an issue of clear importance by mathematics educators (e.g., Bell and Bell, 1985; Connolly and Vilardi, 1989; Shield and Swinson, 1997). Layzer (1989) describes the peculiar synergy between mathematics and ordinary language: "writing and talking about mathematical subject matter stimulates the efforts needed to master abstract mathematical ideas. Conversely, mastery of these ideas enables students to write and speak more articulately about the contents in which these ideas figure" (p. 131).

Mayer and Hillman[5] (1996) state their reasons for having students write in mathematics class. These include having their students: (a) develop positive attitudes towards mathematics: (b) apply mathematics appropriately in problem-solving situations; (c) think about, and reflect on, what they do: and (d) use appropriate mathematical reasons for what they do. These reasons -- while typical goals of math educators -- may sound simplistic but are in fact very complex. This section will explore these and other reasons for the inclusion of journal writing in mathematics classes. We will consider four major topics that demonstrate the importance of writing in mathematics: communication, comprehension, problem-solving and affective purposes.

Writing for Communication

The NCTM (1989) has recommended increased attention to "student communication of mathematical ideas orally and in writing" (p. 129). Elaborating, the NCTM states that, "the very act of communicating clarifies thinking and forces students to engage in doing mathematics. As such, communication is essential to learning and knowing mathematics" (NCTM, 1989, p. 214).

Communication about mathematical thinking allows teachers insights into how students are developing concepts, skills, processes and attitudes. The increased use of language in mathematics class is evident when students discuss their ideas, debate topics, critically analyze arguments, work cooperatively, and write problems and journal entries.

However, speech -- the most common form of communication in learning environments -- does not require the elaboration that is necessary for written communication. In written discourse,

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5 Mayer and Hillman (1996) base their opinions on their work with high school students in Felton, Delaware.

facial and other non-verbal cues are not present. The ideas introduced have to be treated with more depth by the writer because the reader is not able to question the author on ambiguous points. Dougherty (1996) reports, "Writing creates a context in which my students have to integrate algebraic concepts to communicate their ideas. This integration, in turn, forces deeper deliberations and reflections about the algebraic content than could be expected with only class discussion" (p. 556). Thus writing can force a greater exploration of the ideas, and therefore, a deeper understanding of the mathematical concepts. Borasi and Rose (1989) state:

> The assertion that writing, as well as other communication systems, can contribute to learning depends essentially on a Vygotskyan view of the relationship between language and thought as a dialectic one, where language and thought are both transformed in the act of representation....Furthermore, it has been argued that writing can uniquely contribute to the learning process because of a combination of attributes: writing can engage all students actively in the deliberate structuring of meaning; it allows learners to go at their own pace; and it provides unique feedback, since writers can immediately read the product of their own thinking on paper. (p. 348)

Writing for Comprehension

Current research in mathematics concerns students developing an awareness of their own thinking processes and how they conceptualize mathematical symbols into meaningful language (Connolly and Vilardi, 1989). The strongest understanding occurs when students are actively involved in their own learning. "Writing is an active process that promotes students' procedural and conceptual understanding of mathematics. Students often find out what they think when they write" (Miller, 1992, p.354). Borasi and Rose (1989) claim that restating concepts and rules in one's own words can facilitate internalization. An increased knowledge of mathematical content can be gained as writing about the material covered in class provides a better and more personal understanding of the same, as well as stimuli for new inquiries. Dougherty (1996) explains:

> As students analyze arguments, compare and contrast ideas, and synthesize or assimilate information in writing tasks, they are forming a cohesive knowledge base that pulls together otherwise isolated fragments. They are better able to connect ideas within and outside mathematics, which helps them retain ideas and apply them in appropriate situations. (p. 556)

Grossman, Smith and Miller (1993) concur, stating "the ability to articulate and express concerns in written discourse is crucial in students' ability to comprehend with understanding, rather than resort to memorization or imitation of the process" (p. 4).

Journal writing can also serve as a form of self-assessment. "By asking the students to report in their journals how they solved a problem or approached the study of a topic, they can be encouraged to become introspective of how they do and learn mathematics" (Borasi and Rose, 1989, p. 356). The students become aware of their own strengths and difficulties, and because these are on paper, the students can then reflect on their merits and shortcomings, and act accordingly.

Clarke, Waywood and Stephens (1993) suggest that the key to self-assessment through journal writing is to encourage students to question themselves when they do not understand rather than be dependent upon a teacher to tell them whether they understand. This requires an internalization of authority, responsibility and control, and is empowering to the student.

Stempien and Borasi (1985) made the following observations about the role of writing for the learning of mathematics:

> Writing can give an opportunity to clarify our understanding of a concept or topic, as when putting down ideas on paper, we are faced with them and we are obliged to recognize their eventual shortcomings. This could provide a starting point for analyzing what we have not understood and for trying to work on it. Writing involves an organization of our ideas and notions about a topic. This may prove to be a difficult task, but with a worthwhile pay-off: the realization of relationships and connections existing among the bits and pieces that we have separately learned. (p. 16)

Writing for Problem-Solving

> There is nothing worse than surfacing from a bit of work and having no idea what you are doing or why!
>
> (Mason, Burton and Stacey, 1985, p.11)

Writing plays an integral part in problem-solving. "The view of writing as a process emphasizes brainstorming, clarifying, and revising; this view can readily be applied to solving a mathematical problem" (NCTM, 1989, p. 142).

When solving a problem, students often skip from idea to idea. However, by writing these ideas down, the students can then see what they have already tried, what was successful, and what needs to be modified. More importantly, having a written account of the problem-solving process enables both the teacher and the students to examine and pinpoint where the thought process may be faltering. Mayer and Hillman (1996) report, "through their writing, students are often able to pinpoint their difficulties. They tend to ask more precise questions in class rather than saying 'I don't get it.' " (p. 430).

Writing about the process of solving a problem begins to exercise the capacity of thinking about thought (Marwine, 1989). Kenyon (1989) explains that writing down the thoughts and procedures in problem-solving adds another dimension to the processing. Concepts are identified more clearly and sharply. Students gather, formulate, and organize old and new information and strategies. This material is then synthesized as a new structure in their knowledge networks (Nahrgang and Peterson, 1986). "As students write down, reflect on, and react to their thoughts and ideas, they enhance the executive problem-solving abilities, and the problem-solving process becomes more effective" (Kenyon, 1989, p. 77). The writing process then becomes an integral part of the thought process (McMillen, 1986).

Journal writing often begins with prompts given by the teacher. While the subject of prompts will be elaborated later, the specific topic of process prompts is relevant to problem-solving. "Process prompts offer opportunities for students to reflect on why they choose or prefer particular solution strategies and to consider ways in which they learn" (Dougherty, 1996, p. 558).[6] When students have to explain, in writing, why they chose a particular method, they begin to understand their own problem-solving approaches. With this understanding, students can become stronger, more efficient problem-solvers because they are cognizant of their strategies and methods. Borasi and Rose (1989) report an improvement in learning and problem-

6 Dougherty's (1996) comments are based on her experiences as the teacher of a heterogeneous eighth grade algebra class.

solving can result from the articulation of and reflection on the process of doing mathematics. Students also become more confident in the choices they make when confronting a problem (Dougherty, 1996).

Writing for Affective Purposes

All educators, regardless of subject area, would agree that self-esteem, personal confidence, and attitudes toward a subject all play an extremely important role in the learning process. However, it seems that mathematics teachers, more so than others, have to combat anxiety and past negative experiences on a daily basis. The NCTM (1989) recognizes that journal writing can help students clarify feelings about mathematics or about a particular experience or activity in a mathematics classroom.

By writing in a journal, students have access to personal communication with their teacher. Miller (1992) states that journal writing, "establishes an open channel of communication between teacher and students that promotes a good rapport and a positive classroom environment" (p. 355). Borasi and Rose (1989) found that students seemed clearly comfortable expressing their feelings about mathematics and the course, even when they were negative.

Powell and Lopez (1989)[7] learned that when students were given regular opportunities to engage in a written dialogue with their instructor, their mathematics anxiety was lessened. Borasi and Rose (1989) claim that, "expressing their apprehensions about mathematics, reporting

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7 This study was conducted in one section of a computation course at Rutgers University's Newark College of Arts and Sciences. Most were first year students and had negative feelings about mathematics and their ability to learn mathematics.

past experiences of failure or success, and communicating feelings of incompetence or discomfort about the course could help the journal writers learn about themselves and take steps towards overcoming their perceived difficulties" (p. 354).

The journals provide a context for students to inform teachers of personal concerns which may have affected their work. Some students, while excelling in other areas, may be struggling and becoming increasingly frustrated with mathematics. Chapman[8] (1996) found that journal writing in mathematics class was not only a valuable window on thinking, but was also a forum for students to vent their frustrations, as well as a comfortable way to ask for help. McIntosh (1991) says:

> Although I would never argue that these students should be exempted from mathematics classes, I would argue that appealing to their strength (writing, and so on) in mathematics class may help them feel more positive about mathematics as well as enhance their learning. (p. 432)

"The act of writing about the mathematical task and about one's feelings concerning the context of that task has helped many a self-conscious writer-learner overcome frustration and get back into mathematics" (Lax, 1989, p. 251). Generally, writing about the mathematics they are encountering provides students with a greater potential to control their learning and to monitor their own progress. Being able to manage and regulate themselves is empowering for students and they often feel a sense of satisfaction and accomplishment. These feelings, in turn, have a positive effect on their affective response towards the mathematics they are learning. Self-esteem, so crucial to academic success, grows as students realize they have something to say and

8 Chapman (1996) bases her opinions on her experiences at Arundel Senior High School in Gambrills, Maryland.

others want to listen (Scott, Williams and Hyslip, 1992). As their attitudes become more positive, the students begin to see themselves as learners capable of doing and understanding mathematics (Powell and Lopez, 1989).

The Implementation of a Journal Writing Program

This section of the paper covers the more practical day-to-day concerns of starting and maintaining a successful journal writing program. The first step in implementing journal writing is usually taken by the mathematics teacher who often needs to be convinced of the merit of the program. Other concerns include questions about when and what to write, whether or not to grade journals, and the teachers' and students' roles in a journal writing program.

Concerns of the Mathematics Teacher

McIntosh (1991) claims that the lack of enthusiasm can be attributed to the way that the writing program is presented to mathematics teachers. Many teachers are given the impression that they will have to teach writing as well as mathematics. Some teachers balk at the idea of squeezing something else into an already crowded curriculum. These teachers need to be reassured that using journals does not mean changing the course content, but rather incorporating writing strategies into existing courses. Grossman, Smith and Miller (1993) remark that, "many mathematics teachers may feel reluctant to relinquish valuable class time for writing exercises. Certainly time spent on reading students' writing adds to teachers' probable extremely full schedule of teaching" (p. 6).

Lax (1989) addresses the issue of the amount of class time involved by stating, "the time spent in helping students explain clearly what they mean...will lead to major savings in instruction time later on" (p. 253). Mayer and Hillman (1996) see the time commitment as substantial, but the information they receive from the students' writing is irreplaceable. In fact, the research conducted by Gopen and Smith (1990) indicates that "mathematics teachers can incorporate writing assignments into their courses with significant success and without unduly burdensome extra effort" (p. 18).

When to Write

The question of when to write can best be answered by each individual teacher who is familiar with their own classes. Since students are sometimes more resistant to new processes and procedures once the school year has started, Norwood and Carter (1994) recommend introducing journal writing at the beginning of the school year. Some teachers prefer to reserve one class in the schedule for journal writing. Chapman, (1996) who uses journal writing about twice a week, varies the timing of journal activities depending on its role in the day's lesson. She found that journal writing was, "an excellent warm-up for a new topic as well as a quick assessment of learning at the end of class" (p. 589).

Elliott (1996) also uses journals at different times during class. "During the first few minutes, students can respond or reply to a review question....The responses can begin discussion, be recycled for a later time, or not be discussed in lieu of simply having caused students to think" (Elliott, 1996, p. 92). Elaborating, Elliott claims that journal writing becomes an excellent tool for evaluating conceptual understanding when spontaneous opportunities arise in the middle of the class. She also has students use the journal entries at the end of class to summarize the day's lesson.

Miller (1992) argues that the first few minutes of class, "this generally nonproductive period of class be turned into a constructive time for learning" (p. 354), through journal writing. She suggests that writing helps her, and the students, make the transition to mathematics class.

What to Write

While the students are obviously affected by the choice of what to write, Borasi and Rose (1989) suggest that it is also important to consider that teachers may also be affected in their teaching as a result of reading their students' journals. The relationship between students and teachers may also be influenced by the dialogue created through journal writing. This section of the paper will report on (a) the use of prompts, and (b) how teachers can benefit from what students write.

Most teachers use prompts to start the journal writing process (see Appendix). The prompts can be organized into three types: (a) mathematical content, (b) process, and (c) affective/attitudinal (Dougherty, 1996). While these three categories are useful for organizational purposes, they are not mutually exclusive. Some prompts bridge two or more of these classifications. Mathematical content prompts focus on mathematical topics and relationships. Process prompts cause students to reflect on their strategies and their problem-solving methods. Affective/attitudinal prompts are those that reveal more about the personal aspects of the student such as their fears, beliefs, outlooks. Chapman (1996) found that whatever the task, it was important to be explicit, sometimes even modelling responses. The journal writing experience will be different for each teacher and will continue to evolve as classes become more comfortable with the concept.

Through the use of prompts, a teacher can gain some insight into the 'person' who is their student. Chapman (1996) found that journals revealed abilities and mathematical awareness that had been hidden by low grades. Elliott (1996) states that getting to know students was an extra bonus that came from using journals. Gordon and MacInnis[9] (1993) saw that, "personal feelings and emotions were readily explored and expressed as trusting and personal relationships were built in the journal communication" (p. 41). Using journals is also a realistic way of listening to each student individually. Borasi and Rose (1989) show that:

> The journals certainly allowed the teacher as a reader to get to know students individually, and to realize their specific problems and difficulties -- whether they were of a cognitive or affective nature. As a consequence, the teacher became more aware of the individual needs of each student, and could respond better to them -- both individually and corporately. (p. 358)

Teachers can also learn more about the effectiveness of the instructional strategies they utilize. "Teachers who are really brave may ask students to evaluate their teaching techniques. Most students will offer constructive criticism" (Elliott, 1996, p. 93). If students are comfortable with the journal writing process, they will offer information that the teacher could use to improve the instruction of the course. Receiving contrasting feedback could also be helpful since it could remind the teachers of the variability of the students in the class (Borasi and Rose, 1989).

Chapman (1996) found journals extremely useful for diagnosing misconceptions, and the

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9 This research was conducted over the course of a year at an upper elementary school in a large western Canadian city. One hundred and eighty students from grades four to six took part in the study.

journal entries became the basis for the revision of teaching strategies. Waywood[10] (1992) found that through the implementation of a journal writing program, teachers became more aware of themselves as communicators. They paid more attention to how their teaching was organized: "For example, how students take notes, how an overview of a topic is developed, how an important example is recognised. The journal, as a teaching and learning tool, brings these elements of instruction into focus for the teacher" (Waywood, 1992, p. 40).

Asking students about their study patterns and behaviours is always revealing to both the student and the teacher. Knowing how students study or approach problems allows common and individual weaknesses to be identified and explicitly addressed in future classes. Often students are asked to predict and later respond to their grades. This prompt causes students to reflect on their study habits and places the responsibility of improving with the student (Elliott, 1996).

How to Encourage Writing

There are many ways to encourage students to take part in the journal writing process. However, there are some facets of the process which are essential for success. First, the journals have to be easily accessible. Many teachers keep the journals in their classrooms away from public viewing. This ensures that the journals are never lost, damaged or forgotten at home. The students should have separate notebooks for their journals. This enables them, and their teacher, to periodically look back at their work without searching through several books. This basic

10 This research was conducted at Vaucluse College in Melbourne, Australia over the course of four years. The school is a secondary school (grades seven to eleven) for Catholic girls. There are about 500 students from diverse socio-economic backgrounds and nationalities. The teachers and the students were studied during this research.

organization encourages students to see journal writing as an integral part of their course.

Conversely, Marwine (1989) found it more useful if students did not keep separate notebooks from their class notes because it was more helpful to him to see both. He suggests that students simply date each entry. He does recommend, however, that students write only on one side of the page, because the space is needed for his response and any ensuing dialogue.

The students need to know why they are writing. They have to understand the two basic reasons for writing in mathematics class -- to enhance and support their own learning, and to help the teacher monitor their progress (Burns, 1995). Mathematics teachers also have to establish themselves as the audience. The students need to see journal writing as an opportunity for one-on-one conversation with their teacher.

Students who are intimidated by writing need to be reassured that there are no incorrect entries in a journal. All students should be encouraged to use diagrams, tables, lists, flow-charts, etc., to communicate their ideas. Burns (1995) supports primary students by encouraging the use of pictures and at times, takes dictation for the students.

For journal writing to be effective, teachers should participate in the writing assignments and share their own entries with the class (McMillen, 1986; Norwood and Carter, 1994). Marwine (1989) explains that writing along with the students and sharing what is written are critical aspects of community building.

It is also appropriate to have students write in pairs or small groups. Depending on the class and topic, teachers may feel it is of benefit to everyone to have small groups discuss their ideas before each person makes a journal entry (Burns, 1995).

Responding and Evaluating

> The added benefit of teacher as reader and responder adds an incentive to write the journal consistently. When students write and teachers read and write back, there is a unique relationship established between each student and the teacher. Students eagerly anticipate the teacher's answers to their questions or comments on their entries. Students and teachers find something to talk about and the classroom becomes more cooperative and humanized as each see the other in a new and personalized light. (Rose, 1989, p. 25).

Gordon and MacInnis (1993) found, "one of the driving forces for the students' writing appeared to be their keen interest in the teacher's response and the need to maintain that communicative bond" (p. 38). McIntosh (1991) reports, "most students, once they begin writing *and getting their teacher's responses* will continue to write, both in and out of class" (p. 430). Nahrgang and Petersen (1986) and Scott, Williams and Hyslip (1992) agree that teachers' responses to their students' journals are vital to the success of journal writing in mathematics class. Many teachers, however, find reading and responding to journals a daunting task. Chapman (1996) reports how time-consuming it is to look for misconceptions, give positive feedback and revise teaching strategies. In contrast, McIntosh (1991) suggests that journals do not have to be collected every day, but teachers should read entries and respond in writing, however briefly, at least within a week of the entry being made. Marwine (1989) states, "I look forward to reading journals and responses. It rarely becomes a chore because I enjoy the more personal relationship that such exchange brings" (p. 68).

While the majority of teachers make written comments, some teachers choose not to respond in writing, but to speak to individual students privately. Miller (1992) warns that students should never be singled out in front of everyone. Since a set of journal entries may necessitate a response to the whole class, the teacher then needs to be careful to address everyone without naming individual students.

Teacher responses need to be regular and sincere, but not judgmental or evaluative (Gordon and MacInnis, 1993). The responses could be in the form of questions, comments, or notes of encouragement. What is important is that the student realizes that the teacher has taken the time to respond. While no attempt needs to be made to 'teach' mathematical concepts through responses, an effort should be made to inform students that their misconceptions or concerns have been noted and will be addressed (Borasi and Rose, 1989; Gordon and MacInnis, 1993; and others). Journal entries should receive constructive criticism designed to focus the students' attention on effective writing for communicating mathematics (Mayer and Hillman, 1996).

Miller (1992) suggests not grading the writing for content, spelling or grammatical errors. Journal writing should be a non-threatening opportunity for students to write. Although improving the quality of students' writing is not the primary objective, it can become a benefit over a period of time.

However, some educators do use journal writing as an alternate form of assessment. Most students see their academic world only through grades and report cards. Marwine (1989) reports, "telling students that you will read and respond to what they write but will not grade it is not enough because many students equate being graded with being taken seriously, desiring to work only when they can receive a grade" (p. 62). To some, a task ungraded is often unheeded. Chapman (1996) describes her criteria on which journals are evaluated:

> Currently I set such standards as the correct usage of English and

> *punctuation, thoroughness, and neatness. A correct response is not* one of the evaluative criteria, but the justification of assertions and conclusions is. Students are compelled to explore their thinking and to state explicitly the results of such exploration. (p. 589)

Chapman also found that by raising expectations and the point value of the journals, more students became actively engaged in the quality of the journal as a means of communication.

If journals are to be evaluated, the teacher needs to be clear on what is expected. When journals will be graded, modelling responses for the students may help to clarify any ambiguous points. It would be unfair to grade the journals without first explaining to the students the process and expectations. By grading journals, "those who have been successful with traditional methods are challenged in a new way, and those who have not are given the opportunity to succeed in a different venue" (Mayer and Hillman, 1996, p. 432). When asked if the use of writing will frustrate those students who have mastered the mathematical content, Marwine (1989) replied, "it may for those who are not used to questioning what it is they think they know" (p. 69).

Conclusion

This paper suggests a shift from traditional ways of teaching mathematics and offers the belief that written discourse -- specifically in the form of journals -- when used in conjunction with more familiar teaching practices, can be an effective teaching and learning method.

One purpose of this paper was to present some practical ideas for the implementation of journals in the mathematics classroom. For journal writing to be successful, it has to be embedded in the mathematics curriculum. It is imperative that students do not conclude that journal writing is an incidental extra activity. The journal work must be seen to be valued as highly as more traditional aspects of mathematical learning (Waywood, 1992).

Journals are more effective when implemented at the departmental level rather than at the classroom level. Ideally, journals would be introduced in every mathematics class in the school, and journal writing would continue throughout the students' school career. Waywood (1992) reports that there is evidence that a number of years of journal use changes students' attitudes towards mathematics in general, and also towards the learning of mathematics. Students also come to recognize journals as an integral part of their mathematics curriculum.

At first, students may resist writing in mathematics class. However, as the year progresses, they do begin to see the value of the writing process. "Their logic becomes clearer and more focused. They also begin to realize the importance of being able to communicate their knowledge effectively" (Mayer and Hillman, 1996, p. 429). When teachers implement a journal writing program and take the time to respond to each student individually, students may start to see teachers and schools in a new light.

> While teachers partially lose their evaluative role for a more supportive one, students may feel encouraged to be more daring in their attempts to learn. If their respect and trust for the teacher increases, they will be more willing to put effort in the course and to engage in learning activities, even if they may not initially see their worth. (Borasi and Rose, 1989, p. 362)

The increased mutual trust and respect may bring to teachers a new energy for the task which probably originally drew them to the teaching profession -- to help students grow.

Journal writing also adds a different dimension to the assessment process. The NCTM

(1995) states the four overall goals or purposes of assessment should be to: (a) monitor students' progress, (b) help the teacher make instructional decisions, (c) evaluate students' achievement, and (d) evaluate programs. Using journals helps to accomplish all of these goals. The methods of assessment should be open, coherent processes which promote equity among students and enhance mathematical learning (NCTM, 1995). Journals, as one of many forms of assessment, meets these standards.

Some teachers and students may feel that writing in mathematics class is just a phase and will eventually outlive its usefulness. Is writing a fad? Birken (1989) replies, "The answer is yes, for those who blindly imitate others' assignments without tailoring them to their own curriculum and students" (p. 34). Elaborating, Birken cautions that unless students are allowed to explore, think, take risks and learn through journal use, then writing in math class is no more beneficial than traditional tests. Also, for those teachers who are, "forced by a higher level of administration to implement writing assignments of any type without being asked for input" (Birken, 1989, p. 34), journal use will be seen as a fad, a passing pedagogical experiment.

Another focus of this report was to examine how writing influenced the learning of mathematics. Gopen and Smith, (1990) state that thought and writing are so mutually involved and supportive that improving one will improve the other. Taking it a step further, Stempien and Borasi (1985) propose, "while writing can be a tool for learning mathematics, we should also consider the role that mathematical content can have in improving writing" (p. 17). It is suggested that choosing technical content, such as mathematics, could in some cases be of help in learning how to write (Stempien and Borasi, 1985). This could be a direction for future research into the connections between mathematics and writing.

Another hypothesis for related work is presented by Waywood (1992). He suggests that every student has a pattern of organization for learning that dictates their compositions in journals. He questions if a future direction of research should be to discover if there are more or less appropriate 'patterns of organization of learning' for the learning of mathematics.

However, what is most important in any instructional situation is the student and their overall well-being. Using journals in mathematics class, while very valuable on several levels, is not the answer to every educational dilemma. As with any tool, journals have to be used judiciously and should not be expected to deliver results for which they have not been designed.

> Writing in math class is not a panacea. Still, by learning and writing about related topics, by writing about problems which puzzle them, by writing about their fears and feelings, students begin to see math in more human terms. For me it is a way to get to know more about those varied and wonderful people who are my students. (McIntosh, 1991, p. 432)

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Appendix

Journal Writing Prompts

Some of the following prompts have been compiled from a number of sources: Borasi and Rose, 1989: Dougherty. 1996: A Hundred and a Half Ideas for Mathematical Writing Prompts, n.d.: Linn. 1987: Norwood and Carter, 1994: Powell and Lopez. 1989: Stewart and Chance. 1995: Tobias. 1989: Vukovich, 1985. This list is by no means exhaustive, and is given here as suggestions for getting started in a journal writing program.

Affective Prompts

1. Explain how you feel about mathematics now as compared to before you took this class.
2. My best kept secret about math is...
3. If math could be a colour (shape, sound), it would be...because...
4. Write a letter to the newspaper editor explaining why more importance needs to be given to math education.
5. My parents feel that math is...
6. I want to become better at math so that I...
7. People who are good at math...
8. My best experience with math was when...
9. My worst experience with math was when...
10. When it comes to math, I find it difficult to...
11. When I hear someone say math is fun, I...
12. Draw a picture of a mathematician and describe what a mathematician does.
13. Is journal writing helpful? Do you like writing in your journal?
14. If I were better at math, I would...
15. What kind of math figure are you? (Circle, square, triangle, parallelogram, etc.) Why did you choose that figure?

16. Which trigonometric function are you? Why did you choose that function?
17. Write a story. "If I Were a Centimetre Tall".
18. Describe your feelings about showing your work on the board.
19. What images come to mind when you think about (math teachers. tests. jobs involving mathematics, etc.)...
20. Does mathematics or math class scare you in any way?
21. Project yourself ten years into the future and describe your life as you imagine it at that time. Describe the role of math in your life at that time. and describe your math experiences in the years between now and then.
22. My three personal goals for this term are...
23. Describe how today's math class will affect your day.
24. What is your favourite single digit positive number? List the reasons for your choice. Elaborate as much as possible on why it is your very favourite number.
25. What did you like most about your previous math class. What did you like the least?
26. I think I am a _______ math student because...
27. My math grade now is ...because...
28. This is how I feel about Calculus (Algebra, Trigonometry. Fractions. etc.)
29. Draw a cartoon of the 'Math Monster' and write what the 'Math Monster' is saying to you.
30. One mathematics activity I really enjoy is...because...
31. This is how I used math this week (outside of school)...
32. Write a letter to a student who will be taking this class next year, giving some advice about this class.
33. Design two mathematical bumper stickers, one funny, one serious.
34. If you could meet any mathematician/scientist, who would it be and why?

Mathematical Content Prompts

35. The difference between undefined slope and zero slope is...
36. I think a function is... (I thought a function was...)
37. How would you describe a square root?

38. What patterns do you notice in ... (Fractions, Geometry, Trigonometry, Derivatives, etc.)
39. How do you use fractions in your life?
40. Write a poem about numerators and denominators.
41. Make a list of objects or figures in the room which have symmetry. How can you tell?
42. What is a reference angle. Why is it necessary?
43. Write your own definition of a polynomial.
44. Explain how the first derivative can indicate where a relative maximum or minimum occurs on a graph.
45. Write all you know about (exponents, the Cartesian plane, vectors, standard deviations, etc.).
46. How many squares are there on a chess board? Describe your strategy for solving this problem.
47. Describe the mathematics seen in a photograph. (Photograph may need to be provided).
48. Write and solve a word problem whose solution involves multiplying/dividing two or more fractions.
49. Find a shortcut for adding the numbers between 1 and 100.
50. Explain the Pythagorean Theorem. How could it be used to remember the distance formula?
51. Describe practical uses for each of the conic sections.
52. Compare and contrast the terms median, altitude, perpendicular bisector of a triangle.
53. Explain everything you know about imaginary numbers.
54. Write an explanation about the differences between area and perimeter.
55. How many dimensions does a pencil have? Explain your answer.
56. In geometry, what is a degree? What is a radian? Which do you prefer to use?
57. How are the graphs of $y = 1/x^2$ and $y = x^2$ related? How could you predict the behaviour of the second from that of the first?
58. What is the difference between combinations and permutations and by what clues do you distinguish problems involving one or the other?
59. Explain the FOIL method.

60. What are the differences between a circle and an ellipse?
61. Compare and contrast the meanings of the terms 'parallel' and 'perpendicular'.
62. Why can't you divide by zero?
63. How can you find a number with thirteen factors?
64. What is a prime number? Write all you can about prime numbers.
65. How can you tell which is the larger of two fractions?
66. What is scientific notation? Write all you can about it.
67. How do you simplify a radical expression?
68. What is an asymptote? Explain as much as you can.
69. Distinguish between congruent and similar triangles. Write all you can about them.
70. Why do we need proofs in mathematics?

Process Prompts

71. The most important part of solving a problem is...
72. What does it mean to solve an equation?
73. Write instructions for a fifth grader to follow when (adding fractions, finding percentages, calculating averages, etc.)
74. Write a lesson plan on how you would teach a specific math topic.
75. Find something that you learned today that is similar to something you already knew. Write about these similarities.
76. Do you use tables or diagrams when solving a problem? Why or why not?
77. You know several ways to....(solve an equation, factor a quadratic, add fractions, etc.) Which method is your favourite? Why?
78. Write a multiple choice question about _____ and explain how each of the wrong answers could be logical.
79. How important is being neat and organized to you in general, and when you are doing math?
80. When I study for a test, I...
81. Write a letter to your teacher explaining what you do understand about the topic, and what

needs to be clarified.

82. When I read a math textbook. and see a word I don't know. I...

83. The key idea of the lesson today was...

84. Describe the graph of...as if you were explaining it to a friend over the phone.

85. When I see a word problem, the first thing I do is...Then I...

86. Write a word problem using 'OF'. What does 'OF' mean as a math procedure?

87. What are the benefits of journal writing for mathematics classes?

88. How could journal writing be changed to be more effective?

89. When you get a test back, do you make corrections or ask questions? Why or why not?

90. How do I read my math textbook?

91. Describe any computational procedure that you invented.

92. How should we use class time to the best advantage?

93. Write possible test questions for this unit.

94. What is the most significant thing you learned today?

95. What questions are still unanswered at the end of class today?

96. Explain how you can improve your communication and cooperation in the mathematics classroom.

97. Describe any discoveries you make about mathematics (patterns, relationships, procedures, etc.).

98. Describe the process you undertook to solve this problem. (Problem needs to be provided.)

99. Write WHO, WHAT, WHERE, WHEN, WHY, and HOW across the top of your page. Answer these questions based on today's class.

100. Describe the steps you take to prove something in geometry.

Paper Three

An Introduction to Ethnomathematics.

> There is no doubt that mathematics is a valuable aspect of human understanding, and that it is worth pursuing. There is also no doubt that the role it currently plays in many countries and cultures is a narrow version of its potential. Mathematics education is aimed at furthering mathematics understanding for everyone. To accomplish this it is necessary to change the status and functions of mathematics in our society. An ethnomathematical conception to the mathematics task assists this change. (Barton, 1996, p. 229)

Introduction

This paper is an attempt to introduce the field of ethnomathematics to the mathematics educator. The field of ethnomathematics is fairly new, yet is extremely diverse. While many people -- mathematics educators or otherwise -- have never been exposed to ethnomathematics, the subject is growing in popularity and relevance. In fact, in 1990, the International Study Group of Ethnomathematics [ISGEm] officially became an affiliate of the National Council of Teachers of Mathematics [NCTM]. "ISGEm strives to increase our understanding of the cultural diversity of mathematical practices, and to apply this knowledge to education and development" (ISGEm, 1998b, p. 1).

However, there have been some fundamental difficulties with the field of ethnomathematics. There has been epistemological confusion, that is -- problems with the meanings of the words used to explain ideas about culture and mathematics. There are also confusion and philosophical debates surrounding the extent to which mathematics is universal and how mathematical ideas can transcend cultures (Barton, 1996).

The purpose of this paper is to shed some light on the topic of ethnomathematics. The paper is specifically intended for teachers in the field of mathematics. This report is organized in two distinct sections -- the first part is mainly theoretical, while the second is more practical. In an effort to introduce ethnomathematics, a retrospective view is taken and the origin of ethnomathematics is explored. Through a step-by-step inspection of the last few decades, a definition of ethnomathematics is sought. Here the work of Ubiratan D'Ambrosio, Marcia Ascher and Paulus Gerdes is prevalent as they have been pioneers in this field.

It is seen that the definition of ethnomathematics is dependent on several variables. In order to further understand this area of research, a model of classification for ethnomathematical studies is given. Also, the profession of ethnomathematician is defined, in so much as a 'definition' can be stated and is relevant. Finally, some thought is given to the more practical aspects of ethnomathematics in today's classrooms.

What is Ethnomathematics?

A Brief History

At the end of the 1970s and the beginning of the 1980s, there was an increasing awareness among mathematicians of the societal and cultural aspects of mathematics and mathematics education (Gerdes, 1994). Since then, the field of ethnomathematics has been reshaped and redefined many times. A brief study of the history of ethnomathematics reveals its diversity and evolving definition.

At the time, teachers and didacticians of mathematics in developing countries, and later in other countries, began to resist the. "racist and (neo)colonial prejudices related to mathematics, against the eurocentrism in mathematics and its history" (Gerdes, 1994: see also Joseph, 1991). It was stressed that there were other mathematics beyond the formal mathematics taught in schools.

Initially, these other mathematics were conceived as being the mathematics of particular cultural groups. These ideas were called by various names. Zaslavsky's (1973) ground-breaking research into African mathematics is often referred to as the 'sociomathematics' of Africa. Posner (1982) studied the development of mathematical thinking in two West African groups of children. She found that their culture, rather than their schooling, affected mathematical strategy use. The mathematical knowledge displayed by the unschooled children was referred to as 'informal' mathematics. When Gay and Cole (1967) studied the Western-oriented mathematical education of the Kpelle people of Liberia, they found that the students were being taught concepts which had no point nor meaning in their culture. Gay and Cole proposed using the 'indigenous' mathematics as a starting point for mathematics education. These concepts were all eventually united under the term 'ethnomathematics' as coined by Ubiritan D'Ambrosio.

At first, the concept of ethnomathematics was closely linked to culture. In 1984, D'Ambrosio defined ethnomathematics as the way different cultural groups mathematise: count, measure, relate, classify and infer (as cited in Barton, 1996). Just a year later, the term 'ethnomathematics' had already started to evolve. While it was still culturally influenced, ethnomathematics was now used to refer to an emerging form of knowledge which is manifested in practices which may change with time: "the mathematics which is practiced among identifiable cultural groups, such as national-tribal societies, labor groups, children of a certain age bracket, professional classes, and so on" (D'Ambrosio, 1985, p. 45).

In another year. the ideas involved in ethnomathematics were still diverging, but occasionally intertwining. In 1986. Gerdes wrote about "recognising the mathematical character" (p. 10) in various practices. For him, ethnomathematics was initially the mathematics implicit in a certain practice. At the same time, Ascher[1] (in Ascher and Ascher. 1986) defined ethnomathematics as the study of mathematical ideas of non-literate peoples. Her definition. in contrast to D'Ambrosio's idea of identifiable cultural groups. restricted the cultures from which examples could be drawn.

By 1987. D'Ambrosio was referring to ethnomathematics as the codification which allowed a cultural group to describe, manage and understand reality (as cited in Barton. 1996). Two years later, Gerdes was describing ethnomathematics as a movement, an active reclaiming of a mathematical point of view as part of indigenous culture (as cited in Barton, 1996). Concurrently, D'Ambrosio (1989) was defining ethnomathematics as a research programme which encompassed the history of mathematics. It would appear that it was around this time that ethnomathematics began to take on more of a political and socio-historical flavour.

Ascher (1991) remained firm on her conception of ethnomathematics, reiterating that ethnomathematics involved the study and presentation of mathematical ideas of traditional peoples. A year later, D'Ambrosio strengthened his 1987 view of ethnomathematics by suggesting that ethnomathematics was, "the arts or techniques developed by different cultures to explain, to understand, to cope with their environment" (D'Ambrosio, 1992, p. 1184; as cited in Vithal and Skovsmose, 1997).

By 1994, the vast extent of the reach of ethnomathematics began to be acknowledged. Now more universal, all-encompassing views were expressed, and the impact of ethnomathematics on the

1 Ascher here refers to Marcia Ascher and will continue to do so throughout this paper unless otherwise stated.

field of education was beginning to be recognized. Gerdes (1994) referred to ethnomathematics as the cultural anthropology of mathematics and mathematical education. At this time, D'Ambrosio was stating that ethnomathematics had become more global -- in a sense encompassing the history and philosophy, "not only of mathematics, but of everything" (Ascher and D'Ambrosio, 1994, p. 43). In contrast, Ascher (Ascher and D'Ambrosio, 1994), while acknowledging that mathematical ideas of a culture do resonate throughout that culture, saw ethnomathematics as being bounded by mathematics itself.

From this brief retrospective view of the evolution of ethnomathematics, the dynamic and often confusing nature of the topic is obvious. Barton (1996) explains:

> The early difficulties of identifying the subject of ethnomathematics occurred because this subject is assumed to be located within another culture. This causes difficulty because there is no consideration given to the appropriateness of using the term mathematics to describe practices or concepts in a culture which may not contain mathematics as a category of knowledge. (p. 209)

Is Ethnomathematics a Part of Mathematics?

The answers to this question are as varied as the people in the field of ethnomathematics. That is, most people have a different, but equally valid, opinion on the subject. D'Ambrosio (1985) states that ethnomathematics and mathematics are parallel and different. Gerdes (1988) links ethnomathematics to 'folk' or 'indigenous' mathematics, implying a distinction from 'world' mathematics. However, he elaborates, stating that world mathematics is the union of all possible ethnomathematics.

Ascher on the other hand. (Ascher and Ascher. 1986: Ascher. 1991) sees mathematics and ethnomathematics as separate fields of study. Ethnomathematics, in her view, is the study of mathematical ideas of cultures which do not have a category of knowledge labelled 'mathematics' (as cited in Barton, 1996). For Ascher, the difference between mathematics and ethnomathematics is defined culturally.

In order to communicate effectively, and to aid comprehension, this paper proposes a compromise among the differing points of view: ethnomathematics will refer to both culturally specific mathematics and the study thereof. However, while this is useful, it is by no means a definition.

Towards a Definition of Ethnomathematics

Despite the differences among the leaders in the field of ethnomathematics, there is a common thread in their ideas. Each concept of ethnomathematics can be seen as a window. D'Ambrosio sees ethnomathematics as a window on knowledge itself: for Gerdes ethnomathematics is a cultural window on mathematics; and for Ascher, ethnomathematics is the mathematical window on other cultures (Barton, 1996). Therefore, it seems that the definition of ethnomathematics depends heavily upon: (a) what is being viewed, (b) through what 'window' of observation it is being seen, and (c) by whom it is being observed.

Knijnik (1993) and Gerdes (1994) both see ethnomathematics as involving research, yet their definitions differ in the purpose of that research. Knijnik (1993) uses the term ethnomathematics to designate:

> *research* into the conceptions, traditions, and mathematical practices of

> a specific subordinated social group and *pedagogical work* involved in making the group members realize that: 1. they do have knowledge: 2. they can codify and interpret their knowledge: 3. they are capable of acquiring academic knowledge; 4. they are capable of establishing comparisons between these two different types of knowledge in order to choose the more suitable one when they have real problems to solve. (p. 24)

Gerdes (1994) defines ethnomathematics as a field of research reflecting "the consciousness of the existence of many mathematics, particular in certain ways to a variety of (sub) cultures" (p. 20). While Knijnik (1993) acknowledges the pedagogical aspect of ethnomathematics, Gerdes (1994) does not explicitly recognize it.

For Vithal and Skovsmose, (1997) ethnomathematics refers to. "a cluster of ideas concerning the history of mathematics, the cultural roots of mathematics, the implicit mathematics in everyday settings, and mathematics education" (p. 133). Such a broad definition strives to be an 'umbrella' of sorts to all defining possibilities.

While the efforts to define ethnomathematics continue, one very important point needs to be made clear. The issue of the universality of mathematics and its implications for ethnomathematics need to be addressed:

> Ethnomathematics is the study of mathematics which takes into consideration the culture in which mathematics arises. Mathematics is often associated with the study of 'universals'....It is important to remember that something we think of as universal is merely universal to those who share our cultural and historical perspectives." (Ethnomathematics, 1998, p. 1).

One definition that manages to encompass most of the ideas was offered by Barton (1996) who proposed the *following culturally specific definition of ethnomathematics:*

> Ethnomathematics is a research programme of the way in which cultural groups understand, articulate and use the concepts and practices which we describe as mathematical, whether or not the cultural group has a concept of mathematics. (p. 214)

What appears to be critical to the comprehension and acceptance of this definition are the meanings of certain terms. 'Mathematics' refers to the concepts and practices in the work of those people who call themselves mathematicians (Barton, 1996). This mathematics is what is referred to as 'school mathematics' or 'world mathematics' by Carraher, Carraher, and Schliemann, 1985; and Gerdes, 1988, respectively. Therefore, for each group of people -- ethnic, social, professional, etc. -- the concept of mathematics will be different. Ascher (Ascher and D'Ambrosio, 1994) agrees, stating that, "the word 'mathematics' for many people is linked very closely to what they learned in school. Depending on their level of schooling, the word means quite different things to different people" (p. 37).

'Mathematical', on the other hand, refers to those concepts and practices which are identified as being related in some ways to mathematics (Barton, 1996). However, Barton cautions that 'mathematics' and 'mathematical' are culturally specific depending on who is using the terms. For example, some mathematicians may disagree on what can legitimately be included as mathematics.

Barton (1996) uses 'we' in the definition to refer to a group of people who share an understanding of mathematics and who are interested in ethnomathematics. The 'we' also refers to members of a culture which contains the category 'mathematics'. 'Cultural' is taken in the very broadest sense and assumes the meaning as used above by D'Ambrosio (1984, as cited in Barton, 1996), and refers to:

> people who have "developed practices, knowledge, and in particular, jargons and codes, which clearly encompass the way they mathematise, that is the way they count, measure, relate and classify, and the way they infer"....Such a group may be an ethnic group, a national group, a historical group, or a social group within a wider culture. (p. 215)

A major problem encountered by writers on the subject of ethnomathematics is that they have tried to be universal in the scope of their definitions. However, such an attempt goes directly against the doctrine of ethnomathematics itself! Barton (1996) explains why an all-inclusive definition cannot be reached: "Part of the purpose of ethnomathematics is to challenge the universal nature of mathematics, and to expose different mathematical conceptions...Thus a universal definition is not possible" (p. 216).

Implications of the Definition of Ethnomathematics

Returning to the analogy of the window, there are clear implications of the definition of ethnomathematics as proposed by Barton (1996). The definition itself depends on who is stating it and will vary with use. Therefore the practices which it describes will be culturally specific. By this definition, ethnomathematics is not a mathematical study as much as it is an anthropology. Like an anthropological study or an ethnography, one of the difficulties of ethnomathematics is to describe another person's world with one's own codes, languages and concepts (Barton, 1996). Ethnomathematics, then, implies some form of relativism for mathematics.

Another implication of Barton's (1996) definition is that cultures without a category to call 'mathematics' cannot have an activity called ethnomathematics. There are also implications for education. Abraham and Bibby (1988) see possible changes as the study of ethnomathematics allows the possibility of redefining legitimate mathematical knowledge and practices. Similarly. Borba (1990) notes that since. "different people produce different kinds of mathematics. then it is not possible to think about education as being a uniform process to be developed in the same way for different groups" (p. 41). The subject of ethnomathematics and education will be examined and developed later in this report.

The classification of ethnomathematical studies and activities are also affected by the definition of ethnomathematics. An accepted convention of categorizing ethnomathematical research is necessary if people in the field are to be able to communicate effectively. Also. the definition of ethnomathematics directly influences who an 'ethnomathematician' is. and what they do.

Classifying Ethnomathematical Studies

Barton (1996) proposes that there are three dimensions according to which ethnomathematical studies can be classified -- time. culture and mathematics:

> On the time dimension. ethnomathematics may be concerned with the conceptions of an ancient or a contemporary cultural group....The culture dimension of the definition extends from a distinct ethnic group. to a purely social or vocational group....The mathematical dimension of ethnomathematics is determined by the relationship of the mathematical ideas to mathematics itself, i.e. ethnomathematics is a study which may be internal to mathematics, or conceptually removed from existing mathematical conventions. (p. 220)

Barton (1996) arranges these dimensions as axes in three dimensional space (see Figure

1). Each dimension is a continuum and an ethnomathematical study can be located at any point in this space. For example, a historical, ethnic, internal study could feature research into the mathematical practices and conceptions of the ancient Hindu (Point A). A contemporary, external, social group study could focus on the extent that knitting patterns are a mathematical symbol system (Point B) (Barton, 1996). Under this classification system, Zaslavsky's (1973) research into the applications of mathematics in the lives of the African people would be categorized as contemporary, ethnic and internal (Point C).

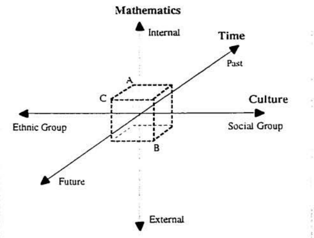

Figure 1 (as adapted from Barton, 1996.) A method of classifying ethnomathematical studies.

A further taxonomy of ethnomathematical activities concerns how the studies are done. Again Barton (1996) suggests a comprehensive method. He claims that four divisions are necessary: descriptive, archaeological, mathematising, and analytic. An ethnomathematical study might include any or all of these divisions.

Every ethnomathematical study will describe the concepts or practices under consideration. The description should be, as much as it is possible, within the context of the culture being studied. Ascher's (1991) research into the Tshokwe and their drawings in the sand, is a good example of a descriptive activity. Ascher describes the culture of the Tshokwe, their rituals, rites of passage, and their tradition of storytelling of which the sand drawings are an important part. Only then does she elaborate, illustrating how graph theory -- in particular the notion of isomorphism -- is illustrated in the drawings.

The archaeological aspect of an ethnomathematical activity involves tracing backwards in time to uncover the mathematics which lie behind the current practice or concept (Barton, 1996). For example, Gerdes (1994) refers to the attempt "to reconstruct or unfreeze the mathematical thinking that is hidden or frozen in old techniques, like, e.g., that of basketmaking" (p. 20).

Another way of exposing the mathematical aspects of an ethnomathematical study is "by mathematising, i.e. by translating the cultural material into mathematical terminology, and relating it to existing mathematical concepts" (Barton, 1996, p. 223). Ascher (1991) does this type of interpretive activity with respect to Warlpiri kin relationships. She uses group theory to show that part of the Warlpiri kin system has a structure called a dihedral group of order eight.

The symmetries of a square -- also a dihedral group of order eight -- are a useful analogy to the Warlpiri kin system. When a square is rotated about its centre and through any multiple of ninety degrees, its shape stays the same although the placement of its corners may be different. Likewise, reflections across a horizontal centre line, a vertical centre line, or a diagonal would also result in the shape of a square possibly with a different orientation. No matter how the rotations and reflections are combined, there are only eight possible resulting orientations of the square.

Similarly, the Warlpiri kin system has eight sections and each person belongs to one of them. Each section is linked to every other section through a descent relationship. Ascher (1991) found that all relationships can be resolved into eight relationships regardless of the extent of the descent line.

If a goal of ethnomathematics is to understand the perceptions of another group, then the influences on the development of a particular phenomenon need to be examined (Barton, 1996). That is, researchers need to ascertain why practices are the way they are. Barton (1996) explains that this analytic activity is more socio-historical than it is mathematical. For example, Gay and Cole's (1967) study of the Kpelle of Liberia revealed some non-mathematical reasons why the Kpelle children had difficulty learning 'Western' mathematics. These reasons included the misuse of the English language, failure to use logical patterns, and perhaps most importantly, not having any use for what they were learning. Similarly, Ascher's (1991) research revealed how the design of the sailing boats used by the navigators of the Central Caroline Islands in Micronesia affect their navigational practices. While the navigation itself has inherent mathematical qualities, the design of the boats -- while imperative to the navigation -- has no mathematical qualities per se.

An Ethnomathematician Is...

The descriptions of who an ethnomathematician is, and what one does, are as varied as the definition of ethnomathematics. However an attempt will be made here to, as concisely as possible, consolidate a host of differences.

The works of Bishop (1991) and Zaslavsky (1991) suggest that ethnomathematicians use liberal definitions of mathematical conceptions, especially for counting, designing, playing and explaining. Ascher (1991), D' Ambrosio (1984, as cited in Barton, 1996), Gay and Cole (1967), Gerdes (1994) and others emphasize the fact that ethnomathematicians are concerned with the socio-cultural factors pertinent to the teaching, learning and development of mathematics.

> Ethnomathematicians draw attention to the fact that mathematics (its techniques and truths) is a *cultural product.* They stress that people -- every culture and every subculture -- develops its own particular mathematics....Under certain economic, social, and cultural conditions, it [mathematics] emerged and developed in certain directions; under other conditions, it emerged and developed in other directions (Gerdes, 1994, p. 20).

Ethnomathematicians emphasize that 'school mathematics' falsely appears to be foreign to the cultural traditions of non-European or non-Western civilizations. There are still reactions against the fact that, "different styles, forms, and modes of thought aiming at explaining and dealing with reality were developed in different natural and cultural environments and run throughout history in parallel with the development of Western mathematics" (D'Ambrosio, 1997, p. 14). In fact, a substantial part of the contents of 'school mathematics' is of African and Asian origin (Gerdes, 1994; Joseph, 1991).

Educationally, ethnomathematicians encourage a critical mathematics that allows students to reflect on the reality in which they live, and that empowers them to create and manipulate mathematics in an emancipatory way (Gerdes, 1994; see also Zaslavsky, 1985, 1991, 1994; and others).

Ethnomathematics in the Classroom

Ascher (Ascher and D'Ambrosio, 1994) believes there are two distinct facets of ethnomathematics which, while related, sometimes need to be more clearly separated:

> "One aspect is seeking understanding of the relationship between mathematical ideas and culture....Once there is this deeper understanding, *then* can come the educational aspect which addresses the question of how to incorporate it: how should we or how do we modify education?" (p. 40)

This paper has already addressed the first aspect of ethnomathematics -- that is, its theoretical roots, its connections to culture and mathematics, and its dynamic definitions and the implications thereof.

This section of the paper will establish: (a) why an ethnomathematical approach is necessary in the mathematics curriculum, (b) why it is important to recognize and incorporate the students' prior cultural development, (c) the changes necessary to the education of prospective teachers, and (d) practical suggestions for including ethnomathematics in the classroom.

Why is Ethnomathematics Necessary?

Zaslavsky (1994) establishes why it is important to introduce ethnomathematical perspectives into the mathematics curriculum:

> Students should recognize that mathematical practices and ideas arose out of the real needs and interests of human beings. They should know that a great deal of the mathematics they learn in elementary and secondary school originated in Asia and Africa centuries before Europeans were aware of more than the most elementary aspects of mathematics. Students of many different backgrounds can take pride

> in the achievements of their people, whereas the failure to include such contributions in the curriculum implies that they do not exist....Most important, they should have the opportunity to see the relevance of mathematics to their own lives and to their community, to research their own ethnomathematics. (p. 6)

Bishop (1994) states that, "many young people in the world are experiencing a dissonance between the cultural tradition represented outside school (for example in their home or their community) and that represented inside the school" (p. 16). Among the groups for whom conflict with, and alienation from, school mathematics exists are: (a) ethnic minority children in Westernised societies; (b) girls in many societies; (c) second language learners; (d) fundamental religious groups, often of a non-Christian nature; and (e) indigenous 'minorities' in Westernised societies. Elaborating, he says the conflicts vary and could concern some of the following factors: (a) language; (b) symbolic representations; (c) attitudes, goals, and cognitive preferences; and (d) values and beliefs.

To combat this perceived estrangement, the NCTM (1989) states, "students should have numerous and varied experiences related to the cultural, historical and scientific evolution of mathematics in the development of our contemporary society and explore relationships among mathematics and the disciplines it serves: the physical and life sciences, the social sciences, and the humanities" (p. 5).

But how can an ethnomathematical perspective be incorporated into the curriculum? The following factors must be considered (Zaslavsky, 1994): (a) the entire mathematics curriculum must be restructured so that mathematical concepts and ethnomathematical aspects are synthesized, (b) teachers at all levels must be well-grounded in mathematics and at the same time be familiar with the interface between mathematics and other subject areas, and (c) research must be conducted and the results made available to teachers on the ways in which underserved and underrepresented students, particularly females and people of colour, can best learn mathematics.

This last consideration has been emphasized by the NCTM (1989) declaring, "it is crucial that conscious efforts be made to encourage all students, especially young women and minorities, to pursue mathematics" (p. 68). Zaslavsky (1985) agrees, claiming that integrating mathematics education with the study of culture and history may motivate more girls to become involved in mathematics.

Incorporating Students' Personal Knowledge

When children go to school, they carry with them their personal history, full of experiences accumulated in living in a particular family and community, and these varying environments result in experiences which may include customs, perceptions, explanations and mysteries (D'Ambrosio, 1990). Therefore, "cognitive work for all students is culturally dependent because students bring to each lesson their past experiences and the diverse facets of their cultural identities" (NCTM, 1995, p. 2). Boaler (1993) suggests that when students' social and cultural values are encouraged and supported in the mathematics class, their learning has more meaning for them (see also Pinxten, 1994). Vithal and Skovsmose (1997) concur, stating "ethnomathematics, as an educational idea, suggests that the content of mathematics education be rooted in the mathematics implicit in the culture with which the children are familiar" (p. 133).

The implications of these statements are almost overwhelming considering the vast variety of students in classrooms -- varying on ethnic, socioeconomic, physical, mental, emotional, and religious levels. However, Bishop (1985) advises that mathematics classroom teaching should take into account, "the pupil's existing knowledge, abilities and feeling" (p. 26). "Instruction that capitalizes, and builds, on what students bring to a problem situation can motivate them to struggle with, and make sense of, the problem" (NCTM, 1995, p.2).

Closely tied to the need to acknowledge and utilize a student's prior knowledge is the necessity to make connections between 'school mathematics' and real-life. "A crucial feature of ethnomathematics is that the ideas or concepts are put to use for practical purposes" (Cobb, 1986, p. 7). As previously mentioned, this lack of connection between school and real-life played a great role in the inability of the Kpelle of Liberia to learn mathematics.

Gerdes (1985) speaks of *problemising reality*, i.e., using real-life problems in mathematics instruction. He states that problemising reality leads to an understanding of mathematics as a tool to transform reality: "problemising reality *motivates* the pupils and gives an entrance to powerful mathematical models" (p. 17). Masingila (1993) maintains that the curriculum should include a "wide variety of rich problems that: (a) build upon the mathematical understanding students have from their everyday experiences, and (b) engage students in doing mathematics in ways that are similar to doing mathematics in out-of-school situations" (p. 19). For example, for students living in a village with a fish-processing plant, their lives -- and the lives of their families -- may revolve around the fishing industry. Their mathematics experience should incorporate as much of this culture as possible.

The Role of Teacher Education

> Teacher education is the key to cultural preservation and development.
> (Bishop, 1988, p. 190)

D'Ambrosio (1990) argues that "teachers can hardly play their role in building up a democratic and just society if they do not foster democratic and just behaviours. Mathematics is an essential component in this entire process" (p. 23). Similarly, Bishop (1988) declares that mathematics teachers "need to know about the cultural history of their subject, and they need to be aware of how their teaching contributes not just to the mathematical development of their pupils, but also to the development of mathematics in their culture" (p. 190).

The ISGEm (1998a) acknowledges that there is often, "teacher and student resistance to emphasizing cultural differences" (p. 5). This can probably be attributed to the dynamic codes of political correctness and the ever-present fear of inadvertently offending someone. To combat this reluctance, the ISGEm (1998a) makes the following recommendations: (a) prospective teachers need to be educated on methods of teaching students from different cultures, especially those who are underprepared in mathematics, and (b) multiculturalism should be embedded within the teacher education curriculum and not taught separately. This integration of multiculturalism and mathematics is also essential in the regular school curriculum.

Consolidating Ethnomathematics and the Curriculum

> Multicultural mathematics should not be taught as a form of mathematical oddity, or as a relief from the real mathematics of the curriculum. (ISGEm, 1998a, p. 6)

The NCTM (1995) states its vision of a reformed curriculum involves shifting, "toward a balanced variety of rich problem situations that encourage students to make connections among the various mathematical topics and that encourage cultural diversity" (p. 2). Katz (1994) urges that it is vitally important that modern curricula, particularly at the secondary level, incorporate the materials necessary to broaden students' understanding not only of mathematics but of the world. The ISGEm (1998a) explains:

> In a pluralistic society, all students need to be exposed to multicultural aspects of mathematics as part of having them interact with students from a variety of cultures. Multicultural aspects of mathematics should be blended throughout the mathematics curriculum irrespective of minority culture. (p. 6)

However, Ascher and D'Ambrosio (1994) caution that introducing ethnomathematics to the curriculum is not an attempt to replace mathematics. They claim that the degree of modification of education depends largely on the goals of the educator and the setting of the education. Every situation will be different.

The merging of ethnomathematics with the present mathematics will have repercussions across the curriculum. It is important to remember that mathematical practices, "include not only formal symbolic systems, but also spatial designs, practical construction techniques, calculation methods, measurement in time and space, specific ways of reasoning and inferring, and other cognitive and material activities" (ISGEm, 1998b, p. 1). Zaslavzky (1991) asserts that through, "linking the study of mathematics with history, language arts, fine arts and other subjects, all the disciplines take on more meaning" (p. 36).

From this it is clear that incorporating ethnomathematics does not only mean introducing the mathematics of varying cultures. It also means assuming a cross-curricular vision of mathematics within the inherent culture of the students. As previously mentioned, it is imperative that the students' personal knowledge be considered an integral part of their education.

Multicultural Ideas for Mathematics Class

Given that changing the curriculum often proves to be very difficult, Bishop (1994) questions what can be done at the classroom level. The suggestions for incorporating ethnomathematical concepts into mathematics class are limited only by the educator. However, the following paragraphs are recommendations for beginning the process.

Questioning what cultural factors drove the early development of mathematics could lead to mathematical discourse on trade, agriculture, astronomy, navigation, religion, and arts and crafts (Seven Views, 1998). For example, an investigation of styles in housing in different cultures is a valuable source of experiences with shapes and sizes as well as perimeter and area concepts, and the skills of approximation and estimation (Zaslavsky, 1985).

Zaslavsky (1985) suggests asking students to imagine how and why people first found it necessary to use number words. Some theories are: (a) to keep track of possessions or livestock, (b) for ritual or religious purposes, and (c) for trade. Elaborating, she says that comparing the number words in a foreign language with the corresponding English words encourages analysis of the structure of the numeration systems. She cites, as an example, that the Yoruba (Nigerian) word for forty-five means 'take five and ten from three times twenty', while in English, forty-five means 'four times ten plus five'. Students can also investigate the numerals of ancient societies (Zaslavsky, 1985; Joseph, 1991).

Another recommendation is to discuss some of the human activities which require some form of mathematics, such as architecture, weaving and sewing, and ornamentation with beads (Ethnomathematics, 1998). Patterns, such as those found in beadwork, knitting, architecture, and so on, hold a myriad of mathematical properties -- shapes, translations, symmetries, etc.. Coburn et al. (1993) recognize that "examining various patterns from the multicultural world in which we live helps children make mathematical connections. These connections enrich our lives and are bridges from the classroom to the outside world" (p. 38).

The ISGEm (1998a) suggests that, "students might write a paper about their cultural background, using topics from the history of mathematics about people from their culture, or by using library or family resources" (p. 5). Many children's books have been written which combine mathematics and culture (see Birch, 1988; Feelings, 1971; Grossman and Long, 1991; Haskins, 1987, 1989a, 1989b; and others). The ISGEm (1998a) proposes that students may also be interested in reviewing movies related to mathematics and culture, for example *Stand and Deliver* by Musca and Menendez (1988).

Andersen Schools (1998) lists several types of resources which can be used to create a multicultural environment. The list promotes the use of: (a) art calendars displaying pictures of African textiles, (b) games from many civilizations (see also Krause, 1983; Zaslavsky, 1985), (c) Japanese origami boxes, (d) tangrams, and (e) folk art, to name a few.

Another means of engaging students in an ethnomathematical experience is to invite into the classroom a professional who uses mathematics as part of their career -- for example, a stock broker, a carpenter, a chef, a scientist, or an engineer. Multicultural Ideas (1998) adds the following suggestions: (a) use recipes from different countries to teach fractions and ratios; (b) use maps of various areas to teach about distance and scale; (c) begin a collection of pictures of architecture and artifacts from other cultures to enrich geometry lessons; (d) use various world statistics whenever possible; (e) include the history of mathematics in lessons (Joseph, 1991); and (f) include women and the contributions they have made to science and mathematics in particular.

Conclusion

This paper attempted to introduce the subject of ethnomathematics to mathematics teachers. From the debates about defining ethnomathematics to the suggestions for classroom practice, the dynamic character of ethnomathematics is evident. One point that is worth reiterating is that it is essential for teachers to link mathematics to everyday life and to the culture of the students. Gerdes, (1988) somewhat idealistically, proposes that with the integration of a regional tradition into a national curriculum, "the knowledge it reveals and its *mathematical potential*, will become less monopolised, less regional, less tribal and...will contribute to the development of a truly national culture" (p. 19).

Ethnomathematics is not without its critics. D'Ambrosio (1997) notes there is an unjustifiable wariness about the growing presence and acceptance of ethnomathematics in school systems:

> There is general acceptance of and interest in the mathematical ideas of other cultures....But when we try to view these facts as complex forms of knowledge and try to relate them to the way these cultures think...there is incredulity. And when we take the same questioning stance and try to identify similar complexities and relationships in Western mathematics, the reaction met with is disdain and even scorn." (p. 14).

Mellin-Olsen (1986, as cited in Abraham and Bibby, 1988) comments on how, "the Norwegian Social Democratic government resisted ethnomathematics in the curriculum on the grounds that it contravened the principle of equality of opportunity in the form of equal curricular content for all" (p. 4). Similarly, Brinkworth (1995) warns that "any emphasis on ethnomathematics in the curriculum is likely to fall foul of the demands for academic mathematics, the so-called 'real maths' " (p. 7). Likewise, Abraham and Bibby (1988) question if a 'ghettoising' or segregation of the curriculum would be the result of giving too much recognition to alternative mathematical thinking.

There are still many questions about ethnomathematics and its practice that need to be answered. Bishop (1994) asks:

> What outside-school mathematics knowledge do teachers recognize as legitimate inside the classroom? What knowledge about the learner's cultures can help mathematics teachers with their classroom decision-making? More fundamentally, how do teachers recognize cultural conflict in their classrooms?...What teaching strategies do

> mathematics teachers adopt if they recognize their classrooms as being multicultural? (p. 18).

Specifically, the issue of the connections among ethnomathematics, culture and power has not been addressed explicitly in this paper. Yet, Vithal and Skovsmose (1997) assert that understanding culture in relation to power is important for education. They claim one difficulty with ethnomathematics is that while, "it identifies culture as an important point of orientation, the research in ethnomathematics usually does not specify much about the relation between culture and power" (p. 139). This is an area that should be targeted for future research.

Seven Views (1998) questions if we can learn more about various cultures by understanding their mathematical ideas. In other words, the apparent symbiotic relationship between mathematics and culture should be more closely studied.

Ascher (in Ascher and D'Ambrosio, 1994) believes that the teaching of the adult learner merits special attention as an issue of growing importance and relevance:

> At all levels of schooling, the teaching of subject matter is intertwined with the teaching of culture. But these people are already fully enculturated. As a result, ethnomathematics may have even more important ramifications for their education. ...And, since they have had many life experiences, some of which may have involved utilizing mathematical ideas, they could make significant contributions to our understanding. (p. 43)

That is, not only will ethnomathematics be more useful and relevant to adults who have more life experience, but these very experiences, and the sharing thereof, could lead to a greater comprehension of the mathematics involved.

Another direction for future research is proposed by Masingila (1993) who suggests that the focus should remain on mathematical practices in everyday situations and children's mathematical practices in out-of-school situations. "Research in both these areas can enable the mathematics community to develop and teach school mathematics in a way that builds upon the students' out-of-school mathematical knowledge" (p. 21).

Vithal and Skovsmose (1997), too, acknowledge the importance of the consideration of the students' background. However, they claim that of equal importance, is the students' *foreground*. "Foreground may be described as the set of opportunities that the learner's social context makes accessible to the learner to perceive as his or her possibilities for the future" (Skovsmose, 1994, as cited in Vithal and Skovsmose, 1997, p. 147). They suggest future ethnomathematical research should include some investigation and recognition of the learners' foreground.

Are we to conclude that schools ought to allow children simply to develop their own computational routines without trying to impart the conventional systems developed in a particular culture? No, of course not. Instead, Carraher, Carraher and Schliemann (1985) suggest educators seek ways of introducing these systems in contexts which allow them to be sustained by daily human sense. The connection to the everyday life of the student is what is vital.

> Ethnomathematical research will oblige everyone to reconsider the history of mathematics; to reconsider cognitive models of learning mathematics; to reconsider the goals, contents, and means of mathematical education; to reconsider the cultural role of mathematics; to reconsider what mathematics is all about. (Gerdes, 1994, p. 21)

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