MATHEMATICAL PROBLEM SOLVING IN A GRADE 2 CLASSROOM: A REPORT OF AN INTERNSHIP

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0-612-54922-4

MATHEMATICAL PROBLEM SOLVING IN A GRADE 2 CLASSROOM:

A REPORT OF AN INTERNSHIP

by

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An internship report submitted to the School of Graduate Studies

In partial fulfillment of the requirements for the degree of Masters of Education

Faculty of Education

Memorial University of Newfoundland

May, 2000

St. John's Newfoundland

ACKNOWLEDGMENTS

I would like to thank Dr. David Reid, my internship faculty supervisor, for his support, direction and advice which made successful completion of this report and Masters of Education degree possible.

I wish to thank Mr. Kevin Sharpe, my internship field supervisor, for allowing me access to Bishop Feild Elementary School and its students. I would also like to express my gratitude to Ms. Rosalyn Hudson for inviting me into her classroom to observe and work with her students. Her enthusiasm for and dedication to learning are exemplary.

A special thanks also to my family, friends and colleagues who have helped make completion of this degree possible.

ABSTRACT

This report describes the internship undertaken by the author, a high school mathematics teacher, in a Grade 2 class at Bishop Feild Elementary School in 1998. Much of this report is anecdotal in nature and details the investigations and explorations of the students as they engaged in mathematical problem solving. The children's mathematical learning through games, children's literature and manipulatives is considered in terms of the reforms proposed by the NCTM *Standards* documents (especially NCTM 1989). Connections between primary and high school problem solving strategies are made, with some suggestions for improvement at the high school level.

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CHAPTER I

INTRODUCTION

I chose to follow the internship route to complete my degree of Masters of Education (Teaching and Learning - Mathematics) to get a closer look at mathematics teaching, particularly problem solving, at the primary level.

The primary level was a deliberate and logical choice in which to spend the 10 weeks of my internship, as the learning environment allowed me the opportunity to incorporate many of the reforms suggested by the NCTM *Curriculum and Evaluation Standards for School Mathematics* (1989). In general, the primary curriculum is less directive and as a result there is more freedom and flexibility to explore various teaching techniques unique to this area. The nature of the primary curriculum is such that it lends itself well to cross-curricular teaching and learning. It is also an excellent environment for mathematical problem solving.

In particular, Ms. Rosalyn Hudson, the teacher who so graciously invited me into her classroom, has a strong interest in mathematics reform and unlike many primary teachers, also has a fairly strong mathematics background. Working with her was both exciting and challenging. She had numerous ideas for mathematical problem solving and was very creative in her attempt to link mathematics to various other areas of the curriculum. Some of these ideas are described in detail in Chapter II.

Having taught high school for the past seven years, this experience was quite an interesting, refreshing and enlightening change. It provided me with the opportunity to work closely with Grade 2 students, to watch and listen to them work, to interact with them and to question and inquire as to the reasons why they did certain things.

The main focus or goal of my study was mathematical problem solving and its application at the Grade 2 level. In conjunction with this, I hoped to gain experience in various teaching techniques not commonly used at the high school level, to make connections and comparisons between problem solving situations, strategies and learning environments in the primary classroom with those of the high school classroom. In addition, I wanted to examine and analyze student thought processes as they engaged in problem solving activities and to introduce, other, more advanced mathematical concepts as a source of enrichment.

For the most part I was successful in meeting the goals set out in my proposal. Recent trends in educational reform stress the importance of problem solving in a constructivist environment in promoting mathematical understanding. Problem solving and its lack of rigidity is thought to be more closely connected to a child's informal, rather than formal, system of mathematics and therefore lends itself to the constructivist approach to learning. I was able to work with the children in different problem solving situations in which mathematics was being constructed. Sometimes this occurred in small group settings and sometimes it was on an individual basis.

Through both my participation and observation I was able to note some similarities and differences between how students at the primary level worked in comparison with high school students. For example, both groups systematically relied on previous knowledge when working on unfamiliar problems, however, primary students were generally more persistent in their efforts. I was also given ample opportunity to observe and participate in alternate teaching techniques associated with games, manipulatives, stories, artwork and learning centers not common in high school.

Analyzing student thought processes as they engage in problem solving activities is a difficult task. I hope that through the many descriptions of classroom experiences included in this report, I will give the reader an idea of the variety of strategies and learning tools used and the thought processes of the students as they worked through the activities.

In addition to working with the prescribed curriculum, I was able to introduce some advanced mathematical concepts including fractions and tessellations. Fully aware that not all students would comprehend such enrichment, I kept it fairly simple. Some students experienced much success while others did not. Unfortunately, our biggest obstacle was the availability of time, as most of the days were spent covering the prescribed curriculum.

Although not specifically mentioned in my proposal, the opportunity to explore the connection between mathematics and children's literature also presented itself. Specifically, the use of mathbags provided a context for many mathematical ideas and allowed students to realize the variety of situations in which people use mathematics for real purposes. Problem solving activities that stem from sources that are not traditionally connected with mathematics allow children to develop flexibility in their thinking and gain a greater level of understanding. Because there are no algorithms to follow and no examples to model, students must choose and carry out their own problem solving strategy (Jaberg, 1995).

Also, having spent 10 weeks with the Grade 2 students in a mathematics learning environment, I think that I was able to gain some insight into possible difficulties experienced by high school students in mathematics. These difficulties are usually related to the type of understanding (instrumental versus relational) that develops. My observations regarding these difficulties are stated in Chapter V.

The remainder of my report has been organized as follows: Chapter II - Review of the Literature, Chapter III - Classroom Experiences, Chapter IV - Mathematics Enrichment, Chapter V - Connections Between Primary and High School, Chapter VI - Reflection.

Chapter II - Review of the Literature, offers an overview of previous research in the area of primary mathematics. It has been broken down into three subsections: Understanding, Problem Solving, and Contexts of Learning Characteristic of Primary. The section on Understanding briefly looks at how understanding is defined in relation to mathematics, the importance of well-connected networks, constructivism and the accumulation of knowledge. The Problem Solving section emphasizes the importance of diverse problem solving situations for the improved learning of mathematics by connecting mathematics to real life occurrences. It also describes the general process of how problem solving should work and makes reference to some specific problem solving strategies. The third section, Contexts of Learning Characteristic of Primary, examines the literature on teaching strategies utilized in the primary setting that are not common at the high school level. It includes the use of games and children's literature as two very effective contexts for both learning and reinforcing mathematical concepts.

Chapter III - Classroom Experiences, has been divided into two subsections: A General Description of the Context and Patterns of the Class and The Learning Activities That Took Place as They Relate to Problem Solving. The activities that took place include those introduced in the literature review, i.e., Games, Children's Literature and Manipulatives. Included also, is a section on Problem Solving that briefly describes some specific problem solving sessions, the problem solving processes used by the students and what they learned.

Chapter IV - Mathematics Enrichment, details work with mathematics content from later in the curriculum. It describes the activity, what the students learned, and whether these efforts were successful.

Chapter V - Connections between Primary and High School, discusses my observations on possible similarities and differences between problem solving at the primary level and the high school level. It includes the type of understanding that students develop and how this understanding influences student success. It also highlights some of the teaching techniques that, with modifications, could be useful at the high school level.

Chapter VI - Reflection, recaps what I feel I have learned from doing this internship at the Grade 2 Level and comments on how I have grown both personally and professionally.

CHAPTER II

REVIEW OF THE LITERATURE

'Teaching for understanding' is a phrase that has been used more frequently in mathematics education during the last decade or so. This is not meant to imply that prior to that time teachers were teaching for misunderstanding, but rather to encapsulate a wide range of reforms in mathematics teaching, including a shift in the focus of classrooms from teacher centered to student centered. The teacher is no longer seen as the 'sage on the stage' rather the 'guide on the side'.

Changes have come about primarily in response to a concern that students are not succeeding at mathematics or developing the necessary understanding that they need. Reform in mathematics education is required in the material taught, how it is taught and the desired learning outcomes.

In 1989, the National Council of Teacher of Mathematics (NCTM) published a document called the *Curriculum and Evaluation Standards for School Mathematics* (hereafter referred to as the *Standards*). This document called for major changes in mathematics education by placing an increased emphasis on problem solving, qualitative reasoning skills and creative and flexible thinking, with a decreased emphasis on mindless drill and practice. "How well children come to understand mathematical ideas is far more important than how many skills they acquire" (p. 16). Later success depends heavily on the quality of the foundation developed. Basic computational skills are no longer sufficient. It was hoped that these changes would bring about increased understanding. The NCTM will soon be releasing *Principles and Standards for School Mathematics*, a revised version of the NCTM *Standards*, which calls for similar reforms. Since it was not yet available in its final form when this was written, all references here are to the 1989 *Standards*.

Understanding

To determine if improved understanding is accomplished, it is helpful to look a little more closely at how understanding is defined. According to Hiebert and Carpenter (1992), understanding is defined

> in terms of the way information is represented and structured. A mathematical idea or procedure or fact is understood if it is part of an internal network. More specifically, the mathematics is understood if its mental representation is part of a network of representations. The degree of understanding is determined by the number and the strength of the connections. (p. 67)

As these networks become larger and more organized, one's understanding grows. Well-connected knowledge is remembered better. Better recall will then be linked with more successful mathematical experiences. The formation of these networks begins very early in a child's life. Their size and complexity gradually increase as children encounter new information and relate it, in a meaningful way, to things that they already know, a process referred to as construction of knowledge.

Piaget has long been thought of as one of the most influential proponents of constructivism. Constructivism is very closely associated with the work of Piaget's theory of intellectual development and has become widely accepted as a legitimate model of learning (Brooks & Brooks, 1993; Coles & Price, 1998; Littledyke & Huxford, 1998; Hiebert & Carpenter, 1992, Richardson, 1997). Piaget contends that learning occurs through a process of assimilation and accommodation. Learners actively construct their own mathematical knowledge and understanding by making connections between new facts and procedures and existing internal networks. Students must internalize new information and relate it to their own experiences for it to become meaningful. Knowledge is built from within one's mind rather than from the outside in.

Children do not enter school void of mathematical knowledge. They have developed/constructed an informal system of mathematics in much the same way the they have developed the ability to talk. Therefore, it is important that teachers build upon this informal and intuitive knowledge, not by teaching formal mathematics, but by helping to construct connections between new information and what is already known (Ginsburg & Baron, 1993).

In some cases, individual's internal representations are flawed and new information does not fit into existing networks. As a result, networks must sometimes be reorganized or rearranged. New connections are formed and old connections may be adjusted or rejected (Hiebert & Carpenter, 1992). This adaptation of one's present networks and connections is what Piaget referred to as accomodation.

Because individuals know the world only through their experiences, the knowledge they construct is very subjective. The question of which knowledge is truth often arises. "In constructivism the concept of truth is replaced with viability" (Wheatley, 1991, p.10). von Glasersfeld (as cited in Bodner, 1986) describes the construction of knowledge as a search for a 'fit' rather than a match with reality. To maintain a constant fit, individual knowledge is continuously being tested and revised as knowledge construction is a dynamic process. Therefore it is possible for individuals to possess the same knowledge at the same time because only specific knowledge will 'work' in a specific context.

Constructivism and the accumulation of knowledge and understanding can also be related to the work of Skemp (1989), Ausubel (1985), Hiebert & Lefevre (1986) and others. Skemp outlines two different models of understanding: relational understanding (intelligent learning) and instrumental understanding (habit learning). Relational understanding involves knowing both what to do and why, whereas instrumental understanding involves using rules without reasons or true understanding of why. When students learn in contexts designed to facilitate their construction of relations between concepts, rather than being expected to receive ideas passively, they are more likely to develop relational understanding.

Similarly, Ausubel (1985) speaks of two types of learning: meaningful learning and rote learning. Meaningful learning occurs if the learning task can be related specifically to what the learner already knows. Rote learning occurs if the learning task consists of purely arbitrary associations that are unrelated to prior knowledge and are therefore meaningless. When meaningful learning occurs, one's existing networks are modified, along with the newly linked information, to form more accurate and relevant networks.

Hiebert and Lefevre (1986) and Hiebert and Carpenter (1992) describe a similar situation referring instead to knowledge. They distinguish between conceptual knowledge (knowing why) and procedural knowledge (knowing how). Conceptual knowledge is rich in relationships and connected networks of understanding, whereas procedural knowledge is a sequence of actions needing minimal internal connections. Student performance based solely on procedural knowledge is likely to be unsuccessful as there are no well formed internal networks with which to make connections.

> If conceptual understandings are linked to procedures, children will not perceive of mathematics as an arbitrary set of rules; will not need to learn or memorize as many procedures; and will have the foundation to apply, re-create, and invent new ones when needed. (NCTM, 1989, p. 32)

Arguments emphasizing the importance of both types of understanding/learning/knowledge have been put forth (Byrnes & Wasik, 1991; Hiebert & Carpenter, 1992; Miller & Kandl, 1991), and rightly so. Both types of knowledge are essential for mathematical expertise. Understanding why a process works is of little use if you do not also know how to perform the required task.

"Procedures in mathematics always depend upon principles represented conceptually.... If the learner connects the procedure with some of the conceptual knowledge on which it is based, then the procedure becomes part of a larger network, closely related to conceptual knowledge" (Hiebert & Carpenter, 1992, p.78). This interconnectedness leads to flexibility in mathematical problem solving.

Initial exposure to new mathematical concepts should focus first on building relationships (conceptual knowledge) rather than on becoming proficient executors of procedures. Unfortunately this series of events usually occurs in reverse order in most classrooms. Proficiency through drill and practice and rote exercises is followed at some later point with trying to connect bits and pieces of previous knowledge into networks. No wonder these connections are not always made successfully by either the students or the teacher.

Problem Solving

With so much talk about how students gain knowledge and understanding, it would be beneficial to include some suggestions for accomplishing this task. Much recent research (Broomes & Petty, 1995; Franke & Carey, 1997; Hembree & Marsh, 1993; McLeod & Adams, 1989; NCTM, 1989; National Research Council, 1989; Schoenfeld, 1992; Taback, 1992; Woods & Sellers, 1997; Worth, 1990) suggests that this goal can be achieved within a problem solving classroom environment. Problem solving opportunities allow students to explore and discover how mathematical concepts are connected. It invokes the ability to think, reason, create, and seek solutions to problems that are meaningful to the students. It should be noted that a problem is any task in which the solution or goal is not immediately attainable or apparent, and what may be seen as a problem for one student may not be problematic for another.

Problem solving is a dynamic, ongoing process. "It is the means by which an individual uses previously acquired knowledge, skills, and understanding to satisfy the demands of an unfamiliar situation" (Krulik & Rudnick, 1989, p. 5). Problem solving is a very fundamental and spontaneous occurrence, however, one that requires practice to achieve proficiency. There really is no right way to teach problem solving. It involves the use of many and varied strategies called heuristics. A single problem may be tackled in a variety of ways each leading to an acceptable solution. Individual strategies, such as 'working backwards', 'using a diagram, chart or table', 'generalizing', 'looking for patterns', 'guess and check', 'using simpler but related problems', 'modeling', and 'elimination', can be taught and mastered through practice, imitation, memorization, cooperation, reflection and osmosis (Kilpatrick, 1985). It is generally believed that students who follow Polya's (1945) four basic steps to problem solving - (a) understanding the problem, (b) devising a plan, (c) carrying out the plan, and (d) looking back - tend to improve their problem solving abilities. However, problem solving skills take time to develop. Becoming a good problem solver is an active process. Only by solving problems will students become effective problem solvers (Barba, 1990).

It should also be noted that the process of problem solving does not always end with positive results or the correct answers. Very often Polya's four step plan is cyclical rather than linear. Many different solution strategies are tried before the 'looking back' shows that the problem is indeed solved. This trial-and-error process is the very essence of problem solving. How children handle disappointment will greatly influence their willingness and desire to engage in future mathematical problem solving. Therefore, the problem solving process should include learning to cope with being unsuccessful and developing a positive attitude. Failure is just one step toward finding a solution. It should be seen as an incentive to try something new, not as a signal to give up.

One of the most fundamental problem solving processes is modelling, a process that comes relatively naturally to most primary grade children. Young children exhibit intuitive analytic modelling skills which enhance their problem solving abilities and enable them to solve problems which are seemingly beyond their capability. Surprisingly these skills seem to wane as students progress through to high school. Through the use of modelling, young children are able to solve addition, subtraction, multiplication and division problems much earlier than generally thought (Carpenter, Ansell, Franke, Fennema & Weisbeck, 1993). Modelling provides a direct and coherent link to children's thinking patterns.

> Modelling provides a framework in which problem solving becomes a sense - making activity. As a consequence, a focus on problem solving as modelling may do more than just provide cognitive schemes for solving problems. It seems likely that it will also have an impact on children's perception of problem solving and themselves as problem solvers. If from an early age children are taught to approach problem solving as an effort to make sense out of problem situations, they may come to believe that learning and doing mathematics involves the solution of problems in ways that always make sense. (Carpenter, Ansell, Franke, Fennema & Weisbeck, 1993, p. 440)

In order for students to become good problem solvers they must learn to think for themselves. Having the teacher continuously tell students the answers and where these answers came from limits the need for investigative and exploratory thinking. In addition to figuring out the answer, students should know how they came to the solution and why it works. Students are also encouraged to find more than one way to solve a problem. They need to be able to make the mathematics behind the problem more personally meaningful. That is, they need to develop a solution that makes sense to them from their point of view within their own personal reality. Only then will they truly become good problem solvers.

This process is often unintentionally impeded by having students complete and master the problems found in text books. Unfortunately, these are not truly problems at all. For the most part, students are simply applying a model, which has been developed and presented in class, to solve routine problems in which only the numbers have been changed. Very little higher-order thought is required. While such practice is important it should not be confused with problem solving (Krulik & Rudnick, 1989).

Closely related to problem solving is the process of problem posing. Problem posing involves generating new problems and questions to explore about a given situation, as well as reformulating a problem during the course of solving it (Silver, 1994). Problem posing can occur before, during, or after solving a given problem.

Not only does problem posing incorporate all the aspects of problem solving, it allows students to move beyond the boundaries set down by text books, permitting them to explore areas that may be more personally meaningful to them. It changes the locus of control from the teacher/text book to the student. Children tend to become more diverse and flexible in their thinking, more responsible for their own learning, more confident in the mathematical ability and better overall problem solvers.

Situations for using problem posing can be many and varied provided that teachers are aware of how to use questioning effectively to promote problem posing. "Questions that are generative in nature, that is, ones that lead to further ideas and questions, are important in guiding children's problem posing efforts" (English, 1997, p.173). Initially these questions may be posed by the teacher as an example of the types of questions that should be asked. Over time, the students will learn to pose their own questions, gradually modifying their own thinking in ways which will broaden their views on mathematics. They will learn that mathematics can be approached in different ways and that there is often more than one correct answer to a specific problem.

Problem posing can be utilized in relation to other typical non-mathematical situations. Events such as daily rainfall or temperature can provide a meaningful context in which both teachers and students can pose and answer questions. Other avenues that can facilitate problem posing include geography, current events and school populations.

Contexts of Learning Characteristic of Primary

Games

Puzzles, games and recreation in mathematics have been in existence for a very long time. Unfortunately, games have often been associated with frivolity and less than serious pursuits. On the contrary, games can provide a valuable setting for the development of mathematical problem solving strategies and should be incorporated into instruction (Kraus, 1982). Games can be used prior to instruction to introduce or develop a topic or concept, during and as a part of the instruction, or after instruction to practice and reinforce skills depending on the instructional level as well as the taxonomic level (Bright, Harvey & Wheeler, 1985). The latter, maintenance games, are probably used most frequently.

> One of the most important aspects [of games] may be that while students are playing, they have control over the game-playing situation. Because of this control, they are often willing to risk making errors by trying to expand on what they know without fear that they will fail in the eyes of the teacher or their peers. (Bright & Harvey, 1982, p. 205)

After all, it's only a game!

Games generate enthusiasm, excitement and enjoyment. They remove much of the drudgery associated with traditional teaching methods. Students usually become strongly motivated and actively involved in the game, involvement which is necessary for learning to occur (Ernest, 1986). Games also encourage improved student cooperation and stimulate mathematical discussion. Games are also very effective with the slower students providing them with the opportunity to develop their skills and concepts without threat (Williams, 1986). Games, in which chance plays a part, perhaps where a die or a spinner or a coin is used, can be helpful in giving weaker players a better chance (Begg, 1997). Not all games require a winner or a loser. Some games just involve class participation. In general, games can significantly enhance student achievement and success in mathematics if they are selected on the basis of the desired objectives and are incorporated into the teaching program.

Children's Literature

Children's literature also supports the art of problem solving and problem posing. Mathematical stories have details that can be changed to create different dimensions of a problem. The original story or problem thus becomes the basis for a multitude of related problems. The more opportunities children have to alter a particular story problem, the greater their understanding of the underlying mathematical concepts will become.

The NCTM *Standards* encourage the use of children's literature as a medium for communicating mathematical ideas. "Children's literature helps to break down the artificial dichotomy that sometimes exists between learning mathematics and living mathematics" (Whiten & Wilde, 1992, p. 4). Children's literature also helps to alleviate much of the anxiety that children sometimes feel toward mathematics. Because mathematics is approached in a non-threatening positive way, children's attitudes tend to reflect this positiveness. Using children's literature to enhance the teaching and learning of mathematics in primary school is widely encouraged (Braddon, Hall & Taylor, 1993; Griffiths & Clyne, 1991; Halpern, 1996; Welchman-Tischler, 1992; Whiten & Wilde, 1995).

Children's literature also offers an excellent way to promote problem solving in mathematics. It provides a context for many mathematical ideas. It also allows students to realize the variety of situations in which people use mathematics for real purposes. Books portray mathematics not as a bunch of symbols and potentially frustrating mental tasks, but as a tool for making decisions and solving problems. They are also great motivational tools. Children's literature provides a social context for mathematics, and fosters student discussion of mathematical issues. Like all forms of knowledge, mathematics is a consequence of social interaction. "Social constructivism acknowledges and emphasizes how personal constructs are formed within a social context involving exchange, accumulation and challenge of individual's views" (Littledyke, 1998, p. 3).

Children's literature provides rich sources of problem solving situations. Some problems may be posed by the author, others by the teacher and still others by the students. Because the questions are usually open-ended and thus open to multiple interpretations, one piece of literature can effectively be used with an entire class, stimulating the needs and interests of students of various ability levels (Carey, 1992).

CHAPTER III

CLASSROOM EXPERIENCES

A General Description of the Context and Patterns of the Class

I began my 10 week internship in the Grade 2 classroom of Bishop Feild Elementary School with Ms. Rosalyn Hudson by observing the day-to-day operations of the classroom. The class consisted of five boys and 17 girls, aged 7 and 8, who sat in groups of six around the room for their seatwork. Much of the time, however, was spent in large group instruction and discussion on the floor in the middle of the room. The language of instruction in the class was English. Most days were organized as follows: 8:55-10:20 language arts/health; 10:40-12:00 mathematics/science; 1:00-2:15 anthology/spelling; 2:15-3:00 learning centers. My primary involvement was in the daily mathematics lesson and the afternoon mathematics center, however, I observed and assisted with all aspects of the curriculum. Being involved with the students from the beginning to the end of each day enabled me to get to know them (academically and personally) as well as allowed them to get to know me. Initially the students were very shy and refrained from interacting with me in any way. However, after a few weeks they were as comfortable with me as I was with them. The students were generally enthusiastic about mathematics. They were very persistent, not easily frustrated, and frequently experienced satisfaction upon successful completion of personally challenging problems, not unlike their high school counterparts. What most set them apart from the students I normally teach, aside from their small size, was their energy and enthusiasm. They were much more outgoing, inquisitive, open, innocent and always seeking my attention and approval.

As a high school teacher, it was also strange being in a second grade classroom because the furniture was so small. There were little desks and chairs and numerous containers of colorful manipulatives everywhere. There were five learning centers located at various places around the classroom where students worked on a particular task everyday. The walls were plastered with samples of students' work, including writing and artwork. The chalkboard was never used. Only a small white board, in the middle of the room, was used when the students gathered together on the carpet for instruction. Such a set up seems to facilitate the teacher's role as a guide rather than a teller. Because instruction often occurs from within a group or from students themselves with some direction from the teacher, it allows for learning to occur in a manner more in keeping with that suggested in the NCTM *Standards*.

The Learning Activities That Took Place as They Relate to Problem Solving

Problem solving was the focus for both my internship and report. As mentioned earlier, problem solving is a way of thinking and reasoning in a creative manner to solve a given problem. Problem solving can take many forms and can be applied to many areas of the curriculum. While the emphasis of my report is on mathematical problem solving, the problem solving that I witnessed was not confined to this area. There were opportunities to employ one's problem solving skills in literature, language, games and activities. Problem solving was not a label applied to the various tasks, rather it was implicit in each particular situation.

What follows is an account of many of the problem solving activities and observations that I encountered during my time at Bishop Feild School. The activities made use of games, manipulatives, and children's literature to teach many mathematical concepts including numeracy (currency, number sense and place value), estimation, patterning, symmetry, addition and subtraction. Rather than give a daily account of what happened, I will try to organize the descriptions according to the type of activity rather than the mathematical content.

Games

As mentioned previously, the use of games for teaching mathematics through problem solving is very effective. They improve student's motivation and remove the fear often associated with learning new concepts. Games were used frequently as a learning strategy in many content areas including place value, spatial perception and geometry, analytical thinking and numeracy. In some cases the games required the use of manipulatives, however the main focus of the activity was the game, therefore I have described these activities in the section on Games.

The following game required the students to find the total score accumulated by the bean bags and in so doing, to reinforce the students' understanding of place value. Each child was given a bean bag and asked to toss it into a target consisting of three concentric rings which was placed on the floor. The center ring had a value of one hundred, the middle ring was worth 10 and the outer ring was worth one. After each student had thrown his/her bean bag the total amount was added together, starting with the outer ring. Each new bag was added one at a time. This game was repeated several times. When the number of bean bags in each ring was less than 10, there were no problems. However, on occasion, the number of bags in the middle or outer ring was more than 10. This created an interesting problem for some of the students. One particular trial yielded 4 ones, 11 tens and 3 hundreds. Various students volunteered answers. One guessed 314, another guessed 340 and yet another guessed 414. Once the correct answer was reached, an explanation was requested. The child who gave the correct answer was not quite sure how to explain where it came from. She said she took 1 from the 11 and added it to the 3 to get 4, but she was not sure exactly why. Another student explained that you cannot have more than 10 tens because that would make one hundred. Therefore the number of hundreds increased from 3 to 4. There was then 1 ten left over and it was combined with the 4 ones to make 14.

The main goal for this activity was for students to realize that not all bean bags represented the same amount. When attributes like color or position are the only things that signify quantity differences, there is the chance for ambiguity. Therefore, when using beanbags or colored counters in this way (non-proportional modelling) it must be done cautiously. Visually there is no difference between the appearance of a tens bean bag or a ones bean bag, only its location on the target determines its value. However, when using the units (ones), sticks (tens) and flats (hundreds) to model numbers (proportional modelling), it is clear to most students that a stick is ten times as large as a unit and likewise a flat is ten times as large as a stick. It is easier for the students who are having difficulty to see the relationship between these blocks than when other objects are used. Gradually, however, students began to notice how different digits of the three digit number changed depending on where the bean bag landed.

Another game, designed also to improve the concept of place value, was introduced in the mathematics center. The materials for the game included one spinner divided into three equal sections (ones, tens, hundreds), a game card for each student divided into three sections (ones, tens, hundreds) and five counters for each student. The object of the game was to end up with a number in the middle. The students could have a maximum of five spins of the spinner to create their number, however, five spins were not always necessary. They had to decide when to stop spinning based on the numbers the others had come up with so that their number was somewhere in the middle. It took a couple of games before everyone fully understood the idea. It was unusual not to want to have the largest number. This activity required some advanced reasoning skills to help the students decide when to stop spinning. Whether you spun first, last or somewhere in the middle also influenced your decision. To help them make their decisions I would ask questions along the way, such as "Who has the smallest/largest number now?", "Who is currently in the middle?". This game worked better with some children than with others. After having played the game several times through, some children became bored. Some of the brighter students in the group deliberately used all five spins to get the highest number so she/he would lose, thus ending the game sooner.

Games were also used in other areas of the mathematics curriculum. A game called the 'One Way Different Train' also reinforced many geometric concepts. The students were all given a couple of pattern blocks which varied in size (big/small), shape (rectangle/square/circle/triangle), thickness (thick/thin) and color (red/blue/yellow). The game began by placing one block on the floor. In turn, students could then add blocks to form a train, however the piece that was added had to be different from the previous piece in only one way. Hence, the problem posed was to figure out which blocks could and could not be used. This required the students to think about all four characteristics of the blocks at the same time. Sometimes, the students had more than one block that satisfied the conditions of the game and could choose either one. Most students had no trouble with this, however, when necessary, other students offered a little help. In most cases, this game continued until all the blocks had been placed on the floor. Occasionally, it would happen that no student had a suitable block to add to the train and the game would end. The students really enjoyed this game.

Numeracy was another area where games were used. To improve the number sense skills of students, I borrowed a game from "*Math Wise: Hands-On Activities and Investigations for Elementary Students*" (Overholt, 1995), called 'Who has….? I have ….'. A set of cards, each containing one question and one answer, was divided evenly among the entire class. A student would begin the game by asking the question printed on his/her card. The idea was to figure out the answer and then check to see if your card had the correct answer. If so you would respond by saying "I have ...". Then that same player would read out the question portion of his/her card and the process would repeat until all questions and answers had been used up. All the students got the opportunity to ask and answer a question and there were no winners or losers. Sample questions include, "Who has the number of feet on four people?", "Who has the number of days in February?", "Who has 7+8?".

Another game, called 'Guess the Mystery Number' (Overholt, 1995 (with modifications)), was also used to improve numeracy. A mystery number was selected but kept secret. Next, a clue number was chosen and revealed to the rest of the class. This clue number had some characteristic in common with the mystery number. For example, a clue number of 44 might signal a mystery number having a tens digit of 4, a units digit of 4, both digits the same or digits that sum to eight.

Students would take turns guessing the mystery number until the correct answer was reached. A hundreds chart was used to help students with their guesses. Each incorrect answer was covered so it wouldn't be guessed twice. The child who guessed correctly now chose the new mystery number. It was a good idea to secretly record the mystery number as sometimes the students would get so caught up in the game that the number would be forgotten. Games, of course, are always fun, but being able to choose the mystery number increased both the involvement and enjoyment of the students.

Mathbags and Children's Literature

As mentioned previously, children's literature is an excellent way to develop mathematical concepts that are naturally embedded in story situations. Ms. Hudson capitalized on this fact in her Grade 2 classroom and invited her students to explore mathematics in their own lives and the lives of others through the use of mathbags. The mathbags were usually given out on Friday afternoons, thereby allowing the students the weekend as well as the first part of the week to complete the work. Each bag consisted of a book (or two), manipulatives, and a task card detailing the activities required. Although the books were not directly related to mathematics, the questions required the students to make connections between the stories they read and mathematics. (See Appendix A for a list of the books used.)

Here are but a few of the titles and the types of questions that were used to explore mathematics. *The Shopping Basket*, by Burningham, was a story that involved a trip to the market to buy various items: 6 eggs, 5 bananas, 4 apples, 3 oranges, 2 doughnuts and 1 pack of crisps. While returning home the boy encountered hungry animals who demanded the food. Rather than give all of the items away, the boy tricked the animals and ended up losing only one of each item purchased. Numeracy and spatial representation were examined through questions like, "If each item cost 5 cents, how much money was needed to buy the items?", "If the boy was also asked to buy cupcakes, how many do you think he would have to buy? Explain?", and "Draw a map of the route you would take to get to a store near your house.".

Dad's Diet, by Comber, describes a father's attempt at losing some weight. The Dad loved food and would cheat on his diet when Mom wasn't looking. Initially he weighed 189 lb. and in the end he weighed only 162 lb. Both of these amounts were compared to that of the other family members. For example, 189 lb. was approximately equal to 4 sons or 7 daughters or 1.5 mothers and 162 lb. was approximately equal to 3.5 sons or 6 daughters or 1.25 mothers. Measurement, mass and numeracy were emphasized by having the students repeat a similar comparison of their weight to that of their other family members. This story could also be used to explore fractions, if the student felt comfortable with that concept.

Mr. Archimedes' Bath by Allen was one of the most frequently requested mathbags. Mr. Archimedes, who liked to bathe with his kangaroo, goat and wombat, could never figure out why the water in his tub overflowed every time he took a bath. For some reason, each time they all got in the tub, the water level rose. Mr. Archimedes blamed each of the animals but then discovered that he himself also caused the water level to rise. He finally figured out the cause of the overflowing water and then spent the rest of the day jumping in and out of the tub watching the water level rise and fall. Also included in this mathbag was another book by Allen called *Who Sank the Boat?*. Measurement, mass and volume were investigated by having the students mark the water level in their tub before and after they got in and compare the measurements. Similarly, using a toy boat and some plastic animals, the students were asked to add one animal at a time and observe what happened. "At what point did the boat sink?" and "Will adding the animals in a different order change anything?", were just a couple of the questions students were asked to answer. Once out of the tub, students were asked to fill a glass half full and mark the level with tape. Next, they were to add pebbles one at a time and watch the water level rise and record their observations.

These mathbags were meant to be done in cooperation with parents/guardians. They provided an opportunity for parents to see one aspect of the child's mathematics education as well as provided a source of quality parent - child interaction. The students really looked forward to taking the mathbags home each week. As with any other task, varying degrees of energy, enthusiasm and effort were put into the accompanying homework. Some students wrote very complex explanations and included illustrations in their journals while others wrote very little or nothing at all. Some students even extended the exercises and made additional observations above and beyond those expected. It was easy to see the types of reasoning and problem solving that were taking place simply by reading what the students had written.

One student, for example, having completed the tasks for the *Dad's Diet* mathbag, decided to make a comparison of foot size. Each family member traced the outline of his/her foot onto a piece of paper. The feet were then cut out and ordered from largest to smallest. It was determined that a person's overall size was also reflected in his/her foot size - the bigger the person, the bigger the foot. Another mathbag using the book *Counting on Calico* by Tildes, investigated counting using cats. Toward the end of the story, the mother cat was carrying some unborn kittens. The number of kittens was not mentioned, only the number of paws. The students were asked to determine how many kittens were inside the mother cat if there were 20 paws. One student actually drew 20 very nice paw prints in her journal and then drew circles around each group of four. Once the last group of paws was circled, the student was clearly able to see that there were five kittens waiting to be born. This activity illustrates how higher order thinking and tasks (such as division) can be easily and successfully accomplished through the use of problem solving.

The type of problem solving described in the section on Games had the students investigating and reinforcing a single mathematical concept or skill. The problem solving skills necessary for use with the mathbags demanded a much higher level of thinking. Many of the skills acquired from previous problem solving activities were used in combination when the students worked with the mathbags. There was more of a real world application of the mathematics and the activities illustrated to the students that mathematics can be both meaningful and fun.

Manipulatives

The use of manipulative materials in the mathematics classroom supports the development of understanding in students of various ages and developmental levels. Manipulatives may be used to introduce new concepts, verify results and/or provide remedial help. Where possible, a variety of manipulatives should be used to develop each concept, the different representations helping students to generalize mathematical ideas (Nova Scotia Department of Education and Culture, 1996).

Manipulatives were perhaps the most important learning tool used in the Grade 2 classroom, and their application extended to include many areas of the mathematics curriculum. Almost anything could be used as a manipulative, including units, sticks and flats, coins, beans, candy, plastic cups, blocks, and buttons. While manipulatives can be purchased commercially, perhaps the best ones are those that can be found around your own home. Manipulatives allow students to work with something tangible and concrete, providing them with a visual representation of the problem at hand. For those students who cannot think and reason abstractly yet, manipulatives are very beneficial.

The ones, tens and hundreds blocks (units, sticks and flats) were frequently used. These helped the students distinguish between the different quantities and to learn how they were related to place value. Students knew that 10 units could be replaced by 1 stick, and that 10 sticks could be replaced by 1 flat. When using these blocks to model simple addition and subtraction, such as 23 + 45, there were few problems. However, some students were unable to start with the correct representation, or if they did, they combined the blocks in the end as if they all had the same value. In other words, they added the individual digits together, e.g., 2 + 3 + 4 + 5 = 14. This problem was limited to only a few students who still had difficulty distinguishing between the values of the blocks.

When used properly, the students realized there was a combined total of 8 units and 6 sticks. To arrive at the correct result, the students counted by tens up to sixty and then added eight for a total of sixty-eight. Although they eventually arrived at the correct answer, some students did not take advantage of the model when trying to arrive at the final answer and began counting up by ones, rather than tens. Such inconsistency demonstrated that place value still was not solidified in their minds.

Similarly, coins were also used many times to reinforce place value. Not all coins were used however, only dollars, dimes and pennies to represent hundreds, tens and ones. Students were given different numbers of plastic coins and were required to figure out how much money they had. Using their prior knowledge of money, they counted by tens when working with dimes and hundreds when working with dollars. Money provides a much different representation of place value than blocks, colored counters or beanbags because children often have an internal network for understanding money that has been constructed through a variety of experiences, usually out of school. Knowledge represented in the network may support features of money that capture place value principles, such as appropriate values of pennies, dimes and dollars and the 10-for-1 trading that can be done with these pieces (Hiebert & Carpenter, 1992). Once again this extension to money should be limited so as not to confuse students. Some may wonder what role quarters and nickels play in such activities.

Another activity with money, not directly related to place value, was using coins to represent various monetary amounts. For example, students, working in pairs, were given

1 quarter, 4 dimes, 2 nickels and 4 pennies and were asked in how many different ways they could make 37 cents using only the coins they had. This activity presented few problems for most. After about ten minutes, students were invited to share some of the combinations they came up with. Examples included: 3 dimes, 1 nickel and 2 pennies; 1 quarter, 1 dime and 2 pennies. When the coins were arranged in a different order, some students mistakenly thought that this was another possible solution. However, upon closer consideration, they quickly realized that they were mistaken. Many students surprisingly misinterpreted what they were asked to do and I wondered why no one asked for clarification. Despite the confusion all groups demonstrated some worthwhile mathematical thinking.

By posing questions in the above open-ended manner, rather than by asking questions that require unique answers, children are encouraged to think about the many possibilities that are available. This allows all students to participate and contribute, as very often the number of answers are extensive.

Manipulatives can take on many forms; and in this activity, designed to improve the children's estimation skills, the manipulatives were chocolate chip cookies. Students were asked to estimate the number of chocolate chips that were baked inside some cookies. Each student was given two cookies. First they had to estimate the number of chocolate chips by visually examining the cookie. Then they had to count the actual number of chips by breaking the cookie into pieces. Students then brought their actual and estimated amounts to a group discussion. I asked them whether they thought they had made 'good' or 'bad' estimates. Most estimates were very close to the actual number, although others were way off. I also asked them if the number of chips counted in the first cookie influenced their estimate for the second cookie. Surprisingly, students did not seem to use the first amount to help determine the second amount. Perhaps this would have been a good time to discuss how students could use their previous knowledge to help them predict the number of chips they might expect in any future cookies. Since most chip counts centered around a single value, it could also have been pointed out that it would be very unlikely that very small or very large numbers of chips would ever be found, thus touching upon the idea of probability.

As a second part of this activity, students got the chance to work with the data collected and to organize it into a bar graph. The graph indicated the various number of chips found in the cookies. The smallest amount was 19, the greatest was 31, and 25 occurred most often.

This activity took place just before recess which allowed the students to enjoy the remnants of the investigation as a snack. They really seemed to enjoy this investigation and planned to repeat it in the future, perhaps using a different brand of cookies and to compare the results. This activity not only made use of estimation, but included some statistics and probability too, all of which are stressed in the *Standards*.

The study of patterns and relationships is also included in the *Standards*, and makes very good use of manipulatives. "Pattern recognition involves many concepts, such as color and shape identification, direction, orientation, size and number relationships" (NCTM, 1989, p.61). Many activities that focused on patterns were performed as whole group activities and mathematics center tasks throughout my 10 week visit.

Growing patterns were explored by the students in one of the weekly mathematics centers. They investigated how different patterns (triangular, rectangular and others) grew by completing various activities using manipulatives such as blocks, sticks and plastic cups. Students were asked to determine the number of objects needed to create the nth representation of a particular pattern having been shown the first three representations. Some patterns were simpler than others, but in all cases students used the manipulatives provided to build models and count the number of parts. Most counted by ones, whereas others counted by twos or fives and reached the total a little faster. Translating the observed growth of the pattern into an algebraic expression was something I considered doing with the students, however, in my professional judgment, this was beyond their mathematical scope and so the idea was abandoned.

Repeating patterns, involving both numbers and shapes, were also investigated through the use of manipulatives and seat work using activity sheets from their mathematics texts. Students seem to be able to identify and replicate repeating patterns a little easier than growing patterns. Perhaps this is because they have had more exposure to things that repeat and can more easily relate to this concept.

Another mathematics center activity, whose main focus was on manipulatives, had students forming specific patterns using sticks. Each student was given nine sticks of equal length and asked to make as many triangles as he/she could. Not surprisingly, all students initially formed three separate triangles. I told them that there were more triangles possible and urged them to try to find the additional ones. At first they seemed doubtful, but, after learning that the triangles could share a side, they once again started rearranging their sticks to produce the missing triangles. After a short time most students in the group found the desired arrangement. Those who had difficulty took some direction from the work of others and used completed arrangements as models to help aid in the creation of their own. The arrangement that produced the greatest number of triangles was itself a triangle with four smaller triangles inside. In response to the question, "How many triangles are there?", students responded without too much thought that there were four. However, following a little prompting, all students were able to find five triangles.

It was interesting to note how we (both teachers and students) put restrictions on our thinking by creating and imposing conditions that do not really exist. These conditions often limit our thinking and thus interfere with our problem solving success. Students initially thought that the triangles must be separate from one another, which influenced their responses. Having gotten past that misconception, students then assumed that all triangles had to be the same size. This, too, limited their thinking.

Using the arrangement mentioned above, the students were then asked to remove a specific number of sticks such that a certain numbers of triangles would remain. After a little thought and some trial and error attempts at removing the proper sticks, most students came up with the correct configuration. Several questions of this sort were asked and the results were positive. To conclude this activity, I gave each student three additional sticks bringing the total to twelve. The task, now, was to use the sticks to form six triangles. Knowing that the triangles could be joined from previous work, the students set about forming various shapes. Most attempts saw the students running out of sticks before having formed the six triangles. Some tried going in a linear fashion while others appeared to follow no set plan. Following many minutes of 'playing' with the sticks, the desired pattern was formed. It was a regular hexagon with six inner spokes forming the six triangles. Each student then formed the pattern for himself/herself. Students commented that the design resembled a wheel. I then told them the name of the shape and asked them if they had seen it anywhere before. Some students mistakenly thought that it was the same shape as a stop sign. Although a stop sign has eight sides, the regularity of the shape did not go unnoticed as being similar to that of a hexagon. A few students, who were doubtful of the difference, actually counted the number of sides on a stop sign on their way to school the next day and informed me that they did have a different number of sides. This is just one indication that problem solving does not end when students leave school each day, that it is a continuous and life long process that people use everyday.

After the stop sign discussion, I urged the students to look around the classroom for objects that were hexagonal in shape. Students went immediately to the box of pattern blocks to obtain the hexagonal shaped block. Of course they were correct, yet I prompted them to look again. After a little while one of the students noticed that the table on which we were working was hexagonal. Actually, it was made up of two smaller trapezoidal shaped tables that joined together to form a hexagon. Other examples of hexagons included the cross section of the big classroom crayons and some ball-point pens. Overall, the children seemed to enjoy this activity perhaps because it used manipulatives or because it was like a game to see who could find the required patterns and shapes first.

Included in the study of patterns and relationships is the study of geometry. During my time with the students they spent a lot of time working with the idea of symmetry using many different manipulatives. They discussed lines of symmetry and investigated them using mirrors, paper folding and pattern blocks.

One such activity had the students complete the mirror image of a magazine picture. They were given a box full of photographs of animals, people, buildings and trees that were cut in half. They chose one they liked, glued it to a piece of paper and then drew and colored the missing half as much like the original as possible. Students enjoyed this and most did a remarkably good job.

Another exercise had the students creating symmetrical designs using pattern blocks and a line of symmetry formed with tape. Whatever blocks they arranged on the left side of the line had to be mirrored with blocks on the right side. Some children created some very elaborate designs and, in some cases, even went three dimensionally, moving upward as well as outward.

Problem Solving

With many of the previously described learning activities it is difficult to explicitly state the problem being solved. In many cases the problem was implicit in what the children were asked to do. By engaging in problem solving in this manner, it is seen not as a distinct topic, but rather a process that permeates the entire mathematics curriculum.

Perhaps the most commonly employed strategy was the creation of a model to represent the problem, through manipulatives. Units, sticks and flats were used to model numbers when adding, subtracting and estimating, whereas plastic cups and blocks were used when investigating patterns.

Sometimes students were able to analyze how they solved a similar yet simpler problem and applied the same technique. For some students this meant using manipulatives, for others, using the hundreds chart.

Problem solving is occurring anytime a student is able to find a way around an obstacle, to find a way out of a difficulty or to attain a desired end where none was known previously. What follows are some specific examples that illustrate the type of problem solving strategies the students used, as well as what they learned.

Toward the end of my internship, I presented the students with a group of word problems, some routine and others non-routine, to give me an idea of the problem solving strategies that the students possessed (See Appendix B). These problems were presented, one at a time, to students in groups of five or six, during the mathematics learning center. Most students were able to answer at least four questions and some even completed six or more. To help students figure out the answers, I provided them with various manipulatives and gave them some hints when necessary.

Many students drew diagrams to represent the problem situations. For example, when asked to figure out how many of 20 worms each of four birds would get if they were shared equally, some students drew 20 worms and grouped them into four groups by drawing circles around them. Although division is not part of the Grade 2 curriculum, when done through a problem solving activity such as this, the solution is easily attainable.

Similarly, when asked to figure out how many pieces of gum there are in three packages if each package has six pieces, students drew three boxes with six sticks in each one. They then counted the total number of sticks and knew that there must be 18 pieces of gum. Once again, multiplication, a topic not included in the curriculum, can be investigated through problem solving. Both problems mentioned above were solved correctly by other students using manipulatives (blocks), rather that diagrams.

Problems involving more than one operation were also solved in the same manner. For example, to solve the problem, "Mary had three packages of cupcakes. There were four cupcakes in each package. She ate five cupcakes. How many were left?", students drew three boxes, each with four circles inside and then crossed out five of them leaving seven.

Other problems were solved using patterns. One such problem asked students to figure out how many squares there were in a particular diagram. Since the squares ranged in size (small, medium and large), I suggested counting the small ones first, followed by the medium and then large. By systematically counting the squares in this way, students reduced their chances for confusion and error.

Another, more non-routine problem, asked how many different outfits can be made from three skirts and four sweaters. Initially, most students responded that there were three outfits possible with one sweater left over. However, when I explained that each sweater could be worn with each skirt, they once again tried to figure out all the possible combinations. Since sweaters and skirts are fairly simple to draw, I suggested that they use some colored pencils and draw the different outfits. One student drew three skirts, each of a different color, and then surrounded each skirt with the four different colored sweaters. Next she drew line connecting the skirts and sweaters. Having done this, she was easily able to see that there were twelve possible outfits.

Another interesting non-routine problem had students think about seating arrangements on a bus. Twenty-six people had to be seated on a bus with 10 seats such that there was either two or three people in each seat. The students had to figure out how many people had to sit three to a seat and how many could sit two to a seat. Once again students made use of diagrams. They began by drawing 10 boxes to represent the 10 seats. Next they began filling up the boxes with circles. When each box had one circle, a second circle was then added, bringing the total number to twenty. Since there was 26 people, six more circles needed to be added. This meant that six boxes (seats) had three circles (people) and four boxes (seats) had only two circles (people).

There were other problems, however all were done using similar problem solving strategies. By working through these problems, students were able to improve their problem solving skills, particularly, the use of diagrams. They learned how to interpret the questions and organize the information so that it was meaningful to them. At the same time, students were able to discover that there is more than one correct way to solve a problem. This problem solving environment also allowed time for discussion and communication between the students.

Another good activity where the problem was very clearly posed involved estimation. Estimation is also one of the intended learning outcomes included in the NCTM *Standards* (1989) for grades K-4, and should be an ongoing part of the study of numbers, computation and measurement. Children need to be aware that mathematics is not always about exactness. Refining one's estimation skills is necessary for dealing with everyday quantitative situations and for developing an awareness of reasonable results. Children need to be able to tell the difference between 'good' estimates and 'poor' estimates.

One way for students to develop their estimation skills is by estimating the number of items within a given space. One such activity asked the students to estimate the number of beans in a jar. The students had previously worked with jars containing 10 beans and 374 beans. They would use these known amounts to help them decide on a reasonable guess. The jar of beans in question contained an amount somewhere between the two known quantities. This fact was determined by visually comparing the jars and using their knowledge of greater and less than. Some students even went so far as to take the jars in hand and shake them and twist them, hoping to get a better look at the actual number of beans.

Each student then submitted a guess on a small piece of paper. Each estimate was posted along a number line which was drawn on the board. The actual number of beans was then counted out in groups of 10. For each 10 a student was given a stick, thus making the counting process more concrete. There were four beans remaining in the end which were represented using four units blocks. Next the students began to tally the total amount by counting by tens. A flat was substituted for 10 sticks each time they reached one hundred . Eventually there were two flats, zero sticks and four units blocks. All students agreed that this represented the number 204.

Many students had guesses which were very close to the actual number of beans. Some, on the other hand, had guessed extremely high (e.g. 30000, 3731). These 'poor' estimates were discussed at the end. I think that one estimate (30000) was given just for fun, whereas the other (3731) was legitimate, illustrating that some children still need practice in estimation.

Not only did this activity improve the estimation skills of the students, but it also helped reinforce the fundamental ideas of place value by utilizing the blocks for counting the number of beans. On another day, this activity was repeated using this new amount (204) as a reference point along with the jars containing 10 and 374 beans. On Valentine's Day the students estimated the number of chocolate hearts in a plastic bag using 10 hearts as a reference.

Through this problem solving activity, students improved and gained confidence in their ability to estimate the number of items in a given space. They relied upon their previous knowledge of similar problems and used it to generate good estimates, essentially applying the 'guess and check' heuristic.

CHAPTER IV

MATHEMATICS ENRICHMENT

As mentioned in my proposal, I had hoped to provide some mathematics enrichment by introducing some ideas/topics not found in the regular Grade 2 curriculum. As part of a mathematics and science morning, I decided to introduce a mathematics center on fractions. The activities and worksheets (see Appendix C) were taken and modified from a web site by Cynthia Lanius at http://math.rice.edu/~lanius/Patterns/. The activities made use of pattern blocks to investigate how the larger blocks could be constructed using the smaller blocks, thus representing the smaller blocks as fractions of the larger ones.

Students were each given a copy of the worksheets as well as the necessary pattern blocks to complete the activity. They were asked to figure out what fraction of the hexagon the triangle was by covering the hexagon with triangles. They discovered that six triangles could fit, therefore a triangle is one sixth as big as a hexagon. Other shapes and fractions were investigated in the same manner.

The students had no difficulty with placing smaller figures on larger ones and noting how many could fit, however, making the leap from this to fractions was just too difficult for some. They were unable to think of one piece as a fraction of another piece. There were fewer difficulties with the more common fractions such as one half and one quarter, but less familiar ones like one third and one sixth were more confusing.

Unfortunately, due to the structure of the morning, the time spent with each group of students was minimal. This created problems with understanding, as time was not available for discussion and detailed explanations. Those students, who had difficulty in mathematics in general, were most affected. Some of the brighter students and those who understood the concept of fractions, enjoyed the lesson and had little difficulty. I would have liked to return to the work we started on fractions, but there was not enough time.

Maybe I was being a bit ambitious, but in a mathematics center on pattern and shape I had the students investigate and form their own tessellations - another topic not formally included in the prescribed curriculum. Although tessellations may not be widely discussed, their use in the everyday world is frequent. I began by showing the students some examples of Escher Art that used tessellations, some pictures from the Internet and a tessellation that I had made myself. We discussed how the same shape was used over and over to completely cover the paper. It was interesting how each piece fit exactly into the one beside it leaving no holes or spaces at all. We then discussed how and where tessellations could be used outside school. The patterns on wallpaper and fabric were two examples that the students mentioned.

Next I began the explanation and demonstration of how each student would design and create his/her own tessellation. We used a simple method whereby each student started with a square piece of paper and cut off a piece from one side and joined it to the opposite side. This resulted in simple but interesting shapes to use as templates for the tessellation. The template was then traced onto a sheet of paper as many times as possible, each time just sliding it along until it fit nicely into the adjacent image. Students found this part of the activity a little tedious and difficult as the template had a tendency to move as it was being traced thus distorting the tessellation slightly. Once the sheet of paper was full the design was then colored using two colors such that adjacent pieces were not the same.

The results ranged in originality and quality, but all were displayed for everyone to see. Through this lesson, students learned what tessellations were, how to create simple tessellation designs and where tessellations can be used outside the classroom.

CHAPTER V

CONNECTIONS BETWEEN PRIMARY AND HIGH SCHOOL

As I worked closely with the students I had hoped to form some comparisons between problem solving, various teaching methods, thought patterns and behaviors at the primary level with those of the high school level. The two were not as closely linked as I had first imagined, however, I was able to make some comparative analysis concerning some aspects of classroom life. The classroom was a place where students searched for meaning, appreciated uncertainty and inquired responsibly about the world around them. Perhaps the greatest difference was the atmosphere and teaching style. I think that a greater effort is made in primary school to help students construct mathematics for themselves. Relational understanding and conceptual knowledge seem to be of greater concern. Because the mathematics is more concrete and hands-on, students are able to make a personally meaningful connection between what they are learning and the real world. Also, the variety of instructional approaches takes into consideration the many learning styles of the students.

For the high school student, procedural knowledge and instrumental understanding are more commonly evident and often the choice of students. It seems that the majority of students would prefer 'knowing how' without necessarily 'knowing why'. Perhaps this is because they have been accustomed to being rewarded for getting the right answer without necessarily knowing the 'why' behind their calculations. Teachers must work to change this prevailing student attitude by emphasizing that knowing why is just as important as knowing how. This may help to explain many of the difficulties that high school students have with mathematics.

That is not to say that all high school students have difficulties and primary students have none. It was evident from my work with the Grade 2 students that some of them were going to experience many problems in mathematics throughout their schooling. One thing, however, that sets primary students apart from high school students is their willingness and desire to get the correct answer. Many high school students often give up far too easily, deciding that they just aren't able to do mathematics. Of course there are always exceptions and with a concerted effort some students do overcome their difficulties in mathematics. Conversely, there are many high school students who possess the same eagerness to obtain the correct answers and develop conceptual understanding. To help those high school students who have difficulties, it might be of benefit to structure the classroom in a similar manner to those found in primary schools. Teachers might improve the learning outcomes of high school students by providing a greater opportunity for problem solving to take place, by encouraging the construction of mathematical concepts, and by allowing for more investigation, group work and discussion.

Since mathematics must be viewed as a means by which we interpret and make sense of the world around us, it must be regularly connected to meaningful, real world applications. Because mathematics at the primary level is taught largely through problem solving and includes continuous real life references, students there are developing the desired appreciation and need for mathematics. As students approach high school, these connections are made less frequently which sometimes causes students to question the usefulness and need for the mathematics they are learning.

Therefore, problem solving opportunities, where students are able to see the application of their mathematics skills to real life situations, need to be emphasized and pursued at the high school level. Many studies (Biggs, 1985; Bright, Harvey & Wheeler, 1985; Dienes, 1963; Kraus, 1982) have shown that problem solving strategies have been acquired and enhanced through the use of mathematical games. Games also significantly improve mathematical communication, student motivation and attitudes.

The students that I worked with during my time in Grade 2 were living proof that games are effective as a teaching technique. The games were used in small group settings as well as whole class activities. Although the prescribed curriculum objectives were covered, the learning atmosphere was very different from that present on a regular basis. The mental barriers, which all too often restrict mathematics learning, were removed and students became more relaxed and receptive to learning. The students became active participants in their own learning.

I feel that these positive effects of mathematical games could be taken advantage of at the high school level as well, as games are also very appealing to older students. With a little work, many of the games that I used in Grade 2 could be modified to suit the curriculum objectives and interest level of high school students. For example, 'Guess The Mystery Number', could be modified to become 'Guess the Conic Section'. Clues could be given one by one until someone or group came up with the equation of the correct conic section. 'Mathematics Jeopardy' could also be used where questions of different categories and difficulty are answered for various point values. This would best be done in groups with some sort of tangible prize for the winning team.

In accordance with the NCTM *Standards*, there was a lot of mathematical communication and discussion taking place in the Grade 2 classroom.

> Communication is vital to doing mathematics…[T]here is a strong link between language and the way we conceive ideas. We enrich our understanding of mathematical concepts by talking about them in our own language. Often when children attempt to explain their ideas, they realize in midstream a distinction that they had not made, gain a new insight, notice a new piece of information, or spot a flaw in their own logic. (Hyde & Hyde, 1991, p. 34)

Communication was a very natural and frequently occurring component of the constructivist, problem solving learning environment that was in use. Students were required to justify their thinking and share their approaches with other students and with the teacher on a regular basis. By discussing their work and explaining their answers and thought processes to the rest of the class, students were improving their own mathematical understanding. Students were provided with many opportunities to communicate their ideas including group work, learning centers, and whole class discussions. The benefits of this communication were threefold. It served to strengthen the mental connections and structures present within their own minds, to explain and clarify various mathematical concepts for others and to inform the teacher of any underlying difficulties.

The importance and benefits of such communication are very often missing at the high school level. Students rarely get the chance to talk about the mathematics in which they engage, and are rather reluctant to do so when invited. I think this is one component of teaching that high school teachers could make use of to improve students' mathematics understanding. Perhaps, if oral communication before the entire class is unpopular, teachers could get students to express their thoughts, ideas and feelings in the form of a mathematics journal. While some teachers already do this, the majority do not.

In general, I feel that students at the primary level take a much more active role in their study of mathematics. This active participation creates in them the desire to learn, the desire to know more, the desire to do well and a better understanding of the mathematical concepts being taught.

Fortunately, with the implementation of the new Atlantic Provinces Education Foundation (APEF) curriculum, many of the concerns mentioned are being addressed through a scrutinization of the course content, foundation and delivery. The APEF curriculum is outcome-based and coherent, demonstrating to students the interconnectedness of learning and life (Nova Scotia Department of Education and Culture, 1996).

The mathematics curriculum as a whole is focused around four unifying ideas that permeate the entire curriculum and reflect the NCTM *Standards*: (a) problem solving; (b) communication; (c) reasoning; and (d) connections. The reinforcing of new concepts through drill and practice (drill and kill) is being replaced with an increased use of manipulatives and other resources, thus improving students' mathematics understanding and motivation. There is more opportunity for exploration, for students to see the interrelated and multi-representational nature of mathematics and its application to real world situations. Increased mathematical communication and group work is also emphasized with this new curriculum.

It is hoped there will be an increased retention and application of mathematical knowledge and improvements in problem solving abilities. This new curriculum also stresses cross-curricular connections aimed at improving the wholeness of learning.

CHAPTER VI

REFLECTION

The Atlantic Provinces Education Foundation (APEF), which reflects the *Standards* (NCTM, 1989) proposes a new direction in the content and emphasis of the mathematics curriculum at the primary level. Included in this reform are changes in instructional approaches. It is hoped that students will learn to value mathematics, become confident in their own ability, communicate and reason mathematically and become good problem solvers (NCTM, 1989). Problem solving should be the central focus of the mathematics curriculum. By focusing on the development of problem solving skills, it is possible to accomplish all of the other goals at the same time. Through this very different approach to mathematics teaching, it is conceivable for students to see mathematics as a sense making activity, one that can help to explain the world around them.

As problem solving was the main focus of my internship, I am pleased to say that it was alive and well in the Grade 2 classroom in which I spent 10 weeks. As described previously in the many classroom activities, students engaged in problem solving on a daily basis. While their efforts may not have been labeled as problem solving, they were provided with many opportunities to develop their problem solving skills.

Learning mathematics at the primary level is a constructive rather than a passive activity. Students are active participants in their own learning. They are encouraged to raise questions and investigate answers through problem solving. It is evident that children at this age have a real love for learning.

My experience in a primary learning environment, has given me a greater understanding of how children learn. By making the mathematics personally meaningful, through the use of problem solving, games and manipulatives, students develop conceptual as well as procedural knowledge.

Personally and professionally, my life was enriched by getting to know these wonderful young people who had so much zest for learning and life. It gave me a new perspective on the high school students that I usually teach. Sometimes it is easy to forget that they, too, were once in Grade 2 and possessed the same eagerness to learn. Perhaps, through the use of some of the learning tools that I observed and used in Grade 2, I can rekindle the spirit of learning that once glowed inside these young individuals.

I was also made more aware of my own teaching style and ways in which I could improve. The love of mathematics is contagious. If children are to feel successful in mathematics, teachers need to communicate to children that mathematics is exciting, necessary and meaningful in their lives. The students in Grade 2 picked up very quickly on my feelings towards what I was teaching. Since this too must also be true of high school students, I must ensure that I demonstrate an attitude and mood which reflects what I wish my students to exhibit. Generally, students will gain enthusiasm and a desire to learn, if they see these same characteristics displayed by the teacher.

I must also be conscious of the fact that not all students learn in the same way. The primary curriculum is delivered in such a way as to vary the teaching techniques as described throughout this report, thereby ensuring that all students learn. Unfortunately, this type of variation in teaching style does not often occur at the high school level.

To bring closure to my internship I asked the students to think about what mathematics meant to them. Their responses helped me to see that, although they use mathematics every day in many ways, most students don't really see mathematics as something separate and distinct; something that they can define. I realized that because mathematics is so intertwined with many aspects of their daily lives, the students were becoming mathematical problem solvers, almost without notice.

Therefore, we as high school teachers, must continue to foster the development of mathematically literate students that was started several years ago, in primary school, by incorporating problem solving as much as possible into our mathematics instruction.

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APPENDIX A

Children's Literature Used in Mathematics Teaching

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Allen, P. (1991). Mr. Archimedes' bath. NY: Harper Collins Publishers.

Anno, M. (1983). Anno's mysterious multiplying jar. NY: Philomel Books.

Burningham, J. (1980). The shopping basket. London: Cape.

Comber, B. (1987). Dad's diet. NY: Scholastic Inc.

Hutchins, P. (1988). The doorbell rang. NY: Greenwillow Books.

Reid, M. (1990). The button box. NY: Penguin Group.

Schwartz, D. M. (1985). How much is a million? NY: Scholastic Inc.

Tildes, P. L. (1995). Counting on Calico. Watertown, MA: Charlesbridge Publishing Inc.

APPENDIX B

Problems Used with Students for Activities Described in Chapter 2

Wendy had 13 cookies. She ate 6 of them. How many cookies does Wendy have left?

Jennifer has 16 blocks. She has 9 more than Bobby. How many blocks does Bobby have?

Cathy has 5 dolls. Her grandmother gave her 3 more. She left 4 of them at her friend's house. How many does she have now?

Kevin had 5 apples. He ate 2 of them. His father gave him 3 more. How many apples does Kevin have now?

John found 21 rocks at the beach. He put 3 rocks in each box. How many boxes did John use to hold his rocks?

Jack has 3 packages of gum. There are 6 pieces of gum in each package. How many pieces of gum does Jack have?

19 children are going to the circus. 5 children can ride in each car. How many cars will be needed to get all 19 children to the circus?

Mary had 3 packages of cupcakes. There were 4 cupcakes in each package. She ate 5 cupcakes. How many cupcakes are left?

Sally is having a birthday party. She invites 4 friends. Each friend is allowed to invite 4 more friends. How many children, including Sally, are invited to the party?

26 children are taking a bus to the zoo. They will have to sit either 2 or 3 to a seat. The bus has 10 seats. How many children will have to sit three to a seat, and how many can sit two to a seat?

Susan is taller than Betty. Donna is shorter than Betty. Ruth is taller that Betty but shorter than Susan. Who is the tallest? Who is the shortest?

Robert had 12 marbles in his pocket. When he got to school he had only 8. How many marbles did he lose?

Peter has 18 blocks. He has 6 more than Fred. How many blocks does Fred have?

Mr. White needs 16 dollars. He already has 7 dollars. How many more does he need?

4 birds found 20 worms. How many worms will each bird get if they are shared equally?

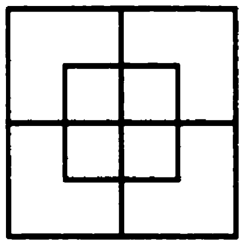
Farmer Brown has 5 chickens and each chicken lays 3 eggs each day. Two of today's eggs fell on the ground and broke. How many eggs does Farmer Brown have now?

Sally bought 3 bags of candy. There were 6 candies in each bag. When she got home she noticed a hole in one of the bags, 2 of the candies had fallen out. How many candy does she have left?

The elevator man went up 7 floors and down 3 floors. What floor is he on now if he started on the second floor?

Mary has 3 skirts and 4 sweaters. How many different outfits can she make?

How many squares are there?



APPENDIX C

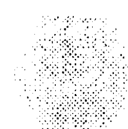
Fractions - Exploring Shapes

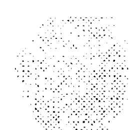
1. Can you name the following shapes?

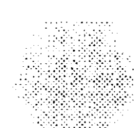
__________ __________ __________ __________

2. How many  are in  ________

3. How many  are in  ________

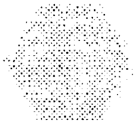
4. How many  are in  ________

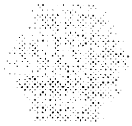
5. How many  are in  ________

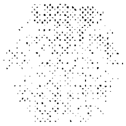
6. How many  are in  ________

7. How many  are in  ________

8. Express the above relationships in fraction form.

- A  is _____ a 

- A  is _____ a 

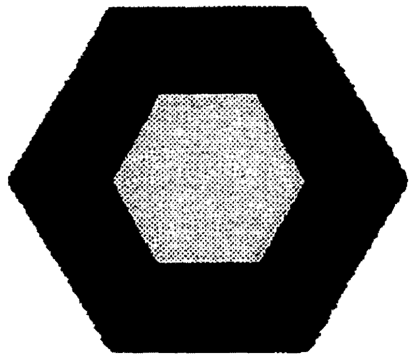
- A  is _____ a 

- A  is _____ a 

- A  is _____ a 

- A  is _____ a 

9. Using the pattern blocks, make a pattern like the one below.



10. What fraction of the design is blue? ________

11. What fraction of the design is red? ________

12. What fraction of the design is yellow? ________

13. What fraction of the design is green? ________