VISCOELASTIC RESPONSE OF FLEXIBLE PAVEMENTS UNDER MOVING DYNAMIC LOADS

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VISCOELASTIC RESPONSE OF FLEXIBLE PAVEMENTS UNDER MOVING DYNAMIC LOADS

by

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This thesis is dedicated to:
my mother
Rose Akua Afriyie

ABSTRACT

This study proposes a mechanistic design model, that analyzes the structural performance of flexible pavements under moving dynamic loads generated by heavy vehicles. It assumes a linear viscoelastic behavior of flexible pavements. The model, expands the framework formulated by the Federal Highway Administration (FHWA) for the mechanistic design program called VESYS. It applies Boltzman's superposition principle to translate the pavement response from a stationary load to pavement response from a moving load of time-dependent amplitude. Experimental dynamic load data obtained using the instrumented vehicle developed by the National Research Council of Canada (NRCC) are used as input in the model.

The variable dynamic responses along the longitudinal axis of the pavement are translated into fatigue cracking and rutting by assuming spatial repeatability of the dynamic loads. The 90th percentile of the cummulative damage values is assumed to represent the overall damage of pavement section. This damage value is then compared with the damage caused by moving loads of constant magnitude (static) through a relationship called Pavement Life Ratio (PLR). This is defined as the ratio of the accumulated number of load repetitions to failure due to static load divided by repetitions to failure due to dynamic loads.

Furthermore, the model is used to investigate the performance of two suspension types (Air and Rubber) on three types of flexible pavements with different structural strengths. The effects of other vehicle characteristics on pavement damage is also investigated.

Results from the analysis show that, moving dynamic loads are more crucial to pavement damage than static loads and the magnitude of the damage is dependent on the suspension type. It was found that, rubber suspension causes greater pavement damage than air suspension and the difference can be as much

as 43%. Moreover pavement surface roughness and vehicle parameters like speed, axle spacing and axle configuration substantially influence the suspension type to increase pavement damage. For multiple axles, unequal load-sharing between the individual axles further increase pavement damage.

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Chapter 1

Introduction

1.1 Background

The early methods of pavement design and analysis were mainly empirical. Such methods were developed based on local conditions and the experience on pavement damage derived through observation. The American Association of State Highway Officials (AASHO) Road Test, conducted between 1958 and 1961 is a notable example of an effort to develop such empirical design methods. One of the major findings of the test was the non-linear relationship between the number of load applications and pavement damage. The empirical methods, though simple and easy to use, are only sufficiently adequate within the limits for which they were developed. With the changing pattern of materials, vehicle types and loads, there have been intensive efforts to get a more realistic understanding of pavement behaviour and vehicle-pavement interaction as a whole. For the past twenty five years, empirical methods of pavement design have been replaced by more rational mechanistic design methods. Several research studies have been conducted on the dynamics of moving vehicles and the mechanics of pavement construction materials.

1.1.1 Mechanistic Methods of Design

The mechanistic methods of design have resulted from the knowledge of pavement material behaviour and simplifying assumptions on the nature of loads applied by moving vehicles on pavements. These methods analyze pavement damage based on material behaviour, loading conditions, vehicle parameters and environmental factors. The design procedure requires the computation of the critical response parameters which are usually the stresses, strains and deflections that develop in the pavement when vehicles pass over them. For flexible pavements, the tensile strain at the bottom of the asphalt concrete layer and the compressive strain at the top of the subgrade are typically considered. These response parameters are calculated using analytical methods or are obtained through in-situ measurements. They are subsequently input into empirical relationships to assess damage. Many of the mechanistic models assume linear elastic behaviour of pavement materials and compute the pavement responses under stationary tire loads (eg. ELSYM5, BISAR, CHEVRL etc.). Some of the models also compute the pavement responses under quasi-static loads or moving loads of constant magnitude, however very few of these (eg. VESYS) consider the viscoelastic behaviour of pavement materials.

In recent years, there have been emphases on viscoelastic pavement material behaviour and the effects of moving dynamic loads on pavements. Several efforts have been made in the last ten years to quantify the dynamic loads of moving vehicles and also to get a better understanding of their impacts on pavements (eg., Sweatman 1983, Woodrooffe, 1986, Cebon, 1990 etc.). The results of these efforts have indicated that, the magnitude of dynamic loads can be considerably different from their stationary values. This is attributed to the interaction between the pavement surface roughness and vehicle parameters like suspension type, speed, axle configuration, tire type, tire inflation pressure etc. However most of the substantial research in this area has been focused on vehicle parameters that will reduce dynamic loads instead of analyzing the impact of dynamic loads on pavements. The consequence is that, most of the available mechanistic pavement design models do not account for dynamic loads. Such deficiencies call for a more detailed and comprehensive study.

1.2 Scope

The scope of this study will focus on the development of a mechanistic pavement response model for a moving load of time-dependent magnitude (ie. dynamic). The model will accommodate single and tandem axles of heavy vehicles. Experimental dynamic load data obtained from measurements under different vehicle speeds and on a wide range of pavement surface roughnesses will serve as the main input to the model. The dynamic load data is obtained with an instrumented vehicle developed by National Research Council of Canada (NRCC). The model will compute pavement responses and then translate them into pavement distress using generally accepted fatigue cracking and rutting damage relationships. The accumulated load applications to failure will be used to compute relative pavement life for two suspension types (Air and Rubber).

Chapter 2

Research Statement and Objectives

2.1 Research Statement

Two important factors need to be considered in the development of any realistic pavement response model. First, bituminous materials exhibit viscoelastic behaviour under load and second, the magnitude of the loads that are exerted on flexible pavements by moving vehicles are time-dependent. In their response to stress, viscoelastic bodies generally exhibit loading rate-dependent behaviour which depend on the frequency of loading and of the time intervals between them. The dynamic loads from moving vehicles also occur at different frequencies with variable intervals between them (Cebon 1990, Addis 1992). Although a lot of progress has been made in this field, a mechanistic model capable of evaluating pavement performance as a function of the viscoelastic material behaviour and moving loads of time-dependent magnitude has not yet been developed.

Many of the early attempts to solve viscoelastic pavement problems have used Laplace and Fourier transforms of elastic solutions. However, their inversions to obtain time-dependent solutions have proved extremely difficult and timeconsuming. The available viscoelastic models consider repeated application of static loads on pavements. A number of studies have also analyzed the effects of dynamic loads in a piece-wise static fashion (eg., Cebon and Hardy 1992). Others also have attempted to solve this problem by developing convolution equations that are used to compute the pavement response parameters (displacement, stresses etc.). However, the process is very tedious and the integrals used to obtain such responses also require several approximations before achieving convergence (eg. Thrower, 1977).

There are also issues regarding the effects of pavement parameters like surface roughness and vehicle characteristics including suspension type, axle spacing and load sharing between multiple axles on the life of flexible pavements. The extent to which these parameters combine to influence pavement performance is not yet fully understood. Nevertheless, all these methods or approaches have been very useful in providing a basic understanding of the problem. They have also drawn attention to the fact that, consideration of dynamic loads and viscoelastic pavement behaviour are important in achieving a realistic pavement analysis. A more reliable method of analyzing pavements as a function of time-dependent material properties and loading will be a useful in pavement analysis, therefore the emphasis on this thesis is directed towards this goal.

2.2 Objectives

The primary objectives of this study include the following:

- Review the pertinent literature on the various approaches taken in quantifying and modelling the impact of dynamic loads on pavements.

- Propose a mechanistic model for flexible pavement response that will take into account moving dynamic loads of heavy vehicles.

- Use this model to analyze experimental dynamic load data to ascertain the impact of dynamic loads.

- Examine the impact of vehicle and pavement parameters on pavement life.

2.3 Methodology

The methodology to be followed includes the following:

- Develop a mathematical model to compute the time histories of dynamic pavement response as a function of dynamic loads.

- Develop a computer algorithm to implement the ideas of the previous step.

- Obtain experimental data for dynamic loads generated by a moving vehicle.

- Select three pavement structural designs for testing the computer algorithms.

- Analyze the interaction between pavements and moving vehicles based on the experimental dynamic load data.

- Compare the performance of two suspension types in terms of pavement damage.

- Finally, select other parameters such as axle spacing, axle configuration, load-sharing etc., and use the dynamic model to investigate their effects.

2.4 Definition of Terms

In order to give a clear understanding of the terminology used, certain definitions will be given concerning the three major load types that act on pavements as they appear in this thesis. These are stationary loads, moving static loads, and moving dynamic loads.

1. A stationary load refers to a concentrated load of constant magnitude acting at a point on the pavement surface.

2. A moving static load refers to a moving load of constant magnitude.

3. A moving dynamic load refers to a moving load of time-dependent magnitude.

Chapter 3

Literature Review

3.1 Quantifying Dynamic Loads

3.1.1 Introduction

The magnitude of the loads exerted by moving vehicles has been a subject of study in recent years. While the factors affecting the axle loads exerted by stationary vehicles are well understood, it is clear that when the vehicle is in motion, its axle loads vary considerably from their stationary values. It has been found from several studies that, the magnitude of dynamic forces is a function of the road roughness and vehicle parameters such as speed, suspension type, tire type, axle configuration and mass distribution of the vehicle.

Early studies on pavement damage considered the effects of static loads in modelling the performance or damage. However, recent studies have found that the contribution of dynamic loads to pavement damage can be substantial and hence must be taken into account. Several research studies have therefore been undertaken to quantify the dynamic loads and the associated pavement damage. Some of these involve measurement of dynamic loads generated by instrumented vehicles, others utilized Weigh-in-Motion (WIM) scales to record dynamic loads from truck loads passing over artificial bumps or obstacles while others utilized analytical vehicle models in attempting to model dynamic vehicle behaviour. Some of the most substantial studies in this area are reviewed next.

3.1.2 Whitemore et al, 1970

Whitemore et al. (1970), conducted the first significant study dealing with dynamic loads at highway speeds. This was part of a National Cooperative Highway Research Program (NCHRP) study to assess dynamic pavement loads. The experimental study employed three alternative methods of measuring dynamic loads:

- A tire pressure transducer.
- A combination of strain gauges and accelerometer on the axles.
- A wheel force transducer made by General Motors (GM).

The strain gauges were used to measure the bending strain as the axle deformed under load. The wheel force transducers were fitted on the hub of one of the wheels. Their output was converted to vertical forces through static calibration. Figures 3.1 and 3.2 show the transducer and the method of strain-gauging the axles. The results of the loads from these alternative methods were compared with those measured by WIM scales and it was found that tire pressure was not in phase with the dynamic loads and hence the tire pressure transducer was not suitable for dynamic load measurements.

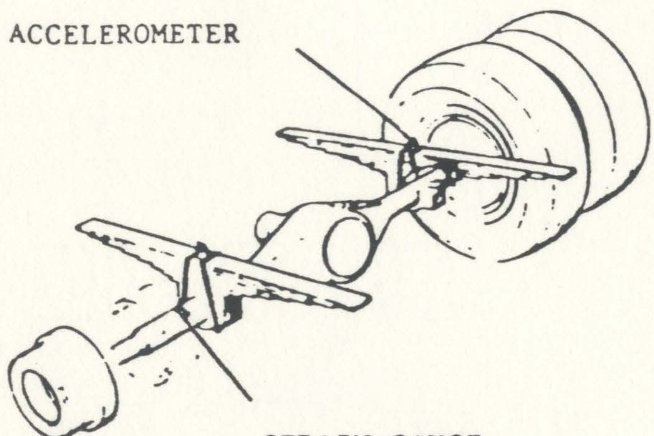


Figure 3.1: Method of Strain Gauging Axles (after Whittemore et al., 1970)

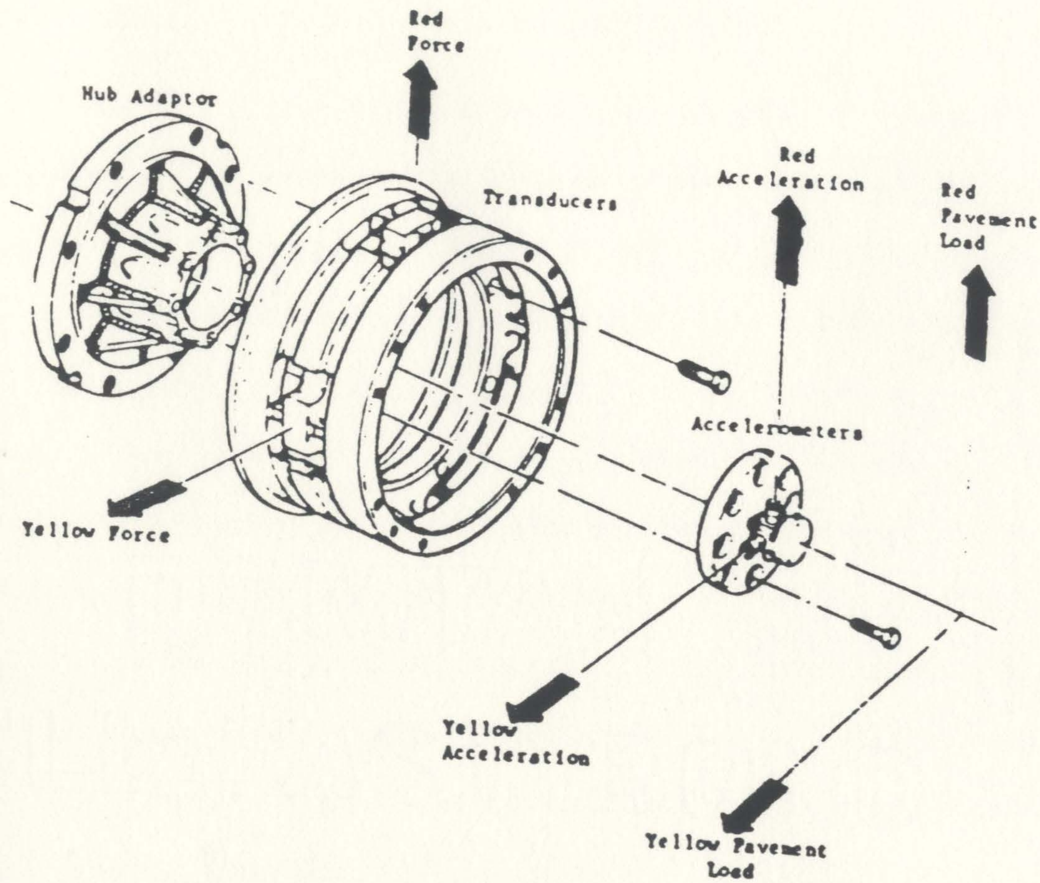


Figure 3.2: GM Wheel Force Transducer (after Whittemore et al., 1970)

3.1.3 Leonard et al, 1974

Leornard, Graigner and Eyre (1979), measured dynamic loads and ground vibrations generated by a series of eight articulated vehicles using a WIM scale embedded in the road surface. The purpose of the study was to determine the effects of gross vehicle weight on pavements and the performance of axle group suspensions. Three suspension groups were tested namely, trailer tridems, trailer tandems, and drive tandem axles. This WIM scale consisted of a rigid platform supported by load cells and arranged so that the top was flush with the road surface. Artificial bumps, 40mm X 250mm in cross section were placed in series along the road. Dynamic loads were measured as the vehicle traversed over the artificial bumps. An impact factor was calculated by dividing the dynamic load by the static loads. Large values of impact factors were obtained especially at higher speeds. It was concluded that, dynamic loads have the higher potential for damaging the road especially at higher speed.

3.1.4 Ullidtz, 1979

Ullidtz used a quarter car model, as shown in Figure 3.3, to simulate dynamic vehicle behaviour. The lower system (M_1, K_1, C_1) represents the masses of the axle and wheel, the spring constant of the tire and tire damping. The upper system represents the mass of the vehicle and the static load transferred to one wheel. The longitudinal profile of the road was first divided into sections and further into subsections of 0.3 meters long. The total dynamic loads exerted on the pavement was taken as the static loads and the dynamic loads caused by the dynamic oscillation of the vehicle.

The dynamic loads for each section were calculated using a linear mathematical model which relates the force at a point to the condition at a preceding point, the vertical position D, vertical velocity V, and the vertical acceleration G at that point (Figure 3.3). The distance between any two consecutive points i and $i+1$ for which the dynamic load is to be calculated was taken to be 50mm. To calculate the force exerted at each point, the wavelength corresponding to the resonance frequency of the mechanical analogue should be longer than 50mm so that, the accerelation can be assumed to be constant between the points. The dynamic forces at point $(i+1)$ on the surface due to vibration from the vehicle are then calculated using the equation:

$$F_{(i+1)} = [D_1(i+1) - D_0(i+1)]\, K_1 + [V_1(i+1) - V_0]\, C_1 \tag{3.1}$$

This procedure is used to calculate loads at subsequent points along the road and the dynamic loads were input in a computer program to predict pavement performance.

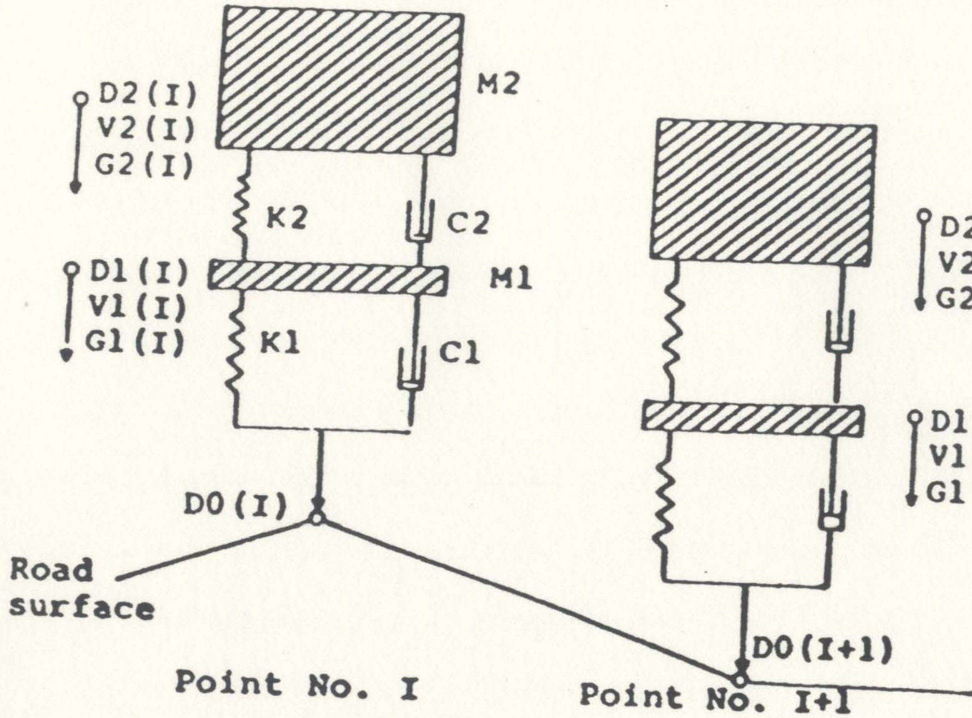


Figure 3.3: Mechanical Analogue Used to Measure Dynamic Loads (After Ullidtz 1979).

3.1.5 Sweatman, 1983

In 1983, Sweatman conducted a study to evaluate the dynamic behaviour of various suspension types. The study was carried out using the General Motors (GM) wheel transducer mentioned earlier in Figure 3.2. The three major parameters considered were the vehicle speed, the road roughness and the tire inflation pressure. It was found that, the method of calibration of the transducer by Whitmore et al., (1970), did not provide sufficient indication of the frequency response during the rolling mode. A new device for dynamic calibration called 'Bump Dynamometer', was therefore developed. 'The bump dynamometer consisted of a pair of unpowered co-axial steel rollers 750mm in diameter and mounted on electronic load cells ...'. This was used to simulate the wheel force amplitudes and frequencies encountered when a vehicle passed over a rough surface. The output of the wheel force transducer were recorded at three dynamometer operating frequencies of 1.2, 2.0, 2.5 Hz. A parameter termed Dynamic Load Coefficient (DLC) was computed to describe the magnitude of variation of the dynamic load. DLC is defined as the root mean square (RMS) of the dynamic wheel force divided by the mean dynamic wheel force.

$$DLC = \frac{RMS(Dynamic)}{Mean(Dynamic)} \tag{3.2}$$

In addition, the distribution of the loads between multiple axles in each suspension group was quantified in terms of the Load Sharing Coefficient (LSC). This was defined as the mean wheel load divided by the static wheel load. It was expressed algebraically as:

$$LSC = \frac{2nZ}{mg} \tag{3.3}$$

where, n=number of axles in group,

Z= mean wheel load (kN),

m=axle group mass,

g=acceleration due to gravity.

Hence, if the load is equally distributed among the axles in a group, (ie, tandem or tridem axles) the LSC will be 1.0. Values below and above 1.0 indicate under-loading and over-loading of the individual axles in the group. Furthermore, regression analysis was performed to determine the relationship between the effects of speed and roughness on the dynamic loads generated by axle group suspension. The general form of equation arrived at was:

$$DLC = VR^{0.5} \tag{3.4}$$

Under the conditions tested, DLC values ranged between 0 and 0.4. LSC values also ranged from $0.791 - 0.983$ for tandem suspensions. It was noted that, the worst suspension in terms of LSC was the Walking Beam while the 4-leaf was the best. Tridem suspension was found to be better than tandem in terms of LSC.

3.1.6 Gorge, 1983

Gorge (1983), conducted an experimental study on the impact of dynamic axle loads on both flexible and rigid pavements. The first of the objectives was to compare the response of pavements due to the various axle configurations. In this case, the vehicles were tested on the smooth portions of a test road to reduce dynamic loads. The other objective was to measure the impact of dynamic loads on pavements and therefore the vehicles were run over pavements with different roughness. The methods used to measure dynamic loads included accelerometer and transducer similar to the one developed by GM, (Whitmore et al. 1970). Strain gauges were embedded in several locations of the road at regular intervals along the wheel path. In other locations, the strain gauges were placed transversely at a particular point in the wheel path. Dynamic loads measured by the transducers and the accelerometers were compared with the strains at each of the sections. Dynamic loads were quantified by computing two parameters namely;

- The Shock Factor
- The Variation Coefficient.

The Shock Factor was defined as the ratio of the instantaneous dynamic load divided by the static axle load. The Variation Coefficient was also defined as the standard deviation of the dynamic load divided by the static load. In this experiment, dynamic loads measured were found to be spatially repetitive along some parts of the road. In their overall dynamic loads study, Shock Factors as high as 1.53 were obtained on the sections with the highest roughness.

3.1.7 Ervin et al., 1983.

An experimental investigation was carried out by Ervin et al [1983], to verify Sweatman's findings. Three vehicles with different suspensions were used and the dynamic wheel forces were measured by strain-gauging the wheel hub. Results similar to Sweatman's were obtained. One significant finding was that, the excitation of all the vehicles were closely related to the roughness features of the road and that the peak forces of each were observed at the same localized area (ie., spatial repeatability). Peak forces up to 2.5 times the static were measured. Dynamic load coefficients were found to be between 0.15 and 0.3 depending on the roughness of the road. However it was concluded that, DLC was not a good index for load impact on the pavement because, certain points in the road with characteristic roughness features experience peak dynamic forces which are repetitive.

3.1.8 Hahn, 1985

A study was made by Hahn (1985), to quantify the dynamic loads of heavy vehicles by considering the acceleration and mass of moving vehicles. The accelerations of the sprung and the unsprung masses of the vehicle were measured and multiplied by their corresponding masses to give the dynamic loads. In addition, Hahn used strain gauge hubs fitted with a GM transducer. The dynamic load was obtained by computing the resultant force in the wheel plane which was taken as the summation of the components of the force vectors. It was however found that, the first method of quantifying dynamic loads was not a reliable one because it was accurate for vehicles with single and tandem axle configurations only.

3.1.9 Addis et al., 1985

Addis, Halliday and Mitchell (1985), measured dynamic loads by relating the tire forces to tire deflection as the vehicle was driven. They used an optical tire deflection device to measure deflection. The dynamic loads measured were compared to strain measurements obtained from gauges embedded in the pavement. The strain gauges were embedded in longitudinal and transverse directions and an articulated truck with leaf spring suspension was driven over the section. Dynamic tire loads of 15% to 20% more than the static force were recorded with a corresponding variations in strain of up to 20% of the those due to the static force. Some spatial repeatability of dynamic loads were also observed.

3.1.10 Woodrooffe et al., 1986

The dynamic wheel load behaviour of heavy vehicle suspensions was investigated by Woodroofe et al. in 1986. The three suspensions examined in this study were, the walking beam, air suspension and spring suspension. Suspension parameters like axle spread, axle load and suspension type were examined in relation to the vehicle speed and road roughness. Test sections representing different levels of roughness were selected and an instrumented vehicle was driven over them at three speeds namely 40, 60 and 80 km/hr.

A combination of strain gauges and accerelometers was installed in the axle housings between the spring-mounting and the wheels brake plates. The strain gauges measured bending moments resulting from vertical forces exerted on the axles as the vehicle traversed. The dynamic loads were resolved into two components, (1) the dynamic axle loads measured by the strain gauges and (2) the inertial components which was obtained by taking the product of the measured vertical accerelation and the inertial mass outboard of the strain gauge. The total dynamic force was expressed algebraically as:

$$F = DA + VA \times EM \tag{3.5}$$

where,

F=the total dynamic wheel force at the pavement surface,

DA=dynamic axle load,

VA=vertical accererlation of the axle,

EM=end of axle mass.

Dynamic load coefficient (DLC) derived by Sweatman [1983], was intended for comparing the suspensions but they found that, DLC was sensitive to the overall mean wheel force. The standard deviation of the dynamic wheel forces were therefore used to analyze their results by expressing it as function of road roughness, vehicle speed and suspension type. Their findings in the study included the following;

- Dynamic wheel loads diminished as the vehicle travelled on smooth roads irrespective of the suspension type.

- Dynamic wheel load increase with vehicle speed at an exponential rate. For example, increasing speed from 40km/hr to 60km/hr increased dynamic loads by 20% while increasing speed from 60km/hr to 80km/hr increased dynamic loads by 150%.

- Dynamic loads are sensitive to axle spread. Shorter axle spread for a suspension group were found to improve axle load equalization.

- For all the three suspensions tested, the air suspension had the best performance while the walking beam was the worst.

3.1.11 Cebon, 1990

Cebon (1990), developed a prototype dynamic wheel force measuring mat equipped with capacitance transducers mounted perpendicular to the wheel path at 0.4m intervals. These transducers were embedded in 13mm thick polymer tiles measuring 1.2 meters by 1.2 meters. This dynamic load measuring device is still in the experimental stage. One of the expected advantages is its portability which will make it easy to place over different road surfaces and that it will be allowed to sample random traffic. Another advantage is that, statistically significant samples of dynamic load histories can be collected for many trucks without any vehicle-mounted instrumentation.

3.2 Modelling Dynamic Load Impact on Flexible Pavements.

Many investigators have provided models to assess the impact of dynamic loads on the pavement structure through theoretical and experimental analysis. Their methodologies differ according to the basic assumptions of material behaviour under load. A brief review of the most significant of these studies is presented next.

3.2.1 Eisenmann, 1975

One of the earlier efforts to model the impact of dynamic loads on pavement was attempted by Eisenmann (1975), based on the "fourth-power law" which has been assumed to summarize the findings of the American Association of State Highway Officials (AASHO) Road Test. The main assumption was that dynamic tire forces exerted on the road are normally distributed (ie. Gaussian), and random in space. The impact of dynamic axle load was quantified by developing a factor termed the Road Stress Factor, Φ;

$$\Phi = P_{stat}^{4}\left[1 + 6S^{2} + 3S^{4}\right] \qquad (3.6)$$

where, P_{stat}=Mean axle load (KN),
S=Coefficient of variation of dynamic load which is identical to Sweatman's DLC. Eisenmann modified Equation 3.6 in 1978 to account for wheel configuration and tire contact pressure by using empirical factors θ_1, θ_{11}. The relationship was:

$$\Phi = V(\theta_1\theta_{11}(P_{stat})^4). \qquad (3.7)$$

where, V=$1 + 6S^2 + 3S^4$

θ_1 and θ_{11} account for tire configuration (single or dual) and tire contact pressure respectively. Typical values for θ_1 and θ_{11} are 1.0 for single tire and 0.9 for twin tires. The Road Stress Factor approach has been followed by many investigators including Hahn, Mitchel and Geynes (1989) in analyzing the dynamic load effects of steel, rubber, and air suspensions.

3.2.2 Brademeyer, 1975

An analytical method of modelling the impact of moving loads was developed by Brademeyer (1975). The pavement response was analyzed using influence functions. The influence function indicates the effect at a given point when a unit load is placed at any point in the structure (Figures 3.4). For example when a stationary load P is placed at a point X_1, the influence at X_0 will be $p(X_1 - X_0)$, (Figure 3.4). For flexible pavement, the response $R(X_0)$ at point X_0 is the product of the influence function and the load at X_1 and is given by:

$$R(X_0) = P(X_1)p(X_1 - X_0) \tag{3.8}$$

The Boltzman's superposition principle (Tschoegl, 1989) was used to superimpose the influence function of a pavement response parameter. The equation was given as:

$$R(t) = \int_0^t [\psi(t - \zeta, \theta)\frac{\partial P}{\partial t}]\partial\zeta \tag{3.9}$$

where, ψ is the pavement response due to a stationary load, θ is environmental history from ζ to t and $\frac{\partial P}{\partial t}$ is the time derivative of the load.

The moving load was modelled as a moving pulse load of constant magnitude with duration D and amplitude A. The loading function is defined as:

$$P(x+t) = Asin^2\left(\frac{\pi}{2} + \frac{\pi t}{D}\right) \tag{3.10}$$

The time derivative of the load pulse becomes,

$$\frac{\partial P}{\partial t} = -\frac{A\pi}{D}\left(sin\frac{2\pi t}{D}\right) \tag{3.11}$$

Substituting Equation 3.11 in Equation 3.9 the pavement response due to a moving load of constant amplitude becomes:

$$R(t) = -\int_0^t (\psi(t-\zeta,\theta)\frac{\partial P}{\partial t})\frac{A\pi}{D}\left(sin\frac{2\pi t}{D}\right)\partial\zeta \tag{3.12}$$

Equation 3.12 was used to calculate critical pavement response parameters (tensile strain at the bottom of the asphalt and compressive strain at the top of the subgrade). This work by Brademeyer formed the basis for the development of the mechanistic pavement design program VESYS (1978) and its subsequent modifications. This approach will also be followed in the mechanistic design model to be formulated in this thesis.

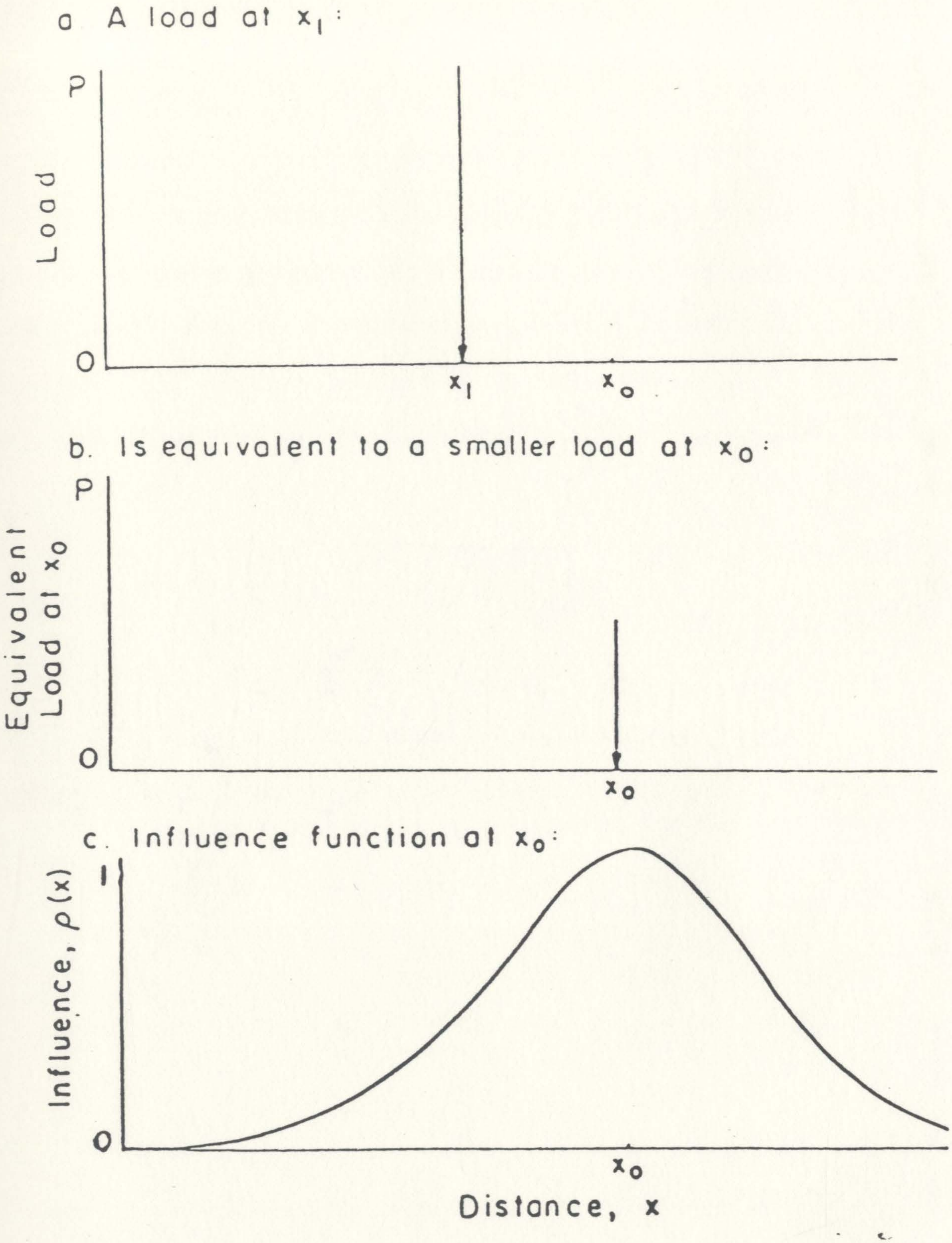


Figure 3.4: Influence Function of Flexible Pavement Response, (After Brademeyer et al. 1975).

3.2.3 Thrower 1977

A study was conducted by Thrower (1977), to provide a method of predicting permanent deformation in flexible pavements. The theoretical study described two approaches namely, Separative Method and the Viscoelastic Method.

The Separative Method assumes that, the relationship between stress and deformation in a pavement structure under moving loads is analogous to the elastic relationship for isotropic materials. For example the elastic stress-strain relationship can be expressed in terms of the shear modulus G and bulk modulus K as:

$$\epsilon_{ij} = \frac{\sigma_m}{3K} + \frac{(9\sigma_{ij} - \sigma_m)}{18G} \tag{3.13}$$

$$\epsilon_{ij} = \frac{\sigma_{ij}}{2G}, \tag{3.14}$$

where ϵ_{ij} is the strain, σ_m=mean stress (ie., $(\sigma_{11}+\sigma_{22}+\sigma_{33})/3$)

Following this analogy, the rate of deformation can be related to the instantaneous stress by ;

$$e_{ij} = \frac{\sigma_m}{3\chi} + \frac{(9\sigma_{ij} - \sigma_m)}{18n} \tag{3.15}$$

$$e_{ij} = \frac{\sigma_{ij}}{2n}, \tag{3.16}$$

where, e_{ij} is the strain rate, n =coefficient of shear viscosity and χ=coefficient of volume viscosity. The general form of this equation is written as:

$$e_{ij} = \psi_{ij}(\sigma_{ij}, n, \chi) \tag{3.17}$$

Therefore the total deformation accumulated for each loading can be given as;

$$e_{ij} = \int_{-\infty}^{\infty} \psi_{ij}(\sigma_{ij}, n, \chi)dt. \tag{3.18}$$

Loads were treated in two different ways. First, the load was considered to be a stationary pulse load and the pavement deformation was given directly in terms of the known stresses as:

$$e_{ij} = \int_{0}^{T} \psi_{ij}(\sigma_{ij}, n, \chi)dt. \tag{3.19}$$

Second, the load was assumed to be a moving load where stress is not constant. However, it was assumed that, vehicle speeds are low enough for a quasi-static analysis to be reasonable. The stress components are then computed through elastic analysis. A slow load moving at a speed c in the x of x,y,z space would produce a response (eg. deformation) expressed as:

$$e_{ij} = \int_{-\infty}^{\infty} \psi_{ij}(\zeta, y, z)dt. \tag{3.20}$$

Where, $\zeta = x - ct$ and The absence of the viscous parameters χ and n can be noted in the above equation due to the elastic solution. The permanent displacement U_i was computed as the integral of the deformation e_{ij} and their spatial derivative between two chosen points as;

$$U_y = \int_{y0}^{y1} (e_{yy})dy. \tag{3.21}$$

$$U_z = \int_{z0}^{z1} (e_{zz})dz. \tag{3.22}$$

In the viscoelastic model, each layer in the multilayered pavement structure was considered as a Maxwell medium. The elastic and viscous parameters were defined by G and n for shear stress and K and χ for hydrostatic stresses. The stress-strain relationship was represented by:

$$G \times S + n \times S = n \times G \times e \tag{3.23}$$

where, S is the stress and e is the strain. When the surface of such a structure is subjected to a pulse load, a joint Laplace-Bessel transform could be written to reflect the actual time-dependent response parameters (stress, strains or displacement). This derivation was not produced because it was considered to be cumbersome and lengthy. Instead, the pavement displacement U_i due to a pulse load was approximated by a residual displacement at infinite time L_t after the removal of the pulse load. This was defined by;

$$U_i = L_{t \to \infty} V_i \tag{3.24}$$

where, V_i is the time dependent displacement. This is identical to the Bessel transform for displacement V_i in the elastic case when the elastic parameters G, and K are replaced by their corresponding viscous parameters n and χ over the duration a, of the pulse. Hence if a vertical stress P is applied uniformly over a circular area of radius r_o, the elastic displacement V_i is given as:

$$V_z(r,z) = \frac{p}{\pi r_o} \int_0^\infty f(\zeta, z, G_i, K_i) J_i(\zeta) J_0(\frac{\zeta r}{r_0}) d\zeta. \tag{3.25}$$

where, J_0 and Ji are Bessel parameters.

Using Equation 3.23, the permanent displacement U_z of a viscoelastic pavement structure subject to a pulse load of duration a, can be given as:

$$V_z(r,z) = \frac{aP}{\pi r_o} \int_0^\infty f(\zeta, z, n_i, \chi_i) J_i(\zeta) J_0(\frac{\zeta r}{r_0}) d\zeta. \tag{3.26}$$

The moving load on a viscoelastic surface was regarded as the superposition of elementary pulses at successive points along the road, therefore the pavement response (eg., permanent displacement) was obtained by integrating the displacement due to a pulse load. For a particular point on the surface of the road $U_z(0,0)$, the equation becomes;

$$V_z(y,z) = \frac{2P}{\pi r_o c}\int_0^\infty \int_0^\infty f(\zeta, 0, n_i, \chi_i)\frac{J_i(\zeta)}{\zeta}d\zeta \tag{3.27}$$

However convergence of the above integral could not be achieved, and therefore a further approximation was made by assuming an upper limit that indicated the relative position of the moving load from the point under consideration. This was done by considering only the deformation when the load is near the particular point and therefore the upper limit of the integral was taken as D, where D is the distance between the load and the point. The vertical displacement at the depth Z then becomes:

$$V_z(y,z) = \frac{2PX}{\pi r_o c}\int_0^\infty f(\zeta, z, n_i, \chi_i)J_i(\zeta)\frac{F(\zeta D/r_0)}{\zeta D/r_0}d\zeta. \tag{3.28}$$

where, $\int_0^a J_0(u)du = F(a)$. These two models, separative and viscoelastic, were used to compute deformation components. Significant differences were observed in their results. It was found out that, the results obtained by the separative method were dependent on the stress values used and the elastic constants assumed for the layers while those of the viscoelastic were totally independent of the elastic parameters.

It was concluded that the viscoelastic analysis was the best approach to be followed in determining pavement response. The result of pulse load and moving loads for both separative and viscoelastic methods showed that, assessing the contribution of each layer to deformation can be misleading when the stationary pulse load is used and that the viscoelastic moving load model is more realistic.

3.2.4 Sweatman, 1983

Sweatman (1983), derived a modified version of the Road Stress Factor initially defined by Eisenmann (1975) and used this to determine the impact of dynamic loads generated by different suspensions. The study expanded Equation 3.6 to non-normally distributed dynamic loads. A modified version of this equation assuming randomness of load distribution was derived as:

$$V = 1 + 6S^2 + 4\gamma_1 S^3 + \gamma_2 S^4 \tag{3.29}$$

where, γ_1 and γ_2 are the skewness and kurtosis coefficients (Beyer 1990). However, after further analysis with this method and comparisons with previous one, it was found that departure from normality in the distribution of load had little effect on the Road Stress Factor V. Sweatman again noted that, due to the spatial repeatability, Eissenman's model would not account for those areas of the pavement that consistently receive higher loads with any reasonable degree of accuracy. The dynamic load impact was therefore estimated as the 95th percentile value, IF_{95} given by:

$$IF_{95} = 1.645 \times DLC \tag{3.30}$$

The Road Stress Factor associated with this 95th percentile value is.

$$\phi_{95} = IF_{95} \tag{3.31}$$

This method was used to analyze the severity of dynamic loads for each of the suspensions and it was observed that dynamic loads on the pavement can exceed the static load by between 3 tons and 7 tons depending on the suspension type. For typical highway conditions, coefficient of variations of dynamic loads ranged between 12.7% and 27% depending on suspension type and the corresponding Road Stress Factor values were in the range 1.11 and 1.46.

3.2.5 Ullidtz, 1983

Ullidtz (1983), developed a mathematical model to analyze the impact of moving vehicle loads on pavements. In this study, a longitudinal section of the road was divided into separate sub-sections 0.3m long. Layer thicknesses and material properties were treated as random variables along the road profile. Rutting and cracking were assumed to result from variations in material properties and the application of dynamic loads. Dynamic loads were analytically modeled as discussed earlier in Section 3.1.4. The pavement response parameters were calculated by combining Boussinesq equations with the method of equivalent layer thickness. A computer program used the equivalent thickness to compute the tensile strains at the bottom of the asphalt concrete layer and stresses at any point in the section. The fatigue behaviour of the asphalt concrete layer was analyzed following the empirical relationship developed by Cooper and Pell [1974] as:

$$log(\epsilon_r) = \frac{14.39(logV_B + 24.2) \times (logSP_i - 42.7 - logN)}{5.3(logV_B + 8.63) \times (logSP_i - 15.8)} \qquad (3.32)$$

where ϵ_r=maximum allowable tensile strain (10^{-6}) for N load applications.

V_B=percentage volume binder in the mix

SP_i=initial ring and ball softening point of the bitumen.

In the case of the permanent deformation, first a critical strain level, ϵ_0 was defined following the relation developed by Bent Lasen (1986)

$$\epsilon_0 = A \times (10^5)^B \times (0.014 \times E)^c \tag{3.33}$$

where E is the Young's Modulus in MPa or,

$$\epsilon_0 = A \times 10^{5B} \times (9.7 \times 10^{-5} \times E)^c \tag{3.34}$$

where E is in psi. The plastic strain, ϵ_p was computed using the plastic stress-strain relation:

$$\epsilon_p = AN^B \left(\frac{\sigma_1}{\sigma_2}\right)^c, for \epsilon_p < \epsilon_0 \tag{3.35}$$

$$\epsilon_p = \epsilon_0 + (N - N_o) \times A^{\frac{1}{B}} \times B \times \epsilon_0^{1-\frac{1}{B}} \times \left(\frac{\sigma_1}{\sigma_2}\right)^c, for \epsilon_p > \epsilon_0 \tag{3.36}$$

where $N_o = \epsilon_0^{\frac{1}{B}} \times A^{\frac{1}{B}} \times \left(\frac{\sigma_1}{\sigma_2}\right)^{\frac{-c}{B}}$,

and A,B,C are constants (Ullidtz 1987).

A critical depth was defined as the depth where load repetitions and stress levels will result in a plastic strain greater than the critical strain. The equation used to compute the change (increase) in plastic deformation, δ between equivalent depths Z_1 and Z_2 caused by load repetition from N_1 to N_2 was given as:

$$\delta = A(\frac{3P}{2\pi\sigma})^c \times \left(Z_1^{1-2c} - Z_2^{1-2c}\right) \times \left(\frac{N_2^B - N_1}{2c - 1}\right) \tag{3.37}$$

for the case where the depth Z_1 is greater than the critical depth and,

$$\delta = A^{\frac{1}{B}} B \epsilon_0^{1-\frac{1}{B}} \left(\frac{3P}{2\pi\sigma}\right)^{(\frac{c}{B})} \left(Z_1^{1-\frac{2c}{B}} - Z_2^{1-\frac{2c}{B}}\right) \frac{N_2 - N_1}{(\frac{2c}{B-1})}. \tag{3.38}$$

for the case where Z_2 is less than the critical depth (Ullidtz 1987), where, ϵ_0=the critical strain level, Z_1 and Z_2 are equivalent depths, P=Load and A, B, C are constants. An index period of one week was used when the pavement evaluation was made (functional and structural conditions of the road were observed). Roughness was obtained by observed variations in permanent deformations along the road profile. Roughness, mean level of rut depth and percentage of cracked subsections were combined to express the pavement performance in terms of Present Serviceability Index (PSI) as used in the AASHO Road Test. The results of this analysis was used to simulate the performance of several AAHSO Road Test sections to a terminal PSI of 2.5. The number of load repetitions to failure in cracking and rutting was compared with the experimental values and was found to be in good agreement.

3.2.6 O'Connell et al, 1986

O'Connel et al. (1986), conducted an experimental study to assess the extent of road damage caused by several types of suspensions namely walking beam, four leaf-spring and air. Dynamic loads were assumed to be randomly distributed in space and therefore any portion of the road have an equal chance of sustaining the same magnitude of loads as a vehicle passes. This also implies that the damage done by any of the axles is independent of the other. A linear elastic model was used to calculated damage parameters like the critical tensile and compressive strains which were then used to derive a modified version of the Road Stress Factor. The major pavement failure modes of rutting and cracking were evaluated. Their conclusions were:

- Dynamic wheel loads have a significant impact on pavement damage and under certain conditions, damage due to such loads can be about 25% higher than that caused by their static loads.

- For all suspension, the walking beam had the most damaging effect while the air suspension produced the least damage.

- Axle spacing is instrumental in decreasing the compressive strain at the subgrade and hence the tendency for rutting.

- The proportionate increase in cracking damage due to an increase in dynamic loads was by far lower than the corresponding reduction in rutting damage.

3.2.7 Cebon, 1987

A theoretical study was conducted by Cebon, (1987), to asses the dynamic load impacts on pavements. Since higher dynamic loads were found to repeat themselves on certain portions on the road while other sections consistently received lower dynamic loads (ie. spatial repeatability of dynamic loads), performance of such sections under higher loads were assumed to determine pavement life. It was argued that, assuming dynamic loads to be randomly distributed in space will overestimate the pavement life. The longitudinal profile of the road was divided into equally spaced subsections. The time histories of the dynamic loads of three axles of different suspension were calculated. The accumulated damage at particular subsections due to axle impact were calculated using a linear model. Five pavement damage criteria were developed namely:

- Aggregate force criterion,

- Fatigue weighted contact stress criterion,

- Tensile strain fatigue criterion.
- Permanent deformation criterion.
- Aggregate fourth power weighted force criterion.

Based on the assumption of spatial repeatability of dynamic loads, it was concluded that dynamic wheel forces have higher impact on pavement fatigue life than previously thought. Fatigue damage due to dynamic loads was found to be four times higher than that due to static loading at locations for a typical highway speed and roughness. Rutting was found to increase as high as 40% more. Damage done by articulated vehicles increased with speed and at certain speeds, pitch coupling between the axles and increased excitation of the vehicle resulted in additional road damage. On smooth roads and at highway speeds, there is a decrease in dynamic road response and therefore the increase in dynamic loads with speed does not have a corresponding effect on pavement fatigue damage.

3.2.8 Papagiannakis et al., 1988

Papagiannakis et al., (1988), used statistical parameters and analysis in modelling dynamic load impacts on pavements. The main aims of the study were two-fold:

1. To investigate the impact of dynamic loads as a means of checking the accuracy of pavement prediction models.
2. To calculate the impact of suspension type on pavements with respect to their dynamic load generation on the pavement performance.

Two experiments were performed. The first was intended to test the issue of spatial repeatability of dynamic loads suggested by earlier researchers like Addis et al. (1986) so that it could be included in modelling the impact on pavements. The second was to examine dynamic load variation under driving conditions over a range of speeds and levels of pavement roughness. Dynamic loads were found to be spatially repeatable in space.

Dynamic waveforms were treated differently in the analysis. The first method considered dynamic loads to be repetitive in space while the second considered dynamic loads to be random in space. A modified version of VESYS-3A was used. VESYS-3A models the pavement structural system as a linearly viscoelastic layered system whose material characteristics are defined in probabilistic terms. To analyze dynamic loads repetitive in space, each section of the road was divided into subsections and components of the dynamic load frequency distribution were applied to them. For the analysis of loads considered to be randomly distributed in space, dynamic loads were input into VESYS 3A as frequency distribution.

The effects of pavement roughness and suspension type on PSI was modeled for a vehicle speed of 80km/h. Tire pressure was fixed at 105 psi and was supposed to vary within 25% within the static under dynamic conditions. Statistical parameters were used to represent the behaviour of dynamic loads from the experimental data. The coefficient of variation due to roughness was estimated and their average was calculated by distinguishing three time intervals. General forms of dynamic load frequency distribution were assumed by observing the trend of the data and as a result coefficient of variations (CVs) less than 15% were considered to follow a normal distribution while those greater than 15% followed a trapezoidal distribution. Using the observation from the AASHO Road Test on load cycles, the impact was confined to a length of 5.2 metres (ie., six increments of 0.86m). It was considered that, the subsections with the highest damage governed the behaviour of the overall section.

This analytical procedure was used to simulate ten AASHO Road test sections. Four of the sections developed relatively low roughness. Two levels of terminal serviceability were considered to reflect pavement failure namely PSI of 2.0 and PSI of 2.5, the rubber suspension showed damage 17% and 22% higher than the above damage. Corresponding values for the air suspension were only 6% and 8%.

3.2.9 Sousa, Monismith et al., 1989

Sousa, Monismith et al., (1989), carried out an analytical and laboratory study to determine the impact of dynamic load on pavements for different suspensions. Dynamic loads were assumed to be normally distributed. The pavement structure was considered as a multilayered system having a viscoelastic response to loads. Dynamic load data was taken from the work done by Gillepsie (1983). A computer program (called SAPSI) that simulates the dynamic response of the layered structure to dynamic loads was developed. The program also incorporates material properties corresponding to the loading frequencies applied. The total impact of a dynamic loads was modeled by first considering the static component and then the dynamic component of the load.

To arrive at the static component, a quasi-static analysis was performed whereby the layers were idealized into thin layers of 0.06 inches each. The reason was that the computer algorithm developed was only capable of computing stresses, strains and deflections in the middle of layers. These thin layers therefore made it possible to calculate the critical tensile strain at the asphalt bottom with a reasonable degree of accuracy (ie. tensile strain at 0.012in from the bottom of the asphalt layer). The fatigue life was estimated following the formula used by Finn et al (1977) as:

$$logN_f = 15.947 - 3.291log(\frac{\epsilon_t}{10^{-1}}) - 0.854log(\frac{S}{10^3}) \tag{3.39}$$

N_f =Number of load application to failure,

ϵ_t= Tensile strain in the asphalt layer,

S=Asphalt mixture stiffness modulus (psi).

The dynamic component of the load was modelled by considering the time variation of the loads and material property variation due to changes in load frequencies. To account for the effect of vehicle speed on the material properties, a linear viscoelastic relation was used which incorporates velocity, to model the pavement response $R(t)$ as:

$$R(t) = LA_1e^{i(w_1)t} \tag{3.40}$$

where t=time

L= constant load applied

A_1=function of pavement thickness

w_1=Constant (function of pavement and velocity)

The dynamic load L(t) was given as:

$$L(t) = S + De^{i(w_2)t} \tag{3.41}$$

where S=static component of the load

$De^{i(w_2)t}$=dynamic component of the load

w_2=predominant frequency of the load

The total impact dynamic loads from the moving vehicle, $R_D(t)$ was therefore modelled by a mathematical equation as a summation of the static and the dynamic responses as:

$$R_D(t) = [S + De^{i(w_2)t}]A_2e^{i(w_1)t} = A_2Se^{i(w_i)t} + A_2De^{i(w_1+w2)t} \tag{3.42}$$

where A_2=constant dependent on pavement structure. This model was used to determine the damaging effect of different types of suspensions tested namely, a torsion bar, a leaf and walking beam. The time histories of the tensile strains caused by each of the suspension was divided into 256 equal parts and the number of load repetitions to failure was calculated by the formula:

$$\frac{1}{N_{f(suspension)}} = \sum_{256}^{i=1}\left(\frac{1}{256N_i}\right) \tag{3.43}$$

The rate of pavement deterioration caused by the dynamic loads was defined by an index termed Reduction of Pavement Life (RPL). This was used to compare the suspensions. A value of 0 denotes an ideal suspension. It was found that, the torsion bar and leaf spring suspensions had RPLs of 19 and 22 respectively while the walking beam had an RPL of 37 and therefore was the most damaging of all the suspension types.

3.2.10 Hardy et al., 1992:

Hardy et al., (1992), developed an integrated vehicle pavement model by providing alternative formulations for calculating pavement response. Three linear models were formulated namely,

- Beam on Winkler foundation,
- Plate on Winkler foundation,
- Layered elastic half space.

Two dynamic response models were also formulated to calculate the response of a moving load from pulse functions.

The first approach termed the "convolution method" models the response of a linear system to a time dependent load as:

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)f(\tau)d\tau \tag{3.44}$$

where $y(t)$ is the response at time t

$f(\tau)$ is the input force at time τ

$h(t)$ is the response at time t to a unit impulse at time t=0

For a moving load of constant speed v with respect to the system, the response at a point x becomes:

$$y(x,t) = \int_{-\infty}^{\infty} h(x-v\tau, t-\tau)f(\tau)d\tau \tag{3.45}$$

where, $y(x,t)$ is the response at position x at time t, $h(x,t)$ is the response at position x and time t to a unit impulse at t=0 (origin).

This equation is then simplified by taking Fourier transforms with respect to time and space variables as:

$$Y(\zeta,\omega) = (\frac{1}{2\pi})^2 \int\int_{-\infty}^{\infty} y(x,t)e^{i\omega t}e^{-i\zeta x}dtdx \tag{3.46}$$

by substituting equation 3.46 in equation 3.44 gives

$$Y(\zeta,\omega) = (\frac{1}{2\pi})^2 \int\int\int_{-\infty}^{\infty} h(x - v\tau, t - \tau) f(\tau) e^{i\omega t} e^{-i\zeta x} d\tau dt dx \tag{3.47}$$

which simplifies to

$$Y(\zeta,\omega) = 2\pi h^2(\zeta,\omega) F(\omega + v\zeta) \tag{3.48}$$

where ω is the angular frequency of loading corresponding to time,

ζ is the wave number corresponding to distance x, Y and F are the transformed functions. This method was used to assess the influence of speed and frequency of the pavement response.

The second approach termed the "influence function method" models the response in a similar way by integrating the pavement response and the impulse function. The response $y(t)$ is given as:

$$y(x,t) = \int_{\infty}^{\infty} h\left(x, v(t - \tau), \tau\right) f(t - \tau) d\tau. \tag{3.49}$$

An assumption is made that, the dynamic load $f(t - \tau)$ is constant within the duration of the impulse over which the integral is calculated and therefore;

$$y(x,t) = I(v, x - vt) f(t) \tag{3.50}$$

where $I(v,x)$=influence function

$$= \int_{0}^{\infty} h\left(x, v(t - \tau), \tau\right) d\tau.$$

3.2.11 Cole et al., 1992

A recent investigation on the measurement and impact of dynamic loads has been conducted by Cole et al., (1992). The main aims of the study included the following:

- To assess the accuracy of a tire force measuring mat developed for the study.
- To formulate a road damage model based on dynamic load data from (1).
- To compare two road damage criteria (spatial repetitiveness and random distribution of dynamic loads).
- To asses damage potential of various suspension types.

The tire force measuring mat consisted of capacitative strip sensors which are enclosed in polyurethane tiles. Each rubber mat is 1.2 meters by 1.2 meters and 13mm thick and containing 3 sensors. The mat is arranged such that the sensors are transverse to the wheel path at 0.4m apart. In the experiment, 47 rubber mats with 141 sensors were laid on the TRL test track over a distance of 56.4m. An instrumented vehicle was driven

over the mat 40 times at speeds ranging from 2m/s to 27m/s. The dynamic load data from the instrumented vehicle was logged by a digital data logger inside the vehicle while the mat sensor measurements were processed by a Marksman M600 data loggers. The two measurements were found to be close for the length except the last 5m of the mat. After this, 14 uninstrumented articulated vehicles, each consisting of a tractor unit and trailer unit with different suspensions and payload were driven over the mat at speeds between 2m/s and 27m/s and the resultant loads recorded.

A model was formulated to take into account the accumulated damage of any of the axles. The aggregate tire force method developed by Cebon [1987] and the Fourth Power Law of the AASHO Road Test were combined to develop 4th Power Aggregate Force expressed as:

$$(A_k)^4 = \sum_{j=1}^{Na} P_{jk}^4 \tag{3.51}$$

where, k=1,2,3...Ns

P_{jk}=force applied by wheel j to sensor k

Na= number of axles on the vehicle

Ns=total number of sensors along the mat.

Two damage criteria were considered in analyzing the impact of the dynamic forces. On the basis of spatial repeatability, the 95th percentile value of the 4th power aggregate force (A_k^4) was

considered representative while the mean value was considered representative of dynamic load random in space. The A_k^4, values of three of the tested vehicles fitted with

different suspensions (rubber, steel,air) were compared. These values were normalized by dividing by the static load. The dynamic force histories of two of the three vehicles fitted with rubber and air suspensions were compared in terms of their normalized A_k^4 values. It was found that, peak forces from each of the vehicles were approximately 3.5 times the static load however, tire forces from the rubber suspension were higher.

The mean and 95th percentile values were computed for the trailer axles, tractor drive axles and whole vehicle. Computations based on the mean dynamic load showed that the drive axles caused similar damage approximately 7%-10% of the static but those based on the 95th percentile values showed peak damage of approximately twice the static load. A similar trend was observed for the trailer axles with the 95th percentile damage being higher than that of the mean. In the case of the whole vehicle model, it was found out that, all the vehicles had similar performance and the effects of the rubber suspension was not as severe as when considered alone. Their conclusion was that,

1. The wheel force measuring mat was sufficiently accurate as a dynamic load measuring device.

2. Assumption of random distribution of dynamic load underestimates the road damage and that damage based on spatial repetitiveness of loads gives the true assessment of damage.

3. The damage caused by suspension depends on whether the damage is in terms of whole vehicle or individual suspensions.

3.2.12 Cole et al., 1992:

The method of quantifying spatial repeatability of dynamic load has been investigated by Cole and Cebon, (1992]), through experimental and analytical techniques. A reference articulated vehicle model was selected and eight parameters of this vehicle were varied to produce a fleet of vehicles. The experimental stage of the study involved the generation of dynamic loads by a fleet of vehicles. These parameters are:

- tire stiffness,

- drive axle spring stiffness and spring friction force for the tractor

- spring stiffness and friction force,

- sprung mass and sprung mass pitch inertia for the trailer.

Thirty six different vehicle types were created by randomly varying these eight parameters. Tire force histories were

measured for each of the vehicle models for a distance of 220m. The effects of all the axles of the moving vehicle at points on the road were modeled by using the" aggregate tire force" method developed by Cebon [1987] and expressed as:

$$(A_k) = \sum_{j=1}^{N_a} P_{jk} \tag{3.52}$$

where, k=1,2,3...N_s,

F_k=aggregate force at point k along the wheel path

P_{jk}=applied force by tire j to point k

N_a=number of axles on the vehicle

N_s=number of points along the mat.

These vehicles were run over a test truck and three methods of spatial repeatability namely Correlation Coefficient, Mean Separation of Peaks and Accumulated Damage, were compared. The basis of using the correlation coefficient to measure spatial repeatability is that, a high repeatability is noticed when peaks of two force histories occur together along the road while a low repeatability occurs when the peak of one force history occurs near

the trough of the other. Thus profiles of force histories should be either in phase or in opposite phase. Correlation coefficient (ρ) is the statistic known to have similar properties and is defined as;

$$\rho = \frac{E[x(t) - m_x][y(t) - m_y]}{\sigma_x \sigma_y} \tag{3.53}$$

where σ_x, σ_y, m_x, m_y are the means and standard deviations of the two load wavelengths x(t) and $y(t)$ (Cole and Cebon 1992). The value ρ ranges from -1 to 1. It implies that a higher correlation between the force histories of the reference vehicle and any of the vehicle indicates a higher repeatability while a lower ρ value indicates low repeatability.

In the experiment, the aggregate tire force histories were calculated for all the thirty six vehicles and the ρ values between them and the reference vehicles computed. A threshold of repeatability defined by a ρ value 0.707 for each was also computed and it was found that, 1/3 of the vehicles had ρ values less than the threshold. However, an investigation of the vehicle parameters showed that, they were ideal cases.

The effects of speed was also investigated by comparing the ρ values of the reference vehicle and similar values from speeds below and above the reference speed of 22m/s. It was found, that as the speed increases above 22m/s, ρ values also increase. A value of 0.925 was computed for a speed of 27m/s. The rational behind the Mean Separation of Peaks is that, the mean value of the differences between the reference aggregate tire force history and another history should be minimum as an indication of high repeatability. For any peak in the reference history considered, the nearest peak in the other history is located and the difference between them is computed. A small mean indicates high repeatability and vice versa. In the experiment, minimum separation of peaks were calculated for nine vehicles at nine different speeds. It was found that the minimum separation calculated for each of the vehicles was not close to the minimum of the reference vehicle (0.0m).

The accumulated damage method measures repeatability in terms of the variation of the accumulated damage relative to the reference vehicle. A high amount of variation of accumulated damage indicates high repeatability while a more uniform distribution of accumulated damage history indicates low repeatability. The coefficient of variation of the time histories of the reference vehicle was calculated as 0.072. The accumulation of damage for all the vehicles were summed up and the coefficient of variation was computed to be 0.060. This value was found to be quite close. It was concluded that, correlation coefficient between aggregate tire force histories was a good method for measuring spatial repeatability and was recommended as superior descriptor of repeatability.

3.2.13 Mitchel and Gyenes 1992:

Mitchel and Gyenes studied the extent of spatial repeatability of dynamic loads on pavements. Their initial approach was to analyze the experimental dynamic load data available at the Transportation Research Laboratory (TRL) of Great Britain. The dynamic load data for two heavy goods vehicles measured at speeds of 48 km/hr and 80 km/hr were analyzed. These loads were measured at 16 month intervals. The analysis showed that, the pattern of load distribution on the pavement did not differ significantly during the two periods.

An attempt was made to further investigate the repeatability of dynamic loads under a mix of traffic at different speeds. The wide range of loads measured for different vehicles with different speeds by TRL were considered. The cumulative wheel loads at several points along the roads were computed. The values were plotted to find the variations of cumulative wheel load along the track. It was found that, cumulative wheel loads varied substantially above the average value for several portions on the track especially at higher especially at higher speeds. A summary of the cumulative wheel loads for steel and air sprung mass trailers and for all axles indicated a regular of wheel load variation of about 15% about the mean load for the entire section with the highest level of roughness. For another test section with medium roughness, a similar pattern was observed with variations of about 9% about the average. Recognizing the repeatability of dynamic loads from the above investigation, an experimental study was undertaken to investigate the distribution of dynamic loads on public roads. It postulated that, since heavy vehicle geometry, axle loads and tire are controlled by regulations, the pattern of loads generated by a fleet of these vehicle will be similar. Moreover experimental evidence showed that most of the heavy vehicles travelled within a small range of speed (between 50 and 65 mph).

In one of their methods to measure dynamic loads, strain gauges were embedded in the public roads at a depth of about 300mm to measure the horizontal strains in the asphalt concrete layer. Analysis of data from this is still in progress. The second method involved the use of WIM sensors spaced at 2.7m intervals. Dynamic loads were collected for over 2000 axle passes between September 1991 and February 1992. Preliminary analysis of these data show cumulative wheel load variation of 20% about the average value. Moreover, analysis of the data collected in September 1991 and February 1992 showed a similar pattern of dynamic load distribution along the road. It was noted that, though the number and type of vehicles analyzed from the TRL data and the experimental data from the public roads, the results provided sufficient evidence to show that, dynamic loads from a wide range of vehicles are not randomly distributed in space.

To quantify the effect of the spatial concentration of dynamic loads on pavement, the fourth power relationship between the load magnitude and road damage was used. The results showed that, for sections with medium roughness where cumulative wheel load variation of 9% about the mean was observed, an extra damage of 41% would occur. For the roughest section with load variations of 15% extra damage of 75% would occur due to spatial concentration of dynamic loads.

3.2.14 Summary

The first part of this chapter presented in brief the various methodologies that have been followed to measure dynamic vehicle loads. Some of the methods involved measurement using WIM sensors embedded in the pavement or measurements by special instrumented vehicles. In most cases, tire pressure transducer, a combination of strain gauges and accelerometer and wheel force transducer developed by GM have been used. The typical method involves strain-gauging the axle mounting between spring-mounting and brake plate. The dynamic loads are either generated from artificial bumps or from the roughness of the road section selected and calibrated through WIM scales. A notable exception was that of Addis et al. who measured dynamic loads by using an optical tire deflection device calibrated to yield tire load from tire deflection. Regardless of the method employed, the dynamic vehicle loads were found to be higher that the static and also a function of the roughness of the road and vehicle parameters.

The second part presented the various approaches that many workers have followed in modelling the impact of dynamic loads on pavements. Many of the models were formulated with the primary aim of examining the impact of vehicle parameters like suspension types. The models and their results differ due to two main factors, (1) the assumptions on load characteristics and (2) assumptions on material behaviour under loads. Many workers, including Ullidtz, Bradmeyer, O'Connel, Sousa et al. have assumed the distribution of dynamic loads to be random in space as a basis for computing pavement damage but, Addis, Cebon, Cole etc. and their workers have considered the spatial repetitiveness of dynamic load. However, Sweatman and Papagiannakis verified both randomness and spatial repetitiveness of dynamic loads in their theoretical and experimental studies and recommended spatial repetitiveness of dynamic loads to govern pavement damage. This has also been confirmed by Mitchell and Gyenes.

In formulating a model for the impact of dynamic loads, almost all the workers have followed one or more of the following methodologies. First, they quantify the dynamic tire force histories of the vehicle and use it to fit a theoretical model to compute the pavement response parameters. Second, data on the dynamic tire force histories are collected and they are then used to calculate the time histories of the pavement response parameters at several points along the road section under consideration. The peak response parameter is used to fit into a damage model which have derived from empirical or experimental analysis and finally the performance of the section is determined by the performance of the areas receiving the peak loads. Others have characterized the magnitude of the dynamic load in terms of the Dynamic Load Coefficient (DLC), and derived a quantity that defines the road damage using the fourth power law of the AASHO Road Test.

Cebon [1990] made a summary of the work done on dynamic load measurements and various models that have been developed to asses vehicle load impacts.

An update of this work are shown in Tables 3.1 and 3.2 respectively. It is clearly evident that much effort continue to be made in the study of vehicle pavement interaction.

Table 3.1: Dynamic Tire Force Measurements, (initial version by Cebon, 1990)

SOURCE	FORCE MEASUREMENT METHODS	TEST SUSPENSIONS	SPEED (km /hr)
Whitemore et al. U.S.A (1970)	Hub strain, Tire pressure	2 leaf spring single, 1 walking beam tandem	54, 88, sweep 32, 48, 64, 80
Leonard et al. U.K. (1974)	WIM Scale (ground vibration)	Leaf spring single, 4 leaf tandem, 6 leaf triaxle, single point tandem	16, 32, 48, 64
Sweatman Australia (1983)	Hub transducer	3 walking Beam, torsion bar tandem, 4 leaf, 6 leaf, air tandem and triaxle, single point tandem	40, 60, 80
Ervin et al. U.S.A (1983)	Hub transducer	Walking beam tandem, 4 leaf tandem, torsion bar	72, 28
Gorge W. Germany (1984), Hahn W. Germany (1984)	Weighted acceleration, hub transducer (pavement strains)	3 leaf spring single, 1 air sinlge, 4 leaf, 6 leaf, air tandem and triaxle, 1 single point tandem	30, 50, 70, 80, 90
Woodrooffe et al. Canada (1985)	Strain (Pavement strains)	2 Walking beam, air single, 2 air tandem, 2 x 4 leaf tandem	40, 60, 80 18, 40
Addis et al. U.K. (1985)	Laser tire deflection, strain (Pavement strains)	2 leaf spring single, 4 leaf (wide) tandem	32, 48, 64
Mitchell et al. U.K. (1989)	Strains (all axles)	1 leaf spring single	32, 48, 64, 80, 96
Cole et al., Cebon and Hardy U.K. (1992)	Tire force mat, tire force histories	Air tandem and triaxle, steel tandem and triaxle, rubber	Range from 7.2-97.2
Cole and Cebon U.K. (1992)	Tire force mat, hub strains, tire force histories	4 axle leaf spring	Range from 7.2-97.2

Table 3.2: Summary of Dynamic Load Models, (initial version by Cebon, 1990)

SOURCE	PAVEMENT MODEL	VEHICLE MODEL PARAMETER EXAMINED	PAVEMENT DAMAGE CRITERIA	INTERACTION ASSUMPTIONS
Ullidtz et al. Denmark (1983)	Flexible, elastic layer Random property changes every 0.3m	Static load	Cracking, Rutting Serviceability (PSI)	Whole life model Damage calculated weekly Profile degradation with time.
Cebon (1985-7) U.K	Flexible, beam on damped elastic foundation, Dynamic	Road Roughness, Speed, Linear/nonlinear	Fatigue, Rutting Simplified criteria using wheel forces only	Single vehicle pass. Damage starts at few locations which experience large strains
O'Connel, Abbo et al. (U.S.A) 1986	Flexible, elastic layer Rigid plate on winkler foundation (FEM)	Road roughness, speed, suspension type and parameters (stiffness, damping friction) Axle spacing, load sharing	Cracking, Rutting PSI DLC	Flexible: single vehicle pass. Modified road stress factor Rigid: Whole life model Joint fault degradation
Brademeyer et al. U.S.A. (1986)	Flexible, elastic layer, Statistical variation of material Modified VESYS 3-A	Static load	Cracking, Rutting PSI, Slope variance (roughness)	Whole life model Damage calculated weekly Road profile spectrum degradation with time
Monismith et al. U.S.A (1988)	Flexible, elastic layer, Fequency dependent stiffness	Suspension type Torsion bar 4 leaf walking beam	Fatigue	Single vehicle pass. Each point on road subjected to the full spectrum of tyre forces
Papagiannakis et al. (1989) Canada	Flexible, elastic layer, Statistical variation of material properties Modified VESYS 3-A	Suspension type Rubber tandem Air tandem	Cracking, Rutting, Slope variance, PSI	Whole life model Spatial repeatability in AASHO road test. Road profile degradation due to cracking
Hardy and Cebon 1992. U.K.	Flexible layered half space, Beam on Winkler f'dn., Plate on Winkler f'dn.	Linear system Harmonic load Air Suspension (Quarter car), Walking beam	Fatigue Cracking	Single vehicle pass Pavement damage due spatial repeatability peak strains
Cole and Cebon 1992. U.K.	Flexible elastic Dynamic, aggregate tire force.	4 axle leaf spring Suspension type and parameters (stiffness, friction force, sprung mass, pitch inertia)	Cracking and cracking Spatial repeatability determined by correlation coefficientbetween tire force histories	Single vehicle pass
Cole et al. 1992	Flexible Dynamic	Suspension type Air, Rubber, Steel	Cracking	Spatial repeatability/Random- damage by 95th percentile and mean values of loads

Chapter 4

Flexible Pavement Response Model

The loads that are exerted on the road pavements by moving vehicles are dynamic. In order to assess the influence of such dynamic loads, a pavement response model capable of simulating dynamic response due to moving loads is required. This section and consequently greater part of this thesis provides a formulation for the response of pavements subjected to moving dynamic loads. This formulation expands the framework established by VESYS (section 3.2.2). The VESYS model uses Boltzman's superposition principle to translate the pavement response from a stationary load to pavement response due to static loads. The following formulation expands this framework to translate the pavement response from a stationary load to pavement response from a moving dynamic load.

4.1 Formulation for Single Axles

This section is devoted to providing a formulation that compute the pavement response due to moving dynamic loads from single axles. It will also provide a basis for formulating a model for tandem or multiple axles in the later sections of this thesis. The moving dynamic load is modelled as a moving pulse load of time-dependent amplitude $A(t)$. The load is assumed to influence a pavement subsection of length 2.25 meters (90 inches). Its loading function is defined as:

$$P(x+t) = A(t) sin^2\left(\frac{\pi}{2} + \frac{\pi t}{D}\right) \tag{4.1}$$

where $A(t)$ is the time-dependent amplitude and D is the duration of the load pulse. The difference between Equation 4.1 and Equation 3.10 is that, the amplitude of the load pulse is a function of time instead of being a constant.

The time derivative of the load pulse therefore becomes:

$$\frac{\partial P}{\partial t} = \frac{\partial A(t)}{\partial t} sin^2\left(\frac{\pi}{2} + \frac{\pi t}{D}\right) - \frac{A(t)\pi}{D} sin\left(\frac{2\pi t}{D}\right) \tag{4.2}$$

Applying Boltzman's superposition principle (equation 3.1), the pavement responses (stress, strain, or deflection) due to a stationary load is transformed into the pavement response due to a moving dynamic pulse load of duration D and time-dependent amplitude $A(t)$ as follows.

$$R(t) = \int_{-D/2}^{D/2} \psi(t-\zeta)\left[\frac{\partial A(t)}{\partial t} sin^2\left(\frac{\pi}{2} + \frac{\pi t}{D}\right) - \frac{A(t)\pi}{D} sin\left(\frac{2\pi t}{D}\right)\right] d\zeta \tag{4.3}$$

where, $\psi(t-\zeta)$ is the pavement response due to the stationary load. The stationary load response is calculated using the multi-layer elastic program ELSYM5. The time-dependent amplitude A(t) is calculated from the experimental dynamic load data collected with the instrumented vehicle developed by the NRCC. It is equal to the ratio of measured dynamic load divided by the stationary load. It must be noted however that, the same stationary load value was input in ELSYM5 to calculate the staionary load response ψ(t) Equation 4.3 is then used to calculate the two critical pavement response parameters, which are the tensile strain at the bottom of the asphalt concrete layer (Exx) and compressive strain at the top of the subgrade (Ezz). An example of the output from Equation 4.3 is shown in Figures 4.1 and 4.2.

The main advantage of this formulation is that, it does not assume the magnitude of the load as constant within the influence zone of the load as assumed by other studies (eg. Cebon and Hardy, 1992). Furthermore, it can be applied to rigid pavements also but, in that case, rigid pavement influence functions must be used which can be derived by any finite element elastic slab model (eg., ABAQUS).

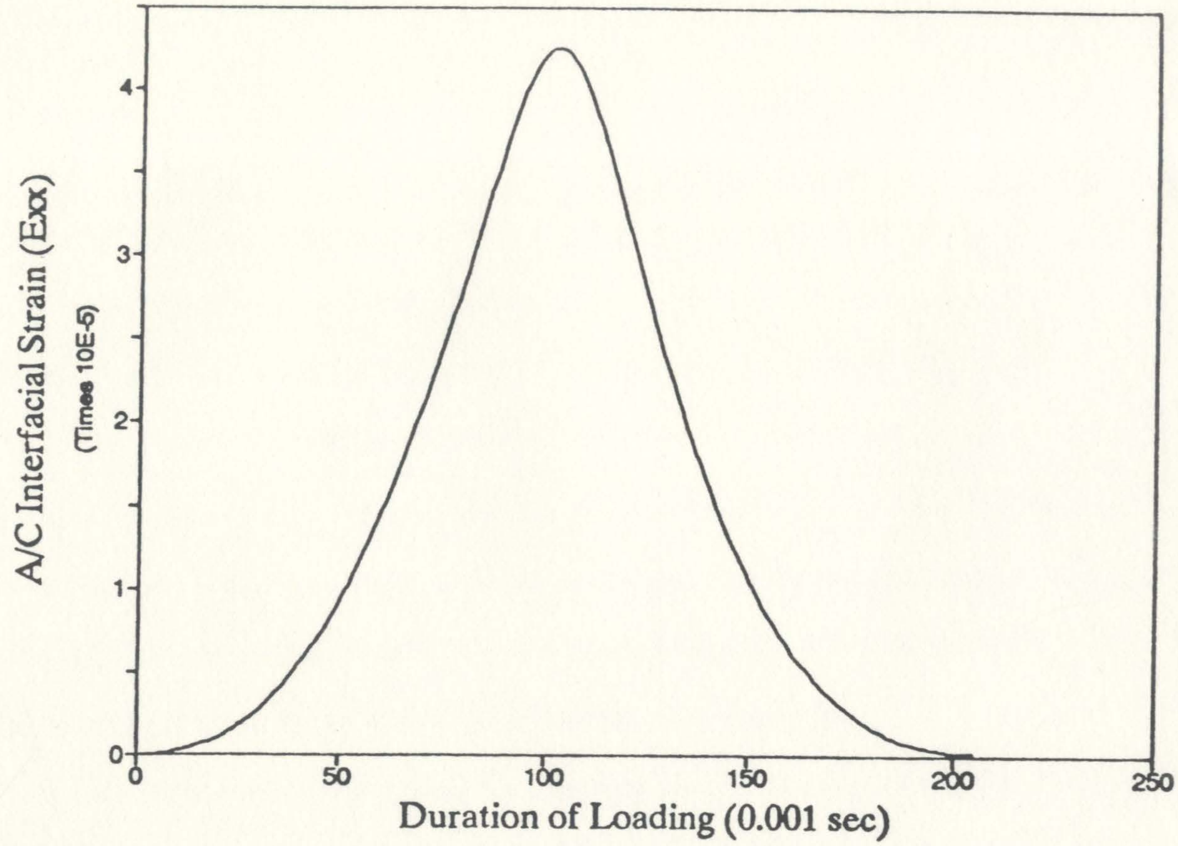


Figure 4.1: Pavement Response Under Moving Dynamic Loads-A/C Interfacial Strains.

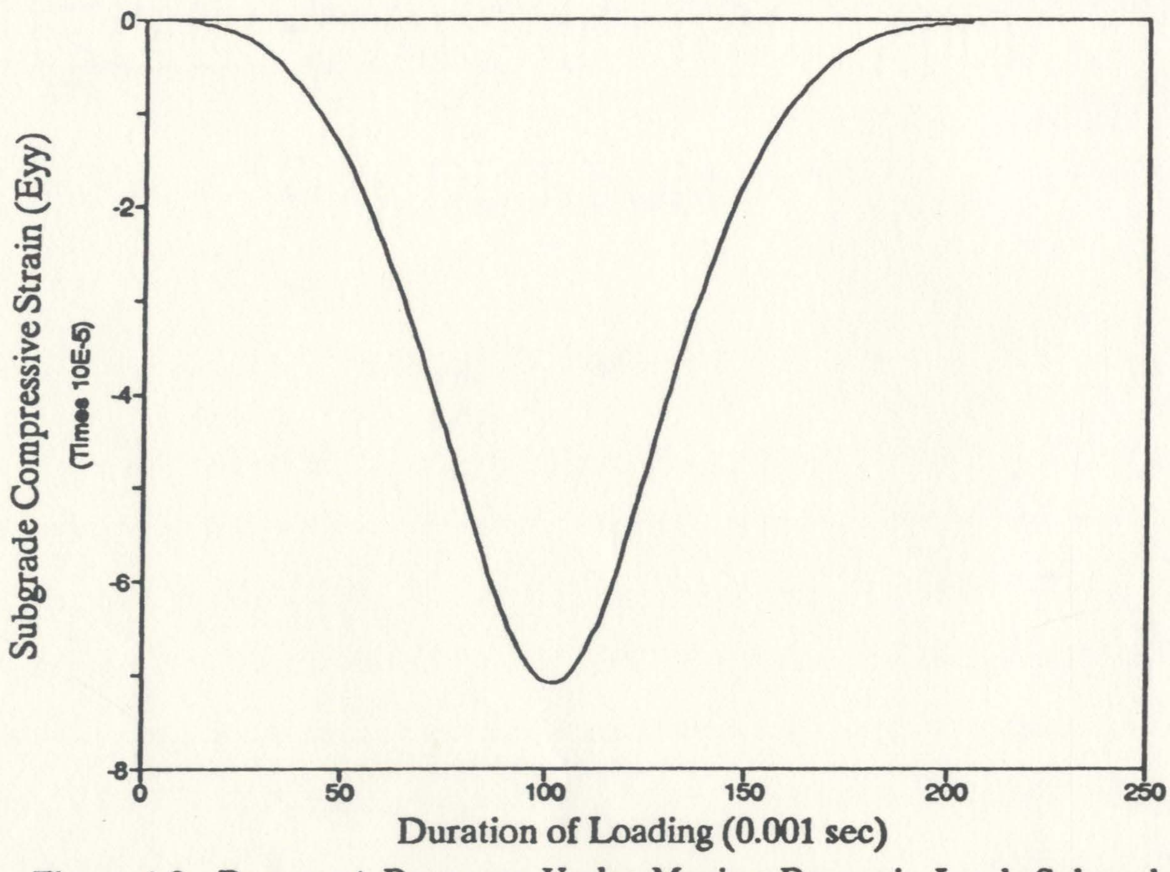


Figure 4.2: Pavement Response Under Moving Dynamic Loads-Subgrade Compressive Strains.

4.2 Formulation for Tandem Axles.

The effects of multiple axles on pavements differ from that of single axles mainly due to the overlapping effects of the loads. Moreover, multiple axles are supposed to share static loads equally. Reports from Sweatman (1983), Mitchell and Simons, (1989) Woodrooffe (1986) and others indicate that, this load equalization is rarely achieved in practice. This section expands on the dynamic load response model developed for single axles to a dynamic response model for loads generated by tandem axles. It will also provide a methodology for studying the dynamics and importance of load sharing and axle spacing in tandems.

The formulation for single axle is applied to tandem axles by superposition of their responses. The amplitudes of the two load pulses generated by the axles depend on the pavement surface roughness and the static load-sharing of the axles and are denoted by $A_1(t)$ and $A_2(t)$. Their loading functions may be represented by the following equations.

$$P(x_1 + t) = A_1(t) sin^2 \left(\frac{\pi}{2} + \frac{\pi t}{D}\right) \tag{4.4}$$

$$P(x_2 + t) = A_2(t) sin^2 \left(\frac{\pi}{2} + \frac{\pi t}{D}\right) \tag{4.5}$$

Each individual axle in a tandem group is assumed to influence a pavement subsection of 2.25 meters (90 inches) length as assumed earlier. The axle spacing is assumed to be of 1.5m (60 inches), hence the zone of influence of the tandem axle becomes 3.75 meters of length along the longitudinal axis of the road (150 inches or 5/3 times the zone of influence of the single axle described above). The responses from the individual axles are calculated and the principle of superposition is used to determine the response in the overlapping zone. That is, the two responses affecting the zone of common influence are summed up and the resulting influence function is obtained. Figure 4.3 shows an influence function of a pavement response under tandem axle load. This arrangement provides an overlapping influence zone of 0.75 meters (30 inches) between the two individual axles in the tandem group. For the parametric study that follows, this axle spacing is considered as a variable, ranging from 1.3 meters to 1.9 meters.

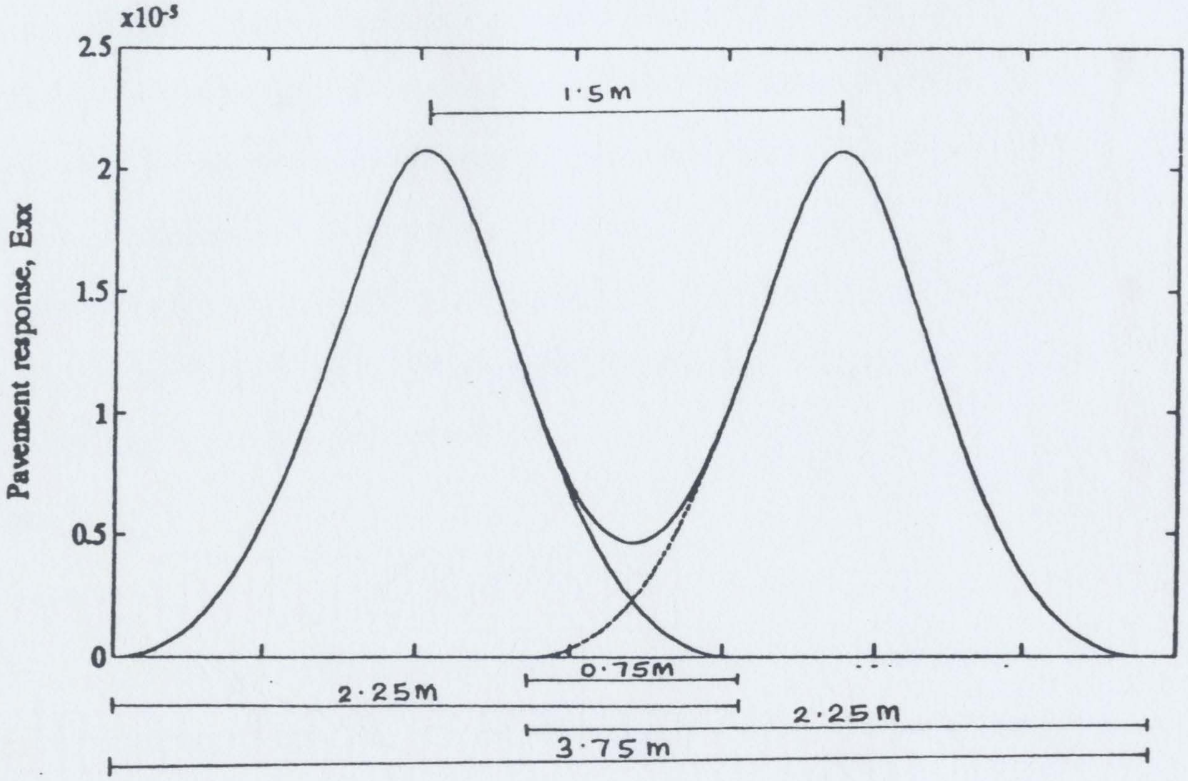


Figure 4.3: Pavement Response Under Tandem Axle Load.

4.3 Modelling Fatigue Cracking and Rutting Damage.

The maximum responses calculated by the formulations presented earlier (for single and tandem axles) are used to estimate the pavement life and also the relative damage potential of the two suspension systems tested. The two main traffic-related failure mechanisms used in pavement life determination are fatigue cracking and permanent deformation (rutting).

In analyzing fatigue cracking damage in a multilayered pavement structure, the conventional relationship between the tensile strain in the asphalt layer and number of load applications is given as:

$$N_i = K_1 \left(\frac{1}{\epsilon_i}\right)^{K_2} \tag{4.6}$$

where N_i is the number of load applications that will cause fatigue failure at the strain level ϵ_i, K_1 and K_2 are experimentally determined constants.
The fatigue constants used in this model were taken from work done by Rauhut et al. (1976), as K_1=7.87E-07 and K_2=3.322 and correspond to 10% area cracked.

Rutting damage is associated with the accumulation of plastic strains in the pavement layers. Experimental analysis and several laboratory investigations (Monismith, 1976, Rauhut et al., 1976, and Eisenman 1977) show that there is a good correlation between subgrade compressive strain and rutting damage. Plastic deformation is therefore assumed to be associated with subgrade compressive strain. A conventional relation which expresses the plastic strain ϵ_p as a function of the number of load repetitions N, is used to calculate the accumulated plastic strain that results in pavement rutting. This relation is given by:

$$\epsilon_p = \epsilon \mu N^{-\alpha} \tag{4.7}$$

where ϵ is the peak elastic strain under pulse load of duration 0.1 seconds and α and μ are constants determined experimentally. Typical values for these constants can be found elsewhere (Rauhut et al., 1976). Alternatively, the number of load repetitions N (pavement damage) that will cause rutting failure at a subgrade compressive strain level ϵ_v can be determined from the relation developed by the U.S Army Corp of Engineers (T.Y Chou, 1976) which has been used by Shahin et al. (1986).

$$\epsilon_v = 5.511 \times 10^{-3} \left(\frac{1}{N^{0.1532}}\right) \tag{4.8}$$

Equation 4.8 will be used later to determine the number of load repetitions to cause rutting failure for different types of pavements.

4.4 Translating Damage into Pavement Life-Single Axles.

The pavement section is first divided into 2.25 meters (90 inches) long subsections to correspond with the influence area of load at a particular point. The number of load applications required to fail each subsection is computed using the maximum responses. The variation of load applications to failure along the pavement section (Figure 4.4) is used to draw a cumulative frequency distribution curve from which the probability of pavement subsection survival is determined (Figure 4.5). The translation of the damage into pavement life is based on the assumption of spatial repeatability of dynamic loads. It has been noted in Sections

3.1.9, 3.2.7, 3.2.8, and 3.2.9 that, there is evidence that moving vehicles apply their loads at approximately the same locations on the pavement surface. This is however, not a very good assumption as far as traffic under "real" highway conditions is concerned. Research is currently in progress to quantify the extent of this spatial repeatability. The 90th percentile of the cumulative frequency distribution of damage values is assumed to represent the pavement failure. This is consistent with the selection of a 10% area-cracked failure relationship (Equation 4.6).

A ratio is defined between the performance under moving static load and that of dynamic loads which is termed Pavement Life Ratio (PLR). It is defined as the number of load repetitions to failure caused by static loads (N_{static}) divided by the number of load repetitions to failure caused by moving dynamic loads, ($N_{dynamic}$). This quantity is expressed algebraically as:

$$PLR = \frac{N_{constant}}{N_{dynamic}} \tag{4.9}$$

The numerical value of PLR indicates the effects of dynamic loads.

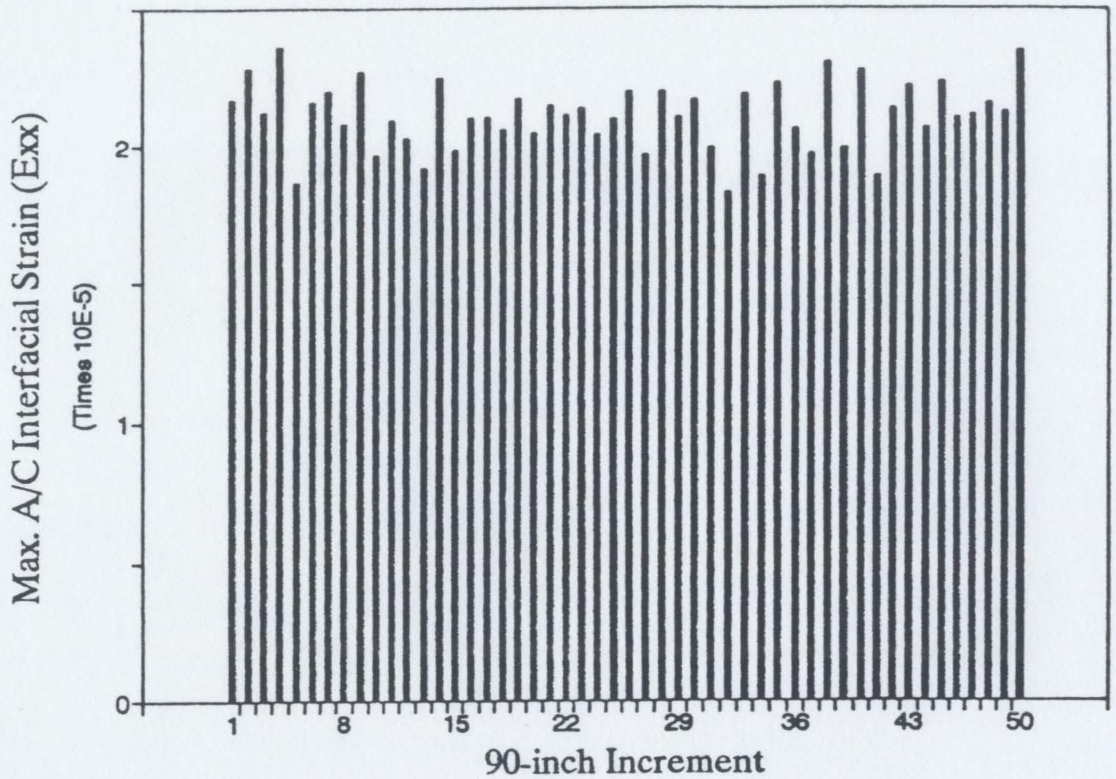


Figure 4.4: Max. A/C Interfacial Strains for Subsections

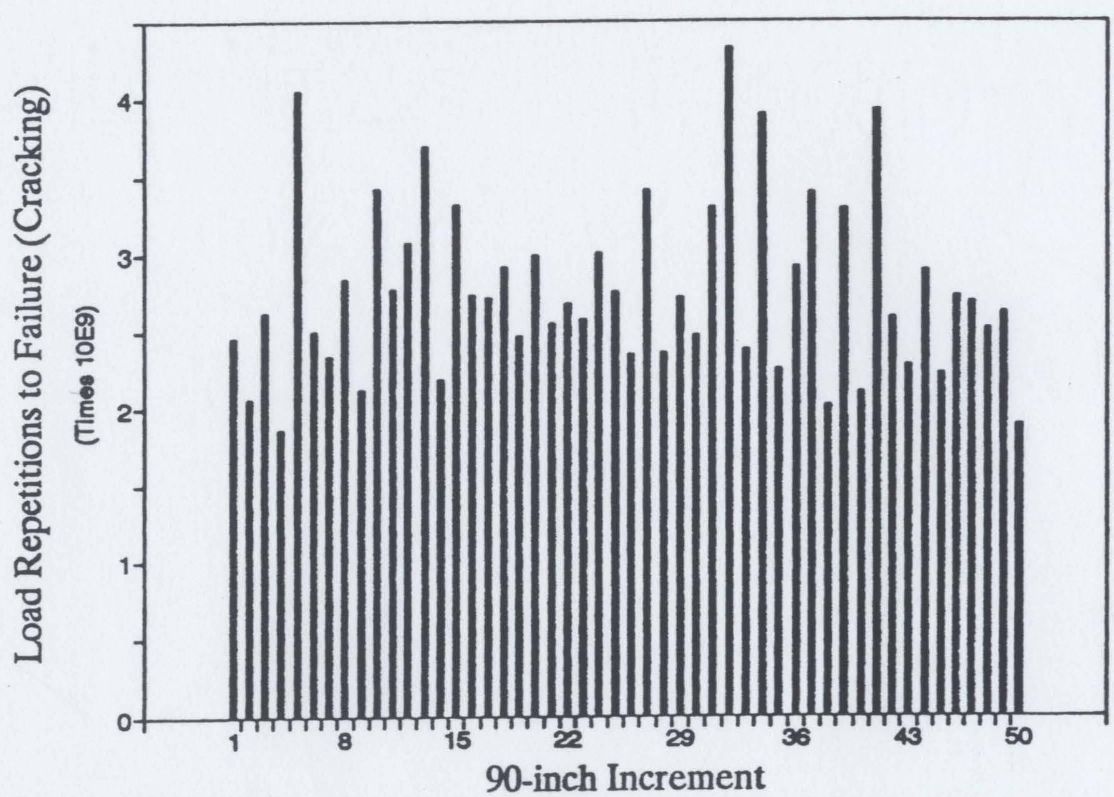


Figure 4.5: Cracking Damage along Pavement Section

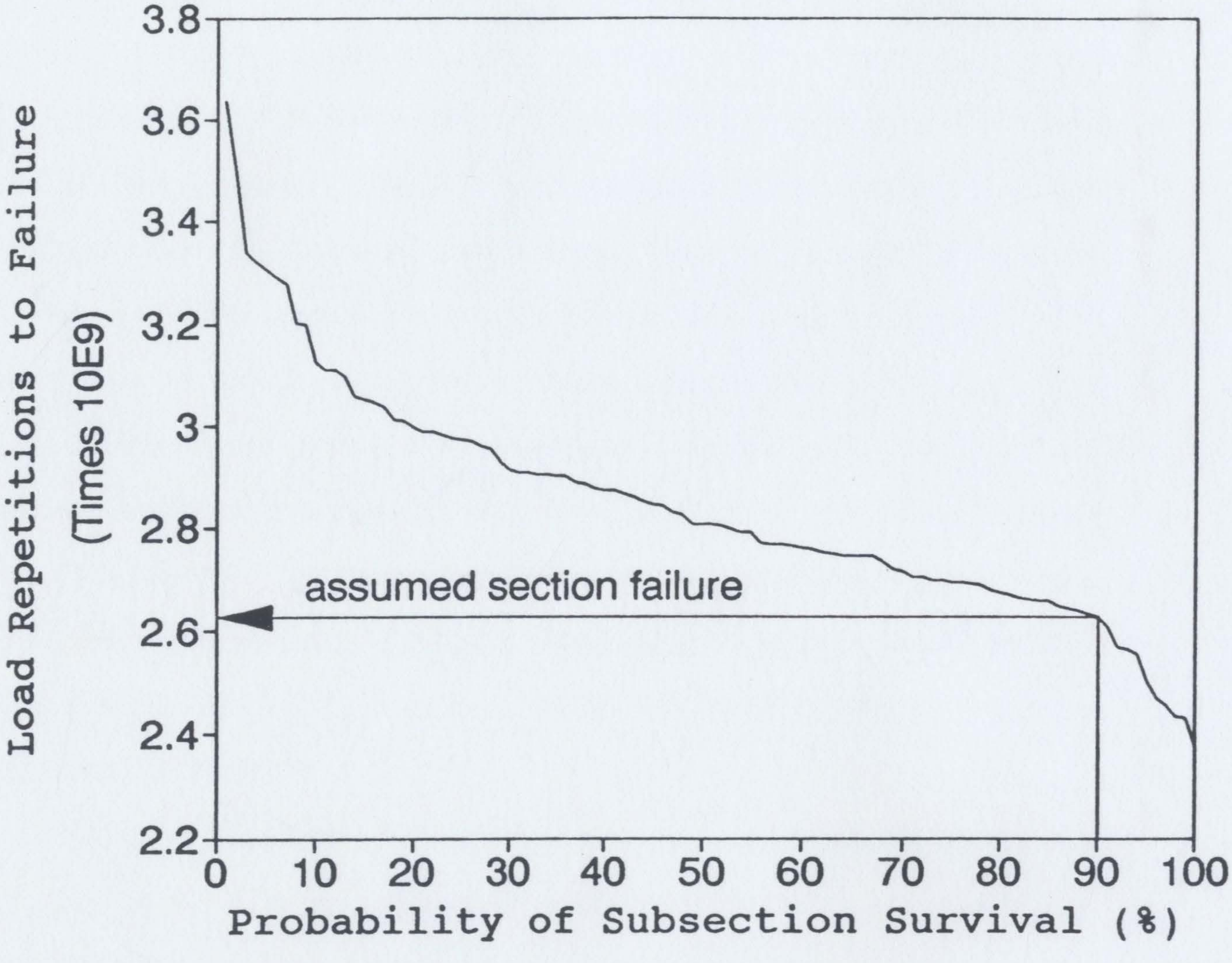


Figure 4.6: Probability of Pavement Subsection Survival

4.5 Translating Damage into Pavement Life-Tandem Axles.

The pavement damage under tandem axle loads, was determined by taking into consideration the strain cycle between the individual axles in the tandem group. Among the methods recommended by the American Society for Testing and Materials (ASTM E1049-85), Papagiannakis et al. (1991), have recommended the rainflow/range-pair method as the best suited for counting strain cycles under multiple axle loads. Following this method, 'Valley-peak-valley cycles only should be counted when considering tensile failure while peak-valley-peak cycles only should be counted for compressive failure, (eg., rutting)'. The maximum strain in a cycle is represented by the range which is defined as the sum of the absolute values between a trough and a peak in a strain cycle. For example, assume the peak responses for the two axles are denoted by P_1 and P_2 and the trough value is V, the response Exx_1 or Ezz_1 is obtained by finding the maximum of P_1 and P_2. The cycle strain Exx_2 or Ezz_2 is also calculated by subtracting V from the minimum of P_1 and P_2.

This method allows the calculation of the response parameters irrespective of which of the axles (leading or trailing) produce the maximum response. Figures 5.8 and 5.9 illustrate how this method was followed in determining the maximum tensile and compressive strains in a strain cycle due to a single pass of tandem axle. The calculated strains were used to compute their corresponding number of load repetitions to failure on each subsection using the three vehicle speeds. Thus for each tandem axle, two different load repetitions were calculated, N_1 corresponding the number of load repetitions due to the maximum of the peak strains from the two axles and N_2 corresponding to maximum inter-axle residual strain. The procedure allows for the consideration of the full effects of tandem axles on the pavement.

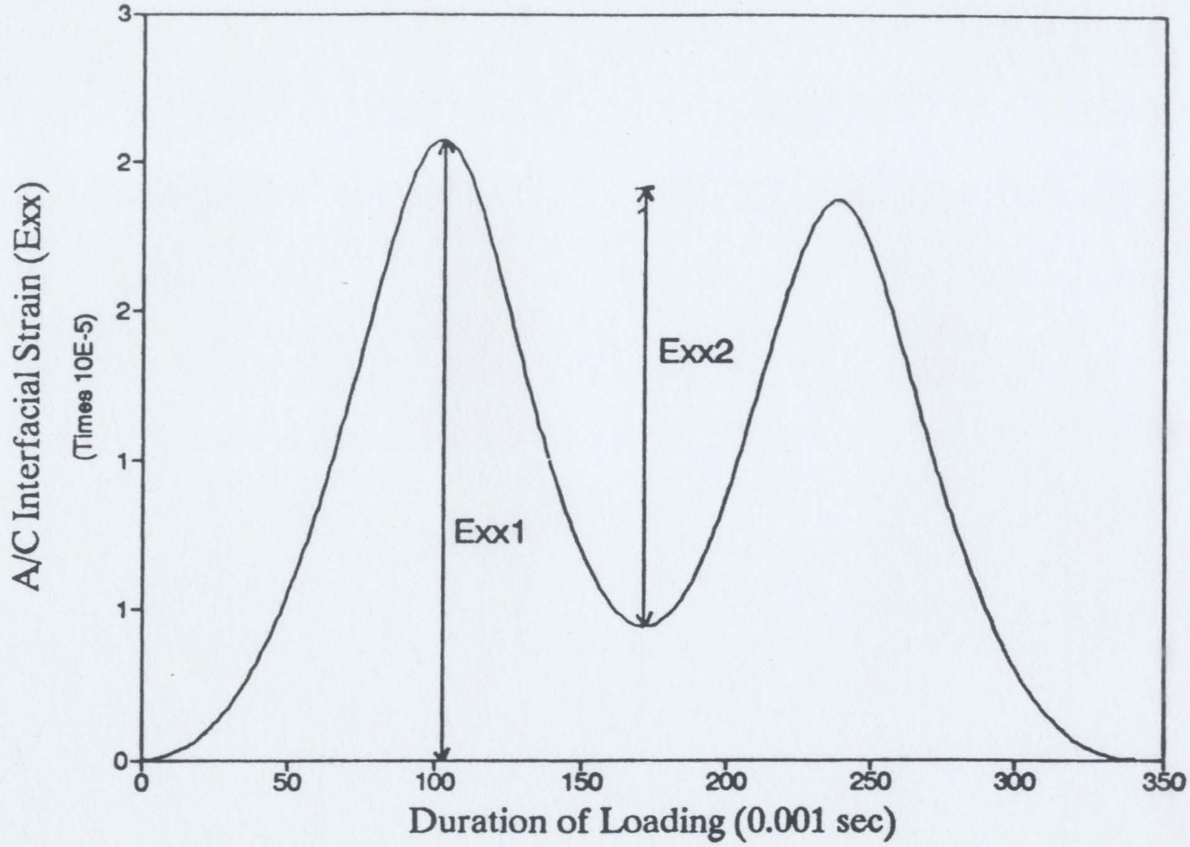


Figure 4.7: Determination of A/C Strains for Calculating Cracking Damage

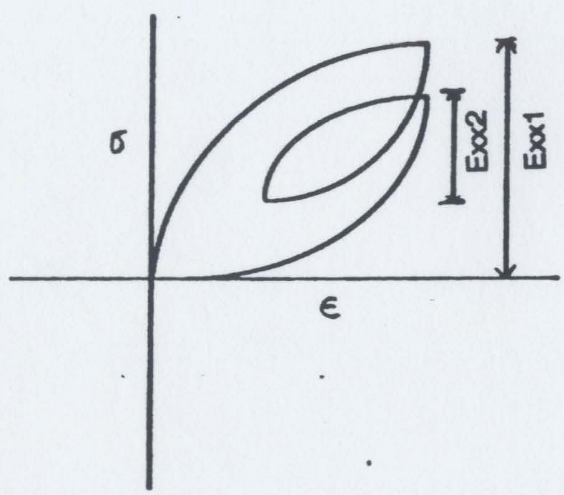


Figure 4.8: A/C Interfacial Strain Cycle Under Tandems.

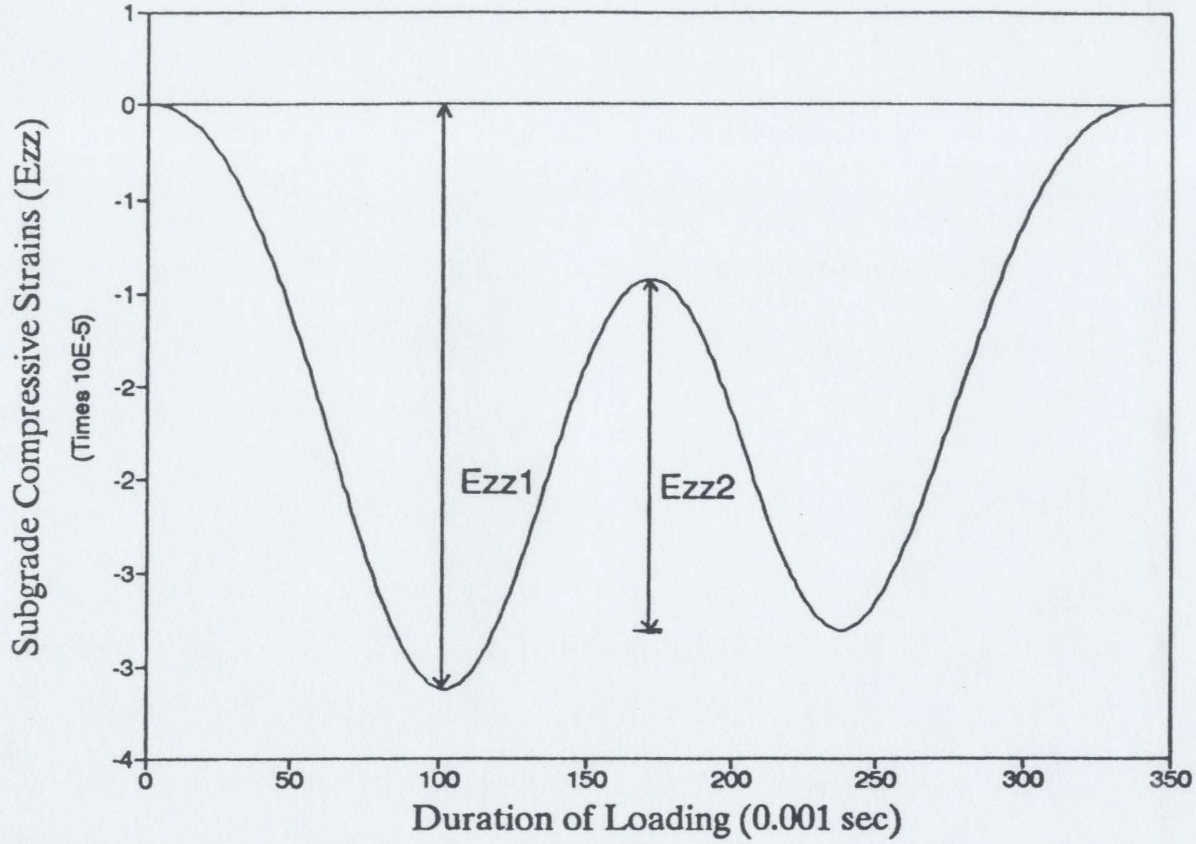


Figure 4.9: Determination of Subgrade Strains for Calculating Rutting Damage

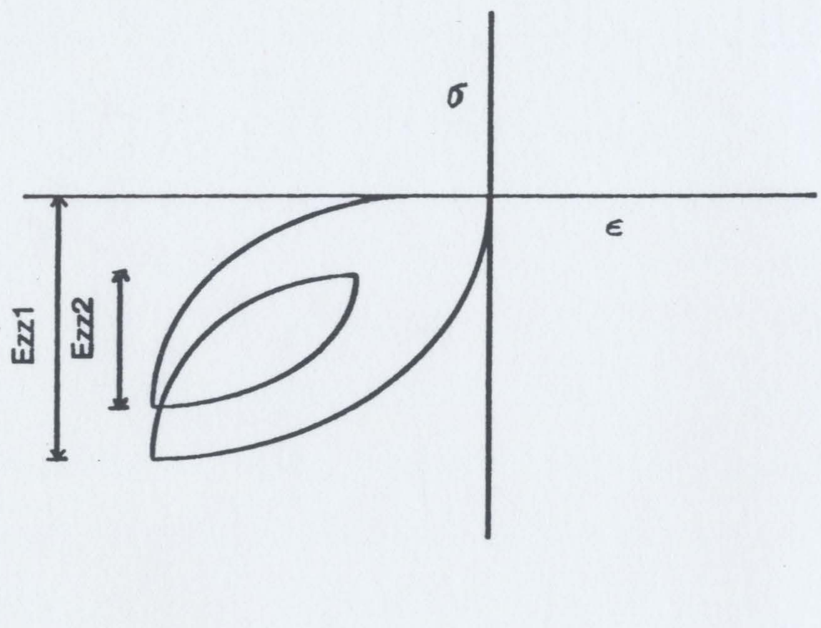


Figure 4.10: Subgrade Compressive Strain Cycle.

Using Miner's rule for cumulative damage, it follows that a single pass of tandem axle consumes pavement life of $\left(\frac{1}{N_1} + \frac{1}{N_2}\right)$, where N_1 and N_2 are the number of repetitions to failure at the strain levels Exx_1 and Exx_2 as indicated in the Figures 5.5 and 5.6. The number of tandem axle load repetitions to failure denoted by (N_{tandem}) is given as:

$$N_{tandem} = \frac{1}{\left(\frac{1}{N_1} + \frac{1}{N_2}\right)} \tag{4.10}$$

The pavement damage models for rutting and cracking (Equations 4.10 and 4.12) were used to calculate the pavement damage due to dynamic tandem axle loads for each of the three pavement types selected. An equation for Pavement Life Ratio was developed for tandem axles by following the procedure for that of single axles. This yielded an equation for Pavement Life Ratio with respect to tandems axle as:

$$PLR_{tandem} = \frac{\left(\frac{1}{N_1} + \frac{1}{N_2}\right)_{dynamic}}{\left(\frac{1}{N_1} + \frac{1}{N_2}\right)_{static}} \tag{4.11}$$

4.6 Implementation of the Pavement Response Model:

Two computer programs, called STATIC and DYNAMIC were developed in FORTRAN to implement the moving static, and the dynamic load response models respectively (Equations 3.12 and 4.3). Subscripts I and II are used in each of the programs to denote the implementation of single and tandem axles respectively. The programs run on IBM PC and compatibles. The stationary load response parameters input in the program are calculated by ELSYM5 at discrete points along the length of the pavement and the program calls a cubic spline subroutine to interpolate the values between any two points. These stationary response parameters are input in the program as a function of time by converting distance to time for a selected vehicle speed.

The program STATIC performs numerical integration of Equation 3.12. (section 3.2.2). Since the load used for obtaining the response under static load is equal to the load used in calculating the influence function of response under stationary load, an amplitude A of 1.0 is input (Equation 3.12). The pavement response at any point within the time interval determined by vehicle speed is calculated using a time increment of 0.001 seconds and the maximum response is combined with the fatigue cracking and rutting damage models to compute the number of load applications to failure (pavement life).

The program DYNAMIC, uses the dynamic load amplitudes measured by the instrumented vehicle developed by the National Research Council of

Canada (NRCC) as additional input. The load amplitudes are converted into amplitude ratios by dividing the measured dynamic loads by the static load of the axle in question as discussed in section 4.1.1. The program then performs numerical integration of Equation 4.3 and computes the responses. The maximum responses are combined with the fatigue cracking and rutting damage models to compute the number of load repetitions to failure. One feature of the program is that, it can calculate the responses by varying starting points of the vehicles. The program first assumes a starting point of the vehicle and calculates the responses for every 2.25 meters increments. Then the program advances to a new starting point and performs the similar calculations for every subsection. This allows for a random location of the dynamic load waveform with respect to the peak of the loading function.The results presented next were obtained using a random starting point. Details of the two programs can be found in Appendices A and B, respectively.

Chapter 5

Database and Analysis

5.1 Experimental Data.

The dynamic load amplitude data input into Equation 4.3 were obtained from an experiment conducted by Papagiannakis et al. (1988) with the instrumented vehicle developed by Vehicle Dynamics Laboratory of the National Research Council of Canada (NRCC). A brief description of the NRCC vehicle is given next.

5.1.1 The NRCC Vehicle

The vehicle used in the experiment was a six-axle semi-trailer tanker truck equipped with two tandem axles, a lift-axle and a steering axle. The two suspension types (air and rubber) tested were located in the drive and trailer tandem axles, respectively. The stationary loads on the tandem groups were 205.52 kN for drive axle and 204.54 kN for trailer axle. Table 5.1 shows a summary of the stationary loads measured on each axle in the tandem group for the two suspensions. From Table 1, it can be seen that, there was no perfect load-sharing in any of the axles groups even under stationary conditions. The air suspension distributed the loads in the ratio of 50.36/49.64 while that of the rubber suspension was 50.84/49.16 Further details of the NRCC vehicle can be found under Woodrooffe et al. 1986.

5.1.2 The Experiment

The main aim of this experiment was to quantify the magnitude of dynamic loads generated by heavy vehicles. The parameters that were taken into account were pavement surface roughness, vehicle speed and suspension type. Five pavement sections were selected to represent five levels of roughness. Two suspension types (air suspension and rubber suspension) were tested at three different vehicle speeds, 40, 60, 80 km/h. Dynamic loads were measured at each of the speeds as the vehicle traversed along the road. The loads were processed as the deviations of the measured dynamic loads from the static values at a sampling frequency of 100 Hz. An example of dynamic load measurements is shown in Table 5.2. The summary of the runs is also shown in Table 5.3 where the dynamic loads are represented by their standard deviations. One unique observation of the pattern of dynamic loads obtained from replicate runs of the NRCC vehicle was their spatial repeatability and this is shown in Figure 5.1.

Table 5.1: Stationary Axle Loads on the NRCC Vehicle
(After Papagiannakis et al. 1988)

AXLE POSITION	TOTAL LOAD ON AXLE (KN)
Steering	55.92
Drive Leading	103.50
Drive Trailing	102.02
Trailer Leading	100.55
Trailer Trailing	103.99

Table 5.2: Typical Processed Data from Dynamic Load Testing, (After Papagiannakis et al. 1988)

Run 31/Site 1/Trailer/Chalmers/80km/hr				
Laser (6)	5th Wheel (7)	Lead (8+9)	Trail (10+11)	Time
0.30	0.90	0.25	1.80	1.67
0.27	1.12	0.66	2.38	1.68
0.25	0.90	1.46	1.72	1.69
0.09	1.01	1.97	2.20	1.70
0.06	1.05	3.73	3.55	1.71
0.06	0.97	3.66	4.51	1.72
0.07	-2.17	4.14	3.88	1.73
0.07	-2.03	3.07	1.94	1.74
0.06	-2.10	2.93	1.10	1.75
0.06	-2.21	1.13	1.54	1.76
0.06	0.90	1.21	1.94	1.77
0.05	0.90	-1.10	1.58	1.78
0.06	0.83	0.40	1.03	1.79
0.06	0.97	1.21	0.15	1.80
0.06	1.16	2.41	0.30	1.81
0.28	1.12	0.00	-0.69	1.82
0.28	1.12	-0.81	-1.46	1.83
0.25	0.94	-2.93	-2.82	1.84
0.07	0.72	-3.23	-2.78	1.85
0.06	0.76	-2.42	-2.49	1.86
0.06	-2.47	-2.71	-2.01	1.87
0.07	-2.17	-3.34	-2.45	1.88
0.27	-2.21	-2.71	-3.11	1.89
0.28	-1.95	-3.19	-3.22	1.90

Table 5.3: Dynamic Load Summary (After Papagiannakis et al. 1988)

RUN #	SITE #	ROUGHNESS (IN/MILE)	SPEED (MPH)	OBSERVATIONS	RUBBER SUSP. SD (KN)	AIR SUSP. SD (KN)
29			25	7069	8.040	8.070
30	1	56	38	4440	10.870	6.940
31			50	3400	14.280	7.400
21			25	7207	11.060	9.440
22	2	87	38	4527	18.760	8.660
24			50	3467	28.170	11.010
13			25	7286	10.860	10.400
14	3	96	38	4576	14.490	11.300
12			50	3505	30.110	13.670
3			25	4913	14.760	12.600
4	4	115	38	3085	16.810	19.070
40			50	2363	33.570	19.460
16			25	6423	22.150	15.650
17	5	201	38	4034	27.060	22.110
20			50	3090	42.550	21.080

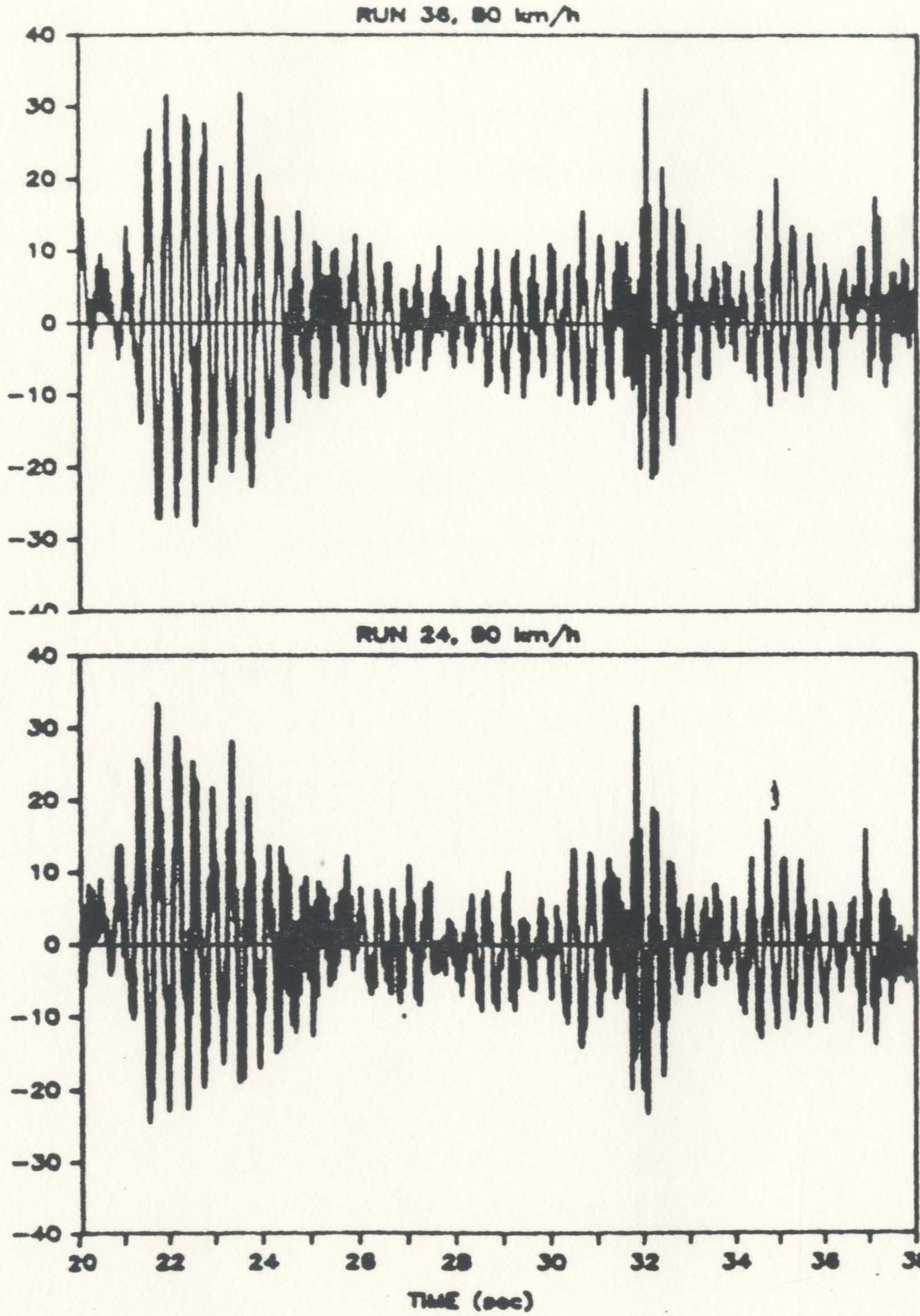
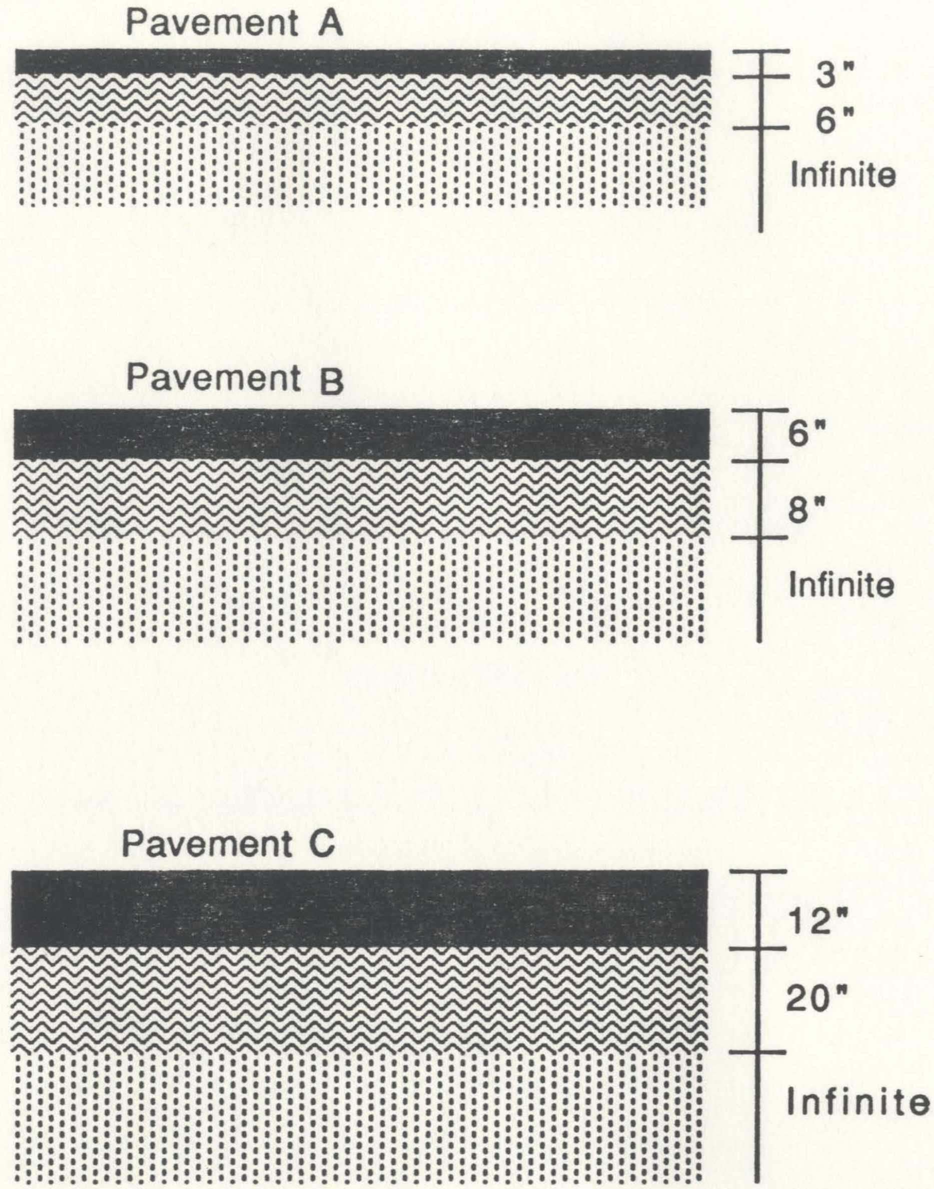


Figure 5.1: Spatial Repeatability of Dynamic Loads (After Papagiannakis 1988).

5.1.3 Stationary Load Response

In order to account for different road pavements with different structural strengths, three different types of pavements were considered in this study for obtaining pavement response influence functions $\psi(t)$. The sections were selected to reflect structurally weak (thin), medium and strong (thick) pavements. Pavement type A, consisted of 3 inches thick asphalt concrete layer on 6 inches granular base that rests on the subgrade. Pavement type B consisted of 6 inches thick asphalt concrete on 8 inches granular base resting on the subgrade, while pavement type C consisted of 12 inches thick asphalt on 20 inches granular base resting on the subgrade. ELSYM5 was used to calculate the responses of each of the three pavements under a stationary load. These responses were calculated at discrete points and a cubic spline routine was fitted to interpolate for values in between. Figure 5.2 shows the sections and their elastic parameters. Figures 5.3 to 5.5 show the stationary load responses (tensile strain at the bottom of the asphalt concrete layer and compressive strain at the top of subgrade) calculated by ELSYM5. The response parameters will serve as input in the static and dynamic response models described in Chapter 4.



Modulus of Elasticity:
A/C Layer=400000 psi
Grannular Base=40000 psi
Subgrade=5000 psi

Poisson ratio:
A/C LAyer=0.4
Grannular Base=0.35
Subgrade=0.35

Figure 5.2: Cross Sections of Flexible Pavements Analyzed:

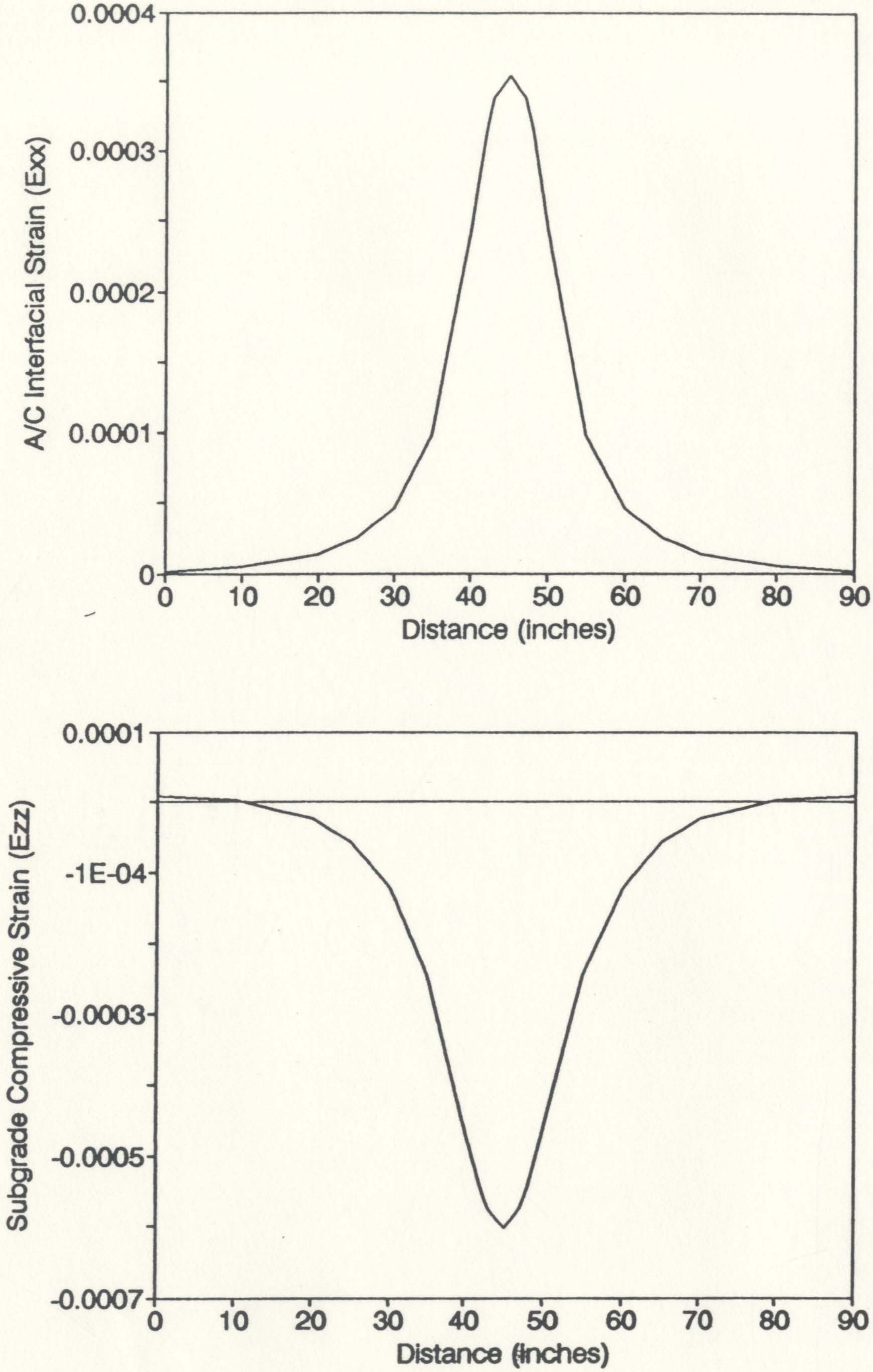


Figure 5.3: Response from Stationary Load-Pavement A

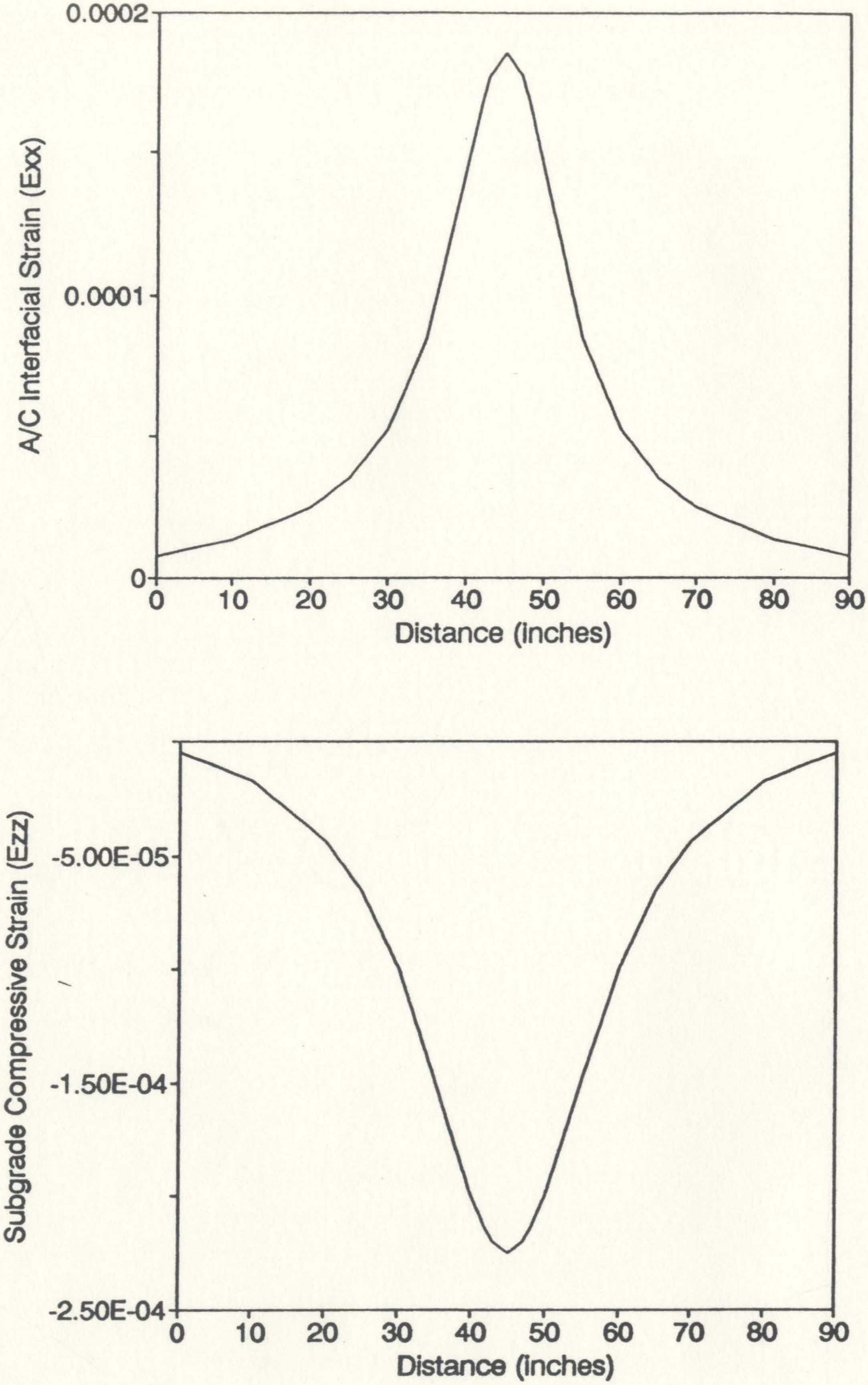


Figure 5.4: Response from Stationary Load-Pavement B

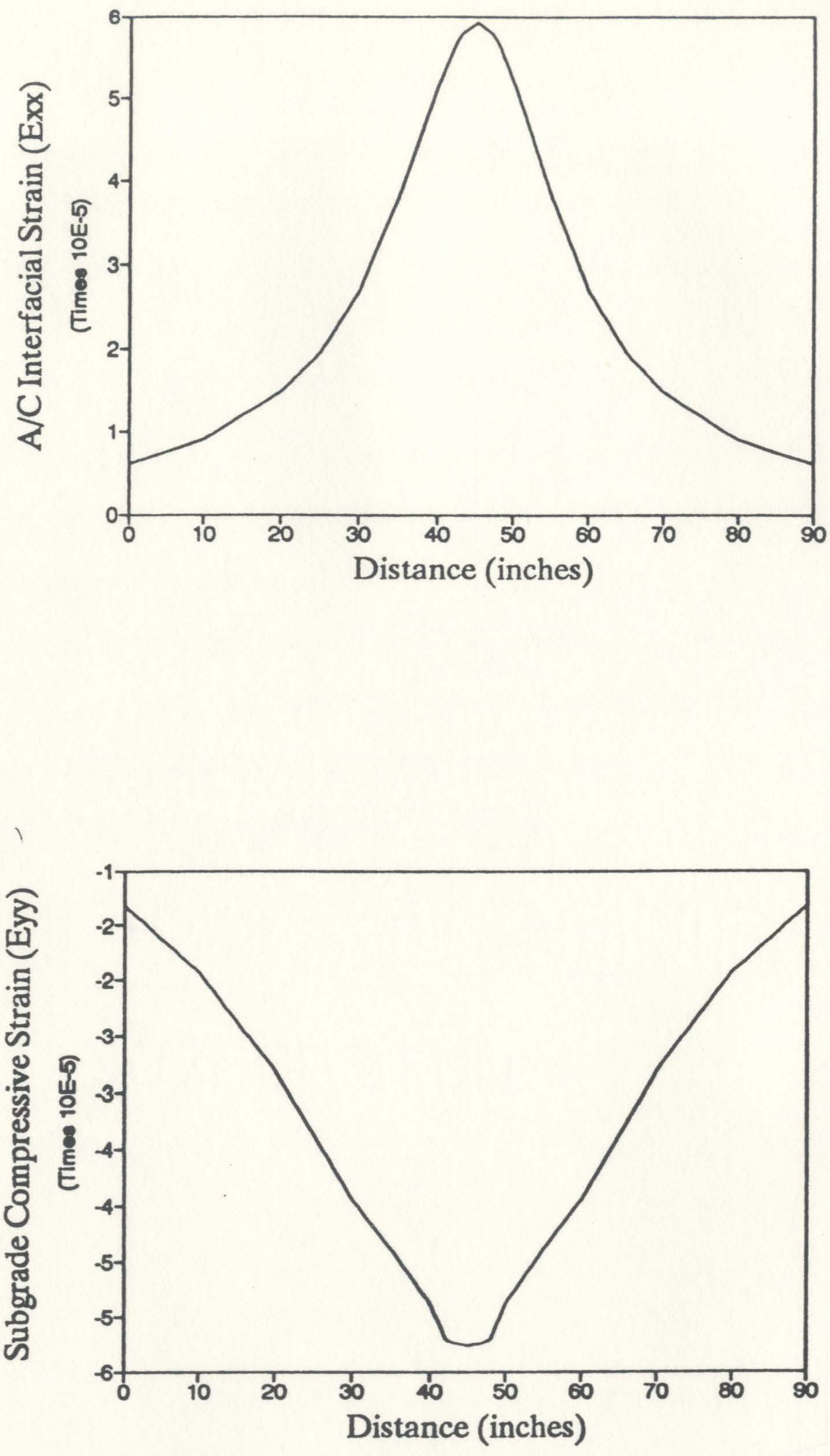


Figure 5.5: Response from Stationary Load-Pavement C

5.2 Impact of Single Axles.

The impact of single axles on pavements were investigated at three vehicle speeds (40, 60, 80 km/h). Data for the trailing axle of the air suspension was not complete therefore in the dynamic analysis, only the dynamic loads measured from the lead drive axle for each of the suspensions (air and rubber) were used as input into DYNAMIC I and the maximum pavement response for every subsection (90-inch increment) was calculated. Equations 4.6 and 4.8, (fatigue cracking and rutting equations) were used to compute the number of load applications to failure. An example of the maximum asphalt concrete interfacial strains (Exx), subgrade compressive strains (Ezz) and their corresponding number of load repetitions to failure for various subsections, calculated for pavement type C is shown in Figures 5.6 to 5.9. respectively.

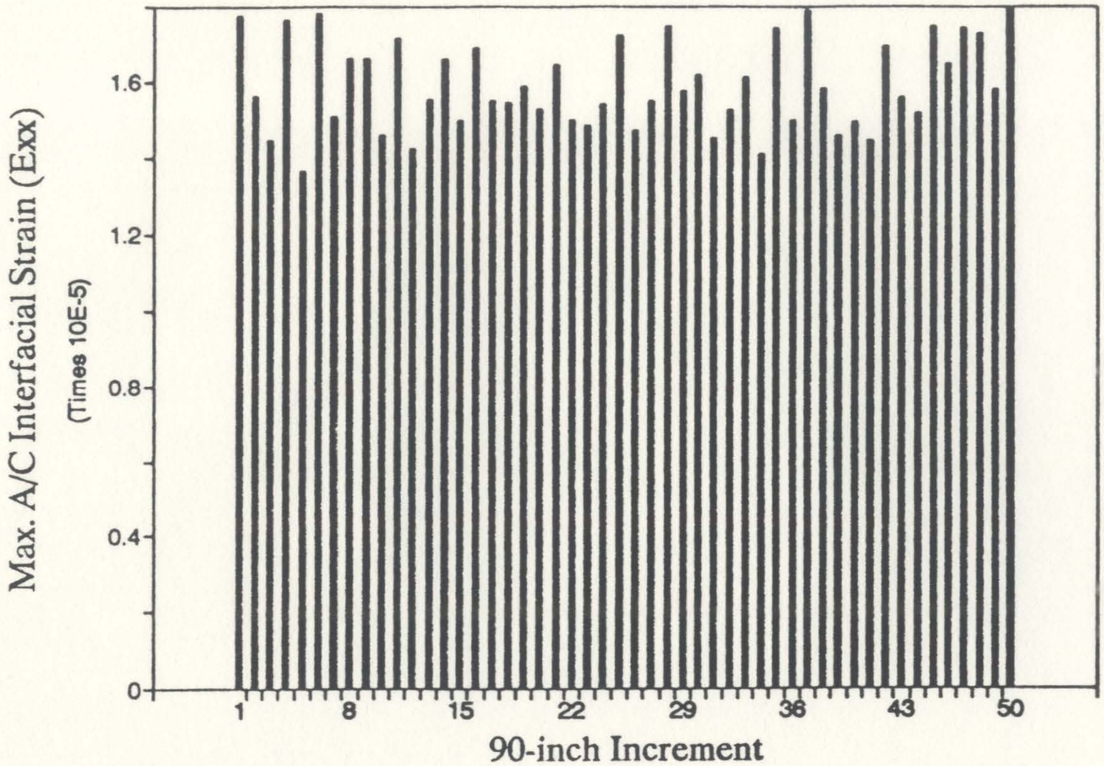


Figure 5.6: Max. A/C Interfacial Strains for Pavement Subsections

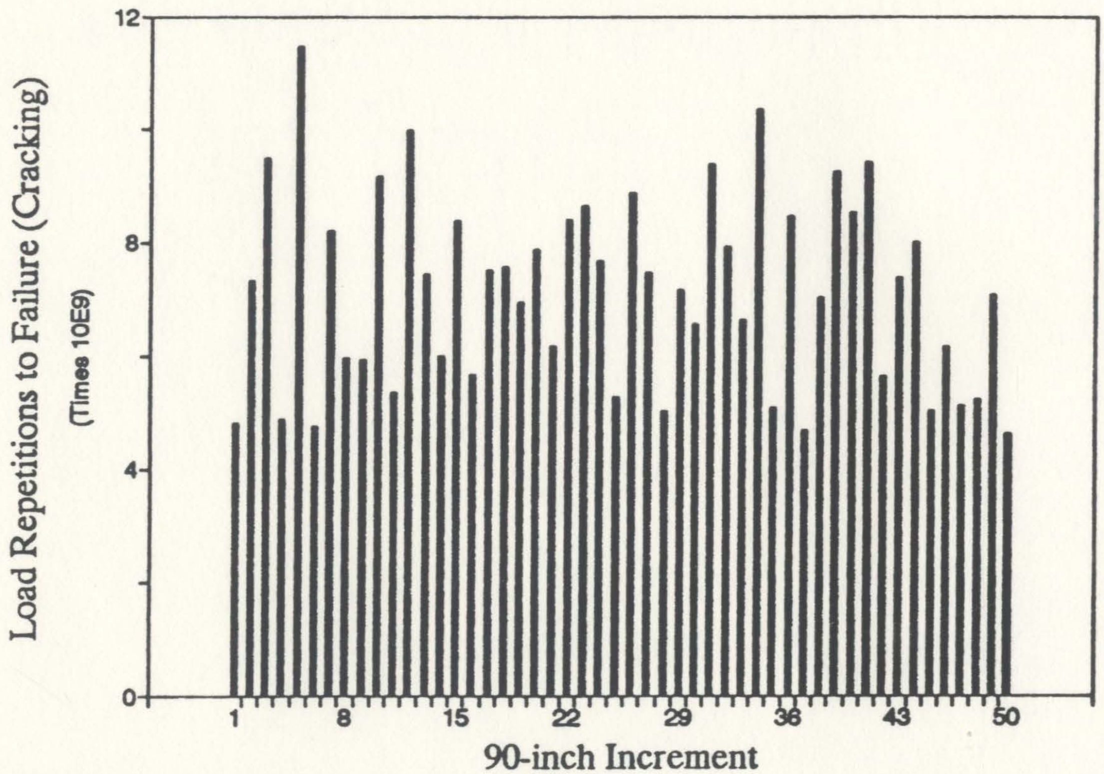


Figure 5.7: Variations in Cracking Damage Along Pavement Section

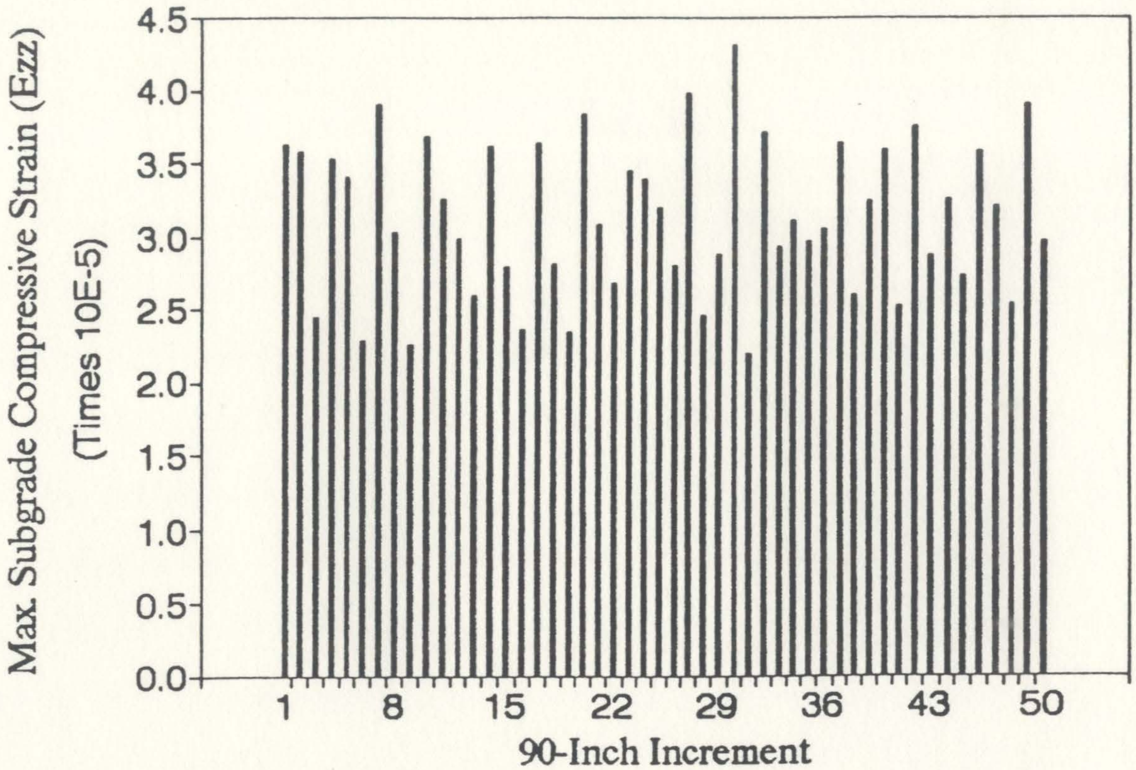


Figure 5.8: Max. Subgrade Compressive Strains for Pavement Subsections

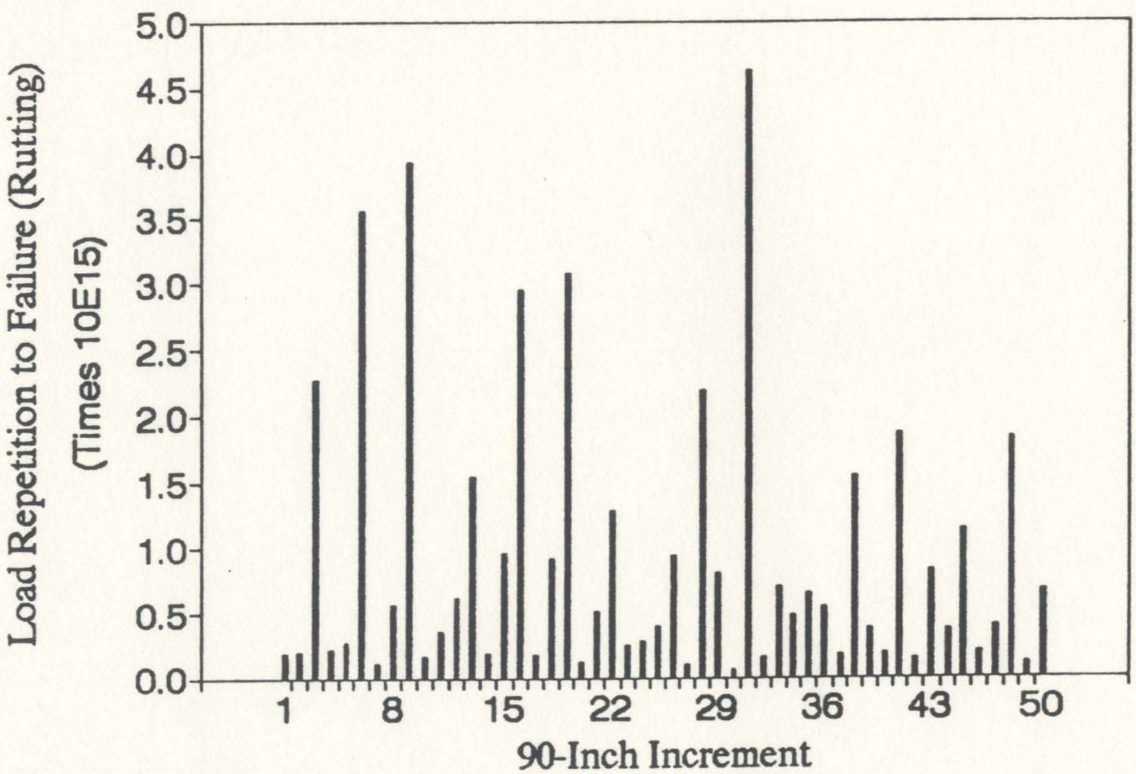
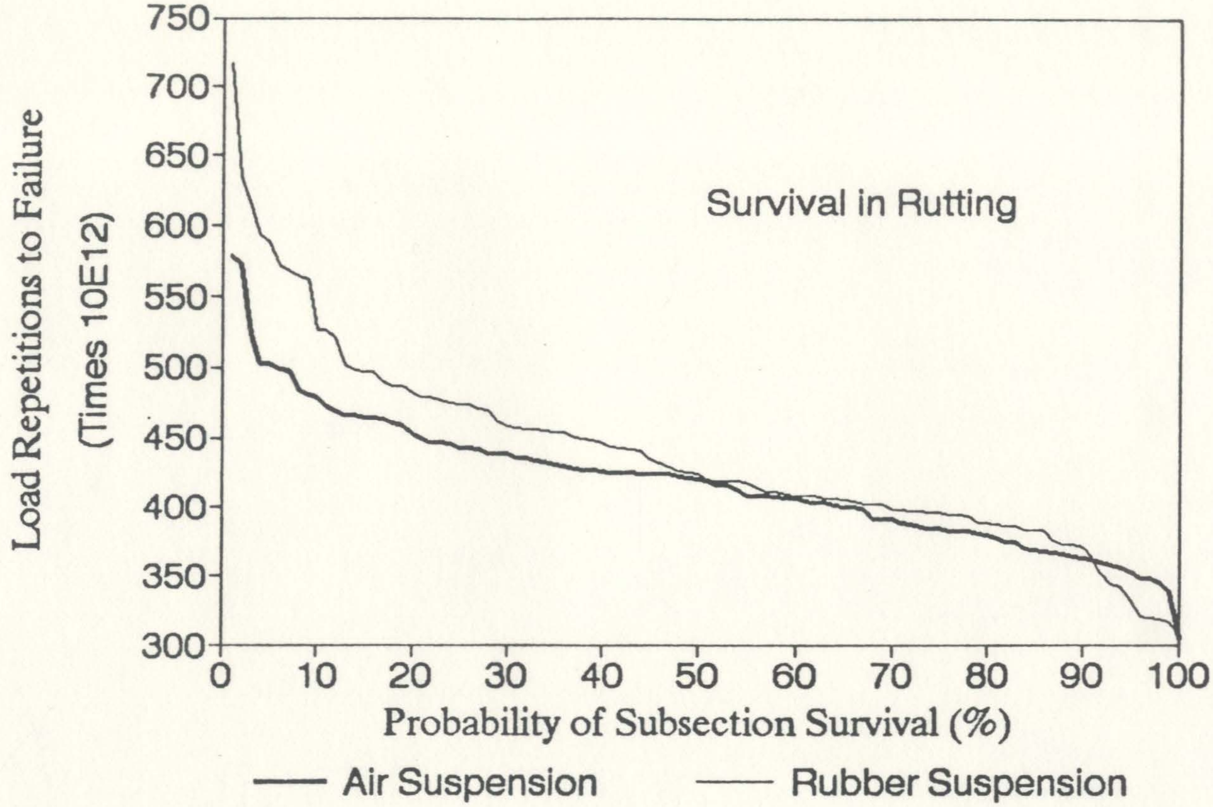


Figure 5.9: Variations in Rutting Damage along Pavement Section

5.2.1 Relative Pavement Life Under Single Axles

The variations of the number of axle load repetitions that will cause rutting and fatigue failure respectively were shown in Figures 5.7 and 5.9. In determining the pavement lives, moving dynamic loads were assumed to be spatially repetitive in space. As mentioned earlier, this assumption was shown to be valid for replicate runs of the NRCC vehicle as shown in Figure 5.1. Additional study will be needed to determine the extent of spatial repeatability of dynamic loads in real 'in-service' traffic (eg. study by Mitchell and Gyenes 1992, Section 3.2.9). The cumulative frequencies that reflects the probability of pavement subsection survival under the two suspension types (Air and Rubber) are shown in Figure 5.10. The 90th percentile values of the load repetitions for the various subsections were assumed to represent the life of the pavement section. Pavement Life Ratios (PLR) were computed for each of the three pavement types and for the two suspensions using Equation 4.13. The results are shown in Tables 5.4 to 5.6.



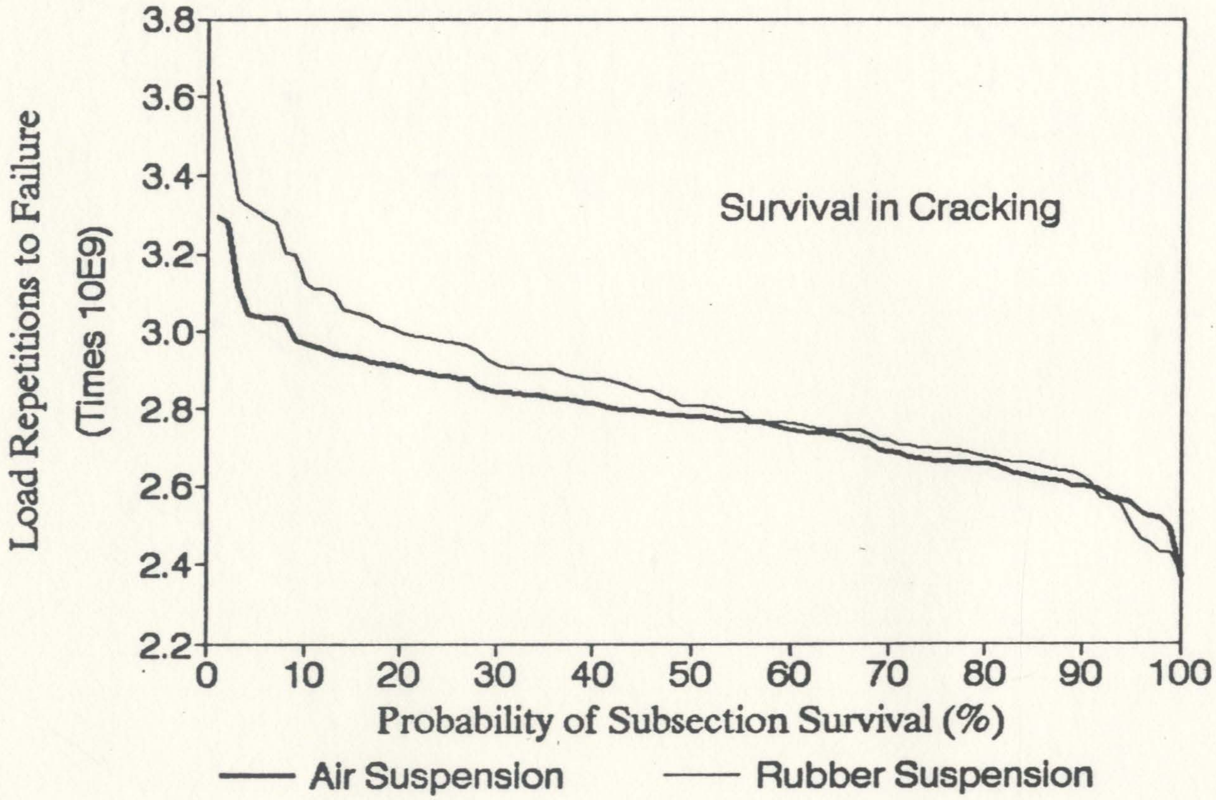


Figure 5.10: Probability of Pavement Subsection Survival for Two Suspension Types.

Table 5.4 Pavement Life Ratio-Single axles on Pavement A

RUN #	SITE #	ROUGH-NESS	SPEED	PLR RUTTING		PLR CRACKING	
		IN/MILE	MPH	RUBBER	AIR	RUBBER	AIR
29			25	1.360	1.298	1.171	1.142
30	1	56	38	1.600	1.355	1.262	1.167
31			50	1.854	1.419	1.376	1.208
21			25	1.628	1.468	1.286	1.202
22	2	87	38	2.126	1.341	1.475	1.158
24			50	3.087	1.680	1.745	1.209
13			25	1.565	1.442	1.262	1.188
14	3	96	38	1.860	1.529	1.371	1.210
12			50	3.140	1.871	1.806	1.253
3			25	1.767	1.501	1.344	1.189
4	4	115	38	1.998	1.681	1.472	1.259
40			50	4.161	2.092	2.058	1.334
16			25	2.230	1.622	1.498	1.267
17	5	201	38	2.722	2.004	1.650	1.301
20			50	5.634	2.153	2.400	1.448

Table 5.5: Pavement Life Ratios-Single Axles on Pavements B.

RUN #	SITE #	ROUGH-NESS	SPEED	PLR RUTTING		PLR CRACKING	
		IN/MILE	MPH	RUBBER	AIR	RUBBER	AIR
29			25	1.372	1.302	1.172	1.143
30	1	56	38	1.591	1.362	1.264	1.168
31			50	1.883	1.555	1.378	1.258
21			25	1.661	1.492	1.287	1.223
22	2	87	38	2.173	1.346	1.479	1.161
24			50	3.028	1.781	1.744	1.212
13			25	1.603	1.498	1.267	1.190
14	3	96	38	1.885	1.536	1.376	1.211
12			50	3.257	1.942	1.818	1.264
3			25	1.840	1.517	1.345	1.191
4	4	115	38	2.059	1.689	1.443	1.269
40			50	4.231	2.111	2.043	1.346
16			25	2.352	1.632	1.521	1.272
17	5	201	38	2.748	2.086	1.664	1.306
20			50	6.031	2.179	2.445	1.568

Table 5.6: Pavement Life Ratios-Single Axles on Pavement C

RUN #	SITE #	ROUGH-NESS	SPEED	PLR RUTTING		PLR CRACKING	
		IN/MILE	MPH	RUBBER	AIR	RUBBER	AIR
29			25	1.403	1.305	1.182	1.121
30	1	56	38	1.607	1.395	1.271	1.136
31			50	1.911	1.403	1.377	1.150
21			25	1.725	1.411	1.304	1.135
22	2	87	38	2.192	1.789	1.486	1.156
24			50	3.245	2.026	1.775	1.218
13			25	1.680	1.466	1.287	1.201
14	3	96	38	1.904	1.892	1.374	1.231
12			50	3.400	2.289	1.843	1.265
3			25	1.988	1.511	1.381	1.202
4	4	115	38	2.224	1.937	1.475	1.286
40			50	5.021	2.373	2.171	1.357
16			25	2.631	1.888	1.587	1.298
17	5	201	38	2.853	2.313	1.688	1.449
20			50	6.976	2.436	2.552	1.624

5.3 Impact of Tandem Axles.

In studying the effects of dynamic loads from tandem axles, only the loads on the tandem axles fitted with rubber suspension were considered because of the problems with the data from the air suspension mentioned earlier (section 5.2). The program DYNAMIC II performs the analysis for every 90-inch increment and for each of the three pavement types. The procedure described in section 4.3.2 was then followed to compute the number of load application to failure and the 90th percentile criterion was again followed to determine the damage for each subsection.

Equation 4.8 is used to compute the relative pavement damage for each of the three pavement structures. Tables 5.6 to 5.8 show the PLR values from this analysis for both fatigue cracking and rutting damage. The values are only for the tandem axles fitted with the rubber suspension.

Table 5.7: Pavement Life Ratios-Tandem Axles on Pavement A

RUN #	SITE #	ROUGHNESS	SPEED	PLR-TANDEM AXLES ON PAVEMENT A	
		IN/MILE	MPH	RUTTING	CRACKING
29			25	3.526	1.140
30	1	56	38	4.621	1.232
31			50	6.407	1.336
21			25	4.247	1.248
22	2	87	38	6.292	1.436
24			50	10.792	1.728
13			25	4.143	1.232
14	3	96	38	6.386	1.329
12			50	11.211	1.780
3			25	4.724	1.309
4	4	115	38	6.039	1.395
40			50	11.776	2.036
16			25	6.279	1.500
17	5	201	38	8.260	1.636
20			50	14.097	2.470

Table 5.8: Pavement Life Ratios-Tandem axles on Pavement B

RUN #	SITE #	ROUGHNESS	SPEED	PLR-TANDEM AXLES ON PAVEMENT B	
		IN/MILE	MPH	RUTTING	CRACKING
29			25	1.820	1.135
30	1	56	38	2.244	1.232
31			50	2.724	1.334
21			25	2.151	1.232
22	2	87	38	2.984	1.412
24			50	4.838	1.739
13			25	1.726	1.219
14	3	96	38	2.018	1.327
12			50	4.957	1.789
3			25	1.798	1.270
4	4	115	38	2.858	1.386
40			50	7.134	2.043
16			25	3.147	1.446
17	5	201	38	3.985	1.627
20			50	10.479	2.491

Table 5.9: Pavement Life Ratios-Tandem Axles on Pavement C

RUN #	SITE #	ROUGHNESS IN/MILE	SPEED MPH	PLR-TANDEM AXLES 0N PAVEMENT C	
				RUTTING	CRACKING
29			25	0.842	1.138
30	1	56	38	1.038	1.223
31			50	1.183	1.332
21			25	1.008	1.234
22	2	87	38	1.390	1.414
24			50	2.081	1.743
13			25	0.969	1.223
14	3	96	38	1.223	1.333
12			50	2.155	1.805
3			25	1.085	1.264
4	4	115	38	1.337	1.380
40			50	3.001	2.035
16			25	1.406	1.438
17	5	201	38	1.871	1.631
20			50	4.518	2.510

5.4 Parametric Study With Respect to Load-Sharing in Tandem Axles.

The effects of suspension type, vehicle speed and pavement surface roughness were considered for single and tandem axle configurations in the preceding sections. This section investigates the effects of another vehicle parameter such as load-sharing among the axles in tandem group. The aim is to examine the extent of pavement damage caused by vehicles with tandem axles which provide poor load equalization.

The unequal load-sharing effect is analyzed using six different load sharing ratios ranging from the ideal situation of perfect loading sharing (50/50) to poor load sharing (65/35) among the axles in the tandem group. The values of the amplitude ratios $A(t)$ corresponding to these ratio range from 1.0/1.0 for perfect load sharing to 1.3/0.7 for poor load-sharing. The different load amplitudes were input in the computer program to perform analysis similar to the preceding ones.

It was decided to investigate whether the pavement damage due to unequal load-sharing was also sensitive to vehicle speed, therefore the effects of the different load-sharing ratios were examined at three vehicle speeds (40, 60 and 80 km/h). The number of load repetitions to failure were calculated following the rainflow/range pair counting method. Table 5.10 gives a summary of the number of repetitions to failure in rutting and cracking under various load-sharing ratios at three different vehicle speeds.

Table 5.10: Load Repetitions to Failure for Different Load-Sharing Ratios

SPEED (KMH)	LOAD RATIO (%)	NUMBER OF VEHICLE PASSES CAUSING FAILURE	
		CRACKING ($x10^9$)	RUTTING ($x10^{14}$)
40	50/50	1.99	2.47
	50.5/49.5	1.96	2.34
	55/45	1.72	1.45
	57.5/42.5	1.57	1.14
	60/40	1.42	0.90
	65/35	1.14	0.57
60	50/50	1.97	2.44
	50.5/49.5	1.94	2.31
	55/45	1.71	1.44
	57.5/42.5	1.56	1.12
	60/40	1.41	0.88
	65/35	1.13	0.57
80	50/50	2.05	2.45
	50.5/49.5	2.02	2.32
	55/45	1.77	1.45
	57.5/42.5	1.61	1.14
	60/40	1.45	0.91
	65/35	1.16	0.58

The effect of unequal load-sharing is related to that of perfect load-sharing by an index which is termed, Load-Sharing Damage Ratio (LSDR). This is defined as the number load repetitions to failure caused by multiple axles with perfect load-sharing (N_{equal}) divided by the number of repetitions to failure caused by multiple axles with unequal load-sharing ($N_{unequal}$). It is expressed algebraically as:

$$LSDR = \frac{N_{equal}}{N_{unequal}} \qquad (5.1)$$

Table 5.11 show the results of the damage ratios obtained using Equation 5.1 above. The trend of pavement cracking and rutting damage due to unequal load ratios in axles moving at different speeds is shown in Figure 5.11. LSDR values are plotted against load split for three vehicle speeds.

Table 5.11: Load-Sharing Damage Ratios (LSDRs).

SPEED (KMH)	LOAD RATIO (%)	LOAD-SHARING DAMAGE RATIO	
		CRACKING	RUTTING
40	50/50	1.000	1.000
	50.5/49.5	1.011	1.057
	55/45	1.152	1.698
	57.5/42.5	1.264	2.175
	60/40	1.400	2.757
	65/35	1.741	4.315
60	50/50	1.000	1.000
	50.5/49.5	1.011	1.057
	55/45	1.152	1.700
	57.5/42.5	1.263	2.177
	60/40	1.398	2.761
	65/35	1.738	4.325
80	50/50	1.000	1.000
	50.5/49.5	1.011	1.056
	55/45	1.158	1.687
	57.5/42.5	1.273	2.152
	60/40	1.411	2.718
	65/35	1.759	4.229

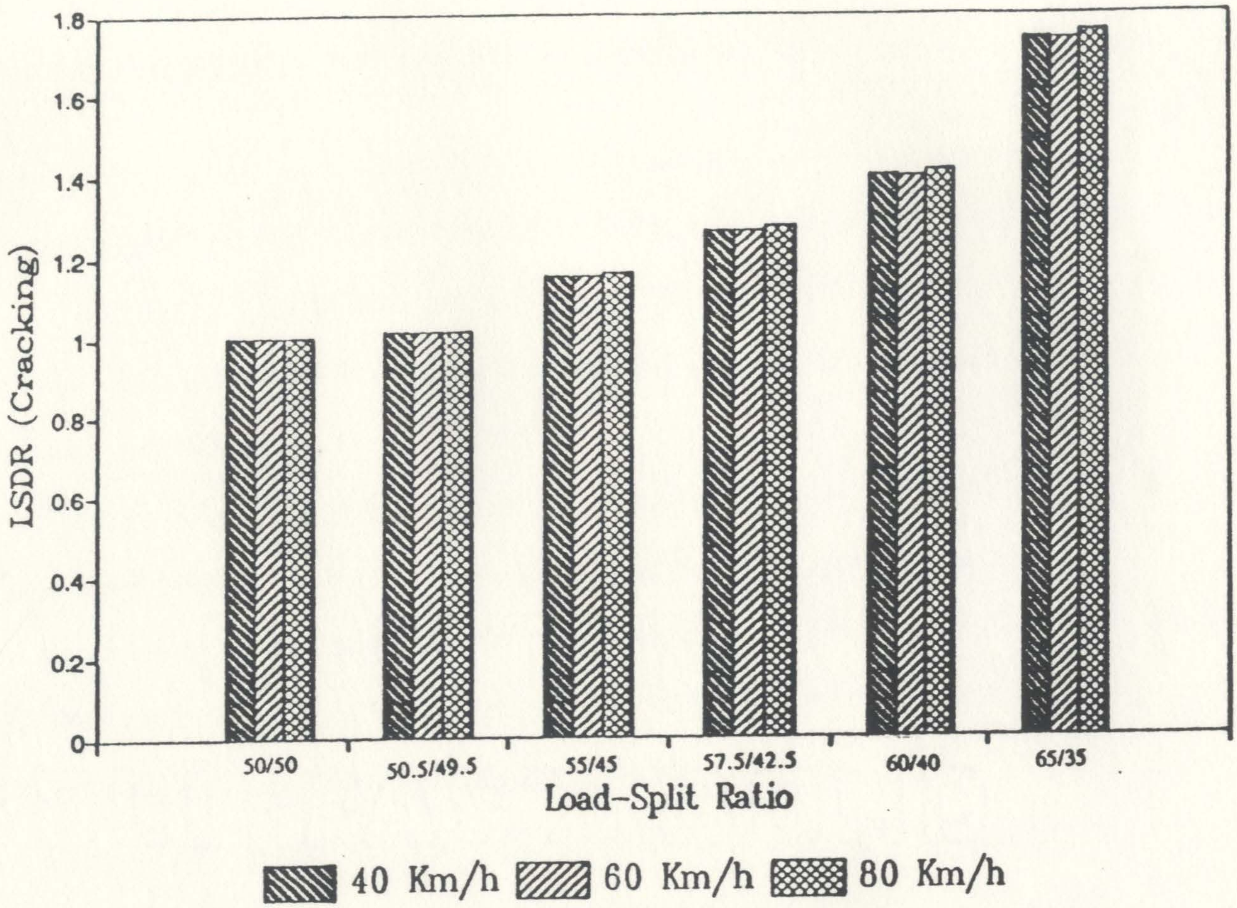


Figure 5.11: Effects of Load-Sharing in Tandems on Pavement Damage.

5.5 Parametric Study With Respect to Axle Spacing in Tandem Axles.

The pavement damage under dynamic loads has so far been analyzed using one type of axle spacing between the individual axles in the tandem group (axles spacing of 1.5m). This section examines the effects of different axle spacings on pavement damage. This investigation is based on the dynamic response model developed in Chapter 4.

The primary aim of this section is to examine the influence of axle spacing on pavement life under dynamic loading conditions. According to a study by Hajek and Agarwal (1990) most of the heavy vehicles with tandem or multiple axle configuration have axle spacings in the range of 1.3 meters and 1.9 meters. Therefore four different axle spacings ranging from 1.3 meters to 1.9 meters were tested. The computer program was modified to perform the pavement damage and life evaluation analysis for these axle spacings by considering all the three pavement types (A,B and C), and for a vehicle speed of 80km/h. The effects of different axle spacings were examined by calculating the number of load repetitions to failure due to that particular spacing holding other parameters constant. The results of this analysis is shown in Table 5.11 and Figure 5.12. A base axle spacing of 1.5 meters is assumed and a ratio is defined to express the relationship between the damage caused by the base axle spacing and that caused by a particular axle spacing. This relationship is defined as Axle Spacing Damage Ratio (ASDR) and expressed algebraically as:

$$ASDR = \frac{N_{1.5m}}{N_x} \tag{5.2}$$

where N_x, is the number of repetition to failure caused by tandem group with an axle spacing of x meters and $N_{1.5m}$ is the damage caused by tandem with axle spacing of 1.5 meters.

Table 5.12: Axle Spacing Damage Ratios (ASDRs) Rubber Suspension.

PAVEMENT TYPE	AXLE SPACING (METERS)	NO. OF VEHICLE PASSES TO FAILURE IN CRACKING (Millions)	DAMAGE RATIO (ASDR)
	1.3	1318.89	0.915
	1.5	1206.92	1.000
A	1.7	1146.35	1.053
	1.9	1111.77	1.085
	1.3	1772.88	0.858
	1.5	1521.07	1.000
B	1.7	1350.53	1.126
	1.9	1232.36	1.234
	1.3	2019.76	0.849
	1.5	1714.83	1.000
C	1.7	1480.59	1.159
	1.9	1304.52	1.315

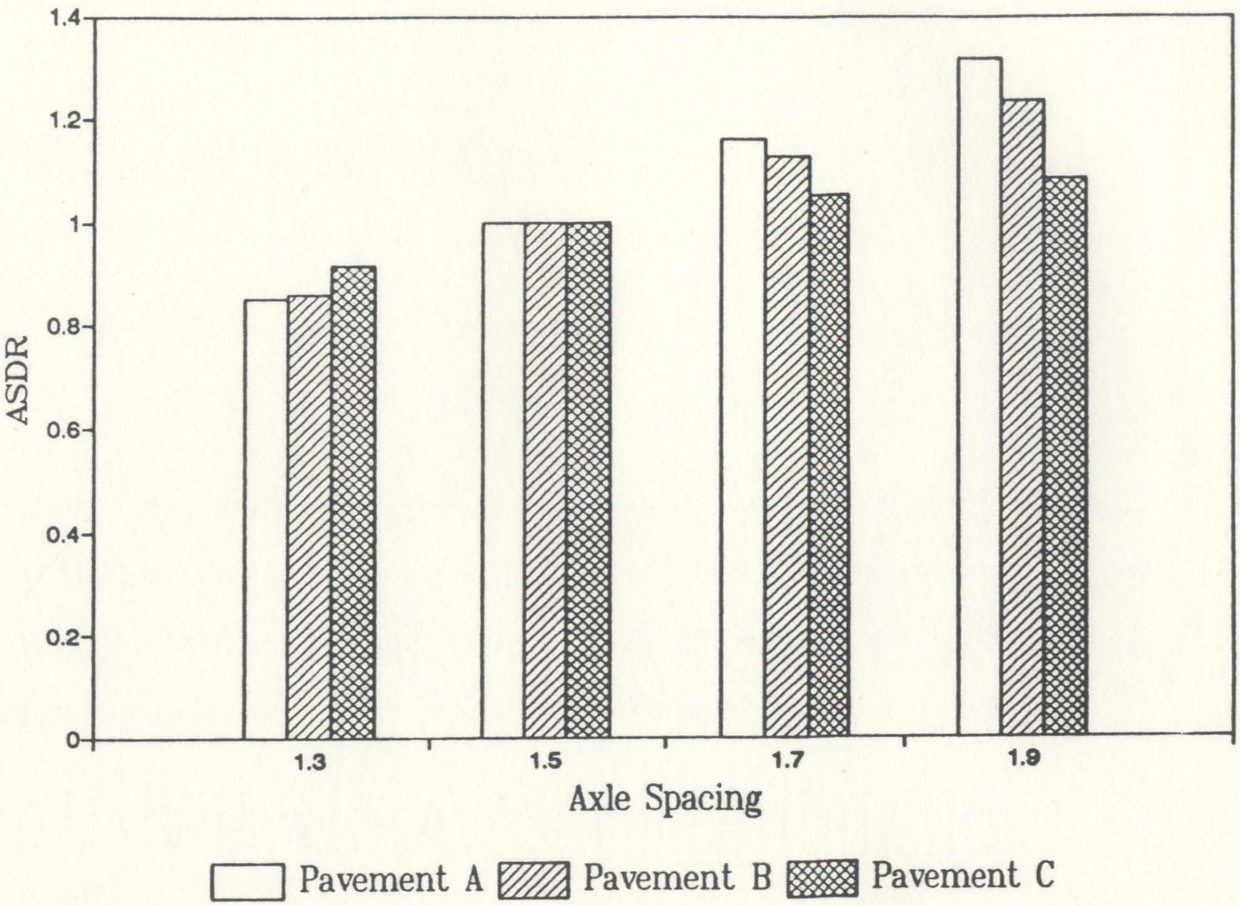


Figure 5.12: Influence of Vehicle Axle Spacing on Pavement Damage.

Chapter 6

Discussion

The damaging effects of moving static and dynamic loads generated by single and tandem axles of heavy vehicles have been analyzed by considering parameters like pavement roughness, vehicle speed, axle spacing, axle configuration and suspension type. This section presents a discussion of the findings.

6.1 Static versus Dynamic Loads

The pavement life ratios (PLRs) were calculated for single and tandem axles loads. It was found that PLR values were greater than 1.0. This suggests a reduction in pavement life due to dynamic loads. For all the pavement types analyzed for cracking damage, PLR value ranged from 1.17 to 2.6 for axles fitted with rubber suspension. For those axles fitted with the air suspension, PLR values ranged between 1.12 and 1.6. In the rutting damage analysis, PLRs for the rubber suspension ranged from 1.3 to 6.9. For the air suspension, the corresponding PLRs ranged from 1.3 to 2.4. This clearly indicated that consideration of moving dynamic load instead of static loads is an important factor in pavement analysis.

The PLR values were found to increase consistently as the roughness and speed of the vehicle increased. At a given level of roughness, PLRs increased with speed. Similarly at a constant speed, PLRs increased with pavement roughness. Figures 6.1 and 6.2 show how vehicle speed affects damage for different types of pavements. Similarly, Figures 6.3 to 6.5 show the effects of roughness on pavement damage by vehicles fitted with air and rubber suspensions and travelling at a particular speed.

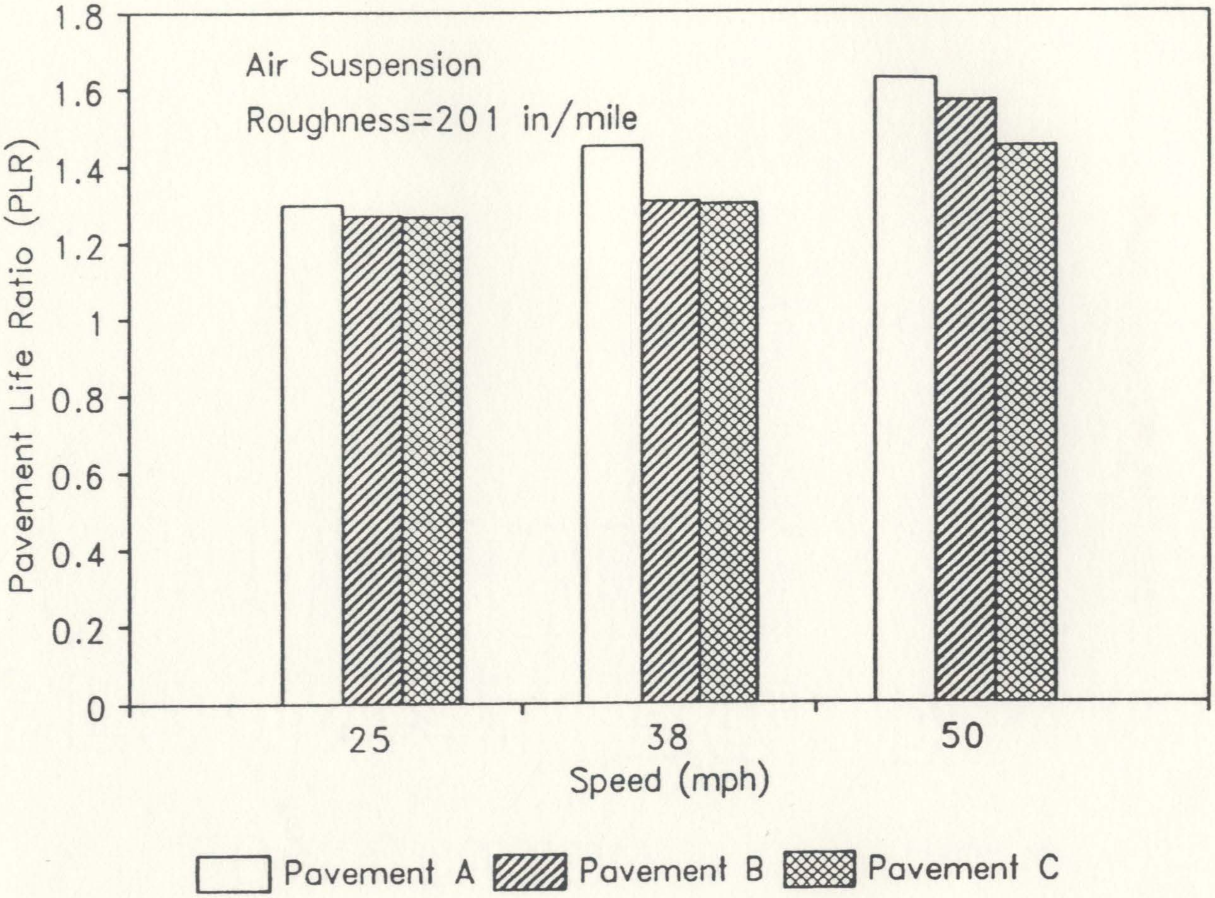


Figure 6.1: Effects of Vehicle Speed on Pavement Life-Air Suspension

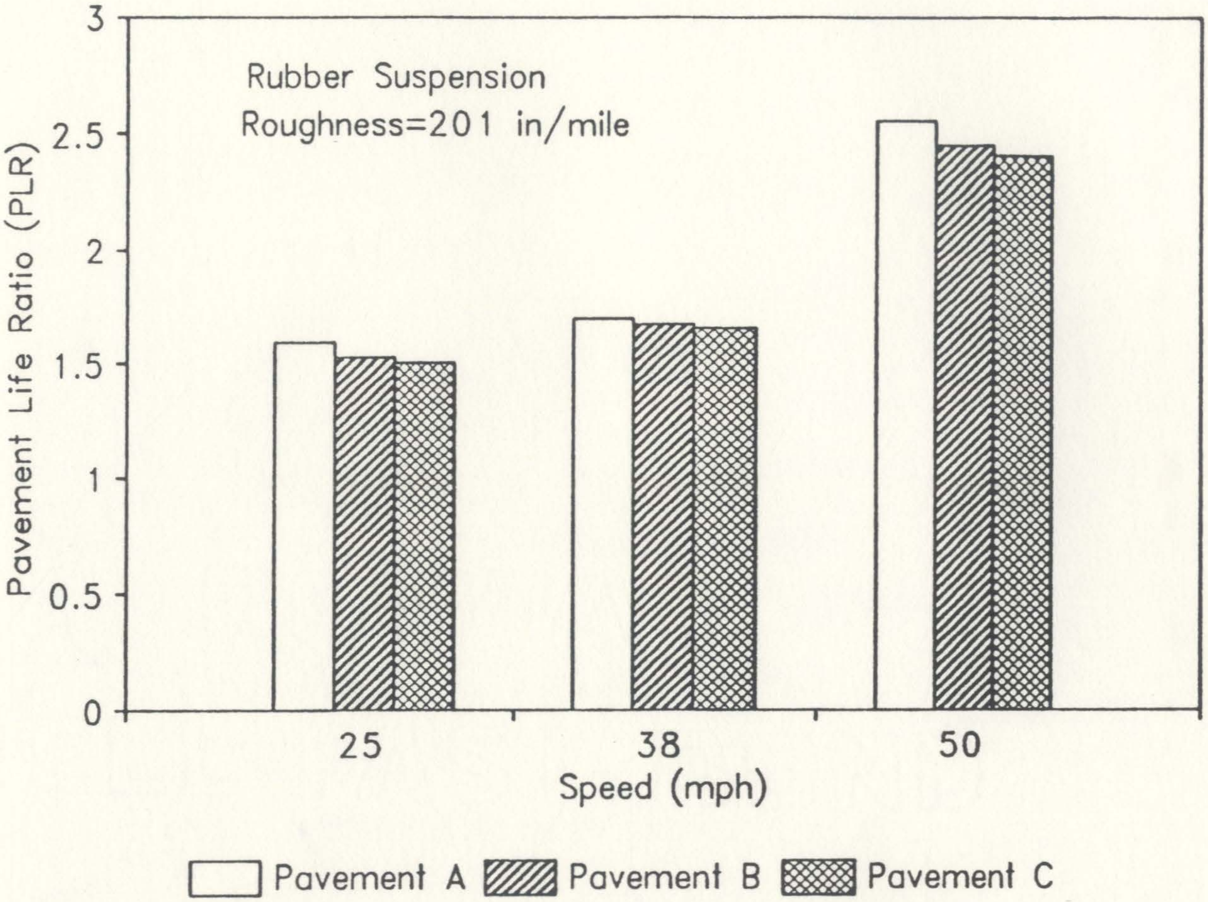


Figure 6.2: Effects of Vehicle Speed on Pavement Life-Rubber Suspension

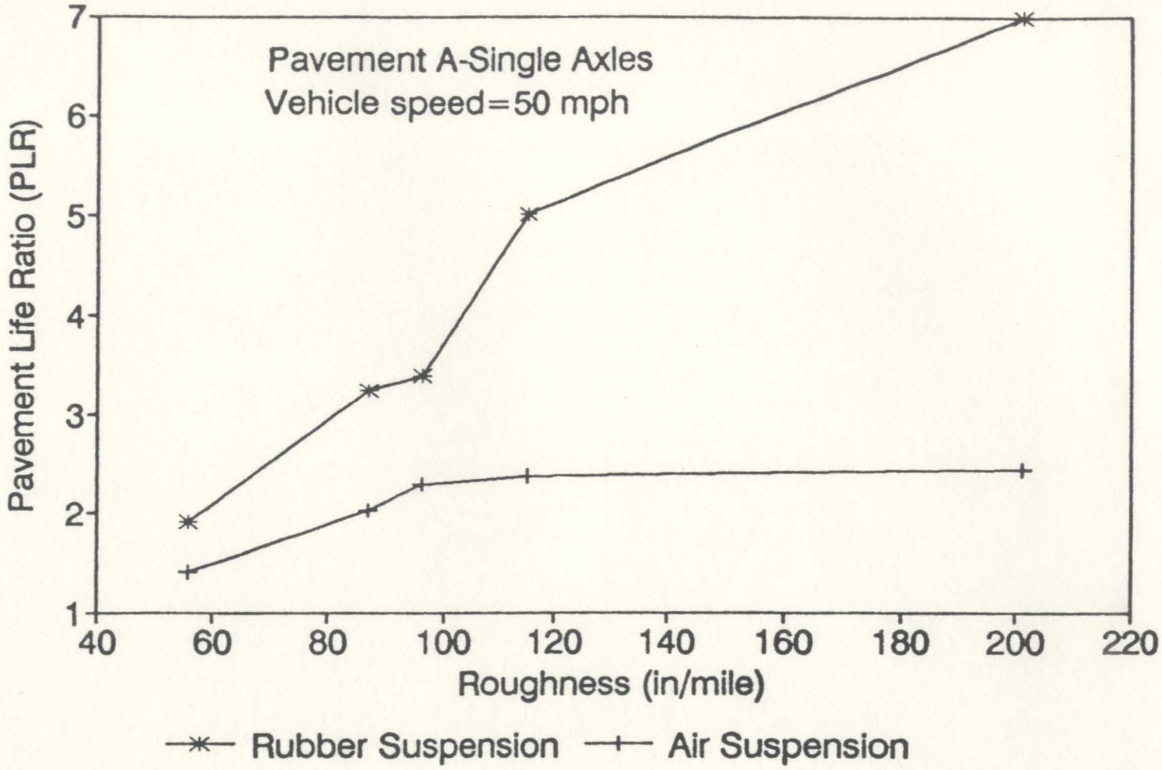


Figure 6.3: Effects of Roughness on Rutting-Pavement A

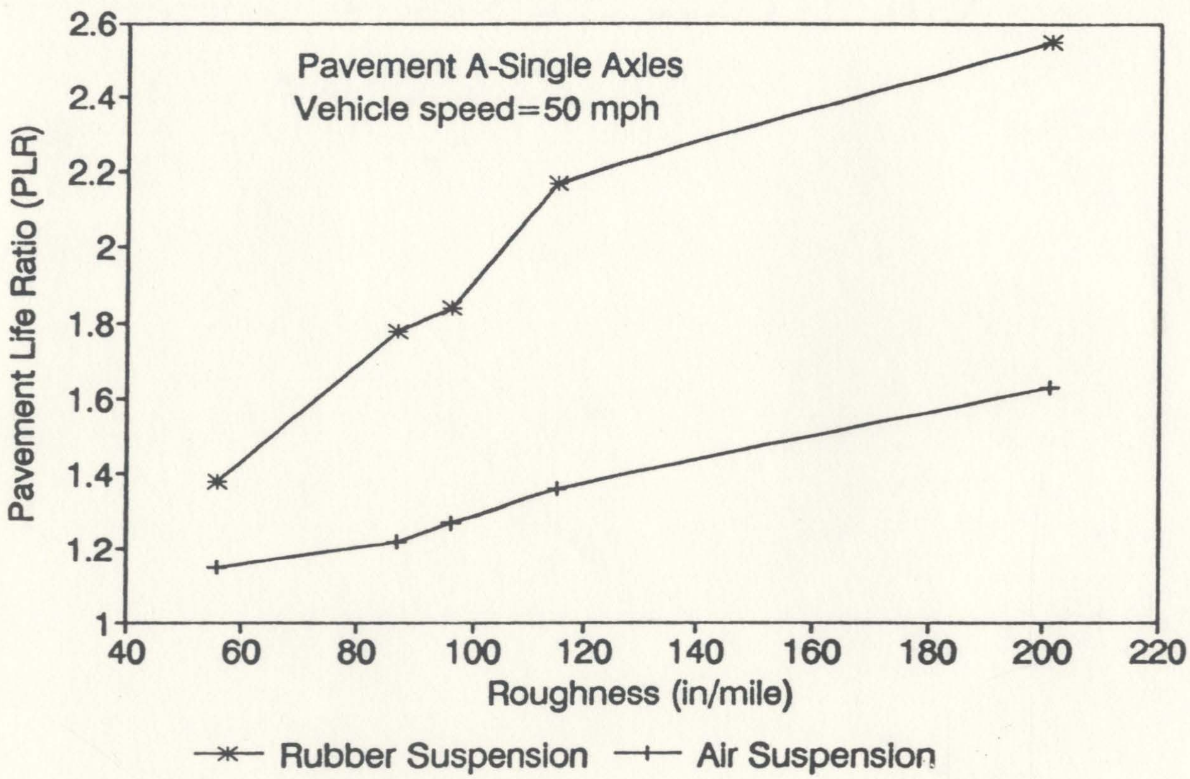


Figure 6.4: Effects of Roughness on Cracking-Pavement A

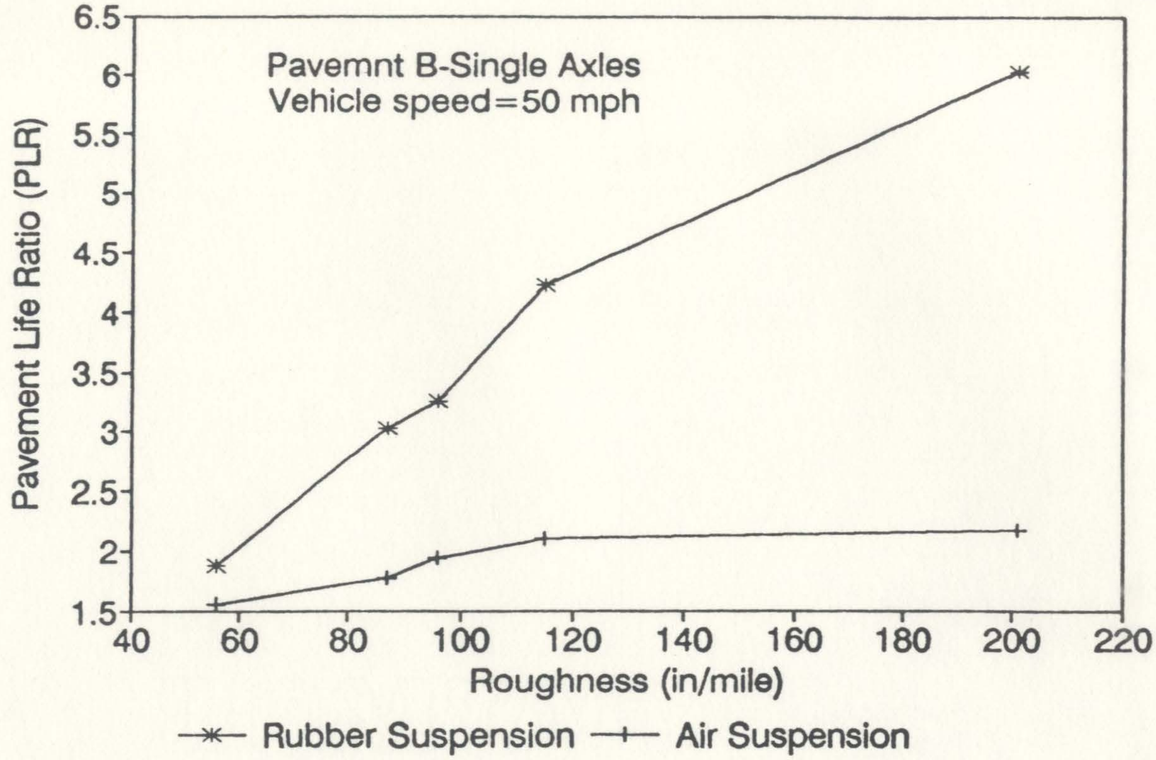


Figure 6.5: Effects of Roughness on Rutting-Pavement B

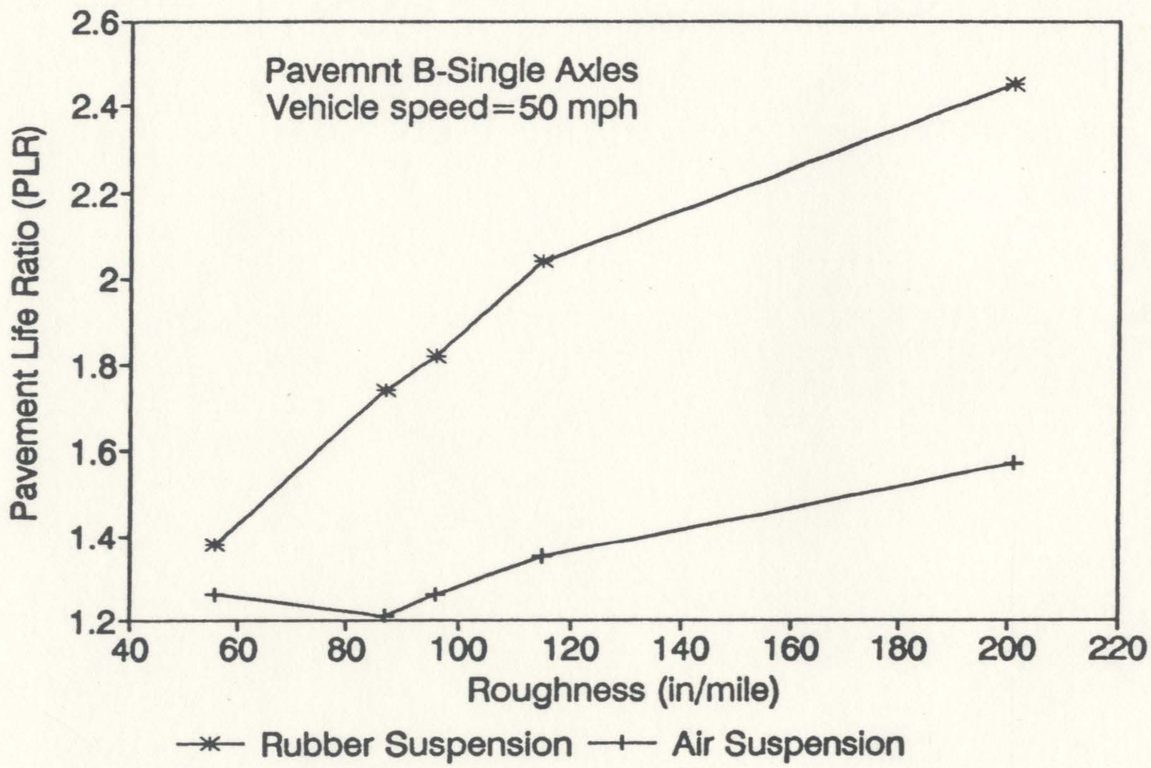


Figure 6.6: Effects of Roughness on Cracking-Pavement B

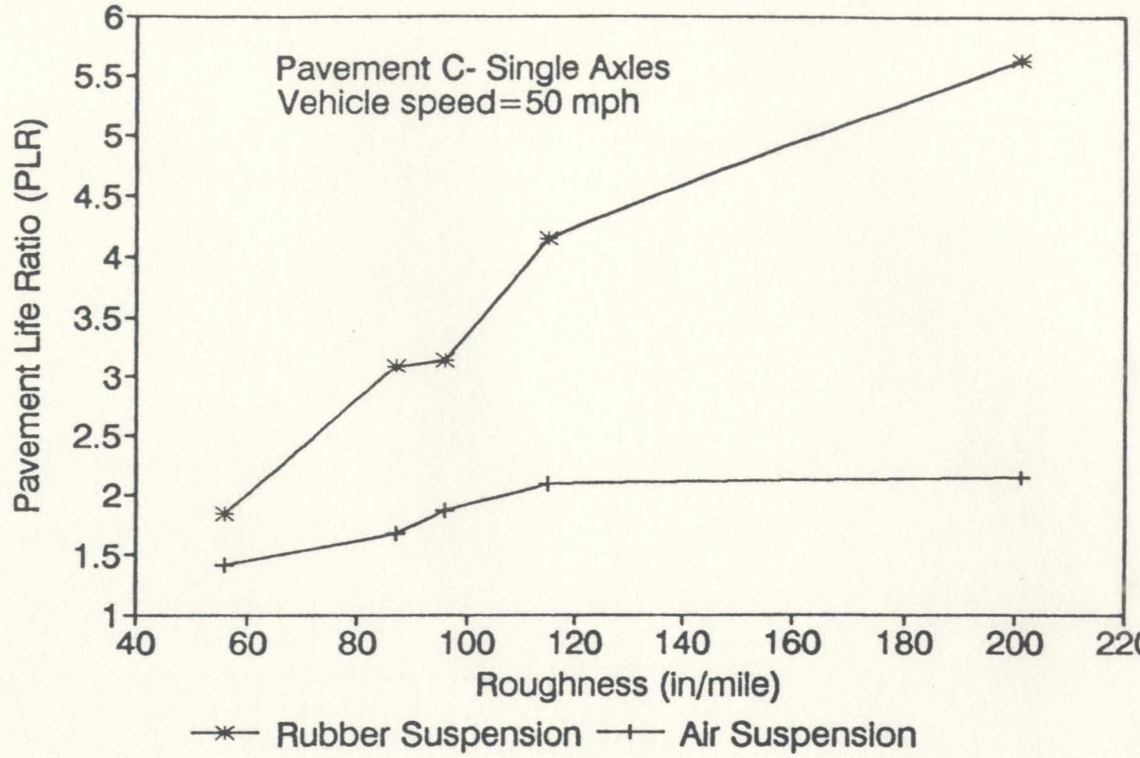


Figure 6.7: Effects of Roughness on Rutting-Pavement C

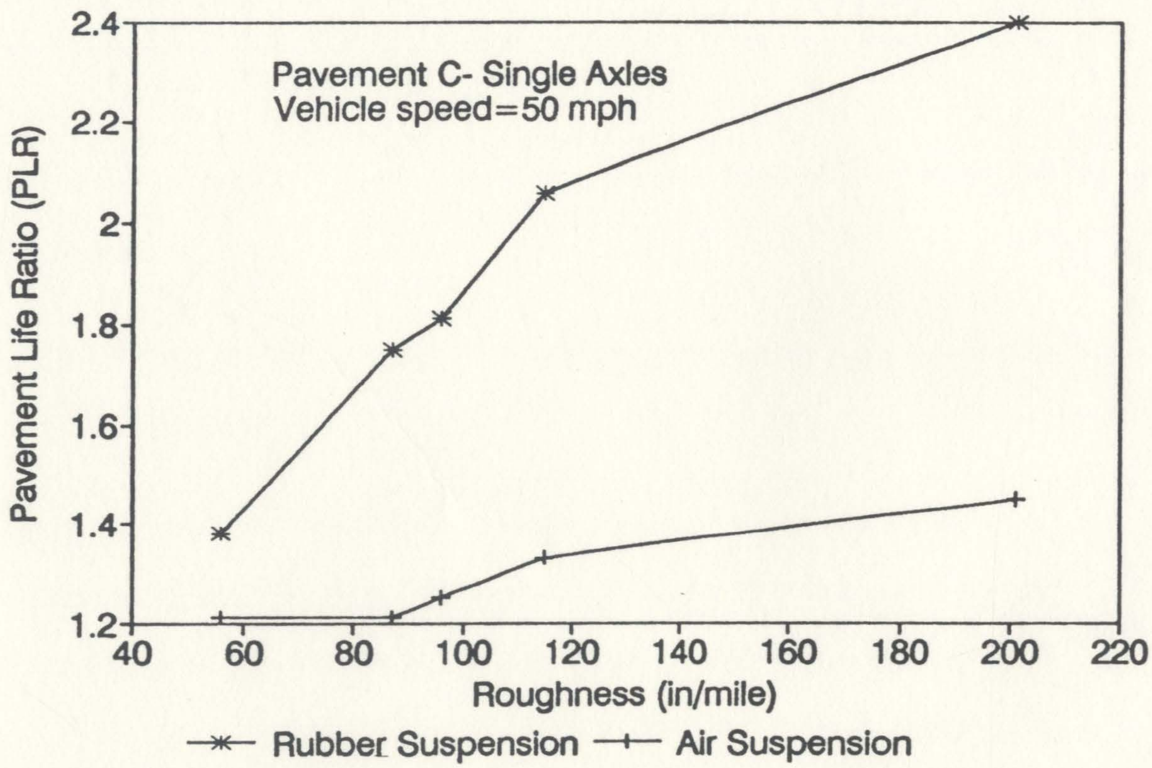


Figure 6.8: Effects of Roughness on Cracking-Pavement C

6.2 Effects of Vehicle Suspension Type on Pavement Damage

The effect of suspension type on pavement damage was very noticeable. For all pavement types considered, PLR values for the rubber suspension were found to be higher than that of the air suspension. The damaging effects of the rubber suspension were also found to be more sensitive to speed and roughness. At the lowest level of surface roughness and vehicle speed, PLR values for the two suspensions were almost the same but as the roughness and speed increased the PLR values of the rubber suspension increased at a higher rate than that of the air suspension. At the highest vehicle speed and surface roughness, the rubber suspension was found to be about 57% more damaging in cracking than the air suspension for pavement A, 56% and 66% for pavements B and C, respectively (Tables 5.4 to 5.6). For rutting damage, the corresponding values are more than twice.

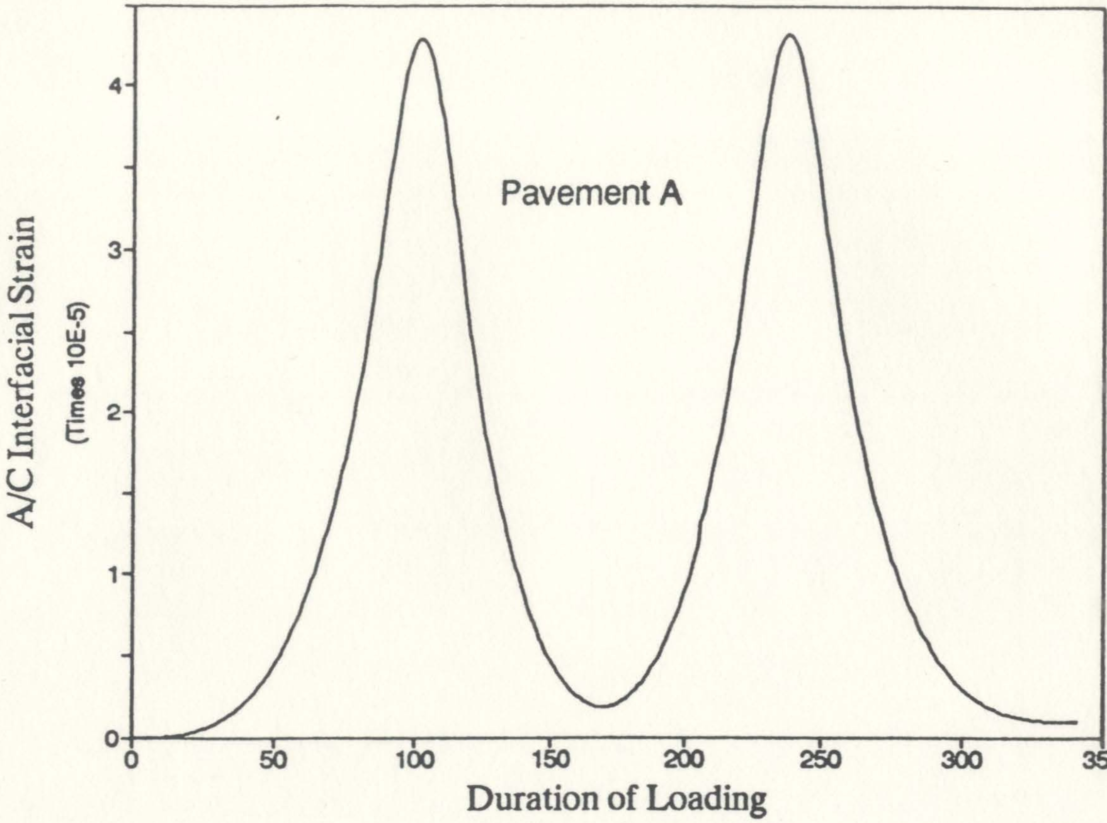
The relative damaging potential of these suspension is also noticeable in Figures 6.1 to 6.8.

6.3 Effects of Axle Configuration on Pavement Damage

The damaging effects of dynamic loads produced by single and tandem axles were examined using the dynamic response models. The computations for pavement damage by single and tandem axles showed that, under the same operating conditions, single axles cause more pavement damage than individual axles in a tandem group for in terms of cracking and rutting. Hence, a single pass of tandem axle consumes pavement life which is less than one pass of two single axles. This trend was noticed in two of pavement the types (B and C) at the various speeds. However, in pavement A (weak pavement), the proportion of damage done by individual axles in the tandem group was either equal to or greater than that of the single axle passes.

These results are explained by the behaviour of both asphalt concrete interfacial strain and subgrade compressive strain for different pavements as shown in Figures 6.9 and 6.10. It is noticeable that, the difference between the peak strains and the inter-axle residual strains increase as the pavement cross-sections or structural strengths decrease. In the weak pavement, the asphalt concrete residual strain reduces to a value close to zero while the subgrade residual strain reduces to a value below zero. Moreover, since the inter-axle residual strains for weak pavements reduce to zero or a value close to zero, calculating damage by tandem axles from strains determined by the rain flow/range pair counting method should give a value approximately equal to or greater than two single axles passes.

A comparison of damage on pavements A and C by single and tandem axles is shown in Table 6.1 in terms of the number of repetitions to cause cracking and rutting failures. These results seem to agree with the trend of AASHO Load Equivalency Factors (LEFs) for various axle configurations and structural numbers and also consistent with current AASHO practices, (Pavement Design Guide, 1986). However, the pavement life ratios (PLRs) calculated for both single and tandem axles for the three different pavement types do not suggest any clear relationship between pavement strength and dynamic load effects on cracking or rutting damage.



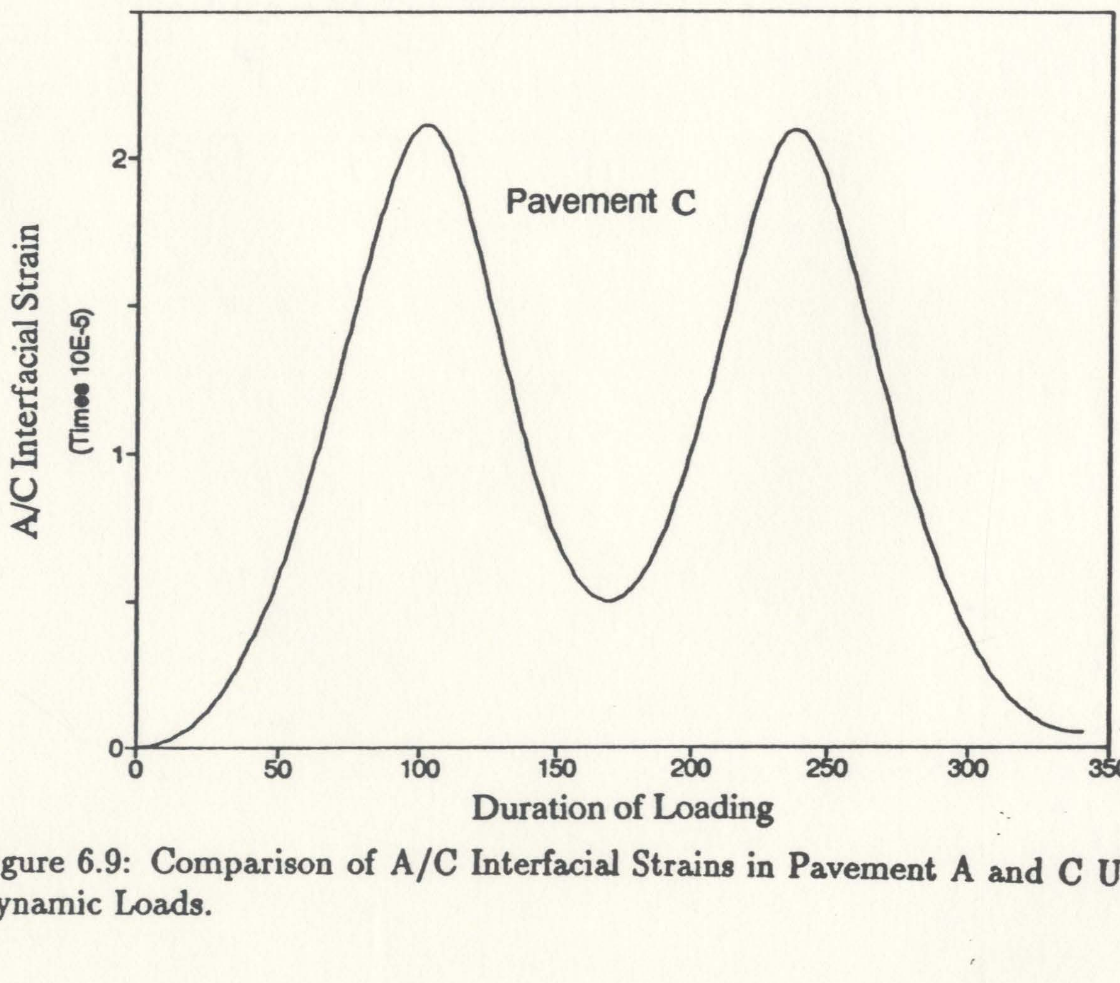
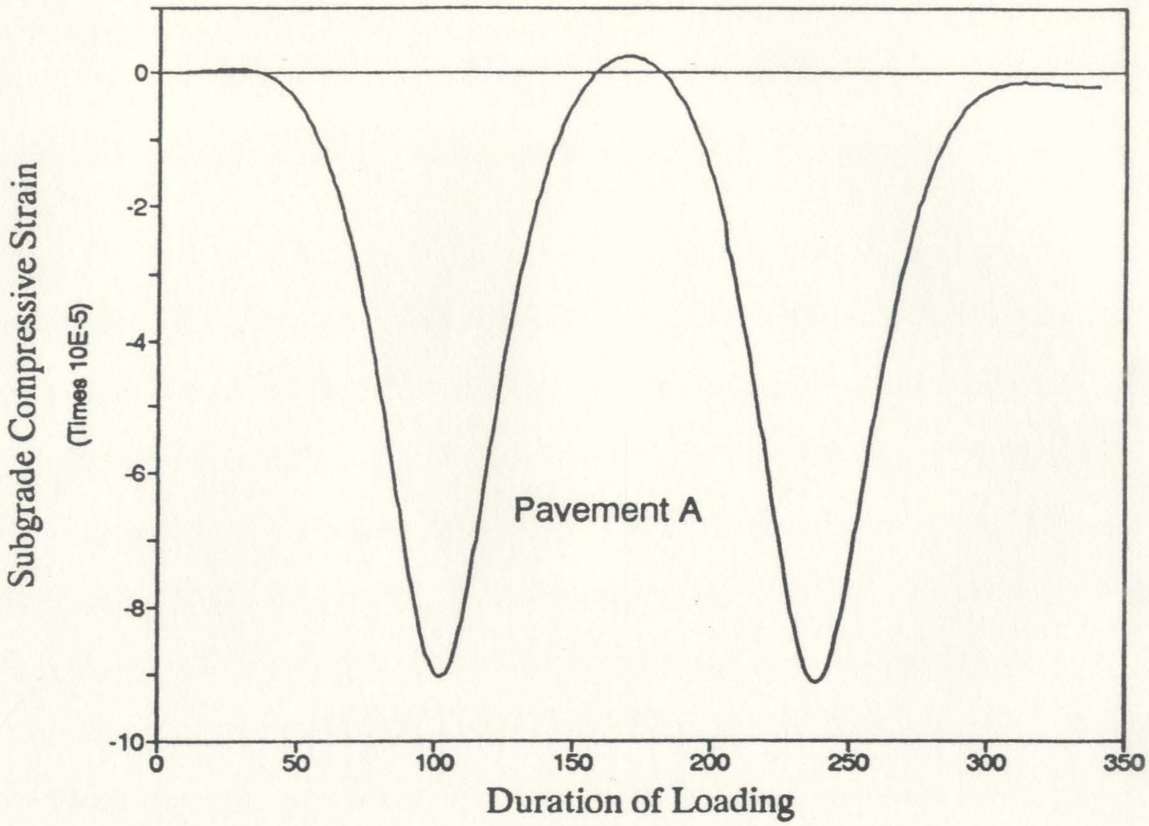


Figure 6.9: Comparison of A/C Interfacial Strains in Pavement A and C Under Dynamic Loads.



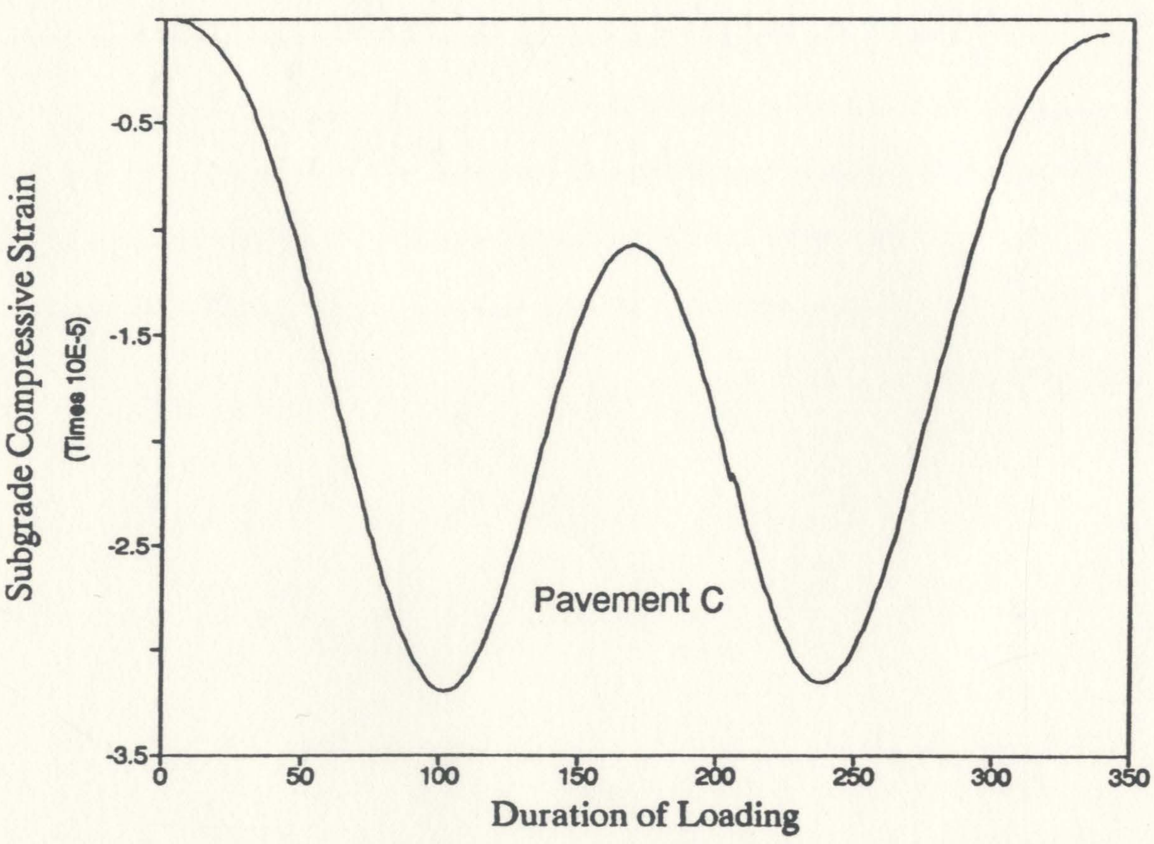


Figure 6.10: Comparison of Subgrade Compressive Strains in Pavements A and C Under Dynamic Loads.

6.4 Load-Sharing and Dynamic Load Variation on Pavement Damage

The effects of load-sharing on pavement cracking and rutting damage were analyzed for three different speeds. The results showed that, unequal load-sharing between the axles produce dynamic load variations which have direct effects on pavement damage. LSDR values for all the split ratios were greater than the ideal case (ie. greater than 1.0) for both cracking and rutting. This indicates that, the net damaging effects of the axles increase as the load-sharing becomes poorer. For a very poor load-sharing ratio of 65/35, LSDR values greater than 1.7 for cracking and 4.3 for rutting were obtained regardless of the vehicle speed tested. In general, the rate of rutting damage was found to be more sensitive to load-sharing than cracking damage. However, the results did not provide any significant evidence on the contribution of speed to the extra damage due to poor load-sharing. For all the three speeds tested, LSDR values were approximately the same as can be seen in Table 5.10 and Figure 5.11. These findings suggest that, load-sharing between axles in multiple axle group is an important factor that must be considered in assessing the effects of multiple axles on pavement damage.

6.5 Effects Axle Group Spacing on Pavement Damage

The effects of axle spacing were quantified in terms of the number of load applications required to cause pavement cracking and rutting failures. All three pavement types were analyzed for the four axle spacings considered that is, 1.3m, 1.5m, 1.7m and 1.9m. Table 5.12 shows the calculated pavement damage for each of the pavement types under the same operating conditions of roughness and speed and their corresponding ASDRs. The general trend observed from the Table 5.11 and Figure 5.11 is that, pavement damage increases as the spacing between the axles in the tandem group increases. However the extent of such damage seems to be related to the pavement structural strength. If axle spacing of 1.5 meters is assumed to be the reference standard spacing, then the computations show that, a similar vehicle with the same parameters but with an axle spacing of 1.9m will reduce the life of pavement A (the thicker pavement) by as much as 32%. The corresponding reduction for pavement B is 24% while the reduction in the life of the structurally weak pavement will only be 8.5% more.

These findings suggest that, the same suspension type fitted to multiple axles can affect the pavement differently depending on the spacing between the axles in the group while the extent of damage depends on the pavement strength. Therefore serious consideration must be given to axle spacing when considering the vehicle parameters affecting the life of medium to strong pavements. The larger the axle spacing the greater the cracking and rutting damage done to the pavement irrespective of the suspension type or other parameters. For structurally weak pavements, axle spacing alone may be assumed to be a less important vehicle parameter.

Chapter 7

Summary and Conclusion

7.1 Summary

The available literature has indicated that moving vehicles exert dynamic axle loads which differ substantially from their stationary values. The magnitude of the dynamic loads and their impacts have been a subject of study in many countries. While relatively much success has been achieved in quantifying the magnitude of the dynamic loads, little has been accomplished in analyzing their impacts on pavements. The several mechanistic pavement design models have not been successful in accounting for the time-dependent behaviour of the dynamic loads and the viscoelastic pavement response. A method has been developed in this thesis for the computation of the time histories of the responses of flexible pavements under moving dynamic loads. The model uses Boltzman's superposition principle to translate the pavement response from a stationary load to a pavement response from a moving load of time-dependent magnitude. The two criteria used to determine pavement life are the tensile strain at the bottom of the asphalt concrete layer and the compressive strain at the top of the subgrade layer. The maximum responses are translated into fatigue cracking and rutting damage respectively in terms the number of load accumulation to cause failure. A comparison is made between the damage (cracking and rutting) due to moving dynamic loads and that due to static loads by the Pavement Life Ratio (PLR).

This model can be used to analyze flexible or rigid pavements using an appropriate influence function of pavement response. Other capabilities include the analysis of loads generated by single and tandem or multiple axles. The experimental dynamic load data measured with an instrumented vehicle developed by the National Research Council of Canada (NRCC) was used as input to the model. Analysis of a typical data measured with vehicles fitted with two suspension types and travelling at three different speeds indicated that, dynamic loads damage pavements more than static loads. The PLR values calculated for all the pavement types considered were in the range of 1.12 and 2.50 for cracking damage while that of rutting ranged between 1.36 and 6.97.

The magnitude of the damage however depends on the suspension type, vehicle speed,pavement surface roughness, vehicle axle configuration, axle spacing etc. In some situations, the rubber suspension was 43% more damaging than the air suspension. The ASDR values used to quantify the effects of axle spacing ranged from 0.85 to 1.32 and the LSDR values indicating the effects of load-sharing also ranged from 1.00 to 4.33. All of these suggested a close relationship between these parameters and pavement damage.

7.2 Conclusion

In summary the following conclusions were drawn and are descibed herein.

- The design life of flexible pavements depends on the magnitude of the moving dynamic loads from vehicles travelling over them but not on the magnitude of their stationary axle loads. Hence mechanistic design models that do not incorporate such loads and the viscoelastic pavement behaviour are inadequate in assessing pavement damage.

- Vehicle suspension type has greater influence on pavement damage. For similar vehicles, those fitted with rubber suspension cause gretare pavement damage than those with air suspension. Moreover, vehicle speed, axle configuration and axle spacing are the primary parameters influencing a particular suspension. On the other hand, higher pavement surface roughness induces higher dynamic load variations which combine with the vehicle parameters to increase pavement damage.

- Considering the effects of dynamic loads and the parameters influencing them, heavy vehicle regulations should be focused on the dynamic load potential of the vehicles. Moreover the regulations should take into consideration the spacing of axles in multiple axle group.

7.3 Recommendations for Further Study

The model developed in this thesis offers a solution for assessing the impact of dynamic loads on pavements. An issue that needs further study to enhance the reliability of mechanistic models is the spatial distribution of dynamic loads on pavements under a mix of in-service traffic. Though experimental data used in this thesis suggests the spatial distribution of dynamic loads under relicate runs (same vehicle, same speed), it is evident that, dynamic loads from a stream of different vehicles travelling with different speeds as occurs in public roads are neither perfectly repeatable (as assumed in this thesis) nor randomly distributed in space (as proposed by Eissenman, 1975). Dynamic load repeatability from real life traffic is somewhere between these extremes. It is therefore necessary to quantify the extent of this concentration of loads under real traffic.

References

Addis, R.R, Halliday, A.R., Mitchell, C.G.B., 1986, *Dynamic Loading of Road Pavements by Heavy Goods Vehicles,* Congress of Engineering Design, Seminar 4A-03, Birmingham, 26 pages, Institution of Mechanical Engineers.

ASTM *Standard Practices for Cycle counting in fatigue Analysis.* ASTM E1049-85, pages 836-848

Beyer, 1981 *Standard Mathematical Tables,* CRC Press, 26th Edition, Florida, USA.

Bent Larsen, K., 1986, *Mathematical Model for Prediction of Functional and Structural Behaviour of Flexible Pavements,* Ph.D. Dissertation, Report No.50, The Institute of Road Transport and Town Planning, Technical University of Denmark.

Brademeyer, B.D., 1975, *Flexible Pavement Systems: Analysis of the Structural Subsystems,* Master's Thesis submitted to Massachussets Institute of Technology, (MIT).

Cebon D., 1990, *Vehicle/Road interaction,* OECD Working Group TR2; Dynamic Loading of Pavements.

Cole, D.J., 1990, *Analysis and Measurements of Dynamic Tire Forces Generated by Lorries,* Ph.D. Dissertation, Cambridge University.

Cooper, K.E. and Pell P.S., 1974, *The Effects of Mix Variables on the Fatigue Strength of Bituminous Materials,* TRRL Report LR663, Berkshire, England.

Chou, T.Y., 1976, *Analysis of Subgrade Rutting in Flexible Airfield Pavements,* Transportation Research Record 616.

Eissenman, J., 1975,*Dynamic Wheel Load Fluctuations, Road Stress,* Strasse und Autobahn 4., pp 233-250

Ervin, R.D., Nisonger, R.L, Sayers, M., Gillepsie, T. D. and Francher, P.S., 1983, *Influence of Truck Size and Weight Variables on the Stability and Control Properties of Heavy Trucks,* University of Michigan Report Number UMTRI-83-10/2.

ELSYM5, *Report No. FHWA-RD-85* Federal Highway Administration, McLean, Virginia, 1985.

Gorge, W., 1984, *Evaluation of Research Reports Concerning the Influence of Commercial Vehicle Development on the Road Fatigue,* English Translation by the International Road Transport Union, Geneva.

Gillepsie, T.D., 1983, *Influence of Size and Weight Variables on Stability and Control of Heavy Trucks,* Report No. UMTRI-83-10/2 University of Michigan.

Hahn, W.D., 1985, *Effects of Commercial Vehicle Design on Road Stress-Vehicle Research Results,* Institut fur Kruftfahrwesen, Universitat Hannover, 12 pages, Translated by Transport and Road Research Laboratory as WP/V,ED/87/38

Hajek, J.J and Agarwal, A.C, 1990, *Influence of Axle Group Spacing on Pavement Damage,* Transportation Research Record No. 1286

Leornard, D.R, Graigner, J.W, and Eyre, R. 1974, *Loads and Vibration caused by Eight Commercial Vehicles with Gross Weight Exceeding 32 tons,* TRR report LR582.

Magnuson, G, Carlsson, H.E., and Ohlsson E. 1986, *The Influence of Heavy Vehicles' Springing Characteristics and Tire Equipment on the Deterioration of the Road,* VTI Report No. 270 1984, Translated by TRRL as WP/V,ED/86/16.

Markow, M.J.,Herdrick, J.K., Brademeyer, B.D. and Abbo, E., 1988, *Analyzing the Interaction between Dynamic Vehicle Loads and Highway Pavements,* Transportation Research Record, No. 1196.

Mitchell, C.G.B. and Gyenes, L. 1989, *Dynamic Pavement Loads Measured for a Variety of Truck Suspensions,* Second International Conference on Heavy Vehicle Weights and Dimensions, Kelowna, British Columbia.

Mitchell, C.G.B. and Gyenes L., 1992, *The Spatial Repeatability of Dynamic Pavement Loads Caused by Heavy Goods Vehicles.* Third International Symposium on Heavy Vehicle Weights and Dimensions, Queen's College, Cambridge.

O'Connel, S., Abbo, E., and Hedrick, K., 1986, *Analysis of Moving Dynamic Loads on Highway Pavements, Part I: Vehicle Response,* Proceedings International Symposuim on Heavy Vehicle Weights and Dimensions, Kelowna, British Columbia.

Papagiannakis, A.T, Haas, R.C.G., Woodroofe, J.H.F., and LeBranc P.A., 1988, *Effects of Dynamic Load on Flexible Pavements,* Transportation Research Record.

Papagiannakis, A.T, Phang, W.A, Woodrooffe, J.H.F., Bergan, A.T., Haas, R.C.G., 1989, *Accuracy of Weigh-in-Motion Scales and Piezoelectric Cables,* Transportation Research Record, No. 1215.

Potter, J.F., 1968, *Dynamic Impact Pressures Generated Under Loads by Sprung and Unsprung Vehicles,* RRL Report LR137.

Rauhut, J.B., Roberts, F.L., Kennedy, T.W., 1979, *Response and Distress Models for Pavements Studies,* Transportation Research Record No. 715

Rauhut, J.B., O'Quin J., and Hudson, W.R., 1976, *Sensitivity Analysis of FHWA Structural Model VESYS-IIM,* Preparatory and Related Studies, Vol. 1, Report No. FHWA-RD-76-23.

Sebaaly, P.E., and Mamlouk, M.S., 1988, *Development of Dynamic failure criterion,* ASCE, J. Transportation Engineering. Vol 114 No. 4.

Shahin, M.Y, Kirchner, K., Blackmon, W., and Tomita Hisao., 1986, *Effects of Layer Slippage on Performance of Asphalt-Concrete Pavements.* Transportation Research Record, No. 1095.

Sousa, J., Monismith, C.L., and Lysmer. J, 1989, *Dynamic Loads: Effects on Pavements.*

Sweatman, P.F., 1983, *A Study of Dynamic Wheel Forces in Axle Group Suspensions of Heavy Vehicles,* Special Report No. 27, Australian Road Research Board.

Sweatman, P.F., 1988, *Dynamic Road Loading: Instrumented vehicles,* FHWA Load Equivalence Workshop, Washington D.C.

Tschoegl N. W., 1989, *The Phenomenological Theory of Linear Viscoelasticity Behaviour, An Introduction.* Springer-Verlag.

Ullidtz, P., and Larsen, B.K, 1979, *Mathematical Model for Predicting Pavement Performance,* Transportation Research Record No. 949,

Ullidtz, P., 1989, *Pavement Analysis,* Elsevier Press,

Whitmore, A.P, Wiley, J.R., Shultz, P.C., and Pollock, D.E., 1970, *Dynamic Pavement Loads of Heavy Vehicles,* NCHRP Report No.105, Highway Research Board.

Woodrooffe, J.H.F., and Leblanc P.A., 1977, *Heavy Vehicle Suspension Variations Affecting Road Life,* Proc. ARRB/FORS Symposium on Heavy Vehicle Suspension Characteristics, Canbera.

Woodrooffe, J.H.F., Leblanc, P.A., and LePiane K.R. 1986, *Effects of Suspension Variation on Vehicle Wheel Loads of a Heavy Articulated Highway Vehicle,* Vehicle Weights and Dimension Study, Vol 11, Road and Transportation Association of Canada.

Appendix A

Program Listing-STATIC

```
C*************************************************************
C     PROGRAM=STATIC I
C*************************************************************
C     PROGRAM CALCULATES GENERAL PAVEMENT RESPONSE
C     DUE TO A MOVING LOAD OF CONSTANT AMPLITUDE
c
C     USES A SPECIFIED TIME INCREMENT [DEFAULT=.001 sec.]
C     TO CALCULATE NUMBER OF REPETITIONS TO FAILURE DUE TO
C     Ezz COMPRESSIVE, EXX TENSILE AND EYY TENSILE STRAINS
C     THE PROGRAM ANALYZES EFFECTS FROM SINGLE AXLES ONLY
C            11 MARCH 1992
C     UPDATED ON 18TH AUGUST 1992
C*************************************************************
C    UPDATED 11TH DEC 1992 (WSU)
C
      DIMENSION SL(500),SZZ(500),SYY(500),SXX(500)
      DIMENSION ST(500),SR1(500),SR3(500),RINTERY(500)
      DIMENSION TINTER(500),RINTER(500),SRZ(500)
      DIMENSION RINTERZ(500)
      CHARACTER*40,COMMENT
      OPEN(UNIT=11,FILE='STRA3IN.DAT',STATUS='OLD')
C      OPEN(UNIT=11,FILE='STRAIN6.DAT',STATUS='OLD')
      OPEN(UNIT=12,FILE='PRIM.DATT',STATUS='NEW')
      OPEN(UNIT=13,FILE='P6Z.DAT',STATUS='NEW')
```

```
       OPEN(UNIT=14,FILE='T6X.DAT',STATUS='NEW')
      READ(11,100) COMMENT
      READ(11,101) (SL(I),SXX(I),SYY(I),SZZ(I),I=1,19)
 100  FORMAT(A40)
 101  FORMAT(F12.0,3E12.3)
C      DO I=1,19
C       PRINT*, ST(I), SXX(I)
C      END DO
C     BEGIN INTERACTIVE DATA INPUT
      PRINT 102
  102 FORMAT(' INPUT SPEED IN km/h')
      ACCEPT 201,SP1
      PRINT 103
  103 FORMAT(' INPUT LOAD AMPL.-> STATIC SOLUT. MULTIPLIER')
      ACCEPT 201,AMPL
  201 FORMAT(F10.0)
      PRINT 104
  104 FORMAT(' INPUT THE TIME INCREMENT "TINCR" IN sec [0.001]')
      ACCEPT 202,TINCR
  202 FORMAT(F10.4)
      IF(TINCR.EQ.0.0)TINCR=0.001
C     CONVERT SPEED TO in/sec AND CALCULATE CONSTANTS
      SP=SP1*.6214*17.6
      DUR=90.0/SP
      START=-DUR/2.
      NUM=DUR/TINCR
C     CONVERT SPACE TO TIME INCREMENTS
      DO 301 I=0,NUM
      RR=NUM
      TI=I/RR
      TINTER(I)=START+TI*DUR
 301  CONTINUE
C     ASSIGN TIME VALUES TO THE 19 KNOWN LOCATIONS
      NDATA=19
      NINTV=18
C AA=ST(NDATA)
DO I =1,NDATA
```

```
ST(I)=START+SL(I)*DUR/90.0
END DO
c PRINT*, 'ST', (ST(i),I=1,19)
C      CALLING CUBIC SPLINE ROUTINES FOR strain=fn(space)
        CALL SPLINE(NDATA,ST,SZZ,RINTERZ,NT)
        CALL SPLINE(NDATA,ST,SXX,RINTER,NT)
        CALL SPLINE(NDATA,ST,SYY,RINTERY,NT)
C@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@@
C      INTEGRATE PRODUCT OF RESPONSE AND PULSE DERIVAT.
       SRZ(0)=0.0
       DO 601 K=1,NUM
       SRZ(K)=SRZ(K-1)-TINCR*RINTERZ(K)*(AMPL*2*ASIN(1.)/DUR)*
      +SIN(2*2*ASIN(1.)*TINTER(K)/DUR)
C      PRINT*,  K,SRZ(K)
C      WRITE(13,*) K,SRZ(K)
 601   CONTINUE
C      DETERMINE MIN RESPONSE (compressive Ezz)
       VALMIN=999.0
       DO 701  I=1,NUM
       IF(SRZ(I).LE.VALMIN)VALMIN=SRZ(I)
 701   CONTINUE
c   COMPUTE THE NUMBER OF REPETIONS TO FAILURE
C BY USING THE RELATIONSHIP DEVELOPED BY THE US ARMY CORP
C   OF ENGINEERS
C------------------------------------------------------
       ZREPS=(5.511E-03/ABS(VALMIN))**(1/0.1532)
c       print*, ' %%%',reps
c--------------------------------------------------------
C********************************************************
C   CALCULATE TENSILE STRAINS EXX
C      INTEGRATE PRODUCT OF RESPONSE AND PULSE DERIVAT.
       SR1(0)=0.0
       DO 605 K=1,NUM
       SR1(K)=SR1(K-1)-TINCR*RINTER(K)*(AMPL*2*ASIN(1.)/DUR)*
      +SIN(2*2*ASIN(1.)*TINTER(K)/DUR)
C       PRINT*, K,SR1(K)
C       WRITE(14,*) K, SR1(K)
```

```
 605  CONTINUE
C     DETERMINE MAX RESPONSE (TENSILE Exx)
      VALMAX=-999.0
      DO 705  I=1,NUM
      IF(SR1(I).GT.VALMAX) VALMAX=SR1(I)
 705  CONTINUE
      print*, valmax
C     CALCULATE NUMBER OF REPETITIONS TO FAILURE
      FK1=7.87E-07
      FK2=3.322
      XREPS=FK1*(1.0/VALMAX)**FK2
C*****************************************************
C  CALCULATE THE RESPPONSE DUE TO Eyy STRAINS
C     INTEGRATE PRODUCT OF RESPONSE AND PULSE DERIVAT.
C      SR3(0)=0.0
C      DO 608 K=1,NUM
C      SR3(K)=SR3(K-1)-TINCR*RINTERY(K)*(AMPL*2*ASIN(1.)/DUR)*
C     +SIN(2*2*ASIN(1.)*TINTER(K)/DUR)
C      WRITE(14,*) K,SR3(K)
C 608  CONTINUE
C     DETERMINE MAX RESPONSE ( Eyy)
C      VALMAY=-999.0
C      DO 708  I=1,NUM
C      IF(SR3(I).GT.VALMAY) VALMAY=SR3(I)
C 708  CONTINUE
C      print*, 'Valmay=',VALMAY
C     CALCULATE NUMBER OF REPS TO FAILURE
C      FK1=7.87E-07
C      FK2=3.322
C      YREPS=FK1*(1/VALMAY)**FK2
C     WRITING ON OUTPUT FILE
      PRINT*,'EXX=',XREPS,'  ','EYY=',YREPS,'  ','EZZ=',ZREPS
      STOP
      END
```

```
C******************************************************
C     PROGRAM=STATIC II

C******************************************************
C     ROGRAM CALCULATES GENERAL PAVEMENT RESPONSE
C     DUE TO A MOVING LOAD OF CONSTANT AMPLITUDE
C     FROM TANDEM AXLES,  CALLS CUBIC SPLINE ROUTINE INTERNALLY
C******************************************************
C     USES A SPECIFIED TIME INCREMENT [DEFAULT=.001 sec]
C     TO CALCULATE ONLY  Exx TENSILE
C            15 MARCH 1992
C     UPDATED ON 14TH AUGUST 1992
C
      DIMENSION S(25),SZZ(25),SYY(25),SXX(25),SRZ1(500)
      DIMENSION SS(25),SR1(500),ST(500),STZ(500)
      DIMENSION SRZ2(500)
      DIMENSION TINTER(500),RINTER(500),SR2(500)
      DIMENSION RINTERZ(500)
      CHARACTER*40,COMMENT
      REAL VALMAX1,VALMAX2
C
      OPEN(UNIT=11,FILE='STRA3IN.DAT',STATUS='OLD')
C      OPEN(UNIT=11,FILE='STRAIN6.DAT',STATUS='OLD')
      OPEN(UNIT=13,FILE='SP1.DAT',STATUS='NEW')
      OPEN(UNIT=15,FILE='STANX.DAT',STATUS='NEW')
      OPEN(UNIT=16,FILE='STANZ.DAT',STATUS='NEW')
      READ(11,100) COMMENT
      READ(11,*) (ST(I),SXX(I),SYY(I),SZZ(I),I=1,19)
 100  FORMAT(A40)
C 101  FORMAT(F12.0,3E12.3)
C     BEGIN INTERACTIVE DATA INPUT
      PRINT 102
  102 FORMAT(' INPUT SPEED IN km/h')
      ACCEPT 201,SP1
      PRINT 103
  103 FORMAT('INPUT FIRST LOAD AMPL.-> STATIC SOLUT. MULTIPLIER')
      ACCEPT 201,AMPL1
```

```
  201 FORMAT(F10.0)
C************************************************************
      PRINT 130
  130 FORMAT('INPUT SECOND LOAD AMPL.-> STATIC SOLUT. MULTIPLIER')
      ACCEPT 133, AMPL2
  133 FORMAT(F10.0)
      PRINT 104
  104 FORMAT(' INPUT THE TIME INCREMENT "TINCR" IN sec [0.001]')
      ACCEPT 202,TINCR
  202 FORMAT(F10.4)
      IF(TINCR.EQ.0.0)TINCR=0.001
C     CONVERT SPEED TO in/sec AND CALCULATE CONSTANTS
      SP=SP1*.6214*17.6
      DUR=90/SP
      START=-DUR/2.
      NUM=DUR/TINCR
C     ASSIGN TIME VALUES TO THE 19 KNOWN LOCATIONS
      DO 251 K=1,19
      SS(K)=START+S(K)/90.*DUR
 251  CONTINUE
C     CONVERT SPACE TO TIME INCREMENTS
      DO 301 I=0,NUM
      RR=NUM
      TI=I/RR
      TINTER(I)=START+TI*DUR
 301  CONTINUE
C     ASSIGN TIME VALUES TO THE 19 KNOWN LOCATIONS
      NDATA=19
      NINTV=18
AA=ST(NDATA)
DO I =1,NDATA
ST(I)=START+ST(I)*DUR/AA
END DO
c PRINT*, 'ST', (ST(I),I=1,19)
C CALL CUBIC SPLINE SUBROUTINE TO PERFORM THE INTERPOLATION FOR STRAINS

       CALL SPLINE(NDATA,ST,SXX,RINTER,NUM)
```

```
c      CALL SPLINE(NDATA,ST,SYY,RINTERY,NUM)
C INTEGRATE PRODUCT OF RESPONSE AND PULSE DERIVATIVE.
C--------------------------------------------------------------
      SR1(0)=0.0
      DO 601 K=1,NUM
      SR1(K)=SR1(K-1)-TINCR*RINTER(K)*(AMPL1*2*ASIN(1.)/DUR)*
     +SIN(2*2*ASIN(1.)*TINTER(K)/DUR)
c      WRITE(13,*) K, SR1(K)
c      print*,K,SR1(K)
 601  CONTINUE
C     DETERMINE Max RESPONSE (TENSILE, Exx1)
      VALMAX1=-999.0
      DO 701  I=1,NUM
      IF(SR1(I).GT.VALMAX1) VALMAX1=SR1(I)
 701  CONTINUE
C
C     CALCULATE NUMBER OF REPS TO FAILURE
      FK1=7.87E-07
      FK2=3.322
      REPS1=FK1*(1.0/VALMAX1)**FK2
C*************************************************************
      SR2(0)=0.0
      DO 606 K=1,NUM
      SR2(K)=SR2(K-1)-TINCR*RINTER(K)*(AMPL2*2*ASIN(1.)/DUR)*
     +SIN(2*2*ASIN(1.)*TINTER(K)/DUR)
c     WRITE(13,*) K, SR2(K)
 606  CONTINUE
C     DETERMINE Max RESPONSE (TENSILE, Exx2)
C
      VALMAX2=-999.0
      DO 705  I=1,NUM
      IF(SR2(I).GT.VALMAX2) VALMAX2=SR2(I)
 705  CONTINUE
C      print*, valmax
C     CALCULATE NUMBER OF REPS TO FAILURE
      FK1=7.87E-07
      FK2=3.322
```

```
      REPS2=FK1*(1.0/VALMAX2)**FK2
C--------------------------------------------------------
C*********************************************************
c CALCULATE SUBGRADE RESPONSE, EZZ
C     CALLING CUBIC SPLINE ROUTINES TO INTEPOLATE EZZ STRAIN=fn(space)
       CALL SPLINE(NDATA, ST, SZZ,RINTERZ,NUM)

C     INTEGRATE PRODUCT OF RESPONSE AND PULSE DERIVATIVE
      SRZ1(0)=0.0
      DO 610 K=1,NUM
      SRZ1(K)=SRZ1(K-1)-TINCR*RINTERZ(K)*(AMPL1*2*ASIN(1.)/DUR)*
     +SIN(2*2*ASIN(1.)*TINTER(K)/DUR)
c      PRINT*,  K,SRZ1(K)
c      WRITE(13,*) K,SRZ1(K)
 610  CONTINUE
C     DETERMINE MIN RESPONSE (compressive Ezz1)
      VALMIN1=999.0
      DO 720  I=1,NUM
      IF(SRZ1(I).LE.VALMIN1)VALMIN1=SRZ1(I)
 720  CONTINUE
c  COMPUTE THE NUMBER OF REPETIONS TO FAILURE
C BY USING THE RELATIONSHIP DEVELOPED BY THE US ARMY CORPS OF ENGINEERS
C-----------------------------------------------------
      REPZ1=(5.511E-03/ABS(VALMIN1))**(1/0.1532)
C      print*, ' %%%',repz1
c------------------------------------------------------
C     INTEGRATE PRODUCT OF RESPONSE AND PULSE DERIVAT.
C     FOR THE SECOND LOAD AMPLITUDE FOR EZZ
      SRZ2(0)=0.0
      DO 609 K=1,NUM
      SRZ2(K)=SRZ2(K-1)-TINCR*RINTERZ(K)*(AMPL2*2*ASIN(1.)/DUR)*
     +SIN(2*2*ASIN(1.)*TINTER(K)/DUR)
c      PRINT*,  K,SRZ2(K)
c      WRITE(13,*) K,SRZ2(K)
 609  CONTINUE
C     DETERMINE MIN RESPONSE (compressive Ezz)
      VALMIN2=999.0
```

```
      DO 709  I=1,NUM
      IF(SRZ2(I).LE.VALMIN2)VALMIN2=SRZ2(I)
 709  CONTINUE
c  COMPUTE THE NUMBER OF REPETIONS TO FAILURE
C BY USING THE RELATIONSHIP DEVELOPED BY THE US ARMY CORP
C  OF ENGINEERS
C---------------------------------------------------
      REPZ2=(5.511E-03/ABS(VALMIN2))**(1/0.1532)
C      print*, ' %%%',repz2
c-----------------------------------------------------
c%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
c DETERMINE THE RESPONSE FOR TANDEM AXLES FOR STATIC LOADING
C%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
      K1=0
      K2=0
      DO  K=1,(5*NUM/3.0)
       IF(K .GT. NUM) THEN
         RX1=0.0
       ELSE
         K1=K1+1
         RX1=SR1(K1)
       END IF
        IF(K.LT.2*NUM/3.0) THEN
         RX2=0.0
        ELSE
         K2=K2+1
         RX2=SR2(K2)
        END IF
        ST(K) =RX1+RX2
        WRITE(15,*) K,ST(K)
C        WRITE(13,*) K, ST(K)
C        PRINT*, K, ST(K)
      END DO
C-----------------------------------------------------------
c  CALCULATE THE EXX STRAINS DUE TO LOAD CYCLES APPLIED BY
C  USING RAINFLOW/RANGE PAIR COUNTING METHOD.
C
```

```
      T1=-999
      T2=999
      T3=-999
      DO M=1,2*NUM/3.0
        IF (ST(M).GT.T1)T1=ST(M)
      END DO
C  CALCULATE THE VALLEY STRAIN
      DO M2=2*NUM/3.0,NUM
        IF(ST(M2).LT.T2)T2=ST(M2)
      END DO
      DO M3=NUM,5*NUM/3.0
        IF(ST(M3).GT.T3)T3=ST(M3)
      END DO
C     CALCULATE THE STRAINS FROM THE CYCLE MOVEMENT
C     (ie. MAX OF THE PEAK AND TROUGH)
      E1=MAX(T1,T3)
      XE2=MIN(T1,T3)
C  FIND THE VALLEY STRAIN
      E2=XE2-T2
C E1 REPRESENTS PEAK STRAIN,  E2 REPRESENTS VALLEY STRAIN MAGNIT
C      PRINT*, '***',E1,E2
C*****************************************************
C     CALCULATE THE NUMBER OF REPETITIONS TO FAILURE FOR SINGLE
C     VEHICLE PASS USING THE TWO Exx STRAINS ALREDAY CALCULATED.
C************************************************************
      FK1=7.87E-07
      FK2=3.322
      REPET1=FK1*(1.0/E1)**FK2
      REPET2=FK1*(1.0/E2)**FK2
C************************************************************
C   RUTTING MODEL  EZZ RESPONSE
      K1=0
      K2=0
      DO  K=1,(5*NUM/3.0)
       IF(K .GT. NUM) THEN
         RZ1=0.0
       ELSE
```

```
          K1=K1+1
          RZ1=SRZ1(K1)
        END IF
         IF(K.LT.2*NUM/3.0) THEN
          RZ2=0.0
         ELSE
          K2=K2+1
          RZ2=SRZ2(K2)
         END IF
         STZ(K) =RZ1+RZ2
         WRITE(16,*) K,STZ(K)
c         WRITE(13,*) K, STZ(K)
C         PRINT*, K, STZ(K)
       END DO
         DO  K=1,(5*NUM/3.0)
         WRITE(13,*) K, ST(K),STZ(K)
         END DO
c-----------------------------------------------------------
c  CALCULATE THE EZZ STRAINS DUE TO LOAD CYCLES APPLIED
C  FOLLOWING THE RAINFLOW/RANGE PAIR CYCLE COUNTING METHOD.
       TZ1=999
       TZ2=-99
       TZ3=999
       DO M=1,2*NUM/3.0
         IF (STZ(M).LT.TZ1)TZ1=STZ(M)
       END DO
C  CALCULATE THE VALLEY STRAIN
       DO M2=2*NUM/3.0,NUM
         IF(STZ(M2).GT.TZ2)TZ2=STZ(M2)
       END DO
       DO M3=NUM,5*NUM/3.0
         IF(STZ(M3).LT.TZ3)TZ3=STZ(M3)
       END DO
C      CALCULATE THE STRAINS FROM THE CYCLICAL MOVEMENT
C      ( PEAK AND TROUGH)
       EZ1=MIN(TZ1,TZ3)
       ZE2=MAX(T1,T3)
```

```
C  FIND THE VALLEY STARIN
      Z2=ZE2-TZ2
C EZ1 REPRESENTS PEAK STRAIN,  Z2 REPRESENTS VALLEY STRAIN MAGNITUDE
      ZTAN1=(5.511E-03/ABS(EZ1))**(1/0.1532)
      ZTAN2=(5.511E-03/ABS(Z2))**(1/0.1532)
        PZZ=((1./ZTAN1)+(1./ZTAN2))
        PXX=((1./REPET1)+(1./REPET2))
         REPZZ=1./PZZ
         REPXX=1./PXX
       PRINT*, REPZZ, REPXX
      STOP
      END
```

Appendix B

Program Listing-DYNAMIC

```
C ********************************************************
C     PROGRAM DYNAMIC I
C *******************************************************
C     PROGRAM CALCULATES GENERAL PAVEMENT RESPONSE
C     FROM A MOVING LOAD OF TIME-DEPENDENT AMPLITUDE
C
C     USES A SPECIFIED TIME INCREMENT [DEFAULT=.001 sec]
C     THE PROGRAM CALLS CUBIC SPLINE ROUTINE TO INTERPOLATE FOR THE REQUIRED
C     VALUES BETWEEN TWO POINTS.
C
C    PROGRAM ASSUMES VARIABLE STARING POINTS OF VEHICLES
C**********************************************************
C           14TH MARCH 1992
C           UPDATED 15TH NOVEMBER 1992
C
      DIMENSION ST(500),SZZ(500),SYY(500),SXX(500)
      DIMENSION SS(500),SRZ(500),SRY(500)
      DIMENSION TINTER(500),RINTER(500),SR(500)
      DIMENSION ZRINTER(500),YRINTER(500), SL(500)
      DIMENSION AMPL(500),TIME(500),STIME(500)
      DIMENSION AMPLINT(500),DAMPLINT(500), AMPLINTY(500)
      DIMENSION AMPLINTZ(500),DAMPLINTZ(500),DAMPLINTY(500)
      CHARACTER*40, COMMENT,C11,C21,C31
```

```
      CHARACTER*10, LOADFILE
C
      PRINT 91
   91 FORMAT('INPUT LOADFILE NAME')
      ACCEPT 92, LOADFILE
   92 FORMAT(A10)
      OPEN(UNIT=11,FILE='STRA3IN.DAT',STATUS='OLD')
c       OPEN(UNIT=11,FILE='strain6.DAT',STATUS='old')
      OPEN(UNIT=13,FILE=LOADFILE,STATUS='OLD')
C       OPEN(UNIT=20,FILE='RET.DAT',STATUS='NEW')
      OPEN(UNIT=21,FILE='R31S.DAT',STATUS='NEW')
C
      READ(11,100) COMMENT
      READ(11,*) (SL(I),SXX(I),SYY(I),SZZ(I),I=1,19)
  100 FORMAT(A40)
C 101  FORMAT(F12.0,3E12.3)
C     BEGIN INTERACTIVE DATA INPUT
      PRINT 102
  102 FORMAT('INPUT SPEED IN km/h')
      ACCEPT 201,SP1
  201 FORMAT(F10.0)
      PRINT 104
  104 FORMAT('INPUT THE TIME INCREMENT "TINCR" IN sec [0.001]')
      ACCEPT 202,TINCR
  202 FORMAT(F10.4)
      IF(TINCR.EQ.0.0)TINCR=0.001
      PRINT 105
  105 FORMAT(' INPUT NUMBER OF DUR INCREMENTS')
      ACCEPT 203, NLOOP
  203 FORMAT(I3)
C     CONVERT SPEED TO in/sec AND CALCULATE CONSTANTS
      SP=SP1*.6214*17.6
      DUR=90/SP
      START=-DUR/2.
      NUM=DUR/TINCR
  204 FORMAT(I1)
      NLOAD=DUR*100+1
```

```
C  SET A COUNTER TO CHANGE THE STARTING POINT OF THE VEHICLE
      NSET=-1
  895 ICOUNT=0
      NSET=NSET+1
      ISET=(NSET*NLOAD/5)+1
      IF(ISET.GE. NLOAD) GO TO 898
      REWIND(13)
C     BEGIN READING-IN DYNAMIC LOAD DATA
C******NOTE, READ ONLY C11,C21 IF WORKING ON B-FILES (AIR SUSPENSION)
      READ(13,100) C11
      READ(13,100) C21
      READ(13,100) C31
      PRINT*, C11
      PRINT*, C21
      PRINT*, C31
C BEGIN THE COMPUTATIONS
      DO 820 L=0,ISET-1
       IF(ISET.LE.1) GO TO 820
      READ(13,888) AMPL(L), TIME(L)
 888  FORMAT(3X,F8.2,20X,F10.2)
C THE SECOND FORMAT READS B- FILES
C  888 FORMAT(3X,F8.2,5X,F8.2)
c      PRINT*, '@@@', AMPL(L), TIME(L)
 820  CONTINUE
C     SAMPLING FREQUENCY OF 100 POINTS/sec
C*********************************
C  START READING IN THE ACTUAL VALUES OF DYNAMIC LOAD DATA
  990 ICOUNT=ICOUNT+1
      DO I=1,NLOAD
      READ(13,891) AMPL(I), TIME(I)
  891 FORMAT(3X,F8.2,20X,F10.2)
C  891 FORMAT(3X,F8.2,5X,F8.2)
C###### THE SECOND FORMAT READS A 'B' RUBBER FILES ONLY
       END DO
C     CONVERT AMPL(I) TO RATIO WRT STATIC TIRE LOAD OF 5,760 LBS
      DO 361 I=1,NLOAD
       AMPL(I)=(((AMPL(I)/2.0)*1000)/4.448)/5760.
```

```
 361  CONTINUE
C     CONVERT SPACE TO TIME INCREMENTS
      DO 301 I=1,NUM
      RR=NUM
      TI=I/RR
      TINTER(I)=START+TI*DUR
 301  CONTINUE
C  ASSIGN TIME VALUES TO THE 19 STRAIN LOCATIONS
      NDATA=19
       DO 123 I =1,NDATA
ST(I)=START+SL(I)*DUR/90.0
 123  CONTINUE

C     CALLING CUBIC SPLINE ROUTINES FOR strain=fn(space)Exx, Ezz
       CALL SPLINE(NDATA,ST,SXX,RINTER,NT)
       CALL SPLINE(NDATA,ST,SZZ,ZRINTER,NT)
       CALL SPLINE(NDATA,ST,SYY,YRINTER,NT)
       NDATA=NLOAD
       NINTV=NLOAD-1
       AW=TIME(NDATA)-TIME(1)
      DO 144 I =1,NDATA
       STIME(I)=START+(TIME(I)-TIME(1))*DUR/AW
 144  CONTINUE
C     CALLING CUBIC SPLINE ROUTINES FOR load_ampl=fn(space)
      CALL SPLINE(NDATA,STIME,AMPL,AMPLINT,NT)
      CALL SPLINE(NDATA, STIME,AMPL,AMPLINTZ,NT)
      CALL SPLINE(NDATA, STIME,AMPL,AMPLINTY,NT)
C-----------------------------------------------------------
C FIND THE AMPLITUDE DERIVATIVE OF THE Exx STRAINS
      DO 503 I=1,NUM
      DAMPLINT(I)=AMPLINT(I)-AMPLINT(I-1)
 503  CONTINUE
C     INTEGRATE PRODUCT OF RESPONSE AND PULSE DERIVAT.
      SR(0)=0.0
      DO 601 K=1,NUM
      SR(K)=SR(K-1)+TINCR*RINTER(K)*(
     +DAMPLINT(K)*(SIN(ASIN(1.)+(2*ASIN(1.)*TINTER(K)/DUR)))**2-
```

```
     +(AMPLINT(K)*2*ASIN(1.)/DUR)*(SIN(2*2*ASIN(1.)*TINTER(K)/DUR))
     +)
C       WRITE(20,*) K, SR(K)
 601  CONTINUE
C     DETERMINE MAX RESPONSE (tensile Exx)
      VALMAX=-999.0
      DO 621  I=1,NUM
      IF(SR(I).GE.VALMAX) VALMAX=SR(I)
C       PRINT*,'DDD', count, valmax
 621  CONTINUE
C     CALCULATE NUMBER OF REPETITIONS TO FAILURE FOR CRACKING
      FK1=7.87E-07
      FK2=3.322
      REPS=FK1*(1./VALMAX)**FK2
c**********************************************************
C FIND THE AMPLITUDE DERIVATIVE THE Ezz STRAINS.
      DO 508 I=1,NUM
      DAMPLINTZ(I)=AMPLINTZ(I)-AMPLINTZ(I-1)
 508  CONTINUE
C  INTEGRATE PRODUCT OF RESPONSE AND PULSE DERIVATIVE.
      SRZ(0)=0.0
      DO 608 K=1,NUM
      SRZ(K)=SRZ(K-1)+TINCR*ZRINTER(K)*(
     +DAMPLINTZ(K)*(SIN(ASIN(1.)+(2*ASIN(1.)*TINTER(K)/DUR)))**2-
     +(AMPLINTZ(K)*2*ASIN(1.)/DUR)*(SIN(2*2*ASIN(1.)*TINTER(K)/DUR))
     +)
C       WRITE(21,*) K, SRZ(K)
 608  CONTINUE
C     DETERMINE MIN RESPONSE (compressive Ezz)
      VALMIN=999.0
      DO 701  I=1,NUM
      IF(SRZ(I).LE.VALMIN) VALMIN=SRZ(I)
 701  CONTINUE
C  CALCULATE THE NUMBER OF REPETITIONS TO FAILURE IN RUTTING.
      ZREPS=(5.511E-03/abs(VALMIN))**(1./0.1532)
C  FIND THE AMPLITUDE DERIVATIVES FOR EYY STRAINS
C     CALCULATE AMPL DERIVATIVES
```

```
      DO 513 I=1,NUM
      DAMPLINTY(I)=AMPLINTY(I)-AMPLINTY(I-1)
 513  CONTINUE
C  INTEGRATE PRODUCT OF RESPONSE AND PULSE DERIVAT.EYY
C       SRY(0)=0.0
      DO 613 K=1,NUM
      SRY(K)=SRY(K-1)+TINCR*YRINTER(K)*(
     +DAMPLINTY(K)*(SIN(ASIN(1.)+2*ASIN(1.)*TINTER(K)/DUR))**2-
     +AMPLINTY(K)*2*ASIN(1.)/DUR*SIN(2*2*ASIN(1.)*TINTER(K)/DUR)
     +)
C       WRITE(15,*) K,SRY(K)
 613  CONTINUE
C     DETERMINE MAX RESPONSE (Tensile, Eyy)
      VALMAY=-999.0
      DO 713  I=1,NUM
      IF(SRY(I).GE.VALMAY) VALMAY=SRY(I)
 713  CONTINUE
C     CALCULATE NUMBER OF REPS TO FAILURE
C      IF(VALMAY .GT.0.0) VAL=VALMAY
      FK1=7.87E-07
      FK2=3.322
C      PRINT*, VALMAX, VALMIN, VALMAY
       WRITE(21,*) ICOUNT,REPS, ZREPS
C       PRINT*,ISET, ICOUNT, REPS, ZREPS
       IF(ICOUNT.LT.NLOOP)GO TO 990
       IF(ICOUNT.EQ.NLOOP) GO TO 895
 898   CONTINUE
       STOP
       END




c************************************************************
C     PROGRAM DYNAMIC II,
C************************************************************
```

```
C     PROGRAM CALCULATES GENERAL PAVEMENT RESPONSE
C     FROM A MOVING LOAD OF TIME-DEPENDENT AMPLITUDE
C     BY TANDEM AXLES, PROGRAM CALLS CUBIC SPLINE INTERNALLY
C**************************************************************
C     USES A SPECIFIED TIME INCREMENT [DEFAULT=.001 sec]
C     TO CALCULATE ONLY  Exx TENSILE, AND EZZ COMPRESSIVE.
C   PROGRAM ASSUMES VARIABLE STARING POINT OF VEHICLES
C************************************************************
C            14 MARCH 1992
C     UPDATED ON I5TH AUGUST 1992
C     UPDATED 2ND DEC 1992 (WSU)
c--------------------------------------------------------------
      DIMENSION SL(500),SXX(500),SYY(500),SZZ(500)
      DIMENSION SS(25)
      DIMENSION TINTER(500),RINTER(500),SR1(500)
      DIMENSION SR2(500),SRZ1(500),SRZ2(500),STZ(500)
      DIMENSION ZRINTER(500)
C
      DIMENSION AMPL1(500),TIME(500),STIME(500)
      DIMENSION AMPLINT1(500),DAMPLINT1(500),ST(500)
      DIMENSION AMPL2(500), DAMPLINT2(500),AMPLINT2(500)
      CHARACTER*40, COMMENT,C1,C2,c3
      CHARACTER*10, LOADFILE
C
      PRINT 91
  91  FORMAT(' INPUT LOADFILE NAME')
      ACCEPT 92, LOADFILE
  92  FORMAT(A10)
C      OPEN(UNIT=11,FILE='STRA12IN.DAT',STATUS='OLD')
      OPEN(UNIT=11,FILE='STRAIN6.DAT',STATUS='OLD')
      OPEN(UNIT=13,FILE=LOADFILE,STATUS='OLD')
      OPEN(UNIT=21,FILE='TR3.DAT',STATUS='NEW')
C      open(unit=15,file='retz.dat',status='new')

      READ(11,100) COMMENT
      READ(11,101) (SL(I),SXX(I),SYY(I),SZZ(I),I=1,19)
 100  FORMAT(A40)
```

```
 101  FORMAT(F12.0,3E12.3)
C     BEGIN INTERACTIVE DATA INPUT
C
      PRINT 102
  102 FORMAT(' INPUT SPEED IN km/h')
      ACCEPT 201,SP1
  201 FORMAT(F10.0)
      PRINT 104
  104 FORMAT(' INPUT THE TIME INCREMENT "TINCR" IN sec [0.001]')
      ACCEPT 202,TINCR
  202 FORMAT(F10.4)
      IF(TINCR.EQ.0.0)TINCR=0.001
      PRINT 105
  105 FORMAT(' INPUT NUMBER OF DUR INCREMENTS')
      ACCEPT 203, NLOOP
  203 FORMAT(I3)
C
C     CONVERT SPEED TO in/sec AND CALCULATE CONSTANTS
      SP=SP1*.6214*17.6
      DUR=90/SP
      START=-DUR/2.
      NUM=DUR/TINCR
      NLOAD=DUR*100+1
C     BEGIN READING-IN DYNAMIC LOAD DATA
C
C  SET A COUNTER TO CHANGE THE STARTING POINT OF THE VEHICLE
       NSET=-1
 895  ICOUNT=0
       NSET=NSET+1
       ISET=(NSET*NLOAD/5.0)+1
       IF(ISET.GE. NLOAD) GO TO 898
      REWIND(13)
      READ(13,100) C1
      READ(13,100) C2
      READ(13,100) C3
      PRINT*, C1,C2,C3
C     SAMPLING FREQUENCY OF 100 POINTS/sec
```

```
      DO 820 L=0,ISET-1
      IF(ISET .LE. 1) GO TO 820
       READ(13,888) AMPL1(L), AMPL2(L), TIME(L)
 888   FORMAT(3X,F9.2,X,2F14.2)
 820  CONTINUE
C*****************************************************
  990 ICOUNT=ICOUNT+1
      DO I=1,NLOAD
      READ(13,891) AMPL1(I), AMPL2(I),TIME(I)
C *** THIS FORMAT READS "A-RUBBER" FILES ONLY
  891 FORMAT(3X,F9.2,X,2F14.2)
c  891 FORMAT(7X,F6.2,9X,F5.2)
CC #####THE SECOND FORMAT READS A 'B' RUBBER FILES ONLY
      END DO
C     CONVERT AMPL(I) TO RATIO WRT STATIC TIRE LOAD OF 5,760 LBS
      DO 361 I=1,NLOAD
      AMPL1(I)=((AMPL1(I)/2.0)*1000/4.448)/5760.
      AMPL2(I)=((AMPL2(I)/2.0)*1000/4.448)/5760.0
 361  CONTINUE
C     CONVERT SPACE TO TIME INCREMENTS
      DO 301 I=1,NUM
      RR=NUM
      TI=I/RR
      TINTER(I)=START+TI*DUR
 301  CONTINUE
C    ASSIGN TIME VALUES TO 19 KNOWN STRAIN LOACTIONS
      NDATA=19
C      NINTV=18
DO I =1,NDATA
ST(I)=START+SL(I)*DUR/90.
  END DO
C     CALLING CUBIC SPLINE ROUTINES FOR strain=fn(space)Exx, Ezz

       CALL SPLINE(NDATA,ST,SXX,RINTER,NT)
       CALL SPLINE(NDATA,ST,SZZ,ZRINTER,NT)
C
      NDATA=NLOAD
```

```
      NINTV=NLOAD-1
AW=TIME(NDATA)-TIME(1)
       DO 811 I=1,NDATA
STIME(I)=start+(TIME(I)-TIME(1))*DUR/AW
 811   CONTINUE
C     CALLING CUBIC SPLINE ROUTINES FOR load_ampl=fn(space)
      CALL SPLINE(NDATA,STIME,AMPL1,AMPLINT1,NT)
      CALL SPLINE(NDATA,STIME,AMPL2,AMPLINT2,NT)

C     CALCULATE **1ST** AMPL DERIVATIVES
      DO 503 I=1,NUM
      DAMPLINT1(I)=AMPLINT1(I)-AMPLINT1(I-1)
 503  CONTINUE
c***********************************************
C     INTEGRATE PRODUCT OF RESPONSE AND PULSE DERIVAT FOR FIRST TIRE.
      SR1(0)=0.0
      SR2(0)=0.0
      DO 601 K=1,NUM
      SR1(K)=SR1(K-1)+TINCR*RINTER(K)*(
     +DAMPLINT1(K)*(SIN(ASIN(1.)+2*ASIN(1.)*TINTER(K)/DUR))**2-
     +AMPLINT1(K)*2*ASIN(1.)/DUR*SIN(2*2*ASIN(1.)*TINTER(K)/DUR)
     +)
C      WRITE(14,*) K,'  ',SR1(K)
 601  CONTINUE
C     DETERMINE MAX RESPONSE (TENSILE Exx)
      VALMAX1=-999.0
      DO 701  I=1,NUM
      IF(SR1(I).GE.VALMAX1) VALMAX1=SR1(I)
 701  CONTINUE
C     CALCULATE NUMBER OF REPS TO FAILURE
      FK1=7.87E-07
      FK2=3.322
      REPS1=FK1*(1./VALMAX1)**FK2
C************************************************************
C     CALCULATE **2ND** AMPLITUDE DERIVATIVES
      DO 506 I=1,NUM
      DAMPLINT2(I)=AMPLINT2(I)-AMPLINT2(I-1)
```

```
 506  CONTINUE
C INTEGRATE PRODUCT OF RESPONSE AND PULSE DERIVAT FOR second TIRE.
      SR2(0)=0.0
      DO 605 K=1,NUM
      SR2(K)=SR2(K-1)+TINCR*RINTER(K)*(
     +DAMPLINT2(K)*(SIN(ASIN(1.)+2*ASIN(1.)*TINTER(K)/DUR))**2-
     +AMPLINT2(K)*2*ASIN(1.)/DUR*SIN(2*2*ASIN(1.)*TINTER(K)/DUR)
     +)
C      Print*, '@@@@', K ,SR2(K)
C      WRITE(15,*) K,SR2(K)
 605  CONTINUE
C---------------------------------------------------------------
C     DETERMINE MAX RESPONSE (TENSILE Exx) FOR SECOND TIRE
      VALMAX2=-999.0
      DO 705  I=1,NUM
      IF(SR2(I).GE.VALMAX2) VALMAX2=SR2(I)
 705  CONTINUE
C     CALCULATE NUMBER OF REPS TO FAILURE
      FK1=7.87E-07
      FK2=3.322
      REPS2=FK1*(1./VALMAX2)**FK2
c%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
c DETERMINE THE RESPONSE FOR TANDEM AXLES DUE TO DYNAMIC LOADING
C%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
      K1=0
      K2=0
      DO  K=1,(5*NUM/3.0)
       IF(K .GT. NUM) THEN
         RX1=0.0
       ELSE
         K1=K1+1
         RX1=SR1(K1)
       END IF
        IF(K.LT.2*NUM/3.0) THEN
         RX2=0.0
        ELSE
         K2=K2+1
```

```
          RX2=SR2(K2)
         END IF
         ST(K) =RX1+RX2
c         PRINT*, K, ST(K)
      END DO
C**************************************************************
C     CALCULATE THE EXX STRAINS DUE TO LOAD CYCLES APPLIED BY
C     FOLLOWING THE RAINFLOW/RANGE PAIR METHOD (ASTM)
      T1=-999.0
      T2=999.0
      T3=-999.0
      DO M=1,2*NUM/3.0
        IF (ST(M).GT.T1)T1=ST(M)
      END DO
C  CALCULATE THE VALLEY STRAIN
      DO M2=2*NUM/3.0,NUM
        IF(ST(M2).LT.T2)T2=ST(M2)
      END DO
      DO M3=NUM,5*NUM/3.0
        IF(ST(M3).GT.T3)T3=ST(M3)
      END DO
C     CALCULATE THE STRAINS FROM THE CYCLICAL MOVEMENT
C     (PEAK AND TROUGH)
      E1=MAX(T1,T3)
      XE2=MIN(T1,T3)
C  FIND THE MAGNITUDE OF THE VALLEY AND MIN PEAK STRAIN
      E2=XE2-T2
C E1 REPRESENTS MAX PEAK STRAIN,  E2 REPRESENTS VALLEY STRAIN MAGNIT
c      PRINT*, '***',E1,E2
C*******************************************************
C     CALCULATE THE NUMBER OF REPETITIONS TO FAILURE FOR SINGLE
C     VEHICLE PASS OF TANDEM AXLE USING THE TWO EXX STRAINS CALCULATED.
C**************************************************************
      FK1=7.87E-07
      FK2=3.322
      REPET1=FK1*(1.0/E1)**FK2
      REPET2=FK1*(1.0/E2)**FK2
```

```
C*************************************************************
c SUBGRADE RESPONSE EZZ
C************************************************************
C     INTEGRATE PRODUCT OF RESPONSE AND PULSE DERIVAT.
C
      SRZ1(0)=0.0
      DO 608 K=1,NUM
      SRZ1(K)=SRZ1(K-1)+TINCR*ZRINTER(K)*(
     +DAMPLINT1(K)*(SIN(ASIN(1.)+2*ASIN(1.)*TINTER(K)/DUR))**2-
     +AMPLINT1(K)*2*ASIN(1.)/DUR*SIN(2*2*ASIN(1.)*TINTER(K)/DUR)
     +)
C      Print*, '#####', K ,SRZ1(K)
 608  CONTINUE
C     DETERMINE MIN RESPONSE (compressive Ezz1)
      VALMIN1=999.0
      DO 730  I=1,NUM
      IF(SRZ1(I).LE.VALMIN1)VALMIN1=SRZ1(I)
 730  CONTINUE
C     PRINT*,'$$$', VALMIN1
C INTEGRATE PRODUCT OF RESPONSE AND PULSE DERIVAT FOR second TIRE.
      DO 607 K=1,NUM
      SRZ2(K)=SRZ2(K-1)+TINCR*ZRINTER(K)*(
     +DAMPLINT2(K)*(SIN(ASIN(1.)+2*ASIN(1.)*TINTER(K)/DUR))**2-
     +AMPLINT2(K)*2*ASIN(1.)/DUR*SIN(2*2*ASIN(1.)*TINTER(K)/DUR)
     +)
C      Print*, '@@@@', K ,SRZ2(K)
C     WRITE(14,*) K,SRZ2(K)
 607  CONTINUE
C     DETERMINE MIN RESPONSE (compressive Ezz2)
      VALMIN2=999.0
      DO 720  I=1,NUM
      IF(SRZ2(I).LE.VALMIN2)VALMIN2=SRZ2(I)
 720  CONTINUE
c  COMPUTE THE NUMBER OF REPETIONS TO FAILURE
C BY  USING THE RELATIONSHIP DEVELOPED BY THE US ARMY CORP
C  OF ENGINEERS
c%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
c DETERMINE THE EZZ RESPONSE FOR TANDEM AXLES DUE TO DYNAMIC LOADING
C%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
      K1=0
      K2=0
      DO  K=1,(5*NUM/3.0)
       IF(K .GT. NUM) THEN
         RZ1=0.0
       ELSE
         K1=K1+1
         RZ1=SRZ1(K1)
       END IF
        IF(K.LT.2*NUM/3.0) THEN
         RZ2=0.0
        ELSE
         K2=K2+1
         RZ2=SRZ2(K2)
        END IF
        STZ(K) =RZ1+RZ2
c       WRITE(16,*) K,STZ(K)
C        PRINT*,'ZZZZ', K, STZ(K)
      END DO
      DO  K=1,(5*NUM/3.0)
C       WRITE(16,*) K,ST(K),STZ(K)
      END DO
C     CALCULATE THE EZZ STRAINS DUE TO LOAD CYCLES APPLIED BY
C     FOLLOWING THE RAINFLOW/RANGE PAIR COUNTING METHOD
      TZ1=999.0
      TZ2=-999.0
      TZ3=999.0
      DO M=1,2*NUM/3.0
        IF (STZ(M).LT.TZ1)TZ1=STZ(M)
      END DO
C  CALCULATE THE VALLEY STRAIN
      DO M2=2*NUM/3.0,NUM
        IF(STZ(M2).GT.TZ2)TZ2=STZ(M2)
      END DO
      DO M3=NUM,5*NUM/3.0
```

```
        IF(STZ(M3).LT.TZ3)TZ3=STZ(M3)
      END DO
C     CALCULATE THE STRAINS FROM THE CYCLE MOVEMENT
C     (PEAK AND TROUGH)
      EZ1=MIN(TZ1,TZ3)
      EZ2=MAX(TZ1,TZ3)
C  FIND THE VALLEY STRAIN
      Z2=EZ2-TZ2
C EZ1 REPRESENTS MAX PEAK STRAIN, EZ2 REPRESENTS VALLEY STRAIN MAGNIT
C      PRINT*,'###',count, TZ1,TZ2,TZ3
C      PRINT*, '***',EZ1,Z2
c-------------------------------------------------------
       ZTAN1=(5.511E-03/ABS(EZ1))**(1/0.1532)
       ZTAN2=(5.511E-03/ABS(Z2))**(1/0.1532)
C DETERMINE THE NUMBER OF REPITIONS TO FAILURE FOR TANDEM AXLES
C FOR BOTH CRACKING AND RUTTING
       TRXX=((1.0/REPET1)+(1.0/REPET2))
       TRZZ=((1.0/ZTAN1)+(1.0/ZTAN2))
       TTX=1.0/TRXX
       TTZ=1.0/TRZZ
       WRITE(21,*) ICOUNT,TTX,TTZ
C       PRINT*,ICOUNT,REPET1,REPET2,ZTAN1,ZTAN2
       PRINT*,ISET, ICOUNT, TTX,TTZ
      IF(ICOUNT.LT. NLOOP)GO TO 990
      IF(ICOUNT.EQ.NLOOP)GO TO 895
 898  CONTINUE
      STOP
      END


c*************************************************************
C  SUBROUTINE SPLINE
C************************************************************
C THIS SUBROUTINE PERFORMS INTERPOLATION BY CUBIC SPLINE METHOD
      SUBROUTINE SPLINE(N,X,Y,Y1, NT)
      REAL A(500,500),S(500),X(500),Y(500),
```

```
     & X1, A1(500),B1(500),C1(500),D(500),R(500),
     &  Y1(500),H(500),GK(500,500),P(500),DISP(500)
      REAL  INT, g(500,500)
       L=N-2
C COMPUTE THE INTERVALS
      M=N-1
      DO K=1,M
      H(K)=X(K+1)-X(K)
      END DO
C GENERATE THE MATRIX
      DO I=1,N-2
        DO J=1,N-2
         A(I,I)=2*(H(I)+H(I+1))
         IF(J.EQ.I-1 .OR.J.EQ.I+1) THEN
         A(I,J)=H(I+1)
         END IF
         R(I)=6.0*((Y(I+2)-Y(I+1))/H(I+1)-((Y(I+1)-Y(I))/H(I)))
C        A(I,N-1)=-6.0*(R(I))
        END DO
       END DO
C SOLVE THIS MATRIX BY GAUSS ELIMINATION METHOD
C USE THE END CONDITION (1) S1=0, SN=0
C DENOTE S1=S(0) AND SN=S(L+1)
C  CALL THE SUBROUTINE SOLVE TO PERFORMN THE GAUSS ELIMINATION
      CALL SOLVE(A,R,L,S)
C STORE THE S1 VALUES IN AN ARRAY CALLED S(I)
      S(L+1)=0.0
      S(0)=0.0
C
C DETERMINE THE COEFFICIENTS A1, B1,C1,D1
      DO I=1,L+1
        A1(I)=(S(I)-S(I-1))/6.0*H(I)
        B1(I)=S(I-1)/2.0
        C1(I)=(Y(I+1)-Y(I))/H(I)-((2*H(I)*S(I-1)+H(I)*S(I))/6.0)
        D(I)=Y(I)
C        PRINT*,'%%D',D(I)
      END DO
```

```
C PERFORM THE INTEPOLATION AT ANY X VALUE FOR THE CORRESPONDING Y
        NT=((X(N)-X(1))/0.001)
        DO K=1,NT
        X1=X(1)+(X(N)-X(1))*(K-1)/(NT-1.)
        DO I=1,N-1
         IF(X1.GE.X(I).AND.X1.LE.X(I+1)) THEN
         Y1(K)=A1(I)*(X1-X(I))**3+B1(I)*(X1-X(I))**2+
     &   C1(I)*(X1-X(I))+ D(I)
         END IF
         IF(K.EQ.NT) Y1(K)=Y(N)
C        WRITE(19,*) X1,Y1(K)
         END DO
        END DO
      RETURN
      END


C -------------------------------------------------------------------
C SUBROUTINE SOLVE. THIS SUBROUTINE SOLVES SIMULTANEOUS EQUSTION BY
C GAUSS ELIMINATION METHOD
C*********************************************************
C DESCRIPTION OF VARIABLES:
C*********************************************************
C FOR AN EQUATION Ax+By+Cz=D;
C GK=COEFICIENT MATRIX, ie., A,B,C etc.
C P=VALUE IN THE RIGHT HAND SIDE, ie., D.
C MEQ=NUMBER OF EQUATIONS
C DISP=SOLVED VALUES
C********************************************************
SUBROUTINE SOLVE(GK,P,MEQ,DISP)
DIMENSION GK(500,500),P(500),DISP(500),G(500,500)
C       PRINT*, 'MEQ=',MEQ
DO 2 I=1,MEQ
DO 3 J=1,MEQ
G(I,J)=GK(I,J)
G(I,MEQ+1)=P(I)
```

```
  3 CONTINUE
  2 CONTINUE
        DO I=1,MEQ
C       PRINT*, (G(I,J),J=1,MEQ+1)
        END DO
C WRITE(5,*)'WRITTING A UNIFIED MATRIX'
C DO 4 I=1,MEQ
C WRITE(5,51)(G(I,J),J=1,MEQ+1)
C  4 CONTINUE
  51 FORMAT(80F10.2)
c READ(5,*)PAUSE2
DO 5 K=1,MEQ-1
DO 6 J=K,MEQ
IF(G(J,K).NE.0) THEN
DO 7 IP=1,MEQ+1
A=G(K,IP)
G(K,IP)=G(J,IP)
g(J,IP)=A
 7 CONTINUE
        GOTO 8
ENDIF
 6 CONTINUE
 8 DO 9 J=K+1,MEQ
C WRITE(5,*)'STEP=',K,'DIVIDER=',G(K,K)
C PRINT*, '**',G(K,K)
        Q=G(J,K)/G(K,K)
  DO 10 IP=K+1,MEQ+1
G(J,IP)=G(J,IP)-Q*G(K,IP)
  10 CONTINUE
  9 CONTINUE
  5 CONTINUE
C        PRINT*,'###',G(MEQ,MEQ)
DISP(MEQ)=G(MEQ,MEQ+1)/G(MEQ,MEQ)
C WRITE(5,*)DISP(MEQ)
I=MEQ-1
  12 SIG=0
DO 11 J=I+1,MEQ
```

```
SIG=SIG+G(I,J)*DISP(J)
  11 CONTINUE
C       PRINT*,'G(I,I)=',G(I,I)
DISP(I)=(G(I,MEQ+1)-SIG)/G(I,I)
I=I-1
IF(I.NE.0) GOTO 12
DO 13 I=1,MEQ
C WRITE(5,*)DISP(I)
  13 CONTINUE
RETURN
END
```