A STUDY OF THE EFFECTIVENESS OF A CALCULATOR-ORIENTED INSTRUCTION UNIT ON STUDENTS' PROBLEM SOLVING ABILITY IN WORKING WITH ROUTINE WORD PROBLEMS

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A STUDY OF THE EFFECTIVENESS OF AN INSTRUCTIONAL UNIT INCORPORATING THE USE OF THE CALCULATOR ON STUDENTS' PROBLEM SOLVING ABILITY IN WORKING WITH ROUTINE WORD PROBLEMS

by

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Abstract

The main purpose of this study was to investigate the effect of an instructional unit incorporating the use of the calculator on students' problem solving ability in working with routine word problems. More specifically, the instructional unit's effect on the number of processes and key processes utilized, the number of correct solutions, and the number of computational errors made was investigated.

The sample consisted of 10 students enrolled in a grade 10 mathematics course designed for low ability students. The textbook for this course was Consumer Related Mathematics. (Kravitz and Brant, 1971)

A test consisting of 10 routine word problems was administered to each student individually as a pretest. Students were asked to verbalize their thoughts as they attempted to reach solutions to the problems, and the interviews were recorded on cassette tapes. Following the pretest the results of each student's performance was coded using the coding sheet.

A calculator orientation unit and an instructional unit were devised for use in the study. The primary purpose of the calculator-orientation unit was to instruct students on proper calculator usage and the calculator's relationship to problem solving. A class set of calculators (Model Tl-1035) was provided for each class period. The main purpose of the instructional unit was to teach students key processes to be

used in problem solving, and how these processes could be applied to different types of problems. The emphasis when teaching the unit was on these key processes with particular attention given to those processes in which students exhibited weaknesses. The duration of the instructional period, including the calculator-orientation unit, was four weeks. Following the instructional unit, students were administered a parallel form of the pretest as a posttest. Students' performances were recorded individually on a cassette tape, and the results coded on the coding sheet.

Significant gains were reported on a number of key processes utilized by students following instruction. Significant gains were also made in the number of correct solutions. Also, a significant decrease in the number of computational errors made was reported.

An important observation made from the study was that allowing students to verbalize their thoughts while solving routine words problems provides a basis for instruction directed towards the specific weaknesses of the students involved. Recommendations were made that the study be replicated in other geographical areas, with larger samples and with students of differing abilities. Also the long-term effects of such units need to be investigated.

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CHAPTER I

BACKGROUND

The area of problem solving has been, and continues to be, a major concern of both mathematics education and teachers. The National Council of Teachers of Mathematics (1981) in its Agenda for Action stressed that problem solving should be the focus of school mathematics for the 1980's and further stated that the mathematics curriculum be organized around problem solving.

More specifically, it has been suggested that emphasis be placed on the ability to solve routine word problems. For example, Barnett, Sowder and Vos (1980) stated that:

> Effective citizenship as a consumer, as a taxpayer, as a wage earner, requires an ability to solve a myriad of routine problems. Checking purchases, calculating interest costs, evaluating best buys, planning meals -- all these are examples of routine problems. (p. 2)

A review of the literature revealed that the ability to solve routine word problems, although a requirement in keeping with societal demands, was an area in which a large number of people exhibited weaknesses. The Second National Assessment of Educational Progress (1981) provided evidence that many people were not very proficient in solving such problems.

A starting point for improvement in the teaching of problem solving requires a detailed analysis of processes employed by students in solving problems. Although written tests have been the most commonly used method of monitoring students' success in problem solving, such tests do not readily lend themselves to an analysis that would determine the specific weaknesses the students possess. Verbalization of thought processes by students as they work through a problem is an alternative method of recording students' specific weaknesses. This method, commonly referred to as the think-aloud technique, has been used extensively by researchers for that purpose. (Zalewski, 1974; Gagne and Smith, 1962)

The analysis of students' protocols obtained by employing the think-aloud method provides directions and guidelines for teaching that focuses on the specific areas of difficulty encountered by the student in problem solving. Here, both technology and research play a major role. Suggestions from research, combined with technological materials, provide a basis for instruction aimed an enhancing the problem-solving situation in the classroom. One such combination exists in the design of instructional units that are designed with the specific weaknesses of the student in mind, and also incorporate the use of teaching aids.

In the area of problem solving, an instructional unit which incorporated the use of the calculator was considered to be one approach aimed at the improvement of problem solving ability. At least one study, (Wheatley, 1980), previously dealt specifically with this topic. In reporting the study Wheatley indicated a need for further research in the area. Furthermore, Suydam (1978) indicated that the calculator's relationship to problem solving was a question of vital concern.

From a review of the literature on the use of the calculator in problem solving situations three advantages of using the calculator were determined. Firstly, an instructional unit incorporating the use of the calculator was considered to be beneficial in that it would allow more time for students to concentrate on the analysis of problems rather than on the computational aspects of the problem. Secondly, such a unit should allow more instructional time for dealing with real world problems and problems oriented to the students' particular needs. Thirdly, provision for a greater number of problems could be made possible in this manner.

In summary, the analysis of processes used by students has provided a basis for more meaningful teaching-learning experiences in the area of problem solving. It seemed feasible that the development of instructional units on problem solving incorporating the use of the hand-held calculator was one possible direction for research. Consequently, research in this area might reveal one way of investigating the practical implications for classroom use.

Purpose of the Study

The purpose of the study was to test the effectiveness of an instructional unit incorporating the use of the calculator on students' problem solving ability in working with routine word problems. The study attempted to seek answers to the following questions.

Question 1. Was an instructional unit incorporating the calculator effective in teaching students to apply key processes when solving routine word problems?

Question 2. Was an instructional unit incorporating the calculator effective in reducing the number of computational errors?

Question 3. Was an instructional unit incorporating the calculator effective in increasing the number of correct solutions?

Limitations to the Study

There were several major limitations to the study. Since the sample studied consisted of only 10 students, the generalizability was limited. The method used to collect the data was another factor considered. Since students had to verbalize their thoughts while solving each problem, it was felt that they might make more, less, or different errors than they normally would given a different situation. The nature of the sample itself was another factor considered to be a limitation to the study. Since all subjects were of below-average ability, it was thought that this might have impeded them in the verbalization of their thought processes.

Definition of Terms

Processes:	procedures used by the students as they work toward the solution of a given problem. For example, disgards irrelevant data, draws a diagram, estimates were processes.
Key Processes:	processes that were specifically developed and taught in the instructional unit. These key processes are outlined in Chapter III.

CHAPTER II

REVIEW OF RELATED LITERATURE

Problem solving is an area which has received extensive study. Recently, the primary focus of research has been on processes employed by students in attempting to solve problems. The principle methodology used in these studies has been to interview students, asking them to think aloud as they attempt to reach a solution. This technique has allowed the investigators to monitor the processes and strategies employed by the student during the problem solving process. Such studies have resulted in the development of coding sheets and checklists to record students' protocols. Studies that have examined the processes and strategies utilized by students have generated suggestions for instruction. Such instructional techniques are aimed at the improvement of students' problem solving performances. Another component of research literature has examined the effect of the calculator on problem solving. The primary purpose of all studies in the area of problem solving has been to provide a basis which should result in, either directly or indirectly more meaningful teaching-learning experience in the classroom.

The chapter is divided into four sections. In Section I studies relating to problem solving models and processes are discussed. In Section II studies that deal with recording and coding students' processes are reported. In Section III studies dealing with instruction in processes are discussed, while studies regarding the use of the calculator in problem solving are reported in Section IV.

Problem Solving Models and Process Studies

Studies that have been conducted on processes exhibited by students in solving word problems have been, for the most part, based on general mathematical models outlined by mathematics educators. One such model was described by Bloom and Broder (1950). The model consisted of four stages which a student passes through in attempting to reach a solution to a problem. These stages include, an understanding of the problem, the understanding of the ideas within the problem, the development of a general approach to the problem, and an attitude towards the solution.

Another general problem solving model was devised by Polya (1957). Polya (1957) identified four phases in the problem-solving process, understanding the problem, devising a plan, execution of the plan, and evaluation of results. Although Polya's model has been used extensively in studies involving novel mathematical problems, a number of studies on routine problem solving also have been based on this model.

Silver (1977) explored one aspect of the devising a plan element of Polya's model. His study centered on the idea of thinking of a similar problem. The eighth graders used in the study classified problems as being similar on the basis of some shared measurable quantity (e.g., time, age). After analysis of the solutions, Silver concluded that, more students used associations based on underlying mathematical structures than on non-underlying mathematical structures.

While thinking of a similar problem can be classified as one of the many processes used by students, another process involves the use of word clues in any given problem. A study to this effect was conducted by Early (1968). He attempted to determine whether the use of word clues had any effect on the correct process used to solve the problem. Early concluded that when word clues are contained in a given problem, students tend to select the correct algorithm more often. He also suggested that the more practice students receive on word problems, the less the dependence on word clues.

Lerch and Hamilton (1966) identified two categories in the solving of routine word problems. These two categories were listed as the ability to determine the correct procedure and the ability to carry out the correct computation. Lerch and Hamilton reported that after students received instruction in writing number sentences which described the problems they were better able to determine the procedure to follow in solving the problem.

This ability was considered by Lerch and Hamilton to be more important than the ability to perform the correct computation.

The trial and error strategy and the ability to estimate are related. Students who use estimation in problem solving quite often also use the trial and error strategy. Several studies have looked at the relationships between trial and error and estimation. Hall (1976) concluded that students who were good estimators were superior in problem solving. Paull (1972) showed that the ability to estimate numerical computations was a significant predictor of the ability to solve problems by the trial and error method.

A useful skill for a good problem solver is the ability to recognize and discard irrelevant data from a problem. Poor problem solvers tend to lack this ability. Bergen (1972) investigated the effect on problem difficulty of adding irrelevant data to a problem. He tested his problems using eighth grade students and found that problems containing only the right amount of information were the least difficult to solve, while problems containing irrelevant data were the most difficult. Bechtold (1965) hypothesized that problems containing irrelevant data could be used to develop students' problem-solving skills. His work with average ability ninth graders confirmed this. He noted that students who train on problems containing irrelevant data transfer their ability to become successful in solving highly complex problems containing no irrelevant data.

From a review of the literature it was concluded that processes can be taught to students and students tend to exhibit these processes following instruction. However, the amount of instruction and the number of processes emphasized may vary with the ability level of the student.

Recording and Coding Processes

Most studies that have dealt with processes have employed the think-aloud method of having students verbalize their thoughts while solving problems. Although some questions exist as to the value of this technique, several researchers have commented on its value. Kilpatrick (1967) stressed the value of this technique but at the same time brought attention to its limitations. He stated:

> The method of thinking aloud has special virtue of being both productive and easy to use. If the subject understands what is wanted, that he is not only to solve the problem but also to tell how he goes about finding the solution ... and if this method is used with the awareness of its limitations, then one can obtain detailed information about thought processes. (p. 8)

The value of the think-aloud method has been investigated in many studies. Zalewski (1974) found that this type of data gathering process captured and classified mathematical problem solving much better than other types of tests. Based on his findings, he suggested that coding schemes can be applied reliably to describe subjects' problem-solving behaviors and that the scoring system permits logical ranking of the subjects.

Gagne and Smith (1962) found that verbalizing thoughts during the problem-solving process actually improved problem-solving performances. They indicated that there were moments of silence when no processes were recorded but suggested that a series of directed questions would probably control this aspect.

Roth (1966) investigated the think-aloud method in terms of the amount of time required to arrive at a solution, and the number of correct solutions. He reported that there was no significant difference in either the number of correct solutions or time factor when students were required to think aloud as compared to the non-verbalization method of problem solving.

Flaherty's (1975) results supported the findings of Roth (1966). He showed that requiring students to think aloud did not significantly influence the problem-solving score or the time needed to complete the problem. He did, however, report that students using the think-aloud method made more computational errors. He attributed this to the difficulty of the problems involved.

The think-aloud method appears to be an effective procedure for monitoring students' processes. The use of the think-aloud technique has resulted in the development of coding schemes and checklists designed to record these processes.

One such coding system was designed by Flaherty (1975). His system consisted of 17 variables on which student processes could be monitored. Kantowski (1977) developed a scheme which gave one point for each of the following: suggesting a plan, persistence, looking back, absence of structural errors, absence of superflurous deductions, and correctness of results. Such coding checklists have been revised and modified by other researchers for particular studies. Days (1978) developed a coding checklist which was a modification of one developed by Kilpatrick (1967). Days (1978) defined process and strategy scores as the number of different problems on which the process or strategy was used. Understanding, representational, and evaluation scores were obtained by summing the process scores under each category.

Researchers who have used the think-aloud method have reported that this technique is a suitable means of obtaining data on students' processes in problem solving. When combined with the use of coding sheets and checklists, the think aloud method allows for accurate monitoring of these processes.

Instruction in Processes

Following the identification of problem-solving processes, many researchers have investigated the effect of instruction with these processes. Brown's (1964) study was aimed at improving instruction in problem solving with ninth grade general mathematics students. In this year-long study, objectives were established initially on which to base future instruction. The teachers participating in the study were encouraged to teach in a manner most comfortable to them, and use materials which they considered would lead to the attainment of the objectives. Although results were inconclusive on the effectiveness of the instruction, Brown (1964) identified a series of logical steps that could be used in the instruction of word problems. These steps included understanding of the problem, looking for what is needed to solve the problem, looking for hidden questions in the problems, deciding what computations to make, estimating a reasonable answer, performing computations, and checking results. Examples of instruction on each of the seven steps were outlined. Brown (1964) concluded from his results, that slower students especially, need to examine the complex operations of problem solving, break these operations into simpler steps, and practice each step separately.

Post and Brennan (1976) proposed a general model for instruction in problem solving. Their model had the following classifications: general heuristic problem-solving procedures, recognition, clarification and understanding of the problem, plan of attack analysis, productive phase and evaluation phase. The tenth graders used in the study received instruction in processes in accordance with the proposed model. Although they did not conclude that instruction in the problem-solving process was effective in promoting problem-solving ability, they did suggest that efforts to improve effectiveness in this area should continue. A possible avenue for mathematics teachers would be the identification of "typical" problem-solving behaviors and attention and maintenance of those behaviors.

Vos (1976) concentrated on five processes for instruction. These included drawing a diagram, approximating and verifying, constructing an equation, classifying data, and constructing a chart. He showed that it was possible to teach the use of such processes but the increase in the ability to solve problems was slight. However, Vos concluded that in selecting five specific problem-solving behaviors many behaviors were ignored. Suggestions were made for further research to identify the most salient problem-solving behaviors and then provide instruction to follow concentrating on those behaviors.

Nelson (1975) investigated one process in instruction, drawing a diagram. It was concluded that students were better able to solve problems presented in diagram form. He also reported that students used the diagram method quite frequently in attempting to solve problems.

Denmark (1965) compared an inductive method versus a deductive method of training students to deduce and use an equation in the solution of a problem. Grade eight and nine classes used a programmed set of lessons. The control group solved problems by trial and error (inductive method) while the experimental group was given instruction in the construction and use of tables to identify and organize the problem data (deductive method). From an analysis of his test results he concluded that both methods had some merit. The deductive approach resulted in students writing a greater number of correct equations, while students using induction produced a greater number of correct solutions.

Palzere (1968) asked secondary school students to verbalize their awareness of the concepts involved in the solution of a verbal problem. If the student verbalized incorrectly, he was told the correct response and asked to reverbalize. The problem-solving performance of students given this treatment was compared to that of students who had not been required to verbalize. When grade level and IQ were held constant, no significant differences were found. However, as grade level increased, the verbalization requirement was significantly increased.

Kantowski (1974) did an exploratory study of problem-solving ability developed around heuristic instruction in geometry. Processes were observed before instruction was initiated and after an initial instructional phase which stressed problem-solving strategies rather than content. By observing the processes employed on pre and posttests, Kantowski indicated that as processes used in geometry changed so did those used in the solution of verbal problems. Not only did the number of problems solved correctly increase, but there was also evidence of the use of processes, of more regular patterns of analysis and synthesis, and of greater persistence.

Smith (1973) studied the effect of general versus specific strategies in mathematical problem-solving tasks. The instructional phase of the study took place during regular class session and was given by means of programmed booklets. He indicated that task-specific instruction was more effective than general instruction in improving problem-solving performance on some learning tasks.

In summary, various instructional techniques aimed at improving problem-solving performance have been investigated. Some of these studies were aimed at general instruction on processes whereas others dealt with the effectiveness of concentrating on one or more processes. From these studies two conclusions were common, namely, processes can be taught to students and instruction aimed at specific behaviors of students in problem solving does improve problem-solving performance. In several reports the need for further research in instructional methods aimed at the specific behaviors exhibited by students in problem-solving situations was suggested.

Problem Solving Utilizing the Calculator

The role of the calculator in problem solving has been examined in several research studies. Studies in which the use of the calculator has been investigated have primarily been concerned with its value as an instructional aid. Suydam (1979) summarized the essence of calculator studies. She stated:

> Almost 100 studies on the effects of calculator use have been conducted during the past four or five years. Many of these studies had one goal: to ascertain whether or not, the use of the calculator would harm students' mathematical achievement. The answer continues to be "No". The calculator does not appear to affect achievement adversely. In all but a few instances, achievement scores are as high or higher when calculators are used for mathematics instruction than when they are not used for instruction. (p. 3)

Studies that have dealt specifically with the role of the calculator in problem solving include a study by Hopkins (1978). Hopkins showed that calculators helped students in a grade nine basic mathematics course to achieve better scores on problem solving than the non-calculator users. Kasnic (1978) provided additional evidence that calculators helped lower ability students to compete successfully with students of higher ability in solving word problems.

In an earlier study, Broussard (1969) investigated the effects of a calculator oriented program combined with flowcharts and other materials for low achievers in junior high. The program incorporated real-world applications and resulted in significant achievement gains. Sixty percent of the students who participated in the program continued to take mathematics courses as compared to 40 percent involved in the control group.

In one study reviewed, the effect of the calculator on problem-solving processes exhibited by students in solving routine word problems was examined (Wheatley, 1980). Her study involved a comparison between problem-solving performance of elementary school pupils using calculators with that of pupils not using calculators. Wheatley sought to identify differences on a range of problem-solving processes, the number of computational errors, and the number of problems solved. Subjects included 46 sixth graders who were randomly assigned to one of the two groups. Both groups studied a unit on operations with decimal fractions, with emphasis on application. Techniques of problem solving were taught as part of the daily schedule. Among the 14 techniques of problem solving taught were: make a list, look for a pattern, make a reasonable estimate, draw a diagram, write mathematical sentences, check work, and retrace steps. The instructional period lasted six weeks. Following instruction students were posttested and results were compared with pretest scores. Of the ten processes used in problem solving, which were analyzed, the calculator group used a total of 152 compared to 104 for the non-calculator group. The calculator group also made fewer computational errors. Differences on production scores and time on task were not significant. Wheatley concluded that calculators allowed students time to focus on problem-solving approaches, and that calculators had a positive effect on children's problem-solving performance.

The value of such studies as Wheatley's (1980) is two-fold. Not only does this type of study provide valuable information on how pupils verbalize their thinking and how best to record and observe such verbalization, but such a study helps in designing instruction suitable to meet the students' needs.

In summary, it was concluded from the studies reviewed that recording the processes used by students during the problem-solving process is one means of attempting to improve students' performance in this area. The think-aloud technique has been determined as a suitable means of studying students' protocols exhibited while solving word problems, and when combined with coding schemes and checklists, allows for accurate recording of these processes. Furthermore, once students' protocols have been recorded and studied, instruction involving the processes proved, for the most part, beneficial in improving the problem-solving behavior of students.

CHAPTER III

THE EXPERIMENTAL DESIGN

In this chapter an explanation of the design used and the materials employed in the study is given. An explanation of the tests and coding scheme used in the study is also given. A detailed description of the procedure followed is included.

Sample

The sample in this study consisted of 10 students enrolled in a grade 10 mathematics course designed for low achievers. The sample are members of a class of grade 10 students enrolled in Practical Mathematics 1202 at the commencement of the school year.

Instruments

The tests used in the study consisted of routine word problems focusing on percent. The problems were selected in light of the processes targeted for analysis as illustrated on the coding sheet. The coding sheet is shown in Figure 1. The coding sheet was designed so that it was possible to keep a record of both the processes and the order in which they were used. The order in which processes were used was recorded in columns from left to right. The coding scheme used to record students' protocols was Wheatley's (1980) with minor modifications. This coding scheme was originally based on the coding system developed by Day (1978).

Figure 1

Coding Sheet

Student Number ____________ Problem Number ________

Date of Coding ____________ Time of Solution ______

Understanding/Representational

Reads Problem	U_1						
Rereads Problem	U_2						
Discards Irrelevant Data	U_3						
Separates Parts of the Condition	U_4						
Draws a Diagram	U_5						

Recall

Recalls a related concept	R_1						
Recalls a related problem	R_2						
Uses method of related problem	R_3						

Production

Reasons deductively (if-then; since-then)	P_1						
Misinterprets problem	P_2						
Selects solution on irrelevant basis	P_3						
Uses, trial and error	P_4						
Guesses	P_5						
Estimates	P_6						
Uses unexpressed equations	P_7						

Evaluation

Makes a routine check	E_1						
Checks conditions	E_2						
Retraces steps	E_3						
Uses another method	E_4						
Questions reasonableness	E_5						
Changes approach	E_6						

Comments about solution

Questions existence of solution	C_1						
Questions necessity/relevance of information	C_2						
Expresses uncertainty about solution	C_3						
Says he/she doesn't know how to solve the problem	C_4						

Computational Errors Tallies Total

This coding sheet was selected primarily for two reasons. Firstly, it included key processes that, based on the literature, were considered to be of importance in allowing students to solve problems most effectively. Some of these processes are rereads problem, discards irrelevant data, draws a diagram, recalls a related problem and estimates.

Secondly, the coding sheet contained processes that students exhibit when solving problems, and that usually lead to an incorrect solution. Examples of these processes are misinterprets the problem and selects solution on irrelevant basis.

Furthermore, the coding sheet was divided into five major categories: Understanding, Recall, Production, Evaluation and Comments about the solution. Under each major category, processes applicable to that category were listed. Processes listed in this order provided a suitable means of coding, since it appeared to be a logical sequence in which processes might be exhibited during problem solving.

The sheet also provided a means of keeping a record of the number of computational errors the students made in solving each problem.

Specific test items were designed to highlight such processes as drawing a diagram, discarding irrelevant data and estimating. More general processes such as rereads problem, check steps, and recalls related problems, were applicable to all problems. Further criteria for the selection of the problems included the appropriate level of difficulty for the students in the sample, and computations that usually required multiplication, division or both. The tests are contained in Appendix A.

Materials

Materials used for the study included the following:

(1) The text used in the Grade 10 course, Practical Mathematics 1202. This text was Consumer Related Mathematics. (Kravitz and Brant, 1971)

(2) Tape Recorders

(3) One Model Tl-1035 L.C.D. Calculator for each student.

For purposes of the instructional portion of the study, two units were developed.

(1) A calculator orientation unit which incorporated published calculator activities selected from:

A. The Calculator Workbook (Sharp, 1977)

B. Calculator Book 1 and 2 (Immerziel and Ockenga, 1979)

C. Calcu-Math Activities (Sydney and Freeman, 1977).

Examples of activities from these sources are contained in Appendix B.

(2) An instructional unit on percents designed to teach students key processes to be used in problem solving.

A sample of selected activities from this unit is included in Appendix C.

The instructional unit was designed prior to conducting the study. Processes considered to be key processes to be used in solving problems were selected. Lessons were designed in which students received instruction in these processes and how they applied to a variety of word problems.

The main objective of the instructional unit was to familiarize students with processes that would help in the solution of word problems. Students were presented with a variety of problems that required the use of certain key processes to arrive at a correct solution. For each problem presented to the students, a series of questions, designed to get students to exhibit suitable processes was asked.

The instructional unit was divided into 16 separate lessons, each focusing on one aspect of percents. Each lesson contained five model problems in which different combinations of processes could be applied. Following each lesson, students were given a set of problems to attempt, and were asked periodically to discuss their procedure and results with the class. Students were also encouraged to write and solve their own problems.

Key processes used in the instructional unit included the following.

Rereads Problem: A student reads a problem a second time while attempting to solve it. If the problem was read more than twice, this was still coded as one instance of rereads problem.

Discards Irrelevant Data: The student eliminates information given in a problem that is not necessarily essential in solving it.

Separates Parts of the Condition: For problems involving two or more steps, the student divides the given information into separate parts for purposes of solving the problem.

Draws a diagram: The student represents a given problem in diagramatic form.

Recalls a related concept: The student recalls a concept from a previous problem which is related to a concept in the given problem.

Recalls a related problem: The student recalls a similar problem encountered previously in problem solving situations.

Recalls Method of Related Problem: Having recalled a similar problem, the student recalls the means by which he solved the previous problem.

Reasons deductively: The student uses deductive reasoning to go from the given information to the required solution.

Estimates: The student gives an approximate answer based on given information in a problem.

Makes a routine check: The student checks numerical calculation performed while solving the problem.

Checks conditions: The student checks the final answer obtained in the original problem statement.

Retraces steps: The student checks separate parts of the processes used to ensure continuity in the total solution.

Procedure

The procedure employed in the study was divided into three phases. Each of the three phases is described in detail in the following paragraphs.

Phase I

A brief instruction period of three class lectures was given. The main purpose of this instruction period was to review the basic concepts of percent. During these class periods, illustrations of the meaning of percent and how percents could be calculated were given.

Phase II

Interview

Each student was taken individually to a room where the tape recorder was already set up. The student was given the opportunity to ask any questions regarding the procedure to be followed. It was emphasized that relating thoughts verbally was of prime importance. As soon as the student felt at ease with the situation, the student was presented with one practice problem. Based on the response to the practice problem, suggestions were made regarding the improvements of the students' verbalization. For example, it might have been suggested that a student speak louder or explain each step. The student was then reminded that he was to verbalize his thoughts in each of the problems to follow. It was explained that problems would be presented one at a time and as soon as the student felt he could go no further or had reached a solution, he could proceed to the next problem.

The student was then presented with a problem and asked to relate the thoughts verbally while attempting to solve it. As soon as the student indicated that he could go no further, or a solution was reached, the student was asked to explain how he arrived at any numbers for which he had not given a verbal explanation. Following this, the student was presented with another problem. This procedure continued until the student had the opportunity to attempt each problem on the pretest. Since at this time no instruction had been given, the students were not permitted to use the calculator when solving the problems. All interviews were conducted in the same manner.

Following completion of the interviews, the tapes were coded. An assistant coded segments of the tapes to check for coding reliability. An analysis of students' strengths and weaknesses were made by studying pretest performances. This analysis gave direction to phase three of the procedure, the instructional period.

Phase III

Instructional Period

Calculators were made available to students for each class period during the instructional phase with students receiving an orientation to proper calculator usage prior to the commencement of the unit on problem solving with percent. A total of four class periods were used for the calculator orientation. During that time, students had an opportunity to become familiar with their calculator and get involved in calculator activities chosen from the previously listed published calculator activity books.

During the instruction period, students received instruction on key processes to be used in problem solving. The content for the instructional unit focused on the unit on percents in the textbook, Consumer Related Mathematics (Kravitz and Brant, 1971). Particular attention was given during this instruction period to the specific weaknesses in processes exhibited by students during the interviews.

The overall organization of the instructional unit was based on Polya's (1957) model for solving problems. The processes taught were incorporated into the four steps, understanding the problem, devising a plan, executing the plan and evaluating the results.

For the duration of the instruction period, the calculator was used in all problem solving situations. The unit included problems which focused on real life situations and the students wrote and solved their own problems. The value of the calculator was stressed in solving real world problems and in areas of approximation and estimation. Students were taught to focus on the procedure for solving the problems.

The duration of the instructional unit, including the calculator orientation component was four weeks, following which, a posttest was administered. Students were permitted to use the calculator on the posttest. The procedure for the posttest was the same as for the pretest. Form B, which was similar in structure to the pretest was used as a posttest.

Analysis of Data

Following completion of the procedure an analysis of the data was conducted. The following null hypotheses were tested.

(1) There is no significant difference between the number of key processes utilized by students before and after instruction.

Key processes were tabulated using the coding sheet. A student received a 1 if he used a key process once in a word problem. While he may have used a key process several times in one problem, he still received a 1 for the use of that process.

A t-test for dependent samples was used to test hypothesis 1.

(2) There is no significant difference between the number of computational errors made by students before and after instruction. Students could make more than one computational error per problem.

(3) There is no significant difference between the number of correct solutions made by students before and after instruction.

A t-test for dependent samples was used to test hypotheses 2 and 3.

Pilot Study

A pilot study, using a similar group of students at a school different from that used in the main study, was conducted approximately one month prior to the study.

The purposes of the pilot were:

(1) To determine whether or not students would exhibit codable processes as they attempted to solve routine word problems.

(2) To familiarize the investigator with the interview procedure.

(3) To obtain practice in coding processes using the coding checklist and make necessary modifications.

(4) To help in the selection of appropriate problems to be used in the main study.

The procedure and analysis used for the pilot were similar to those described for Phase II of the main study.

From the pilot study, it was concluded that students were not reluctant to verbalizing their thoughts and did exhibit codable processes such as: rereads problem, recalls a related concept, and guesses.

As a result of the pilot, it was decided on a standardized format to be used during the interview. It was decided to use one coding sheet containing all the processes as opposed to two. Use of two sheets tended to make coding too difficult.

In light of the pilot study, five out of 15 problems were omitted from the problem set since it appeared that they were too difficult for the group of students in the sample. The remaining 10 problems comprised the pretest for the main study. The posttest consisted of a parallel form of the pretest.

CHAPTER IV

ANALYSIS OF DATA

The chapter is divided into three sections. In Section I, students' performances on the number of processes and key processes exhibited on the pretest and posttest are reported. The first null hypothesis was tested and the results reported in this section. In Section II and III, respectively, computational errors and the number of correct solutions for each test are presented. Hypotheses 2 and 3 were tested and the results reported in these sections. Analysis of the data was based on nine students' performances since one student participating in the study failed to complete the term.

Processes and Key Processes

In Table 1, a summary indicating students' overall utilization of processes on the pretest as compared with the posttest is presented. From the table the number of times a specific process was exhibited by students on the pretest and the posttest can be determined. The numbers listed in each column indicate that the specific process was exhibited by a given student in that many problems on each test. For example, Student 1 has a score of 3 for process U_2 on the pretest. This indicates that process U_2 was used in 3 problems on the pretest.

Table 1

Total Number of Processes Used By Students on the Pretest and Posttest

STUDENT NUMBER

	1		2		3		4		5		6		7		8		9			
	Pr	Po	Pr	Po	Pr	Po	Pr	Po	Pr	Po	Pr	Po	Pr	Po	Pr	Po	Pr	Po	PRE	POST
U_1	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	90	90
* U_2	3	8	4	10	0	2	2	0	1	1	1	2	5	3	3	4	5	1	24	31
* U_3	0	3	1	3	2	3	2	3	0	3	1	3	0	3	2	3	1	2	9	26
* U_4	0	0	0	0	0	0	0	1	0	6	0	1	0	0	1	1	0	4	1	13
* U_5	0	1	1	1	0	1	0	1	1	2	0	1	0	2	0	1	0	1	2	11
* R_1	6	2	1	2	1	4	3	2	4	1	6	2	5	0	3	1	3	1	32	15
* R_2	2	7	3	6	2	5	0	6	2	7	0	6	1	6	1	4	2	5	13	52
* R_3	2	7	3	6	2	4	0	6	2	7	0	5	1	6	1	4	2	5	13	50
* P_1	3	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	4	2
P_2	3	0	5	1	2	3	5	1	4	2	5	3	3	2	4	2	2	2	33	16
P_3	0	0	4	2	3	0	1	0	2	0	1	1	3	0	4	3	1	1	18	7
P_4	0	0	0	0	3	0	2	0	0	0	0	1	1	0	0	1	1	0	7	2
P_5	1	0	0	0	1	0	2	1	0	5	0	1	1	0	0	0	0	0	5	2
* P_6	1	3	0	1	0	1	0	2	0	0	0	0	0	3	0	0	0	1	1	11
P_7	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
* E_1	0	3	0	0	0	0	0	0	0	1	0	0	0	5	0	0	0	1	0	10
* E_2	1	4	1	5	0	0	0	0	1	3	0	0	0	5	0	1	1	1	4	19
* E_3	0	2	0	1	0	1	0	0	0	3	1	0	0	5	0	0	0	0	1	12
E_4	1	0	0	0	0	1	2	0	0	2	1	1	0	0	0	0	1	0	6	3
E_5	1	1	0	0	1	3	0	0	3	1	0	1	0	0	0	0	0	0	5	6
E_6	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
C_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C_2	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0
C_3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
C_4	0	0	2	0	1	0	1	0	2	2	2	1	1	0	0	0	3	1	12	4
Total	34	53	35	48	30	39	28	33	34	53	29	38	32	50	29	35	32	37	Process	Total

* *Key Processes*

Having established the baseline data for the overall processes, the key processes that were emphasized in the instructional unit were then examined.

Hypothesis 1.

There is no significant difference between the number of key processes utilized by students before and after instruction.

The results were analyzed using a t-test for dependent samples, and a 0.05 level of significance. A summary of the results is given in Table 2.

Table 2

Change in the Number of Key Processes
(n = 9)

Score	Mean	S.D.	t-Value
Pretest	11.56	3.40	7.56*
Posttest	27.79	8.36	

* t significant at the 0.05 level

A 0.05 level of significance (df = 8) required a t-value of 2.31. Since the t-value obtained was 7.56, Hypothesis 1 was rejected. There was a significant increase in the total number of key processes utilized by students following the instruction period.

In addition to a significant increase in the total number of key processes, all key processes increased substantially with the exception of process R_1, recalls a related concept, which decreased from 32 to 15, and process P_1, reasons deductively, which remained relatively the same.

Of the key processes that increased, most were applicable to all problems, however, three of these key processes were only applicable to some of the problems on both tests. Discards irrelevant data, U_3, was applicable to three of the 10 problems on each test. Thus, the maximum number of times discards irrelevant data would have been appropriately used was three for each student on each test, giving a possible maximum total of 27. Draws a diagram, U_5, would have been usefully applied to three problems on both tests, giving a maximum of two for each student or a possible maximum total of 18. Estimates, P_6, would have been usefully applied to three of the problems on both tests. Again, the student could have appropriately used that particular process three times on each test giving a possible maximum total of 27.

The results of students' performance on these particular key processes are reported in Table 3.

It should be noted that all nine students gained on process U_3, eight of the nine students gained on process U_5, while five of the nine students gained on process P_6. These results clearly indicate that for all key processes, with the exception of R_1, recalls a related concept, gains were

Table 3

Total Number of Times Problem Specific Key Processes Were Used Appropriately By Students

Process	Code	Max. Appropriate	Pretest	Posttest
Discards Irrelevant Data	U_3	27	9	26
Draws a Diagram	U_5	18	2	11
Estimates	P_6	27	1	11
TOTAL			12	48

found both on the total number of key processes utilized, and on the use of individual key processes. Furthermore, most students increased their use of individual key processes.

While gains were found on most key processes following instructions, an examination of the data presented in Table 1 on the overall utilization of all processes, indicated that a number of processes, not considered to be key processes, was reduced. The number of times P_2, misinterprets the problem, was used decreased from 33 to 16. The number of times P_3, selects solution on irrelevant basis, was used decreased from 18 to 7, while the number of times C_4, says he does not know how to solve the problem, was used decreased from 12 on the pretest to four on the posttest. All of these decreases were in fact desirable behaviors. The number of times other processes such as P_1, P_4, P_7, E_4, E_5, E_6, C_1, C_2 and C_3, were used remained relatively the same.

Not only did the number of times certain processes used by students decrease substantially, but also most students exhibited these processes fewer times. For example, seven of the nine students reduced their use of process P_2, misinterprets the problem, six of the nine students reduced their use of process P_3, selects solution on irrelevant basis, while five of the nine students reduced their use of process C_4, says he doesn't know how to solve the problem. Results of students' utilizations of key processes and general processes are further discussed in Chapter V.

In Table 4 the number of computational errors made by each student on both the pretest and the posttest is given.

Table 4

Total Number of Computational Errors Made
By Students on the Pretest and the Posttest

	Student Number								
	1	2	3	4	5	6	7	8	9
Pretest	3	6	10	2	9	4	7	5	4
Posttest	1	0	0	0	1	5	0	1	0

Hypothesis 2.

There is no significant difference in the number of computational errors made by students before and after instruction.

The results were analyzed using a t-test for dependent samples, and a 0.05 level of significance. A summary of the results is given in Table 5.

Table 5

Change in the Number of Computational Errors

Score	Mean	S.D.	t-Value
Pretest	5.56	2.54	-4.09*
Posttest	0.89	1.52	

* t significant at the 0.05 level

A 0.05 level of significance (df = 8) required a t-value of 2.31. Since the t-value obtained was -4.09, hypothesis 2 was rejected. There were significantly less computational errors made on the posttest than on the pretest. Furthermore, eight of the nine students made fewer computational errors on the posttest than on the pretest. This result is discussed in Chapter V.

In Table 6 the total number of correct solutions made by students on the pretest and the posttest is given.

Table 6

Total Number of Correct Solutions Made By Students on the Pretest and the Posttest

	Student Numbers								
	1	2	3	4	5	6	7	8	9
Pretest	6	2	2	2	3	1	1	1	4
Posttest	7	7	6	9	8	3	9	5	7

* Maximum was 10

Hypothesis 3.

There is no significant difference in the number of correct solutions made by students before and after instruction.

The results were analyzed using a t-test for dependent samples, and a 0.05 level of significance. A summary of the results is given in Table 7.

Table 7

Change in Correct Solutions

Score	Mean	S.D.	t-Value
Pretest	2.44	1.57	7.00*
Posttest	7.00	1.95	

* t significant at the 0.06 level

A 0.05 level of significance (df = 8) required a t-value of 2.31. Since the t-value obtained was 7.00, Hypothesis 3 was rejected. There was a significant difference in the number of correct solutions made on the posttest than on the pretest. In fact, all nine students made gains in the number of correct solutions on the posttest as compared with the pretest. Further discussion of this result is presented in Chapter V.

Summary

In this chapter, the results and an analysis of the testing of the three hypotheses outlined in Chapter III were reported. It was found that on all three variables, significant differences occurred. Students made significant gains on the number of key processes utilized in solving problems, and in the number of correct solutions made on the posttest as compared with the pretest. Computational errors were significantly less on the posttest. Implications of these findings are discussed in Chapter V.

CHAPTER V

SUMMARY, CONCLUSIONS, DISCUSSION, IMPLICATIONS AND RECOMMENDATIONS

Summary

The study was designed to investigate the effect of a calculator-oriented instructional unit on students' problem solving ability when working with routine word problems. Specifically, the effects of an instructional unit incorporating the calculator on the number of processes and key processes, the number of computational errors made, and the number of problems solved correctly were investigated.

The sample consisted of 10 students enrolled in a grade 10 mathematics course designed for low ability students.

A pretest consisting of 10 routine word problems was administered individually to each student. Students were asked to "think-aloud" as they completed each question. The sessions were recorded on cassette tapes and then coded. An instructional unit of approximately one month duration was designed with major emphasis being placed on the weaknesses students exhibited on the pretest. The instructional unit incorporated a calculator-orientation unit of four class periods designed to instruct students in proper calculator usage. Following the instruction period, a parallel form of the pretest was administered as a posttest. The same procedure for administration was used for the posttest as for the pretest. Students' protocols were recorded and coded. A t-test for dependent samples was used to check the three hypotheses for significance at the 0.05 level.

Conclusions

There were three major conclusions from the study. Firstly, it was found that there was a significant increase in the number of key processes utilized by students when solving routine word problems. Secondly, there was a significant decrease in the number of computational errors made by students on the posttest than on the pretest. Thirdly, the number of correct solutions significantly increased following instruction.

Discussion

Significant gains were reported on the total number of key processes utilized by students following instruction. Not only did the total number of processes increase significantly, but all key processes increased in number with the exception of the use of R_1, recalls a related concept, which decreased in number from 32 to 15 and the use of process P_1, reasons deductively, which remained relatively the same. In addition to a significant increase in the total number of key processes utilized, and gains in the use of all but two of the key processes, all nine students increased their use of key processes following instruction. These results clearly indicate that key processes can be taught to students. Since the instructional unit emphasized key processes to be used when solving problems, students appeared to have incorporated the use of such processes in solving the problems on the posttest.

As was previously mentioned, process R_1, recalls a related concept, decreased in number from 32 on the pretest to 15 on the posttest. It was felt that on the pretest students recalled a related concept quite frequently but appeared to be unable to relate that concept to the problems they were solving. Thus students could not completely solve the problem. For example, students would recall how to find the percent of a number, but could not usefully apply this concept in finding the solution of a given problem. On the posttest however, students tended to disregard the use of process R_1, in favor of process R_2, recalls a related problem, and process R_3, recalls the method of the related problem. The use of both these processes enabled students to complete the solution of the problem. Again, this selection of useful key processes can be linked to the effect of the instructional unit. Since students had the opportunity during the instructional unit to solve problems similar in structure to the problems on the pretest and posttest, they were able to recall some of these problems and the method used to solve them when solving the problems on the posttest.

On the three problem specific key processes, gains were reported on the use of all three of these processes on the posttest. Also, all nine students gained on process U_1, discards irrelevant data, eight of the nine students gained on process U_5, draws a diagram, while five of the nine students gained on process P_6, estimates. During the instructional unit, particular attention was given to these problem specific key processes. Students were given an opportunity to deal with a variety of problems where these specific key processes could be appropriately used. As a result, it was felt that students were much better able to identify problems in which these key processes could be appropriately applied and use these processes when solving certain problems on the posttest.

An examination of students overall use of processes indicated that for processes other than key processes, utilization of these processes by students either decreased or remained relatively the same. Since only the key processes were emphasized in the instructional unit, this result could be interpreted as indicating a more effective use of key processes and problem specific key processes when solving problems. This conjecture is supported by the fact that the number of key processes utilized by students increased from 104 on the pretest to 251 on the posttest.

Other ineffective processes, such as misinterprets the problem decreased.

A significant decrease in the number of computational errors made on the posttest than on the pretest was reported. Although the instructional unit did not focus on computational errors, eight of the nine students made fewer computational errors on the posttest than on the pretest. This result could be due to students' use of the calculator when solving the problems on the posttest. Since students were allowed to use the calculator when solving the problems on the posttest but not on the pretest, and had received instruction in proper calculator usage, one would expect the number of computational errors to be reduced. An alternative explanation could be that the ability of students to use more effective procedures for problem solving reduced frustrations that may have led to computational errors, and therefore, the number of errors students made was reduced.

An increase in the number of correct solutions following instruction was found. In fact, all nine students made gains in the number of problems solved correctly on the posttest as compared to the pretest. This result seems to indicate that both the effective use of processes and a reduction in the number of computational errors made, were contributing factors in enabling students to solve more problems correctly. Since on the pretest students most often failed to reach correct solutions because they exhibited weaknesses in the proper approach to the problems and in their ability to compute properly, it was felt that both these criteria led to a significant increase in this area.

Implications

The findings of this study have implications for the classroom teacher in the area of problem solving. A study of the results showed that instructional units, such as the one used in this study, which focus on specific weaknesses of students, are beneficial in increasing the problem solving performances of students. The following points might be considered by the classroom teacher. Firstly, diagnose weaknesses of students when solving problems and secondly, design instructional units to focus on these specific weaknesses.

Proper calculator usage on the part of the student results from instruction in this area. Again the classroom teacher might consider instruction in proper calculator usage as part of their mathematics program.

In summary it is suggested that when the combination of a calculator orientation unit and an instructional unit designed to teach key processes was made available to students, they became more efficient problem solvers, both in their approach to the problems and the number of problems solved correctly. It appears that this combination or similar combinations should be practiced in the classroom in an effort to improve the problem-solving situation.

Recommendations for Further Research

The present study was conducted in one geographical location. A similar study should be conducted in other areas to provide a more detailed basis from which to evaluate the effectiveness of this procedure for designing instructional materials in the mathematics curriculum.

This study was conducted using a small sample consisting of low-ability level students. A similar study should be conducted using larger samples and different ability levels to determine whether this type of instruction is beneficial in improving the problem-solving performances of larger groups and different ability levels of students. A study of this nature would help determine to which ability group, this type of instruction is most beneficial.

In the present study the long-term effects of the instruction period on students' ability to retain correct procedures and effective processes to be used when solving problems was not investigated. Further studies of this type should consider the use of a retention test to determine whether or not this procedure is effective in getting students to retain effective methods of problem solving.

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APPENDIX A

Copy of Tests

Form A

1. At a certain store a roll of fabric 25 metres long sells for $148.75. If Jane bought 40% of the roll of fabric, how many metres did she buy?
2. Two planes leave cities which are 1600 kilometres apart. Plane A has a speed of 500 kph while plane B travels 25% faster than plane A. If the planes are travelling toward each other, how far apart will they be at the end of one hour?
3. A pair of jeans sells for $26.00 at Store A and $45.00 at Store B. Store B is offering a 48% reduction in all sales. Which store offers the best buy?
4. In Store A a certain article sells for $275.00. Store B is offering the same article for $285.00. Store A is reducing the price of the article by 25%, but Store B is offering a 30% discount. How much would a person pay for the article at Store B?
5. An employee earning $16,000.00 yearly is offered a choice between two increases, 25% or $4000.00. Which increase would be the better choice?
6. A child and an adult sat at opposite ends of a balanced see-saw. The adult is one metre from the point of support. The child's distance from the

point of support is four times greater than the adult's distance. What percent of the total length of the see-saw does the child's distance represent?

7. John bought a stero console for $240 by paying 30% down and the remainder in 12 monthly payments. Find the amount of each payment?

8. In a certain city with a population of 50000, 60% of the total population speak English only, 24% speak French only, 7% speak both English and French. The remainder of the population speak languages other than French and English. How many people speak French only?

9. A car travelled 199 km on 25 litres of gasoline. At that rate, would 200 litres of gasoline be sufficient for a 1700 km trip?

10. If car A has a radiator capacity of 12 litres while car B has a capacity of 16 litres. The radiator in both cars contains a mixture of antifreeze and water. If car A has 5 litres of antifreeze while car B has 7 litres, which car has the greater percentage of antifreeze in its radiator?

Form B

1. An Admiral refridgerator sells for \$349.98 at store A and \$702.98 at store B. Store B is offering a 48% reduction in all sales. Which store offers the best buy?

2. Two planes leave cities which are 2500 km apart. Plane A has a speed of 650 kph while plane B is travelling 24% slower than plane A. If the planes are travelling towards each other, how far apart will the planes be at the end of $1\frac{1}{2}$ hours?

3. At a plumbing supply store a roll of copper pipe 28.2 metres long sells for \$44.98. If John bought 15% of the roll of copper pipe, how many metres did he buy?

4. At garage A a car sells for \$9989. Garage B is offering the same type car for \$10102. Garage A is reducing the price of the car by 11%, but garage B is offering a 15% discount. How much would a person pay for the car at garage B?

5. An employee earning 19998 is offered a choice between two increases, 24% or \$6200. Which increase is the better choice?

6. A ship's chain had been in storage for several years. When it was uncoiled it was discovered that a portion of it was rust covered. The rust covered section

ended 50 links from one end. The non-rusted covered section was 4 times greater than the rust-covered section. What percent of the chain was not rust covered?

7. John bought a car selling for $10229 by paying 25% down and the remainder in 36 monthly payments. Find the amount of each payment?

8. In a certain city with a population of 102000, 65.2% of the total population speak English only; 29.4% speak French only; 9.7% speak both English and French. The remainder of the population speak languages other than French and English. How many people speak English only?

9. Plane A travelled 2988 km. on 74.8 L of aviation fuel. At that rate would 160 L of aviation fuel be sufficient for a 13000 km trip?

10. Two oil tanks both contain a mixture of furnace oil and stove oil. Tank A has a capacity of 1200 L and contains 500 L of stove oil. Tank B has a capacity of 1600 L and contains 700 L of stove oil. Which tank has the greater percentage of stove oil?

APPENDIX B

Samples of Calculator Orientation Unit Activities

Activity 1

Write the key entry sequence for each of the following operations.

Function		Example	Key Entry Sequence
Addition	a.	29 + 42	______________
	b.	32.1 + 49.7	______________
Subtraction	a.	29 - 15	______________
	b.	25 - 60	______________
Multiplication	a.	25 x 6	______________
	b	39 x 9.02	______________
Division	a.	144 ÷ 12	______________
	b.	129 ÷ 82	______________
Percentage	a.	25 x 2%	______________
	b.	25 x .02	______________

*This exercise will be done together with the class. The teacher asks individual students to explain the key entry sequence for each item. The teacher writes that sequence on the board, while the student fills it in on the worksheet.

(Adapted from Calcu-Math Activities, p. 7)

Activity 2

Work with a friend taking turns using the calculator, the other his head or paper and pencil. You decide when the calculator is most beneficial.

Examples	Head	Paper/Pencil	Calculator
a. 6 + 8	___	___	___
b. 297 - 89	___	___	___
c. 93 x 80	___	___	___
d. 4212 ÷ 6	___	___	___
e. 1200 ÷ 10 x 10 - 29	___	___	___

(Adapted from Calcu-Math Activities, p. 8)

This activity is followed by a discussion where students relate their personal experience on when or when not to use the calculator. The teacher selects several student examples on when the calculator proved more beneficial than the other methods.

Fun Activities

Use your calculator to solve the following puzzles.

1. Enter a figure which is twice your age, add 5; multiply by 50; add the number of students in your class; subtract the number of days in a year (365); add 115; divide by 100 and press = key. What is significant about the number to the right of the decimal point; to the left?
   (Calcu-Math Puzzles, p. 10)

2. Write down any number of seven or less digits and enter it in the calculator. Add to it the next higher number in the sequence; add 9; divide by 2 and subtract the original number. Press = . The answer will always be the same. How come? Explain your answer.
   (Calcu-Math Puzzles, p. 10)

3. Write down a number of not more than six digits and enter it on the calculator; add 25, multiply by 2, subtract 4, divide by 2, subtract the original and press = . The answer will always be 23. How come? Use an equation to explain your answer.
   (Calcu-Math Puzzles, p. 11)

4. Write down a number of 4 digits and enter it; multiply by 2, add 4; multiply by 5, and add 12, multiply by 10; subtract 320 and press = . Drop the zeroes from the end of the answer and what do you have?
(Calcu-Math Puzzles, p. 11)

5. Write down any 3 digit number in which the digits are all the same (such as 999) but don't enter it in your calculator. Find the sum of the 3 digits, enter it in the calculator and multiply by 37. What do you get? How does the answer relate to the original number? Try some other numbers to see what happens.
(Calcu-Math Puzzles, p. 11)

Calculator Workbox

Activity 2

Which comes first?

Be careful when different mathematical operations are used in a calculation: the order you perform them in will often affect the result. On many machines, the calculation 3.7 + (5.8 x 7.2) must be entered as 5.8 x 7.2 + 3.7 in order to arrive at the correct answer of 45.46. Sometimes you have to "translate" the problem for the machine, and think about what you are "really" doing. For example, $\frac{24}{2 \times 6}$ must be input as 24 ÷ 2 ÷ 6 = 2, not as 24 ÷ 2 x 6 = 72. (Try it and see!) Think carefully about the operations and their correct order as you evaluate the problems. It pays to estimate your answer first! Give your answers to one decimal place.

A $76 + (5.2 \times 9.3)$

B $\dfrac{76}{5.2 \times 9.3}$

C $\dfrac{76 \times 3.5}{5.2 \times 9.3}$

D $\dfrac{76 + 9.3}{5.2}$

E $\dfrac{76}{5.2 + 9.3}$

F $\dfrac{76 \times 3.5}{5.2 + 9.3}$

Activity 54

Percent

A A block of metal alloy is 34.7% copper, 43.3% aluminum and the rest is nickel. If the block has a mass of 17.6 kg, how much of each metal is present?

B A ball, when dropped, will bounce back to 73% of the height from which it falls. The ball is dropped from a height of 7.0 m; find the height at the top of:

(i) the first bounce

(ii) the fifth bounce

C Ty Cobb holds the major league baseball record career average with 4191 hits in 11429 times at bat. What percentage of his "at bats" did he get a hit?

D Cobb also holds the career RBI record. If 53.54% of his hits scored a run, how many RBI's is he credited with?

Activity 58

Depreciation

A A car is a good example of an asset which drops in value during its lifetime. Each year, the resale value is less: the car is said to depreciate in value. A car costing $5645 depreciates each of the first five years according to the schedule shown.

Year	Percent depreciation
1	23%
2	19%
3	16%
4	14%
5	13%

What is the value of the car at the end of five years?

B A machine was purchased for $44175; it was expected to have a life of 15 years and be sold for scrap for $380 at that time. If the depreciation is the same amount each year find the annual depreciation.

Activity 75

Playing With Numbers

A Select a one-digit number, multiply it by 9, multiply the result by 12345679.

Select a second one-digit number, multiply it by 9, then the result by 12345679.

Can you predict the result before you select a third one-digit number?

B 1 Select 3 consecutive whole numbers.

2 Find the product of the 3 numbers.

3 Divide the product by 6.

C 1 Select the middle number of B-1.

2 Cube the number.

3 Subtract the number in 1 from the result in 2.

4 Divide the result in 3 by 6.

Compare the answers for B-3 and C-4.

APPENDIX C

Instructional Unit: Objectives and Sample Lessons

Instructional Unit

Objectives

After completion of the instructional unit, the student will be able to:

1. (a) state problems fluently;

   (b) point out the principal parts of the problem, the unknown, the data and the condition;

   (c) use given data systematically to arrive at a solution;

2. (a) identify essential data in a problem;

   (b) identify irrelevant data in a problem;

   (c) discard irrelevant data from problems containing it;

3. (a) identify problems in which drawing a diagram is useful;

   (b) represent problems diagramatically;

   (c) use diagrams to solve problems and verify solutions to others;

4. (a) recall related concepts from past experiences with problem solving;

   (b) recall related problems from past experiences with problem solving;

5. (a) state whether given data in a problem is in terms of today's standards. For example, dollars measurement;

   (b) re-write given problems in terms of today's standards;

6. (a) estimate answers to given problems;

   (b) identify reasonable answers to given problems;

7. (a) make a routine check of steps used in solving a problem;

   (b) evaluate the solution to the problem in terms of the procedure used;

8. Use the calculator in performing computations for a given problem.

Lesson I

Part One:

Objective: The student will be able to solve problems involving finding percent of a number.

Model: Problem 1

Suppose you were given an exam containing 30 short answer questions. If you got 40% of these questions correct, how many questions did you answer correctly?

The teacher presents this problem on the chalkboard and discussed the problem with the class by asking a series of questions and discussing responses.

Q1. What are we asked to find?

Q2. Could you list your given information?

Q3. Could you recall a related problem to this one? (i.e., one that is similar)

Q4. If so, do you recall the method used for solving such a problem?

Selection of Method

Student discussion on an appropriate method for solving the problem is noted and discussed.

The teacher outlines two methods:

(i) $n = .40 \times 30$

(ii) $\frac{40}{100} = \frac{X}{30}$

How to Perform the Method

(i) mentally

(ii) paper and pencil

(iii) calculator

This aspect of how to perform the computation is briefly discussed, reiterating previous discussion from the calculator orientation unit.

Checking Reasonableness:

(i) Placement of decimal is discussed.

(ii) Since 40% of 30 is what we had to find, 50% is one-half of 30 or 15, so 12 would be reasonable.

Evaluation of Results:

The teacher stresses that it is important that students evaluate their result by:

(i) checking method used in terms of data.

(ii) checking computational procedure or procedures.

(iii) checking for reasonableness in terms of given data.

Model: Problem 2

A block of metal is 34% copper, 48% aluminum and the rest is nickel. If the block has a mass of 16.2 kg. find the amount of aluminum present?

The teacher stresses the importance of reading the problem carefully. The teacher reads the problem and asks students:

Q1. What information is given?

Q2. What are we asked to find?

The teacher emphasizes that re-reading is often essential to understanding the problem.

Q3. In terms of what we are asked to find is there any given information that is not needed to solve the problem?

The teacher stresses that:

(1) Irrelevant data is often found in word problems.

(2) The identification of such data often reduces the amount of tedious computation that a person may encounter if such data is not identified.

Following the identification of the irrelevant data, the teacher points out that such data may be discarded completely for purposes of solving the problem.

After irrelevant data has been identified and discarded the next phase in the discussion of the problem involves selection of method. Here, for this particular problem the teacher stresses:

(1) Estimation: Since we now know that we must know that we must find 48% of 16.2 we can estimate our answer. 48% is close to 50% or ½ of 16.2. Thus, 8.1 would be a close estimate.

The teacher emphasizes that estimation is very important in problem solving for it gives an idea of what the answer should be approximately.

Computation of the Solution:

16.2 x .48 = n

<u>Questioning Reasonableness</u>:

i.e., Based on the estimated answer.

<u>Checking Results</u>:

(i) Procedure

(ii) Computation

Lesson I

Part Two:

Objective: The student will be able to find what percent one number is of another.

Problem 2:

In a large high school there are 320 Grade 12 students, 255 Grade 11's, 346 Grade 10's, and 564 Grade 9 students. In a survey it was determined that 962 of the total 1485 students planned to continue education after high school. What percent of the students plan to complete their education?

This problem is presented and first the teacher asks students to identify given data and what they are asked to find. After doing so, the teacher asks:

> Is there any data in this problem that is not necessary in answering the question asked?
>
> ... Responses are discussed and the students are reminded that the identification of irrelevant data is an important step in solving the problem. (Recall discussion from Lesson I.)

After discarding the irrelevant data the teacher points out that we are trying to find what percent 962 is of the total 1485 students. The teacher then asks:

Q4. Could you give an estimate of approximately what the answer would be?

If a response is given, that response is discussed if not, the teacher explains.

We have the ratio $\frac{962}{1485}$. We could say 962 is close to 1000. While 1485 is close to 1500. Thus we have $\frac{1000}{1500}$ or 2/3. You recall that 2/3 is 66 2/3%.

The teacher emphasizes the value of estimating answers prior to making the actual computation. Most importantly, here, the teacher points out that it is one way we can check reasonableness having computed the real answer.

Students are asked to now perform the computation with the use of the calculator.

$$\frac{962}{1485} = \frac{X}{100} \quad \underline{\underline{\text{or}}} \quad X = 96200 \div 1485$$

Write answer on chalkboard.

Q. Is the answer reasonable in terms of the estimate?

Q. How might we check our result to make certain we are correct?

The teacher allows the students to discuss methods of evaluation and reiterates the importance of evaluating.

Lesson I

Part Two:

Model: Problem 4

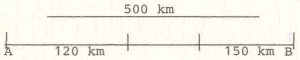
Two cars leave cities which are 500 km apart. Car A is travelling at a speed of 80 kph while car B has a speed of 100 kph. After $1\frac{1}{2}$ hours of travelling toward each other what percentage of the distance remains to be covered?

Q1. What information are we given?

Q2. What are we asked to find?

Having identified the given data and what is asked the teacher points out that a diagram could be helpful in putting this problem in perspective.

The teacher draws a line to represent the total distance between the cities.



The teacher rereads the problem to the class and points out that car A has a speed of 80 kph. Thus in $1\frac{1}{2}$ hours the car will have travelled 120 km. This is denoted on the diagram. Car B is travelling 100 kph. Thus in 1 1/2 hours it will have travelled 150 km. Thus we can see that 120 + 150 gives 270 km covered. Thus we must subtract from 500 to find the remaining distance. This yields

230 km. left to be covered.

We are not looking for what percent 230 is of 500. We could say $\frac{230}{500} = \frac{X}{100}$.

Estimating we know that we have a little under 50% of the distance remaining.

Calculating we have X = 23000 ÷ 500 <u>or</u> 46%.

<u>Evaluation of Results</u>:

The teacher illustrates evaluation of the results obtained by a couple of methods.

(i) Checking results of parts of the condition.

(ii) Checking computations performed.

Lesson I

<u>Part Three</u>:

Objective: The student will be able to find percent of what number.

Model: Problem 4

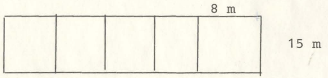
A man wishes to sod a rectangular lawn. Due to financial difficulty he is unable to sod all the lawn at the same time. He decides to divide the lawn up into 5 lengthwise rectangular strips measuring 15 m x 8 m and sod one strip each spring. The cost of sodding one strip is \$388. (i) What % of the lawn will be sodded each year? (ii) What is the total area of the lawn?

The teacher reads the problem and selects a student to read it. The following questions are asked:

1. List the given data.
2. What are we asked to find?
3. Is there any data given in the problem that is not needed to solve the problem.

Based on the response to question 3, the teacher points out that "the cost of sodding one strip" is not needed and thus, may be discarded from the problem.

Next, the teacher emphasizes that the use of a diagram in this problem would be helpful in solving the problem. The teacher represents the problem diagramatically on the board.



The teacher points out that the total rectangular lawn represents 100%. Since the lawn is divided into 5 equal rectangular strips, then each strip represents 20% of the total area, which is the answer to part (1) of the problem.

Next, the teacher asks students if they recall how to calculate the area of one strip. .... Each strip is rectangular thus we use the formula for the area of a rectangle.

$$\begin{aligned} A &= 1 \times w \\ &= 15 \times 8 \\ &= 120 \text{ m}^2 \end{aligned}$$

We are asked to find the total area.

Q. If the area of one strip is 120 m^2, how might we calculate the total area?

Student responses are discussed and the teacher points out that;

(i) We could simply say 120 x 5 or 600m.

or (ii) We could say 120 is 20% of what number?

Thus, $120 = 120 \times n$

$n = 120 \div .20$

<u>or</u> 600 m ?

Evaluation of Problem:

(i) The teacher checks through the data by reading the problem to certify the diagram representation.

(ii) Check area calculation.

(iii) Check total area calculation.