DECYCLING AND DOMINATING CUBES AND GRIDS

YUBO ZOU

DECYCLING AND DOMINATING CUBES AND GRIDS

by

©Yubo Zou

A thesis submitted to the

School of Graduate Studies

in partial fulfilment of the

requirements for the degree of

Doctor of Philosophy

Department of Mathematics and Statistics

Memorial University of Newfoundland

March 2006 Submitted

St. John's Newfoundland Canada

Your file Votre référence
ISBN: 978-0-494-30443-3
Our file Notre référence
ISBN: 978-0-494-30443-3

Abstract

The decycling number of a graph G, denoted $\nabla(G)$, is defined to be the minimum number of vertices which can be removed from G so that the resultant graph contains no cycle. A vertex subset D is a dominating set of a graph G if each vertex in $V(G)$ is either in D or is adjacent to a vertex in D. The domination number $\gamma(G)$ is the minimum cardinality of a dominating set of G. Several variations of domination are formed by combining domination with other graph theoretical properties, giving rise to independent domination, perfect domination, total domination, distance-k domination, connected domination, clique domination and cycle domination.

Since these invariants are normally NP-hard to evaluate, it is useful to have classes of graphs for which their values are known. We focus on studying some popular interconnection topologies, such as the Cartesian product of two cycles, $C_m \square C_n$, the Fibonacci cube of order n, Γ_n, and the n-dimensional hypercube, Q_n.

The exact value of the decycling number of $C_m \square C_n$ for all m and n, and lower and upper bounds of the decycling number for the n-dimensional Fibonacci cubes are obtained. Subsequently, we turn to investigate the domination number of the n-dimensional hypercube. An improved lower bound for the domination number

and an improved upper bound for the independent domination number of Q_n are established. The technique we develop is then extended to establish a lower bound on the distance-k domination number.

Acknowledgements

I would like to express my great appreciation to my supervisor, Dr. David A. Pike, for his careful guidance, patience, insight and invaluable suggestions throughout my PH. D. program. Without his help, this work would not have been possible. My deepest thanks to him is not only for his help of my academic research and kind encouragement, but also for his great effort to help me hone my writing skills. I very much appreciate his financial support for my attendance at some great international conferences and workshops.

Many thanks go to Dr. Peter Booth and Dr. Xiaoqiang Zhao as members of my supervisor committee. I am sincerely grateful to Dr. Nabil Shalaby for teaching me the courses of combinatorial design and enumeration. I also thank them for their encouragement and nice advice.

I would like to take this opportunity to thank the School of Graduate Studies at Memorial University of Newfoundland for supporting me with the graduate fellowship. I am also grateful to the Department of Mathematics and Statistics, Professors Herb Gaskill, Bruce Watson and Chris Radford, the past and present department heads, for providing me with the necessary facilities for research and teaching assistant fellowship. Thanks go to all staff members at the department for their help.

Thanks to Mr. Chao Zhong, Dr. Lin Wang, Dr. Dashun Xu for all their help encouragement when I was new at Memorial University. Thanks also to Ms. Andrea C. Burgess and her family for their kindness and helping me to improve my English.

I deeply thank my parents and my brothers. Their understanding and emotional support have been inspiring me to complete my studies.

Finally, a special thanks to my wife Jiajia, her strength, patience and encouragement are most important for me, specifically when I encounter difficulties. She gives me helpful suggestions and helps me to overcome those frustrations.

Yubo Zou

Table of Contents

List of Tables

List of Figures

List of Appendices

Chapter 1

Introduction

In this chapter, we present some basic definitions and terminology of graph theory. Subsequently, we introduce some popular interconnection topologies, such as grids, Cartesian products of cycles, hypercube and Fibonacci cube of order n. Finally, we give the outline of the organization of the chapters of this thesis.

1.1 Basic Terminology

This section introduces the basic and important graph theory definitions, notation and labelling.

A *graph* G is a collection of *vertices* $\big($denoted by $V(G)\big)$ and *edges* $\big($denoted by $E(G)\big)$ connecting some pairs of vertices. All graphs we consider are assumed to be finite, undirected and simple, i.e., without loops and multiple edges. For a graph G we use $p(G)$ and $q(G)$ to denote its number of vertices and edges respectively, using p and q for abbreviation if there is no confusion.

If $e = uv$ is an edge of G, then u and v are the two *endpoints* of e, are *adjacent* and are *neighbours*. For a vertex v of G, $\deg_G(v)$ denotes the *degree* of v, i.e., the number of neighbours of v in G, using $\deg(v)$ for abbreviation. $\Delta(G)$ and $\delta(G)$ are, respectively, the maximum and minimum degree of a vertex of the graph G. We say that a graph is *regular* if each vertex of G is of the same degree. The *neighbourhood* of v, written $N_G(v)$ or $N(v)$, is the set of vertices adjacent to v.

The *complement* $\overline{G}$ of a graph G is the graph with vertex set $V(G)$ defined by $uv \in E(\overline{G})$ if and only if $uv \notin E(G)$. A *clique* in a graph is a set of pairwise adjacent vertices.

A *path* is a non-empty graph $P = (V, E)$ of the form

$$V = \{v_1, v_2 \ldots, v_n\} \qquad E = \{v_1v_2, v_2v_3 \ldots, v_{n-1}v_n\},$$

where the v_i are all distinct. The vertices v_1 and v_n are *linked* by P and are called its *ends*. The number of edges of a path is its *length*. A *cycle* is a non-empty graph $C = (V, E)$ of the form

$$V = \{v_1, v_2 \ldots, v_n\} \qquad E = \{v_1v_2, v_2v_3 \ldots, v_{n-1}v_n, v_nv_1\}.$$

The path and cycle with n vertices are denoted P_n and C_n, respectively. Noticing that a cycle is a two regular graph.

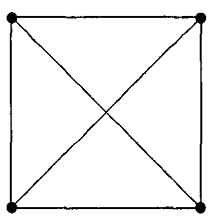
A *subgraph* of a graph G is a graph H such that $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$ and the assignment of endpoints to edges in H is the same as in G. We then write $H \subseteq G$. For a vertex subset S of G, the *induced subgraph* $G[S]$ is the graph having S as its vertex set and all edges whose endpoints are both in S.

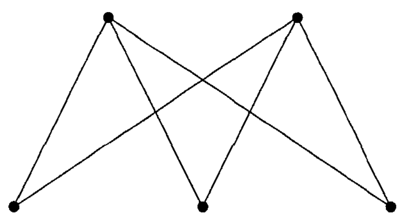
A graph G is *connected* if each pair of vertices in G belongs to a path; otherwise, G is *disconnected*. The *components* of a graph G are its maximal connected subgraphs.

The *girth* of a graph with a cycle is the length of its shortest cycle. A graph without a cycle has infinite girth.

A graph with no cycle is *acyclic.* A *forest* is an acyclic graph. A *tree* is a connected acyclic graph and a *pendant vertex* (or *leaf*) is a vertex of degree 1. A *complete graph* is a simple graph whose vertices are pairwise adjacent; a complete graph with p vertices is denoted K_p. A *bipartite graph* is a set of vertices partitioned into two disjoint sets such that each edge connects vertices in different parts. A *complete bipartite graph* is a bipartite graph such that every pair of vertices in the two sets are adjacent; if there are r and s vertices in the two sets, the complete bipartite graph is denoted $K_{r,s}$, where r and s are positive integers.

K_4





$K_{2,3}$

Figure 1.1: The graphs K_4 and $K_{2,3}$.

An *independent set* (or *stable set*) in a graph is a set of pairwise nonadjacent vertices. The *independence number* $\alpha(G)$ of a graph G is the maximum number of vertices in an independent set. A *vertex cover* of a graph G is a vertex subset $Q \subseteq V(G)$ that contains at least one endpoint of every edge. The *covering number* $\beta(G)$ is the minimum number of vertices in a set that meets every edge. These two parameters are connected by the elementary but useful result that $\alpha(G) + \beta(G) = p(G)$.

An *isomorphism* from a simple graph G to a simple graph H is a bijection $f : V(G) \to V(H)$ such that $uv \in E(G)$ if and only if $f(u)f(v) \in E(H)$. We say that G is *isomorphic* to H, written $G \cong H$, if there is an isomorphism from G to H. Two graphs are *homeomorphic* if they can be made isomorphic by inserting new vertices of degree 2 into edges.

For additional graph theory terminology and notational conventions we follow [56].

1.2 Interconnection Topology

Traditionally, problems are solved by a series of instructions, executed one after the other by the computer. Only one instruction may be executed at any moment in time. With the progress of the multi-processor system and parallel computing algorithm, now, it is possible to deal with problems of huge scale, since parallel processors execute more than one instruction simultaneously. Where each processor has its own individual data stream, it is usually necessary to communicate data between processors. One way for processors to communicate data is to use a shared memory and shared variables. However this is unrealistic for large numbers of processors. A more realistic assumption is that each processor has its own private memory and data communication takes place using message passing via an *interconnection topology* (*network topology*).

The interconnection topology plays a central role in determining the overall performance of a multi-processor system. If the network cannot provide adequate performance, for a particular application, processors will frequently be forced to wait for data to arrive. Subsequently, we introduce some popular and useful interconnection topologies.

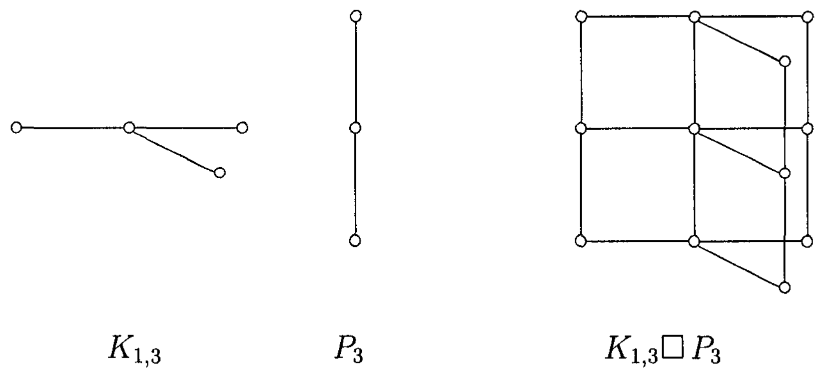


Figure 1.2: An example of graph Cartesian product $K_{1,3}\Box P_3$.

First of all, we want to introduce the *Cartesian product* of graphs G and H, written $G\Box H$, being the graph with vertex set $V(G) \times V(H)$ specified by putting (u, v) adjacent to (u', v') if and only if $u = u'$ and $vv' \in E(H)$, or $v = v'$ and $uu' \in E(G)$. The Cartesian product operation is commutative, i.e., $G\Box H \cong H\Box G$. Figure 1.2 is an example of $K_{1,3}\Box P_3$.

The most powerful interconnection topology is the fully connected network which is the complete graph introduced in Section 1.1. For a fully connected network with n processors, any processor can exchange data with any of the other $n - 1$ processors and there are $n(n-1)/2$ connections between these n processors. Even though this is the best network, in the sense that it has the highest number of connections per processors, to construct such an interconnection topology is infeasible when n is large.

Therefore some form of limited interconnection network must be used.

A *grid graph*, also called a lattice graph, is the Cartesian product of paths on m and n vertices, written $P_m \Box P_n$. A grid graph $P_m \Box P_n$ has mn vertices and $(m-1)n + (n-1)m = 2mn - m - n$ edges (see Figure 1.3 as an example of $P_4 \Box P_6$). Among the mn vertices, $(m-2)(n-2) = mn - 2m - 2n + 4$ vertices are degree four (known as interior vertices), $2m - 4 + 2n - 4$ vertices are degree three and four vertices are degree two.

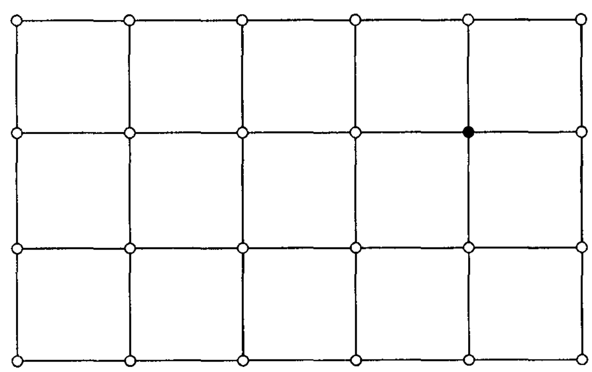


Figure 1.3: An example of grid graph $P_4 \Box P_6$.

The concept of the grid graph can be extended to the Cartesian product of two cycles on m and n vertices, written $C_m \Box C_n$, which has mn vertices and each vertex has degree four.

Following the notation in [54], it will be useful to have a standard labelling for the vertex set for the Cartesian product of graphs $G \Box H$, and we choose one that corresponds to matrix notation: the ith vertex in the jth copy of G will be denoted $v_{i,j}$. Carrying the matrix analogy further, we sometimes speak of the copies of G and

H as the columns and rows, respectively, of $G \Box H$. This idea is significantly useful when we investigate the Cartesian products of two paths or cycles. For instance, the vertex labelled by "•" in Figure 1.3 is denoted by $v_{2,5}$.

Subsequently, we want to introduce a frequently used interconnection topology, the *n-dimensional hypercube* or hypercube of order n, Q_n. There are two standard ways to define the n-dimensional hypercube. One is defined recursively as the Cartesian product $Q_n = K_2 \Box Q_{n-1}$, where $Q_1 = K_2$; another, and also a convenient way to define Q_n, is to treat its vertex set as the set of all binary words of length n, two vertices being adjacent if and only if the Hamming distance between them is exactly 1, where the Hamming distance of two binary words is the number of coordinates that they differ. In Figure 1.4, hypercubes of order $n = 1, 2, 3$ are presented.

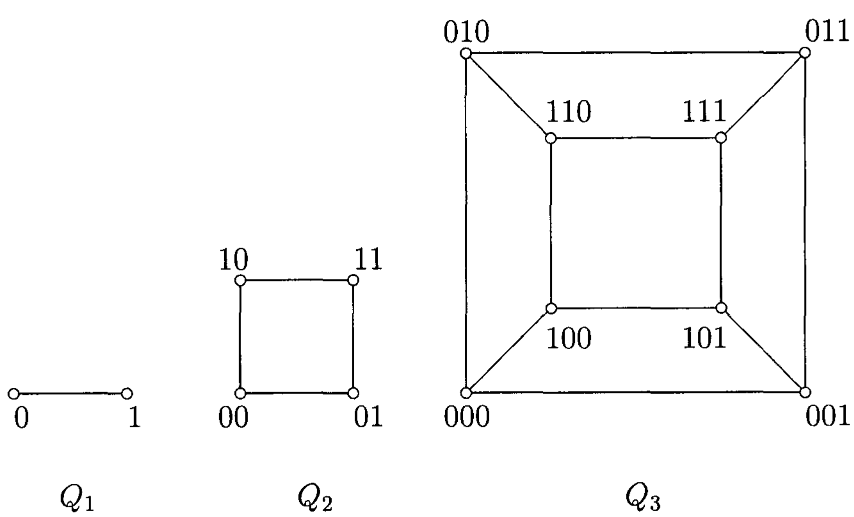


Figure 1.4: The n-dimensional hypercubes Q_n for $n = 1, 2, 3$.

The hypercube is a popular interconnection topology for parallel processing since it has many appealing properties. However, the number of vertices in the hypercube must be a power of 2, which restricts the permissible size of the hypercube. In 1993, Hsu [29] introduced a new interconnection topology — Fibonacci cubes, which have a slower growth rate than hypercubes and can be embedded in hypercubes.

Recall that the Fibonacci numbers form a sequence of positive integers $\{f_n\}_{n=0}^{\infty}$ where $f_0 = 1$, $f_1 = 2$ and $f_n = f_{n-1} + f_{n-2}$ for $n \geqslant 2$. Given that i is a non-negative integer such that $i \leqslant f_n - 1$, then i can be uniquely represented as a sum of distinct non-consecutive Fibonacci numbers (*Zeckendorf's Theorem* [59]) in the form $i = \sum_{j=0}^{n-1} b_j f_j$ where b_j is either 0 or 1, for $0 \leqslant j \leqslant n-1$ with the condition $b_j b_{j+1} = 0$ for $0 \leqslant j < n-1$. The sequence $[b_{n-1}, \ldots, b_1, b_0]$ is called the order-n Fibonacci code of i, and uniquely determines i. For example, $i = 11 = 8 + 3 = f_4 + f_2$ has Fibonacci code 10100.

The Fibonacci cube Γ_n of order n (or n-dimensional Fibonacci cube) is the graph (V_n, E_n) where $V_n = \{0, 1, \ldots, f_n - 1\}$ and two vertices i and j are adjacent if and only if their Fibonacci codes differ in exactly one bit. The Fibonacci cubes for the first few values of n are depicted in Figure 1.5.

1.3 Outline of Thesis

In this thesis, we will deal with two problems in graph theory — decycling and dominating. Chapter 2 discusses the history and development of the decycling problem, and some new results regarding the decycling of graph expansions.

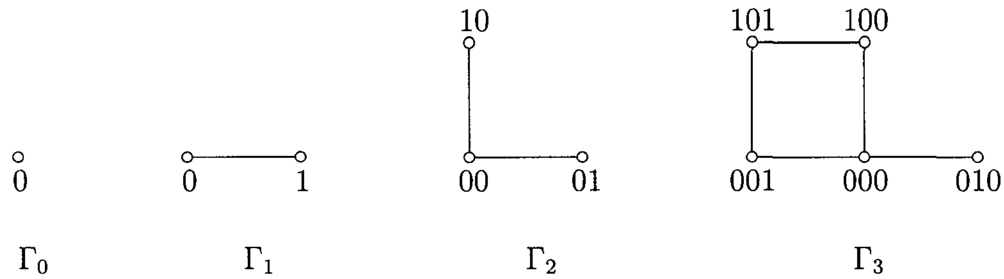


Figure 1.5: The n-dimensional Fibonacci cubes Γ_n for $n = 0, 1, 2, 3$.

Chapter 3 focuses on decycling the Cartesian product of two cycles. We begin by investigating the general lower bound for the decycling number of Cartesian product of two cycles, $C_m \square C_n$ which is a 4-regular graph. An improved lower bound is obtained, and from this bound, the exact value of the decycling number for the Cartesian product of two cycles and decycling sets for all possible cases are also obtained. Moreover, we find a vertex subset that yields a maximum induced tree in Cartesian product of two cycles. Finally, we apply our main results to get an upper bound for the decycling number of Cartesian product of two paths.

Chapter 4 turns to an investigation of the decycling number for the Fibonacci cube of order n. We first review the development for the graph properties of the Fibonacci cubes, and obtain a lower bound for the decycling number of the Fibonacci cube of order n. Then we investigate the exact value of the decycling number for $n \leqslant 7$, and describe a computer search heuristic to find a decycling set for $n \leqslant 9$. Finally, we obtain an upper bound based on the independence number.

Chapter 5 discusses domination numbers for the cubes (Fibonacci cubes and hypercubes). First, we investigate the standard domination number for the n-dimensional Fibonacci cube for $n \leqslant 8$. Then, we turn to study various domination parameters including independent, perfect and distance-k domination number for the hypercubes.

Chapter 6 concludes the thesis and suggests future research.

Chapter 2

Decycling Number

In this chapter, some results regarding the decycling number of graph expansion are presented. We begin with definition of some relevant terms. Subsequently, we give a brief review of the progress on the decycling problem, and follow with discussions on decycling graphs.

2.1 Definition

To design a parallel program by using multiple-processor system and using interconnection topology and exchanging data between processors, sometimes, the following circumstance might happen: two or more processes are waiting for each other to complete before proceeding. The result is that both processes hang and we call it a *deadlock*. To prevent the deadlock, we usually have two approaches, one is to remove some processes, another is to eliminate some connections between processes. For optimization, we want to find the minimum number of processes or connections between processes being deleted. In graph theory, the problem can be reformulated to remove the minimum vertex or edge subset such that the resultant graph is acyclic.

The minimum number of edges whose removal eliminates all cycles in a given graph is already known as the *cycle rank* or sometimes called *Betti number.* This parameter has a simple solution: $b(G) = q - p + \omega(G)$ (see [57, p. 46]), where $\omega(G)$ denotes the number of connected components in graph G. However, the problem of destroying all cycles from a graph by means of deleting vertices (this problem goes back at least to the work of Kirchhoff [34] on spanning trees) does not have a simple solution and the associated decision problem was proved to be *NP*-complete (see [33]).

If G is a graph, then any vertex subset S for which $G - S$ contains no cycles is a *decycling set.* The size of a minimum decycling set S in a graph G is the *decycling number* of G and will be denoted by $\nabla(G)$. A decycling set of this order is called a ∇-set. In the literature decycling sets are sometimes also called *feedback vertex sets,* see [15, 20, 21, 35, 36, 37, 40, 51, 52, 53].

The problem of finding the decycling number can be reformulated as the problem of determining the maximum size $f(G)$ of a vertex subset F that induces a forest, called the *maximum induced forest* of G. Moreover, following Erdős, Saks and Sós [19], we define the size $t(G)$ of a maximum subset T of $V(G)$ that would induce a tree. They also presented the problem of finding the *path number,* i.e., the maximum size of a vertex subset that would induce a path.

It is trivial that $\nabla(G) = 0$ if and only if G is a forest, and $\nabla(G) = 1$ if and only if G has at least one cycle and some vertex is on all of its cycles. Also, it is easy to see that $\nabla(K_{r,s}) = r - 1$ if $1 \leqslant r \leqslant s$, and

$$\nabla(K_p) = \begin{cases} p-2 & \text{if } p \geq 3, \\ 0 & \text{if } p = 1 \text{ or } 2, \end{cases}$$

$$f(K_p) = t(K_p) = \begin{cases} 2 & \text{if } p \geq 2, \\ 1 & \text{if } p = 1. \end{cases}$$

2.2 History

In this section, we present a brief review of the progress on the problem of finding the decycling number, or equivalently finding the maximum induced forest, of graph.

2.2.1 Cubic Graphs

In 1974, Jaeger [30] considered the problem of finding a maximum induced forest in a cubic graph G and he got

$$f(G) \leqslant \left\lfloor \frac{3p-2}{4} \right\rfloor.$$

Bondy, Hopkins and Staton (see [10]) proved that for a connected graph with $\Delta(G) \leqslant 3$ and $p > 4$, then $f(G) \geqslant \left\lceil \frac{5p-2}{8} \right\rceil$. Moreover, if G is triangle-free, this bound was improved to $f(G) \geqslant \left\lceil \frac{2p-1}{3} \right\rceil$ [10]. Lu and Zheng [60] further obtained a sharp bound $f(G) \geqslant \frac{2p}{3}$ for connected triangle-free cubic graphs G where $p(G) \neq 8$.

For a connected cubic graph with girth g, Speckenmeyer proved in [52] that

$$\nabla(G) \leqslant \frac{g+1}{4g-2}p + \frac{g-1}{2g-1}.$$

This improved his earlier results:

$$\nabla(G) \leqslant \frac{3p}{8} + 1,$$

and

$$\nabla(G) \leqslant \frac{p}{4} + \frac{k(G)+1}{2},$$

in [51], where $k(G)$ denotes the maximal number of disjoint cycles in G.

Let $\mathcal{G}$ denote the family of cubic graphs obtained by taking cubic trees (trees in which each vertex has degree 1 or 3) and replacing each vertex of degree 3 by a triangle and also attaching a copy of K_4 with one subdivided edge at every vertex of degree 1. A sharp upper bound for the decycling number of cubic graphs has been obtained in [40]:

Theorem 2.2.1 ([40]) *Let G be a cubic graph with girth g. Then*

$$\nabla(G) \leqslant \frac{g}{4(g-1)}p + \frac{g-3}{2g-2}$$

if $G \notin \{K_4, Q_3, W\} \cup \mathcal{G}$ where W is the Wagner graph (see [18] *and* Figure 2.1).

If $g = 3$ and $G \in \mathcal{G}$, then $\nabla(G) = \dfrac{3p}{8} + \dfrac{1}{4}$.

Corollary 2.2.2 ([40]) *If $g \geqslant 3$ then $\nabla(G) \leqslant \dfrac{3p}{8}$ for $G \notin \{K_4\} \cup \mathcal{G}$. If G is a connected cubic graph with $g \geqslant 4$ and $G \neq Q_3$ or W, then $\nabla(G) \leqslant \dfrac{p}{3}$.*

Theorem 2.2.3 ([40]) *Let G be a connected graph of maximum degree 3. If $G \neq K_4$ then*

$$\nabla(G) \leqslant \left\lfloor \frac{q+1}{4} \right\rfloor.$$

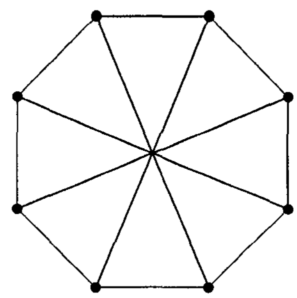


Figure 2.1: The Wagner graph.

The family $\mathcal{G}$ of graphs show the sharpness of this result.

In 1999, Li and Liu [35] presented a polynomial time algorithm to determine the decycling number of any cubic graph.

The following problems regarding the decycling number of cubic graphs are still open (presented in [7]):

Problem 2.2.4 *Which cubic graphs G satisfy $\nabla(G) = \left\lceil \frac{p+1}{2} \right\rceil$?*

Problem 2.2.5 *Which cubic planar graphs G satisfy $\nabla(G) = \left\lceil \frac{p+1}{2} \right\rceil$?*

2.2.2 Planar Graphs

D. West has prepared a detailed summary discussing induced forests in planar graphs at `http://www.math.uiuc.edu/~west/openp/planforest.html`, part of which we reiterate in this section. We begin by presenting three conjectures on finding the maximum induced forest of a planar graph.

Conjecture 2.2.6 ([5]) *Every planar graph has an induced forest with at least half of the vertices.*

Conjecture 2.2.7 ([3]) *Every bipartite planar graph has an induced forest with at least* $\frac{5}{8}$ *of the vertices.*

D. West in his summary also mentioned the following conjecture of Chappell:

Conjecture 2.2.8 *Every planar graph has an induced linear forest with more than* $\frac{4}{9}$ *of the vertices, where a linear forest is a forest in which every component is a path.*

Without using the Four Colour Theorem, Conjecture 2.2.6 would directly imply that every planar graph has an independent set with at least one-quarter of the vertices. Similarly, Conjecture 2.2.8 would imply that every planar graph has an independent set with at least 2/9 of the vertices (Albertson [4]).

The best known result on Conjecture 2.2.6 came from Borodin [12], in which he proved that every planar graph has a proper 5-colouring and every cycle uses at least three colours, from which he deduced that every planar graph has an induced forest with at least 2/5 of the vertices. Then, Akiyama and Watanabe [3] showed that Conjectures 2.2.6 and 2.2.7 are best possible by giving some examples in 1987.

In 1990, Hosono [28] proved that there is always an induced forest with at least 2/3 of the vertices for outerplanar graphs, and this is best possible. Subsequently, Borodin and Glebov [13] proved a stronger result that the vertices of an outerplanar graph with girth at least 5 can be partitioned into an independent set and another set that induces a forest, which answered Conjecture 2.2.6 for outerplanar graphs with girth at least 5.

Poh [50] proved that the vertex set of a planar graph can be partitioned into three sets inducing linear forests, which implied that every planar graph has an induced linear forest with at least 1/3 of the vertices. Three groups ([2, 14, 43]) showed independently that the vertex set of an outerplanar graph can be partitioned into two sets that induce linear forests, and hence there exists a linear forest of size at least half of the vertices of an outerplanar graph. Chappell conjectured that every outerplanar graph has an induced linear forest with more than 4/7 of the vertices and this was proved by Pelsmajer [45], who showed that every outerplanar graph with p vertices has an induced linear forest with at least $\left\lceil \frac{4p+2}{7} \right\rceil$ vertices and this bound is sharp.

2.2.3 Hypercubes

In 1997, Beineke and Vandell [9] investigated the decycling number of hypercubes and two dimensional grids. They gave the bounds $2^{n-1} - \frac{2^{n-1}-1}{n-1} \leqslant \nabla(Q_n) \leqslant \nabla(Q_{n-1}) + 2^{n-2}$. For $n \leqslant 8$, the exact value of $\nabla(Q_n)$ was obtained in [9]. Lower

Table 2.1: Decycling number of Q_n for $n \leqslant 8$.

n	1	2	3	4	5	6	7	8
$\nabla(Q_n)$	0	1	3	6	14	28	56	112

and upper bounds on $\nabla(Q_n)$ for $9 \leqslant n \leqslant 13$ were also obtained in [9], and these bounds were improved in [8]. In [21], the general upper bound was further improved to $\nabla(Q_n) \leqslant 2^{n-1} - \dfrac{2^{n-1}}{2(n-1)}$.

In 2003, Pike [46] made a connection between $\nabla(Q_n)$ and the size $A(n,4)$ of a maximum binary code of length n and minimum distance 4, and improved bounds $2^{n-1} - \dfrac{2^{n-1}-(n+1)}{n-1} \leqslant \nabla(Q_n) \leqslant 2^{n-1} - A(n,4)$ were also obtained. Moreover $\nabla(Q_n) = 2^{n-1} - A(n,4)$ if and only if there exists a minimum decycling set in Q_n that is also an independent set.

These results are listed in Table 2.2, where numbers inside "( )" are from [8], numbers inside "[ ]" are from [21], numbers inside "{ }" are from [46] and other numbers are from [9].

Table 2.2: Bounds on $\nabla(Q_n)$ for $9 \leqslant n \leqslant 13$.

Cubes	Lower bounds	Upper bounds
Q_9	224(225){226}	{236}[240](237)312
Q_{10}	448(456){457}	{472}[483](493)606
Q_{11}	896(922){923}	{952}[972](1005)1184
Q_{12}	1782(1862){1863}	{1904}[1954](2029)2224
Q_{13}	3584(3755){3756}	{3840}[3925](4077)4680

2.2.4 Grids

In 1997, Beineke and Vandell [9] also studied the decycling number of the grid graphs $P_m\Box P_n$. They obtained a general lower bound $\nabla(P_m\Box P_n) \geqslant \left\lfloor \dfrac{mn-m-n+2}{3} \right\rfloor$, and the exact value of $\nabla(P_m\Box P_n)$ for $2 \leqslant m \leqslant 7$ and $n \geqslant 4$ (see Table 2.3). In their paper, they also obtained a general result on determining the decycling number for $\nabla(P_m\Box P_n)$ where $m = 3r+1$ and $n \equiv 0 \pmod 2$,

$$\nabla(P_m\Box P_n) = rn - r + 1\,.$$

Theorem 2.2.9 ([8]) *Suppose S is any minimum decycling set of $P_m\Box P_n$ with*

$$\nabla(P_m\Box P_n) = \left\lceil \frac{mn-m-n+2}{3} \right\rceil .$$

Let

$$S' = \{v_{2i-1,2j-1} \;:\; v_{i,j} \in S\}$$

and

$$T = \{v_{ij} \;:\; i = 2,4,\ldots,2m-2;\; j = 2,4,\ldots,2n-2\}.$$

Then $S' \cup T$ is a minimum decycling set of $P_{2m-1}\Box P_{2n-1}$.

Theorem 2.2.9 is very useful in obtaining the following result which covers a general case other than the result in [9].

Corollary 2.2.10 ([8]) *For any positive integers r and s*

$$\nabla(P_{6r+1}\Box P_{4s-1}) = 8rs - 4r + 1.$$

Table 2.3: $\nabla(P_m \square P_n)$ for $2 \leqslant m \leqslant 7$ and $n \geqslant 4$.

m	$\nabla(P_m \square P_n)$
2	$\left\lfloor \frac{n}{2} \right\rfloor$
3	$\left\lfloor \frac{3n}{4} \right\rfloor$
4	n
5	$\left\lfloor \frac{3n}{2} \right\rfloor - \left\lfloor \frac{n}{8} \right\rfloor - 1$
6	$\left\lfloor \frac{5n}{3} \right\rfloor$
7	$2n - 1$

2.3 Decycling of Graph Expansions

Once a graph parameter has been studied for several classes of graphs, it is often interesting to investigate that parameter when these graphs are combined. In [9], Beineke and Vandell studied the decycling number for combinations of graphs, and several of their results are listed below:

1. For disjoint graphs G and H,

$$\nabla(G + H) = \nabla(G) + \nabla(H),$$

   where $G + H$ denotes the *sum* of graphs G and H with vertex set $V(G) \cup V(H)$ and edge set $E(G) \cup E(H)$.

2. If neither G nor H is a null graph, then

$$\nabla(G \vee H) = \min\{p(G) + \nabla(H),\ p(H) + \nabla(G)\}\,.$$

If G is not a null graph, then

$$\nabla(G \vee \overline{K}_n) = \min\{p(G) - 1, \nabla(G) + n\}\,.$$

Here $G \vee H$ denotes the *join* of graphs G and H having vertex set $V(G) \cup V(H)$ and edge set $E(G) \cup E(H) \cup \{uv \mid u \in V(G) \text{ and } v \in V(H)\}$.

3. For any graph G,

$$2\nabla(G) \leqslant \nabla(K_2 \square G) \leqslant \nabla(G) + \beta(G)\,.$$

In the remainder of this section, we present new results on the decycling number of the graph expansions. First of all, refering to the Meredith graph (see [11, Appendix III] and [42]) which is obtained from the Petersen graph following some expansion rule and is a 4-regular 4-connected non-Hamiltonian graph, we define a first expansion. For a given graph G, and $v \in V(G)$, G_v^B denotes the graph obtained from G by replacing a vertex v of degree d by a $K_{d,d-1}$ then arbitrarily pair the d neighbours of v in G with the d vertices in A_v, A_v and A'_v denote the independent sets of size d and $d-1$ in $K_{d,d-1}$ respectively. In general, we can define G_W^B by applying a similar procedure to each vertex in W, where $W \subseteq V(G)$ (see Figure 2.2 for an example).

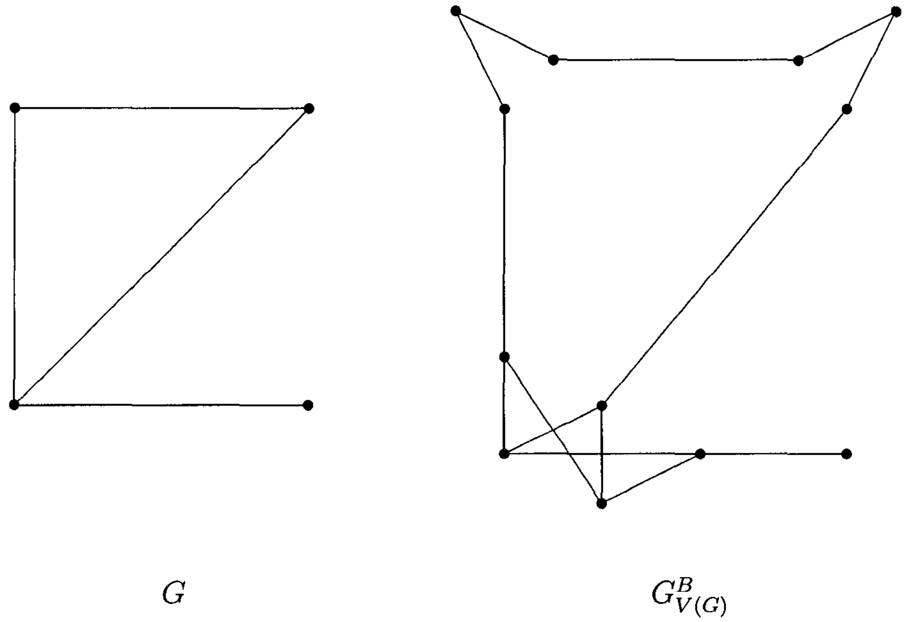


Figure 2.2: An example of $G^B_{V(G)}$.

To study the decycling number of $G^B_{V(G)}$, we need to consider another graph expansion, which we introduce here. For a given graph G, and $v \in V(G)$, suppose $N_G(v) = \{x_1, x_2, \ldots, x_{d_v}\}$, where $d_v = \deg_G(v)$. G^S_v denotes the graph obtained from G by replacing v by a star S_v with center v' and leaves $x'_1, x'_2, \ldots, x'_{d_v}$ and adding edges $x_i x'_i$ for $1 \leqslant i \leqslant d_v$. Similarly, we define G^S_W by applying the above procedure to each vertex in $W \subseteq V(G)$ (see Figure 2.3 for an example).

Theorem 2.3.1 *If G is a connected graph with $\delta(G) \geqslant 2$, then $\nabla\left(G^S_{V(G)}\right) = \nabla(G)$.*

Proof. For each copy of $K_{1,d}$, we call the vertex of degree d the center vertex, and others leaf vertices. Let S' be a minimum decycling set in $G^S_{V(G)}$, if all the vertices in S' are center vertices, then obviously $\nabla(G) = \nabla\left(G^S_{V(G)}\right)$.

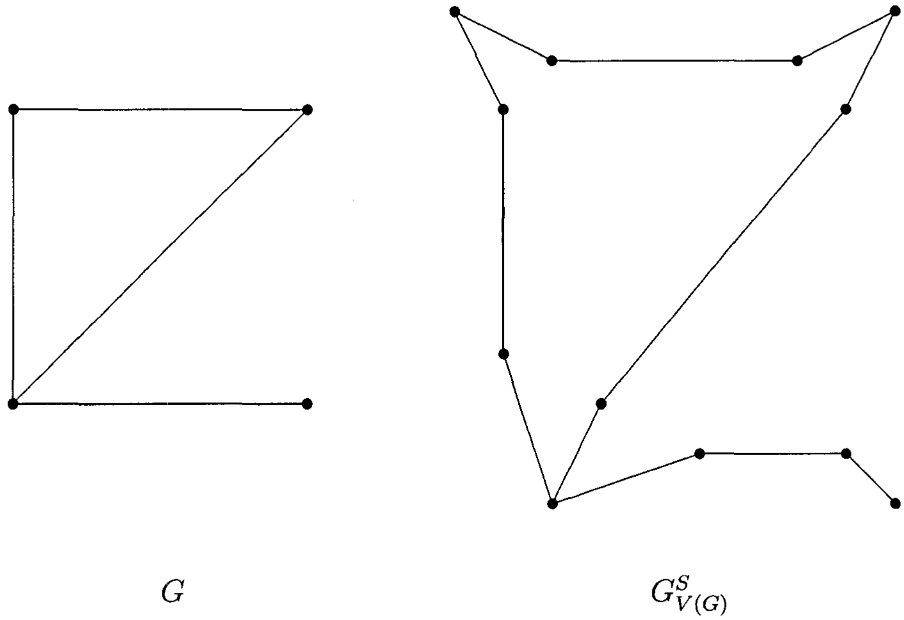


Figure 2.3: An example of $G^S_{V(G)}$.

If not, there exist leaf vertices of some copy of $K_{1,d}$ in S'. If there is only one leaf vertex in S' and the center vertex is not in S', then put the center vertex into S' and take the leaf vertex out of S'. If there are more than two leaf vertices in S', then it is obvious that S' is not a minimum decycling set of $G^S_{V(G)}$ (we can construct a new decycling set with fewer vertices by putting the center vertex into S' and taking the leaf vertices out of S'). □

Theorem 2.3.2 *If G is a connected graph with $\delta(G) \geqslant 2$, then* $\nabla\left(G^B_{V(G)}\right) = \sum_{v\in V(G)} (\deg_G(v) - 2) + \nabla(G)$.

Proof. Assume that S' is a minimum decycling set of $G^B_{V(G)}$. If $\exists\ v \in V(G)$, S' contains vertices in A_v, then, to decycle this copy of $K_{d,d-1}$, we need to remove $d-1$

vertices in A_v, which contradicts that S' is a minimum decycling set (we can choose $d-2$ vertices from A'_v into S'), or we remove vertices in both A_v and A'_v into S', which contradicts that S' is a minimum decycling set of $G^B_{V(G)}$. So we claim that a minimum decycling set of $G^B_{V(G)}$ contains just vertices from A'_v of each $K_{d,d-1}$ and $d-2$ vertices must be in S' for each $K_{d,d-1}$. Therefore, the resultant graph is isomorphic to $G^S_{V(G)}$, and by applying Theorem 2.3.1 we obtain $\nabla\left(G^B_{V(G)}\right) = \sum_{v\in V(G)} (\deg_G(v) - 2) + \nabla(G)$.
□

The last expansion we are interested in is similar to the previous two expansions, but replaces a vertex of degree d with a K_d and then arbitrarily joins the d neighbours of v and the d vertices in K_d; the new graph is denoted by G^K_v (respectively G^K_W, see Figure 2.4 for example). First, we investigate this expansion on regular graphs.

Theorem 2.3.3 *Let r be an integer and $r \geqslant 2$. If G is an r-regular connected graph with p vertices, then*

$$\nabla\left(G^K_{V(G)}\right) = \begin{cases} \sum\limits_{v\in V(G)} (r-2) = 2\big(q(G) - p(G)\big) & \text{if } r > 2, \\ 1 & \text{if } r = 2. \end{cases}$$

(If $v_i \in V(G)$, we use $v_{i,1}, v_{i,2}, \ldots, v_{i,r}$ to denote the corresponding vertices of K_r in $G^K_{V(G)}$.)

Proof. If $r > 2$, there are p vertex disjoint cliques in $G^K_{V(G)}$, so $\nabla\left(G^K_{V(G)}\right) \geqslant \sum_{v\in V(G)} (r-2)$. Then we construct a vertex subset S in $G^K_{V(G)}$ of size $\sum_{v\in V(G)} (r-2)$

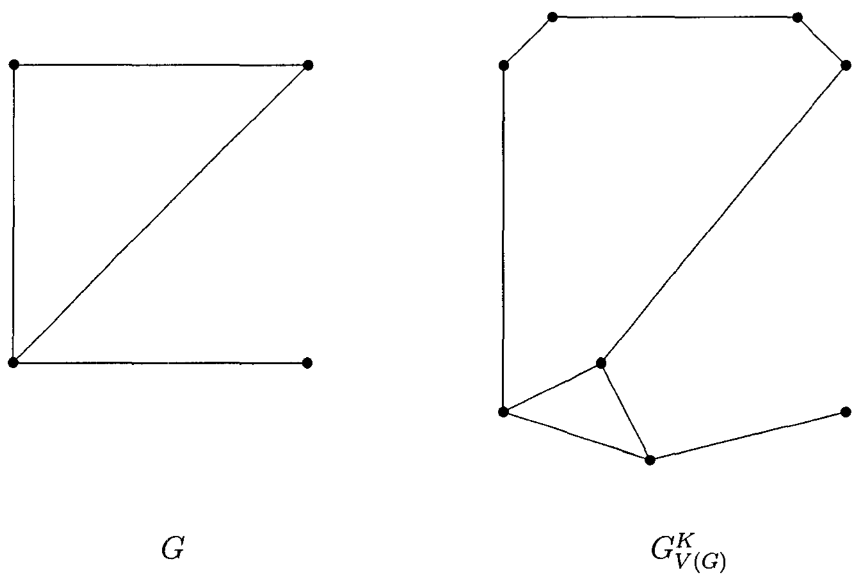


Figure 2.4: An example of $G^K_{V(G)}$.

by choosing any $r-2$ vertices from each clique in $G^K_{V(G)}$. It is easy to see that the left-over graph $G^K_{V(G)} - S$ has $\Delta \leqslant 2$.

If there exists a cycle $v_{1,1}v_{1,2}v_{2,1}v_{2,2}\cdots v_{k,1}v_{k,2}v_{1,1}$ in $G^K_{V(G)} - S$, then there exists a corresponding cycle $v_1v_2\cdots v_kv_1$ in G. It is obvious that all vertices $v_{i,l} \in S (1 \leqslant i \leqslant k,\ 3 \leqslant l \leqslant r)$. For some j, let $S_1 = S - v_{j,h} \cup \{v_{j,1}\}$ (or $S_1 = S - v_{j,h} \cup \{v_{j,2}\}$), where h is an integer between 3 and r. It is obvious that $|S_1| = |S|$. In $G^K_{V(G)} - S_1$ there is a new edge $v_{j,h}v_{j,2}$. Since $\Delta(G^K_{V(G)} - S) \leqslant 2$, we go through $v_{j,h}v_{j,2}v_{j+1,1}v_{j+1,2}\cdots v_{k,1}v_{k,2}v_{1,1}v_{1,2}\cdots v_{j-1,1}v_{j-1,2}$, and there is no cycle containing $v_{j,h}v_{j,2}$, hence the number of cycles is reduced by one. In some step m, we get an acyclic graph $G^K_{V(G)} - S_m$. So $\nabla\left(G^K_{V(G)}\right) = \sum\limits_{v \in V(G)} (r-2) = 2\big(q(G) - p(G)\big)$.

If $r = 2$, C_p (cycle with p vertices) is the only 2-regular connected graph. After applying the expansion, the resultant graph is still a cycle on $2p$ vertices. It is easy to see that $\nabla\left(C_p{}^{K}_{V(C_p)}\right) = 1$. $\square$

More generally, by applying a technique similar to that used in the above theorem, we have the following result:

Theorem 2.3.4 *Let G be a graph with p vertices and $\delta(G) \geqslant 2$, then*

$$\nabla\left(G^K_{V(G)}\right) = \begin{cases} 1 & \textit{if } G \cong C_p\,, \\ \sum\limits_{v \in V(G)} (\deg_G(v) - 2) & \textit{otherwise.} \end{cases}$$

Proof. By applying the similar argument in Theorem 2.3.3, we can easily get the desired result. $\square$

Chapter 3

Decycling Cartesian Products of Two Cycles

In this chapter, we study the decycling number for the family of graphs consisting of the Cartesian product of two cycles. We completely solve the problem of determining the decycling number of $C_m \Box C_n$ for all m and n. Moreover, we find a vertex set T that yields a maximum induced tree in $C_m \Box C_n$. The main results in this chapter are published in [47].

3.1 Introduction

In [9], various introductory results on decycling number were presented, followed by investigations into hypercubes as well as grid graphs $P_m \Box P_n$. Further results concerning $\nabla(P_m \Box P_n)$ were subsequently presented in [8] and summarized in a survey paper on decycling [7].

In [8] and again in [7], determining the decycling number for the Cartesian product of two cycles, i.e. $\nabla(C_m \Box C_n)$, is presented as an open problem. In this chapter we completely solve this problem, determining $\nabla(C_m \Box C_n)$ for all $m \geqslant 3$ and $n \geqslant 3$.

Further, for each combination of m and n other than $m = n = 4$, we show how to construct a minimum decycling set S of size $\nabla(C_m\Box C_n)$, such that $T = V(C_m\Box C_n) - S$ is the vertex set of a maximum induced tree in $C_m\Box C_n$.

Before moving on to our results, we review some of the introductory results that appeared first in [9]:

Lemma 3.1.1 *If G is a connected graph with p vertices ($p > 2$), q edges, and maximum degree Δ, then*

$$\nabla(G) \geqslant \frac{q-p+1}{\Delta-1}.$$

Theorem 3.1.2 *If G and H are homeomorphic graphs, then $\nabla(G) = \nabla(H)$.*

3.2 Decycling $C_m\Box C_n$ (initial cases)

In the next two sections, we investigate the decycling number of the graph $C_m\Box C_n$, the Cartesian product of a cycle C_m on m vertices and a cycle C_n on n vertices. In this section we establish lower bounds on $\nabla(C_m\Box C_n)$ and obtain exact results for several initial cases. In the next section, we obtain a general result that describes a decycling set of the graph $C_m\Box C_n$, for all m and n.

In Chapter 1, we introduced a standard labelling for the Cartesian products of graphs. Figure 3.1 is a simple example of $C_4\Box C_7$, and the vertex labelled by "•" is denoted by $v_{3,4}$. Note that $C_m\Box C_n$ is a 4-regular graph, and hence by Lemma 3.1.1, we have the following lower bound for the size of any decycling set of $C_m\Box C_n$.

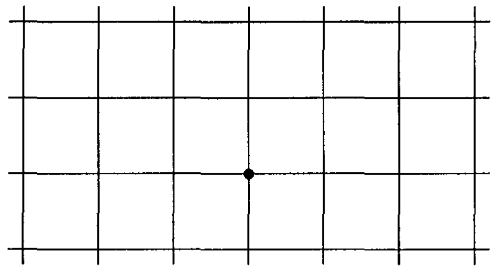


Figure 3.1: $C_4 \Box C_7$, Cartesian product of C_4 and C_7.

Lemma 3.2.1 *For the graph $C_m \Box C_n$,*

$$\nabla(C_m \Box C_n) \geqslant \frac{mn+1}{3}\,. \tag{3.1}$$

In order that decycling will be more readily recognizable in our figures, we frequently emphasize only the vertices of the decycling set.

Because the result for $C_4 \Box C_n$ is different from the other graphs $C_m \Box C_n$, $m \neq 4$, we dispose of this case prior to developing more general results.

Theorem 3.2.2 $\nabla(C_4 \Box C_n) = \left\lceil \frac{3n}{2} \right\rceil$*, for all* $n \geqslant 3$.

Proof. Every column of the graph is a 4-cycle, so we must remove at least one vertex in each column for decycling. For any two adjacent columns in $C_4 \Box C_n$, we need to remove at least 3 vertices to decycle these two columns, and so if n is even, then $\nabla(C_4 \Box C_n) \geqslant \frac{3n}{2}$. When n is even, we can find a decycling set of that size. Let $M = \bigcup_{i=1}^{k-1} \left(\{v_{2,4i-3},\ v_{4,4i-3},\ v_{3,4i-2},\ v_{1,4i-1},\ v_{3,4i-1},\ v_{2,4i}\}\right)$ where $k = \left\lceil \frac{n}{4} \right\rceil$. Then $S = M \cup \{v_{2,n-1},\ v_{4,n-1},\ v_{1,n}\}$ is a decycling set for $C_4 \Box C_n$ where $n \equiv 2 \pmod 4$ (see Figure 3.2 for the case $n = 6$). If $n \equiv 0 \pmod 4$ and $n > 4$, then $S = M \cup$

$\{v_{2,n-3},\ v_{4,n-3},\ v_{1,n-2},\ v_{2,n-1},\ v_{3,n-1},\ v_{1,n}\}$ is a minimum decycling set (see Figure 3.3 for the case $n = 8$).

Observe that in each case the vertices of $V(C_m \Box C_n) - S$ induce a tree. However, for $C_4 \Box C_4$, no minimum decycling set will yield a forest with only one component. In this case, a maximum induced tree has 9 vertices, and is produced by the non-minimum decycling set $\{v_{3,1},\ v_{4,1},\ v_{2,2},\ v_{1,3},\ v_{3,3},\ v_{1,4},\ v_{4,4}\}$. A maximum induced forest having 10 vertices is obtained with the decycling set $\{v_{1,1},\ v_{3,1},\ v_{2,2},\ v_{1,3},\ v_{3,3},\ v_{4,4}\}$.

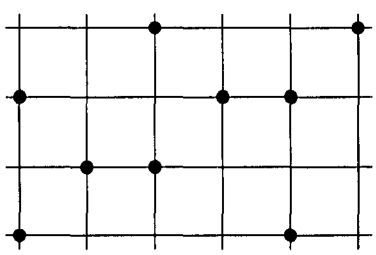


Figure 3.2: A decycling set of $C_4 \Box C_6$

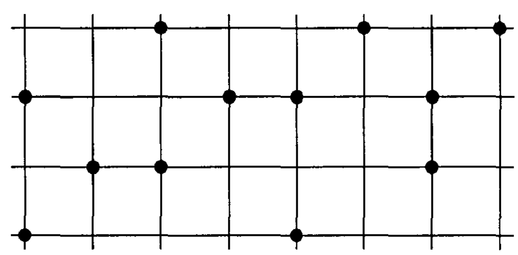


Figure 3.3: A decycling set of $C_4 \Box C_8$

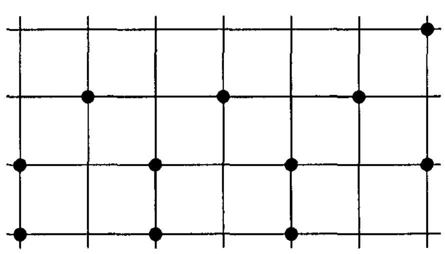


Figure 3.4: A decycling set of $C_4 \Box C_7$

If n is odd, in every two adjacent columns, we still must remove at least 3 vertices. If we attempt to remove exactly 3 vertices from each pair of columns $(i, i+1)$ for $1 \leqslant i < n-1$, then we find that removing only 1 vertex from column n leaves a cycle in the graph (one of the pairs $(n-1, n)$ or $(n, 1)$ will contain only 2 vertices of the decycling set). It follows that at least $3\left(\frac{n-1}{2}\right)+2=\left\lceil\frac{3n}{2}\right\rceil$ vertices will have to be removed in order to decycle $C_4 \square C_n$ where n is odd. A decycling set of this size, and whose complement induces a tree, exists, namely $\bigcup_{i=1}^{k}\left(\{v_{3,2i-1},\ v_{4,2i-1},\ v_{2,2i}\}\right) \cup \{v_{1,n},\ v_{3,n}\}$, where $k=\frac{n-1}{2}$. (See Figure 3.4 for the case $n=7$.) $\square$

Throughout the remainder of this section, we assume that $4 \notin \{m, n\}$.

In [9], the *outlay* of a set S of vertices in a graph G is defined as

$$\theta(S) := \sigma(S) - |S| - \epsilon(S) - \omega(G-S) + 1\,,$$

where $\sigma(S)$ is the sum of the degrees of the vertices in S, $\epsilon(S)$ is the number of edges in the induced subgraph $G[S]$, and $\omega(G-S)$ is the number of components in $G-S$.

By Lemma 1.3 in [9], if G is a connected graph with p vertices and q edges, and S is any decycling set for G, then $\theta(S)=q-p+1$. For $C_m \square C_n$, we can easily compute $mn+1=\theta(S)=3|S|+1-(\epsilon+\omega)$, so

$$3|S| = mn + (\epsilon+\omega)\,. \tag{3.2}$$

Since $\omega \geqslant 1$, therefore $|S| \geqslant \frac{mn+1}{3}$, which in turn yields Inequality 3.1. We will now show that $\epsilon+\omega>1$, thereby obtaining a greater lower bound on $\nabla(C_m \square C_n)$ than is given by Lemma 3.2.1.

Lemma 3.2.3 *For $G = C_m \square C_n$, if S is a minimum decycling set, then*

$$\epsilon(S) + \omega(G-S) \geqslant 2\,.$$

Proof. By the symmetry of the graph $C_m \Box C_n$, we can consider the following 6 cases only:

① $m \equiv 0 \pmod 3$, $n \equiv 0 \pmod 3$;

② $m \equiv 1 \pmod 3$, $n \equiv 0 \pmod 3$;

③ $m \equiv 1 \pmod 3$, $n \equiv 1 \pmod 3$;

④ $m \equiv 2 \pmod 3$, $n \equiv 0 \pmod 3$;

⑤ $m \equiv 2 \pmod 3$, $n \equiv 1 \pmod 3$;

⑥ $m \equiv 2 \pmod 3$, $n \equiv 2 \pmod 3$.

By considering Equation 3.2, modulo 3, we quickly find that $\epsilon + \omega \geqslant 2$ for cases ③ and ⑥, and $\epsilon + \omega \geqslant 3$ for cases ①, ②, ④. We therefore now consider only case ⑤.

For each 4-cycle (a, b, c, d) in $C_m \Box C_n$, we now create the block $\{a, b, c, d\}$, and we let $\mathcal{B}$ be the set of all such blocks. Let $\mathcal{H}$ be the bipartite graph with bipartition $(V, \mathcal{B})$, where $V = V(C_m \Box C_n)$ and in which $v \in V$ is adjacent to $B \in \mathcal{B}$ if and only if $v \in B$.

Given a subset S of V, we also consider the related bipartite graph $\mathcal{H} - S$ having bipartition $(V - S, \mathcal{B} - S)$, where $\mathcal{B} - S = \{B - S \,:\, B \in \mathcal{B}\}$ and in which $v \in V - S$ is adjacent to $B \in \mathcal{B} - S$ if and only if $v \in B$. Whenever S is a decycling set of $C_m \Box C_n$, each vertex $v \in V - S$ will have degree 4 in $\mathcal{H} - S$, yet the mn vertices of

$\mathcal{B} - S$ must each have degree at most 3 (as blocks, the mn elements of $\mathcal{B} - S$ must each have size at most 3).

Assuming that $\epsilon + \omega = 1$, then $\epsilon = 0$, $\omega = 1$, and so Equation 3.2 yields

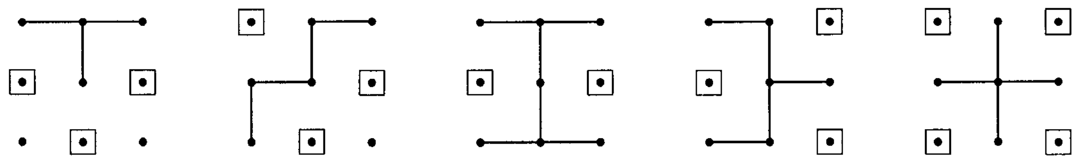
$$\nabla(C_m \square C_n) = \frac{mn+1}{3}. \tag{3.3}$$

Since $\varepsilon = 0$, it follows that each of the mn blocks of $\mathcal{B} - S$ must have size of 2 or 3. Let's assume that x of the vertices of $\mathcal{B} - S$ have degree 2 in $\mathcal{H} - S$, so the remaining $mn - x$ vertices of $\mathcal{B} - S$ each have degree 3 in $\mathcal{H} - S$. Counting the number of edges in $\mathcal{H} - S$, we have

$$4(mn - \nabla) = 2x + 3(mn - x)$$
$$x = \frac{mn+4}{3}.$$

At this point, each vertex of $(C_m \square C_n) - S$ must have degree 1, 2, 3, or 4 (a vertex of degree 0 is impossible because $\omega = 1$). Those of degree 2 can be further classified, depending on whether the 2 neighbours are in orthogonal directions or in the same lateral direction. Thus we can now identify each vertex of $(C_m \square C_n) - S$ as being of Type-1, 2o, 2l, 3, or 4, as illustrated in Figure 3.5. (Since $\epsilon = 0$, there exist only these 5 types of vertices in $(C_m \square C_n) - S$.) Throughout this lemma, in order that the vertices in the decycling set will be more readily differentiable from the vertices of $(C_m \square C_n) - S$ in the figure, we use "•" to denote the vertices in $C_m \square C_n$, and surround a vertex by "□" to indicate that it is in the decycling set.

Every Type-1 vertex of $G - S$ is in two blocks of size 2 in $\mathcal{B} - S$ and two blocks



1 2o 2l 3 4

Figure 3.5: The 5 types of vertices in $G - S$ (the vertex in the center is the vertex that we mean for the type).

of size 3, every Type-2o vertex is in one block of size 2 and three blocks of size 3, and each remaining vertex of $(C_m \Box C_n) - S$ is in four blocks of size 3. Suppose that we have x_1, x_{2o}, x_{2l}, x_3, x_4 vertices of Type-1, 2o, 2l, 3, 4 respectively. Counting the number of edges in $\mathcal{H} - S$ which are incident to the blocks of size 2 in $\mathcal{B} - S$, we have

$$2x_1 + x_{2o} = 2x = \frac{2mn + 8}{3} . \tag{3.4}$$

Similarly, counting the number of edges which are incident to the blocks of size 3, we have

$$\begin{aligned} 2x_1 + 3x_{2o} + 4x_{2l} + 4x_3 + 4x_4 &= 3(mn - x) \\ &= 3mn - 3\left(\frac{mn + 4}{3}\right) \\ &= 2mn - 4 . \end{aligned} \tag{3.5}$$

Counting the number of edges of $C_m \Box C_n$ with one endpoint in S and the other in $V - S$, we have

$$3x_1 + 2x_{2o} + 2x_{2l} + 1x_3 + 0x_4 = 4\nabla . \tag{3.6}$$

Solving the system of equations consisting of Equation 3.4 through Equation 3.6, we find that

$$x_4 = 1 + x_{2l}\,.$$

Since $x_{2l} \geqslant 0$, then there is at least one Type-4 vertex. Note that each Type-4 vertex can only be adjacent in $G-S$ to vertices that are Type-1 or Type-$2l$. Since $\omega(G-S) = 1$, each Type-4 vertex must have at least one Type-$2l$ neighbour. Moreover, since $x_4 = 1 + x_{2l}$, it follows that the Type-$2l$ and Type-4 vertices induce a tree in which the vertices are alternately Type-4 and Type-$2l$. The neighbours of this tree in $G-S$ are all Type-1. But $\omega(G-S) = 1$, and hence we now deduce that $x_{2o} = x_3 = 0$, yielding a solution to Equations 3.4 through Equation 3.6:

$$\begin{cases} x_1 = \dfrac{mn+4}{3}\,, \\ x_{2l} = \dfrac{mn-8}{6}\,, \\ x_4 = \dfrac{mn-2}{6}\,. \end{cases}$$

Consider the vertices of an arbitrary row in G. Since $\varepsilon = 0$, each vertex of S in the row will be incident with 2 horizontal edges of G that do not appear in $G-S$. The horizontal edges that do appear in $G-S$ form a collection of horizontal paths, each of which has two endpoints that are both Type-1 vertices, whereas the interior vertices are alternately Type-4 and Type-$2l$. Each horizontal path thus contains an even number of edges. Of the n horizontal edges that appeared in this row in G, an even number do not appear in $G-S$ while an even number do. Hence n must be even, or else we have a contradiction. By considering an arbitrary column of G, we similarly find that m must be even.

Now we consider the vertices in the decycling set. At this point, the vertices in S must have 0, 1, or 2 Type-$2l$ neighbours. Those having no Type-$2l$ neighbour can be further classified, depending on the number of Type-4 diagonal neighbours, where we define a *diagonal neighbour* of a vertex u to be any vertex that is distance 2 from u in a 4-cycle of G. The resultant 5 possible types of vertices in S are illustrated in Figure 3.6.

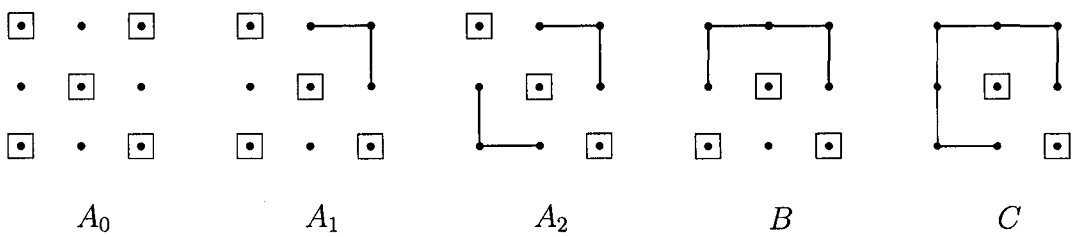


Figure 3.6: The 5 types of vertices in S (the vertex in the center is the vertex that we mean for the type).

For each Type-A_k vertex ($k = 0, 1, 2$), its 4 neighbours are all Type-1 vertices. Each Type-B vertex has one Type-$2l$ neighbour and the other three are Type-1. For each Type-C vertex, two of its neighbours (in orthogonal directions) are Type-$2l$ and the other two are Type-1. Since both m and n are even, then G has an even number of rows and even number of columns, so we label the rows and columns of G such that all of the Type-4 vertices are in rows and columns with even parity. Consequently, all Type-A_0 vertices are in rows and columns with even parity while all vertices of Type-A_1, A_2, B, or C are in rows and columns with odd parity. Type-1 and Type-$2l$ vertices have row and column labels with different parities. Hence all of the vertices of S that are diagonal neighbours of any vertex of Type-A_1, A_2, B, or C must be of Type-A_0.

Note that all $\frac{mn}{4}$ of the vertices in rows and columns each having odd parity (i.e. the Type-A_1, A_2, B and C vertices) are in the decycling set. And since $\varepsilon = 0$, all $\frac{mn}{2}$ of the vertices in rows and columns having different parities (i.e. the Type-1 and $2l$ vertices) are not in S. Thus, if we let T denote the set of all vertices that are Type-A_1, A_2, B or C, then $(C_m \square C_n) - T$ is homeomorphic to $C_{\frac{m}{2}} \square C_{\frac{n}{2}}$, and so by Theorem 3.1.2,

$$\begin{aligned} \nabla(C_m \square C_n) &= \nabla\big((C_m \square C_n) - T\big) + |T| \\ &= \nabla(C_{\frac{m}{2}} \square C_{\frac{n}{2}}) + \frac{mn}{4}\,. \end{aligned} \tag{3.7}$$

More generally, we find that

$$\nabla(C_{m_i} \square C_{n_i}) = \nabla(C_{m_{i-1}} \square C_{n_{i-1}}) + m_{i-1}n_{i-1}\,,$$

where $m_i = \frac{m}{2^{k-i}}$ and $n_i = \frac{n}{2^{k-i}}$ for $i = 0, 1, \cdots, k$, and k is the least non-negative integer such that at least one of $\frac{m}{2^k}$ or $\frac{n}{2^k}$ is either odd or equal to 4. Further, observe that $\nabla(C_{m_i} \square C_{n_i}) = \frac{m_i n_i + 1}{3}$ if and only if $\nabla(C_{m_{i-1}} \square C_{n_{i-1}}) = \frac{m_{i-1}n_{i-1} + 1}{3}$. But $\nabla(C_{m_0} \square C_{n_0}) \neq \frac{m_0 n_0 + 1}{3}$, since either one of m_0 or n_0 is odd or else Theorem 3.2.2 applies. We therefore have a contradiction to Equation 3.3 and hence $\varepsilon(S) + \omega(G - S) \geqslant 2$ for case ⑤. By again considering Equation 3.2, modulo 3, we now conclude that $\varepsilon(S) + \omega(G - S) \geqslant 4$. □

Theorem 3.2.4 $\nabla(C_m \square C_n) \geqslant \left\lceil \frac{mn+2}{3} \right\rceil$.

Proof. By Lemma 3.2.3 and Equation 3.2, we can easily get the desired result. $\Box$

The reduction technique described towards the end of the proof of Lemma 3.2.3 can also be employed as a doubling construction, which we now present.

Theorem 3.2.5 *Suppose S is any minimum decycling set of $C_m \Box C_n$ with $\nabla(C_m \Box C_n) = \left\lceil \frac{mn+2}{3} \right\rceil$. Let $S' = \{v_{2i-1,2j-1} : v_{i,j} \in S\}$ be a vertex set in $C_{2m} \Box C_{2n}$, and $T = \{v_{i,j} : i = 2, 4, \cdots, 2m;\ j = 2, 4, \cdots, 2n\}$. Then $S' \cup T$ is a minimum decycling set of $C_{2m} \Box C_{2n}$. Furthermore, if $\omega\big((C_m \Box C_n) - S\big) = 1$, then there exists a minimum decycling set S'' of $C_{2m} \Box C_{2n}$ such that $\omega\big((C_{2m} \Box C_{2n}) - S''\big) = 1$.*

Proof. Note that $(C_{2m} \Box C_{2n}) - T$ is a graph homeomorphic to $C_m \Box C_n$. Hence, $\big((C_{2m} \Box C_{2n}) - T\big) - S' = (C_{2m} \Box C_{2n}) - (S' \cup T)$ is acyclic. Therefore, $S' \cup T$ is a decycling set of $C_{2m} \Box C_{2n}$.

Since by Theorem 3.2.4,

$$\begin{aligned} \nabla(C_{2m} \Box C_{2n}) &\geqslant \left\lceil \frac{4mn+2}{3} \right\rceil \\ &= mn + \left\lceil \frac{mn+2}{3} \right\rceil \\ &= |T| + |S'| \, , \end{aligned}$$

$S' \cup T$ is a minimum decycling set of $C_{2m} \Box C_{2n}$.

Clearly $\varepsilon(S' \cup T) = 0$. In order to obtain a maximum induced tree in $C_{2m} \Box C_{2n}$, we therefore must transform the decycling set $S' \cup T$ into a new decycling set S'' such that $\varepsilon(S'') = \varepsilon(S)$.

Suppose that for some i and j, both of $v_{i,j}$ and $v_{i,j+1}$ are in S. Then $v_{2i-1,2j}$ is an isolated vertex in $(C_{2m} \square C_{2n}) - (S' \cup T)$. Also, $v_{2i,2j} \in T$ and $v_{2i+1,2j} \notin (S' \cup T)$. By removing $v_{2i,2j}$ from the decycling set and replacing it with $v_{2i+1,2j}$, we obtain a new minimum decycling set. If $\varepsilon(S) \geqslant 1$, we consider the following cases to perform similar transformations:

i. If for some i and j, both of $v_{i,j}$ and $v_{i,j+1}$ are in S, then $v_{2i-1,2j}$ is an isolated vertex in $(C_{2m} \square C_{2n}) - (S' \cup T)$. Also, $v_{2i,2j} \in T$ and $v_{2i+1,2j} \notin (S' \cup T)$. By removing $v_{2i,2j}$ from the decycling set and replacing it with $v_{2i+1,2j}$, we obtain a new minimum decycling set.

ii. If for some i and j, $v_{i,j}, v_{i+1,j}$ and $v_{i+1,j+1}$ are all in the decycling set S, then $v_{2i,2j-1}$ and $v_{2i+1,2j}$ are isolated vertices in $(C_{2m} \square C_{2n}) - (S' \cup T)$. Also $v_{2i,2j} \in T$ and $v_{2i,2j+1} \notin (S' \cup T)$. By removing $v_{2i,2j}$ from the decycling set and replacing it with $v_{2i,2j+1}$, we obtain a new minimum decycling set.

iii. If for some i and j, $v_{i,j}, v_{i+1,j}$, $v_{i+1,j+1}$ and $v_{i+2,j}$ are all in the decycling set S of $C_m \square C_n$, then $v_{2i,2j-1}$, $v_{2i+1,2j}$ and $v_{2i+2,2j-1}$ are isolated vertices in $(C_{2m} \square C_{2n}) - (S' \cup T)$. Also $v_{2i,2j-2}$, $v_{2i+2,2j} \in T$ and $v_{2i,2j-3}$, $v_{2i+3,2j} \notin (S' \cup T)$. By removing $v_{2i,2j-2}$, and $v_{2i+2,2j}$ from the decycling set and replacing them with $v_{2i,2j-3}$, and $v_{2i+3,2j}$, we obtain a new minimum decycling set.

By performing such transformations, we obtain a minimum decycling set S'' for which $\varepsilon(S'') = \varepsilon(S)$ and $\omega\big((C_{2m} \square C_{2n}) - S''\big) = 1$. $\square$

We now carry on with several initial cases:

Lemma 3.2.6 $\nabla(C_3 \square C_n) = \left\lceil \dfrac{3n+2}{3} \right\rceil$.

Proof. By Theorem 3.2.4, we get $\nabla(C_3 \square C_n) \geqslant \left\lceil \dfrac{3n+2}{3} \right\rceil = n+1$. Let $k = \left\lfloor \dfrac{n}{3} \right\rfloor$, $M = \bigcup_{i=1}^{k} \{v_{1,3i-2},\ v_{2,3i-1},\ v_{3,3i}\}$, $S_0 = \{v_{2,n}\}$, $S_1 = \{v_{2,n},\ v_{3,n}\}$, $S_2 = \{v_{1,n-1},\ v_{2,n-1},\ v_{2,n}\}$. Then $M \cup S_t$ is a decycling set of size $n+1$ for $C_3 \square C_n$ with the property that $\omega\big((C_3 \square C_n) - (M \cup S_t)\big) = 1$, where $n \equiv t \pmod 3$. □

Lemma 3.2.7 $\nabla(C_8 \square C_n) = \left\lceil \dfrac{8n+2}{3} \right\rceil$.

Proof. By Theorem 3.2.4, we have the lower bound $\nabla(C_8 \square C_n) \geqslant \left\lceil \dfrac{8n+2}{3} \right\rceil$. We should divide the number of columns into 6 cases modulo 6. Let $k = \left\lfloor \dfrac{n}{3} \right\rfloor$, $M = \bigcup_{i=1}^{k} \{v_{1,3i-2},\ v_{4,3i-2},\ v_{7,3i-2},\ v_{2,3i-1},\ v_{5,3i-1},\ v_{3,3i},\ v_{6,3i},\ v_{8,3i}\}$. For $n \equiv 3 \pmod 6$, $M \cup \{v_{1,n}\}$ is a minimum decycling set. For $n \equiv 1 \pmod 6$, $M \cup \{v_{2,n},\ v_{5,n},\ v_{6,n},\ v_{8,n}\}$ is a minimum decycling set. For $n \equiv 4 \pmod 6$, $M \cup \{v_{1,n},\ v_{3,n},\ v_{5,n},\ v_{7,n}\}$ is a minimum decycling set. For $n \equiv 5 \pmod 6$, $M \cup \{v_{2,n-1},\ v_{5,n-1},\ v_{7,n-1},\ v_{3,n},\ v_{5,n},\ v_{8,n}\}$ is a minimum decycling set. For $n \equiv 0 \pmod 6$, $\big(M - \{v_{2,n-1}\}\big) \cup \{v_{3,n-1},\ v_{2,n}\}$ is a minimum decycling set. For $n \equiv 2 \pmod 6$, $\bigcup_{i=1}^{k-1} \{v_{1,3i-2},\ v_{4,3i-2},\ v_{7,3i-2},\ v_{2,3i-1},\ v_{5,3i-1},\ v_{3,3i},\ v_{6,3i},\ v_{8,3i}\} \cup \{v_{1,n-4},\ v_{4,n-4},\ v_{7,n-4},\ v_{2,n-3},\ v_{5,n-3},\ v_{1,n-2},\ v_{4,n-2},\ v_{7,n-2},\ v_{2,n-1},\ v_{6,n-1},\ v_{8,n-1},\ v_{3,n},\ v_{5,n},\ v_{8,n}\}$ is a minimum decycling set. Note that in each of these six cases, the corresponding maximum induced forest consists of a single tree. □

Lemma 3.2.8 *If* $m = 3r$, *then* $\nabla(C_m \square C_n) = rn + 1$.

Proof. By Theorem 3.2.4, we have the lower bound, $\nabla(C_{3r}\Box C_n) \geqslant \left\lceil \frac{3rn+2}{3} \right\rceil = rn+1$. Now if n is odd, then let $M = \bigcup_{i=1}^{r} \left(\{v_{3i-2,1}\} \cup \bigcup_{j=1}^{k}\{v_{3i-1,2j},\ v_{3i,2j+1}\}\right)$, where $n = 2k+1$. Considering the graph $(C_{3r}\Box C_n) - M$, rows $3i-2$, $3i-1$ and $3i$ have a path from $v_{3i-2,2}$ to $v_{3i,2}$ for each $1 \leqslant i \leqslant r$. By joining these paths, we have a cycle C of length $r(n+3)$ starting and ending at $v_{1,2}$. Each vertex not on cycle C and not in M has one neighbour in C and three neighbours in M. So $(C_{3r}\Box C_n) - M$ consists of the cycle C with $r(n-3)$ pendant edges and $6r$ vertices of degree 2. (See Figure 3.7 for an example of the left-over graph of $(C_6\Box C_{11}) - M$.)

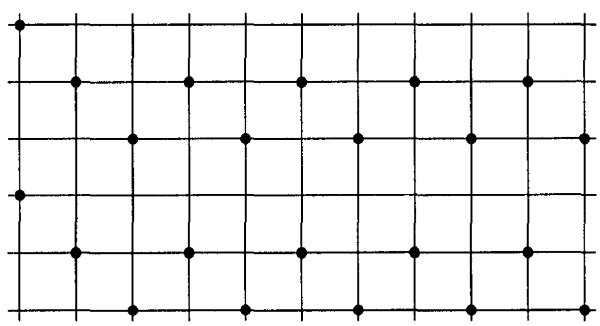


Figure 3.7: The left-over graph (with one cycle) of $(C_{3r}\Box C_n) - M$, where n is odd.

If n is even and r is odd, let $M = \bigcup_{i=1}^{r} \left(\{v_{3i-2,1},\ v_{3i-2,n-2}, v_{3i-1,n},\ v_{3i,n-1}\} \cup \bigcup_{j=1}^{k}\{v_{3i-1,2j+1},\ v_{3i,2j}\}\right)$, where $n = 2k+4$. Using a similar method to the above case, $(C_{3r}\Box C_n) - M$ consists a cycle C of length $r(n+6)$ with $r(n-6)$ pedant edges and $12r$ vertices of degree 2. (See Figure 3.8 for an example of the left-over graph of $(C_9\Box C_{14}) - M$.)

In either of the above cases, $|M| = rn$ and $(C_m\Box C_n) - M$ is connected and

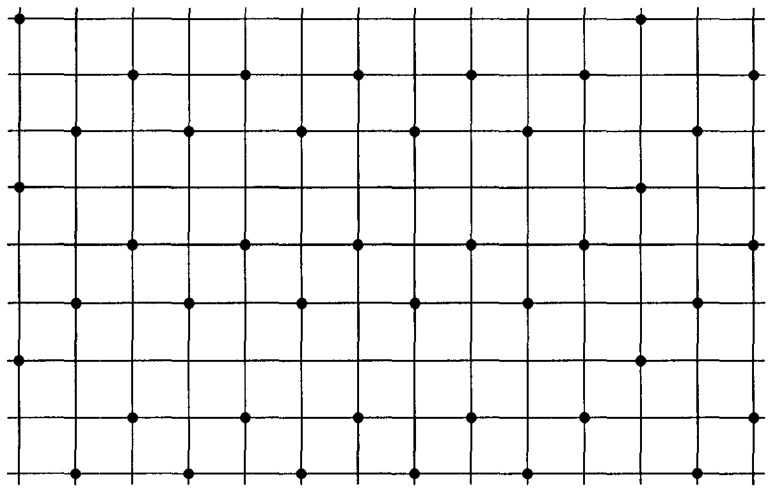


Figure 3.8: The left-over graph (with one cycle) of $(C_{3r} \Box C_n) - M$, where n is even and r is odd.

contains only one cycle. Now by letting S consist of M plus any degree 2 vertex of the cycle C of $(C_m \Box C_n) - M$, we obtain a minimum decycling set, the complement of which induces a tree.

Now suppose that r and n are both even, and let k be the least non-negative integer such that at least one of $\frac{r}{2^k}$ or $\frac{n}{2^k}$ is odd or $\frac{n}{2^k}$ equals 8, and let $m_i = \frac{m}{2^{k-i}}$ and $n_i = \frac{n}{2^{k-i}}$ for each $i = 0, 1, \cdots, k$. Then we can find a minimum decycling set S_0 of cardinality $\frac{rn}{2^{2k}} + 1$ in $C_{m_0} \Box C_{n_0}$ such that $\omega\big((C_{m_0} \Box C_{n_0}) - S_0\big) = 1$. Now, for each $i = 0, 1, \cdots, k-1$, apply Theorem 3.2.5 to $C_{m_i} \Box C_{n_i}$ to construct a minimum decycling set S_{i+1} of size $\nabla(C_{m_0} \Box C_{n_0}) + \frac{mn}{2^{2k}} \sum_{j=0}^{i} 4^j$ in $C_{m_{i+1}} \Box C_{n_{i+1}}$ such that $\omega\big((C_{m_{i+1}} \Box C_{n_{i+1}}) - S_{i+1}\big) = 1$. It follows that $\nabla(C_m \Box C_n) = \nabla(C_{m_0} \Box C_{n_0}) + \frac{mn}{2^{2k}} \sum_{j=0}^{k-1} 4^j = rn + 1$. $\square$

By the symmetry of the graph $C_m\Box C_n$, in the rest of this chapter, we do not need to consider the case when $3 \mid mn$.

Lemma 3.2.9 $\nabla(C_5\Box C_n) = \left\lceil \dfrac{5n+2}{3} \right\rceil$.

Proof. By Theorem 3.2.4, we have $\nabla(C_5\Box C_n) \geqslant \left\lceil \dfrac{5n+2}{3} \right\rceil$. Let $k = \left\lfloor \dfrac{n}{3} \right\rfloor$, $M = \bigcup_{i=1}^{k}\{v_{1,3i-2},\ v_{3,3i-2},\ v_{2,3i-1},\ v_{4,3i-1},\ v_{5,3i}\}$, $S_1 = \{v_{1,n},\ v_{3,n},\ v_{4,n}\}$, and $S_2 = \{v_{1,n-1},\ v_{3,n-1},\ v_{4,n},\ v_{5,n}\}$. We can easily verify that $M \cup S_t$ is a decycling set of size $\left\lceil \dfrac{5n+2}{3} \right\rceil$ for $C_5\Box C_n$ such that $\omega\big((C_5\Box C_n) - (M \cup S_t)\big) = 1$, where $n \equiv t \pmod 3$.
□

Lemma 3.2.10 $\nabla(C_7\Box C_n) = \left\lceil \dfrac{7n+2}{3} \right\rceil$.

Proof. By Theorem 3.2.4, we have $\nabla(C_7\Box C_n) \geqslant \left\lceil \dfrac{7n+2}{3} \right\rceil$. Let $k = \left\lfloor \dfrac{n}{3} \right\rfloor$. For $n \equiv 1 \pmod 3$, let $S = \bigcup_{i=1}^{k-1}\{v_{2,3i-2},\ v_{6,3i-2},\ v_{3,3i-1},\ v_{5,3i-1},\ v_{7,3i-1},\ v_{1,3i},\ v_{4,3i}\} \cup \{v_{3,n-3},\ v_{5,n-3},\ v_{7,n-3},\ v_{2,n-2},\ v_{5,n-2},\ v_{1,n-1},\ v_{3,n-1},\ v_{6,n-1},\ v_{4,n},\ v_{7,n}\}$. For $n \equiv 2 \pmod 3$, let $S = \bigcup_{i=1}^{k}\{v_{1,3i-2},\ v_{5,3i-2},\ v_{2,3i-1},\ v_{4,3i-1},\ v_{7,3i-1},\ v_{3,3i},\ v_{6,3i}\} \cup \{v_{1,n-1},\ v_{4,n-1},\ v_{7,n-1},\ v_{1,n},\ v_{3,n},\ v_{6,n}\}$. In both cases, S is a decycling set of size $\left\lceil \dfrac{7n+2}{3} \right\rceil$ and $\omega\big((C_7\Box C_n) - S\big) = 1$. □

According to Lemmas 3.2.6–3.2.10, we have the following theorem:

Theorem 3.2.11 *Let m and n be integers such that $m \in \{5, 7, 8\} \cup \{3, 6, 9, \cdots\}$, $n \geqslant 3$, and $n \neq 4$. Then $\nabla(C_m\Box C_n) = \left\lceil \dfrac{mn+2}{3} \right\rceil$, and $t(C_m\Box C_n) = mn - \nabla(C_m\Box C_n)$.*

3.3 Decycling $C_m \Box C_n$ (the general cases)

In the previous section, we discussed the decycling set of $C_m \Box C_n$ for small values of m. In this section, we investigate all of the remaining cases. Since we have already solved the problem for $m \equiv 0 \pmod 3$, we only need to consider four cases (modulo 6), i.e. $m \equiv 1 \pmod 6$, $m \equiv 2 \pmod 6$, $m \equiv 4 \pmod 6$, $m \equiv 5 \pmod 6$, and by the symmetry of $C_m \Box C_n$, for each case, we do not need to consider the case of $n \equiv 0 \pmod 3$. Unless stated otherwise, throughout this section we assume that $4 \notin \{m, n\}$.

Lemma 3.3.1 *If $m = 6r + 1$, then $\nabla(C_m \Box C_n) = 2rn + \left\lceil \frac{n+2}{3} \right\rceil$.*

Proof. If $r = 1$, then the result follows from Lemma 3.2.10. For $r > 1$, we employ an iterative construction in which we add 6 new rows at a time, starting with the graph $C_7 \Box C_n$. This iterative technique takes advantage of a particular configuration of three consecutive rows; this configuration is initially present in the decycling set described in Lemma 3.2.10 for $C_7 \Box C_n$, and a copy of the configuration is produced with each iteration. After $(r-1)$ iterations, we will have constructed a decycling set of size $2rn + \left\lceil \frac{n+2}{3} \right\rceil$ in $C_m \Box C_n$. From Theorem 3.2.4, we have $\nabla(C_m \Box C_n) \geqslant \left\lceil \frac{mn+2}{3} \right\rceil = \left\lceil \frac{(6r+1)n+2}{3} \right\rceil = 2rn + \left\lceil \frac{n+2}{3} \right\rceil$, which tells us that the decycling set we construct is optimal.

We now consider two subcases, each of which employs its own configuration of three consecutive rows from Lemma 2.10:

i. $n \equiv 1 \pmod 3$.

Beginning with the decycling set of $C_7 \square C_n$, we say that a row is Type-α if its deleted vertices are in the same columns as those of the fifth row of $C_7 \square C_n$ which was described in Lemma 3.2.10 (i.e. row j is Type-α if the set of vertices removed from it is $\bigcup_{i=1}^{t}\{v_{j,3i-1}\} \cup \{v_{j,n-3}\}$ where $n = 3t+1$). Similarly, Type-β (resp. Type-γ) rows are those with a configuration identical to that of the sixth $\left(\bigcup_{i=1}^{t-1}\{v_{j,3i-2}\} \cup \{v_{j,n-1}\}\right)$ (resp. seventh $\left(\bigcup_{i=1}^{t-1}\{v_{j,3i-1}\} \cup \{v_{j,n-3},\, v_{j,n}\}\right)$) row of $C_7 \square C_n$.

Focusing on the 3 consecutive rows that are, in order, of Type-α, β, γ in $C_{7+6k} \square C_n$, for some $k \geqslant 0$, we now describe how to insert six new rows and obtain a minimum decycling set of $C_{7+6(k+1)} \square C_n$. Following the row of Type-α in $C_{7+6k} \square C_n$, insert three new rows, the first 2 being of Type-β and Type-γ respectively. For the third of these new rows, add the vertices in columns $3i$ (where $i = 1,\ 2, \cdots, t-1$) and $n-2$ to the decycling set.

Now, following the original Type-β row, insert another three new rows. For the first of these three new rows, we select the vertices in columns $3i$ (where $i = 1,\ 2, \cdots, t-1$), $n-2$ and n to add to the decycling set. For the second row, select the vertices in columns $3i-1$ (where $i = 1,\ 2, \cdots, t-1$) and $n-3$. For the third row, delete vertices so that it is a Type-β row (see Figure 3.9 for the expansion pattern where $n = 13$). We now have a minimum decycling set of $C_{7+6(k+1)} \square C_n$.

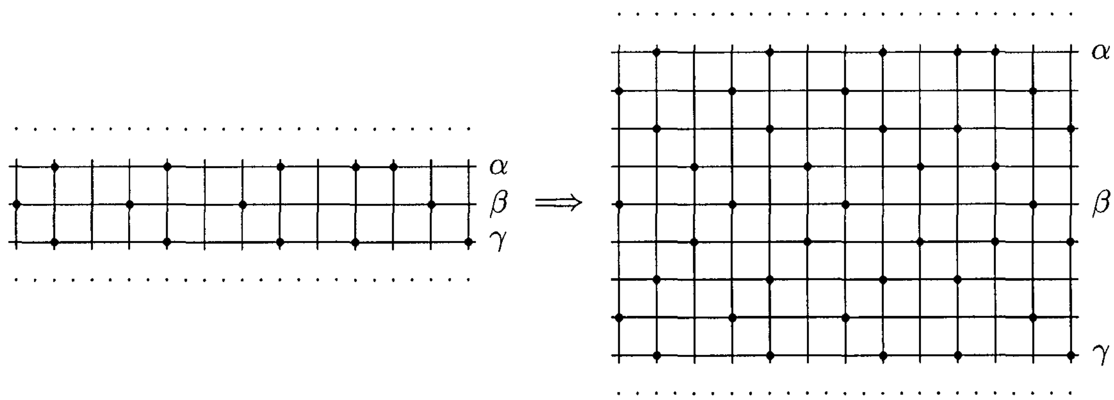


Figure 3.9: Expansion of decycling set for $m \equiv 1 \pmod 6$, $n \equiv 1 \pmod 3$

Note that the new graph, $C_{7+6(k+1)} \square C_n$, contains 3 consecutive rows that are of Type-α, β, γ, and hence we can iterate this expansion procedure.

ii. $n \equiv 2 \pmod 3$.

Similar to the previous case, start from the decycling set of $C_7 \square C_n$. A row is Type-α if its deleted vertices are in the same columns as those of the fourth row of $C_7 \square C_n$ (i.e. row j is Type-α if the set of vertices removed from it is $\bigcup_{i=1}^{t}\{v_{j,3i-1}\} \cup \{v_{j,n-1}\}$, where $n = 3t+2$). Similarly, Type-β (resp. Type-γ) rows are those with a configuration identical to that of the fifth $\left(\bigcup_{i=1}^{t}\{v_{j,3i-2}\}\right)$ (resp. sixth $\left(\bigcup_{i=1}^{t}\{v_{j,3i}\} \cup \{v_{j,3t+2}\}\right)$) row of $C_7 \square C_n$.

Focusing on the 3 consecutive rows that are, in order, of Type-α, β, γ in $C_{7+6k} \square C_n$, for some $k \geqslant 0$, we insert six new rows and obtain a minimum decycling set of $C_{7+6(k+1)} \square C_n$. Following the row of Type-α in $C_{7+6k} \square C_n$, insert three new rows, being of Type-β, Type-α and Type-γ in that order, and

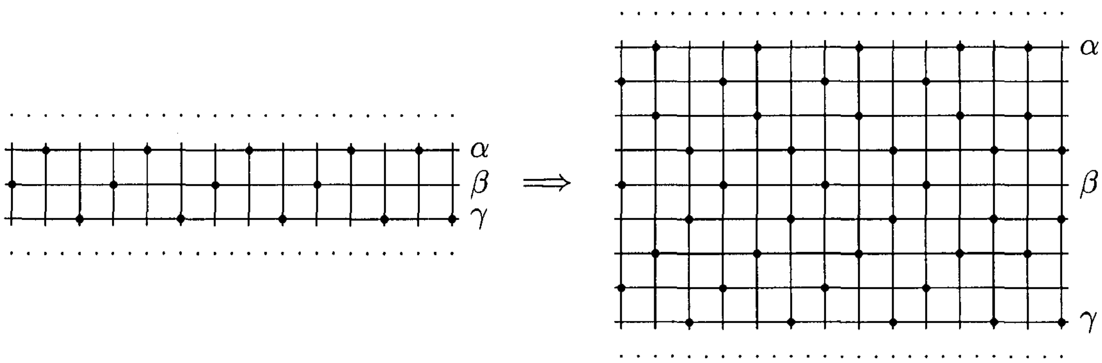


Figure 3.10: Expansion of decycling set for $m \equiv 1 \pmod 6$, $n \equiv 2 \pmod 3$

now following the original Type-β row, insert another three new rows, being of Type-γ, Type-α and Type-β in order (see Figure 3.10 for the expansion pattern where $n = 14$). We now have the decycling set of $C_{7+6(k+1)} \square C_n$.

Note that the new graph, $C_{7+6(k+1)} \square C_n$, contains three consecutive rows that are of Type-α, β, γ, so we can iterate the expansion procedure.

In each case we obtain a decycling set S of size $2rn + \left\lceil \frac{n+2}{3} \right\rceil$ in $C_m \square C_n$. Moreover, the iteration procedure does not contribute to $\varepsilon(S)$ and hence we conclude that $\omega\big((C_m \square C_n) - S\big) = 1$. $\square$

Lemma 3.3.2 *If $m = 6r + 5$, then $\nabla(C_m \square C_n) = 2rn + \left\lceil \frac{5n+2}{3} \right\rceil$.*

Proof. Similar to Lemma 3.3.1, we have the lower bound of the decycling set, $\nabla(C_{6r+5} \square C_n) = 2rn + \left\lceil \frac{5n+2}{3} \right\rceil$. We can start from the decycling set for $C_5 \square C_n$ described in Lemma 3.2.9, and insert $6r$ rows of vertices to get the decycling set for $C_{6r+5} \square C_n$. We divide this case into 2 subcases:

i. $n \equiv 1 \pmod 3$.

Starting from the decycling set of $C_5\Box C_n$, a row is Type-α if its deleted vertices are in the same columns as those of the third row of $C_5\Box C_{3t+1}$ (i.e. row j is Type-α if the set of vertices removed from it is $\bigcup_{i=1}^{t+1}\{v_{j,3i-2}\}$), where $n = 3t+1$. Similarly, Type-β (resp. Type-γ) rows are those with a configuration identical to that of the fourth $\left(\bigcup_{i=1}^{t}\{v_{j,3i-1}\}\cup\{v_{j,n}\}\right)$ (resp. fifth $\left(\bigcup_{i=1}^{t}\{v_{j,3i}\}\right)$) row of $C_5\Box C_{3t+1}$.

Focus on 3 consecutive rows that are, in order, of Type-α, β, γ in $C_{5+6k}\Box C_{3t+1}$ for some $k \geqslant 0$. We now insert three new rows following the Type-α row in $C_{5+6k}\Box C_{3t+1}$, the first two being of Type-β, Type-γ respectively. For the third of these new rows, add the vertices in columns $3i-2$ ($i = 1, 2, \cdots, t$) to the decycling set.

Then following the original Type-β row, insert another three new rows. For the first of these rows, we select the vertices in columns $3i+1$ ($i = 1, 2, \cdots, t-1$) and $3t$ to add to the decycling set. For the second row, we select the vertices in columns 1 and $3i$ ($i = 1, 2, \cdots, t-1$), for the third one, we delete vertices so that it is a Type-β row (see Figure 3.11 for the expansion pattern where $n = 13$). We now have a minimum decycling set of $C_{5+6(k+1)}\Box C_{3t+1}$.

ii. $n \equiv 2 \pmod 3$.

Beginning with the decycling set of $C_5\Box C_n$, a row is Type-α if its deleted

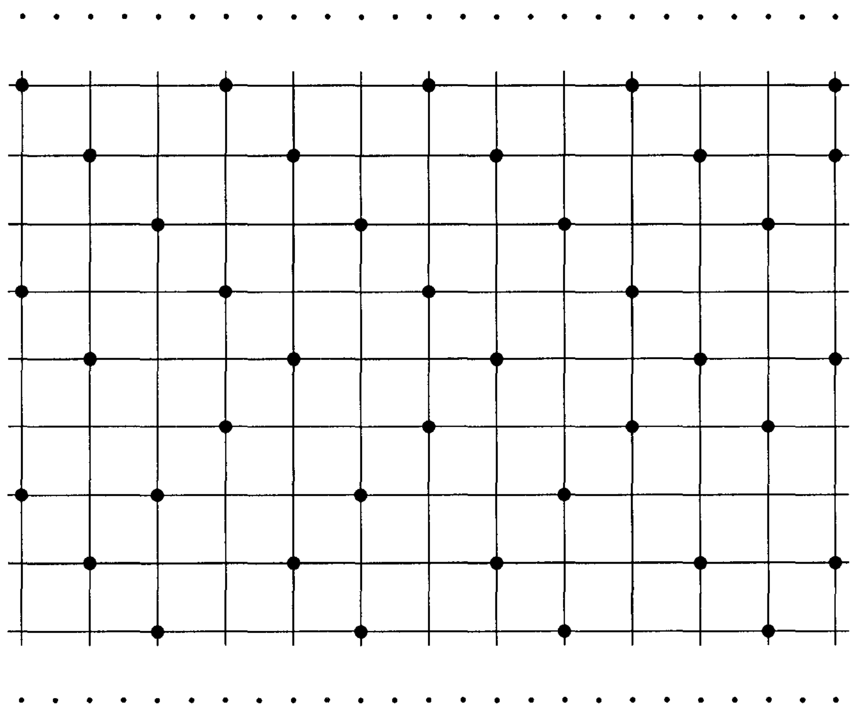


Figure 3.11: Expansion result for $m \equiv 5 \pmod 6$, $n \equiv 1 \pmod 3$

vertices are in the same columns as those of the second row of $C_5 \Box C_{3t+2}$ which was described in Lemma 3.2.9 (i.e. row j is Type-α if the set of vertices removed from it is $\bigcup_{i=1}^{t}\{v_{j,3i-1}\}$ where $n = 3t+2$). Similarly, Type-β (resp. Type-γ) rows are those with a configuration identical to that of the third $\left(\bigcup_{i=1}^{t+1}\{v_{j,3i-2}\}\right)$ (resp. fourth $\left(\bigcup_{i=1}^{t+1}\{v_{j,3i-1}\}\right)$) row of $C_5 \Box C_{3t+2}$.

Focus on any three consecutive rows that are, in order, of Type-α, β, γ in $C_{5+6k} \Box C_n$, for some $k \geqslant 0$. Following the Type-α row, insert three new rows, the first 2 being Type-β and Type-γ respectively. For the third of these new rows, add the vertices in columns $3i$ (where $i = 1, 2, \cdots, t$) to the decycling

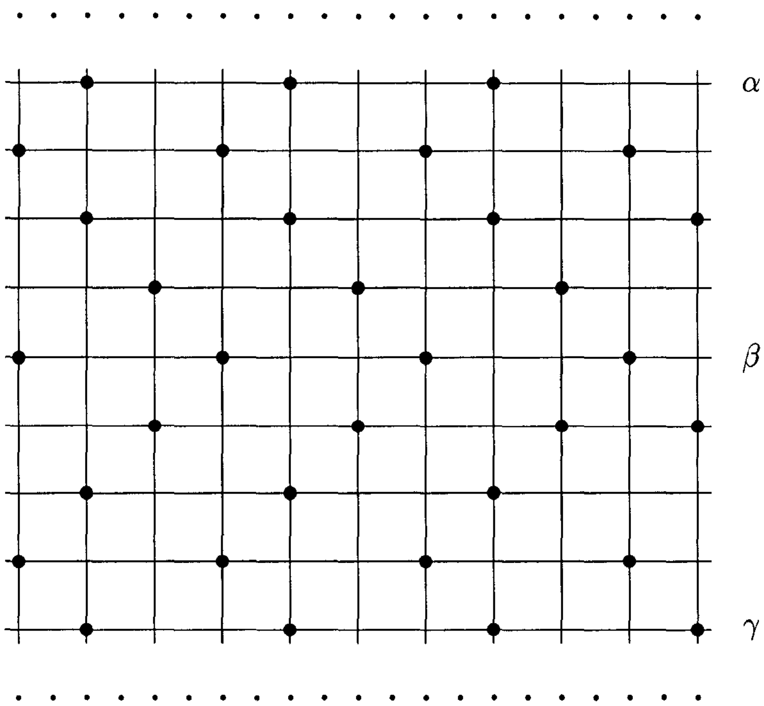


Figure 3.12: Expansion result for $m \equiv 5 \pmod 6$, $n \equiv 2 \pmod 3$

set.

Then, following the original Type-β row, insert another three new rows. For the first of these rows, we select the vertices in columns $3i$ (where $i = 1, 2, \cdots, t$) and n to add to the decycling set. For the remaining two, delete vertices so that they are of Type-α and Type-β in order (see Figure 3.12 for the expansion pattern where $n = 11$). We now have a minimum decycling set for $C_{5+6(k+1)} \Box C_n$.

Note that in both cases, the new graph, $C_{5+6(k+1)} \Box C_n$, contains 3 consecutive rows that are of Type-α, β, γ, and therefore we can iterate the expansion procedure to obtain a decycling set S of size $2rn + \left\lceil \frac{5n+2}{3} \right\rceil$ in $C_m \Box C_n$. Moreover $\omega\big((C_m \Box C_n) - S\big) = 1$ and so $(C_m \Box C_n) - S$ is a maximum induced tree in $C_m \Box C_n$. $\Box$

We use another method to deal with the remaining cases.

Lemma 3.3.3 *If* $m \equiv 2$ *or* $4 \pmod 6$ *and* $n \equiv 2$ *or* $4 \pmod 6$, *then* $\nabla(C_m \Box C_n) = \left\lceil \frac{mn+2}{3} \right\rceil$.

Proof. For these cases, both m and n are even, so we use the similar method described in Lemma 3.2.8. Let k be the least non-negative integer such that at least one of $\frac{m}{2^k}$ or $\frac{n}{2^k}$ is odd or equals 8, and let $m_i = \frac{m}{2^{k-i}}$ and $n_i = \frac{n}{2^{k-i}}$ for each $i = 0, 1, \cdots, k$. Then we can find a minimum decycling set S_0 of cardinality $\left\lceil \frac{m_0 n_0 + 2}{3} \right\rceil$ in $C_{m_0} \Box C_{n_0}$ such that $\omega\big((C_{m_0} \Box C_{n_0}) - S_0\big) = 1$. Now, for each $i = 0, 1, \cdots k-1$, apply Theorem 3.2.5 to $C_{m_i} \Box C_{n_i}$ to construct a minimum decycling set S_{i+1} of size $\nabla(C_{m_0} \Box C_{n_0}) + \frac{mn}{2^{2k}} \sum_{j=0}^{i} 4^j$ in $C_{m_{i+1}} \Box C_{n_{i+i}}$ such that $\omega\big((C_{m_{i+1}} \Box C_{n_{i+1}}) - S_{i+1}\big) = 1$. It follows that $\nabla(C_m \Box C_n) = \nabla(C_{m_0} \Box C_{n_0}) + \frac{mn}{2^{2k}} \sum_{j=0}^{k-1} 4^j = \left\lceil \frac{mn+2}{3} \right\rceil$. $\Box$

We have now completely solved not only the problem of finding a minimum decycling set in $C_m \Box C_n$, but also the problem of finding a maximum induced tree. We summarize the cardinalities of each set of vertices:

Theorem 3.3.4 *Let* $m \geqslant 3$ *and* $n \geqslant 3$ *be integers. Then*

$$\nabla(C_m \Box C_n) = \begin{cases} \left\lceil \frac{3n}{2} \right\rceil & \textit{if } m = 4, \\ \left\lceil \frac{3m}{2} \right\rceil & \textit{if } n = 4, \\ \left\lceil \frac{mn+2}{3} \right\rceil & \textit{otherwise} \end{cases}$$

and

$$t(C_m \square C_n) = \begin{cases} 9 & \text{if } m = n = 4, \\ mn - \nabla(C_m \square C_n) & \text{otherwise.} \end{cases}$$

3.4 Remarks

By removing the row and column which have the maximum number of vertices in the minimum decycling set of $C_{m+1} \square C_{n+1}$, we obtain a (not necessarily minimum) decycling set for $P_m \square P_n$. So we have the following corollary.

Corollary 3.4.1 *Let $m, n > 3$ be integers. Then*

$$\nabla(P_m \square P_n) \leqslant \begin{cases} \left\lceil \dfrac{(m+1)(n+1)+2}{3} \right\rceil - \dfrac{m+1}{2} - \dfrac{n+1}{2} & \text{if both } m \text{ and } n \text{ are odd,} \\ \left\lceil \dfrac{(m+1)(n+1)+2}{3} \right\rceil - \left\lceil \dfrac{m+1}{3} \right\rceil - \left\lceil \dfrac{n+1}{3} \right\rceil & \text{otherwise.} \end{cases}$$

Proof. If both m and n are odd, we apply the technique described in Theorem 3.2.5. Consider a minimum decycling set for $C_{\frac{m+1}{2}} \square C_{\frac{n+1}{2}}$, then use the doubling construction to get a decycling set for $C_{m+1} \square C_{n+1}$ in which there exist a row and a column such that half of the vertices in those row and column are in the decycling set of $C_{m+1} \square C_{n+1}$. Hence $\nabla(P_m \square P_n) \leqslant \nabla(C_{m+1} \square C_{n+1}) - \dfrac{m+1}{2} - \dfrac{n+1}{2}$.

Otherwise, in the minimum decycling set for $C_{m+1} \square C_{n+1}$ which was described in Sections 3.2 and 3.3, we can select a row and a column, one third of the vertices of which are deleted for decycling. □

We observe that the upper bound for $\nabla(P_m \Box P_n)$ obtained in Corollary 3.4.1 is comparable to the upper bound of Beineke and Vandell [9, Theorem 5.4], and when m and n are both odd, it is as good as the upper bound obtained by Caragiannis et al. [15, Theorem 6].

Chapter 4

Decycling Fibonacci Cubes

In this chapter, we discuss the decycling number for the n-dimensional Fibonacci cubes Γ_n, which are a family of graphs with applications as interconnection topologies. We present lower and upper bounds of the decycling number for the Fibonacci cubes and the exact value of the decycling number for the Fibonacci cubes of order $n \leqslant 7$. The main results in this chapter have been accepted for publication (see [48]). To simplify notation, we write ∇_n for $\nabla(\Gamma_n)$ throughout this chapter.

4.1 Introduction

In Chapter 1, we introduced an interconnection topology called the n-dimensional Fibonacci cube. It was first introduced by Hsu (see [29]) in 1993. More than 20 papers were subsequently published discussing the graph properties of the Fibonacci cubes.

In [29], Hsu gave the formula to compute the number of edges and studied the embedding relation between the hypercube and Fibonacci cube. Also, Hsu presented a very useful *Fibonacci cube decomposition lemma* in [29], which will be reviewed in the next section of this chapter.

Chung, Hsu and Liu [39] extended the standard Fibonacci cube to the k-th order Fibonacci cube of dimension n (denoted Γ_n^k and sometimes called generalized Fibonacci cube, which can be viewed as the graph obtained by removing every node in Q_{n-k} that has k or more consecutive 1's in its binary expression) and studied the longest cycles in Γ_n^k. Below is the main result in [39]

1. For $k = 2$, Γ_n is Hamiltonian if $n = 3i$ for some integer $i \geqslant 2$; otherwise, the length of the longest cycle is $p(\Gamma_n) - 1$.

2. For $k \geqslant 3$, Γ_n^k is Hamiltonian if n can be expressed as one of the following forms: $(k+1)i, (k+1)i-3, (k+1)i-4, \ldots, (k+1)i-k$ for some integer $i \geqslant 2$; otherwise, the length of the longest cycle is $p(\Gamma_n^k) - 1$.

In 1996, Zagaglia Salvi [58] investigated the problem of the existence of cycles of every even length in Γ_n^k.

Theorem 4.1.1 ([58]) *For $n \geqslant 7$, every edge of the Fibonacci cube Γ_n belongs to cycles of every even length.*

Theorem 4.1.2 ([58]) *For $k \geqslant 3$ and $n \geqslant k+2$, every edge of Γ_n^k belongs to cycles of every even length.*

An edge colouring of G is vertex-distinguishing if distinct vertices are assigned distinct colour sets, where the *colour set* of a vertex v is the set of colours assigned to the edges incident to v. The *observability* of G is the minimum number of colours in a proper edge colouring that is vertex-distinguishing [16]. In 2002, Dedó, Torri and Zagaglia Salvi ([17]) studied the observability of the Fibonacci cubes, and proved that the observability of Γ_n is n for $n \geqslant 4$.

Table 4.1: Decycling number of Γ_n.

n	$\lvert V_n \rvert$	$\lvert E_n \rvert$	∇_n
1	2	1	0
2	3	2	0
3	5	5	1
4	8	10	1
5	13	20	3
6	21	38	6
7	34	71	11
8	55	130	19
9	89	235	33
10	144	420	53-55
11	233	744	86-94
12	377	1308	139-158
13	610	2285	225-264
14	987	3970	364-439

In [44], Munarini and Zagaglia Salvi obtained the independence number $\alpha(\Gamma_n) =$

$\left\lceil \frac{p(\Gamma_n)}{2} \right\rceil = \left\lceil \frac{f_n}{2} \right\rceil$.

In Section 4.2, we will discuss lower bounds on ∇_n as well as the exact value of ∇_n for small values of n. In Section 4.3, we present upper bounds on ∇_n. Our results, as they apply to Γ_n for $1 \leqslant n \leqslant 14$, are summarized in Table 4.1.

4.2 Lower Bound

To establish our first lower bound, we will take advantage of some properties of Γ_n that we now review.

Lemma 4.2.1 ([29], Decomposition of Fibonacci cubes) *Let $\Gamma_n = (V_n, E_n)$ denote the Fibonacci cube of order n where $n \geqslant 2$. Let $\mathrm{LOW}(n)$ $\big($resp. $\mathrm{HIGH}(n)\big)$ denote the subgraph induced by the set of vertices $\{0, 1, \cdots, f_{n-1} - 1\}$ $\big($resp. $\{f_{n-1}, f_{n-1} + 1, \cdots, f_n - 1\}\big)$. Then*

1. $\mathrm{LOW}(n) \cong \Gamma_{n-1}$;

2. $\mathrm{HIGH}(n) \cong \Gamma_{n-2}$.

Moreover, if $\mathrm{LINK}(n) = \big\{\{i,j\} \,:\, \big|i-j\big| = f_{n-1}, \{i,j\} \in E_n\big\}$, then the two disjoint subgraphs $\mathrm{LOW}(n)$ and $\mathrm{HIGH}(n)$ are connected exactly by the set of edges $\mathrm{LINK}(n)$.

By Lemma 4.2.1, the Fibonacci cube Γ_n of order n can be decomposed into two disjoint subgraphs Γ_{n-1} and Γ_{n-2}. Therefore, to decycle Γ_n, we need to decycle each of the two subgraphs, yielding the following lower bound for the decycling number of Γ_n.

Corollary 4.2.2 $\nabla_n \geqslant \nabla_{n-1} + \nabla_{n-2}$.

In [29], Hsu gave a formula to calculate the number of edges in Γ_n in terms of the Fibonacci numbers $\left(|E_n| = \dfrac{2(n+1)f_n - (n+2)f_{n-1}}{5}\right)$. In order to establish lower bounds and exact values of ∇_n for small n, we will also want to know about the degree sequence of Γ_n. If we let $d_n(i)$ denote the degree of vertex i in Γ_n, then we have the following lemma:

Lemma 4.2.3 *For $n \geqslant 2$,*

$$d_n(i) = \begin{cases} d_{n-1}(i) + 1, & 0 \leqslant i < f_{n-2}, \\ d_{n-1}(i), & f_{n-2} \leqslant i < f_{n-1}, \\ d_{n-2}(i - f_{n-1}) + 1, & f_{n-1} \leqslant i < f_n, \end{cases}$$

where $d_0(0) = 0$, $d_1(0) = 1$, and $d_1(1) = 1$. The maximum degree $\Delta(\Gamma_n) = n$, and vertex 0 is the only vertex of degree n. For $n \geqslant 4$, vertices 1 and f_{n-1} are the only vertices of degree $n - 1$.

Proof. For $d_n(i)$, the proof follows immediately from Lemma 4.2.1. The remaining statements easily follow by induction, again using Lemma 4.2.1. □

We next prove a sequence of lemmas that will provide the decycling number of the n-dimensional Fibonacci cubes for $n \leqslant 7$.

Lemma 4.2.4 *The decycling number $\nabla_4 = 1$, and there is only one ∇-set (for which the corresponding maximum induced forest is isomorphic to the path P_7).*

Proof. The value of ∇_4 follows from Corollary 4.2.2 and Figure 4.1 in which the vertex labelled by "o" is in S. It is easy to see $S = \{0\}$ is the unique ∇-set for Γ_4, and the left-over graph $\Gamma_4 - S$ is a path of order 7, so we also find the path number of Γ_4. □

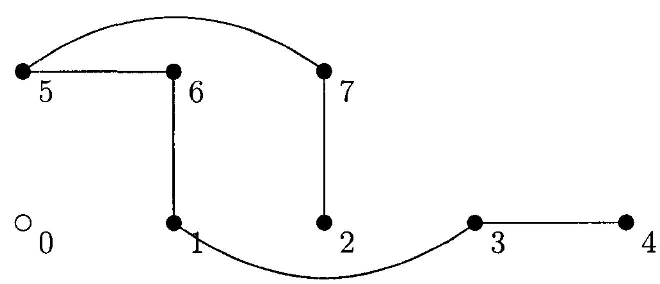


Figure 4.1: A decycling set of Γ_4.

Lemma 4.2.5 $\nabla_5 = 3$.

Proof. By Corollary 4.2.2, $\nabla_5 \geqslant 2$. To decycle Γ_5, we need to decycle the two disjoint subgraphs LOW(5) and HIGH(5). If $\nabla_5 = 2$, then we need to remove the unique ∇-set in LOW(5), and one more vertex in HIGH(5) to decycle HIGH(5), i.e. vertex 8, 9, 11 or 12. In each case it is a simple exercise to see that Γ_5 contains a cycle even after deleting both vertex 0 and vertex i, for $i \in \{8, 9, 11, 12\}$. Hence $\nabla_5 \geqslant 3$. Figure 4.2 shows that there exists a decycling set of size 3 in Γ_5. Note in Figure 4.2, the maximum induced forest is a path of order 10, so we also get the path number for Γ_5. □

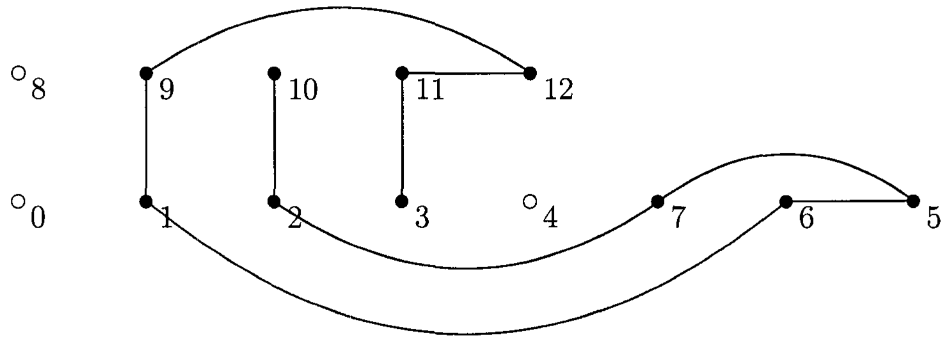


Figure 4.2: A decycling set of Γ_5.

By using computers to do an exhaustive search in Γ_5, we found 6 decycling sets of size 3. Two of them $\big(\{0,4,9\}$ and $\{0,9,11\}\big)$ are independent; the other four are $\{0,1,11\}$, $\{0,1,12\}$, $\{0,4,8\}$, and $\{0,8,12\}$.

Lemma 4.2.6 $\nabla_6 = 6$.

Proof. By Corollary 4.2.2, $\nabla_6 \geqslant 4$. If $\nabla_6 = 4$, then by Lemma 4.2.3 at most $\Delta(\Gamma_6) + 2(\Delta(\Gamma_6) - 1) + (\Delta(\Gamma_6) - 2) = 20$ edges are removed, and at least 18 edges remain, which is too many for a forest on 17 vertices. Hence $\nabla_6 \geqslant 5$.

If $\nabla_6 = 5$, we count the number of vertices and edges in the left-over graph. Suppose that S is a decycling set of size 5 in Γ_6. If the degree sequence of the vertices in S is $6,5,5,4,4$, then observe that both vertices of degree 5 are adjacent to the vertex of degree 6. Also it is easy to verify that each vertex of degree 4 in Γ_6 has at least one neighbour of degree 5 or 6. So $\varepsilon(S) \geqslant 4$ (where $\varepsilon(S)$ denotes the number of edges in the induced subgraph $\Gamma_n[S]$), and at least 18 edges are left in $\Gamma_6 - S$, which is too many for a forest on 16 vertices. If the degree sequence is $6,5,4,4,4$, we have

$\varepsilon(S) \geqslant 1$, and at least 16 edges remaining, which is still too many for a forest on 16 vertices. Any other degree sequence of length 5 will also result in $\Gamma_6 - S$ having at least 16 edges. So we have $\nabla_6 \geqslant 6$. $\{0, 4, 7, 8, 14, 18\}$ is a decycling set of size 6 for Γ_6. □

By computer search, we found 19 decycling sets of size 6 for Γ_6 (5 are independent decycling sets). More details are available in Appendix 1.

Lemma 4.2.7 $\nabla_7 = 11$.

Proof. By Corollary 4.2.2, we have $\nabla_7 \geqslant 3 + 6 = 9$. By using techniques similar to those described in Lemma 4.2.6, we can easily improve the lower bound to get $\nabla_7 \geqslant 10$.

Assume that $\nabla_7 = 10$ and S is a decycling set of size 10 in Γ_7. Since $|V_7| = 34$, $|E_7| = 71$, and $\Gamma_7 - S$ has 24 vertices, that means at most 23 edges are left in $\Gamma_7 - S$. Define $E' = E_7 - E(\Gamma_7 - S)$. In order for $\Gamma_7 - S$ to be a forest, it is necessary that $|E'| \geqslant 71 - 23 = 48$. By observation, we also have the following useful information in Γ_7: all the neighbours of vertex 0 have degree 6 or degree 5; there are 8 vertices of degree 5; each of vertices 1 and 21 has 2 neighbours of degree 5; $\{3, 4\}, \{8, 29\} \in E_7$. There are the following 5 cases:

1. All the vertices of degree 7 and 6 are in S. There are two subcases:

   (a) There exists no degree 4 vertex in S. Notice that each vertex of degree 5 has a neighbour of degree 6 or 7, then $\varepsilon(S) = |E(\Gamma_7[S])| > 6$. Hence

$|E'| \leqslant 47$.

(b) There exists at least one degree 4 vertex in S. We have $\varepsilon(S) \geqslant 6$. Hence $|E'| \leqslant 47$.

2. Vertex 0 is in S, exactly one of vertices 1 or 21 is not in S.

(a) All vertices of degree 5 are in S. Then $\varepsilon(S) \geqslant 6$ and $|E'| \leqslant 47$.

(b) At least one of degree 5 vertex is not in S. Then $|E'| \leqslant 47$.

3. Vertex 0 and exactly one of vertices 1 or 21 are not in S.

(a) All vertices of degree 5 are in S.

(b) At least one vertex of degree 5 is not in S.

For above two subcases, it is easy to verify that $|E'| \leqslant 47$.

4. Vertices 1 and 21 are in S, vertex 0 is not in S. There are several subcases:

(a) All vertices of degree 5 are in S, which means that $\Gamma_7 - S$ has at least 2 components, and $\varepsilon(S) \geqslant 4$. Hence $|E'| \leqslant 48$, and at least 23 edges remain in $\Gamma_7 - S$, which is too many for a forest on 24 vertices with 2 components.

(b) Exactly one vertex of degree 5 is not in S. Then $\varepsilon(S) \geqslant 4$. Hence $|E'| \leqslant 47$.

(c) Exactly two vertices of degree 5 are not in S. Then either $\Gamma_7 - S$ has at least two components and $|E'| \leqslant 48$, or $\varepsilon(S) \geqslant 4$ which means $|E'| \leqslant 46$.

(d) Exactly three vertices of degree 5 are not in S. Either $\varepsilon(S) \geqslant 2$ which means $|E'| \leqslant 47$, or $\Gamma_7 - S$ has at least 3 components and $|E'| \leqslant 49$. Hence 22 edges remain which is too many for a forest with 3 components and 24 vertices.

(e) Exactly four vertices of degree 5 are not in S. Either $\Gamma_7[S]$ has an edge, so $|E'| \leqslant 47$, or this forces using vertices 12, 25, 30, and 32 (all of degree 4) to avoid any edges in $\Gamma_7[S]$, but then this isolates, for example, vertex 9 or 24, and hence increases the number of components.

(f) More than 5 vertices of degree 5 are not in S. In this case, $|E'| \leqslant \sum_{v \in S} d_7(v) \leqslant 47$.

5. Vertices 0, 1 and 21 are not in S. The only possibility is all the vertices of degree 5 are in S. Hence $\varepsilon(S) = 2$ and $|E'| \leqslant 46$.

In all of the above cases, there are not enough edges removed for decycling, and therefore $\nabla_7 \geqslant 11$. By a computer search, we find 192 decycling sets of size 11 in Γ_7 (18 are independent). These decycling sets are listed in Appendix 1. □

Computer Search Heuristics. For $n \leqslant 7$, an exhaustive computer search to find all the decycling sets of size ∇_n in Γ_n is possible. For $n \geqslant 8$, Lemma 4.2.1 suggests the following heuristic to find $\nabla(\Gamma_n)$

Step 1. Find all minimal decycling sets of size $\nabla_{n-2}+i$ in Γ_{n-2}, where $i = 0, 1, \cdots$. Do the same thing in Γ_{n-1}, obtaining all minimal decycling sets of size $\nabla_{n-1}+j$

$(j = 0, 1, \cdots)$.

Step 2. By the Decomposition Lemma(4.2.1), if S is a decycling set in Γ_n, then $S \cap V(\mathrm{LOW}(n))$ must be a decycling set in Γ_{n-1}, and $S \cap V(\mathrm{HIGH}(n))$ must be a decycling set in Γ_{n-2}. Therefore, to find a decycling set of size $\nabla_{n-1} + \nabla_{n-2} + k$ in Γ_n, we test all the combinations of a decycling set of size $\nabla_{n-2} + i$ in Γ_{n-2} and a decycling set of size $\nabla_{n-1} + j$ in Γ_{n-1} together with x more vertices in the left-over graph of Γ_n, where $i + j + x = k$, and i, j, x are non-negative integers.

Such a computer search shows that there is no decycling set of size less than 19 and 33 for Γ_8 and Γ_9 respectively, so $\nabla_8 = 19$ and $\nabla_9 = 33$. The details of the results are listed in Table 4.2, for $n \leqslant 9$. Blank entries in the table correspond to computational tasks that are prohibitively time consuming.

Table 4.2: Computer search results of decycling sets for Γ_n

Γ_n	$\lvert V_n \rvert$	∇_n	Number of minimal decycling sets of size $\nabla_n + i$								
			$i=0$	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$
5	13	3	6	58	36	4	0	0	0	0	0
6	21	6	19	704	1933	639	65	0	0	0	0
7	34	11	192	8528	113175	454916	453985	112185	7996	96	0
8	55	19	33	10649	778540	21836552					
9	89	33	58								

Based on the computer search, we found that $\nabla_n = \nabla_{n-1} + \nabla_{n-2} + \mu(n)$ where $\mu(n)$ is small integer value (for $n = 4, 5, 6, 7, 8, 9$ it would be $0, 1, 2, 2, 2, 3$ respectively). This observation leads to the following conjecture:

Conjecture 4.2.8 $\nabla_n = \nabla_{n-1} + \nabla_{n-2} + \mu(n)$*, where* $\mu(n)$ *is a non-decreasing function of* n*.*

From Table 4.2, we found that there are still too many choices to test to feasibly find the decycling number of Γ_n if $n \geqslant 10$. In order to decrease the number of choices, we then consider the independent decycling sets, denoting the independent decycling number of Γ_n by ∇_n^0. We have already noted that $\nabla_5^0 = 3$, $\nabla_6^0 = 6$ and $\nabla_7^0 = 11$. By Corollary 4.2.2, we know that $\nabla_8^0 \geqslant 6 + 11 = 17$, and from the results summarized in Table 4.2, $\nabla_8^0 \geqslant 19$. However, it can also be argued theoretically that $\nabla_8^0 = 19$ by considering a line of reasoning similar to that presented in the proof of Lemma 4.2.7.

As with ∇_n, determining exact values of ∇_n^0 becomes increasingly difficult as n increases. We can, however, make some progress by using the same method described earlier, and noting that if S is an independent decycling set of Γ_n, then $S \cap \mathrm{LOW}(n)$ $\big($resp. $S \cap \mathrm{HIGH}(n)\big)$ must be an independent decycling set for Γ_{n-1} (resp. Γ_{n-2}).

By such an exhaustive search, there is no independent decycling set of size less than 55 and 94 for Γ_{10} and Γ_{11} respectively, so $\nabla_{10}^0 = 55$ and $\nabla_{11}^0 = 94$. The details of the results for the independent decycling sets for Γ_n are presented in Table 4.3 and the corresponding decycling sets are listed in Appendix 2.

Table 4.3: Computer search results of independent decycling sets for Γ_n

Γ_n	$\lvert V_n\rvert$	∇_n^0	Number of minimal independent decycling sets of size $\nabla_n^0 + i$								
			$i=0$	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$	$i=6$	$i=7$	$i=8$
5	13	3	2	11	7	1	0	0	0	0	0
6	21	6	5	16	7	0	1	0	0	0	0
7	34	11	18	53	34	4	1	0	0	0	0
8	55	19	6	162	586	392	92	4	0	0	0
9	89	33	6	818	6325	10437	7358	2455	394	13	0
10	144	55	1	135	9100	102334	444544	899721	273817		
11	233	94	166								

Based on Table 4.3, we found that the decycling number and the independent decycling number of Γ_n are equal for $n \leqslant 9$, which suggests this conjecture:

Conjecture 4.2.9 *For the n-dimensional Fibonacci cube* Γ_n,$\nabla_n = \nabla_n^0$.

4.3 Upper Bound

In Section 4.2, we studied the lower bound of the decycling number of Fibonacci cubes, and got the exact decycling number of Γ_n when n is small. In this section, we will discuss the upper bound of the decycling number of Γ_n.

By Lemma 4.2.1, we have the following upper bound:

Lemma 4.3.1 $\nabla_n \leqslant \nabla_{n-1} + \left\lfloor \frac{f_{n-2}}{2} \right\rfloor$.

Proof. Γ_n can be decomposed into two disjoint subgraphs which are isomorphic to Γ_{n-1} and Γ_{n-2} respectively. And these two subgraphs are connected by LINK(n) (Lemma 4.2.1). Consider the subgraph induced by a maximum induced forest in LOW(n) and a maximum independent set in HIGH(n); such a graph is clearly acyclic. Since Γ_n is a bipartite graph, it contains a maximum independent set of size at least $\left\lceil \frac{f_n}{2} \right\rceil$, and in [44] it was proved that Γ_n has independence number $\alpha(\Gamma_n) = \left\lceil \frac{f_n}{2} \right\rceil$. So $\nabla_n \leqslant \nabla_{n-1} + f_{n-2} - \left\lceil \frac{f_{n-2}}{2} \right\rceil$, and the upper bound follows. □

Furthermore, we found several independent decycling sets of size 94 in Γ_{11}. After analyzing these 166 decycling sets, we observed that there always exist 19 components in the left-over graph $\Gamma_{11} - S$. This may be used to decrease the upper bound of ∇_n when $n \geqslant 12$. Here is the idea:

Consider the induced forest F formed by the method described in Lemma 4.3.1. The number of components in F cannot be less than the number of components in the induced forest in LOW(n). Let (X, Y) be the bipartition of HIGH(n), in which one of X or Y consists of n-bit strings (each beginning with 10) having an odd number of 1's (i.e. having odd Hamming weight), and the other set consists of n-bit strings having an even number of 1's. Let $Z \in \{X, Y\}$, such that each vertex of Z is in F and let $Z' = V\big(\text{HIGH}(n)\big) - Z$ be the other set in $\{X, Y\}$. For each vertex $v \in Z'$, if no two of the undeleted neighbours of v in Γ_n are located in the same component of F, then we can undelete v, and the resultant graph clearly remains acyclic. This decreases the upper bound of ∇_n by one. The vertices in Z' whose neighbours are in different components of F form a vertex subset we denote by A. Let C denote the components of F and then form a bipartite graph B with bipartition (A, C), where each component in F and each vertex in A will be a vertex in B, and $E(B) = \{(i, C_j) : i \in A, C_j \in C, |N_{\Gamma_n}(i) \cap V(C_j)| = 1\}$. Here $N_G(v)$ denotes the neighbours of vertex v in G. We then find a minimum decycling set S_A of B with the property that $S_A \subseteq A$. Then in Γ_n, remove the vertices in $A - S_A$ from S; the resultant graph is acyclic.

Since B has fewer vertices than Γ_n, we can easily find a minimum decycling set of B by choosing vertices in A. Also we noticed that the size of A is typically smaller than the size of C in $B = (A, C)$.

By pursuing this idea we were able to reduce the upper bound on ∇_{12} to 158 from the value of 166 obtained from Lemma 4.3.1; this new upper bound is reflected in Table 4.1. Examples of decycling sets matching the presented upper bound for Γ_{12}, Γ_{13} and Γ_{14} are archived in Appendix 3.

Chapter 5

Dominating Cubes

In this chapter, we deal with the problem of dominating cubes. Let $\gamma(G)$, $\gamma_i(G)$, $\gamma_p(G)$ and $\gamma_{\leqslant k}(G)$ denote the domination, the independent domination, the perfect domination and the distance-k domination numbers of the graph G, respectively. We first investigate the domination number for the Fibonacci cube of order n. Then we turn to discuss the domination parameters for the hypercube of order n. An improved lower bound for the domination number $\gamma(Q_n)$, an improved upper bound for the independent domination number $\gamma_i(Q_n)$ and a lower bound for the distance-k domination number $\gamma_{\leqslant k}(Q_n)$ are obtained. Also, we discuss the exact values for the domination, the perfect domination and the independent domination numbers for Q_8. The main results in Section 5.3 have been submitted for possible publication ([49]).

5.1 Introduction

A vertex subset D is a *dominating* set of a graph G if each vertex in $V(G)$ is either in D or is adjacent to a vertex in D. A vertex in D is said to *dominate* itself and all its neighbours. In [55], Weakley briefly mentioned the concept of excess cover, which we now formalize as *over-domination*: the *over-domination* of a graph G with respect to a dominating set D of G is defined as: $OD_G(D) = \left(\sum_{v\in D}\left(\deg_G(v)+1\right)\right) - |V(G)|$, using $OD(D)$ for abbreviation if there is no confusion. For example, if a vertex not in D is dominated by two vertices in D, then it contributes 1 to the over-domination. We develop this idea further in the following lemma.

Lemma 5.1.1 *Let G be a connected graph with p vertices. If D is a dominating set, then*

$$\sum_{v\in D}(\deg(v)+1) \geqslant p\,.$$

Proof. The proof follows that the over-domination of a graph with respect to a dominating set is a non-negative integer. □

The idea of domination has various applications in design and analysis of communication networks, social sciences, optimization, bioinformatics, computational complexity, and algorithm design [24, 25].

Several variations of domination exist. For instance, an *independent dominating* set is a dominating set which is also an independent set. A dominating set D is called a *perfect dominating* set if every vertex in $V - D$ is adjacent to exactly one vertex in D.

The concept of domination can also be extended to distance-k domination. A vertex subset D is a *distance-k dominating* set if every vertex of V is distance at most k from some vertex in D. For this case, a vertex is said to dominate itself and its k-neighbourhood, where the *k-neighbourhood* of a vertex $v \in V(G)$ is a vertex subset

$N_{k,G}(v) = \{u : u \in V(G) \text{ and } d(u,v) \leqslant k\}$. We will abbreviate this to $N_k(v)$ if there is no confusion. If $d_G(u,v) = k$, then u is a distance-k neighbour of v in the graph G.

The *domination number* $\gamma(G)$ is the minimum cardinality of a dominating set of G. Similarly, we define the *independent domination number* $\gamma_i(G)$, the *perfect domination number* $\gamma_p(G)$ and the *distance-k domination number* $\gamma_{\leqslant k}(G)$ to be the minimum cardinality of an independent dominating set, a perfect dominating set and a distance-k dominating set, respectively. Notice that the distance-1 domination number $\gamma_1(G)$ is the standard domination number $\gamma(G)$.

The topic of domination was experiencing significant growth in the last few decades. Haynes, Hedetniemi and Slater have two books, [24, 25], that deal exclusively with domination in graphs. Also, there exist several survey articles on domination in graphs, i.e. [23], [26] and [27].

In Section 5.2, we will discuss the domination number for the Fibonacci cube of order at most 8. In Section 5.3, we focus on domination, independent domination, perfect domination and distance-k domination numbers for the hypercube of order n.

5.2 Dominating Fibonacci Cubes

In Chapter 4, we discussed the decycling number for the n-dimensional Fibonacci cubes. Here, we investigate the domination number of the Fibonacci cube of order n. Recalling Lemma 4.2.1, the Fibonacci cube Γ_n can be decomposed into two vertex disjoint subgraphs Γ_{n-1} and Γ_{n-2}. Dominating both of the two subgraphs is quite enough to dominate Γ_n, which yields the following bounds for the domination number of Γ_n.

Corollary 5.2.1 (a) $\gamma(\Gamma_n) \leqslant \gamma(\Gamma_{n-1}) + \gamma(\Gamma_{n-2})$.

(b) $\gamma(\Gamma_{n-1}) \leqslant \gamma(\Gamma_n)$.

We next prove a sequence of lemmas that provide the domination number of the n-dimensional Fibonacci cubes for $n \leqslant 6$.

Lemma 5.2.2 $\gamma(\Gamma_0) = \gamma(\Gamma_1) = \gamma(\Gamma_2) = 1$.

Proof. It is obvious after refering to Figure 1.5. □

Lemma 5.2.3 $\gamma(\Gamma_3) = 2$

Proof. Refering to Figure 5.1 in which each vertex labelled by "•" is in D, the result is obvious. □

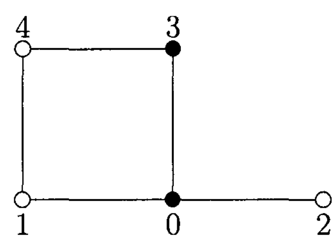


Figure 5.1: A dominating set of Γ_3.

By an exhaustive search in Γ_3, there are 4 dominating sets of size 2 ({0, 1}, {0, 3}, {0, 4} and {2, 4}).

Lemma 5.2.4 $\gamma(\Gamma_4) = 3$.

Proof. By Corollary 5.2.1, $\gamma(\Gamma_4) \geqslant 2$. Assume that $\gamma(\Gamma_4) = 2$ and let D be a putative dominating set of size 2 in Γ_4. Applying Lemma 5.1.1, there are the following cases:

1. The degree sequence of D is 4, 3. Observe that all the vertices of degree 3 have the vertices of degree 4 being one of their neighbours and $OD(D) \geqslant 2$. Hence, the number of dominated vertices is less than 8.

2. The degree sequence of D is 4, 2. Observe that any vertex of degree 2 is distance at most 2 from the vertex of degree 4, which forces the number of dominated vertices to be less than 8.

3. The degree sequence of D is 3, 3. Then the two vertices in D have two common neighbours. There still exists two vertices in Γ_4 undominated.

In all of the above cases, there exists some vertex which was undominated, and therefore $\gamma(\Gamma_4) \geqslant 3$. By a computer search, we find 12 dominating sets of size 3 in Γ_4. They are {0, 1, 2}, {0, 1, 5}, {0, 1, 7}, {0, 3, 5}, {0, 4, 5}, {1, 3, 7}, {1, 4, 7}, {2, 3, 6}, {2, 4, 5}, {2, 4, 6}, {3, 6, 7} and {4, 5, 7}. Figure 5.2 shows a dominating set of size 3 in Γ_4. $\square$

Lemma 5.2.5 $\gamma(\Gamma_5) = 4$.

Proof. By Corollary 5.2.1, $\gamma(\Gamma_5) \geqslant 3$. If $\gamma(\Gamma_5) = 3$, we consider the following cases:

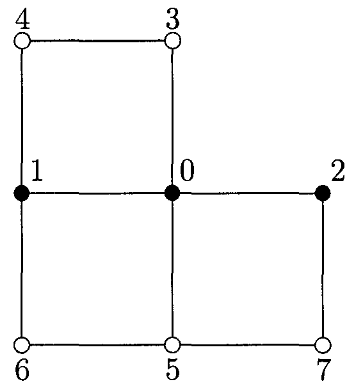


Figure 5.2: A dominating set of Γ_4.

1. The degree sequence in D is 5, 4, 4. There are exactly two vertices of degree 4 in Γ_5, and both of them are neighbours of vertex 0, which is the only vertex of degree 5 in Γ_5 (Lemma 4.2.3). There are just 12 vertices dominated.

2. The degree sequence in D is 5, 4, 3. The vertex of degree 5 and one of the vertices of degree 4 in Γ_5 are adjacent, which requires that the vertex of D with degree 3 must be distance at least 3 from the other vertices of D. We observe that vertex 12 is the only vertex having distance at least 3 from vertex 0 in Γ_5. However, vertex 12 is distance 2 from any vertex of degree 4, and so the number of dominated vertices is less than 13.

3. The degree sequence in D is 5, 4, 2. We know that the vertex of D with degree 4 must be adjacent to vertex 0, and then $OD(D) \geqslant 2$. It is impossible to construct a dominating set with this degree sequence.

4. The degree sequence in D is 5, 3, 3. If one vertex of degree 3 is adjacent to

vertex 0, then $OD(D) \geqslant 2$. Therefore, the number of dominated vertices is less than 13. If one vertex of degree 3, say v, is distance 2 from vertex 0, then $OD(D) \geqslant 1$, which requires that another vertex of degree 3 must be distance at least 3 from vertices 0 and v. However, that is impossible in Γ_5.

5. The degree sequence in D is 5, 3, 2. This requires the three vertices in D must be distance at least 3 from each other. However, all vertices of degree 2 in Γ_5 (i.e. vertices 6, 7 and 10) are distance 2 from vertex 0.

6. The degree sequence in D is 4, 4, 3 or 4, 4, 2. The two vertices of degree 4 have two common neighbours which yields $OD(D) \geqslant 2$. So, the number of dominated vertices is less than 13.

7. The degree sequence in D is 4, 3, 3, which requires that these three vertices must be distance at least 3 from each other. In Γ_5, vertices 1 and 8 are the only two vertices that have degree 4. Considering the symmetry of the Fibonacci code (reading the Fibonacci code from left to right or from right to left), we assume without loss of generality that vertex 8, which has degree 4, is in D. Then $\{4, 7, 8\}$ is the only set of three vertices are distance at least 3 from each other. However, $\deg_{\Gamma_5}(7) = 2$.

In all of above cases, there does not exist a dominating set of size 3 in Γ_5. Therefore, $\gamma(\Gamma_5) \geqslant 4$. By computer search, we found 24 dominating sets of size 4 in Γ_5, and all these sets are listed here: $\{0, 1, 2, 11\}$, $\{0, 1, 2, 12\}$, $\{0, 2, 5, 12\}$, $\{0, 2, 6, 12\}$, $\{0,$

4, 5, 8}, {0, 5, 8, 12}, {0, 5, 10, 12}, {1, 2, 5, 11}, {1, 2, 6, 11}, {1, 2, 7, 11}, {1, 4, 7, 8}, {1, 5, 10, 11}, {1, 7, 8, 11}, {1, 7, 10, 11}, {2, 3, 5, 9}, {2, 3, 6, 9}, {2, 4, 5, 8}, {2, 4, 6, 8}, {2, 6, 11, 12}, {3, 5, 9, 10}, {4, 5, 7, 8}, {4, 5, 8, 10}, {4, 5, 10, 12} and {4, 6, 7, 8}. □

Lemma 5.2.6 $\gamma(\Gamma_6) = 5$.

Proof. Applying Corollary 5.2.1, we have $\gamma(\Gamma_6) \geqslant 4$. Assume that $\gamma(\Gamma_6) = 4$ and let D be a dominating set of size 4 in Γ_6. There are the following cases:

1. Vertex 0 and both the vertices of degree 5 are all in D. Then $OD(D) \geqslant 5$, and therefore the number of dominated vertices is at most 20.

2. Vertex 0 and just one of the vertices of degree 5 are in D. Since these two vertices are adjacent, we have $OD(D) \geqslant 2$. D is a dominating set if and only if the other two vertices in D are degree 4 and distance at least 3 from vertex 0. However, all the vertices of degree 4, i.e. vertices 2, 3, 4, 5, 8, 14, and 18, are distance at most 2 from vertex 0.

3. Vertex 0 is not in D and both the vertices of degree 5 are in D. Since the two vertices of degree 5 have two common neighbours, i.e. vertices 0 and 14, the number of dominated vertices is at most 20.

4. Vertex 0 is not in D and just one vertex of degree 5 is in D, which requires that the other three vertices in D must have degree 4 and all four vertices in D must

be distance at least 3 from each other. In Γ_6, vertices 1 and 13 are the only two vertices that have degree 5. We assume without loss of generality that vertex 13, which has degree 5, is in D, then vertex 4 is the only vertex that is degree 4 and distance at least 3 from vertex 13, which is a contradiction.

Hence, there does not exist a dominating set of size 4 in Γ_6, which implies $\gamma(\Gamma_6) \geqslant 5$. By computer search, we found 4 dominating sets of size 5 in Γ_6 (i.e. {1, 2, 11, 16, 18}, {1, 2, 11, 17, 18}, {4, 6, 7, 8, 13}, {4, 7, 8, 13, 19}). □

By applying techniques similar to those described in Lemma 5.2.6, we can prove that $\gamma(\Gamma_7) \geqslant 7$. Using exhaustive search, there is no dominating set of size 7 in Γ_7; therefore $\gamma(\Gamma_7) \geqslant 8$. By computer search, there are 71 dominating sets of size 8 in Γ_7. Similarly, using computers to do the exhaustive search, there exist 510 dominating sets of size 12 in Γ_8. The results about the domination number of Γ_n, $1 \leqslant n \leqslant 8$, are summarized in Table 5.1 (see Table 4.1 for other information about Γ_n).

Table 5.1: Domination number of Γ_n.

n	1	2	3	4	5	6	7	8
$\gamma(\Gamma_n)$	1	1	2	3	4	5	8	12

5.3 Dominating Hypercubes

When considering the n-cube Q_n, it will be convenient to treat its vertex set as the set of all binary words of length n, two vertices being adjacent if and only if the

Hamming distance between them is 1. Additionally, by $e_1, e_2, \ldots, e_n$ (e_i has ith bit 1 and all other bits 0) we denote the words of Hamming weight 1, where the Hamming weight of a binary word w is the number of non-zero coordinates in the word. If t and w are two binary words, then by tw we denote the word obtained by concatenating t and w. More generally, if S is any set of binary words, then $tSw = \{tsw : s \in S\}$.

Refering now to concepts in coding theory, an (n, d) code is a code of length n and minimum Hamming distance d. The maximum cardinality of a binary (n, d) code is $A(n, d)$. Much work has been done on the problem of determining $A(n, d)$ for various combinations of n and d (see [38, 41] for instance). In this chapter, we use $A(n, 3)$ $\big($resp. $A(n, 2k+1)\big)$ to establish a new lower bound on the domination number (resp. the distance-k domination number) for the n-cube.

Stout obtained $\gamma(Q_n) \geqslant \dfrac{2^n}{n+1}$ with equality holding if $n = 2^k - 1$ (see [22, p. 279]). By using an algorithmic method, in [31], Jha proved that $\dfrac{2^n}{n+1} \leqslant \gamma(Q_n) \leqslant \dfrac{2^n}{2^{\lfloor \log_2 (n+1) \rfloor}}$. If $n = 2^k - 1$, the two bounds coincide and hence $\gamma(Q_n) = 2^{n-k}$. These bounds are correct also for $\gamma_i(Q_n)$ and $\gamma_p(Q_n)$ in general. Subsequently, in [6], Arumugam and Kala independently proved the above results theoretically. They also obtained the exact values of $\gamma(Q_n)$, $\gamma_p(Q_n)$ and $\gamma_i(Q_n)$ for $n \leqslant 7$, showed $\gamma(Q_n) \leqslant 2^{n-3}$ for $n \geqslant 7$, and established $\gamma_p(Q_n) \leqslant 2^{n-3}$ for $n \geqslant 7$ and $\gamma_i(Q_n) \leqslant 2^{n-2}$ for $n \geqslant 3$. Weakley [55] gave a correct proof of a theorem of Johnson [32], $\gamma(Q_n) \geqslant \left\lceil \dfrac{2^n}{n} \right\rceil$ for n even, with equality when $n = 2^k$ for $k > 0$.

5.3.1 Domination

Although the domination number of Q_8 was known from a result of Johnson [32], we give an independent proof of $\gamma(Q_8) = 32$ by applying a new technique which can be extended to study the distance-k domination number of hypercubes.

Theorem 5.3.1 $\gamma(Q_8) = 32$.

Proof. Let D be a minimum dominating set of Q_8, C be a maximal $(8, 3)$ code among D, and $\bar{C} = D - C$. From [38] we know that $A(8, 3) \leqslant 20$, so $|C| \leqslant 20$. Hence $\forall v \in \bar{C}$, $\exists u \in C$ such that $d(u, v) \leqslant 2$, where $d(u, v)$ denotes the Hamming distance between u and v. If $d(u, v) = 1$, then these two vertices will contribute 2 to the over-domination and we call v a Type-1 vertex (see Figure 5.3). If $d(u, v) = 2$,

Figure 5.3: $u \in C$, $v \in \bar{C}$, v is a Type-1 vertex.

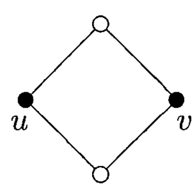


Figure 5.4: $u \in C$, $v \in \bar{C}$, v is a Type-2 vertex and u is a Type-2 mate.

any pair of distance 2 vertices will have exactly 2 common neighbours in Q_n, then $N(u) \cap N(v)$ will contribute 2 to the over-domination and we say that v is a Type-2 vertex, and that u is a Type-2 mate (see Figure 5.4). Each vertex of $\bar{C}$ is either a Type-1 vertex or a Type-2 vertex.

From [6, 22, 31] we have $\gamma(Q_8) \geqslant \dfrac{256}{8+1} > 28$. If $|D| = 29$, then $|\bar{C}| \geqslant 9$ since $|C| \leqslant 20$. Hence $OD(D) \geqslant 2|\bar{C}| \geqslant 18$. However, a dominating set of size 29 yields $OD(D) = 29 \times 9 - 2^8 = 5$, which is a contradiction. So $\gamma(Q_8) \geqslant 30$. By using a similar argument, we obtain $\gamma(Q_8) \geqslant 31$.

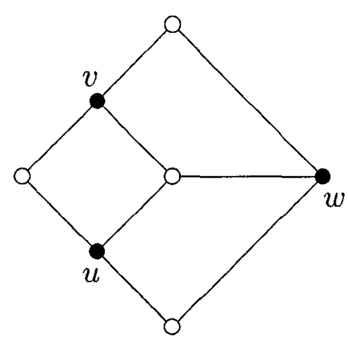


Figure 5.5: Configuration respect to vertices u, v and w.

If $\gamma(Q_8) = 31$, then $OD(D) = 23$. Necessarily $|C| = 20$ and $|\bar{C}| = 11$, since $|\bar{C}| \geqslant 12$ would imply that $OD(D) \geqslant 24$. Also, there exist $u, v \in \bar{C}$ and $w \in C$, such that the distance between any pair of these three vertices is exactly 2. Therefore, w has 3 non-private neighbours and u, v have 4 non-private neighbours in total (see Figure 5.5 for detail), where a *private neighbour* of vertex $v \in D$ is a vertex which is dominated by v and is not dominated by any other vertex of D and a *non-private neighbour* is a vertex which is dominated by at least two vertices of D [25]. The other vertices in $\bar{C}$ will form an $(8, 3)$ code, and each vertex of $\bar{C} - \{u, v\}$ is distance at least 3 from each of u and v. Suppose there are x Type-1 vertices in $\bar{C} - \{u, v\}$. Then we count the number of private neighbours dominated by C, which is all neighbours of all vertices of C except the following:

1. x Type-1 vertices;

2. 3 neighbours of w;

3. 2 neighbours for each Type-2 mate in $C - \{w\}$.

So there are $20 \times 8 - x - 3 - 2(9 - x) = 139 + x$ private neighbours dominated by C. Subsequently, we count the number of private neighbours dominated by $\bar{C}$, which is the total number of neighbours of vertices of $\bar{C}$, except for:

1. 1 neighbour for each Type-1 vertex;

2. 4 neighbours of u, v in total;

3. 2 neighbours for each Type-2 vertex in $\bar{C} - \{u, v\}$.

So there are $11 \times 8 - x - 4 - 2(9 - x) = 66 + x$ private neighbours dominated by $\bar{C}$. The number of non-private neighbours dominated by D is $2(9-x)+4$. By adding the number of private neighbours dominated by D, the number of non-private neighbours dominated by D and the number of vertices in D, we will have the total number of vertices in Q_8:

$$(139 + x) + (66 + x) + 22 - 2x + 31 = 258.$$

But $|V(Q_8)| = 2^8 = 256$, and so there is a contradiction. Therefore $\gamma(Q_8) \geqslant 32$.

In [6], it was proved that $\gamma(Q_n) \leqslant 2^{n-3}$ for $n \geqslant 7$. When $n = 8$, $\gamma(Q_8) \leqslant 32$, so that $\gamma(Q_8) = 32$. $\square$

Theorem 5.3.2 $\gamma(Q_n) \geqslant \dfrac{2^n - 2A(n,3)}{n-1}$. *Moreover, equality holds when* $n = 2^k - 1$.

Proof. Suppose D is a minimum dominating set for Q_n, so that $|D| = \gamma(Q_n)$. Let C be a maximal $(n,3)$ code among D and $\bar{C} = D - C$. Since $|C| \leqslant A(n,3)$, we have $|\bar{C}| \geqslant \gamma(Q_n) - A(n,3)$. Noticing that $OD(D) \geqslant 2|\bar{C}|$, we have

$$
\begin{aligned}
2\big(\gamma(Q_n) - A(n,3)\big) &\leqslant 2|\bar{C}| \leqslant (n+1)\gamma(Q_n) - 2^n \\
\gamma(Q_n) &\geqslant \frac{2^n - 2A(n,3)}{n-1}.
\end{aligned}
$$

Moreover, when $n = 2^k - 1$, $\gamma(Q_n) = 2^{n-k} = A(n,3)$, and any minimum dominating set D is both a perfect and independent dominating set. Hence $|C| = A(n,3)$ and $|\bar{C}| = 0$, and so all the inequalities hold with equality. $\square$

We now compare the lower bound from Theorem 5.3.2 with the lower bound of $\gamma(Q_n) \geqslant \dfrac{2^n}{n+1}$ from [6, 22, 31]. Observe that $A(n,4) \leqslant \dfrac{2^{n-1}}{n}$ (see [46, Lemma 5]), and $A(n-1, 2i-1) = A(n, 2i)$ [41]. So $A(n,3) \leqslant \dfrac{2^n}{n+1}$, and hence

$$
\begin{aligned}
& (n+1)A(n,3) \leqslant 2^n \\
\iff & n2^n - 2^n \leqslant n2^n + 2^n - 2(n+1)A(n,3) \\
\iff & \frac{2^n}{n+1} \leqslant \frac{2^n - 2A(n,3)}{n-1}.
\end{aligned}
$$

Therefore the lower bound, $\gamma(Q_n) \geqslant \dfrac{2^n - 2A(n,3)}{n-1}$, from Theorem 5.3.2 is no worse than the lower bound, $\gamma(Q_n) \geqslant \dfrac{2^n}{n+1}$, in [6, 22, 31]. That it constitutes an improvement is shown in Table 5.2, in which we compare these two lower bounds on $\gamma(Q_n)$; we also present the known bounds on $A(n,3)$ as found in [1], for $8 \leqslant n \leqslant 16$. For n

even, the bound $\gamma(Q_n) \geqslant \left\lceil \frac{2^n}{n} \right\rceil$ established by Johnson [32] and Weakley [55] is the best of these three lower bounds.

Table 5.2: Comparison of lower bound on $\gamma(Q_n)$.

n	$\left\lceil \frac{2^n}{n+1} \right\rceil$	$\left\lceil \frac{2^n - 2A(n,3)}{n-1} \right\rceil$	$A(n,3)$	$\gamma(Q_n)$
8	29	31	20	32
9	52	54	40	54–64
10	94	98	72	98–128
11	171	176	144	176–256
12	316	326	256	326–512
13	586	598	512	598–1024
14	1093	1103	1024	1103–2048
15	2048	2048	2048	2048
16	3856	3933-4007	2720-3276	3933–4096

5.3.2 Independent and Perfect Domination

Theorem 5.3.3 $\gamma_p(Q_8) = 32$.

Proof. By Theorem 5.3.1, we have $\gamma_p(Q_8) \geqslant \gamma(Q_8) = 32$. From [6, Theorem 3.3], we have $\gamma_p(Q_8) \leqslant 2^{8-3} = 32$. Therefore $\gamma_p(Q_8) = 32$. □

Theorem 5.3.4 $\gamma_i(Q_n) \leqslant 2\gamma_i(Q_{n-1})$ *for* $n \geqslant 2$.

Proof. Let D_{n-1} be an independent dominating set in Q_{n-1}, D^O_{n-1} be the subset of D consisting of all vertices in D_{n-1} with odd Hamming weight, and D^E_{n-1} be the subset of D consisting of all vertices in D_{n-1} with even Hamming weight. Let $D'_{n-1} = \{u + e_{n-1} : u \in D_{n-1}\}$. It is easy to verify that D'_{n-1} is also an independent dominating set in Q_{n-1}. Let $D_n = 0D_{n-1} \cup 1D'_{n-1}$. It is a simple exercise to verify that D_n is a dominating set in Q_n.

Since D_{n-1} is an independent set, $d_{Q_{n-1}}(u_i, u_j) \geqslant 2$, where $u_i, u_j \in D^O_{n-1}$, then we have $d_{Q_n}(0u_i, 0u_j) \geqslant 2$, $d_{Q_n}(0u_i, 1(u_i + e_{n-1})) \geqslant 2$, and $d_{Q_n}(0u_i, 1(u_j + e_{n-1})) \geqslant 2$. Similarly, $d_{Q_{n-1}}(u, v) \geqslant 3$, where $u \in D^O_{n-1}$ and $v \in D^E_{n-1}$; hence $d_{Q_n}(0u, 1(v + e_{n-1})) \geqslant 3$ and $d_{Q_n}(1(u + e_{n-1}), 1(v + e_{n-1})) \geqslant 3$. Therefore D_n is an independent dominating set in Q_n. So $\gamma_i(Q_n) \leqslant 2\gamma_i(Q_{n-1})$. □

Corollary 5.3.5 $\gamma_i(Q_8) = 32$.

Proof. By Theorem 5.3.1, we have $\gamma_i(Q_8) \geqslant \gamma(Q_8) = 32$. From Theorem 5.3.4 and [6, 31], $\gamma_i(Q_8) \leqslant 2\gamma_i(Q_7) = 2 \times 16 = 32$. □

Corollary 5.3.6 $\gamma_i(Q_n) \leqslant 2^{n-3}$ *for all* $n \geqslant 7$.

Proof. The proof follows from Theorem 5.3.4 and Corollary 5.3.5. □

5.3.3 Distance-k Domination

The technique used in Section 5.3.1 to prove Theorem 5.3.2 can be extended to find a lower bound for the distance-k domination number of Q_n. Given two vertices in

$V(Q_n)$, say u and v, that are distance d apart, where $1 \leqslant d \leqslant 2k$, we need to determine $|N_{k,Q_n}(u) \cap N_{k,Q_n}(v)|$. Since this value is constant for any pair of distance d vertices in Q_n, we denote it by $N_{k,Q_n}(d)$.

Lemma 5.3.7 *Let* $a = \left\lfloor \frac{d}{2} \right\rfloor$ *and* $i_0 = \begin{cases} 0 & \text{if } 1 \leqslant d \leqslant k \\ d-k & \text{if } k < d \leqslant 2k \end{cases}$. *Then*

$$N_{k,Q_n}(d) = \begin{cases} 2\sum_{i=i_0}^{a-1}\left[\binom{d}{i}\sum_{j=0}^{k-d+i}\binom{n-d}{j}\right] + \binom{d}{a}\sum_{j=0}^{k-a}\binom{n-d}{j} & \text{if } d = 2a, \\ 2\sum_{i=i_0}^{a}\left[\binom{d}{i}\sum_{j=0}^{k-d+i}\binom{n-d}{j}\right] & \text{if } d = 2a+1. \end{cases}$$

Proof. Let u and v be distinct vertices in $V(Q_n)$ such that u and v are distance d apart. Without loss of generality, let $u + v = e_1 + e_2 + \cdots + e_d$, meaning that any shortest path between u and v uses the differences $e_1, e_2, \ldots, e_d$. For $1 \leqslant d \leqslant k$, it is easy to verify that there are $\binom{d}{i}$ vertices being distance i from u and distance $d - i$ from v, where $0 \leqslant i \leqslant \left\lfloor \frac{d}{2} \right\rfloor$. For $k < d \leqslant 2k$, any vertex at distance i from u (resp. v) and distance $d - i$ from v (resp. u), where $0 \leqslant i < d - k$, is in $N_{k,Q_n}(u)$ (resp. $N_{k,Q_n}(v)$), but not in $N_{k,Q_n}(u) \cap N_{k,Q_n}(v)$; it is obvious that there are $\binom{d}{i}$ vertices being distance i from u and distance $d - i$ from v, where $d - k \leqslant i \leqslant \left\lfloor \frac{d}{2} \right\rfloor$. So for either case, and for $i_0 \leqslant i \leqslant \left\lfloor \frac{d}{2} \right\rfloor$, there are $\binom{d}{i}$ vertices being distance i from u and distance $d - i$ from v, and all of these $\binom{d}{i}$ vertices are in $N_{k,Q_n}(u) \cap N_{k,Q_n}(v)$.

For any vertex $w \in V(Q_n)$ such that $i_0 \leqslant d(w, u) = i \leqslant \left\lfloor \frac{d}{2} \right\rfloor$ and $d(w, v) = d - i$, $w + \delta_{d+1}e_{d+1} + \delta_{d+2}e_{d+2} + \cdots + \delta_n e_n \in N_{k,Q_n}(u) \cap N_{k,Q_n}(v)$ if and only if $\vec{\delta} = (\delta_{d+1}, \delta_{d+2}, \ldots, \delta_n)$ is a binary vector with Hamming weight at most $k - d + i$. If the Hamming weight of $\vec{\delta}$ is precisely j $(0 \leqslant j \leqslant k-d+i)$, then $\vec{\delta}$ corresponds to a unique vertex in $N_{k,Q_n}(u) \cap N_{k,Q_n}(v)$, and vice versa; for fixed j and w, there are $\binom{n-d}{j}$ such vertices. Each vertex at distance i $(i_0 \leqslant i \leqslant \left\lfloor \frac{d}{2} \right\rfloor)$ from u and distance $d-i$ from v will contribute a distinct set of $\sum_{j=0}^{k-d+i} \binom{n-d}{j}$ vertices to $N_{k,Q_n}(d)$. By the symmetry of Q_n, so does each vertex at distance $d-i$ from u and distance i $(i_0 \leqslant i \leqslant \left\lfloor \frac{d}{2} \right\rfloor)$ from v. For $d = 2a$, each vertex at distance a from u and distance a from v will contribute $\sum_{j=0}^{k-a} \binom{n-d}{j}$ to $N_{k,Q_n}(d)$. □

In order to use the technique described in Section 5.3.2, we will want to use

$$N_{k,Q_n} = \min_{\substack{u,v \in V(Q_n) \\ 1 \leqslant d(u,v) \leqslant 2k}} \left\{ N_{k,Q_n}\big(d(u,v)\big) \right\}.$$

Evaluating N_{k,Q_n} is not a trivial exercise, but for $k = 1$ we find that $N_{1,Q_n}(1) = 2\binom{1}{0}\binom{n-1}{0} = 2$ and $N_{1,Q_n}(2) = \binom{2}{1}\binom{n-2}{0} = 2$, so that $N_{1,Q_n}(1) = N_{1,Q_n}(2)$. For $k \geqslant 2$,

$$N_{k,Q_n}(1) = 2\left[\binom{n-1}{0} + \binom{n-1}{1} + \cdots + \binom{n-1}{k-1}\right] \quad \text{and}$$

$$N_{k,Q_n}(2) = 2\left[\binom{n-2}{0} + \binom{n-2}{1} + \cdots + \binom{n-2}{k-2}\right] + \binom{2}{1}\left[\binom{n-2}{0} + \binom{n-2}{1} + \cdots + \binom{n-2}{k-1}\right].$$

By applying the fundamental binomial identity

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1},$$

and noticing that $\binom{n-1}{0} = \binom{n-2}{0} = 1$, we have $N_{k,Q_n}(1) = N_{k,Q_n}(2)$ for all $k \in \mathbb{N}$. By observation, we also found that $N_{k,Q_n}(2i-1) = N_{k,Q_n}(2i)$, and $N_{k,Q_n}(d)$ is a non-increasing function of d for $n \leqslant 10$ and any choices of k and i. These observations lead to the following questions:

Question 5.3.1 *For any choices of k and n, does $N_{k,Q_n}(2i-1) = N_{k,Q_n}(2i)$, for $1 \leqslant i \leqslant k$?*

Question 5.3.2 *For fixed k and n, is $N_{k,Q_n}(d)$ a non-increasing function of d?*

By extending the technique used in Theorem 5.3.2, we have the following result:

Theorem 5.3.8 $\gamma_{\leqslant k}(Q_n) \geqslant \dfrac{2^n - N_{k,Q_n} \cdot A(n, 2k+1)}{s(n,k) - N_{k,Q_n}}$, *where* $s(n,k) = \binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{k}$.

Proof. Suppose D is a minimum distance-k dominating set for Q_n, so that $|D| = \gamma_{\leqslant k}(Q_n)$. Let C be a maximal $(n, 2k+1)$ code among D and $\bar{C} = D - C$. Since $|C| \leqslant A(n, 2k+1)$, we have $|\bar{C}| \geqslant \gamma_{\leqslant k}(Q_n) - A(n, 2k+1)$.

Note that the number of distance j neighbours of any vertex $v \in V(Q_n)$ is $\binom{n}{j}$ (see [8, Theorem 3.1]). Therefore, any vertex $v \in D$ dominates $\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{k} = s(n,k)$ vertices. Similarly, we define the *distance-k over-domination* of a graph G with respect to a distance-k dominating set D as : $OD_{k,G}(D) = \left(\sum_{v \in D} s(n,k)\right) - |V(G)|$, using $OD_k(D)$ for abbreviation if there is no confusion.

Noticing that $OD_k(D) \geqslant N_{k,Q_n} \cdot |\bar{C}|$, we have

$$N_{k,Q_n} \cdot \big(\gamma_{\leqslant k}(Q_n) - A(n, 2k+1)\big) \leqslant N_{k,Q_n} \cdot |\bar{C}| \leqslant s(n,k)\gamma_{\leqslant k}(Q_n) - 2^n$$

$$\gamma_{\leqslant k}(Q_n) \geqslant \frac{2^n - N_{k,Q_n} \cdot A(n, 2k+1)}{s(n,k) - N_{k,Q_n}}.$$

□

Chapter 6

Conclusion

In this thesis we have mainly considered two problems in graph theory — decycling and dominating, specifically for Cartesian product of two cycles, Fibonacci cubes and hypercubes.

In Chapter 1, the background and definitions used in this thesis were given. Then, we presented the definitions of some of the classes of graphs which are studied in this thesis, i.e. Cartesian product of cycles, n-dimensional hypercube and the Fibonacci cube of order n.

Chapter 2 introduced the history and progress of finding the decycling number of graphs, or equivalently finding the size of a maximum induced forest in the graphs. Some known results and open problems in several classes of graphs, i.e. cubic graphs, planar graphs, hypercubes and grids, were illustrated. Then, we gave definitions for three graph expansions and obtained some new results regarding the decycling number of those graph expansions.

Chapter 3 focused on finding the decycling number of the Cartesian product of two cycles, $C_m \Box C_n$. Noticing that the graph $C_m \Box C_n$ is 4-regular, we established a lower bound for the decycling number of $C_m \Box C_n$ by applying a known result. Then, by analyzing the structure of $C_m \Box C_n$, we got an improved lower bound. And from this lower bound, the exact value of the decycling number for the Cartesian product of two cycles and a corresponding decycling set for each possible case is also obtained. Moreover, we found a vertex subset that yields a maximum induced tree in the graph $C_m \square C_n$. Finally, our main results can be applied to get an upper bound for the decycling number of Cartesian product of two paths, $P_m \square P_n$.

Chapter 4 turned to investigate the decycling number for the Fibonacci cube of order n. First of all, we reviewed the progress for the graph properties of the Fibonacci cubes, and obtained a lower bound for the decycling number of the Fibonacci cube of order n. Then, we presented a sequence of theoretical proofs to obtain the exact value of the decycling number for $n \leqslant 7$, and described a computer search heuristic to find the decycling number for $n \leqslant 9$ and the independent decycling number for $n \leqslant 11$. Finally, we established an upper bound based on the independence number.

Chapter 5 discussed domination parameters for the cubes (Fibonacci cubes and hypercubes). We began by studying the domination number for the n-dimensional Fibonacci cube for $n \leqslant 8$. Then, we investigated various domination parameters including independent, perfect and distance-k domination number for the hypercube. An improved lower bound for the domination number $\gamma(Q_n)$, making connection between the domination number and maximum (n, d) code, was obtained. Also, we presented an improved upper bound for the independent domination number $\gamma_i(Q_n)$ and a new lower bound for the distance-k domination number $\gamma_{\leqslant k}(Q_n)$.

For further study, there is a wealth of existing literature focusing on finding the minimum decycling set or minimum feedback vertex set for various families of graphs. Some families of graphs, such as cubic graphs, permutation graphs, and interval and comparability graphs, are known to have polynomial solutions. On the other hand, determining the decycling number of planar graphs, bipartite graphs, perfect graphs, and comparability graphs has been shown to be *NP*-hard. But it is conjectured that every planar graph G has a decycling set of size at most half of the vertices of G, and furthermore, that every bipartite planar graph has a decycling set of size at most $3/8$ of the vertices of G.

We have already presented some results on the decycling number of Fibonacci cubes. These results can be further studied to revise the lower and upper bounds. Also, it is interesting to study the connection between the decycling number of the hypercube and the coding theory.

Domination parameters are a popular topic for graph theory; there are more than 2,000 papers focusing on this problem. Some families of graphs had been proved to have polynomial solutions. But there are lots of graphs that are NP-complete. It is useful to have classes of graphs for which the values are known. Thus, investigation on the domination parameters of the hypercube, Fibonacci cube, Cartesian product of cycles and other interesting interconnection topologies is a prospective direction. In addition, studying the exact value of some domination parameters for fixed order of hypercube, Fibonacci cube and Cartesian product of cycles is also an interesting problem. Again, the domination number of the n-dimensional hypercube has a close connection with coding theory, specifically, with a maximum binary code of length n and minimum Hamming distance d. This concept can be extended to the distance-k domination number and maximum binary code of length n and minimum Hamming distance $2k + 1$.

Bibliography

[1] Erik Agrell, Alexander Vardy, and Kenneth Zeger. A table of upper bounds for binary codes. *IEEE Trans. Inform. Theory*, 47(7):3004–3006, 2001.

[2] Jin Akiyama, Hiroshi Era, Severino V. Gervacio, and Mamoru Watanabe. Path chromatic numbers of graphs. *J. Graph Theory*, 13(5):569–575, 1989.

[3] Jin Akiyama and Mamoru Watanabe. Maximum induced forests of planar graphs. *Graphs Combin.*, 3:201–202, 1987.

[4] Michael O. Albertson. A lower bound for the independence number of a planar graph. *J. Combin. Theory Ser. B*, 20(1):84–93, 1976.

[5] Michael O. Albertson and David Berman. A conjecture on planar graphs. In J. Adrian Bondy and U. S. R. Murty, editors, *Graph Theory and Related Topics*, page 357, New York, 1979. Academic Press [Harcourt Brace Jovanovich Publishers].

[6] S. Arumugam and R. Kala. Domination parameters of hypercubes. *J. Indian Math. Soc. (N.S.)*, 65(1-4):31–38, 1998.

[7] Sheng Bau and Lowell W. Beineke. The decycling number of graphs. *Australas. J. Combin.*, 25:285–298, 2002.

[8] Sheng Bau, Lowell W. Beineke, Genmin Du, Zhishan Liu, and Robert C. Vandell. Decycling cubes and grids. *Util. Math.*, 59:129–137, 2001.

[9] Lowell W. Beineke and Robert C. Vandell. Decycling graphs. *J. Graph Theory*, 25(1):59–77, 1997.

[10] J. Adrian Bondy, Glenn Hopkins, and William Staton. Lower bounds for induced forests in cubic graphs. *Canad. Math. Bull.*, 30(2):193–199, 1987.

[11] J. Adrian Bondy and U. S. R. Murty. *Graph Theory with Applications.* American Elsevier Publishing Co., Inc., New York, 1976.

[12] O. V. Borodin. A proof of B. Grünbaum's conjecture on the acyclic 5-colorability of planar graphs. *Dokl. Akad. Nauk SSSR*, 231(1):18–20, 1976.

[13] O. V. Borodin and A. N. Glebov. On the partition of a planar graph of girth 5 into an empty and an acyclic subgraph. *Diskretn. Anal. Issled. Oper. Ser. 1*, 8(4):34–53, 2001.

[14] I. Broere and C. M. Mynhardt. Generalized colorings of outerplanar and planar graphs. In *Graph Theory with Applications to Algorithms and Computer Science (Kalamazoo, Mich., 1984)*, Wiley-Intersci. Publ., pages 151–161. Wiley, New York, 1985.

[15] Ioannis Caragiannis, Christos Kaklamanis, and Panagiotis Kanellopoulos. New bounds on the size of the minimum feedback vertex set in meshes and butterflies. *Inform. Process. Lett.*, 83(5):275–280, 2002.

[16] Ján Černý, Mirko Horňák, and Roman Soták. Observability of a graph. *Math. Slovaca*, 46(1):21–31, 1996.

[17] Ernesto Dedó, Damiano Torri, and Norma Zagaglia Salvi. The observability of the Fibonacci and the Lucas cubes. *Discrete Math.*, 255(1-3):55–63, 2002. Combinatorics '98 (Palermo).

[18] Reinhard Diestel. *Graph Theory*, volume 173 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 2000.

[19] Paul Erdős, Michael Saks, and Vera T. Sós. Maximum induced trees in graphs. *J. Combin. Theory Ser. B*, 41(1):61–79, 1986.

[20] Paola Festa, Panos M. Pardalos, and Mauricio G. C. Resende. Feedback set problems. In *Handbook of Combinatorial Optimization, Supplement Vol. A*, pages 209–258. Kluwer Acad. Publ., Dordrecht, 1999.

[21] Riccardo Focardi, Flaminia L. Luccio, and David Peleg. Feedback vertex set in hypercubes. *Inform. Process. Lett.*, 76(1-2):1–5, 2000.

[22] Frank Harary, John P. Hayes, and Horng-Jyh Wu. A survey of the theory of hypercube graphs. *Comput. Math. Appl.*, 15(4):277–289, 1988.

[23] Teresa W. Haynes. Domination in graphs: a brief overview. *J. Combin. Math. Combin. Comput.*, 24:225–237, 1997.

[24] Teresa W. Haynes and Stephen T. Hedetniemi, editors. *Domination in Graphs*, volume 209 of *Monographs and Textbooks in Pure and Applied Mathematics.* Marcel Dekker Inc., New York, 1998. Advanced topics.

[25] Teresa W. Haynes, Stephen T. Hedetniemi, and Peter J. Slater. *Fundamentals of Domination in Graphs*, volume 208 of *Monographs and Textbooks in Pure and Applied Mathematics.* Marcel Dekker Inc., New York, 1998.

[26] S. T. Hedetniemi and R. C. Laskar, editors. *Topics on Domination*, volume 48 of *Annals of Discrete Mathematics.* North-Holland Publishing Co., Amsterdam, 1991. Reprint of Discrete Math. **86** (1990), no. 1-3.

[27] Michael A. Henning. Domination in graphs: a survey. *Congr. Numer.*, 116:139–172, 1996. Surveys in graph theory (San Francisco, CA, 1995).

[28] Kiyoshi Hosono. Induced forests in trees and outerplanar graphs. *Proc. Fac. Sci. Tokai Univ.*, 25:27–29, 1990.

[29] Wen-Jing Hsu. Fibonacci cubes—a new interconnection topology. *IEEE Trans. Parallel Distributed Systems*, 4:3–12, 1993.

[30] F. Jaeger. On vertex-induced forests in cubic graphs. In *Proceedings of the Fifth Southeastern Conference on Combinatorics, Graph Theory and Computing*

(Florida Atlantic Univ., Boca Raton, Fla., 1974), pages 501–512. Congressus Numerantium, No. X, Winnipeg, Man., 1974. Utilitas Math.

[31] Pranava K. Jha. *Hypercubes, median graphs and products of graphs: Some algorithmic and combinatorial results.* Ph.D. dissertation, Department of Computer Science, Iowa State University, 1990.

[32] S. M. Johnson. A new lower bound for coverings by rook domains. *Utilitas Math.*, 1:121–140, 1972.

[33] Richard M. Karp. Reducibility among combinatorial problems. In *Complexity of Computer Computations (Proc. Sympos., IBM Thomas J. Watson Res. Center, Yorktown Heights, N.Y., 1972)*, pages 85–103. Plenum, New York, 1972.

[34] G. Kirchhoff. Über die auflösung der gleichungen, auf welche man bei der untersuchung der linearen verteilung galvanischer ströme geführt wird. *Ann. Phys. Chem.*, 72:497–508, 1847.

[35] Deming Li and Yanpei Liu. A polynomial algorithm for finding the minimum feedback vertex set of a 3-regular simple graph. *Acta Math. Sci. (English Ed.)*, 19(4):375–381, 1999.

[36] Y. Daniel Liang. On the feedback vertex set problem in permutation graphs. *Inform. Process. Lett.*, 52(3):123–129, 1994.

[37] Y. Daniel Liang and Maw-Shang Chang. Minimum feedback vertex sets in cocomparability graphs and convex bipartite graphs. *Acta Inform.*, 34(5):337–346, 1997.

[38] Simon Litsyn. An updated table of the best binary codes known. In *Handbook of Coding Theory, Vol. I, II*, pages 463–498. North-Holland, Amsterdam, 1998.

[39] Jenshiuh Liu, Wen-Jing Hsu, and Moon Jung Chung. Generalized Fibonacci cubes are mostly Hamiltonian. *J. Graph Theory*, 18(8):817–829, 1994.

[40] Jiping Liu and Cheng Zhao. A new bound on the feedback vertex sets in cubic graphs. *Discrete Math.*, 148(1-3):119–131, 1996.

[41] F. J. MacWilliams and N. J. A. Sloane. *The Theory of Error-Correcting Codes.* North-Holland Publishing Co., Amsterdam, 1977. North-Holland Mathematical Library, Vol. 16.

[42] G. H. J. Meredith. Regular n-valent n-connected nonHamiltonian non-n-edge-colorable graphs. *J. Combinatorial Theory Ser. B*, 14:55–60, 1973.

[43] Peter Mihók. On vertex partition numbers of graphs. In *Graphs and Other Combinatorial Topics (Prague, 1982)*, volume 59 of *Teubner-Texte Math.*, pages 183–188. Teubner, Leipzig, 1983.

[44] Emanuele Munarini and Norma Zagaglia Salvi. Structural and enumerative properties of the Fibonacci cubes. *Discrete Math.*, 255(1-3):317–324, 2002. Combinatorics '98 (Palermo).

[45] Michael J. Pelsmajer. Maximum induced linear forests in outerplanar graphs. *Graphs Combin.*, 20(1):121–129, 2004.

[46] David A. Pike. Decycling hypercubes. *Graphs Combin.*, 19(4):547–550, 2003.

[47] David A. Pike and Yubo Zou. Decycling Cartesian products of two cycles. *SIAM J. Discrete Math.*, 19(3):651–663, 2005.

[48] David A. Pike and Yubo Zou. Decycling of Fibonacci cubes. *Australas. J. Combin.*, 35:31–40, 2006.

[49] David A. Pike and Yubo Zou. Dominating hypercubes. submitted.

[50] K. S. Poh. On the linear vertex-arboricity of a planar graph. *J. Graph Theory*, 14(1):73–75, 1990.

[51] Ewald Speckenmeyer. Bounds on feedback vertex sets of undirected cubic graphs. In *Algebra, Combinatorics and Logic in Computer Science, Vol. I,II (Győr, 1983)*, volume 42 of *Colloq. Math. Soc. János Bolyai*, pages 719–729. North-Holland, Amsterdam, 1986.

[52] Ewald Speckenmeyer. On feedback vertex sets and nonseparating independent sets in cubic graphs. *J. Graph Theory*, 12(3):405–412, 1988.

[53] Shuichi Ueno, Yoji Kajitani, and Shin'ya Gotoh. On the nonseparating independent set problem and feedback set problem for graphs with no vertex degree exceeding three. In *Proceedings of the First Japan Conference on Graph Theory and Applications (Hakone, 1986)*, volume 72, pages 355–360, 1988.

[54] Robert C. Vandell. *Integrity of digraphs.* Ph.D. dissertation, Department of Mathematics and Statistics, Western Michigan University, 1996.

[55] William D. Weakley. Minimum dominating sets in hypercubes. In *Combinatorics, graph theory, and algorithms, Vol. I, II (Kalamazoo, MI, 1996)*, pages 861–866. New Issues Press, Kalamazoo, MI, 1999.

[56] Douglas B. West. *Introduction to Graph Theory.* Prentice Hall Inc., Upper Saddle River, NJ, 2nd edition, 2001.

[57] Robin J. Wilson. *Introduction to Graph Theory.* Addison-Wesley/Longman, Reading, MA, 4th edition, 1996.

[58] Norma Zagaglia Salvi. On the existence of cycles of every even length on generalized Fibonacci cubes. *Matematiche (Catania)*, 51(suppl.):241–251 (1997), 1996.

[59] E. Zeckendorf. Représentation des nombres naturels par une somme de nombres de Fibonacci ou de nombres de Lucas. *Bull. Soc. Roy. Sci. Liège*, 41:179–182, 1972.

[60] Mao Lin Zheng and Xiaoyun Lu. On the maximum induced forests of a connected cubic graph without triangles. *Discrete Math.*, 85(1):89–96, 1990.

Appendix 1

Minimum Decycling Sets of Fibonacci Cubes

Minimum decycling sets (size 3) for Γ_5:

1. $\{0, 1, 11\}$
2. $\{0, 1, 12\}$
3. $\{0, 4, 8\}$
4. $\{0, 4, 9\}$
5. $\{0, 8, 12\}$
6. $\{0, 9, 11\}$

Minimum decycling sets (size 6) for Γ_6:

1. $\{0, 2, 4, 9, 14, 18\}$
2. $\{0, 2, 4, 11, 14, 18\}$
3. $\{0, 4, 7, 8, 14, 18\}$
4. $\{0, 4, 7, 11, 14, 18\}$
5. $\{0, 4, 8, 14, 15, 18\}$
6. $\{0, 4, 9, 14, 15, 18\}$
7. $\{1, 2, 3, 5, 8, 13\}$
8. $\{1, 2, 3, 5, 9, 13\}$
9. $\{1, 2, 3, 5, 11, 13\}$
10. $\{1, 2, 3, 5, 12, 13\}$
11. $\{1, 2, 3, 8, 13, 18\}$
12. $\{1, 2, 3, 12, 13, 18\}$
13. $\{1, 2, 4, 5, 8, 13\}$
14. $\{1, 2, 4, 5, 11, 13\}$
15. $\{1, 3, 5, 7, 8, 13\}$
16. $\{1, 3, 5, 8, 13, 15\}$
17. $\{1, 3, 5, 8, 13, 20\}$
18. $\{1, 3, 7, 8, 13, 18\}$
19. $\{1, 4, 5, 8, 13, 20\}$

Minimum decycling sets (size 11) for Γ_7:

1. $\{0, 2, 4, 7, 11, 14, 18, 22, 26, 29, 33\}$
2. $\{0, 2, 4, 9, 11, 14, 18, 22, 24, 26, 29\}$
3. $\{0, 2, 4, 11, 14, 18, 22, 23, 26, 29, 33\}$
4. $\{0, 4, 6, 7, 9, 11, 13, 22, 23, 24, 29\}$
5. $\{0, 4, 6, 7, 11, 13, 14, 22, 23, 29, 33\}$
6. $\{0, 4, 6, 7, 11, 13, 18, 22, 23, 29, 33\}$
7. $\{0, 4, 6, 7, 11, 14, 18, 22, 23, 29, 33\}$
8. $\{0, 4, 7, 11, 14, 18, 22, 23, 26, 29, 33\}$
9. $\{1, 2, 3, 4, 5, 6, 8, 13, 21, 30, 32\}$
10. $\{1, 2, 3, 4, 5, 7, 8, 13, 21, 30, 32\}$
11. $\{1, 2, 3, 4, 5, 8, 13, 18, 21, 30, 32\}$
12. $\{1, 2, 3, 4, 5, 8, 13, 19, 21, 30, 32\}$
13. $\{1, 2, 3, 4, 5, 8, 13, 20, 21, 30, 32\}$
14. $\{1, 2, 3, 4, 5, 8, 13, 21, 26, 30, 32\}$
15. $\{1, 2, 3, 4, 5, 8, 13, 21, 27, 30, 32\}$
16. $\{1, 2, 3, 4, 5, 8, 13, 21, 27, 30, 33\}$
17. $\{1, 2, 3, 4, 5, 8, 13, 21, 27, 32, 33\}$
18. $\{1, 2, 3, 4, 5, 8, 13, 21, 28, 30, 32\}$
19. $\{1, 2, 3, 4, 5, 8, 13, 21, 28, 30, 33\}$
20. $\{1, 2, 3, 4, 5, 8, 13, 21, 28, 32, 33\}$
21. $\{1, 2, 3, 4, 5, 8, 13, 22, 26, 29, 33\}$
22. $\{1, 2, 3, 4, 5, 8, 13, 22, 28, 29, 33\}$
23. $\{1, 2, 3, 4, 5, 8, 14, 18, 21, 30, 32\}$
24. $\{1, 2, 3, 4, 5, 8, 14, 20, 21, 30, 32\}$
25. $\{1, 2, 3, 4, 5, 9, 13, 21, 27, 29, 33\}$
26. $\{1, 2, 3, 4, 5, 9, 13, 21, 28, 29, 33\}$
27. $\{1, 2, 3, 4, 5, 11, 13, 21, 27, 29, 33\}$
28. $\{1, 2, 3, 4, 5, 11, 13, 21, 28, 29, 33\}$
29. $\{1, 2, 3, 4, 6, 7, 8, 13, 21, 30, 32\}$
30. $\{1, 2, 3, 4, 8, 13, 18, 21, 26, 30, 32\}$
31. $\{1, 2, 3, 5, 6, 8, 12, 13, 21, 25, 29\}$
32. $\{1, 2, 3, 5, 6, 8, 12, 13, 21, 25, 30\}$
33. $\{1, 2, 3, 5, 6, 8, 13, 21, 25, 30, 32\}$
34. $\{1, 2, 3, 5, 6, 12, 13, 21, 25, 29, 30\}$
35. $\{1, 2, 3, 5, 7, 8, 12, 13, 21, 25, 29\}$
36. $\{1, 2, 3, 5, 7, 8, 12, 13, 21, 25, 30\}$
37. $\{1, 2, 3, 5, 7, 8, 13, 21, 25, 30, 32\}$
38. $\{1, 2, 3, 5, 7, 12, 13, 21, 25, 29, 30\}$
39. $\{1, 2, 3, 5, 8, 11, 13, 19, 21, 25, 30\}$
40. $\{1, 2, 3, 5, 8, 11, 13, 21, 25, 28, 30\}$
41. $\{1, 2, 3, 5, 8, 12, 13, 18, 21, 25, 29\}$
42. $\{1, 2, 3, 5, 8, 12, 13, 18, 21, 25, 30\}$
43. $\{1, 2, 3, 5, 8, 12, 13, 19, 21, 22, 32\}$
44. $\{1, 2, 3, 5, 8, 12, 13, 19, 21, 22, 33\}$
45. $\{1, 2, 3, 5, 8, 12, 13, 19, 21, 25, 29\}$
46. $\{1, 2, 3, 5, 8, 12, 13, 19, 21, 25, 30\}$
47. $\{1, 2, 3, 5, 8, 12, 13, 19, 21, 29, 33\}$

48. $\{1, 2, 3, 5, 8, 12, 13, 19, 21, 30, 32\}$
49. $\{1, 2, 3, 5, 8, 12, 13, 20, 21, 25, 29\}$
50. $\{1, 2, 3, 5, 8, 12, 13, 20, 21, 25, 30\}$
51. $\{1, 2, 3, 5, 8, 12, 13, 21, 22, 28, 32\}$
52. $\{1, 2, 3, 5, 8, 12, 13, 21, 22, 28, 33\}$
53. $\{1, 2, 3, 5, 8, 12, 13, 21, 25, 26, 29\}$
54. $\{1, 2, 3, 5, 8, 12, 13, 21, 25, 26, 30\}$
55. $\{1, 2, 3, 5, 8, 12, 13, 21, 25, 27, 29\}$
56. $\{1, 2, 3, 5, 8, 12, 13, 21, 25, 27, 30\}$
57. $\{1, 2, 3, 5, 8, 12, 13, 21, 25, 27, 32\}$
58. $\{1, 2, 3, 5, 8, 12, 13, 21, 25, 27, 33\}$
59. $\{1, 2, 3, 5, 8, 12, 13, 21, 25, 28, 29\}$
60. $\{1, 2, 3, 5, 8, 12, 13, 21, 25, 28, 30\}$
61. $\{1, 2, 3, 5, 8, 12, 13, 21, 25, 28, 32\}$
62. $\{1, 2, 3, 5, 8, 12, 13, 21, 25, 28, 33\}$
63. $\{1, 2, 3, 5, 8, 12, 13, 21, 27, 29, 33\}$
64. $\{1, 2, 3, 5, 8, 12, 13, 21, 27, 30, 32\}$
65. $\{1, 2, 3, 5, 8, 12, 13, 22, 24, 26, 29\}$
66. $\{1, 2, 3, 5, 8, 12, 13, 22, 24, 28, 29\}$
67. $\{1, 2, 3, 5, 8, 12, 13, 22, 25, 26, 29\}$
68. $\{1, 2, 3, 5, 8, 12, 13, 22, 25, 28, 29\}$
69. $\{1, 2, 3, 5, 8, 12, 14, 18, 21, 25, 29\}$
70. $\{1, 2, 3, 5, 8, 12, 14, 18, 21, 25, 30\}$
71. $\{1, 2, 3, 5, 8, 12, 14, 20, 21, 25, 29\}$
72. $\{1, 2, 3, 5, 8, 12, 14, 20, 21, 25, 30\}$
73. $\{1, 2, 3, 5, 8, 13, 18, 21, 25, 30, 32\}$
74. $\{1, 2, 3, 5, 8, 13, 19, 21, 22, 32, 33\}$
75. $\{1, 2, 3, 5, 8, 13, 19, 21, 25, 29, 33\}$
76. $\{1, 2, 3, 5, 8, 13, 19, 21, 25, 30, 32\}$
77. $\{1, 2, 3, 5, 8, 13, 19, 21, 25, 30, 33\}$
78. $\{1, 2, 3, 5, 8, 13, 19, 21, 30, 32, 33\}$
79. $\{1, 2, 3, 5, 8, 13, 20, 21, 25, 30, 32\}$
80. $\{1, 2, 3, 5, 8, 13, 21, 22, 28, 32, 33\}$
81. $\{1, 2, 3, 5, 8, 13, 21, 25, 26, 30, 32\}$
82. $\{1, 2, 3, 5, 8, 13, 21, 25, 27, 29, 33\}$
83. $\{1, 2, 3, 5, 8, 13, 21, 25, 27, 30, 32\}$
84. $\{1, 2, 3, 5, 8, 13, 21, 25, 28, 30, 32\}$
85. $\{1, 2, 3, 5, 8, 13, 21, 25, 28, 30, 33\}$
86. $\{1, 2, 3, 5, 8, 13, 21, 27, 30, 32, 33\}$
87. $\{1, 2, 3, 5, 8, 13, 22, 24, 26, 29, 33\}$
88. $\{1, 2, 3, 5, 8, 13, 22, 24, 28, 29, 33\}$
89. $\{1, 2, 3, 5, 8, 13, 23, 25, 26, 30, 32\}$
90. $\{1, 2, 3, 5, 8, 13, 23, 25, 27, 30, 32\}$
91. $\{1, 2, 3, 5, 8, 14, 18, 21, 25, 30, 32\}$
92. $\{1, 2, 3, 5, 8, 14, 20, 21, 25, 30, 32\}$
93. $\{1, 2, 3, 5, 9, 11, 13, 19, 21, 25, 29\}$
94. $\{1, 2, 3, 5, 9, 11, 13, 19, 21, 29, 33\}$

95. $\{1, 2, 3, 5, 9, 11, 13, 21, 25, 27, 29\}$
96. $\{1, 2, 3, 5, 9, 11, 13, 21, 27, 29, 33\}$
97. $\{1, 2, 3, 5, 9, 12, 13, 21, 25, 27, 29\}$
98. $\{1, 2, 3, 5, 9, 12, 13, 21, 25, 28, 29\}$
99. $\{1, 2, 3, 5, 9, 13, 19, 21, 25, 29, 33\}$
100. $\{1, 2, 3, 5, 9, 13, 21, 25, 28, 29, 33\}$
101. $\{1, 2, 3, 5, 10, 12, 13, 19, 21, 30, 32\}$
102. $\{1, 2, 3, 5, 10, 12, 13, 21, 27, 30, 32\}$
103. $\{1, 2, 3, 5, 11, 12, 13, 21, 25, 27, 29\}$
104. $\{1, 2, 3, 5, 11, 12, 13, 21, 25, 28, 29\}$
105. $\{1, 2, 3, 5, 11, 13, 19, 21, 22, 29, 33\}$
106. $\{1, 2, 3, 5, 11, 13, 19, 21, 25, 29, 30\}$
107. $\{1, 2, 3, 5, 11, 13, 21, 22, 28, 29, 33\}$
108. $\{1, 2, 3, 5, 11, 13, 21, 25, 28, 29, 30\}$
109. $\{1, 2, 3, 5, 12, 13, 18, 21, 25, 29, 30\}$
110. $\{1, 2, 3, 5, 12, 13, 19, 21, 22, 29, 32\}$
111. $\{1, 2, 3, 5, 12, 13, 19, 21, 22, 29, 33\}$
112. $\{1, 2, 3, 5, 12, 13, 19, 21, 25, 29, 30\}$
113. $\{1, 2, 3, 5, 12, 13, 19, 21, 29, 30, 32\}$
114. $\{1, 2, 3, 5, 12, 13, 19, 21, 30, 31, 32\}$
115. $\{1, 2, 3, 5, 12, 13, 20, 21, 25, 29, 30\}$
116. $\{1, 2, 3, 5, 12, 13, 21, 22, 28, 29, 32\}$
117. $\{1, 2, 3, 5, 12, 13, 21, 22, 28, 29, 33\}$
118. $\{1, 2, 3, 5, 12, 13, 21, 25, 26, 29, 30\}$
119. $\{1, 2, 3, 5, 12, 13, 21, 25, 27, 29, 30\}$
120. $\{1, 2, 3, 5, 12, 13, 21, 25, 27, 29, 32\}$
121. $\{1, 2, 3, 5, 12, 13, 21, 25, 27, 29, 33\}$
122. $\{1, 2, 3, 5, 12, 13, 21, 25, 28, 29, 30\}$
123. $\{1, 2, 3, 5, 12, 13, 21, 25, 28, 29, 32\}$
124. $\{1, 2, 3, 5, 12, 13, 21, 25, 28, 29, 33\}$
125. $\{1, 2, 3, 5, 12, 13, 21, 27, 29, 30, 32\}$
126. $\{1, 2, 3, 5, 12, 13, 21, 27, 30, 31, 32\}$
127. $\{1, 2, 3, 6, 7, 8, 12, 13, 21, 25, 29\}$
128. $\{1, 2, 3, 6, 7, 8, 12, 13, 21, 25, 30\}$
129. $\{1, 2, 3, 6, 7, 8, 13, 21, 25, 30, 32\}$
130. $\{1, 2, 3, 8, 12, 13, 18, 21, 25, 26, 29\}$
131. $\{1, 2, 3, 8, 12, 13, 18, 21, 25, 26, 30\}$
132. $\{1, 2, 3, 8, 13, 18, 21, 25, 26, 30, 32\}$
133. $\{1, 2, 4, 5, 6, 8, 13, 21, 25, 30, 32\}$
134. $\{1, 2, 4, 5, 7, 8, 13, 21, 25, 30, 32\}$
135. $\{1, 2, 4, 5, 8, 11, 13, 19, 21, 22, 33\}$
136. $\{1, 2, 4, 5, 8, 11, 13, 19, 21, 25, 30\}$
137. $\{1, 2, 4, 5, 8, 11, 13, 19, 21, 29, 33\}$
138. $\{1, 2, 4, 5, 8, 11, 13, 19, 21, 30, 32\}$
139. $\{1, 2, 4, 5, 8, 11, 13, 21, 22, 28, 33\}$
140. $\{1, 2, 4, 5, 8, 11, 13, 21, 24, 27, 30\}$
141. $\{1, 2, 4, 5, 8, 11, 13, 21, 24, 27, 33\}$

142. {1, 2, 4, 5, 8, 11, 13, 21, 24, 28, 30}
143. {1, 2, 4, 5, 8, 11, 13, 21, 24, 28, 33}
144. {1, 2, 4, 5, 8, 11, 13, 21, 25, 28, 30}
145. {1, 2, 4, 5, 8, 11, 13, 21, 27, 29, 33}
146. {1, 2, 4, 5, 8, 11, 13, 21, 27, 30, 32}
147. {1, 2, 4, 5, 8, 12, 13, 19, 21, 22, 32}
148. {1, 2, 4, 5, 8, 12, 13, 19, 21, 30, 32}
149. {1, 2, 4, 5, 8, 12, 13, 21, 22, 28, 32}
150. {1, 2, 4, 5, 8, 12, 13, 21, 25, 27, 32}
151. {1, 2, 4, 5, 8, 12, 13, 21, 25, 28, 32}
152. {1, 2, 4, 5, 8, 12, 13, 21, 27, 30, 32}
153. {1, 2, 4, 5, 8, 13, 18, 21, 25, 30, 32}
154. {1, 2, 4, 5, 8, 13, 19, 21, 22, 32, 33}
155. {1, 2, 4, 5, 8, 13, 19, 21, 24, 29, 33}
156. {1, 2, 4, 5, 8, 13, 19, 21, 24, 30, 32}
157. {1, 2, 4, 5, 8, 13, 19, 21, 24, 30, 33}
158. {1, 2, 4, 5, 8, 13, 19, 21, 25, 30, 32}
159. {1, 2, 4, 5, 8, 13, 19, 21, 30, 32, 33}
160. {1, 2, 4, 5, 8, 13, 20, 21, 25, 30, 32}
161. {1, 2, 4, 5, 8, 13, 21, 22, 28, 32, 33}
162. {1, 2, 4, 5, 8, 13, 21, 24, 27, 29, 33}
163. {1, 2, 4, 5, 8, 13, 21, 24, 27, 30, 32}
164. {1, 2, 4, 5, 8, 13, 21, 24, 28, 30, 33}
165. {1, 2, 4, 5, 8, 13, 21, 25, 26, 30, 32}
166. {1, 2, 4, 5, 8, 13, 21, 25, 27, 30, 32}
167. {1, 2, 4, 5, 8, 13, 21, 25, 28, 30, 32}
168. {1, 2, 4, 5, 8, 13, 21, 27, 30, 32, 33}
169. {1, 2, 4, 5, 11, 13, 19, 21, 22, 29, 33}
170. {1, 2, 4, 5, 11, 13, 21, 22, 28, 29, 33}
171. {1, 2, 5, 8, 12, 13, 16, 21, 25, 27, 32}
172. {1, 2, 5, 8, 12, 13, 16, 21, 25, 28, 32}
173. {1, 2, 5, 8, 12, 13, 17, 21, 25, 27, 32}
174. {1, 2, 5, 8, 12, 13, 17, 21, 25, 28, 32}
175. {1, 3, 4, 5, 7, 8, 13, 21, 23, 30, 32}
176. {1, 3, 4, 5, 8, 13, 20, 21, 23, 30, 32}
177. {1, 3, 5, 7, 8, 12, 13, 21, 23, 25, 29}
178. {1, 3, 5, 7, 8, 12, 13, 21, 23, 25, 30}
179. {1, 3, 5, 7, 8, 13, 21, 23, 25, 30, 32}
180. {1, 3, 5, 8, 12, 13, 20, 21, 23, 25, 29}
181. {1, 3, 5, 8, 12, 13, 20, 21, 23, 25, 30}
182. {1, 3, 5, 8, 13, 20, 21, 23, 25, 30, 32}
183. {2, 3, 4, 5, 6, 8, 9, 13, 21, 22, 32}
184. {2, 3, 4, 5, 6, 8, 9, 13, 21, 22, 33}
185. {2, 3, 4, 5, 6, 8, 13, 21, 22, 29, 33}
186. {2, 3, 4, 5, 6, 8, 13, 21, 22, 30, 32}
187. {2, 3, 4, 5, 8, 9, 13, 19, 21, 22, 32}
188. {2, 3, 4, 5, 8, 9, 13, 19, 21, 22, 33}

189. {2, 3, 4, 5, 8, 13, 19, 21, 22, 29, 33}
190. {2, 3, 4, 5, 8, 13, 19, 21, 22, 30, 32}
191. {2, 3, 5, 6, 8, 12, 13, 14, 21, 25, 30}
192. {2, 3, 5, 8, 12, 13, 14, 21, 25, 27, 30}

Minimum decycling sets (size 19) for Γ_8:

1. {0, 2, 4, 6, 7, 9, 11, 14, 16, 18, 21, 25, 29, 35, 37, 39, 42, 47, 54}
2. {0, 2, 4, 6, 7, 9, 11, 14, 16, 18, 21, 29, 33, 35, 37, 39, 42, 47, 49}
3. {0, 2, 4, 6, 7, 9, 11, 14, 16, 18, 21, 29, 33, 35, 37, 39, 42, 47, 54}
4. {1, 2, 3, 5, 6, 8, 12, 13, 17, 20, 21, 25, 30, 34, 38, 41, 45, 48, 52}
5. {1, 2, 3, 5, 6, 8, 12, 13, 17, 20, 21, 30, 32, 34, 38, 41, 42, 48, 52}
6. {1, 2, 3, 5, 6, 8, 12, 13, 17, 20, 21, 30, 32, 34, 38, 41, 45, 48, 52}
7. {1, 2, 3, 5, 6, 8, 12, 13, 17, 21, 22, 32, 34, 38, 41, 43, 48, 49, 52}
8. {1, 2, 3, 5, 6, 8, 12, 13, 17, 21, 30, 32, 34, 38, 41, 42, 48, 49, 52}
9. {1, 2, 3, 5, 6, 8, 12, 13, 17, 21, 30, 32, 34, 38, 41, 43, 48, 49, 52}
10. {1, 2, 3, 5, 6, 8, 12, 13, 20, 21, 22, 32, 34, 38, 41, 43, 48, 50, 52}
11. {1, 2, 3, 5, 6, 8, 12, 13, 20, 21, 25, 30, 34, 38, 41, 42, 48, 50, 52}
12. {1, 2, 3, 5, 6, 8, 12, 13, 20, 21, 25, 30, 34, 38, 41, 45, 48, 50, 52}
13. {1, 2, 3, 5, 6, 8, 12, 13, 20, 21, 30, 32, 34, 38, 41, 42, 48, 50, 52}
14. {1, 2, 3, 5, 6, 8, 12, 13, 20, 21, 30, 32, 34, 38, 41, 43, 48, 50, 52}
15. {1, 2, 3, 5, 8, 12, 13, 17, 19, 21, 22, 32, 34, 38, 41, 43, 48, 49, 52}
16. {1, 2, 3, 5, 8, 12, 13, 17, 19, 21, 30, 32, 34, 38, 41, 42, 48, 49, 52}
17. {1, 2, 3, 5, 8, 12, 13, 17, 19, 21, 30, 32, 34, 38, 41, 43, 48, 49, 52}
18. {1, 2, 3, 5, 8, 12, 13, 17, 20, 21, 25, 30, 34, 36, 38, 40, 45, 48, 52}
19. {1, 2, 3, 5, 8, 12, 13, 17, 20, 21, 25, 30, 34, 38, 40, 41, 45, 48, 52}
20. {1, 2, 3, 5, 8, 12, 13, 17, 20, 21, 30, 32, 34, 36, 38, 40, 42, 48, 52}
21. {1, 2, 3, 5, 8, 12, 13, 17, 20, 21, 30, 32, 34, 36, 38, 40, 45, 48, 52}
22. {1, 2, 3, 5, 8, 12, 13, 17, 20, 21, 30, 32, 34, 38, 40, 41, 42, 48, 52}
23. {1, 2, 3, 5, 8, 12, 13, 17, 20, 21, 30, 32, 34, 38, 40, 41, 45, 48, 52}
24. {1, 2, 3, 5, 8, 12, 13, 19, 20, 21, 22, 32, 34, 38, 41, 43, 48, 50, 52}
25. {1, 2, 3, 5, 8, 12, 13, 19, 20, 21, 30, 32, 34, 38, 41, 42, 48, 50, 52}
26. {1, 2, 3, 5, 8, 12, 13, 19, 20, 21, 30, 32, 34, 38, 41, 43, 48, 50, 52}
27. {1, 2, 3, 5, 8, 12, 13, 19, 21, 22, 32, 34, 38, 41, 43, 48, 49, 50, 52}
28. {1, 2, 3, 5, 8, 12, 13, 19, 21, 30, 32, 34, 38, 41, 42, 48, 49, 50, 52}
29. {1, 2, 3, 5, 8, 12, 13, 19, 21, 30, 32, 34, 38, 41, 43, 48, 49, 50, 52}
30. {1, 2, 3, 5, 8, 12, 13, 20, 21, 25, 30, 34, 36, 38, 40, 42, 48, 50, 52}
31. {1, 2, 3, 5, 8, 12, 13, 20, 21, 25, 30, 34, 36, 38, 40, 45, 48, 50, 52}
32. {1, 2, 3, 5, 8, 12, 13, 20, 21, 25, 30, 34, 38, 40, 41, 42, 48, 50, 52}
33. {1, 2, 3, 5, 8, 12, 13, 20, 21, 25, 30, 34, 38, 40, 41, 45, 48, 50, 52}

Minimum decycling sets (size 33) for Γ_9:

1. {0, 4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 26, 29, 33, 35, 36, 37, 39, 46, 47, 56, 57, 58, 60, 63, 68, 74, 76, 80, 85, 87}
2. {0, 4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 26, 29, 33, 35, 36, 37, 46, 47, 52, 56, 57, 58, 60, 63, 68, 74, 76, 80, 85, 87}

3. {0, 4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 29, 33, 35, 36, 37, 39, 46, 47, 56, 57, 58, 60, 63, 68, 74, 76, 80, 81, 85, 87}
4. {0, 4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 26, 29, 33, 35, 36, 37, 39, 46, 47, 56, 57, 58, 60, 63, 68, 74, 76, 78, 80, 85, 87}
5. {0, 4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 26, 29, 33, 35, 36, 37, 46, 47, 52, 56, 57, 58, 60, 63, 68, 74, 76, 78, 80, 85, 87}
6. {0, 4, 6, 7, 9, 10, 11, 14, 15, 17, 18, 22, 23, 26, 29, 33, 35, 36, 37, 39, 46, 47, 56, 57, 58, 60, 63, 68, 74, 76, 80, 85, 87}
7. {0, 4, 6, 7, 9, 10, 11, 14, 15, 17, 18, 22, 23, 26, 29, 33, 35, 36, 37, 46, 47, 52, 56, 57, 58, 60, 63, 68, 74, 76, 80, 85, 87}
8. {0, 4, 6, 7, 9, 10, 11, 14, 16, 18, 20, 22, 23, 26, 29, 33, 35, 36, 37, 39, 46, 47, 56, 57, 58, 60, 63, 68, 74, 76, 80, 85, 87}
9. {0, 4, 6, 7, 9, 10, 11, 14, 16, 18, 20, 22, 23, 26, 29, 33, 35, 36, 37, 46, 47, 52, 56, 57, 58, 60, 63, 68, 74, 76, 80, 85, 87}
10. {0, 4, 6, 7, 9, 10, 11, 14, 16, 18, 20, 22, 23, 29, 33, 35, 36, 37, 39, 46, 47, 56, 57, 58, 60, 63, 68, 74, 76, 80, 81, 85, 87}
11. {0, 4, 6, 7, 9, 10, 11, 14, 16, 18, 22, 23, 26, 29, 33, 35, 36, 37, 46, 47, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 85, 87}
12. {0, 4, 6, 7, 9, 10, 11, 14, 16, 18, 22, 23, 29, 33, 35, 36, 37, 46, 47, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 81, 85, 87}
13. {0, 4, 6, 7, 9, 10, 11, 14, 16, 18, 22, 26, 29, 33, 35, 36, 37, 46, 47, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 78, 80, 85, 87}
14. {0, 4, 6, 7, 9, 10, 11, 14, 17, 18, 22, 23, 26, 29, 33, 35, 36, 37, 46, 47, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 85, 87}
15. {0, 4, 6, 7, 9, 11, 14, 15, 16, 18, 22, 23, 26, 29, 33, 35, 36, 37, 39, 42, 47, 56, 57, 58, 60, 63, 67, 68, 74, 76, 80, 86, 87}
16. {0, 4, 6, 7, 9, 11, 14, 15, 16, 18, 22, 23, 26, 29, 33, 35, 36, 37, 42, 47, 52, 56, 57, 58, 60, 63, 67, 68, 74, 76, 80, 86, 87}
17. {0, 4, 6, 7, 9, 11, 14, 15, 16, 18, 22, 23, 26, 29, 33, 35, 36, 38, 39, 42, 47, 56, 57, 58, 60, 63, 67, 68, 74, 76, 80, 86, 87}
18. {0, 4, 6, 7, 9, 11, 14, 15, 16, 18, 22, 23, 29, 33, 35, 36, 37, 39, 42, 47, 56, 57, 58, 60, 63, 67, 68, 74, 76, 80, 81, 86, 87}
19. {0, 4, 6, 7, 9, 11, 14, 15, 17, 18, 22, 23, 26, 29, 33, 35, 36, 37, 39, 42, 47, 56, 57, 58, 60, 63, 67, 68, 74, 76, 80, 86, 87}
20. {0, 4, 6, 7, 9, 11, 14, 15, 17, 18, 22, 23, 26, 29, 33, 35, 36, 37, 42, 47, 52, 56, 57, 58, 60, 63, 67, 68, 74, 76, 80, 86, 87}
21. {0, 4, 6, 7, 9, 11, 14, 16, 18, 20, 22, 23, 26, 29, 33, 35, 36, 37, 39, 42, 47, 56, 57, 58, 60, 63, 67, 68, 74, 76, 80, 86, 87}
22. {0, 4, 6, 7, 9, 11, 14, 16, 18, 20, 22, 23, 26, 29, 33, 35, 36, 37, 42, 47, 52, 56, 57, 58, 60, 63, 67, 68, 74, 76, 80, 86, 87}
23. {0, 4, 6, 7, 9, 11, 14, 16, 18, 20, 22, 23, 26, 29, 33, 35, 36, 38, 39, 42, 47, 56, 57, 58, 60, 63, 67, 68, 74, 76, 80, 86, 87}
24. {0, 4, 6, 7, 9, 11, 14, 16, 18, 20, 22, 23, 29, 33, 35, 36, 37, 39, 42, 47, 56, 57, 58, 60, 63, 67, 68, 74, 76, 80, 81, 86, 87}

25. {0, 4, 6, 7, 9, 11, 14, 16, 18, 22, 23, 26, 29, 33, 35, 36, 37, 42, 47, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 86, 87}
26. {0, 4, 6, 7, 9, 11, 14, 16, 18, 22, 23, 26, 29, 33, 35, 36, 38, 42, 47, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 86, 87}
27. {0, 4, 6, 7, 9, 11, 14, 16, 18, 22, 23, 29, 33, 35, 36, 37, 42, 47, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 81, 86, 87}
28. {0, 4, 6, 7, 9, 11, 14, 17, 18, 22, 23, 26, 29, 33, 35, 36, 37, 42, 47, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 86, 87}
29. {1, 2, 3, 5, 6, 8, 12, 13, 17, 20, 21, 25, 28, 30, 31, 34, 38, 43, 48, 49, 52, 55, 59, 62, 65, 66, 69, 73, 77, 79, 81, 84, 88}
30. {1, 2, 3, 5, 6, 8, 12, 13, 17, 20, 21, 25, 28, 30, 32, 34, 38, 43, 48, 49, 52, 55, 59, 62, 65, 66, 69, 73, 77, 78, 81, 84, 88}
31. {1, 2, 3, 5, 6, 8, 12, 13, 17, 20, 21, 25, 30, 32, 34, 38, 41, 43, 48, 49, 52, 55, 59, 62, 65, 66, 69, 73, 77, 78, 81, 84, 88}
32. {1, 2, 3, 5, 6, 8, 12, 13, 17, 20, 21, 25, 30, 32, 34, 38, 41, 43, 48, 50, 52, 55, 59, 62, 65, 66, 69, 73, 77, 78, 81, 84, 88}
33. {1, 2, 3, 5, 6, 8, 12, 13, 17, 20, 21, 25, 30, 32, 34, 41, 43, 45, 48, 50, 52, 55, 59, 62, 65, 66, 69, 73, 77, 78, 81, 84, 88}
34. {1, 2, 3, 5, 6, 8, 12, 13, 17, 20, 21, 25, 30, 34, 41, 43, 45, 48, 50, 52, 55, 59, 62, 65, 66, 69, 73, 77, 78, 79, 81, 84, 88}
35. {1, 2, 3, 5, 6, 8, 12, 13, 17, 20, 21, 28, 30, 31, 32, 34, 38, 43, 48, 49, 52, 55, 59, 62, 65, 66, 69, 73, 77, 79, 81, 84, 88}
36. {1, 2, 3, 5, 6, 8, 12, 13, 17, 20, 21, 28, 30, 32, 34, 38, 41, 45, 48, 52, 55, 59, 64, 65, 69, 70, 73, 77, 78, 79, 81, 84, 88}
37. {1, 2, 3, 5, 6, 8, 12, 13, 17, 20, 21, 28, 30, 32, 34, 38, 43, 48, 49, 52, 55, 59, 62, 65, 66, 69, 73, 77, 78, 79, 81, 84, 88}
38. {1, 2, 3, 5, 6, 8, 12, 13, 17, 20, 21, 28, 30, 32, 34, 38, 43, 48, 49, 52, 55, 59, 62, 65, 66, 69, 73, 77, 78, 79, 82, 84, 88}
39. {1, 2, 3, 5, 6, 8, 12, 13, 17, 20, 21, 30, 32, 34, 38, 41, 43, 48, 49, 52, 55, 59, 62, 65, 66, 69, 73, 77, 78, 79, 81, 84, 88}
40. {1, 2, 3, 5, 6, 8, 12, 13, 17, 20, 21, 30, 32, 34, 38, 41, 43, 48, 50, 52, 55, 59, 62, 65, 66, 69, 73, 77, 78, 79, 81, 84, 88}
41. {1, 2, 3, 5, 6, 8, 12, 13, 20, 21, 25, 30, 32, 34, 38, 41, 43, 48, 50, 52, 55, 59, 62, 65, 66, 69, 71, 73, 77, 78, 81, 84, 88}
42. {1, 2, 3, 5, 8, 12, 13, 17, 19, 20, 21, 25, 28, 30, 31, 34, 38, 43, 48, 49, 52, 55, 59, 61, 62, 65, 66, 68, 77, 79, 81, 84, 88}
43. {1, 2, 3, 5, 8, 12, 13, 17, 19, 20, 21, 25, 28, 30, 32, 34, 38, 43, 48, 49, 52, 55, 59, 61, 62, 65, 66, 68, 77, 78, 81, 84, 88}
44. {1, 2, 3, 5, 8, 12, 13, 17, 19, 20, 21, 25, 30, 32, 34, 38, 41, 43, 48, 49, 52, 55, 59, 61, 62, 65, 66, 68, 77, 78, 81, 84, 88}
45. {1, 2, 3, 5, 8, 12, 13, 17, 19, 20, 21, 25, 30, 32, 34, 38, 41, 43, 48, 50, 52, 55, 59, 61, 62, 65, 66, 68, 77, 78, 81, 84, 88}
46. {1, 2, 3, 5, 8, 12, 13, 17, 19, 20, 21, 25, 30, 32, 34, 41, 43, 45, 48, 50, 52, 55, 59, 61, 62, 65, 66, 68, 77, 78, 81, 84, 88}

47. $\{1, 2, 3, 5, 8, 12, 13, 17, 19, 20, 21, 25, 30, 34, 41, 43, 45, 48, 50, 52, 55, 59, 61, 62, 65, 66, 68, 77, 78, 79, 81, 84, 88\}$
48. $\{1, 2, 3, 5, 8, 12, 13, 17, 19, 20, 21, 25, 31, 32, 34, 38, 40, 43, 49, 50, 52, 55, 59, 61, 62, 64, 66, 68, 77, 78, 81, 84, 88\}$
49. $\{1, 2, 3, 5, 8, 12, 13, 17, 19, 20, 21, 27, 28, 30, 32, 34, 38, 43, 48, 49, 52, 55, 59, 61, 62, 65, 66, 68, 77, 78, 79, 84, 88\}$
50. $\{1, 2, 3, 5, 8, 12, 13, 17, 19, 20, 21, 28, 30, 31, 32, 34, 38, 43, 48, 49, 52, 55, 59, 61, 62, 65, 66, 68, 77, 79, 81, 84, 88\}$
51. $\{1, 2, 3, 5, 8, 12, 13, 17, 19, 20, 21, 28, 30, 32, 34, 38, 43, 48, 49, 52, 55, 59, 61, 62, 65, 66, 68, 77, 78, 79, 81, 84, 88\}$
52. $\{1, 2, 3, 5, 8, 12, 13, 17, 19, 20, 21, 30, 32, 34, 38, 41, 43, 48, 49, 52, 55, 59, 61, 62, 65, 66, 68, 77, 78, 79, 81, 84, 88\}$
53. $\{1, 2, 3, 5, 8, 12, 13, 17, 19, 20, 21, 30, 32, 34, 38, 41, 43, 48, 50, 52, 55, 59, 61, 62, 65, 66, 68, 77, 78, 79, 81, 84, 88\}$
54. $\{1, 2, 3, 5, 8, 12, 13, 19, 20, 21, 25, 30, 32, 34, 38, 41, 43, 48, 50, 52, 55, 59, 61, 62, 65, 66, 68, 72, 77, 78, 81, 84, 88\}$
55. $\{4, 6, 7, 9, 10, 11, 12, 14, 15, 16, 18, 22, 23, 24, 26, 29, 35, 36, 37, 39, 42, 47, 56, 57, 58, 60, 63, 68, 74, 76, 80, 85, 87\}$
56. $\{4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 47, 56, 57, 58, 60, 63, 67, 68, 74, 76, 80, 84\}$
57. $\{4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 47, 56, 57, 58, 60, 63, 67, 68, 74, 76, 80, 85\}$
58. $\{4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 35, 36, 37, 39, 42, 47, 56, 57, 58, 60, 63, 67, 68, 74, 76, 80, 85, 87\}$

Appendix 2

Minimum Independent Decycling Sets of Fibonacci Cubes

Minimum independent decycling sets (size 3) for Γ_5:

1. $\{0, 4, 9\}$
2. $\{0, 9, 11\}$

Minimum independent decycling sets (size 6) for Γ_6:

1. $\{0, 4, 7, 11, 14, 18\}$
2. $\{0, 4, 9, 14, 15, 18\}$
3. $\{1, 2, 3, 5, 8, 13\}$
4. $\{1, 2, 3, 5, 12, 13\}$
5. $\{1, 3, 5, 8, 13, 20\}$

Minimum independent decycling sets (size 11) for Γ_7:

1. $\{0, 4, 6, 7, 11, 14, 18, 22, 23, 29, 33\}$
2. $\{0, 4, 7, 11, 14, 18, 22, 23, 26, 29, 33\}$
3. $\{1, 2, 3, 5, 8, 12, 13, 19, 21, 25, 30\}$
4. $\{1, 2, 3, 5, 8, 12, 13, 19, 21, 30, 32\}$
5. $\{1, 2, 3, 5, 8, 12, 13, 20, 21, 25, 30\}$
6. $\{1, 2, 3, 5, 8, 12, 13, 21, 25, 27, 30\}$
7. $\{1, 2, 3, 5, 8, 12, 13, 21, 25, 27, 32\}$
8. $\{1, 2, 3, 5, 8, 12, 13, 21, 25, 28, 30\}$
9. $\{1, 2, 3, 5, 8, 12, 13, 21, 25, 28, 32\}$
10. $\{1, 2, 3, 5, 8, 12, 13, 21, 27, 30, 32\}$
11. $\{1, 2, 3, 5, 8, 13, 19, 21, 25, 30, 32\}$
12. $\{1, 2, 3, 5, 8, 13, 20, 21, 25, 30, 32\}$
13. $\{1, 2, 3, 5, 8, 13, 21, 25, 27, 30, 32\}$
14. $\{1, 2, 3, 5, 8, 13, 21, 25, 28, 30, 32\}$

15. $\{1, 2, 3, 5, 12, 13, 19, 21, 30, 31, 32\}$
16. $\{1, 2, 3, 5, 12, 13, 21, 27, 30, 31, 32\}$
17. $\{1, 2, 5, 8, 12, 13, 17, 21, 25, 27, 32\}$
18. $\{1, 2, 5, 8, 12, 13, 17, 21, 25, 28, 32\}$

Minimum independent decycling sets (size 19) for Γ_8:

1. $\{1, 2, 3, 5, 8, 12, 13, 17, 19, 21, 30, 32, 34, 38, 41, 43, 48, 49, 52\}$
2. $\{1, 2, 3, 5, 8, 12, 13, 17, 20, 21, 25, 30, 34, 38, 40, 41, 45, 48, 52\}$
3. $\{1, 2, 3, 5, 8, 12, 13, 17, 20, 21, 30, 32, 34, 38, 40, 41, 45, 48, 52\}$
4. $\{1, 2, 3, 5, 8, 12, 13, 19, 20, 21, 30, 32, 34, 38, 41, 43, 48, 50, 52\}$
5. $\{1, 2, 3, 5, 8, 12, 13, 19, 21, 30, 32, 34, 38, 41, 43, 48, 49, 50, 52\}$
6. $\{1, 2, 3, 5, 8, 12, 13, 20, 21, 25, 30, 34, 38, 40, 41, 45, 48, 50, 52\}$

Minimum independent decycling sets (size 33) for Γ_9:

1. $\{0, 4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 26, 29, 33, 35, 36, 37, 39, 46, 47, 56, 57, 58, 60, 63, 68, 74, 76, 80, 85, 87\}$
2. $\{0, 4, 6, 7, 9, 10, 11, 14, 16, 18, 22, 23, 26, 29, 33, 35, 36, 37, 46, 47, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 85, 87\}$
3. $\{0, 4, 6, 7, 9, 11, 14, 15, 16, 18, 22, 23, 26, 29, 33, 35, 36, 37, 39, 42, 47, 56, 57, 58, 60, 63, 67, 68, 74, 76, 80, 86, 87\}$
4. $\{0, 4, 6, 7, 9, 11, 14, 16, 18, 22, 23, 26, 29, 33, 35, 36, 37, 42, 47, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 86, 87\}$
5. $\{4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 47, 56, 57, 58, 60, 63, 67, 68, 74, 76, 80, 85\}$
6. $\{4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 35, 36, 37, 39, 42, 47, 56, 57, 58, 60, 63, 67, 68, 74, 76, 80, 85, 87\}$

Minimum independent decycling sets (size 55) for Γ_{10}:

1. $\{0, 4, 6, 7, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 53, 56, 57, 58, 60, 63, 67, 68, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 106, 108, 109, 110, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141\}$

Minimum independent decycling sets (size 94) for Γ_{11}:

1. $\{4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232\}$
2. $\{4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 184, 187, 193, 194, 196, 199, 203, 205, 206, 210, 213, 217, 221, 222, 223, 225, 228, 232\}$

3. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 120, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232}
4. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 165, 172, 174, 175, 176, 178, 182, 185, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232}
5. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 120, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232}
6. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 119, 121, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 184, 187, 193, 194, 196, 199, 203, 205, 206, 210, 213, 217, 221, 222, 223, 225, 228, 232}
7. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 82, 85, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 120, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232}
8. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 80, 82, 83, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 172, 174, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 214, 217, 221, 222, 223, 228, 232}
9. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 82, 85, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 117, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232}
10. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97,

101, 102, 109, 110, 114, 116, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232}

11. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 116, 119, 121, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232}
12. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 193, 194, 196, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232}
13. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 117, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232}
14. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 165, 169, 172, 175, 176, 178, 182, 185, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232}
15. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 116, 119, 121, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232}
16. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 116, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
17. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 116, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145,

146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232}

18. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 116, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
19. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 119, 121, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 210, 213, 217, 221, 222, 223, 225, 228, 232}
20. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 210, 213, 217, 221, 222, 223, 225, 228, 232}
21. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 210, 213, 217, 221, 222, 223, 225, 228, 232}
22. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 116, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
23. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 110, 116, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
24. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 116, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182,

187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}

25. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 117, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
26. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232}
27. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 87, 90, 91, 92, 94, 97, 101, 102, 110, 114, 117, 119, 123, 127, 129, 130, 134, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 209, 210, 213, 217, 221, 222, 225, 228, 232}
28. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 130, 132, 133, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
29. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 120, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 209, 210, 213, 217, 221, 222, 225, 228, 232}
30. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 188, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
31. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 108, 110, 114, 116, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}

32. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 108, 110, 114, 116, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 214, 215, 217, 221, 222, 223, 225, 228, 232}
33. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 108, 110, 116, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
34. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 108, 110, 116, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 214, 215, 217, 221, 222, 223, 225, 228, 232}
35. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 188, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 214, 217, 221, 222, 225, 228, 232}
36. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 74, 75, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 164, 165, 169, 171, 175, 176, 178, 182, 184, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 215, 217, 221, 222, 225, 228, 232}
37. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 74, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 164, 165, 169, 171, 175, 176, 178, 182, 184, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 214, 215, 217, 221, 222, 225, 228, 232}
38. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 188, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
39. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97,

101, 102, 106, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 188, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 215, 217, 221, 222, 225, 228, 232}

40. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 130, 132, 133, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 215, 217, 221, 222, 225, 228, 232}
41. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 185, 187, 189, 192, 194, 196, 199, 203, 205, 206, 208, 209, 210, 213, 215, 217, 221, 222, 225, 228, 232}
42. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 185, 187, 189, 192, 194, 196, 199, 203, 205, 206, 208, 209, 210, 213, 217, 221, 222, 225, 228, 232}
43. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 215, 217, 221, 222, 225, 228, 232}
44. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 214, 215, 217, 221, 222, 225, 228, 232}
45. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 215, 217, 221, 222, 225, 228, 232}
46. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 116, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145,

146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 215, 217, 221, 222, 225, 228, 232}

47. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 116, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 215, 217, 221, 222, 225, 228, 232}
48. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 116, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 214, 215, 217, 221, 222, 225, 228, 232}
49. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
50. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 208, 209, 210, 213, 217, 221, 222, 225, 228, 232}
51. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232}
52. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232}
53. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182,

185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232}

54. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 188, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232}
55. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 116, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232}
56. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 214, 217, 221, 222, 225, 228, 232}
57. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 214, 217, 221, 222, 225, 228, 232}
58. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 74, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 164, 165, 169, 171, 175, 176, 178, 182, 184, 185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 215, 217, 221, 222, 225, 228, 232}
59. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 116, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 214, 217, 221, 222, 225, 228, 232}
60. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 208, 209, 210, 213, 214, 217, 221, 222, 225, 228, 232}

61. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
62. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 208, 209, 210, 213, 215, 217, 221, 222, 225, 228, 232}
63. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232}
64. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232}
65. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 67, 68, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 214, 215, 217, 221, 222, 223, 225, 228, 232}
66. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 130, 132, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 187, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
67. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 68, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 214, 215, 217, 221, 222, 225, 228, 232}
68. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97,

101, 102, 108, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}

69. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 74, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 164, 165, 171, 174, 175, 176, 178, 182, 184, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232}
70. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232}
71. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232}
72. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 80, 83, 85, 86, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 188, 189, 192, 196, 199, 203, 205, 206, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232}
73. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232}
74. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 209, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232}
75. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 137, 139, 141, 145, 146,

147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 188, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232}

76. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 108, 110, 114, 116, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232}
77. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 108, 110, 116, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232}
78. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 74, 75, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 164, 165, 171, 174, 175, 176, 178, 182, 184, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
79. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 74, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 164, 165, 171, 174, 175, 176, 178, 182, 184, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 214, 215, 217, 221, 222, 223, 225, 228, 232}
80. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 209, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
81. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 74, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 164, 165, 169, 171, 175, 176, 178, 182, 184, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 214, 217, 221, 222, 225, 228, 232}
82. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 74, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 164, 165, 169, 171, 175, 176, 178, 182, 184,

185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232}

83. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232}
84. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 208, 209, 210, 213, 217, 221, 222, 225, 228, 232}
85. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 114, 119, 123, 127, 130, 132, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 187, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 215, 217, 221, 222, 225, 228, 232}
86. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 130, 132, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 187, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 215, 217, 221, 222, 225, 228, 232}
87. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 114, 119, 123, 127, 130, 132, 134, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 187, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 215, 217, 221, 222, 225, 228, 232}
88. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 80, 83, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 208, 213, 214, 217, 221, 222, 223, 225, 228, 232}
89. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 80, 83, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 188, 189, 192, 196, 199, 203, 205, 206, 208, 213, 214, 217, 221, 222, 223, 225, 228, 232}

90. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 213, 214, 217, 221, 222, 223, 225, 228, 232}
91. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 116, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 213, 214, 217, 221, 222, 223, 225, 228, 232}
92. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 74, 76, 80, 83, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 164, 165, 171, 174, 175, 176, 178, 182, 184, 185, 189, 192, 196, 199, 203, 205, 206, 208, 213, 214, 215, 217, 221, 222, 223, 225, 228, 232}
93. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 210, 213, 217, 221, 222, 223, 225, 228, 232}
94. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
95. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 133, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
96. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 82, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 208, 213, 215, 217, 221, 222, 223, 225, 228, 232}
97. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 83, 85, 90, 91, 92, 94, 97,

101, 102, 106, 108, 110, 119, 121, 123, 127, 129, 130, 132, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}

98. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 108, 110, 114, 119, 120, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
99. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 108, 110, 114, 120, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
100. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 83, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 208, 213, 215, 217, 221, 222, 223, 225, 228, 232}
101. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 83, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 188, 189, 192, 196, 199, 203, 205, 206, 208, 213, 215, 217, 221, 222, 223, 225, 228, 232}
102. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 208, 213, 215, 217, 221, 222, 223, 225, 228, 232}
103. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 76, 80, 83, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 213, 214, 215, 217, 221, 222, 223, 225, 228, 232}
104. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 76, 80, 83, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 116, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145,

146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 213, 214, 215, 217, 221, 222, 223, 225, 228, 232}

105. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 185, 187, 189, 192, 194, 196, 199, 203, 205, 206, 208, 209, 210, 213, 217, 221, 222, 225, 228, 232}
106. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 82, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 228, 232}
107. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 82, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 209, 210, 213, 214, 217, 221, 222, 228, 232}
108. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 117, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 209, 210, 213, 217, 221, 222, 225, 228, 232}
109. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232}
110. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
111. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 119, 121, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182,

187, 188, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232}

112. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 119, 121, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
113. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 117, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 209, 210, 213, 217, 221, 222, 225, 228, 232}
114. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 117, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 209, 210, 213, 215, 217, 221, 222, 225, 228, 232}
115. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 117, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 209, 210, 213, 217, 221, 222, 225, 228, 232}
116. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 117, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 209, 210, 213, 215, 217, 221, 222, 225, 228, 232}
117. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 129, 130, 132, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 215, 217, 221, 222, 225, 228, 232}
118. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 114, 119, 120, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 215, 217, 221, 222, 225, 228, 232}

119. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 114, 119, 123, 127, 130, 132, 133, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 215, 217, 221, 222, 225, 228, 232}
120. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 185, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
121. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 114, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
122. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 108, 110, 114, 119, 123, 127, 129, 130, 134, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
123. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
124. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232}
125. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 80, 83, 85, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 129, 130, 132, 137, 138, 139, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232}
126. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 80, 83, 85, 90, 91, 92, 94, 97,

101, 102, 108, 110, 117, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232}

127. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 174, 175, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232}
128. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 114, 116, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 174, 175, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232}
129. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 116, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 174, 175, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 209, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232}
130. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232}
131. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}
132. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 184, 187, 193, 194, 196, 199, 203, 205, 206, 210, 213, 217, 221, 222, 223, 225, 228, 232}
133. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146,

147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 184, 187, 193, 194, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232}

134. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 82, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 208, 209, 213, 215, 217, 221, 222, 223, 225, 228, 232}
135. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 82, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 117, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 209, 213, 215, 217, 221, 222, 223, 225, 228, 232}
136. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 83, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 114, 119, 120, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 213, 215, 217, 221, 222, 223, 225, 228, 232}
137. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 83, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 116, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 213, 215, 217, 221, 222, 223, 225, 228, 232}
138. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 82, 85, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 117, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 209, 213, 215, 217, 221, 222, 223, 225, 228, 232}
139. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232}
140. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 184,

185, 189, 192, 193, 199, 203, 205, 206, 208, 209, 210, 213, 217, 221, 222, 225, 228, 232}

141. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 209, 210, 213, 217, 221, 222, 228, 232}
142. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 117, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232}
143. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 129, 130, 132, 137, 138, 139, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232}
144. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 129, 130, 132, 137, 138, 139, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 214, 217, 221, 222, 225, 228, 232}
145. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 117, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232}
146. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 117, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 209, 210, 213, 214, 217, 221, 222, 225, 228, 232}
147. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 74, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232}

148. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 74, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 206, 208, 210, 213, 214, 215, 217, 221, 222, 225, 228, 232}
149. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 74, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
150. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 133, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
151. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 193, 194, 196, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232}
152. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 119, 121, 123, 127, 129, 130, 133, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
153. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 193, 194, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
154. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 119, 121, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 193, 194, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
155. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92,

94, 97, 101, 102, 106, 109, 110, 119, 121, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 193, 194, 196, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232}

156. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
157. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 117, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
158. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 117, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
159. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 185, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
160. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 185, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
161. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232}
162. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 129, 130, 134, 137, 139, 141, 145,

146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232}

163. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 114, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
164. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
165. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 114, 119, 123, 127, 129, 130, 134, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}
166. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 119, 121, 123, 127, 129, 130, 134, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232}

Appendix 3

Decycling of Fibonacci Cubes (Upper Bound)

Decycling sets (size 158) for Γ_{12}:

1. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 74, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 164, 165, 169, 171, 175, 176, 178, 182, 184, 185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 253, 254, 258, 260, 261, 263, 264, 267, 273, 276, 277, 278, 281, 282, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 303, 306, 310, 312, 314, 317, 321, 322, 326, 328, 329, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 373, 375, 376}
2. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 74, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 164, 165, 169, 171, 175, 176, 178, 182, 184, 185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 253, 254, 258, 260, 261, 263, 264, 267, 273, 276, 277, 278, 281, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 373, 375, 376}
3. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 263, 264, 267, 271, 273, 276, 277, 278, 282, 283, 285, 288, 292, 294, 297, 299,

302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 332, 333, 336, 337, 338, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 369, 373, 375, 376}

4. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 263, 264, 267, 271, 273, 277, 278, 282, 283, 285, 288, 292, 294, 297, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 369, 373, 375, 376}
5. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 263, 264, 267, 273, 276, 277, 278, 281, 282, 283, 285, 288, 292, 294, 297, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 332, 333, 336, 337, 338, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 369, 373, 375, 376}
6. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 260, 261, 263, 264, 265, 267, 273, 276, 278, 281, 282, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 303, 306, 310, 311, 312, 314, 321, 322, 326, 328, 329, 332, 333, 336, 337, 338, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 369, 373, 375, 376}
7. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 260, 261, 263, 264, 265, 267, 273, 276, 278, 281, 282, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 303, 306, 310, 312, 314, 317, 321, 322, 326, 328, 329, 332, 333, 336, 337, 338, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 369, 373, 375, 376}

8. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 260, 261, 263, 264, 265, 267, 271, 273, 276, 278, 282, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 303, 306, 310, 311, 312, 314, 321, 322, 326, 328, 329, 332, 333, 336, 337, 338, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 369, 373, 375, 376}
9. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 260, 261, 263, 264, 265, 267, 271, 273, 276, 278, 282, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 303, 306, 310, 312, 314, 317, 321, 322, 326, 328, 329, 332, 333, 336, 337, 338, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 369, 373, 375, 376}
10. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 263, 264, 267, 273, 276, 278, 281, 282, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 303, 306, 310, 312, 314, 317, 321, 322, 326, 328, 329, 332, 333, 336, 337, 338, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 369, 373, 375, 376}
11. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 263, 264, 267, 271, 273, 276, 278, 282, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 303, 306, 310, 312, 314, 317, 321, 322, 326, 328, 329, 332, 333, 336, 337, 338, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 369, 373, 375, 376}
12. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145,

146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 260, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 278, 282, 283, 285, 288, 292, 294, 295, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 364, 368, 369, 373, 375, 376}

13. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 260, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
14. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 120, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 260, 261, 263, 265, 267, 271, 273, 274, 276, 277, 278, 282, 283, 285, 288, 292, 294, 295, 298, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 364, 368, 369, 373, 375, 376}
15. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 165, 172, 174, 175, 176, 178, 182, 185, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 246, 250, 252, 254, 258, 260, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 278, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 364, 368, 369, 373, 375, 376}
16. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 120, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 260, 261, 263, 265,

267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 298, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}

17. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 246, 250, 252, 254, 258, 260, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 278, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 364, 368, 369, 373, 375, 376}
18. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 120, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 246, 250, 252, 254, 258, 260, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 278, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 364, 368, 369, 373, 375, 376}
19. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 184, 187, 193, 194, 196, 199, 203, 205, 206, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 246, 250, 252, 254, 258, 260, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 278, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 364, 368, 369, 373, 375, 376}
20. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 120, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 260, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 278, 282, 283, 285, 288, 292, 294, 295, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 364, 368, 369, 373,

375, 376}

21. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 120, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 246, 250, 252, 254, 258, 260, 261, 263, 265, 267, 271, 273, 274, 276, 277, 278, 282, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 303, 306, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 364, 368, 369, 373, 375, 376}
22. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 120, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 260, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
23. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 116, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 246, 250, 252, 253, 254, 258, 260, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 278, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 345, 346, 348, 351, 355, 357, 358, 364, 368, 369, 373, 375, 376}
24. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 116, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 214, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 246, 250, 252, 253, 254, 258, 260, 261, 263, 264, 265, 267, 271, 273, 276, 277, 278, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 345, 346, 348, 351, 355, 357, 358, 361, 364, 368, 369, 373, 375, 376}
25. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97,

101, 102, 106, 110, 114, 116, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 304, 306, 310, 311, 312, 314, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}

26. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 116, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 304, 306, 310, 311, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
27. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 116, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 304, 306, 310, 311, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
28. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 116, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 304, 306, 310, 311, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
29. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 116, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225,

228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 304, 306, 310, 311, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}

30. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 116, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 304, 306, 310, 311, 312, 314, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
31. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 246, 250, 252, 254, 258, 260, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 278, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 364, 368, 369, 373, 375, 376}
32. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 116, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 283, 285, 288, 292, 294, 295, 298, 299, 302, 304, 306, 310, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
33. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 116, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 261, 264, 265, 267, 271, 273, 274, 276, 277, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 304, 306, 310, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336,

337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}

34. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 116, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 261, 264, 265, 267, 271, 273, 274, 276, 277, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 304, 306, 310, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
35. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 116, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 261, 264, 265, 267, 273, 274, 276, 277, 278, 281, 283, 285, 288, 292, 294, 295, 297, 302, 303, 304, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
36. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 116, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 261, 264, 265, 267, 271, 273, 274, 276, 277, 278, 283, 285, 288, 292, 294, 295, 297, 302, 303, 304, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
37. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 116, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 261, 264, 265, 267, 271, 273, 274, 276, 277, 278, 283, 285, 288, 292, 294, 295, 297, 298, 302, 304, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
38. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46,

47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 116, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 261, 264, 265, 267, 273, 274, 276, 277, 278, 281, 283, 285, 288, 292, 294, 295, 297, 298, 302, 304, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}

39. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 116, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 261, 264, 265, 267, 271, 273, 274, 276, 277, 283, 285, 288, 292, 294, 295, 297, 298, 302, 304, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
40. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 117, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 209, 210, 213, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 304, 306, 310, 311, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
41. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 117, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 209, 210, 213, 215, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 304, 306, 310, 311, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
42. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182,

184, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 264, 265, 267, 273, 274, 276, 277, 278, 281, 283, 285, 288, 292, 294, 295, 297, 302, 303, 304, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}

43. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 264, 265, 267, 273, 274, 276, 277, 278, 281, 283, 285, 288, 292, 294, 295, 297, 298, 302, 304, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
44. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 119, 121, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 263, 264, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
45. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 76, 80, 83, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 213, 214, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 260, 261, 263, 264, 265, 267, 273, 276, 278, 281, 282, 283, 285, 288, 292, 294, 295, 297, 298, 302, 303, 304, 306, 310, 312, 314, 317, 321, 322, 326, 328, 329, 332, 333, 336, 337, 338, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 369, 373, 375, 376}
46. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 76, 80, 83, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 213, 214, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 260, 261, 263, 264, 265, 267, 271, 273, 276, 278, 281, 282, 285, 288, 292, 294, 295, 297,

298, 302, 303, 304, 306, 310, 312, 314, 317, 321, 322, 326, 328, 329, 332, 333, 336, 337, 338, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 369, 373, 375, 376}

47. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 304, 306, 310, 311, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
48. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
49. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
50. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 304, 306, 310, 311, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}

51. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 76, 80, 83, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 114, 119, 123, 127, 130, 132, 133, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 171, 174, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 213, 214, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 260, 261, 263, 264, 265, 267, 271, 273, 276, 278, 282, 283, 285, 288, 292, 294, 295, 297, 298, 302, 303, 304, 306, 310, 312, 314, 317, 321, 322, 326, 328, 329, 332, 333, 336, 337, 338, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 369, 373, 375, 376}
52. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 294, 295, 297, 299, 302, 303, 304, 306, 310, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
53. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
54. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
55. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145,

146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}

56. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 294, 295, 297, 299, 302, 303, 304, 306, 310, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
57. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 294, 295, 297, 299, 302, 304, 306, 310, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
58. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
59. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261,

264, 265, 267, 271, 273, 276, 277, 282, 283, 285, 288, 294, 295, 297, 299, 302, 304, 306, 310, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 369, 373, 375, 376}

60. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 260, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
61. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 176, 178, 182, 187, 188, 192, 193, 196, 199, 203, 205, 206, 209, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
62. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 184, 187, 193, 194, 196, 199, 203, 205, 206, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
63. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 184, 187, 193, 194, 196, 199, 203, 205, 206, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373,

375, 376}

64. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 67, 68, 72, 75, 76, 80, 82, 85, 86, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 172, 174, 175, 176, 178, 182, 184, 187, 193, 194, 196, 199, 203, 205, 206, 210, 213, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 294, 295, 297, 299, 302, 303, 304, 306, 310, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
65. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 110, 117, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 263, 264, 267, 273, 276, 277, 278, 281, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 373, 375, 376}
66. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 129, 130, 132, 137, 138, 139, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 263, 264, 267, 276, 277, 278, 281, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 332, 333, 336, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 369, 373, 375, 376}
67. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 76, 80, 83, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 108, 109, 110, 119, 121, 123, 127, 129, 130, 132, 137, 138, 139, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 185, 189, 192, 196, 199, 203, 205, 206, 208, 210, 213, 214, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 263, 264, 267, 276, 277, 278, 281, 282, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 303, 306, 310, 312, 314, 317, 321, 322, 326, 328, 329, 332, 333, 336, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 369, 373, 375, 376}
68. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 83, 85, 86, 87, 90, 91, 92,

94, 97, 101, 102, 108, 110, 119, 121, 123, 127, 130, 132, 137, 138, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 175, 176, 178, 182, 184, 185, 189, 192, 193, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 263, 264, 267, 273, 276, 277, 278, 281, 282, 283, 285, 288, 292, 294, 297, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 361, 364, 368, 373, 375, 376}

69. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 74, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 295, 297, 299, 302, 303, 304, 306, 310, 311, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
70. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 74, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 139, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 171, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 278, 282, 283, 285, 288, 292, 295, 297, 299, 302, 303, 304, 306, 310, 311, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 364, 368, 369, 373, 375, 376}
71. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 193, 194, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 304, 306, 310, 311, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
72. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 133, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225,

228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 304, 306, 310, 311, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}

73. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 119, 121, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 193, 194, 196, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 263, 264, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
74. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 193, 194, 196, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 304, 306, 310, 311, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
75. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 193, 194, 196, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 310, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
76. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 72, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 193, 194, 196, 199, 203, 205, 206, 208, 210, 213, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 254, 258, 260, 261, 263, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 306, 311, 312, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336,

337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}

77. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 117, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 304, 306, 310, 311, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
78. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 110, 114, 117, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 304, 306, 310, 311, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
79. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 264, 265, 267, 273, 274, 276, 277, 278, 281, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 304, 306, 310, 311, 314, 317, 321, 322, 326, 328, 329, 331, 332, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
80. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 264, 265, 267, 273, 274, 276, 277, 278, 281, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 304, 306, 310, 311, 314, 317, 321, 322, 326, 328, 329, 331, 332, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}
81. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46,

47, 51, 53, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 185, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 298, 299, 302, 304, 306, 310, 311, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}

82. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 53, 56, 57, 58, 60, 63, 68, 75, 76, 80, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 109, 110, 114, 119, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 175, 176, 178, 182, 185, 187, 192, 193, 196, 199, 203, 205, 206, 208, 210, 213, 215, 217, 221, 222, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 264, 265, 267, 271, 273, 274, 276, 277, 282, 283, 285, 288, 292, 294, 295, 297, 299, 302, 303, 304, 306, 310, 311, 314, 317, 321, 322, 328, 329, 331, 332, 333, 336, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376}

Decycling sets (size 264) for Γ_{13}:

1. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 264, 265, 267, 273, 274, 276, 277, 278, 281, 283, 285, 288, 292, 294, 295, 297, 302, 303, 304, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376, 378, 379, 380, 382, 385, 389, 390, 394, 396, 397, 398, 402, 404, 405, 408, 409, 411, 417, 418, 420, 421, 422, 425, 427, 429, 432, 436, 438, 439, 441, 442, 443, 446, 447, 448, 450, 454, 455, 456, 458, 461, 465, 466, 470, 472, 473, 475, 476, 481, 482, 484, 488, 489, 490, 492, 495, 499, 501, 502, 503, 508, 512, 513, 517, 519, 520, 521, 525, 527, 528, 530, 531, 532, 536, 537, 539, 543, 545, 547, 550, 554, 556, 557, 558, 560, 563, 567, 568, 572, 574, 575, 577, 578, 584, 588, 589, 593, 595, 596, 597, 601, 603, 604, 606, 607, 608}
2. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260,

261, 264, 265, 267, 273, 274, 276, 277, 278, 281, 283, 285, 288, 292, 294, 295, 297, 302, 303, 304, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376, 378, 379, 380, 382, 385, 389, 390, 394, 396, 397, 398, 402, 404, 405, 407, 408, 409, 411, 417, 418, 420, 421, 422, 425, 427, 429, 432, 436, 438, 439, 441, 442, 443, 446, 447, 448, 450, 454, 455, 456, 458, 465, 466, 470, 472, 473, 475, 476, 481, 482, 484, 488, 489, 490, 492, 495, 499, 501, 502, 503, 508, 512, 513, 517, 519, 520, 521, 525, 527, 528, 530, 531, 532, 536, 537, 539, 543, 545, 547, 550, 554, 556, 557, 558, 560, 563, 567, 568, 572, 574, 575, 577, 578, 584, 588, 589, 593, 595, 596, 597, 601, 603, 604, 606, 607, 608}

3. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 264, 265, 267, 273, 274, 276, 277, 278, 281, 283, 285, 288, 292, 294, 295, 297, 298, 302, 304, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376, 378, 379, 380, 382, 385, 389, 390, 394, 396, 397, 398, 402, 404, 405, 408, 409, 411, 417, 418, 420, 421, 422, 425, 427, 429, 432, 436, 438, 439, 441, 442, 443, 446, 447, 448, 450, 454, 455, 456, 458, 461, 465, 466, 470, 472, 473, 475, 476, 481, 482, 484, 488, 489, 490, 492, 495, 499, 501, 502, 503, 508, 512, 513, 517, 519, 520, 521, 525, 527, 528, 530, 531, 532, 536, 537, 539, 543, 545, 547, 550, 554, 556, 557, 558, 560, 563, 567, 568, 572, 574, 575, 577, 578, 584, 588, 589, 593, 595, 596, 597, 601, 603, 604, 606, 607, 608}
4. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 264, 265, 267, 273, 274, 276, 277, 278, 281, 283, 285, 288, 292, 294, 295, 297, 298, 302, 304, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376, 378, 379, 380, 382, 385, 389, 390, 394, 396, 397, 398, 402, 404, 405, 407, 408, 409, 411, 417, 418, 420, 421, 422, 425, 427, 429, 432, 436, 438, 439, 441, 442, 443, 446, 447, 448, 450, 454, 455, 456, 458, 465, 466, 470, 472, 473, 475, 476, 481, 482, 484, 488, 489, 490, 492, 495, 499, 501, 502, 503, 508, 512, 513, 517, 519, 520, 521, 525, 527, 528, 530, 531, 532, 536, 537, 539, 543, 545, 547, 550, 554, 556, 557, 558, 560, 563, 567, 568, 572, 574, 575, 577, 578, 584, 588, 589, 593, 595, 596, 597, 601, 603, 604, 606, 607, 608}

Decycling sets (size 439) for Γ_{14}:

1. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 264, 265, 267, 273, 274, 276, 277, 278, 281, 283, 285, 288, 292, 294, 295, 297, 302, 303, 304, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376, 378, 379, 380, 382, 385, 389, 390, 394, 396, 397, 398, 402, 404, 405, 408, 409, 411, 417, 418, 420, 421, 422, 425, 427, 429, 432, 436, 438, 439, 441, 442, 443, 446, 447, 448, 450, 454, 455, 456, 458, 461, 465, 466, 470, 472, 473, 475, 476, 481, 482, 484, 488, 489, 490, 492, 495, 499, 501, 502, 503, 508, 512, 513, 517, 519, 520, 521, 525, 527, 528, 530, 531, 532, 536, 537, 539, 543, 545, 547, 550, 554, 556, 557, 558, 560, 563, 567, 568, 572, 574, 575, 577, 578, 584, 588, 589, 593, 595, 596, 597, 601, 603, 604, 606, 607, 608, 611, 612, 613, 615, 618, 623, 627, 629, 630, 631, 635, 637, 638, 640, 641, 642, 644, 648, 650, 651, 653, 654, 655, 658, 659, 660, 662, 665, 669, 671, 672, 674, 675, 676, 679, 680, 681, 683, 687, 688, 689, 691, 698, 699, 703, 705, 706, 708, 709, 710, 713, 714, 715, 717, 721, 722, 723, 725, 728, 732, 734, 735, 736, 738, 741, 745, 746, 750, 752, 753, 754, 758, 760, 761, 763, 764, 765, 768, 769, 770, 776, 778, 780, 783, 787, 790, 791, 793, 796, 800, 801, 805, 807, 808, 810, 811, 814, 817, 821, 822, 826, 828, 829, 830, 834, 836, 837, 839, 840, 841, 843, 847, 849, 850, 852, 853, 854, 857, 858, 861, 865, 866, 872, 876, 878, 879, 880, 882, 889, 890, 894, 896, 897, 900, 901, 903, 906, 910, 911, 915, 917, 919, 923, 925, 926, 928, 929, 930, 933, 935, 937, 940, 944, 945, 949, 951, 952, 953, 957, 959, 960, 962, 963, 964, 966, 970, 972, 973, 975, 976, 977, 980, 981, 982, 984}
2. {4, 6, 7, 9, 10, 11, 14, 15, 16, 18, 22, 23, 24, 26, 29, 33, 35, 36, 37, 39, 42, 46, 47, 51, 54, 56, 57, 58, 60, 63, 67, 68, 75, 76, 82, 85, 86, 87, 90, 91, 92, 94, 97, 101, 102, 106, 108, 110, 119, 121, 123, 127, 129, 130, 134, 137, 138, 141, 145, 146, 147, 149, 152, 156, 157, 161, 163, 164, 165, 169, 172, 174, 175, 178, 182, 184, 187, 192, 193, 196, 199, 203, 205, 206, 210, 213, 215, 217, 221, 222, 223, 225, 228, 232, 234, 235, 236, 238, 241, 245, 246, 250, 252, 253, 254, 258, 260, 261, 264, 265, 267, 273, 274, 276, 277, 278, 281, 283, 285, 288, 292, 294, 295, 297, 302, 303, 304, 306, 310, 311, 312, 314, 317, 321, 322, 326, 328, 329, 331, 332, 337, 338, 340, 344, 345, 346, 348, 351, 355, 357, 358, 359, 364, 368, 369, 373, 375, 376, 378, 379, 380, 382, 385, 389, 390, 394, 396, 397, 398, 402, 404, 405, 407, 408, 409, 411, 417, 418, 420, 421, 422, 425, 427, 429, 432, 436, 438, 439, 441, 442, 443, 446, 447, 448, 450, 454, 455, 456, 458, 465, 466, 470, 472, 473, 475, 476, 481, 482, 484, 488, 489, 490, 492, 495, 499, 501, 502, 503, 508, 512, 513, 517, 519, 520, 521, 525, 527, 528, 530, 531, 532, 536, 537, 539, 543, 545, 547, 550, 554, 556, 557, 558, 560, 563, 567, 568, 572, 574, 575, 577, 578, 584, 588, 589, 593, 595, 596, 597, 601, 603, 604, 606, 607, 608, 611, 612, 613,

615, 618, 623, 627, 629, 630, 631, 635, 637, 638, 640, 641, 642, 644, 648, 650, 651, 653, 654, 655, 658, 659, 660, 662, 665, 669, 671, 672, 674, 675, 676, 679, 680, 681, 683, 687, 688, 689, 691, 698, 699, 703, 705, 706, 708, 709, 710, 713, 714, 715, 717, 721, 722, 723, 725, 728, 732, 734, 735, 736, 738, 741, 745, 746, 750, 752, 753, 754, 758, 760, 761, 763, 764, 765, 768, 769, 770, 776, 778, 780, 783, 787, 790, 791, 793, 796, 800, 801, 805, 807, 808, 810, 811, 814, 817, 821, 822, 826, 828, 829, 830, 834, 836, 837, 839, 840, 841, 843, 847, 849, 850, 852, 853, 854, 857, 858, 861, 865, 866, 872, 876, 878, 879, 880, 882, 889, 890, 894, 896, 897, 900, 901, 903, 906, 910, 911, 915, 917, 919, 923, 925, 926, 928, 929, 930, 933, 935, 937, 940, 944, 945, 949, 951, 952, 953, 957, 959, 960, 962, 963, 964, 966, 970, 972, 973, 975, 976, 977, 980, 981, 982, 984}