PROBABILISTIC OCEAN OUTFALL DESIGN

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by

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Abstract

Factors affecting initial dilution and bacterial concentration at an area near the outfall discharge, e.g. sewage flow rate, seawater currents and bacterial decay, are highly variable. Because of this, a probabilistic approach for ocean outfall design and analysis is essential in predicting the performance of the outfall and in reflecting the probabilistic nature of the initial dilution and bacterial concentration.

The intention of this thesis is to develop and apply a design procedure using a probabilistic method to calculate initial dilution and bacterial concentration at a location of interest. The scope of the study is directed at design and analysis for a horizontal buoyant round jet in a density unstratified seawater environment. Uncertainty of five parameters of design, i.e. sewage flow rate, tidal height, seawater currents, decay parameter, and bacterial concentration in the sewage before discharge into seawater are taken into account in this study.

A comparison of the probabilistic approach with the deterministic approach shows that the probabilistic approach may provide a full range of possible values of the parameters of interest other than a fixed value. Associated probability values for the parameters of interest can also be obtained using the probabilistic methods. The procedure for outfall design using a probabilistic approach is straight forward, and may work in practice because the analysis of an existing outfall (the Spaniard's Bay Outfall, Newfoundland, Canada) has resulted in good agreement with field data.

Comparison among the various probabilistic methods studied shows that all methods generally give the same answers for the case of initial dilution, except for a small probability of failure which is typically less 4 %. It is found that First Order Second Moment (FOSM), Improved Mean-First Order Second Moment (IM-FOSM) and Advanced First Order Second Moment (AFOSM) with assumed normal parameters work well for use in analysis of initial dilution. In practice, the use of FOSM is recommended for its simplicity.

For the case of bacterial concentration, FOSM gives poor results because the performance function in this case is complex and non linear, but AFOSM with assumed non-normal parameters is recommended. Monte Carlo Simulations (MCS) may also be used if a fast computer and software are available. It should be remembered that the choice of the probability method should consider the problem under investigation as well as the cost and facilities available.

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Finally, I would like to say, "*Alhamdulilaahi rabbil'aalamiin*", Praise be to God, Allah SWT, The Cherisher and Sustainer of the Worlds, who makes it possible to do this work. *Wa Allahu A'lam.*

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B : discharge specific buoyancy flux (m^4/sec^3), $B = Q\ g\ (\rho_a - \rho_e)/\rho_e$;

BOD_5 : five-day biochemical oxygen demand;

b : initial width of waste field (m);

C_1 : Lee's model constant (coefficient) for buoyancy-dominated near field;

C_2 : Lee's model constant (coefficient) for buoyancy-dominated far field;

C_i : initial concentration of indicator bacteria;

C_o : initial effluent concentration of surface waste field;

C_{max} : maximum concentration of effluent at a distance of x;

$Cov(X_i, X_j)$: covariance of basic variable X_i and X_j;

CV(X) : coefficient of variation of variable X;

D : nozzle diameter of the outfall (m);

D_m : molecular diffusion coefficient;

E(X) : expectation of the variable X;

e : error model term;

erf(y) : error function of y;

F : densimetric Froude number (dimensionless);

g : gravitational acceleration (m/sec^2);

g(X) : functional description of system performance function X;

H : total available depth between outfall port to the free surface (m);

H(X) : function representing the corrected model output;

h : tidal height (m) above Lowest Normal Water, LNW;

k_d : decay rate-constant (s^{-1});

k_o : diffusion coefficient at $x = 0$;

k_x : turbulent diffusion coefficient in x direction;

k_y : turbulent diffusion coefficient in y direction;

k_z : turbulent diffusion coefficient in z direction;

L : length scale (taken as the surface plume with);

n : power exponent;

m_x : sample mean of variable X;

P_f : probability of failure;

p_i : Blom's plotting possition;

Q : wastewater flow rate (m^3/sec);

q_c : solute mass flux of concentration C in direction of x;

S_d : concentration reduction factor in bacterial decay;

S_i : initial dilution in general, either in still or moving water (dimensionless);

S_m : initial dilution for moving waters (dimensionless);

S_o : initial dilution for still waters (dimensionless);

S_s : secondary dilution (dimensionless);

S_t : total compounded concentration reduction factor;

s_x : sample standard deviation of variable X;

T_{90} : decay parameter (hours);

T_m : threshold level in performance function for moving water initial dilution;

T_L : threshold level in a performance function;

T_o : threshold level in performance function for still water initial dilution;

U : ambient seawater current (m/sec);

u : surface current speed (m/s);

V_j : jet velocity (m);

Var(X) : variance of the variable X;

X_i : basic variable i having random values;

x : distance downstream from outfall discharge (m);

Y : seawater depth above discharge (m), i.e. the distance between outfall port to the bottom of surface layer;

Y_i : transformed variable (for the Box-Cox transformation of the power normal distribution);

Z : standardized variable Z;

Z_i : inverses of the cumulative distribution function of the normal distribution;

α : coefficient of proportionality;

β : reliability index;

δ : difference between the calculated probability of failure using method i and that using MCS;

ρ_a : density of ambient seawater (kg/m^3);

ρ_e : density of wastewater (kg/m^3);

λ : transformation parameter (for the Box-Cox transformation of power normal distribution);

μ_i : mean value of variable i;

μ_{Xi} : mean value of basic variable X_i.

σ_i : standard deviation of variable i;

Δ : difference between the calculated and critical probability of failure (%);

$(\partial g)/(\partial X_i)$: first partial derivative of system performance function with respect to basic variable i evaluated at μ_{Xi};

$(\partial^2 g)/(\partial X_i\, \partial X_j)$: second partial derivative of system performance function with respect to basic variable i evaluated at μ_{Xi}, and basic variable j evaluated at μ_{Xj};

Chapter 1

Introduction

1.1 Background and Objectives of Thesis

Throughout the history of modern engineering, modeling and analysis using quantitative methods have been the most important tools for design and assessment purposes. Some of these methods have become quite elaborate and include sophisticated analysis; however, irrespective of the level of sophistication in the models, including experimental models, they are developed under some assumptions to simplify problems so that they may not reflect the actual conditions of the problems under investigation. Furthermore, people have rarely provided *complete information* including ranges of values and uncertainties in data.

Ang and Tang (1975) noted that one cannot predict with certainty the occurrence (or nonoccurrence) of specific events. The underlying uncertainty may be due to: a) the inherent randomness of the natural phenomenon, b) the inaccuracies in estimation of the parameters and in choice of the distribution, and c) the inaccuracies of modeling which is based on idealized assumptions. As a result, the use of a deterministic approach, which does not take these uncertainty factors into account in solving engineering problems, although sometimes useful, is unrealistic. This may lead to a partial loss of information, misleading results, and incorrect solutions. Properly, the tools of engineering analysis should therefore include methods and concepts for evaluating the significance of uncertainty on system performance and design.

Ocean outfall design is, in fact, a very small subset of the engineering designs necessary to make the world an environmentally-safe place to live in. The main purpose of ocean outfall design is to reduce the concentration of contaminants in wastewater (i.e. usually sewage) to a level which is acceptable to the environment by utilizing natural processes which are available in the ocean to dilute, disperse and assimilate the wastewater. Following some degree of land-based treatment (either primary or secondary treatment), together with dilution and dispersion in the ocean, the wastes are then stabilized by bio-chemical processes. The processes include oxidation of organic-chemical material and decay of *solar-energy-sensitive* bacteria. Therefore, the proper design should be *environmental-and-cost effective* (Sharp & Allen, 1987; Marine Treatment Working Group, 1990; Sharp, 1991).

Many ocean outfalls have been constructed in marine waters over the world as discussed by Gunnerson (1975), Grace, R. A. (1978), and Toms(1986). The main structures in an ocean outfall system are the outfall itself which is the pipeline or the tunnel that transports the wastewater from the land to its disposal point in relatively open costal waters, generally beyond the surf zone . The length of the pipe (or the tunnel) varies from case to case depending upon the treatment capacity of the outfall and the environmental behavior of sea waters. Good examples may be the Spaniard's Bay Outfall in Newfoundland for handling sewage in small communities (Sharp, 1991) and the Boston Outfall in Massacusetts for large outfall systems (French, 1989).

Located on the east coast of Newfoundland, Canada, the Spaniard's Bay Outfall was designed to handle a peak flow design of about 3347 to 4426 m^3/day discharged through two 0.1 m diameter nozzles at approximately 0.1 km offshore in about 7 m depth of water. On the other hand, the Boston Outfall was designed for an average flow of 18 x 10^6 m^3/day using approximately 16 km of offshore tunnel.

The primary concern of outfall design is to ensure that the wastewater is well assimilated in the ocean by using the assimilative capacity of the ocean. Referring to Goldberg, Wolfe (1988) defined the assimilative capacity as "*a concept for waste management in which the waste inputs to an environment are balanced against natural environmental processes of dilution, dispersion, and degradation to maintain the potentially adverse environmental impacts within acceptable bounds.*" Thus, the assimilative capacity of the ocean reflects the extent to which the ocean can receive wastes discharged from the outfall installation without unacceptable impacts such as extremes in oxygen concentration deficit, bacteria concentration, and aesthetic impacts.

Whereas dispersion and degradation (i.e. chemical oxidation and bacterial decay) are entirely controlled by the natural condition of the ocean environment, the initial dilution of wastewater with ambient seawater may be increased by properly designing the geometry of the jet and controlling the flow of the effluent. Because parameters affecting initial dilution, dispersion, and bacterial decay such as wastewater flow rate, seawater depth above discharge, ambient seawater currents, and intensity of solar radiation are highly random variables, probabilistic concepts are essential in estimating the performance of an ocean outfall and in reflecting the probabilistic nature of the dilution and the concentration of effluent in the vicinity of a target area, e.g. bathing or shellfish area.

As a result, the current widely used practice involving simple deterministic models in designing ocean outfall e.g. Lee and Neville-Jones (1987-a), Brooks (1960) needs to be complemented by probabilistic analysis. In fact, probability methods have been a scientific, workable alternative tool in solving engineering problems. Accordingly, this thesis research is to develop and apply a design procedure using a probabilistic method to calculate initial dilution and bacterial concentration in the vicinity of a target area. The problem may then be formulated and a methodology would be determined to solve problems as discussed in the following section.

1.2. Scope of Study, Problem Formulation, and Methodology

Designing an ocean outfall system may be done either using deterministic or probabilistic approaches. In the deterministic approach, for given typical input parameters, the value of the output-parameter of interest, e.g. initial dilution or bacterial concentration at a specific location in the vicinity the outfall, is estimated as a fixed single value. Unlike the deterministic approach, because of inevitable uncertainty in parameters involved in the system, the probabilistic approach expresses the parameter of interest in terms of its occurrence (e.g. a specific value) and its associated probability.

In a probabilistic design, for example, a designer of an outfall system no longer simply expects that the design would provide an initial dilution of, for example, 100. Instead, it specifies that the probability is, say 5 %, that the design would have an initial dilution less than 100. This indicates the probability of failure, i.e. 5%, that the specified dilution criterion, i.e. 100, cannot be satisfied. This approach would provides a more realistic estimation to comply with the government regulations as discussed in Chapters Two and Three.

A wide range of aspects, ranging from social and environmental surveys to technical aspects such as hydraulics and construction, are present in designing an outfall system. A great deal of work is needed to accomplish a probabilistic analysis on all the aspects. As in the previous discussion, important aspects of the design are to calculate initial dilution and bacterial concentration at a distance from the discharge so that a discharge structure, proper sitting strategies, and effluent transport estimations may be then determined (Jirka and Lee, 1994).

Although it is possible to increase initial dilution by using multiport diffusers, particularly for large outfall systems, the diffusers provide increased initial dilution only within a small mixing zone near the diffusers. Gunnerson (1988) recommends the use of a simple open end in cases where adequate initial dilution will be provided to meet water quality standards, and in cases where plume submergence due to a diffuser is unattainable or undesirable. In addition, the simple open end is the easiest terminus to build and maintain.

Dilution and transport of the effluent are affected by the seawater environment which may be stratified or unstratified with regard to density. Many outfall systems can be categorized or assumed to be unstratified water density, for example, the Spaniard Bay Outfall in Newfoundland (Sharp, 1991) and Miami-Central Outfall in Florida (Huang, 1994). For these reasons, this research topic is directed at the probabilistic hydraulic design to calculate initial dilution, secondary dilution, and bacterial decay for a horizontal buoyant round jet in a density unstratified seawater environment.

Having concentrated on the selected topic, the problem can then be formulated as follows:-

1. Choose deterministic models appropriate for the ocean outfall under investigation.
2. Classify the degree of variability of parameters affecting initial dilution, secondary dilution, and bacterial decay.
3. Calculate sample moments and fitting probability distributions to the parameters of interest.

4. Define performance functions, their threshold level, and probability of failure.
5. Formulate government regulations relating to the sewage discharge in terms of probability statements (referred herein as the critical probability of failure).
6. Compute probability of failure for a given threshold level using several probabilistic methods, e.g. first order second moment (FOSM), advance first order second moment (AFOSM), and Monte Carlo Simulation (MCS).
7. Evaluate whether or not that the final design is acceptable.

The methodology for the probabilistic design of ocean outfalls is therefore the answer to the problems in the above formulation. More detailed discussion is given in Chapter Three which presents a procedure for probabilistic ocean outfall design.

1.3 Outline of Thesis

Chapter Two presents a background review of the basic concepts of designing ocean outfalls. After discussing the purpose and description of ocean outfalls, this chapter reviews principles of outfall design, the concept of dilution and bacterial decay. This chapter also briefly reviews factors affecting performance of an ocean outfall system including effluent outlet geometries, characteristics of urban wastewater and the marine environment.

Chapter Three will introduce types and measures of uncertainty, and discuss a procedure for probabilistic ocean outfall design particularly for a horizontal buoyant round jet in still or moving waters. After reviewing previous works on the application of probabilistic methods in ocean outfall design, Chapter Four presents the development and application of the probabilistic methods for the outfall design. Formula derivations and the procedure of the simulation are given in this chapter. The four probabilistic methods discussed are First Order Second Moment (FOSM), Improved Mean-First Order Second Moment (IM-FOSM), Advance First Order Second Moment (AFOSM) and Monte Carlo Simulations (MCS).

Chapter Five deals with a case study of Spaniard's Bay Outfall located on the east cost of Newfoundland, Canada. A comparison between calculated results and field test data, and a comparison between a deterministic and probabilistic approaches are then discussed in Chapter Six. Discussion and Conclusions, including recommendations for use of probabilistic ocean outfall design, are given in Chapter Seven.

Chapter 2

Basic Concepts in Designing Ocean Outfalls

2.1. Introduction: reason for designing an ocean outfall

Most large cities of the world are located along estuaries, embayments, or the open ocean. As a consequence, the coastal region is the most heavily used part of the world's ocean for serving national security, commerce, industry, fisheries, recreation, water supply, and waste disposal activities. Appropriately planned and constructed systems for wastewater disposal are therefore crucial in maintaining the value of the environment and other resources of the coastal region (Arlosorroff , 1988).

Discharging wastewater to the marine environment has been a controversial issue for several years (Baalsrud, 1975; Calvert, 1975; Pearson, 1975; Clough and Canon, 1980; Sharp and Allen, 1987; Sharp, 1991; Wood, et. al., 1993). This is mostly because of different ways of looking at problems in treating sewage waste and whether or not it is considered acceptable to be discharged in the seawater body. However, such discharges are commonly done, and are usually directed to comply with wastewater disposal regulations. The purpose of design is to ensure compliance with the environmental quality objectives by which it is required that a body of seawater should be suitable for its designed purpose such as recreation or shellfish. Design should also consider the most economical and reliable alternatives.

Other methods of dealing with sewage and wastewater also exist. For example, the aim of secondary treatment is to reduce the concentration of contaminants to a level which is acceptable to the environment into which they are discharged (Marine Treatment Working Group, 1990). Unlike the other methods of treating wastewater, ocean outfalls utilize natural processes which are available in the ocean to dilute, disperse and assimilate wastewater following an appropriate level of land-based treatment (either primary or secondary treatment).

When sewage is discharged into the ocean, it mixes with the ambient sea water and many phenomena occur, such as dispersion, advection and bacterial decay. As wastewater rises between the discharge point and the terminal height, it is diluted by mixing with ambient seawater. For the region close to the discharge point, the mixing process is dynamically affected by the discharge (Jirka and Lee, 1994), and the dilution experienced by the wastewater in this active dispersal region is referred to as initial dilution.

Following completion of the initial dilution, the established waste field is advected by ocean currents and diluted further by oceanic turbulence. This further dilution is usually called secondary dilution. Whereas the initial dilution occurs in a region closed to the discharge point in which the dilution is controlled by geometry of jet, ambient water and wastewater characteristics, the secondary dilution occurs in a region in which the mixing is entirely controlled by natural processes.

The combination of ocean outfall and land-based treatment is sometime referred to as marine treatment. The marine treatment of wastewater utilizes the capacity of the sea to complete the treatment carried out on land. Comparisons between fully land-based treatment and marine treatment are available elsewhere, for example Sharp & Allen (1987), Marine Treatment Working Group (1990), and Sharp (1991). The following brief description is cited from Marine Treatment Working Group (1990).

In marine treatment, the preliminary treatment carried out at the headworks usually includes the fine screening of wastewater down to 6 mm or less with the screenings being removed from the site. This, together with high dilution of the effluent, would reduce the impact of sewage debris and would reduce the size of any visible slick on the sea surface. Initial dilution of the effluent can reduce faecal coliform concentration by a factor of, typically, 200 (i.e. from 10 million/100ml to 50,000/100ml). The reduction of bacteria concentration is further increased through the natural processes of dispersion and decay such that after approximately 4 hours of advection from the discharge site bacterial concentration would be typically 10,000 times lower than in the effluent from the secondary treatment plant.

With this ability to create dilution it is acceptable to design marine treatment schemes to give acceptably low bacterial concentrations in bathing areas as well as other target areas. Comparing marine treatment with full secondary treatment on land only, one may find that the main perceived advantage of inland secondary treatment is that the discharge mass of contaminants would be relatively small. However, this is only significant for inland watercourses and in poorly flushed marine waters where eutrophication might otherwise occur. Elsewhere the organic fraction and nutrients are readily assimilated by the marine environment.

Tables 2.1 and 2.2 summarize comparison between on-land treatment facilities and marine treatment (pre-discharge treatment and ocean outfall) in terms of costs and the performance.

Table 2.1. Typical Estimated Cost ($) for On-land Treatment Plant and Marine Outfall (Consultants' estimate in 1987, after Sharp, 1991)*

Item	On-land treatment plant ($)	Marine outfall ($)	
		100 m length	300 m length
Site development	150,000.-	50,000.-	50,000.-
Sewage treatment pant	750,000.-	-	-
Grit & screen chamber	-	25,000.-	25,000.-
Comminutor	-	25,000.-	25,000.-
Pumping station	-	75,000.-	75,000.-
Outfall	-	100,000.-	300,000.-
Capital Cost:	**900,000.-**	**275,000.-**	**475,000.-**
Manpower	7,500.-	5,000.-	5,000.-
Power	45,000.-	6,500.-	6,500.-
Chemical	10,000.-	-	-
Sludge removal	4,000.-	-	-
Maintenance	10,000.-	3,500.-	3,500.-
Debris removal	-	2,500.-	2,500.-
Debt repayment (interest rate of 12%)	120,000.-	36,000.-	62,500.-
Annual Operations & Maintenance:	**196,500.-**	**53,500.-**	**80,000.-**

*) estimated of construction cost proved fairly accurate, with the final cost being within 5 % of the estimate. The estimated life time for the two systems is 20 years. The treatment plant and the outfall were designed for treating sewage waste from a small town in Spaniard's Bay (peak flow of about 3347 to 4426 m^3/day).

Table 2.2. Performance Comparison between On-land Treatment Facility and Combined Pre-discharge and Ocean Outfall (after Allen & Sharp, 1987)

On-land treatment facility		Pre-discharge treatment and ocean outfall	
BOD (mg/l):			
raw sewage	220		220
primary treatment	200	after kinetic dilution	12
secondary treatment	30	after buoyant dilution	<1.5 above ambient
tertiary treatment	12-15	after 6 hours difficult to detect	
Suspended solid (mg/l):			
raw sewage	220		220
primary treatment	90	after kinetic dilution	13
secondary treatment	30	after buoyant dilution	<1 above ambient
tertiary treatment	12-15	after 6 hours difficult to detect	
Total coliforms per 100 ml:			
raw sewage	1×10^8		1×10^8
primary treatment	5×10^6	after kinetic dilution	5×10^6
secondary treatment	4×10^6	after buoyant dilution	4×10^5
tertiary treatment	2×10^6	after 6 hours	5×10^2
Sterilization	1×10^6		
Additional comments:			
Land use problems	significant	small	
Odor problems	a concern	controllable	
Disposal problems	significant	minor	
Nutrient Enhancement	a concern	minor	

2.2. Principles of Outfall Design

In engineering design it is important to determine whether the designed system will satisfy requirements which are set up to ensure that impacts resulting from the operation of the designed system are within acceptable bounds. The requirements for ocean outfall design are usually regulated by the government, and they vary from case to case depending upon the sea environment and the degree of risk being within acceptable limits.

A great deal of work must be conducted to properly design an ocean outfall system. The works lie on a range from social and environmental surveys to technical aspects. The social issues would essentially involve "*a public use survey of coastal and the determination of the extent and possible impact of public opinion on the final site selection decision*" (William, 1985). Once preliminary information on an acceptable site has been obtained, detailed oceanographic and biological resource surveys are undertaken to rank alternatives and provide the information base for the selection of construction methods, outfall design parameters, and effluent dilution calculations.

Beside the mechanical or construction aspects such as type of pipe (or tunnel) material and joints, estimation of wave and current effects on the pipe, anchoring of the pipe, and so on, hydraulic aspects of the proposed outfall should be well designed so that effluent concentrations anywhere in the vicinity of the outfall discharge are within acceptable bounds as given in the regulation. To mitigate any harmful local effects and to anticipate the global large scale degradation and transformation processes, two principal means may be used, i.e first, the choice and design of a discharge structure, and secondly, proper sitting strategies and effluent transport estimations (Jirka and Lee, 1994).

Although these two means are essentially connected with each other, each has different tasks. The discharge structure should achieve considerably high initial dilutions, for example well above 100, whereas the site condition should ensure that the effluent is effectively removed from the vicinity discharge point by maintaining a reasonable secondary dilutions. Usually, the bacterial decay process in the wastewater is also considered in the hydraulic design to estimate bacterial concentrations along the coastline near the discharge locality.

2.3. Initial Dilution

By definition, initial dilution is the ratio of the pollutant concentration in the wastewater to the maximum concentration at the terminal height or boil. For still homogenous seawater, the terminal height is near to the surface. Wood, et. al. (1993) stated, "*the initial dilution is that obtained by the entrainment from the surrounding fluid during the rise of the effluent from the outfall ports to its equilibrium level or the free surface.*" This rising motion occurs because of buoyancy resulting from the density difference, because the density of wastewater is usually less than that of seawater. Typical geometry of a buoyant jet following discharge from a horizontal, round nozzle is given in figure 2.1. Williams (1985) reported that in the surface field formed over the diffuser (or nozzle for figure 2.1), the wastewater is typically diluted by a factor of 50 - 150 in situations where coastal currents are weak.

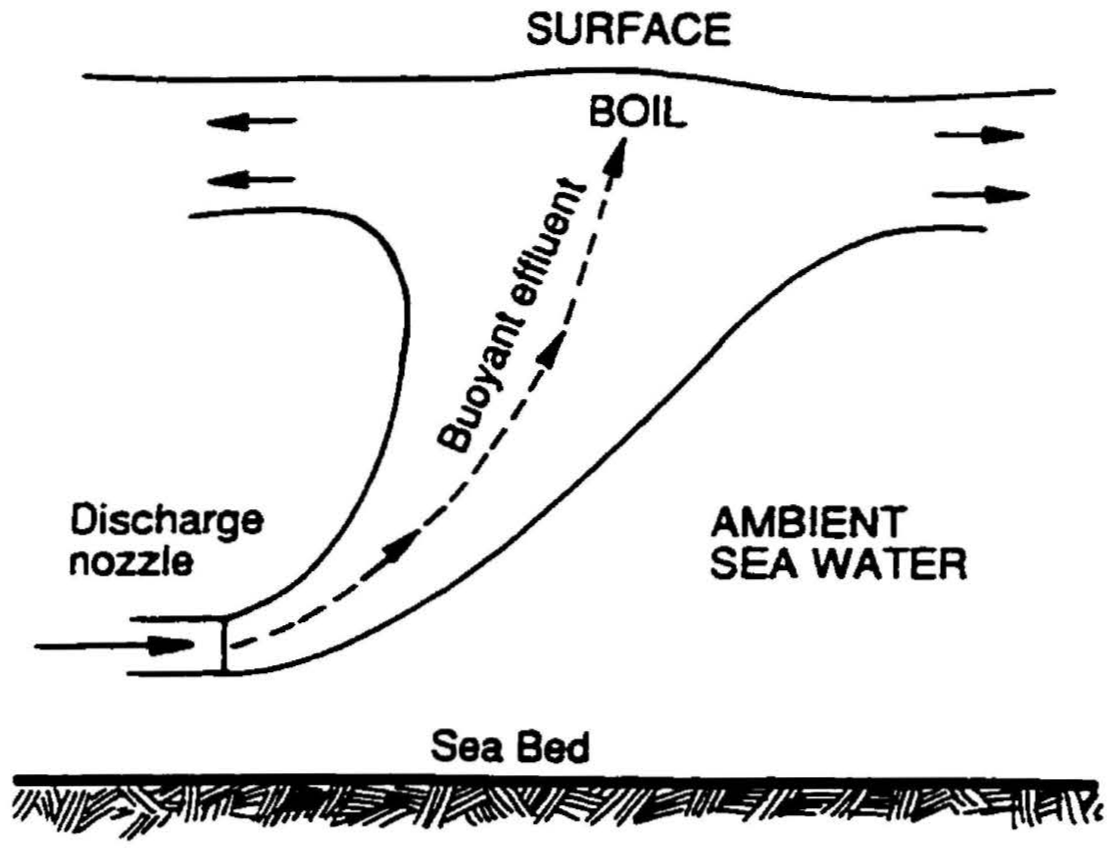

Figure 2.1. Typical geometry of buoyant jet following discharge from horizontal, round nozzle (after Moore et. al., 1991)

The degree of initial dilution depends on the depth above discharge, the design of the effluent outlet and the strength of the current flow at the discharge location. The presence of the current would increase the initial dilution and alter the thickness of the surface field. If the coastal water is density stratified, the effluent may form a subsurface field resulting from the neutral buoyancy at an intermediate depth. In many cases, however, the density stratification is not significant such as in Spaniard Bay outfall (Sharp, 1991) and in most of New Zealand coastal waters (Williams, 1985).

Large numbers of mathematical and experimental models are available for initial dilution calculations as discussed by Wood, et. al. (1993) and Jirka & Lee (1994). These models are presented in the following sections. Details of the derivations can be found elsewhere e.g. Fischer et al., 1979; Wood, et. al., 1993.

2.3.1. Initial Dilution in Still Waters

For a stagnant condition, many deterministic models have been available either for the case of a pure jet (density difference zero), a pure plume (initial momentum is very small compared to buoyancy), or a buoyant jet in which both buoyancy and momentum are important. The first two cases, however, are very rare for an ocean outfall system, except for discharging sewage using a submerged outfall in a fresh water reservoir (momentum dominated) or a point source releasing sewage in the very deep ocean water (buoyancy dominated). In most cases, it is necessary to consider both buoyancy and momentum and a wide variety of solutions is available (Sharp, 1989-a).

One of the solutions is given by Cederwall (1968) which provides a good, simple, empirical solution. Cederwall's (1968) model values cover a full range of horizontally discharged buoyant jets, from buoyancy dominated to momentum dominated. The calculations agree with other theoretically and experimentally derived results (Sharp, 1989-a; Wood, 1993). Cederwall's solutions are based on the equations:-

$$S_o = 0.54 F^{9/16} \left(\frac{Y}{D}\right)^{7/16} \qquad \textit{for the range of} \ \left(\frac{Y}{D}\right) < 0.5 F \tag{2.1}$$

$$S_o = 0.54 F \left[0.38 \ \frac{(Y/D)}{F} + 0.66\right]^{5/3} \qquad \textit{for the range of} \ \left(\frac{Y}{D}\right) > 0.5 F \tag{2.2}$$

where S_o is initial dilution of the wastewater in still waters (dimensionless), D is outfall diameter (m), Y is seawater depth above discharge (m), and F is densimetric Froude number (dimensionless), defined by:-

$$F = \frac{4Q}{3.14 \ D^{5/2}} \left(\frac{g(\rho_a - \rho_e)}{\rho_e}\right)^{-1/2} \tag{2.3}$$

where Q is wastewater flow rate (m^3/s), g is gravitational acceleration(m/s^2), ρ_a and ρ_e are density (kg/m^3) of ambient seawater and effluent, respectively. It is emphasized here that if a surface layer is typically formed, Y is the total available water depth above discharge (H) subtracted by the depth of the surface layer. The upper layer thickness for the stagnant condition may be approximated to be 10-15% of H for vertical round buoyant jet (Lee & Jirka, 1981) or 25-30% of H for buoyant jets from a multiport diffuser (William, 1985). The closer estimate of the layer thickness for the horizontal round buoyant jet is assumed to be that given by Lee & Jirka, (1981).

2.3.2. Initial Dilution in Moving Waters

When the effluent is discharged into a current, increased dilution may be expected. The increase is not, however, easy to calculate accurately, and few theoretical studies have been backed up by field measurements. Sharp & Moore (1987) and Lee and Neville-Jones (1987-a) proposed approaches to such problems. Sharp & Moore's (1987) model is based on modifications of the still water dilution given by Chederwall's (1968) model. For a seawater current U, their equation is:-

$$S_m = S_o + S_o^{1.33}\, 1.57 \left(\frac{U}{V_j}\right)^{0.359} \tag{2.4}$$

where S_m is initial dilution in moving waters, S_o is initial dilution in still water under identical conditions of depth and density difference, and V_j is jet velocity.

The approach given by Lee and Neville-Jones (1987-a) is based on all available field data on initial dilution at that time. They re-analyzed those data for horizontal round jets in cross-flow to provide a better estimation (more general) than the previous site-specific empirical models. Their equations are:

$$S_m = 0.31 \left(\frac{B^{1/3} H^{5/3}}{Q}\right) \qquad \textit{for the range of } H\left(\frac{U^3}{B}\right) < 5 \tag{2.5}$$

$$S_m = 0.32 \left(\frac{U H^2}{Q}\right) \qquad \textit{for the range of } H\left(\frac{U^3}{B}\right) > 5 \tag{2.6}$$

where Q is wastewater flow rate (volume flux of jet discharge), and B is discharge specific buoyancy flux (m^4/s^3) defined by:

$$B = \frac{Q\ g(\rho_a - \rho_e)}{\rho_a} \tag{2.7}$$

Discussion and comparison between the two models is given by Sharp (1989-b). However, as indicated above, Lee and Neville-Jones' (1987-a) model is based on all available field data at the time, and has been generally accepted as providing a good estimation of the initial dilution. For this reason, Sharp (personal communication) suggested the use of Lee and Neville-Jones' (1987-a) model for determining initial dilution in moving waters.

2.4. Secondary Dilution and Bacterial Decay

2.4.1. Secondary Dilution

After rising to the surface, the effluent spreads laterally in a fashion governed by turbulent diffusion and surface currents. Although the seawater body may essentially be still water, the presence of surface currents can be significant as shown, for example, at Spaniard's Bay (Sharp, 1991). Advection and diffusion cause a secondary dilution which will further decrease effluent concentration. The discussion of the both diffusive and advective components is widely published, for example in Fischer, et. al.(1979) and Wood, et. al. (1993).

For a one-dimensional diffusion process, mass transport is usually defined by an analogy to Fick's Law (Fischer, et. al.,1979; Wood, et. al.,1993) given by:-

$$q_c = -D_m \frac{\partial C}{\partial x} \tag{2.8}$$

where q_c is the solute mass flux of concentration C in direction of x, D_m is the molecular diffusion coefficient, and the minus sign indicates that the transport is from high to low concentration.

The rate of mass transport through a unit area in the y-z plane by the component velocity in the x direction u is the quantity (uC), i.e. the rate at which fluid volume passes through the unit area (velocity x unit area = volume per unit time) multiplied by the concentration of mass in that volume. This rate is referred to as the advective flux (Fischer, et. al., 1979). The total mass transport is the diffusive and advective flux, i.e.:-

$$q = u\,C - D_m \frac{\partial C}{\partial x} \tag{2.9}$$

The relationship between the flux q(x,t) and concentration C(x,t), where t is unit time, is given by the equation for conservation of mass (Fischer, et. al., 1979), and for one-dimension the equation is:-

$$\frac{\partial C}{\partial t} + \frac{\partial q}{\partial x} = 0 \tag{2.10}$$

Substituting equation 2.9 into equation 2.10 for constant D would give:-

$$\frac{\partial C}{\partial t}+\frac{\partial}{\partial x}(u\,C)=D\,\frac{\partial^2 C}{\partial x^2} \tag{2.11}$$

Written out fully in Cartesian coordinates, the equation is then:-

$$\frac{\partial C}{\partial t}+u\,\frac{\partial C}{\partial x}+v\,\frac{\partial C}{\partial y}+w\,\frac{\partial C}{\partial z}=D\left[\frac{\partial^2 C}{\partial x^2}+\frac{\partial^2 C}{\partial y^2}+\frac{\partial^2 C}{\partial z^2}\right] \tag{2.12}$$

This equation is often referred to as the "advective diffusion" equation.

When the flow is turbulent such as in most buoyant jets, the molecular diffusion coefficient must be replaced by a turbulent diffusion coefficient. However, in practical problems the turbulence is often not homogenous, and it is common to find the advective diffusion equation written with spatially variable coefficients in the form (Fischer, et. al., 1979):-

$$\frac{\partial C}{\partial t}+u\,\frac{\partial C}{\partial x}+v\,\frac{\partial C}{\partial y}+w\,\frac{\partial C}{\partial z}=\frac{\partial}{\partial x}\left(k_x\,\frac{\partial C}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y\,\frac{\partial C}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z\,\frac{\partial C}{\partial z}\right) \tag{2.13}$$

where k_x, k_y, and k_z are turbulent diffusion coefficients in x, y, and z directions.

Of importance here is the determination of the diffusion coefficient. As reported (Sterregaard, 1975; Talbot, 1975), the horizontal diffusion coefficient is not constant. In fact, it increases at some power of the length scale and is usually formulated as:-

$$k_x = \alpha\, L^n \tag{2.14}$$

where α is a coefficient of proportionality, L is length scale (usually taken as the surface plume width), and n is a power exponent ($1 \leq n \leq 4/3$ for coastal waters). In open waters, where eddy growth is not limited, equation (2.14) may be expressed by Richardson's law or "four third" law, i.e:-

$$k_x = \alpha L^{4/3} \tag{2.15}$$

The value of α lies generally between 0.002 and 0.01 $cm^{2/3}/s$, and $\alpha = 0.01\ cm^{2/3}/s$ ($= 0.0005\ m^{2/3}/s$) is commonly used (Williams, 1985; Markham, 1993).

In estimating effluent concentration after experiencing secondary dilution, Brooks (1960) used equation 2.13 and 2.15 simplified by the following assumptions, i.e. a) the diffusion process follows the linear diffusion law with variable k (diffusion coefficient); b) the diffusion coefficient k is function of length scale L (usually waste field width), which is in turn a function of longitudinal direction x (and not lateral direction, y); c) vertical mixing is negligible; d) longitudinal mixing (in direction of current) is negligible; and e) the flow is steady state. Under these circumstances, equation 2.13 reduces to:-

$$u\frac{\partial C}{\partial x} = \frac{\partial}{\partial x}\left(k_x \frac{\partial C}{\partial x}\right) \tag{2.16}$$

with k_x given by equation 2.15.

A practical solution given by Brooks (1960) is:-

$$\frac{C_{max}}{C_o} = erf\left(\sqrt{\frac{1.5}{\left(1 + 8\frac{k_o x}{u b^2}\right)^3 - 1}}\right) \tag{2.17}$$

where C_{max} is the maximum concentration of effluent at a distance of x from the discharge point, C_o is the initial concentration of surface waste field which may be estimated from initial concentration of the effluent above the outfall at the beginning of the surface spread, u is the velocity of a uniform surface current in the x direction, k_o is the diffusion coefficient at x = 0 (obtained from equation 2.15 with L=b), b is the initial width of waste-field which is often taken as one third of the depth if the plumes do not intermix (Sharp, 1989-a), and erf is the error function defined by:-

$$erf(y) = \frac{2}{\sqrt{\pi}} \int_0^y e^{-v^2} dv \tag{2.18}$$

This function can be calculated using Simpson's rule (Markham, 1993). Alternatively, it can be evaluated from statistical tables of the area under the Normal Distribution curve by a change in variable such that:-

$$erf(y) = 2A(z) \tag{2.19}$$

where z = 1.414 y , and A(z) is the area of the Standard Normal Distribution from 0 to z along the abscissa (Williams, 1985). The solution of equation (2.18) is practically the same as given by the commonly used error function table, and it is easy to solve when using computer simulations.

2.4.2. Bacterial Decay

In addition to the reduction of bacterial concentration resulting from physical processes (dilution, dispersion), the reduction of bacterial concentration with time after discharge through a marine outfall is also caused by a loss of viability involving many factors including solar radiation, osmotic stress, water clarity and predation by natural microbiota (Bell, et. al., 1992). The effective concentration reduction due to bacterial decay is generally approximated by first order group population kinetics.

Based on the first order group population kinetics (Bell, et. al., 1992; Wood, 1993), the rate of decay is proportional to the concentration, C, of indicator bacteria, i.e.:-

$$\frac{\partial C}{\partial t} = -k_d \, C \tag{2.20}$$

where k_d is the decay rate-constant. The effluent concentration C_t at time t is then:-

$$C_t = C_i \, e^{-k_d \, t} \tag{2.21}$$

where C_i is the initial indicator bacteria, which will depend on the level of effluent treatment. Travel time, t, from outfall discharge to the target area with a distance of x may be approximated by $t = x/u$ (u is surface current speed).

The rate-constant, k_d is obtained from experimental data by linear regression of log bacterial counts on time. It is conventionally expressed in terms of the time required for the bacteria to decrease to one-tenth of their original number, excluding physical dilution. This value is defined as the T_{90} value (i.e. 90 percent reduction) and is related to the decay rate-constant k_d by equation (2.22):-

$$k_d = \frac{\ln 10}{T_{90}} = \frac{2.3}{T_{90}} \tag{2.22}$$

Equation (2.21) can then be expressed as:-

$$C_t = C_i \, e^{-\frac{2.3\, x}{u T_{90}}} \tag{2.23}$$

2.4.3. Bacterial Concentration at a Target Area

Secondary dilution and bacterial decay can be expressed in terms of dilution or a concentration reduction factor (because of bacterial decay). From Brooks' equation (equation 2.17), secondary dilution, S_s is then:-

$$S_s = \frac{C_o}{C_{max}} = \frac{1}{erf\left(\sqrt{\frac{1.5}{\left(1 + 8\frac{k_o x}{ub^2}\right)^3 - 1}}\right)} \tag{2.24}$$

And from equation 2.23, the concentration reduction factor, S_d (because of bacterial decay) is:-

$$S_d = \frac{C_i}{C_t} = e^{\frac{2.3\,x}{u T_{90}}} \tag{2.25}$$

The total compounded concentration reduction, S_t resulted from a combination of initial dilution, secondary dilution, and bacterial decay is:-

$$S_t = S_i \times S_s \times S_d \tag{2.26}$$

The maximum constituent concentration at the target area with a distance of x from the discharge point can also be determined from the relationship:-

$$S_t = \frac{(C_e - C_b)}{(C_x - C_b)} \tag{2.27}$$

where C_e is effluent concentration in the outfall pipe, C_b is background concentration, and C_x is maximum concentration at the target area such as at bathing or shellfish areas. If the background concentration (C_b) is negligible, the compounded total dilution (S_t) can be thus expressed as a function of (C_e) and (C_x) only. Including equation (2.26), the concentration at the target area is then given by:-

$$C_x = \frac{C_e}{S_i\ S_s\ S_d} \tag{2.28}$$

or

$$C_x = \frac{C_e}{S_i}\ e^{\frac{-2.3\ x}{u T_{90}}}\ erf\left(\sqrt{\frac{1.5}{\left(1 + 8\frac{k_o x}{u b^2}\right)^3 - 1}}\right) \tag{2.29}$$

For conservative constituents, k_d in equation 2.22 equals zero. The term, which contains T_{90}, is therefore not included in equation 2.29 when calculating the concentration of conservative constituents.

2.5. Effluent Outlet Geometries

Initial dilution, secondary dilution, and bacterial decay are greatly affected by natural processes. However, whereas secondary dilution and bacterial decay are entirely beyond design control, initial dilution can be increased by proper design of the geometry of the jet given a typical characteristic of wastewater and ambient seawater. The geometry of the jet is determined by the shape and size of effluent outlet. This may consist of a simple open end, perhaps with a slight upward turn, or may consist of a multiport diffuser, containing a regularly spaced line of relatively small ports.

Most small outfalls, as well as ones built before 1950, have simple open ends (Gunnerson, 1988). For large-diameter outfalls, the multiport diffuser has become a conventional design feature. In this design, the end of the pipe is capped off and wastewater flow enters the sea through a series of small holes spaced along the sides of the outfall over most of the offshore section. The length of pipe through which effluent leaves the outfall is called the diffuser and is typically a hundred to a thousand meters in length (Grace, 1978).

The purpose of such multiport diffusers is to ensure a much greater initial interception of ambient dilution water by the effluent stream in order to obtain greater initial dilution. However, a multiport diffuser provides increased initial dilution only within a small mixing zone near the diffuser. At the distance of a few lengths downstream, particularly for density unstratified conditions, the plume dilution distribution becomes independent of the diffuser length (Gunnerson, 1988).

For this reason, use of a simple open end is recommended in cases where it will provide adequate initial dilution to meet water quality standards, and in cases where plume submergence due to a diffuser is unattainable or undesirable. A simple open end is the easiest terminus to build and maintain (Gunnerson, 1988).

2.6. Characteristics of Wastewater and Water Quality Standards

Characteristics of wastewater, in particular sewage, are very important to identify in designing ocean outfalls. This includes wastewater flow rate for calculating initial dilution (equation 2.1 - 2.7) and concentration of the contaminant of interest (e.g. total coliforms) for estimating the maximum concentration at the target area such as at bathing or shellfish areas (equation 2.21). In this section, these characteristics are briefly reviewed along with water quality standards which are regulated by relevant government bodies.

2.6.1. Wastewater Flow Rate

Domestic sewage flow varies throughout the day, and varies from case to case depending upon the water consumption in residential areas. It has been recognized that the sewage flow tends to increase as the living standards of a community are improved. However, there is a common pattern for sewage flow that reaches its peak in the morning, typically about 8-10 a.m.(Grace, 1978; Williams, 1985). Figure 2.2 shows a typical measured sewage flow on 30 August 1988 from 07.30 a.m. to 07.00 p.m. in a small residential area. The flows were measured every 15 minutes at the pumphouse just before entering the Spaniard's Bay Outfall, Newfoundland, Canada (Sharp, 1989-c).

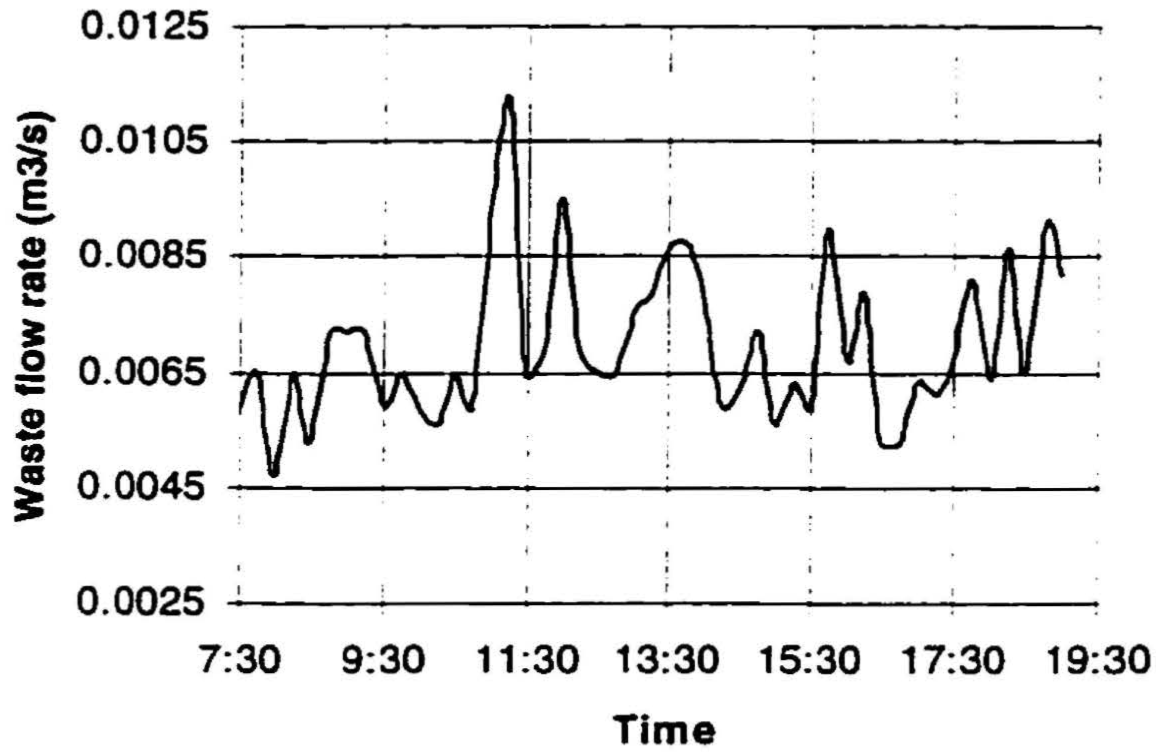

Figure 2.2 Variation of sewage flow rate during the day for a typical small residential area (Data from Sharp, 1989-c).

Because of a lack of measured data, household sewage quantity is usually estimated using standard per capita waste generation allowances applied against present and future population projections. This is typically about 180 liters per capita per day. Small quantities of infiltration and stormwater inflow increase the average annual daily flow of the domestic sewage to between 200 and 270 liters per capita per day. Excessive domestic sewage quantities indicate excessive infiltration and inflow (Williams, 1985). Other estimates of per capita flow and contaminant load are shown in Table 2.3.

Table 2.3. Typical Estimate of Per-Capita Daily Waste Loading with City Population (After Mueller & Anderson, 1983)

Population range (in thousand)	Flow (m^3/cap-day)	Suspended solids (g/cap-day)	BOD_5*	Number of cities
Rural	0.27	51	60	-
< 20	0.48	86	72	20
20 - 50	0.45	77	82	20
50 - 150	0.57	113	86	20
> 150	0.49	113	118	18

*) 5-day biochemical oxygen demand

2.6.2. Physical, Chemical and Biological Characteristics

Sewage characteristics differ from region to region, depending upon the lifestyle and habits of residents and on the presence of commercial or industrial activity in the community. Generally, fresh sewage has a density approximately the same as that of freshwater. It has a pronounced grey tint which would be lost with reasonable dilution in the ocean (Williams, 1985). Sewage contains solids of variable size, with a total solids of approximately 700 mg/l (for a nominal medium-strength), 1200 mg/l (strong), and 350 mg/l (weak). Of the 700 mg/l of the total solids in medium sewage, about 30% can be classified as suspended solids (Grace, 1978).

Sewage also contains organic (e.g., carbohydrates, proteins, and fats) and inorganic (e.g., ammonia nitrogen and metallic ion) substances. Table 2.4. indicates typical composition of untreated domestic wastewater. In addition to the physical and chemical substances, bacteria from the gut of warm blooded animals and people occur in large numbers in sewage (Wood, 1993). Faecal coliform and faecal streptococci are two indicator organisms which are usually taken to indicate the potential presence and survival of pathogens in sewage.

Table 2.4. Typical Composition of Untreated Domestic Sewage (data from Metcalf & Eddy Inc, 1979, and Wood, 1993)

Constituent	Concentration (mg/l)		
	Strong	Medium	Weak
Total solids	1200	720	350
Biochemical Oxygen Demand (BOD_5)	400	220	110
Total Organic Carbon (TOC)	290	160	80
Chemical Oxygen Demand (COD)	1000	500	250
Total Nitrogen (as N)	85	40	20
Total Phosphorus (as P)	15	8	4
Grease	150	100	50

Citing from Mara, Sinton (1993) reported that in animal feces, the faecal streptococci generally outnumber faecal coliform, although the overall concentration appears to differ markedly between species. For example, sheep faeces contains approximately 3.8×10^7 faecal streptococci per gram compared to 1.6×10^7 faecal coliform. In contrast, streptococcal concentrations in human faeces - which is typically around 3.0×10^6 per gram- are generally less than those for faecal coliform, which are typically around 1.7 x 10^7 per gram. Typical concentration of bacteria indicators in wastewater is given in Table 2.5.

Table 2.5. Typical Concentration of Faecal Indicator Bacteria (per 100 ml) (after Wood, 1993)

Source of wastewater	Total coliform	Faecal coliform	Faecal Streptococci
Raw sewage[1)]	22 x 10^6	8 x 10^6	1.6 x 10^6
Meatwork[2)]	1 x 10^8	4.2 x 10^7	9.5 x 10^6

Note 1). data obtained from several communities in USA

2). typical data from New Zealand

2.6.3. Wastewater Impacts and Water Quality Standards

Introducing an excessive amount of wastewater into seawater may result in a number of problems for both human and marine life. A notable example was the July 1976 oxygen depletion (anoxia) in marine waters of the middle Atlantic Bight. The event was attributed to a variety of both natural and human-related (high-nutrient loading) causes (Myers, 1983).

Health problems have been associated with microbial pathogens from sewage or from their indicators. It has been reported (Stevenson in Salas, 1986) that statistically significant epidemiologically detectable health effects occur at levels of around 2300 to 2700 coliforms per 100 ml. This was demonstrated by studies on Lake Michigan at Chicago in 1948 and on the Ohio River at Dayton, Kentucky in 1949.

To maintain an acceptable quality of seawater, standards are often set by safeguarding the marine ecosystem for particular uses of the coastal region. For example, areas may be defined for use by water sports, bathing, or for shellfish harvesting. Although other contaminants are regulated, for most domestic sewage the water standards are determined by indicator bacteria (Wood, et. al., 1993). In many part of the world, the regulations are based on the broader scale of conditions in the receiving waters around the outfall rather than those in the effluent as it leaves the discharge conduit (Allen & Sharp, 1987). The standards or regulations are either in deterministic (e.g. Table 2.6) or probabilistic (e.g. Table 2.7) terms.

Table 2.6. Limiting Standard in Yugoslavian Adriatic (after Allen & Sharp, 1987)

Constituent	Class 1*	Class 2*	Class 3*	Class 4*
MPN total coliform (per 100 ml)	100	5000	200,000	>200,000
Suspended solids (mg/L)	10	20	60	-

*) Class 1. Waters containing shellfish nurseries
Class 2. Waters used for recreational purposes
Class 3. Waters used by fishing industry
Class 4. All other waters, including closed harbors

Table 2.7. European Economic Community (EEC) Microbial Standards for Bathing Water (from Wood, et. al., 1993)

Organism	Guideline Value	Mandatory Value
Total coliform (per 100 ml)	500(80)*	10,000(95)*
Faecal coliform (per 100 ml)	100(80)*	2,000(95)*
Faecal streptococci (per 100 ml)	100(90)*	-

*) Numbers in parentheses indicate percentage of samples in which the counts must not be exceeded.

More detailed descriptions may also be defined. For example, standards for public bathing waters have been defined as, "*the faecal coliform concentration based on a minimum of not less than five samples for any 30-day period, shall not exceed a log mean of 200 per 100 ml nor shall more than 10 percent of the total samples during any 60-day period exceed 400 per 100 ml*" [The California Ocean Plan (1988) in Wood, et. al. (1993)] or, alternatively, as, "*the median faecal coliform bacterial shall not exceed 200 per 100 ml based on a minimum of one water sample taken on each five separate days over not more than a 30-day period; nor shall more than 10% of samples taken on separate days during any 3-day period exceed 400 faecal coliform per 100 ml*" [Water Quality Criteria Working Party (1981) in Williams (1985)].

More stringent requirements have been set for shellfish harvesting waters, e.g."*the median faecal coliform bacterial shall not exceed 14 MPN per 100 ml based on a* *minimum of one water sample taken on each of 10 consecutive days when the risk of contamination is greatest, and not more than 10% of samples shall exceed 43 MPN per 100 ml*" [Water Quality Criteria Working Party (1981) in Williams (1985)].

2.7. The Marine Environment

The characteristics of the marine environment play an important role in governing the various physical and microbiological processes involved in the marine discharge of wastewater. This includes seawater depth, which is important for accomplishing initial dilution, and seawater current, which governs both initial and secondary dilution processes. Furthermore, seawater density may be stratified or unstratified. However, as discussed in Chapter One, this thesis considers the case of homogeneous seawater density. This consideration is typically applicable when temperature and salinity have insignificant stratification as shown, for example, in Spaniard's Bay, Newfoundland, Canada, during the field test program (Sharp, 1991) where the density difference ratio was about 0.027.

2.7.1. Water Depth and Tides

The water depth above discharge affects the degree of initial dilution achievable in an outfall system. As shown in equations 2.1 and 2.3 (for still waters) and equations 2.5 and 2.6 (for moving waters), the larger the water depth above discharge the greater the initial dilution. For this reason, the length of the outfall must be designed to ensure sufficient depth to give adequate initial dilution, and also to ensure acceptable bacterial concentrations in sensitive areas.

The seawater depth varies because of the tide which is the gradual rise and fall of the sea surface during a day due to gravitational attractive forces between the sun, the moon and the earth (Grace, 1978). The significance effect of this variation on the calculation of initial dilution is somewhat controversial. Huang (1994) used the hourly tidal height (above mean lower low water (MLLW) level) in calculating the variation of the initial dilution. However, Lee and Koenig (1995) suggested that it was unnecessary there to consider tidal variations because the variation was small relative to the total depth, and because the deterministic calculations for tide is often accurate. A detailed theory of the tides is not given here, but can be found elsewhere, for example in Godin (1972) and Grace (1978). A discussion of the subject related to the uncertainty of the depth will be presented in more detail in Chapter Three, and in a case study (Chapter Five).

2.7.2. Seawater Currents

For outfall design purposes, the speed and the direction of the currents are particularly important as they affect the initial dilution process and the overall circulation and flushing of the area. Typical applications of current data in outfall design fall into four categories (Robert, 1986), i.e., the estimation of initial dilution, the quantifying of coastal hydrodynamics and circulation, the estimation of dispersion and transport and the calculation of probabilities of wastewater field shoreline impact levels.

A cumulative current speed distribution, as given by Wood, et. al. (1993), is very useful for computing initial dilution and for extracting the engineering design velocity, especially for data gathered over a few months. The method may be reproduced with some modification as shown in figure 2.3.

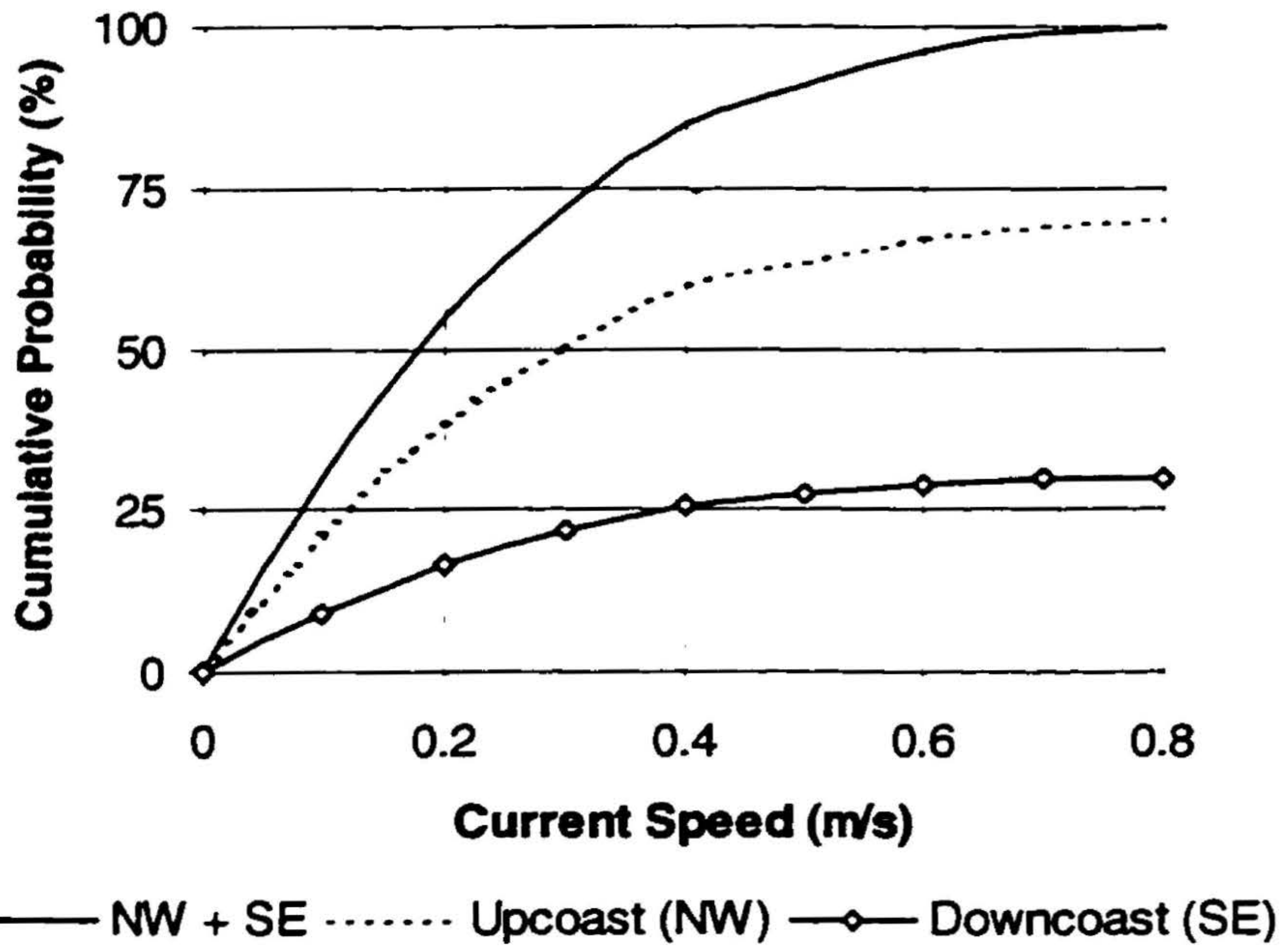

Figure 2.3. Typical Current Speed Distribution (after Wood, et.al., 1993)

From this figure, current speed and its associated probability are ready to read. For example, the percentages of time for which the currents are less than 0.4 m/s, are 25 %, 60%, and 85 % for down-coast (SE) direction, up-coast (NW) direction, and for all direction (SW+NW), respectively.

Chapter 3

Procedure for Probabilistic Ocean Outfall Design

3.1. Introduction: types and measures of uncertainty

The models discussed in Chapter Two are deterministic models which consider the use of single values instead of random values of the parameters. However, real world problems are complex, interconnected, and random. Therefore, when designers estimate the value of a specific output of a model, questions pertaining to the determination of the input parameters, including their value(s), and the reliability of such estimations are of interest. Answers to these questions are often based on observational or experimental data. Furthermore, even when the data are available, designers cannot predict with certainty the value of the output of models. At best, they could say that an event will occur with an associated probability. The uncertainty information is therefore required for the probabilistic design and reliability analysis.

For a typical engineering system, the uncertainty may be associated: (1) with inherent variability of the physical process or (2) with the imperfection in the modeling of the physical process (Ang and Tang, 1984). Whereas the first type of the uncertainty occurs because it is inherently not possible to ascertain the realization or outcome of the natural process, the potential errors of the imperfect model cannot be entirely corrected deterministically. From an engineering design stand point, the inherent variability is essentially a state of nature and the resulting uncertainty cannot be controlled. On the other hand, the uncertainty associated with the modeling may be reduced through the use of more accurate models or the acquisition of additional data (Ang and Tang, 1984).

Inherent natural variability includes the random temporal and spatial fluctuations inherent in natural processes (Melching, 1995). For the case of outfall design, a good example of the effect of natural variability is in determining T_{90} (the time required for bacteria to decrease to one-tenth of their original number, excluding physical dilution). It has been found that the value of T_{90} for bacteria in any set of experiments always varies (Bell, et. al., 1992; Wood, et. al., 1993), and differs from place to place depending upon the intensity of solar radiation which depends on the weather condition or seasonal variations. The fact that there is significant scatter in the measured T_{90} from several typical experimental runs gives rise to uncertainty in the actual value of T_{90}.

Errors in estimations or modeling, which include the measurement error (such as statistical sampling error) and the imperfection of models used, also gives rise to uncertainty. For example, the mean value of T_{90} is usually estimated from observed experimental data. Conceivably, this estimate of the "true" mean value would contain error. If the experiment is repeated and other sets of data are obtained, the sample mean estimated from the other sets of data would most likely be different. The collection of all the means from the different samples will also have a mean value which may well be different from the individual mean values and may be assumed to be closer to the "true" mean value (which remains unknown).

Many empirical hydraulic models are applied in a deterministic fashion to hydraulic design problems (Tung, 1994). Empirical equations are often presented without explicitly stating or elaborating the associated uncertainty. In the applications of regression analysis to develop such equations, for example, the results presented are usually the values of regression coefficients or best-fitted parameter values with some goodness-of-fit indicators such as a coefficient of determination. In this practice, the intrinsic uncertainty associated with the empirical equations is lost, and the information contained in the data is not fully utilized. As a result, some information in the data is unused resulting in unnecessary waste of information and perhaps flawed design decisions (Tung, 1994).

To include uncertainty effects in the analysis and design of an ocean outfall system, quantification of uncertainty measures is therefore important. The uncertainty information required by the reliability analysis and probabilistic design includes the mean, variance, and, in some cases, the probability density function (PDF) or cumulative distribution function

(CDF) of the basic variables describing the system under investigation. If the basic variables are significantly correlated, the covariance matrix and, in some cases, joint PDF's may also be needed (Melching 1995; Ang and Tang, 1984).

3.2. Design Procedure Using a Probabilistic Approach

The procedure proposed for probabilistic ocean outfall design is described in the following subsections which essentially provide the answers to the problems formulated in Chapter One. This procedure can be applied to design ocean outfall systems, particularly in calculating initial dilution, secondary dilution and bacterial decay for horizontal buoyant round jet in still and moving waters with density unstratified. The application of this procedure is shown in Chapter Five dealing with a case study in which probabilistic analysis is applied to an existing ocean outfall.

3.2.1. Choosing a Deterministic Model

When using a probabilistic approach, one cannot totally leave out deterministic models. Although probabilistic methods provide a scientific, workable alternative tool to solve engineering problems, they are actually complementary to physically based deterministic models. The deterministic models for initial dilution and bacterial concentration at a target area located x (m) from the outfall discharge, e.g. bathing or shellfish area, are used in this probabilistic outfall design and analysis. This is because initial dilution calculation is usually used to estimate whether significant slick formed at the surface above the discharge, and because bacterial concentration at the target area should be designed to comply with relevant regulations.

In the case of a horizontal buoyant round jet in still and moving waters with seawater density unstratified, as discussed in Chapter Two, Cederwall's (1968) model and Lee and Neville-Jones' (1987-a) model are suitable for use in their respective condition. Recall equation 2.1 and 2.2 for Cederwall's solutions of initial dilution in still waters:-

$$S_o = 0.54F^{9/16}\left(\frac{Y}{D}\right)^{7/16} \qquad \textit{for the range of } \left(\frac{Y}{D}\right) < 0.5F \qquad (2.1)$$

$$S_o = 0.54F\left[0.38\,\frac{(Y/D)}{F} + 0.66\right]^{5/3} \qquad \textit{for the range of } \left(\frac{Y}{D}\right) > 0.5F \qquad (2.2)$$

And recall equation 2.5 and 2.6 for Lee and Neville-Jones' solutions of initial dilution in moving waters:-

$$S_m = 0.31\left(\frac{B^{1/3}H^{5/3}}{Q}\right) \qquad \textit{for the range of } H\left(\frac{U^3}{B}\right) < 5 \qquad (2.5)$$

$$S_m = 0.32\left(\frac{UH^2}{Q}\right) \qquad \textit{for the range of } H\left(\frac{U^3}{B}\right) > 5 \qquad (2.6)$$

Improvement for the initial dilution model in moving waters may be made because observational data are available (Lee & Neville-Jones, 1987-b; Lee & Koenig, 1995). The data indicate that significant variabilities were found in the observed initial dilution although there was little uncertainty associated with the measured input variables such as the depth of free surface above discharge, the ambient current speed, and the volume flux of the jet discharge. This suggested that the constants in equations 2.5 and 2.6 (i.e 0.31 in equation 2.5 and 0.32 in equation 2.6) may be variable (Lee and Neville-Jones, 1987-b; Lee and Koenig, 1995).

In order to include the variability of the coefficients, equations 2.5 and 2.6 may be modified as follows:-

$$S_m = C_1\left(\frac{B^{1/3}H^{5/3}}{Q}\right) \qquad \textit{for the range of } H\left(\frac{U^3}{B}\right)<5 \tag{3.1}$$

$$S_m = C_2\left(\frac{UH^2}{Q}\right) \qquad \textit{for the range of } H\left(\frac{U^3}{B}\right)>5 \tag{3.2}$$

where C_1 and C_2 are the model coefficients which might be described in terms of probabilistic basis (further discussion is given in subsection 3.2.2).

Bacterial concentration at a target area located at x from the discharge point has been formulated in equation 2.29 which may be used as the deterministic model for probabilistic analysis. Recall equation 2.29:-

$$C_x = \frac{C_e}{S_i} e^{\frac{-2.3x}{uT_{90}}} erf\left(\sqrt{\frac{1.5}{\left(1 + 8\frac{k_o x}{ub^2}\right)^3 - 1}}\right) \tag{2.29}$$

3.2.2. Identifying Uncertain Parameters

Parameters affecting the system can be classified whether they should be considered as ones having random values or having constant values. This classification is a starting point to include the effects of natural variability and the uncertainty in the deterministic models used. Whereas the effects of natural variability are taken into account by introducing the uncertainty of input-variables of interest (for example by using their mean, variance or probability density function), the uncertainty of the deterministic models used are represented by, whenever possible, converting the deterministic-model coefficients into variable coefficients with specified uncertainty.

For a horizontal buoyant round jet in density unstratified seawater environment, discussions on the model inputs and model coefficients, which are considered to be uncertain parameters, are given in the following:-

• ***Wastewater Flow Rate***

As discussed in 2.6.1, for a given residential area domestic sewage flow varies, and the estimated value of the flow would therefore be significantly uncertain. Huang et. al. (1994) noted that the distribution of effluent discharge rate is typically much skewed to high values. However, the measures of the uncertainty, i.e., mean, standard deviation, coefficient of variation, and probability density function, may be different from case to case. Therefore, the use of local data is preferable whenever possible.

For example, the wastewater flow rate produced by a small residential area at Spaniard's Bay is available in a time series data which was sampled quarter-hourly (Sharp, 1989-c). The time series (figure 2.2) was analyzed to determine the distribution of wastewater flow rate. It was found that the power-normal distribution (see next subsection 3.2.3) fits the data fairly well.

• ***Seawater Currents***

Seawater currents are highly variable in both strength and direction, and should be estimated based on field measurements. A typical representation of seawater currents was shown in figure 2.3 which depicts current speed cumulative distribution and specifies the proportion of the direction of the currents as it is up-coast or down-coast. The following provide other examples of current data used in ocean outfall analysis.

Webb (1987) presented the ocean current speeds obtained from field experiments using drogue tracking techniques. The data were intended to design a new outfall for a coastal town in New South Wales. The measured currents, from more than 50 data sets, ranged from 0.02 to 0.6 m/s and were found to be well fitted using the log-normal distribution. The current direction was characterized as heading either toward the beach (80% of the time in that case) or away from the beach.

When assessing alternatives for wastewater disposal in the City of Tijuana, Mexico, Orlob and Tumeo (1986) analyzed data for three sets of measurements off Point Loma at different times. These indicated that the mean velocities parallel to the shore at depths of about 20 m were in the range of 0.08 to 0.12 m/s, while currents normal to the shoreline (East-West) were less than half as great. The measured currents were found to be normally distributed with standard deviations in the range of 0.02 to 0.06 m/s.

In some cases, the main body of the seawater may be essentially stagnant. Slight surface currents may still be presence, however. At Spaniard's Bay, Sharp (1994) reported that although there were slight surface currents of the order of 0.03 to 0.06 m/s during field measurement tests, the main body of the seawater was essentially still. If field data are not available, the surface currents may be assumed to be induced by winds, and the value of the surface current can be taken to be about 2 to 3 percent of the wind speed (Sullivan & Vithanage, 1994).

To take into account the direction of the surface current in the analysis, it is important to have some idea of the distribution of the current direction. It is easier to simplify the problem into two cases, i.e. whether or not the effluent discharge from the outfall is going towards the location of interest (Webb, 1987; Sharp, 1989-c). In such a condition, the Binomial or Poisson distribution may be applicable (Devore, 1995). In any binomial experiment in which n is large (say, number of trial $n \geq 100$), and p is small (say, probability of occurrence $p \leq 0.01$), the Binomial distribution may be close to the Poisson distribution defined as (Devore, 1995):-

$$p(x;\lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!} \tag{3.3}$$

where λ is the parameter of the distribution, and x = 0, 1, 2, ... indicating the number of success among trials, and $p(x,\lambda)$ is the probability of having x success during the trial when the process is following the Poisson distribution with parameter of λ (an example is shown in subsection 5.3.3).

• ***Seawater Depth***

As discussed in subsections 2.3.1 and 2.3.2, seawater depth for initial dilution calculations refers to either the total water depth above discharge (H) or the total seawater depth above discharge subtracted by the waste field thickness (Y). Whereas the first type of seawater depth is calculated from the effluent outlet to the seawater surface and is used in moving water conditions, the later one is used in still water situations. Regardless of the variation of the waste surface layer, the two types of seawater depth are, especially for ocean outfalls at relatively shallow waters, sensitive to the variation of the tide which rises and falls gradually.

Huang et. al. (1994) recently took tidal variation into account when calculating initial dilution using a time domain simulation. Although they were unable to fit the data for tidal height into a theoretical probability density function (PDF), they noted that the distribution of the tidal height might be bimodal, and that the distribution might be approximated using the uniform distribution. They found that the difference between the mean water level and the mean lower low water level was typically 1.4 m for the Miami-Central Outfall at the east coast of South Florida. They also calculated that the 10 percentile on the cumulative distribution for the tidal height was 1.0 m.

In cases where tidal variation must be considered, e.g. Spaniard's Bay which has tidal variations were approximately 20 % of the mean water level, it is necessary to have a measure of the uncertainty of the tidal height, and may be expressed in terms of mean, standard deviation, coefficient of variation, or probability density function. For example, Huang et. a. (1994) suggested that the standard deviation of tidal height at Miami-Central Outfall was 0.3 m with an approximately uniform distribution.

• ***Decay Parameter, T_{90}***

Wood, et. al. (1993) noted that in the past T_{90} has often been assumed to be constant with a single mean value T_{90} of 4 hours being commonly used in outfall studies. Field experiments carried out in various parts of the world have indicated that the values vary within wide limits ranging from 0.6 to 24 hours in daylight to values about 60 to 100 hours at night. Bell, et. al. (1992) proposed a deterministic model for diurnal cycle of hourly T_{90}. Summary of measurement of T_{90} for coliform groups is given in Table 3.1.

Table 3.1. Summary of measured T_{90} (hours) for coliform group

Location	Measures of uncertainty	References
Santos coastal waters, Brazil	Almost log-normally distributed, no correlation to season & temperature, with median of 1.4.	Occhipinti in discussing Mitchell & Chamberlin (1975)
Fortaleza outfall, Brazil	Mean = 1.32, Coeff. of Variation = 13.1	Agudo & Santos (1986)
Santa Barbara, California, USA	Mean = 2.35, Standard dev. = 1.1	Mitchell & Chamberlin (1975)
Sidmouth & Bridport, England	Mean = 3.7, Standard dev. = 2.5	Mitchell & Chamberlin (1975)
St. John's, NF, Canada	Log-normally distributed, with mean = 4.7, Standard dev.= 0.997)	Data from Thoms (not published)

It can be seen in Table 3.1 that the T_{90} values vary from place to place. Hedgland (in discussion of Mitchell & Chamberlin, 1975) also noted that results of individual T_{90} studies were often highly unreliable, partly, because of absence of steady-state velocities, variations in mixing with weather conditions, differences in die-off rates between day and night, and difficulties arising from variations in background coliform counts with tidal movement. Therefore, whenever possible, data from locations having similar climatological and oceanographical conditions to that of the proposed outfall location should be used for the outfall design. When faecal streptococci is also used as indicator bacteria, the ratio of median values of T_{90} for faecal streptococci (FC) to faecal coliforms (FC) may be approximated to be 1.3 to 2.0 with a mean value of 1.5 (Wood, et. al.,1993).

• ***Effluent Concentration***

As discussed in 2.6.2, sewage contains physical, chemical, and biological contaminants. Concentration of each of these varies from case to case, and typical concentrations of faecal coliform bacteria at raw sewage were shown in Table 2.5. However, the total coliforms concentration (22 x 10^6 per 100 ml) in that table is not always correct for sewage from other locations. Even if the coliforms concentration is measured at the same location, it is not likely to have one fixed value in different measurements. At best, the coliforms concentration must therefore be expressed in term of measures of uncertainty.

Webb (1987) reported that the coliform concentration was log-normally distributed. However, the value of the distribution parameters, i.e. mean and standard deviation were dependent on the degree of treatment of the sewage. Orlob and Tumeo (1986) found that, for the effluent from the Tijuana Plant, Mexico, the median concentration of total coliforms was about 4.3×10^7 per 100 ml with a 90 percentile of 2.4×10^8 per 100 ml. For outfall design purpose, the measures of the uncertainty of the effluent concentration may be obtained from local surveys.

• ***Model Coefficients***

In hydraulic design, it is most difficult to include the uncertainty in the model coefficients, partly because empirical hydraulic models are often used in a deterministic manner without explicitly considering their intrinsic uncertainties, and partly because enough original data to accompany the deterministic models are often not published. Tung (1994) gave an example of pipe outlet-riprap system (figure 3.1) to demonstrate the use of information provided by statistical regression analysis about uncertainty features in an empirical hydraulic model. A more detailed discussion is given in his paper.

In his example, Tung (1994) modified a deterministic riprap model (equation 3.4) into a probabilistic riprap model by considering uncertainty of the model coefficients and introducing the error term. The deterministic riprap model is:-

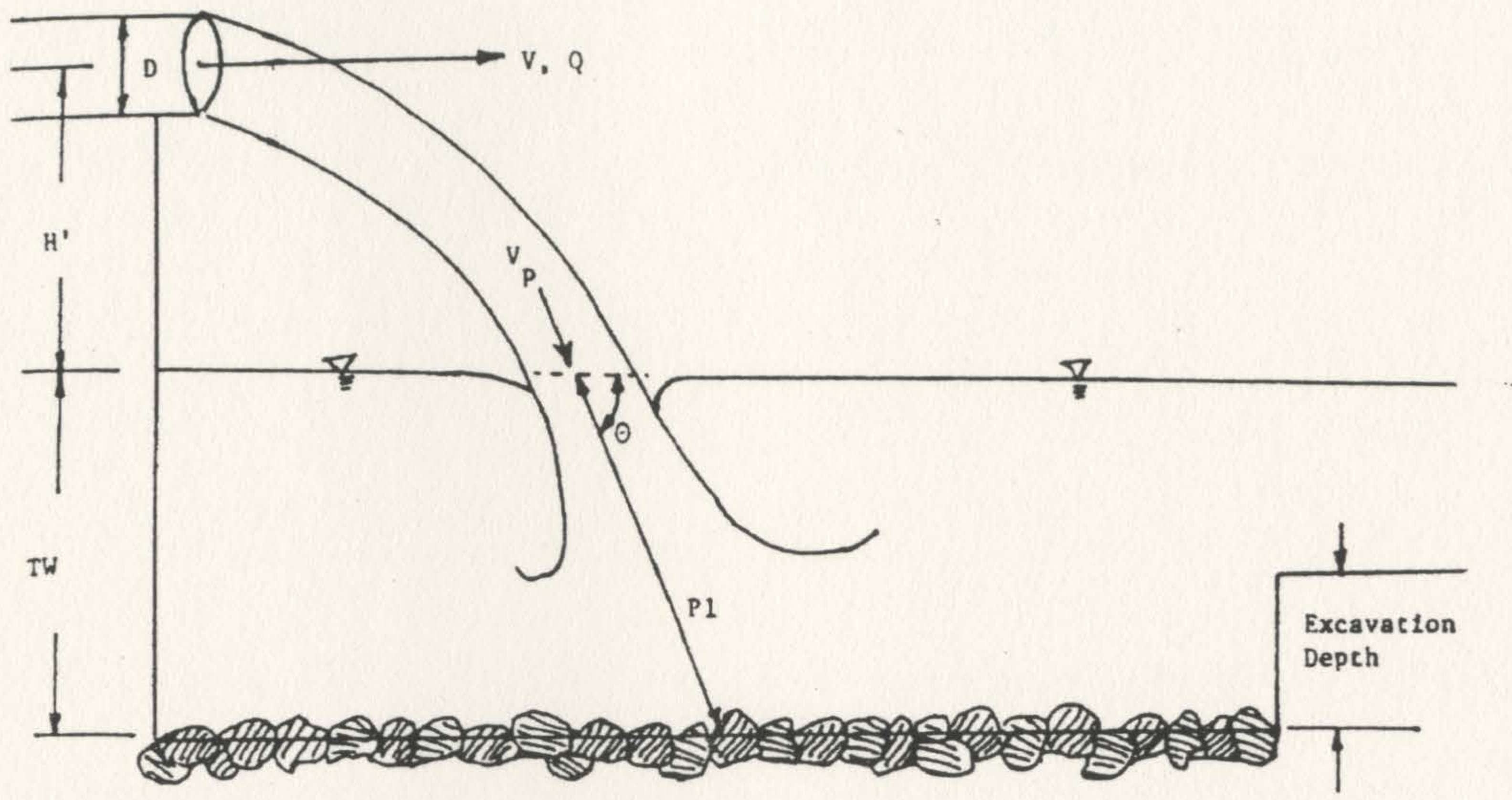

Figure 3.1. Definition Sketch of Pipe Outlet-Riprap System (after Tung, 1994)

$$SN_{D_s} = c\left(\frac{Pl}{D}\right) \tag{3.4}$$

where c is regression coefficient, and SN_{Ds} is a stability number which is defined:-

$$SN_{D_s} = \frac{V}{\sqrt{g(S_s - 1)D_s}} \tag{3.5}$$

with V is the exit pipe flow velocity, g is gravitational acceleration, S_s is the specific gravity of riprap material, D_s is the characteristic size of riprap material, D is the pipe diameter, and Pl is the penetration depth of the jet from the pipe outlet. In figure 3.1, TW is tailwater depth, and H' is vertical distance from the center of the pipe to tailwater surface.

Equation 3.4, does not include any relevant statistical information with regard to the regression coefficients and error associated with the regression equation. Reanalyzing this problem, Tung (1994) proposed the following regression model:-

$$SN_{D_s} = a + b\left(\frac{Pl}{D}\right) + e \tag{3.6}$$

in which a and b are regression coefficients, and e is a model error term. For a typical condition, Tung (1994) found the regression results as follows:-

$$\mu_a = 0.2274;\ \ \mu_b = 0.3820;\ \ \sigma_a = 0.1338;\ \ \sigma_b = 0.0102;\ \ \sigma_e = 0.6958$$

where μ_a and μ_b are the mean values of regression coefficients a and b; σ_a and σ_b are the standard deviations associated with the regression coefficients a and b; and σ_e is the model standard error representing the uncertainty associated with the regression model.

When sufficient data are available for outfall systems, the deterministic initial dilution, secondary dilution, and bacterial decay could also be modified into probabilistic outfall models similar in format to equation 3.6. The required data for these purposes are not however presently available. Therefore, the modification of the deterministic models in the outfall design is limited to the initial dilution model in moving water as data for this are available as shown in Figures 3.2 and 3.3 (Neville-Jones, 1987-b; Lee & Koenig, 1995).

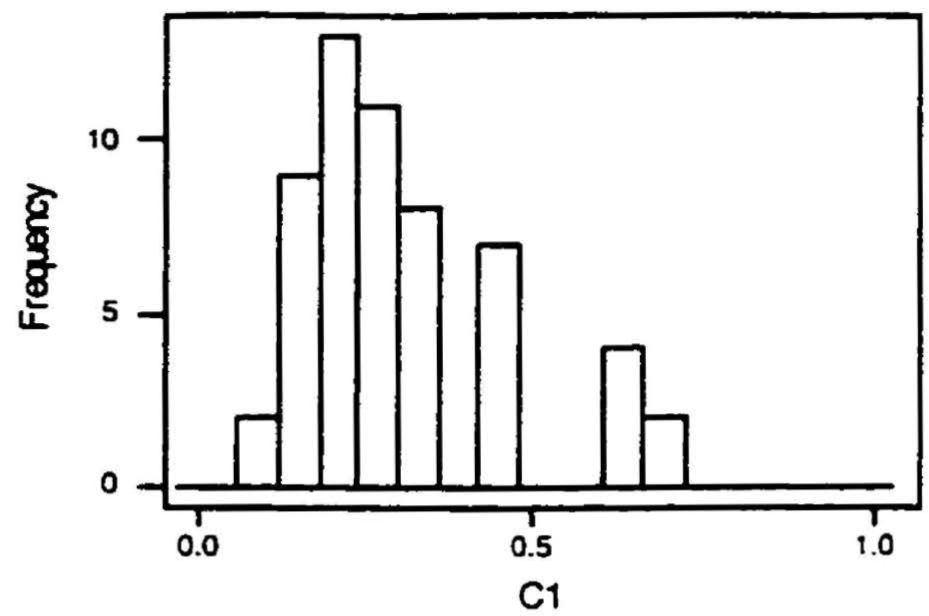

Figure 3.2. Measured Dilution Constants at Hasting Outfall
for BDNF (56 observations) with average of $C_1 = 0.31$

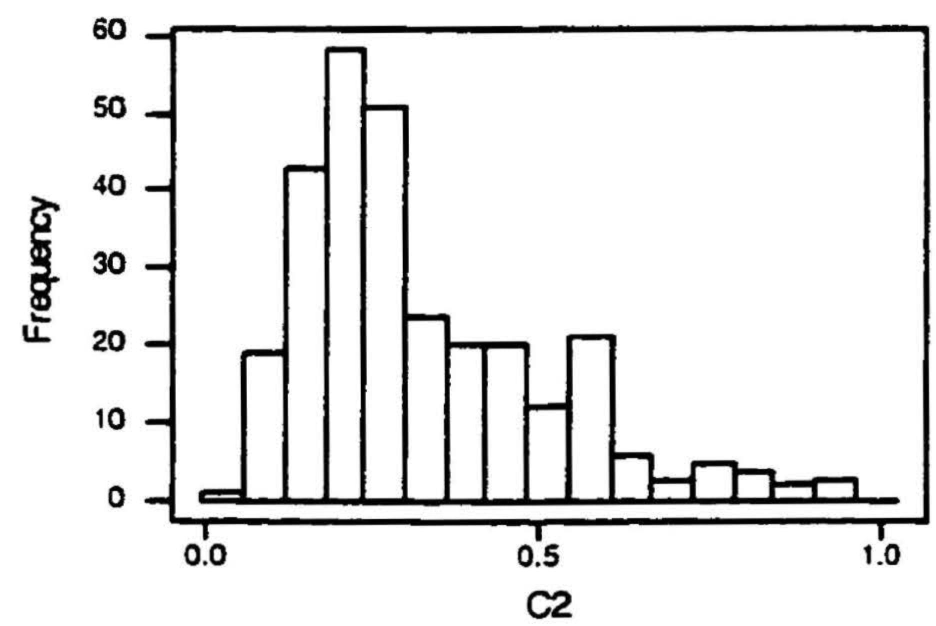

Figure 3.3. Measured Dilution Constants at Hasting Outfall
for BDFF (292 observations) with the average of C_2=0.3

The model coefficients, C_1 and C_2 given in Figure 3.2 and 3.3 are well fitted with log-normal distribution with a mean and standard deviation of -1.274 and 0.492 for $\ln(C_1)$; and a mean and standard deviation of -1.288 and 0.574 for $\ln(C_2)$. More detailed discussion is given in the next subsection 3.2.3.

3.2.3. Fitting Probability Distribution to the Parameters

Probabilistic characteristics of parameters discussed in the previous section would be described completely if the form of the distribution function (e.g. probability density or mass function) can be specified. In practice, however, the form of the distribution function, or even the descriptive statistics (e.g. range, mean, standard deviation, median and skewness), may not be exactly known. Approximate solutions are therefore necessary, and these can be obtained from historical data samples with the assumption of ergodicity, i.e. the available data are assumed to represent the actual condition under investigation.

For this reason, the methods of inferential statistics are used when the data is a sample (i.e. the population data is not available), and the objective is to go beyond the sample to draw conclusions about the population based on sample information (Devore, 1995). Further discussion of the inferential statistics is not given here, but can be found elsewhere (e.g. Ang & Tang, 1975; and Devore, 1995). In this section, methods for calculating the first and second moments (i.e. the mean and variance) as well as for fitting the probability density function from sample data are briefly reviewed.

The mean and variance are the most important statistics to describe uncertainty of the parameters in the models. Whereas the mean reflects the central value of the parameters, the variance is a measure of the dispersion of their values. The third moment (skewness) may also be important when the underlying distribution is known to be non-symmetric.

When using the sample data, the sample moments can be used as estimates of the corresponding moments of the parameters (Ang & Tang, 1975; 1986; Devore, 1995).

For a parameter X having random values, the sample mean (m_x) and sample variance (Var(X)) can be defined as (Ang & Tang, 1975; Smith, 1986; Devore, 1995):-

$$m_x = \frac{1}{n} \sum_{i=1}^{n} x_i \tag{3.7}$$

$$Var(X) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - m_x)^2 \tag{3.8}$$

Where m_x and Var(X) are the point estimates of the population mean, μ, and population variance σ^2, respectively, with sample size of n, namely $x_1, x_2, \ldots, x_n$. More convenient measures of dispersion are the square root of the variance, i.e. the standard deviation, and the relative dispersion to the central value, i.e. coefficient of variation (CV). The standard deviation and coefficient of variation of X are defined:-

$$s_x = \sqrt{Var(X)} \tag{3.9}$$

$$COV(X) = \frac{s_x}{m_x} \tag{3.10}$$

where s_x and CV(X) are the standard deviation and coefficient of variation of X respectively. After the moments of the parameters involved in the system (e.g. mean and variance of seawater currents) have been estimated, the parameters of their probability distribution may then be determined. The probability distribution can be in either discrete or continuous forms, but the most commonly used in engineering is the bell-shape distribution curve known as the normal distribution. The probability density function (pdf) for the normal distribution is defined as:-

$$f(x) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp\left(-\frac{(x_i - \mu_x)^2}{2\sigma_x^2} \right) \qquad -\infty \leq x \leq \infty \tag{3.10}$$

Where mean μ_x and variance σ_x^2 may be replaced with the sample mean and sample variance respectively. In abbreviated form, equation (3.10) is written as:-

$$X \sim N\,(\mu_x, s_x) \tag{3.11}$$

The normal pdf is symmetrical about the mean and has various distribution shapes depending on the mean and standard deviation. It is therefore often advantageous to use the so-called standardized variable Z:

$$Z = \frac{X - \mu_x}{\sigma_x} \tag{3.12}$$

where Z is the standardized variable which has a mean of 0 and a standard deviation of 1, regardless of the distribution of X. If X is normally distributed, Z is also normally distributed but with a mean of 0 and a standard deviation of 1, and with pdf and cdf (cumulative distribution function) given in equation 3.13 and 3.14 respectively.

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2} \right) \qquad -\infty \leq z \leq \infty \tag{3.13}$$

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} \exp[-\frac{z^2}{2}]dz \qquad -\infty \le z \le \infty \tag{3.14}$$

F(z) is now a function of only a single variable z, and its value has been tabulated in most statistical books (e.g. Ang & Tang, 1975; Smith, 1986; Olkin, et. al, 1994; Devore, 1995). A typical depiction of standard normal distribution is given in figure 3.4.

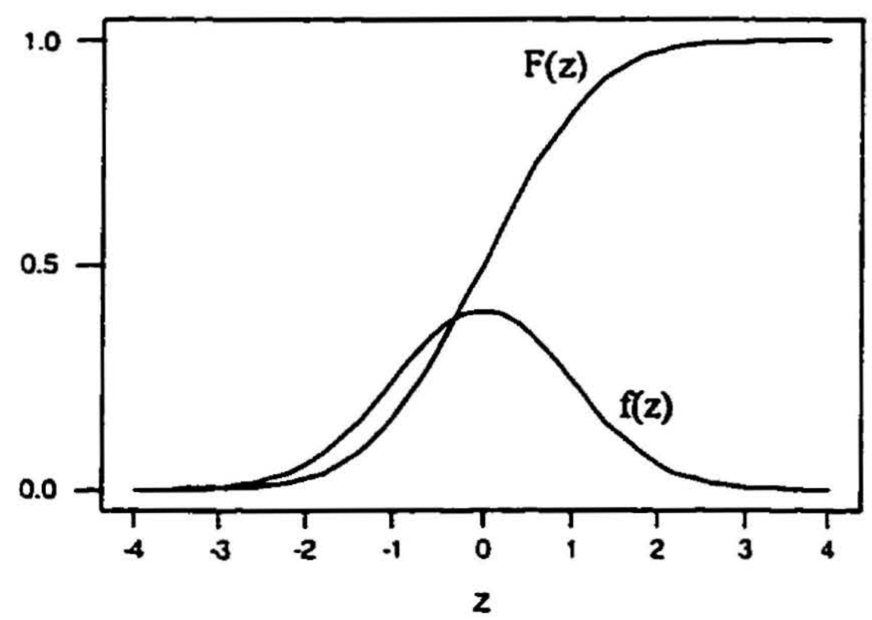

Figure 3.4. Probability Density Function and
Cumulative Distribution Function for Standard Normal

Under general conditions the distribution of the sum of n random values parameters approaches a normal distribution when n is large, regardless of the shape of the contributing parameters. That is, X is normally distributed if:-

$$X = \sum_{i=1}^{n} Y_i \tag{3.15}$$

where Y_i is the contributing parameter. This principle is called Central Limit Theorem.

In many phenomena, the causative factors are not additive, however. Instead, they are multiplicative such as:-

$$X = \prod_{i=1}^{n} W_i \tag{3.16}$$

Where W_i is i^{th} nonnegative contributing random values parameter. Each of the contributing parameters is fairly independent and does not dominate the value of the product. Taking logarithms, equation 3.16 becomes:-

$$Y = \ln(X) = \sum_{i=1}^{n} \ln(W_i) \tag{3.17}$$

By virtue of the Central Limit Theorem, Y is now (approximately) normally distributed. In general, a nonnegative parameter X has a lognormal distribution whenever Y=ln(X) is normally distributed. With this transformation, the pdf of the lognormal distribution is then:

$$f(x) = \frac{1}{x\sigma_y\sqrt{2\pi}} \exp\left(-\frac{[\ln(x_i)-\mu_x]^2}{2\sigma_y^2}\right) \qquad x_i \geq 0 \tag{3.18}$$

where μ_y and σ_y are the parameters of the lognormal distribution, i.e. the mean and standard deviation of Y = ln(X). The relationship between these parameters and those of X is as follows:-

$$\mu_y = \ln(\mu_x) - \frac{1}{2}\sigma_y^2 \tag{3.19}$$

$$\sigma_y^2 = \ln\left[CV(X)^2 + 1\right] \tag{3.20}$$

Because of its relationship with the normal distribution, probabilities associated with a lognormal variate can also be determined using the table of standard normal probabilities. This makes it easy to calculate probabilities of the lognormal distribution. Furthermore, because the values of parameters having a lognormal distribution are always positive, the lognormal distribution may be useful in those applications where the values of the variate are known to be strictly positive (Lye, 1992), for example effluent concentration, decay parameter (T_{90}), seawater speed, etc.

Theoretical continuous probabilistic models other than the normal and lognormal distributions are available. However, very view of these other distributions are as well known or as widely tabulated as the normal distribution. Indeed, many distributions after a suitable transformation would follow an approximately normal distribution. For example, wastewater flow rate of the Spaniard's Bay Outfall can be transformed using the Box-Cox transformation with a transformation parameter λ of -1.1 (see equation 3.24 and figure 3.7). Other examples are given by Lye (1992), i.e. if X is gamma distribution, then cube-root transformation, $X^{1/3}$, would be approximately normally distributed. If X is log-gamma distributed, then $[\ln(x)]^{1/3}$ is approximately normal.

Other distributions that may not be suitable to be transformed into the normal distribution may still exist. However, this report does not intend to describe them in depth but they can be found elsewhere (e.g. Evans, et. al., 1993; Olkin et. al, 1994; Devore, 1995). The functional forms of the density and cumulative distribution of some non-normal

Table 3.2. Summary of Non-normal Distributions (after Olkin, et. al., 1994)

Distribution & parameters	Probability Density Function	Cumulative Distribution	Mean	Variance	Range
Exponential ($\theta > 0$)	$\theta e^{-\theta x}$	$1 - e^{-\theta x}$	$\frac{1}{\theta}$	$\frac{1}{\theta^2}$	$x > 0$
Gamma ($r > 0, \theta > 0$)	$\frac{\theta^r x^{r-1} e^{-\theta r}}{\Gamma(r)}$	$I_r(\theta x)$	$\frac{r}{\theta}$	$\frac{r}{\theta^2}$	$x > 0$
Weibull ($-\infty < \nu < \infty$, $\alpha > 0, \beta > 0$)	$\frac{\beta}{\alpha}\left(\frac{x-\nu}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x-\nu}{\alpha}\right)^{\beta}}$	$1 - e^{-\left(\frac{x-\nu}{\alpha}\right)^{\beta}}$	$\nu + \alpha\,\Gamma\left(1 + \frac{1}{\beta}\right)$	$\alpha^2\left[\Gamma\left(1+\frac{1}{\beta}\right) - \Gamma^2\left(1+\frac{1}{\beta}\right)\right]$	$x > \nu$
Uniform ($-\infty < a < b < \infty$)	$\frac{1}{b-a}$	$\frac{x-a}{b-a}$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$x \geq a$ $x \leq b$
Beta ($r, s > 0$)	$\frac{x^{r-1}(1-x)^{s-1}}{\beta(r,s)}$	$I_x(r,s)$	$\frac{r}{r+s}$	$\frac{rs}{(r+s)^2(r+s+1)}$	$0 \leq x \leq 1$
Log-normal ($-\infty < \mu_Y < \infty$, $\sigma_Y^2 > 0$)	$\frac{e^{-\frac{(\ln(x)-\mu_Y)^2}{2\sigma_Y^2}}}{\sqrt{2\pi x^2 \sigma_Y^2}}$	$F_z\left(\frac{\ln(x)-\mu_Y}{\sigma_Y}\right)$	$e^{\left(\mu_Y + \frac{\sigma_Y^2}{2}\right)}$	$\left(e^{\sigma_Y^2} - 1\right)e^{\left(2\mu_Y + \sigma_Y^2\right)}$	$x > 0$

distributions, together with the appropriate ranges for the argument x, are summarized in table 3.2. The table also shows the mean and variance for each type of distribution.

Having the above theoretical distributions, the random phenomena associated with the parameters of the system under investigation may then be formulated mathematically. However, sometimes the probabilistic characteristics of the parameters are not readily amenable to theoretical deduction or formulation. In such cases the functional form of the required probability distribution may not be easy to derive or ascertain. The approach to these situations is to determine the required probabilistic model empirically based on available observational data. Goodness-of-fit test may be used to test and verify an assumed probability distribution in the light of available data (Ang & Tang, 1975).

For a normal distribution, the normal probability plot is used, based on the standard normal distribution function, to test the acceptability of fitting the available data. Whereas the y-axis (in arithmetic scale) of the normal probability plot represents the value of the variate X, the x-axis are the values of the standard normal variate z_i, and/or the cumulative probabilities F(z) corresponding to the indicated values of z_i. If the data points plot is approximately on a straight line, the data are normally distributed.

The probability plot correlation coefficient (PPCC) test provides an objective way of testing the linearity of the plotted points on the normal probability plot. For most practical purposes, the values of z_i can be approximated as (Schemeiser, 1979):-

$$z_i = 5.0633 \left[p_i^{0.135} - (1 - p_i)^{0.135} \right] \tag{2.21}$$

where p_i is the probability value assigned to x_i ($P(X \leq x_i)$ using an appropriate plotting position. If Blom's plotting position (Blom, 1958) is used, p_i is defined:-

$$p_i = \frac{(m - 0.375)}{(n + 0.25)} \tag{2.22}$$

where m is rank (the smallest value is ranked 1, the largest value is ranked n), and n is the number of values. Having this formulation, the probability plot correlation coefficient is just the sample correlation coefficient (r) between the ordered x_i and z_i (where i = 1, 2, ..., n), that is:-

$$r = \frac{\sum_{i=1}^{n} \left[(x_i - m_x)(z_i - m_z) \right]}{n\, s_x s_z} \tag{3.23}$$

where m_x and m_z are the respective mean values of the x_i's and z_i's, and s_x and s_z are the respective standard deviations of x_i's and z_i's. If the calculated r value is greater than its critical value, then the data would be considered to come from a normal distribution. The critical value for a sample size n at a significant level α can be found from the statistical table (for example in Lye, 1993), or alternatively it can be determined from simulations. If α = 5 % and n = 50, for example, this means that it would be expected that only 5 % of many normally distributed sample sizes of 50 would have a value of the sample correlation coefficient less than the critical value.

When the data is not normal but transformable, the normal probability plot may also be applied to the transformed data. For an example, the measured initial dilution constants (C_1 and C_2) for moving water conditions are known to have skewed distribution as shown in figure 3.2 and 3.3 (see previous section 3.2.2). After taking log of the data and calculating the parameters of the log-normal distribution using equation 3.19 and 3.20, it was found that the parameters are $\mu_y = -1.274$ and $\sigma_y = 0.492$ [for $\ln(C_1)$], and $\mu_y = -1.288$ and $\sigma_y = 0.574$ [for $\ln(C_2)$]. The normal probability plot for $\ln(C_1)$ can be shown in figure 3.5. Similarly for $\ln(C_2)$, the normal probability plot for C_2 is given in figure 3.6.

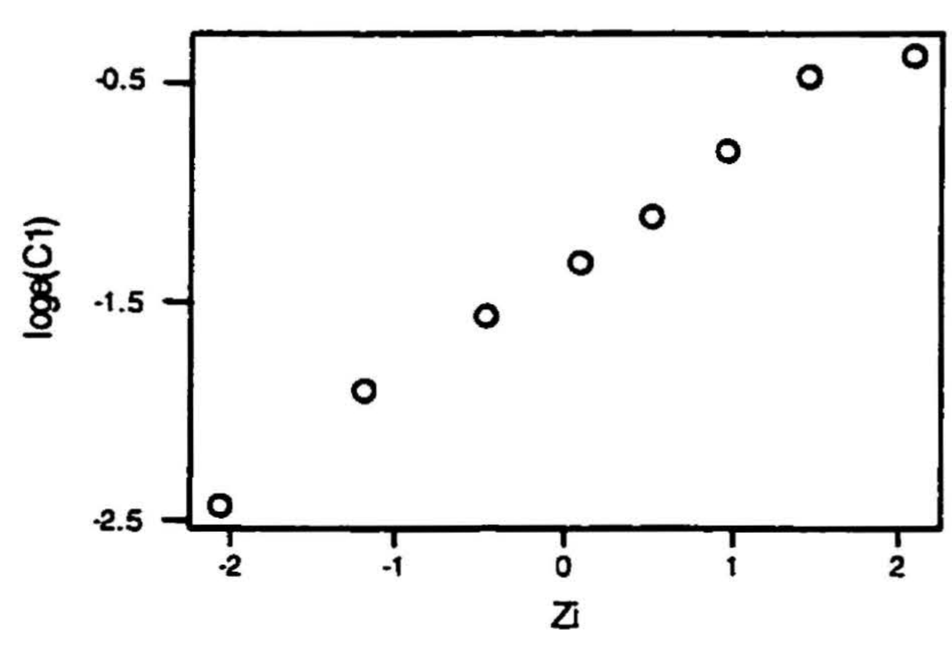

Figure 3.5. Normal Probability Plot for $\ln(C_1)$ with PPCC of 0.994

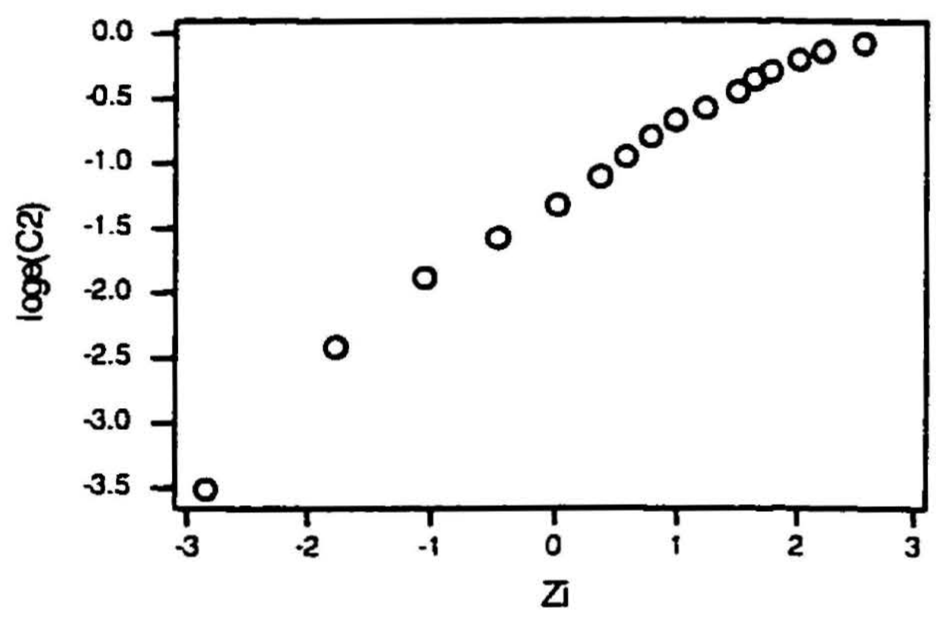

Figure 3.6. Normal Probability Plot for $\ln(C_2)$, with PPCC of 0.996

It can be shown that $\ln(C_1)$ and $\ln(C_2)$ are normally distributed, because their PPCC is greater than its critical value at significant level of 5% [$r_{crit} = 0.979$ for $\ln(C_1)$ and $r_{crit} = 0.995$ for $\ln(C_2)$]. For this reason, the measured initial dilution constants (C_1 and C_2) for moving water conditions can be considered to be log-normally distributed with respective a mean and standard deviation of -1.274 and 0.492 for $\ln(C_1)$, and -1.288 and 0.574 for $\ln(C_2)$.

As another example of fitting non-normal distributions, the wastewater flow rate produced by a small residential area flowing into the Spaniard's Bay (Sharp, 1989-c) was re-analyzed to determine its probability distribution function. A typical time series of the data during the day is depicted in figure 2.2 (see section 2.6.1). The data are neither normally nor log-normally distributed so that another transformation must be tried to fit.

In this case the Box-Cox transformation is used to find the most appropriate approximation of the distribution. An easy way of selecting the Box-Cox transformation is found in Lye (1993). The transformation is given by:-

$$Y_i = \frac{x_i^{\lambda} - 1}{\lambda} \qquad with\ \lambda \neq 0 \tag{3.24}$$

where x_i is the data of wastewater flow rate and λ Box-Cox transformation parameter to be determined such that Y_i (the transformed data of wastewater flow rate) are approximately normally distributed. Simulations are used to find the most appropriate λ (lambda) which gives maximum coefficient correlation. Simulation results are given in Figures 3.7 and 3.8. The figure shows that the maximum correlation coefficient is given by $\lambda = -1.1$ and $r_{max} = 0.9974$. Using $\lambda = -1.1$ and equation 3.24, the mean and standard deviation of Y_i are then -219.95 and 37.73 respectively.

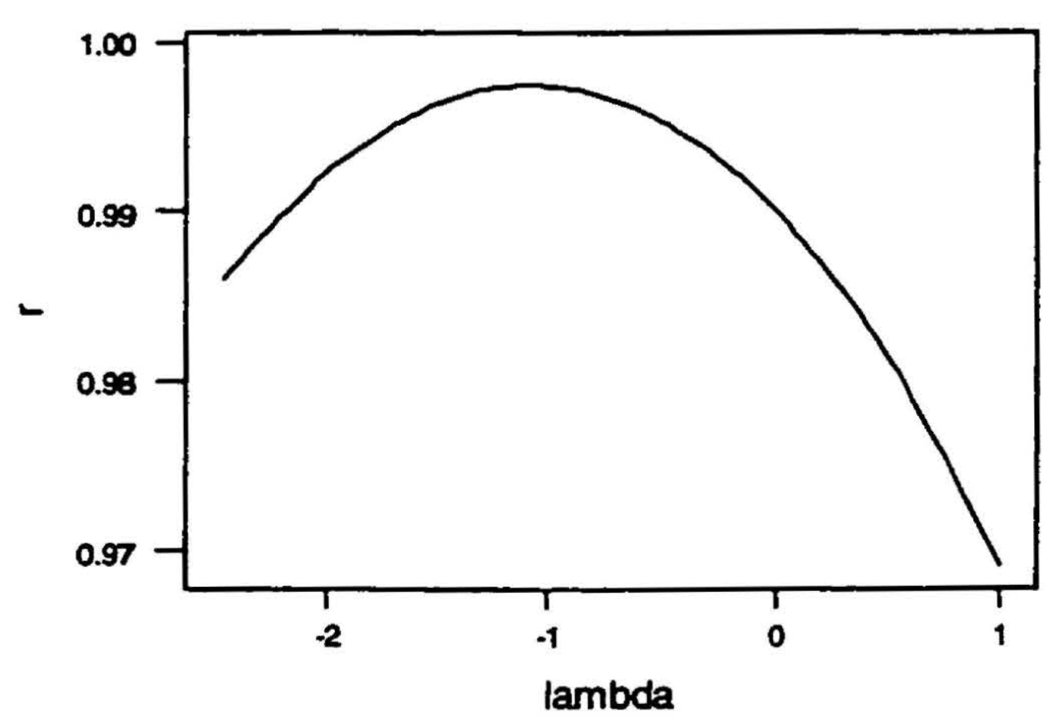

Figure 3.7. Typical simulation results to find out λ (lambda); r_{max} is for $\lambda \rightarrow -1.1$.

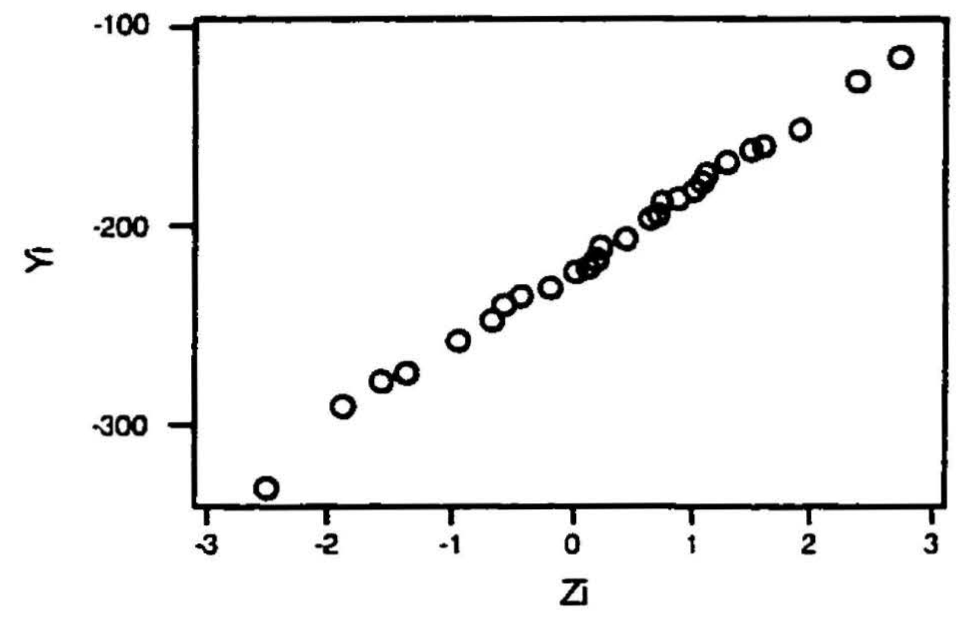

Figure 3.8. Normal Probability Plot for Yi (equation 3.3) with $\lambda = -1.1$ for wastewater flow rate at Spaniard's Bay Outfall; PPCC = 0.9974.

The critical value of the correlation coefficient for a sample size of 188 and significance level of 5 % is 0.9950. Because the normal probability plot coefficient correlation, PPCC, given by this transformation (i.e. 0.9974) is greater than its critical value, it can be said that Y_i are normally distributed with mean and standard deviation of -219.95 and 37.73 respectively. This means that the wastewater flow rate is well fitted using the power normal distribution (Box-Cox transformation).

3.2.4. Defining Performance Function, Threshold Level and Probability of Failure

In the reliability analysis and probabilistic design, a convenient way to consider the uncertainty in the design output for a particular case is to assess the probability that the output parameter of interest (e.g. initial dilution, concentration of total coliforms, etc.) would exceed some limit state or threshold level (Melching, 1995). The design output may be in terms of performance function (also called safety margin) which is the difference between the capacity (or in this case the threshold level) and the value calculated for the design loading. Failure occurs when the performance function is less than zero. For a given performance function Z, the risk of failure or the probability of failure is defined by:-

$$P_f = P(Z<0) \tag{3.25}$$

where P_f is the probability of failure, and Z is the performance function which reflects the difference between the capacity and the loading..

Melching (1995) described the performance function in the form of:-

$$Z = T_L - H(X) = g(X) \tag{3.26}$$

where T_L is the threshold level, H(X) is a function representing the corrected model output, g(X) is a functional form of the performance function, and X is the basic parameter.

H(X) may be a deterministic model but the basic random values parameters are used to include the uncertainty. If the model coefficients are also uncertain, they can be included in the parameters describing the model uncertainty. For example, if the threshold level is the maximum concentration of total coliforms, H(X) is then the model for calculating the concentration of total coliforms as given in equation 2.29 (see section 2.4). In that case, the basic parameters consist of effluent concentration, decay parameter (T_{90}) and initial dilution, or alternatively, wastewater flow rate, seawater current, and depth above discharge.

In the above example, the interest is to define probability of failure, i.e. the probability that a specified criterion for effluent concentration at vicinity of the outfall discharge cannot be achieved using a designed ocean outfall. If the EEC Microbial Standard for bathing waters (see table 2.7 in subsection 2.6.3) is applied, for instance, 95 % of the concentration of total coliforms in samples taken at a bathing area near the designed ocean outfall should be equal to or less than 10,000 per 100 ml. In this case, the concentration of total coliform of 10,000 per 100 ml is taken as the threshold level, and the failure occurs when the total coliform concentration resulting from the outfall discharge is more than 10,000 per 100 ml.

Because of uncertainty in the parameters affecting the initial dilution and effluent concentration, values of initial dilution and effluent concentration will also be uncertain. In the above case, the concentration of total coliforms at a given location may be less or more than 10,000 per 100 ml. Therefore, there is a possibility that the value of performance function is less than zero. The probability of failure is then the probability of the value of the performance function being less than zero, or in this case, the probability that the concentration of total coliforms is higher than 10,000 per 100 ml.

3.2.5. Determining Critical Probability of Failure

To assess whether a particular design is acceptable or not, criteria need to be established. The criteria may be given in terms of the maximum probability of failure using a specified value which is referred to here as the critical probability of failure. The critical probability of failure is taken or derived from the government regulation which in turn considers the use of the receiving water, either for bathing area, shellfish area, or others. Therefore, the value of the critical probability of failure may vary from case to case depending upon the sea environment and the extent to risk being acceptable bounds.

For the deterministic standards (regulations) such as those which apply in the Yugoslavian Adriatic (table 2.6), the probabilistic approach may not be applied directly. However, the approach can be used to assess several alternative designs which are based on a deterministic standard. For example, if some alternatives are acceptable under the deterministic regulation, the probability of failure for each alternative may be calculated, and the choice would fall on the alternative with the minimum probability of failure.

When the regulations are in the forms of a probabilistic standard (e.g. table 2.7 or in the detailed descriptions discussed in subsection 2.6.3), a way of deriving the value of probability of failure from such the regulations must be established. Indeed, Ang & Tang (1975) indicated that in monitoring the daily water quality there are two alternatives of concern, i.e. whether or not the water tested meet the pollution control standards. In such cases, the Bernoulli model may be applied. Ching (1997) and Huang (1997) also suggested that it could be assumed that a Bernoulli process was involved if the events of the measured pollutant concentrations exceeded the water quality standard.

If the probability of occurrence of an event in each trial is p (and probability of non occurrence is 1-p), then the probability of exactly x occurrences among n trials in a Bernoulli sequence is given by the Binomial distribution, as follows (Ang & Tang, 175; Smith, 1986; Devore, 1995; Huang, 1997):-

$$P(X=x) = \left[\frac{n!}{x!\,(n-x)!}\right] p^{x}(1-p)^{n-x} \tag{3.28}$$

where x and p are parameters of the Binomial distribution.

For example, if the interest is to formulate the probability of failure based on the regulation given by the Water Quality Criteria Working Party '81 (William, 1985) for shellfish harvesting waters (see subsection 2.6.3), the problem can be broken into two cases, i.e., one in which the median of faecal coliforms bacterial shall not exceed 14 MPN per 100 ml based on a minimum of one water sample taken on each 10 consecutive days, or the other in which not more than 10% of samples shall exceed 43 MPN per 100 ml. The problem may then be reanalyzed by employing the principles of Bernoulli events.

If X is the number of failures, then for the highest risk, $P(X=0) = 0.5$ (median reflects the 50% of the cumulative distribution), and for regular conditions, $P(X=0) = 0.1$ (10% from the samples). When it is assumed that there is no overlap of the 10 consecutive day periods, the possible number of trials during a year period is given by $n= 37$ (365/10 rounded to 37 as n should be an integer). Using equation 3.28 with $n=37$ and $x=0$, the critical probability of failure is then $p = 0.0188$ (for highest risk), and $p = 0.0611$ (for regular conditions). The critical probability of failure may be converted to percentages, i.e. 1.88 % and 6.11% respectively. The procedure may be applied for other cases.

3.2.6. Computation of Probability of Failure

The definition of the probability of failure is given by equation 3.25. Nevertheless, methods of obtaining that probability of failure have not yet been shown. Many probabilistic methods are available for such a purpose, including the method of First Order Second Moment (with first or second order approximation for the mean), Advance First Order Second Moment, and Monte Carlo Simulations. Discussion of these methods is given in Chapter Four.

3.2.8. Evaluation of Results

When the probability of failure has been calculated, and a specified value for the critical probability of failure has been set up, the acceptability of the design can be evaluated by simply looking at the difference between the two, defined as:-

$$\Delta = P_{critical} - P_f \tag{3.29}$$

where Δ is the difference between the calculated and critical probability of failure. If Δ is less than zero, the design is not acceptable.

A convenient way of finding alternatives for design purposes is to evaluate P_f for a certain range of design parameters. For example, for a given outfall location, the designer may look at P_f as a function of the pipe diameter, the seawater depth, or the distance of the outfall discharge to the location of interest so that the reliability of the system can be compared for each combination of the design parameters. Detailed examples are given in the case study in Chapter Five.

Chapter 4

Development and Application of Probabilistic Methods

4.1. Introduction: previous work on probabilistic ocean outfall design

Probabilistic approaches are relatively new for assessing and designing ocean outfalls. Huang *et. al.* (1994) proposed an approach which employs a time domain simulation using actual data sets to generate a time series of initial dilution. Huang presented input parameters (i.e., ambient seawater currents, seawater depth above discharge and wastewater flow rate) in term of time series. He then used them as input into a semi-empirical equation, applicable to the Miami Outfall, to produce a time series of hourly initial dilution. Values of all the parameters at a given time, $t = t_4$ for example, were used to calculate the initial dilution at that time.

Although this approach takes into account the variability of the input parameters, its application for estimating the frequency of low dilution events can be misleading. This is because it does not account for the possibility of initial dilution resulting from the combinations of parameter values at different times (for example the initial dilution resulting from the input variables: current at $t = t_1$, seawater depth at $t = t_2$, and wastewater flow rate at $t = t_3$). It is unreasonable to suggest, for example, that the value of wastewater flow rate at $t = t_1$ always corresponds to the values of other variables at that time. Furthermore, because the probabilistic description in this approach is in terms of an accumulated time fraction, the return period of a low dilution event cannot be deduced from this method.

In addition to the use of time domain simulation, Monte Carlo simulations (MCS) have been used in designing ocean outfalls, as shown in Orlob and Tumeo (1986), Webb (1987) and Bale et. al (1990). These works are generally interested in distribution of bacterial concentration at a distance from an outfall discharge. Initial dilution distribution, or probability that a specified value of the dilution would not be achieved by the outfall, is not well emphasized. In fact, as discussed in Chapter Two, both initial dilution and bacterial concentration are essential in outfall design and analysis. Beside, these simulations do not include tidal variation which, as discussed in section 3.2.2, is important for small outfalls.

Furthermore, it is considered useful to have a relatively simple probabilistic method to design ocean outfalls, although a simple method might give only approximate solutions. As noted by Cornell (Melching 1995), it is still better to have an approximate solution to the whole problem than an exact solution of only a portion of it. For this reason the method of first order second moment (FOSM) and advance first order second moment (AFOSM) have been applied to the design of ocean outfalls, and have been compared with MCS to determine the performance of these methods for use in outfall design.

4.2. First Order Second Moment (FOSM) Method

The probability distribution of random values of a variable can, theoretically, be derived from the probability distribution of the basic variables. Accordingly, the probability distribution of the initial dilution or the effluent concentration at a distance from the outfall discharge would be obtained when the probability distributions of the input parameters and the model coefficients are available. However, in many applications the probability distributions of the parameters involved may not be known. Information may be limited to the mean and variance of the parameters. Furthermore, even if the probability distributions are available, such derivations are generally difficult to perform, especially when the function is nonlinear.

In such circumstances, the moments - particularly the mean and variance - may be sufficient for practical purposes even if the correct probability distribution must be left undetermined (Ang and Tang, 1975). The FOSM method provides a methodology for obtaining an estimate of the mean and variance of random values of a performance function which is a function of one or several other parameters. The FOSM method is therefore a useful tool in solving the problem under consideration. The following is the general methodology of the method.

If T_L is a threshold level, and H (X) is the model operations that estimate the output parameter of interest, the resulting performance function is (from equation 3.26):-

$$Z = T_L - H(X) = g(X) \tag{3.26}$$

where g (X) is the functional form of the performance function with respect to the basic variables, X, and the constant (threshold level), T_L. In this case, the threshold level is the value of initial dilution specified by the designer or the effluent concentration set up by the government agency (see section 3.2.4 for detailed discussion). Because many input parameters are involved in the model operation, equation 3.26 can be written as:-

$$Z = g(X) = g(X_1, X_2, X_3, \ldots, X_n, T_L) \tag{4.1}$$

The mean and variance of Z can be obtained by first expanding equation 4.1 in a Taylor series about the mean values of X_1, X_2, X_3, X_4,...,X_n (threshold level has a constant value), thus:-

$$Z = g(\mu_{x1}, \mu_{x2}, \mu_{x3}, \ldots, \mu_{xn}, T_L) + \sum_{i=1}^{n} (x_i - \mu_{xi}) \left(\frac{\partial g}{\partial x_i}\right)$$

$$+ \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i - \mu_{xi})(x_j - \mu_{xj}) \frac{\partial^2 g}{\partial x_i x_j} + \ldots \tag{4.2}$$

where the derivatives are evaluated at the mean values of the X's. Truncating the series at the linear terms and taking expectations, the first order mean and variance of Z are:-

$$E_1(Z) \approx g(\mu_{x1}, \mu_{x2}, \mu_{x3}, \ldots, \mu_{xn}) \tag{4.3}$$

$$Var(Z) \approx \sum_{i=1}^{n} \left(\frac{\partial g}{\partial x_i} \right)^2 Var(x_i) + \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial g}{\partial x_i} \frac{\partial g}{\partial x_j} Cov(x_i, x_j) \tag{4.4}$$

To improve the performance of the method, Lye (personal communication) suggested that for practical purposes, it is possible to use the second-order approximation for the mean. Using this improvement, equation 4.4 becomes:-

$$E_2(Z) \approx g(\mu_{x1}, \mu_{x2}, \mu_{x3}, \ldots, \mu_{xn}, T_L) + \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{\partial^2 g}{\partial x_i x_j} Cov(x_i, x_j) \tag{4.5}$$

If the X's are uncorrelated (or statistically independent) for i and j, then the expressions for the mean and the variance are:-

$$E_2(Z) \approx g(\mu_{X1}, \mu_{X2}, \mu_{X3}, \ldots, \mu_{Xn}, T_L) + \frac{1}{2} \sum_{i=1}^{n} \frac{\partial^2 g}{\partial x_i^2} Var(x_i) \tag{4.6}$$

$$Var(Z) = \sigma_z^2 \approx \sum_{i=1}^{n} \left(\frac{\partial g}{\partial x_i} \right)^2 Var(x_i) \tag{4.7}$$

The uncertainty of the system may be measured in terms of a reliability index, β, which is defined as:-

$$\beta = \frac{E(Z)}{\sigma_z} \tag{4.8}$$

where E(Z) can be either $E_1(Z)$ or $E_2(Z)$. As can be seen, β is the reciprocal of the coefficient of variation, and can be directly used to compare the reliability of various alternatives for the system under consideration.

In many engineering cases, the reliability of the alternative is measured using the probability of failure. Typically, it is assumed that Z is normally distributed, and thus the probability of failure for a given T_L is defined as:-

$$P_f(T_L) = 1 - \phi(\beta) \tag{4.9}$$

where $P_f(T_L)$ for a given T_L, and $\phi(\beta)$ is the standard normal integral.

When the performance function cannot be assumed to be normally distributed, equation 4.9 cannot be directly applied. For such a case, the solution may be found provided the assumed probability distribution of the parameters are available. Whenever possible, the simplest way is to use the equivalent normal distribution. For example, when H(X) in equation 3.26 is assumed to be log-normally distributed with a mean of μ_Y and a standard deviation of σ_Y, equation 4.9 can be used with a modification of the reliability index, i.e. $\beta = [\ln(T_L) - \mu_Y]/\sigma_Y$, in which T_L is the threshold level (see table 3.2 for summary of non-normal distributions).

The greatest advantage of the FOSM method is its simplicity. In fact, only the mean and variance of the input parameters are necessary to obtain an estimate of the mean and variance of the performance function. Using a reasonable assumption of probability distribution of the performance function, the probability of failure may then be determined. However, this method has problems (Melching and Anmangandla, 1992; Melching, 1995) because failure events generally happen at extreme values rather than near the central values of the basic variables. As a result, the quality of the calculation obtained from the FOSM method would be degraded at extreme probabilities.

4.2.1. Mean and Variance for Initial Dilution in Still Waters

Cederwall's (1968) model was chosen as the deterministic model for initial dilution of horizontal round jet in still waters (see section 3.2.1). For probabilistic design, the input parameters, i.e. densimetric Froude number F, and seawater depth above discharge Y, are modified to include their uncertainty. The model defines the dilution under two conditions, i.e. high depth-diameter ratio (Y/D > 0.5 F), and low depth-diameter ratio (Y/D < 0.5 F). From equations 2.1 and 2.2 :-

$$S_o = 0.54 F^{9/16} \left(\frac{Y}{D}\right)^{7/16} \qquad for\ the\ range\ \left(\frac{Y}{D}\right) < 0.5 F \qquad (2.1)$$

$$S_o = 0.54 F \left[0.38\ \frac{(Y/D)}{F} + 0.66\right]^{5/3} \qquad for\ the\ range\ \left(\frac{Y}{D}\right) > 0.5 F \qquad (2.1)$$

For a given threshold level for initial dilution in still waters T_o, the performance function can be written as follows:-

$$Z_o = S_o - T_o \tag{4.10}$$

In equation 4.10, the negative threshold level and positive initial dilution are to satisfy the definition of failure probability as given in equation 3.15. Using performance function in equation 4.10, probability of failure is the probability that the initial dilution is less than threshold level T_o.

For S_0 given in equation 2.1, the first and second partial derivations of equation 4.10 are then:-

$$\frac{\partial Z_o}{\partial F} = 0.30375 \left(\frac{Y}{FD}\right)^{7/16} \qquad \frac{\partial^2 Z_o}{\partial F^2} = \frac{0.13289}{F}\left(\frac{Y}{FD}\right)^{7/16} \tag{4.11}$$

$$\frac{\partial Z_o}{\partial Y} = \frac{0.23625}{D^{7/16}}\left(\frac{F}{Y}\right)^{9/16} \qquad \frac{\partial^2 Z_o}{\partial Y^2} = \frac{0.13289}{H\ D^{7/16}}\left(\frac{F}{Y}\right)^{9/16} \tag{4.12}$$

The first order expectation and variance for the performance function can be calculated using equation 4.3 and 4.4, and the results are:-

$$E_1(Z_o) = 0.54\ \mu_F^{9/16}\left(\frac{\mu_Y}{D}\right)^{7/6} - T_o \tag{4.13}$$

$$Var(Z_o) = 0.09226\left(\frac{\mu_Y}{D\mu_F}\right)^{7/8} Var(F) + \frac{0.05581}{D^{7/8}}\left(\frac{F}{Y}\right)^{9/8} Var(Y) \tag{4.14}$$

where E_1 (Z_o) and Var (Z_o) are the first order expectation and variance of the performance function Z_o, respectively. The mean (the first order expectation) in equation 4.14 can be improved into second order, i.e.:-

$$E_2(Z_o) = 0.54\mu_F^{9/16}\left(\frac{\mu_Y}{D}\right)^{7/6} - T_o + \frac{0.06645}{D^{7/16}}\left(\frac{\mu_Y^{7/16}}{\mu_F^{23/16}} Var(F) + \frac{\mu_F^{9/16}}{\mu_Y^{25/16}} Var(Y)\right) \tag{4.15}$$

where E_2 (Z_o) is the second order mean of the performance function Z_o. When the available data is in terms of wastewater flow rate Q, the relationship between F and Q is defined in equation 2.3, that is :-

$$F = \frac{4Q}{3.14\ D^{5/2}}\left(\frac{g(\rho_a - \rho_e)}{\rho_e}\right)^{-1/2} \tag{2.3}$$

Assuming that D, g, ρ_a ρ_e are constant, the mean and standard deviation of F are then:-

$$\mu_F = \frac{1.27323}{D^{5/2}}\left(\frac{g(\rho_a - \rho_e)}{\rho_e}\right)^{-1/2} \mu_Q \tag{4.16}$$

$$Var(F) = \left(\frac{1.62114\ \rho_e}{D^5 g(\rho_a - \rho_e)} \right) Var(Q) \tag{4.17}$$

The procedure used for the case of (Y/D < 0.5 F) can also be employed to get the mean and standard variance of Z_o for the case of (Y/D > 0.5 F). The solutions are:-

$$E_1(Z_o) = 0.54\ \mu_F \left(\frac{0.38\mu_Y}{\mu_F\ D} + 0.66 \right)^{5/3} - T_o \tag{4.18}$$

$$Var(Z_o) = \zeta_1 + \zeta_2 \tag{4.19}$$

where $\zeta_1 = \left(0.36 \frac{\mu_Y}{D\ \mu_F} + 0.66 \right)^{4/3} \frac{0.1169\ Var(Y)}{D^2}$, and

$$\zeta_2 = \left(0.38 \frac{\mu_Y}{D\mu_F} + 0.66 \right)^{10/3} 0.2916\ Var(F) + \left(0.38 \frac{\mu_Y}{D\mu_F} + 0.66 \right)^{4/3} \frac{0.1169\ \mu_Y^2 Var(F)}{D^2 \mu_F^2}$$

The second order improved mean is:-

$$E_2(Z_o) = \left(\frac{0.263\mu_Y}{\mu_F^{2/5}\ D} + 0.456\mu_F^{3/5} \right)^{5/3} - T_o + \frac{0.04332 \left(\frac{\sigma_Y^2}{\mu_F} + \frac{\mu_Y^2 \sigma_F^2}{\mu_F^3} \right)}{D^2 \left(\frac{0.38\mu_Y}{\mu_F\ D} + 0.66 \right)^{1/3}} \tag{4.20}$$

4.2.2. Mean and Variance for Initial Dilution in Moving Waters

The Lee-Neville Jones (1987-a) model was chosen as the deterministic model for initial dilution of horizontal round jet in moving waters (see section 3.2.1). For probabilistic design, the input parameters (seawater current, seawater depth above discharge, and wastewater flow rate) and the model coefficients (for both BDNF and BDFF) were taken as uncertain parameters. From equations 3.1 and 3.2, Lee-Neville Jones (1987) model is rewritten as-:

$$S_m = C_1 \left(\frac{B^{1/3} H^{5/3}}{Q} \right) \qquad \textit{for the range} \quad H \left(\frac{U^3}{B} \right) < 5 \tag{3.1}$$

$$S_m = C_2 \left(\frac{U H^2}{Q} \right) \qquad \textit{for the range} \quad H \left(\frac{U^3}{B} \right) > 5 \tag{3.2}$$

Similar to the still water conditions, the performance function becomes:-

$$Z_m = S_m - T_m \tag{4.21}$$

where Z_m and T_m are the performance function and the threshold level for initial dilution in moving water conditions. For the case of BDNF, $H(U^3/B) < 5$, the partial derivations of Z_m with respect to C_1, B, H, and Q are as follows:-

$$\frac{\partial Z_m}{\partial C_1} = \frac{B^{1/3} H^{5/3}}{Q} \qquad \frac{\partial^2 Z_m}{\partial C_1^2} = 0 \tag{4.22}$$

$$\frac{\partial Z_m}{\partial B} = \frac{C_1 H^{5/3}}{3 B^{2/3} Q} \qquad \frac{\partial^2 Z_m}{\partial B^2} = - \frac{2 C_1 H^{5/3}}{9 B^{5/3} Q} \tag{4.23}$$

$$\frac{\partial Z_m}{\partial H} = \frac{5\ C_1\ B^{1/3}H^{2/3}}{3\ Q} \qquad \frac{\partial^2 Z_m}{\partial H^2} = \frac{5\ C_1\ B^{1/3}}{9\ H^{1/3}\ Q} \tag{4.24}$$

$$\frac{\partial Z_m}{\partial Q} = -\frac{C_1\ B^{1/3}H^{5/3}}{Q^2} \qquad \frac{\partial^2 Z_m}{\partial Q^2} = \frac{2\ C_1\ B^{1/3}H^{5/3}}{Q^3} \tag{4.25}$$

Evaluating at mean values, the first order expectation and variance of the performance function are given in equation 4.26 and 4.27, respectively. The improved second order mean is given in equation 4.28.

$$E_1(Z_m) = \left(\frac{\mu_{C_1}\mu_B^{1/3}\mu_H^{5/3}}{\mu_O} \right) - T_m \tag{4.26}$$

$$Var(Z_m) = \left(\frac{\mu_B^{1/3}\mu_H^{5/3}}{\mu_Q} \right)^2 Var(C_1) + \left(\frac{\mu_{C_1}\mu_H^{5/3}}{3\mu_B^{2/3}\mu_Q} \right)^2 Var(B) + \left(\frac{5\mu_{C_1}\mu_B^{1/3}\mu_H^{2/3}}{\mu_Q} \right)^2 Var(H) + \left(\frac{\mu_{C_1}\mu_B^{1/3}\mu_H^{5/3}}{\mu_Q^2} \right)^2 Var(Q) \tag{4.27}$$

$$E_2(Z_m) = \left(\frac{\mu_{C_1}\mu_B^{1/3}\mu_H^{5/3}}{\mu_Q} \right) - T_m + \frac{1}{2}\left(-\frac{2\mu_{C_1}\mu_H^{5/3}}{9\mu_B^{5/3}\mu_Q} Var(B) + \frac{5\mu_{C_1}\mu_B^{1/3}}{9\mu_H^{1/3}\mu_Q} Var(H) + \frac{2\mu_{C_1}\mu_B^{1/3}\mu_H^{5/3}}{\mu_Q^3} Var(Q) \right) \tag{4.28}$$

The relationship between the buoyancy flux, B, and the wastewater flow rate is expressed as in equation 2.7, that is:-

$$B = \frac{g(\rho_a - \rho_e)}{\rho_e} Q \tag{2.7}$$

In many cases, the gravitational acceleration and the density ratio can be assumed to be constant. For a typical case of $g = 9.8\ m/s^2$ and $(\rho_a-\rho_e)/\rho_e = 0.027$, the buoyancy flux is simply a linear function of the waste flow rate, i.e. $B = 0.2646\ Q$; and the mean and variance of B are $\mu_B = 0.2646\ \mu_Q$ and $Var(B) = 0.07\ Var(Q)$ respectively.

The other expression of the performance function can be obtained by replacing the buoyancy flux in terms of the waste flow rate. With this modification, equation 4.26, 4.27, and 4.28 become equation 4.29, 4.30, and 4.31, respectively.

$$E_1(Z_m) = \left(\frac{0.642\ \mu_{C_1} \mu_H^{5/3}}{\mu_Q^{2/3}} \right) - T_m \tag{4.29}$$

$$Var(Z_m) = \left(\frac{0.642 \mu_H^{5/3}}{\mu_Q^{2/3}} \right)^2 Var(C_1) + \left(\frac{1.070 \mu_{C_1} \mu_H^{2/3}}{\mu_Q^{2/3}} \right)^2 Var(H) + \left(\frac{0.428 \mu_{C_1} \mu_H^{5/3}}{\mu_Q^{5/3}} \right)^2 Var(Q) \tag{4.30}$$

$$E_2(Z_m) = \left(\frac{0.642\ \mu_{C_1} \mu_H^{5/3}}{\mu_Q^{2/3}} \right) - T_m + \frac{1}{2} \left(\frac{0.713 \mu_{C_1}}{\mu_H^{1/3} \mu_Q^{2/3}} Var(H) + \frac{0.713 \mu_{C_1} \mu_H^{5/3}}{\mu_Q^{8/3}} Var(Q) \right) \tag{4.31}$$

For the case of BDFF, $H(U^3/B) > 5$, the mean and variance of the performance function can be obtained using the same procedure. The solutions are given in the following equations:-

$$E_1(Z_m) = \left(\frac{\mu_{C_2} \mu_U \mu_H^2}{\mu_Q} \right) - T_m \tag{4.32}$$

$$Var(Z_m) = \left(\frac{\mu_U \mu_H^2}{\mu_Q} \right)^2 Var(C_2) + \left(\frac{\mu_{C_2} \mu_H^2}{\mu_Q} \right)^2 Var(U) + \left(\frac{2\mu_{C_2} \mu_U \mu_H}{\mu_Q} \right)^2 Var(H) + \left(\frac{\mu_{C_2} \mu_U \mu_H^2}{\mu_Q^2} \right)^2 Var(Q) \tag{4.33}$$

$$E_2(Z_m) = \left(\frac{0.642\ \mu_{C_1} \mu_H^{5/3}}{\mu_Q^{2/3}} \right) - T_m + \frac{1}{2} \left(\frac{0.713 \mu_{C_1}}{\mu_H^{1/3} \mu_Q^{2/3}} Var(H) + \frac{0.713 \mu_{C_1} \mu_H^{5/3}}{\mu_Q^{8/3}} Var(Q) \right) \tag{4.34}$$

4.2.3. Mean and Variance for Bacterial Concentration at a Target Area

As can be seen in the above discussion, the mean and variance of the performance function for initial dilution can be estimated using the simple FOSM method. The solutions for the mean may be in the forms of the first or second order approximation of the Taylor series. However, the formulation for the approximation using the second order or the higher one may be prohibitive because of the complexity of the performance function under consideration.

This is particularly true when the performance function consists of many variables with a non-linear relationship such as equation 2.29 for calculating effluent concentration at a target area (e.g. bathing or shellfish areas) near the outfall discharge. Furthermore, the analysis of initial dilution showed (as will be demonstrated in Chapter 5) that the first and second order approximations give approximately the same answer. Because of this and because of the extreme complexity of equation 2.29, only a first order approximation was evaluated for equation 2.29.

From equation 3.16, for a given threshold level T_c the performance function for the effluent concentration is defined:-

$$Z_c = T_c - C_x \tag{4.35}$$

where C_x is the maximum effluent concentration at the target area of interest given by equation 2.28. The first partial derivations of equation 4.35 are:-

$$\frac{\partial Z_c}{\partial C_e} = -\frac{e^{\frac{-2.3\,x}{u\,T_{90}}}}{S_i}\, erf\left(\sqrt{\frac{1.5}{a^3-1}}\right) \tag{4.36}$$

$$\frac{\partial Z_c}{\partial S_i} = -\,C_e\,\frac{e^{\frac{-2.3\,x}{u\,T_{90}}}}{S_i^2}\, erf\left(\sqrt{\frac{1.5}{a^3-1}}\right) \tag{4.37}$$

$$\frac{\partial Z_c}{\partial T_{90}} = -\frac{2.3\,C_e}{S_i u T_{90}^2}\, e^{\frac{-2.3\,x}{u\,T_{90}}}\, erf\left(\sqrt{\frac{1.5}{a^3-1}}\right) \tag{4.38}$$

$$\frac{\partial Z_c}{\partial u} = -2.3\, C_e\, e^{\left(\frac{-2.3\,x}{u\,T_{90}}\right)} \left[\frac{erf\left(\sqrt{\frac{1.5}{a^3 - 1}}\right)}{u^2\, T_{90}\, S_i} + 7.21 \frac{a^2\, k_o\, x\, e^{-\left(\frac{1.5}{a^3-1}\right)}}{u^2\, b^2\, S_i \left(a^3 - 1\right)^{1.5}} \right] \quad (4.39)$$

where $a = \left(1 + \frac{8\, k_o\, x}{u\, b^2}\right)$

Evaluating at mean values of the variables involved, the mean and variance of the performance function are then:-

$$E_1(Z_c) = T_c - \left[\frac{\mu_{C_e}}{\mu_{S_i}}\, e^{\left(\frac{-2.3\,x}{\mu_u\, \mu_{T_{90}}}\right)} erf\left(\sqrt{\frac{1.5}{a_\mu^3 - 1}}\right) \right] \quad (4.40)$$

$$Var(Z_c) = \xi\, \frac{\mu_{Ce}^2}{\mu_{Si}^2}\, e^{\left(\frac{-4.6\,x}{\mu_u\, \mu_{T_{90}}}\right)} \left(erf\left(\sqrt{\frac{1.5}{a_\mu^3 - 1}}\right)\right)^2 \quad (4.41)$$

where $a_\mu = \left(1 + \frac{8\, k_o\, x}{\mu_u\, b^2}\right)$, and ξ is defined as:-

$$\xi = \sigma_{Ce}^2 + \frac{\mu_{Ce}^2\, \sigma_{S_i}^2}{\mu_{S_i}^2} + \frac{5.29\, \mu_{Ce}^2\, \sigma_{T_{90}}^2}{\mu_u^2\, \mu_{T_{90}}^4} + \frac{5.29\, \mu_{Ce}^2\, \sigma_u^2}{\mu_u^2} \left(\frac{1}{\mu_u^2 \mu_{T_{90}}^2} + \frac{51.98\, a_\mu^4 k_o^2 x^2 e^{\left(\frac{-3}{a^3-1}\right)}}{\mu_u^2 b^4 \left(a_\mu^3 - 1\right)^3} \right)$$

It can be seen in equation 4.40 and 4.41 that it is necessary to calculate the mean and variance of the initial dilution in order to obtain the mean and variance of the bacterial concentration. This may, in turn, be calculated using the same FOSM method. Whereas the variance of the initial dilution can be taken to be the same as the variance of the associated performance function (because of constant T_c), the mean of the initial dilution is given by equations 4.42 to 4.45.

For initial dilution in still waters with ($Y/D < 0.5\ F$):

$$\mu_{S_i} = \mu_{S_o} = 0.54\ \mu_F^{9/16} \left(\frac{\mu_H}{D} \right)^{7/6} \tag{4.42}$$

For initial dilution in still waters with ($Y/D \geq 0.5\ F$):

$$\mu_{S_i} = \mu_{S_o} = 0.54\ \mu_F \left(\frac{0.38\mu_H}{\mu_F\ D} + 0.66 \right)^{5/3} \tag{4.43}$$

For initial dilution in moving waters , BDNF:

$$\mu_{S_i} = \mu_{S_m} = \left(\frac{\mu_{C_1} \mu_B^{1/3} \mu_H^{5/3}}{\mu_Q} \right) \tag{4.44}$$

For initial dilution in moving waters , BDFF:

$$\mu_{S_i} = \mu_{S_m} = \left(\frac{\mu_{C_2} \mu_U \mu_H^2}{\mu_Q} \right) \tag{4.45}$$

4.3. Advance First Order Second Moment (AFOSM) Method

Although the FOSM method is a promising approach to simplify a complicated problem, its solution could vary depending on how the performance function is expressed. Different expressions for the performance function would produce different values of reliability index. Smith (1986) indicated that an invariant reliability index is obtained if the point chosen for the linear approximation is actually on the failure boundary. The design point, which has maximum probability of failure, lies somewhere along the boundary.

Smith (1986) described the method of AFOSM as an improvement on the FOSM method. The essence is to linearize the performance function via a Taylor series expansion at the likely failure points (x_1*, x_2*, x_3*,..., x_n*) at the failure surface, i.e. g (x_1*, x_2*, x_3*,..., x_n*) = 0. From equation 4.2, the performance function of Z is then:-

$$Z = g(x_1^*, x_2^*, x_3^*, \ldots, x_n^*) + \sum_{i=1}^{n} (x_i - x_i^*) \left(\frac{\partial g}{\partial x_i}\right)$$

$$+ \frac{1}{2}\sum_{i=1}^{n} \sum_{j=1}^{n} (x_i - x_i^*)(x_j - x_j^*) \frac{\partial^2 g}{\partial x_i x_j} + \ldots \qquad (4.46)$$

where the derivatives are evaluated at x_i*. As g (x_1*, x_2*, x_3*,..., x_n*) = 0, truncating the series at the linear terms, equation 4.46 would be:-

$$Z = \sum_{i=1}^{n} (x_i - x_i^*)\ g'(x_i^*) \qquad (4.47)$$

where $g'(x_i^*)$ is the first derivative of $g(X)$ (from equation 4.1) evaluated at the point x^* equals $(x_1^*, x_2^*, x_3^*, \ldots, x_n^*)$. Therefore, (for independent variables):-

$$\mu_Z = \sum_{i=1}^{n} (\mu_i - x_i^*)\ g'(x_i^*) \tag{4.48}$$

$$Var(Z) = \sum_{i=1}^{n} [g'(x_i^*)]^2\ Var(x_i) \tag{4.49}$$

The relative contribution of any variable, X_i, to the value of variance of Z is defined as a sensitivity factor, α_i, which is:-

$$\alpha_i = \frac{g'(x_i^*)\ \sigma_i}{\sigma_z} \tag{4.50}$$

From equation 4.49 and 4.50, the standard deviation of Z is then:-

$$\sigma_z = \sum_{i=1}^{n} \alpha_i\ g'(x_i^*)\ \sigma_i \tag{4.51}$$

It seems to be fairly simple and straightforward to solve the problem using this approach because from equations 4.48 and 4.49 the mean and variance of the performance function are readily obtained for further use in determining the probability of failure. However, it must be pointed out that the determination of the failure point is not a simple task. A commonly used approach is to use a computer iteration as shown by Rackwitz (1976), Smith (1986) and Melching (1992).

Smith (1986) proposed an approach using standardized variables which have a mean and standard deviation of zero and one respectively. When derivation the performance function is not a problem, the standardization would make it easy to calculate the probability of failure.

In the standardized variables, if x_1 is a particular value of a variable with a mean of μ and a standard deviation of σ_1, then the corresponding reduced variable, y_1, is given by:-

$$y_1 = \frac{x_1 - \mu_1}{\sigma_1} \tag{4.52}$$

By this transformation, the failure surface can now be expressed as:-

$$Z = h(y) = (y_1, y_2, y_3, \ldots, y_n) \tag{4.53}$$

Equation 4.47 now becomes:-

$$Z = \sum_{i=1}^{n} (y_i - y_i^*)\, h'(y_i^*) \tag{4.54}$$

where $h'(y_i^*)$ is the first derivative of $h(y)$ evaluated at the point y^* $(y_1^*, y_2^*, y_3^*, \ldots, y_n^*)$. As the mean and standard deviation of the standardized variable are zero and one respectively, the mean and standard deviation of Z are then:-

$$\mu_Z = \sum_{i=1}^{n} (\mu_i - y_i^*)\, h'(y_i^*) = -y_i \sum_{i=1}^{n} h'(y_i^*) \tag{4.55}$$

$$\sigma_z = \sum_{i=1}^{n} \alpha_i \, h'(y_i^*) \, \sigma_i \quad = \sum_{i=1}^{n} \alpha_i \, h'(y_i^*) \tag{4.56}$$

The reliability index now is:-

$$\beta = \frac{\mu_z}{\sigma_z} \quad = \frac{-y_i \sum_{i=1}^{n} h'(y_i^*)}{\sum_{i=1}^{n} \alpha_i \, h'(y_i^*)} \tag{4.57}$$

Therefore:-

$$\sum_{i=1}^{n} h'(y_i^*) \; [-y_i^* \; - \; \alpha_i \, \beta] \; = 0 \tag{4.58}$$

The solution, in terms of the standardized variables is then:-

$$y_i^* = - \alpha_i \, \beta \qquad \text{for all i} \tag{4.59}$$

Trial and error calculations on these equations or iteration using a computer program may be used to find the value of failure point.

4.3.1. Reliability Index

The estimation of the failure point, and hence of the reliability index may be obtained using a computer iteration given by Smith (1986). The solution techniques given use either non-standardized or standardized variables. A modification of the iteration procedure is as follows:-

For non-standardized variables:

(1) Set initial value of the reliability index, $\beta = 0$

(2) Set $x_i^* = m_i$ for all i values, where m_i is the mean of variable i

(3) Compute $\partial g / \partial x_i$ for all i, at $x_i = x_i^*$

(4) Compute α_i for all i , where α_i is defined as:-

$$\alpha_i = \frac{g'(x^*)\,\sigma_i}{\sqrt{\sum_{i=1}^{n}\left[g'(x^*)\,\sigma_i\right]^2}} \tag{4.60}$$

(5) Compute new x* for each variable, using the equation:-

$$x_i^* = m_i - \alpha_i\,\beta\,\alpha_i \tag{4.61}$$

(6) Repeat steps 3 to 5 until a stable value of x_i^* are achieved

(7) Evaluate $Z = g(x_i^*)$

(8) Define a new value of the reliability index, β using the following equation:-

$$\beta = \frac{m_Z}{\sigma_Z} = \frac{\sum_{i=1}^{n}\left(m_i - x_i^*\right) g'(x_i^*)}{\sum_{i=1}^{n}\alpha_i\, g'(x_i^*)\,\sigma_i} \tag{4.62}$$

(9) Repeat steps 3 to 8 to achieve $Z = 0$

For standardized variables:

(1) Determine an expression for g(X)

(2) Evolve an expression for h(y)

(3) Determine expressions for all first partial derivative of h(y), h'(y_i)

(4) Set $y_i = 0$ and $\beta = 0$

(5) Evaluate all h'(y_i)

(6) Evaluate h(y_i)

(7) Evaluate standard deviation of Z from:-

$$\sigma_Z = \sum_{i=1}^{n} \alpha_i \, h'(y_i) = \sqrt{\sum_{i=1}^{n} \left[h'(yi)\right]^2} \tag{4.63}$$

(8) Define new values for y_i from:-

$$y_i = -\frac{h_i'}{\sigma_Z}\left[\beta + \frac{h(y_i)}{\sigma_Z}\right] \tag{4.64}$$

(9) Calculate the new reliability index, β:-

$$\beta = \frac{m_Z}{\sigma_Z} = \frac{-\sum_{i=1}^{n} y_i \, h'(y)}{\sigma_Z} \tag{4.65}$$

(10) Repeat step 5 to 9 until the values converge.

4.3.2. Problems with Non-normal Parameters involved

Having calculated the reliability index, the probability of failure can now be calculated directly using equation 4.9 when all parameters involved and the performance function of interest are assumed to be normally distributed. However, if the performance function is known to have a non-normal distribution, the calculation of the probability of failure should take into account the effect of the non-normal distribution. For a given reliability index, the essence of calculating the probability of failure is just the same as that used in the FOSM method, i.e. by transforming into a normal distribution or using table 3.2.

Similarly, when the input parameters involved are known to be not normally distributed, the parameters may be transformed into the normal distribution. The transformed form for each parameter is then used in the iteration to obtain the reliability index. Having the reliability index, the standard normal cumulative distribution can then be used to determine the probability of failure (Melching, 1995).

Rackwitz and Fiessler (1978) proposed a method for the treatment of independent non-normal parameters which is widely used in the reliability analysis, for example in Melching and Anmangandla (1992) and Melching (1995). In the proposed method, a non-normally distributed parameter, X_i, can be approximated at any point to a normally distributed one provided that both the cumulative density functions and the probability density functions are equal at the point chosen. For the purpose of analysis the point chosen is the failure point.

With these conditions the values of the mean and standard deviation of the normalized parameters can be found from the expression:-

$$\sigma_i^N = \frac{f^N\left[\phi^{-1}\left(F_{xi}(x_i^*)\right)\right]}{f_{xi}(x_i^*)} \tag{4.66}$$

$$m_i^N = x_i^* - \phi^{-1}\left(F_{xi}(x_i^*)\right)\sigma_i^N \tag{4.67}$$

where σ_i^N is the standard deviation of the normalized parameter, X_i, m_i^N is the mean of the normalized parameter, X_i, $F_{xi}(x_i^*)$ is the cumulative probability of X_i at x_i^*, $f_{xi}(x_i^*)$ is the probability density of X_i at x_i^*, $\phi^{-1}(.)$ is the inverse normal distribution, and $f^N(.)$ is the standardized normal density function.

The method becomes easy to apply when using standardized variables as shown by Smith (1986). The essence of the method (Smith, 1986) is to normalize the non-normal parameters, and to replace the mean and standard deviation of each parameter (see equation 4.52) with the mean and standard deviation of the normalized parameters (from equation 4.66 and 4.67). The application of this method to the probability analysis of an ocean outfall is shown in Chapter Five.

4.4. Monte Carlo Simulations (MCS) Method

Ang and Tang (1984) defined simulation as the process of replicating the real world based on a set of assumptions and conceived models of reality. The simulation can be used to estimate or evaluate the performance or response of an engineering system. For example, in estimating initial dilution for an outfall system, with a particular random value of the parameters (i.e. input variables and model coefficients), the simulation would give a specific result for the initial dilution. Repeating this procedure would provide a set of values for initial dilution which could be statistically analyzed.

When problems under investigation involve random values of the parameters with known (or assumed) probability distributions, the simulation is referred to as a *Monte Carlo Simulation* (MCS). In each simulation, MCS use a particular set of random values generated in accordance with the corresponding probability distribution function of the parameters (Melching and Anmangandla, 1992). Then, for each simulation, the performance function is calculated using the appropriate values of these parameters and the threshold level for the output parameter of interest. Detailed discussion of MCS is not given here but can be found elsewhere, e.g. Ang and Tang (1984), and Melching (1995).

In the ocean outfall design, a set of values of initial dilution or effluent concentration would be obtained by using MCS with a deterministic model and the generated random values for each parameter involved. In other words, the main task in a MCS is to generate random values from a prescribed probability distribution. For a given set of generated random values, the simulation is deterministic (Ang and Tang, 1984).

Figure 4.1 shows a diagram of MCS which could be used for designing ocean outfalls. In the diagram, the pipe diameter is varied and the probability of failure is calculated for each value of diameter. Having a certain range of pipe diameters, it permits the depiction of a relationship between probability of failure and pipe diameter. From the relationship, a suitable value of diameter may be chosen. For other purposes, pipe diameter may be specified as a constant value, and other parameters may then be varied, e.g. mean seawater depth or mean wastewater flow rate.

Monte Carlo simulation (MCS) is a flexible technique for a great variety of problems. Melching, C. S. and Anmangandla, S. (1992) noted that it may be the only method that can estimate the cumulative distribution function of the performance function for cases with highly nonlinear or complex system relationships. In practice, however, MCS may be limited because of cost and computer capability. As a general rule, it should therefore be used only as a last resort when analytical solution methods are not available or are ineffective (Ang and Tang, 1984). Further discussion of the method is given in Chapter Six.

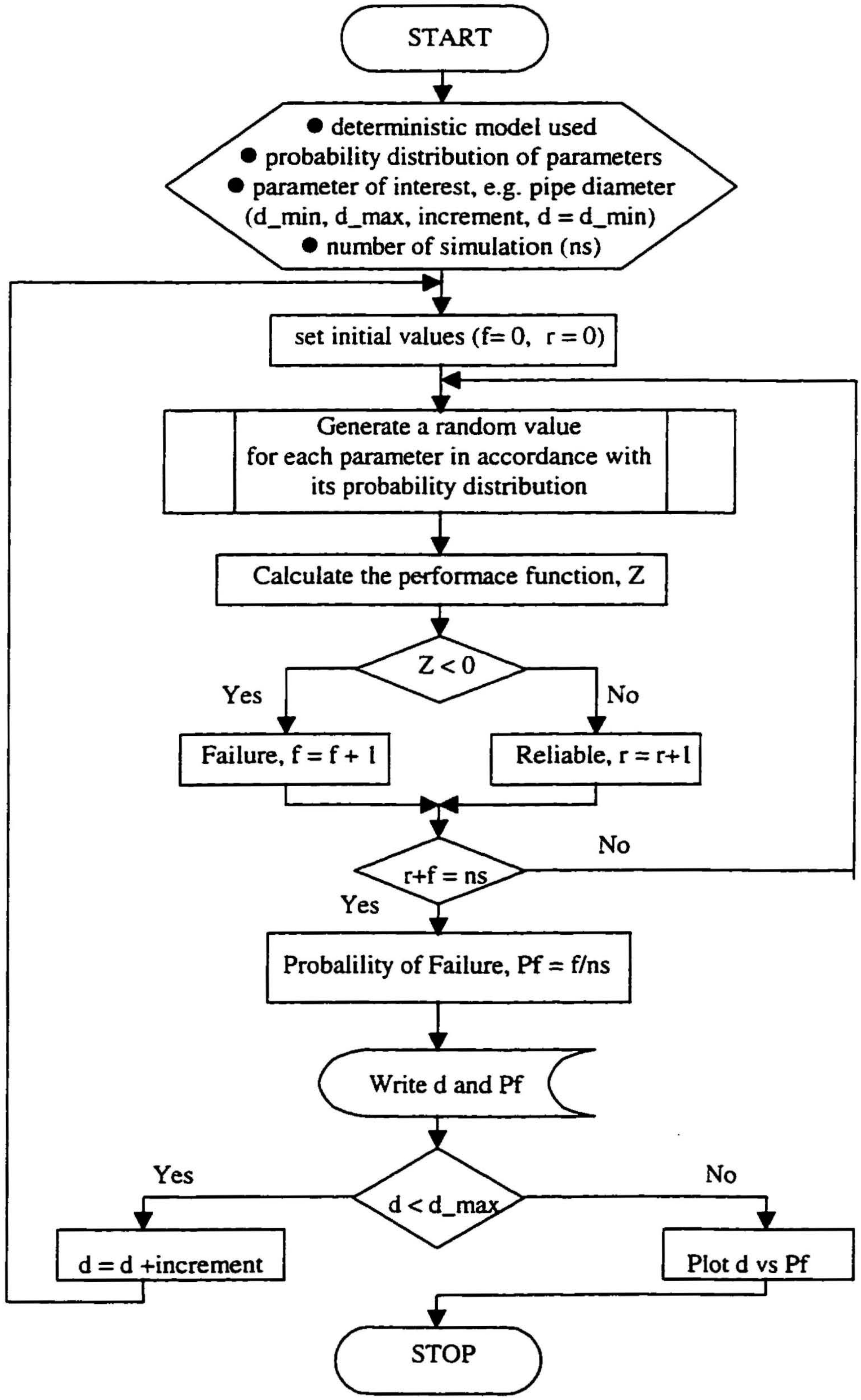

Figure 4.1. Diagram of MCS for Ocean Outfall Design

Chapter 5

Cases Study: the Spaniard's Bay Outfall, Newfoundland, Canada

5.1. Introduction

The data from the Spaniard's Bay Outfall are used here to show the application of the probabilistic approach discussed in the previous chapters. The step-by-step procedure given in Chapter Three is applied to a case study of the outfall. In fact, this study is not really a probabilistic design because the outfall is already in operation. Instead, the data of the outfall have been re-analyzed using the probabilistic approach, the intent being to show how the proposed approach may work under actual conditions as discussed further in Chapter Six. Although only some parts of the whole subject under consideration are covered in this case study, the others may be done using the same methods.

5.2. General Conditions of the Ocean Outfall

Spaniard's Bay Outfall is located on the east coast of Newfoundland, Canada, and serves a town that is predominantly nonindustrial and that has a population of around 1100 (Sharp, 1991). The outfall was designed to handle the sewage wastes from the town of Tilton and some of the waste from town of Spaniard's Bay. The peak flow design of the outfall is about 3347 to 4426 cubic meters per day (Sharp, 1989-c). The outfall consists of a 200 mm pipe discharging the waste horizontally through two 100 mm diameter nozzles about 100 m offshore in about 5 m deep of water. The general layout of the outfall and the sampling location during the Monitoring Study (Sharp, 1991) are shown in figures 5.1 and 5.2 respectively. Figures 5.3 and 5.4 depict the outfall details and its typical nozzle arrangement.

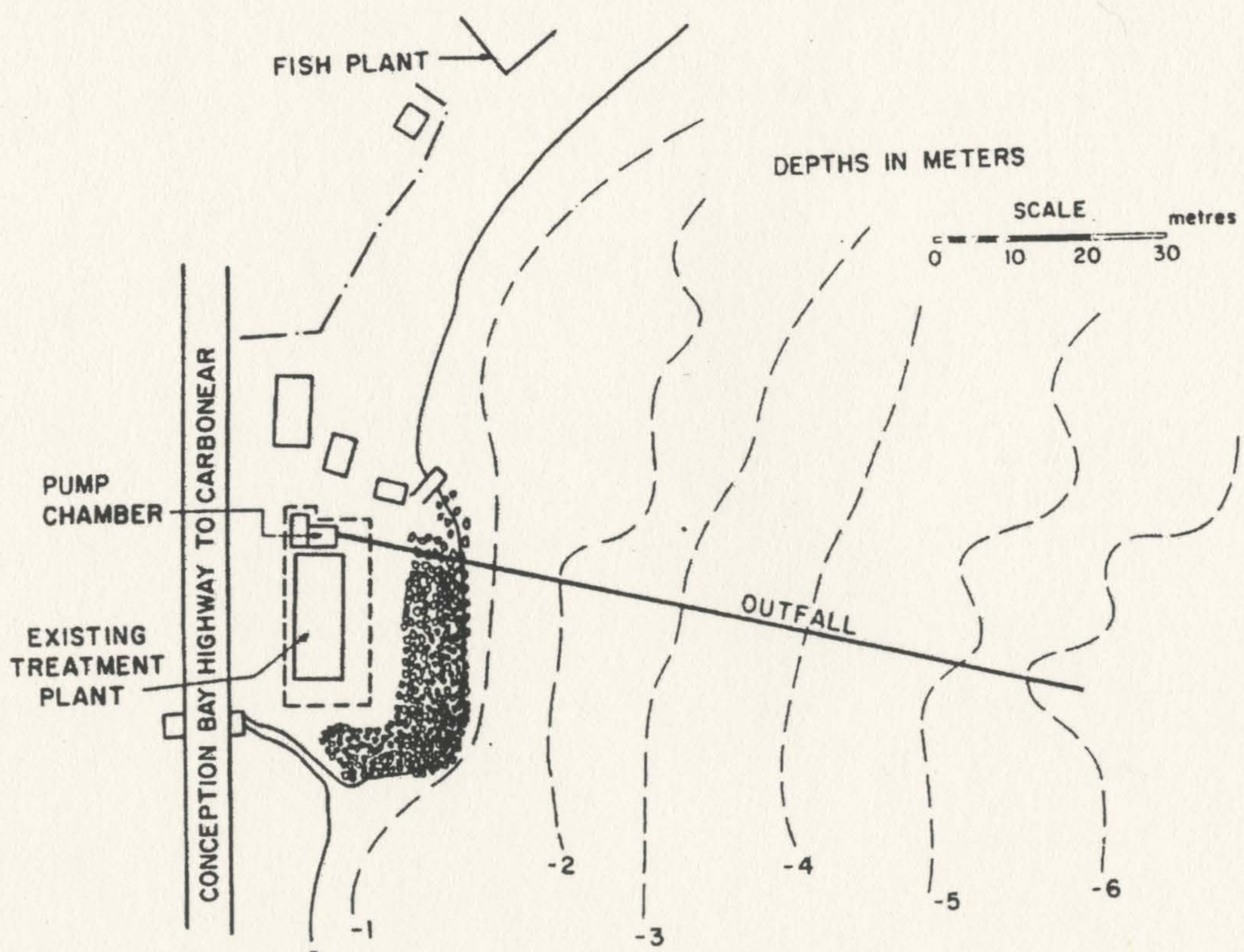

Figure 5.1. General Layout of Spaniard's Bay Outfall (after Sharp, 1991)

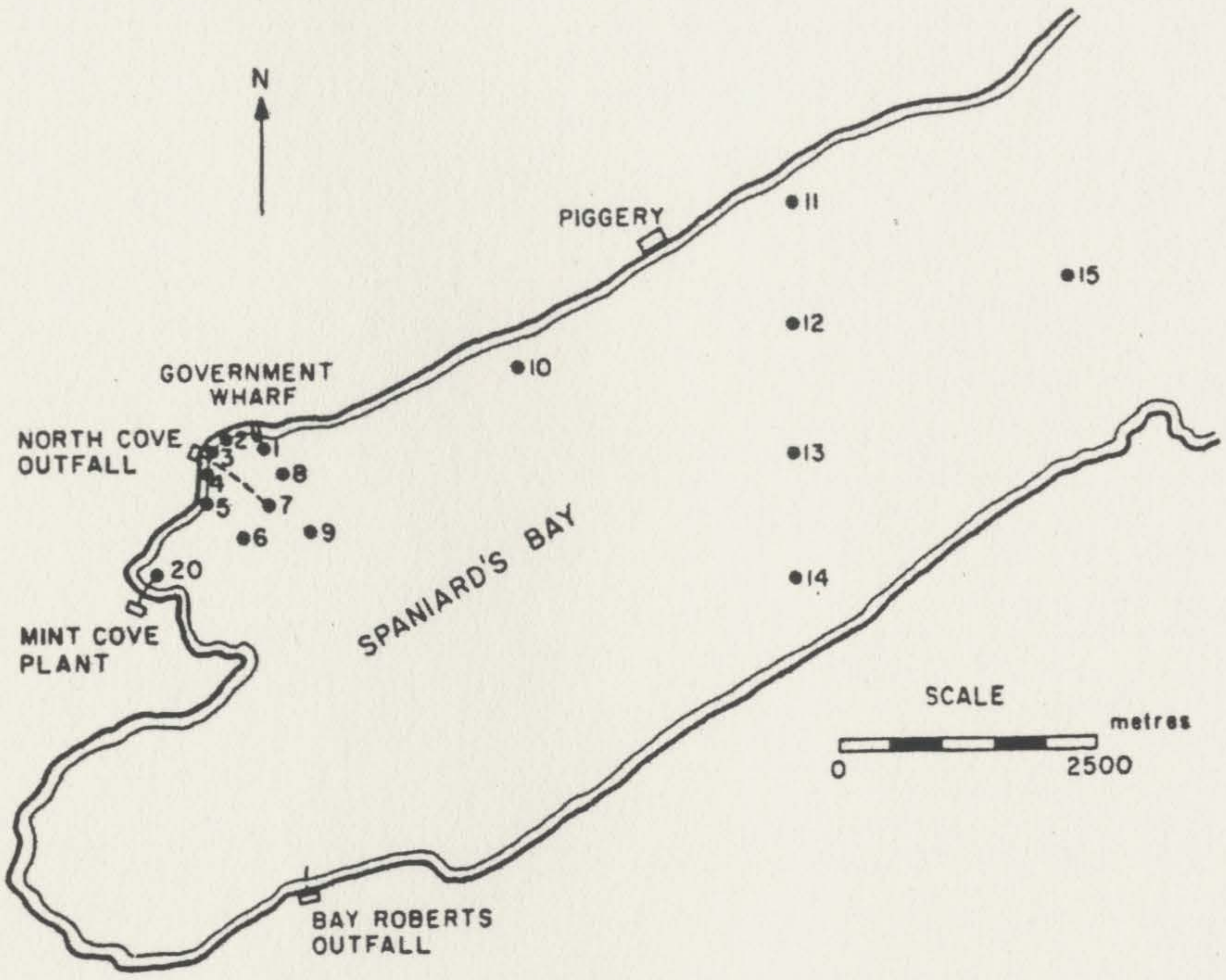

Figure 5.2. Sampling Locations during the Monitoring Study (after Sharp, 1991)

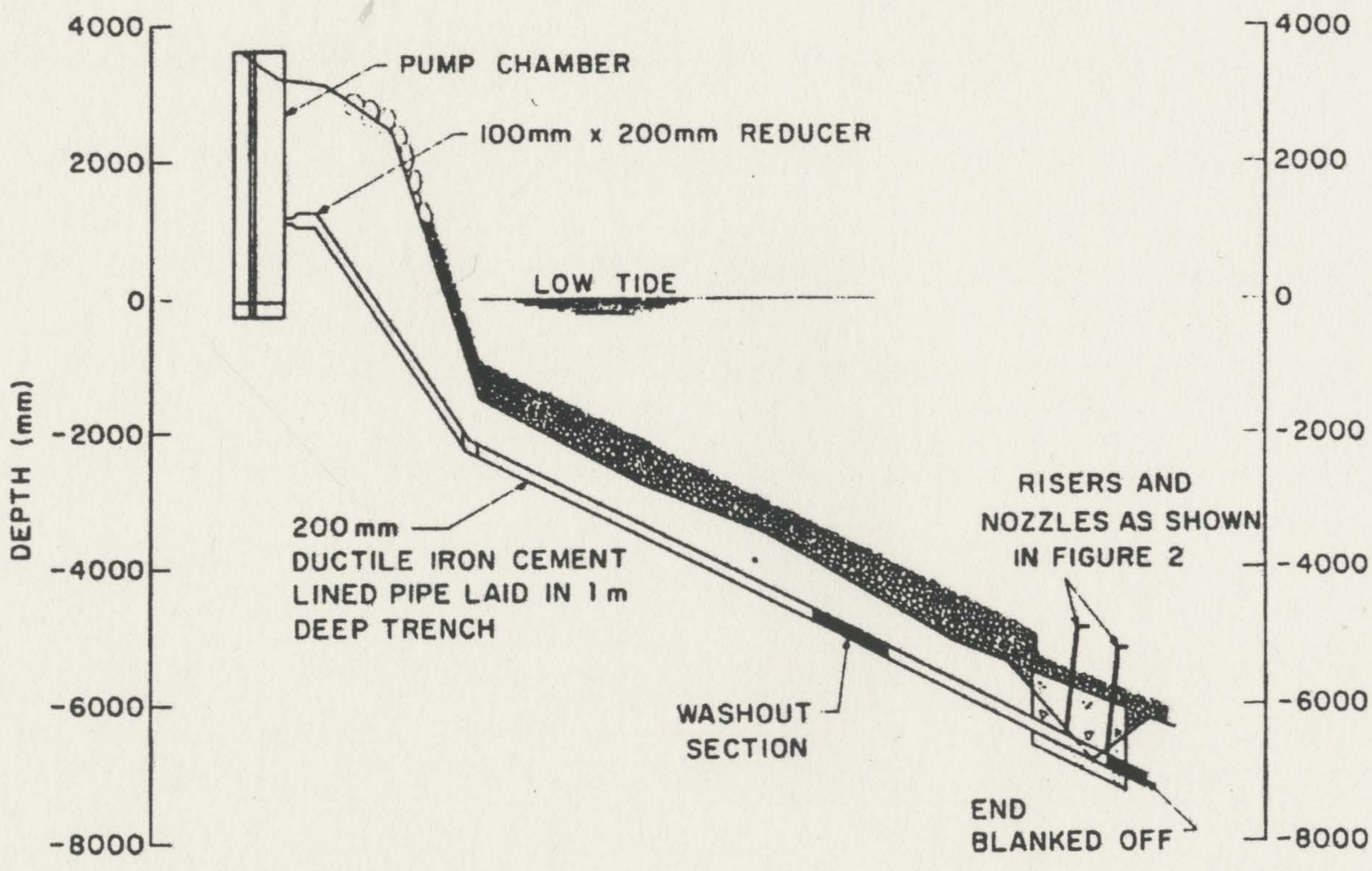

Figure 5.3. Typical Details of the Outfall (not to scale, after Sharp, 1991)

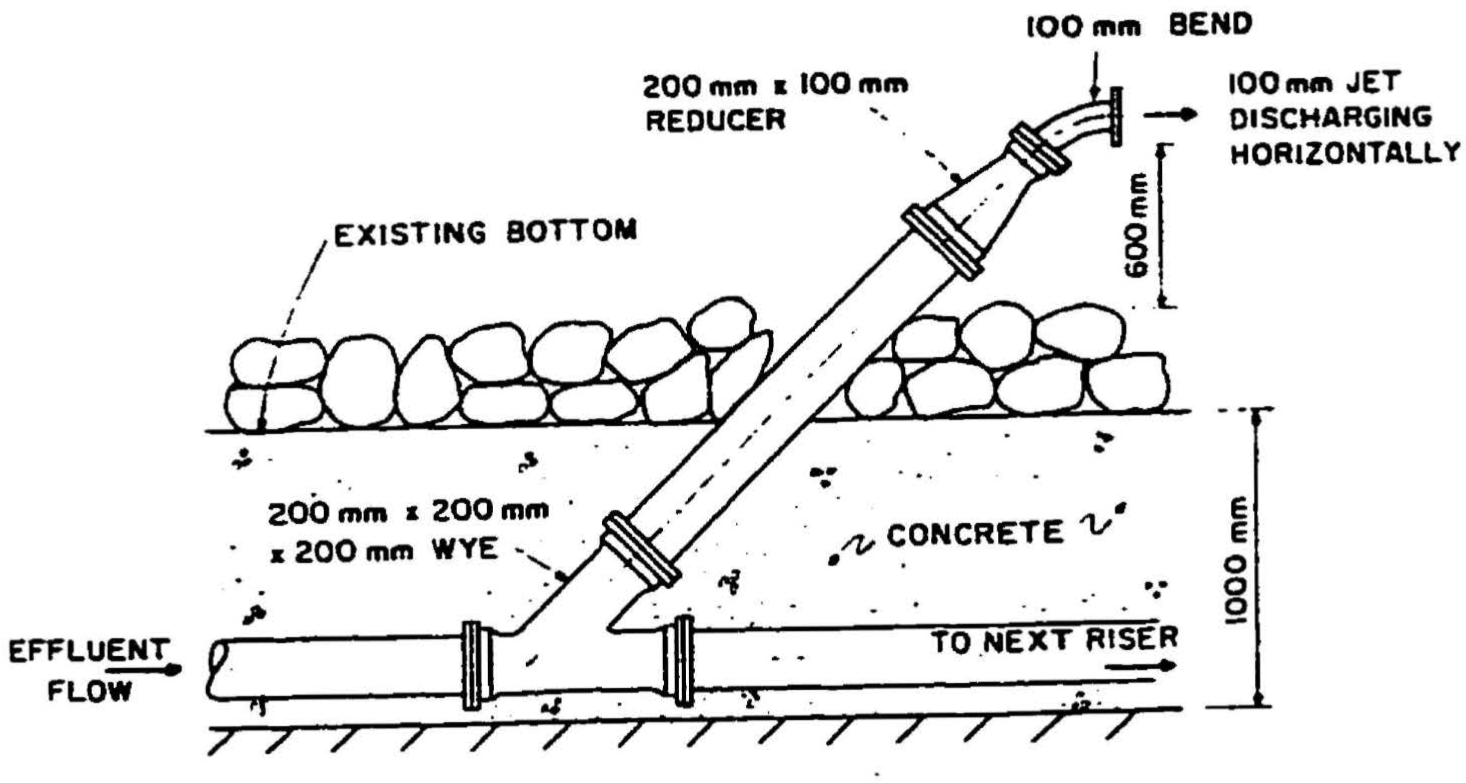

Figure 5.4. Typical Nozzle Arrangement for the Outfall (after Sharp, 1991)

The other important information is that temperature and salinity measurement showed little stratification (Sharp, 1991). On November 1986, Newfoundland and Labrador Consulting Engineers Ltd. (1987) conducted a field study which measured the variation in salinity and temperature with depth. A direct reading salinometer was used and recordings were made from a small boat. Results showed that the seawater at Spaniard's Bay was a homogeneous water mass. For outfall design and analysis purposes, the density difference ratio may then be taken constant, $(\rho_a - \rho_e)/\rho_e = 0.027$ (Sharp, 1994).

Sharp (1994) also reported that although there were slight surface currents driven by the prevailing wind, the main body of receiving water was essentially still. This suggested that it was not necessary to include current effects in the analysis of initial dilution.

Furthermore, it was reported that the observed surface layer of the effluent was very thin outside the boil (Sharp, 1994). This is different from larger outfall discharges which may develop a surface layer to about one third of the total depth above the discharge (see section 2.3.1). This may be due the small discharge involved (Sharp, 1994). More detailed descriptions of the outfall may be found elsewhere (Newfoundland and Labrador Consulting Engineers Ltd., 1987; Sharp, 1989-c; and Sharp1991)

5.3. Deterministic Models for the System and Their Parameters

As indicated in the previous section, the main-body of seawater at the Spaniard's Bay Outfall was essentially still water. Therefore, Cederwall's (1968) model could be used to analyze initial dilution. For this outfall, Sharp(1994) suggested using equation 2.2:-

$$S_o = 0.54\,F\left[\frac{0.38\,Y}{F\,D} + 0.66\right]^{5/3} \qquad \text{for } Y/D > 0.5\,F \qquad (2.2)$$

$$\text{in which } F = \frac{4\,Q}{3.14\,D^{5/2}}\left(\frac{g(\rho_a - \rho_e}{\rho_e}\right)^{-1/2} \qquad \text{(all are in metric units)}$$

It is noted here that equation 2.2 uses D to represent the outfall diameter. However, the Spaniard's Bay Outfall involves two nozzles discharging wastewater horizontally so that D is used to represent diameter of each nozzle, and Q is the wastewater flowing in each nozzle (Gowda, 1992). Therefore, the measured wastewater flow rate should be divided by two before use in the Cederwall's model.

Furthermore, as discussed previously, the observed surface layer of effluent was very thin outside the boil (Sharp, 1994), and it is therefore considered to be negligible. For this reason, Y in equation 2.2 is taken as the total seawater depth available above the discharge. Typically this would be 4.5 m, the distance from the outfall port to lowest normal water (LNW), plus tidal height above LNW.

The other model necessary for the analysis is that for estimating the effluent concentration after the outfall spreads on the surface. This is given in equation 2.29:-

$$C_x = \frac{C_e}{S_i} e^{\frac{-2.3x}{uT_{90}}} erf\left(\sqrt{\frac{1.5}{\left(1 + 8\frac{k_a x}{ub^2}\right)^3 - 1}}\right) \tag{2.29}$$

where $S_i = S_o$ (equation 2.2), and u is the speed of the surface current (m/s).

Using these two models, from Chapter Three (section 3.2.2), the uncertain parameters involved in the analysis are:-

(1) Wastewater flow rate, Q

(2) Tidal variation above LNW, h

(3) Surface current driven by the prevailing wind, u

(4) Decay parameter, T_{90}

(5) Effluent concentration before discharged into seawater, C_e

This study was unable to obtain any data which could be used to an uncertainty analysis of other parameters. This includes the estimations of the value of b (initial width of waste-field), and k_o (diffusion coefficient at x = 0). As discussed in section 2.4.1, for the purpose of the case study b is taken as one third of the depth (Sharp, 989-a), and $k_o = 0.0005\ b^{4/3}$ (in metric units) (Williams, 1985; Markham, 1993). Similar problems were encountered in taking into account the uncertainty of the model coefficients (i.e. 0.54, 0.38, and 0.66 in equation 2.2). For this reason, the model itself is treated as deterministic with no variation in the coefficients. Probabilistic analysis then uses the deterministic models (i.e. equation 2.2 and 2.9) with variability in the five variable listed in the previous page, i.e. Q, h, u, C_e, and T_{90}.

5.4. Sample Moments and Probability Distribution of the Parameters

The available data for the Spaniard's Bay were analyzed to calculate the sample moments and the probability distribution of the parameters. Data for wastewater flow for the Spaniard's Bay Outfall were analyzed in section 3.2.3, and the results are summarized in Table 5.1. In the table, analyzed results (as will be discussed in the following paragraphs) for other parameters are also shown.

Values of the decay parameter T_{90} are assumed to be the same as those measured near St. John's, the capital city of the province. St. John's waters are geographically similar to the location of the outfall. Using data from Thoms (unpublished), the decay parameter was found to be log-normally distributed. This distribution is almost the same as that of other data from other sources (see table 3.1). Data statistics are summarized in table 5.1. To conform with the field test data (Sharp, 1989-c), only data obtained during summer period are used in the analysis.

Table 5.1. Parameters for Analysis of the Spaniard's Bay Outfall

Parameter	mean (stdev)	Distribution	Source(s) of Data for Analysis
Wastewater flow rate (m^3/s)	0.00691 (0.0012)	Power-normal (λ= -1.1, μ_Y = -219.95, σ_Y=37.73)	Data from Sharp (1989-c)
Surface currents: • magnitude (m/s)	0.016 (0.016)	Exponential (θ = 62.5)	Assumption based on Huang (1994)
• direction (to offshore or not)	1.1394 (1.0674)	Poisson (np=λ=1.1394)	Data from Newf. & Lab. Cons. Eng., Ltd (1987)
Tide (above LNW) (m)	0.70 (0.404)	Uniform (a = 0, b = 1.4)	Assumption based on Huang (1994)
Decay parameter, T_{90} (hours)	4.7 (0.997)	Log-normal (μ_Y = 1.527, σ_Y=0.196)	Data from Joe Thoms (not published)
Effluent concentration (per 100 ml)	8.4×10^6 (2.1×10^6)	Log-normal (μ_Y = 15.913, σ_Y=0.246)	The mean is from Newf. & Lab. Cons. Eng., Ltd (1987)

Bacterial concentration in the sewage before discharge into seawater is, in general, log-normally distributed. The mean and standard deviation of the concentration depend on the degree of the treatment of the wastewater (Webb, 1987). Newfoundland and Labrador Consulting Engineers, Ltd (1987) reported concentrations of 8.4 x 10^6 and 4.5 x 10^6 per 100 ml for total and faecal coliforms respectively at raw sewage in the wet well of the North Cove sewage treatment plant, which was replaced by the Spaniard's Bay outfall. No statistical information was available for this data, however. For this reason, the characteristics of the effluent concentration before entering seawater were assumed to be the same as those given in table 5.1.

Surface current data were obtained from Newfoundland and Labrador Consulting Engineers Ltd. (1987) who conducted field studies at Spaniard's Bay on 25th - 27th November 1986. Drogues consisted of four aluminum vanes, each approximately 450 mm x 600 mm set at right angles to each other. These were suspended by rope from a surface float with a marker flag attached and were used to measure the current. Unfortunately, detailed current data, which could have been statistically analyzed, were not available in publications. The only information was that the surface current was of the order of 100 to 200 meters per hour (Newfoundland and Labrador Consulting Engineers Ltd., 1987; and Sharp, 1991).

To define the distribution of the magnitude of the surface current, it was therefore necessary to assume that the current fitted some distribution model. For approximation purposes, the surface current at Spaniard Bay Outfall site was assumed to be exponentially distributed with a mean of 0.016 m/sec. This assumption was taken by considering: a) the current value is always positive; b) although the probability is very small, there is a probability that the ratio of current relative to the mean could be large; c) Exponential distribution is a special case of the Weibull distribution which is known to be the best fit in a similar case, i.e. Miami-Central Outfall (Huang, 1994).

It was also reported that the direction of the surface current followed that of the prevailing wind direction which was offshore for approximation of 68 % of the time (Newfoundland and Labrador Consulting Engineers Ltd., 1987; Sharp 1989-c). As discussed in subsection 3.2.2, the problem in determining the distribution of current direction can be simplified by considering only whether, or not, the effluent discharged by the outfall would be going to the location of interest (Webb, 1987; Sharp, 1989-c). The Poisson distribution (equation 3.3) is then applicable to the case under investigation.

As an example, if in this case study the location of interest or target area is the point 5 with boil location at the point 7 (see figure 5.2), it is then important to know whether, or not, the current is going to the shore (point 5). Assuming that the currents have the same direction as that of the wind direction (Sharp, 1989-c), 68 % time the surface current is offshore, and otherwise is going to the shore (32%). For the Poisson distribution, if the probability of the current going offshore is $p(x,\lambda)$, then the probability of the current going onshore is $p(0,\lambda)$. From equation 3.3 with $p(0,\lambda) = 0.32$, λ is then equal to 1.1394.

Tide data for the location were not available. However, Fisheries and Oceans Canada (1997) provides tide tables which estimate the calculated data for tidal height at Bell Island, the area closest to Spaniard's Bay. As shown by Godin(1972), methods of calculating such a table are usually deterministic, and in fact, data provided by the table seem to have a bimodal distribution (figure 5.5).

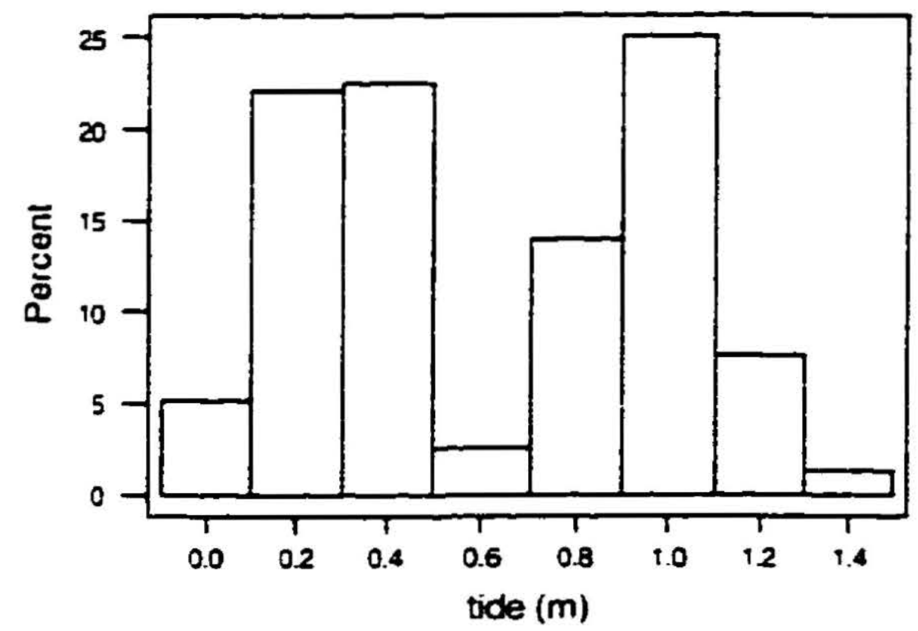

Figure 5.5. Typical Calculated Tide for The Bell Island
(from Fisheries and Oceans Canada, 1997)

These data were not used for the analysis of the Spaniard Bay Outfall. The reason was that the calculated data in the table are only an approximation, and it is not easy to analyze such a bimodal distribution. An analysis should be performed only if there is some profit to be gained by doing it. In other words, a complicated analysis which is not applicable to the problem is not of interest.

In encountering similar problems, Huang (1994) estimated that the distribution of tidal height for the Miami-Central Outfall was uniform with a standard deviation of 0.3 m. This study assumes that the distribution provided from the similar outfall site (Miami-Central

Outfall) may be used as a guide to estimate the distribution of tidal height in Spaniard's Bay. For this reason, the distribution of tidal height, h, above LNW is assumed to be uniform from 0. m to 1.4 m.

5.5. Calculating Probability of Failure

Using the procedure given in Chapter Three, a performance function for the analysis may be set up in terms of initial dilution S_o or effluent concentration in vicinity of the outfall discharge C_x, i.e.:-

$$Z_o = 0.54\, F \left[\frac{0.38\, Y}{F\, D} + 0.66 \right]^{5/3} - T_o \tag{5.1}$$

and

$$Z_c = T_c - \frac{C_e}{S_o} e^{\frac{-2.3x}{uT_{90}}}\, erf\left(\sqrt{\frac{1.5}{\left(1 + 8\frac{k_o x}{ub^2}\right)^3 - 1}} \right) \tag{5.2}$$

where Z_o and Z_c are the performance functions for the analysis of the initial dilution and effluent concentration respectively. T_o and T_c are the threshold levels for the analysis of the initial dilution and effluent concentration respectively.

5.5.1. Initial Dilution

During the design of the outfall, efforts were extended to achieve an initial dilution of about 30 (Gowda, 1992; Newfoundland & Labrador Consulting Engineers, Ltd., 1987). In a probabilistic analysis, this value may be used as the threshold level and the importance of the analysis is to estimate the probability of failure, i.e. the probability that the actual initial dilution will be less than 30. In general, the probability of failure is the probability that the performance function defined in equation 3.26 will have a value less than or equal to zero.

The calculation of probability of failure is performed using the following probabilistic methods:-

(1) First Order Second Moment (FOSM)

(2) Improved Mean FOSM (IM-FOSM), i.e. FOSM with second order of mean

(3) Advanced First Order Second Moment (AFOSM)

(4) Monte Carlo Simulations (MCS)

- ***Initial Dilution using FOSM***

If the performance function Z_0 is assumed to be normally distributed, the probability of failure (i.e. the probability of initial dilution being less than 30) for the existing ocean outfall can be calculated using equation 4.9 with $T_L = 30$:-

$$P_f(30) = 1-\phi(\beta) \tag{5.3}$$

where $\beta = \dfrac{E_1(Z_o)}{\sqrt{\mathrm{Var}(Z_o)}}$ and Z_o is the performance function defined in equation 5.1.

From equation 4.18 and 4.19, the solution for β can be obtained. Rewriting equation 4.18 and 4.19 with threshold level $T_o = 30$:-

$$E_1(Z_o) = 0.54\ \mu_F \left(\frac{0.38\mu_Y}{\mu_F D} + 0.66 \right)^{5/3} - 30 \tag{5.4}$$

$$\mathrm{Var}(Z_o) = \zeta_1 + \zeta_2 \tag{5.5}$$

where $\zeta_1 = \left(0.36 \dfrac{\mu_Y}{D\,\mu_F} + 0.66 \right)^{4/3} \dfrac{0.1169\,\mathrm{Var}(Y)}{D^2}$, and

$$\zeta_2 = \left(0.38 \frac{\mu_Y}{D\mu_F} + 0.66 \right)^{10/3} 0.2916\,\mathrm{Var}(F) + \left(0.38 \frac{\mu_Y}{D\mu_F} + 0.66 \right)^{4/3} \frac{0.1169\,\mu_Y^2\,\mathrm{Var}(F)}{D^2\mu_F^2}$$

Using the values given in table 5.1, the solutions for the existing outfall are:-

$E_1(Z_o) = 16.4330$

$\mathrm{Var}(Z_0) = 33.4419$

$\beta = 2.8417$

$\phi(2.8417) = 0.997756$ (interpolated from standard normal integral, which can be found in statistical books, or from Appendix A).

$P_f(30) = 1-\phi(2.8417)$

$P_f = 0.002244$, or $P_f = 0.2244$ %.

If the initial dilution is assumed to be log-normally distributed, the reliability index is $\beta = -[\ln(T_o) - \mu_Y]/\sigma_Y$. The minus sign is to satisfy that the failure occurs when initial dilution is less than T_o. Using equations given in table 3.2 (page 64), it is found that:-

Mean of $\ln(S_0) = \mu_Y =$ 3.8303

Standard deviation of $\ln(S_o) = \sigma_Y =$ 0.1241

$$\beta = -\frac{[\ln(30) - 3.8303]}{0.1241} = 3.4577$$

$\phi(3.4577) =$ 0.999729

$P_f = 1 - 0.999729 = 0.000271$

or $P_f =$ 0.0271 %

If the problem involves the design a new ocean outfall, the designer may be interested in the relationship between some parameter of interest, e.g. nozzle diameter or mean seawater water level, and the probability of failure. As an example, the calculation has been repeated using the same procedure as above but for calculating the probability of failure as a function of nozzle diameter. Calculated results are shown in Table 5.2.

For constructing a cumulative distribution of initial dilution, the probability of the dilution being less than a specified value is of interest. For this purpose, the probability of failure has also been calculated by varying the threshold level, and calculated results are shown in Table 5.3.

Table 5.2. Calculated Results of FOSM by Varying Nozzle Diameter

(Case of initial dilution with threshold level $T_o = 30$)

Nozzle Diameter (m)	Probability of Failure (%)	
	Zo assumed normal	So assumed lognormal
0.05	0.0002	0.0000
0.10	0.2244	0.0271
0.15	0.8340	0.2441
0.20	1.3777	0.5369

Table 5.3. Calculated Results of FOSM by Varying Threshold Level

(Case of initial dilution with nozzle diameter = 0.1 m)

Threshold level (To)	Probability of Failure (%)	
	Zo assumed normal	So assumed lognormal
20	0.0002	0.0000
25	0.0105	0.0000
30	0.2244	0.0271
35	2.4018	1.3334
40	13.2978	12.7138
45	40.2143	42.4400
50	73.1322	74.4923
55	93.0755	92.3186
60	99.0513	98.3339
65	99.9338	99.7226

- ***Initial Dilution using IM-FOSM***

This method is essentially the same as FOSM, except the mean of the performance function is calculated up to the second order approximation. The solution for the mean is given by equation 4.20. Recall equation 4.20 with threshold level $T_0 = 30$:-

$$E_2(Z_o) = \left(\frac{0.263\mu_Y}{\mu_F^{2/5} D} + 0.456\mu_F^{3/5} \right)^{5/3} - 30 + \frac{0.04332\left(\frac{\sigma_Y^2}{\mu_F} + \frac{\mu_Y^2 \sigma_F^2}{\mu_F^3} \right)}{D^2 \left(\frac{0.38\mu_Y}{\mu_F D} + 0.66 \right)^{1/3}} \tag{5.6}$$

If Z_o is assumed to be normally distributed, using the values given in table 5.1, the solutions for the existing outfall are:-

$E_2(Z_o) = 16.2845$

$Var(Z_o) = 33.4419$

$\beta = 2.8160$

$\phi(2.8160) = 0.997569$ (interpolated from standard normal integral, which can be found in statistical books, or from Appendix A).

$P_f(30) = 1 - \phi(2.8160)$

$P_f = 0.002431$, or $P_f = 0.2431$ %.

If the initial dilution is assumed to be log-normally distributed:-

Mean of $\ln(S_0) = \mu_Y =$ 3.8271

Standard deviation of $\ln(S_o) = \sigma_Y =$ 0.0155

$$\beta = -\frac{[\ln(30) - 3.8271]}{0.1245} = 3.4209$$

$\phi(3.4209) = 0.999729$

$P_f = 1 - 0.999689 = 0.000311$

or $P_f = 0.0311$ %

The relationship between nozzle diameter and the probability of failure calculated using IM-FOSM are shown in Table 5.4. Using the same procedure, cumulative distribution of initial dilution is developed by varying the threshold level as shown in Table 5.5.

Table 5.4. Calculated Results of IM-FOSM by Varying Nozzle Diameter
(Case of initial dilution with threshold level $T_o = 30$)

Nozzle Diameter (m)	Probability of Failure (%)	
	Zo assumed normal	So assumed lognormal
0.05	0.0003	0.0000
0.10	0.2431	0.0311
0.15	0.8964	0.2734
0.20	1.4754	0.5958

Table 5.5. Calculated Results of IM-FOSM by Varying Threshold Level

(Case of initial dilution with nozzle diameter = 0.1 m)

Threshold level (To)	Probability of Failure (%)	
	Zo assumed normal	So assumed lognormal
20	0.0003	0.0000
25	0.0116	0.0001
30	0.2431	0.0311
35	2.5507	1.4511
40	13.8575	13.3440
45	41.2109	43.4900
50	73.9725	75.2530
55	93.4110	92.6553
60	99.1148	98.4126
65	99.9395	99.7370

- ***Initial Dilution using AFOSM***

The standardized procedure requires the conversion of all parameters involved to a standardized form (see equation 4.52). The standardized form of the Froude number, y_F, and the tide, y_h, are then defined as:-

$$y_F = \frac{F - \mu_F}{\sigma_F} \quad \text{and} \quad y_h = \frac{h - \mu_h}{\sigma_h} \tag{5.7}$$

The performance function, Z_o, given in equation 5.1 becomes:-

$$Z_o = h(y) = 0.54\left(\sigma_F y_F + \mu_F\right)\left[0.38\frac{Y_{LNW} + \sigma_h y_h + \mu_h}{D\left(\sigma_F y_F + \mu_F\right)} + 0.66\right]^{5/3} - 30 \qquad (5.8)$$

in which the mean μ_h and standard deviation σ_h of tide are obtained from table 5.1. The mean μ_F and standard deviation σ_F of Froude number are obtained using equation 4.16 and 4.17 along with Table 5.1. Y_{LNW} is the distance between the port of the outfall to the surface for lowest normal water, and for the Spaniard's Bay is 4.5 m.

The partial derivatives of the performance function to y_h and y_F are:-

$$h'(y_h) = \frac{0.342\,\sigma_h}{D}0\left[0.38\frac{Y_{LNW} + \sigma_h y_h + \mu_h}{D\left(\sigma_F y_F + \mu_F\right)} + 0.66\right]^{2/3} \qquad (5.9)$$

and

$$h'(y_F) = 0.54\,\sigma_F\left[0.38\frac{Y_{LNW} + \sigma_h y_h + \mu_h}{D\left(\sigma_F y_F + \mu_F\right)} + 0.66\right]^{5/3}$$

$$-\left(\frac{0.342\,(Y_{LNW} + \sigma_h y_h + \mu_h)\,\sigma_F}{D\left(\sigma_F y_F + \mu_F\right)}\right)\left[0.38\frac{Y_{LNW} + \sigma_h y_h + \mu_h}{D\left(\sigma_F y_F + \mu_F\right)} + 0.66\right]^{2/3} \qquad (5.10)$$

The iteration procedure can then now be performed. For a threshold value $T_o = 30$ and nozzle diameter 0.1 m, the typical iteration results (using EXCEL) are given in Table 5.6. As can be seen in Table 5.6, the iteration converges after the fourth iteration, and the probability of failure is 0.0731 %.

Table 5.6. Typical Iteration Results using AFOSM

(assumed normally distributed parameters)

y_F	y_h	β	Z_o=h(y)	h'(yF)	h'(yh)	μ_Z	σ_Z	P_f (%)
0.0000	0.0000	0.0000	16.116	-0.6815	5.4714	0.0000	5.5137	-
0.3613	-2.9007	2.9215	1.2110	-0.4238	4.6615	13.674	4.6807	0.1742
0.2879	-3.1671	3.1802	0.0084	-0.4049	4.5925	14.661	4.6104	0.0736
0.2794	-3.1697	3.1820	0.0000	-0.4049	4.5924	14.670	4.6103	0.0731
0.2794	-3.1697	3.1820	0.0000	-0.4049	4.5924	14.670	4.6103	0.0731
0.2794	-3.1697	3.1820	0.0000	-0.4049	4.5924	14.670	4.6103	0.0731

If the parameters involved are known to be non-normally distributed, the mean and standard deviation should be replaced with the mean and standard deviation of the normalized parameters (Smith, 1986) as discussed in section 4.3.2. Recall equation 4.66 and 4.67:-

$$\sigma_i^N = \frac{f^N\left[\phi^{-1}\left(F_{xi}(x_i^*)\right)\right]}{f_{xi}(x_i^*)} \tag{4.66}$$

$$m_i^N = x_i^* - \phi^{-1}\left(F_{xi}(x_i^*)\right)\sigma_i^N \tag{4.67}$$

Here σ_i^N is the standard deviation of the normalized parameter X_i, m_i^N is the mean of the normalized parameter X_i, $F_{xi}(x_i^*)$ is the cumulative probability of X_i at x_i^*, $f_{xi}(x_i^*)$ is the probability density of X_i at x_i^*, $\phi^{-1}(a)$ is the inverse normal distribution of a, and $f^N(a)$ is the standardized normal density function of a.

Table 5.1 shows that the distribution of the distribution of the tidal height above LNW is uniform (a = 0, b =1.4) with a mean and standard deviation of 0.7 and 0.404, respectively. From Table 3.2 the probability density function and cumulative distribution for uniform distribution is given by (all are in metric units):-

$$f_x(x) = \frac{1}{b-a} = \frac{1}{1.4-0} = 0.7143 \tag{5.11}$$

$$F_x(x) = \frac{x-a}{b-a} = 0.7143\,x \tag{5.12}$$

In equation 5.12, for x = 0.7, then $F_x(0.7) = 0.5$.

From Table of Standard Normal Integral (appendix A), $\phi^{-1}(0.5) = 0$

From Table of Ordinates of Standard Normal Curve (Appendix B), $f^N(0) = 0.3989$

Using equations 4.66 and 4.77 along with these above values, the results are:-

$$\sigma_h^N = \frac{0.3989}{0.7143} = 0.55846$$

$$m_h^N = 0.7 - (0) = 0.7$$

Using the same procedure, the mean and standard deviation of the normalized wastewater flow rate can be obtained, i.e. 0.006302 and 0.000859 respectively. The mean and standard deviation of each nozzle are half of these values. The mean and standard deviation of the Froude number are calculated using equations 4.16 and 4.17, and the results are 2.4639 and 0.0549 respectively.

Having obtained the mean and standard deviation of the normalized parameters, the iteration can now be performed. Table 5.7 shows typical iteration results using AFOSM with non-normal parameters. As can be seen in the table, for the case of non-normal parameter the iteration also converge after the fourth iteration, and the probability of failure is 0.5449 %.

Table 5.7. Typical Iteration Results using AFOSM
(non-normal parameter distribution as given in Table 5.1)

y_F	y_h	β	$Z_o = h(y)$	$h'(y_F)$	$h'(y_h)$	μ_Z	σ_Z	$P_f(\%)$
0.0000	0.0000	0.0000	18.7756	-0.5870	8.0667	0.0000	8.0880	
0.1685	-2.3153	2.3209	1.5095	-0.3509	6.7630	15.7175	6.7721	1.0146
0.1318	-2.5404	2.5438	0.0140	-0.3314	6.6349	16.8990	6.6432	0.5482
0.1270	-2.5428	2.5459	0.0000	-0.3312	6.6339	16.9105	6.6422	0.5449
0.1270	-2.5428	2.5459	0.0000	-0.3312	6.6339	16.9105	6.6422	0.5449
0.1270	-2.5428	2.5459	0.0000	-0.3312	6.6339	16.9105	6.6422	0.5449

The calculated probabilities of failure as a function of nozzle diameter and threshold level are shown in Table 5.8 and 5.9 respectively. The tables show the calculated results for both cases of normal and non-normal parameters.

Table 5.8. Calculated Results of AFOSM by Varying Nozzle Diameter

(Case of initial dilution with threshold level T_o = 30)

Nozzle Diameter (m)	Probability of Failure (%)	
	normal parameters	non-normal parameters
0.05	0.000	0.004
0.10	0.073	0.545
0.15	0.409	1.404
0.20	0.778	2.019

Table 5.9. Calculated Results of AFOSM by Varying Threshold Level

(Case of initial dilution with nozzle diameter = 0.1 m)

Threshold level (To)	Probability of Failure (%)	
	normal parameters	non-normal parameters
20	0.000	0.002
25	0.001	0.044
30	0.073	0.545
35	1.650	3.463
40	12.662	12.956
45	41.932	31.759
50	75.558	55.986
55	93.948	77.357
60	99.124	90.803
65	99.924	97.044

• ***Initial Dilution using MCS***

The procedure of the simulations is shown in figure 4.1 and a typical Macro program is shown in Appendix C. Unlike in the other methods, MCS is flexible and results can be to presented either in terms of cumulative distribution or probability distribution of initial dilution. The simulations were performed using MINITAB software and for the existing outfall the mean and standard deviation of the simulated initial dilution are 46.9 and 6. 84 respectively. Probability of the initial dilution being less than 30 is 0.1310 %. The distribution of initial dilution from MCS is shown in figure 5.6.

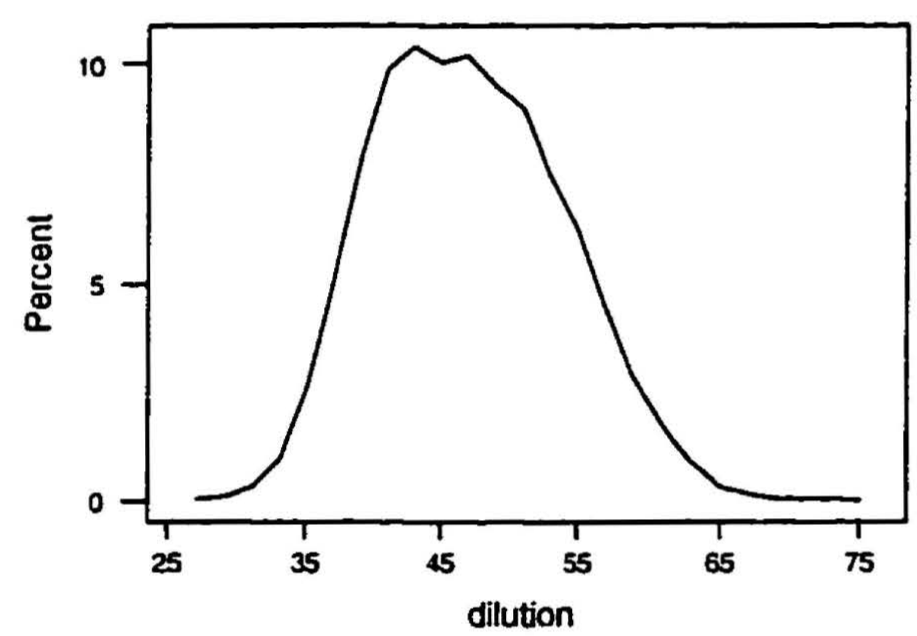

Figure 5.6. Distribution of Initial Dilution from MCS [using the simulated sewage flow based on the data given in Sharp (1989-c)]

The calculated results given by MCS, FOSM, IM-FOSM, and AFOSM are then depicted together in Figure 5.7 and 5.8.

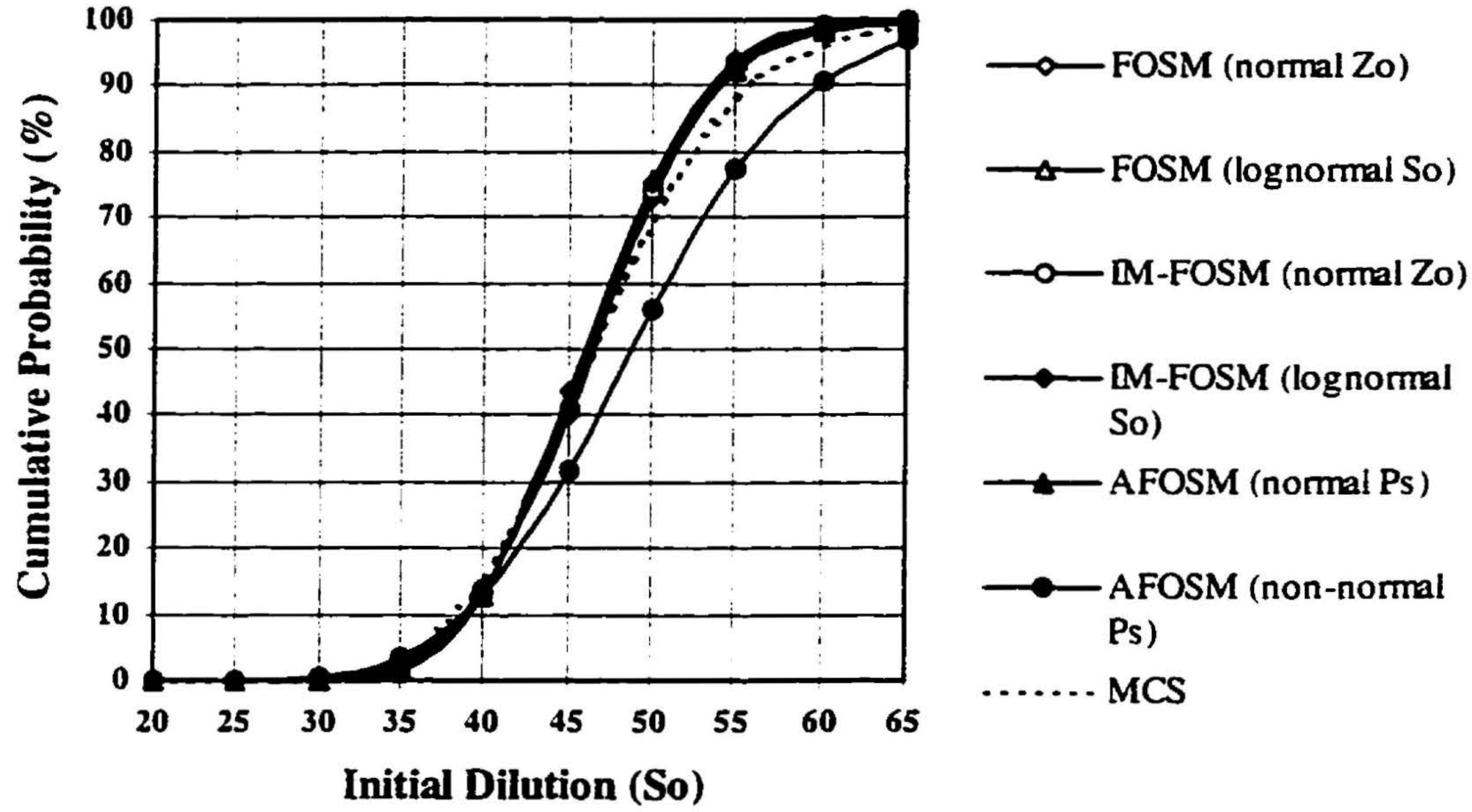

Figure 5.7. Cumulative Distribution of Initial Dilution

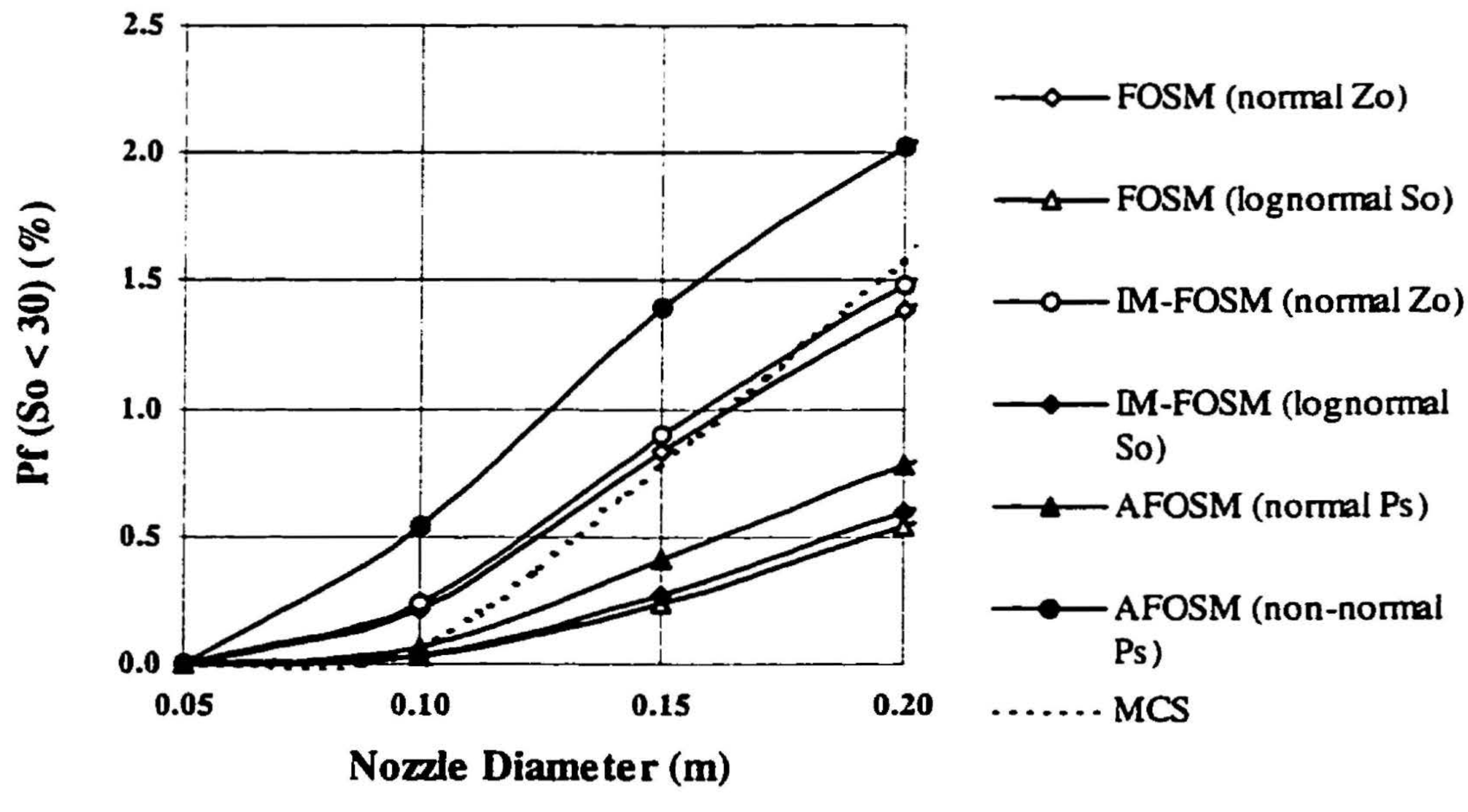

Figure 5.8. Probability of Failure (%) vs Nozzle Diameter (m)

5.5.1. Effluent Concentration at a Target Area

In this study, interest is in the concentration of total coliforms. Because of limited information about the use of the area near the outfall discharge, the location of interest is taken as 100 m onshore. This distance is used only for example purposes. The current which causes the movement of the effluent to this location is an onshore current which occurs for about 32 % of the time (from section 5.4).

If the standard for total coliforms is taken from Dept. of Environment (Canadian), the threshold level T_c for recreational water is then 500 per 100 ml (Newfoundland & Labrador Consulting Engineers, Ltd, 1987). Failure thus occurs when the concentration of total coliforms in that area is higher than 500 per 100 ml. As the Canadian regulation does not provide a probabilistic standard, the critical probability of failure cannot be specified using the standard. The probability of failure calculated in this study is used as an example only.

Indeed, the probability of failure can be calculated for different alternatives. For example, the effect of different distance between the outfall discharge and the location of interest can be compared during the design process. In this study, probability of failure was calculated by varying the distance between the outfall discharge and the location of interest. Furthermore, the exceedance probability, i.e. the probability that total coliform concentration exceeds a specified value was also calculated by varying the threshold level.

• ***Effluent Concentration using FOSM***

If the performance function Z_c (equation 5.2) is assumed to be normally distributed, the probability of failure (i.e. the probability of total coliform concentration exceeding 500 per 100 ml) for the existing ocean outfall can be calculated using equation 4.9 with T_L = Tc. For $T_c = 500$:-

$$P_f(500) = 1-\phi(\beta) \tag{5.13}$$

where $\beta = \dfrac{E_1(Z_c)}{\sqrt{Var(Z_c)}}$.

From equation 4.40 and 4.41, the solution for β can be obtained. Rewriting equation 4.40 and 4.41 with threshold level $T_c = 500$:-

$$E_1(Z_c) = 500 - \left[\frac{\mu_{C_e}}{\mu_{S_i}} e^{\left(\frac{-2.3x}{\mu_u \mu_{T90}}\right)} erf\left(\sqrt{\frac{1.5}{a_\mu^3 - 1}}\right)\right] \tag{5.14}$$

$$Var(Z_c) = \xi \frac{\mu_{Ce}^2}{\mu_{Si}^2} e^{\left(\frac{-4.6x}{\mu_u \mu_{T90}}\right)} \left(erf\left(\sqrt{\frac{1.5}{a_\mu^3 - 1}}\right)\right)^2$$

(5.15) where $a_\mu = \left(1 + \dfrac{8 k_o x}{\mu_u b^2}\right)$, and ξ is defined as:-

$$\xi = \sigma_{C_e}^2 + \frac{\mu_{C_e}^2 \sigma_{S_1}^2}{\mu_{S_1}^2} + \frac{5.29\,\mu_{C_e}^2 \sigma_{T_{90}}^2}{\mu_u^2 \mu_{T_{90}}^4} + \frac{5.29\,\mu_{C_e}^2 \sigma_u^2}{\mu_u^2}\left(\frac{1}{\mu_u^2 \mu_{T_{90}}^2} + \frac{51.98\,a_\mu^4 k_o^2 x^2 e^{\left(\frac{-3}{a^3-1}\right)}}{\mu_u^2 b^4 \left(a_\mu^3 - 1\right)^3}\right)$$

Using the values given in table 5.1, the solution for the existing outfall are:-

$E_1(Z_c) = -688$

$Var(Z_c) = 7.8886 \times 10^{11}$

$\beta = -0.000000245$

$\phi(-0.000000245) = 0.5$ (interpolated from standard normal integral, which can be found in statistical books, or from Appendix A).

$P_f(500) = 1 - \phi(-0.000000245)$

$P_f(500) = 0.5$

Because the probability of the currents being onshore direction is 32 % or 0.32, the probability of failure is then:

$P_f(500) = 0.5 \times 0.32 = 0.1600$ (or 16.00 %)

If the effluent concentration is assumed to be log-normally distributed, the probability of failure is 0.1728 %, and if exponentially distributed 21.0085 %.

As an example, the calculation was done using the same procedure as the above solution for calculating the probability of failure as a function of the distance (X) between the outfall discharge and the location of interest. It was assumed that the depth of seawater is proportional to X (i.e. constant slope). Typical calculated failure probabilities are shown in Table 5.10.

Table 5.10. Calculated Results (FOSM) by Varying Discharge Distance
(Case of total coliform concentration with threshold level Tc = 500 per 100 ml)

X	Probability of Failure (%)		
(Onshore, m)	normal Z_c	log-normal C_x	Exponential C_x
50	16.000	0.372	29.381
75	16.000	0.247	26.059
100	16.000	0.173	21.009
125	16.000	0.125	14.632
150	15.999	0.092	8.190
175	15.999	0.069	3.326
200	15.999	0.052	0.849
225	15.999	0.040	0.111
250	15.999	0.031	0.006
275	15.999	0.029	0.001
300	15.999	0.019	0.000
325	15.999	0.015	0.000

For developing the exceedance probability of effluent concentration, the probability of failure was calculated by varying the threshold level. Typical calculated results are shown in Table 5.11.

Table 5.11. Calculated Results (FOSM) by Varying Threshold Level
(Case of total coliform concentration with nozzle diameter of 0.1 m)

T_c (per 100 ml)	Probability of Failure (%)		
	normal Z_c	log-normal C_x	Exponential C_x
10	16.000	1.083	31.732
100	16.000	0.389	29.417
200	16.000	0.277	27.043
300	16.000	0.226	24.860
400	16.000	0.194	22.853
500	16.000	0.173	21.009
600	16.000	0.157	19.313
700	16.000	0.144	17.754
800	16.000	0.134	16.321
900	16.000	0.126	15.003
1000	16.000	0.119	13.792
2000	15.999	0.081	5.945

• ***Effluent Concentration using AFOSM***

In the standardized procedure, all parameters involved are converted to a standardized form (see equation 4.52). From the performance function defined in equation 5.2, there are four uncertain parameters involved, i.e, Effluent concentration before discharge into the ocean C_e, initial dilution S_o, decay parameter T_{90}, surface current u.

In the standardized form, the performance function, Z_c becomes:-

$$Z_c = h(y) = 500 - \left[\frac{(\sigma_{Ce} y_{Ce} + \mu_{C_e})}{(\sigma_{So} y_{So} + \mu_{So})} e^{\left(\frac{-2.3x}{(\sigma_u y_u + \mu_u)(\sigma_{T90} y_{T90} + \mu_{T90})} \right)} erf\left(\sqrt{\frac{1.5}{y_a^3 - 1}} \right) \right] \quad (5.16)$$

where $$y_a = \left(1 + \frac{8 k_o x}{(\sigma_u y_u + \mu_u) b^2} \right)$$

and y_{Ce}, y_{SO}, y_{T90}, and y_u are the standardized forms of effluent concentration before discharge into the ocean C_e, initial dilution S_o, decay parameter T_{90}, and surface current u, respectively.

The partial derivatives of the performance functions for each parameter are:-

$$h'(y_{C_e}) = \frac{-\sigma_{Ce}}{(\sigma_{So} y_{So} + \mu_{So})} e^{\left(\frac{-2.3x}{(\sigma_u y_u + \mu_u)(\sigma_{T90} y_{T90} + \mu_{T90})} \right)} erf\left(\sqrt{\frac{1.5}{y_a^3 - 1}} \right) \quad (517)$$

$$h'(y_{S_o}) = \frac{\sigma_{So} (\sigma_{Ce} y_{Ce} + \mu_{C_e})}{(\sigma_{So} y_{So} + \mu_{So})^2} e^{\left(\frac{-2.3x}{(\sigma_u y_u + \mu_u)(\sigma_{T90} y_{T90} + \mu_{T90})} \right)} erf\left(\sqrt{\frac{1.5}{y_a^3 - 1}} \right) \quad (5.18)$$

$$h'(y_{T_{90}}) = \frac{-2.3x\,\sigma_{T90} (\sigma_{Ce} y_{Ce} + \mu_{C_e})}{(\sigma_{So} y_{So} + \mu_{So})(\sigma_{T90} y_{T90} + \mu_{T90})^2} e^{\left(\frac{-2.3x}{(\sigma_u y_u + \mu_u)(\sigma_{T90} y_{T90} + \mu_{T90})} \right)} erf\left(\sqrt{\frac{1.5}{y_a^3 - 1}} \right)$$

(5.19)

$$h'(y_u) = \frac{-2.3\,x\,\sigma_u(\sigma_{Ce}y_{Ce}+\mu_{C_e})}{(\sigma_u y_u+\mu_u)^2(\sigma_{So}y_{So}+\mu_{So})(\sigma_{T90}y_{T90}+\mu_{T90})} e^{\left(\frac{-2.3x}{(\sigma_u y_u+\mu_u)(\sigma_{T90}y_{T90}+\mu_{T90})}\right)} erf\left(\sqrt{\frac{1.5}{y_u^3-1}}\right)$$

$$-\frac{29.3939\,x\,y_u^2 k_o \sigma_u(\sigma_{Ce}y_{Ce}+\mu_{C_e})}{(\sigma_{So}y_{So}+\mu_{So})(\sigma_u y_u+\mu_u)^2 b^2\sqrt{\frac{\pi}{y_u^3+1}}} e^{\left(\frac{-2.3x}{(\sigma_u y_u+\mu_u)(\sigma_{T90}y_{T90}+\mu_{T90})}\right)} e^{\left(\frac{1.5}{y_u^3-1}\right)} \qquad (5.20)$$

If the distribution functions of the parameters involved are not known, it is usually assumed that the parameters are normally distributed. Using this assumption, the iteration procedure given in section 4.3.1 can then be used to solve the problem provided values are available for the means and standard deviations of each parameter in the above equations. In this case, the means and standard deviations of each parameter were taken from Table 5.1, except for those for initial dilution which is calculated using FOSM as discussed in the previous section.

The iteration procedure can be performed using a computer program or a spreadsheet. For threshold T_c = 500, D = 0.1 m, X =100 m onshore with the probability of currents being in an onshore direction of 32%, the typical iteration results (using EXCEL) are given in table 5.12. As can be seen in the table, the iteration converges after the fifth iteration, and the probability of failure, i.e. that total coliform concentration will be more than 500, is 19.72 %.

Table 5.12. Typical Iteration Results using AFOSM
(Case of bacterial concentration with normal parameters, X = 100 m and Tc = 500)

y_u	y_{Ce}	y_{T90}	y_{Si}	β	Z =h(y)	h'(y_u)	h'(y_{Ce})	h'(y_{T90})	h'(y_{Si})	P_f (%)
0.000	0.000	0.000	0.000	0.000	-688.189	-2703.4	-297.0	-214.1	148.665	-
-0.249	-0.027	-0.020	0.014	-0.252	-86.128	-2014.2	-147.5	-141.9	73.210	19.18
-0.292	-0.021	-0.021	0.011	-0.294	-2.710	-1884.9	-126.4	-129.2	62.815	19.70
-0.294	-0.020	-0.020	0.010	-0.296	-0.003	-1881.0	-125.6	-128.8	62.483	19.72
-0.294	-0.020	-0.020	0.010	-0.296	0.000	-1881.0	-125.6	-128.8	62.483	19.72
-0.294	-0.020	-0.020	0.010	-0.296	0.000	-1881.0	-125.6	-128.8	62.483	19.72
-0.294	-0.020	-0.020	0.010	-0.296	0.000	-1881.0	-125.6	-128.8	62.483	19.72

If the parameters involved are known to be non-normally distributed, the means and standard deviations for the parameters involved should be replaced with the mean and standard deviation of the normalized parameters (Smith, 1986) as discussed in section 4.3.2. Using the same procedure as in the case of initial dilution, the means and standard deviations of the normalized parameters were calculated. These are given in Table 5.13.

Table 5.13. Mean and Standard Deviation of the Normalized Parameters for Analysis of the Spaniard's Bay Outfall using AFOSM

Parameter	Parameter with Original Fitted Distribution (Table 5.1)		Normalized Parameters	
	mean	standard dev.	mean	standard dev.
Wastewater flow rate (m^3/s)	0.00691	0.0012	0.00630	0.00086
Surface currents (m/s)	0.01600	0.0160	0.01047	0.01639
Tide (above MWL) (m)	0.70000	0.4040	0.7000	0.55846
Decay parameter, T_{90} (hrs)	4.70000	0.9970	4.6034	0.9212
Effluent concentration (per 100 ml)	8.4×10^6	2.1×10^6	8.141×10^6	2.073×10^6

Having obtained the means and standard deviations of the normalized parameters, iteration can now be performed using the same procedure as case of normal parameters (see Table 5.12). The calculated probabilities of failure, as a function of the distance between the outfall discharge and a target area of interest, are shown in Table 5.14. The table shows the calculated results for both normal and non-normal parameters.

Table 5.14. Calculated Results (AFOSM) by Varying Discharge Distance
(Case of total coliform concentration with threshold level Tc = 500 per 100 ml)

X	Probability of Failure (%)	
(Onshore, m)	Normal Parameters	Non-normal Parameters
50	23.944	19.824
75	22.001	17.546
100	19.720	15.045
125	17.160	12.440
150	14.431	9.872
175	11.673	7.488
200	9.045	5.406
225	6.688	3.703
250	4.705	2.400
275	3.140	1.469
300	1.985	0.848
325	1.187	0.473

For developing the exceedance probability of effluent concentration, the probability of failure was calculated by varying the threshold level Tc. Table 5.15 shows typical calculated results using AFOSM for both normal and non-normal parameters.

Table 5.15. Calculated Results (AFOSM) by Varying Threshold Level
(Case of total coliform concentration with discharge distance of 100 m)

T_c (per 100 ml)	Probability of Failure (%)	
	Normal Parameters	Non-normal Parameters
2	25.403	21.648
100	23.055	18.775
200	21.985	17.536
300	21.135	16.581
400	20.393	15.767
500	19.720	15.045
600	19.095	14.388
700	18.508	13.781
800	17.951	13.216
900	17.419	12.685
1000	16.909	12.184
2000	12.642	8.287

- ***Effluent Concentration using MCS***

The procedure of the simulations is shown in figure 4.1, and a typical Macro program is shown in the appendix C. The 28,000 simulations were performed using MINITAB software. For the existing outfall, it was found that the mean and standard deviation of the simulated total coliform concentration (per 100 ml) are 2368 and 4537.5 respectively. The probability of bacterial concentration being more than 500 per 100 ml is 15.85 %.

The simulation results are depicted in figure 5.9 an 5.10 a long with a comparison with the calculated results given by the other methods (FOSM and AFOSM).

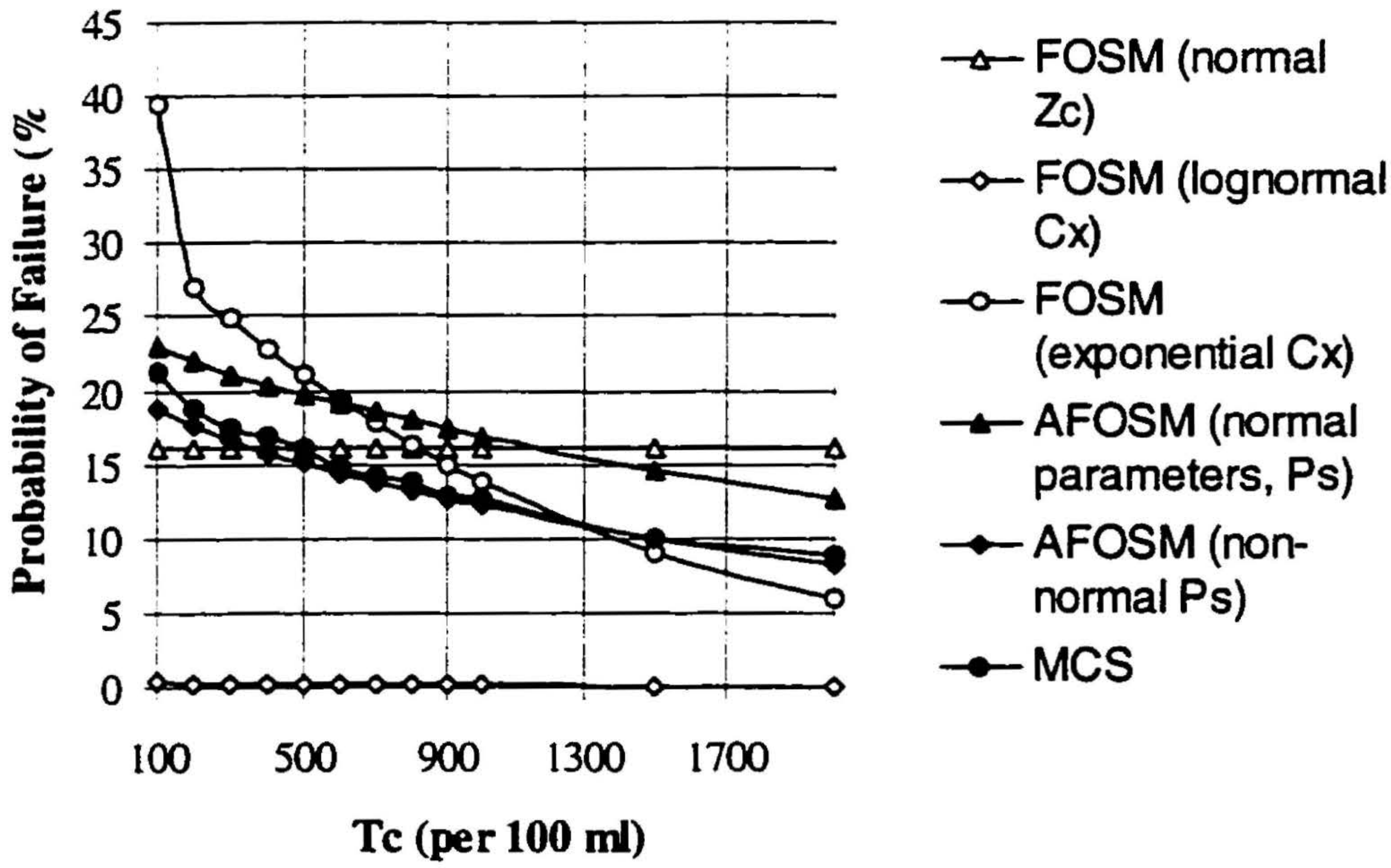

Figure 5.9. Probability of Failure vs Threshold Level, Tc

(Case of bacterial concentration for the existing outfall)

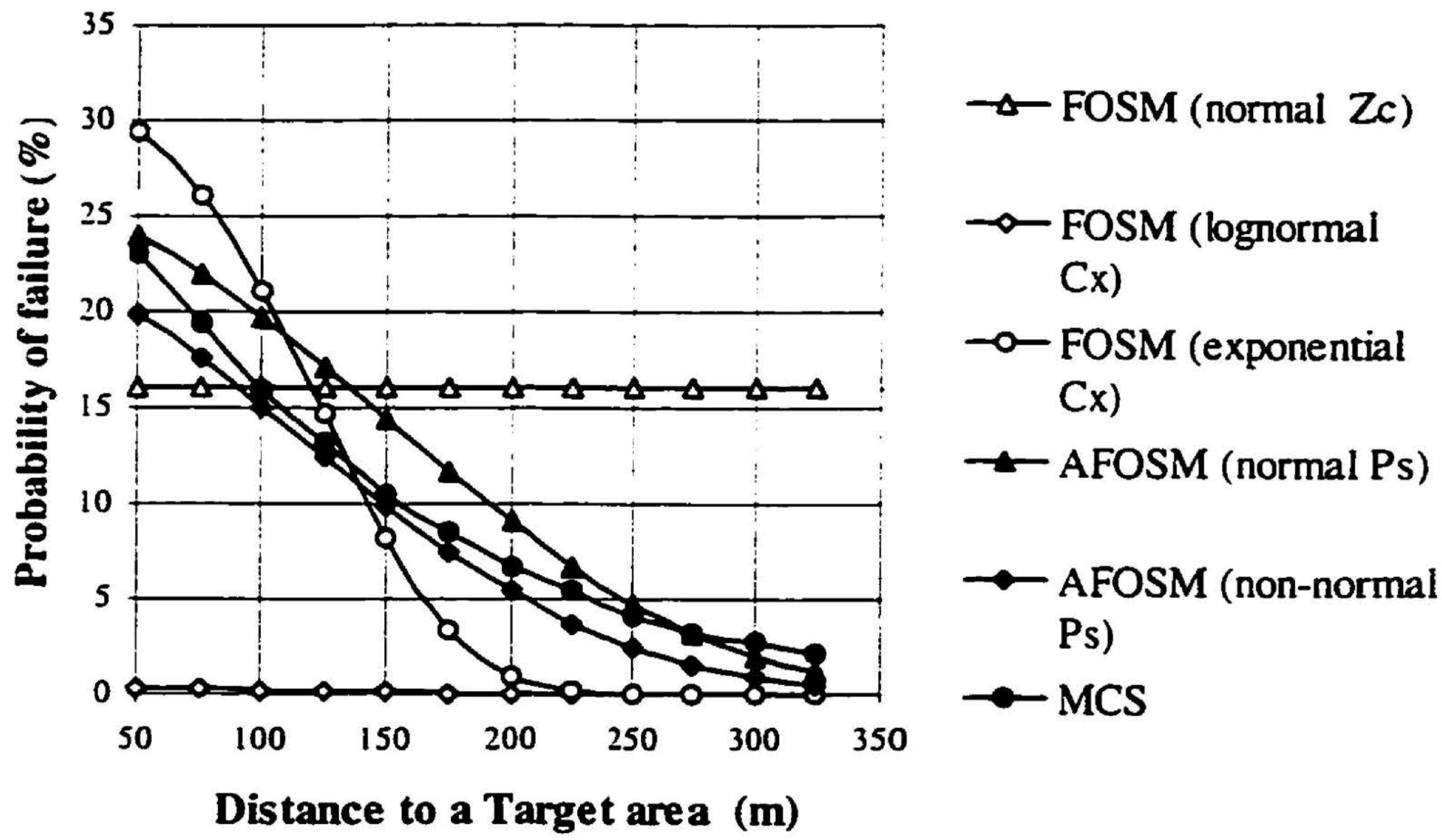

Figure 5.10. Probability of Failure vs Distance from a Target Area

(Case of bacterial concentration with threshold level of 500 per 100 ml)

Chapter 6

Comparison Between

Calculated Results and Available Data

6.1. Introduction

The application of the probabilistic methods for an ocean outfall analysis was shown in Chapter 5. In order to compare the methods in terms of their applicability to a case of interest, a criterion must be provided so that it would be possible to choose a suitable method for that case. Ideally, all calculated results should be compared with field data. When the calculated results given by a probabilistic method and the field data are the same, or when the difference is within an acceptable limit, say 10 %, the method is then applicable for use. When several methods are applicable, the simplest one may be beneficial for use in practice.

In this chapter, calculated results shown in Chapter 5 are analyzed in order to determine the applicability of the probabilistic methods to ocean outfall design and analysis. Available field data obtained from Sharp (1989-c) and Gowda (1992) were used in this analysis. Because data from the deterministic design at Spaniard's Bay Outfall are also available (Newfoundland and Labrador Consulting Engineer Ltd, 1987), a comparison between the deterministic and the probabilistic approach is also given in this chapter to show the advantage of the use of probabilistic approach in ocean outfall design and analysis.

6.2. Field Test Data and Calculated Results

Field data for initial dilution at the Spaniard's Bay Outfall were obtained from dye studies which were a part of the Spaniard's Bay Outfall Monitoring Study (Sharp, 1989-c). In that study, red dye was injected into the pumping station and traced after it left the outfall. Samples were taken in the pump chamber and above the outfall. These were used to calculate initial dilution. The dye studies were conducted on 7 October 1988 and 28 June 1989. Detailed description of the studies can be found in (Sharp,1989-c, Gowda, 1992).

Before injecting dye, the flow into the pumphouse was estimated. Flow measurement was undertaken by recording the time taken for the water level to rise a set distance in the rectangular section of the pumping chamber. Similar measurements were made for the flow out of the chamber under the action of one pump (Sharp, 198-c).

The procedure used in the dye studies was such that the sewage flowing out from the chamber into the outfall pipe was always greater than that flowing into the chamber. Table 6.1 shows a comparison between inflow and outflow rate in the pump chamber during the dye study in 1989.

Table 6.1. Comparison Between Inflow and Outflow of Sewage (liter per sec) in the Pump Chamber During the Dye Studies, 1989 (Gowda, 1992)[1)]

No. Test	Inflow (to the chamber)	Outflow (into outfall)	Excess (%) [(2)]
Morning 1	6.72	11.60	72.62
Morning 2	6.46	11.80	82.66
Afternoon 1	6.80	11.26	65.59
Afternoon 2	5.80	11.45	97.41
Afternoon 3	6.26	11.58	84.98
Afternoon 5	8.03	13.11	63.26
Mean (std dev.)	6.67 (0.76)	11.80 (0.67)	78.15 (13.32)

1). No. test of Afternoon 4 is not shown because the duration of pumping was very short, 74 sec.

2). Excess is calculated using formula: Excess = [(Outflow - Inflow)/Inflow] x 100 %.

Initial data obtained from the dye studies (Sharp, 1989-c) were reanalyzed to determine the distribution of the dilution. The histogram of the data (with sample size of 53) is shown in figure 6.1. The data have a mean and standard deviation of 33.0 and 10.93, respectively.

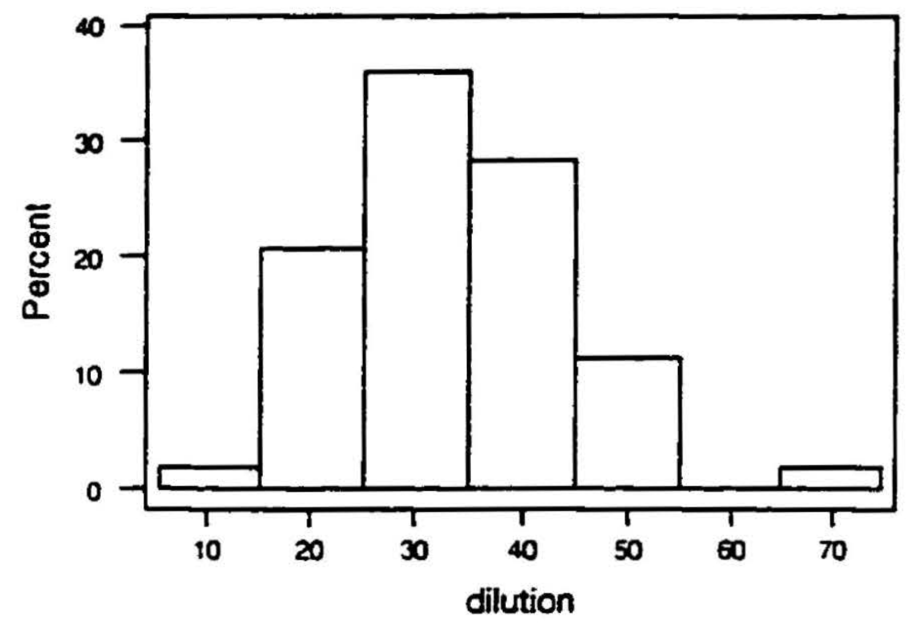

Figure 6.1. Histogram of Initial Dilution from Field Test Data
(average dilution is calculated ignoring extreme values, after Sharp, 1989-c)

Using the procedure given in section 3.2.3, the data may be well fitted with a log-normal distribution. The normal probability plot for log of initial dilution, ln(S), is shown in figure 6.2. At a significance level α of 5% with a sample size of 53, the initial dilution data for the Spaniard's Bay Outfall can be assumed to come from a log-normal distribution, with μ_y = 10.93 and $\sigma_y = 0.3345$.

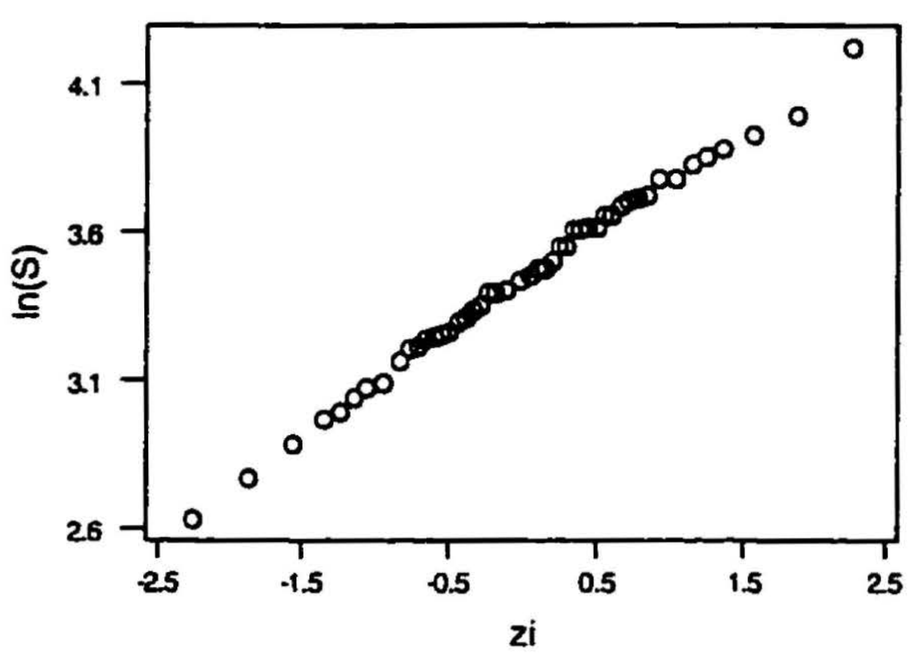

Figure 6.2. Normal Probability Plot for ln (S), with PPCC of 0.997

Good agreement between calculated dilution using the Cederwall's model and the field test data for the Spaniard's Bay Outfall was reported by Sharp (1991) and Gowda (1992). Although there were scattered field test data, the calculated dilution gave a good agreement with the averaged data. Furthermore, Gowda(1992) indicated that there was difficulty to ensure whether the sampling conducted using a boat took samples exactly at the boil of the outfall. As a result, the use of uncertainty analysis without taking into account the uncertainty in the sampling is likely to result in a smaller standard deviation.

The analysis given in Chapter 5 used of sewage flow from (Sharp, 1989-c), which was approximately the same as the sewage flow into the chamber, rather than the sewage flows obtained from the dye studies. This was because the sewage flow recorded during the dye studies was very limited. In fact, only seven data points from the "1989 test" were reported (Gowda, 1992), and this is a very small sample size for further statistical analysis. Furthermore, the case study which used the sewage flow data during normal operation may reflect the actual operation of the outfall.

From Table 6.1, it can be seen that the sewage flowing out from the chamber into the outfall pipe was always greater than that flowing into the chamber with the average excess of 78.15 %. As the greater sewage flow would tend to give less initial dilution (from Cederwall's equation), the dilution given by calculations using the actual sewage flow may be greater than that obtained from the dye studies.

For this reason, the distribution of initial dilution is assumed to be the same as that obtained in the dye studies but with corrected parameters of the distribution. For comparison purposes, the Monte Carlo Simulations (MCS) were performed with input data given in table 5.1. The simulation procedure was essentially the same as that for calculating probability of failure (Figure 4.1) except the interest here was to record possible values from the simulations, and to develop a distribution of these values. A modification of Webb's (1987) procedure was used as follows :-

(1) Generate a value of each parameter involved in accordance with its probability distribution

(2) Calculate initial dilution

(3) Record the calculated values from step 2

(4) Repeat steps 1 to 3 many times to ensure the accuracy of the statistics and probability distribution obtained, typically tens of thousands of simulations (Melching, 1995).

The simulation were performed using a MINITAB software. Initial dilution was found to have a mean and standard deviation of 46.9 and 6.84 respectively. Simulation results are depicted in figure 6.3. As expected, the simulated values are higher than those obtained from the dye studies, which had a mean and standard deviation of 35.18 and 8.41 respectively.

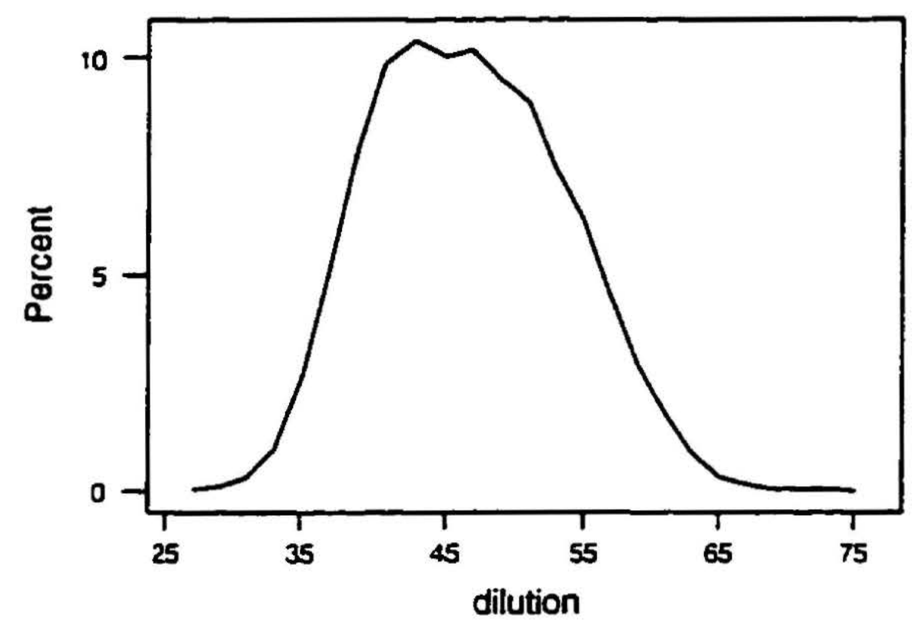

Figure 6.3. Distribution of Initial Dilution from MCS [using the simulated sewage flow based on the data given in Sharp (1989-c)]

Figure 6.3 suggests that the distribution of the simulated initial dilution has a positive skewness. To test the agreement with the distribution of field data, which are log-normally distributed, the simulated data was fitted using log-normal distribution. After taking the log of the simulated dilution, a normal probability plot can be developed (see figure 6.4). The log of simulated data tends to lie on a straight line on the normal probability plot, with correlation coefficient of 0.9983, p-value of 0.0258, $\mu_Y = 3.8394$, and $\sigma_Y = 0.1165$.

From the above discussion, it can be said that the calculation approach using Monte Carlo Simulations (MCS) can reproduce the distribution of initial dilution given by the field test data although distribution parameters for the condition of sewage flow between the two are different. The mean and standard deviation of simulated initial dilution were 46.9 and 6.84, respectively. The simulated values are generally higher than those obtained from the dye studies, which had a mean and standard deviation of 35.18 and 8.41 respectively.

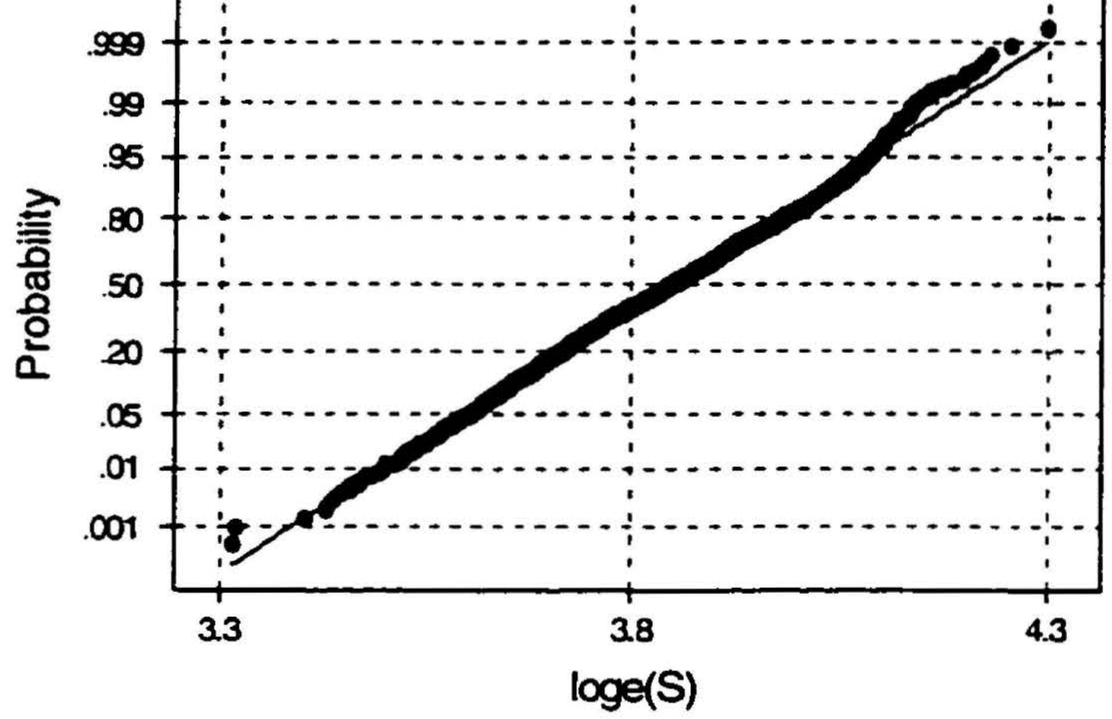

Figure 6.4. Normal Probability Plot for Log of Simulated Initial Dilution

The difference is acceptable because, as above discussion, because of the difference of sewage flow by an excess of 78.15 (see Table 6.1), and because of difficulty in taking samples exactly at the boil of the outfall. Comparisons between the field test data and the results of other probabilistic methods are not given here, but they were compared with MCS as indicated in the discussion (see Chapter 7).

The data available for effluent concentration around the shoreline at the outfall location make it difficult for such comparisons to be done because the data are very limited (Sharp, 1989-c). Furthermore, the MSC have been used evaluating coastline contamination (Orlob and Tumeo, 1986; Webb, 1987; and Balle, et. al, 1990) as discussed in section 4.1. Because of this, and because the MCS method has shown good results as discussed above, it was assumed that the method would be applicable for probabilistic analysis of effluent concentration around the shoreline.

6.3. Deterministic and Probabilistic Approach

Before discussing the calculated results given in previous chapters, it is important to mention here that the calculation of initial dilution and effluent concentration near the outfall discharge is part of the hydraulic design. In the design, requirements other than initial dilution and effluent concentration are also important to consider (Sharp, 1989-a), for example, flushing velocities (0.6 m/s to 1 m/s) in the manifold pipe at least once per day to inhibit settlement of solids, and densimetric Froude number more than 1 to prohibit seawater intrusion. The actual design should also consider other hydraulic factors which may be found elsewhere, e.g. Grace (1978) and Sharp (1989-a).

A deterministic approach for designing an ocean outfall typically uses the average and the peak flow rate during the period of design. For still water conditions, pipe/nozzle diameter and mean seawater level are determined by evaluating initial dilutions for each possible combination of the parameters involved. For example, Table 6.2 shows a typical deterministic approach used in the design process for the Spaniard's Bay Outfall (Newfoundland and Labrador Consulting Engineers Ltd., 1987). It can be seen in Table 6.2 that the values of initial dilution vary depending on extreme values of variables (e.g. wastewater flow rate 1572 m^3/day and 4426 m^3/day for no 1 and 2).

Unlike the deterministic approach, the probabilistic design estimates a full range of possible values of initial dilution and bacterial concentration at locations of interest. Associated probability values are also obtained. All probabilistic methods discussed in the previous chapters provide a cumulative distribution of the output parameter of interest and probability of failure for a given design scenario (see Figures 5.7, 5.8, 5.9, and 5.10).

Table 6.2. Initial Dilution for Each Nozzle using Deterministic Approach*)

No	Depth, Y_o (m)	Flow (m^3/s)	Flow each nozzle (m^3/s)	Froude number	Y_o/D	Initial Dilution
1	7	0.0182	0.0091	6.9	69	44
2	7	0.0512	0.0256	19.4	69	33
3	5	0.0067	0.0035	2.7	50	43
4	5	0.0121	0.0061	4.8	50	33

*) No 1 and 2 are obtained from Newfoundland and Labrador Consulting Engineers Ltd., 1987 where nozzle diameter D is taken 0.102 m. No 3 and 4 are calculated using data from the existing outfall (Sharp, 1989-c), and D = 0.1 m.

For conditions given in the third row of Table 6.2, the deterministic approach gives an initial dilution of 43. This does not give any information on the possibility that this dilution will not be achieved by the outfall. However, a probabilistic design would also estimate the probability of initial dilution being less than 43. From Figure 5.6, probabilistic methods provide a probability, which is about 28 %.

From this example, it is clear that the probabilistic approach is more realistic than the deterministic one in reflecting the uncertainty of the initial dilution. This is particularly useful when the designer realizes that the operational condition of the outfall will vary with time depending on the condition of wastewater and seawater environment. As a result, a fixed value of dilution estimation can be misleading. Furthermore, the field test data of initial dilution usually spread out over a certain range rather than a fixed value (Moore, et. al. 1991), which indicate the uncertainty of the dilution. In this situation, the solutions given by the probabilistic methods are applicable.

In the case study (Chapter 5), the cumulative probability of the initial dilution is typically about 28 %, $P(S_o \leq 43) = 0.28$. In other cases, however, $P(S_o \leq 43)$ may not be the same. This is because of jet geometry, characteristics of the effluent and the ambient seawater which may vary from case to case. Therefore the outfall design based on the deterministic approach may result in unequal marine environmental protection for different outfalls.

Suppose there are two outfalls designed using the deterministic approach at $S_o = 43$, and it is assumed that the two outfall can meet this design criterion, i.e. $S_o = 43$. However, the two outfalls may have different $P(S_o \leq 43)$; say $P_1(S_o \leq 43) = 5$ % and $P_2(S_o \leq 43) = 10\%$. If this were the case, although based on the same deterministic design, the first outfall would be better than the second in protecting the marine environment. This is because, in the first outfall, the probability of initial dilution being equal or less than 43 is only half of that given by the second outfall.

In addition, if the design is conducted using the probabilistic approach, and for example, the design criterion is set up at $P(S_o \leq 43) = 10\%$, the first outfall can be modified by, for example, reducing the length of the outfall to achieve the same design criterion for initial dilution. This means that in the first outfall, the construction cost would be cheaper than that in the deterministic design.

However, reducing the length of the outfall would be able to increase the probability of getting higher effluent concentration at a target area located onshore as shown Figure 5.9. Before deciding to reduce the length of the designed outfall, one should therefore consider the effect of this length reduction on the probability of failure, i.e. the probability of effluent concentration being more than a specified value or threshold level. As an example, figure 5.9 provides information on the relationship between the distance of the outfall discharge to the probability of failure. Using this information, if the government regulation is given in probabilistic terms, one may find a suitable distance so that the designed outfall would comply with the regulation. Otherwise, one may look at the minimum probability of failure for the same cost.

Chapter 7

Discussion and Conclusions

7.1 Discussions

7.1.1. Probabilistic Methods

Chapter Five shows that second moment methods (FOSM, IM-FOSM, and AFOSM with normal parameters) give approximately the same answer most of the time for calculating probability of failure in the case of initial dilution, but not in the case of effluent concentration. Table 7.1 shows the difference between calculated results given by the second moment methods and those given by MCS for the case of initial dilution. In the table, difference is defined as:-

$$\delta = \frac{ABS\left(P_{f\_i} - P_{f\_MCS}\right)}{P_{f\_MCS}} \text{ x } 100\% \tag{7.1}$$

where δ is the difference between the calculated probability of failure using method i and that using MCS, P_{f_i} is the probability of failure calculated using method of i, and P_{f_MCS} is the probability of failure simulated using MCS. ABS indicates the use of absolute value.

Table 7.1. Difference (%) of Calculated Results for Each Method Relative to the MCS (Case of initial dilution, D = 0.1 m)

Threshold Level, To	FOSM		IM-FOSM		AFOSM	
	Normal Zo	Lognormal So	Normal Zo	Lognormal So	Normal Parameters	Non-normal Parameters
20.0	0.0	0.0	0.0	0.0	0.0	0.0
25.0	0.0	0.0	0.0	0.0	0.0	0.0
30.0	69.2	76.9	84.6	76.9	46.2	315.4
35.0	7.0	48.4	1.2	43.8	36.0	34.1
40.0	12.8	16.7	9.2	12.6	17.0	15.1
45.0	3.2	2.1	0.8	4.6	0.9	23.6
50.0	5.1	7.0	6.3	8.1	8.5	19.6
55.0	5.9	5.0	6.3	5.4	6.9	12.0
60.0	3.0	2.2	3.1	2.3	3.1	5.6
65.0	1.0	0.7	1.0	0.8	0.9	2.0
average	**10.7**	**15.9**	**11.2**	**15.5**	**12.0**	**42.7**

However, for the case of effluent concentration, only AFOSM with non-normal parameters gives a good agreement with MCS. Table 7.2 shows that the average difference between calculated results given by AFOSM with non-normal parameters and those given MCS is only about 5 %.

Table 7.2. Difference (%) of Calculated Results for Each Method Relative to the MCS (Case of effluent concentration, D = 0.1 m, X = 100 m onshore)

Threshold Level, Tc	FOSM			AFOSM	
	Normal Zc	Log-normal Cx	Exponential Cx	Normal Parameters	Non-normal Parameters
2	36.0	100.0	100.0	1.5	13.5
100	24.9	98.2	85.1	8.3	11.8
200	15.0	98.5	43.6	16.7	6.9
300	8.4	98.7	42.4	21.0	5.0
400	5.0	98.8	35.7	21.1	6.4
500	0.5	98.9	30.6	22.6	6.5
600	8.0	98.9	30.3	28.9	2.9
700	12.7	99.0	25.0	30.3	2.9
800	15.5	99.0	17.8	29.6	4.6
900	25.5	99.0	17.6	36.6	0.5
1000	25.8	99.1	8.4	33.0	4.2
1500	60.6	99.0	9.1	46.7	0.6
2000	81.6	99.1	32.5	43.5	5.9
AVERAGE	**22.8**	**99.0**	**34.2**	**24.3**	**5.1**

For the case of effluent concentration, the calculated probability of failure given by FOSM is poor as shown in Figures 5.8 and 5.9. Even for FOSM with an assumed normal performance function or assumed log-normal effluent concentration the probability of failure is approximately constant regardless of the value of the threshold level or the distance of the outfall discharge to a target area of interest.

It is also noted that there are relatively large differences between these methods and MCS particularly when calculating small probabilities of failure, i.e. typically for failure probability less than 4 %. This is not unexpected because the calculated results given by the second moment methods (i.e. FOSM and AFOSM) are only approximate solutions obtained by ignoring the higher order terms in the Taylor series (see Chapter Four). Furthermore, many other engineering applications of the methods have shown similar results, i.e the quality of the second moment method is degraded for calculating small probabilities (e.g. Melching and Anmangandla, 1992; Melching, 1995).

However, for approximation purposes, the use of the methods is applicable (Smith, 1986). This is because the difference between the calculated results with MCS is relatively small when the methods are applied for conditions under which they were developed. Therefore, the choice of the method depends on the systems under investigation, i.e. the variability of the parameters involved, and the complexity of the performance function.

As shown in Tables 7.1 and 7.2, different methods provide different benefits for different cases. Table 7.1 (for the case of initial dilution) shows that FOSM has a good agreement with MCS for case of initial dilution with an average difference of about 10 %, and that FOSM and I-FOSM give practically the same answer.On the other hand, it is clear in Table 7.2 that AFOSM with non normal parameters is a better method than FOSM for calculating the probability of failure for effluent concentration at a target area. The difference between the calculated results given by AFOSM and those given by MCS is only about 5 %.

Although many researchers have reported that AFOSM gives good agreement with MCS, and even better agreement than FOSM for the tails of a probability distribution (e.g. Melching and Anmangandla, 1992; Smith, 1986), it is found for the case of initial dilution that FOSM performs as accurately, and sometimes more accurately, than AFOSM. Although this conclusion may be different from other case, this can happen particularly when the non-linearity effects are not large (Melching, 1995). This may be the reason for the results, and the following is intended to show that the initial dilution model used in the analysis cannot be said to have a large nonlinearity

Recall the initial dilution model used (equation 2.2):-

$$S_o = 0.54\, F \left[\frac{0.38\, Y}{F\, D} + 0.66 \right]^{5/3} \qquad \text{for } Y/D > 0.5\ F \quad (2.2)$$

In this case Y/D >> F, it is therefore possible to drop the constant (0.66) in equation 2.2. to give:-

$$S_o = \frac{0.1077}{D^{1.7}}\left(\frac{Y^{1.7}}{F^{0.7}}\right) \tag{6.2}$$

indicating that, for a given outfall diameter, the initial dilution is a function of Y and F with a simple relationship. Therefore, it is reasonable to say that the non-linearity effects may not be large. The finding, that FOSM gives a closer solution to the MCS than does AFOSM in the case of initial dilution is therefore acceptable.

On the other hand, AFOSM with non-normal parameters gives a better approximation for a more complex problem such as effluent concentration calculation. The performance function for the case of effluent concentration cannot be assumed linear and simple. For a comparison with the equation of initial dilution, recall equation 2.28 as follows:-

$$C_x = \frac{C_e}{S_i} e^{\frac{-2.3x}{uT_{90}}} erf\left(\sqrt{\frac{1.5}{\left(1+8\frac{k_o x}{ub^2}\right)^3 - 1}}\right) \tag{2.28}$$

Therefore, the results of the analysis agree with the general rule that AFOSM give better solutions for non-linear and complex systems such as, in this case, calculation of effluent concentration.

It is also noted that the results given by IM-FOSM are practically the same as those given by FOSM. Therefore, the use of FOSM is preferable for designing initial dilution in still water situations. Furthermore, Melching (1995) indicated that the use of IM-FOSM is incorrect and misleading because the key to improving estimates offered by FOSM is to provide a better estimate of the variance of the performance function. The complete second order approach (for both mean and variance) is very accurate but quite computationally intensive (Melching, 1995).

7.1.2. Application of the Probabilistic Methods in Outfall Design

It has been shown that FOSM and AFOSM may be applied in calculating probability of failure for the cases of initial dilution and effluent concentration. When the distributions of parameters involved are known, AFOSM with non-normal parameters would give a good solution. These would be significant advantages in applying these methods in outfall design because the calculation can be performed faster and easier using a spreadsheet, e.g. EXCEL, than by using MCS which is time consuming, typically about 20 hours using a computer with medium quality. The equations derived in the previous chapters are readily available for use in such applications.

For practical purposes, FOSM and AFOSM are applicable for use in outfall design. However, FOSM should not be applied for the case of effluent concentration in the vicinity of a target area, as the calculated probability of failure given by FOSM is poor. Even for FOSM with an assumed normal performance function Z_o, the probability of failure is approximately constant regardless of the value of the threshold level or the distance of the outfall discharge to a target area of interest.

However, MCS method can also be a good method for outfall design. This method can generate directly the probability distribution that cannot be developed using other methods discussed here. The method is also flexible, even when the critical probability of failure is not easy to derive from government regulations.When the distributions of parameters involved in the system under investigation are known, MCS may be preferable if a fast computer is available. However, it should be remembered that the choice of the probabilistic methods should consider the problem under investigation along with the cost and facilities available.

7.1.3. Data Collection

It is important to note that the data used in the analyses, in this thesis, were limited. Some were actually taken from other outfall data, or were estimated using assumptions. Therefore, this analysis may not exactly represent the actual condition of Spaniard's Bay. Instead, this work shows how the probabilistic approach may work for outfall design and analysis.

7.2 Conclusions

The probabilistic approach for outfall analysis and design is a realistic approach because it takes into account the effect of variability of parameters involved in the system. Using probabilistic methods, one may find a reasonable outfall design criterion which reflect the statistical nature of parameters affecting initial dilution and bacterial concentration near the the outfall discharge.

A comparison of the probabilistic approach with the deterministic approach shows that the probabilistic approach shows that the probabilistic approach may provide a full range of possible values of the parameters of interest other than a fixed value. Associated probability values for the parameters of interest can also be obtained using the probabilistic methods. The procedure for outfall design using a probabilistic approach is straight forward as shown in Chapter Three. This may work in the field and the analysis of an existing outfall (the Spaniard's Bay Outfall) has resulted in good agreement with field data.

Comparison among the various probabilistic methods studied shows that all methods generally give the same answers for the case of initial dilution, except for a small probability of failure which is typically less 4 %. FOSM and AFOSM are a relative simple technique but give accurate results relative to the MCS. Although IM-FOSM is more complicated than FOSM, its performance is practically the same as given by FOSM. It is found that FOSM, IM-FOSM and AFOSM with assumed normal parameters work well for use in analysis and design of initial dilution. In practice, the use of FOSM is recommended for its simplicity.

For the case of effluent concentration, FOSM gives poor results because the performance function in this case is complex and non linear, but AFOSM may be applied in this case. When distributions of the parameters involved are known, the use of AFOSM with assumed non-normal parameters is recommended as the derived equations are available (at Chapter Four and Five) and give accurate results.

MCS may be used if a fast computer and software are available. However, it should be remembered that the choice of the probability method should consider the problem under investigation along the cost and facilities available.

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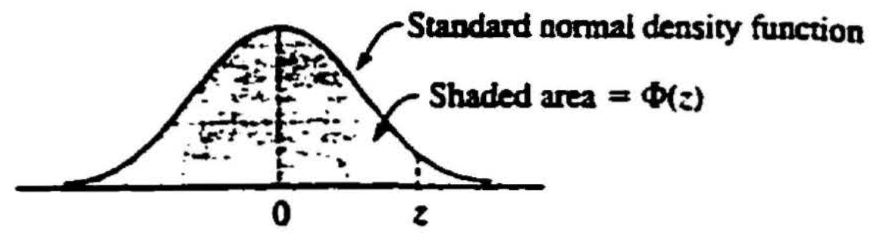
APPENDIX A

Table of Standard Normal Integral

Table A. The Standard Normal Integral-$\phi(y)$

(after Devore, 1995)

$\Phi(z) = P(Z \leq z)$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
−3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
−3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
−3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
−3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
−3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
−2.9	0.0019	0.0018	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
−1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
−1.8	0.0359	0.0352	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
−1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
−1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
−1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
−1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0722	0.0708	0.0694	0.0681
−1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
−1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
−1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
−1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
−0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

Table A. The Standard Normal Integral-$\phi(y)$

(continued)

$\Phi(z) = P(Z \leq z)$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9278	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

APPENDIX B

Table of Ordinates of Standard Normal Curve

Table B. Ordinates of the Standard Normal Curve-$f_Y(y)$

(after Smith, 1986)

y	0	1	2	3	4	5	6	7	8	9
0.0	0.3989	0.3989	0.3989	0.3988	0.3986	0.3984	0.3982	0.3989	0.3977	0.3973
0.1	0.3970	0.3965	0.3961	0.3956	0.3951	0.3945	0.3939	0.3932	0.3925	0.3918
0.2	0.3910	0.3902	0.3894	0.3885	0.3876	0.3867	0.3857	0.3847	0.3836	0.3825
0.3	0.3814	0.3802	0.3790	0.3778	0.3765	0.3752	0.3739	0.3725	0.3712	0.3697
0.4	0.3683	0.3668	0.3653	0.3637	0.3621	0.3605	0.3589	0.3572	0.3555	0.3538
0.5	0.3521	0.3503	0.3485	0.3467	0.3448	0.3429	0.3410	0.3391	0.3372	0.3352
0.6	0.3332	0.3312	0.3292	0.3271	0.3251	0.3230	0.3209	0.3187	0.3166	0.3144
0.7	0.3123	0.3101	0.3079	0.3056	0.3034	0.3011	0.2989	0.2966	0.2943	0.2929
0.8	0.2987	0.2874	0.2850	0.2827	0.2803	0.2780	0.2756	0.2732	0.2709	0.2685
0.9	0.2661	0.2637	0.2613	0.2589	0.2565	0.2541	0.2516	0.2492	0.2468	0.2444
1.0	0.2420	0.2396	0.2371	0.2347	0.2323	0.2299	0.2275	0.2251	0.2227	0.2203
1.1	0.2179	0.2155	0.2131	0.2107	0.2083	0.2059	0.2036	0.2012	0.1989	0.1965
1.2	0.1942	0.1919	0.1895	0.1872	0.1849	0.1826	0.1804	0.1781	0.1758	0.1736
1.3	0.1714	0.1691	0.1669	0.1647	0.1626	0.1604	0.1582	0.1561	0.1539	0.1518
1.4	0.1497	0.1476	0.1456	0.1435	0.1415	0.1394	0.1374	0.1354	0.1334	0.1315
1.5	0.1295	0.1276	0.1257	0.1238	0.1219	0.1200	0.1182	0.1163	0.1145	0.1127
1.6	0.1109	0.1092	0.1074	0.1057	0.1040	0.1023	0.1006	0.0989	0.0973	0.0957
1.7	0.0940	0.0925	0.0909	0.0893	0.0878	0.0863	0.0848	0.0833	0.0818	0.0804
1.8	0.0790	0.0775	0.0761	0.0748	0.0734	0.0721	0.0707	0.0694	0.0681	0.0669
1.9	0.0656	0.0644	0.0632	0.0620	0.0608	0.0596	0.0584	0.0573	0.0562	0.0551
2.0	0.0540	0.0529	0.0519	0.0508	0.0498	0.0488	0.0478	0.0468	0.0459	0.0449
2.1	0.0440	0.0431	0.0422	0.0413	0.0404	0.0396	0.0387	0.0379	0.0371	0.0363
2.2	0.0355	0.0347	0.0339	0.0332	0.0325	0.0317	0.0310	0.0303	0.0297	0.0290
2.3	0.0283	0.0277	0.0270	0.0264	0.0258	0.0252	0.0246	0.0241	0.0235	0.0229
2.4	0.0224	0.0219	0.0213	0.0208	0.0203	0.0198	0.0194	0.0189	0.0184	0.0180
2.5	0.0175	0.0171	0.0167	0.0163	0.0158	0.0154	0.0151	0.0147	0.0143	0.0139
2.6	0.0136	0.0132	0.0129	0.0126	0.0122	0.0119	0.0116	0.0113	0.0110	0.0107
2.7	0.0104	0.0101	0.0099	0.0096	0.0093	0.0091	0.0088	0.0086	0.0084	0.0081
2.8	0.0079	0.0077	0.0075	0.0073	0.0071	0.0069	0.0067	0.0065	0.0063	0.0061
2.9	0.0060	0.0058	0.0056	0.0055	0.0053	0.0051	0.0050	0.0048	0.0047	0.0046
3.0	0.0044	0.0043	0.0042	0.0040	0.0039	0.0038	0.0037	0.0036	0.0035	0.0034
3.1	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026	0.0025	0.0025
3.2	0.0024	0.0023	0.0022	0.0022	0.0021	0.0020	0.0020	0.0019	0.0018	0.0018
3.3	0.0017	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014	0.0013	0.0013
3.4	0.0012	0.0012	0.0012	0.0011	0.0011	0.0010	0.0010	0.0010	0.0009	0.0009
3.5	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007	0.0007	0.0007	0.0006
3.6	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005	0.0005	0.0005	0.0005	0.0004
3.7	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003
3.8	0.0003	0.0003	0.0003	0.0003	0.0003	0.002	0.0002	0.0002	0.0002	0.0002
3.9	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.0001

APPENDIX C

MINITAB Program

for

Monte Carlo Simulations (MCS)

C.1. Computer Program for the Case of Effluent Concentration

C.1.1. MACRO (MAC file) for Varying the distance of the outfall discharge from a target area

```
GMACRO
MCS
# To execute type: % 'c:\myfiles\thesis\simulasi\monte_X.mac'
# Nozzle diameter is 0.1 m.
# Threshold level is 500 per 100 ml.
# Distance X from outfall discharge is in k2 (onshore):
let k2=50
DO k1=1:12
let k20=1
  do K3=1:28000
  # Generating random sewage flow in C1:
  name c1 'flow'
  random 1 c1;
  normal -219.95 37.73.
  let c1=(1+((-1.1)*c1))**(1/-1.1)
  let k4=c1/2
  # Generating random surface current in c2:
  name c2 'current'
  random 1 c2;
  exponential 0.016.
  # Indentifying current direction, randomly, in c3:
  # if c3=0, then the current in onshore direction
  name c3 'direct_u'
  random 1 c3;
  poisson 1.1394.
  copy c3 k10
  # Generating random tide in C4:
  name c4 'tide'
  random 1 c4;
  uniform 0 1.4.
  # Generating decay parameter in c5:
  name c5 'T_90(hr)'
  random 1 c5;
  lognormal 1.527 0.196.
  let k5=c5*3600
  # Generating effluent concentration in c6:
  name c6 'C_e'
  random 1 c6;
  lognormal 15.913 0.246.
  # Calculating y in the error function, x(m)in k6,
  # and b=(1/3)*0.045*k2 in k100 & ko=0.0005 b^(4/3) in k110:
  let k100=0.045*k2/3
  let k110=0.0005*(k100**(4/3))
  let k6=k2
  name c7 'y'
  let C7=sqrt(1.5/(((1+((8*(k110)*k6)/(c2*(k100**2))))**3)-1))
```

```
   name c8 'z'
   let c8=C7*(sqrt(2))
   name c9 'erf(y)'
   cdf c8 c9
   let c9=2*(c9-0.5)
   # Calculating initial dilution in c10:
   name c10 'So'
   let k17=(2.47269*k4/(0.1**(5/2)))
   let
c10=0.54*k17*(((0.38*((0.045*k2)+c4)/(k17*0.1))+0.66)**(5/3))
   name c11 'Cx'
   let k30=c9*(c6/c10)*exp((-2.3*k6)/(c2*k5))
   let c11(k3)=k30
   If k10> 0
      #name c12 'offshore'
      #let c12(k20)=k30
      #let k20=k20+1
   else
      name c12 'onshore'
      let c12(k20)=k30
      let k20=k20+1
   endif
   enddo
# For threshold level To in k8:
let k8=500
name c13 'failure'
let C13=(C12>k8)
name c14 'X(m)'
let c14(k1)=k2
name c15 'Pf(%)'
let c15(k1)=100*(sum(c13))/k3
let k2=k2+25
ENDDO
ENDMACRO
```

C.1.2. MACRO (MAC file) for Varying threshold level, Tc

```
GMACRO
MCS
# To execute type: % 'c:\myfiles\thesis\simulasi\monte_X.mac'
# Nozzle diameter is 0.1 m
# Distance of the outfall discharge from a target area is 100 m.
# threshold level Tc in k2:
let k2=100
DO k1=1:20
let k20=1
  do K3=1:28000
  # Generating random sewage flow in C1:
  name c1 'flow'
  random 1 c1;
  normal -219.95 37.73.
  let c1=(1+((-1.1)*c1))**(1/-1.1)
  let k4=c1/2
  # Generating random surface current in c2:
  name c2 'current'
  random 1 c2;
  exponential 0.016.
  # Indentifying current direction, randomly, in c3:
  # if c3=0, then the current in onshore direction
  name c3 'direct_u'
  random 1 c3;
  poisson 1.1394.
  copy c3 k10
  # Generating random tide in C4:
  name c4 'tide'
  random 1 c4;
  uniform 0 1.4.
  # Generating decay parameter in c5:
  name c5 'T_90(hr)'
  random 1 c5;
  lognormal 1.527 0.196.
  let k5=c5*3600
  # Generating effluent concentration in c6:
  name c6 'C_e'
  random 1 c6;
  lognormal 15.913 0.246.
  # Calculating y in the error function, x(m)in k6,
  # and b=(1/3)*4.5 m in k100 & ko=0.0005 b^(4/3) in k110:
  let k100=4.5/3
  let k110=0.0005*(k100**(4/3))
  let k6=100
  name c7 'y'
  let C7=sqrt(1.5/(((1+((8*(k110)*k6)/(c2*(k100**2))))**3)-1))
  name c8 'z'
  let c8=C7*(sqrt(2))
  name c9 'erf(y)'
  cdf c8 c9
  let c9=2*(c9-0.5)
  # Calculating initial dilution in c10:
```

```
   name c10 'So'
   let k17=(2.47269*k4/(0.1**(5/2)))
   let c10=0.54*k17*(((0.38*(4.5+c4)/(k17*0.1))+0.66)**(5/3))
   name c11 'Cx'
   let k30=c9*(c6/c10)*exp((-2.3*k6)/(c2*k5))
   let c11(k3)=k30
   If k10> 0
      #name c12 'offshore'
      #let c12(k20)=k30
      #let k20=k20+1
   else
      name c12 'onshore'
      let c12(k20)=k30
      let k20=k20+1
   endif
   enddo
# For threshold level To in k8:
let k8=k2
name c13 'failure'
let C13=(C12>k8)
name c14 'Tc/100ml'
let c14(k1)=k2
name c15 'Pf(%)'
let c15(k1)=100*(sum(c13))/k3
let k2=k2+100
ENDDO
```

C.2. Computer Program for the Case of Initial Dilution

C.2.1. Varying Nozzle Diameter

◘ Main Program (file: FAIL-SD.MTB)

```
# This Program is to calculate probability of failure Pf(So<30)
# with varying nozzle diameter D (m).
# For 4 values of diameter, type:(the smallest D is 0.05 in k2)
# MTB >let k1=1
# MTB > let k2=0.05
# MTB > exec 'C:\MYFILES\THESIS\SIMULASI\FAIL-SD.MTB' 4
name c6 'D, To=30'
name c7 'Pf(%)'
LET K5=1
# number of simulations is k75
let k75=28000
EXEC 'c:\myfiles\thesis\simulasi\vary-d.mtb' k75
let c6(k1)=k2
let c7(k1)=(100*sum(c5))/k75
name c8 'mean(so)'
let c8(k1)=mean(c3)
LET K2=K2+0.05
let k1=k1+1
END
```

◘ Sub_Program (file: VARY-D.MTB)

```
# generating random flow in c1:
name c1 'flow'
random 1 c1;
normal -219.95 37.73.
let c1=(1+((-1.1)*c1))**(1/(-1.1))
# flow for each nozzle k3:
let k3=c1/2
# generating random tide:
name c2 'tide'
random 1 c2;
uniform 0 1.4.
let c2=c2+ 4.5
# densimetic Froude number (k3)with nozzle diameter of k2 m:
let k4= k3/((3.14/4)*(((k2)**5)*9.8*0.027)**(0.5))
# dilution:
name c3 'dilution'
let C3(k5)=(0.54*k4*(((0.38*c2)/(k2*k4) + 0.66)**(5/3)))
# performance function Z with To=30
name c4 'Z'
let C4=(0.54*k4*(((0.38*c2)/(k2*k4) + 0.66)**(5/3)))-30
# identify whether Z is less than zero:
name c5 '(Z<0) ?'
let c5(k5)=(c4<0)
let k5=k5+1
END
```

C.2.2. Varying Threshold Level

◘ Main Program (file: FAIL-ST.MTB)

```
# The Program is to calculate probability of failure
# with varying threshold level To for the existing outfall
# For 10 values of To(the smallest To is 20 in k2), type:
# MTB >let k1=1
# MTB > let k2=20
# MTB > exec 'C:\MYFILES\THESIS\SIMULASI\FAIL-ST.MTB' 10
name c6 'To,D=0.1'
name c7 'Pf(%)'
LET K5=1
# number of simulations is k75
let k75=28000
EXEC 'c:\myfiles\thesis\simulasi\Vary-To.mtb' k75
let c6(k1)=k2
let c7(k1)=(100*sum(c5))/k75
LET K2=K2+5
let k1=k1+1
END
```

◘ Sub_Program (file: VARY-To.MTB)

```
# generating random flow in c1:
name c1 'flow'
random 1 c1;
normal -219.95 37.73.
let c1=(1+((-1.1)*c1))**(1/(-1.1))
# flow for each nozzle k2:
let k3=c1/2
# generating random tide:
name c2 'tide'
random 1 c2;
uniform 0 1.4.
let c2=c2+ 4.5
# densimetic Froude number (k3)with nozzle diameter of 0.1 m:
let k4= k3/((3.14/4)*(((0.1)**5)*9.8*0.027)**(0.5))
# dilution:
name c3 'dilution'
#let C3(k5)=(0.54*k4*(((0.38*c2)/(0.1*k4) + 0.66)**(5/3)))
# performance function Z with To=k2
name c4 'Z'
let C4=(0.54*k4*(((0.38*c2)/(0.1*k4) + 0.66)**(5/3)))-k2
# identify whether Z is less than zero:
name c5 '(Z<0) ?'
let c5(k5)=(c4<0)
let k5=k5+1
END
```