THE ALPHA-MODIFIED QUASI-SECOND ORDER NEWTON-RAPHSON METHOD FOR LOAD FLOW SOLUTIONS IN RECTANGULAR FORM

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by

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ABSTRACT

In this thesis a new class of methods for solving power system load flows is developed, which is called the alpha-modified quasi-second order Newton-Raphson (alpha - M.Q.S.O.N.R.) class.

The theory is derived from the Taylor expansion for multivariable functions and is developed using rectangular coordinates. The first part of the theory is actually the Newton-Raphson algorithm. This is expanded by including the second order terms from the Taylor expansion to produce a quasi-second order Newton-Rahpson solution method. The theory is then further modified to yield the more general alpha-modified method.

Extensive testing is performed using the new method for seven different power systems. The same systems are rerun with the Newton-Raphson so that a comparison can be made and the new method's relative merit judged. The rate of convergence is studied, as well as the mismatch values which occur during the load flows. This is done for many different values of alpha and for the different systems. The results are analysed and discussed in detail with the main result being that the alpha - M.Q.S.O.N.R. method tested, with alpha equal to zero, is superior to the Newton-Raphson.

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Because there is quite a large number of equations in this thesis, the author was finding it extremely difficult to locate someone willing to type the manuscript. He is thankful to Karen Murphy who did attempt the job, but was unable to do more than one chapter. However, rescue did come at the last minute when four typists within the Engineering Department offered to share the typing load. This touching gesture has earned the author's eternal gratitude for the work performed by Brenda Walsh, Edwina Newhook, Pat Gibson, and Janet Coffen.

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LIST OF SYMBOLS

B_{pq}	Active portion of the sum of the admittances connected to bus p between it and bus q, on a single line representation of the power system.
e_p	Active portion of the complex bus voltage of bus p.
Δe_i	Voltage correction for the active portion of the voltage at bus i.
$\Delta e_i'$	Updated voltage correction for the active portion as calculated using the second order correction factors.
f_p	Reactive portion of the complex bus voltage of bus p.
Δf_i	Voltage correction for the reactive portion.
$\Delta f_i'$	Updated voltage correction for the reactive portion.
f_i	The set of simultaneous load flow equations for the Taylor expansion.
G_{pq}	Reactive portion of the sum of the admittances connected to bus p between it and bus q, on a single line representation of the power system.
$\underline{H_i}$	Hessian matrix of second order derivatives.
I_p	Sum of currents flowing into bus p.
I_p^*	Conjugate of I_p.
$\underline{J}$	Jacobian matrix of first order derivatives
M.Q.S.O.N.R.	Modified quasi-second order Newton-Raphson.
N.R.	Newton-Raphson.
n	Number of buses in system.
no V.C.B.	There are no. voltage controlled buses other than the slack bus.
P_p	Total injected active power into bus p.
P_{psched}	Scheduled active power at bus p.

ΔP_p	Active power mismatch for bus p.
$\Delta P'$	Updated active power bus mismatch matrix as calculated using the second order correction factors.
Q_p	Total calculated reactive power at bus p.
Q_{psched}	Scheduled reactive power at bus p.
ΔQ_p	Reactive power mismatch for bus p.
$\Delta Q'$	Updated reactive power bus mismatch matrix as calculated with the second order correction factor.
Q.S.O.N.R.	Quasi-second order Newton-Raphson.
R_p	Active power second order correction factor for bus p.
S_p	Total calculated complex power at bus p.
T_p	Reactive power second order correction factor for bus p.
U_p	Square of voltage magnitude second order correction factor for bus p.
V_p	Voltage at bus p.
$V_p{}^*$	Conjugate of V_p.
$\|V_p\|$	Magnitude of V_p.
$\|V_p\|_{sched}$	Scheduled voltage magnitude for bus p.
$\Delta\|V_p\|^2$	Square of voltage magnitude mismatch for bus p.
$\Delta\|V\|^{2'}$	Updated square of voltage matnitude matrix as calculated using second order ocrrection factors.
x_i	Variables in Taylor expansion.
Y_{pq}	Complex sum of all admittances connected to bus p of a line between it and bus q, on a single line representation of the power system. This is also the p-q th element of the bus admittance matrix.
α	Alpha, the variable whose value determines what portion of the second order correction factors are applied to the updating of the bus mismatches and what portion applied to the updating of the Jacobian submatrices diagonal elements.

Δi, Δx_i	Variable corrections of the Taylor expansion.
$\underline{\Delta}^T$	Transpose of the variable correction matrix.
ξ	Tolerance required in order that convergence criterion be met.

CHAPTER I

INTRODUCTION

1.1 Background

1.1.1 Definition of Load Flow

Load flow (or power flow) is the most frequently performed of routine digital-computer power network calculations. A load flow program is used to determine the powerflow, both real and imaginary, over transmission lines and the corresponding bus voltages across the system. This calculation is a steady state solution or snapshot of the system under one specific condition and configuration. A configuration describes the system connections, such as the particular lines connected and generating stations on line. The system conditions describe the magnitude of load and the generation schedule to meet the load. Simply put, load flow is therefore defined as the solution for the static operating condition of an electric power transmission system.

1.1.2 Uses for Load Flow Calculations

Load flow calculations are performed in power system planning, operational planning, and operation control. It is necessary to use them to see the effects on the whole system of, and help determine, network changes involving:

1. plant site selection
2. plant size and number of units
3. operation of plant (i.e., base load or peak)
4. routing of new transmission lines
5. location, and voltage level, of new area interconnections

6. bulk power transfer capability
7. size and location of capacitors and reactors for reactive compensation
8. contingency evaluation due to temporary loss of generation and/or transmission circuits

Over the past decade, load flow calculations have also been utilized more and more in optimization and stability studies in electric power systems.

1.1.3 A Brief History of Load Flow

Twenty-Five years ago all electrical networks were solved either by hand or by a network analyser. The analyser was simply an electrical analogue device which scaled down the electrical quantities of a system. It is now obsolete as a tool for routine studies of power systems and is relegated to the worthwhile role of an educational tool.

This rapid demise of the "analogue calculator" started in 1956 when the first truly successful (i.e., practical) automatic digital simultaneous quadratic equations solution method was developed by Ward and Hale [71]. Their method was an approximation to the Newton iterative technique using rectangular form which allowed it to be run on the small-memory computer by neglecting the off-diagonal elements of the Jacobian submatrices and also by using the admittance matrix formulation. Using this as a basis, the programs, which immediately followed, implemented the Gauss-Seidel algorithm, introduced by Glimn and Stagg [33], and were accepted by the power industry. Other methods were also proposed such as Jordan's relaxation method [39] and zero mismatch methods, but Gauss-Seidel's became the industry standard.

Unfortunately, Gauss-Siedel requires a large number of iterations to obtain a solution and this number increases with system size. Also, the algorithm is such, that any adjustments made in an iteration take several more iterations to propagate their effect throughout the system. This sometimes leads to convergence problems.

In the early 1960's the Bonneville Power Administration (B.P.A.) was engaged in research involving the Newton-Raphson method, which had been shown to have very powerful convergence properties [72], [73], [74], but poor computational efficiency. In 1963, Sato and Tinney of B.P.A. publicly introduced the concept of optimally ordered elimination for the solution of large, general, sparse networks, and showed such methods were very efficient for solving large power system problems [53]. The mathematics behind the concept are concisely summarized in [63] and [65]. This constituted a major break-through in power system network computation, with the original application being to dramatically improve the computing speed and storage requirements of the Newton-Raphson method [64]. As a result, this method is now widely regarded as the preeminent general-purpose load flow approach, and has been adopted by much of the power industry.

1.1.4 Impetus for This Thesis

With the stimulus of increasing problem sizes, on-line applications, system optimization, and the fact that there is no "best" method for all systems, the development of faster and more efficient algorithms for solving load flows continues to be the object of many research efforts.

Recently Sachdev and Medicherla introduced the second order Newton-

Raphson (S.O.N.R.) method for load flow solutions [46]. It was applied in polar form. In discussing that paper, El-Hawary and Vetter [29] suggest that employing the rectangular form may enhance convergence. This seems like a reasonable assumption. For quadratic functions, as the load flow equations are, the derivatives of order higher than two are zero. Thus, a second order Taylor expansion-based iteration, such as Newton-Raphson, will converge in one iteration in the single variable problem. The load flow problem is a multi-variable problem that normally requires more than one iteration to solve the nonlinear equations. The use of the polar form introduces trignometric functions, but with the rectangular form, terms beyond second order are zero, thus making the second order scheme exact, and likely superior.

1.2 Scope of the Thesis

In this thesis. a quasi-second order Newton-Raphson (Q.S.O.N.R.) load flow method in retangular coordinates, as suggested in [29], is developed. This then forms the basis for the formulation of the alpha-modified quassi-second order Newton-Raphson (alpha-M.Q.S.O.N.R.) method.

Chapter II provides the theory upon which this thesis is based . First, the load flow problem is formulated in rectangular coordinates. Then, starting with the Taylor expansion for multivariable functions, the Newton-Raphson technique is evolved.

The derivation of the Q.S.O.N.R. is detailed in section 3.3. This is followed by the formulation of a number of alpha-modified second order schemes in section 3.4. The alpha - M.Q.S.O.N.R. method number one, derived in subsection 3.4.2, is the algorithm upon which most of the testing was performed.

Chapter IV details the application of this method to solving load flows for a number of test systems, which are described in section 4.3. The next section presents and analyses the results obtained.

The last chapter summarizes the conclusions reached as a result of the testing, and suggests some areas in which further efforts could be applied. Seven appendices are included. These describe many aspects related to the development reported in the text. Among these are descriptions of test systems, derivative evaluations, program listings to mention a few.

CHAPTER II

SOME BASIC DEVELOPMENTS

2.1 Introduction

In this chapter, some background theory is developed which will form the basis for work reported in this thesis. The following section briefly introduces the theory of the Newton-Raphson technique which is the foundation for the methods developed in chapter 3. Section 2.3 illustrates how a general power system is represented so that its parameters and variables can be used in the load flow solution methods effectively. Section 2.4 sets up the problem by formulating the static load flow equations of the system modeled in the previous section. The last section then derives the Newton-Raphson method in rectangular coordinates to the load flow equations.

2.2 Introduction to the Newton-Raphson Method

It is well known that the Newton-Raphson method is based on a Taylor expansion of multi-variable functions, where only the first order terms are considered. For a system of nonlinear equations, f_i, the Taylor expansion is,

$$
\begin{aligned}
f_i(x_1+\Delta_1,\ x_2+\Delta_2,\ \ldots) = f_i(x_1,\ x_2,\ \ldots) &+ \frac{1}{1!}\sum_{j=1}^{n}\Delta_j\frac{\partial f_i}{\partial x_j} \\
&+ \frac{1}{2!}\sum_{j=1}^{n}\sum_{k=1}^{n}\Delta_j\,\Delta_k\frac{\partial^2 f_i}{\partial x_j\partial x_k} \\
&+ \frac{1}{3!}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{\ell=1}^{n}\Delta_j\,\Delta_k\,\Delta_\ell\frac{\partial^3 f_i}{\partial x_j\partial x_k\partial x_\ell} \\
&+ \ldots \qquad ,\ i=\ell,\ \ldots n \qquad (2.1)
\end{aligned}
$$

The Newton-Raphson method uses only the first order approximation of the above expression, which leaves

$$f_i(x_1+\Delta_1, x_2+\Delta_2, \ldots) - f_i(x_1, x_2, \ldots) = \sum_{j=1}^{n} \Delta_j \frac{\partial f_i}{\partial x_j}, \quad i=1, \ldots, n \tag{2.2}$$

In vector form this can be written as,

$$\underline{\Delta f} = \underline{J}\ \underline{\Delta}$$

and manipulated to obtain

$$\underline{\Delta} = \underline{J}^{-1}\ \underline{\Delta f} \tag{2.3}$$

with which we can solve for the increments, Δ_i. Here, $\underline{J}$ denotes the Jacobian matrix of first order partial derivatives. In the last section of this chapter, this method is employed in solving the static load flow equations for a general power system.

2.3 Modeling of the power system

Before developing the static load flow equations and the solution methods of concern, modeling of the power system must be discussed.

A balanced three-phase power system is assumed, and the transmission system is represented by its positive-phase-sequence network of linear lumped series and shunt branches. Figure 2.1 represents a simple 3-bus system and illustrates the main types of buses as well as the main parameters and variables associated with each line and bus.

The symbols used in figure 2.1 are defined as follows:

$S_p = P_p+jQ_p$	Power generation or power demand at bus p
$V_p = V_p\angle\theta_p$	Voltage at bus p (polar coordinates)
$Z_{pq} = R_{pq}+jX_{pq}$	Series impedance of the transmission line between bus p and bus q
X_{po}	Shunt reactors and or static capacitors at bus p

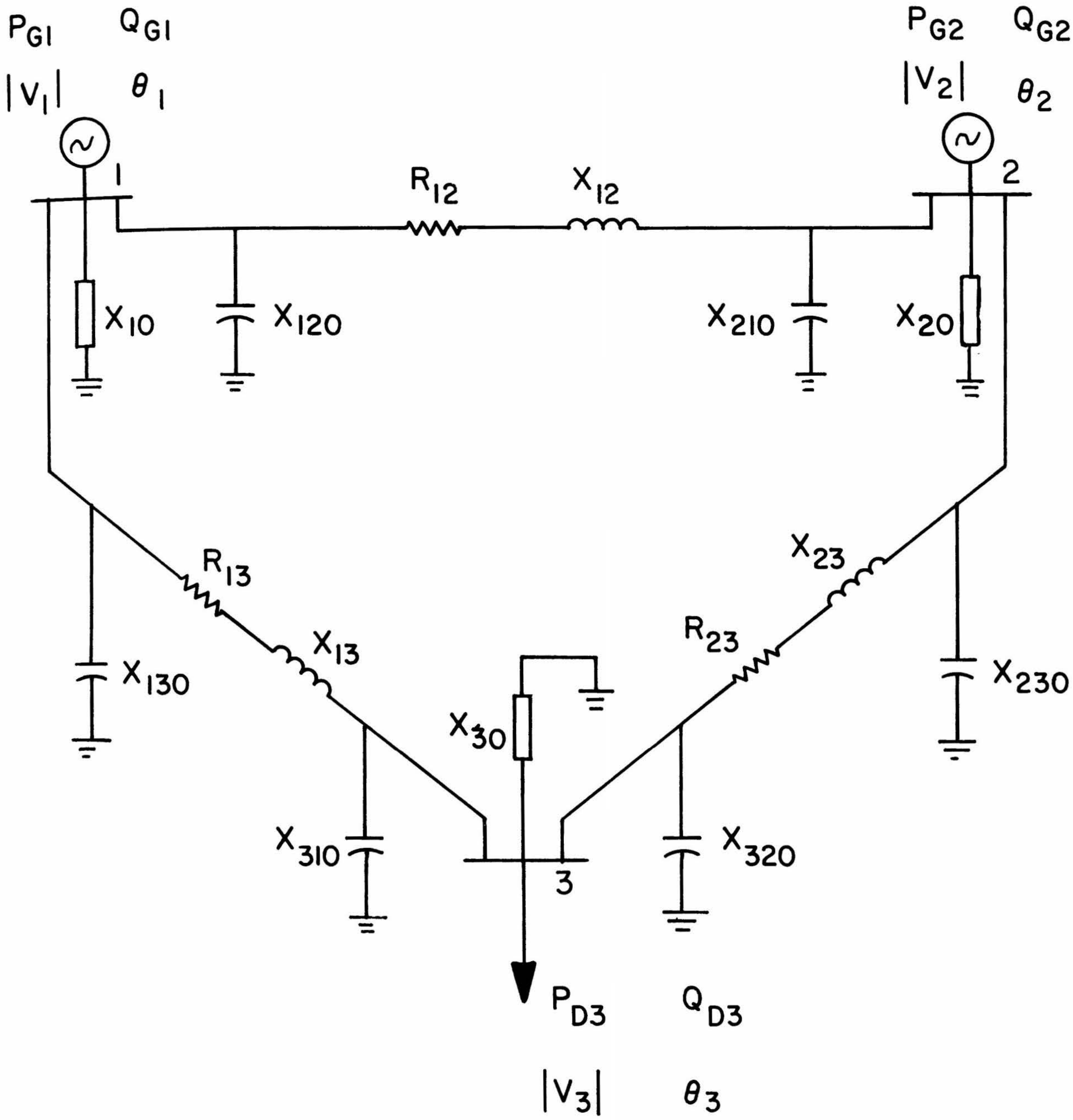


FIGURE 2.1 MODEL OF A 3-BUS POWER SYSTEM

X_{pqo}, X_{qpo} Transmission line shunt reactance. In a normal line, $X_{pqo} = X_{qpo}$ and their sum is the total line shunt reactance. $X_{pqo} \neq X_{qpo}$ if there is a tap setting transformer in tandem with the line.

Buses are categorized into three main types for load flow solution purposes:

(i) A load bus, such as bus 3, has the total injected power, P, and the reactive power, Q, specified, while the voltage magnitude and angle are the unknowns to be solved for.

(ii) A voltage regulated bus (a generator bus), such as bus 1 or bus 2, is one at which the total injected active power, P, is specified, and the voltage magnitude is maintained at a specified value by the reactive power injection, Q. Therefore Q and the voltage angle, θ, must be solved for.

(iii) The slack (or swing) bus. One of the generator buses, such as bus 1, is taken as the slack bus. Because total line losses are not known before a load flow study, one bus must have the active and reactive power unspecified so that it can pick up "the slack"--the difference between the total injected power of all the other buses and the requirements of the system loads. The slack bus voltage angle is assigned as the system phase reference and is considered a known quantity. Being a voltage controlled bus, the voltage magnitude is also set and known.

If a line has a fixed tap setting transformer as shown in figure 2.2, the transmission line variables are manipulated in order to obtain

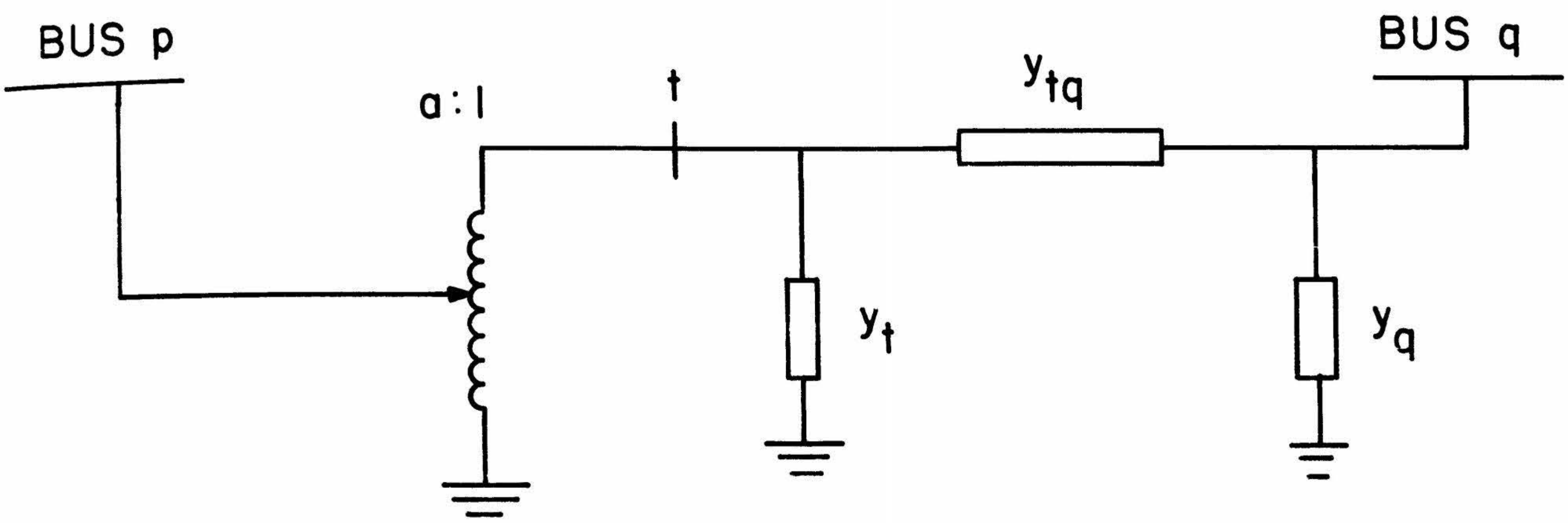


FIGURE 2.2. FIXED TAP SETTING TRANSFORMER IN TANDEM WITH A LINE.

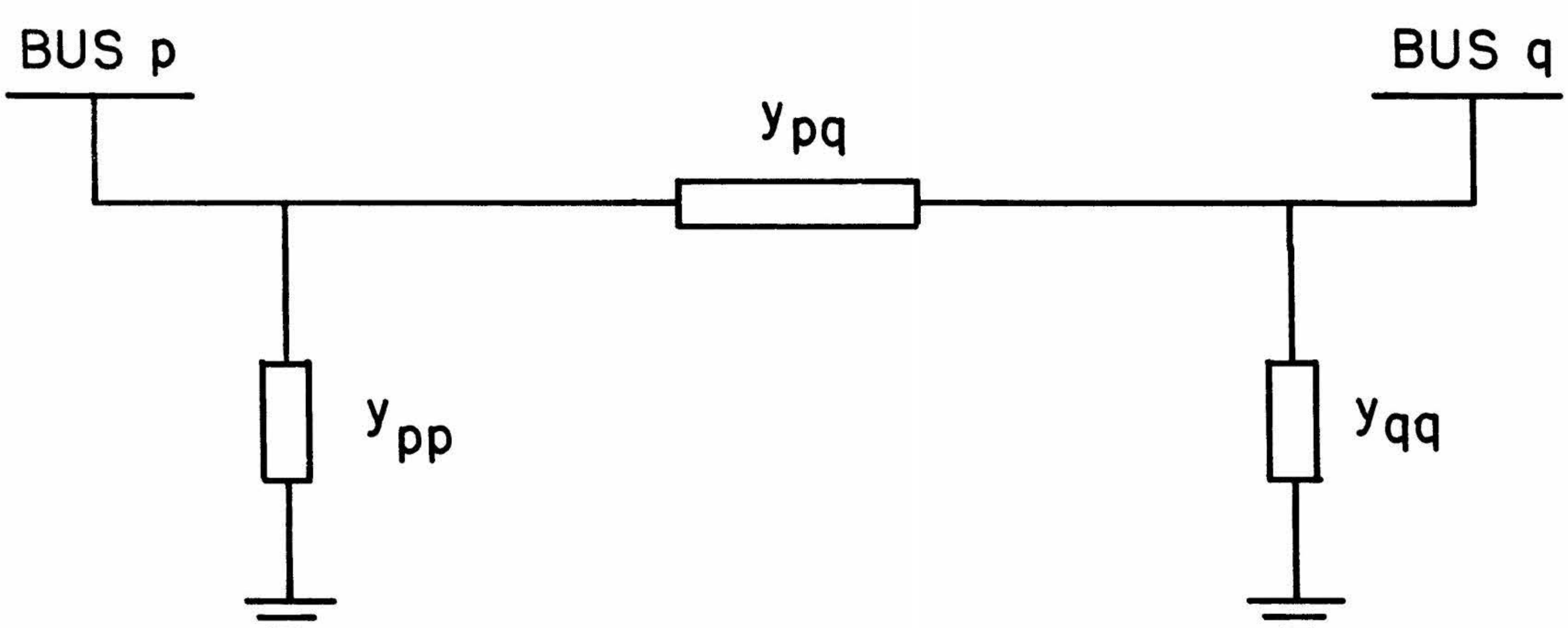


FIGURE 2.3 EQUIVALENT MODEL OF THE LINE WITH THE TRANSFORMER OF FIGURE 2.2.

the equivalent model of figure 2.3 so that it will be compatible with the load flow solution procedure. The manipulation is performed in the following way.

The transformer performance expressed using the transmission form of the two-port network presentation is

$$\begin{bmatrix} e_p \\ i_p \end{bmatrix} = \begin{bmatrix} a & o \\ o & 1/a \end{bmatrix} \begin{bmatrix} e_t \\ i_t \end{bmatrix} \qquad (2.4)$$

This describes the relationship between the nodal voltages and currents at node p in terms of those at node t on the line side of the transformer.

For the transmission line the following relationship holds:

$$\begin{bmatrix} e_t \\ i_t \end{bmatrix} = \begin{bmatrix} [1 + (y_q/y_{tq})] & (-1/y_{tq}) \\ [y_t + y_q + (\frac{y_t y_q}{y_{tq}})] & [-(1 + \frac{y_t}{y_{tq}})] \end{bmatrix} \begin{bmatrix} e_q \\ i_q \end{bmatrix} \qquad (2.5)$$

Substituting equation (2.5) into equation (2.4) yields

$$\begin{bmatrix} e_p \\ i_p \end{bmatrix} = \begin{bmatrix} a[1 + (y_q/y_{tq})] & (-a/y_{tq}) \\ \frac{1}{a}[y_t + y_q + (\frac{y_t y_q}{y_{tq}})] & -\frac{1}{a}(1 + \frac{y_t}{y_{tq}}) \end{bmatrix} \begin{bmatrix} e_q \\ i_q \end{bmatrix} \qquad (2.6)$$

In order for the transformer and line of figure 2.2 to be replaced with the equivalent π networks of figure 2.3, equation (2.6) must be equivalent to the transmission relationship of figure 2.3, which is,

$$\begin{bmatrix} e_p \\ i_p \end{bmatrix} = \begin{bmatrix} (1 + \frac{y_{qq}}{y_{pq}}) & (-1/y_{pq}) \\ y_{pp} + y_{qq} + (\frac{y_{pp} y_{qq}}{y_{pq}}) & -(1 + \frac{y_{pp}}{y_{pq}}) \end{bmatrix} \begin{bmatrix} e_q \\ i_q \end{bmatrix} \qquad (2.7)$$

Equating the two relationships for the elements (1, 2) leads to

$$-1/y_{pq} = {}^{-a}/y_{tq}$$

which reduces to

$$y_{pq} = y_{tq}/a \qquad (2.8)$$

Also we have for the (1, 1) elements:

$$1 + (y_{qq}/y_{pq}) = a\,[1 + (y_q/y_{tq})]$$

which reduces to,

$$y_{qq} = y_{tq} + y_q - y_{pq} \qquad (2.9)$$

The equality of the (2, 2) elements results in

$$1 + (y_{pp}/y_{pq}) = \frac{1}{a}\,(1 + y_t/y_{tq})$$

which reduces to

$$y_{pp} = \frac{y_t}{a} + (\frac{1}{a} - 1)\; y_{pq} \qquad (2.10)$$

With equations (2.8), (2.9), and (2.10) the transmission line with a transformer of figure 2.2 is transformed to the "standard line" representation of figure 2.3 whose parameters can now be used in the formulation of the static load flow equations.

2.4 Formulation of the Static Load Flow Equations

2.4.1 Introduction

Nodal analysis is almost universally preferred in the formulation of the load flow problem for analysis. This is because of the simplicity of data preparation and the ease with which the bus admittance matrix can be formed and modified for network changes in subsequent cases. It is centered around the nodal equation,

$$\underline{I} = \underline{Y} \cdot \underline{E} \qquad (2.11)$$

where the vector $\underline{I}$ is the set of currents flowing into each bus and the vector $\underline{E}$ is the set of bus voltages, and the vector $\underline{Y}$ is the bus admittance matrix.

2.4.2 Derivation of Load Bus Equations

The power at bus p is defined as

$$S_p = V_p I_p^* \tag{2.12}$$

Also,

$$S_p = P_p + jQ_p \tag{2.13}$$

Therefore,

$$P_p + jQ_p = V_p I_p^* \tag{2.14}$$

Taking the conjugate of both sides of (2.14), so that I_p can be evaluated more conveniently, results in,

$$P_p - jQ_p = V_p^* I_p \tag{2.15}$$

Referring to figure 2.1 it can be easily seen that, applying ohm's law, the total current flowing from a bus p, would be,

$$I_p = V_p\,(y_{po}) + \sum_{\substack{q=1\\ q\neq p}}^{m} (V_p - V_q)\, y_{pq} \tag{2.16}$$

where,

$V_p \triangleq$ voltage at bus p

$V_q \triangleq$ voltage at a bus, q, which is connected to bus p

$m \triangleq$ number of buses connected to bus p

$y_{pq} \triangleq$ the series admittance of the line connecting bus p and bus q

$y_{po} \triangleq$ the sum of all the shunt admittances to ground connected to bus p, including that of bus shunt reactors and/or static capacitors and that portion of the line shunt admittance delegated to the leg of the π-equivalent model of the line which is closest to bus p.

Equation (2.16) can be reduced to

$$I_p = V_p \, Y_{pp} - \sum_{\substack{q=1 \\ q \neq p}}^{m} + V_q \, y_{pq}$$

where Y_{pp} is the sum of all admittances connected to bus p.

$$Y_{pp} = y_{po} + \sum_{\substack{q=1 \\ q \neq p}}^{m} y_{pq}$$

Letting $Y_{pq} = -y_{pq}$, the general form will be

$$I_p = \sum_{q=1}^{n} V_q \, Y_{pq} \qquad (2.17)$$

where n is the number of buses in the system. Note that for any bus q not connected to bus p, the term Y_{pq} is zero.

Substituting this result into equation (2.15) gives,

$$P_p - jQ_p = V_p^{*} \sum_{q=1}^{n} V_q \, Y_{pq} \qquad (2.18)$$

Equation (2.18) defines the load flow problem to be solved. The real and reactive portions of the power are separated to form a set of 2n equations. The method that will be used for solving these requires the use of rectangular coordinates. Thus, with

$$V_p = e_p + jfp \qquad (2.19)$$

and

$$Y_{pq} = G_{pq} - jB_{pq} \qquad (2.20)$$

The expressions in rectangular coordinates are substituted into equation (2.18) to give,

$$P_p - jQ_p = (e_p - jf_p) \sum_{q=1}^{n} [(e_q + jf_q)(G_{pq} - jB_{pq})]$$

$$= (e_p - jf_p) \sum_{q=1}^{n} [(e_q G_{pq} + f_q B_{pq}) + j(f_q G_{pq} - e_q B_{pq})]$$

Separating the real and imaginary parts gives,

$$P_p = e_p[\sum_{q=1}^{n} (e_q G_{pq} + f_q B_{pq})] + f_p[\sum_{q=1}^{n} (f_q G_{pq} - e_q B_{pq})]$$

$$Q_p = f_p[\sum_{q=1}^{n} (e_q G_{pq} + f_q B_{pq})] - e_p[\sum_{q=1}^{n} (f_q G_{pq} - e_q B_{pq})]$$

Simplified, the static load flow equations in rectangular coordinate form are,

$$P_p = \sum_{q=1}^{n} [e_p(e_q G_{pq} + f_q B_{pq}) + f_p(f_q G_{pq} - e_q B_{pq})] \qquad (2.21)$$

$$Q_p = \sum_{q=1}^{n} [f_p(e_q G_{pq} + f_q B_{pq}) - e_p(f_q G_{pq} - e_q B_{pq})] \qquad (2.22)$$

2.4.3 Derivation of the Voltage Controlled Bus Equations

Equations (2.21) and (2.22) specify the static operating state at the load buses. However, at a voltage controlled bus, the specified variables are active power and voltage magnitude. The two equations governing such a bus are equation (2.21) and

$$|V_p|^2 = e_p^{\,2} + f_p^{\,2} \qquad (2.23)$$

2.4.4 The Slack Bus

At the slack bus the voltage magnitude and angle are specified. The active and reactive powers are determined at the conclusion of a load flow solution, since the purpose of the slack bus is to provide the additional power to supply the transmission losses which are not known initially. As a result there are no equations to be formally solved iteratively for this bus.

2.4.5 Summary

The load flow problem for an n-node power system consists of a set of 2(n-1) nonlinear simultaneous algebraic equations to be satisfied.

At a load bus, p,

$$P_p = \sum_{q=1}^{n} e_p(e_q G_{pq} + f_q B_{pq}) + f_p(f_q G_{pq} - e_q B_{pq}) \tag{2.21}$$

$$Q_p = \sum_{q=1}^{n} f_p(e_q G_{pq} + f_q B_{pq}) - e_p(f_q G_{pq} - e_q B_{pq}) \tag{2.22}$$

At a voltage controlled bus, r,

$$P_r = \sum_{q=1}^{n} e_r(e_q G_{rq} + f_q B_{rq}) + f_r(f_q G_{rq} - e_q B_{rq}) \qquad (2.21)$$

$$|V_r|^2 = e_r^{\,2} + f_r^{\,2} \tag{2.23}$$

2.5 Formulation of the Newton-Raphson Method in Rectangular Form

2.5.1 Solution Criteria

The equations defining the static operating state of the power system, derived in the previous section, must be satisfied for an exact load flow solution. The criteria for solution will be set up as,

$$|\Delta P_p| = |P_{psched} - P_{pcal}| \le \xi \tag{2.24}$$

$$|\Delta Q_p| = |Q_{psched} - Q_{pcal}| \le \xi \tag{2.25}$$

for load buses and

$$|\Delta P_p| = |P_{psched} - P_{pcal}| \le \xi \qquad (2.24)$$

$$|\Delta|V_p|^2| = ||V_p|^2_{sched} - |V_p|^2_{cal}| \le \xi \qquad (2.25)$$

for voltage controlled buses, where P_{psched}, Q_{psched}, and $|V_p|_{sched}$ are the scheduled variables; P_{cal}, Q_{cal}, and $|V_p|^2_{cal}$ are the variable values as calculated by equations (2.21), (2.22), and (2.23), respectively;

ΔP_p, ΔQ_p, and $\Delta|V_p|^2$ are called the bus mismatches; and ξ is the tolerance within which the bus mismatches must fall to constitute a solution--normally of the order .01 to 10 MW. or MVar or $(kv)^2$.

2.5.2 Formulation For the Load Buses

The Newton-Raphson (N.R.) method requires that a set of linear equations be formed expressing the relationship between the changes in the active and reactive powers and the bus voltage components during the iterative solution procedure. This is achieved using the Taylor expansion. For the system of nonlinear, multivariable equations, f_i, the Taylor expansion is

$$\begin{aligned} f_i\,(x_1+\Delta_1,\ x_2+\Delta_2,\ \ldots) &= f_i\,(x_1,\ x_2,\ \ldots) \\ &+ \frac{1}{1!}\sum_{j=1}^{n} \Delta_j \frac{\partial f_i}{\partial x_j} \\ &+ \frac{1}{2!}\sum_{j=1}^{n}\sum_{k=1}^{n} \Delta_j \Delta_k \frac{\partial^2 f_i}{\partial x_j \partial x_k} \\ &+ \frac{1}{3!}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{\ell=1}^{n} \Delta_j \Delta_k \Delta_\ell \frac{\partial^3 f_i}{\partial x_j \partial x_k \partial x_\ell} \\ &+ \ldots \qquad ,\ i=1,\ 2,\ \ldots n \end{aligned} \tag{2.1}$$

The N.R. method is based on taking the first order approximation of expression (2.1), which is the first two terms

$$f_i\,(x_1 + \Delta_1,\ X_2 + \Delta_2,\ \ldots) = f_i\,(x_1,\ X_2,\ \ldots) + \sum_{j=1}^{n} \Delta_j \frac{\partial f_i}{\partial x_j}$$

$$i=1,\ \ldots,\ n \tag{2.27}$$

Rearranging (2.27) to get it in the form of the mismatch criteria relationships of section 2.5.1,

$$f_i(x_1+\Delta_1,\ x_2+\Delta_2,\ \ldots) - f_i(x_1,\ x_2,\ \ldots) = \sum_{j=1}^{n} \Delta_j \frac{\partial f_i}{\partial x_j}$$

or

$$\Delta f_i = \sum_{j=1}^{n} \Delta_j \frac{\partial f_i}{\partial x_j} \qquad (2.28)$$

Applying this to the load flow equations, the function, f_i, becomes that of the active bus power P_p, or the reactive bus power Q_p, both being functions of the voltages of the system's buses. Thus,

$$P_p(V_1 + \Delta V_1,\ V_2 + \Delta V_2,\ \ldots) - P_p(V_1,\ V_2,\ \ldots) = \sum_{i=1}^{n} \left(\frac{\partial P_p}{\partial V_i} \Delta V_i\right) \qquad (2.29)$$

$$Q_p(V_1 + \Delta V_1,\ V_2 + \Delta V_2,\ \ldots) - Q_p(V_1,\ V_2,\ \ldots) = \sum_{i=1}^{n} \left(\frac{\partial Q_p}{\partial V_i} V_i\right) \qquad (2.30)$$

If,

$$\Delta P_p = P_p(V_1 + \Delta V_1,\ V_2 + \Delta V_2,\ \ldots) - P_p(V_1,\ V_2.\ \ldots)$$

$$\Delta Q_p = Q_p(V_1 + \Delta V_1,\ V_2 + \Delta V_2,\ \ldots) - Q_p(V_1,\ V_2,\ \ldots)$$

and

$$V_i = e_i + jf_i \qquad (2.19)$$

then,

$$\Delta P_p = \sum_{i=1}^{n} \left(\frac{\partial P_p}{\partial e_i} \Delta e_i\right) + \sum_{i=1}^{n} \left(\frac{\partial P_p}{\partial f_i} \Delta f_i\right) \qquad (2.31)$$

$$\Delta Q_p = \sum_{i=1}^{n} \left(\frac{\partial Q_p}{\partial e_i} \Delta e_i\right) + \sum_{i=1}^{n} \frac{\partial Q_p}{\partial f_i} \Delta f_i) \qquad (2.32)$$

(2.31) and (2.32) is the needed set of linear equations expressing the relationship between the changes in active and reactive powers and the components of the bus voltages. In matrix form, for an n-bus system (the nth bus being the slack bus),

$$
\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \vdots \\ \Delta P_{n-1} \\ \hline \Delta Q_1 \\ \Delta Q_2 \\ \vdots \\ \Delta Q_{n-1} \end{bmatrix}
=
\left[\begin{array}{cccc|cccc}
\frac{\partial P_1}{\partial e_1} & \frac{\partial P_1}{\partial e_2} & \cdots & \frac{\partial P_1}{\partial e_{n-1}} & \frac{\partial P_1}{\partial f_1} & \frac{\partial P_1}{\partial f_2} & \cdots & \frac{\partial P_1}{\partial f_{n-1}} \\
\frac{\partial P_2}{\partial e_1} & \frac{\partial P_2}{\partial e_2} & \cdots & \frac{\partial P_2}{\partial e_{n-1}} & \frac{\partial P_2}{\partial f_1} & \frac{\partial P_2}{\partial f_2} & \cdots & \frac{\partial P_2}{\partial f_{n-1}} \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
\frac{\partial P_{n-1}}{\partial e_1} & \frac{\partial P_{n-1}}{\partial e_2} & \cdots & \frac{\partial P_{n-1}}{\partial e_{n-1}} & \frac{\partial P_{n-1}}{\partial f_1} & \frac{\partial P_{n-1}}{\partial f_2} & \cdots & \frac{\partial P_{n-1}}{\partial f_{n-1}} \\
\hline
\frac{\partial Q_1}{\partial e_1} & \frac{\partial Q_1}{\partial e_2} & \cdots & \frac{\partial Q_1}{\partial e_{n-1}} & \frac{\partial Q_1}{\partial f_1} & \frac{\partial Q_1}{\partial f_2} & \cdots & \frac{\partial Q_1}{\partial f_{n-1}} \\
\frac{\partial Q_2}{\partial e_1} & \frac{\partial Q_2}{\partial e_2} & \cdots & \frac{\partial Q_2}{\partial e_{n-1}} & \frac{\partial Q_2}{\partial f_1} & \frac{\partial Q_2}{\partial f_2} & \cdots & \frac{\partial Q_2}{\partial f_{n-1}} \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
\frac{\partial Q_{n-1}}{\partial e_1} & \frac{\partial Q_{n-1}}{\partial e_2} & \cdots & \frac{\partial Q_{n-1}}{\partial e_{n-1}} & \frac{\partial Q_{n-1}}{\partial f_1} & \frac{\partial Q_{n-1}}{\partial f_2} & \cdots & \frac{\partial Q_{n-1}}{\partial f_{n-1}}
\end{array}\right]
\begin{bmatrix} \Delta e_1 \\ \Delta e_2 \\ \vdots \\ \Delta e_{n-1} \\ \hline \Delta f_1 \\ \Delta f_2 \\ \vdots \\ \Delta f_{n-1} \end{bmatrix}
\tag{2.33}
$$

The coefficient matrix of first order partial derivatives is called the Jacobian matrix. In compact form, we have,

$$
\begin{bmatrix} \Delta P \\ \hline \Delta Q \end{bmatrix} = \left[\begin{array}{c|c} J_1 & J_2 \\ \hline J_3 & J_4 \end{array}\right] \begin{bmatrix} \Delta e \\ \hline \Delta f \end{bmatrix} \tag{2.34}
$$

The elements of the Jacobian are derived in Appendix A.

2.5.3 Formulation for the Voltage Controlled Buses

In section 2.4.3 it was noted that the equations governing the voltage controlled buses were,

$$P_p = \sum_{q=1}^{n} [e_p(e_q G_{pq} + f_q B_{pq}) + f_p(f_q G_{pq} - e_q B_{pq})] \tag{2.21}$$

and

$$|V_p|^2 = e_p^2 + f_p^2 \tag{2.23}$$

In the previous section the linear relationship for ΔP in terms of Δe and Δf was found. In the same way, using the Taylor expansion as a basis, the linear relationship for $\Delta|V|^2$ is

$$\Delta|V_p|^2 = \sum_{i=1}^{n} (\frac{\partial|V_p|^2}{\partial e_i} \Delta e_i) + \sum_{i=1}^{n} (\frac{\partial|V_p|^2}{\partial f_i} \Delta f_i) \tag{2.35}$$

This leads us to the complete linear relationships for the system of n buses of which there are m voltage controlled buses (one of which is the slack bus). This is given by (2.36). In compact form, (2.36 is represented as

$$\begin{bmatrix} \Delta P \\ \hline \Delta Q \\ \hline \Delta|V|^2 \end{bmatrix} = \left[\begin{array}{c|c} J_1 & J_2 \\ \hline J_3 & J_4 \\ \hline J_5 & J_6 \end{array}\right] \begin{bmatrix} \Delta e \\ \hline \Delta f \end{bmatrix} \tag{2.37}$$

where J_1, J_2, J_3, and J_4 are the same as in equation (2.34). The Jacobian elements for J_5 and J_6 are derived in Appendix A, along with those of J_1, J_2, J_3, and J_4.

2.5.4 Step by Step Procedure

Having obtained the relationship (2.37), the basic step by step procedure of the N.R. method can be listed, briefly as follows:

1) Form the bus admittance matrix, [Y]
2) Initialize the bus voltage components, e_i and f_i
3) Calculate P_i, Q_i, and $|V_j|^2$ using equations (2.21), (2.22), and (2.23), respectively.

$$
\left[\begin{array}{c}
\Delta P_1 \\
\Delta P_2 \\
\vdots \\
\Delta P_{n-1} \\
\hdashline
\Delta Q_1 \\
\vdots \\
\Delta Q_{n-m} \\
\hdashline
\Delta |V_{n-m+1}|^2 \\
\vdots \\
\Delta |V_{n-1}|^2
\end{array}\right]
=
\left[\begin{array}{cccc:cccc}
\frac{\partial P_1}{\partial e_1} & \frac{\partial P_1}{\partial e_2} & \cdots & \frac{\partial P_1}{\partial e_{n-1}} & \frac{\partial P_1}{\partial f_1} & \frac{\partial P_1}{\partial f_2} & \cdots & \frac{\partial P_1}{\partial f_{n-1}} \\
\frac{\partial P_2}{\partial e_1} & \frac{\partial P_2}{\partial e_2} & \cdots & \frac{\partial P_2}{\partial e_{n-1}} & \frac{\partial P_2}{\partial f_1} & \frac{\partial P_2}{\partial f_2} & \cdots & \frac{\partial P_2}{\partial f_{n-1}} \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
\frac{\partial P_{n-1}}{\partial e_1} & \frac{\partial P_{n-1}}{\partial e_2} & \cdots & \frac{\partial P_{n-1}}{\partial e_{n-1}} & \frac{\partial P_{n-1}}{\partial f_1} & \frac{\partial P_{n-1}}{\partial f_2} & \cdots & \frac{\partial P_{n-1}}{\partial f_{n-1}} \\
\hdashline
\frac{\partial Q_1}{\partial e_1} & \frac{\partial Q_1}{\partial e_2} & \cdots & \frac{\partial Q_1}{\partial e_{n-1}} & \frac{\partial Q_1}{\partial f_1} & \frac{\partial Q_1}{\partial f_2} & \cdots & \frac{\partial Q_1}{\partial f_{n-1}} \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
\frac{\partial Q_{n-m}}{\partial e_1} & \frac{\partial Q_{n-m}}{\partial e_2} & \cdots & \frac{\partial Q_{n-m}}{\partial e_{n-1}} & \frac{\partial Q_{n-m}}{\partial f_1} & \frac{\partial Q_{n-m}}{\partial f_2} & \cdots & \frac{\partial Q_{n-m}}{\partial f_{n-1}} \\
\hdashline
\frac{\partial |V_{n-m+1}|^2}{\partial e_1} & \frac{\partial |V_{n-m+1}|^2}{\partial e_2} & \cdots & \frac{\partial |V_{n-m+1}|^2}{\partial e_{n-1}} & \frac{\partial |V_{n-m+1}|^2}{\partial f_1} & \frac{\partial |V_{n-m+1}|^2}{\partial f_2} & \cdots & \frac{\partial |V_{n-m+1}|^2}{\partial f_{n-1}} \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\
\frac{\partial |V_{n-1}|^2}{\partial e_1} & \frac{\partial |V_{n-1}|^2}{\partial e_2} & \cdots & \frac{\partial |V_{n-1}|^2}{\partial e_{n-1}} & \frac{\partial |V_{n-1}|^2}{\partial f_1} & \frac{\partial |V_{n-1}|^2}{\partial f_2} & \cdots & \frac{\partial |V_{n-1}|^2}{\partial f_{n-1}}
\end{array}\right]
\left[\begin{array}{c}
\Delta e_1 \\
\Delta e_2 \\
\vdots \\
\Delta e_{n-1} \\
\hdashline
\Delta f_1 \\
\vdots \\
\Delta f_{n-m} \\
\hdashline
\Delta f_{n-m+1} \\
\vdots \\
\Delta f_{n-1}
\end{array}\right]
\qquad (2.36)
$$

4) Calculate ΔP_i, ΔQ_i and $\Delta|V_j|^2$ with

$$\Delta P_i = P_{i\,sched} - P_i \tag{2.38}$$

$$\Delta Q_i = Q_{i\,sched} - Q_i \tag{2.39}$$

$$\Delta|V_j|^2 = |V_{j\,sched}|^2 - |V_j|^2 \tag{2.40}$$

5) Check to see if ΔP_i, ΔQ_i, and $\Delta|V_j|^2$ are within the specified tolerance, ξ. If so, solution has been achieved. If not, continue.

6) Calculate the Jacobian matrix elements

7) Solve the set of equations (2.36) for Δe_i and Δf_i. That is, compute

$$\begin{bmatrix} \Delta e \\ \hline \\ \Delta f \end{bmatrix} = \left[\begin{array}{c|c} J_1 & J_2 \\ \hline J_3 & J_4 \\ \hline J_5 & J_6 \end{array}\right]^{-1} \begin{bmatrix} \Delta P \\ \hline \Delta Q \\ \hline \Delta|V|^2 \end{bmatrix} \tag{2.41}$$

8) Update e_i and f_i,

$$e_i^{(k+1)} = e_i^{(k)} + \Delta e_i^{(k)} \tag{2.42}$$

$$f_i^{(k+1)} = f_i^{(k)} + \Delta f_i^{(k)} \tag{2.43}$$

9) Go to step 3.

CHAPTER III

DERIVATION OF SOME ALPHA-MODIFIED QUASI-SECOND ORDER NEWTON RAPHSON METHODS

3.1 Introduction

In the previous chapter theory was developed upon which the work reported in this thesis is based. Section 3.2 briefly illustrates how the Newton-Raphson method is extended to obtain a second order Newton-Raphson technique (S.O.N.R.) and then an alpha-modified second order Newton Raphson procedure for solving a set of nonlinear simultaneous equations. Section 3.3 extends the Newton-Raphson derivation outlined in the previous chapter, by detailing the derivation of the S.O.N.R. and its application to the load flow problem. This is what we can call a Quasi-Second Order Newton Raphson (Q.S.O.N.R.) method in retangular form. Section 3.4 builds upon this by developing a number of alpha-modified quasi-second order Newton-Raphson (M.Q.S.O.N.R.) methods, one of which is programmed and tested on a number of power systems, the results of which are presented and analysed in chapter 4.

3.2 Description of the method

A second order Newton-Raphson is an obvious extension of the Newton-Raphson method. It is obtained by simply taking the second order terms of the Taylor expansion, for a multivariable function, into account. This results in

$$f_i(x_1+\Delta_1,\ x_2+\Delta_2,\ \ldots) = f_i(x_1,\ x_2,\ \ldots) + \sum_{j=1}^{n} \Delta_j \frac{\partial f_i}{\partial x_j}$$

$$+ \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} \Delta_j \Delta_k \frac{\partial^2 f_i}{\partial x_j \partial x_k} \tag{3.1}$$

we now have to solve a system of second-order equations in the increments Δ. Unless n=1, this will not have an explicit solution. The above expression can be written in vector form as,

$$\Delta f_i = \underline{\nabla f_i^T} \cdot \underline{\Delta} + \frac{1}{2} \underline{\Delta}^T \underline{H_i} \Delta \tag{3.2}$$

Here $\underline{\Delta f_i}$ is the gradient of f_i, and $\underline{H_i}$ is the Hessian, which is the matrix of second-order derivatives.

Many alternative forms exist for implementing the above relationship. In the next section, the Q.S.O.N.R. method is presented and, in section 3.4, the alpha-modified algorithms are derived. The basic idea of an alpha-modified method can be shown by rewriting equation (3.2) in the following form,

$$\Delta f_i - \alpha_i \left[\frac{1}{2} \underline{\Delta}^T \underline{H_i} \underline{\Delta}\right] = \left[\underline{\nabla f_i} + (1-\alpha_i)\left(\frac{1}{2} \underline{\Delta}^T \underline{H_i}\right)\right] \underline{\Delta} \tag{3.3}$$

This is in fact the basis for the M.Q.S.O.N.R. method number one which is derived and applied to the load flow problem in subsection 3.4.2.

3.3 Formulation of the Quasi-Second Order Newton-Raphson method in Rectangular Form

3.3.1 Derivation of the active power second order correction factors

With the Newton Raphson method, only the first order terms of the Taylor expansion are used [equation (2.27)]. Now the second order terms will be ultilized. This results in,

$$f_i(x_1+\Delta_1, x_2+\Delta_2, \ldots) - f_i(x_1, x_2, \ldots) = \sum_{j=1}^{n} \left(\frac{\partial f_i}{\partial x_j} \Delta_j\right)$$

$$+ \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} \left(\Delta_j \Delta_k \frac{\partial^2 f_i}{\partial x_j \partial x_k}\right)$$

$$+ \frac{1}{2} \sum_{j=1}^{n} (\Delta_j^2 \frac{\partial^2 f_i}{\partial x_j^2}) \tag{3.4}$$

Equation (3.4) is of the form,

$$\Delta f_i = D_1 + D_2 + D_3 \tag{3.5}$$

where, D_1 represents the first order terms and D_2 and D_3 represent the second order terms.

To obtain the active power correction factors we write

$$\Delta P_i = D_{1p} + D_{2p} + D_{3p} \tag{3.6}$$

this relates the change in active powers with the change in the bus voltages, Δe_i and Δf_i.

The first order terms are the same as those derived in Section 2.5. For the active power component, they are,

$$D_{1p} = \sum_{\substack{q=1 \\ q \neq p}}^{n-1} \frac{\partial P_p}{\partial e_q} (\Delta e_q) + \frac{\partial P_p}{\partial e_p} (\Delta e_p)$$

$$+ \sum_{\substack{q=1 \\ q \neq p}}^{n-1} \frac{\partial P_p}{\partial f_p} (\Delta f_q) + \frac{\partial P_p}{\partial f_p} (\Delta f_p) \tag{3.7}$$

The second order terms are D_{2p} and D_{3p}.

$$D_{2p} = \frac{1}{2} \sum_{\substack{q=1 \\ q \neq p}}^{n-1} \frac{\partial^2 P_p}{\partial e_q^2} (\Delta e_q)^2 + \frac{1}{2} \sum_{\substack{q=1 \\ 1 \neq p}}^{n-1} \frac{\partial^2 P_p}{\partial f_q^2} (\Delta f_q)^2$$

$$+ \frac{1}{2} \frac{\partial^2 P_p}{\partial e_p^2} (\Delta e_p)^2 + \frac{1}{2} \frac{\partial^2 P_p}{\partial f_p^2} (\Delta f_p)^2$$

, which reduces to

$$D_{2p} = \frac{1}{2}\frac{\partial^2 P_p}{\partial e_p^2}(\Delta e_p)^2 + \frac{1}{2}\frac{\partial^2 P_p}{\partial f_p^2}(\Delta f_p)^2$$

Evaluating the second order terms gives

$$D_{2p} = G_{pp}\,(\Delta e_p)^2 + G_{pp}\,(\Delta f_p)^2 \tag{3.8}$$

All the second order partial derivatives (Hessian terms) are derived in Appendix B.

Now for the rest of the second order terms, D_{3p}

$$D_{3p} = A_p + B_p + C_p \tag{3.9}$$

where

$$A_p = \sum_{q=1}^{n-1}\sum_{r=1}^{n-1}\frac{\partial^2 P_p}{\partial e_q \partial f_r}(\Delta e_q)\,(\Delta f_r)$$

$$B_p = \sum_{q=1}^{n-2}\sum_{r=q+1}^{n-1}\frac{\partial^2 P_p}{\partial e_q \partial e_r}(\Delta e_q)\,(\Delta e_r)$$

$$C_p = \sum_{q=1}^{n-2}\sum_{r=q+1}^{n-1}\frac{\partial^2 P_p}{\partial f_q \partial f_r}(\Delta f_q)\,(\Delta f_r)$$

Now, expand and reduce the above terms. Starting with,

$$A_p = \sum_{q=1}^{n-1}\sum_{r=1}^{n-1}\frac{\partial^2 P_p}{\partial e_q \partial f_r}(\Delta f_r)\,(\Delta e_q)$$

This is equivalent to

$$A_p = \sum_{\substack{q=1\\ q\neq p}}^{n-1}\Delta e_q\left[\sum_{r=1}^{n-1}\frac{\partial^2 P_p}{\partial e_q \partial f_r}\Delta f_r\right] + \Delta e_p\sum_{r=1}^{n-1}\frac{\partial^2 P_p}{\partial e_p \partial f_r}\Delta f_r$$

Expanding,

$$A_p = \sum_{\substack{q=1\\q\neq p}}^{n-1} \Delta e_q \left[\sum_{\substack{r=1\\r\neq p}}^{n-1} \frac{\partial^2 P_p}{\partial e_q \partial f_r} \Delta f_r \right] + \sum_{\substack{q=1\\q\neq p}}^{n-1} \Delta e_q \ \Delta f_p \frac{\partial^2 P_p}{\partial e_q \partial f_p}$$

$$+ \Delta e_p \sum_{\substack{r=1\\r\neq p}}^{n-1} \frac{\partial^2 P_p}{\partial e_p \partial f_r} \Delta f_r + \Delta e_p \ \Delta f_p \frac{\partial^2 P_p}{\partial e_p \partial f_p}$$

This reduces to,

$$A_p = \sum_{\substack{q=1\\q\neq p}}^{n-1} \Delta e_p \ \Delta f_p \ (-B_{pq}) + \Delta e_p \sum_{\substack{r=1\\r\neq p}}^{n-1} B_{pr} \ \Delta f_r$$

In more compact form, the above expression is,

$$A_p = \sum_{\substack{r=1\\r\neq p}}^{n-1} B_{pr} \ [\Delta e_p \ \Delta f_r - \Delta f_p \ \Delta e_r] \tag{3.10}$$

Next, the term,

$$B_p = \sum_{q=1}^{n-2} \sum_{r=q+1}^{n-1} \frac{\partial^2 P_p}{\partial e_q \partial e_r} \ (\Delta e_q) \ (\Delta e_r)$$

This term can be expanded in the following way:

$$B_p = \sum_{q=1}^{p-1} \sum_{\substack{r=q+1\\r\neq p}}^{n-1} \frac{\partial^2 P_p}{\partial e_q \partial e_r} \ (\Delta e_q) \ (\Delta e_r) + \sum_{q=1}^{p-1} \frac{\partial^2 P_p}{\partial e_q \partial e_p} \ (\Delta e_q) \ (\Delta e_p)$$

$$+ \sum_{q=p+1}^{n-2} \sum_{r=q+1}^{n-1} \frac{\partial^2 P_p}{\partial e_q \partial e_r} \ (\Delta e_q) \ (\Delta e_r) + \sum_{r=p+1}^{n-1} \frac{\partial^2 P_p}{\partial e_p \partial e_r} \ (\Delta e_p) \ (\Delta e_r)$$

As a result of the following relationship,

$$\frac{\partial^2 P_p}{\partial e_q \partial e_r} = 0 \text{ for } r \neq p \text{ or } q \neq p$$

the expression for B_p reduces to,

$$B_p = \sum_{q=1}^{p-1} \frac{\partial^2 P_p}{\partial e_q \partial e_p} (\Delta e_q)\ (\Delta e_p) + \sum_{r=p+1}^{n-1} \frac{\partial^2 P_p}{\partial e_p \partial e_r} (\Delta e_p)\ (\Delta e_r)$$

Substituting the values of the Hessian terms yields,

$$B_p = \Delta e_p \sum_{q=1}^{p-1} G_{pq}\ \Delta e_q + \Delta e_p \sum_{r=p+1}^{n-1} G_{pr}\ \Delta e_r$$

which is,

$$B_p = \Delta e_p \sum_{\substack{r=1 \\ r \neq p}}^{n-1} G_{pr}\ \Delta e_r \tag{3.11}$$

Now for the last part of equation (3.9) which is

$$C_p = \sum_{q=1}^{n-2} \sum_{r=q+1}^{n-1} \frac{\partial^2 P_p}{\partial f_q \partial f_p} (\Delta f_q)\ (\Delta f_r)$$

$$= \sum_{q=1}^{p-1} \sum_{\substack{r=q+1 \\ r \neq p}}^{n-1} \frac{\partial^2 P_p}{\partial f_q \partial f_r} (\Delta f_q)\ (\Delta f_r)$$

$$+ \sum_{q=1}^{p-1} \frac{\partial^2 P_p}{\partial f_q \partial f_p} (\Delta f_q)\ (\Delta f_p)$$

$$= \sum_{q=p+1}^{n-2} \sum_{r=q+1}^{n-1} \frac{\partial^2 P_p}{\partial f_q \partial f_r} (\Delta f_q)\ (\Delta f_r)$$

$$+ \sum_{r=p+1}^{n-1} \frac{\partial^2 P_p}{\partial f_p \partial f_r} (\Delta f_p)\ (\Delta f_r)$$

Since

$$\frac{\partial^2 P_p}{\partial f_q \partial f_r} = 0 \text{ for } r \neq p \text{ or } q \neq p$$

the expression for C_p reduces to,

$$C_p = \sum_{q=1}^{p-1} \frac{\partial^2 P_p}{\partial f_q \partial f_p} (\Delta f_q)(\Delta f_p)$$

$$+ \sum_{r=p+1}^{n-1} \frac{\partial^2 P_p}{\partial f_p \partial f_r} (\Delta f_p)(\Delta f_r)$$

Substituting the values of the Hessian terms yields

$$C_p = \Delta f_p \sum_{q=1}^{p-1} G_{pq} \, \Delta f_q + \Delta f_p \sum_{r=p+1}^{n-1} G_{pr} \, \Delta f_r$$

which, in more compact form, is

$$C_p = \Delta f_p \sum_{\substack{r=1 \\ r \neq p}}^{n-1} G_{pr} \, \Delta f_r \tag{3.12}$$

Thus, equation (3.9) is reduced, using (3.10), (3.11), and (3.12), to

$$D_{3p} = \Delta e_p \sum_{\substack{r=1 \\ r \neq p}}^{n-1} [B_{pr} \, \Delta f_r + G_{pr} \, \Delta e_r]$$

$$+ \Delta f_p \sum_{\substack{r=1 \\ r \neq p}}^{n-1} [G_{pr} \, \Delta f_r - B_{pr} \, \Delta e_r] \tag{3.13}$$

Combining equations (3.8) and (3.13) yields the second order terms of the Taylor expansion in the form,

$$R_p = D_{2p} + D_{3p} = \Delta e_p \sum_{r=1}^{n-1} [B_{pr} \, \Delta f_r + G_{pr} \, \Delta e_r]$$

$$+ \Delta f_p \sum_{r=1}^{n-1} [G_{pr} \, \Delta f_r - B_{pr} \, \Delta e_r] \tag{3.14}$$

which are the active power second order correction factors.

3.3.2 Derivation of the reactive power second order correction factors

From equation (3.4), we write,

$$\Delta Q_i = D_{1Q} + D_{2Q} + D_{3Q} \tag{3.15}$$

The first order terms are,

$$D_{1Q} = \sum_{\substack{q=1 \\ q \neq p}}^{n-1} \frac{\partial Q_p}{\partial e_q} \Delta e_q + \frac{\partial Q_p}{\partial e_p} (\Delta e_p)$$

$$+ \sum_{\substack{q=1 \\ q \neq p}}^{n-1} \frac{\partial Q_p}{\partial f_q} (\Delta f_q) + \frac{\partial Q_p}{\partial f_p} (\Delta f_p) \tag{3.16}$$

The second order terms, D_{2Q} and D_{3Q}, are

$$D_{2Q} = \frac{1}{2} \sum_{\substack{q=1 \\ q \neq p}}^{n-1} \frac{\partial^2 Q_p}{\partial e^2_q} (\Delta e_q)^2 + \frac{1}{2} \sum_{\substack{q=1 \\ q \neq p}}^{n-1} \frac{\partial^2 Q_p}{\partial f^2_q} (\Delta f_q)^2$$

$$+ \frac{1}{2} \frac{\partial^2 Q_p}{\partial e^2_p} (\Delta e_p)^2 + \frac{1}{2} \frac{\partial^2 Q_p}{\partial f^2_p} (\Delta f_p)^2$$

which reduces to

$$D_{2Q} = \frac{1}{2} \frac{\partial^2 Q_p}{\partial e^2_p} (\Delta e_p)^2 + \frac{1}{2} \frac{\partial^2 Q_p}{\partial f^2_p} (\Delta f_p)^2$$

which, in turn, is

$$D_{2Q} = B_{pp} (\Delta e_p)^2 + B_{pp} (\Delta f_p)^2 \tag{3.17}$$

As well,

$$D_{3Q} = A_Q + B_Q + C_Q \tag{3.18}$$

where,

$$A_Q = \sum_{q=1}^{n-1} \sum_{r=1}^{n-1} \frac{\partial^2 Q_p}{\partial e_q \partial f_r} (\Delta e_q)(\Delta f_r)$$

$$B_Q = \sum_{q=1}^{n-2} \sum_{r=q+1}^{n-1} \frac{\partial^2 Q_p}{\partial e_q \partial e_r} (\Delta e_q)(\Delta e_r)$$

$$C_Q = \sum_{q=1}^{n-2} \sum_{r=q+1}^{n-1} \frac{\partial^2 Q_p}{\partial f_q \partial f_r} (\Delta f_q)(\Delta f_r)$$

First, expanding A_Q,

$$A_Q = \sum_{\substack{q=1 \\ q\neq p}}^{n-1} \Delta e_q \left[\sum_{r=1}^{n-1} \frac{\partial^2 Q_p}{\partial e_q \partial f_r} \Delta f_r \right]$$

$$+ \Delta e_p \sum_{r=1}^{n-1} \frac{\partial^2 Q_p}{\partial e_p \partial f_r} \Delta f_r$$

Expanding again yields,

$$A_Q = \sum_{\substack{q=1 \\ q\neq p}}^{n-1} \Delta e_q \sum_{\substack{r=1 \\ r\neq p}}^{n-1} \frac{\partial^2 Q_p}{\partial e_p \partial f_r} \Delta f_r + \sum_{\substack{q=1 \\ q\neq p}}^{n-1} \Delta e_q \, \Delta f_p \frac{\partial^2 Q_p}{\partial e_q \partial f_p}$$

$$+ \Delta e_p \sum_{\substack{r=1 \\ r\neq p}}^{n-1} \frac{\partial^2 Q_p}{\partial e_p \partial f_r} \Delta f_r + \Delta e_p \, \Delta f_p \frac{\partial^2 Q_p}{\partial e_p \partial f_p}$$

Substituting the values of the Hessian terms reduces the above expression to,

$$A_Q = \Delta f_p \sum_{\substack{q=1 \\ q\neq p}}^{n-1} G_{pq} \Delta e_q + \Delta e_p \sum_{\substack{r=1 \\ r\neq p}}^{n-1} - G_{pr} \Delta f_r$$

which is, in a more compact form,

$$A_Q = \sum_{\substack{r=1 \\ r\neq p}}^{n-1} G_{pr} \left[\Delta f_p \, \Delta e_r - \Delta e_p \, \Delta f_r\right] \tag{3.19}$$

Now for the next term,

$$B_Q = \sum_{q=1}^{n-2} \sum_{r=q+1}^{n-1} \frac{\partial^2 Q_p}{\partial e_q \partial e_r} (\Delta e_q)(\Delta e_r)$$

Expanding,

$$B_Q = \sum_{q=1}^{p-1} \sum_{\substack{r=q+1 \\ r\neq p}}^{n-1} \frac{\partial^2 Q_p}{\partial e_q \partial e_r} (\Delta e_q)(\Delta e_r)$$

$$+ \sum_{q=1}^{p-1} \frac{\partial^2 Q_p}{\partial e_q \partial e_p} (\Delta e_q)(\Delta e_p)$$

$$+ \sum_{q=p+1}^{n-2} \sum_{r=q+1}^{n-1} \frac{\partial^2 Q_p}{\partial e_q \partial e_r} (\Delta e_q)(\Delta e_r)$$

$$+ \sum_{r=p+1}^{n-1} \frac{\partial^2 Q_p}{\partial e_p \partial e_r} (\Delta e_p)(\Delta e_r)$$

Since

$$\frac{\partial^2 Q_p}{\partial e_q \partial e_r} = 0 \text{ for } r \neq p \text{ or } q \neq p$$

we have,

$$B_Q = \sum_{q=1}^{p-1} \frac{\partial^2 Q_p}{\partial e_q \partial e_p} (\Delta e_q)(\Delta e_p)$$

$$+ \sum_{r=p+1}^{n-1} \frac{\partial^2 Q_p}{\partial e_p \partial e_r} (\Delta e_p)(\Delta e_r)$$

Substituting in the values of the Hessian terms yields,

$$B_Q = \Delta e_p \sum_{q=1}^{p-1} B_{pq} \, \Delta e_q + \Delta e_p \sum_{r=p+1}^{n-1} B_{pr} \, \Delta e_r$$

and in compact form

$$B_Q = \Delta e_p \sum_{\substack{r=1 \\ r\neq p}}^{n-1} B_{pr} \, \Delta e_r \tag{3.20}$$

Finally,

$$C_Q = \sum_{q=1}^{n-2} \sum_{r=q+1}^{n-1} \frac{\partial^2 Q_p}{\partial f_q \partial f_r} (\Delta f_q)\ (\Delta f_r)$$

Expanding,

$$C_Q = \sum_{q=1}^{p-1} \sum_{\substack{r=q+1 \\ r \neq p}}^{n-1} \frac{\partial^2 Q_p}{\partial f_q \partial f_r} (\Delta f_q)\ (\Delta f_r)$$

$$+ \sum_{q=1}^{p-1} \frac{\partial^2 Q_p}{\partial f_q \partial f_p} (\Delta f_q)\ (\Delta f_p)$$

$$+ \sum_{q=p+1}^{n-2} \sum_{r=q+1}^{n-1} \frac{\partial^2 Q_p}{\partial f_q \partial f_r} (\Delta f_q)\ (\Delta f_r)$$

$$+ \sum_{r=p+1}^{n-1} \frac{\partial^2 Q_p}{\partial f_p \partial f_r} (\Delta f_p)\ (\Delta f_r)$$

Since

$$\frac{\partial^2 Q_p}{\partial f_q \partial f_r} = 0 \text{ for } r \neq p \text{ or } q \neq p$$

the above is essentially,

$$C_Q = \sum_{q=1}^{p-1} \frac{\partial^2 Q_p}{\partial f_q \partial f_p} (\Delta f_q)\ (\Delta f_p)$$

$$+ \sum_{r=p+1}^{n-1} \frac{\partial^2 Q_p}{\partial f_p \partial f_r} (\Delta f_p)\ (\Delta f_r)$$

Using the derivative expressions we then have

$$C_Q = \Delta f_p \sum_{q=1}^{p-1} B_{pq}\ \Delta f_q + \Delta f_p \sum_{r=p+1}^{n-1} B_{pr}\ \Delta f_r$$

More compactly,

$$C_Q = \Delta f_p \sum_{\substack{r=1 \\ r \neq p}}^{n-1} B_{pr}\ \Delta f_r \tag{3.21}$$

Thus, equation (3.18) is reduced, using (3.19), (3.20) and (3.21), to

$$D_{3Q} = \Delta f_p \sum_{\substack{r=1 \\ r\neq p}}^{n-1} (G_{pr}\,\Delta e_r + B_{pr}\,\Delta f_r)$$

$$+ \Delta e_p \sum_{r\neq p}^{n-1} [B_{pr}\,\Delta e_r - G_{pr}\,\Delta f_r] \qquad (3.22)$$

Combining equation (3.17) with equation (3.22) yields the second order terms of the Taylor expansion in the form,

$$T_p = D_{2Q} + D_{3Q} = \Delta f_p \sum_{r=1}^{n-1} [G_{pr}\,\Delta e_r + B_{pr}\,\Delta f_r]$$

$$+ \Delta e_p \sum_{r=1}^{n-1} [B_{pr}\,\Delta e_r - G_{pr}\,\Delta f_r] \qquad (3.23)$$

which are the reactive power second order correction factors.

3.3.3 Derivation of the voltage magnitude second order correction factors

Again, for voltage controlled buses a relationship is needed that relates the change in the bus voltage magnitudes with the changes in the components of the bus voltages, Δe_i and Δf_i. Here we desire the relationship to include second order terms. This is a very simple derivation to perform. From the Taylor expansion it can be written that,

$$\Delta|V_i|^2 = \sum_{j=1}^{n} \frac{\partial|V_i|^2}{\partial e_j}\Delta e_j + \sum_{j=1}^{n} \frac{\partial|V_i|^2}{\partial f_j}\Delta f_j$$

$$+ \frac{1}{2}\sum_{j=1}^{n}\sum_{k=1}^{n} \Delta e_j\,\Delta f_k\,\frac{\partial^2|V_i|^2}{\partial e_j \partial f_k}$$

$$+ \frac{1}{2} \sum_{j=1}^{n} \sum_{k=1}^{n} \Delta f_j \; \Delta e_k \frac{\partial^2 |V_i|^2}{\partial f_j \partial e_k}$$

$$+ \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} \Delta e_j \; \Delta e_k \frac{\partial^2 |V_i|^2}{\partial e_j \partial e_k}$$

$$+ \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} \Delta f_j \; \Delta f_k \frac{\partial^2 |V_i|^2}{\partial f_j \partial f_k} \qquad (3.24)$$

Since the relationship for $|V_i|^2$ is simply,

$$|V_i|^2 = e_i^{\,2} + f_i^{\,2} \qquad (2.23)$$

the first and second order partial derivatives of $|V_i|^2$ will be derived here. Doing this will show how easily (3.24) may then be reduced.

$$\frac{\partial |V_i|^2}{\partial e_j} = 2e_j \quad , \quad j = i$$
$$= 0 \quad , \quad j \neq i$$

$$\frac{\partial |V_i|^2}{\partial f_j} = 2f_j \quad , \quad j = i$$
$$= 0 \quad , \quad j \neq i$$

$$\frac{\partial^2 |V_i|^2}{\partial e_j \partial f_k} = 0 \quad , \quad \text{for all } j \text{ and } k$$

$$\frac{\partial^2 |V_i|^2}{\partial f_j \partial e_k} = 0 \quad , \quad \text{for all } j \text{ and } k$$

$$\frac{\partial^2 |V_i|^2}{\partial e_j \partial e_k} = 2 \quad , \quad j = k = i$$
$$= 0 \quad , \quad \text{otherwise}$$

$$\frac{\partial^2 |V_i|^2}{\partial f_j \partial f_k} = 2 \quad , \quad j = k = i$$
$$= 0 \quad , \quad \text{otherwise}$$

With these derivations, (3.24) is quickly reduced to,

$$\Delta|V_i| = 2e_i\,\Delta e_i + 2f_i\,\Delta f_i + \Delta e_i^{\,2} + \Delta f_i^{\,2} \tag{3.25}$$

with the second order factors being

$$U_i = \Delta e_i^{\,2} + \Delta f_i^{\,2} \tag{3.26}$$

3.3.4 Step by step procedure for the Quasi-second order Newton-Raphson method

Having derived the second order correction factors in equations (3.14), (3.23), and (3.26) for ΔP_i, ΔQ_i, and $\Delta|V_i|^2$ respectively, the basic step by step procedure of the Q.S.O.N.R. is now listed, briefly, as follows:

1) Form the bus admittance matrix, [Y]

2) Initialize the bus voltages components, e_i and f_i

3) Calculate P_i, Q_i, and $|V_j|^2$ using equations (2.21), (2.22) and (2.23), respectively

4) Calculate the power and voltage magnitudes mismatches, ΔP_i, ΔQ_i, and $\Delta|V_j|^2$, with,

$$\Delta P_i = P_{i\,SCHED} - P_i \tag{2.38}$$

$$\Delta Q_i = Q_{i\,SCHED} - Q_i \tag{2.39}$$

$$\Delta|V_j|^2 = |V_{j\,SCHED}|^2 - |V_j|^2 \tag{2.40}$$

5) Check to see if ΔP_i, ΔQ_i, and $\Delta|V_j|^2$ are within the specified tolerance, ξ. If so, solution is achieved. If not, continue to step 6.

6) Calculate the Jacobian matrix elements

7) Solve the set of equations (2.36) for Δe_i and Δf_i. That is, compute

$$\begin{bmatrix} \Delta e \\ \hline \\ \Delta f \end{bmatrix} = \left[\begin{array}{c|c} J_1 & J_2 \\ \hline J_3 & J_4 \\ \hline J_5 & J_6 \end{array}\right]^{-1} \begin{bmatrix} \Delta P \\ \hline \Delta Q \\ \hline \Delta |V|^2 \end{bmatrix} \tag{2.41}$$

8) Check to see if a solution has been achieved. If so, procedure stops, only needing to go through one half of the present iteration. If not, continue.

9) Calculate the second order correction factors, R_p, T_p, and U_i with equations (3.14), (3.23), and (3.26) respectively, using the values of Δe_j and Δf_j obtained in step (7)

10) Update the power and voltage magnitude mismatches by subtracting the correction factors of step (9) from the original mismatches of step (4)

$$\Delta P_p' = \Delta P_p - R_p$$

$$\Delta Q_p' = \Delta Q_p - T_p$$

$$\Delta |V_i|^{2'} = \Delta |V_i|^2 - U_i$$

11) With the new mismatch values from step (10), solve (3.36) for new values of Δe_i and Δf_i. The same Jacobian found in step (6) and used in step (7) is again used here.

$$\begin{bmatrix} \Delta e' \\ \hline \\ \Delta f' \end{bmatrix} = \left[\begin{array}{c|c} J_1 & J_2 \\ \hline J_3 & J_4 \\ \hline J_5 & J_6 \end{array}\right]^{-1} \begin{bmatrix} \Delta P' \\ \hline \Delta Q' \\ \hline \Delta |V|^{2'} \end{bmatrix}$$

12) With the values of the voltage changes found in step (11), update the bus voltages.

$$e_i^{(k+1)} = e_i^{(k)} + \Delta e_i'^{(k)}$$

$$f_i^{(k+1)} = f_i^{(k)} + \Delta f_i'^{(k)}$$

13) Go to step (3).

It is worth noting here that steps (1) to (7), which constitutes one half iteration of the Q.S.O.N.R., is, in fact, the Newton-Raphson method (when combined with steps (12) and (13)).

3.4 Alpha-Modified Quasi-Second Order Newton-Raphson Method

3.4.1 Introduction

The most direct way of incorporating the second order correction factors, derived in subsections 3.3.1, 3.3.2, and 3.6.3, into a load flow procedure is with the Q.S.O.N.R. method described in section 3.3.4. Now, in this section, four alternative means of applying the factors are presented. The first two of these alternatives will each be shown to be simply a special case of a general alpha-modified technique. The other two will be immediately written in general alpha-modified forms. In each of the first two general techniques, the Q.S.O.N.R. is also shown to be a special case.

The first alpha-modified method was studied, applied, and analysed in detail. The results are presented and discussed in Chapter 4.

The other three alpha-modified methods were not persued beyond the formulation stage, even though they are fairly straightforward, due to time limitations.

3.4.2 M.Q.S.O.N.R. method number 1

First, we will rewrite the linear relationships which includes the second order terms and which are solved in the Q.S.O.N.R. for the voltage changes [step (11) of Q.S.O.N.R.].

From the N.R. procedure, we have

$$\Delta P_p = NRCP \tag{2.31}$$

$$\Delta Q_p = NRCQ \tag{2.32}$$

$$\Delta |V_p|^2 = NRCV \tag{2.35}$$

where,

$$NRCP = \sum_{i=1}^{n-1} \frac{\partial P_p}{\partial e_i} \Delta e_i + \sum_{i=1}^{n-1} \frac{\partial P_p}{\partial f_i} \Delta f_i$$

$$NRCQ = \sum_{i=1}^{n-1} \frac{\partial Q_p}{\partial e_i} \Delta e_i + \sum_{i=1}^{n-1} \frac{\partial Q_p}{\partial f_i} \Delta f_i$$

$$NRCV = \sum_{i=1}^{n-1} \frac{\partial |V_p|^2}{\partial e_i} \Delta e_i + \sum_{i=1}^{n-1} \frac{\partial |V_p|^2}{\partial f_i} \Delta f_i$$

Also,

$$\Delta P_p = P_{pSCHED} - P_p \tag{2.38}$$

$$\Delta Q_p = Q_{pSCHED} - Q_p \tag{2.39}$$

$$\Delta |V_p|^2 = |V_{pSCHED}|^2 - |V_p|^2 \tag{2.40}$$

The second order correction factors are,

$$R_p = \Delta e_p\ (CR1) + \Delta f_p\ (CR2) \tag{3.14}$$

$$T_p = \Delta f_p\ (CR1) - \Delta e_p\ (CR2) \tag{3.23}$$

$$U_p = \Delta e_p\ (\Delta e_p) + \Delta f_p\ (\Delta f_p) \tag{3.26}$$

where,

$$CR1 = \sum_{r=1}^{n-1} (B_{pr}\ \Delta f_r + G_{pr}\ \Delta e_r) \tag{3.27}$$

$$CR2 = \sum_{r=1}^{n-1} (G_{pr}\ \Delta f_r - B_{pr}\ \Delta e_r) \tag{3.28}$$

Based on the Taylor expansion, the mismatch relationships with the correction factors added become

$$\Delta P_p = NRCP + R_p \tag{3.29}$$

$$\Delta Q_p = NRCQ + T_p \tag{3.30}$$

$$\Delta|V_p|^2 = NRCV + U_p \tag{3.31}$$

In the S.O.N.R. method, R_p, T_p, and U_p are calculated using the Δe_i and Δf_i as determined from the N.R. portion of the procedure. The factors are then incorporated in the following way [step (10) of Q.S.O.N.R.]:

$$\Delta P_p' = NRCP \tag{3.32}$$

$$\Delta Q_p' = NRCQ \tag{3.33}$$

$$\Delta|V_p|^{2'} = NRCV \tag{3.34}$$

where,

$$\Delta P_p' = \Delta P_p - R_p \tag{3.35}$$

$$\Delta Q_p' = \Delta Q_p - T_p \tag{3.36}$$

$$\Delta |V_p|^{2'} = \Delta |V_p|^2 - U_p \tag{3.37}$$

Equations (3.35), (3.36), and (3.37) are then solved for new values of voltage changes, $\Delta e_i'$ and $\Delta f_i'$.

The M.Q.S.O.N.R. method is the same as the Q.S.O.N.R. method until the point where the second order correction factors are calculated. Rather than calculate those factors entirely, we now calculate only CR1 and CR2 with equations (3.27) and (3.28), using Δe_i and Δf_i as calculated with the N.R. relationship [step (7) of Q.S.O.N.R.].

We can see how the correction factors will be used by writing out and expanding equations (3.29), (3.30) and (3.31), as follows,

$$\Delta P_p = \sum_{\substack{i=1 \\ i \neq p}}^{n-1} \frac{\partial P_p}{\partial e_i} \Delta e_i' + \sum_{\substack{i=1 \\ i \neq p}}^{n-1} \frac{\partial P_p}{\partial f_i} \Delta e_i' + \left(\frac{\partial P_p}{\partial e_p} + CR1\right) \Delta e_p' + \left(\frac{\partial P_p}{\partial f_p} + CR2\right) \Delta f_p' \tag{3.38}$$

$$\Delta Q_p = \sum_{\substack{i=1 \\ i \neq p}}^{n-1} \frac{\partial Q_p}{\partial e_i} \Delta e_i' + \sum_{\substack{i=1 \\ i \neq p}}^{n-1} \frac{\partial Q_p}{\partial e_i} \Delta e_i' + \left(\frac{\partial Q_p}{\partial e_p} - CR2\right) \Delta e_p' + \left(\frac{\partial Q_p}{\partial f_p} + CR1\right) \Delta f_p' \tag{3.39}$$

$$\Delta|V_p|^2 = \sum_{\substack{i=1 \\ i\neq p}}^{n-1} \frac{\partial|V_p|^2}{\partial e_i} \Delta e_i' + \sum_{\substack{i=1 \\ i\neq p}}^{n-1} \frac{\partial|V_p|^2}{\partial f_i} \Delta f_i$$

$$+ \left(\frac{\partial|V_p|^2}{\partial e_p} + \Delta e_p\right) \Delta e_p' + \left(\frac{\partial|V_p|^2}{\partial f_i} + \Delta f_p\right) \Delta f_p' \qquad (3.40)$$

Equations (3.38), (3.39), and (3.40) are solved in the given forms with the second order correction factors modifying only the diagonal elements of J_1, J_2, J_3, J_4, J_5, and J_6,

$$\begin{bmatrix} \Delta e' \\ \hline \\ \Delta f' \\ \\ \end{bmatrix} = \left[\begin{array}{c:c} J_1' & J_2' \\ \hdashline J_3' & J_4' \\ \hdashline J_5' & J_6' \end{array}\right]^{-1} \begin{bmatrix} \Delta P \\ \hdashline \Delta Q \\ \hdashline \Delta|V|^2 \end{bmatrix} \qquad (3.41)$$

The load flow procedure then continues on, as before [step (12) of Q.S.O.N.R.]. The main changes from the Q.S.O.N.R. occur in steps (9), (10), and (11).

It is easily shown that the Q.S.O.N.R. and the M.Q.S.O.N.R., number one, are specific cases of a general scheme, if we write equations (3.38), (3.39), and (3.40) in the following forms,

$$\Delta P_p - \alpha_p[(\Delta e_p)\ CR1 + \Delta f_p\ (CR2)] = \sum_{i=1}^{n-1} \frac{\partial P_p}{\partial e_i} \Delta e_i'$$

$$+ \sum_{i=1}^{n-1} \frac{\partial P_p}{\partial f_i} \Delta f_i'$$

$$+ (1 - \alpha_p)\ [CR1\ (\Delta e_p') + CR2\ (\Delta f_p')]$$

$$(3.42)$$

$$\Delta Q_p - \alpha_p \left[(\Delta f_p)\ CR1 - (\Delta e_p)\ CR2 \right] = \sum_{i=1}^{n-1} \frac{\partial Q_p}{\partial e_i} \Delta e_i{'}$$

$$+ \sum_{i=1}^{n-1} \frac{\partial Q_p}{\partial f_i} \Delta f_i{'}$$

$$+ (1 - \alpha_p) \left[CR1\ (\Delta f_p{'}) - CR2\ (\Delta e_p{'}) \right] \tag{3.43}$$

$$\Delta |V_p|^2 - \alpha_p \left[(\Delta e_p)^2 + (\Delta e_f)^2 \right] = \sum_{i=1}^{n-1} \frac{\partial |V_p|^2}{\partial e_i} \Delta e_i{'}$$

$$+ \sum_{i=1}^{n-1} \frac{\partial |V_p|^2}{\partial f_i} \Delta f_i{'}$$

$$+ (1 - \alpha_p) \left[\Delta e_p\ (\Delta e_p{'}) + \Delta f_p\ (\Delta f_p{'}) \right] \tag{3.44}$$

If we set alpha equal to 1, then these equations reduce to the equations employed in step (11) of the Q.S.O.N.R. method. If alpha is set to 0, then (3.42), (3.43), and (3.44) reduce to equations (3.38), (3.39), and (3.40) which constitute the modified Q.S.O.N.R. method presented in this subsection. This is the alpha-modified Q.S.O.N.R. which was programmed and tested. The programming is such, that, given the proper flagging, a particular power system under consideration can be solved by N.R., Q.S.O.N.R. (i.e., M.Q.S.O.N.R. with α_p = 1) or M.Q.S.O.N.R. with any other alpha value that is desired.

3.4.3 M.Q.S.O.N.R. method number 2

As stated in the introduction to this section, the three alpha-modified second order methods described in this and the next subsections were not programmed. However they represent major variants of the alpha-modified idea and, as such, should be presented.

As with the M.Q.S.O.N.R. method number 1, the load flow solution procedure for this method is the same as the Q.S.O.N.R. until we come to the point where the second order correction factors are calculated. Instead of calculating at this point, the correction factor expressions of equations (3.14), (3.23), and (3.26) are incorporated into mismatch equations (2.31), (2.32), and (2.35) to arrive at the forms,

$$\Delta P_p = \sum_{i=1}^{n-1} \frac{\partial P_p}{\partial e_i} \Delta e_i' + \sum_{i=1}^{n-1} \frac{\partial P_p}{\partial f_i} \Delta f_i'$$

$$+ \sum_{i=1}^{n-1} (G_{pi} \Delta e_p - B_{pi} \Delta f_p) \Delta e_i'$$

$$+ \sum_{i=1}^{n-1} (B_{pi} \Delta e_p + G_{pi} \Delta f_p) \Delta f_i'$$

which reduces to

$$\Delta P_p = \sum_{i=1}^{n-1} (\frac{\partial P_p}{\partial e_i} + G_{pi} \Delta e_p - B_{pi} \Delta f_p) \Delta e_i'$$

$$+ \sum_{i=1}^{n-1} (\frac{\partial P_p}{\partial f_i} + B_{pi} \Delta e_p + G_{pi} \Delta f_p) \Delta f_i' \qquad (3.45)$$

Similarly,

$$\Delta Q_p = \sum_{i=1}^{n-1} (\frac{\partial Q_p}{\partial e_i} + G_{pi} \Delta f_p + B_{pi} \Delta e_p) \Delta e_i'$$

$$+ \sum_{i=1}^{n-1} (\frac{\partial Q_p}{\partial f_i} + B_{pi} \Delta f_p - G_{pi} \Delta e_p) \Delta f_i' \qquad (3.46)$$

and,

$$\Delta|V_p|^2 = \sum_{i=1}^{n-1} \frac{\partial|V_p|^2}{\partial e_i} \Delta e_i{}' + \sum_{i=1}^{n-1} \frac{\partial|V_p|^2}{\partial f_i} \Delta f_i{}'$$

$$+ (\Delta e_p)\, \Delta e_p{}' + (\Delta f_p)\, \Delta f_p{}' \tag{3.47}$$

From (3.45), (3.46), and (3.47) we can see that every non-zero Jacobian element will be modified. These equations are used in this form to solve,

$$\begin{bmatrix} \Delta e' \\ \hline \Delta f' \end{bmatrix} = \begin{bmatrix} J_1{}' & J_2{}' \\ \hline J_3 & J_4 \\ \hline J_5 & J_6 \end{bmatrix}^{-1} \begin{bmatrix} \Delta P \\ \hline \Delta Q \\ \hline \Delta|V|^2 \end{bmatrix}$$

From here the procedure continues on as with the Q.S.O.N.R.

In a general alpha-modified form the mismatch equations can be written as

$$\Delta P_p - \alpha_p \sum_{i=1}^{n-1} [(G_{pi}\, \Delta e_p - B_{pi}\, \Delta f_p)\, \Delta e_i + (B_{pi}\, \Delta e_p + G_{pi}\, \Delta f_p)\, \Delta f_i]$$

$$= \sum_{i=1}^{n-1} [\frac{\partial P_p}{\partial e_i} + (1 - \alpha)\, (G_{pi}\, \Delta e_p - B_{pi}\, \Delta f_p)\, \Delta e_i{}']$$

$$= \sum_{i=1}^{n-1} [\frac{\partial P_p}{\partial f_i} + (1 - \alpha)\, (B_{pi}\, \Delta e_p + G_{pi}\, \Delta f_p)\, \Delta f_i{}'] \tag{3.48}$$

$$\Delta Q_p - \alpha_p \sum_{i=1}^{n-1} [(G_{pi}\, \Delta f_p + B_{pi}\, \Delta e_p)\, \Delta e_i + (B_{pi}\, \Delta f_p - G_{pi}\, \Delta e_p)\, \Delta f_i]$$

$$= \sum_{i=1}^{n-1} [\frac{\partial Q_p}{\partial e_i} + (1 - \alpha)\, (G_{pi}\, \Delta f_p + B_{pi}\, \Delta e_p)\, \Delta e_i{}']$$

$$+ \sum_{i=1}^{n-1} [\frac{\partial Q_p}{\partial f_i} + (1 - \alpha)\ (B_{pi}\ \Delta f_p - G_{pi}\ \Delta e_p)\ \Delta f_i{}'] \qquad (3.49)$$

$$\Delta |V_p|^2 - \alpha_p\ (\Delta e_p{}^2 + \Delta e_f{}^2) = \sum_{i=1}^{n-1} \frac{\partial |V_p|^2}{\partial e_i}\ \Delta e_i{}' + \sum_{i=1}^{n-1} \frac{\partial |V_p|^2}{\partial f_i}\ \Delta f_i{}'$$

$$+ (1 - \alpha)\ (\Delta e_p)\ \Delta e_p{}'$$

$$+ (1 - \alpha)\ (\Delta f_p)\ \Delta f_p{}' \qquad (3.50)$$

By setting alpha equal to one we obtain the equations for the Q.S.O.N.R. method. With alpha equal to 0 we get the specific case, illustrated earlier in this subsection, of all the Jacobian elements being altered while the mismatch values, ΔP_p, ΔQ_p, and $\Delta|V_p|^2$, remain unaltered.

Thus, we have M.Q.S.O.N.R. number 2 method for solving a load flow problem.

3.4.4 M.Q.S.O.N.R. methods numbers 3 and 4

These two methods change the basic Q.S.O.N.R. method a little more drastically than the first two alpha-modified methods. However, they are, in fact, variants of those two methods since they use the same "second order" mismatch relationships.

Basically, the change from the previous two procedures is the elimination, except during the first iteration, of the Newton-Raphson voltage correction calculations [step (7) of Q.S.O.N.R.] which constitutes the first half iteration.

All the derivations needed for these two new methods have been evaluated previously. Therefore the proposed procedures for both can be presented now, and are as follows, (unless noted, each step is for both methods).

1) Form the bus admittance matrix, [Y]

2) Initialize the bus voltage, $e_i + jf_i$

3) Calculate P_p, Q_p, $|V_p|^2$ with equation (2.21), (2.22), and (2.23)

4) Calculate the bus mismatches with

$$\Delta P_p = P_{psched} - P_p - \alpha_p \sum_{i=1}^{n-1} [\Delta e_p (B_{pi} \Delta f_i + G_{pi} \Delta e_i) + \Delta f_p (G_{pi} \Delta f_i - B_{pi} \Delta e_i)] \qquad (3.51)$$

$$\Delta Q_p = Q_{psched} - Q_p - \alpha_p \sum_{i=1}^{n-1} [\Delta f_p (B_{pi} \Delta f_i + G_{pi} \Delta e_i) - \Delta e_p (G_{pi} \Delta f_i - B_{pi} \Delta e_i)] \qquad (3.52)$$

$$\Delta |V_p|^2 = |V_{psched}|^2 - |V_p|^2 - \alpha_p [(\Delta e_p)^2 + (\Delta f_p)^2] \qquad (3.53)$$

Note that during the first iteration, Δe_i and Δf_i will be zero, which will make the "alpha terms" of the above relationship, zero.

5) Check to see if the power and voltage magnitude mismatches, calculated in step (4), are within the specified tolerance. If so, solution reached. If not, continue.

6) Calculate the "modified Jacobian" elements. For M.Q.S.O.N.R. number three, we take the expression from equations (3.42), (3.43), and (3.44), resulting in elements for J, given by

$$J_1(p,i) = \frac{\partial P_p}{\partial e_i} \qquad , \quad i \neq p$$

$$= \frac{\partial P_p}{\partial e_i} + (1 - \alpha_p)\ CR1 \qquad , \quad i = p$$

for J_2:

$$J_2(p,i) = \frac{\partial P_p}{\partial f_i} \qquad , \quad i \neq p$$

$$= \frac{\partial P_p}{\partial f_i} + (1 - \alpha_p)\ CR2 \qquad , \quad i = p$$

for J_3:

$$J_3(p,i) = \frac{\partial Q_p}{\partial e_i} \qquad , \quad i \neq p$$

$$= \frac{\partial Q_p}{\partial e_i} - (1 - \alpha_p)\ CR2 \qquad , \quad i = p$$

for J_4:

$$J_4(p,i) = \frac{\partial Q_p}{\partial f_i} \qquad , \quad i \neq p$$

$$= \frac{\partial Q_p}{\partial f_i} + (1 - \alpha_p)\ CR1 \qquad , \quad i = p$$

for J_5:

$$J_5(p,i) = \frac{\partial |V_p|^2}{\partial e_i} = 0 \qquad , \quad i \neq p$$

$$= \frac{\partial |V_p|^2}{\partial e_i} + (1 - \alpha_p)(\Delta e_p) \qquad , \quad i = p$$

for J_6:

$$J_6(p,i) = \frac{\partial |V_p|^2}{\partial f_i} = 0 \qquad , \quad i \neq p$$

$$= \frac{\partial |V_p|^2}{\partial f_i} + (1 - \alpha_p)(\Delta f_p) \qquad , \quad i = p$$

For the M.Q.S.O.N.R. number four, we utilize the expressions from equations (3.48), (3.49), and (3.50), resulting in the elements being for J_1:

$$J_1(p,i) = \frac{\partial P_p}{\partial e_i} + (1 - \alpha_p)(G_{pi}\ \Delta e_p - B_{pi}\ \Delta f_p)$$

for J_2:

$$J_2(p,i) = \frac{\partial P_p}{\partial f_i} + (1 - \alpha_p)(B_{pi}\ \Delta e_p + G_{pi}\ \Delta f_p)$$

for J_3:

$$J_3(p,i) = \frac{\partial Q_p}{\partial e_i} + (1 - \alpha_p)(G_{pi}\ \Delta f_p + B_{pi}\ \Delta e_p)$$

for J_4:

$$J_4(p,i) = \frac{\partial Q_p}{\partial f_i} + (1 - \alpha_p)(B_{pi}\ \Delta f_p - G_{pi}\ \Delta e_p)$$

for J_5:

$$J_5(p,i) = \frac{\partial |V_p|^2}{\partial e_i} + (1 - \alpha_p)(\Delta e_p)$$

for J_6:

$$J_6(p,i) = \frac{\partial |V_p|^2}{\partial f_i} + (1 - \alpha_p)(\Delta f_p)$$

note that, for both methods, during the first iteration Δe_i and Δf_i are both zero. This leaves the coefficient matrix composed of true Jacobian elements.

7) With the mismatch values found in step (4) and the modified Jacobian elements found in step (6), solve the following set of simultaneous equations for the voltage corrections

$$\begin{bmatrix} \Delta e \\ \hline \\ \Delta f \end{bmatrix} = \left[\begin{array}{c|c} J_1 & J_2 \\ \hline J_3 & J_4 \\ \hline J_5 & J_6 \end{array}\right]^{-1} \begin{bmatrix} \Delta P \\ \hline \Delta Q \\ \hline \Delta |V|^2 \end{bmatrix}$$

8) Update the bus voltages of the K^{th} iteration,

$$e_i^{(K+1)} = e_i^{(K)} + \Delta e_i^{(K)}$$

$$f_i^{(K+1)} = f_i^{(K)} + \Delta f_i^{(K)}$$

9) Go to step (3).

With the alpha-M.Q.S.O.N.R. methods number 3 and number 4, the main advantage over the other second order methods presented previously is the elimination of the need to solve a second set of simultaneous equations during a single iteration. As a result the storage requirements should be slightly less and the computer CPU time per iteration will be substantially reduced. And, since the second order correction factors are employed, it would appear that the rate of convergence to the load flow solution would be very similar. Certainly these two methods are worth investigating and will be mentioned again in the concluding chapter.

CHAPTER IV

APPLICATION AND ANALYSIS OF THE ALPHA-MODIFIED QUASI SECOND ORDER NEWTON-RAPHSON METHOD FOR SOLVING LOAD FLOW PROBLEMS

4.1 Sparsity

As stated in the introductory chapter, the powerful convergence properties of the Newton-Raphson technique in solving load flows can only be utilized competitively through the use of sparsity programming. For the author's programs, a package of subroutines for the solution of sparse sets of linear equations obtained from the Atomic Energy Research Establishment (U.K.) in Harwell, England [14] was adopted. These basicaly employ Gaussian elimination and aim at maintaining both stability and sparseness. It also has the feature which, after an equation set has been solved once, allows further systems with the same coefficient matrix, or one having the same sparsity pattern, to be solved more economically. Load flow solutions do, of course, benefit immensely from this feature. Only slight modifications were required to make these subroutines compatible with the program as it stood at the stage of incorporating sparsity techniques, and with the computer system being used. The subroutine specification sheets are given in Appendix C, while their listings are in appendix E, along with the main body of the program.

Incorporating the sparsity subroutine required major modifications to the program. Maximizing overall efficiency and compatibility dictated inclusion of programming modifications to (1) reorder the lines and their associated data into an optimum format, (2) decompose the bus admittance matrix into sparse vectors, (3) construct the Jacobian matrix sparsely, and (4) change almost all of the remaining program which worked on these sparse matrices. However, after all was said and done the benefits were immediately seen. The load flow, with the largest system being tested (IEEE 118-Bus, which is not really that big), had a reduction of 75% in core requirements and a 63% saving in CPU time, for an overall cost reduction of 91%. This improved even more as the program continued to be made more efficient. Of course, with systems of about 25 buses or less, the sparsity programming caused an increase in CPU time and storage requirements.

4.2 The Computer Program

4.2.1 General

The program was written in the FORTRAN IV language and was run on an IBM 370/158 model II with SVS operating system owned by Newfoundland and Labrador Computer Services.

It should be noted here that the program, as it presently exists, is an experimental testing program, rather than one for production runs. As such, it does not yet contain the capability to handle many of the special features of a "real-life" power system, such as under load tap changing, phase shifting transformers, and tie line control.

Figure 4.1 gives the general flow chart which illustrates, in a basic manner, how the alpha-modified quasi second order Newton-Raphson procedure is programmed. It also shows that an ordinary Newton-Raphson

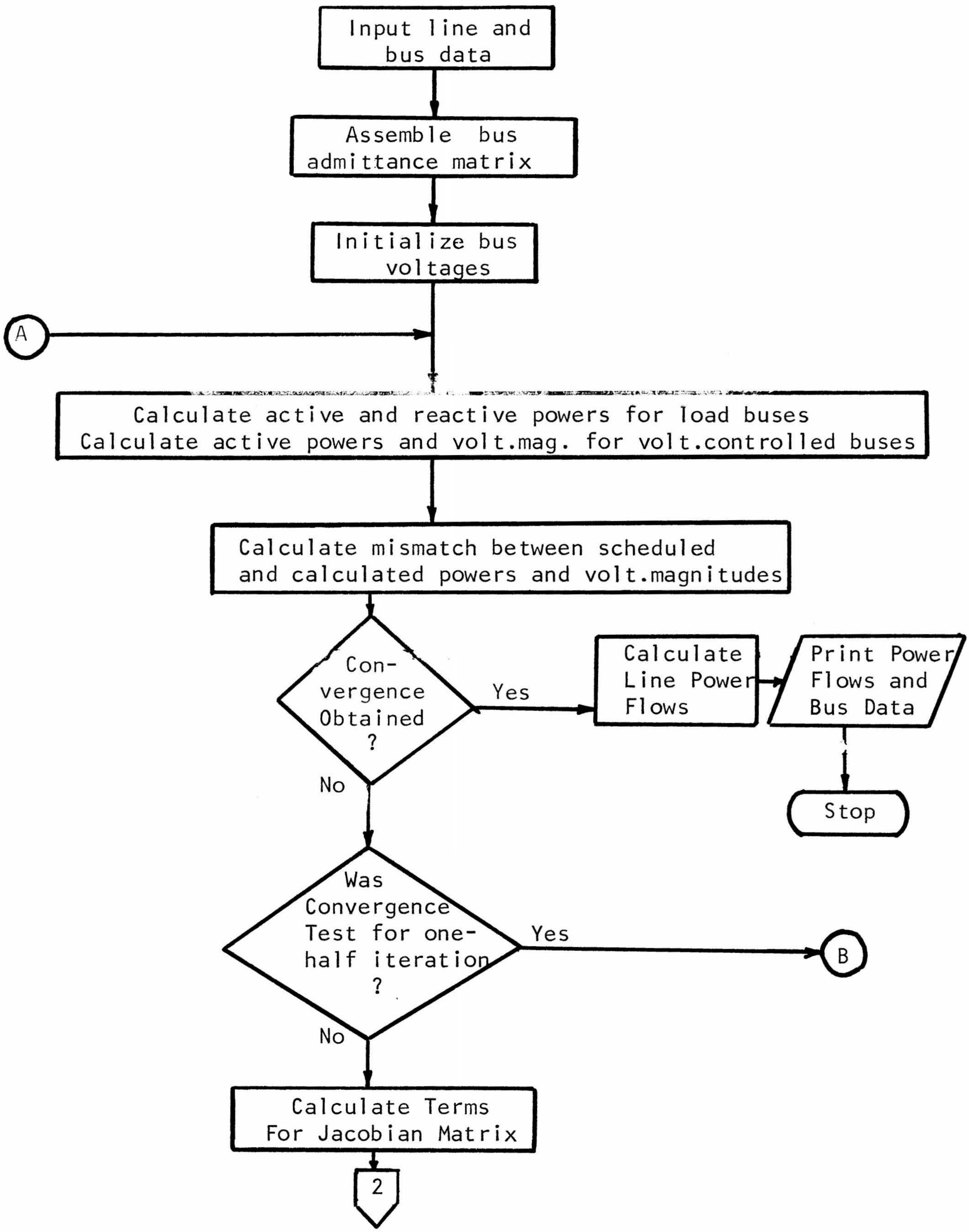
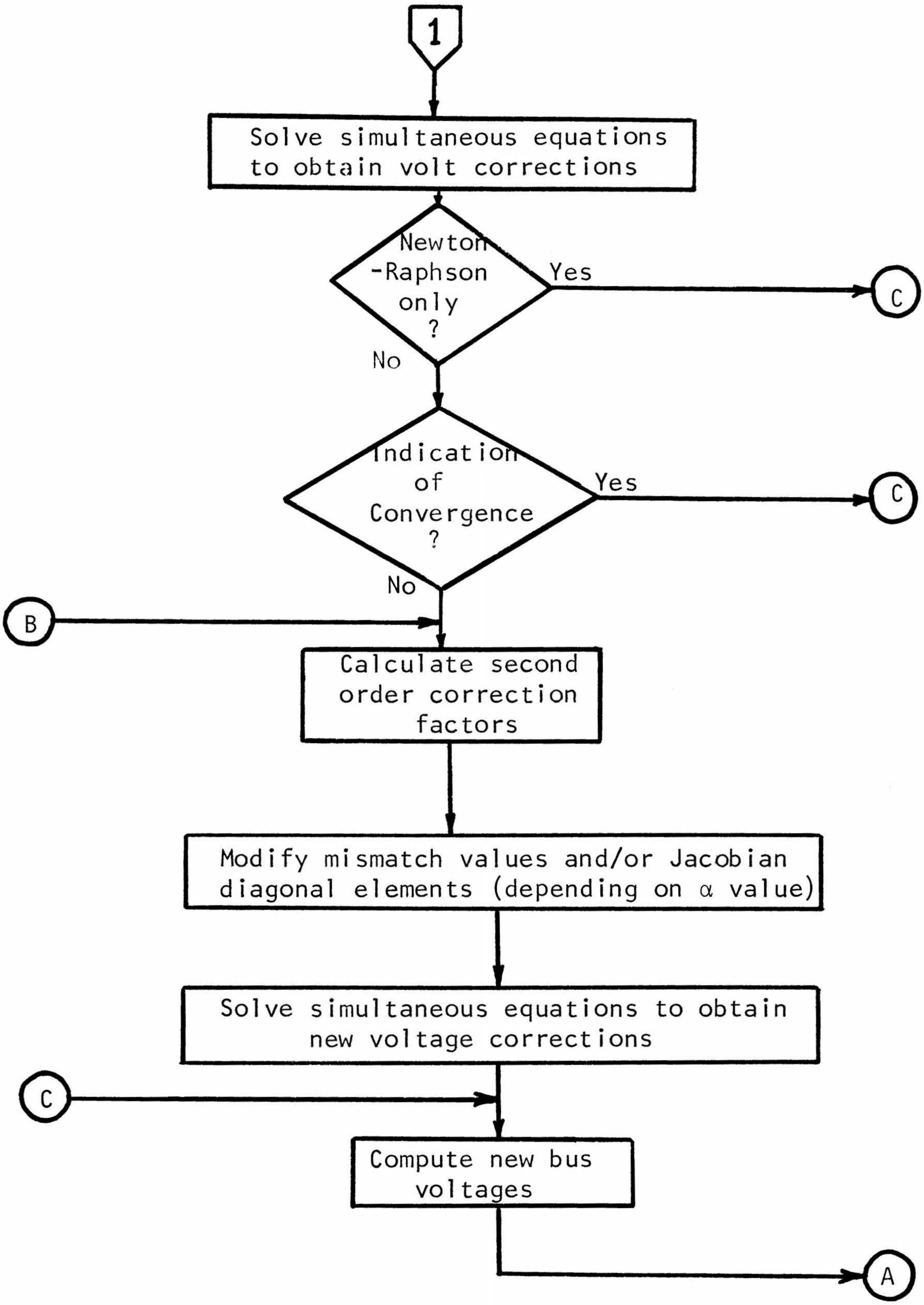


Figure 4.1: General flow chart for the alpha-modified quasi-second order Newton Raphson method.


1
Solve simultaneous equations to obtain volt corrections
Newton -Raphson only ?
Yes
C
No
Indication of Convergence ?
Yes
C
No
B
Calculate second order correction factors
Modify mismatch values and/or Jacobian diagonal elements (depending on α value)
Solve simultaneous equations to obtain new voltage corrections
C
Compute new bus voltages
A

solution procedure is able to be performed. In fact, one of the advantages of second order type extensions of the Newton-Raphson method is that a subroutine with the additional programming can be simply added to existing production programs with only a few minor changes necessary.

Apendix D contains the documentation defining the computer variable names, as well as how they should be dimensioned. The complete program listing is contained in appendix E.

4.2.2 Definition of One-Half Iteration

As will be seen in the results, we use the term a half iteration. This will be fully explained here. The second order load flow technique entails twice solving a set of simultaneous equations for bus voltage corrections. Once, with the power and voltage magnitude mismatches calculated with first order correction terms and later, with second order correction factors as well. It is possible, however, that on the last iteration needed for convergence, the convergence criterion could be met with the "first order" voltage corrections. It may not be necessary to continue on and obtain an updated set of voltage corrections.

Therefore, before attempting to solve the second set of equations, there is a check to see if this step can be skipped. One way this could have been implimented would have been to calculate, at this point, new voltages using the "first order" voltage corrections, new bus powers and voltage magnitudes with these new voltages, and new mismatches. Then, perform the convergence test to see if these mismatches are within the specified tolerance. However, doing this during every iteration would possibly involve as much or more computation time than if there was no attempt to eliminate the last half of the last iteration.

A much simpler way of determining the necessity for the last half iteration is utilized. With the "first order" voltage corrections, the second order power and voltage magnitude correction factors are calculated. If these factors are all less than the given tolerance, then the new mismatches would be all much the same as the previous mismatches used in the calculation to obtain the "first order" voltage corrections. In most cases, this would lead to the updated voltage corrections being much the same as the non-updated values, which is a fairly good indication that convergence has been reached.

So, if all the factors are lower than the tolerance, new voltages are calculated, then new bus variables and mismatches, followed by the convergence test. In the majority of cases, when this occurs, it has been found that convergence has indeed been obtained. If, as is sometimes the case, it has not, then the program returns to the original departure point and continues on with the second half of the iteration.

4.2.3. Program Limitations

The computer program is set up to perform a load flow study of a straightforward normal power system with no extras. It can handle load buses and generator, or voltage controlled, buses with static capacitors or shunt reactors; lines with series impedence and line charging susceptances; transformers with off-nominal turns ratios.

It is not limited as to system size, though with each system a new set of dimension cards must be typed (the sizes of which are given in appendix D). There can be any number of buses, lines, transformers, shunt reactors or capacitors.

Buses must be numbered consecutively, starting at one. The voltage controlled buses have to have the higher numbers, with the slack (or swing) bus having the highest. The lines and their associated parameters are reordered by the program.

4.3 Test Systems

4.3.1 5-Bus Power System

The 5-bus system is given in [55], and is the smallest system tested. It consists of three load buses, two voltage controlled buses, one of which acts as the slack bus, and seven lines. The system data consists of (1) the scheduled generation and loads and assumed bus voltages, (2) line impedances and line charging susceptances, and (3) Regulated bus data. The other systems also have transformer data and static capacitor or shunt reactor data. The data for all the test systems is in appendix F.

4.3.2 23-Bus Power System

The 23-bus system is based on the one used in [11] for testing a solution method for minimum loss and economic dispatch problems. In our case, all buses but one (the slack bus) are treated as load buses. There are thirty lines whose data, along with the bus information, is presented in appendix F, tables F.4 and F.5.

4.3.3 IEEE 57-Bus Test System

This was originally a part of the American Electric Power Corporation's

1962 system made available to the electric utility industry as a standard test case for evaluating various analytical methods and computer programs for the solution of power networks. It eventually became one of the standard IEEE test systems. It consists of fifty load buses, seven voltage controlled buses, eighty lines, seventeen transformers with off-nominal turns ratios, and three static capacitors. All the data is contained in appendix F, tables F.6 to F.10.

4.3.4 IEEE 118-Bus Test System

This system came to be a standard test case in the same manner as the previous one. It is made up of sixty-four load buses, fifty-four voltage regulated buses, one hundred and eighty-six lines, nine transformers with off-nominal turns ratios, and fourteen static capactors and shunt reactors. All this data is in appendix F, along with the data for the other systems, tables F.11 to F.15.

4.3.5 Other Test Systems

The other three systems used for testing are simply the 5, 57, and 118 bus systems with only one voltage controlled bus each (the slack buses), with their solution bus powers being used as scheduled bus loads. This was done as a convenient way to obtain some new test systems. And, indeed they are new with different convergence patterns and different solutions than the systems from which they are derived. The data for these systems, bus power, assumed voltage, line, transformer and static capacitor and reactor information is the same as for the original systems. They are referred to as the 5-bus (no.V.C.B.), the 57-bus (No. V.C.B.) and the 118-bus (No. V.C.B.) systems, with V.C.B. standing for Voltage Controlled Buses.

4.4 Results

4.4.1 General

The developed load flow technique has been extensively tested using the seven power systems described in the previous section. These systems cover a wide range in terms of size-5 to 118 buses - and the larger systems with only one voltage controlled bus provide an extra challenge in that in such cases, convergence is usually more difficult to achieve.

Many variations of the alpha-M.Q.S.O.N.R. method were tried such as adding second order corrections only in the first iteration or applying them only to load buses. However, it was found that the best overall results occur with the straight forward application of the method. The improvement is in terms of convergence rate and computation time. Thus, this variant will be the main focus of attention in this section.

The results of studies for all the test systems are presented graphically and discussed in detail. For comparison purposes, all system load flows were also solved with the Newton-Raphson method.

4.4.2 Iterations to Convergence

From the description of Chapter 3 treating the alpha-modified Q.S.O.N.R., it can be seen that any value of alpha can be chosen and used. It would appear from the theory that the choice of alpha will be important. Its value determines what proportion of the second order correction factors is subtracted from the bus mismatches and what portion is added to the Jacobian elements in the setting up of the simultaneous set of equations to be solved for the updated voltage corrections.

Therefore, we are interested in studying what effect if any, do different values of alpha have. After this is done we determine which is the best alpha to use. With this best alpha, we can then see how the alpha-M.Q.S.O.N.R. method stacks up against the Newton-Raphson method.

The above was accomplished by taking each of the test systems and computing load flows with many different alpha values (ranging from -2.0 to +2.0 in increments of 0.1) as well as with the Newton-Raphson method. The results of all this testing are tabulated in table 4.1, which gives the number of iterations to convergence for each alpha value for all the systems. Convergence was achieved when the maximum bus mismatches after an iteration (or half an iteration) were below the given tolerance. These results have been plotted for each system in figures 4.2 to 4.9 as graphs of iterations to convergence versus alpha values for tolerances of 0.0001 and 0.001 p.u. On the far right hand side of each graph there are two dots which tell the number of iterations required for the two tolerances for the Newton-Raphson (N.-R.) method.

Some general observations about the alpha-M.Q.S.O.N.R. method are immediately evident from the graphs. With all the systems, except the 118-Bus systems, and with every value of alpha, the number of iterations required for the smaller tolerance is either the same or only one-half an iteration more. This indicates that the method could best prove its worth against other techniques when low mismatch tolerances are specified. With the 118-Bus systems, 0.0001 p.u. is about the lowest mismatch value that can be achieved. This makes it more difficult to reach convergence, which shows up on the graphs as a fairly erratic plot (figure 4.5), or as no con-

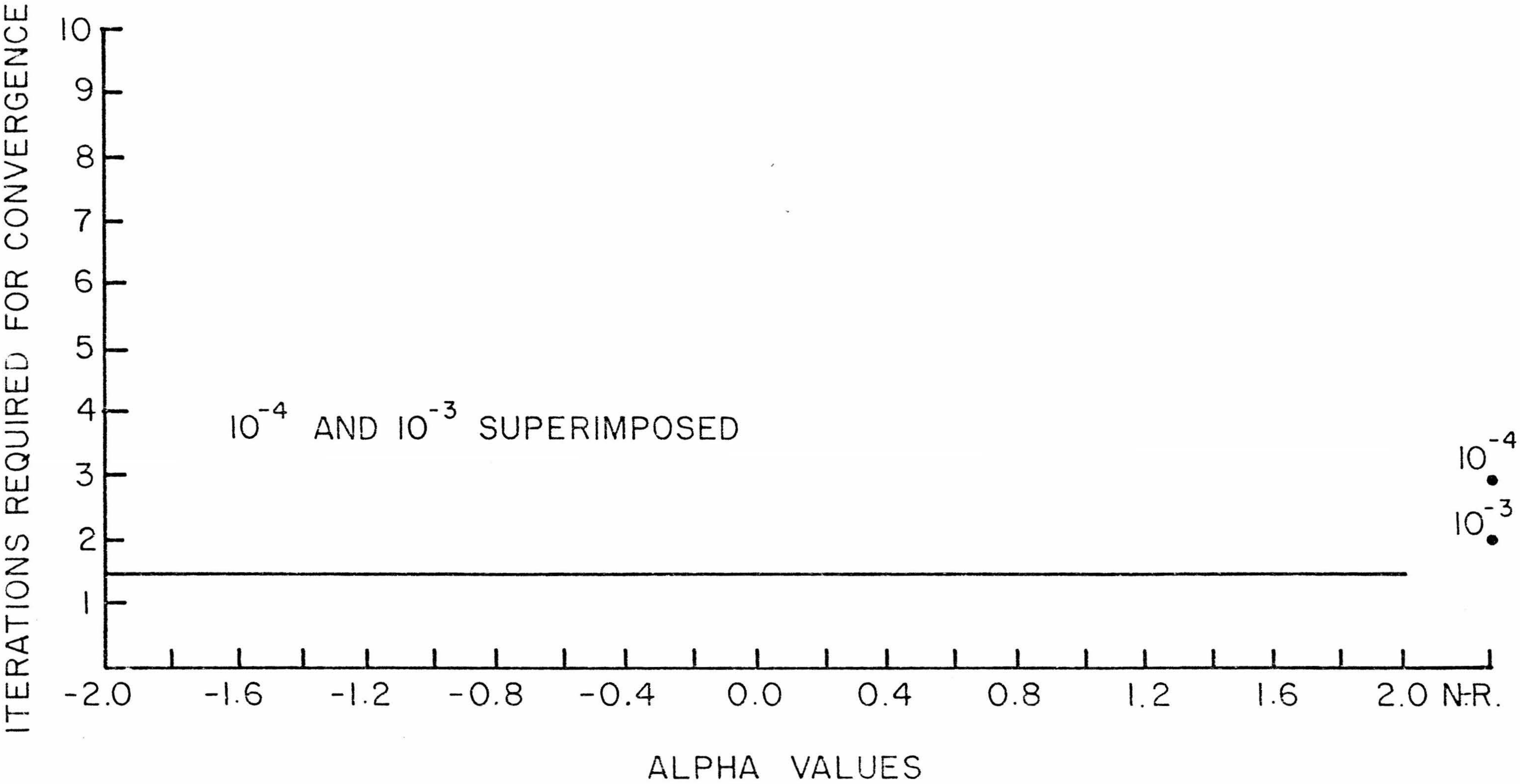


Figure 4.2: Number of Iterations to Convergence For the 5-Bus System with Tolerances of 10^{-4} p.u. and 10^{-3} p.u.

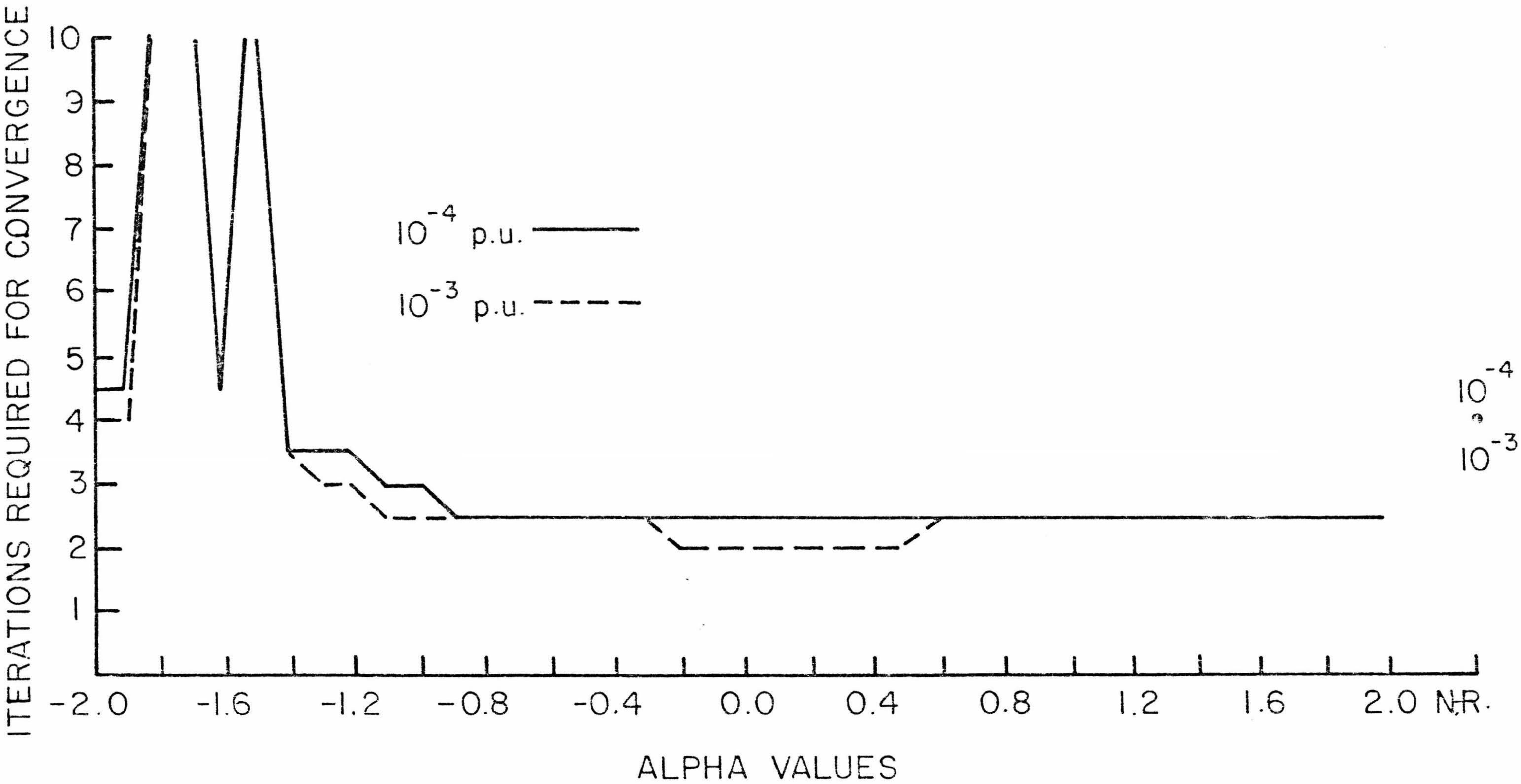


Figure 4.3: Number of Iterations to Convergence for the 23-Bus System with Tolerances of 10^{-4} p.u. and 10^{-3} p.u.

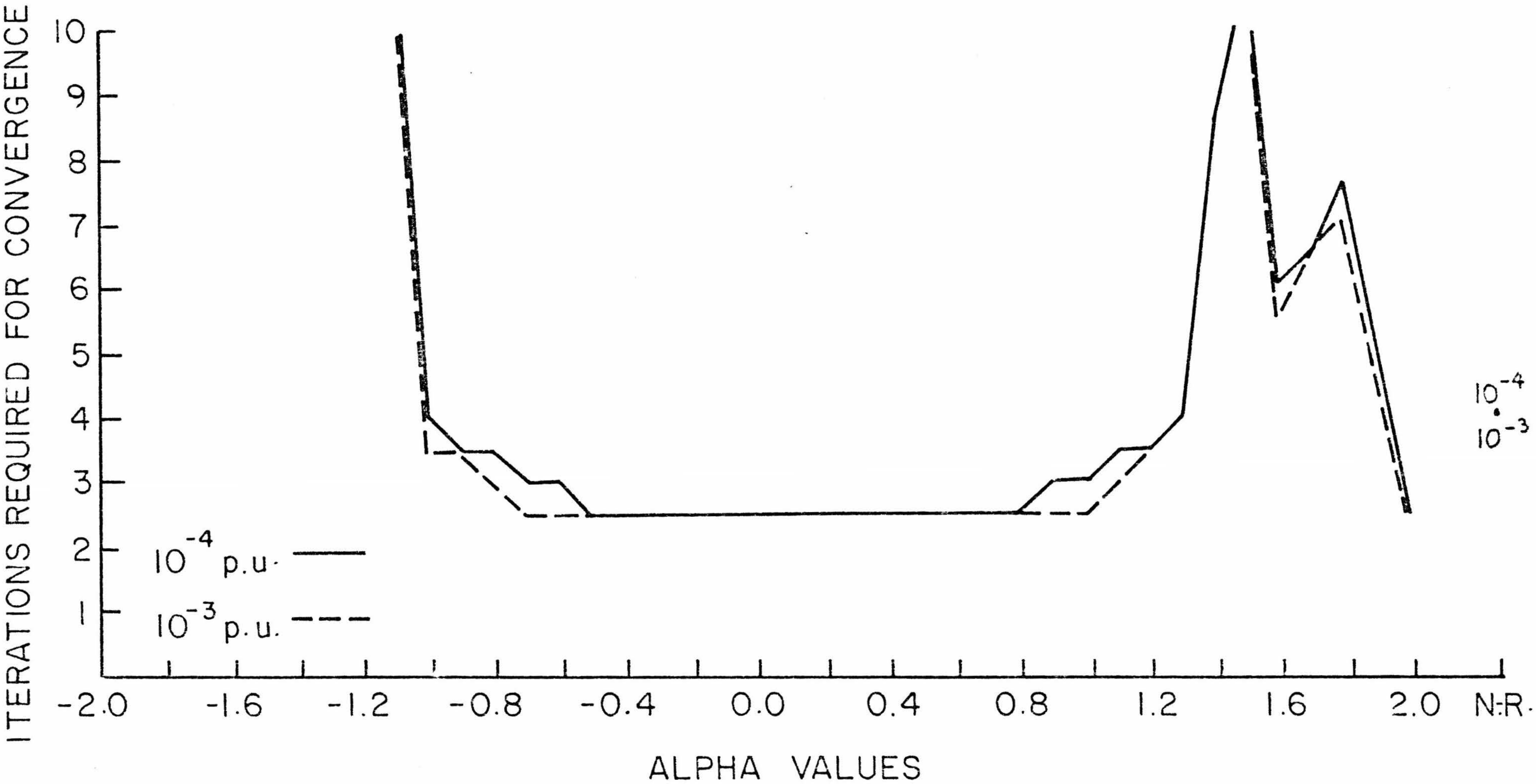


Figure 4.4: Number of Iterations to Convergence for the IEEE 57-Bus System with Tolerances of 10^{-4} p.u. and 10^{-3} p.u.

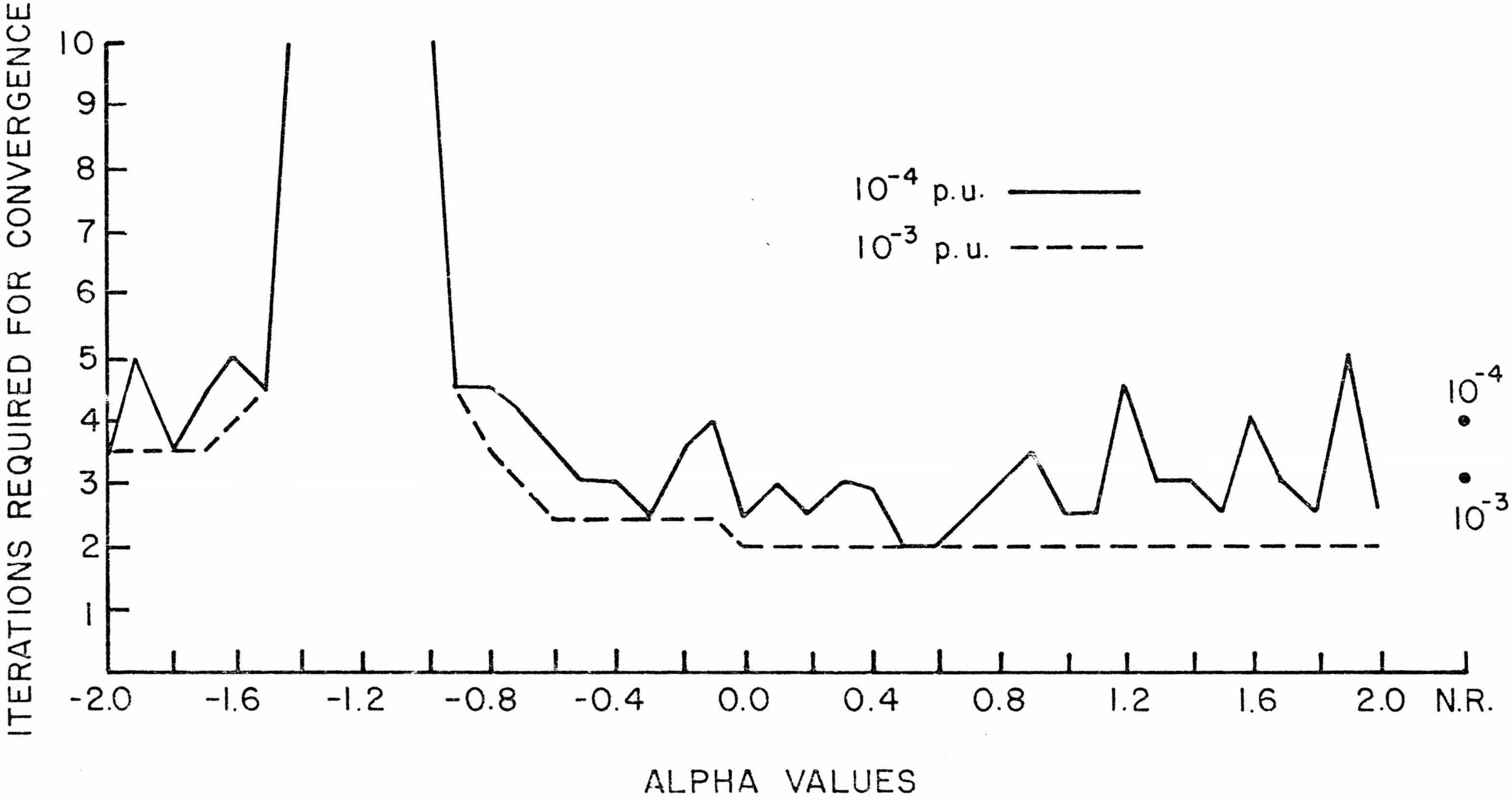


Figure 4.5: Number of Iterations to Convergence for the IEEE 18-Bus System with $V_{start} = 1\ 30^{0}$

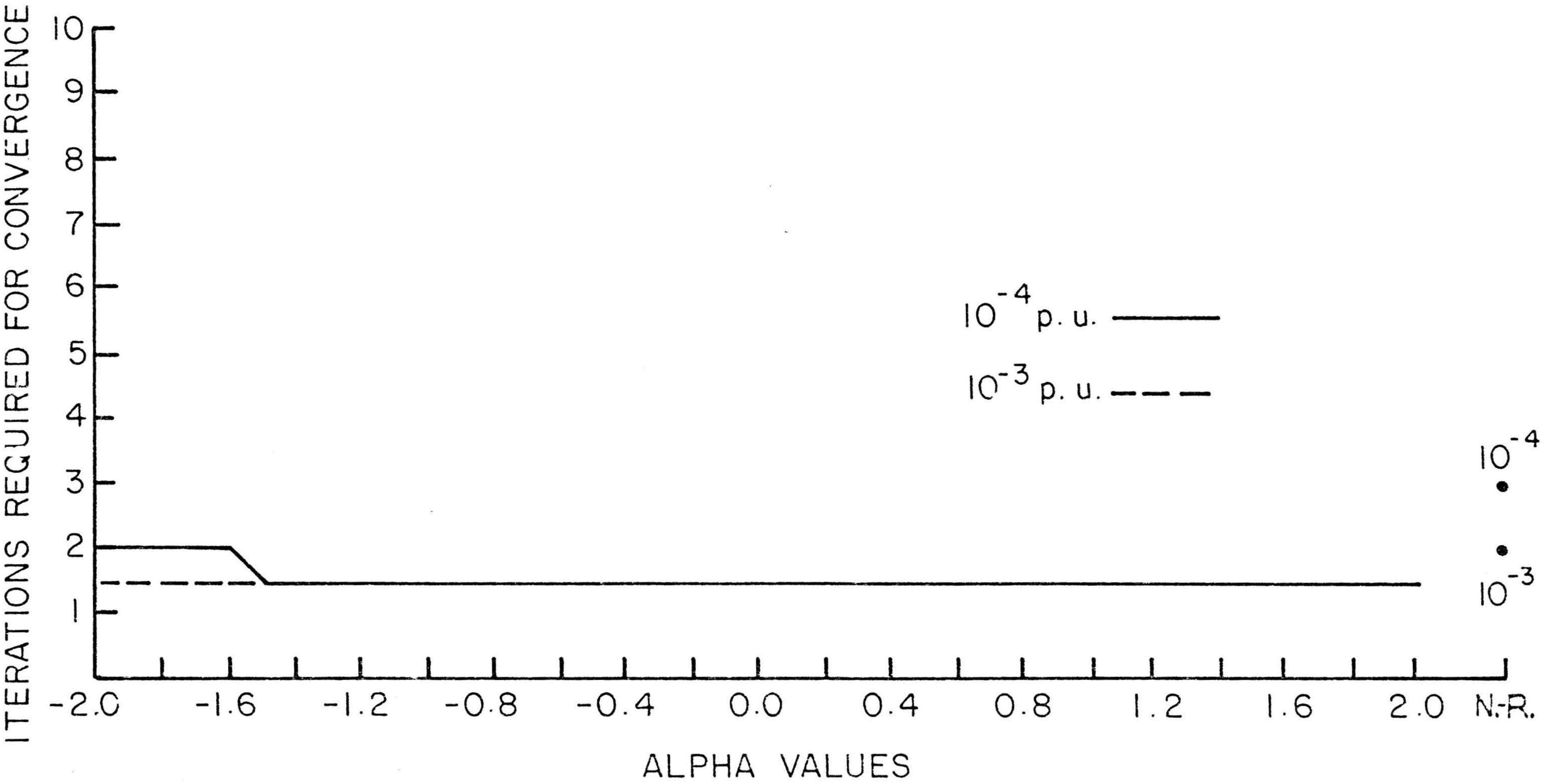


Figure 4.6: Number of Iterations to Convergence for the 5-Bus (No V.C.B.) System with Tolerances of 10^{-4} p.u. and 10^{-3} p.u.

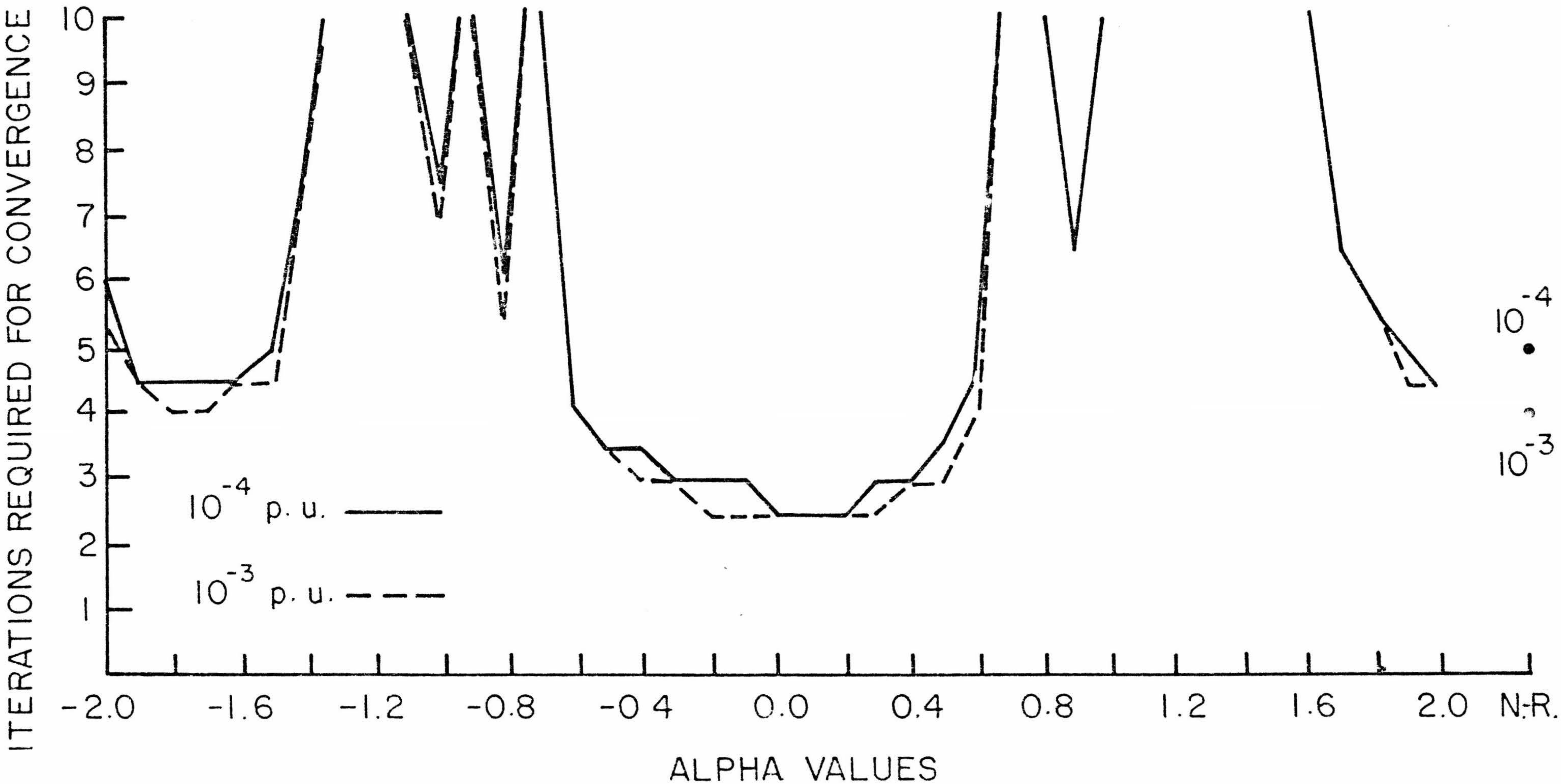


Figure 4.7: Number of Iterations to Convergence for the 57-Bus (No V.C.B.) System with Tolerances of 10^{-4} p.u. and 10^{-3} p.u.

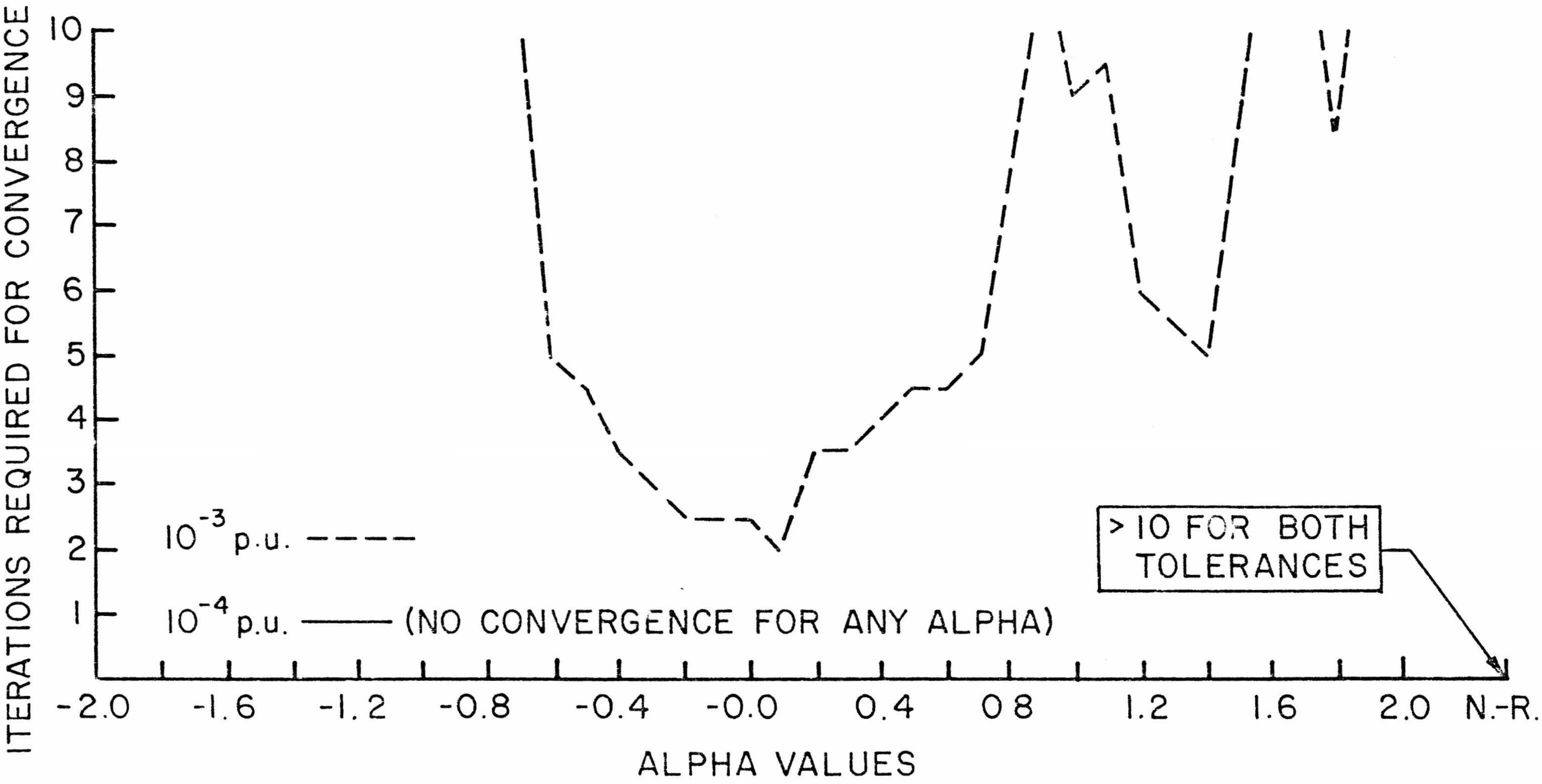


Figure 4.8: Number of Iterations to Convergence for the 118-Bus (No. V.C.B.) System with Tolerances of 10^{-4} p.u. and 10^{-3} p.u.

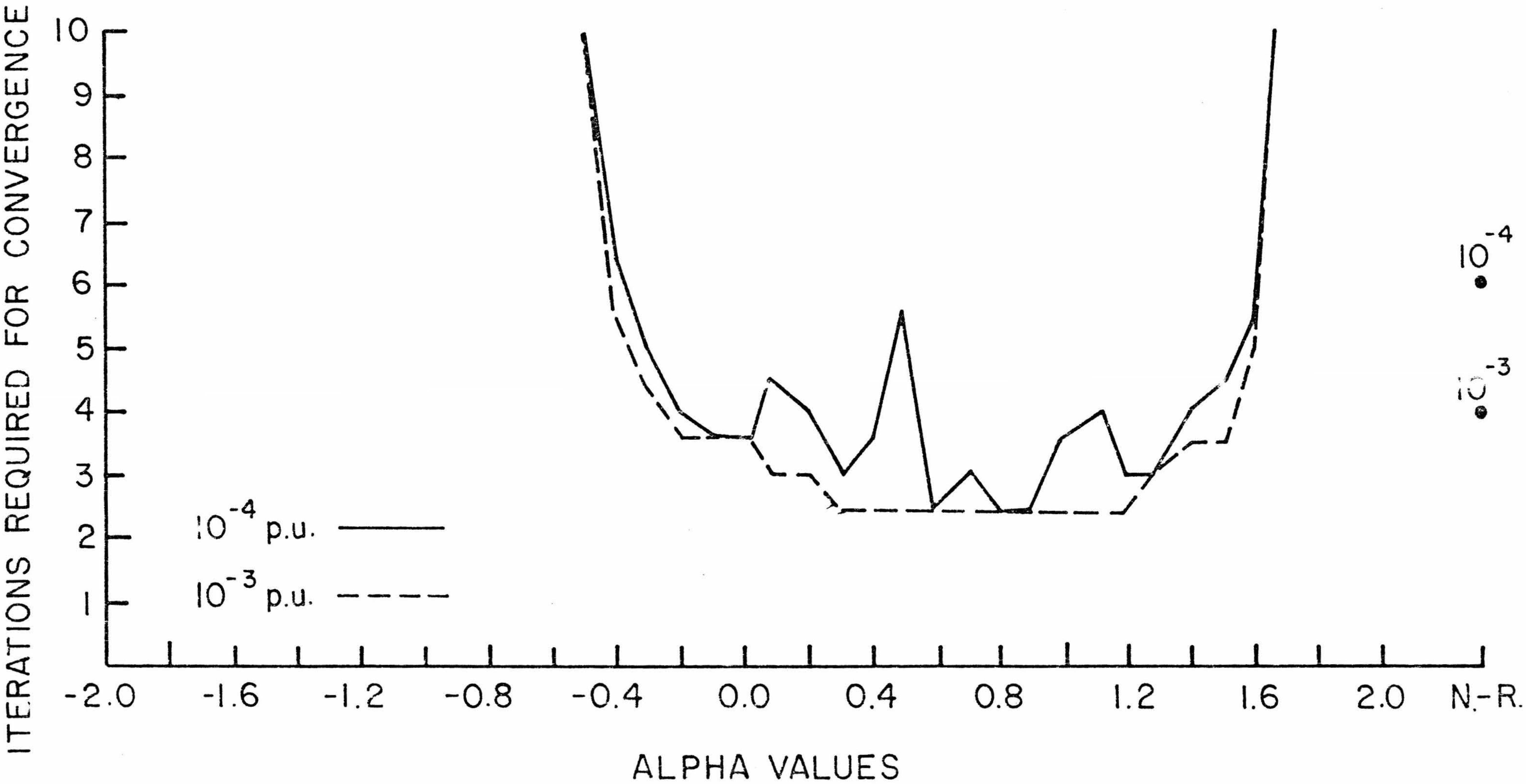


Figure 4.9: Number of Iterations to Convergence for the 118-Bus System with Initial Bus Angles Set to 0.0 Degrees.

vergence at all (figure 4.8). This is in contrast to the well-behaved plots for the 0.001 p.u. tolerance in the same graphs.

In all cases, except for the 5 bus system, larger negative values of alpha generally result in a higher number of iterations required for convergence at best or no convergence at all (that is, at least not within 10 iterations). Except for the three smallest systems the same is true for larger positive values of alpha. But, when the value of alpha approaches zero, the method is consistently successful in bringing the load flow to a quick and successful solution.

The results lead us to the conclusion that the best convergence properties will in all likelihood occur when the alpha value is zero. This is reasonable since alpha having the value of zero causes the full second order correction factors to be added to the diagonal elements of the Jacobian submatrices and no portion subtracted from the bus mismatches. In effect, this results in the second order factors being calculated partly using the updated voltage corrections which are being solved for, thus moving those updated values further towards the correct figures. This can be seen by looking at equations (3.42), (3.43), and (3.44).

An additional advantage of alpha being equal to zero is that, because it is only necessary to modify the Jacobian, diagonal elements, there is less computational effort required than with other alpha values (except for alpha equal to one, where only the bus mismatches are modified).

Now, having determined that zero is the best value which alpha can have, the method with this alpha value must be compared with the Newton-Raphson in order to judge its merit. The number of iterations required for two tolerances with the Newton-Raphson procedure is shown on the right hand side of the figures 4.2 to 4.5. In every case, but one, the alpha M.Q.S.O.N.R. load flow converges in half, or one-half more than half, the number of iterations of the Newton-Raphson load flow. The only exception is the 118-bus (No V.C.B.) system for which the Newton-Raphson fails to converge while the new method converges in 2.5 iterations for a tolerance of 0.001 p.u.

The first half of an alpha-M.Q.S.O.N.R. iteration is the same as one N.-R. iteration. The second half iteration requires very little extra storage and less C.P.U. time than the first half (or one N.-R. iteration). This is so because bus mismatches calculated in the first half only require a simple modification in the second half. Also, the Jacobian elements calculated in the first half are used again in the second half, with only the diagonal elements undergoing a computationally simple modification. As a result, all the load flows converge, not only at a much faster rate, but also in less time than the Newton-Raphson load flows and with almost the same storage requirements.

4.4.3 Bus Mismatches during Load Flows

The comparison between the alpha-M.Q.S.O.N.R. algorithm and the Newton-Raphson method is illustrated in Figures 4.10 to 4.16 from the point of view of the bus mismatches during load flows. The largest per unit mismatches of the bus active power, reactive power, and square of the voltage magnitude are plotted for each iteration for (1) the Newton-Raphson method, (2) the alpha-M.Q.S.O.N.R. method with alpha equal to zero, and (3) the alpha - M.Q.S.O.N.R. method with alpha equal to one (which is equivalent to

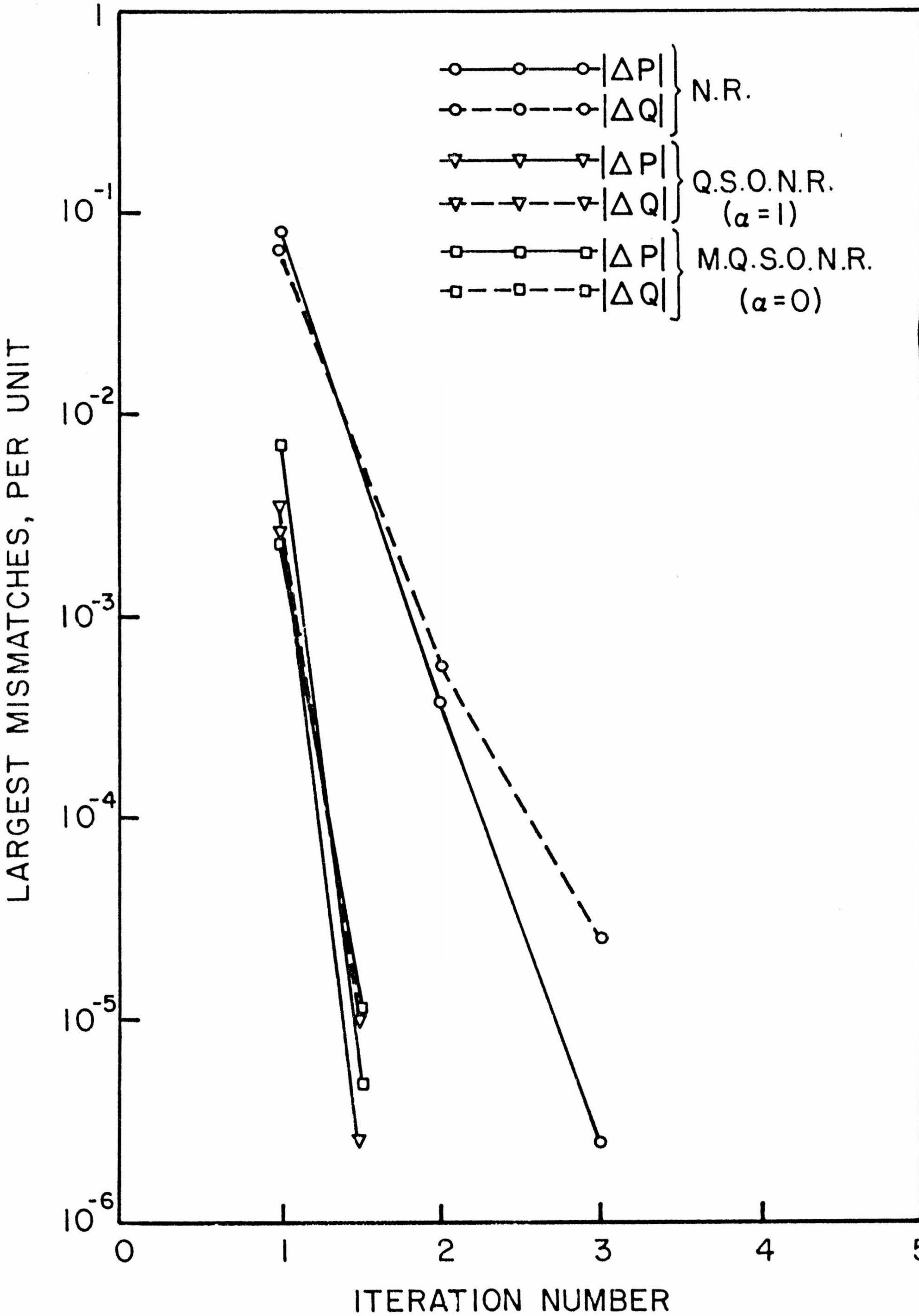


Figure 4.10: Convergence Patterns of Load Flows of the 5-Bus System.

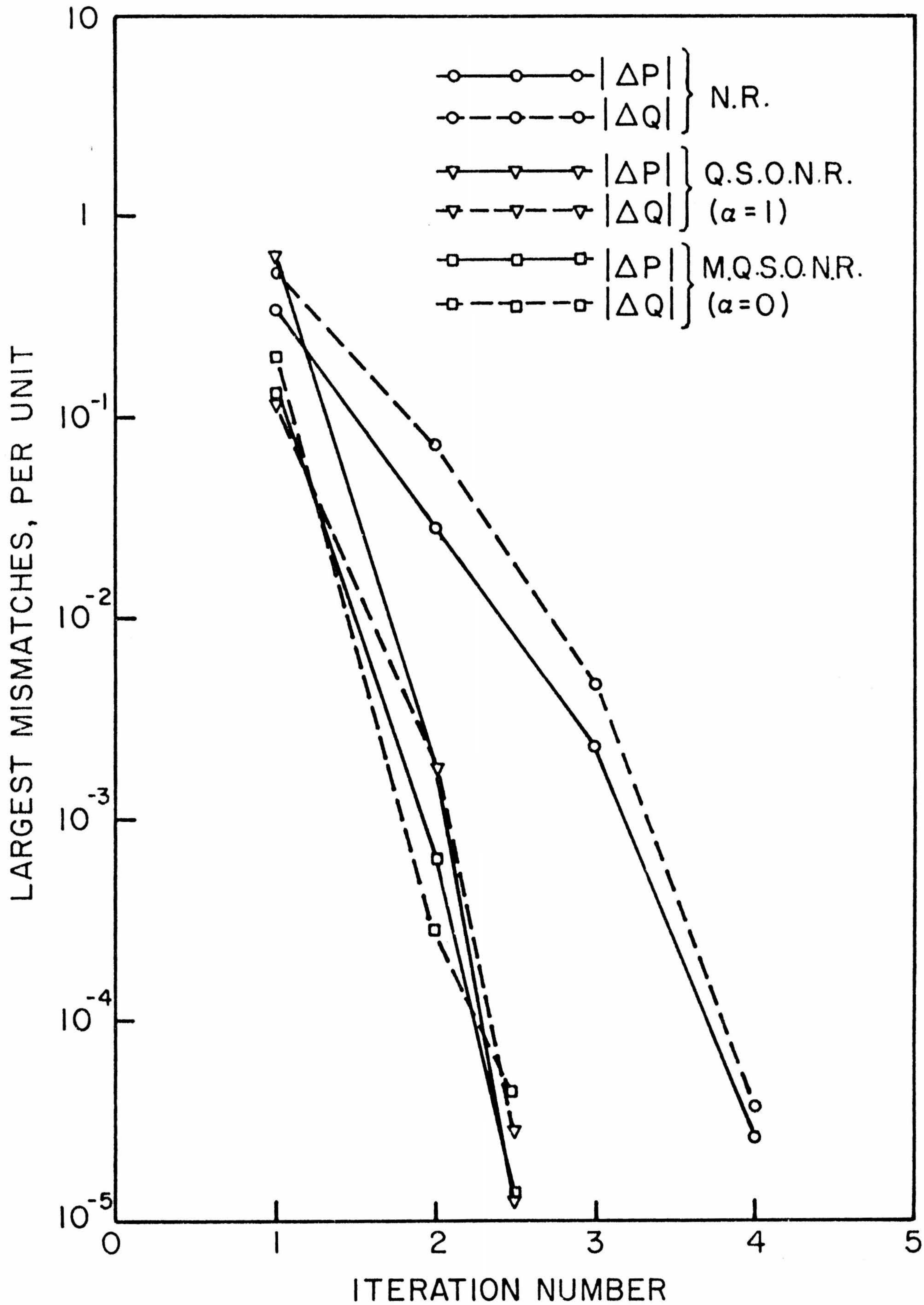


Figure 4.11: Convergence Patterns of Load Flows of the 23-Bus System

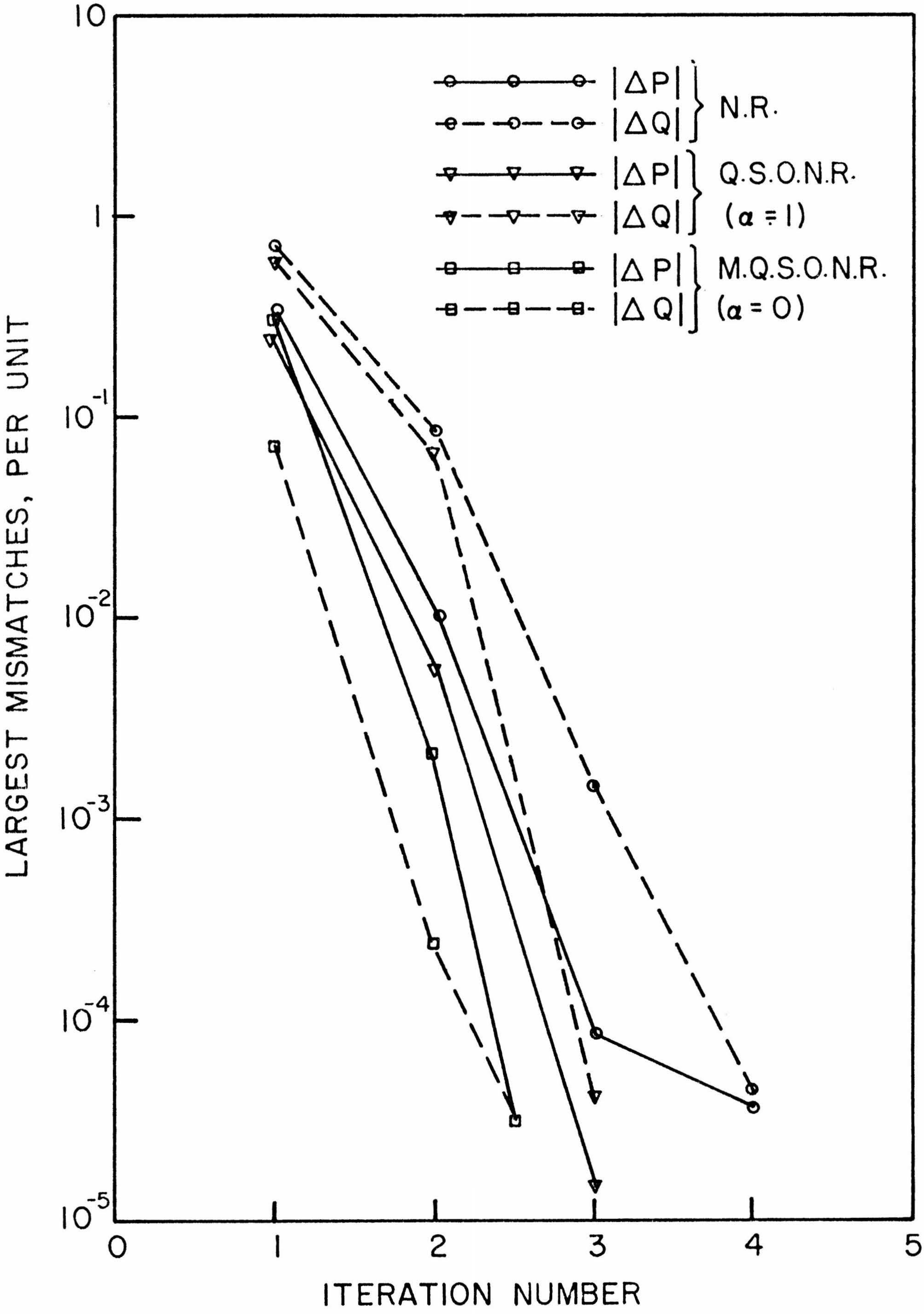


Figure 4.12: Convergence Patterns of Load Flows of the IEEE 57-Bus Test System.

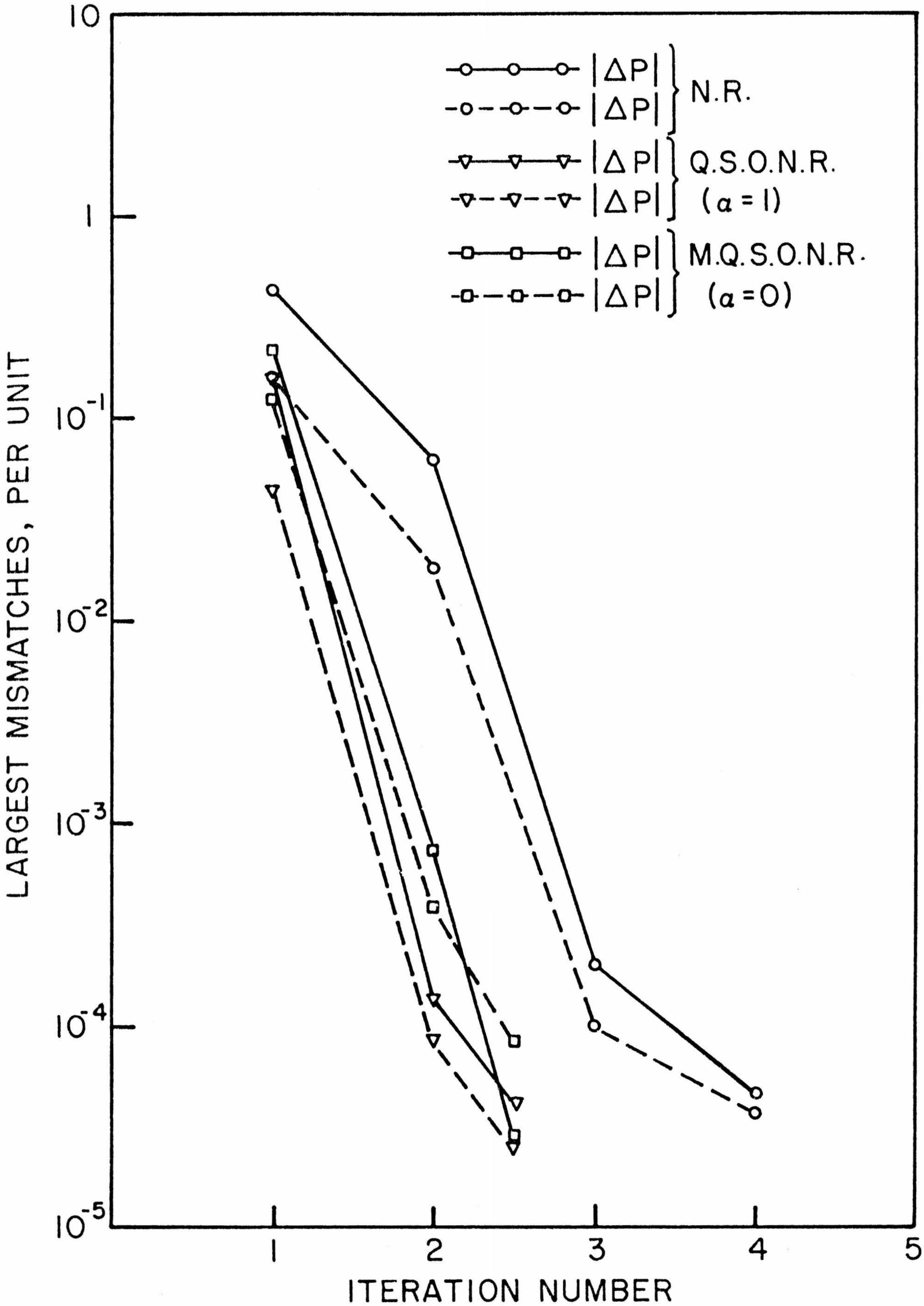


Figure 4.13: Convergence Patterns of Load Flows of the IEEE 118-Bus Test System.

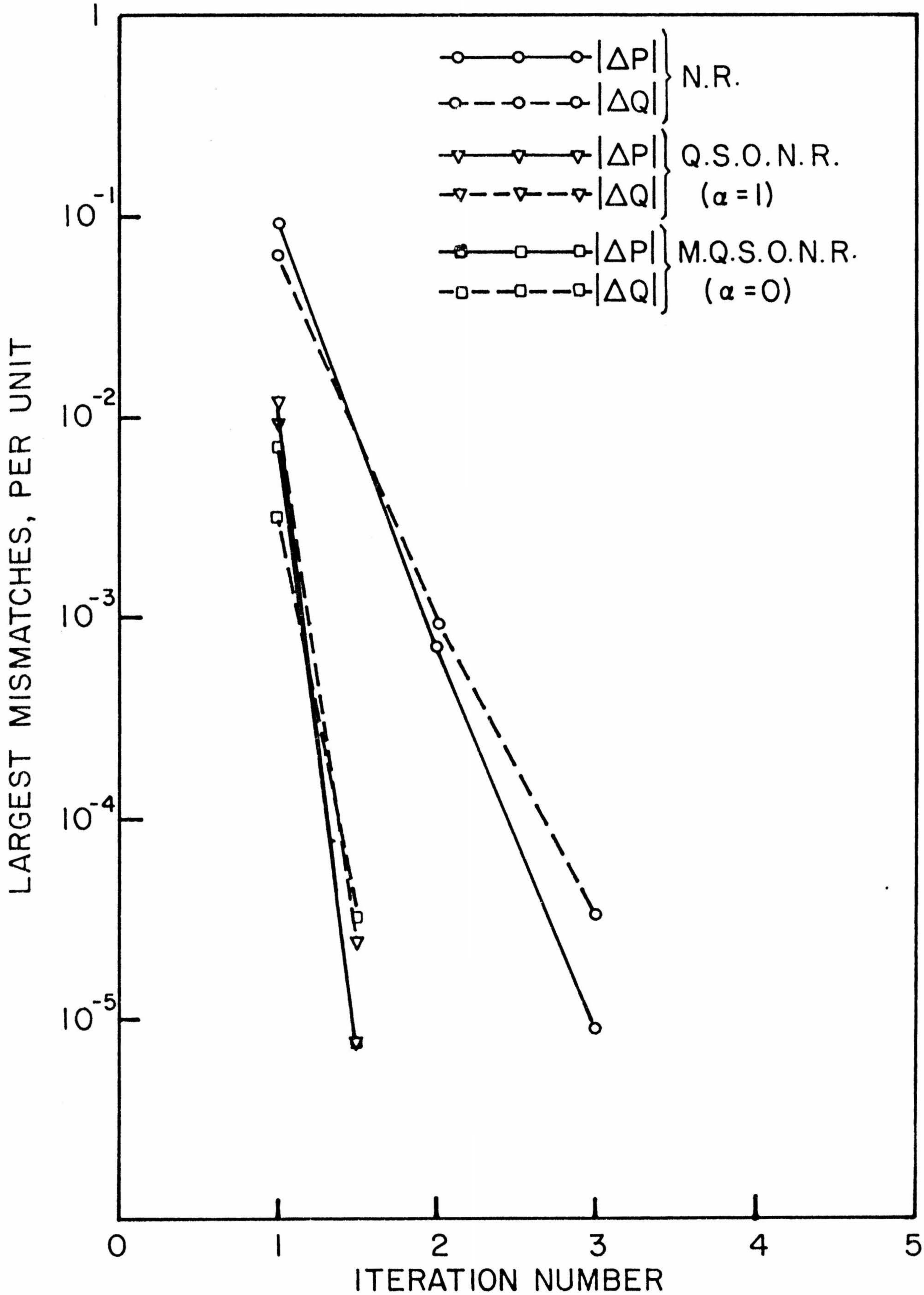


Figure 4.1 : Convergence Patterns of Load Flows of the 5-Bus (No Voltage Regulated Buses) System

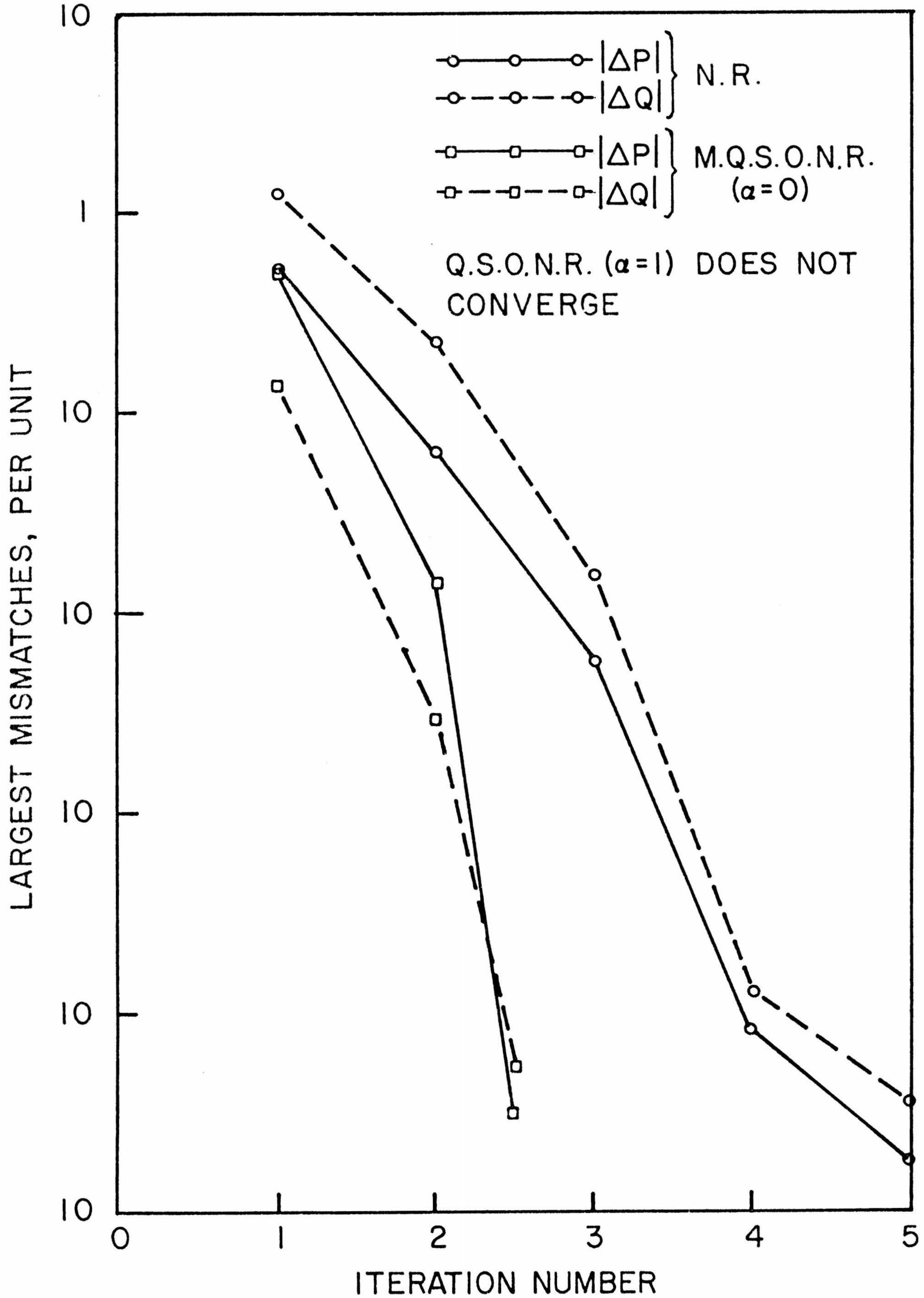


Figure 4.15: Convergence Patterns of Load Flows of the 57-Bus (No Voltage Regulated Buses) System

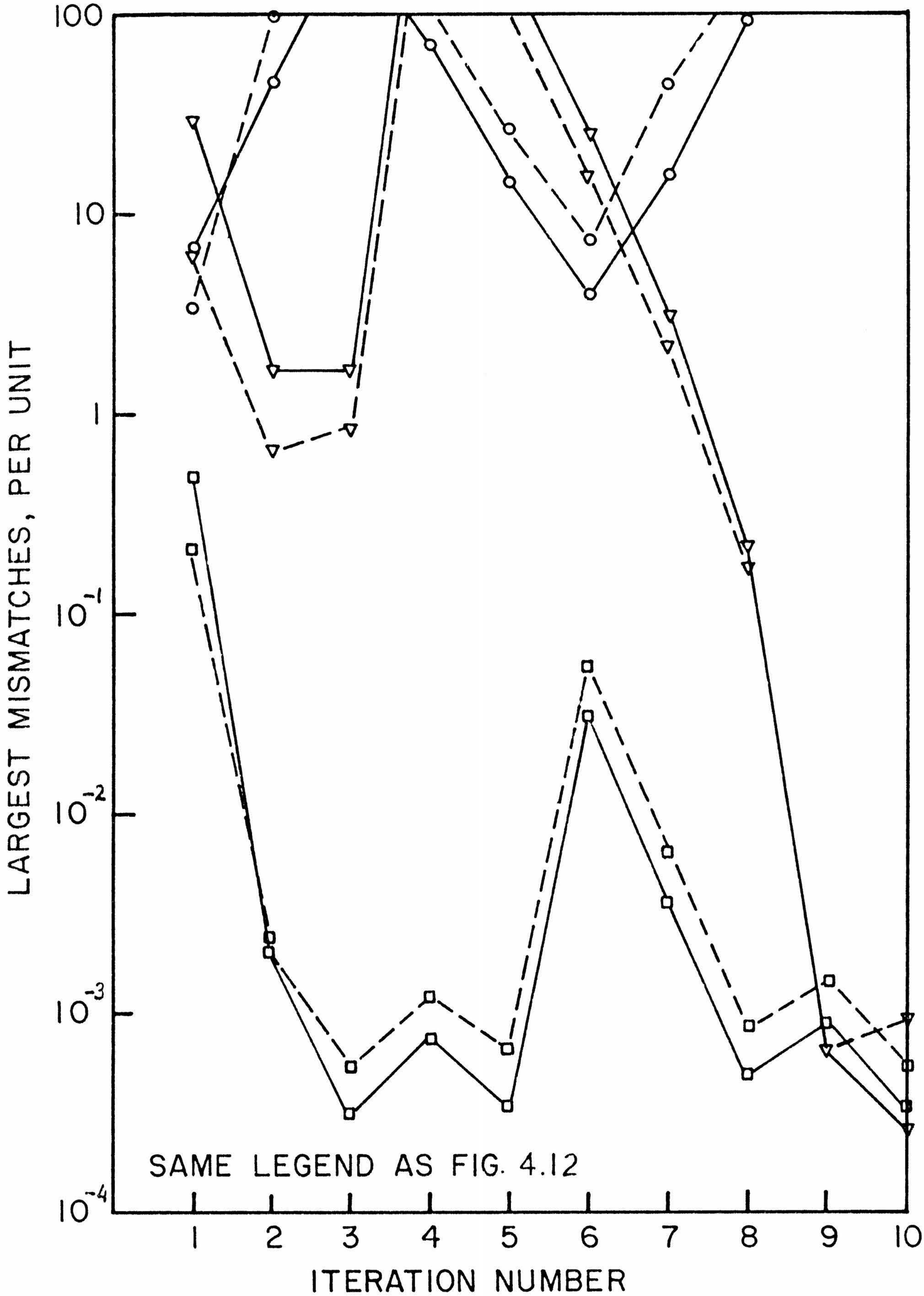


Figure 4.16: Convergence Patterns of Load Flows of the 118-Bus (No Voltage Controlled Buses) System.

normal Q.S.O.N.R. method). The complete sets of maximum mismatch values for each system and for every alpha value between -2.0 and +2.0 (in 0.1 increments) are tabulated in Appendix G.

On the graphs we can again see how much faster is the rate of convergence with the alpha - M.Q.S.O.N.R. method with alpha equal to zero in comparison with the Newton-Raphson method. Also, in almost every case, the former method converges to a more accurate solution than does the latter.

Comparing the new method with alpha equal to zero and with alpha equal to one shows that in the 5-bus, 23-bus, 118-bus, and the 5-bus (no V.C.B.) systems, the latter converges to a slightly more accurate solution in the same number of iterations as the former. However, with the 118-bus (no V.C.B.) system the method with alpha equal to zero is a great deal better than when alpha equals one. With the 57-bus system it is also much better in that it converges faster. And, with the 57-bus (no V.C.B.) system the method with alpha equal to one does not even converge. Therefore, as an overall result, the alpha-M.Q.S.O.N.R. method with alpha equal to zero is superior to the Q.S.O.N.R. method (which is the alpha-M.Q.S.O.N.R. method with alpha equal to one).

4.4.4. Importance of the Initial Values for the Bus Voltages

Using the second order terms in a load flow allows convergence to occur over a larger range of initial guesses of the load bus voltages than with a first order technique such as Newton-Raphson. However, it was found that only when the bus voltages started from a flat start did the alpha-M.Q.S.O.N.R. method perform at its best. A flat start means that the initial voltage magnitudes at all load buses are set to one per unit and the initial voltage angle is set to be the same as that of the slack bus (which serves as the reference bus).

To illustrate this point, the 118-bus system is used. It has a slack bus voltage angle of thirty degrees. Figure 4.5 shows the convergence pattern for a normal flat start with each load bus having an initial voltage magnitude of one per unit and an angle of thirty degrees. Figure 4.16 shows the convergence pattern when the load buses have their initial voltage magnitudes the same, at one per unit, but their initial voltage angles set at zero degrees. The difference between the two graphs is substantial. The first shows the lowest number of iterations to convergence to be two, while the other shows two and one-half as the lowest. The range of alpha values with which convergence occurs is much reduced in the second graph, and the load flow when alpha equals zero is not the best. Also, convergence for the Newton-Raphson method is not as good. So, there is no doubt that a flat start should be employed for the best results.

TABLE 4.1: Number of Iterations to Convergence for Various Alpha Values and Various Systems.

System	Tolerance (p.u.)	Newton -Raphson	ALPHA VALUES								
			-2.0	-1.9	-1.8	-1.7	-1.6	-1.5	-1.4	-1.3	-1.2
5 - Bus	.0001	3.0	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
	.001	2.0	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
23 - Bus	.0001	4.0	4.5	4.5	>10.0	>10.0	4.5	>10.0	3.5	3.5	3.5
	.001	4.0	4.0	4.0	>10.0	>10.0	4.5	>10.0	3.4	3.0	3.0
57 - Bus	.0001	4.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0
	.001	4.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0
118 - Bus	.0001	4.0	3.5	5.0	3.5	4.5	5.0	4.5	>10.0	>10.0	>10.0
	.001	3.0	3.5	3.5	3.5	3.5	4.0	4.5	>10.0	>10.0	>10.0
5 - Bus (No.V.C.B)	.0001	3.0	2.0	2.0	2.0	2.0	2.0	1.5	1.5	1.5	1.5
	.001	2.0	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
57 - Bus (No. V.C.B.)	.0001	5.0	6.0	4.5	4.5	4.5	4.5	5.0	7.5	>10.0	>10.0
	.001	4.0	5.5	4.5	4.0	4.0	4.5	4.5	7.5	>10.0	>10.0
118 - Bus	.0001	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0
	.001	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0

TABLE 4.1 (Continued)

SYSTEM	Tolerance (p.u.)	ALPHA VALUES									
		-1.1	-1.0	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2
5 - Bus	.0001	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
	.001	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
23 - Bus	.0001	3.0	3.0	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
	.001	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.0
57 - Bus	.0001	>10.0	4.0	3.5	3.5	3.0	3.0	2.5	2.5	2.5	2.5
	.001	>10.0	3.5	3.5	3.0	2.5	2.5	2.5	2.5	2.5	2.5
118 - Bus	.0001	>10.0	>10.0	4.5	4.5	4.0	3.5	3.0	3.0	2.5	3.5
	.001	>10.0	>10.0	4.5	3.5	3.0	2.5	2.5	2.5	2.5	2.5
5 - Bus (No. V.C.B)	.0001	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
	.001	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
57 - Bus (No V.C.B)	.0001	>10.0	7.5	>10.0	6.0	>10.0	4.0	3.5	3.5	3.0	3.0
	.001	>10.0	7.0	>10.0	5.5	>10.0	4.0	3.5	3.0	3.0	2.5
115 - Bus (No V.C.B)	.0001	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0
	.001	>10.0	>10.0	>10.0	>10.0	>10.0	5.0	4.5	3.5	3.0	2.5

TABLE 4.1 (Continued)

SYSTEM	Tolerance (p.u.)	ALPHA VALUES -0.1	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
5 - Bus	.0001	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
	.001	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
23 - Bus	.0001	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
	.001	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.5	2.5	2.5
57 - Bus	.0001	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
	.001	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
118 - Bus	.0001	4.0	2.4	3.0	2.5	3.0	3.0	2.0	2.0	2.5	3.0
	.001	2.5	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
5 - Bus (No V.C.B.)	.0001	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
	.001	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
57 - Bus (No V.C.B.)	.0001	3.0	2.5	2.5	2.5	3.0	3.0	3.5	4.5	>10.0	>10.0
	.001	2.5	2.5	2.5	2.5	2.5	3.0	3.0	4.0	>10.0	>10.0
118 - Bus (No V.C.B.)	.0001	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0
	.001	2.5	2.5	2.0	3.5	3.5	4.0	4.5	4.5	5.0	7.5
118 - Bus ($V_{start}=1\underline{/0°}$)	.0001										
	.001										

TABLE 4.1 (Continued)

SYSTEM	Tolerance (p.u.)	ALPHA VALUES 0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
5 - Bus	.0001	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
	.001	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
23 - Bus	.0001	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
	.001	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
57 - Bus	.0001	3.0	3.0	3.5	3.5	4.0	8.5	>10.0	6.0	6.5	7.5	>10.0	2.5
	.001	2.5	2.5	3.0	3.5	4.0	8.5	>10.0	5.5	6.5	7.0	>10.0	2.5
118 - Bus	.0001	3.5	2.5	2.5	4.5	3.0	3.0	2.5	4.0	3.0	2.5	5.0	2.5
	.001	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
5 - Bus (No V.C.B)	.0001	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
	.001	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5	1.5
57 - Bus (No V.C.B.)	.0001	6.5	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	6.5	5.5	5	4.5
	.001	6.5	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	6.5	5.5	4.5	4.5
118 - Bus (No V.C.B.)	.0001	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0	>10.0
	.001	>10.0	9.0	9.5	6.0	5.5	5.0	8.5	>10.0	>10.0	8.5	>10.0	>10.0

CHAPTER V

CONCLUDING REMARKS

5.1 Conclusions

Although much time was spent developing and testing the alpha-modified quasi-second order Newton-Raphson technique in rectangular coordinates, the results were consistent enough so as to make precise conclusions possible.

First, it was found that, for all systems tested, load flow convergence characteristics are best when alpha is equal to zero. With this value, the method was shown to have a convergence rate about twice as fast as the Newton-Raphson technique. The additional computing effort envolved does not correspond to twice as much per iteration. Therefore, the method is also faster in terms of computer time. Also, there is only a slight increase in storage requirements. Though not discussed previously, it does appear that this new method does have better convergence properties than the second order load flow method in polar coordinates reported by Sachdev and Medicherla [46].

The merit of this method is enhanced by the fact that by simply adding a subroutine, load flow programs presently in use can be extended to include second order terms without any major modifications.

As a final remark, it might be noted that there is an indication that this method would be successful with ill-conditioned systems. However much more testing is required in this area before any conslusions can be stated.

5.2 Future Work

In every test case conducted for this thesis, convergence with the alpha-M.Q.S.O.N.R. (alpha equal to zero) occurs in 1.5, 2.0, or 2.5 iterations. Obviously, it would be very difficult to converge in fewer iterations, no matter how good a technique was applied. In order to better assess the capability of the new method, it should be tested with much larger systems and with difficult networks (including underground transmission). An indication of its power is given by the rapid convergence for the 118-Bus (no V.C.B.) system for which the Newton-Raphson fails to converge. Similar results were obtained for another system not reported here.

There is virtually an unlimited number of things that can be done with the technique presented in this thesis to try and achieve a better load flow method. I will suggest here some ideas to pursue and directions to follow.

First on the agenda would be to perform the modifications necessary to obtain further alpha-modified methods such as those mentioned in Chapter III.

As the theory for the alpha-modified methods illustrates, alpha can take on a different value for every bus of a system being tested. Therefore, an effort should be made to develop (and to test the worth of) a procedure to choose the best values of alpha to operate on their corresponding buses.

The present program could be expanded to include the ability to solve systems with D.C. lines and links. Also, additional programming could be added to allow optimal load flows to be performed.

In an attempt to generally decrease the number of iterations required for this method (as well as for other load flow methods), it may be worth utilizing a different, equally valid, convergence criterion. Perhaps the average of all the absolute mismatch values, subject to limiting the maximum value that a mismatch could have, could be used. This would eliminate the very common situation where convergence is delayed only because one or two bus mismatches are not quite within the given tolerance. If the average were used, there would probably be substantial savings in computing time.

Another way of reducing storage requirements, computational effort and computing time would be to develop, from the present algorithm, a decoupled load flow method. This would, of course, be based upon exploiting the weak coupling between real power and voltage magnitude, and reactive power and voltage phase angles. There is a good possibility though, that adding second order terms may be of insignificant value in the presence of the major approximations used in decoupling.

A further area of investigation could look into ways of getting better initial estimates of the voltages at load buses, such as a variation of that reported by Stott [58].

This is only a brief list of areas worth pursuing. The number of major and minor modifications that could be applied to the alpha-M.Q.S.O.N.R. method, is limited only by ones imagination.

In closing, the method in its own right should be applied to any practical problem involving nonlinear equations. Examples in the power systems area include, but are not limited to, various formulations of the economy operation problem, stability analysis, automatic generation control, and hysteresis motor dynamic performance and optimization evaluation.

BIBLIOGRAPHY AND REFERENCES

1. Baumann, "Some New Aspects of Load Flow Calculation", IEEE Trans. on PAS, Vol. 85, pp. 1164-1176, November 1966.

2. Bennett, "Digital Computers and the Load-flow Problem", Proc. of the IEE (London), Vol. 103 B, Supplement 1, p. 16, 1956.

3. Billinton, Bollinger, and Dhar, "A New Modified Newton's Method for Load-Flow Analysis", Conference Paper No. C 72 004-5 for the IEEE Winter Power Meeting, 1972.

4. Boneparte and W.W. Maslin, "Simplified Load Flow", Proc. of the 1967 Power Industry Computer Application Conference, Pittsburgh, PA, pp. 385-394, May 15-17, 1967.

5. Brameller and Denmead, "Some Improved Methods for Digital Network Analysis", Proc. of IEE (London) Vol. 109, Pt. A, pp. 109-116, 1962.

6. Britton, "Improved Area Interchange Control for Newton's Method Load Flows", IEEE Trans. on PAS, Vol. 88, pp. 1577-1581, October 1969.

7. Britton, "Improved Load Flow Performance Through a More General Equation Form", IEEE Trans. on PAS, Vol. 90, pp. 109-116, January 1971.

8. Brown, Carter, Happ, and Person, "Power Flow Solution by Matrix Iterative Method", AIEE Trans. of PAS, Vol. 82, Part III, pp. 1-10, April 1963.

9. Brown and Tinney, "Digital Solutions for Large Power Networks", AIEE Trans. on PAS, Vol. 76, p. 347, June 1957.

10. Carré, "Solution of Load-Flow Problems by Partitioning Systems into Trees", IEEE Trnas. on PAS, Vol. 87, pp. 1931-1938, November 1968.

11. Cory, B.J., and P.B. Henser, "Economic Dispatch with Security using Non-linear Programming, Proceedings", Fourth Power Systems Computations Conference, Grenoble, France, paper 2.1/5, September 1972.

12. Conner, D.A., "Representative Bibliography on Load-Flow Analysis and Related Topics", Paper C73-104-7, IEEE P.E.S. Winter Meeting, New York, January, 1973.

13. Cronin, J.H. and M.B. Newman, "Digital Load-Flow Program for 1000 Bus Systems", Trans. IEEE on P.A.S., Vol. 83, pp. 718-720, 1964.

14. Curtis, A.R., and J.K. Reid, "Fortran Subroutines for the Solution of Sparse sets of Linear Equations", Theoretical Physics Division, Atomic Energy Research Establishment, Harwell, Oxfordshire, 1971.

15. Delson, "The Spurious Solution in Load Flow Calculations", AIEE Conference Paper 62-277, 1962.

16. Despotović, Babić, and Mastilović, "A Rapid and Reliable Method for Solving Load Flow Problems", IEEE Trans. on PAS, Vol. 90, pp. 123-130, January 1971.

17. Dommel, Hermann, W., "Newton's Method", T-PAS 68, October, pp. 1866-1877.

18. Dommel and Tinney, "Optimal Power Flow Soltuions", IEEE Trans. on PAS, Vol. 87, pp. 1866-1876, October 1968.

19. Dommel, Tinney, and Powell, "Further Developments in Newton's Method for Power System Application", Confernece Paper No. 70 CP 161 for the IEEE Winter Power Meeting, 1970.

20. Dopazo, Klitin, Stagg, and Watson, "Optimization Technique for Real and Reactive Power Allocation", Proc. of the IEEE, Vol. 55, pp. 1877-1885, November 1967.

21. Dunstan, "Digital Load Flow Studies", AIEE Trans. on PAS, Vol. 73, Part III-A, p. 825, 1954.

22. Dusonchet, Talukdar, Sinnot, and El-Abaid, "Load Flows Using a Combination of Point Jacobi and Newton's Methods", IEEE Trans. on PAS, Vol. 90, pp. 941-949, May 1971.

23. Dyrkaez and Maginniss, "A New Automatic Program for Load Flow Studies on the IBM 704", AIEE Trans. on PAS, Vol. 78, Part III-A, p. 52, 1959.

24. El-Abiad and Jaimes, "A Method for Optimum Scheduling of Power and Voltage Magnitude", IEEE Trans. on PAS, Vol. 88, pp. 413-422, April 1969.

25. El-Abiad, Watson, and Stagg, "The Load-flow Problem - Its Formulation and Solution - Part I", AIEE Conference Paper 61-1054, 1961.

26. Elgerd, "Electric Energy Systems Theory: An Introduction", McGraw-Hill Book Co., pp. 200-265, 1971.

27. El-Hawary, M.E., Discussion of [42], Discussion A 76 491-1, IEEE Summer Power Meeting, Portland, Oregon, 1976.

28. El-Hawary, M.E., O.K. Wellon, and P.R. Hamilton, "The Modified Qausi-Second Order Newton-Raphson Method for Low Flow Solutions", Paper No. 77015 - A, International Electrical and Electronics Conference and Exposition, Toronto, Sept. 1977.

29. El-Hawary, M.E., and W.J. Vetter, Discussion [46], IEEE Trans. on Power Apparatus and Systems, Vol. PAS-96, pp. 196, 1977.

30. Freris, L.L., and A.M. Sasson, "Investigations on the Load-Flow Problem", Proc. Inst. Elec. Eng., Vol. 114, pp. 1960, December 1967.

31. Galloway, Taylor, Hogg, and Scott, "New Approach to Power-System Load-Flow Analysis in a Digital Computer", Proc. of the IEE (London), Vol. 117, pp. 165-196, January 1970.

32. Glimn, Habermann, Henderson, Kirchmayer, "Digital Calculations of Network Impedances", AIEE Trans. on PAS, Vol. 74, Part III, p. 1285, 1955.

33. Glimm and Stagg, "Automatic Calculation of Load Flows", AIEE Trans. on PAS, Vol. 76, pp. 817-825, October 1957.

34. Gupta and Davis, "Digital Computers in Power System Analysis", Proc. of the IEE (London), Vol. 108 A, pp. 383-398, 1961.

35. Hale and Goodrich, "Digital Computation of Power Flow - Some New Aspects", AIEE Trans. on PAS, Vol. 78 A, p. 919, October 1959.

36. Henderson, "Automatic Digital Computer Solution of Load-Flow Studies", AIEE Trans. on PAS, Vol. 73, Part III, pp. 1696-1702, February 1955.

37. Hurbert and Haynes, "A Rapid Digital Computer Solution for Power System Network Load Flow", IEEE Trans. on PAS, Vol. 90, 934-940, May 1971.

38. Humpage, "Analytical Formulation of the Load Flow Problem", Proc. of the 1967 Power System Load Flow Analysis Symposium, London, 1967.

39. Jordan, "Rapid Converging Digital Load Flow", AIEE Trans. on PAS, Vol. 76, pp. 1433-1438, February 1957.

40. Laughton and Davis, "Numerical Techniques in Solution of Power System Load-Flow Problems", Proc. of the IEE (London), Vol. III, p. 1575, September 1964.

41. Marriage, E.A., and J.E.D. Northcote-Green, "Load Flow and Stability Studies for Transmission Planning", Transactions, Engineering and Operating Division, CEA, Vol. 13, 1974.

42. Medicherla, T.K.P., and M.S. Sachdev, "A Second Order Decoupled Load Flow Technique", paper A 76 491-1, IEEE Summer Power Meeting, Portland, Oregon, 1976.

43. Meisel and Barnard, "Application of Fixed-Point Techniques to Load-Flow Studies", IEEE Trans. on PAS, Vol. 89, pp. 136-140, January 1970.

44. Ogbuobiri, E.C., W.F. Tinney, and J.W. Walker, "Sparsity Directed Decomposition from Gaussian Elimination of Matrices, <u>IEEE Trans. Power App. & Syst.</u>, Vol. PAS-89, No. 1, pp. 141, 149, January 1970.

45. Peterson, N.M., and W.S. Meyer, "Automatic Adjustment of Transformer and Phase-shifter Taps in the Newton Power Flow", <u>IEEE Trans. Power App. & Systems</u>, Vol. PAS-90, pp. 103-1088, January/February 1971.

46. Sachdev, M.S., and T.K.P. Medicherla, "A Second Order Load Flow Technique", IEEE Trans. on Power Apparatus and Systems, Vol. PAS-96, pp. 189-197, 1977.

47. Sachdev, M.S., T.K.P. Medicherla, and R. Billinton, "Second Order Load Flow Techniques", Transactions, Engineering and Operating Division, CEA, 1977.

48. Sasson, "Combined Use of the Powell and Fletcher-Powell Nonlinear Programming Methods for Optimal Load Flows", IEEE Trans. on PAS, Vol. 88, pp. 1530-1537, October 1969.

49. Sasson and Jaimes, "Digital Methods Applied to Power Flow Studies", IEEE Trans. on PAS, Vol. 86, pp. 860-867, July 1967.

50. Sasson, Trevino, and Aboytes, "Improved Newton's Load Flow Through a Minimization Technique", IEEE Trans. on PAS, Vol. 90, pp. 1974-1981, September 1971.

51. Sasson, Viloria, and Aboytes, "Optimal Load Flow Solution Using the Hessian Matrix", Proceedings of the 1971 Power Industry Computer Applications Confernece, pp. 203-209, May 23-26, 1971.

52. Sasson, Viloria, and Aboytes, "Optimal Load Flow Soltuion the Hessian Matrix", Conference Paper No. T 72 198-5 for the IEEE Winter Power Meeting, 1972.

53. Sato and Tinney, "Techniques for Exploiting the Sparsity of the Network Admittance Matrix", IEEE Trans. on PAS, Vol. 82, Part pp. 944-950, December 1963.

54. Sato, N., and W.F. Tinney, "Techniques for Exploiting the Sparsity of the Network Admittance Matrix", IEEE Trans. Power Apparatus and Systems, Vol. 82, pp. 944-950, December 1963.

55. Stagg, G.W., and A.H. El-Abiad, "Computer Methods in Power System Analysis", McGraw Hill, New York, 1968.

56. St. Clair and Stagg, "Experience in Computation of Load Flow Studies Using High Speed Computer", AIEE Conference Paper 58-1100, 1958.

57. Stott, B., "Decoupled Newton Load Flow", IEEE Trans. PAS, Vol. PAS-91, pp. 1955-1959, September/October, 1972.

58. Stott, B., "Effective Starting Process for N.R. Load Flows", Proc. Inst. Elec. Eng., Vol. 118, pp. 983-987, August 1971.

59. Stott, B., "Review of Load-Flow Calculation Methods", Proceedings of IEEE (Special issue on Computer in the Power Industry), Vol. 62, No. 7, pp. 916-929, July 1974.

60. Stott, B., and O. Alsac, "Fast Decoupled Load Flow", IEEE Trans. PAS, Vol. 93, pp. 859-869, May/June 1974.

61. Stott, "Decoupled Newton Load Flow", Conference Paper No. T 72 135-7 for the IEEE Winter Power Meeting, 1972.

62. Taylor and Treece, "Load Flow Analysis by the Gauss-Seidel Method", Proc. of the 1967 Power System Load Flow Analysis Symposium, London, 1967.

63. Tinney, W.F., "Notes on Network Solutions by Optimally Ordered Triangular Factorization", Notes to supplement [Tinney, Walker, November, 1967] [65].

64. Tinney and Hart, "Power Flow Solution by Newton's Method", IEEE Trans. on PAS, Vol. 86, pp. 1449-1460, November 1967.

65. Tinney, W.F., and J.W. Walker, "Direct Solutions of Sparse Network Equations by Optimally Ordered Triangular Factorization", Proceedings of IEEE, Vol. 55, pp. 1801-1809, November 1967.

66. Tinney, W.F., W.L. Powell, N.M. Peterson, "Sparsity-oriented Network Reduction", IEEE PICA Conference Proceedings, Minneapolis, pp. 384-390, July 1973.

67. Tinney, W.F., and W.S. Meyer, "Solution of Large Sparse Systems by Ordered Triangular Factorization", IEEE Trans. Automatic Control, Vol. Ac-18, No. 4, pp. 333-346, August 1973.

68. Walker, J.W., "Renumbering Subroutine", July, 1967, A computer subroutine implementing theory of [65].

69. Wallach and Even, "Application of Newton's Method to Load-Flow Calculations", Proc. of the IEE (London), Vol. 114, pp. 372-374, March 1967; Discussion Vol. 114, pp. 1163-1164, August 1967.

70. Wallach, Even, and Yavin, "Improved Methods for Load-Flow Calculations", IEEE Trans. on PAS, Vol. 90, pp. 116-122, January 1971.

71. Ward and Hale, "Digital Computer Solution of Power-Flow Problems", AIEE Trans. on PAS, Vol. 75, pp. 398-404, June 1956.

72. Van Ness, "Convergence of Iterative Load Flow Studies", AIEE Trans. on PAS, Vol. 78B, pp. 1590-1597, February 1960.

73. Van Ness, "Iteration Methods for Digital Load-Flow Studies", AIEE Trans. on PAS, Vol. 78A, Part III, pp. 583-588, August 1959.

74. Van Ness and Griffin, "Elimination Methods for Load-Flow Studies", AIEE Trans. on PAS, Vol. 80, pp. 299-304, June 1961.

75. Wu, Felix, F., "Theoretical Study of the Convergence of the fast Decoupled Load Flow", IEEE Trans. on Power Apparatus & Systems, Vol. PAS-96, pp. 268-275, January 1977.

APPENDIX A

JACOBIAN ELEMENTS

The Jacobian elements are the first order derivatives of the load flows equations, (2.21), (2.22), and (2.23).

The first order partial derivatives of the active bus powers are:

$$\frac{\partial P_p}{\partial e_q} = e_p G_{pq} - f_p B_{pq} \qquad q \neq p$$

$$\frac{\partial P_p}{\partial e_p} = 2e_p G_{pp} + \sum_{\substack{q=1 \\ q\neq p}}^{n} (e_q G_{pq} + f_q B_{pq})$$

$$\frac{\partial P_p}{\partial f_q} = e_p B_{pq} + f_p G_{pq} \qquad q \neq p$$

$$\frac{\partial P_p}{\partial f_p} = 2f_p G_{pp} + \sum_{\substack{q=1 \\ q\neq p}}^{n} (f_q G_{pq} - e_p B_{pq})$$

The first order partial derivatives of the reactive bus powers are:

$$\frac{\partial Q_p}{\partial e_q} = e_p B_{pq} + f_p G_{pq} \qquad q \neq p$$

$$\frac{\partial Q_p}{\partial e_p} = 2e_p B_{pp} - \sum_{\substack{q=1 \\ q\neq p}}^{n} (f_q G_{pq} - e_q B_{pq})$$

$$\frac{\partial Q_p}{\partial f_q} = -e_p G_{pq} + f_p B_{pq} \qquad q \neq p$$

$$\frac{\partial Q_p}{\partial f_p} = 2f_p B_{pp} + \sum_{\substack{q=1 \\ q\neq p}}^{n} (e_q G_{pq} + f_q B_{pq})$$

The first order partial derivatives of the square of the bus voltage magnitudes are:

$$\frac{\partial |V_p|^2}{\partial e_q} = 0$$

$$\frac{\partial |V_p|^2}{\partial e_p} = 2e_p$$

$$\frac{\partial |V_p|^2}{\partial f_q} = 0$$

$$\frac{\partial |V_p|^2}{\partial f_p} = 2f_p$$

APPENDIX B

HESSIAN ELEMENTS

The Hessian elements are the second order derivatives of the load flow equations, (2.21), (2.22) and (2.23). As such, we may obtain them by simply taking the derivatives of the Jacobian elements found in appendix A.

Based on the first order partial derivatives, the second order partial derivaties of the active bus powers are:

$$\frac{\partial^2 P_p}{\partial e_q \partial e_p} = \frac{\partial^2 P_p}{\partial e_p \partial e_q} = G_{pq}$$

$$\frac{\partial^2 P_p}{\partial e_q^2} = 0$$

$$\frac{\partial^2 P_p}{\partial e_p^2} = 2\ G_{pp}$$

$$\frac{\partial^2 P_p}{\partial f_q \partial e_p} = \frac{\partial^2 P_p}{\partial e_p \partial f_q} = B_{pq}$$

$$\frac{\partial^2 P_p}{\partial f_q^2} = 0$$

$$\frac{\partial^2 P_p}{\partial f_q \partial f_p} = \frac{\partial^2 P_p}{\partial f_p \partial f_q} = G_{pq}$$

$$\frac{\partial^2 P_p}{\partial f_p^2} = 2\ G_{pp}$$

$$\frac{\partial^2 P_p}{\partial f_p \partial e_q} = \frac{\partial^2 P_p}{\partial e_q \partial f_p} = -\ B_{pq}$$

$$\frac{\partial^2 P_p}{\partial e_q \partial f_q} = \frac{\partial^2 P_p}{\partial f_q \partial e_q} = 0$$

$$\frac{\partial^2 P_p}{\partial f_p \partial e_p} = \frac{\partial^2 P_p}{\partial e_p \partial f_p} = 0$$

$$\frac{\partial^2 P_p}{\partial e_r \partial e_q} = 0$$

$$\frac{\partial^2 P_p}{\partial f_r \partial f_q} = 0$$

$$\frac{\partial^2 P_p}{\partial e_r \partial f_q} = 0$$

$$\frac{\partial^2 P_p}{\partial f_r \partial e_q} = 0$$

The second order partial derivatives for the reactive bus powers are:

$$\frac{\partial^2 Q_p}{\partial e_q \partial e_p} = \frac{\partial^2 Q_p}{\partial e_p \partial e_q} = B_{pq}$$

$$\frac{\partial^2 Q_p}{\partial e_q^2} = 0$$

$$\frac{\partial^2 Q_p}{\partial e_p^2} = 2\ B_{pp}$$

$$\frac{\partial^2 Q_p}{\partial f_q \partial e_p} = \frac{\partial^2 Q_p}{\partial e_p \partial f_q} = -G_{pq}$$

$$\frac{\partial^2 Q_p}{\partial f_q^{\,2}} = 0$$

$$\frac{\partial^2 Q_p}{\partial f_q \partial f_p} = \frac{\partial^2 Q_p}{\partial f_p \partial f_q} = B_{pq}$$

$$\frac{\partial^2 Q_p}{\partial f_p^{\,2}} = 2\ B_{pp}$$

$$\frac{\partial^2 Q_p}{\partial f_p \partial e_q} = \frac{\partial^2 Q_p}{\partial e_q \partial f_p} = G_{pq}$$

$$\frac{\partial^2 Q_p}{\partial e_q \partial f_q} = \frac{\partial^2 Q_p}{\partial f_q \partial e_q} = 0$$

$$\frac{\partial^2 Q_p}{\partial f_p \partial e_p} = \frac{\partial^2 P_p}{\partial e_p \partial f_p} = 0$$

$$\frac{\partial^2 Q_p}{\partial e_r \partial e_q} = 0$$

$$\frac{\partial^2 Q_p}{\partial f_r \partial e_q} = 0$$

$$\frac{\partial^2 Q_p}{\partial e_r \partial f_q} = 0$$

$$\frac{\partial^2 Q_p}{\partial f_r \partial e_q} = 0$$

The second order partial derivatives for the square of the bus voltage magnitudes are:

$$\frac{\partial^2 |V_p|^2}{\partial e_q \partial e_p} = \frac{\partial^2 |V_p|^2}{\partial e_p \partial e_q} = 0$$

$$\frac{\partial^2 |V_p|^2}{\partial e_q^2} = 0$$

$$\frac{\partial^2 |V_p| 2}{\partial e_p^2} = 2$$

$$\frac{\partial^2 |V_p|^2}{\partial f_q \partial e_p} = \frac{\partial^2 |V_p|^2}{\partial e_p \partial f_q} = 0$$

$$\frac{\partial^2 |V_p|^2}{\partial f_q^2} = 0$$

$$\frac{\partial^2 |V_p|^2}{\partial f_q \partial f_p} = \frac{\partial^2 |V_p|^2}{\partial f_p \partial f_q} = 0$$

$$\frac{\partial^2 |V_p|^2}{\partial^2 f_p^2} = 2$$

$$\frac{\partial^2 |V_p|^2}{\partial f_p \partial e_q} = \frac{\partial^2 |V_p|^2}{\partial e_q \partial f_p} = 0$$

$$\frac{\partial^2 |V_p|^2}{\partial e_q \partial f_q} = \frac{\partial^2 |V_p|^2}{\partial f_q \partial e_q} = 0$$

$$\frac{\partial^2 |V_p|^2}{\partial f_p \partial e_p} = \frac{\partial^2 |V_p|^2}{\partial e_p \partial f_p} = 0$$

$$\frac{\partial^2 |V_p|^2}{\partial e_r \partial e_q} = 0$$

$$\frac{\partial^2 |V_p|^2}{\partial f_r \partial f_q} = 0$$

$$\frac{\partial^2 |V_p|^2}{\partial e_r \partial f_q} = 0$$

$$\frac{\partial^2 |V_p|^2}{\partial f_r \partial e_q} = 0$$

APPENDIX C

SUBROUTINE SPECIFICATION SHEETS

In this appendix a description of the sparsity directed programs for solving linear sets of equations is given. These were developed at the U.K. Atomic Energy Establishments' Horwell Laboratories.

1. Purpose

To calculate equilibration factors for the rows and columns of a sparce nxn matrix A, which, if applied before Gaussian elimination with pivoting, will make the choice of pivots more likely to lead to low growth of round-off errors. The scaling factors are integral powers 16**R_i (for row i) and 16**C_j (for column j) of 16 (but this base can easily be changed to suit a computer which uses a different radix for floating-point operations). Thus pivots should be chosen as if the matrix elements had been

$$b_{ij} = a_{jj} * 16**(R_i + C_j) \qquad (1)$$

The matrix A is stored in the condensed form used by MA18A. Here the non-zero elements are stored linearly by columns, with row numbers in a parallel INTEGER*2 array and with pointers to the start and finish of each column in a smaller INTEGER*2 array.

2. Calling sequence and argument list

```
CALL MC12A(A,IRN,IP,N,NP,ISC,WS,ISING)
```

where

A — is a REAL array which contains the non-zero elements of the matrix to be scaled, with all elements of a column consecutive.

IRN — is an INTEGER*2 array in which IRN(K) contains i if A(K) holds a_{ij}.

IP — is an INTEGER*2 array of dimension (NP,2), where NP $\geq$ N+1. IP(m,2) contains j, the column number of the column which is stored mth in sequence, and IP (j,1) contains k, where A(k) is the first element of column j. IP(IP(m+1,2),1) thus points to the first element of A beyond those for column j. IP(N+1,2) contains N+1, and IP(N+1,1) points to the first unused element of array A.

N — is the order of A.

NP — is the first dimension of IP.

ISC — is an INTEGER*2 array of dimensions (NP,2) in which integer scaling powers are returned, R_i in ISC(j,1) and C_j in ISC(j,2) (see §3 below).

WS — is a workspace array holding 4*N REAL*4 numbers.

ISING — (INTEGER*4) is set on return to 0 normally, but if row I or column J of A is found to consist only of zero elements, ISING is set to I or -J respectively. The scaling factors returned for non-zero rows and columns are correct. For several zero rows or columns, only the last one detected is returned in ISING.

Note: The output and workspace requirements are identical for single - and double-precision versions, only array A being declared REAL*8 in the latter.

3. Method

The variables p_i and c_j are chosen to minimize the function

$$\phi = \sum_{i,j} (f_{ij}-p_i-c_j)^2 \tag{2}$$

where -

$$f_{ij} = \log |a_{ij}|/\log 16 \tag{3}$$

and the summation is over pairs i,j for which $a_{ij} \neq 0$. This is done to sufficient accuracy in only a few matrix-by vector multiplications. Then R_i and C_j are obtained by rounding $-p_i$ and $-c_j$ to integers. See Curtis and Reid (1971) for further information.

Use of this method gives far better results on sparse matrices than scali to equilibrate row and column norms, and MC12A/AD is called by MA18A/AD before factorising a matrix.

Reference

Curtis, A.R. and Reid, J.K. "On the automatic scaling of matrices for Gaussian elimination", AERE note TP.444 (1971).

1. Purpose

This subroutine solves a general sparse NXN system of linear equations

$$\sum_{j=1}^{N} a_{ij} x_j = b_i, \quad i=1,2,\ldots.N$$

(i.e. find $x = A^{-1}b$) or related problems.

There are four entries:

(a) MA18A decomposes A into triangular factors using a pivotal strategy designed to compromise between maintaining sparsity and controlling loss of accuracy through roundoff.

(b) MA18B uses the factors produced by MA18A (or MA18C) to find $A^{-1}b$, $(A^T)^{-1}b$, Ab or A^Tb.

(c) MA18C factorises a new matrix A of the same pattern, using the pivotal sequence determined by an earlier entry to MA18A.

(d) MA18D loads the elements of a new matrix A into a storage array in the sequence required by MA18C, calling a user-supplied subroutine to obtain each column of A and using indexing information stored by MA18A.

It is envisaged that MA18C may be called many times for one call of MA18A, so it is much faster. Also it is expected that MA18B may be called with many different vectors for the same matrix A.

2. Argument lists

CALL MA18A (A,IRN,IP,N,NP,G,U,IA)

CALL MA18B(A,IRN,IP,N,NP,W,B,MTYPE)

CALL MA18C(A,IRN,IP,N,NP,G)

CALL MA18D(A,IRN,IP,N,NP,W,NAME)

A — is a REAL*4 (or REAL*8 for the D versions) array of dimension IA holding the non-zero elements of the matrix A on entry to MA18A or MA18C and the elements of the triangular factors on exit. Elements are stored by columns. For entry to MA18A, they must be in natural row order within each column and the columns must be in natural order; that is a_{ij} precedes $a_{k\ell}$ if $j < \ell$ or if $j = \ell$ and $i < k$. Thus a typical order might be a_{11}, a_{31}, a_{12}, a_{23}, a_{53}, a_{54}, a_{45}, a_{55}. Before entry to MA18C, the elements of A should be set by calling MA18D. A is altered by MA18A, by MA18C and by MA18D.

IRN — is an INTEGER*2 array of dimension IA*2, whose first IA elements are used to hold row numbers and whose remaining elements are used for workspace by MA18A only. If a_{ij} is held in A(K) then IRN(K) must contain i; for the above example IRN would contain 1,3,5,1,4,2,5,5,4,5, IRN is altered by MA18A.

IP — is an INTEGER*2 array of dimensions (NP,13) where NP $\geq$ 1. Before entry to MA18A the values of IP(J,1), J-1,...N should be set to the subscript in array A of the first element of column J of the matrix and IP(N+1,1) should be set to the subscript of the first unused location in A; thus in the above example IP would contain

1,4,6,8,9,11. The contents of IP are altered by MA18A. ((IP (I,J), I=1,N+1), J=1,5) should be left undisturbed between a MA18A entry and a subsequent entry to MA18B/C/D, or (for J=3,4) to MA18A if the previous scaling factors are to be used (see §4). The rest of IP is available as workspace. An equivalence should be used to ensure that IP starts on a 4-byte boundary. MA18C uses the whole of IP as workspace if it obtains new scaling factors (see §4).

N (INTEGER*4) is the order of the matrix A.

NP (INTEGER*4) is the first dimension of the array IP and should be at least N+1.

G (REAL*4 or REAL*8 for the D version) is an output parameter used to indicate the possible growth of errors during the elimination. Normally MA18A and MA18C scale the rows and columns of the matrix (see §4) so that the comparisons used in choosing each pivot will be reasonable. The maximum difference between the floating-point exponent of any element at any stage of the elimination and the floating-point exponent of the initial largest element in its column is evaluated; G is set to the computer rounding error times 16 to the power of this integer. It is thus an estimate of the relative perturbation on the elements of A. It is set to <u>-1</u> in the event of an error, such as singularity of the matrix or lack of space, preventing successful execution.

U (REAL*4 or REAL*8 for the D version) is a number set by the user in the range $0 < U \leq 1$ to control the choice of pivots: if $U > 1$

it is reset to 1 and if $U \leq 0$ it is reset to the relative floating-point accuracy. When searching a row/column for a pivot any element less than U times the largest element in the row/column is excluded. Thus decreasing U biases the algorithm towards maintaining sparsity at the expense of G and vice-versa. The value 0.25 has been found satisfactory in test examples.

IA (INTEGER*4) indicates the size of arrays A and IRN. The number of elements in the decomposed form of A is limited to IA which may not exceed (32767-N) because of the use of INTEGER*2 indices.

W is a REAL*4 (or REAL*8 for the D version) working array of dimension at least N. W(1) may be equivalenced with an element of IP beyond IP(N+1,5) to save space.

B is a REAL*4 (or REAL*8 for the D version) array of dimension N used to hold b on entry and $A^{-1}b$, $(A^T)^{-1}b$, Ab or A^Tb on exit.

MTYPE is an INTEGER*4 variable controlling the action of MA18B. It should have the value 1,2,3, or 4 according to whether $A^{-1}b$, $(A^T)^{-1}b$, Ab or A^Tb is required. If MA18B is called erroneously, it sets MTYPE=0 before return.

NAME is the name of a user supplied subroutine called by MA18D. It must be declared EXTERNAL by the calling program. It is called by

CALL NAME (N,W,J)

and should return all the non-zero elements a_{IJ} of column J of the new matrix in W(I), without altering other elements of W. MA18D will call NAME N times (with J values <u>not</u> in sequence), and load the new matrix into array A.

3. Use of the entries

Between a call of MA18A and a subsequent call of MA18B, MA18C or MA18D, the contents of (IRN(I), I=1, IA), ((IP(I,J), I=1,N+1), J=1,5) N,NP should not be altered. IP(N+1,2) is set to N+1 on successful completion of MA18A, or to zero if an error has been detected. IP(N+1,3) is set to zero normally but to 1 if MA18C detects a zero pivot.

By examining IP(N+1,2) and for MA18B, IP(N=L,3) subsequent entry points can conclude that MA18A has not been previously entered, or that it or MA18C diagnosed an error. This causes an error diagnostic from the new entry. G is set to -1 by MA18A or MA18C if an error is found, so that the calling program can test for success by examining the sign of G on return. MA18B detects two types of error: those due to invalid entry as described above, which it signals to the calling program by setting MTYPE=0; or if it finds MTYPE out of range, when it leaves it unaltered.

Execution of MA18C is much faster than MA18A, but it is important to check the value of G on successful return from MA18C, in case the old pivotal sequence is poor, for the new matrix, from the roundoff point of view. If G is too large to be acceptable as a relative perturbation on A, the arrays A, IRN, IP should be reset and MA18A re-entered. IF there is ample space in arrays A and IRN, it may be worth while to increase the value of U. The number of locations used in A or IRN is (IP(N+1,1)-1). G should in any case be monitored to detect error returns.

4. Subroutines called and common block

MA18A (and in some circumstances MA18C) calls subroutine MC12A to obtain row and column scaling factors for the matrix. The application of this scaling is controlled by a parameter in a named common block, which also contains the output stream number for diagnostic messages. The common statement is

```
COMMON/MA18E/JP,JSCALE
```

and the default values are JP-6, JSCALE=1.

By including this statement in his program, the user can, if he wishes, charge the stream number JP or the scaling parameter JSCALE.

NB: This should be done by Fortran instructions, not by BLOCK DATA.

The significance of JP is obvious; that of JSCALE is as follows:

JSCALE	Scaling action
< 0	Scaling factors determined during an earlier call to MA18A (or MA18C) are applied to the current matrix.
= 0	No scaling is done (i.e. all scaling factors are set to 1.0).
= 1	MC12A is called by MA18A to obtain scaling factors, but the action with MA18C is as for JSCALE < 0.
> 1	MC12A is called both by MA18A and by MA18C.

If JSCALE > 1, MA18C will change all 13 columns of array IP, otherwise it will use only the first 5 columns.

The time overhead for calling MC12A is significant on MA18C, but not usually on MA18A. Moreover, use of scaling factors with MA18C affects only the value of G, not the pivotal sequence as with MA18A.

5. Error diagnostics

A number of error conditions are diagnosed which prevent successful completion. Most are detected by MA18A, since the other entry points are used only after this one has either succeeded with one set of matrix elements in the prescribed sparsity pattern, or has recorded its failure for them. The following messages may be printed on stream JP by MA18:

(i) ERROR RETURN FROM MA18A BECAUSE THE ELEMENT HELD IN A(k) IS OUT OF ORDER

This message covers sequence errors in the indexing information supplied in IRN, IP; k gives the location at which the error was detected.

(ii) ERROR RETURN FROM MA18A BECAUSE THE MATRIX IS SINGULAR. COLUMN (or ROW) j IS DEPENDENT ON THE REST.

This message covers two cases, (a) where the indexing information specifies no non-zero elements in column (or row) j, (b) where after elimination on the column or row all the elements eligible as pivots are zero.

(iii) ERROR RETURN FROM MA18A BECAUSE IA IS TOO SMALL. SPACE RAN OUT WHEN ELIMINATING ON PIVOT i.

This message appears when there is insufficient room to store a new non-zero element generated in elimination operations using the ith pivot. Thus if $i \ll N$ much more space will probably be needed, but if i is nearly equal to N just a little more may suffice.

(iv) ERROR RETURN FROM MA18B BECAUSE MTYPE = m WHICH IS OUT OF RANGE.

This message needs no comment.

(v) ERROR RETURN FROM MA18B (or MA18C or MA18D) BECAUSE PREVIOUS ENTRY TO MA18A (or MA18C) GAVE ERROR RETURN.

This message is given with MA18C in the second position only if MA18B occurs in the first position. In that case, the error detected by MA18C was a zero pivot, which may not occur on subsequent re-entry to MA18C with a further new matrix.

(vi) ERROR RETURN FROM MA18B(or MA18C or MA18D) BECAUSE NO PREVIOUS ENTRY TO MA18A.

This message is given if IP(N+1,2) is found to have a value which is neither N+1 (after a successful exit from MA18A) nor 0 (after an error return from MA18A).

(vii) ERROR RETURN FROM MA18C BECAUSE ZERO PIVOT (i j).

This message signifies that a_{ij} was found to be zero when it was due to be used as a pivot by MA18C.

(viii) ERROR RETURN FROM MA18C BECAUSE MC12A HAS GIVEN ERROR RETURN WITH IS = j.

This message isgnifies that all the elements of the new matrix in row i (if $i > 0$) or in column (-i) (if $i < 0$) have been found to be zero.

6. Method

The subroutine is described in detail in AERE Report R.6844, which should be consulted for certain coding details if it is planned to transfer the subroutine to a computer other than the System/360.

```
C       SUBROUTINE VSRTPM (A,LA,IR)                                     VSRT0010
C                                                                       VSRT0020
C-VSRTPM --------S/D-----LIBRARY L----------------------------------VSRT0030
C-VSORTP                                                                VSRT0040
C                                                                       VSRT0050
C   FUNCTION        VSRTPM - SORT ARRAYS BY ABSOLUTE VALUE -             VSRT0060
C                              PERMUTATIONS RETURNED                    VSRT0070
C                   VSORTP - SORT ARRAYS BY ALGEBRAIC VALUE-            VSRT0080
C                              PERMUTATIONS RETURNED                    VSRT0090
C   USAGE                  - CALL VSRTPM (A.LA.IR)                      VSRT0100
C                          - CALL VSORTP (A.LA.IR)                      VSRT0110
C   PARAMETERS      A      - ON INPUT, CONTAINS THE ARRAY TO BE         VSRT0120
C                            SORTED ON OUTPUT, A CONTAINS THE           VSRT0130
C                            SORTED ARRAY                               VSRT0140
C                   LA     - INPUT VARIABLE CONTAINING THE NUMBER VSRT0150
C                            OF ELEMENTS IN THE ARRAY TO BE SORTED VSRT0160
C                   IR(LA) - ON INPUT, IR CONTAINS THE INTEGER          VSRT0170
C                            VALUES 1,2,....LA. SEE PROGRAMMING         VSRT0180
C                            NOTES.                                     VSRT0190
C                          - ON OUTPUT, IR CONTAINS A RECORD OF THE VSRT0200
C                            PERMUTATIONS MADE ON THE VECTOR A.         VSRT0210
C   PRECISION              - SINGLE/DOUBLE                              VSRT0220
C   LANGUAGE               - FORTRAN                                    VSRT0230
C                                                                       VSRT0240
C-------------------------------------------------------------------VSRT0250
```

CALL VSRTPM (A,LA,IR)

CALL VSORTP (A,LA,IR)

Purpose

VSRTPM sorts any LA consecutive elements of a vector into ascending sequence by absolute value, keeping a record in IR of the permutations to the vector A. That is, the elements of IR are moved in the same manner as are the elements in A as A is being sorted.

VSORTP sorts any LA consecutive elements of a vector into ascending sequence by algebraic value, keeping a record in IR of the permutations to the vector A. That is, the elements of IR are moved in the same manner as are the elements in A as A is being sorted.

Algorithm

VSRTPM/VSORTP uses the algorithm declared in IMSL routine VSORTM/VSORTA.

Programming Notes

1. IR and A must have dimension at least LA.
2. The vector IR must be initialized before entering VSRTPM/VSORTP. Ordinarily, IR(1) = 1, IR(2) = 2,..., IR(LA) = LA. For wider applicability, any integer that is to be associated with A(I) for I = 1,2...,LA may be entered into IR(I).
3. If entry VSRTPM is used, A is replaced by the sorted absolute values of its elements, on output.

Example

CALL VSORTP (A,LA,IR)

Input:

A = (10.,9.,8.,7.,6.,5.,4.,3.,2.,1.)

LA = 10

IR = (1, 2, 3, 4, 5, 6, 7, 8, 9, 10)

Output:

A = (1.,2.,3.,4.,5.,6.,7.,8.,9.,10.)

IR = 10, 9, 8, 7, 6, 5, 4, 3, 2, 1)

APPENDIX D

DEFINITIONS AND DIMENSIONS OF COMPUTER VARIABLE NAMES

DEFINITIONS OF COMPUTER VARIABLE NAMES

For definitions of the following ten variables, see the sparsity subroutine specification sheets in Appendix C:

A, (IA + IEXTRA), G, IP, IRN, MTYPE, N, NP, U, and W.

The other definitions are:

NAME	DESCRIPTION
AA	INPUTED FLAG WHICH TELLS IF TOTAL LINE shunt admittance to ground is inputed (AA = 0) or only one-half of total value is inputed (AA = 1).
A1	a vector which holds the row numbers of the non-zero elements of the upper (or lower) triangle of the bus admittance matrix.
A2	a vector which tells us how many non-zero upper triangle bus admittance matrix elements that have been at the commencement of a given column.
ALPHA	the value determines the portion of the second order correction factor which is used to update the bus mismatches and the portion used to update the diagonal elements of the Jacobian submatrices.
BB	inputed flag which indicates whether the transformer inputed information is the tap setting (BB = 1) or the turns ratio (BB = 0).
CC	inputed flag which determines whether a Newton-Raphson load flow is to be solved (CC = 0) or one using the alpha - M.Q.S.O.N.R. method (CC = 1).

D	diagonal elements of the bus admittance matrix.
D1, D2, D3, D4	vectors which contain the locations of the diagonal elements of the Jacobian submatrices within the vector of the non-zero elements of the Jacobian, JACOB.
DELK	CONTAINS the iteration number for which the corresponding maximum mismatch values are stored.
DELMAX	the specified tolerance required in order for convergence to occur.
DELMXP	maximum active power mismatch.
DELMXQ	maximum reactive power mismatch.
DELMXV	maximum voltage squared mismatch.
DELP	variable which stores the value which DELMXP has at the end of an iteration.
DELQ	variable which stores the value which DELMXQ contains at the end of an iteration.
DELS	temporary storage for updated bus mismatches; needed when there is an unsuccessful convergence check at the one-half iteration point.
DELTAP	active power bus mismatch.
DELTAQ	reactive power bus mismatch.
DELTAS	vector which contains all the bus mismatches.
DELTAV	voltage squared bus mismatch.
DELV	stores the value which DELMXV contains after an iteration.
DESERY, DSHTY, DSHTYD, DTFMR	temporary storage for line data when the line numbers are being rearranged.
E	real component of bus voltage.

EB	bus number at the receiving end of a line.
EB1	temporary storage for EB's when line numbers are being rearranged.
ERROR1	voltage corrections after the first half of an iteration.
F	imaginary component of the bus voltage.
IA	the number of non-zero elements in the Jacobian matrix.
IEXTRA	workspace required for the sparsity subroutine, MAIBA.
JACA1	vector which contains the row numbers of all the non-zero elements of the Jacobian matrix.
JACA2	vector which tells how many non-zero Jacobian matrix elements there are up to the commencement of a given column.
JACOB	non-zero elements of the Jacobian matrix.
K	iteration counter.
KO	the number of voltage regulated buses, not including the slack bus.
K1	the number of load buses.
K2	the number of buses, not including the slack bus.
KDEL	records the number of times that a convergence check is performed.
KKK	if one-half an iteration is successful in converging, value is set to 5. Otherwise its value is 0.
KMAX	maximum number of iterations allowed.
KNECT	records the number of lines connected to the slack bus.
KTOT	the number of non-zero elements in the bus admittance matrix.

LINE	contains line numbers.
LINE1	temporary storage for line numbers during line renumbering sequence.
MB	the number of voltage controlled buses.
MAXY	total number of non-zero elements in the upper (or lower) tirangle of the bus admittance matrix.
NB	the total number of buses.
NEWA1	vector containing the row numbers of all the non-zero elements of the bus admittance matrix.
NEWA2	vector containing running totals, column by column, of the number of non-zero elements in the bus admittance matrix.
NEWY	vector containing all the non-zero elements of the bus admittance matrix.
NJACOB	the number of non-zero elements in the Jacobian matrix.
NL	the total number of lines.
P	calculated active bus power.
PSCHED	shceduled active bus power.
Q	calculated reactive bus power.
QSCHED	scheduled reactive bus power.
R	second order correction factor for active power.
RECORD	records the number of lines connected to the sending buses.
S	complex conjugate bus power.
SB	bus number at the sending end of a line.

SBCAP	static capacitor or shunt reactor connected to the sending bus of a line.
SERY	line series admittance.
SERZ	line series impedance.
SHTY	line charging susceptance.
SHTYB	line charging represented by admittance shunted to ground at the received end of a line.
SUM1, SUM2	summation portions of the first order mismatch correction factors.
SUM3, SUM4	summation portions of the second order mismatch correction factors.
T	second order correction factors for reactive power.
TFMR	inputed off-nominal tap setting of a transformer.
U2	second order correction feactors for the square of the bus voltage.
V	complex bus voltage.
VSCHED	sheduled bus voltage magnitude.
VV	complex conjugate of the bus voltage.
Y	non-diagonal elements of the upper (or lower) triangle of the bus admittance matrix.

If a series of load flows with different alpha values is solved during one computer run, then there are also:

AANT	input of the initial alpha value.
AINT2, AINT3 ...	other alpha values.
IFLAG	counter to determine if a table output may be terminated.

JFLAG	counts the number of tables printed on one page.
KFLAG	counts the number of tables with less than 2.5 iterations, on a given page.
KKSAVE	indicator as to whether the load flow solution ends in a half iteration.
KSAVE	number of iterations to convergence for a particular alpha value.
LIMIT	number of tables desired to be outputed; there are six alphas per table.
SAVEP	the maximum active power mismatch is stored for every iteration.
SAVEQ	the maximum reactive power mismatch is stored for every iteration.
SAVEV	the maximum voltage squared mismatch is stored for every iteration.

DIMENSIONS OF VARIABLES

We have,

NB $\triangleq$ number of buses

NL $\triangleq$ number of lines

MB $\triangleq$ number of voltage controlled buses, including the slack bus.

VARIABLE	DIMENSION
A	IEXTRA + (NB - 1)
A1	NL
A2	NB
CR1	NB - 1
CR2	NB - 1
D	NB
D1	NB - 1
D2	NB - 1
D3	NB - 1
D4	NB - 1
DELK	2*(KMAX)
DELP	2*(KMAX)
DELQ	2*(KMAX)
DELS	2*(NB - 1)
DELTAS	2*(NB - 1)
DELV	2*(KMAX)
DSERY	NL
DSHTY	NL

DTFMR	NL
E	NB
EB	NL
EB1	NL
ERROR1	2X(NB - 1)
F	NB
± BUS	NB
IP	[(2*NB - 1), 13]
IRN	2*(IEXTRA + dimension of JACOB)
JACA1	8*(number of lines not connected to the slack bus) + 4*(NB - 1)
JACA2	NB + 1
JACOB	same as for JACA1
LINE	NL
LINE1	NL
NEWA1	(2*NL) + NB
NEWA2	NB + 1
NEWY	(2XNL) + NB
P	NB
PSCHED	NB
Q	NB
QSCHED	NB
RECORD	NB
SB	NL
SBCAP	NL
SERY	NL

SHTY	NL
SHTYB	NL
SUM1	NB - 1
SUM2	NB - 1
SUM3	NB - 1
SUM4	NB - 1
TFMR	NL
V	NB
VSCHED	MB - 1
W	2*(NB - 1)
Y	NL

If a series of load flows with different alpha values is solved in the one run, then there are also:

KSAVE	number of alpha values used + 1
KKSAVE	number of alpha values used + 1
SAVEP	(number of alpha values)*(KMAX)
SAVEQ	(number of alpha values)*(KMAX)
SAVEV	(number of alpha values)*(KMAX)

APPENDIX E

PROGRAM LISTING

```
      DO 210 J=1,NB
      RECORD(J)=0
      D(J)=0.0
210   CONTINUE
C
C     READ LINE NUMBER, STARTING BUS, END BUS, LENGTH, SHUNT ADMITTANCE
C     IN P.U. PER UNIT LENGTH,SERIES IMPEDANCE IN P.U. PER UNIT LENGTH
C
      WRITE(6,102)
102   FORMAT(///,50X,10(1H*),' L I N E  D A T A ',10(1H*),/,/,27X,'LINE'
     1,2X,'SB',3X,'EB',2X,'TFMR RATIO',2X,'SHUNT ADMITTANCE',6X,'SERIES
     2IMPEDANCE',6X,'SB STATIC CAP.',/)
      DO 108 I=1,NL
      READ(5,100)J,SB(I),EB(I),SERZ,SHTY(I),TFMR(I),SBCAP(I)
100   FORMAT(3I5,5F10.4,F1.0,F9.4)
      IF(AA.EQ.0.)GO TO 75
      SHTY(I)=2*SHTY(I)
75    WRITE(6,104)I,SB(I),EB(I),TFMR(I),SHTY(I),SERZ,SBCAP(I)
104   FORMAT(' ',24X,3I5,F9.3,4X,2F9.4,4X,2F9.4,7X,F3.1,F8.4)
      IF(TFMR(I).EQ.0.)GO TO 106
      IF(BB.EQ.0.)GO TO 80
      B=TFMR(I)
      GO TO 85
80    B=1.+TFMR(I)/100.
85    SERY(I)=1./(B*SERZ)
      SHTYB(I)=(1.0/SERZ)*(1.0-(1.0/B))+SHTY(I)/2.
      SHTY(I)=SHTY(I)/B**2-SERY(I)+SERY(I)/B
      SHTY(I)=2.*SHTY(I)
      GO TC 108
  106 SERY(I)=1./SERZ
      SHTYB(I)=SHTY(I)/2.
108   CONTINUE
C    ------------------------------------------------------------------
C
C     SPARSE STORAGE OF ADMITTANCE MATRIX
C     ( INCLUDES REARRANGING LINE NUMBERS IF NECESSARY )
C
C
C
C        ASSEMBLE DIAGONAL ELEMENTS
C
      DO 119 I=1,NL
      Y(I)=0.0
      L=SB(I)
      M=EB(I)
      D(L)=D(L)+SERY(I)+SHTY(I)/2.+SBCAP(I)
      D(M)=D(M)+SERY(I)+SHTYB(I)
119   CONTINUE
      D(107)=D(107)+.06
      KNECT=0
      DO 220 I=1,NL
C
```

```
//E30111CK JOB (3011,10KC,1,02),WELLON,CLASS=C
/*JOBPARM FORMS=0234
//  EXEC  FORTGCLG,REGION.GO=129K
//FORT.SYSIN DD *
C
      COMPLEX Y,V,S,SS,SR,IBUS,SUMS,VV,D,NEWY
      COMPLEX SHTY,DSHTY,ZSER,SERY,DSERY,SHTYB,DSHTYB,SBCAP,SERZ
      INTEGER DUMMY,DUM1,DUM2,RECORD,EB1,LINE,LINE1
      INTEGER SB,EB,A1,A2,D1,D2,D3,D4,AA,BB,CC
      INTEGER*2 JACA1,JACA2,IP(235,13),IRN(6248)
      REAL JACOB,MAGN
      REAL*8 ALPHA
      DIMENSION D1(117),D2(117),D3(117),D4(117),CR1(117),CR2(117)
      DIMENSION E(118),F(118),V(118),P(118),Q(118),D(118),A2(118)
      DIMENSION PSCHED(118),QSCHED(118),IBUS(118),RECORD(118)
      DIMENSION SHTY(186),SHTYB(186),SERY(186),DSHTY(186),DSHTYB(186)
      DIMENSION DSERY(186),LINE(186),LINE1(186),EB1(186),Y(186),A1(179)
      DIMENSION TFMR(186),DTFMR(186),SBCAP(186)
      DIMENSION SB(186),EB(186),W(234),DELTAS(234),ERROR1(234),A(3124)
      DIMENSION NEWY(476),NEWA1(476),NEWA2(119),VSCHED(53)
      DIMENSION JACOB(2124),JACA1(2124),JACA2(235)
      DIMENSION DELK(20),DELP(20),DELQ(20),DELV(20)
      DIMENSION SAVEP(500),SAVEQ(500),SAVEV(500),KSAVE(45),KKSAVE(45)
      DIMENSION DELS(234)
      COMMON /MEMORY/ JACOB,JACA1,JACA2
      EXTERNAL RELOAD
      WRITE(6,2)
2     FORMAT(/,//,5X,' 118 BUS TEST SYSTEM ')
C
C     READ NUMBER OF BUSSES, NUMBER OF LINES, NUMBER OF VOLTAGE CONTROL
C     BUSSES INCLUDING SLACK BUS, AND FLAGS
C
      READ(5,74)ALPHA
74    FORMAT(F10.3)
      READ(5,100)AA,BB,CC
      READ(5,100)NB,NL,MB
      WRITE(6,76)ALPHA,AA,BB,CC,NB,NL,MB
76    FORMAT(' ',//,5X,'ALPHA   AA   BB   CC    NB    NL     MB',/,F8.
     11,2X,3I5,3I7)
C
C        PRINTOUT OF CHANGES MADE IN BUS NUMBERS
C
      WRITE (6,151)
151   FORMAT(' ',//,5X,'THE FOLLOWING BUS NUMBERS WERE CHANGED TO ALLOW
     1THE',/,'REGULATED BUSES TO HAVE THE LARGEST NUMBERS :',//,20X,' BE
     2COMES',/)
      DO 155 I=1,10
      READ(5,153)J1,J2,J3,J4,J5,J6,J7,J8,J9,J10,J11,J12,J13,J14,J15,J16
153   FORMAT(16I5)
      WRITE(6,154)J1,J2,J3,J4,J5,J6,J7,J8,J9,J10,J11,J12,J13,J14,J15,J16
154   FORMAT(' ',8(14X,I5,'    "    ',I4,/))
155   CONTINUE
```

```
C     THE SENDING AND END BUS DESIGNATIONS ARE REVERSED IF THE S.B.
C       NUMBER IS LARGER THAN THE E.B. NUMBER
C
      IF(SE(I).LT.EB(I))GO TO 30
      IF(TFMR(I).NE.0.0) TFMR(I)=1./TFMR(I)
      DUMMY=SB(I)
      SB(I)=EB(I)
      EB(I)=DUMMY
30    CONTINUE
C
C       RECORD THE NUMBER OF LINES CONNECTED TO EACH SENDING BUS
C
      RECORD(SB(I))=RECORD(SB(I))+1
      IF(EB(I).EQ.NB)KNECT=KNECT+1
      LINE(I)=I
C
  220 CONTINUE
      KNECT=KNECT+RECORD(NE)
C
C       REARRANGE THE LINES IN AN ORDER ACCEPTABLE FOR THE FORMATION
C       OF THE SPARSE BUS ADMITTANCE VECTOR
C
      CALL VSORTP(SB,NL,LINE)
      DO 31 I=1,NL
31    EB1(I)=EB(LINE(I))
      DUM1=0
      DUM2=0
      DO 35 I=1,NB
      IF(RECORD(I).EQ.0)GO TO 35
      DUMMY=RECORD(I)
      DO 32 J=1,DUMMY
      DUM1=DUM1+1
      EB1(J)=EB1(DUM1)
32    LINE1(J)=LINE(DUM1)
      CALL VSORTP(EB1,DUMMY,LINE1)
      DO 34 K=1,DUMMY
      DUM2=DUM2+1
      EB(DUM2)=EB1(K)
34    LINE(DUM2)=LINE1(K)
35    CONTINUE
C
C       REARRANGE THE LINE DATA TO BE COMPATIBLE WITH THE
C       NEW LINE NUMBERS
C
      DO 36 I=1,NL
      DTFMR(I)=TFMR(I)
      DSHTY(I)=SHTY(I)
      DSHTYB(I)=SHTYB(I)
36    DSERY(I)=SERY(I)
      DO 37 I=1,NL
      TFMR(I)=DTFMR(LINE(I))
      SHTY(I)=DSHTY(LINE(I))
```

```
      SHTYB(I)=DSHTYB(LINE(I))
37    SERY(I)=DSERY(LINE(I))
C
      WRITE(6,38)
38    FORMAT(///,40X,10(1H*),' R E O R D E R E D   L I N E   D A T A ',1
     10(1H*),//,18X,'NEW',5X,'OLD',/,18X,'LINE',4X,'LINE',5X,'SB',3X,'EB
     2',2X,'TFMR RATIO',2X,'SHUNT ADMITTANCE',6X,'SERIES IMPEDENCE',6X,'
     3SB STATIC CAP.',/)
      DO 40 I=1,NL
      SERZ=1./SERY(I)
      WRITE(6,39)I,LINE(I),SB(I),EB(I),TFMR(I),SHTY(I),SERZ,SBCAP(LINE(I
     1))
39    FORMAT(' ',12X,3I8,2X,I3,F9.3,3X,2F9.4,4X,2F9.4,7X,F3.1,F8.4)
40    CONTINUE
C
      J1=1
      LL=0
      MM=0
      A2(1)=1
      DO 128 I=1,NL
      L=SB(I)
      M=EB(I)
      IF(LL.EQ.0) GO TO 122
      IF(L.EQ.LL)GO TO 124
      IF(L.EQ.LL+1)GO TO 122
      DUM1=L-2
      DO 120 II=LL,DUM1
120   A2(II+1)=J1
122   A2(L)=J1
      GO TO 126
124   IF(M.EQ.MM) J1=J1-1
126   A1(J1)=M
      Y(J1)=Y(J1)-SERY(I)
      J1=J1+1
      LL=L
      MM=M
128   CONTINUE
142   MAXY=J1-1
144   IF(L.EQ.NB-1) GO TO 134
      DUM1=NB-1
      DO 132 I=L,DUM1
132   A2(I+1)=J1
134   CONTINUE
      A2(NB)=J1
      NRL=MAXY
      WRITE(6,1665)MAXY,A2(NB)
1665  FORMAT(' ',///,5X,'MAXY=',I4,5X,'A2(NB)=',I4,///)
      WRITE(6,1331)(I,D(I),I=1,NB)
1331  FORMAT(' ',40(3(4X,'* D(',I3,')=',2F12.4),//))
      KB=0
      K2=NB-1
      DO 1334 I=1,K2
```

```
      KA=A2(I+1)-A2(I)
      IF(KA.EQ.0)GO TO 1334
      DO 1333 J=1,KA
      KB=KB+1
      IF(A2(I).NE.KB)GO TO 1332
      WRITE(6,1335)I,A2(I)
1335  FORMAT(' ',/,65X,'A2(',I3,')=',I4)
1332  WRITE(6,1330)KB,Y(KB),KB,A1(KB)
1330  FORMAT(' ',5X,'Y(',I3,')=',2F12.4,4X,'* A1(',I3,')=',I4,//)
1333  CONTINUE
1334  CONTINUE
C
C  RESTORAGE OF SPARSE ADMITTANCE ELEMENTS IN A MANNER MORE EASILY
C     USED WHEN THE JACOBIAN IS FORMED.
C
      NEWY(1)=D(1)
      NEWA1(1)=1
      NEWA2(1)=1
      IA=A2(2)-A2(1)
      DO 136 I1=1,IA
      KA=I1+1
      NEWY(KA)=Y(I1)
136   NEWA1(KA)=A1(I1)
      NEWA2(2)=KA+1
      DO 152 I=2,NB
      DUM1=I-1
      DO 144 I3=1,DUM1
      IF(A2(I3).EQ.A2(I3+1)) GO TO 144
      KB=A2(I3+1)-1
      DUM2=A2(I3)
      DO 142 I2=DUM2,KB
      IF(A1(I2).NE.I) GO TO 138
      KA=KA+1
      NEWY(KA)=Y(I2)
      NEWA1(KA)=I3
      GO TO 144
138   IF(A1(I2).GT.I) GO TO 144
142   CONTINUE
144   CONTINUE
      KA=KA+1
      NEWY(KA)=D(I)
      NEWA1(KA)=I
      IF(I.EQ.NB)GO TO 148
      IF(A2(I).EQ.A2(I+1)) GO TO 148
      KC=A2(I+1)-1
      DUM2=A2(I)
      DO 146 I4=DUM2,KC
      KA=KA+1
      NEWY(KA)=Y(I4)
146   NEWA1(KA)=A1(I4)
148   NEWA2(I+1)=KA+1
152   CONTINUE
```

```
      MDUM=2*NRL+NB
      WRITE(6,1442)(I,NEWY(I),NEWA1(I),I=1,MDUM)
1442  FORMAT(' ',3X,'I=',I4,3X,'NEWY=',2F14.4,8X,'NEWA1=',I3,/)
      NNB=NB+1
      WRITE(6,1443)(I,NEWA2(I),I=1,NNB)
1443  FORMAT(' ',//,6(2X,'* NEWA2(',I3,')=',I4))
C
C ------------------------------------------------------------
C
C     READ IN SPECIFIED BUS DATA; REAL POWER, REACTIVE POWER, REFERENCE
C     VOLTAGE V(SB), VOLTAGE CONTROL BUS MAGNITUDES AND REACTIVE
C     POWER LIMITS
C
44    K0=MB-1
      K1=NB-MB
      K2=NB-1
      READ(5,150)(PSCHED(I),I=1,K2)
      READ(5,150)(QSCHED(I),I=1,K1)
      READ(5,150)E(NB),F(NB),(VSCHED(I),I=1,6)
      IF(K0.GT.6) READ(5,150)(VSCHED(I),I=7,K0)
150   FORMAT(8F10.3)
      WRITE(6,160)(PSCHED(I),I=1,K2)
160   FORMAT(1H1,//,' P(SCHEDULED)(I) =',11F10.3,//,50(18X,11F10.3,//))
      WRITE(6,161)(QSCHED(I),I=1,K1)
161   FORMAT(1H0,//,' Q(SCHEDULED)(I) =',11F10.3,//,50(18X,11F10.3,//))
      WRITE(6,162)E(NB),F(NB),(VSCHED(I),I=1,K0)
162   FORMAT(//,10X,' V(SB) =',2F10.3,///6X,' VSCHED(I) =',11F10.3,//,50
     1(18X,11F10.3,//))
      READ(5,105)KMAX,DELMAX
105   FORMAT(I5,F10.7)
      READ(5,1550)IEXTRA
1550  FORMAT(I10)
      WRITE(6,1663)IEXTRA
1663  FORMAT(' ',///,10X,'IEXTRA=',I5,///)
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
C     INITIALIZE UNKNOWN VOLTAGES AND REACTIVE POWERS
C
      IJ=0
      IJK=0
      DO 1019 I=1,45
      KSAVE(I)=0
1019  CONTINUE
101   CONTINUE
      DO 250 I=1,K2
      E(I)=.866025404
      F(I)=0.5
250   CONTINUE
      K5=K1+1
      DO 251 I=K5,K2
251   Q(I)=0.0
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
```

```
C
      R=0.0
      T=0.0
      L=0
      K=0
      KKK=0
      KDEL=0
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
900   CONTINUE
      DO 255 I=1,K2
      CR1(I)=0.0
      CR2(I)=0.0
255   CONTINUE
901   CONTINUE
C
C     CALCULATE THE REAL AND REACTIVE POWERS
C
      DELMXP=0.0
      DELMXQ=0.0
      DELMXV=0.0
      IB=0
      DO 261 I=1,K2
      IA=NEWA2(I+1)-NEWA2(I)
      IF(IA.EQ.0) GO TO 261
      SUM1=0.0
      SUM2=0.0
      DO 262 J=1,IA
      IB=IB+1
      IC=NEWA1(IB)
      SUM1=SUM1+E(IC)*REAL(NEWY(IB))-F(IC)*AIMAG(NEWY(IB))
      SUM2=SUM2+F(IC)*REAL(NEWY(IB))+E(IC)*AIMAG(NEWY(IB))
262   CONTINUE
      P(I)=E(I)*SUM1+F(I)*SUM2
      Q(I)=F(I)*SUM1-E(I)*SUM2
C
C     CALCULATE MAXIMUM REAL AND REACTIVE POWER ERRORS
C
      DELTAP=PSCHED(I)-P(I)
      DELTAS(I)=DELTAP
      IF(ABS(DELTAP).GT.DELMXP)DELMXP=ABS(DELTAP)
      IF(I.GT.K1)GO TO 261
      DELTAQ=QSCHED(I)-Q(I)
      DELTAS(I+K2)=DELTAQ
      IF(ABS(DELTAQ).GT.DELMXQ)DELMXQ=ABS(DELTAQ)
261   CONTINUE
C
      WRITE(6,351)
351   FORMAT(1H0,15X,5(1H*),' POWER ERROR1 MATRIX ',5(1H*),//,15X,'DELTA
     1 P',10X,'DELTA Q',5X,'DELTA V-SQUARED',/)
      DO 353 I=1,K1
      WRITE(6,352)I,DELTAS(I),DELTAS(I+K2)
352   FORMAT(' ',I5,2F17.7,/)
```

```
353   CONTINUE
C
C       IF THERE ARE VOLTAGE CONTROLLED BUSSES
C
      IF(KC.EQ.0)GO TO 273
      DO 272 I=K5,K2
      V2=E(I)**2+F(I)**2
      DELTAV=VSCHED(I-K1)**2-V2
      DELTAS(I+K2)=DELTAV
      IF(ABS(DELTAV).GT.DELMXV)DELMXV=ABS(DELTAV)
      WRITE(6,271)I,DELTAS(I),DELTAS(I+K2)
271   FORMAT(' ',I5,F17.7,17X,F17.7)
272   CONTINUE
273   CONTINUE
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
C     TEST FOR CONVERGENCE OF SYSTEM
C
      KDEL=KDEL+1
      DELK(KDEL)=FLOAT(K)+FLOAT(KKK)/10.
      DELP(KDEL)=DELMXP
      DELQ(KDEL)=DELMXQ
      DELV(KDEL)=DELMXV
      WRITE(6,303) DELMXP,DELMXQ,DELMXV
303   FORMAT(' ',///,5X,'DELTAP MAX. =',F15.8,/,5X,'DELTAQ MAX. =',F15.8
     1,/,5X,'DELTAV MAX. =',F15.8,/)
      IF(DELMXP.LE.DELMAX.AND.DELMXQ.LE.DELMAX.AND.DELMXV.LE.DELMAX)GO T
     1O 950
      IF(KKK.NE.5) GO TO 222
      KDEL=KDEL-1
      KKK=0
      K=K+1
      DO 221 I=1,K2
      E(I)=E(I)-ERROR1(I)
      F(I)=F(I)-ERROR1(I+K2)
221   CONTINUE
      GO TO 1066
222   CONTINUE
      IF(K.GE.KMAX)GO TO 1000
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C
C     CALCULATE BUS CURRENTS
C     CALCULATE ELEMENTS OF THE JACOBIAN MATRIX
C
      K3=K2*2
      K=K+1
      WRITE(6,279)K,KKK
279   FORMAT(1H1,/,4X,'ITERATION NO. ',I2,'.',I1,//,9X,'CONJUGATE',/,9X,
     1'OF  POWER',24X,'CURRENT')
      DO 320 I=1,K2
      S=CMPLX(P(I),-Q(I))
      VV=CMPLX(E(I),-F(I))
```

```
      IBUS(I)=S/VV
      WRITE(6,281)S,IBUS(I)
281   FORMAT(' ',/,2F12.5,7X,2F12.5)
320   CONTINUE
C
      KB=0
      KTOT=NEWA2(NB+1)-(2*(NEWA2(NB+1)-NEWA2(NB))-1)-1
      DO 186 I=1,K2
      JACA2(I)=KB+1
      KA=NEWA2(I+1)-NEWA2(I)
      KD=0
      IF(KA.EQ.NB) KA=KA-1
180   CONTINUE
C
C     FORMULATE J1 AND J4
C
      DUMMY=NEWA2(I)
      IF(NEWA1(DUMMY+KA-1).EQ.NB) KA=KA-1
      DUM1=DUMMY+KA-1
      DO 182 I1=DUMMY,DUM1
      KB=KB+1
      KG=2*KTOT+KB+KA
      JACOB(KB)=E(NEWA1(I1))*REAL(NEWY(I1))+F(NEWA1(I1))*AIMAG(NEWY(I1))
     1JACOB(KG)=-JACOB(KB)
      IF(NEWA1(I1).GT.K1) JACOB(KG)=0.0
      JACA1(KB)=NEWA1(I1)
      JACA1(KG)=NEWA1(I1)+K2
      IF(NEWA1(I1).NE.I) GO TO 182
      JACOB(KB)=JACOB(KB)+REAL(IBUS(NEWA1(I1)))
      JACOB(KG)=JACOB(KG)+REAL(IBUS(NEWA1(I1)))
      IF(NEWA1(I1).GT.K1) JACOB(KG)=2.*F(NEWA1(I1))
      D1(I)=KB
      D4(I)=KG
182   CONTINUE
C
C     FORMULATE J2 AND J3
C
      DUMMY=NEWA2(I)
      DO 184 I1=DUMMY,DUM1
      KB=KB+1
      KH=2*KTOT+KB-KA
      JACOB(KH)=-E(NEWA1(I1))*AIMAG(NEWY(I1))+F(NEWA1(I1))*REAL(NEWY(I1)
     1)
      JACOB(KB)=JACOB(KH)
      IF(NEWA1(I1).GT.K1) JACOB(KB)=0.0
      JACA1(KB)=NEWA1(I1)+K2
      JACA1(KH)=NEWA1(I1)
      IF(NEWA1(I1).NE.I) GO TO 184
      JACOB(KB)=JACOB(KB)-AIMAG(IBUS(NEWA1(I1)))
      IF(NEWA1(I1).GT.K1) JACOB(KB)=2.*E(NEWA1(I1))
      JACOB(KH)=JACOB(KH)+AIMAG(IBUS(NEWA1(I1)))
      D2(I)=KH
```

```
      D3(I)=KB
184   CONTINUE
      JACA2(I+K2+1)=KG+1
186   CONTINUE
      JACA2(NB)=2*KTOT+1
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
C       PRINT JACOBIAN VECTOR
C
      WRITE(6,403)
403   FORMAT(1H1,//,5X,' THE JACOBIAN SPARSITY VECTOR AND ROW LOCATION V
     1ECTOR, JACA1',//,2(4X,'I',5X,'JACOB(I)',5X,'JACA1(I)'),/)
      NJACOB=8*(NRL-KNECT)+(NB-1)*4
      N2=NJACOB/2
      DO 404 I=1,N2
      N3=I+N2
      WRITE(6,405)I,JACOB(I),JACA1(I),N3,JACOB(N3),JACA1(N3)
405   FORMAT(' ',/,1X,I4,F14.6,I10,'*',I6,F14.6,I10)
404   CONTINUE
      MNB=2*NB-1
      WRITE(6,1444)(I,JACA2(I),I=1,MNB)
1444  FORMAT(' ',//,6(3X,'| JACA2(',I3,')=',I4))
      WRITE(6,1448)
1448  FORMAT(' ',////,4X,'I',4X,'D1(I)',3X,'D2(I)',3X,'D3(I)',3X,'D4(I)'
     1,/)
      DO 1447 I=1,K2
      WRITE(6,1446)I,D1(I),D2(I),D3(I),D4(I)
1446  FORMAT(' ',/,2X,I3,4I8)
1447  CONTINUE
C - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -
      IA=JACA2(2*K2+1)-1
      WRITE(6,1664)NJACOB,IA
1664  FORMAT(' ',///,5X,'NJACOB=',I5,5X,'IA=',I5,///)
      IF((K+KKK/5).LE.2) GO TO 1770
C
C       USE MA18D TO LOAD THE JACOBIAN NONZERO ELEMENTS INTO THEIR
C       PROPER PLACES IN THE ORIGINAL 'A' MATRIX. THEN MA18C FACTORS
C       THIS MATRIX.
C
      DO 1549 J=1,IA
1549  A(J)=JACOB(J)
      CALL MA18D(A,IRN,IP,N,NP,W,RELOAD)
      CALL MA18C(A,IRN,IP,N,NP,G)
      WRITE(6,1554)G
      GO TO 1559
1770  CONTINUE
C
      N=2*(NB-1)
      NP=N+1
      DO 1551 I=1,IA
      A(I)=JACOB(I)
      IRN(I)=JACA1(I)
1551  CONTINUE
```

```
      DO 1552 I=1,NP
      IP(I,1)=JACA2(I)
1552  CONTINUE
C
C       SET PARAMETER, U, CALL MA18A TO FACTOR JACOB AND CHECK FOR ERRO
C
      U=0.25
      CALL MA18A(A,IRN,IP,N,NP,G,U,IA+IEXTRA)
      WRITE(6,1554)G
1554  FORMAT('0VALUE OF G( RELATIVE PERTURBATION OF A ):',/,41('*'),//,E
     115.7,///)
      IF((G.EQ.-1.0).OR.(IP(N+1,2).EQ.0)) GO TO 1553
C
C       SOLVE FOR THE VOLTAGE CORRECTIONS (FIRST ORDER)
C
1559  MTYPE=1
      CALL MA18B(A,IRN,IP,N,NP,W,DELTAS,MTYPE)
      IF(MTYPE.EQ.0) GO TO 1556
      GO TO 1558
C
1553  WRITE(6,1555)
1555  FORMAT('-INVALID RETURN FROM MA18A')
      STOP
1556  WRITE(6,1557)
1557  FORMAT('-INVALID RETURN FROM MA18B')
      STOP
C
1558  CONTINUE
C
      WRITE(6,366)
366   FORMAT(1H0,///,5(1H*),' VOLTAGE ERROR1 MATRIX ',5(1H*),//,15X,'DEL
     1TA E',10X,'DELTA F',/)
      DO 367 I=1,K2
      ERROR1(I)=DELTAS(I)
      ERROR1(I+K2)=DELTAS(I+K2)
  367 WRITE(6,352) I,ERROR1(I),ERROR1(I+K2)
      IF(CC.EQ.0.)GO TO 411
C
C     CALCULATE SECOND ORDER CORRECTION FOR REAL AND REACTIVE POWERS
C
71    CONTINUE
      WRITE(6,407)
407   FORMAT(1H0,///,'  NO.',15X,'SUM3',17X,'SUM4',15X,'R',18X,'T',15X,'
     1R*ALPHA',12X,'T*ALPHA',7X,'L')
      L=0
      IB=0
      DO 1662 I=1,K2
      IA=NEWA2(I+1)-NEWA2(I)
      IF(IA.EQ.0)GO TO 1662
      SUM3=0.0
      SUM4=0.0
      DO 1661 J=1,IA
```

```
      IB=IB+1
      IC=NEWA1(IB)
      IF(IC.EQ.NB) GO TO 1661
      SUM3=SUM3-AIMAG(NEWY(IB))*ERROR1(IC+K2)+REAL(NEWY(IB))*ERROR1(IC)
      SUM4=SUM4+REAL(NEWY(IB))*ERROR1(IC+K2)+AIMAG(NEWY(IB))*ERROR1(IC)
1661  CONTINUE
      CR1(I)=SUM3
      CR2(I)=SUM4
      R=SUM3*ERROR1(I)+SUM4*ERROR1(I+K2)
      T=-SUM4*ERROR1(I)+SUM3*ERROR1(I+K2)
      IF(ABS(R).LT.DELMAX.AND.ABS(T).LT.DELMAX) L=L+1
      RO=R
      TO=T
      R=R*ALPHA
      T=T*ALPHA
      WRITE(6,408)I,SUM3,SUM4,RO,TO,R,T,L
408   FORMAT(1H0,/,3X,I3,F20.8,F21.8,2F19.8,2F19.8,I6)
 603  DELTAR=PSCHED(I)-P(I)-R
 607  DELTAS(I)=DELTAP
      DELS(I)=DELTAP
      IF(I.GT.K1) GO TO 1662
 604  DELTAQ=QSCHED(I)-Q(I)-T
      DELTAS(I+K2)=DELTAQ
      DELS(I+K2)=DELTAQ
1662  CONTINUE
      IF(L.EQ.K2)GO TO 412
1066  CONTINUE
      IF(L.NE.K2) GO TO 1067
      DO 333 I=1,K2
      DELTAS(I)=DELS(I)
      DELTAS(I+K2)=DELS(I+K2)
333   CONTINUE
1067  CONTINUE
      IF(ALPHA.EQ.0.AND.KO.EQ.0)GO TO 433
C
      WRITE(6,368)
368   FORMAT(1H1,15X,5(1H*),' POWER ERROR2 MATRIX ',5(1H*),//,15X,'DELTA
     1 P',10X,'DELTA Q',5X,'DELTA V-SQUARED',/)
      DO 369 I=1,K1
 369  WRITE(6,352) I,DELTAS(I),DELTAS(I+K2)
C
C      IF THERE ARE VOLTAGE CONTROLLED BUSSES
C
441   IF(KO.EQ.0)GO TO 444
      DO 442 I=K5,K2
      U2=ERROR1(I)**2+ERROR1(I+K2)**2
      U2=U2*ALPHA
      DELTAV=VSCHED(I-K1)**2-E(I)**2-F(I)**2-U2
      DELTAS(I+K2)=DELTAV
      WRITE(6,271)I,DELTAS(I),DELTAS(I+K2)
442   CONTINUE
444   CONTINUE
```

```
      IF(ALPHA.EQ.1.)GO TO 414
433   DO 33 I=1,K2
      JACOB(D1(I))=JACOB(D1(I))+(1.-ALPHA)*CR1(I)
      JACOB(D4(I))=JACOB(D4(I))+(1.-ALPHA)*CR1(I)
      JACOB(D2(I))=JACOB(D2(I))+(1.-ALPHA)*CR2(I)
      JACOB(D3(I))=JACOB(D3(I))-(1.-ALPHA)*CR2(I)
33    CONTINUE
C
C       IF THERE ARE VOLTAGE CONTROLLED BUSES
C
      IF(MB.EQ.1)GO TO 1706
      DO 1705 I=K5,K2
      JACOB(D3(I))=JACOB(D3(I))+(1.-ALPHA)*ERROR1(I)+(1.-ALPHA)*CR2(I)
      JACOB(D4(I))=JACOB(D4(I))+(1.-ALPHA)*ERROR1(I+K2)-(1.-ALPHA)*CR1(I
     1)
1705  CONTINUE
1706  CONTINUE
      IA=JACA2(2*K2+1)-1
      IF(K.EQ.1) GO TO 1703
      DO 1649 I=1,IA
1649  A(I)=JACOB(I)
      CALL MA18D(A,IRN,IP,N,NP,W,RELOAD)
      CALL MA18C(A,IRN,IP,N,NP,G)
      WRITE(6,1554)G
      GO TO 414
C
1703  CONTINUE
      N=2*(NB-1)
      NP=N+1
      DO 1700 I=1,IA
      A(I)=JACOB(I)
      IRN(I)=JACA1(I)
1700  CONTINUE
      DO 1701 I=1,NP
      IP(I,1)=JACA2(I)
1701  CONTINUE
C
C       SET PARAMETER, U, CALL MA18A TO FACTOR JACOB AND CHECK FOR ERRO
C
      U=0.25
      CALL MA18A(A,IRN,IP,N,NP,G,U,IA+IEXTRA)
      WRITE(6,1554)G
      IF((G.EQ.-1.0).OR.(IP(N+1,2).EQ.0)) GO TO 1553
C
C       SOLVE FOR THE SECOND ORDER VOLTAGE CORRECTIONS
C
414   MTYPE=1
      CALL MA18B(A,IRN,IP,N,NP,W,DELTAS,MTYPE)
      IF(MTYPE.EQ.0) GO TO 1556
C
      WRITE(6,371)
```

```
371   FORMAT(1H0,///,5(1H*),' VOLTAGE ERROR2 MATRIX ',5(1H*),//,15X,'DEL
     1TA E',10X,'DELTA F',/)
      DO 372 I=1,K2
 372  WRITE(6,352) I,DELTAS(I),DELTAS(I+K2)
C
C     DETERMINE THE NEW REAL AND IMAGINARY BUS VOLTAGES
C
411   DO 420 I=1,K2
      E(I)=E(I)+DELTAS(I)
420   F(I)=F(I)+DELTAS(I+K2)
C
      GO TO 900
C
C   IF DOING A HALF ITERATION
C
412   DO 413 I=1,K2
      E(I)=E(I)+ERROR1(I)
413   F(I)=F(I)+ERROR1(I+K2)
      IF(KKK.EQ.0)K=K-1
      KKK=5-KKK
      GO TO 901
C
C     CONVERGENCE OBTAINED - CALCULATE SLACK BUS POWER
C
950   SUM5=CMPLX(0.0,0.0)
      DUMMY=NEWA2(NB+1)-1
      DUM1=NEWA2(NB)
      DO 509 I=1,NB
509   V(I)=CMPLX(E(I),F(I))
      DO 510 I=DUM1,DUMMY
      DUM2=NEWA1(I)
      SUM5=SUM5+NEWY(I)*V(DUM2)
510   CONTINUE
      P(NB)=REAL(SUM5*CONJG(V(NB)))
      Q(NB)=-AIMAG(SUM5*CONJG(V(NB)))
C
C     WRITE OUT BUS DATA
C
      IF(CC.EQ.1.)GO TO 503
      WRITE(6,502)K
502   FORMAT(1H1,/,5(1H*),'  FIRST ORDER NEWTON-RAPHSON ITERATIVE TECHNI
     1QUE CONVERGED IN',I2,' ITERATIONS ',5(1H*),//////,31X,'BUS',7X,'VO
     2LTAGE',8X,'MAGNITUDE',2X,'PHASE(DEGS)',2X,'REAL POWER',2X,'REACTIV
     3E POWER',/)
      GO TO 505
503   WRITE(6,511)K,KKK
511   FORMAT(1H1,/,5(1H*),' SECOND ORDER NEWTON-RAPHSON ITERATIVE TECHNI
     1QUE CONVERGED IN',I2,'.',I1,' ITERATIONS.',//////,31X,'BUS',7X,'VO
     2LTAGE',8X,'MAGNITUDE',2X,'PHASE(DEGS)',2X,'REAL POWER',2X,'REACTIV
     3E POWER',/)
505   DO 512 I=1,NB
      PHASE=ATAN2(F(I),E(I))*57.29578
```

```
      MAGN=CABS(V(I))
512   WRITE(6,513)I,V(I),MAGN,PHASE,P(I),Q(I)
513   FORMAT(' ',26X,I7,2X,2F8.4,4X,F7.4,4X,F9.5,6X,F8.4,4X,F8.4)
C
C     CALCULATE AND WRITE OUT LINE FLOWS
C
      WRITE(6,514)
514   FORMAT(1H1,///,45X,10(1H*),' LINE FLOWS ',10(1H*),//,38X,'LINE',4X
     1,'SB',3X,'EB',5X,'REAL POWER',3X,'REACTIVE POWER',/)
      DO 515 I=1,NL
      L=SB(I)
      M=EB(I)
      SS=CONJG(V(L))*(V(L)-V(M))*SERY(I)+CONJG(V(L))*V(L)*(SHTY(I)/2.0)
      SR=CONJG(V(M))*(V(M)-V(L))*SERY(I)+CONJG(V(M))*V(M)*(SHTY(I)/2.0)
      SS=CONJG(SS)
      SR=CONJG(SR)
      WRITE(6,516)I,L,M,SS
515   WRITE(6,516)I,M,L,SR
516   FORMAT(' ',37X,3I5,2F13.4)
      GO TO 1005
1000  WRITE(6,1001)KMAX,DELMAX
 1001 FORMAT(/,/,/,/,3X,'THE MAXIMUM ALLOWED NUMBER OF ITERATIONS HAS BE
     1EN PERFORMED ','(',I2,')','WITHOUT ACHIEVING CONVERGENCE TO THE DE
     2SIRED ACCURACY ','(',F8.5,')','.')
1005  CONTINUE
      WRITE(6,1004)DELMAX
1004  FORMAT(/,/,/,/,5X,'THE REQUIRED ACCURACY WAS',F8.5)
      WRITE(6,1011)
1011  FORMAT(///,5X,'THE MAXIMUM MISMATCHES AT EACH CONVERGENCE CHECK WE
     1RE :',/,3X,'ITERATION',8X,'DELTA P',12X,'DELTA Q',12X,'DELTA V**2'
     2,/,3X,9('_'),8X,7('_'),12X,7('_'),12X,10('_'),/)
C
C
      WRITE(6,1012)DELK(1),DELP(1),DELQ(1),DELV(1)
1012  FORMAT(6X,F3.1,3X,F17.8,2X,F17.8,2X,F17.8,/,' ________________',/)
C
C     SAVING MAXIMUM MISMATCH VALUES OF EACH ITERATION
C                WITH THE PRESENT ALPHA VALUE
C
      DO 1013 I=2,KDEL
      IJ=IJ+1
      SAVEP(IJ)=DELP(I)
      SAVEQ(IJ)=DELQ(I)
      SAVEV(IJ)=DELV(I)
      WRITE(6,1012)DELK(I),DELP(I),DELQ(I),DELV(I)
1013  CONTINUE
      IJK=IJK+1
      KSAVE(IJK)=KDEL-1
      KKSAVE(IJK)=KKK
      IF(CC.NE.1) GO TO 1070
      WRITE(6,1009)ALPHA
1009  FORMAT(///,5X,' THE VALUE OF ALPHA WAS',F6.2,///)
```

```
      IF(ALPHA.GE.2.0) GO TO 1006
      ALPHA=ALPHA+0.1D0
      GO TO 101
1006  CONTINUE
C
C       READ THE INITIAL ALPHA VALUE AND THE NUMBER OF TABLES
C                     TO BE PRINTED
C
      READ(5,1015) AANT,LIMIT
1015  FORMAT(F10.2,I5)
      DO 1075 JJJ=1,5
      AINT=AANT
C
C       PRINTING THE MAXIMUM MISMATCH VALUES IN TABLES
C
      WRITE(6,1014)
1014  FORMAT(1H1,/,/,/,/,/,/,14X,'TABLE    : MAXIMUM PER UNIT MISMATCHES
     1AFTER EACH ITERATION FOR THE IEEE 118-BUS SYSTEM',/)
      IAA=1
      JFLAG=0
      KFLAG=0
      DO 1061 I=1,LIMIT
      IFLAG=0
      AINT2=AINT+0.1
      AINT3=AINT+0.2
      AINT4=AINT+0.3
      AINT5=AINT+0.4
      AINT6=AINT+0.5
      WRITE(6,1016)AINT,AINT2,AINT3,AINT4,AINT5,AINT6
1016  FORMAT(14X,85('-'),/,13X,'I',7X,'I',5X,'I',14X,'MAXIMUM MISMATCHES
     1 FOR VARIOUS ALPHA VALUES',14X,'I',/,13X,'I',' ITER. I   *   I',71('
     2-'),'I',/,13X,'I  NO.  I  ',3X,'I',3X,F4.1,5(4X,'I',3X,F4.1),4X,'I
     3',/,13X,'I',85('-'),'I')
      WRITE(6,1036)
1036  FORMAT(1H+,20X,'I',5X,'I',6(11X,'I'))
      IX=(I-1)*6+1
      DO 1055 II=1,10
C
C       PRINT THE ACTIVE POWER MISMATCHES
C
      WRITE(6,1017)
1017  FORMAT(13X,'I',7X,'I',2X,'P  I ')
      IF(KSAVE(IX).EQ.0) GO TO 1021
      IF(II.LE.KSAVE(IX)) WRITE(6,1018)SAVEP(IAA)
1018  FORMAT(1H+,28X,E9.3)
      IF(II.EQ.(KSAVE(IX)+1).AND.KKSAVE(IX).EQ.5) WRITE(6,1031)
1031  FORMAT(1H+,31X,'***',4X,'I ')
      IAB=IAA+KSAVE(IX)
      IF(KSAVE(IX+1).EQ.0) GO TO 1021
      IF(II.LE.KSAVE(IX+1))WRITE(6,1020)SAVEP(IAB)
1020  FORMAT(1H+,40X,E9.3)
      IF(II.EQ.(KSAVE(IX+1)+1).AND.KKSAVE(IX+1).EQ.5) WRITE(6,1033)
```

```
1033  FORMAT(1H+,43X,'***',4X,'I')
      IAB=IAB+KSAVE(IX+1)
      IF(KSAVE(IX+2).EQ.0) GO TO 1021
      IF(II.LE.KSAVE(IX+2))WRITE(6,1022)SAVEP(IAB)
1022  FORMAT(1H+,52X,E9.3)
      IF(II.EQ.(KSAVE(IX+2)+1).AND.KKSAVE(IX+2).EQ.5) WRITE(6,1035)
1035  FORMAT(1H+,55X,'***',4X,'I')
      IAB=IAB+KSAVE(IX+2)
      IF(KSAVE(IX+3).EQ.0) GO TO 1021
      IF(II.LE.KSAVE(IX+3))WRITE(6,1024)SAVEP(IAB)
1024  FORMAT(1H+,64X,E9.3)
      IF(II.EQ.(KSAVE(IX+3)+1).AND.KKSAVE(IX+3).EQ.5) WRITE(6,1037)
1037  FORMAT(1H+,67X,'***',4X,'I')
      IAB=IAB+KSAVE(IX+3)
      IF(KSAVE(IX+4).EQ.0) GO TO 1021
      IF(II.LE.KSAVE(IX+4))WRITE(6,1026)SAVEP(IAB)
1026  FORMAT(1H+,76X,E9.3)
      IF(II.EQ.(KSAVE(IX+4)+1).AND.KKSAVE(IX+4).EQ.5) WRITE(6,1039)
1039  FORMAT(1H+,79X,'***',4X,'I')
      IAB=IAB+KSAVE(IX+4)
      IF(KSAVE(IX+5).EQ.0) GO TO 1021
      IF(II.LE.KSAVE(IX+5))WRITE(6,1028)SAVEP(IAB)
1028  FORMAT(1H+,88X,E9.3)
      IF(II.EQ.(KSAVE(IX+5)+1).AND.KKSAVE(IX+5).EQ.5) WRITE(6,1041)
1041  FORMAT(1H+,91X,'***',4X,'I')
1021  WRITE(6,1023)
1023  FORMAT(1H+,27X,6(11X,'I'))
C
C     PRINT THE REACTIVE POWER MISMATCHES
C
      WRITE(6,1030) II
1030  FORMAT(13X,'I',2X,I2,3X,'I    Q I ')
      IF(KSAVE(IX).EQ.0) GO TO 1027
      IF(II.LE.KSAVE(IX)) WRITE(6,1018)SAVEQ(IAA)
      IF(II.EQ.KSAVE(IX)) IFLAG=IFLAG+1
      IAB=IAA+KSAVE(IX)
      IF(II.LE.KSAVE(IX+1))WRITE(6,1020)SAVEQ(IAB)
      IF(II.EQ.KSAVE(IX+1)) IFLAG=IFLAG+1
      IAB=IAB+KSAVE(IX+1)
      IF(II.LE.KSAVE(IX+2))WRITE(6,1022)SAVEQ(IAB)
      IF(II.EQ.KSAVE(IX+2)) IFLAG=IFLAG+1
      IAB=IAB+KSAVE(IX+2)
      IF(II.LE.KSAVE(IX+3))WRITE(6,1024)SAVEQ(IAB)
      IF(II.EQ.KSAVE(IX+3)) IFLAG=IFLAG+1
      IAB=IAB+KSAVE(IX+3)
      IF(II.LE.KSAVE(IX+4))WRITE(6,1026)SAVEQ(IAB)
      IF(II.EQ.KSAVE(IX+4)) IFLAG=IFLAG+1
      IAB=IAB+KSAVE(IX+4)
      IF(II.LE.KSAVE(IX+5))WRITE(6,1028)SAVEQ(IAB)
      IF(II.EQ.KSAVE(IX+5)) IFLAG=IFLAG+1
1027  WRITE(6,1023)
C
```

```
C     PRINT THE MAXIMUM MISMATCHES FOR THE SQUARE OF THE
C     VOLTAGE MAGNITUDES
C
      WRITE(6,1043)
1043  FORMAT(13X,'I',7X,'I',2X,'V  I ')
      IF(KSAVE(IX).EQ.0) GO TO 1032
      IF(II.LE.KSAVE(IX)) WRITE(6,1018)SAVEV(IAA)
      IAB=IAA+KSAVE(IX)
      IF(KSAVE(IX+1).EQ.0) GO TO 1032
      IF(II.LE.KSAVE(IX+1))WRITE(6,1020)SAVEV(IAB)
      IAB=IAB+KSAVE(IX+1)
      IF(KSAVE(IX+2).EQ.0) GO TO 1032
      IF(II.LE.KSAVE(IX+2))WRITE(6,1022)SAVEV(IAB)
      IAB=IAB+KSAVE(IX+2)
      IF(KSAVE(IX+3).EQ.0) GO TO 1032
      IF(II.LE.KSAVE(IX+3))WRITE(6,1024)SAVEV(IAB)
      IAB=IAB+KSAVE(IX+3)
      IF(KSAVE(IX+4).EQ.0) GO TO 1032
      IF(II.LE.KSAVE(IX+4))WRITE(6,1026)SAVEV(IAB)
      IAB=IAB+KSAVE(IX+4)
      IF(KSAVE(IX+5).EQ.0) GO TO 1032
      IF(II.LE.KSAVE(IX+5))WRITE(6,1028)SAVEV(IAB)
1032  WRITE(6,1023)
      WRITE(6,1044)
1044  FORMAT(13X,'I',85('-'),'I')
      WRITE(6,1036)
      IAA=IAA+1
      IF(IFLAG.NE.6) GO TO 1055
      WRITE(6,1045)
1045  FORMAT(1H+,/)
      IF(II.EQ.KSAVE(IX).AND.KKSAVE(IX).EQ.5) WRITE(6,1046)
      IF(II.EQ.KSAVE(IX+1).AND.KKSAVE(IX+1).EQ.5) WRITE(6,1047)
      IF(II.EQ.KSAVE(IX+2).AND.KKSAVE(IX+2).EQ.5) WRITE(6,1048)
      IF(II.EQ.KSAVE(IX+3).AND.KKSAVE(IX+3).EQ.5) WRITE(6,1049)
      IF(II.EQ.KSAVE(IX+4).AND.KKSAVE(IX+4).EQ.5) WRITE(6,1050)
      IF(II.EQ.KSAVE(IX+5).AND.KKSAVE(IX+5).EQ.5) WRITE(6,1051)
1046  FORMAT(1H+,31X,'***')
1047  FORMAT(1H+,43X,'***')
1048  FORMAT(1H+,55X,'***')
1049  FORMAT(1H+,67X,'***')
1050  FORMAT(1H+,79X,'***')
1051  FORMAT(1H+,91X,'***')
      JFLAG=JFLAG+1
      IF(II.LE.2) KFLAG=KFLAG+1
      GO TO 1056
1055  CONTINUE
1056  CONTINUE
      AINT=AINT+0.6
      IAA=1+KSAVE(IX)+KSAVE(IX+1)+KSAVE(IX+2)+KSAVE(IX+3)+KSAVE(IX+4)+KS
     1AVE(IX+5)+IAA-II-1
      IF(II.LE.4.AND.JFLAG.EQ.1) GO TO 1059
      IF(II.LE.2.AND.KFLAG.EQ.2.AND.JFLAG.EQ.2) GO TO 1059
```

```
      WRITE(6,1057)
 1057 FORMAT(//,22X,'* - V REPRESENTS THE MAXIMUM MISMATCHES OF THE SQUA
     1RE OF THE VOLTAGE MAGNITUDES FOR',/,32X,'VOLTAGE CONTROLLED BUSES'
     2,//,20X,'*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED')
      WRITE(6,1058)
 1058 FORMAT(1H1,/,/,/,/,/,/,14X,'TABLE      ( CONTINUED )',/)
      KFLAG=0
      JFLAG=0
      GO TO 1061
 1059 WRITE(6,1060)
 1060 FORMAT(/)
 1061 CONTINUE
 1075 CONTINUE
 1070 STOP
      END
      SUBROUTINE RELOAD(N,W,J)
      DIMENSION W(234)
      REAL JACOB(2124)
      INTEGER*2 JACA1(2124),JACA2(235)
      COMMON /MEMORY/ JACOB,JACA1,JACA2
      II1=JACA2(J)
      II2=JACA2(J+1)-1
      DO 1772 I=II1,II2
 1772 W(JACA1(I))=JACOB(I)
      RETURN
      END
      SUBROUTINE ISRANK (IX,IO,N)
C
C       THIS SUBROUTINE WILL SORT THE INTEGER*2 ARRAY IX INTO
C       ASCENDING ORDER SUCH THAT   IX(IO(I)) < IX(IO(I+1)) FOR
C       I = 1,N .
C       THE CODE IS A NEAR COPY OF THE ISRANK ROUTINE OF WATFIV
C       AND IS DESIGNED TO TAKE THE PLACE OF THE HARWELL ROUTINE
C       KE10AS IN THE SPARSITY ROUTINE MA18AD.
C
C                                                      G. SOMERTON
C
C
      IMPLICIT INTEGER*2 (I-M)
      DIMENSION IX(N),IO(N)
      IONE = 1
      ITWO = 2
      IS1 = IONE
      IF1 = N
C
C       THE FOLLOWING CODE IS ENTIRELY THE WATFIV ROUTINE ISRANK
C       EXCEPT EVERYTHING IS IN INTEGER*2 ARITHMETIC.
C
      IO(IS1) = IS1
      IS2 = IS1+IONE
      DO 40 I=IS2,IF1
      IS = IX(I)
```

```
      M = I-IONE
      DO 10 J=IS1,M
      IOJ = IO(J)
      IF (IS.GT.IX(IOJ)) GOTO 20
 10   CONTINUE
      IO(I) = I
      GOTO 40
 20   IM = J+1
      IZ = I+IM
      DO 30 IB = IM,I
      IA = IZ-IB
 30   IO(IA) = IO(IA-IONE)
      IO(J) = I
 40   CONTINUE
C
C       THE FOLLOWING SECTION OF CODE CHANGES THE IO POINTERS FROM
C       DESCENDING TO ASCENDING ORDER.
C
      N2 = N/ITWO
      DO 50 I=1,N2
      II = I-IONE
      ITEMP = IO(I)
      IO(I) = IO(IF1-II)
 50   IO(IF1-II) = ITEMP
      RETURN
      END
      SUBROUTINE MA18A (A,IND,IW,N,NP,G,U,IA)
      REAL*4 EPS/1E-6/,ZERO/0.0/,ONE/1.0/
      REAL*8 ROWCOL(2)/8H     ROW ,8H COLUMN /
C     EPS IS THE RELATIVE ACCURACY OF FLOATING-POINT COMPUTATION.
      LOGICAL*1 LFAC,MFAC(4)
      INTEGER*2 IND(IA,2),IW(NP,13),IFAC(2)
       EQUIVALENCE(IFAC(1),FAC,LFAC,DAK),(JFAC,MFAC(1))
      DIMENSION A(IA),IK(2),IC(2),JC(2),JP(2)
      COMMON/MA1 8ED/LP,JSCALE
C     MATRIX ELEMENTS ARE HELD IN A(K),K=1,2,...,KA.
C      ON ENTRY IND(K,1) HOLDS THE ROW NUMBER OF THE ELEMENT HELD IN
C A(K). IN THE MAIN BODY OF THE SUBROUTINE IND(K,1),IND(K,2) HOLD THE
C ADDRESS OF THE NEXT ELEMENT IN THE ROW/COLUMN IF THERE IS ONE AND
C FOR THE LAST ELEMENT IN THE ROW/COLUMN HOLD (IA+ THE ROW/COLUMN
C NUMBER). THESE NUMBERS ARE NEGATED IF THEY POINT TO ELEMENTS THAT
C HAVE BEEN IN A PIVOTAL COLUMN/ROW. FINALLY IND(K,1) IS RESET TO THE
C ROW NUMBER OF THE ELEMENT HELD IN A(K).
C  ON ENTRY AND ON EXIT IW(I,1) CONTAINS THE ADDRESS OF THE FIRST
C ELEMENT OF COLUMN I AND IW(N+1,1) CONTAINS THE ADDRESS OF THE FIRST
C UNUSED ELEMENT IN A. ON EXIT IW(I,2) HOLDS THE COLUMN NUMBERS IN
C PIVOTAL ORDER. AFTER A SUCCESSFULL ENTRY IW(N+1,2)=N+1 AND IW(N+1,3)=0
C AFTER AN UNSUCCESSFUL ENTRY IW(N+1,2)=0. IN THE MAIN BODY OF THE
C SUBROUTINE IW(I,1),IW(I,2) HOLD THE ADDRESS OF THE FIRST ELEMENT OF
C THE I TH ROW/COLUMN AND ARE NEGATED IF THE FIRST ELEMENT HAS BEEN
C IN A PIVOTAL COLUMN/ROW.
      KA=IW(N+1,1)-1
```

```
C     IW(I,3),IW(I,4) HOLD THE LOGS TO BASE 16 OF THE ROW/COLUMN
C SCALING FACTORS USED.
C     IW(I,5),IW(I,6) HOLD THE POSITION IN THE ORDERING BY NUMBER OF
C NON-ZEROS OF THE LAST ROW/COLUMN TO HAVE LESS THAN I NON-ZERO
C ELEMENTS OR ZERO IF NONE HAVE LESS THAN I NON-ZERO ELEMENTS.
C ON EXIT IW(I,5) HOLDS THE POSOTOON OF THE I TH ROW IN THE PIVOTAL
C ORDERING.
C     IW(I,7),IW(I,8) HOLD THE NUMBER OF NON-ZEROS IN THE I TH ROW/
C COLUMN.
C     IW(I,9),IW(I,10) HOLD THE POSITION OF THE I TH ROW/COLUMN
C IN THE ORDERING BY NUMBER OF NON-ZEROS.
C     IW(I,11),IW(I,12) HOLD ROW/COLUMN NUMBERS IN PIVOTAL ORDER FOR
C I<IP AND IN ORDER OF INCREASING NUMBERS OF NON-ZEROS OTHERWISE.
C     IW(I,13) HOLDS THE EXPONENT OF THE MAXIMAL ELEMENT IN THE I TH
C COLUMN OF THE SCALED VERSION OF THE ORIGINAL MATRIX.
      U=AMIN1(ONE,AMAX1(U,EPS*ONE))
      N1=N+1
C     FIND SCALING FACTORS.
      DO 1 I=1,N1
1     IW(I,2)=I
      J1=3
      IF(JSCALE)6,8,2
2     J=2*(KA/2)+3
      CALL MC12A (A,IND,IW,N,NP,IW(1,3),IW(1,5),I)
3     L=1
      IR=IABS(I)
      IF(I.LT.0)L=2
      IF(I.NE.0) GO TO 560
6     J1=5
8     DO 10 I=1,N1
      IW(I,2)=IW(I,1)
      IW(I,11)=I
      IW(I,12)=I
      IW(I,13)=0
      DO 10 J=J1,9
10    IW(I,J)=0
C     SCALE THE MATRIX, SET ROW AND COLUMN LINKS, FIND FIRST ELEMENTS
C OF THE ROWS, COUNT THE NUMBER OF NON-ZEROS IN THE ROWS AND COLUMNS
C AND FIND EXPONENTS OF MAXIMAL COLUMN ELEMENTS.
      IG=0
      DO 30 J=1,N
      FAC=ONE
C     TEMPORARILY WE USE IW(I,9) TO HOLD THE ADDRESS OF THE LAST NON-
C ZERO ENCOUNTERED IN THE I TH ROW.
      K1=IW(J,2)
      K2=IW(J+1,2)-1
      IW(J,8)=K2-K1+1
      IL=0
      AMAX=ZERO
      DO 20 K=K1,K2
      I=IND(K,1)
      IF(I.LE.IL) GO TO 520
```

```
      IL=I
C     ON THE IBM 360 THE FOLLOWING TWO INSTRUCTIONS ARE EQUIVALENT TO
C     FAC=16.**( IW(I,3)+IW(J,4) )
      JFAC=65+IW(I,3)+IW(J,4)
      LFAC=MFAC(4)
      A(K)=A(K)*FAC
      AMAX=AMAX1(AMAX, ABS(A(K)))
      IND(K,1)=I+IA
      IND(K,2)=K+1
      KL=IW(I,9)
      IF(KL.LE.0)IW(I,1)=K
      IF(KL.GT.0)IND(KL,1)=K
      IW(I,9)=K
20    IW(I,7)=IW(I,7)+1
      IND(K2,2)=IA+J
      DAK=AMAX
C     ON THE IBM 360 THE FOLLOWING INSTRUCTION SETS JFAC TO THE
C     SMALLEST INTEGER GREATER THAN ALOG16(DAK)+64
      MFAC(4)=LFAC
30    IW(J,13)=JFAC
C     SET UP THOSE VECTORS IN IW ASSOCIATED WITH ORDERING BY NUMBERS
C OF NON-ZEROS.
      DO 130 L=1,2
      CALL ISRANK (IW(1,L+6),IW(1,L+10),N)
C     CHECK FOR NULL ROW OR COLUMN.
      IR=IW(1,L+10)
      IF(IW(IR,L+6).LE.0)GO TO 560
      DO 110 I=1,N
      J=IW(I,L+10)
      IW(J,L+8)=I
      NZ=IW(J,L+6)
110   IF(NZ.NE.N)IW(NZ+1,L+4)=I
      J=0
      DO 130 I=1,N
      IF(IW(I,4+L).EQ.0)IW(I,L+4)=J
130   J=IW(I,L+4)
C
C     NOW PERFORM THE MAIN ELIMINATION.
      DO 298 IP=1,N
C     FIND THE PIVOT. WE DO THIS BY SEARCHING A ROW/COLUMN. THE
C NEXT ROW/COLUMN TO BE USED IS JC(L)=IW(IK(L),L+10).
133   IK(1)=IP
      IK(2)=IP
C     JCOST IS THE COST OF THE CHEAPEST PIVOT SO FAR FOUND.
      JCOST=N*N
135   DO 140 L=1,2
      JC(L)=IW(IK(L),L+10)
140   IC(L)=IW(JC(L),L+6)
C     ICOST IS THE MINIMAL POSSIBLE COST OF A PIVOT NOT SO FAR FOUND.
      ICOST=(IC(1)-1)*(IC(2)-1)
      IF(JCOST.LE.ICOST)GO TO 160
```

```
      L=1
      IF(IC(1).GT.IC(2))L=2
      IR=JC(L)
C     FIND THE MAXIMAL ELEMENT IN ROW/COLUMN UNDER CONSIDERATION.
      AMAX=ZERO
      K=IW(IR,L)
      GO TO 143
142   K=IND(-K,L)
143   IF(K.LT.0)GO TO 142
      KK=K
      GO TO 145
144   AMAX=AMAX1(AMAX, ABS(A(KK)))
      KK=IND(KK,L)
145   IF(KK.LE.IA) GO TO 144
      IF(AMAX.EQ.ZERO) GO TO 560
      AU=AMAX*U
      L3=3-L
      IK(L)=IK(L)+1
C     NOW CONSIDER THE ELEMENTS IN THE ROW/COLUMN IN TURN.
146   IF( ABS(A(K)).LT.AU) GO TO 150
      KK=K
147   KK=IND(KK,L3)
      IF(KK.LE.IA) GO TO 147
      KCOST=(IC(L)-1)*(IW(KK-IA,9-L)-1)
      IF(KCOST.GE.JCOST) GO TO 150
      JCOST=KCOST
      KP=K
      JP(L)=IR
      JP(L3)=KK-IA
      IF(JCOST.LE.ICOST) GO TO 160
150   K=IND(K,L)
      IF(IA-K)135,146,146
C
C     REARRANGE THE LINKS SO THAT THE PIVOTAL ROW AND COLUMN ARE IN
C CORRECT PIVOTAL SEQUENCE.
160   DO 188 L=1,2
C     "MOVE" THE PIVOTAL COLUMN FIRST AND THEN THE PIVOTAL ROW.
      IK(3-L)=JP(3-L)
      K=IABS(IW(IK(3-L),3-L)+0)
C     K POINTS TO AN ELEMENT IN THE PIVOTAL COLUMN/ROW.
C     KM POINTS TO ITS PREDESSOR IN ITS ROW/COLUMN.
C     KL POINTS TO THE LAST ELEMENT THAT HAS BEEN PIVOTAL IN ITS
C ROW/COLUMN.
165   KO=K
170   KO=IABS(IND(KO,L)+0)
      IF(KO.LE.IA)GO TO 170
      IK(L)=KO-IA
      KL=KO
      KO=IW(KO-IA,L)
      IF(IW(IK(L),L+6).LE.0)GO TO 174
C     ON THE REMOVAL OF THE ELEMENT A(IK(I),IK(2)) THE FOLLOWING
```

```
C INSTRUCTIONS ARE USED TO UPDATE THE NUMBERS OF ELEMENTS IN THE
C CORRESPONDING ROW AND COLUMN AND MAKE CONSEQUENT CHANGES TO THE
C ORDERING BY NUMBER OF NON-ZEROS.
      DO 171 LM=1,2
      IR=IK(LM)
      NZ=IW(IR,LM+6)-1
      IW(IR,LM+6)=NZ
      JPOS=IW(NZ+1,LM+4)+1
      IPOS=IW(IR,LM+8)
      IF(IPOS.EQ.JPOS)GO TO 171
      JR=IW(JPOS,LM+10)
      JJ=IW(IPOS,LM+10)
      IW(IPOS,LM+10)=IW(JPOS,LM+10)
      IW(JPOS,LM+10)=JJ
      JJ=IW(IR,LM+8)
      IW(IR,LM+8)=IW(JR,LM+8)
      IW(JR,LM+8)=JJ
171   IW(NZ+1,LM+4)=JPOS
      GO TO 174
172   KL=-KO
      KO=IND(KL,L)
174   IF(KO.LT.0)GO TO 172
      KM=KL
      GO TO 178
176   KM=KO
      KO=IND(KO,L)
178   IF(KO.NE.K) GO TO 176
      IF(KL.EQ.KM) GO TO 182
      IND(KM,L)=IND(K,L)
      IF(KL.LE.IA)GO TO 183
      IND(K,L)=IW(IK(L),L)
180   IW(IK(L),L)=-K
      GO TO 186
182   IF(KM-IA)184,184,180
183   IND(K,L)=IND(KL,L)
184   IND(KL,L)=-K
186   K=IABS(IND(K,3-L)+0)
      IF(K.LE.IA) GO TO 165
188   CONTINUE
C
C     OVERWRITE THE ELEMENTS OF THE PIVOTAL COLUMN BY MULTIPLIERS
C AND PERFORM THE ELIMINATION.
      K=IND(KP,2)
      GO TO 295
190   M=K
193   M=IND(M,1)
      IF(M.LE.IA)GO TO 193
      L=M-IA
      A(K)=A(K)/A(KP)
      KI=KP
      KL=K
      GO TO 280
```

```
195   M=KL
200   M=IND(M,2)
      IF(M.LE.IA)GO TO 200
      JL=M-IA
      IF(JI-JL)210,275,270
C
C     CREATE A NEW NON-ZERO IN POSITION (L,JI).
210   KA=KA+1
      IF(KA.GT.IA)GO TO 580
      A(KA)=ZERO
      IK(1)=L
      IK(2)=JI
      IND(KA,1)=IND(KLAST,1)
      IND(KLAST,1)=KA
C     THE ORDER OF THE COLUMN LINKS DOES NOT MATTER SO WE PUT THE
C NEW ELEMENT AS THE SECOND IN ITS COLUMN.
      IND(KA,2)=IND(KI,2)
      IND(KI,2)=KA
C     ON THE ADDITION OF THE ELEMENT A(IK(1),IK(2)) THE FOLLOWING
C INSTRUCTIONS ARE USED TO UPDATE IW.
      DO 250 LM=1,2
      IR=IK(LM)
      NZ=IW(IR,LM+6)
      IW(IR,LM+6)=NZ+1
      JPOS=IW(NZ+1,LM+4)
      JR=IW(JPOS,LM+10)
      IF(IR.EQ.JR)GO TO 250
      IPOS=IW(IR,LM+8)
      JJ=IW(IPOS,LM+10)
      IW(IPOS,LM+10)=IW(JPOS,LM+10)
      IW(JPOS,LM+10)=JJ
      JJ=IW(IR,LM+8)
      IW(IR,LM+8)=IW(JR,LM+8)
      IW(JR,LM+8)=JJ
250   IW(NZ+1,LM+4)=JPOS-1
      KL=KA
      GO TO 275
265   M=KI
267   M=IND(M,2)
      IF(M.LE.IA)GO TO 267
      JI=M-IA
270   KLAST=KL
      KL=IND(KL,1)
      IF(IA-KL)210,195,195
275   A(KL)=A(KL)-A(K)*A(KI)
      DAK= ABS(A(KL))
C     ON THE IBM 360 THE FOLLOWING INSTRUCTION SETS JFAC TO THE
C FLOATING-POINT EXPONENT OF DAK.
      MFAC(4)=LFAC
      IG=MAX0(IG,JFAC-IW(JI,13))
280   KI=IND(KI,1)
      IF(KI.LE.IA)GO TO 265
```

```
290   K=IND(K,2)
295   IF(K.LE.IA)GO TO 190
298   CONTINUE
C
C     SCAN BY ROWS REPLACING ROW LINKS BY ROW NUMBERS.
      DO 310 I=1,N
      K=IABS(IW(I,1)+0)
300   KK=IND(K,1)
      IND(K,1)=I
      K=IABS(KK)
      IF(K.LE.IA)GO TO 300
310   CONTINUE
C
C     SCAN BY COLUMNS REPLACING COLUMN LINKS BY ORDERING NUMBERS.
      J=1
      DO 330 I=1,N
      K=IABS(IW(IW(I,12),2)+0)
      IW(IW(I,12),1)=J
320   KK=IND(K,2)
      IND(K,2)=J
      J=J+1
      K=IABS(KK)
      IF(K.LE.IA)GO TO 320
330   CONTINUE
      IW(N+1,1)=J
C     REORDER.
      KA=J-1
      DO 360 I=1,KA
      IF(I.EQ.IND(I,2))GO TO 360
      A1=A(I)
      I1=IND(I,1)
      J=I
350   K=IND(J,2)
      IND(J,2)=J
      A2=A(K)
      I2=IND(K,1)
      A(K)=A1
      IND(K,1)=I1
      A1=A2
      I1=I2
      J=K
      IF(K.NE.I)GO TO 350
360   CONTINUE
C
C     SET REMAINING VECTORS IN PREPARATION FOR FACTOR AND OPERATE.
      DO 370 I=1,N1
      IW(I,2)=IW(I,12)
370   IW(I,5)=IW(I,9)
C
C     RESTORE THE MATRIX TO ITS UNEQUILIBRATED STATE.
```

```
      FAC=ONE
      DO 400 J=1,N
      JCOL=IW(J,2)
      K1=IW(JCOL,1)
      K2=IW(IW(J+1,2),1)-1
      IO=IW(JCOL,4)
      DO 400 K=K1,K2
      JRO=IND(K,1)
C     ON THE IBM 360 THE FOLLOWING TWO INSTRUCTIONS ARE EQUIVALENT TO
C     FAC=16.0**(-IO-IW(JRO,3))
      JFAC=65-IO-IW(JRO,3)
      LFAC=MFAC(4)
      A(K)=A(K)*FAC
400   IF(IW(JRO,5).EQ.J)IO=-IW(JRO,3)
      IW(N+1,3)=0
      G=EPS*16D0**IG
      RETURN
C
C     THE FOLLOWING INSTRUCTIONS IMPLEMENT THE FAILURE EXITS.
500   WRITE(LP,510)
510   FORMAT('+ERROR RETURN FROM MA18A BECAUSE')
      IW(N1,2)=0
      G=-ONE
      RETURN
520   WRITE(LP,530)K
      GO TO 500
530   FORMAT(//34X,'THE ELEMENT HELD IN A(',I5,') IS OUT OF ORDER')
560   WRITE(LP,570)ROWCOL(L),IR
570   FORMAT(//34X
     1,'THE MATRIX IS SINGULAR.',A8,I4,' IS DEPENDENT IN THE REST')
      WRITE (LP,510)
      IW(N1,2) = 0
      G=-2.0
      RETURN
580   WRITE(LP,590)IP
590   FORMAT(//34X,'IA IS TOO SMALL. SPACE RAN OUT WHEN ELIMINATING'
     1,' ON PIVOT',I5)
      GO TO 500
      END
      BLOCK DATA
      COMMON /MA18ED/ JP,JSCALE
      DATA JP/6/
      DATA JSCAL E/1/
      END
      SUBROUTINE MA18B (A,IRN,IP,N,NP,AWS,AVECT,MTYPE)
C
C
C     THIS ENTRY OPERATES ON THE VECTOR AVECT WITH THE FOLLOWING
C     MATRICES ACCORDING TO THE VALUE OF MTYPE
C
C        MTYPE = 1          A**(-1)
```

```
C      MTYPE = 2         A**(-T)
C      MTYPE = 3         A
C      MTYPE = 4         A**T
      COMMON /MA18ED/ JP,JSCALE
C
C     JP IS STREAM NUMBER FOR DIAGNOSTICS
C     JSCALE CONTROLS EQUILIBRATION
C
C      ARND IS ROUNDING ERROR ESTIMATE
      DATA ARND/1.E-6/
      REAL ZERO /0.0/
C      ANAME CONTAINS NAMES FOR USE IN DIANOSTICS
      REAL*8 ANAME(5)/' MA18A ',' MA18B  ',' MA18C  ',' MA18D  ',
     X' MC12A  '/
      LOGICAL LOG
C      ON S/360 NEXT TWO STATEMENTS PERMIT ACCES TO EXPONENT OF REAL
C      EQUIV BETWEEN AU AND JU IS ALSO USED IN ALL IMPLEMENTATIONS
      LOGICAL*1 LU,LUU(4)
      EQUIVALENCE (AU,JU,LU),(JUU,LUU(1))
C      DIMENSIONS OF ARGUMENT ARRAYS COMMON TO ALL ENTRIES
      INTEGER*2 IRN(1),IP(NP,5)
      DIMENSION A(1)
C      DIMENSIONS SPECIAL TO THIS ENTRY
      DIMENSION AWS(1)
      DIMENSION AVECT(1)
      KERR=1
      MT=MTYPE
      MTYPE=0
      KENTRY=2
C      TEST FOR INVALID ENTRY
      IF(IP(N+1,2).EQ.0) GO TO 2205
      IF(IP(N+1,2).NE.N+1)GO TO 2207
      KERR=3
      IF(IP(N+1,3).NE.0) GO TO 2205
      LOG=.FALSE.
      MTYPE=MT
C      JUMP ACCORDING TO REQUIRED OPERATION
      IF(MTYPE.LE.0.OR.MTYPE.GT.4) GO TO 2204
      GO TO (101,102,103,104),MTYPE
C      SET UP FOR MTYPE =1,4, PERMUTING AVECT TO AWS
104   LOG=.TRUE.
101   DO 110 J=1,N
110   AWS(IP(J,5))=AVECT(J)
      AU=ZERO
C      FIRST DIVIDE BY L OR MULTIPLY BY L**T
      DO 140 J=1,N
      J1=IP(IP(J+1,2),1)
141   J1=J1-1
      JSEQ=IP(IRN(J1),5)
      IF(JSEQ.EQ.J) GO TO 111
      IF(LOG)GO TO 1412
      AWS(JSEQ)=AWS(JSEQ)-AWS(J)*A(J1)
```

```
      GO TC 141
1412  AWS(J)=AWS(J)+AWS(JSEQ)*A(J1)
      GO TC 141
111   IF(.NOT.LOG)JU=J1
C      SAVE PIVOT POSITION FOR MTYPE=1
C      SET ZERO FOR MTYPE=4
140   AVECT(IP(J,2))=AU
C     NOW DIVIDE BY U OR MULTIPLY BY U**T
      J=N
143   JCOL=IP(J,2)
      J1=IP(JCOL,1)
      IF(LCG)GO TO 144
C      RECCVER PIVOT POSITION IN JU
      AU=AVECT(JCOL)
      AVECT(JCOL)=AWS(J)/A(JU)
144   JSEQ=IP(IRN(J1),5)
      IF(LOG)GO TO 1442
      IF(JSEQ.EQ.J)GO TO 119
      AWS(JSEQ)=AWS(JSEQ)-AVECT(JCOL)*A(J1)
      GO TO 1443
1442  AVECT(JCOL)=AVECT(JCOL)+AWS(JSEQ)*A(J1)
      IF(JSEQ.EQ.J)GO TO 119
1443  J1=J1+1
      GO TC 144
119   J=J-1
      IF(J.GT.0) GO TO 143
      GO TC 3004
C      ENTRY FOR MTYPE=2,3
103   LOG=.TRUE.
C      FIRST DIVIDE BY U**T OR MULTIPLY BY U
102   DO 120 J=1,N
      JCOL=IP(J,2)
      J1=IP(JCOL,1)
121   JSEQ=IP(IRN(J1),5)
      IF(JSEQ.EQ.J) GO TO 122
      IF(LCG) GO TO 1212
      AVECT(JCOL)=AVECT(JCOL)-AWS(JSEQ)*A(J1)
      GO TC 1213
1212  AWS(JSEQ)=AWS(JSEQ)+A(J1)*AVECT(JCOL)
1213  J1=J1+1
      GO TC 121
122   IF(LCG) GO TO 132
      AWS(JSEQ)=AVECT(JCOL)/A(J1)
      GO TC 120
132   AWS(JSEQ)=AVECT(JCOL)*A(J1)
120   CONTINUE
C      NOW DIVIDE BY L**T CR MULTIPLY BY L
      J=N
123   J1=IP(IP(J+1,2),1)
124   J1=J1-1
      JRO=IRN(J1)
      IF(IP(JRO,5).EQ.J) GC TO 125
```

```
      IF(LOG) GO TO 1242
      AWS(J)=AWS(J)-AVECT(JRO)*A(J1)
      GO TO 124
1242  AVECT(JRO)=AVECT(JRO)+AWS(J)*A(J1)
      GO TO 124
125   AVECT(JRO)=AWS(J)
      J=J-1
      IF(J.GT.0) GO TO 123
      GO TO 3004
C
      ENTRY MA18C (A,IRN,IP,N,NP,    AGRO)
C      ON ENTRY, THE ARRAYS ARE AS SET UP BY MA18AD, EXCEPT THAT A
C     CONTAINS THE ELEMENTS OF A NEW MATRIX IN UNFACTORISED FORM.
C     ON EXIT, THE CONTENTS OF A HAVE BEEN REPLACED BY THE L/U DECOMP-
C     OSITION OF THIS NEW MATRIX, EXACTLY AS ON EXIT FROM MA18AD.
C      THE ARGUMENT AGRO IS SET TO THE GROWTH ESTIMATE AS FOR MA18AD.
C     NEW SCALING FACTORS ARE CALCULATED AND STORED IF JSCAL.GT.1, OR
C     IF JSCALE=0 WHEN THEY WILL ALL BE UNITY.        OTHERWISE, THE OLD
C     SCALING FACTORS ARE USED.     THE PIVOTAL SEQUENCE SET BY MA18AD
C     IS ALWAYS USED.
      IP(N+1,3)=0
      KERR=1
      AGRO=-1D0
      KENTRY=3
C      TEST VALIDITY OF ENTRY
      IF(IP(N+1,2).EQ.0)GO TO 2205
      IF(IP(N+1,2).NE.N+1)GO TO 2207
C      GET SCALING FACTORS IF INDICATED
      IF(JSCALE.LE.1)GO TO 991
C      SAVE COLUMN 5 IN COLUMN 13
      DO 898 I=1,N
898   IP(I,13)=IP(I,5)
C     MC12AD USES COLUMNS 5 TO 12 AS WORKSPACE
C      RESTORE COLUMN 5
      DO 897 I=1,N
897   IP(I,5)=IP(I,13)
      IF(IS.NE.0)GO TO 2208
991   IF(JSCALE.NE.0) GO TO 892
C      SET ZERO SCALING POWERS IF JSCALE=0
      DO 895 L=1,2
      DO 895 K=1,N
895   IP(K,L+2)=0
892   JUU=0
      IAG=0
C      OPERATE ON COLUMNS IN SEQUENCE
      DO 56 J=1,N
      JST=IP(IP(J,2),1)
      JND=IP(IP(J+1,2),1)-1
C      FIND APPROX LOG OF MAX ELEMENT IN COLUMN
      IA0=0
      DO 57 J1=JST,JND
      AU= ABS(A(J1))
```

```
C     ON SYSTEM/360, NEXT STATEMENT SETS JUU SO THAT
C      16**(JUU-64).GT.AU.GE.16**(JUU-65)
      LUU(4)=LU
57    IAC=MAX0(IAO,JUU+IP(IRN(J1),3))
C      SCAN THROUGH COLUMN, ELIMINATING WITH ELEMENTS OF U AND
C     BRANCHING WHEN PIVOT FOUND
      ASSIGN 581 TO JPIV
      DO 58 J1=JST,JND
      GO TO JPIV,(581,582)
581   KSEQ=IP(IRN(J1),5)
C      BRANCH ON PIVOT
      IF(KSEQ.EQ.J) GO TO 59
C      ELEMENT OF U - GET OLD PIVOTAL COLUMN
      KST=IP(IP(KSEQ,2),1)
      KND=IP(IP(KSEQ+1,2),1)-1
      AMULT=A(J1)
      L1=J1+1
C      SCAN DOWN COLUMNS KEEPING IN STEP
      DO 60 K1=KST,KND
      KRO=IRN(K1)
C      SKIP IRRELEVANT ELEMENTS
      IF(IP(KRO,5).LE.KSEQ)GO TO 60
62      IF(KRO.EQ.IRN(L1))GO TO 63
      L1=L1+1
      GO TO 62
C      ELIMINATION STEP
63    A(L1)=A(L1)-AMULT*A(K1)
      AU= ABS(A(L1))
C      ON S/360,NEXT STATEMENT SETS JUU AS ABOVE
      LUU(4)=LU
C      UPDATE LOG OF GROWTH ESTIMATE
      IAG=MAX0(IAG,JUU+IP(IRN(L1),3)-IAO)
      L1=L1+1
60    CONTINUE
      GO TO 58
C      TEST FOR ZERO PIVOT
59    IF(A(J1).EQ.ZERO)GO TO 1011
      AMULT=A(J1)
      ASSIGN 582 TO JPIV
      GO TO 58
C      DIVIDE ELEMENTS OF L BY PIVOT
582   A(J1)=A(J1)/AMULT
58    CONTINUE
56    CONTINUE
C      SET GROWTH ESTIMATE
      AGRO=ARND*16D0**IAG
      GO TO 3004
      ENTRY MA18D (A,IRN,IP,N,NP,AWS,NAME)
      KERR=1
      KENTRY=4
C      TEST VALIDITY OF ENTRY
      IF(IP(N+1,2).EQ.0)GO TO 2205
```

```
      IF(IP(N+1,2).NE.N+1)GO TO 2207
C      INITIALLY CLEAR COLUMN
2402  DO 301 J=1,N
      AWS(J)=ZERO
301   CONTINUE
C      COLUMNS IN PIVOTAL ORDER
      DO 303 J=1,N
      JCOL=IP(J,2)
C      LOAD COLUMN
      CALL NAME(N,AWS,JCOL)
C      COPY TO REQUIRED PLACE AND RESET TO ZERO
      J1=IP(JCOL,1)
      J2=IP(IP(J+1,2),1)-1
      DO 302 L1=J1,J2
      JRO=IRN(L1)
      A(L1)=AWS(JRO)
302   AWS(JRO)=ZERO
303   CONTINUE
      GO TO 3004
C      DIAGNOSTIC PRINTING
2204  WRITE(JP,2304)ANAME(2),MTYPE
2304  FORMAT('0ERROR RETURN FROM',A8,'BECAUSE MTYPE =',I5,' WHICH IS OUT
     X OF RANGE')
      GO TO 3004
2205  WRITE(JP,2305)ANAME(KENTRY),ANAME(KERR)
2305  FORMAT('0ERROR RETURN FROM',A8,'BECAUSE PREVIOUS ENTRY TO',A8,
     X'GAVE ERROR RETURN')
      GO TO 3004
2207  WRITE(JP,2307) ANAME(KENTRY),ANAME(1)
2307  FORMAT('0ERROR RETURN FROM',A8,'BECAUSE NO PREVIOUS ENTRY TO',A8)
      GO TO 3004
1011  WRITE(JP,2309)ANAME(3),IRN(J1),IP(J,2)
2309  FORMAT('0ERROR RETURN FROM',A8,'BECAUSE ZERO PIVOT (',2I4,')')
      IP(N+1,3)=1
      GO TO 3004
2208  WRITE(JP,2308) ANAME(3),ANAME(5),IS
2308  FORMAT('0ERROR RETURN FROM',A8,'BECAUSE',A8,'HAS GIVEN ERROR ',
     X'RETURN WITH IS=',I4)
3004  RETURN
      END
      SUBROUTINE MC12A (A,IND,IP,N,NP,DIAG,RES,IS)
      REAL A(1)
      INTEGER*2 IND(1),IP(NP,2),DIAG(NP,2)
      REAL*4 RES(N,4)
C      DIAG IS USED TO RETURN INTEGER SCALING BOWERS, AND TO HOLD
C     COUNTS OF NON-ZEROS IN ROWS AND COLUMNS DURING EXECUTION.
C     IT IS SET TO 0 ON SUCCESSFUL COMPLETION, TO I IF ROW I HAS ONLY
C       ZERO ELEMENTS, TO -I IF COLUMN I HAS ONLY ZERO ELEMENTS
C      RES IS A WORKSPACE ARRAY. COLUMNSI AND 2 HOLD NON-ZERO HALF
C        OF RESIDUAL VECTOR FOR TWO CONSECUTIVE ITERATIONS.   COLUMN
C        3 HOLDS COLUMN SCALING POWERS, AND COLUMN 4 HOLDS THEIR
```

```
C       CHANGES OVER A DOUBLE ITERATION.
C
        DATA SMIN/.01/
C       SMIN IS USED IN A CONVERGENCE TEST ON (RESIDUAL NORM1**2
        INTEGER*2 JU(2)
        LOGICAL*1 IU,IW(3)
        EQUIVALENCE (UU,IW(1)),(U,IU,JU(1))
        UU=100.
        IS=0
C       INITIALISE FOR ACCUMULATION OF SUMS AND PRODUCTS
        DO 2 L=1,2
        DO 2 I=1,N
        RES(I,L)=0.
        RES(I,L+2)=0.
      2 DIAG(I,L)=0
        DO 3 J=1,N
        I2=IP(J,2)
        K1=IP(I2,1)
        K2=IP(IP(J+1,2),1)-1
        IF(K1.GT.K2) GO TO 3
        DO 4 K=K1,K2
        I1=IND(K)
        U=ABS(A(K))
        IF(U.EQ.0)GO TO 4
C       ON THE IBM 360 THE FOLLOWING TWO INSTRUCTIONS FIND THE SMALLEST
C        INTEGER GREATER THAN ALOG16(U).
        IW(2)=IU
        U=UU-64.
C       COUNT NON-ZEROS IN ROW AND COLUMN
        DIAG(I1,1)=DIAG(I1,1)+1
        DIAG(I2,2)=DIAG(I2,2)+1
        RES(I1,1)=RES(I1,1)+U
52      RES(I2,3)=RES(I2,3)+U
4       CONTINUE
      3 CONTINUE
C       COMPUTE RHS VECTORS TESTING FOR ZERO ROW OR COLUMN
        J=0
        JU(1)=17920
        DO 8 I=1,N
        J=J+DIAG(I,1)
        DO 9 L=1,2
        IF(DIAG(I,L).GT.0 ) GO TO 13
        DIAG(I,L)=1
        IS=I*(3-2*L)
     13 CONTINUE
C       ON IBM 360 NEXT INSTRUCTION SETS 0 TO VALUE OF POSITIVE INTEGER
        JU(2)=DIAG(I,L)
      9 RES(I,2*L-1)=RES(I,2*L-1)/U
8       CONTINUE
        SM=SMIN*J
C       SWEEP TO COMPUTE INITIAL RESIDUAL VECTOR
        DO 10 J1=1,N
```

```
      J=IP(J1,2)
      K1=IP(J,1)
      K2=IP(IP(J1+1,2),1)-1
      IF(K1.GT.K2)GO TO 10
      DO 12 K=K1,K2
      IF(A(K).EQ.0.0) GO TO 12
      I=IND(K)
C     ON IBM 360 NEXT INSTRUCTION SETS U TO VALUE OF PSITIVE INTEGER
      JU(2)=DIAG(I,1)
      RES(I,1)=RES(I,1)-RES(J,3)/U
12    CONTINUE
   10 CONTINUE
C     INITIALISE ITERATION
      E=0.
      E1=0.
      Q=1.
      S=0.
      DO 11 I=1,N
C     ON IBM 360 NEXT INSTRUCTION SETS U TO VALUE OF POSITVE INTEGER
      JU(2)=DIAG(I,1)
   11 S=S+U*RES(I,1)**2
      L=2
      IF(S.LE.SM) GO TO 101
C     ITERATION STEP
   20 EM=E*E1
C    SWEEP THROUGH MATRIX TO UPDATE RESIDUAL VECTOR
      DO 22 J3=1,N
      J1=IP(J3,2)
      K1=IP(J1,1)
      K2=IP(IP(J3+1,2),1)-1
      IF(K1.GT.K2) GO TO 22
      DO 28 K=K1,K2
      IF(A(K).EQ.0.0) GO TO 28
      J2=(2-L)*(J1-IND(K))
      I=J1-J2
      J=IND(K)+J2
      RES(I,L)=RES(I,L)+RES(J,3-L)
28    CONTINUE
   22 CONTINUE
      S1=S
      S=0.
      DO 23 I=1,N
      V=-RES(I,L)/Q
C     ON IBM 360 NEXT INSTRUCTION SETS U TO VALUE OF POSITVE INTEGER
      JU(2)=DIAG(I,L)
      RES(I,L)=V/U
   23 S=S+V*RES(I,L)
      E1=E
      E=Q*S/S1
      Q1=Q
      Q=1.-E
      M=3-L
```

```
      IF(S.GT.SM) GO TO 27
      E=M-1
      M=1
      Q=1.
   27 IF(L.EQ.2) GO TO 25
      QM=Q*Q1
      DO 26 I=1,N
      RES (I,4)=(EM*RES(I,4)+RES(I,2))/QM
   26 RES(I,3)=RES(I,3)+RES(I,4)
   25 L=M
      DO 24 I=1,N
C      ON IBM 360 NEXT INSTRUCTION SETS U TO VALUE OF POSITIVE INTEGER
      JU(2)=DIAG(I,L)
   24 RES(I,L)=RES(I,L)*U*E
      IF(S.GT.SM)GO TO 20
C      SWEEP THROUGH MATRIX TO GET ROW SCALING POWERS
  101 DO 103 J1=1,N
      J=IP(J1,2)
      K1=IP(J,1)
      K2=IP(IP(J1+1,2),1)-1
      IF(K1.GT.K2)GO TO 103
      DO 105 K=K1,K2
      U=ABS(A(K))
      IF(U.EQ.0.)GO TO 105
C      ON IBM 360 NEXT TWO INSTRUCTIONS FIND THE SMALLEST INTEGER
C     LESS THAN ALOG16(U)
      IW(2)=IU
      U=UU-64.
      I=IND(K)
      RES(I,1)=RES(I,1)+RES(J,3)-U
105   CONTINUE
  103 CONTINUE
C      CONVERT POWERS TO INTEGERS
      JU(1)=17920
      DO 104 I=1,N
C     ON IBM 360 NEXT INSTRUCTION SETS 0 TO VALUE OF PASITIVE INTEGER
      JU(2)=DIAG(I,1)
      V=RES(I,1)/U
      DIAG(I,1)=V+SIGN(0.5,V)
  104 DIAG(I,2)=-(RES(I,3)+SIGN(0.5,RES(I,3)))
      RETURN
      END
/*
//LKED.SYSLIB DD DSN=SYS1.FORTLIB,DISP=SHR
//      DD DSN=NLCS.IMSLLIB,DISP=SHR
//   DD DSN=F30005.SPARSE.ROUTINES,DISP=SHR
//GO.SYSIN   DD   *
```

APPENDIX F

TEST SYSTEMS DATA

(i) 5 - Bus

(ii) 23 - Bus

(iii) IEEE 57 - Bus

(iv) IEEE 118 - Bus

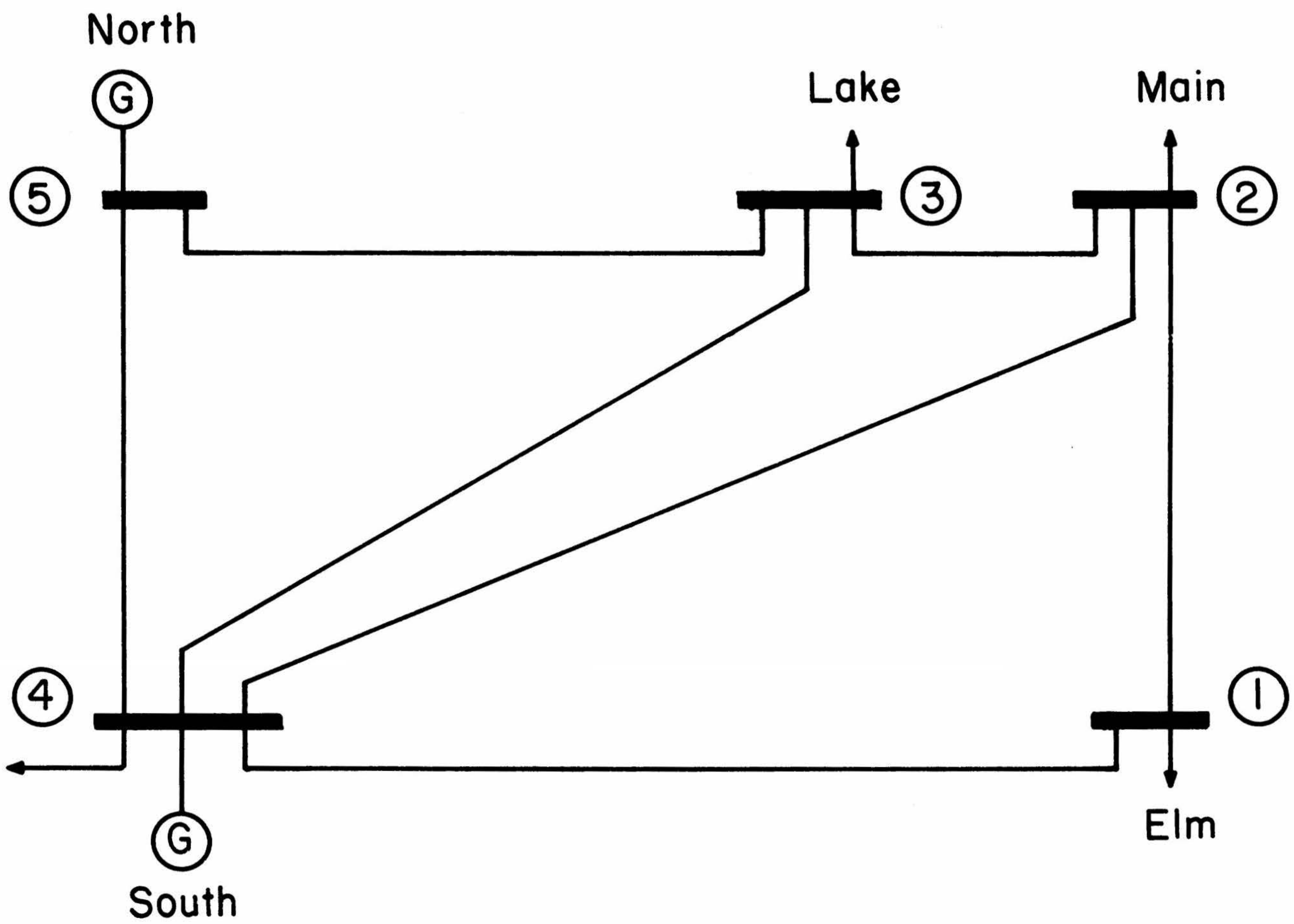


FIGURE F.1 5-BUS POWER SYSTEM OF STAGG AND EL-ABIAD

TABLE F.1: Impedance and Line-Charging Data for the 5-Bus Test System

LINE DESIGNATION	RESISTANCE p.u.*	REACTANCE p.u.*	LINE CHARGING p.u.*
2-1	0.08	0.24	0.025
3-2	0.01	0.03	0.010
4-1	0.04	0.12	0.015
4-2	0.06	0.18	0.020
4-3	0.06	0.18	0.020
5-3	0.08	0.24	0.025
5-4	0.02	0.06	0.030

* Impedance and line charging susceptance in per unit on a 100 MVA base.
Line Charging: one-half of total charging of line.

TABLE F.2: Scheduled Loads and Generation and Assumed Bus Voltages for the 5-Bus Test System

Bus Number	Starting Bus Volt. Mag. p.u.	Starting Bus Volt. Angle deg.	Generation MW	Generation MVAr	Load MW	Load MVAr
1	1.0	0.0	0	0	60	10
2	1.0	0.0	0	0	40	5
3	1.0	0.0	0	0	45	15
4	1.0	0.0	40	30	20	10
5**	1.0	1.06	0	0	0	0

** Slack Bus

TABLE F.3: Regulated Bus Data for the 5-Bus Test System

Bus Number	Voltage Magnitude p.u.	Minimum MVAr Capability	Maximum MVAr Capability
4	1.047	-10	50

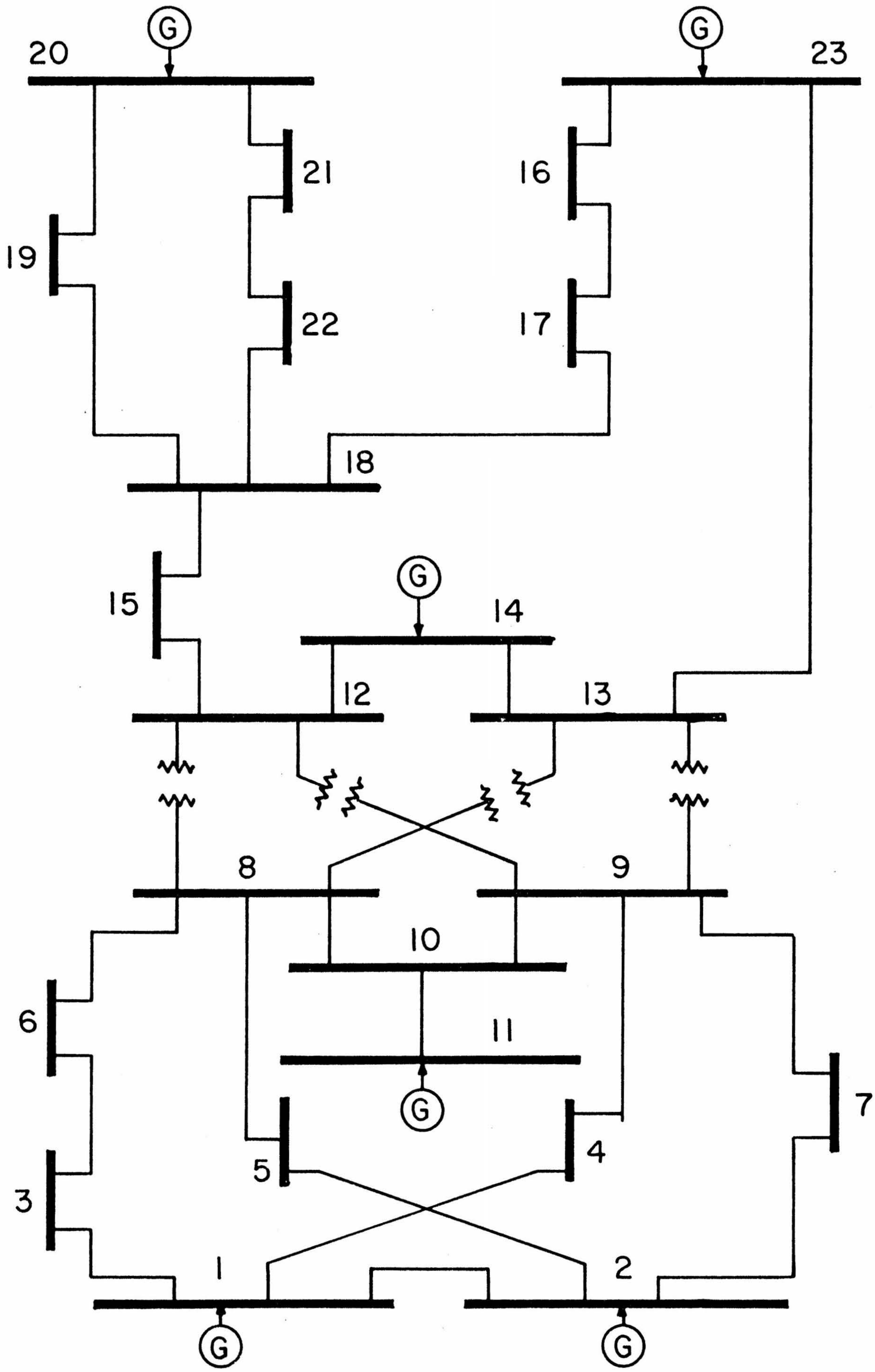
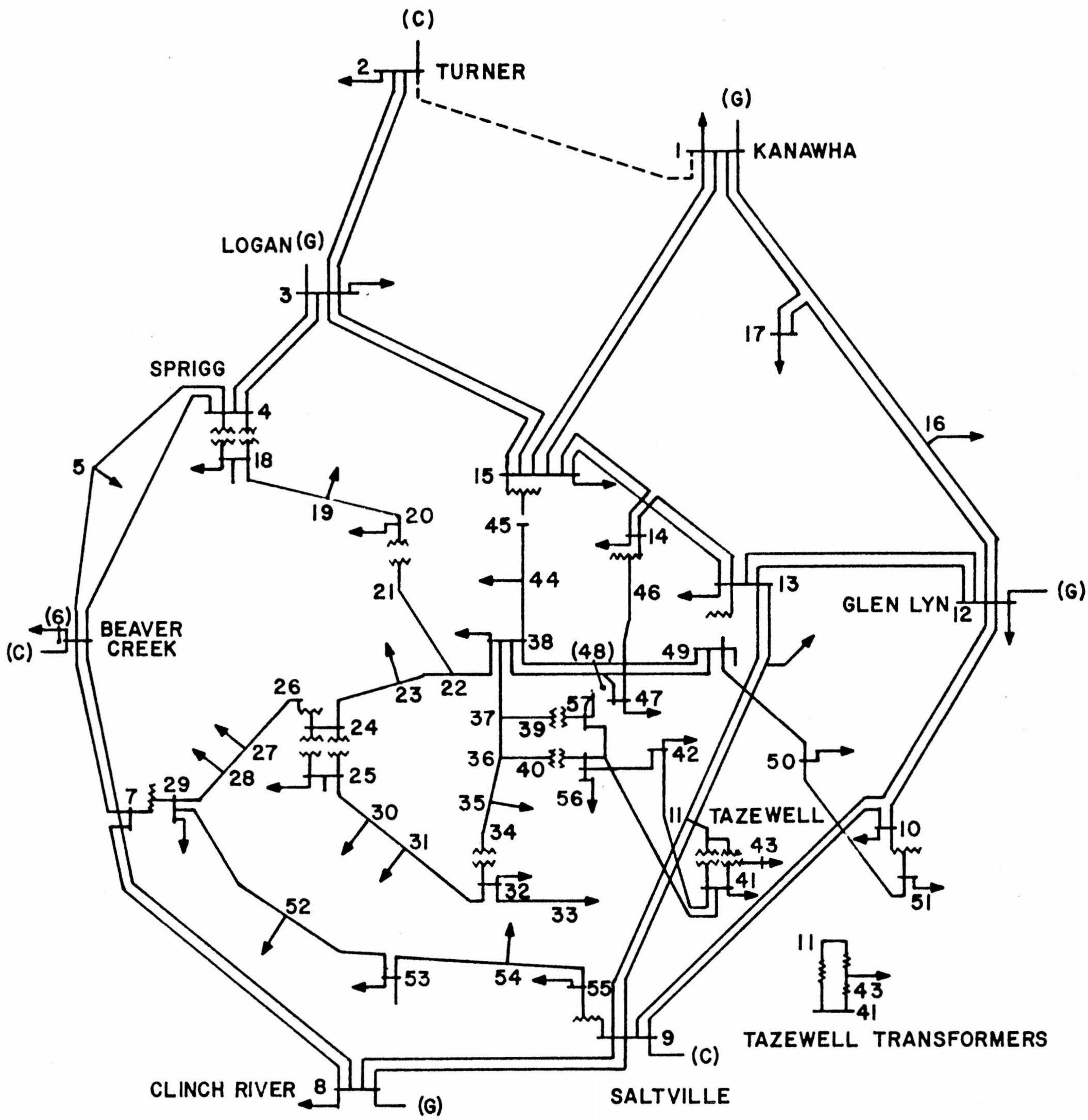


FIGURE F.2 23 - BUS POWER SYSTEM.



Bus-Code Diagram

(C) Synchronous Compensators
(G) Generators

FIGURE F.3 IEEE 57-BUS POWER SYSTEM

TABLE F.4: Impedance and Line Charging Data for the 23-Bus Test System

Line Designition	Resistance p.u.*	Reactance p.u.*	Line Charging p.u.*
1-3	0.0242	0.0540	0.0118
1-4	0.0309	0.0693	0.0151
2-5	0.0404	0.0888	0.0197
5-8	0.0325	0.0709	0.0157
2-7	0.0615	0.1620	0.0342
3-6	0.0576	0.1520	0.0320
4-9	0.0266	0.0700	0.0148
9-7	0.0229	0.0504	0.0112
8-6	0.0446	0.1003	0.0218
1-10	0.0233	0.0514	0.0456
8-10	0.0597	0.1315	0.0291
9-10	0.0597	0.1315	0.0291
13-14	0.0043	0.0351	0.2373
14-12	0.0043	0.0351	0.2373
15-12	0.0038	0.0307	0.2078
18-15	0.0035	0.0288	0.1951
23-13	0.0089	0.0720	0.4871
16-17	0.0010	0.0080	0.0543
17-18	0.0021	0.0167	0.1133
19-18	0.0016	0.0127	0.0862
20-19	0.0045	0.0362	0.2451
22-18	0.0024	0.0192	0.1298
20-21	0.0019	0.0156	0.1056
21-22	0.0014	0.0114	0.0770
23-16	0.0020	0.0164	0.1109
12-8	0.0023	0.0839	0.0000
13-8	0.0023	0.0839	0.0000
12-9	0.0019	0.1300	0.0000
13-9	0.0023	0.0839	0.0000
1-2	0.0025	0.2000	0.0000

* Impedance and line charging susceptance in per unit on a 100 MVA base. Line charging: total charging of line.

TABLE F.5: Scheduled Loads and Generation and Assumed Bus Voltages for the 23-Bus Test System.

Bus Number	Starting Bus Volt. Mag. p.u.	Starting Bus Volt. Angle deg.	Generation MW	Generation MVAr	Load MW	Load MVAr
1	1.0	0.0	0.663	0.143	0.0	0.0
2	1.0	0.0	0.817	0.0	0.0	0.236
3	1.0	0.0	0.0	0.0	0.0	0.0
4	1.0	0.0	0.0	0.0	0.470	0.120
5	1.0	0.0	0.0	0.0	0.510	0.130
6	1.0	0.0	0.0	0.0	0.410	0.100
7	1.0	0.0	0.0	0.0	0.480	0.120
8	1.0	0.0	0.0	0.0	0.010	0.0
9	1.0	0.0	0.0	0.0	1.500	0.380
10	1.0	0.0	0.0	0.0	1.770	0.440
11	1.0	0.0	1.030	0.121	0.0	0.0
12	1.0	0.0	0.0	0.0	0.600	0.0
13	1.0	0.0	0.400	0.0	0.0	0.0
14	1.0	0.0	0.204	0.0	0.0	0.132
15	1.0	0.0	0.0	0.0	2.010	0.500
16	1.0	0.0	0.0	0.0	1.320	0.330
17	1.0	0.0	0.0	0.0	3.440	0.860
18	1.0	0.0	0.0	0.0	1.040	0.260
19	1.0	0.0	0.0	0.0	3.760	0.940
20	1.0	0.0	8.030	0.984	0.0	0.0
21	1.0	0.0	0.0	0.0	3.750	0.940
22	1.0	0.0	2.100	0.520	0.0	0.0
23**	1.050	0.0	8.100	1.505	0.0	0.0

** Slack bus

TABLE F.6: Impedance and Line-Charging Data for the IEEE 57-Bus Test System.

Line Designation	Resistance p.u.*	Reactance p.u.*	Line Charging p.u.*
57-2	0.0083	0.0280	0.0645
2-3	0.0298	0.0850	0.0409
3-4	0.0112	0.0366	0.0190
4-5	0.0625	0.1320	0.0129
4-6	0.0430	0.1480	0.0174
6-7	0.0200	0.1020	0.0138
6-8	0.0339	0.1730	0.0235
8-9	0.0099	0.0505	0.0274
9-10	0.0369	0.1679	0.0220
9-11	0.0258	0.0848	0.0109
9-12	0.0648	0.2950	0.0386
9-13	0.0481	0.1580	0.0203
13-14	0.0132	0.0434	0.0055
13-15	0.0269	0.0869	0.0115
57-15	0.0178	0.0910	0.0494
57-16	0.0454	0.2060	0.0273
57-17	0.0238	0.1080	0.0143
3-15	0.0162	0.0530	0.0272
4-18	0	0.555	0
4-18	0	0.43	0
5-6	0.0302	0.0611	0.0062
7-8	0.0139	0.0712	0.0097
10-12	0.0277	0.1262	0.0164
11-13	0.0223	0.0732	0.0094
12-13	0.0178	0.0580	0.0302
12-16	0.0180	0.0813	0.0108
12-17	0.0397	0.1790	0.0238
14-15	0.0171	0.0547	0.0074
18-19	0.4610	0.6850	0
19-20	0.2830	0.4340	0
20-21	0	0.7767	0
21-22	0.0736	0.1170	0
22-23	0.0099	0.0152	0
23-24	0.1660	0.2560	0.0042
24-25	0	1.182	0
24-25	0	1.23	0
24-26	0	0.0473	0
26-27	0.1650	0.2540	0
27-28	0.0618	0.0954	0
28-29	0.0418	0.0587	0
7-29	0	0.0648	0
25-30	0.1350	0.2020	0
30-31	0.3260	0.4970	0
31-32	0.5070	0.7550	0
32-33	0.0392	0.0360	0
32-34	0	0.9530	0

TABLE F.6: CONT'd

Line Designation	Resistance p.u*	Reactance p.u*	Line Charging p.u.*
34-35	0.0520	0.0780	0.0016
35-36	0.0430	0.0537	0.0008
36-37	0.0290	0.0366	0
37-38	0.0651	0.1009	0.0010
37-39	0.0239	0.0379	0
36-40	0.0300	0.0466	0
22-38	0.0192	0.0295	0.0010
11-41	0	0.7490	0
41-42	0.2070	0.3520	0
41-43	0	0.4120	0
38-44	0.0289	0.0585	0.0010
15=45	0	0.1042	0
14-46	0	0.0735	0
46-47	0.0230	0.0680	0.0016
47-48	0.0182	0.0233	0
48-49	0.0834	0.1290	0.0024
49-50	0.0801	0.1280	0
50-51	0.1386	0.2200	0
10-51	0	0.0712	0
13-49	0	0.1910	0
29-52	0.1442	0.1870	0
52-53	0.0762	0.0984	0
53-54	0.1878	0.2320	0
54-55	0.1732	0.2265	0
11-43	0	0.1530	0
44-45	0.0624	0.1242	0.0020
40-56	0	1.1950	0
56-41	0.5530	0.5490	0
56-42	0.2125	0.3540	0
39-1	0	1.3550	0
1-56	0.1740	0.2600	0
38-49	0.1150	0.1770	0.0030
38-48	0.0312	0.0482	0
9-55	0	0.1205	0

* Impedance and line charging susceptance in p.u. on a 100000kVA base
Line charging: one-half of total charging of line.

TABLE F.7: Operating Conditions for the IEEE 57-Bus Test System

Bus Number *	Starting Bus Volt. Mag. p.u.	Angle deg.	Generation MW	MVAr	Load MW	MVAr
57**	1.04	0	0	0	55.0	17.0
2	1.0	0	0	0	3.0	88.0
3	1.0	0	40	0	41.0	21.0
4	1.0	0	0	0	0	0
5	1.0	0	0	0	13.0	4.0
6	1.0	0	0	0	75.0	2.0
7	1.0	0	0	0	0	0
8	1.0	0	450	0	150.0	22.0
9	1.0	0	0	0	121.0	26.0
10	1.0	0	0	0	5.0	2.0
11	1.0	0	0	0	0	0
12	1.0	0	310	0	377.0	24.0
13	1.0	0	0	0	18.0	2.3
14	1.0	0	0	0	10.5	5.3
15	1.0	0	0	0	22.0	5.0
16	1.0	0	0	0	43.0	3.0
17	1.0	0	0	0	42.0	8.0
18	1.0	0	0	0	27.2	9.8
19	1.0	0	0	0	3.3	0.6
20	1.0	0	0	0	2.3	1.0
21	1.0	0	0	0	0	0
22	1.0	0	0	0	0	0
23	1.0	0	0	0	6.3	2.1
24	1.0	0	0	0	0	0
25	1.0	0	0	0	6.3	3.2
26	1.0	0	0	0	0	0
27	1.0	0	0	0	9.3	0.5
28	1.0	0	0	0	4.6	2.3
29	1.0	0	0	0	17.0	2.6
30	1.0	0	0	0	3.6	1.8
31	1.0	0	0	0	5.8	2.9
32	1.0	0	0	0	1.6	0.8
33	1.0	0	0	0	3.8	1.9
34	1.0	0	0	0	0	0
35	1.0	0	0	0	6.0	3.0
36	1.0	0	0	0	0	0
37	1.0	0	0	0	0	0
38	1.0	0	0	0	14.0	7.0
39	1.0	0	0	0	0	0
40	1.0	0	0	0	0	0
41	1.0	0	0	0	6.3	3.0
42	1.0	0	0	0	7.1	4.4
43	1.0	0	0	0	2.0	1.0
44	1.0	0	0	0	12.0	1.8
45	1.0	0	0	0	0	0
46	1.0	0	0	0	0	0
47	1.0	0	0	0	29.7	11.6
48	1.0	0	0	0	0	0
49	1.0	0	0	0	18.0	8.5

TABLE F.7: CONT'D

Bus Number*	Starting Bus Volt. Mag. p.u.	Angle deg.	Generation MW	MVAr	Load MW	MVAr
50	1.0	0	0	0	21.0	10.5
51	1.0	0	0	0	18.0	5.3
52	1.0	0	0	0	4.9	2.2
53	1.0	0	0	0	20 0	10.0
54	1.0	0	0	0	4.1	1.4
55	1.0	0	0	0	6.8	3.4
56	1.0	0	0	0	7.6	2.2
1	1.0	0	0	0	6.7	2.0

* This is prior to the Bus Renumbering Necessary to Enable the Regulated Buses to have the higher numbers.

** Swing Machine

TABLE F.8: Transformer Data For the IEEE 57-Bus Test System

Transformer designation	Tap Setting*
4-18	0.97
4-18	0.978
7-29	0.967
9-55	0.94
10-51	0.93
11-41	0.955
11.43	0.958
13-49	0.895
14-46	0.9
15-45	0.955
21-20	1.043
24-25	1.000
24-25	1.000
24-26	1.043
34-32	0.975
39-1	0.98
40-56	0.958

* Off-nominal turns ratio, as determined by the actual transformer-tap positions and the voltage baiss. In the case of nominal turns ratio, this would equal 1.

TABLE F.9: Regulated Bus Data For the IEEE 57-Bus Test System.

Bus Number	Voltage Magnitude p.u	Minimum MVAr capability	Maximum MVAr capability
2	1.01	-17	50
3	0.985	-10	60
6	0.98	-8	25
8	1.005	-140	200
9	0.98	-3	9
12	1.015	-50	155

TABLE F.10: Static Capacitor Data For the IEEE 57-Bus Test System

Bus Number	Susceptance* p.u.
18	0.1
25	0.059
53	0.063

* Susceptance in p.u. on a 100000 kVA Base.

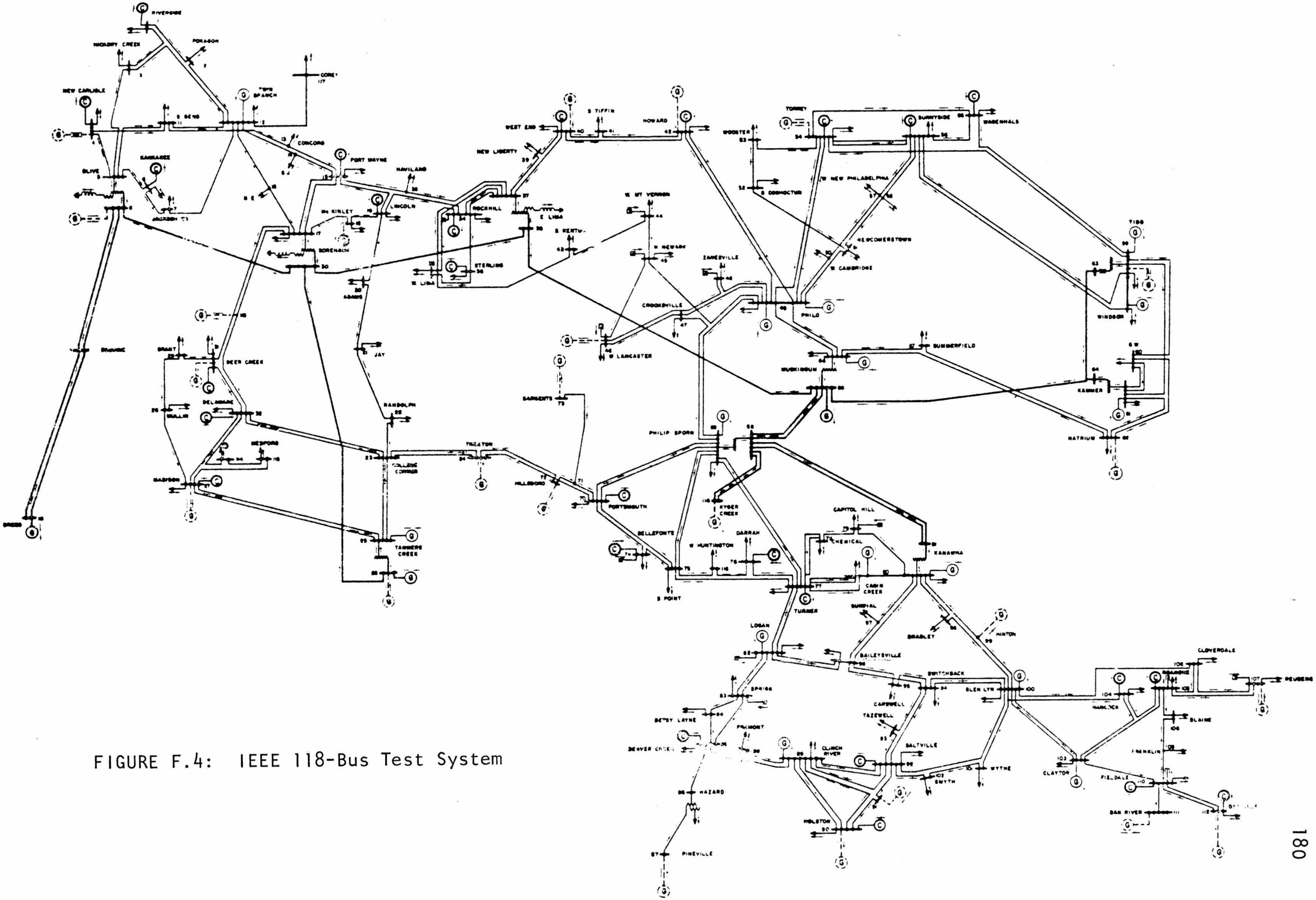


FIGURE F.4: IEEE 118-Bus Test System

TABLE F.11: CONT'D

Line Designation	Resistance Per Unit**	Reactance Per Unit**	Line Charging Per Unit **
34-37	.00256	.00940	.00492
37-38	0	.03750	0
37-39	.03210	.10600	.01350
37-40	.05930	.16800	.02100
30-38	.00464	.05400	.21100
39-40	.01840	.06050	.00776
40-41	.01450	.04870	.00611
40-42	.05550	.18300	.02330
41-42	.04100	.13500	.01720
43-44	.06080	.24540	.03034
34-43	.04130	.16810	.02113
44-45	.02240	.09010	.01120
45-46	.04000	.13560	.01660
46-47	.03800	.12700	.01580
46-48	.06010	.18900	.02360
47-49	.01910	.06250	.00802
42-49	.07150	.32300	.04300
42-49	.07150	.32300	.04300
45-49	.06840	.18600	.02220
48-49	.01790	.05050	.00629
49-50	.02670	.07520	.00937
49-51	.04860	.13700	.01710
51-52	.02030	.05880	.00698
52-53	.04050	.16350	.02029
53-54	.02630	.12200	.01550
49-54	.07300	.28900	.03690
49-54	.08690	.29100	.03650
54-55	.01690	.07070	.01010
54-56	.00275	.00955	.00366
55-56	.00488	.01510	.00187
56-57	.03430	.09660	.01210
50-57	.04740	.13400	.01660
56-58	.03430	.09660	.01210
51-58	.02550	.07190	.00894
54-59	.05030	.22930	.02990
56-59	.08250	.25100	.02845
56-59	.08030	.23900	.02680
55-59	.04739	.21580	.02823
59-60	.03170	.14500	.01880
59-61	.03280	.15000	.01940
60-61	.00264	.01350	.00728
60-62	.01230	.05610	.00734
61-62	.00824	.03760	.00490
59-63	0	.03860	0
63-64	.00172	.02000	.10800
61-64	0	.02680	0
38-65	.00901	.09860	.52300
64-65	.00269	.03020	.19000
49-66	.01800	.09190	.01240
49-66	.01800	.09190	.01240
62-66	.04820	.21800	.02890
62-67	.02580	.11700	.01550

TABLE F.11: Impedance and Line Charging Data for the IEEE 118-Bus Test System.

Line Designation	Resistance Per Unit**	Reactance Per Unit**	Line Charging Per Unit **
1-2	.03030	.09990	.01270
1-3	.01290	.04240	.00541
4-5	.00176	.00798	.00105
3-5	.02410	.10800	.01420
5-6	.01190	.05400	.00713
6-7	.00459	.02080	.00275
8-9	.00244	.03050	.58100
5-8	0	.02670	0
9-10	.00258	.03220	.61500
4-11	.02090	.06880	.00874
5-11	.02030	.06820	.00869
11-12	.00595	.01960	.00251
2-12	.01870	.06160	.00786
3-12	.04840	.16000	.02030
7-12	.00862	.03400	.00437
11-13	.02225	.07310	.00938
12-14	.02150	.07070	.00908
13-15	.07440	.24440	.03134
14-15	.05950	.19500	.02510
12-16	.02120	.08340	.01070
15-17	.01320	.04370	.02220
16-17	.04540	.18010	.02330
17-18	.01230	.05050	.00649
18-19	.01119	.04930	.00571
19-20	.02520	.11700	.01490
15-19	.01200	.03940	.00505
20-21	.01830	.08490	.01080
21-22	.02090	.09700	.01230
23-24	.01350	.04920	.02490
23-25	.01560	.08000	.04320
25-26	0	.03820	0
25-27	.03180	.16300	.08820
27-28	.01913	.08550	.01080
28-29	.02370	.09430	.01190
17-30	0	.03880	0
8-30	.00431	.05040	.25700
26-30	.00799	.08600	.45400
17-31	.04740	.15630	.01995
29-31	.01080	.03310	.00415
23-32	.03170	.11530	.05865
31-32	.02980	.09850	.01255
27-32	.02290	.07550	.00963
15-33	.03800	.12440	.01597
19-34	.07520	.24700	.03160
35-36	.00224	.01020	.00134
35-37	.01100	.04970	.00659
33-37	.04150	.14200	.01830
34-36	.00871	.02680	.00284
22-23	.03420	.15900	.02020

TABLE F.11: CONT'D

Line Designation	Resistance Per Unit**	Reactance Per Unit**	Line Charging Per Unit **
65-66	0	.03700	0
66-67	.02240	.10150	.01341
65-68	.00138	.01600	.31900
47-118	.08440	.27780	.03546
49-118	.09850	.32400	.04140
68-118	0	.03700	0
118-70	.03000	.12700	.06100
24-70	.10221	.41150	.05099
70-71	.00882	.03550	.00439
24-72	.04880	.19600	.02440
71-72	.04460	.18000	.02222
71-73	.00866	.04540	.00589
70-74	.04010	.13230	.01684
70-75	.04280	.14100	.01800
118-75	.04050	.12200	.06200
74-75	.01230	.04060	.00517
76-77	.04440	.14800	.01840
118-77	.03090	.10100	.05190
75-77	.06010	.19990	.02489
77-78	.00376	.01240	.00632
78-79	.00546	.02440	.00324
77-80	.01700	.04850	.02360
77-80	.02940	.10500	.01140
79-80	.01560	.07040	.00935
68-81	.00175	.02020	.40400
80-81	0	.03700	0
77-82	.02980	.08530	.04087
82-83	.01120	.03665	.01898
83-84	.06250	.13200	.01290
83-85	.04300	.14800	.01740
84-85	.03020	.06410	.00617
85-86	.03500	.12300	.01380
86-87	.02828	.20740	.02225
85-88	.02000	.10200	.01380
88-89	.01390	.07120	.00967
89-90	.05180	.18800	.02640
89-90	.02380	.09970	.05300
90-91	.02540	.08360	.01070
89-92	.00990	.05050	.02740
89-92	.03930	.15810	.02070
91-92	.03870	.12720	.01634
92-93	.02580	.08480	.01090
92-94	.04810	.15800	.02030
93-94	.02230	.07320	.00938
94-95	.01320	.04340	.00555
80-96	.03560	.18200	.02470
85-89	.02390	.17300	.02350

TABLE F.11: CONT'D

Line Designation	Resistance Per Unit**	Reactance Per Unit**	Line Charging Per Unit**
82-96	.01620	.05300	.02720
94-96	.02690	.08690	.01150
80-97	.01830	.09340	.01270
80-98	.02380	.10800	.01430
80-99	.04540	.20600	.02730
92-100	.06480	.29500	.03860
94-100	.01780	.05800	.03020
95-96	.01710	.05470	.00737
96-97	.01730	.08850	.01200
98-100	.03970	.17900	.02380
99-100	.01800	.08130	.01080
100-101	.02770	.12620	.01640
92-102	.01230	.05590	.00732
101-102	.02460	.11200	.01470
100-103	.01600	.05250	.02680
100-104	.04510	.20400	.02705
103-104	.04660	.15840	.02035
103-105	.05350	.16250	.02040
100-106	.06050	.22900	.03100
104-105	.00994	.03780	.00493
105-106	.01400	.05470	.00717
105-107	.05300	.18300	.02360
105-108	.02610	.07030	.00922
106-107	.05300	.18300	.02360
108-109	.01050	.02880	.00380
103-110	.03906	.18130	.02305
109-110	.02780	.07620	.01010
110-111	.02200	.07550	.01000
110-112	.02470	.06400	.03100
17-113	.00913	.03010	.00384
32-113	.06150	.20300	.02590
32-114	.01350	.06120	.00814
27-115	.01640	.07410	.00986
114-115	.00230	.01040	.00138
68-116	.00034	.00405	.08200
12-117	.03290	.14000	.01790
75-69	.01450	.04810	.00599
76-69	.01640	.05440	.00678

* Based on AEP System for Total Loss Formula June 1962.

** Impedance and line-charging susceptance in per unit on a 100,000 kva base

Line charging one-half of total charging of line

TABLE F.12: Operating Conditions for the IEEE 118-Bus Test System

Bus Number	Starting Bus Voltage Mag. p.u.	Angle Deg.	Generation MW	Generation MVAr	Load MW	Load MVAr
1	1.0	0	0	0	51	27
2	1.0	0	0	0	20	9
3	1.0	0	0	0	39	10
4	1.0	0	-9	0	30	12
5	1.0	0	0	0	0	0
6	1.0	0	0	0	52	22
7	1.0	0	0	0	19	2
8	1.0	0	-28	0	0	0
9	1.0	0	0	0	0	0
10	1.0	0	450	0	0	0
11	1.0	0	0	0	70	23
12	1.0	0	85	0	47	10
13	1.0	0	0	0	34	16
14	1.0	0	0	0	14	1
15	1.0	0	0	0	90	30
16	1.0	0	0	0	25	10
17	1.0	0	0	0	11	3
18	1.0	0	0	0	60	34
19	1.0	0	0	0	45	25
20	1.0	0	0	0	18	3
21	1.0	0	0	0	14	8
22	1.0	0	0	0	10	5
23	1.0	0	0	0	7	3
24	1.0	0	-13	0	0	0
25	1.0	0	220	0	0	0
26	1.0	0	314	0	0	0
27	1.0	0	-9	0	62	13
28	1.0	0	0	0	17	7
29	1.0	0	0	0	24	4
30	1.0	0	0	0	0	0
31	1.0	0	7	0	43	27
32	1.0	0	0	0	59	23
33	1.0	0	0	0	23	9
34	1.0	0	0	0	59	26
35	1.0	0	0	0	33	9
36	1.0	0	0	0	31	17
37	1.0	0	0	0	0	0
38	1.0	0	0	0	0	0
39	1.0	0	0	0	27	11
40	1.0	0	-46	0	20	23
41	1.0	0	0	0	37	10
42	1.0	0	-59	0	37	23

TABLE F.12: CONT'D

Bus Number	Starting Bus Voltage Mag. p.u.	Angle Deg.	Generation MW	Generation MVAr	Load MW	Load MVAr
43	1.0	0	0	0	18	7
44	1.0	0	0	0	16	8
45	1.0	0	0	0	53	22
46	1.0	0	19	0	28	10
47	1.0	0	0	0	34	0
48	1.0	0	0	0	20	11
49	1.0	0	204	0	87	30
50	1.0	0	0	0	17	1
51	1.0	0	0	0	17	8
52	1.0	0	0	0	18	5
53	1.0	0	0	0	23	11
54	1.0	0	48	0	113	32
55	1.0	0	0	0	63	22
56	1.0	0	0	0	84	18
57	1.0	0	0	0	12	3
58	1.0	0	0	0	12	3
59	1.0	0	155	0	277	113
60	1.0	0	0	0	78	3
61	1.0	0	160	0	0	0
62	1.0	0	0	0	77	14
63	1.0	0	0	0	0	0
64	1.0	0	0	0	0	0
65	1.0	0	391	0	0	0
66	1.0	0	392	0	39	18
67	1.0	0	0	0	28	7
68	1.0	0	0	0	0	0
118	1.035	30	516.4	0	0	0
70	1.0	0	0	0	66	20
71	1.0	0	0	0	0	0
72	1.0	0	-12	0	0	0
73	1.0	0	-6	0	0	0
74	1.0	0	0	0	68	27
75	1.0	0	0	0	47	11
76	1.0	0	0	0	68	36
77	1.0	0	0	0	61	28
78	1.0	0	0	0	71	26
79	1.0	0	0	0	39	32
80	1.0	0	477	0	130	26
81	1.0	0	0	0	0	0
82	1.0	0	0	0	54	27
83	1.0	0	0	0	20	10
84	1.0	0	0	0	11	7
85	1.0	0	0	0	24	15
86	1.0	0	0	0	21	10

TABLE F.12: CONT'D

Bus Number	Starting Bus Voltage Mag. p.u.	Angle Deg.	Generation MW	Mvar	Load MW	Mvar
87	1.0	0	4	0	0	0
88	1.0	0	0	0	48	10
89	1.0	0	607	0	0	0
90	1.0	0	-85	0	78	42
91	1.0	0	-10	0	0	0
92	1.0	0	0	0	65	10
93	1.0	0	0	0	12	7
94	1.0	0	0	0	30	16
95	1.0	0	0	0	42	31
96	1.0	0	0	0	38	15
97	1.0	0	0	0	15	9
98	1.0	0	0	0	34	8
99	1.0	0	-42	0	0	0
100	1.0	0	252	0	37	18
101	1.0	0	0	0	22	15
102	1.0	0	0	0	5	3
103	1.0	0	40	0	23	16
104	1.0	0	0	0	38	25
105	1.0	0	0	0	31	26
106	1.0	0	0	0	43	16
107	1.0	0	-22	0	28	12
108	1.0	0	0	0	2	1
109	1.0	0	0	0	8	3
110	1.0	0	0	0	39	30
111	1.0	0	36	0	0	0
112	1.0	0	-43	0	25	13
113	1.0	0	-6	0	0	0
114	1.0	0	0	0	8	3
115	1.0	0	0	0	22	7
116	1.0	0	-184	0	0	0
117	1.0	0	0	0	20	8
69	1.0	0	0	0	33	15

* Swing Machine

TABLE F.13: Regulated Bus Data For the IEEE 118-Bus Test System

Bux Number	Voltage Magnitude Per Unit	Minimum Mvar Capability	Maximum Mvar Capability
1	.955	-5	15
4	.998	-300	300
6	.99	-13	50
8	1.015	-300	300
10	1.05	-147	200
12	.99	-35	120
15	.97	-10	30
18	.973	-16	50
19	.962	-8	24
24	.992	-300	300
25	1.05	-47	140
26	1.015	-1000	1000
27	.968	-300	300
31	.967	-300	300
32	.963	-14	42
34	.984	-8	24
36	.98	-8	24
40	.97	-300	300
42	.985	-300	300
46	1.005	-100	100
49	1.025	-85	210
54	.955	-300	300
55	.952	-8	23
56	.954	-8	15
59	.985	-60	180
61	.995	-100	300
62	.998	-20	20
65	1.005	-67	200
66	1.05	-67	200
70	.984	-10	32
72	.98	-100	100
73	.991	-100	100
74	.958	-6	9
76	.943	-8	23
77	1.006	-20	70
80	1.04	-165	280
85	.985	-8	23
87	1.015	-100	1000
89	1.005	-210	300
90	.985	-300	300
91	.98	-100	100
92	.99	-3	9
99	1.01	-100	100

TABLE F.13: CONT'D

Bus Number	Voltage Magnitude Per Unit	Minimum Mvar Capability	Maximum Mvar Capability
100	1.017	-50	155
103	1.01	-15	40
104	.971	-8	23
105	.965	-8	23
107	.952	-200	200
110	.973	-8	23
111	.98	-100	1000
112	.975	-100	1000
113	.993	-100	200
116	1.005	-1000	1000

TABLE F.14: Transformer Data for the IEEE 118-Bus Test System

TRANSFORMER DESIGNATION	TAP SETTING*
8-5	.985
26-25	.96
30-17	.96
38-37	.935
63-59	.96
64-61	.985
65-66	.935
68-69	.935
81-80	.935

* Off-nominal turns ratio, as determined by the actual transformer tap positions and the voltage bases. In the case of nominal turns ratio, this would equal 1.

TABLE F.15: Static Capacitor Data for the IEEE 118-Bus Test System

BUS NUMBER	SUSCEPTANCE PER UNIT*
5	-.4
17	0
34	.14
37	-.25
44	.1
45	.1
46	.1
48	.15
74	.12
79	.2
82	.2
83	.1
105	.2
107	.06
110	.06

* Susceptance in per unit on a 100,00-kva base

APPENDIX G

MAXIMUM MISMATCHES DURING LOAD FLOWS

TABLE G1: MAXIMUM PER UNIT MISMATCHES AFTER EACH ITERATION FOR THE 5-BUS SYSTEM

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-2.0	-1.9	-1.8	-1.7	-1.6	-1.5
1	P	0.196E-01	0.188E-01	0.181E-01	0.174E-01	0.167E-01	0.161E-01
	Q	0.410E-02	0.384E-02	0.360E-02	0.337E-02	0.317E-02	0.298E-02
	V	0.744E-04	0.639E-04	0.505E-04	0.381E-04	0.305E-04	0.191E-04
2	P	0.468E-04	0.368E-04	0.456E-04	0.327E-04	0.327E-04	0.246E-04
	Q	0.816E-04	0.776E-04	0.688E-04	0.669E-04	0.567E-04	0.581E-04
	V	0.286E-05	0.954E-06	0.286E-05	0.954E-06	0.286E-05	0.954E-06
		***	***	***	***	***	***

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-1.4	-1.3	-1.2	-1.1	-1.0	-0.9
1	P	0.154E-01	0.148E-01	0.141E-01	0.135E-01	0.129E-01	0.123E-01
	Q	0.282E-02	0.267E-02	0.254E-02	0.242E-02	0.233E-02	0.224E-02
	V	0.105E-04	0.286E-05	0.477E-05	0.105E-04	0.181E-04	0.229E-04
2	P	0.246E-04	0.254E-04	0.244E-04	0.232E-04	0.212E-04	0.121E-04
	Q	0.489E-04	0.426E-04	0.404E-04	0.326E-04	0.289E-04	0.316E-04
	V	0.954E-06	0.954E-06	0.954E-06	0.954E-06	0.954E-06	0.954E-06
		***	***	***	***	***	***

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-0.8	-0.7	-0.6	-0.5	-0.4	-0.3
1	P	0.117E-01	0.111E-01	0.105E-01	0.989E-02	0.931E-02	0.874E-02
	Q	0.217E-02	0.212E-02	0.208E-02	0.206E-02	0.205E-02	0.206E-02
	V	0.267E-04	0.315E-04	0.353E-04	0.391E-04	0.391E-04	0.420E-04
2	P	0.727E-05	0.989E-05	0.697E-05	0.972E-05	0.134E-04	0.381E-05
	Q	0.277E-04	0.314E-04	0.203E-04	0.239E-04	0.406E-04	0.126E-04
	V	0.954E-06	0.954E-06	0.954E-06	0.954E-06	0.954E-06	0.954E-06
		***	***	***	***	***	***

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G1 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-0.2	-0.1	0.0	0.1	0.2	0.3
1	P	0.817E-02	0.760E-02	0.704E-02	0.648E-02	0.593E-02	0.538E-02
	Q	0.208E-02	0.211E-02	0.217E-02	0.222E-02	0.229E-02	0.238E-02
	V	0.439E-04	0.429E-04	0.439E-04	0.420E-04	0.420E-04	0.391E-04
2	P	0.160E-04	0.572E-05	0.483E-05	0.381E-05	0.560E-05	0.250E-05
	Q	0.402E-04	0.103E-04	0.114E-04	0.125E-04	0.261E-04	0.951E-05
	V	0.954E-06	0.954E-06	0.954E-06	0.954E-06	0.954E-06	0.954E-06
		***	***	***	***	***	***

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		0.4	0.5	0.6	0.7	0.8	0.9
1	P	0.484E-02	0.429E-02	0.375E-02	0.321E-02	0.267E-02	0.233E-02
	Q	0.248E-02	0.259E-02	0.272E-02	0.286E-02	0.301E-02	0.317E-02
	V	0.391E-04	0.353E-04	0.343E-04	0.296E-04	0.267E-04	0.210E-04
2	P	0.340E-05	0.127E-04	0.709E-05	0.554E-05	0.834E-05	0.103E-04
	Q	0.993E-05	0.361E-04	0.305E-04	0.910E-05	0.298E-04	0.155E-04
	V	0.954E-06	0.954E-06	0.286E-05	0.954E-06	0.286E-05	0.286E-05
		***	***	***	***	***	***

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		1.0	1.1	1.2	1.3	1.4	1.5
1	P	0.240E-02	0.246E-02	0.250E-02	0.253E-02	0.255E-02	0.257E-02
	Q	0.336E-02	0.354E-02	0.375E-02	0.396E-02	0.419E-02	0.443E-02
	V	0.172E-04	0.114E-04	0.572E-05	0.954E-06	0.858E-05	0.153E-04
2	P	0.268E-05	0.536E-05	0.411E-05	0.501E-05	0.185E-05	0.435E-05
	Q	0.972E-05	0.201E-04	0.921E-05	0.258E-04	0.969E-05	0.260E-04
	V	0.954E-06	0.954E-06	0.954E-06	0.954E-06	0.954E-06	0.954E-06
		***	***	***	***	***	***

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G1 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		1.6	1.7	1.8	1.9	2.0	2.1
1	P	0.258E-02	0.256E-02	0.271E-02	0.326E-02	0.380E-02	0.435E-02
	Q	0.469E-02	0.495E-02	0.524E-02	0.553E-02	0.584E-02	0.615E-02
	V	0.229E-04	0.324E-04	0.391E-04	0.486E-04	0.591E-04	0.677E-04
2	P	0.548E-05	0.590E-05	0.346E-05	0.811E-05	0.596E-05	0.530E-05
	Q	0.102E-04	0.247E-04	0.969E-05	0.110E-04	0.248E-04	0.100E-04
	V	0.954E-06	0.954E-06	0.954E-06	0.954E-06	0.954E-06	0.954E-06
		***	***	***	***	***	***

TABLE G2: MAXIMUM PER UNIT MISMATCHES AFTER EACH ITERATION FOR THE 23-BUS SYSTEM

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-2.0	-1.9	-1.8	-1.7	-1.6	-1.5
1	P	0.310E+01	0.267E+01	0.229E+01	0.197E+01	0.171E+01	0.149E+01
	Q	0.383E+01	0.212E+01	0.134E+01	0.937E+00	0.703E+00	0.559E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.428E+00	0.340E+00	0.380E+00	0.281E+02	0.118E+01	0.295E+01
	Q	0.109E+01	0.735E+00	0.129E+01	0.678E+02	0.365E+01	0.511E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.436E-01	0.825E-01	0.474E+00	0.659E+01	0.195E+00	0.135E+01
	Q	0.888E-01	0.398E-01	0.134E+01	0.204E+01	0.156E+00	0.148E+01
	V	0.0	0.0	0.0	0.0	0.0	0.0
4	P	0.272E-03	0.907E-03	0.222E+01	0.295E+01	0.121E-02	0.462E+01
	Q	0.631E-04	0.588E-03	0.760E+01	0.114E+02	0.713E-03	0.496E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
5	P	0.763E-05	0.105E-04	0.808E+02	0.376E+01	0.153E-04	0.347E+01
	Q	0.188E-04	0.194E-04	0.133E+03	0.661E+01	0.303E-04	0.345E+02
	V	0.0	0.0	0.0	0.0	0.0	0.0
6	P	***	***	0.424E+03	0.360E+01	***	0.169E+01
	Q			0.107E+04	0.186E+02		0.224E+02
	V			0.0	0.0		0.0
7	P			0.522E+01	0.504E+02		0.120E+02
	Q			0.127E+02	0.128E+03		0.315E+02
	V			0.0	0.0		0.0
8	P			0.857E+04	0.145E+02		0.716E+02
	Q			0.350E+05	0.143E+02		0.285E+03
	V			0.0	0.0		0.0
9	P			0.302E+05	0.229E+04		0.304E+02
	Q			0.941E+05	0.617E+04		0.134E+03
	V			0.0	0.0		0.0
10	P			0.865E+05	0.209E+04		0.134E+04
	Q			0.273E+06	0.568E+04		0.134E+05
	V			0.0	0.0		0.0

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G2 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-1.4	-1.3	-1.2	-1.1	-1.0	-0.9
1	P	0.131E+01	0.114E+01	0.100E+01	0.877E+00	0.764E+00	0.662E+00
	Q	0.465E+00	0.401E+00	0.356E+00	0.323E+00	0.298E+00	0.279E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.393E+00	0.158E+00	0.784E-01	0.425E-01	0.242E-01	0.143E-01
	Q	0.567E-01	0.176E-01	0.748E-02	0.401E-02	0.280E-02	0.204E-02
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.973E-02	0.823E-03	0.102E-03	0.172E-04	0.787E-05	0.238E-04
	Q	0.286E-02	0.206E-03	0.361E-04	0.280E-04	0.357E-04	0.703E-04
	V	0.0	0.0	0.0	0.0	0.0	0.0
4	P	0.200E-04	0.105E-04	0.124E-04			***
	Q	0.314E-04	0.384E-04	0.209E-04			
	V	0.0	0.0	0.0			
		***	***	***			

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-0.8	-0.7	-0.6	-0.5	-0.4	-0.3
1	P	0.567E+00	0.480E+00	0.399E+00	0.322E+00	0.250E+00	0.215E+00
	Q	0.264E+00	0.252E+00	0.242E+00	0.233E+00	0.225E+00	0.218E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.867E-02	0.540E-02	0.346E-02	0.229E-02	0.159E-02	0.115E-02
	Q	0.136E-02	0.860E-03	0.511E-03	0.257E-03	0.129E-03	0.932E-04
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.210E-04	0.200E-04	0.858E-05	0.219E-04	0.181E-04	0.112E-04
	Q	0.225E-04	0.197E-04	0.148E-04	0.302E-04	0.376E-04	0.313E-04
	V	0.0	0.0	0.0	0.0	0.0	0.0
		***	***	***	***	***	***

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G2 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-0.2	-0.1	0.0	0.1	0.2	0.3
1	P	0.187E+00	0.161E+00	0.136E+00	0.112E+00	0.134E+00	0.194E+00
	Q	0.211E+00	0.204E+00	0.198E+00	0.192E+00	0.185E+00	0.178E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.890E-03	0.722E-03	0.627E-03	0.576E-03	0.550E-03	0.542E-03
	Q	0.124E-03	0.197E-03	0.275E-03	0.373E-03	0.490E-03	0.603E-03
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.153E-04	0.143E-04	0.134E-04	0.954E-05	0.954E-05	0.175E-04
	Q	0.347E-04	0.174E-04	0.465E-04	0.253E-04	0.263E-04	0.159E-04
	V	0.0	0.0	0.0	0.0	0.0	0.0
		***	***	***	***	***	***

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		0.4	0.5	0.6	0.7	0.8	0.9
1	P	0.253E+00	0.312E+00	0.371E+00	0.431E+00	0.492E+00	0.553E+00
	Q	0.171E+00	0.164E+00	0.157E+00	0.149E+00	0.140E+00	0.131E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.557E-03	0.583E-03	0.761E-03	0.981E-03	0.124E-02	0.155E-02
	Q	0.750E-03	0.899E-03	0.107E-02	0.124E-02	0.143E-02	0.161E-02
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.162E-04	0.172E-04	0.114E-04	0.143E-04	0.172E-04	0.153E-04
	Q	0.333E-04	0.251E-04	0.204E-04	0.245E-04	0.389E-04	0.213E-04
	V	0.0	0.0	0.0	0.0	0.0	0.0
		***	***	***	***	***	***

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G2 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES 1.0	1.1	1.2	1.3	1.4	1.5
1	P	0.616E+00	0.680E+00	0.746E+00	0.815E+00	0.886E+00	0.960E+00
	Q	0.122E+00	0.125E+00	0.137E+00	0.150E+00	0.163E+00	0.177E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.189E-02	0.229E-02	0.275E-02	0.327E-02	0.387E-02	0.457E-02
	Q	0.181E-02	0.201E-02	0.220E-02	0.237E-02	0.256E-02	0.274E-02
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.124E-04	0.172E-04	0.954E-05	0.124E-04	0.210E-04	0.210E-04
	Q	0.284E-04	0.315E-04	0.225E-04	0.249E-04	0.280E-04	0.179E-04
	V	0.0	0.0	0.0	0.0	0.0	0.0
		***	***	***	***	***	***

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES 1.6	1.7	1.8	1.9	2.0	2.1
1	P	0.104E+01	0.112E+01	0.120E+01	0.129E+01	0.139E+01	0.149E+01
	Q	0.192E+00	0.208E+00	0.224E+00	0.242E+00	0.261E+00	0.281E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.540E-02	0.637E-02	0.754E-02	0.896E-02	0.107E-01	0.130E-01
	Q	0.290E-02	0.304E-02	0.316E-02	0.326E-02	0.330E-02	0.327E-02
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.267E-04	0.124E-04	0.124E-04	0.181E-04	0.153E-04	0.191E-04
	Q	0.246E-04	0.308E-04	0.229E-04	0.452E-04	0.243E-04	0.285E-04
	V	0.0	0.0	0.0	0.0	0.0	0.0
		***	***	***	***	***	***

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G3: MAXIMUM PER UNIT MISMATCHES AFTER EACH ITERATION FOR THE IEEE 57-BUS SYSTEM

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-2.0	-1.9	-1.8	-1.7	-1.6	-1.5
1	P	0.171E+01	0.191E+01	0.221E+01	0.271E+01	0.364E+01	0.592E+01
	Q	0.323E+00	0.397E+00	0.543E+00	0.858E+00	0.167E+01	0.464E+01
	V	0.111E-01	0.141E-01	0.188E-01	0.273E-01	0.461E-01	0.108E+00
2	P	0.151E+01	0.281E+01	0.111E+02	0.930E+01	0.495E+01	0.550E+01
	Q	0.395E+00	0.175E+01	0.401E+02	0.169E+02	0.545E+01	0.691E+01
	V	0.240E-02	0.163E-01	0.486E+00	0.236E+00	0.755E-01	0.781E-01
3	P	0.418E+01	0.193E+01	0.654E+01	0.964E+01	0.109E+02	0.151E+02
	Q	0.227E+01	0.297E+01	0.491E+02	0.209E+02	0.237E+02	0.406E+02
	V	0.248E-01	0.328E-01	0.454E+00	0.236E+00	0.300E+00	0.483E+00
4	P	0.301E+01	0.189E+01	0.453E+01	0.120E+02	0.126E+02	0.175E+02
	Q	0.359E+01	0.276E-01	0.128E+03	0.322E+02	0.318E+02	0.517E+02
	V	0.407E-01	0.173E-03	0.850E+00	0.329E+00	0.334E+00	0.493E+00
5	P	0.908E+01	0.729E+01	0.494E+01	0.143E+02	0.148E+02	0.193E+02
	Q	0.207E+02	0.113E+02	0.102E+03	0.440E+02	0.431E+02	0.629E+02
	V	0.233E+00	0.169E+00	0.757E+00	0.419E+00	0.401E+00	0.512E+00
6	P	0.580E+01	0.699E+01	0.145E+02	0.166E+02	0.171E+02	0.212E+02
	Q	0.594E+02	0.135E+02	0.332E+03	0.573E+02	0.558E+02	0.754E+02
	V	0.278E+00	0.169E+00	0.229E+01	0.509E+00	0.480E+00	0.562E+00
7	P	0.418E+02	0.101E+02	0.108E+02	0.189E+02	0.195E+02	0.234E+02
	Q	0.438E+03	0.266E+02	0.256E+03	0.714E+02	0.700E+02	0.900E+02
	V	0.295E+01	0.307E+00	0.184E+01	0.596E+00	0.566E+00	0.637E+00
8	P	0.397E+02	0.120E+02	0.109E+04	0.213E+02	0.221E+02	0.259E+02
	Q	0.409E+03	0.367E+02	0.996E+04	0.865E+02	0.854E+02	0.106E+03
	V	0.325E+01	0.394E+00	0.600E+02	0.680E+00	0.652E+00	0.721E+00
9	P	0.362E+02	0.143E+02	0.375E+04	0.237E+02	0.246E+02	0.285E+02
	Q	0.115E+03	0.494E+02	0.340E+05	0.102E+03	0.102E+03	0.124E+03
	V	0.467E+00	0.489E+00	0.209E+03	0.765E+00	0.738E+00	0.805E+00
10	P	0.376E+02	0.164E+02	0.113E+03	0.262E+02	0.276E+02	0.312E+02
	Q	0.171E+03	0.625E+02	0.922E+03	0.119E+03	0.119E+03	0.142E+03
	V	0.652E+00	0.575E+00	0.516E+01	0.849E+00	0.822E+00	0.888E+00

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G3 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES -1.4	-1.3	-1.2	-1.1	-1.0	-0.9
1	P	0.184E+02	0.645E+01	0.402E+01	0.245E+01	0.170E+01	0.128E+01
	Q	0.354E+02	0.913E+02	0.810E+01	0.303E+01	0.165E+01	0.106E+01
	V	0.689E+00	0.152E+01	0.120E+00	0.420E-01	0.227E-01	0.153E-01
2	P	0.169E+02	0.103E+02	0.882E+01	0.323E+01	0.688E+00	0.257E+00
	Q	0.369E+02	0.838E+02	0.401E+02	0.214E+01	0.140E+00	0.317E-01
	V	0.375E+00	0.164E+01	0.554E+00	0.240E-01	0.125E-02	0.294E-03
3	P	0.159E+02	0.234E+04	0.753E+01	0.405E+01	0.296E-01	0.159E-02
	Q	0.406E+02	0.225E+05	0.398E+02	0.594E+01	0.514E-02	0.216E-03
	V	0.299E+00	0.175E+03	0.433E+00	0.658E-01	0.602E-05	0.954E-06
4	P	0.172E+02	0.200E+04	0.830E+01	0.755E+01	0.630E-04	0.402E-04
	Q	0.513E+02	0.187E+05	0.622E+02	0.240E+02	0.410E-04	0.304E-04
	V	0.368E+00	0.144E+03	0.537E+00	0.290E+00	0.954E-06	0.191E-05
5	P	0.193E+02	0.302E+05	0.760E+01	0.694E+01		***
	Q	0.665E+02	0.286E+06	0.653E+02	0.269E+02		
	V	0.473E+00	0.204E+04	0.535E+00	0.270E+00		
6	P	0.207E+02	0.764E+07	0.793E+01	0.844E+01		
	Q	0.825E+02	0.724E+08	0.858E+02	0.452E+02		
	V	0.566E+00	0.517E+06	0.666E+00	0.416E+00		
7	P	0.196E+02	0.326E+06	0.772E+01	0.832E+01		
	Q	0.983E+02	0.309E+07	0.934E+02	0.516E+02		
	V	0.647E+00	0.221E+05	0.710E+00	0.449E+00		
8	P	0.781E+04	0.146E+05	0.807E+01	0.895E+01		
	Q	0.400E+05	0.139E+06	0.113E+03	0.677E+02		
	V	0.746E+01	0.992E+03	0.826E+00	0.554E+00		
9	P	0.287E+03	0.112E+04	0.870E+01	0.898E+01		
	Q	0.180E+04	0.103E+05	0.124E+03	0.774E+02		
	V	0.213E+01	0.721E+02	0.876E+00	0.608E+00		
10	P	0.289E+03	0.313E+06	0.966E+01	0.925E+01		
	Q	0.187E+04	0.290E+07	0.143E+03	0.930E+02		
	V	0.108E+02	0.186E+05	0.976E+00	0.698E+00		

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G3 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-0.8	-0.7	-0.6	-0.5	-0.4	-0.3
1	P	0.100E+01	0.816E+00	0.679E+00	0.577E+00	0.498E+00	0.435E+00
	Q	0.749E+00	0.554E+00	0.420E+00	0.320E+00	0.240E+00	0.173E+00
	V	0.118E-01	0.987E-02	0.871E-02	0.795E-02	0.742E-02	0.702E-02
2	P	0.118E+00	0.605E-01	0.333E-01	0.192E-01	0.115E-01	0.709E-02
	Q	0.111E-01	0.484E-02	0.240E-02	0.128E-02	0.683E-03	0.325E-03
	V	0.117E-03	0.539E-04	0.251E-04	0.113E-04	0.376E-05	0.286E-05
3	P	0.149E-03	0.919E-04	0.310E-04	0.301E-04	0.292E-04	0.198E-04
	Q	0.318E-04	0.594E-04	0.341E-04	0.851E-04	0.355E-04	0.266E-04
	V	0.191E-05	0.191E-05	0.954E-06	0.107E-05	0.954E-06	0.954E-06
4	P	0.446E-04			***	***	***
	Q	0.420E-04					
	V	0.191E-05					
		***					

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-0.2	-0.1	0.0	0.1	0.2	0.3
1	P	0.386E+00	0.346E+00	0.314E+00	0.289E+00	0.269E+00	0.253E+00
	Q	0.113E+00	0.850E-01	0.722E-01	0.601E-01	0.947E-01	0.146E+00
	V	0.671E-02	0.644E-02	0.621E-02	0.598E-02	0.576E-02	0.554E-02
2	P	0.452E-02	0.299E-02	0.208E-02	0.156E-02	0.127E-02	0.111E-02
	Q	0.160E-03	0.961E-04	0.234E-03	0.416E-03	0.690E-03	0.115E-02
	V	0.304E-05	0.286E-05	0.286E-05	0.286E-05	0.221E-05	0.381E-05
3	P	0.423E-04	0.356E-04	0.299E-04	0.261E-04	0.267E-04	0.225E-04
	Q	0.625E-04	0.443E-04	0.309E-04	0.322E-04	0.287E-04	0.184E-04
	V	0.954E-06	0.954E-06	0.954E-06	0.954E-06	0.191E-05	0.954E-06
		***	***	***	***	***	***

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G3 (CONTINUED)

ITER. NO.	*	0.4	0.5	0.6	0.7	0.8	0.9
		MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
1	P	0.241E+00	0.233E+00	0.228E+00	0.226E+00	0.227E+00	0.232E+00
	Q	0.198E+00	0.252E+00	0.309E+00	0.369E+00	0.434E+00	0.503E+00
	V	0.535E-02	0.514E-02	0.490E-02	0.465E-02	0.437E-02	0.407E-02
2	P	0.101E-02	0.898E-03	0.698E-03	0.280E-03	0.575E-03	0.230E-02
	Q	0.196E-02	0.343E-02	0.606E-02	0.108E-01	0.196E-01	0.361E-01
	V	0.381E-05	0.286E-05	0.477E-05	0.322E-05	0.381E-05	0.376E-05
3	P	0.135E-04	0.232E-04	0.373E-04	0.193E-04	0.268E-04	0.293E-04
	Q	0.225E-04	0.379E-04	0.474E-04	0.461E-04	0.772E-04	0.440E-04
	V	0.954E-06	0.954E-06	0.954E-06	0.191E-05	0.191E-05	0.191E-05
		***	***	***	***	***	

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G3 (CONTINUED)

ITER. NO.	*	1.0	1.1	1.2	1.3	1.4	1.5
		MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
1	P	0.239E+00	0.250E+00	0.265E+00	0.283E+00	0.307E+00	0.335E+00
	Q	0.577E+00	0.657E+00	0.742E+00	0.834E+00	0.929E+00	0.103E+01
	V	0.381E-02	0.351E-02	0.319E-02	0.284E-02	0.245E-02	0.204E-02
2	P	0.580E-02	0.132E-01	0.299E-01	0.717E-01	0.181E+00	0.676E+01
	Q	0.681E-01	0.134E+00	0.277E+00	0.614E+00	0.122E+01	0.108E+03
	V	0.262E-05	0.286E-05	0.172E-04	0.658E-04	0.359E-03	0.239E-01
3	P	0.154E-04	0.226E-04	0.241E-03	0.648E-02	0.873E+02	0.906E+00
	Q	0.297E-04	0.257E-03	0.312E-02	0.834E-01	0.119E+04	0.161E+02
	V	0.191E-05	0.954E-06	0.954E-06	0.114E-04	0.123E+00	0.270E-01
4	P		0.314E-04	0.215E-04	0.379E-04	0.123E+02	0.274E+00
	Q		0.533E-04	0.321E-04	0.625E-04	0.177E+03	0.210E+01
	V		0.191E-05	0.191E-05	0.954E-06	0.194E-01	0.278E-02
5	P		***	***		0.206E+01	0.321E-01
	Q					0.260E+02	0.144E+00
	V					0.190E-02	0.286E-04
6	P					0.367E+00	0.625E-02
	Q					0.353E+01	0.446E-01
	V					0.104E-03	0.191E-05
7	P					0.252E-01	0.704E+01
	Q					0.291E+00	0.147E+02
	V					0.339E-04	0.596E-03
8	P					0.118E-03	0.106E+01
	Q					0.160E-02	0.222E+01
	V					0.954E-06	0.107E-03
9	P					0.525E-04	0.137E+00
	Q					0.459E-04	0.330E+00
	V					0.954E-06	0.655E-04
10	P					***	0.359E-01
	Q						0.496E-01
	V						0.200E-04

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G3 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		1.6	1.7	1.8	1.9	2.0	2.1
1	P	0.369E+00	0.410E+00	0.456E+00	0.502E+00	0.523E+00	0.527E+00
	Q	0.111E+01	0.117E+01	0.114E+01	0.891E+00	0.237E+00	0.301E+01
	V	0.159E-02	0.111E-02	0.707E-03	0.125E-02	0.200E-02	0.299E-02
2	P	0.582E+01	0.207E+02	0.892E+01	0.250E+01	0.115E-01	0.103E+00
	Q	0.723E+02	0.285E+03	0.950E+02	0.134E+02	0.192E-02	0.286E+00
	V	0.779E-02	0.513E-01	0.361E-02	0.746E-02	0.923E-04	0.519E-03
3	P	0.411E+00	0.215E+01	0.145E+01	0.705E+01	0.233E-04	0.154E+00
	Q	0.109E+02	0.445E+02	0.145E+02	0.126E+03	0.390E-04	0.351E+00
	V	0.401E-02	0.180E-01	0.316E-02	0.109E+01	0.191E-05	0.689E-04
4	P	0.592E-01	0.134E+01	0.444E-01	0.186E+02	***	0.254E-01
	Q	0.135E+01	0.683E+01	0.201E+01	0.244E+03		0.592E-01
	V	0.231E-03	0.673E-03	0.322E-02	0.215E+01		0.112E-04
5	P	0.139E-02	0.255E+00	0.192E-01	0.385E+01		0.394E+00
	Q	0.562E-01	0.729E+00	0.128E+00	0.541E+02		0.148E+01
	V	0.668E-05	0.751E-03	0.545E-04	0.399E+00		0.582E-04
6	P	0.280E-04	0.321E-02	0.475E-02	0.274E+01		0.129E+00
	Q	0.365E-04	0.187E-01	0.162E-01	0.122E+02		0.208E+00
	V	0.954E-06	0.519E-05	0.954E-06	0.286E-01		0.753E-04
7	P		0.655E-04	0.258E-03	0.353E+02		0.119E-01
	Q		0.691E-04	0.736E-03	0.103E+03		0.175E-01
	V		0.954E-06	0.191E-05	0.657E+00		0.572E-05
8	P		***	0.181E-04	0.639E+02		0.983E-04
	Q			0.261E-04	0.392E+03		0.699E-04
	V			0.954E-06	0.149E+01		0.191E-05
9	P			***	0.129E+02		
	Q				0.641E+02		
	V				0.170E+00		
10	P				0.985E+00		
	Q				0.212E+02		
	V				0.611E-01		

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G4: MAXIMUM PER UNIT MISMATCHES AFTER EACH ITERATION FOR THE IEEE 118-BUS SYSTEM

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-2.0	-1.9	-1.8	-1.7	-1.6	-1.5
1	P	0.126E+01	0.129E+01	0.136E+01	0.148E+01	0.168E+01	0.203E+01
	Q	0.281E+00	0.294E+00	0.317E+00	0.353E+00	0.413E+00	0.517E+00
	V	0.735E-01	0.791E-01	0.879E-01	0.101E+00	0.124E+00	0.170E+00
2	P	0.530E+00	0.523E+00	0.541E+00	0.585E+00	0.656E+00	0.724E+00
	Q	0.886E-01	0.881E-01	0.106E+00	0.135E+00	0.190E+00	0.300E+00
	V	0.146E-01	0.110E-01	0.832E-02	0.501E-02	0.313E-01	0.124E+00
3	P	0.127E-01	0.139E-01	0.184E-01	0.308E-01	0.718E-01	0.306E+00
	Q	0.264E-02	0.281E-02	0.375E-02	0.665E-02	0.178E-01	0.833E-01
	V	0.830E-04	0.772E-04	0.105E-03	0.200E-03	0.519E-03	0.139E-02
4	P	0.557E-04	0.127E-03	0.963E-04	0.155E-03	0.543E-04	0.288E-02
	Q	0.322E-04	0.279E-03	0.949E-04	0.299E-03	0.164E-03	0.717E-03
	V	0.954E-05	0.954E-06	0.229E-04	0.954E-06	0.954E-06	0.277E-04
5	P	***	0.328E-04	***	0.479E-04	0.543E-04	0.775E-04
	Q		0.429E-04		0.810E-04	0.412E-04	0.804E-04
	V		0.954E-06		0.954E-06	0.954E-06	0.954E-06
					***		***

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G4 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-1.4	-1.3	-1.2	-1.1	-1.0	-0.9
1	P	0.273E+01	0.440E+01	0.107E+02	0.166E+03	0.202E+02	0.264E+01
	Q	0.719E+00	0.121E+01	0.307E+01	0.492E+02	0.624E+01	0.132E+01
	V	0.293E+00	0.605E+00	0.189E+01	0.371E+02	0.574E+01	0.940E+00
2	P	0.157E+01	0.981E+02	0.738E+02	0.292E+03	0.483E+02	0.268E+01
	Q	0.959E+00	0.417E+02	0.216E+02	0.860E+02	0.118E+02	0.123E+01
	V	0.700E+00	0.285E+02	0.157E+02	0.656E+02	0.137E+02	0.450E+00
3	P	0.395E+02	0.605E+02	0.126E+03	0.422E+03	0.499E+02	0.347E+00
	Q	0.149E+02	0.363E+02	0.354E+02	0.122E+03	0.108E+02	0.290E+00
	V	0.777E+01	0.388E+02	0.271E+02	0.943E+02	0.143E+02	0.456E-01
4	P	0.456E+02	0.107E+03	0.162E+03	0.516E+03	0.582E+02	0.830E-02
	Q	0.158E+02	0.638E+02	0.438E+02	0.146E+03	0.117E+02	0.558E-02
	V	0.894E+01	0.738E+02	0.348E+02	0.115E+03	0.165E+02	0.658E-04
5	P	0.620E+02	0.859E+02	0.190E+03	0.599E+03	0.595E+02	0.368E-04
	Q	0.205E+02	0.539E+02	0.495E+02	0.167E+03	0.131E+02	0.358E-04
	V	0.123E+02	0.645E+02	0.406E+02	0.133E+03	0.166E+02	0.381E-05
6	P	0.792E+02	0.105E+03	0.214E+03	0.666E+03	0.637E+02	***
	Q	0.250E+02	0.648E+02	0.541E+02	0.183E+03	0.151E+02	
	V	0.158E+02	0.783E+02	0.455E+02	0.147E+03	0.177E+02	
7	P	0.963E+02	0.916E+02	0.236E+03	0.723E+03	0.656E+02	
	Q	0.293E+02	0.585E+02	0.579E+02	0.196E+03	0.165E+02	
	V	0.193E+02	0.726E+02	0.499E+02	0.159E+03	0.190E+02	
8	P	0.113E+03	0.102E+03	0.257E+03	0.767E+03	0.677E+02	
	Q	0.332E+02	0.658E+02	0.614E+02	0.205E+03	0.179E+02	
	V	0.227E+02	0.837E+02	0.540E+02	0.168E+03	0.204E+02	
9	P	0.129E+03	0.949E+02	0.277E+03	0.806E+03	0.697E+02	
	Q	0.368E+02	0.629E+02	0.645E+02	0.212E+03	0.193E+02	
	V	0.251E+02	0.818E+02	0.579E+02	0.176E+03	0.219E+02	
10	P	0.145E+03	0.988E+02	0.297E+03	0.840E+03	0.712E+02	
	Q	0.402E+02	0.665E+02	0.678E+02	0.219E+03	0.206E+02	
	V	0.293E+02	0.882E+02	0.619E+02	0.182E+03	0.232E+02	

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G4 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-0.8	-0.7	-0.6	-0.5	-0.4	-0.3
1	P	0.854E+00	0.470E+00	0.345E+00	0.308E+00	0.279E+00	0.256E+00
	Q	0.748E+00	0.517E+00	0.392E+00	0.312E+00	0.257E+00	0.216E+00
	V	0.347E+00	0.179E+00	0.123E+00	0.930E-01	0.759E-01	0.656E-01
2	P	0.257E+00	0.591E-01	0.220E-01	0.109E-01	0.590E-02	0.340E-02
	Q	0.148E+00	0.472E-01	0.182E-01	0.786E-02	0.364E-02	0.192E-02
	V	0.265E-01	0.511E-02	0.142E-02	0.489E-03	0.200E-03	0.973E-04
3	P	0.145E-02	0.114E-03	0.394E-04	0.487E-04	0.439E-04	0.365E-04
	Q	0.134E-02	0.217E-03	0.137E-03	0.955E-04	0.403E-04	0.960E-04
	V	0.315E-04	0.954E-06	0.954E-06	0.954E-06	0.954E-06	0.954E-06
4	P	0.203E-03	0.552E-04	0.322E-04			***
	Q	0.326E-03	0.748E-04	0.403E-04			
	V	0.954E-06	0.954E-06	0.954E-06			
5	P	0.699E-04		***			
	Q	0.410E-04					
	V	0.954E-06					
		***					

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G4 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-0.2	-0.1	0.0	0.1	0.2	0.3
1	P	0.238E+00	0.223E+00	0.211E+00	0.201E+00	0.192E+00	0.185E+00
	Q	0.184E+00	0.158E+00	0.136E+00	0.118E+00	0.102E+00	0.879E-01
	V	0.586E-01	0.537E-01	0.501E-01	0.474E-01	0.453E-01	0.436E-01
2	P	0.199E-02	0.123E-02	0.744E-03	0.478E-03	0.299E-03	0.173E-03
	Q	0.114E-02	0.674E-03	0.394E-03	0.239E-03	0.139E-03	0.754E-04
	V	0.553E-04	0.353E-04	0.219E-04	0.134E-04	0.900E-05	0.668E-05
3	P	0.918E-04	0.101E-03	0.282E-04	0.534E-04	0.410E-04	0.424E-04
	Q	0.211E-03	0.224E-03	0.816E-04	0.624E-04	0.477E-04	0.890E-04
	V	0.954E-06	0.954E-06	0.179E-06	0.954E-06	0.954E-06	0.954E-06
4	P	0.435E-04	0.316E-04	***		***	
	Q	0.296E-04	0.619E-04				
	V	0.954E-06	0.954E-06				
		***					

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		0.4	0.5	0.6	0.7	0.8	0.9
1	P	0.179E+00	0.173E+00	0.169E+00	0.165E+00	0.161E+00	0.158E+00
	Q	0.756E-01	0.644E-01	0.543E-01	0.450E-01	0.397E-01	0.423E-01
	V	0.422E-01	0.412E-01	0.402E-01	0.393E-01	0.386E-01	0.381E-01
2	P	0.239E-03	0.791E-04	0.381E-04	0.694E-04	0.988E-04	0.811E-04
	Q	0.428E-03	0.884E-04	0.674E-04	0.102E-03	0.121E-03	0.116E-03
	V	0.668E-05	0.572E-05	0.477E-05	0.423E-05	0.489E-05	0.596E-05
3	P	0.450E-04			0.330E-04	0.439E-04	0.112E-03
	Q	0.547E-04			0.326E-04	0.482E-04	0.218E-03
	V	0.954E-06			0.954E-06	0.954E-06	0.954E-06
4	P				***		0.432E-04
	Q						0.680E-04
	V						0.119E-06
							***

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G4 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		1.0	1.1	1.2	1.3	1.4	1.5
1	P	0.156E+00	0.154E+00	0.152E+00	0.159E+00	0.175E+00	0.191E+00
	Q	0.447E-01	0.468E-01	0.488E-01	0.507E-01	0.524E-01	0.541E-01
	V	0.375E-01	0.371E-01	0.367E-01	0.364E-01	0.361E-01	0.359E-01
2	P	0.131E-03	0.134E-03	0.197E-03	0.231E-03	0.276E-03	0.295E-03
	Q	0.681E-04	0.108E-03	0.825E-04	0.272E-03	0.123E-03	0.960E-04
	V	0.739E-05	0.978E-05	0.122E-04	0.145E-04	0.171E-04	0.197E-04
3	P	0.410E-04	0.410E-04	0.722E-04	0.280E-04	0.552E-04	0.378E-04
	Q	0.262E-04	0.754E-04	0.170E-03	0.890E-04	0.748E-04	0.420E-04
	V	0.954E-06	0.954E-06	0.954E-06	0.954E-06	0.954E-06	0.179E-06
4	P	***	***	0.988E-04			***
	Q			0.225E-03			
	V			0.954E-06			
5	P			0.457E-04			
	Q			0.683E-04			
	V			0.179E-06			
				***			

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G4 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		1.6	1.7	1.8	1.9	2.0	2.1
1	P	0.208E+00	0.227E+00	0.246E+00	0.266E+00	0.288E+00	0.311E+00
	Q	0.558E-01	0.574E-01	0.592E-01	0.610E-01	0.629E-01	0.676E-01
	V	0.358E-01	0.365E-01	0.381E-01	0.398E-01	0.416E-01	0.436E-01
2	P	0.340E-03	0.394E-03	0.472E-03	0.533E-03	0.567E-03	0.627E-03
	Q	0.298E-03	0.168E-03	0.937E-04	0.285E-03	0.109E-03	0.145E-03
	V	0.224E-04	0.252E-04	0.279E-04	0.310E-04	0.342E-04	0.377E-04
3	P	0.134E-03	0.440E-04	0.689E-04	0.149E-03	0.353E-04	0.305E-04
	Q	0.293E-03	0.410E-04	0.677E-04	0.250E-03	0.960E-04	0.745E-04
	V	0.954E-06	0.954E-06	0.179E-06	0.954E-06	0.179E-06	0.119E-06
4	P	0.391E-04		***	0.163E-03	***	***
	Q	0.339E-04			0.381E-03		
	V	0.954E-06			0.954E-06		
5	P				0.381E-04		
	Q				0.352E-04		
	V				0.954E-06		

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G5: MAXIMUM PER UNIT MISMATCHES AFTER EACH ITERATION FOR THE 5-BUS (NO VOLTAGE CONTROLLED BUSES) SYSTEM

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-2.0	-1.9	-1.8	-1.7	-1.6	-1.5
1	P	0.276E-01	0.265E-01	0.253E-01	0.242E-01	0.231E-01	0.221E-01
	Q	0.299E-01	0.284E-01	0.269E-01	0.255E-01	0.241E-01	0.227E-01
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.103E-04	0.148E-04	0.495E-05	0.120E-04	0.168E-04	0.431E-04
	Q	0.295E-04	0.430E-04	0.338E-04	0.432E-04	0.429E-04	0.966E-04
	V	0.0	0.0	0.0	0.0	0.0	0.0
							***

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-1.4	-1.3	-1.2	-1.1	-1.0	-0.9
1	P	0.210E-01	0.200E-01	0.189E-01	0.179E-01	0.169E-01	0.159E-01
	Q	0.213E-01	0.199E-01	0.186E-01	0.172E-01	0.159E-01	0.146E-01
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.437E-04	0.397E-04	0.278E-04	0.259E-04	0.227E-04	0.221E-04
	Q	0.926E-04	0.789E-04	0.764E-04	0.649E-04	0.637E-04	0.525E-04
	V	0.0	0.0	0.0	0.0	0.0	0.0
		***	***	***	***	***	***

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-0.8	-0.7	-0.6	-0.5	-0.4	-0.3
1	P	0.149E-01	0.139E-01	0.129E-01	0.120E-01	0.110E-01	0.100E-01
	Q	0.133E-01	0.120E-01	0.107E-01	0.944E-02	0.820E-02	0.689E-02
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.190E-04	0.162E-04	0.111E-04	0.119E-04	0.108E-04	0.727E-05
	Q	0.740E-04	0.404E-04	0.364E-04	0.286E-04	0.454E-04	0.282E-04
	V	0.0	0.0	0.0	0.0	0.0	0.0
		***	***	***	***	***	***

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G5 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-0.2	-0.1	0.0	0.1	0.2	0.3
1	P	0.905E-02	0.809E-02	0.712E-02	0.650E-02	0.699E-02	0.749E-02
	Q	0.565E-02	0.437E-02	0.313E-02	0.232E-02	0.250E-02	0.269E-02
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.882E-05	0.960E-05	0.751E-05	0.101E-04	0.108E-04	0.578E-05
	Q	0.200E-04	0.203E-04	0.333E-04	0.241E-04	0.240E-04	0.213E-04
	V	0.0	0.0	0.0	0.0	0.0	0.0
		***	***	***	***	***	***

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		0.4	0.5	0.6	0.7	0.8	0.9
1	P	0.803E-02	0.859E-02	0.918E-02	0.980E-02	0.105E-01	0.111E-01
	Q	0.291E-02	0.321E-02	0.449E-02	0.578E-02	0.708E-02	0.839E-02
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.916E-05	0.608E-05	0.566E-05	0.328E-05	0.381E-05	0.542E-05
	Q	0.274E-04	0.212E-04	0.205E-04	0.171E-04	0.172E-04	0.230E-04
	V	0.0	0.0	0.0	0.0	0.0	0.0
		***	***	***	***	***	***

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		1.0	1.1	1.2	1.3	1.4	1.5
1	P	0.118E-01	0.126E-01	0.133E-01	0.141E-01	0.150E-01	0.158E-01
	Q	0.966E-02	0.110E-01	0.123E-01	0.137E-01	0.150E-01	0.164E-01
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.751E-05	0.519E-05	0.882E-05	0.120E-04	0.187E-04	0.296E-04
	Q	0.244E-04	0.277E-04	0.185E-04	0.283E-04	0.200E-04	0.196E-04
	V	0.0	0.0	0.0	0.0	0.0	0.0
		***	***	***	***	***	***

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G5 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		1.6	1.7	1.8	1.9	2.0	2.1
1	P	0.167E-01	0.177E-01	0.186E-01	0.197E-01	0.207E-01	0.218E-01
	Q	0.178E-01	0.192E-01	0.206E-01	0.220E-01	0.235E-01	0.250E-01
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.277E-04	0.291E-04	0.340E-04	0.470E-04	0.449E-04	0.563E-04
	Q	0.268E-04	0.498E-04	0.330E-04	0.450E-04	0.466E-04	0.542E-04
	V	0.0	0.0	0.0	0.0	0.0	0.0
		***	***	***	***	***	***

TABLE G6: MAXIMUM PER UNIT MISMATCHES AFTER EACH ITERATION FOR THE 57-BUS (NO VOLTAGE CONTROLLED BUSES) SYSTEM

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-2.0	-1.9	-1.8	-1.7	-1.6	-1.5
1	P	0.168E+01	0.167E+01	0.169E+01	0.174E+01	0.185E+01	0.202E+01
	Q	0.807E+00	0.720E+00	0.671E+00	0.638E+00	0.623E+00	0.639E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.147E+01	0.951E+00	0.815E+00	0.807E+00	0.906E+00	0.117E+01
	Q	0.224E+00	0.119E+00	0.102E+00	0.160E+00	0.267E+00	0.513E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.114E+01	0.396E+00	0.172E+00	0.150E+00	0.216E+00	0.599E+00
	Q	0.157E+01	0.248E-01	0.107E-01	0.107E-01	0.168E-01	0.246E-01
	V	0.0	0.0	0.0	0.0	0.0	0.0
4	P	0.463E+00	0.122E-01	0.961E-03	0.625E-03	0.176E-02	0.344E-01
	Q	0.148E+00	0.116E-02	0.827E-04	0.629E-04	0.219E-03	0.626E-02
	V	0.0	0.0	0.0	0.0	0.0	0.0
5	P	0.262E-01	0.165E-04	0.176E-04	0.181E-04	0.133E-04	0.277E-04
	Q	0.501E-02	0.354E-04	0.446E-04	0.286E-04	0.365E-04	0.789E-04
	V	0.0	0.0	0.0	0.0	0.0	0.0
6	P	0.249E-04	***	***	***	***	
	Q	0.501E-04					
	V	0.0					

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G6 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-1.4	-1.3	-1.2	-1.1	-1.0	-0.9
1	P	0.229E+01	0.274E+01	0.361E+01	0.569E+01	0.155E+02	0.420E+02
	Q	0.712E+00	0.916E+00	0.149E+01	0.355E+01	0.198E+02	0.294E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.184E+01	0.412E+01	0.209E+02	0.640E+01	0.154E+02	0.428E+02
	Q	0.134E+01	0.911E+01	0.375E+02	0.711E+01	0.438E+02	0.250E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.397E+01	0.219E+02	0.156E+02	0.406E+01	0.262E+01	0.370E+04
	Q	0.120E+02	0.117E+03	0.505E+02	0.709E+02	0.743E+01	0.216E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
4	P	0.150E+01	0.261E+01	0.424E+01	0.112E+02	0.246E+01	0.317E+03
	Q	0.175E+02	0.125E+02	0.117E+02	0.912E+02	0.123E+01	0.135E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
5	P	0.139E+01	0.127E+01	0.496E+01	0.463E+02	0.924E+00	0.297E+03
	Q	0.506E+00	0.290E+01	0.253E+02	0.329E+03	0.621E+00	0.123E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
6	P	0.339E+00	0.268E+02	0.195E+01	0.297E+03	0.131E+00	0.110E+04
	Q	0.124E+00	0.226E+02	0.265E+02	0.691E+03	0.475E-02	0.458E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
7	P	0.815E-02	0.126E+02	0.104E+02	0.692E+03	0.301E-03	0.974E+03
	Q	0.115E-02	0.147E+02	0.126E+02	0.167E+04	0.595E-04	0.398E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
8	P	0.146E-04	0.242E+02	0.871E+01	0.233E+04	0.182E-04	0.114E+03
	Q	0.339E-04	0.249E+02	0.162E+02	0.584E+04	0.366E-04	0.442E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0
9	P	***	0.438E+02	0.379E+01	0.190E+03	***	0.214E+02
	Q		0.154E+03	0.855E+01	0.560E+03		0.640E+02
	V		0.0	0.0	0.0		0.0
10	P		0.122E+04	0.205E+01	0.139E+06		0.625E+02
	Q		0.153E+04	0.107E+02	0.387E+06		0.123E+03
	V		0.0	0.0	0.0		0.0

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G6 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-0.8	-0.7	-0.6	-0.5	-0.4	-0.3
1	P	0.313E+01	0.214E+01	0.152E+01	0.115E+01	0.918E+00	0.758E+00
	Q	0.108E+02	0.376E+01	0.201E+01	0.128E+01	0.882E+00	0.620E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.860E+02	0.114E+03	0.101E+01	0.316E+00	0.136E+00	0.677E-01
	Q	0.260E+03	0.329E+03	0.224E+00	0.352E-01	0.101E-01	0.492E-02
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.675E+01	0.123E+03	0.694E-01	0.287E-02	0.240E-03	0.281E-04
	Q	0.190E+02	0.353E+03	0.403E-01	0.138E-02	0.119E-03	0.362E-04
	V	0.0	0.0	0.0	0.0	0.0	0.0
4	P	0.508E+00	0.303E+03	0.458E-04	0.127E-04	0.150E-04	
	Q	0.146E+01	0.187E+04	0.356E-04	0.518E-04	0.369E-04	
	V	0.0	0.0	0.0	0.0	0.0	
5	P	0.137E-01	0.340E+03		***	***	
	Q	0.242E-01	0.180E+04				
	V	0.0	0.0				
6	P	0.135E-04	0.356E+04				
	Q	0.339E-04	0.141E+05				
	V	0.0	0.0				
7	P		0.293E+04				
	Q		0.126E+05				
	V		0.0				
8	P		0.478E+07				
	Q		0.224E+08				
	V		0.0				
9	P		0.849E+07				
	Q		0.397E+08				
	V		0.0				
10	P		0.202E+07				
	Q		0.940E+07				
	V		0.0				

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G6 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES -0.2	-0.1	0.0	0.1	0.2	0.3
1	P	0.645E+00	0.565E+00	0.507E+00	0.466E+00	0.438E+00	0.421E+00
	Q	0.427E+00	0.270E+00	0.134E+00	0.554E-01	0.108E+00	0.223E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.368E-01	0.216E-01	0.140E-01	0.106E-01	0.958E-02	0.977E-02
	Q	0.325E-02	0.230E-02	0.287E-02	0.532E-02	0.106E-01	0.240E-01
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.220E-04	0.183E-04	0.213E-04	0.305E-04	0.184E-04	0.244E-04
	Q	0.653E-04	0.508E-04	0.526E-04	0.357E-04	0.446E-04	0.499E-04
	V	0.0	0.0	0.0	0.0	0.0	0.0
				***	***	***	

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G6 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES 0.4	0.5	0.6	0.7	0.8	0.9
1	P	0.413E+00	0.411E+00	0.416E+00	0.512E+00	0.625E+00	0.761E+00
	Q	0.334E+00	0.443E+00	0.547E+00	0.638E+00	0.703E+00	0.713E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.942E-02	0.467E-02	0.226E-01	0.213E+00	0.258E+02	0.412E+02
	Q	0.600E-01	0.163E+00	0.461E+00	0.434E+00	0.184E+03	0.298E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.154E-04	0.174E-03	0.152E-01	0.536E+02	0.392E+01	0.562E+01
	Q	0.369E-04	0.999E-03	0.792E-01	0.541E+02	0.248E+02	0.410E+02
	V	0.0	0.0	0.0	0.0	0.0	0.0
4	P		0.153E-04	0.629E-04	0.720E+01	0.586E+03	0.722E+00
	Q		0.473E-04	0.157E-03	0.723E+01	0.222E+04	0.544E+01
	V		0.0	0.0	0.0	0.0	0.0
5	P		***	0.125E-04	0.101E+01	0.246E+04	0.806E-01
	Q			0.217E-04	0.102E+01	0.110E+05	0.568E+00
	V			0.0	0.0	0.0	0.0
6	P			***	0.206E+00	0.136E+06	0.371E-02
	Q				0.211E+00	0.525E+06	0.146E-01
	V				0.0	0.0	0.0
7	P				0.913E+00	0.481E+05	0.324E-04
	Q				0.108E+01	0.186E+06	0.827E-04
	V				0.0	0.0	0.0
8	P				0.347E+00	0.816E+04	***
	Q				0.195E+01	0.313E+05	
	V				0.0	0.0	
9	P				0.969E-01	0.163E+04	
	Q				0.370E+00	0.708E+04	
	V				0.0	0.0	
10	P				0.144E+00	0.330E+03	
	Q				0.142E+00	0.116E+04	
	V				0.0	0.0	

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G6 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		1.0	1.1	1.2	1.3	1.4	1.5
1	P	0.928E+00	0.114E+01	0.143E+01	0.184E+01	0.244E+01	0.800E+01
	Q	0.608E+00	0.494E+00	0.701E+00	0.324E+01	0.110E+02	0.459E+02
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.125E+00	0.394E+00	0.920E+00	0.106E+04	0.158E+03	0.511E+02
	Q	0.501E+00	0.357E+00	0.736E+00	0.149E+04	0.931E+02	0.671E+02
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.112E+03	0.178E+01	0.807E+01	0.157E+03	0.916E+02	0.108E+03
	Q	0.185E+03	0.301E+01	0.316E+02	0.219E+03	0.574E+02	0.235E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0
4	P	0.157E+02	0.344E+00	0.119E+01	0.242E+02	0.830E+05	0.424E+03
	Q	0.260E+02	0.548E+00	0.497E+01	0.325E+02	0.158E+06	0.725E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0
5	P	0.216E+01	0.448E+02	0.141E+01	0.438E+01	0.138E+05	0.302E+03
	Q	0.365E+01	0.355E+02	0.357E+01	0.459E+01	0.264E+05	0.580E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0
6	P	0.260E+00	0.352E+02	0.425E+02	0.181E+04	0.469E+06	0.205E+04
	Q	0.485E+00	0.523E+02	0.579E+02	0.206E+04	0.891E+06	0.763E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
7	P	0.462E+00	0.424E+05	0.587E+02	0.126E+07	0.339E+07	0.119E+05
	Q	0.884E+00	0.191E+06	0.842E+02	0.240E+07	0.646E+07	0.342E+05
	V	0.0	0.0	0.0	0.0	0.0	0.0
8	P	0.398E+02	0.763E+04	0.317E+04	0.546E+06	0.631E+06	0.285E+04
	Q	0.750E+02	0.330E+05	0.707E+04	0.983E+06	0.120E+07	0.849E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
9	P	0.557E+01	0.124E+04	0.207E+04	0.196E+06	0.101E+06	0.466E+03
	Q	0.106E+02	0.612E+04	0.424E+04	0.330E+06	0.191E+06	0.136E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
10	P	0.762E+00	0.324E+03	0.365E+03	0.221E+07	0.173E+05	0.703E+02
	Q	0.157E+01	0.161E+04	0.777E+03	0.555E+07	0.330E+05	0.209E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G6 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		1.6	1.7	1.8	1.9	2.0	2.1
	P	0.116E+03	0.127E+03	0.194E+02	0.874E+01	0.540E+01	0.387E+01
1	Q	0.602E+03	0.630E+03	0.933E+02	0.414E+02	0.253E+02	0.180E+02
	V	0.0	0.0	0.0	0.0	0.0	0.0
	P	0.113E+02	0.216E+02	0.380E+01	0.216E+01	0.211E+01	0.781E+01
2	Q	0.912E+02	0.979E+02	0.143E+02	0.623E+01	0.379E+01	0.845E+01
	V	0.0	0.0	0.0	0.0	0.0	0.0
	P	0.269E+03	0.203E+01	0.678E+00	0.301E+00	0.286E+00	0.115E+01
3	Q	0.963E+03	0.162E+02	0.196E+01	0.783E+00	0.411E+00	0.287E+02
	V	0.0	0.0	0.0	0.0	0.0	0.0
	P	0.469E+03	0.108E+00	0.441E-01	0.971E-02	0.524E-02	0.338E+01
4	Q	0.111E+04	0.230E+01	0.178E+00	0.405E-01	0.147E-01	0.679E+01
	V	0.0	0.0	0.0	0.0	0.0	0.0
	P	0.985E+02	0.223E-01	0.360E-03	0.193E-04	0.296E-04	0.117E+01
5	Q	0.298E+03	0.200E+00	0.139E-02	0.356E-04	0.909E-04	0.439E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
	P	0.563E+20	0.374E-03	0.276E-04		***	0.820E-01
6	Q	0.217E+21	0.159E-02	0.384E-04			0.292E-02
	V	0.0	0.0	0.0			0.0
	P	0.133E+20	0.210E-04	***			0.711E-04
7	Q	0.513E+20	0.297E-04				0.573E-04
	V	0.0	0.0				0.0
	P	0.405E+19	***				
8	Q	0.156E+20					
	V	0.0					
	P	0.950E+20					
9	Q	0.366E+21					
	V	0.0					
	P	0.362E+21					
10	Q	0.140E+22					
	V	0.0					

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G7: MAXIMUM PER UNIT MISMATCHES AFTER EACH ITERATION FOR THE 118-BUS (NO VOLTAGE CONTROLLED BUSES) SYSTEM

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-2.0	-1.9	-1.8	-1.7	-1.6	-1.5
1	P	0.390E+01	0.392E+01	0.382E+01	0.434E+01	0.619E+01	0.215E+02
	Q	0.433E+01	0.153E+01	0.183E+01	0.223E+01	0.446E+01	0.276E+02
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.662E+01	0.464E+04	0.698E+02	0.251E+04	0.409E+02	0.310E+03
	Q	0.997E+01	0.417E+04	0.502E+02	0.650E+04	0.843E+02	0.152E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.309E+02	0.329E+04	0.163E+03	0.351E+05	0.182E+03	0.322E+03
	Q	0.718E+02	0.386E+04	0.379E+03	0.955E+05	0.224E+03	0.938E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0
4	P	0.548E+02	0.639E+03	0.221E+03	0.249E+09	0.136E+02	0.258E+03
	Q	0.614E+02	0.351E+04	0.647E+03	0.135E+10	0.464E+02	0.990E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0
5	P	0.256E+03	0.283E+05	0.211E+05	0.207E+09	0.230E+03	0.214E+03
	Q	0.110E+04	0.417E+05	0.472E+05	0.113E+10	0.556E+03	0.806E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0
6	P	0.464E+06	0.257E+05	0.354E+04	0.105E+10	0.653E+03	0.123E+04
	Q	0.139E+07	0.362E+05	0.198E+05	0.564E+10	0.251E+04	0.248E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
7	P	0.334E+06	0.274E+05	0.628E+04	0.907E+10	0.251E+03	0.806E+04
	Q	0.100E+07	0.387E+05	0.347E+05	0.486E+11	0.132E+04	0.167E+05
	V	0.0	0.0	0.0	0.0	0.0	0.0
8	P	0.646E+03	0.199E+05	0.475E+04	0.139E+09	0.693E+03	0.164E+05
	Q	0.193E+04	0.384E+05	0.262E+05	0.747E+09	0.338E+04	0.663E+05
	V	0.0	0.0	0.0	0.0	0.0	0.0
9	P	0.103E+06	0.236E+07	0.145E+06	0.113E+09	0.152E+04	0.324E+05
	Q	0.339E+06	0.588E+07	0.793E+06	0.604E+09	0.539E+04	0.134E+06
	V	0.0	0.0	0.0	0.0	0.0	0.0
10	P	0.453E+07	0.205E+07	0.137E+06	0.890E+11	0.254E+05	0.873E+05
	Q	0.151E+08	0.512E+07	0.750E+06	0.140E+13	0.745E+05	0.443E+06
	V	0.0	0.0	0.0	0.0	0.0	0.0

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G7 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-1.4	-1.3	-1.2	-1.1	-1.0	-0.9
1	P	0.110E+02	0.190E+01	0.137E+01	0.213E+01	0.486E+01	0.275E+03
	Q	0.185E+02	0.517E+01	0.686E+01	0.118E+02	0.408E+02	0.153E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.154E+03	0.197E+01	0.317E+02	0.905E+02	0.117E+03	0.854E+03
	Q	0.124E+03	0.417E+01	0.434E+02	0.680E+02	0.975E+02	0.186E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.256E+02	0.150E+01	0.811E+01	0.437E+02	0.239E+03	0.608E+04
	Q	0.171E+02	0.516E+01	0.406E+02	0.374E+02	0.554E+03	0.203E+05
	V	0.0	0.0	0.0	0.0	0.0	0.0
4	P	0.534E+02	0.353E+01	0.355E+02	0.312E+02	0.526E+02	0.520E+04
	Q	0.261E+02	0.701E+01	0.895E+02	0.229E+03	0.112E+03	0.787E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
5	P	0.272E+05	0.166E+02	0.420E+03	0.259E+04	0.180E+03	0.718E+04
	Q	0.275E+05	0.284E+02	0.996E+03	0.484E+04	0.127E+03	0.887E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
6	P	0.223E+05	0.217E+03	0.176E+07	0.219E+05	0.576E+02	0.762E+04
	Q	0.221E+05	0.248E+03	0.468E+07	0.426E+05	0.203E+03	0.943E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
7	P	0.876E+05	0.340E+02	0.191E+07	0.559E+05	0.167E+03	0.365E+06
	Q	0.150E+06	0.351E+02	0.514E+07	0.109E+06	0.361E+03	0.507E+06
	V	0.0	0.0	0.0	0.0	0.0	0.0
8	P	0.878E+05	0.251E+05	0.302E+07	0.470E+04	0.491E+03	0.763E+06
	Q	0.140E+06	0.431E+05	0.812E+07	0.911E+04	0.119E+04	0.100E+07
	V	0.0	0.0	0.0	0.0	0.0	0.0
9	P	0.108E+06	0.202E+05	0.218E+06	0.105E+04	0.202E+04	0.716E+06
	Q	0.154E+06	0.347E+05	0.585E+06	0.205E+04	0.670E+04	0.941E+06
	V	0.0	0.0	0.0	0.0	0.0	0.0
10	P	0.101E+06	0.274E+07	0.203E+05	0.246E+04	0.394E+06	0.214E+07
	Q	0.139E+06	0.485E+08	0.466E+05	0.484E+04	0.999E+06	0.288E+07
	V	0.0	0.0	0.0	0.0	0.0	0.0

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G7 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-0.3	-0.7	-0.6	-0.5	-0.4	-0.3
1	P	0.258E+03	0.140E+01	0.228E+01	0.233E+01	0.207E+01	0.171E+01
	Q	0.337E+03	0.539E+01	0.184E+01	0.110E+01	0.789E+00	0.587E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.416E+04	0.171E+03	0.138E+01	0.399E+00	0.344E+00	0.134E+00
	Q	0.404E+04	0.410E+03	0.253E+01	0.307E+01	0.607E+00	0.203E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.775E+03	0.164E+02	0.555E+00	0.416E+00	0.154E-02	0.135E-03
	Q	0.146E+04	0.371E+02	0.336E+00	0.363E-01	0.225E-02	0.730E-03
	V	0.0	0.0	0.0	0.0	0.0	0.0
4	P	0.112E+04	0.640E+01	0.212E-02	0.877E-02	0.303E-03	0.328E-03
	Q	0.270E+04	0.349E+02	0.126E-02	0.497E-02	0.548E-03	0.547E-03
	V	0.0	0.0	0.0	0.0	0.0	0.0
5	P	0.158E+04	0.266E+02	0.314E-03	0.152E+00	0.352E-03	0.302E-03
	Q	0.340E+04	0.466E+02	0.571E-03	0.266E+00	0.587E-03	0.521E-03
	V	0.0	0.0	0.0	0.0	0.0	0.0
6	P	0.769E+03	0.343E+02	0.323E-03	0.138E-01	0.361E-03	0.295E-03
	Q	0.260E+04	0.490E+02	0.543E-03	0.242E-01	0.635E-03	0.528E-03
	V	0.0	0.0	0.0	0.0	0.0	0.0
7	P	0.290E+04	0.548E+03	0.639E-03	0.133E-02	0.561E-03	0.357E-03
	Q	0.107E+05	0.806E+03	0.116E-02	0.232E-02	0.956E-03	0.639E-03
	V	0.0	0.0	0.0	0.0	0.0	0.0
8	P	0.204E+05	0.143E+03	0.871E-03	0.317E-03	0.126E-02	0.818E-03
	Q	0.341E+05	0.215E+03	0.145E-02	0.557E-03	0.220E-02	0.140E-02
	V	0.0	0.0	0.0	0.0	0.0	0.0
9	P	0.156E+07	0.552E+04	0.412E-03	0.844E-03	0.288E-03	0.363E-03
	Q	0.355E+07	0.829E+04	0.760E-03	0.146E-02	0.519E-03	0.633E-03
	V	0.0	0.0	0.0	0.0	0.0	0.0
10	P	0.122E+06	0.671E+03	0.383E-02	0.393E-03	0.180E-02	0.151E-02
	Q	0.278E+06	0.100E+04	0.669E-02	0.711E-03	0.315E-02	0.267E-02
	V	0.0	0.0	0.0	0.0	0.0	0.0

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G7 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-0.2	-0.1	0.0	0.1	0.2	0.3
1	P	0.126E+01	0.724E+00	0.483E+00	0.714E+00	0.169E+01	0.292E+01
	Q	0.409E+00	0.306E+00	0.211E+00	0.219E+00	0.488E+00	0.819E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.343E-01	0.659E-02	0.240E-02	0.406E-02	0.238E-01	0.829E-01
	Q	0.474E-01	0.890E-02	0.217E-02	0.479E-02	0.257E-01	0.835E-01
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.195E-01	0.817E-03	0.300E-03	0.491E-03	0.879E-03	0.152E-02
	Q	0.327E-01	0.143E-02	0.549E-03	0.908E-03	0.153E-02	0.260E-02
	V	0.0	0.0	0.0	0.0	0.0	0.0
4	P	0.205E-02	0.340E-03	0.737E-03	0.789E-03	0.334E-03	0.281E-03
	Q	0.357E-02	0.608E-03	0.124E-02	0.135E-02	0.558E-03	0.546E-03
	V	0.0	0.0	0.0	0.0	0.0	0.0
5	P	0.322E-03	0.138E-02	0.350E-03	0.375E-03	0.120E-02	0.315E-02
	Q	0.578E-03	0.242E-02	0.634E-03	0.643E-03	0.211E-02	0.545E-02
	V	0.0	0.0	0.0	0.0	0.0	0.0
6	P	0.666E-03	0.271E-03	0.313E-01	0.571E-02	0.289E-03	0.462E-03
	Q	0.112E-02	0.512E-03	0.547E-01	0.999E-02	0.517E-03	0.843E-03
	V	0.0	0.0	0.0	0.0	0.0	0.0
7	P	0.491E-03	0.299E-03	0.355E-02	0.754E-03	0.136E-02	0.720E-03
	Q	0.918E-03	0.487E-03	0.623E-02	0.131E-02	0.239E-02	0.126E-02
	V	0.0	0.0	0.0	0.0	0.0	0.0
8	P	0.167E-02	0.568E-03	0.491E-03	0.397E-03	0.312E-03	0.307E-03
	Q	0.290E-02	0.979E-03	0.834E-03	0.667E-03	0.530E-03	0.532E-03
	V	0.0	0.0	0.0	0.0	0.0	0.0
9	P	0.305E-03	0.707E-03	0.819E-03	0.831E-01	0.951E-03	0.538E-02
	Q	0.509E-03	0.124E-02	0.145E-02	0.145E+00	0.168E-02	0.940E-02
	V	0.0	0.0	0.0	0.0	0.0	0.0
10	P	0.296E-03	0.428E-03	0.316E-03	0.963E-02	0.303E-03	0.738E-03
	Q	0.542E-03	0.754E-03	0.578E-03	0.168E-01	0.491E-03	0.132E-02
	V	0.0	0.0	0.0	0.0	0.0	0.0

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G7 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		0.4	0.5	0.6	0.7	0.8	0.9
1	P	0.449E+01	0.652E+01	0.915E+01	0.126E+02	0.171E+02	0.230E+02
	Q	0.122E+01	0.171E+01	0.232E+01	0.308E+01	0.403E+01	0.521E+01
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.207E+00	0.428E+00	0.762E+00	0.116E+01	0.143E+01	0.143E+01
	Q	0.203E+00	0.431E+00	0.853E+00	0.154E+01	0.221E+01	0.183E+01
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.255E-02	0.440E-02	0.703E-02	0.534E-01	0.347E+00	0.252E+03
	Q	0.442E-02	0.765E-02	0.166E-01	0.150E+00	0.342E+00	0.302E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0
4	P	0.432E-03	0.655E-03	0.113E-02	0.212E-02	0.413E+00	0.342E+02
	Q	0.752E-03	0.116E-02	0.197E-02	0.387E-02	0.265E+00	0.413E+02
	V	0.0	0.0	0.0	0.0	0.0	0.0
5	P	0.179E-02	0.363E-03	0.297E-03	0.406E-03	0.285E-01	0.807E+04
	Q	0.316E-02	0.590E-03	0.516E-03	0.689E-03	0.246E-01	0.235E+05
	V	0.0	0.0	0.0	0.0	0.0	0.0
6	P	0.367E-03	0.950E-02	0.269E-01	0.129E-02	0.276E-03	0.119E+04
	Q	0.618E-03	0.166E-01	0.470E-01	0.228E-02	0.815E-03	0.340E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
7	P	0.747E+01	0.132E-02	0.359E-02	0.328E-03	0.811E-04	0.176E+03
	Q	0.131E+02	0.229E-02	0.629E-02	0.569E-03	0.365E-03	0.470E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0
8	P	0.930E+00	0.305E-03	0.568E-03	0.767E-01	0.133E-03	0.468E+04
	Q	0.163E+01	0.537E-03	0.101E-02	0.134E+00	0.666E-03	0.661E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
9	P	0.116E+00	0.761E-02	0.442E-03	0.103E-01	0.204E-03	0.151E+04
	Q	0.203E+00	0.133E-01	0.784E-03	0.180E-01	0.402E-03	0.236E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
10	P	0.145E-01	0.109E-02	0.964E-03	0.145E-02	0.255E-03	0.420E+03
	Q	0.254E-01	0.187E-02	0.166E-02	0.254E-02	0.487E-03	0.597E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G7 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		1.0	1.1	1.2	1.3	1.4	1.5
1	P	0.304E+02	0.394E+02	0.495E+02	0.594E+02	0.669E+02	0.694E+02
	Q	0.667E+01	0.840E+01	0.104E+02	0.125E+02	0.147E+02	0.169E+02
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.169E+01	0.228E+01	0.255E+01	0.335E+01	0.432E+01	0.142E+02
	Q	0.648E+00	0.246E+01	0.469E+01	0.383E+01	0.299E+01	0.438E+01
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.163E+01	0.151E+05	0.516E+00	0.365E+00	0.939E+00	0.129E+04
	Q	0.867E+00	0.106E+05	0.190E+01	0.186E+00	0.423E+00	0.937E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0
4	P	0.130E+04	0.216E+04	0.701E+00	0.354E-02	0.147E+00	0.214E+03
	Q	0.810E+03	0.151E+04	0.368E+00	0.681E-02	0.116E+00	0.150E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0
5	P	0.182E+03	0.309E+03	0.428E-01	0.617E-03	0.347E-03	0.326E+02
	Q	0.114E+03	0.214E+03	0.669E-01	0.109E-02	0.644E-03	0.246E+02
	V	0.0	0.0	0.0	0.0	0.0	0.0
6	P	0.251E+02	0.432E+02	0.450E-03	0.331E-03	0.116E-02	0.355E+01
	Q	0.162E+02	0.311E+02	0.809E-03	0.555E-03	0.205E-02	0.314E+01
	V	0.0	0.0	0.0	0.0	0.0	0.0
7	P	0.314E+01	0.558E+01	0.670E-03	0.340E-02	0.319E-03	0.105E+00
	Q	0.223E+01	0.443E+01	0.114E-02	0.596E-02	0.538E-03	0.632E+00
	V	0.0	0.0	0.0	0.0	0.0	0.0
8	P	0.212E+00	0.479E+00	0.319E-03	0.591E-03	0.661E-02	0.224E-02
	Q	0.186E+00	0.457E+00	0.578E-03	0.106E-02	0.115E-01	0.330E-02
	V	0.0	0.0	0.0	0.0	0.0	0.0
9	P	0.678E-03	0.482E-02	0.524E-02	0.324E-03	0.108E-02	0.314E-03
	Q	0.664E-03	0.558E-02	0.919E-02	0.542E-03	0.183E-02	0.558E-03
	V	0.0	0.0	0.0	0.0	0.0	0.0
10	P	0.258E-03	0.213E-03	0.854E-03	0.359E-02	0.297E-03	0.227E-02
	Q	0.917E-03	0.823E-03	0.146E-02	0.626E-02	0.543E-03	0.396E-02
	V	0.0	0.0	0.0	0.0	0.0	0.0

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G7 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		1.6	1.7	1.8	1.9	2.0	2.1
1	P	0.653E+02	0.542E+02	0.362E+02	0.220E+02	0.236E+04	0.152E+02
	Q	0.194E+02	0.233E+02	0.314E+02	0.537E+02	0.649E+03	0.107E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0
2	P	0.220E+03	0.456E+02	0.417E+02	0.809E+02	0.195E+07	0.182E+02
	Q	0.382E+02	0.112E+02	0.137E+02	0.277E+02	0.799E+06	0.771E+02
	V	0.0	0.0	0.0	0.0	0.0	0.0
3	P	0.131E+03	0.573E+03	0.171E+02	0.147E+02	0.318E+06	0.118E+02
	Q	0.303E+02	0.173E+03	0.703E+01	0.774E+01	0.130E+06	0.161E+02
	V	0.0	0.0	0.0	0.0	0.0	0.0
4	P	0.518E+02	0.427E+04	0.107E+02	0.444E+01	0.508E+05	0.556E+02
	Q	0.125E+02	0.121E+04	0.266E+01	0.211E+01	0.207E+05	0.677E+02
	V	0.0	0.0	0.0	0.0	0.0	0.0
5	P	0.761E+02	0.790E+03	0.109E+01	0.169E+01	0.993E+04	0.271E+03
	Q	0.187E+02	0.240E+03	0.696E+00	0.274E+01	0.384E+04	0.602E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0
6	P	0.230E+03	0.155E+04	0.437E+00	0.229E+02	0.167E+04	0.105E+05
	Q	0.585E+02	0.411E+04	0.410E+01	0.269E+02	0.614E+03	0.244E+05
	V	0.0	0.0	0.0	0.0	0.0	0.0
7	P	0.484E+02	0.313E+03	0.227E+00	0.199E+02	0.149E+06	0.180E+04
	Q	0.124E+02	0.843E+03	0.396E+00	0.128E+03	0.462E+05	0.416E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0
8	P	0.840E+01	0.750E+04	0.473E-02	0.586E+02	0.243E+05	0.301E+03
	Q	0.247E+01	0.125E+05	0.101E-01	0.215E+03	0.754E+04	0.695E+03
	V	0.0	0.0	0.0	0.0	0.0	0.0
9	P	0.516E+00	0.116E+04	0.449E-03	0.104E+02	0.393E+04	0.152E+05
	Q	0.750E+00	0.194E+04	0.744E-03	0.501E+02	0.121E+04	0.390E+05
	V	0.0	0.0	0.0	0.0	0.0	0.0
10	P	0.116E+01	0.779E+07	0.437E-03	0.347E+01	0.630E+03	0.252E+04
	Q	0.520E+01	0.129E+08	0.800E-03	0.952E+01	0.193E+03	0.643E+04
	V	0.0	0.0	0.0	0.0	0.0	0.0

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G3: MAXIMUM PER UNIT MISMATCHES AFTER EACH ITERATION FOR THE IEEE 118-BUS SYSTEM

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-2.0	-1.9	-1.8	-1.7	-1.6	-1.5
1	P	0.370E+03	0.734E+02	0.455E+02	0.216E+02	0.127E+02	0.105E+02
	Q	0.934E+03	0.173E+02	0.144E+03	0.199E+02	0.881E+01	0.575E+01
	V	0.465E+02	0.269E+02	0.142E+02	0.337E+01	0.314E+01	0.380E+01
2	P	0.271E+05	0.677E+02	0.183E+04	0.424E+02	0.590E+01	0.557E+01
	Q	0.172E+05	0.156E+02	0.245E+04	0.917E+01	0.377E+01	0.250E+01
	V	0.186E+05	0.484E+02	0.266E+04	0.404E+01	0.306E+01	0.436E+01
3	P	0.105E+07	0.551E+02	0.185E+05	0.284E+02	0.448E+03	0.419E+02
	Q	0.871E+05	0.154E+02	0.458E+04	0.215E+02	0.325E+02	0.109E+02
	V	0.169E+06	0.386E+02	0.590E+04	0.220E+01	0.936E+02	0.168E+02
4	P	0.136E+10	0.151E+03	0.349E+06	0.284E+02	0.690E+03	0.322E+02
	Q	0.128E+10	0.169E+02	0.632E+05	0.721E+02	0.492E+02	0.894E+01
	V	0.780E+09	0.866E+02	0.814E+05	0.168E+02	0.963E+02	0.138E+02
5	P	0.164E+10	0.174E+03	0.271E+06	0.344E+02	0.878E+03	0.739E+02
	Q	0.155E+10	0.206E+02	0.480E+05	0.447E+02	0.613E+02	0.167E+02
	V	0.939E+09	0.935E+02	0.619E+05	0.137E+02	0.135E+03	0.255E+02
6	P	0.215E+12	0.209E+03	0.928E+13	0.654E+04	0.888E+03	0.595E+02
	Q	0.159E+12	0.255E+02	0.685E+12	0.149E+04	0.606E+02	0.141E+02
	V	0.885E+11	0.106E+03	0.208E+13	0.263E+04	0.110E+03	0.217E+02
7	P	0.447E+12	0.204E+03	0.669E+11	0.173E+05	0.852E+03	0.900E+02
	Q	0.331E+12	0.249E+02	0.494E+10	0.395E+04	0.569E+02	0.198E+02
	V	0.184E+12	0.994E+02	0.150E+11	0.698E+04	0.176E+03	0.304E+02
8	P	0.251E+11	0.256E+03	0.133E+11	0.441E+05	0.858E+03	0.781E+02
	Q	0.185E+11	0.312E+02	0.979E+09	0.251E+04	0.559E+02	0.177E+02
	V	0.103E+11	0.120E+03	0.297E+10	0.291E+05	0.150E+03	0.273E+02
9	P	0.203E+15	0.226E+03	0.109E+11	0.617E+07	0.879E+03	0.100E+03
	Q	0.150E+15	0.270E+02	0.806E+09	0.279E+06	0.561E+02	0.218E+02
	V	0.834E+14	0.103E+03	0.245E+10	0.402E+07	0.205E+03	0.337E+02
10	P	0.320E+15	0.344E+03	0.498E+11	0.822E+05	0.932E+03	0.927E+02
	Q	0.236E+15	0.394E+02	0.368E+10	0.372E+04	0.582E+02	0.204E+02
	V	0.131E+15	0.150E+03	0.111E+11	0.536E+05	0.176E+03	0.316E+02

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G3 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-1.4	-1.3	-1.2	-1.1	-1.0	-0.9
1	P	0.608E+01	0.182E+02	0.788E+03	0.257E+03	0.472E+02	0.232E+02
	Q	0.482E+01	0.897E+01	0.129E+03	0.250E+02	0.756E+01	0.115E+02
	V	0.590E+01	0.141E+02	0.185E+03	0.319E+02	0.916E+01	0.168E+02
2	P	0.453E+04	0.329E+02	0.935E+03	0.394E+04	0.597E+03	0.223E+02
	Q	0.281E+03	0.772E+01	0.149E+03	0.383E+03	0.627E+02	0.115E+02
	V	0.138E+04	0.123E+02	0.215E+03	0.504E+03	0.869E+02	0.170E+02
3	P	0.332E+04	0.229E+02	0.690E+04	0.348E+04	0.495E+03	0.252E+02
	Q	0.582E+03	0.673E+01	0.105E+04	0.334E+03	0.537E+02	0.139E+02
	V	0.287E+04	0.108E+02	0.151E+04	0.442E+03	0.751E+02	0.208E+02
4	P	0.763E+04	0.536E+02	0.210E+05	0.174E+05	0.996E+03	0.250E+02
	Q	0.477E+03	0.188E+02	0.320E+04	0.166E+04	0.110E+03	0.153E+02
	V	0.227E+04	0.299E+02	0.459E+04	0.221E+04	0.155E+03	0.231E+02
5	P	0.148E+04	0.391E+02	0.156E+04	0.201E+05	0.850E+03	0.257E+02
	Q	0.922E+02	0.155E+02	0.242E+03	0.192E+04	0.958E+02	0.171E+02
	V	0.449E+03	0.249E+02	0.346E+03	0.255E+04	0.136E+03	0.263E+02
6	P	0.136E+04	0.982E+02	0.514E+04	0.221E+05	0.100E+04	0.257E+02
	Q	0.862E+02	0.310E+02	0.759E+03	0.212E+04	0.115E+03	0.185E+02
	V	0.417E+03	0.498E+02	0.108E+04	0.282E+04	0.165E+03	0.289E+02
7	P	0.360E+04	0.839E+02	0.655E+04	0.186E+05	0.914E+03	0.258E+02
	Q	0.164E+03	0.278E+02	0.996E+03	0.177E+04	0.107E+03	0.200E+02
	V	0.628E+03	0.449E+02	0.141E+04	0.233E+04	0.154E+03	0.317E+02
8	P	0.851E+04	0.103E+03	0.156E+04	0.304E+05	0.942E+03	0.256E+02
	Q	0.416E+03	0.333E+02	0.238E+03	0.287E+04	0.112E+03	0.213E+02
	V	0.173E+04	0.540E+02	0.338E+03	0.379E+04	0.162E+03	0.343E+02
9	P	0.449E+03	0.981E+02	0.149E+04	0.252E+05	0.901E+03	0.254E+02
	Q	0.223E+02	0.323E+02	0.232E+03	0.238E+04	0.110E+03	0.225E+02
	V	0.899E+02	0.526E+02	0.331E+03	0.314E+04	0.159E+03	0.370E+02
10	P	0.376E+04	0.107E+03	0.155E+04	0.968E+05	0.897E+03	0.253E+02
	Q	0.232E+03	0.349E+02	0.245E+03	0.906E+04	0.111E+03	0.237E+02
	V	0.573E+03	0.571E+02	0.350E+03	0.119E+05	0.162E+03	0.396E+02

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G3 (CONTINUED)

ITER. NO.	*	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3
		MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
1	P	0.188E+03	0.103E+04	0.728E+02	0.271E+02	0.149E+02	0.972E+01
	Q	0.420E+02	0.147E+03	0.169E+02	0.915E+01	0.664E+01	0.535E+01
	V	0.850E+02	0.331E+03	0.182E+02	0.565E+01	0.271E+01	0.160E+01
2	P	0.162E+03	0.252E+04	0.129E+06	0.699E+02	0.400E+01	0.961E+00
	Q	0.379E+02	0.185E+03	0.742E+04	0.582E+01	0.177E+01	0.791E+00
	V	0.765E+02	0.893E+03	0.353E+05	0.229E+02	0.270E+01	0.691E+00
3	P	0.345E+03	0.232E+04	0.251E+08	0.281E+04	0.621E+01	0.525E+00
	Q	0.913E+02	0.561E+03	0.143E+07	0.217E+03	0.520E+00	0.738E-01
	V	0.173E+03	0.798E+03	0.685E+07	0.835E+03	0.831E+00	0.334E-01
4	P	0.295E+03	0.802E+03	0.216E+07	0.513E+05	0.617E+00	0.149E-02
	Q	0.811E+02	0.138E+04	0.123E+06	0.385E+04	0.204E+00	0.293E-03
	V	0.154E+03	0.118E+03	0.590E+06	0.152E+05	0.912E-01	0.267E-04
5	P	0.323E+03	0.218E+04	0.206E+06	0.117E+08	0.152E-01	0.448E-04
	Q	0.888E+02	0.352E+04	0.117E+05	0.883E+06	0.209E-02	0.618E-04
	V	0.169E+03	0.433E+03	0.562E+05	0.349E+07	0.239E-03	0.954E-06
6	P	0.300E+03	0.623E+05	0.229E+05	0.114E+07	0.196E-03	***
	Q	0.854E+02	0.375E+05	0.131E+04	0.854E+05	0.312E-03	
	V	0.162E+03	0.171E+04	0.627E+04	0.338E+06	0.954E-06	
7	P	0.302E+03	0.219E+06	0.335E+04	0.143E+06	0.410E-04	
	Q	0.889E+02	0.129E+06	0.186E+03	0.107E+05	0.411E-04	
	V	0.167E+03	0.589E+04	0.903E+03	0.423E+05	0.954E-06	
8	P	0.297E+03	0.761E+06	0.874E+03	0.260E+05	***	
	Q	0.901E+02	0.632E+06	0.497E+02	0.196E+04		
	V	0.169E+03	0.284E+05	0.240E+03	0.774E+04		
9	P	0.296E+03	0.267E+07	0.866E+03	0.854E+04		
	Q	0.928E+02	0.240E+07	0.452E+02	0.631E+03		
	V	0.174E+03	0.107E+06	0.226E+03	0.251E+04		
10	P	0.293E+03	0.377E+07	0.369E+06	0.234E+06		
	Q	0.950E+02	0.339E+07	0.189E+05	0.172E+05		
	V	0.177E+03	0.317E+06	0.976E+05	0.688E+05		

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G8 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		-0.2	-0.1	0.0	0.1	0.2	0.3
1	P	0.764E+01	0.638E+01	0.541E+01	0.465E+01	0.410E+01	0.382E+01
	Q	0.452E+01	0.389E+01	0.336E+01	0.289E+01	0.244E+01	0.200E+01
	V	0.106E+01	0.752E+00	0.559E+00	0.425E+00	0.324E+00	0.242E+00
2	P	0.623E+00	0.410E+00	0.274E+00	0.186E+00	0.127E+00	0.868E-01
	Q	0.404E+00	0.222E+00	0.128E+00	0.756E-01	0.452E-01	0.269E-01
	V	0.252E+00	0.112E+00	0.553E-01	0.290E-01	0.156E-01	0.834E-02
3	P	0.639E-01	0.933E-02	0.155E-02	0.278E-03	0.634E-04	0.362E-04
	Q	0.975E-02	0.144E-02	0.230E-03	0.339E-03	0.157E-03	0.674E-04
	V	0.256E-02	0.277E-03	0.381E-04	0.572E-05	0.954E-06	0.954E-06
4	P	0.232E-04	0.401E-04	0.487E-04	0.127E-03	0.257E-04	
	Q	0.539E-04	0.360E-04	0.955E-04	0.245E-03	0.618E-04	
	V	0.954E-06	0.191E-05	0.954E-06	0.954E-06	0.954E-06	
5	P		***	***	0.552E-04		
	Q				0.748E-04		
	V				0.954E-06		
					***		

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G3 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		0.4	0.5	0.6	0.7	0.8	0.9
1	P	0.357E+01	0.335E+01	0.316E+01	0.299E+01	0.282E+01	0.267E+01
	Q	0.155E+01	0.109E+01	0.596E+00	0.303E+00	0.489E+00	0.110E+01
	V	0.172E+00	0.109E+00	0.585E-01	0.756E-01	0.973E-01	0.133E+00
2	P	0.588E-01	0.338E-01	0.240E-01	0.125E-01	0.403E-02	0.105E-01
	Q	0.157E-01	0.881E-02	0.460E-02	0.433E-02	0.430E-02	0.479E-02
	V	0.420E-02	0.182E-02	0.604E-03	0.297E-03	0.654E-03	0.990E-03
3	P	0.113E-03	0.422E-04	0.433E-04	0.481E-04	0.617E-04	0.428E-04
	Q	0.252E-03	0.116E-03	0.810E-04	0.445E-04	0.542E-04	0.276E-04
	V	0.954E-06	0.954E-06	0.572E-05	0.954E-06	0.954E-06	0.954E-06
4	P	0.355E-04	0.200E-03	***		***	***
	Q	0.261E-04	0.448E-03				
	V	0.179E-06	0.954E-06				
5	P	***	0.212E-03				
	Q		0.408E-03				
	V		0.954E-06				
6	P		0.333E-04				
	Q		0.312E-04				
	V		0.954E-06				
			***				

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G3 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		1.0	1.1	1.2	1.3	1.4	1.5
1	P	0.251E+01	0.234E+01	0.221E+01	0.228E+01	0.234E+01	0.237E+01
	Q	0.177E+01	0.251E+01	0.333E+01	0.425E+01	0.527E+01	0.643E+01
	V	0.189E+00	0.255E+00	0.327E+00	0.404E+00	0.488E+00	0.579E+00
2	P	0.234E-01	0.498E-01	0.106E+00	0.228E+00	0.503E+00	0.117E+01
	Q	0.682E-02	0.122E-01	0.300E-01	0.719E-01	0.174E+00	0.437E+00
	V	0.165E-02	0.422E-02	0.111E-01	0.273E-01	0.663E-01	0.166E+00
3	P	0.781E-04	0.705E-04	0.965E-04	0.991E-04	0.101E-02	0.209E-01
	Q	0.115E-03	0.170E-03	0.878E-04	0.612E-04	0.255E-03	0.580E-02
	V	0.954E-06	0.954E-06	0.149E-05	0.101E-04	0.150E-03	0.324E-02
4	P	0.346E-04	0.504E-04			0.416E-04	0.113E-03
	Q	0.609E-04	0.946E-04			0.819E-04	0.148E-03
	V	0.954E-06	0.954E-06			0.954E-06	0.954E-06
5	P	***					0.410E-04
	Q						0.347E-04
	V						0.954E-06
							***

*** - ONLY ONE-HALF OF THE LAST ITERATION WAS PERFORMED

TABLE G8 (CONTINUED)

ITER. NO.	*	MAXIMUM MISMATCHES FOR VARIOUS ALPHA VALUES					
		1.6	1.7	1.8	1.9	2.0	2.1
1	P	0.235E+01	0.261E+01	0.363E+01	0.512E+01	0.784E+01	0.165E+02
	Q	0.772E+01	0.916E+01	0.107E+02	0.122E+02	0.132E+02	0.119E+02
	V	0.676E+00	0.776E+00	0.867E+00	0.922E+00	0.902E+00	0.742E+00
2	P	0.306E+01	0.105E+02	0.849E+02	0.204E+04	0.252E+04	0.808E+04
	Q	0.118E+01	0.455E+01	0.571E+02	0.100E+04	0.164E+03	0.318E+04
	V	0.425E+00	0.107E+01	0.491E+02	0.403E+03	0.677E+02	0.466E+03
3	P	0.162E+01	0.279E+05	0.157E+09	0.121E+09	0.691E+03	0.421E+04
	Q	0.460E+00	0.600E+04	0.188E+08	0.651E+08	0.207E+02	0.130E+04
	V	0.267E+00	0.258E+04	0.166E+08	0.207E+07	0.165E+02	0.146E+03
4	P	0.276E+00	0.385E+08	0.249E+08	0.199E+08	0.140E+04	0.137E+04
	Q	0.634E-01	0.830E+07	0.298E+07	0.107E+08	0.820E+02	0.534E+03
	V	0.317E-01	0.357E+07	0.264E+07	0.343E+06	0.123E+03	0.627E+02
5	P	0.136E-03	0.853E+07	0.629E+07	0.324E+07	0.101E+04	0.355E+03
	Q	0.103E-03	0.184E+07	0.753E+06	0.175E+07	0.268E+03	0.128E+03
	V	0.343E-04	0.792E+06	0.666E+06	0.556E+05	0.181E+03	0.122E+02
6	P	0.496E-04	0.297E+07	0.134E+10	0.742E+09	0.298E+06	0.586E+02
	Q	0.955E-04	0.589E+06	0.161E+09	0.399E+09	0.124E+06	0.216E+02
	V	0.179E-06	0.254E+06	0.142E+09	0.124E+08	0.642E+05	0.253E+01
7	P	***	0.314E+08	0.110E+15	0.127E+09	0.535E+05	0.221E+02
	Q		0.676E+07	0.129E+14	0.683E+08	0.204E+05	0.246E+01
	V		0.291E+07	0.117E+14	0.212E+07	0.106E+05	0.788E+00
8	P		0.566E+07	0.689E+18	0.210E+08	0.944E+05	0.593E+01
	Q		0.126E+07	0.802E+17	0.113E+08	0.997E+05	0.216E+00
	V		0.544E+06	0.732E+17	0.351E+06	0.341E+05	0.541E+00
9	P		0.966E+05	0.237E+18	0.391E+07	0.367E+07	0.583E+00
	Q		0.208E+06	0.276E+17	0.210E+07	0.221E+07	0.112E+00
	V		0.897E+05	0.252E+17	0.646E+05	0.984E+06	0.249E-01
10	P		0.167E+06	0.211E+20	0.208E+07	0.771E+07	0.141E-02
	Q		0.342E+05	0.245E+19	0.112E+07	0.451E+07	0.590E-02
	V		0.147E+05	0.224E+19	0.312E+05	0.205E+07	0.169E-03

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