A STUDY INVESTIGATING A HIERARCHICAL MODEL RELATING THE CONCEPT OF CONSERVATION OF MECHANICAL ENERGY

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by

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ABSTRACT

The major purpose of this study was to identify a learning hierarchy leading to the learning of the concept of conservation of mechanical energy, to the level normally found in high school physics courses. A secondary purpose was to determine students' misconceptions relating to the specific skills hypothesized to lead to the attainment of this concept.

The main sample consisted of 156 grade-ten physics students in two senior high schools. A test instrument, which was designed to test for skills of the hypothesized hierarchy, was administered to these subjects soon after instruction of the topic was completed by the teacher involved. A similar test was administered one week after the initial test. An instructional booklet, which was intended to remediate for skills which subjects failed to learn during regular classroom instruction, was administered after the initial test.

Two psychometric methods, namely the ordering-theoretic method and the Dayton and Macready method were used to analyze the data. The results of this analysis indicated that the hypothesized hierarchy was not supported in its entirety. However, an alternative hierarchy containing eight of the ten skills of the hypothesized hierarchy was considered valid. Most connections between component skills within this alternative hierarchy were also validated by both methods. However, further testing of the relationship between some skills was considered desirable. Consequently, these relationships were further tested with a supplementary sample of 123 grade-ten students. Analysis

of these data resulted in agreement between the ordering-theoretic and Dayton and Macready methods, and a psychometrically validated hierarchy was presented.

A further test was also applied to the data in order to determine if transfer of learning existed between subordinate and superordinate skills. Connections between three of the upper skills in the hierarchy were validated in terms of the learning transfer relationship. Learning transfer relationships for the other skills could not be determined because of a limitation in the test of transfer applied.

The report concludes with a discussion of subjects' misconceptions of the skills involved, as revealed by analysis of the test items.

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Chapter 1

THE PROBLEM

Introduction to the Problem

Most curriculum developers stress the importance of providing for proper sequencing of content in the curriculum. However, there is no general agreement as to how content should be sequenced. Doll (1978, p. 135) notes the many different attempts to establish sequence as: movement from the simple to complex, new learning based on prerequisite learning, movement from part to whole or from whole to part, chronological ordering of events, movement from the present into the past, concentric movement in ever-widening circles of understanding or involvement and movement from concrete experiences to concepts.

Posner and Strike (1976) point to the need to establish the different ways that content "can" be sequenced before deciding which way it "should" be sequenced. They divide content sequencing into five major categories. These are world-related, concept-related, inquiry-related, learning-related and utilization-related. They suggest that curriculum developers use these categories as an aid to choosing the most appropriate method of sequencing. The present study would fall under their category of learning-related sequencing.

Orlosky and Smith (1978) note that curriculum includes courses but it must also include issues such as scope, balance and sequence. They suggest three major conceptions of sequencing. One is that the learner orders his own learning as he deals with a situation from moment to moment. Thus, planned sequencing is not necessary. The learner selects what he wants to know as the need arises. As the individual learns to cope with situations, he acquires whatever knowledge is needed in whatever sequence is appropriate to his needs. A second major conception of sequencing is termed macrosequencing by Orlosky and Smith. Macrosequencing is described as the organization of knowledge and the formulation of instruction to coincide with the different stages of an individual's development. Piaget's theory of intellectual development would be included in the category of macrosequencing. Orlosky and Smith suggest the importance of macrosequencing in arranging a program of studies but not for day-to-day tasks of sequencing content. Their third conception is microsequencing. This conception makes the assumption that for any learning task there is a hierarchy extending from the very simple to the more abstract and complex elements. According to Orlosky and Smith, this notion of sequencing is useful to the teacher as he deals with the students from moment to moment in the classroom. The present study falls under the category of microsequencing as described by Orlosky and Smith.

Probably the most well known advocate of microsequencing as described by Orlosky and Smith is Robert Gagné. Gagné (1965) has developed a theory of cumulative learning based on the premise that learning of prerequisite capabilities enhances learning of superordinate capabilities. More recent forms of Gagné's theory (Gagné, 1970, 1977) restrict this model to particular kinds of learning. The details of this restriction will be elaborated upon in the next section.

Which of the above models, if any, is superior has not been established. At present the choice of a particular model seems to depend upon theoretical persuasion rather than empirical evidence.

None has been investigated enough for an unequivocal decision. Each merits further research. In the present study the model provided by Gagné is applied to the learning of a physics concept, namely, the concept of conservation of mechanical energy.

Gagné's Hierarchical Model of Learning

Gagné's learning hierarchy model is based on the idea that learning of a complex skill requires prior learning of prerequisite skills. Thus, a hierarchy can be developed leading through a number of prerequisite skills until the terminal skill in the hierarchy is encountered. Since Gagné (1962) first proposed a hierarchical arrangement of "learning sets" for "finding formulas for the sum of n terms in a number series" his emphasis on the importance of learning hierarchies has not changed. However, adjustments have been made to his original model. These adjustments are evident from the changes made between the first to the third editions of his book The Conditions of Learning (1965, 1970, 1977). The major adjustments concern the separation of learning into different domains, the restriction of hierarchies to one of these domains, and a decrease in the amount of content which should be covered by a hierarchy.

Gagné (1972, 1977) indicates the need to recognize five domains of learning. These represent motor skills, verbal information, intellectual skills, cognitive strategies and attitudes, respectively. Each of these domains requires different instructional treatments and different methods of evaluation. The major adjustment that Gagné has made to his hierarchical learning model is that he now restricts it to the domain of intellectual skills, where an intellectual skill is "knowing how" to do something as opposed to "knowing that" about something. For example, the ability to "calculate the velocity of an object given displacement and time" would be an intellectual skill, whereas being able to simply state that "velocity is the rate of change of displacement with time" represents verbal information.

Although the hierarchical model now appears restricted because it includes only one of five domains, this domain represents a major part of school learning. In addition, the learning of intellectual skills, according to Gagné, is crucial for learning in the other domains.

Gagné (1973) suggests that learning hierarchies are best suited for single lessons. Griffiths (1979) suggests that what constitutes the optimum amount of content is not yet answered. In a study of the mole concept in chemistry, Griffiths used content normally requiring several lessons for completion.

Gagné (1977) distinguishes several distinct types of learning which are related hierarchically within the domain of intellectual skills. These are presented in Figure 1. Although all these types of learning may occur in the school setting, most instruction in school is concerned with the intellectual skills relating to discriminations, concepts, rules and higher-order rules. Gagné suggests that more basic forms of learning, for example associations and chains, are generally applicable only to preschool children.

Most learning hierarchies have been developed using the question suggested by Gagné (1962, p. 358), "What would the individual have to be able to do in order that he can attain successful performance on this task, provided he is given only instructions?" This question is asked of the terminal skill, and prerequisite skills are developed for this terminal skill. Once these skills have been determined, the same question is asked of these new skills. This procedure is

Higher-order Rules
require as prerequisites

Rules
which require as prerequisites

Concepts
which require as prerequisites

Discriminations
which require as prerequisites

Basic Forms of Learning:
Associations and Chains

Figure 1. Gagné's (1977) representation of types of intellectual skills.

continued until the hierarchy is developed to the stage where no further reasonable skills are identified. The resulting hierarchy may be linear or branched, such that in the latter case several independent skills may be directly prerequisite to a higher skill. Independent prerequisites to a higher skill can be learned in any order as long as all have been acquired before the higher skill. There is also the possibility of disjunctive branches where more than one, but not simultaneously all, subordinate skills would be prerequisite to a particular superordinate skill. However, in the literature such connections tend to be limited in number.

If two skills are hierarchically related and an individual has acquired the superordinate skill, then he must also have acquired the prerequisite or subordinate skill. Furthermore, the learning of the subordinate skill should have helped him learn the superordinate skill. Hence, there are two basic definitions to a hierarchical relationship. Firstly, where a prerequisite skill(s) is necessary for learning a superordinate skill a psychometric relationship is said to exist. Possession of the prerequisite skill(s) does not guarantee learning of the superordinate skill. However, the superordinate skill cannot be learned without the prerequisite skill(s). Secondly, a transfer of learning relationship may exist. In this case, learning the prerequisite skill(s) mediates transfer to the related superordinate skill. Ideally, a hierarchy should be valid in terms of both definitions. In practice, most studies have concentrated on one or the other definition. The present study focuses on both.

Alternative Hierarchy Theories

Two other theories which imply that learning is hierarchical but in a different fashion are the developmental theory of Piaget (1964) and the meaningful verbal learning theory of Ausubel (1963). The main points of each are presented below.

Piaget's Theory of Intellectual Development. Piaget's theory is hierarchical in that the learner progresses through different stages of intellectual development, with each stage being a prerequisite to the next stage. Piaget suggests that individuals pass through stages described as sensori-motor, pre-operational, concrete-operational and formal-operational, respectively. According to Piaget, each is qualitatively distinct from the others. Further, progression through these stages is invariant, although the rate of progression may vary substantially. In this sense, development is hierarchical. In addition, according to Piaget, within stages certain behaviors are always exhibited before others.

Although Piaget's theory is hierarchical, it differs in at least one fundamental respect from Gagné's theory. According to Piaget (1964), specific learning is considered to be dependent upon intellectual development. In contrast, according to Gagné (1968), development is considered to be dependent upon prior learning.

There are arguments that Piaget's theory and Gagné's theory are too disparate to be combined (Strauss, 1972, p. 102). However, arguments have also been made that these theories may be used eclectically. For example, Griffiths (1978, p. 12) suggests that if it can be shown that hierarchical dependencies exist within curriculum content this should not be ignored. Correspondingly, he suggests that if it can be shown that developmental limitations might inhibit the learning of such content then this too should not be ignored. Thus, it would be inappropriate to ignore evidence from either model in the application of the other.

Ausubel's Theory of Meaningful Verbal Learning. According to Ausubel, the most important factor affecting learning is what the learner already knows (Ausubel, 1963, p. 76). Meaningful learning can only occur if new information can be linked to these existing concepts.

Existing relevant concepts are considered to be anchoring posts for new concepts and are called "subsumers." The process of integrating the new concepts with the existing concepts is called "subsumption." Subsumption alters the existing concepts and makes them more comprehensive since they now include more knowledge. To facilitate this learning process, Ausubel places emphasis on the use of "advance organizers" to link the new learning to the existing knowledge of the learner. An advance organizer is a more general idea or representation than the new material to be learned. It uses ideas with which the students will have some familiarity and it is intended to bridge the gap between the existing knowledge of the learner and new information. It is, as the name suggests, presented just prior to new learning. Ausubel (1977) suggests learning is progressive differentiation. The major concept is presented first followed by the specific details and smaller concepts that comprise the major concept. This he calls subsumptive learning.

Whether Ausubel's theory or Gagné's theory is superior as an aid to designing instruction has not been determined. Both are still being researched. However, although there are discrepancies between these theories there are also important similarities. In particular, Heimer (1969) notes that both Ausubel's theory and Gagné's theory depend on what the learner already knows as a key element to learning new material. It also seems reasonable that Gagné's prerequisite skills may act as anchoring posts (in the Ausubelian sense) for new information. Ausubel (1963, p. 86) has noted the importance of Gagné's theory for this purpose.

Definition of Terms

Capability: the ability to perform a specific function under specified conditions, e.g., a capability might be the ability to calculate the amount of work done in lifting an object to a specified height.

Gagné-type task analysis: deriving a hierarchy by asking Gagné's question, "What would the individual have to be able to do in order that he can attain successful performance on this task, provided he is given only instructions?" of each skill in turn, from the terminal skill downward. All connections that seem reasonably possible are included in the hierarchy.

Gravitational Potential energy: the energy which an object possesses by virtue of its position above some point below it. Mathematically, this is represented by the equation P.E. = mgh.

Hierarchical connection: a connection between two skills such that the learning of the lower skill is necessary and/or enhances the learning of the upper skill.

Hypothesized hierarchy: a learning hierarchy leading to learning of the concept of conservation of mechanical energy. The hierarchy contains 10 intellectual skills.

Instructional booklet: a booklet designed to teach all the skills of the hypothesized hierarchy. However, each student using the booklet need study only those skills which gave him difficulty as indicated by the results of a pretest.

Intellectual skill: knowing "how" as contrasted with knowing "that" of information (Gagné, 1977). For example, the ability to apply the equation K.E. = $\frac{1}{2}mv^2$ in appropriate circumstances rather than merely being able to state it.

Kinetic energy: energy of motion, as indicated by the equation K.E. = $\frac{1}{2}mv^2$ where m is an object's mass and v is its speed.

Learning hierarchy: an arrangement of intellectual skills in which skills are related to other skills in subordinate-superordinate relationships, such that the subordinate skill in each pair is necessary for the learning of the superordinate skill and/or exhibits transfer of learning to the superordinate skill.

Mechanical energy: for this study, mechanical energy includes kinetic energy and gravitational potential energy.

Nonvertical motion: motion that has, simultaneously, a horizontal component and a vertical component.

Posttest: a test given subsequent to the use of the instructional booklet. It is composed of 10 sub-tests of two items each. Each sub-test represents one of the intellectual skills in the hypothesized hierarchy. The 20 items are arranged randomly throughout the test.

Pretest: a test given prior to the use of the instructional booklet but after classroom instruction. It is composed of 10 sub-tests of two items each. Each sub-test represents one of the intellectual skills in the hypothesized hierarchy. The 20 items are arranged randomly throughout the test.

Subordinate skill: the lower skill in a hierarchical connection between two skills.

Superordinate skill: the upper skill in a hierarchical connection between two skills.

Validated hierarchy: a hierarchy containing a set of intellectual skills in superordinate-subordinate relationships such that subordinate skills are found to be empirically necessary for, and/or to significantly enhance, learning of related superordinate skills leading to the concept of conservation of mechanical energy.

Vertical motion: motion which is directly upward or directly downward. This motion does not have a horizontal component.

Need for the Study

The organization of content for instruction and for curriculum materials has been an important issue in education for many years (Bruner, 1960; Dewey, 1916; Rugg, 1927; Tyler, 1950). However, no satisfactory answer has yet been reached. Heimer (1969) notes that in constructing curricula, important decisions must be made for structuring the content and ordering instructional tasks. He further indicates the need for "empirically testable sets of hypotheses for identifying and guiding those instructional decisions."

Jones and Russell (1979) note the continuous search by teachers for more effective ways to transmit knowledge, skills and attitudes to students. They further indicate the continuous search by theoreticians to provide models or theories to explain learning and thus provide answers for teachers. They suggest that Gagné's theory is such a model because it "provides science curriculum developers the rare opportunity to conceptualize agreed upon science goals and objectives in a reality-oriented, learner-centered way." In addition they suggest it provides opportunities to identify valid, ordered sequences of instruction, permits the teacher to diagnose each student's limitations and strengths more effectively, and provides ideas for teaching techniques and strategies.

Perhaps the most important aspect of Gagné's theory is its optimism for learning. In Gagné's theory, learning a new intellectual skill depends upon the learner having acquired related prerequisite skills. This encourages teachers to help students acquire these prerequisite skills instead of attributing failures to developmental limitations. Further, its greater specificity in terms of learning conditions and sequencing of particular skills suggests possible advantages over Ausubel's model.

However, Gagné's theory has not had the degree of application in classroom practice that might be expected. In part, at least, this may be attributed to the practical difficulties involved in validating learning hierarchies (White, 1973; White & Gagné, 1974). However, renewed interest in techniques for validating hierarchies (Bart & Krus, 1973; Dayton & Macready, 1976a; White & Clark, 1973) suggests that the time is appropriate for new learning hierarchy studies. The present study represents one such attempt.

For the present study, the topic of conservation of mechanical energy was chosen because it is an important part of what is probably the most important and pervasive principle in all of physics, namely, the conservation of energy. Conservation of mechanical energy includes the important concepts of kinetic energy, potential energy and conservation, which a student will encounter in many areas of physics other than the topic of conservation of mechanical energy as presented in a typical high school course.

Purpose of the Study

The major purpose of the study is to identify a learning hierarchy leading to the learning of the concept of conservation of mechanical energy, to the level normally found in high school physics courses. The hierarchy will be considered both in terms of its psychometric and transfer characteristics.

A secondary purpose is to determine particular misconceptions which high school students hold with respect to the intellectual skills leading to the learning of the concept of conservation of mechanical energy.

Research Questions

Question 1. Does the arrangement of intellectual skills represented in the hypothesized hierarchy represent a learning hierarchy which is valid psychometrically?

If the answer to question one is negative, question two will be considered.

Question 2: Does some other arrangement of the intellectual skills represented in the hypothesized hierarchy represent a learning hierarchy which is valid psychometrically?

Question 3: Do any connections between pairs of intellectual skills represented in the hypothesized hierarchy represent connections which are valid in terms of transfer of learning?

Question 4: What misconceptions do high school students hold with respect to the intellectual skills represented in the hypothesized hierarchy?

Delimitations of the Study

Restriction of the study to a small number of schools represents a potentially serious delimitation. It is possible that students in other schools may respond differently, although there is no particular reason to believe this.

The study is restricted to one particular topic in physics. Identification of a valid hierarchy for this topic, or failure to identify such a hierarchy, does not necessarily imply anything about the hierarchical nature of physics in general or any other science.

Another delimitation is the use of only one grade level. Evidence gathered at one grade level may not be valid at higher or lower grade levels.

Finally, any hierarchy developed may not represent the only valid hierarchy for the concept under study.

Limitations of the Study

A limitation exists in the fact that the investigator had no control over sample selection. The sample represented a relatively narrow academic range which tended towards high ability. A wider range of sample would have been desirable because in testing for hierarchical relationships a substantial variation of performance for different skills is most useful for analysis. Partly for this reason, a supplementary sample was studied with respect to the relationship between some of the skills.

Another limitation is present in the procedure used to remediate any subordinate skills which subjects did not exhibit on the pretest. Although an instructional booklet designed for individual remediation of the skills comprising the hypothesized hierarchy was given to each student, there was no control over whether the booklet was used. As the intent of the instructional booklet was to promote learning of skills lacking by each individual at the time of the pretest, and hence to contribute to evidence for transfer of learning, the observed transfer effect may be inhibited.

Finally, since no measuring instrument is free of errors, the instruments used in this study represent a further limitation. Ideally, subjects should get both items for a skill correct or both incorrect. However, since items representing the same skill may not be identical in structure or presentation, some variation in response from the ideal situation was evident, although this variation was generally small.

Summary

The importance of sequencing for learning has been discussed and some different modes of sequencing have been presented. Gagné's theory of learning has been suggested as a practical theory for use in the classroom. The potential usefulness of this theory and the inadequacy of past research in this area indicate that further research is necessary. This study proposes to test Gagné's hierarchical learning theory for a topic covered in high school physics courses, namely, the conservation of mechanical energy.

Overview

Chapter two, which follows, considers some of the initial techniques which have been used to identify learning hierarchies and presents in more detail some recent methods which were used in the present study. The chapter also includes a description of empirical studies relating to hierarchies in science and mathematics. Chapter three presents the design of the study and a description of the test instruments and procedures. Chapter four describes the analysis of data and the results obtained from the study. The final chapter contains a summary of the study and the major implications and recommendations for further research.

Chapter 2

RELATED RESEARCH

Introduction

White (1973) reviewed research into learning hierarchies and concluded that most of the studies which support or do not support learning hierarchies suffer from "one or more of the following weaknesses: small sample size, imprecise specification of component elements, use of only one question per element, and placing of tests at the end of the learning program or even the omission of instruction altogether." White and Gagné (1974) note that past research on learning hierarchies has been plagued by deficiencies in design and validation methods. White and Gagné also note that much research has not included all facets of the learning hierarchy model, such as the need to show positive transfer within skills of the hierarchy.

In order to overcome these weaknesses in design, White (1974a) recommended the following nine stage procedure for the identification and validation of learning hierarchies:

1. Define, in behavioral terms, the element that is to be the pinnacle of the hierarchy.

2. Derive the hierarchy by asking Gagné's question ("What must the learner be able to do in order to learn this new element, given only instructions?") of each element in turn, from the pinnacle element downward. Include all connections that seem reasonably possible since the validation process can only destroy postulated connections, not create them. Avoid verbalized knowledge elements, they can be included in the instructions.

3. Check the reasonableness of the postulated hierarchy with experienced teachers and subject matter experts.

4. Invent possible divisions of the elements of the hierarchy, so that very precise definitions are obtained.

5. Carry out an investigation of whether the invented divisions do in fact represent different skills. One way of doing this is to write two or more questions for each division and give them to a sample of Ss. Whenever any Ss are observed to answer correctly the set of questions for one division, while answering incorrectly the set for another, the divisions are taken to be separate skills. White has given a description of the practical arrangements of such an investigation.

6. Write a learning program for the elements, embedding in it test questions for the elements. The questions for an element should follow immediately after the frames that teach the element. There must be two or more questions for each element to allow for an estimate of their reliability.

7. Have at least 150 Ss, suitably chosen, work through the program, answering the questions as they come to them.

8. Analyze the results to see whether any of the postulated connections between elements should be rejected. A suitable test of a hierarchical relationship has been developed by White and Clark. The hypotheses compared in the test are Ho: the proportion of the population from which the sample was drawn who can learn the higher element without the lower element is zero; and Ha: the above proportion is greater than zero. The test provides estimates of the probabilities of the observed results given that Ho is true or given specific values of the proportion under Ha.

9. Remove from the hierarchy all connections for which the probability under Ho is small, say 0.05 or less.

The need for stages six and seven was tested later in a study by White and Gagné (1978). Subjects were tested on all the skills of a hypothesized hierarchy after they had received regular classroom instruction, hence eliminating these two stages. The results were consistent with a study by White (1974b) which tested the same hierarchy but used stages six and seven as in White's above model. Hence, programmed instruction and testing during instruction may not be essential.

The elimination of stages six and seven of White's model was also suggested by Griffiths (1979). Griffiths argues that White's recommendation of a programmed instruction format restricts the applicability of the learning hierarchy to one mode of instruction. He suggests that if the hierarchy does exist, it should exist regardless of the mode of instruction. Further, contrary to White and Gagné, in a study of the mole concept in chemistry, Griffiths (1979) found that different hierarchical arrangements were obtained when the final test data were used for analysis compared to using data obtained during instruction. He notes that it may be possible for students to gain a skill after being tested on it due to testing effects or through benefit from downward transfer from later instruction. Also, testing during instruction encourages confounding through short-term retention of learning. Therefore, Griffiths suggests, it is better to test the elements of a hierarchy after instruction of all the elements of the hierarchy is complete rather than during instruction of the elements of the hierarchy.

Stage eight of White's model involves statistical analysis of the empirical evidence obtained from the study. Some of the early statistical methods used to validate learning hierarchies are presented briefly in the next section. After this, more recent methods which were used in the present study are presented in greater detail.

Early Methods of Hierarchy Validation

Gagné initiated investigation of his learning hierarchy model in a series of studies in the nineteen sixties (Gagné & Paradise, 1961; Gagné, Mayor, Garstens & Paradise, 1962; Gagné & staff, 1965). In these studies, Gagné used an index called proportion positive transfer to provide a measure of the validity of the hierarchy. This index may be illustrated by reference to Figure 2 in which the letters A to D reflect the frequencies of subjects exhibiting the pass-fail relationships described below:

A = number of subjects who achieve the upper skill and all related lower skills.

B = number of subjects who fail to achieve the upper skill but achieve all related lower skills.

C = number of subjects who fail to achieve the upper skill and at least one related lower skill.

D = number of subjects who achieve the upper skill and fail to achieve all related lower skills.

The index is defined by the following expression:

$$\text{Proportion positive transfer} = \frac{A+C}{A+C+D}$$

According to Gagné, subjects in groups A and C are supportive of the hypothesized hierarchy whereas subjects in group D are contrary to it. Subjects in group B are considered neutral. For a valid connection, the number in group D should be zero and the proportion positive transfer is then equal to 1.0. In Gagné's studies a criterion level of 0.90 was set.

White (1974c) has clearly indicated that this index is unacceptable for validation of learning hierarchies. In particular, he demonstrated that the index can take values close to 1.0 even if there is no hierarchical relationship between the skills. In addition, he noted that the index does not take account of errors of measurement and lacks a sampling distribution.

Eisenberg and Walbesser (1971) suggested the use of other indices based upon the frequencies represented in Figure 2. Five indices were developed represented by the following expressions:

		UPPER SKILL	
		Fail	Pass
LOWER SKILL(S)	Pass	B	A
	Fail	C	D

Figure 2. Contingency table for calculation of proportion positive transfer.

1. Consistency $= \frac{A}{A+D}$

2. Adequacy $= \frac{A}{A+B}$

3. Inverse consistency $= \frac{C}{C+B}$

4. Inverse adequacy $= \frac{C}{C+D}$

5. Completeness $= \frac{A}{A+C}$

White (1974c) also reviewed these indices and again was critical. Eisenberg and Walbesser recommend that three of the indices, consistency, adequacy and completeness, must have values greater than 0.85 for a connection between two skills to be hierarchical. However, White showed that to have a minimum value of 0.85, more than 65% of the subjects must achieve the highest skill in the hierarchy. If such a large proportion achieves the highest skill in the hierarchy, nearly all subjects will achieve the lower skills. Hence, it is impossible to say that subjects who do not learn the lower skills cannot learn higher ones because these subjects are not there to be observed.

The phi correlation coefficient has also been suggested as a measure of the relationship between pairs of skills in a hierarchy (Capie & Jones, 1971). However, as White indicates, a positive correlation between two skills does not necessarily imply a hierarchical relationship, even though a hierarchical relationship implies a positive correlation. A significant positive correlation between two skills in a hypothesized hierarchy is a necessary but not a sufficient condition for a valid connection.

Griffiths and Cornish (1978) divided methods of validation of learning hierarchies into two classes: those which reflect the transfer properties of hierarchies and those which reflect the notion of a relatively inviolate sequence. The latter are often termed psychometric methods. Two recent psychometric methods were used in the present study and these are presented in some detail below. One of these methods, the "ordering-theoretic" method (Airasian & Bart, 1975; Bart & Krus, 1973) focuses upon pairs of skills. The second method (Dayton & Macready, 1976a) considers the hierarchy as a whole. Because of its recent application in a number of studies, another test which compares skills in pairs (White & Clark, 1973) is also described in detail. At present, there does not appear to be a generally acceptable method for testing the transfer properties of hierarchies. However, a recent transfer method developed by Griffiths (1979) shows promise. It was used in the present study and is also described below.

The Ordering-Theoretic Method

In the ordering-theoretic method, the relationship between pairs of skills in the hypothesized hierarchy is determined. From this a composite hierarchy is identified.

Referring back to Figure 2 (p. 21), A, B, C, D represent the number of students who fall in each cell as determined by empirical data. A student who fails the lower skill but passes the upper skill would be included in the 'D' cell. If the postulated lower skill is prerequisite to the postulated higher skill, then the number of students who fall in the 'D' cell should be zero and therefore this cell is often called the critical cell. However, Airasian and Bart (1975) suggest that numbers higher than zero should be allowed in the critical cell because the method does not take account of errors of measurement. Airasian and Bart suggest using a preset tolerance level to overcome this limitation. For example, a preset tolerance level of 2%

in a study of 200 students would mean that the critical cell could contain up to four students before the hierarchical connection is rejected.

One of the major limitations of the ordering-theoretic method is setting the appropriate tolerance level. Obviously, different tolerance levels may lead to different decisions regarding hierarchical connections. The choice of an appropriate tolerance level appears to be a matter of personal preference at this time. In contrast, another method (White & Clark, 1973) involving comparison of skills in pairs is not deterministic. This will be considered next.

White and Clark Test of Inclusion

In this method a skill by skill matrix of scores is formed from student responses to questions testing the skills of the hypothesized hierarchy. This is illustrated in Figure 3 for two questions per skill. To test for a hierarchical connection between Skill I and Skill II, White and Clark use the null hypothesis that the proportion of the population from which the sample is drawn that will possess Skill II only is zero. This can be stated as Ho: $P_{II} = 0$ where P_{II} equals the proportion of the population with Skill II only. This hypothesis means that there should be no entries in the critical cell of the matrix (marked with a C) except for those representing errors of measurement. Entries in the critical cell represent those students who have failed both questions for Skill I and passed both questions for Skill II, thereby violating the null hypothesis.

Assuming that the null hypothesis is true, there is a probability that some entries may still be found in the critical cell due to errors of measurement. White and Clark (1973) give this

SKILL II (upper) Questions Correct

SKILL I (lower) Questions Correct		0	1	2
	2			
	1			
	0			C

Figure 3. Matrix for White and Clark test.

probability by the following:

$$P_{02} = P_0(1-\Theta_b)^2\Theta_d^{\,2}+P_I(1-\Theta_a)^2\Theta_d^{\,2}+P_{II}(1-\Theta_b)^2\Theta_c^{\,2}+P_B(1-\Theta_a)^2\Theta_c^{\,2}$$

where

P_{02} = the probability of entries in the critical cell

P_0 = the proportion of the population with neither skill

P_I = the proportion of the population with skill one only

P_{II} = the proportion of the population with skill two only

P_B = the proportion of the population with both skills

Θ_a = the probability of someone with Skill I answering correctly any Skill I question

Θ_b = the probability of someone without Skill I answering correctly any Skill I question

Θ_c, Θ_d are the corresponding probabilities for Skill II.

Estimated values of the P and Θ parameters are determined from the marginal totals and P_{02} then calculated. The number of entries allowed in the critical cell due to measurement error alone depends on the value of this probability. If the number of entries in the critical cell exceeds those due to errors of measurement this would violate the null hypothesis. That is, Skill I is not prerequisite to Skill II because too many students possess Skill II without possessing Skill I.

Linke (1975) suggests the White and Clark test would be more realistic if it allowed some entries in the critical cell other than those from measurement error. Linke allowed 1% and 2% exceptions, respectively, in the critical cell. In another study, Beeson (1977) extended this conception of substantial rather than absolute hierarchical dependency by allowing 5% exceptions in addition to measurement error. Finally, whereas White and Clark describe the application of their test only for two and three questions per skill, the test has been extended by Griffiths and Cornish (1978) to allow any number of questions per skill.

Unfortunately, undesirable restrictions must be placed on the model to obtain the parameter estimates from the marginal totals. In particular, Θ_b is assumed to equal zero and Θ_c is assumed to equal one. According to White and Clark, this restriction reduces the possibility of a type I error (rejection of a true hypothesis), but correspondingly reduces the power of the test. A further complication was found by Beeson (1977) and Linke (1975). When few subjects possess the higher skill the power of the test is expected to be high, and when almost all subjects possess the lower skill the power of the test is expected to be low. However, each of these investigators found that the power of the test against the alternative hypothesis of 10% exceptions was lower than expected when few subjects possessed the higher skill and higher than expected when almost all subjects possessed the lower skill. The model of Dayton and Macready was considered by this investigator to be more desirable than the model of White and Clark because it partly overcomes these deficiencies through maximum likelihood estimation. Hence, the Dayton and Macready model was used in the present study and is described below.

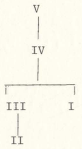
The Dayton and Macready Model

The ordering-theoretic method and the White and Clark test consider hierarchical connections between pairs of skills. From these connections a composite hierarchy is produced. The Dayton and Macready model is different from these methods because it considers the hierarchy as a whole.

The Dayton and Macready model is a deterministic extension of Guttman scalogram analysis (Guttman, 1944). It also extends scalogram analysis to hierarchies of any configuration and not just linear examples. In addition, the model allows for misclassification parameters and provides for statistical tests to determine goodness of fit between the data and the postulated hierarchy.

The Dayton and Macready model considers the probability of all possible response patterns for a postulated hierarchy under the assumption that the hierarchy is valid. If the hierarchy is valid, only some of these possible response patterns are acceptable.

Consider the following branched hierarchy containing five skills:



There are 32 possible response patterns which could be obtained from subjects tested on these skills. However, only eight of these are true response patterns in that they would satisfy the hierarchical connections implied. These are (00000), (10000), (01000), (11000), (01100), (11100), (11110), (11111) where 1 and 0 represent possession and nonpossession, respectively, of a skill, and where the skills are considered in the hypothesized sequence.

If the hierarchy is valid, the probability of a subject producing any one specific response pattern "u" for this hierarchy is given by:

$$\text{(i)}\quad P(u) = \sum_{j=1}^{q} P(u|v_j)\cdot\Theta_j$$

where v_j represents the set of q true response patterns, and

$$\text{(ii)}\quad P(u|v_j) = \prod_{i=1}^{k} \alpha_i^{a_{ij}} (1-\alpha_i)^{b_{ij}} \beta_i^{c_{ij}} (1-\beta_i)^{d_{ij}}$$

In equation (ii), α_i and β_i are the misclassification parameters referred to previously. The probability that a subject produces a correct response to a skill which he should not have completed correctly if the hierarchy is valid is represented by α_i. The probability that a subject produces an incorrect response to a skill which he should have completed correctly if the hierarchy is valid is represented by β_i.

Essentially what is implied by the use of these two equations is that the values of the misclassification parameters α_i and β_i are respectively raised to a power necessary to fit all 'true' response pattern vectors to an observed data vector. Similarly $(1-\alpha_i)$ and $(1-\beta_i)$ are raised to the number of 'correct' responses in each case. The product of these over all possible response patterns is equation (ii). Multiplying this by the probability that the j^{th} true pattern vector occurs (Θ_j), and summing this for all true pattern vectors (q) yields the probabilistic model represented by equation (1).

Maximum likelihood estimates of the various parameters in the probabilistic model are obtained and these are used to compute the number of expected responses for each possible response pattern. The goodness of fit between the data and the hypothesized hierarchy is then calculated by both a Pearson chi-square test and a likelihood ratio expressed in the form of a chi-square.

In order to use the Dayton and Macready model a hierarchy must be postulated a priori. The complete hierarchy is accepted or rejected and no decisions can be made concerning individual connections within the hierarchy. However, different composite hierarchies of the same skills can be tested to obtain the hierarchy most consistent with the data.

Methods of Testing for Transfer

As indicated previously, Gagné's index of proportion positive transfer and others like it are not considered to be acceptable as tests of the degree of transfer between skills in a hierarchy. One possible method which can be used to show evidence of transfer is to teach the skills in a hierarchy in all possible sequences (Uprichard, 1970). However, this method is not practical for hierarchies containing more than two or three skills since the number of possible sequences would be too large for research purposes. Okey and Gagné (1970) showed evidence of positive transfer by comparing two different groups of subjects. One group was instructed first and was used to identify those subordinate skills in the hierarchy which gave subjects difficulty. The second group then received revised instructions intended to overcome these difficulties. Okey and Gagné were able to show that the attainment of the final task of the hierarchy and, collectively, the tasks testing the subordinate skills increased significantly more for the second group. Hence, they concluded, the hierarchy showed transfer of learning. However, the method does not provide for testing of specific transfer effects between skills of the hierarchy.

White and Gagné (1974) suggest the following steps for investigating positive transfer. Select a hierarchy which has been validated psychometrically. For each connection to be studied obtain two groups of learners, all of whom are immediately ready to learn the lower skill.

Teach one group the lower skill, then the higher skill. Teach the other group only the higher skill. Positive transfer is demonstrated if more of the first group than the second group acquire the superordinate skill. White and Gagné acknowledge that there may be difficulty in the practical application of this design. Certainly it would require substantial interference with normal classroom teaching. At present there do not appear to have been any attempts to use this method.

Bergan (1980) has suggested a structural equation model as a means of testing Gagné's positive transfer hypothesis. It is similar to Gagné's hierarchical model except that several other variables in addition to the learning of prerequisite skills are considered to affect the learning of higher level skills. According to Bergan, a structural equation model would show the relationships among exogenous variables determined by causes operating outside the model, endogenous variables determined by exogenous variables or other endogenous variables within the model, and disturbances including all unspecified sources of variation affecting model variables but not explicitly identified in the model. A set of simultaneous equations based on multiple regression techniques is used to describe these relationships. According to Bergan, the coefficients in these equations represent an indication of the extent to which independent model variables mediate transfer with respect to dependent variables. Bergan's approach considers that other variables as well as prerequisite skills may be responsible for positive transfer between skills. This possibility seems reasonable and warrants further investigation. However, the present study was concerned only with the transfer effects of subordinate skills. Therefore a more direct test of transfer was needed.

Griffiths (1979) developed the method of testing for positive transfer which was used in the present study. This method is practical for research with intact classes and is suitable for hierarchies with any number of skills. The following steps are involved in Griffiths' method:

1. Each skill in the hypothesized hierarchy which has one or more subordinate skills is identified. The steps which follow refer to each individual set of subordinate-superordinate skills.
2. Following regular instruction, those subjects who failed to exhibit any of these subordinate skills and the superordinate skill in each set are identified. This group forms the sub-sample for each set of skills under test. The size of this sub-sample varies from skill to skill.
3. All subjects are administered a self-instructional remedial unit in which each subject is directed to those skills not learned during regular instruction.
4. Following remediation, those subjects in each sub-sample who gained and failed to gain the subordinate skill in the particular set under test are identified. These subjects are designated 'gain' and 'no gain' respectively.
5. Similarly, subjects in the sub-samples identified in step two are designated 'pass' or 'fail' with respect to their performance on the superordinate skill in each set of skills.
6. The significance of the relationship between Gain/No Gain and Pass/Fail for each set is determined by application of a chi-square test with one degree of freedom in each case.

The next section is concerned with past attempts to validate learning hierarchies in science and mathematics. Most of the validation methods described above have been used in one or more of these studies.

Learning Hierarchies in Science and Mathematics

Since Gagné first proposed his idea of learning hierarchies in 1961, most investigations have been in the area of science and mathematics. Gagné used hierarchies consisting of mathematical skills in his early attempts to support his theory. Unfortunately, according to White (1973), Gagné's initial studies had several weaknesses. White's criticism of three of Gagné's pioneering studies are summarized below.

Gagné and Paradise (1961) developed a hierarchy leading to the terminal skill "solving linear equations." The numbers of subjects who learned the higher skills without possessing the relevant subordinate skills were small. However, in 11 of 15 connections these numbers were not zero, which they should have been if the hierarchy was completely valid. Gagné and Paradise suggested that these numbers may have resulted from errors of measurement. White (1973) criticized this study on several grounds. Firstly, the use of only one question per skill which eliminates any estimation of errors of measurement; secondly, errors in the construction of some connections; and thirdly, the use of the "index of positive transfer." White rejected this index on the basis that it can be high even when the elements of a hierarchy are independent. Thus, when Gagné used a high index as evidence of a hierarchical connection, the results were not necessarily dependable.

Gagné, Mayor, Garstens and Paradise (1962) devised a mathematics hierarchy with two final tasks, namely "adding integers" and "stating, using specific numbers, the series of steps necessary to formulate a definition of addition of integers, using whatever properties are needed, assuming those not previously established." They used from two to nine items to test each skill of the hierarchy and various indices, including the index of positive transfer, to test its validity. Again White showed that these indices can be misleading.

Gagné and staff (1965) found few exceptions to a hierarchy relating to "sets" in mathematics. However, White suggested that, in addition to faults found in previous studies by Gagné, the definitions of skills in this study were too general and many could have been divided into several subskills.

Okey and Gagné (1970) used a different approach in attempting to provide support for Gagné's model. They designed a study based on the premise that any inability to perform a task should be attributable to failure on skills subordinate to it as predicted by Gagné's model. A sample of 135 tenth, eleventh, and twelfth-grade chemistry students was divided into two treatment groups. Each group was given a pretest and posttest on the final task which was "Solve solubility product problems." In addition, each group was given a pretest and posttest on the subordinate skills in the learning hierarchy. One group received instructions and were tested. The results of the test were analyzed to determine student weaknesses in skills of the hierarchy. The second group were given a revised instructional program designed to eliminate these weaknesses. A significant difference in the level of performance was confirmed for the second group compared to the first group. Okey and Gagné concluded that improved performance on subordinate skills led to a significantly improved performance on the final task.

Despite the attractiveness of this study, Griffiths (1979) has criticized it on several grounds. Firstly, skills were not defined as precisely as they might have been. In some cases a skill such as "solve solubility product problems" might encompass a wide range of outcomes. Secondly, there was a lack of investigation of specific transfer effects between skills within the hierarchy. Finally, the percentage of individuals successful on some subordinate skills, even after remediation, was low.

At least two studies have provided support for Gagné's model by comparing the results of teaching a number of skills in all possible sequences. Uprichard (1970) taught the concepts 'equals, greater than and less than' in all six possible sequences to six groups of nursery school children. Uprichard found that 'equals' was prerequisite to 'greater than' and 'less than'. The second study (Resnick, Siegel & Kresh, 1971) tested kindergarten children on two multiple classification skills which were hypothesized to be hierarchically related. Children who could perform neither of the two skills on a pretest were divided into two groups and were taught the skills in the two possible orders. Results generally confirmed the hypothesized hierarchical relationship between the two skills. The method used in these studies is possible only for hierarchies consisting of a few skills. Hierarchies consisting of larger numbers of skills would require too many different sequences of instruction for practical purposes. In addition, in some circumstances, it could be misleading and therefore unethical to teach subjects a set of skills in a different order than that which was hypothesized, because the hypothesized order is expected to produce greater success.

Wiegand (1969) compared the maturational view of Piaget with the hierarchical learning model of Gagné. She developed a hierarchy for a variation of Piaget's inclined plane task (Inhelder & Piaget, 1958), with the inclined plane task serving as a test of transfer. The final task in the hierarchy was to determine the relationship between the height and weight of a car on an inclined plane, the weight of a block resting at the bottom of the incline, and the distance the block was pushed by the car after the car rolled down the incline. Thirty subjects of approximately 12 years of age who had failed both a pretest for the final task and the transfer task were assigned to one of three groups. The groups were demonstration-test-retest, test-retest and test, respectively. This allowed Wiegand to account for the extraneous influences of the demonstration of the task and possible effects of the initial test. Wiegand found that subjects who could not perform either the final task or the transfer task did so readily when they learned the subordinate skills.

Raven (1968) also compared the models developed by Piaget and Gagné, respectively, in a study of the concept of momentum. A learning hierarchy and a developmental hierarchy were tested on 160 children between five and eight years of age. The results favored the order represented by the developmental hierarchy. However, Griffiths (1979) argues strongly against the results of this study. One of the more important criticisms concerns the validity of the learning hierarchy developed by Raven. Griffiths suggests that the learning hierarchy may not be a learning hierarchy at all but simply a re-combining, in a 'logical' order, of the elements of the developmental hierarchy. Griffiths argues that a more precisely defined learning hierarchy may yield different results. A replication of Raven's study by Murray (1981) offers some support for this view.

Linke (1975) emphasized obtaining replicative results and larger samples to test Gagné's theory. He tested 204 grade-eight students in Brisbane, Australia and 212 grade-eight students in Adelaide, Australia. The students were given a program of instruction based on a hypothesized hierarchy relating to graphical interpretation skills. Linke applied the White and Clark test to the data, but allowed 1 and 2% exceptions in addition to measurement error. In addition, he calculated the statistical power of the test for each connection using the alternative hypothesis of 10% exceptions as an indication of no hierarchical dependence. Linke found that most of the connections postulated were valid, and that similar results were obtained for both studies. According to Linke, the latter represented an important finding especially since there were differences in formal curricular background between the groups. However, Linke's results must be treated with caution. In particular, Linke notes that the alternative hypothesis was insensitive to subordinate skill difficulty levels and too sensitive to superordinate skill difficulty levels, which often led to unrealistically high and low levels, respectively, of the power of the test. Another weakness of Linke's study concerns the number of exceptions allowed in the critical cell. In many cases these numbers seem abnormally high considering the number of subjects involved in the study. For example, application of the test to one connection allowed for 16 exceptions at the absolute level of hierarchical dependence. Linke suggested that the high numbers obtained were due to guessing and chance mistakes which resulted in response inconsistency. It is argued by the present investigator that these response inconsistencies were frequent enough to raise serious concern for Linke's results. It is desirable that the data should be tested with methods other than the White and Clark test.

Partly because most previous applications of Gagné's theory were related to mathematics, Beeson (1977) chose to investigate its application to an area of science curriculum. The final task in the hierarchy concerned the determination of quantities in electric circuits. In addition, Beeson investigated the distinction between the place of intellectual skills and verbal information in the hierarchy. Gagné and others have claimed that the elements of learning hierarchies should consist only of intellectual skills. The White and Clark test, allowing 0, 1 and 5% exceptions, was used to test connections between intellectual skills. Beeson suggested that one question was sufficient to test verbal information elements. Therefore the White and Clark test, which requires at least two questions per element, could not be used for any connections involving verbal information elements. In these cases the validity of each connection was decided by subjective judgement. Beeson found that although most connections between intellectual skills were validated, the same was not true of connections involving verbal information elements. All connections with verbal information as the superordinate element were rejected, with the exception of one connection which was accepted at the weaker (5%) level. Apart from this one case, it was found that all validated connections involving verbal information elements in the hierarchy were cases where verbal information was subordinate to the intellectual skill.

In another study, Beeson (1981) was concerned with the suggestion that students learned intellectual skills in a mechanical way rather than in a meaningful way. He suggested that the mechanical learning of intellectual skills may be a result of teaching the skills in a relatively isolated manner such that students are unable to combine skills already learned in order to achieve mastery of higher level skills, and also noted the possibility that a more elaborate context of verbal information may be required to learn the nature of some intellectual skills. A third possible explanation, suggested by Beeson was that the learner may be unable to recall the relevant subordinate skills at the appropriate time. In order to investigate these explanations, a total of 188 grade-ten students were taught the hierarchy of intellectual skills validated in the original study (Beeson, 1977). The students were divided into three groups and taught in one of three different contexts: one in which the skills were taught in isolation, one in which additional verbal instructions were provided and a third in which the skills were taught in relation to a relevant anchoring idea, as suggested by Ausubel's theory of meaningful verbal learning (Ausubel, 1963). Following instruction, students were tested after an interval of approximately two days and again after an interval of seven weeks for achievement of the final task of the hierarchy, lateral transfer and subordinate skills. According to Beeson, results indicated that the 'anchoring-idea-group' did significantly better on the short-term tests of lateral transfer than the other two groups. However, these results did not persist after seven weeks. The same group did significantly better on the test of the final task given after seven weeks than the other two groups, but there was no significant difference on the short-term test. Beeson claims that these results provide evidence that students tend to learn intellectual skills in a mechanical way and that more meaningful learning would result if students learned a hierarchy of intellectual skills in the context of an anchoring idea.

Griffiths (1979) compared the theories of Gagné and Piaget in a study of the "mole" concept in grade-ten chemistry and found evidence of a learning hierarchy for this concept. In addition, he found that the availability of subordinate intellectual skills accounted for much more of the variance on related superordinate skills than did developmental test scores. Application of the White and Clark test and the ordering-theoretic method to the same data yielded good agreement. Application of the Dayton and Macready test to the same data yielded a basically similar hierarchy and allowed direct comparison of alternative hierarchies which had been difficult to distinguish by application of the White and Clark and ordering-theoretic methods. Griffiths also obtained evidence of significant positive transfer from subordinate to superordinate skills. The method he used to obtain evidence of positive transfer has been described in the previous section and it is the method used in the present study.

Bergan and Jeska (1980) were also concerned to identify transfer. Their study related to a seriation hierarchy. The sample consisted of 72 children between the ages of 2 years 9 months and 5 years 10 months, who were selected because pretesting revealed that they possessed none of the skills of the hypothesized hierarchy. Structural analysis (Bergan, 1980) was applied to the data in an attempt to show evidence of vertical transfer between skills. The results of the structural analysis were then compared to the results of the White and Clark test applied to the same data. In several cases skills validated by the White and Clark test as being prerequisite to higher level skills did not mediate transfer to these higher level skills according to the structural analysis. These results led Bergan and Jeska to suggest that validation of a hierarchy cannot be complete until both prerequisite relations and vertical transfer among skills have been established.

Gagné has consistently stated the need to be concerned with this dual nature of hierarchical connections, but it has been ignored in many studies in favor of identifying prerequisite relationships only. However, the more recent studies of Griffiths, and Bergan and Jeska seem to suggest a trend toward renewed efforts to validate hierarchies with respect to both dimensions.

Summary

This chapter has indicated the weaknesses of some of the early methods used to validate learning hierarchies. It has presented in some detail several more recent methods which have been developed to overcome the weaknesses of these early methods. The chapter concludes with a summary of several studies in science and mathematics related to the learning hierarchy model. Although the number of studies reporting well validated learning hierarchies is small, these studies offer sufficient support to warrant further research into the application of the hierarchy model.

Chapter 3

DESIGN, INSTRUMENTATION AND PROCEDURE

This chapter describes the practical aspects involved in the present study. These aspects include the construction of the hypothesized hierarchy, development of questions to test the skills of the hypothesized hierarchy, development of an instructional booklet for remediation of skills and the procedure used to administer the test questions and instructional booklet.

Construction of the Hierarchy

In chapter one it was indicated that it was decided to investigate the concept of conservation of mechanical energy because of its importance to the understanding of the general concept of conservation of energy. It was also noted in chapter one that the topic of conservation of mechanical energy includes the important concepts of kinetic energy, gravitational potential energy and conservation. The concept of friction was not included in the development of the hierarchy. Since mechanical energy is only conserved in an ideal frictionless system, the effects of friction are probably best considered after instruction relating to this ideal situation has been completed. The addition of frictional forces means that mechanical energy is no longer conserved and the more general concept of conservation of energy must be considered. This is beyond the scope of this study. For the present study, the *ability to calculate total mechanical energy, potential energy and kinetic energy for objects which follow a nonvertical path, given situations where mechanical energy* *is conserved* was taken as the terminal skill in the hierarchy.

The hypothesized hierarchy was derived by asking the question "What would the individual have to be able to do in order that he can attain successful performance on this task, provided he is given only instructions?" of each skill in turn, from the terminal skill downward. This Gagné-type task analysis was continued until skills were reached which could be assumed to be already learned by the population represented. The lower skills of the hierarchy resulting from this analysis were hypothesized to be skills relating to a basic understanding of potential energy and kinetic energy. With regard to the sequence of concepts covered in high school physics courses, these skills are usually taught immediately following instruction related to the concept of work.

Application of the task analysis, followed by modification after discussion with content experts and analysis of pilot test data, led to the identification of the following skills for the concept of conservation of mechanical energy:

10. *Conservation of Mechanical Energy--nonvertical motion [terminal skill.]*

    Calculate total mechanical energy, potential energy and kinetic energy for objects which follow a nonvertical path, given situations where mechanical energy is conserved.

9. *Conservation of Mechanical Energy--vertical motion.*

    Calculate total mechanical energy, potential energy and kinetic energy for objects which follow a vertical path, given situations where mechanical energy is conserved.

8. *Conservation of Mechanical Energy--basic application.*

Calculate either the potential energy or the kinetic energy of an object at one position, given either its kinetic energy or potential energy, respectively, at this position and given its potential energy and kinetic energy at some other position.

7. *Potential Energy--nonvertical reference point.*

Calculate the potential energy of an object with respect to a given point when the object is not <u>directly</u> above that point.

6. *Change in Potential Energy.*

Calculate the change in potential energy of an object between different heights above the same reference level, given that the object rises or falls vertically.

5. *Potential Energy--different reference levels.*

Calculate the potential energy of an object with respect to different reference levels, given the mass of the object and its height above the reference levels.

4. *Potential Energy--basic application.*

Calculate the potential energy of an object, given its mass and height.

3. *Change in Kinetic Energy.*

Calculate the change in kinetic energy of an object when its speed changes, given the mass of the object and different speeds of the object.

2. *Kinetic Energy--basic application.*

Calculate the kinetic energy of an object, given its mass and speed.

7. *Discrimination between Potential Energy and Kinetic Energy.* Discriminate between objects that have potential energy and objects that have kinetic energy.

The arrangement of the above skills in their hypothesized hierarchical relationship is shown in Figure 4. Several comments relating to the hierarchy in Figure 4 are appropriate at this point.

Firstly, skills representing only mathematical competence are not included in the hierarchy. It is argued here that mathematical difficulties should not be allowed to interfere with determination of understanding of the physics concepts involved in the hierarchy. Therefore the need for mathematical competence in the solution of test items was minimized by using numerical values which were easy to manipulate. In addition, students were required to show all calculations. In this way it was possible to identify mathematical difficulties or errors as opposed to difficulties with the physics concepts.

Secondly, a justification for the selection of the terminal skill should be mentioned. Class or textbook problems designed to test understanding of the conservation of mechanical energy usually require that students calculate the final speed of an object or the final height it attains, by application of the conservation of mechanical energy. It is argued here that this type of problem goes beyond what is required to test understanding of the concept of mechanical energy. The solution to this type of problem usually requires a calculation of the final kinetic energy or final potential energy from which the speed or height, respectively, is calculated. The position taken in this study is that a student has already acquired an understanding of the conservation of mechanical energy when he uses it to determine the final kinetic or

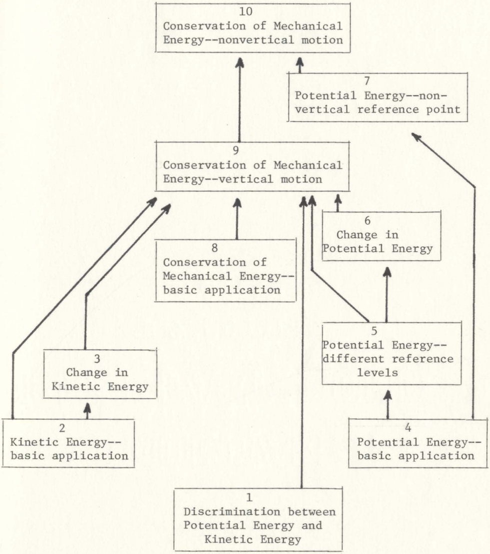


Figure 4. The hypothesized hierarchy.

potential energy. Thus, the calculation of speed or height in these problems is a skill which follows application of the conservation of mechanical energy to the problem. Since the aim of this study was to identify a hierarchy for the concept of conservation of mechanical energy only, the ability to calculate speed or height was considered superfluous.

Thirdly, it was suggested by one of the subject matter experts consulted that students may interpret conservation of mechanical energy in two different ways. One interpretation is that the sum of the potential energy and kinetic energy is a constant value. Problems can then be solved by equating the sum of the potential energy and kinetic energy at one position to the sum of the potential energy and kinetic energy at a second position. The missing variable is then calculated from this equation without any necessity to calculate the <u>change</u> in kinetic energy or the <u>change</u> in potential energy. A second interpretation utilizes the notion that a loss or gain in one form of mechanical energy results in an equivalent gain or loss, respectively, in the other form of mechanical energy. Problems can then be solved by equating the change in kinetic energy between two positions of an object to the change in potential energy between the same two positions. The missing variable is then calculated from this equation without any necessity to calculate the total mechanical energy.

This second interpretation requires the student to calculate a change in kinetic energy or a change in potential energy. Therefore it appeared necessary to include Skills 3 and 6 to account for this. This decision is consistent with White's (1974a, 1974b) suggestion that all connections that seem reasonably possible be included in the hierarchy since the validation process can destroy postulated connections but cannot create them.

Fourthly, the use of elements representing only verbal information was avoided in constructing the hypothesized hierarchy. Gagné (1977) restricts the use of hierarchies to the domain of intellectual skills. White (1973) has suggested that some attempts to validate hierarchies have not been successful because verbal information elements have been included in the hierarchy. It is possible an individual may be able to state the definition of a concept without understanding it. It is also possible that an individual may show understanding of that concept without being able to adequately define it. Consequently, for these reasons, the hierarchy hypothesized in the present study attempts to test understanding of each element of the hierarchy through application rather than mere verbalization.

Sample

The main sample consisted of 156 grade-ten students enrolled in an introductory physics program in two senior high schools under the jurisdiction of the Avalon Consolidated School Board in St. John's, Newfoundland. There were 97 boys and 59 girls. The mean age was 15.75 years with a standard deviation of 0.46 years. An additional sample of subjects of similar age from two other schools was used to further examine the relationship between several of the skills. This part of the procedure is explained in chapter four.

Instruments

Three instruments were administered during the course of the study. The composition of each will be described briefly in this section. Each is presented in detail in appendices 1-3.

Pretest. This is a 20-item test designed by the investigator to test mastery of the 10 skills in the hypothesized hierarchy. Each skill of the hypothesized hierarchy is represented by two questions. Ten questions, each representing a different skill, were randomly selected and numbered one to 10. The remaining 10 questions were also randomly selected and numbered 11 to 20. This randomization was used to prevent bias in favor of the hierarchy. The test questions were discussed with two science educators and three physics teachers in order to determine content validity.

All questions with the exception of those representing Skill 1 were free-response items. The questions representing Skill 1 were multiple-choice items which required an explanation for the choice selected. Subjects were required to answer each question in the space provided. Eighty minutes were allowed for completion of the test. In no case did it appear that there was insufficient time for all subjects to complete the test.

Posttest. The posttest was the same test as the pretest except for two changes. Each of these changes was made to reduce the effect of the pretest on posttest results. Firstly, the questions were presented in a different order. The 10 questions given on the second half of the pretest were scrambled and given on the first half of the posttest. The 10 questions given on the first half of the pretest were also scrambled and given on the second half of the posttest. Secondly, numerical values were changed in all questions except those testing Skill 1, where numerical values were not used. Hence, the two tests may be considered to be parallel forms.

Instructional booklet. The instructional booklet was designed by the investigator to allow remediation of the skills represented in the hypothesized hierarchy. The contents of the instructional booklet were discussed with two science educators and one physics teacher. These discussions resulted in several revisions before the final version was accepted.

Each subject was required to complete only those sections of the instructional booklet which covered the skills he or she had failed to demonstrate on the pretest. The sections to be studied by each individual subject were indicated in the instructional booklets by the investigator after marking of the pretest. Thus, each booklet represented a self-instructional remedial package for each individual subject.

Procedure

The hypothesized hierarchy was discussed with two science educators and three physics teachers. As a result of these discussions some changes were made. Following these discussions the test of skills in the hypothesized hierarchy was field-tested with an intact class of 28 grade-ten physics students from a school which was not participating in the main study. An analysis of the pilot test data resulted in some further changes to the hypothesized hierarchy, as well as some changes in some of the questions. The instructional booklet was also field-tested with this same class. The students followed the same instructions used in the main study and were asked to note any areas of difficulty as they worked through the booklet. One minor change was made to the instructional booklet as a result of student comments. In general, students indicated that the instructions in the booklet were easy to follow and understand. After these changes were made, the hypothesized hierarchy, test questions and instructional booklet were in the final form used in the study.

It has been suggested in chapter two that the method of instruction should not be a factor in whether or not the hierarchy exists. Hence, there was no interference with the teaching methods used by the teachers involved in the study. Only intact classes were used. After the teacher had covered the topic in class, each class was administered the pretest. In order to provide immediate feedback to subjects, the pretest was marked by the investigator the same day as it was written. For analysis purposes it was marked again later. Each subject was given the instructional booklet during his or her next physics class. In most cases this was the day immediately following the writing of the pretest. In no case was it more than two days following the writing of the pretest. Subjects were asked to study the instructional booklet in order to remediate skills which, according to the results of the pretest, were missing. Those skills which each subject missed were indicated on his or her copy of the instructional booklet. The posttest was administered one week after the pretest.

A regularly scheduled 80-minute double period was used to administer the pretest and posttest to each class. Subjects were advised, either by the investigator or the classroom teacher, that the tests represented part of a research study. After the initial introduction the investigator and the classroom teacher shared responsibility for supervision of the tests. All questions on the tests were answered on the test paper in spaces provided after each question. Subjects were instructed to show all of their calculations.

Chapter 4

ANALYSIS OF RESULTS

Introduction

The validation of a learning hierarchy is determined from the responses of subjects to tests of the intellectual skills represented in the hierarchy. Hence, the validity and reliability of these tests is an important consideration. This chapter begins with a discussion of the validity and reliability of the tests used in the present study. This is followed by application of the ordering-theoretic method (Airasian & Bart, 1975; Bart & Krus, 1973) and the Dayton and Macready (1976a) method to the data collected from the tests. The Dayton and Macready method is considered the main psychometric test of the data. The ordering-theoretic method was useful for an initial sifting of the data to suggest the most appropriate connections between skills for use with the Dayton and Macready test. As a result of these tests a psychometrically validated hierarchy is presented. The hypothesized hierarchy is also tested for positive transfer from subordinate to superordinate skills using the method suggested by Griffiths (1979).

For the application of the Dayton and Macready method the computer program developed by Dayton and Macready (1976b) was used. All other statistical procedures were performed using the SPSS 300 statistical package (Nie, Hull, Jenkins, Steinbrenner & Bent, 1975).

Validity of the Test Items

The test items used in the present study were designed to determine mastery or nonmastery of specific skills relating to the concept of conservation of mechanical energy as taught in high school physics. Hence, content validity is an important consideration in the construction of these tests. Consequently, the test items were discussed with two science educators and three physics teachers until agreement was reached about the content validity of the items. The items were also field-tested with one class of grade-ten physics students who were not participating in the main study. Analysis of these data resulted in some minor changes to the items.

Reliability of the Test Items

As explained in chapter three, both the pretest and posttest consist of 10 two-item tests, each representing a different skill from the hypothesized hierarchy. Since each test of a skill consists of only two items, conventional reliability statistics are not meaningful. However, it is reasonable to assume that if two items reliably test the same skill there should be a positive correlation between subject responses to these items. An appropriate correlation coefficient for use with nominal dichotomous data, as collected in the present study, is the phi coefficient. In theory, a correlation coefficient of unity should be obtained between two items testing the same skill. In practice, such a perfect correlation is seldom obtained since individual items, while representing the same skill, may not be identical in structure or presentation.

The phi coefficient presents an additional problem. Only in exceptional circumstances can the phi coefficient assume a value of unity. Gulliksen (1945) clearly indicated that the maximum value that can be assumed by the phi coefficient is usually less than this because it is extremely sensitive to the difficulty levels of the items being compared. Hence, interpretation of phi coefficients presents some difficulty. Nevertheless, phi coefficients significantly greater than zero should be obtained through two items testing the same skill. The values of the phi coefficients for the pretest and posttest are represented in Tables 1 and 2, respectively. The significance level reported in these tables represent the level at which the null hypothesis of no correlation between the test items can be rejected.

The values of the phi coefficients for the items in the pretest were all significantly different from zero at the .001 significance level. However, in almost all cases, the phi coefficients were reduced for the posttest. At first glance this seems surprising because the items being compared for the pretest and posttest are essentially the same items. However, Gulliksen's arguments applied to the present study suggests the improved performance of the subjects may be the reason for these reduced values. The formula for the maximum possible correlation that can be assumed between any two items is presented by Gulliksen (p. 79) as

$$r_{ij}^{\max} = \sqrt{\frac{P_j(1-P_i)}{P_i(1-P_j)}}\,, \quad (P_i \geqq P_j)$$

Table 1

Phi Coefficients Between Items Testing the Same Skill on the Pretest

Skill	Test Items	No. of Subjects	Phi Coefficient	Significance Level
10. Conservation of Mechanical Energy--nonvertical motion	9,18	149	.81	.001
9. Conservation of Mechanical Energy--vertical motion	6,14	147	.56	.001
8. Conservation of Mechanical Energy--basic application	3,16	149	.95	.001
7. Potential Energy--nonvertical reference point	5,11	149	.70	.001
6. Change in Potential Energy	7,19	148	.46	.001
5. Potential Energy--different reference levels	4,15	142	.55	.001
4. Potential Energy--basic application	2,17	152	.92	.001
3. Change in Kinetic Energy	1,20	148	.76	.001
2. Kinetic Energy--basic application	8,13	151	.94	.001
1. Discrimination between Potential Energy and Kinetic Energy	10,12	151	.38	.001

Table 2

Phi Coefficients Between Items Testing the Same Skill on the Posttest

Skill	Test Items	No. of Subjects	Phi Coefficient	Significance Level
10. Conservation of Mechanical Energy--nonvertical motion	6,15	139	.75	.001
9. Conservation of Mechanical Energy--vertical motion	8,12	141	.59	.001
8. Conservation of Mechanical Energy--basic application	2,11	140	.57	.001
7. Potential Energy--nonvertical reference point	3,17	140	.55	.001
6. Change in Potential Energy	10,13	141	.31	.001
5. Potential Energy--different reference levels	5,18	138	.34	.001
4. Potential Energy--basic application	9,20	142	.66	.001
3. Change in Kinetic Energy	4,14	142	.18	.05
2. Kinetic Energy--basic application	1,19	141	1.00	.001
1. Discrimination between Potential Energy and Kinetic Energy	7,16	140	.19	.05

where P_i = the proportion of subjects answering item i correctly

P_j = the proportion of subjects answering item j correctly

r_{ij}^{max} = the maximum possible correlation that could be obtained from these proportions.

The maximum correlation coefficient can only achieve unity when $P_i = P_j$. For any other proportions of P_i and P_j, the correlation coefficient is reduced. However, as Gulliksen demonstrates, the reduction of this coefficient is more sensitive to differences in the proportions answering each item correctly when these proportions depart from a 50% level. For example, if 50% and 45% of the subjects correctly answer items i and j, respectively, the maximum correlation coefficient that can be obtained for these items is 0.90. However, if 95% and 90% of the subjects correctly answer items i and j, respectively, the maximum correlation coefficient that can be obtained for these items is only 0.69. Hence, when items are answered correctly by a high proportion of subjects, even small variations in these items could lead to misleadingly low correlation coefficients. This characteristic of the phi coefficient is particularly relevant to the posttest items because a high proportion of subjects answered correctly most of these items. In spite of this, the phi coefficient for the items on the posttest were still significant at the .001 level of significance, with only two exceptions.

For each of these two exceptions, the correlation coefficients were significant at the .05 level. These two exceptions can be rationalized in terms of the difficulty level explanation described above. Hence, for Skill 1 (Discrimination between Potential Energy and Kinetic Energy), 92.1% answered item 16 correctly, 98.6% answered item 7 correctly and 91.4% answered both items correctly. These figures suggest a greater agreement than the phi coefficient (0.19) indicates. For Skill 3 (Change in Kinetic Energy), 94.4% answered item 14 correctly, 97.9% answered item 4 correctly and 93.0% answered both items correctly. Again these figures suggest greater agreement than the phi coefficient (0.18) indicates. Hence, all items were considered acceptable for the present study.

Application of the Ordering-Theoretic Method

Research question one asks "Does the arrangement of intellectual skills represented in the hypothesized hierarchy represent a learning hierarchy which is valid psychometrically?" If the answer to question one is negative question two will be considered. Research question two asks "Does some other arrangement of intellectual skills represented in the hypothesized hierarchy represent a learning hierarchy which is valid psychometrically?" The ordering-theoretic method was used as the initial test of the data in pursuit of the answer to these questions. This was followed by application of the Dayton and Macready method. Each of these methods was applied to the pretest data but not to the posttest data. It was felt that the posttest data could not be used to psychometrically validate the hypothesized hierarchy, as most subjects had to complete some section(s) in the instructional booklet before the administration of the posttest. It seems a reasonable assumption that the use of this booklet, which was designed to teach the skills of the hypothesized hierarchy, may bias the posttest data in favor of the hierarchy.

As indicated in chapter two, the ordering-theoretic method tests the relationship between pairs of skills. The results of applying this test to the pretest data are indicated in Table 3. This table contains the percentage of exceptions to the existence of a hierarchical

Table 3

Ordering-Theoretic Method: Percentage of Exceptions to Hierarchical Connections

		UPPER SKILL									
		10	9	8	7	6	5	4	3	2	1
LOWER SKILL	10	-	7.6 (144)	28.1 (146)	11.6 (146)	21.4 (145)	29.5 (139)	34.2 (149)	28.3 (145)	30.4 (148)	21.6 (148)
	9	3.5 (144)	-	16.7 (144)		13.3 (143)	17.3 (139)	21.1 (147)	16.8 (143)	18.5 (146)	14.4 (146)
	8	0.0 (146)	0.0 (144)	-							
	7	6.8 (146)			-			24.2 (149)			
	6	0.0 (145)	0.0 (143)			-	1.4 (140)	1.4 (148)			
	5	0.0 (139)	0.0 (139)			0.7 (140)	-	1.4 (142)			
	4	0.0 (149)	0.0 (147)		0.0 (149)	0.0 (148)	0.0 (142)	-			
	3	0.0 (145)	0.0 (143)						-	0.0 (147)	
	2	0.0 (148)	0.0 (146)						0.0 (147)	-	
	1	0.0 (148)	0.0 (146)								-

NOTE: Numbers in parentheses represent the number of subjects included in the analysis for the particular pair of skills under test.

relationship for each pair of skills both in the hypothesized direction and in the direction opposite to that hypothesized. The number of subjects for whom data were included varies from a minimum of 139 to a maximum of 149 for different skills, even though the original sample consisted of 156 subjects. This is because the responses of some subjects were difficult to interpret and were consequently recorded as missing data. In addition, four subjects participating in the overall study did not write the pretest.

The interpretation of Table 3 may be aided by an illustrative example. Consider Skills 10 and 5 in the table. If Skill 10 is suggested to be superordinate to Skill 5 there are no exceptions indicated for this arrangement. This means that none of the subjects tested exhibited Skill 10 without also exhibiting Skill 5. Thus, a hierarchical relationship is suggested. Considering the reverse connection of Skill 5 superordinate to Skill 10 there are 29.5% exceptions. This means that 29.5% of the subjects tested exhibited Skill 5 but did not exhibit Skill 10. These results offer strong support for the hypothesis that Skill 10 is superordinate to Skill 5.

The hierarchical relationships which emerge from the results in Table 3 are expressed in Table 4. Table 4 indicates those skills subordinate to each of the other skills allowing for 1, 2 and 5% exceptions, respectively. Clearly, slightly different hierarchies would emerge from analysis of the data at different levels of stringency. However, at all three levels of stringency 17 out of 21 hypothesized hierarchical dependencies were considered valid. Except for Skill 7 (Potential Energy--nonvertical reference point) and Skill 9 (Conservation of Mechanical Energy--vertical motion) no subjects exhibited the

Table 4

Summary of Hierarchical Connections Identified After Application of the Ordering-Theoretic Method

Level	10	9	8	7	6	5	4	3	2	1
1%	8	8			5	4		2	3	
	6	6		4	4					
	5	5								
	4	4								
	3	3								
	2	2								
	1	1								
2%	8	8			5	6	6	2	3	
	6	6		4	4	4	5			
	5	5								
	4	4								
	3	3								
	2	2								
	1	1								
5%	9	8			5	6	6	2	3	
	8	6		4	4	4	5			
	6	5								
	5	4								
	4	3								
	3	2								
	2	1								
	1									

Note: The interpretation of the above table may be aided by an illustrative example. Table 4 indicates that Skill 10 is superordinate to Skills 1, 2, 3, 4, 5, 6 and 8 at the 1%, 2% and 5% level and also superordinate to Skill 9 at the 5% level.

terminal skill, Skill 10 (Conservation of Mechanical Energy--non-vertical motion), without also exhibiting those skills hypothesized to be subordinate to Skill 10. The connection between Skill 10 and Skill 9 was found to be valid at the 5% level of exceptions. Skill 7, although exhibited by more subjects than Skill 10, was not found to be subordinate to it. However, this failure was marginal (6.8% exceptions). Examination of the test items for Skill 7 and Skill 10, respectively (Appendix 1), reveals a possible explanation for the failure to show a stronger hierarchical connection between these skills. Items testing Skill 7 included redundant information in the form of extra distances not actually needed to solve the item. This information was deliberately included to determine if subjects understood that gravitational potential energy depends on the vertical distance above a reference level. As expected, many subjects selected the wrong distance for the calculation of the potential energy. However, items testing Skill 10 included only those distances necessary to solve the items, as is common practice in classroom instruction and textbooks. It is possible that if extra distances had been given for the Skill 10 items, some subjects may have used these distances incorrectly. As it was, it seems possible that they may have used the only distances available without fully understanding the Skill 10 items. This argument is supported by analysis of the 6.8% of subject responses which were exceptions to the hierarchical connection. All of these subjects answered Skill 7 items incorrectly because they selected the distance along the slope instead of the vertical distance. It seems reasonable that these subjects may have selected the wrong distance again for Skill 10 items if these distances were available. Although the connections between Skill 10 and Skills 7 and 9, respectively, are not as strong as expected, they are sufficiently strong to warrant inclusion in the application of the Dayton and Macready test to the data. Also, some additional data were collected to further test the relationship between these skills. These data are discussed later in this chapter.

All of the hypothesized hierarchical relationships involving Skill 9 as the superordinate skill were strongly supported by the data. No subjects exhibited Skill 9 without also exhibiting the seven skills hypothesized to be subordinate to Skill 9. The hypothesis that Skill 7 is superordinate to Skill 4 is also strongly supported, with no exceptions.

Skills 4, 5 and 6 are related as hypothesized at the 1% level only. At the 2% and 5% levels neither skill is superordinate to the other. It appears that the distinction between these three skills is not sufficient to warrant all three in the hierarchy. Omission of Skill 6 (Change in Potential Energy) from the hierarchy can be justified. It was indicated in chapter three that Skill 6 was included as an alternative or disjunctive branch to Skill 9 which could be avoided by subjects. More importantly, Skill 6 can be viewed as two applications of Skill 4 (Potential Energy--basic application) followed by a subtraction of the values obtained. This distinction does not seem to be sufficient to justify Skill 6 as a separate skill. Hence, Skill 6 was eliminated from the hierarchy.

The failure to show a hierarchical connection between Skill 4 and Skill 5 at the 2% and 5% levels of exceptions is more difficult to understand. Skill 5 (Potential Energy--different reference levels) requires the understanding that potential energy depends on the particular reference level designated whereas Skill 4 does not require this understanding. This perceived distinction together with the tenuous evidence provided by the ordering-theoretic method seemed sufficient to warrant further testing. Thus, Skill 4 and Skill 5 were retained for the application of the Dayton and Macready test. Furthermore, a high percentage of subjects in the present sample had acquired both of Skills 4 and 5. Although this has no effect on the validity of a hierarchical connection, it makes this connection more difficult to validate empirically. Thus, additional data were collected relating to these skills in an attempt to clarify any possible hierarchical connection. These additional data will be discussed later in this chapter.

Skill 2 (Kinetic Energy--basic application) and Skill 3 (Change in Kinetic Energy) appear to be equivalent skills. No subjects exhibited Skill 2 without also exhibiting Skill 3; however, the same results are found when the reverse order is considered. Again, as with Skill 6, Skill 3 was included as a disjunctive branch of the hierarchy which could be avoided. Indeed, the relationship between Skill 2 and Skill 3 is similar to the relationship between Skill 6 and Skill 4. Skill 3 can be considered as two applications of Skill 2 followed by a subtraction of the two values obtained. It is understandable that this distinction may not be sufficient to justify them as separate skills for the hierarchy under study. Hence, Skill 3 was eliminated from the hypothesized hierarchy.

As indicated previously, most of the hypothesized connections were acceptable at all criterion levels and represent strong connections in terms of the ordering-theoretic method. Application of the ordering-theoretic method in conjunction with the arguments presented above suggest Hierarchy 1, which is presented in Figure 5. This hierarchy was further

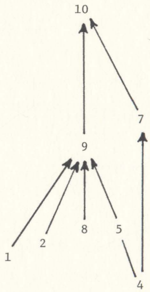


Figure 5. Hierarchy 1, the hierarchy remaining after application of the ordering-theoretic method.

investigated by application of the Dayton and Macready test.

Application of the Dayton and Macready Test

The Dayton and Macready test was described in detail in chapter two. Goodness of fit between the data and a hypothesized hierarchy is determined by both a chi-square analysis and the determination of a likelihood ratio expressed as a chi-square.

In computing the value of the likelihood ratio the Dayton and Macready test yields estimates of the misclassification parameters needed to provide a fit between the data and the hypothesized hierarchy. As these values increase, confidence in the particular hierarchy decreases. Hence, it is very important that subjects are properly classified as masters or nonmasters of each component skill. For this reason subjects who scored correctly on only one of the two items testing a particular skill were dropped from the analysis since it could not be determined whether or not they possessed the skill.

At present, the computer program which is essential to the application of the Dayton and Macready test can only accommodate small hierarchies. It cannot accommodate the number of true-response patterns implied by Hierarchy 1. Consequently, Hierarchy 1 was divided into two parts for application of the test. For convenience, these two parts will be labelled Sub-Hierarchy A and Sub-Hierarchy B, respectively. They are represented in Figure 6. Sub-Hierarchy A represents the connections between Skills 1, 2, 8, 9 and 10. Sub-Hierarchy B represents the connections between Skills 4, 5, 7, 9 and 10. The null hypothesis under test is that there is no significant difference between the observed frequencies of response patterns and those acceptable under the assumption that the hierarchy is valid. Large values for the

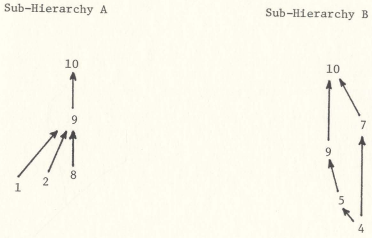


Figure 6. Sub-Hierarchies A and B.

maximum likelihood estimate and/or the misclassification parameters indicate rejection of a hierarchy under test. A significance level of less than .05 for the maximum likelihood estimate would indicate lack of sufficient correspondence between hierarchy and data. Table 5 indicates the value of the misclassification parameters and the likelihood estimates for Sub-Hierarchies A and B.

Clearly, the results presented in Table 5 indicate that both Sub-Hierarchies A and B are consistent with the data. The misclassification parameter, α, is zero for both sub-hierarchies and the misclassification parameter, β equals .02 and .04 for Sub-Hierarchies A and B, respectively. The significance level of the likelihood estimate is greater than .05 for both sub-hierarchies. The significance level for Sub-Hierarchy A is greater than .70, while for Sub-Hierarchy B it is greater than .30. Hence, Hierarchy 1 has been validated by the Dayton and Macready test.

Table 5 also indicates that the significance level obtained for Sub-Hierarchy A is higher than that obtained for Sub-Hierarchy B. Sub-Hierarchy B contains the relationship between Skill 7 and Skill 10 which has been discussed extensively in this chapter because the hypothesized connection between these skills narrowly missed acceptance for the ordering-theoretic method. Here, again, the results are suggestive that additional data could be helpful to clarify the question of validity of this connection. These additional data will now be discussed.

Supplementary Data

Earlier in this chapter it was indicated that the hypothesized hierarchical connection between Skill 7 and Skill 10 was not as strong

Table 5

Dayton and Macready Test: Likelihood and Misclassification Parameter Estimates for Sub-Hierarchies A and B

Sub-Hierarchy	Misclassification Parameter α	Misclassiciation Parameter β	Maximum Likelihood Estimate	Degrees of Freedom	Significance of Data
A	.00	.02	14.7	20	> .70
B	.00	.04	21.5	22	> .30

as anticipated. Arguments were presented which suggested that the items testing Skill 10 may be partially at fault. In particular, an alternative choice of distances was not included in these items. There was also discussion concerning the fact that, for the ordering-theoretic method, Skill 5 was superordinate to Skill 4 only at the 1% level of exceptions. It was suggested that a stronger hierarchical relationship may be hidden because a large percentage of subjects had no difficulty with either skill. The supplementary data and analysis which follow is an attempt to confirm the arguments presented regarding these connections and thereby strengthen the interpretation of the data. Skill 9 was included because, for the ordering-theoretic method, the hypothesized connection between Skill 9 and Skill 10 was accepted only at the 5% level of exceptions. For the purpose of the supplementary data the arrangement of skills under test is represented by Sub-Hierarchy B as shown previously in Figure 6 (p. 68). Sub-Hierarchy A was not further tested. Relationships involving the skills in this component of the complete hierarchy were strongly confirmed in the original analysis and did not require further testing.

For the supplementary study, Skill 10 items were changed from the original items by adding alternative distances. A minor change was made to the diagram for one Skill 5 item and one Skill 7 item, respectively, in order to improve their clarity. Items testing Skill 4 and Skill 9 remained unchanged.

The additional sample consisted of 123 grade-ten students enrolled in the same introductory physics program as those students involved in the main study. There were 87 boys and 36 girls. Five intact classes from two senior high schools which did not participate in the main study were selected. Two classes were from one school and three from the other school. In each case, all classes in each school were administered the test on the same day. As with the main data, these data were analyzed using the ordering-theoretic and Dayton and Macready methods.

The results of applying the ordering-theoretic method to the supplementary data are shown in Table 6. As explained previously in this chapter, the numbers in the table represent the percentage of exceptions to a hierarchical connection between each particular pair of skills. The number of subjects considered for each pair of skills under consideration is shown in parentheses in the table. Because of difficulty in interpreting the responses of some subjects, these numbers vary slightly for each pair of skills considered. Several important issues have been clarified by the supplementary data.

Firstly, the hypothesized connections between Skill 9 and Skill 10, and Skill 7 and Skill 10, are stronger for the supplementary data than for the main data. Skill 10 is superordinate to Skill 9 with no exceptions, compared to 3.5% exceptions for the main data. Skill 10 is also superordinate to Skill 7 with only 1.7% exceptions, compared to 6.8% exceptions for the main data. It appears that the arguments presented that stronger connections exist between these skills than was indicated by the main data have been supported.

Secondly, the hypothesized connection between Skill 5 and Skill 4 has been strengthened, but it still represents the weakest connection in the hierarchy. Both sets of data indicate no exceptions to the hypothesis that Skill 5 is superordinate to Skill 4. There are exceptions in both sets of data if Skill 4 is considered superordinate to Skill 5. However, these skills could be considered equivalent at the

Table 6

Ordering-Theoretic Method:
Percentage of Exceptions to Hierarchical Connections:
Supplementary Data

		UPPER SKILL 10	9	7	5	4
	10	-	7.0 (114)	40.7 (118)	57.0 (114)	81.2 (117)
	9	0.0 (114)	-		31.2 (109)	51.8 (112)
LOWER SKILL	7	1.7 (118)		-		32.5 (117)
	5	0.0 (114)	0.0 (109)		-	3.5 (113)
	4	0.0 (117)	0.0 (112)	0.0 (117)	0.0 (113)	-

NOTE: Numbers in parentheses represent the number of subjects included in the analysis for the particular pair of skills under test.

5% and 2% levels of exceptions for the supplementary and main data, respectively. As suggested previously in this chapter, the high percentage of subjects who acquired both skills makes validation difficult. For the main sample, 77.5% showed mastery of both skills while 63.7% showed mastery of both skills for the supplementary sample. In each case the percentage is high enough to be detrimental to the determination of a hierarchical relationship. However, the hypothesized connection is more strongly supported by the supplementary data which has the smaller percentage with mastery of both skills. It can be speculated that a study containing below average subjects or some other method of analysis (such as used by Uprichard, 1970) may show a stronger hierarchical connection between Skill 4 and Skill 5 than is indicated in the present study. However, there was no opportunity to pursue this in the present study.

Thirdly, the hypotheses that Skill 9 is superordinate to Skill 5 and that Skill 7 is superordinate to Skill 4 are again strongly supported, with no exceptions to the relationship in each case.

The Dayton and Macready test applied to the supplementary data yielded misclassification parameters of .04 and .00 for α and β, respectively, for Sub-Hierarchy B. The maximum likelihood estimate was 20.5 with 22 degrees of freedom. The significance of this estimate is greater than .50. Again, the arrangement of skills represented by Sub-Hierarchy B has been validated by the Dayton and Macready test. In addition, the significance of the likelihood estimate has increased from the value obtained for the main sample.

Sub-Hierarchy B has been strongly validated by application of the Dayton and Macready test to both the main data and the supplementary data. Sub-Hierarchy A was already strongly validated by application of this test to the main data. Connections between each particular pair of skills within these sub-hierarchies have also been validated by application of the ordering-theoretic method at one or more levels of stringency. Hence, research question one has been answered negatively but research question two has been answered positively. The hypothesized hierarchy was not supported in its entirety. However the composite of Sub-Hierarchies A and B which has been previously designated Hierarchy 1 is considered to be a psychometrically valid hierarchy and is shown in more detail in Figure 7.

Transfer of Learning Within the Hypothesized Hierarchy

Research question three asks "Do any connections between pairs of intellectual skills represented in the hypothesized hierarchy represent connections which are valid in terms of transfer of learning?" As indicated in chapter two, Gagné's index of proportion positive transfer and other indices related to it are not considered to be acceptable tests of the degree of transfer between skills in a hierarchy. Several other methods of testing for transfer were described in chapter two and were also concluded to be inappropriate for the present study. Hence, a method suggested by Griffiths (1979) was applied to the main data. A test of transfer could not be applied to the supplementary data because instructional booklets and posttests were not administered to these subjects. The details of Griffiths' method of transfer were described in chapter two. Essentially, the method involves testing subjects on all skills of the hypothesized hierarchy followed by instruction designed to remediate missing skills. This is followed by a posttest of the skills. A chi-square test is then used to determine

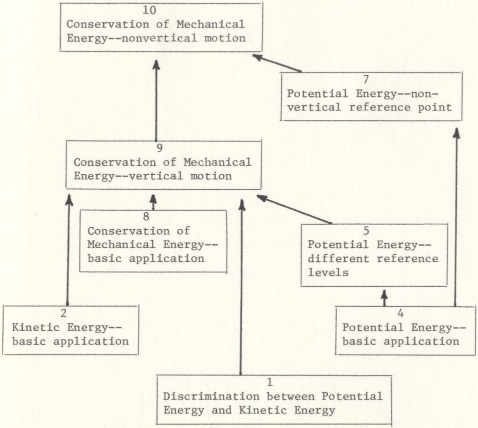


Figure 7. Hierarchy 1, the psychometrically valid hierarchy.

if those subjects who gained prerequisite skills between tests are more successful on posttest items testing the superordinate skills than are those subjects who failed to gain the prerequisite skills.

Two major difficulties became apparent from application of Griffiths' test of transfer to the data in the present study. First, the test requires that a substantial number of subjects must fail to show mastery of the hypothesized skills on the pretest because these subjects represent the sample for application of the test. This is, of course, less likely for the lower skills in a hierarchy than for the upper skills. In the present study, the pilot data did not suggest an acute problem. However, subjects in the main sample were more successful, thereby making it more difficult to test for transfer. A second major difficulty arises when two or more skills are conjointly prerequisite to another skill. Griffiths' test of transfer does not provide a means of determining the proportional effect of transfer which occurs from each prerequisite skill.

These difficulties were very evident in the present study. In particular, only the connection between Skill 7 and Skill 10 and the connection between Skill 9 and Skill 10 contained sufficient numbers for a meaningful test of transfer. The results are reported in Table 7.

Significant transfer of learning was found from Skill 9 to Skill 10 ($p < .001$). Thus, learning of Skill 9 should significantly enhance learning of Skill 10. Significant transfer ($p < .05$) was also indicated for the relationship of Skill 7 subordinate to Skill 10. This implies that learning of Skill 7 should also significantly enhance learning of Skill 10. In the present study, research question three could only be answered for the hypothesized connections between Skills

Table 7

Griffiths' Test of Transfer from
Subordinate to Superordinate Skills

Connection	No-Gain Fail (N)	No-Gain Pass (N)	Gain Fail (N)	Gain Pass (N)	χ^2	df	Significance
7 to 10*	3	0	4	18	5.18	1	< .05
9 to 10+	6	2	0	12			< .001

*Corrected chi-square.

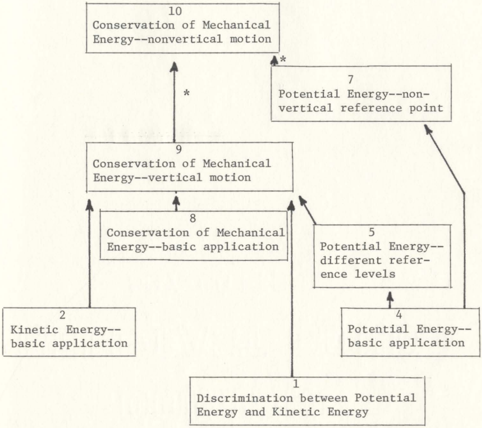
+Fisher's exact test applied.

7, 9 and 10 and has been answered positively. Hierarchy 2, which includes the relationships validated in the psychometric and transfer sense, is presented in Figure 8 and is discussed in the next section.

The Structure of the Validated Hierarchy

Hierarchy 2 is similar to the hypothesized hierarchy. All nine skills hypothesized to be subordinate to Skill 10 (Conservation of Mechanical Energy--nonvertical motion) were found empirically to be subordinate in terms of the psychometric definition of hierarchical dependency. However, although Skill 3 (Change in Kinetic Energy) and Skill 6 (Change in Potential Energy) were subordinate to Skill 10, they were eliminated from the hypothesized hierarchy. It was clearly indicated from application of the ordering-theoretic method that Skill 3 was not sufficiently different from Skill 2 (Kinetic Energy--basic application) to warrant inclusion as a separate skill. Similarly, Skill 6 was eliminated because it was not sufficiently different from Skill 4 (Potential Energy--basic application). It appears that the ability to calculate changes in kinetic energy (Skill 3) or changes in potential energy (Skill 6) should require little, if any, instruction when learners can already exhibit Skills 2 and 4, respectively.

Two relationships hypothesized to exist between pairs of skills were validated using the psychometric and transfer definitions of hierarchical dependency. These were the relationships between Skill 7 and Skill 10 and between Skill 9 and Skill 10. The ability to exhibit Skill 7 (Potential Energy--nonvertical reference point) was necessary for the learning of Skill 10 (Conservation of Mechanical Energy--nonvertical motion). Further, Skill 7 enhanced this learning as evidenced by a significant transfer effect. Similarly, Skill 9 (Conservation



NOTE: *indicates that the connection is valid in terms of both the psychometric and transfer definitions of hierarchical dependency.

Figure 8. Hierarchy 2, the validated hierarchy.

of Mechanical Energy--vertical motion) was necessary for, and enhanced learning of Skill 10.

Finally, all other hierarchical relationships represented by Hierarchy 2 have been substantiated in terms of the psychometric definition. None of these relationships were denied in terms of the transfer definition. However, they could not be tested in terms of this definition because the number of subjects failing to exhibit subordinate skills in these relationships was too small to allow a meaningful interpretation of the results.

The final section of this chapter contains a discussion of the misconceptions held by subjects regarding the concept of conservation of mechanical energy.

Subjects' Misconceptions Pertaining to the Concept of Conservation of Mechanical Energy and Related Subordinate Skills

Some intellectual skills, such as those represented by terminal skills in learning hierarchies, require the mastery of several subordinate skills. According to Gagné, all of these subordinate skills must be mastered before the terminal skill can be mastered. When students have difficulty with a terminal skill, it is important that the reason(s) for this difficulty is identified. However, classroom testing often concentrates on testing for achievement of the terminal skill only. Hence, when several skills are prerequisite to a particular terminal skill, the analysis of only those items testing the terminal skill may not reveal all of the underlying misconceptions. This is especially true for weaker students who often cannot attempt, in any meaningful way, the items testing complex skills.

One of the advantages of hierarchically related learning is that it lends itself readily to analysis of misconceptions. Each subordinate skill can be analyzed separately for misconceptions that may not be revealed by analysis of only the test items representing a terminal skill. The identification of these misconceptions allows for their possible elimination during the development of curriculum materials and instruction. This could be accomplished by designing materials and instruction to avoid these misconceptions or by advising students about the misconceptions that are possible during the learning of each new skill.

It should be noted that the instructional booklet used in the present study to remediate missing skills did not deliberately attempt to eliminate the misconceptions reported in this section. The format of the present study required that subjects be given the instructional booklet almost immediately following the pretest. Hence, the instructional booklet required preparation well in advance of the analysis of subject misconceptions. However, it was apparent that during the interim between pretest and posttest many of the misconceptions were eliminated. It is possible that the instructional booklet eliminated many of these misconceptions, but the design of the study did not include direct testing for this.

The importance and potential usefulness of this aspect of hierarchical learning led to the consideration of research question four. Research question four asks "What misconceptions do high school students hold with respect to the intellectual skills represented in the hypothesized hierarchy?" In order to answer this question, all items which were answered incorrectly on the pretest were scrutinized to uncover subjects' misconceptions. In cases where a subject exhibited more than one misconception for a particular skill, each separate misconception is reported. In cases where a subject exhibited the same misconception on both items testing the same skill, this misconception was counted only once. In many other cases it was impossible to interpret a subject's approach to a particular item. Further, it was arbitrarily decided that only those misconceptions which were held by at least four subjects would be reported. Table 8 summarizes the misconceptions observed for each skill. The percentages reported in Table 8 are based on the 152 subjects who wrote the pretest as part of the main study. The misconceptions for each skill will now be discussed.

For Skill 1 (Discrimination between Potential Energy and Kinetic Energy), the form(s) of energy associated with a moving object seemed to confuse some subjects. In particular, eight subjects (5.3%) indicated that a moving object has both kinetic and potential energy because it is moving and will continue to move. In addition, other subjects were confused with the relationship between work and energy. Five subjects (3.3%) indicated that a force pushing the object gave it kinetic and potential energy. Under particular circumstances, a force which does work on an object can give that object kinetic energy or potential energy or both. However, these circumstances were not present in items testing Skill 1. Finally, for Skill 1, there was some confusion concerning the interpretation of the definition of energy. Energy is usually defined as the capacity to do work. It seems that four subjects (2.6%) incorrectly interpreted this definition to mean that an object must be doing work in order to have energy. It is possible that they do not understand the meaning of the word "capacity" in this definition.

Table 8

Misconceptions Relating to the Skills of the Hypothesized Conservation of Mechanical Energy Hierarchy

Skill	Misconception	No. of Subjects	%
1. (a)	Stating that an object has both kinetic energy and potential energy because it is moving and will continue to move.	8	5.3
(b)	Stating that a force pushing an object gives it kinetic and potential energy.	5	3.3
(c)	Stating that an object has neither kinetic energy nor potential energy because it is not doing any work.	4	2.6
2.	Use of an incorrect formula $\frac{1}{2}mgv^2$ for the calculation of kinetic energy.	11	7.2
3. (a)	Use of an incorrect formula $\frac{1}{2}mgv^2$ for the calculation of kinetic energy (same as Skill 2).	12	7.9
(b)	Use of an incorrect formula $\frac{1}{2}m(\Delta V)^2$ for the calculation of the change in kinetic energy.	6	3.9
4.	No misconceptions reported.		
5.	Calculating the potential energy with respect to the bottom level in diagram instead of the level designated.	17	11.2
6.	Calculating the correct change in potential energy between the two levels designated but then subtracting this value from the potential energy at one of the two levels.	19	12.5
7. (a)	Using the distance along the slope to calculate the potential energy.	47	30.9
(b)	Stating the potential energy equals zero.	5	3.3
8.	Using ratio of potential energy over kinetic energy of an object at one position to be equal to the ratio of potential energy over kinetic energy of the object at a second position.	5	3.3

Table 8 (Continued)

Skill	Misconception	No. of Subjects	%
9. (a)	Claiming that the final kinetic energy of an object equals its potential energy.	14	9.2
(b)	Calculating the potential energy of an object using the given speeds as heights (h) in the formula for potential energy (mgh).	13	8.6
(c)	Claiming the potential energy of a falling object at a designated level equals its kinetic energy at this level.	11	7.2
(d)	Calculating the kinetic energy using the given heights as speed (v) in the formula for kinetic energy ($\frac{1}{2}mv^2$).	8	5.3
(e)	Claiming the initial kinetic energy of an object equals its potential energy at any other position as the object moves.	4	2.6
(f)	Use of an incorrect formula $\frac{1}{2}mgv^2$ for the calculation of kinetic energy (same as Skill 2).	5	3.3
10. (a)	Claiming the kinetic energy of the object at its initial position equals its kinetic energy at the final position.	32	21.1
(b)	Omitting the initial kinetic energy of the object.	10	6.6
(c)	Using ratio of the given heights of the object to be equal to the ratio of the initial and final speeds of the object.	7	4.6
(d)	Use of an incorrect formula $\frac{1}{2}mgv^2$ for the calculation of kinetic energy (same as for Skill 2).	5	3.3

For Skill 2 (Kinetic Energy--basic application) and Skill 3 (Change in Kinetic Energy) the misconception, use of an incorrect formula $\frac{1}{2}mgv^2$ for the calculation of kinetic energy, was observed for 11 subjects (7.2%) and 12 subjects (7.9%), respectively. A given mass of an object must be multiplied by the acceleration due to gravity in order to calculate potential energy, but not to calculate kinetic energy. This misconception may be related to confusion between the formulas for potential energy and kinetic energy or it may be a misunderstanding of the definition of kinetic energy. An additional misconception is peculiar to Skill 3. Six subjects (3.9%) were not aware that the square of the difference between two numbers does not equal the difference between the squares of the same two numbers. For example,

$$(6-2)^2 = 16 \text{ whereas}$$

$$6^2-2^2 \text{ should equal } 32.$$

When calculating the change in kinetic energy, these subjects subtracted the two speeds and then squared this value, whereas they should have squared the speeds and then subtracted the values obtained.

There was little difficulty with Skill 4 items (Potential Energy--basic application) and no misconceptions are reported. Seventeen subjects (11.2%) held the single misconception found in the items testing Skill 5 (Potential Energy--different reference levels). There are two possible explanations suggested for this misconception. Firstly, some subjects appeared to believe that potential energy is always calculated with respect to the lowest point indicated in the problem. Secondly, subjects may not understand the term "with respect to" as used in potential energy problems, even though this is the expression used in the textbook for the course and is also the expression used during instruction in the classroom.

The one misconception relating to Skill 6 (Change in Potential Energy) is difficult to explain. Nineteen subjects (12.5%) calculated the change in potential energy between the two levels indicated, but they were not aware that this was the answer required. Consequently, these subjects subtracted this answer from the potential energy at one of these two levels with respect to some other level. There was no apparent reason for doing this.

A misconception concerning Skill 7 (Potential Energy--nonvertical reference point) was held by more subjects than any other misconception for any skill. Forty-seven subjects (30.9%) used the distance along the slope in the calculation of potential energy. This represents a serious deficiency in the understanding of potential energy for these subjects because it limits their ability to apply the conservation of mechanical energy to vertical motion only. Five subjects (3.3%) indicated that the potential energy of the object was zero with respect to the level designated, even though the object was clearly above this level. Again it seems that the term, "with respect to," had been misunderstood.

One misconception was identified concerning Skill 8 (Conservation of Mechanical Energy--basic application). Five subjects (3.3%) conceived the notion that the ratio of potential energy over kinetic energy of an object at one position equals the ratio of the potential energy over kinetic energy at other positions. These subjects were aware that some form of relationship existed between potential energy and kinetic energy. It appears that they have misunderstood the meaning of "conservation" as it pertains to the concept of conservation of mechanical energy.

For Skill 9 (Conservation of Mechanical Energy--vertical motion), the misconceptions labelled 9(a), 9(c) and 9(e) in Table 8 and exhibited by 9.2%, 7.2% and 2.6% of the subjects, respectively, may be considered as one general misconception. These subjects seemed to be under the illusion that the calculation of the value of one of the two forms of mechanical energy also yields the value of the other form. Consequently, the possibility of an object simultaneously having potential energy and kinetic energy seemed to confuse them. Also relating to Skill 9, 13 subjects (8.6%) substituted given speeds of the objects as heights in the formula for potential energy. Similarly, eight subjects (5.3%) substituted given heights of the object as speed in the formula for kinetic energy. These subjects appear to be "plugging" into the formula for potential and kinetic energy, using whatever information is available. The mechanical approach used by these subjects suggests little, if any, understanding of the conservation of mechanical energy. Lastly, the misconception which was initially indicated for Skill 2 (use of an incorrect formula $\frac{1}{2}mgv^2$ for the calculation of kinetic energy) shows up once again in Skill 9, being exhibited by five subjects (3.3%).

The major misconception associated with the terminal skill, Skill 10 (Conservation of Mechanical Energy--nonvertical motion), is that 32 subjects (21.1%) simply calculated the kinetic energy of the cart at position A and claimed that this was also the kinetic energy at position B. These subjects did not seem to understand that the potential energy, and therefore the kinetic energy, changes as the cart moves from A to B. It is likely that many of these subjects did not reach the level of understanding of the concept of conservation of mechanical energy required to proceed further than this initial calculation of the kinetic energy at position A. Hence, they simply stated that the kinetic energy at A was also the kinetic energy at B. Ten subjects (6.6%) ignored the initial speed of the cart at position A. It seems likely that some of these subjects would have correctly answered the item if the cart was initially stationary at position A. Again, as with some misconceptions relating to Skill 9, subjects seemed confused when objects simultaneously have potential and kinetic energy. Seven subjects (4.6%) equated the ratio of the given heights to the ratio of the initial and final speeds. The kinetic energy was then calculated using the value of the final speed obtained from this ratio. This procedure seems to represent an intuitive approach which avoids application of the conservation of mechanical energy. However, this procedure is incorrect and represents a serious misconception for these subjects. Lastly, a misconception (use of an incorrect formula $\frac{1}{2}mgv^2$ for the calculation of kinetic energy) which was prevalent in Skills 2, 3 and 9 is again evident in Skill 10, being exhibited by five subjects (3.3%).

It is apparent from the above discussion that many of the misconceptions which were evident from subordinate skill items were not evident from analysis of the terminal skill only, although some were. Generally, each separate skill contained misconceptions peculiar to that skill. Hence, any one misconception or combination of misconceptions could lead to a student's inability to perform the terminal skill. In particular, many misconceptions would have been missed if only Skill 10 items were analyzed. The ability to identify problems in the learning of intellectual skills through identification of misconceptions relating to subordinate skills represents a potentially important application of the hierarchical learning model.

Summary of Results

In this chapter the research questions which form the basis of this study have been investigated. Research question one was answered negatively. The hypothesized hierarchy was not supported in its entirety. The answer to research question two was positive. An alternative hierarchy was identified which was validated psychometrically by application of the ordering-theoretic method and the Dayton and Macready test. Some marginally insignificant connections within this hierarchy were supported more strongly after collection and analysis of additional data. The identification of transfer between related skills was hindered by large percentages of subjects showing mastery of lower level skills on the pretest of the hypothesized hierarchy. Consequently, positive transfer could only be investigated for some connections at the highest level within the hierarchy where numbers were sufficient to provide an appropriate test of the data. The existence of positive transfer obtained for these connections supported the psychometrically validated hierarchy. Finally, misconceptions relating to the skills in the hypothesized hierarchy were reported and discussed.

Chapter 5

SUMMARY, IMPLICATIONS AND RECOMMENDATIONS

Summary

The major purpose of this study was to identify a learning hierarchy leading to learning the concept of conservation of mechanical energy, to the level normally found in high school physics courses. The identification of this hierarchy was considered from both a psychometric and transfer point of view. As discussed in chapter two, many of the methods of validating learning hierarchies have been the subject of appropriate criticisms. Several more recent methods show promise but have not been used extensively for hierarchy validation. The application of several of these methods was considered an important aspect of this study. It is hoped that the application of these methods within the present study will contribute to the information available with respect to the practical application of these methods. A secondary purpose of the study was to determine particular misconceptions which high school students hold with respect to the intellectual skills leading to the learning of the concept of conservation of mechanical energy. Case (1978) suggests a combination of task analysis and identification of individuals' misconceptions to be a profitable combination for the production of better instructional materials, especially in a remedial setting. The determination of misconceptions for the concept of conservation of mechanical energy represents an attempt to provide useful information for future instructional development relating to this concept.

A learning hierarchy for the concept of conservation of mechanical energy was derived using a Gagné-type task analysis. This hierarchy was discussed with experts in the subject area and field-tested with an intact class of grade-ten students. As a result, some modifications were made to the hierarchy. Subjects from five intact grade-ten classes were administered a pretest developed by the investigator to test the skills of the hypothesized hierarchy. This was followed by an instructional booklet designed by the investigator to remediate missing skills. Missing skills were identified for each individual subject from the analysis of the pretest. After remediation the subjects were administered a posttest which was identical to the pretest except for minor numerical changes in most items and a change in the order of presentation of the items. The test and the instructional booklet were discussed with content experts and field-tested prior to their administration.

Two psychometric tests were applied to the data. These were the ordering-theoretic method and the Dayton and Macready method. As a result of these analyses, two skills were eliminated from the hypothesized hierarchy. In addition, some connections were identified which marginally missed acceptance by the ordering-theoretic method. These connections were retested with a different sample of grade-ten students. Finally, the arrangement of skills represented in Figure 7 (p. 76) is considered to represent a psychometrically valid hierarchy. A test of transfer developed by Griffiths was also applied to the data. However, the small numbers failing lower skills limited the application of this method to the connections between skills at the upper levels of the hierarchy. In these cases, the results were supportive of the psychometrically validated hierarchy and are represented in Figure 8 (p. 80).

Lastly, all items which were answered incorrectly on the pretest were scrutinized in order to uncover possible misconceptions relating to each skill within the hypothesized hierarchy. These were reported and discussed.

Implications

The major implication of this study concerns the support for Gagné's model of hierarchical learning. A number of skills have been identified, each of which is a necessary prerequisite to an understanding of the conservation of mechanical energy. Where it could be tested, it was also evident that the learning of subordinate skills provides positive transfer to the learning of superordinate skills.

A number of hierarchy validation methods are presently available but no one method seems entirely acceptable. The application of different methods to the same data is perhaps the safest procedure at this time. Two psychometric methods were applied in the present study. These were the ordering-theoretic method which considers skills in pairs and the Dayton and Macready method which considers the composite hierarchy. Each has limitations. The ordering-theoretic method does not allow for errors of measurement. Consequently, an arbitrary tolerance level is usually allowed. Unfortunately, the arrangement of skills may vary depending on the choice of tolerance level. This method also requires a mastery decision for each subject for each skill. However, the method is simple to apply and, for the present study, yielded results which were usually consistent with the Dayton and Macready method. Hence, the method seems convenient for an initial sifting of the data from which some marginal connections may be investigated by more sophisticated analysis. The Dayton and Macready method also requires a mastery decision for each subject for each skill. However, it considers the composite hierarchy and provides for an appropriate statistical test of the goodness of fit between the hypothesized hierarchy and the data. This represents a major advantage over the arbitrarily chosen tolerance levels of the ordering-theoretic method.

Testing for transfer between skills in a hierarchy is a problem. In the present study a method developed by Griffiths was utilized. Two disadvantages of it became apparent. Firstly, the method requires a mastery decision for each subject for each skill on a pretest and a posttest. Analysis of the pretest provides an initial sample for the test of transfer which includes only those subjects who are classified as nonmasters for the skills under test. The size of this sample may be small if one of the skills under test is a relatively easy skill which most subjects have acquired. Mastery decisions necessary for these same subjects on the posttest can further reduce the sample to be tested. Within the context of the present study, the size of the sample was often reduced to such an extent that the test of transfer between skills was not meaningful. Hence, the application of this method seems to require larger numbers of unsuccessful subjects than were available in the present study. Secondly, this method does not provide a means to determine the proportional effect of transfer which can be attributed to each prerequisite skill when two or more skills are prerequisite to the same skill. However, other available methods appear to be even less satisfactory. Given the importance of establishing transfer of learning between skills, this is a pressing problem for learning hierarchy researchers.

The ability to uncover misconceptions peculiar to subordinate skills, which were not evident from analysis of only the terminal skill, represents a major implication. The hierarchical learning model is usually considered for its application to the sequencing of instruction. The results of the present study suggest it may also serve a valuable role in the identification of student difficulties. In addition, these discovered misconceptions could influence the design of new instructional materials. For example, the misconceptions reported in this study should be an aid to anyone interested in designing instruction for the concept of conservation of mechanical energy.

The final implication of the present study is related to the addition of information, within a test item, which is not necessary for the solution of the item. The present study indicates that it is sometimes possible to solve test items and still have serious misconceptions relating to the skill under test. It is common for examiners and textbook writers to provide only the information necessary to solve a test item. It is uncommon to provide excess information. However, the present study implies that some excess information in test items may be beneficial to the determination of the subject's understanding.

Suggestions for Further Research

1. Some success has been attained in the few studies that have applied the hierarchical model to science concepts. It is suggested that this model be applied to other concepts in science.

2. The results of applying different methods of validation to common data should be further investigated.

3. The Dayton and Macready method should be extended to include the testing of larger hierarchies.

4. The development of more effective methods of testing for transfer between subordinate and superordinate skills in learning hierarchies should be investigated.

5. The application of the hierarchical model to help uncover students' misconceptions should be further investigated.

6. The effect of excess information on test performance should be investigated.

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APPENDICES

NOTE: In each test the spaces left for answers have been decreased from the original test.

APPENDIX 1. Conservation of Mechanical Energy Pretest

STUDENT INSTRUCTIONS

This is a closed book test. You are asked to answer all of the questions in the space provided on this paper. If you do not have enough space to answer a question, place the remainder of the answer on the reverse side of the paper.

It is very important that you show all of your work, as you will lose no marks for any work which is incorrect, only for incorrect or incomplete answers.

If you find that you are having difficulty answering any one question, do not spend too long on it. Proceed to the next question.

Attempt as many questions as you can, even if you cannot complete it.

NAME ____________________

SCHOOL ____________________

DATE ____________________

1. A 2 kg ball is thrown upward with a speed of 30 m/s. How much has the kinetic energy of the ball changed when it has slowed to 10 m/s?

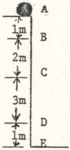
2. A 5 kg box is lifted vertically upward for a distance of 10 meters above the floor. What is its potential energy with respect to the floor?

3. A brick falls off a high building. At a certain point during its fall, the kinetic energy of the brick is 800 J and its potential energy with respect to the ground is 1,000 J. As it falls further, its potential energy is reduced to 600 J with respect to the ground. What is its kinetic energy at this point?

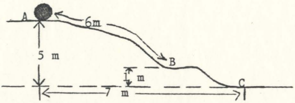
4. A stone of mass 2 kg rests on a ledge as shown.

   (a) What is its potential energy at A with respect to B?

   (b) What is its potential energy at A with respect to D?



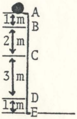
5.



A ball of mass 2 kg rolls from A to C as shown above. What was its potential energy at A with respect to position B?

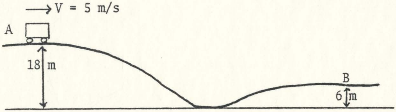
6. A stone of mass 2 kg is thrown vertically upward with a speed of 6 m/s. What is its potential energy when it slows down to 2 m/s?

7. A 2 kg stone rests on a ledge as shown. If the stone falls from the ledge, what is its change in potential energy between levels B and D?



8. A cart of mass 100 kg has a speed of 5 m/s. What is its kinetic energy?

9.



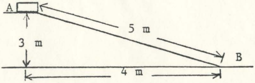
A cart of mass 20 kg rolls past point A with a speed of 5 m/s as shown above. Find its kinetic energy at B. (Assume the track is frictionless).

10. A box slides across a horizontal floor at a speed of 10 m/s. The box has:

(a) kinetic energy
(b) potential energy
(c) kinetic energy and potential energy
(d) neither kinetic energy nor potential energy

Briefly explain your choice.

11.



A box of mass 3 kg rests at the top of an incline as shown above. What is its potential energy with respect to B?

12. A stone rests on a ledge 100 meters above the ground below. With respect to the ground below, the stone has:

(a) kinetic energy
(b) potential energy
(c) kinetic and potential energy
(d) neither kinetic nor potential energy

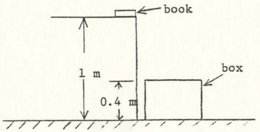
Briefly explain your choice.

13. A 2 kg stone is dropped from a building. What is its kinetic energy when it reaches a speed of 2 m/s?

14. A boy standing on a bridge drops a 4 kg stone into the water below him. If the boy dropped the stone from 50 meters above the water, what was its kinetic energy when it was 20 meters above the water?

15. A book of mass 1 kg rests on the top of a table which is 1 meter above the floor. A box which is 0.4 meters high rests on the floor. Find:

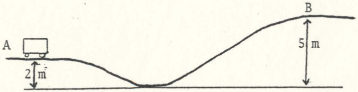
(a) potential energy of the book with respect to the floor
(b) potential energy of the book with respect to the top of the box



16. A boy throws a stone in the air. At one point as the stone rises, its potential energy is 400 J and its kinetic energy is 800 J. As it rises further, its kinetic energy is reduced to 300 J. What is its potential energy at this point?

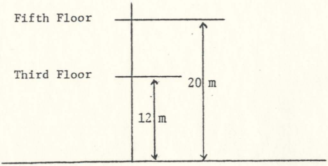
17. A man of mass 70 kg is about to jump from a window 20 meters above the ground. What is his potential energy with respect to the ground?

18.



A cart of mass 2 kg passes point A with a speed of 10 m/s and rolls up a hill past point B as shown above. Find the kinetic energy of the cart as it passes point B. (Assume the track is frictionless).

19. A 70 kg man rides an elevator from the third floor to the fifth floor of a large building. The distance from the ground to the third floor is 12 meters and the distance from the ground to the fifth floor is 20 meters. What is the man's <u>change</u> in potential energy when he moves from the third floor to the fifth floor?



20. A 6 kg stone is thrown from a mountain ledge with a speed of 4 m/s. A little while later its speed is 9 m/s. How much has the kinetic energy of the stone <u>changed</u> when it increases speed to 9 m/s?

APPENDIX 2: Conservation of Mechanical Energy Posttest

STUDENT INSTRUCTIONS

This is a closed book test. You are asked to answer all of the questions in the space provided on this paper. If you do not have enough space to answer a question, place the remainder of the answer on the reverse side of the page.

It is very important that you show all of your work, as you will lose no marks for any work which is incorrect, only for incorrect or incomplete answers.

If you find that you are having difficulty answering any one question, do not spend too long on it. Proceed to the next question.

Attempt as many questions as you can, even if you cannot complete it.

NAME ______________________

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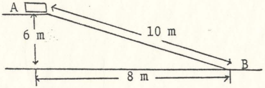
1. A 4 kg stone is dropped from a building. What is its kinetic energy when it reaches a speed of 5 m/s?

2. A boy throws a stone in the air. At one point as the stone rises, its potential energy is 300 J and its kinetic energy is 800 J. As it rises further, its kinetic energy is reduced to 500 J. What is its potential energy at this point?

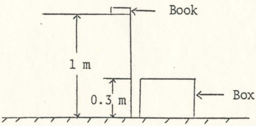
3. 

A box of mass 4 kg rests at the top of an incline as shown above. What is its potential energy with respect to B?

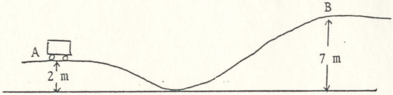
4. A 4 kg stone is thrown from a mountain ledge with a speed of 5 m/s. A little while later its speed is 7 m/s. How much has the kinetic energy of the stone <u>changed</u> when it increases speed to 7 m/s?

5. A book of mass 2 kg rests on the top of a table which is 1 meter above the floor. A box which is 0.3 meters high rests on the floor. Find:

(a) potential energy of the book with respect to the floor
(b) potential energy of the book with respect to the top of the box



6.



A cart of mass 4 kg passes point A with a speed of 20 m/s and rolls up a hill past point B as shown above. Find the kinetic energy of the cart as it passes point B. (Assume the track is frictionless).

7. A stone rests on a ledge 100 meters above the ground below. With respect to the ground below, the stone has:

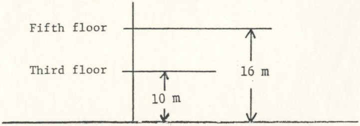
(a) kinetic energy
(b) potential energy
(c) kinetic and potential energy
(d) neither kinetic nor potential energy

Briefly explain your choice.

8. A boy standing on a bridge drops a 4 kg stone into the water below him. If the boy dropped the stone from 40 meters above the water, what was its kinetic energy when it was 10 meters above the water?

9. A man of mass 80 kg is about to jump from a window 30 meters above the ground. What is his potential energy with respect to the ground?

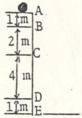
10. A 60 kg man rides an elevator from the third floor to the fifth floor of a large building. The distance from the ground to the third floor is 10 meters, and the distance from the ground to the fifth floor is 16 meters. What is the man's change in potential energy when he moves from the third to the fifth floor?



11. A brick falls of a high building. At a certain point during its fall, the kinetic energy of the brick is 600 J, and its potential energy with respect to the ground is 1,000 J. As it falls further, its potential energy is reduced to 200 J with respect to the ground. What is its kinetic energy at this point?

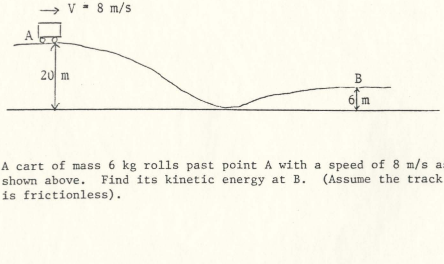
12. A stone of mass 2 kg is thrown vertically upward with a speed of 8 m/s. What is its <u>potential</u> energy when it slows down to 3 m/s?

13. A 3 kg stone rests on a ledge as shown. If the stone falls from the ledge, what is its <u>change</u> in potential energy between levels B and D?



14. A 2 kg ball is thrown upward with a speed of 50 m/s. How much has the kinetic energy of the ball <u>changed</u> when it has slowed to 20 m/s?

15.

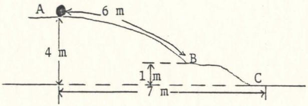


A cart of mass 6 kg rolls past point A with a speed of 8 m/s as shown above. Find its kinetic energy at B. (Assume the track is frictionless).

16. A box slides across a horizontal floor at a speed of 10 m/s. The box has:

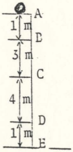
(a) kinetic energy
(b) potential energy
(c) kinetic energy and potential energy
(d) neither kinetic nor potential energy

17.



A ball of mass 5 kg rolls from A to C as shown above. What was its potential energy at A with respect to position B?

18. A stone of mass 5 kg rests on a ledge as shown.



(a) What is its potential energy at A with respect to B?

(b) What is its potential energy at A with respect to D?

19. A cart of mass 80 kg has a speed of 3 m/s. What is its kinetic energy?

20. A 3 kg box is lifted vertically upward for a distance of 8 meters above the floor. What is its potential energy with respect to the floor?

APPENDIX 3: Instructional Booklet

You have recently completed a test which included the concepts of kinetic energy, gravitational potential energy and conservation of mechanical energy. Your results indicate that you had difficulty with some of the questions. It is important that you overcome these difficulties <u>now</u> if you are to have success with some future topics and future physics courses.

The material in this booklet will help you overcome these difficulties. However, it can only do this if you read the instructions carefully and if you are prepared to use a little "energy"!

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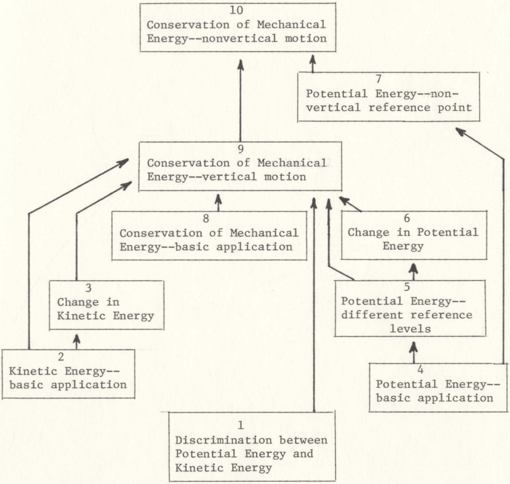
Conservation of Mechanical Energy

Introduction

The material covered in this instructional booklet is summarized in the chart below:



Each of the numbered blocks in the chart represent a "skill" which you need to acquire if you are to have success with conservation of mechanical energy problems. You have recently written a test which covered these skills. These results indicate that you are having difficulty with some of the skills listed in the chart. This instructional booklet is intended to help you overcome these difficulties. Since you are not having difficulty with all of the skills in the chart, you do not have to cover all the material in this booklet. You just cover the material related to the skills which are giving you difficulty!

The skills which gave you difficulty are indicated below by a check (√) mark in front of these skills.

NOTE: P.E. means potential energy
K.E. means kinetic energy

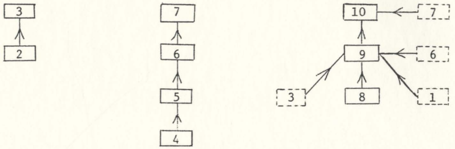
The skills with the check marks (√) in front of them are the skills you need to cover in this booklet. Start with the <u>lowest numbered</u> skill which has a check mark in front of it. Cover all the material for this skill. When you have done this go to the next lowest numbered skill that has a check mark in front of it. Continue this procedure until you have covered all the skills that have check marks in front of them.

The chart of the skills shows you how the skills are connected. Do not be too concerned if you do not fully understand the chart. Its main purpose is to show you that to master a more complex skill like skill number 10, it is necessary to master other skills first.

For each skill there is a question which tests that skill called a representative question. The solution to this question is then given in the instructions for the skill. After the instructions, there is an additional question given for practice. The answers for these additional questions are given on page 23.

The chart of skills can be divided into three sections as follows:

<u>Section I</u> (K.E.) <u>Section II</u> (P.E.) <u>Section III</u> (Conservation of Mechanical energy)



You can see from the diagram for each section that the skills are arranged vertically. If you have mastered <u>all</u> the skills in Section I and especially Skill No. 3 which is the top skill, then you have a good understanding of kinetic energy.

If you have mastered <u>all</u> the skills in Section II and especially the top skills you have a good understanding of gravitational potential energy.

Section III includes the higher level skills (Nos. 8, 9 and 10). Section III skills involve using the skills you have learned in Sections I and II, so that you can solve conservation of mechanical energy problems. The diagram of Section III skills shows you how the skills from Sections I and II lead into Section III skills.

Skill No. 1 is an introductory skill which covers the difference between gravitational potential energy and kinetic energy. This is a very necessary skill for an understanding of conservation of mechanical energy.

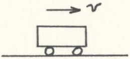
Some skills you have already mastered. The skills listed in the key to skills on page 2 with check marks in front of them are giving you difficulty. When you master these skills, you should have a good understanding of K.E., P.E. and conservation of mechanical energy.

The work you put into this booklet will be energy well used!

SKILL I	You should be able to distinguish between objects that have kinetic energy and those which have gravitational potential energy

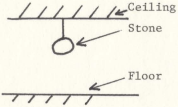
Representative Question: Skill I

(a)



Car moving to the right with speed v

(b)



Stone suspended above the floor

From the above diagrams, which object has gravitational potential energy? Which object has kinetic energy? Why?

Instructions: Skill I

Scientists define energy as the capacity to do work. If the cart in diagram (a) above struck another movable object in its path, it would exert a force on this object, move the object and thus do work. This energy which the cart has because it is moving is called kinetic energy.

Other examples of kinetic energy are:

1. Water flowing in a river
2. An apple falling from a tree
3. A car travelling on the highway

Whereas an object with kinetic energy can do work because it is moving, other objects can have gravitational potential energy. An object with gravitational potential energy has the "potential" to do work due to its height above a level below it. In diagram (b) above, if the cord holding the stone were cut, the stone would fall because of the force of gravity. Thus, the stone has gravitational potential energy because if released it would fall and could then do work on something (perhaps a large spike needs hammering) below it.

Other examples of gravitational potential energy are:

1. A rock resting on the top of a cliff
2. Water being held by a 'dam'
3. A flower pot resting on a window ledge

It is common for objects to have potential energy and kinetic energy at the same time. Some examples are:

1. Water coming down a waterfall
2. A car moving up or down a hill
3. A ball falling through the air

These have kinetic energy because they are moving and potential energy because they are above some level to which they can fall.

PRACTICE PROBLEM (Answer on page 23)

Indicate which one of the following has (a) gravitational potential energy, (b) kinetic energy and (c) both gravitational potential energy and kinetic energy.

1. A bowling ball moving toward the pins
2. A parachutist falling through the air
3. A man about to jump off a ledge

Instruction

Once you have completed the practice problem, turn back to the key on page 2. Find the next lowest numbered skill which has a check mark in front of it. Then locate the page number where the material related to this skill begins in the booklet. Turn to that page and proceed to complete the material under this skill.

SECTION I

SKILL 2	You should be able to calculate the kinetic energy of an object given its mass and speed

Representative Question: Skill 2

Calculate the kinetic energy of a 0.5 kg bullet if its speed is 600 m/s.



$V = 600$ m/s

"BANG!!"

m = .05 kg

Instructions: Skill 2

The kinetic energy of any object is given by the formula:

$$K.E. = \tfrac{1}{2}mV^2$$

where m = the mass of the object
V = the speed of the object

Since energy is the capacity to do work, it has the same units as work, that is, joules. If the mass of the object is expressed in kilograms and the speed in m/s, then the kinetic energy will be in joules.

Therefore, to solve the above question for Skill 3:

$$\begin{aligned} K.E. &= \tfrac{1}{2}mV^2 \\ &= \tfrac{1}{2}\ (.05\ \text{kg})\ (600\ \text{m/s})^2 \\ &= 9000\ \text{joules} \end{aligned}$$

PRACTICE PROBLEM: Skill 2 (Answer on page 23)

A girl throws her physics textbook at her boyfriend with a speed of 5 m/s. If the book has a mass of one kilogram, what is its kinetic energy?

Instruction

Once you have completed the practice problem, turn back to the key on page 2. Find the next lowest numbered skill which has a check mark in front of it. Then locate the page number where the material related to this skill begins in the booklet. Turn to that page and proceed to complete the material under this skill.

SKILL 3	You should be able to calculate the change in kinetic energy by an object given V_1, V_2 and its mass

Representative Question: Skill 3

A 2000 kg Ford Thunderbird car passes a small foreign car. In doing this it speeds up from 10 m/s to 30 m/s. What is its change in kinetic energy?

Instructions: Skill 3

Since the car has changed its speed, it has also changed its kinetic energy. However, the kinetic energy depends on the speed squared and also the mass of the car. There are two methods of finding the change in kinetic energy.

Method 1 K.E. at 30 m/s $= \frac{1}{2}$ (2000) $(30)^2$ = 900,000 joules
K.E. at 10 m/s $= \frac{1}{2}$ (2000) $(10)^2$ = 100,000 joules
Change in kinetic energy = 900,000 - 100,000 = 800,000 joules

Method 2 Change in K.E. = K.E. at 30 m/s - K.E. at 10 m/s

$$= \tfrac{1}{2}(2000)(30)^2 - \tfrac{1}{2}(2000)(10)^2$$
$$= \tfrac{1}{2}(2000)(30^2 - 10^2)$$
$$= \tfrac{1}{2}(2000)(900 - 100)$$
$$= 800{,}000 \text{ joules}$$

NOTE: Either method is acceptable.

The same methods would be used for an object slowing down except that the object would lose kinetic energy.

PRACTICE PROBLEM: Skill 3 (Answer on page 23)

A driver of a 2000 kg Ford Thunderbird passes a small foreign car, and notices a big, local policeman behind the wheel. The Thunderbird slows down from 30 m/s to 20 m/s. What is its change in kinetic energy?

Instruction

Once you have completed the practice problem, turn back to the key on page 2. Find the next lowest numbered skill which has a check mark in front of it. Then locate the page number where the material related to this skill begins in the booklet. Turn to that page and proceed to complete the material under this skill.

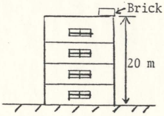
SECTION II

Skills 2 and 3 of Section I covered skills concerning kinetic energy. Skills 4, 5, 6, 7, 8 and 9 of this section cover another form of energy called gravitational potential energy.

SKILL 4	You should be able to calculate the gravitational potential energy of an object given its mass and height above a given level

Representative Question: Skill 4

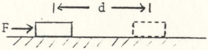
The top of a building is 20 meters above the ground. A brick with a mass of 2 kg sits on top of the building. What is the potential energy of the brick with respect to the ground?



Instructions: Skill 4

You should have learned previously (classroom lectures or textbooks) that if a force acts on an object and succeeds in moving it, then that force does work on the object. The formula for this work is given by:

W = F X d
Where W = work done
F = force acting on the object
d = distance the object is moved by the force



When an object falls through the air, the force causing it to fall is the force of gravity commonly called the object's weight. Thus, when the formula above is used on a falling object, F equals the weight of the object.

But weight = mass X acceleration due to gravity
= m X g
∴ W = F X d can be expressed as
W = m X g X d for a falling object

This is the amount of work an object could do if it fell a distance "d". When it is at a distance "d" above some level, it has the "potential" to do this work. This is what is called gravitational potential energy. Thus, gravitational potential energy is given by the following formula:

$\boxed{P.E. = m\ g\ d}$

where m = mass of the object
g = acceleration due to gravity
d = height above given level

Therefore, to solve the above question for Skill 4:

$$P.E. = m\ g\ d = 2\ kg \times 9.8\ m/s^2 \times 20\ m$$
$$= 392\ \text{joules}$$

PRACTICE PROBLEM: Skill 4 (Answer on page 23)

A one kg book rests on a shelf. The shelf is two meters above the floor. Find the potential energy of the book with respect to the floor.

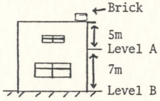
Instruction

Once you have completed the practice problem, turn back to the key on page 2. Find the next lowest numbered skill which has a check mark in front of it. Then locate the page number where the material related to this skill begins in the booklet. Turn to that page and proceed to complete the material under this skill.

SKILL 5	You should be able to calculate the potential energy of an object for different reference levels

Representative Question: Skill 5

If the brick resting on top of the building has a mass of 3 kg, find its potential energy with respect to (i) level A, (ii) level B and (iii) the top of the building.



Instructions: Skill 5

The potential energy of the brick depends on the reference level you designate. Once you have designated the reference level, then the potential energy of the brick depends on its height above this level.

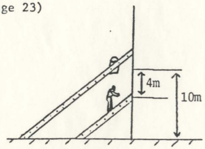
$\therefore$ P.E. with respect to Level A = m g d = 3 kg X 9.8 m/s^2 X 5m = 147 J
However, the brick is 12 meters above Level B, therefore,
P.E. with respect to Level B = m g d = 3 kg X 9.8 m/s^2 X 12 m = 353 J
The brick is zero meters above the top of the building, therefore,
P.E. with respect to the top of the building = m g d = 3 kg X 9.8 m/s^2 X 0m = 0 J

Thus, the potential energy is different depending on the reference level you chose to measure it from. If the brick were to fall to Level A, it could do 147 joules of work, thus, it has 147 joules of potential energy with respect to Level A. However, if the brick were to fall to Level B, it could do 353 joules of work, therefore, its potential energy with respect to Level B is 353 joules.

Since the brick is already resting on the top of the building, it cannot fall to the top of the building and therefore, it has zero potential energy with respect to the top of the building.

PRACTICE PROBLEM: Skill 5 (Answer on page 23)

A bucket of paint with a mass of 2 kg rests dangerously on a ladder 10 meters above the ground. A painter on another ladder has foolishly positioned himself such that his head is 4 meters below the bucket of paint. Calculate the potential energy of the bucket of paint with respect to (a) the ground and (b) the painter's head.



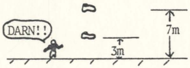
Instruction

Once you have completed the practice problem, turn back to the key on page 2. Find the next lowest numbered skill which has a check mark in front of it. Then locate the page number where the material related to this skill begins in the booklet. Turn to that page and proceed to complete the material under this skill.

SKILL 6	You should be able to calculate the change in potential energy (ΔP.E.) of an object between different heights above the same reference level as it rises or falls vertically

Representative Question: Skill 6

A basketball player after losing the championship game throws his running shoe (or sneaker) into the air. If the running shoe has a mass of 0.2 kg, what is its change in potential energy when it rises from 3 meters to 7 meters above the gym floor?



Instructions: Skill 6

There are three method of solving this question.

Method 1 P.E. with respect to gym floor at 3 meters = m g d = 0.2 kg X 9.8 m/s^2 X 3 m = 5.9 J
P.E. with respect to gym floor at 7 meters = m g d = 0.2 kg X 9.8 m/s^2 X 7m = 13.7 J
Therefore, change in P.E. from 3 meters to 7 meters = 13.7 J - 5.9 J = 7.8 J

Method 2 Since P.E. = m g d, any change in d results in a change in P.E. When the running shoe rises upward from 3m to 7m, this is a change in height (Δd) of 4m. Therefore, Change in P.E. from 3m to 7m = m g Δ d = 0.2 kg X 9.8 m/s^2 X 4m = 7.8 J

Method 3 Designate the 3m level as the reference level. When the running shoe is 7m above the floor, it is 4m above the designated reference level. Therefore,
P.E. at 7 m level with respect to 3 m level = m g d = 0.2 kg X 9.8 m/s^2 X 4m = 7.8 J

<u>PRACTICE PROBLEM:</u> Skill 6 (Answer on page 23)

A 70 kg physics teacher after a frustrating day in the classroom, decides to end it all by jumping out the window of a high building. Being a devout physicist, he calculates the change in his potential energy between when he is 20 meters and 5 meters above the ground. What answer did he get?

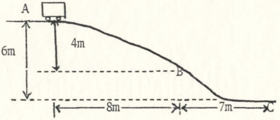
<u>Instruction</u>

Once you have completed the practice problem, turn back to the key on page 2. Find the next lowest numbered skill which has a check mark in front of it. Then locate the page number where the material related to this skill begins in the booklet. Turn to that page and proceed to complete the material under this skill.

SKILL 7	You should be able to calculate the potential energy of an object above a given point when the object is not directly above that point

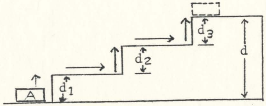
Representative Question: Skill 7

A 5 kg cart rolls from A to C. What was its P.E. at B with respect to C?



Instructions: Skill 7

Notice in the above question that the cart does not move only vertically as did the objects in all previous examples. Fortunately, this doesn't make any difference to the calculation of potential energy. The reasoning which follows may help you understand this.



A box A is raised to the top of a set of stairs as shown above. The box gains potential energy only when it is moved vertically. The horizontal movements do not affect its potential energy. Objects which move horizontally do not have work done on them against gravity and therefore, horizontal motion does not affect gravitational potential energy.

Therefore, the potential energy given to box A with respect to its former location at the bottom of the stairs is $m\ g\ d_1 + m\ g\ d_2 + m\ g\ d_3 = m\ g\ (d_1 + d_2 + d_3) = m\ g\ d$. This means that to calculate the potential energy of the box when it is carried up the stairs, the vertical distance only is required.

The potential energy of an object above any point depends only on its vertical distance above that point (assuming that its mass is constant).

Solution to Representative Question

$$\begin{aligned} \text{P.E. at B with respect to C} &= m\ g\ d \\ &= 5\ \text{kg} \times 9.8\ \text{m/s}^2 \times (6\text{m} - 4\text{m}) \\ &= 98\ \text{J} \end{aligned}$$

Notice that the vertical distance between B and C is used and it was not necessary to know the other distances.

PRACTICE PROBLEM: Skill 7 (Answer on page 23)

In the above Representative Question for Skill 11, what was the potential energy of the cart at position A with respect to C?

Instruction

Once you have completed the practice problem, turn back to the key on page 2. Find the next lowest numbered skill which has a check mark in front of it. Then locate the page number where the material related to this skill begins in the booklet. Turn to that page and proceed to complete the material under this skill.

SECTION III

Skills 8, 9 and 10 which complete this booklet, cover conservation of mechanical energy.

In order to understand conservation of mechanical energy, you must first understand kinetic energy and gravitational potential energy. Therefore, at this point in this booklet, you should have completed any skills in Sections I and II which gave you difficulty (as indicated by the check marks in the key to skills).

SKILL 8	If you are given the P.E. and K.E. of an object at one position, you should be able to apply the conservation of mechanical energy to find the P.E. of the object at a second position given its K.E. at this position or find its K.E. at a second position given its P.E. at this position

Representative Question: Skill 8

A ball is dropped from a high building. At one position as it falls its potential energy is 10,000 joules and its kinetic energy is 2,000 joules. At another position as it falls, its kinetic energy is 8,000 joules. What is the potential energy at this position?

Instructions: Skill 8

Mechanical energy is defined as the sum of an object's potential energy and kinetic energy. In many situations, if one can neglect frictional forces and if changes in the speed of an object are due to the force of gravity, then mechanical energy is conserved.

This means that the sum of the potential energy and kinetic energy is a constant. Stated another way, any loss of potential energy results in an equal gain of kinetic energy, or any gain of potential energy results in an equal loss of kinetic energy such that the total mechanical energy is always constant.

In the question above for Skill 8, if friction is neglected, the mechanical energy is conserved. Therefore,

Total mechanical energy = P.E. + K.E. at 1st position
= 10,000 J + 2,000 J
= 12,000 J

at 2nd position total mechanical energy must be 12,000 J since mechanical energy is conserved. Therefore,

P.E. + K.E. = 12,000 J at 2nd position
∴ P.E. = 12,000 J − K.E.
P.E. = 12,000 J − 8,000 J
P.E. = 4,000 J

PRACTICE PROBLEM: Skill 8 (Answer on page 23)

A stone is thrown upward and at a certain height it has a potential energy of 1,000 J and a kinetic energy of 5,000 J. What is the kinetic energy of the stone when its potential energy is 4,000 J?

Instruction

Once you have completed the practice problem, turn back to the key on page 2. Find the next lowest numbered skill which has a check mark in front of it. Then locate the page number where the material related to this skill begins in the booklet. Turn to that page and proceed to complete the material under this skill.

SKILL 9	You should be able to calculate the total mechanical energy, P.E. and K.E. for objects which follow a vertical path, i.e., objects which fall vertically or objects which are thrown vertically upward

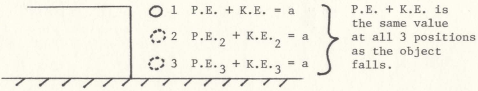
Representative Question: Skill 9

A 2 kg stone is dropped from a window ledge which is 20m above the ground below. When it has fallen to a point 5m above the ground, find (a) its total mechanical energy with respect to ground level
(b) its P.E. with respect to ground level
(c) its K.E.

Instructions: Skill 9

Mechanical energy is conserved in the above situation (if we ignore the friction caused by the object passing through the air).

If mechanical energy is conserved, this means the sum of the P.E. and K.E. is constant for all positions of the object as it moves up or down (depending on the problem you get). This does not mean that P.E. is constant or that K.E. is constant, only that the sum of the two is constant. Therefore, all losses of P.E. result in equal gains in K.E. (or the reverse in the case of rising objects), such that the total of the two is always constant. The diagram below illustrates what happens when an object falls.



As an object falls, it loses P.E. but gains K.E. As an object rises, it loses K.E. but gains P.E.

N.B. Pages 28 and 29 in Unit 3 of Physics: A Human Endeavour, gives a partial proof that mechanical energy is conserved in the absence of friction. You may, if you wish, read the following numerical example to show that mechanical energy is conserved for an object which is falling due to gravity.

A 3 kg object is dropped from a height of 100 meters above the ground. What is its total mechanical energy at the moment it is released and what is its total mechanical energy after it has been falling for one second?

P.E. when released = m g d = 3 kg X 9.8 m/s^2 X 100m = 2940 J
K.E. when released = 0 since it is not moving
Therefore, <u>total mechanical energy at start</u> = <u>2940 J</u>

<u>1 second later</u>

Its speed can be found from $V_f = V_i + at = 0 + 9.8\ m/s^2 \times 1\ s$
$= 9.8\ m/s$

Therefore, its K.E. = ½ (3 kg) $(9.8\ m/s)^2$ = 144.1 J

The distance it has fallen in 1 second can be found using:
$S = V_i\ t + \frac{1}{2} at^2 = 0 + \frac{1}{2} (9.8\ m/s^2)\ (1\ s)^2 = 4.9\ m$
If it has fallen 4.9m, it is still 100m - 4.9m = 95.1m above the ground.
Therefore, its potential energy = 3 kg X 9.8 m/s^2 X 95.1m
= 2795.9 J

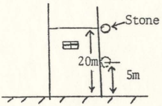
<u>Total mechanical energy after object has fallen for 1 second</u>

K.E. + P.E. = 144.1 J + 2795.9 J = <u>2940 J</u>

The total mechanical energy has stayed constant.

<u>Solution to Question: Skill 9</u>

At the instant the stone is dropped, it has only P.E. since it is not moving. Therefore, its total mechanical energy is all P.E. As the stone falls, it loses P.E. but increases speed thereby gaining K.E. such that the total mechanical energy is constant.



(a) Total mechanical energy = P.E. + K.E.
but initial K.E. = 0
∴ Total mechanical energy = P.E. + 0 = 2 kg X 9.8 m/s^2 X 20m
= 392 J with respect to the ground.
At 5m above the ground, the stone must still have 392 J of mechanical energy but now it has both P.E. and K.E.
∴ at 5m above the ground
P.E. + K.E. = 392 J

(b) P.E. at 5m above the ground = m g d = 2 kg X 9.8 m/s^2 X 5m = 98 J

(c) P.E. + K.E. = 392 J
∴ K.E. at 5m above the ground = 392 J - 98 J = 294 J

PRACTICE PROBLEM: Skill 9 (Answer on page 23)

A 2 kg ball is thrown vertically upward with an initial speed of 30 m/s. When it has risen 6m calculate:

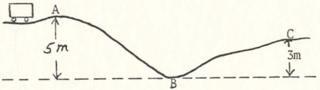
(a) its total mechanical energy
(b) its P.E.
(c) its K.E.

Instruction

Once you have completed the practice problem, turn back to the key on page 2. Find the next lowest numbered skill which has a check mark in front of it. Then locate the page number where the material related to this skill begins in the booklet. Turn to that page and proceed to complete the material under this skill.

SKILL 10	In situations where mechanical energy is conserved, you should be able to calculate the total mechanical energy, potential energy and kinetic energy for objects which follow a nonvertical path

Representative Question: Skill 10



A cart of mass 4 kg rolls past position A with a speed of 10 m/s.

(a) What is its total mechanical energy?
(b) What is its K.E. at position B?
(c) What is its K.E. at position C?

Instructions: Skill 10

If we assume there is no friction present, then mechanical energy is conserved in this problem. This means that the sum of the potential energy and kinetic energy of the cart is always constant as it rolls from A to C. Of course, we have to choose a reference level for potential energy. The choice of a reference level depends on the questions you are asked, but the most convenient place to take is usually the lowest level the object reaches.

As the cart rolls from A to B, it loses P.E., but gains the same amount of K.E. As it travels from B to C, it regains some of its P.E. and loses some K.E. At all locations during its motion, the sum of P.E. + K.E. is constant.

The total mechanical energy can be found from information about the cart as it passes position A.

B is used here as the reference level for zero potential energy.

(a) Total mechanical energy $= P.E._A + K.E._A$

$$= m\ g\ d + \tfrac{1}{2}mv^2$$
$$= 4\text{ kg} \times 9.8\text{ m/s}^2 \times 5\text{m} + \tfrac{1}{2}\ (4\text{ kg})\ (10\text{ m/s})^2$$
$$= 196\text{ J} + 200\text{ J}$$
$$= 396\text{ J}$$

(b) At location B, the P.E. equals zero, but total mechanical energy must still equal 396 J

$\therefore$ $P.E._B + K.E._B$ = total mechanical energy

$0 + K.E._B = 396$ J

$\therefore K.E._B = 396$ J

(c) At location C, the ball has P.E. and K.E.

$P.E._C + K.E._C = 396$ J

At C, the ball has regained P.E. given by

$P.E._C = m\ g\ d = 4\text{ kg} \times 9.8\text{ m/s}^2 \times 3\text{m}$

$= 118$ J

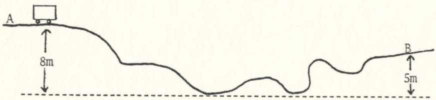
Since $P.E._C + K.E._C = 396$ J

and $P.E._C = 118$ J

Therefore $K.E._C = 396\text{ J} - 118\text{ J}$

$= 278$ J

<u>PRACTICE PROBLEM:</u> Skill 10 (Answer on page 23)



A cart of mass 2 kg rolls past A with a speed of 4 m/s, rolls down the hill and over the track shown above and eventually passes position B. What is the kinetic energy of the cart as it passes position B?

Answers to Practice Problems (using $g = 9.8\ m/s^2$)

Skill 1, page 4:
1. kinetic energy
2. kinetic energy and potential energy
3. potential energy

Skill 2, page 6: K.E. = 12.5 J

Skill 3, page 7: Δ K.E. = 500,000 J

Skill 4, page 8: P.E. = 19.6 J

Skill 5, page 10: (a) P.E. = 196 J with respect to the ground
(b) P.E. = 78.4 J with respect to the painter's head

Skill 6, page 12: Δ P.E. = 10,290 J

Skill 7, page 14: P.E. = 294 J

Skill 8, page 16: K.E. = 2,000 J

Skill 9, page 18: (a) at the instant it is thrown upward, its total mechanical energy is all kinetic energy

$$\begin{aligned} \therefore \text{total mechanical energy} &= \text{K.E.} + \text{P.E.} \\ &= \tfrac{1}{2}mV^2 + 0 \\ &= \tfrac{1}{2}\,(2\ \text{kg})\,(30\ \text{m/s})^2 \\ &= 900\ \text{J} \end{aligned}$$

(b) P.E. = 118 J
(c) K.E. = 900 J - 118 J = 782 J

Skill 10, page 21: K.E. = 75 J

APPENDIX 4: Supplementary Test

STUDENT INSTRUCTIONS

This is a closed book test. You are asked to answer all of the questions in the space provided on this paper. If you do not have enough space to answer a question, place the remainder of the answer on the reverse side of the page.

It is very important that you show all of your work, as you will lose no marks for any work which is incorrect, only for incorrect or incomplete answers.

If you find that you are having difficulty answering any one question, do not spend too long on it. Proceed to the next question.

Attempt as many questions as you can, even if you cannot complete it.

NAME____________________________

SCHOOL__________________________

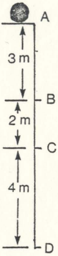
DATE____________________________

1. A 5 kg box is lifted vertically upward for a distance of 10 meters above the floor. What is its potential energy with respect to the floor?

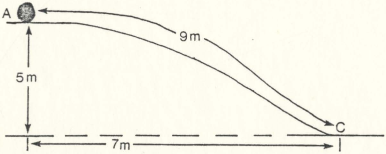
2. A stone of mass 2 kg rests on a ledge as shown.

   (a) What is its potential energy at A with respect to B?

   (b) What is its potential energy at A with respect to C?



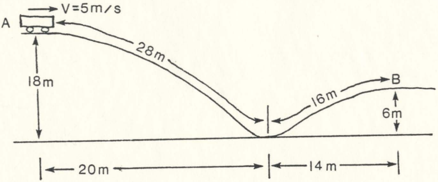
3.



A ball of mass 2 kg rolls from A to C as shown above. What was its potential energy at A with respect to position C?

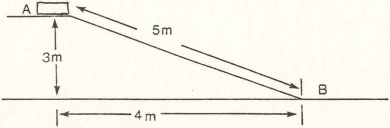
4. A stone of mass 2 kg is thrown vertically upward with a speed of 6 m/s. What is its potential energy when it slows down to 2 m/s?

5.



A cart of mass 20 kg rolls past point A with a speed of 5 m/s as shown above. Find its kinetic energy at B. (Assume the track is frictionless).

6.

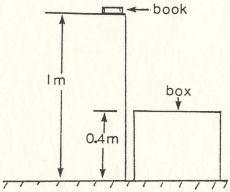


A box of mass 3 kg rests at the top of an incline as shown above. What is its potential energy with respect to B?

7. A boy standing on a bridge drops a 4 kg stone into the water below him. If the boy dropped the stone from 50 meters above the water, what was its kinetic energy when it was 20 meters above the water?

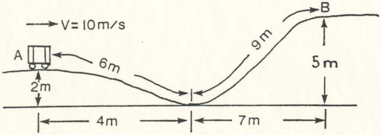
8. A book of mass 1 kg rests on the top of a table which is 1 meter above the floor. A box which is 0.4 meters high rests on the floor. Find:

(a) potential energy of the book with respect to the floor.
(b) potential energy of the book with respect to the top of the box.



9. A man of mass 70 kg is about to jump from a window 20 meters above the ground. What is his potential energy with respect to the ground?

10.



A cart of mass 2 kg passes point A with a speed of 10 m/s and rolls up a hill past point B as shown above. Find the kinetic energy of the cart as it passes point B. (Assume the track is frictionless).

APPENDIX 5: Test Scores on the Pretest, the Posttest and the Supplementary Test

(1 = correct response; 0 = incorrect response; 9 = missing information)

Pretest

Student No.	Skill	1		2		3		4		5		6		7		8		9		10	
	Item	10	12	8	13	1	20	2	17	4	15	7	19	5	11	3	16	6	14	9	18
1		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5		1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	9	1
6		1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1
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Posttest

Student No.	Skill	1		2		3		4		5		6		7		8		9		10	
	Item	16	7	19	1	14	4	20	9	18	5	13	10	17	3	11	2	12	8	15	6
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4		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
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23		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
24		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
25		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
26		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
27		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
28		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
29		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
30		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
31		9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
32		1	1	1	1	1	1	1	1	9	1	1	1	1	1	1	1	1	1	1	1
33		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
34		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1
35		1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

APPENDIX 5: (Continued)

Pretest

Student No.	Skill 1		2		3		4		5		6		7		8		9		10	
Item	10	12	8	13	1	20	2	17	4	15	7	19	5	11	3	16	6	14	9	18
36	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1
37	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
38	1	1	1	1	1	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0
39	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
40	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
41	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	0	0
42	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1
43	0	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	0	0	0	0
44	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
45	1	1	0	0	9	0	1	1	1	1	0	1	0	0	0	0	0	0	0	0
46	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
47	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
48	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1
49	1	1	1	1	1	1	1	1	1	1	1	9	1	9	1	1	1	1	0	0
50	1	1	1	1	1	1	1	1	9	1	1	1	0	0	1	1	0	0	1	1
51	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1
52	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0
53	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
54	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
55	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
56	1	1	1	1	0	1	1	1	9	1	0	1	1	0	1	1	9	0	0	1
57	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	0	1
58	1	1	1	1	1	1	1	1	1	1	0	1	9	1	1	1	0	1	1	0
59	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
60	0	1	1	1	1	1	1	1	1	1	0	0	1	1	9	9	0	0	0	0
61	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
62	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
63	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	9
64	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
65	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	0	1	1
66	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
67	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
68	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
69	1	1	1	1	1	1	1	1	9	0	9	0	0	0	1	1	1	1	0	0
70	0	1	1	1	1	1	1	1	0	1	1	1	0	0	1	1	0	1	0	1

Posttest

Student No.	Skill 1		2		3		4		5		6		7		8		9		10	
Item	16	7	19	1	14	4	20	9	18	5	13	10	17	3	11	2	12	8	15	6
36	1	1	1	1	1	1	1	1	1	1	9	1	1	1	1	1	0	0	1	1
37	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
38	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
39	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
40	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
41	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
42	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	1	1
43	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
44	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
45	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	1	9	0
46	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1
47	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0
48	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
49	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
50	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
51	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
52	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
53	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
54	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
55	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
56	1	1	1	1	1	1	1	1	0	1	1	0	0	0	1	1	0	1	0	1
57	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
58	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
59	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
60	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
61	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
62	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
63	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
64	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
65	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
66	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
67	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
68	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
69	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
70	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

APPENDIX 5: (Continued)

Student No.	Pretest																				Posttest																			
Skill	1		2		3		4		5		6		7		8		9		10		1		2		3		4		5		6		7		8		9		10	
Item	10	12	8	13	1	20	2	17	4	15	7	19	5	11	3	16	6	14	9	18	16	7	19	1	14	4	20	9	18	5	13	10	17	3	11	2	12	8	15	6
71	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
72	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
73	0	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	1	0	0
74	0	0	1	1	1	1	1	1	1	1	0	0	0	0	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1
75	0	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	1	0	0
76	0	1	1	1	1	1	1	1	0	1	1	1	0	0	1	1	0	1	0	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	1	1
77	0	0	1	1	1	1	1	1	1	1	0	1	0	0	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
78	1	1	1	1	1	1	1	1	1	0	1	0	0	0	1	1	0	0	0	0	0	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
79	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	9	0	0	0	1	0	1	1	0	1	1	1	9	0	1	1	1	0	1	1	0	0	0	0
80	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
81	0	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
82	1	1	0	0	0	0	1	1	1	1	1	1	0	1	1	1	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
83	1	1	1	9	0	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
84	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
85	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1	1	1
86	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
87	0	1	0	0	0	0	1	1	1	1	0	1	0	0	0	0	0	0	0	0	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
88	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	0	1	0	0	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
89	1	1	0	0	1	0	1	1	1	1	0	1	1	1	1	1	0	0	0	0	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	9	1	1
90	1	1	0	0	0	0	1	1	1	0	1	0	0	0	1	1	0	1	0	0	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
91	0	0	1	1	1	1	1	1	1	0	1	1	0	0	0	1	0	0	0	0	9	9	1	1	0	1	0	1	0	0	0	0	0	0	1	1	1	0	0	0
92	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
93	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
94	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
95	0	1	1	1	1	1	1	1	1	1	0	1	0	0	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1
96	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
97	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
98	0	0	1	1	1	1	1	1	0	0	0	1	0	0	1	1	0	0	0	0	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0
99	9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
100	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
101	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
102	1	1	0	0	0	0	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
103	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
104	1	1	0	0	0	0	1	1	1	1	1	1	0	1	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	9	0	0	9	0

APPENDIX 5: (Continued)

Pretest

Student No.	Skill 1		2		3		4		5		6		7		8		9		10	
	Item 10	12	8	13	1	20	2	17	4	15	7	19	5	11	3	16	6	14	9	18
105	0	0	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	0	0	0
106	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0
107	0	0	0	0	0	0	0	0	9	0	9	0	0	0	1	1	0	0	0	0
108	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	0	0
109	1	1	1	1	1	1	1	1	1	1	0	1	1	0	9	9	1	1	0	1
110	0	1	1	0	1	9	1	1	1	1	1	1	1	1	1	1	0	0	0	0
111	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
112	1	1	1	1	1	1	1	1	1	0	1	1	0	1	1	1	1	1	1	1
113	0	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1
114	0	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	0	0	0
115	0	1	1	1	1	1	1	1	1	1	0	1	0	0	1	1	1	1	1	1
116	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1	0	0
117	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0
118	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1
119	0	1	0	0	0	0	1	1	1	1	1	1	0	0	1	1	0	0	0	0
120	1	0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
121	0	1	1	1	1	1	1	1	9	0	1	1	1	0	1	1	1	0	0	1
122	0	1	0	0	0	0	0	1	1	0	1	0	0	0	0	0	0	0	0	1
123	1	1	1	1	1	1	1	1	1	1	0	1	0	0	1	1	1	9	0	0
124	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0
125	1	1	1	1	1	0	1	1	0	9	1	1	0	1	0	0	0	9	0	0
126	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
127	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
128	0	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	1
129	0	0	1	1	1	1	1	1	1	1	1	1	0	1	0	0	0	1	0	0
130	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	1	1
131	1	1	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0	0	0	0
132	1	1	1	1	1	1	1	1	9	1	1	1	9	1	1	1	1	1	1	0
133	1	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0
134	0	0	1	1	1	1	1	1	1	0	1	0	1	1	1	1	0	1	0	1
135	1	1	1	1	1	1	1	1	1	9	1	1	0	0	1	1	0	0	1	1
136	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
137	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Posttest

Student No.	Skill 1		2		3		4		5		6		7		8		9		10	
	Item 16	7	19	1	14	4	20	9	18	5	13	10	17	3	11	2	12	8	15	6
105	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
106	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	0	0	0	0
107	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
108	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
109	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
110	1	1	1	1	1	1	1	1	1	1	1	1	1	1	9	9	0	0	1	1
111	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
112	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
113	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
114	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
115	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
116	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
117	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	0	0
118	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
119	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	0	1	1
120	1	1	1	1	9	9	1	1	1	1	1	1	1	1	1	1	0	0	0	0
121	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
122	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	0	1	1
123	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
124	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
125	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
126	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
127	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
128	0	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	0	0	1	1
129	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
130	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0
131	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	1	1	1
132	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
133	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
134	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
135	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1
136	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	0	0	0	1
137	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

APPENDIX 5: (Continued)

	Pretest																				Posttest																			
Student No. / Skill	1		2		3		4		5		6		7		8		9		10		1		2		3		4		5		6		7		8		9		10	
Item	10	12	8	13	1	20	2	17	4	15	7	19	5	11	3	16	6	14	9	18	16	7	19	1	14	4	20	9	18	5	13	10	17	3	11	2	12	8	15	6
138	1	1	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1
139	1	1	0	0	0	0	0	0	0	0	0	0	0	0	9	0	0	0	0	0	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
140	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
141	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
142	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	1	9	1	1	1	1	1	9	9	0	1	9	1	1	1	0	0	9	1
143	1	1	1	1	1	1	1	1	1	1	1	9	1	0	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
144	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1
145	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	1	1	1	1	1	1
146	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
147	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	0	0	1	1	1	1	1	0	9	1	0	0	1	1	0	1	1	1	0	1
148	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
149	1	1	1	1	9	9	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	9	1	9	0	9	1	0	1	0	1	0	0	0	0
150	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9
151	1	1	1	1	9	1	1	1	1	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0
152	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
153	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
154	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	9	9	0	1	0	0	0	0	0	1	0	1	1	0	0	0	0	0
155	1	1	1	1	0	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
156	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	1	1	1	0	0	0	0	0	0

APPENDIX 5: (Continued)

SUPPLEMENTARY TEST

Student	Skill	4		5		7		9		10	
No.	Item	1	9	2	8	3	6	4	7	5	10
1		1	1	1	0	0	0	0	0	0	0
2		1	0	1	0	0	0	0	0	0	0
3		1	1	0	1	1	0	0	1	0	0
4		0	1	0	0	0	0	0	0	0	0
5		1	1	1	1	0	0	0	1	0	0
6		1	0	1	1	0	1	0	0	0	0
7		1	1	1	1	0	0	1	1	0	0
8		0	0	0	0	0	0	0	0	0	0
9		0	1	0	0	0	0	0	0	0	0
10		1	0	1	0	0	0	0	0	0	0
11		0	0	0	0	0	0	0	0	0	0
12		1	1	9	0	1	0	0	0	0	0
13		1	0	0	1	0	0	0	0	0	0
14		1	1	1	1	0	0	0	0	0	0
15		1	1	1	1	1	1	0	0	0	0
16		1	1	1	1	0	0	0	1	0	0
17		9	9	9	9	9	9	9	9	9	9
18		1	1	1	1	1	1	0	0	0	0
19		1	1	1	1	0	0	0	0	0	0
20		1	1	0	0	0	0	0	1	0	0
21		1	1	1	1	0	1	0	1	0	0
22		1	1	1	1	0	0	0	1	0	0
23		1	1	1	1	1	1	0	0	0	0
24		1	1	1	1	1	1	1	1	1	1
25		1	1	1	1	0	0	0	1	0	0
26		1	1	1	1	0	0	0	0	0	0
27		1	1	9	1	0	1	0	0	0	0
28		1	1	1	1	1	1	1	1	0	0
29		1	1	1	0	1	0	0	1	0	0
30		1	1	1	1	0	1	1	1	0	0
31		1	1	1	1	0	0	0	1	0	0
32		1	1	1	1	0	0	0	0	0	0

Student	Skill	4		5		7		9		10	
No.	Item	1	9	2	8	3	6	4	7	5	10
33		1	1	1	1	1	1	0	1	0	0
34		1	1	1	0	0	0	0	0	0	0
35		1	1	1	1	1	1	0	0	0	0
36		1	1	1	1	0	0	0	1	0	0
37		1	1	1	1	1	1	0	1	0	0
38		1	1	9	1	0	0	0	0	0	0
39		1	1	1	1	0	0	0	1	0	0
40		1	1	1	1	1	1	0	0	0	0
41		1	1	1	1	0	0	0	0	0	0
42		1	1	1	1	0	0	0	1	0	0
43		1	1	1	1	0	0	1	0	0	0
44		1	1	1	0	1	1	0	1	0	0
45		1	1	1	1	0	1	1	1	0	0
46		1	1	1	1	0	0	1	1	0	0
47		1	1	1	1	0	0	0	1	0	0
48		1	1	1	1	0	0	0	1	0	0
49		1	1	1	1	0	0	0	0	0	0
50		1	1	1	1	1	1	0	9	1	1
51		1	1	1	1	1	1	9	0	1	9
52		1	1	1	0	0	0	1	1	0	0
53		1	1	1	1	1	1	1	1	0	0
54		1	1	1	1	0	0	1	1	1	1
55		1	1	1	1	1	1	0	9	0	0
56		1	1	1	0	1	1	0	0	0	0
57		1	1	1	1	1	1	0	0	0	0
58		1	1	0	1	1	1	0	0	0	0
59		1	1	1	1	1	1	0	1	0	0
60		1	1	1	1	1	1	0	0	0	0
61		1	9	1	9	1	1	0	9	0	9
62		1	1	1	1	1	0	0	0	0	0
63		1	1	1	1	1	1	0	0	0	0
64		1	1	1	0	1	1	0	9	0	0

APPENDIX 5: (Continued)

Student No.	Skill 4		5		7		9		10	
	Item 1	9	2	8	3	6	4	7	5	10
65	1	1	1	0	1	1	0	0	0	0
66	1	1	0	0	1	1	0	0	0	0
67	1	1	1	0	1	1	0	0	0	0
68	1	1	1	1	1	1	0	0	0	0
69	1	1	1	0	0	0	0	0	0	0
70	9	9	9	9	9	9	9	9	9	9
71	1	1	1	1	1	1	0	0	0	0
72	1	1	1	1	1	1	0	0	0	0
73	1	1	1	1	0	0	0	1	0	0
74	1	9	0	0	0	1	0	0	0	0
75	1	1	1	1	1	1	0	1	0	1
76	1	1	0	0	1	1	0	0	0	0
77	1	1	1	1	1	1	0	0	0	0
78	1	1	1	0	1	1	0	0	0	0
79	1	1	1	1	1	1	0	9	0	0
80	1	1	1	0	0	0	0	0	0	0
81	1	1	1	1	1	1	0	0	0	0
82	1	1	1	1	0	0	0	0	0	0
83	1	1	1	1	1	1	0	0	0	0
84	1	1	1	1	0	0	1	1	1	1
85	1	1	1	1	1	1	0	1	0	0
86	1	1	1	1	1	1	0	0	0	0
87	1	0	1	0	1	0	0	0	0	0
88	1	1	1	1	1	1	0	0	0	0
89	1	1	0	1	1	1	0	0	0	0
90	1	1	1	1	1	1	0	0	0	0
91	1	1	1	1	1	0	9	9	0	0
92	1	1	1	0	0	0	0	0	0	0
93	1	1	1	1	1	1	0	0	0	0
94	1	1	1	0	1	1	0	1	0	0
95	1	1	1	0	0	0	0	0	0	0

Student No.	Skill 4		5		7		9		10	
	Item 1	9	2	8	3	6	4	7	5	10
96	9	1	1	0	1	1	0	0	0	0
97	1	1	0	1	1	1	0	0	0	0
98	1	1	1	1	0	1	0	0	0	0
99	1	1	1	1	1	1	0	0	0	0
100	1	1	1	0	1	1	0	0	0	0
101	1	1	9	1	1	1	0	0	0	0
102	1	1	1	1	0	0	0	0	0	0
103	1	1	1	1	1	1	1	1	1	1
104	1	1	1	0	1	1	0	0	0	0
105	1	1	1	1	1	1	1	1	1	1
106	1	1	1	1	1	1	0	0	0	0
107	1	1	0	0	1	1	0	0	0	0
108	0	0	0	0	0	0	0	0	0	0
109	1	0	1	1	0	0	0	1	0	0
110	1	1	1	1	0	0	0	0	0	0
111	1	1	9	1	1	1	0	0	0	0
112	1	1	1	1	1	1	0	1	0	0
113	1	1	1	1	9	1	0	1	0	0
114	1	1	1	1	0	0	1	1	0	0
115	1	1	1	1	0	0	0	0	0	0
116	1	1	1	0	1	1	0	0	0	0
117	1	1	1	1	0	1	0	1	0	0
118	1	1	1	1	0	0	0	0	0	0
119	1	1	1	0	0	0	0	1	0	0
120	1	1	1	1	1	1	0	1	1	1
121	1	1	1	0	1	1	0	0	0	0
122	1	0	1	1	0	0	1	0	0	0
123	1	0	0	0	0	0	0	0	0	0