A STUDY TO COMPARE THE EFFECTIVENESS OF INDIVIDUAL MANIPULATION OF INSTRUCTIONAL MATERIALS IN LEARNING GRADE FOUR FRACTIONS VERSUS A TEACHER DEMONSTRATION METHOD USING THE SAME MATERIALS

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By

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ABSTRACT

Purpose of the Study

The main purpose of this study was to compare the achievement of Grade IV students taught a unit on fractions by a method whereby they individually manipulated concrete materials with a method in which there was only teacher demonstration using the same instructional materials. Another purpose was to compare results achieved in selected levels of Bloom's Taxonomy by the two methods of instruction.

Procedure

The investigation was carried out in four Grade IV classes in one school in St. John's, Newfoundland. Two of these classes, consisting of 63 students, comprised the Experimental Group, and the other two classes, consisting of 67 students, formed the Control Group. Intact classes were used and the method of instruction was randomly assigned to the classes. The only difference in instruction for the two groups was the manner in which they concrete materials were used.

Objectives for a unit in Grade IV fractions were written in behavioral terms by the investigator. The unit was taught for ten consecutive periods of forty-five minutes duration. Three other days were used for testing.

Students' achievement in the unit on fractions taught in the experiment was measured by a specifically constructed twenty-item test. The items were designed to evaluate the

attainment of each objective of the unit and were classified according to the levels of The Taxonomy. The reliability of the test (.74) was found by using a Pearson product-moment correlation between the posttest and retention test scores. The achievement test was given as a pretest, posttest, and retention test. However, the order of the items on the test was changed each time the test was administered. The pretest was given three days previous to the instruction and the posttest was given one day after the completion of instruction. A retention test was administered four weeks later.

Analysis of covariance and the chi square test were used to analyze the data obtained from the posttest and retention test. The pretest scores were used as a covariate in the Analysis of Covariance. The level of significance was set at .05 for all statistical tests.

Conclusions

1. Grade IV students who individually manipulated concrete materials scored significantly higher on a posttest and a retention test designed to measure achievement in a unit on fractions than those taught by a teacher demonstration method.
2. Grade IV students who individually manipulated concrete materials did not show a significant difference in achievement in questions on a posttest and retention test designed to measure Knowledge and Comprehension, compared with those taught by a teacher demonstration method.

3. Grade IV students who individually manipulated concrete materials achieved significantly higher on posttest questions designed to measure Application, compared with those taught by a teacher demonstration method. However, there was no difference between the two treatments on retention test questions designed to measure Application.

4. Grade IV students who individually manipulated concrete materials scored significantly higher on questions designed to measure Analysis, than did students taught by a teacher demonstration method.

5. Grade IV students who individually manipulated concrete materials scored significantly higher on posttest questions designed to measure Synthesis, than those taught by a teacher demonstration method. However, there was no significant difference between the two treatments on a retention test question designed to measure Synthesis.

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CHAPTER I

INTRODUCTION

Rationale for the Study

Many changes have taken place within the past twenty years in elementary school mathematics. These changes have been so extensive that they are called by some a revolution (Price, 1961). Others, among them, Davis (1967), are of the opinion that we are just passing through an evolution in school mathematics and the real revolution has not yet taken place.

During the 60's, the scope and sequence of the conente of school mathematics and the objectives of instruction were mainly agreed upon and implemented by new programs (Davis, 1967). An increased interest in the way students learn mathematics, especially in the elementary grades, has given some mathematics educators the feeling that a change in content is only a partial answer in improving mathematics achievement. There is an indication that a change in methods of instruction is necessary as well. Hence, those concerned with the mathematics achievement of their students are directing their attention to the "how" of teaching mathematics (Brousseau, 1973).

Many modern learning theories and research associated with them have also influenced educators to look at new methods of instruction. These theories contend that students can have a better understanding of concepts if they discover these concepts by themselves through experience related to the physical world (Bruner, 1966; Dienes, 1964; Piaget,1964).

One of the recognized authorities on the cognitive development of children and the origin of their concepts is the Swiss psychologist, Jean Piaget. His many experiments, relating not only to the way children form concepts in general, but also relating specifically to the formation of mathematical concepts,are described in his many publications.

According to Piaget, children in the elementary grades are at a stage of development where a predominance of actions is needed before new ideas can be added to their structure. Shulman (1970) writes:

> Piaget's emphasis upon action as a prerequisite to the internalization of cognitive operations has stimulated the focus upon direct manipulation of mathematically relevant materials in the early grades (Shulman, 1970, p. 42).

Copeland (1972), describing Piaget's stages of development in children, says:

> The child at the concrete operational level should have concrete objects as a basis for abstracting mathematical ideas....As the child manipulates objects he is at some point able to disengage the mathematical idea or structure to begin learning about abstract mathematics inductively by using objects in the physical world. It is not sufficient to "tell" or "explain" or "show". The child should disengage the mathematics from the objects themselves (Copeland, 1972, p. 12).

Piaget emphasizes the fact that mathematical concepts do not arise from the objects themselves but from the individual's actions performed on the objects:

> But there is a second type of experience which I shall call logical-mathematical experience where the knowledge is not drawn from the objects, but it is drawn by the actions effected upon the

> objects. This is not the same thing. When one acts upon objects, the objects are indeed there, but there is also the set of actions which modify the objects (Piaget, 1964, pp.11-12).

Bruner also suggests that children should use concrete materials and manipulative devices in the elementary classroom. He says that children at the elementary school age are "capable of grasping intuitively and concretely a great many of the basic ideas of mathematics. But he can do so only in terms of concrete operations (Bruner, 1966, p. 38)."

The theories of Piaget and Bruner are supported by Dienes. He concludes, from his many studies conducted in the classroom, that children under twelve years of age need to manipulate concrete materials that manifest mathematical concepts to help them abstract these concepts. Dienes (1964) says:

> In the large majority of cases, what students communicate by writing down or uttering mathematical signs is merely the signs themselves and not the structure for which the signs are supposed to act as symbols. One way of overcoming this is by setting up mathematical situations in which children learn mathematical structures in much the same way as they learn about structures in the real world; that is, by manipulating actual objects (Dienes, 1964, pp. 25-26).

If the theoretical discussions of Piaget, Bruner, and Dienes about using concrete materials are sound, then teaching students by a method based on the use of concrete materials should be more effective. Van Engen (1953) emphasizes the importance of instructional materials to learning and the formation of concepts. He says:

> Reactions to the world of concrete objects are the foundation stones from which the structure of abstract ideas arises. These reactions are refined, reorganized, and integrated so that they become even more useful and even more powerful than the original responses (Van Engen, 1953, p. 86).

The literature seems to indicate that though there is much theoretical discussion concerning the value of using manipulative materials, well defined and controlled research is just beginning on the efficacy of the approach.

Kieren (1969) reports that the bulk of the studies on manipulative activities reported between 1964 and 1969 are oversimplified or pilot studies and suffered from numerous methodological defects.

Bernard (1972) who did a study on the historical development of the laboratory approach to elementary school mathematics, found that during the period 1966-1971, the active use of instructional materials was used in more programs, discussed in more publications, and advocated by more educators that had been noted at any previous time. According to Bernard (1972) there is a need for research to ascertain the true value of using this approach in teaching mathematics to elementary school children.

Besides the issue of whether or not to use concrete materials, studies done by Bisio (1971) and Toney(1968) questioned whether children gain better understanding of a mathematical concept when each child has the opportunity to individually manipulate materials or if a teacher demonstration using the same materials is equally effective.

If a teacher demonstration type of instruction is as effective as each child individually manipulating instructional materials, then it would be possible to provide a wider variety of materials for the instructional program with the same amount of funds. It could also be more economical with respect to the time required for collecting and distributing materials in the classroom and the space needed for storage.

The investigation by Bisio (1971) indicated that the passive use of materials may be as effective as the active use. He states that, "inasmuch as the passive use of materials is far more economical of both teacher time and money, further research should be conducted to verify this inference (p. 120)."

Toney (1968) indicated a trend toward greater achievement by the group using the individually manipulated materials and concluded that this was a more effective means for building understanding than was a teacher demonstration.

Carmody (1971) tested the effectiveness of three instructional approaches; concrete, semi-concrete, and symbolic, in elementary grades. Her study suggested the need for formulating specific behavioral objectives that were expected to be achieved in the instruction using concrete materials. Carmody (1971) cites one benefit of stating behavioral objectives for the unit of instruction as "the criteria used for evaluating the instructional procedures could then be in terms of objectives (p. 30)."

Research by Bierden (1968) and Morford (1969) on behavioral objectives in the cognitive domain supports the use of these objectives in helping to improve teaching and in evaluation of learner achievement.

The Taxonomy of Educational Objectives, The Classification of Educational Goals, Handbook I: Cognitive Domain, (Bloom, Engelhart, Furst, Hell, Krathwohl, 1956), is a valuable aid in classifying educational objectives and test items of cognitive processes. (Hereafter this taxonomy will be referred to as The Taxonomy.)

The Taxonomy, as explained by Bloom (1954), classifies "the intended behavior of students - the ways in which individuals are to act, think, or feel as the result of participating in some unit of instruction (p. 12)." These intended behaviors are arranged in hierarchical order of the different classes of objectives from simple to complex and in order of difficulty.

The six major classes of The Taxonomy, from simple to complex are:

(1) Knowledge, (2) Comprehension, (3) Application,
(4) Synthesis, and (6) Evaluation.

Bloom explains this classification of objectives:

> Our attempt to arrange educational behaviors from simple to complex was based on the idea that a particular simple behavior may become integrated with other equally simple behaviors to form a more complex behavior (Bloom, 1956, p.18).

Bloom (1956) also asserts that there is evidence from studying problems on examinations to support the hypothesis that as the order of objectives goes from simple to complex so also does the order of difficulty. Bloom says:

> Thus, problems requiring knowledge of specific facts are generally answered correctly more frequently than problems requiring a knowledge of the universals and abstractions in a field. Problems requiring knowledge of principles and concepts are correctly answered more frequently than problems requiring both knowledge of the principle and some ability to apply it in new situations. Problems requiring analysis and synthesis are more difficult than problems requiring comprehension (Bloom, 1956, pp. 18-19).

Since the major purpose in constructing a taxonomy of educational objectives is, as explained by Bloom, to facilitate communication, The Taxonomy was subjected to a number of checks to see if there could be agreement on classification of specific educational objectives and test materials. When members of the group compiling The Taxonomy tried to classify a large number of test items according to the six major classes described previously, they found that it was necessary to know the examinees' prior educational experiences.

Bloom says:

> This suggests that, in general, test material can be satisfactorily classified by means of the taxonomy only when the context in which the test problems were used is known or assumed (Bloom, 1956, p. 21).

The Taxonomy refers to the classifying of text exercises to be somewhat more complicated than that of classifying educational objectives. This is how Bloom describes the task of classifying test items:

> Before the reader can classify a particular test exercise he must know, or at least make some assumptions about, the learning situations which have preceded the test. He must also actually

attempt to solve the test problems and note the mental processes he utilizes (Bloom, 1956, p. 51).

The present investigator did not find any literature on the use of manipulative concrete materials in learning a mathematical concept where an attempt was made to compare students' achievement on items categorized according to the different levels of The Taxonomy.

Research is also needed on which concepts in mathematics can be learned better by using manipulative materials. Only one research study reviewed by the investigator concerned the use of manipulative materials in the learning of fractions. Students when they encounter fractions are at what Piaget calls the concrete operational stage where they do not have the cognitive structure required to deal with abstract mathematical notions (Adler, 1966). There seems to be a need for more research on whether the manipulation of concrete materials will be beneficial to students in learning fractions.

Purpose of the Study

The main purpose of this study was to compare the achievement of Grade IV students taught a unit on fractions by a method whereby they individually manipulate concrete materials with a method in which there is only teacher demonstration using the same instructional materials.

A second purpose of the study was to compare results achieved in selected levels of The Taxonomy by the two methods of instruction.

Plan of the Study

To achieve the purposes of the study objectives for an instructional unit in Grade IV fractions were written in behavioral terms by the investigator.

Items for an achievement test to be used to evaluate the instructional unit were then designed by the investigator. At least one item was designed to evaluate the attainment of each objective for the instructional unit. The test items were then classified according to The Taxonomy. They were judged to be on the level stated by a panel consisting of two mathematics education professors and the investigator. The test items were also judged by the same persons to see if they evaluated the objectives they were designed to evaluate.

The concrete materials for the instructional unit were constructed by the investigator. They consisted of a Fraction Kit, cardboard markers, and paper for folding. Fraction Games for drill purposes were designed for the study. The same materials were made in a larger size and also in felt to be used for demonstration by the teacher.

Daily lesson plans were written by the investigator for the entire instructional unit with the objectives of each lesson stated in behavioral terms.

The instructional unit was taught by the investigator to four Grade IV classes in one school situated in St.John's, Newfoundland. Intact classes were used and the method of instruction was randomly assigned to the classes.

The unit was taught for ten consecutive periods of forty-five minutes duration. A pretest was given three days before instruction for the unit commenced, and a posttest was given one day after the completion of instruction. A retention test was administered four weeks later. The same test was given as the pretest, posttest and retention test. However, the order of questions was changed each time the test was administered.

Limitations of the Study

Several limitations, found in both the design and methodology of the study, may have influenced the results. The findings of the study were restricted by the inherent reliability and validity of the instruments used. Also, the generalizations from the conclusion of the investigation were limited by the population from which the sample was drawn.

The following delimitations were placed on the study:

1. The sample in the study was not a random sample. A school was chosen that had at least four Grade IV classes; also,intact classes were used.
2. The study was only concerned with one concept in mathematics, and so generalizations could not be made about the effectiveness of these methods of instruction with other concepts.
3. No attempt was made to evaluate the effectiveness of the concrete objects used in the study.

Definition of Terms

The following terms were defined for use in the study:

Behavioral objective is a proposed change in the observable behavior of a learner.

Concrete is something real which can be seen and felt.

Demonstration means to show an article or process involving concrete materials to help the viewer know and understand.

Instructional materials are those media used in teaching which contribute to the learning process.

Manipulate is to move, treat or operate with the hands.

Manipulative materials are those concrete objects used for instruction that have movable, attached, or separate parts that can be assembled into some structure.

Original learning is that which has taken place when measured immediately after the unit is taught without any further formal instruction on another unit or units in mathematics.

Retention is how much of the unit is remembered four weeks after the posttest is given.

Hypotheses

1. Grade-four students who are given the opportunity to individually manipulate concrete materials will show a significant

difference in achievement in original learning in a unit on fractions compared with those taught by a teacher demonstration using the same material.

II. Grade-four students who are given the opportunity to individually manipulate concrete materials will show a significant difference in retention of a unit on fractions compared with those taught by a teacher demonstration method.

III. Grade-four students who are given the opportunity to individually manipulate concrete materials will show a significant difference in achievement in questions designed to measure knowledge, as defined in The Taxonomy, compared with those taught by a teacher demonstration method.

IV. Grade-four students who are given the opportunity to individually manipulate concrete materials will show a significant difference in achievement in questions designed to measure comprehension, as defined in The Taxonomy, compared with those taught by a teacher demonstration method.

V. Grade-four students who are given the opportunity to individually manipulate concrete materials will show a significant difference in achievement in questions designed to measure application, as defined in The Taxonomy, compared with those taught by a teacher demonstration method.

VII. Grade four students who are given the opportunity to individually manipulate concrete materials will show a significant difference in achievement in questions designed to measure synthesis, as defined in <u>The Taxonomy</u>, compared with those taught by a teacher demonstration method.

CHAPTER II

REVIEW OF RELATED RESEARCH AND LITERATURE

Manipulative Materials in the Classroom

The literature and research of recent years on mathematics education indicates an increased interest and attention to instructional materials and their uses in learning mathematics.

The October 1968 issue of The Arithmetic Teacher included a fifteen-page bibliography of manipulative materials for use in the classroom, and the Thirty-fourth Yearbook of the National Council of Teachers of Mathematics was devoted entirely to instructional aids in mathematics.

This concern for the use of concrete manipulative materials in learning mathematics has a long history. The well known and often quoted Chinese proverb, "I hear, and I forget, I see and I remember, I do, and I understand", has been used as a motto by several proponents of the active learning approach.

According to Grossman (1971),"Educators have been advised since 1855 to employ manipulative materials in teaching specific concepts in mathematics (p. 230)."

The British Mathematician, John Perry, in a famous address before the British Society in 1901, advocated a change from the pure abstract approach in teaching to the use of scaled drawings, graphs, models, physical apparatus, and practical applications.

E.H.Moore supported Perry's view in an address before the American Mathematical Society in 1902. Moore (1902) advocated the use of materials which necessitate direct learning participation such that, "Students consider that they are studying the subject itself, and not the words, either printed or oral, of any authority on the subject (p. 371)."

Davis (1966) states that a great need in elementary school mathematics is for a greater use of physical materials in mathematics classes. He reported that although this need has been recognized since the early part of the century, a large scale observation in American classrooms showed that the majority of mathematics classes were being taught by means of teacher talk, listening, reading, and writing. Children very rarely had any physical objects to manipulate or observe to enhance the learning experience.

A study by Harshman, Wells, and Payne (1962) investigated three groups of first grade students for one year using different sets of manipulative materials. The three experimental programs for the groups were labeled Program A, Program B, and Program C, and involved twenty-six teachers. Program A consisted of a commercial set of materials called Numberaid, while Program B used a set of inexpensive commercial materials. The third experimental program, Program C, used only homemade materials chosen or made by the teacher.

Achievement tests and an attitude survey were given to the 654 pupils involved in the experiment, and test results were analyzed using the analysis of variance technique.

Results showed the mean attitude score was highest for Program C, and lowest for Program A. No significant differences were found to exist between Programs A, B, and C, in arithmetic computation and reasoning, and total arithmetic achievement. When comparisons were made among students within particular IQ ranges, significant differences did occur in total arithmetic achievement at the .01 level, all in favour of Program C.

Harshman et al, (1962) concluded on the basis of data from this study, that "the highest expenditure for manipulative materials for arithmetic instruction does not seem to be justified (p. 191)." They caution, however, that there should be more control of the other variables in the study to find the true effect of the materials. One of these they mention in particular is the teacher variable. They conclude that, "individual differences in classroom management and teacher participation in the classroom activities among the twenty-six teachers were contributing variables throughout the study (p. 192)."

Sole (1957) found that the use of a variety of aids did not produce any better achievement in mathematics than the use of only one aid. He thought that the results of his study might have been different if time was not a factor when using concrete materials.

Biggs (1966) claimed superiority for a method which used many types of manipulative materials in the learning of mathematics over a method which used only one type of material such as Cuisenaire rods. This is similar to the view of Dienes (1964) in which he emphasizes that a child needs many models of a particular mathematical concept in order to learn that concept.

Bernstein (1963) gives certain principles that have evolved regarding the selection and use of materials. He says that there should be a direct correlation between the operations which are carried on with a device and the operations which are carried on in doing the same mathematics with paper and pencil. Also, the use of manipulative aids should exploit as many senses as possible and any aid used to abstract a mathematical concept should have some moving part or parts.

Van Engen (1953) emphasized the importance of instructional materials in learning and their part in the formation of concepts. He states "Reactions to the world of concrete objects are the foundation stones from which the structure of abstract ideas arises (p. 86)."

Experimental Research on Manipulative Materials

Experimental studies on the learning of mathematics using concrete materials seem to be of three types. In one type, the experimental group uses only Cuisenaire rods as the concrete material. The control group or groups are identified either by the use of some type of concrete aid, other than the Cuisenaire rods, or by the complete absence of materials, or both. Another type of experimental study reviewed is one comparing groups using a concrete, semi-concrete, or symbolic treatment. The third group of studies of special interest, and bearing more directly than the others on the investigator's study, is those involving individual manipulation of concrete material versus teacher demonstration using the same materials.

There was little research found that directly related to the need to manipulate concrete materials in the study of fractions. However, according to Anderson (1969), concrete experiences have produced results, in the study of fractions, which are superior to those secured through the use of drill.

A review of the experimental research on activity learning was made by Kieren (1969) for the period from January 1964, through December 1968. He concluded that during this period "there was no dearth of theoretical discussion on the value of manipulative learning in mathematics, but the quality of the actual research was questionable (p. 513)."

According to Kieren (1969), "Most of the studies were small in scale, and perhaps far too lacking in control and in potential generalizability to be considered good research (p. 516)."

Another such review by Vance and Kieren says:

> While there seem to be ever increasing amounts of manipulative and mathematics laboratory materials available, there are a relatively small number of good research or evaluation efforts (Vance & Kieren, 1971, p. 585).

In summarizing their findings, they state:

> The research and evaluation literature suggests that laboratory approaches can be used practically and effectively...Furthermore, laboratory approaches are not a panacea, but appear to be an effective

> instructional methodology in a teacher's repertoire (Vance and Kieren, 1971, p. 589).

Studies Using Cuisenaire Rods

There are many different types of concrete materials that can be used to facilitate the learning of mathematics. Regardless of which materials are used, the main purpose in using them should be to provide a model of some mathematical concept to be learned. One such material that serves as models for many concepts and which can be manipulated by the student is Cuisenaire rods.

Several studies have been reported which were designed to delve into the effectiveness of the Cuisenaire method of teaching elementary school mathematics. This method of instruction was named for its Belgian founder, Georges Cuisenaire, and has been developed and promoted by Dr. C. Gattegno of the University of London. It is sometimes referred to as the Cuisenaire-Gattegno method. Different colored rods varying in length from one to ten centimeters are supplied to each child, or small group of children. Through manipulation of the rods, the children are led to discover mathematical relationships.

A study of the Cuisenaire-Gattegno mathod was conducted in Canada by Lucow (1963) with rural and urban grade three students in Manitoba schools. He found it an effective method in the teaching of multiplication and division to these students but noted that non-Cuisenaire methods in the hands of experienced teachers yield results that are just as good.

Other studies on the Cuisenaire method were done by Nasca (1966), Crowder (1965), and Hollis (1965). These, as well as the study of Lucow (1963), used groups that were not randomly selected, were initially different, and were taught by different teachers. Nevertheless, all three tentatively concluded that the Cuisenaire group did as well on a traditional test as the control groups, but did better on a test designed to be specifically sensitive to the Cuisenaire method.

Nasca (1966) also concluded that the ability of second grade pupils to assimilate mathematical concepts has been underestimated by those who stay with the traditional methodology. He asserts:

> All too often, curriculum specialists have been handicapped in content selection by traditional procedures for teaching that content. With broader methodologies available, it becomes essential to re-evaluate terminal behaviors in light of the increased competencies that can be generated. A procedure which provides children with the opportunity to perform operations with concrete materials and encourages them to abandon such models in favour of mental or "abstract" manipulation can obviously provide superior gains in achievement (Nasca, 1966 p. 225).

The findings of Passy (1963) about the use of Cuisenaire materials were somewhat different from that of the above studies. He reported a study which involved three groups of third grade children. One group consisted of 990 subjects using the Cuisenaire program. A second group of 375 students participated in a "meaningful" program but did not use

Cuisenaire material. The third group consisted of 500 children who had been in third grade before the Cuisenaire materials were in use in the school district. The three groups were matched and compared on the basis of reading ability, mental ability, length of the child's attendance in the district, and the teacher's experience in the district. No significant differences at the 2% level were found.

In analysing the data at the end of the year's study, Passy found that the group utilizing the Cuisenaire materials scored significantly less on the achievement test than the two samples not using the Cuisenaire method. The results were significant at the five percent level on both skill in computation and in mathematical reasoning.

In 1962, Brownell conducted a study using pupils completing grade three. In this study he compared the conventional program with the Cuisenaire program in Scotland and the Conventional, Cuisenaire, and Dienes programs in England. The Conventional program was described by Brownell as being similar to the textbook - drill type of instruction. The Cuisenaire program used the Cuisenaire rods, and the Dienes program used the multi-base-arithmetic blocks.

Brownell found that students in the Scottish study taught by the Cuisenaire method demonstrated much greater maturity of thought processes in finding answers for the number combinations than did the children of the Conventional program. The students that used the Cuisenaire rods also had much more ability to explain the rationale of computation.

The results of the English study were just the reverse of the Scottish results. In the English studies, students taught by the Conventional program ranked highest for effectiveness in promoting conceptual maturity and in thought processes with combinations. They also ranked first in explaining the rationale of computation and problem attacks.

Brownell concluded that multi-base-arithmetic blocks produced little evidence of special value. He felt this was not due to deficiencies of the blocks, but rather to the way in which they were used. To be effective the blocks must be used in conjunction with the regular learning instead of being used superficially.

Brownell's conclusion that the blocks must be used as a part of the regular learning in order to have an effect, gave support to the investigator's decision to use concrete materials to introduce the concept of fraction. The materials are not used to reinforce the concept or to show it after it has been taught, but are an integral part of the learning of the concept.

Studies Comparing Concrete, Semi-Concrete and Symbolic Methods of Instruction

Many studies have been done to evaluate the impact of selected concrete materials on the understanding of certain mathematical concepts as compared to a control group or groups which are characterized by either nonuse of materials or by use of semi-concrete materials or both. However, very few of these studies dealt with the use of materials in learning fractions.

Carmody (1971) and Curry (1971) did similar studies on the effectiveness of three instructional approaches in the elementary grades. The three approaches consisted of a group that used concrete materials, a group that used semi-concrete materials, and a group that used no materials and were classified as symbolic or abstract.

The purpose of the study done by Carmody (1971) was to investigate both theoretically and experimentally the assumption that the use of concrete and semi-concrete materials can contribute significantly to the learning of mathematics at the elementary school level. The experiment done in three sixth-grade classes was taught by Carmody for forty-five minutes a day for eleven days. Typical classroom conditions were maintained, with the exception of the introduction of the materials.

The results of a test designed to measure transfer, supported the use of concrete or semi-concrete materials. The group using semi-concrete materials scored significantly higher than the symbolic group on a numeration test, but no significant differences were found between the groups using concrete or semi-concrete materials. Carmody (1971) reported that many of the theoretical discussions on the use of concrete materials emphasized the importance of having specific behavioral objectives for instruction using concrete aids and for helping in the choice of aids to use.

The objectives of the investigation by Curry (1971) were to determine the most effective of three different methods of teaching clock arithmetic to third-grade students and to identify method by ability level interactions, if they existed. The methods of teaching were called concrete, semi-concrete and abstract. Each child in the concrete treatment was given a clock. In the semi-concrete treatment the teacher referred to pictures of clocks and only verbal reference was made to clocks in the abstract method. Intact classes and different instructors were used in all treatments. Three-factor analysis of variance was used to analyze the data obtained from two posttests that were given. These tests, constructed by Curry, were designed to test computation and understanding of principles. It was concluded that methods providing concrete materials or pictures, resulted in greater computational skill and greater understanding of properties by third graders than did a verbal method. Also, the analysis showed that there were no ability level differences on any of the tests and no method by ability level interactions either.

Davidson (1973) did a study designed to measure the impact of concrete materials, when used in conjunction with the textbook, on the understanding of mathematical concepts by grade three and grade four children.

The study involved 432 children during the 1969-70 school year. During this time the children in the experimental group used concrete materials and those in the control group used the text and drill materials.

The Lorge-Thorndike Intelligence Test was used to obtain an I.Q. score for each child and two different forms of the Iowa Test of Educational Achievement were used as a pretest and posttest. Piaget-type conservation tests were also administered to a sample of 160 of the students.

The findings showed that on the Iowa Test there were no significant differences between the two groups. Among grade three children, the experimental average-low I.Q. group had significantly greater conservation responses than did the corresponding control group. These variances were significant at the .05 level on the Conservation of Weight and Length tests, and at the .01 level on the Conservation of Mass test. The grade four high I.Q. experimental group had significantly greater conservation responses at the .01 level , especially in geometry. At this level the concrete materials seemed to enrich the geometry of the textbook.

Studies done by Swick (1959) and Ekman (1967) gave strong support to the desirability of using multi-sensory aids in teaching both arithmetic computation and reasoning. There also seemed to be indication from Swick's study that the attitude of both teachers and students improved toward arithmetic during the experimental period.

Ekman's study (1967) compared the effectiveness of three methods of presenting addition and subtraction ideas to third-grade students in 27 classrooms in Minnesota selected by simple random sampling. The first method presented the concept immediately in algorithm form. The second method used pictures to develop the concepts before the algorithm was presented and the third method used individually manipulated cardboard discs to develop the concepts before presenting the algorithms.

Nine classes were taught by each method for 18 days. A test, designed to measure understanding, transfer, and computational skill, was administered as a pretest before the instructional period, as a posttest immediately following the instructional period, and as a retention test about six and one-half weeks after the learning period.

Three covariance analyses were computed for each of the areas of understanding, transfer, and computational skill as measured by the test. When a whole class was used as the experimental unit, no significant differences were found due to treatment. When a single pupil was used as the experimental unit, some differences were found. The third method, where students individually manipulated discs, was found to be superior to the other two methods at the three and one-half percent level of significance at the end of the learning period on the understanding scale. On the transfer scale, the third method, again, was superior to the other methods at the four percent level of significance at the end of the entire period. But the first method, presenting the algorithm immediately, was found to be superior for retention to the other two methods at the four percent level of significance. From the analysis of the data, Ekman (1967) concluded that manipulative materials were helpful in increasing understanding and transfer ability with these third-grade students in the learning of addition and subtraction.

Fennema (1972) also examined the question of whether there are differences in the learning of groups of students who learned a specified principle represented by a concrete model or by a symbolic model. Ninety-five second grade students were randomly assigned to eight groups, each of which was then given either a concrete or symbolic treatment.

After 14 instructional sessions, a test of recall and two tests of transfer were used to assess learning. Analysis of variance was used to analyze the results.

The results of the recall test indicated that it was possible for children between seven and eight years of age to learn a mathematical idea to the point of direct recall using either model. However, when learning was defined in a broader way and included transfer or extension of the principle, children who used the symbolic model performed at a higher level than those who used the concrete model.

Fennema (1972) suggests that a possible reason for these results might be that children in the study had a program the previous year which emphasized the manipulation of concrete objects. So, since they had already had presymbolic experiences, use of the symbolic model with its greater generalizability was more effective for them.

Fennema cites another limitation of her study to be the use of Cuisenaire rods only. No other concrete materials were investigated for their effectiveness in the study.

A conclusion that Fennema (1972) draws that might be of great importance to all studies of this type is, "More empirical data must be collected to determine in which situations concrete models contribute most to the learning of mathematical ideas (p. 238)."

Two studies, Johnson (1970) and Ropes (1973) investigated the effects of manipulative materials on the attitudes of students using them. They found no significant overall attitude change toward mathematics, but there was indication of a greater awareness of the enjoyment to be derived from mathematics and an increased liking for that subject by students using manipulative materials. Johnson cautions that it is difficult to conclude whether the apparent changes in attitude were produced by the differences in instruction, the role of the teacher, the topic studied, other school related variables, or variables unrelated to school.

Studies Comparing Individual Manipulation of Materials with Teacher Demonstration

Bisio (1971), Jameison (1964), Toney (1968) and Trueblood (1967), did similar studies investigating the effectiveness of individual manipulation of instructional material as compared to a teacher demonstration using the same materials in developing understanding in mathematics.

The study by Bisio (1971) was designed to test by experimental means the comparative effectiveness of three methods of teaching addition and subtraction of like fractions to fifth-grade pupils. In Treatment A, neither the teacher nor the students used manipulative materials. In Treatment B, the teacher demonstrated for the students using the same materials as in Treatment C. In Treatment C, both teachers and pupils used the manipulative materials.

The study was conducted in 29 fifth-grade classes in California public elementary schools involving 501 pupils. All subjects were administered a pretest and a posttest of addition and subtraction of like fractions designed by Bisio. Initial reading and arithmetic achievement was measured by the Stanford Achievement Test, Intermediate Form 1.

The scores for the three groups were subjected to analysis of covariance and a t-test to test significance of differences between treatment groups.

Bisio (1971) concluded from the study that children taught to add and subtract like fractions using manipulative materials (both actively using them and by teacher demonstration) were at least equal to children taught by a method not involving manipulative materials, and there were no indications of unfavourable results from the use of manipulative materials. He also concluded that while the actual use of manipulative materials appears to be beneficial to most students, and is better than nonuse, the passive use of materials appears equally effective. Bisio says: "In as much as the passive use of materials is far more economical of both teacher time and money, further research should be conducted to verify this inference (Bisio, 1971, p. 120)."

Toney's study (1968) was carried out in two grade four classes in a laboratory school of a midwestern university in the United States. The classes were formed into two equivalent groups on the basis of social, emotional, physical, and intellectual characteristics of the students. The only difference in the course of study for the two groups was the manner in which instructional materials were used. The students in the experimental group were given instructional materials to handle and manipulate individually, while the students in the control group only observed a demonstration by the teacher with the same instructional materials.

Data were collected by the administration of the Edwards Test of Arithmetic Meaning, and the Arithmetic Section of the California Achievement Test. The Edwards Test was designed to measure understanding of mathematical principles while the California Achievement Test was designed to measure all areas of general mathematical achievement including computation as well as understanding of mathematical concepts.

An analysis of variance was used to analyze the data. There were no statistically significant differences between class means as determined by the test for understanding of basic mathematical principles or for general mathematical achievement. The group using individually manipulated materials made greater gains in proficiency on both measuring instruments than the group seeing only a teacher demonstration.

Toney concluded that, "the data indicated a trend toward greater achievement by the group using the individually manipulated materials, and the use of these materials seems to be a somewhat more effective means for building understanding than does a teacher demonstration (p. 86)."

Jameison (1964) tested the effectiveness of three methods of teaching numeration systems to seventh-grade students. Three classes of students were taught for five days. One class received instruction which involved the use of a large variable-base abacus which was demonstrated by the instructor. Another class received instruction in which both the large abacus and the smaller student manipulated abaci were used. Only the blackboard and chalk were used with the third class.

A pretest and a posttest, designed and validated especially for the experiment, were given at the beginning and end of the experiment. Pupils' gain scores, obtained from difference in pretest and posttest scores were subjected to a simple randomized analysis of variance. This analysis resulted in acceptance of the hypothesis of no difference in mean gains of each group. An analysis of covariance used on the data also showed no apparent difference in the mean gains of the groups.

The time allotted to the treatments may have been an important factor in the outcome of the study. Jameison suggests that he is "not entirely certain that the time allotted to the treatments was an ideal length of time for observing the optimum effect of the teaching aids (p. 84)."

Trueblood (1967) conducted an experiment to see if students ages eight to eleven would achieve and retain more by manipulating visual tactual aids (T-1) or by a treatment in which the students observed the teacher use such devices (T-2)

(T-2). The students in seven fourth-grade classes were randomly assigned to T-1 or T-2. Analysis of covariance was used to analyze results of the posttest and the retention test with Mental Age as the covariate.

Results from the analysis showed that pupils taught by T-2 scored higher on the posttest than did pupils taught by T-1 at the .01 level of significance. There was no significant difference between means on the retention test.

The results of these four studies, Bisio (1971), Jameison (1964), Toney (1968) and Trueblood (1967), on the comparison of the effectiveness of two ways of using materials to help learn mathematical concepts are far from conclusive. There seems to be a need for much more research to evaluate an instructional approach where students individually manipulate concrete materials compared with one where the teacher demonstrates to the pupils using the same instructional materials.

Piaget

Recent learning theories, which recognize that both the child and the environment play an important role in the learning process, have influenced the use of manipulative materials in the elementary grades.

Although there has been valuable research by many authorities on certain aspects of children's intellectual growth and development, the present investigator reviewed the contribution of Jean Piaget, and the interpretations and the implications of his theories by Adler (1966), Elkind (1967), Flavell (1963), and Isaacs (1968) for this study.

The extensive investigations and experiments of Jean Piaget, relating not only to how preschool and school age children form concepts in general, but also relating specifically to the formation of mathematical concepts, are described in his books and articles. These investigations and experiments have led Piaget to hypothesize that there is an evolution from the thought world of the child to that of the adult.

Piaget (1967) regards the child as an organism inexperienced in the organization and structure that characterizes most adult thinking. To him there seems to be a contrast between the "instability and incoherence of childhood ideas with the systemization of adult reasoning (p. 3)."

Piaget, as interpreted by Flavell (1963) sees the dual process of assimilation and accommodation as the chief controlling factor of intellectual growth or functioning. Flavell explains that the term "cognitive assimilation" refers to the fact that:

> every cognitive encounter with an environmental object necessarily involves some kind of cognitive structuring (or restructuring) of that object in accord with the nature of the organism's existing intellectual organization (Flavell, 1963, p. 48).

However, assimilation is always being modified by an accompanying process of accommodation. Adler (1966) describes accommodation as:

> the process of perpetual modification of mental structures to meet the requirements of each

> particular experience. Accommodation is the tendency of mental structures to change under the influence of the environment (Adler, 1966, p. 578)."

As the child progresses from an infant to adulthood, Piaget conceives of the child's ways of acting and thinking as being "changed several times as new mental structures emerge out of old ones, modified by accumulated accommodations (Adler, 1966, p. 578)."

Piaget identified four major stages in the development of intelligence. The child's stage of development indicates the level of thought of which he is capable. The order of these stages is constant, but chronological age at which each stage is reached varies.

Piaget (1969) says about the stages:

> Their order of succession is constant, although the average ages at which they occur may vary with the individual, according to his degree of intelligence or with the social milieu. Thus, the unfolding of the stages may give rise to accelerations or retardations, but their sequence remains constant in the areas (operations, etc.) in which such stages have been shown to exist (Piaget, 1969, p. 153).

Flavell (1963) does an extensive review of the four stages of intellectual development described by Piaget. The child is in the <u>sensori-motor</u> stage from birth to one and a half to two years. From eighteen months to the age of six or seven years, the child is in what Piaget calls the <u>preoperational</u> stage. During this stage the child is egocentric in his view of objects and events. His thinking is very much influenced by the present. He tends to attend to only one event at a time. Because of this, the child, during this stage meets many contradictions but this does not seem to concern him at all (Isaacs, 1968, p. 23).

As the child enters the concrete operational stage, at approximately seven to eight years, his thinking becomes more systematic and structured. It is during this stage that the child is able to organize his thoughts into interrelated systems. Assimilation and accommodation begin to operate as a team, and actions are now reversible for the child (Adler, 1966, p. 579).

However, Piaget's writings remind us that the thinking of the child in this concrete operational stage is oriented toward observation and manipulation of concrete events and objects in his environment. Since most elementary school children are in the concrete operational stage, Piaget's theory lends support to the use of concrete materials in the learning of mathematics in the elementary school.

Piaget discussing the concrete operational stage says:

> It signifies that at this level, the level of the beginning of logic proper, the operations are not as yet concerned with propositions or verbal declarations but with objects themselves ....(Piaget, 1967, p. 124).

The fourth stage, called formal operational or "the stage of adult reasoning" by Adler (1966) begins at around eleven to twelve years. Piaget (1967) says about this stage: "Hypothetical- deductive reasoning thus becomes possible and with it the constitution of a 'formal' logic, i.e. a logic applicable to any kind of content (p. 125)."

Throughout Piaget's writings, there is an emphasis on the dependence of one stage of development on the preceding stages. Flavell (1963) summarizes Piaget's views on the implication of this belief for educating children:

> In trying to teach a child some general principle or rule, one should so far as it is feasible parallel the developmental process of internalization of actions, that is, the child should first meet with the principle in the most concrete and action-oriented content possible; he should be allowed to manipulate objects himself and "see" the principle operate in his own actions. Then, it should become progressively more internalized and schematic by reducing perceptual and motor supports, e.g. moving from objects to symbols of objects, from motor action to speech (Flavell, 1963, p. 84).

Adler (1966) suggests that Piaget's findings imply that many opportunities must be provided for physical action in learning. To learn effectively, Adler maintains that children must be participants and not merely onlookers. They need to touch, move and manipulate things. He cautions, however, that such actions are only the foundation for the development of a mental operation. Children need to be guided toward less dependence on the physical action (Adler, 1966, p. 583).

Summary

A review of the research and literature published within recent years in elementary school mathematics, indicates a growing interest by educators and psychologists in the manner in which students learn most effectively. One of the more recent learning theories suggests that learning at certain stages of the child's cognitive development proceeds from the concrete to the abstract. This theory is stimulating teachers to ask questions about the most effective method of teaching mathematics. Proponents of the theory that learning proceeds from the concrete to the abstract suggest that adequate experiences with concrete materials must be provided for the learner in order for him to learn effectively on the abstract level of mathematics. Experts in the field of mathematics as well as psychology have expressed the desirability of the manipulation of concrete materials in the early stages of the development of a mathematical concept.

This interest in the use of concrete materials in the learning of mathematics has brought a tremendous increase in the quantity and variety of instructional materials available for use in the mathematics instructional program. Research studies have been conducted in an attempt to evaluate the effectiveness of such materials. One problem with studies of this type is that traditional evaluative instruments may not be subtle enough to evaluate learning and understanding of mathematics taught using concrete materials (Carmody, 1971, p. 28).

The findings of studies concerned with the use of manipulative materials are far from conclusive. Studies done by Bernstein (1963), Biggs (1966), and Dienes (1964) stress the importance of selecting materials to meet the objectives of the mathematics being taught. They also cite the need for having a variety of materials while learning a mathematical concept.

It was concluded from a study done by Harshmen et al (1962) that expensive commercial concrete materials were no better than homemade ones for students in learning mathematics. No attempt was made by Harshmen et al to control the teacher variable or the time factor, and these may have influenced the results of the study. Similar flaws were reported in a study carried out by Sole (1957) designed to judge the effectiveness of a variety of manipulative aids.

In general, the experimental studies reviewed in this chapter, which compared methods of instruction characterized by concrete materials, semiconcrete materials, or nonuse of materials, tend to support the use of semiconcrete or concrete objects in the teaching of elementary school mathematics. However, there is no significant difference in their use as measured by instruments designed to measure traditional methods of teaching.

Studies involving the use of Cuisenaire rods in teaching mathematics are far from conclusive. Investigations by Crowder (1965), Hollis (1965), Lucow (1963), and Nasca (1966), showed favourable results in favor of the Cuisenaire method. A complete reversal of results, however, such as was found in studies by Brownell (1968) and Passy (1963), makes one wary of attaching great significance to the various findings. The many variables involved in each of the studies, especially students not being randomly selected,and different teachers being used for experimental and control groups, must be considered.

Studies by Carmody (1971) and Curry (1971) to compare the effectiveness of three instructional approaches in elementary grades, found no significant differences between the groups using concrete or semi-concrete materials, but both methods better than the nonuse of materials. Carmody (1971) and Curry (1971) cautioned that in doing studies similar to theirs there should be more consideration given to the choice of materials used, the teacher variable, and the length of time taken to carry out the study.

Similar studies to those of Carmody (1971) and Curry (1971) reviewed by the investigator did not produce conclusive evidence in favor of concrete materials over semi-concrete or no materials at all. Again, all the research stressed the need for longitudinal studies with broader samples. An attempt should also be made to determine in which situations concrete materials should be used. Another consideration, cited in the experimental studies on the use of concrete materials, is that standardized achievement tests or tests of basic skills may not be suitable in measuring the effectiveness of a concrete approach to teaching mathematics.

Although the experimental studies on the use of concrete materials give few guidelines to teachers and others interested in the way children learn mathematics, a broader perspective or sounder guidelines can be found in the theoretical positions hypothesized on the subject.

Piaget's theory relative to the role of concrete materials in the teaching of elementary school mathematics was summarized in this chapter. For this study the contributions of Piaget are considered through the interpretations and implications of his theories by Adler (1966), Elkind (1967), Flavell (1963), and Isaacs (1968).

The learning theory of Jean Piaget, as pointed out by Adler (1966), gives a rationale for the use of physical materials and experiences in teaching mathematics. Piaget's theory, based upon many years of study and experimentation with children in Geneva, Switzerland, is that intelligence is the interaction between the organism and the surrounding world.

Piaget views mathematical concepts, at least those of number and operation with numbers, as having their origin in experience involving actions with concrete objects. This relationship of concrete materials to the formation of mathematical concepts decreases as the child proceeds through the successive stages of cognitive developed described by Piaget (Flavell, 1963).

It is also implied in the theories of Piaget that if mathematical concepts are abstracted from concrete materials rather than being learned in a purely symbolic form, it is less likely that the processes of mathematics would become merely a matter of symbol manipulation. Adler (1966) cites another advantage of using concrete materials in that the child can actually test or verify a mathematical principle in a concrete situation and learn to view mathematics as something that is reasonable or verifiable.

It seems that although the relationship of learning mathematical concepts and the use of concrete objects which has characterized educational thinking during recent years has been confirmed by many theorists, there is still a need for experimental testing of the role of concrete materials in specific mathematical concepts, and in what stage of the child's development. As Van Engen (1971) has indicated "The study of those experiences that enhance the development of mathematical concepts is sorely needed (p. 50)."

Neither is there much research to provide guidelines for the most effective ways in which to use concrete materials. Experiments were done by Bisio (1970), Jameison (1964), Toney (1968) and Trueblood (1967) to test whether the manipulation of concrete materials by each individual child produces better understanding than when the child only sees a demonstration of the materials in the development of a concept. Again, the results of these studies were not conclusive.

Also, the literature on the use of concrete materials in the learning of mathematics shows that limited experimental work has been done with the use of materials in learning fractions by elementary school students.

The present experiment is an attempt to add to the experimental data on teaching mathematics with the use of manipulative materials. In particular, the present study is an effort to see if the actual manipulation of materials is a more effective way for grade-four students to learn fractions than a teacher demonstration using the same materials.

CHAPTER III

DESIGN AND PROCEDURES

This study investigated the achievement of grade-four students taught a unit on fractions by a method whereby the students individually manipulated concrete materials compared with a method in which there was only a teacher demonstration using the same instructional materials.

This chapter describes the manner in which the investigation was conducted. It includes a description of the population and sample used in the study, the experimental design, the instructional treatments for the experimental and control groups, the instructional unit and materials, and the experimental variables. It also describes the manner in which the instrument for collecting the data was developed and how it was administered.

Population and Sample

The population for the study consisted of fourth-grade students who had not been exposed to either method of instruction used in the investigation previous to the study.

The sample consisted of 130 students in four grade-four classes in Mary Queen of Peace Elementary School, situated in St. John's, Newfoundland. The school is classified as an Elementary Public School, and is under the jurisdiction of the Roman Catholic School Board for St. John's. Mary Queen of Peace Elementary School has an enrollment of 538 students in grades four to eight. The classes used in the study were described by the principal as having a great variety in student ability and background.

This particular school was chosen because it contained at least four co-educational grade four classes, which were more difficult to find in other elementary schools in the area. Fourth grade classes were chosen because it is ordinarily in this grade that students first receive a formal introduction to fractions.

Permission was received from the Roman Catholic School Board for St. John's to conduct the study at Mary, Queen of Peace School. A meeting was then arranged with the principal of the school and teachers of the four grade-four classes involved. Both the principal and teachers were enthusiastic about the study and were very co-operative.

Experimental Design

The two methods of instruction used in the investigation were assigned to the four available classes in the following manner:

I. Two intact classes, randomly selected from the four available, were taught by a method whereby the students, working in groups of two, manipulated the concrete materials used in teaching the instructional unit. This method of instruction was called Treatment A. The two classes formed the Experimental Group.

II. The two remaining classes were taught by the teacher demonstration method using the same materials as Treatment A. This method of instruction was called Treatment B. These two classes formed the Control Group.

Ten lessons of forty-five minute duration were taught on consecutive teaching days, extending from November 5, 1973 to November 19, 1973. The experimental and control groups were taught by the investigator. Three other teaching days were used for testing. A pretest was given three days before the start of the study, a posttest was given one day after the completion of the instructional unit, and the retention test four weeks later. The four classes were taught the same lessons and given the same assignments and tests. The instructional periods were of equal length for all classes.

The instructional unit on fractions used in the study followed closely the content on fractions outlined in the prescribed textbook used in the Newfoundland schools, Elementary School Mathematics (Eicholz, O'Daffer, Brumfiel, and Shanks, 1969, p.p.252-279). Behavioral objectives were written by the investigator for the unit and were the same for both groups. The only difference in Treatment A and Treatment B was the way in which the materials were used.

Typical classroom conditions were maintained during the experiment with the exception that children in the Experimental Group worked in groups of two while using the materials.

The general design of the experiment was modeled on Design 10 in Experimental and Quasi-Experimental Designs for Research (Campbell and Stanley, 1963, p. 47-50). Campbell and Stanley (1963) suggest this design for use in educational research involving an experimental and a control group where both are given a pretest and a posttest but the two groups do not have pre-experimental sampling equivalence. The groups are such that are found when an investigator has to use intact classrooms for his experimental and control groups and cannot randomly assign students to groups. The assignment of the experimental variable to the groups is assumed to be random (Campbell and Stanley, 1963, p. 47).

Instructional Treatment for the Experimental Group

The method of instruction developed for use with the Experimental Group was characterized by manipulation of concrete materials by the students working in groups of two. The students manipulated the materials in response to teacher direction. The teacher worked with individual students when necessary.

The exercises assigned to the students for practice work were ones selected from the prescribed textbook used in the school, supplemented by worksheets and fraction games for further drill.

Lesson plans were designed to meet the objectives of each day's lesson (Appendix B).

Instructional Treatment for the Control Group

The instruction for the Control Group differed from that of the Experimental Group in the use of materials. With the Control Group, the teacher demonstrated using the same materials as in the Experimental Group, but made in a larger size. Materials of felt were also used for demonstration on a flannel board.

The objectives of the teaching unit, the lesson plans, and assignments were the same for both groups, except with the Control Group only the teacher did what the students were directed to do in the Experimental Group. The teaching approach was expository in nature, with students responding to teacher instruction.

Description of the Instructional Unit

The instructional unit for the study was a unit on fractions. It was the same for the four classes used in the study. The unit consisted of (a) a formal introduction to the concept of fractional number and (b) an introduction to the concept of equivalent fractions.

The unit included such topics as the relationship between the concept of a number pair and that of a fraction, the concept of fractional parts of an object, the concept of fractions to compare part of an object with the whole object, the concept of fraction to compare part of a set with a whole set, sets of equivalent fractions, and simple problems involving fractions.

The instructional unit, used in the study, followed the program outlined in the textbook Elementary School Mathematics (Eicholz et al, 1969, pp. 252-279), presently used in most Newfoundland schools. The investigator stated objectives, in behavioral terms, for the unit taught. The objectives were stated as suggested in Stating Behavioral Objectives (Mager, 1962). These objectives are listed in Appendix A.

Lesson plans were then written to help teach each objective. Assignments were included with the lesson plan (Appendix B).

Description of Instructional Materials

The main instructional material used in the study was a fraction kit made by the investigator. The items in the kit were six-inch circles made of tagboard divided into halves, thirds, sixths, and twelfths. Five of the items in each kit were whole circles. Each fractional part was of a different colour which made it easy for the students to recognize the parts.

A fraction kit similar to the one used by the students, but with ten-inch circles, was made for demonstration by the teacher. A similar one was also made from felt for demonstration purposes.

Another instructional material used in the study to help develop the concept of equivalent fractions was different coloured one-inch circular markers. For Treatment B these markers were made of felt, but of the same colour as those used in Treatment A. Paper for folding was also used with both groups.

Fraction games were used for extra drill .

The Experimental Variable

The experimental variable in the investigation was the method of instruction used which was aimed at developing skill and understanding of fractional number and equivalent fractions. The method of instruction used was based on the use of manipulative materials.

None of the pupils in the investigation were told they were part of an experiment. The introduction of a new teacher and the novelty of materials seemed to bring more than normal enthusiasm in the classes, especially among students in the Experimental Group. Though the attitude of the students appeared to be very good in each of the groups throughout the experiment, no claim is being made for any of the instructional methods for the purpose of improving pupil attitude.

Non-Experimental Variables

When educational research is carried out in the real classroom situation, there are many non-experimental variables. Perhaps the most important one affecting many studies is the variation resulting from different teacher ability. In the present study, the teachers of the grade-four classes involved were reluctant to teach either the experimental or control groups because of their unfamiliarity with the instructional materials. Thus, the investigator taught the four classes. This might have added another bias in that the investigator may have unwittingly favoured one or the other method of instruction.

There are many other non-experimental variables which might have influenced the study, such as, general ability, school achievement, sex, time of day, and many others. Because intact classes were used in the study, the investigator did not have control over many of these variables. However, students had been randomly assigned to classes by the principal at the beginning of the school year, so all the classes contained heterogeneous groups. Also, the distribution of boys and girls was approximately the same for both groups. To help equate the experimental and control groups in case there were any initial differences between them, a pretest was given three days previous to the study, and the results of this test were used as a covariate in the analysis of data obtained.

Each item for the achievement test was then categorized according to the levels of The Taxonomy by the investigator and two mathematics education professors. There was total agreement on the placement of items categorized. None of the items were found to be on the Evaluation level.

Twenty items from those that were categorized were selected by the investigator for the achievement test. This test was used as a pretest, posttest and retention test. The order of the questions changed for each test.

Administration of the Instrument

Students' achievement in the unit on fractions taught in the experiment was measured by a specifically constructed twenty-item written examination. The items on the test were arranged on four pages and students were given space to write their answers on the test paper. No time limit was imposed on the students, as it was desired to eliminate time as a factor in the study.

The test was administered to students in the four classes used in the investigation three days previous to the teaching of the instructional unit. There were 128 students present for the pretest. One student from each of the Experimental and Control groups was absent.

The same test was given as a posttest the next teaching day after completion of the instructional unit on fractions. Two students who were not present for the pretest were present for the posttest and four other students were not present for the posttest. Three of these students were from the Experimental Group and one from the Control Group. Two students were absent for the Retention Test; one from each Group.

Statistics involving pretest, posttest and retention scores were computed with 120 subjects. Fifty-seven of the subjects were in the Experimental Group and 63 were in the Control Group.

The test was administered and scored by the investigator. An item was scored either right or wrong. Each correct answer received one point.

Analysis of Data

The data obtained from the instrument used to measure achievement on the unit on fractions were subjected to several statistical techniques.

An analysis of covariance was used to test Hypotheses I, II, IV, V, and VI. These hypotheses are stated in Chapter I. The criterion variables used to test these hypotheses were the scores on the posttest and retention tests and the three subtests classified as Comprehension, Application and Analysis contained in the posttest and retention test. The pretest scores were used as a covariate.

The computer program ANCVIO (University of Alberta, 1969) titled One Way Analysis of Covariance was used for the analysis. This program proposes to give an analysis of covariance using single or multiple covariates. The analysis is computed on the basis of a pooled regression equation.

Researchers usually use one of two methods to control variability due to experimental error: direct and statistical. Direct control can be obtained by randomly assigning subjects to experimental and control groups, making the conditions under which the experiment is conducted as uniform as possible, or increasing the accuracy, reliability and validity of the instruments used for measurement.

It is not always possible to use direct control in educational research. Subjects cannot always be randomly assigned to experimental conditions, so intact groups, such as is found in the classroom, must be used. Therefore, to eliminate potential sources of bias, statistical controls must be used.

In the present experiment, analysis of covariance was used to statistically control initial differences in the experimental and control groups which were comprised of intact classes. The pretest scores were used as a covariate for purposes of adjusting the measurements on the criterion variables, the scores obtained from the posttest and retention tests and the three subtests.

Kerlinger (1964) describes the technique of analysis of covariance as a method of analyzing intact groups in the following way:

> Analysis of covariance is a form of analysis of variance that tests the significance of the difference between means of final experimental data by taking into account and adjusting initial differences in the data... (Kerlinger, 1964, p. 348).

One assumption underlying the analysis of covariance is that the samples must be independent random samples from normally distributed and equally variable populations having the same means. Roscoe (1969) states:

> Generally, when the investigator is working with samples of the same size or nearly the same size, he may ignore this assumption unless he has reason to believe that his measures deviate greatly from it. Of course, the assumption of normality may be ignored if the samples are of adequate size, due to the benefits of the central limit theory (Roscoe, 1969, p. 236).

In using analysis of covariance,homogeneity of regression is also assumed, and the relationship between the criterion variable and the covariate should be linear.

Since the ANCVIO program used to obtain the analysis of covariance was a combination of the regression model with the analysis of variance model, Winer (1971) says,"An assumption with respect to additivity of treatment and regression effects is implied (p. 764)."

The chi square was used to analyze Hypotheses II and VI. The criterion variables used to analyze these two hypotheses were the number of correct responses on the Knowledge and Synthesis subtests. The data received from the tabulation of correct responses were entered in a two-way contingency table for chi square analysis of difference in instructional groups on the Knowledge and Synthesis subtests due to treatment. Since both the knowledge and Synthesis subtests contained only one item each, it did not seem meaningful to use analysis of covariance to analyze the data from these two subtests.

The number of correct responses for each item on the posttest and retention test was also tabulated. A chi square was then calculated on the data collected for each item on the test, the entire posttest and retention test, and the other three subtests. The purpose of doing these chi squares was to see if any further information than that obtained from the analysis of covariance could be found.

The requirements for using the chi square are a) that each observation or frequency is independent of all other observations, b) the expected frequency in all cells should be equal to or greater than five when the degrees of freedom equal one. When degrees of freedom are greater than one, the expected frequency in all cells should be equal to or greater than five in at least 80% of the cells (Runyon & Haber, 1970). These requirements were met in the present study.

The reliability of the instrument used to obtain the data was done by a process known as the stability method (Ahman & Glock, 1967, p. 315). According to the stability method, a test is administered to a group of pupils once, then after a certain time interval it is administered a second time to the same group of pupils. A coefficient of reliability is computed from the two sets of test scores. This correlation coefficient to test the reliability of the test was found by computing a Pearson product moment correlation coefficient. The computer program PEARSON CORR described in the Statistical Package for the Social Studies (Nie, 1970) was used to obtain this coefficient.

The Pearson product moment correlation gives the strength of association between two variables. The two variables in this study were the posttest and retention test scores obtained on the instrument whose reliability was being tested. The instrument used for the posttest and retention test was identical except for the order of the items. The posttest was administered one day after the completion of the instructional unit and the retention test was administered four weeks later. During this four week period, no instruction in fractions was given.

The percentage of correct responses to each question was also calculated for each item. The purpose of this was to obtain the percentage of correct responses to items on each level of The Taxonomy. It was expected that the number of correct responses on the Knowledge, Comprehension and

Application levels would be much higher than the correct responses on the Analysis and Synthesis levels. The percentage of correct responses for each item could also be compared for Treatments A and B.

CHAPTER IV

ANALYSIS OF DATA

The results of the statistical analysis of the data for each hypothesis in the study are reported in this chapter.

An achievement test in fractions, administered as a posttest one day after the completion of the instructional unit, and as a retention test four weeks later, was used as the criterion measure. The same test was divided into five subtests according to the levels of The Taxonomy to give Knowledge, Comprehension, Application, Analysis, and Synthesis subtests. The scores on these subtests were also used as criterion measures.

The tests were administered to all students who were present on the day the tests were given. Data are reported for the 120 subjects who completed all phases of the experiment.

Analysis of covariance and chi square analysis were employed to test the hypotheses of the study. Additional information concerning the performance of the experimental and control groups was obtained by using a chi square test on each item of the achievement test. Also, the percentage of correct responses by students on each item of the test for both experimental and control groups was calculated.

The analysis of covariance was used to test hypotheses I, II, IV, V, and VI. This was a suitable technique for this study since students were not randomly assigned to groups.

Intact classes were used. A pretest was given to the students prior to instruction on the unit. The pretest scores were used as a covariate for purposes of adjusting the measurements on the criterion variables.

The level of significance for the F-ratio for this study was set at .05.

A chi square test was used to test hypotheses II and VII. The level of significance chosen to test these two hypotheses was .05.

A Pearson product-moment correlation coefficient between the posttest and retention test scores was calculated to test the reliability of the achievement test used in the study. The reliability of the test was found to be .74.

Testing Hypothesis I

Hypothesis I

Grade-four students who are given the opportunity to individually manipulate concrete materials will show a significant difference in achievement in original learning in a unit on fractions compared with those taught by a teacher demonstration using the same materials.

This hypothesis was examined using an analysis of covariance test on scores obtained from the posttest. The pretest scores were used as a covariate.

Table 1 shows the means and standard deviations for the pretest and posttest scores for the experimental and control groups which received two different methods of instruction, Treatment A and Treatment B, respectively. The adjusted mean scores for the posttest are also given.

TABLE 1

Means and Standard Deviations for Pretest and Posttest Scores for Treatment A and Treatment B

	Mean*			Standard Deviation	
	Pretest	Posttest	Adjusted Posttest	Pretest	Posttest
Treatment A	3.7	16.1	16.2	2.9	2.1
Treatment B	4.1	14.7	14.6	3.2	1.9

*Maximum Mean Score 20

Table 1 shows that there was a substantial increase in mean scores from pretest to posttest. The increase in mean scores indicates that both methods of instruction were beneficial to the student in learning fractions.

Table 2 shows the results of the analysis of covariance for the test of Hypothesis 1 using scores obtained from the posttest, given one day after completion of the instructional unit.

The F-ratio obtained by using analysis of covariance was 22.2. This was significant at the .05 level. On the basis of the F-ratio, it was found that there was a significant difference in achievement between students in Treatment A and Treatment B.

Therefore, Hypothesis 1 was not rejected.

TABLE 2

Analysis of Covariance of Effects of Instructional Treatment for Posttest

Source of Variance	SS	df	MS	Adj F	P
Treatments	79.6	1	79.6	22.2	.05
Error	421.2	117	3.6		

Testing Hypothesis II

Hypothesis II

Grade-four students who are given the opportunity to individually manipulate concrete materials will show a significant difference in retention of a unit on fractions compared with those taught by a teacher demonstration method.

The means and standard deviations for the pretest and retention test scores are given in Table 3. There was very little adjustment made in the retention test mean score as can be seen from Table 3.

The results of the analysis of covariance for the retention test are given in Table 4. Pretest scores were used as a covariate.

TABLE 3

Means and Standard Deviations for Pretest and Retention Test for Treatment A and Treatment B

	Mean*			Standard Deviation	
	Pretest	Retention	Adjusted Retention	Pretest	Retention
Treat. A	3.7	14.9	14.97	2.9	3.9
Treat. B	4.1	13.0	12.97	3.2	2.4

*Maximum Score for Pretest and Retention Test

TABLE 4

Analysis of Covariance of the Effects of Instructional Treatments for Retention Test

Source of Variance	SS	df	MS	Adj F	P
Treatments	118.9	1	118.9	18.7	<.05
Error	737.1	117	6.3		

Table 4 shows the results of the data analysis for the test of Hypothesis II using scores obtained from the test used to measure retention.

The F-ratio of 18.7 was significant at the .05 level. Therefore, Hypothesis II was not rejected.

Testing Hypothesis III

Hypothesis III

Grade-four students who are given the opportunity to individually manipulate concrete materials will show a significant difference in achievement in questions designed to measure Knowledge, as defined in The Taxonomy, compared with those taught by a teacher demonstration method.

This hypothesis was tested using a chi square test, since there was only one item on the test classified as Knowledge. This was Question 14 on the achievement test (see Appendix C). The number of students that answered the Knowledge question correctly was tabulated for the posttest and retention test. The correct responses, called the observed frequency, were entered in two-way contingency tables for chi square analysis of differences in the experimental and control groups due to treatment.

The values of the chi square statistic for the Knowledge question on the posttest and retention test are presented in Table 5. The probability for one degree of freedom is stated.

The chi square statistic of 1.22 for the Knowledge question on both the posttest and retention test was not significant at the .05 level. It was concluded that there was not a significant difference in the number of correct responses by students in Treatment A compared with the number of correct responses by students in Treatment B on the

Knowledge question.

Therefore, Hypothesis III was rejected.

TABLE 5

Chi Square Analysis of Posttest and Retention Knowledge Subtest

	Posttest Knowledge	Retention Knowledge
χ^2	1.22	1.22
	> .05	> .05

The percentages of students with correct responses on the Knowledge level question for the posttest and retention test are shown in Table 6.

TABLE 6

Percentage of Students with Correct Responses on Knowledge Subtest

	Posttest Knowledge	Retention Knowledge
Treatment A	84.2	82.5
Treatment B	90.5	79.4

Although the chi square statistic indicated no significant difference in the two treatments on the Knowledge level question, the percentage of students with correct responses on the posttest was higher for Treatment B than for Treatment A. In the Knowledge question on the retention test, the

the percentage of students with correct responses was higher for Treatment A. There was a substantial decline, as can be seen in Table 6, in the percentage of correct responses in Treatment B for the Knowledge question from the posttest to the retention test.

Testing Hypothesis IV

Hypothesis IV

Grade-four students who are given the opportunity to individually manipulate concrete materials will show a significant difference in achievement in questions designed to measure Comprehension, as defined in The Taxonomy, compared with those taught by a teacher demonstration method.

Hypothesis IV was tested using the scores obtained from the Comprehension subtest on both the posttest and retention test. The Comprehension subtest contained items 1,2,4,6,7, 8,13,16 of the instrument used for the posttest and retention test (See Appendix C). Hypothesis IV was analyzed to test a) if there were differences in the two treatments when the Comprehension posttest scores were used as the criterion variable and b) if there were differences in the two treatments when the Comprehension retention test scores were used as the criterion variable.

Table 7 shows the means and standard deviations for the Comprehension subtest for the posttest and retention test.

The means for the posttest and the retention Comprehension subtests for Treatment A were slightly higher than the means for Treatment B. An analysis of covariance on the scores of both treatments shows that there was no significant difference between the two treatments.

TABLE 7

Means and Standard Deviation for Posttest and Retention Comprehension Subtests

	Mean*		Standard Deviation	
	Posttest Comprehen	Retention Comprehen	Posttest Comprehen	Retention Comprehen
Treatment A	7.0	6.7	.8	1.3
Treatment B	6.9	6.4	1.0	1.1

*Posttest and Retention Comprehension Subtests Maximum Score 8

The results of the analysis of covariance for the posttest and retention Comprehension subtests are given in Table 8.

TABLE 8

Analysis of Covariance of the Effects of Instructional Treatments for Posttest and Retention Comprehension Subtests

Source of Variance	SS	df	MS	F	P
Posttest					
Treatments	.59	1	.59	.64	> .05
Error	108.9	117	.93		
Retention					
Treatments	4.6	1	4.6	3.7	> .05
Error	104.4	117	1.2		

On the basis of the data from the posttest and retention Comprehension subtests, it was found that there was no significant difference between the achievement of students in Treatment A and Treatment B.

Therefore, Hypothesis IV was rejected on both the post-test and retention test.

Testing Hypothesis V

Hypothesis V

Grade IV students who are given the opportunity to individually manipulate concrete materials will show a significant difference in achievement in questions designed to measure Application, as defined in The Taxonomy, compared with those taught by a teacher demonstration method.

This hypothesis was tested using scores on the Application subtest obtained from the posttest and retention test. Items 5, 11, 17, 18 of the test were classified as Application.

Hypothesis V was tested for differences in the two treatments when a) the Application posttest scores were used as the criterion variable and b) the application retention test scores were used as the criterion variable.

Table 9 shows the means and standard deviations for the posttest and retention Application subtest.

The mean score for Treatment A was slightly higher on both the retention and posttest Application subtest than the mean score for Treatment B.

TABLE 9

Means and Standard Deviations for Posttest and Retention Application Subtests

	Mean*		Standard Deviation	
	Posttest Applicat	Retention Applicat	Posttest Applicat	Retention Applicat
Treatment A	3.2	2.6	.95	.85
Treatment B	2.9	2.4	.81	.80

*Maximum Score for Application Subtest 4

Table 10 presents the results of the analysis of covariance on the Application subtest scores.

TABLE 10

Analysis of Covariance of the Effects of Instructional Treatments for Posttest and Retention Application Subtests

Source of Variance	SS	df	MS	F	P
Posttest					
Treatment	4.9	1	4.9	7.1	$<.05$
Error	80.7	117	0.69		
Retention					
Treatment	2.8	1	2.8	1.76	$>.05$
Error	187.2	117	1.6		

On the basis of the F-ratio for the posttest Application subtest, shown in Table 10, it was concluded that students in Treatment A scored significantly higher than those students in

Treatment B. When the F-ratio was found using the retention Application subtest, it was concluded that there was no significant difference between the two treatments.

Hypothesis V was accepted when the posttest Application subtest was used as the criterion variable. However, it was rejected when the criterion variable was the retention Application subtest.

Testing Hypothesis Vl

Hypothesis Vl

Grade-four students who are given the opportunity to individually manipulate concrete materials will show a significant difference in achievement in questions designed to measure Analysis, as defined in *The Taxonomy*, compared with those taught by a teacher demonstration method.

The questions designed to measure Analysis, items 3, 9, 10, 12, 15, 19 on the instrument described in Appendix C, are classified as the Analysis subtest. Scores for the Analysis subtest are tabulated for the posttest and retention test.

Hypothesis Vl was analysed to test a) if there were differences in the two treatments when the Analysis posttest scores were used as criterion variables and b) if there were differences in the two treatments when the Analysis retention subtest scores were used as criterion variables.

Table 11 shows the means and standard deviations for the posttest and retention Analysis subtests.

The mean for Treatment A on both the posttest and retention Analysis subtest was higher than the mean for

Treatment B.

TABLE 11

Mean and Standard Deviations of Posttest and Retention Analysis Subtests

	Mean*		Standard Deviation	
	Posttest Analysis	Retention Analysis	Posttest Analysis	Retention Analysis
Treatment A	4.5	4.0	0.37	1.6
Treatment B	3.4	2.9	1.1	1.4

*Maximum Score for Analysis Subtest 6

Table 12 presents the results for the analysis of covariance for the posttest and retention Analysis subtests.

TABLE 12

Analysis of Covariance of the Effects of Instructional Treatments for Posttest and Retention Analysis Subtests

Source of Variance	SS	df	MS	Adj F	P
Posttest					
Treatments	43.9	1	43.9	54.5	$< .05$
Error	94.8	117	0.81		
Retention					
Treatments	45.9	1	45.9	33.9	$< .05$
Error	163.8	117	1.4		

It was concluded on the basis of the F-ratios in Table 12, that students in Treatment A scored significantly higher than those in Treatment B on a posttest and retention Analysis subtest.

Therefore, Hypothesis VI was accepted.

Testing Hypothesis VII

Hypothesis VII

Grade-four students who are given the opportunity to individually manipulate concrete materials will show a significant difference in achievement in questions designed to measure Synthesis, as defined in The Taxonomy, compared with those taught by a teacher demonstration method.

This hypothesis was tested by using a chi square test. The number of correct responses on the Synthesis subtest was used as the criterion variable to test Hypothesis Vll. The Synthesis subtest contained only one item, Question 20, from the test instrument (See Appendix C).

The hypothesis was tested using a) the number of correct responses for both treatments on the Synthesis subtests when given as part of the posttest, and b) the number of correct responses for both treatments on the Synthesis subtests when given as part of the retention test. The number of correct responses and the number of incorrect responses for both treatments were used as the observed frequencies. The observed frequencies were entered in a two-way contingency table to obtain the chi square statistic.

Two chi square statistics were produced. One was calculated for the posttest Synthesis subtest, and one for the retention Synthesis subtest. The results of the chi square analysis of Hypothesis Vll using the posttest and retention subtests as criterion variables are presented in Table 13.

TABLE 13

Chi Square Analysis of Posttest and Retention Synthesis Subtests

	Posttest Synthesis	Retention Synthesis
χ	9.0	1.9
p	$< .05$	$> .05$

It was concluded on the basis of the chi square statistic that students in Treatment A had significantly more correct responses than those in Treatment B on a question in the posttest designed to measure Synthesis.

When the number of correct responses from the retention Synthesis subtest was used as the criterion variable to calculate the chi square statistic, it was concluded that no difference existed between the two treatment groups.

Therefore, Hypothesis Vll was accepted for the posttest and rejected for the retention test.

Valuable information may be obtained from the percentage of students answering the Synthesis question correctly on the posttest and retention test. Table 14 presents this information on the percentage answering the Synthesis question correctly.

TABLE 14

Percentage of Students with Correct Responses on Synthesis Subtest

	Posttest Synthesis	Retention Synthesis
Treatment A	52.6	38.6
Treatment B	26.9	26.9

It is noted that on this higher level of cognitive development, the percentage of students answering the question correctly was less than 50% in all cases, except the 52.6% of the students on the posttest in Treatment A. There was a considerable decline in the percentage of students in Treatment A answering the question correctly on the post-test compared with the retention test. However, the percentage of students in Treatment B answering the question correctly (26.9%) remained constant for the posttest and retention test.

Supplementary Data

The instrument used to measure achievement of students on the unit of fractions was constructed so that it could be scored on the responses to the Knowledge level items, the Comprehension level items, the Application level items, the Analysis level items, the Synthesis level items, and to the test as a whole. The different levels of items were called subtests.

The percentage of students giving correct responses for each of the subtests was calculated and these percentages presented in Table 15. The percentage of students giving correct responses suggests that differences exist in the degree of mastery of the unit on fractions as one moves upward through the levels of cognitive development as defined in The Taxonomy. This was true for both treatments on both posttest and retention test. A slight departure from this decrease was made by the experimental group on the Knowledge and Comprehension levels. The percentage of students in Treatment A responding correctly was slightly lower on the Knowledge subtest than on the Comprehension subtest. Also, the percentage of students in the experimental group giving correct answers on the Retention Application subtest was lower than for the posttest Analysis subtest.

TABLE 15

Percentage of Correct Responses on Knowledge Comprehension, Application, Analysis and Synthesis Subtests

	Know	Comp	Appl	Anal	Synt
Treatment A					
Posttest	84.2	85.7	79.8	74.9	52.6
Retention	82.5	84.4	63.6	68.7	38.6
Treatment B					
Posttest	90.5	84.5	69.8	57.9	26.9
Retention	79.4	77.6	62.3	54.8	26.9

A chi square test was also performed on each of the posttest and retention tests and on the entire test. The significance level for all these statistics was .05. The results of the chi square analysis for the posttest and its subtests are given in Table 16. The probability level is also stated in the table.

TABLE 16

Chi Square Analysis for Posttest and Knowledge, Comprehension, Application, Analysis and Synthesis Subtests

df = 1

	Posttest	Know	Comp	App	Anal	Synt
χ^2	18.0	1.22	.145	5.06	22.33	9.0
ρ*	< .05	> .05	> .05	> .05	>.05	>.05

*ρ = .05, df = 1 then χ^2 = 3.841

The results of the chi square analysis for the retention test and its subtests are given in Table 17.

TABLE 17

Chi Square Analysis for Retention Test and Knowledge, Comprehension, Application, Analysis and Synthesis Subtests

df = 1

	Retention	Know	Comp	Appl	Anal	Synt
	22.09	1.22	2.27	1.28	26.3	1.865
ρ *	< .05	> .05	>.05	> .05	>.05	>.05

*ρ = .05, df = 1, then χ^2 = 3.841

No further information than that obtained from the analysis of covariance technique of testing the hypotheses was obtained by using the chi square.

The results of the chi square analysis of each item on the test is presented in Table 18. The table shows that differences in seven of the items on the test, 5, 9, 10, 15, 17, 19, 20, were significant at the .05 level.

TABLE 18

Chi Square Analysis of Correct Responses to Questions on Posttest

Question	1	2	3	4	5	6	7
χ^2	.10	.10	.07	.26	4.2	2.8	1.45
p	$>.05$	$>.05$	$>.05$	$>.05$	$<.05$	$>.05$	$>.05$
Question	8	9	10	11	12	13	14
χ^2	.27	4.8	9.03	1.05	3.34	.31	1.22
p	$>.05$	$<.05$	$<.05$	$>.05$	$>.05$	$>.05$	$>.05$
Question	15	16	17	18	19	20	
χ^2	10.92	.74	4.36	0	5.09	9.0	
p	$<.05$	$>.05$	$<.05$	$<.05$	$<.05$	$<.05$	

CHAPTER V

SUMMARY AND CONCLUSIONS

This chapter includes a summary of the study, conclusions that were drawn from the analysis of the data, and recommendations for further investigation.

Summary

Within the past decade, interest has grown in the ways in which children learn mathematics. Educators are trying to find effective means to help children develop understanding of mathematical concepts. Modern learning theories and research associated with them contend that children, especially in the elementary grades, can have a better understanding of concepts if they discover these concepts by themselves through experiences related to the physical world. Interest in manipulative materials to increase understanding in the early stages of the development of mathematical concepts has grown.

A theoretical base for the use of manipulative materials in teaching mathematics comes from the learning theory of Jean Piaget which suggests that intelligence is the interaction between the organism of the child and his surrounding world.

However, in spite of the interest favourable to the use of manipulative materials, the research reported to support the subject is far from conclusive. Neither does the research provide guidelines for the most effective ways to use such materials. Teachers of mathematics would find it both interesting and useful to know if the manipulation of concrete materials by each individual child produces better understanding than the observation by the child of a demonstration of the materials. Also of interest for educators is whether concrete materials are more effective in learning certain mathematical concepts.

The main purpose of the present study was to compare the achievement of grade-four students taught a unit on fractions by an instructional method whereby the students individually manipulate concrete materials with a method in which there was only teacher demonstration using the same instructional materials. A second purpose of the study was to compare results achieved in each level of The Taxonomy by the two methods of instruction.

The population for the study, all chosen from the same school, consisted of 130 fourth grade students who had not been exposed to either method of instruction previous to the study. Statistics were computed using 120 students who had completed all phases of the experiment.

Four intact classes were used in the study. Two of these classes were randomly assigned the method of instruction characterized by the manipulation of concrete materials. These two classes formed the experimental group. The other two classes were taught by the teacher demonstration method. This was the control group. The content, assignments, lessons, plans and instructional materials were the same for both groups. The main difference in instruction for the two groups was the ways in which the materials were used. The four classes were taught by the investigator for the entire study which lasted for two weeks.

The instructional unit on fractions used in the study followed closely the content outlined in the prescribed textbook for the school. The content consisted of a) a formal introduction to the concept of fractional number and b) an introduction to the concept of equivalent fractions. Behavioral objectives were written for the unit and were the same for both groups.

The instructional materials used in the study were a fraction kit, consisting of different coloured tagboard circles cut into fractional parts, cardboard markers and paper for folding. All materials for the demonstration method were similar but larger in size, with some materials made of felt for flannel board demonstration to the students. Fractional games were also used in the study for the purpose of drill.

The instrument used in the study to measure achievement was a specifically constructed 20 item achievement test. One test item for each stated objective of the study was constructed and then categorized according to the levels of <u>The Taxonomy</u> by the investigator and two other mathematics educators. Twenty items from those that were categorized were selected for the achievement test. The test was administered to the students in the study as a pretest three days before the beginning of the instructional program, and as a posttest one day after the completion of the instructional program.

Four weeks after the completion of the instructional program, the instrument was again administered by the investigator to measure retention. During this four week period, no instruction in fractions was given. Each student's test was scored for the entire test and for each of the subtests categorized as Knowledge, Comprehension, Application, Analysis and Synthesis according to the levels of The Taxonomy. Each correct item on the test scored one point.

The reliability of the instrument was found by using the stability method. A Pearson product movement correlation coefficient was calculated by using scores on the posttest and retention test for each student. This was found to be .74.

The hypotheses in the study were tested using the statistical technique of analysis of covariance and the chi square test. The level of significance for both statistical tests was set at .05. Scores on the posttest and retention test and each of the subtests were used as criterion variables in testing the hypotheses. The mean of the present scores was used as a covariate in calculating the analysis of covariance.

Conclusions

Based upon the statistical analysis of the data obtained from the instrument used to measure achievement on the unit on fractions, the following conclusions were drawn:

1. Hypothesis 1 was accepted. With posttest scores as the criterion variable and pretest scores as

the covariate, an analysis of covariance, used to test Hypothesis 1, showed significance at the .05 level. It was concluded that grade-four students who individually manipulated concrete materials scored significantly higher, when measured in original learning in a unit on fractions, than those taught by a teacher demonstration method.

2. Hypothesis II was accepted. Grade-four students who individually manipulated concrete materials showed significantly higher retention on a unit of fractions compared with those taught by a teacher demonstration method. This conclusion was based upon an analysis of covariance performed on the retention test scores for students subjected to the two methods of instruction. The analysis of covariance test showed significance at the .05 level.

3. Hypothesis III was rejected. Grade-four students who individually manipulated concrete materials did not show a significant difference in achievement in questions designed to measure Knowledge, as defined in The Taxonomy, compared with those taught by a teacher demonstration method. This hypothesis was tested by tabulating the number of correct responses for the Knowledge subtest for each method of instruction, and then analyzing this data by

using a chi square test. Although the chi square test showed no significant difference between the two groups, the number of correct responses for the Knowledge subtest was higher for the control group when measured on the posttest. However, on the retention test, the number of correct responses for the control group was lower than that of the experimental group.

4. Hypothesis IV was rejected. Grade-four students who individually manipulated concrete materials did not show a significant difference in achievement in questions designed to measure Comprehension, as defined in The Taxonomy, compared with those taught by a teacher demonstration method. An analysis of covariance performed on the data obtained from the Comprehension subtest showed significance at the .05 level when both posttest scores and retention test scores were used as criterion variables.

5. Hypothesis V was tested using scores from questions designed to measure Application as defined in The Taxonomy, on a) the posttest and b) the retention test. When Application posttest scores were used, Hypothesis V was accepted. Grade-four students who individually manipulated concrete materials achieved significantly higher on posttest

questions designed to measure Application compared with those taught by a teacher demonstration method. However, grade-four students who individually manipulated concrete materials did not show a significant difference in achievement on retention test questions designed to measure Application, compared with those taught by a teacher demonstration method. An analysis of covariance performed on the post-test Application scores, showed significance at the .05 level. An analysis of covariance performed on the Application scores on the retention test was not significant at the .05 level.

6. Hypothesis VI was supported. Grade-four students who individually manipulated concrete materials scored significantly higher on questions designed to measure analysis, as defined in <u>The Taxonomy</u> than did students taught by a teacher demonstration method. The difference was significantly higher for scores on both the posttest and retention test. The analysis of covariance performed on both the posttest and the retention test showed significance at the .05 level.

7. Hypothesis VII was supported when the scores from the Synthesis subtest given as a posttest were used as the criterion variable. However, Hypothesis VII was rejected when the retention scores were used. The chi square statistic for the posttest Synthesis

subtest was significant at the .05 level. The chi square statistic for the Synthesis subtest calculated from the retention test was not significant at the .05 level. It was concluded that grade-four students who individually manipulated concrete materials scored significantly higher on a Synthesis subtest obtained from a post-test given to measure original learning compared with those taught by a teacher demonstration method. However, when the Synthesis subtest scores were obtained from a retention test, there was no significant difference between the two groups. Another conclusion was that the percentage of students correctly answering the question on the Synthesis level was much lower than for questions on any other level.

8. Within the limitations of this study, there is support for the use of concrete materials in the teaching of a unit on fractions. There seems to be more favourable results when students manipulate the materials themselves than when they just see the teacher using them, especially when measured on the higher levels of The Taxonomy.

Recommendations for Further Research

Based upon the findings and the conclusions drawn from the study, the following recommendations for further research are suggested:

1. It is recommended that concrete instructional approaches in which students are allowed to manipulate the materials themselves be used as a teaching method in mathematics instruction for students in the elementary school.
2. There should be a revision of the instrument used in the present study so more items on each of the levels of <u>The Taxonomy</u> are included. Since students in Treatment A in the present study seemed to achieve significantly higher than those in Treatment B on questions designed to measure the higher levels of cognitive development, there should be more items especially on the Analysis and Synthesis subtests so that more decisive conclusions may be drawn.
3. Additional research should be carried out on the manipulation of concrete materials as an instructional approach compared with a teacher demonstration method utilizing a broader sample and a longer treatment period.
4. Further research is needed on the age or grade when this approach would be suitable. Factors such as the stage of the child's development and the nature of the mathematics being taught would influence these studies.
5. There should be further investigation of the interaction among instructional methods that

use concrete materials, student's performance on the different levels of cognitive development, and the aptitude level of the student.

6. It is recommended that the present study be replicated using an instrument to measure changes in student's attitudes towards mathematics. The observations by the investigator during the study seemed to indicate that the enthusiasm displayed by the experimental group was greater than that of the control group.

7. Further research should be done on the efficacy of other concrete materials than those used in the present study in teaching fractions to grade IV students.

8. It is recommended that an instrument be be developed to measure achievement that did not involve the use of the written language as extensively as the one used in the present study. Some of the students in the study were below the fourth-grade level in reading ability and their performance on the test may not have been a good indication of their achievement on the unit on fractions.

APPENDIX A

OBJECTIVES FOR THE INSTRUCTIONAL UNIT ON FRACTIONS

OBJECTIVES FOR THE INSTRUCTIONAL UNIT ON FRACTIONS

1. Given a picture of an object or set of objects such that the fractional part of the object or set is indicated, the child is able to state the number pair for the indicated part of the object or set.
2. Given a picture of an object or set such that the fractional part of the object or set is indicated, the child is able to write a fraction for the indicated part of the object or set.
3. Given a fraction, the child is able to read it.
4. Given a number pair story, the child is able to write a fraction to represent it.
5. Given an object or set, so that the fractional part of the object or set is the whole object or set, the child is able to write a fraction to represent it.
6. Given a fraction, the child is able to draw a picture to represent it.
7. Given a number pair story, the child is able to draw a picture to represent it.
8. When considering a part of a set of objects, the child recognizes that the objects of the set do not all have to be the same size.
9. Given a region or set of objects, the child will recognize that parts of an object, like halves, thirds, or fourths, must be congruent.
10. The child can recognize and find 1/2, 1/4, 1/3, 1/5 of an object or set of objects.

11. The child can recognize and find several parts of an object or set of objects.
12. When given a picture comparing part of an object with the whole object, the child is able to write more than one fraction to express the comparison.
13. Given a set of objects, such that the fractional part is indicated, the child is able to write more than one fraction to compare part of the set with the whole set.
14. Given a picture depicting equivalent fractions, the child is able to recognize and use the phrases "equivalent fractions" and "is equivalent to".
15. Given a picture which illustrates a pair of equivalent fractions, the child can write the pair of fractions.
16. Given a pair of equivalent fractions, the child is able to draw a picture to illustrate that the fractions are equivalent.
17. Given a fraction, the child is able to write one equivalent to it.
18. Given charts showing sets of equivalent fractions, the child will be able to list the next three or four fractions in the set.
19. Given the first three or four fractions of a set of equivalent fractions, the child is able to list other members of the set.
20. Given a fraction, the child will be able to identify the numerator and denominator of the fraction.

21. Given the numerator and denominator, the child will be able to write the fraction.
22. Given a picture that illustrates a fraction with zero numerator, the child will recognize it.
23. Given a set of equivalent fractions with zero numerator, the child will be able to write other fractions in the set.
24. Given two fractions, the child will be able to determine if they are equivalent by using the product method.
25. Given simple word problems, the child is able to apply the concept of fraction as part of a whole or part of a set of objects, to solve the problem.
26. Given a word problem expressing a relationship between numerator and denominator of a fraction the child will be able to identify the fraction.

APPENDIX B

LESSON PLANS

Introduction

The lesson plans for Treatment A and B are designed to meet the objectives for the Instructional Unit on Fractions as outlined in Appendix A. The same lesson plans are to be used for both treatments. In Treatment A, the students are given directions by the teacher on how to manipulate the instructional materials. The same manipulations are done by the teacher only in Treatment B. The teacher who uses Treatment B demonstrates to the students with instructional materials larger in size than those used by the students in Treatment A. The teacher in Treatment B also uses the flannel board for demonstration.

Both instructional methods need to make use of drawings, diagrams, chalkboard, overhead and textbook when necessary.

The assignments given in the lesson plans are the same for Treatment A and B.

All instructions are given orally to the student. The teacher may use drawings, demonstrations, or overheads to make instructions clearer to the student.

Lesson plans are to be used in conjunction with Elementary School Mathematics, book 4, second edition, by Eicholz and O'Daffer, published by Addison Wesley (Canada) Limited, Don Mills, Ontario, 1969.

Lesson 1

Objectives: 1-4, 6, 7

Materials: Fraction Kits, Markers

Textbook: Pages 252-253

(a) In the first lesson, students working in groups of two, familiarize themselves with the fraction kit by sorting sectors of the same size, e.g. halves together, fourths together, and so on. Students may be unfamiliar with the names of the fractional parts, so reference can be made to the different sectors by colour.

(b) Have students cover a black circle with two sectors. They will notice two of these exactly cover the whole circle. Then ask students to remove one sector.

We say one of the two parts is.....

We say one-half of the circle is...

We write 1/2 of the circle is yellow.

Give students the opportunity to continue using the kits to compare sectors with the whole circle, having them say the associated number pair and write the fraction.

(c) Write a fraction on the chalkboard or overhead and have students represent it using their kits. Then the teacher writes a fraction on the board and the students make drawings to show the fraction, shading or coloring the fractional part.

(d) Have students place four red and two blue markers on their desks.

Ask: How many are there in all?

How many are red?

What fractional part of the set of markers is red?

What fractional part is blue?

Do drawings on the board and ask similar questions to the ones listed above.

What part of the set is shaded?

What part of the set is not shaded?

Exercises on pages 252-253.

Lesson 2

Objectives: 5, 8

Materials: Fraction Kits

Textbook: Pages 254-255

(a) Have students cover a whole circle with sectors the same size, e.g. fourths.

Ask: How many equal parts?

What fractional part of the whole circle is the four parts?

How do you write this fraction?

Students should write the names for other fractions such as 2/2, 3/3, 6/6 etc.

(b) This activity is designed to help children recognize that, when they are considering a part of a set of objects, the objects of the set do not all have to be the same size.

Ask students to arrange any number of sectors from their kits on their desks. It may be better to omit twelfths to keep the number of sectors from becoming too clumsy to handle. Then have the students count the number of sectors they have placed on the desk. Ask questions such as what fractional part of the set of sectors is green, or blue, etc. For instance, if a student has on his desk four fourths, three thirds, and two halves, the teacher could ask what part of the set of sectors is white. The answer in this case would be two ninths.

Exercises on pages 254-255.

Lesson 3

Objectives: 9, 10, 11

Materials: Fraction kits

Textbook: Pages 256-257

(a) Students examine sectors of circles to see that parts of an object, such as halves, thirds, or fourths, are the same size.

(b) Have the students arrange different sets of sectors from their kits to show fractional parts of sets of objects.

Have the students place twelve white sectors, for example, on their desks, and ask them to show 1/2, 1/3, 2/3, etc. of the set. Further practice in finding parts of an object or set of objects can be given by having students draw pictures on their books and then draw loops around certain fractional parts.

Exercises on pages 256-257.

Lesson 4

Objectives: 12, 13

Materials: Six red, six blue markers;
paper for folding.

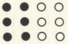
Textbook: Pages 258-261.

This activity is designed to provide additional practice working with fractions as part of a set of objects and introducing the concept of equivalent fractions.

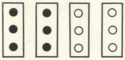
Have the students group the twelve markers in as many ways as possible to show that different fractional numerals may be used to compare the red markers with the whole set.

Overheads made by the teacher beforehand help the students with this exercise.

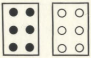
<u>Example of Overhead</u> - colour circles the same as the student' markers.



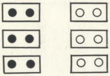
$$\frac{6}{12}$$



$$\frac{2}{4}$$



$$\frac{1}{2}$$



$$\frac{3}{6}$$

This activity could be continued by having the students use 2 red markers and 6 blue markers.

Ask: How many fractions can you write to compare the red markers to the whole set?

What fractional part of the whole set of markers is the red markers?

Have students draw 24 circles, or triangles, or any other picture on their paper with 8 of them shaded or coloured. Have students draw loops around the circles to show 8/24, 1/3, 2/6, 4/12.

Exercise on page 259, No. 2.

The next activity leads toward the development of an understanding of the equivalent-fraction when the fraction is considered as part of an object.

The students should see that there are many fractional names for the same coloured part of each piece of paper they fold.

Give each child a strip of rectangular paper.

First have the children fold the paper into two equal parts. Colour one part.

Name the fractional part coloured.

Then fold the same piece of paper into four equal parts.

Ask: How many parts are coloured?

What fractional part is coloured?

Then have the students fold the paper into eight equal parts.

Ask: How many parts are coloured?

What fractional part is coloured?

Exercises on pages 261, No. 2.

Lesson 5

Objectives: 14, 15, 16, 17.

Materials: Fraction Kit.

Textbook: Pages 262-263

Ask students to place three yellow sixths over a black circle. Find one sector from the kit that will exactly cover the three-sixths.

Ask: What is this fraction?

We say 3/6 is equivalent to 1/2.

We say 3/6 and 1/2 are equivalent fractions.

Use the kits to find other pairs of equivalent fractions and name these.

Exercise on pages 262-263.

Lesson 6

Objectives: 18, 19.

Materials: Overheads with charts depicting equivalent fractions.

Textbook: Pages 264-265.

The charts should be drawn with torn-off ends to introduce children to the concept that the number of fractions in the set of equivalent fractions is unlimited. Show on the overhead charts of equivalent fractions, such as the one-half chart illustrated in the textbook Elementary School Mathematics, book 4, second edition, published by Addison Wesley (Canada), page 264.

Lesson 7

Objectives: 19 (continued)

Textbook: Page 266

Students continue to practice building sets of equivalent fractions when they are given the first three or four fractions in the sets. Students should be allowed to devise their own method for building the sets of equivalent fractions. Some will find a number pattern in the numerator and denominator; others may see that numerator and denominator are multiplied by the same number. Further practice with paper folding may help students if they are still having problems with building the sets of equivalent fractions.

Exercises on page 267, No. 2.

Lesson 8

Objectives: 20, 21, 22, 23, 26

Materials: Fraction Kits, Markers

Textbook: Pages 268-269, 274-275.

Have the children depict fractions with the sections of the kit to see that the numerator tells how many parts of an object are being considered and the denominator tells the total number of parts in which the object is divided.

Do the same with the markers so that the students can see that the numerator tells how many parts of a set of objects are being considered and the denominator tells the total number of parts in the set.

Ask the students to depict a fraction using the fraction kit, then write the fraction, finally name the numerator and denominator. After the numerator and denominator of a fraction have been named, have the students make the fractions with their kits and then write the fraction. Have the students depict fractions with zero denominators, using their kits.

Exercises on pages 268, 269, 274.

Lesson 9

Objective: 24.

Materials: Fraction Kit.

Textbook: Pages 276-277.

On the board (or overhead), write several pairs of fractions,some equivalent, others not equivalent. Allow the students time to attempt to discover whether or not the two fractions are equivalent. To do this, some students may use their kits, others may use pictures, some may discover other ways. It is important that students have a method to check whether or not two fractions are equivalent.

If none of the students have discovered the "cross product" method of checking for equivalence then show how this method is used.

Exercises on pages 276-277.

Lesson 10

Objectives: Review of Objectives 1-26

Materials: Games for drill

Play Fraction Bingo and Dominoes with the students to provide further practice in equivalent fractions.

APPENDIX C

FRACTION TEST

Fraction Test Instructions

1. Check to see that students have completed all necessary information about name and classroom number. Students should write first and last name.
2. When given as a pretest, explain to the students that the test is given to find out how much they know about fractions. Explain that they have not studied fractions so they might not do too well in it. The results of this test will in no way affect their mark in mathematics for their mid-term report.
3. The teacher can read a word for a student if he does not know it. The teacher is not to explain the meaning of any word or symbol.
4. Explain that Question No. 7 requires a Yes or No answer.
5. Answers are to be placed in the space provided.
6. Students may use scratch paper.
7. Students can take as long as they wish to finish the test.
8. Tests are to be collected when the student indicates he is finished.
9. The items on the test are classified according to levels in The Taxonomy. The items in each of the levels constitute subtests. Item 14 is Knowledge level. Items 1, 2, 4, 6, 7, 9, 13, 16, are classified as Comprehension. Items 5, 11, 17, 18 are classified as Application. Items 3, 8, 10, 12, 15, 19 are classified as Analysis. Item 20 is classified as Synthesis.

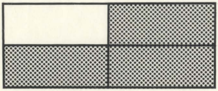
Besides a score on the entire test, scores will be calculated for each student on each of the subtests.

FRACTION TEST Grade IV

NAME: ____

CLASS: ____

1. Write the fraction suggested by the shaded part of this region



1.__________

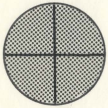
2. Write the fraction that tells which part of the dots is black

2.__________

3. A pie is cut into six equal pieces. Mary took five pieces. Write a fraction to tell how many pieces Mary took.

3.__________

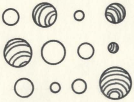
4. Write a fraction to tell what part of this circle is shaded.



4.__________

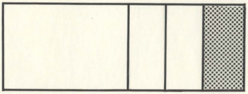
5. Draw a picture to show the fraction 5/9.

6. Write the fraction which tells what part of the set of balls has stripes.



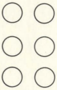
6.___________

7. Is 1/4 of this region shaded?

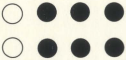


7.__________

8. Draw a loop around 2/3 of the balls.



9. Write two fractions to tell what part of the set of balls is black.



9.______ , ______

10. Draw a picture to show that 3/4 and 9/12 are equivalent fractions.

11. Write a fraction equivalent to 2/3

11.__________

12. Give the missing denominator:

15/20 is equivalent to 3/ 12.__________

13. Give the next three fractions in the set

(1/2, 2/4, 3/6, 4/8 ....)

13.____,____,____

14. The numerator of the fraction 2/3

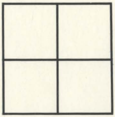
is 14.__________

15. Choose the correct answer:

If the denominator of a fraction is 3 times its numerator, the fraction is equivalent to

a) 3/6 b) 1/3 c) 2/3 d) 6/9 15.__________

16. Write a fraction that tells what part of this square is shaded.



16.__________

17. Use the product method to check if 4/10 and 10/25 are equivalent fractions.

18. What fraction of a week is 2 days?

18.__________

19. A piece of string is 10 inches long, how long is 1/5 of the string?

19.__________

20. What is 2/3 of 12? 20.__________

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